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Z BOSON PRODUCTION AND DECAY VIA GLUONS

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We perform a complete $O(\alpha_s^3)$ calculation of the coupling of the Z boson to three gluons via a quark loop, keeping the full quark-mass dependence. This involves the evaluation of the fourth rank AVVV polarisation tensor. We give expressions for the helicity amplitudes. The results are applied to the decay $Z \to ggg$ and to the production of the Z at large transverse momentum at hadron colliders. The total $Z \to ggg$ decay rate is in the range 4–8 keV for $m_{top} > 40$ GeV, of which the axial coupling accounts for approximately 15%. The contribution of the gluon fusion process $gg \to gZ$ via a quark loop is a 1–3% correction to the tree level $\bar{q}q \to gZ$ and $gq(\bar{q}) \to Zq(\bar{q})$ processes. About 80% of the $gg \to Zg$ cross section is due to the axial coupling.

1. Introduction

In quantum field theory, interactions that are not present at the tree level can be generated via loop effects. The classical example of this is the scattering of photons by photons. Since the strong and the weak interactions are also described by gauge theories effects analogous to photon scattering are possible. Particularly interesting is the case where purely "weakly" interacting particles couple to purely "strongly" interacting particles. An example of this is the coupling of the Z boson to three gluons, which is generated via a quark loop. This coupling is the subject of this paper.

The effects of the $Z \to ggg$ coupling are present in a number of presently planned experiments. At LEP a large number of Z bosons will be produced and one can in principle look for the decay $Z \to ggg$. This will certainly be very difficult because of the large background from $Z \to \bar{q}qg$. Even if the decay $Z \to ggg$ is not directly measurable it is important to know its size because it is a $O(\alpha_s^3)$ correction to the ratio $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ at the Z peak, which may be used to extract a precise value of α_s at the m_Z scale.

Another possibility to look for the existence of the Zggg coupling is the production of the Z boson at large p_T at hadron machines via the gluon fusion subprocess $gg \to Zg$. At supercollider energies, one probes the low Feynman x region of the structure functions, where the gluon structure function is large. Gluon fusion subprocesses are therefore enhanced and a priori it is not clear whether this

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enhancement is sufficient to compensate for the extra factors of α_s compared to the $\bar{q}q \to Zg$ and $gq \to Zq$ subprocesses.

The coupling of the Z boson to the gluons contains two colour structures. One is a piece proportional to the structure constants f^{abc} of SU(3) and comes from the axial vector coupling of the Z to the quarks (AVVV). When the quarks inside a doublet are degenerate the contribution from the axial coupling is zero since the "up" and "down" type quarks contribute with opposite sign. The other piece is proportional to the completely symmetric tensor d^{abc} of SU(3) and comes from the vector coupling of the Z to the quarks (VVVV). Because of the different colour structures of the vector and axial couplings, there is no interference between them and they add incoherently.

The vector Zggg coupling has been mostly studied in the literature, since, except for the colour factor, the treatment is exactly the same as for the four photon coupling in QED, which is reviewed in ref. [1]. The results of ref. [1] have been applied to the γ^*ggg coupling in ref. [2]. Similar calculations as ref. [2], allowing for the vector coupling of the Z to the quarks, were performed repeatedly in refs. [3–6]. The first three of these papers give a result that is a factor six too large, which was corrected in the last paper. Allowing for this factor refs. [3–5] agree with refs. [2, 6]. The numerical results of ref. [7] are about a factor three smaller than the others. The dependence of the Z decay on the mass of the top quark has been studied in these papers for $m_t < \frac{1}{2}m_Z$. Due to the different colour structures, these calculations, which ignore the axial coupling give a lower bound on the total Zggg coupling. An estimate of the contribution of the axial coupling, giving the f^{abc} part of the Zggg coupling, has been performed in ref. [8], which considers the contribution of the triangle graphs in fig. 1. It was found that the contribution of the axial current was comparable to the piece from the vector current. However this calculation is not satisfactory, because the triangle graphs are not by themselves gauge invariant.

In this paper we study the complete $O(\alpha_s^3)$ coupling of the Z boson to three on-shell gluons. This involves the construction of the AVVV polarisation tensor, which has not appeared in the literature before. This tensor is described in sect. 2. In

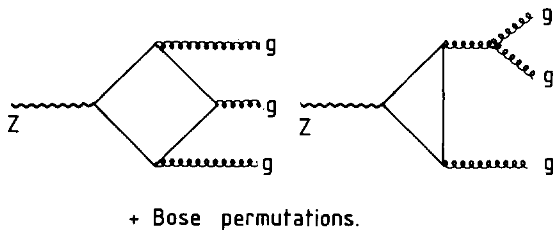

Fig. 1. Feynman diagrams contributing to the coupling of the Z boson to three gluons. The triangle graph contributes only to the f^{abc} colour structure via the $\gamma_5\gamma_\mu$ coupling of the Z to the quarks. The box graph contributes both to the f^{abc} and the d^{abc} colour structures.

sect. 3 we give the contribution of the axial vector current to the helicity amplitudes. In sect. 4 we discuss the decay $Z \to ggg$. In sect. 5 we consider the production of Z bosons at large p_T at the SSC. Sect. 6 contains our conclusions while appendix A contains some definitions of integrals appearing in the calculation. In appendix B we give the helicity amplitudes coming from the vector current in the same notation as sect. 3.

2. The AVVV polarisation tensor

The coupling of the Z boson to the quarks is given by $g_Z Z \bar{\psi}(v_q + a_q \gamma_5)\gamma_\mu \psi$, with

$$v_u = \tfrac{1}{4} - \tfrac{2}{3}\sin^2\theta_w; \qquad a_u = \tfrac{1}{4}; \tag{2.1}$$

$$v_d = -\tfrac{1}{4} + \tfrac{1}{3}\sin^2\theta_w; \qquad a_d = -\tfrac{1}{4}; \tag{2.2}$$

$$g_Z = \frac{e}{\sin\theta_w \cos\theta_w}. \tag{2.3}$$

The Zggg coupling is generated by the box and triangle graphs shown in fig. 1. From charge conjugation it is clear that the vector coupling of the Z boson only gives a contribution to the symmetric SU(3) d^{abc} tensor. This contribution is entirely due to the box graphs of fig. 1. The axial vector coupling only gives a contribution to the anti-symmetric colour tensor f^{abc}. This term is generated both by the triangle graphs and the box graphs. The contribution of a (u, d) quark doublet to the axial tensor can be written as

$$A^{\alpha\mu\nu\rho} = \frac{g_s^3 g_Z}{(2\pi)^4} \frac{f^{abc}}{4}\left[a_u A^{\alpha\mu\nu\rho}(m_u) + a_d A^{\alpha\mu\nu\rho}(m_d)\right]. \tag{2.4}$$

We denote the momenta and Lorentz indices of the gluons by p_1^μ, p_2^ν, p_3^ρ. The momentum of the Z boson is denoted by

$$p_4^\alpha = -p_1^\alpha - p_2^\alpha - p_3^\alpha.$$

We define

$$s = 2p_1 \cdot p_2, \qquad t = 2p_2 \cdot p_3, \qquad u = 2p_1 \cdot p_3,$$

$$s_1 = s - m_Z^2, \qquad t_1 = t - m_Z^2, \qquad u_1 = u - m_Z^2.$$

Using the fact that the gluons are on-shell allows us to put

$$p_1^\mu = p_2^\nu = p_3^\rho = p_1 \cdot p_1 = p_2 \cdot p_2 = p_3 \cdot p_3 = 0.$$

After these conditions are put in, the axial vector polarisation tensor $A^{\alpha\mu\nu\rho}$ is transversal:

$$p_1^{\mu} A^{\alpha\mu\nu\rho} = p_2^{\nu} A^{\alpha\mu\nu\rho} = p_3^{\rho} A^{\alpha\mu\nu\rho} = 0 . \tag{2.5}$$

The transversality of the tensor is strictly true only if one sums over the two quarks in the doublet. If one keeps only one quark there is a mass independent non-transversal contribution that depends on the renormalisation procedure. This is a manifestation of the anomaly. Because this term is independent of the mass it cancels when summing over the quarks in a doublet.

Taking into account momentum conservation and after repeated use of the Schouten identity, which says that an anti-symmetric tensor with five indices vanishes in four dimensions, and using the methods of ref. [9], we find

$$\begin{aligned}
&A^{\alpha\mu\nu\rho}(m_f)\\
&\quad= (p_1\cdot p_2 A_1(p_1,p_2,p_3,m_f) + p_2\cdot p_3 A_2(p_1,p_2,p_3,m_f))\epsilon^{\alpha\mu\nu p_1} p_1^{\rho}\\
&\quad\quad -\left(\frac{p_1\cdot p_2\, p_1\cdot p_3}{p_2\cdot p_3} A_1(p_1,p_2,p_3,m_f) + p_1\cdot p_3 A_2(p_1,p_2,p_3,m_f)\right)\epsilon^{\alpha\mu\nu p_1} p_2^{\rho}\\
&\quad\quad + A_1(p_1,p_2,p_3,m_f)\epsilon^{\alpha\mu p_1 p_2} p_1^{\nu} p_1^{\rho} - \frac{p_1\cdot p_3}{p_2\cdot p_3} A_1(p_1,p_2,p_3,m_f)\epsilon^{\alpha\mu p_1 p_2} p_1^{\nu} p_2^{\rho}\\
&\quad\quad + A_2(p_1,p_2,p_3,m_f)\epsilon^{\alpha\mu p_1 p_2} p_3^{\nu} p_1^{\rho} + A_3(p_1,p_2,p_3,m_f)\epsilon^{\alpha\mu p_1 p_2} p_3^{\nu} p_2^{\rho}\\
&\quad\quad - (p_1\cdot p_3 A_2(p_1,p_2,p_3,m_f) + p_2\cdot p_3 A_3(p_1,p_2,p_3,m_f))\epsilon^{\alpha\mu p_1 p_2}\delta^{\nu\rho}\\
&\quad\quad + \text{Bose permutations}.
\end{aligned} \tag{2.6}$$

$\epsilon^{\alpha\mu p_1 p_2}$ and $\epsilon^{\alpha\mu\nu p_1}$ are a shorthand notation for $\epsilon^{\alpha\mu ab} p_1^a p_2^b$ and $\epsilon^{\alpha\mu\nu a} p_1^a$ respectively. For the Bose permutations, we have to take into account the antisymmetry of f^{abc} in eq. (2.4). The functions A_1, A_2, A_3 are given by

$$\begin{aligned}
&A_1(p_1,p_2,p_3,m_f)\\
&\quad= \frac{16}{us^2}\Bigg[-\frac{2ut(s-u_1)}{u_1^2}B_1(u) - \frac{2ut(s-t_1)}{t_1^2}B_1(t)\\
&\quad\quad + 2m_f^2\left(s_1 C_1(s) + \frac{(2s^2-t_1^2)}{t_1}C_1(t) + \frac{(2s^2-u_1^2)}{u_1}C_1(u)\right)\\
&\quad\quad - m_f^2(stD(s,t) + suD(s,u) - 3utD(u,t)) - \frac{ut}{s}E(u,t)\Bigg],
\end{aligned} \tag{2.7}$$

$$A_2(p_1, p_2, p_3, m_f)$$

$$= +16\left(\frac{(s_1^2+2ut)}{s_1^2 ut}B_1(s) + \frac{(u_1^2+4st)}{u_1^2 st}B_1(u) + \frac{(3t_1^2-2s^2)}{t_1^2 su}B_1(t)\right)$$

$$+64m_f^2\left(\frac{C_1(s)}{ss_1} + \frac{C_1(t)}{tt_1} + \frac{2C_1(u)}{uu_1}\right)$$

$$+\frac{16m_f^2}{s}\left(\frac{(u-s)}{u}D(s,t) - \frac{(t+s)}{t}D(s,u) - 2D(u,t)\right)$$

$$+8\left(\frac{1}{u^2}E(s,t) + \frac{1}{t^2}E(s,u) + \frac{2}{s^2}E(u,t)\right), \tag{2.8}$$

$$A_3(p_1, p_2, p_3, m_f)$$

$$= -32\left(\frac{1}{t^2}B_1(s) + \frac{1}{t_1^2}B_1(t) + \left(\frac{1}{st} - \frac{u}{u_1 t^2} + \frac{u}{u_1^2 t}\right)B_1(u)\right)$$

$$+32m_f^2\left(-\frac{s_1}{st^2}C_1(s) + \left(\frac{t_1}{st^2} - \frac{2}{tt_1}\right)C_1(t) - \left(\frac{u_1}{st^2} - \frac{2}{su_1}\right)C_1(u)\right)$$

$$+16m_f^2\left(\frac{1}{s}D(s,t) + \left(\frac{1}{t} - \frac{1}{s} + \frac{3u}{t^2}\right)D(s,u) + \left(\frac{1}{s} + \frac{t_1}{st}\right)D(u,t)\right)$$

$$-8\left(\frac{(2u+t)}{t^3}E(s,u) + \frac{1}{s^2}E(u,t)\right). \tag{2.9}$$

The functions B_1, C, C_1, D, E are defined in appendix A.

We notice that in the limit $m_f = 0$ the functions A_1, A_2, A_3 go to a constant. As a consequence a light generation does not contribute to the AVVV tensor even if there is a large ratio between the masses of the doublet, as is for instance the case for the charm and strange quark. Ratios like $\log(m_c^2/m_s^2)$ are always suppressed by the charm quark mass.

3. The AVVV helicity amplitudes

Using the Zggg polarisation tensor from sect. 2 we can construct the axial vector helicity amplitudes. We work in the rest frame of the p_1 and p_2 system. The momenta are given in this frame by

$$p_1^\mu = (0,0,-p;-p),$$
$$p_2^\mu = (0,0,p;-p),$$
$$p_3^\mu = (q\sin\theta,0,q\cos\theta;q),$$
$$p_4^\mu = (-q\sin\theta,0,-q\cos\theta;E), \tag{3.1}$$

where $E = \sqrt{q^2+m_Z^2}$. The helicity vectors are

$$e_1^+ = e_2^- = (1/\sqrt{2})(-i,1,0;0),$$
$$e_1^- = e_2^+ = (1/\sqrt{2})(i,1,0;0),$$
$$e_3^+ = e_4^- = (1/\sqrt{2})(i\cos\theta,1,-i\sin\theta;0),$$
$$e_3^- = e_4^+ = (1/\sqrt{2})(-i\cos\theta,1,i\sin\theta;0),$$
$$e_0 = (1/m_Z)(-E\sin\theta,0,-E\cos\theta;q). \tag{3.2}$$

$e_i^\pm$ is the $\pm$ polarisation vector of particle i and e_0 is the longitudinal polarisation of the Z boson. We define the quantity

$$\Delta = \sqrt{-m_Z^2/2stu}\,.$$

As in sect. 2 we leave out a factor $g_s^3 g_Z a_q f^{abc}/4(2\pi)^4$ in the definition of the helicity amplitudes.

We find that the axial Zggg helicity amplitudes, $A_{\lambda_1\lambda_2\lambda_3\lambda_4}$, where λ_i denotes the polarisation of particle i, for a single quark of mass m_f are:

$$\begin{aligned}A_{++++}(s,t,u) = {} & +16\left(\frac{(tu_1+us)}{u_1^2}B_1(u) - \frac{(ut_1+ts)}{t_1^2}B_1(t)\right)\\ & +32m_f^2 s\left(\frac{1}{u_1}C_1(u) - \frac{1}{t_1}C_1(t)\right)\\ & +8m_f^2\left(s_1[D(s,u)-D(s,t)] - (u-t)D(u,t)\right)\\ & +\frac{4(u-t)}{s}E(u,t),\end{aligned} \tag{3.3}$$

$$A_{+-++}(s,t,u) = A_{+---}(s,u,t)$$

$$= +\frac{16m_Z^2 u}{s_1}\left(\frac{1}{s_1}B_1(s) - \frac{t}{u_1^2}B_1(u)\right)$$

$$+\frac{16m_f^2}{s_1}\left((u-t)C_1(s) + t_1C_1(t) - \frac{\left(u_1^2+2ut\right)}{u_1}C_1(u)\right)$$

$$+8m_f^2\left(-t_1D(s,t) + \frac{\left(s+m_Z^2\right)u}{s_1}D(s,u) - \frac{ut_1}{s_1}D(u,t)\right)$$

$$-\frac{4um_Z^2}{s_1t}E(s,u), \tag{3.4}$$

$$A_{+-+-}(s,t,u) = A_{+--+}(s,u,t)$$

$$= +\frac{16s}{s_1}\left(B_1(s) + \frac{(s_1u_1+ut)}{u_1^2}B_1(u)\right)$$

$$+\frac{16m_f^2}{s_1}\left(s_1C_1(s) - t_1C_1(t) + \frac{\left(u_1^2-2t^2\right)}{u_1}C_1(u)\right)$$

$$+\frac{8m_f^2}{s_1}\left(s(s_1-2u)D(s,u) - tt_1D(u,t)\right) - \frac{4s(s_1-u)}{s_1t}E(s,u), \tag{3.5}$$

$$A_{+++0}(s,t,u)/\Delta = -16ut\left(\frac{(2s+t)}{u_1^2}B_1(u) + \frac{(2s+u)}{t_1^2}B_1(t)\right)$$

$$-\frac{16m_f^2}{m_Z^2}\left(s_1^2C_1(s) - \frac{\left(t_1^2s_1+2s^2t\right)}{t_1}C_1(t) - \frac{\left(u_1^2s_1+2s^2u\right)}{u_1}C_1(u)\right)$$

$$+\frac{8m_f^2}{m_Z^2}\left(sts_1D(s,t) + sus_1D(s,u) + ut\left(3m_Z^2-s\right)D(u,t)\right)$$

$$-\frac{8ut}{s}E(u,t), \tag{3.6}$$

$$A_{+-+0}(s,t,u)/\Delta = A_{+--0}(s,u,t)$$

$$= +\frac{16us}{s_1}\left(\frac{u}{s_1}B_1(s) + \frac{(s_1s-2ut)}{u_1^2}B_1(u)\right)$$

$$-\frac{16m_f^2}{m_Z^2 s_1}\left\{(s_1^2t_1 + 2sut)C_1(s) - (t_1^2s_1 - 2stt_1)C_1(t)\right.$$

$$\left. - \frac{\left(2su(tt_1 + m_Z^2u_1) + t_1s_1u_1^2\right)}{u_1}C_1(u)\right\}$$

$$+\frac{8m_f^2}{m_Z^2}\left(stt_1D(s,t) + \frac{su(3um_Z^2 - st)}{s_1}D(s,u)\right.$$

$$\left. + \frac{utt_1(s+m_Z^2)}{s_1}D(u,t)\right)$$

$$-\frac{8u^2s}{s_1t}E(s,u), \tag{3.7}$$

$$A_{+++-} = A_{++-+} = A_{++--} = A_{++-0} = 0. \tag{3.8}$$

The other helicity amplitudes follow from parity, i.e. $A_{----} = -A_{++++}$. In order to get the complete helicity amplitudes the above formulas have to be summed over the different quarks.

To calculate the complete Zggg coupling, the helicity amplitudes coming from the vector coupling of the Z to the quarks, $V_{\lambda_1\lambda_2\lambda_3\lambda_4}$, are also needed. These amplitudes have been described in refs. [1–6] in a different notation. To give a complete set of amplitudes, we write the VVVV helicity amplitudes in our notation in appendix B.

4. The decay Z → ggg

Using the helicity amplitudes, $A_{\lambda_1\lambda_2\lambda_3\lambda_4}$ and $V_{\lambda_1\lambda_2\lambda_3\lambda_4}$, from sect. 3 and appendix B it is now straightforward to calculate the decay rate Z → ggg:

$$\Gamma(\mathrm{Z}\to \mathrm{ggg}) = \frac{8\alpha_s^3(m_Z^2)\alpha_Z}{3.3!m_Z(2\pi)^4}\int \sum_{\lambda_1\ldots\lambda_4}\left(\tfrac{5}{6}\left|\sum_q v_q V_{\lambda_1\lambda_2\lambda_3\lambda_4}\right|^2 + \tfrac{3}{2}\left|\sum_q a_q A_{\lambda_1\lambda_2\lambda_3\lambda_4}\right|^2\right)$$

$$\times \prod_{i=1}^{3}\frac{d^3p_i}{2E_i(2\pi)^3}(2\pi)^4\delta_4\left(p_Z - \sum_{j=1}^{3}p_j\right). \tag{4.1}$$

The factors $\frac{5}{6}$ and $\frac{3}{2}$ are the colour factors for the symmetric and anti-symmetric colour structures respectively. The factor $1/3!$ is the identical particle factor for the gluons. For α_s we take $\alpha_s(m_Z^2) = 12\pi/(23\log(m_Z^2/\Lambda^2)) = 0.134$ for $\Lambda =$ 200 MeV. For the masses of the light quarks we took $m_u = m_d = m_s = m_c = 1$ GeV, $m_b = 4.6$ GeV and finally, $m_Z = 92$ GeV and $\sin^2\theta_w = 0.23$. As already mentioned in refs. [1–6] there are no infrared or collinear divergences in the integration over the gluon momenta, because there is no lower order process $Z \to gg$. The resulting decay rate as a function of the top mass is given in fig. 2. The decay rate is of the order of a few keV, which corresponds to a $Z \to ggg/Z \to \mu\mu$ ratio of $5\text{–}9 \times 10^{-5}$. Assuming a yearly integrated luminosity of 10^{38} cm^{-2} at LEP, one expects 6–11 $Z \to ggg$ events. However, there are of the order 10^4 more tree level three jet events. It is therefore clear that there is no way to separate the $Z \to ggg$ decay from the background. This would only be possible if one could distinguish gluon jets from quark jets on an event by event basis.

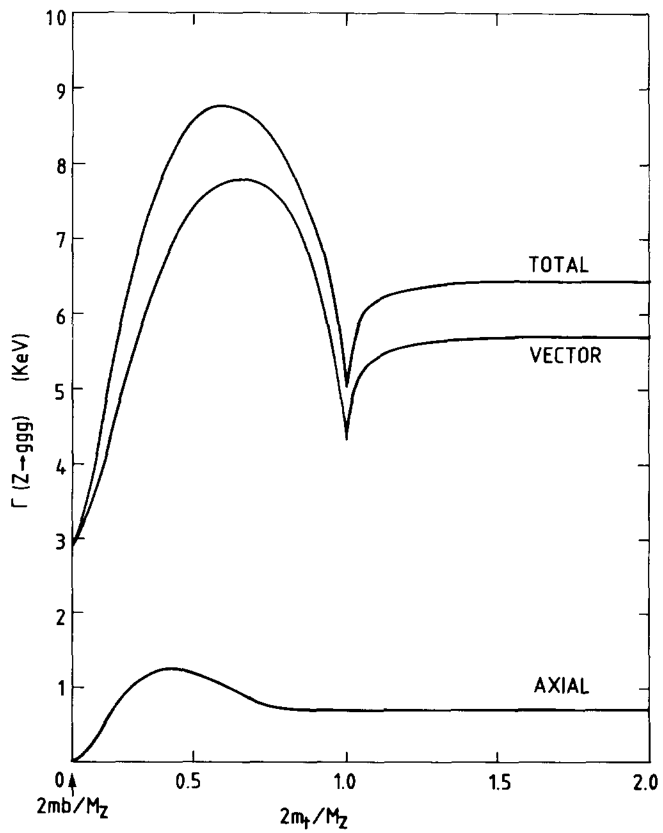

Fig. 2. The decay rate of the Z boson into three gluons as a function of the top quark mass. The contribution of the axial vector part, the vector part and the total are shown separately.

For the vector piece we expect that as $m_t \to \infty$ the top quark decouples. That this is indeed the case can be seen as follows. If $m_t = 0$ there are six massless quarks contributing to the amplitude, each weighted by its weak charge; if the top quark decouples there are only five quarks contributing. We therefore expect for the vector part

$$\frac{\Gamma(m_t \to \infty)}{\Gamma(m_t = 0)} = \frac{(3v_d + 2v_u)^2}{(3v_d + 3v_u)^2} = 2.0. \tag{4.2}$$

As can be seen from fig. 2, this relation is indeed satisfied.

For the axial vector piece the decay rate goes to a constant as $m_t \to \infty$. The top quark does therefore not decouple from the theory. If one would remove the top quark completely, one would be left with a Z boson coupling to a $\gamma_5\gamma_\mu$ current which is not traceless. Such a theory is anomalous. Therefore the heavy quark, which acts as a regulator for this anomalous theory, has to generate an effective pointlike interaction in the low-energy effective lagrangian, giving a constant decay rate of the Z boson.

5. Z production at large p_T

The production process gg → Zg is described by the same amplitudes as the decay Z → ggg, as given in sect. 3 and appendix B. The only difference is that now u and t are negative. In order to study the structure of the amplitudes we show in fig. 3 the spin-averaged matrix element squared for right angle scattering as a function of the top mass for a fixed value of $\hat{s}$. The graph is normalized to the matrix element squared for only five quark flavours. For a light top quark the vector part of the matrix element squared is a factor 0.5 smaller than if the top quark is infinitely heavy, showing the decoupling of the top quark as discussed in sect. 4. At around $m_t \approx \frac{1}{2}m_Z$ the slope of the curves changes due to the possibility that two real top quarks combine to make a Z boson. At $m_t \approx \frac{1}{2}\sqrt{\hat{s}}$ there is another change of slope due to the possibility of making two real top quarks. After this value the curve approaches 1 for $m_t \to \infty$. The axial part, on the other hand, does not approach 1 as $m_t \to \infty$ since the top quark does not decouple, as discussed in sect. 4.

For the physical production rate in proton collisions one has to fold in the integration over the gluon structure functions

$$\mathrm{d}\sigma(\mathrm{pp} \to \mathrm{gg} \to \mathrm{Zg}) = \mathrm{d}x_1\,\mathrm{d}x_2\, f_g(x_1, Q^2) f_g(x_2, Q^2)\, \mathrm{d}\hat{\sigma}(\hat{s}), \tag{5.1}$$

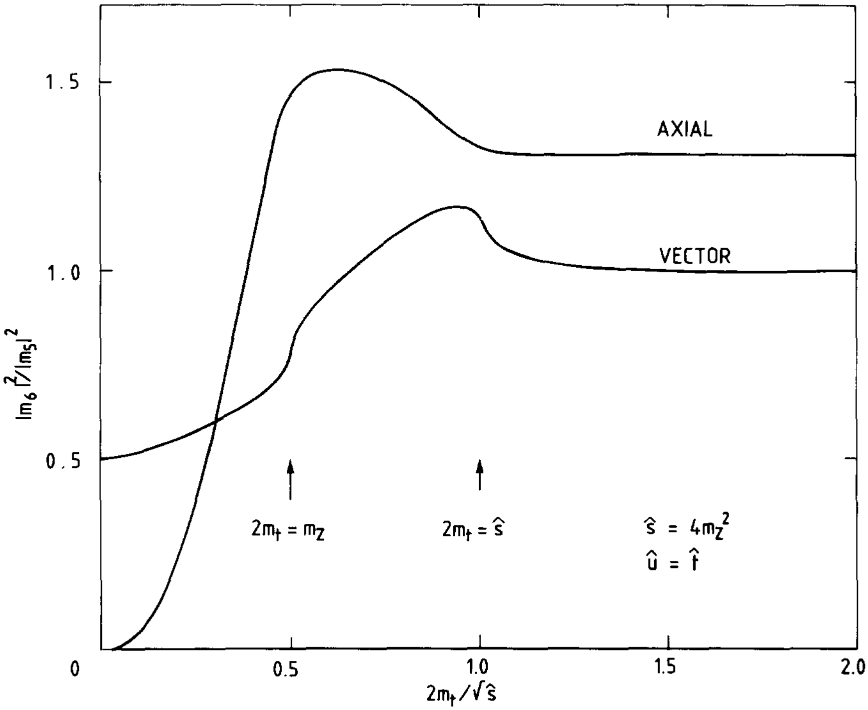

Fig. 3. The exact gg → Zg matrix elements squared as a function of the top quark mass at $\hat{s} = 4m_Z^2$ and $\hat{u} = \hat{t}$. Both the contributions of the axial vector part and the vector part are shown. The curves have been normalised to the case where only five quarks contribute.

with $\hat{s} = x_1 x_2 s$ and

$$\mathrm{d}\hat{\sigma}(\hat{s}) = \frac{\alpha_s^3(Q^2)\alpha_Z}{32\hat{s}(2\pi)^4} \sum_{\lambda_1 \ldots \lambda_4} \left(\tfrac{5}{6} \left| \sum_q v_q V_{\lambda_1\lambda_2\lambda_3\lambda_4} \right|^2 + \tfrac{3}{2} \left| \sum_q a_q A_{\lambda_1\lambda_2\lambda_3\lambda_4} \right|^2 \right)$$

$$\times \prod_{i=3,Z} \frac{\mathrm{d}^3 p_i}{2E_i(2\pi)^3} (2\pi)^4 \delta_4(p_Z + p_3 - p_1 - p_2). \tag{5.2}$$

In fig. 4 we show the gg → Zg cross section as a function of the top quark mass for pp collisions at $\sqrt{s} = 40$ TeV. We used Duke–Owens structure functions [10] with $\Lambda = 0.2$ GeV evolved to a scale $Q^2 = \frac{1}{4}\hat{s}$ and $\alpha_s = 12\pi/[23 \log(Q^2/\Lambda^2)]$. To give an idea of an observable cross section, both gluon and Z are required to have rapidity, $|y| < 2.5$, and transverse momentum $p_T > 40$ GeV. The structure visible in

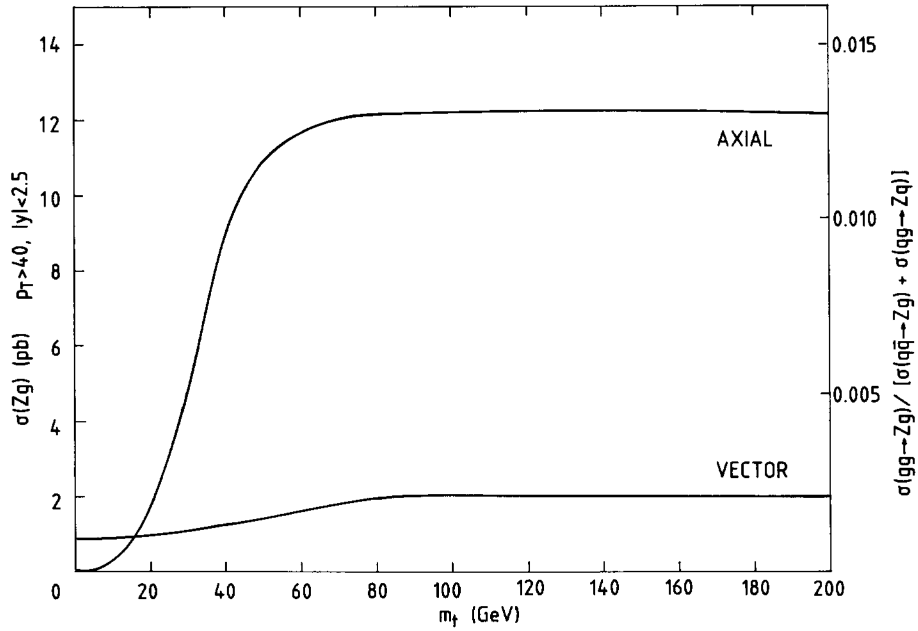

Fig. 4. The cross section for gg → Zg in pp collisions at $\sqrt{s} = 40$ TeV as a function of the top quark mass. The cuts $|y| < 2.5$ and $p_T > 40$ GeV have been imposed. The right-hand scale shows the gg → Zg cross section normalised to the sum of the q$\bar{\text{q}}$ → Zg and gq($\bar{\text{q}}$) → Zq($\bar{\text{q}}$) cross sections with the same cuts.

fig. 3 is smeared out by the integration over the gluon structure functions, however, there is still a marked rise in the cross section at $m_t \sim \frac{1}{2}m_Z$. The right-hand scale shows the gg → Zg cross section normalised to the sum of the q$\bar{\text{q}}$ → Zg and gq($\bar{\text{q}}$) → Zq($\bar{\text{q}}$) cross sections with the same cuts.

It is remarkable that the gg → Zg cross section is now dominated by the axial vector part of the cross section. In the axial coupling the Z boson is predominantly longitudinally polarised and the cross section rises like $\hat{s}/m_Z^2$ while for the vector coupling it is mainly transversely polarised. This is because at high energies, the longitudinal polarisation vector of the Z boson is

$$e_0^\alpha = \frac{p_4^\alpha}{m_Z} + \mathrm{O}(m_Z/E). \tag{5.3}$$

The first term, which gives rise to the $\hat{s}/m_Z^2$ enhancement for the axial piece, vanishes for the vector part due to current conservation. Still, however, the gluon fusion cross section is only a small part of the total. For $m_t > 40$ GeV, the gluon initiated process is 1–3% of the sum of the tree level processes. From the above

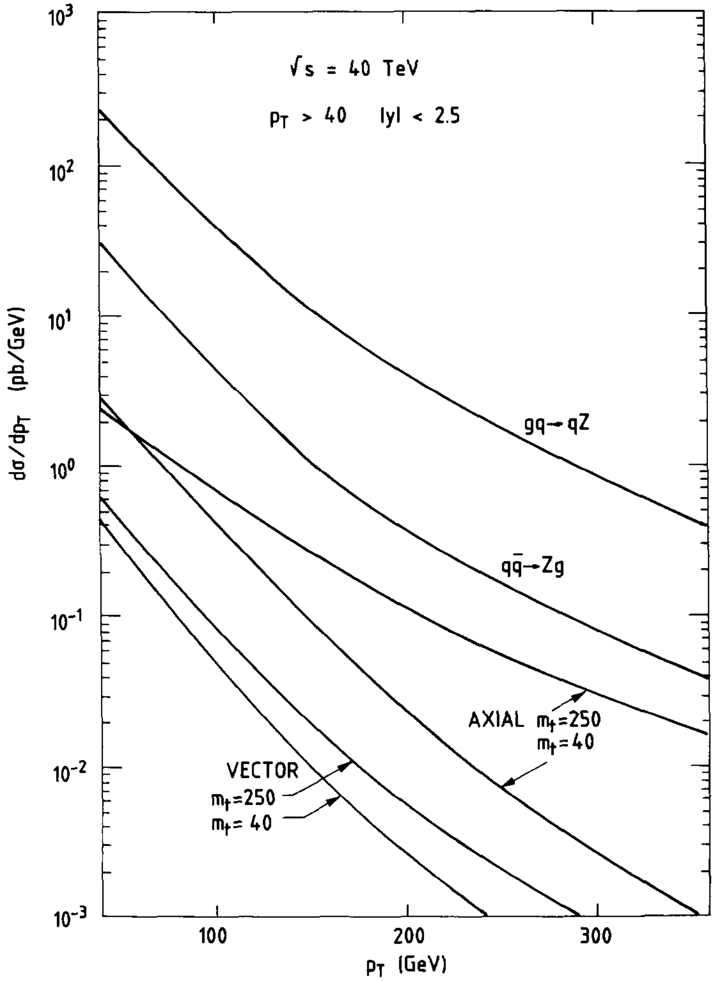

Fig. 5. The transverse momentum distribution for gg → Zg, $q\bar{q}$ → Zg and the sum of qg → Zq and $\bar{q}$g → Z$\bar{q}$ in pp collisions at $\sqrt{s} = 40$ TeV. As in fig. 4, the cuts $|y| < 2.5$ in $p_T > 40$ GeV have been imposed on the final state particles. Curves are shown for $m_t = 40$ GeV and 250 GeV.

argument, and also because of the smaller gluon luminosity, the gg → Zg process is an even smaller effect at the LHC.

The transverse momentum, p_T, and invariant mass, M(Zg), distributions are shown in figs. 5 and 6. As in fig. 4, both final-state particles have $|y| < 2.5$ and $p_T > 40$ GeV. It is seen that for higher values of p_T or M(Zg), i.e. large $\hat{s}$, the axial vector part gets somewhat further enhanced over the vector part, especially for large masses of the top quark. However over the whole range the tree level processes dominate.

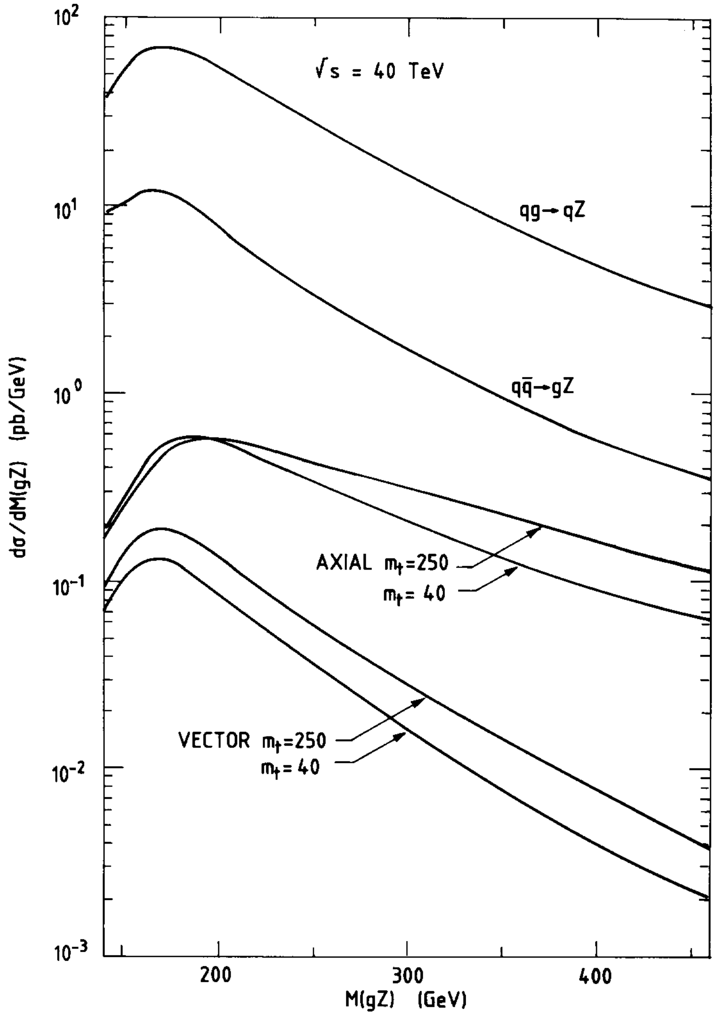

Fig. 6. The invariant mass distribution for $gg \to Zg$, $q\bar{q} \to Zg$ and the sum of $qg \to Zq$ and $\bar{q}g \to Z\bar{q}$ in pp collisions at $\sqrt{s} = 40$ TeV. As in fig. 4, the cuts $|y| < 2.5$ and $p_T > 40$ GeV have been imposed on the final state particles. Curves are shown for $m_t = 40$ GeV and 250 GeV.

6. Conclusions

We have calculated the full polarisation tensor for the axial coupling of the Z with three gluons via box and triangle quark loops. Full dependence of the quark mass has been kept. The tensor may be written in terms of three independent functions, which depend on complex Spence functions.

We have applied this result to two physical processes, namely, Z decay to three gluons and Z production at large transverse momentum via gluon fusion. To facilitate the calculation of the squared matrix elements, we have given analytic forms for the five independent non-zero helicity amplitudes. Since the vector coupling of the Z boson also couples with three gluons, we have given the nine independent non-zero helicity amplitudes for completeness. The axial and vector couplings have different colour structures and can be added incoherently.

The $Z \to ggg$ decay is dominated by the vector coupling. The total width is 4–8 keV for $m_t > 40$ GeV of which the axial coupling contributes about 15%. Although this corresponds to O(10) events per year at LEP, there is a large background from $Z \to q\bar{q}g$ three jet events which, unless it is possible to separate gluon jets from quark jets on an event by event basis, make the $Z \to ggg$ decay unobservable.

Z production at large transverse momentum from the Zggg coupling is dominated by the axial coupling. This is because the Z is preferentially produced with a longitudinal polarisation by the axial coupling. The axial coupling contributes about six times as much as the vector coupling for $m_t > 40$ GeV. However, in this case the tree level $q\bar{q} \to Zg$, $qg \to Zq$ and $\bar{q}g \to Z\bar{q}$ subprocesses are a much more copious source of Z bosons at large p_T. The Zggg coupling contributes at most 3% to the total cross section for Z bosons with large transverse momentum. Therefore we conclude that the prospects for observing the Zggg coupling through either of these processes is rather remote.

Appendix A

In this appendix we define the integrals appearing in the calculation. We use the notation $(p_1+p_2)^2 = s$, $(p_1+p_3)^2 = u$, $(p_2+p_3)^2 = t$, $s_1 = s - m_Z^2$, $t_1 = t - m_Z^2$, $u_1 = u - m_Z^2$. There is the two-point function $B(s)$:

$$B(s) = -i\pi^2 \int_0^1 \mathrm{d}x \log\left(m_f^2 - i\epsilon - sx(1-x)\right)$$

$$= -i\pi^2 \left[\log\left(m_f^2\right) - 2 + \sqrt{1 - 4\left(m_f^2 - i\epsilon\right)/s}\, \log\left(\frac{-z}{1-z}\right)\right], \tag{A.1}$$

with

$$z = \tfrac{1}{2}\left(1 + \sqrt{1 - 4\left(m_f^2 - i\epsilon\right)/s}\right). \tag{A.2}$$

In the amplitudes only the following combination is present:

$$B_1(s) = B(s) - B\left(m_Z^2\right). \tag{A.3}$$

The next integral that appears is the three-point function $C(s)$ with two external massless lines $p_1^2 = p_2^2 = 0$, $(p_1 + p_2)^2 = s$:

$$C(s) = \int \frac{\mathrm{d}^4 q}{(q^2 - m_\mathrm{f}^2)\big((q+p_1)^2 - m_\mathrm{f}^2\big)\big((q+p_1+p_2)^2 - m_\mathrm{f}^2\big)}$$

$$= i\pi^2 \int_0^1 \frac{\mathrm{d}x}{sx} \log\left(1 - i\epsilon - \frac{s}{m_\mathrm{f}^2} x(1-x)\right) = \frac{i\pi^2}{2s}\left[\log\left(\frac{-z}{1-z}\right)\right]^2, \quad \text{(A.4)}$$

with z given by (A.2).

The three-point function $C_1(s)$ with one external massless line, $p_1^2 = 0$, $p_2^2 = m_\mathrm{Z}^2$, $(p_1 + p_2)^2 = s$ is given by

$$s_1 C_1(s) = sC(s) - m_\mathrm{Z}^2 C(m_\mathrm{Z}^2). \quad \text{(A.5)}$$

Finally, there is the four-point function with three massless and one massive external line $p_1^2 = p_2^2 = p_3^2 = 0$, $p_4^2 = m_\mathrm{Z}^2$:

$$D(s,t) = \int \frac{\mathrm{d}^4 q}{(q^2 - m_\mathrm{f}^2)\left[(q+p_1)^2 - m_\mathrm{f}^2\right]\left[(q+p_1+p_2)^2 - m_\mathrm{f}^2\right]\left[(q-p_4)^2 - m_\mathrm{f}^2\right]}$$

$$= \frac{i\pi^2}{st} \int_0^1 \frac{\mathrm{d}x}{x(1-x) + m_\mathrm{f}^2 u/ts} \left\{ -\log\left[1 - i\epsilon - \frac{m_\mathrm{Z}^2}{m_\mathrm{f}^2} x(1-x)\right]\right.$$

$$\left. + \log\left[1 - i\epsilon - \frac{s}{m_\mathrm{f}^2} x(1-x)\right] + \log\left[1 - i\epsilon - \frac{t}{m_\mathrm{f}^2} x(1-x)\right]\right\}. \quad \text{(A.6)}$$

This result can be expressed in terms of Spence functions via the relation

$$\int_0^1 \frac{\mathrm{d}x}{x(1-x) + m_\mathrm{f}^2 u/ts} \log\left[1 - i\epsilon - \frac{v}{m_\mathrm{f}^2} x(1-x)\right]$$

$$= \frac{1}{\sqrt{1 + 4m_\mathrm{f}^2 u/ts}} \left[\mathrm{Sp}\left(\frac{x_-}{x_- - y}\right) - \mathrm{Sp}\left(\frac{x_+}{x_+ - y}\right) + \mathrm{Sp}\left(\frac{x_-}{y - x_+}\right)\right.$$

$$\left. - \mathrm{Sp}\left(\frac{x_+}{y - x_-}\right) + \log\left(\frac{-x_-}{x_+}\right) \log\left(1 - i\epsilon - \frac{v}{m_\mathrm{f}^2} x_- x_+\right)\right], \quad \text{(A.7)}$$

where

$$x_{\pm} = \tfrac{1}{2}\left(1 \pm \sqrt{1 + 4m_f^2 u/ts}\right), \tag{A.8}$$

$$y = \tfrac{1}{2}\left(1 + \sqrt{1 - 4\left(m_f^2 - i\epsilon\right)/v}\right). \tag{A.9}$$

As an auxiliary function we define

$$E(s,t) = sC(s) + tC(t) + s_1 C_1(s) + t_1 C_1(t) - stD(s,t). \tag{A.10}$$

Appendix B

THE VVVV HELICITY AMPLITUDES

In this appendix we give the contributions to the different amplitudes coming from the vector coupling of the Z in the notation of sect. 3. We leave out the factor

$$g_s^3 g_Z v_q d^{abc}/4(2\pi)^4.$$

These amplitudes are also described in refs. [1–6] in a different notation. As in sect. 3 we denote the vector helicity amplitudes by $V_{\lambda_1\lambda_2\lambda_3\lambda_4}$ where λ_i is the polarisation of particle i:

$$\begin{aligned}
V_{++++}(s,t,u) &= V_{+---}(s,t,u) \\
&= +\frac{8ut\left(s+m_Z^2\right)}{s_1 t_1 u_1} + \frac{8}{s_1}\left(\frac{u(2tu_1+su)}{u_1^2}B_1(u) + \frac{t(2ut_1+st)}{t_1^2}B_1(t)\right) \\
&\quad + \frac{8m_f^2}{ss_1}\left\{2m_Z^2 sC(s) + \left(m_Z^2 - 2s\right)\left[uC(u) + tC(t)\right]\right. \\
&\qquad \left. + \frac{m_Z^2(s^2+u^2) - 2stt_1}{t_1}C_1(t) + \frac{m_Z^2(s^2+t^2) - 2suu_1}{u_1}C_1(u)\right\} \\
&\quad + 8m_f^2\left\{-s_1\left[D(s,u) + D(s,t)\right] + \frac{\left(u^2u_1 + t^2t_1 + sut\right)}{ss_1}D(u,t)\right\} \\
&\quad + 16m_f^4\left[D(s,u) + D(s,t) + D(u,t)\right] + \frac{4(u^2+t^2)}{ss_1}E(u,t),
\end{aligned} \tag{B.1}$$

$$V_{+++-}(s,t,u) = -\frac{8ut(s+m_Z^2)}{s_1t_1u_1} + \frac{8utm_Z^2}{ss_1}\left(\frac{(s-2u_1)}{u_1^2}B_1(u) + \frac{(s-2t_1)}{t_1^2}B_1(t)\right)$$

$$+8m_f^2\left\{\frac{s(u^2+t^2)}{uts_1}C(s) + \frac{u(su+2tm_Z^2)}{sts_1}C(u)\right.$$

$$+\frac{t(st+2um_Z^2)}{sus_1}C(t)$$

$$+\frac{s_1^2}{ut}C_1(s) + \left[\frac{2um_Z^2}{s_1t_1} - \frac{u_1}{s_1} + \frac{s}{u} + \frac{2u}{s}\right]C_1(t)$$

$$\left.+\left[\frac{2tm_Z^2}{s_1u_1} - \frac{t_1}{s_1} + \frac{s}{t} + \frac{2t}{s}\right]C_1(u)\right\}$$

$$+8m_f^2\left(\frac{su}{t}D(s,u) + \frac{st}{u}D(s,t) + \frac{ut(s-4m_Z^2)}{ss_1}D(u,t)\right)$$

$$+16m_f^4(D(s,u)+D(s,t)+D(u,t)) + \frac{8utm_Z^2}{s^2s_1}E(u,t), \quad \text{(B.2)}$$

$$V_{++-+}(s,t,u) = -8 + 8m_f^2\left[\frac{s(u^2+t^2)}{uts_1}C(s) + \frac{(s_1^2+t^2)}{s_1t}C(u) + \frac{(s_1^2+u^2)}{s_1u}C(t)\right.$$

$$\left.+\frac{s_1^2}{ut}C_1(s) + \frac{uu_1}{s_1t}C_1(u) + \frac{tt_1}{s_1u}C_1(t)\right]$$

$$+8m_f^2\left[\frac{su}{t}D(s,u) + \frac{st}{u}D(s,t) + \frac{ut}{s_1}D(u,t)\right]$$

$$+16m_f^4[D(s,u)+D(s,t)+D(u,t)], \quad \text{(B.3)}$$

$$V_{++--}(s,t,u) = -8 + 16m_f^2m_Z^2/s_1C(s)$$

$$+16m_f^4(D(s,u)+D(s,t)+D(u,t)) + \frac{8m_f^2m_Z^2}{ss_1}E(u,t), \quad \text{(B.4)}$$

$V_{+-++}(s,t,u)$

$$= -\frac{8su}{s_1u_1} + \frac{8m_Z^2ut}{u_1^2s_1}B_1(u)$$

$$+8m_f^2\left[\frac{ss_1}{ut}C(s) + \frac{u^2}{s_1t}C(u) + \frac{(s_1^2+u^2)}{s_1u}C(t)\right.$$

$$\left. + \frac{(u^2+t^2)}{ut}C_1(s) + \frac{tt_1}{s_1u}C_1(t) + \frac{(2t^2m_Z^2 + t_1tu_1 + s_1su_1)}{s_1u_1t}C_1(u)\right]$$

$$+\frac{8m_f^2}{s_1}\left[\frac{u(s_1s - m_Z^2t)}{t}D(s,u) + \frac{(sts_1 - u^2m_Z^2)}{u}D(s,t) + ut_1D(u,t)\right]$$

$$+16m_f^4[D(s,u) + D(s,t) + D(u,t)] + \frac{4m_Z^2u}{s_1t}E(s,u), \qquad \text{(B.5)}$$

$V_{+-+-}(s,t,u)$

$$= V_{+--+}(s,u,t)$$

$$= +\frac{8su}{s_1u_1} + \frac{8s}{s_1t}\left[(u-s_1)B_1(s) + \frac{u(tu+2u_1s_1)}{u_1^2}B_1(u)\right]$$

$$+8m_f^2\left[-\frac{2s}{t}C(s) + \frac{u(tm_Z^2 - 2ss_1)}{s_1ts}C(u) + \frac{tm_Z^2}{ss_1}C(t)\right.$$

$$\left. -\frac{2s_1}{t}C_1(s) + \frac{(m_Z^2t(t^2+s^2) - 2s^2uu_1)}{s_1u_1ts}C_1(u) + \frac{t_1m_Z^2}{ss_1}C_1(t)\right]$$

$$+8m_f^2\left[s\left(\frac{4u}{t} - \frac{t}{s_1}\right)D(s,u) - \frac{st}{s_1}D(s,t) - \frac{t(s^2+m_Z^2u)}{ss_1}D(u,t)\right]$$

$$+16m_f^4[D(s,u) + D(s,t) + D(u,t)] + \frac{4s(t^2 - 2us_1)}{s_1t^2}E(s,u), \qquad \text{(B.6)}$$

$$V_{+++0}(s,t,u)/\Delta = +\frac{16sut(u-t)}{s_1 t_1 u_1}$$
$$+\frac{16ut}{s_1}\left[\frac{(2uu_1+m_Z^2 t)}{u_1^2}B_1(u)-\frac{(2tt_1+m_Z^2 u)}{t_1^2}B_1(t)\right]$$
$$+\frac{16m_f^2}{s_1}\left\{s(u-t)C(s)+ut[C(u)-C(t)]\right.$$
$$\left.+\frac{t(t_1^2-2s^2)}{t_1}C_1(t)-\frac{u(u_1^2-2s^2)}{u_1}C_1(u)\right\}$$
$$+8m_f^2\left[suD(s,u)-stD(s,t)+\frac{3ut(u-t)}{s_1}D(u,t)\right]$$
$$+\frac{8ut(t-u)}{s_1 s}E(u,t), \tag{B.7}$$

$$V_{++-0}(s,t,u)/\Delta$$
$$=+\frac{16m_f^2}{s_1}((t-u)sC(s)+ut[C(u)-C(t)]+tt_1C_1(t)-uu_1C_1(u))$$
$$+8m_f^2\left(stD(s,t)-suD(s,u)+\frac{ut(u-t)}{s_1}D(u,t)\right), \tag{B.8}$$

$$V_{+-+0}(s,t,u)/\Delta$$
$$=V_{+--0}(s,u,t)/\Delta$$
$$=+\frac{16sut}{u_1 s_1}+\frac{16su}{s_1 u_1^2}\left[u_1^2 B_1(s)+(s_1 u_1+ut)B_1(u)\right]$$
$$+\frac{16m_f^2}{s_1}\left\{-ss_1C(s)+ut[C(u)-C(t)]+tt_1C_1(t)+\frac{u(u_1^2-2t^2)}{u_1}C_1(u)\right\}$$
$$+\frac{8m_f^2}{s_1}[(u-t)stD(s,t)-(3u+t)suD(s,u)+(2s+u-t)utD(u,t)]$$
$$+\frac{8su^2}{ts_1}E(s,u). \tag{B.9}$$

The other helicity amplitudes follow from parity, i.e. $V_{----} = V_{++++}$. To get the complete amplitudes one has to sum over the quarks.

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