Building structure functions at higher orders

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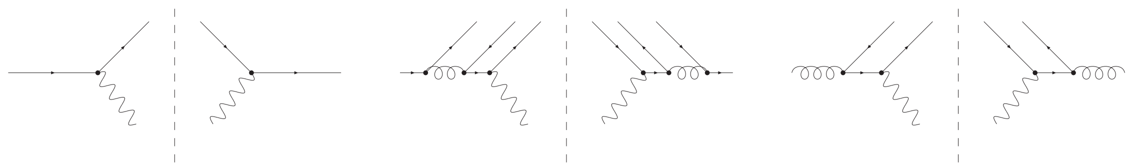
Theoretical setup

For Higgs production in VBF, we need the DIS structure functions for scattering off a Z-boson (neutral current) as well as off a $W^{\pm}$-boson (charged current), i.e. F_i^V with $i = 1, 2, 3$ and $V \in \{Z, W^{\pm}\}$. To NLO, this has been documented well in Ref. [1] (see also the review Ref. [2]). Below, we present formulae for the relevant structure functions to second order in QCD. For both, neutral- and charged-current structure functions we employ the PDG conventions [3].

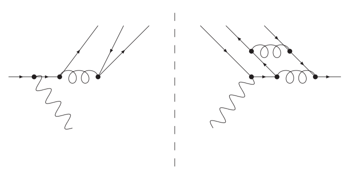
Beyond NLO, there are three issues to address here.

- We need to implement the correct dependence of the PDFs on the flavor quantum numbers.
- We need to separate the flavor non-singlet, pure-singlet and gluon contributions, see Fig. 1.
- We need to implement the correct scale dependence keeping $\mu_r \neq \mu_f$.



(a) (b) (c)

Figure 1: Sample diagrams for the non-singlet, pure-singlet and gluon contribution to vector-boson ($W^{\pm}, Z$) production. The dashed line indicates the final state cut.



(a)

Figure 2: Sample diagram for the a new contribution beyond NNLO to vector-boson ($W^{\pm}, Z$) production in the quark sector (non-singlet and singlet). The dashed line indicates the final state cut.

Neutral-current Z-exchange

We expand the DIS neutral current structure functions for Z-exchange F_k^Z with $k = 1,2,3$ as follows:

$$
\begin{aligned}
F_i^Z(x,Q^2) &= a_i(x) \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \\
&\quad \times \sum_{j=1}^{n_f} \left(v_j^2 + a_j^2\right) \left\{ \Big(q_j(y) + \bar{q}_j(y)\Big) C^+_{i,\mathrm{ns}}(z) + \sum_{k=1}^{n_f} \Big(q_k(y) + \bar{q}_k(y)\Big) C_{i,\mathrm{ps}}(z) + g(y)\, C_{i,\mathrm{g}}(z) \right\}, \qquad (1) \\
F_3^Z(x,Q^2) &= \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \sum_{i=1}^{n_f} 2\, v_i\, a_i \Big(q_i(y) - \bar{q}_i(y)\Big) C^-_{3,\mathrm{ns}}(z), \qquad (2)
\end{aligned}
$$

where $i = 1,2$ and the pre-factors are $a_1(x) = 1/2$, $a_2(x) = x$.

Here, the (anti)-quark and gluon distributions are denoted q_i, $\bar{q}_i$ and g and taken at the factorization scale μ_f. The singlet distribution q_s and the (non-singlet) valence distribution q^v_ns are given by

$$
\begin{aligned}
q_\mathrm{s} &= \sum_{i=1}^{n_f} \Big(q_i + \bar{q}_i\Big), \qquad (3) \\
q^\mathrm{v}_\mathrm{ns} &= \sum_{i=1}^{n_f} \Big(q_i - \bar{q}_i\Big). \qquad (4)
\end{aligned}
$$

The non-singlet part of F_k^Z ($k = 1,2$) evolves like a flavor asymmetry of the type q^+_ns. The most general definition of these asymmetries reads, see e.g. [4],

$$
q^\pm_{\mathrm{ns},ij} = \Big(q_i \pm \bar{q}_i\Big) - \Big(q_j \pm \bar{q}_j\Big). \qquad (5)
$$

We can use these relation to define

$$
\begin{aligned}
q^+_{\mathrm{ns},i} &= \Big(q_i + \bar{q}_i\Big) - q_\mathrm{s} = \sum_{j=1}^{n_f} q^+_{\mathrm{ns},ij} \qquad (6) \\
q^-_{\mathrm{ns},i} &= \Big(q_i - \bar{q}_i\Big) - q^\mathrm{v}_\mathrm{ns} = \sum_{j=1}^{n_f} q^-_{\mathrm{ns},ij}\,. \qquad (7)
\end{aligned}
$$

With these relations, we arrive at the following alternative expressions for F_i^Z,

$$
\begin{aligned}
F_i^Z(x,Q^2) &= a_i(x) \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \qquad (8) \\
&\quad \times \sum_{j=1}^{n_f} \left(v_j^2 + a_j^2\right) \left\{ q^+_{\mathrm{ns},j}(y)\, C^+_{i,\mathrm{ns}}(z) + q_\mathrm{s}(y)\, C_{i,\mathrm{q}}(z) + g(y)\, C_{i,\mathrm{g}}(z) \right\},
\end{aligned}
$$

where $i = 1,2$ and $C_{i,\mathrm{q}} = C^+_{i,\mathrm{ns}} + C_{i,\mathrm{ps}}$, also q_s and $q^+_{\mathrm{ns},i}$ of Eqs. (3) and (6). Note, that $C_{i,\mathrm{ps}} \neq 0$ starting at two-loop order. Likewise

$$F_3^Z(x,Q^2) \;=\; \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \sum_{i=1}^{n_f} 2\, v_i\, a_i \left\{ q^-_{\mathrm{ns},i}(y)\, C^-_{3,\mathrm{ns}}(z) + q^\mathrm{v}_\mathrm{ns}(y)\, C^\mathrm{v}_{3,\mathrm{ns}}(z) \right\}, \tag{9}$$

where q^v_ns and $q^-_{\mathrm{ns},i}$ of Eqs. (4) and (7) have been used. The coefficient function is defined as $C^\mathrm{v}_{3,\mathrm{ns}} = C^-_{3,\mathrm{ns}} + C^\mathrm{s}_{3,\mathrm{ns}}$. Note, that $C^\mathrm{v}_{3,\mathrm{ns}} = C^-_{3,\mathrm{ns}}$ up to two-loop order, i.e. $C^\mathrm{s}_{3,\mathrm{ns}} \neq 0$ starting only at three-loop order. Thus, for all practical purposes, the form of F_3^Z as given in Eq. (9) suffices.

Coupling constants

The coupling constants are given by

$$v_i^2 + a_i^2 \;=\; \begin{cases} \left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_w\right)^2 & u\text{-type quarks}, \\[2ex] \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w\right)^2 & d\text{-type quarks}, \end{cases} \tag{10}$$

and, likewise,

$$2 v_i a_i \;=\; \begin{cases} \frac{1}{2} - \frac{4}{3}\sin^2\theta_w & u\text{-type quarks}, \\[2ex] \frac{1}{2} - \frac{2}{3}\sin^2\theta_w & d\text{-type quarks}. \end{cases} \tag{11}$$

Coefficient functions

The coefficient functions C_i parameterize the hard partonic scattering process. They depend only on scaling variable x, and dimensionless ratios of Q^2, μ_f and the renormalization scale μ_r. Their complete scale dependence, i.e. the logarithmic towers in $R = \mu_r^2/\mu_f^2$ and $M = Q^2/\mu_f^2$ (keeping $\mu_r \neq \mu_f$) is easily derived by renormalization group methods. The perturbative expansion of C_i in the strong coupling α_s up to two loops reads in the non-singlet sector,

$$\begin{aligned} C^+_{i,\mathrm{ns}}(x) \;=\;& \delta(1-x) + a_s \Big\{ c^{(1)}_{i,\mathrm{q}} + L_M P^{(0)}_\mathrm{qq} \Big\} \qquad (12) \\ &+ a_s^2 \Big\{ c^{(2),+}_{i,\mathrm{ns}} + L_M \Big(P^{(1),+}_\mathrm{ns} + c^{(1)}_{i,\mathrm{q}} (P^{(0)}_\mathrm{qq} - \beta_0) \Big) + L_M^2 \Big(\frac{1}{2} P^{(0)}_\mathrm{qq} (P^{(0)}_\mathrm{qq} - \beta_0) \Big) \\ &+ L_R \beta_0 c^{(1)}_{i,\mathrm{q}} + L_R L_M \beta_0 P^{(0)}_\mathrm{qq} \Big\}, \\ C^-_{3,\mathrm{ns}}(x) \;=\;& \delta(1-x) + a_s \Big\{ c^{(1)}_{3,\mathrm{q}} + L_M P^{(0)}_\mathrm{qq} \Big\} \qquad (13) \\ &+ a_s^2 \Big\{ c^{(2),-}_{3,\mathrm{ns}} + L_M \Big(P^{(1),-}_\mathrm{ns} + c^{(1)}_{3,\mathrm{q}} (P^{(0)}_\mathrm{qq} - \beta_0) \Big) + L_M^2 \Big(\frac{1}{2} P^{(0)}_\mathrm{qq} (P^{(0)}_\mathrm{qq} - \beta_0) \Big) \\ &+ L_R \beta_0 c^{(1)}_{3,\mathrm{q}} + L_R L_M \beta_0 P^{(0)}_\mathrm{qq} \Big\}, \qquad (14) \end{aligned}$$

and in the singlet sector [1]

$$C_{i,\mathrm{q}}(x) \;=\; \delta(1-x) + a_s \Big\{ c^{(1)}_{i,\mathrm{q}} + L_M P^{(0)}_\mathrm{qq} \Big\} \tag{15}$$

[1] All coefficient functions can be taken e.g. from Ref. [5]. Note, however, that both the pure-singlet and the gluon coefficient functions need to be divided by a factor n_f (due to the conventions of Ref. [5] with $\langle e^2 \rangle = 1/n_f \sum_i e_i^2$ in the case of photon exchange).

$$+a_s^2\Big\{c_{i,\mathrm{q}}^{(2)}+L_M\Big(P_{\mathrm{qq}}^{(1)}+c_{i,\mathrm{q}}^{(1)}(P_{\mathrm{qq}}^{(0)}-\beta_0)+c_{i,\mathrm{g}}^{(1)}P_{\mathrm{gq}}^{(0)}\Big)+L_M^2\Big(\frac{1}{2}P_{\mathrm{qq}}^{(0)}(P_{\mathrm{qq}}^{(0)}-\beta_0)+\frac{1}{2}P_{\mathrm{qg}}^{(0)}P_{\mathrm{gq}}^{(0)}\Big)$$

$$+L_R\beta_0 c_{i,\mathrm{q}}^{(1)}+L_R L_M\beta_0 P_{\mathrm{qq}}^{(0)}\Big\},$$

$$C_{i,\mathrm{ps}}(x) = a_s^2\Big\{c_{i,\mathrm{ps}}^{(2)}+L_M\Big(P_{\mathrm{ps}}^{(1)}+c_{i,\mathrm{g}}^{(1)}P_{\mathrm{gq}}^{(0)}\Big)+L_M^2\frac{1}{2}P_{\mathrm{qg}}^{(0)}P_{\mathrm{gq}}^{(0)}\Big\}, \tag{16}$$

$$C_{i,\mathrm{g}}(x) = a_s\Big\{c_{i,\mathrm{g}}^{(1)}+L_M P_{\mathrm{qg}}^{(0)}\Big\} \tag{17}$$

$$+a_s^2\Big\{c_{i,\mathrm{g}}^{(2)}+L_M\Big(P_{\mathrm{qg}}^{(1)}+c_{i,\mathrm{q}}^{(1)}P_{\mathrm{qg}}^{(0)}+c_{i,\mathrm{g}}^{(1)}(P_{\mathrm{gg}}^{(0)}-\beta_0)\Big)+L_M^2\Big(\frac{1}{2}P_{\mathrm{qq}}^{(0)}P_{\mathrm{qg}}^{(0)}\frac{1}{2}P_{\mathrm{qg}}^{(0)}(P_{\mathrm{gg}}^{(0)}-\beta_0)\Big)$$

$$+L_R\beta_0 c_{i,\mathrm{g}}^{(1)}+L_R L_M\beta_0 P_{\mathrm{qg}}^{(0)}\Big\},$$

where $i=1,2$ and $a_s=\alpha_s(\mu_r)/(4\pi)$. We abbreviate $L_M=\ln(Q^2/\mu_f^2)$ and $L_R=\ln(\mu_r^2/\mu_f^2)$ and all products are understood as Mellin convolutions. Moreover, we have $P_{\mathrm{qq}}^{(1)}=P_{\mathrm{ns}}^{(1),+}+P_{\mathrm{ps}}^{(1)}$ and $c_{i,\mathrm{q}}^{(2)}=c_{i,\mathrm{ns}}^{(2),+}+c_{i,\mathrm{ps}}^{(2)}$.

Our expansion parameter is always $a_s=\alpha_s/(4\pi)$ and the conventions for the running coupling are

$$\frac{d}{d\ln\mu^2}\frac{\alpha_s}{4\pi} \equiv \frac{d\,a_s}{d\ln\mu^2} = -\beta_0\,a_s^2-\beta_1\,a_s^3-\beta_2\,a_s^4-\dots\,, \tag{18}$$

where β_n denote the usual four-dimensional expansion coefficients of the beta function in QCD, i.e. starting with

$$\beta_0=\frac{11}{3}C_A-\frac{2}{3}n_f\,. \tag{19}$$

In QCD, the color coefficients are $C_A=3$ and $C_F=4/3$. The splitting functions $P_{ij}^{(l)}$ can be taken e.g. from Ref. [4, 6]. At leading order they read

$$P_{\mathrm{qq}}^{(0)}(x) = C_F\Big(\frac{4}{1-x}-2-2x+3\delta(1-x)\Big), \tag{20}$$

$$P_{\mathrm{qg}}^{(0)}(x) = 2n_f(1-2x+2x^2), \tag{21}$$

$$P_{\mathrm{gq}}^{(0)}(x) = C_F\Big(\frac{4}{x}-4+2x\Big), \tag{22}$$

$$P_{\mathrm{gg}}^{(0)}(x) = C_A\Big(\frac{4}{1-x}+\frac{4}{x}-8+4x-4x^2+\frac{11}{3}\delta(1-x)\Big)-\frac{2}{3}n_f\,\delta(1-x), \tag{23}$$

Please also note the explicit factor of $(2n_f)$ in Eq. (21) which is due to the definition of $P_{\mathrm{qg}}^{(0)}$ in Eq. (21) (and, likewise for $P_{\mathrm{qg}}^{(1)}$) in Ref. [6]. This factor originates from summation over all quarks and anti-quarks. [2]

Charged-current $W^\pm$-exchange

For the charged current structure functions $F_i^{W^\pm}$ with $W^\pm$-exchange with $i=1,2,3$ we have at leading order in QCD in terms of the parton densities

$$F_2^{W^+}(x) = 2x\Big(\bar{u}+d+s+\bar{c}+b\Big), \tag{24}$$

[2] In analogy to the pure-singlet and the gluon coefficient functions (see footnote 1) also the splitting functions $P_{\mathrm{qg}}^{(0)}$ and $P_{\mathrm{qg}}^{(1)}$ in Eqs. (15)–(17) need to be divided by a factor n_f.

$$F_2^{W^-}(x) = 2x\Big(u+\bar{d}+\bar{s}+c+\bar{b}\Big), \tag{25}$$
$$F_3^{W^+}(x) = 2\Big(-\bar{u}+d+s-\bar{c}+b\Big), \tag{26}$$
$$F_3^{W^-}(x) = 2\Big(u-\bar{d}-\bar{s}+c-\bar{b}\Big). \tag{27}$$

With these expressions, we can construct to leading order the structure functions for the sum and differences, $W^+ \pm W^-$. The latter have well defined transformation properties under the standard OPE of DIS.

$$\begin{aligned} F_2^{W^+ + W^-}(x) &= 2x\Big[(u+\bar{u})+(d+\bar{d})+(s+\bar{s})+(c+\bar{c})+(b+\bar{b})\Big] \\ &= 2x q_{\rm s}\,, \end{aligned} \tag{28}$$
$$\begin{aligned} F_2^{W^+ - W^-}(x) &= 2x\Big[-(u-\bar{u})+(d-\bar{d})+(s-\bar{s})-(c-\bar{c})+(b-\bar{b})\Big] \\ &= -2x\delta q_{\rm ns}^-\,, \end{aligned} \tag{29}$$
$$\begin{aligned} F_3^{W^+ + W^-}(x) &= 2\Big[(u-\bar{u})+(d-\bar{d})+(s-\bar{s})+(c-\bar{c})+(b-\bar{b})\Big] \\ &= 2q_{\rm ns}^{\rm v}\,, \end{aligned} \tag{30}$$
$$\begin{aligned} F_3^{W^+ - W^-}(x) &= 2\Big[-(u-\bar{u})+(d-\bar{d})+(s-\bar{s})-(c-\bar{c})+(b-\bar{b})\Big] \\ &= -2\delta q_{\rm ns}^-\,, \end{aligned} \tag{31}$$

where the asymmetry $\delta q_{\rm ns}^-$ parametrizes the iso-triplet component of the proton, i.e. $u \neq d$ and so on. It arises from Eq. (5) as

$$\delta q_{\rm ns}^- = \sum_{i\in u-{\rm type}}\ \sum_{j\in d-{\rm type}} q_{{\rm ns},ij}^-\,. \tag{32}$$

In order to identify definite flavor representations for the PDFs and the respective coefficient functions we expand the DIS charged current structure functions for $W^+ \pm W^-$-exchange $F_k^{W^+\pm W^-}$ with $k = 1,2,3$ as follows:

$$F_i^{W^+ + W^-}(x,Q^2) = 2a_i(x)\int_0^1 dz \int_0^1 dy\,\delta(x-yz)\sum_{j=1}^{n_f}\left(v_j^2+a_j^2\right)\Big\{q_{\rm s}(y)C_{i,{\rm q}}(z)+g(y)C_{i,{\rm g}}(z)\Big\}\,, \tag{33}$$
$$F_i^{W^+ - W^-}(x,Q^2) = 2a_i(x)\int_0^1 dz \int_0^1 dy\,\delta(x-yz)\sum_{j=1}^{n_f}\left(v_j^2+a_j^2\right)\Big(-\delta q_{\rm ns}^-(y)\Big)C_{i,{\rm ns}}^-(z)\,, \tag{34}$$
$$F_3^{W^+ + W^-}(x,Q^2) = 2\int_0^1 dz \int_0^1 dy\,\delta(x-yz)\sum_{i=1}^{n_f} 2v_i a_i\, q_{\rm v}(y)C_{3,{\rm ns}}^{\rm v}(z)\,, \tag{35}$$
$$F_3^{W^+ - W^-}(x,Q^2) = 2\int_0^1 dz \int_0^1 dy\,\delta(x-yz)\sum_{i=1}^{n_f} 2v_i a_i \Big(-\delta q_{\rm ns}^-(y)\Big)C_{i,{\rm ns}}^-(z)\,. \tag{36}$$

Here we have used the relations for $q_{\rm s}$, $q_{\rm ns}^{\rm v}$ and $\delta q_{\rm ns}^-$ of Eqs. (3) and (4) and (32).

Taking the sum and the difference, we obtain for the structure functions $F_k^{W^\pm}$ with $k = 1,2$ which describe individual $W^\pm$-exchange,

$$
\begin{aligned}
F_i^{W^-}(x,Q^2) &= a_i(x) \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \\
&\quad \times \sum_{j=1}^{n_f} \left(v_j^2 + a_j^2\right) \left\{ \delta q_{\rm ns}^-(y)\, C_{i,\rm ns}^-(z) + q_{\rm s}(y)\, C_{i,\rm q}(z) + g(y)\, C_{i,\rm g}(z) \right\}, \qquad (37) \\
F_3^{W^-}(x,Q^2) &= \int_0^1 dz \int_0^1 dy\, \delta(x-yz) \sum_{i=1}^{n_f} 2\, v_i\, a_i \left\{ \delta q_{\rm ns}^-(y)\, C_{3,\rm ns}^-(z) + q_{\rm ns}^{\rm v}(y)\, C_{3,\rm ns}^{\rm v}(z) \right\}. \qquad (38)
\end{aligned}
$$

The respective results for $F_i^{W^+}$ are obtained from Eqs. (37), (38) with the simple replacement $\delta q_{\rm ns}^- \to -\delta q_{\rm ns}^-$.

Please recall, that the functions $C_{i,\rm ns}^+$ and $C_{i,\rm q}$ start to differ only at two-loop order; up to NLO there is no difference (cf. the simple replacement rules in Ref. [1]). Recall also, that $C_{3,\rm ns}^{\rm v} = C_{i,\rm ns}^-$ up to two-loop order.

This implies, that the iso-triplet component of the proton $\delta q_{\rm ns}^-$ enters in a non-trivial way for the first time at NNLO. It numerical impact is expected to be small though.

Coupling constants

The coupling constants are given by

$$
v_i = a_i = \frac{1}{\sqrt{2}}\,. \qquad (39)
$$

Electromagnetic γ-exchange

This interaction gives contributions to the F_i^γ structure function for $i = 1,2$ only, because the γ-exchange is not a CP-violating interaction.

Just as a reminder we recall that the structure function F_2^γ at leading order takes the following form

$$F_2^\gamma(x) = x\sum_{i=1}^{n_f} e_i^2\Big(q_i+\bar{q}_i\Big), \tag{40}$$

where n_f is the number of active flavors and the electromagnetic charges are $e_i = 2/3$ for a u-type quark and $e_i = -1/3$ for a d-type quark.

At higher orders we have the following structure, e.g. for F_2^γ, see e.g. Ref. [5],

$$x^{-1}F_2^\gamma = C_{2,\mathrm{ns}}^+ \otimes q_{\mathrm{ns}}^\gamma + \langle e^2\rangle\Big(C_{2,\mathrm{q}}\otimes q_{\mathrm{s}} + C_{2,\mathrm{g}}\otimes g\Big)\,, \tag{41}$$

where $\otimes$ denotes the Mellin convolution and

$$\langle e^2\rangle = \frac{1}{n_f}\sum_{i=1}^{n_f} e_i^2\,. \tag{42}$$

Finally according to the notation in Ref. [5],

$$C_{i,\mathrm{q}} = C_{i,\mathrm{ns}} + C_{i,\mathrm{ps}}\,. \tag{43}$$

Recall that for an even number of n_f, we have $\langle e^2\rangle = 5/18$. According to Eqs. (40), (41) we can easily check that the quark combination q_{ns}^γ is

$$q_{\mathrm{ns}}^\gamma = \Delta(e^2)\left\{\sum_{i=u-\mathrm{type}}(q_i+\bar{q}_i) - \sum_{i=d-\mathrm{type}}(q_i+\bar{q}_i)\right\}, \tag{44}$$

where $\Delta(e^2) = 1/2(e_u^2 - e_d^2) = 1/6$. Note that q_{ns}^γ evolves like a q_{ns}^+ quark combination, see also [7]. Recall $F_3^\gamma = 0$.

Up to two-loop order we can recover the cases for neutral- and charged current interactions discussed above with the following set of substitutions [3] in Eq. (41). E.g. for F_2^Z we have, cf. Eq. (8),

$$x^{-1}F_2^Z = x^{-1}F_2^\gamma \begin{cases} e_u^2 \to \left(\frac{1}{2} - \frac{4}{3}\sin^2(\theta_W)\right)^2 \\[2ex] e_d^2 \to \left(\frac{1}{2} - \frac{2}{3}\sin^2(\theta_W)\right)^2 \end{cases}, \tag{45}$$

and the PDFs remain unchanged.

Likewise, e.g. for $F_2^{W^-}$ we have, cf. Eq. (37)

$$x^{-1}F_2^{W^-} = x^{-1}F_2^\gamma \begin{cases} \langle e^2\rangle \to 1 \\[2ex] q_{\mathrm{ns}}^\gamma \to \delta q_{\mathrm{ns}}^- = [(u-\bar{u})-(d-\bar{d})]+[(c-\bar{c})-(s-\bar{s})]+\ldots \end{cases}, \tag{46}$$

where substitution of the nonsinglet PDF accounts for the proper quark-flavor dependence.

[3]The pure-singlet and the gluon coefficient functions of Ref. [5] need to be divided by a factor n_f due to the conventions of Eq. 42.

References

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