Phenomenology of a barophilic Z'

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1 Introduction

Nature at its most elementary level is described by the Standard Model (SM). It is a quantum field theory of the fundamental particles and their interactions which has been successfully tested to a very high accuracy for many processes in the last fourty years. One of the most eloquent example of that is the computation of $g-2$ for the magnetic moment of the electron. It matches the observed value with a relative precision of 10^{-12}. So the small quantum corrections to the Born approximation are essential in comparing theory and experiment. Along with this, the great coherence of the SM allowed people to predict many new particles before they were even observed, such as the Z boson or the top quark.

The Standard Model is a gauge theory with symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$. Each of these gauge groups is connected to vector gauge bosons, the photon for $U(1)_{em}$[1], the weak bosons W^+,W^- and Z for $SU(2)_L$ and the gluons for $SU(3)_c$. Noether's theorem also associate a conserved charge (electric, weak and color respectively) to each of them. $U(1)_{em}$ and $SU(3)_c$ are exact, so photons and gluons are massless and the force they carry has an infinite range. The weak bosons acquire their mass from ElectroWeak Symmetry Breaking (EWSB) through the Higgs mechanism.

Many physicists believe that all fundamental interactions must have a common root. They suppose that strong and electroweak interactions can be described by one simple gauge group G at very high energy $E > E_{GUT}$. Such theories are called Grand Unified Theories (GUT's). From the experimental bounds on the proton decay, we must have $E_{GUT} > 10^{15}$ GeV. It is important also that this scale is smaller than the Planck Mass $M_P \approx 10^{19}$ GeV where gravity is expected to become as strong as the other forces. E_{GUT} is also predicted as the meeting point of the three running gauge coupling constants. For energies $E \ll E_{GUT}$ the gauge group G must be broken to retain the SM gauge symmetry structure. The smallest group G that can be broken in this way is $SO(5)$[2]. For any larger group, like SO(10), the breaking scheme will necessary end up with at least one extra neutral gauge boson, a new particle called Z'. This particle is the object of this thesis. It is often associated to an extension $U(1)'$ to the SM gauge group. Since we want a massive Z', this new gauge group should be broken as well at some scale $E_{Z'}$. Roughly speaking, it means that processes at energy $E < E_{Z'}$ do not feel the new symmetry $U(1)'$ so that we have not observed them yet. The Left-Right (LR) model is another common model starting from an extended gauge symmetry, $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_Y$, which also introduce a Z' from the additional $SU(2)_R$ broken gauge symmetry. This family of models is one way to introduce a Z' which however highly constraints its couplings. In particular, they ask for universal coupling of the Z' to quarks. We will discuss later other models with more freedom in this regard.

To set up the framework, it is worth now to recall the amazingly concise expression of the SM, embedded in the three little lines of its Lagrangian.

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F^a_{\mu\nu} F^{a\ \mu\nu} + i\bar{\psi}\not{D}\psi & (1.1) \\ &+ \lambda_d \bar{\psi}_L \phi d_R + \lambda_u \bar{\psi}_L \tilde{\phi} u_R + h.c. & (1.2) \\ &+ |D_\mu \phi|^2 - V(\phi) & (1.3) \end{aligned}$$

The flavor,gauge and spinor indices are left understood which contributes to the apparent simplicity of the expression above. Each line of the SM Lagrangian is a different sector of the theory. The gauge sector (line 1.1) has been extensively tested for years. More recently, the Higgs mechanism allowed a mass term for fermions in the flavor sector (line 1.2) via EWSB induced by the potential $V(h)$ in the Higgs sector (line 1.3). The EWSB sector is only constrained by indirect observations at LEP, and the existence of the Higgs itself is still an open question that LHC might settle in a close future. This sector is the most permeable to new extensions and it is where new physics is expected to come into play. In particular there is so far no direct evidence for the Higgs, which makes it the only particle of the Standard Model that has never been directly observed yet. So, before even thinking about theories Beyond the Standard Model (BSM), one

[1] Since $U(1)_Y \neq U(1)_{em}$ the photon γ and the weak boson Z are combinations of the two neutral generators of $SU(2)_L$ and $U(1)_Y$.

[2] $SO(5)$ is ruled out because it predicts a scale E_{GUT} which is incompatible with the bound on the observed proton life-time.

would like to have an experimental evidence that the Higgs mechanism is indeed how particles acquire their mass. This is the main goal of the Large Hadron Collider (LHC), a supercollider built in Geneva which plans to collide two protons at 14 TeV rest-frame enery. Electroweak precision measurements show that the Higgs mass is less than 182 GeV at 95% confidence level and it must be heavier than 114 GeV because of the lack of direct signal at the Large Electron Collider (LEP). Within this mass range, the SM Higgs will necessarly be observed if it exists. If it does not, something else will show up, simply because then the W interactions are no longer unitary at TeV energies. So the LHC will either confirm the existence of the Higgs or at least give insights to the new physics at the TeV scale.

In any case, the Standard Model cannot be the end of the story. First of all, it does not include gravity and no one knows how to conciliate quantum field theories with general relativity. It is also incompatible with the recent observation of neutrino oscillations. In cosmology, dark matter evidence remains problematic since the Standard Model does not provide any candidate for it. Moreover, the impressive success of the SM becomes conceptually puzzling when considering the fermion masses which are spread over a large range of values and the Standard Model simply takes them as inputs. The quadratically divergent radiative corrections to the Higgs mass suggest a TeV-scale cutoff. If we exclude anthropic principle to explain this fine-tuning then a more complete and natural theory of electroweak symmetry breaking would include a stabilization mechanism for the Higgs mass through new physics.

For these reasons, the Standard Model is believed not to be a fundamental theory and there is a need for new physics at higher energies. Speculations on new models are however limited by the fact that there is still no evidence for new physics above the Fermi scale. In addition to that, the unexpected effectiveness of the SM suggests that any new TeV-scale physics is also perturbative. The two main streams of investigation for TeV-scale new *perturbative* theories are Supersymmetry on one side and Dynamical Electroweak Symmetry Breaking on the other, the latter meaning that the Higgs is a composite particle.

In supersymmetric theories, every Standard Model field has a superpartner of opposite statistics. The quadratic sensitivity of the Higgs mass to the TeV scale is removed by cancellations of the radiative corrections of Standard Model fields with those of their superpartners.

The second class of models are ultimately based on the Higgs degrees of freedom being composite at very high energies. They rely on the discretization of these extra dimensions introduced. The Higgs field then appears as a pseudo-Goldstone[3] boson at TeV energies and below, with conventional gauge, Yukawa and self-couplings. The quadratic sensitivity to the cutoff scale that these couplings normally induce are cancelled by particle of the *same* statistics, unlike supersymmetry. Global symmetries of the theory ensure these cancellations. The symmetry breaking giving rise to the pseudo-Goldstone bosons typically occurs at $\sim$10 TeV.

In this last kind of models [2], new particles are introduced in the sector of composite degrees of freedom. Because of the partial compositeness of the SM fields, these new states can couple non-democratically to the SM fermions. In particular it would be possible to have a new neutral vector boson Z', analog to the weak Z boson of the standard model but predominantly coupling to heavy quarks. It is a very special feature of this model to allow and justify non-universal coupling of the Z' to quarks.

Like the SM Z boson, the Z' is expected to be a very short-lived particle. It can only be observed through its decay products or through indirect interference effects. This can occur either in very high energy processes (hadron colliders) or in high precision measurements at lower energies (lepton colliders). In the first kind of process the energy must not only be high enough to produce a Z' but also such that it can be detected over the SM background. This background is always present since the Z' share the same couplings as the other SM neutral gauge bosons. In precision measurements, the combined experimental and theoretical errors must be smaller than the expected deviation due to a Z'.

The special affinity of the Z' with heavy quarks in the model discussed above might play a crucial role when considering Z' direct detection. Indeed, heavy quarks are not present in the initial state of a proton collider, so the Drell-Yan production channel is suppressed by the low coupling constants of the light quarks. The other decay channels, either via a loop or with more particles in the final state, are of the same order of

[3] Technicolor models are less interesting for the purpose of this work since the composite particle can be very heavy and so broad that they could even not be experimentally identified.

magnitude and a complete computation of them is required to decide which one prevales. This is the subject of this thesis which will give prospects for the observation of such a peculiar Z'. The outcome depends on the value of its mass and couplings so the results presented in this thesis should let specific models decide wether their predicted Z' can be directly detected at LHC or not.

2 Motivation

The SM Higgs has a very peculiar behaviour towards quarks since it couples to fermions proportionally to their mass. Then, the main production channel is gluon fusion via a loop of heavy quarks ($gg \to H$) and not a direct $q\bar{q} \to H$ process. In other words, the Higgs couples so much stronger to the heavy quarks that it is worth paying the price of a loop factor to include them. The Higgs only couples to heavy particles and at the same time these are difficult to produce on-shell and hard to detect so that a big part of the Higgs phenomenology is dictated by loop processes.

In the introduction we mentioned the existence of models predicting a Z' which, like the Higgs, couples predominantly to heavy fermions. One of them [2], based on the deconstruction of a warped space, is discussed in the next section. The bottom and top quark are significantly heavier than the fermions from the two other families and the SM does not provide an explanation for that. One can speculate that such a new boson might be involved in the mechanism responsible for this difference. It would then interact directly with the fermions of the third generation and only much more weakly with the lighter quarks and leptons, via radiatively created mixings. A scheme with the breaking of universality in weak interactions can be linked with the problem of fermion mass hierarchy. A neutral gauge boson interacting differently with the heaviest fermion can naturally lead to a realistic mass hierarchy and Kobayashi-Maskawa mixing matrix. These kind of models often need to include more unknown heavy fermions and scalar Higgs boson. Otherwise, quantum anomalies break the unitarity and the mass pattern for heavy fermions will violate experimental bounds on deviation from the Standard Model in the low energy data.

It is even possible to further break universality of Z' couplings by considering interactions only with the top quark. Such a Z' would also break chirality. An example of such a model is given in the appendix 7.3. The fermion set added to cure the spoiled anomaly cancellations and mass spectrum is somewhat *ad-hoc*, hence diminishing its naturalness. Due to its specific couplings, the experimental bound on such a Z' is weak. In fact, only the vector coupling $Z'b\bar{b}$ has a bound from the decay of Υ states into $\tau^+\tau^-$. It has been estimated by P.Osland [7] that a Z' with such a coupling must have a mass $m_{Z'} > 50$ GeV. No restriction can be obtained from the pure axial $Z'b\bar{b}$ vertex or the interaction with the top quark, which leaves a large window for the existence of this Z'.

The production mechanisms for such an on-shell Z' are rather different than those for a Z' with universal couplings. The Drell-Yan (*i.e.* $q\bar{q} \to Z'$) production is suppressed by the small coupling constant, so we are left with gluon fusion only (we do not consider Z-Z' mixing, see [4] for that). Gluon fusion can occur at tree-level with two heavy quarks in the final state, a process only in α_s^2 but suppressed by the parton distribution function. Gluons can also fuse through loop, but unlike the Higgs, the Z' cannot be produced in the $gg \to Z'$ channel, because the cross-section for such a process is analytically null. Indeed, Lee-Yang theorem [4] prevents a massive colorless vector particle from decaying to two massless identical vector particles. Of course, the vice-versa for production is also true. So either the fermion in the loop or one of the gluon legs has to radiate an extra gluon to allow the process. This, however, comes at the price of another α_s and it is not clear a priori how it compares with tree-level diagrams. To answer this question, we need the direct computation of the $gg \to Z'g$ cross-section to decide. This computation, however, is not an easy task and is the aim of this thesis.

A heavy Z' will decay directly into $b\bar{b}$ or $t\bar{t}$. The matrix element of $gg \to Z'g$ can also be involved here when considering three gluon-jets decay. If the Z' is lighter than twice the mass of the top quark, the decay into leptons could occur with a sizeable branching ratio, providing clean signature for its detection.

[4] see Appendix 7.1. Also note that if the Z' is off-shell, this result does not apply an a *small* contribution can come from diagrams like $gg \to Z' \to q\bar{q}$.

3 Models

3.1 Simplified warped effective field theory

In the previous section, some models introducing a Z' particles were briefly mentionned. The simplified warped effective field theory by R.Contino *et al.* [4] should retain more attention because it gives a natural explanation for an enhanced coupling to heavy quarks. It considers warped compactifications of higher dimensional spacetime to construct a purely four-dimensional two-sector effective field theory describing the Standard Model fields and just their first Kaluza-Klein (KK) excitations [5]. In the effective theory, the higher dimensional warped compactification is realized through strong dynamics. In this picture, the theory is of the form

$$\mathcal{L} = \mathcal{L}_{elementary} + \mathcal{L}_{composite} + \mathcal{L}_{mixing} \tag{3.1}$$

The first sector $\mathcal{L}_{elementary}$ consists in weakly-coupled elementary particles. The second one, $\mathcal{L}_{composite}$, describes tightly bound composite states, including the Higgs doublet. The couplings are not at all of the same orders in this sector. The intra-composite forces are by far the strongest and it is assumed that the weaker inter-composite coupling $g_\star$ is still larger that the coupling in the elementary sector g_{el}. Typically, one has g_{el} 1 and $1 < g_\star << 4\pi$ (to stay in the perturbative range). The mass of the composite states is expected to be large, at the TeV scale so that they have not been observed yet. The last piece of Lagrangian, $\mathcal{L}_{mixing}$ enables the two sectors to couple to each other which results in mass-mixing. Consequently, mass eigenstates are superpositions of elementary and composite particles. The lightest mass eignestates obtained after diagonalization of the mass-matrix are then identified with the SM fields. This mixing of the elementary and composite states can be parametrized in term of an angle ϕ_n which measures the degree of compositeness [6]

$$\langle \mathrm{SM}_n| = \cos\phi_n \langle \mathrm{el}_n| + \sin\phi_n \langle \mathrm{comp}_n| \tag{3.2}$$

And the corresponding orthogonal admixture constitute the TeV-scale new physics.

$$\langle \mathrm{Heavy}_n| = -\sin\phi_n \langle \mathrm{el}_n| + \cos\phi_n \langle \mathrm{comp}_n| \tag{3.3}$$

So now, all the SM particles have some degree of compositeness, except for the Higgs which is taken as fully composite. This is precisely what helps to answer[7] the hierarchy problem, because the SM particles now only couple to the Higgs through their composite part, suppressing this coupling by a factor $\sin^2\phi_n$. The Yukawa of the SM are given in term of the Yukawa of the composite sector through the relation

$$(Y_{SM})_{ij} = \sin\left(\phi_{\psi_{L_i}}\right) (Y_\star)_{ij} \sin\left(\phi_{\psi_{L_j}}\right) \tag{3.4}$$

Where $(Y_\star)_{ij}$ are the Yukawa couplings in the composite sector, taking value in the same range as $g_\star$, say between 1 and 4. From 3.4, we see that the degree of compositeness of the SM particles is proportional to their mass so that the heavy top is expected to have a large ϕ_t. However, the bottom and the top quark have very different mass so they cannot have the same degree of compositeness. In addition to that, the isopsin symmetry requires that $\phi_{t_L} = \phi_{b_L}$ so one solution is to take ϕ_{t_L} minimal and ϕ_{t_R} maximal to compensate. To avoid that ϕ_{t_R} gives a divergent contribution to the Higgs mass, it is taken as a full chiral[8] member of the composite sector, so we have the extreme case of full t_R compositeness

$$\sin(\phi_{t_R}) = 1 \tag{3.5}$$

[5] which correspond to composite states in the strong dynamics picture.

[6] What happens here is analogous to the case in the SM of a pion which is mostly a QCD condensate with a tiny admixture of Higgs pseudoscalar, since both break $SU(2)_L$.

[7] Of course, in this simplified model, the ϕ_n are just inputs but in the underlying warped compactification theory, their value follow an exponential serie

[8] So that it participates in the composite dynamics that it is also responsible for the classical V(H) in the effective theory and will therefore not enter in the measure of finetuning.

The order of magnitude of the couplings between two elementary states and a composite one can be easily derived from the rule that three elementary components interact with strength g_{el} and three composite components with strength $g_\star$

$$g_{SM_1 SM_2 heavy_3} \sim -g_{el}\cos\phi_1\cos\phi_2\sin\phi_3 + g_\star\sin\phi_1\sin\phi_2\cos\phi_3 \tag{3.6}$$

The composite sector contains an *excited* version of all the SM fermions and gauge bosons to provide a small composite component to all of them. In particular, there is a composite partner to the Z boson, which we call Z'_1. It is heavier, and only couples to the composite part of the SM particles. The higher degree of compositeness of the heavy fermions now gives a natural explanation for their enhanced coupling to Z'_1.

The composite sector has a symmetry $\{SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X\}^{comp}$ even larger than the SM one. So there is another Z'_2 candidate from the additional gauged $SU(2)_R$ which does not mix with the SM gauge bosons. Therefore, the coupling of the top quark to Z'_2 (which is purely composite) will be, according to Eq. (3.6) and Eq. (3.5), of order g_*. The other quarks, being much lighter than the t-quark, will have their Z'-coupling suppressed by $\sin{\phi_q}^2 = \frac{Y_{SM}}{Y_\star}$.
This model brings the most serious proposal for a Z' predominantly coupling to heavy quarks.

3.2 Elementar Z' coupling to the top quark only

In the model above, the Z' appear in an effective field theory and the selectivity of its couplings is due to new inputs coming from the underlying fundamental theory. It is also possible to stay in the framework of the Standard Model and simply inroduce a single new vector boson Z' by adding another $U(1)_X$ to the gauge symmetries of the SM. When setting $X = B - L$, it is called the B-L extension to the SM [5]. The B-L tag refers to the $U(1)_X$ quantum numbers given to the fermions, with B and L their Baryonic and Leptonic number. With this choice of quantum numbers the anomaly cancellation still holds and no SM mass term is forbidden. The counterpart is that this Z' then couples democratically to all quarks.

Now, if only the top quark recieved an X charge, no other SM particle would couple to Z' and we would be left in the specific case studied in this thesis. However, the mass term for the top quark becomes problematic, as well as the anomaly cancellation (if we still want to think of the theory as *fundamental* and including only elementary particles). In fact, it is possible to cure the model by adding a set of new colored quarks. Although this is an *ad-hoc* solution, the final picture is consistent. A short review of this model is given in Appendix 7.3.

4 Z' Production channels

4.1 A *Standard* Z'

The aim here is to study Z' production in the most model-independent possible way. Any Z' will have either an axial or vector coupling to the fermions, so we can write it generically as follows

$$\Delta\mathcal{L} = Z'^{\mu} J_\mu \text{ and } J_\mu = g_{Z'}\bar{\psi}(v_q + a_q\gamma_5)\gamma_\mu\psi \tag{4.1}$$

Throughout this analysis, we consider a *Standard Z'*, i.e. a particle that has exactly the same couplings as the SM Z-boson but is simply heavier. Of course, since we assume our Z' to predominantly couple to heavy quarks, only the top and bottom quark are taken into account. Other specific models can then easily extrapolate their matter of interest from what will be presented. For such a Z' and in the notation of

Eq. (4.1), we have

$$\begin{aligned}
v_f &= \frac{1}{2}T_{3L} - Q\sin^2(\theta_w) \text{ and } a_f = \frac{1}{2}T_{3L}, \text{ so} \\
v_u &= \frac{1}{4} - \frac{2}{3}\sin^2(\theta_w) \text{ and } a_u = \frac{1}{4} \\
v_d &= -\frac{1}{4} + \frac{1}{3}\sin^2(\theta_w) \text{ and } a_d = -\frac{1}{4} \\
g_{Z'} &= \frac{e}{\sin(\theta_w)\cos(\theta_w)}
\end{aligned} \tag{4.2}$$

where θ_w is the Weinberg angle.
The Drell-Yan production from the annihilation of *light* quarks which was the main production mechanism for a universal Z' is now suppressed by low coupling constants. So only the production channels involving heavy quarks are now relevant. If the Z' would mix with the SM Z boson, it could also be produced via the production of a Z that would later turn into a Z'. This mixing would affect the Z-pole which is precisely measured at LEP, so it is expected to be rather small. For our purpose here, we will consider a Z' completely decoupled from the weak bosons of the SM[9].

Heavy quarks are not present[10] in the incoming protons of the hardon colliders, so we will create them from gluons. Two different kinds of channel are then possible. First, the loop production with gluon fusion and radiation of an additional gluon (because of Lee-Yang theorem 7.1) or the open production $gg \to Z'Q\bar{Q}$. The gluon fusion channel is in α_s^3 and suppressed by a loop factor whereas the open production is in α_s^2. However, the open diagram produces two on-shell top quarks and this costs a big price in the Parton Distribution Function (PDF). To investigate which of the two production mechanisms prevales, one needs a complete computation of the cross-section of both processes.

4.2 Open production

The representative Feynman diagrams for the open production are

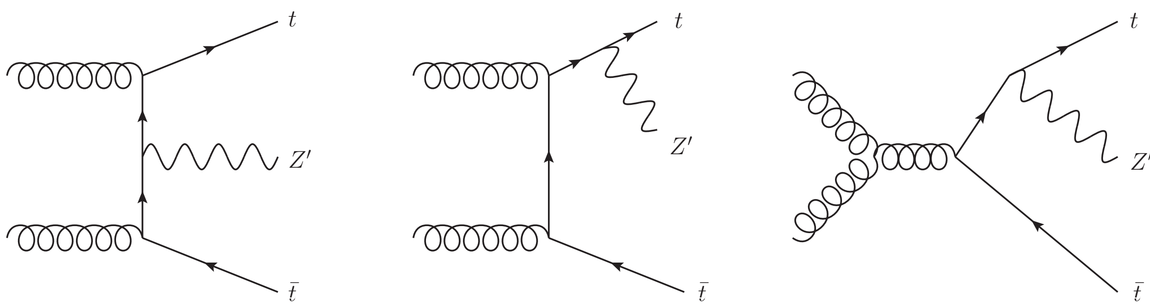

Figure 4.1: Three out of the eight Feynman diagrams contributing to the open production. The others can be obtained by changing the quark legs that emits a Z' and/or exchanging the two initial gluons.

When squared, there are possible interferences[11] between the axial and vector couplings of the Z'. As we will see, the vector contribution is much smaller than the axial one, so we could safely neglect their interference. Numerically, the relative contribution of this interference turns out to be less than one thousandth,

[9]So that the production channel $W^+W^- \to Z'$ is not considered either.
[10]The Drell-Yan production from sea $b\bar{b}$ of the protons must not be taken into accunt here since we worked in the *four-flavour* proton scheme where this production channel corresponds to the open diagram $gg \to Z'b\bar{b}$. (*i.e.* registering both production channel would be a double-counting).
[11]Only the massive part of the propagators is expected to interfere, however this still need to be analytically verified.

so out of the accuracy of the Monte-Carlo computation. The computation of the cross-section is performed using MadGraph [8] and MadEvent [9], with an *ad-hoc* model generated by FeynRules [10]. In FeynRules, the default SM Model is modified with the definition of a new gauge group $U(1)_X$ and its associated gauge boson Z'. The following piece of Lagrangian giving the Z' couplings to the fermions is introduced

$$\Delta\mathcal{L} = -\frac{1}{4}Z'^{\mu\nu}Z'_{\mu\nu} - \frac{1}{2}m_{Z'}^2 Z'^{\mu}Z'_{\mu} + g_{Z'}Z'_{\mu}\left(v_u\bar{u}\gamma^\mu u + v_d\bar{d}\gamma^\mu d + a_u\bar{u}\gamma^\mu\gamma^5 u + a_d\bar{d}\gamma^\mu\gamma^5 d\right) \tag{4.3}$$

where $Z'^{\mu\nu}$ is the field strength of the gauge boson Z'. The parameters $m_{Z'}$, $g_{Z'}$, v_u, v_d, a_u and a_d are added to the list of external parameters in the model file. Then, in MadGraph, the process `gg>Zptt~` or `gg>Zpbb~` is specified in the `proc_card.dat`. We also indicate here that the ZPrime model has to be used. The running and factorization scale are set in the `run_card.dat`. This is also where the cuts can be disabled. Finally, the `param_card.dat` is modified accordingly to the parameters[12] of the computation, in GeV,

- $m_u = m_d = m_c = m_s = 1$
  $m_b = 4.6$; $m_t = 171.2$
  $m_{Z'} = 91.188$,if not varying in the plot.
- $\sin^2(\theta_w(m_Z)) = 0.23122$; $\alpha_{EM}(m_Z) \simeq \frac{1}{128}$
- Fixed factorization and running scale Q set to
  ⋄ $m_{Z'}$ for the $gg \to Z'g$
  ⋄ $m_{Z'} + 2m_q$ for the $gg \to Z'q\bar{q}$
- $\alpha_s(Q)$ running at first order with $\alpha_s(m_Z) = 0.13$
- PDF set used: cteq6l1
- Proton-proton collision energy
  ⋄ 14 TeV for LHC
  ⋄ 1.96 TeV for Tevatron
- No cuts

4.3 Production via gluon fusion through loop

The loop production channel is somehow more cumbersome, especially when one wants to track the dependence on the mass of the fermion running in the loop. There are two possible diagrams for Z' production through gluon fusion via a loop. The loop is either a box or a triangle depending on wether the extra gluon is emitted from one of the gluon legs or from the quark in the loop.

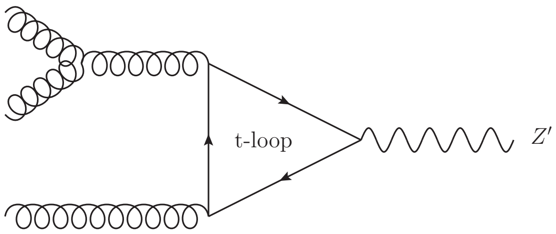

Figure 4.2: Z' production through a triangle loop.

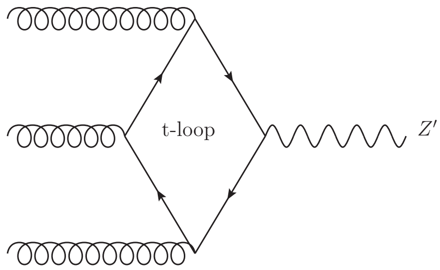

Figure 4.3: Z' production through a box loop.

These two classes of diagrams 4.2 and 4.3 will carry different tensor structures. The triangle loop contains a trace of only two $SU(3)_C$ generators and the box loop three of them. So the color part of the triangle diagram has a f^{abc} structure whereas the box diagram contains both f^{abc} and d^{abc} since the loop factor includes $Tr(t^a t^b t^c)$

$$Tr\left(t^a t^b t^c\right) = \frac{1}{2}Tr\left(\left\{t^a, t^b\right\}t^c + \left[t^a, t^b\right]t^c\right) = \frac{1}{4}\left(d^{abc} + if^{abc}\right) \tag{4.4}$$

The VVV coupling of the triangle loop is zero by Furry's theorem (easy to see by applying charge conjugation to the external vector currents). Instead, the box loop includes both AVVV and VVVV coupling. From

[12]For running constants, the reference point is $m_Z = 91.188$ GeV.

charge conjugation (but on the fermion in the loop this time) it is also clear that the vector coupling of Z' will only contribute to the fully symmetric color tensor d^{abc} while the axial only goes with the anti-symmetric f^{abc}. So the total amplitude can be split into a vector part which only comes from the box diagram and an axial part induced by both loops. Notice also that the color part of the tensor can be completely factored out of the amplitude, since it is decoupled from the Lorentz part. Eventually, we may write the amplitude in terms of the axial and vector polarization tensors $A^{\alpha\mu\nu\rho}$ and $V^{\alpha\mu\nu\rho}$

$$\mathcal{A}^{abc\alpha\mu\nu\rho} = f^{abc}\tilde{A}^{\alpha\mu\nu\rho} + d^{abc}\tilde{V}^{\alpha\mu\nu\rho} \tag{4.5}$$

And for practical purposes, the coupling constants $g_{Z'}$ and g_s can be factored out by redifining these polarization tensors.

$$\tilde{A}^{\alpha\mu\nu\rho}(m_f, m_{Z'}, s, t, u) = \frac{g_s^3 g_{Z'}}{(2\pi)^4}\frac{f^{abc}}{4}a_q A^{\alpha\mu\nu\rho}(m_q, m_{Z'}, s, t, u) \tag{4.6}$$

$$\tilde{V}^{\alpha\mu\nu\rho}(m_f, m_{Z'}, s, t, u) = \frac{g_s^3 g_{Z'}}{(2\pi)^4}\frac{d^{abc}}{4}v_q V^{\alpha\mu\nu\rho}(m_q, m_{Z'}, s, t, u) \tag{4.7}$$

Notice right away that there is no possible interference between the two terms in the decomposition above. Indeed, the color part of the first term is fully anti-symmetric and the other symmetric. Therefore, we can treat the axial contribution separately from the vector one, even at the cross-section level.

$$|\mathcal{A}|^2 = 16\frac{\alpha_s^3\alpha_Z}{(2\pi)^4}\left[\frac{5}{6}v_q\,|V|^2 + \frac{3}{2}a_q\,|A|^2\right] \tag{4.8}$$

One typically does not directly work with the full expression of the Lorentz tensor, but rather choose a basis for the polarizations of the gluons and the Z' and projects it onto the vectors of this basis to get a set of functions called *Helicity Amplitudes*. In principle there is one helicity amplitude for each combination of polarizations of the four particles. Fortunately, they are not all independent and using parity operations and special properties of the polarization basis chosen[13], we can reduce the set of independent helicity amplitudes to a limited number[14]. It is what Glover *et al.* did in their paper [3].

Their computation for the Axial amplitude has been completely double-checked and proven to be correct. The rederivation of the polarization tensor $A^{\alpha\mu\nu\rho}$ of Eq. (4.6) has been performed independently by F.Tramontano [14], using Passarino-Veltman decomposition methods. He also derived the final results (i.e. not the polarization tensor) using a modern method [12]. This expression of the tensor contains two-, three- and four-point scalar integrals which have been analytically continued in order to be implemented for numeric computations. Using Mathematica, the polarization tensor given by Glover *et al.* (See Eq. (7.23) in Appendix 4.6) was projected onto the specific basis of Eq. (7.18) to *analytically* check the helicity amplitudes. To complete the double-check of Glover *et al.* results, their plot[15] of the decay width of $Z \to ggg$ has been reproduced (see Appendix 7.2). In this process, four typos were identified in [3]:

- In the four-point scalar integral $D(s,t)$, the expression of the integral (A.7) had to be modified[16].
- Their polarization tensor need to be contracted with $(-,+,+,+)$, the unusual convention for the metric.
- The pieces of their vector helicity amplitudes which are independent of the scalar integrals should all take an additional factor $i\pi^2$
- The equalities $V_{++++}(s,t,u) = V_{+---}(s,u,t)$ and $A_{+-+0}(s,t,u)/\Delta = A_{+--0}(s,u,t)$ are typos. They should be $V_{+---}(s,t,u) = V_{+-++}(s,u,t)$ and $A_{+-+0}(s,t,u)/\Delta = A_{+--0}(s,u,t)/\Delta$

[13]Note that the optimal basis minimizing the number of independant helicity amplitude is the *cyclic basis*.
[14]a specific example given in 7.4.
[15]Fig. 2 in [?].
[16]See the correction in Eq. (7.33).

The final axial and vector helicity amplitudes in terms of the Mandelstamm variables are given in the Appendix 7.4, along with the specification of the helicity basis chosen.

For the physical production rate in proton collisions, one has to fold in the integration over the gluon structure functions.

$$d\sigma(pp \to gg \to Z'g) = \int_0^1 dx_1 \int_0^1 dx_2 f_g(x_1, Q^2) f_g(x_2, Q^2) d\hat{\sigma}(\underbrace{x_1 x_2 s}_{\hat{s}}) \tag{4.9}$$

where we will note Q^2 both the factorization and running scale. To build the differential cross-section $d\hat{\sigma}$, one simply sums the square of the helicity amplitudes over the different quarks running in the loop and their polarizations and average over colors and polarizations of the incoming particles. Including the phase-space we can write:

$$\begin{aligned} d\hat{\sigma}(\hat{s}) &= \frac{\alpha_s^3(Q^2)\alpha_{Z'}}{32\hat{s}(2\pi)^4} \sum_{\lambda_1 \ldots \lambda_4} \left(\frac{5}{6} \Big| \sum_q v_q V_{\lambda_1\lambda_2\lambda_3\lambda_4} \Big|^2 + \frac{3}{2} \Big| \sum_q a_q A_{\lambda_1\lambda_2\lambda_3\lambda_4} \Big|^2 \right) \\ &\times \prod_{i=3,Z'} \frac{d^3 p_i}{2E_i(2\pi)^3} (2\pi)^4 \delta^4(p_{Z'} + p_3 - p_1 - p_2) \end{aligned} \tag{4.10}$$

This expression has been implemented in a f77 program with an embedded phase space generator. It uses integration variables which are different as the ones above. For the fractions x_1 and x_2 we use

$$\tau = x_1 x_2, \quad y = \frac{1}{2} \log \frac{x_1}{x_2}$$

These variables have the advantage of a having a flat distribution for the incoming energy $\hat{s} = \tau s$ and symmetric expressions $x_1 = \sqrt{\tau} e^y$ and $x_2 = \sqrt{\tau} e^{-y}$ so that the Jacobian of the transformation is one (unitary transformation). It also has the advantage that the integration limits on τ and y are easy to express in terms of s_{min}. The phase-space generetor uses four random variables, two for τ and y and two for ϕ and $\cos\theta$, which generate the four-momenta of the gluons and the Z' from which the Mandelstamm variables s,t and u can be retrieved to compute the amplitude. The integration is then carried on using adaptative Monte-Carlo methods with the VEGAS [11] package. The next section presents the results.

4.4 Cross-sections comparison

The following two plots show the total (*i.e.* axial plus vector) Z' production cross-section from each of the dominant channels we identified. In the loop production, the anomalous contribution has been taken out with the procedure described in the Appendix 7.4. This is irrelevant in the cases where the two quarks of a given generation are included in the loop, but it has noticeable effects when considering the contribution from a single quark. For the Drell-Yan production, all the four light quarks u, d, c and s are included.

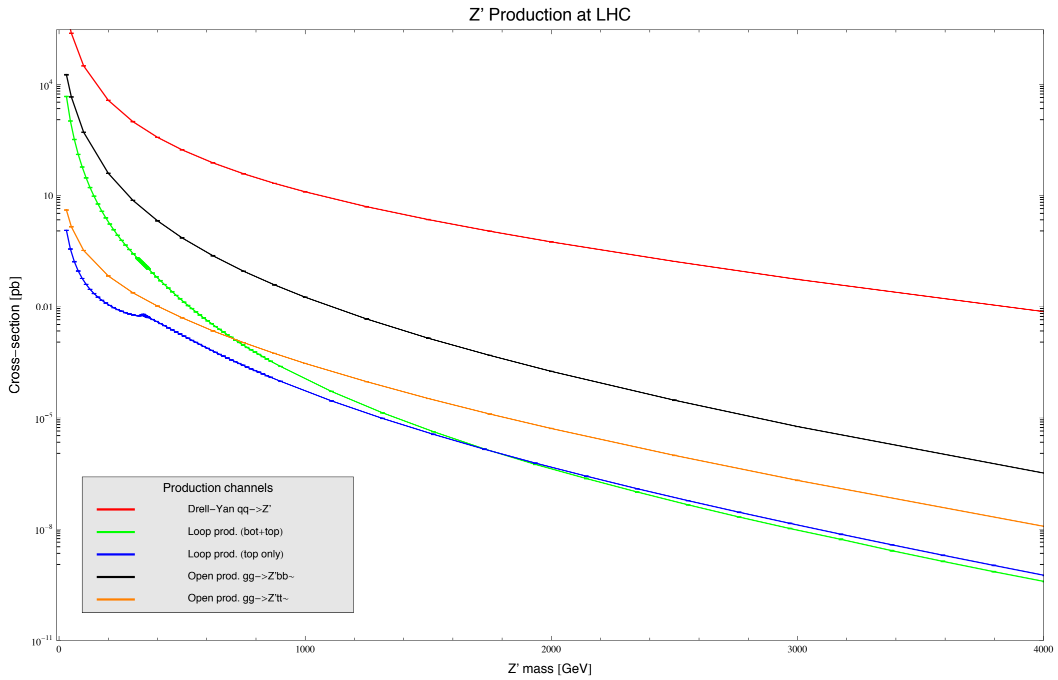

Figure 4.4: Cross section of Z' production at LHC

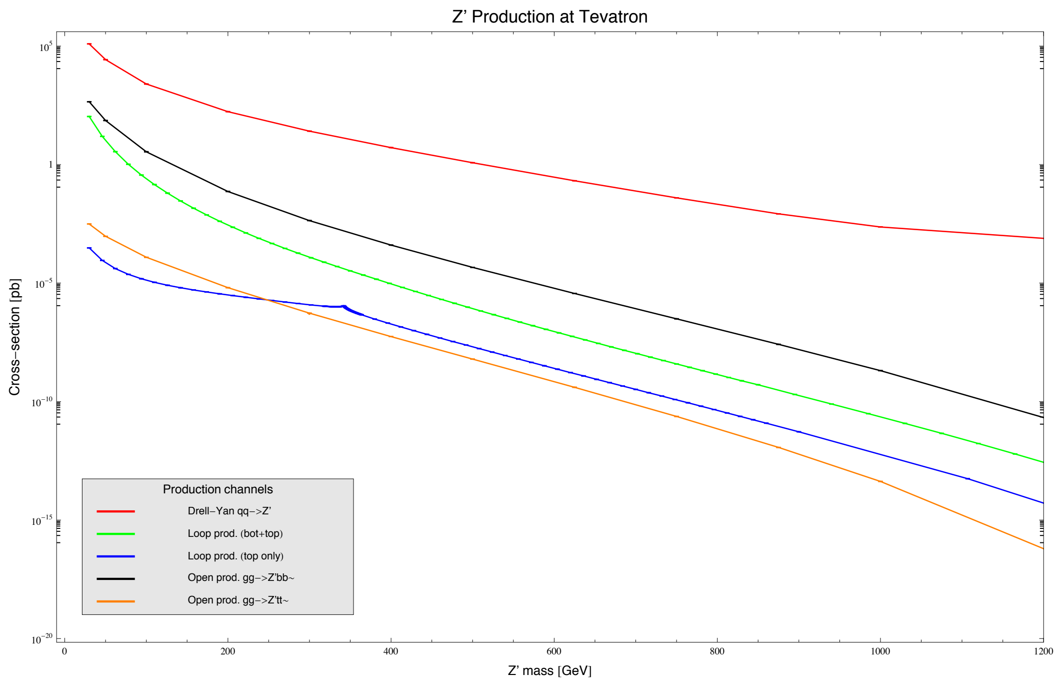

Figure 4.5: Cross section of Z' production at TEV

When considering a Z' coupling equally srongly to the top and bottom quark, then $gg \to Z'b\bar{b}$ dominates. The model [4] presented in 3.1 introduced the case of a Z' with an enhanced coupling to the top especially (since $m_t \gg m_b$). At Tevatron, the cross section of loop production is larger than for open production for $m_{Z'} > 300GeV$ whereas at LHC $gg \to Z't\bar{t}$ is still always smaller by one order of magnitude. However, there are composite replica of the SM fermions which are heavier than the top quark but lighter, or not much heavier, than the composite Z'. These extra fermions couple with full strength to a pure composite Z' and might play a role when included in the loop. Their contribution is very model-dependent because they add[17] coherently or incoherently depending on their relative axial and vector couplings.

In any case, in order for these channels to be relevant, the Drell-Yan production must be suppressed by roughly five order of magnitudes. This would be the case if the couplings a_q and v_q of the light fermions to the Z' are suppressed by a factor at least $3 \cdot 10^{-3}$. In the model by R.Contino *et al.*, if we consider the Z' that does not couple with the SM weak bosons and assume $\phi_{q_{L_i}} = \phi_{q_{R_i}} = \phi_{q_i}$ we obtain a suppression factor

$$\alpha_{q_i} \sim \sin^2 \phi_{q_i} = \frac{Y_{SM_{q_i}}}{Y_{\star_{q_i}}}$$

which is maximal for the charm quark, $\alpha_{q_c} = \frac{m_c\sqrt{2}}{vY_{\star_{q_c}}} \simeq 10^{-3}$, but still enough.[18] Of course, open $gg \to Z'b\bar{b}$ also falls down below production channels involving the t-quark.

In the loop production, the light fermion generations are not included simply because of the selectivity of the Z' couplings we are interested in. In the case of democtratic Z' couplings, the light fermions would contribute. The next plot enlights this by giving the contribution of only one up-type and one down-type quark of equal mass m_q running in the loop. It also gives insights to what would be the contribution of a "generation" of heavy composite fermions.

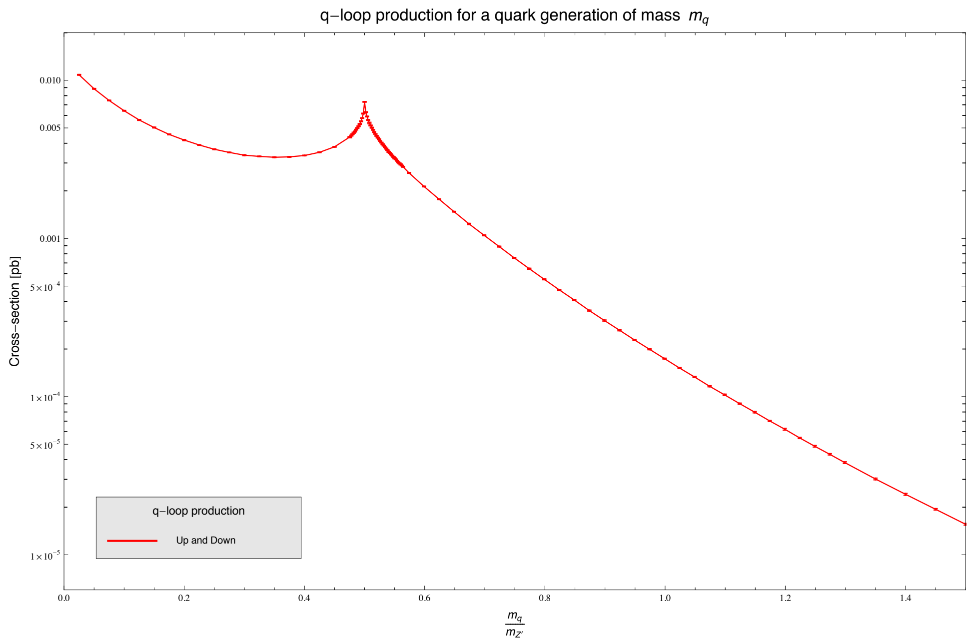

Figure 4.6: Contribution to loop production of a family of heavy quarks with $m_u = m_d$.

[17]See Fig. 4.6 for the contribution of such a "generation" of composite fermions.

[18]When considering the Z' that mixes with the SM weak bosons, it couples to light fermions with strength $g_{Z'}^{[\psi]} = g_{SM} \tan\theta \simeq g_{SM}(g_{SM}/g_\star)$ which is only of order 10^{-2} and might not be enough to suppress Drell-Yan production from light quarks.

The conclusions from this plot need to be taken with some care since there is no contribution from the axial part in the cross-section above. The two quarks have opposite axial coupling $a_u = -a_d$ and the same mass, so that their respective axial contribution cancel each other. Only the vector part contributes here, through the difference $-\frac{1}{3}\sin^2(\theta_w)$ in their vector couplings.

The axial amplitudes quickly go to a constant in the limit $(m_t/m_{Z'}) \to \infty$, so that even if $m_c \gg m_s$ the difference in the axial contribution of m_c and m_s is still suppressed by the ratio $(m_c/m_{Z'})$. As a consequence, a light generation does not contribute to the AVVV tensor. Contrary to that, for the top and bottom quark, both (m_t/m_b) and $(m_t/m_{Z'})$ are sizeable so that in plots (4.4) and (4.5) the axial part contributes more than the vector part, even when both the t and b quark run in the loop.

The couplings of this Z' are those of a standard Z-boson and of course, any model will end up with either axial or vector couplings to the fermions. What could differ from the SM is the ratio between the amount of axial and vector coupling. It is then interesting to plot separately the axial and vector contribution of the relevant channels.

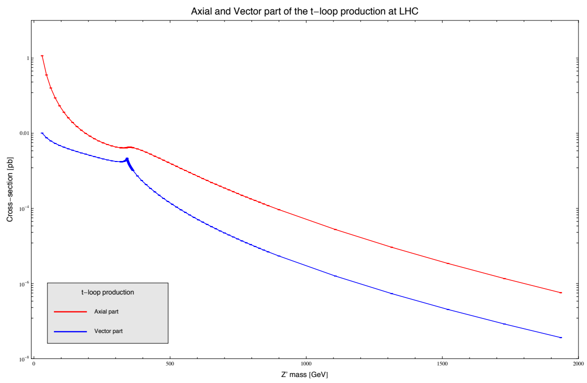

Figure 4.7: Axial and Vector contribution to the top loop production channel at LHC

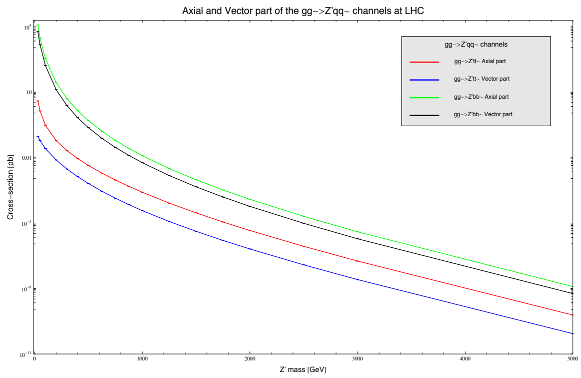

Figure 4.8: Axial and Vector contribution to the open $gg \to Z'q\bar{q}$ production channel at LHC

The vector contribution is always much smaller than the axial one, and the fact that the $v_q^{(SM)}$ is roughly twice smaller than $a_q^{(SM)}$ is not enough to explain this. This observation might be of special interest for peculiar models with large ratios v_q/a_q. In all these plots, there is a bump at the treshold $2m_t = m_{Z'}$ due to the possibility that two on-shell top quarks combine to make a Z' boson.

The very heavy extra fermions will never be directly observed but they also couple to Z' and can run in the loop of gluon fusion to affect its amplitude. The following plot shows how the t-loop production cross section evolves as the mass of the top increases.

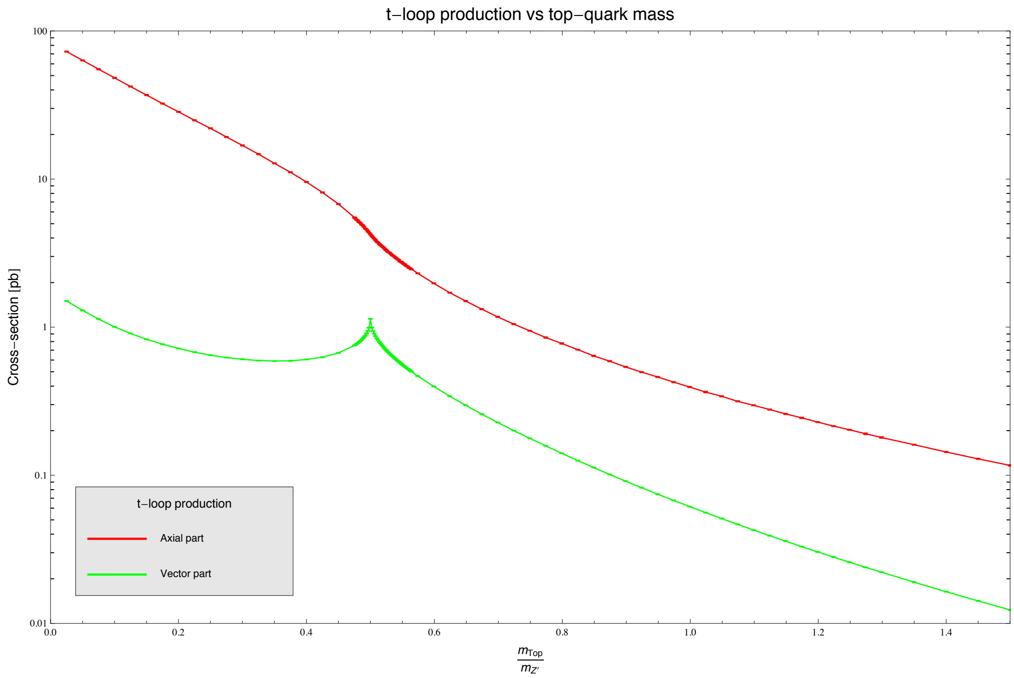

Figure 4.9: Evolution of the loop production cross-section at LHC with the mass of the running top-quark.

In the limit $m_t \to \infty$, the top quark is expected to decouple from the theory. This naturally happens for the vector part. The axial part couples to a $\gamma_\mu\gamma_5$ current which is not traceless, hence leading to anomalous terms independent of the quark mass. Therefore, a very heavy quark acts as a regulator for the anomalous theory and generates a non-vanishing effective point-like interaction when integrated out. This anomaly will always be cancelled in a fundamental theory and in the Standard Model it happens independently in each fermion generation. This is why the anomalous terms have been taken out from the axial amplitudes, so that the top quark decouples *both* in the axial and vector part. In plot 4.4, when the ratio $(m_t/m_{Z'})$ decreases, the matrix element for the axial and vector part gets smaller as well. However, because of the rapidly increasing parton distribution function, the resulting cross-section still increases.

The diagrams with different quarks running in the loop interfere so it is never easy to extrapolate the effect of including a new quark in the loop from its single contribution to it. This last plot gives a good example of that, since it might look counterintuitive to have a lower cross section with both the top and the bottom included than with only the top or only the bottom.

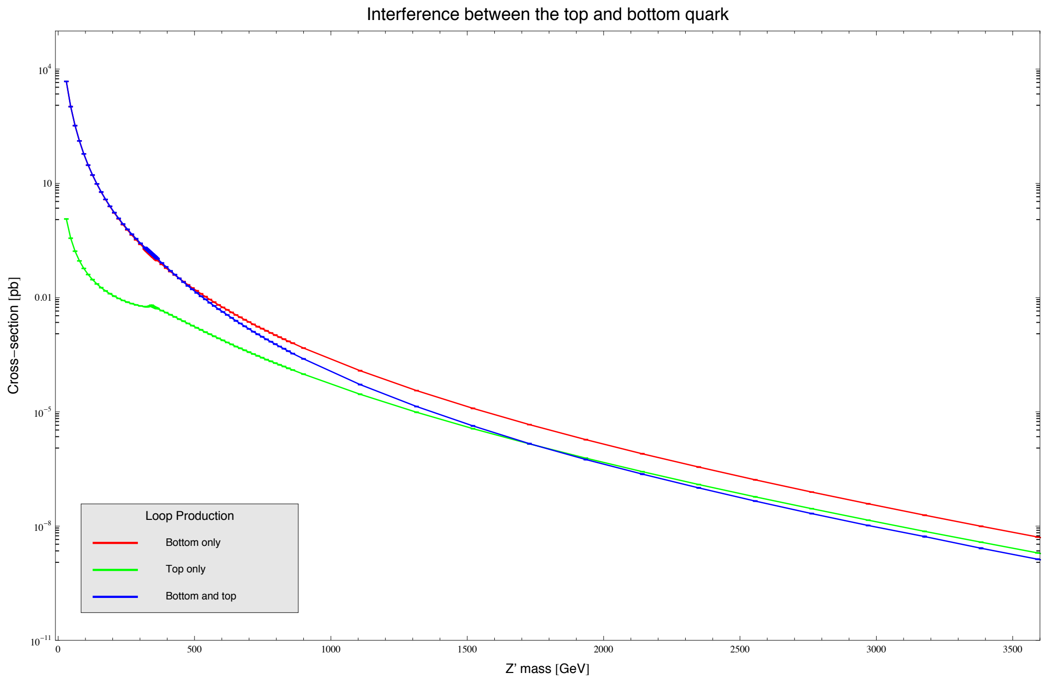

Figure 4.10: Gluon fusion cross-section with only the top, or the bottom, or both running in the loop

The bottom and top quark curves are not as different as the plot suggests it. In fact, we observe here the same threshold $2m_q = m_{Z'}$ as discussed before, but simply shifted in the case of the light bottom quark.

5 Z' Decay

Once produced on-shell, the Z' decays into particles that can be detected and betray its presence. Many models predicting a Z' also introduce new particles coupling to it, so cascade decays could matter, but are model dependent. For a Z' with sizeable mixing with the SM weak bosons and the Higgs, decays such as $Z' \to W^+W^-$ or $Z' \to hZ$ have the largest branching ratios (See [4] for a study of this case in the framework of the model by R.Contino *et al.* [4]).

When considering couplings only to fermions and proportionaly to their mass, the Z' will almost only decay to $t\bar{t}$ if it is heavier than $2m_t$. The branching ratios of the decay to other lighter fermions of mass m_f are negligeable because suppressed by $(m_q/m_t)^2$. This happens almost immediatly after the Z' mass hits the threshold $m_{Z'} = 2m_t$. Indeed, the decay width of a vector particle with mass $m_{Z'}$ to two fermions of mass m_f is given by [7]

$$\Gamma_f = \frac{g_{Z'}^2 m_{Z'}}{12\pi}\sqrt{1-(2m_f/m_{Z'}^2)}\left\{v_f^2\left(1+\frac{2m_f^2}{m_{Z'}^2}\right)+a_f^2\left(1-\frac{4m_f^2}{m_{Z'}^2}\right)\right\} \tag{5.1}$$

which goes very quickly to its asymptotical value $\Gamma(m_f = 0)$ when the ratio $(2m_f/m_{Z'})$ becomes larger. In the case of a Z' lighter than $2m_t$ we can consider two situations.

First, the remaining fermions still couple proportionnally to their mass. The branching ratios of decays into leptons $e\bar{e}$, $\mu\bar{\mu}$ and the light quarks $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ are still suppressed and only the decays into $\tau\bar{\tau}$, $b\bar{b}$ and $c\bar{c}$ remain. These events are overhelmed by the QCD background, so the Z' would be hard, if not impossible, to detect.

Secondly, a model could predict that all the couplings to other fermions than the top are *equally* suppressed. As a consequence, as soon as $m_{Z'}$ is smaller than $2m_t$, the branching ratios would become exactly the same as in the case of a SM Z-boson, hence providing a clean signature for its detection in the leptonic decay. In this last case, the experimental bounds on such a Z' are very weak since it can only be produced via processes involving a top quark so a Z' mass smaller than $2m_t$ =350 GeV is not excluded. Parrallel to that, it has a very clean decay signature, so this lets a large open window for discovery at LHC. However, models predicting an enhanced coupling to the top quark only and without relating this to the large top mass would be rather singular[19].

The $Z' \to \gamma\gamma\gamma$ channel is negligible since it is of order α_{EM}^3. The analog $Z' \to ggg$ decay is of order α_s^3 and also not expected to give a contribution even close to be as large as the direct decay into two fermions. However, this decay is in the scope of this thesis and has been computed with the same tools as for the production, apart that the decay rate is now given by

$$\begin{aligned}\Gamma(Z' \to ggg) &= \frac{8\alpha_s^3(m_{Z'}^2)\alpha_{Z'}}{3(3!)m_{Z'}(2\pi)^4} \int \sum_{\lambda_1 \ldots \lambda_4} \left(\frac{5}{6} \left| \sum_q v_q V_{\lambda_1\lambda_2\lambda_3\lambda_4} \right|^2 + \frac{3}{2} \left| \sum_q a_q A_{\lambda_1\lambda_2\lambda_3\lambda_4} \right|^2 \right) \\ &\times \prod_{i=1}^{3} \frac{d^3p_i}{2E_i(2\pi)^3} (2\pi)^4 \delta^4(p_{Z'} - \sum_{j=1}^{3} p_j)\end{aligned} \tag{5.2}$$

What differs from the differential cross-section is the flux factor and the integration measure. There is also an additional 3! in the denominator for the identical particle factor of the three outcoming gluons. The parameters used here are the same as listed in (4.2).

In this case, the phase space is directly generated from the Mandelstamm variable s and t. Using these variables, expression (5.3) becomes

$$\Gamma(Z' \to ggg) = \frac{\alpha_s^3(m_{Z'}^2)\alpha_{Z'}}{36 m_{Z'}^3 (2\pi)^7} \int_0^{m_{Z'}^2} ds \int_0^{m_{Z'}^2 - s} dt \sum_{\lambda_1 \ldots \lambda_4} \left(\frac{5}{6} \left| \sum_q v_q V_{\lambda_1\lambda_2\lambda_3\lambda_4} \right|^2 + \frac{3}{2} \left| \sum_q a_q A_{\lambda_1\lambda_2\lambda_3\lambda_4} \right|^2 \right)$$

The following plots gives the application of Eq. (5.3) to the $Z' \to ggg$ decay with only the top quark running in the loop.

[19]And it would not provide any explanation for the fermion mass spectrum. The SM extension described in 7.3 might be an example, although it still needs to be further investigated.

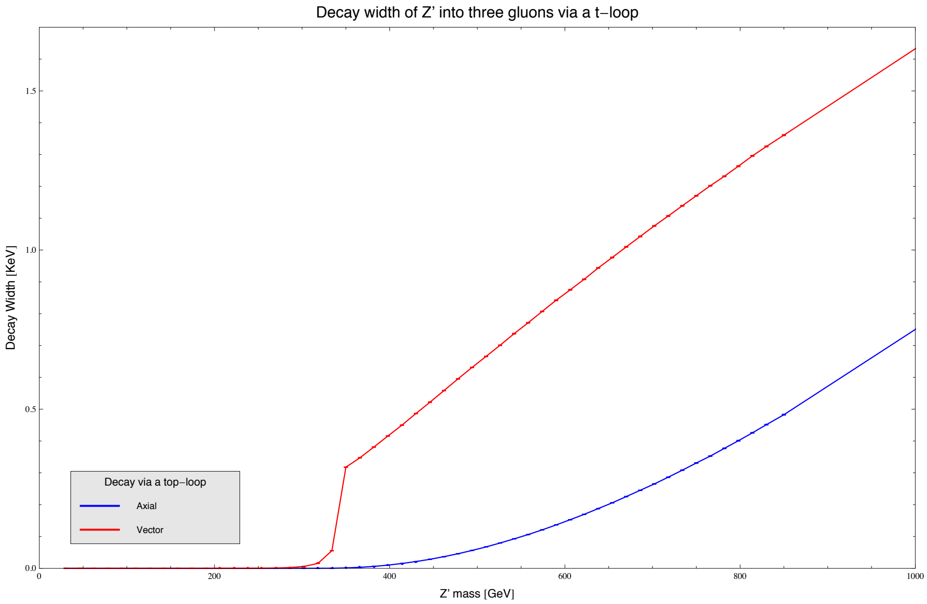

Figure 5.1: Decay width of Z' into three gluons via a t-loop.

For $m_{Z'} < 2m_t$ the matrix element is very small and the decay rate is negligible. Since very heavy quarks decouple from the theory, it is expected that $\Gamma(Z' \to ggg)$ vanishes in the limit $(m_t/m_Z) \to 0$. The transition occurs abruptly and then, alike with $Z' \to Q\bar{Q}$, the decay rate linearly increases with $m_{Z'}$. This is because the amplitudes go to a constant in this limit and the integration $\frac{2}{m^3_{Z'}} \int_0^{m^2_{Z'}} ds \int_0^{m^2_{Z'}-s} dt$ brings a prefactor m_Z to it.

The two quarks of a given generation interfere destructively and when both of them are included in the loop, the decay rate goes to a constant in the large $m_{Z'}$ limit.

6 Conclusion

Although many models introduce a Z' with democratic couplings to all fermions, there are special theories where it couples proportionally to the fermion masses. The experimental bounds on this Z' are much weaker [7] than for a universal one. An attractive theory in this regard considers partial compositeness of the SM fields which naturally predicts this peculiar Z' behaviour [4]. It is also possible to introduce a Z' coupling only to t_R through an extension $U(1)_X$ of the SM gauge symmetries. Such a model is consistent as a *fundamental* theory, but very unnatural because of the *ad-hoc* set of fermion added.

The possible production channels for such a Z' are investigated. In particular, the cumbersome computation of the production $gg \to Z'g$ through gluon fusion via a fermionic loop is carried on. The axial and vector amplitudes for this process, which can be added incoherently in this case, have been derived keeping the full dependence to the quark mass. We showed that at LHC the open production $gg \to Z't\bar{t}$ always dominates the loop production. In order for this channel to be relevant, the Drell-Yan production from the annihilation of light quarks must be suppressed by five orders of magnitude. This corresponds to a suppression factor of $3 \cdot 10^{-3}$ in their couplings, which is consistent within model [2].

The Z' considered does not mix with the SM weak bosons and therefore almost only decays into $t\bar{t}$ when $m_{Z'} > 2m_t$. In the case of $m_{Z'} < 2m_t$, the decay is invisible if the couplings to the three remaining fermions are suppressed proportionally to their masses. If the suppression factor is the same for all fermions except the top quark, then the branching ratios are those of the case of a SM Z boson and the Z' has a clean signature in the leptonic channel.

As a follow-up to this work, the model outlined in (7.3) might be investigated further[20] to derive the experimental bounds on it and see if the new degrees of freedom introduced can have any explanatory power towards open problems.

A similar analysis can be undertaken for a Z' coupling only to the top quark and also weakly mixing with the SM weak bosons. Then of course, LEP observables are affected and give significant bounds [2] on the Z' mass. It is interesting to see what is the maximum possible mixing angle[21] θ so that $m_{Z'} < 2m_t$ is not excluded and that the decays $Z' \to hZ$ and $Z' \to W^+W^-$ still happen with sizeable branching ratios.

The entertaining story of the barophilic Z' does not end with this thesis and will hopefully meet a turning point with the longed for first LHC data.

[20] In particular, see under which condition the diagonalization of the fermion mass matrix gives back eigenstates which can be identified with the observed SM states.

[21] The Z' from the neutral gauge boson in $\rho^\star$ in model 3.1 is not of this type, because it mixes with an angle $\tan\theta = \frac{g_{el}}{g_*}$, ratio which cannot be much smaller than $\frac{1}{6}$.

7 Appendix

7.1 Lee-Yang Theorem

Lee-Yang theorem states that a massive spin-1 vecor-like particle cannot decay into two *identical vector-like massless bosons*. To prove this, let's try to construct a spin-1 vector state $|v\rangle_i$ from two massless vector states. Such a state behaves like a vector $\mathbf{v}$ under space transformations and also carry an index i from 1 to 3, making a total of three J=1 states. If we place ourself in the center of mass frame and use the ladder operators, we may write

$$|v\rangle_i = \int \frac{d^3p}{(2\pi)^6} \frac{1}{2\sqrt{E_{\boldsymbol{p}} E_{-\boldsymbol{p}}}} X_i^{jk}(\boldsymbol{p}) a_j^\dagger(\boldsymbol{p}) a_k^\dagger(-\boldsymbol{p}) |0\rangle \tag{7.1}$$

where $X_i^{jk}(\boldsymbol{p})$ is a rank-three tensor which depends only on $\boldsymbol{p}$. Its most general form is

$$X_{ijk}(\boldsymbol{p}) = A\epsilon_{ijk} + Bp_i\delta_{jk} + Cp_j\delta_{ik} + Dp_k\delta_{ij} + B'p_i\epsilon_{jkl}p^l + C'p_j\epsilon_{kil}p^l + D'p_k\epsilon_{ijl}p^l + Ep_ip_jp_k \tag{7.2}$$

where all the functions A,...,D' only depend on $|\boldsymbol{p}|$. The particles created by $a^\dagger$ are massless here, so from their equation of motion, we have $p_i a_i^\dagger = 0$ which means that the C,D,C' and D' terms above don't contribute. The expression of X_{ijk} is then left with

$$X_{ijk}(\boldsymbol{p}) = A\epsilon_{ijk} + Bp_i\delta_{jk} + B'p_i\epsilon_{jkl}p^l \tag{7.3}$$

which satisfies the following relation

$$X_{ijk}(\boldsymbol{p}) = -X_{ikj}(-\boldsymbol{p}) \tag{7.4}$$

Taking advantage of the fact that we assumed the two initial particles to obey Bose statistics, we can commute the two creation operators in 7.1 at no cost

$$|v\rangle_i = \int \frac{d^3p}{(2\pi)^6} \frac{1}{2\sqrt{E_{\boldsymbol{p}} E_{-\boldsymbol{p}}}} X_i^{jk}(\boldsymbol{p}) a_k^\dagger(-\boldsymbol{p}) a_j^\dagger(\boldsymbol{p}) |0\rangle$$

and re-label $\boldsymbol{p}$ as $-\boldsymbol{p}$ and (j,k) as (k,j)

$$|v\rangle_i = \int \frac{d^3p}{(2\pi)^6} \frac{1}{2\sqrt{E_{-\boldsymbol{p}} E_{\boldsymbol{p}}}} X_i^{kj}(-\boldsymbol{p}) a_j^\dagger(\boldsymbol{p}) a_k^\dagger(-\boldsymbol{p}) |0\rangle \tag{7.5}$$

By adding 7.1 with 7.5 and using 7.4 we get

$$|v\rangle_i = \frac{1}{2}\int \frac{d^3p}{(2\pi)^6} \frac{1}{2\sqrt{E_{\boldsymbol{p}} E_{-\boldsymbol{p}}}} \underbrace{\left(X_i^{jk}(\boldsymbol{p}) + X_i^{kj}(-\boldsymbol{p})\right)}_{=0 \text{ by } 7.4} a_j^\dagger(\boldsymbol{p}) a_k^\dagger(-\boldsymbol{p}) |0\rangle = 0 \tag{7.6}$$

That means it's not possible to create a massive J=1 state from two identical massless vector-like bosons. Notice that if the initial state was colored, then this restriction does not hold anymore because an aditional color tensor structure goes with X_{ikj} and its antisymmetric part can compensate for the relation 7.4. This is what happens in the three-gluon vertex. This does not affect the phenomenology discussed in this thesis where only a Z' neutral under $SU(3)_c$ is considered.

7.2 Check with Glover's plot for Z decay into three gluons

In [3], E.W.N. Glover *et al.* present a plot (Fig. 2 of their paper) for the decay of the SM Z boson into three gluons via a loop including all the six quark of the Standard Model. The mass of the top-quark varies on the x-axis. All the parameters were indicated in the paper and no PDF enters in the computation, so the reproduction of this plot provides a clean check of the computations carried on in this thesis.

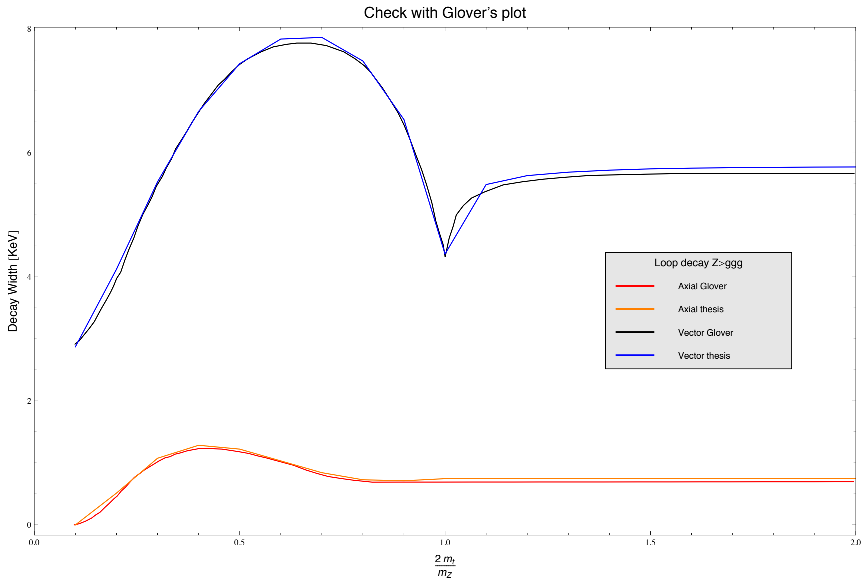

Figure 7.1: Reproduction of Glover *et al.* plot for the decay $Z \to ggg$ via a loop including all quarks.

7.3 Fundamental theory for a Z' coupling to the top-quark only

In this model, the Z' is directly introduced by adding an additional $U(1)_X$ gauge group to the symmetries of the Standard Model. The charges X of the SM particles have to be specified. The requirement of having an anomaly-free theory and vanishing Flavor Changing Neutral Currents (FCNC) severely contraints them and one simple consistent choice is the assignment of B-L (Baryonic - Leptonic number) to the new charge. Also, the Higgs sector includes now a new complex scalar singlet (under all gauge groups) which spontaneously break $U(1)_X$ and gives mass to the Z' boson. With these features, the model is known as the B-L extension to the SM, and has already retained some interest[5].
Many studies [4][2] show that the B-L extension has strong experimental bounds. The strongest come from the indirect searches at LEPII while the direct searches already ruled out any B-L type of Z' lighter than 600 GeV.

In the case of a Z' coupling to the top quark only, the current experimental constraints are much weaker and a large window remains open for discovery at LHC. However, breaking generation and chiral universality for the Z' couplings while keeping the model anomaly free[22] is subtle. An example of a model accomplishing this is outilined in the next sections.

7.3.1 Overview

This model lies in the same framework as the B-L extension to the SM mentionned earlier. So the Lagrangian has a $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge symmetry and the Higgs sector contains an additional complex scalar singlet S. S is neutral towards all gauge groups but $U(1)_X$, so it decouples from the SM gauge bosons, preventing any direct mixing of Z' with the neutral W^3 and B^0.

In the Higgs potential, there will be a term mixing the SM Higgs with S, opening interesting possibilities for the model. The right-handed[23] top-quark t_R is the only SM particle with a $U(1)_X$ charge X=1, so it

[22] We insist ton having a fundamental theory here. Otherwise, there are still difficult problems arising from the fermion mass terms.

[23] Since the generator T_X of $U(1)_X$ commutes with the isospin operator T^3, the charge of the two quarks of the left-handed doublets should be the same. Therefore, if the left-handed t-quark had an X charge, this would enable the Z' coupling of the

will be the only one to couple with the associated Z' boson. To cancel the anomalies assiociated with the new $U(1)_X$ gauge group, four left and right primed colored quarks are added, which in turn, also enable a mass term for the right-handed top. The new gauge group does maximum violation of chirality in the third fermion generation, and it is why it is somehow difficult to find a set of new *massive* fermions fixing the anomalies.

Particle	T^3	Y	X
ϕ	-1/2	+1/2	0
S	0	0	-1
t_L	+1/2	+1/6	0
t_R	0	+2/3	+1
t'_L	0	+2/3	0
t'_R	0	+2/3	-1
u'_R	0	+2/3	0
u'_L	0	+2/3	+1
c'_R	0	+2/3	0
c'_L	0	+2/3	-1
x'_R	0	+2/3	0
x'_L	0	+2/3	0

Table 1: Charges of the relevant particles

7.3.2 Higgs Sector

The most general renormalizable potential for the Higgs doublet ϕ and the singlet S one is

$$V(\phi, S) = -\mu_H^2\phi\phi^\dagger + \frac{\lambda_H}{2}(\phi\phi^\dagger)^2 - \mu_S^2 SS^\dagger + \frac{\lambda_S}{2}(SS^\dagger)^2 + \lambda SS^\dagger\phi\phi^\dagger \tag{7.7}$$

The are no cubic terms here, since it is impossible to construct such a term invariant under $SU(2)_Y$ and $U(1)_X$. Minimizing the potential gives the vacuum expectation values (vev) v and s of the fields ϕ and S.

$$\frac{\partial V}{\partial(\phi\phi^\dagger)} = \frac{\partial V}{\partial(SS^\dagger)} = 0 = \begin{cases} -\mu_H^2 + \lambda_H v^2 + \lambda s^2 = 0 \\ -\mu_S^2 + \lambda_S s^2 + \lambda v^2 = 0 \end{cases}$$

$$s^2 = \frac{\lambda\mu_H^2 - \mu_S^2\lambda_H}{\lambda^2 - \lambda_S\lambda_H} \text{ and } v^2 = \frac{\lambda\mu_S^2 - \mu_H^2\lambda_S}{\lambda^2 - \lambda_H\lambda_S} \tag{7.8}$$

We can now expand the kinetic term of S around its vev and see how the Z' gauge boson acquires mass. The covariant derivative of S only contains Z' with the $U(1)_X$ coupling constant g_z

$$S(x) \to s + \frac{1}{\sqrt{2}}(S_1(x) + iS_2(x))$$

$$\begin{aligned} |D_\mu S|^2 &\to \frac{1}{2}(\partial_\mu + ig_z Z'_\mu)(s + S_1 + iS_2)(\partial^\mu - ig_z Z'^\mu)(s + S_1 - iS_2) \\ &= \frac{1}{2}(\partial_\mu S_1)^2 + \frac{1}{2}(\partial_\mu S_2)^2 + \sqrt{2}g_z s Z'_\mu\partial^\mu S_2 + g_z^2 s^2 Z'_\mu Z'^\mu + ... \end{aligned}$$

Where we omitted terms cubic and quartic in the fields S_1, S_2 and Z'^μ. The third term in the expression above modifies the Z' propagator to make it properly tranverse whereas the fourth one is a mass term for the Z'.

$$m_{Z'} = \frac{1}{2}g_z s \tag{7.9}$$

b-quark.

Expanding the potential in the same way, one finds the mass terms for the Higgs. The mixing term in λ introduces non-diagonal terms in the mass matrix [5] of ϕ and S. In our analysis we will consider the Higgses as completely decoupled (very small λ). Then, the component H of the Higgs doublet and the real component[24] S_1 of S, which we will call H' from now on, acquire a mass. The other degrees of freedom are realized as massless Goldstone bosons and are eaten to give mass to the gauge bosons.

$$m_H = \sqrt{2\lambda_H} v \ , \ m_{H'} = \sqrt{2\lambda_S} s \tag{7.10}$$

7.3.3 t_R coupling

The right-handed top quark is the only SM particle with an X charge. It is the bridge between the Standard Model observables and the additional states introduced[25]. Therefore, it's the root of all the phenomenology related to this new model. t_R couples to Z' through its kinematic term.

$$\Delta\mathcal{L} = \bar{t}_R(i\not{D})t_R + \bar{t'}_R(i\not{D})t'_R \bar{u'}_R(i\not{D})u'_R + \bar{c'}_R(i\not{D})c'_R \supset g_z Z'_\mu J^\mu_X \tag{7.11}$$

$$J^\mu_X = \bar{t}_R(+1)\gamma^\mu t_R + \bar{t'}_R(-1)\gamma^\mu t'_R + \bar{u'}_R(+1)\gamma^\mu u'_R + \bar{c'}_R(-1)\gamma^\mu c'_R \tag{7.12}$$

where (+1) and (-1) are the $U(1)_X$ charges of the new quarks. If we want to consider this theory as fundamental, we want that axial-vector anomalies cancel. The six new states charged under $U(1)_Y$ might affect the natural anomaly cancellation occuring in the SM. But it is trivial to check that they don't since they always appear in left and right[26] colored pair with the same Y charge. But there are also new anomalies from triangle diagrams involving $U(1)_X$ which have to cancel as well. These diagrams are

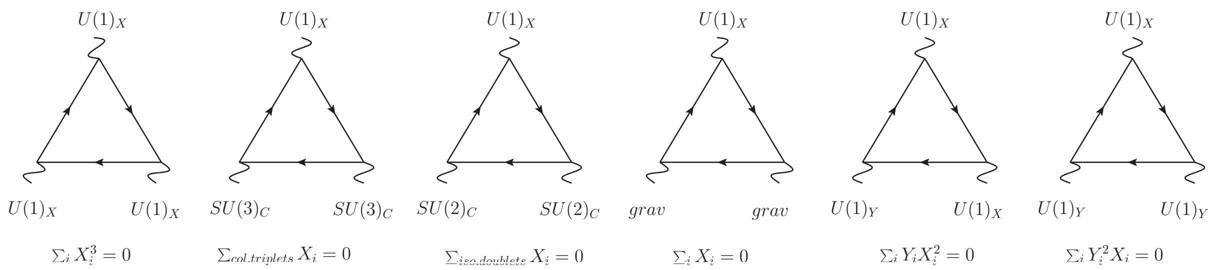

Figure 7.2: Triangle-loop diagrams leading to anomalies along with the associated condition to cancel them.

The condition assiociated to the $U(1)^3_X$ as well as the colored and gravitational anomaly are trivially satisfied

$$\sum_i = X_i^3 = (+1)^3 + (-1)^3 - (+1)^3 - (-1)^3 = 0 \tag{7.13}$$

$$\sum_i = X_i = (+1) + (-1) - (+1) - (-1) = 0 \tag{7.14}$$

All the new states are singlet under $SU(2)_L$, so the third diagram is not problematic either. The two conditions from the contribution of the $U(1)_Y$ and $U(1)_X$ gauge currents are less explicitly fulfilled

$$\sum_i Y_i X_i^2 = (+2/3)(+1)^2 + (+2/3)(-1)^2 - (+2/3)(+1)^2 - (+2/3)(-1)^2 = 0 \tag{7.15}$$

[24] In general, the massive state is a linear combination of S_1 and S_2 but with the specific choice made for the vacuum state of S, only S_1 acquires a mass and S_2 is the Nambu-Goldstone boson.

[25] Of course, these new fermions also have an hypercharge, but they are expected to be very heavy and not directly influencing phenomenology

[26] Remember to flip the sign of the contribution of the left-handed fermions

$$\sum_i Y_i^2 X_i = (+2/3)^2(+1) + (+2/3)^2(-1) - (+2/3)^2(+1) - (+2/3)^2(-1) = 0 \quad (7.16)$$

The particle set added here is the minimal one to cancel the anomalies assiociated to a peculiar Z' coupling only to the top quark. But there remains some freedom within this set. For instance, one could choose the u' and c' Weyl spinors to be singlet under $SU(3)$ or put the X charge on the left-handed u' and c' instead. However, this would prevent u' and c' to mix with the other $Y = +2/3$ states, then spoiling the diagonalization of the fermion mass matrix. The chirality choice for the new fields is also open, and it is possible to chose more left-handed fields than right-handed ones. The resulting set of new particles looks akward but does not lead to any inconsistency. An example of such a set is given in the chart below.

Particle	T^3	Y	X
ϕ	-1/2	+1/2	0
S	0	0	-1
t_L	+1/2	+1/6	0
t_R	0	+2/3	+1
t'_L	0	+2/3	0
t''_L	0	+2/3	-1
u'_L	0	-2/3	0
u'_R	0	-2/3	-1
c'_R	0	+2/3	0
c'_L	0	-2/3	+1
x'_R	0	-2/3	0
x'_L	0	-2/3	0

Table 2: An alternative set of additional fermions, with more left-handed than right-handed states.

Right-handed neutrinos could also come into play and recieve an X charge, hence enabling a natural heavy dynamical mass term for them through the scalar S. Of course adding more primed fermions is possible and lead to other solutions where their charges X_i and Y_i are usually no longer fractional since they appear quadratically in (7.15) and (7.16). However, it is worth to notice that if we keep unitary (plus or minus one) charges for the new spinors, on can freely add n sets of four fields $\{u'_L, u'_R, c'_L, c'_R\}$ and simply give $Y^{(n)}$ to each set so that $\sum_n Y^{(n)} = +4/3$. In this paper, only the minimal extension of table (1) is considered. But what ultimately motivates one or the other is the compatibility with an eventual fundamental theory breaking down to this one. The current experimental bounds on the new predicted states also play a role, and lead us to the next section about mass terms.

7.3.4 Mass terms

There are now three kinds of different mass term for the $Q = +2/3$ particles.

- A mass term from the $U(1)_x$ symmetry breaking (denoted †). $-\lambda \bar{t'_L} S t_R$
- A mass term from the $SU(2)_L$ symmetry breaking (denoted $\star$). $-\lambda \bar{t_L} \phi u'_R$
- A Dirac mass (denoted X). $-m^2 \bar{t'_L} u'_R$

The resulting mass matrix for $Q = +2/3$ particles takes the following form.

$$\left(\begin{array}{c|ccccccc} & u_R & c_R & t_R & t'_R & u'_R & c'_R & x'_R \\ \hline U_L & \star & \star & 0 & 0 & 0 & 0 & \star \\ C_L & \star & \star & 0 & 0 & 0 & 0 & \star \\ t_L & \star & \star & 0 & 0 & 0 & 0 & \star \\ t'_L & X & X & \dagger & \dagger & X & X & X \\ u'_L & \dagger & \dagger & X & 0 & \dagger & \dagger & \dagger \\ c'_L & \dagger & \dagger & 0 & X & \dagger & \dagger & \dagger \\ x'_L & X & X & \dagger & \dagger & X & X & X \end{array}\right)$$

The determinant of this matrix is not zero, so it can be diagonalized and all eigenvectors have a non-zero eigenvalue. This is important, because a natural way to explain why we didn't see the additional fermions yet is that they are very massive.

In rotating the space of particles $Q = +3/2$ it is important to find back the eigenstates which match the physcal observed u_R,c_R and t_R states. The diagonalization of the matrix above is not the only source of mixing. A second one comes from the Higgs sector and has already been discussed whereas a third one can be introduced when considering a term $Z^{\mu\nu} F_{\mu\nu}$ in the kinetic terms of the gauge bosons. $G_{\mu\nu}$ and $F_{\mu\nu}$ are the field strength of the photon and Z' respectively.

The development of these mixings is not carried on in this paper, but would lead to the phenomenology and the associated free parameters of the theory.

7.4 Helicity Amplitudes

We work in the rest frame of the p_1 and p_2 system, so the momenta are

$$\begin{aligned} p_1^\mu &= (-p, 0, 0, -p) \\ p_2^\nu &= (-p, 0, 0, p) \\ p_3^\rho &= (q, q\sin\theta, 0, q\cos\theta) \\ p_4^\alpha &= (E, -q\sin\theta, 0, -q\cos\theta) \end{aligned} \tag{7.17}$$

The polarization amplitude is projected onto the vectors of a chosen basis for the polarizations of the three gluons and the Z'. This basis is

$$\begin{aligned} e_1^+ = e_2^- &= 1/\sqrt{2}(0, -i, 1, 0) \\ e_1^- = e_2^+ &= 1/\sqrt{2}(0, i, 1, 0) \\ e_3^+ = e_4^- &= 1/\sqrt{2}(0, i\cos\theta, 1, -i\sin\theta) \\ e_3^- = e_4^+ &= 1/\sqrt{2}(0, -i\cos\theta, 1, i\sin\theta) \\ e_4^L &= (1/m_Z)(q, -E\sin\theta, 0, -E\cos\theta) \end{aligned} \tag{7.18}$$

It is more convenient to use the Mandelstamm variable defined as

$$s = 2p_1 \cdot p_2, \quad t = 2p_2 \cdot p_3, \quad u = 2p_1 \cdot p_3 \tag{7.19}$$

With four other auxiliary definitions

$$s_1 = s - m_Z^2, \quad t_1 = t - m_Z^2, \quad u_1 = u - m_Z^2, \quad \Delta = \sqrt{-m_Z^2/(2stu)} \tag{7.20}$$

Since the gluons are massless on-shell particles and because of energy-momentum conservation, we have

$$p_1^2 = p_2^2 = p_3^2 = 0, \quad s + t + u = m_Z^2 \tag{7.21}$$

All the vectors from the basis above can be expressed in terms of s, t and u using these relations

$$p = \frac{1}{2}\sqrt{s}, \quad q = p - \frac{m_Z^2}{4p}, \quad \cos\theta = \frac{2t}{m_Z^2 - s} - 1, \quad E = 2p - q \tag{7.22}$$

This basis (7.18) has the special property that if gluon one is exchanged with gluon two, their helicity state is still an eigenvector of the basis but corresponding to an helicity of opposite sign. This special relation will allows to express some helicity amplitudes in term of another one. For instance, one can apply the parity operator to the helicity amplitude $A_{+-++}(s,t,u)$ to obtain, up to an irrelevant phase, $A_{-+--}(s,t,u)$. Then, exchanging gluon one and two gives $A_{+-++}(s,t,u) = A_{+---}(s,u,t)$ which is one of the relation used in the expressions of the axial amplitudes. Notice that s and t also need to be switched when interchanging gluons.

In the notation of Eq. 4.6, the polarization tensor obtained by Glover *et al.* in [3] and independently rederived by F.Tramontano [14] is

$$\begin{aligned} A^{\alpha\mu\nu\rho}(s,t,u,m_f,m_{Z'}) \quad = \quad & (p_1\cdot p_2 A_1 + p_2\cdot p_3 A_2)\,\epsilon^{\alpha\mu\nu p_1} p_1^\rho - \left(\frac{p_1\cdot p_2 p_1\cdot p_3}{p_2\cdot p_3} A_1 + p_1\cdot p_3 A_2\right)\epsilon^{\alpha\mu\nu p_1} p_2^\rho \\ & + A_1 \epsilon^{\alpha\mu p_1 p_2} p_1^\nu p_1^\rho - \frac{p_1\cdot p_3}{p_2\cdot p_3} A_1 \epsilon^{\alpha\mu p_1 p_2} p_1^\nu p_2^\rho + A_2 \epsilon^{\alpha\mu p_1 p_2} p_3^\nu p_1^\rho + A_3 \epsilon^{\alpha\mu p_1 p_2} p_3^\nu p_2^\rho \\ & + (p_1\cdot p_3 A_2 + p_2\cdot p_3 A_3)\,\epsilon^{\alpha\mu p_1 p_2}\delta^{\nu\rho} + \text{ Bose permutations} \end{aligned} \tag{7.23}$$

where $\epsilon^{\alpha\mu p_1 p_2}$ and $\epsilon^{\alpha\mu\nu p_1}$ are a shorthand notations for $\epsilon^{\alpha\mu ab} p_1^a p_2^b$ and $\epsilon^{\alpha\mu\nu a} p_1^a$ respectively. The metric convention for contracting the tensor is $(+,-,-,-)$. For the Bose permutations, the Lorentz indices μ,ν and ρ have to be interchanged since each of them is attached to a given particle tag (as explicit in 7.17). It means that the polarization vectors contracted with $A^{\alpha\mu\nu\rho}$ also need to be switched. To account for the antisymmetric f^{abc} *factored out* in Eq. (4.6), the odd permutations take a minus sign. When performing the Bose permutations, do not forget to also accordingly exchange the variable s, t and u of the functions A_1,A_2 and A_3 defined as

$$\begin{aligned} A_1(s,t,u) \quad = \quad & -\frac{32t}{s^2}\left(\frac{(s-t_1)}{t_1^2} B_1(t) + \frac{(s-u_1)}{u_1^2} B_1(u)\right) \\ & + \frac{32 m_f^2}{s^2 u}\left(s_1 C_1(s) + \frac{2s^2 - t_1^2}{t_1} C_1(t) + \frac{2s^2 - u_1^2}{u_1} C_1(u)\right) \\ & - \frac{16 m_f^2}{s^2 u}(stD(s,t) + suD(s,u) - 3tuD(u,t)) - \frac{16t}{s^3} E(u,t) \end{aligned} \tag{7.24}$$

$$\begin{aligned} A_2(s,t,u) \quad = \quad & 16\left(\frac{(s_1^2 + 2tu)}{s_1^2 tu} B_1(s) + \frac{(3t_1^2 - 2s^2)}{s t_1^2 u} B_1(t) + \frac{(u_1^2 + 4st)}{s t u_1^2} B_1(u)\right) \\ & + 64 m_f^2 \left(\frac{C_1(s)}{s s_1} + \frac{C_1(t)}{t t_1} + \frac{2C_1(u)}{u u_1}\right) \\ & + \frac{16 m_f^2}{s}\left(\frac{(u-s)}{u} D(s,t) - \frac{(s+t)}{t} D(s,u) - 2D(u,t)\right) \\ & + 8\left(\frac{E(s,t)}{u^2} + \frac{E(s,u)}{t^2} + \frac{2E(u,t)}{s^2}\right) \end{aligned} \tag{7.25}$$

$$\begin{aligned}
A_3(s,t,u) &= -32\left(\frac{1}{t^2}B_1(s)+\frac{1}{t_1^2}B_1(t)+\left(-\frac{u}{t^2u_1}+\frac{u}{tu_1^2}+\frac{1}{st}\right)B_1(u)\right)\\
&+32m_f^2\left(-\frac{s_1}{st^2}C_1(s)+\left(\frac{t_1}{st^2}-\frac{2}{tt_1}\right)C_1(t)-\left(\frac{u_1}{st^2}-\frac{2}{su_1}\right)C_1(u)\right)\\
&+16m_f^2\left(\frac{1}{s}D(s,t)+\left(\frac{3u}{t^2}-\frac{1}{s}+\frac{1}{t}\right)D(s,u)+\left(\frac{t_1}{st}+\frac{1}{s}\right)D(u,t)\right)\\
&-8\left(\frac{(t+2u)}{t^3}E(s,u)+\frac{1}{s^2}E(u,t)\right) \qquad (7.26)
\end{aligned}$$

The functions B_1, C, C_1, D and E correspond to combinations of two-, three- and four-points scalar integrals. The two-point function $B(s)$ is

$$\begin{aligned}
\frac{i}{\pi^2}B(s) &= \int_0^1 dx\log\left(m_f^2-i\epsilon-sx(1-x)\right)\\
&= \log\left(m_f^2\right)-2+\sqrt{1-\frac{4(m_f^2-i\epsilon)}{s}}\log\left(\frac{-z}{1-z}\right) \qquad (7.27)
\end{aligned}$$

With

$$z=\frac{1}{2}\left(1+\sqrt{1-\frac{4(m_f^2-i\epsilon)}{s}}\right) \qquad (7.28)$$

But in the amplitudes, only the following combination appears

$$B_1(s)=B(s)-B(m_Z^2) \qquad (7.29)$$

The three-point integral with two massless lines $p_1^2=p_2^2=0$ and $(p_1+p_2)^2=s$ is

$$\begin{aligned}
C(s) &= \int\frac{d^4q}{\left(q^2-m_f^2\right)\left((q+p_1)^2-m_f^2\right)\left((q+p_1+p_2)^2-m_f^2\right)}\\
&= i\pi^2\int_0^1\frac{dx}{sx}\log\left(1-i\epsilon-\frac{s}{m_f^2}x(1-x)\right)\\
&= \frac{i\pi^2}{2s}\left(\log\left(\frac{-z}{1-z}\right)\right)^2 \qquad (7.30)
\end{aligned}$$

with z given by Eq. (7.28). The three-point function $C_1(s)$ with one external massless and on massive external line, $p_1^2=0$, $p_2^2=m_Z^2$ and $(p_1+p_2)^2=s$ is given by

$$s_1C_1(s)=sC(s)-m_Z^2C(m_Z^2) \qquad (7.31)$$

Finally, there is the four-point function with three massless external lines and one massive external line $p_1^2=p_2^2=p_3^2=0$ and $p_4^2=m_Z^2$

$$\begin{aligned}
D(s,t) &= \int\frac{d^4q}{\left(q^2-m_f^2\right)\left((q+p_1)^2-m_f^2\right)\left((q+p_1+p_2)^2-m_f^2\right)\left((q-p_4)^2-m_f^2\right)}\\
&= \frac{i\pi^2}{st}\int_0^1\frac{dx}{x(1-x)+m_f^2u/(ts)}\Big\{-\log\left(1-i\epsilon-\frac{m_Z^2}{m_f^2}x(1-x)\right)\\
&+\log\left(1-i\epsilon-\frac{s}{m_f^2}x(1-x)\right)+\log\left(1-i\epsilon-\frac{t}{m_f^2}x(1-x)\right)\Big\} \qquad (7.32)
\end{aligned}$$

This result can be expressed in terms of dilogs[27] via the relation

$$\int_0^1 \frac{dx}{x(1-x)+m_f^2 u/(ts)} \log\left(1-\frac{v}{m_f^2}x(1-x)\right) =$$
$$\frac{2}{x_- - x_+}\Big\{ \log\left(1-\frac{1}{x_+}\right)\left(\log\left(v/m_f^2\right)+\log\left(x_+-y_+\right)+\log\left(x_+-y_-\right)\right)$$
$$+Li_2\left(\frac{x_+}{x_+-y_+}\right)+Li_2\left(\frac{x_+}{x_+-y_-}\right)-Li_2\left(\frac{x_+-1}{x_+-y_+}\right)-Li_2\left(\frac{x_+-1}{x_+-y_-}\right)\Big\} \tag{7.33}$$

where

$$x_+ = \frac{1}{2}\left(1+\sqrt{1+4m_f^2 u/(ts)}\right), \quad x_- = \frac{1}{2}\left(1-\sqrt{1+4m_f^2 u/(ts)}\right)$$
$$y_+ = \frac{1}{2}\left(1+\sqrt{1-4m_f^2/v}\right), \quad y_- = \frac{1}{2}\left(1-\sqrt{1-4m_f^2/v}\right)$$

The auxiliary function $E(s,t)$ is defined as

$$E(s,t) = sC(s)+tC(t)+s_1C_1(s)+t_1C_1(t)-stD(s,t) \tag{7.34}$$

The analytic continuation of these expressions is performed by adding a regulator $-i\epsilon$ to each fermion mass m_f^2 and propagate[28] it until it reaches a function with a branch cut. Then the limiting cases are defined and the appropriate expression is derived for each analytical region identified.

[27] $Li_2(t) = -\int_0^t \frac{\log|1-t|}{t}dt$

[28] It means that it is taken out of the expression $f(m_f^2)$ where m_f^2 appears by substituing it in the first order taylor expansion of f.

The helicity amplitudes are denoted $A_{\lambda_1\lambda_2\lambda_3\lambda_4}$ with λ_i being the polarization of particle i. In terms of the scalar integrals above, the set of axial helicity amplitudes, as defined in Eq. (4.6), is

$\boldsymbol{A_{++++}(s,t,u)} = 32m_f^2 s\left(\frac{C_1(u)}{u_1} - \frac{C_1(t)}{t_1}\right) + 8m_f^2\left(s_1(D(s,u) - D(s,t)) - (u-t)D(u,t)\right) +$
$16\left(\frac{(su+tu_1)B_1(u)}{u_1^2} - \frac{(st+t_1u)B_1(t)}{t_1^2}\right) + \frac{4(u-t)E(u,t)}{s}$

$\boldsymbol{A_{+-++}(s,t,u)} = \frac{16m_f^2}{s_1}\left((u-t)C_1(s) + t_1C_1(t) - \frac{(u_1^2+2tu)C_1(u)}{u_1}\right) +$
$8m_f^2\left(-t_1D(s,t) + \frac{(m_Z^2+s)uD(s,u)}{s_1} - \frac{t_1uD(u,t)}{s_1}\right) + \frac{16m_Z^2u}{s_1}\left(\frac{B_1(s)}{s_1} - \frac{tB_1(u)}{u_1^2}\right) - \frac{4m_Z^2uE(s,u)}{s_1t}$

$\boldsymbol{A_{+---}(s,t,u)} = A_{+-++}(s,u,t)$

$\boldsymbol{A_{+-+-}(s,t,u)} = \frac{16m_f^2}{s_1}\left(s_1C_1(s) - t_1C_1(t) + \frac{(u_1^2-2t^2)C_1(u)}{u_1}\right) + \frac{8m_f^2}{s_1}\left(s\left(s_1-2u\right)D(s,u) - tt_1D(u,t)\right) +$
$\frac{16s}{s_1}\left(B_1(s) + \frac{(tu+s_1u_1)B_1(u)}{u_1^2}\right) - \frac{4s}{s_1t}\left(s_1-u\right)E(s,u)$

$\boldsymbol{A_{+--+}(s,t,u)} = A_{+-+-}(s,u,t)$

$\boldsymbol{A_{+++0}(s,t,u)}/\Delta = -\frac{16m_f^2}{m_Z^2}\left(C_1(s)s_1^2 - \frac{(2ts^2+s_1t_1^2)C_1(t)}{t_1} - \frac{(2us^2+s_1u_1^2)C_1(u)}{u_1}\right) +$
$\frac{8m_f^2}{m_Z^2}\left(ss_1tD(s,t) + ss_1uD(s,u) + \left(3m_Z^2 - s\right)tuD(u,t)\right) - 16tu\left(\frac{(2s+u)B_1(t)}{t_1^2} + \frac{(2s+t)B_1(u)}{u_1^2}\right) - \frac{8tu}{s}E(u,t)$

$\boldsymbol{A_{+-+0}(s,t,u)}/\Delta = -\frac{16m_f^2}{m_Z^2s_1}\left(\left(t_1s_1^2 + 2stu\right)C_1(s) - \left(s_1t_1^2 - 2stt_1\right)C_1(t) - \frac{\left(s_1t_1u_1^2+2su\left(u_1m_Z^2+tt_1\right)\right)C_1(u)}{u_1}\right) +$
$\frac{8m_f^2}{m_Z^2}\left(stt_1D(s,t) + \frac{su\left(3m_Z^2u-st\right)D(s,u)}{s_1} + \frac{(m_Z^2+s)tt_1uD(u,t)}{s_1}\right) + \frac{16su}{s_1}\left(\frac{uB_1(s)}{s_1} + \frac{(ss_1-2tu)B_1(u)}{u_1^2}\right) - \frac{8su^2}{s_1t}E(s,u)$

$\boldsymbol{A_{+--0}(s,t,u)} = A_{+-+0}(s,u,t)$

$\boldsymbol{A_{+++-}} = \boldsymbol{A_{++-+}} = \boldsymbol{A_{++--}} = \boldsymbol{A_{++-0}} = 0$

In the limit $m_f \to \infty$ the fermion is expected to decouple from the theory. We observe that for the axial piece, this is not true since the amplitudes go to a constant. This is a manifestation of the anomaly which ultimately always cancel with the contribution of other fermions. In the SM case, the anomaly of the top and bottom quark cancel, because $a_u = -a_d$. However, if we are interested in the contribution of the top quark only, it is necessary to take out the anomaly. To identify the anomalous part, we observe that only terms in $m_f^0B_1(s,m_f)$, $m_f^2C_1(s,m_f)$, $m_f^2D(s,t,m_f)$ and $m_f^0E(s,t,m_f)$ appear in the axial helicity amplitudes. Out of this four combinations, only one, $m_f^2C_1(s,m_f)$, has a leading term independent of the mass of the quark, as shown in the following expansion

$$m_f^2C_1\left(s,m_f,m_Z\right) = -\frac{i\pi^2}{2} - \frac{i\pi^2}{24}\left(m_Z^2+s\right)\frac{1}{m_f^2} + \mathcal{O}\left(\frac{1}{m_f^4}\right) \quad (7.35)$$

So taking out the anomalous part can be done by redifining $C_1(s,m_f,m_Z)$ to $C_1(s,m_f,m_Z) + \frac{i\pi^2}{2m_f^2}$. Of course, $E(s,t,m_f,m_Z)$ must still be expressed in term of the initially defined $C(s,m_f,m_Z)$ and $C_1(s,m_f,m_Z)$.

To make things clearer, we provide here the analytical expression of the anomalous part $\bar{A}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ for each helicity amplitude.

$$
\begin{aligned}
\bar{\boldsymbol{A}}_{++++}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) &= 16i\pi^2 s\left(\frac{1}{t_1}-\frac{1}{u_1}\right)\\
\bar{\boldsymbol{A}}_{+-++}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) &= \frac{16i\pi^2 tu}{s_1u_1}\\
\bar{\boldsymbol{A}}_{+---}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) &= \bar{A}_{+-++}(s,u,t)\\
\bar{\boldsymbol{A}}_{+-+-}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) &= -\frac{16i\pi^2 st}{s_1u_1}\\
\bar{\boldsymbol{A}}_{+--+}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) &= \bar{A}_{+-+-}(s,u,t)\\
\bar{\boldsymbol{A}}_{+++\boldsymbol{0}}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u})/\Delta &= \frac{-16i\pi^2 stu\left(m_Z^2+s\right)}{m_Z^2t_1u_1}\\
\bar{\boldsymbol{A}}_{+-+\boldsymbol{0}}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u})/\Delta &= \frac{-16i\pi^2 stut_1}{m_Z^2s_1u_1}\\
\bar{\boldsymbol{A}}_{+--\boldsymbol{0}}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) &= \bar{A}_{+-+0}(s,u,t)\\
\bar{\boldsymbol{A}}_{+++-}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) &= \bar{\boldsymbol{A}}_{++-+}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) = \bar{\boldsymbol{A}}_{++--}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) = \bar{\boldsymbol{A}}_{++-\boldsymbol{0}}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) = 0
\end{aligned}
\tag{7.36}
$$

The vector helicity amplitudes, as defined in 4.7, are

$$
\begin{aligned}
&\boldsymbol{V}_{++++}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) = 16m_f^4(D(s,t)+D(s,u)+D(u,t)) +\\
&\tfrac{8m_f^2}{ss_1}\left(2sC(s)m_Z^2 + \tfrac{\left(m_Z^2\left(s^2+u^2\right)-2stt_1\right)C_1(t)}{t_1} + \tfrac{\left(m_Z^2\left(s^2+t^2\right)-2suu_1\right)C_1(u)}{u_1} + \left(m_Z^2-2s\right)\left(tC(t)+uC(u)\right)\right) +\\
&8\left(\tfrac{\left(t_1t^2+sut+u^2u_1\right)D(u,t)}{ss_1} - s_1(D(s,t)+D(s,u))\right)m_f^2 + \tfrac{8}{s_1}\left(\tfrac{t(st+2t_1u)B_1(t)}{t_1^2} + \tfrac{u(su+2tu_1)B_1(u)}{u_1^2}\right) +\\
&\tfrac{4}{ss_1}\left(t^2+u^2\right)E(u,t) + \tfrac{8i\pi^2\left(m_Z^2+s\right)tu}{s_1t_1u_1}
\end{aligned}
$$

$$
\begin{aligned}
&\boldsymbol{V}_{+++-}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) = 16m_f^4(D(s,t)+D(s,u)+D(u,t)) +\\
&8m_f^2\left(\tfrac{C_1(s)s_1^2}{tu} + \left(\tfrac{2um_Z^2}{s_1t_1} + \tfrac{2u}{s} - \tfrac{u_1}{s_1} + \tfrac{s}{u}\right)C_1(t) + \left(\tfrac{2tm_Z^2}{s_1u_1} + \tfrac{2t}{s} - \tfrac{t_1}{s_1} + \tfrac{s}{t}\right)C_1(u)\right) +\\
&8m_f^2\left(\tfrac{s\left(t^2+u^2\right)C(s)}{tus_1} + \tfrac{t\left(2um_Z^2+st\right)C(t)}{sus_1} + \tfrac{u\left(2tm_Z^2+su\right)C(u)}{sts_1}\right) + 8m_f^2\left(\tfrac{stD(s,t)}{u} + \tfrac{suD(s,u)}{t} + \tfrac{\left(s-4m_Z^2\right)tuD(u,t)}{ss_1}\right) +\\
&\tfrac{8m_Z^2tu}{ss_1}\left(\tfrac{(s-2t_1)B_1(t)}{t_1^2} + \tfrac{(s-2u_1)B_1(u)}{u_1^2}\right) + \tfrac{8m_Z^2tu}{s^2s_1}E(u,t) - \tfrac{8i\pi^2\left(m_Z^2+s\right)tu}{s_1t_1u_1}
\end{aligned}
$$

$$
\begin{aligned}
&\boldsymbol{V}_{++-+}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) = 16m_f^4(D(s,t)+D(s,u)+D(u,t)) +\\
&8m_f^2\left(\tfrac{C_1(s)s_1^2}{tu} + \tfrac{tt_1C_1(t)}{us_1} + \tfrac{uu_1C_1(u)}{ts_1} + \tfrac{s\left(t^2+u^2\right)C(s)}{tus_1} + \tfrac{\left(s_1^2+u^2\right)C(t)}{us_1} + \tfrac{\left(s_1^2+t^2\right)C(u)}{ts_1}\right) +\\
&8m_f^2\left(\tfrac{stD(s,t)}{u} + \tfrac{suD(s,u)}{t} + \tfrac{tuD(u,t)}{s_1}\right) - 8i\pi^2
\end{aligned}
$$

$$
\boldsymbol{V}_{++--}(\boldsymbol{s},\boldsymbol{t},\boldsymbol{u}) = 16m_f^4(D(s,t)+D(s,u)+D(u,t)) + \tfrac{16m_Z^2m_f^2}{s_1}C(s) + \tfrac{8m_Z^2m_f^2}{ss_1}E(u,t) - 8i\pi^2
$$

$$\boldsymbol{V_{+-++}(s,t,u)} = 16m_f^4(D(s,t)+D(s,u)+D(u,t)) + 8m_f^2\left(\frac{C(u)u^2}{s_1 t} + \frac{\left(2m_Z^2t^2+t_1u_1t+ss_1u_1\right)C_1(u)}{s_1tu_1} + \frac{\left(t^2+u^2\right)C_1(s)}{tu} + \frac{tt_1C_1(t)}{s_1u} + \frac{ss_1C(s)}{tu} + \frac{\left(s_1^2+u^2\right)C(t)}{s_1u}\right) + \frac{8m_f^2}{s_1}\left(\frac{\left(ss_1t_1u^2\right)D(s,t)}{u} + \frac{\left(ss_1-m_Z^2t\right)uD(s,u)}{t} + t_1uD(u,t)\right) + \frac{8m_Z^2tu}{s_1u_1^2}B_1(u) + \frac{4m_Z^2u}{s_1t}E(s,u) - \frac{8i\pi^2su}{s_1u_1}$$

$$\boldsymbol{V_{+---}(s,t,u)} = V_{+-++}(s,u,t)$$

$$\boldsymbol{V_{+-+-}(s,t,u)} = 16m_f^4(D(s,t)+D(s,u)+D(u,t)) + 8m_f^2\left(\frac{t_1C_1(t)m_Z^2}{ss_1} + \frac{tC(t)m_Z^2}{ss_1} - \frac{2s_1C_1(s)}{t} + \frac{\left(m_Z^2t\left(s^2+t^2\right)-2s^2uu_1\right)C_1(u)}{ss_1tu_1} - \frac{2sC(s)}{t} + \frac{\left(m_Z^2t-2ss_1\right)uC(u)}{ss_1t}\right) + 8m_f^2\left(-\frac{stD(s,t)}{s_1} + s\left(\frac{4u}{t}-\frac{t}{s_1}\right)D(s,u) - \frac{t\left(um_Z^2+s^2\right)D(u,t)}{ss_1}\right) + \frac{8s}{s_1t}\left(\left(u-s_1\right)B_1(s) + \frac{u(tu+2s_1u_1)B_1(u)}{u_1^2}\right) + \frac{4s\left(t^2-2s_1u\right)}{s_1t^2}E(s,u) + \frac{8i\pi^2su}{s_1u_1}$$

$$\boldsymbol{V_{+--+}(s,t,u)} = V_{+-+-}(s,u,t)$$

$$\boldsymbol{V_{+++0}(s,t,u)}/\Delta = \frac{16m_f^2}{s_1}\left(\frac{t\left(t_1^2-2s^2\right)C_1(t)}{t_1} - \frac{u\left(u_1^2-2s^2\right)C_1(u)}{u_1} + s(u-t)C(s) + tu(C(u)-C(t))\right) + 8m_f^2\left(-stD(s,t) + suD(s,u) + \frac{3tu(u-t)D(u,t)}{s_1}\right) + \frac{16i\pi^2stu(u-t)}{s_1t_1u_1} + \frac{16tu}{s_1}\left(\frac{\left(tm_Z^2+2uu_1\right)B_1(u)}{u_1^2} - \frac{\left(um_Z^2+2tt_1\right)B_1(t)}{t_1^2}\right) + \frac{8t(t-u)u}{ss_1}E(u,t)$$

$$\boldsymbol{V_{++-0}(s,t,u)}/\Delta = \frac{16m_f^2}{s_1}\left(tt_1C_1(t) - uu_1C_1(u) + s(t-u)C(s) + tu(C(u)-C(t))\right) + 8m_f^2\left(stD(s,t) - suD(s,u) + \frac{tu(u-t)D(u,t)}{s_1}\right)$$

$$\boldsymbol{V_{+-+0}(s,t,u)}/\Delta = \frac{16m_f^2}{s_1}\left(tt_1C_1(t) + \frac{u\left(u_1^2-2t^2\right)C_1(u)}{u_1} - ss_1C(s) + tu(C(u)-C(t))\right) + \frac{16su}{s_1u_1^2}\left(B_1(s)u_1^2 + \left(tu+s_1u_1\right)B_1(u)\right) + \frac{8m_f^2}{s_1}\left(st(u-t)D(s,t) - su(t+3u)D(s,u) + tu(2s-t+u)D(u,t)\right) + \frac{8su^2}{s_1t}E(s,u) + \frac{16i\pi^2stu}{s_1u_1}$$

$$\boldsymbol{V_{+--0}(s,t,u)} = V_{+-+0}(s,u,t)$$

The other twelve axial and vector helicity amplitudes follow from parity, i.e. $A_{++++} = -A_{----}$.

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