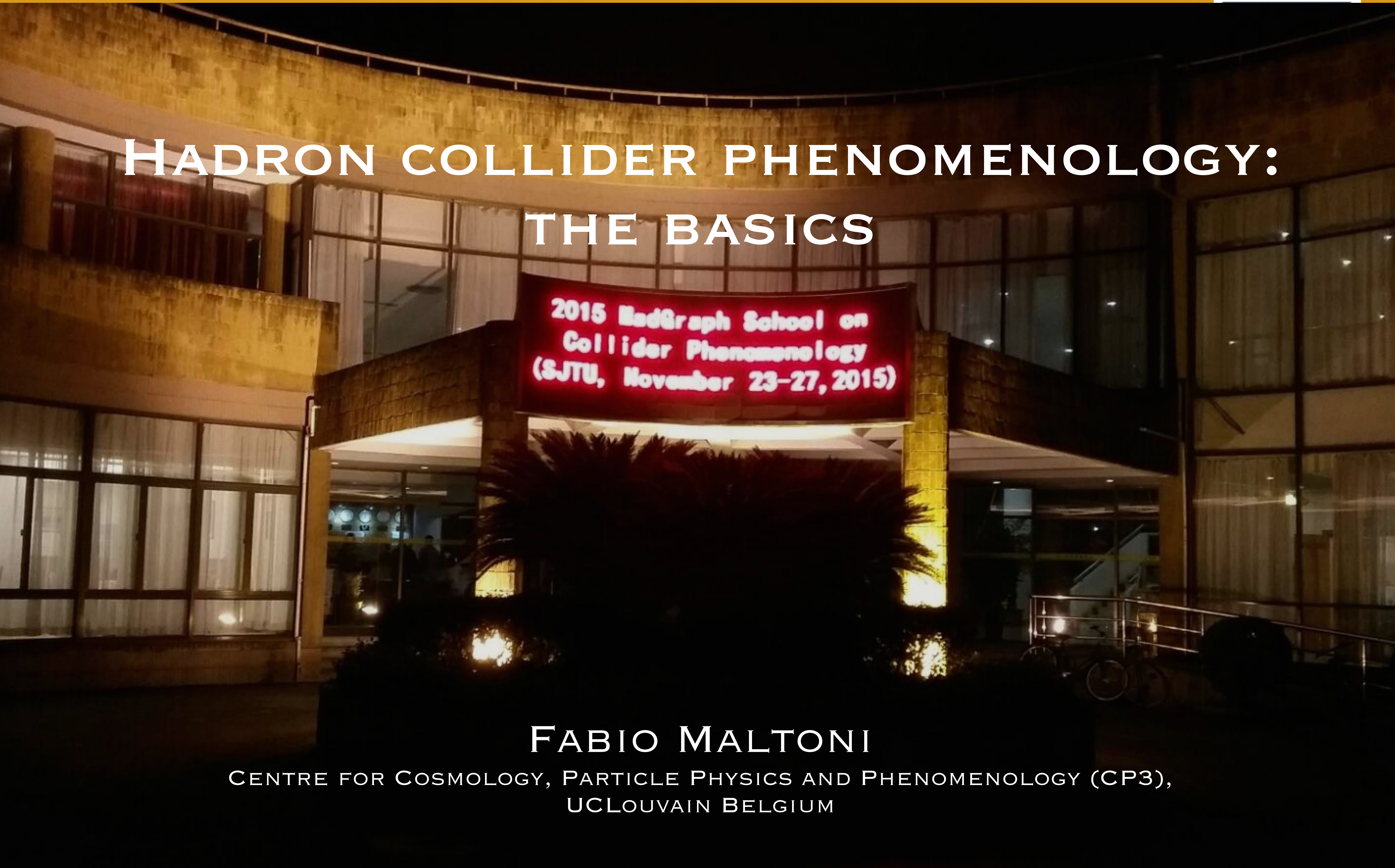
Hadron collider phenomenology: the basics



Fabio Maltoni
Centre for Cosmology, Particle Physics and Phenomenology (CP3),
UCLouvain Belgium

NEWS FROM THE LHC

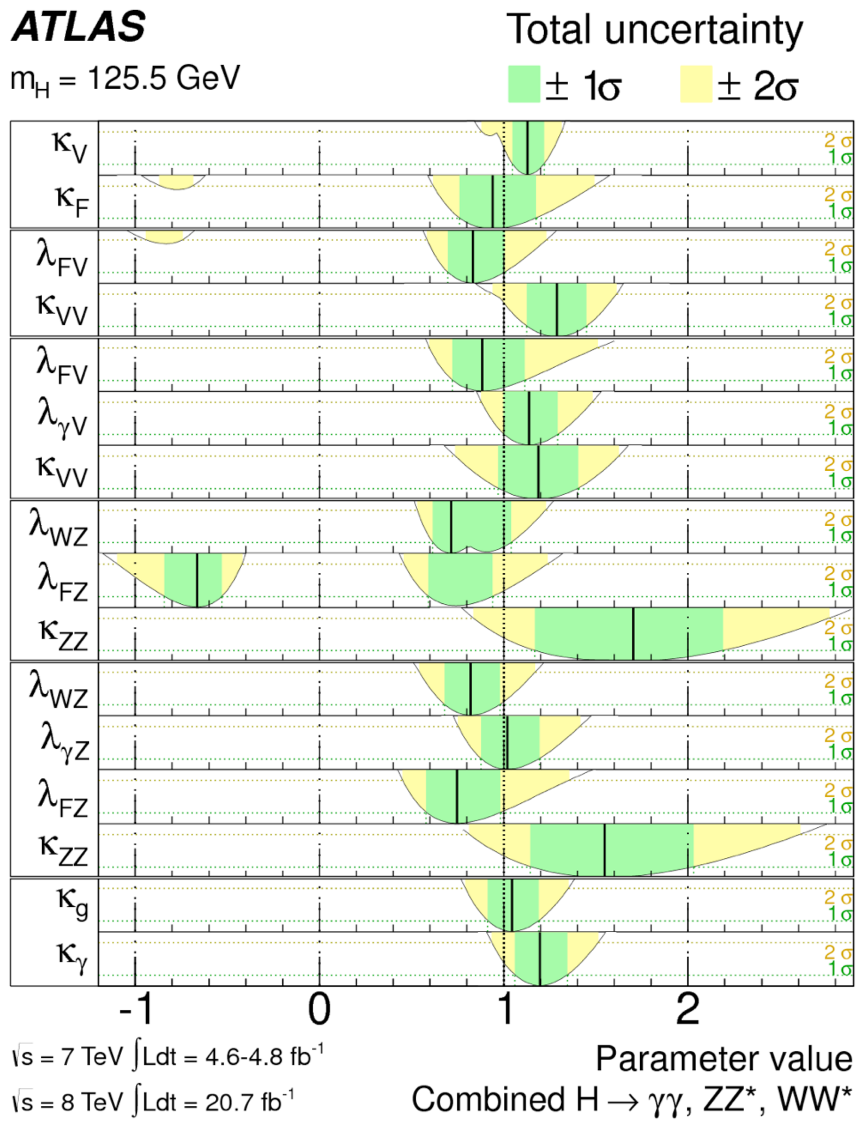
NEWS FROM THE LHC

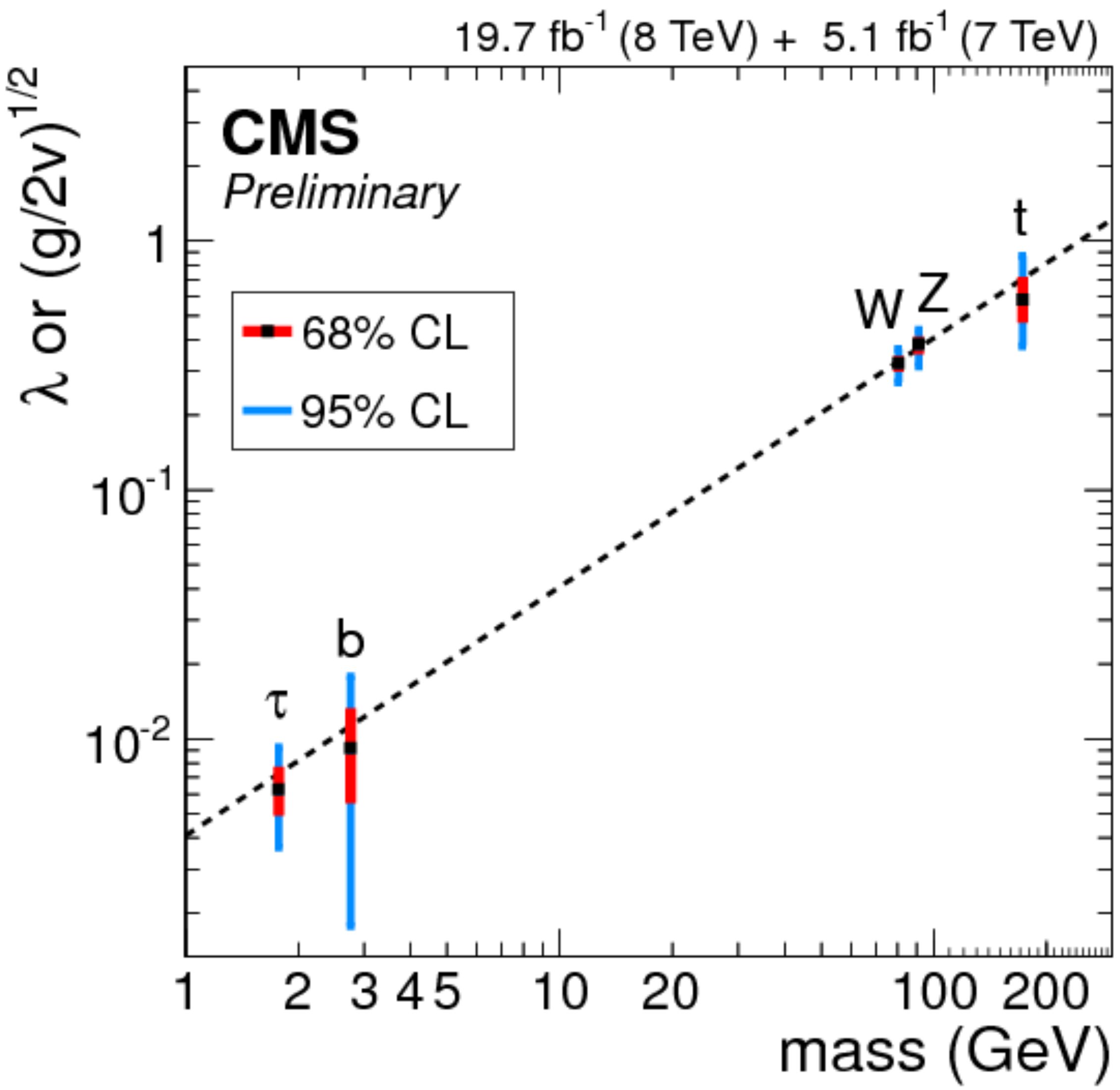
- A NEW FORCE HAS BEEN DISCOVERED, THE FIRST EVER SEEN* NOT RELATED TO A GAUGE SYMMETRY.

*fundamental, ie with elementary mediators.

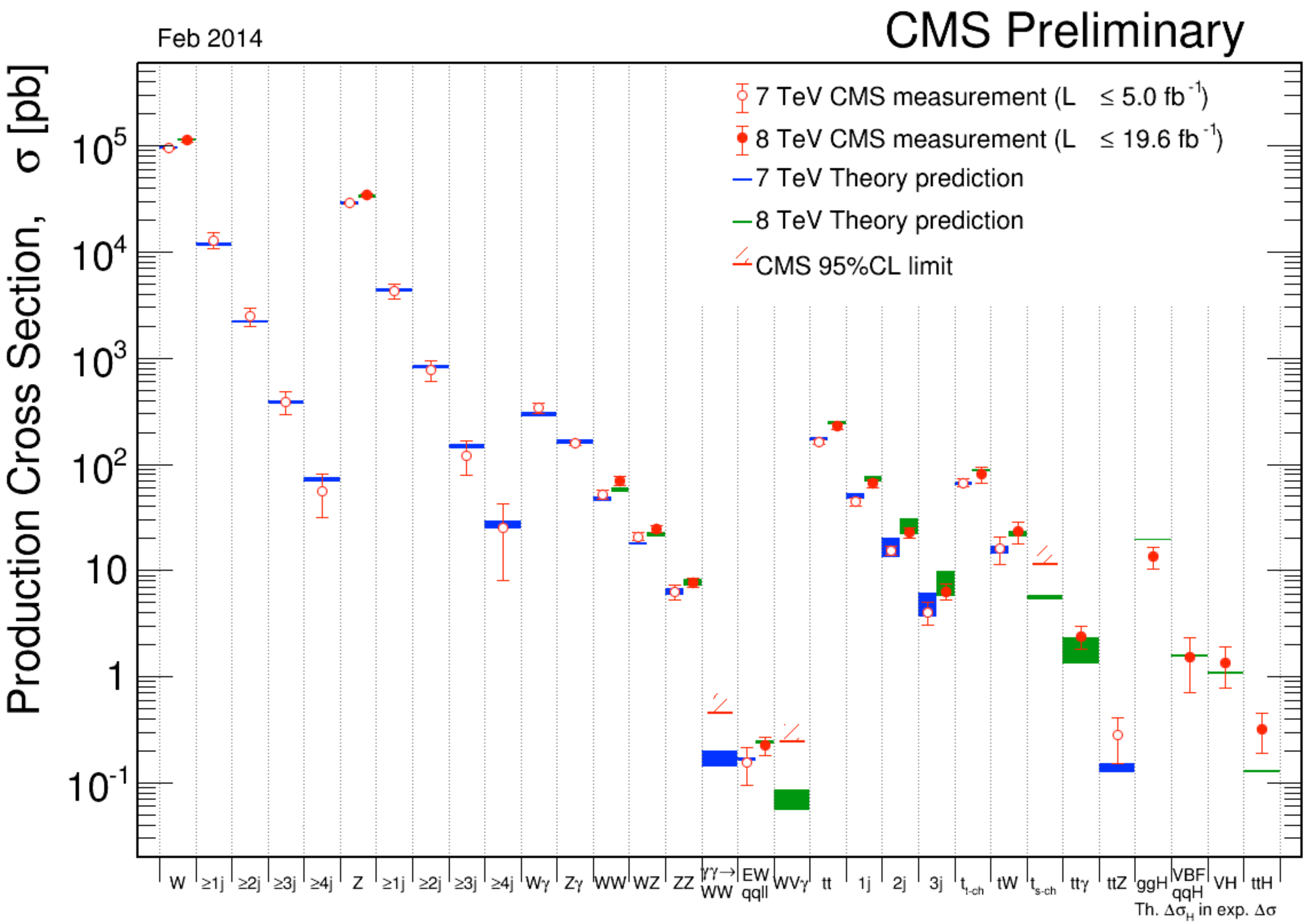
NEWS FROM THE LHC

- Its mediator looks a lot like the SM scalar: H-universality of the couplings





NEWS FROM THE LHC



Even this plot actually needs theory input (and the total quoted uncertainty in the measurements does have a contribution from theory)!!!

ATLAS Exotics Searches* - 95% CL Exclusion

Status: ICHEP 2014

ATLAS Preliminary

$\int\mathcal{L}\,dt = (1.0 - 20.3)\ \text{fb}^{-1}$ $\sqrt{s} = 7, 8$ TeV

	Model	ℓ, γ	Jets	E_T^{miss}	$\int\mathcal{L}\,dt[\text{fb}^{-1}]$	Mass limit		Reference
Extra dimensions	ADD $G_{KK} + g/q$	–	1-2 j	Yes	4.7	M_D 4.37 TeV	$n = 2$	1210.4491
	ADD non-resonant $\ell\ell$	$2e, \mu$	–	–	20.3	M_S 5.2 TeV	$n = 3$ HLZ	ATLAS-CONF-2014-030
	ADD QBH $\to \ell q$	1 e, μ	1 j	–	20.3	M_{th} 5.2 TeV	$n = 6$	1311.2006
	ADD QBH	–	2 j	–	20.3	M_{th} 5.82 TeV	$n = 6$	to be submitted to PRD
	ADD BH high N_{trk}	2 μ (SS)	–	–	20.3	M_{th} 5.7 TeV	$n = 6$, $M_D = 1.5$ TeV, non-rot BH	1308.4075
	ADD BH high $\sum p_T$	≥ 1 e, μ	≥ 2 j	–	20.3	M_{th} 6.2 TeV	$n = 6$, $M_D = 1.5$ TeV, non-rot BH	1405.4254
	RS1 $G_{KK} \to \ell\ell$	2 e, μ	–	–	20.3	G_{KK} mass 2.68 TeV	$k/\overline{M}_{Pl} = 0.1$	1405.4123
	RS1 $G_{KK} \to WW \to \ell\nu\ell\nu$	2 e, μ	–	Yes	4.7	G_{KK} mass 1.23 TeV	$k/\overline{M}_{Pl} = 0.1$	1208.2880
	Bulk RS $G_{KK} \to ZZ \to \ell\ell qq$	2 e, μ	2 j / 1 J	–	20.3	G_{KK} mass 730 GeV	$k/\overline{M}_{Pl} = 1.0$	ATLAS-CONF-2014-039
	Bulk RS $G_{KK} \to HH \to b\bar{b}b\bar{b}$	–	4 b	–	19.5	G_{KK} mass 590-710 GeV	$k/\overline{M}_{Pl} = 1.0$	ATLAS-CONF-2014-005
	Bulk RS $g_{KK} \to t\bar{t}$	1 e, μ	≥ 1 b, $\geq$ 1J/2j	Yes	14.3	g_{KK} mass 2.0 TeV	BR = 0.925	ATLAS-CONF-2013-052
	S^1/Z_2 ED	2 e, μ	–	–	5.0	$M_{KK} \approx R^{-1}$ 4.71 TeV		1209.2535
	UED	2 γ	–	Yes	4.8	Compact. scale R^{-1} 1.41 TeV		ATLAS-CONF-2012-072
Gauge bosons	SSM $Z' \to \ell\ell$	2 e, μ	–	–	20.3	Z' mass 2.9 TeV		1405.4123
	SSM $Z' \to \tau\tau$	2 τ	–	–	19.5	Z' mass 1.9 TeV		ATLAS-CONF-2013-066
	SSM $W' \to \ell\nu$	1 e, μ	–	Yes	20.3	W' mass 3.28 TeV		ATLAS-CONF-2014-017
	EGM $W' \to WZ \to \ell\nu\,\ell'\ell'$	3 e, μ	–	Yes	20.3	W' mass 1.52 TeV		1406.4456
	EGM $W' \to WZ \to qq\ell\ell$	2 e, μ	2 j / 1 J	–	20.3	W' mass 1.59 TeV		ATLAS-CONF-2014-039
	LRSM $W'_R \to t\bar{b}$	1 e, μ	2 b, 0-1 j	Yes	14.3	W' mass 1.84 TeV		ATLAS-CONF-2013-050
	LRSM $W'_R \to t\bar{b}$	0 e, μ	≥ 1 b, 1 J	–	20.3	W' mass 1.77 TeV		to be submitted to EPJC
CI	CI $qqqq$	–	2 j	–	4.8	Λ 7.6 TeV	$\eta = +1$	1210.1718
	CI $qq\ell\ell$	2 e, μ	–	–	20.3	Λ 21.6 TeV	$\eta_{LL} = -1$	ATLAS-CONF-2014-030
	CI $uutt$	2 e, μ (SS)	≥ 1 b, ≥ 1 j	Yes	14.3	Λ 3.3 TeV	$\lvert C\rvert = 1$	ATLAS-CONF-2013-051
DM	EFT D5 operator (Dirac)	0 e, μ	1-2 j	Yes	10.5	M_* 731 GeV	at 90% CL for $m(\chi) < 80$ GeV	ATLAS-CONF-2012-147
	EFT D9 operator (Dirac)	0 e, μ	1 J, ≤ 1 j	Yes	20.3	M_* 2.4 TeV	at 90% CL for $m(\chi) < 100$ GeV	1309.4017
LQ	Scalar LQ 1[st] gen	2 e	≥ 2 j	–	1.0	LQ mass 660 GeV	$\beta = 1$	1112.4828
	Scalar LQ 2[nd] gen	2 μ	≥ 2 j	–	1.0	LQ mass 685 GeV	$\beta = 1$	1203.3172
	Scalar LQ 3[rd] gen	1 e, μ, 1 τ	1 b, 1 j	–	4.7	LQ mass 534 GeV	$\beta = 1$	1303.0526
Heavy quarks	Vector-like quark $TT \to Ht + X$	1 e, μ	≥ 2 b, ≥ 4 j	Yes	14.3	T mass 790 GeV	T in (T,B) doublet	ATLAS-CONF-2013-018
	Vector-like quark $TT \to Wb + X$	1 e, μ	≥ 1 b, ≥ 3 j	Yes	14.3	T mass 670 GeV	isospin singlet	ATLAS-CONF-2013-060
	Vector-like quark $TT \to Zt + X$	2/$\geq$3 e, μ	$\geq$2/$\geq$1 b	–	20.3	T mass 735 GeV	T in (T,B) doublet	ATLAS-CONF-2014-036
	Vector-like quark $BB \to Zb + X$	2/$\geq$3 e, μ	$\geq$2/$\geq$1 b	–	20.3	B mass 755 GeV	B in (B,Y) doublet	ATLAS-CONF-2014-036
	Vector-like quark $BB \to Wt + X$	2 e, μ (SS)	≥ 1 b, ≥ 1 j	Yes	14.3	B mass 720 GeV	B in (T,B) doublet	ATLAS-CONF-2013-051
Excited fermions	Excited quark $q^* \to q\gamma$	1 γ	1 j	–	20.3	q^* mass 3.5 TeV	only u^* and d^*, $\Lambda = m(q^*)$	1309.3230
	Excited quark $q^* \to qg$	–	2 j	–	20.3	q^* mass 4.09 TeV	only u^* and d^*, $\Lambda = m(q^*)$	to be submitted to PRD
	Excited quark $b^* \to Wt$	1 or 2 e, μ	1 b, 2 j or 1 j	Yes	4.7	b^* mass 870 GeV	left-handed coupling	1301.1583
	Excited lepton $\ell^* \to \ell\gamma$	2 e, μ, 1 γ	–	–	13.0	ℓ^* mass 2.2 TeV	$\Lambda = 2.2$ TeV	1308.1364
Other	LSTC $a_T \to W\gamma$	1 e, μ, 1 γ	–	Yes	20.3	a_T mass 960 GeV		to be submitted to PLB
	LRSM Majorana ν	2 e, μ	2 j	–	2.1	N^0 mass 1.5 TeV	$m(W_R) = 2$ TeV, no mixing	1203.5420
	Type III Seesaw	2 e, μ	–	–	5.8	$N^\pm$ mass 245 GeV	$\lvert V_e\rvert$=0.055, $\lvert V_\mu\rvert$=0.063, $\lvert V_\tau\rvert$=0	ATLAS-CONF-2013-019
	Higgs triplet $H^{\pm\pm} \to \ell\ell$	2 e, μ (SS)	–	–	4.7	$H^{\pm\pm}$ mass 409 GeV	DY production, BR($H^{\pm\pm} \to \ell\ell$)=1	1210.5070
	Multi-charged particles	–	–	–	4.4	multi-charged particle mass 490 GeV	DY production, $\lvert q\rvert = 4e$	1301.5272
	Magnetic monopoles	–	–	–	2.0	monopole mass 862 GeV	DY production, $\lvert g\rvert = 1g_D$	1207.6411

$\sqrt{s} = 7$ TeV $\sqrt{s} = 8$ TeV

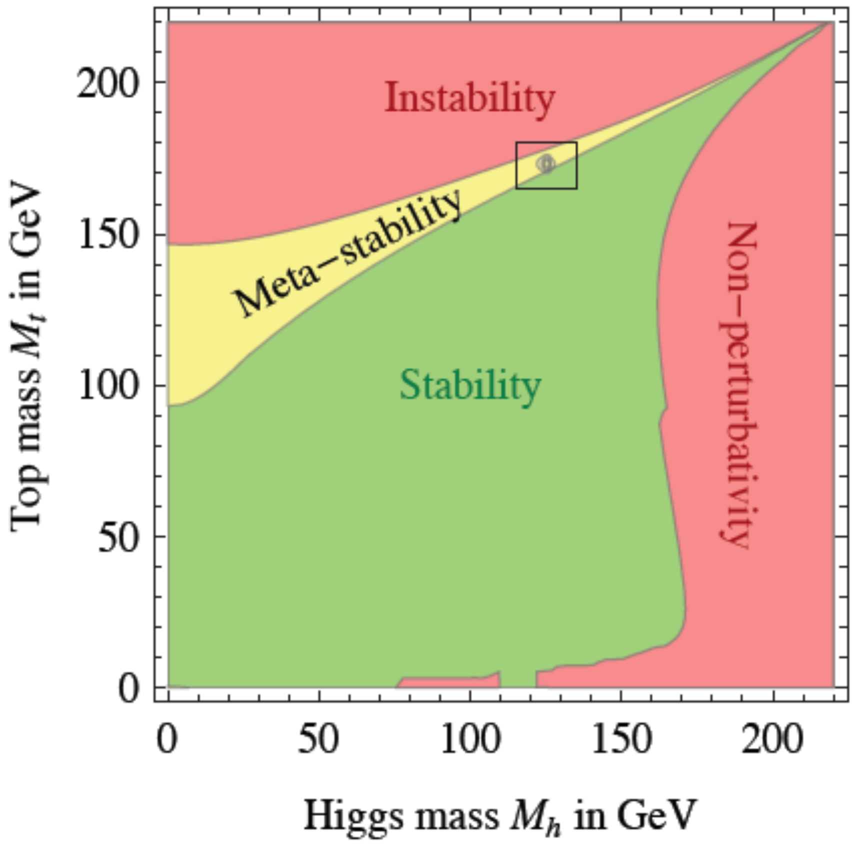
10^{-1} 1 10 **Mass scale [TeV]**

*Only a selection of the available mass limits on new states or phenomena is shown.

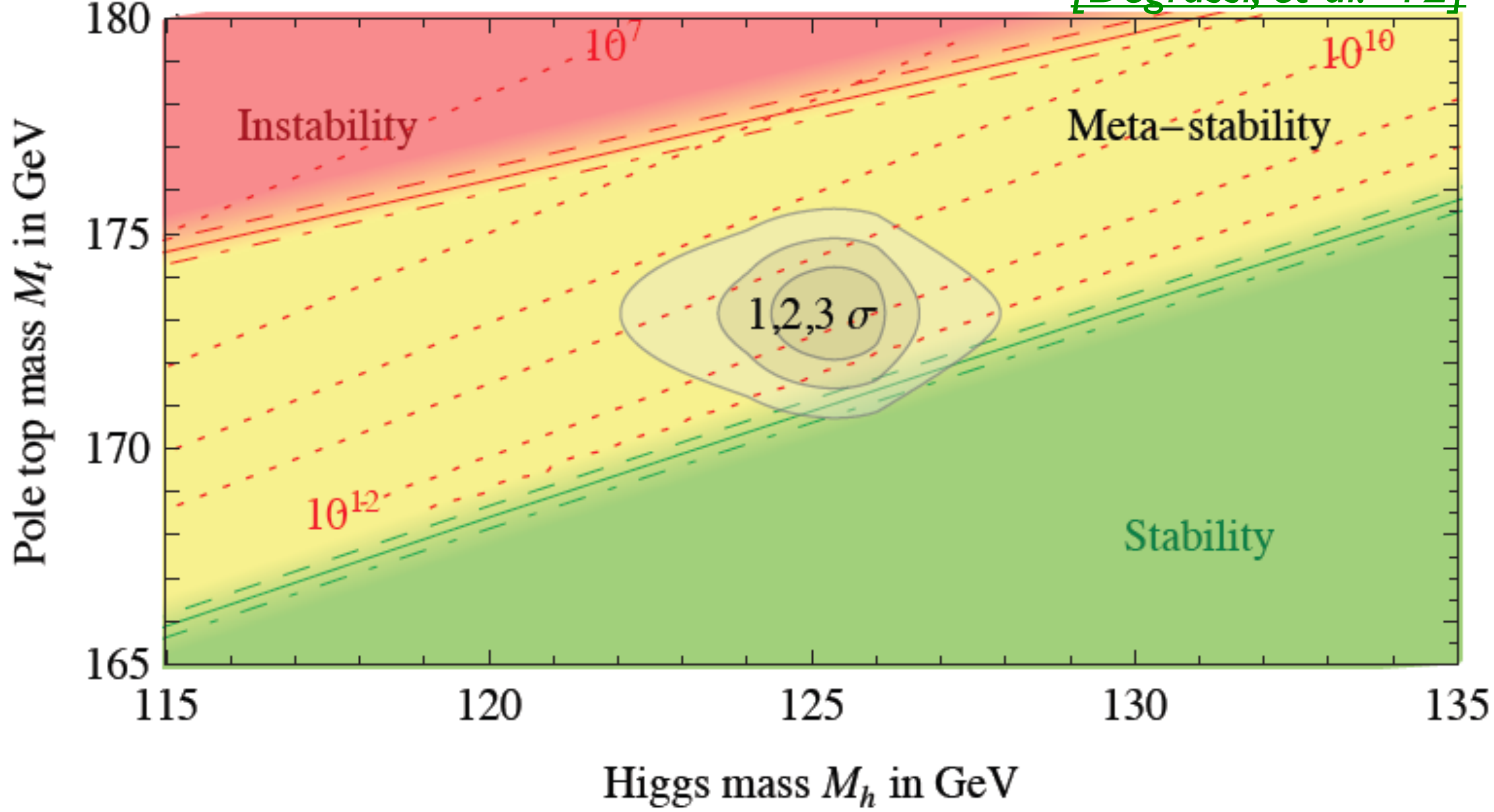
No sign of new physics (so far)!

NEWS FROM THE LHC

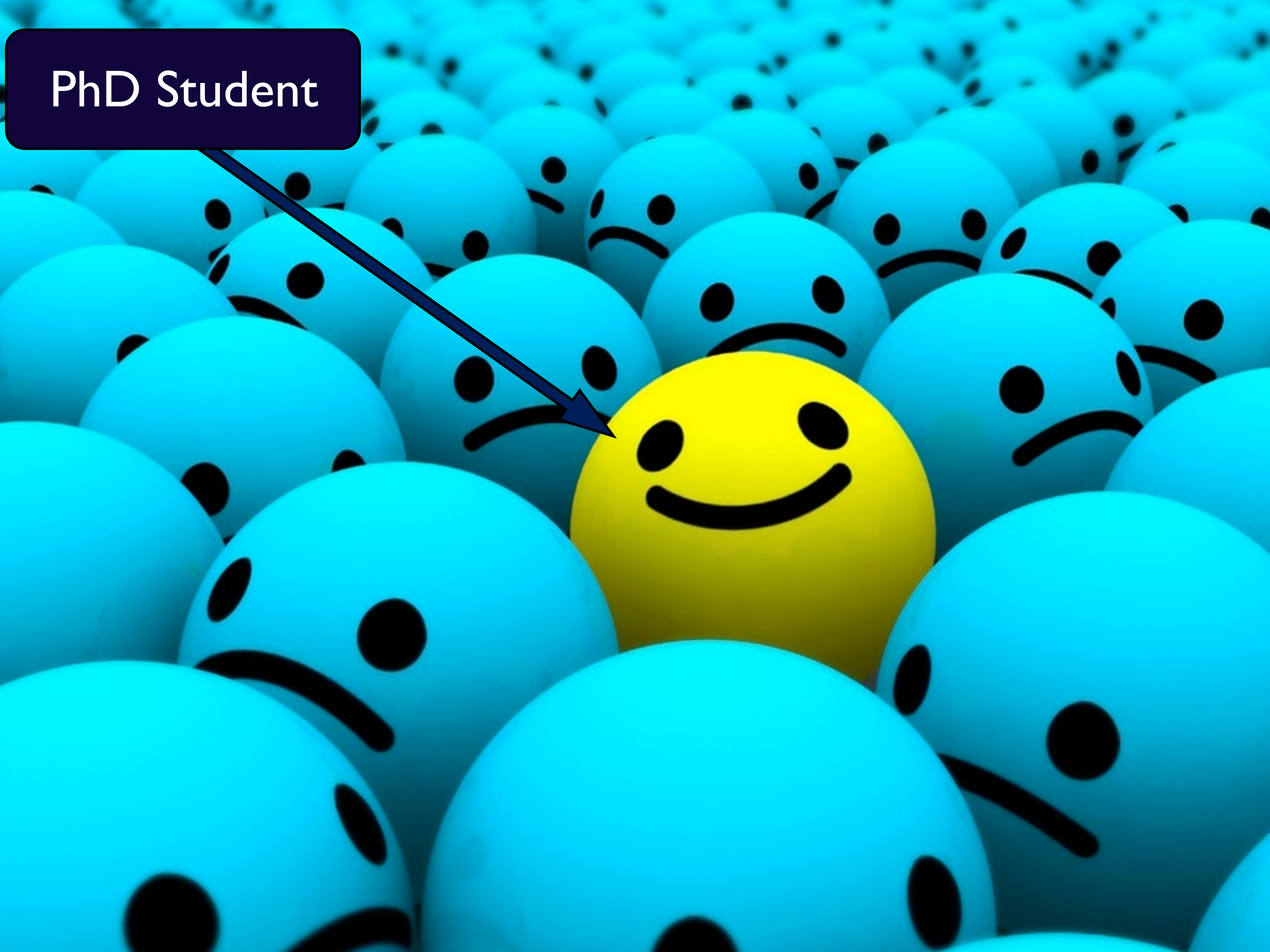
NO SIGN OF NEW PHYSICS....EVERYTHING LOOKS CONSISTENT UP TO VERY HIGH SCALE...EVEN THE FATE OF THE UNIVERSE.



[Degrassi, et al. '12]



$$y_t(M_t) = 0.93587 + 0.00557\left(\frac{M_t}{\mathrm{GeV}} - 173.15\right)\ldots \pm 0.00200_{\mathrm{th}}$$


PhD Student

WHY HAPPY?

Why happy?

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- **Massification** (the practice of making luxury products available to the mass market) : MC's in the hands of every th/exp might turn out to be the best overall strategy for discovering the Unexpected. accurate simulations for both SM and BSM are a must.

SEARCHING FOR NEW PHYSICS

Statement #1

The most promising approach to look for NP at the LHC is to cover the widest range of TH- and/or EXP-motivated searches.

Searches should aim at being sensitive to the

highest-possible scales of energy

SEARCHING FOR NEW PHYSICS

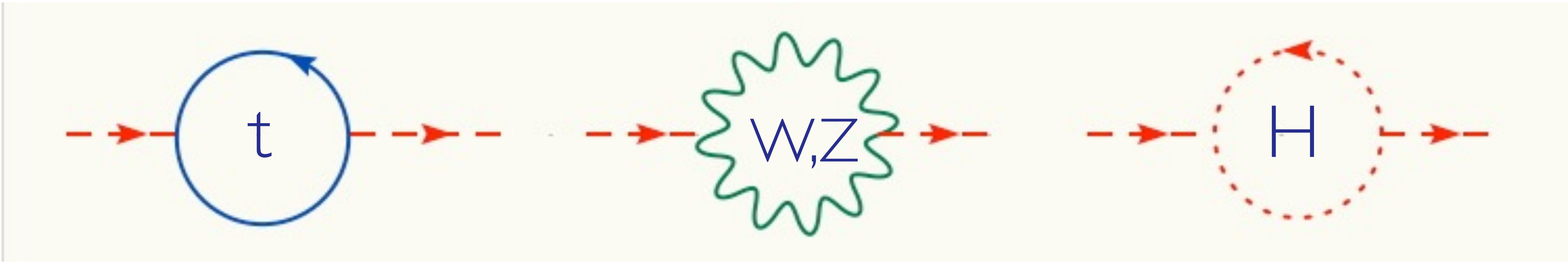
Statement #2

The Higgs provides a privileged searching ground

- It has just been discovered. Some of its properties are either just been measured or completely unknown.
- A plethora of production and decay modes available.
- First "elementary" scalar ever : carrier of a new Yukawa force, whose effects still need to be measured.
- $(\Phi^\dagger . \Phi)$ dim=2 singlet object $\Longrightarrow$ Higgs portal to a new sector.
- Several motivations to have a reacher scalar sector with more doublets or higher representations $\Longrightarrow$ Higgs= might

SEARCHING FOR NEW PHYSICS

Quantum corrections affect the stability of the Higgs mass. Consider the SM as an effective field theory valid up to scale Λ:



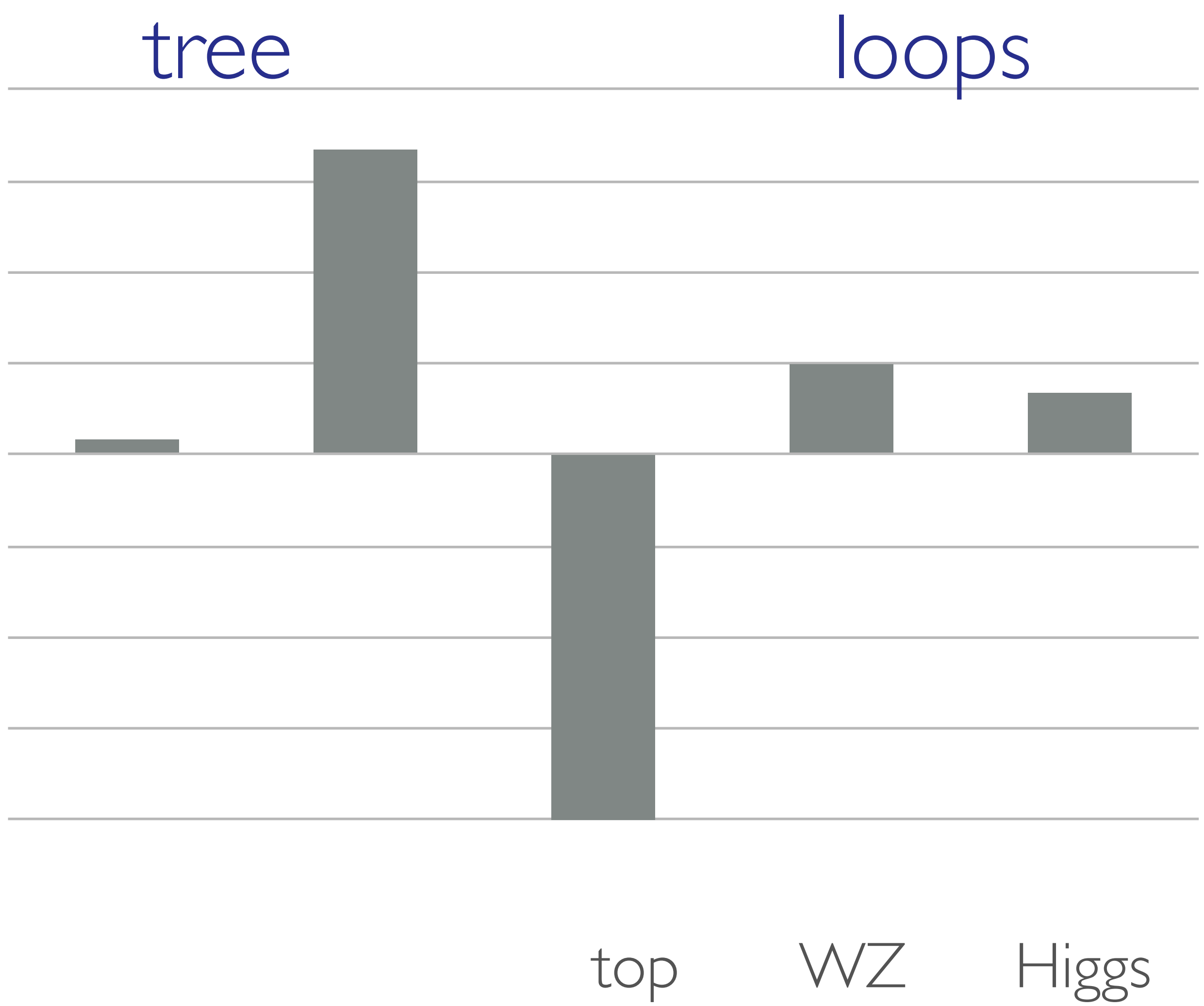
$$m_H^2 = m_{H0}^2 \quad - \frac{3}{8\pi^2} y_t \Lambda^2 \quad + \frac{1}{16\pi^2} g^2 \Lambda^2 \quad + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

Putting numbers, one gets:

$$(125\,\mathrm{GeV})^2 = m_{H0}^2 + \left[-(2\,\mathrm{TeV})^2 + (700\,\mathrm{GeV})^2 + (500\,\mathrm{GeV})^2\right] \left(\frac{\Lambda}{10\mathrm{TeV}}\right)^2$$

SEARCHING FOR NEW PHYSICS

$m_H^2 \sim (125\ \mathrm{GeV})^2$



$$(125\,\mathrm{GeV})^2 = m_{H0}^2 + \left[-(2\,\mathrm{TeV})^2 + (700\,\mathrm{GeV})^2 + (500\,\mathrm{GeV})^2\right]\left(\frac{\Lambda}{10\mathrm{TeV}}\right)^2$$

Definition of naturalness: less than 90% cancellation:

$$\Lambda_t < 3\,\mathrm{TeV}$$

Ex1= $\delta m_H^2 = \frac{3y_t^2}{8\pi^2}\tilde{m}_t^2 \ln \frac{\tilde{m}_t^2}{\Lambda^2}$

Ex2= $\delta m_H^2 \approx \frac{\lambda_\Phi}{16\pi^2} M^2 \ln \frac{M^2}{\Lambda^2}$

SEARCH FOR NEW PHYSICS AT THE LHC

Model-dependent	⟷	Model-independent
SUSY, 2HDM, ED,…		simplified models, EFT, …
Search for new states	⟷	Search for new interactions
specific models, simplified models		anomalous couplings, EFT…
Exotic signatures	⟷	Standard signatures
precision measurements		rare processes

SEARCH FOR NEW PHYSICS AT THE LHC

Search for new states

SUSY, EXOTICS, BSM HIGGS

Search for new interactions

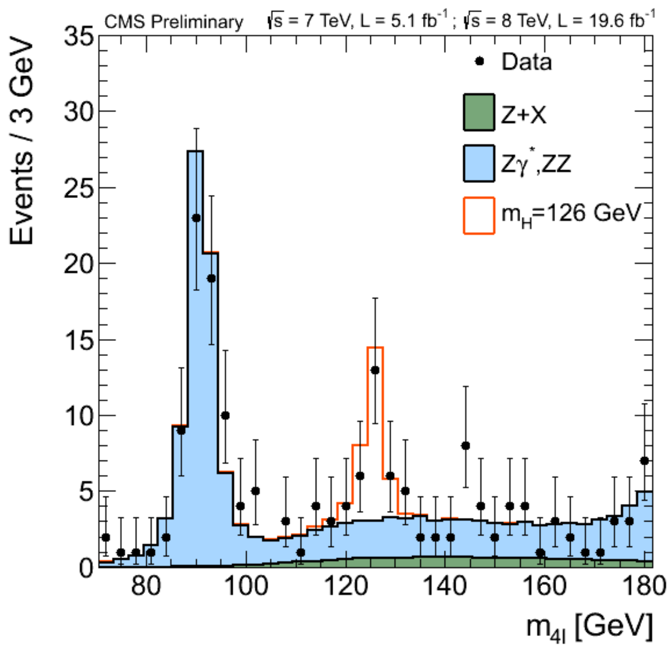
SM

Search for new states at the LHC

SEARCH FOR NEW STATES AT THE LHC

peak

pp→H→4l



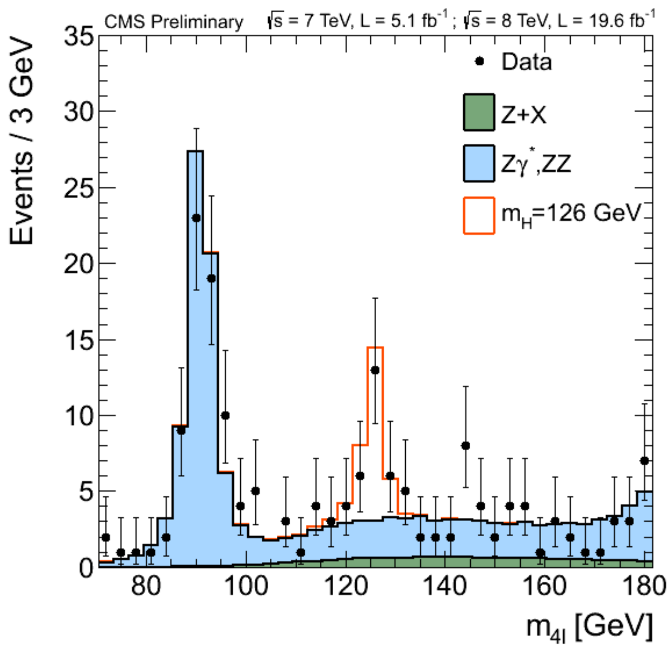
"easy"

Background directly measured from data. TH needed only for parameter extraction (Normalization, acceptance,...)

Search for new states at the LHC

peak

$pp \to H \to 4l$



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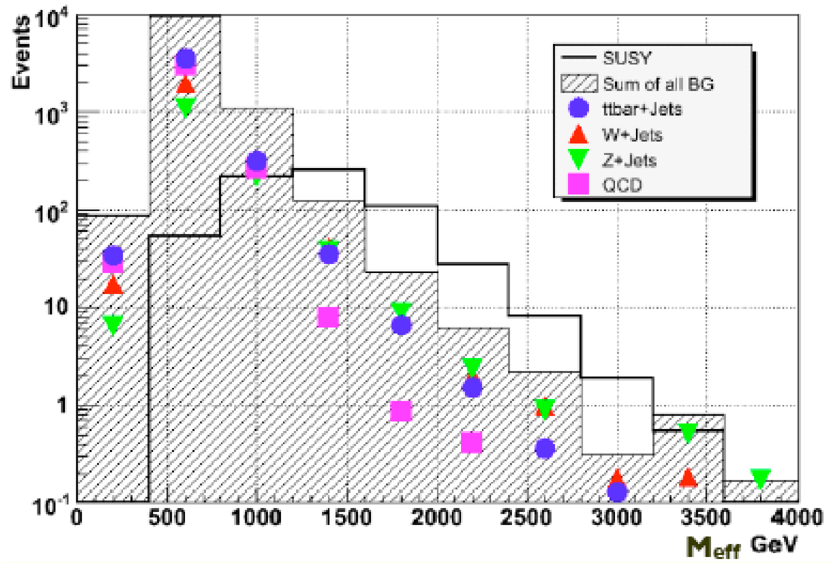
Background directly measured from data. TH needed only for parameter extraction (Normalization, acceptance,...)

shape

$pp \to \tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q} \to jets + \not{E}_T$



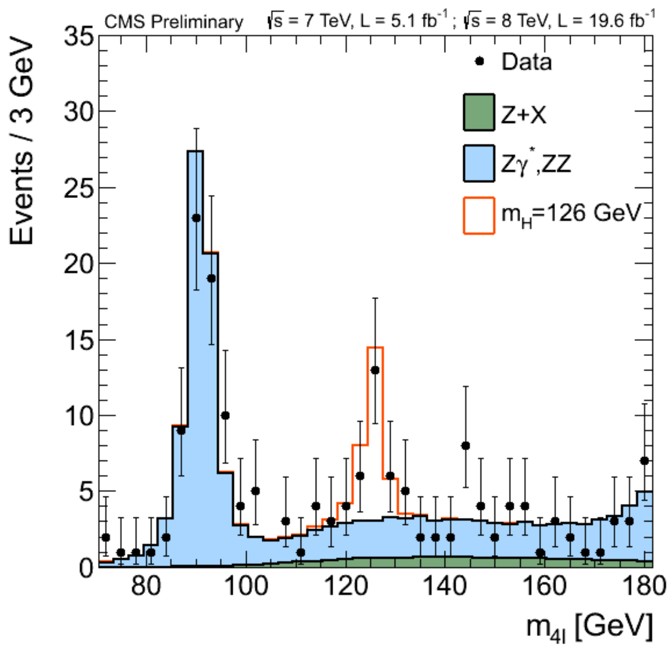
hard

Background shapes needed. Flexible MC for both signal and background tuned and validated with data.

Search for new states at the LHC

peak

$pp \to H \to 4l$



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Background directly measured from data. TH needed only for parameter extraction (Normalization, acceptance,...)

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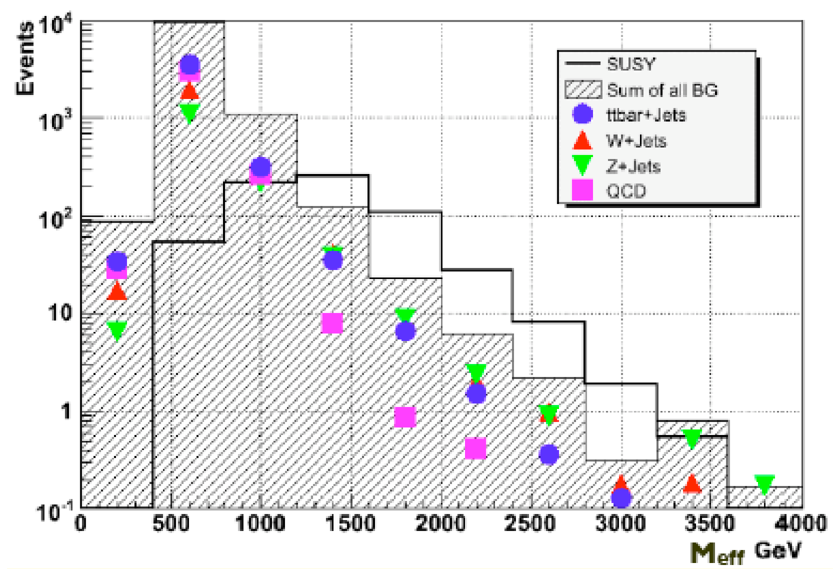
$pp \to \tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q} \to \text{jets} + \not{E}_T$

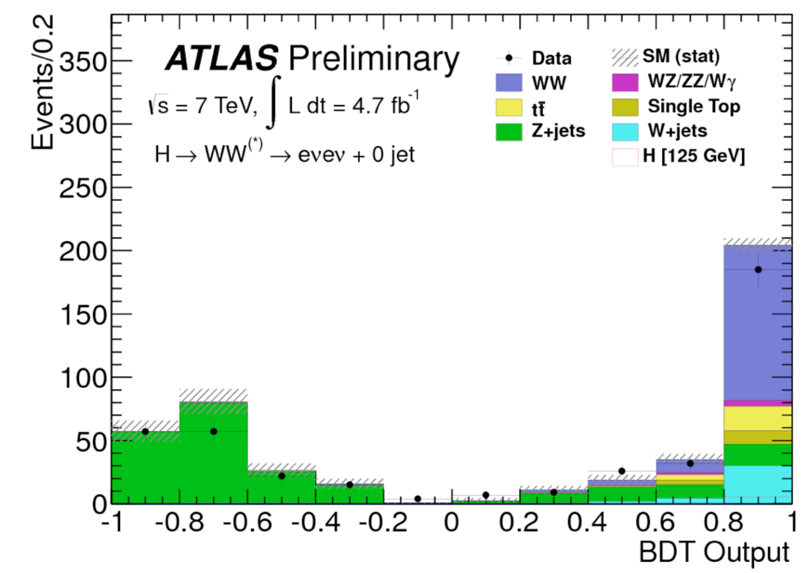


hard

Background shapes needed. Flexible MC for both signal and background tuned and validated with data.

discriminant

$pp \to H \to W^+W^-$



very hard

Background normalization and shapes known very well. Interplay with the best theoretical predictions (via MC) and data.

Search for new interactions at the LHC

Two main strategies for searching new physics

Search for new states ⟷ Search for new interactions

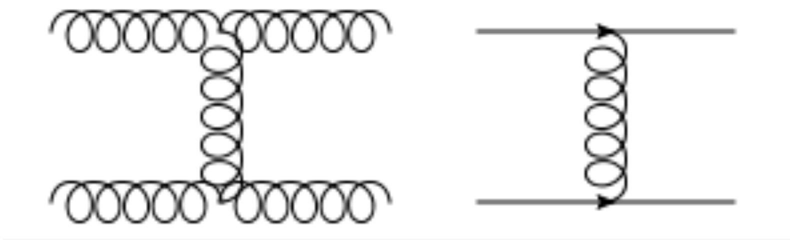
$$\mathcal{L}_{SM}^{(6)} = \mathcal{L}_{SM}^{(4)} + \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \ldots$$

The BSM ambitions of the SM programme can be recast in a powerful way in terms of one simple statement:

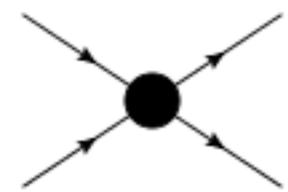
BSM goal of the SM LHC program:

determination of the couplings of the SM Lagrangian at DIM=6

Search for new interactions at the LHC



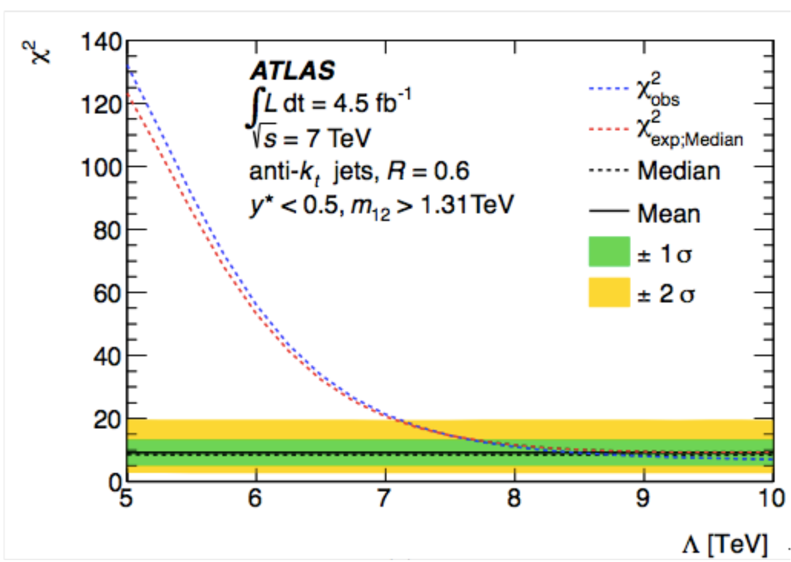
VS

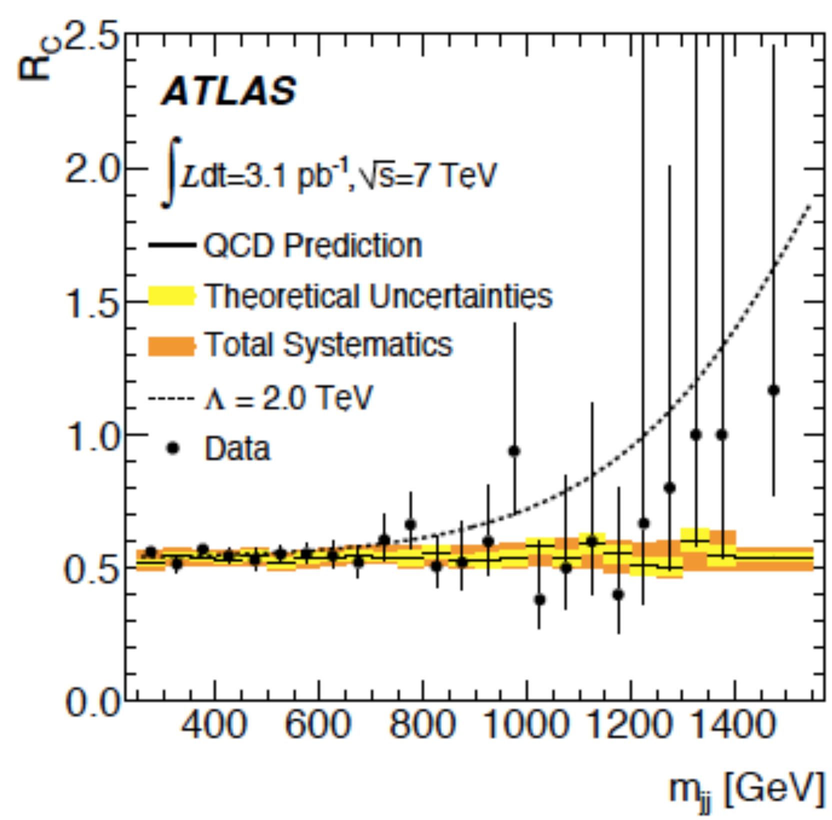


$$\mathcal{L}_{NP} = \frac{1}{2\Lambda^2}(c_1 O_1 + c_2 O_2)$$

$$|M|^2 \sim \frac{s^2}{t^2} \qquad t = -\frac{s}{2}(1 - \cos\theta) \qquad\qquad |M|^2 \sim \frac{s^2}{\Lambda^2}$$

$$R_C = \frac{\#\text{central jets with} |\eta_{1,2}| < 0.7)}{\#\text{forward jets with } 0.7 < |\eta_{1,2}| < 1.4)}$$





Rapidity and pseudorapidity

$$y = \frac{1}{2}\log\frac{E+p_z}{E-p_z} = \frac{1}{2}\log\frac{p^+}{p^-}$$

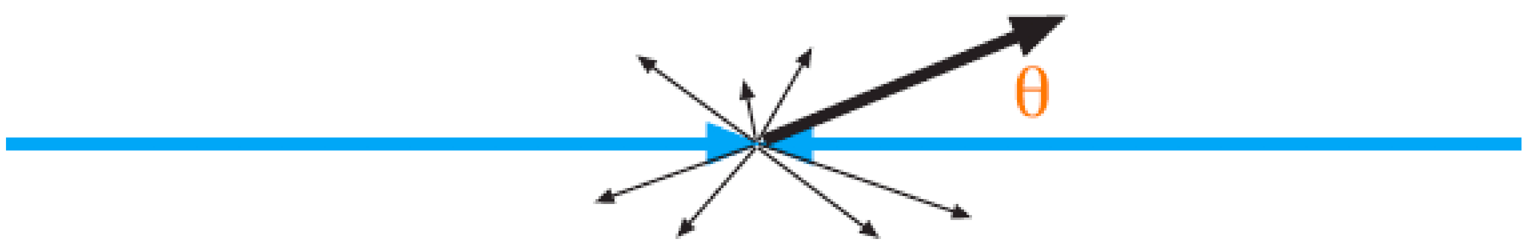
RAPIDITY

$$\eta = -\log(\tan(\theta/2))$$

PSEUDORAPIDITY

with

$$\tan\theta = \frac{p_T}{p_z}$$



1. Rapidity transforms additively under a Lorentz boost : Y→Y'=Y+Ω
2. Rapidity differences are Lorentz invariants : ΔY→ΔY'
3. Pseudo rapidity has a direct experimental definition but no specia properties under the Lorentz boosts.
4. For massless particles rapidity and pseudo rapidity are the same.

Search for new physics : Summary

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- In the search and characterisation of new physics, accurate and experimental friendly predictions for collider physics range from being very useful to strictly necessary.

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- Confidence on possible excesses, evidences and eventually discoveries builds upon an intense (and often non-linear) process of description/prediction of data via MC's.

Search for new physics : Summary

- In the search and characterisation of new physics, accurate and experimental friendly predictions for collider physics range from being very useful to strictly necessary.
- Confidence on possible excesses, evidences and eventually discoveries builds upon an intense (and often non-linear) process of description/prediction of data via MC's.
- Both **measurements** and **exclusions** rely on accurate predictions.

New generation (LHC) of MC tools

Theory

Lagrangian
Gauge invariance
QCD
Partons
NLO
Resummation
...

Detector simulation
Pions, Kaons, ...
Reconstruction
B-tagging efficiency
Boosted decision tree
Neural network
...

Experiment

New generation (LHC) of MC tools

Theory

Lagrangian
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...

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Neural network
...

Experiment

New generation (LHC) of MC tools

Idea

Lagrangian

FeynRules

ME Generator

Signal & Bkg

Events

PS+Had

Detect. Sim.

Data

Aims of the week

- Master the basic concepts of the physics of the LHC
- Learn about the latest techniques that allow to make accurate and predictions for events at the LHC in the SM and Beyond.
- Install the full chain of tools on your laptop.
- Apply and use the tools to make your own LHC search, simulating signal and background.

Aims of the week

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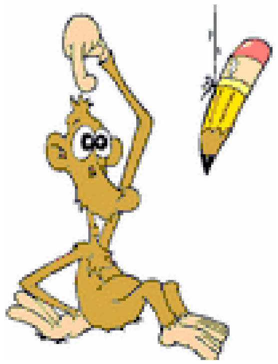
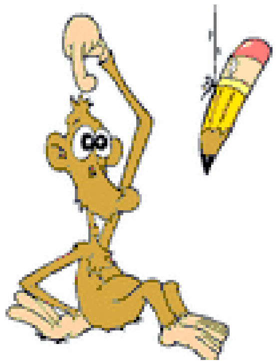


Think

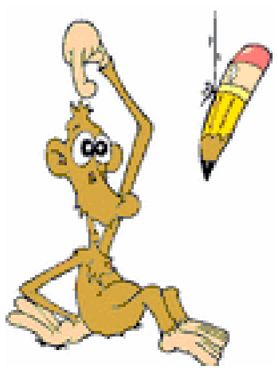
Aims of the week



Think

Participate

Aims of the week

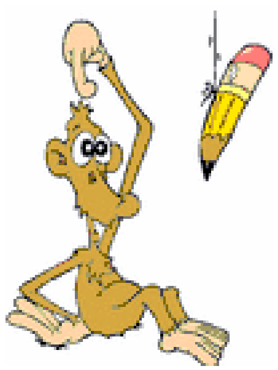


Think

Participate

Work

Aims of the week

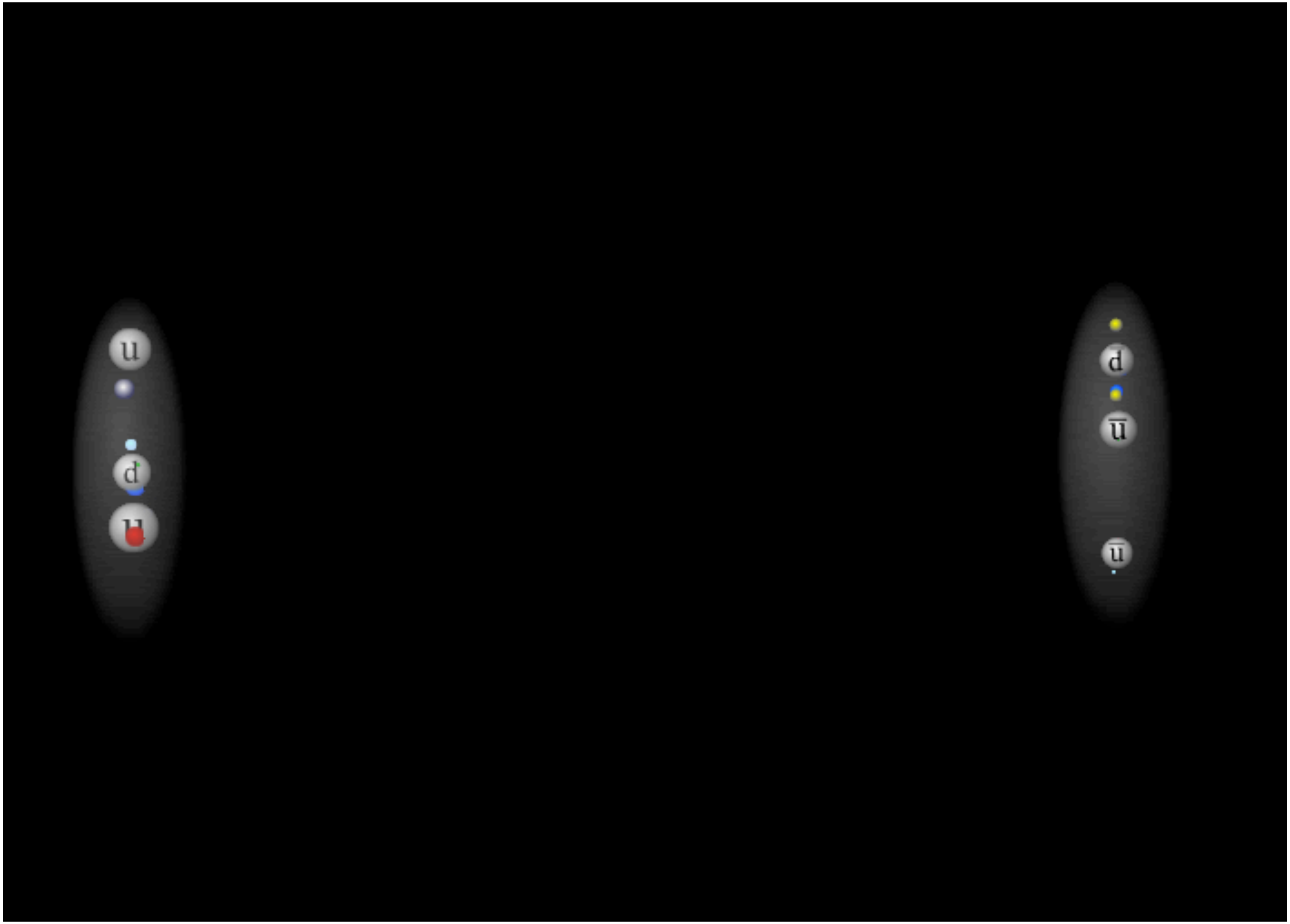


Think

Participate

Work

- The morning lectures for reviewing or introducing new concepts
- The afternoons, the most important part of the school, will be devoted to the tutorials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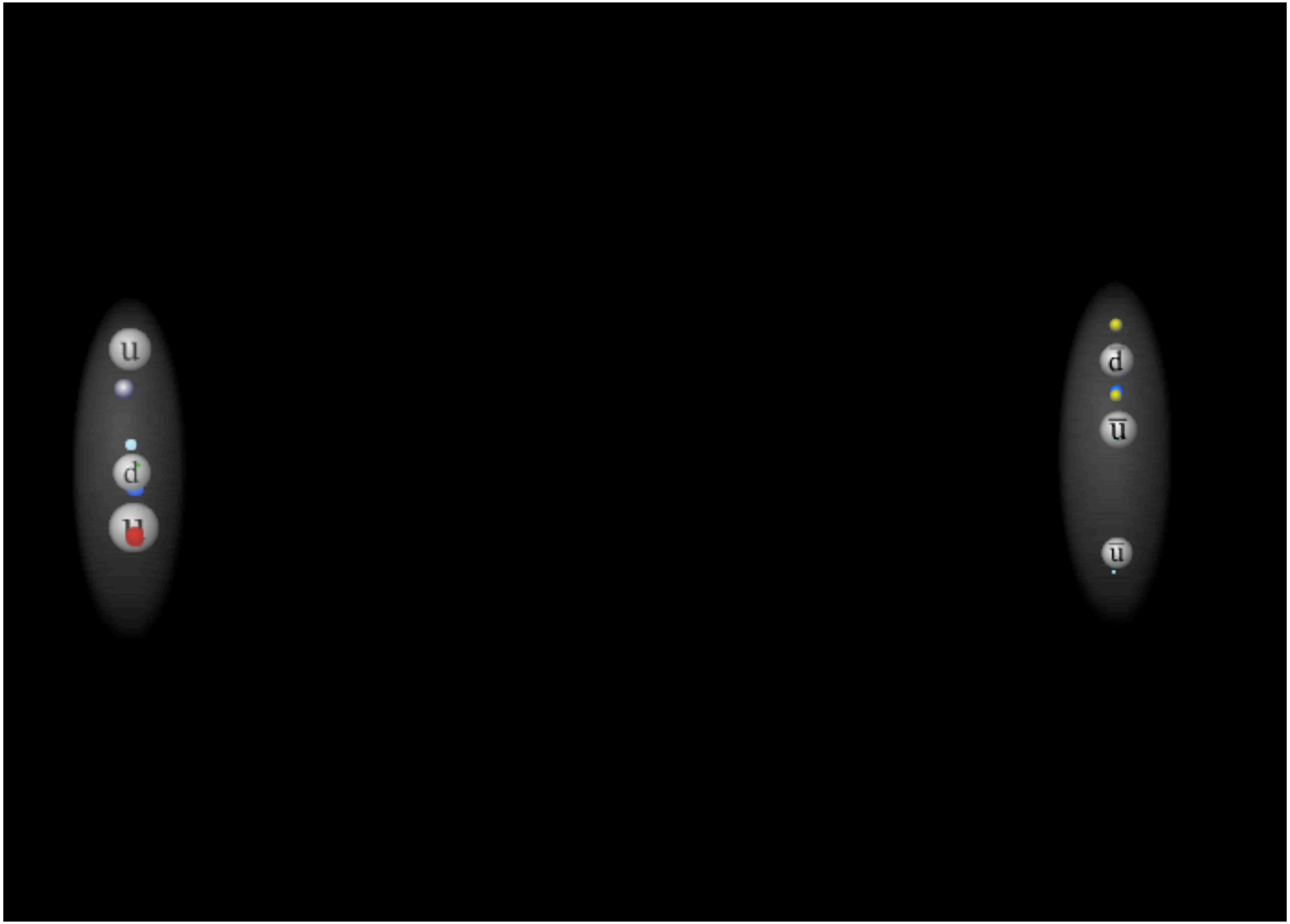
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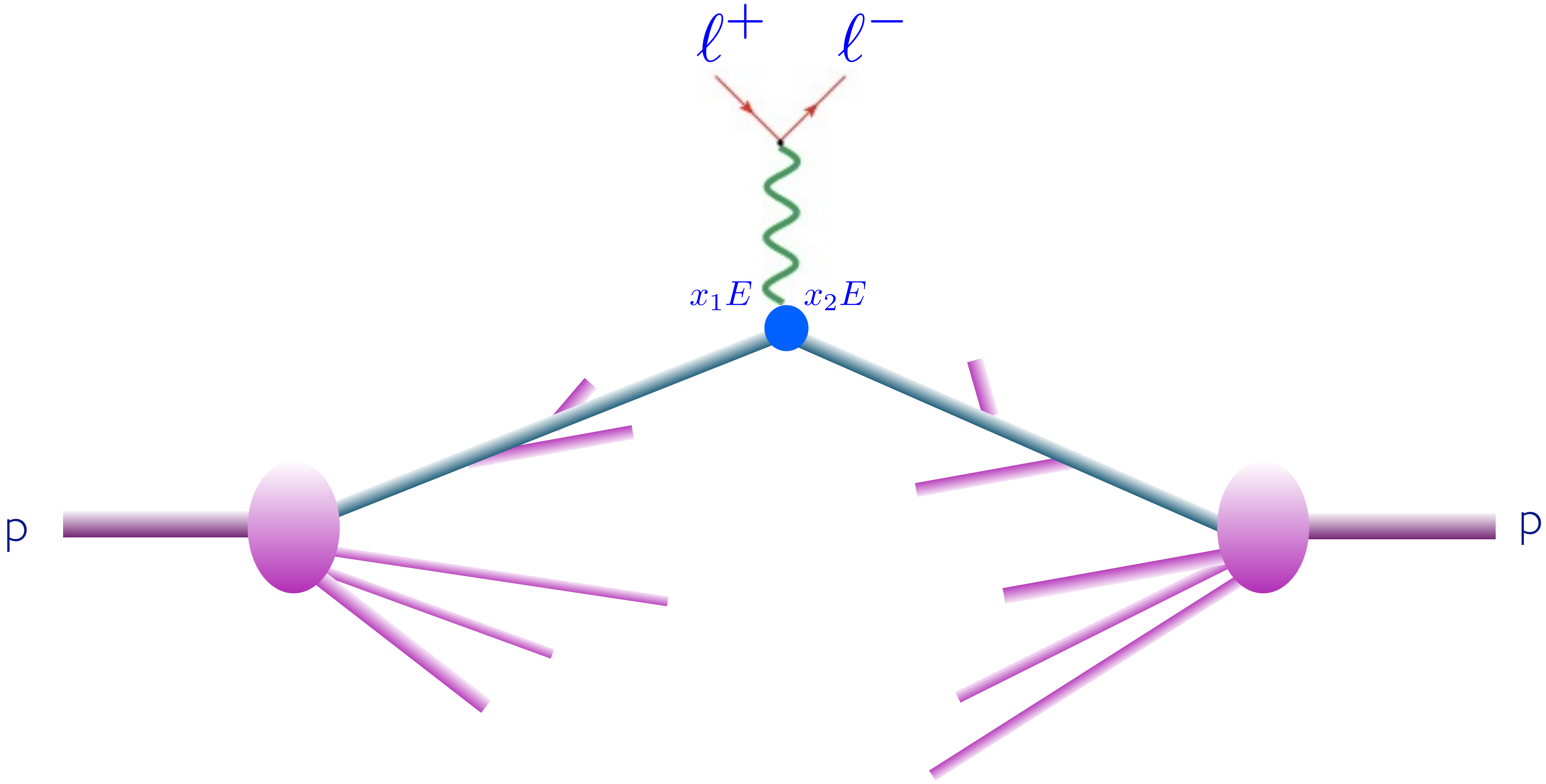
Master formula for the LHC

Master formula for the LHC



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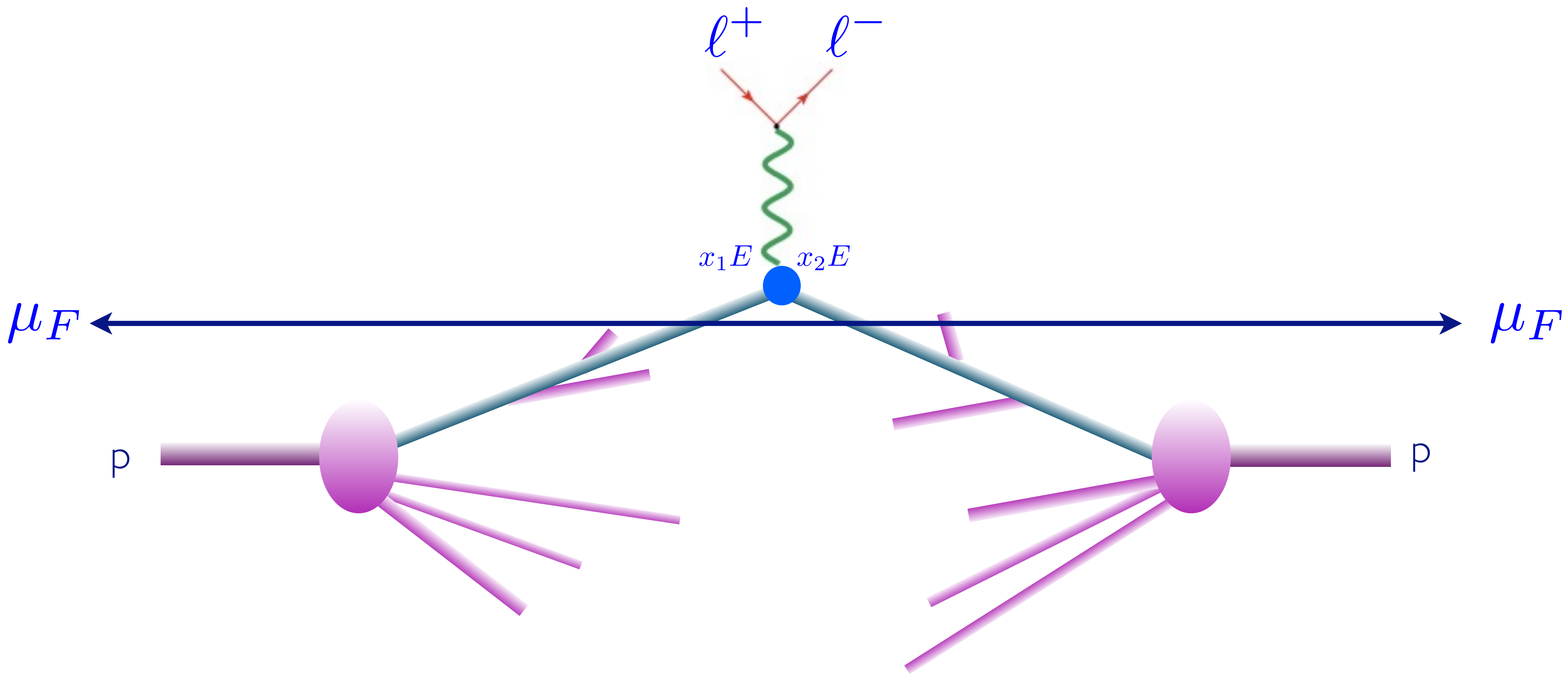

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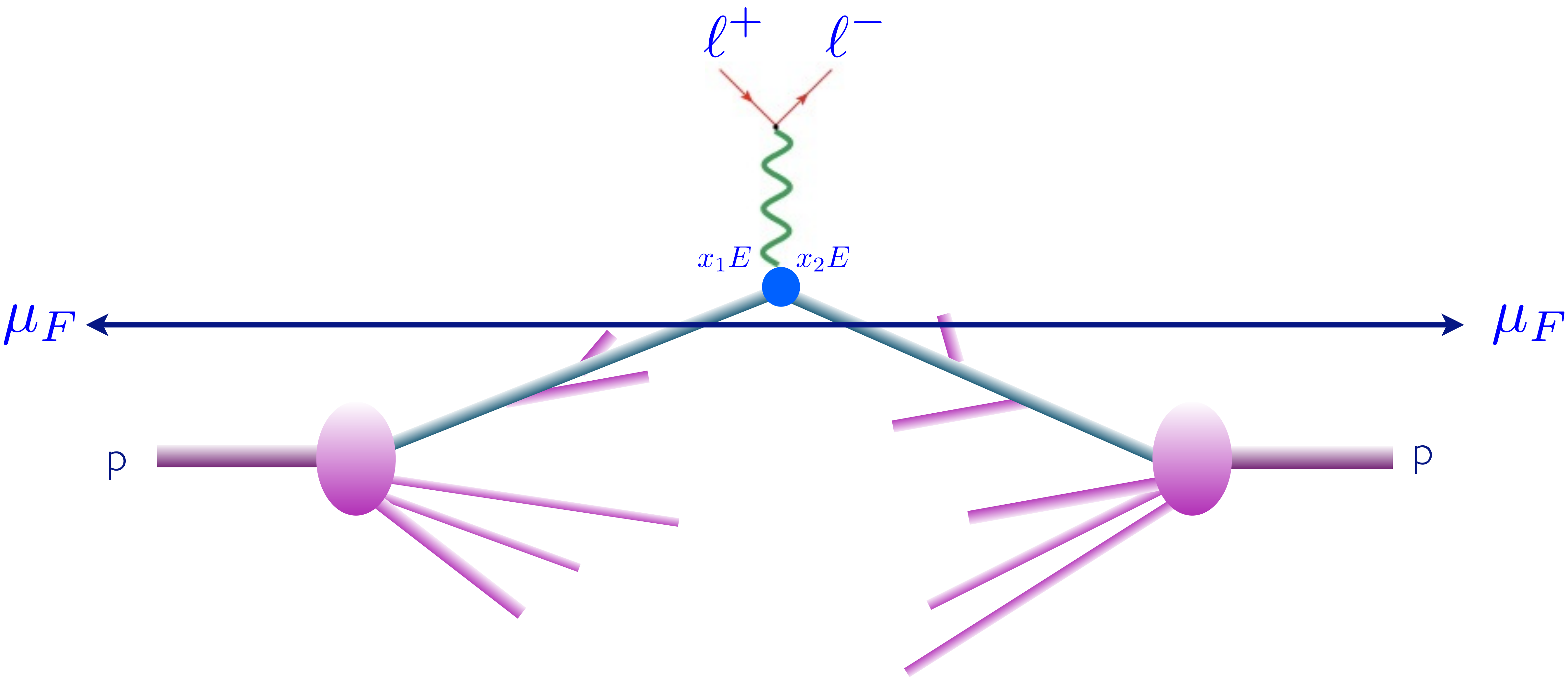

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

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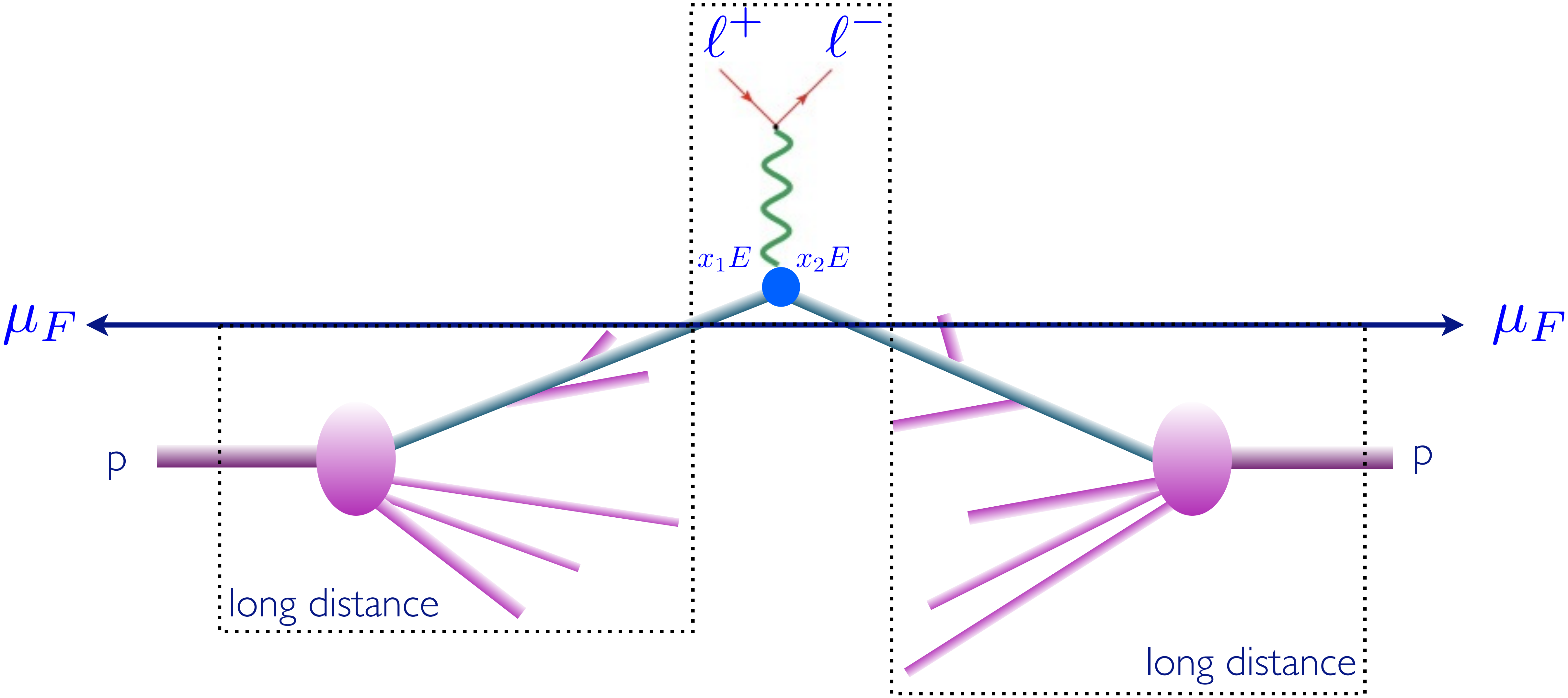

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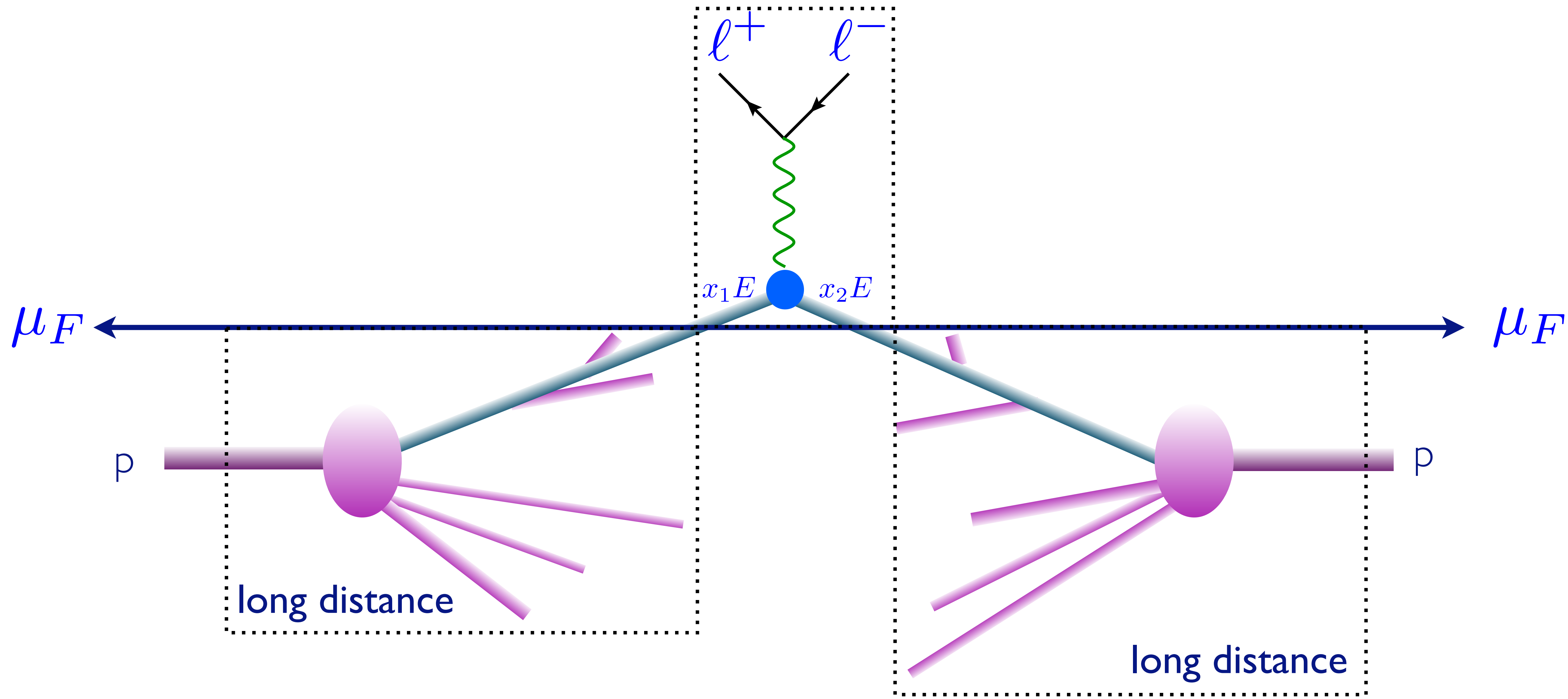


$$d\sigma = \sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

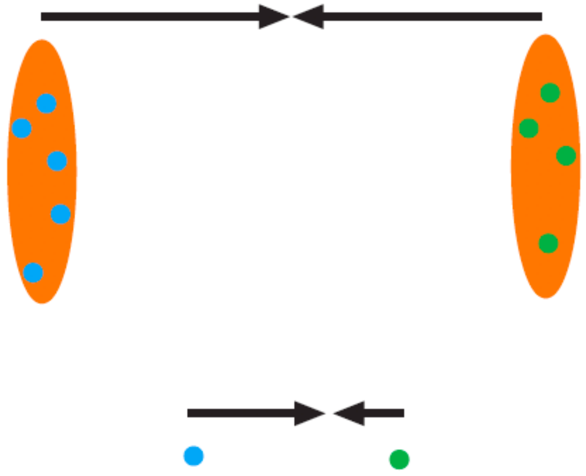
Parton density functions

Parton-level cross section

PP KINEMATICS

We describe the collision in terms of parton energies

$E_1 = x_1$ Ebeam
$E_2 = x_2$ Ebeam

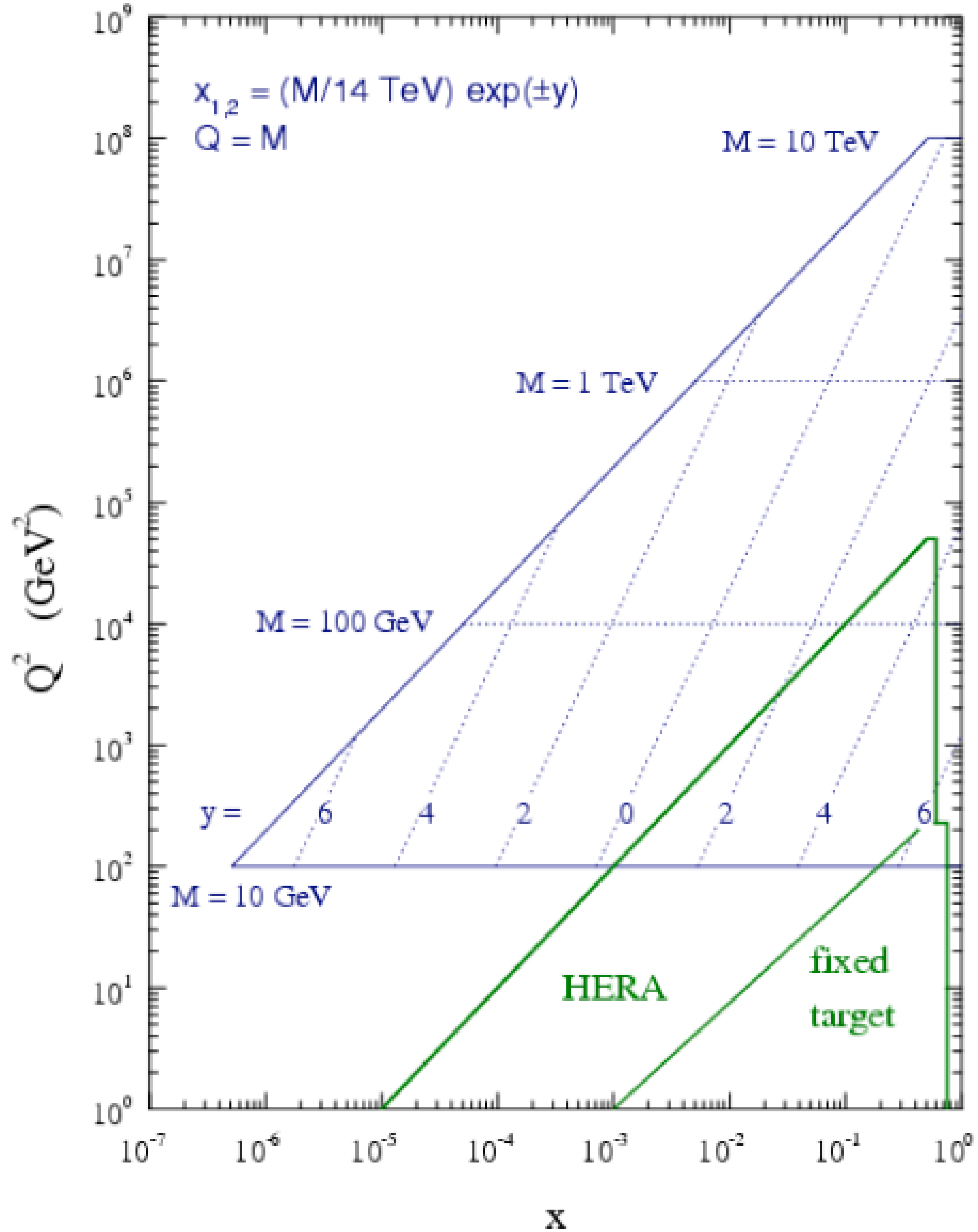


Obviously the partonic c.m.s. frame will be in general boosted. Let us say that the two partons annihilate into a particle of mass M.

$$M^2 = x_1 x_2 S = x_1 x_2 4E_{\text{beam}}^2$$

$$y = \frac{1}{2} \log \frac{x_1}{x_2}$$

$$x_1 = \frac{M}{\sqrt{S}} e^{y} \quad x_2 = \frac{M}{\sqrt{S}} e^{-y}$$



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Parton-level cross section

- How do I justify the use of such a simple formula for something so complicated as a pp collision.
- How is that possible that I can write cross sections in terms of the product of a short-distance coefficient times a long distance?
- The PDF's cannot be computed. Why? So what do we do? Do they depend on the scale?
- The parton level cross sections can be computed in perturbation theory (PT)? Why? Is the master formula valid at any order in PT?

Strong interactions

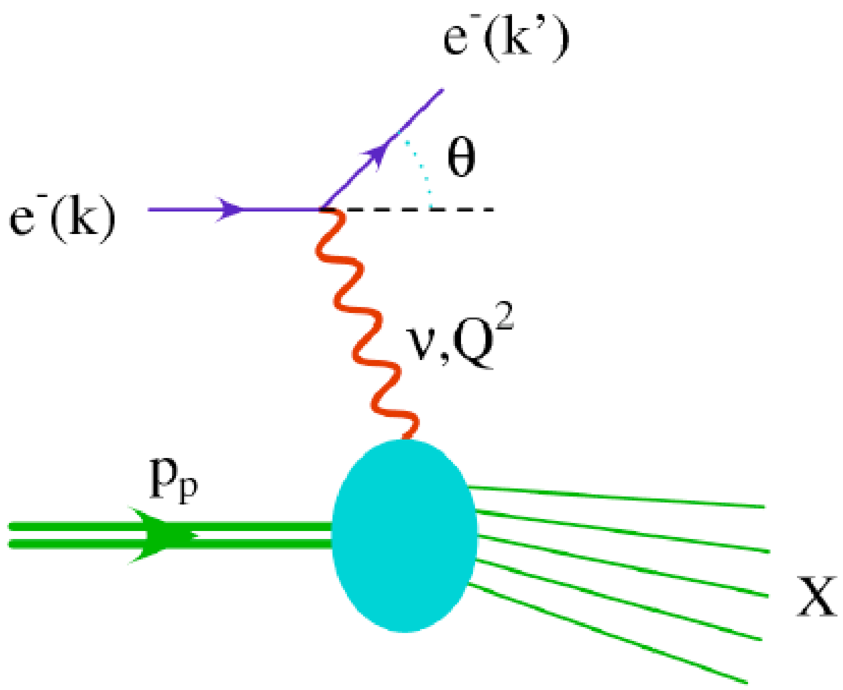
Strong interactions are characterized at moderate energies by a single* dimensionful scale, Λ_S , of few hundreds of MeV:

$$\sigma_h \cong 1/\Lambda_s^2 \cong 10 \text{ mb}$$
$$\Gamma_h \cong \Lambda_s$$
$$R \cong 1/\Lambda_s \cong 1 \text{ fm}$$

No hint to the presence of a small parameter! Very hard to understand and many attempts...

*neglecting quark masses..!!!

Scaling



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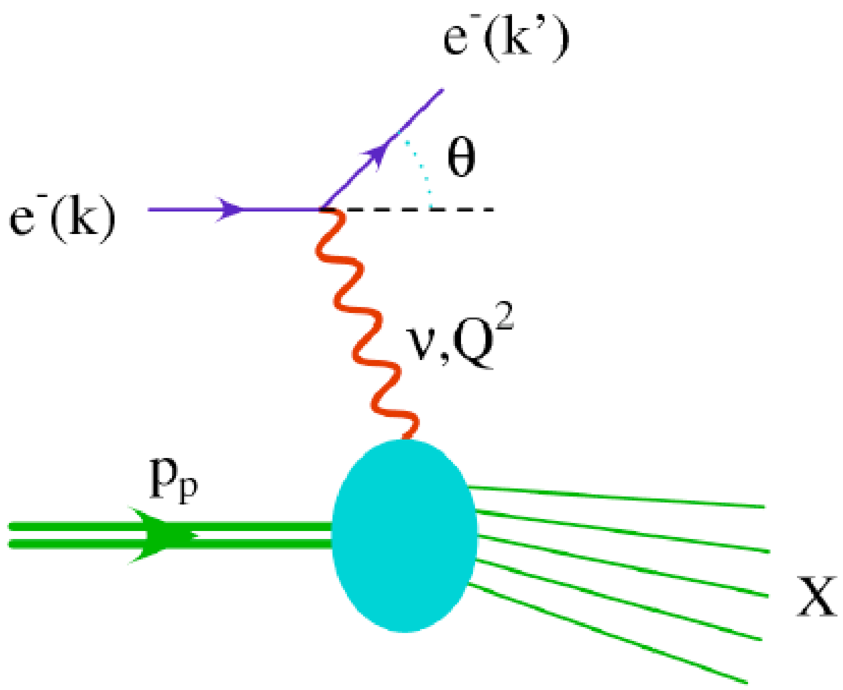

$$s = (P+k)^2$$ cms energy^2

$$Q^2 = -(k-k')^2$$ momentum transfer^2

$$x = Q^2/2(P\cdot q)$$ scaling variable

$$\nu = (P\cdot q)/M = E - E'$$ energy loss

$$y = (P\cdot q)/(P\cdot k) = 1 - E'/E$$ rel. energy loss

$$W^2 = (P+q)^2 = M^2 + \frac{1-x}{x}Q^2$$ recoil mass

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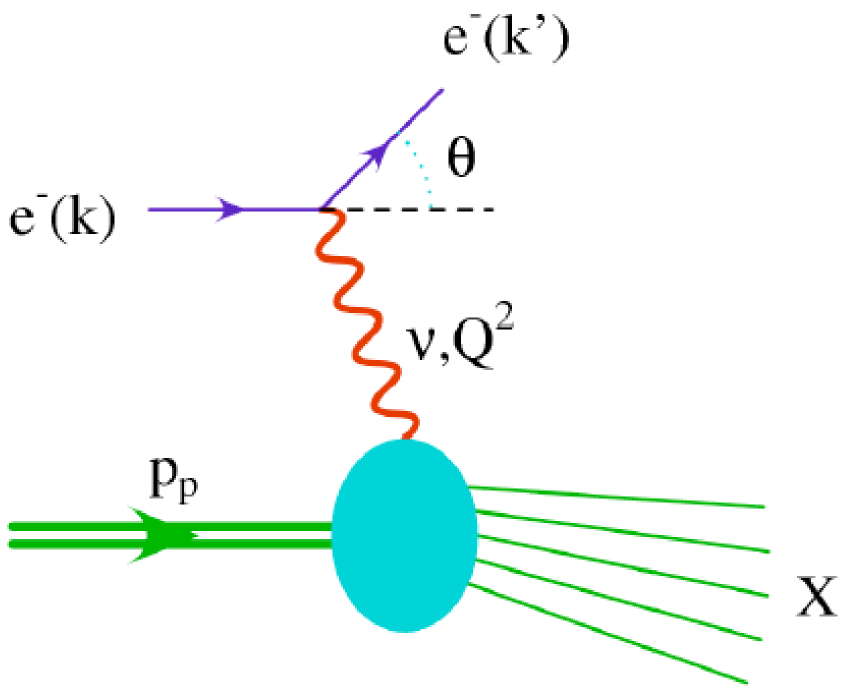

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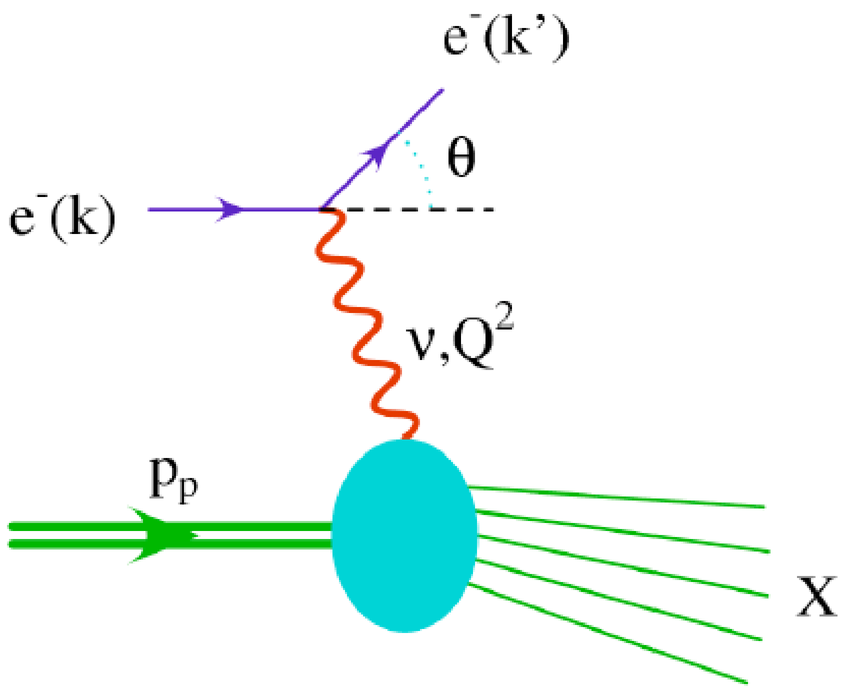

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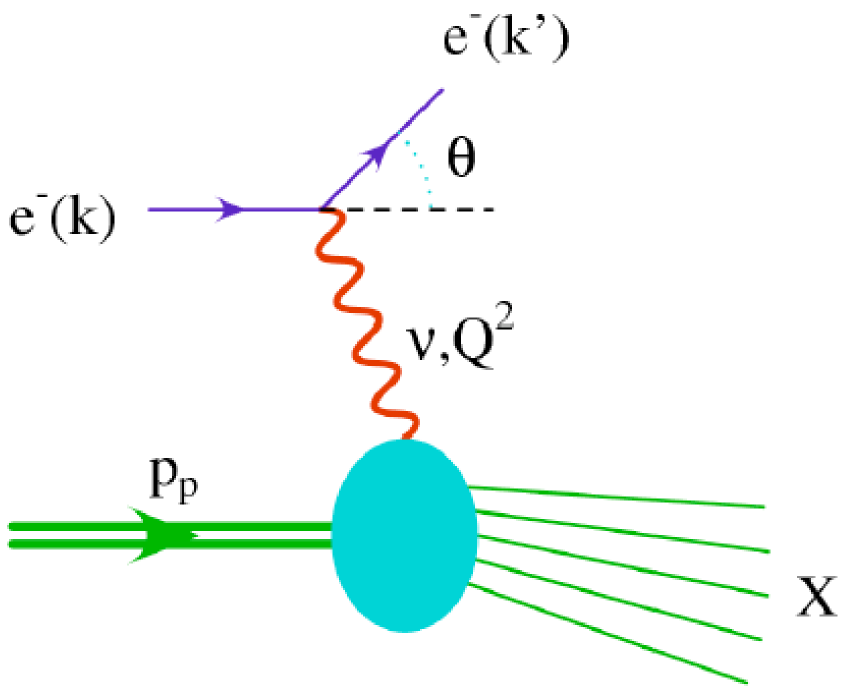
SCALING



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What should we expect for $F(q^2,x)$?

Scaling

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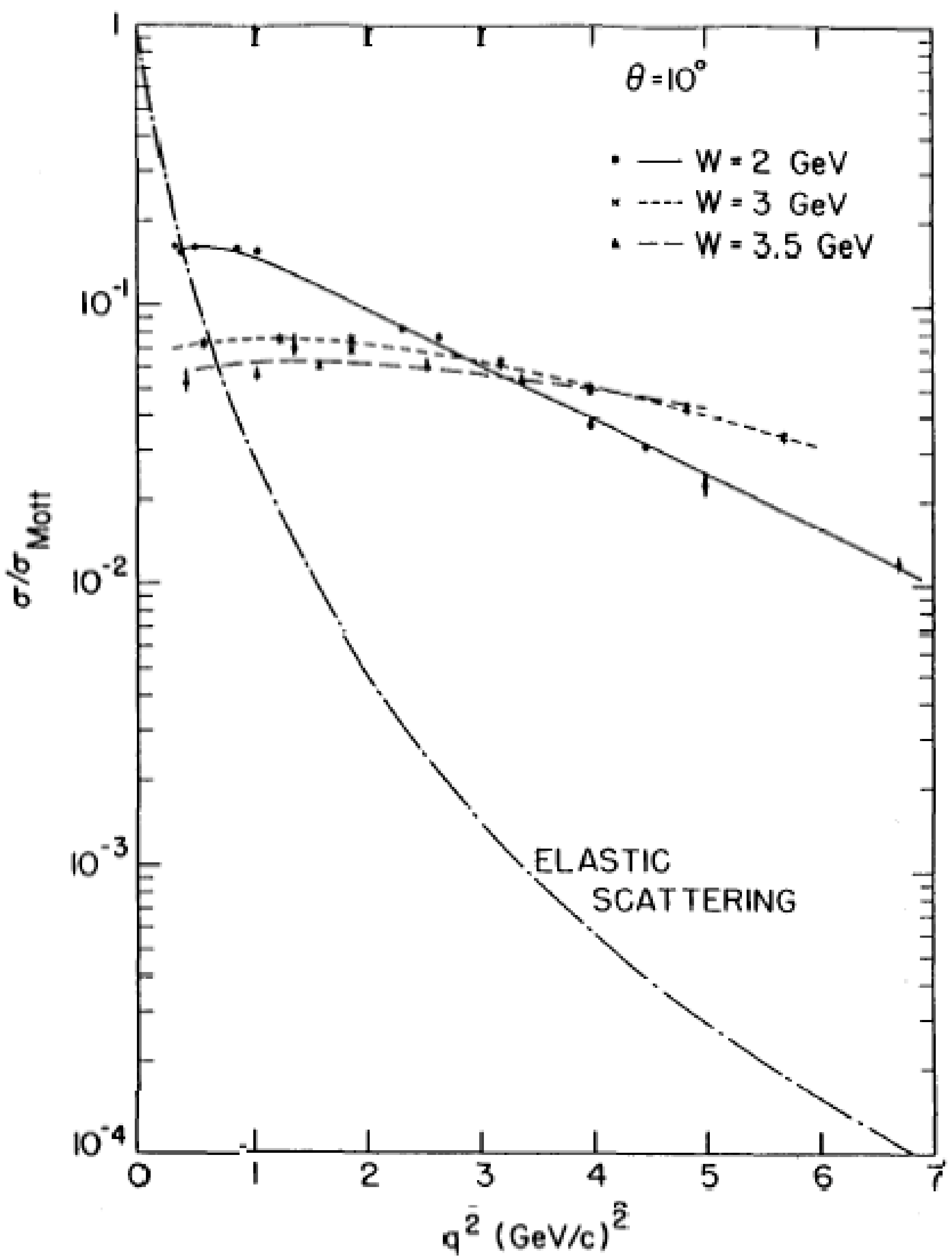
$$F^2_{elastic}(q^2) << 1 \text{ and } F^2_{inelastic}(q^2) \sim 1$$

i.e., there are points (quarks!) inside the protons, however the hit quark behaves as a free particle that flies away without feeling or caring about confinement!!!

SCALING



$$\frac{d^2\sigma^{\mathrm{EXP}}}{dxdy} \sim \frac{1}{Q^2}$$

Remarkable!!! Pure dimensional analysis!

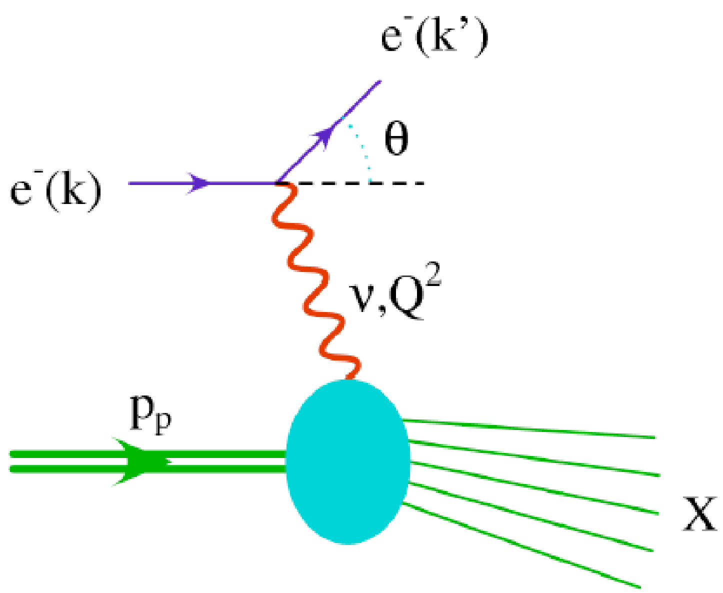
The right hand side does not depend on Λ_S !

This is the same behaviour one may find in a renormalizable theory like in QED.

Other stunning example is again $e^+e^- \to$ hadrons.

This motivated the search for a weakly-coupled theory at high energy!

DEEP-INELASTIC SCATTERING: TOWARDS THE PARTON MODEL



$$\sigma^{ep\to eX} = \sum_X \frac{1}{4ME}\int d\Phi \, \frac{1}{4}\sum_{\text{spin}} |\mathcal{M}|^2$$

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{[1+(1-y)^2]F_1(x,Q^2) + \frac{1-y}{x}\left[F_2(x,Q^2) - 2xF_1(x,Q^2)\right]\right\}$$

Comments:

* Different y dependence can differentiate between F_1 and F_2
* The first term represents the absorption of a transversely polarized photon, the second of a longitudinal one.
* Bjorken scaling ⇒ F_1 and F_2 obey scaling themselves, i.e. they do not depend on Q.

Deep-inelastic scattering: towards the parton model

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The space-time picture suggests the possibility of separating short- and long-distance physics ⇒ factorization! Turned into the language of Feynman diagrams DIS looks like:

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where

$f_{i/h}(\xi)$ is the probability to find a parton with flavor i in an hadron h carrying a light-cone momentum ξp+

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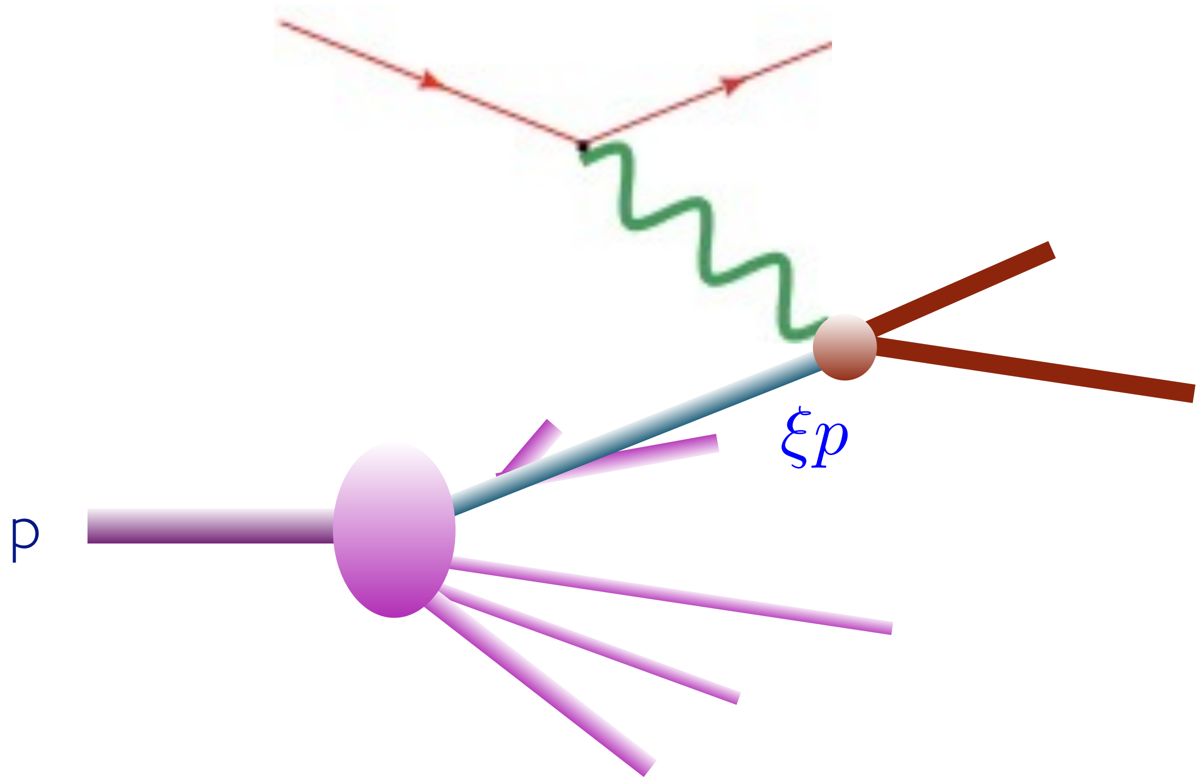

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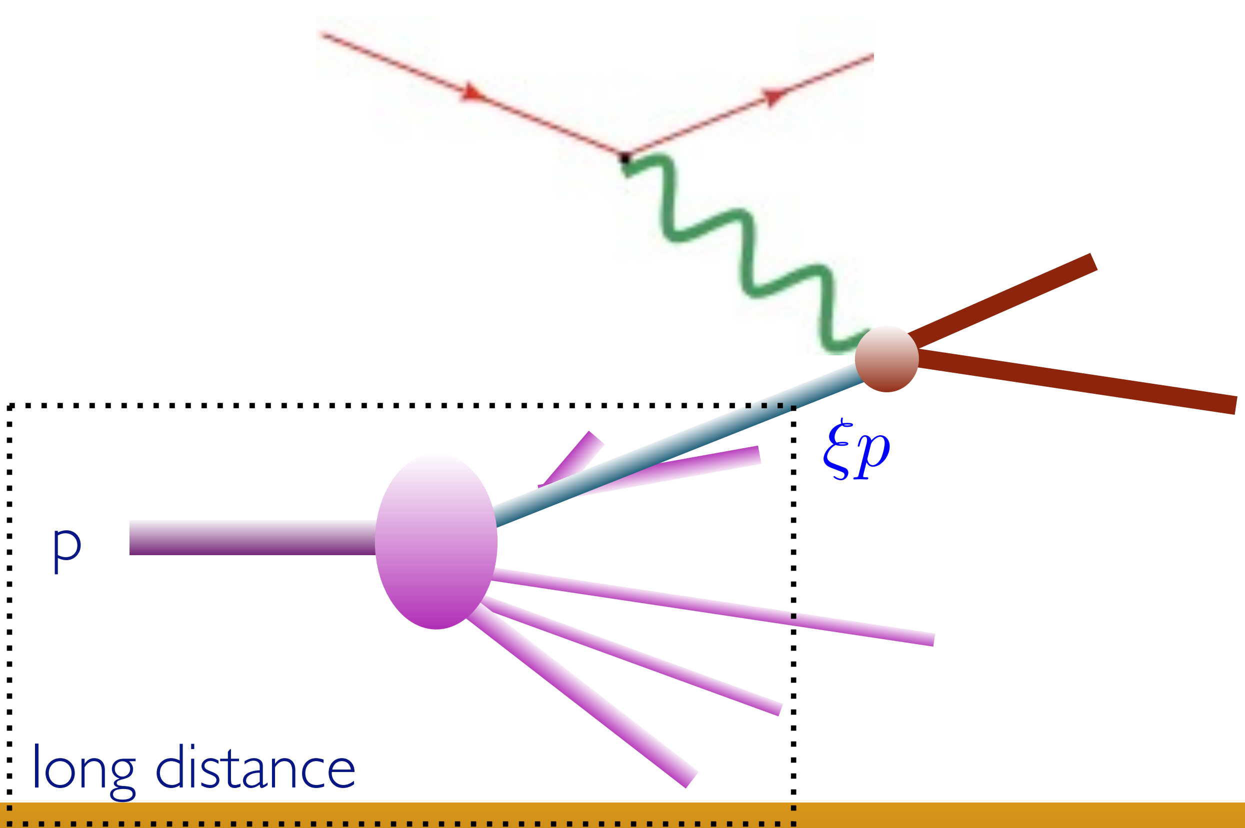

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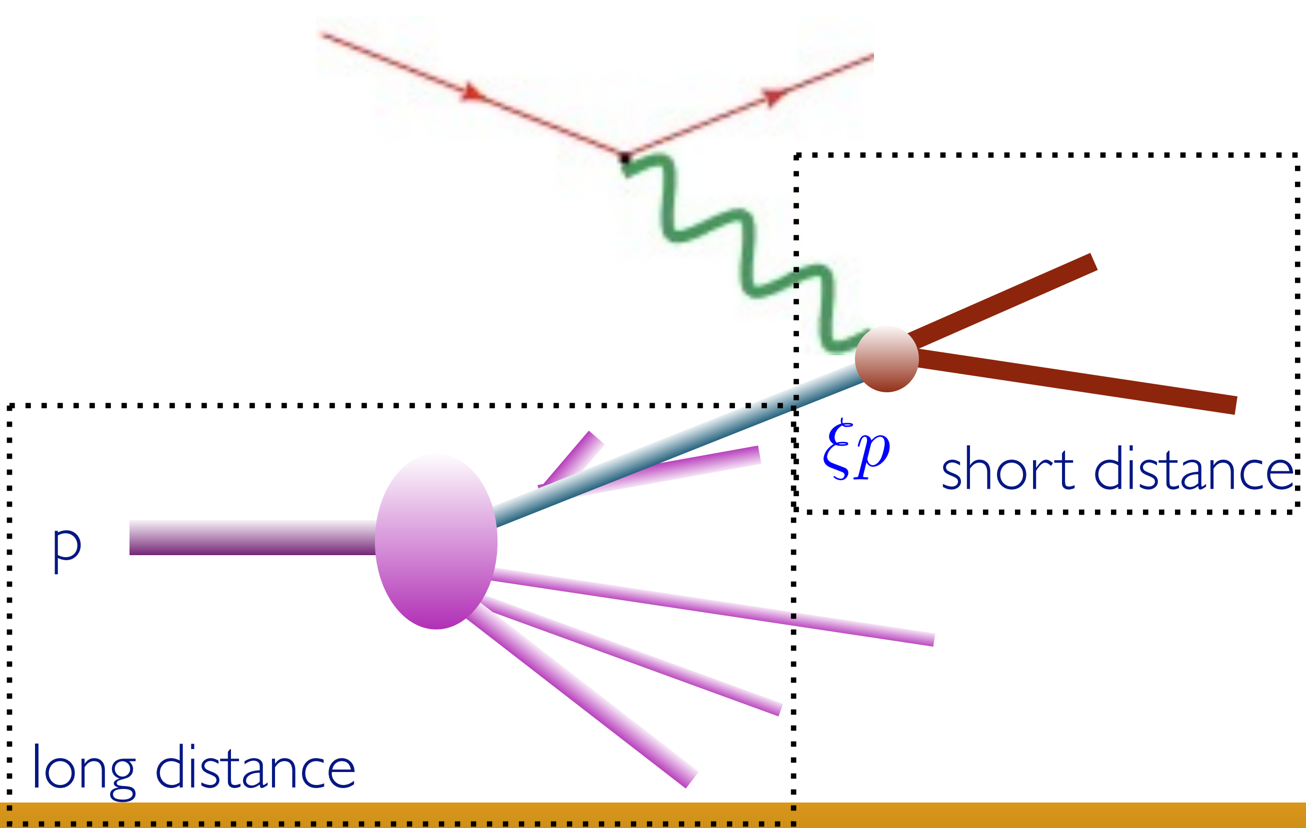

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Deep-inelastic scattering: towards the parton model

We can now compare with our "inclusive" description of DIS in terms of structure functions (which, BTW, are physical measurable quantities),

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{[1+(1-y)^2]F_1(x,Q^2) + \frac{1-y}{x}\left[F_2(x,Q^2) - 2xF_1(x,Q^2)\right]\right\}$$

with our parton model formulas:

$$\frac{d^2\sigma}{dxdQ^2} = \int_0^1 \frac{d\xi}{\xi}\sum_i f_i(\xi)\frac{d^2\sigma}{d\hat{x}dQ^2}(\frac{x}{\xi},Q^2) \quad \text{with} \quad \frac{d^2\hat{\sigma}}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\frac{1}{2}\left[1+(1-y)^2\right]e_q^2\,\delta(x-\xi)$$

we find (be careful to distinguish x and ξ)

$$F_2(x) = 2xF_1 = \sum_{i=q,\bar{q}}\int_0^1 d\xi\, f_i(\xi)\, xe_q^2\delta(x-\xi) = \sum_{i=q,\bar{q}} e_q^2\, x f_i(x)$$

* So we find the scaling is true: no dependence on Q^2.
* q and qbar enter together : no way to distinguish them with NC. Charged currents are needed.
* $F_L(x)$ = $F_2(x)$ - 2 $F_1(x)$ vanishes at LO (Callan-Gross relation), which is a test that quarks are spin 1/2 particles! In fact if the quarks where scalars we would have had $F_1(x)$ = 0 and F_2=F_L .

Deep-inelastic scattering: towards the parton model

Probed at scale Q, sea contains all quarks flavours with mq less than Q.
For Q ~ 1 we expect

$$\begin{aligned} u(x) &= u_V(x) + \bar{u}(x) \\ d(x) &= d_V(x) + \bar{d}(x) \\ s(x) &= \bar{s}(x) \end{aligned}$$

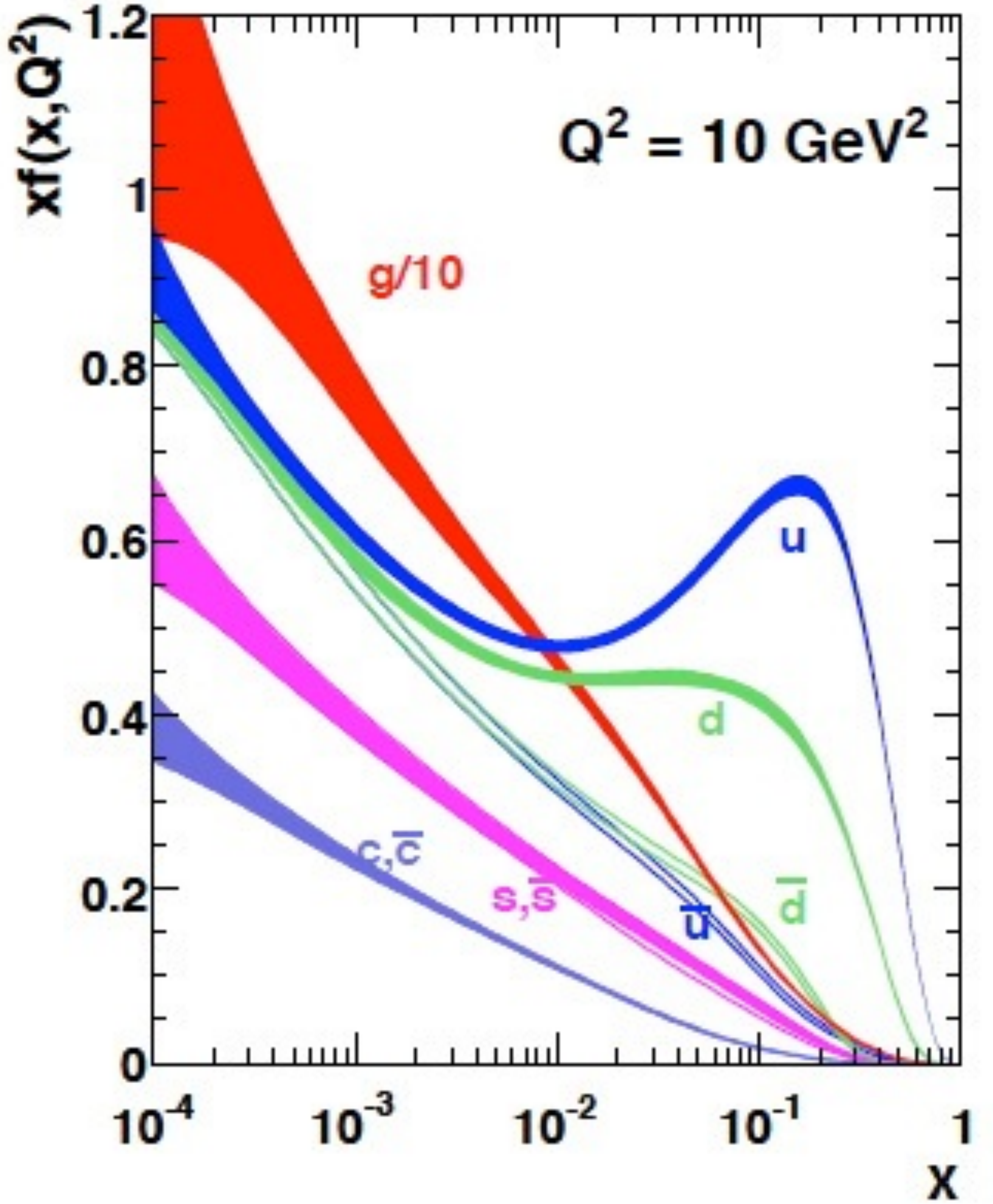
$$\int_0^1 dx\, u_V(x) = 2\,, \quad \int_0^1 dx\, d_V(x) = 1\,.$$

And experimentally one finds

$$\sum_q \int_0^1 dx\, x[q(x) + \bar{q}(x)] \simeq 0.5\,.$$

Thus quarks carry only about 50% of proton's momentum. The rest is carried by gluons. Although not directly measured in DIS, gluons participate in other hard scattering processes such as large-pt and prompt photon production.

Quark and gluon distribution functions



Comments:

The sea is NOT SU(3) flavor symmetric.

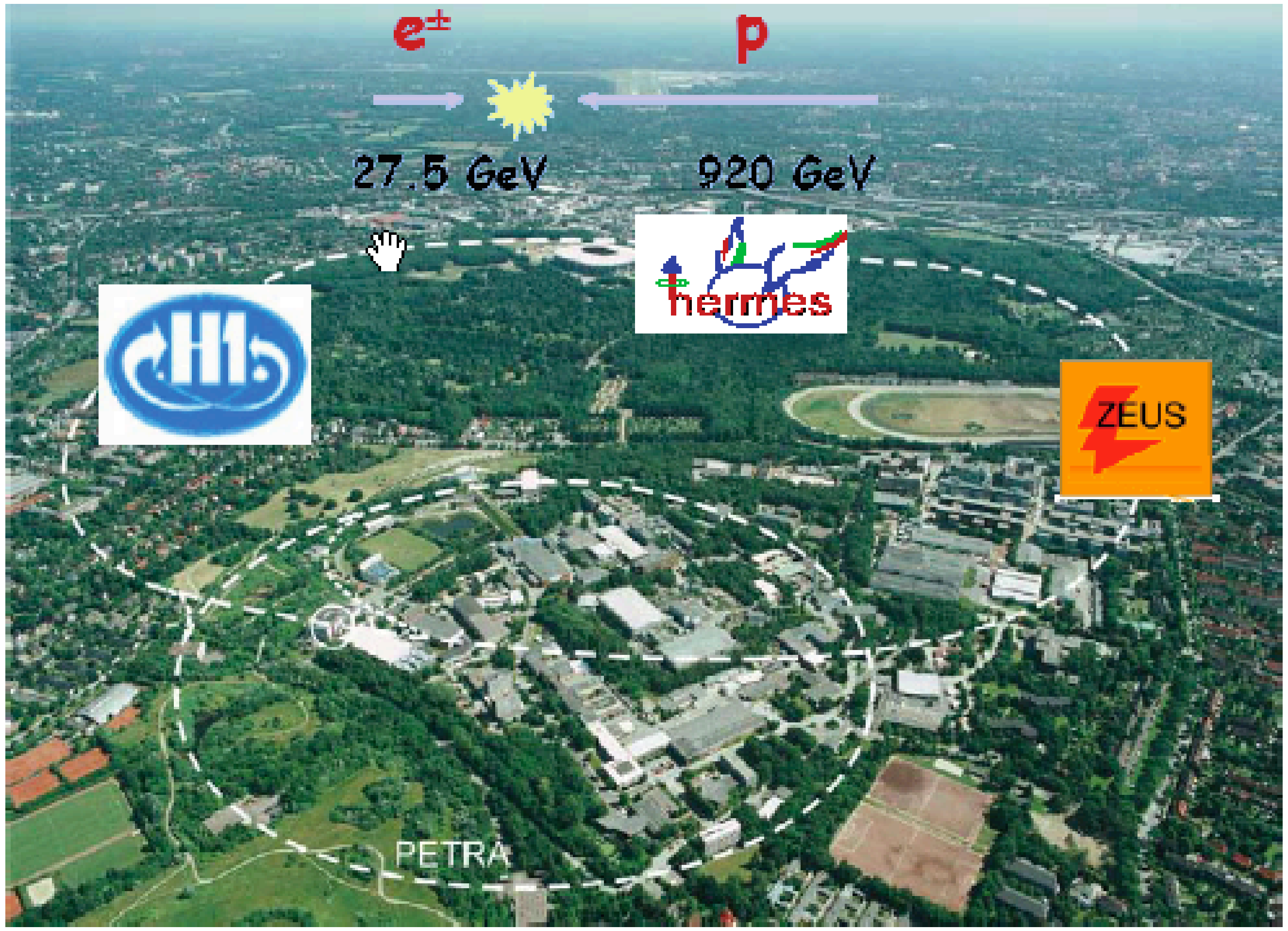
The gluon is huge at small x

There is an asymmetry between the ubar and dbar quarks in the sea.

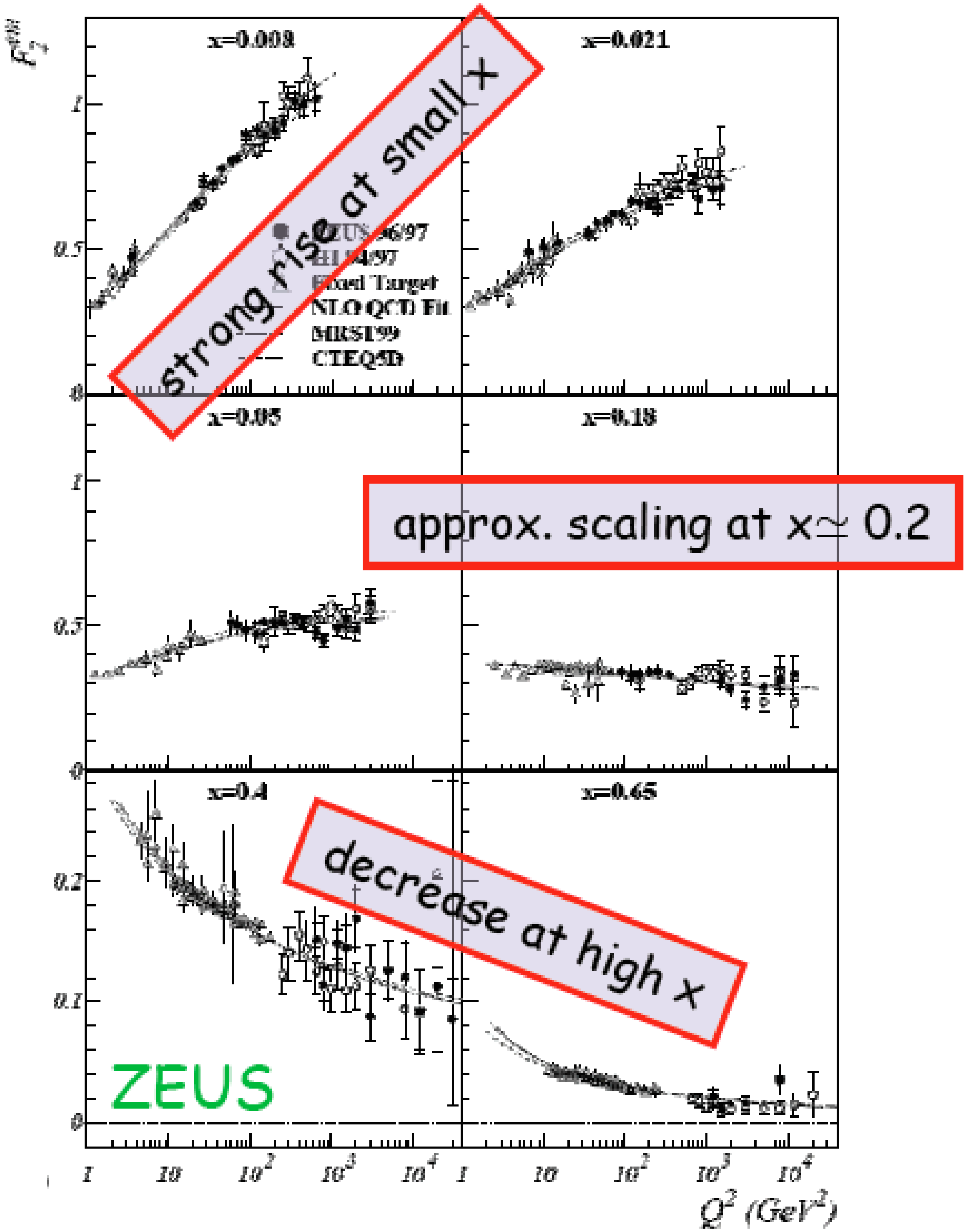
Note that there are uncertainty bands!!

Scaling violations

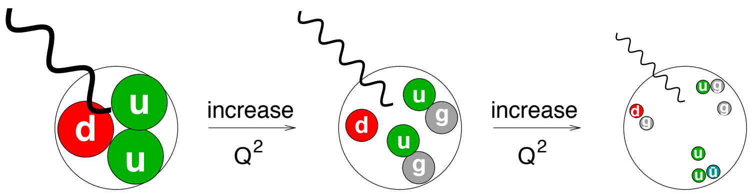
first ep collider



At HERA scaling violations were observed!



EVOLUTION



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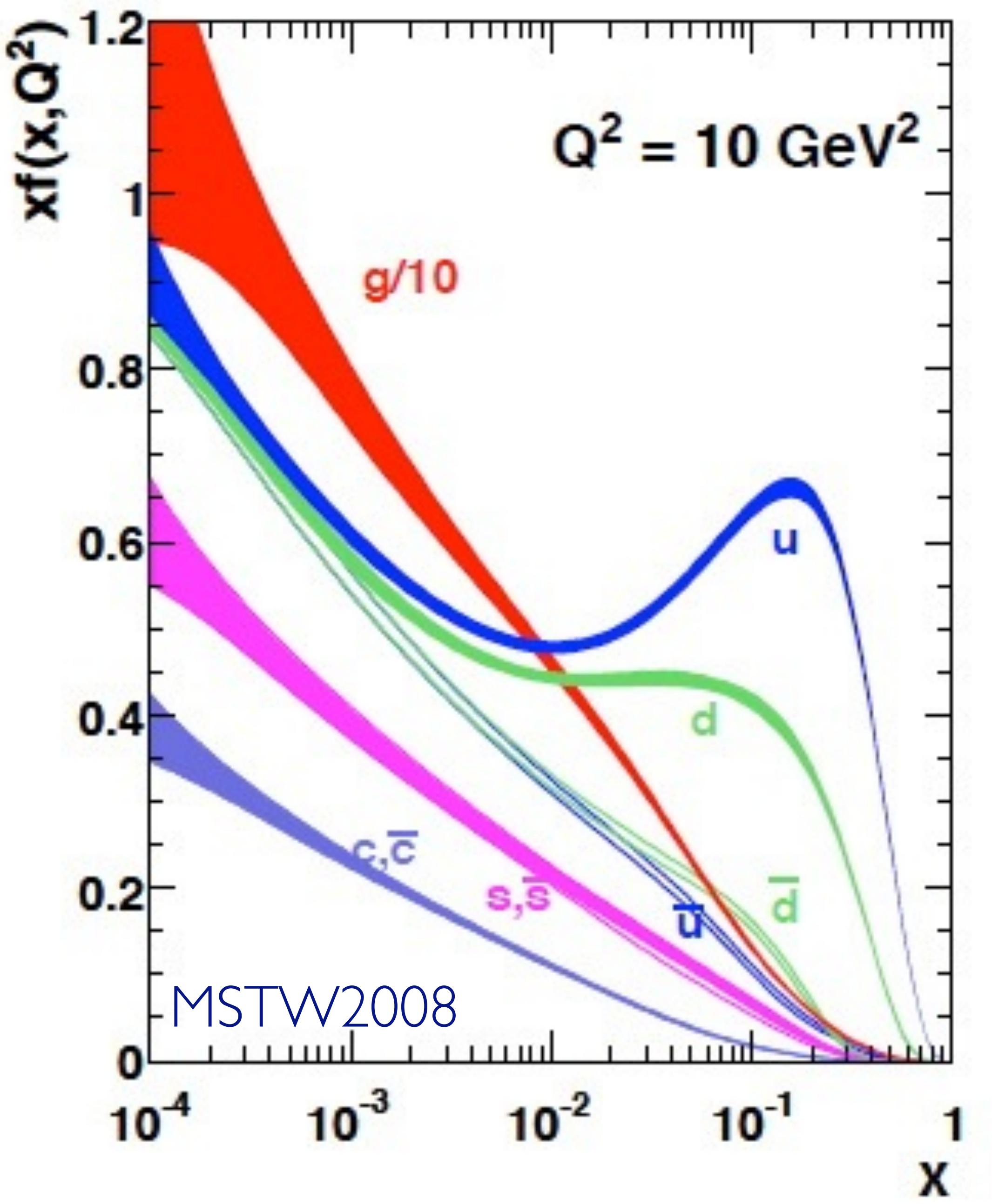
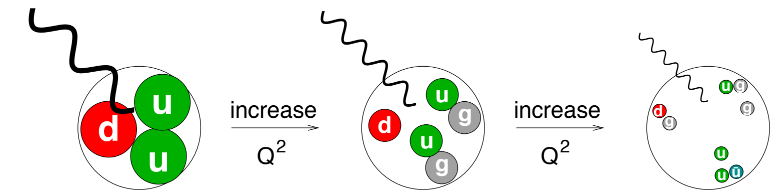


$xf(x,Q^2)$
$Q^2 = 10\ \mathrm{GeV}^2$
g/10
u
d
$\bar{d}$
$\bar{u}$
$s,\bar{s}$
$c,\bar{c}$
MSTW2008
1.2
1
0.8
0.6
0.4
0.2
0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

EVOLUTION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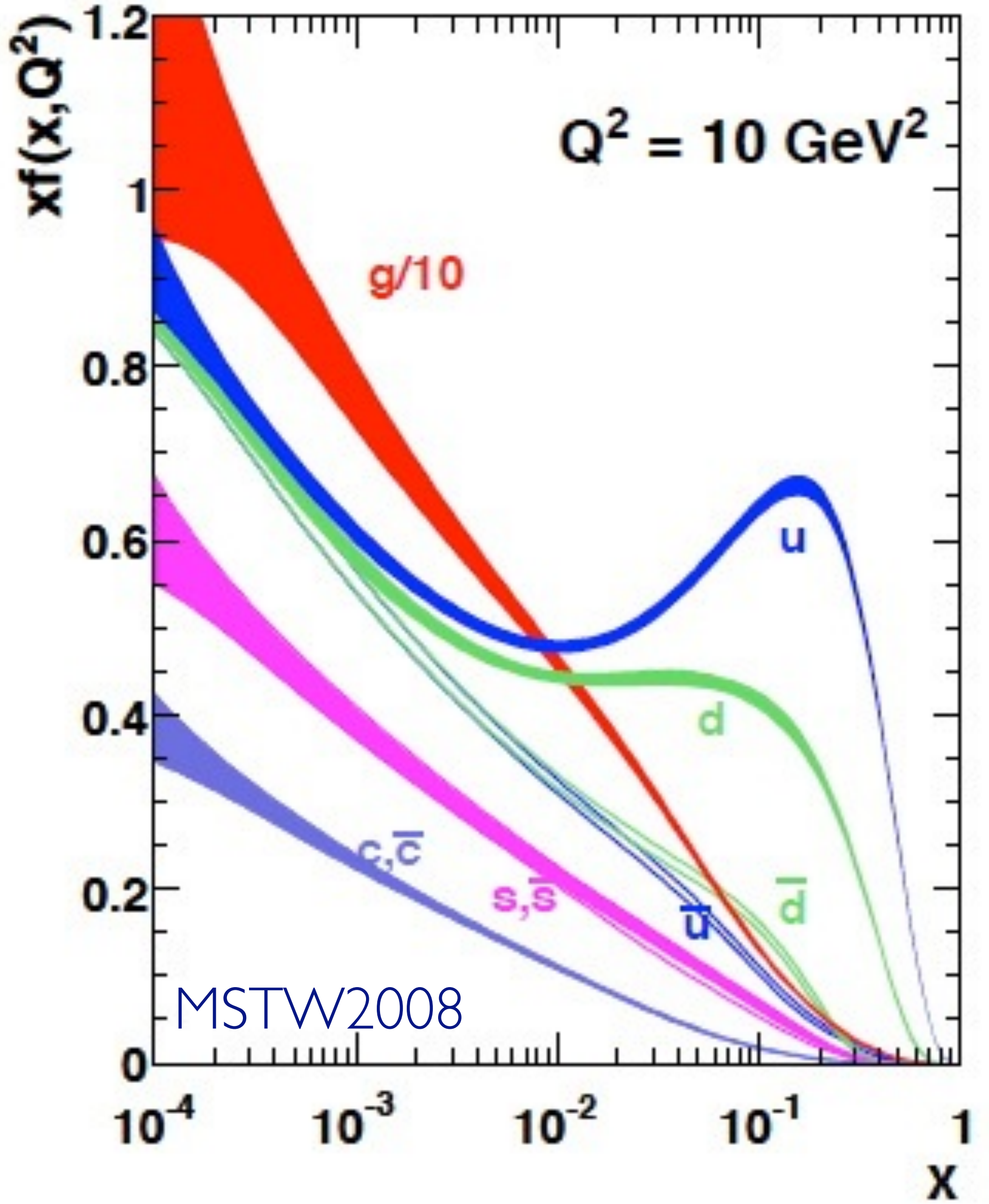
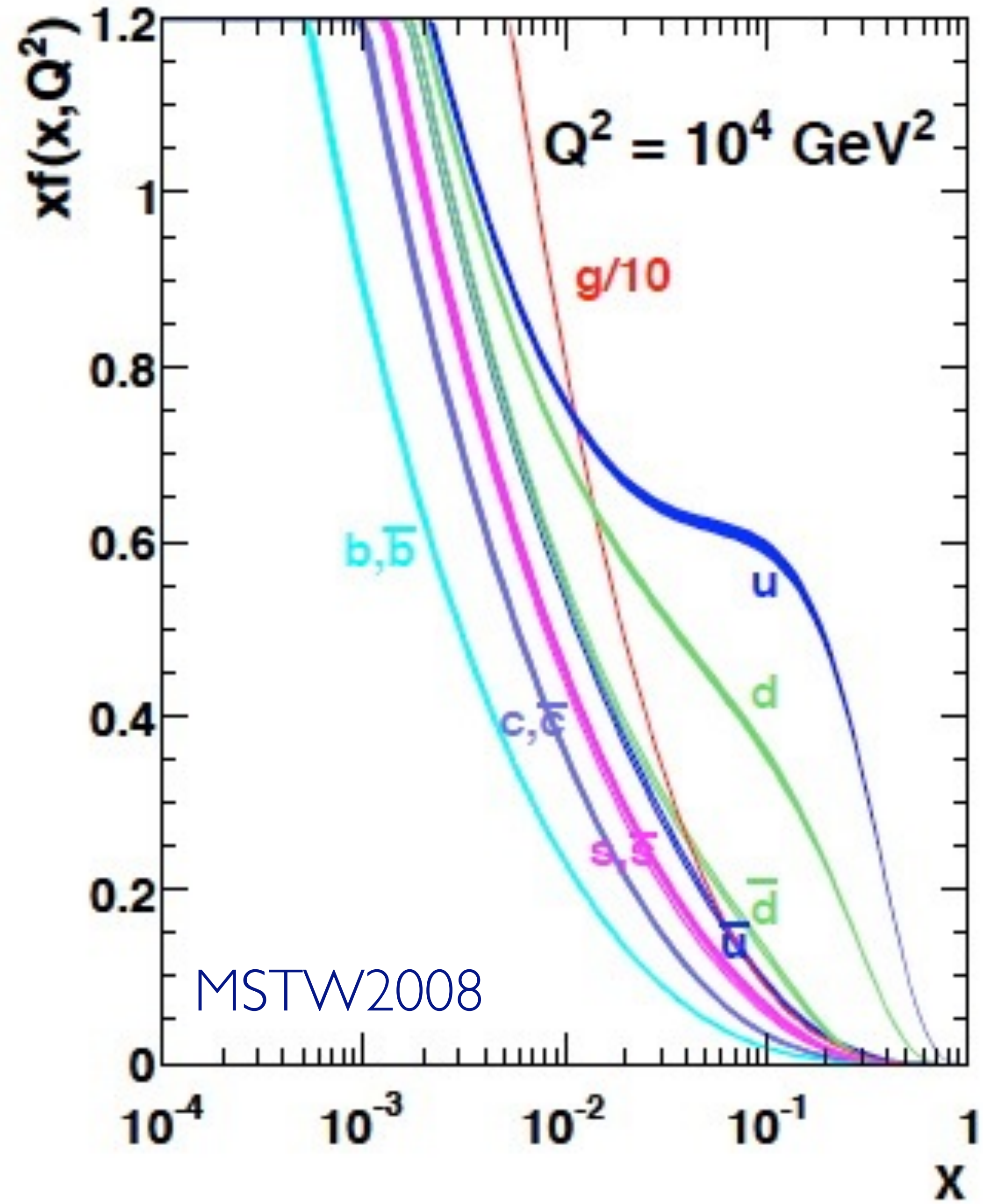
increase
Q^2


$xf(x,Q^2)$
$Q^2 = 10\ \mathrm{GeV}^2$
g/10
u
d
$\bar{d}$
$\bar{u}$
$s,\bar{s}$
$c,\bar{c}$
MSTW2008
0
0.2
0.4
0.6
0.8
1
1.2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x


$xf(x,Q^2)$
$Q^2 = 10^4\ \mathrm{GeV}^2$
g/10
$b,\bar{b}$
u
d
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$s,\bar{s}$
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MSTW2008
0
0.2
0.4
0.6
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1.2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

PDFs

NON-PERTURBATIVE INFORMATION THAT IS FITTED FROM A WEALTH OF EXPERIMENTAL DATA

- THE PDF IS PARAMETRISED AT A GIVEN LOW SCALE IN TERMS OF AN ANALYTIC OR NN FUNCTION AND MOMENTUM SUM RULES ARE IMPOSED.
- THEY ARE EVOLVED THROUGH THE DGLAP EQUATIONS:

$$Q^2 \frac{\partial f_a(x, Q^2)}{\partial Q^2} = \int_x^1 \frac{dz}{z} P_{ab}(\alpha_{\mathrm{S}}(Q^2), z) f_b(x/z, Q^2)$$

$$P_{ab}(\alpha_S, z) = \underbrace{\frac{\alpha_S}{2\pi} P_{ab}^{(0)}(z)}_{\text{LO (1974)}} + \underbrace{\left(\frac{\alpha_S}{2\pi}\right)^2 P_{ab}^{(1)}(z)}_{\text{NLO (1980)}} + \underbrace{\left(\frac{\alpha_S}{2\pi}\right)^3 P_{ab}^{(2)}(z)}_{\text{NNLO (2004)}} + \ldots..$$

PDFs

GLOBAL FITS: RECENT PROGRESS IN METHODOLOGY AND DATA SETS:

- NNPDF3.0 1410.8849
- MMHTCT14 1412.3989
- CT14 1506.07443

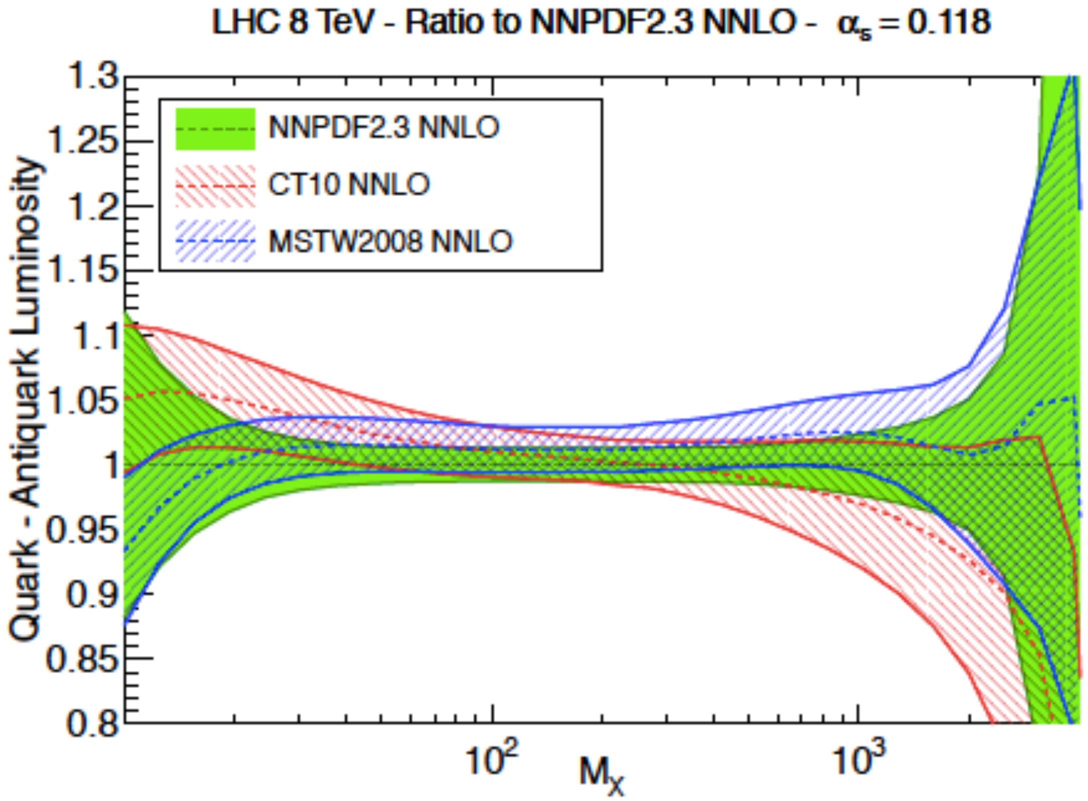
StefanoForte®

	NNPDF3.0	MMHT14	CT14
NO. OF FITTED PDFS	7	7	6
PARAMETRIZATION	NEURAL NETS	$x^a(1-x)^b \times$ CHEBYSCHEV	$x^a(1-x)^b \times$ BERNSTEIN
FREE PARAMETERS	259	37	30-35
UNCERTAINTIES	REPLICAS	HESSIAN	HESSIAN
TOLERANCE	NONE	DYNAMICAL	DYNAMICAL
CLOSURE TEST	✔	✗	✗
REWEIGHTING	REPLICAS	EIGENVECTORS	EIGENVECTORS

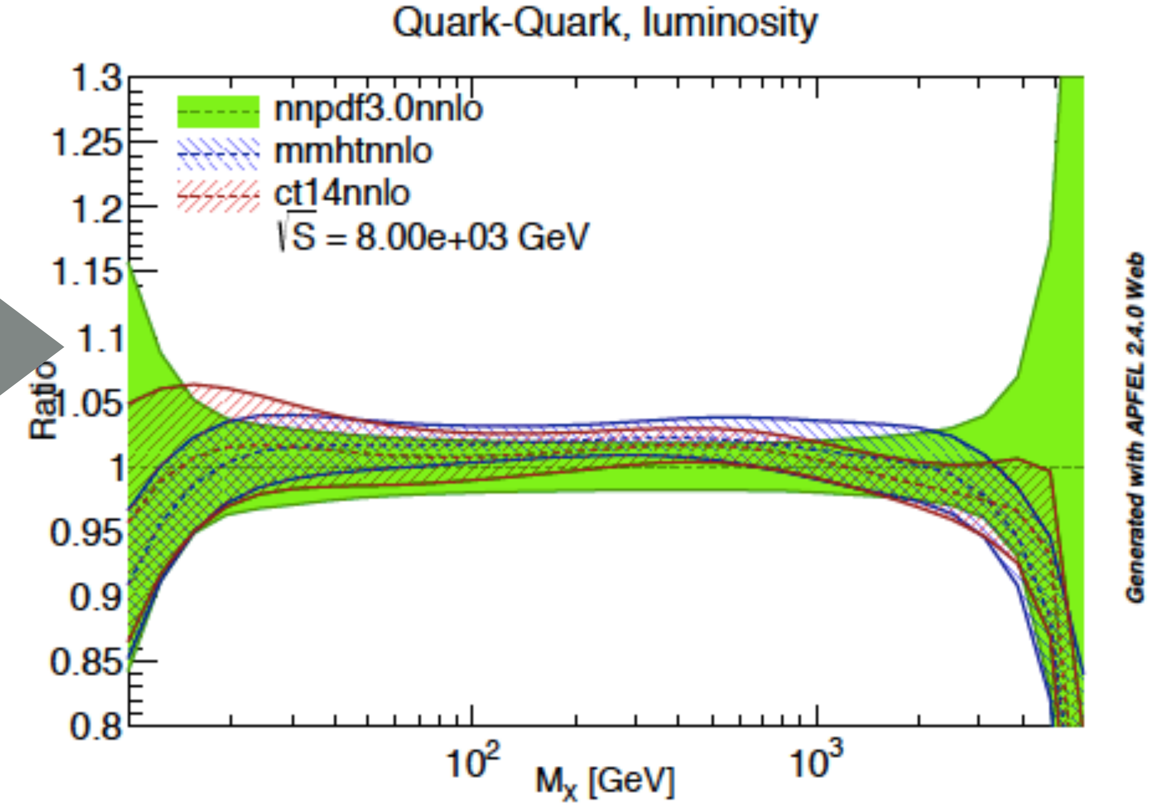
Other non-global sets: HeraPDF, ABM14, GJR

QUARK-QUARK

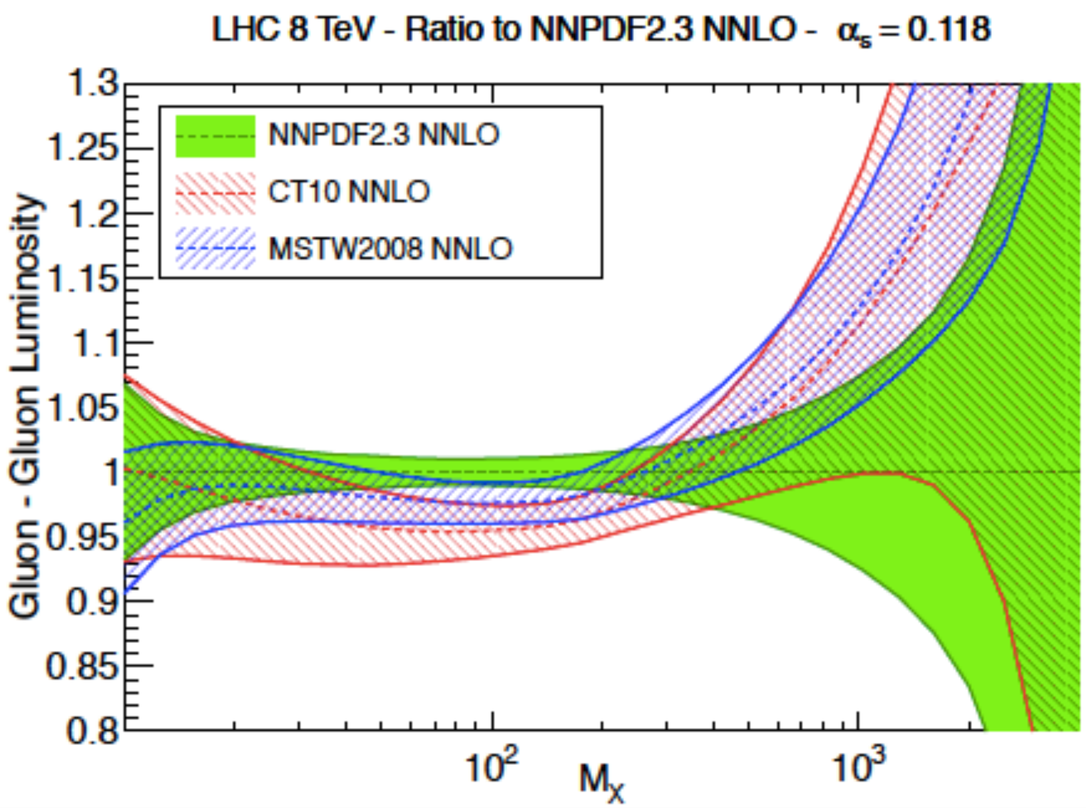
2012

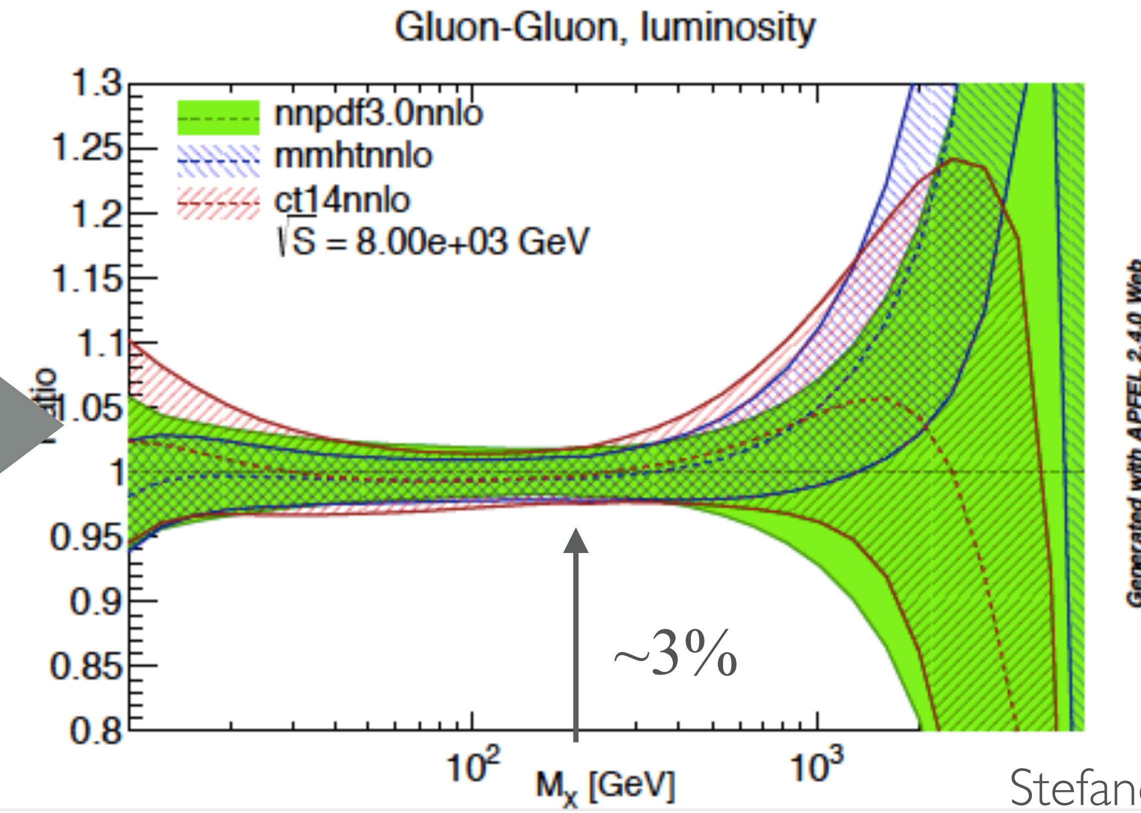


2015



GLUON-GLUON





StefanoForte

Asymptotic freedom

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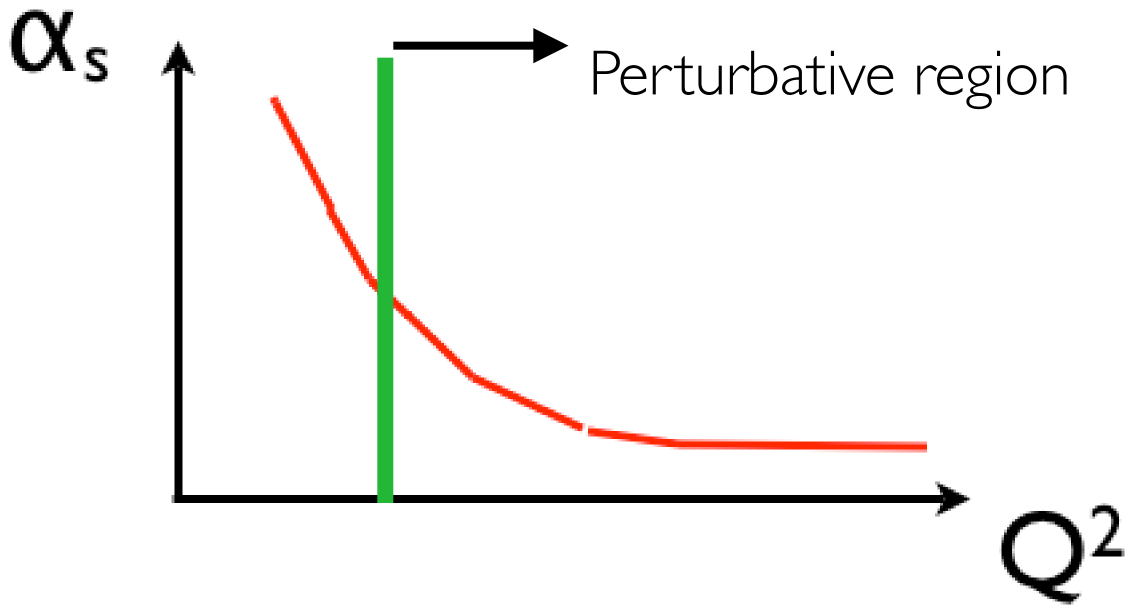

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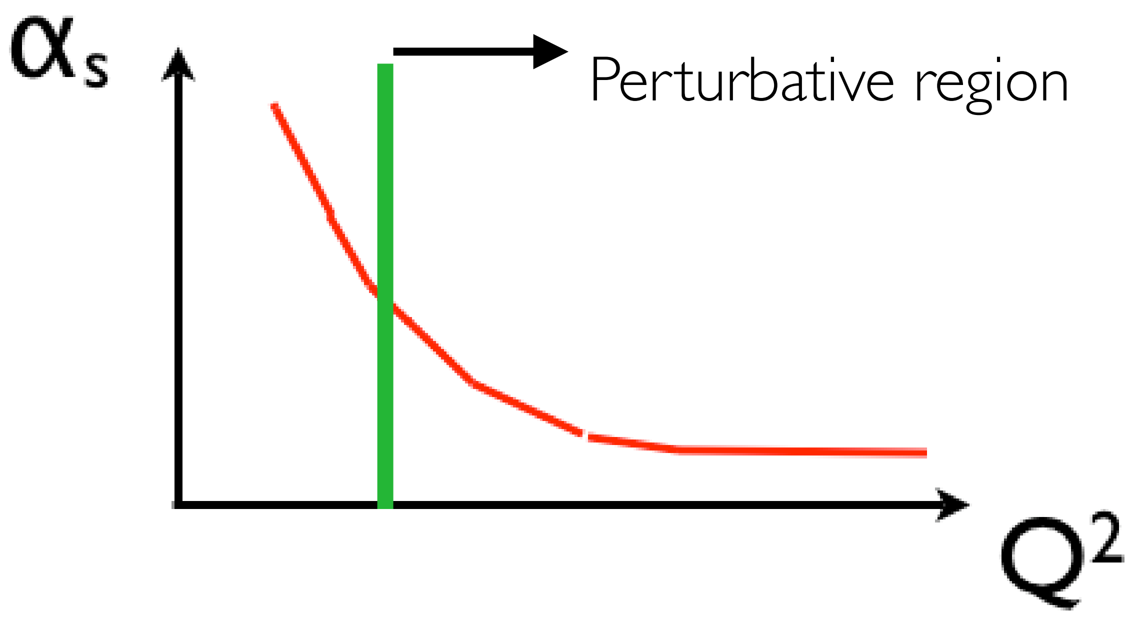

In renormalizable QFT's scale invariance is broken by the renormalization procedure and couplings depend logarithmically on scales.

The QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_f \bar{\psi}_i^{(f)} (i \not{\partial} - m_f) \psi_i^{(f)} - \bar{\psi}_i^{(f)} (g_s t^a_{ij} \not{A}_a) \psi_j^{(f)}$$

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Gauge Fields

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Gauge Fields

Matter

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Gauge Fields

Matter

Interaction

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$$[t^a, t^b] = i f^{abc} t^c$$ → Algebra of SU(N)

$$\mathrm{tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$$ → Normalization

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$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

How many colors?

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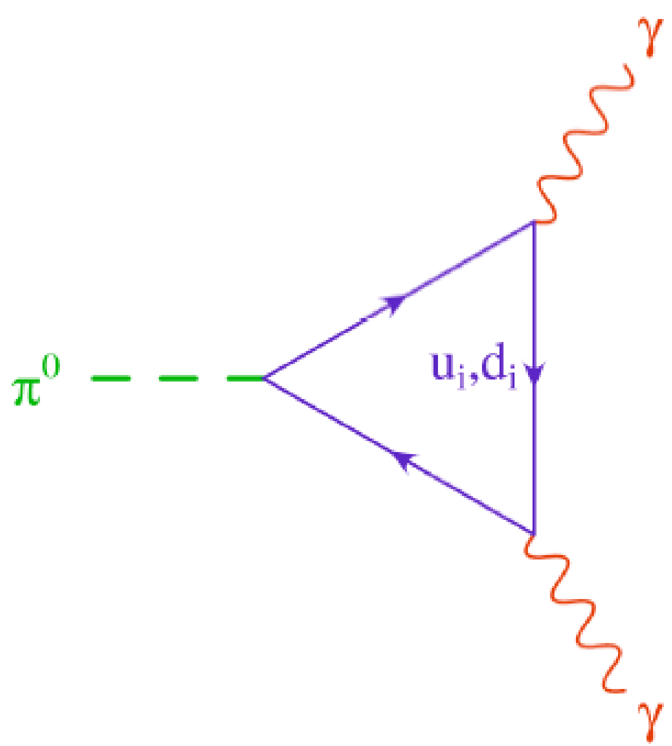

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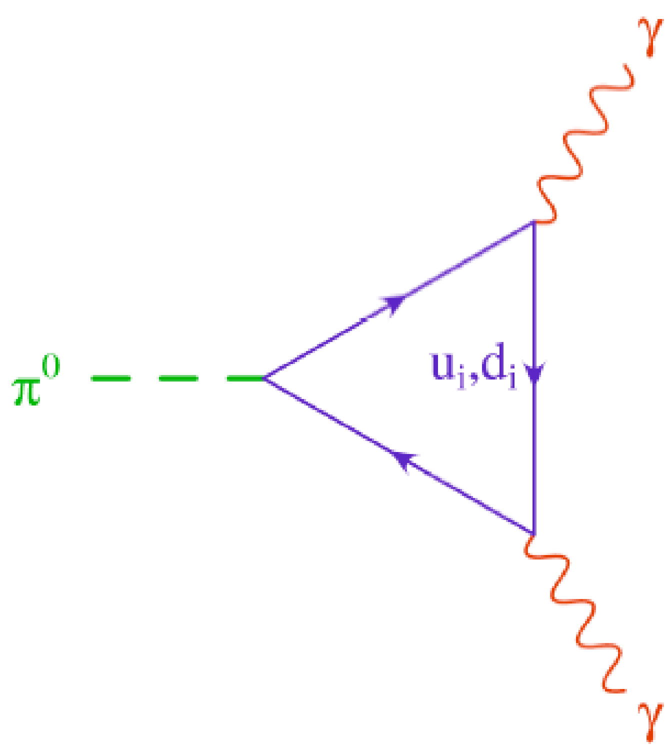

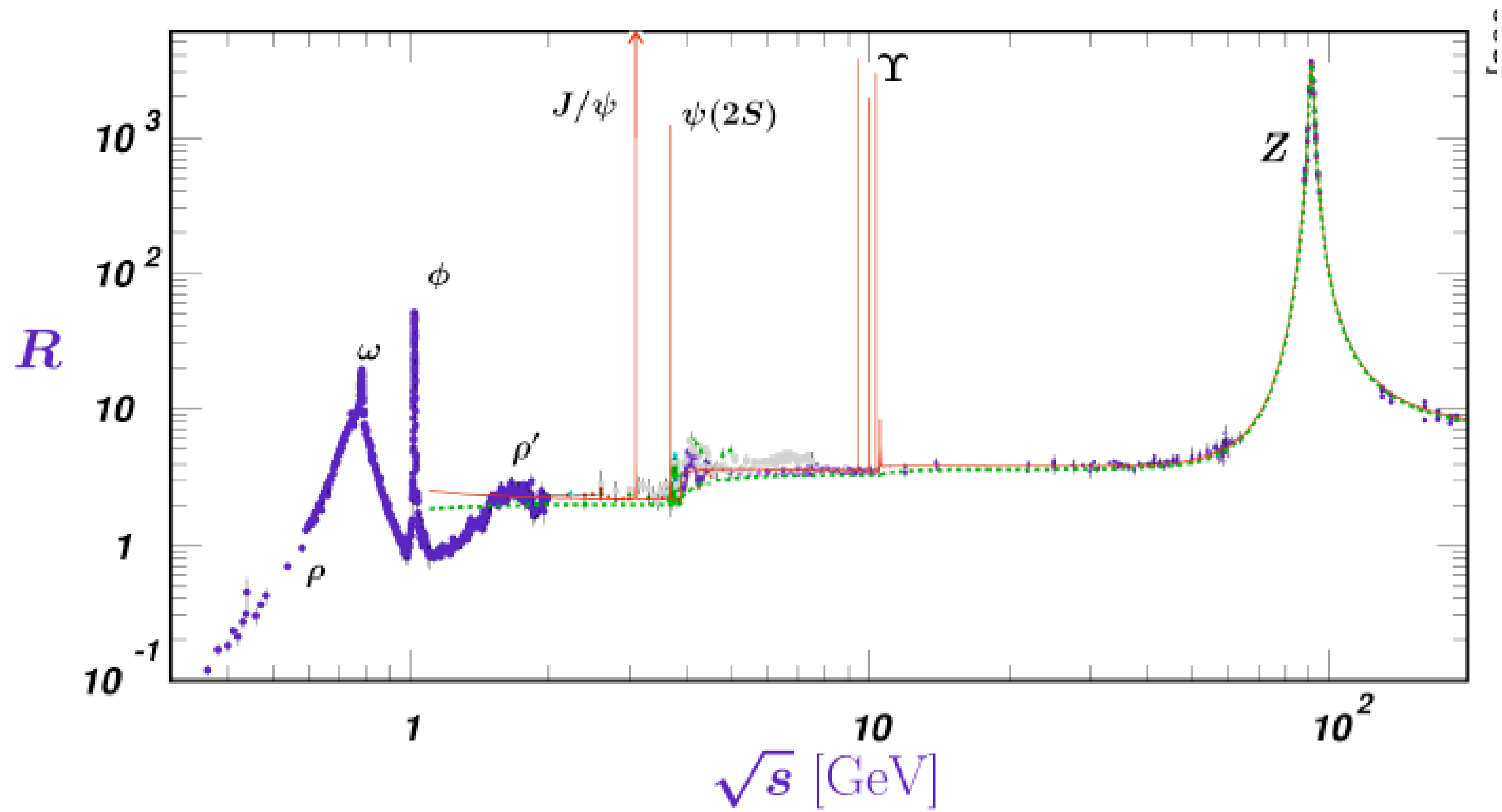

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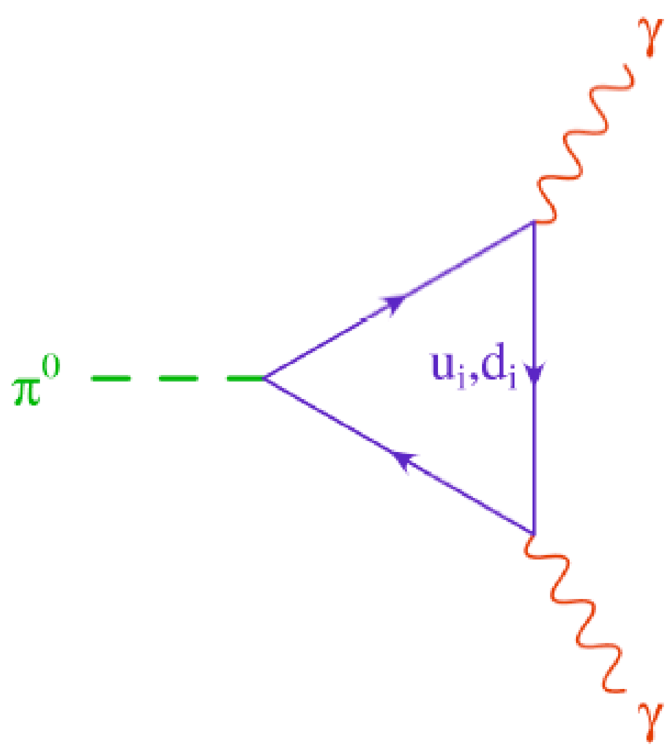

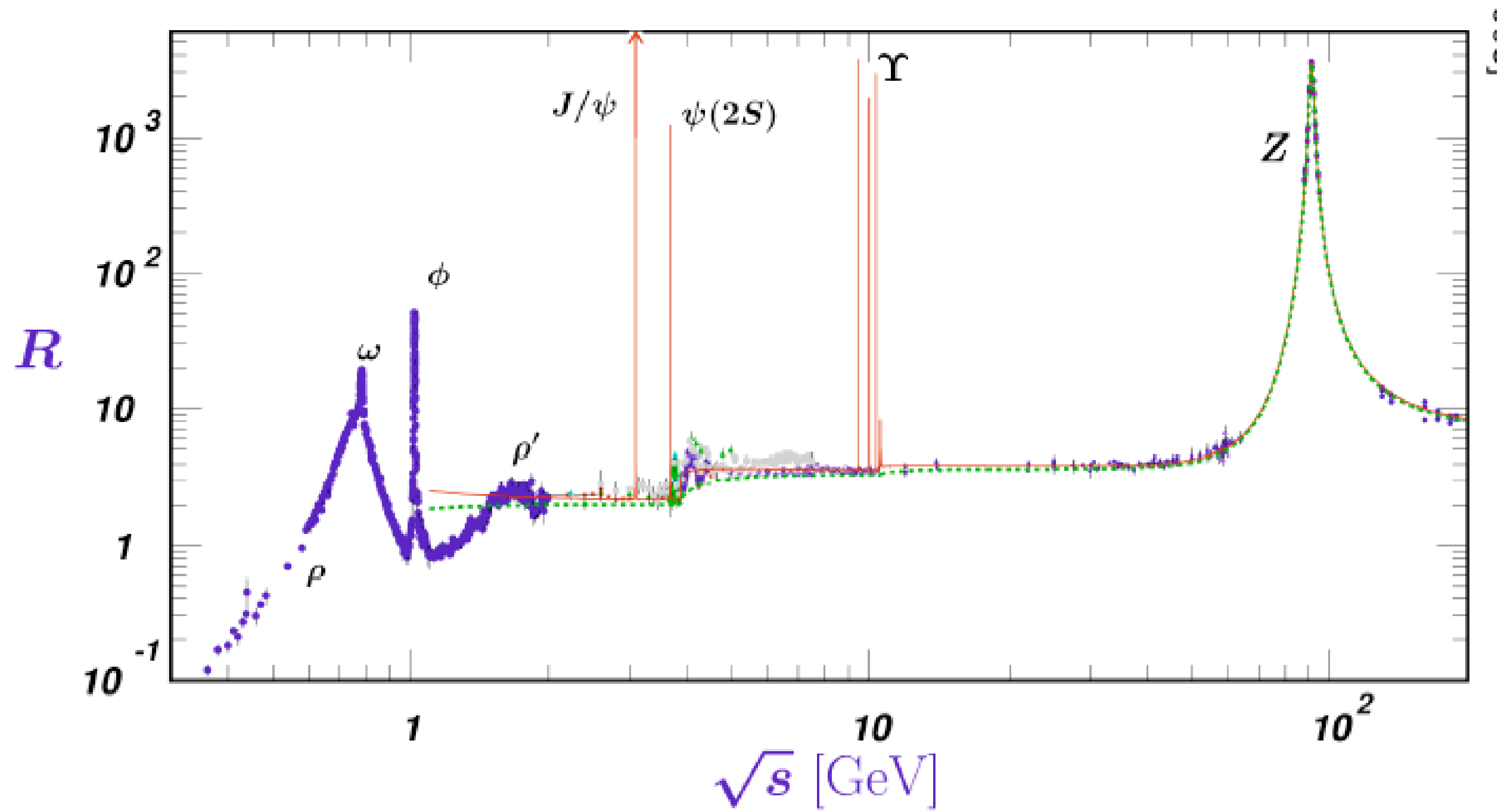

$$\Gamma \sim N_c^2 \left[Q_u^2 - Q_d^2\right]^2 \frac{m_\pi^3}{f_\pi^2}$$

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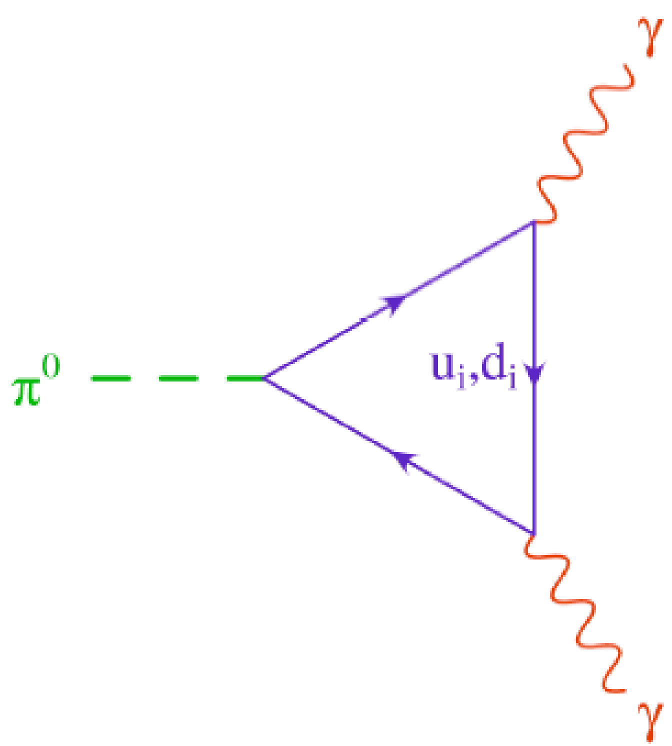

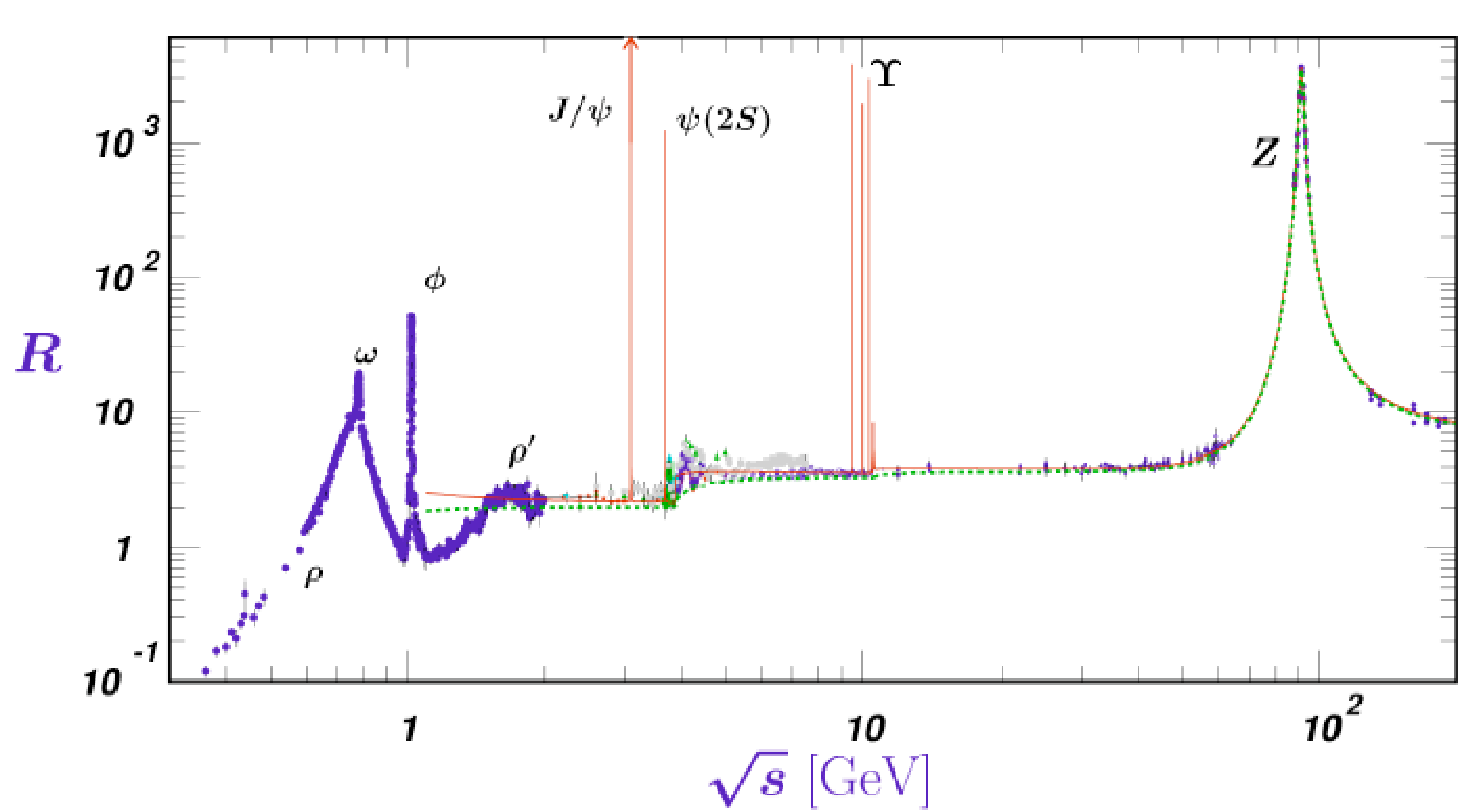

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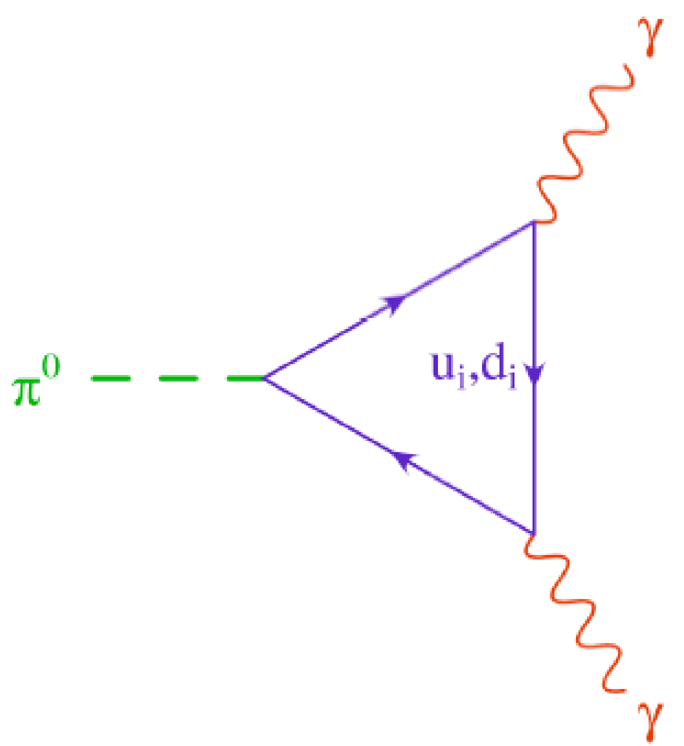

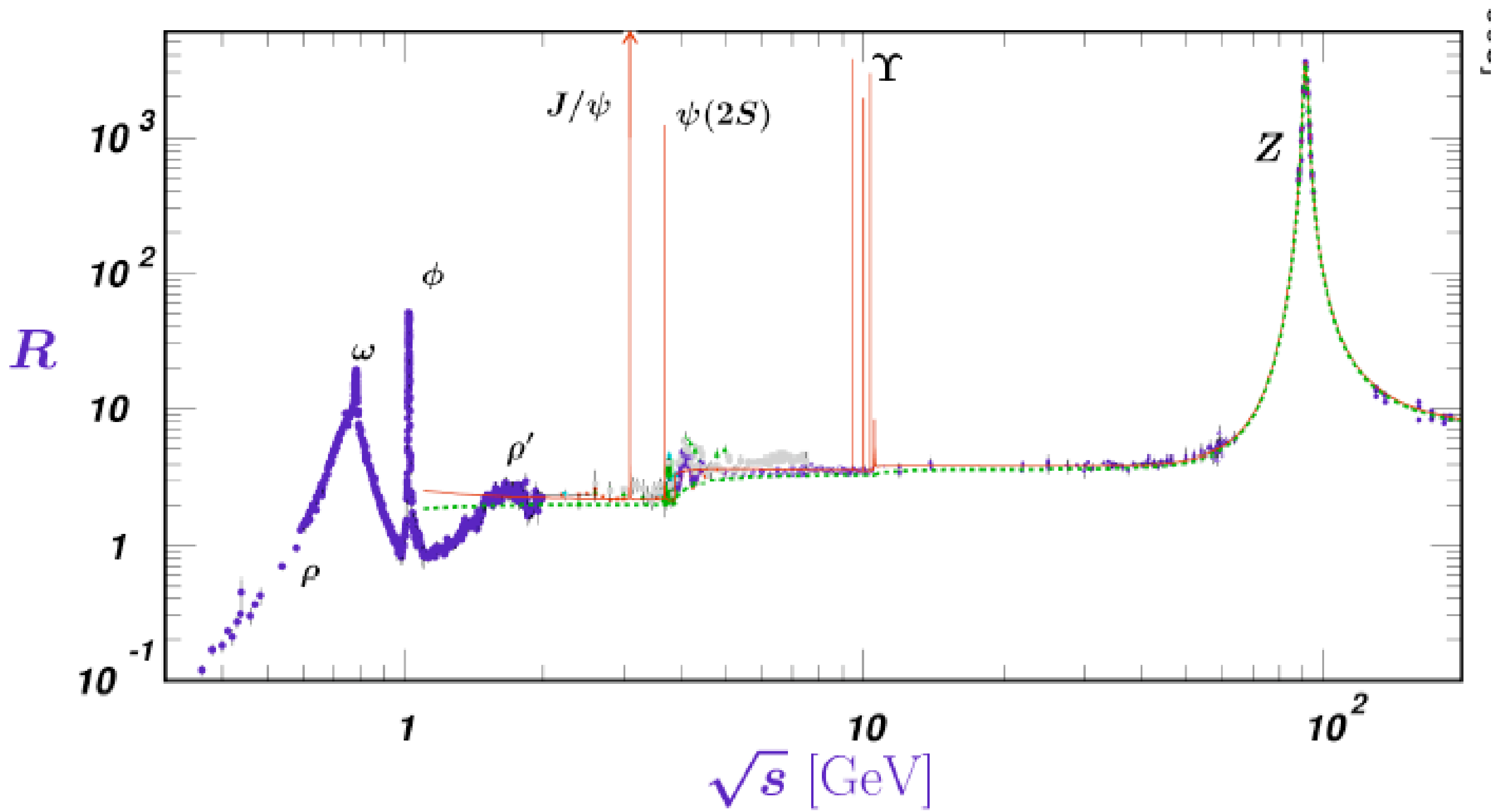

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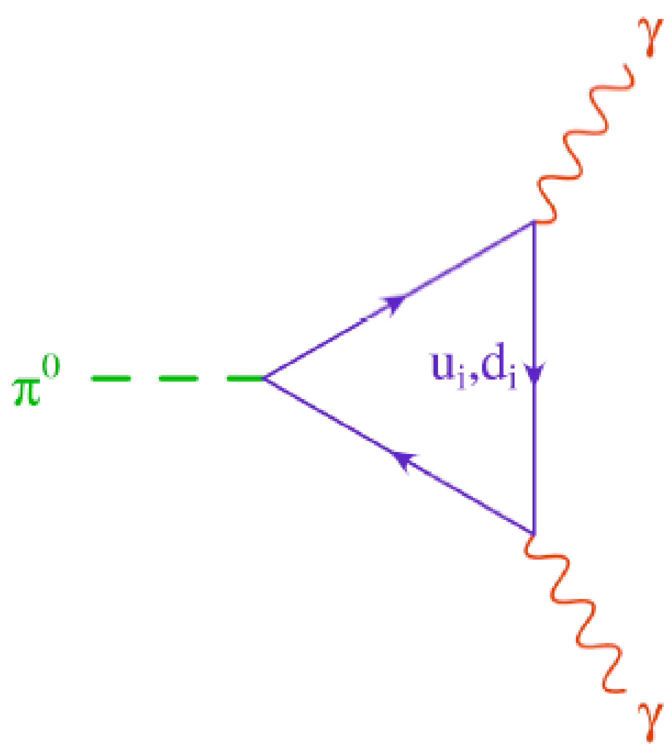

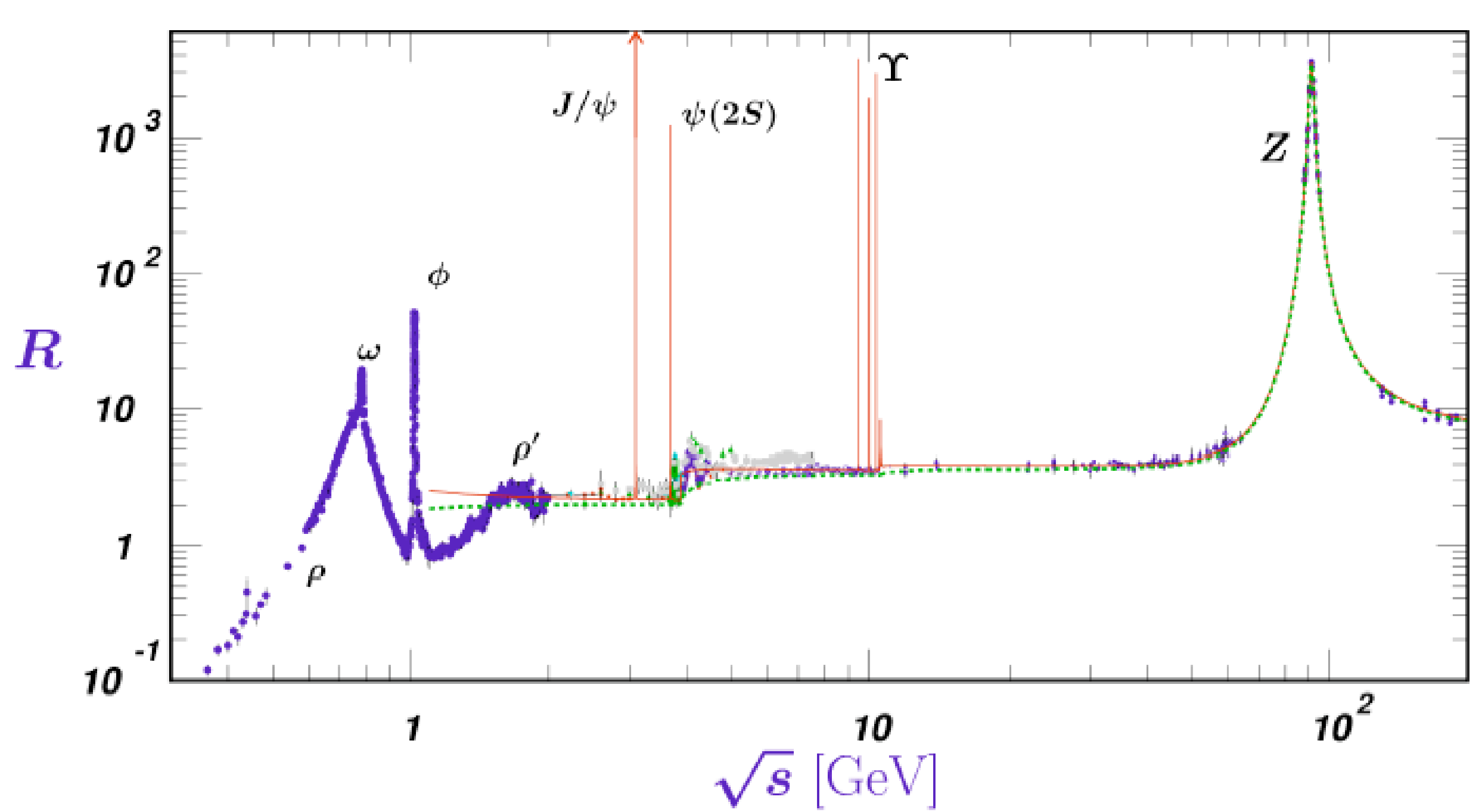

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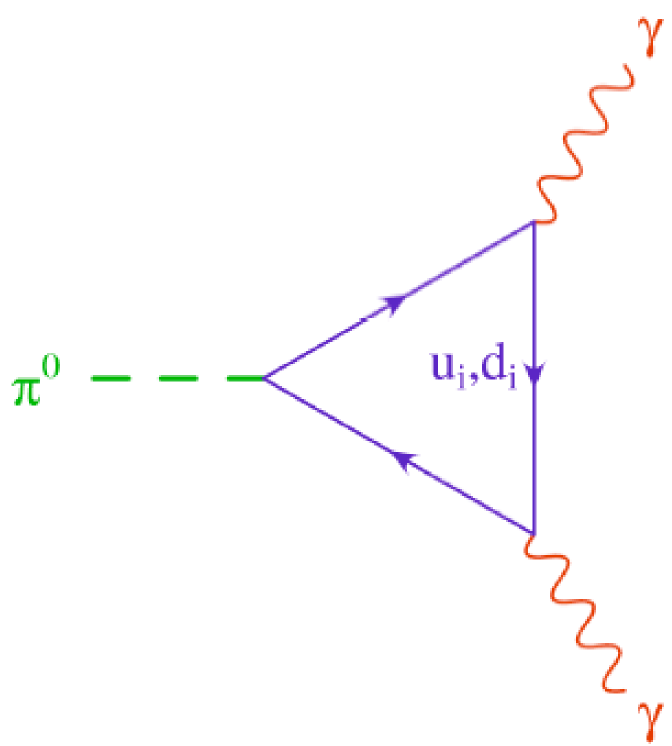

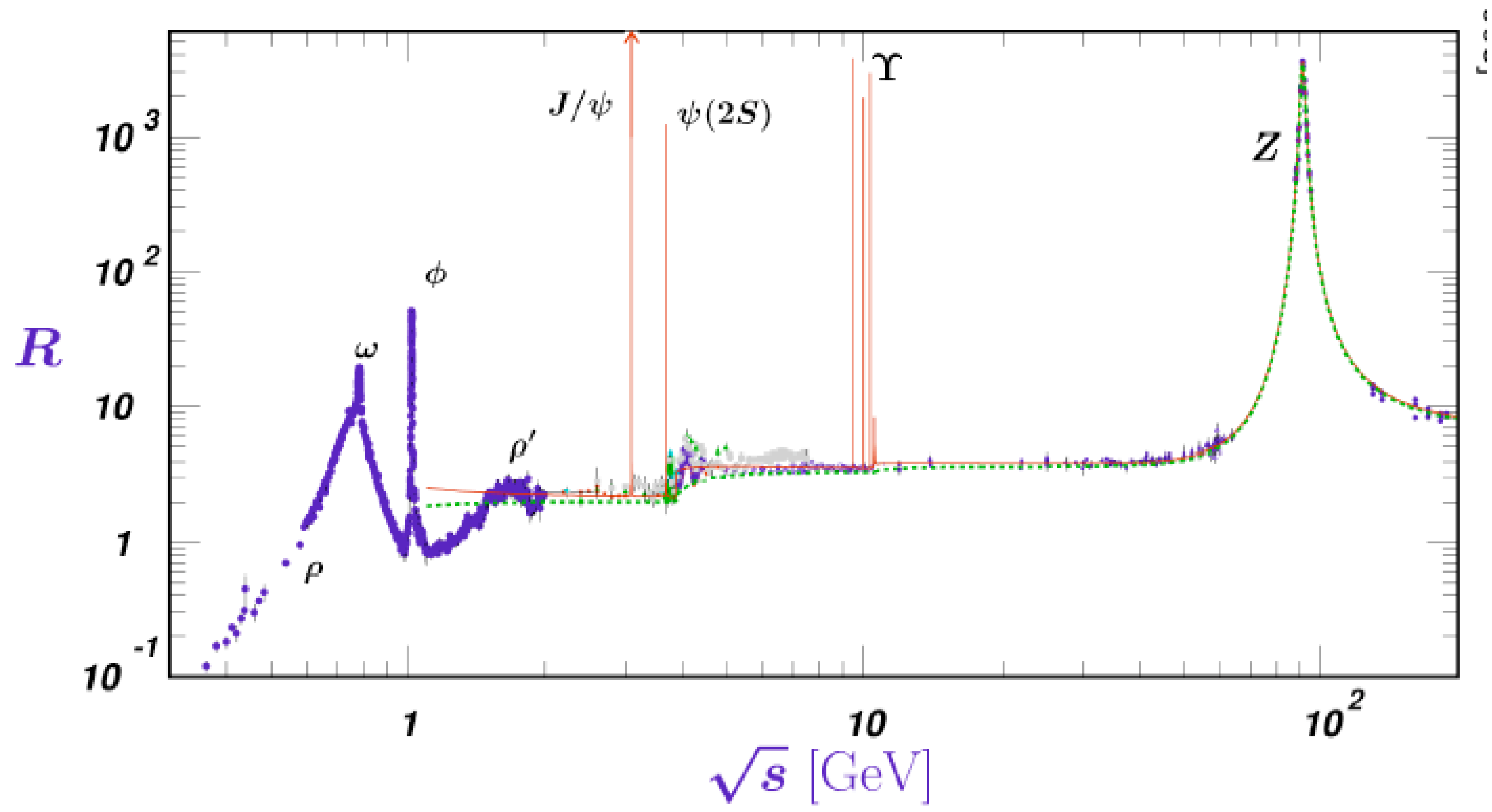

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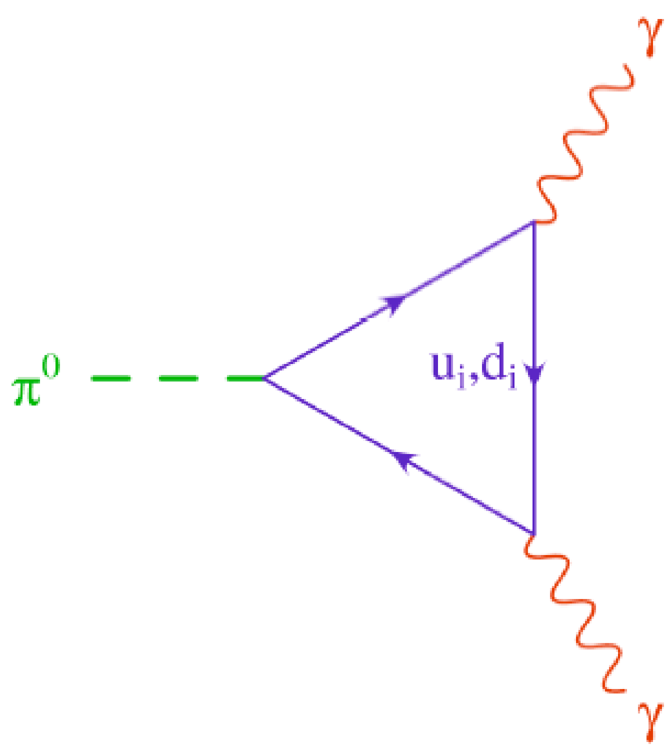

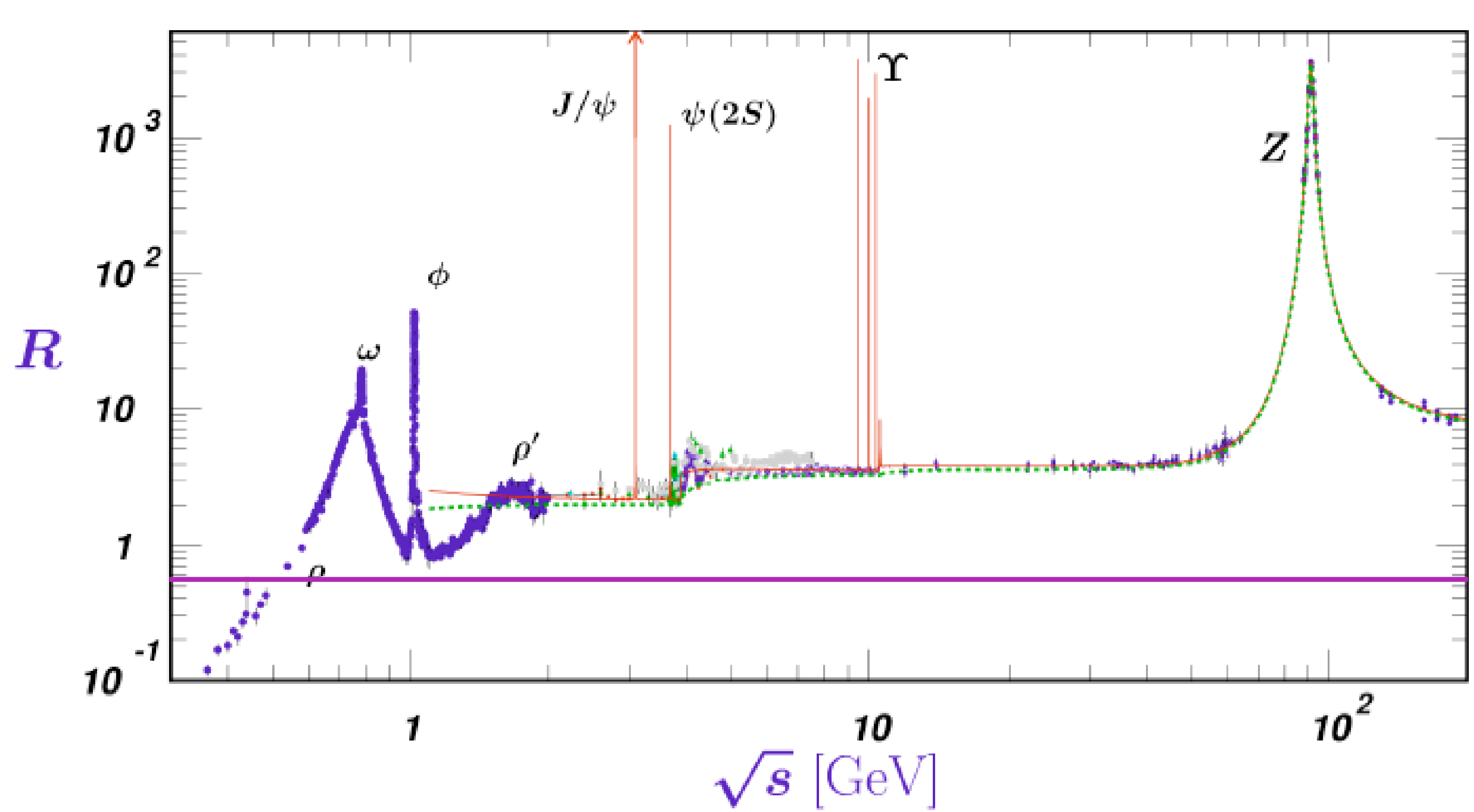

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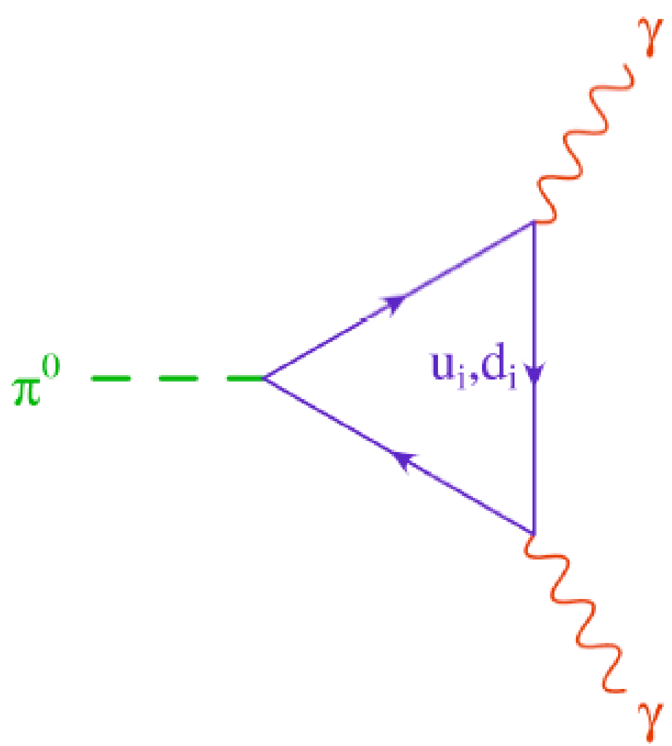

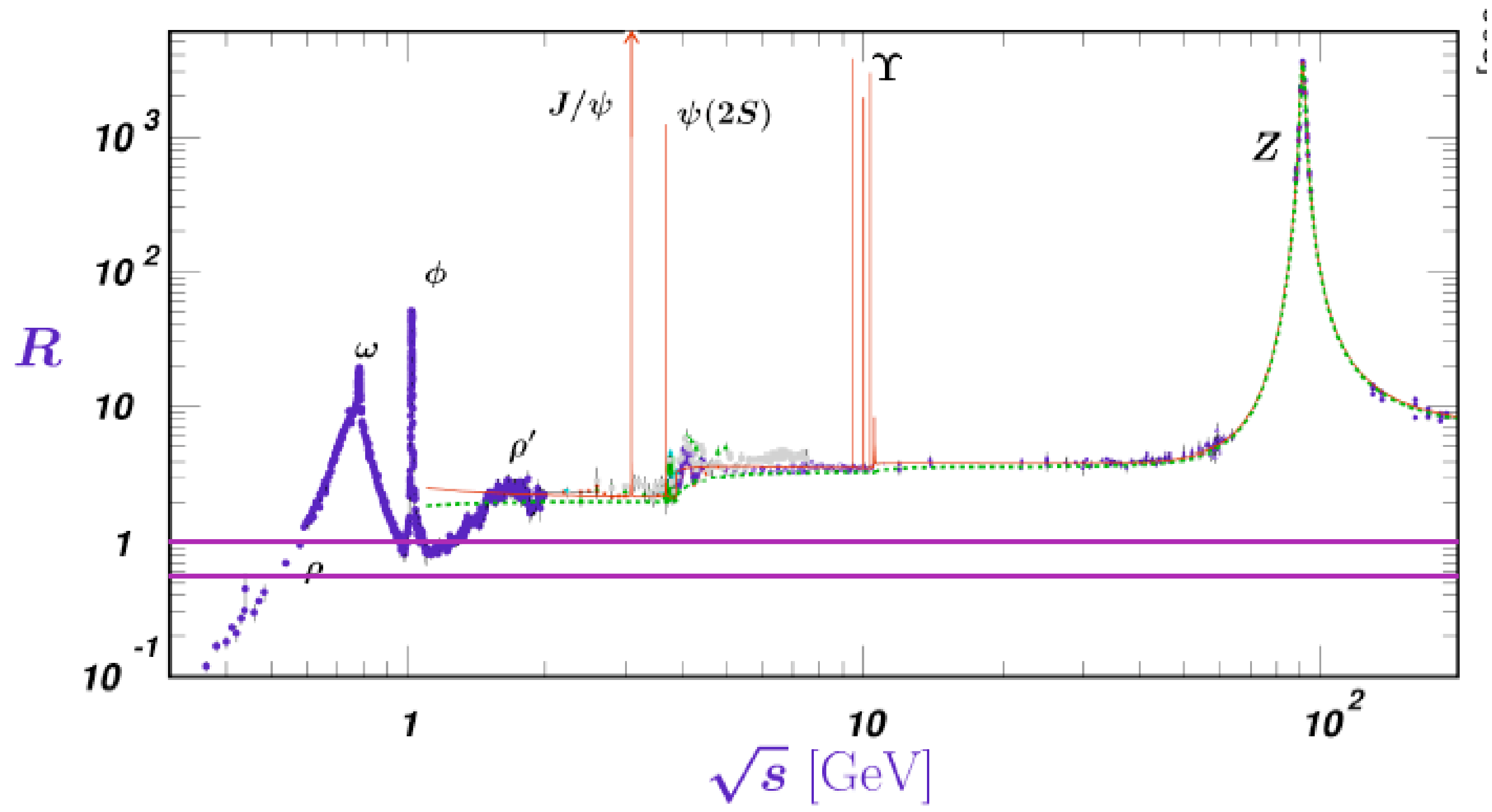

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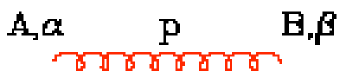
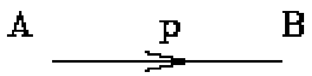
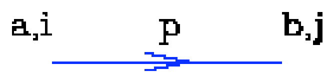
THE FEYNMAN RULES OF QCD



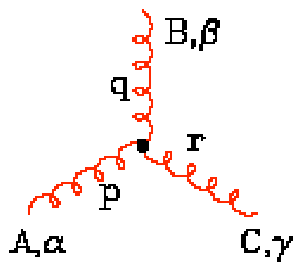
$$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^{\alpha}p^{\beta}}{p^{2}+i\epsilon}\right]\frac{i}{p^{2}+i\epsilon}$$



$$\delta^{AB}\,\frac{i}{(p^{2}+i\epsilon)}$$

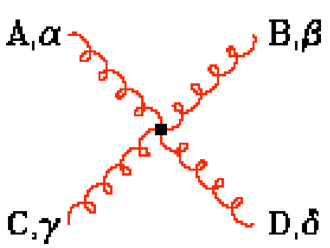


$$\delta^{ab}\,\frac{i}{(\not{p}-m+i\epsilon)_{ji}}$$

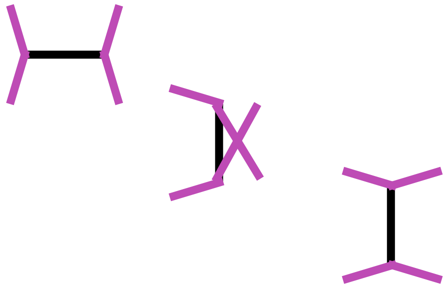


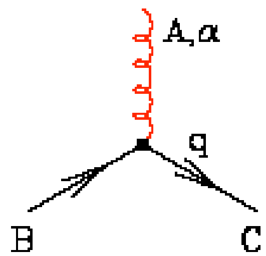
$$-g\;f^{ABC}[(p-q)^{\gamma}g^{\alpha\beta}+(q-r)^{\alpha}g^{\beta\gamma}+(r-p)^{\beta}g^{\gamma\alpha}]$$

(all momenta incoming)

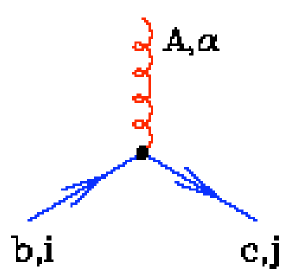


$$-ig^{2}\;f^{XAC}f^{XBD}\;[g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$
$$-ig^{2}\;f^{XAD}f^{XBC}\;[g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$$
$$-ig^{2}\;f^{XAB}f^{XCD}\;[g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$





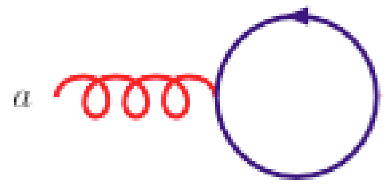
$$g\;f^{ABC}q^{\alpha}$$



$$-ig\;(t^{A})_{cb}\;(\gamma^{\alpha})_{ji}$$

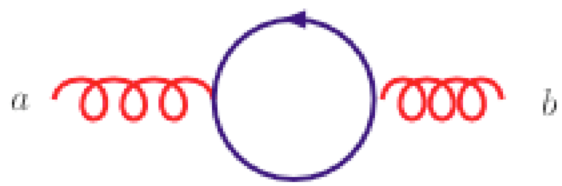
THE COLOR ALGEBRA

$$\mathrm{Tr}(t^a) = 0$$



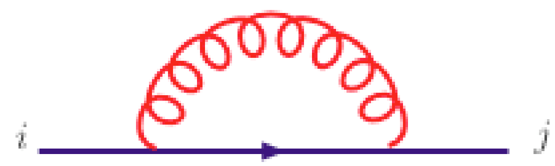
= 0

$$\mathrm{Tr}(t^a t^b) = T_R \delta^{ab}$$



= T_R *

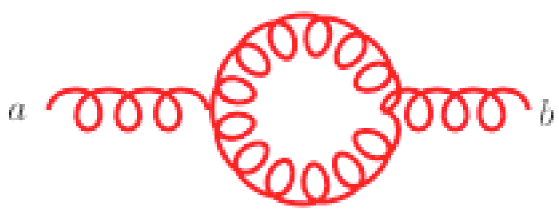
$$(t^a t^a)_{ij} = C_F \delta_{ij}$$



= C_F *

$$\sum_{cd} f^{acd} f^{bcd}$$

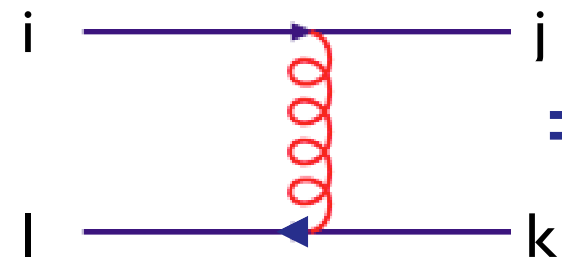
$$= (F^c F^c)_{ab} = C_A \delta_{ab}$$



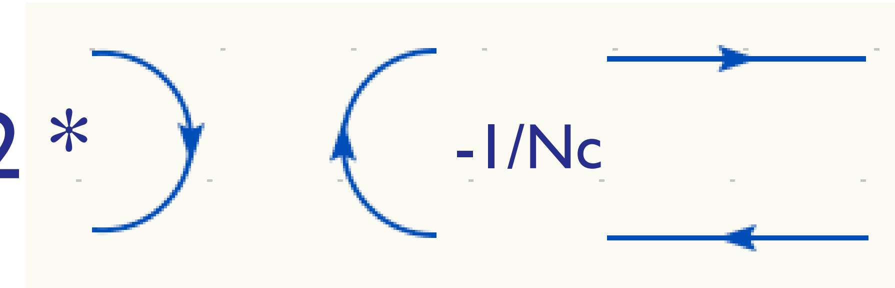
= C_A*

The color algebra

$$t^a_{ij}t^a_{kl} = \frac{1}{2}(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl})$$



= 1/2 *



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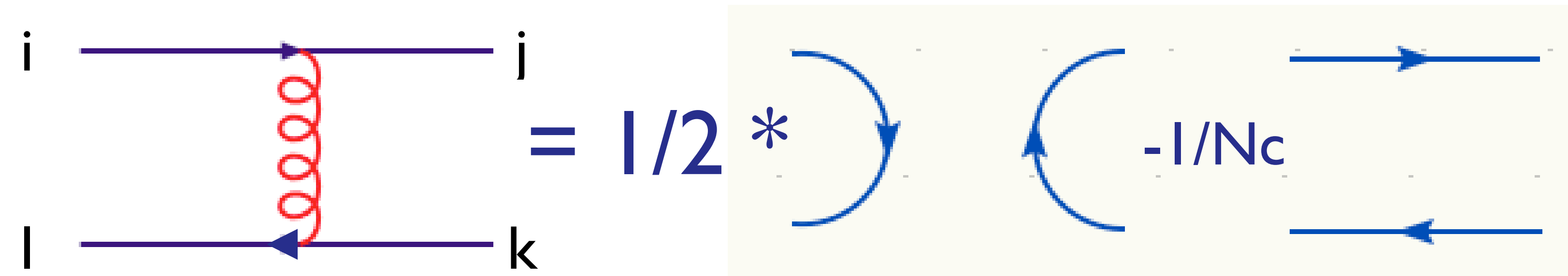

Problem: Show that the one-gluon exchange between quark-antiquark pair can be attractive or repulsive. Calculate the relative strength.

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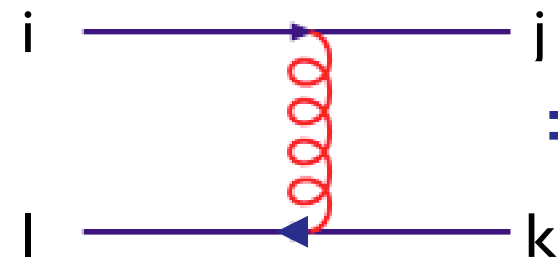
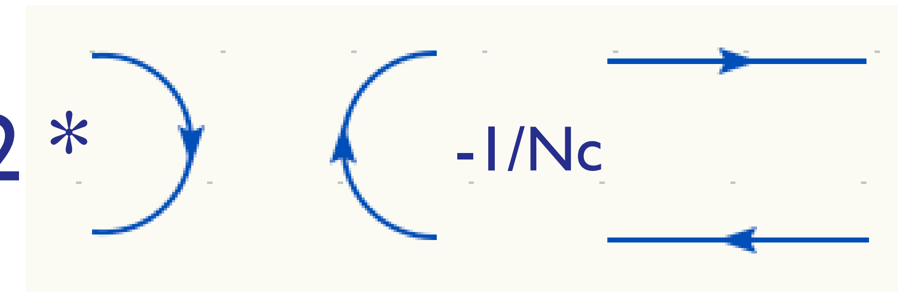


= 1/2 *



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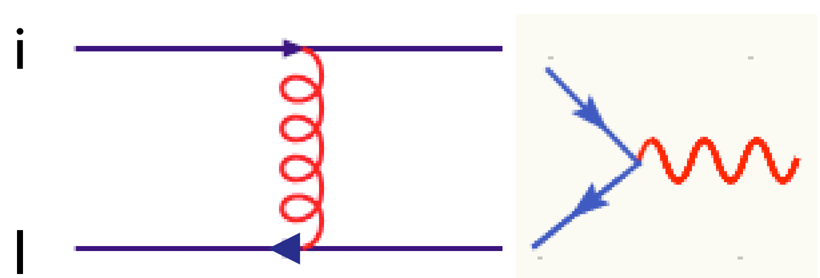
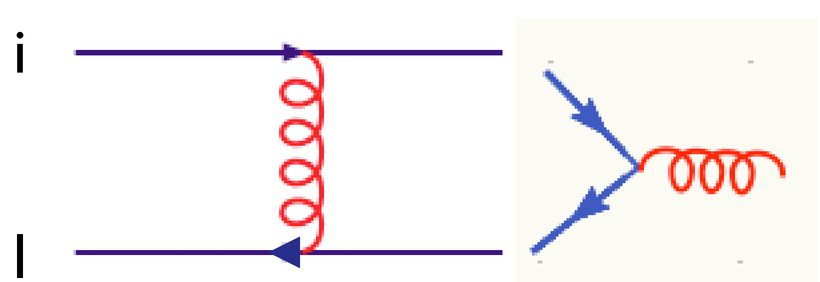
Solution: a q qb pair can be in a singlet state (photon) or in octet (gluon) : $3 \otimes \bar{3} = 1 \oplus 8$



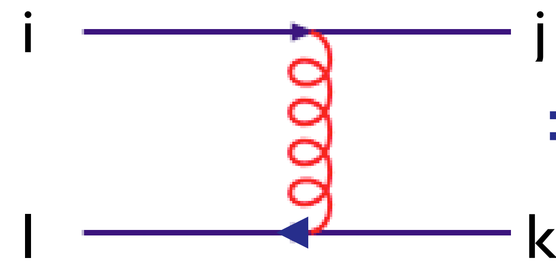
$$\frac{1}{2}(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{lk})\delta_{ki} = \frac{1}{2}\delta_{lj}(N_c - \frac{1}{N_c}) = C_F\delta_{lj}$$



$$\frac{1}{2}(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{lk})t^a_{ki} = -\frac{1}{2N_c}t^a_{lj}$$

The color algebra

$$t^a_{ij}t^a_{kl} = \frac{1}{2}(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl})$$



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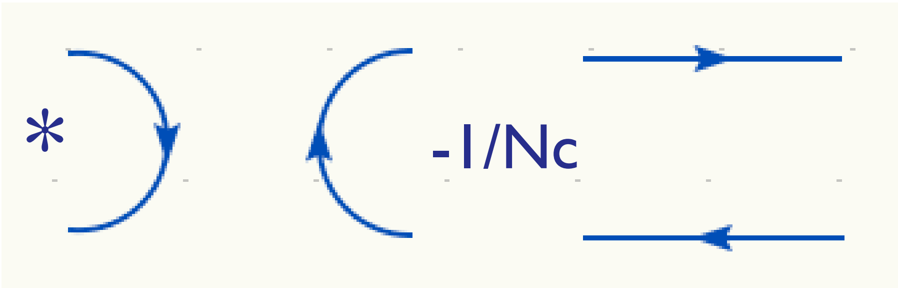

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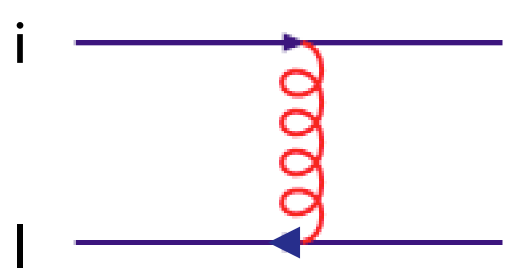

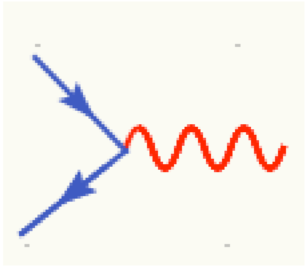

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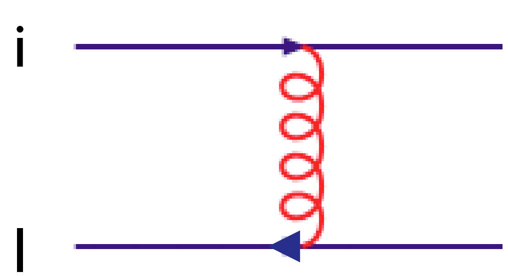

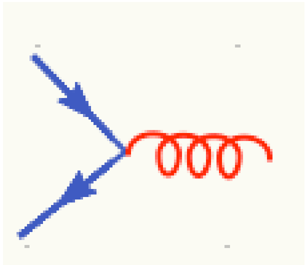

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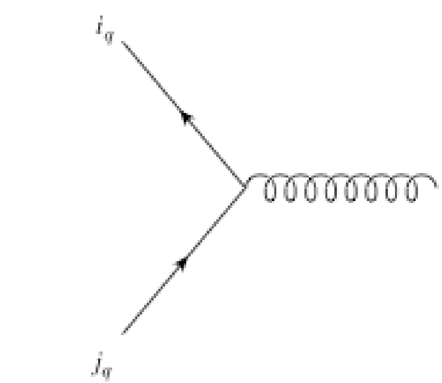
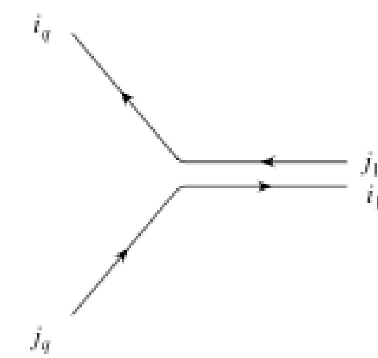
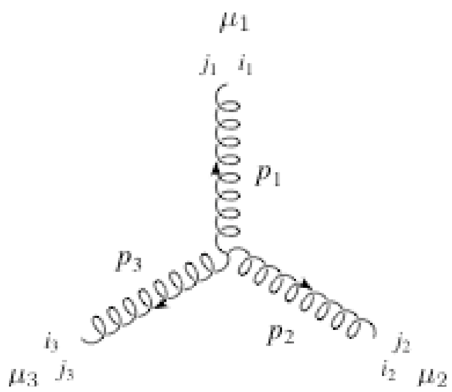
<0, repulsive

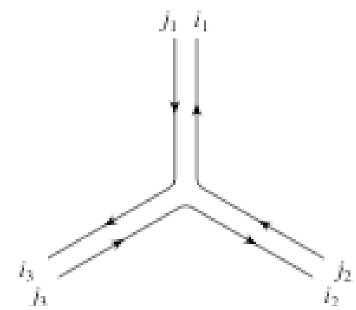
Color algebra: 't Hooft double line





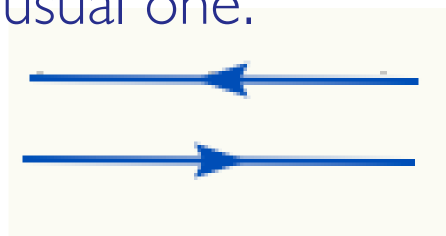
$$i\frac{g}{\sqrt{2}}\gamma_1^{\mu}\delta_{j_1}^{i_q}\delta_{j_q}^{i_1}$$





$$i\frac{g}{\sqrt{2}}\sum K^{\mu_1\mu_2\mu_3}\delta_{j_1}^{i_3}\delta_{j_2}^{i_1}\delta_{j_3}^{i_2}$$

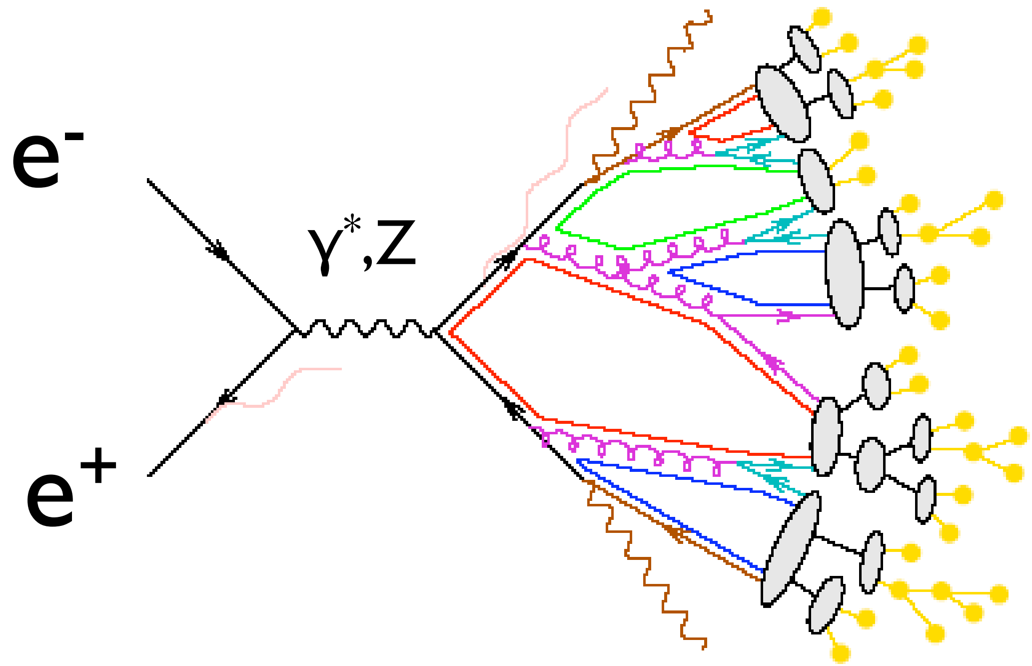
This formulation leads to a graphical representation of the simplifications occuring in the large Nc limit, even though it is exactly equivalent to the usual one.

 $\approx$ 1/2 

In the large Nc limit, a gluon behaves as a quark-antiquark pair. In addition it behaves classically, in the sense that quantum interference, which are effects of order $1/Nc^2$ are neglected. Many QCD algorithms and codes (such a the parton showers) are based on this picture.

Ren. group and asymptotic freedom

Let us consider the process:
$e^-e^+ \to$ hadrons and for a $Q^2 >> \Lambda_S$.
At this pont (though we will!) we don't have an idea how to calculate the details of such a process.
So let's take the most inclusive approach ever: we just want to count how many events with hadrons in the final state there are wrt to a pair of muons.



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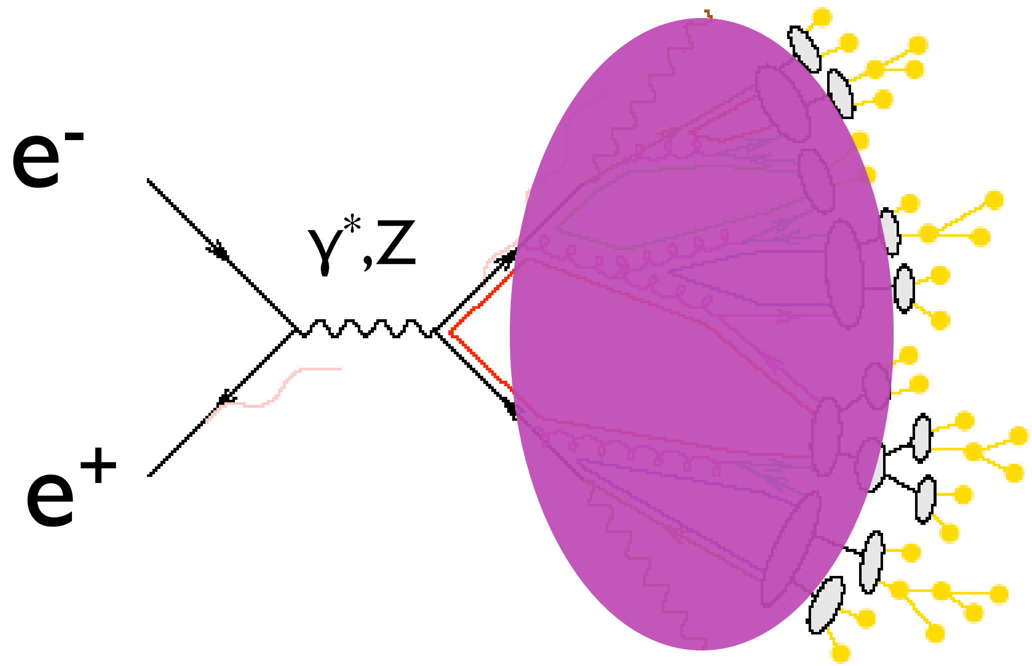

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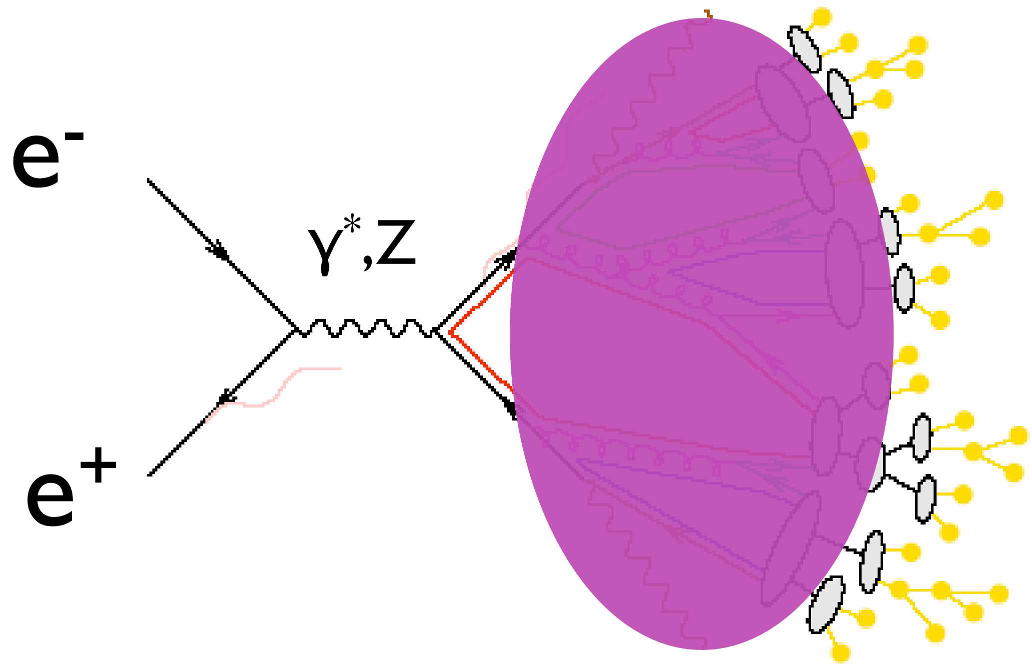
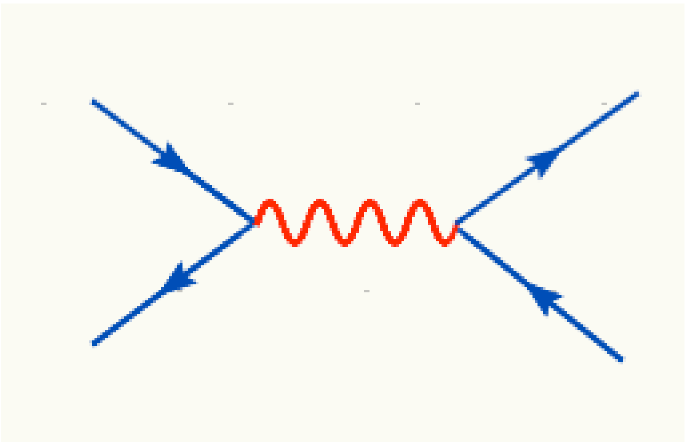


Zeroth Level: e+ e- → qq

$$R_0 = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_f Q_f^2$$



Very simple exercise. The calculation is exactly the same as for the $\mu+\mu$-.

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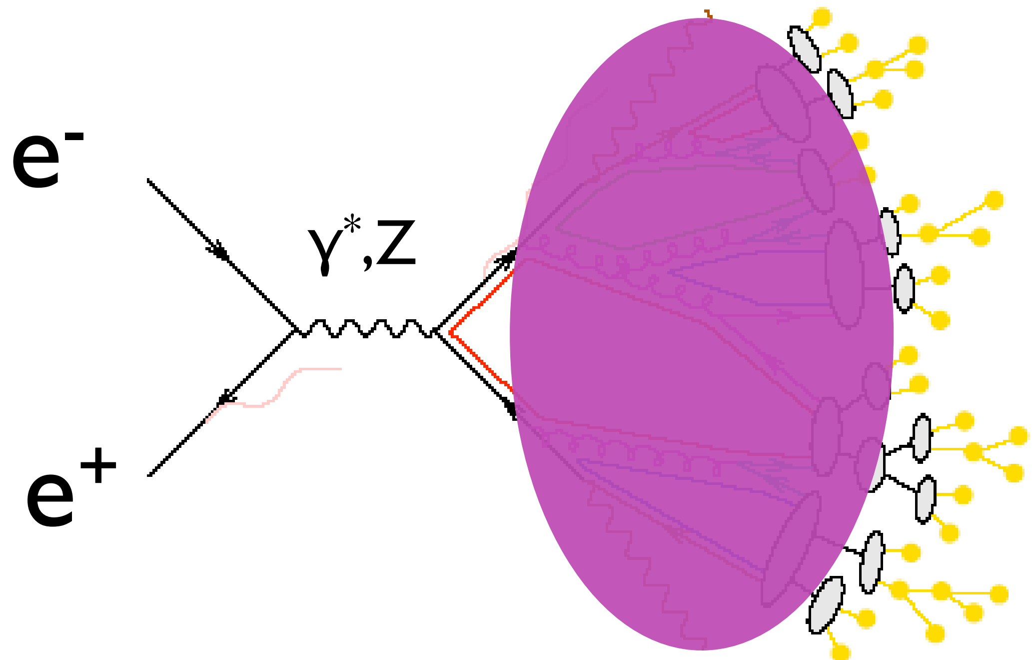
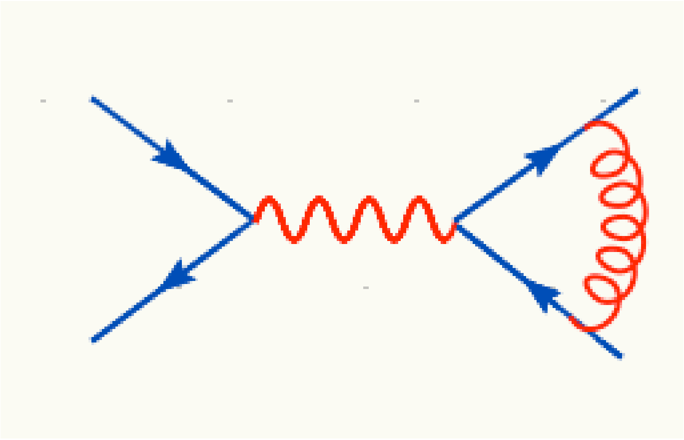


First improvement: e+ e- → qq at NLO
Already a much more difficult calculation!
There are real and virtual contributions. There are:
* UV divergences coming from loops
* IR divergences coming from loops and real diagrams. Ignore the IR for the moment (they cancel anyway) We need some kind of trick to regulate the divergences. Like dimensional regularization or a cutoff M. At the end the result is VERY SIMPLE:



$$R_1 = R_0 \left(1 + \frac{\alpha_S}{\pi}\right)$$

No renormalization is needed! Electric charge is left untouched by strong interactions!

Ren. group and asymptotic freedom

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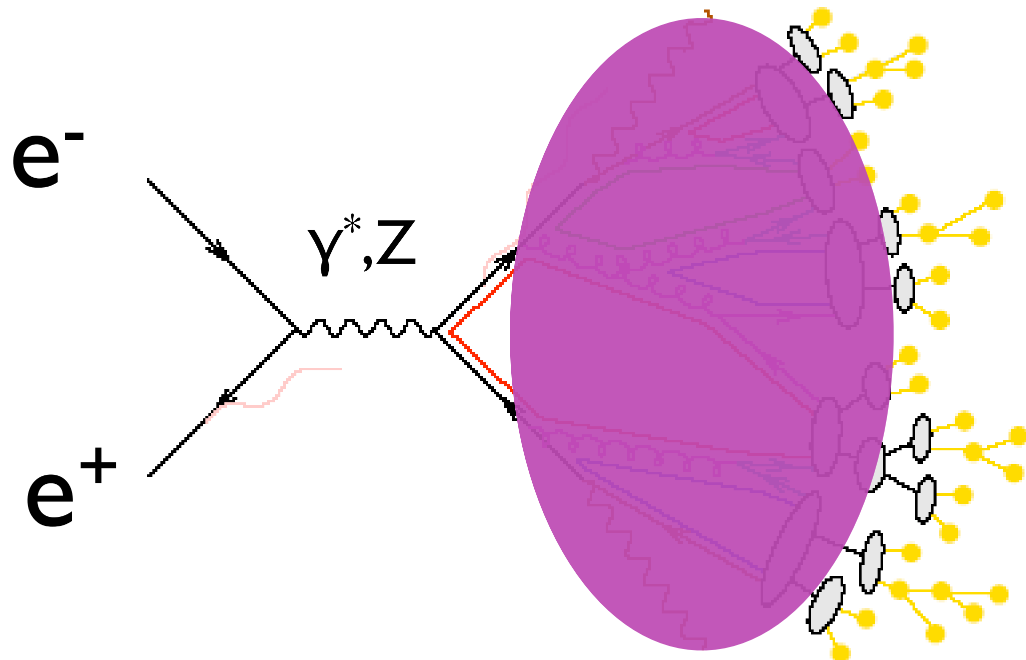
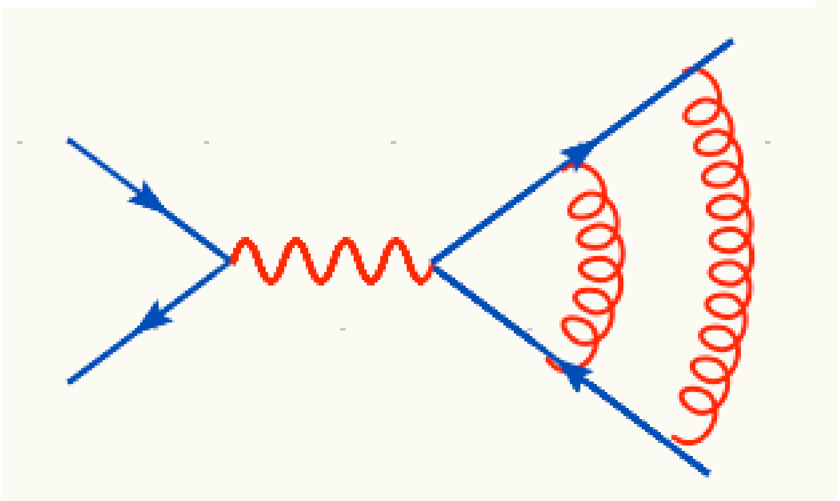


Second improvement: e+ e- → qq at NNLO
Extremely difficult calculation!
Something new happens:

$$R_2 = R_0\left(1+\frac{\alpha_S}{\pi}+\left[c+\pi b_0\log\frac{M^2}{Q^2}\right]\left(\frac{\alpha_S}{\pi}\right)^2\right)$$



The result is explicitly dependent on the arbitrary cutoff scale. We need to perform normalization of the coupling and since QCD is renormalizable we are guaranteed that this fixes all the UV problems at this order.

$$\alpha_S(\mu) = \alpha_S + b_0\log\frac{M^2}{\mu^2}\alpha_S^2$$

Ren. group and asymptotic freedom

(1) $$R_2^{\rm ren}(\alpha_S(\mu), \frac{\mu^2}{Q^2}) = R_0 \left(1 + \frac{\alpha_S(\mu)}{\pi} + \left[c + \pi b_0 \log \frac{\mu^2}{Q^2} \right] \left(\frac{\alpha_S(\mu)}{\pi} \right)^2 \right)$$

(2) $$\alpha_S(\mu) = \alpha_S + b_0 \log \frac{M^2}{\mu^2} \alpha_S^2 \qquad b_0 = \frac{11 N_c - 2 n_f}{12 \pi} > 0$$

Comments:

1. Now R_2 is finite but depends on an arbitrary scale **μ**, directly and through **α_s**. We had to introduce **μ** because of the presence of M.

2. Renormalizability guarantees than any physical quantity can be made finite with the SAME substitution. If a quantity at LO is $A\alpha_s^N$ then the UV divergence will be N A b_0 log M^2 α_s^{N+1}.

3. R is a physical quantity and therefore cannot depend on the arbitrary scale **μ**!! One can show that at order by order:

$$\mu^2 \frac{d}{d\mu^2} R^{\rm ren} = 0 \Rightarrow R^{\rm ren}(\alpha_S(\mu), \frac{\mu^2}{Q^2}) = R^{\rm ren}(\alpha_S(Q), 1)$$

which is obviously verified by Eq. (1). Choosing **μ** ≈ Q the logs ...are resummed!

Ren. group and asymptotic freedom

(2) $\alpha_S(\mu) = \alpha_S + b_0 \log \frac{M^2}{\mu^2} \alpha_S^2$ $\qquad$ $b_0 = \frac{11 N_c - 2 n_f}{12\pi}$ >0

4. From (2) one finds that:

$$\beta(\alpha_S) \equiv \mu^2 \frac{\partial \alpha_S}{\partial \mu^2} = -b_0 \alpha_S^2 \qquad \Rightarrow \qquad \alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}}$$

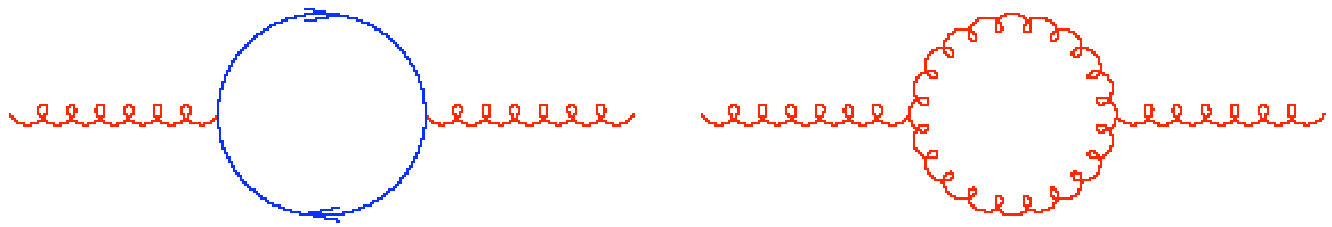
This gives the running of α_S. Since $b_0 > 0$, this expression make sense for all scale $\mu > \Lambda$.
In general one has:

$$\frac{d\alpha_S(\mu)}{d \log \mu^2} = -b_0 \alpha_S^2(\mu) - b_1 \alpha_S^3(\mu) - b_2 \alpha_S^4(\mu) + \ldots$$

where all b_i are finite (renormalization!). At present we know the b_i up to b_3 (4 loop calculation!!). b_1 and b_2 are renormalization scheme independent. Note that the expression for $\alpha_S(\mu)$ changes accordingly to the loop order. At two loops we have:

$$\alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}} \left[1 - \frac{b_1}{b_0^2} \frac{\log \log \mu^2/\Lambda^2}{\log \mu^2/\Lambda^2} \right]$$

Why is the beta function negative in QCD?



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Roughly speaking, quark loop diagram (a) contributes a negative N_f term in b_0, while the gluon loop, diagram (b) gives a positive contribution proportional to the number of colors N_c, which is dominant and make the overall beta function negative.

$$b_0 = \frac{11N_c - 2n_f}{12\pi} \quad >0 \quad \Rightarrow \quad \beta(\alpha_S)<0 \text{ in QCD}$$

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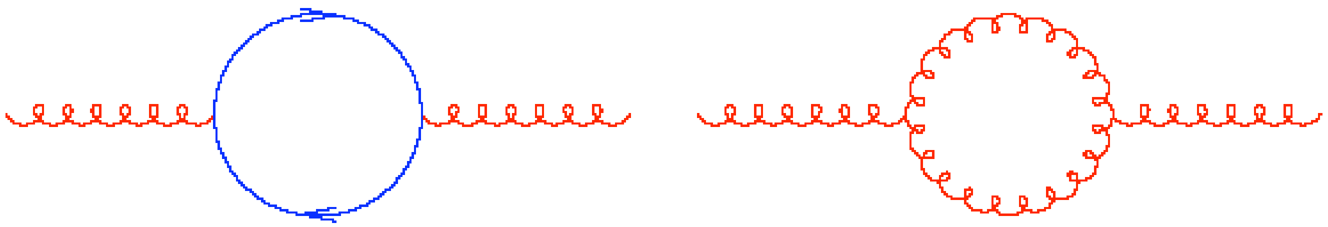

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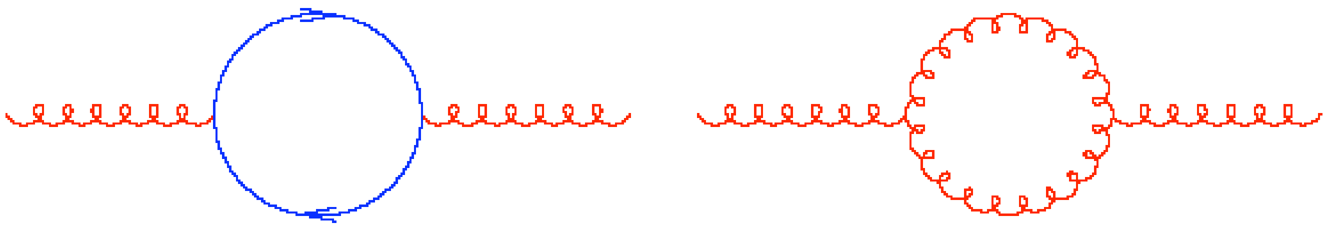

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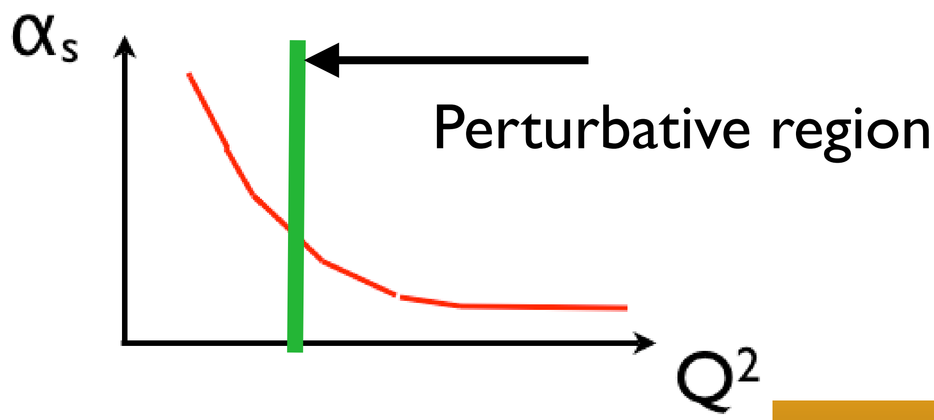

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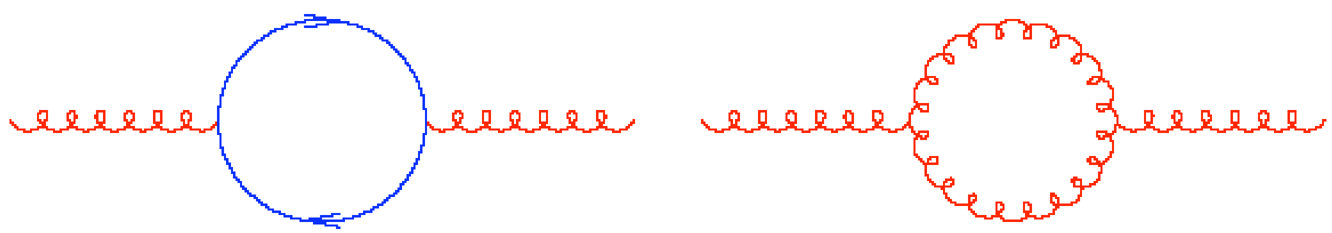

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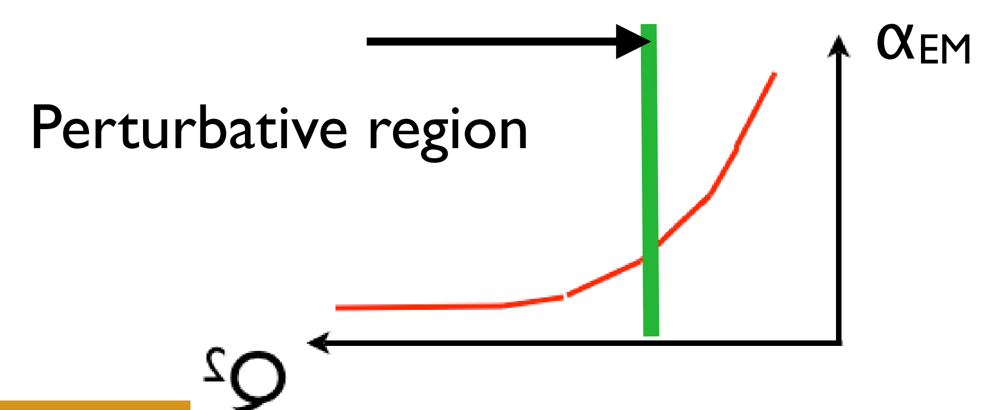

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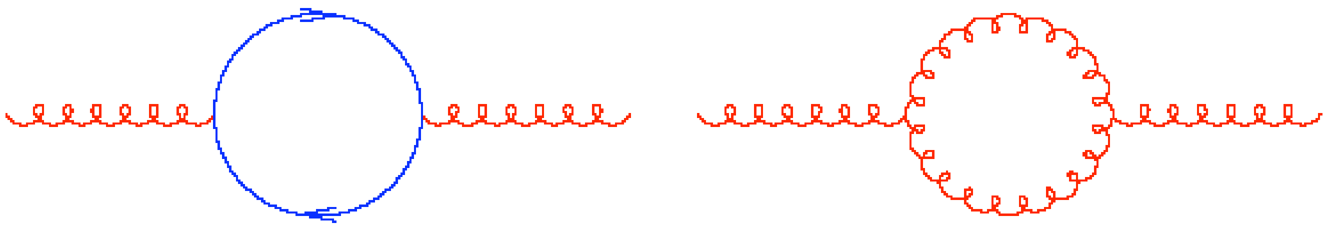

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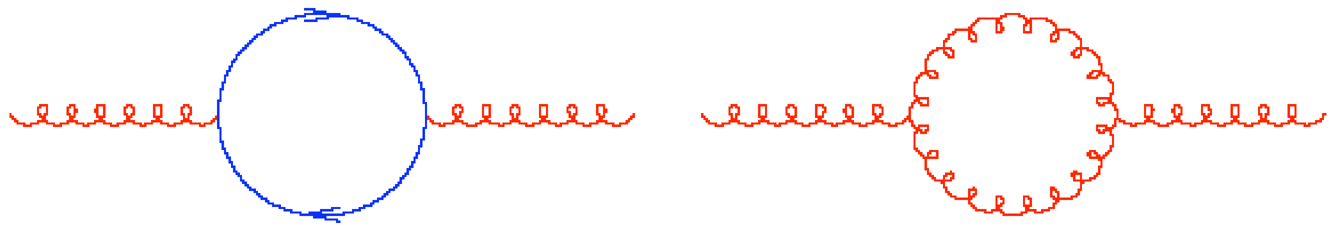

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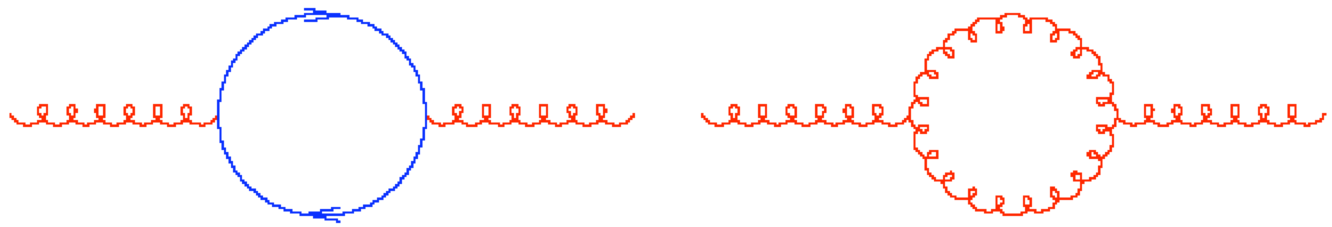

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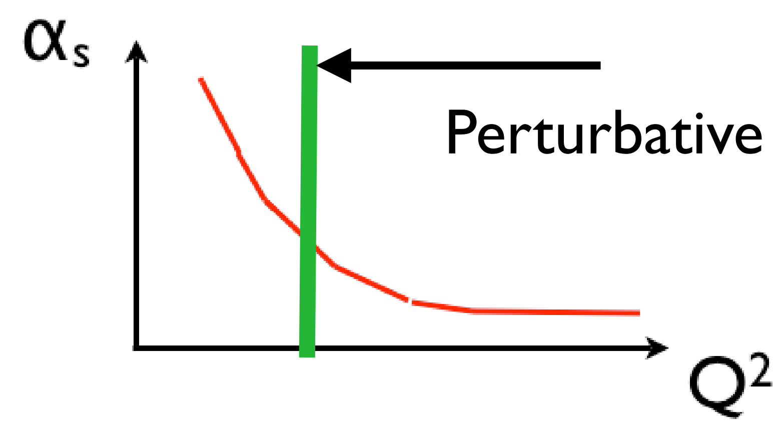

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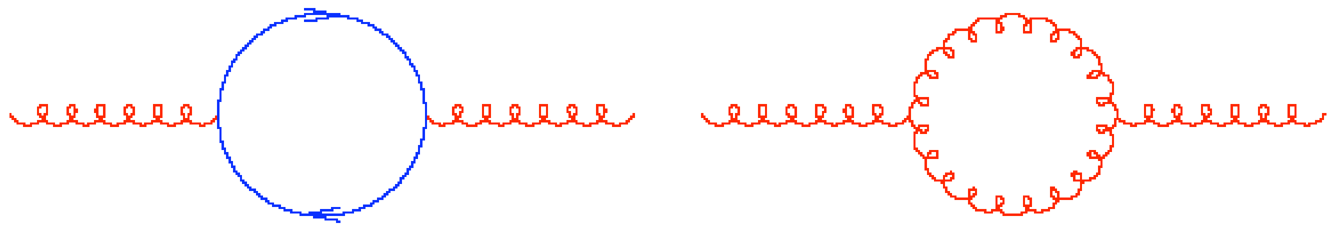

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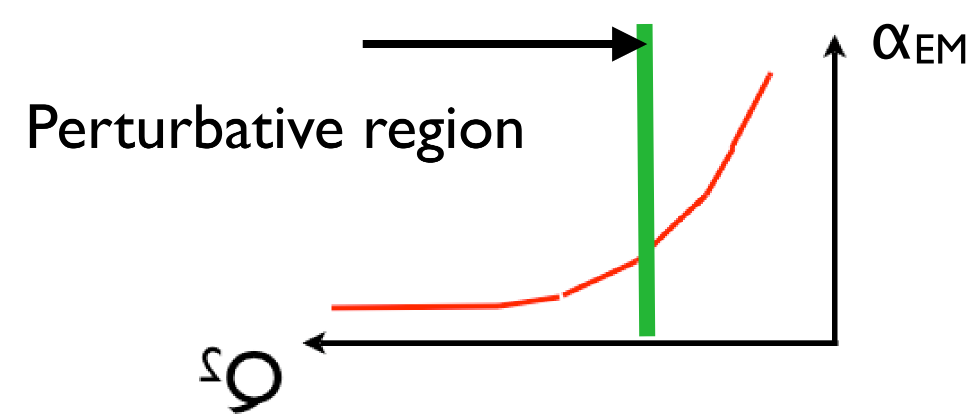

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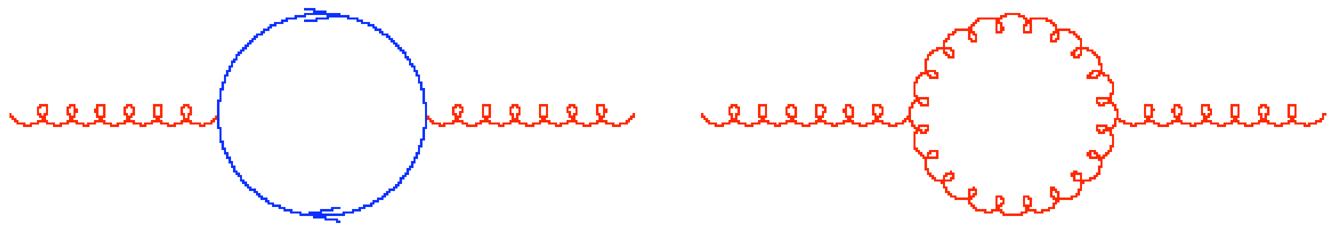

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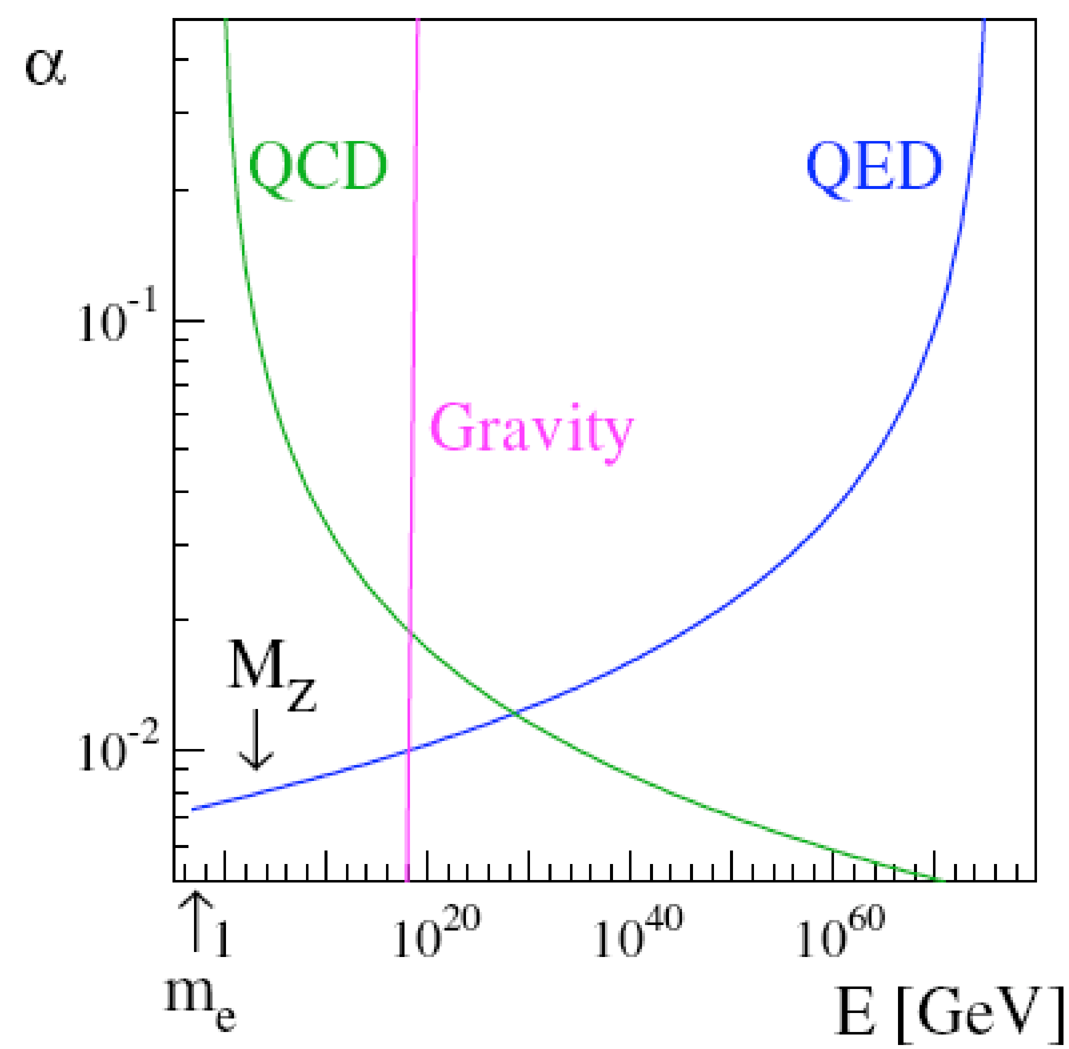
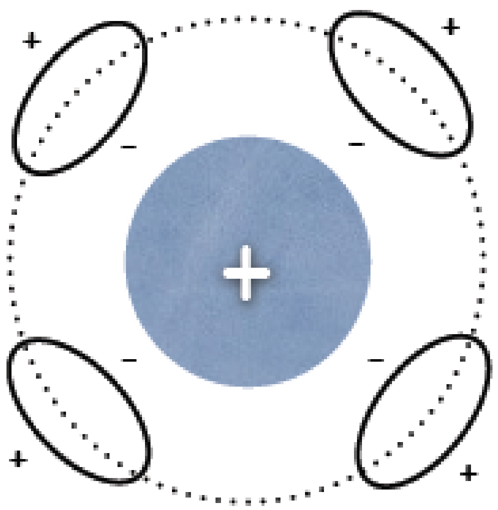


Why is the beta function negative in QCD?

QED

charge screening



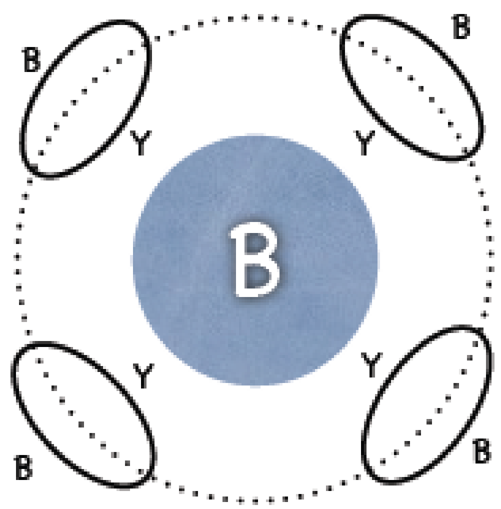
as a result the charge increases as you get closer to the center

DIELECTRIC $\varepsilon > 1$

Why is the beta function negative in QCD?

QCD

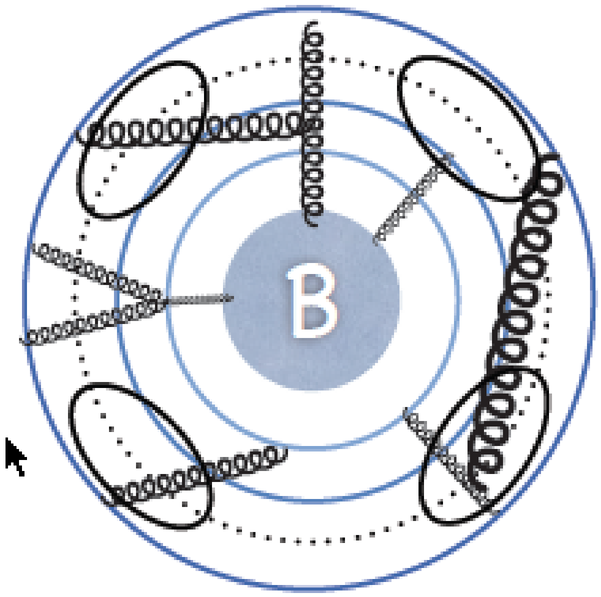
charge screening
from quarks



DIAMAGNETIC **μ**<1
(=DIELECTRIC **ε**>1, SINCE **με**=1)

$$\delta\mu = -(-1/3 + (2 \times \frac{1}{2})^2)q^2 = -\frac{2}{3}q^2$$

charge anti-screening
from gluons



gluons align as little magnets along the color lines and make the field increase at larger distances.

PARAMAGNETIC **μ**>1

$$\delta\mu = (-1/3 + 2^2)q^2 = \frac{11}{3}q^2$$

Ren. group and asymptotic freedom

Given

$$\alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}} \qquad b_0 = \frac{11N_c - 2n_f}{12\pi}$$

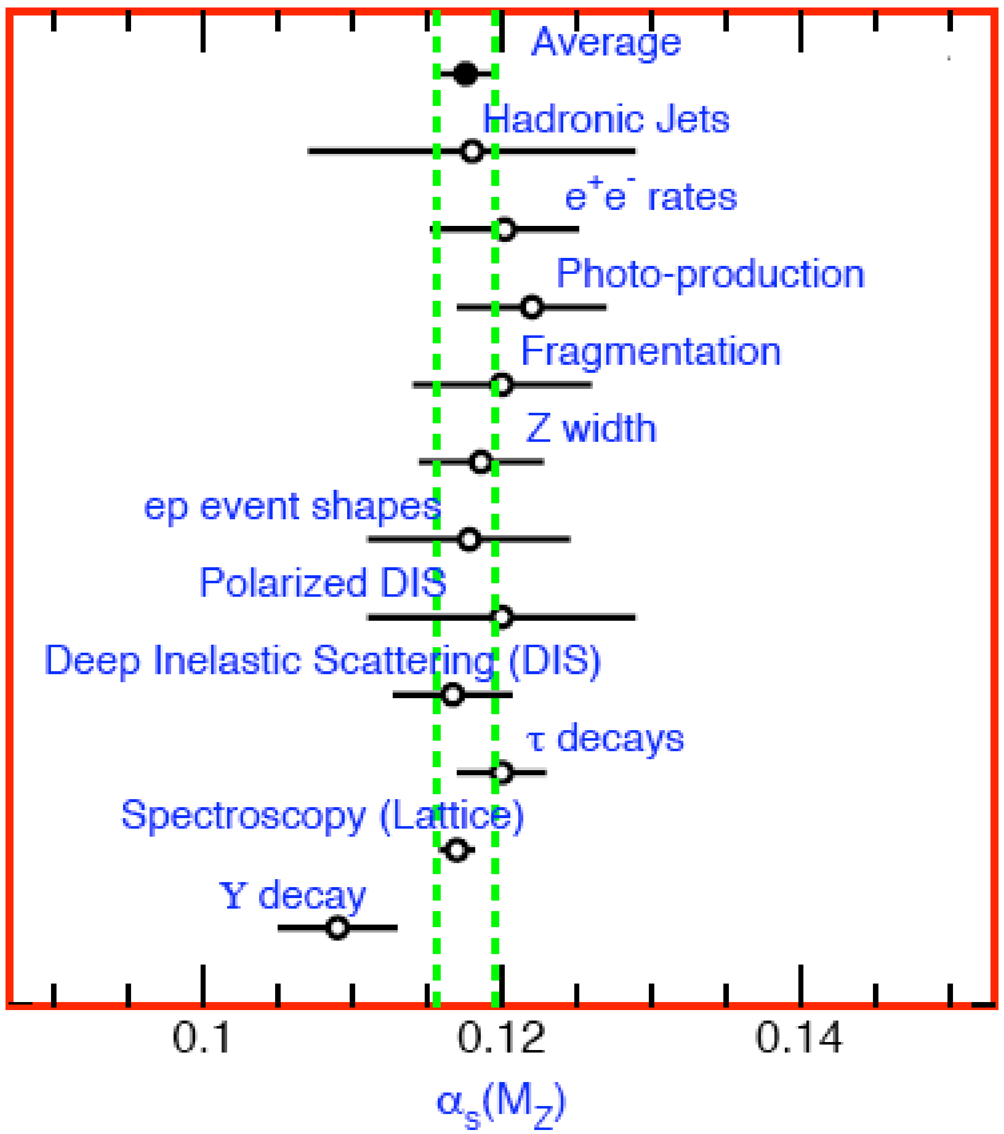
It is tempting to use identify Λ with Λ_S=300 MeV and see what we get for LEP I

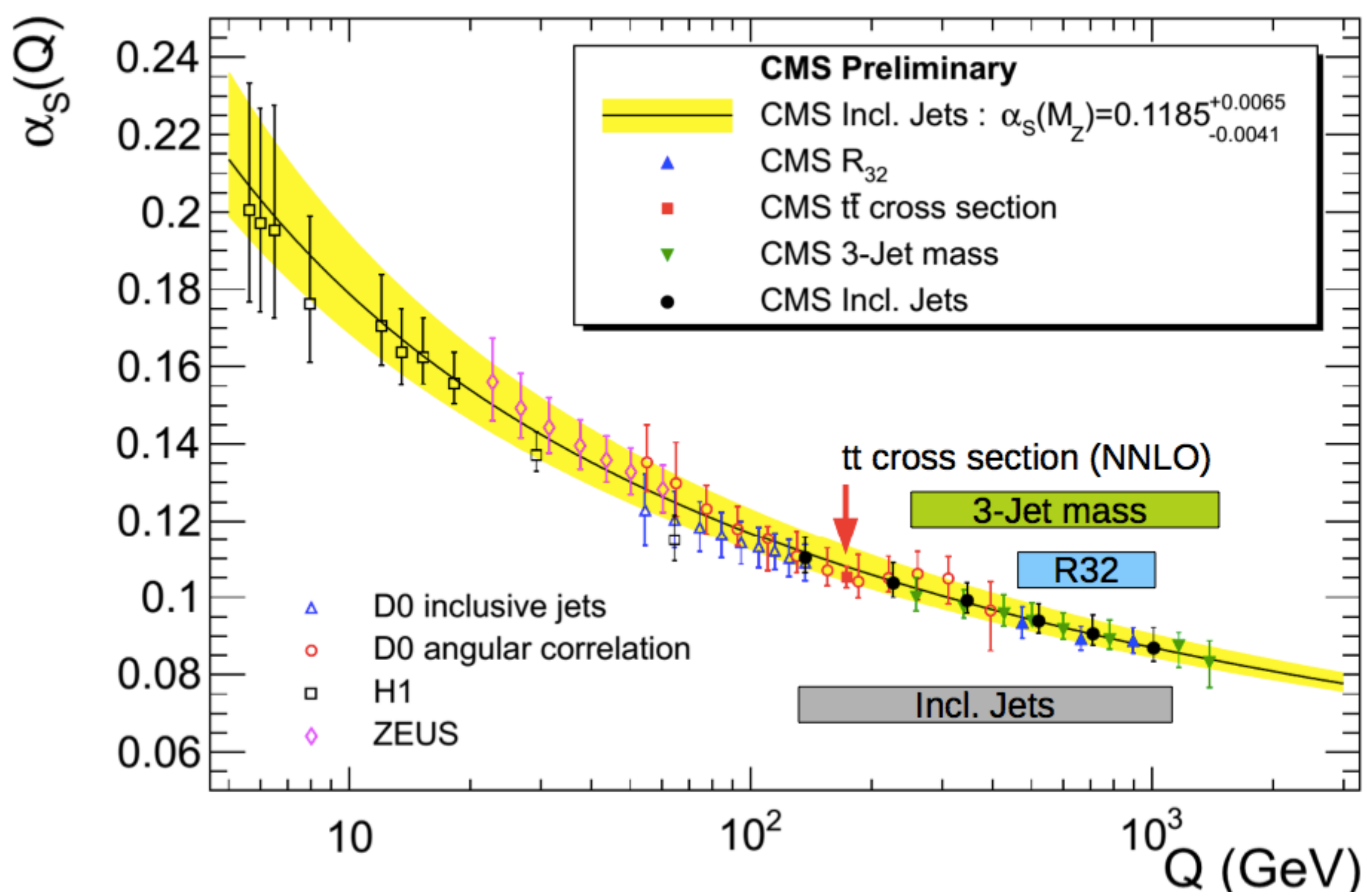
$$R(M_Z) = R_0\left(1 + \frac{\alpha_S(M_Z)}{\pi}\right) = R_0(1 + 0.046)$$

which is in very reasonable agreement with LEP.

This example is very sloppy since it does not take into account heavy flavor thresholds, higher order effects, and so on. However it is important to stress that had we measured 8% effect at LEP I we would have extracted Λ= 5 GeV, a totally unacceptable value...

aS: Experimental results

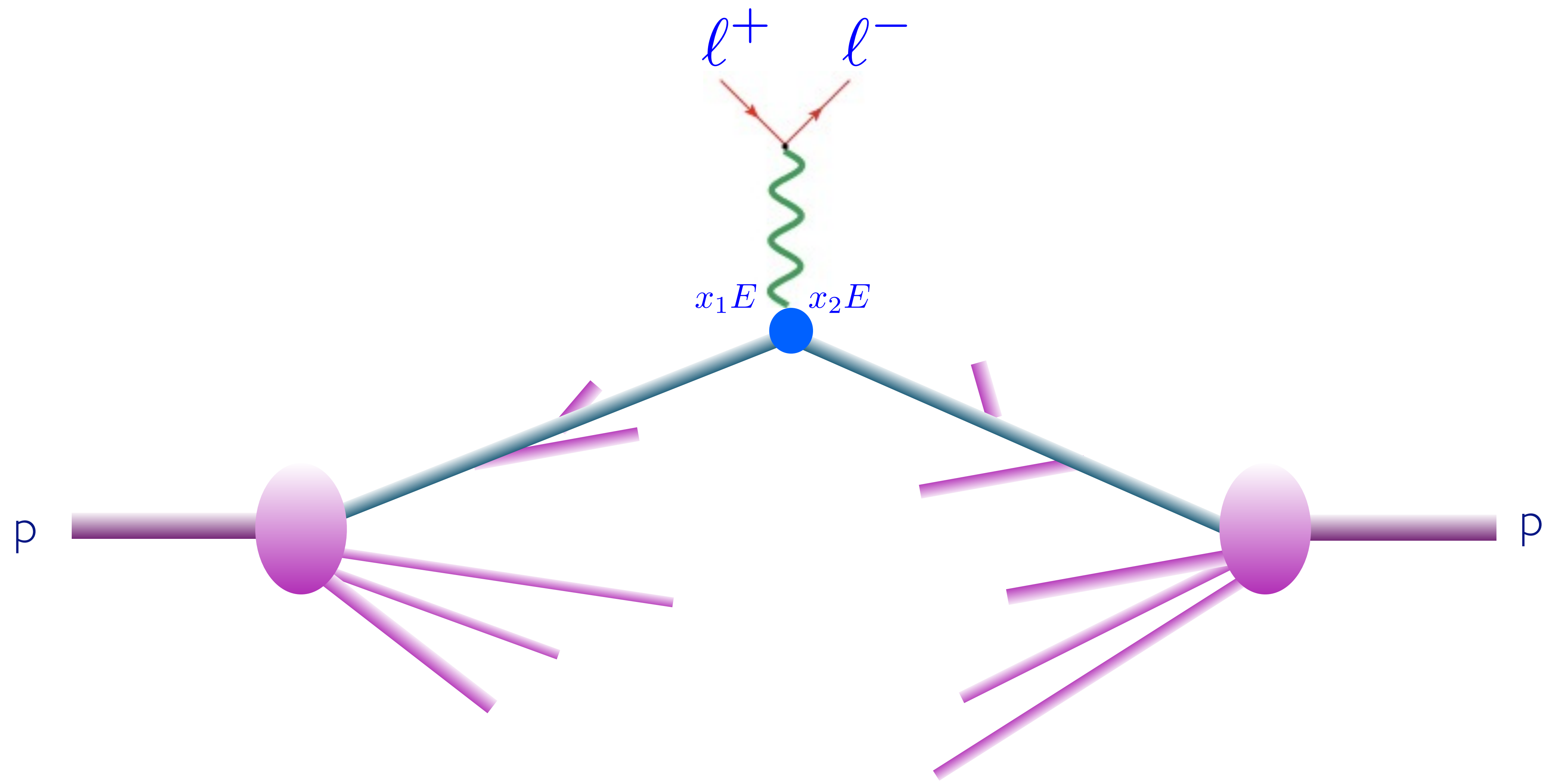




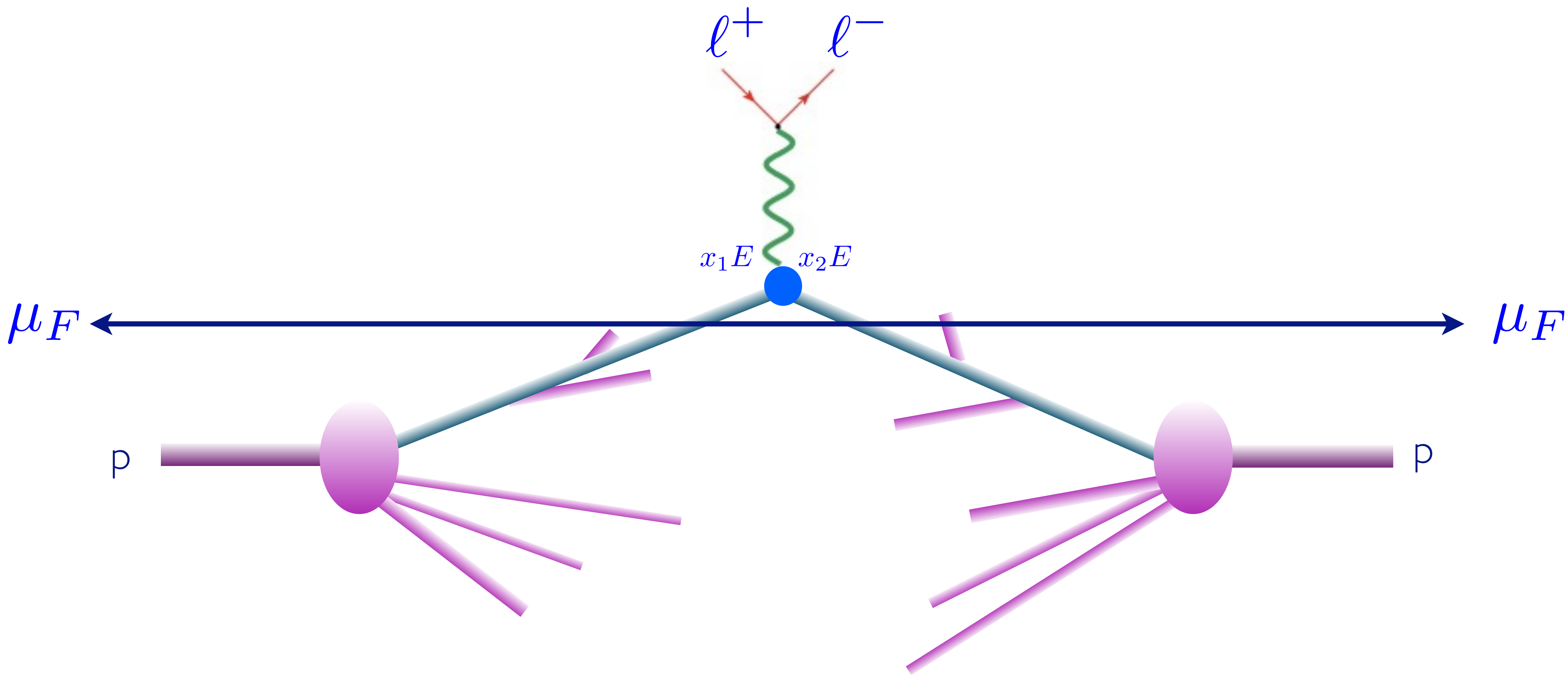
Many measurements at different scales all leading to very consistent results once evolved to the same reference scale, M_Z.

Master formula for the LHC

Master formula for the LHC



Master formula for the LHC



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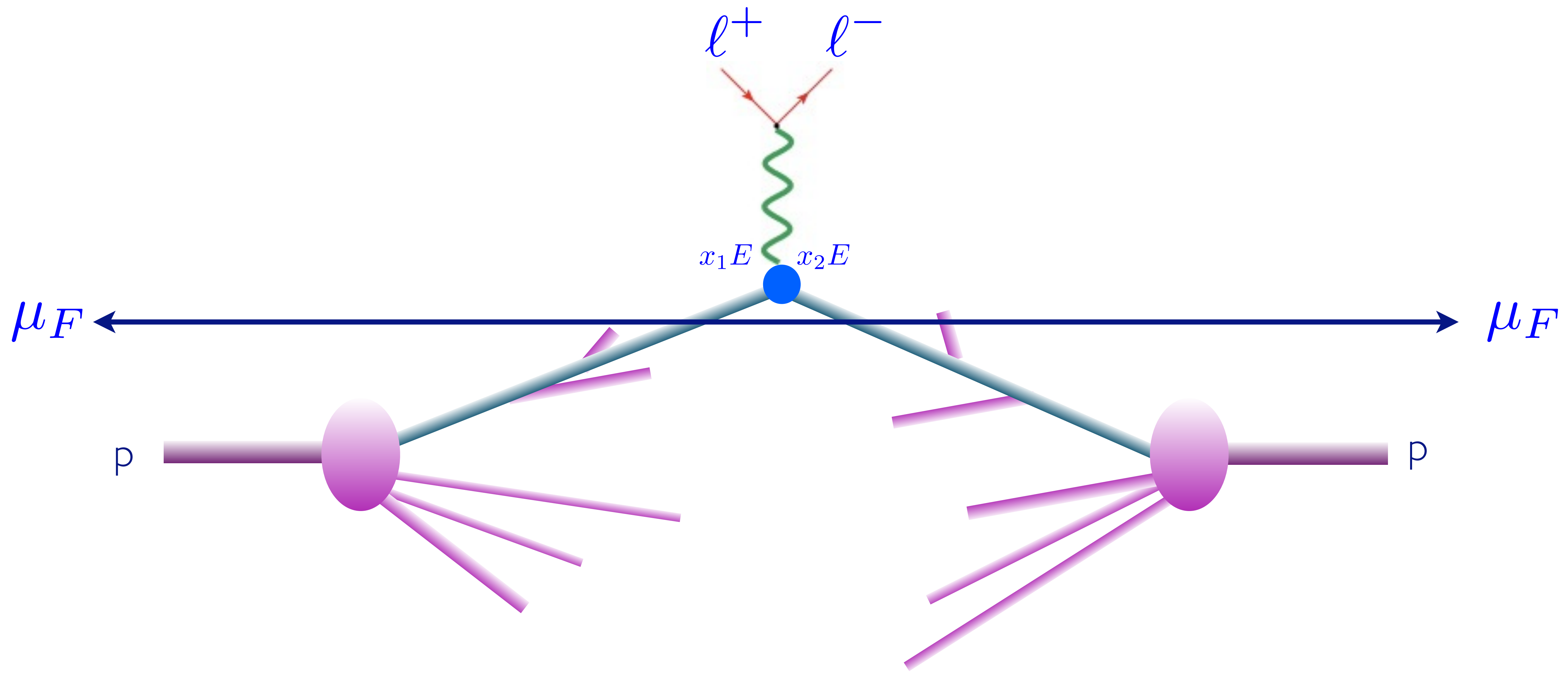

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

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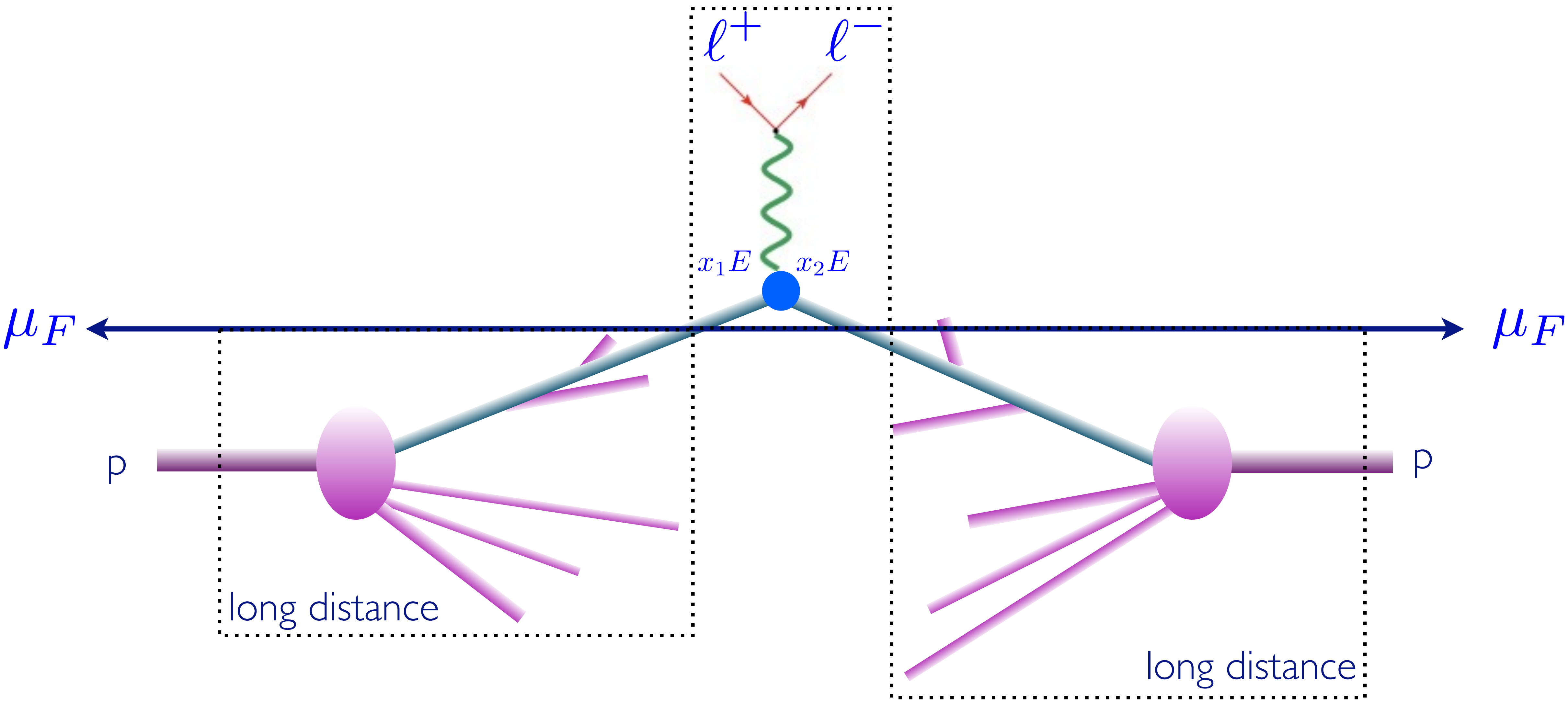

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Example: t tbar production

Let's see how to calculate the cross section for a simple process such as pp → ttbar. There are two initial states possible, gg and qqbar. For gg (which will dominate at the LHC) we obtain:

$$\frac{d\sigma}{d\hat{s}} = \int_0^1 \int_0^1 dx_1 dx_2 \, g(x_1, \mu_F) g(x_2, \mu_F) \, \hat{\sigma}(\hat{s}) \delta(\hat{s} - x_1 x_2 s)$$

We introduce the variable tau, that is proportional to x1 and x2:

$$\tau \equiv \frac{\hat{s}}{s} = x_1 x_2$$

and obtain

$$\frac{d\sigma}{d\tau} = \int_0^1 \int_0^1 dx_1 dx_2 \, g(x_1, \mu_F) g(x_2, \mu_F) \, \frac{\hat{\sigma}(\hat{s})}{\tau} \delta\left(1 - \frac{x_1 x_2}{\tau}\right)$$

Example: t tbar production

$$\frac{d\sigma}{d\tau} = \frac{\hat{\sigma}(\hat{s})}{\tau} \boxed{\int_\tau^1 \frac{dx_1}{x_1} g(x_1) g(\frac{\tau}{x_1})}$$

We define the dimensionless partonic luminosity Lgg:

$$\frac{dL_{gg}}{d\tau} \equiv \int_\tau^1 \frac{dx_1}{x_1} g(x_1) g(\frac{\tau}{x_1})$$

and calculate the total cross section as:

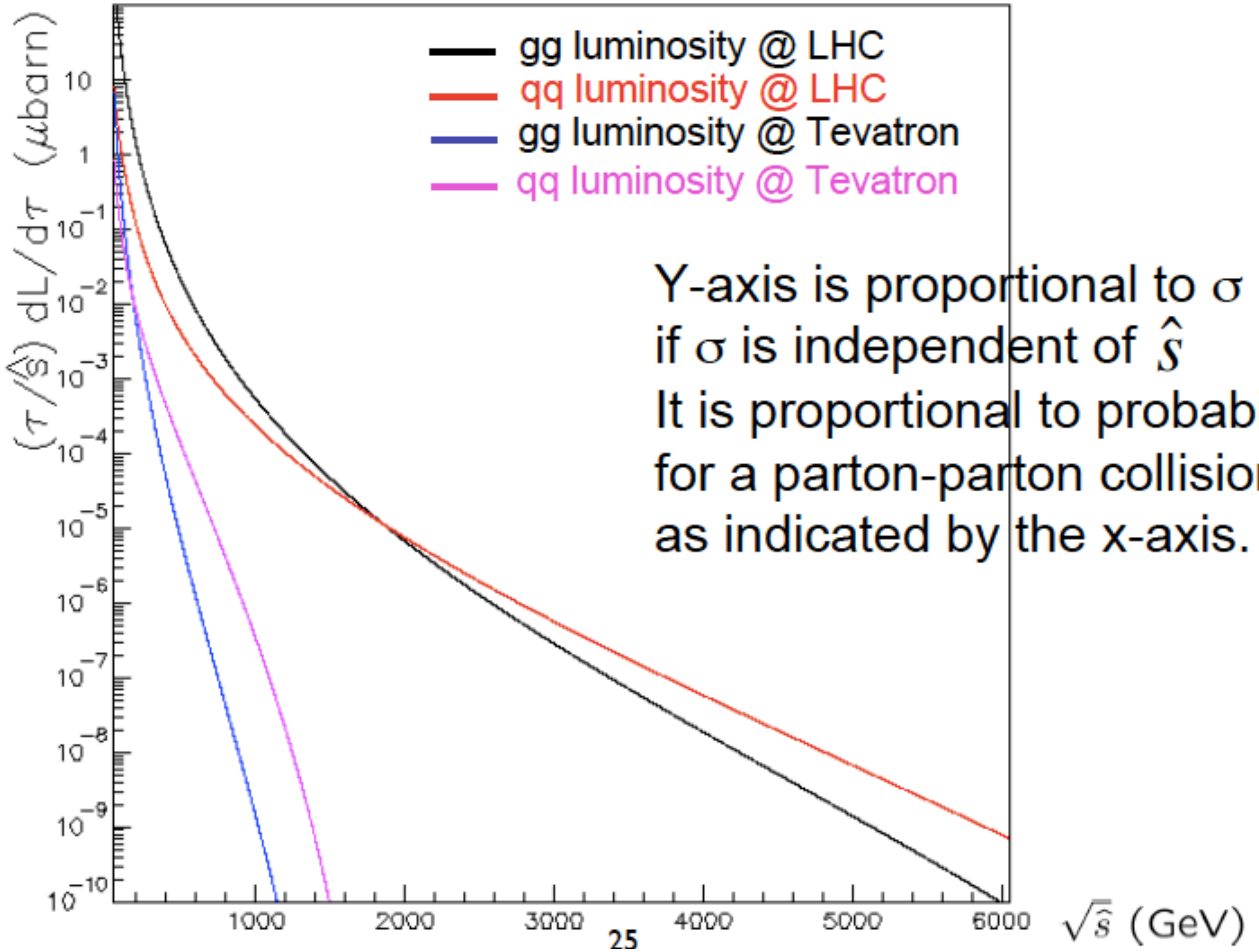
$$\sigma(pp \to t\bar{t} + X) = \int_{\tau_{\min}}^1 d\tau \cdot \hat{\sigma}_{gg\to t\bar{t}}(s\tau) \cdot \frac{dL}{d\tau}$$

$$= \int_{\tau_{\min}}^1 \frac{d\tau}{\tau} \cdot [\hat{s}\hat{\sigma}_{gg\to t\bar{t}}(\hat{s})] \cdot \frac{\tau dL}{\hat{s} d\tau}$$

close to a constant

"cross section"

Example: t tbar production



Y-axis is proportional to σ
if σ is independent of $\hat{s}$
It is proportional to probability
for a parton-parton collision with $\hat{s}$
as indicated by the x-axis.

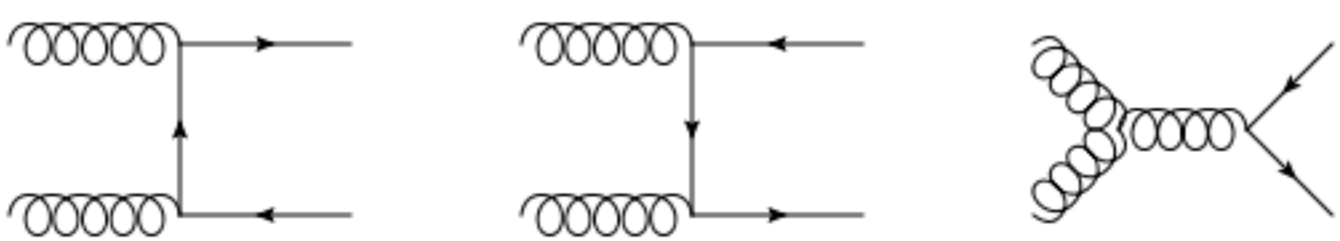
Example: t tbar production

$$\frac{dL_{gg}}{d\tau} \equiv \int_{\tau}^{1} \frac{dx_1}{x_1} g(x_1) g(\frac{\tau}{x_1})$$

If we take for simplicity $g(x) = \frac{1}{x^{1+\delta}} \Rightarrow \frac{dL_{gg}}{d\tau} = \frac{1}{\tau^{1+\delta}} \log \tau$

i.e. the total "cross section" will scale as a power of $1/M_T^{1+\delta} \log M_T$

The short distance coefficient can be easily calculated at LO via the Feynman diagrams:



EXAMPLE: T TBAR PRODUCTION

$$\frac{1}{256}|M|^2 = \frac{3g_s^4}{4}\frac{(m^2-t)(m^2-u)}{s^2} - \frac{g_s^4}{24}\frac{m^2(s-4m^2)}{(m^2-t)(m^2-u)} + \frac{g_s^4}{6}\frac{tu-m^2(3t+u)-m^4}{(m^2-t)^2}$$
$$+\frac{g_s^4}{6}\frac{tu-m^2(t+3u)-m^4}{(m^2-u)^2} - \frac{3g_s^4}{8}\frac{tu-2m^2t+m^4}{s(m^2-t)} - \frac{3g_s^4}{8}\frac{tu-2m^2u+m^4}{s(m^2-u)}$$

3 DIAGRAMS SQUARED + THE INTERFERENCES. THIS AMPLITUDE IS INTEGRATE OVER THE PHASE SPACE AT FIXED SHAT:

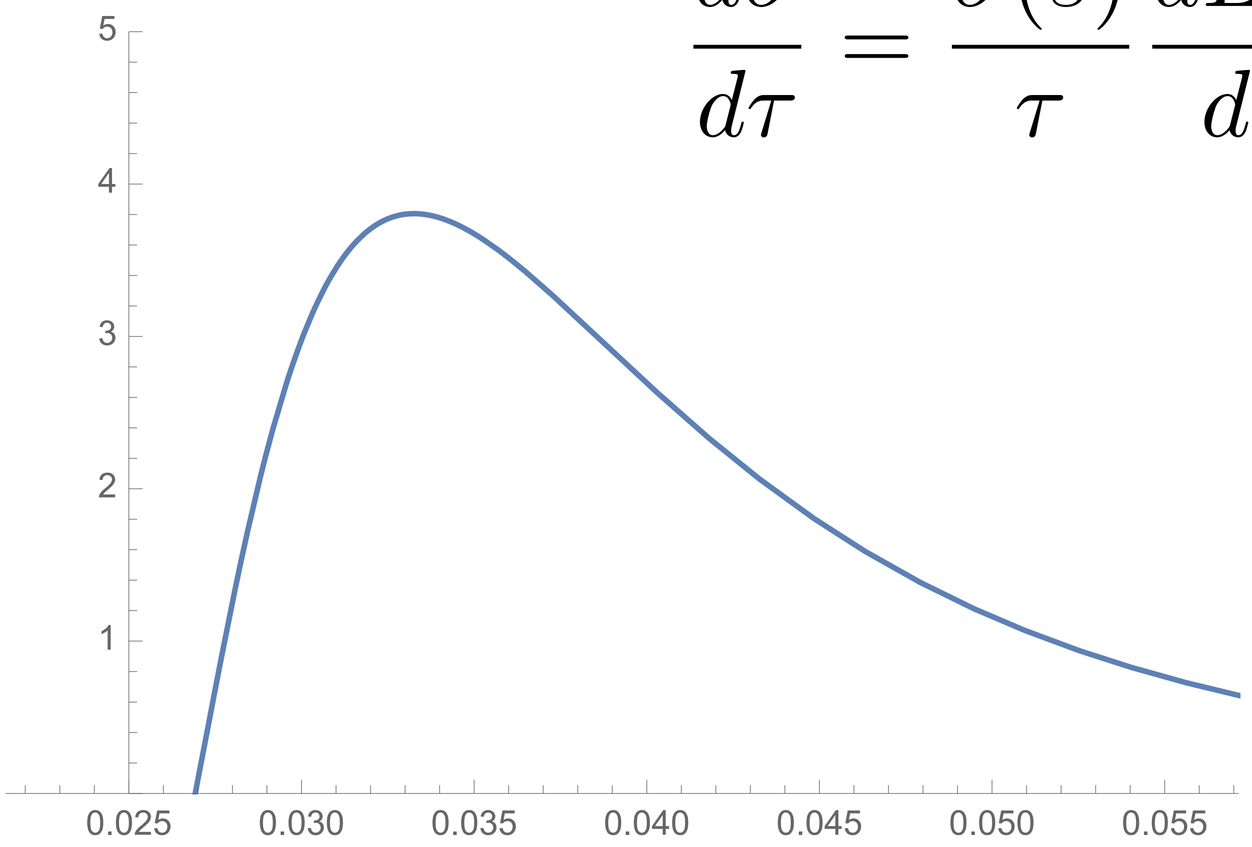
$$\hat{\sigma}_{gg\to t\bar{t}} = \frac{1}{2\hat{s}}\,\beta\,2\pi\int_{-1}^{+1} d\cos\theta^*\;|M|^2/256$$

EVENTUALLY GIVING:

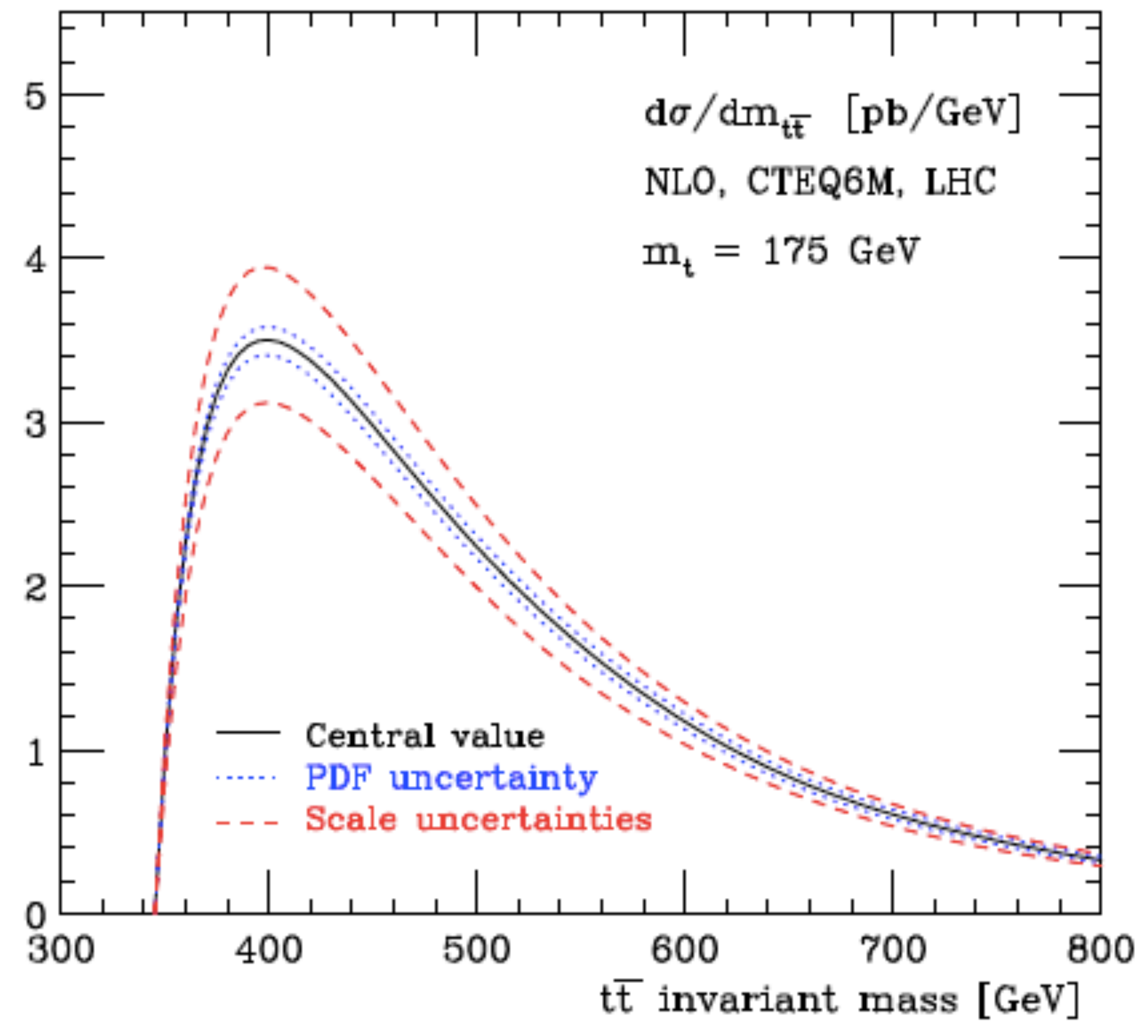
$$\beta = \sqrt{1-4m_t^2/\hat{s}}$$
$$\hat{\sigma}_{gg\to t\bar{t}} = \frac{\pi\alpha_s^2\beta}{48\hat{s}}\left(31\beta + \left(\frac{33}{\beta} - 18\beta + \beta^3\right)\ln\left[\frac{1+\beta}{1-\beta}\right] - 59\right)$$

Example: t tbar production

$$\frac{d\sigma}{d\tau} = \frac{\hat{\sigma}(\hat{s})}{\tau}\frac{dL_{gg}}{d\tau}$$

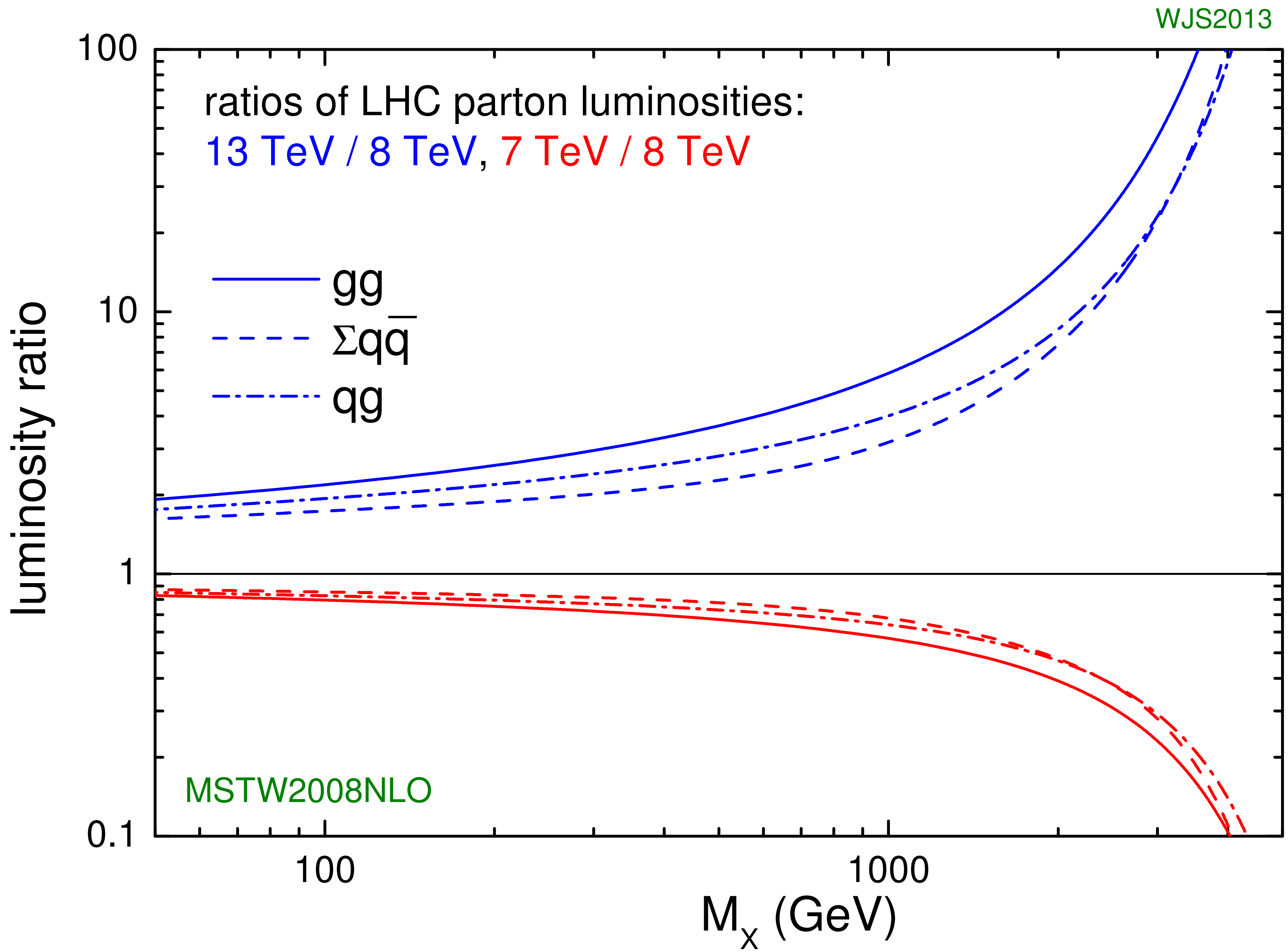


LO estimation with toy PDF

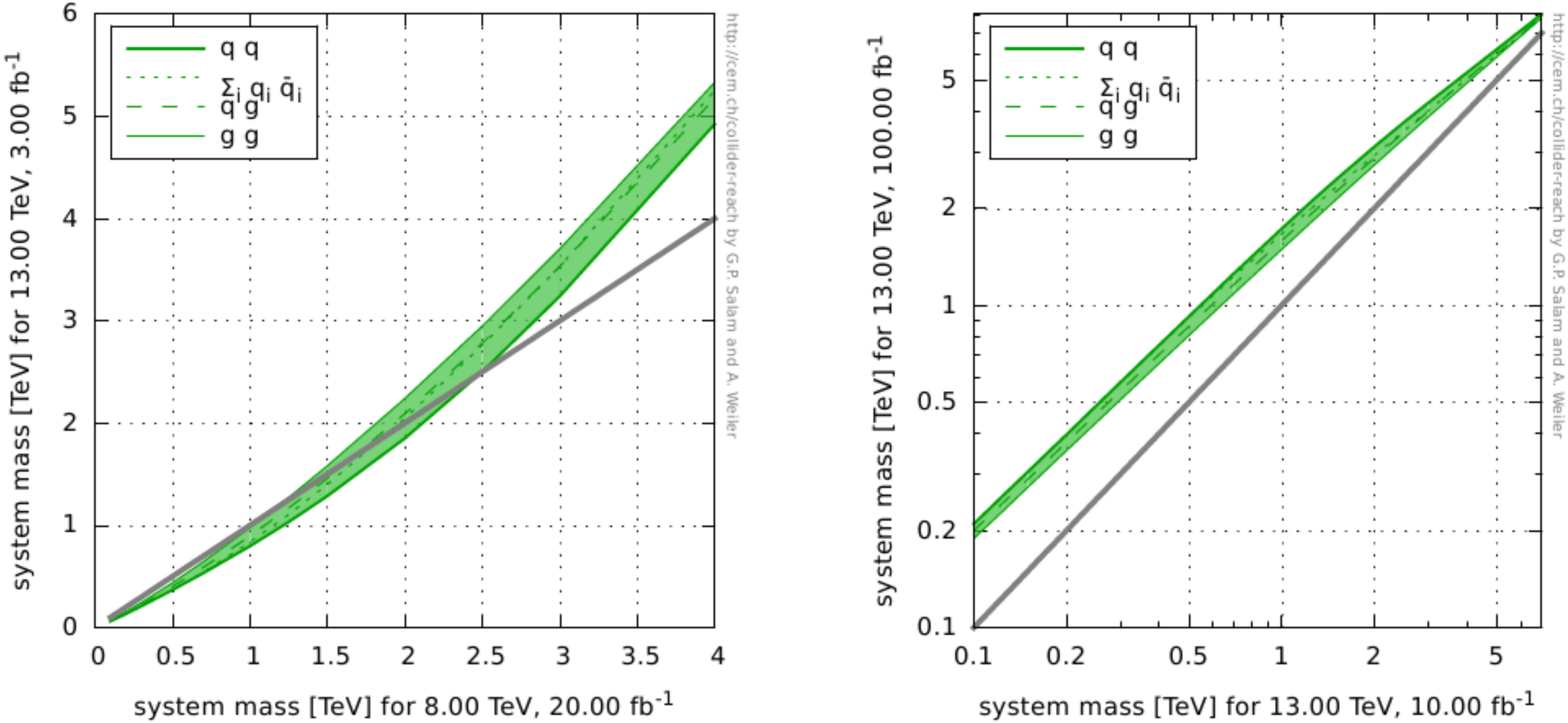


NLO result with proper MC

Parton luminosities

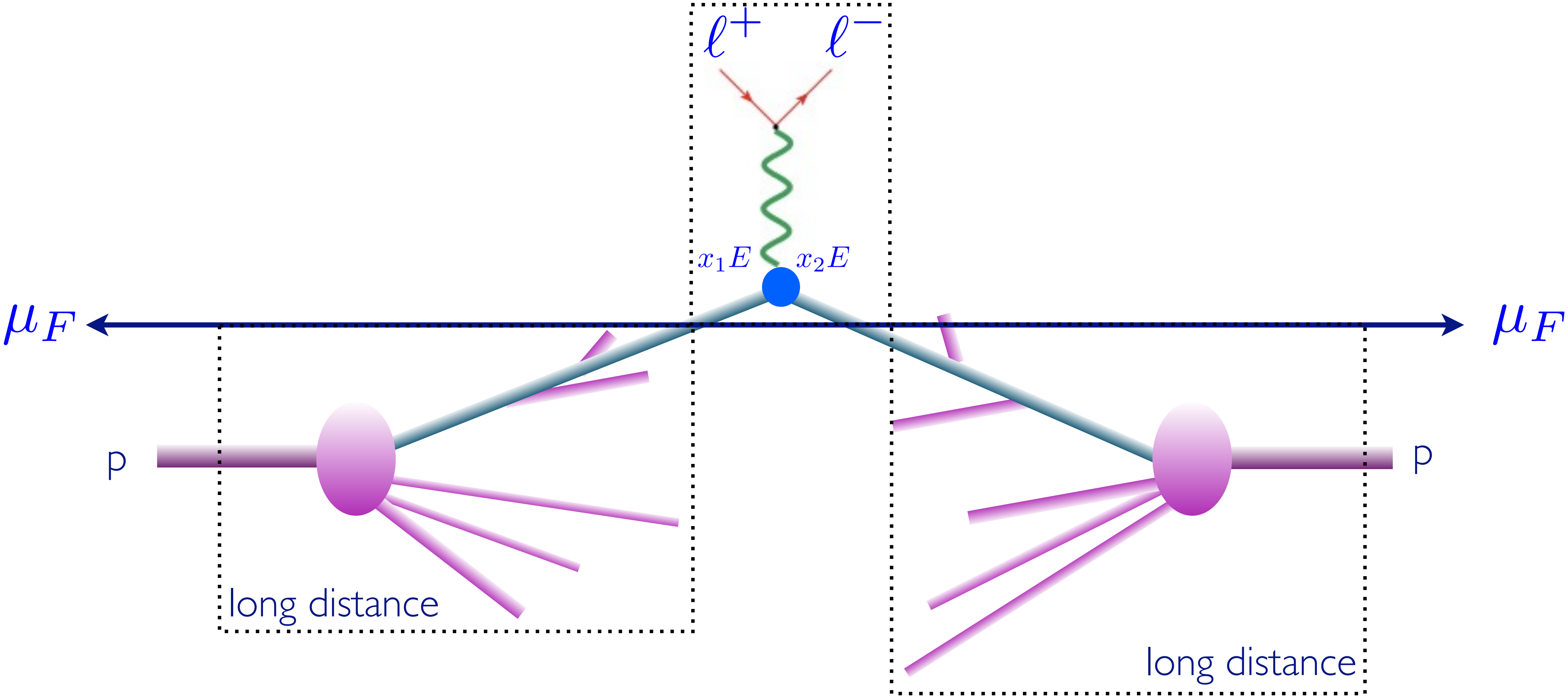


COLLIDER REACH



http://collider-reach.web.cern.ch/collider-reach/

Master formula for the LHC



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Remark on our master formula

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

- By calculating the short distance coefficient at tree-level we obtain the first estimate of rates for inclusive final states.

- Even at LO extra radiation is included: it is described by the PDF's in the initial state and by the definition of a final state parton, which at LO represents all possible final state evolutions.

- Due to the above approximations a cross section at LO can strongly depend on the factorization and renormalization scales.

- Predictions can be systematically improved, at NLO and NNLO, by including higher order corrections in the short distance and in the evolution of the PDF's.

Perturbative expansion

$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ Parton-level cross section

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$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ **Parton-level cross section**

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$$\hat{\sigma} = \sigma^{\rm Born}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \ldots\right)$$

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LO predictions

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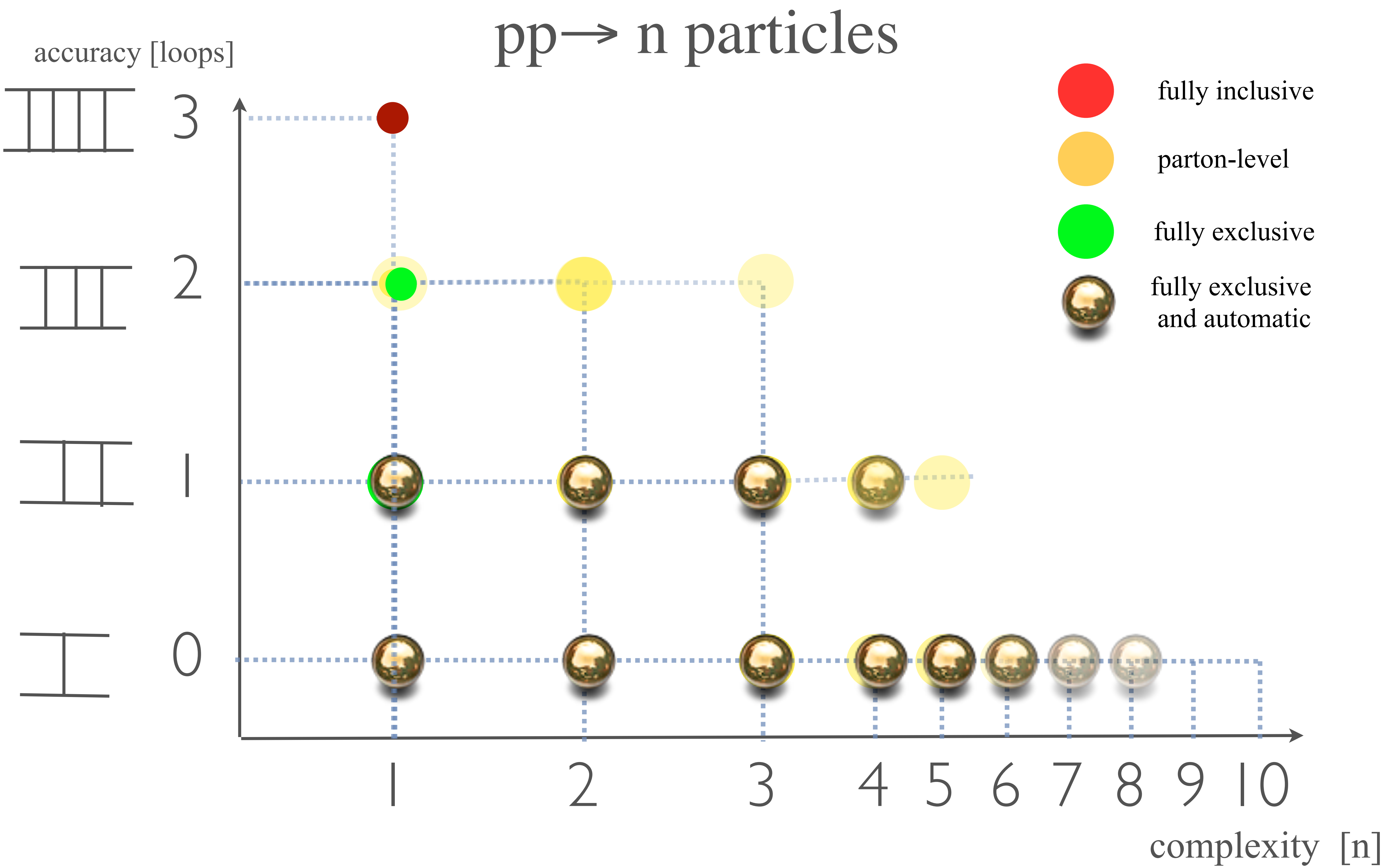
LO predictions | **NLO corrections** | **NNLO corrections** | **NNNLO corrections**

- Including higher corrections improves predictions and reduces theoretical uncertainties

PREDICTIONS IN QCD FOR THE LHC:



Conclusions

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Having a good understanding and control of QCD phenomenology at colliders is a necessary condition to make any measurement or search for new phenomena.

Enjoy and maximally profit from the school!

Hands-on!

EXPERIENCE A "SIMPLE" NLO CALCULATION YOURSELF

pp→Higgs+x at NLO

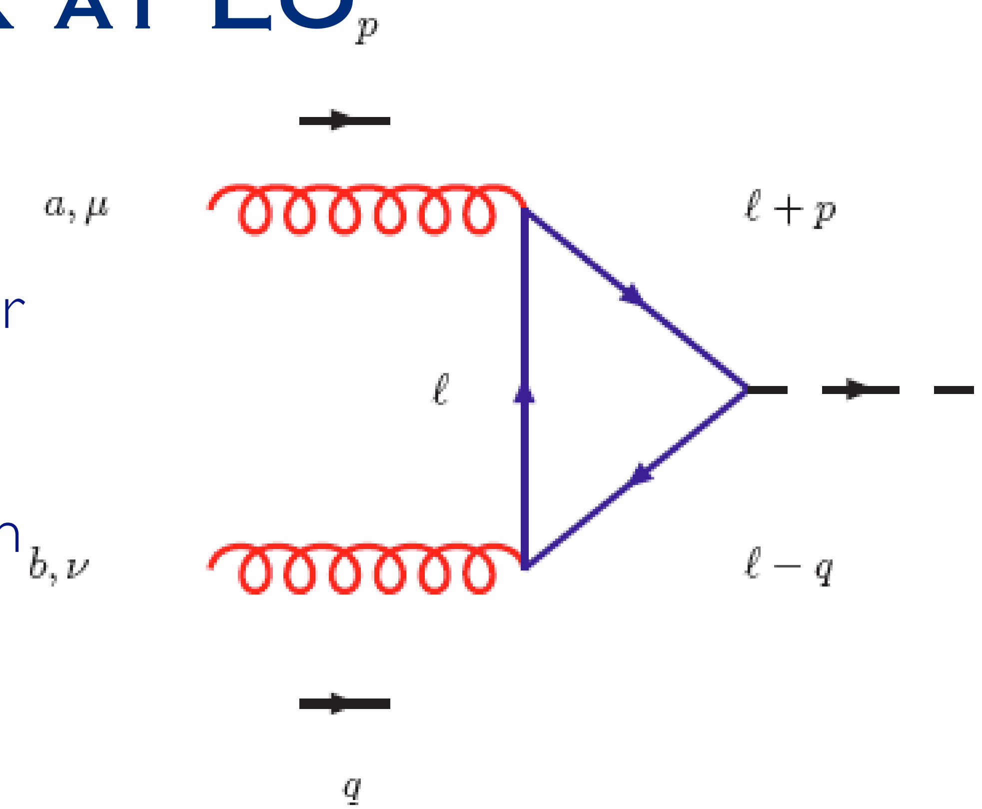
- LO : 1-loop calculation and HEFT
- NLO in the HEFT
  - Virtual corrections and renormalization
  - Real corrections and IS singularities
- Cross sections at the LHC

pp→H+x at LO

This is a "simple" 2→1 process.

However, at variance with pp→W, the LO order process already proceeds through a loop.

In this case, this means that the loop calculation has to give a finite result!



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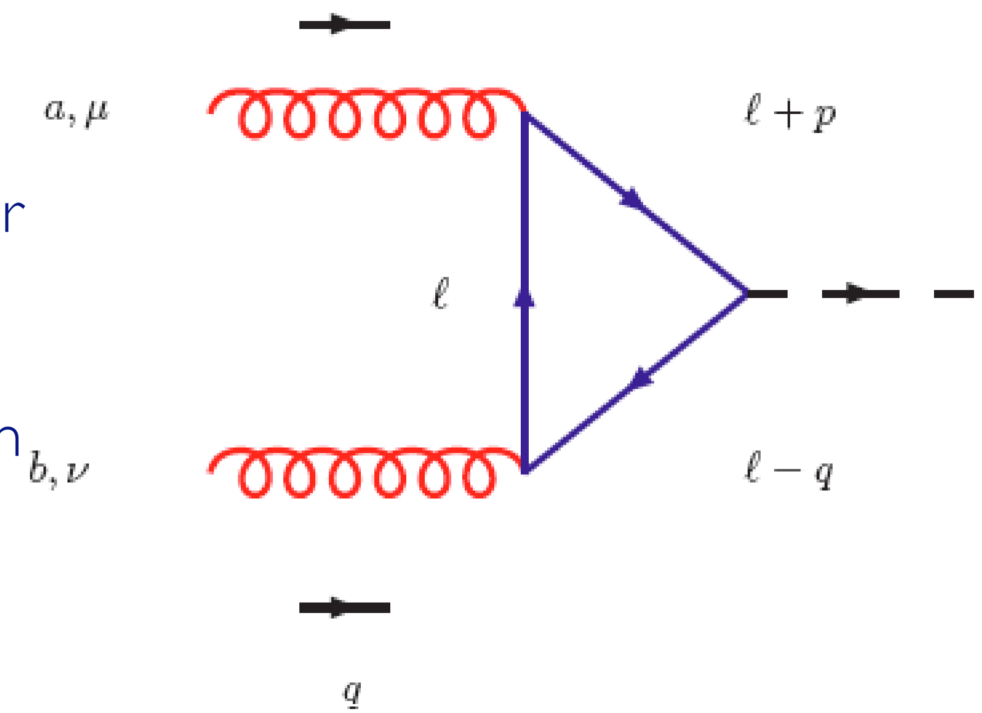

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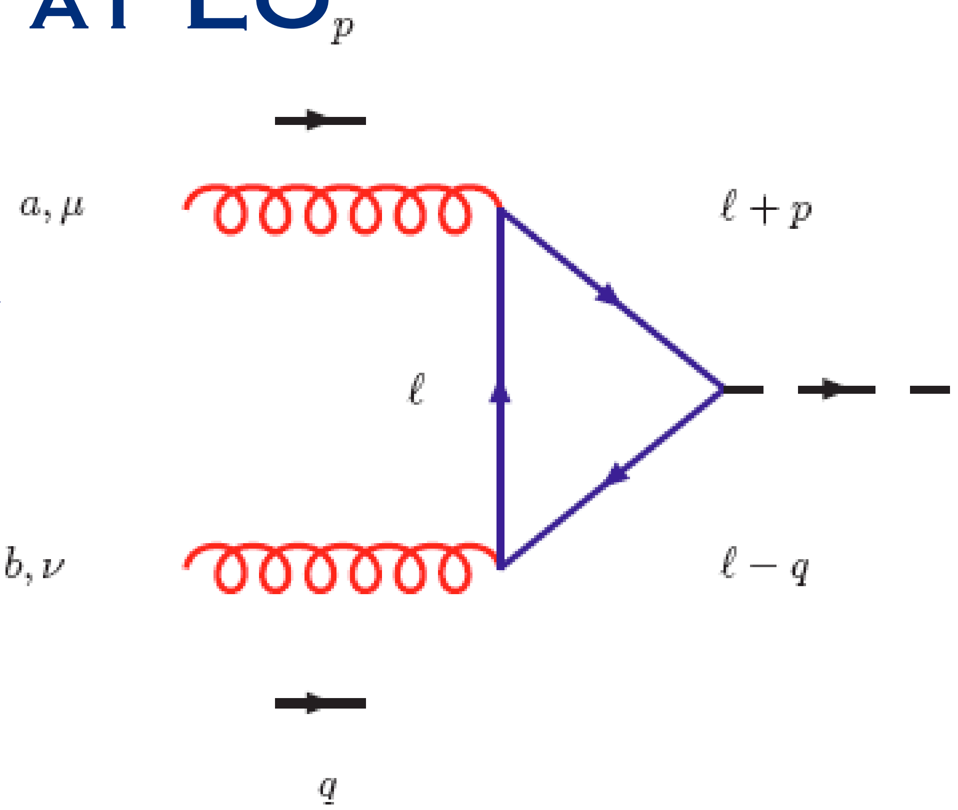

$$i\mathcal{A} = -(-ig_s)^2 \mathrm{Tr}(t^a t^b)\left(\frac{-im_t}{v}\right)\int \frac{d^d\ell}{(2\pi)^n}\frac{T^{\mu\nu}}{\mathrm{Den}}(i)^3 \epsilon_\mu(p)\epsilon_\nu(q)$$

where

$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$$

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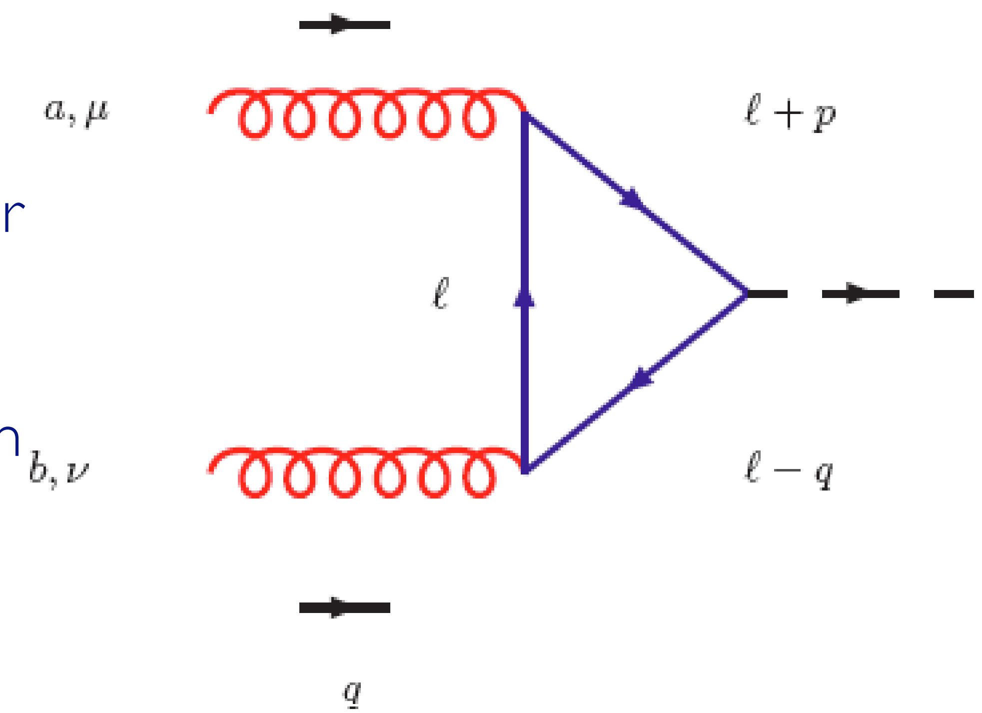

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where

$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$$

We combine the denominators into one by using $\displaystyle \frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1-x-y)]^3}$

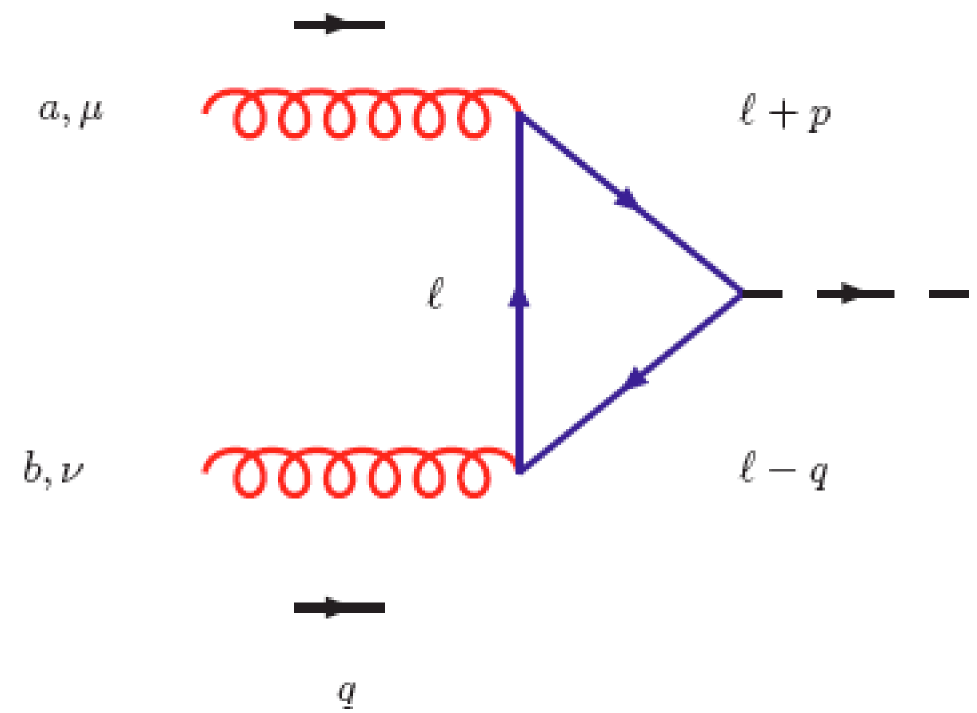
$$\frac{1}{\mathrm{Den}} = 2\int dx\ dy \frac{1}{[\ell^2 - m_t^2 + 2\ell \cdot (px - qy)]^3}.$$

pp→H+x at LO$_p$

We shift the momentum:

$$\ell' = \ell + px - qy$$

$$\frac{1}{\text{Den}} \to 2\int dx\ dy \frac{1}{[\ell'^{\,2} - m_t^2 + M_H^2 xy]^3}.$$



PP→H+X AT LO

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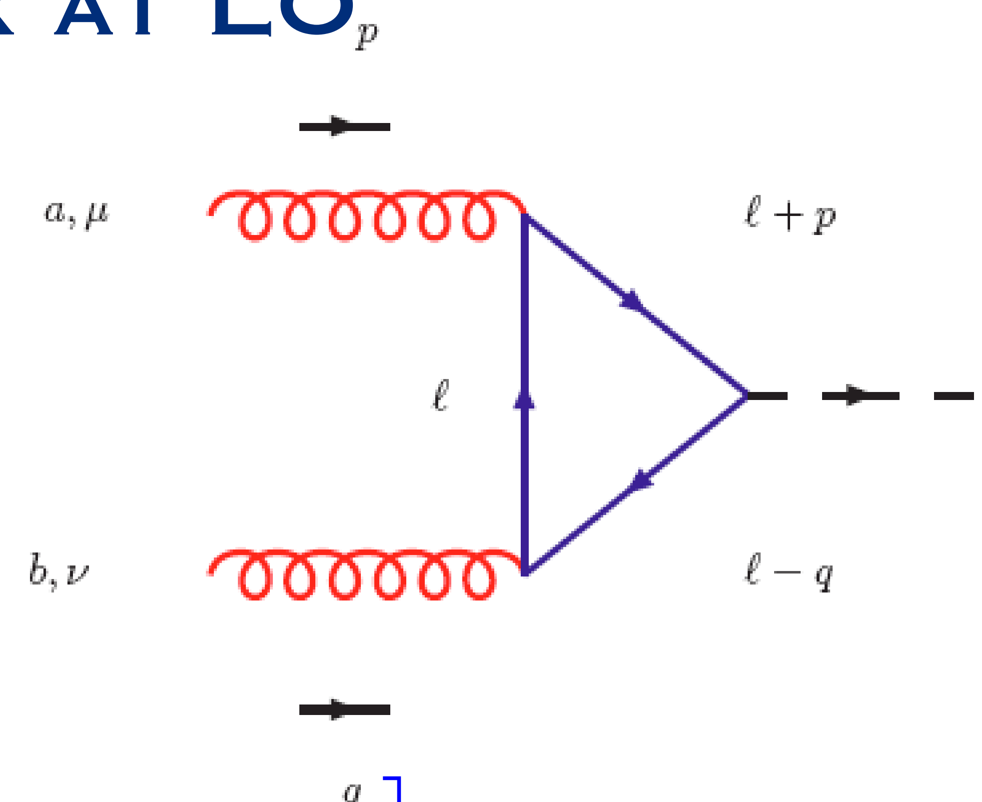

And now the tensor in the numerator:

$$T^{\mu\nu} = \mathrm{Tr}\left[(\ell + m_t)\gamma^\mu(\ell + p + m_t)(\ell - q + m_t)\gamma^\nu)\right]$$

$$= 4m_t\left[g^{\mu\nu}(m_t^2 - \ell^2 - \frac{M_H^2}{2}) + 4\ell^\mu\ell^\nu + p^\nu q^\mu\right]$$

where I used the fact that the external gluons are on-shell. This trace is proportional to mt ! This is due to the spin flip caused by the scalar coupling.

pp→H+x at LO

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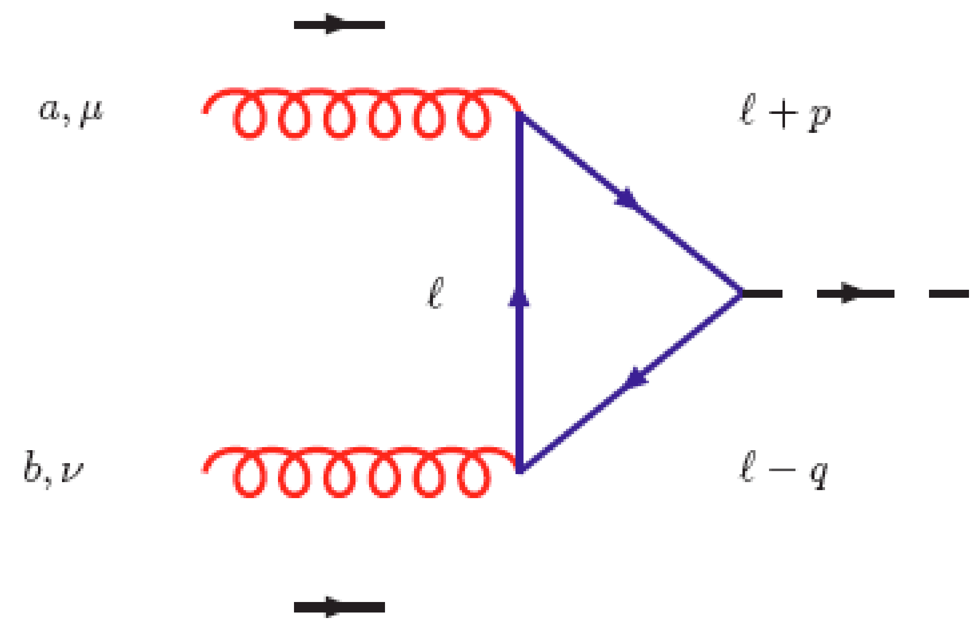

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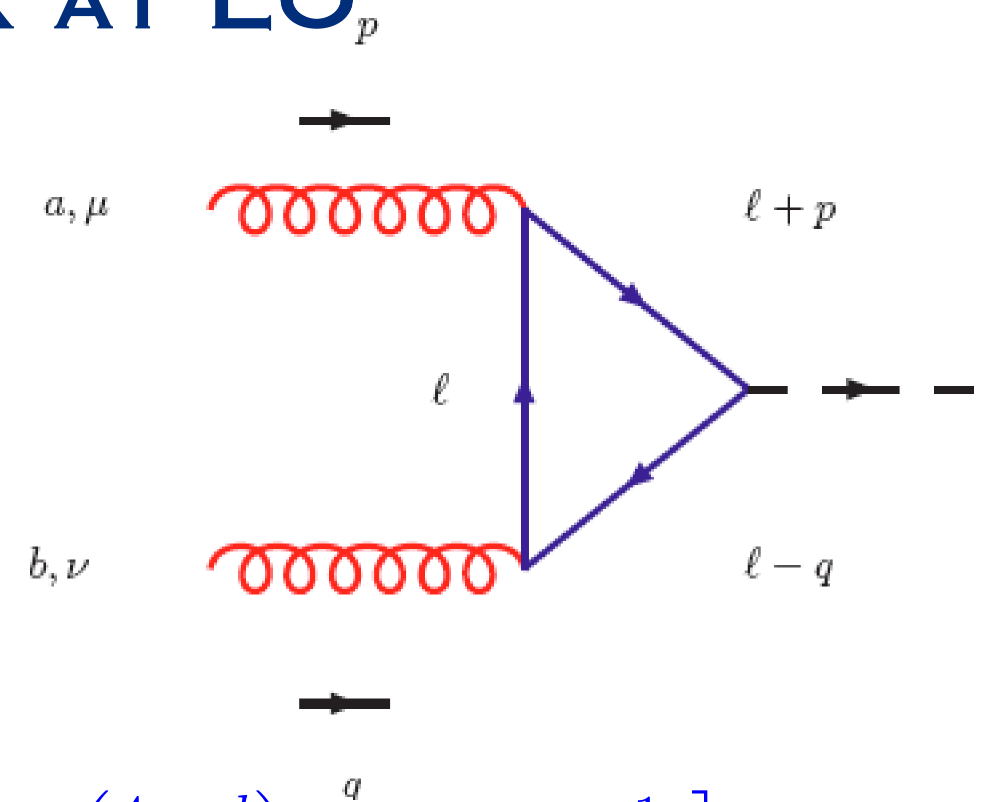
where I used the fact that the external gluons are on-shell. This trace is proportional to mt ! This is due to the spin flip caused by the scalar coupling.

Now we shift the loop momentum also here, we drop terms linear in the loop momentum (they are odd and vanish) and

pp→H+x at LO

We perform the tensor decomposition using:

$$\int d^dk \frac{k^\mu k^\nu}{(k^2-C)^m} = \frac{1}{d} g^{\mu\nu} \int d^dk \frac{k^2}{(k^2-C)^m}$$



So I can write an expression which depends only on scalar loop integrals:

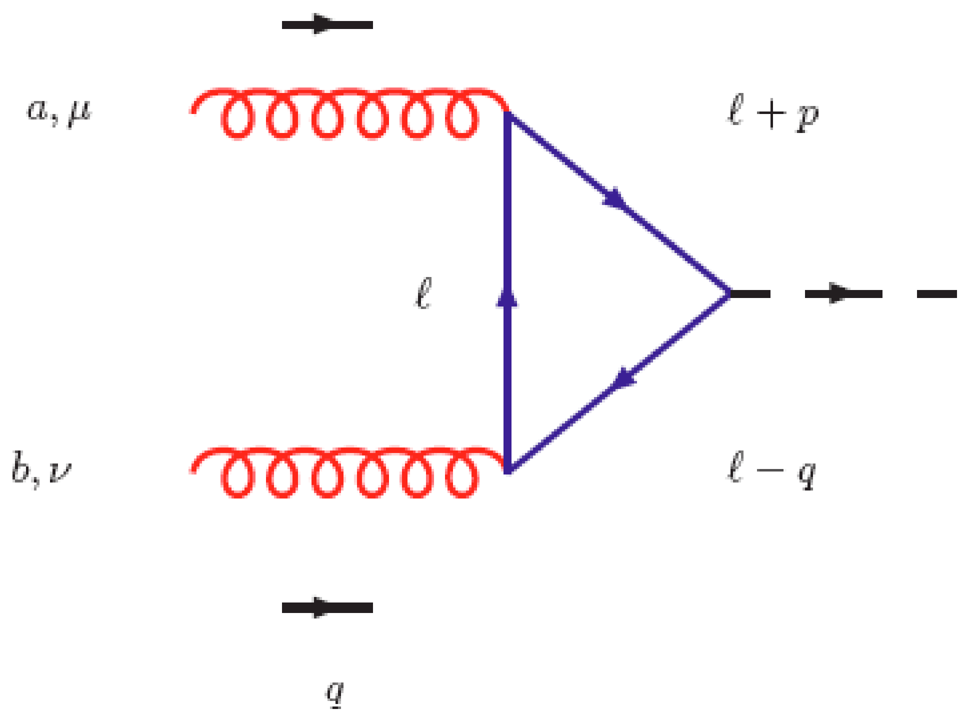
$$i\mathcal{A} = -\frac{2g_s^2 m_t^2}{v}\delta^{ab}\int \frac{d^d\ell'}{(2\pi)^d}\int dxdy \Big\{ g^{\mu\nu}\left[m^2 + \ell'^2\left(\frac{4-d}{d}\right) + M_H^2(xy - \frac{1}{2})\right]$$
$$+p^\nu q^\mu (1-4xy)\Big\} \frac{2dxdy}{(\ell'^2 - m_t^2 + M_H^2 xy)^3}\epsilon_\mu(p)\epsilon_\nu(q).$$

There's a term which apparently diverges....??
Ok, Let's look the scalar integrals up in a table (or calculate them!)

pp→H+x at LO$_p$

$$\int \frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^2-C)^3} = \frac{i}{32\pi^2}(4\pi)^\epsilon\frac{\Gamma(1+\epsilon)}{\epsilon}(2-\epsilon)C^{-\epsilon}$$
$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-C)^3} = -\frac{i}{32\pi^2}(4\pi)^\epsilon\Gamma(1+\epsilon)C^{-1-\epsilon}.$$

where d=4-2eps. By substituting we arrive at a very simple final result!!



$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

Comments:

* The final dependence of the result is m_t^2 : one from the Yukawa coupling, one from the spin flip.
* The tensor structure could have been guessed by gauge invariance.
* The integral depends on m_t and m_h.

LO cross section

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \, g(x_1, \mu_f) g(x_2, \mu_f) \, \hat{\sigma}(gg \to H)$$

$$x_1 \equiv \sqrt{\tau} e^{y} \quad x_2 \equiv \sqrt{\tau} e^{-y} \quad \tau = x_1 x_2 \qquad \tau_0 = M_H^2/S \quad z = \tau_0/\tau$$

$$= \frac{\alpha_S^2}{64\pi v^2} \mid I\left(\frac{M_H^2}{m^2}\right) \mid^2 \tau_0 \int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy g(\sqrt{\tau_0} e^{y}) g(\sqrt{\tau_0} e^{-y})$$

The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

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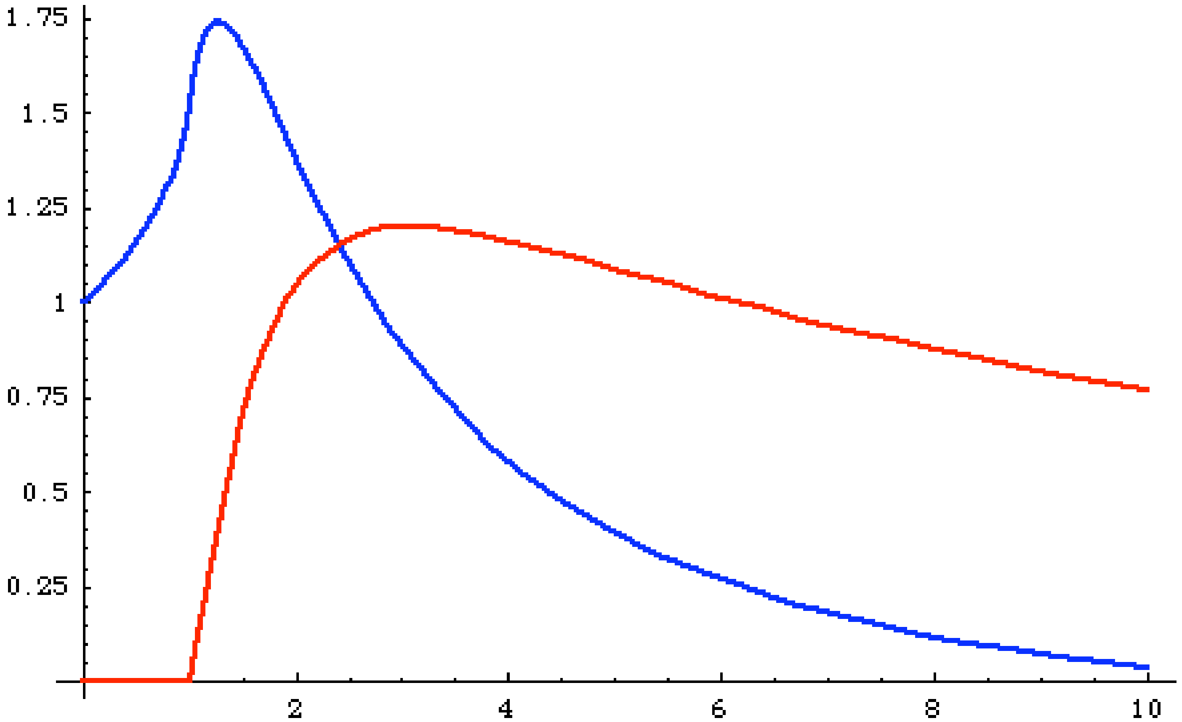

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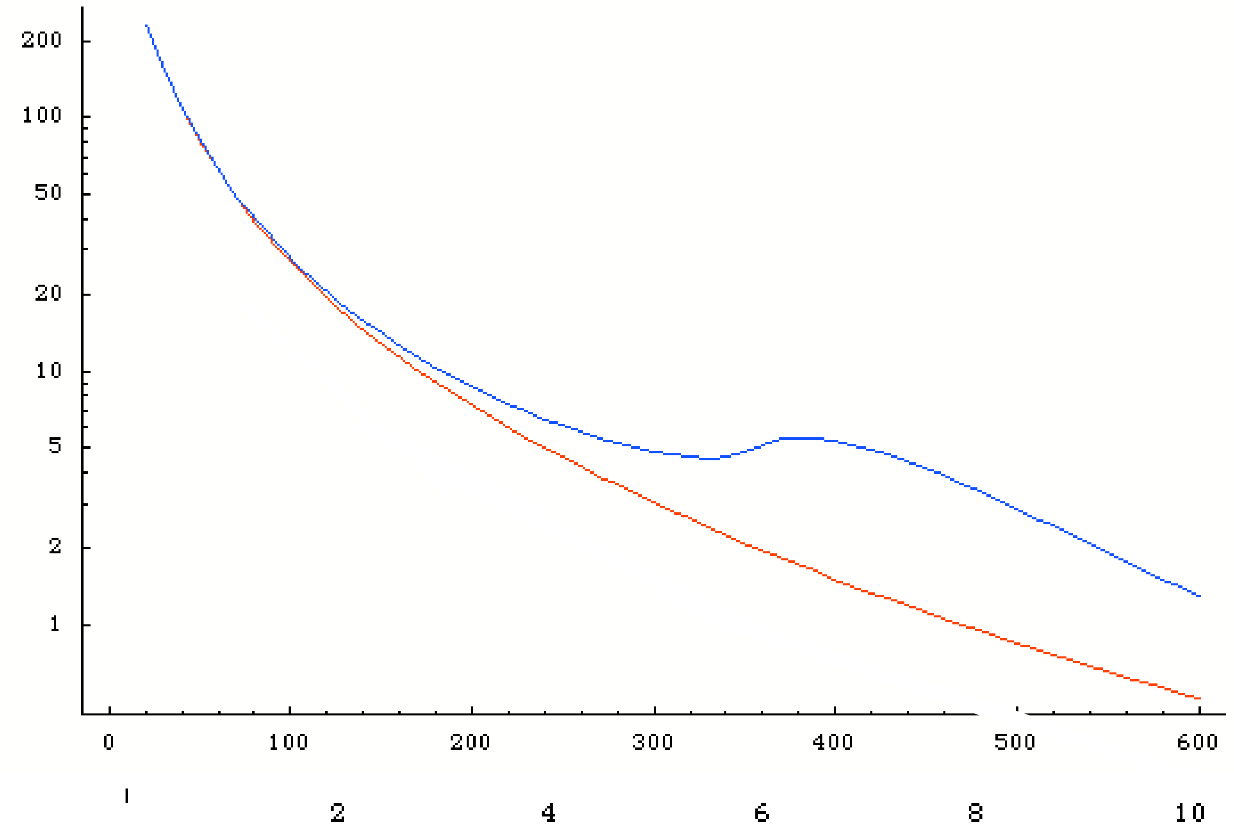
The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

I(x) has both a real and imaginary part, which develops at $m_h = 2m_t$.

This causes a bump in the cross section.

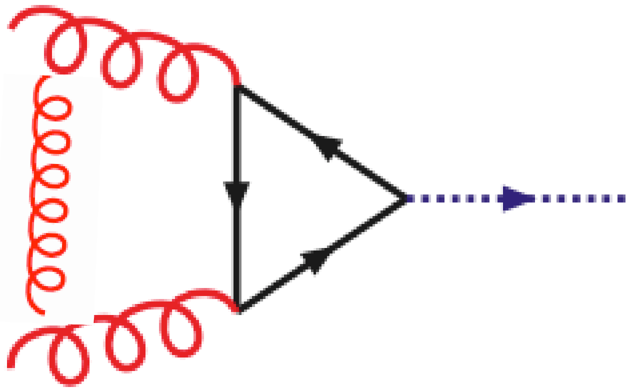


pp → H+x @ NLO

At NLO we have to include an extra parton (virtual or real).

The virtuals will become a two-loop calculation!!

Can we avoid that?



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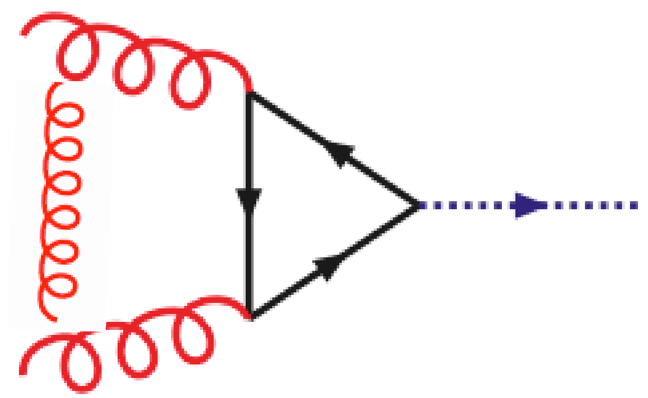

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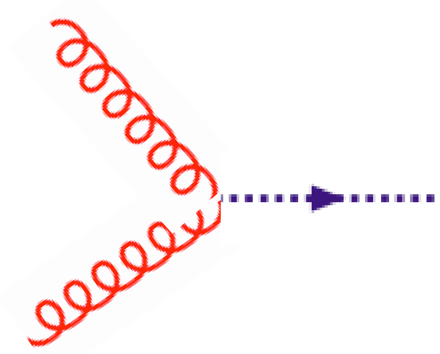
Can we avoid that?

Let's consider the case where the Higgs is light:

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

$$\xrightarrow{m \gg M_H} -\frac{\alpha_S}{3\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\epsilon_\mu(p)\epsilon_\nu(q).$$



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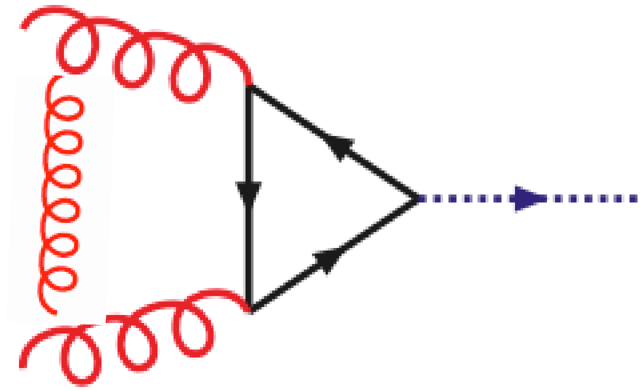

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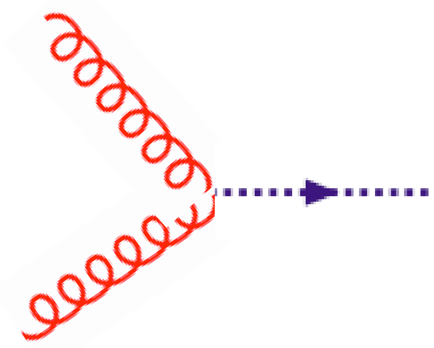


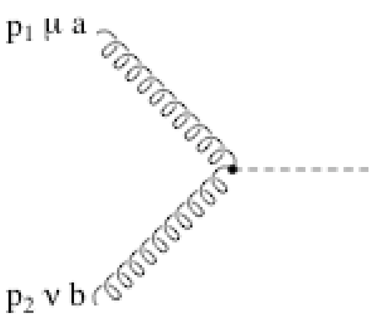
This looks like a local vertex, ggH.

The top quark has disappeared from the low energy theory but it has left something behind (non-decoupling).

Higgs effective field theory

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}\left(1 - \frac{\alpha_S}{3\pi}\frac{H}{v}\right) G^{\mu\nu}G_{\mu\nu}$$

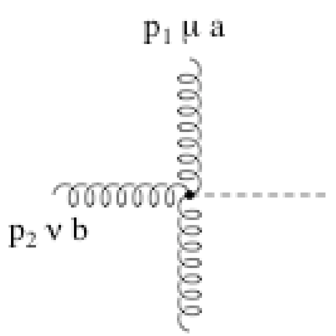
This is an effective non-renormalizable theory (no top) which describes the Higgs couplings to QCD.



(a)

$iA\delta^{ab}H^{\mu\nu}(p_1,p_2)$

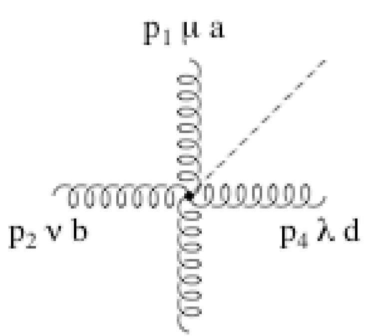
$$H^{\mu\nu}(p_1,p_2) = g^{\mu\nu}p_1\cdot p_2 - p_1^{\nu}p_2^{\mu}.$$



(b)

$-Agf^{abc}V^{\mu\nu\sigma}(p_1,p_2,p_3)$

$$V^{\mu\nu\rho}(p_1,p_2,p_3) = (p_1-p_2)^{\rho}g^{\mu\nu} + (p_2-p_3)^{\mu}g^{\nu\rho} + (p_3-p_1)^{\nu}g^{\rho\mu},$$



(c)

$-iAg^2X^{abcd}_{\mu\nu\sigma\lambda}$

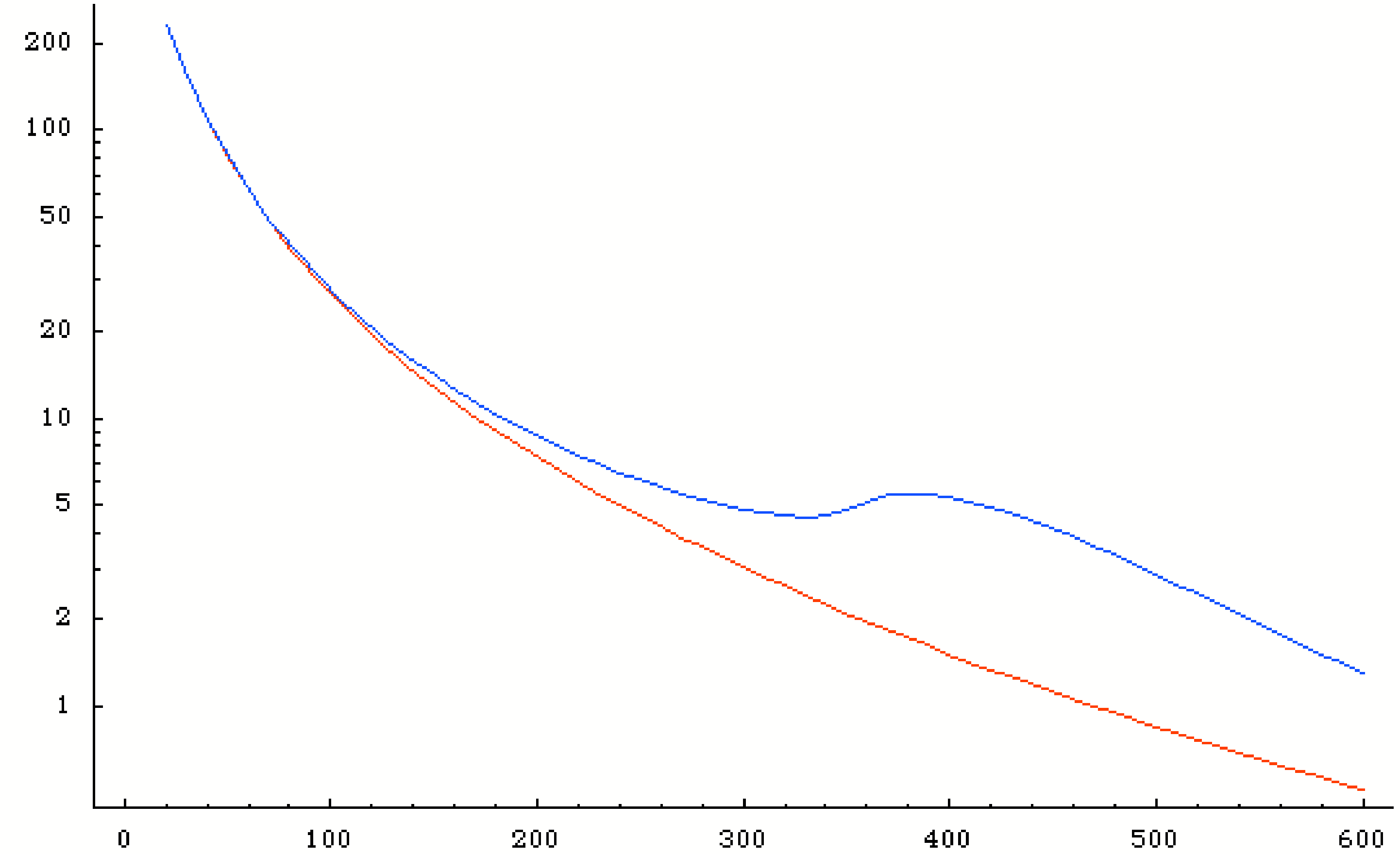
$$\begin{aligned} X^{\mu\nu\rho\sigma}_{abcd} &= f_{abe}f_{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f_{ace}f_{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f_{ade}f_{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma}). \end{aligned}$$

LO cross section: full vs HEFT

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, g(x_1,\mu_f) g(x_2,\mu_f)\, \hat{\sigma}(gg \to H)$$

The accuracy of the calculation in the HEFT calculation can be directly assessed by taking the limit $m \to \infty$.

For light Higgs is better than 10%.

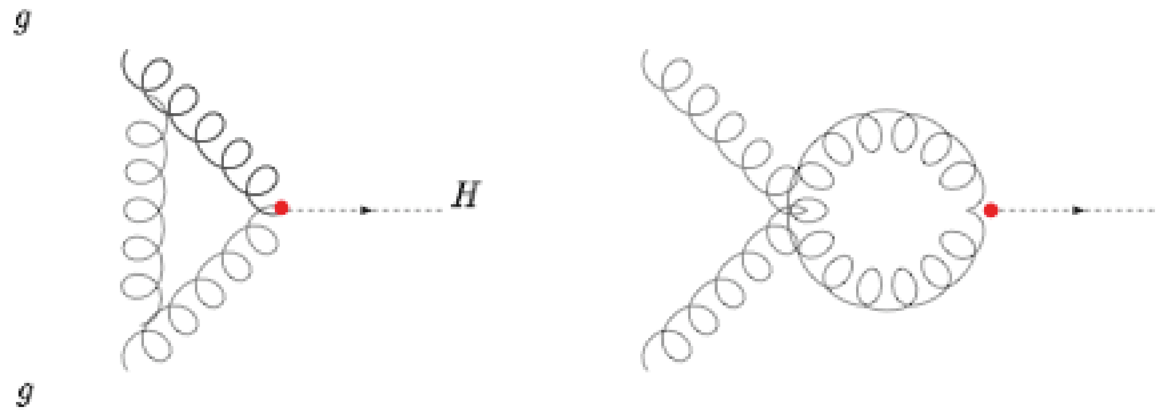


So, if we are interested in a light Higgs we use the HEFT and simplify our life. If we do so, the NLO calculation becomes a standard 1-loop calculation, similar to Drell-Yan at NLO.

We can (try to) do it!!

Virtual contributions

Virtual contributions



Out of 8 diagrams, only two are non-zero (in dimensional regularization), a bubble and a triangle.

They can be easily written down by hand.

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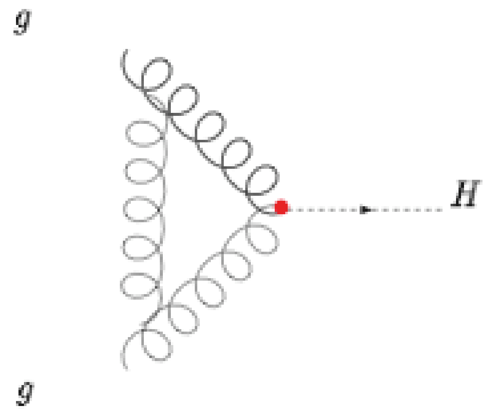

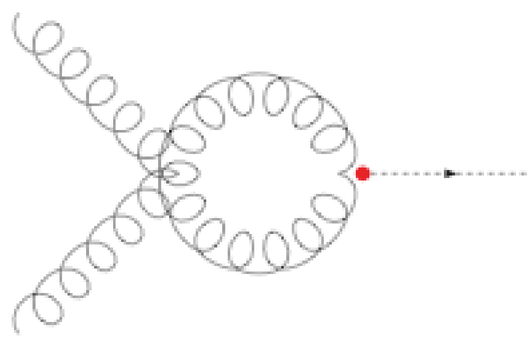

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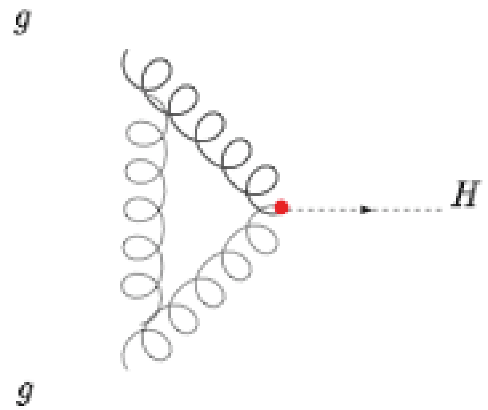

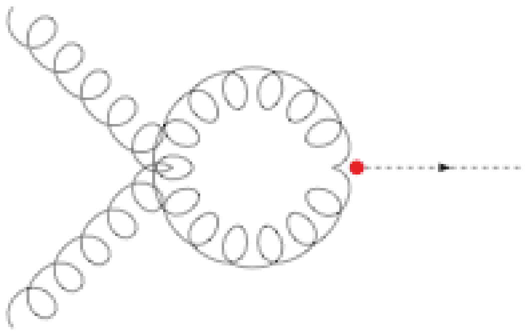

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$$\mathcal{L}_{\mathrm{eff}}^{\mathrm{NLO}} = \left(1 + \frac{11}{4}\frac{\alpha_S}{\pi}\right)\frac{\alpha_S}{3\pi}\frac{H}{v}G^{\mu\nu}G_{\mu\nu}$$

One also have to consider that the coefficient of the HEFT receive corrections which have to be included in the result.

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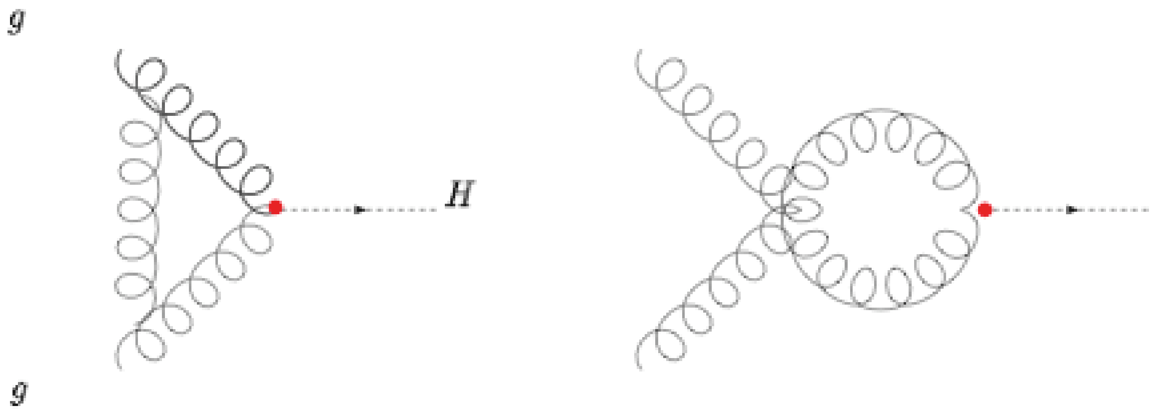

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One also have to consider that the coefficient of the HEFT receive corrections which have to be included in the result.

The result is:

$$\sigma_{\rm virt} = \sigma_0\,\delta(1-z)\,\left[1 + \frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma\left(-\frac{2}{\epsilon^2} + \frac{11}{3} + \pi^2\right)\right]\,,$$

$$\sigma_{\rm Born} = \frac{\alpha_S^2}{\pi}\frac{m_H^2}{576 v^2 s}(1+\epsilon+\epsilon^2)\mu^{2\epsilon}\,\delta(1-z) \equiv \sigma_0\,\delta(1-z) \qquad z = m_H^2/s$$

Real contributions I

$$\sigma^{\overline{\rm MS}}(qg) = \sigma_{\rm real} + \sigma^{\rm coll.}_{\rm c.t.}$$
$$= \sigma_0 \frac{\alpha_S}{2\pi} C_F \left[p_{gq}(z) \log \frac{m_H^2}{\mu_F^2} + p_{gq}(z) \log \frac{(1-z)^2}{z} + \frac{(1-z)(7z-3)}{2z} \right]$$

$$\sigma^{\rm coll.}_{\rm c.t.} = \sigma_0 \frac{\alpha_S}{2\pi} \left[\left(\frac{\mu^2}{\mu_F^2} \right)^{\epsilon} \frac{c_\Gamma}{\epsilon} P_{gq}(z) \right]$$

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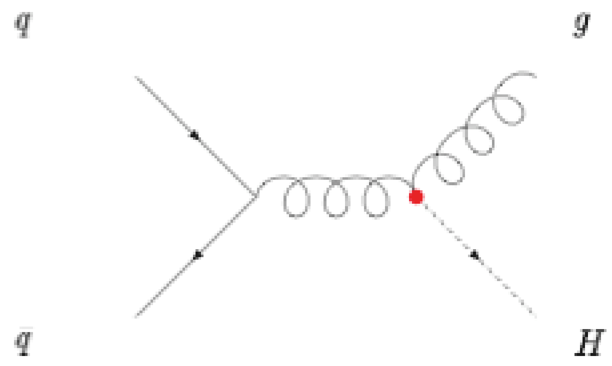

$$\overline{|\mathcal{M}|^2} = \frac{4}{81}\frac{\alpha_S^3}{\pi v^2}\frac{(\hat{u}^2+\hat{t}^2)-\epsilon(\hat{u}+\hat{t})^2}{\hat{s}}$$

Integrating over phase space (cms angle theta)

$$\hat{t} = -\hat{s}(1-z)(1-\cos\theta)/2$$
$$\hat{u} = -\hat{s}(1-z)(1+\cos\theta)/2$$

$$\sigma_{\rm real}(q\bar{q}) = \sigma_0\frac{\alpha_S}{2\pi}\frac{64}{27}\frac{(1-z)^3}{z}$$

$$\sigma^{\rm coll.}_{\rm c.t.} = \sigma_0\frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu_F^2}\right)^\epsilon\frac{c_\Gamma}{\epsilon}P_{gq}(z)\right]$$

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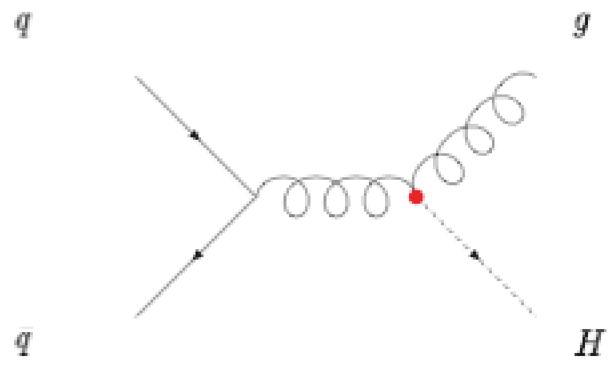

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finite!

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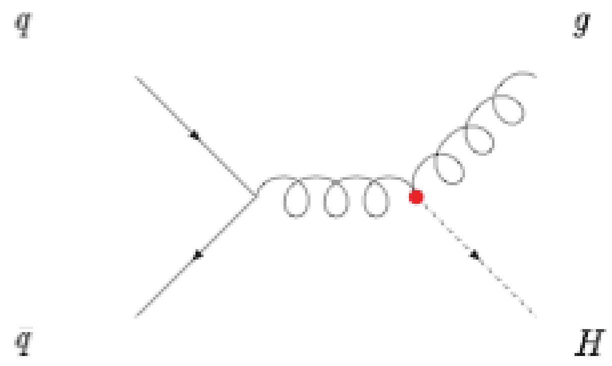

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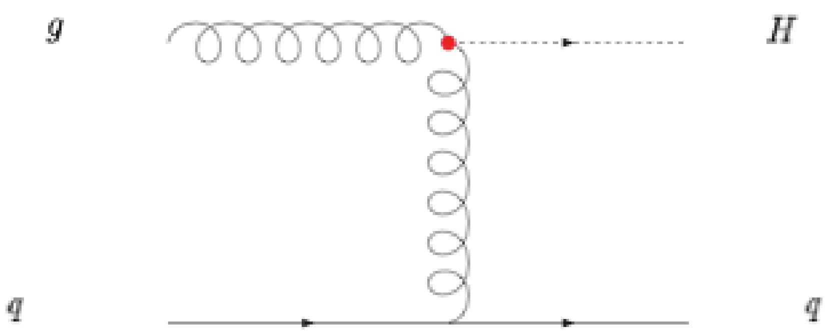

$$\overline{|\mathcal{M}|^2} = -\frac{1}{54(1-\epsilon)}\frac{\alpha_S^3}{\pi v^2}\,\frac{(\hat{u}^2+\hat{s}^2)-\epsilon(\hat{u}+\hat{s})^2}{\hat{t}}$$

Integrating over the D-dimensional phase space the collinear singularity manifests a pole in 1/eps

$$\sigma_{\rm real} = \sigma_0\,\frac{\alpha_S}{2\pi}C_F\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma\,\left[-\frac{1}{\epsilon}p_{gq}(z)+\frac{(1-z)(7z-3)}{2z}+p_{gq}(z)\log\frac{(1-z)^2}{z}\right]$$

$$\sigma^{\rm coll.}_{\rm c.t.} = \sigma_0\frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}\frac{c_\Gamma}{\epsilon}P_{gq}(z)\right]$$

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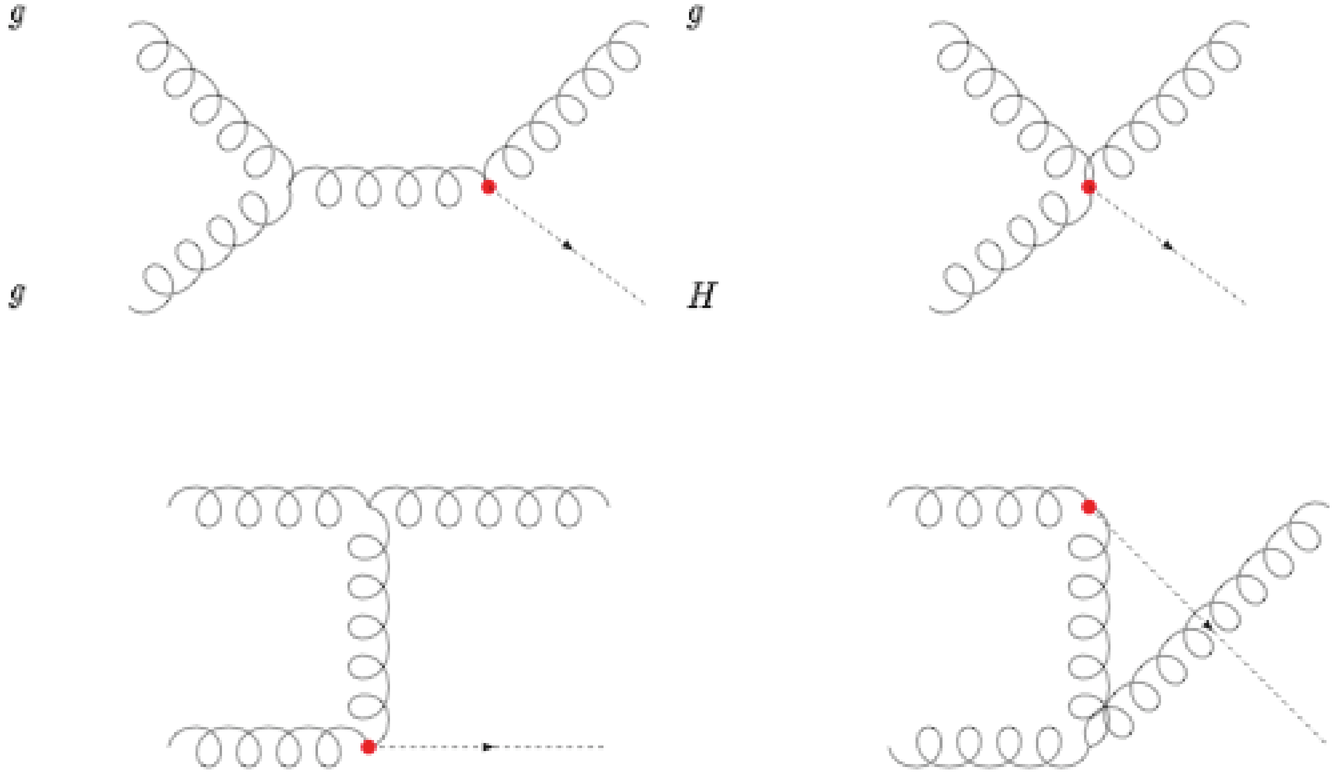
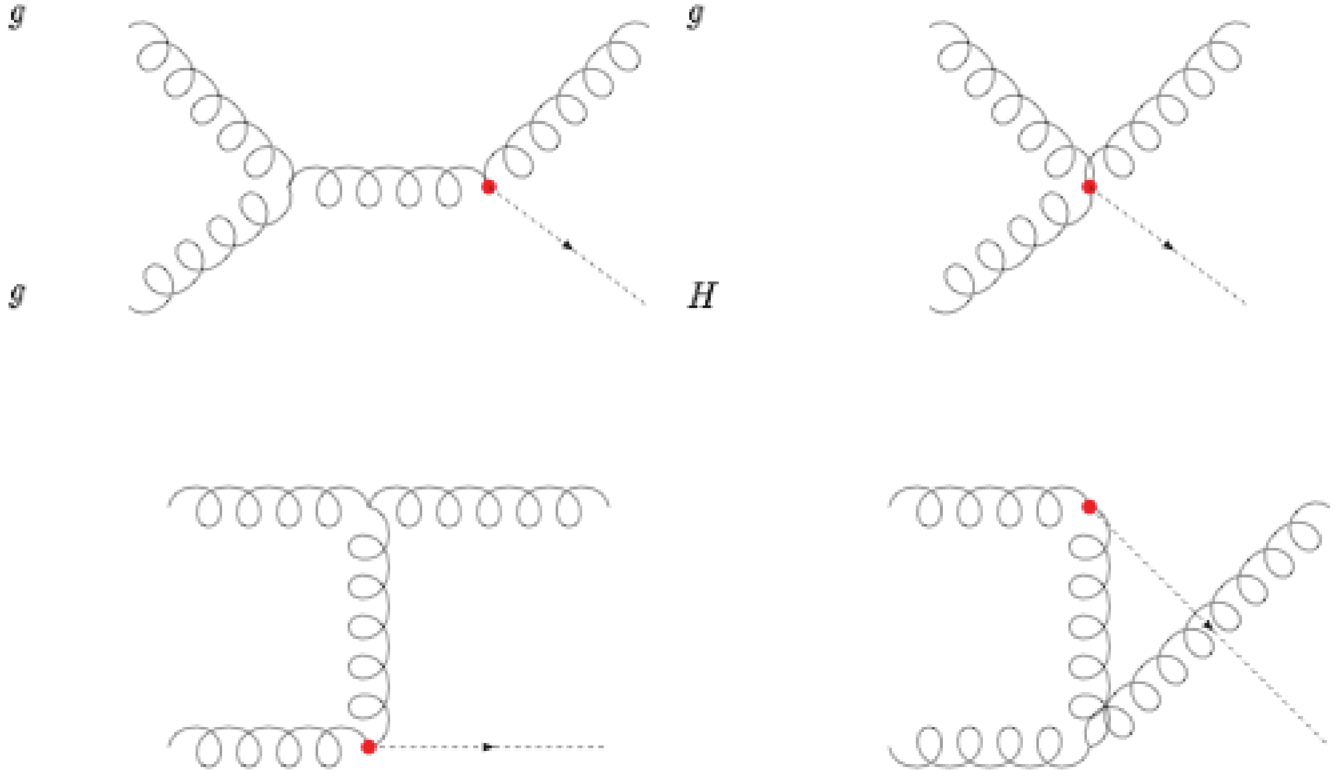
Real contributions II



This is the last piece: the result at the end must be finite!

REAL CONTRIBUTIONS II



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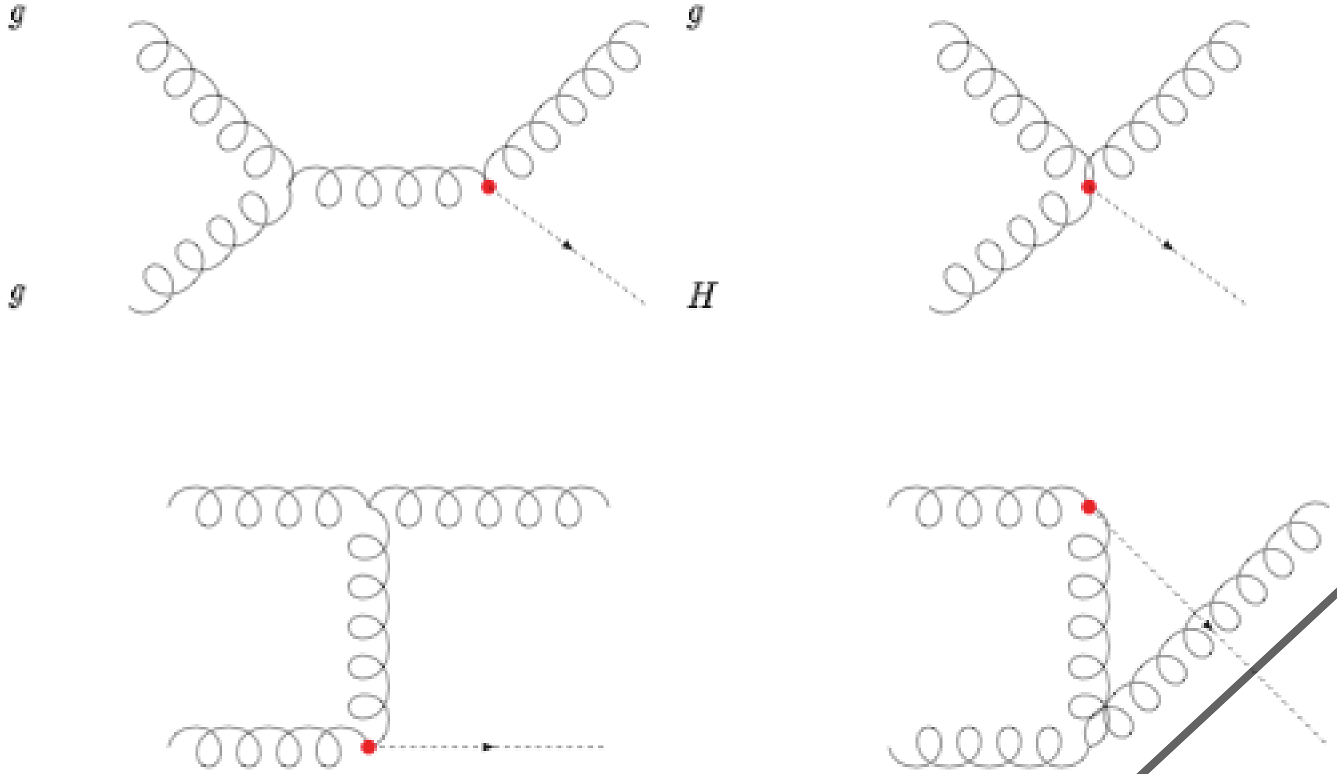
$$
\begin{aligned}
\sigma_{\rm real} \;=\; & \sigma_0 \frac{\alpha_S}{2\pi} C_A \left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma \left[\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\frac{b_0}{C_A} - \frac{\pi^2}{3}\right)\delta(1-z) \right. \\
& - \frac{2}{\epsilon} p_{gg}(z) - \frac{11}{3}\frac{(1-z)^3}{z} - 4\frac{(1-z)^2(1+z^2)+z^2}{z(1-z)}\log z \\
& \left. + 4\frac{1+z^4+(1-z)^4}{z}\left(\frac{\log(1-z)}{1-z}\right)_+ \right] .
\end{aligned}
$$

Real contributions II



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2/eps cancels with the virtual contribution ✓

$$\sigma_{\rm real} = \sigma_0 \frac{\alpha_S}{2\pi} C_A \left(\frac{\mu^2}{m_H^2}\right)^\epsilon c_\Gamma \left[\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\frac{b_0}{C_A} - \frac{\pi^2}{3} \right) \delta(1-z) \right.$$
$$- \frac{2}{\epsilon} p_{gg}(z) - \frac{11}{3}\frac{(1-z)^3}{z} - 4\frac{(1-z)^2(1+z^2)+z^2}{z(1-z)} \log z$$
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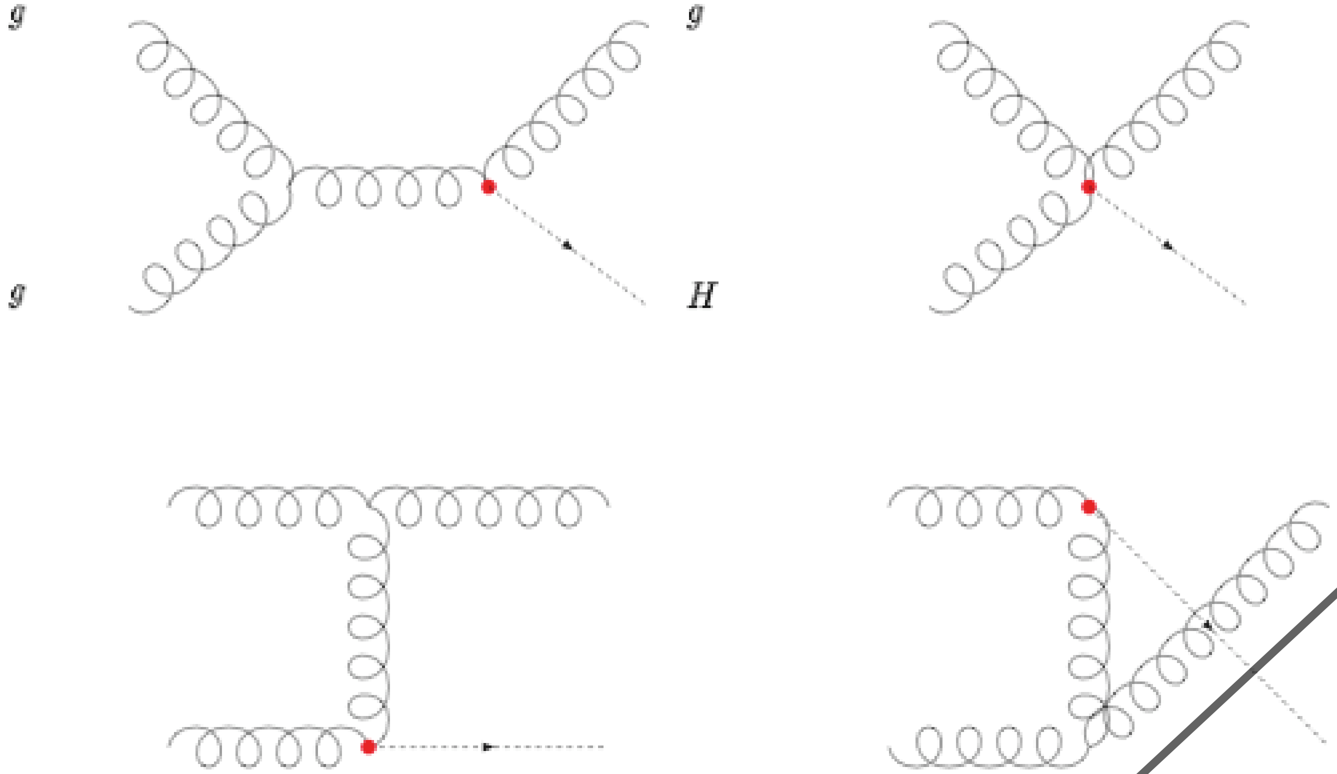

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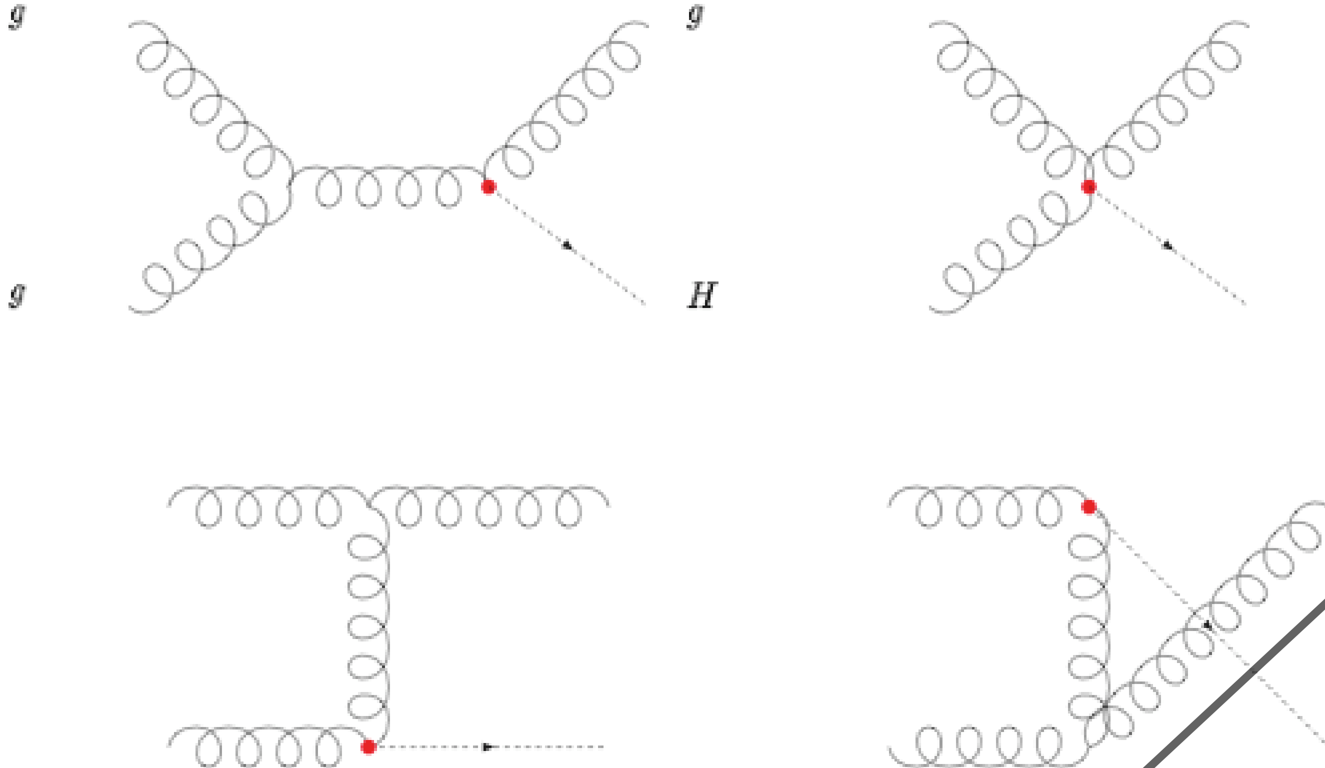
This is the renormalization of the coulping!!

$$\sigma^{\rm UV}_{\rm c.t.} = 2\ \sigma_{\rm Born} \frac{\alpha_S}{2\pi} \left[- \left(\frac{\mu^2}{\mu^2_{\rm UV}} \right)^\epsilon c_\Gamma \frac{b_0}{\epsilon} \right] ✓$$

$$\begin{aligned}\sigma_{\rm real} \quad = \quad & \sigma_0 \frac{\alpha_S}{2\pi} C_A \left(\frac{\mu^2}{m_H^2} \right)^\epsilon c_\Gamma \left[\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \frac{b_0}{C_A} - \frac{\pi^2}{3} \right) \delta(1-z) \right. \\ & - \frac{2}{\epsilon} p_{gg}(z) - \frac{11}{3} \frac{(1-z)^3}{z} - 4 \frac{(1-z)^2 (1+z^2) + z^2}{z(1-z)} \log z \\ & \left. + 4 \frac{1 + z^4 + (1-z)^4}{z} \left(\frac{\log(1-z)}{1-z} \right)_+ \right] .\end{aligned}$$

Real contributions II



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$$\sigma^{\rm UV}_{\rm c.t.} = 2\,\sigma_{\rm Born}\frac{\alpha_S}{2\pi}\left[-\left(\frac{\mu^2}{\mu^2_{\rm UV}}\right)^{\epsilon} c_\Gamma \frac{b_0}{\epsilon}\right] ✓$$

This is an initial-state divergence to be reabsorbed in the pdf

$$\sigma^{\rm coll.}_{\rm c.t.} = 2\,\sigma_0\frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}\frac{c_\Gamma}{\epsilon}P_{gg}(z)\right] ✓$$

$$\begin{aligned}\sigma_{\rm real} &= \sigma_0\,\frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma \left[\left(\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\frac{b_0}{C_A}-\frac{\pi^2}{3}\right)\delta(1-z)\right.\\ &\quad -\frac{2}{\epsilon}p_{gg}(z)-\frac{11}{3}\frac{(1-z)^3}{z}-4\frac{(1-z)^2(1+z^2)+z^2}{z(1-z)}\log z\\ &\quad \left.+4\frac{1+z^4+(1-z)^4}{z}\left(\frac{\log(1-z)}{1-z}\right)_+\right].\end{aligned}$$

Final results = you made it!!

$$\sigma(pp \to H) = \sum_{ij} \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 f_i(x_1, \mu_f) f_j(x_2, \mu_f) \hat{\sigma}(ij)[\mu_f/m_h, \mu_r/m_h, \alpha_S(\mu_r)]$$

The final cross section is the sum of three channels: q qbar, q g, and g g.

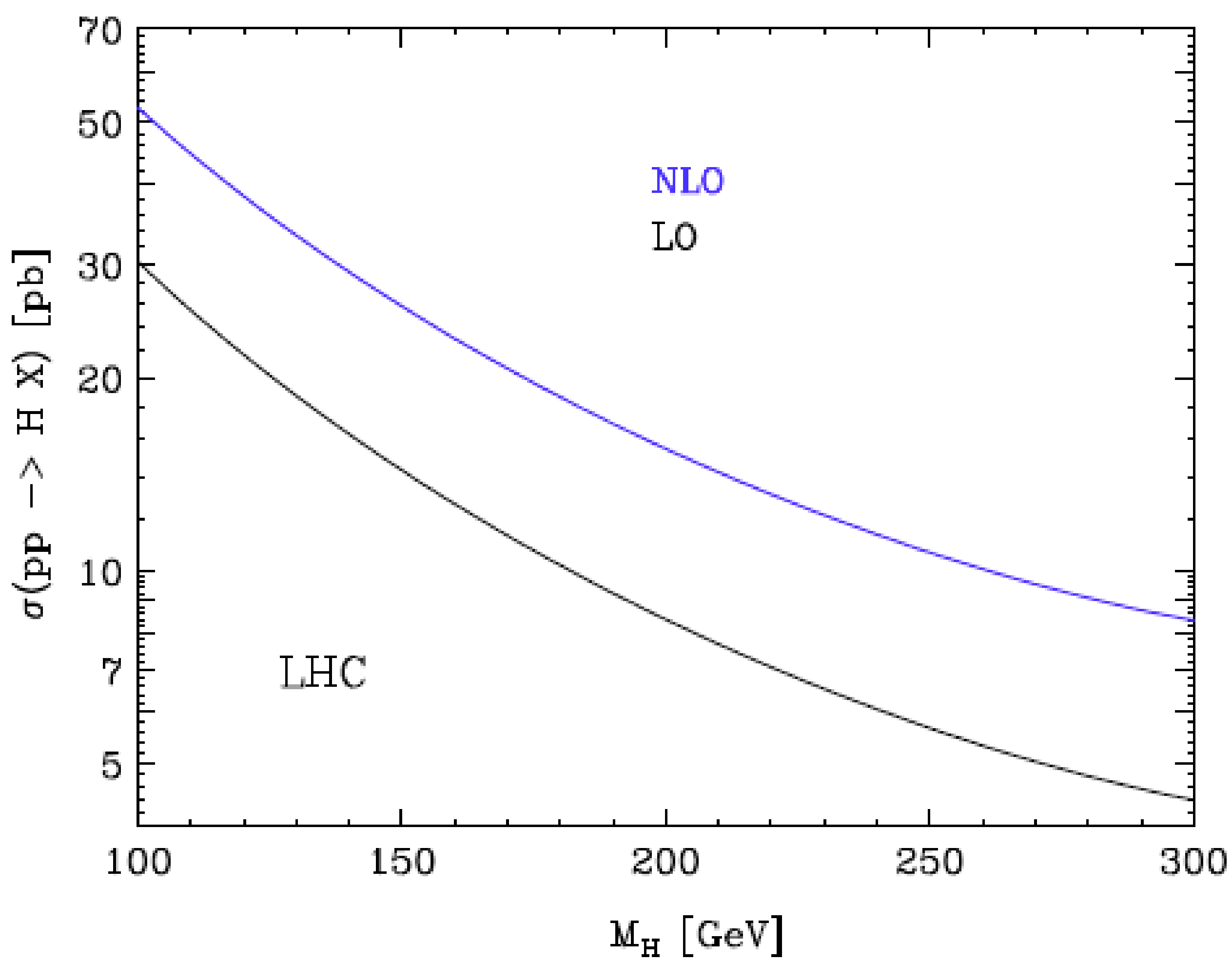
The short distance cross section at NLO depends explicitly on the subtraction scales (renormalization and factorization).

The explicit integration over the pdf's is trivial (just mind the plus distributions).

The result is that the corrections are huge!

K factor is ~2 and scale dependence not really very much improved.

Is perturbation theory valid? NNLO is mandatory...



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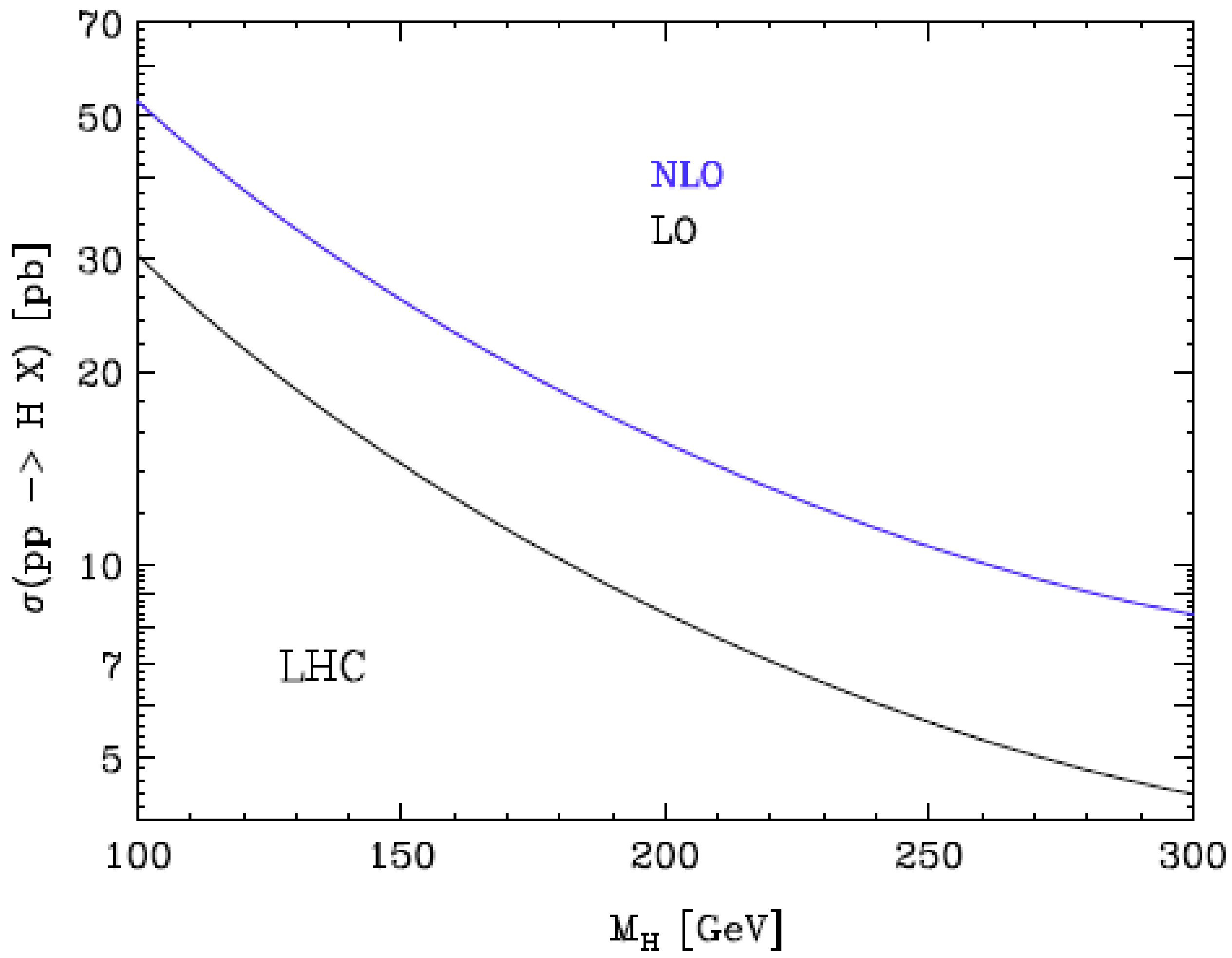

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