Predictive Monte Carlo tools for the LHC

Fabio Maltoni

Centre for Cosmology, Particle Physics and Phenomenology (CP3), BelgiuM

Lecture III

PLAN

- Basics : LO predictions and event generation
- Fixed-order calculations : from NLO to NNLO
- Exclusive predictions : Parton Shower
- Merging ME+PS
- Matching NLO with PS

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Today

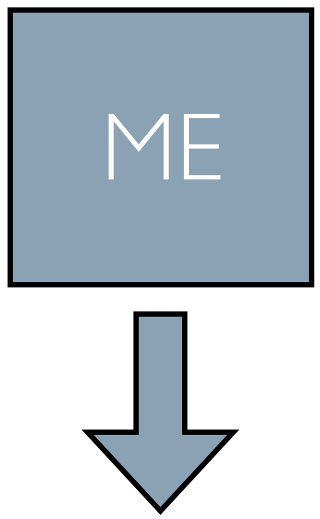
Predictive MC's

- There are better ways to describe hard radiation: matrix elements!
- There are two ways to improve a Parton Shower Monte Carlo event generator with matrix elements:
  - ME+PS merging: Include matrix elements with more final state partons to describe hard, well-separated radiation better
  - NLO+PS matching: Include full NLO corrections to the matrix elements to reduce theoretical uncertainties in the matrix elements. The real-emission matrix elements will describe the hard radiation

Merging ME+PS

Matrix elements vs. Parton showers

Matrix elements vs. Parton showers



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2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

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Difficulty: avoid double counting, ensure smooth distributions

Goal for ME/PS merging

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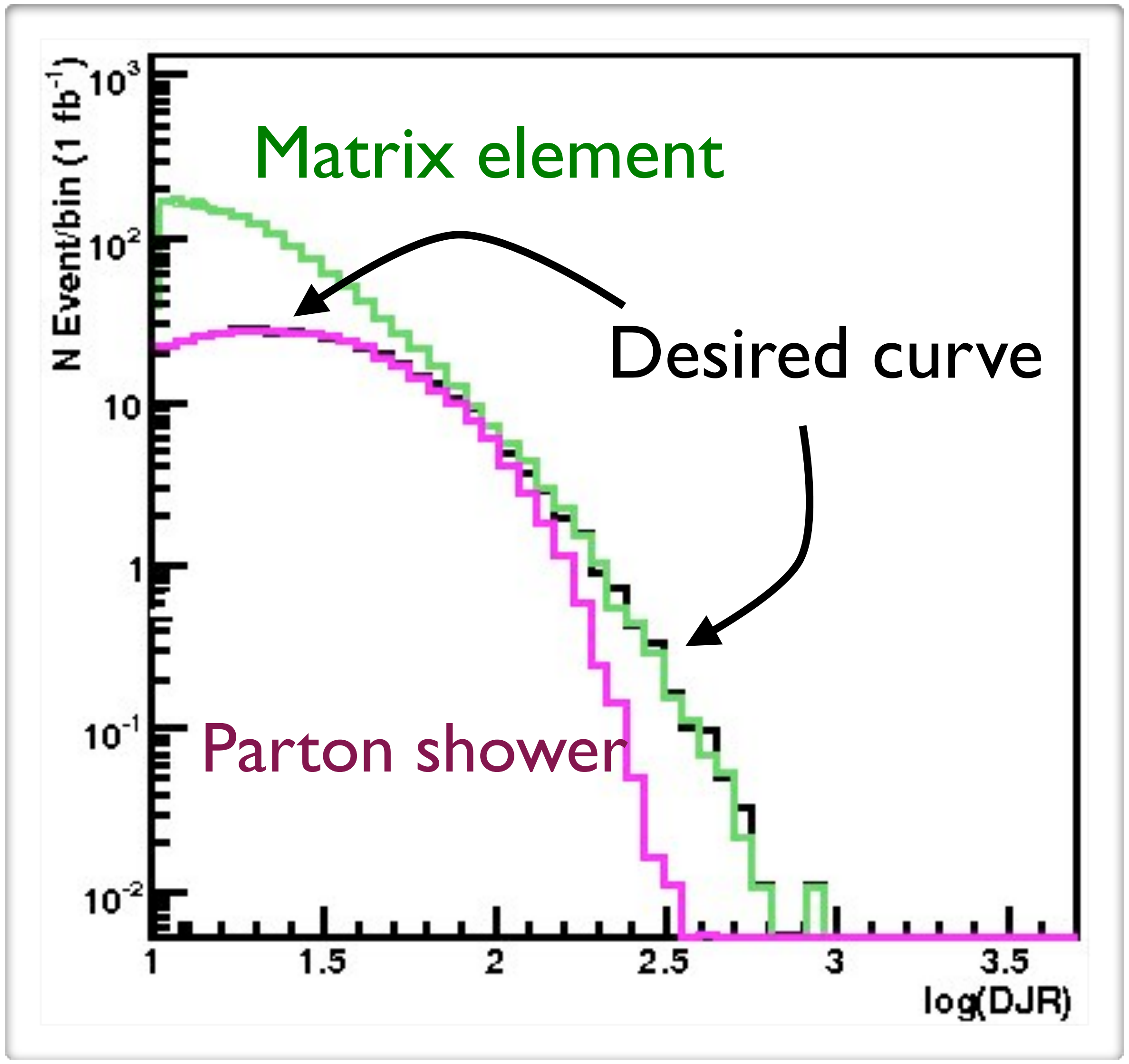
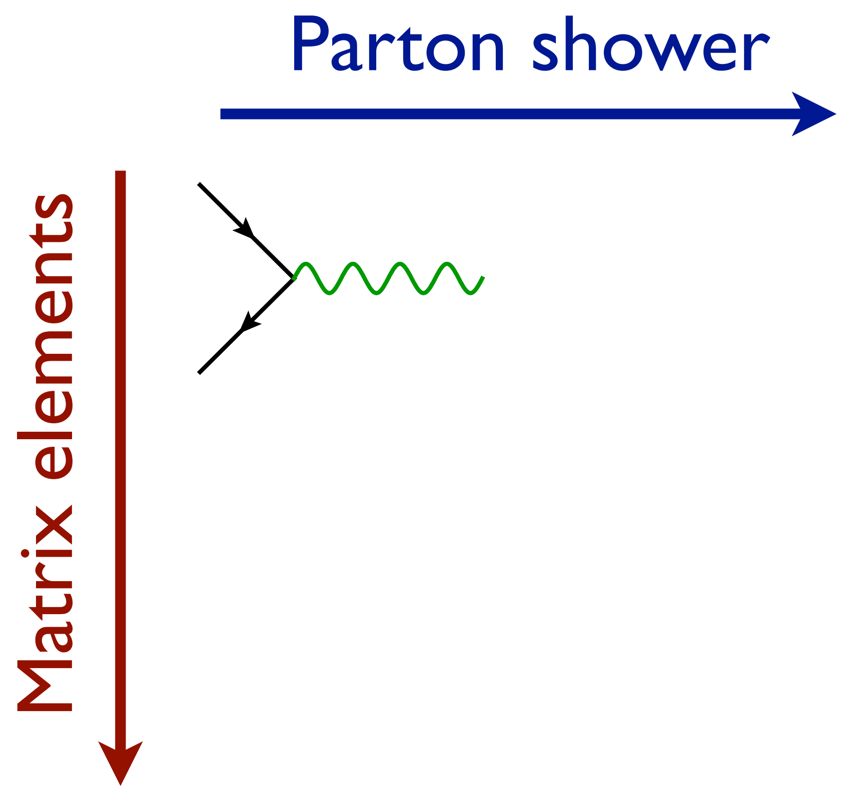


2nd QCD radiation jet in top pair production at the LHC

Possible double counting



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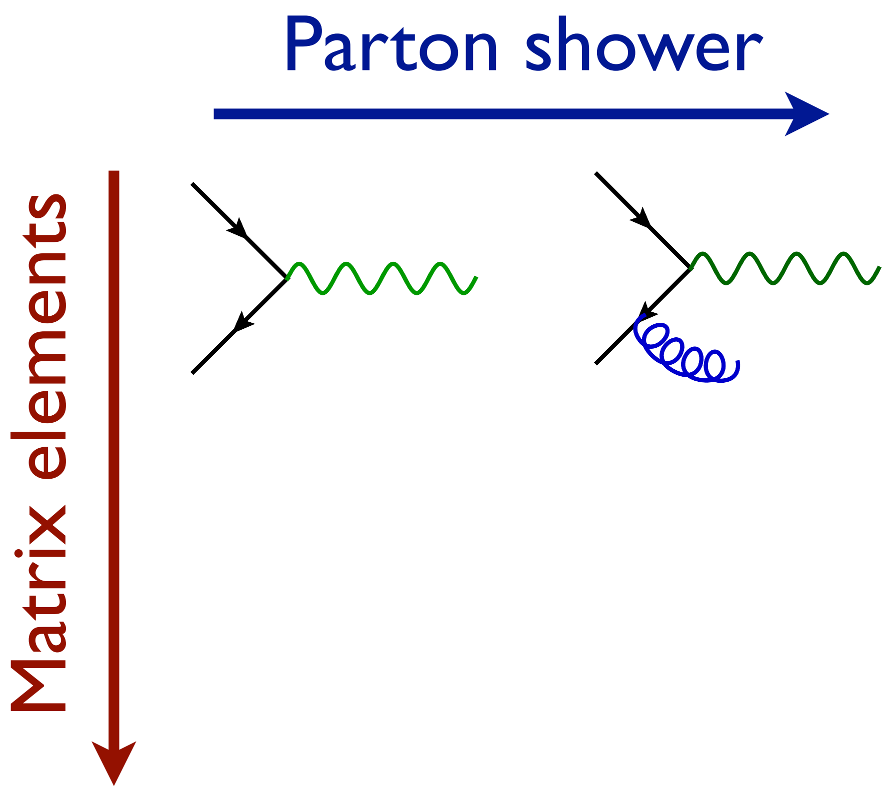

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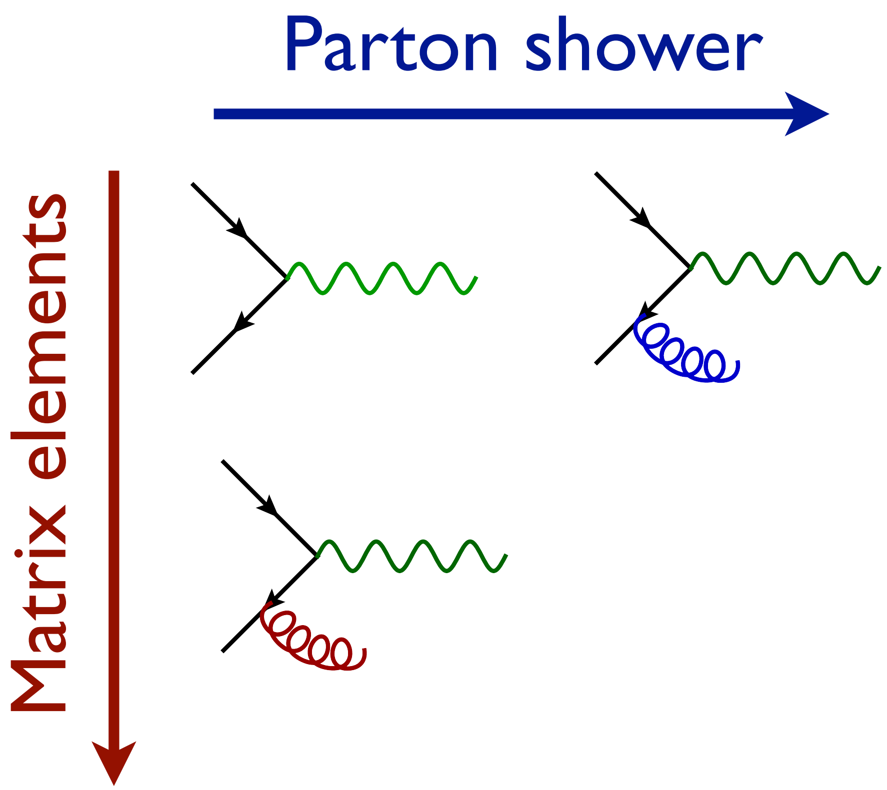

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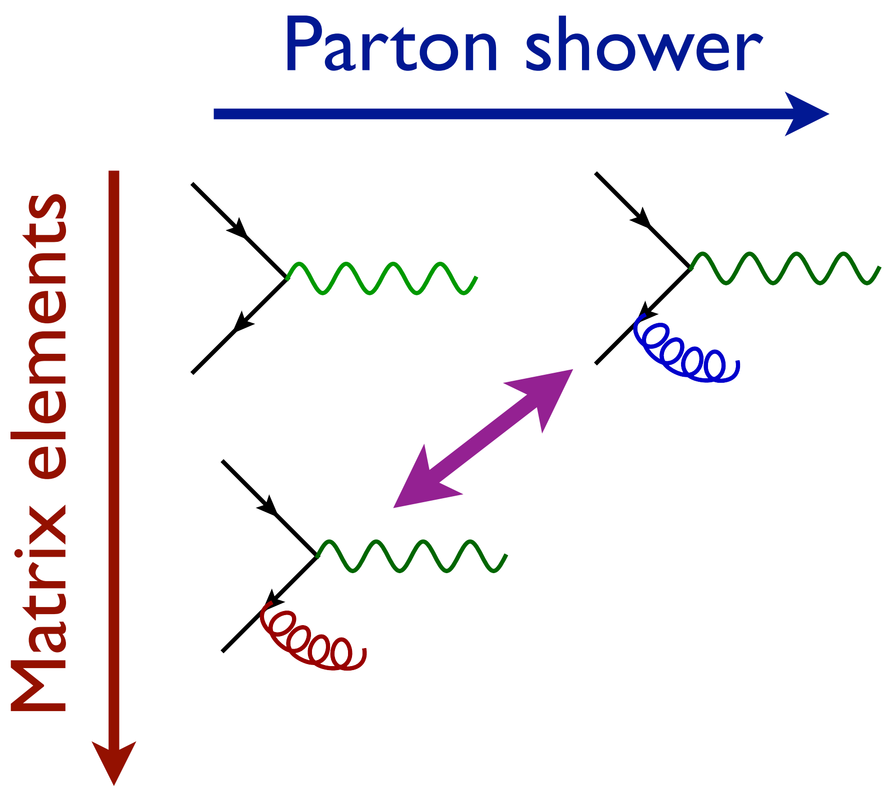

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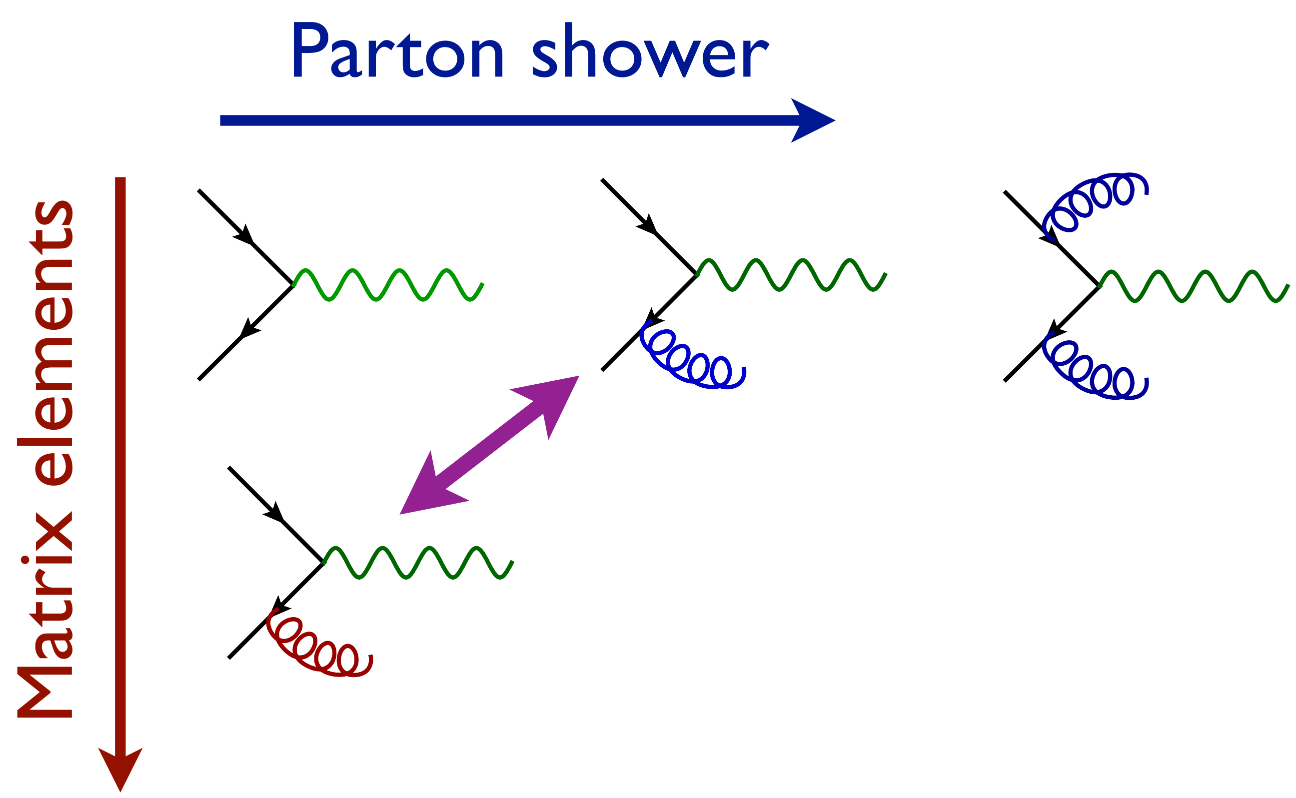

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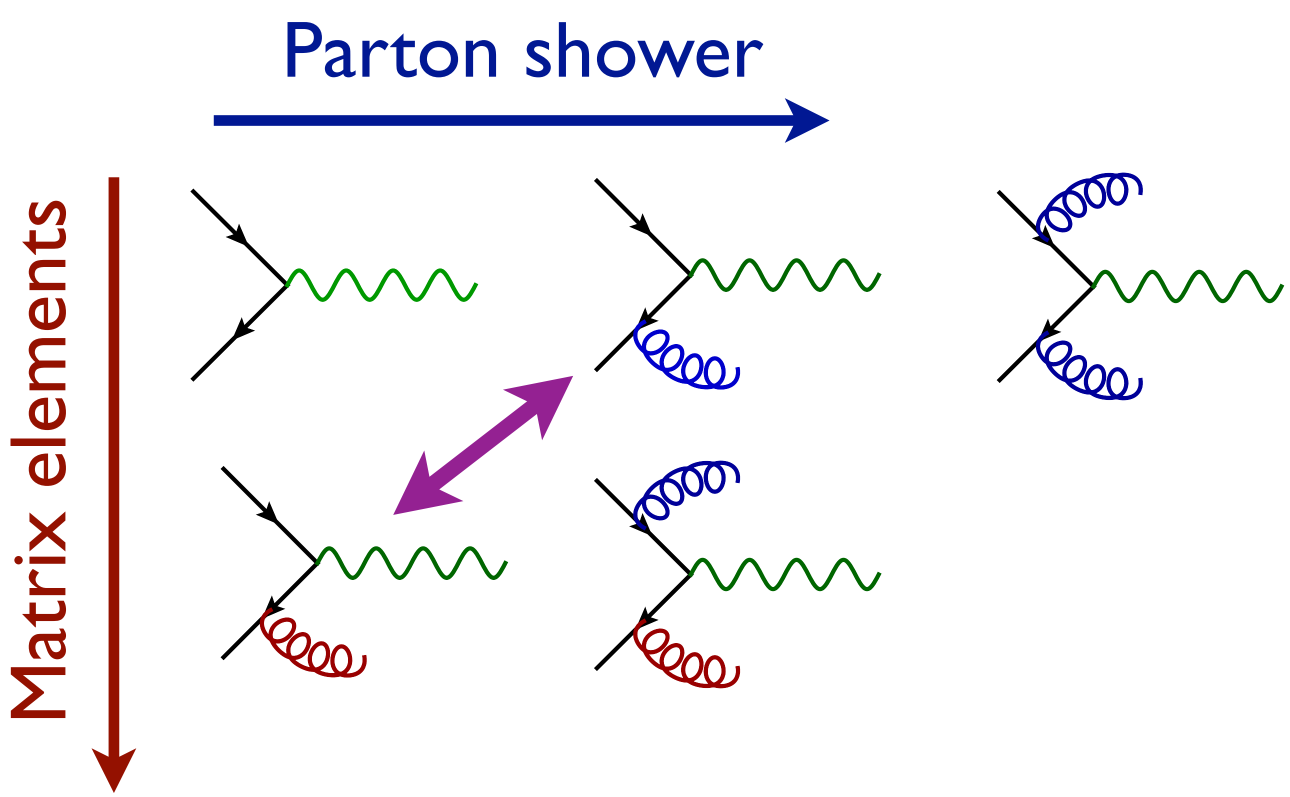

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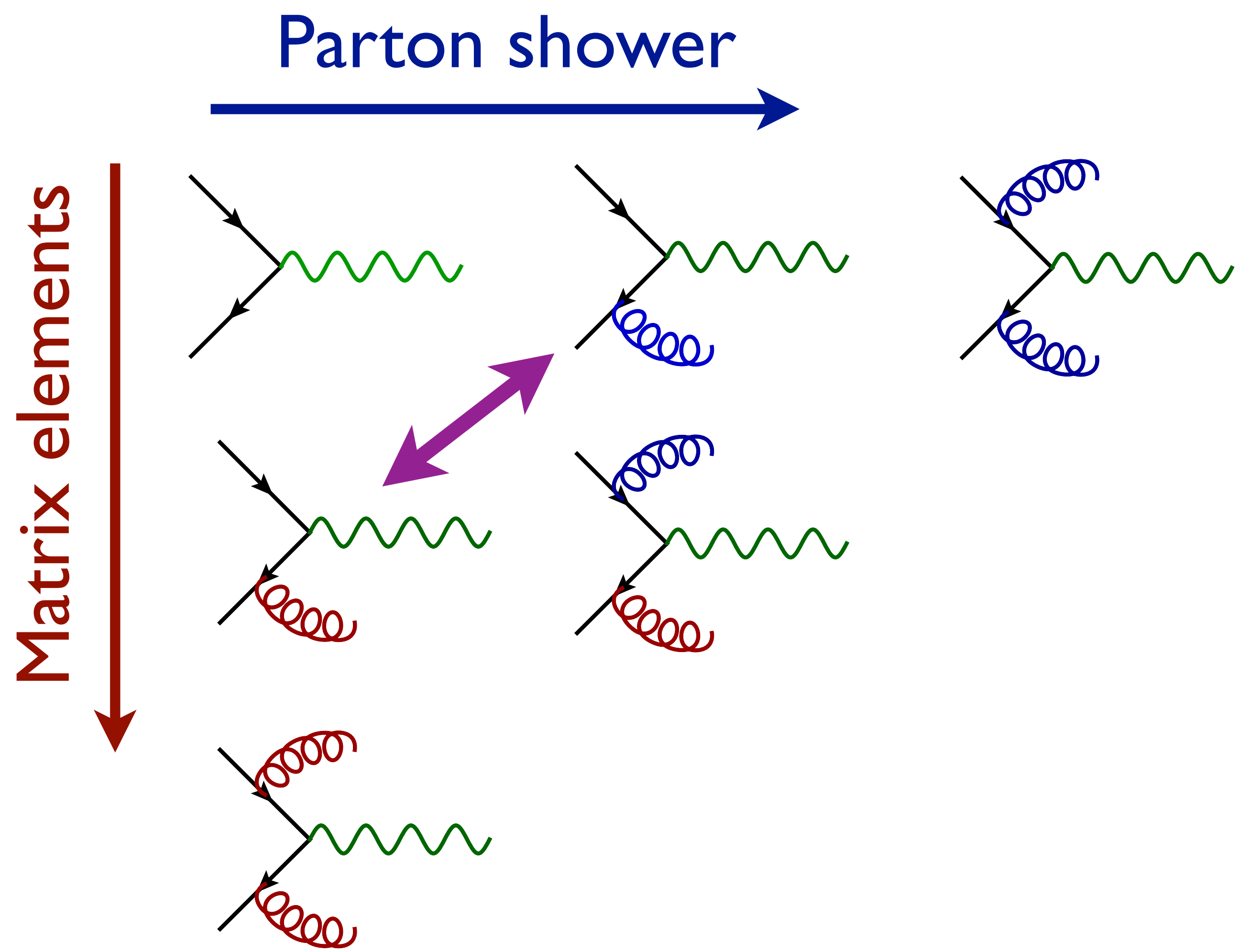

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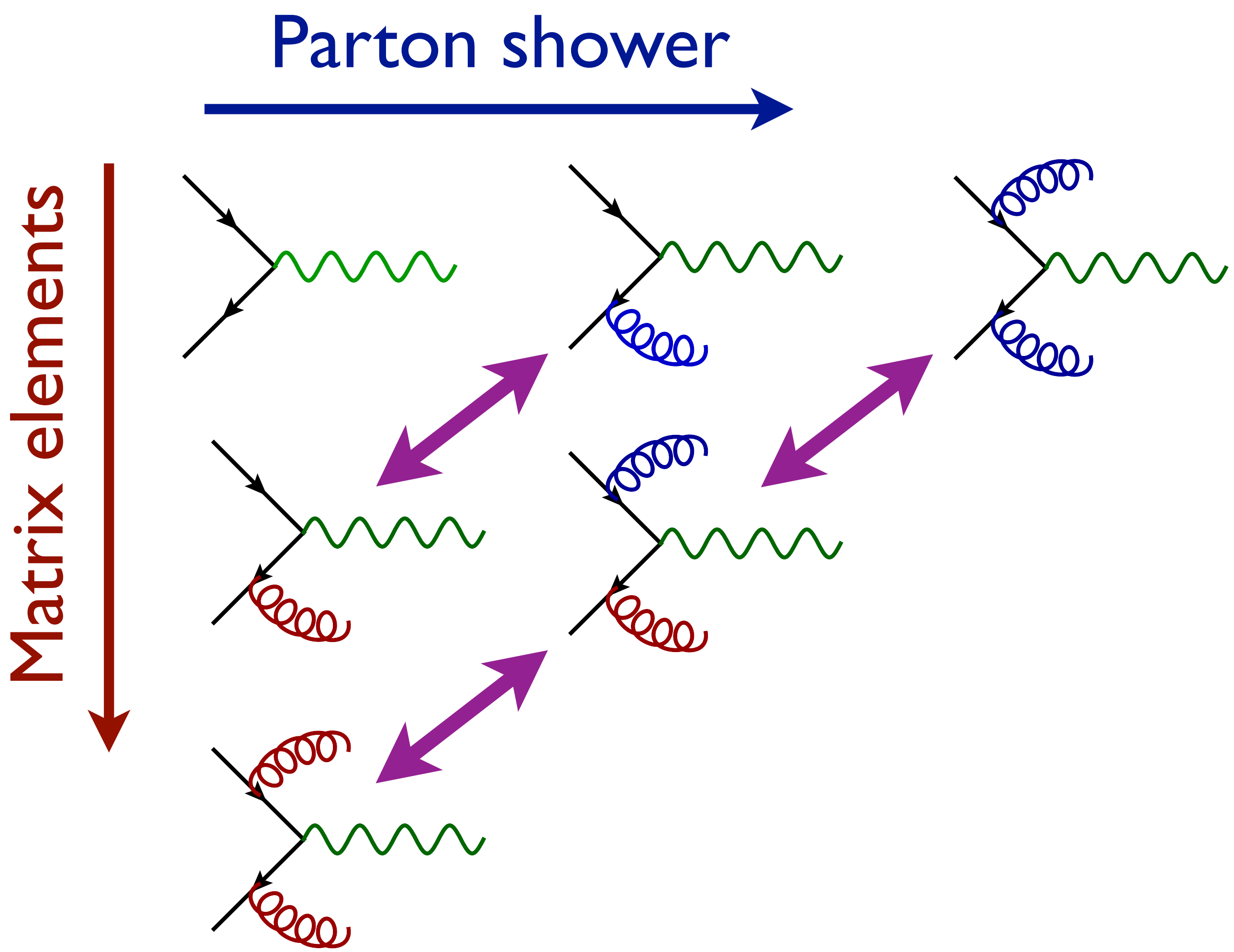

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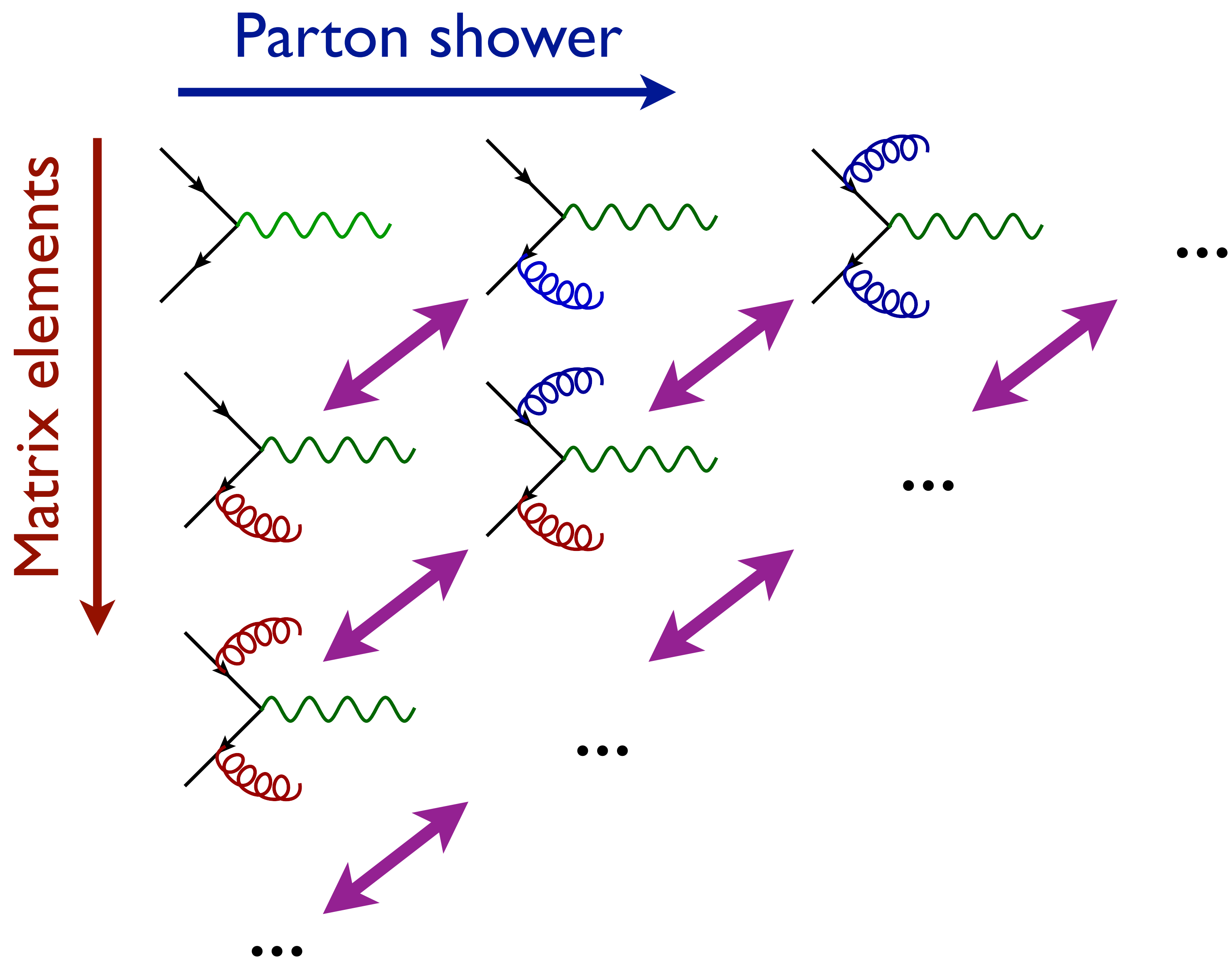

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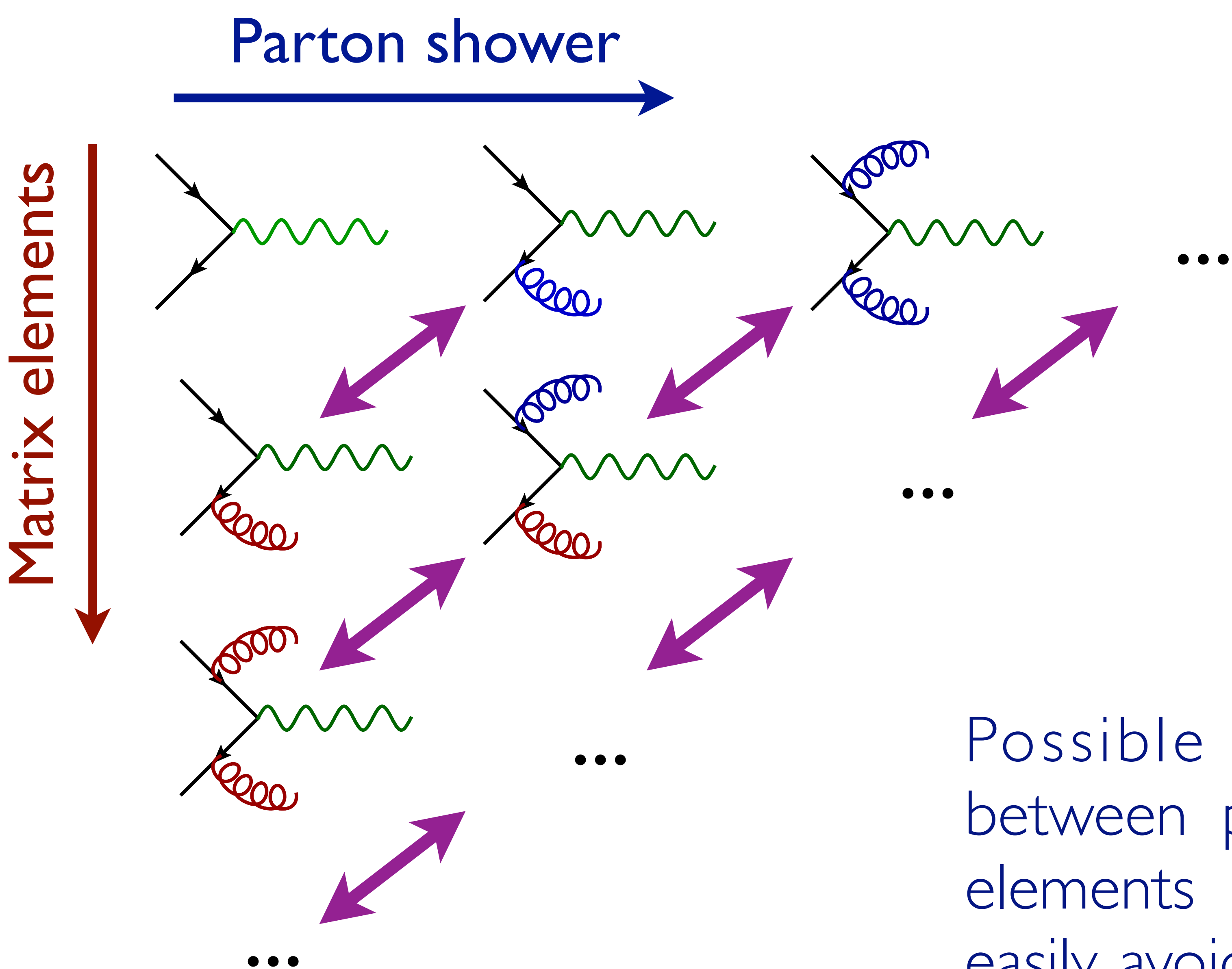

Possible double counting between partons from matrix elements and parton shower easily avoided by applying a cut in phase space

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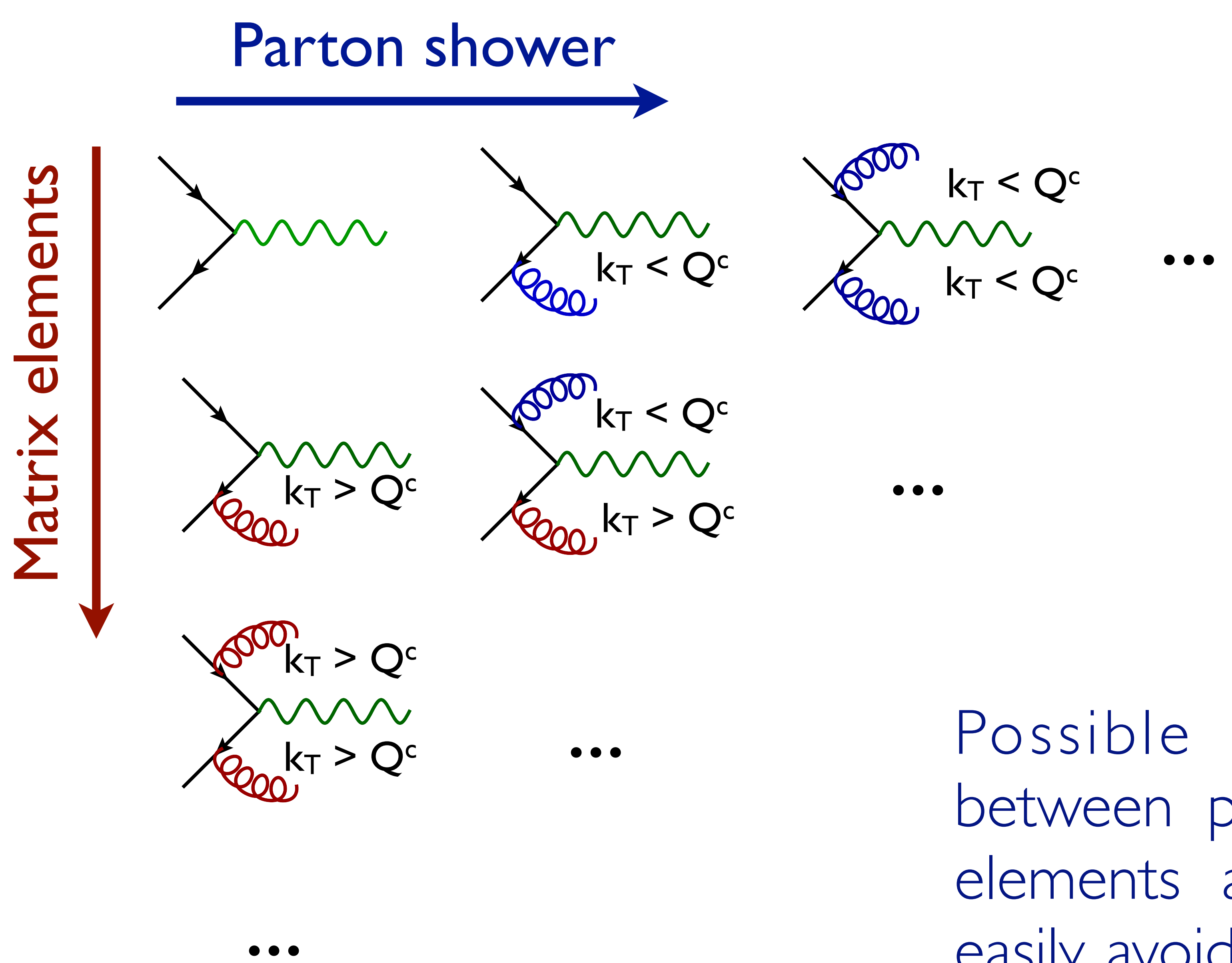
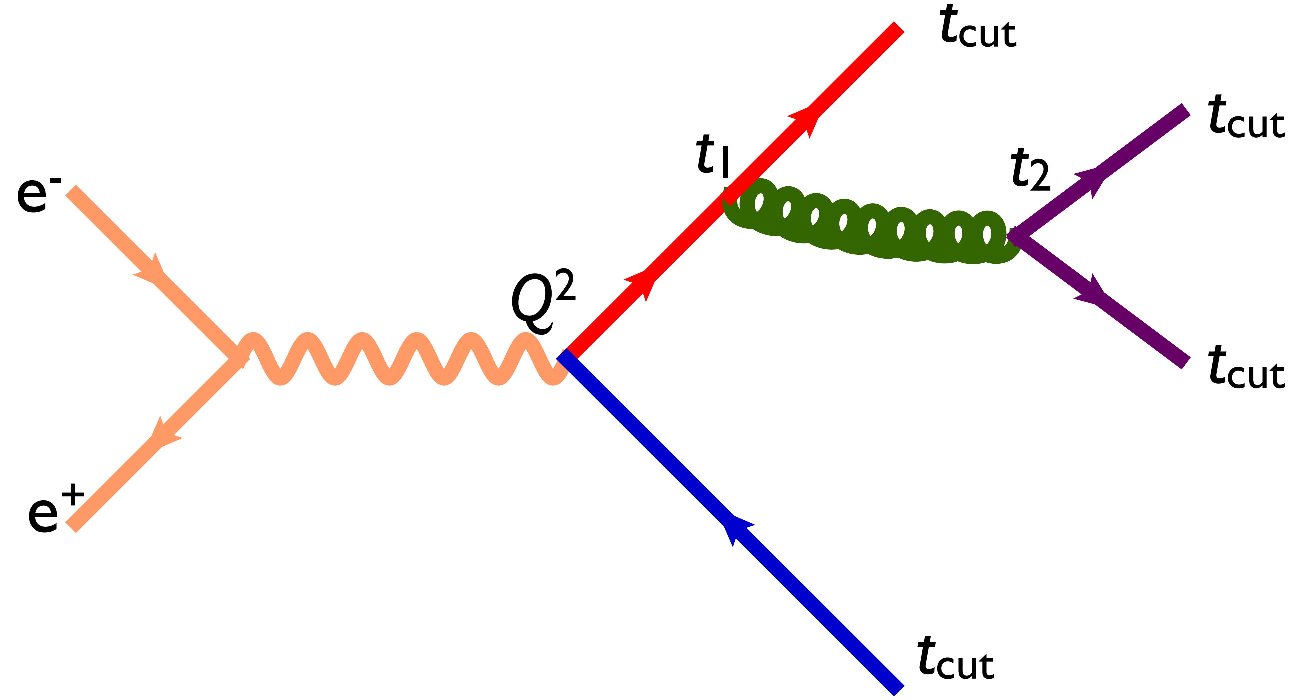


Possible double counting between partons from matrix elements and parton shower easily avoided by applying a cut in phase space

Merging ME with PS

- So double counting no problem, but what about getting smooth distributions that are independent of the precise value of Q^c?
- Below cutoff, distribution is given by PS
  - need to make ME look like PS near cutoff
- Let's take another look at the PS!

Merging ME with PS



- How does the PS generate the configuration above (i.e. starting from e^+e^- -> qqbar events)?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(Q^2,t_1))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree (remember Δ(A,B) = Δ(A,C) Δ(C,B))

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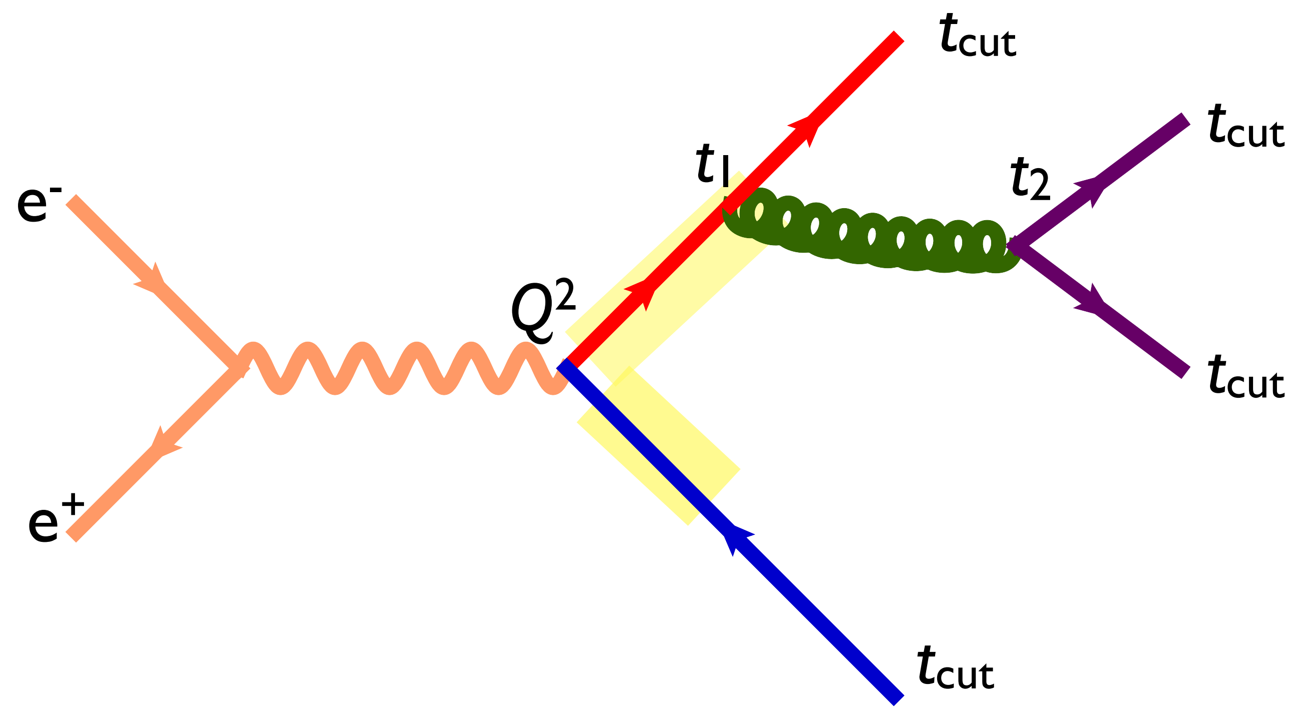

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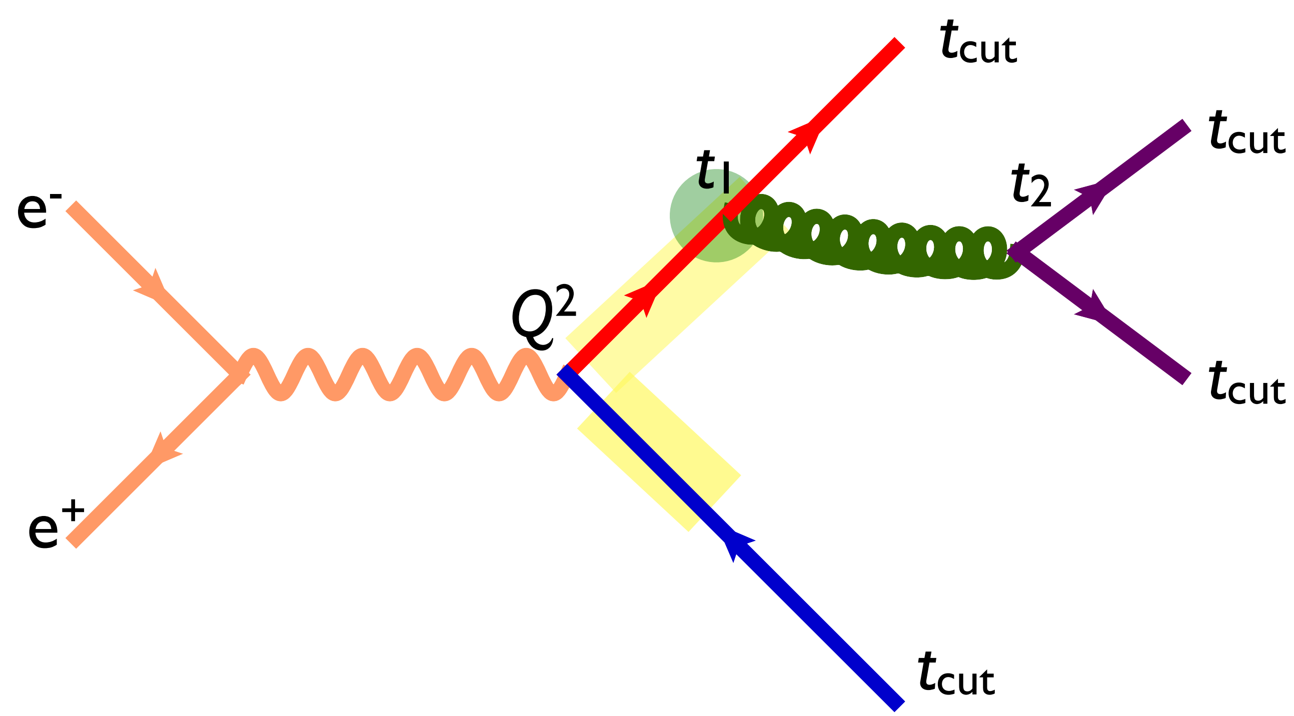

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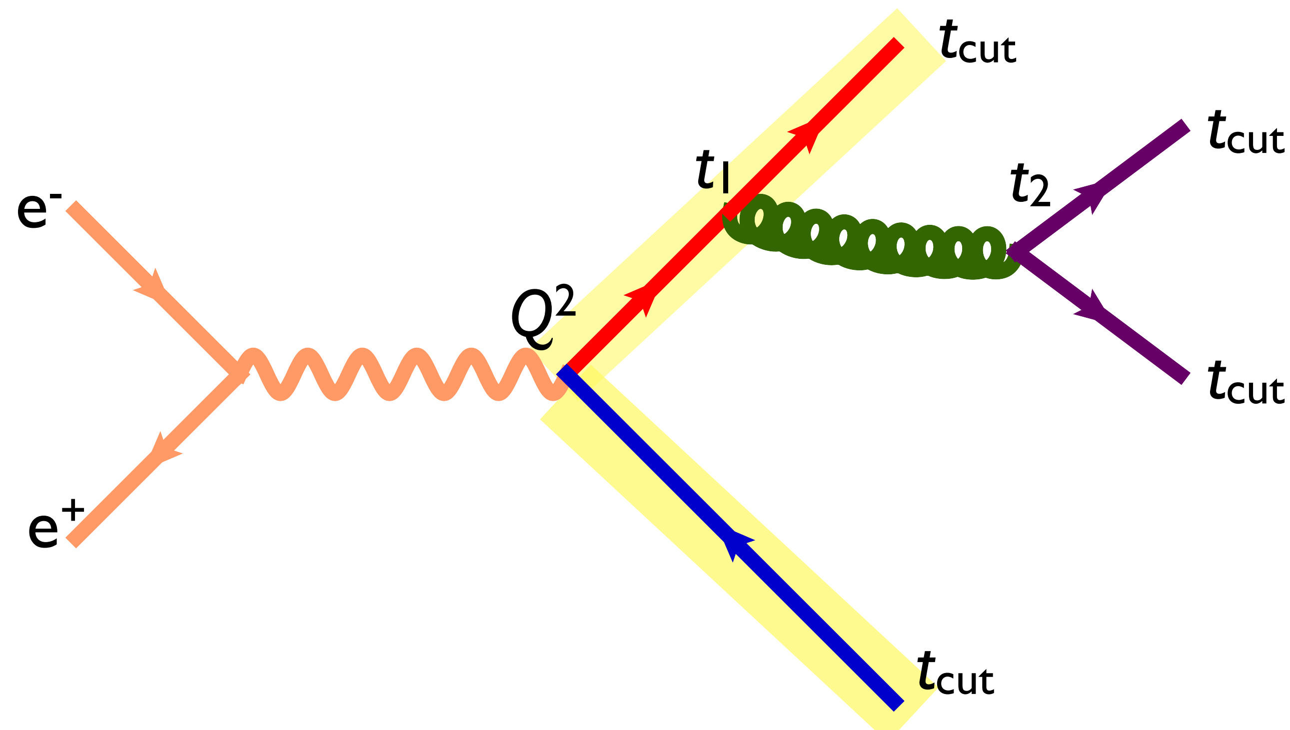

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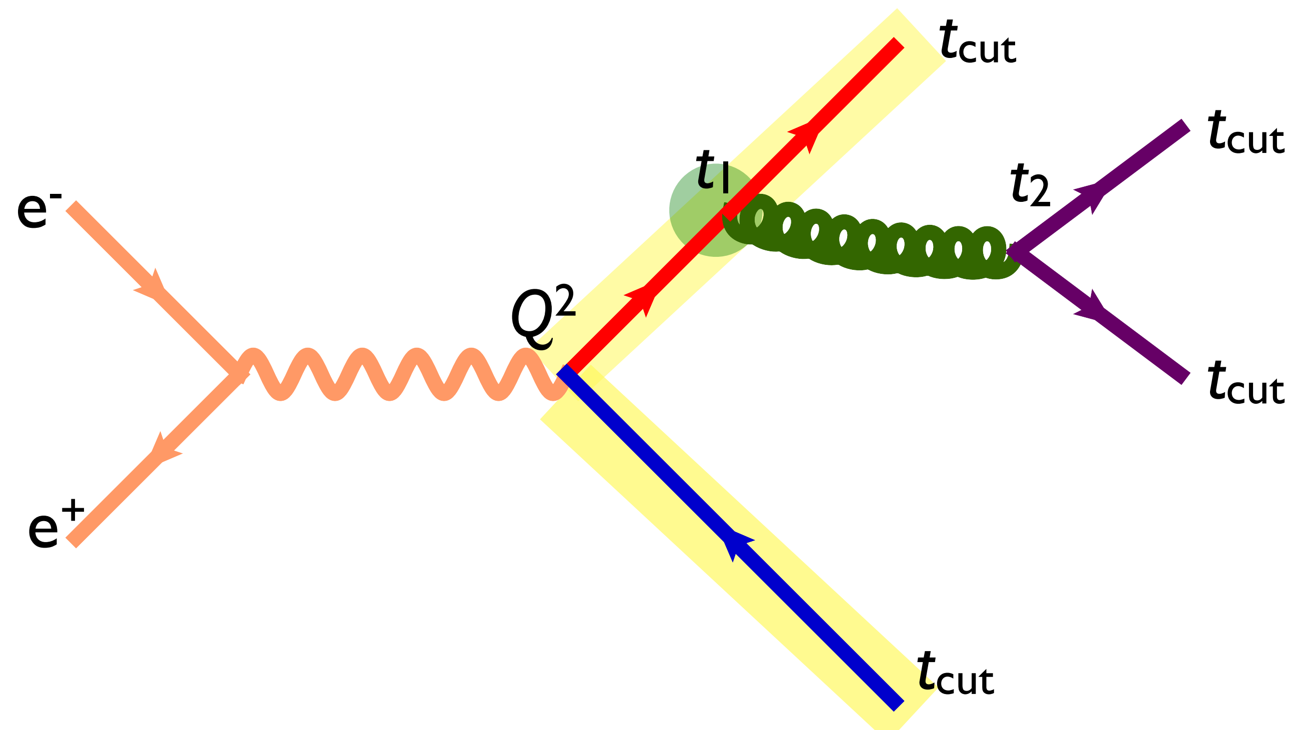

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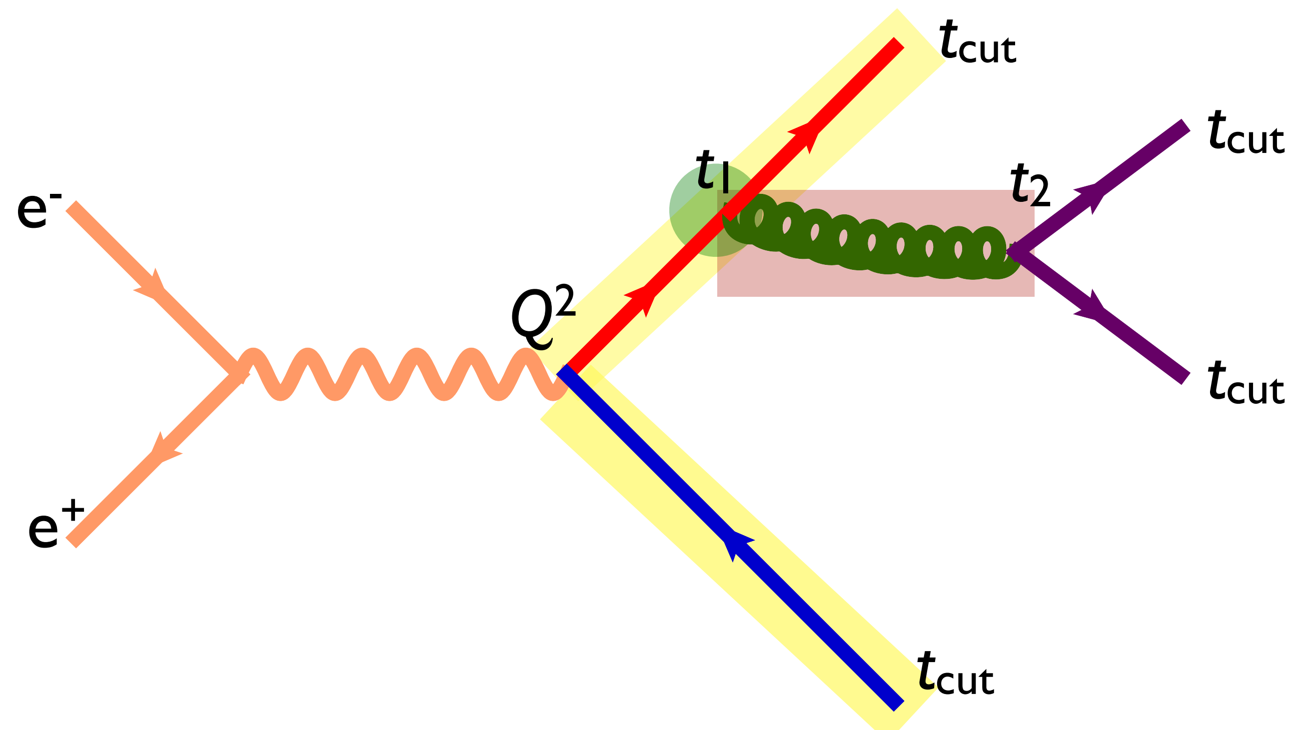

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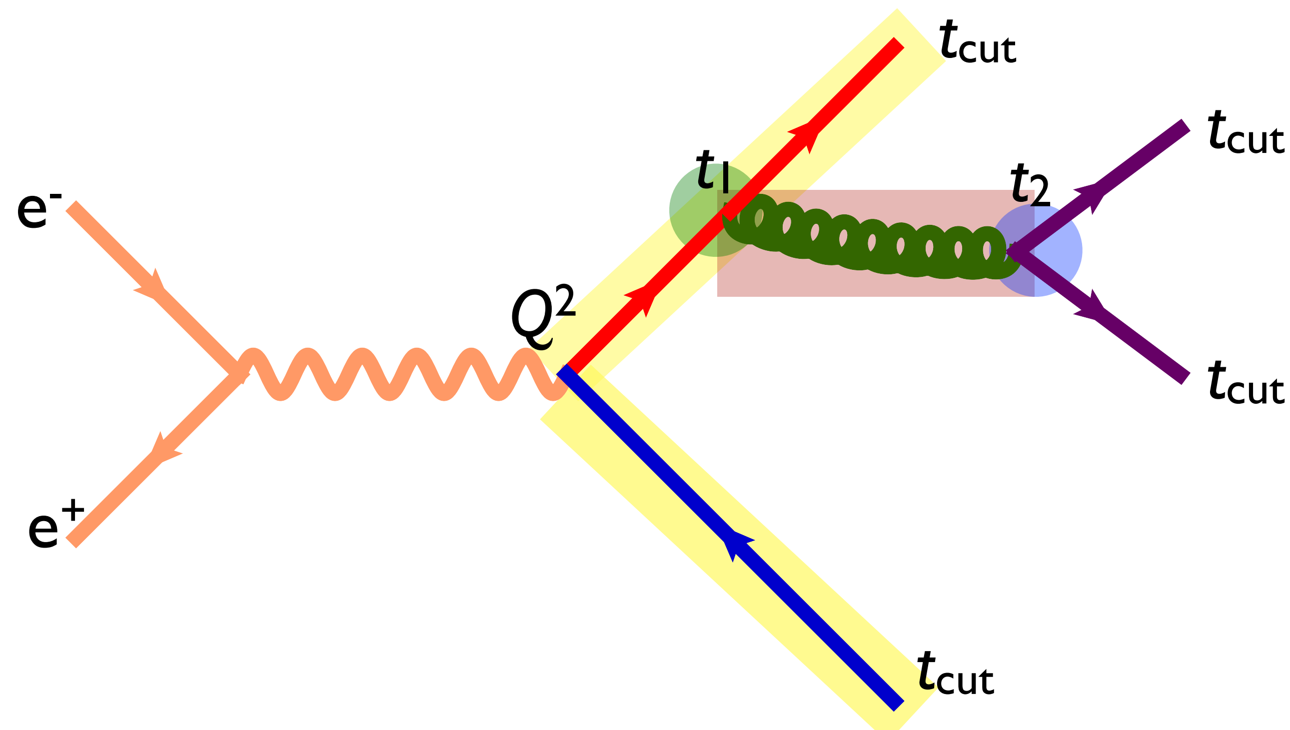

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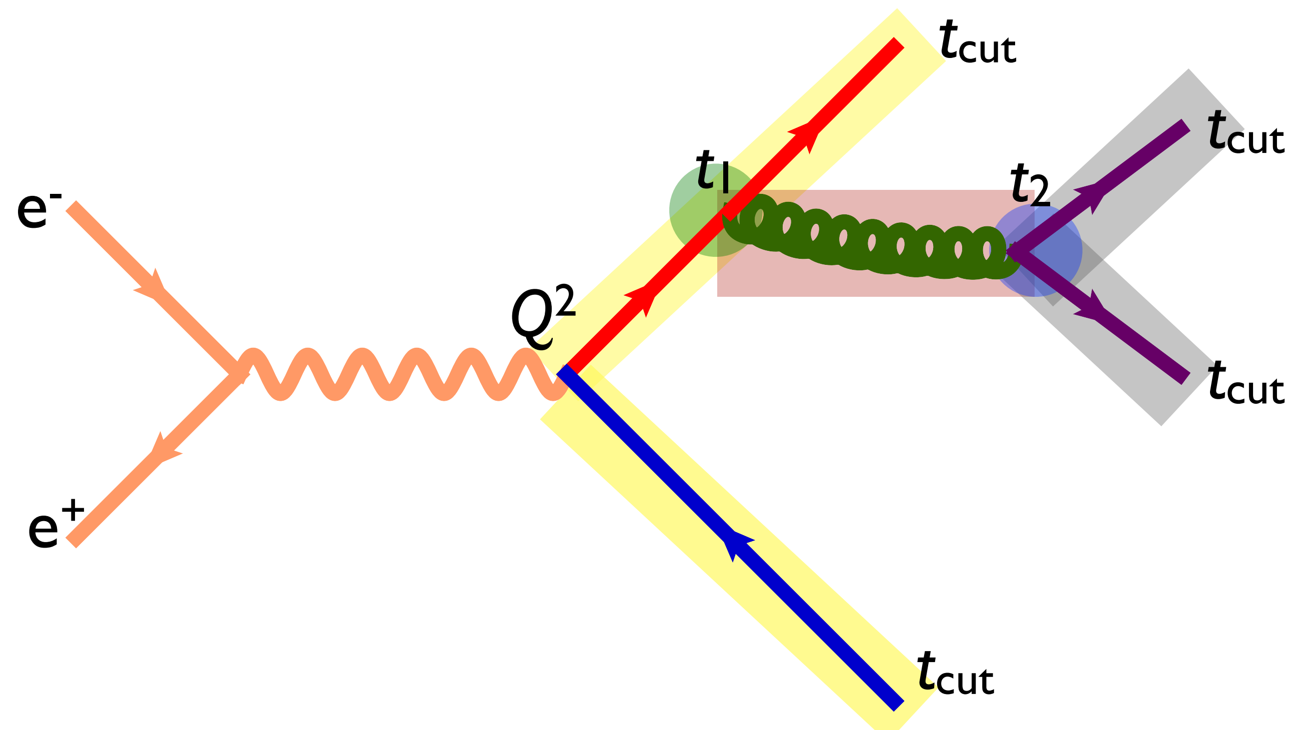

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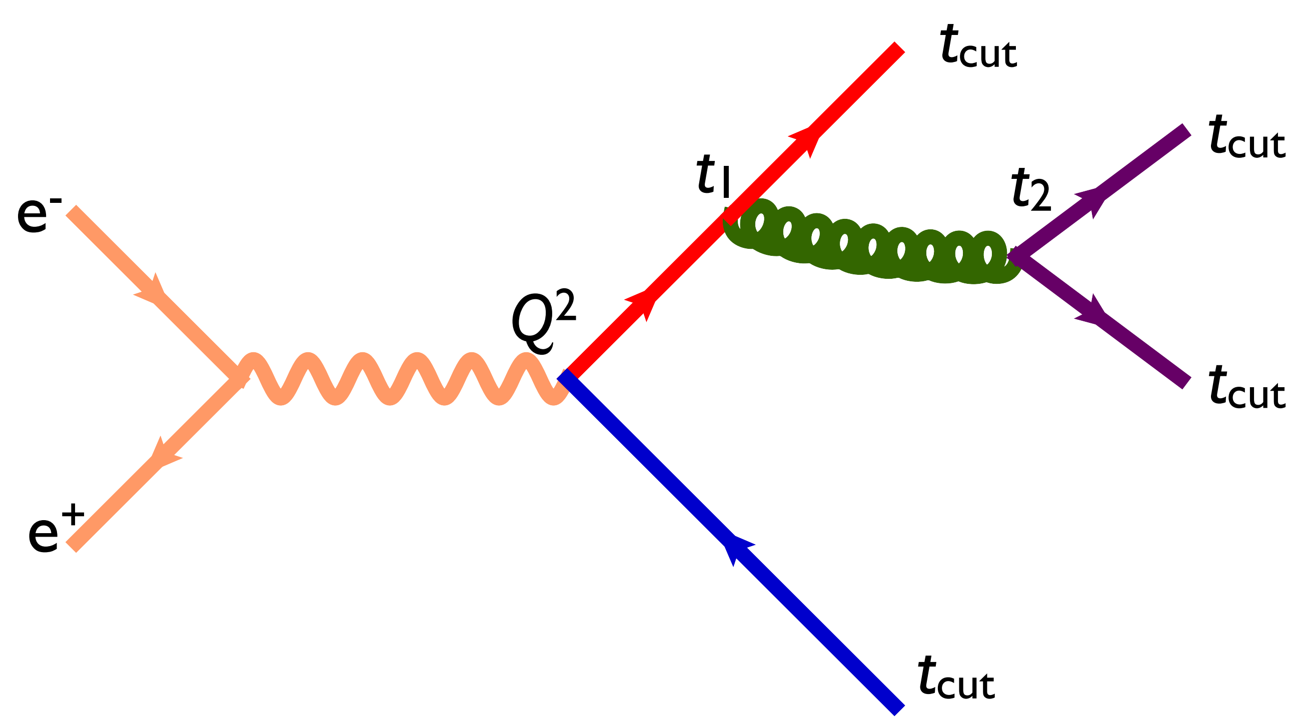

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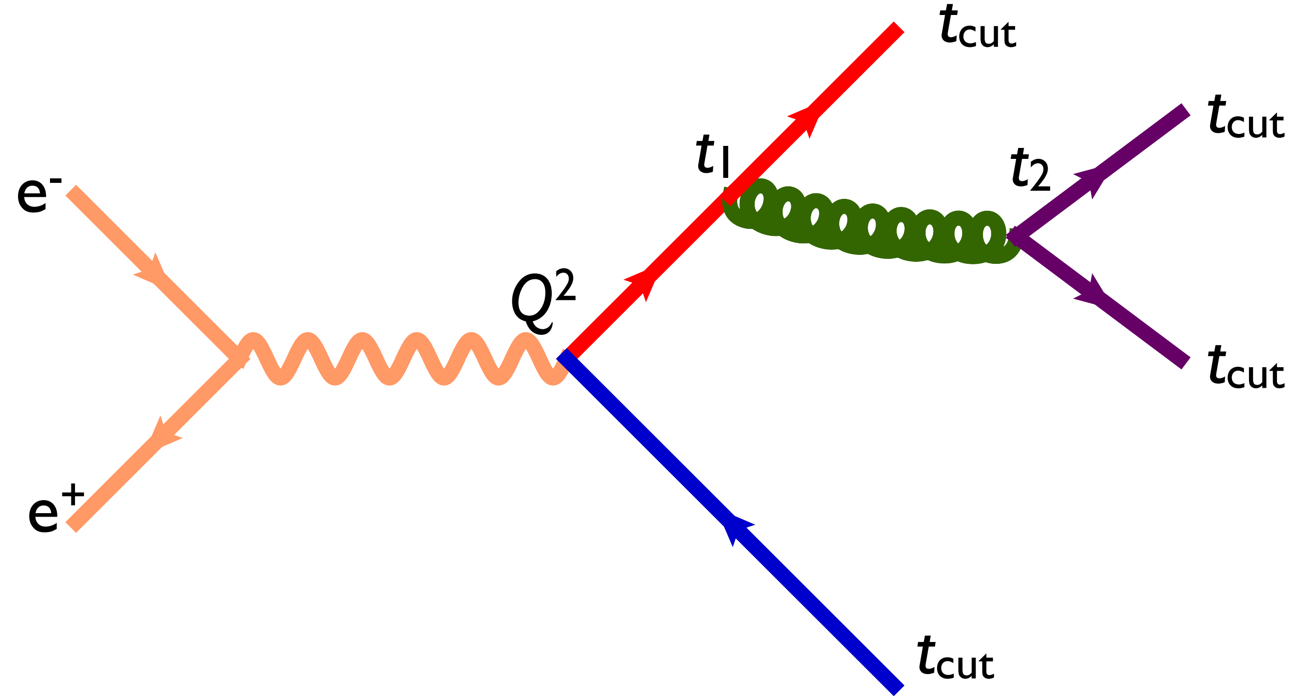

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Corresponds to the matrix element
BUT with α_s evaluated at the scale of each splitting

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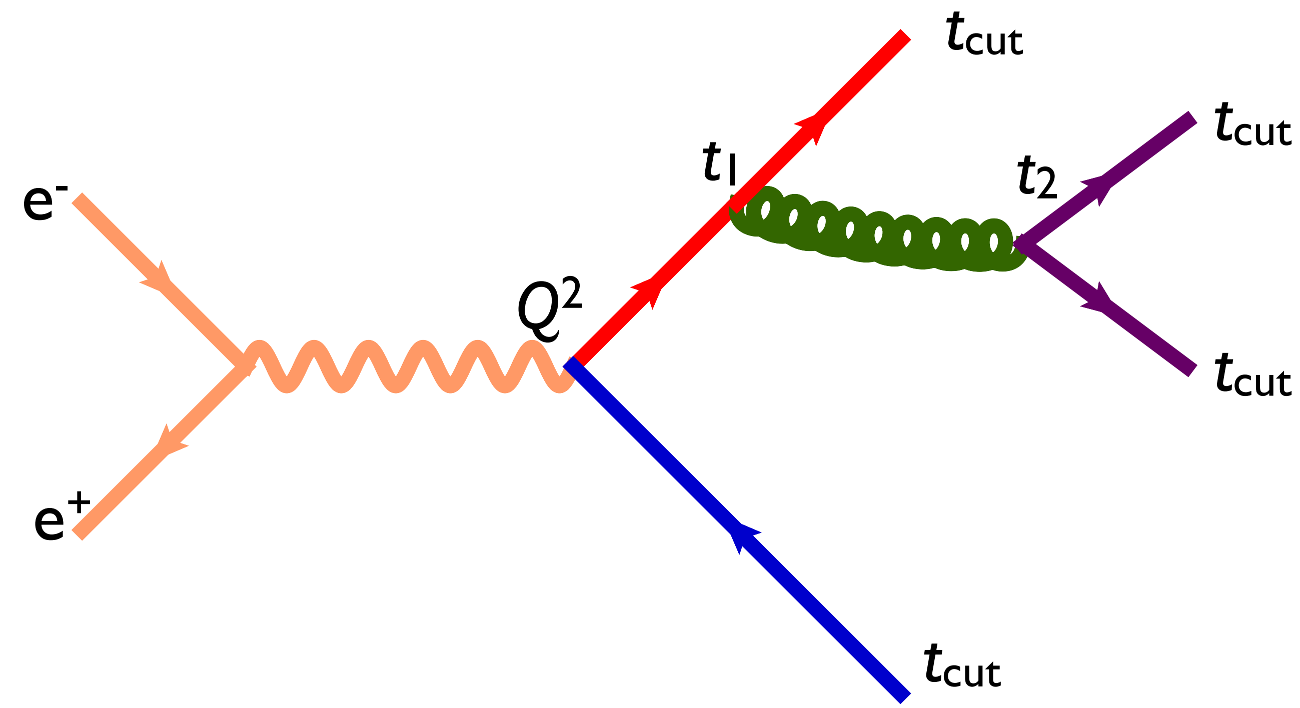

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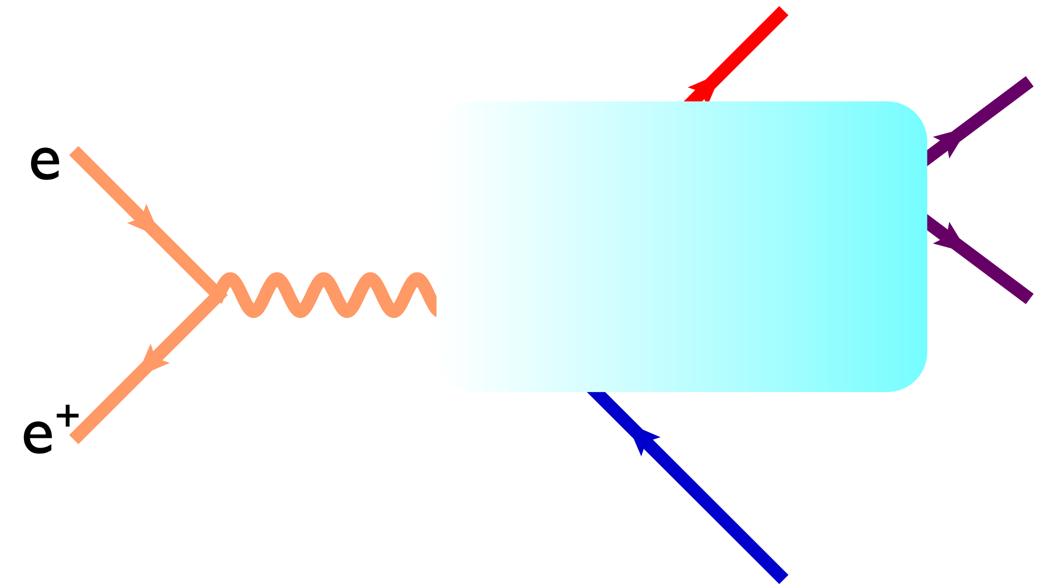
Sudakov suppression due to disallowing additional radiation above the scale t_{cut}

Merging ME with PS



To get an equivalent treatment of the corresponding matrix element, do as follows:

1. Cluster the event using some clustering algorithm
   - this gives us a corresponding "parton shower history"
2. Reweight α_s in each clustering vertex with the clustering scale

$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(Q^2)} \frac{\alpha_s(t_2)}{\alpha_s(Q^2)}$$

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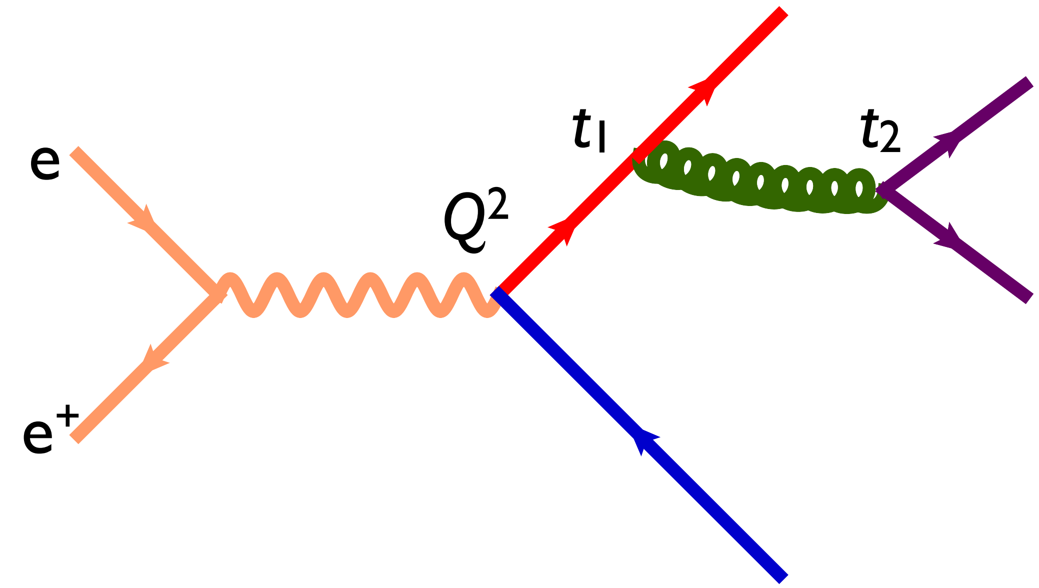

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MLM matching

[M.L. Mangano, 2002, 2006]
[J. Alwall et al 2007, 2008]

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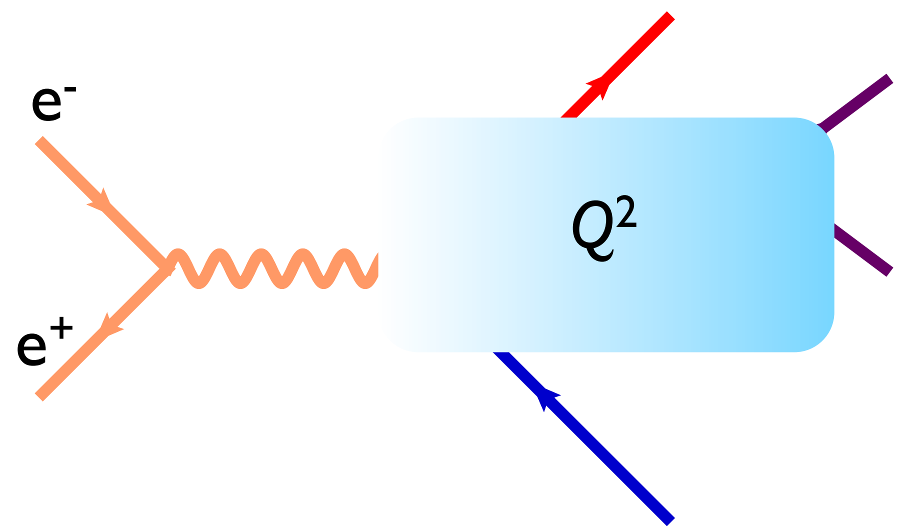

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- The suppression for this is $(\Delta_q(Q^2, t_{\rm cut}))^4$ so the internal structure of the shower history is ignored. In practice, this approximation is still pretty good
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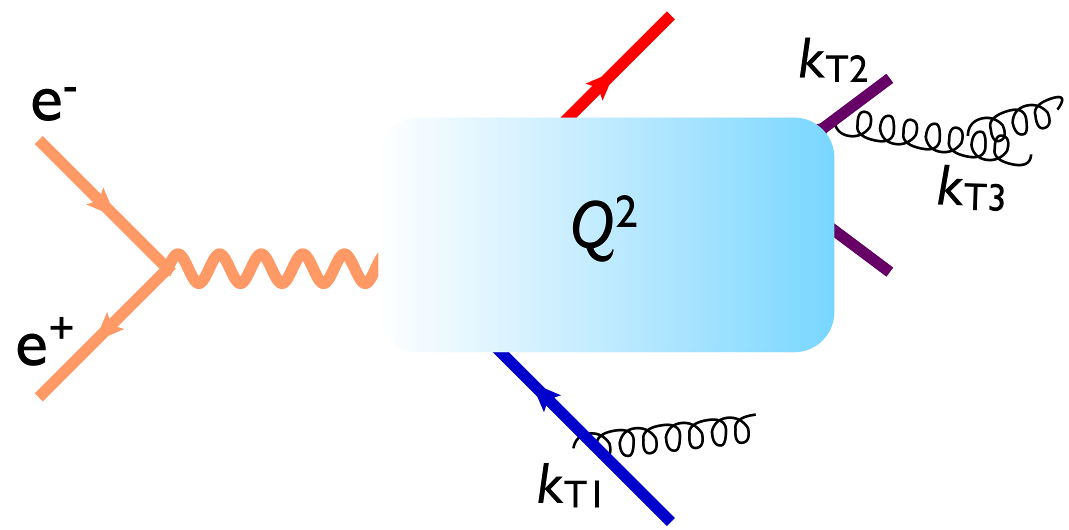

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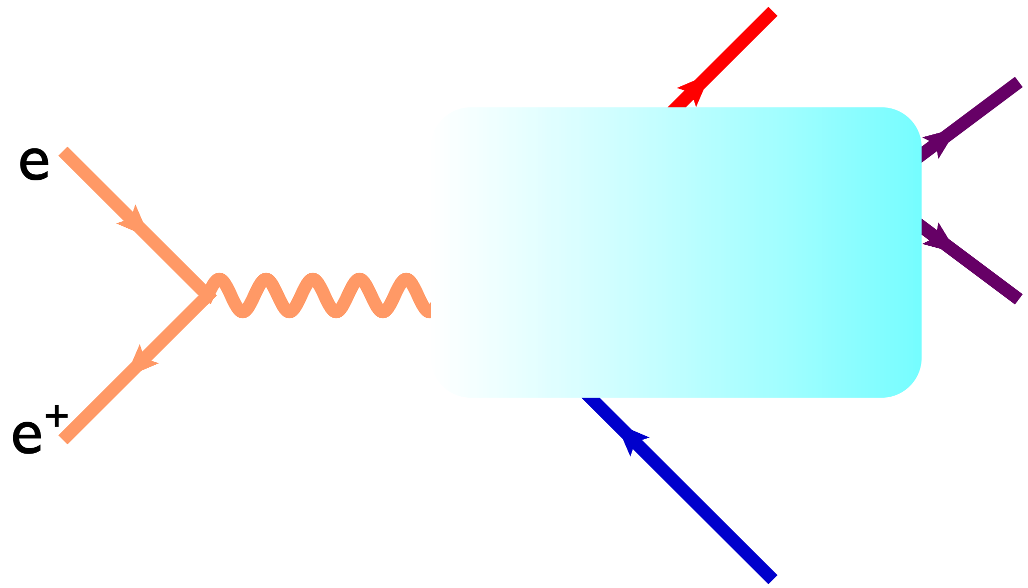
Monday 10 June 2013

CKKW MATCHING

[Catani, Krauss, Kuhn, Webber, 2001]



$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$$

- Once the 'most-likely parton shower history' has been found, one can also reweight the matrix element with the Sudakov factors that give that history

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- To do this correctly, must use same variable to cluster and define this sudakov as the one used as evolution parameter in the parton shower. Parton shower can start at $t_{\rm cut}$.

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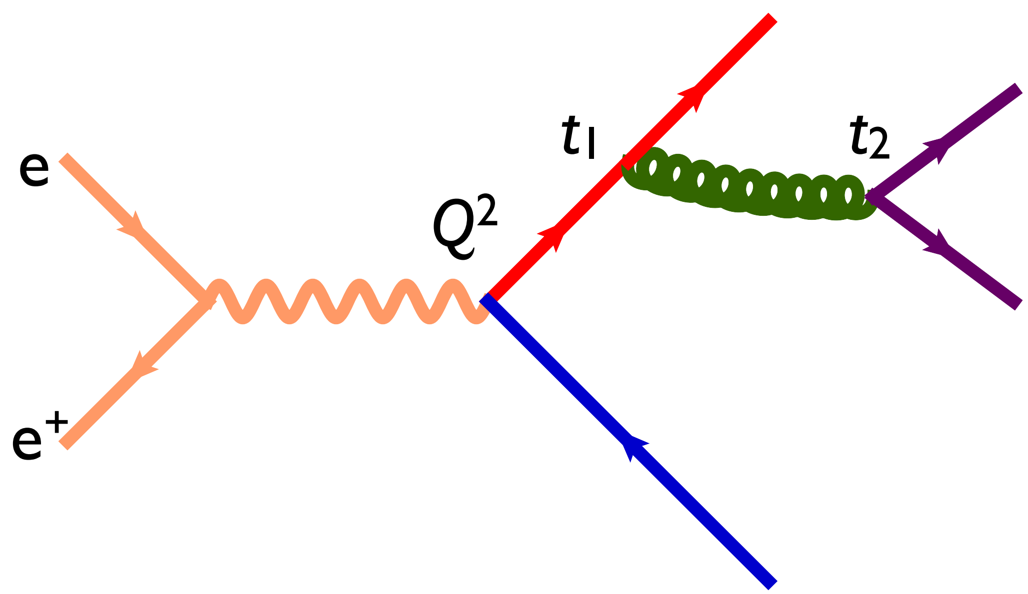

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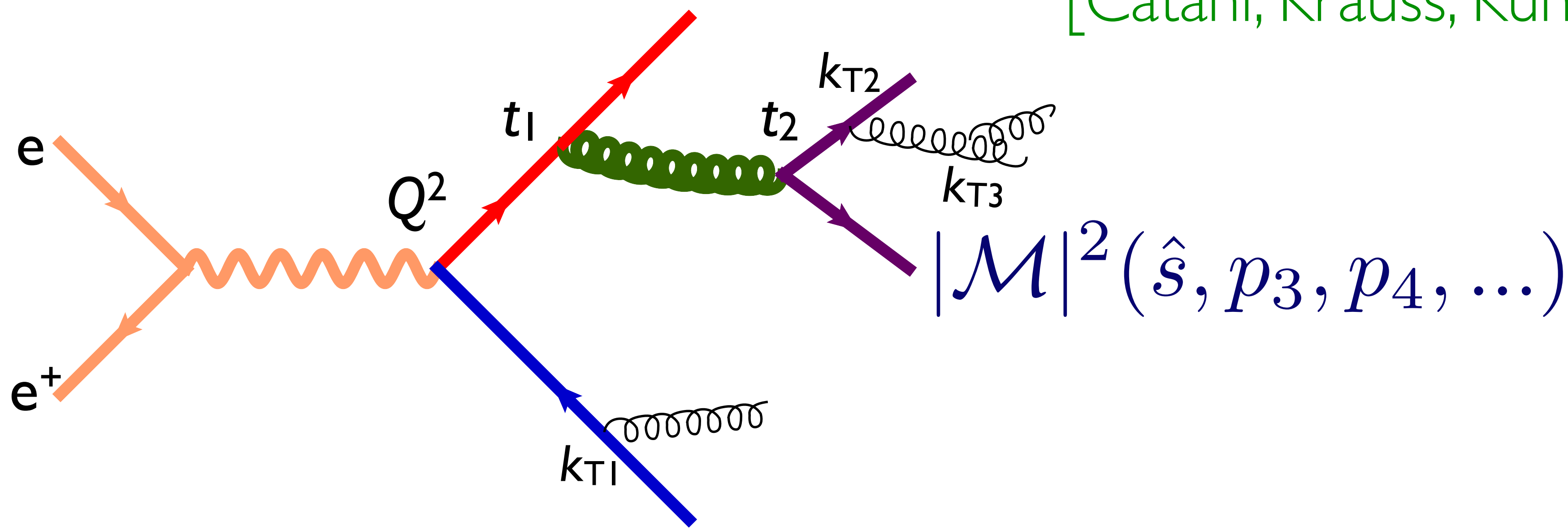

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MATCHING FOR INITIAL STATE RADIATION

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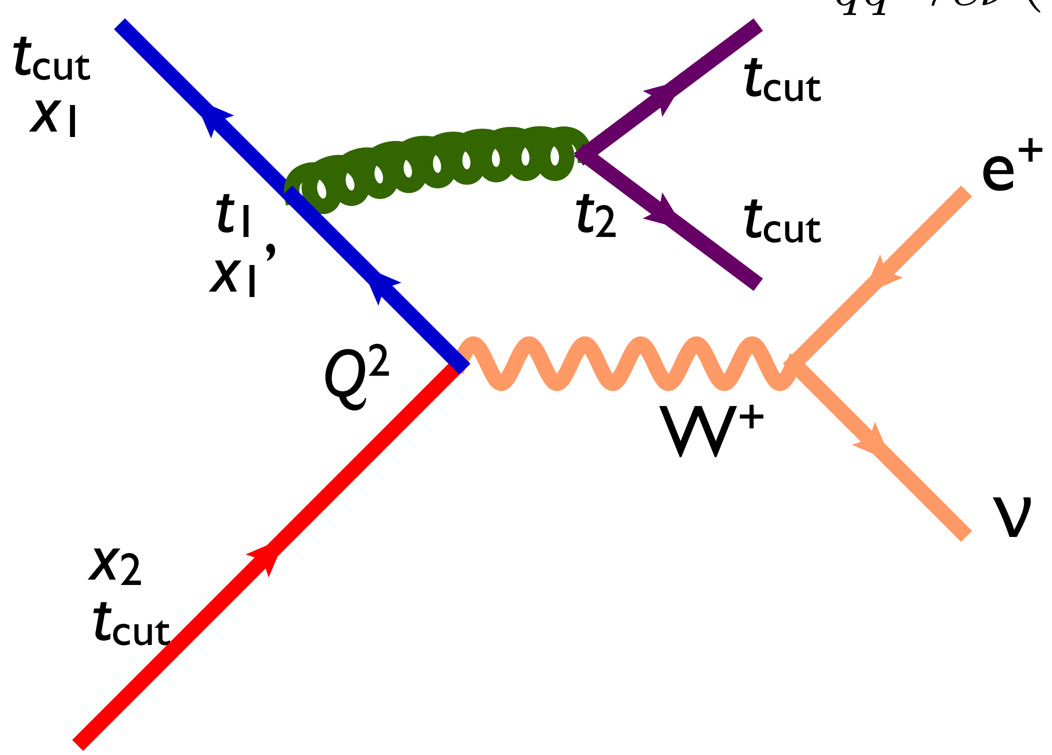
- Let's do the same exercise as before:

$$\mathcal{P} = (\Delta_{Iq}(Q^2, t_{\rm cut}))^2 \Delta_g(t_1, t_2)(\Delta_q(t_2, t_{\rm cut}))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x_1', t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$
$$\times \hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s}, \dots) f_q(x_1', Q^2) f_{\bar{q}}(x_2, Q^2)$$



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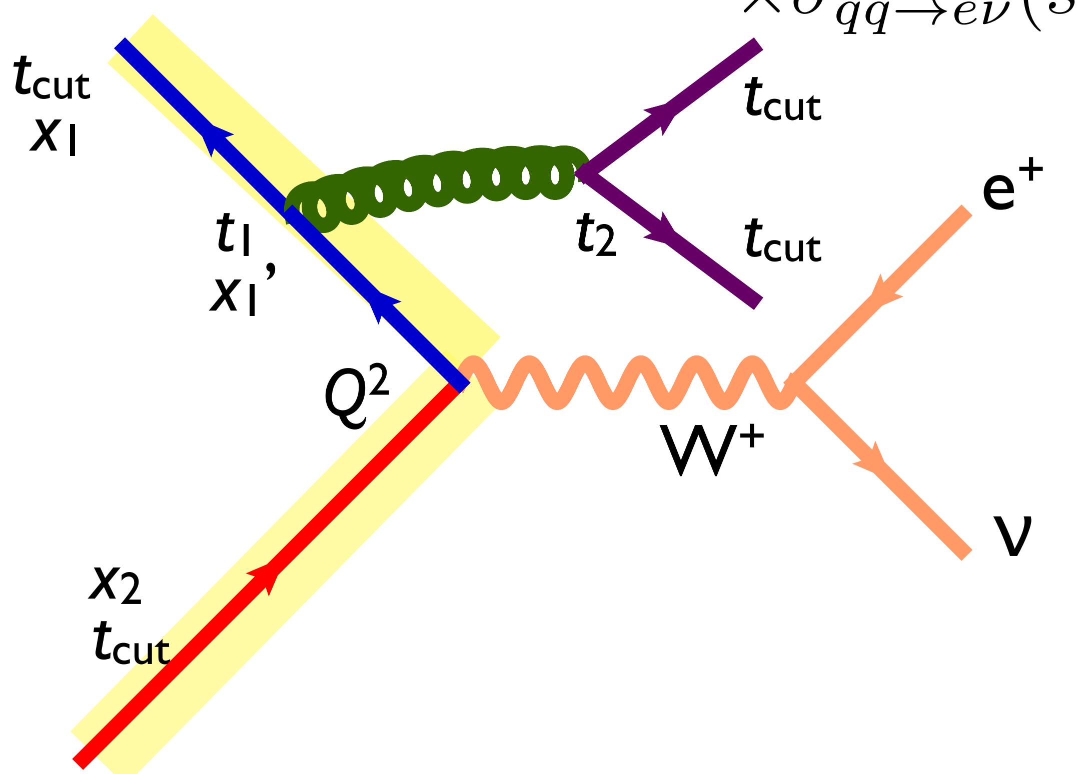

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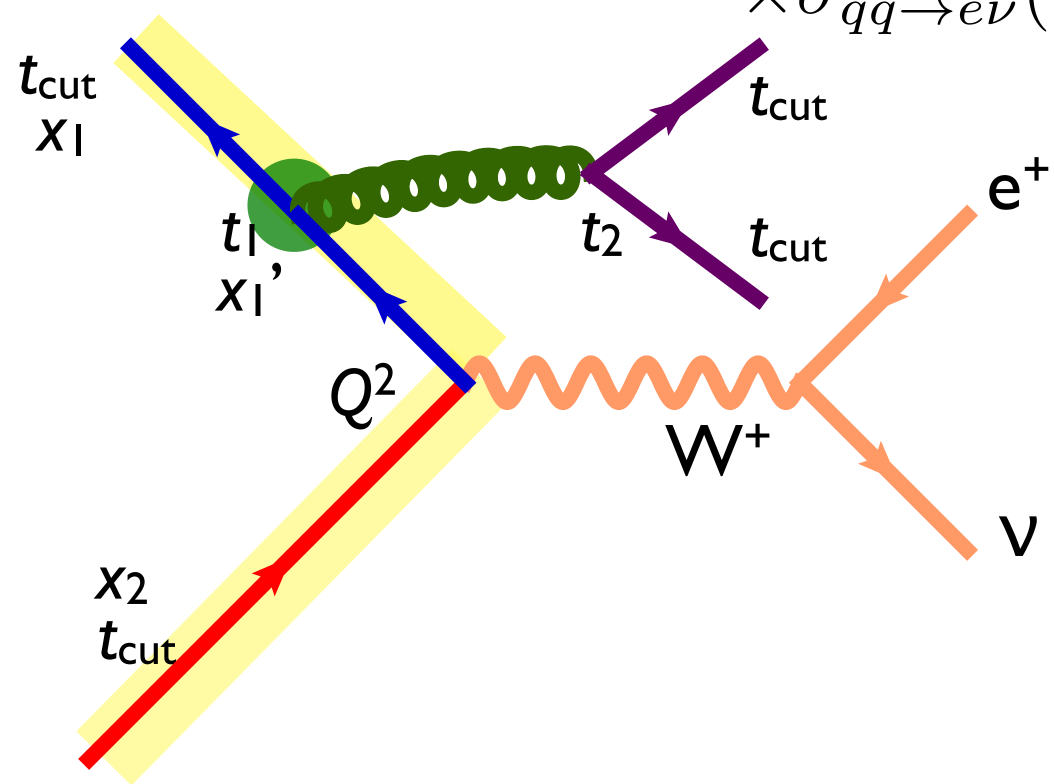

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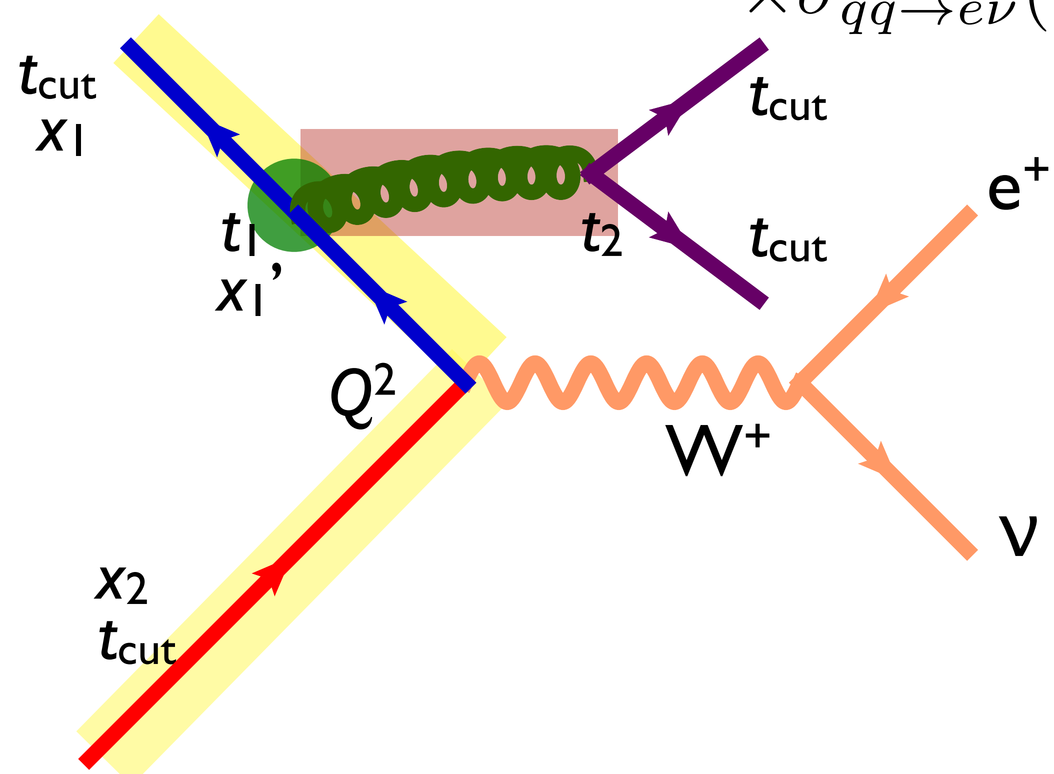

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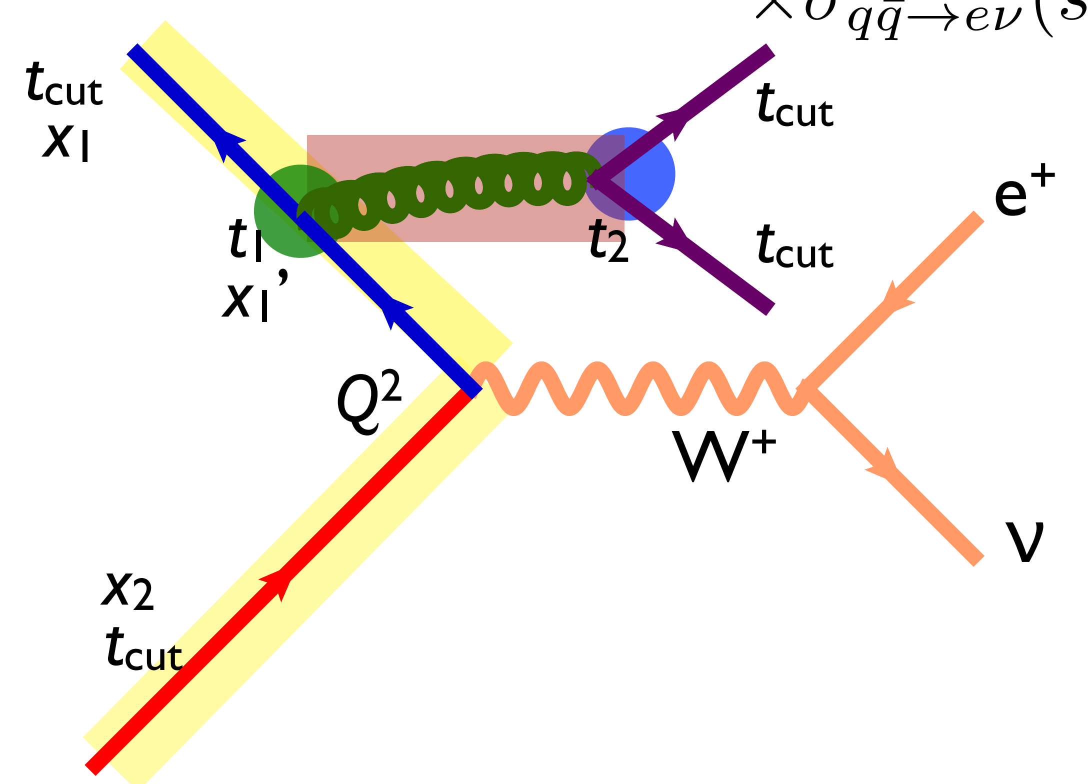

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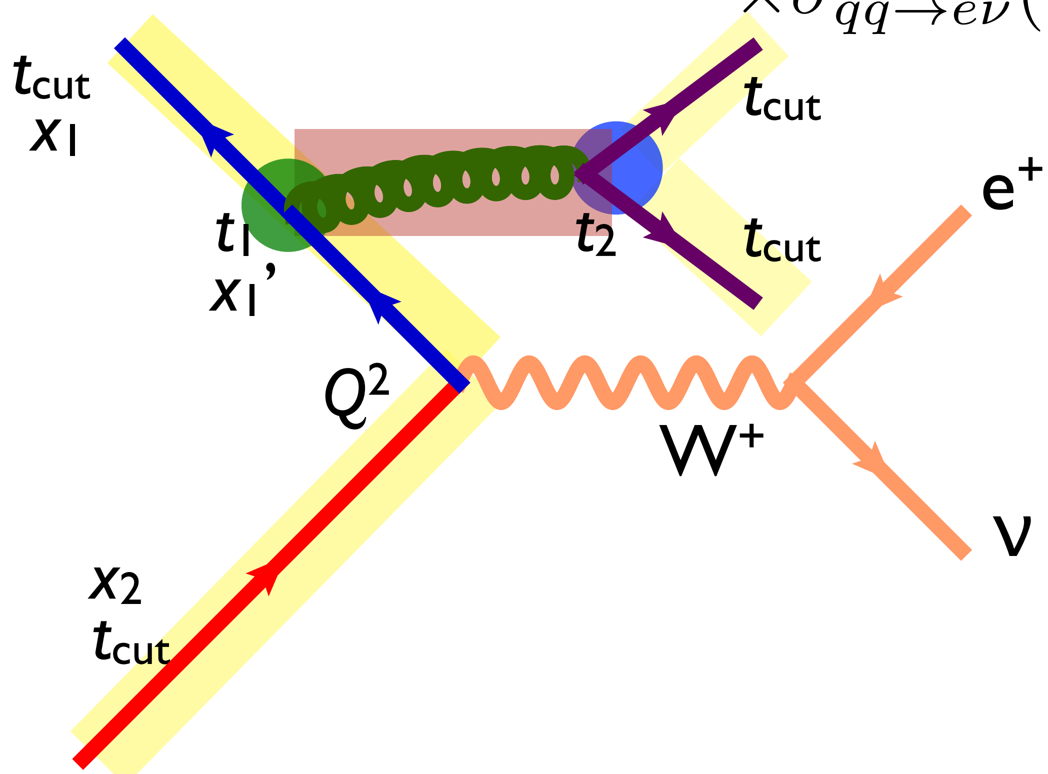

MATCHING FOR INITIAL STATE RADIATION

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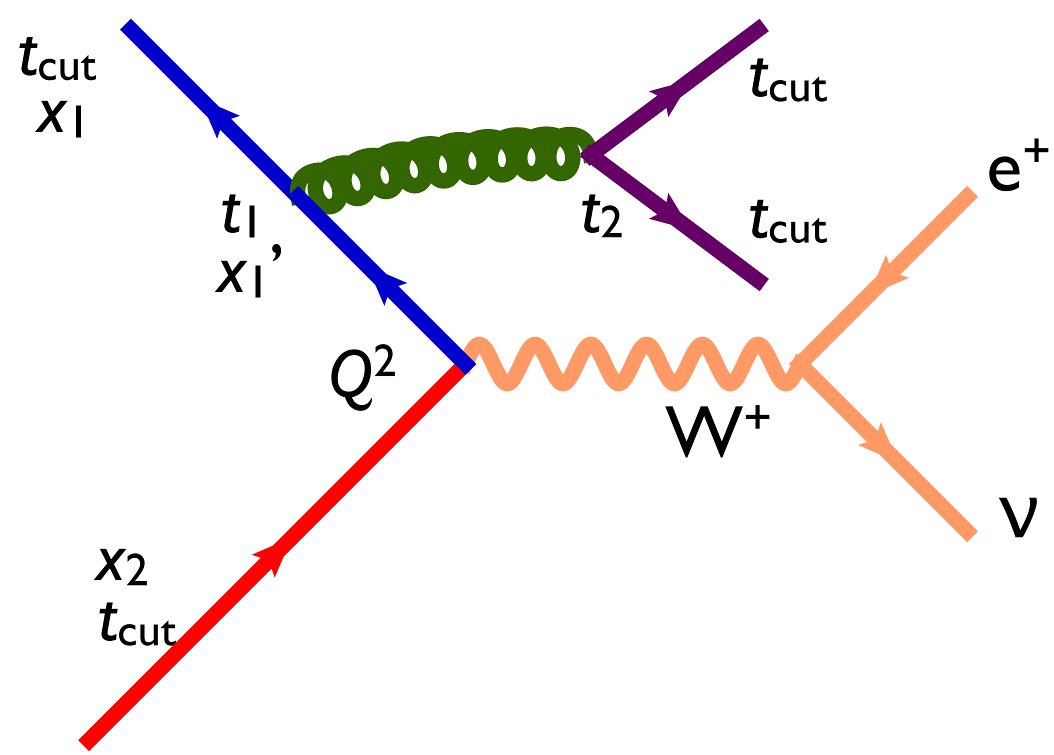

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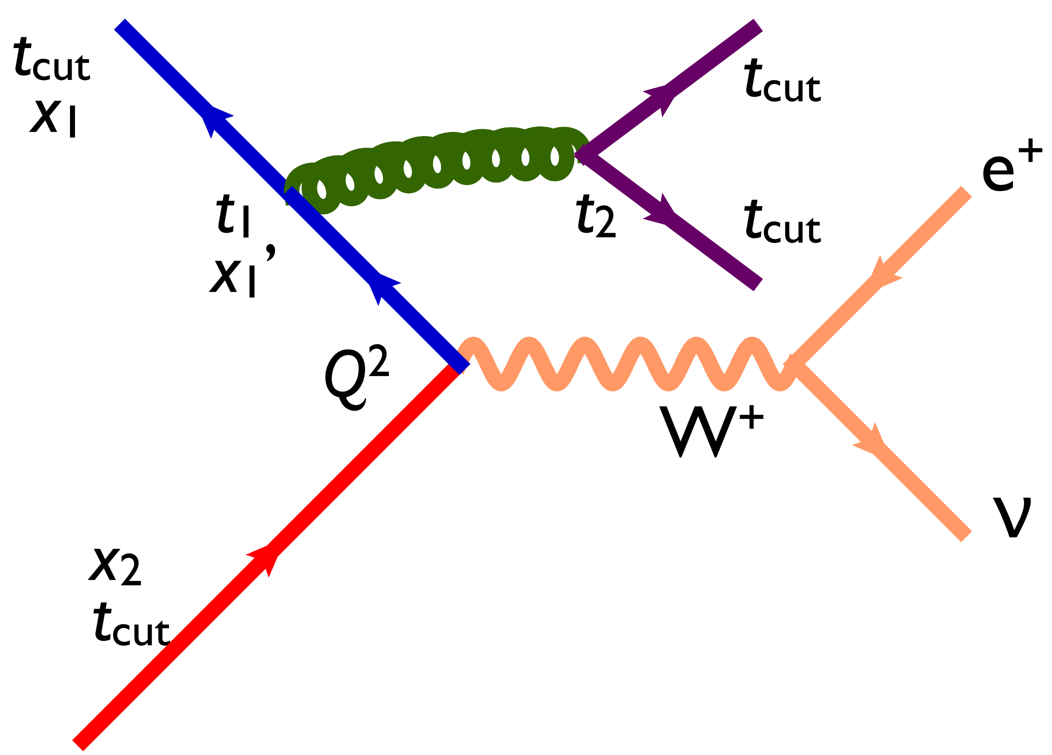
ME with α_s evaluated at the scale of each splitting



MATCHING FOR INITIAL STATE RADIATION

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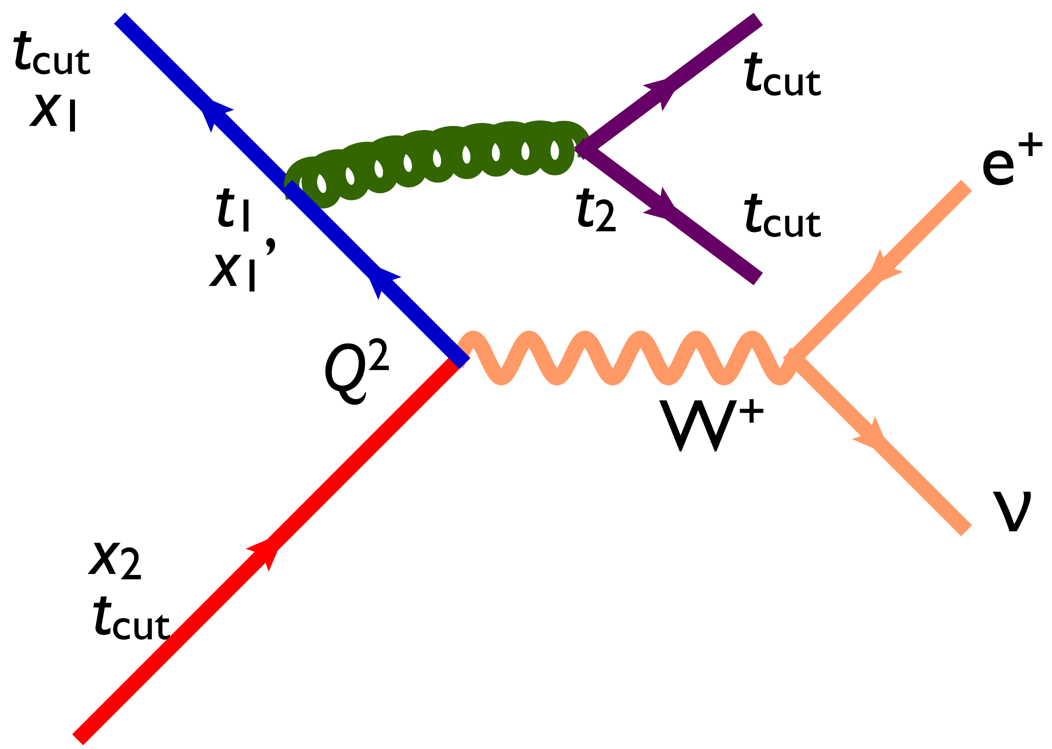
PDF reweighting



Monday 10 June 2013

MATCHING FOR INITIAL STATE RADIATION

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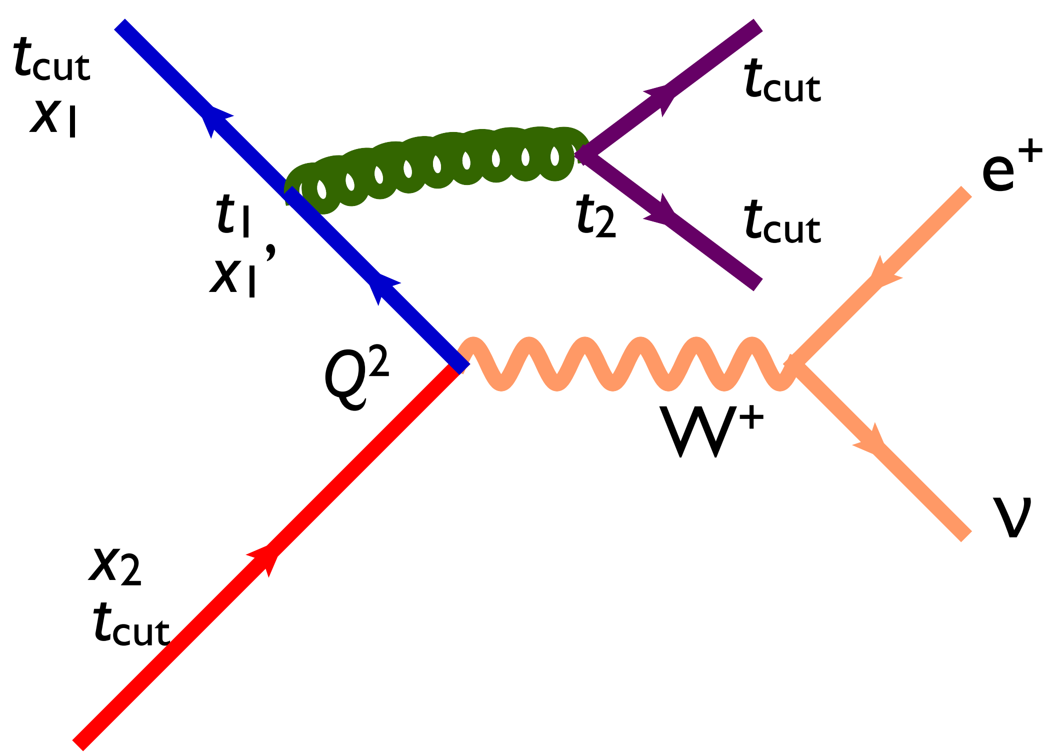
ME with α_s evaluated at the scale of each splitting

PDF reweighting

Sudakov suppression due to non-branching above scale t_{cut}



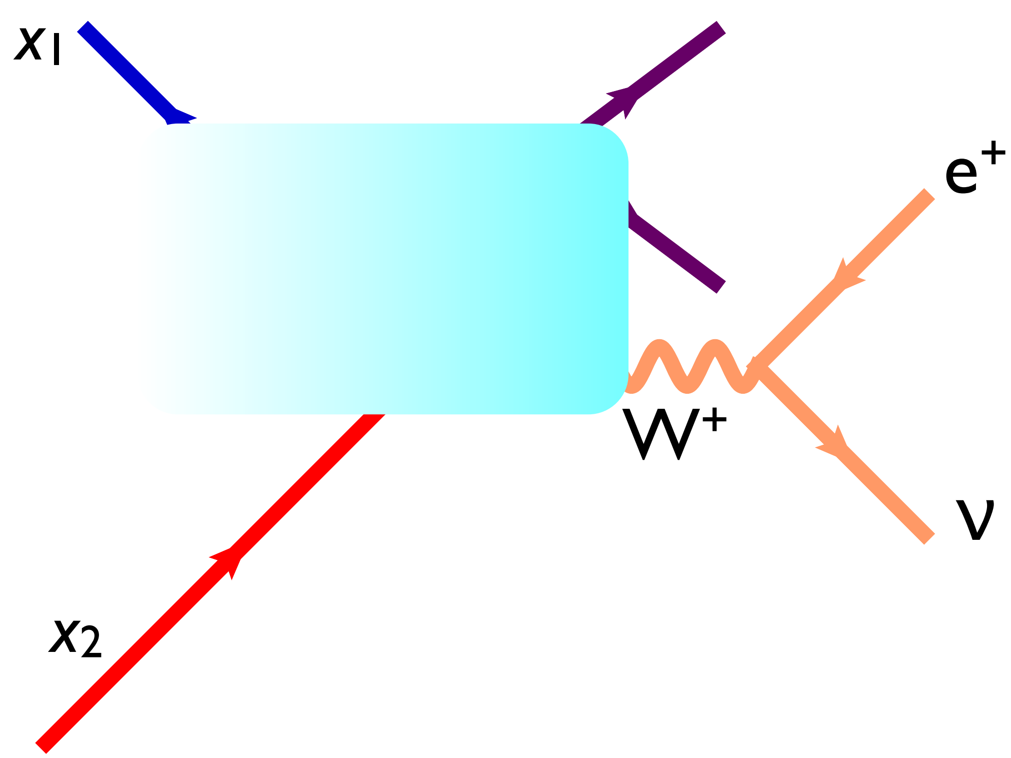
MATCHING FOR INITIAL STATE RADIATION

- Again, use a clustering scheme to get a parton shower history, but now reweight both due to α_s and PDF

$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(Q^2)} \frac{\alpha_s(t_2)}{\alpha_s(Q^2)} \frac{f_q(x_1', Q^2)}{f_q(x_1', t_1)}$$

- Remember to use first clustering scale on each side for PDF scale:

$$\mathcal{P}_{\text{event}} = \hat{\sigma}(x_1, x_2, p_3, p_4, \dots) f_q(x_1, t_1) f_{\bar{q}}(x_2, Q^2)$$



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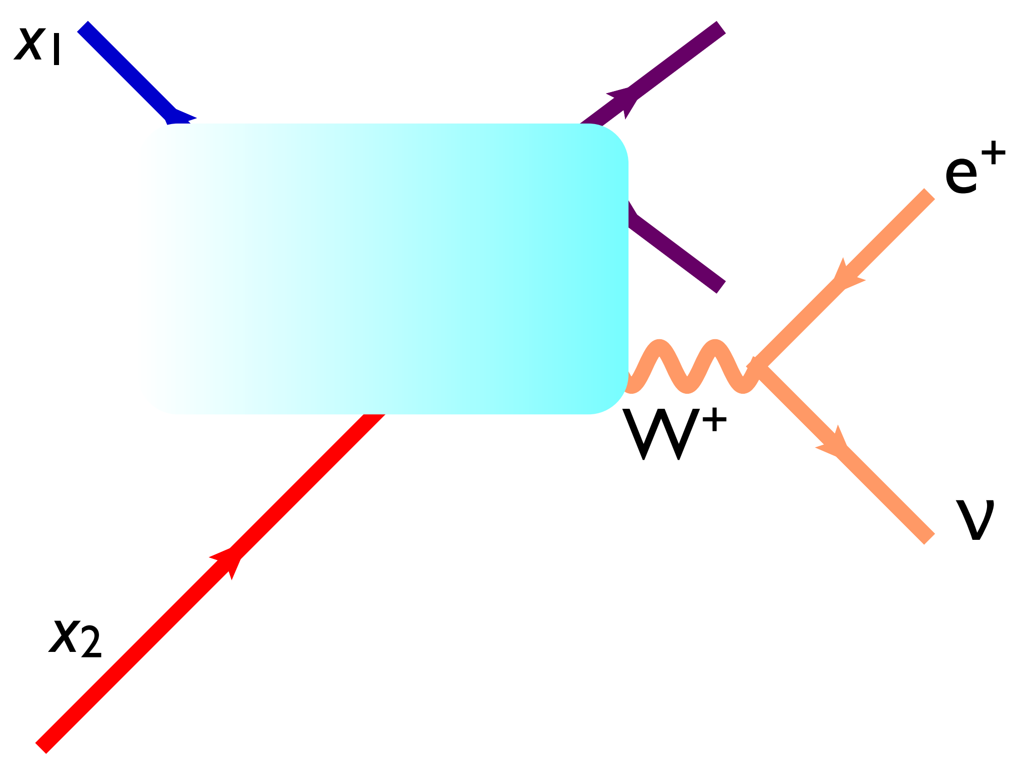
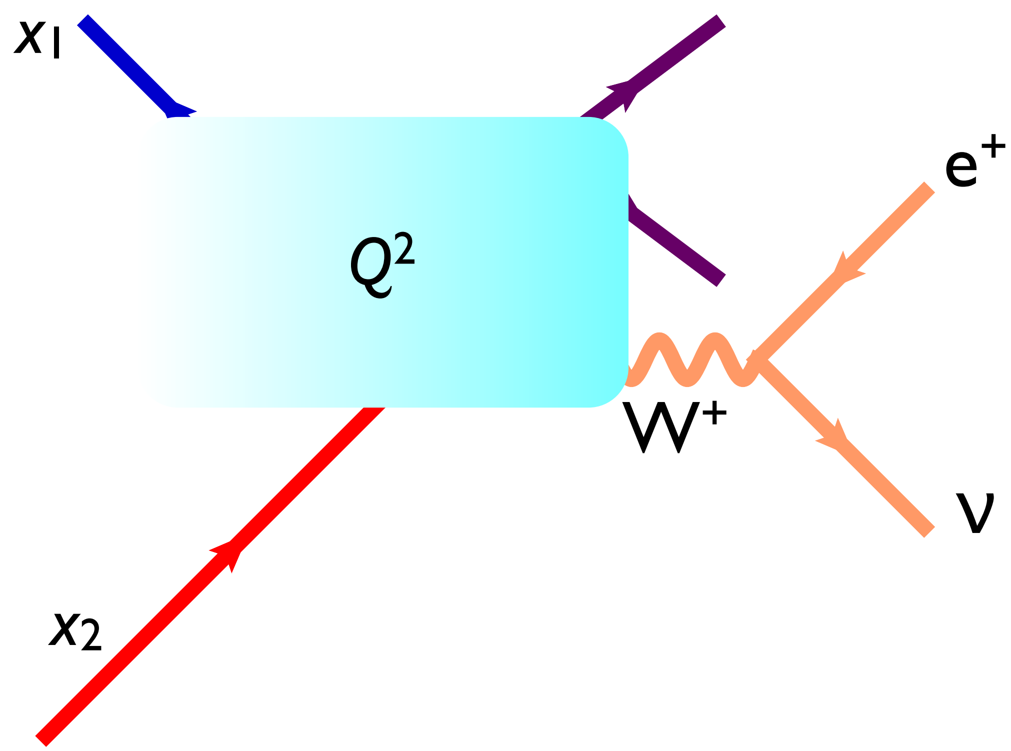


MATCHING FOR INITIAL STATE RADIATION

- And again, run the shower and then veto events if the hardest shower emission scale $k_{T1} > t_{\text{cut}}$.
- The resulting Sudakov suppression from the procedure is

$$(\Delta_{Iq}(Q^2, t_{\text{cut}}))^2(\Delta_q(Q^2, t_{\text{cut}}))^2$$

- which again is a good enough approximation of the correct expression (much better than $(\Delta_{Iq}(Q^2, t_{\text{cut}}))^2\Delta_g(t_1, t_2)(\Delta_q(t_2, t_{\text{cut}}))^2$ in e^+e^-, since the main suppression here is from Δ_{Iq})



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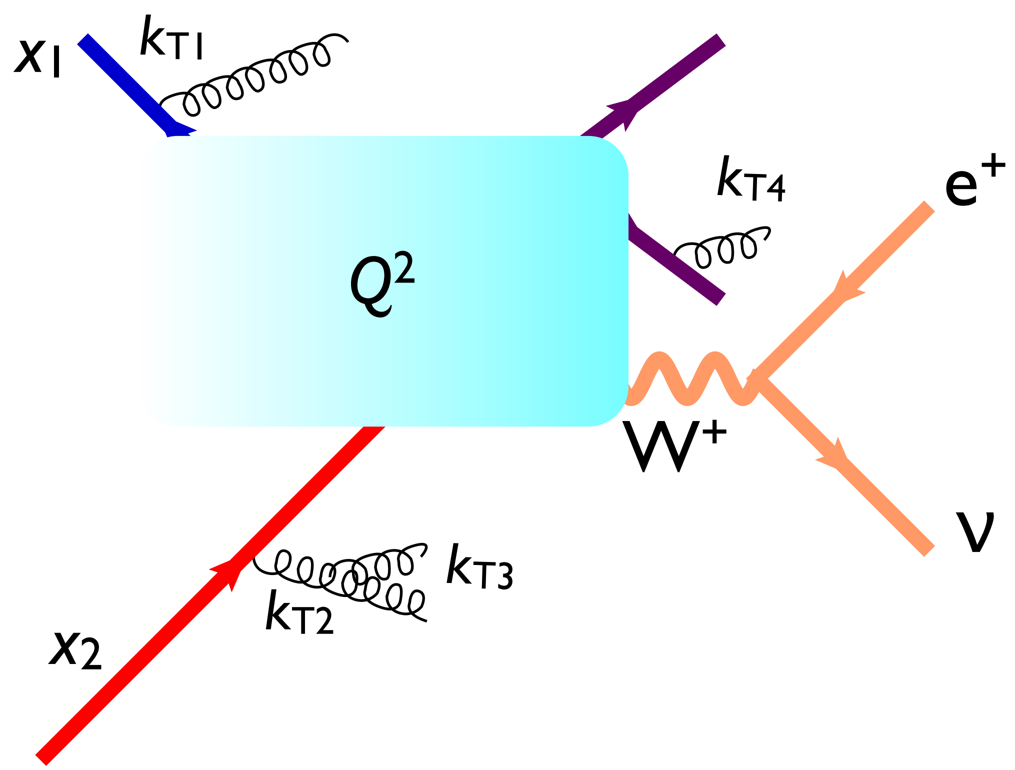
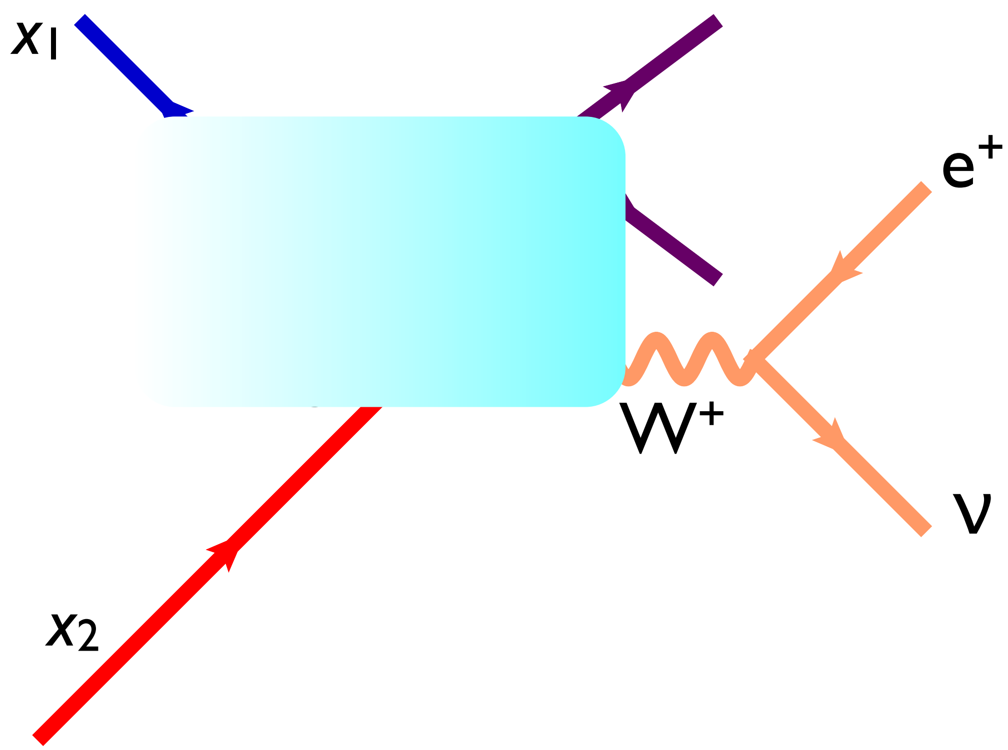


MATCHING FOR INITIAL STATE RADIATION

- Like before, for CKKW we reweight the matrix elements with the Sudakov factors given by the ‘most-likely parton shower history’
- Again, if we cluster correctly we can start the shower at the scale t_{cut}



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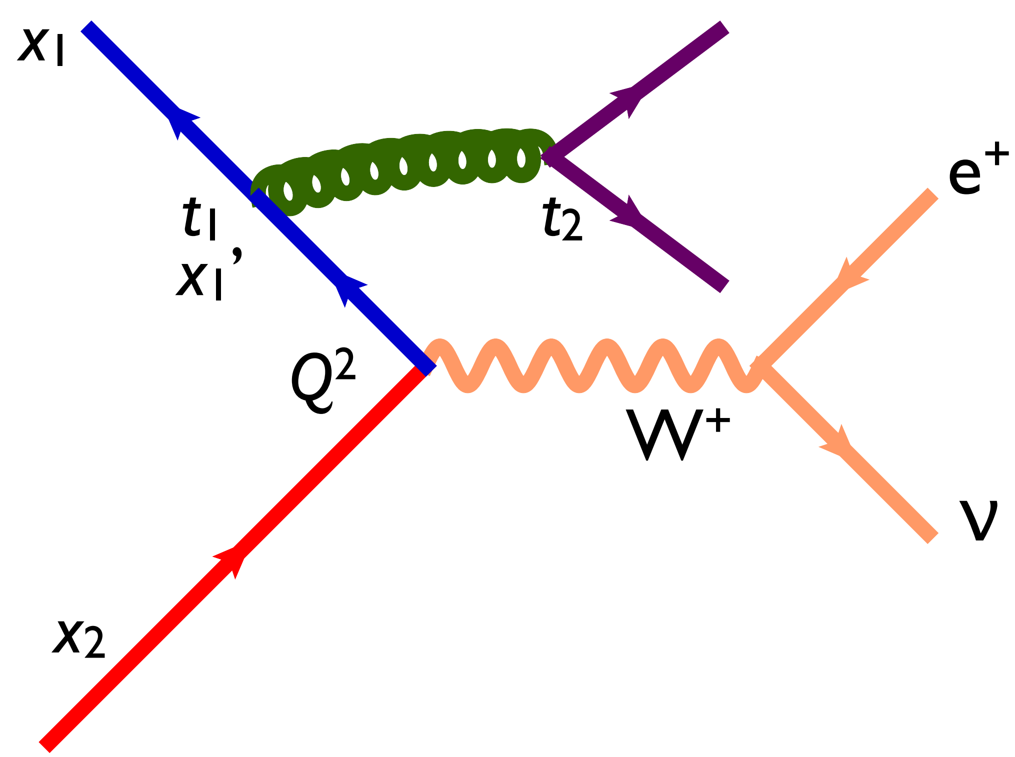

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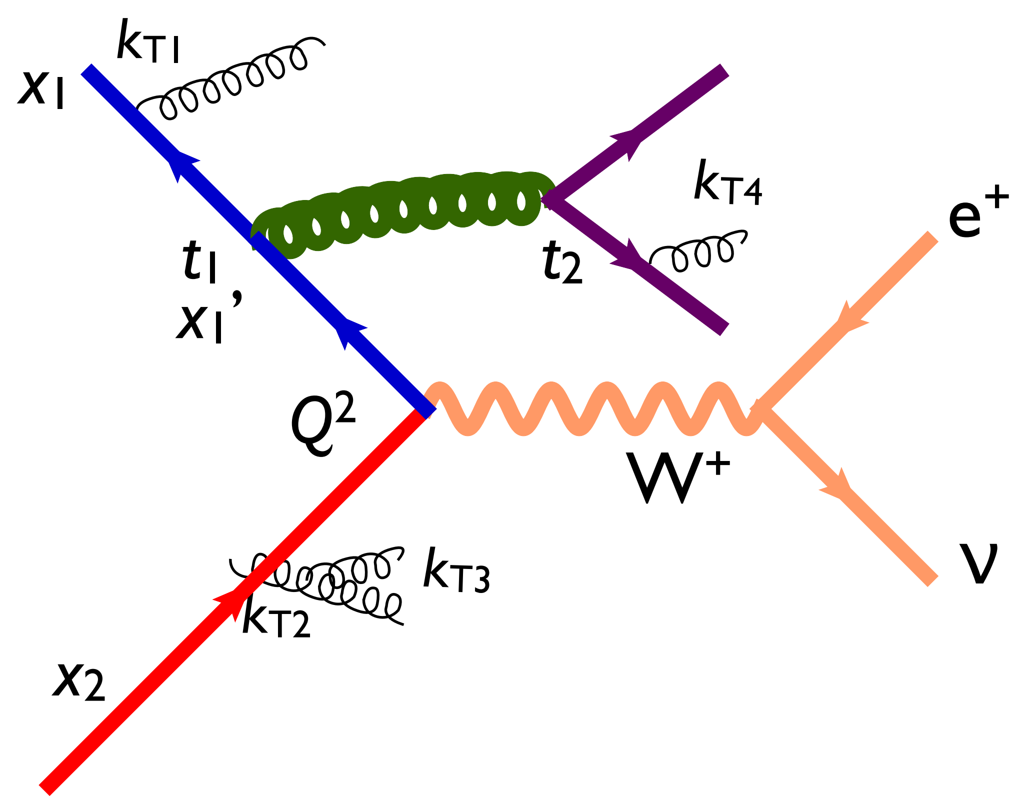


Matching schemes in existing codes

- AlpGen: MLM (cone)
- MadGraph: MLM (cone, k_T, shower-k_T)
- Sherpa: CKKW

MATCHING SCHEMES "FREEDOM"

- We have a number of choices to make in the above procedure. The most important are:

1. The clustering scheme used to determine the parton shower history of the ME event
2. What to use for the scale Q^2 (factorization scale)
3. How to divide the phase space between parton showers and matrix elements

Cluster schemes

1. The clustering scheme used inside MadGraph and Sherpa to determine the parton shower history is the Durham k_T scheme. For e^+e^-:

$$k_{Tij}^2 = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$$

and for hadron collisions, the minimum of:

$$k_{Ti\mathrm{beam}} = m_i^2 + p_{Ti}^2 = (E_i + p_{zi})(E_i - p_{zi})$$

and

$$k_{Tij}^2 = \min(p_{Ti}^2, p_{Tj}^2)R_{ij}$$

with

$$R_{ij} = 2[\cosh(y_i - y_j) - \cos(\phi_i - \phi_j)] \simeq (\Delta y)^2 + (\Delta\phi)^2$$

Find the smallest k_{Tij} (or $k_{Ti\mathrm{beam}}$), combine partons *i* and *j* (or *i* and the beam), and continue until you reach a 2 → 2 (or 2 → 1) scattering.

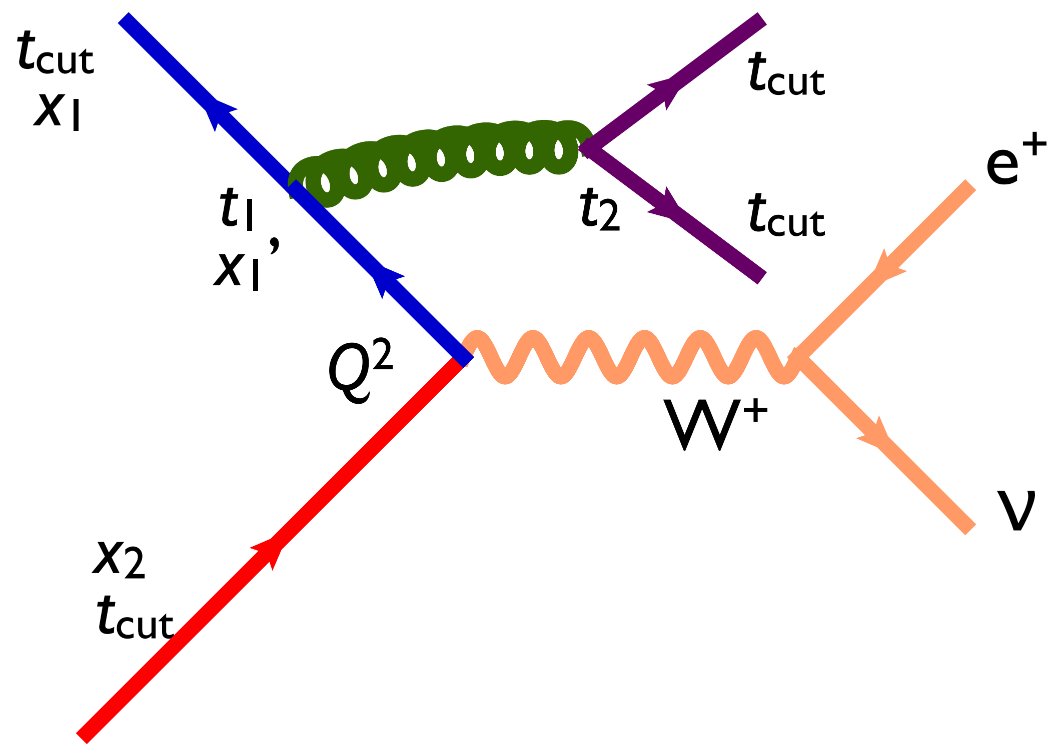
2. In AlpGen a more naive cone algorithm is used.

Cluster schemes

- Cannot use the standard k_T clustering:
  - MadGraph and Sherpa only allow clustering according to valid diagrams in the process. This means that, e.g., two quarks or quark-antiquark of different flavor are never clustered, and the clustering always gives a physically allowed parton shower history.
  - If there is an on-shell propagator in the diagram (e.g. a top quark), only clustering according to diagrams with this propagator is allowed.

Hard scale

2. The clustering provides a convenient choice for factorization scale Q^2:



Cluster back to the 2 → 2 (here qq → W⁻g) system, and use the W boson transverse mass in that system.

Phase-space division

Phase-space division

3. How to divide the phase space between PS and ME:
   This is where the schemes really differ:

   AlpGen: MLM Cone
   MadGraph: MLM Cone, k_T or shower-k_T
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   Sherpa: CKKW

   a. Cone jet MLM scheme (better suited for angular ordered showers, i.e. herwig, but works for all showers):
      - Use cuts in p_T (p_T^{ME})and ΔR between partons in ME
      - Cluster events after parton shower using a cone jet algorithm with the same ΔR and $p_T^{match} > p_T^{ME}$
      - Keep event if all jets are matched to ME partons (i.e., all ME partons are within ΔR of a jet)

Phase-space division

3. How to divide the phase space between PS and ME:
   b. k_T-jet MLM scheme (better suited for k_T ordered showers, i.e. pythia, but works for all showers):
   - Use cut in the Durham k_T in ME
   - Cluster events after parton shower using the same k_T clustering algorithm into k_T jets with $k_T^{match} > k_T^{ME}$
   - Keep event if all jets are matched to ME partons (i.e., all partons are within k_T^{match} to a jet)

   c. Shower-k_T scheme (works only with pythia, i.e. k_T ordered shower):
   - Use cut in the Durham k_T in ME
   - After parton shower, get information from the PS generator about the k_T^{PS} of the hardest shower emission
   - Keep event if $k_T^{PS} < k_T^{match}$

Phase-space division

3. How to divide the phase space between PS and ME:
   d. CKKW Scheme (Need special veto'ed shower):
      - Use cut in the Durham k_T in ME (k_T^{match})
      - Because the Durham k_T is not the same as the evolution parameter of the shower, we might miss contributions, therefore
      - Start the shower at the original scale, and after each emission, check the value of t_i:
      - if $t_i > k_T^{match}$ veto that emission, i.e. continue the shower as if that emission never happened

Summary of matching algorithms

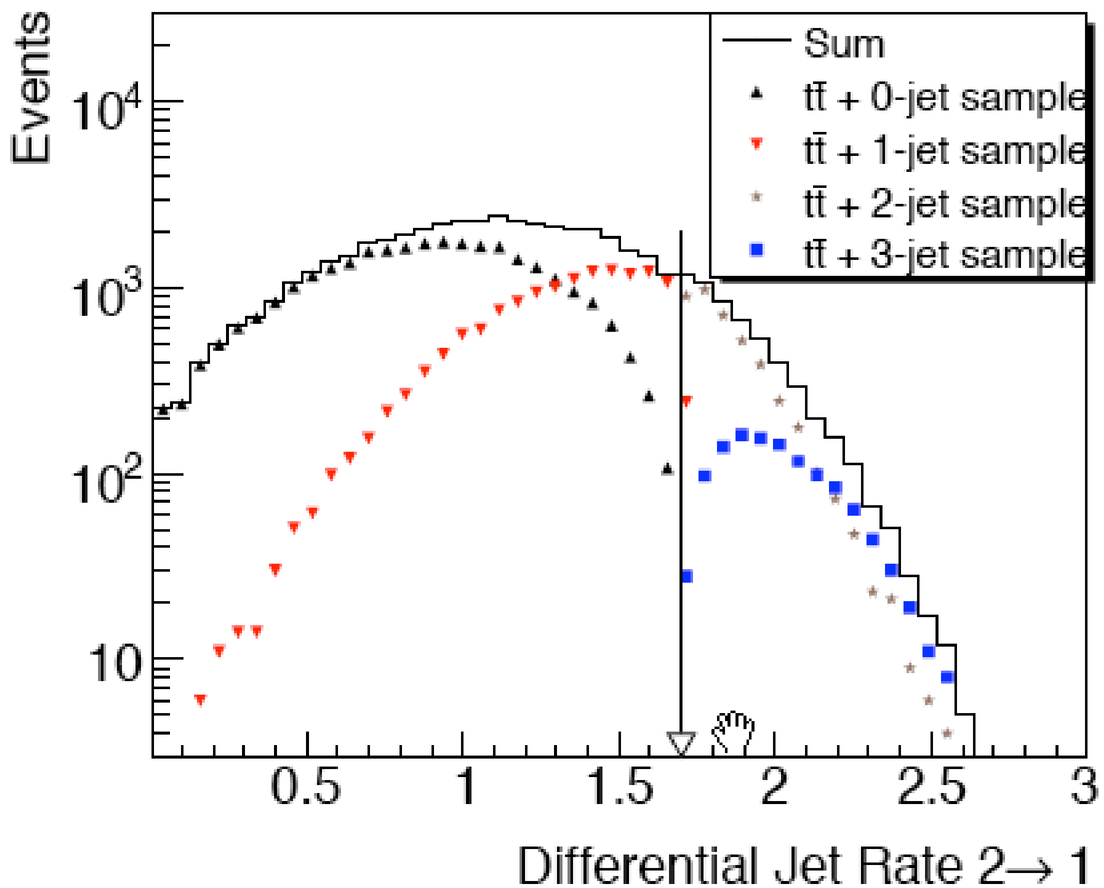
1. Generate ME events (with different parton multiplicities) using parton-level cuts ($p_T^{ME}/\Delta R$ or k_T^{ME})
2. Cluster each event and reweight α_s and PDFs based on the scales in the clustering vertices
3. Run the parton shower with starting scale $Q^2 = m_T^2$.

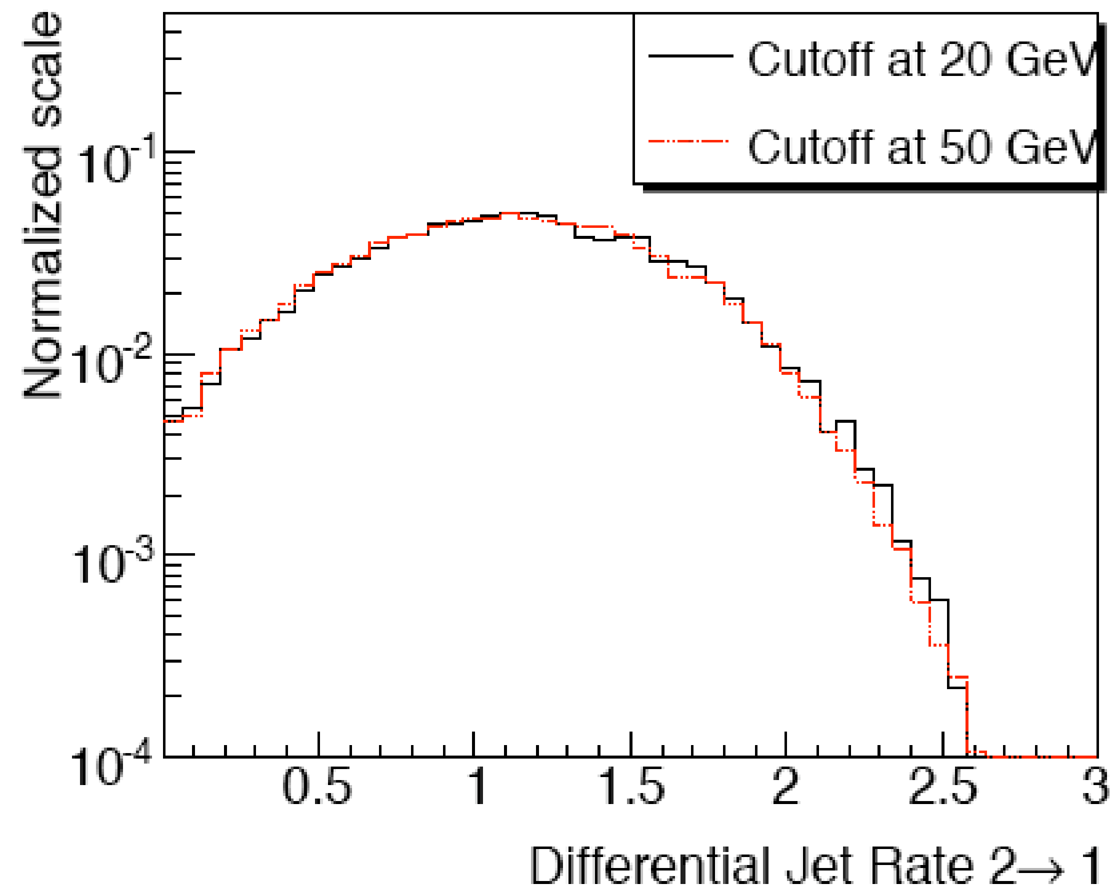
Summary of matching algorithm

4. a) For MLM: Check that the number of jets after parton shower is the same as ME partons, and that all jets after parton shower are matched to the ME partons (using one of the schemes in the last slides) at a scale Q^{match}. If yes, keep the event. If no, reject the event. Q^{match} is called the *matching scale*.

   b) For CKKW: Reweight the matrix elements with the Sudakovs related to the "most-likely parton shower history". Start the shower at the at the scale Q^2, but veto emissions which are already taken care of by the matrix elements.

Sanity checks: differential jet rates

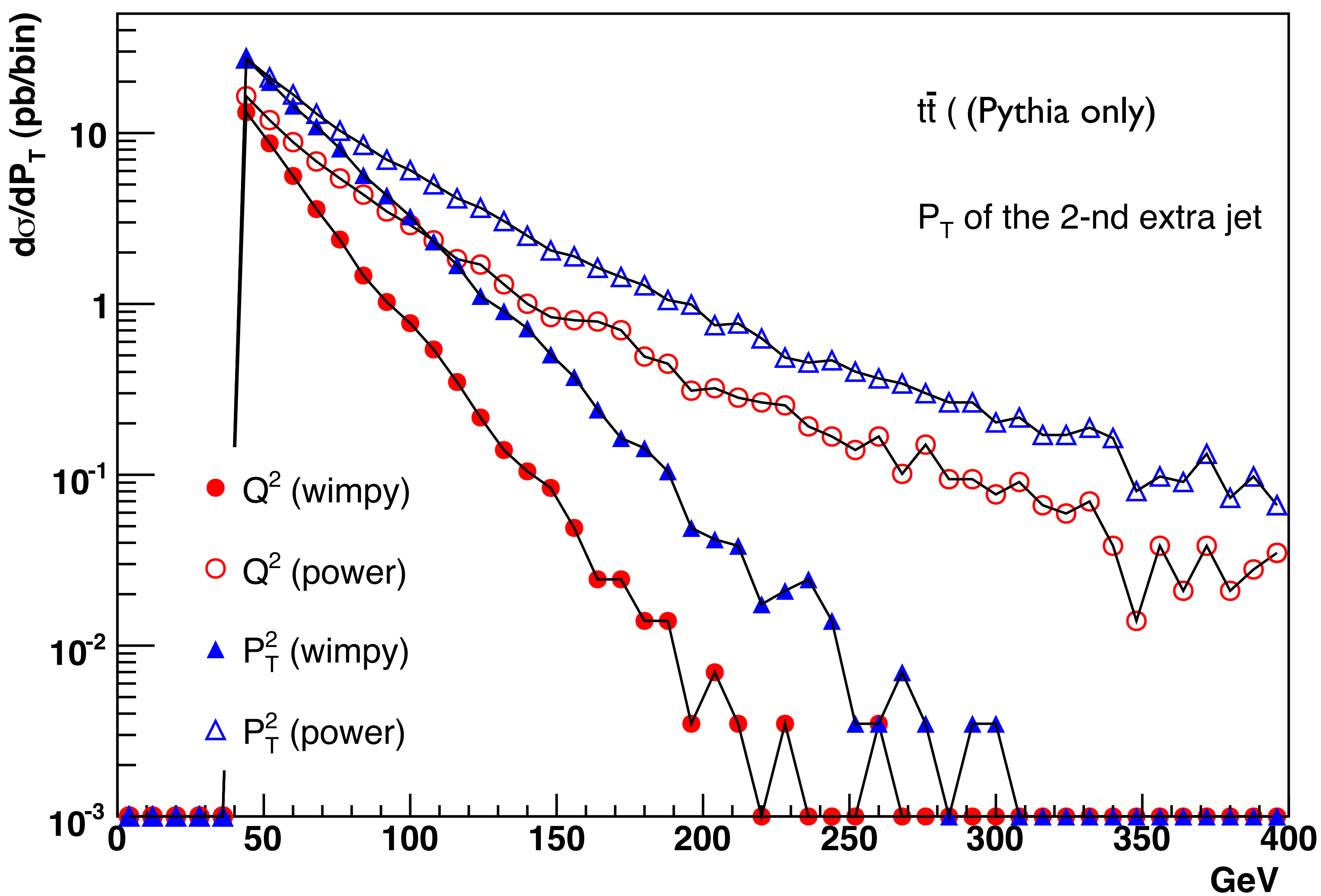




Jet rates are independent of and smooth at the cutoff scale

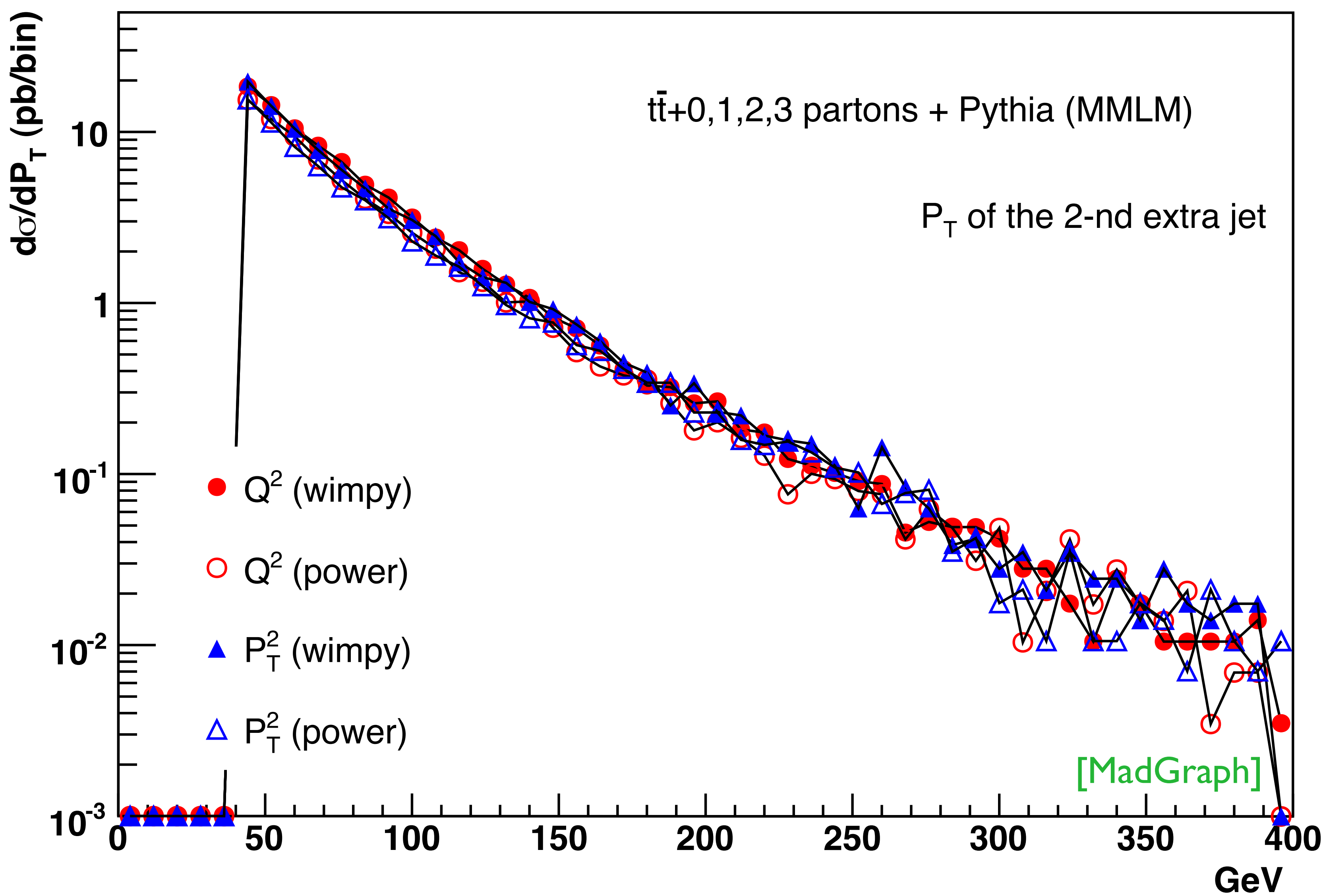
PS alone vs. Matched sample

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)

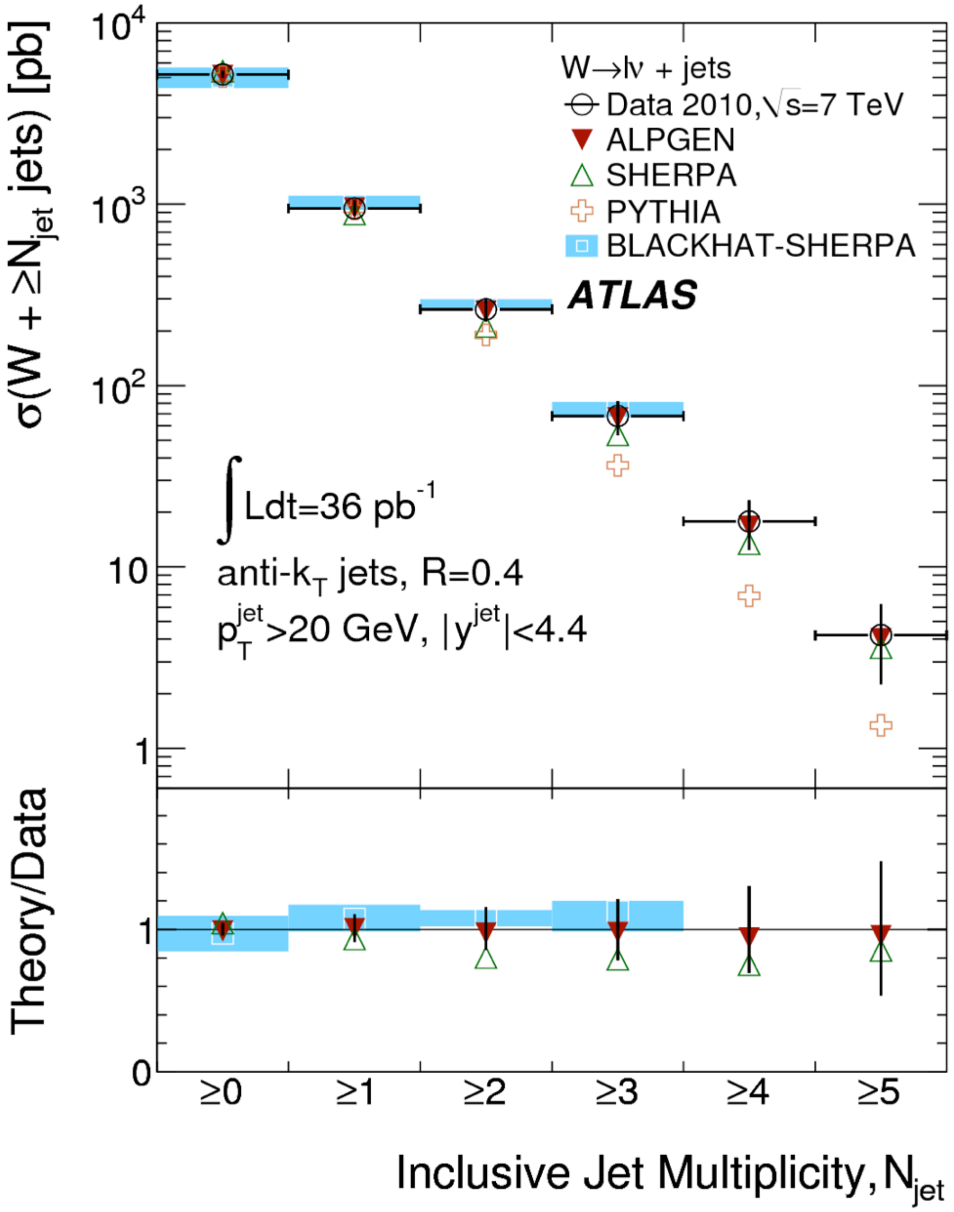


PS alone vs. Matched sample

In a matched sample these differences are irrelevant since the behavior at high pt is dominated by the matrix element.



TH/EXP comparison at the LHC



Bonus: Even rates in outstanding agreement with data and NLO

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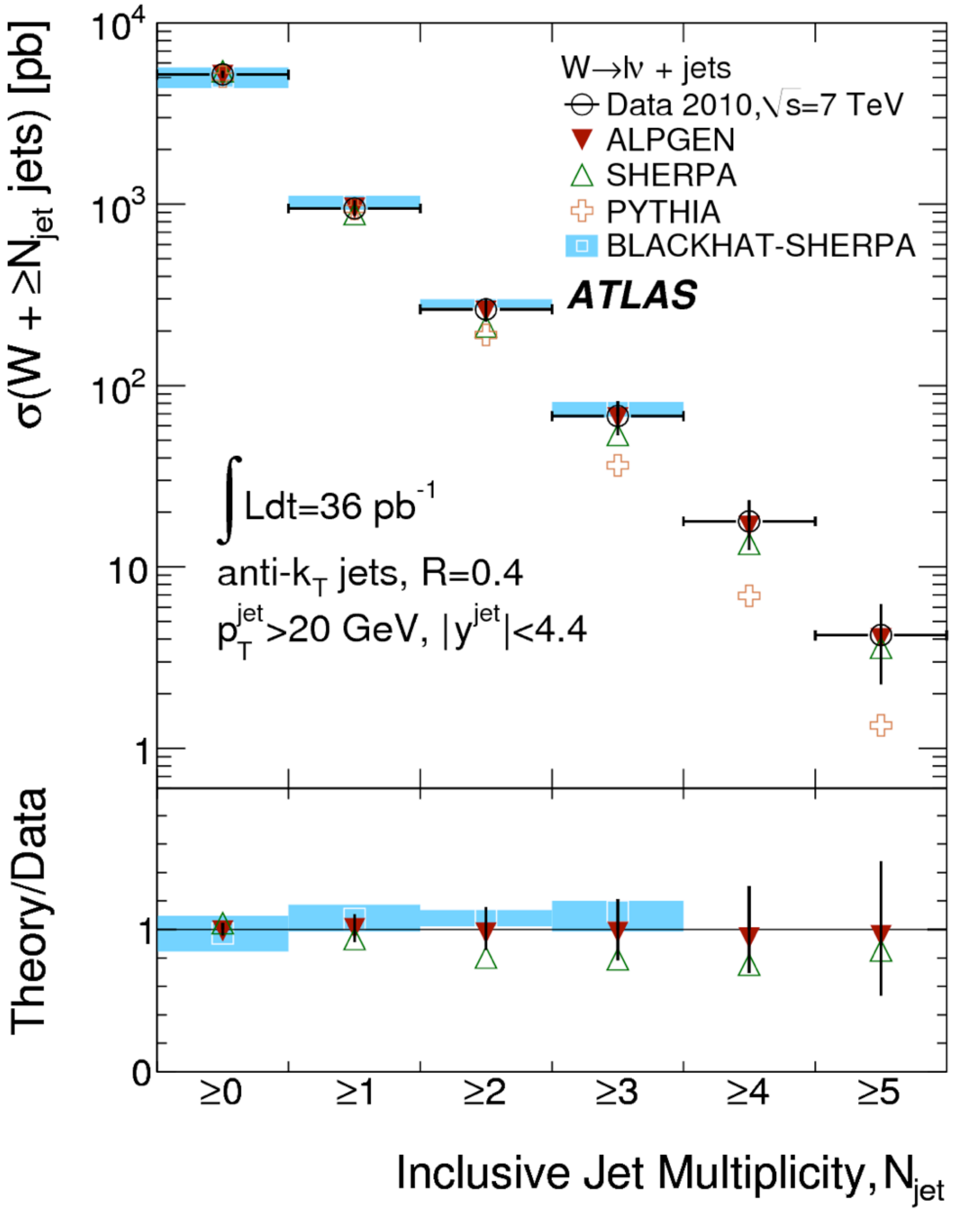

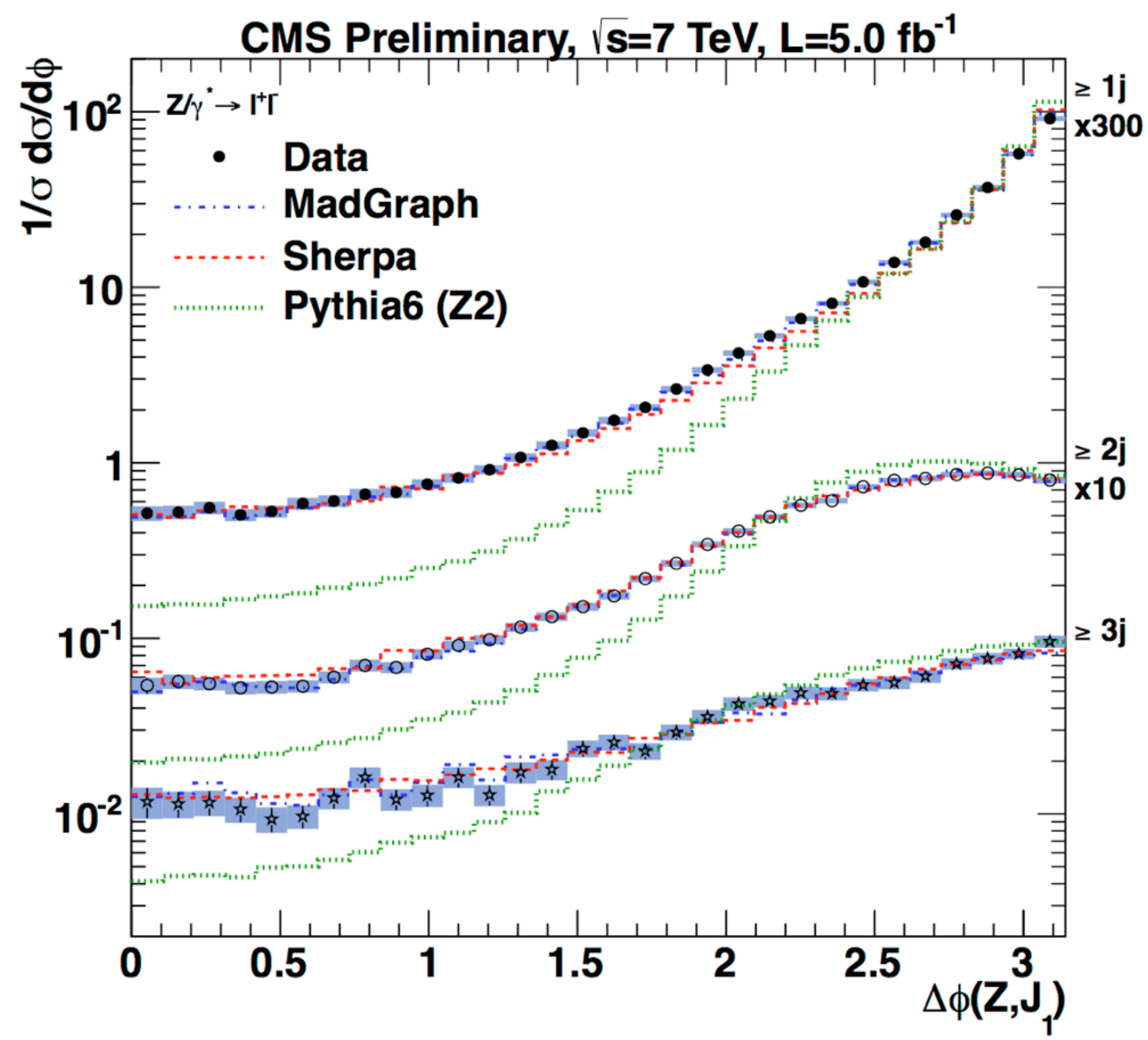

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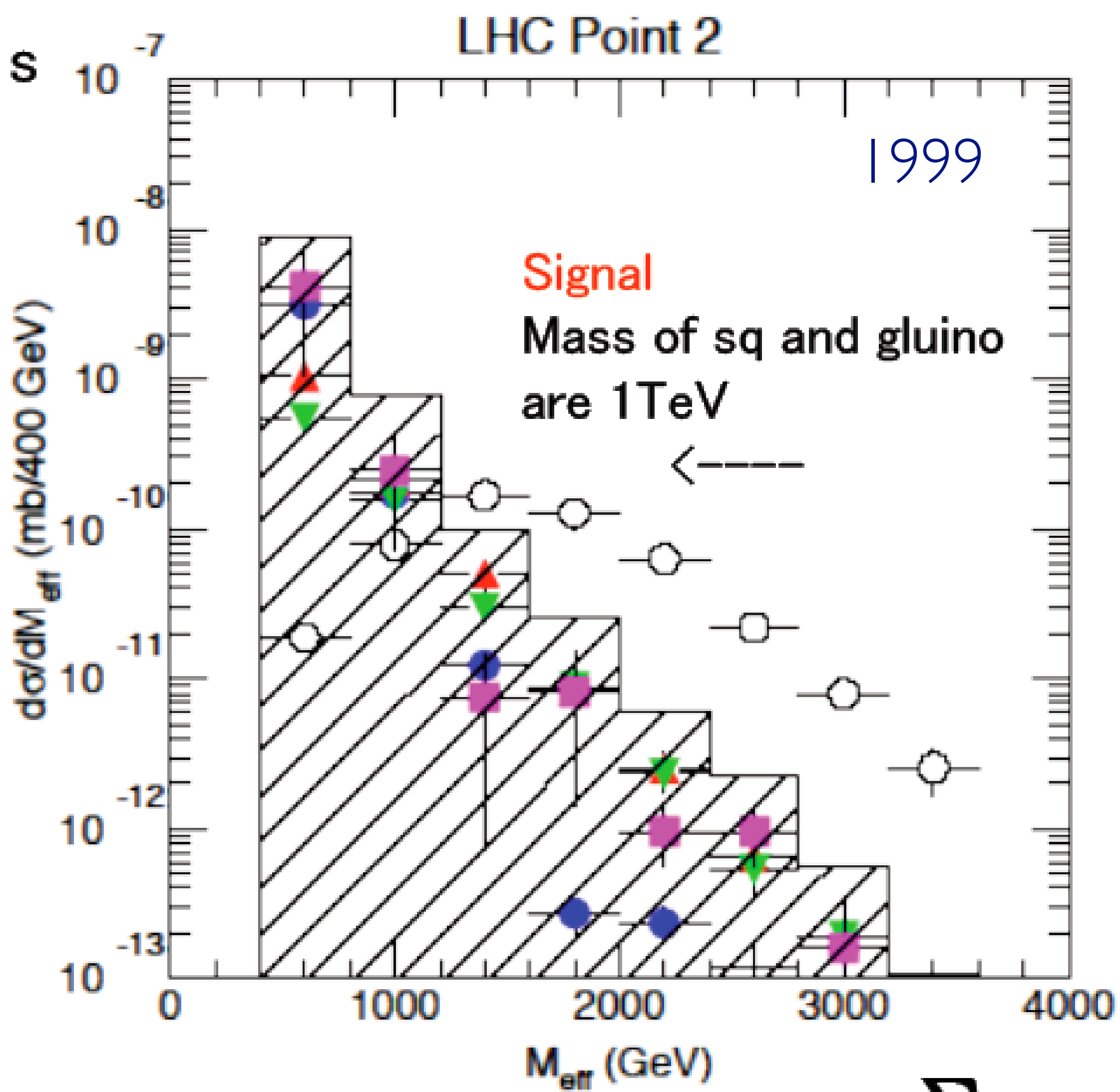
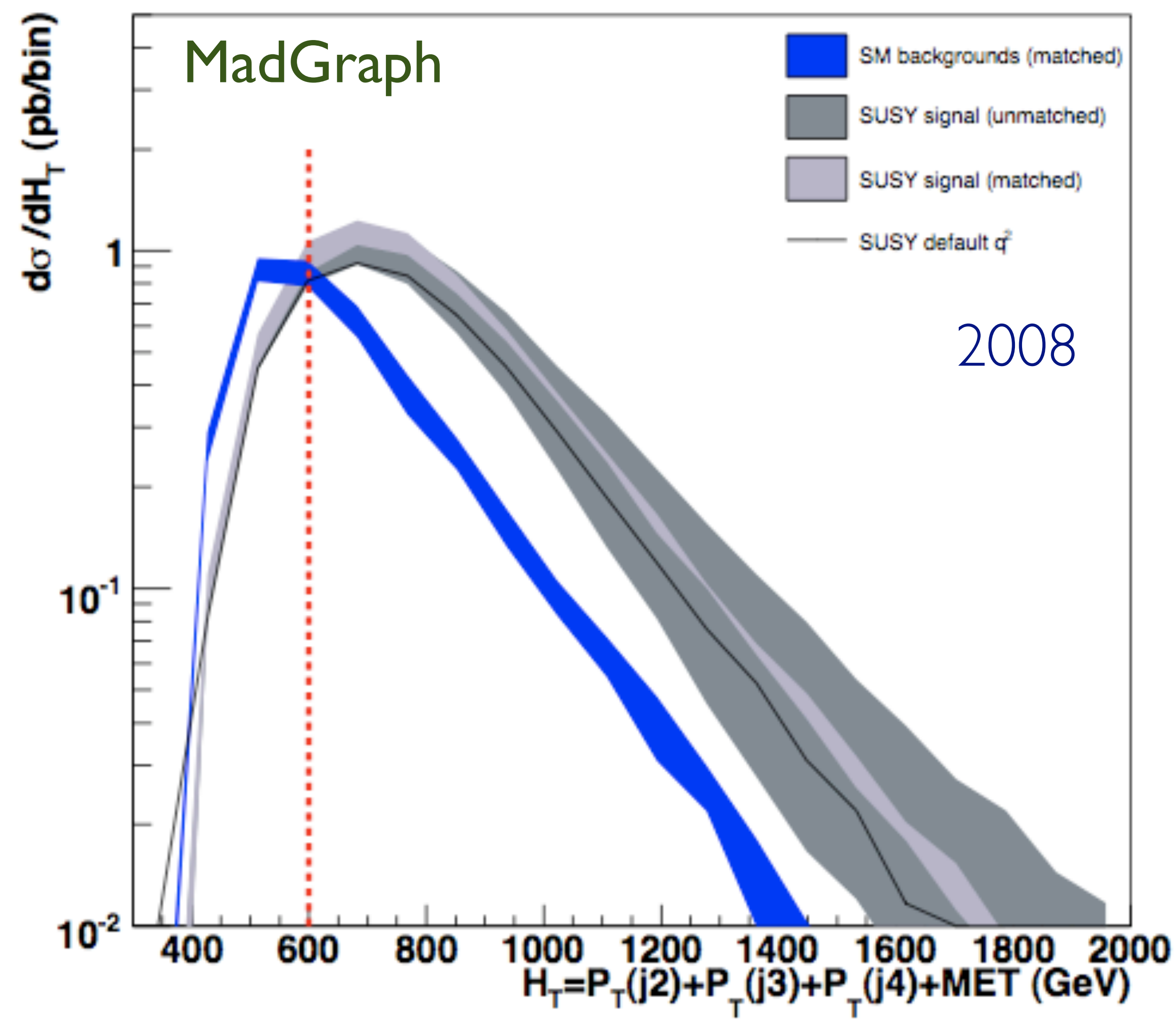
SUSY matched samples



$= \not{E}_T + \sum P_T$



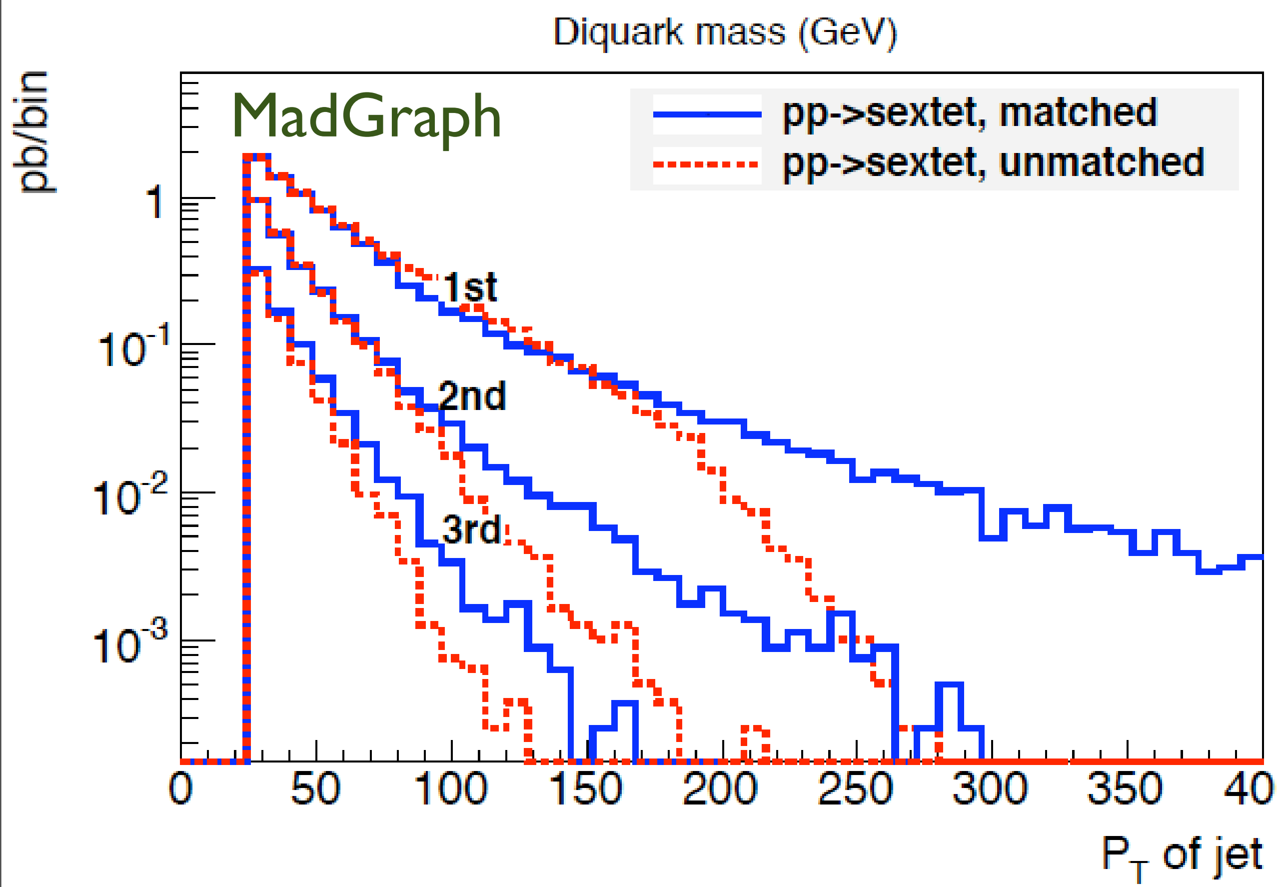
Both signal and background matched!

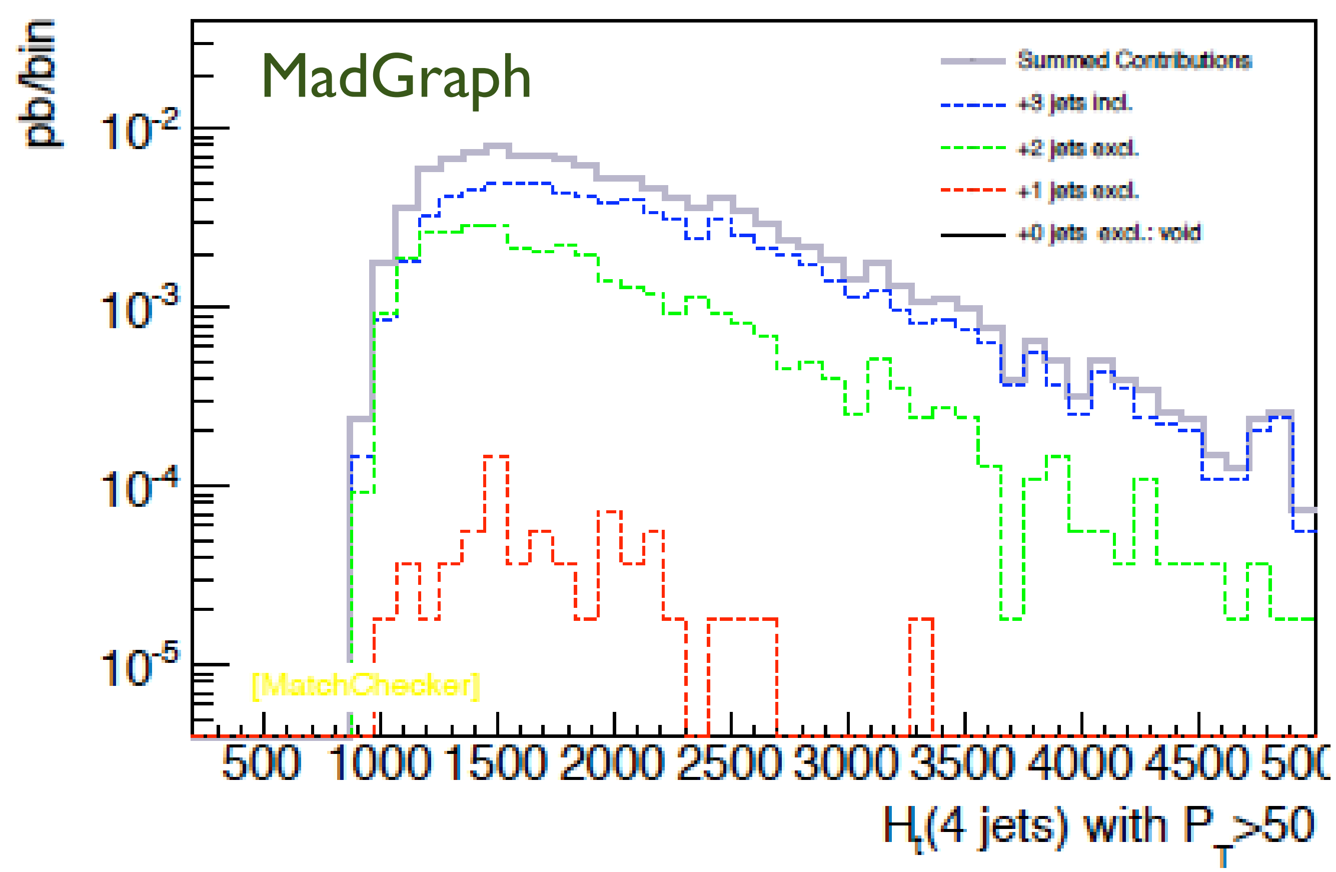
Sizable reduction of the uncertainties and simulation consistency .

Example: BSM multijet final states

pp→X6 +jets

pp→Graviton (ADD&RS) +jets





New Physics models can be easily included in Matrix Element generators via FeynRules and results automatically for multi-jet inclusive final state obtained at the same level of accuracy that for the SM.

Summary of ME/PS merging

- Merging matrix elements of various multiplicities with parton showers improves the predictive power of the parton shower outside the collinear/soft regions.
  - These matched samples give excellent prescription of the data (except for the total normalization).
- There is a dependence on the parameters responsible for the cut in phase-space (i.e. the matching scale).
- By letting the matrix elements mimic what the parton shower does in the collinear/soft regions (PDF/alpha$_s$ reweighting and including the Sudakov suppression) the dependence is greatly reduced.
- In practice, one should check explicitly that this is the case by plotting differential jet-rate plots for a couple of values for the matching scale.

NLO+PS matching

ME

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Shower MC

1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are collinear and/or soft
5. Partial interference through angular ordering
6. Needed for hadronization

Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

NLO+PS MATCHING

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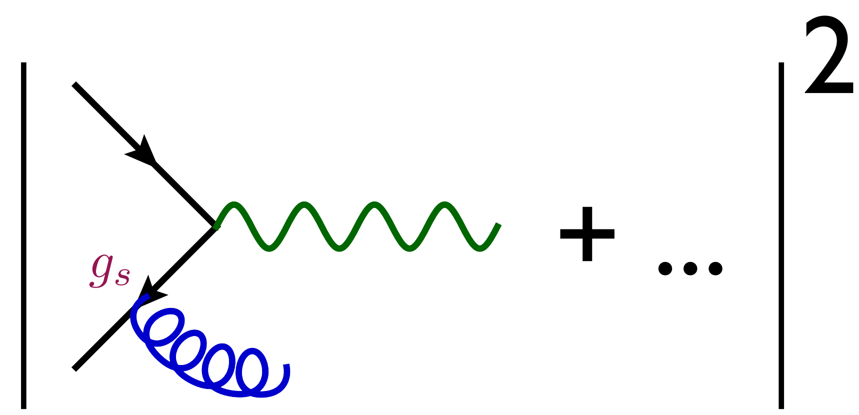
5. Partial interference through angular ordering
6. [illegible]

No longer true at NLO!

Approaches are complementary: merge them!

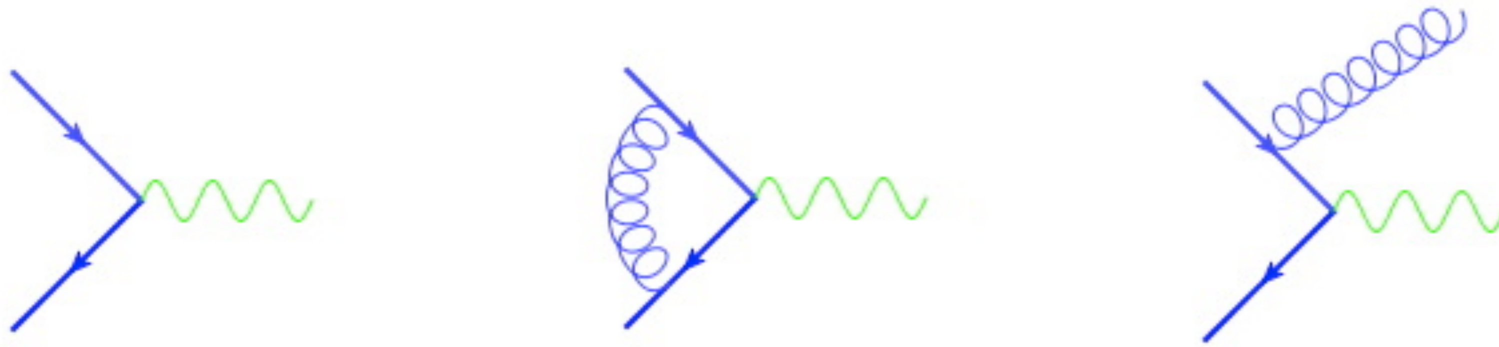
Difficulty: avoid double counting, ensure smooth distributions

At NLO



- We have to integrate the real emission over the **complete** phase-space of the one particle that can go soft or collinear to obtain the infra-red poles that will cancel against the virtual corrections
- We cannot use the same matching procedure: requiring that all partons should produce separate jets is not infrared safe
- We have to invent a new procedure to match NLO matrix elements with parton showers

Naive (wrong) approach



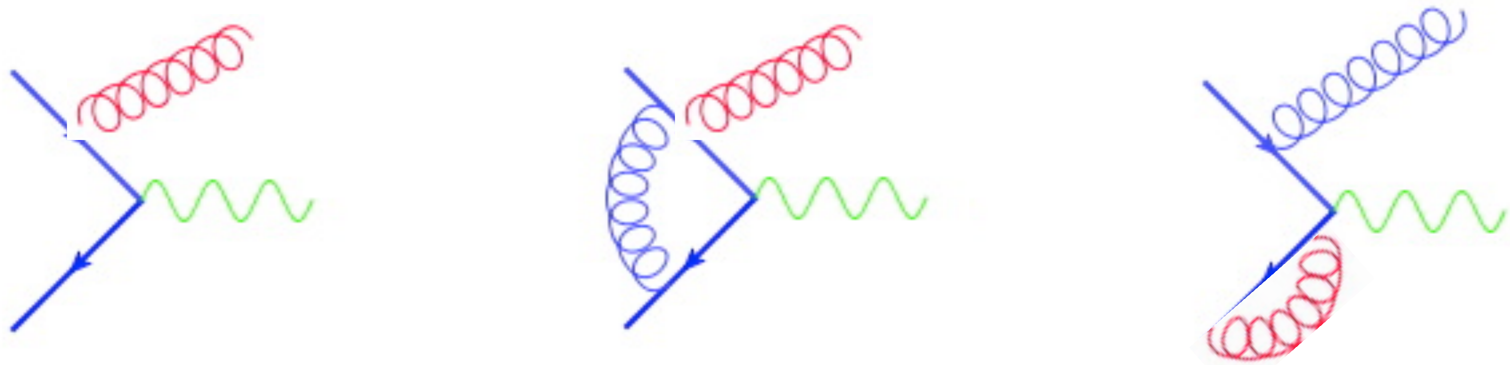
- In a fixed order calculation we have contributions with *m* final state particles and with *m*+1 final state particles

$$\sigma^{\mathrm{NLO}} \sim \int d^4\Phi_m\, B(\Phi_m) + \int d^4\Phi_m \int_{\mathrm{loop}} d^d l\, V(\Phi_m) + \int d^d\Phi_{m+1}\, R(\Phi_{m+1})$$

- We could try to shower them independently
- Let $I_{\mathrm{MC}}^{(k)}(O)$ be the parton shower spectrum for an observable O, showering from a k-body initial condition
- We can then try to shower the *m* and *m*+1 final states independently

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m\left(B + \int_{\mathrm{loop}} V\right)\right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} R\right] I_{\mathrm{MC}}^{(m+1)}(O)$$

NAIVE (WRONG) APPROACH



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Double counting

$$\frac{d\sigma_{\text{NLOwPS}}}{dO} = \left[d\Phi_m(B + \int_{\text{loop}} V)\right] I_{\text{MC}}^{(m)}(O) + \left[d\Phi_{m+1}R\right] I_{\text{MC}}^{(m+1)}(O)$$

- But this is wrong!
- If you expand this equation out up to NLO, there are more terms then there should be and the total rate does not come out correctly
- Schematically $I_{\text{MC}}^{(k)}(O)$ for 0 and 1 emission is given by

$$I_{\text{MC}}^{(k)}(O) \sim \Delta_a(Q^2, Q_0^2) + \Delta_a(Q^2, t) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s(t)}{2\pi} P_{a\to bc}(z)$$

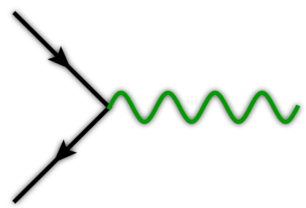
- And Δ is the Sudakov factor

$$\Delta_a(Q^2, t) = \exp\left[-\sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_s(t')}{2\pi} P_{a\to bc}\right]$$

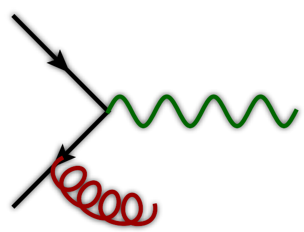
Sources of double counting

Parton shower →

Born+Virtual:



Real emission:



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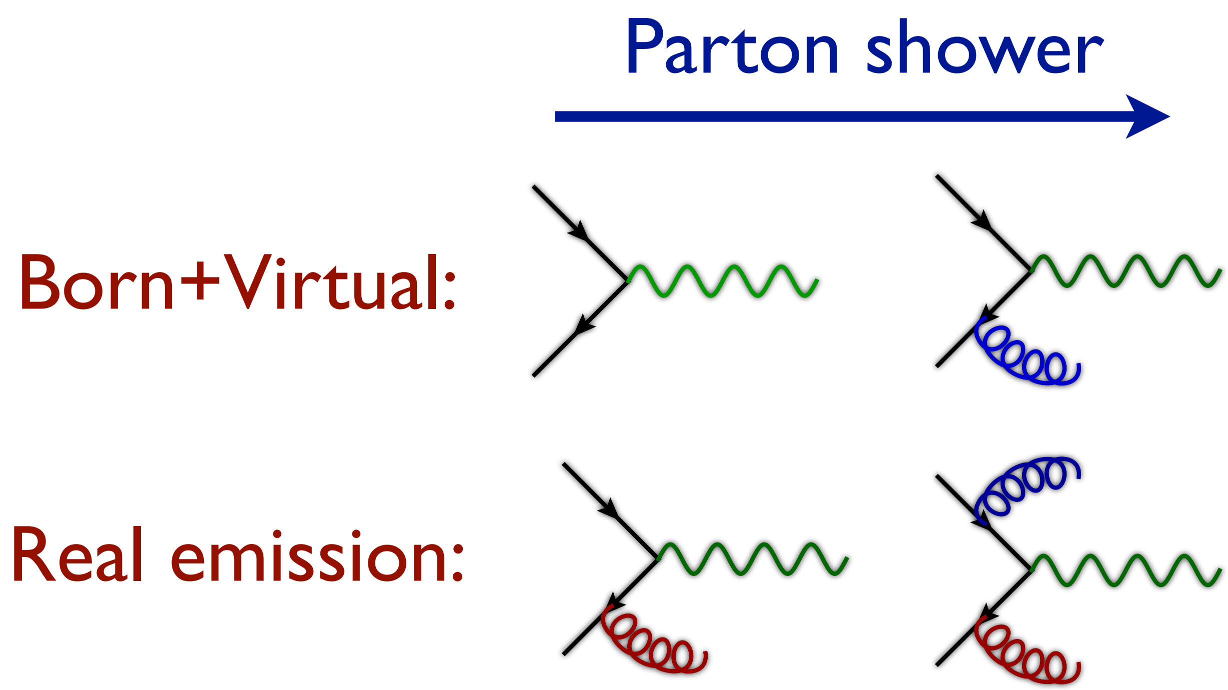

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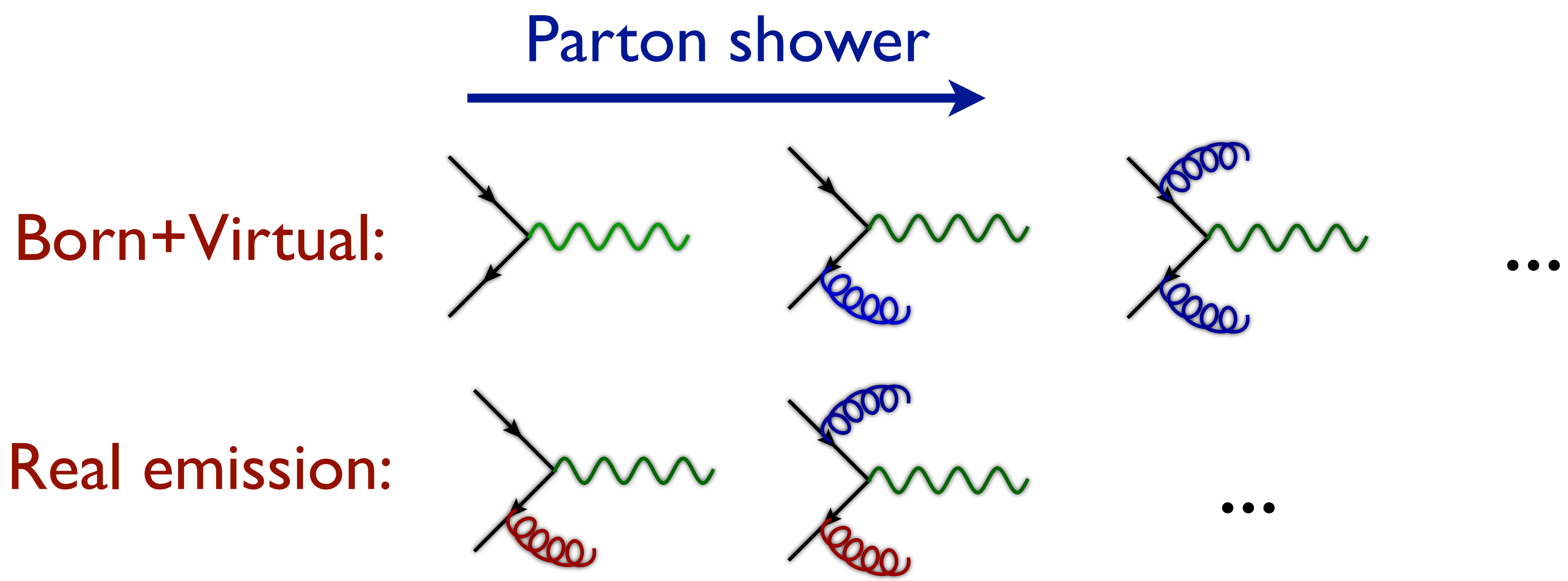

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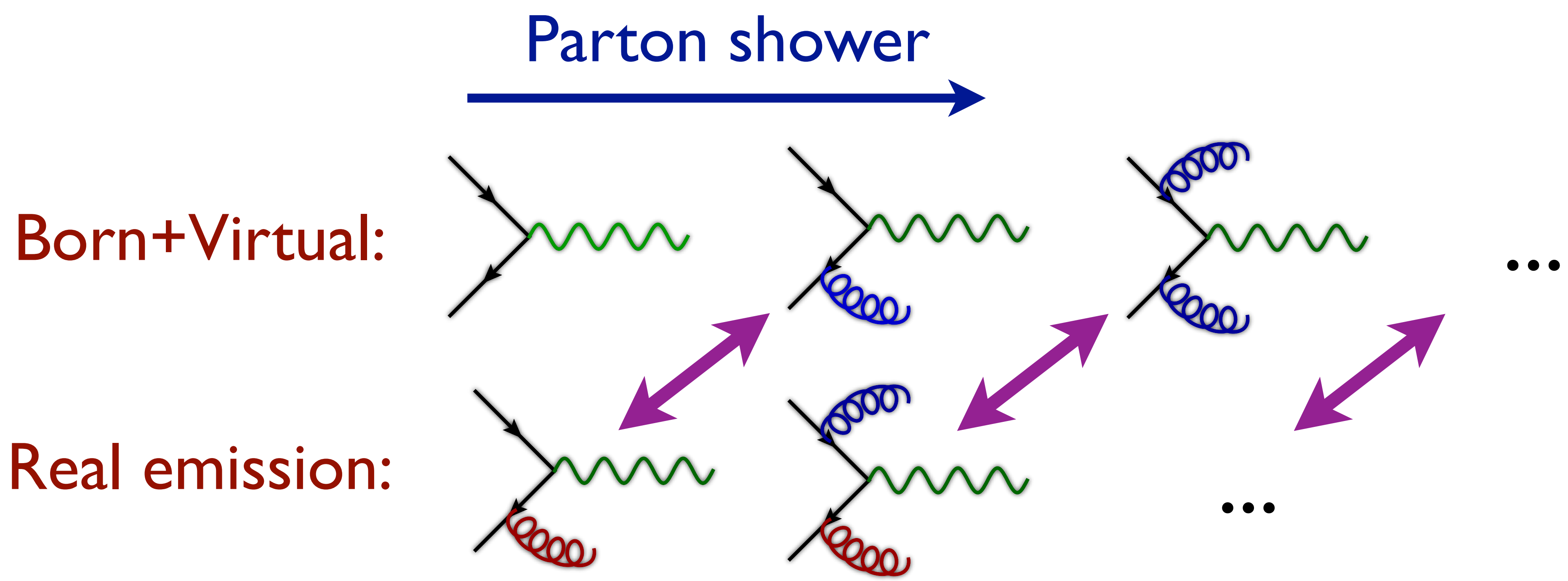

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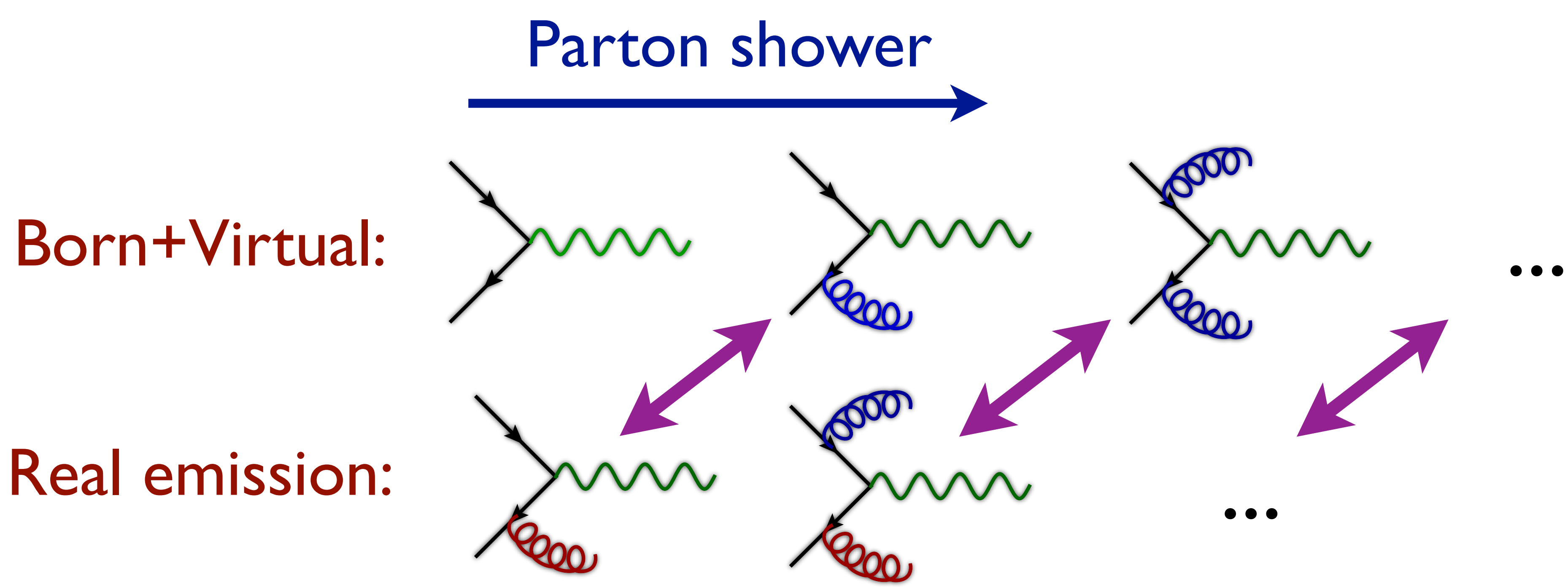


- There is double counting between the real emission matrix elements and the parton shower: the extra radiation can come from the matrix elements or the parton shower
- There is also an overlap between the virtual corrections and the Sudakov suppression in the zero-emission probability

Double counting in virtual/Sudakov

- The Sudakov factor **Δ** (which is responsible for the resummation of all the radiation in the shower) is the no-emission probability
- It's defined to be **Δ** = 1 - *P*, where P is the probability for a branching to occur
- By using this conservation of probability in this way, **Δ** contains contributions from the virtual corrections implicitly
- Because at NLO the virtual corrections are already included via explicit matrix elements, **Δ** is double counting with the virtual corrections
- In fact, because the shower is unitary, what we are double counting in the real emission corrections is exactly equal to what we are double counting in the virtual corrections (but with opposite sign)!

Avoiding double counting

- There are two methods to circumvent this double counting
  - MC@NLO (Frixione & Webber)
  - POWHEG (Nason)

MC@NLO PROCEDURE

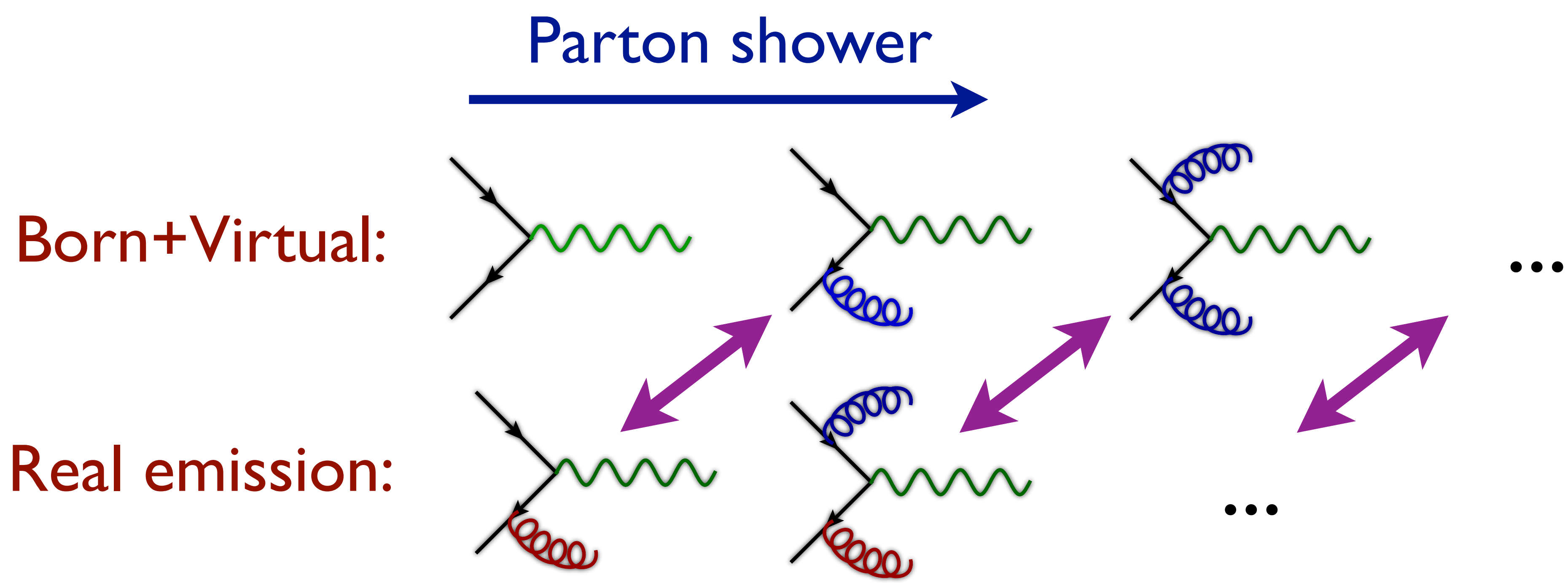
[Frixione & Webber (2002)]

- To remove the double counting, we can add and subtract the same term to the m and m+1 body configurations

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

- Where the *MC* are defined to be the contribution of the parton shower to get from the m body Born final state to the m+1 body real emission final state

MC@NLO PROCEDURE



$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC \right) \right] I_{\mathrm{MC}}^{(m)}(O)$$
$$+ \left[d\Phi_{m+1} \left(R - MC \right) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

- Double counting is explicitly removed by including the "shower subtraction terms"

MC@NLO PROPERTIES

- Good features of including the subtraction counter terms

  1. **Double counting avoided**: The rate expanded at NLO coincides with the total NLO cross section

  2. **Smooth matching**: MC@NLO coincides (in shape) with the parton shower in the soft/collinear region, while it agrees with the NLO in the hard region

  3. **Stability:** weights associated to different multiplicities are separately finite. The *MC* term has the same infrared behavior as the real emission (there is a subtlety for the soft divergence)

- Not so nice feature (for the developer not for the user..!)

  1. **Parton shower dependence:** the form of the *MC* terms depends on what the parton shower does exactly. Need special subtraction terms for each parton shower to which we want to match

Double counting avoided

$$\frac{d\sigma_{\rm NLOwPS}}{dO} = \left[d\Phi_m (B + \int_{\rm loop} V + \int d\Phi_1 MC) \right] I_{\rm MC}^{(m)}(O) + \left[d\Phi_{m+1} (R - MC) \right] I_{\rm MC}^{(m+1)}(O)$$

- Expanded at NLO

$$I_{\rm MC}^{(m)}(O) dO = 1 - \int d\Phi_1 \frac{MC}{B} + d\Phi_1 \frac{MC}{B} + \ldots$$

$$d\sigma_{\rm NLOwPS} = \left[d\Phi_m (B + \int_{\rm loop} V + \int d\Phi_1 MC) \right] I_{\rm MC}^{(m)}(O) dO + \left[d\Phi_{m+1} (R - MC) \right]$$

$$\simeq d\Phi_m (B + \int_{\rm loop} V) + d\Phi_{m+1} R = d\sigma_{\rm NLO}$$

Smooth matching

$$\frac{d\sigma_{\rm NLOwPS}}{dO} = \left[d\Phi_m (B + \int_{\rm loop} V + \int d\Phi_1 MC) \right] I_{\rm MC}^{(m)}(O)$$
$$+ \left[d\Phi_{m+1}(R - MC) \right] I_{\rm MC}^{(m+1)}(O)$$

- Smooth matching:
  - Soft/collinear region: $R \simeq MC \Rightarrow d\sigma_{\rm MC@NLO} \sim I_{\rm MC}^{(m)}(O) dO$
  - Hard region, shower effects suppressed, ie.

    $MC \simeq 0 \quad I_{\rm MC}^{(m)}(O) \simeq 0 \quad I_{\rm MC}^{(m+1)}(O) \simeq 1$

    $\Rightarrow d\sigma_{\rm MC@NLO} \sim d\Phi_{m+1} R$

STABILITY & UNWEIGHTING

$$\frac{d\sigma_{\rm NLOwPS}}{dO} = \left[d\Phi_m \left(B + \int_{\rm loop} V + \int d\Phi_1 MC\right)\right] I_{\rm MC}^{(m)}(O)$$
$$+ \left[d\Phi_{m+1}(R - MC)\right] I_{\rm MC}^{(m+1)}(O)$$

- The MC subtraction terms are defined to be what the shower does to get from the m to the m+1 body matrix elements. Therefore the cancellation of singularities is exact in the (R - MC) term: there is no mapping of the phase-space in going from events to counter events as we saw in the FKS subtraction

- The integral is bounded all over phase-space; we can therefore generate unweighted events!
  - "S-events" (which have m body kinematics)
  - "H-events" (which have m+1 body kinematics)

Negative weights

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC \right) \right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1} \left(R - MC \right) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

- We generate events for the two terms between the square brackets (S- and H-events) separately
- There is no guarantee that these contributions are separately positive (even though predictions for infra-red safe observables should always be positive!)
- Therefore, when we do event unweighting we can only unweight the events **up to a sign**. These signs should be taken into account when doing a physics analysis (i.e. making plots etc.)
- **The events are only physical when they are showered.**

Example : ttbar production

A NLO calculation always refers to an IR-safe observable.

An NLO code will, in general, be able to produce results for several quantities and distributions, only some of which will be at NLO accuracy.

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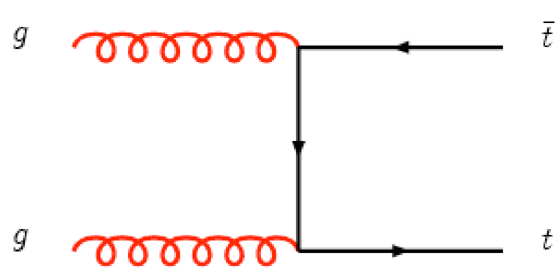
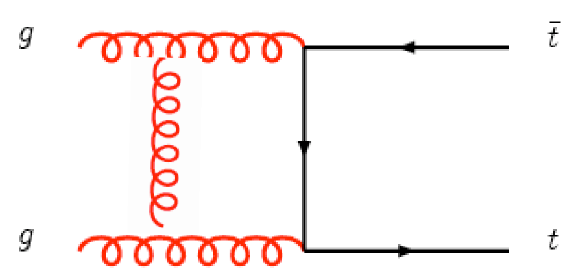


LO



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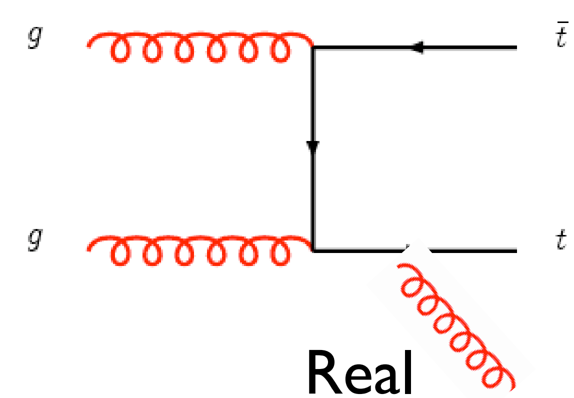

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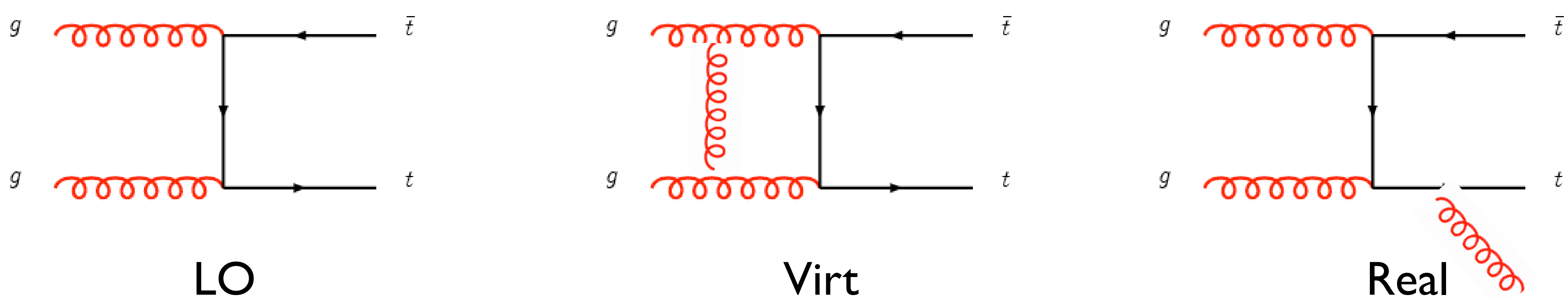

LO Virt Real

- Total cross section, $\sigma(t\bar{t})$
- P_T of one top quark
- P_T of the $t\bar{t}$ pair
- P_T of the jet
- $t\bar{t}$ invariant mass, $m(t\bar{t})$
- $\Delta\Phi(t\bar{t})$

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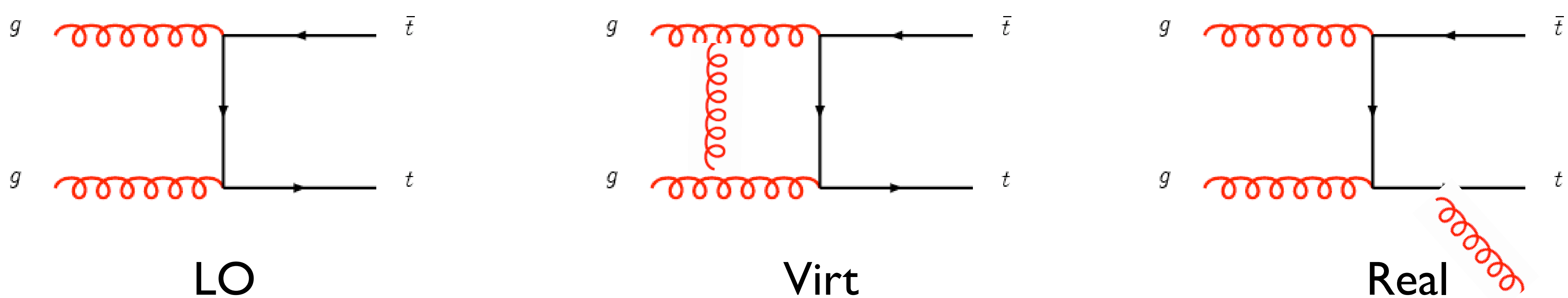

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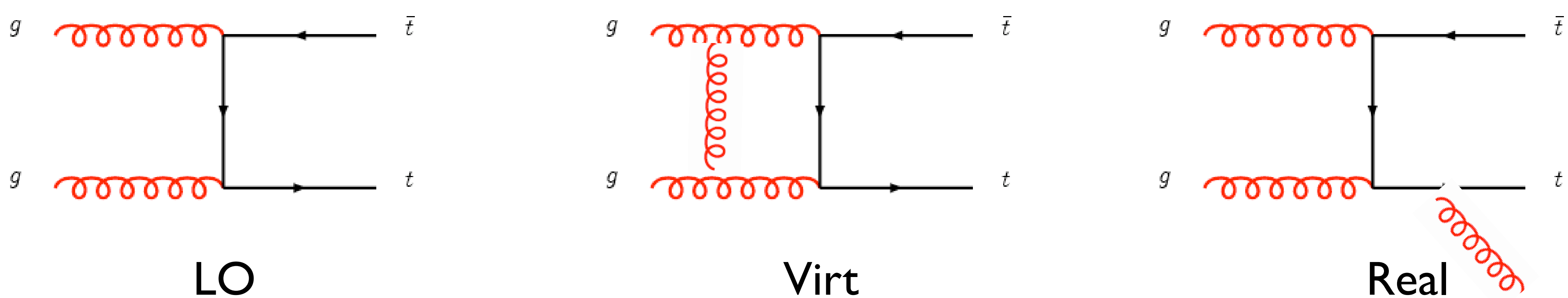

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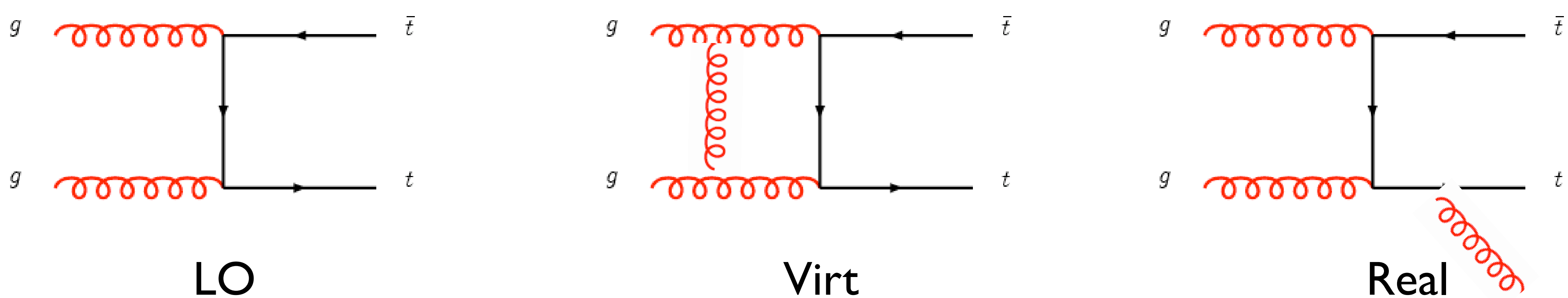

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EXAMPLE : TTBAR PRODUCTION

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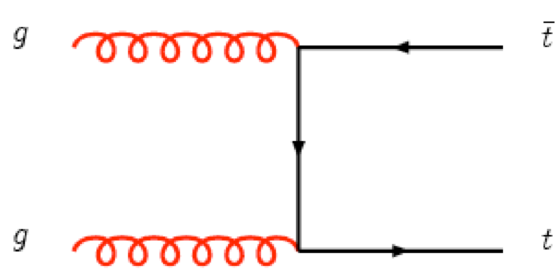

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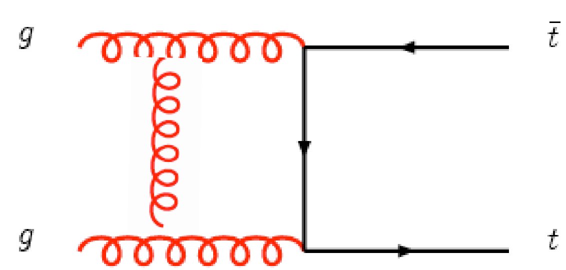

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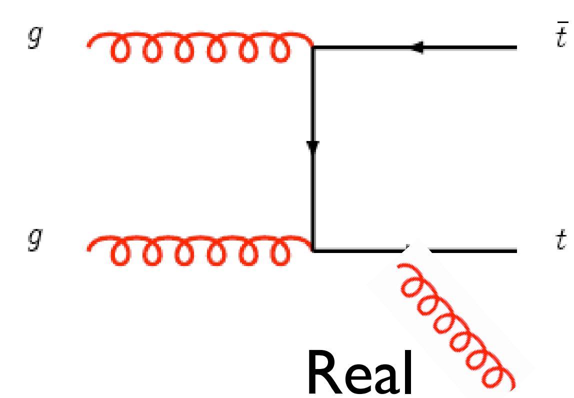

Real

- ☞ Total cross section, σ(tt) .... ✓
- ☞ P_T of one top quark .... ✓
- ☞ P_T of the tt pair .... ✗
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- ☞ tt invariant mass, m(tt)
- ☞ $\Delta\Phi$(tt)

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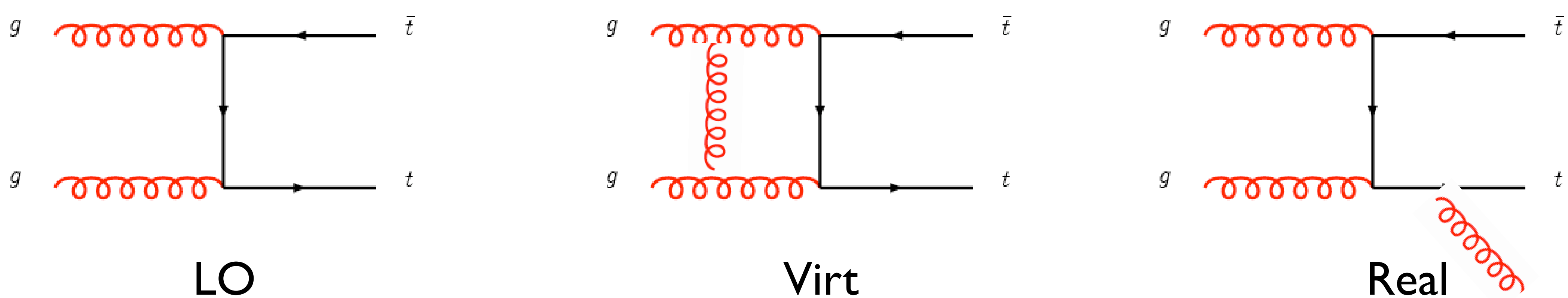

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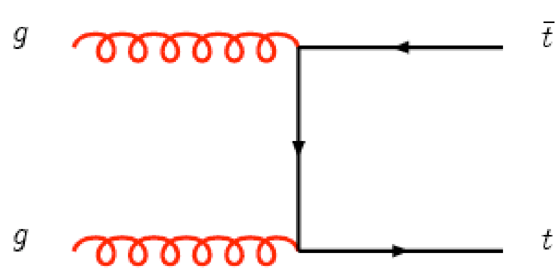

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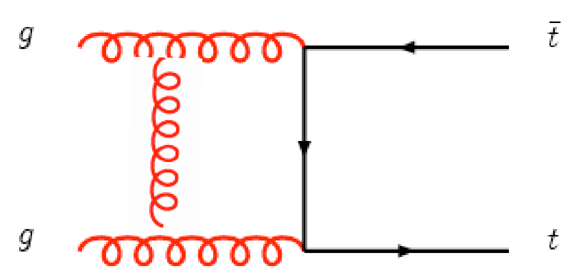

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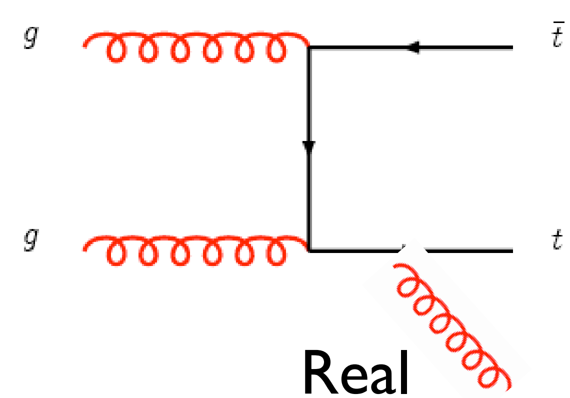

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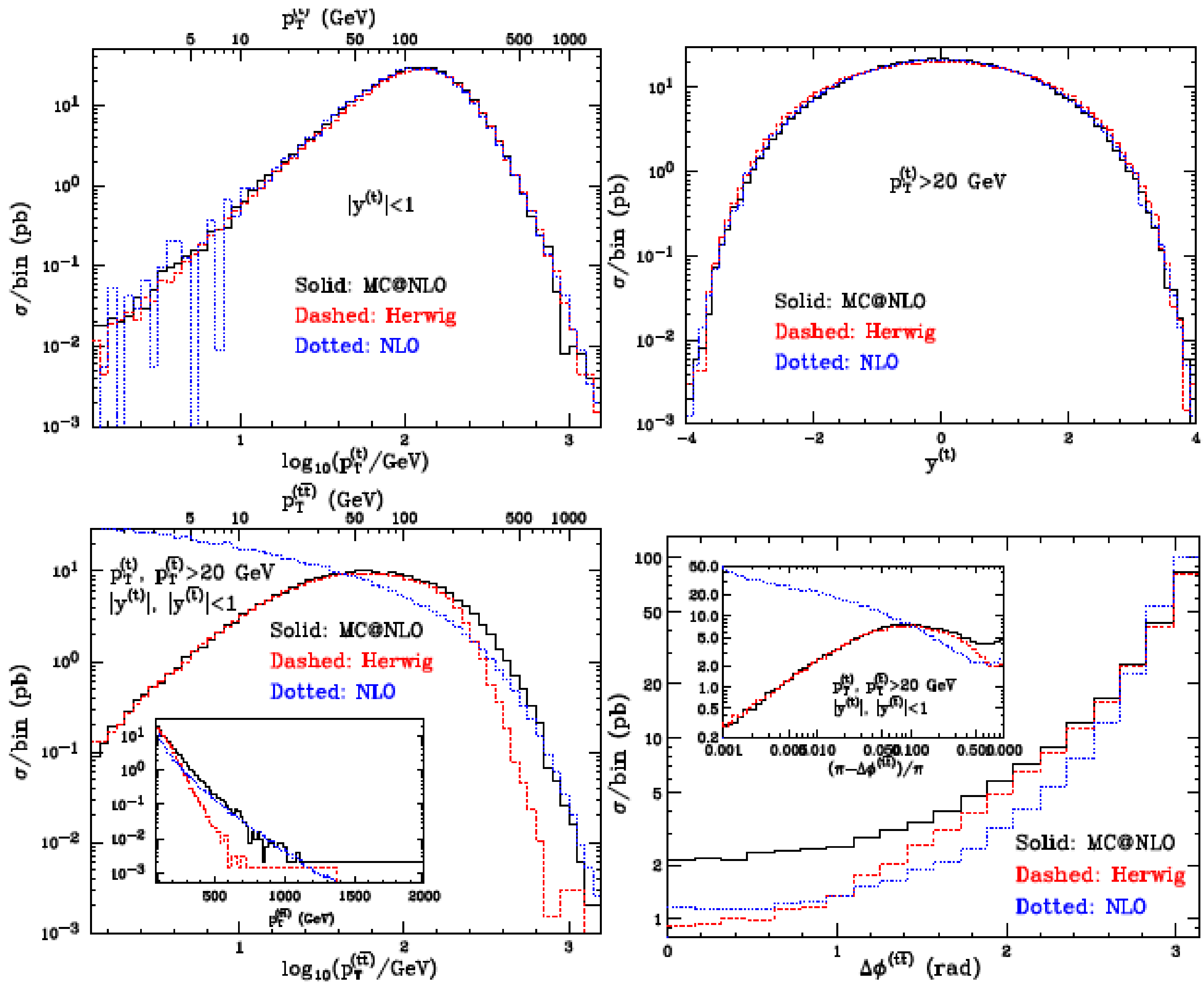
Example : ttbar production



POWHEG

Nason (2004)

- Consider the probability of the first emission of a leg (inclusive over later emissions)

$$d\sigma = d\sigma_m d\Phi_m \left[\Delta(Q^2, Q_0^2) + \Delta(Q^2, t) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) \right]$$

- In the notation used here, this is equivalent to

$$d\sigma = d\Phi_m \, B \left[\Delta(Q^2, Q_0^2) + \Delta(Q^2, t) d\Phi_{(+1)} \frac{MC}{B} \right]$$

- One could try to get NLO accuracy by replacing B with the NLO rate (integrated over the extra phase-space)

$$B \to B + V + \int d\Phi_{(+1)} \, R$$

- This naive definition is not correct: the radiation is still described only at leading logarithmic accuracy, which is not correct for hard emissions.

POWHEG

- This is double counting.
  To see this, expand the equation up to the first emission
  $$d\Phi_B \left[B + V + \int d\Phi_{(+1)} R\right] \left[1 - \int d\Phi_{(+1)} \frac{MC}{B} + d\Phi_{(+1)} \frac{MC}{B}\right]$$
  which is not equal to the NLO

- In order to avoid double counting, one should replace the definition of the Sudakov form factor with the following:

$$\Delta(Q^2, Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2} d\Phi_{(+1)} \frac{MC}{B}\right] \to \tilde{\Delta}(Q^2, Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2} d\Phi_{(+1)} \frac{R}{B}\right]$$

  corresponding to a modified differential branching probability

$$d\tilde{p} = d\Phi_{(+1)} R/B$$

- Therefore we find for the POWHEG differential cross section

$$d\sigma_{\rm POWHEG} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R\right] \left[\tilde{\Delta}(Q^2, Q_0^2) + \tilde{\Delta}(Q^2, t)\ d\Phi_{(+1)} \frac{R}{B}\right]$$

Properties

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[\tilde{\Delta}(Q^2, Q_0^2) + \tilde{\Delta}(Q^2, t)\ d\Phi_{(+1)} \frac{R}{B} \right]$$

- The term in the square brackets integrates to one (integrated over the extra parton phase-space between scales Q_0^2 and Q^2)
  (this can also be understood as unitarity of the shower below scale t)
  POWHEG cross section is normalized to the NLO

- Expand up to the first-emission level:

  $$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[1 - \int d\Phi_{(+1)} \frac{R}{B} + d\Phi_{(+1)} \frac{R}{B} \right] = d\sigma_{\text{NLO}}$$

  so double counting is avoided

- Its structure is identical an ordinary shower, with normalization rescaled by a global K-factor and a different Sudakov for the first emission: no negative weights are involved.

MC@NLO AND POWHEG

MC@NLO and POWHEG

$$d\sigma^{\rm NLO+PS} = d\Phi_B \bar{B}^s(\Phi_B)\left[\Delta^s(p_\perp^{\rm min}) + d\Phi_{R|B}\frac{R^s(\Phi_R)}{B(\Phi_B)}\Delta^s(p_T(\Phi))\right] + d\Phi_R R^f(\Phi_R)$$

MC@NLO and POWHEG

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integrates to 1 (unitarity)

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integrates to 1 (unitarity)

with

$$\bar{B}^s = B(\Phi_B) + \left[V(\Phi_B) + \int d\Phi_{R|B} R^s(\Phi_{R|B})\right]$$

Full cross section at fixed Born kinematics (If F=1).

$$R(\Phi_R) = R^s(\Phi_R) + R^f(\Phi_R)$$

MC@NLO and POWHEG

$$d\sigma^{\mathrm{NLO+PS}} = d\Phi_B \bar{B}^s(\Phi_B)\left[\Delta^s(p_\perp^{\mathrm{min}}) + d\Phi_{R|B}\frac{R^s(\Phi_R)}{B(\Phi_B)}\Delta^s(p_T(\Phi))\right] + d\Phi_R R^f(\Phi_R)$$

integrates to 1 (unitarity)

with

$$\bar{B}^s = B(\Phi_B) + \left[V(\Phi_B) + \int d\Phi_{R|B} R^s(\Phi_{R|B})\right]$$

Full cross section at fixed Born kinematics (If F=1).

$$R(\Phi_R) = R^s(\Phi_R) + R^f(\Phi_R)$$

This formula is valid both for both MC@NLO and POWHEG

MC@NLO and POWHEG

$$d\sigma^{\rm NLO+PS} = d\Phi_B \bar{B}^s(\Phi_B) \left[\Delta^s(p_\perp^{\rm min}) + d\Phi_{R|B} \frac{R^s(\Phi_R)}{B(\Phi_B)} \Delta^s(p_T(\Phi)) \right] + d\Phi_R R^f(\Phi_R)$$

integrates to 1 (unitarity)

with

$$\bar{B}^s = B(\Phi_B) + \left[V(\Phi_B) + \int d\Phi_{R|B} R^s(\Phi_{R|B}) \right]$$

Full cross section at fixed Born kinematics (If F=1).

$$R(\Phi_R) = R^s(\Phi_R) + R^f(\Phi_R)$$

This formula is valid both for both MC@NLO and POWHEG

MC@NLO: $R^s(\Phi) = P(\Phi_{R|B})\, B(\Phi_B)$

Needs exact mapping (ΦB,ΦR) →Φ

POWHEG: $R^s(\Phi) = F R(\Phi)\, , R^f(\Phi) = (1-F) R(\Phi)$

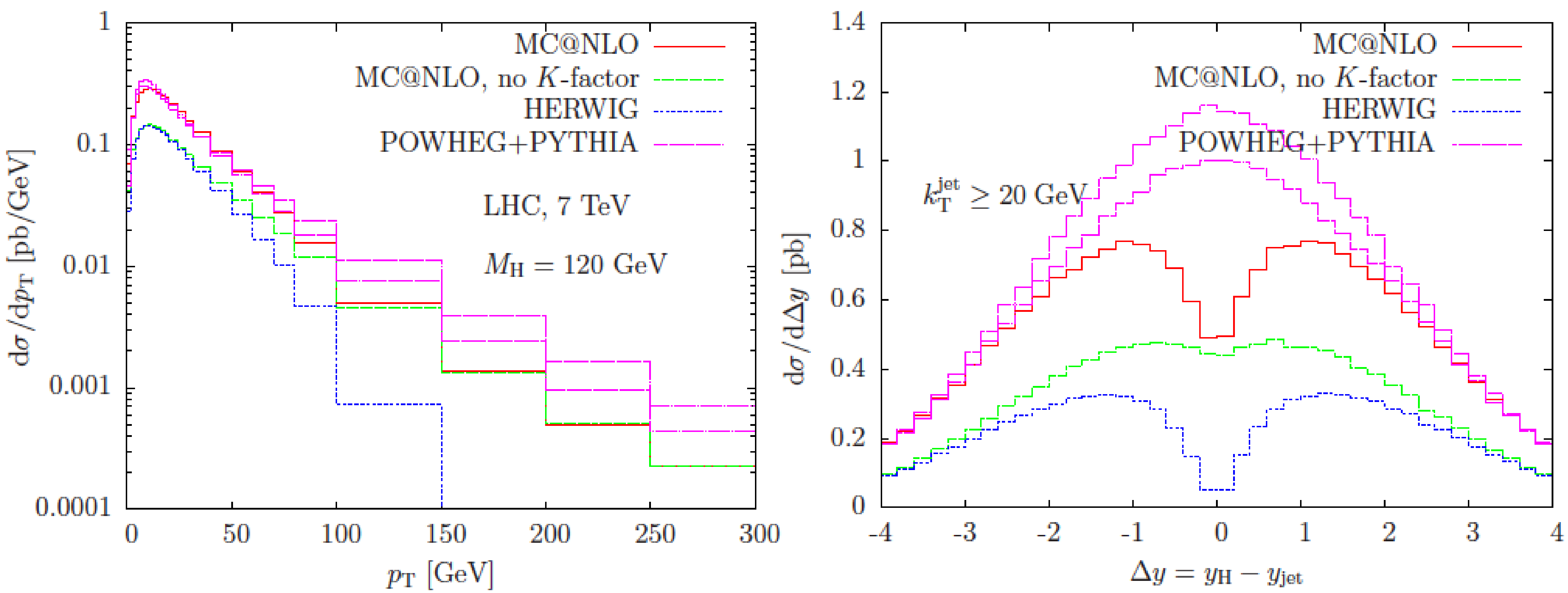
F=1 = Exponentiates the Real. It can be damped by hand.

MC@NLO and POWHEG

	MC@NLO	POWHEG
MC@NLO does not exponentiate the non-singular part of the real emission amplitudes	☺	☹
MC@NLO does not require any tricks for treating Born zeros	☺	☹
POWHEG is independent from the parton shower (although, in general the shower should be a truncated vetoed)	☹	☺
POWHEG is (almost) negative weighted events free	☹	☺
Automation of the method: http://amcatnlo.cern.ch http://powhegbox.mib.infn.it/	☺	☺

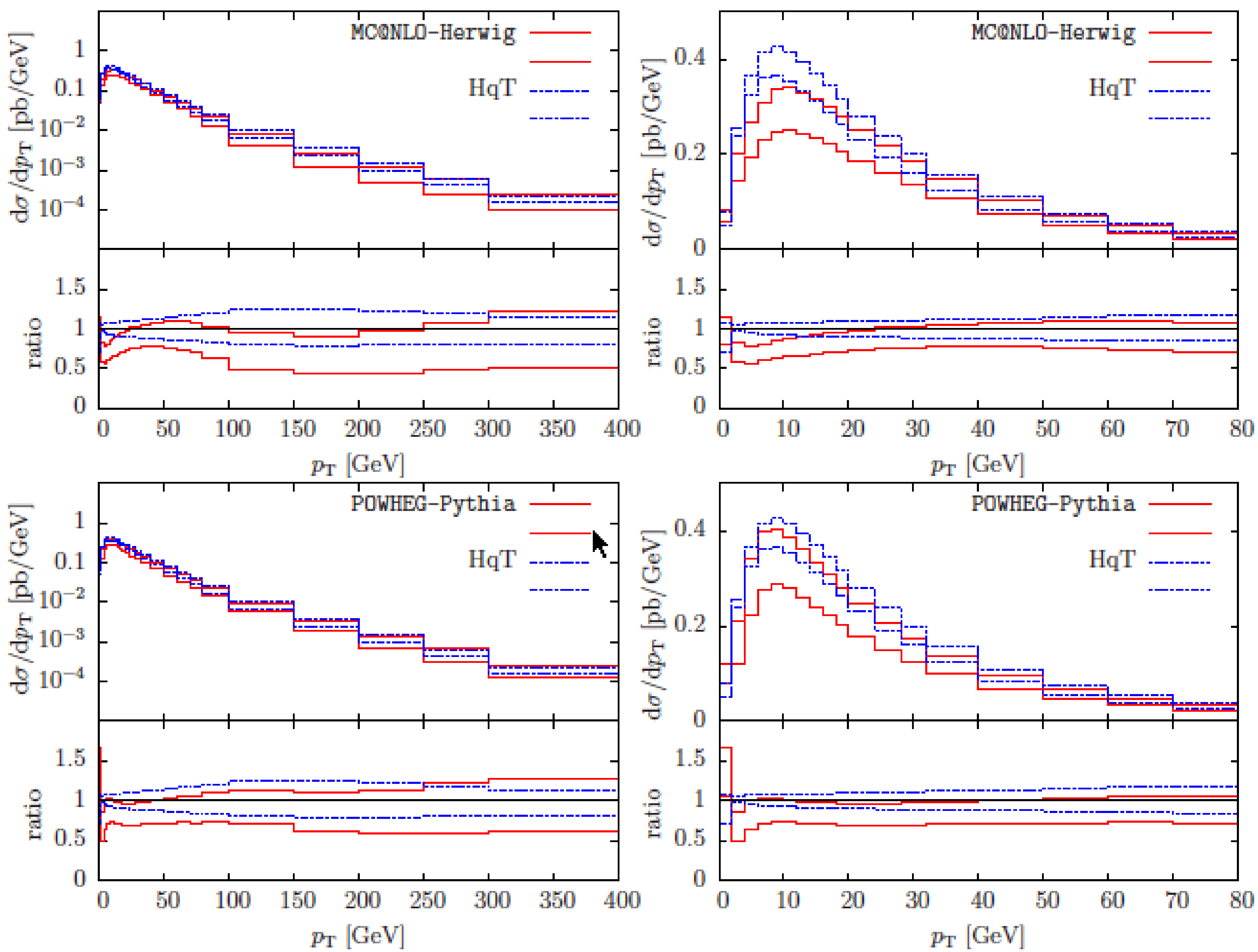
MC@NLO and POWHEG

Nason and Webber 2012



Pt of the Higgs in ggH

MC@NLO and POWHEG



SUMMARY

- We want to match NLO computations to parton showers to keep the good features of both approximations

  - In the MC@NLO method:
    by including the shower subtraction terms in our process we avoid double counting between NLO processes and parton showers

  - In the POWHEG method:
    apply an overall K-factor, and modify the (Sudakov of the) first emission to fill the hard region of phase-space according to the real-emission matrix elements

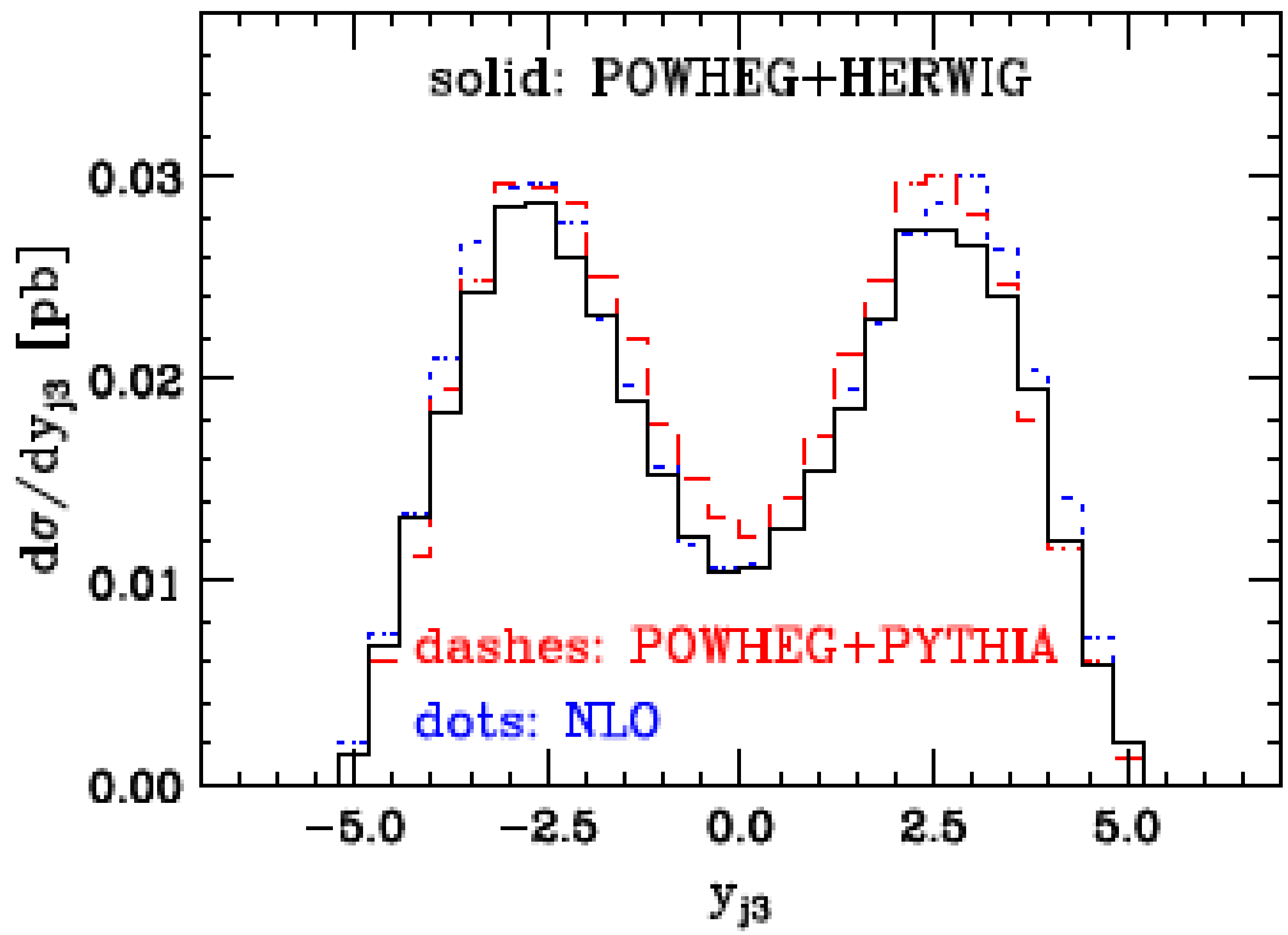
- First studies to combine NLO+PS matching with ME+PS merging have been made, but nothing 100% satisfactory has come out yet...

STATE OF THE ART

POWHEG BOX

Public framework to promote any NLO calculation into NLO+PS via the POWHEG method. Several processes implemented and available now:

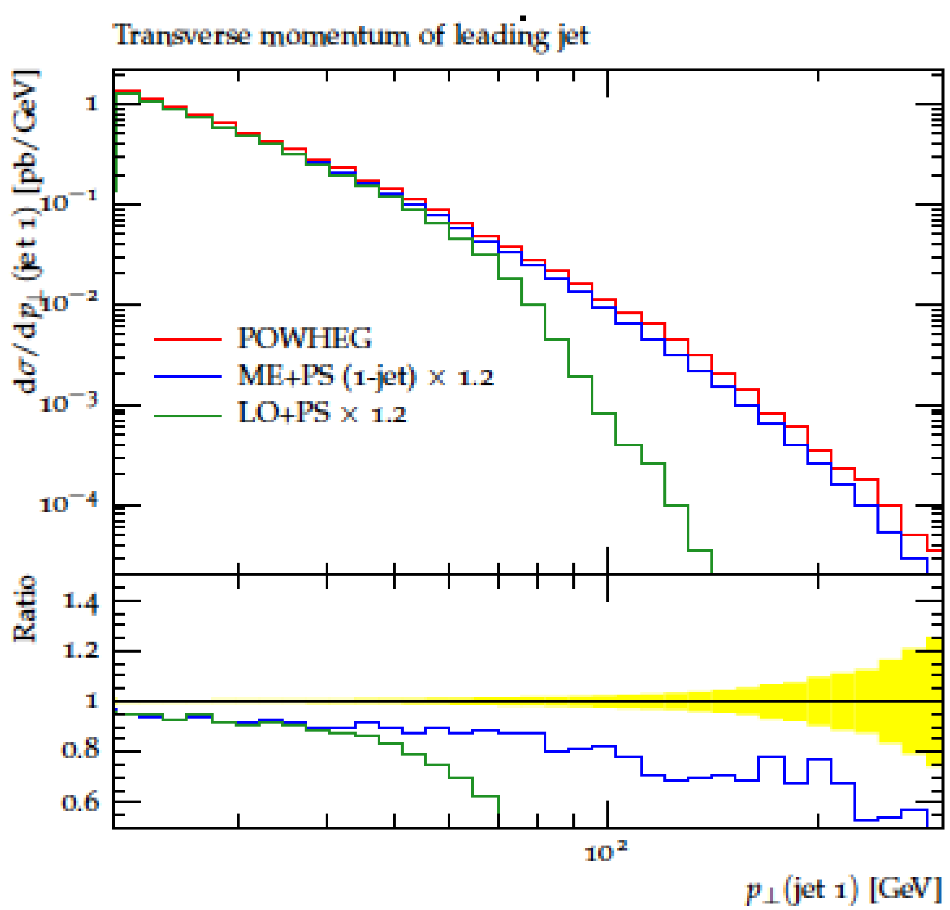
- jj, QQ
- W, Z inclusive
- Wj,Zj
- Zjj
- Wbb
- WW,WZ,ZZ
- W±W±jj
- single top
- H (with hvq loops)
- Hj, Hjj
- VBF
- tH^+



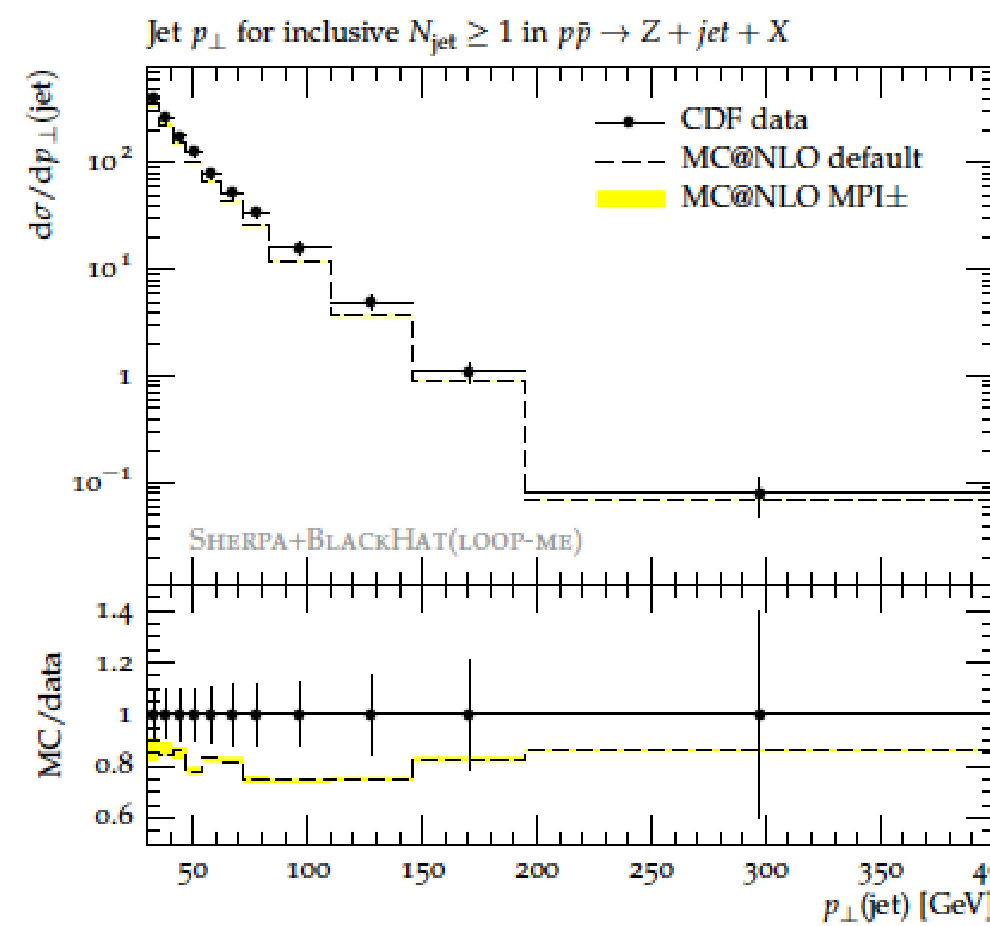
pp→Hjj

SHERPA

SHERPA has implemented both MC@NLO and POWHEG methods. It uses external loop amplitudes, while the rest is automatic. Several processes available now in particular with extra jets.



W



Z+1 jet

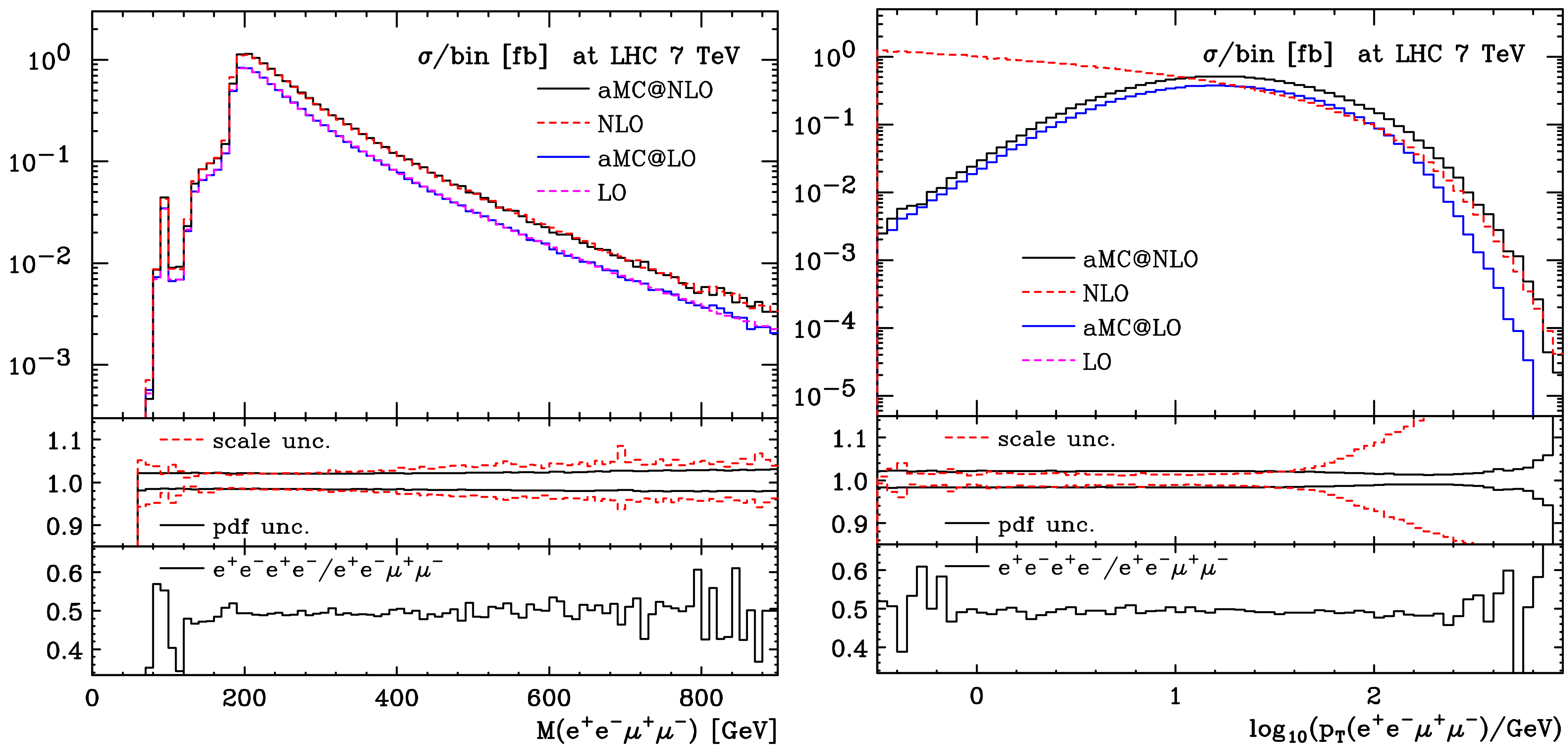
aMC@NLO

Fully automatic implementation of the MC@NLO method using MadLoop and MadFKS.

- Large class of processes available as they can be generated automatically.
- Automatic scale and PDF uncertainties without need of rerunning.
- NLO+PS for processes with n-jets tested and validated.
- Public release coming via MG5 soon..

Let's see a few examples in detail...

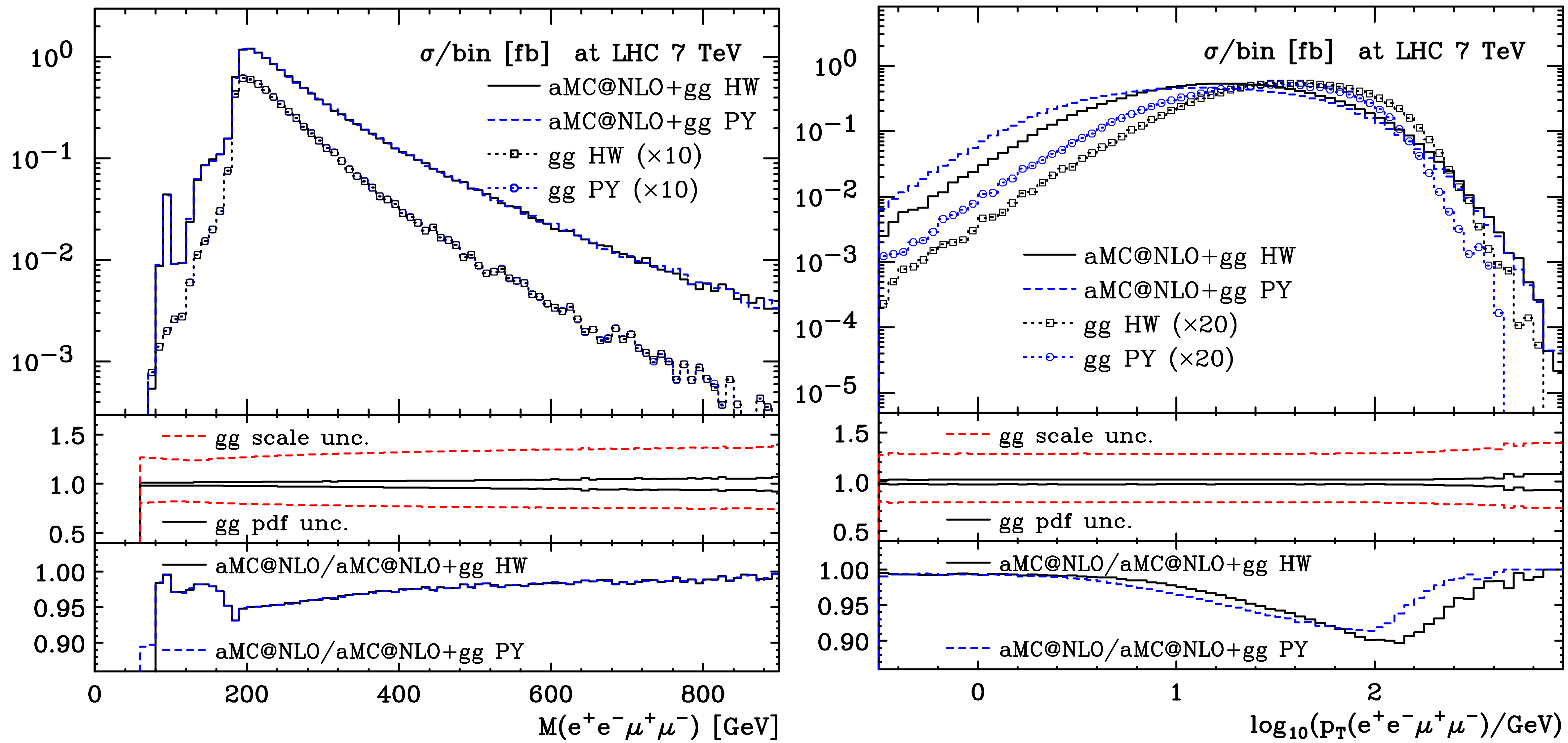
Four-lepton production



- 4-lepton invariant mass is almost insensitive to parton shower effects. 4-lepton transverse moment is extremely sensitive
- Including scale uncertainties

Monday 10 June 2013

Four-lepton production



- Differences between Herwig (black) and Pythia (blue) showers large in the Sudakov suppressed region (much larger than the scale uncertainties)
- Contributions from gg initial state (formally NNLO) are of 5-10%

pp → HTT/ATT

- Top pair production in association with a (pseudo-)scalar Higgs boson
- Three scenarios

I) scalar Higgs H, with $m_H = 120$ GeV

II) pseudo-scalar Higgs A, with $m_A = 120$ GeV

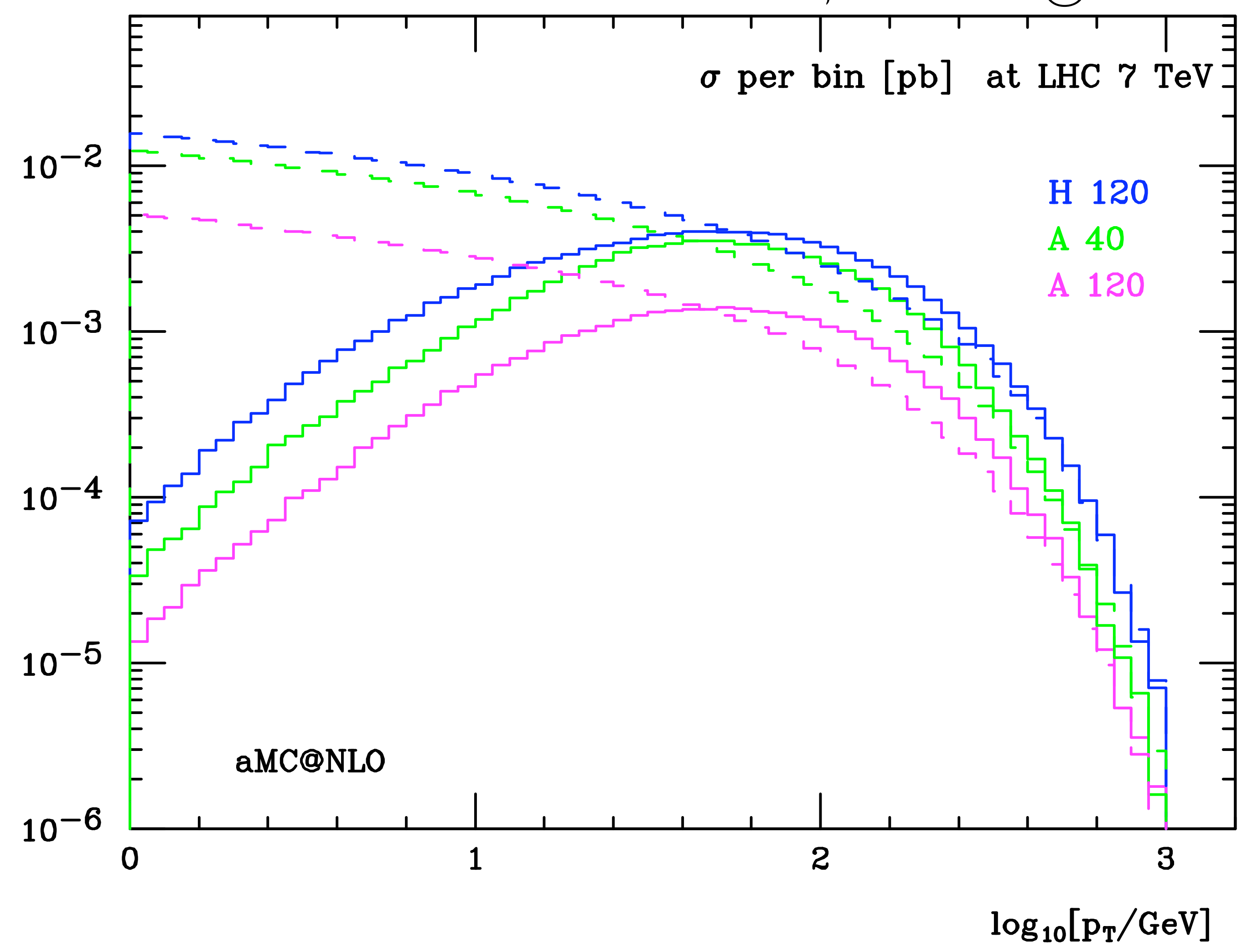
III) pseudo-scalar Higgs A, with $m_A = 40$ GeV

- SM-like Yukawa coupling, $y_t/\sqrt{2} = m_t/v$
- Renormalization and factorization scales $\mu_F = \mu_R = \left(m_T^t m_T^{\bar{t}} m_T^{H/A}\right)^{\frac{1}{3}}$ with $m_T = \sqrt{m^2 + p_T^2}$ and $m_t^{pole} = m_t^{MS} = 172.5$ GeV
- Note: first time that pp → ttA has been computed beyond LO

PP → HTT/ATT

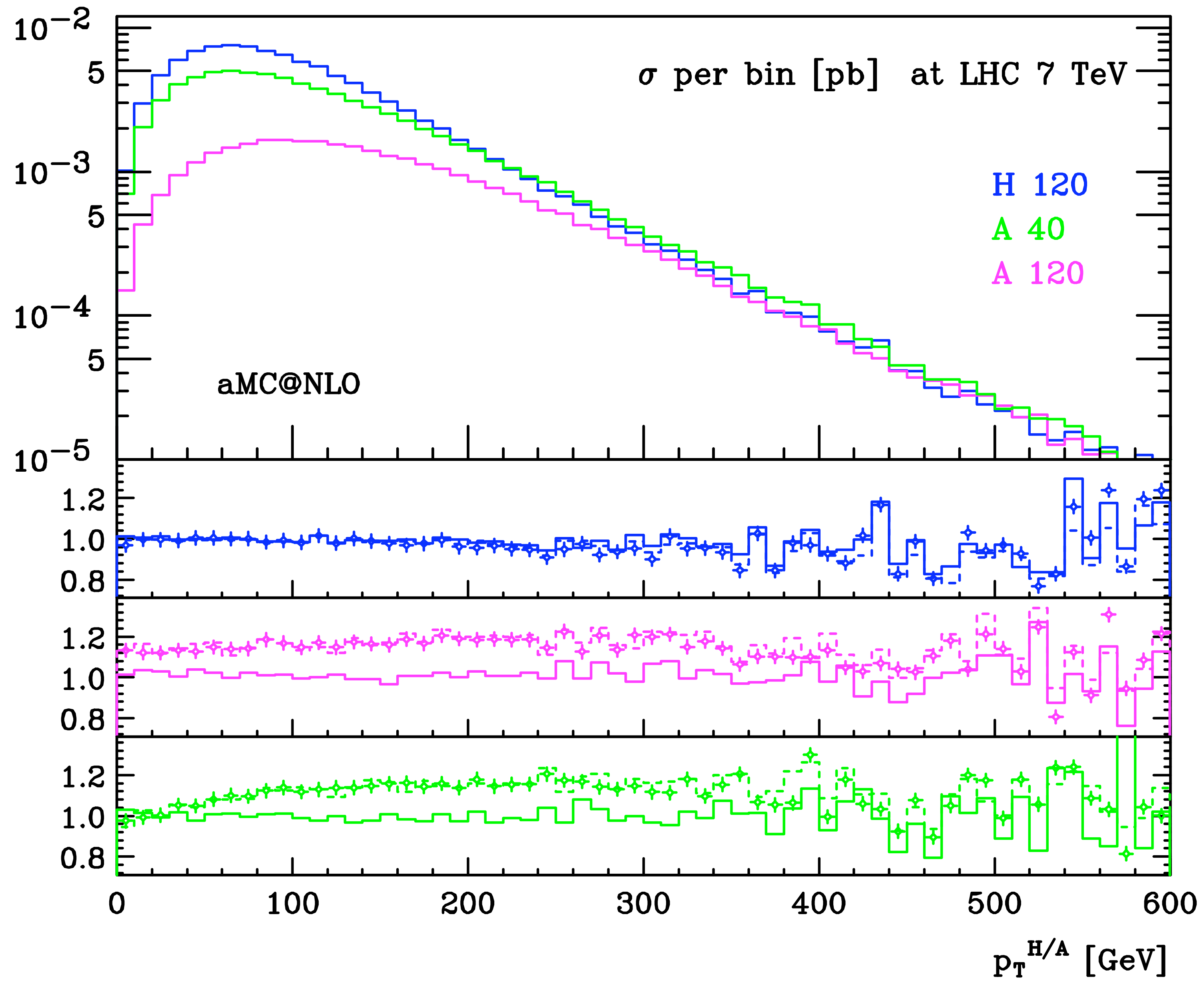
- Three particle transverse momentum, p_T(H/A t tbar), is obviously sensitive to the impact of the parton shower
- Infrared sensitive observable at the pure-NLO level for $p_T \to 0$
- aMC@NLO displays the usual Sudakov suppression
- At large p_T's the two descriptions coincide in shape and rate

Dashed: NLO, Solid: aMC@NLO



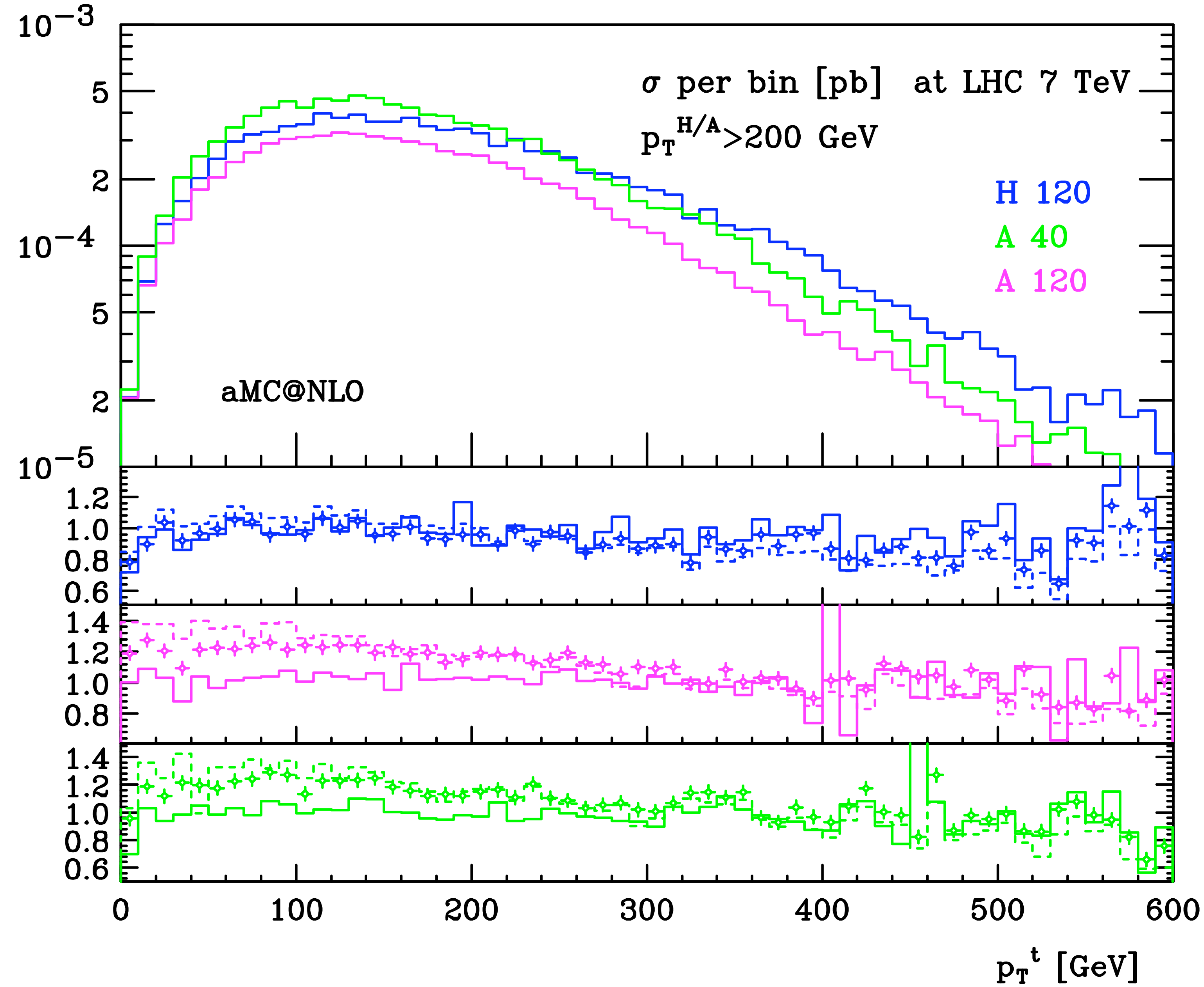
PP → HTT/ATT

- Transverse momentum of the Higgs boson
- Lower panels show the ratio with LO (dotted), NLO (solid) and aMC@LO (crosses)
- Corrections are small and fairly constant
- At large p_T, scalar and pseudo-scalar production coincide: boosted Higgs scenario [*Butterworth et al., Plehn et al.*] should work equally well for pseudo-scalar Higgs



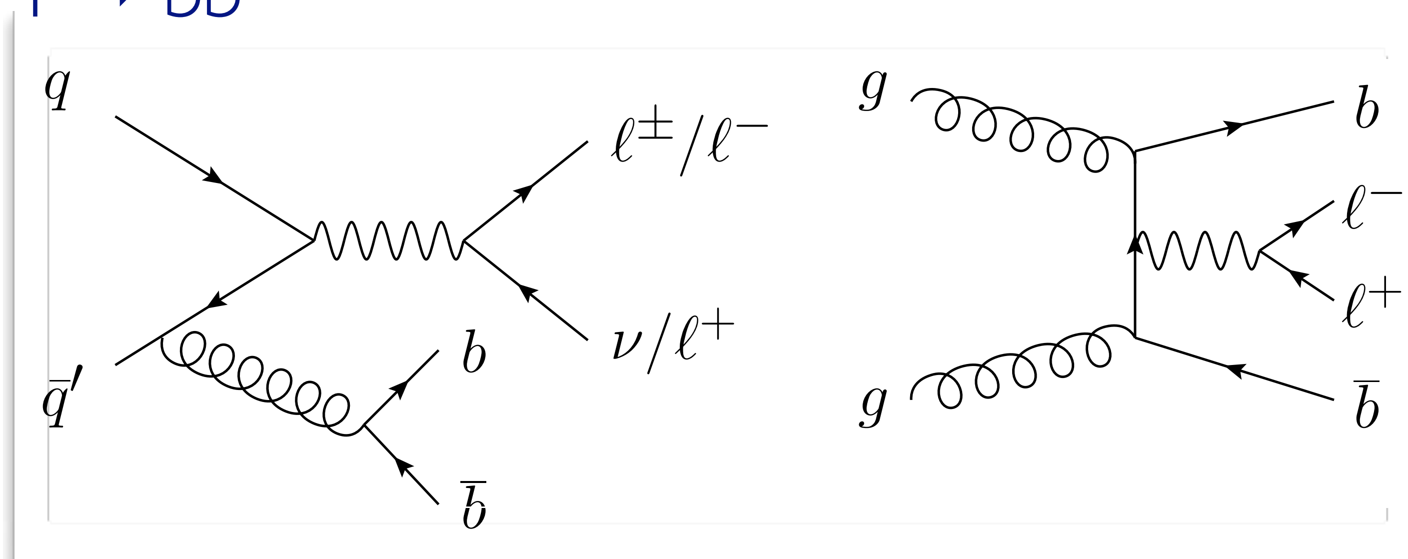
PP → HTT/ATT

- Boosted Higgs: $p_T^{H/A} > 200$ GeV
- Transverse momentum of the top quark
- Corrections compared to (MC@)LO are significant and cannot be approximated by a constant K-factor



pp → Wbb/Zbb

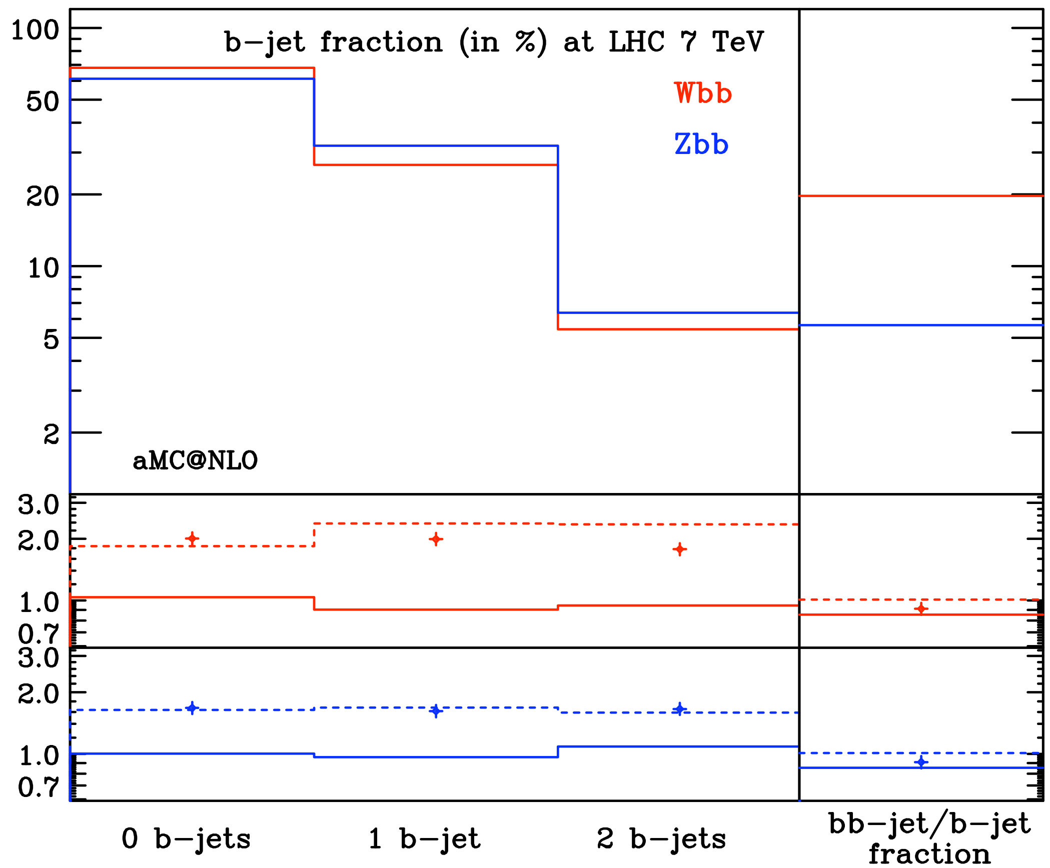
- Background to pp → HW/HZ, H → bb
- 4 Flavor scheme calculations
  - Massive b quarks
  - No initial state b quarks
  - Born is finite: no generation cuts are needed
- At LO, Wbb is purely qq induced, while Zbb has also contributions from gg initial states
- Cross sections for Zbb and Wbb are similar at LHC 7 TeV

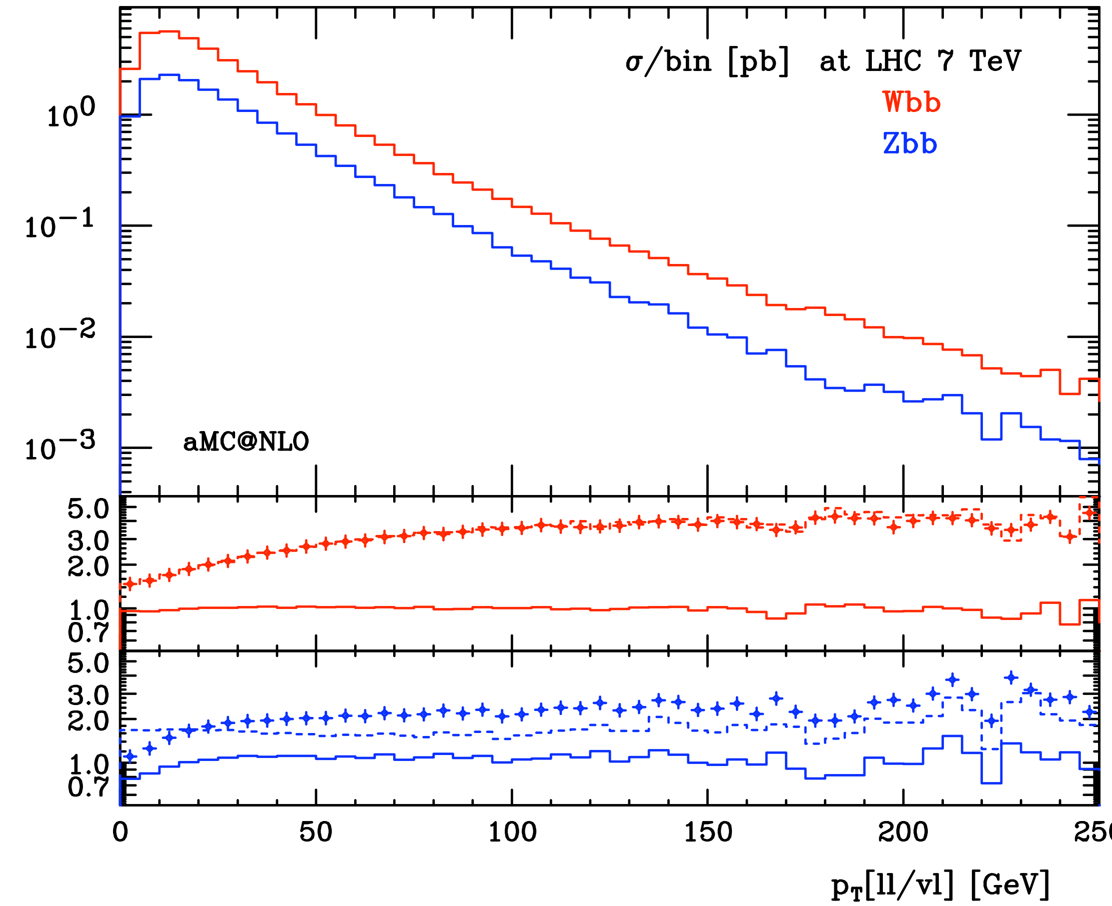


	Cross section (pb)					
	Tevatron $\sqrt{s}$ =1.96 TeV			LHC $\sqrt{s}$ =7 TeV		
	LO	NLO	K factor	LO	NLO	K factor
$\ell\nu b\bar{b}$	4.63	8.04	1.74	19.4	38.9	2.01
$\ell^+\ell^- b\bar{b}$	0.860	1.509	1.75	9.66	16.1	1.67

pp → Wbb/Zbb

aMC@NLO team

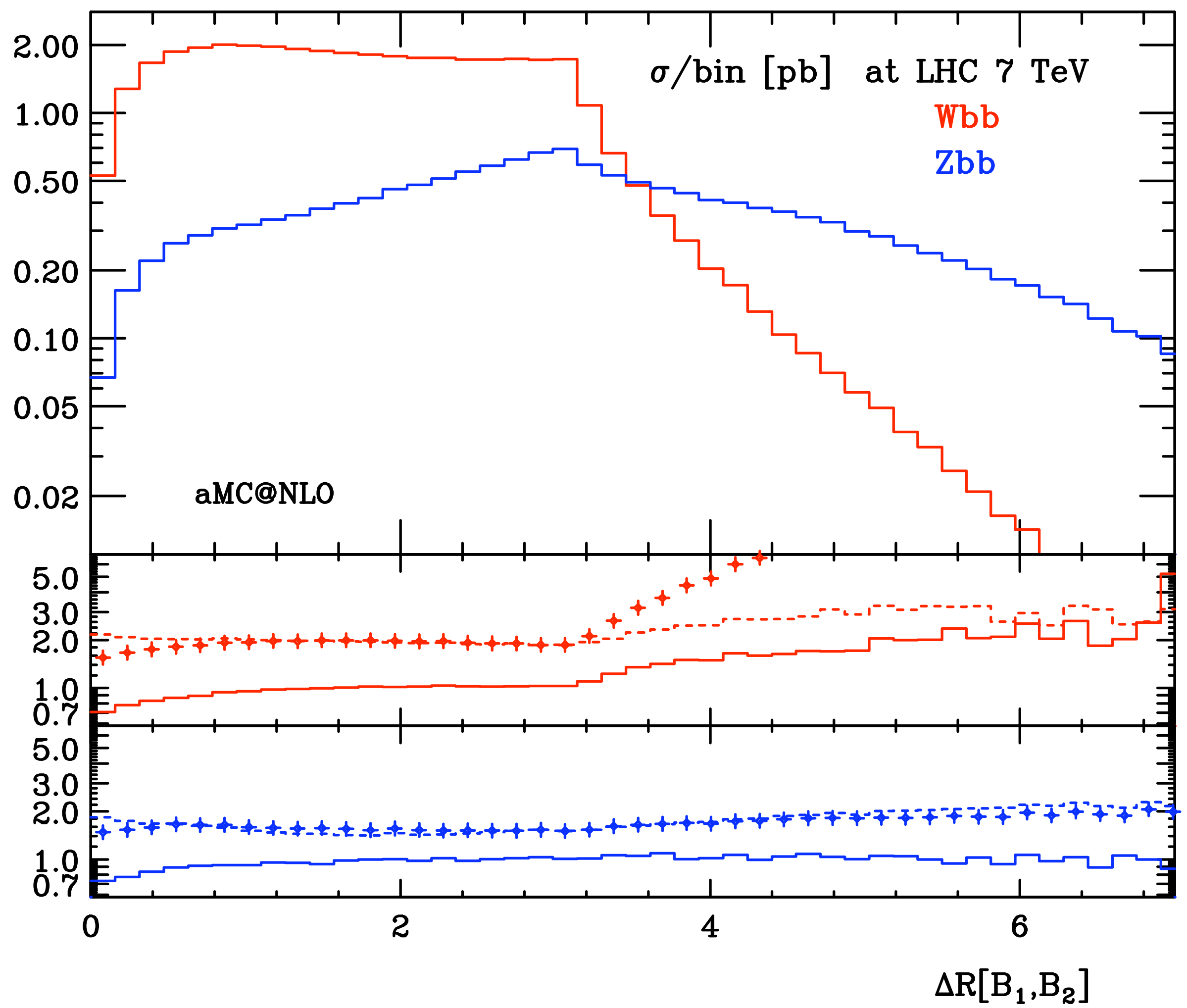




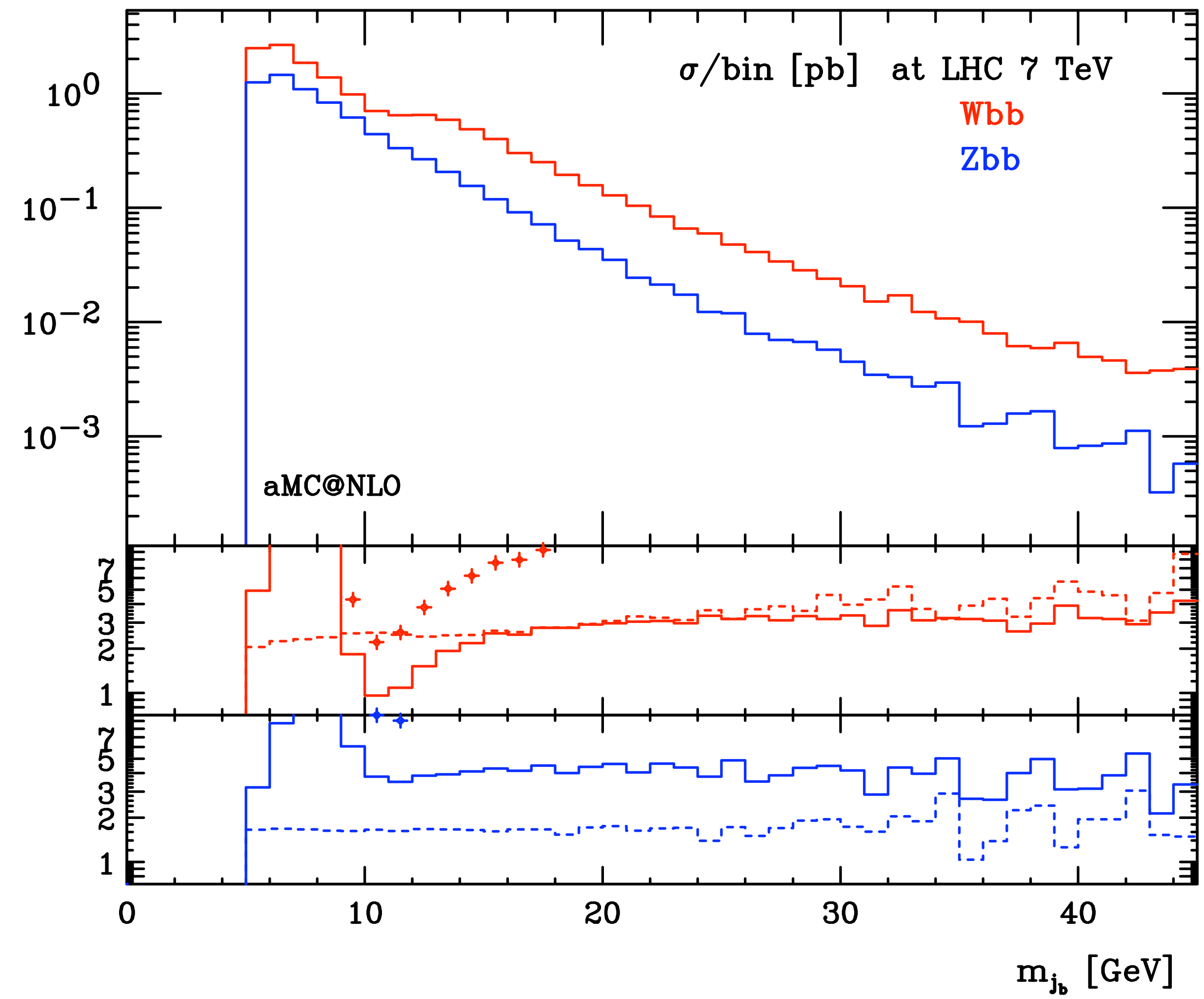
- In Wbb, ~20% of b-jets are bb-jets; for Zbb only ~6%
  - Jets defined with anti-k_T and R=0.5, with $p_T(j)$>20 GeV and $|\eta|$<2.5
- Lower panels show the ratio of aMC@NLO with LO (crosses), NLO (solid) and aMC@LO (dotted)
- NLO and aMC@NLO very similar and consistent

PP → WBB/ZBB

Distance between B-mesons

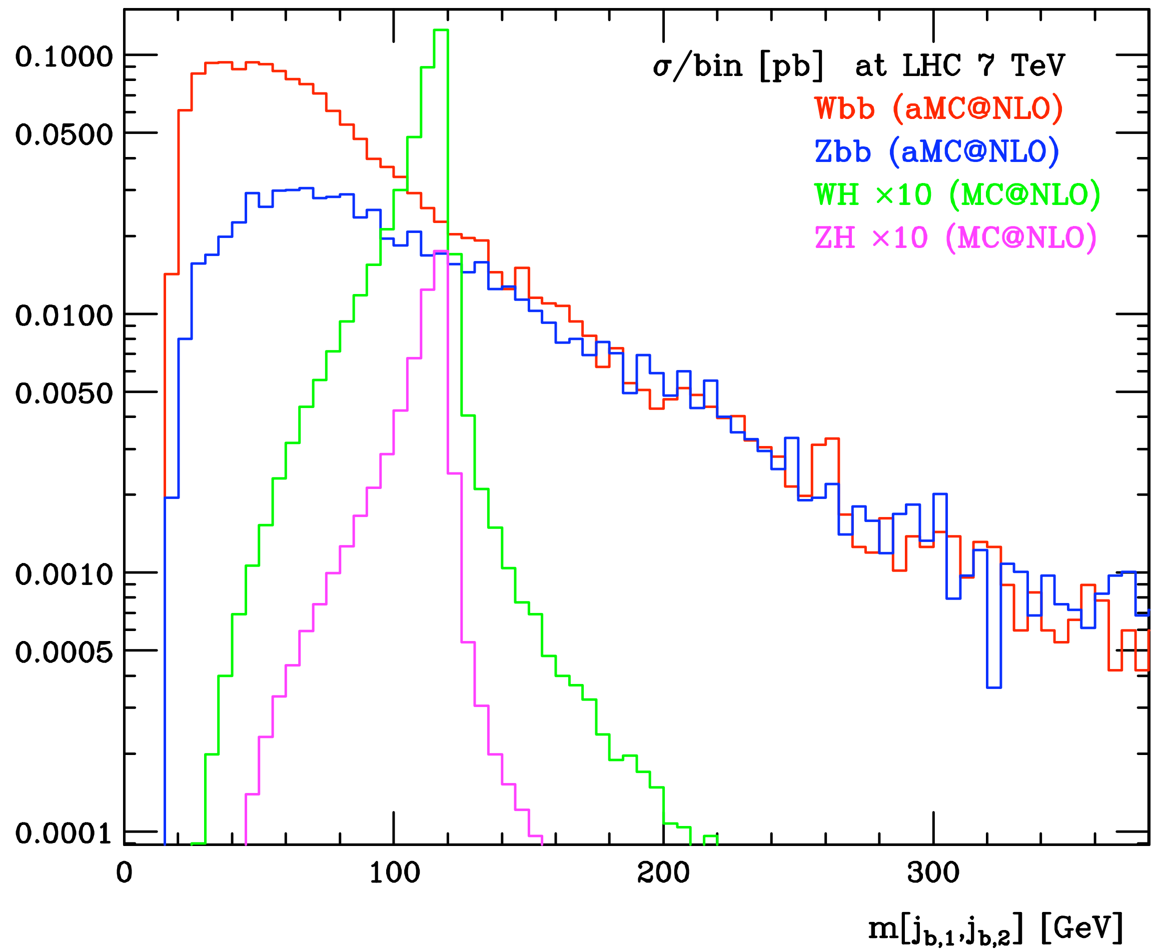


b-jet mass



- For some observables NLO effects are large and/or parton showering has large effects

Signal + background



Using (a)MC@NLO both signal and background for Vector boson production in association with a Higgs boson (where the Higgs decays to b anti-b) can be produced at the same NLO accuracy, including showering and hadronization effects

NLO+PS

"Best" tools when NLO calculation is available (i.e. low jet multiplicity).

* Main points:
  * NLO+PS provide a consistent to include K-factors into MC's
  * Scale dependence is meaningful
  * Allows a correct estimates of the PDF errors.
  * Non-trivial dynamics beyond LO included for the first time.

N.B. : The above is true for observables which are at NLO to start with!!!

* Current developments:
  * Upgrading of all available NLO computations to MC's in progress
  * Extendable to BSM without hurdles.
  * No merging with different multiplicities available yet (CKKW@NLO)

SM Status circa 2002

pp → n particles

SM Status circa 2002

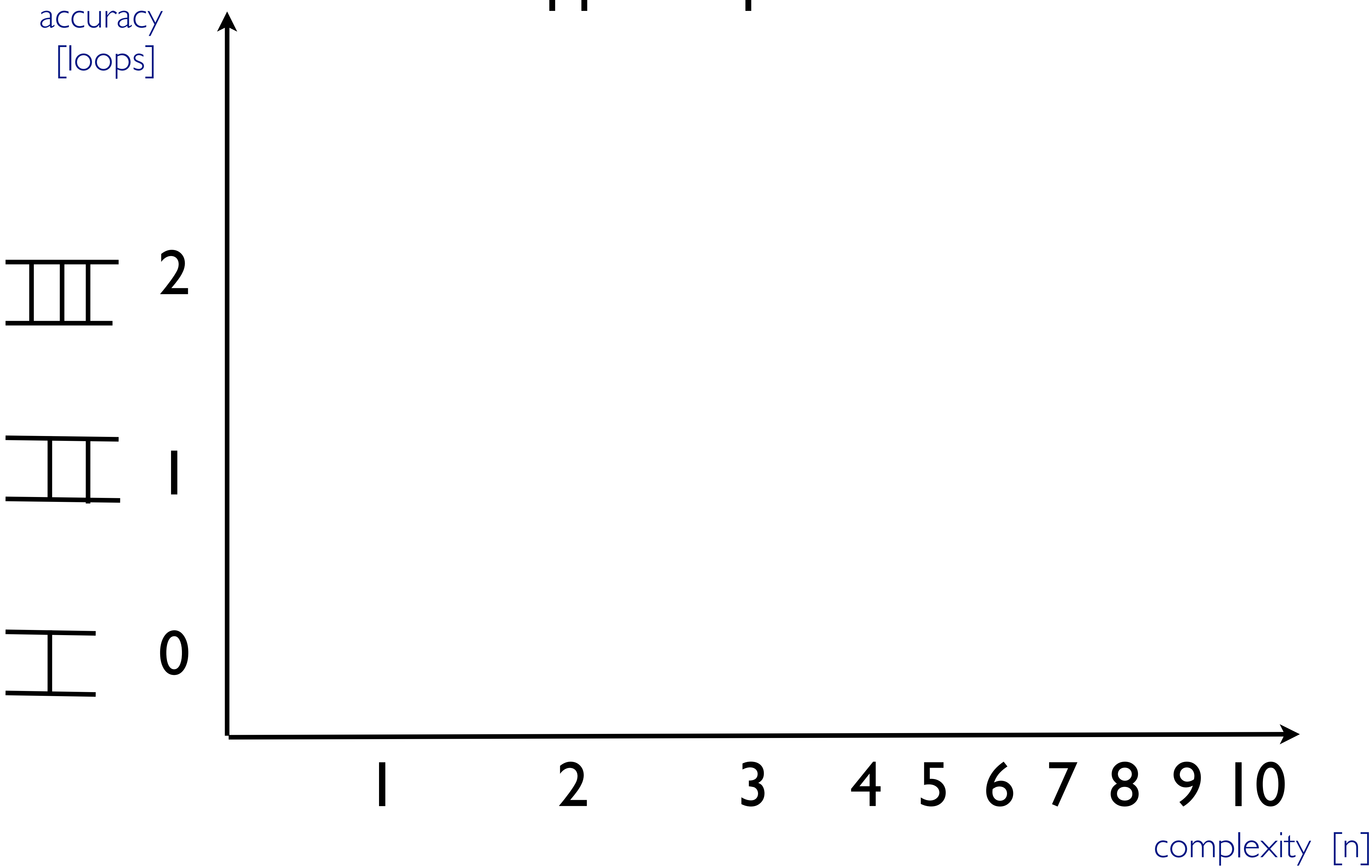
pp→ n particles

1 2 3 4 5 6 7 8 9 10

complexity [n]

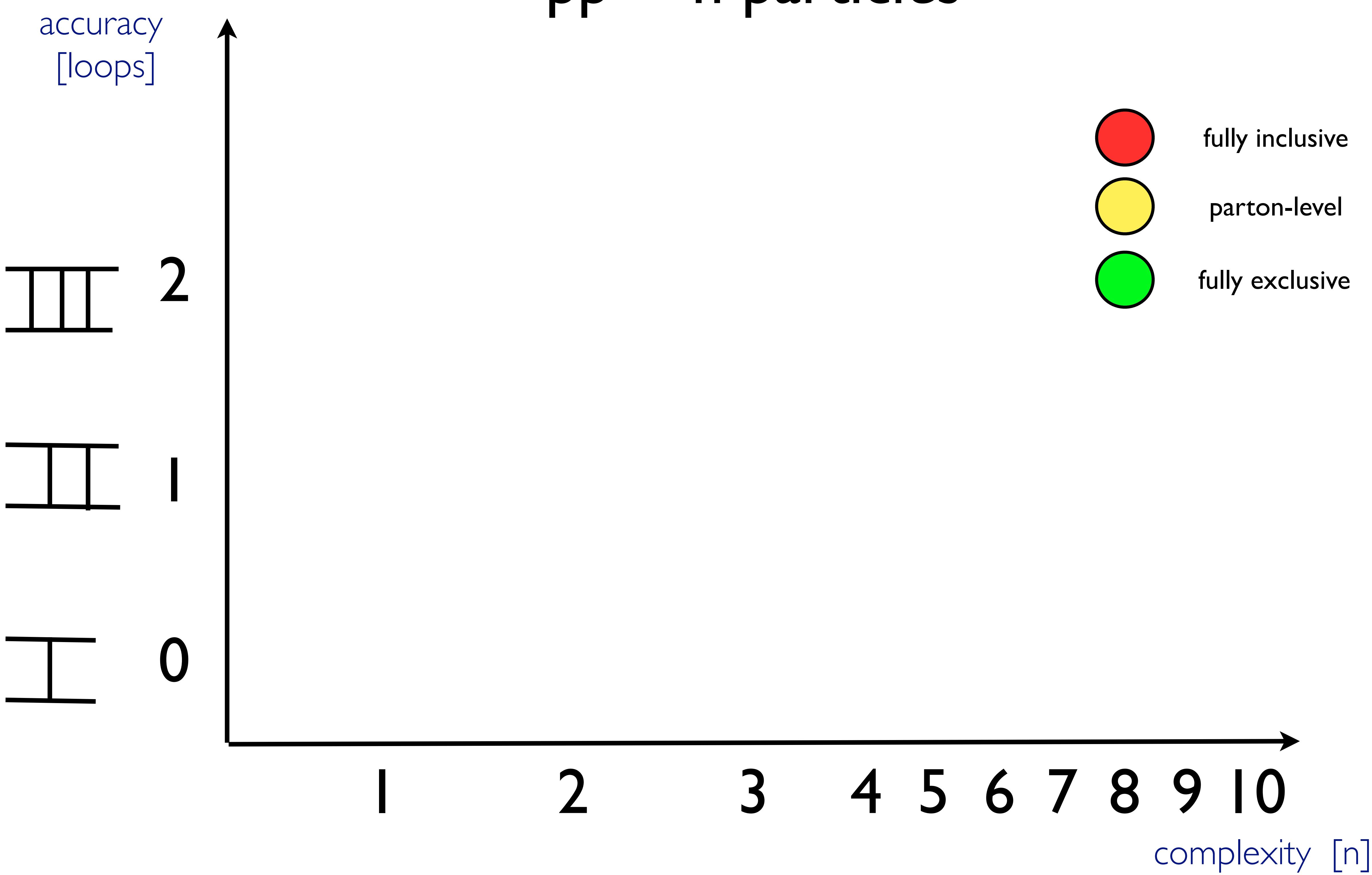
SM Status circa 2002

pp→ n particles

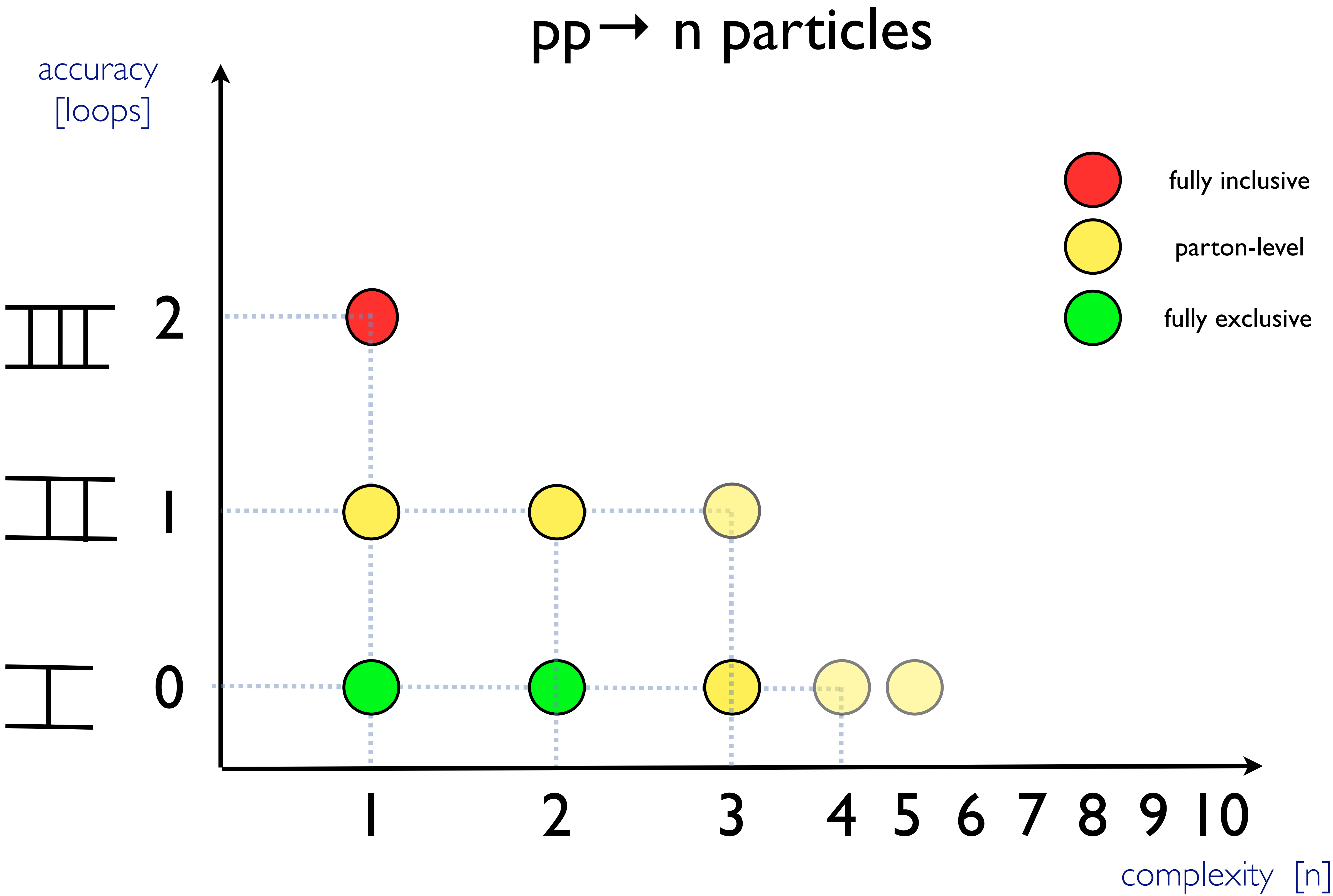


SM Status circa 2002

$pp \rightarrow$ n particles

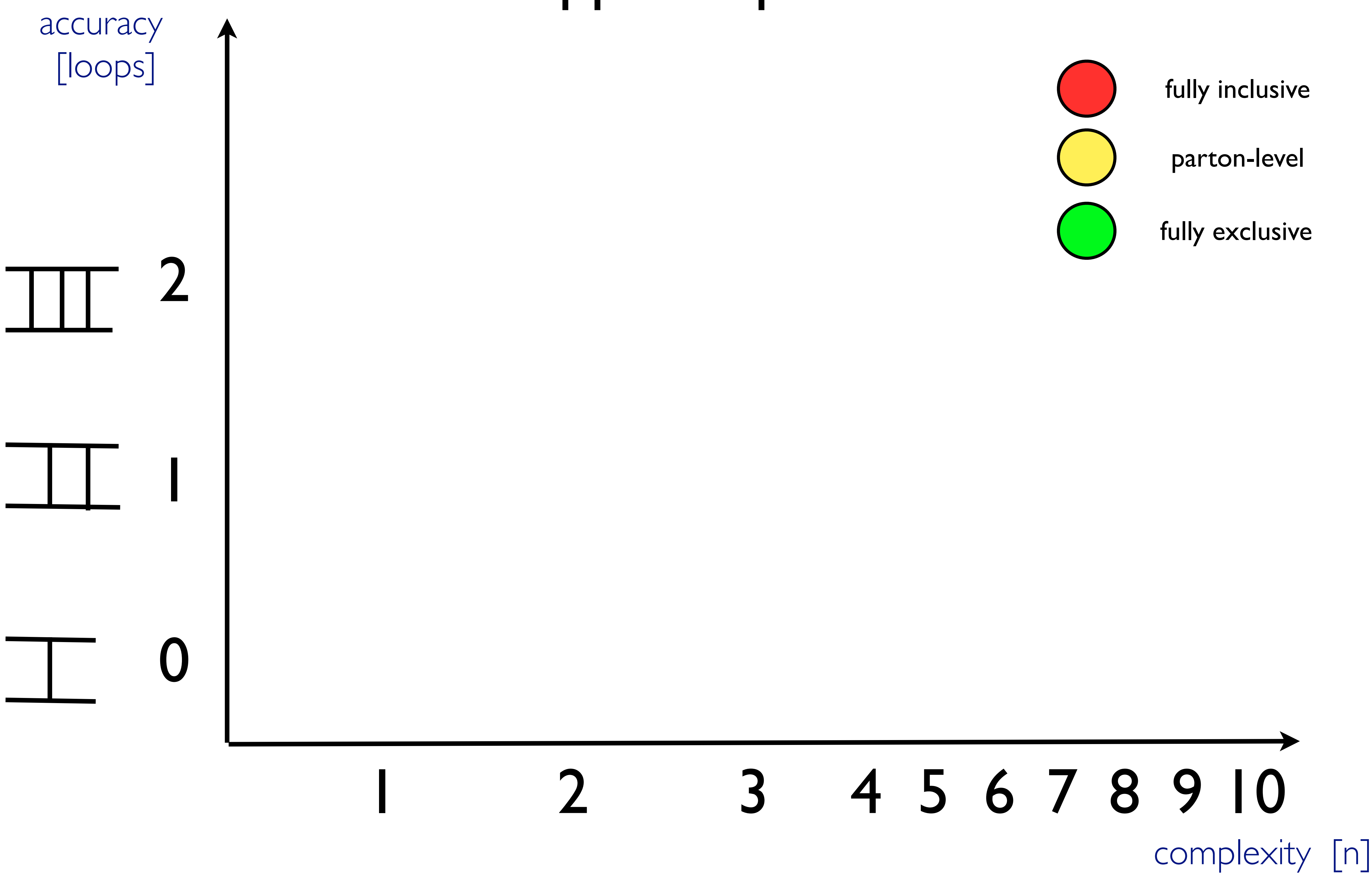


SM STATUS CIRCA 2002

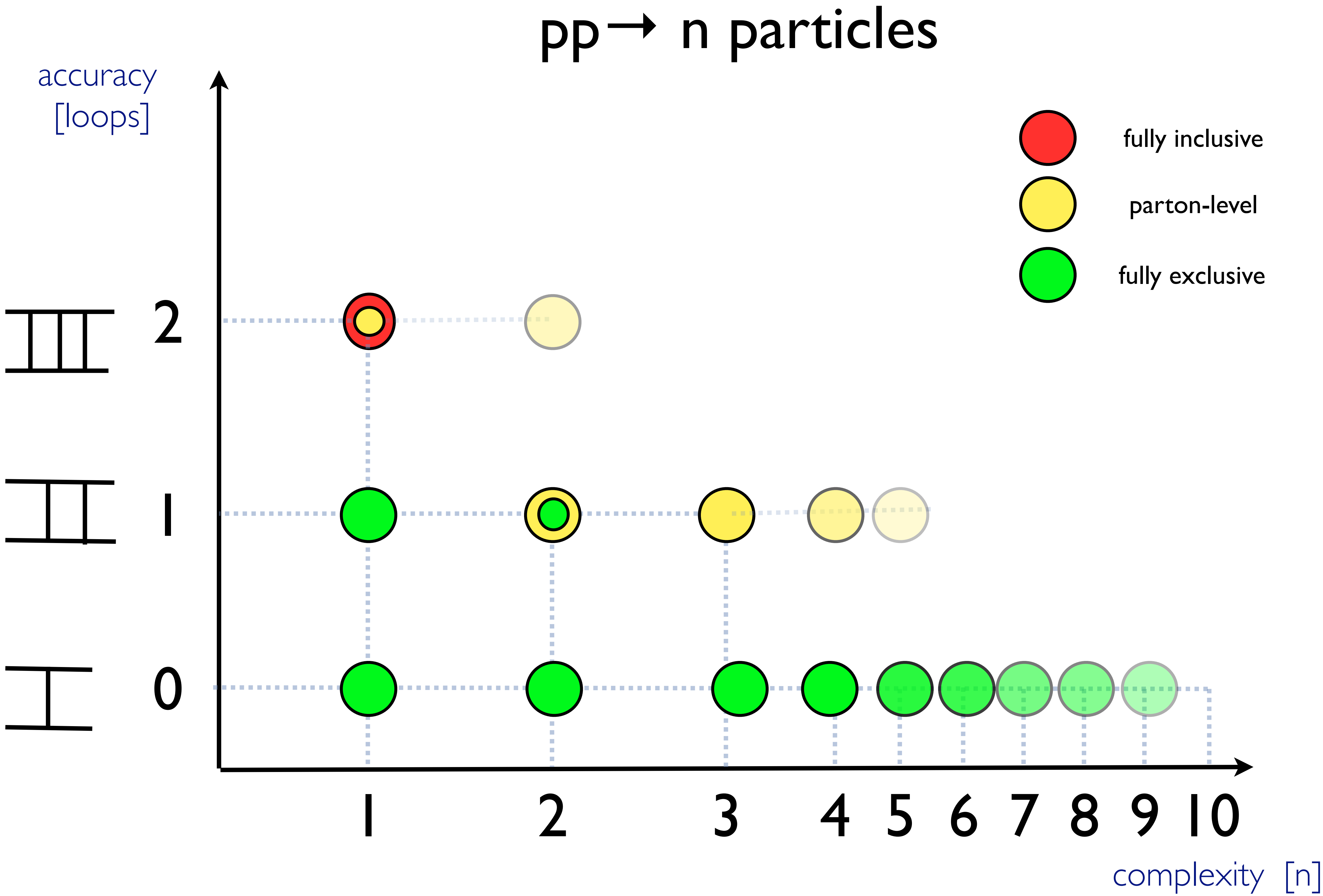


SM Status : since 2007

pp→ n particles



SM Status : since 2007



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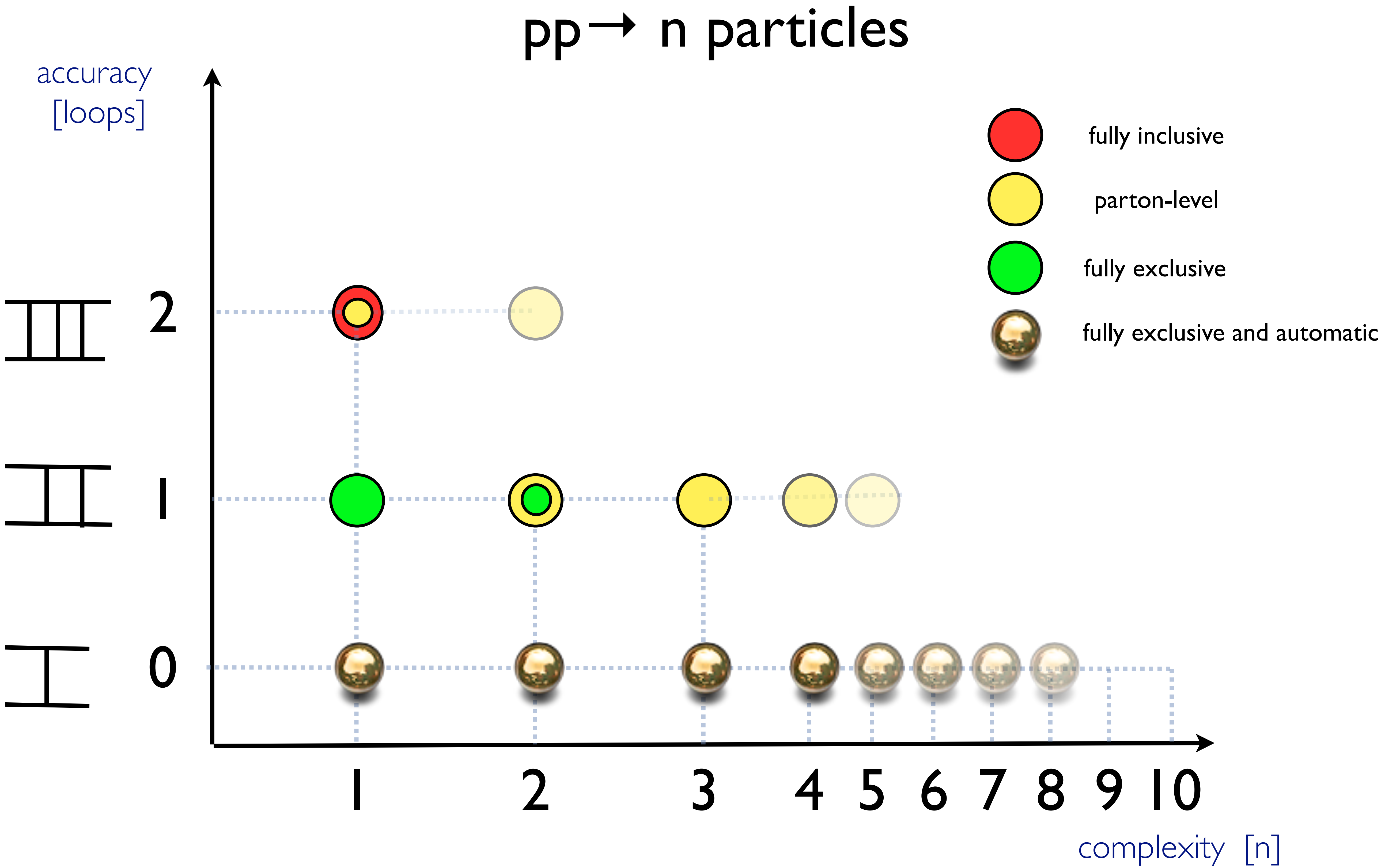
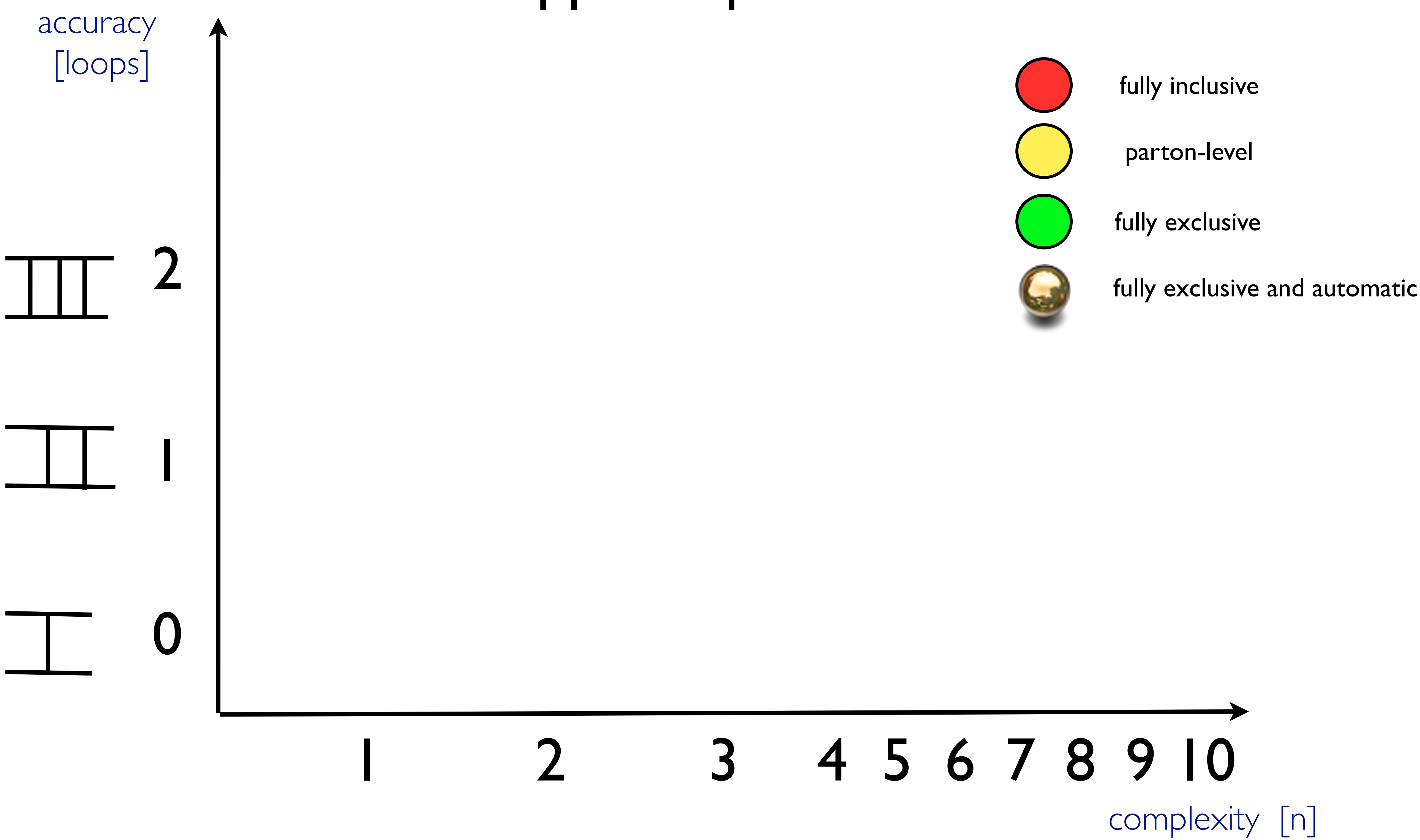


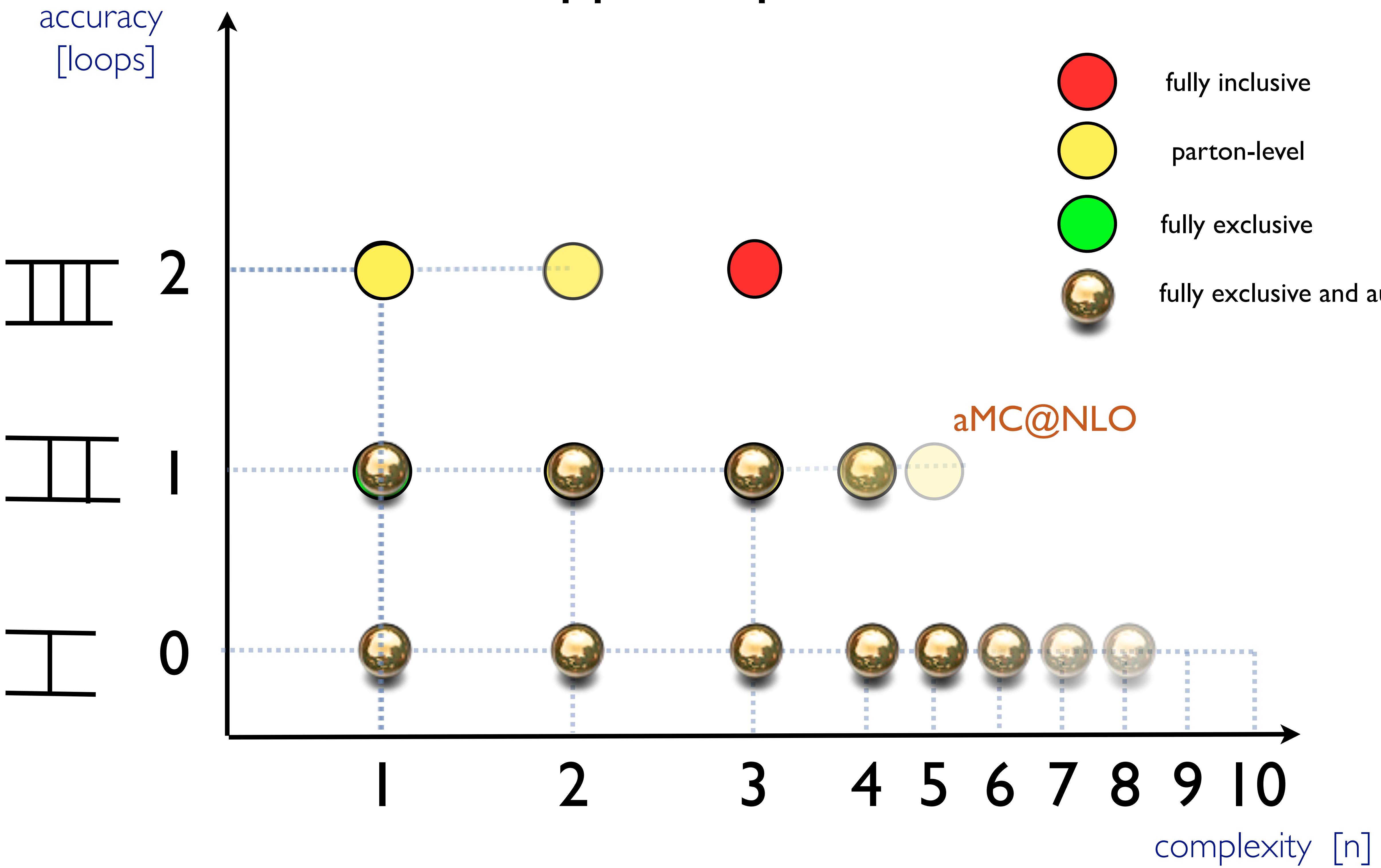
SM Status: now

pp→ n particles



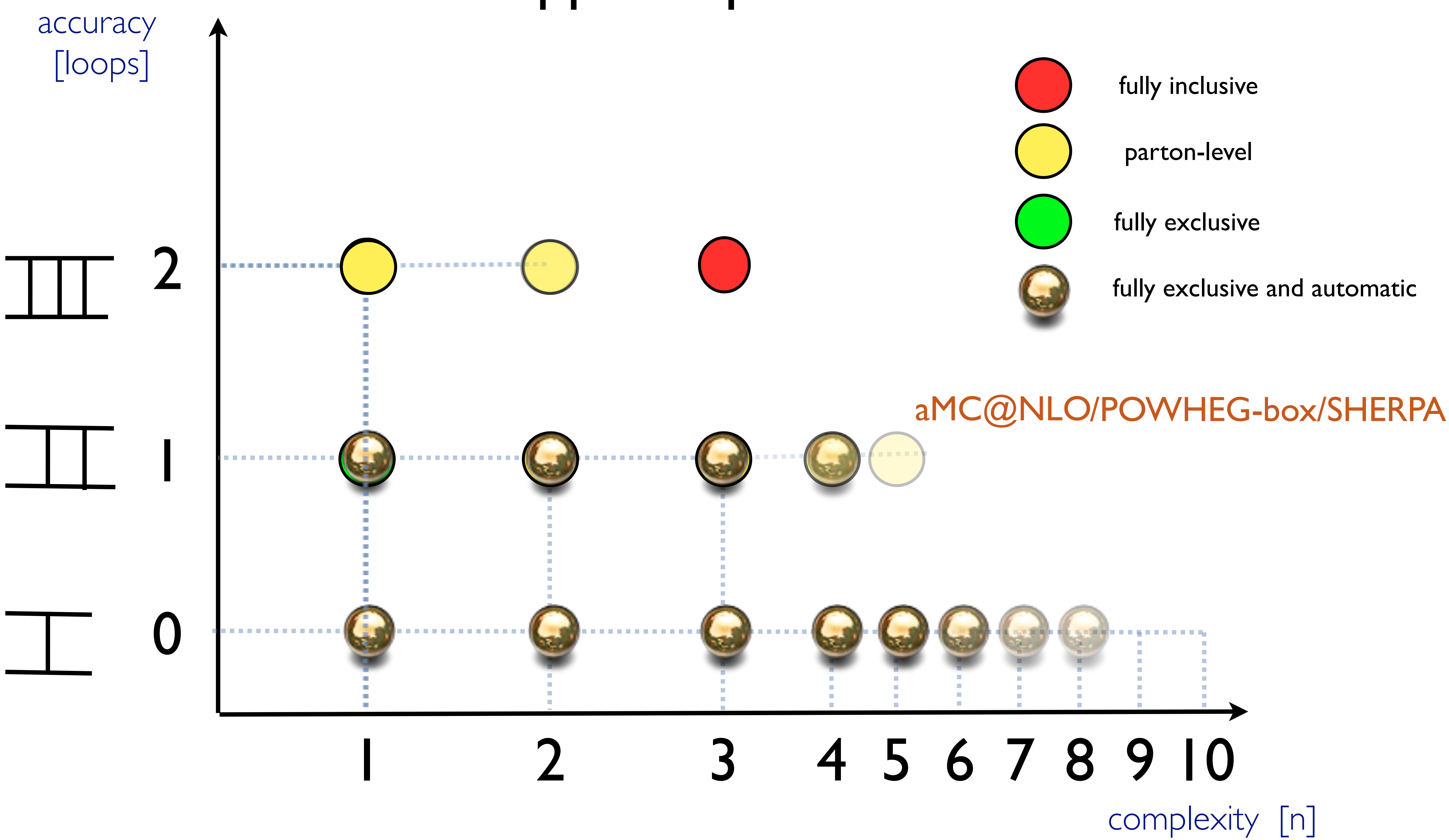
SM Status: now

pp→ n particles



SM Status: now

pp→ n particles



Conclusions

- The need for better description and more reliable predictions for SM processes for the LHC has motivated a significant increase of theoretical and phenomenological activity in the last years, leading to several important achievements in the field of QCD and MC's.
- A new generation of tools and techniques is now available.
- A complete set of NLO computations is available, even in fully automatic form. Several NNLO results are being used already now and will be extended in the future.
- New techniques and codes available for interfacing at LO and NLO computations at fixed order to parton-shower has been proven for SM (and BSM).
- Unprecedented accuracy and flexibility achieved.
- EXP/TH interactions enhanced by a new framework where exps and theos speak the same language.

Credits

To organize this presentation I have benefited from lectures (and actual slides), talks and discussions with many people.
In particular:

- Mike Seymour (MC basics)
- Claude Duhr (FeynRules)
- Johan Alwall (ME+PS merging)
- Rikkert Frederix, Paolo Torrielli (NLO+PS)
- Stefano Frixione, Michelangelo Mangano, Paolo Nason (for QCD, PS, LO, NLO, and more...)
- ....

Whom I all warmly thank!!