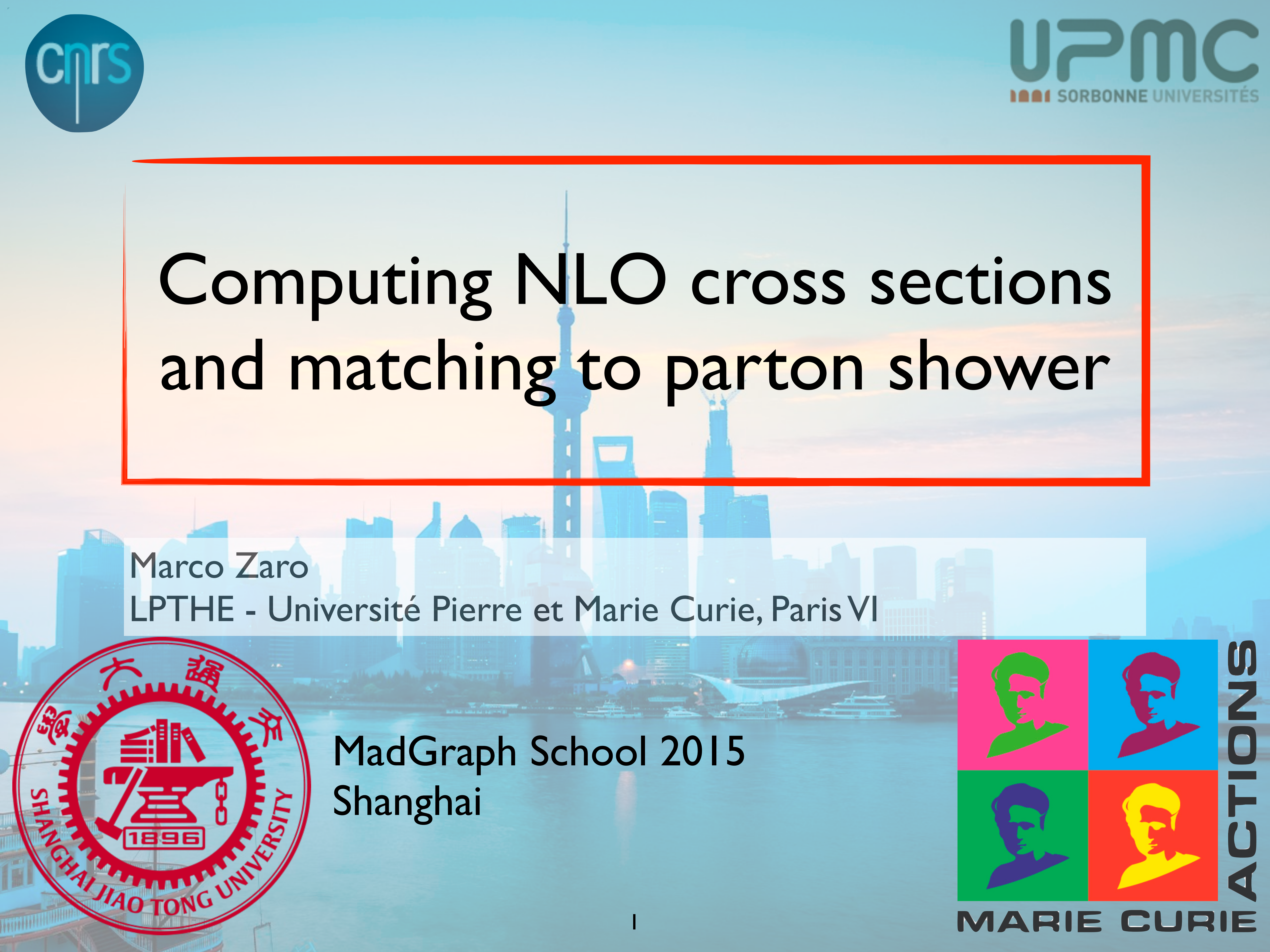

Computing NLO cross sections and matching to parton shower

Marco Zaro
LPTHE - Université Pierre et Marie Curie, Paris VI

MadGraph School 2015
Shanghai

Introduction:
Why do we need N$^{(k)}$LO?

1) Discoveries at hadron colliders

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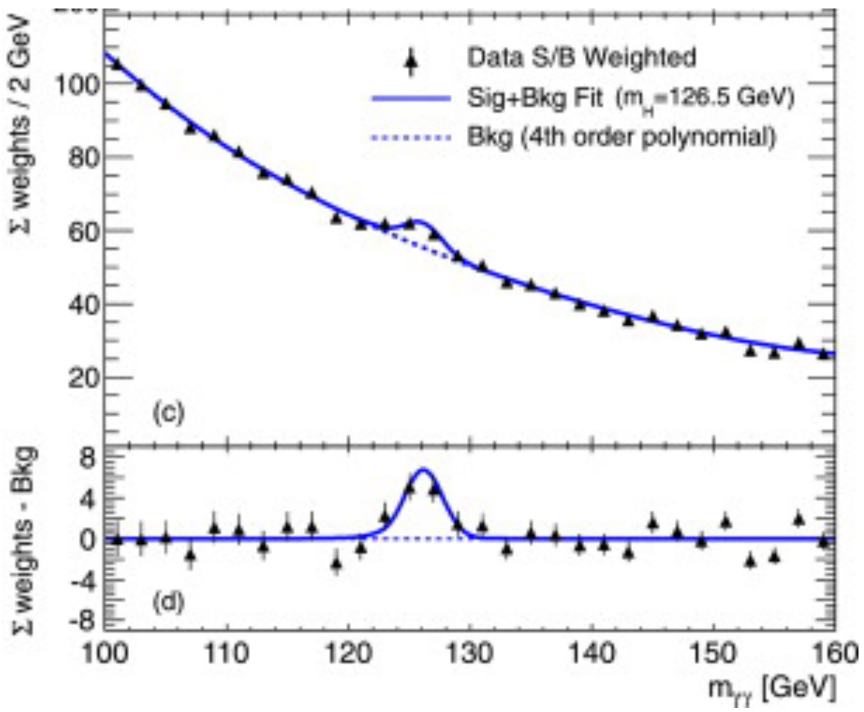
Peak

$H \to \gamma\gamma$



EASY

Background directly measured from data.
Theory needed only for parameter extraction

1) Discoveries at hadron colliders

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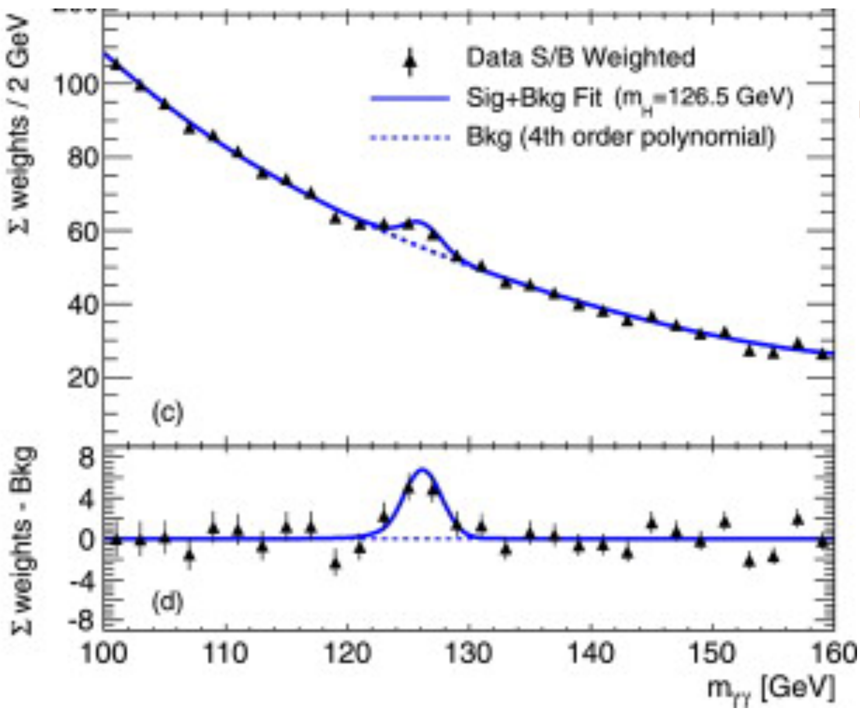
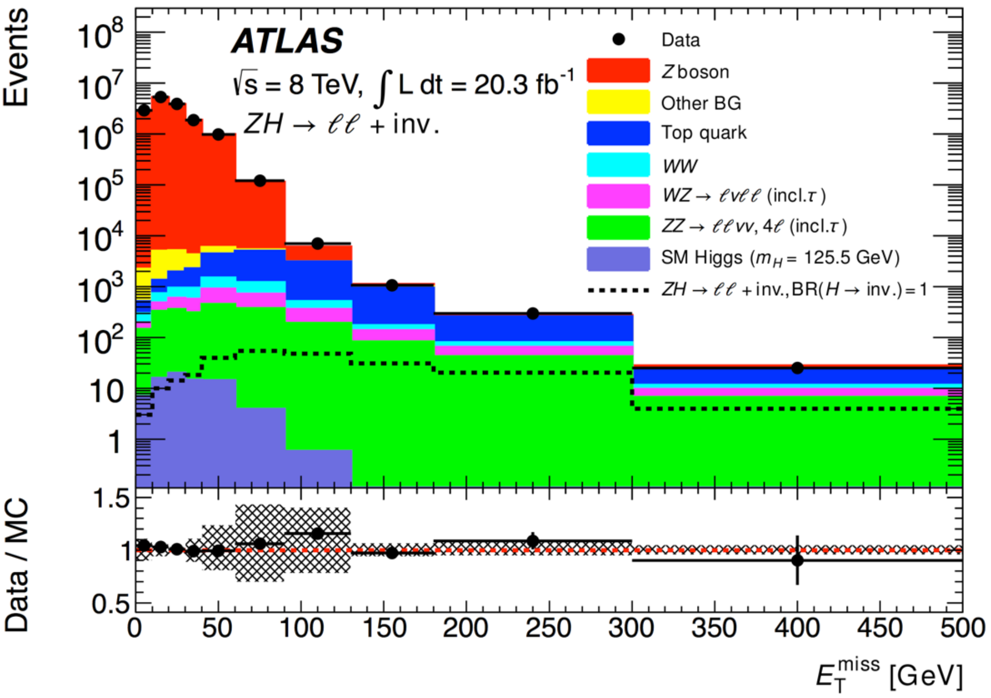
$H \to \gamma\gamma$

Shape

$Z H \to l\, l + inv.$





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HARD

Background **SHAPE** needed.
Flexible MC for both signal and background validated and tuned to data

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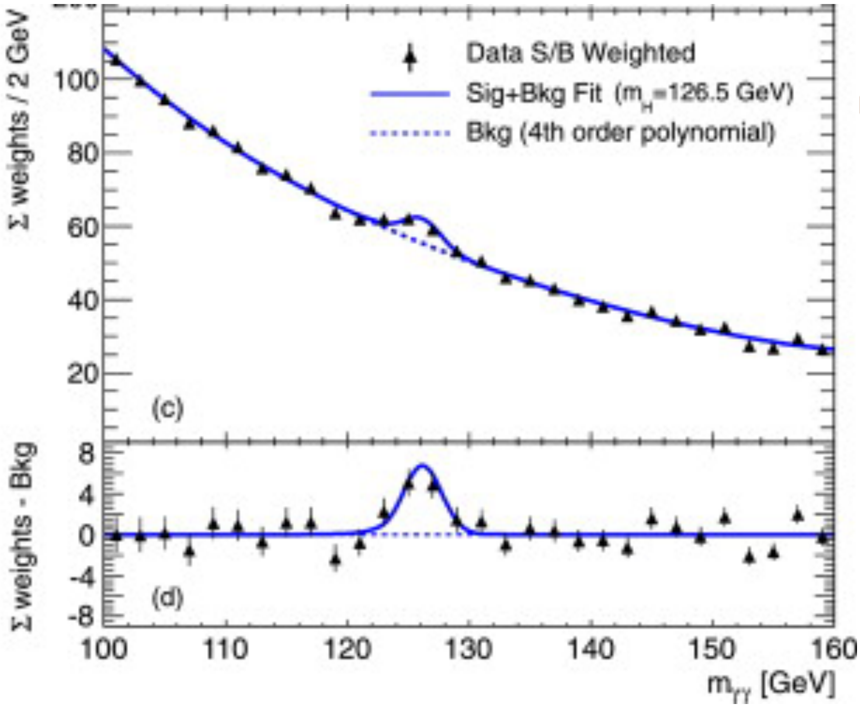

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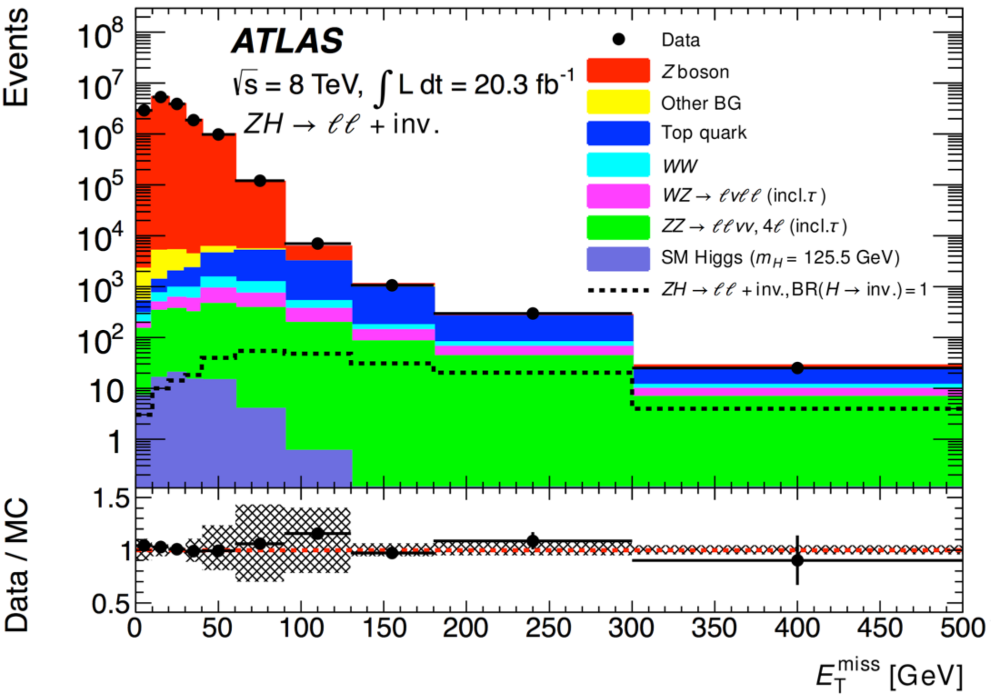
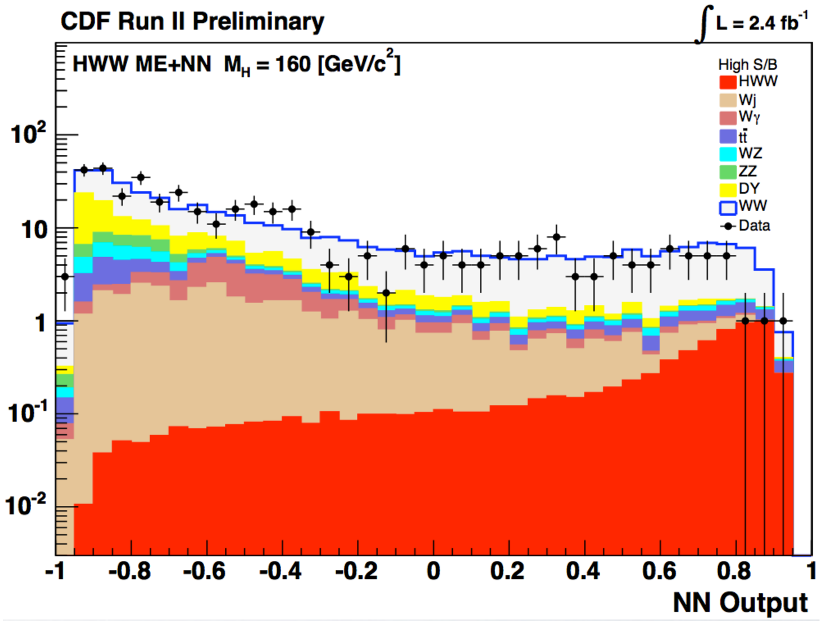


HARD

Background **SHAPE** needed.
Flexible MC for both signal and background validated and tuned to data

Rate

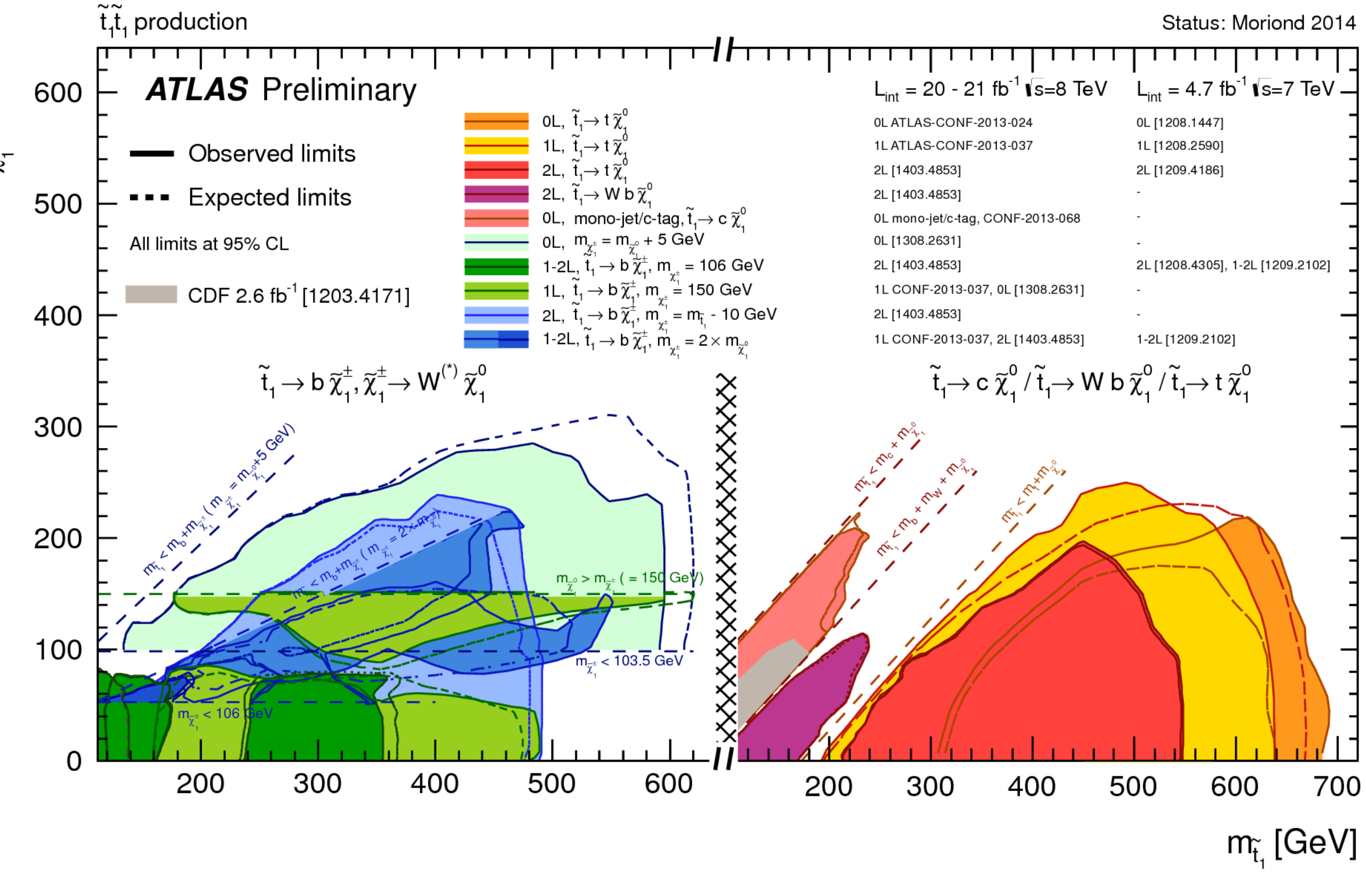
$H \rightarrow W^+ W^-$

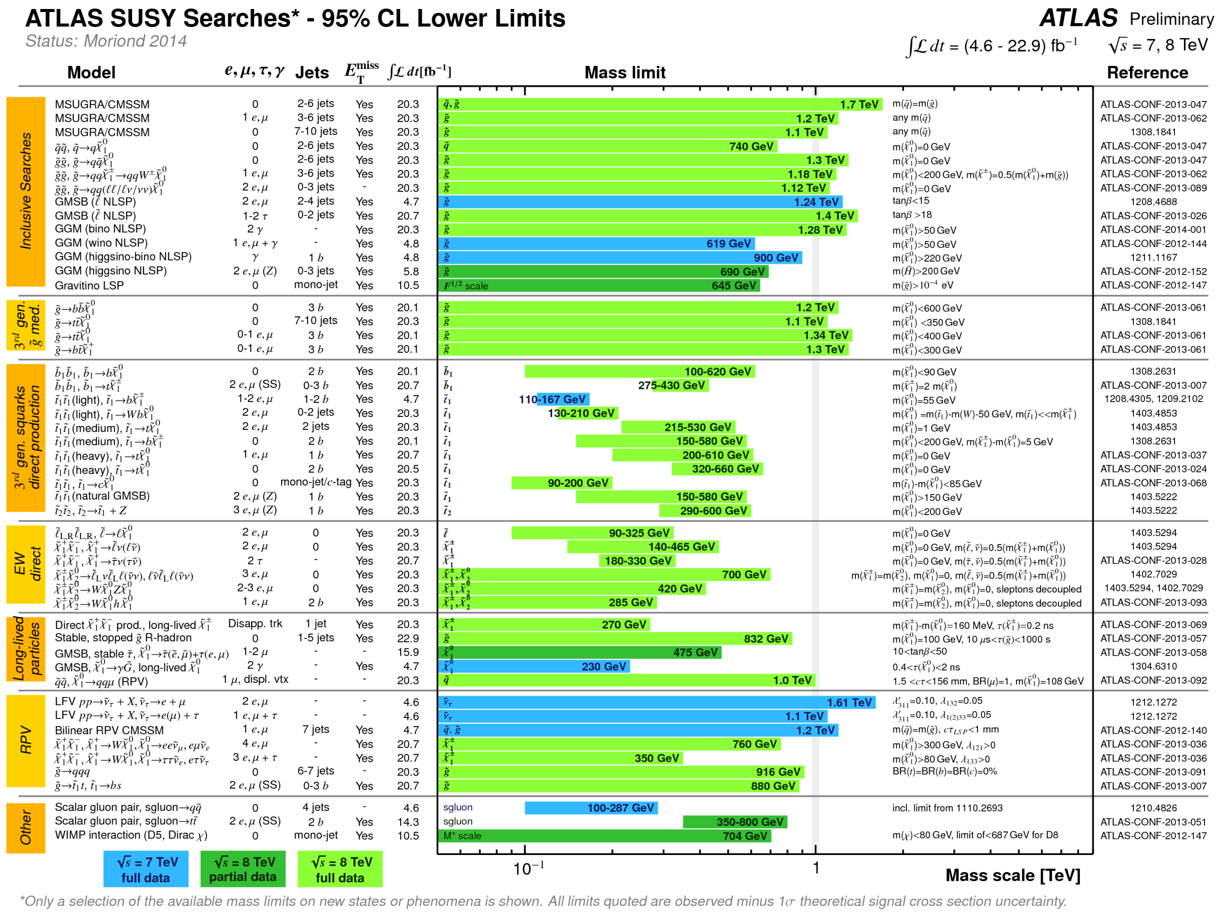


VERY HARD

Relies on prediction for both **shape** and **normalization**.
Complicated interplay of best simulations and data

New physics?





- No NP has been discovered yet
- Either there is no NP, or it is hiding very well
- If it is there, it will be a ‘Hard’ or ‘very Hard’ discovery
  - Need for accurate predictions for signal and background

2) Measurement of parameters

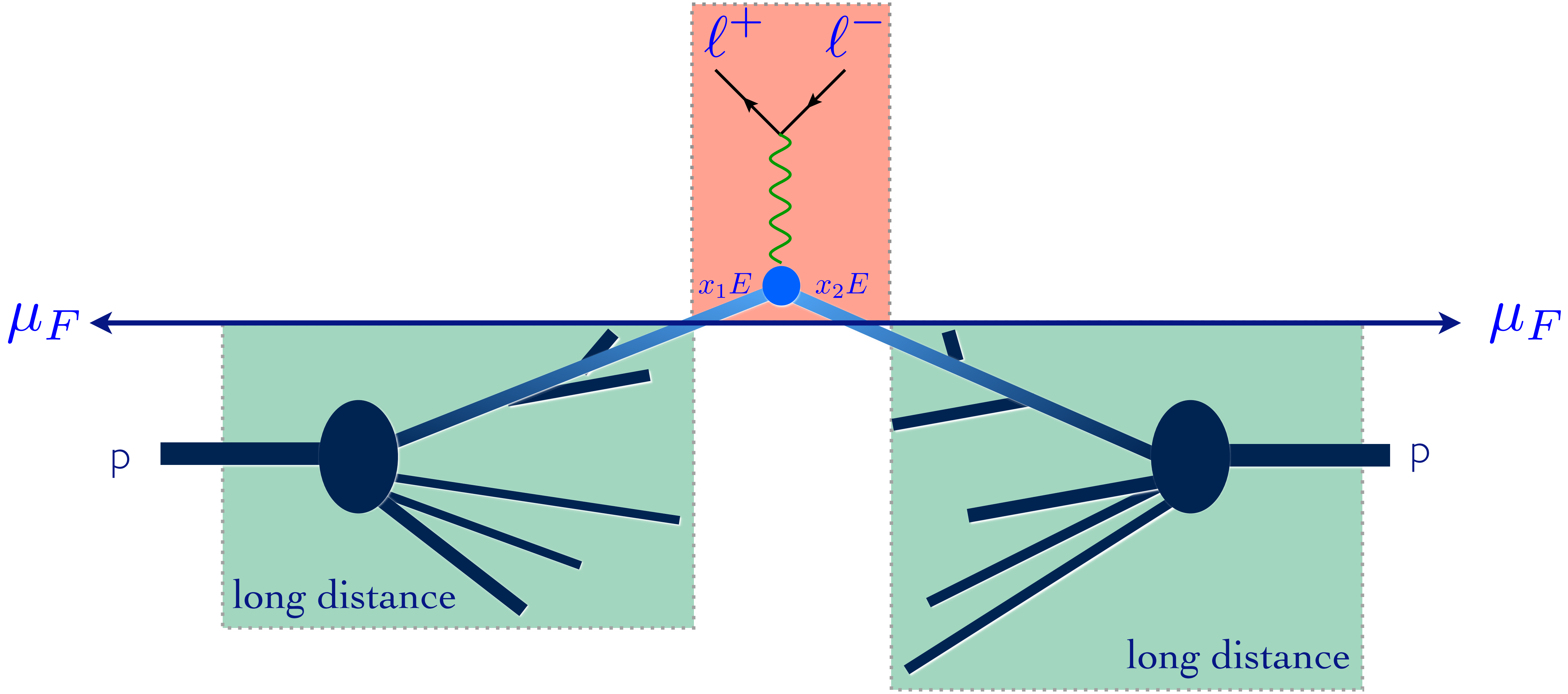
- E.g.: Extracting the top mass from leptonic observables
  - Start with pseudo-data with m_t^{pd}=174.3 GeV
  - Use theoretical predictions with different accuracy

TH. ACC.	m_t
NLO+PS+MS	$174.48^{+0.73}_{-0.77}[5.0]$
LO+PS+MS	$175.98^{+0.63}_{-0.69}[16.9]$
NLO+PS	$175.43^{+0.74}_{-0.80}[29.2]$
LO+PS	$187.90^{+0.6}_{-0.6}[428.3]$
fNLO	$174.41^{+0.72}_{-0.73}[96.6]$
fLO	$197.31^{+0.42}_{-0.35}[2496.1]$

- Large differences appear in the reconstructed m_t, due to different TH accuracies
- Better TH simulations improve central value *and reliability of uncertainties*

Frixione, Mitov arXiv:1407.2763

How to compute a cross-section



$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

Perturbation theory at work

$\hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$ Parton-level cross section

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter

$$\hat{\sigma} = \alpha_s^b\sigma_0 + \alpha_s^{b+1}\sigma_1 + \alpha_s^{b+2}\sigma_2 + \alpha_s^{b+3}\sigma_3 + \dots$$

- Remember:

$$\alpha_s = \alpha_s(\mu_R) \qquad \sigma_i = \sigma_i(\mu_R,\mu_F)$$

Both coupling and cross section depend on *unphysical* scales

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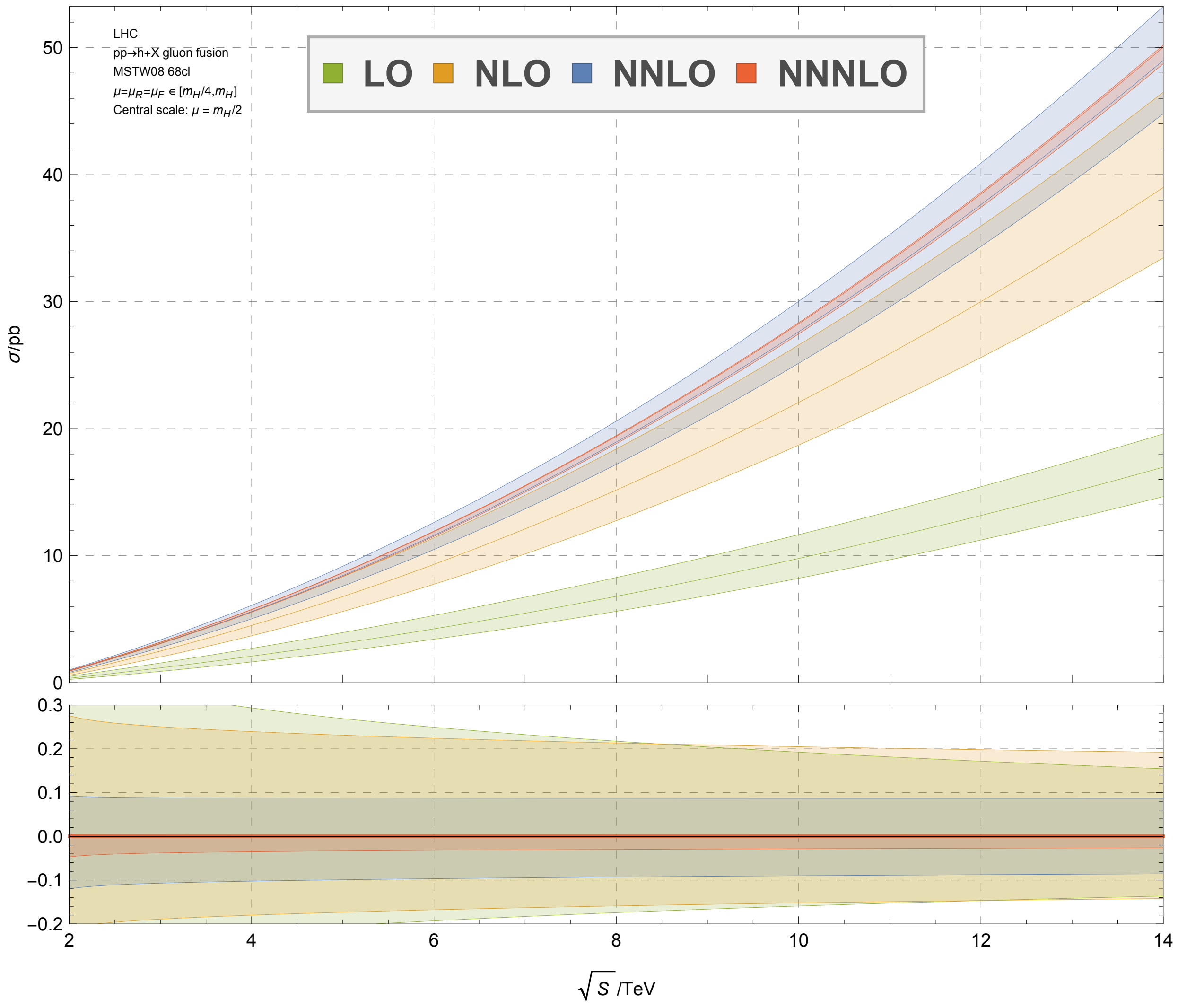
Both coupling and cross section depend on *unphysical* scales

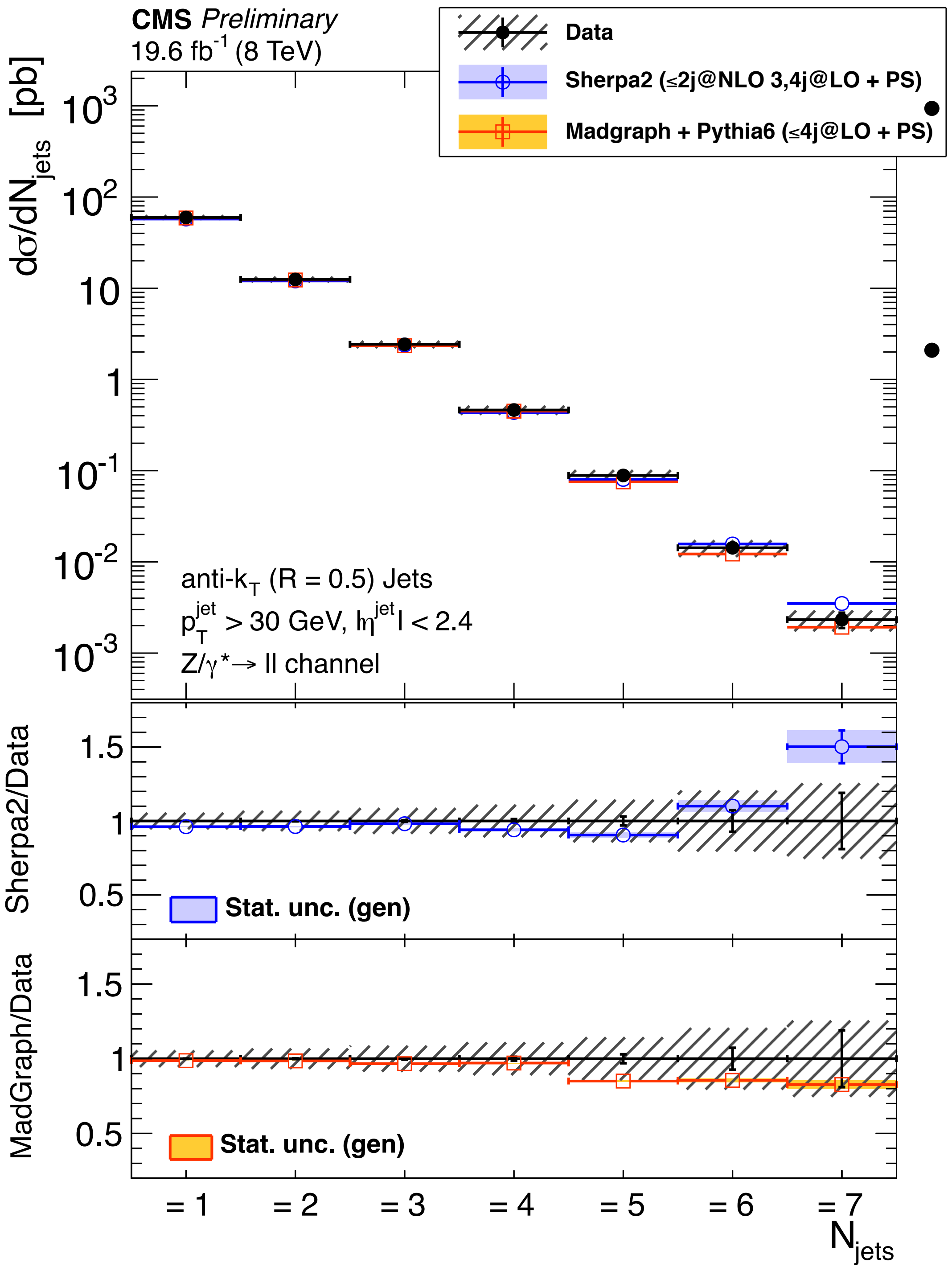
Perturbation theory at work

- The inclusion of higher orders improves the reliability of a given computation
  - More reliable description of total rates and shapes
  - Residual uncertainties related to the arbitrary scales in the process decrease
  - The computational complexity grows exponentially
  - NLO is mandatory for LHC physics (in particular at RunII)!



Anastasiou, Duhr, Dulat, Herzog, Mitslberger arXiv:1503.06056

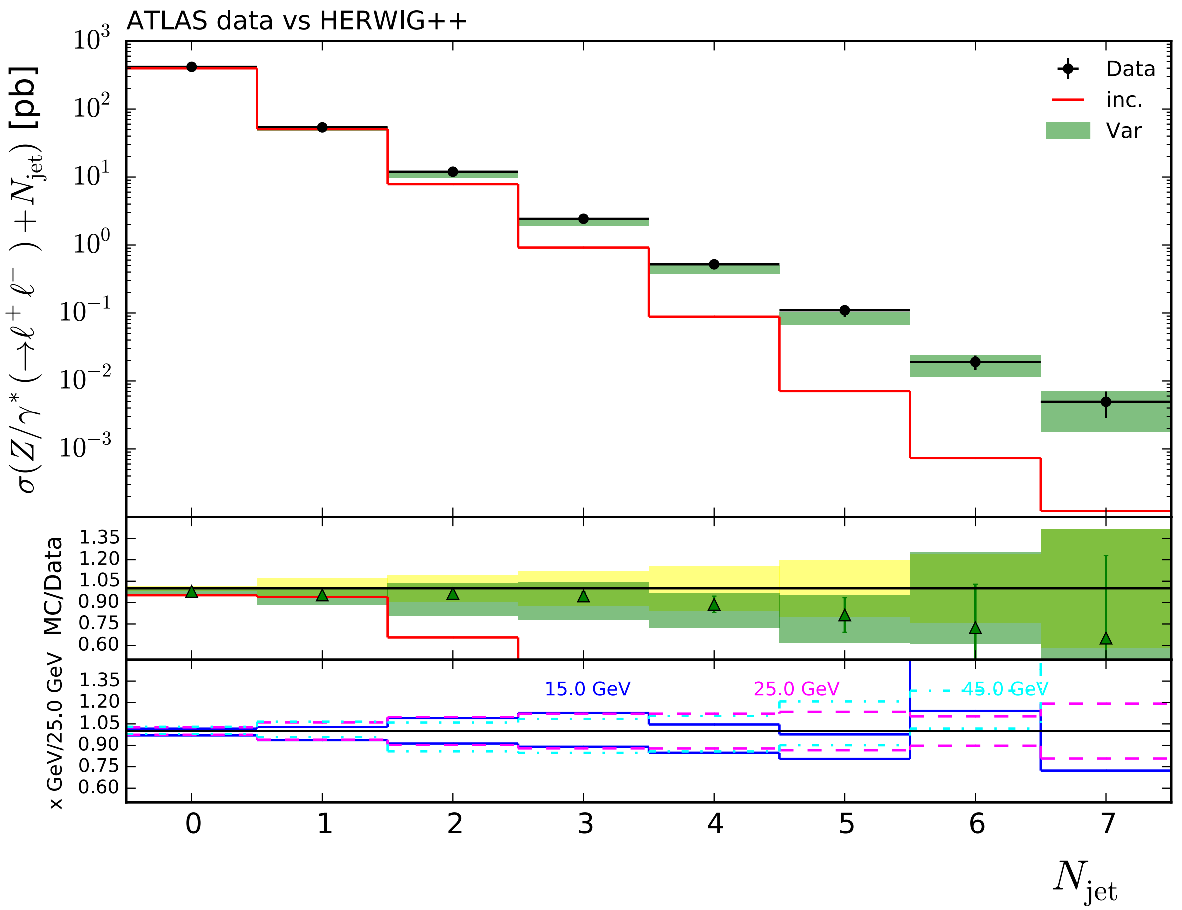
Perturbation theory at work



CMS PAS SMP-13-007

- Tree level with parton shower computed with MADGRAPH 5 [1] interfaced with PYTHIA 6, for parton shower and hadronization, with the same configuration as described in section 3. The total cross section is normalised to the NNLO cross section computed with FEWZ. This NNLO normalisation is not applied to the other prediction that follows.
- Multileg NLO with parton shower computed with Sherpa 2 [2] and Blackhat [33, 34] for the one-loop corrections. The matrix elements include the five processes pp $\to Z + N$ jet, $N = 0 \ldots 4$, with an NLO accuracy for $N \leq 2$ and LO accuracy for $N = 3, 4$. The CT10 PDF is used. The merging of parton shower and matrix elements is done with the MEPS@NLO method [35] and QCUT parameter set to 20 GeV.

Frederix, Frixione, Papaefstathiou, Prestel, Torrielli, arXiv:1511.00847



In these lectures:

- How to compute effectively a NLO cross section?
  - How to compute loops?
  - How to deal with infrared divergences?
- How to generate events to be showered at NLO?

NLO

- NLO evolution:
  - e.g. pp→W+*n* jets

year	1978	1989	2002	2009	2010	2013
	Altarelli, Ellis, Martinelli	Arnold, Ellis, Reno	Campbell, Ellis	BlackHat+Sherpa Ellis, Melnikov, zanderighi	BlackHat+Sherpa	BlackHat+Sherpa
n=	0	1	2	3	4	5
#virt diag $u\bar{d}\to W^+$ ng	2	43	416	4489	57026	…

NLO

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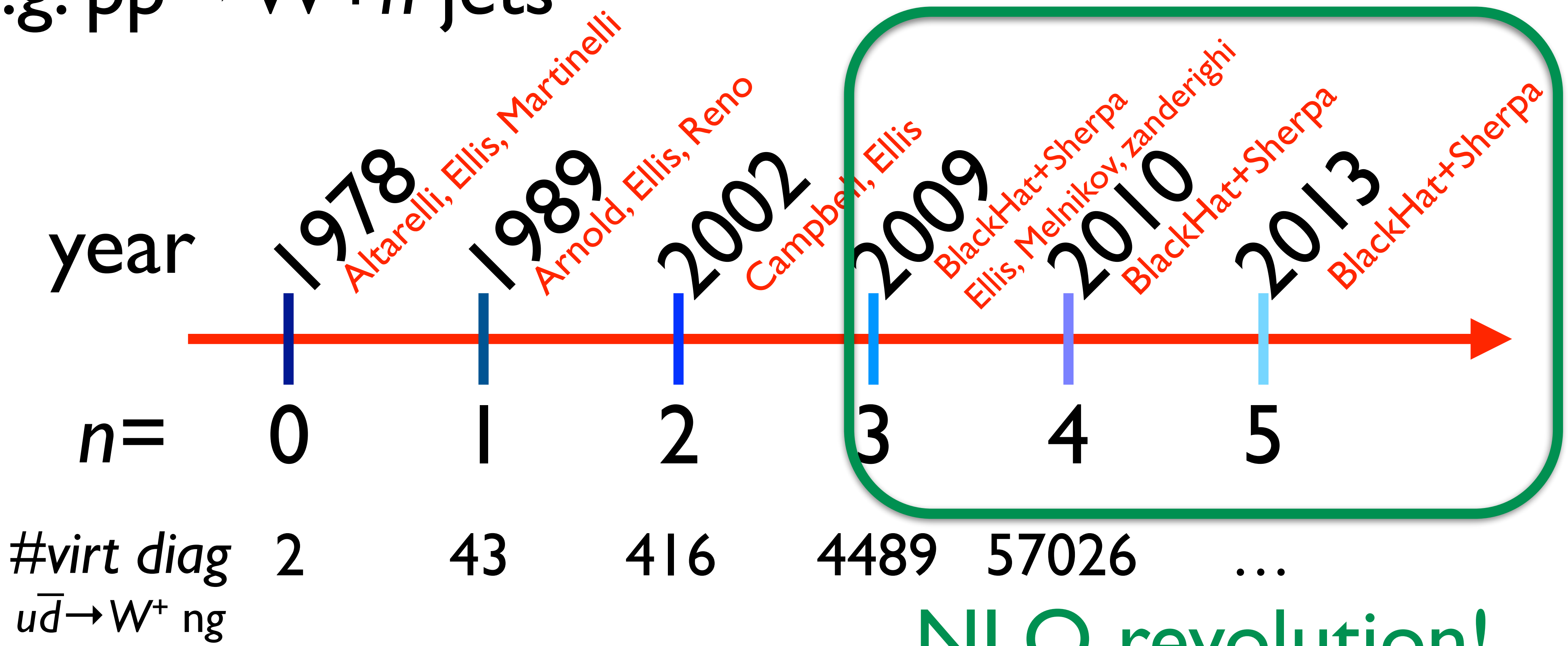

year	1978 Altarelli, Ellis, Martinelli	1989 Arnold, Ellis, Reno	2002 Campbell, Ellis	2009 BlackHat+Sherpa Ellis, Melnikov, zanderighi	2010 BlackHat+Sherpa	2013 BlackHat+Sherpa
n=	0	1	2	3	4	5
#virt diag $u\bar{d}\to W^+ ng$	2	43	416	4489	57026	…

NLO revolution!

NLO revolution

- Amazing development of computational techniques to tackle *any* process at NLO
  - Local subtraction
  - Computation of loop MEs
    - Tensor reduction
    - Generalized unitarity
    - Integrand reduction

Frixione, Kunszt, Signer, hep-ph/9512328
Catani, Seymour, hep-ph/9605323

Passarino, Veltman, 1979
Denner, Dittmaier, hep-ph/509141
Binoth, Guillet, Heinrich, Pilon, Reiter, arXiv:0810.0992

Bern, Dixon, Dunbar, Kosower, hep-ph/9403226 + …
Ellis, Giele, Kunszt, arXiv:0708.2398
+ Melnikov, arXiv:0806.3467

Ossola, Papadopoulos, Pittau, hep-ph/0609007
Del Aguila, Pittau, hep-ph/0404120
Mastrolia, Ossola, Reiter, Tramontano, arXiv:1006.0710

Going NLO

$$\hat{\sigma} = \alpha_s^b \sigma_0 + \alpha_s^{b+1} \sigma_1 + \alpha_s^{b+2} \sigma_2 + \alpha_s^{b+3} \sigma_3 + \dots$$

- NLO is the first order where the scale dependence in α_s and PDFs is compensated by loop corrections
  - First reliable predictions for rates and uncertainties
- Better description of final state (inclusion of extra radiation)
- Opening of new partonic channels from real emissions

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NLO: how to?

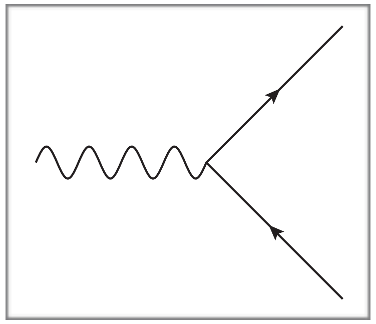
- Three ingredients need to be computed at NLO

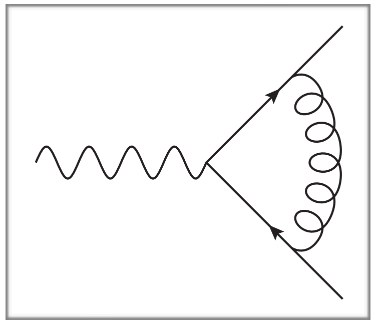
$$\sigma_{NLO} = \int_n \alpha_s^b d\sigma_0 + \int_n \alpha_s^{b+1} d\sigma_V + \int_{n+1} \alpha_s^{b+1} d\sigma_R$$

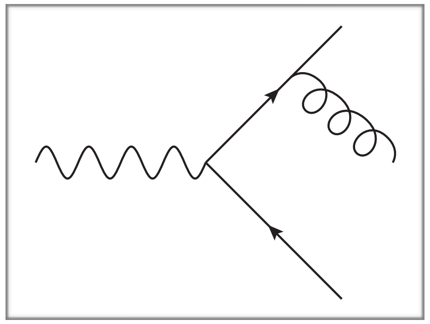
Born cross section

Virtual corrections

Real-emission corrections

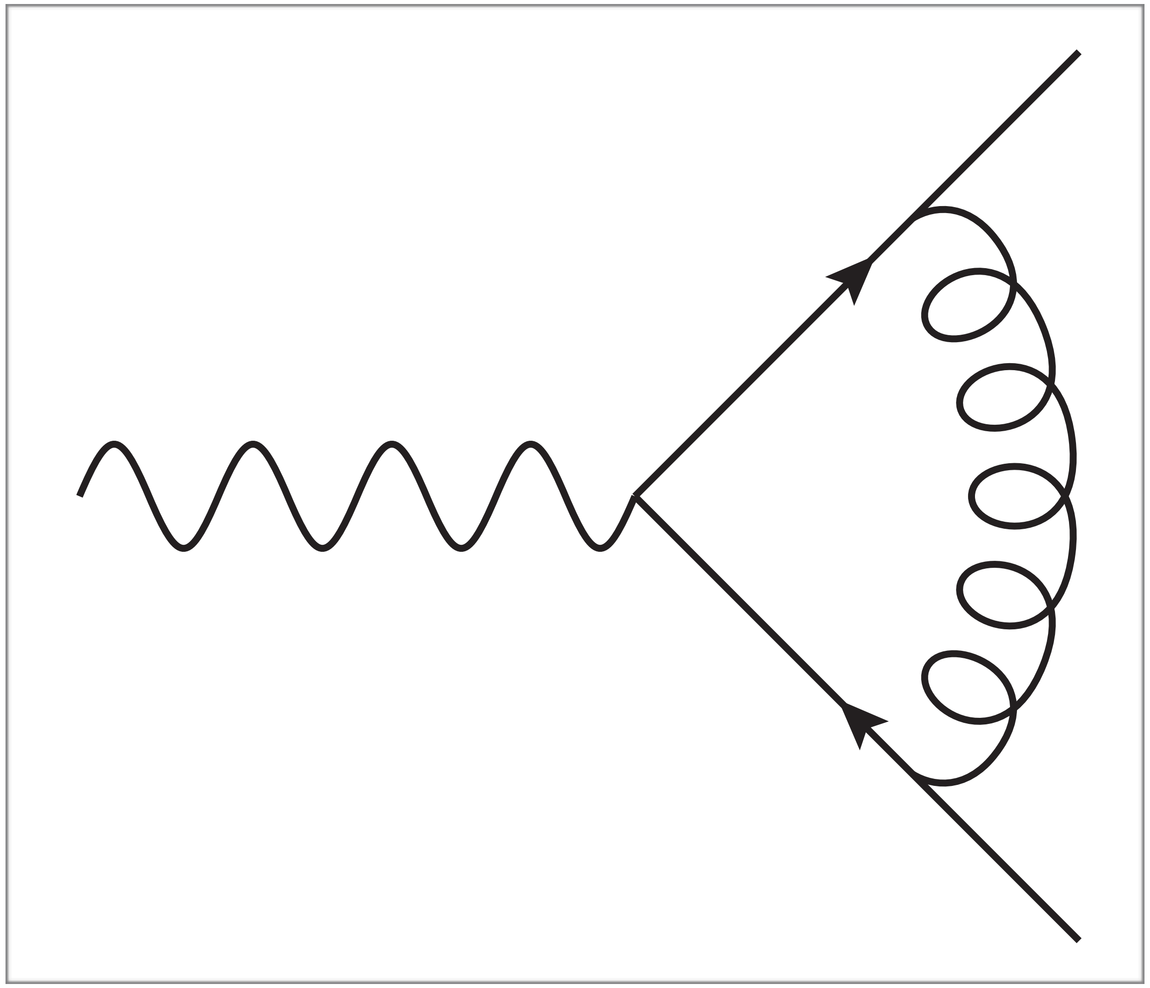






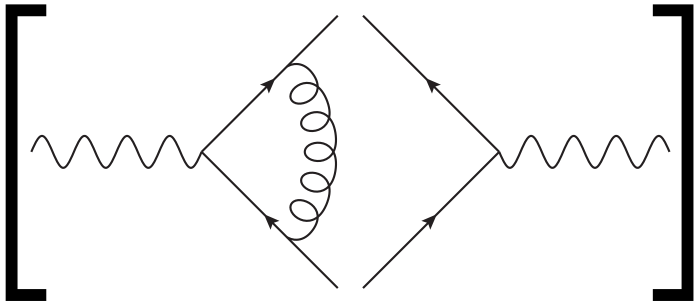
- Remember: virtual and reals are not separately finite, but their sum is (KLN theorem). Divergences have to be subtracted before numerical integration. We will shortly see how

How to compute loops?

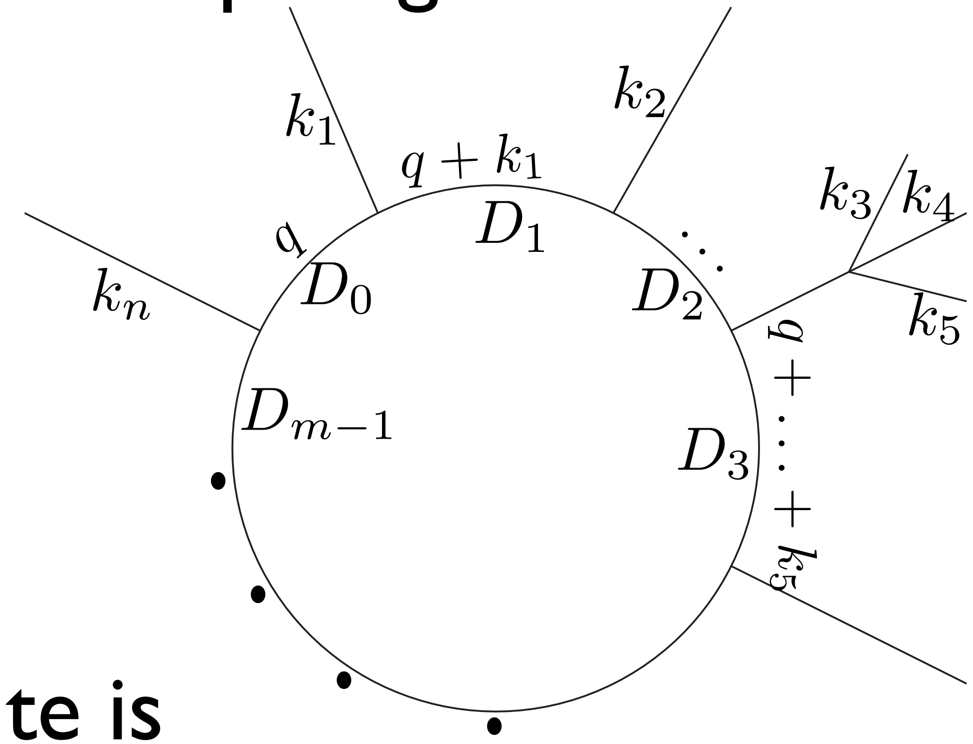


Computing loops numerically

$$d\sigma_V = 2\Re\Big[\qquad\Big]$$



- Consider a m-point one-loop diagram with n external momenta



- The integral to compute is

$$\int d^d l \frac{N(l)}{D_0 D_1 \dots D_{m-1}} \qquad D_i = (l + p_i)^2 - m_i^2$$

A hint…

- Any one-loop integral can be cast in the form

$$\int d^d l \frac{N(l)}{D_0 D_1 \ldots D_{m-1}} = \sum \mathrm{coeff}_i \int d^d l \frac{1}{D_{i_0} D_{i_1} \ldots}$$

- that is, a linear combination of scalar integrals
- Only scalar integrals with up to 4 denominators are needed → the basis is finite!
- The coefficients depend only on external momenta and parameters.

Scalar integrals

$$\mathcal{M}^{1\text{loop}} = \sum_{i_0,i_1,i_2,i_3} d_{i_0 i_1 i_2 i_3} \mathcal{D}_{i_0 i_1 i_2 i_3} + \sum_{i_0,i_1,i_2} c_{i_0 i_1 i_2} \mathcal{C}_{i_0 i_1 i_2} + \sum_{i_0,i_1} b_{i_0 i_1} \mathcal{B}_{i_0 i_1} + \sum_{i_0} a_{i_0} \mathcal{A}_{i_0} + R + \mathcal{O}(\varepsilon)$$

Box $\mathcal{D}_{i_0 i_1 i_2 i_3} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2} D_{i_3}}$

Triangle $\mathcal{C}_{i_0 i_1 i_2} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2}}$

Bubble $\mathcal{B}_{i_0 i_1} = \int d^d l \frac{1}{D_{i_0} D_{i_1}}$

Tadpole $\mathcal{A}_{i_0} = \int d^d l \frac{1}{D_{i_0}}$

- Scalar integrals are known and available as libraries

  FF (van Oldenborgh, CPC 66,1991)
  QCDLoop (Ellis, Zanderighi, arXiv:0712.1851)
  OneLOop (Van Hameren, arXiv:1007.4716)

How to compute the coefficients?

- Several techniques exist
  - Computation of loop MEs
    - Tensor reduction
    - Generalized unitarity
    - Integrand reduction

In these lectures

Passarino, Veltman, 1979
Denner, Dittmaier, hep-ph/509141
Binoth, Guillet, Heinrich, Pilon, Reiter, arXiv:0810.0992

Bern, Dixon, Dunbar, Kosower, hep-ph/9403226 + …
Ellis, Giele, Kunszt, arXiv:0708.2398
+ Melnikov, arXiv:0806.3467

Ossola, Papadopoulos, Pittau, hep-ph/0609007
Del Aguila, Pittau, hep-ph/0404120
Mastrolia, Ossola, Reiter, Tramontano, arXiv:1006.0710

Integrand reduction

$$\int d^d l \frac{N(l)}{D_0 D_1 \ldots D_{m-1}} = \sum \mathrm{coeff}_i \int d^d l \frac{1}{D_{i_0} D_{i_1} \ldots}$$

- Can we take away the integral?

$$\frac{N(l)}{D_0 D_1 \ldots D_{m-1}} \neq \sum \mathrm{coeff}_i \frac{1}{D_{i_0} D_{i_1} \ldots}$$

- Of course not, we must take into account for terms which integrate to 0, the so-called spurious terms:

$$\frac{N(l)}{D_0 D_1 \ldots D_{m-1}} = \sum \left(\mathrm{coeff}_i + \mathrm{spurious}_i(l)\right) \frac{1}{D_{i_0} D_{i_1} \ldots}$$

Spurious terms

- The functional form of the spurious terms is known and depends on the rank (powers of l in the numerator) and on the number of denominators. Del Aguila, Pittau, hep-ph/0404120
  - E.g. a rank-1 box

$$\tilde{d}_{i_0 i_1 i_2 i_3}(l) = \tilde{d}_{i_0 i_1 i_2 i_3}\, \epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma$$

  - The integral is 0

$$\int d^d l \frac{\tilde{d}_{i_0 i_1 i_2 i_3}(l)}{D_0 D_1 D_2 D_3} = \tilde{d}_{i_0 i_1 i_2 i_3} \int d^d l \frac{\epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma}{D_0 D_1 D_2 D_3} = 0$$

OPP decomposition

Ossola, Papadopoulos, Pittau, hep-ph/0609007

$$\frac{N(l)}{D_0 D_1 \ldots D_{m-1}} = \sum \left(\mathrm{coeff}_i + \mathrm{spurious}_i(l)\right) \frac{1}{D_{i_0} D_{i_1} \ldots}$$

- If we multiply both sides times $D_0 D_1 \ldots D_{m\text{-}1}$ we get

$$\begin{aligned} N(l) = & \sum_{i_0,i_1,i_2,i_3} \left(d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}\right) \prod_{i \neq i_0,i_1,i_2,i_3} D_i \\ & + \sum_{i_0,i_1,i_2} \left(c_{i_0 i_1 i_2} + \tilde{c}_{i_0 i_1 i_2}\right) \prod_{i \neq i_0,i_1,i_2} D_i \\ & + \sum_{i_0,i_1} \left(b_{i_0 i_1} + \tilde{b}_{i_0 i_1}\right) \prod_{i \neq i_0,i_1} D_i \\ & + \sum_{i_0} \left(a_{i_0} + \tilde{a}_{i_0}\right) \prod_{i \neq i_0} D_i \\ & + \tilde{P}(l) \prod_i D_i + \mathcal{O}(\varepsilon) \end{aligned}$$

Getting the coefficients

- *N(l)* is known from the diagrams and the functional form of spurious terms is known too
  - We can sample *N(l)* at various values of the loop momentum, and get a system of linear equations
  - The sampling can be done numerically
  - By choosing smart values of l (in the complex plane), the system can be greatly simplified
  - E.g. we can choose l such that

$$D_1(l^{\pm}) = D_2(l^{\pm}) = D_3(l^{\pm}) = D_4(l^{\pm}) = 0$$

$$\Downarrow$$

$$N(l^{\pm}) = (d_{1234} + \tilde{d}_{1234}(l^{\pm})) \prod_{i \neq 1,2,3,4} D_i(l^{\pm})$$

Getting the coefficients

- Two values of *l* and the knowledge of the spurious terms functional form are enough to extract the box coefficient

$$d_{1234} = \frac{1}{2}\left(\frac{N(l^+)}{\prod_{i\neq 1,2,3,4} D_i(l^+)} + \frac{N(l^-)}{\prod_{i\neq 1,2,3,4} D_i(l^-)}\right)$$

- Similarly, all the box coefficients can be determined
- Then one can move on to the triangles (choosing *l* such that 3 denominators vanish)
- Then to the bubbles, and finally to the tadpoles

Getting the coefficient: recap

- For each PS point, we have to solve a system of equations numerically
- The system reduces when special values of the loop momentum are chosen
- $N(l)$ can be the numerator of the full matrix element, of a single diagram or anything in between
- For a given PS point, the numerator has to be sampled several times (~50 for a 4-point diagrams)

The evil is in the *d*etails: Complications in *d* dimensions

- So far, we did not care much about the number of dimensions we were using
- In general, external momenta and polarisations are in 4 dimensions; only the loop momentum is in d
- To be more rigorous, we compute the integral

$$\int d^d l \frac{N(l,\tilde{l})}{\bar{D}_0 \bar{D}_1 \ldots \bar{D}_{m-1}}$$

$$\underset{d\text{-dim}}{\bar{l}} = \underset{4\text{-dim}}{l} + \underset{\varepsilon\text{-dim}}{\tilde{l}}$$

$$\bar{D}_i = (\bar{l} + p_i)^2 - m_i = (l + p_i)^2 - m_i^2 + \tilde{l}^2 = D_i + \tilde{l}^2$$

$$l \cdot \tilde{l} = 0 \qquad \bar{l} \cdot p_i = l \cdot p_i \qquad \bar{l} \cdot \bar{l} = l \cdot l + \tilde{l} \cdot \tilde{l}$$

Implications

- The reduction should be consistently done in d dimensions

$$
\begin{aligned}
\mathcal{M}^{1\mathrm{loop}} = & \sum_{i_0,i_1,i_2,i_3} d_{i_0 i_1 i_2 i_3} \bar{\mathcal{D}}_{i_0 i_1 i_2 i_3} \\
& + \sum_{i_0,i_1,i_2} c_{i_0 i_1 i_2} \bar{\mathcal{C}}_{i_0 i_1 i_2} \\
& + \sum_{i_0,i_1} b_{i_0 i_1} \bar{\mathcal{B}}_{i_0 i_1} \\
& + \sum_{i_0} a_{i_0} \bar{\mathcal{A}}_{i_0} \\
& + \mathcal{O}(\varepsilon)
\end{aligned}
$$

Implications

- The reduction should be consistently done in d dimensions

$$
\begin{aligned}
\mathcal{M}^{1\text{loop}} = & \sum_{i_0,i_1,i_2,i_3} d_{i_0 i_1 i_2 i_3} \bar{\mathcal{D}}_{i_0 i_1 i_2 i_3} \\
& + \sum_{i_0,i_1,i_2} c_{i_0 i_1 i_2} \bar{\mathcal{C}}_{i_0 i_1 i_2} \\
& + \sum_{i_0,i_1} b_{i_0 i_1} \bar{\mathcal{B}}_{i_0 i_1} \\
& + \sum_{i_0} a_{i_0} \bar{\mathcal{A}}_{i_0} \\
& + \mathcal{O}(\varepsilon)
\end{aligned}
$$

$$\Rightarrow$$

$$
\begin{aligned}
\mathcal{M}^{1\text{loop}} = & \sum_{i_0,i_1,i_2,i_3} d_{i_0 i_1 i_2 i_3} \mathcal{D}_{i_0 i_1 i_2 i_3} \\
& + \sum_{i_0,i_1,i_2} c_{i_0 i_1 i_2} \mathcal{C}_{i_0 i_1 i_2} \\
& + \sum_{i_0,i_1} b_{i_0 i_1} \mathcal{B}_{i_0 i_1} \\
& + \sum_{i_0} a_{i_0} \mathcal{A}_{i_0} \\
& + \textcircled{R} + \mathcal{O}(\varepsilon)
\end{aligned}
$$

That is why the *rational terms* are needed

The rational terms

OPP, arXiv:0802.1876

- In the OPP method, two types of rational terms are there: $R=R_1+R_2$
- Both originate from the UV part of the model, but only R_1 can be computed in the OPP decomposition
- R_1 originates from the *denominators* (propagators) in the loops

$$\frac{1}{\bar{D}_i} = \frac{1}{D_i}\left(1 - \frac{\tilde{l}^2}{D_i}\right)$$

- The denominator structure is known, so these terms can be directly included in the OPP reduction
- R_1 contributions are proportional to

$$\int d^d l \frac{\tilde{l}^2}{\bar{D}_i \bar{D}_j} = -\frac{i\pi^2}{2}\left[m_i^2 + m_j^2 - \frac{(p_i - p_j)^2}{2}\right] + \mathcal{O}(\varepsilon)$$

$$\int d^d l \frac{\tilde{l}^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\varepsilon)$$

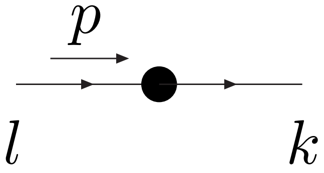
$$\int d^d l \frac{\tilde{l}^2}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\varepsilon)$$

R_2

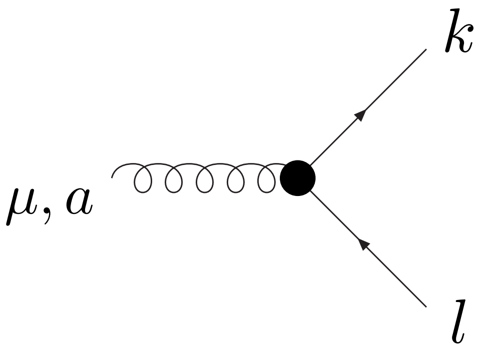
- R_2 terms originate from the numerator.
  Integrals with rank ≥2 can have terms in the numerator ~ to $\tilde{l}^2$
- This dependence can be quite hidden and become explicit only after having done the Clifford algebra
- Since we want a fully numerical approach, these terms cannot be obtained directly with the OPP reduction
- Within a given (renormalizable) model, only a finite set of terms that can give rise to these terms exists. They can be identified and computed as the "R_2 counterterms"

R_2 Feynman rules

- In a renormalizable theory, only up to 4-point integrals contribute to the R_2 terms
- They can be included in the computation using special Feynman rules (as it is done for the UV renormalisation). For example:



$$= \frac{ig^2}{16\pi^2}\frac{N_{col}^2-1}{2N_{col}}\delta_{kl}(-\not{p}+2m_q)\,\lambda_{HV}$$



$$= \frac{ig^3}{16\pi^2}\frac{N_{col}^2-1}{2N_{col}}\,t^a_{kl}\gamma_\mu\,(1+\lambda_{HV})$$

Draggiotis, Garzelli, Papadopoulos, Pittau, arXiv:0903.0356

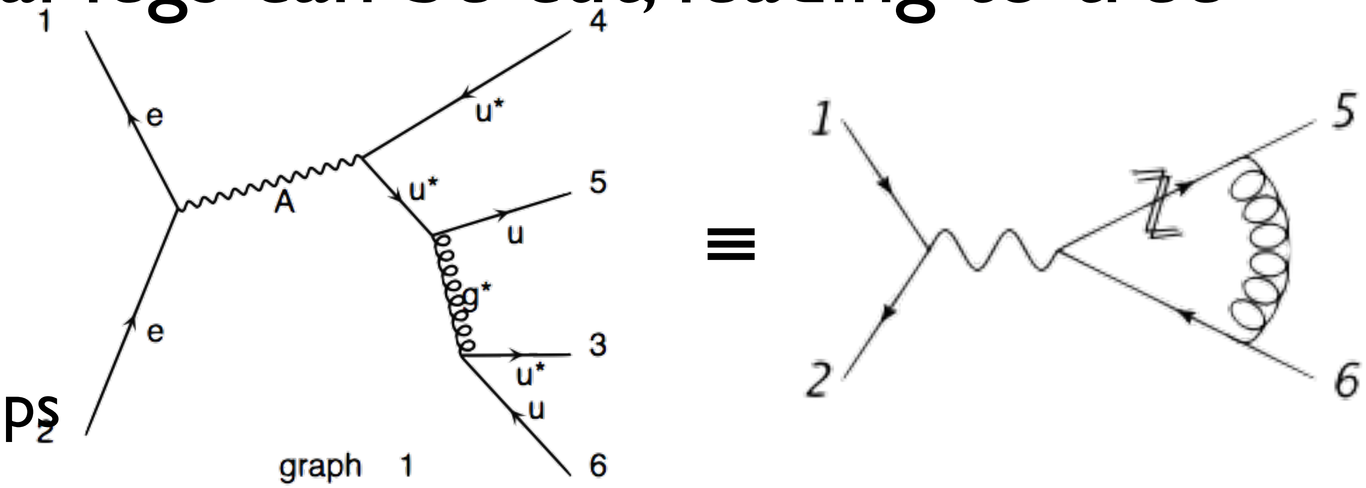
- Similarly to the UV counterterms, the R_2 terms are model dependent and need to be explicitly computed for BSM models This is now automated for renormalizable theories

Degrande, arXiv:1406.3030

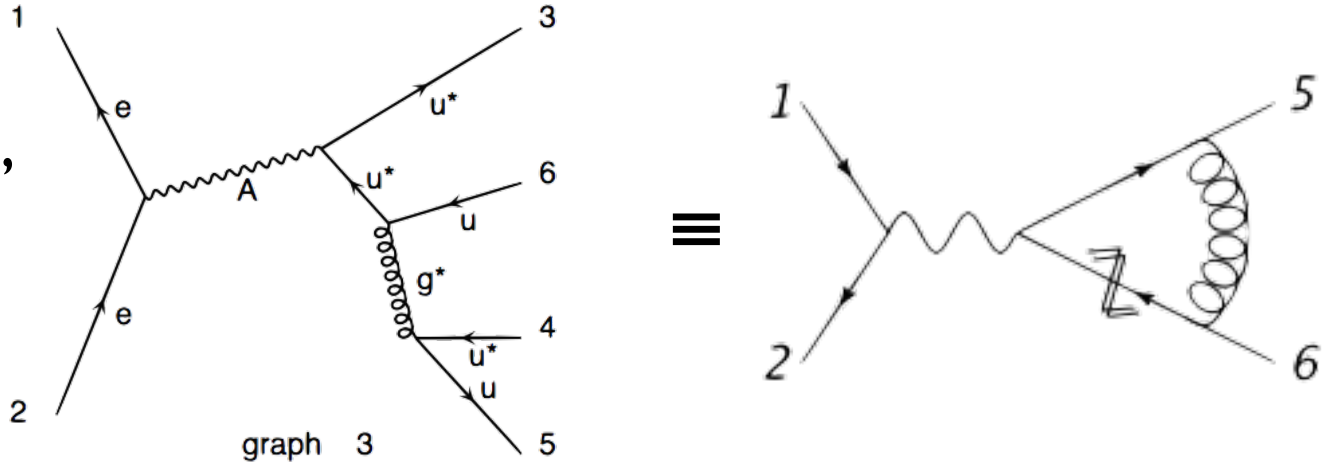
MadLoop

Hirschi et al, arXiv:1103.0621

- How to automate loop computation?
- Exploit MadGraph's capabilities to generate tree-level diagrams
- Loop diagrams with n external legs can be cut, leading to tree diagrams with n+2 legs
  - All diagrams with 2 extra particles are generated, those which are needed are filtered out
  - Each diagram is assigned a tag, which helps removing mirror/cyclic configurations
  - Additional filters to remove tadpole/ bubbles on external legs
  - Contract with Born, do the color algebra, re-glue the cut particle, etc…
  - Add UV and R2 counterterms as extra vertices

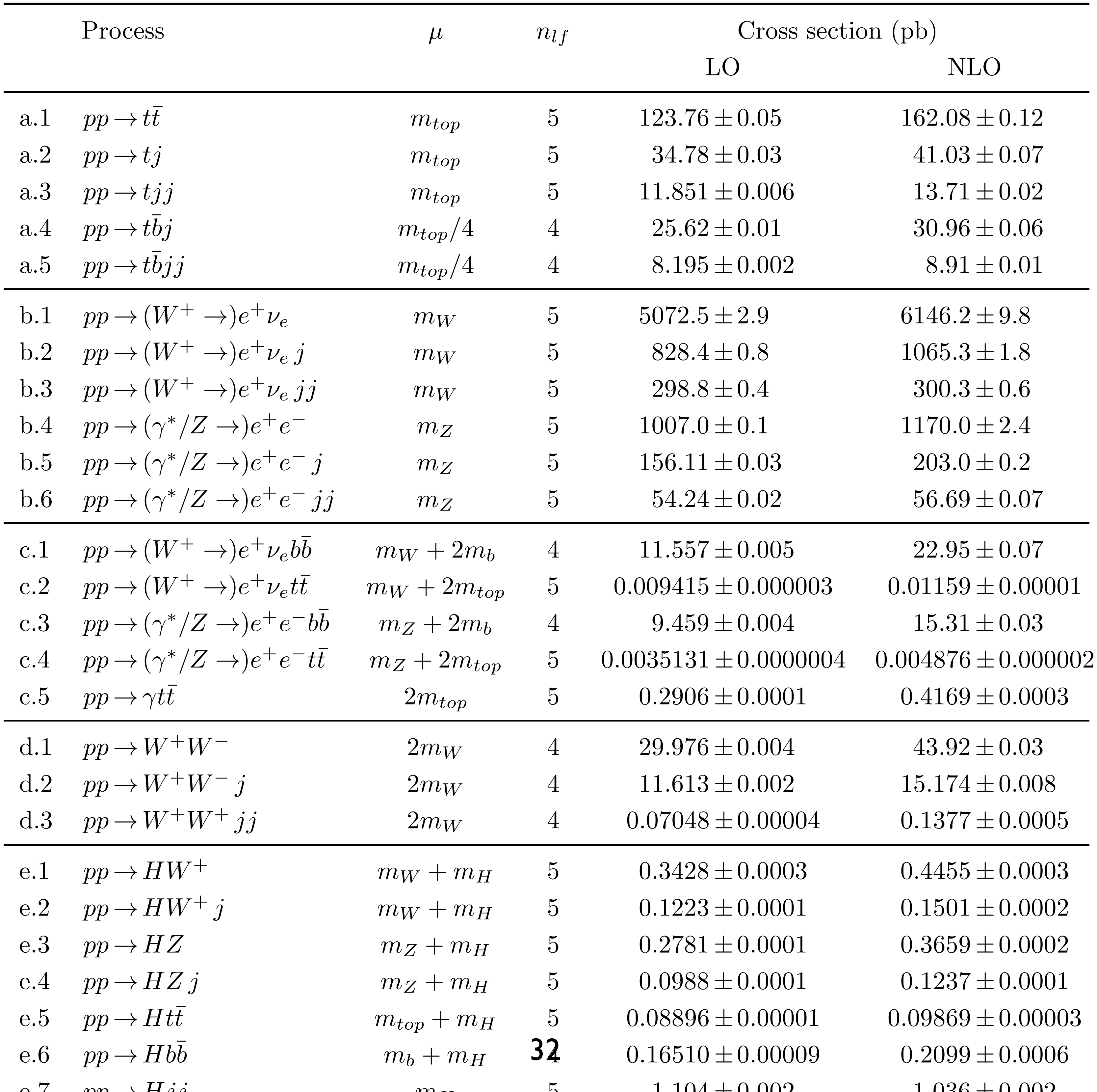


$\text{Diag}_{\ 1} = [u^*(6)g^*(5)u^*(A)]$



$\text{Diag}_{\ 3} = [u^*(A)u^*(6)g^*(5)]$

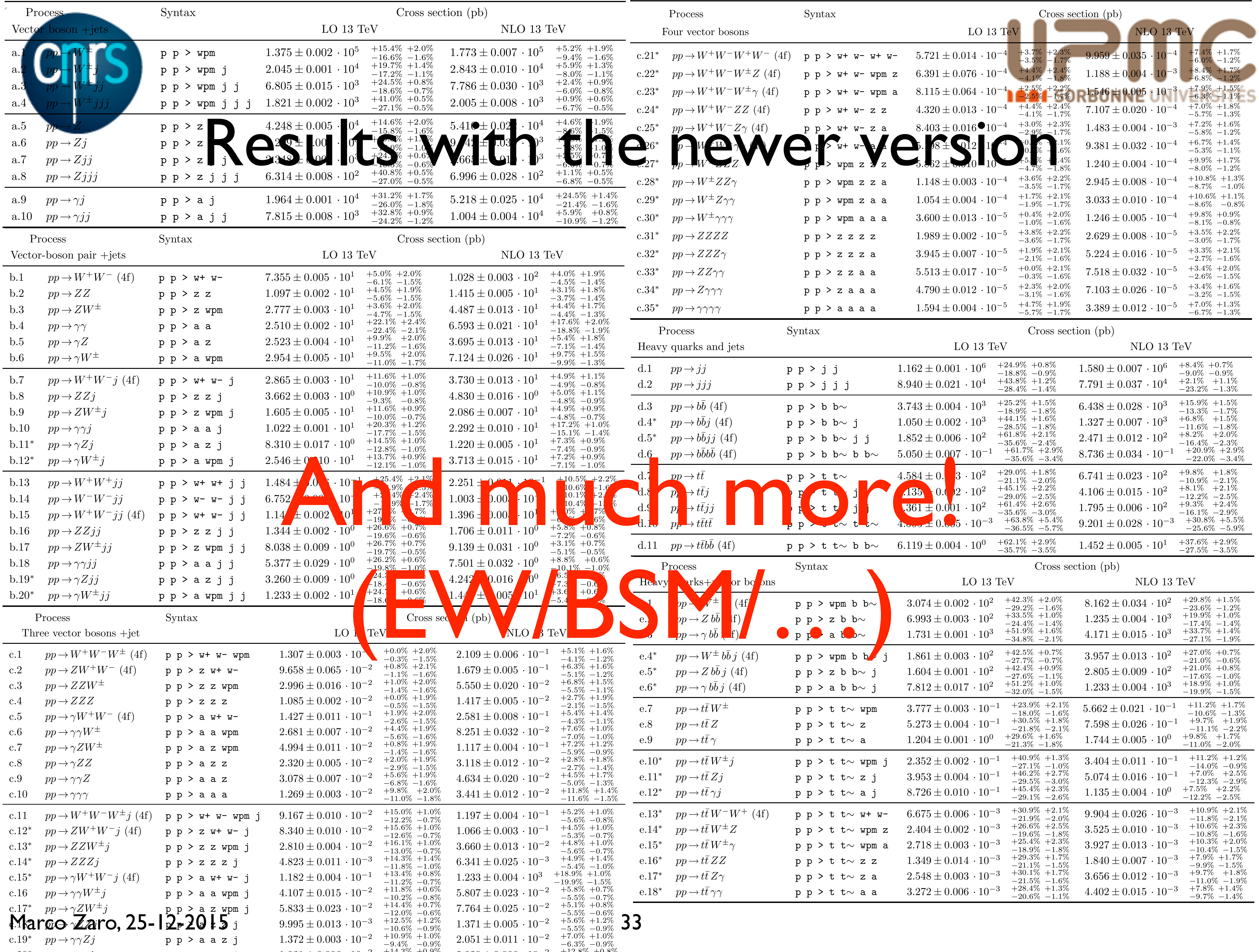
Results with the older (MG4) version



	Process	μ	n_{lf}	Cross section (pb)	
				LO	NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

Marco Zaro, 25-12-2015

Results with the newer version



Process		Syntax	Cross section (pb)					
Vector boson +jets			LO 13 TeV			NLO 13 TeV		
a.1	$pp\to W^\pm$	`p p > wpm`	$1.375 \pm 0.002 \cdot 10^5$	$^{+15.4\%}_{-16.6\%}$	$^{+2.0\%}_{-1.6\%}$	$1.773 \pm 0.007 \cdot 10^5$	$^{+5.2\%}_{-9.4\%}$	$^{+1.9\%}_{-1.6\%}$
a.2	$pp\to W^\pm j$	`p p > wpm j`	$2.045 \pm 0.001 \cdot 10^4$	$^{+19.7\%}_{-17.2\%}$	$^{+1.4\%}_{-1.1\%}$	$2.843 \pm 0.010 \cdot 10^4$	$^{+5.9\%}_{-8.0\%}$	$^{+1.3\%}_{-1.1\%}$
a.3	$pp\to W^\pm jj$	`p p > wpm j j`	$6.805 \pm 0.015 \cdot 10^3$	$^{+24.5\%}_{-18.6\%}$	$^{+0.8\%}_{-0.7\%}$	$7.786 \pm 0.030 \cdot 10^3$	$^{+2.4\%}_{-6.0\%}$	$^{+0.9\%}_{-0.8\%}$
a.4	$pp\to W^\pm jjj$	`p p > wpm j j j`	$1.821 \pm 0.002 \cdot 10^3$	$^{+41.0\%}_{-27.1\%}$	$^{+0.5\%}_{-0.5\%}$	$2.005 \pm 0.008 \cdot 10^3$	$^{+0.9\%}_{-6.7\%}$	$^{+0.6\%}_{-0.5\%}$
a.5	$pp\to Z$	`p p > z`	$4.248 \pm 0.005 \cdot 10^4$	$^{+14.6\%}_{-15.8\%}$	$^{+2.0\%}_{-1.6\%}$	[illegible]	$^{+4.6\%}_{-8.6\%}$	$^{+1.9\%}_{-1.5\%}$
a.6	$pp\to Zj$	`p p > z` [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
a.7	$pp\to Zjj$	`p p > z` [illegible] `j`	[illegible]	[illegible]	$^{+0.6\%}_{-0.6\%}$	[illegible]	[illegible]	[illegible]
a.8	$pp\to Zjjj$	`p p > z j j j`	$6.314 \pm 0.008 \cdot 10^2$	$^{+40.8\%}_{-27.0\%}$	$^{+0.5\%}_{-0.5\%}$	$6.996 \pm 0.028 \cdot 10^2$	$^{+1.1\%}_{-6.8\%}$	$^{+0.5\%}_{-0.5\%}$
a.9	$pp\to \gamma j$	`p p > a j`	$1.964 \pm 0.001 \cdot 10^4$	$^{+31.2\%}_{-26.0\%}$	$^{+1.7\%}_{-1.8\%}$	$5.218 \pm 0.025 \cdot 10^4$	$^{+24.5\%}_{-21.4\%}$	$^{+1.4\%}_{-1.6\%}$
a.10	$pp\to \gamma jj$	`p p > a j j`	$7.815 \pm 0.008 \cdot 10^3$	$^{+32.8\%}_{-24.2\%}$	$^{+0.9\%}_{-1.2\%}$	$1.004 \pm 0.004 \cdot 10^4$	$^{+5.9\%}_{-10.9\%}$	$^{+0.8\%}_{-1.2\%}$

Process		Syntax	Cross section (pb)					
Vector-boson pair +jets			LO 13 TeV			NLO 13 TeV		
b.1	$pp\to W^+W^-$ (4f)	`p p > w+ w-`	$7.355 \pm 0.005 \cdot 10^1$	$^{+5.0\%}_{-6.1\%}$	$^{+2.0\%}_{-1.5\%}$	$1.028 \pm 0.003 \cdot 10^2$	$^{+4.0\%}_{-4.5\%}$	$^{+1.9\%}_{-1.4\%}$
b.2	$pp\to ZZ$	`p p > z z`	$1.097 \pm 0.002 \cdot 10^1$	$^{+4.5\%}_{-5.6\%}$	$^{+1.9\%}_{-1.5\%}$	$1.415 \pm 0.005 \cdot 10^1$	$^{+3.1\%}_{-3.7\%}$	$^{+1.8\%}_{-1.4\%}$
b.3	$pp\to ZW^\pm$	`p p > z wpm`	$2.777 \pm 0.003 \cdot 10^1$	$^{+3.6\%}_{-4.7\%}$	$^{+2.0\%}_{-1.5\%}$	$4.487 \pm 0.013 \cdot 10^1$	$^{+4.4\%}_{-4.4\%}$	$^{+1.7\%}_{-1.3\%}$
b.4	$pp\to \gamma\gamma$	`p p > a a`	$2.510 \pm 0.002 \cdot 10^1$	$^{+22.1\%}_{-22.4\%}$	$^{+2.4\%}_{-2.1\%}$	$6.593 \pm 0.021 \cdot 10^1$	$^{+17.6\%}_{-18.8\%}$	$^{+2.0\%}_{-1.9\%}$
b.5	$pp\to \gamma Z$	`p p > a z`	$2.523 \pm 0.004 \cdot 10^1$	$^{+9.9\%}_{-11.2\%}$	$^{+2.0\%}_{-1.6\%}$	$3.695 \pm 0.013 \cdot 10^1$	$^{+5.4\%}_{-7.1\%}$	$^{+1.8\%}_{-1.4\%}$
b.6	$pp\to \gamma W^\pm$	`p p > a wpm`	$2.954 \pm 0.005 \cdot 10^1$	$^{+9.5\%}_{-11.0\%}$	$^{+2.0\%}_{-1.7\%}$	$7.124 \pm 0.026 \cdot 10^1$	$^{+9.7\%}_{-9.9\%}$	$^{+1.5\%}_{-1.3\%}$
b.7	$pp\to W^+W^-j$ (4f)	`p p > w+ w- j`	$2.865 \pm 0.003 \cdot 10^1$	$^{+11.6\%}_{-10.0\%}$	$^{+1.0\%}_{-0.8\%}$	$3.730 \pm 0.013 \cdot 10^1$	$^{+4.9\%}_{-4.9\%}$	$^{+1.1\%}_{-0.8\%}$
b.8	$pp\to ZZj$	`p p > z z j`	$3.662 \pm 0.003 \cdot 10^0$	$^{+10.9\%}_{-9.3\%}$	$^{+1.0\%}_{-0.8\%}$	$4.830 \pm 0.016 \cdot 10^0$	$^{+5.0\%}_{-4.8\%}$	$^{+1.1\%}_{-0.9\%}$
b.9	$pp\to ZW^\pm j$	`p p > z wpm j`	$1.605 \pm 0.005 \cdot 10^1$	$^{+11.6\%}_{-10.0\%}$	$^{+0.9\%}_{-0.7\%}$	$2.086 \pm 0.007 \cdot 10^1$	$^{+4.9\%}_{-4.8\%}$	$^{+0.9\%}_{-0.7\%}$
b.10	$pp\to \gamma\gamma j$	`p p > a a j`	$1.022 \pm 0.001 \cdot 10^1$	$^{+20.3\%}_{-17.7\%}$	$^{+1.2\%}_{-1.5\%}$	$2.292 \pm 0.010 \cdot 10^1$	$^{+17.2\%}_{-15.1\%}$	$^{+1.0\%}_{-1.4\%}$
b.11*	$pp\to \gamma Zj$	`p p > a z j`	$8.310 \pm 0.017 \cdot 10^0$	$^{+14.5\%}_{-12.8\%}$	$^{+1.0\%}_{-1.0\%}$	$1.220 \pm 0.005 \cdot 10^1$	$^{+7.3\%}_{-7.4\%}$	$^{+0.9\%}_{-0.9\%}$
b.12*	$pp\to \gamma W^\pm j$	`p p > a wpm j`	$2.546 \pm 0.010 \cdot 10^1$	$^{+13.7\%}_{-12.1\%}$	$^{+0.9\%}_{-1.0\%}$	$3.713 \pm 0.015 \cdot 10^1$	$^{+7.2\%}_{-7.1\%}$	$^{+0.9\%}_{-1.0\%}$
b.13	$pp\to W^+W^+jj$	`p p > w+ w+ j j`	$1.484 \pm$ [illegible] $\cdot 10^{-1}$	[illegible]	[illegible]	$2.251 \pm$ [illegible]	[illegible]	[illegible]
b.14	$pp\to W^-W^-jj$	`p p > w- w- j j`	6.752 [illegible]	[illegible]	$^{+2.4\%}_{-1.7\%}$	$1.003 \pm$ [illegible]	$^{+10.1\%}_{-10.4\%}$	[illegible]
b.15	$pp\to W^+W^-jj$ (4f)	`p p > w+ w- j j`	[illegible] $\pm 0.002 \cdot 10^1$	[illegible]	[illegible]	$1.396 \pm$ [illegible] $\cdot 10^1$	[illegible]	[illegible]
b.16	$pp\to ZZjj$	`p p > z z j j`	$1.344 \pm 0.002 \cdot 10^0$	$^{+26.6\%}_{-19.6\%}$	$^{+0.7\%}_{-0.6\%}$	$1.706 \pm 0.011 \cdot 10^0$	$^{+5.8\%}_{-7.2\%}$	$^{+0.8\%}_{-0.6\%}$
b.17	$pp\to ZW^\pm jj$	`p p > z wpm j j`	$8.038 \pm 0.009 \cdot 10^0$	$^{+26.7\%}_{-19.7\%}$	$^{+0.7\%}_{-0.5\%}$	$9.139 \pm 0.031 \cdot 10^0$	$^{+3.1\%}_{-5.1\%}$	$^{+0.7\%}_{-0.5\%}$
b.18	$pp\to \gamma\gamma jj$	`p p > a a j j`	$5.377 \pm 0.029 \cdot 10^0$	$^{+26.2\%}_{-19.8\%}$	$^{+0.6\%}_{-1.0\%}$	$7.501 \pm 0.032 \cdot 10^0$	$^{+8.8\%}_{-10.1\%}$	$^{+0.6\%}_{-1.0\%}$
b.19*	$pp\to \gamma Zjj$	`p p > a z j j`	$3.260 \pm 0.009 \cdot 10^0$	[illegible]	[illegible]	4.242 [illegible] $0.016 \cdot 10^0$	[illegible]	[illegible]
b.20*	$pp\to \gamma W^\pm jj$	`p p > a wpm j j`	$1.233 \pm 0.002 \cdot 10^1$	[illegible]	[illegible]	[illegible] $0.005 \cdot 10^1$	[illegible]	[illegible]

Process		Syntax	Cross section (pb)					
Three vector bosons +jet			LO 13 TeV			NLO 13 TeV		
c.1	$pp\to W^+W^-W^\pm$ (4f)	`p p > w+ w- wpm`	$1.307 \pm 0.003 \cdot 10$ [illegible]	$^{+0.0\%}_{-0.3\%}$	$^{+2.0\%}_{-1.5\%}$	$2.109 \pm 0.006 \cdot 10^{-1}$	$^{+5.1\%}_{-4.1\%}$	$^{+1.6\%}_{-1.2\%}$
c.2	$pp\to ZW^+W^-$ (4f)	`p p > z w+ w-`	$9.658 \pm 0.065 \cdot 10^{-2}$	$^{+0.8\%}_{-1.1\%}$	$^{+2.1\%}_{-1.6\%}$	$1.679 \pm 0.005 \cdot 10^{-1}$	$^{+6.3\%}_{-5.1\%}$	$^{+1.6\%}_{-1.2\%}$
c.3	$pp\to ZZW^\pm$	`p p > z z wpm`	$2.996 \pm 0.016 \cdot 10^{-2}$	$^{+1.0\%}_{-1.4\%}$	$^{+2.0\%}_{-1.6\%}$	$5.550 \pm 0.020 \cdot 10^{-2}$	$^{+6.8\%}_{-5.5\%}$	$^{+1.5\%}_{-1.1\%}$
c.4	$pp\to ZZZ$	`p p > z z z`	$1.085 \pm 0.002 \cdot 10^{-2}$	$^{+0.0\%}_{-0.5\%}$	$^{+1.9\%}_{-1.5\%}$	$1.417 \pm 0.005 \cdot 10^{-2}$	$^{+2.7\%}_{-2.1\%}$	$^{+1.9\%}_{-1.5\%}$
c.5	$pp\to \gamma W^+W^-$ (4f)	`p p > a w+ w-`	$1.427 \pm 0.011 \cdot 10^{-1}$	$^{+1.9\%}_{-2.6\%}$	$^{+2.0\%}_{-1.5\%}$	$2.581 \pm 0.008 \cdot 10^{-1}$	$^{+5.4\%}_{-4.3\%}$	$^{+1.4\%}_{-1.1\%}$
c.6	$pp\to \gamma\gamma W^\pm$	`p p > a a wpm`	$2.681 \pm 0.007 \cdot 10^{-2}$	$^{+4.4\%}_{-5.6\%}$	$^{+1.9\%}_{-1.6\%}$	$8.251 \pm 0.032 \cdot 10^{-2}$	$^{+7.6\%}_{-7.0\%}$	$^{+1.0\%}_{-1.0\%}$
c.7	$pp\to \gamma ZW^\pm$	`p p > a z wpm`	$4.994 \pm 0.011 \cdot 10^{-2}$	$^{+0.8\%}_{-1.4\%}$	$^{+1.9\%}_{-1.6\%}$	$1.117 \pm 0.004 \cdot 10^{-1}$	$^{+7.2\%}_{-5.9\%}$	$^{+1.2\%}_{-0.9\%}$
c.8	$pp\to \gamma ZZ$	`p p > a z z`	$2.320 \pm 0.005 \cdot 10^{-2}$	$^{+2.0\%}_{-2.9\%}$	$^{+1.9\%}_{-1.5\%}$	$3.118 \pm 0.012 \cdot 10^{-2}$	$^{+2.8\%}_{-2.7\%}$	$^{+1.8\%}_{-1.4\%}$
c.9	$pp\to \gamma\gamma Z$	`p p > a a z`	$3.078 \pm 0.007 \cdot 10^{-2}$	$^{+5.6\%}_{-6.8\%}$	$^{+1.9\%}_{-1.6\%}$	$4.634 \pm 0.020 \cdot 10^{-2}$	$^{+4.5\%}_{-5.0\%}$	$^{+1.7\%}_{-1.3\%}$
c.10	$pp\to \gamma\gamma\gamma$	`p p > a a a`	$1.269 \pm 0.003 \cdot 10^{-2}$	$^{+9.8\%}_{-11.0\%}$	$^{+2.0\%}_{-1.8\%}$	$3.441 \pm 0.012 \cdot 10^{-2}$	$^{+11.8\%}_{-11.6\%}$	$^{+1.4\%}_{-1.5\%}$
c.11	$pp\to W^+W^-W^\pm j$ (4f)	`p p > w+ w- wpm j`	$9.167 \pm 0.010 \cdot 10^{-2}$	$^{+15.0\%}_{-12.2\%}$	$^{+1.0\%}_{-0.7\%}$	$1.197 \pm 0.004 \cdot 10^{-1}$	$^{+5.2\%}_{-5.6\%}$	$^{+1.0\%}_{-0.8\%}$
c.12*	$pp\to ZW^+W^-j$ (4f)	`p p > z w+ w- j`	$8.340 \pm 0.010 \cdot 10^{-2}$	$^{+15.6\%}_{-12.6\%}$	$^{+1.0\%}_{-0.7\%}$	$1.066 \pm 0.003 \cdot 10^{-1}$	$^{+4.5\%}_{-5.3\%}$	$^{+1.0\%}_{-0.7\%}$
c.13*	$pp\to ZZW^\pm j$	`p p > z z wpm j`	$2.810 \pm 0.004 \cdot 10^{-2}$	$^{+16.1\%}_{-13.0\%}$	$^{+1.0\%}_{-0.7\%}$	$3.660 \pm 0.013 \cdot 10^{-2}$	$^{+4.8\%}_{-5.6\%}$	$^{+1.0\%}_{-0.7\%}$
c.14*	$pp\to ZZZj$	`p p > z z z j`	$4.823 \pm 0.011 \cdot 10^{-3}$	$^{+14.3\%}_{-11.8\%}$	$^{+1.4\%}_{-1.0\%}$	$6.341 \pm 0.025 \cdot 10^{-3}$	$^{+4.9\%}_{-5.4\%}$	$^{+1.4\%}_{-1.0\%}$
c.15*	$pp\to \gamma W^+W^-j$ (4f)	`p p > a w+ w- j`	$1.182 \pm 0.004 \cdot 10^{-1}$	$^{+13.4\%}_{-11.2\%}$	$^{+0.8\%}_{-0.7\%}$	$1.233 \pm 0.004 \cdot 10^{3}$	$^{+18.9\%}_{-19.9\%}$	$^{+1.0\%}_{-1.5\%}$
c.16	$pp\to \gamma\gamma W^\pm j$	`p p > a a wpm j`	$4.107 \pm 0.015 \cdot 10^{-2}$	$^{+11.8\%}_{-10.2\%}$	$^{+0.6\%}_{-0.8\%}$	$5.807 \pm 0.023 \cdot 10^{-2}$	$^{+5.8\%}_{-5.5\%}$	$^{+0.7\%}_{-0.7\%}$
c.17*	$pp\to \gamma ZW^\pm j$	`p p > a z wpm j`	$5.833 \pm 0.023 \cdot 10^{-2}$	$^{+14.4\%}_{-12.0\%}$	$^{+0.7\%}_{-0.6\%}$	$7.764 \pm 0.025 \cdot 10^{-2}$	$^{+5.1\%}_{-5.5\%}$	$^{+0.8\%}_{-0.6\%}$
[illegible]	[illegible]	[illegible]	$9.995 \pm 0.013 \cdot 10^{-3}$	$^{+12.5\%}_{-10.6\%}$	$^{+1.2\%}_{-0.9\%}$	$1.371 \pm 0.005 \cdot 10^{-2}$	$^{+5.6\%}_{-5.5\%}$	$^{+1.2\%}_{-0.9\%}$
c.19*	$pp\to \gamma\gamma Zj$	`p p > a a z j`	$1.372 \pm 0.003 \cdot 10^{-2}$	$^{+10.9\%}_{-9.4\%}$	$^{+1.0\%}_{-0.9\%}$	$2.051 \pm 0.011 \cdot 10^{-2}$	$^{+7.0\%}_{-6.3\%}$	$^{+1.0\%}_{-0.9\%}$

Process		Syntax	Cross section (pb)					
Four vector bosons			LO 13 TeV			NLO 13 TeV		
c.21*	$pp\to W^+W^-W^+W^-$ (4f)	`p p > w+ w- w+ w-`	$5.721 \pm 0.014 \cdot 10^{-4}$	$^{+3.7\%}_{-3.5\%}$	$^{+2.3\%}_{-1.7\%}$	$9.959 \pm 0.035 \cdot 10^{-4}$	$^{+7.4\%}_{-6.0\%}$	$^{+1.7\%}_{-1.2\%}$
c.22*	$pp\to W^+W^-W^\pm Z$ (4f)	`p p > w+ w- wpm z`	$6.391 \pm 0.076 \cdot 10^{-4}$	$^{+4.4\%}_{-4.1\%}$	$^{+2.4\%}_{-1.8\%}$	$1.188 \pm 0.004 \cdot 10^{-3}$	$^{+8.4\%}_{-6.8\%}$	$^{+1.7\%}_{-1.2\%}$
c.23*	$pp\to W^+W^-W^\pm\gamma$ (4f)	`p p > w+ w- wpm a`	$8.115 \pm 0.064 \cdot 10^{-4}$	$^{+2.5\%}_{-2.5\%}$	$^{+2.2\%}_{-1.7\%}$	$1.546 \pm 0.005 \cdot 10^{-3}$	$^{+7.9\%}_{-6.3\%}$	$^{+1.5\%}_{-1.1\%}$
c.24*	$pp\to W^+W^-ZZ$ (4f)	`p p > w+ w- z z`	$4.320 \pm 0.013 \cdot 10^{-4}$	$^{+4.4\%}_{-4.1\%}$	$^{+2.4\%}_{-1.7\%}$	$7.107 \pm 0.020 \cdot 10^{-4}$	$^{+7.0\%}_{-5.7\%}$	$^{+1.8\%}_{-1.3\%}$
c.25*	$pp\to W^+W^-Z\gamma$ (4f)	`p p > w+ w- z a`	$8.403 \pm 0.016 \cdot 10^{-4}$	$^{+3.0\%}_{-2.9\%}$	$^{+2.3\%}_{-1.7\%}$	$1.483 \pm 0.004 \cdot 10^{-3}$	$^{+7.2\%}_{-5.8\%}$	$^{+1.6\%}_{-1.2\%}$
c.26*	[illegible]	[illegible]	[illegible]	[illegible]	$^{+2.1\%}_{-1.6\%}$	$9.381 \pm 0.032 \cdot 10^{-4}$	$^{+6.7\%}_{-5.3\%}$	$^{+1.4\%}_{-1.1\%}$
c.27*	[illegible]	`p p > wpm z` [illegible] `z`	[illegible]	[illegible]	[illegible]	$1.240 \pm 0.004 \cdot 10^{-4}$	$^{+9.9\%}_{-8.0\%}$	$^{+1.7\%}_{-1.2\%}$
c.28*	$pp\to W^\pm ZZ\gamma$	`p p > wpm z z a`	$1.148 \pm 0.003 \cdot 10^{-4}$	$^{+3.6\%}_{-3.5\%}$	$^{+2.2\%}_{-1.7\%}$	$2.945 \pm 0.008 \cdot 10^{-4}$	$^{+10.8\%}_{-8.7\%}$	$^{+1.3\%}_{-1.0\%}$
c.29*	$pp\to W^\pm Z\gamma\gamma$	`p p > wpm z a a`	$1.054 \pm 0.004 \cdot 10^{-4}$	$^{+1.7\%}_{-1.9\%}$	$^{+2.1\%}_{-1.7\%}$	$3.033 \pm 0.010 \cdot 10^{-4}$	$^{+10.6\%}_{-8.6\%}$	$^{+1.1\%}_{-0.8\%}$
c.30*	$pp\to W^\pm\gamma\gamma\gamma$	`p p > wpm a a a`	$3.600 \pm 0.013 \cdot 10^{-5}$	$^{+0.4\%}_{-1.0\%}$	$^{+2.0\%}_{-1.6\%}$	$1.246 \pm 0.005 \cdot 10^{-4}$	$^{+9.8\%}_{-8.1\%}$	$^{+0.9\%}_{-0.8\%}$
c.31*	$pp\to ZZZZ$	`p p > z z z z`	$1.989 \pm 0.002 \cdot 10^{-5}$	$^{+3.8\%}_{-3.6\%}$	$^{+2.2\%}_{-1.7\%}$	$2.629 \pm 0.008 \cdot 10^{-5}$	$^{+3.5\%}_{-3.0\%}$	$^{+2.2\%}_{-1.7\%}$
c.32*	$pp\to ZZZ\gamma$	`p p > z z z a`	$3.945 \pm 0.007 \cdot 10^{-5}$	$^{+1.9\%}_{-2.1\%}$	$^{+2.1\%}_{-1.6\%}$	$5.224 \pm 0.016 \cdot 10^{-5}$	$^{+3.3\%}_{-2.7\%}$	$^{+2.1\%}_{-1.6\%}$
c.33*	$pp\to ZZ\gamma\gamma$	`p p > z z a a`	$5.513 \pm 0.017 \cdot 10^{-5}$	$^{+0.0\%}_{-0.3\%}$	$^{+2.1\%}_{-1.6\%}$	$7.518 \pm 0.032 \cdot 10^{-5}$	$^{+3.4\%}_{-2.6\%}$	$^{+2.0\%}_{-1.5\%}$
c.34*	$pp\to Z\gamma\gamma\gamma$	`p p > z a a a`	$4.790 \pm 0.012 \cdot 10^{-5}$	$^{+2.3\%}_{-3.1\%}$	$^{+2.0\%}_{-1.6\%}$	$7.103 \pm 0.026 \cdot 10^{-5}$	$^{+3.4\%}_{-3.2\%}$	$^{+1.6\%}_{-1.5\%}$
c.35*	$pp\to \gamma\gamma\gamma\gamma$	`p p > a a a a`	$1.594 \pm 0.004 \cdot 10^{-5}$	$^{+4.7\%}_{-5.7\%}$	$^{+1.9\%}_{-1.7\%}$	$3.389 \pm 0.012 \cdot 10^{-5}$	$^{+7.0\%}_{-6.7\%}$	$^{+1.3\%}_{-1.3\%}$

Process		Syntax	Cross section (pb)					
Heavy quarks and jets			LO 13 TeV			NLO 13 TeV		
d.1	$pp\to jj$	`p p > j j`	$1.162 \pm 0.001 \cdot 10^6$	$^{+24.9\%}_{-18.8\%}$	$^{+0.8\%}_{-0.9\%}$	$1.580 \pm 0.007 \cdot 10^6$	$^{+8.4\%}_{-9.0\%}$	$^{+0.7\%}_{-0.9\%}$
d.2	$pp\to jjj$	`p p > j j j`	$8.940 \pm 0.021 \cdot 10^4$	$^{+43.8\%}_{-28.4\%}$	$^{+1.2\%}_{-1.4\%}$	$7.791 \pm 0.037 \cdot 10^4$	$^{+2.1\%}_{-23.2\%}$	$^{+1.1\%}_{-1.3\%}$
d.3	$pp\to b\bar{b}$ (4f)	`p p > b b~`	$3.743 \pm 0.004 \cdot 10^3$	$^{+25.2\%}_{-18.9\%}$	$^{+1.5\%}_{-1.8\%}$	$6.438 \pm 0.028 \cdot 10^3$	$^{+15.9\%}_{-13.3\%}$	$^{+1.5\%}_{-1.7\%}$
d.4*	$pp\to b\bar{b}j$ (4f)	`p p > b b~ j`	$1.050 \pm 0.002 \cdot 10^3$	$^{+44.1\%}_{-28.5\%}$	$^{+1.6\%}_{-1.8\%}$	$1.327 \pm 0.007 \cdot 10^3$	$^{+6.8\%}_{-11.6\%}$	$^{+1.5\%}_{-1.8\%}$
d.5*	$pp\to b\bar{b}jj$ (4f)	`p p > b b~ j j`	$1.852 \pm 0.006 \cdot 10^2$	$^{+61.8\%}_{-35.6\%}$	$^{+2.1\%}_{-2.4\%}$	$2.471 \pm 0.012 \cdot 10^2$	$^{+8.2\%}_{-16.4\%}$	$^{+2.0\%}_{-2.3\%}$
d.6	$pp\to b\bar{b}b\bar{b}$ (4f)	`p p > b b~ b b~`	$5.050 \pm 0.007 \cdot 10^{-1}$	$^{+61.7\%}_{-35.6\%}$	$^{+2.9\%}_{-3.4\%}$	$8.736 \pm 0.034 \cdot 10^{-1}$	$^{+20.9\%}_{-22.0\%}$	$^{+2.9\%}_{-3.4\%}$
d.7	$pp\to t\bar{t}$	`p p > t t~`	$4.584 \pm$ [illegible] $\cdot 10^2$	$^{+29.0\%}_{-21.1\%}$	$^{+1.8\%}_{-2.0\%}$	$6.741 \pm 0.023 \cdot 10^2$	$^{+9.8\%}_{-10.9\%}$	$^{+1.8\%}_{-2.1\%}$
d.8	[illegible]	[illegible]	[illegible]	$^{+45.1\%}_{-29.0\%}$	$^{+2.2\%}_{-2.5\%}$	$4.106 \pm 0.015 \cdot 10^2$	$^{+8.1\%}_{-12.2\%}$	$^{+2.1\%}_{-2.5\%}$
d.9	[illegible]	[illegible]	[illegible] $\pm 0.001 \cdot 10^2$	$^{+61.4\%}_{-35.6\%}$	$^{+2.6\%}_{-3.0\%}$	$1.795 \pm 0.006 \cdot 10^2$	$^{+9.3\%}_{-16.1\%}$	$^{+2.4\%}_{-2.9\%}$
d.10	$pp\to t\bar{t}t\bar{t}$	[illegible]	[illegible] $\cdot 10^{-3}$	$^{+63.8\%}_{-36.5\%}$	$^{+5.4\%}_{-5.7\%}$	$9.201 \pm 0.028 \cdot 10^{-3}$	$^{+30.8\%}_{-25.6\%}$	$^{+5.5\%}_{-5.9\%}$
d.11	$pp\to t\bar{t}b\bar{b}$ (4f)	`p p > t t~ b b~`	$6.119 \pm 0.004 \cdot 10^0$	$^{+62.1\%}_{-35.7\%}$	$^{+2.9\%}_{-3.5\%}$	$1.452 \pm 0.005 \cdot 10^1$	$^{+37.6\%}_{-27.5\%}$	$^{+2.9\%}_{-3.5\%}$

Process		Syntax	Cross section (pb)					
[illegible]			LO 13 TeV			NLO 13 TeV		
[illegible]	[illegible]	`p p > wpm b b~`	$3.074 \pm 0.002 \cdot 10^2$	$^{+42.3\%}_{-29.2\%}$	$^{+2.0\%}_{-1.6\%}$	$8.162 \pm 0.034 \cdot 10^2$	$^{+29.8\%}_{-23.6\%}$	$^{+1.5\%}_{-1.2\%}$
[illegible]	[illegible]	`p p > z b b~`	$6.993 \pm 0.003 \cdot 10^2$	$^{+33.5\%}_{-24.4\%}$	$^{+1.0\%}_{-1.4\%}$	$1.235 \pm 0.004 \cdot 10^3$	$^{+19.9\%}_{-17.4\%}$	$^{+1.0\%}_{-1.4\%}$
[illegible]	[illegible]	[illegible]	$1.731 \pm 0.001 \cdot 10^3$	$^{+51.9\%}_{-34.8\%}$	$^{+1.6\%}_{-2.1\%}$	$4.171 \pm 0.015 \cdot 10^3$	$^{+33.7\%}_{-27.1\%}$	$^{+1.4\%}_{-1.9\%}$
e.4*	$pp\to W^\pm b\bar{b}j$ (4f)	`p p > wpm b b~ j`	$1.861 \pm 0.003 \cdot 10^2$	$^{+42.5\%}_{-27.7\%}$	$^{+0.7\%}_{-0.7\%}$	$3.957 \pm 0.013 \cdot 10^2$	$^{+27.0\%}_{-21.0\%}$	$^{+0.7\%}_{-0.6\%}$
e.5*	$pp\to Zb\bar{b}j$ (4f)	`p p > z b b~ j`	$1.604 \pm 0.001 \cdot 10^2$	$^{+42.4\%}_{-27.6\%}$	$^{+0.9\%}_{-1.1\%}$	$2.805 \pm 0.009 \cdot 10^2$	$^{+21.0\%}_{-17.6\%}$	$^{+0.8\%}_{-1.0\%}$
e.6*	$pp\to \gamma b\bar{b}j$ (4f)	`p p > a b b~ j`	$7.812 \pm 0.017 \cdot 10^2$	$^{+51.2\%}_{-32.0\%}$	$^{+1.0\%}_{-1.5\%}$	$1.233 \pm 0.004 \cdot 10^3$	$^{+18.9\%}_{-19.9\%}$	$^{+1.0\%}_{-1.5\%}$
e.7	$pp\to t\bar{t}W^\pm$	`p p > t t~ wpm`	$3.777 \pm 0.003 \cdot 10^{-1}$	$^{+23.9\%}_{-18.0\%}$	$^{+2.1\%}_{-1.6\%}$	$5.662 \pm 0.021 \cdot 10^{-1}$	$^{+11.2\%}_{-10.6\%}$	$^{+1.7\%}_{-1.3\%}$
e.8	$pp\to t\bar{t}Z$	`p p > t t~ z`	$5.273 \pm 0.004 \cdot 10^{-1}$	$^{+30.5\%}_{-21.8\%}$	$^{+1.8\%}_{-2.1\%}$	$7.598 \pm 0.026 \cdot 10^{-1}$	$^{+9.7\%}_{-11.1\%}$	$^{+1.9\%}_{-2.2\%}$
e.9	$pp\to t\bar{t}\gamma$	`p p > t t~ a`	$1.204 \pm 0.001 \cdot 10^0$	$^{+29.6\%}_{-21.3\%}$	$^{+1.6\%}_{-1.8\%}$	$1.744 \pm 0.005 \cdot 10^0$	$^{+9.8\%}_{-11.0\%}$	$^{+1.7\%}_{-2.0\%}$
e.10*	$pp\to t\bar{t}W^\pm j$	`p p > t t~ wpm j`	$2.352 \pm 0.002 \cdot 10^{-1}$	$^{+40.9\%}_{-27.1\%}$	$^{+1.3\%}_{-1.0\%}$	$3.404 \pm 0.011 \cdot 10^{-1}$	$^{+11.2\%}_{-14.0\%}$	$^{+1.2\%}_{-0.9\%}$
e.11*	$pp\to t\bar{t}Zj$	`p p > t t~ z j`	$3.953 \pm 0.004 \cdot 10^{-1}$	$^{+46.2\%}_{-29.5\%}$	$^{+2.7\%}_{-3.0\%}$	$5.074 \pm 0.016 \cdot 10^{-1}$	$^{+7.0\%}_{-12.3\%}$	$^{+2.5\%}_{-2.9\%}$
e.12*	$pp\to t\bar{t}\gamma j$	`p p > t t~ a j`	$8.726 \pm 0.010 \cdot 10^{-1}$	$^{+45.4\%}_{-29.1\%}$	$^{+2.3\%}_{-2.6\%}$	$1.135 \pm 0.004 \cdot 10^0$	$^{+7.5\%}_{-12.2\%}$	$^{+2.2\%}_{-2.5\%}$
e.13*	$pp\to t\bar{t}W^-W^+$ (4f)	`p p > t t~ w+ w-`	$6.675 \pm 0.006 \cdot 10^{-3}$	$^{+30.9\%}_{-21.9\%}$	$^{+2.1\%}_{-2.0\%}$	$9.904 \pm 0.026 \cdot 10^{-3}$	$^{+10.9\%}_{-11.8\%}$	$^{+2.1\%}_{-2.1\%}$
e.14*	$pp\to t\bar{t}W^\pm Z$	`p p > t t~ wpm z`	$2.404 \pm 0.002 \cdot 10^{-3}$	$^{+26.6\%}_{-19.6\%}$	$^{+2.5\%}_{-1.8\%}$	$3.525 \pm 0.010 \cdot 10^{-3}$	$^{+10.6\%}_{-10.8\%}$	$^{+2.3\%}_{-1.6\%}$
e.15*	$pp\to t\bar{t}W^\pm\gamma$	`p p > t t~ wpm a`	$2.718 \pm 0.003 \cdot 10^{-3}$	$^{+25.4\%}_{-18.9\%}$	$^{+2.3\%}_{-1.8\%}$	$3.927 \pm 0.013 \cdot 10^{-3}$	$^{+10.3\%}_{-10.4\%}$	$^{+2.0\%}_{-1.5\%}$
e.16*	$pp\to t\bar{t}ZZ$	`p p > t t~ z z`	$1.349 \pm 0.014 \cdot 10^{-3}$	$^{+29.3\%}_{-21.1\%}$	$^{+1.7\%}_{-1.5\%}$	$1.840 \pm 0.007 \cdot 10^{-3}$	$^{+7.9\%}_{-9.9\%}$	$^{+1.7\%}_{-1.5\%}$
e.17*	$pp\to t\bar{t}Z\gamma$	`p p > t t~ z a`	$2.548 \pm 0.003 \cdot 10^{-3}$	$^{+30.1\%}_{-21.5\%}$	$^{+1.7\%}_{-1.6\%}$	$3.656 \pm 0.012 \cdot 10^{-3}$	$^{+9.7\%}_{-11.0\%}$	$^{+1.8\%}_{-1.9\%}$
e.18*	$pp\to t\bar{t}\gamma\gamma$	`p p > t t~ a a`	$3.272 \pm 0.006 \cdot 10^{-3}$	$^{+28.4\%}_{-20.6\%}$	$^{+1.3\%}_{-1.1\%}$	$4.402 \pm 0.015 \cdot 10^{-3}$	$^{+7.8\%}_{-9.7\%}$	$^{+1.4\%}_{-1.4\%}$

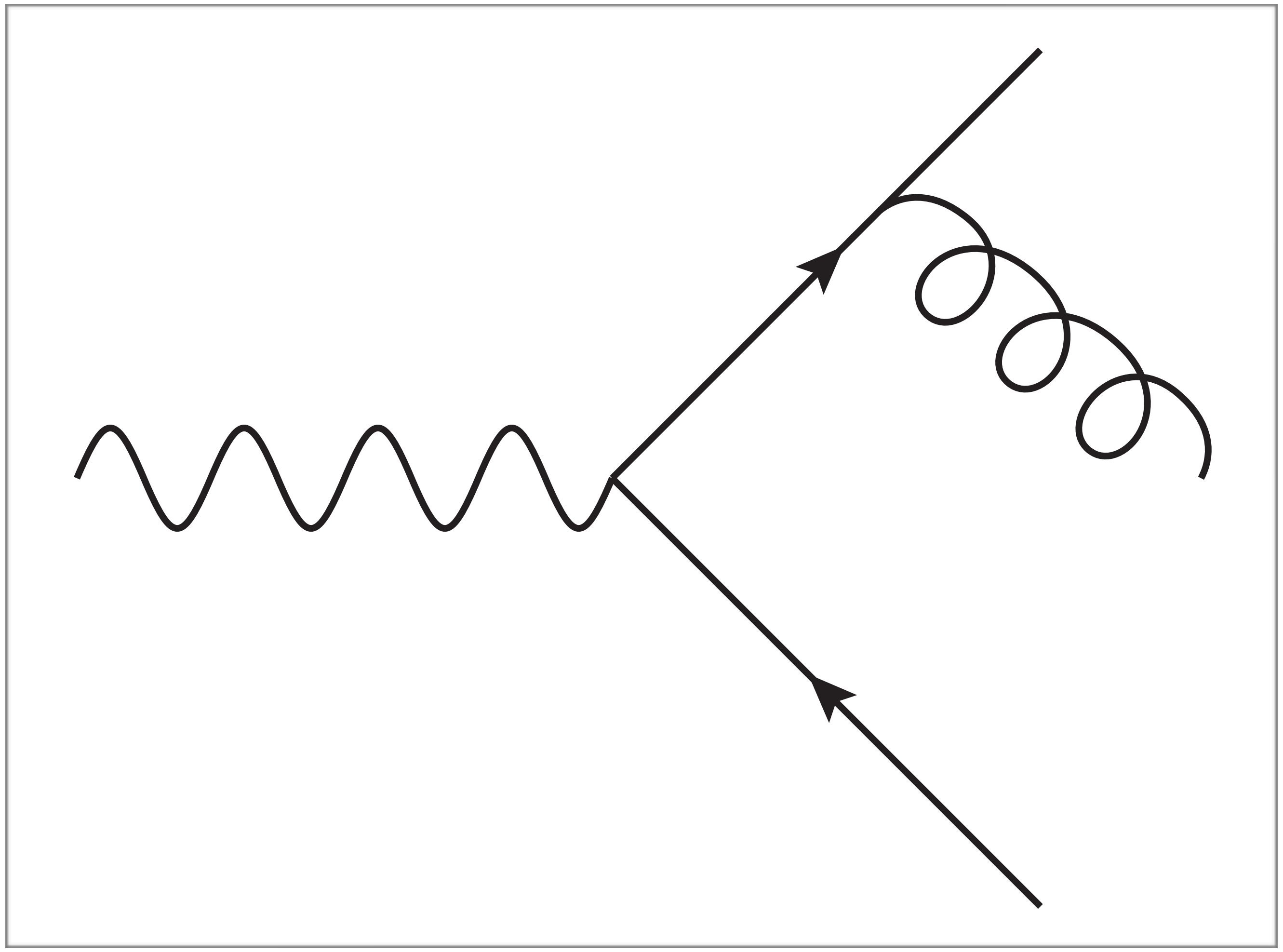
And much more!

(EW/BSM/...)

How to compute loops: Summary

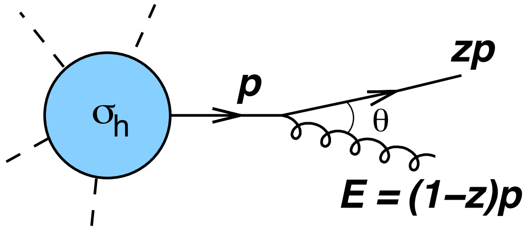
- There has been an enormous progress in loop computation techniques in the recent years
- For one-loop computation, we need to find the coefficient which multiply the scalar integrals
- OPP is a powerful method to compute the coefficients numerically. Some cares need to be taken because of dimensional regularisation

Infrared divergences



Branching

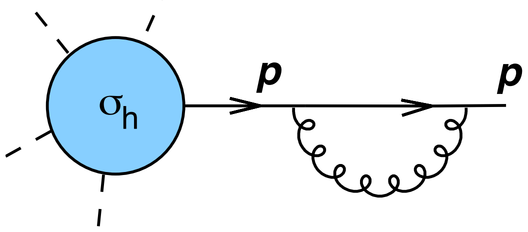
- Let us consider the branching of a gluon from a quark



$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

Where k_t is the transverse momentum of the gluon $k_t = E\sin\theta$. It diverges in the soft ($z \to 1$) and collinear ($k_t \to 0$) region

- These singularities cancel with the virtual contribution, which comes from the integration of the loop momentum



$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

- The cancelation happens if we cannot distinguish between the case of no branching, and of a soft or collinear branching

Cancelation of divergences

- The KLN theorem tells us that divergences from the virtual and real emission cancel in the sum if observables are insensitive to soft and collinear branchings (IR-safety)
- When doing an analytic computation in dimensional regularisation, divergences appear as poles in the regularisation parameter ε
- In the real emissions, poles appear *after* the phase space integration in d dimension

Infrared safety

- In order to have meaningful predictions in fixed-order perturbation theory, observables must be IR-safe, *i.e.* not sensitive to the emission of soft or collinear partons.
- In particular, if an observable depends on the momentum p_i, it must not be sensitive on the branching $p_i \rightarrow p_j + p_k$, where either p_j is soft or p_j and p_k are collinear
- For example
  - The number of gluons in an event is not IR-safe
  - The number of jets with $p_T > p_T^{min}$ is IR-safe

Phase space integration

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} + \int d^4\Phi_n \mathcal{V} + \int d^4\Phi_{n+1} \mathcal{R}$$

contains $\int d^d l$

- For complicated processes the integrations have to be done via MonteCarlo techniques, in an integer number of dimensions
- Divergences have to be canceled explicitly
- Slicing/Subtraction methods have been developed to extract divergences from the phase-space integrals

Example

- Suppose that we can cast the phase space integral in the form

$$\int_0^1 dx f(x) \quad \text{with} \quad f(x) = \frac{g(x)}{x} \quad \text{and } g(x) \text{ a regular function}$$

- We introduce a regulator which renders the integral finite

$$\int_0^1 dx x^\varepsilon f(x) = \int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- The divergence shows as a pole in ε. How can we extract the pole?

Phase space slicing

$$\lim_{\varepsilon\to 0}\int_0^1 dx x^\varepsilon f(x) = \lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- We introduce a small parameter δ«1:

$$\begin{aligned}\lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}} &= \lim_{\varepsilon\to 0}\left(\int_0^\delta dx \frac{g(x)}{x^{1-\varepsilon}} + \int_\delta^1 dx \frac{g(x)}{x^{1-\varepsilon}}\right)\\ &\simeq \lim_{\varepsilon\to 0}\left(\int_0^\delta dx \frac{g(0)}{x^{1-\varepsilon}} + \int_\delta^1 dx \frac{g(x)}{x^{1-\varepsilon}}\right)\\ &= \lim_{\varepsilon\to 0}\frac{\delta^\varepsilon}{\varepsilon} g(0) + \int_\delta^1 dx \frac{g(x)}{x}\\ &= \lim_{\varepsilon\to 0}\left(\frac{1}{\varepsilon} + \log\delta\right) g(0) + \int_\delta^1 dx \frac{g(x)}{x}\end{aligned}$$

Phase space slicing

$$\lim_{\varepsilon\to 0}\int_0^1 dx x^{\varepsilon} f(x) = \lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- We introduce a small parameter δ«1:

$$\lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}} = \lim_{\varepsilon\to 0}\left(\int_0^{\delta} dx \frac{g(x)}{x^{1-\varepsilon}} + \int_{\delta}^1 dx \frac{g(x)}{x^{1-\varepsilon}}\right)$$

$$\simeq \lim_{\varepsilon\to 0}\left(\int_0^{\delta} dx \frac{g(0)}{x^{1-\varepsilon}} + \int_{\delta}^1 dx \frac{g(x)}{x^{1-\varepsilon}}\right)$$

$$= \lim_{\varepsilon\to 0}\frac{\delta^{\varepsilon}}{\varepsilon} g(0) + \int_{\delta}^1 dx \frac{g(x)}{x}$$

$$= \lim_{\varepsilon\to 0}\left(\boxed{\frac{1}{\varepsilon}} + \log\delta\right) g(0) + \int_{\delta}^1 dx \frac{g(x)}{x}$$

pole in ε

Phase space slicing

$$\lim_{\varepsilon\to 0}\int_0^1 dx x^\varepsilon f(x) = \lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- We introduce a small parameter δ«1:

$$\begin{aligned}\lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}} &= \lim_{\varepsilon\to 0}\left(\int_0^\delta dx \frac{g(x)}{x^{1-\varepsilon}} + \int_\delta^1 dx \frac{g(x)}{x^{1-\varepsilon}}\right)\\ &\simeq \lim_{\varepsilon\to 0}\left(\int_0^\delta dx \frac{g(0)}{x^{1-\varepsilon}} + \int_\delta^1 dx \frac{g(x)}{x^{1-\varepsilon}}\right)\\ &= \lim_{\varepsilon\to 0}\frac{\delta^\varepsilon}{\varepsilon}g(0) + \int_\delta^1 dx \frac{g(x)}{x}\\ &= \lim_{\varepsilon\to 0}\left(\boxed{\frac{1}{\varepsilon}} + \log\delta\right)g(0) + \boxed{\int_\delta^1 dx \frac{g(x)}{x}}\end{aligned}$$

finite integral
(can be computed numerically)

pole in ε

Subtraction method

$$\lim_{\varepsilon\to 0}\int_0^1 dx x^\varepsilon f(x) = \lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- Add and subtract $g(0)/x$

$$\begin{aligned}\lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}} &= \lim_{\varepsilon\to 0}\int_0^1 dx x^\varepsilon \left(\frac{g(0)}{x} + \frac{g(x)}{x} - \frac{g(0)}{x}\right)\\ &= \lim_{\varepsilon\to 0}\int_0^1 dx \left(\frac{g(0)}{x^{1-\varepsilon}} + \frac{g(x)-g(0)}{x^{1-\varepsilon}}\right)\\ &= \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}g(0) + \int_0^1 dx \frac{g(x)-g(0)}{x}\end{aligned}$$

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$$= \lim_{\varepsilon\to 0}\int_0^1 dx \left(\frac{g(0)}{x^{1-\varepsilon}} + \frac{g(x)-g(0)}{x^{1-\varepsilon}}\right)$$

$$= \lim_{\varepsilon\to 0}\boxed{\frac{1}{\varepsilon}g(0)} + \int_0^1 dx \frac{g(x)-g(0)}{x}$$

pole in ε

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$$= \lim_{\varepsilon\to 0}\boxed{\frac{1}{\varepsilon}g(0)} + \int_0^1 dx \boxed{\frac{g(x)-g(0)}{x}}$$

finite integral
(can be computed numerically)

pole in ε

Slicing vs Subtraction

- In both cases the pole is extracted and we end up with a finite remainder:

$$g(0)\log\delta + \int_\delta^1 dx\,\frac{g(x)}{x} \qquad \int_0^1 dx\,\frac{g(x)-g(0)}{x}$$

- Subtraction acts like a plus distribution
- Slicing works only for small δ, and one has to prove the δ-independence of cross section and distribution; subtraction is exact
- In both methods there are cancelation between large numbers. If for a given observable $\lim_{x\to 0} O(x) \neq O(0)$ or we choose a too small bin size, instabilities will arise (we cannot ask for an infinite resolution)
- Subtraction is more flexible: good for automation

NLO with subtraction

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} + \int d^4\Phi_n \mathcal{V} + \int d^4\Phi_{n+1} \mathcal{R}$$

- Including the subtraction terms the expression becomes

$$\begin{aligned} \sigma_{NLO} = & \int d^4\Phi_n \mathcal{B} \\ & + \int d^4\Phi_n \left(\mathcal{V} + \int d^d\Phi_1 \mathcal{C} \right)_{\varepsilon\to 0} \\ & + \int d^4\Phi_{n+1} \left(\mathcal{R} - \mathcal{C} \right) \end{aligned}$$

- Terms in brackets are finite and can be integrated numerically in *d*=4 and independently one from another

NLO with subtraction

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Poles cancel from
d-dim integration

- Terms in brackets are finite and can be integrated numerically in *d*=4 and independently one from another

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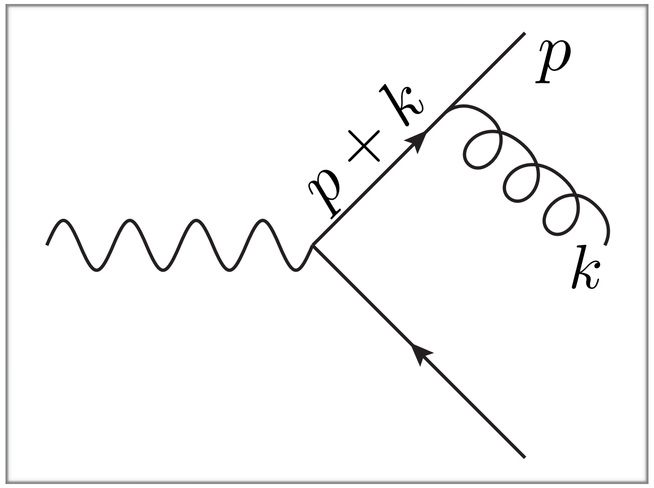
Poles cancel from *d*-dim integration

Integrand is finite in 4 dimension

- Terms in brackets are finite and can be integrated numerically in *d*=4 and independently one from another

The subtraction term

- The subtraction term C should be chosen such that:
  - It exactly matches the singular behaviour of R
  - It can be integrated numerically in a convenient way
  - It can be integrated exactly in d dimension, leading to the soft and/or collinear poles in the dimensional regulator
  - It is process independent (overall factor times Born)
- QCD comes to help: structure of divergences is universal:



$$(p+k)^2 = 2E_pE_k(1-\cos\theta_{pk})$$

- Collinear singularity:

$$\lim_{p//k} |M_{n+1}|^2 \simeq |M_n|^2 \; P^{AP}(z)$$

- Soft singularity:

$$\lim_{k\to 0} |M_{n+1}|^2 \simeq \sum_{ij} |M_n^{ij}|^2 \frac{p_ip_j}{p_ik \; p_jk}$$

Two subtraction methods

Dipole subtraction

Catani, Seymour, hep-ph/9602277 & hep-ph/9605323

- Most used method
- Recoil taken by one parton
  → N^3 scaling
- Method evolves from cancelation of soft divergences
- Proven to work for simple and complicated processes
- Automated in MadDipole, AutoDipole, Sherpa, Helac-NLO, …

FKS subtraction

Frixione, Kunszt, Signer, hep-ph/9512328

- Less known method
- Recoil distributed among all particles
  → N^2 scaling
- Probably (?) more efficient because less subtraction terms are needed
- Method evolves from cancelation of collinear divergences
- Proven to work for simple and complicated processes
- Automated in MadGraph5_aMC@NLO and in the Powheg box/Powhel

FKS subtraction #1
Phase space partition

- Let us consider the real emission

$$d\sigma_R = \left|M^{n+1}\right|^2 d\Phi_{n+1}$$

- The matrix element $|M^{n+1}|^2$ diverges as

$$\left|M^{n+1}\right| \sim \frac{1}{\xi_i^2}\frac{1}{1-y_{ij}}$$

$$\xi_i = E_i\sqrt{\hat{s}}$$

$$y_{ij} = \cos\theta_{ij}$$

- Partition the phase space in order to have at most one soft and one collinear singularity

$$d\sigma_R = \sum_{ij} S_{ij}\left|M^{n+1}\right|^2 d\Phi_{n+1} \qquad \sum_{ij} S_{ij} = 1$$

$$S_{ij} \to 1 \text{ if } k_i \cdot k_j \to 0 \qquad S_{ij} \to 0 \text{ if } k_{m\neq i} \cdot k_{n\neq j} \to 0$$

FKS subtraction #2
Plus prescriptions

- Use plus prescriptions in y_{ij} and ξ_i to subtract the divergences

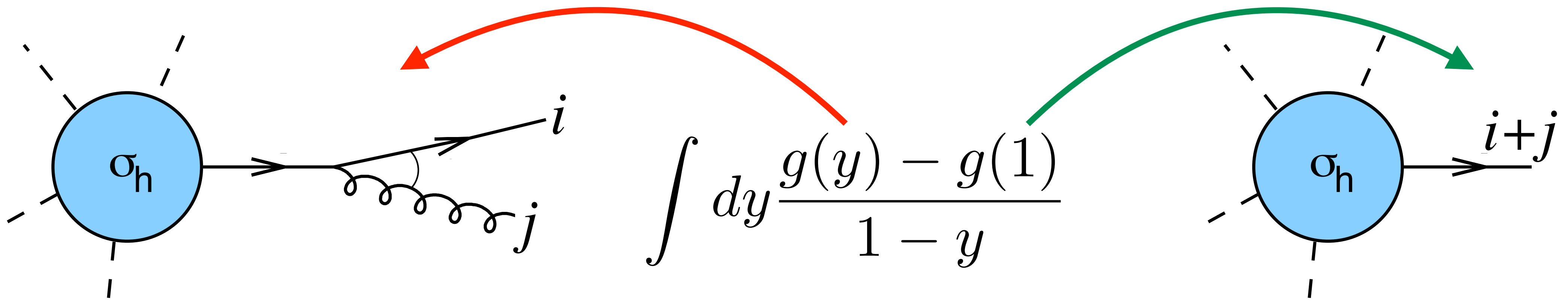
$$d\sigma_{\tilde{R}} = \sum_{ij} \left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i (1-y_{ij}) S_{ij} \left|M^{n+1}\right|^2 d\Phi_{n+1}$$

- Plus prescriptions are defined as

$$\int d\xi \left(\frac{1}{\xi}\right)_+ f(\xi) = \int d\xi \frac{f(\xi)-f(0)}{\xi} \qquad \int dy \left(\frac{1}{1-y}\right)_+ g(y) = \int dy \frac{g(y)-g(1)}{1-y}$$

- Maximally three counterevents are needed
  - Soft counterevent ($\xi_i \to 0$)
  - Collinear counterevents ($y_{ij} \to 1$)
  - Soft-collinear counterevents ($\xi_i \to 0$ and $y_{ij} \to 1$)

Kinematics of counterevents



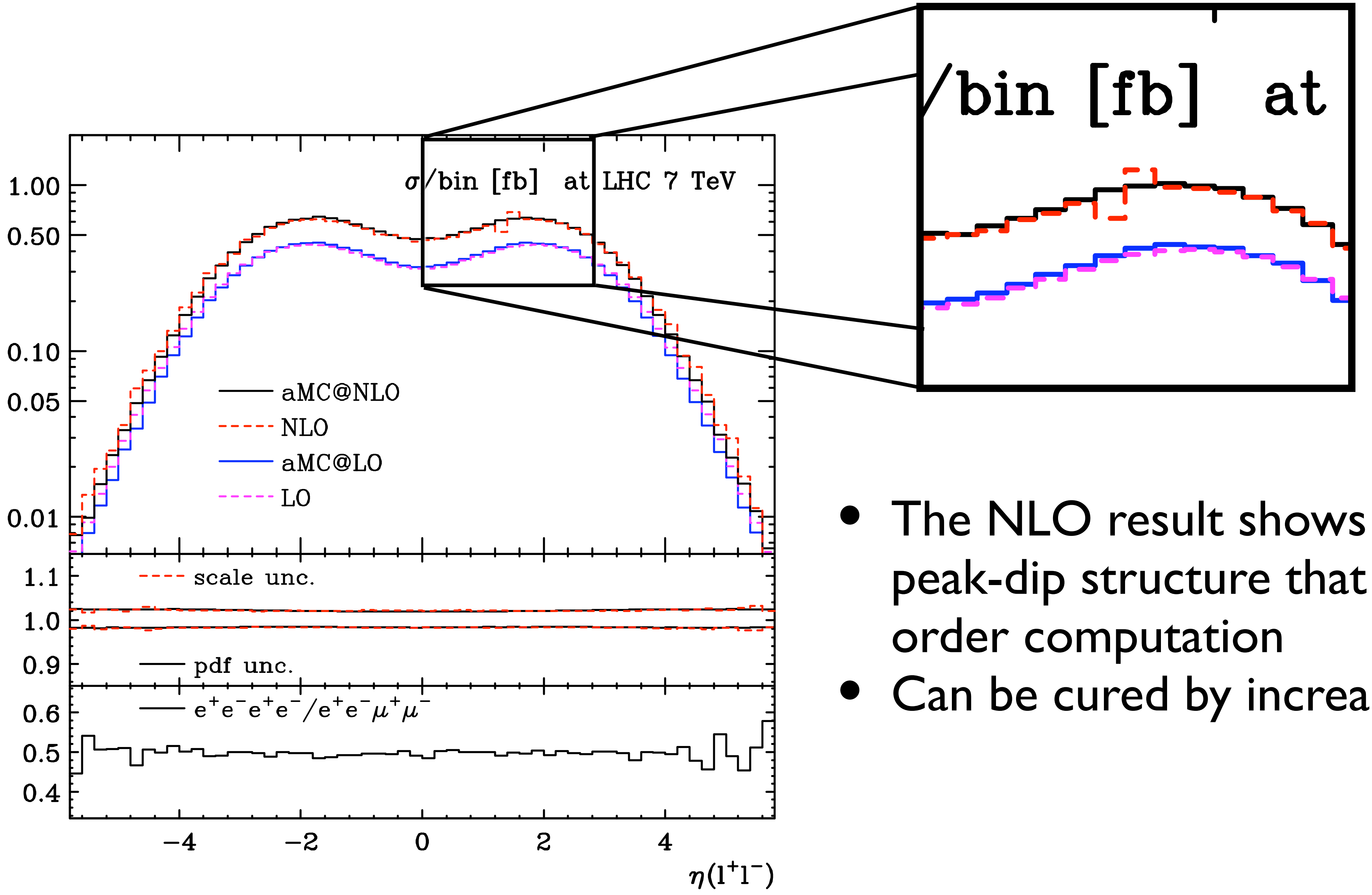
$$\int dy \frac{g(y) - g(1)}{1 - y}$$

Real emission

Subtraction term

- If i and j are on-shell in the event, for the counterevent the combined particle $i+j$ must be on shell
- $i+j$ can be put on shell only be reshuffling the momenta of the other particles
- It can happen that event and counterevent end up in different histogram bins
  - Use IR-safe observables and don't ask for infinite resolution!
  - Still, these precautions do not eliminate the problem…

An example in 4-lepton production



- The NLO result shows the typical peak-dip structure that hampers fixed-order computation
- Can be cured by increasing the statistics

Can we generate unweighted events at NLO?

- Another consequence of the kinematic mismatch is that we cannot generate events at NLO
- n+1-body contribution and n-body contribution are not bounded from above → unweighting not possible
- Further ambiguity on which kinematics to use for the unweighted events

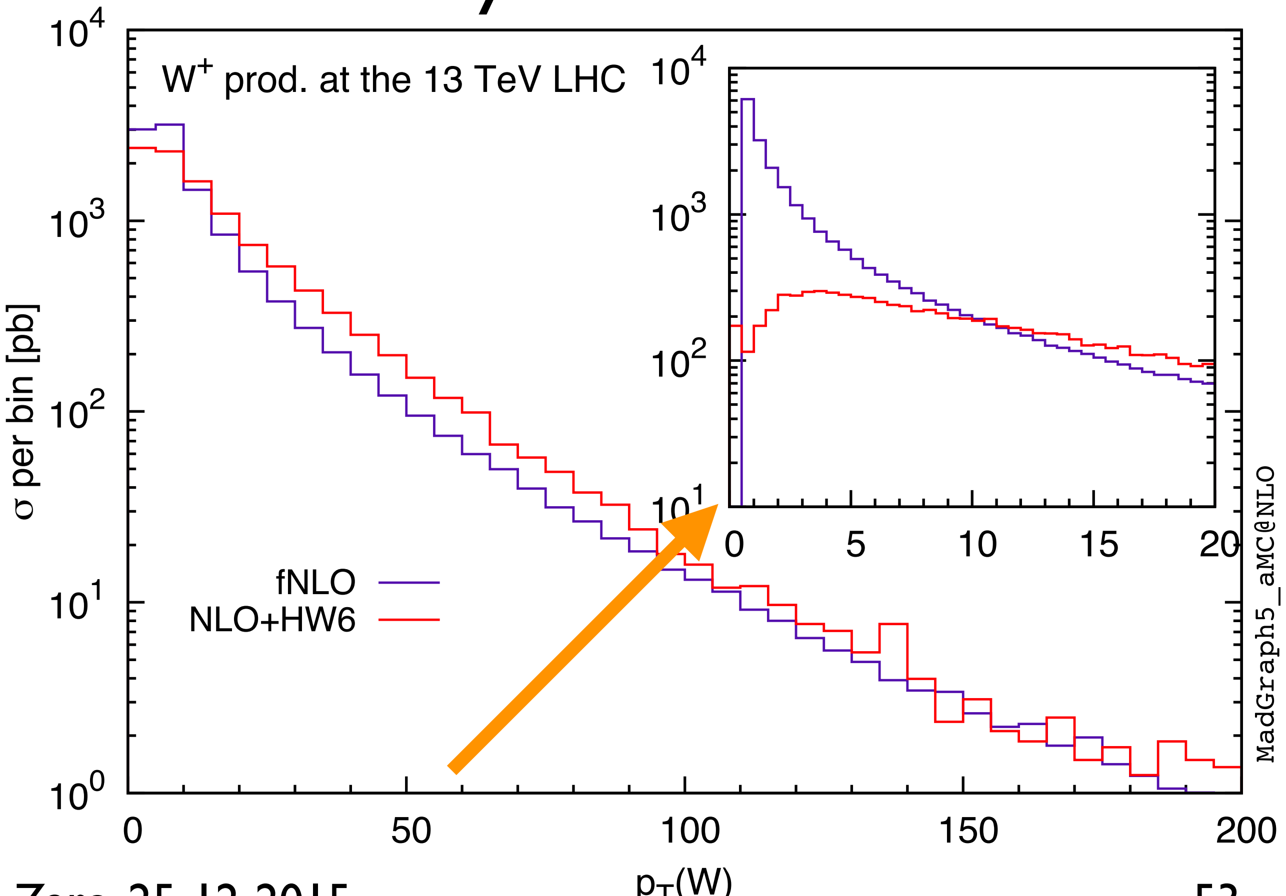
Filling histograms on-the-fly

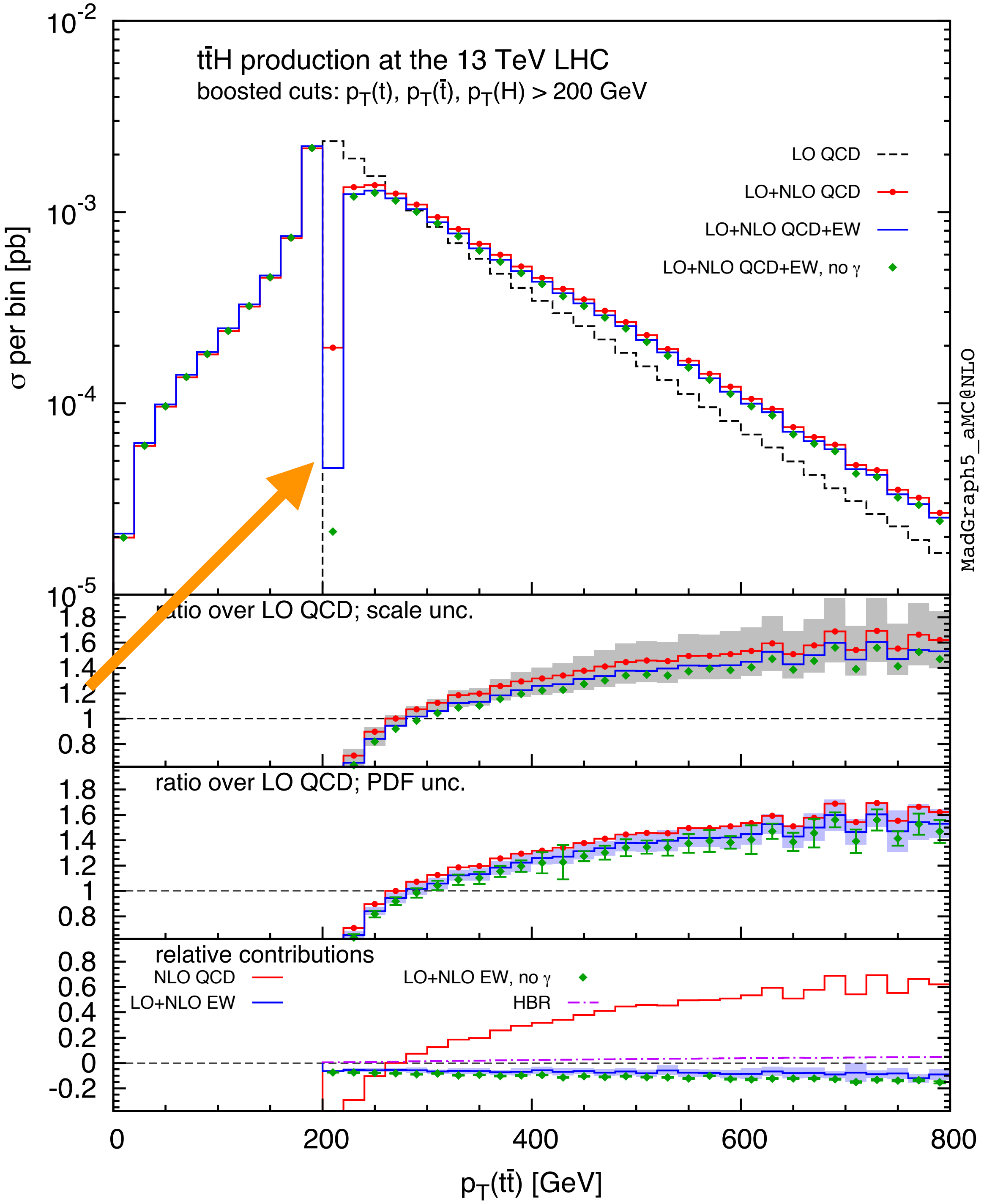
$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} \\ + \int d^4\Phi_n \left(\mathcal{V} + \int d^d\Phi_1 \mathcal{C} \right)_{\varepsilon \to 0} \\ + \int d^4\Phi_{n+1} \left(\mathcal{R} - \mathcal{C} \right)$$

- In practice, two set of momenta are generated during the MC integration
  - A n-body set, for Born, virtuals and counterterms
  - A $n+1$-body set, for the real emission
- The various terms are computed. Cuts are applied on the corresponding momenta and histograms are filled with the weight and kinematics of each term

Instabilities at fixed order

- Besides the mis-binning problem, the kinematics mismatch can lead to odd behaviours of certain observables, in particular when some constraint coming from the n-body kinematics is relaxed in the $n+1$-body one





Subtracting IR divergences: Summary

- Virtual and real matrix element are not finite, but their sum is. Subtraction methods can be used to extract divergences for real-emission matrix elements and cancel explicitly the poles from the virtuals
- Event and counterevents have different kinematics. Unweighting is not possible, we need to fill plots on-the-fly with weighted events
- For plots, only IR-safe observable with finite resolution must be used!

Intermezzo:
Is it all at NLO?

- Suppose we have a code for pp→$t\bar{t}$ @NLO. Are all the following (IR-safe) variables described at NLO?
  - top p_T
  - $t\bar{t}$ pair p_T
  - $t\bar{t}$ pair invariant mass
  - jet p_T
  - $t\bar{t}$ azimuthal distance

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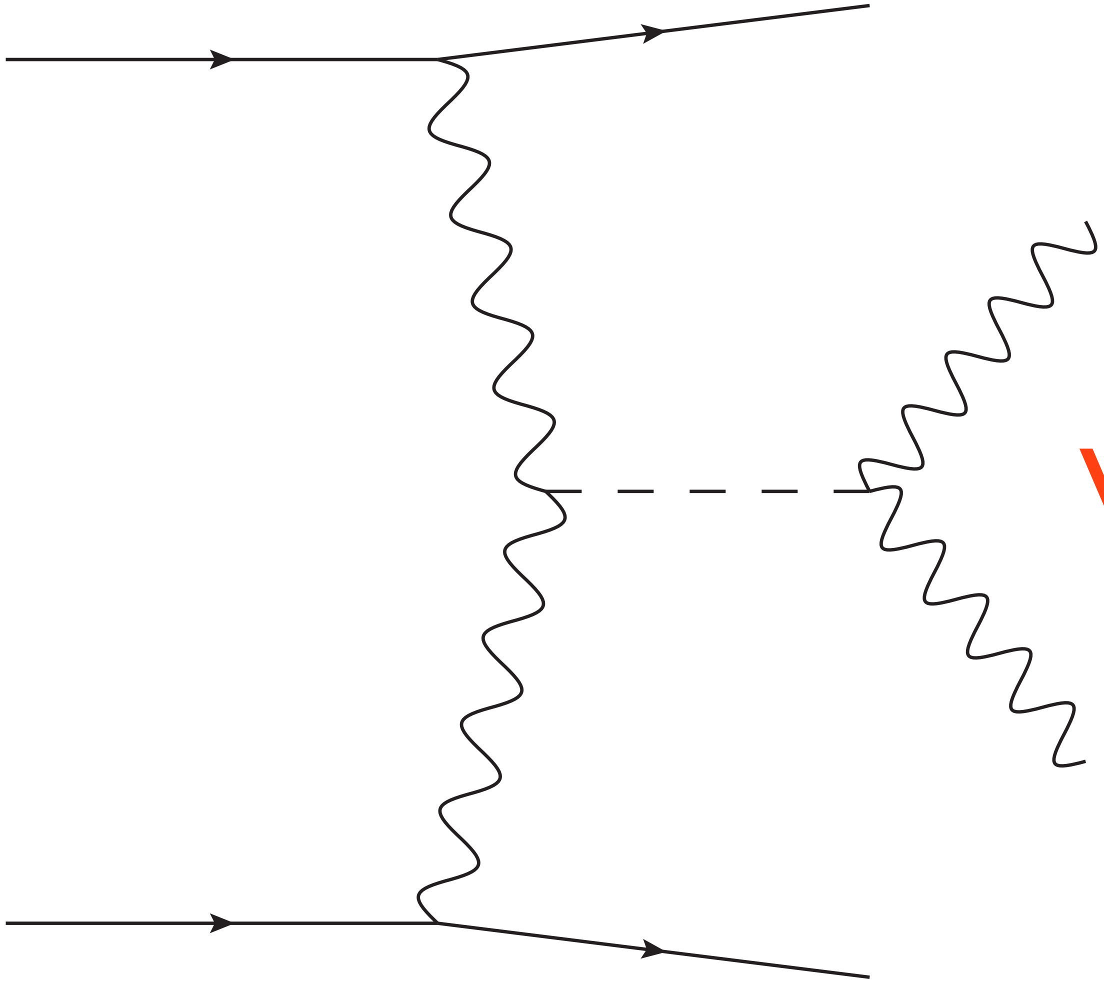
Intermezzo:
Is it all at NLO?

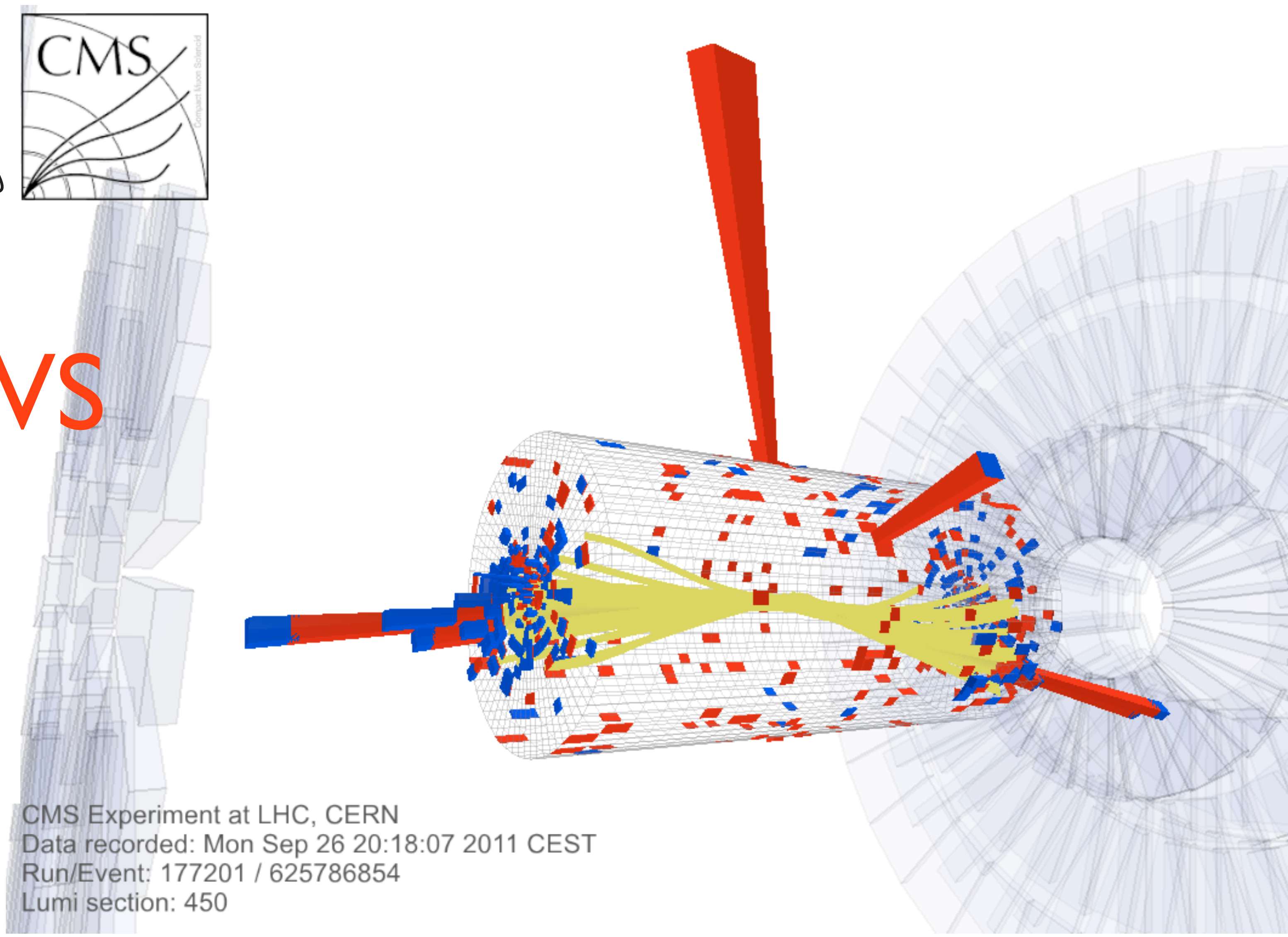
- Suppose we have a code for $pp \to t\bar{t}$ @NLO. Are all the following (IR-safe) variables described at NLO?

top p_T	YES
$t\bar{t}$ pair p_T	NO
$t\bar{t}$ pair invariant mass	YES
jet p_T	NO
$t\bar{t}$ azimuthal distance	NO

Matching NLO predictions and parton showers



VS



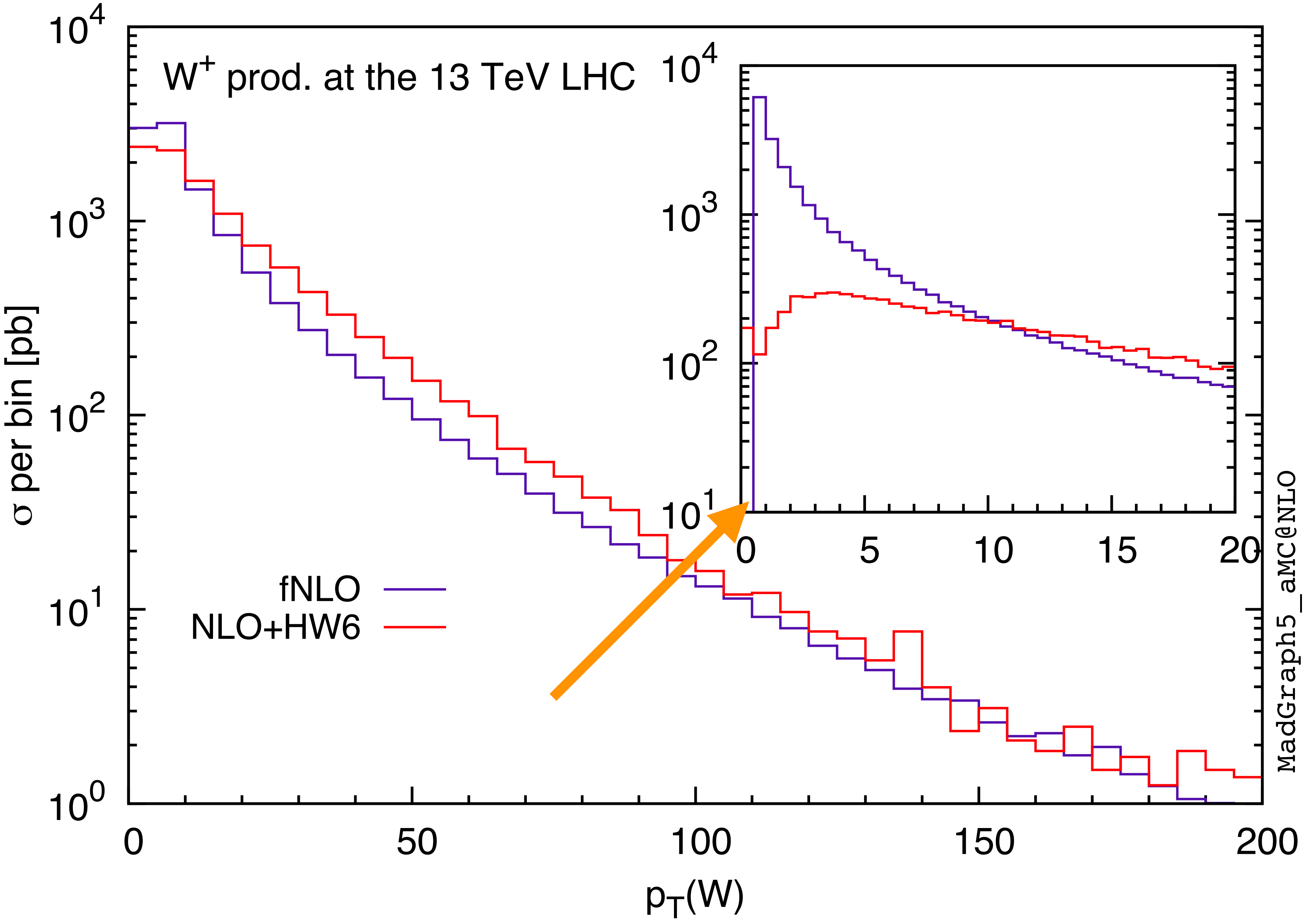
CMS Experiment at LHC, CERN
Data recorded: Mon Sep 26 20:18:07 2011 CEST
Run/Event: 177201 / 625786854
Lumi section: 450

Matching NLO predictions and parton showers

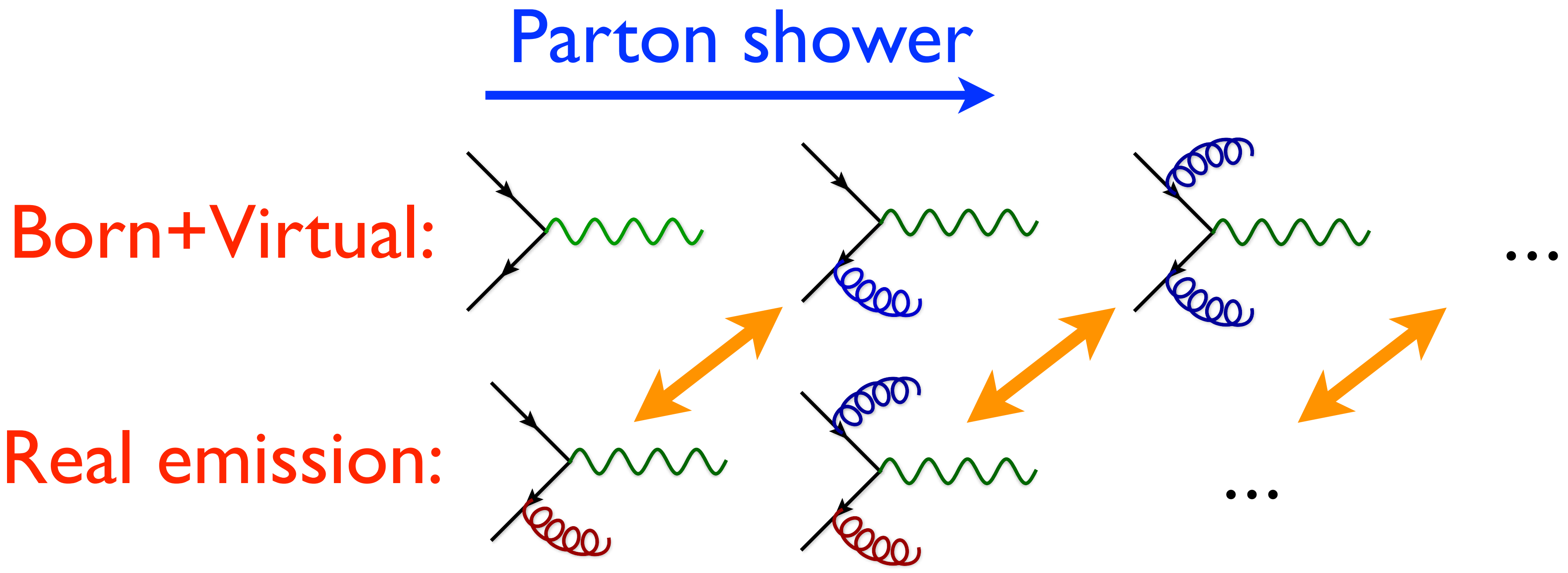
- Parton showers evolve hard partons by emitting extra QCD radiation down to a more realistic final state made of hadrons
- This resums the effect of soft gluon radiations, and cures fixed-order instabilities
- After the parton shower, a fully exclusive description of the event is available
- NLO corrections are inclusive by definition, but they provide the first reliable estimate of rates and uncertainties

Can we attach a parton shower to NLO simulations?

Fixed order instabilities, again



Warning: double counting!


Parton shower

Born+Virtual:

Real emission:

…

…

- There is a double counting between real emission and the parton shower
- There is also double counting between the virtuals and the non-emission probability from the Sudakov factor

Double counting the virtuals

- The Sudakov factor Δ, which is responsible for the resummation performed by the shower, is the no-emission probability (1-P, P being the emission probability)
- Δ therefore contains implicitly contributions from the virtual corrections
- We should therefore avoid to double counting the contribution from the virtuals in the matrix element and in the Sudakov
- Because of unitarity, what is double counted in the virtuals is exactly opposite to what is double counted by the reals

How to avoid double counting at NLO?

- Two methods exist:
  - MC@NLO Frixione, Webber hep-ph/0204244
  - Powheg Nason, hep-ph/0409146

Naive (wrong) matching

- Let us *assume* we can generate events separately for Born, virtuals and real emissions, and that we pass them to a parton shower $\frac{d\sigma_{\text{``}NLO+PS\text{''}}}{dO} = \left[\mathcal{B}+\mathcal{V}\right] d\Phi_n\, I^n_{MC}(O) + d\Phi_{n+1}\mathcal{R}\, I^{n+1}_{MC}(O)$
- Do we get the NLO cross section?
    - Let us expand the shower operator at order α_S (0 or 1 emission)

$$I_{MC} = \Delta_a\left(Q, Q_0\right) + \Delta_a\left(Q, Q_0\right) d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

$$\Delta_a\left(Q, Q_0\right) = \exp\left[-\int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}\right] \simeq 1 - \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

$$I_{MC} \simeq 1 - \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc} + d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

Naive (wrong) matching

- At order α_S we get

$$\frac{d\sigma_{\text{“}NLO+PS\text{”}}}{dO} = [\mathcal{B}+\mathcal{V}]\,d\Phi_n + d\Phi_{n+1}\mathcal{R}$$

$$-\,\mathcal{B}\,d\Phi_n \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc} + \mathcal{B}\,d\Phi_n d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

- Which is not the NLO

MC@NLO matching

- In the MC@NLO formalism, double counting can be cured by the so-called Monte Carlo counterterms, defined as

$$\Delta(Q^2, Q_0) = \exp\left(\int d\Phi_1 \frac{MC}{\mathcal{B}}\right) \qquad MC = \left|\frac{\partial\Phi_1^{MC}}{\partial\Phi_1}\right| \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

- The MC@NLO cross section is defined as

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n \, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- Again, if we expand up to α_S we recover the NLO

$$I_{MC} = 1 - \int d\Phi_1 MC + d\Phi_1 MC$$

$$\frac{d\sigma_{\text{``}MC@NLO\text{''}}}{dO} = \left[\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right] d\Phi_n + d\Phi_{n+1}\left[\mathcal{R} - MC\right]$$

$$+ \mathcal{B}\left[- \int d\Phi_1 MC + d\Phi_1 MC\right] d\Phi_n$$

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- Again, if we expand up to α_S we recover the NLO

$$I_{MC} = 1 - \int d\Phi_1 MC + d\Phi_1 MC$$

$$\frac{d\sigma_{\text{“}MC@NLO\text{”}}}{dO} = \left[\mathcal{B}+\mathcal{V}+\cancel{\int d\Phi_1 MC}\right] d\Phi_n + d\Phi_{n+1}\left[\mathcal{R}-MC\right]$$

$$+\,\mathcal{B}\left[\cancel{-\int d\Phi_1 MC} + d\Phi_1 MC\right] d\Phi_n$$

MC@NLO matching

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$$I_{MC} = 1 - \int d\Phi_1 MC + d\Phi_1 MC$$

$$\frac{d\sigma_{\text{“}MC@NLO\text{”}}}{dO} = \left[\mathcal{B} + \mathcal{V} + \int \cancel{d\Phi_1 MC}\right] d\Phi_n + d\Phi_{n+1}\left[\mathcal{R} - \cancel{MC}\right]$$

$$+ \mathcal{B}\left[- \int \cancel{d\Phi_1 MC} + \cancel{d\Phi_1 MC}\right] d\Phi_n$$

The *MC* counterterm

- *MC* has some remarkable properties:
  - It avoids double counting when matching to PS ←Just shown
  - It matches the singular behaviour of the real-emission ME, making it possible to unweight events (some special cares are needed for the soft region)
  - It ensures a smooth matching: NLO+PS has the same shape of the shower in the soft/collinear region; in the hard region, it approaches the NLO
  - It is PS dependent, as it depends on the PS details. For each PS, we need its own *MC* counterterms

Unweighting

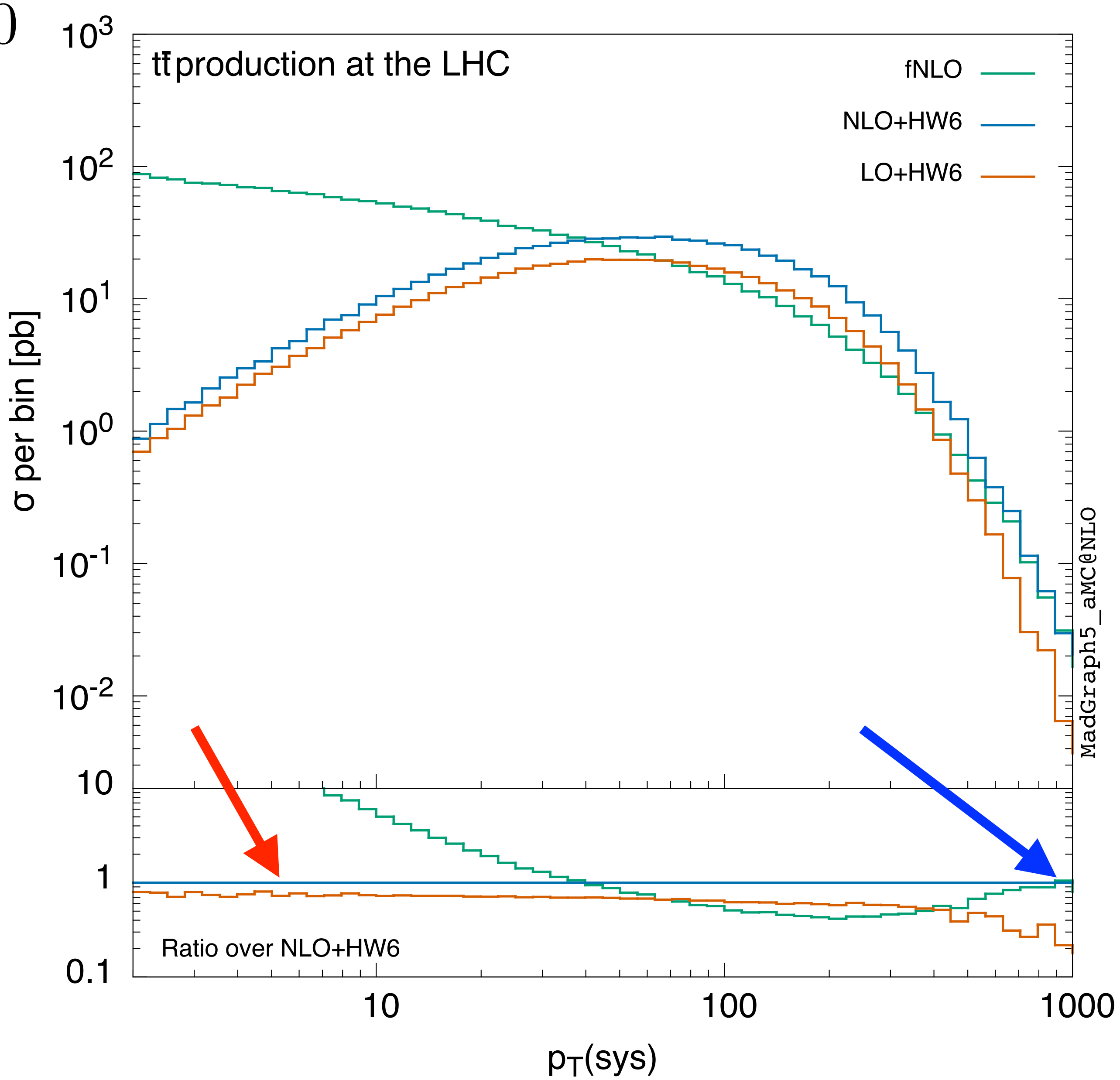
$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n\, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- MC is by construction what the shower does to go from n to n+1. It matches exactly R in the soft-collinear region. Furthermore, it has the same kinematics as R, therefore there is no reshuffling needed. The n and n+1 body contributions are separately finite and bounded. Unweighted events can be generated!
  - S-events, with n-body kinematics
  - H-events, with n+1-body kinematics

Smooth matching

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n\, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- In the soft/collinear region, $\mathcal{R} - MC \sim 0$ so that
$$\frac{d\sigma_{MC@NLO}}{dO} \simeq I^n_{MC}(O)$$
- In the hard region, MC=0 (it is bound to be zero far from singular regions). The only contribution comes from the real-emission ME



The MC counterterms and the FKS subtraction

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n\, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- The MC counterterms already make the cross-section finite. Are the local counterterms still needed?
- Yes, because we cannot integrate MC analytically to extract the poles
- In practice, we have

$$\frac{d\sigma_{MC@NLO}}{dO} = \left[\mathcal{B} + \left(\mathcal{V} + \int d\Phi_1 \mathcal{C}\right) + \int d\Phi_1 \left(MC - \mathcal{C}\right)\right] d\Phi_n\, I^n_{MC}(O)$$
$$+ (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

Negative weights

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n \, I^n_{MC}(O) + (\mathcal{R} - MC) \, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- Events are generated for n- and n+1-body kinematics separately
- Nothing guarantees that the two contributions are separately positive
- The unweighting has to be done up to a sign, and the sign should be taken into account when filling plots
- Remember: results are physical only after having showered the events!

Powheg

- Let us consider the LO+PS cross-section expanded up to the first emission:

$$d\sigma_{LO+PS} = \mathcal{B}\, d\Phi_n \left[\Delta(Q,Q_0) + \Delta(Q,Q_0) d\Phi_1 \frac{\alpha_s(t)}{2\pi} P\right]$$

- We could think of going NLO by replacing the Born with the NLO cross section

$$d\sigma_{\text{“}NLO+PS\text{”}} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[\Delta(Q,Q_0) + \Delta(Q,Q_0) d\Phi_1 \frac{\alpha_s(t)}{2\pi} P\right]$$

- Of course, there is double counting. This is in particular due by the fact that the integral in the Sudakov does not contain R

A modified Sudakov

- In order to avoid double counting one could use a modified Sudakov

$$\tilde{\Delta}(Q,Q_0) = \exp\left(-\int_{Q_0^2}^{Q^2} d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right)$$

- Such that

$$d\sigma_{\text{“}NLO+PS\text{”}} = \left(\mathcal{B}+\mathcal{V}+\int d\Phi_1\mathcal{R}\right) d\Phi_n \left[\tilde{\Delta}(Q,Q_0) + \tilde{\Delta}(Q,Q_0) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right]$$

- But the total rate is not the NLO! The second parentheses does not integrate to 1. It has to be modified to

$$d\sigma_{\text{“}NLO+PS\text{”}} = \left(\mathcal{B}+\mathcal{V}+\int d\Phi_1\mathcal{R}\right) d\Phi_n \left[\tilde{\Delta}(Q,Q_0) + \tilde{\Delta}(Q,t) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right]$$

- Where t is the scale at which R/B is evaluated

Properties

$$d\sigma_{Powheg} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[\tilde{\Delta}(Q, Q_0) + \tilde{\Delta}(Q, t) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right]$$

- Note that

$$\tilde{\Delta}(Q, t)\frac{\mathcal{R}}{\mathcal{B}} = \frac{d\tilde{\Delta}(Q, t)}{dt}$$

- Therefore

$$\int_{Q_0}^{Q} dt \tilde{\Delta}(Q, t)\frac{\mathcal{R}}{\mathcal{B}} = \tilde{\Delta}(Q, Q) - \tilde{\Delta}(Q, Q_0) = 1 - \tilde{\Delta}(Q, Q_0)$$

- So the [] integrates to 1. The NLO normalisation is kept
- If one expands at order α_S:

$$d\sigma_{Powheg} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[1 - \int d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}} + d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right] = d\sigma_{NLO}$$

- Double counting is avoided

Comments

$$d\sigma_{Powheg} = \left(\mathcal{B}+\mathcal{V}+\int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[\tilde{\Delta}(Q,Q_0) + \tilde{\Delta}(Q,t) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right]$$

- The Powheg cross section has the same structure as an ordinary shower, with a global K-factor correction and a different Sudakov for the first emission
- Note that when matching to PS one has to veto emissions harder than t (in the Powheg formalism, is has to be interpreted as transverse momentum), even for showers with a different ordering variable
  - Formula to be modified for angular-ordered PS in order to keep color coherence
- MC@NLO and Powheg are formally equivalent at NLO level. In practice, there are many differences between the two

MC@NLO vs Powheg

- The two matching procedure can be cast in a single formula

$$d\sigma_{NLO+PS} = d\Phi_n \bar{\mathcal{B}}^s \left[\Delta^s(Q, Q^0) + d\Phi_1 \frac{\mathcal{R}^s}{\mathcal{B}} \Delta^s(Q, t) \right] + d\Phi_{n+1} \mathcal{R}^f$$

- With

$$\bar{\mathcal{B}}^s = \mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}^s$$

- And the real-emission ME has been split in a singular and non-singular (finite) part

$$\mathcal{R} = \mathcal{R}^s + \mathcal{R}^f$$

- The difference between the two methods is in R^s:

MC@NLO $\mathcal{R}^s = \frac{\alpha_s}{2\pi} P \mathcal{B} = MC$

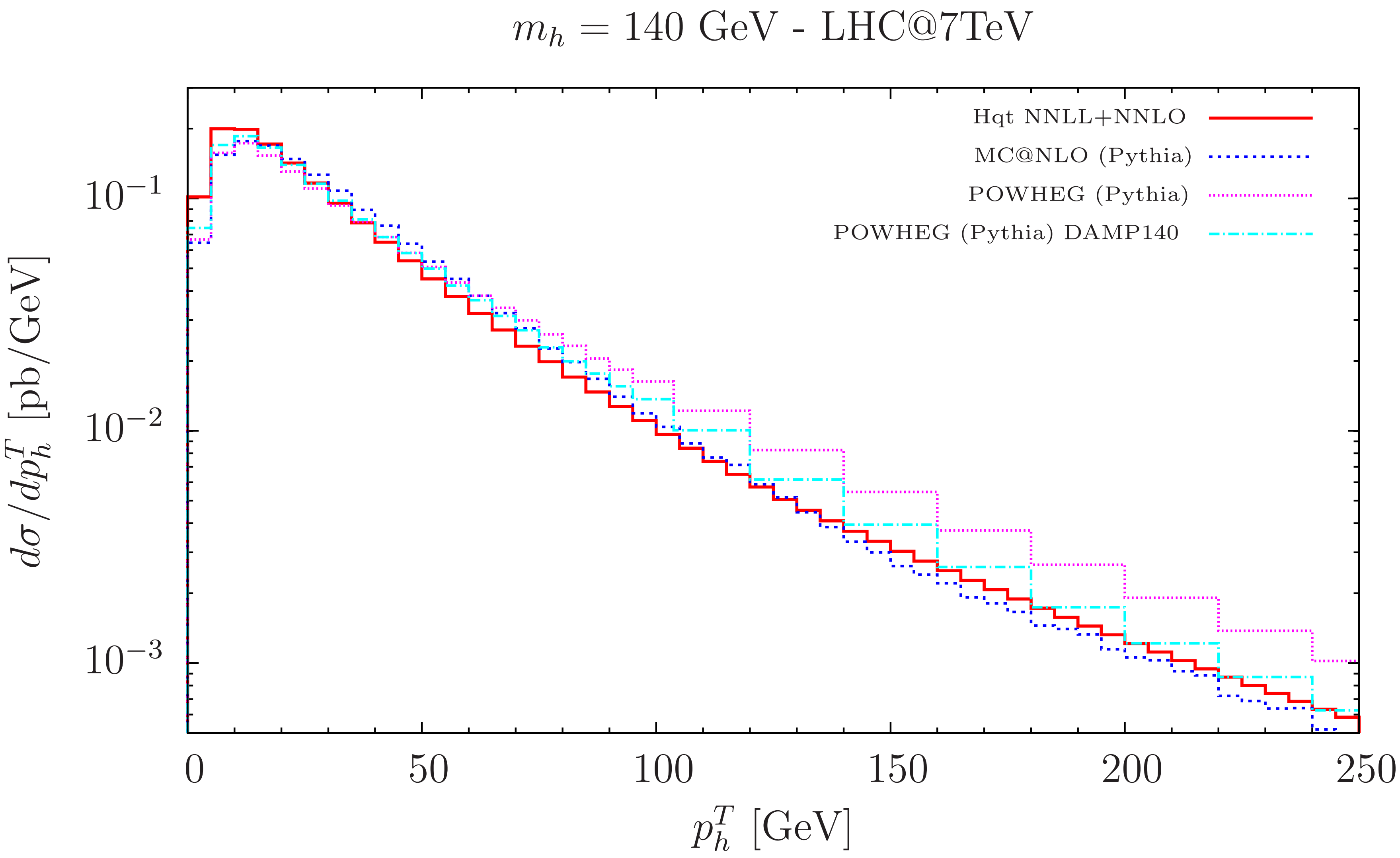
Powheg $\mathcal{R}^s = F\mathcal{R} \quad \mathcal{R}^f = (1 - F)\mathcal{R}$

default F=1,
but can be tuned in order to
suppress non-singular part of R

Effect of F

$$F = \frac{h^2}{h^2 + p_T^2}$$

p_T≫h are suppressed



MC@NLO naturally matches analytic resummation+FO curve at large p_T
Powheg (without damping) overshoots the FO
Damping recovers matching at large p_T

Comparison and summary

	MC@NLO	POWHEG
Parton showers are (usually) not exact in the soft limit: MC@NLO needs an artificial smoothing	☹	☺
MC@NLO does not exponentiate the non-singular part of the real emission amplitudes	☺	☹
MC@NLO does not require any tricks for treating Born zeros	☺	☹
POWHEG is independent from the parton shower (although, in general the shower should be a truncated vetoed)	☹	☺
POWHEG has (almost) no negatively weighted events	☹	☺
Automation of the methods: http://amcatnlo.cern.ch, http://powhegbox.mib.infn.it, http://www.sherpa-mc.de	☺	☺