Predictive Monte Carlo tools for the LHC

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Lecture II

PLAN

- Basics : LO predictions and event generation
- Fixed-order calculations : from NLO to NNLO
- Exclusive predictions : Parton Shower
- Merging ME+PS
- Matching NLO with PS

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Today

LO predictions : remarks

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1,x_2,\alpha_S(\mu_R^2),\frac{Q^2}{\mu_F^2},\frac{Q^2}{\mu_R^2})$$

- By calculating the short distance coefficient at tree-level we obtain the first estimate of rates for inclusive final states.

- Even at LO **extra radiation is included:** it is described by the PDF's in the initial state and by the definition of a final state parton, which at LO represents all possible final state evolutions.

- Due to the above approximations a cross section at LO can strongly depend on the factorization and renormalization scales.

- Predictions can be systematically improved, at NLO and NNLO, by including higher order corrections in the short distance and in the evolution of the PDF's.

NLO PREDICTIONS

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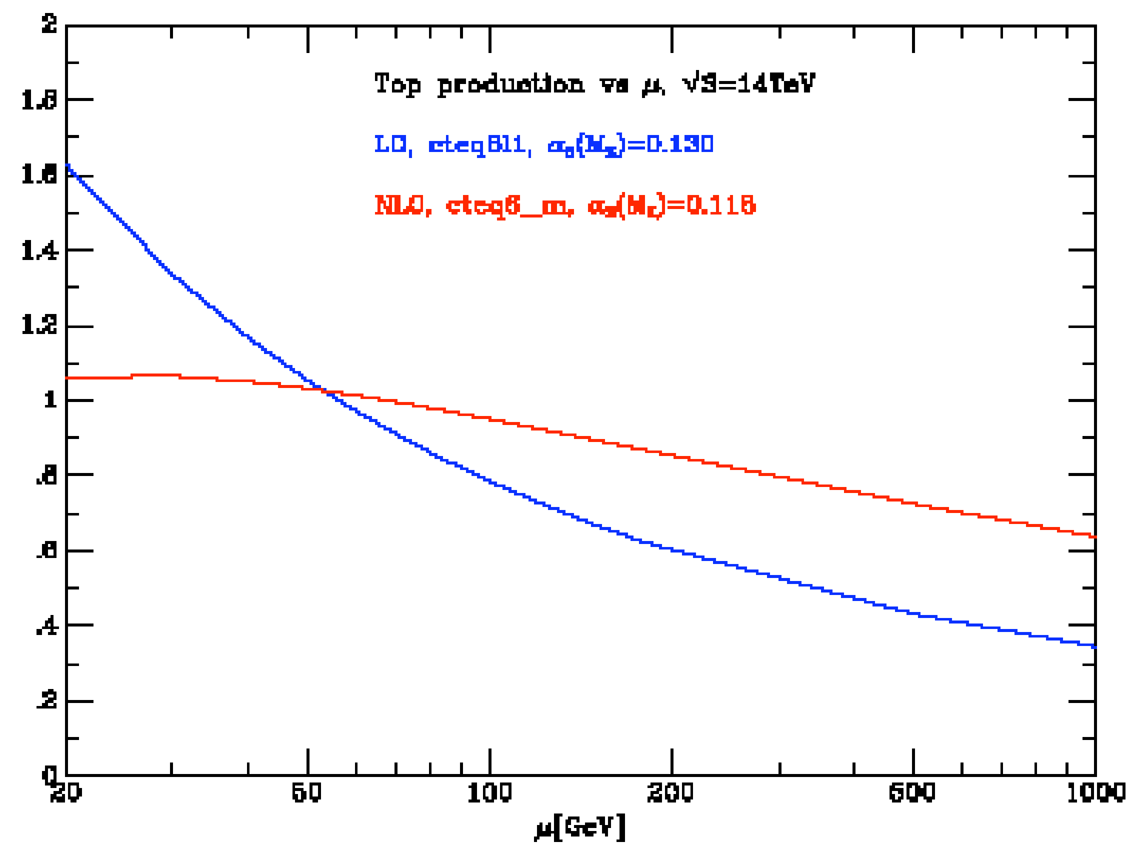
$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Why?

1. First order where scale dependences are compensated by the running of α_S and the evolution of the PDF's: FIRST RELIABLE ESTIMATE OF THE TOTAL CROSS SECTION.

2. The impact of extra radiation is included. For example, jets now have a structure.

3. New effects coming up from higher order terms (e.g., opening up of new production channels or phase space dimensions) can be evaluated.



Elements of a NLO computation

NLO contributions have three parts

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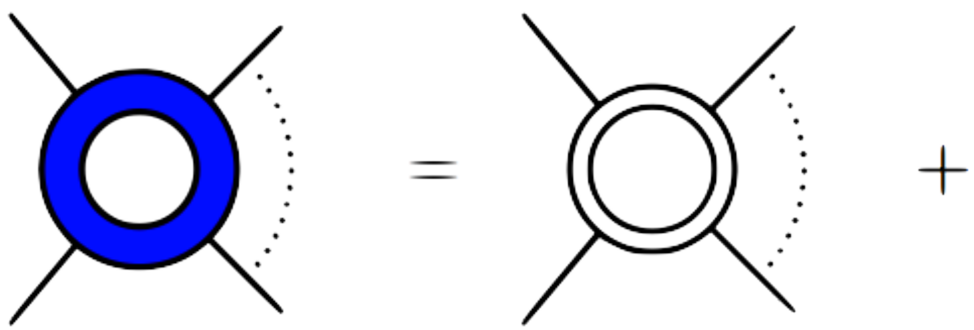

$$\sigma^{\mathrm{NLO}} = \int_m d^{(d)} \sigma^V +$$

Virtual part

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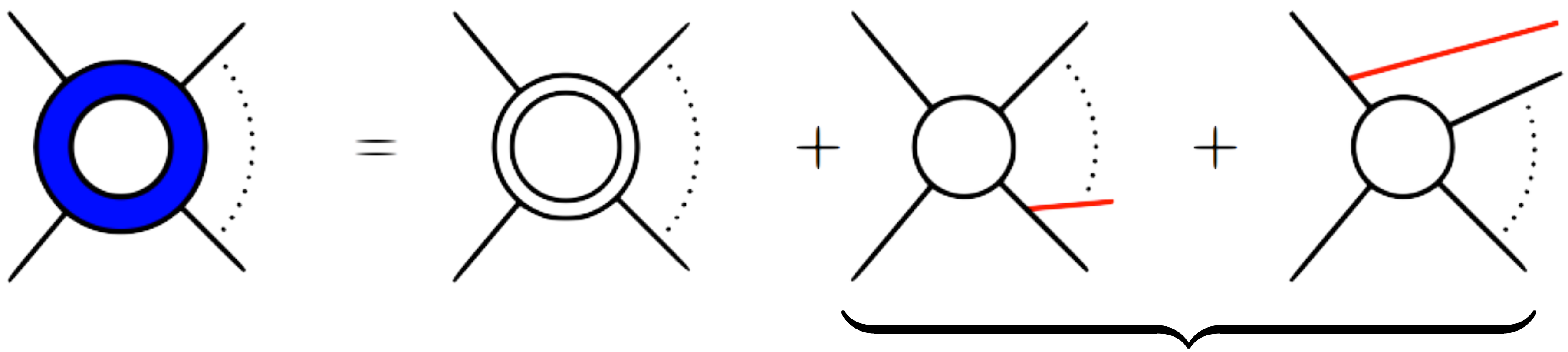

$$\sigma^{\mathrm{NLO}} = \int_m d^{(d)}\sigma^V + \int_{m+1} d^{(d)}\sigma^R +$$

Virtual part

Real emission part

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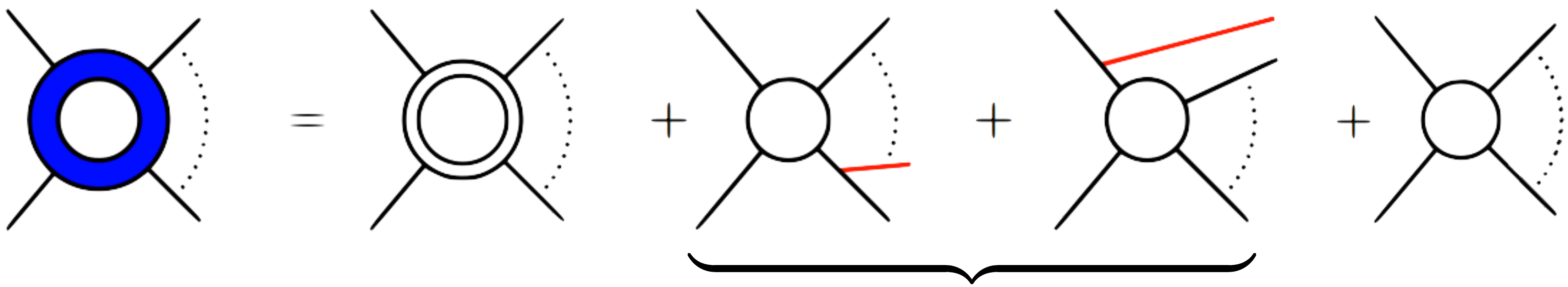

$$\sigma^{\text{NLO}} = \int_m d^{(d)}\sigma^V + \int_{m+1} d^{(d)}\sigma^R + \int_m d^{(4)}\sigma^B$$

Virtual part — Real emission part — Born

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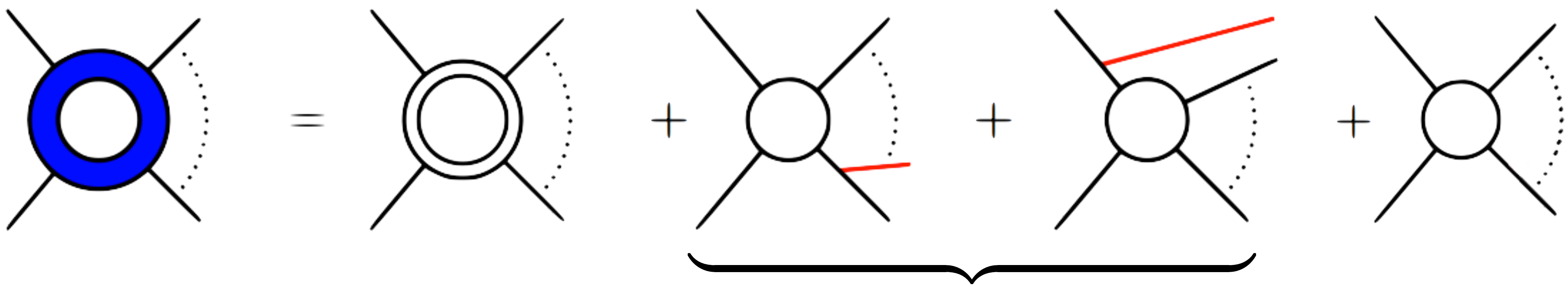

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Virtual part Real emission part Born

- Loops have been for long the bottleneck of NLO computations
- Virtuals and Reals are each divergent and subtraction scheme need to be used (Dipoles, FKS, Antenna's)
- A lot of work is necessary for each computation

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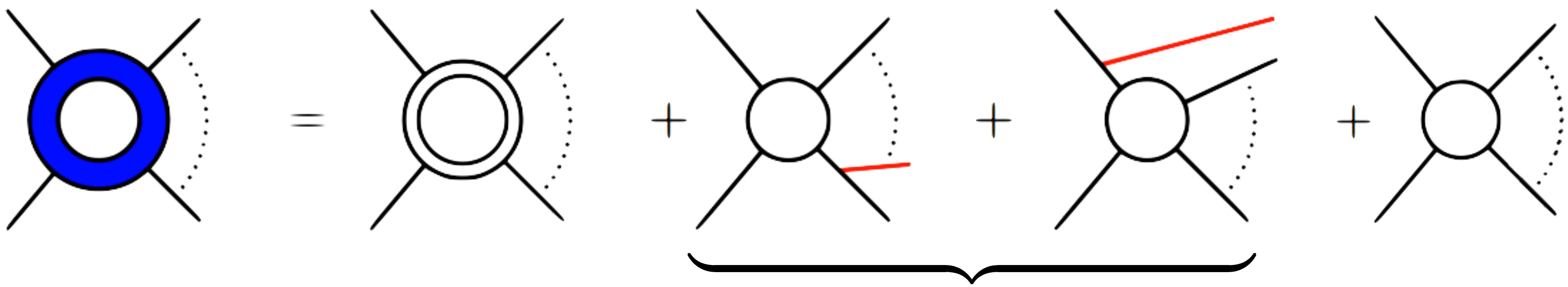

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The cost of a new prediction at NLO could easily exceed 100k euro/dollar.

Best example: MCFM

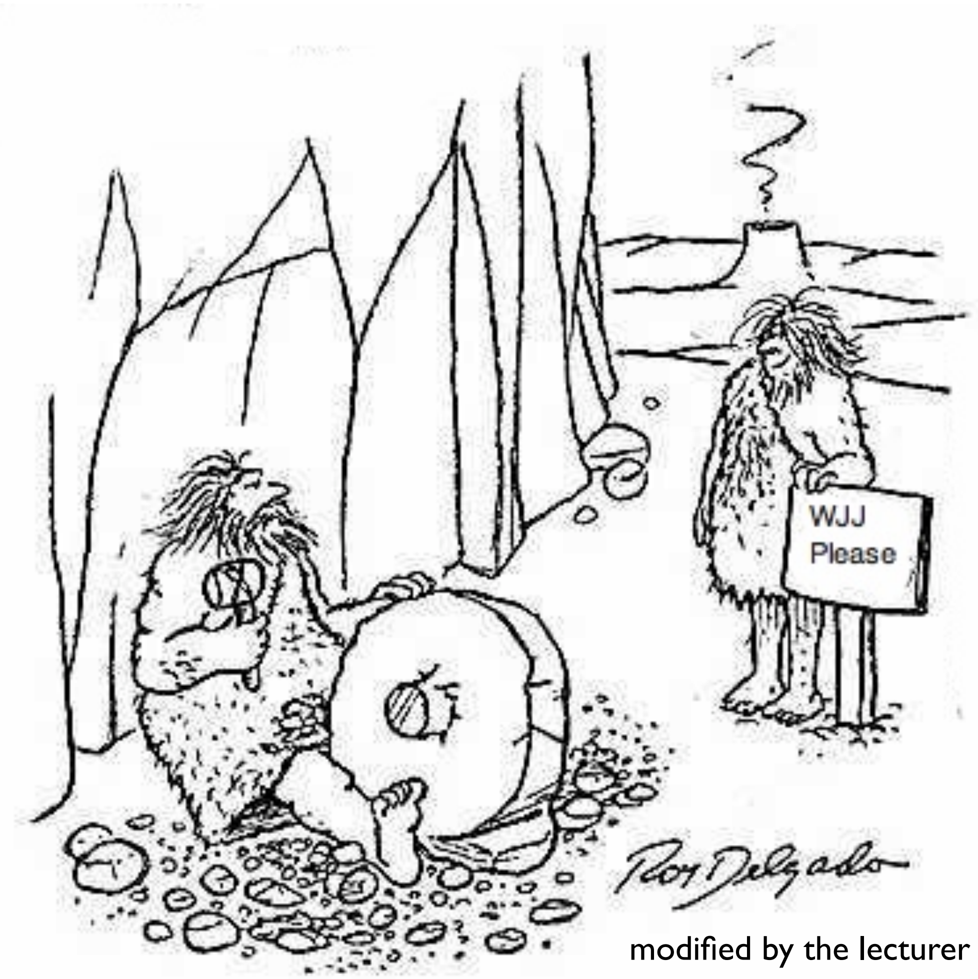
Downloadable general purpose NLO code [Campbell, Ellis, Williams+collaborators]

Final state	Notes	Reference
W/Z		
diboson (W/Z/γ)	photon fragmentation, anomalous couplings	hep-ph/9905386, arXiv:1105.0020
Wbb	massless b-quark massive b quark	hep-ph/9810489 arXiv:1011.6647
Zbb	massless b-quark	hep-ph/0006304
W/Z+1 jet		
W/Z+2 jets		hep-ph/0202176, hep-ph/0308195
Wc	massive c-quark	hep-ph/0506289
Zb	5-flavour scheme	hep-ph/0312024
Zb+jet	5-flavour scheme	hep-ph/0510362

Final state	Notes	Reference
H (gluon fusion)		
H+1 jet (g.f.)	effective coupling	
H+2 jets (g.f.)	effective coupling	hep-ph/0608194, arXiv:1001.4495
WH/ZH		
H (WBF)		hep-ph/0403194
Hb	5-flavour scheme	hep-ph/0204093
t	s- and t-channel (5F), top decay included	hep-ph/0408158
t	t-channel (4F)	arXiv:0903.0005, arXiv:0907.3933
Wt	5-flavour scheme	hep-ph/0506289
top pairs	top decay included	

- >30 processes
- First results implemented in 1998 ...this is 13 years worth of work of several people (~4M$)
- Cross sections and parton-level distributions at NLO are provided
- One general framework. However, each process implemented by hand. Now automation is possible.

Predictions at NLO



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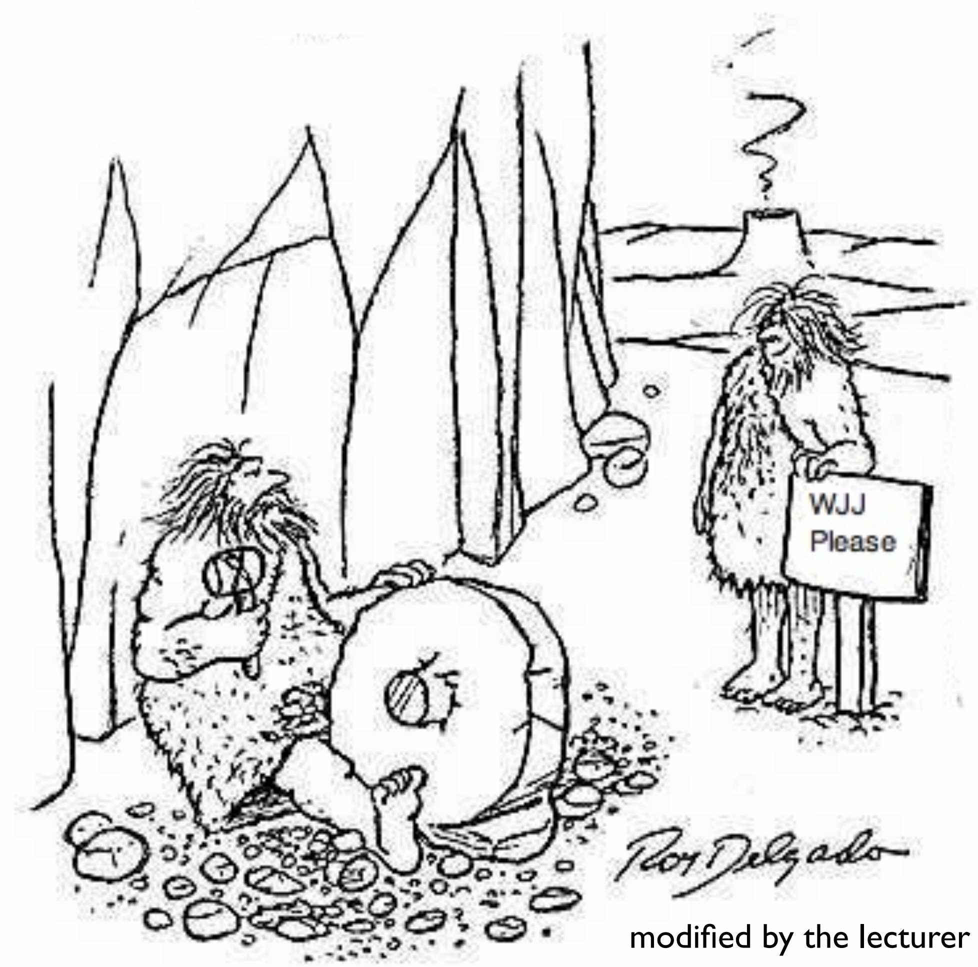
Predictions at NLO



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Generalized Unitarity
(ex. BlackHat, Rocket,...)

Integrand Reduction
(ex. CutTools, Samurai)

Tensor Reduction
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Thanks to new amazing results, some of them inspired by string theory developments, now the computation of loops has been extended to high-multiplicity processes or/and automated.

EXAMPLE OF AUTOMATIC NLO: MADLOOP+MADFKS

[Hirshi, Frederix, Frixione, FM, Garzelli, Pittau, Torrielli, 1103.0621].

Total cross sections at the LHC for 26 sample procs

Running time (Generate the code, and run it): Two weeks on a 150+ node cluster

Other approaches available (GoSam, MadGolem, HELAC-NLO, ...)

Room for improvement (speed, user-friendliness), yet a wall has been broken.

	Process	μ	n_{lf}	Cross section (pb) LO	NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

Infrared-safe quantities

DEFINITION: quantities are that are insensitive to soft and collinear branching.

For these quantities, an extension of the general theorem (KLN) exists which proves that infrared divergences cancel between real and virtual or are simply removed by kinematic factors.

EXAMPLES: total rates & cross sections, jet distributions, shape variables...

NLO codes return histograms of IR safe quantities (not events!)

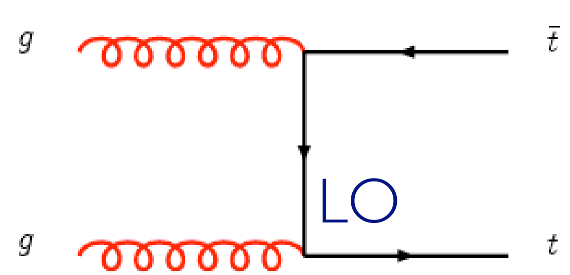
Predictions at NLO

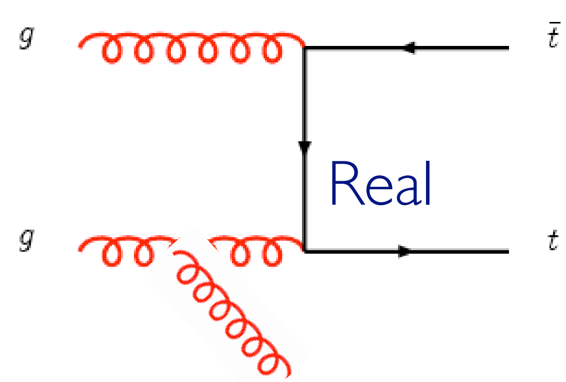
Warning!

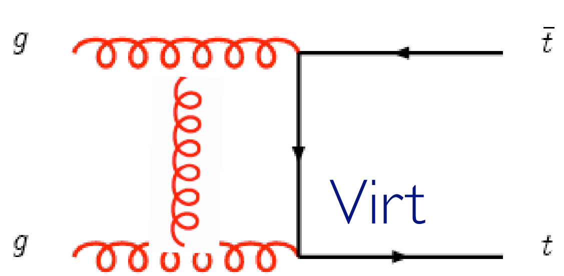
Calling a code "a NLO code" is an abuse of language and can be confusing.
A NLO calculation always refers to an IR-safe observable, when the genuine α_S corrections to this observable on top of the LO estimate are known.

An NLO code will, in general, be able to produce results for several quantities and distributions, only some of which will be at NLO accuracy.

Example: Suppose we use the NLO code for pp $\rightarrow$ tt







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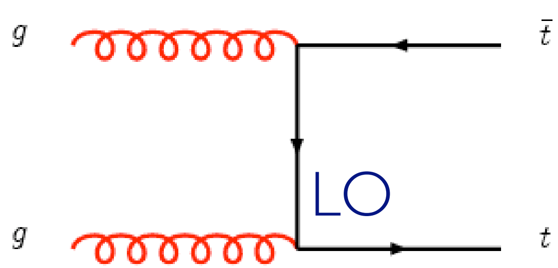

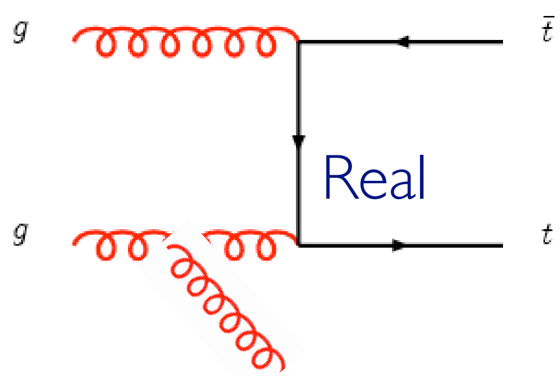

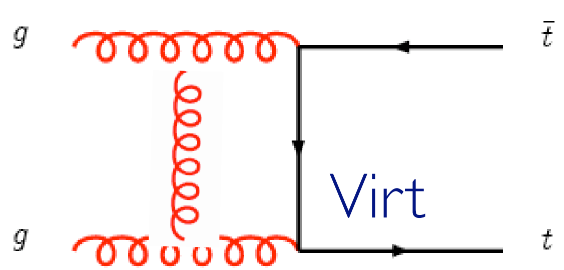

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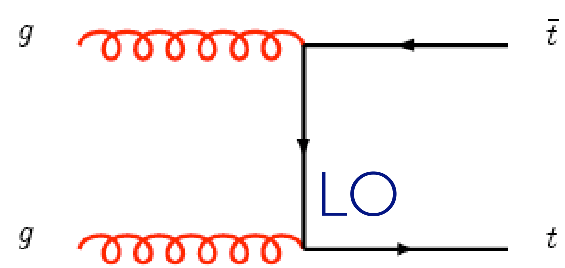

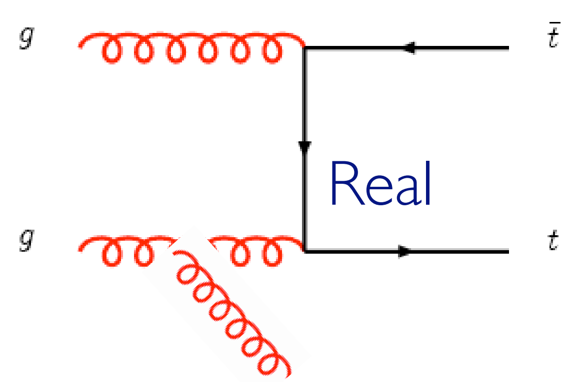

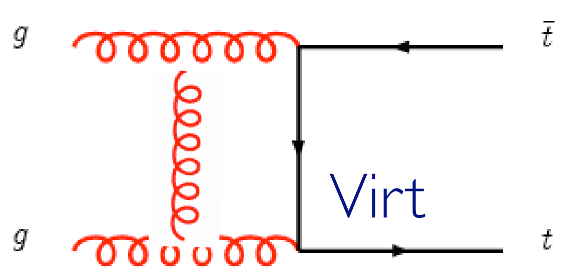

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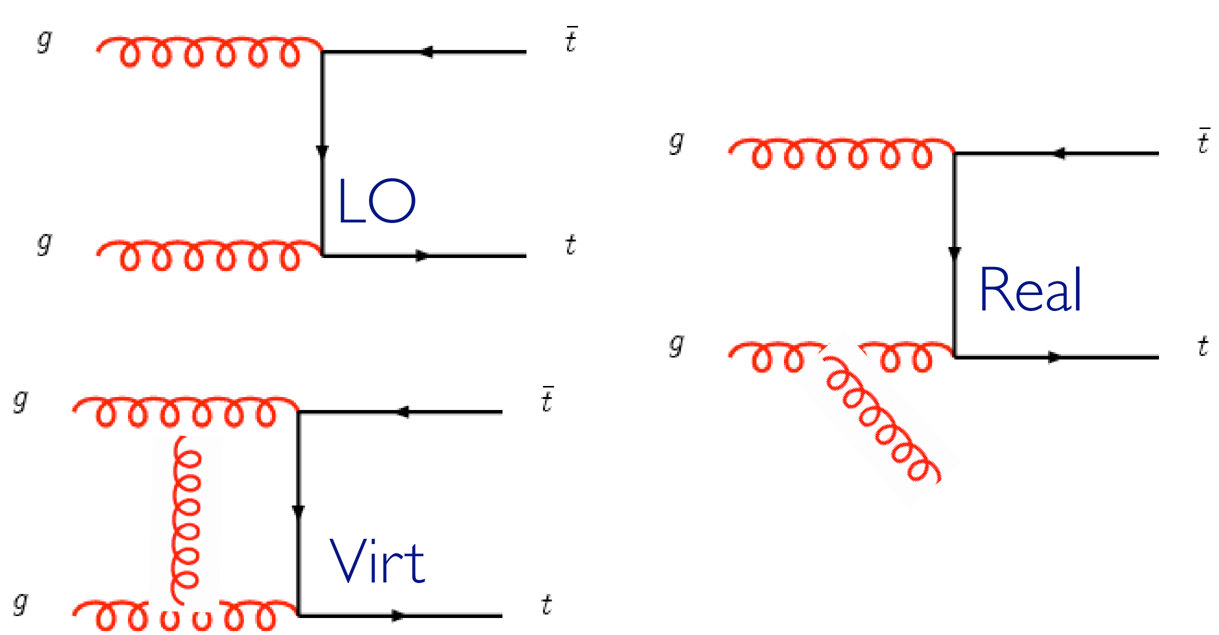

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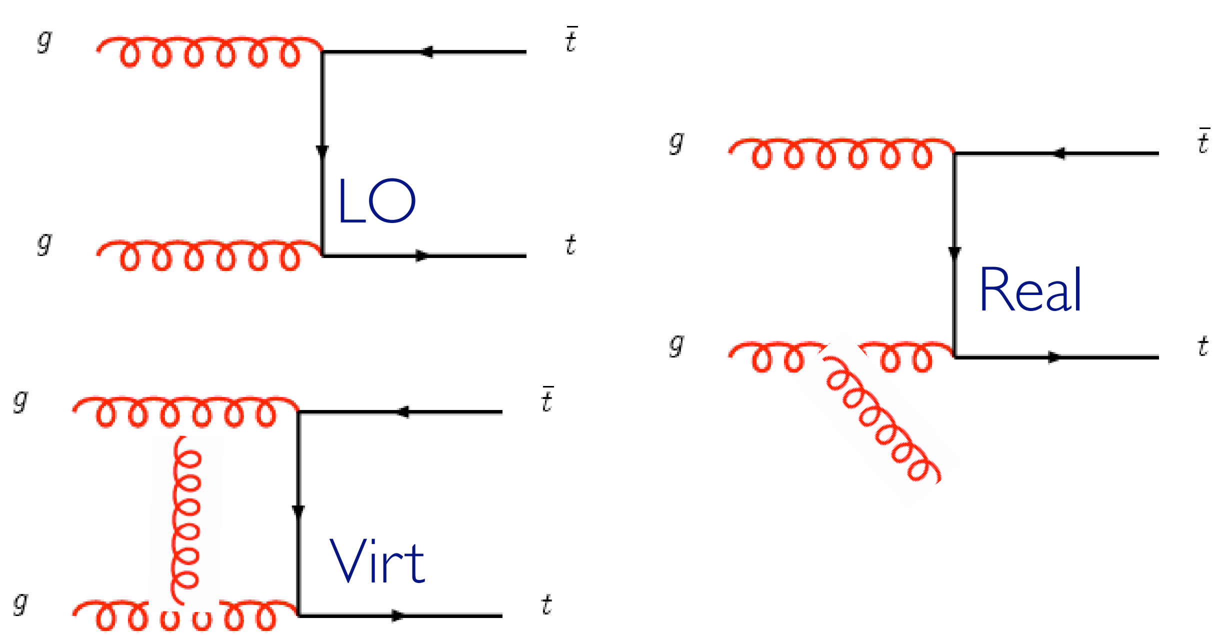

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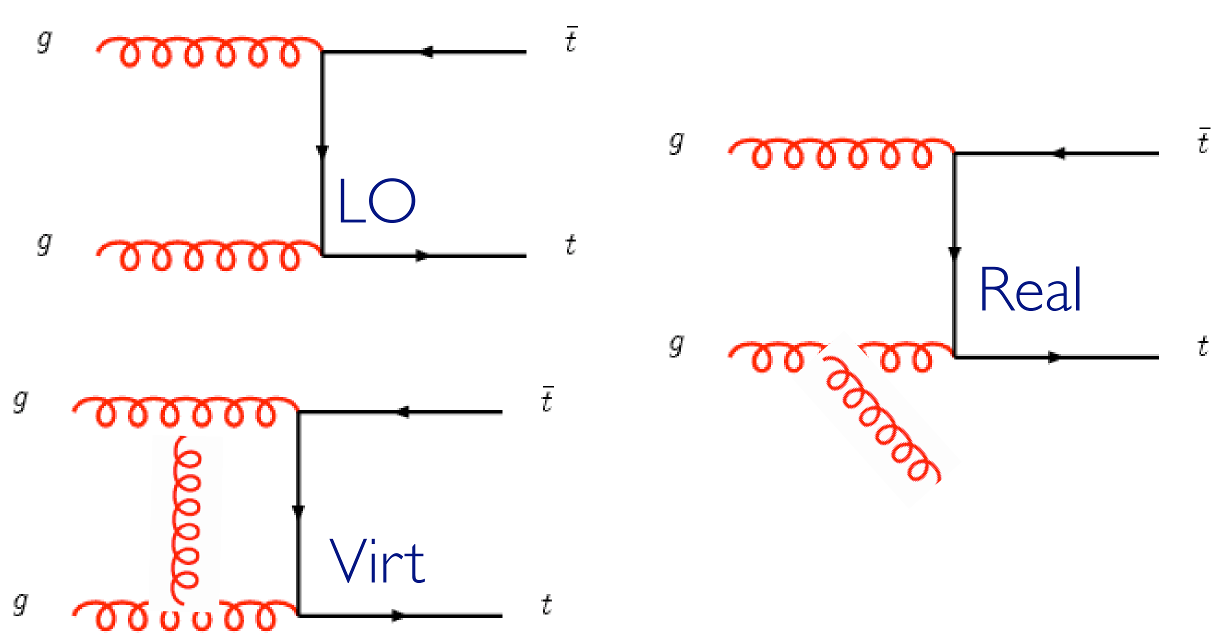

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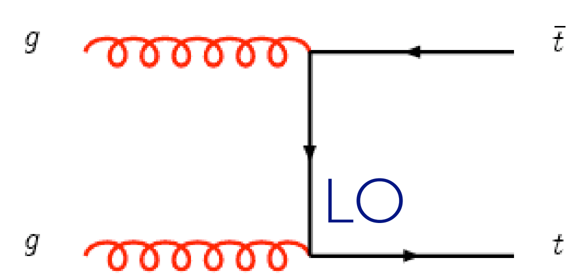

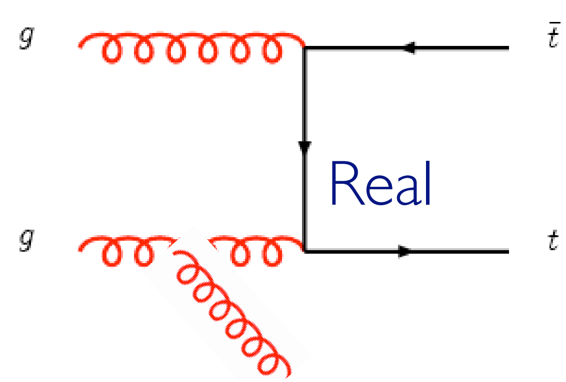

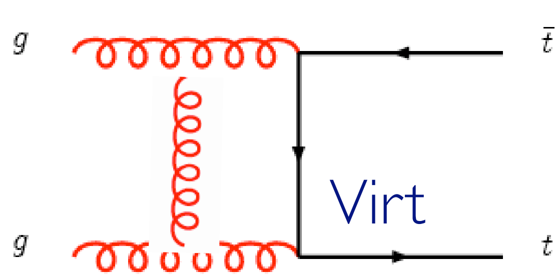

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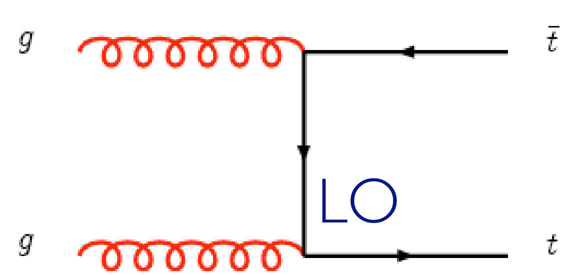

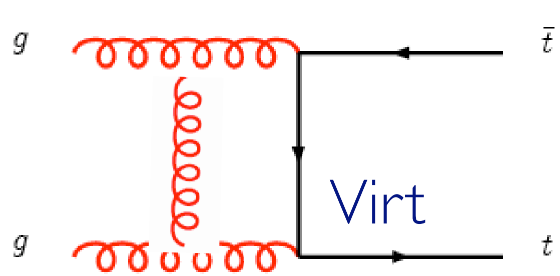

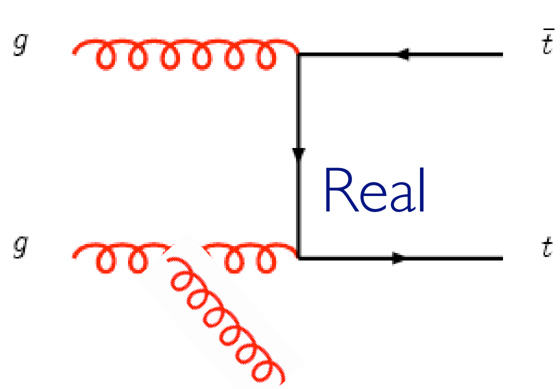

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PREDICTIONS AT NNLO

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

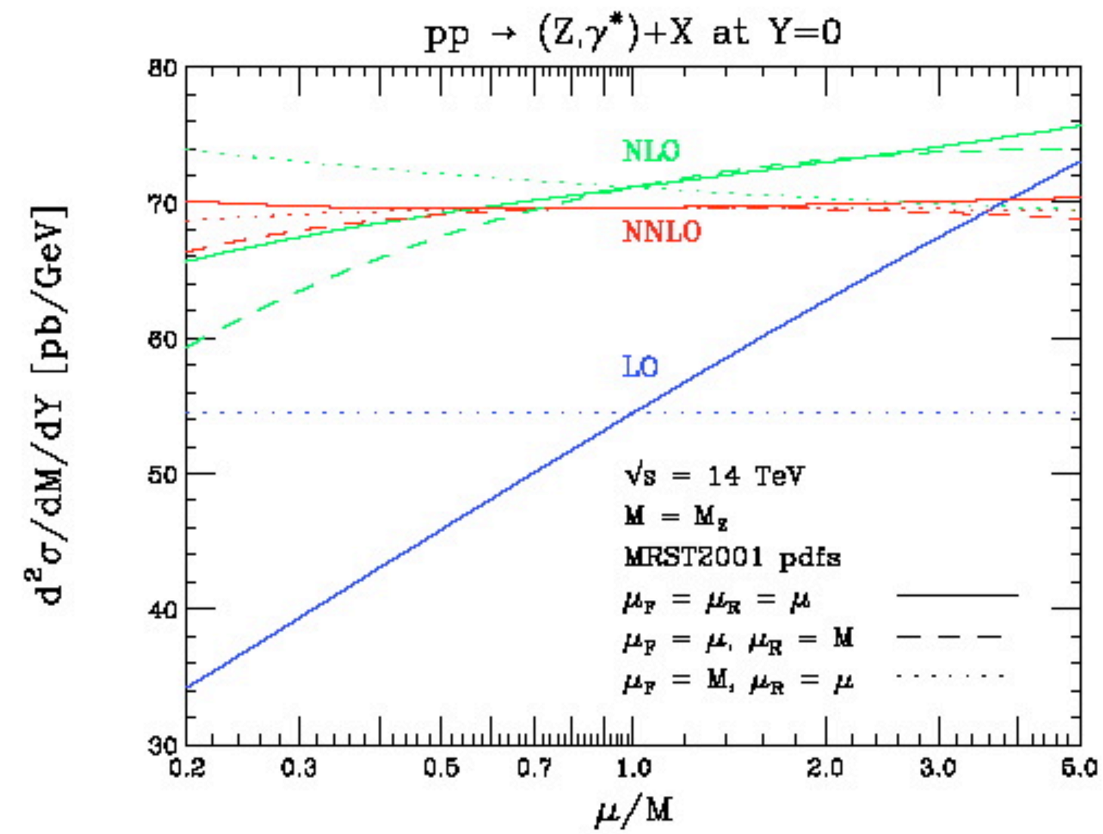
$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \ldots$$

Why?

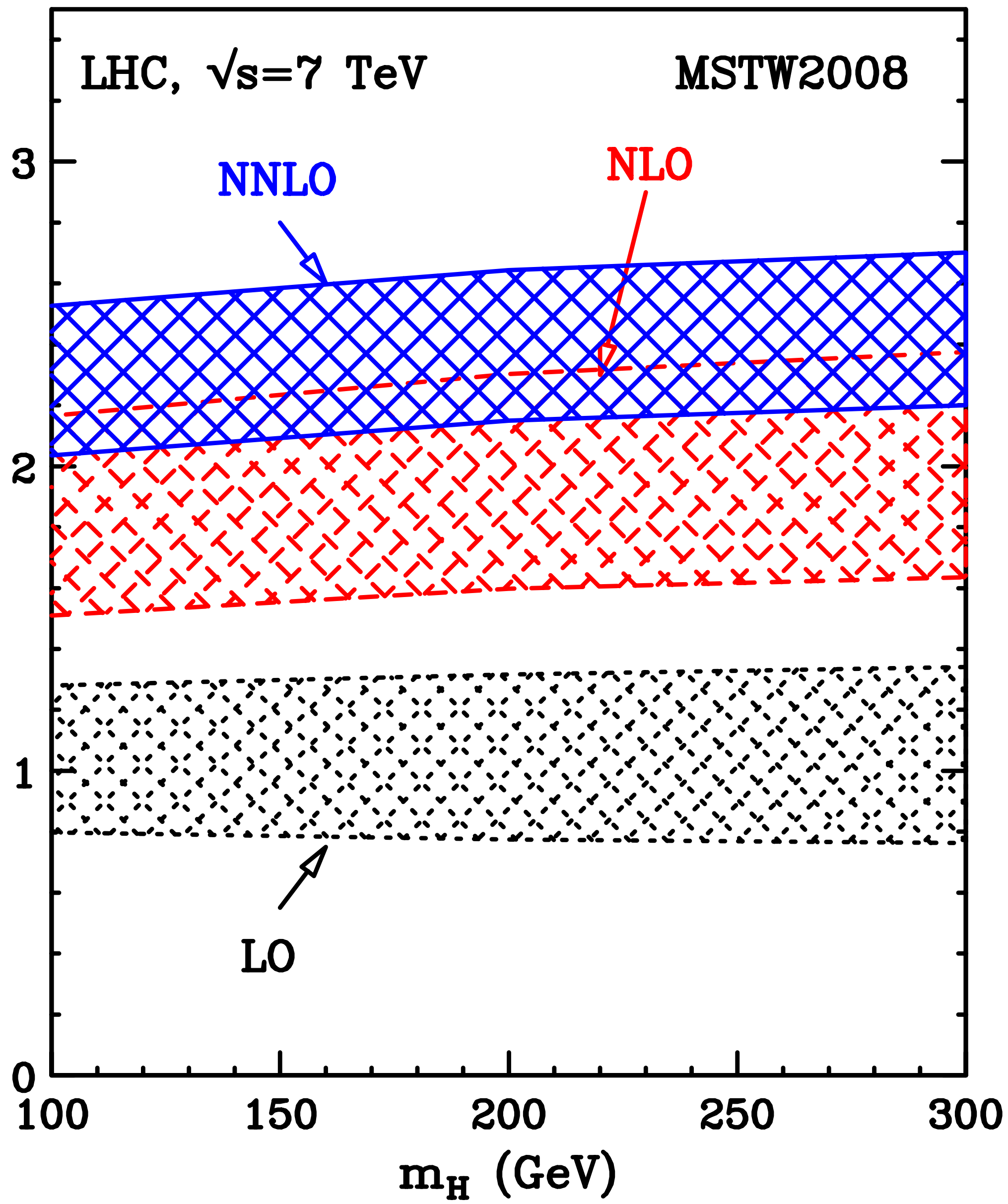
- A NNLO computation gives control on the uncertainties of a perturbative calculation.

- It's "mandatory" if NLO corrections are very large to check the behaviour of the perturbative series

- It's the best we have! It is needed for Standard Candles and for really exploiting all the available information, for example that of NNLO PDF's.

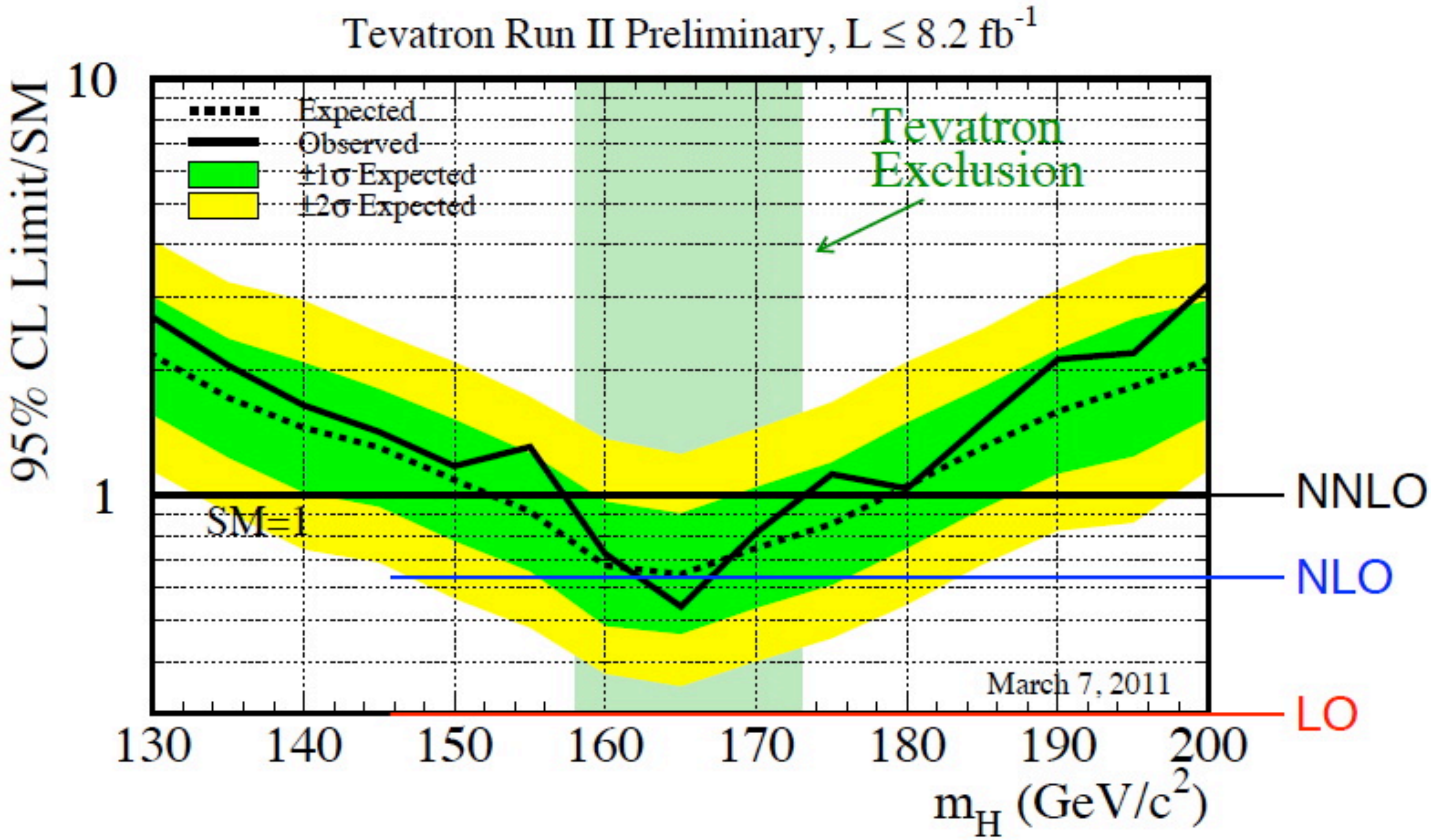


Higgs predictions at NNLO



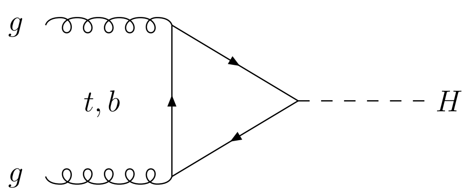
- LO calculation is not reliable.
- The perturbative series stabilizes.
- NLO estimation of higher orders effects by scale uncertainty works reasonably well.

Higgs predictions at NNLO

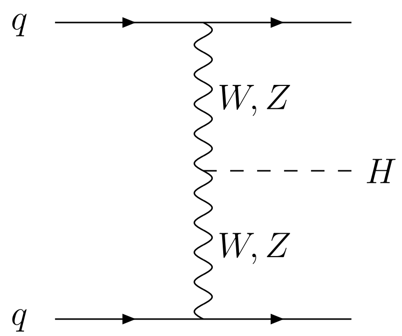


be careful : just illustrative example, not very precise

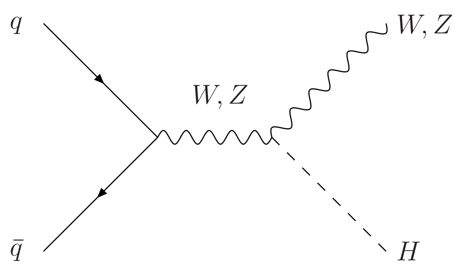
Higgs predictions at 7 TeV



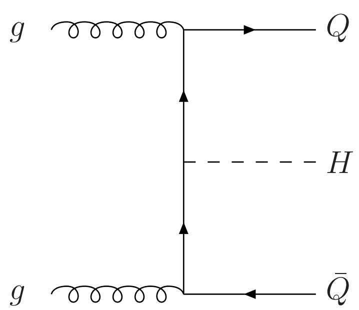
GluonFusion



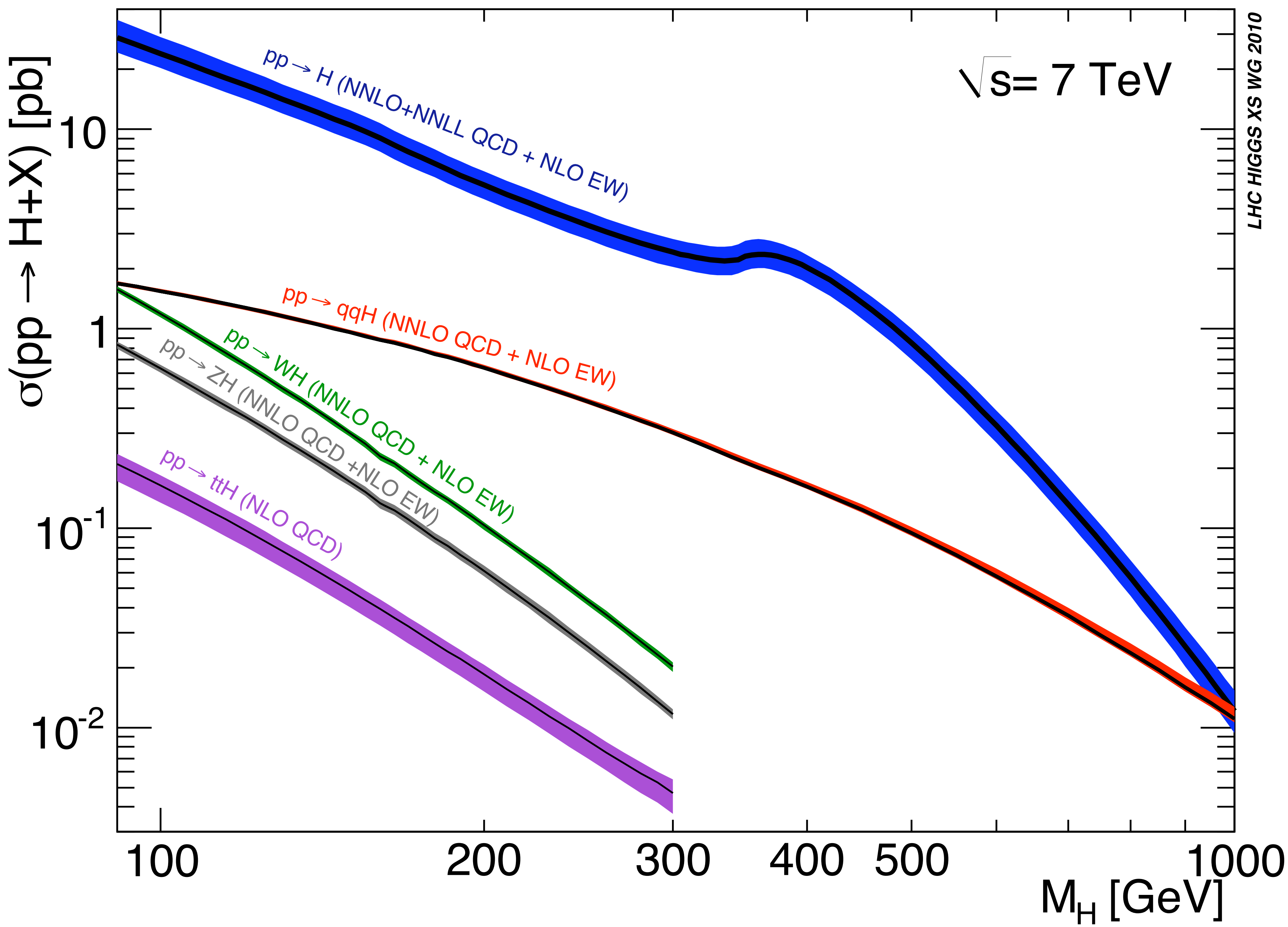
vector boson fusion (VBF)



associated production with vector bosons



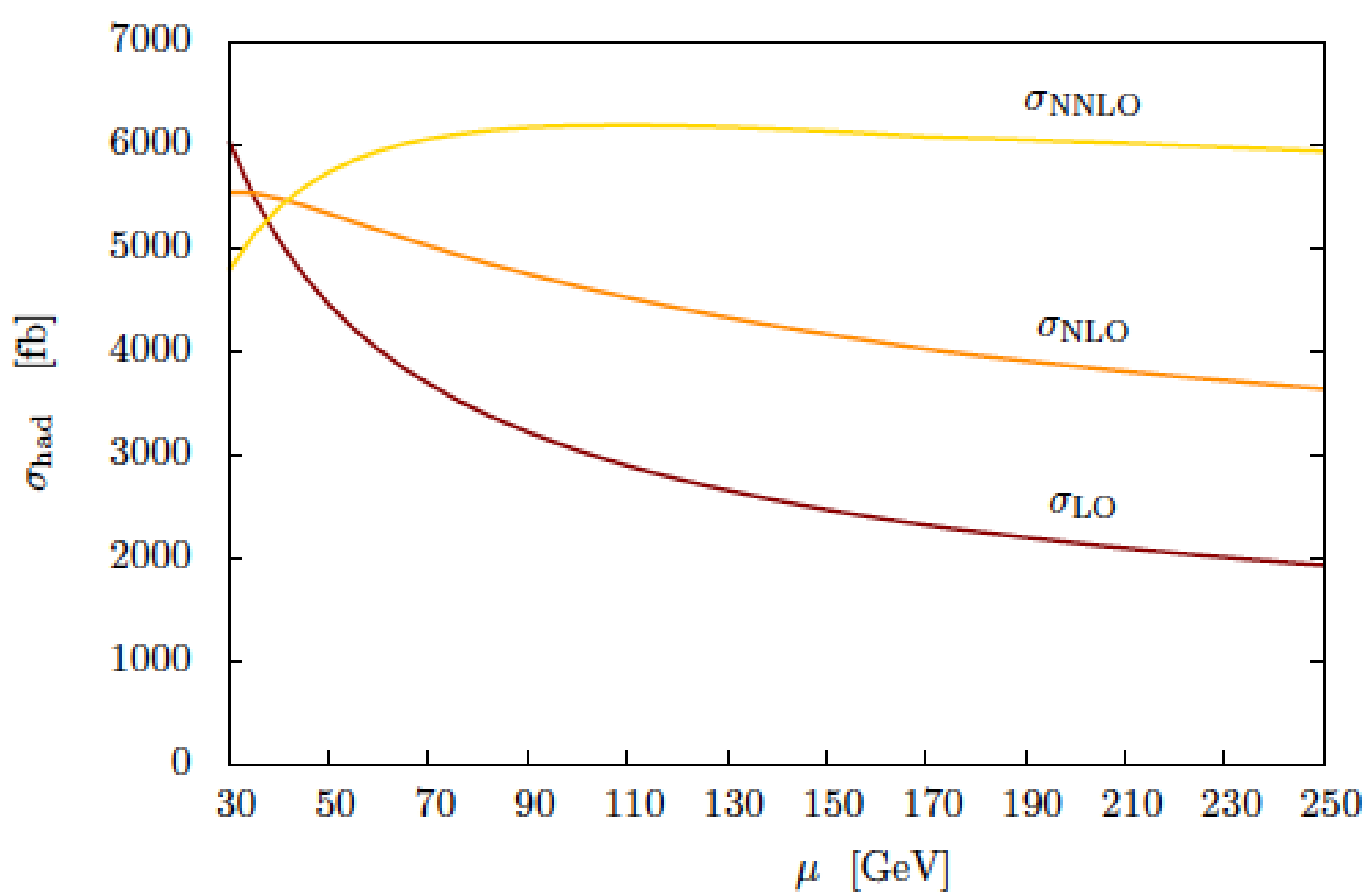
associated production with heavy quarks



Monday 10 June 2013

Higgs predictions at NNLO

pp→H+1 jet @ NNLO (only gluons)



Radja Boughezal, Fabrizio Caola, Kirill Melnikov, Frank Petriello1, and Markus Schulze. arxiv:1302.6216

MILESTONE RESULT!

Predictions at NNLO : final remarks

- Handful of precious predictions at NNLO now available for Higgs and Drell-Yan processes at the parton level for distributions.

- ttbar at NNLO now available

- General schemes possible yet not fully tested and available.

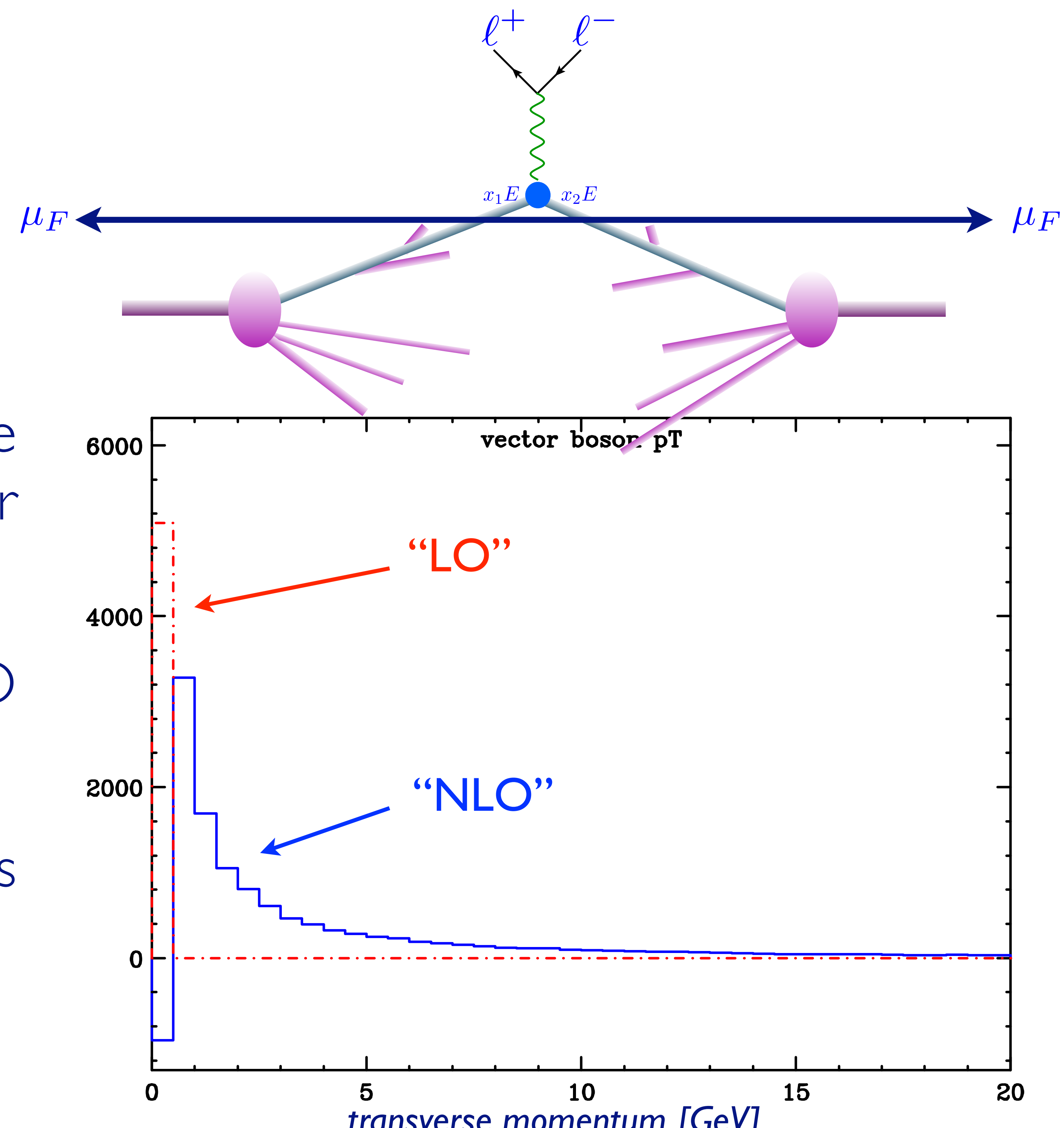
NNLO stays to the LHC era
as
NLO stayed to the Tevatron era

Limits of fixed-order predictions

- There are lots of observables that are perfectly well-behaved in this perturbative approach, i.e. that show a good convergence behavior. In particular, sufficiently inclusive observables over well-separated objects are well described.
- But more exclusive observables will, in general, be poorly described in perturbation theory

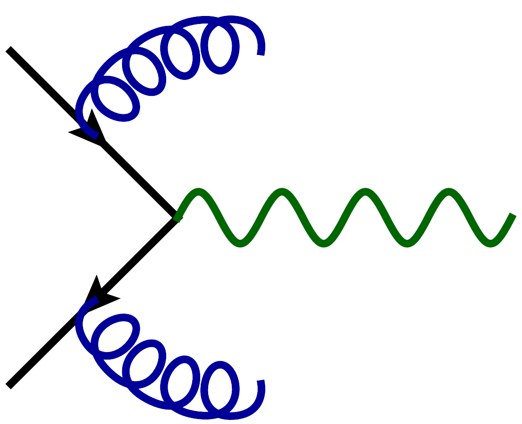
Limits of fixed-order predictions

- Consider Drell-Yan production:
  $pp \rightarrow \gamma^{*}/Z \rightarrow e^{+}e^{-} + X$
- What happens if we plot the transverse momentum of the vector boson?
- Both the LO and the NLO distributions are non-physical
- Low-transverse momentum regions is very sensitive to emissions

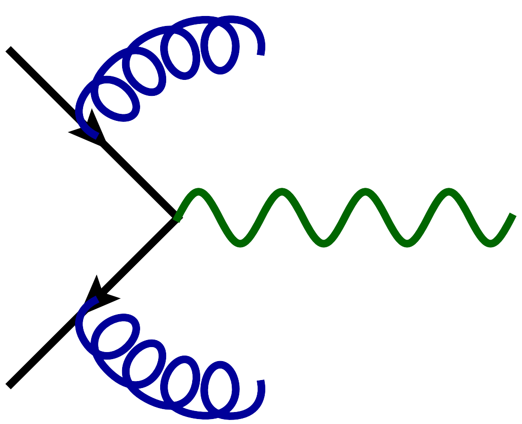


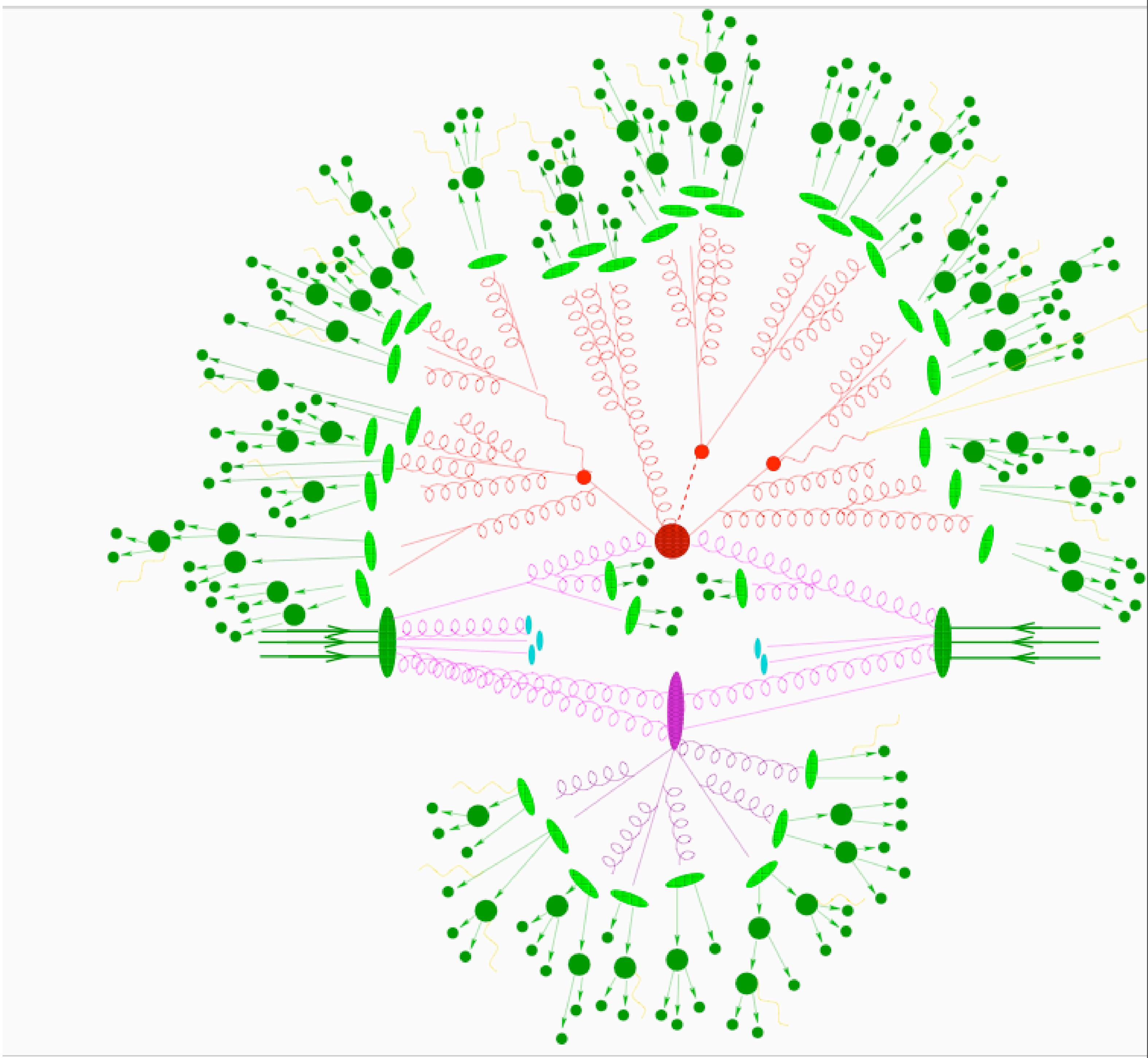
Limits of fixed-order predictions

Limits of fixed-order predictions

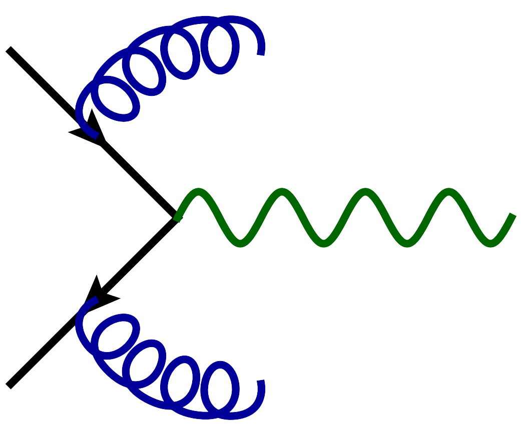


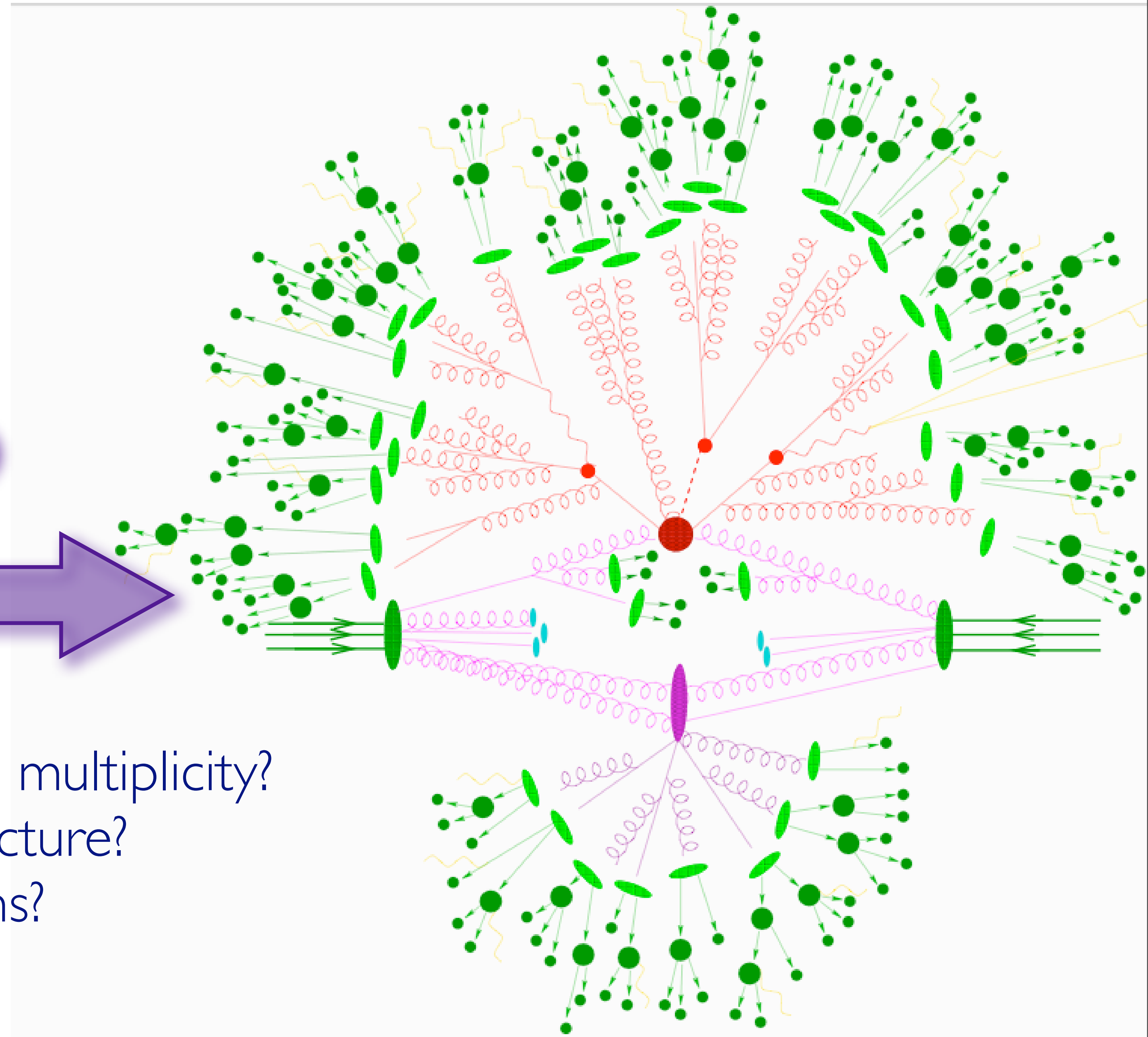
Limits of fixed-order predictions





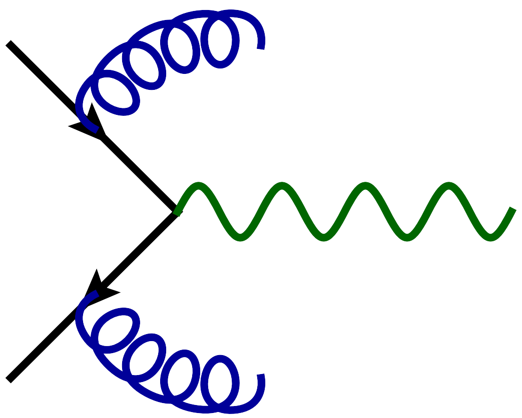
Limits of fixed-order predictions

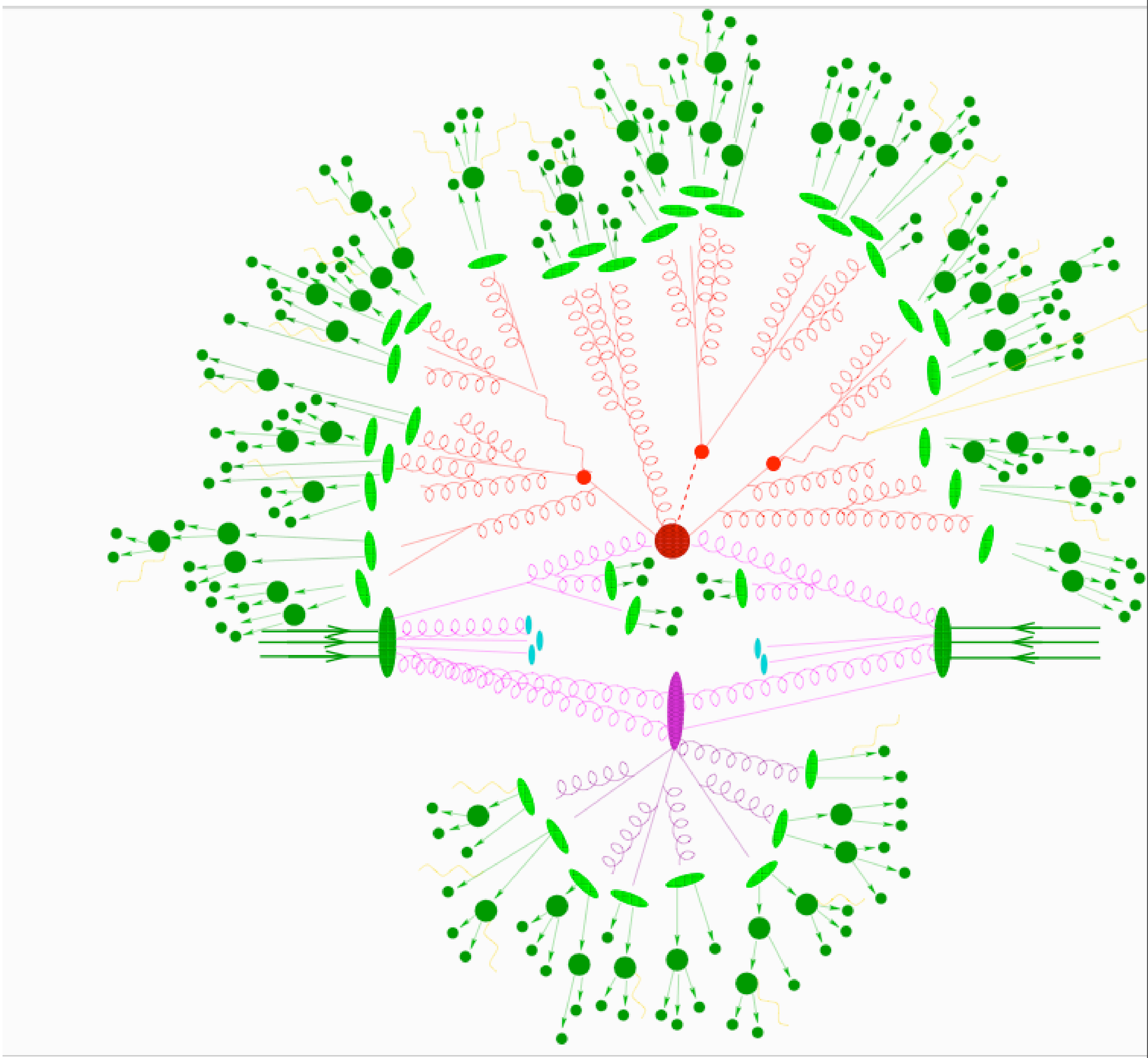




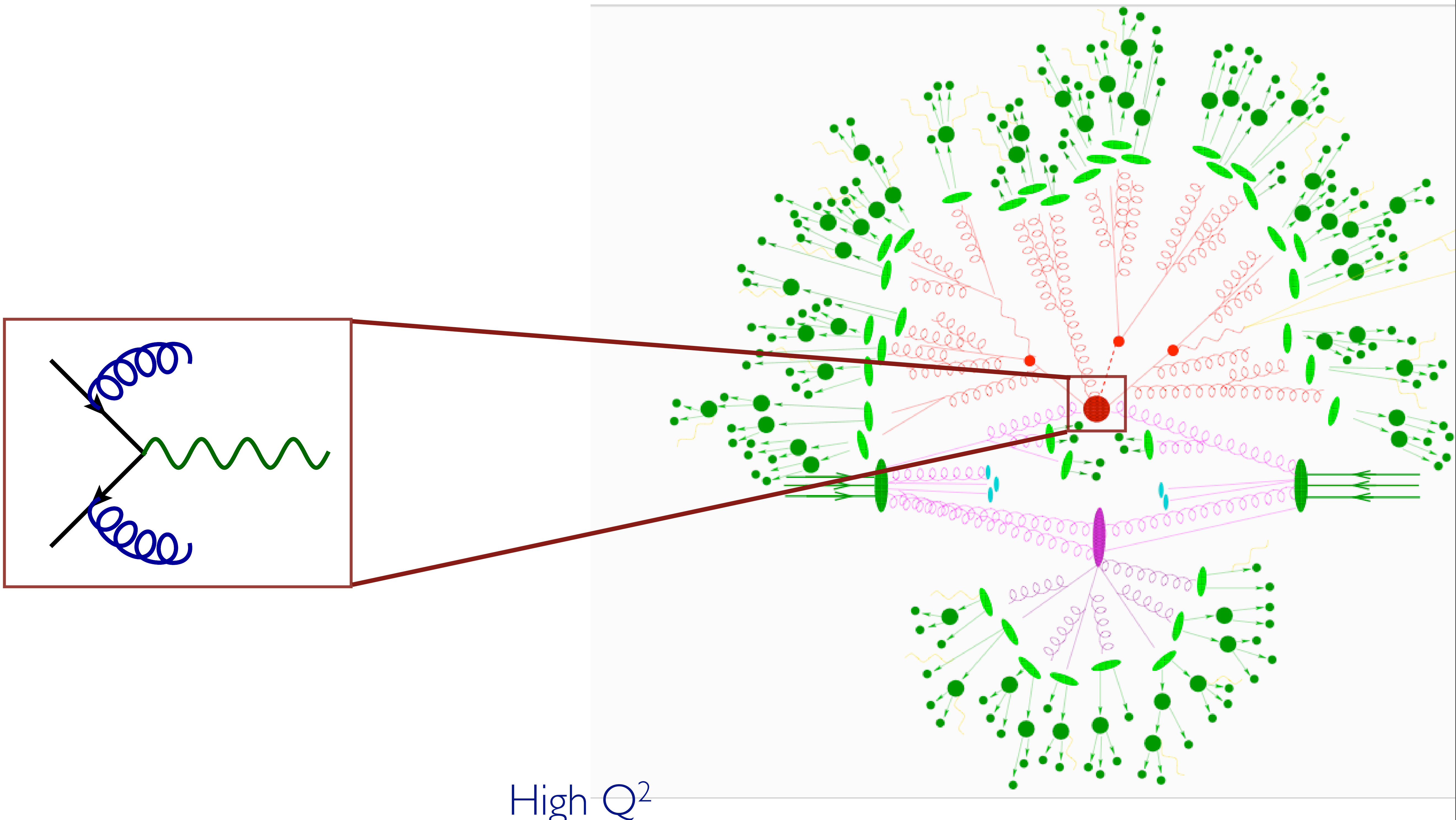
- Particle multiplicity?
- Jet structure?
- Hadrons?

Limits of fixed-order predictions





Limits of fixed-order predictions



Limits of fixed-order predictions

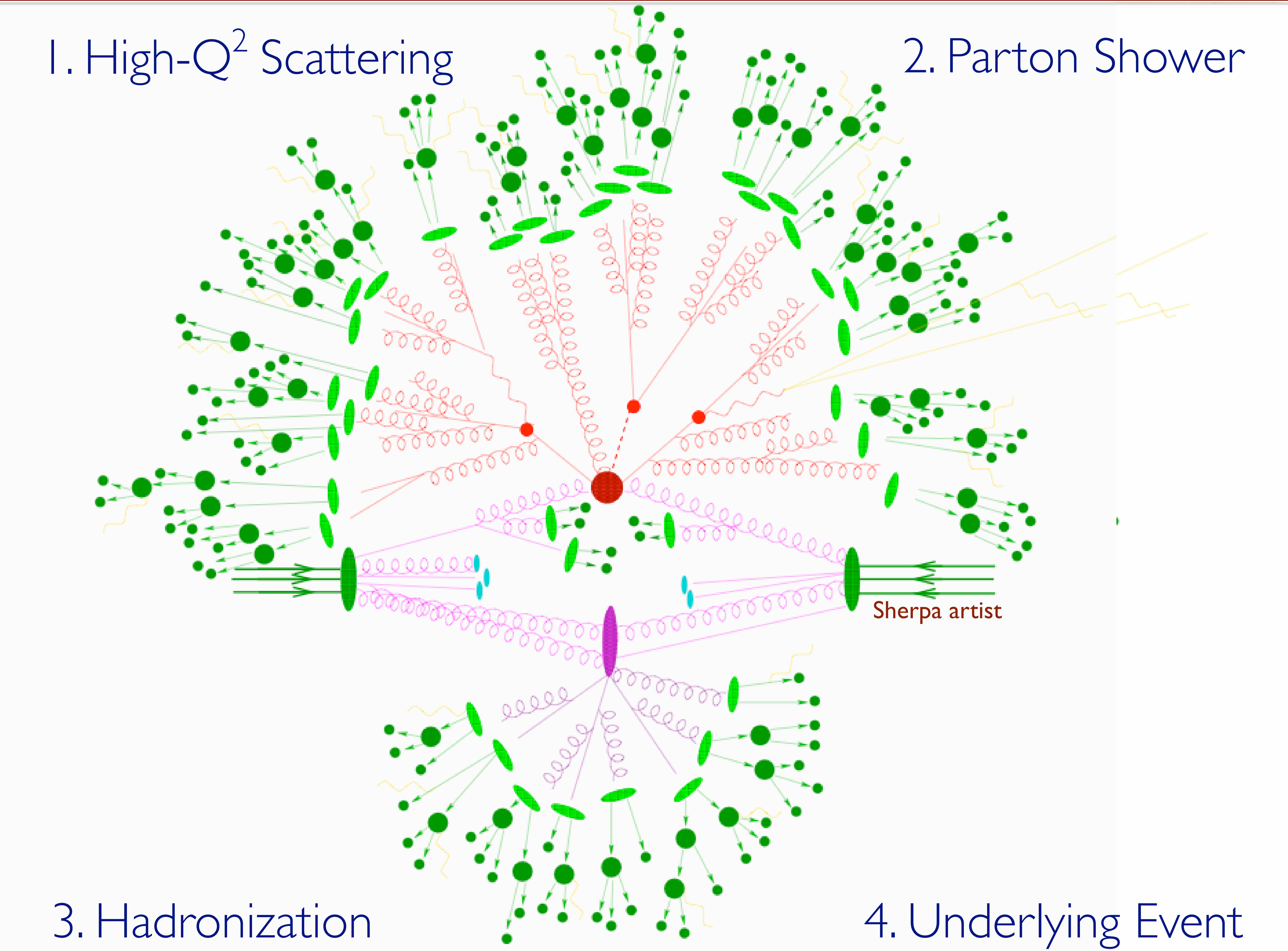
- Parton level calculations (NLO and NNLO) can be done only for an handful of partons.

- In an (N)NLO calculation, only a limited set of observables is at (N)NLO accuracy.

- In fixed-order calculations many observables (such as jets) have a hypersimplified structure (certainly not realistic).

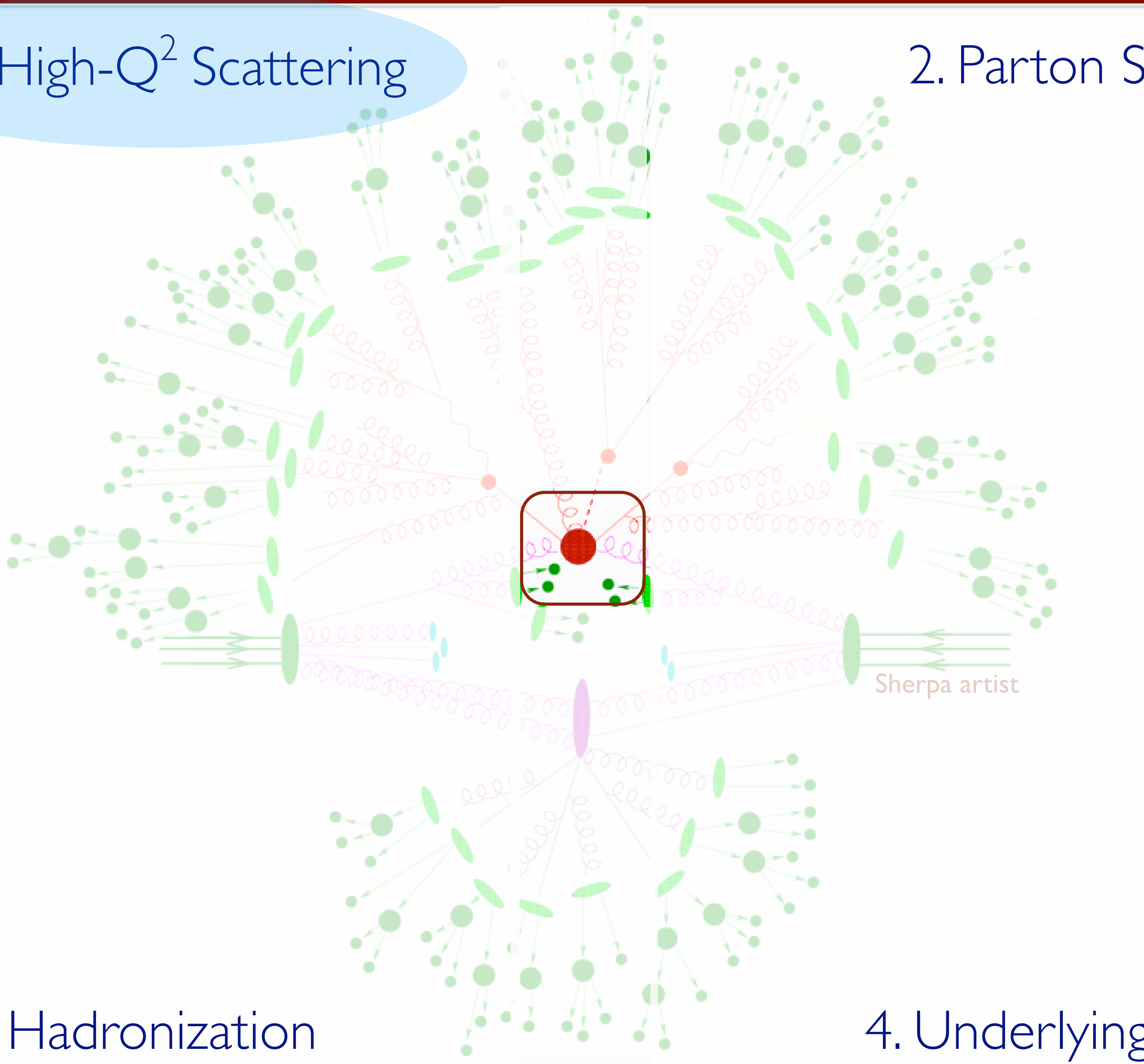
- In fixed-order calculations many observables (such as those dominated by soft and collinear effects) are not reliable.

- (N)NLO calculations contain local infinities that cancels in IR-safe observables yet make unweghting impossible $\Rightarrow$ no event generation!


1. High-Q^2 Scattering
2. Parton Shower
Sherpa artist
3. Hadronization
4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower

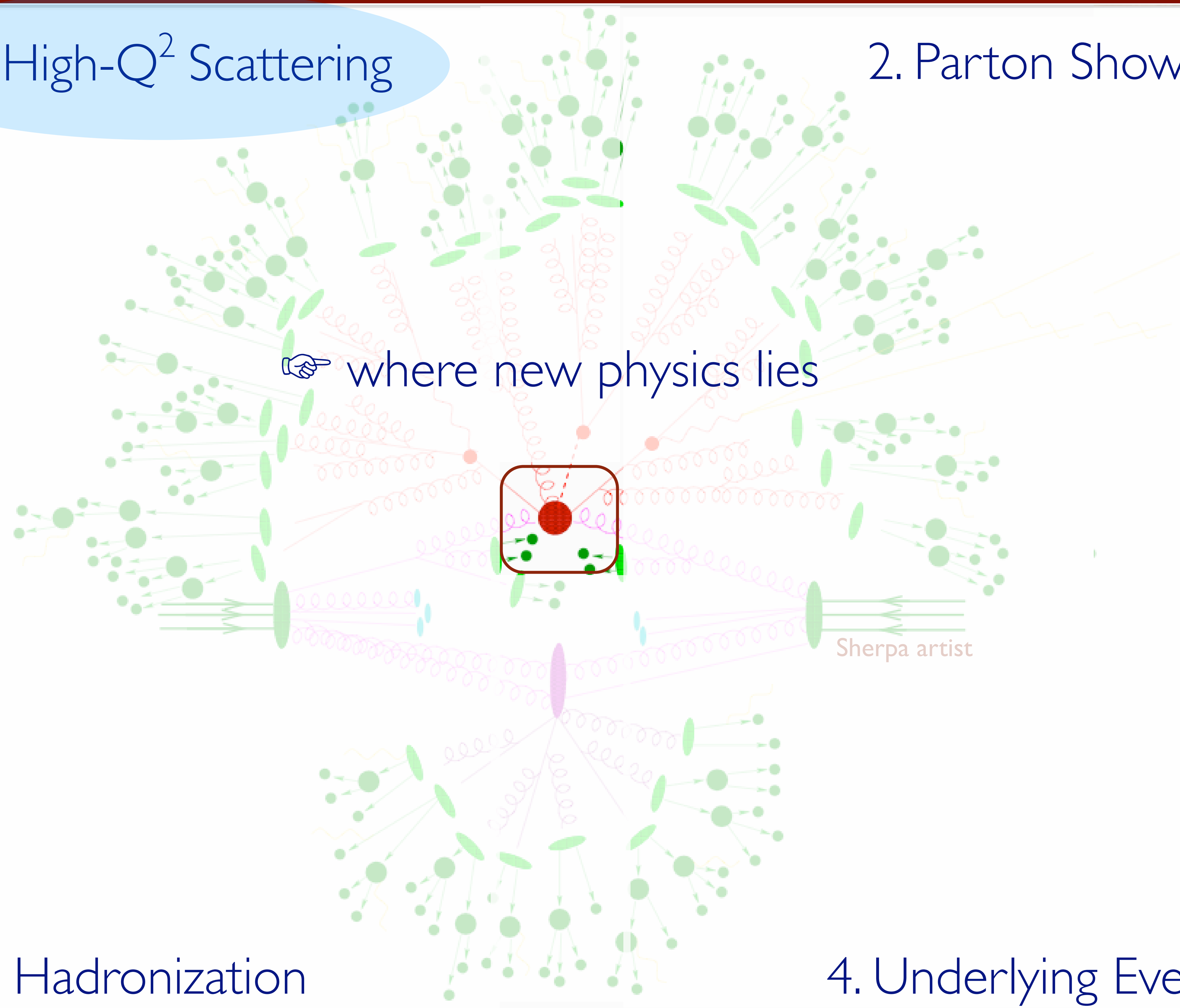


3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



☞ where new physics lies

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

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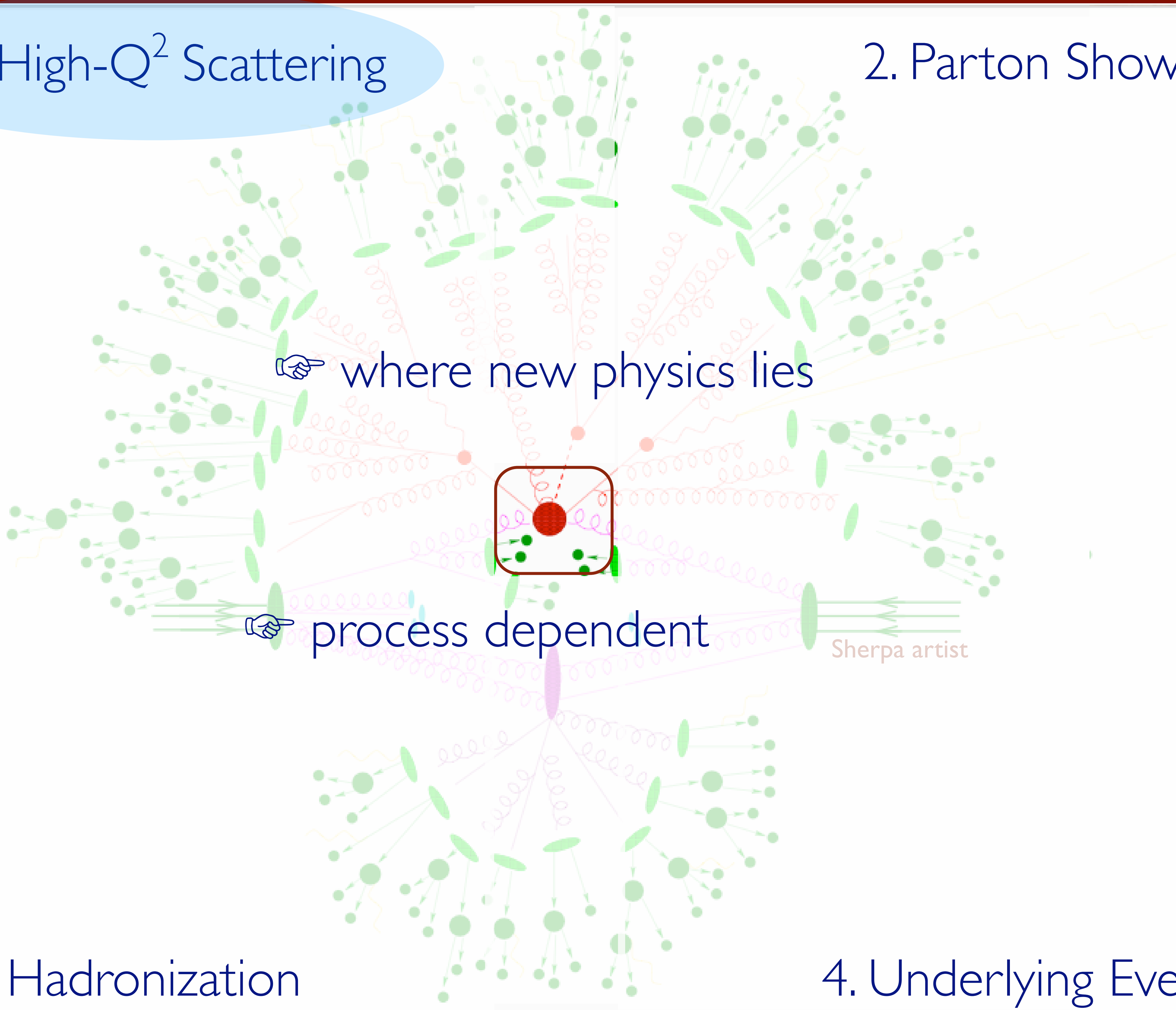

☞ where new physics lies

☞ process dependent

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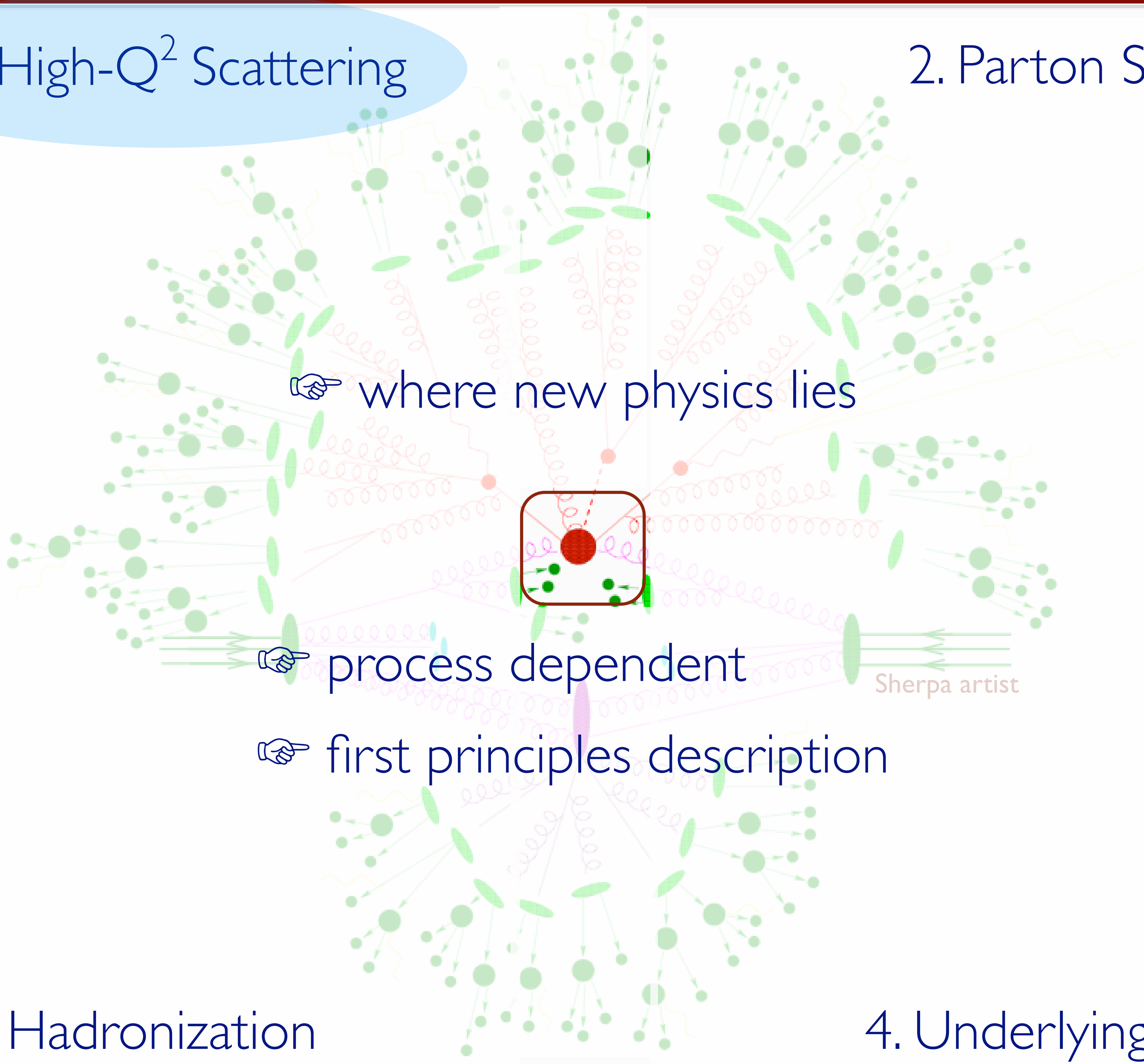

Sherpa artist

☞ where new physics lies

☞ process dependent

☞ first principles description

3. Hadronization

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1. High-Q^2 Scattering

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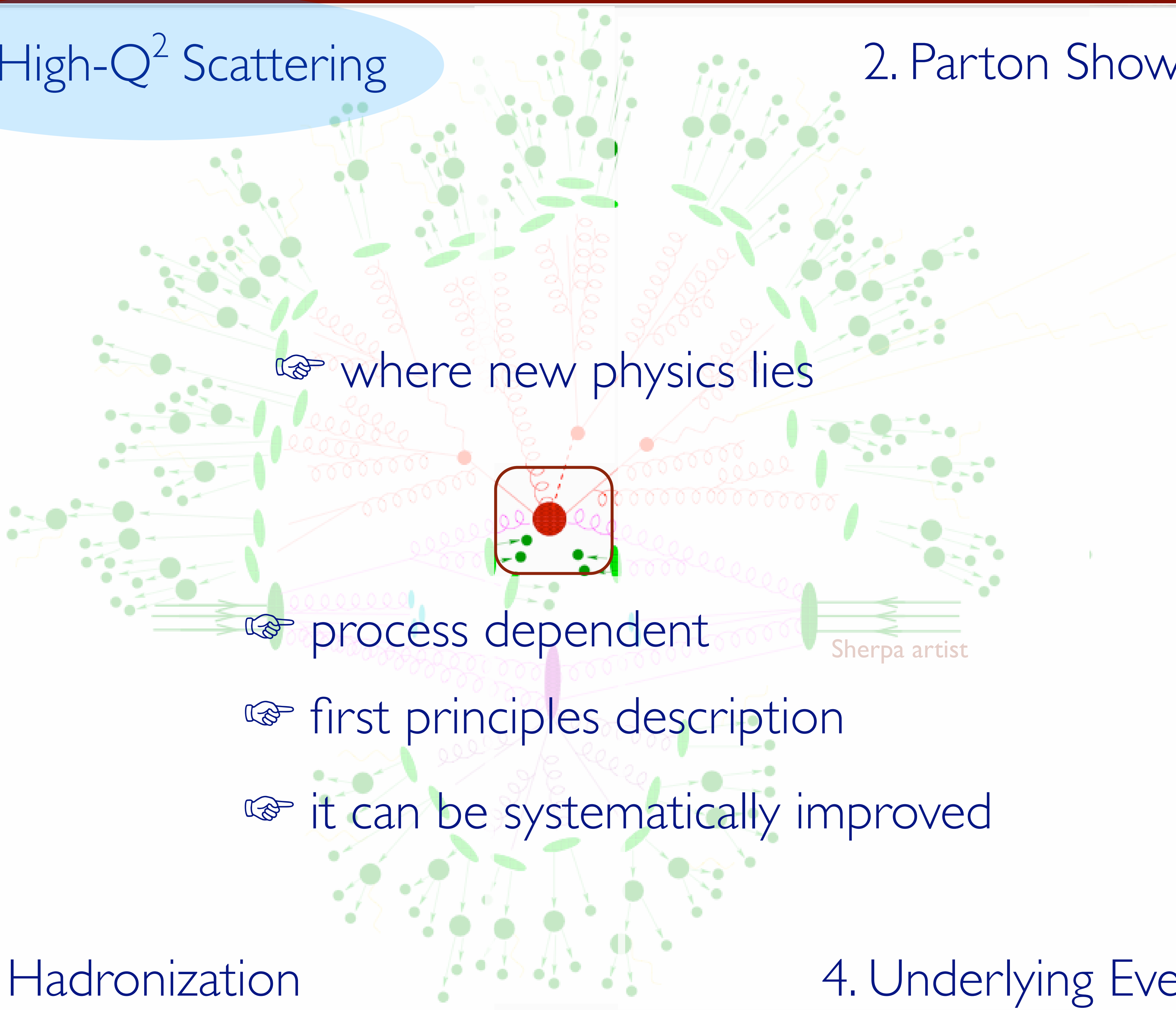
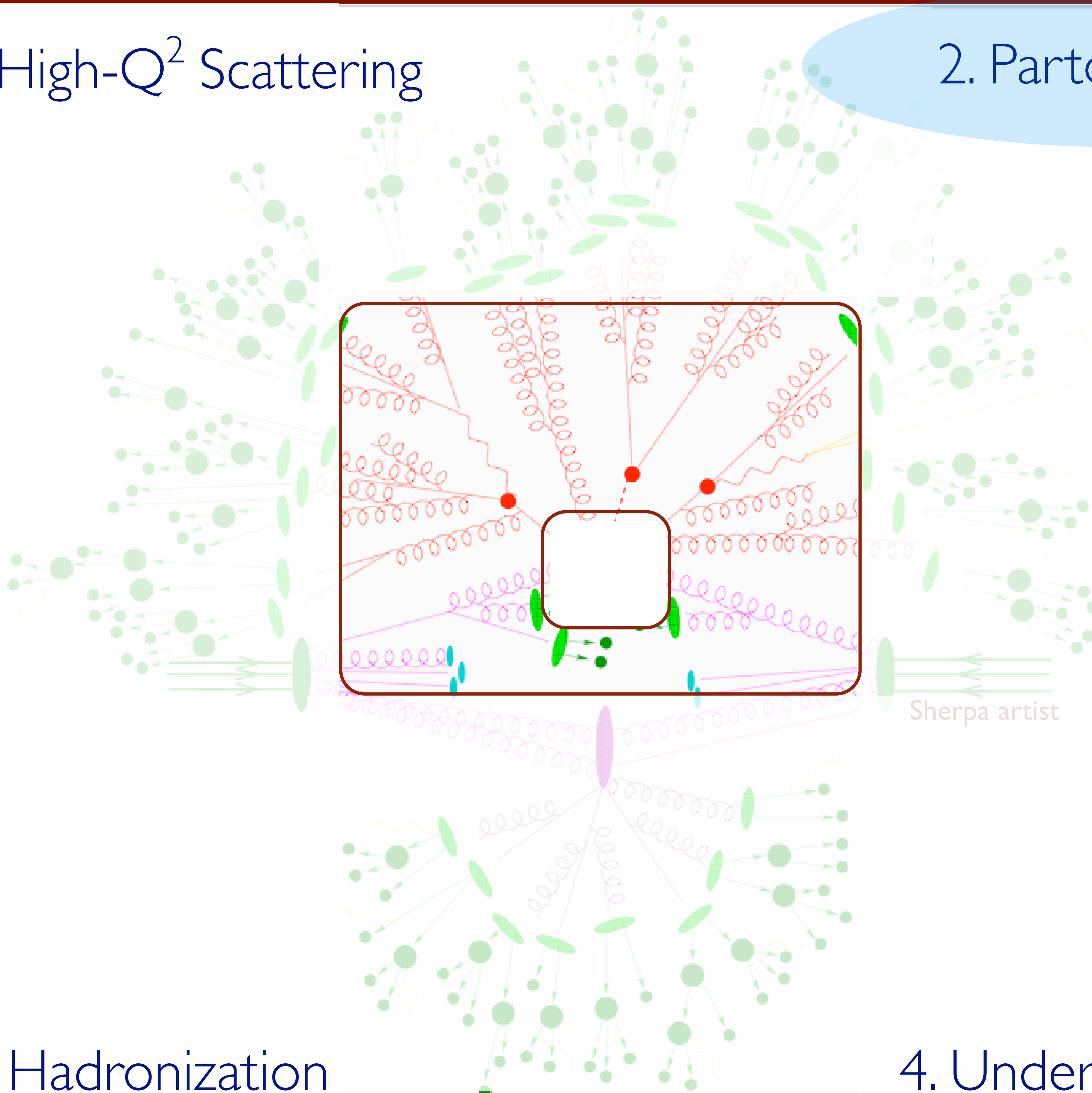
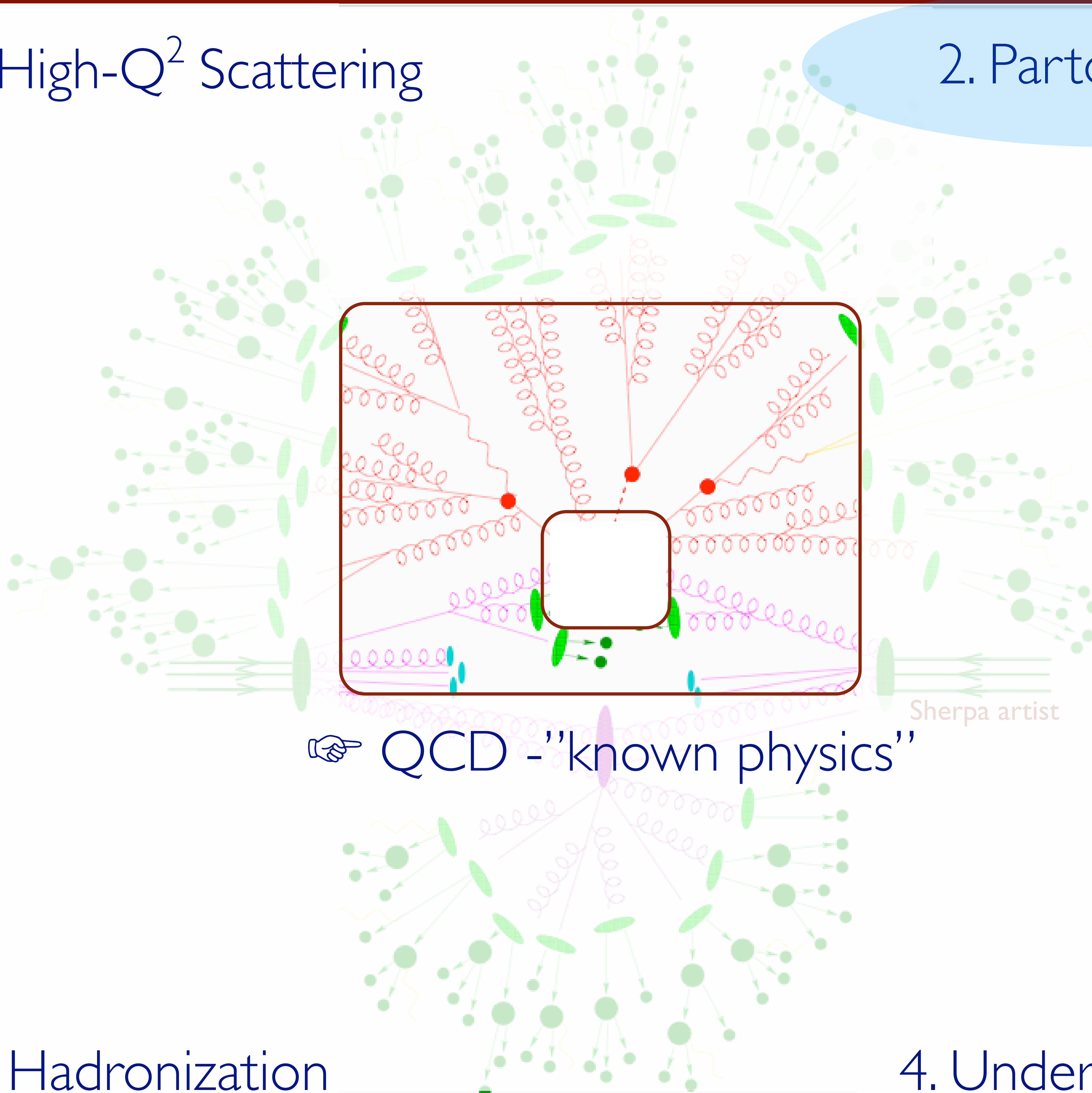


- ☞ where new physics lies
- ☞ process dependent
- ☞ first principles description
- ☞ it can be systematically improved

3. Hadronization

4. Underlying Event

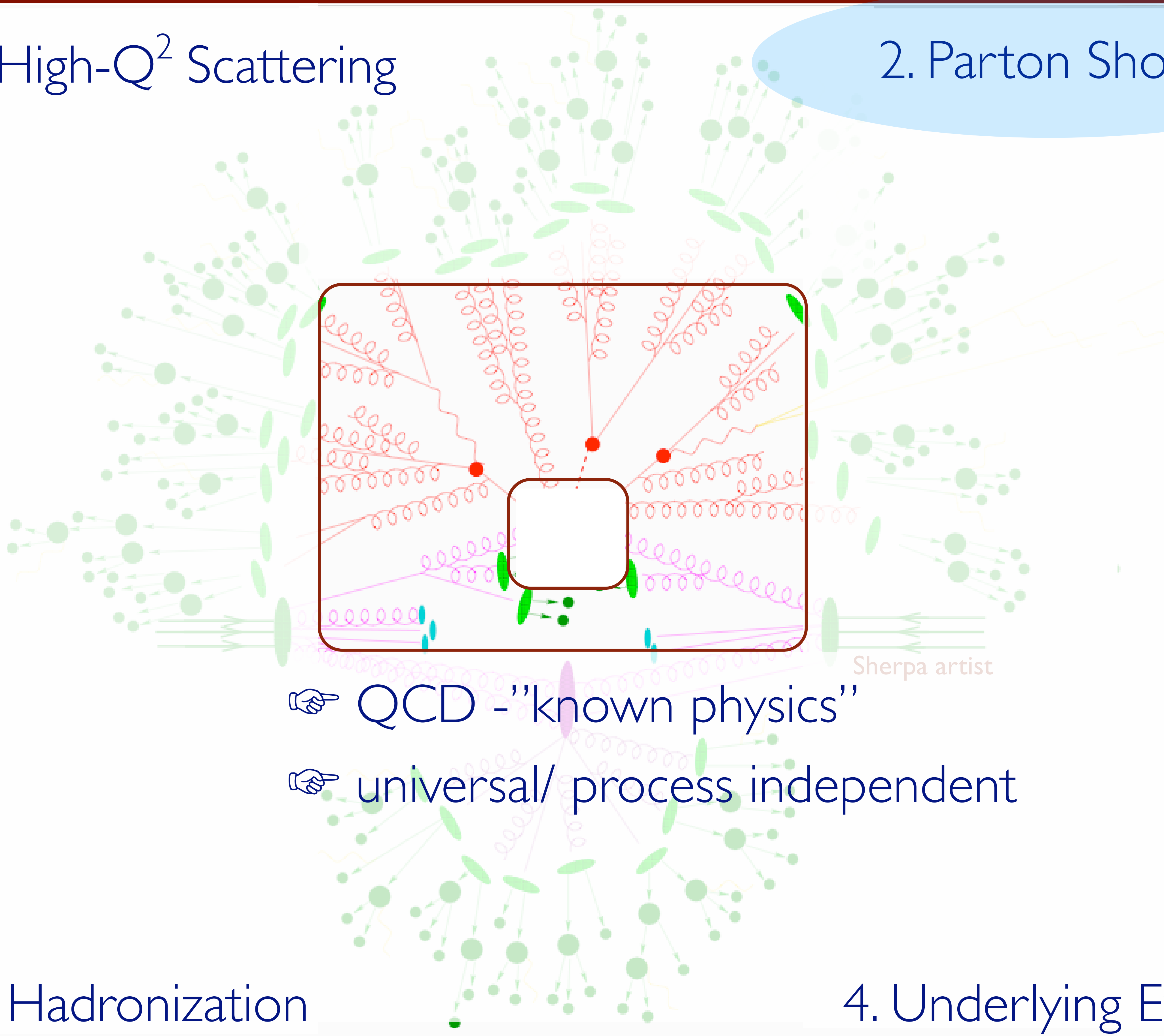

Sherpa artist



☞ QCD -"known physics"

1. High-Q^2 Scattering

2. Parton Shower



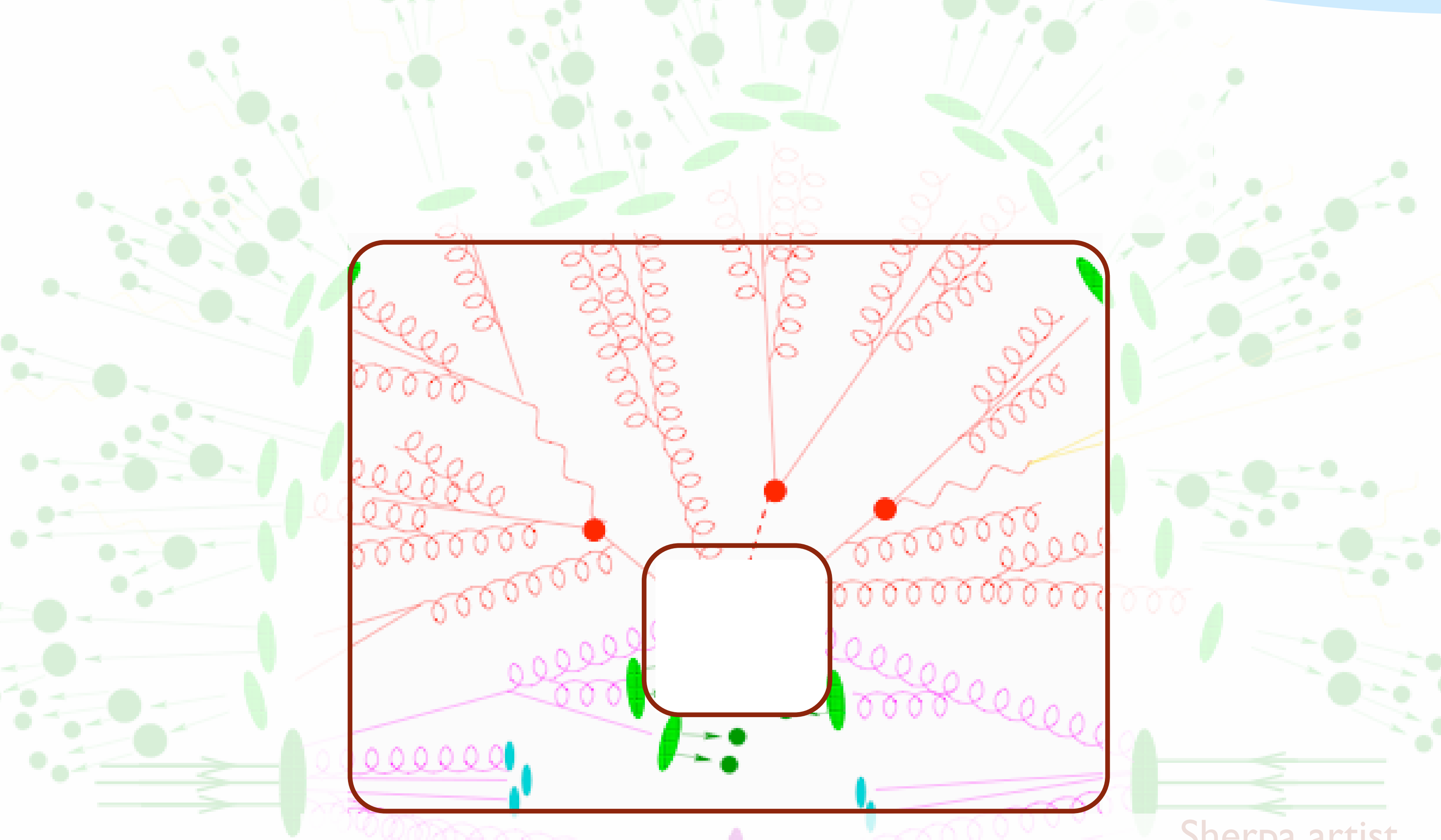
- ☞ QCD -"known physics"
- ☞ universal/ process independent

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



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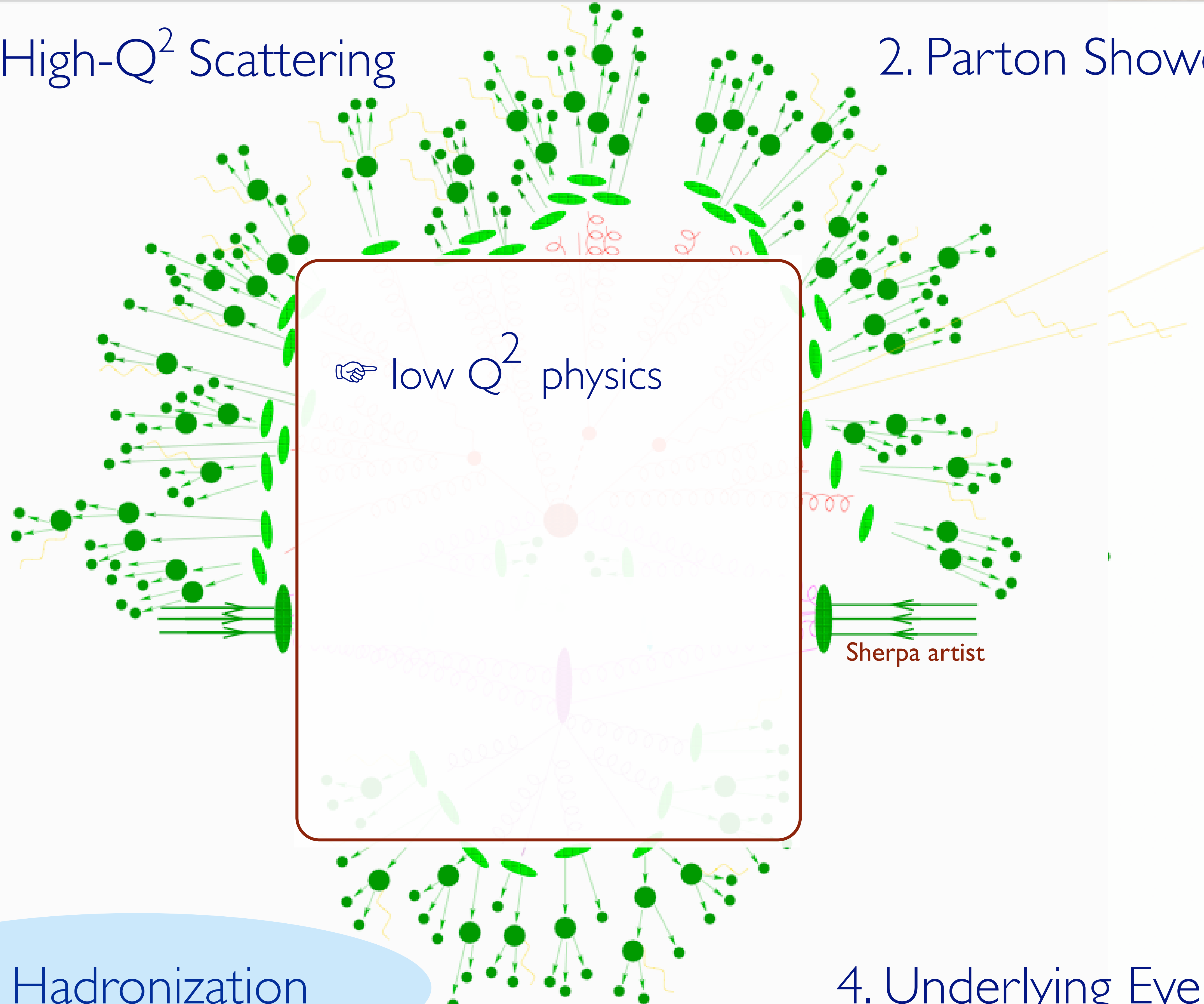


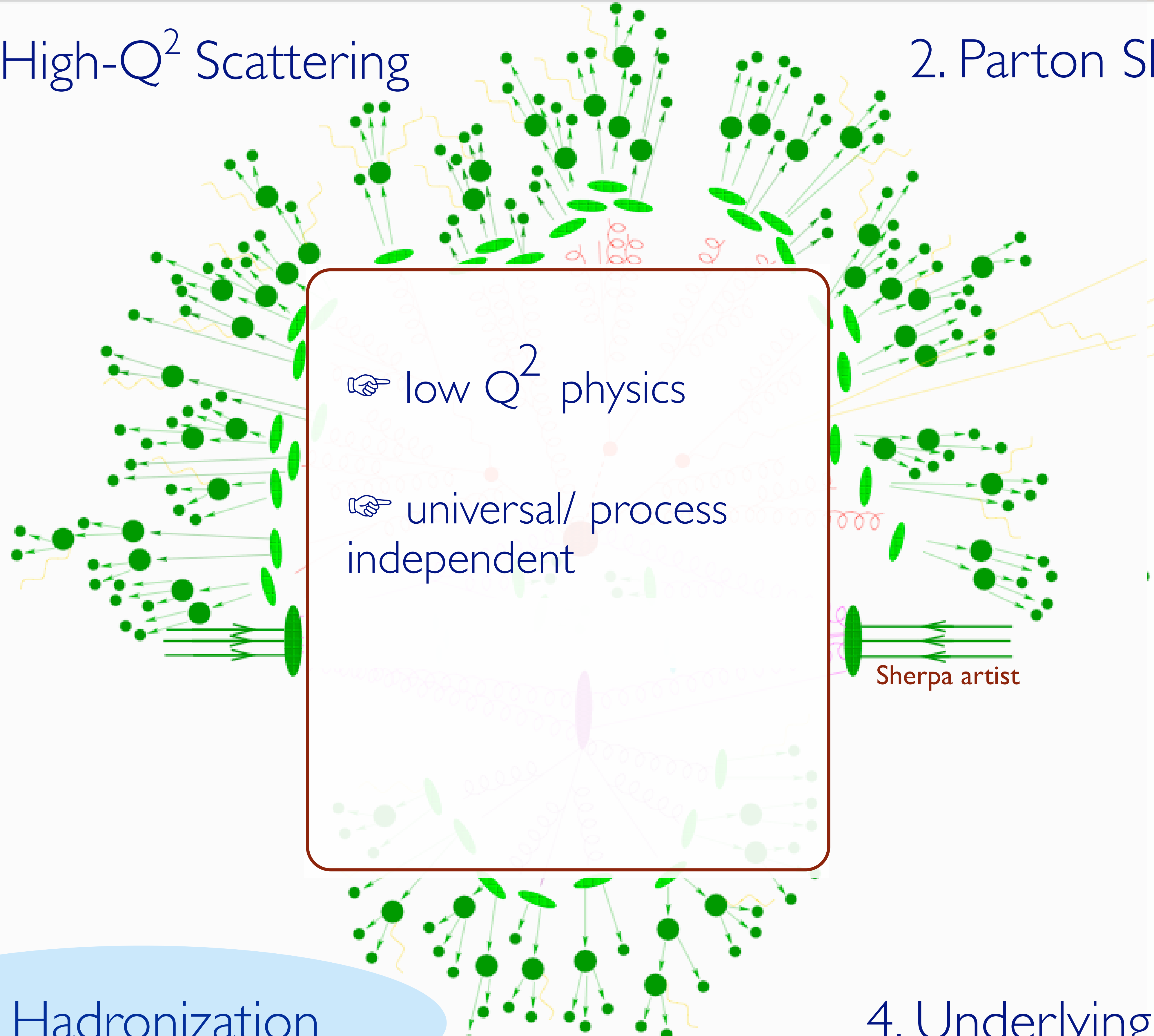
☞ low Q^2 physics

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



☞ low Q^2 physics

☞ universal/ process independent

3. Hadronization

4. Underlying Event

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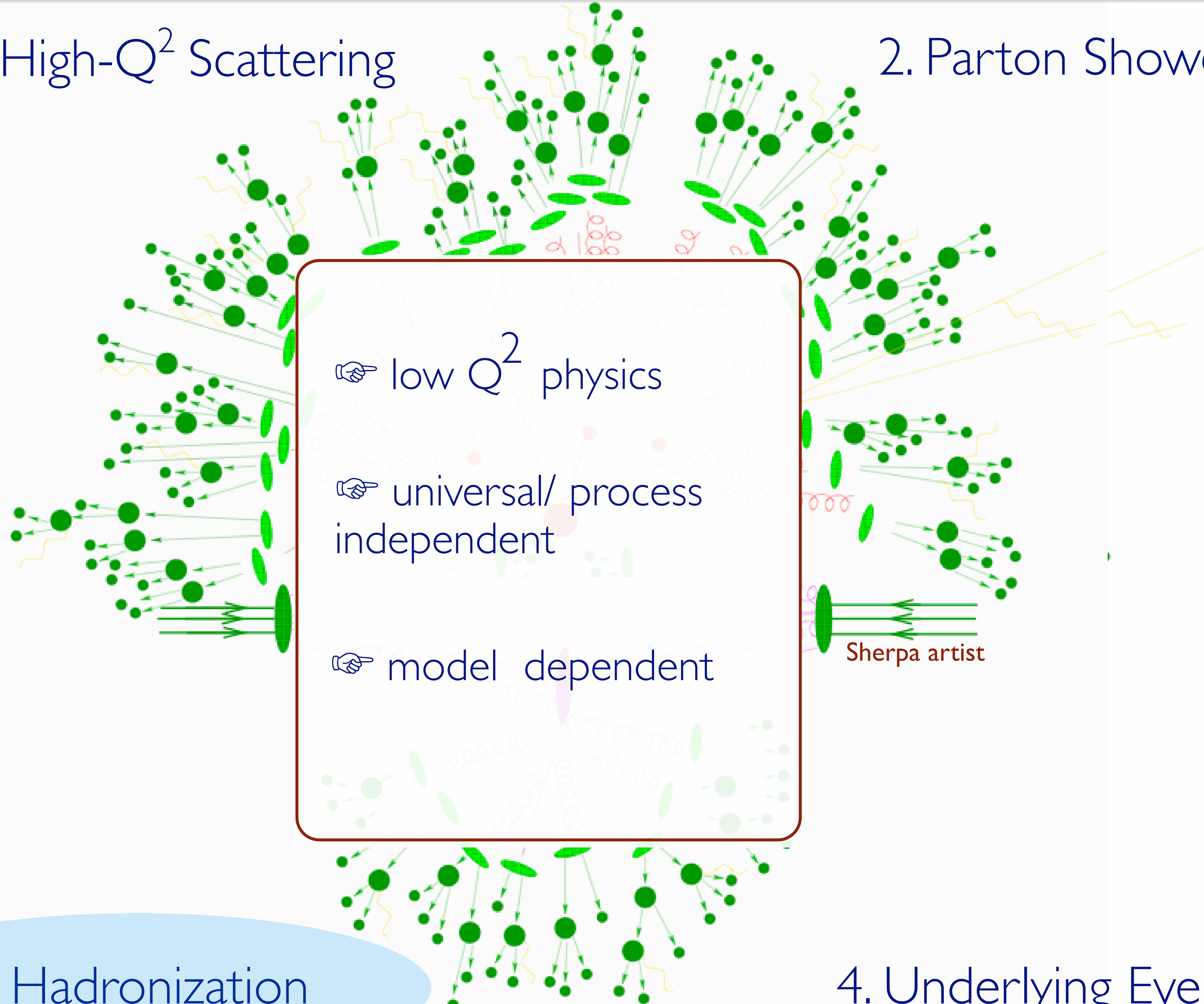

- ☞ low Q^2 physics
- ☞ universal/ process independent
- ☞ model dependent

3. Hadronization

4. Underlying Event

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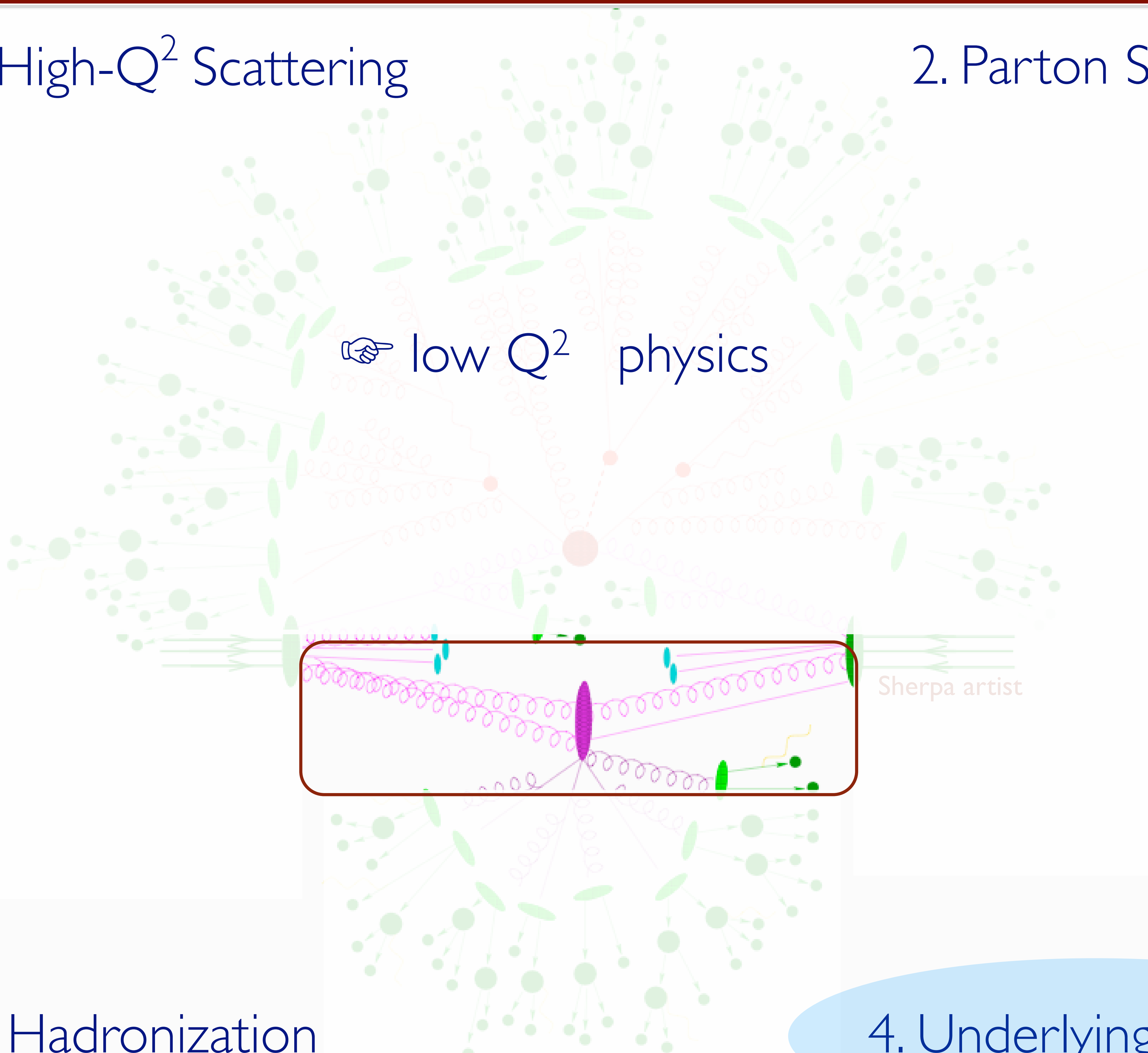
2. Parton Shower

☞ low Q^2 physics

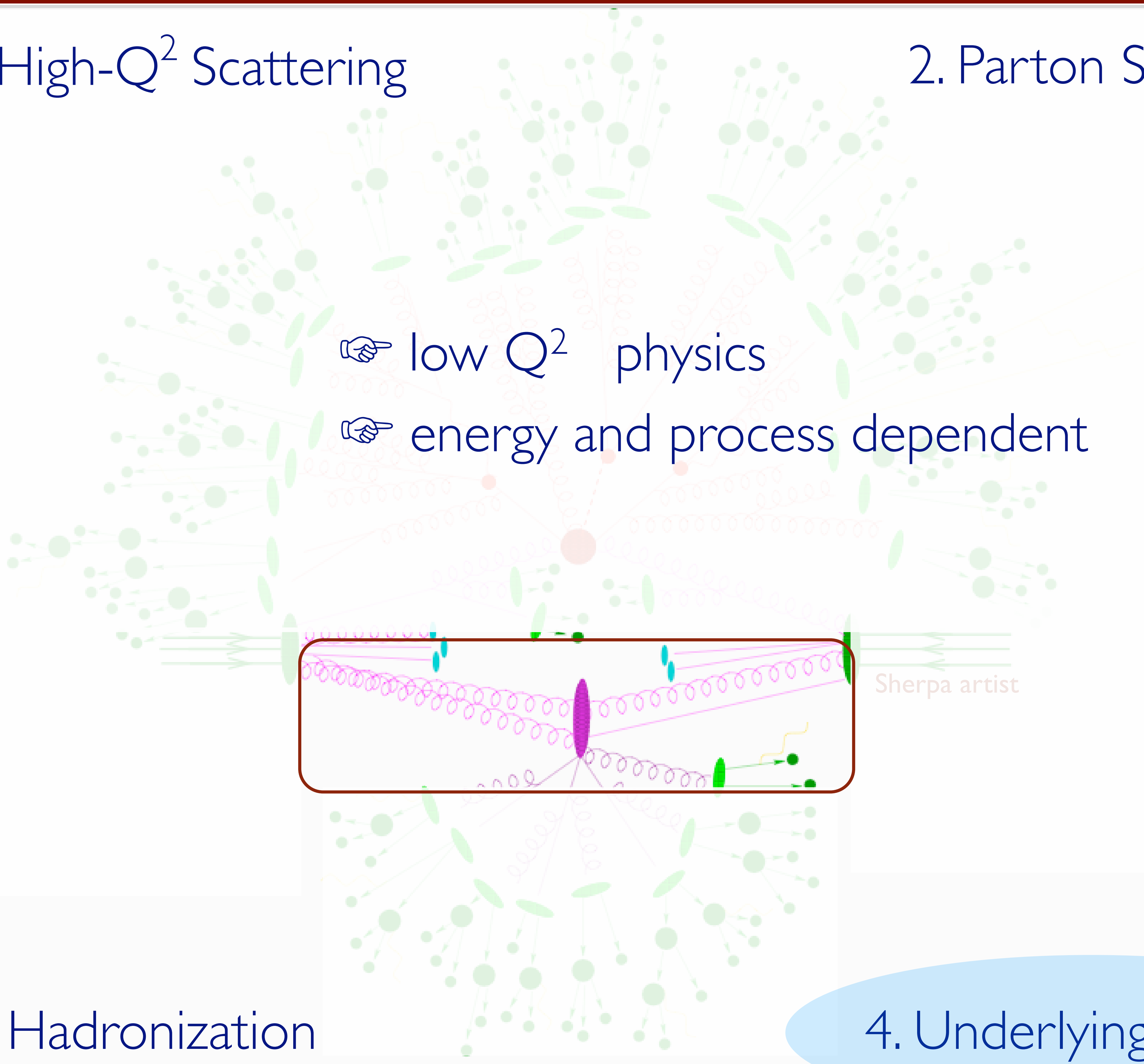


3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



☞ low Q^2 physics

☞ energy and process dependent

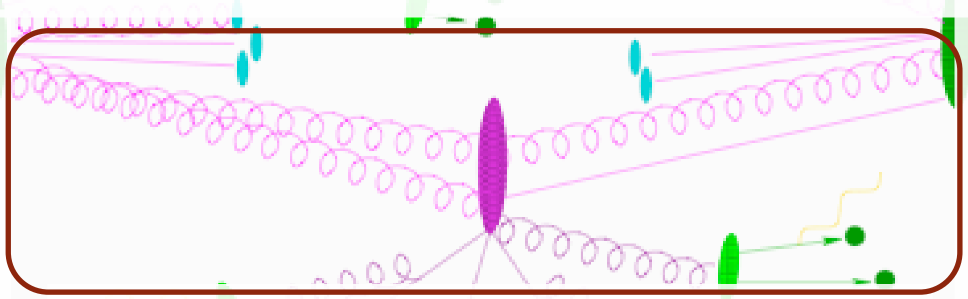
3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower

- ☞ low Q^2 physics
- ☞ energy and process dependent
- ☞ model dependent



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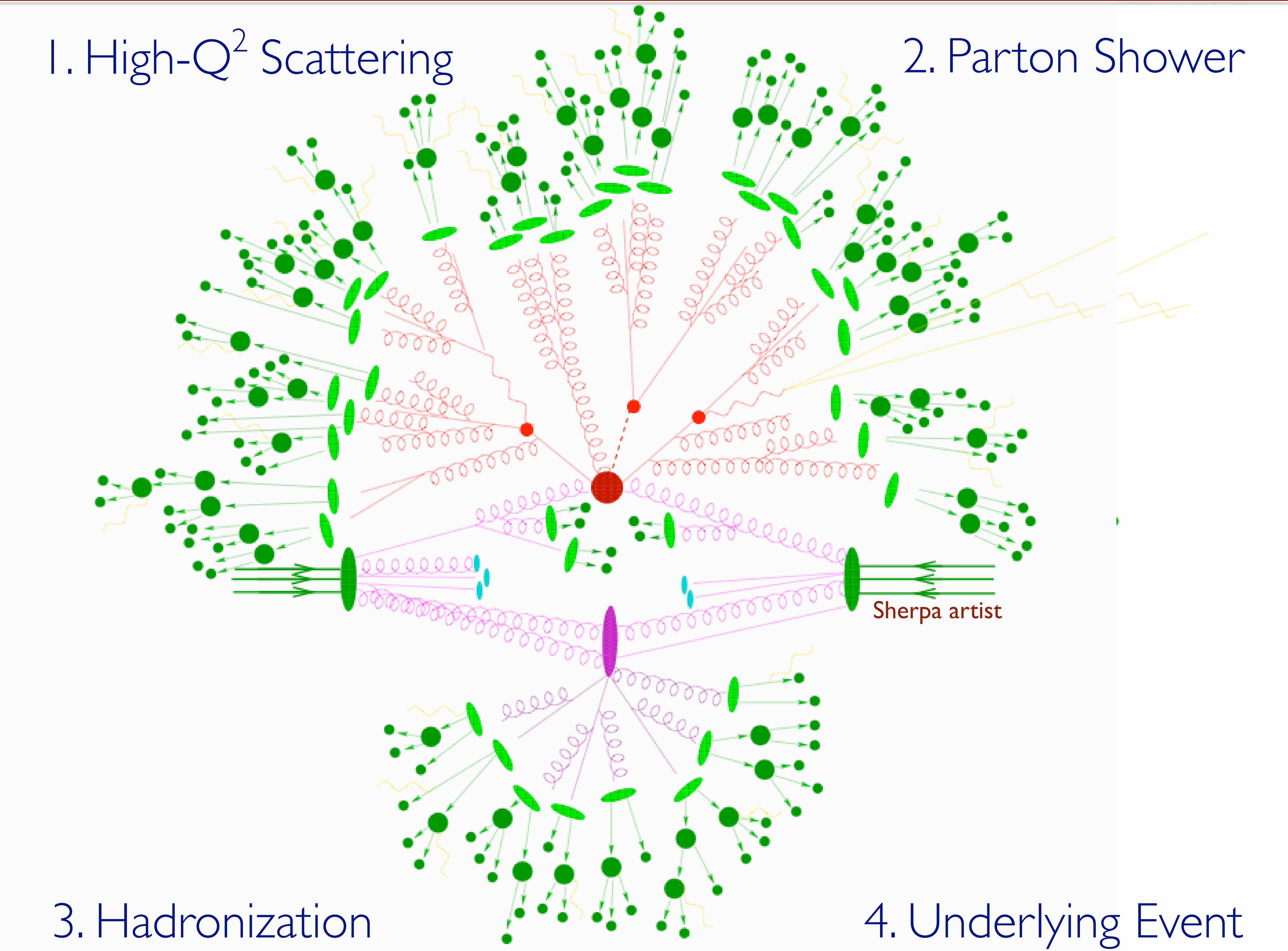
4. Underlying Event


1. High-Q^2 Scattering
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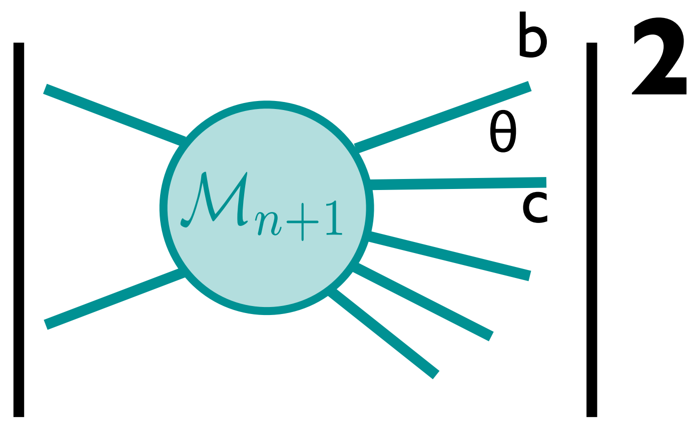
Sherpa artist

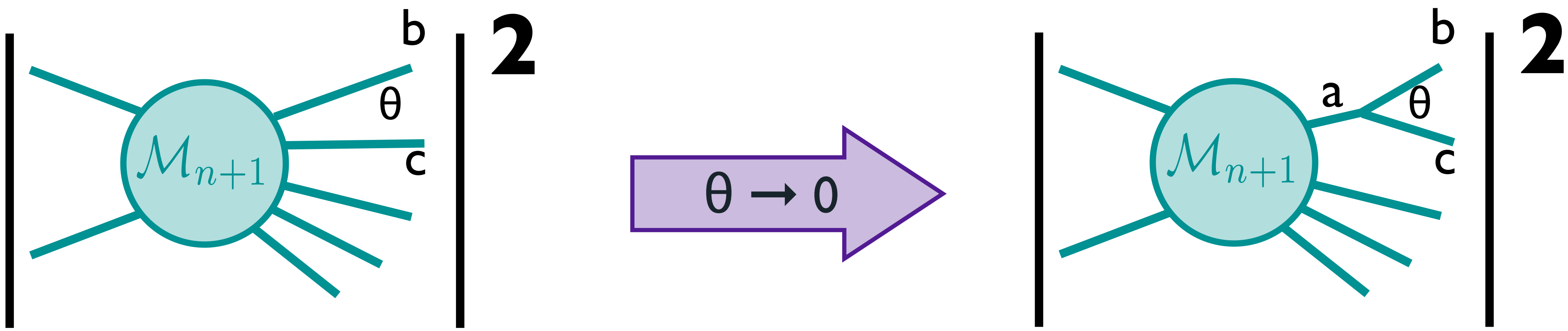
Parton shower

- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation
- This effect should be unitary: the inclusive cross section shouldn't change when extra radiation is added
- Remember that parton-level cross sections for a hard process are inclusive in anything else.
  E.g. for LO Drell-Yan production **all** radiation is included via PDFs (apart from non-perturbative power corrections)
- And finally we want to turn partons into hadrons (hadronization)....

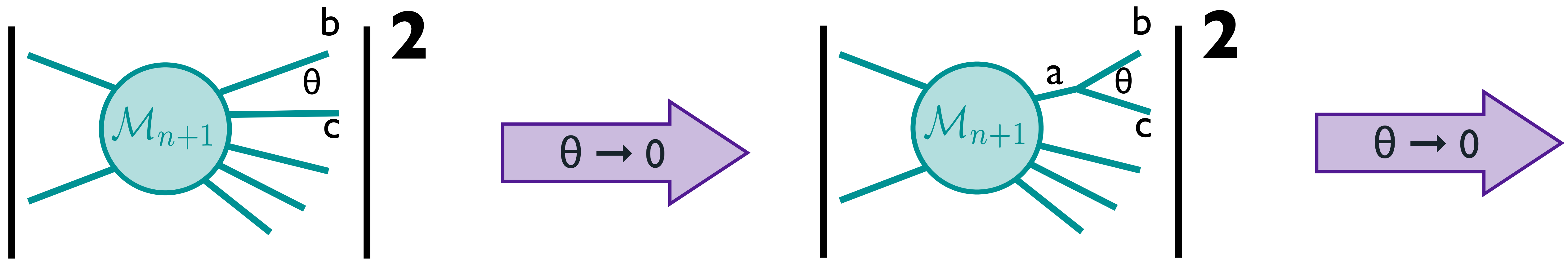
Collinear factorization



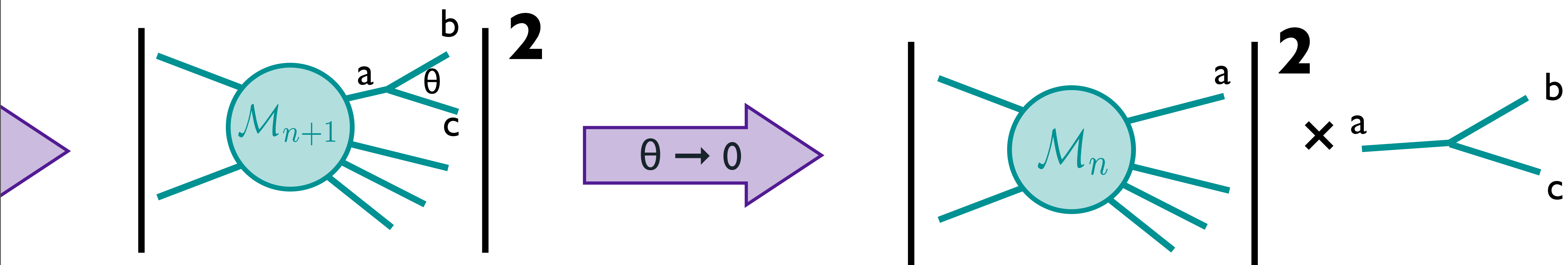
Collinear factorization



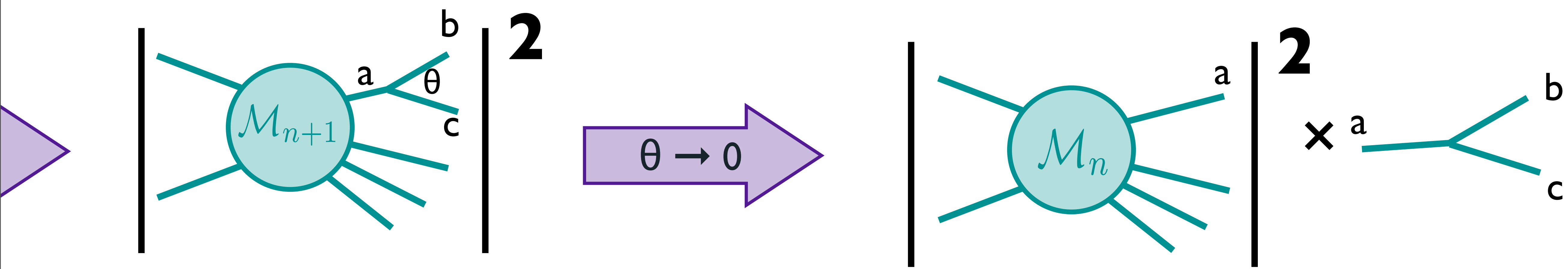
Collinear factorization



Collinear factorization

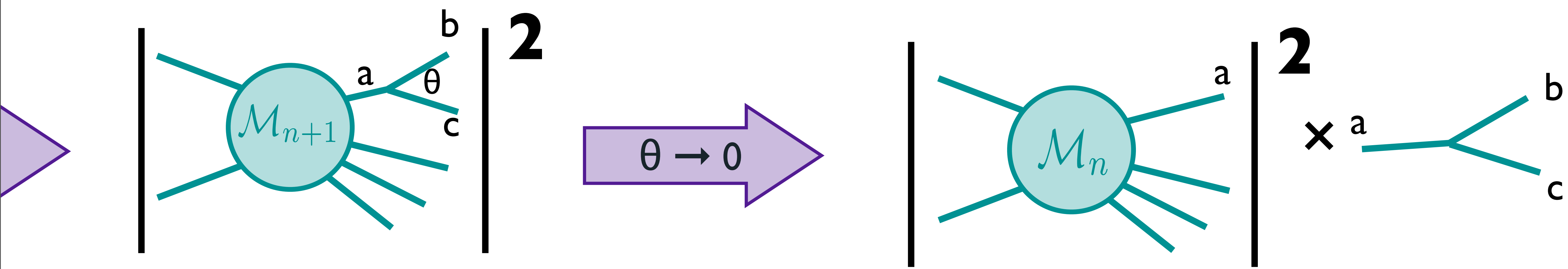


Collinear factorization



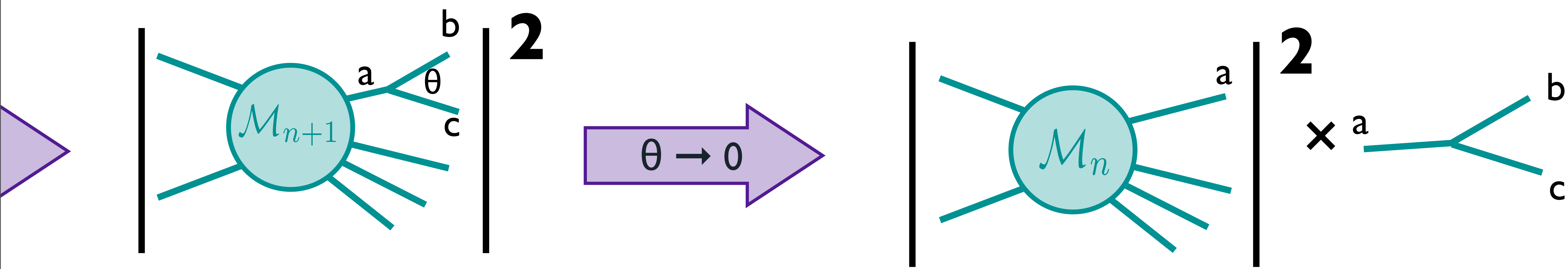
- Consider a process for which two particles are separated by a small angle θ.

Collinear factorization



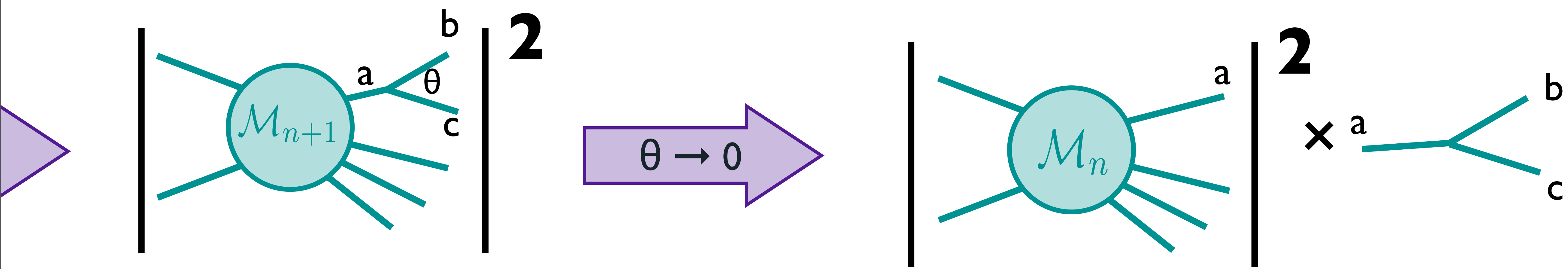
- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.

Collinear factorization



- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.
- The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability.

Collinear factorization



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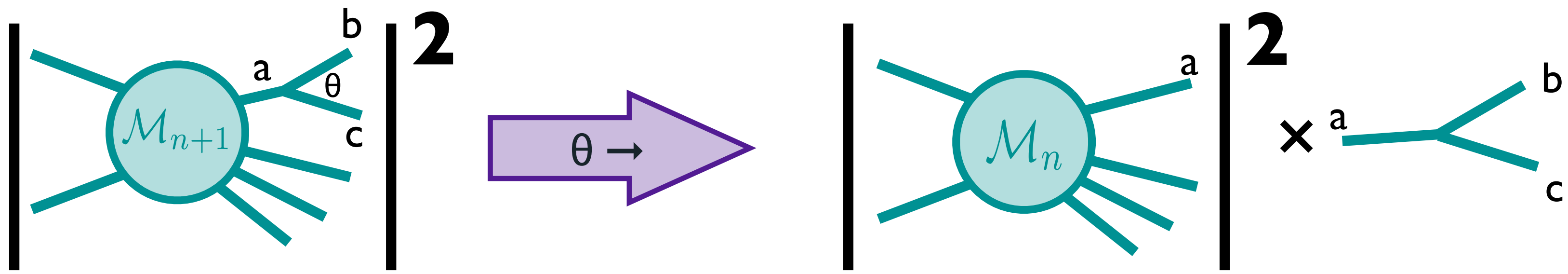
- The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability.
- The first task of Monte Carlo physics is to make this statement quantitative.

Collinear factorization



- The process factorizes in the collinear limit. This procedure it universal!

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \;\simeq\; |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_{\rm S}}{2\pi} P_{a\to bc}(z)$$

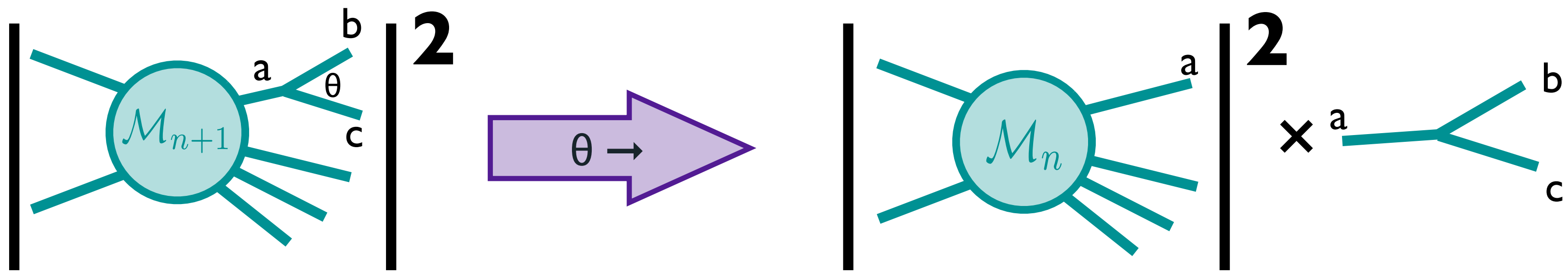
- Notice that what has been roughly called 'branching probability' is actually a singular factor, so one will need to make sense precisely of this definition.

- At the leading contribution to the (n+1)-body cross section the Altarelli-Parisi splitting kernels are defined as:

$$P_{g\to qq}(z) = T_R \left[z^2 + (1-z)^2\right], \qquad P_{g\to gg}(z) = C_A \left[z(1-z) + \frac{z}{1-z} + \frac{1-z}{z}\right],$$

$$P_{q\to qg}(z) = C_F \left[\frac{1+z^2}{1-z}\right], \qquad P_{q\to gq}(z) = C_F \left[\frac{1+(1-z)^2}{z}\right].$$

Collinear factorization



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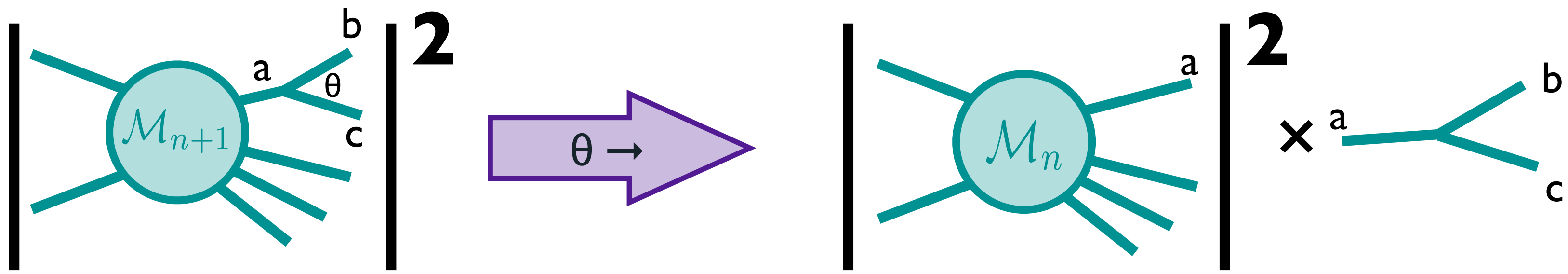
$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- **t** can be called the 'evolution variable' (will become clearer later): it can be the virtuality m^2 of particle a or its p_T^2 or $E^2\theta^2$ ...
  - It represents the hardness of the branching and tends to 0 in the collinear limit.
- Indeed in the collinear limit one has: so that the factorization takes place for all these definitions:

$$m^2 \simeq z(1-z)\theta^2 E_a^2$$
$$p_T^2 \simeq z m^2$$

$$d\theta^2/\theta^2 = dm^2/m^2 = dp_T^2/p_T^2$$

Collinear factorization



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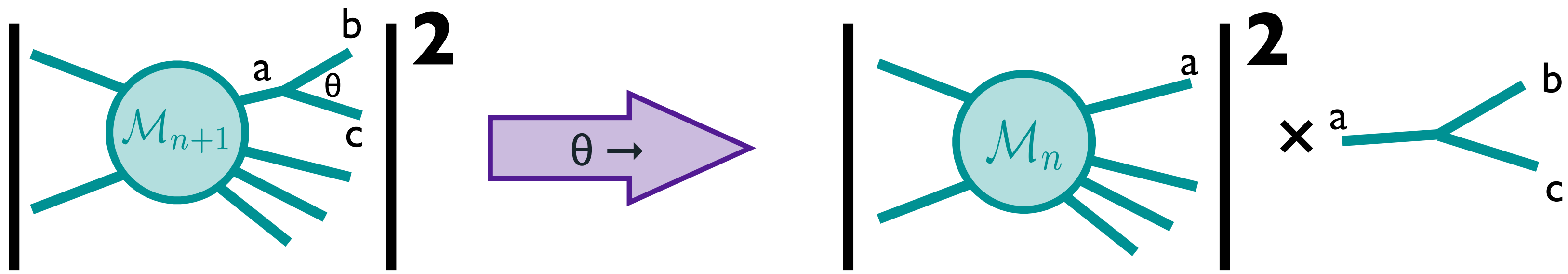
- **z** is the "energy variable": it is defined to be the energy fraction taken by parton **b** from parton **a**. It represents the energy sharing between **b** and **c** and tends to 1 in the soft limit (parton c going soft)

- **Φ** is the azimuthal angle. It can be chosen to be the angle between the polarization of a and the plane of the branching.

Collinear factorization



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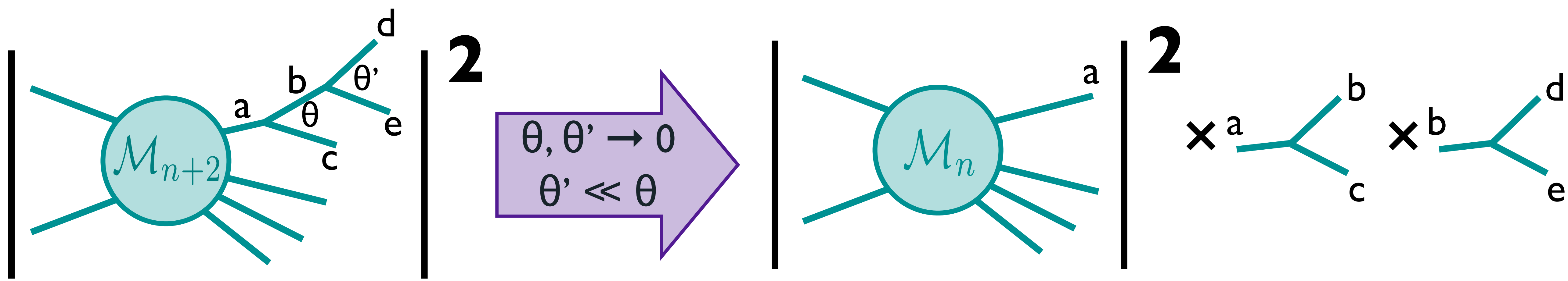
$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- This is an amplitude squared: naively one would maybe expect **1/t²** dependence. Why is the square not there?
    - It's due to angular-momentum conservation.
      E.g., take the splitting **q ⟶ qg**: helicity is conserved for the quarks, so the final state spin differs by one unity with respect to the initial one. The scattering happens in a p-wave (orbital angular momentum equal to one), so there is a suppression factor as **t ⟶ 0**.
    - In fact, a factor **1/t** is always cancelled in an explicit computation

Multiple emission

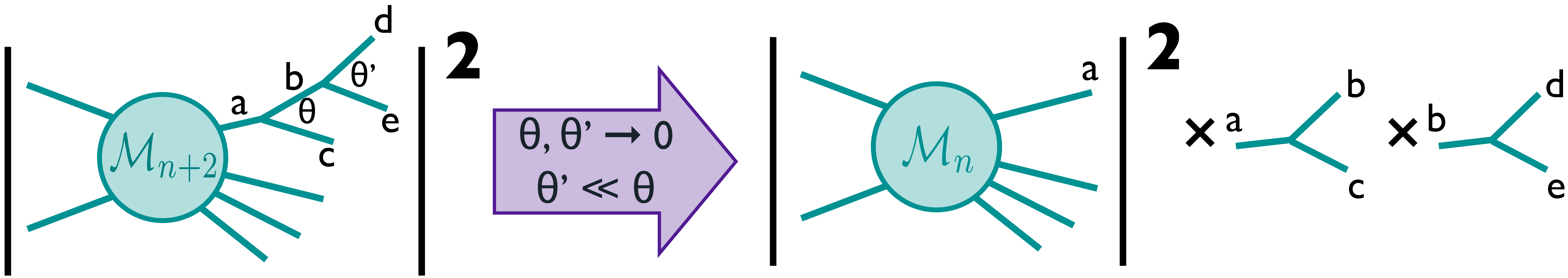


- Now consider $\mathbf{M}_{n+1}$ as the new core process and use the recipe we used for the first emission in order to get the dominant contribution to the (n+2)-body cross section: add a new branching at angle much smaller than the previous one:

$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) \times \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_S}{2\pi} P_{b\to de}(z')$$

- This can be done for an arbitrary number of emissions. The recipe to get the leading collinear singularity is thus cast in the form of an iterative sequence of emissions whose probability does not depend on the past history of the system: a 'Markov chain'. No interference!!!

Multiple emission



- The dominant contribution comes from the region where the subsequently emitted partons satisfy the strong ordering requirement: θ » θ' » θ''...
  For the rate for multiple emission we get

$$\sigma_{n+k} \propto \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \cdots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

  where Q is a typical hard scale and Q_0 is a small infrared cutoff that separates perturbative from non perturbative regimes.

- Each power of α_s comes with a logarithm. The logarithm can be easily large, and therefore it can lead to a breakdown of perturbation theory.

Absence of interference

- The collinear factorization picture gives a branching sequence for a given leg starting from the hard subprocess all the way down to the non-perturbative region.

- Suppose you want to describe two such histories from two different legs: these two legs are treated in a completely uncorrelated way. And even within the same history, subsequent emissions are uncorrelated.

- The collinear picture completely misses the possible interference effects between the various legs. The extreme simplicity comes at the price of quantum inaccuracy.

- Nevertheless, the collinear picture captures the leading contributions: it gives an excellent description of an arbitrary number of (collinear) emissions:

  - it is a "resummed computation"

  - it bridges the gap between fixed-order perturbation theory and the non-perturbative hadronization.

Sudakov form factor

The differential probability for the branching **a ⟶ bc** between scales **t** and **t+dt** knowing that no emission occurred before:

$$dp(t) = \sum_{bc} \frac{dt}{t} \int dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

The probability that a parton does NOT split between the scales **t** and **t+dt** is given by **1-dp(t).**

Probability that particle a does not emit between scales $\mathbf{Q^2}$ and **t**

$$\Delta(Q^2,t) = \prod_k \left[1 - \sum_{bc} \frac{dt_k}{t_k} \int dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)\right] =$$

$$\exp\left[-\sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)\right] = \exp\left[-\int_t^{Q^2} dp(t')\right]$$

- **$\Delta(Q^2,t)$** is the Sudakov form factor
  - Property: **$\Delta(A,B) = \Delta(A,C)\,\Delta(C,B)$**

Parton shower

- The Sudakov form factor is the heart of the parton shower. It gives the probability that a parton does not branch between two scales
- Using this no-emission probability the branching tree of a parton is generated.
- Define **dP_k** as the probability for k ordered splittings from leg a at given scales

$$
\begin{aligned}
dP_1(t_1) &= \Delta(Q^2,t_1)\; dp(t_1)\Delta(t_1,Q_0^2),\\
dP_2(t_1,t_2) &= \Delta(Q^2,t_1)\; dp(t_1)\; \Delta(t_1,t_2)\; dp(t_2)\; \Delta(t_2,Q_0^2)\Theta(t_1-t_2),\\
\ldots &= \ldots\\
dP_k(t_1,\ldots,t_k) &= \Delta(Q^2,Q_0^2)\prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1}-t_l)
\end{aligned}
$$

- **Q_0^2** is the hadronization scale (~1 GeV). Below this scale we do not trust the perturbative description for parton splitting anymore.
- This is what is implemented in a parton shower, taking the scales for the splitting **t_i** randomly (but weighted according to the no-emission probability).

Unitarity

$$dP_k(t_1,...,t_k) \quad = \quad \Delta(Q^2,Q_0^2)\prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1}-t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

$$P_k \equiv \int dP_k(t_1,...,t_k) = \Delta(Q^2,Q_0^2)\frac{1}{k!}\left[\int_{Q_0^2}^{Q^2} dp(t)\right]^k, \quad \forall k=0,1,...$$

- Summing over all number of emissions

$$\sum_{k=0}^{\infty} P_k = \Delta(Q^2,Q_0^2)\sum_{k=0}^{\infty}\frac{1}{k!}\left[\int_{Q_0^2}^{Q^2} dp(t)\right]^k = \Delta(Q^2,Q_0^2)\exp\left[\int_{Q_0^2}^{Q^2} dp(t)\right] = 1$$

- Hence, the total probability is conserved

Choice of evolution parameter

$$\Delta(Q^2,t) = \exp\left[-\sum_{bc}\int_t^{Q^2}\frac{dt'}{t'}dz\frac{d\phi}{2\pi}\frac{\alpha_S}{2\pi}P_{a\to bc}(z)\right]$$

- There is a lot of freedom in the choice of evolution parameter t. It can be the virtuality m^2 of particle a or its $p_T{}^2$ or $E^2\theta^2$ ... For the collinear limit they are all equivalent
- However, in the soft limit ($z \longrightarrow 1$) they behave differently
- Can we chose it such that we get the correct soft limit?

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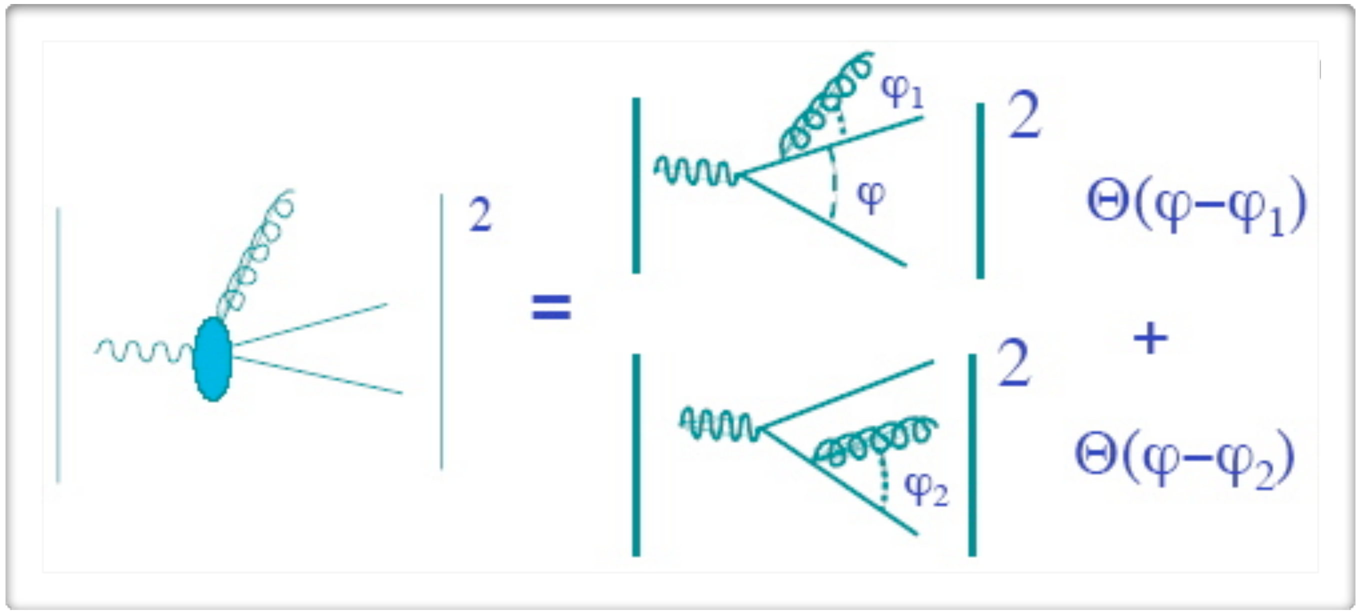
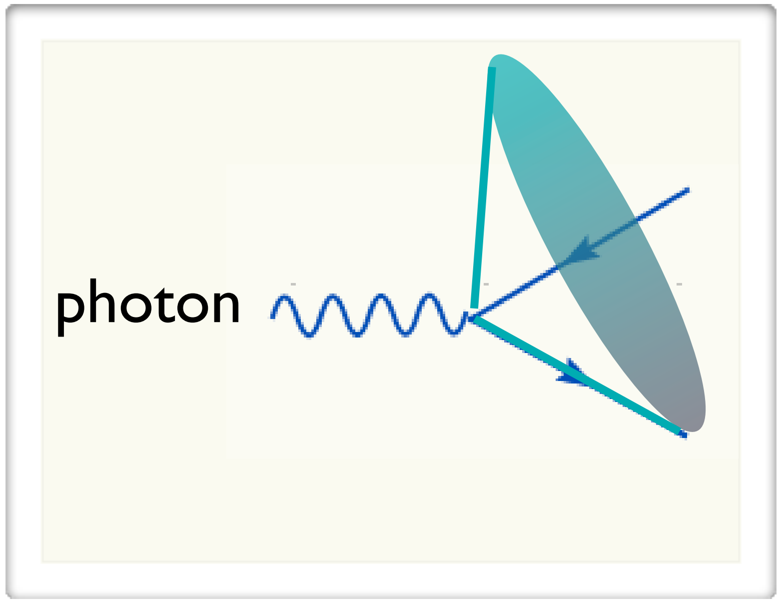
YES! It should be (proportional to) the angle θ

Angular ordering



Radiation inside cones around the orginal partons is allowed (and described by the eikonal approximation), outside the cones it is zero (after averaging over the azimuthal angle)



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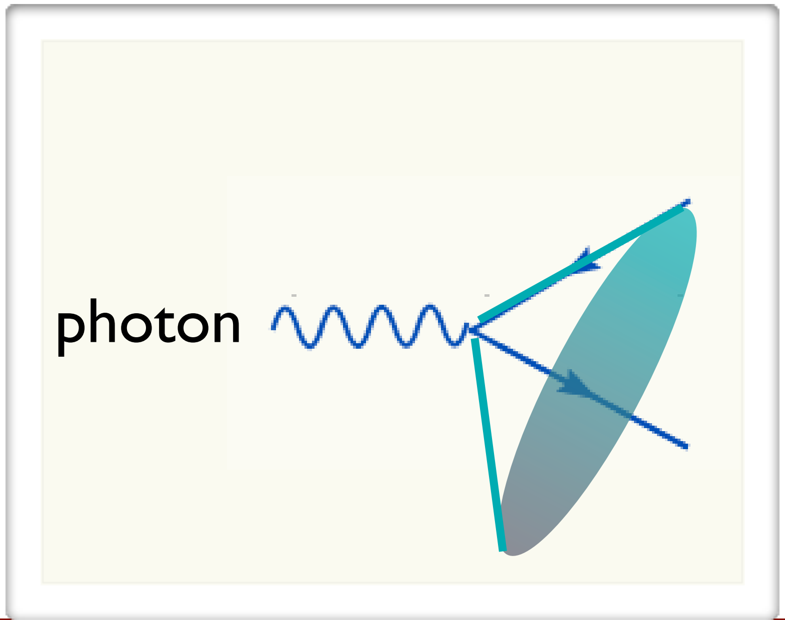
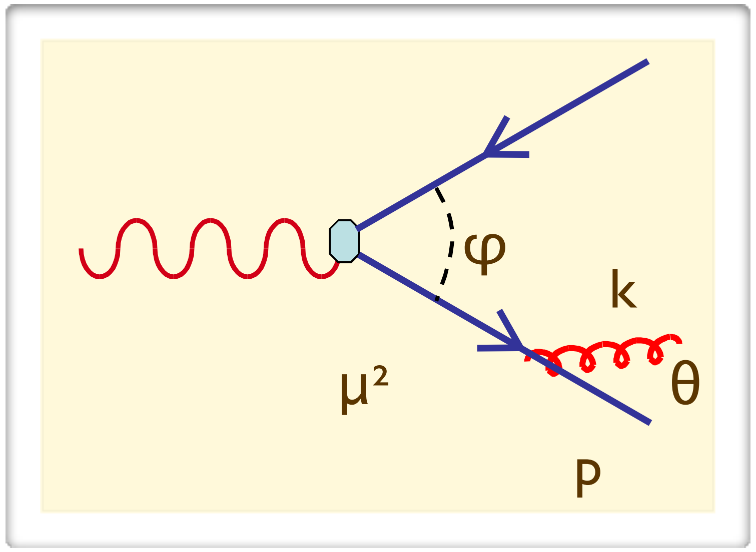


Intuitive explanation



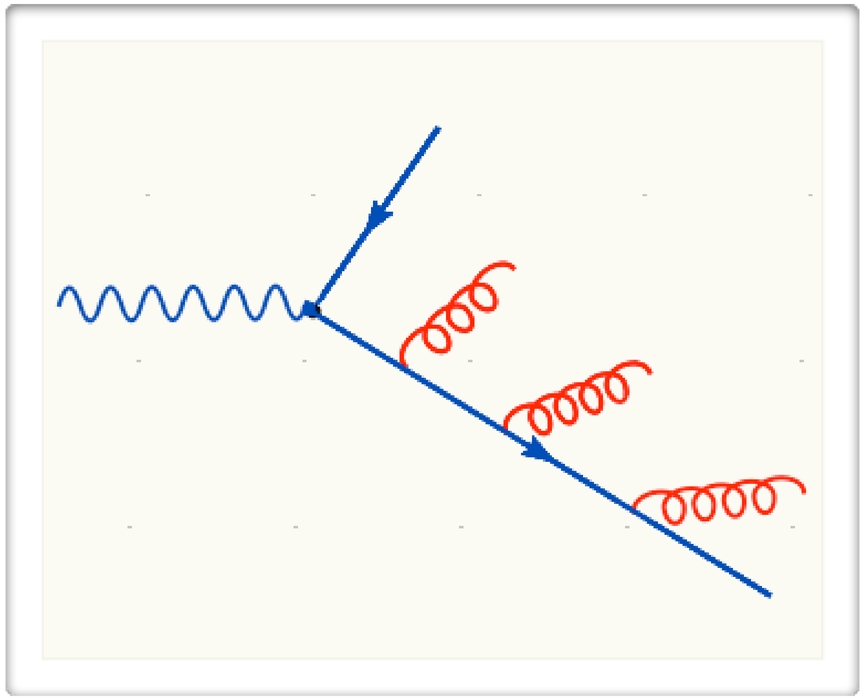
- Lifetime of the virtual intermediate state:
  $\tau < \gamma/\mu = E/\mu^2 = 1/(k_0\theta^2) = 1/(k_\perp\theta)$
- Distance between q and qbar after τ:
  $d = \varphi\tau = (\varphi/\theta)\ 1/k_\perp$

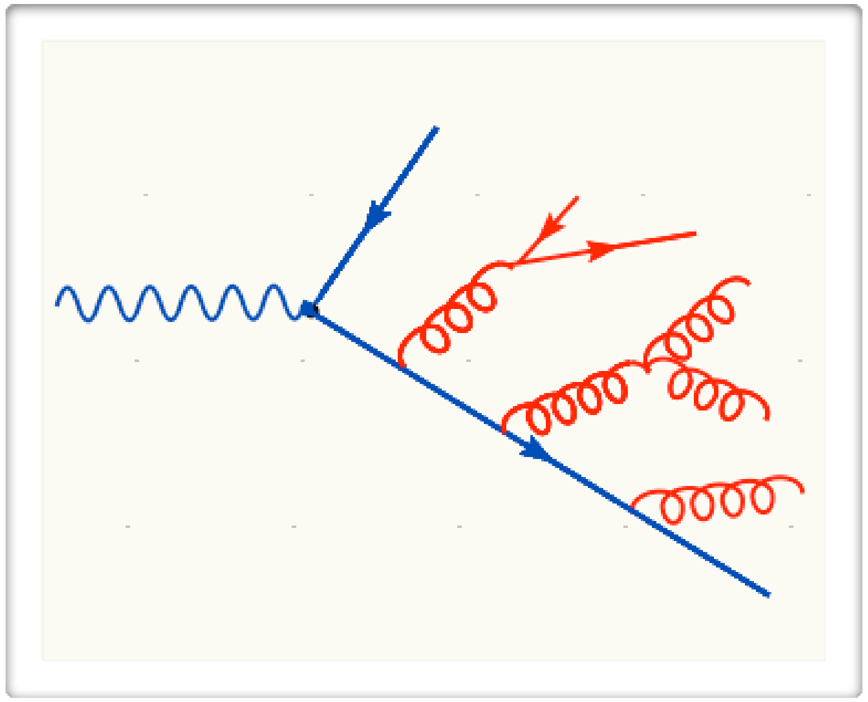
$\mu^2 = (p+k)^2 = 2E\, k_0\, (1\text{-}\cos\theta)$
$\sim E\, k_0\, \theta^2 \sim E\, k_\perp\, \theta$

If the transverse wavelength of the emitted gluon is longer than the separation between q and qbar, the gluon emission is suppressed, because the q qbar system will appear as colour neutral (i.e. dipole-like emission, suppressed)

Therefore $d > 1/k_\perp$, which implies $\theta < \varphi$.

Angular ordering





- The construction can be iterated to the next emission, with the result that the emission angles keep getting smaller and smaller.
- One can generalize it to a generic parton of color charge Q_k splitting into two partons i and j, $Q_k=Q_i+Q_j$. The result is that inside the cones i and j emit as independent charges, and outside their angular-ordered cones the emission is coherent and can be treated as if it was directly from color charge Q_k.
- KEY POINT FOR THE MC!
- Angular ordering is automatically satisfied in θ ordered showers! (and easy to account for in p_T ordered showers).

Angular ordering

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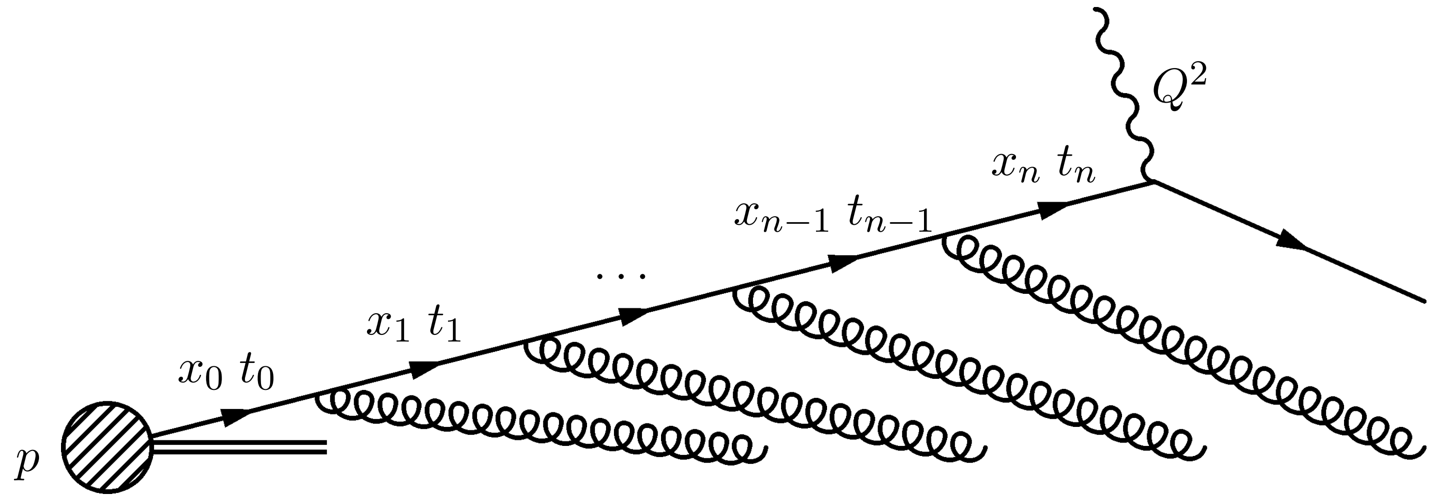
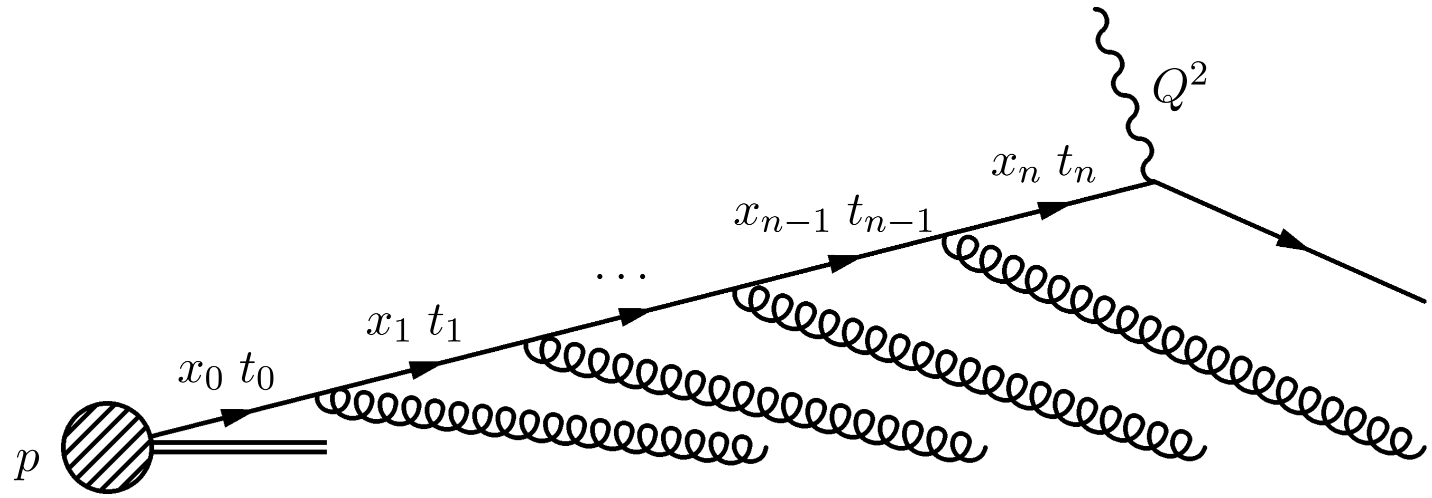
3. It is not an exclusive property of QCD (i.e., it is also present in QED) but in QCD produces very non-trivial effects, depending on how particles are color connected.

Initial-state parton splittings



- So far, we have looked at final-state (time-like) splittings. For initial state, the splitting functions are the same
- However, there is another ingredient: the parton density (or distribution) functions (PDFs). Naively: Probability to find a given parton in a hadron at a given momentum fraction **$x = p_z/P_z$** and scale **t**.

Initial-state parton splittings



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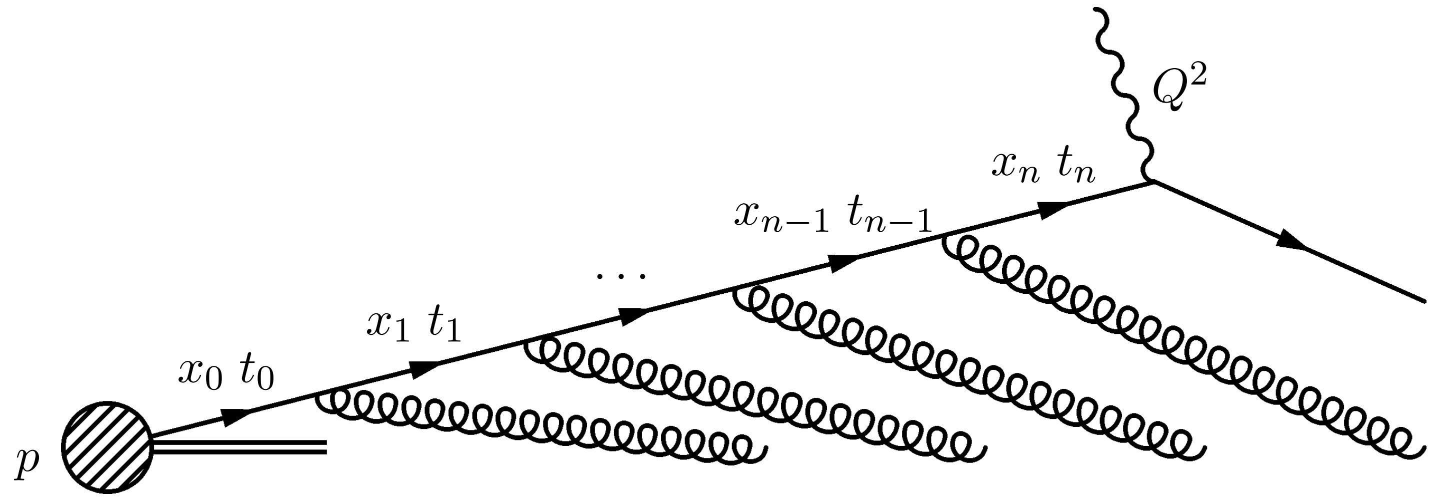
- How do the PDFs evolve with increasing $\mathbf{t}$?

Initial-state parton splittings



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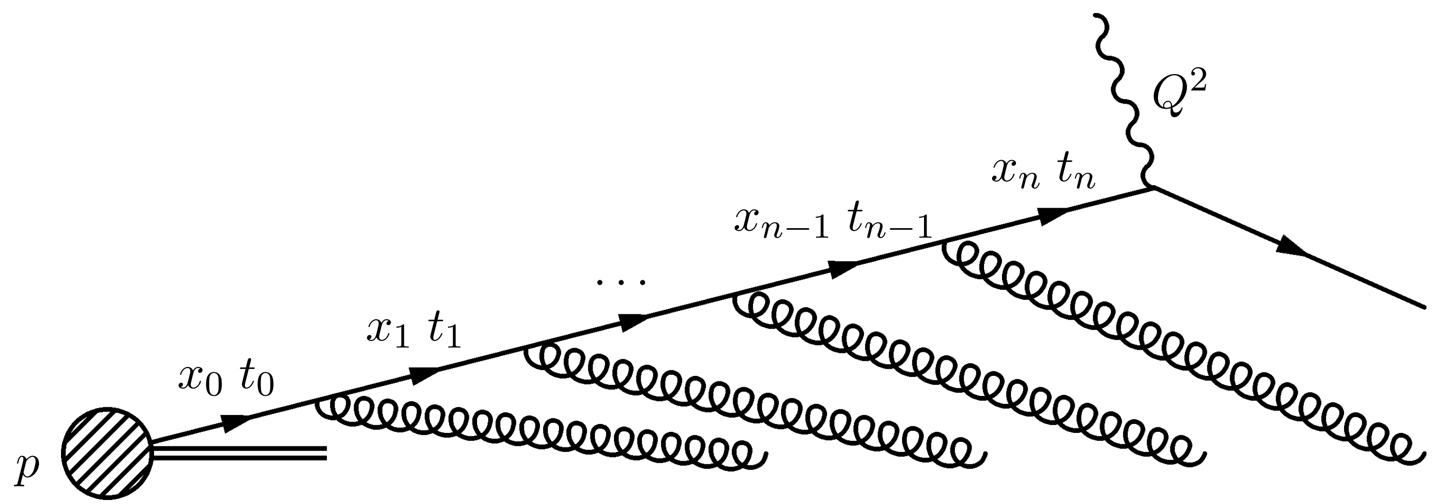
- How do the PDFs evolve with increasing **t**?

$$t\frac{\partial}{\partial t} f_i(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P_{ij}(z) f_j\left(\frac{x}{z}, t\right) \qquad \text{DGLAP}$$

Initial-state parton splittings



- Start with a quark PDF **$f_0(x)$** at scale **t_0**. After a single parton emission, the probability to find the quark at virtuality **$t > t_0$** is

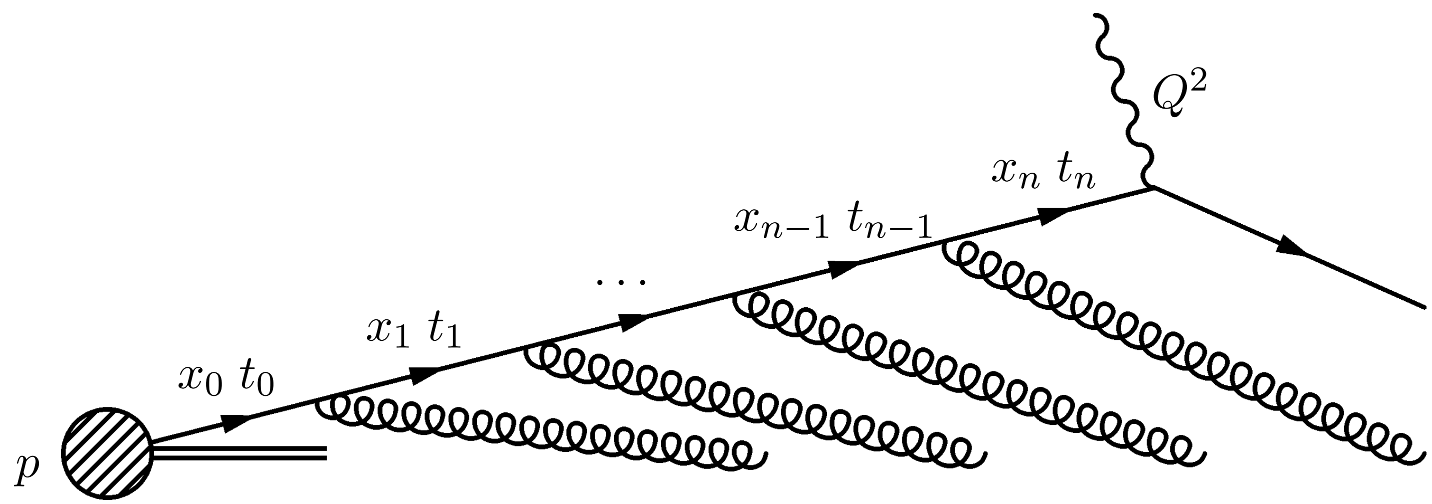
$$f(x,t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) f_0\left(\frac{x}{z}\right)$$

- After a second emission, we have

$$f(x,t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) \Big\{ f_0\left(\frac{x}{z}\right) + \int_{t_0}^{t'} \frac{dt''}{t''} \frac{\alpha_s}{2\pi} \int_{x/z}^1 \frac{dz'}{z'} P(z') f_0\left(\frac{x}{zz'}\right) \Big\}$$

f(x/z, t')

The DGLAP equation



- So for multiple parton splittings, we arrive at an integral-differential equation:

$$t\frac{\partial}{\partial t} f_i(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P_{ij}(z) f_j\left(\frac{x}{z}, t\right)$$

- This is the famous DGLAP equation (where we have taken into account the multiple parton species i, j). The boundary condition for the equation is the initial PDFs $f_{i0}(x)$ at a starting scale t_0 (around 2 GeV).

- These starting PDFs are fitted to experimental data.

Initial-state parton showers

- To simulate parton radiation from the initial state, we start with the hard scattering, and then "deconstruct" the DGLAP evolution to get back to the original hadron: backwards evolution!
  - i.e. we undo the analytic resummation and replace it with explicit partons (e.g. in Drell-Yan this gives non-zero p_T to the vector boson)
- In backwards evolution, the Sudakovs include also the PDFs -- this follows from the DGLAP equation and ensures conservation of probability:

$$\Delta_{Ii}(x,t_1,t_2) = \exp\left\{-\int_{t_1}^{t_2} dt' \sum_j \int_x^1 \frac{dx'}{x'}\frac{\alpha_s(t')}{2\pi} P_{ij}\left(\frac{x}{x'}\right)\frac{f_i(x',t')}{f_j(x,t')}\right\}$$

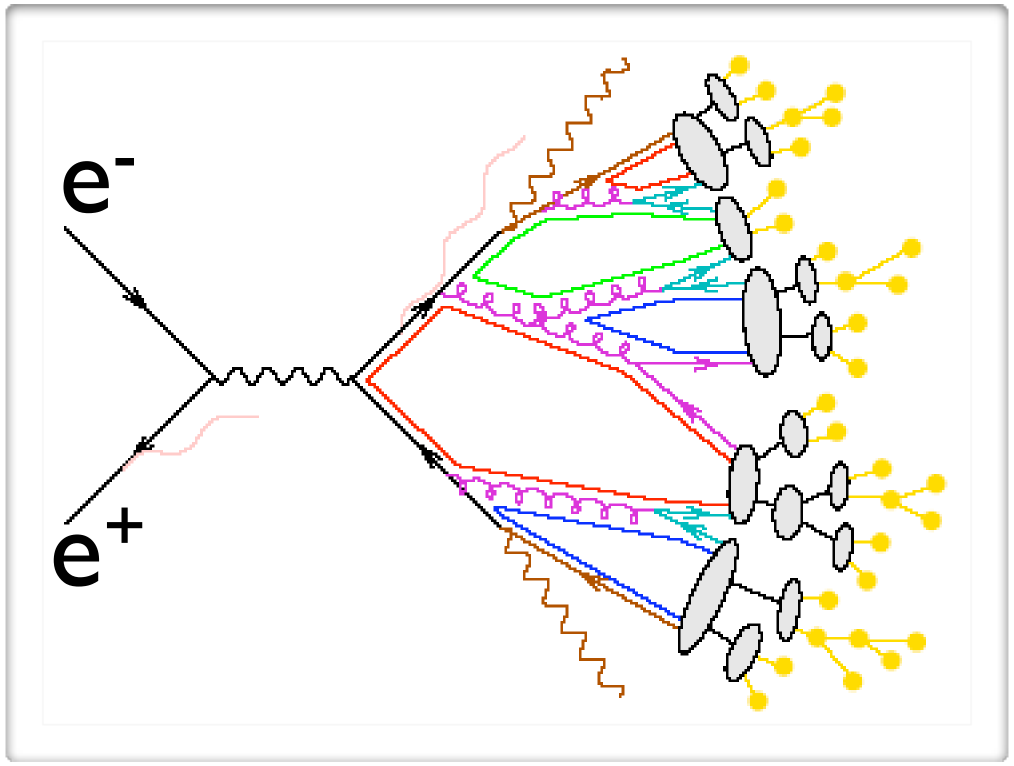
  This represents the probability that parton **i** will stay at the same **x** (no splittings) when evolving from $\mathbf{t_1}$ to $\mathbf{t_2}$.
- The shower simulation is now done as in a final state shower!

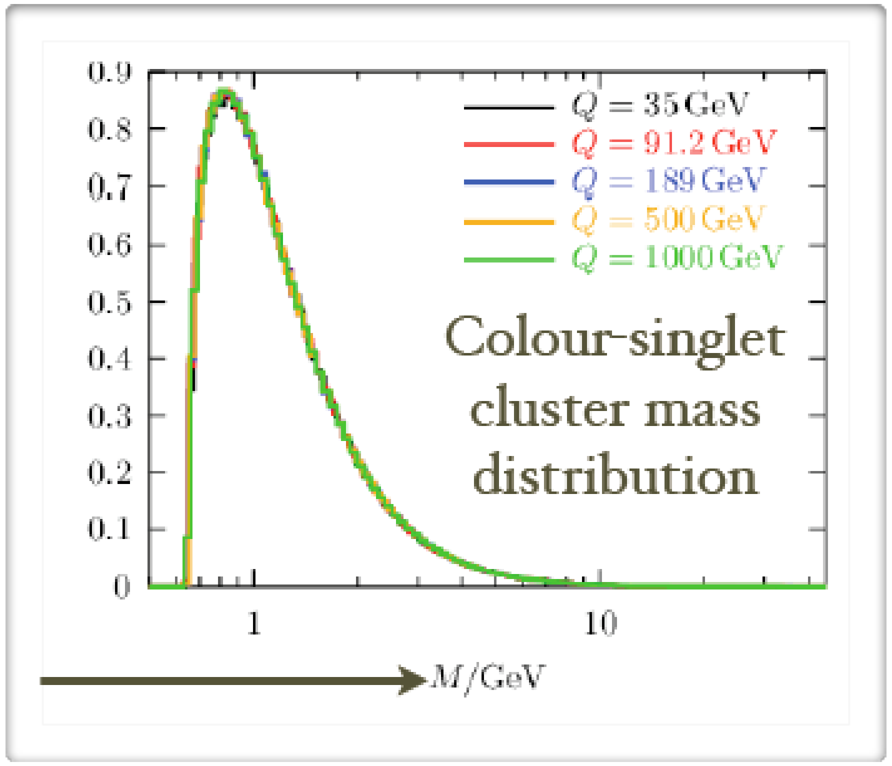
HADRONIZATION

- The shower stops if all partons are characterized by a scale at the IR cut-off: $Q_0 \sim 1$ GeV.
- Physically, we observe hadrons, not (colored) partons.
- We need a non-perturbative model in passing from partons to colorless hadrons.
- There are two models (string and cluster), based on physical and phenomenological considerations.

Cluster model

The structure of the perturbative evolution including angular ordering, leads naturally to the clustering in phase-space of color-singlet parton pairs (preconfinement). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass color singlet clusters.





String model

From lattice QCD one sees that the color confinement potential of a quark-antiquark grows linearly with their distance: V(r) ~ kr, with k ~ 0.2 GeV. This is modeled with a string with uniform tension (energy per unit length) k that gets stretched between the qq pair.

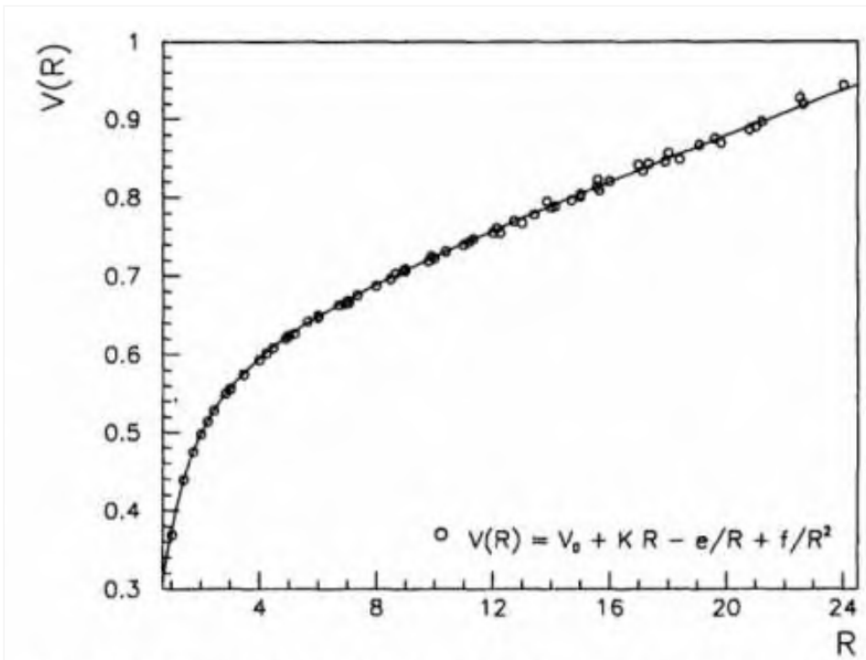
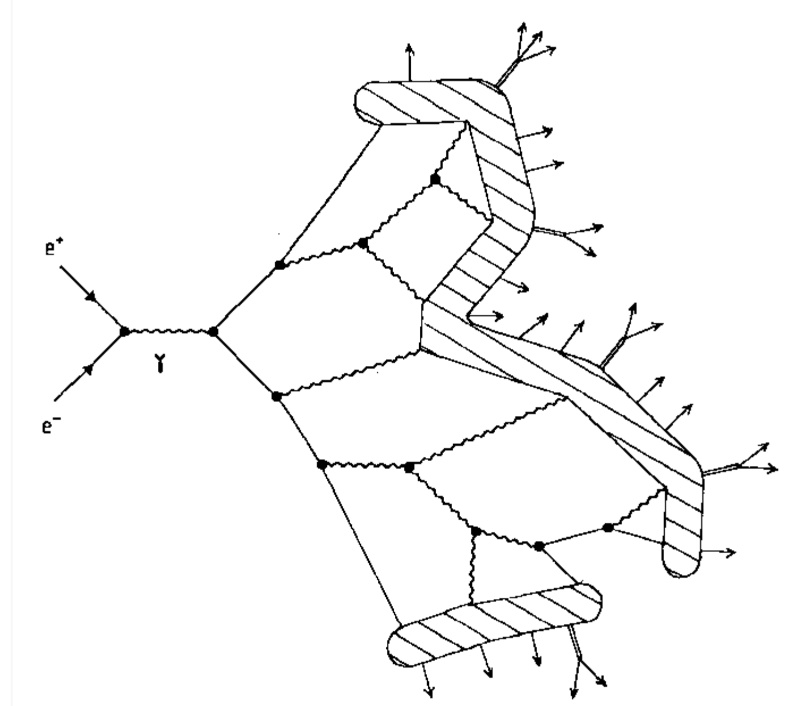


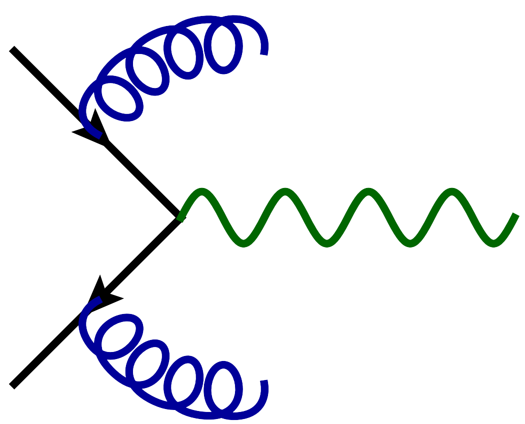
Fig. 2.9. QCD potential *vs.* *R* (in lattice units) from lattice QCD. Figure from ref. [23].

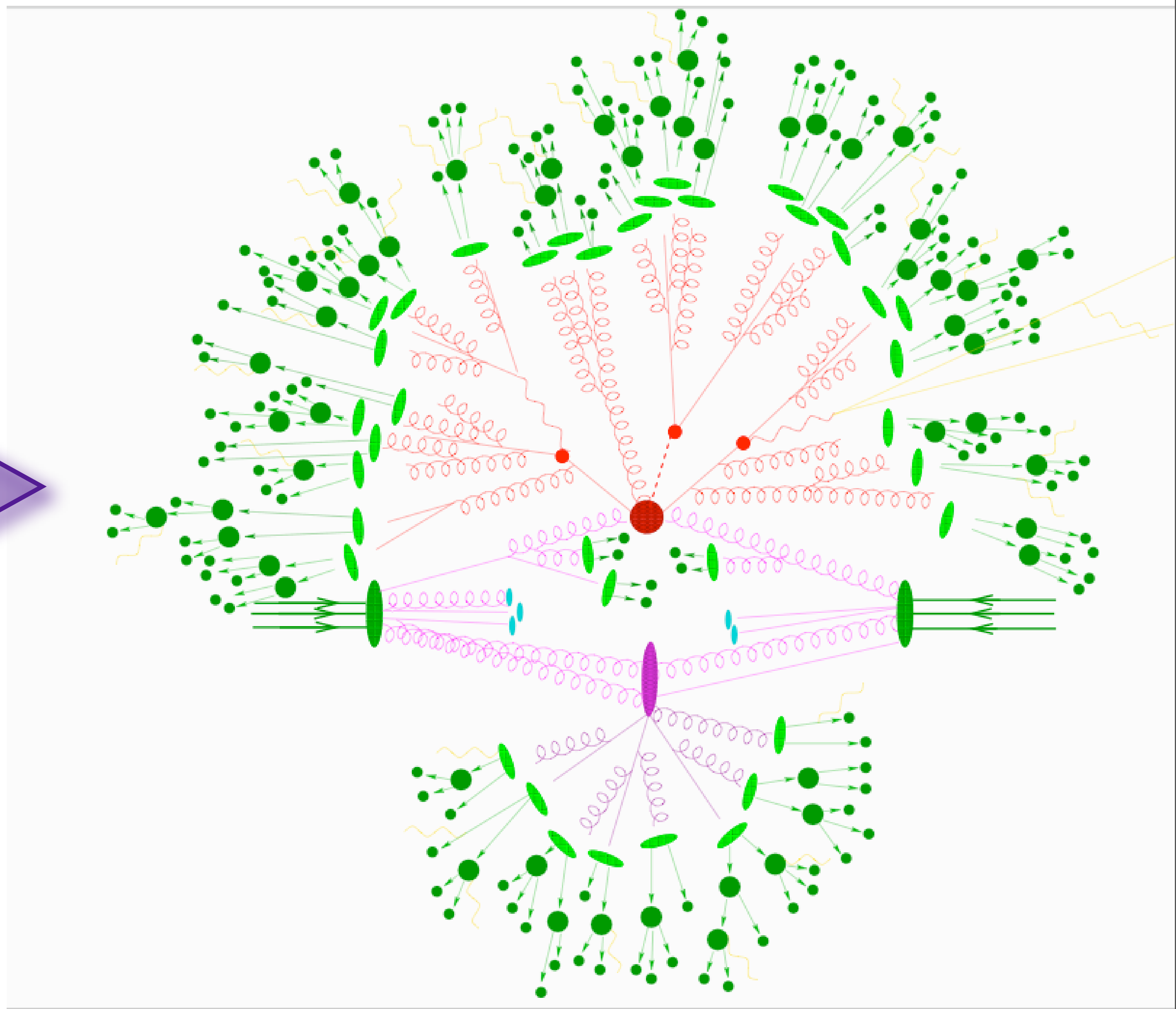


When quark-antiquarks are too far apart, it becomes energetically more favorable to break the string by creating a new qq pair in the middle.

Monday 10 June 2013

Exclusive observable

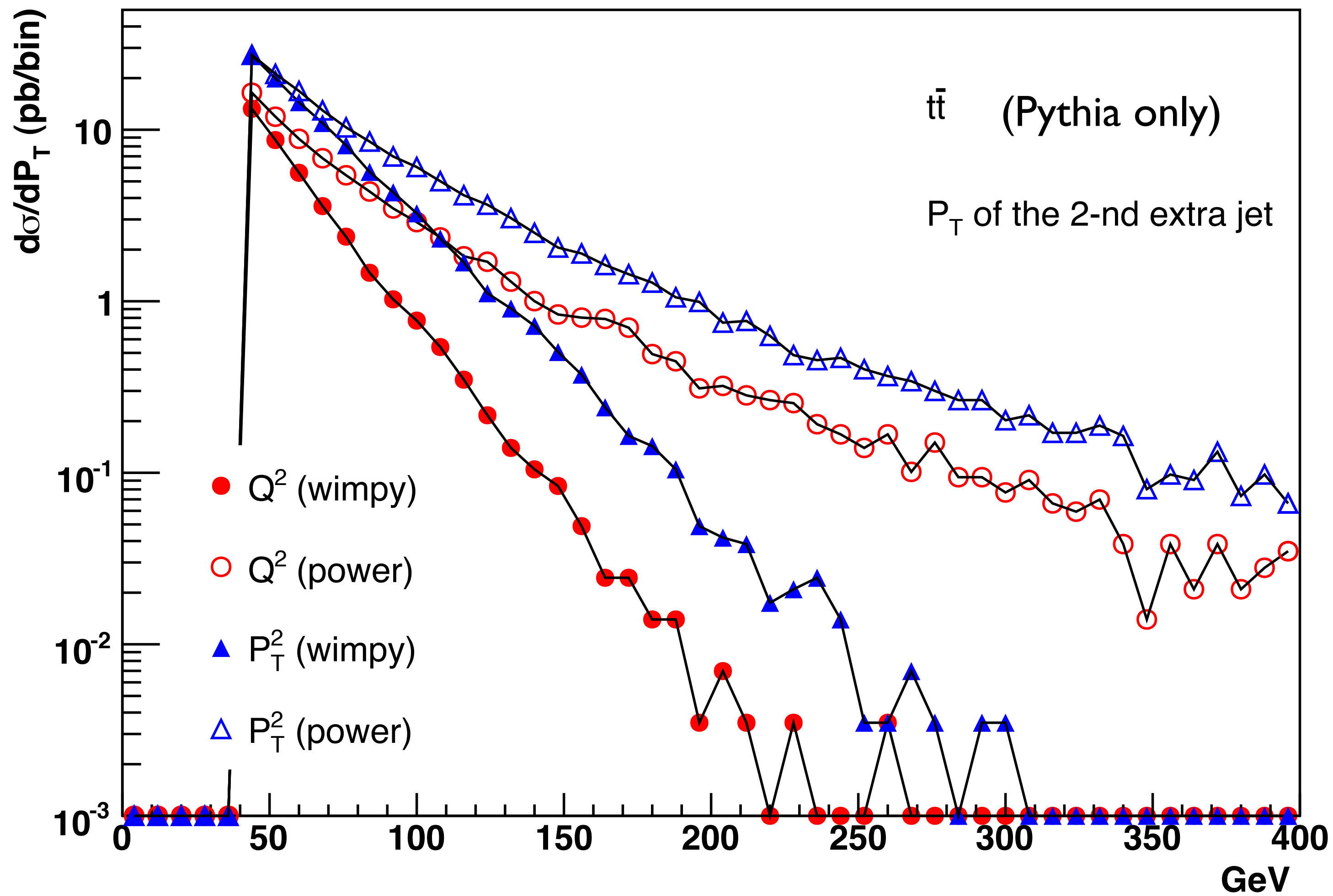




A parton shower program associates one of the possible histories (and pre-histories in case of pp collisions) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

Shower starting scale

Varying the shower starting scale ('wimpy' or 'power') and the evolution parameter ('Q^2' or '$p_T{}^2$') a whole range of predictions can be made:



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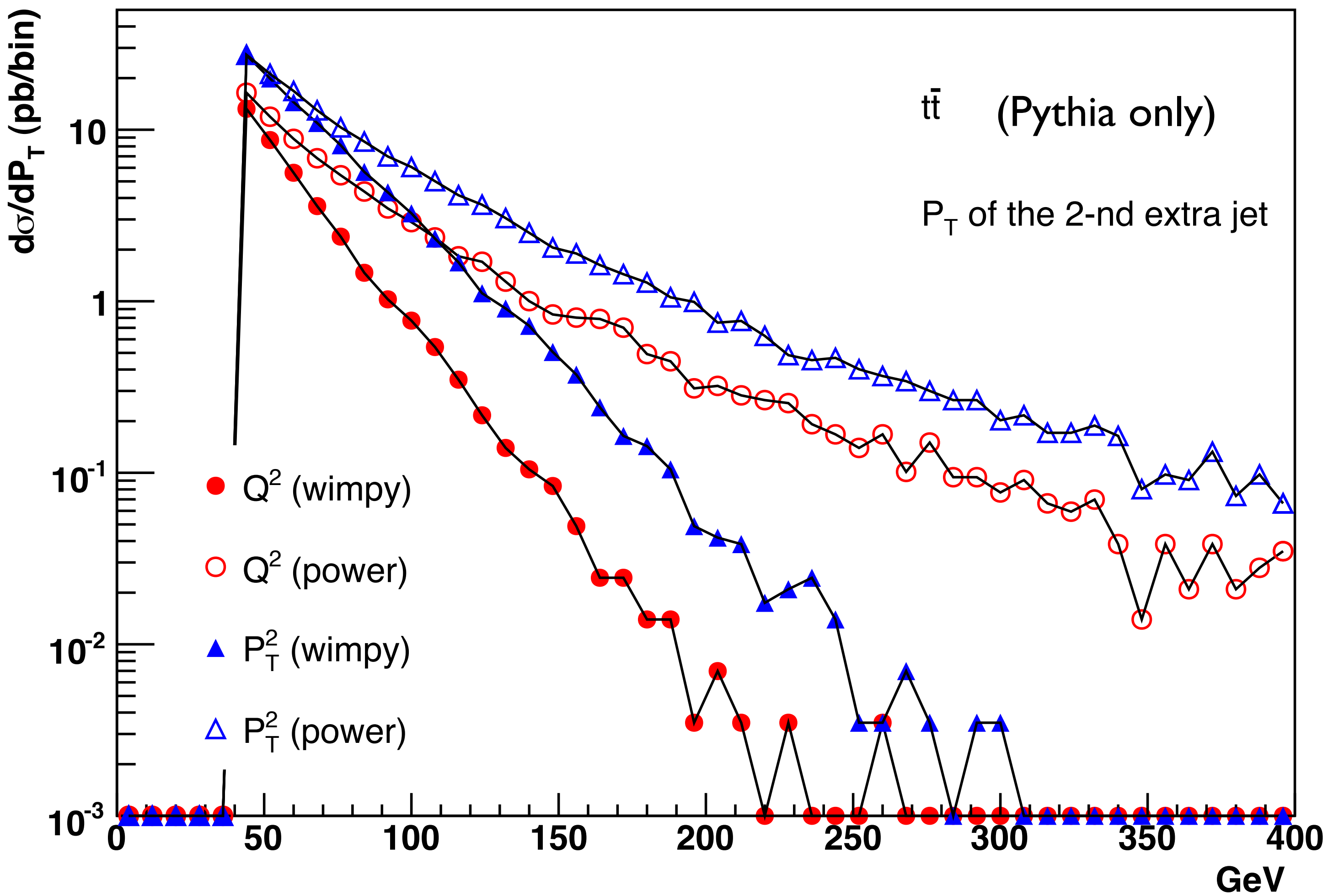

Ideal to describe the data: one can tune the parameters and fit it!
But is this really what we want...Does it work for other procs?

Parton shower MC event generators

A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

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- Always the first experimental choice
- Complete exclusive description of the events: hard scattering, showering & hadronization (and underlying event)
- Reliable and well-tuned tools
- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD

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Shower MC Generators: PYTHIA, HERWIG, SHERPA

Parton shower : summary

- The parton shower dresses partons with radiation. This makes the inclusive parton-level predictions (i.e. inclusive over extra radiation) completely exclusive
    - In the soft and collinear limits the partons showers are exact, but in practice they are used outside this limit as well.
    - Partons showers are universal (i.e. independent from the process)
- There is a cut-off in the shower (below which we don't trust perturbative QCD) at which a hadronization model takes over
    - Hadronization models are universal and independent from the energy of the collision

Credits

To organize this presentation I have benefited from lectures (and actual slides), talks and discussions with many people.
In particular:

- Mike Seymour (MC basics)
- Claude Duhr (FeynRules)
- Johan Alwall (ME+PS merging)
- Rikkert Frederix, Paolo Torrielli (NLO+PS)
- Stefano Frixione, Michelangelo Mangano, Paolo Nason (for QCD, PS, LO, NLO, and more...)
- ....

Whom I all warmly thank!!

Herwig

- All HERWIG versions implement the angular-ordering: subsequent emissions are characterized by smaller and smaller angles.

HERWIG 6: $$t = \frac{p_b \cdot p_c}{E_b E_c} \simeq 1 - \cos\theta$$

HERWIG++: $$t = \frac{(p_{b\perp})^2}{z^2(1-z)^2} = t(\theta)$$

- With angular ordering the parton shower does not populate the full phase space: empty regions of the phase space, called "dead zones", will arise.

- It may seem that the presence of dead zones is a weakness, but it is not so: they implement correctly the collinear approximation, in the sense that they constrain the shower to live uniquely in the region where it is reliable.
Matrix element corrections (MLM/CKKW matching) remove the dead-zones

- Hadronization: cluster model.

Pythia

- Choice of evolution variables for Fortran and C++ versions:

PYTHIA 6: $t = (p_b + p_c)^2 \sim z(1-z)\theta^2 E_a^2$

PYTHIA 8: $t = (p_b)_\perp^2$

- Simpler variables, but decreasing angles not guaranteed: PYTHIA rejects the events that do not respect the angular ordering. In practice equivalent to angular ordering (in particular for Pythia 8)

- Not implementing directly angular ordering, the phase space can be filled entirely (even without matrix element corrections), so one can have the so called "power shower" (use with a certain care: it uses the collinear/soft approximation for from the region where it is valid)

- Hadronization: string model.

SHERPA

- SHERPA uses a different kind of shower not based on the collinear $1 \longrightarrow 2$ branching, but on more complex $2 \longrightarrow 3$ elementary process: emission of the daughter off a color dipole

- The real emission matrix element squared is decomposed into a sum of terms $D_{ij,k}$ (dipoles) that capture the soft and collinear singularities in the limits i collinear to j, i soft (k is the spectator), and a factorization formula is deduced in the leading color approximation:

$$D_{ij,k} \to B \frac{\alpha_S}{p_i \cdot p_j} K_{ij,k}$$

- The shower is developed from a Sudakov form factor

$$\Delta = \exp\left(- \int \frac{dt}{t} \int dz \; \alpha_S \; K_{ij,k} \right)$$

- It treats correctly the soft gluon emission off a color dipole, so angular ordering is built in.

- Hadronization: cluster model (default) and string model