Building structure functions at higher orders

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January 19, 2010

Theoretical setup

For Higgs production in VBF, we need the DIS structure functions for scattering off a Z-boson (neutral current) as well as off a $W^{\pm}$-boson (charged current), i.e. F_i^V with $i = 1,2,3$ and $V \in \{Z, W^{\pm}\}$. To NLO, this has been documented well in Ref. [1] (see also the review Ref. [2]). Below, we present formulae for the relevant structure functions to second order in QCD. For both, neutral- and charged-current structure functions we employ the PDG conventions [3].

Beyond NLO, there are three issues to address here.

- We need to implement the correct dependence of the PDFs on the flavor quantum numbers.
- We need to separate the flavor non-singlet, pure-singlet and gluon contributions, see Fig. 1.
- We need to implement the correct scale dependence keeping $\mu_r \neq \mu_f$.

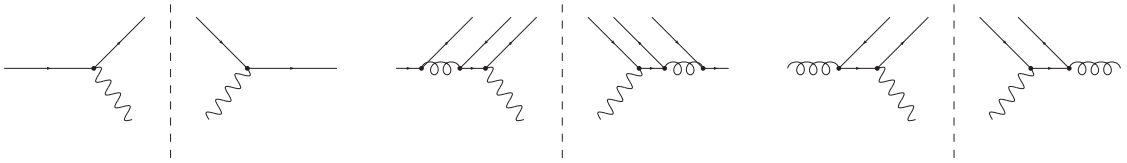

Figure 1: Sample diagrams for the non-singlet, pure-singlet and gluon contribution to vector-boson ($W^{\pm}, Z$) production. The dashed line indicates the final state cut.

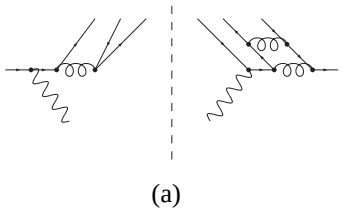

Figure 2: Sample diagram for the a new contribution beyond NNLO to vector-boson ($W^{\pm}, Z$) production in the quark sector (non-singlet and singlet). The dashed line indicates the final state cut.

Neutral-current Z-exchange

We expand the DIS neutral current structure functions for Z-exchange F_k^Z with $k = 1,2,3$ as follows:

$$\begin{aligned} F_i^Z(x,Q^2) &= a_i(x) \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \\ &\quad \times \sum_{j=1}^{n_f} \left(v_j^2 + a_j^2\right) \left\{ \left(q_j(y) + \bar{q}_j(y)\right) C_{i,\mathrm{ns}}^+(z) + \sum_{k=1}^{n_f} \left(q_k(y) + \bar{q}_k(y)\right) C_{i,\mathrm{ps}}(z) + g(y)\, C_{i,\mathrm{g}}(z) \right\}, \end{aligned} \tag{1}$$

$$F_3^Z(x,Q^2) = \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \sum_{i=1}^{n_f} 2\, v_i\, a_i \left(q_i(y) - \bar{q}_i(y)\right) C_{3,\mathrm{ns}}^-(z), \tag{2}$$

where $i = 1,2$ and the pre-factors are $a_1(x) = 1/2$, $a_2(x) = x$.

Here, the (anti)-quark and gluon distributions are denoted q_i, $\bar{q}_i$ and g and taken at the factorization scale μ_f. The singlet distribution q_s and the (non-singlet) valence distribution q_ns^v are given by

$$q_\mathrm{s} = \sum_{i=1}^{n_f} \left(q_i + \bar{q}_i\right), \tag{3}$$

$$q_\mathrm{ns}^\mathrm{v} = \sum_{i=1}^{n_f} \left(q_i - \bar{q}_i\right). \tag{4}$$

The non-singlet part of F_k^Z ($k = 1,2$) evolves like a flavor asymmetry of the type q_ns^+. The most general definition of these asymmetries reads, see e.g. [4],

$$q_{\mathrm{ns},ij}^\pm = \left(q_i \pm \bar{q}_i\right) - \left(q_j \pm \bar{q}_j\right). \tag{5}$$

We can use these relation to define

$$q_{\mathrm{ns},i}^+ = \left(q_i + \bar{q}_i\right) - q_\mathrm{s} = \sum_{j=1}^{n_f} q_{\mathrm{ns},ij}^+ \tag{6}$$

$$q_{\mathrm{ns},i}^- = \left(q_i - \bar{q}_i\right) - q_\mathrm{ns}^\mathrm{v} = \sum_{j=1}^{n_f} q_{\mathrm{ns},ij}^-. \tag{7}$$

With these relations, we arrive at the following alternative expressions for F_i^Z,

$$\begin{aligned} F_i^Z(x,Q^2) &= a_i(x) \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \\ &\quad \times \sum_{j=1}^{n_f} \left(v_j^2 + a_j^2\right) \left\{ q_{\mathrm{ns},j}^+(y)\, C_{i,\mathrm{ns}}^+(z) + q_\mathrm{s}(y)\, C_{i,\mathrm{q}}(z) + g(y)\, C_{i,\mathrm{g}}(z) \right\}, \end{aligned} \tag{8}$$

where $i = 1,2$ and $C_{i,\mathrm{q}} = C^+_{i,\mathrm{ns}} + C_{i,\mathrm{ps}}$, also q_s and $q^+_{\mathrm{ns},i}$ of Eqs. (3) and (6). Note, that $C_{i,\mathrm{ps}} \neq 0$ starting at two-loop order. Likewise

$$F_3^Z(x,Q^2) = \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \sum_{i=1}^{n_f} 2\, v_i\, a_i \left\{ q^-_{\mathrm{ns},i}(y)\, C^-_{3,\mathrm{ns}}(z) + q^\mathrm{v}_\mathrm{ns}(y)\, C^\mathrm{v}_{3,\mathrm{ns}}(z) \right\}, \tag{9}$$

where q^v_ns and $q^-_{\mathrm{ns},i}$ of Eqs. (4) and (7) have been used. The coefficient function is defined as $C^\mathrm{v}_{3,\mathrm{ns}} = C^-_{3,\mathrm{ns}} + C^\mathrm{s}_{3,\mathrm{ns}}$. Note, that $C^\mathrm{v}_{3,\mathrm{ns}} = C^-_{3,\mathrm{ns}}$ up to two-loop order, i.e. $C^\mathrm{s}_{3,\mathrm{ns}} \neq 0$ starting only at three-loop order. Thus, for all practical purposes, the form of F_3^Z as given in Eq. (9) suffices.

Coupling constants

The coupling constants are given by

$$v_i^2 + a_i^2 = \begin{cases} \left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_w\right)^2 & u\text{-type quarks}, \\ \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w\right)^2 & d\text{-type quarks}, \end{cases} \tag{10}$$

and, likewise,

$$2 v_i a_i = \begin{cases} \frac{1}{2} - \frac{4}{3}\sin^2\theta_w & u\text{-type quarks}, \\ \frac{1}{2} - \frac{2}{3}\sin^2\theta_w & d\text{-type quarks}. \end{cases} \tag{11}$$

Coefficient functions

The coefficient functions C_i parameterize the hard partonic scattering process. They depend only on scaling variable x, and dimensionless ratios of Q^2, μ_f and the renormalization scale μ_r. Their complete scale dependence, i.e. the logarithmic towers in $R = \mu_r^2/\mu_f^2$ and $M = Q^2/\mu_f^2$ (keeping $\mu_r \neq \mu_f$) is easily derived by renormalization group methods. The perturbative expansion of C_i in the strong coupling α_s up to two loops reads in the non-singlet sector,

$$\begin{aligned} C^+_{i,\mathrm{ns}}(x) &= \delta(1-x) + a_s\left\{c^{(1)}_{i,\mathrm{q}} + L_M P^{(0)}_{\mathrm{qq}}\right\} \\ &\quad + a_s^2\left\{c^{(2),+}_{i,\mathrm{ns}} + L_M\left(P^{(1),+}_{\mathrm{ns}} + c^{(1)}_{i,\mathrm{q}}(P^{(0)}_{\mathrm{qq}} - \beta_0)\right) + L_M^2\left(\frac{1}{2}P^{(0)}_{\mathrm{qq}}(P^{(0)}_{\mathrm{qq}} - \beta_0)\right)\right. \\ &\quad \left. + L_R\beta_0 c^{(1)}_{i,\mathrm{q}} + L_R L_M \beta_0 P^{(0)}_{\mathrm{qq}}\right\}, \end{aligned} \tag{12}$$

$$\begin{aligned} C^-_{3,\mathrm{ns}}(x) &= \delta(1-x) + a_s\left\{c^{(1)}_{3,\mathrm{q}} + L_M P^{(0)}_{\mathrm{qq}}\right\} \\ &\quad + a_s^2\left\{c^{(2),-}_{3,\mathrm{ns}} + L_M\left(P^{(1),-}_{\mathrm{ns}} + c^{(1)}_{3,\mathrm{q}}(P^{(0)}_{\mathrm{qq}} - \beta_0)\right) + L_M^2\left(\frac{1}{2}P^{(0)}_{\mathrm{qq}}(P^{(0)}_{\mathrm{qq}} - \beta_0)\right)\right. \\ &\quad \left. + L_R\beta_0 c^{(1)}_{3,\mathrm{q}} + L_R L_M \beta_0 P^{(0)}_{\mathrm{qq}}\right\}, \end{aligned} \tag{13}$$

and in the singlet sector [1]

$$C_{i,\mathrm{q}}(x) = \delta(1-x) + a_s\left\{c^{(1)}_{i,\mathrm{q}} + L_M P^{(0)}_{\mathrm{qq}}\right\} \tag{14}$$

[1] All coefficient functions can be taken e.g. from Ref. [5, 6]. Note, however, that both the pure-singlet and the gluon coefficient functions need to be divided by a factor n_f (due to the conventions of Ref. [5] with $\langle e^2\rangle = 1/n_f \sum_i e_i^2$ in the case of photon exchange).

$$
\begin{aligned}
&+a_s^2\Big\{c_{i,\rm q}^{(2)} + L_M\Big(P_{\rm qq}^{(1)} + c_{i,\rm q}^{(1)}(P_{\rm qq}^{(0)} - \beta_0) + c_{i,\rm g}^{(1)} P_{\rm gq}^{(0)}\Big) + L_M^2\Big(\frac{1}{2}P_{\rm qq}^{(0)}(P_{\rm qq}^{(0)} - \beta_0) + \frac{1}{2}P_{\rm qg}^{(0)}P_{\rm gq}^{(0)}\Big)\\
&+L_R\beta_0 c_{i,\rm q}^{(1)} + L_R L_M \beta_0 P_{\rm qq}^{(0)}\Big\},\\
C_{i,\rm ps}(x) &= a_s^2\Big\{c_{i,\rm ps}^{(2)} + L_M\Big(P_{\rm ps}^{(1)} + c_{i,\rm g}^{(1)} P_{\rm gq}^{(0)}\Big) + L_M^2 \frac{1}{2} P_{\rm qg}^{(0)} P_{\rm gq}^{(0)}\Big\}, \qquad (15)\\
C_{i,\rm g}(x) &= a_s\Big\{c_{i,\rm g}^{(1)} + L_M P_{\rm qg}^{(0)}\Big\} \qquad (16)\\
&+a_s^2\Big\{c_{i,\rm g}^{(2)} + L_M\Big(P_{\rm qg}^{(1)} + c_{i,\rm q}^{(1)} P_{\rm qg}^{(0)} + c_{i,\rm g}^{(1)}(P_{\rm gg}^{(0)} - \beta_0)\Big) + L_M^2\Big(\frac{1}{2}P_{\rm qq}^{(0)}P_{\rm qg}^{(0)}\frac{1}{2}P_{\rm qg}^{(0)}(P_{\rm gg}^{(0)} - \beta_0)\Big)\\
&+L_R\beta_0 c_{i,\rm g}^{(1)} + L_R L_M \beta_0 P_{\rm qg}^{(0)}\Big\},
\end{aligned}
$$

where $i = 1,2$ and $a_s = \alpha_s(\mu_r)/(4\pi)$. We abbreviate $L_M = \ln(Q^2/\mu_f^2)$ and $L_R = \ln(\mu_r^2/\mu_f^2)$ and all products are understood as Mellin convolutions. Moreover, we have $P_{\rm qq}^{(1)} = P_{\rm ns}^{(1),+} + P_{\rm ps}^{(1)}$ and $c_{i,\rm q}^{(2)} = c_{i,\rm ns}^{(2),+} + c_{i,\rm ps}^{(2)}$.

Our expansion parameter is always $a_s = \alpha_s/(4\pi)$ and the conventions for the running coupling are

$$
\frac{d}{d\ln\mu^2}\frac{\alpha_s}{4\pi} \equiv \frac{d\,a_s}{d\ln\mu^2} = -\beta_0\,a_s^2 - \beta_1\,a_s^3 - \beta_2\,a_s^4 - \dots\,, \qquad (17)
$$

where β_n denote the usual four-dimensional expansion coefficients of the beta function in QCD, i.e. starting with

$$
\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f\,. \qquad (18)
$$

In QCD, the color coefficients are $C_A = 3$ and $C_F = 4/3$. The splitting functions $P_{ij}^{(l)}$ can be taken e.g. from Ref. [4, 7]. At leading order they read

$$
\begin{aligned}
P_{\rm qq}^{(0)}(x) &= C_F\Big(\frac{4}{1-x} - 2 - 2x + 3\delta(1-x)\Big), \qquad (19)\\
P_{\rm qg}^{(0)}(x) &= 2\,n_f\,(1 - 2x + 2x^2), \qquad (20)\\
P_{\rm gq}^{(0)}(x) &= C_F\Big(\frac{4}{x} - 4 + 2x\Big), \qquad (21)\\
P_{\rm gg}^{(0)}(x) &= C_A\Big(\frac{4}{1-x} + \frac{4}{x} - 8 + 4x - 4x^2 + \frac{11}{3}\delta(1-x)\Big) - \frac{2}{3}\,n_f\,\delta(1-x), \qquad (22)
\end{aligned}
$$

Please also note the explicit factor of $(2n_f)$ in Eq. (20) which is due to the definition of $P_{\rm qg}^{(0)}$ in Eq. (20) (and, likewise for $P_{\rm qg}^{(1)}$) in Ref. [7]. This factor originates from summation over all quarks and anti-quarks. [2]

Charged-current $W^{\pm}$-exchange

For the charged current structure functions $F_i^{W^+}$ with W^+-exchange with $i = 1,2,3$ we have at leading order in QCD in terms of the parton densities

$$
F_2^{W^+}(x) = 2\,x\Big(\bar{u} + d + s + \bar{c} + b\Big), \qquad (23)
$$

[2] In analogy to the pure-singlet and the gluon coefficient functions (see footnote 1) also the splitting functions $P_{\rm qg}^{(0)}$ and $P_{\rm qg}^{(1)}$ in Eqs. (14)–(16) need to be divided by a factor n_f.

$$F_2^{W^-}(x) = 2x\Big(u+\bar{d}+\bar{s}+c+\bar{b}\Big), \tag{24}$$

$$F_3^{W^+}(x) = 2\Big(-\bar{u}+d+s-\bar{c}+b\Big), \tag{25}$$

$$F_3^{W^-}(x) = 2\Big(u-\bar{d}-\bar{s}+c-\bar{b}\Big). \tag{26}$$

With these expressions, we can construct to leading order the structure functions for the sum and differences, $W^+ \pm W^-$. The latter have well defined transformation properties under the standard OPE of DIS.

$$\begin{aligned} F_2^{W^+ + W^-}(x) &= 2x\Big[(u+\bar{u})+(d+\bar{d})+(s+\bar{s})+(c+\bar{c})+(b+\bar{b})\Big] \\ &= 2x\,q_{\rm s}\,, \end{aligned} \tag{27}$$

$$\begin{aligned} F_2^{W^+ - W^-}(x) &= 2x\Big[-(u-\bar{u})+(d-\bar{d})+(s-\bar{s})-(c-\bar{c})+(b-\bar{b})\Big] \\ &= -2x\,\delta q_{\rm ns}^-\,, \end{aligned} \tag{28}$$

$$\begin{aligned} F_3^{W^+ + W^-}(x) &= 2\Big[(u-\bar{u})+(d-\bar{d})+(s-\bar{s})+(c-\bar{c})+(b-\bar{b})\Big] \\ &= 2\,q_{\rm ns}^{\rm v}\,, \end{aligned} \tag{29}$$

$$\begin{aligned} F_3^{W^+ - W^-}(x) &= 2\Big[-(u+\bar{u})+(d+\bar{d})+(s+\bar{s})-(c+\bar{c})+(b+\bar{b})\Big] \\ &= -2\,\delta q_{\rm ns}^+\,, \end{aligned} \tag{30}$$

where the asymmetry $\delta q_{\rm ns}^-$ parametrizes the iso-triplet component of the proton, i.e. $u \neq d$ and so on. It arises from Eq. (5) as

$$\delta q_{\rm ns}^{\pm} = \sum_{i\in u-{\rm type}} \sum_{j\in d-{\rm type}} q_{{\rm ns},ij}^{\pm}\,. \tag{31}$$

In order to identify definite flavor representations for the PDFs and the respective coefficient functions we expand the DIS charged current structure functions for $W^+ \pm W^-$-exchange $F_k^{W^+\pm W^-}$ with $k = 1,2,3$ as follows:

$$F_i^{W^+ + W^-}(x,Q^2) = 2\,a_i(x) \int_0^1 dz \int_0^1 dy\,\delta(x-yz)\,\frac{1}{n_f}\sum_{j=1}^{n_f}\Big(v_j^2+a_j^2\Big)\Big\{q_{\rm s}(y)\,C_{i,{\rm q}}(z)+g(y)\,C_{i,{\rm g}}(z)\Big\}, \tag{32}$$

$$F_i^{W^+ - W^-}(x,Q^2) = 2\,a_i(x) \int_0^1 dz \int_0^1 dy\,\delta(x-yz)\,\frac{1}{n_f}\sum_{j=1}^{n_f}\Big(v_j^2+a_j^2\Big)\Big(-\delta q_{\rm ns}^-(y)\Big)\,C_{i,{\rm ns}}^-(z), \tag{33}$$

$$F_3^{W^+ + W^-}(x,Q^2) = 2 \int_0^1 dz \int_0^1 dy\,\delta(x-yz)\,\frac{1}{n_f}\sum_{i=1}^{n_f} 2\,v_i\,a_i\,q_{\rm v}(y)\,C_{3,{\rm ns}}^{\rm v}(z), \tag{34}$$

$$F_3^{W^+ - W^-}(x,Q^2) = 2 \int_0^1 dz \int_0^1 dy\,\delta(x-yz)\,\frac{1}{n_f}\sum_{i=1}^{n_f} 2\,v_i\,a_i\Big(-\delta q_{\rm ns}^+(y)\Big)\,C_{3,{\rm ns}}^+(z). \tag{35}$$

Here we have used the relations for $q_{\rm s}$, $q_{\rm ns}^{\rm v}$ and $\delta q_{\rm ns}^{\pm}$ of Eqs. (3) and (4) and (31).

Taking the sum and the difference, we obtain for the structure functions $F_k^{W^\pm}$ with $k = 1,2$ which describe individual $W^\pm$-exchange,

$$\begin{aligned} F_i^{W^-}(x,Q^2) &= a_i(x) \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \\ &\quad \times \frac{1}{n_f} \sum_{j=1}^{n_f} \left(v_j^2 + a_j^2\right) \left\{ \delta q_{\rm ns}^-(y)\, C_{i,\rm ns}^-(z) + q_{\rm s}(y)\, C_{i,\rm q}(z) + g(y)\, C_{i,\rm g}(z) \right\}, \end{aligned} \tag{36}$$

$$F_3^{W^-}(x,Q^2) = \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \frac{1}{n_f} \sum_{i=1}^{n_f} 2\, v_i\, a_i \left\{ \delta q_{\rm ns}^+(y)\, C_{3,\rm ns}^+(z) + q_{\rm ns}^{\rm v}(y)\, C_{3,\rm ns}^{\rm v}(z) \right\}, \tag{37}$$

where in this case $C_{i,q} = C_{i,ns}^- + C_{i,ps}$. The respective results for $F_i^{W^+}$ are obtained from Eqs. (36), (37) with the simple replacement $\delta q_{\rm ns}^\pm \to -\delta q_{\rm ns}^\pm$.

Please recall, that the functions $C_{i,\rm ns}^+$ and $C_{i,\rm q}$ start to differ only at two-loop order; up to NLO there is no difference (cf. the simple replacement rules in Ref. [1]). Recall also, that $C_{3,\rm ns}^{\rm v} = C_{3,\rm ns}^-$ up to two-loop order.

This implies, that the iso-triplet component of the proton $\delta q_{\rm ns}^-$ enters in a non-trivial way for the first time at NNLO. It numerical impact is expected to be small though.

Coupling constants

The coupling constants are given by

$$v_i = a_i = \frac{1}{\sqrt{2}}. \tag{38}$$

Note that in this case

$$\frac{1}{n_f} \sum_{j=1}^{n_f} \left(v_j^2 + a_j^2\right) = \frac{1}{n_f} \sum_{i=1}^{n_f} 2\, v_i\, a_i = 1. \tag{39}$$

Coefficient functions

The dependence on the factorization and the renormalization scales of $C_{i,ns}^+, C_{i,q}$ and C_3^+ can be obtained from Eqs.(12)-(16) performig the substitutions $c_{i,ns}^{(2),+} \leftrightarrow c_{i,\rm ns}^{(2),-}$, $c_{3,\rm ns}^{(2),+} \leftrightarrow c_{3,\rm ns}^{(2),-}$ and $P_{\rm ns}^{(1),+} \leftrightarrow P_{\rm ns}^{(1),-}$. [3]

[3] In this case the pure-singlet and the gluon coefficient functions of Ref. [5] need **not** to be divided by a factor n_f and also **not** the splitting functions $P_{qg}^{(0)}$ and $P_{qg}^{(1)}$.

Electromagnetic γ-exchange

This interaction gives contributions to the F_i^γ structure function for $i = 1,2$ only, because the γ-exchange is not a CP-violating interaction.

Just as a reminder we recall that the structure function F_2^γ at leading order takes the following form

$$F_2^\gamma(x) \;=\; x\sum_{i=1}^{n_f} e_i^2\Big(q_i + \bar{q}_i\Big), \tag{40}$$

where n_f is the number of active flavors and the electromagnetic charges are $e_i = 2/3$ for a u-type quark and $e_i = -1/3$ for a d-type quark.

At higher orders we have the following structure, e.g. for F_2^γ, see e.g. Ref. [5],

$$x^{-1}F_2^\gamma \;=\; C_{2,\mathrm{ns}}^{+}\otimes q_{\mathrm{ns}}^{\gamma} + \langle e^2\rangle\Big(C_{2,\mathrm{q}}\otimes q_{\mathrm{s}} + C_{2,\mathrm{g}}\otimes g\Big)\,, \tag{41}$$

where $\otimes$ denotes the Mellin convolution and

$$\langle e^2\rangle \;=\; \frac{1}{n_f}\sum_{i=1}^{n_f} e_i^2\,. \tag{42}$$

Finally according to the notation in Ref. [5],

$$C_{i,\mathrm{q}} = C_{i,\mathrm{ns}} + C_{i,\mathrm{ps}}\,. \tag{43}$$

Recall that for an even number of n_f, we have $\langle e^2\rangle = 5/18$. According to Eqs. (40), (41) we can easily check that the quark combination q_{ns}^γ is

$$q_{\mathrm{ns}}^{\gamma} \;=\; \Delta(e^2)\Big\{\sum_{i=u-\mathrm{type}}(q_i+\bar{q}_i) - \sum_{i=d-\mathrm{type}}(q_i+\bar{q}_i)\Big\}, \tag{44}$$

where $\Delta(e^2) = 1/2(e_u^2 - e_d^2) = 1/6$. Note that q_{ns}^γ evolves like a q_{ns}^+ quark combination, see also [8]. Recall $F_3^\gamma = 0$.

Up to two-loop order we can recover the cases for neutral- and charged current interactions discussed above with the following set of substitutions in Eq. (41). E.g. for F_2^Z we have, cf. Eq. (8),

$$x^{-1}F_2^Z = x^{-1}F_2^\gamma \left\{ \begin{array}{l} e_u^2 \to \left(\frac{1}{2} - \frac{4}{3}\sin^2(\theta_W)\right)^2 \\ \\ e_d^2 \to \left(\frac{1}{2} - \frac{2}{3}\sin^2(\theta_W)\right)^2 \end{array} \right. , \tag{45}$$

and the PDFs remain unchanged.

Likewise, e.g. for $F_2^{W^-}$ we have, cf. Eq. (36)

$$x^{-1}F_2^{W^-} = x^{-1}F_2^\gamma \left\{ \begin{array}{l} \langle e^2\rangle \to 1 \\ \\ q_{\mathrm{ns}}^{\gamma} \to \delta q_{\mathrm{ns}}^{-} = [(u-\bar{u}) - (d-\bar{d})] + [(c-\bar{c}) - (s-\bar{s})] + \ldots \end{array} \right. , \tag{46}$$

where substitution of the nonsinglet PDF accounts for the proper quark-flavor dependence.

References

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