

LPHY2224: Suggested Exercises and Projects

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1. The rho-parameter¹

a) Show that for an arbitrary number of Higgs multiplets ($\langle\varphi_i\rangle_0 \neq 0, (i = 1, \dots)$), the rho-parameter becomes

$$\rho_0 = \frac{\sum_i ((I_W)_i^2 + (I_W)_i - (I_{W3}^2)_i) \langle\varphi_i\rangle_0^2}{2 \sum_i (I_{W3}^2)_i \langle\varphi_i\rangle_0^2}.$$

b) Given two Higgs fields, respectively with quantum numbers $I_W = -I_{W3} = 1/2$ and $I_W = 1, I_{W3} = 0$, and having the nonvanishing vacuum expectation values $\langle\varphi_{1/2}\rangle$ and $\langle\varphi_1\rangle$, obtain a bound for $|\langle\varphi_1\rangle/\langle\varphi_{1/2}\rangle|$ assuming an experimental value $\rho_0 = 0.993 \pm 0.004$.

2. Neutrino masses and the see-saw mechanism

The Minimal Standard Model (MSM) with massless neutrinos may be extended to include masses for the neutrinos. In order to account for the (unknown) small masses of the observed neutrinos, a so-called “see-saw” mechanism has been proposed, initially within Grand Unified Theories (GUT’s), which is also feasible when extending the MSM with right-handed neutrinos for each generation. Discuss this see-saw mechanism within the Standard Model.

3. The gauge anomaly cancellation mechanism in the Standard Model

Discuss and consider the requirement for gauge anomaly cancellation in gauge theories, and establish how within the Standard Model all potential gauge anomalies do indeed cancel.

4. The neutrino mixing matrix in the Standard Model

Discuss how the Pontecorvo-Maki-Nagakawa-Sakata mixing matrix for massive neutrinos arises within the Standard Model.

5. Left-Right Symmetric Models

Discuss how the gauge symmetries of the Standard Model could be extended into the gauge symmetries of so-called Left-Right Symmetric Models based on the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Consider some of the ensuing phenomenological consequences in terms of extra massive gauge bosons and the existence of Higgs particles beyond the neutral Higgs particle of the Standard Model.

¹Problem XVI-1, page 467, Chapter XVI of, *Dynamics of the Standard Model*, John F. Donoghue, E. Golowich and B. R. Holstein (Cambridge University Press, 1992, pbk 1994).