MadGraph5

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Plan

- Details of the computation
  - Evaluation of matrix-element
  - Phase-Space integration
- What is MG5_aMC

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism

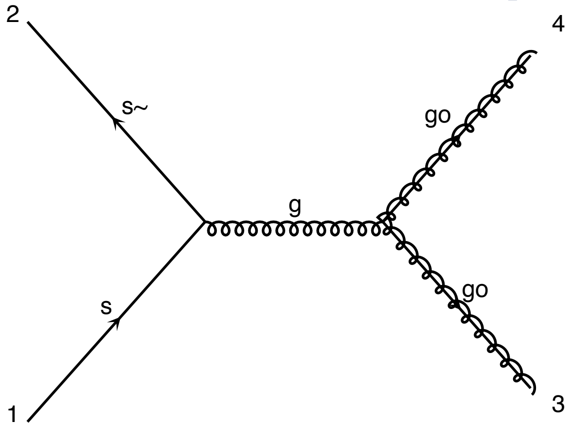


diagram 1 QCD=2, QED=0

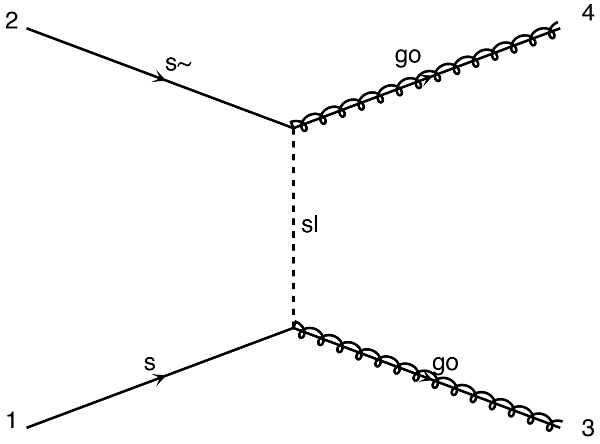


diagram 2 QCD=2, QED=0

- Evaluate the matrix-element

$$|\mathcal{M}|^2$$

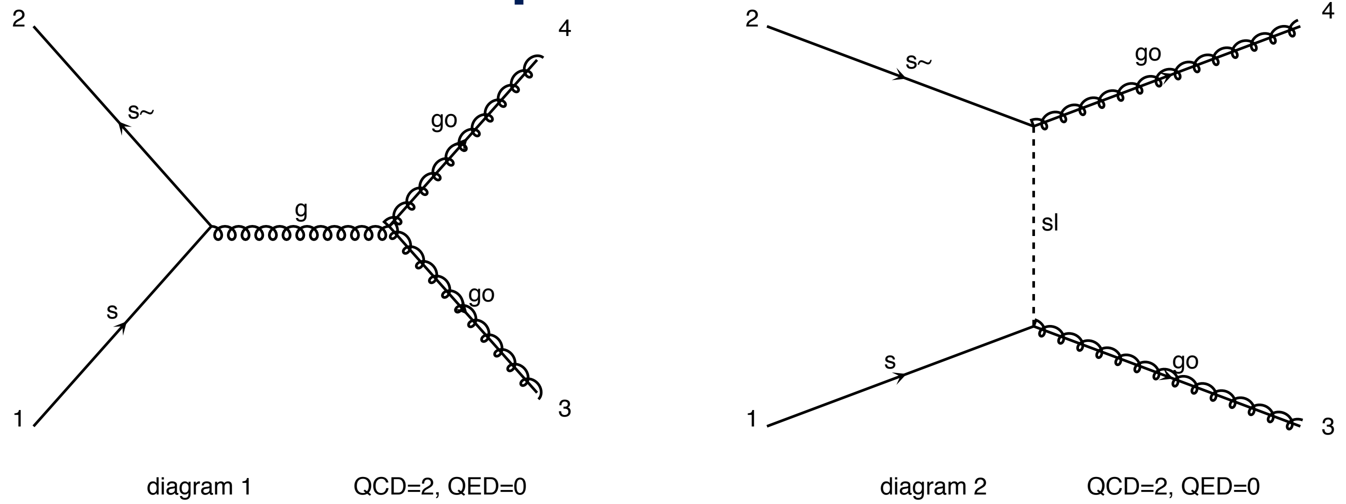
- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



Easy enough

- Evaluate the matrix-element

$$|\mathcal{M}|^2$$

Hard

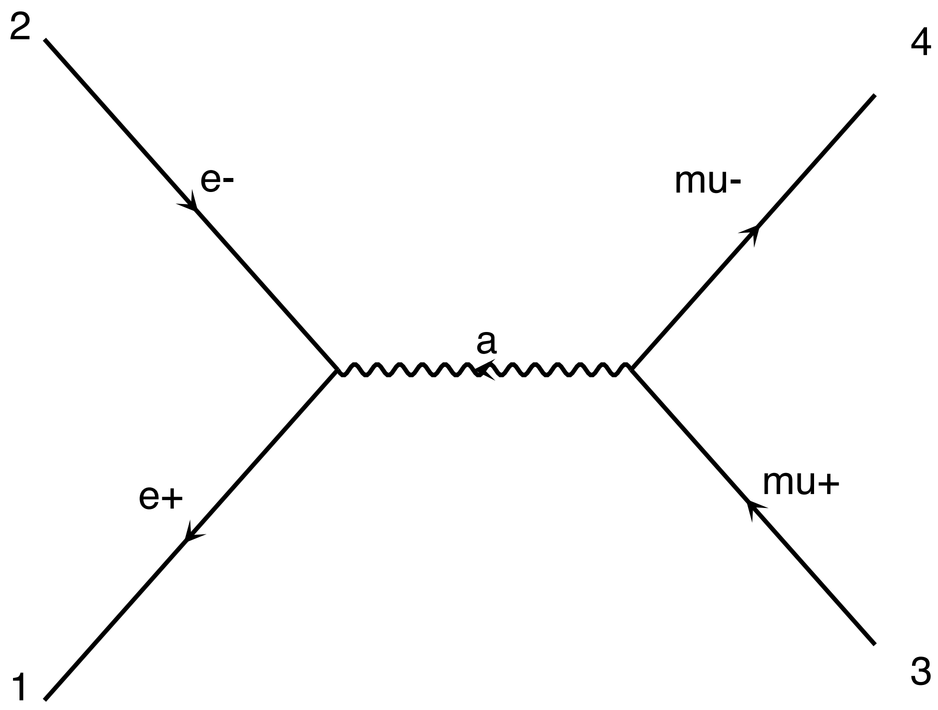
- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Very Hard

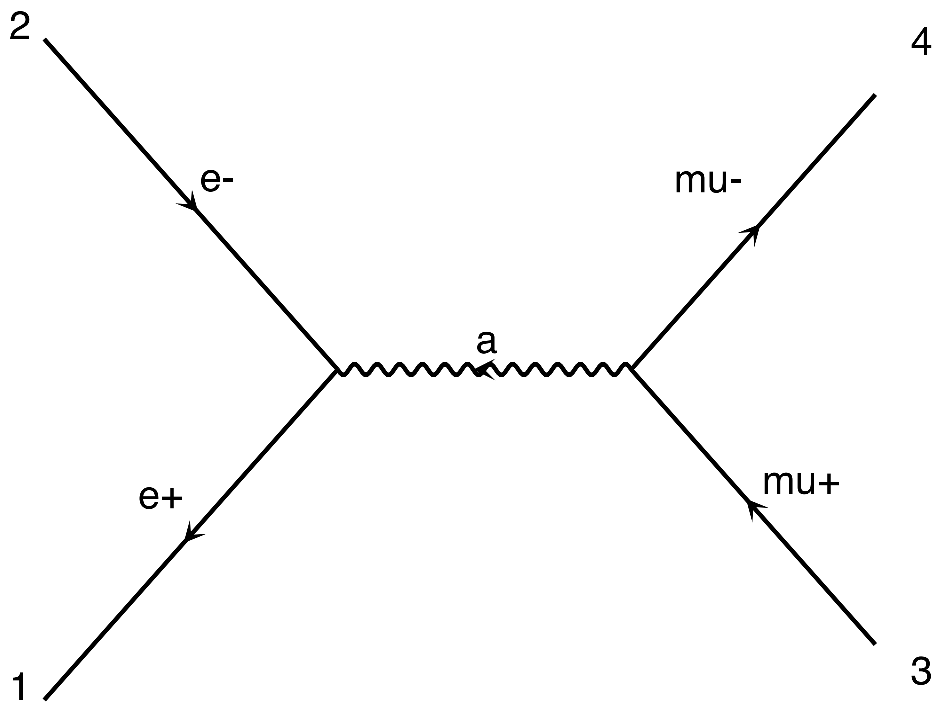
(in general)

Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

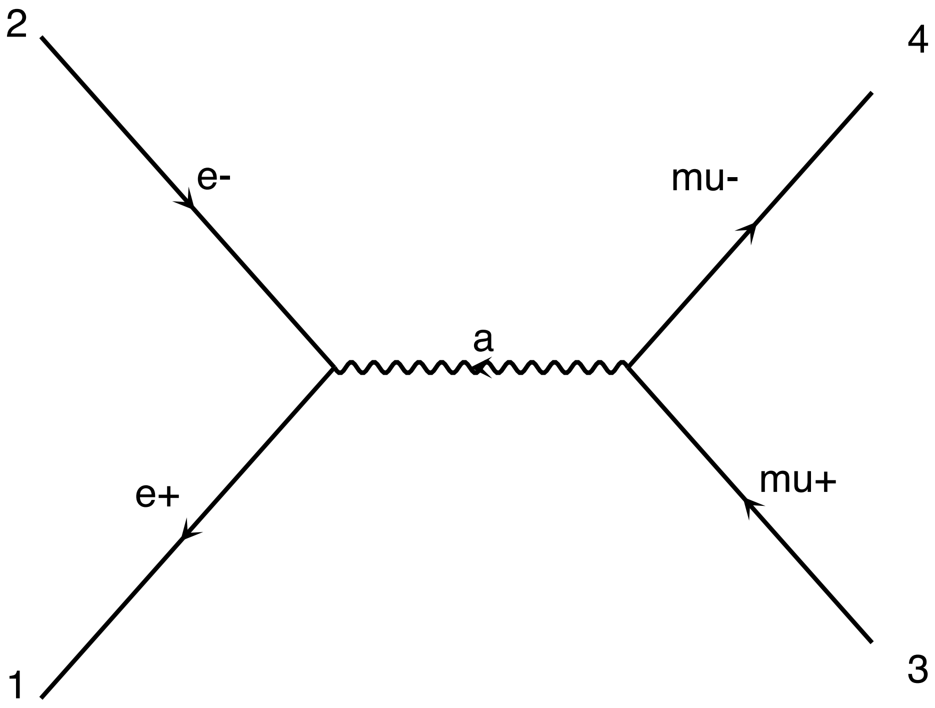
Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

Matrix Element



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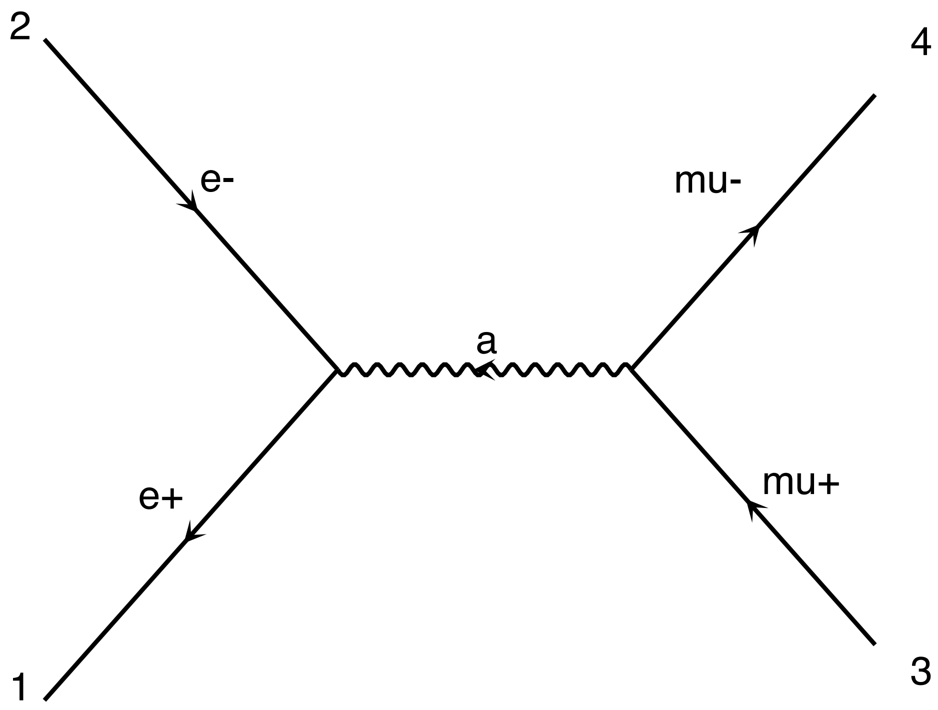
$$\sum_{pol}\bar{u}u = \not{p} + m$$

Matrix Element



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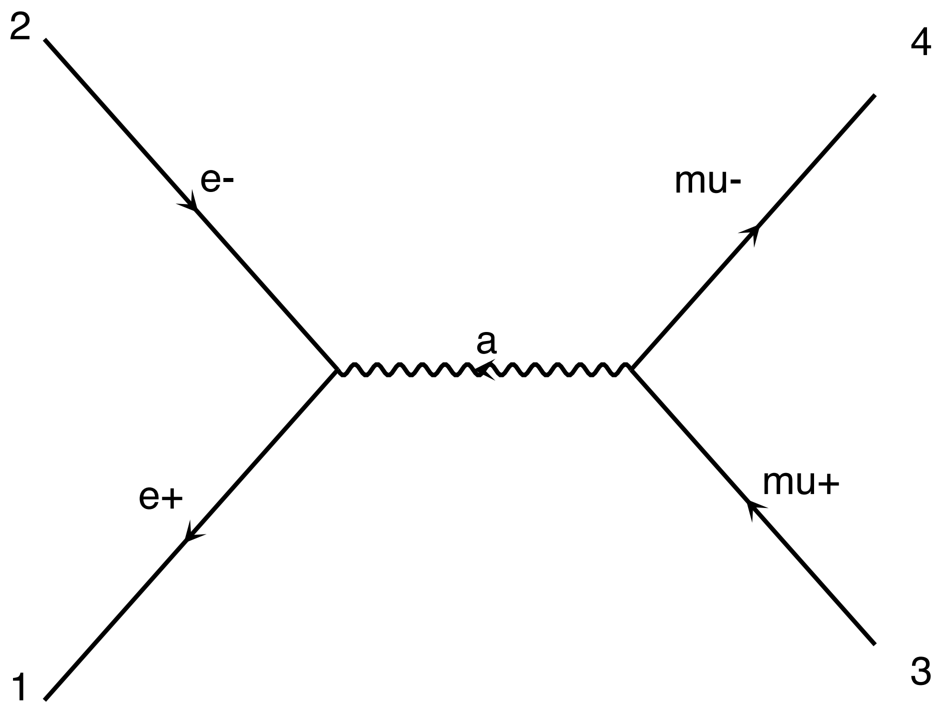
$$\rightarrow \frac{e^4}{4q^4}Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

Matrix Element



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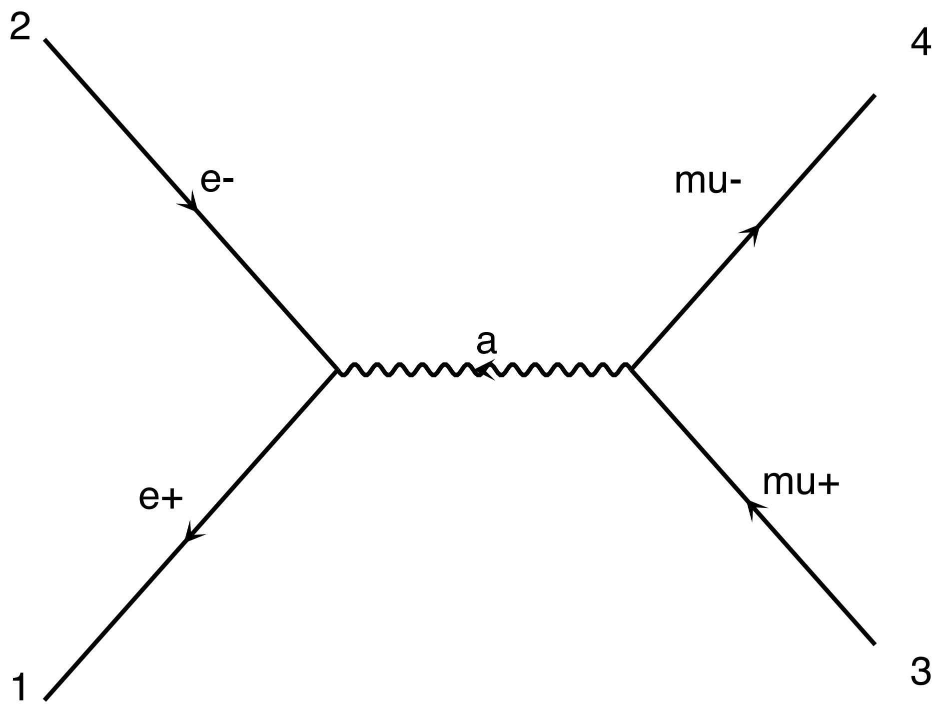
$$\rightarrow \frac{8e^4}{q^4}[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Matrix Element



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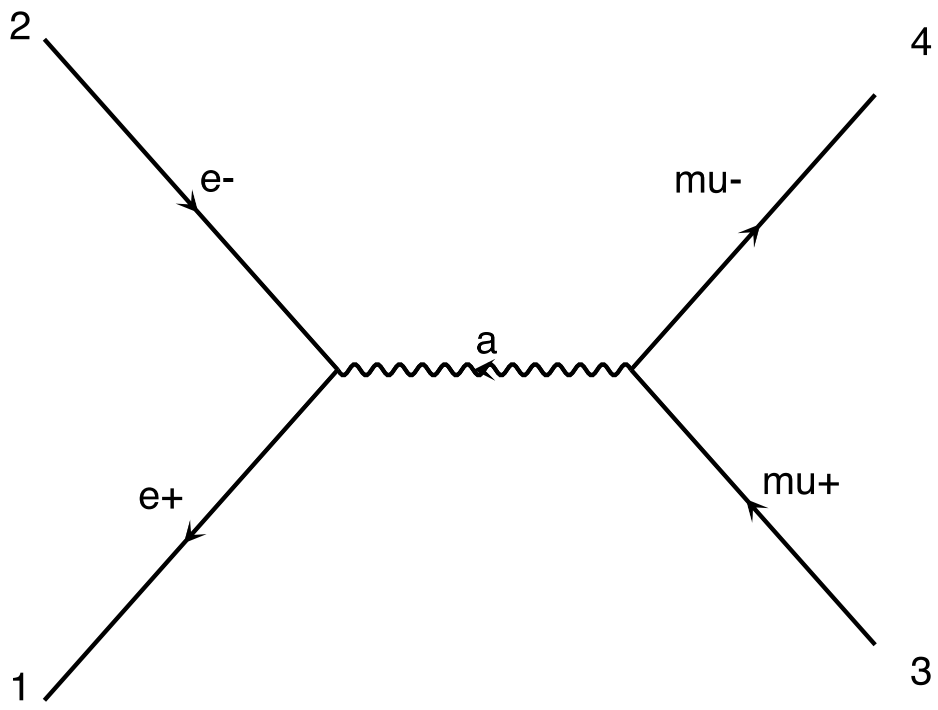
Very Efficient !!!

Matrix Element



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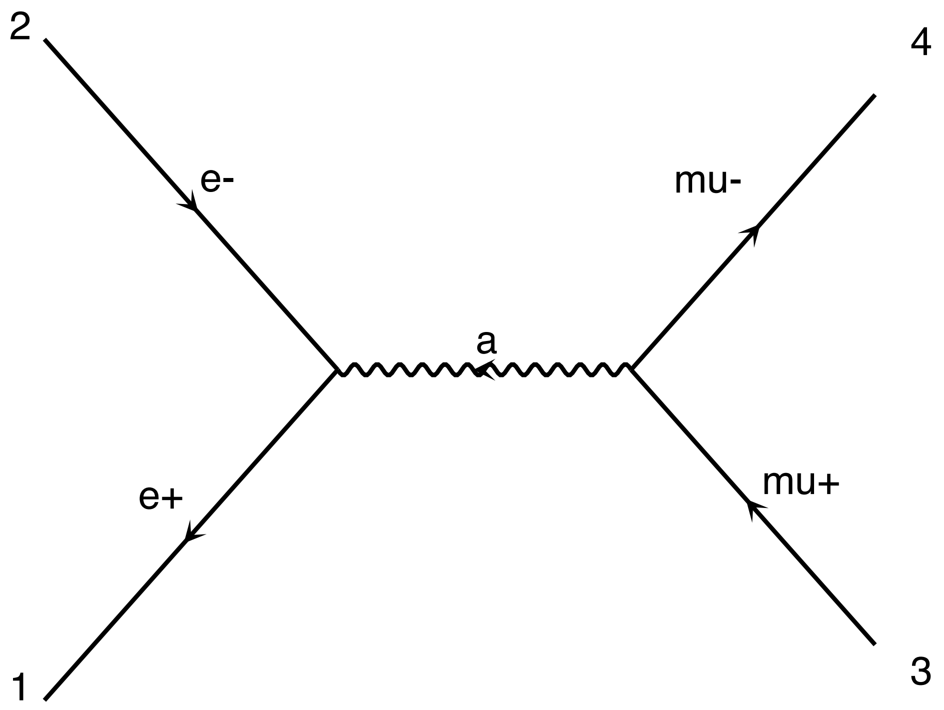
Very Efficient !!!

Only for 2→2 and 2→3

Matrix Element



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$$\rightarrow \frac{8e^4}{q^4}[(p_1.p_3)(p_2.p_4)+(p_1.p_4)(p_2.p_3)]$$

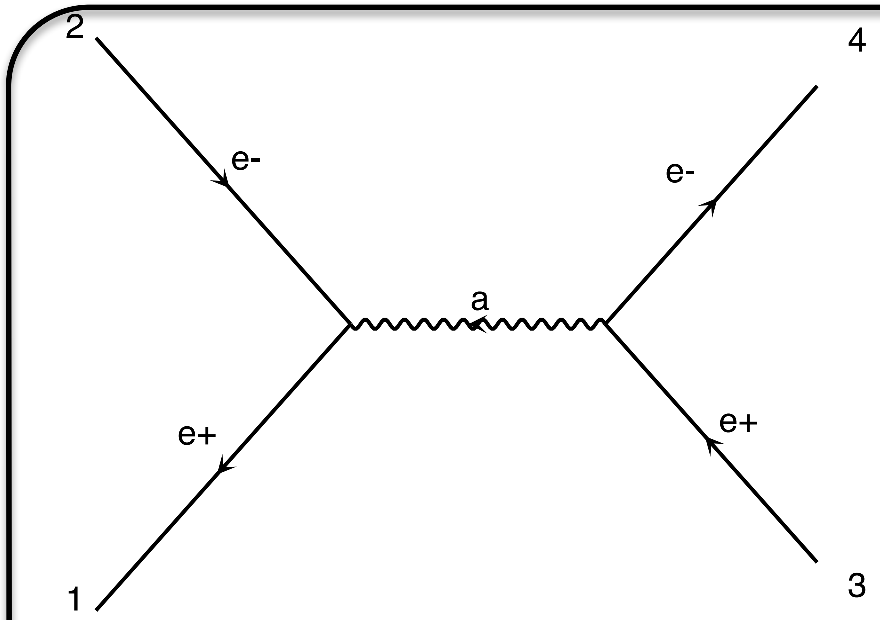
Very Efficient !!!

Only for 2→2 and 2→3

Because the number of terms rises as N^2

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu}\, v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\, e\, \gamma^{\nu}\, u)$$

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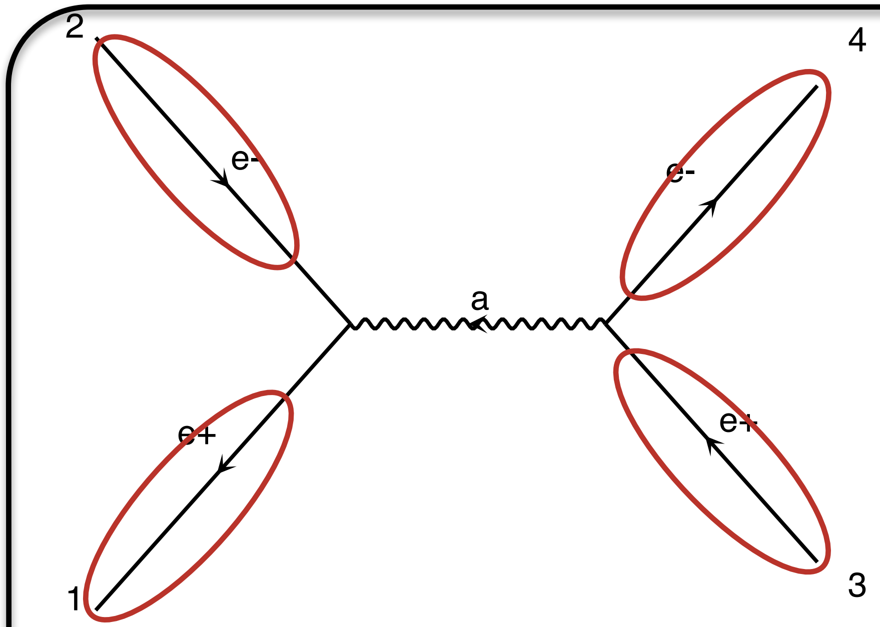

$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v)\, \frac{g_{\mu\nu}}{q^2}\, (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

Idea

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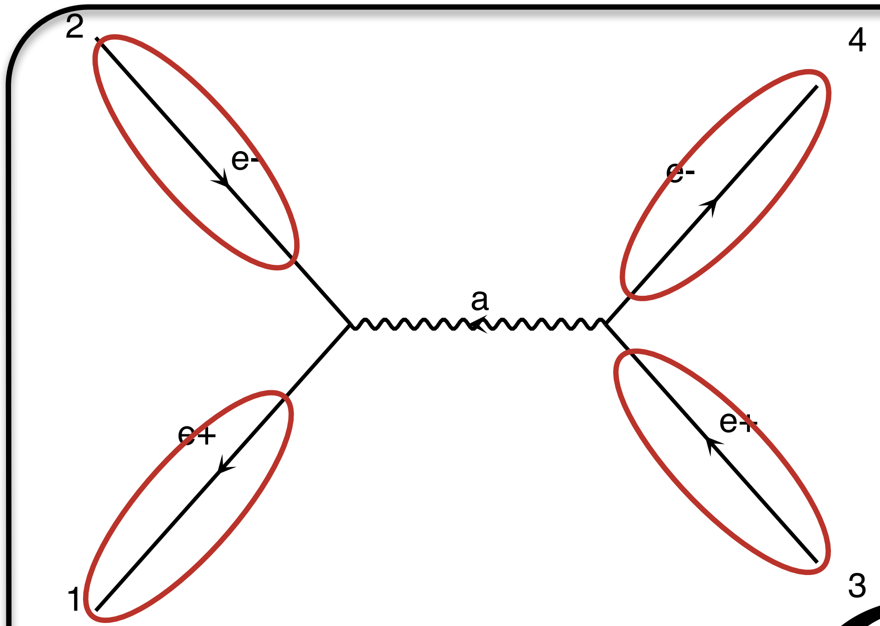

$$\mathcal{M} = (\boxed{\bar{u}}\, e\, \gamma^{\mu} \boxed{v})\, \frac{g_{\mu\nu}}{q^2}\, (\boxed{\bar{v}}\, e\, \gamma^{\nu} \boxed{u})$$

Numbers for given helicity and momenta

Lines present in the code.

$$\left\{ \begin{array}{l} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{array} \right.$$

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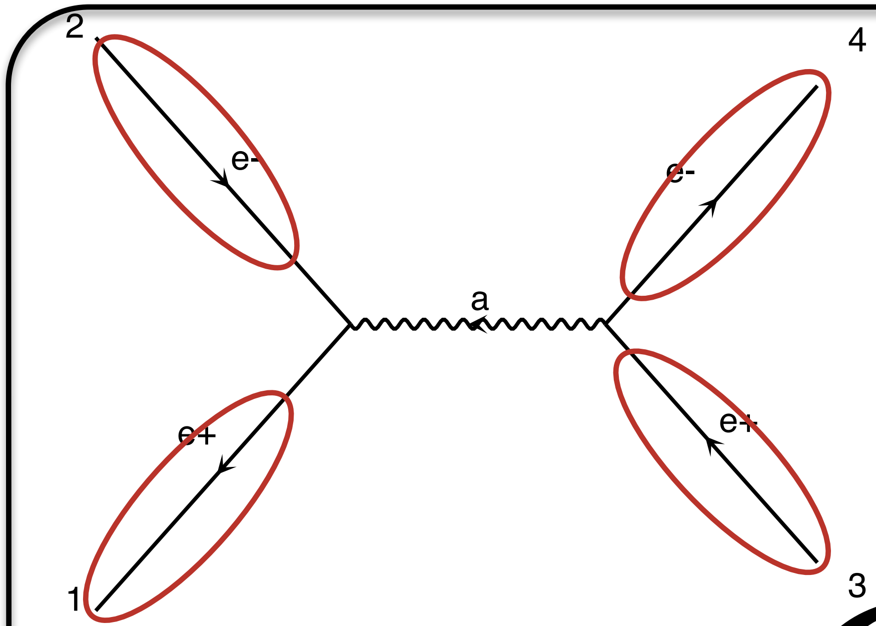

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$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_\lambda(\vec{p}) \\ \omega_\lambda(p)\chi_\lambda(\vec{p}) \end{pmatrix}$$

$$\omega_\pm(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} |\vec{p}| + p_z \\ p_x + ip_y \end{pmatrix},$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} -p_x + ip_y \\ |\vec{p}| + p_z \end{pmatrix}.$$

Idea

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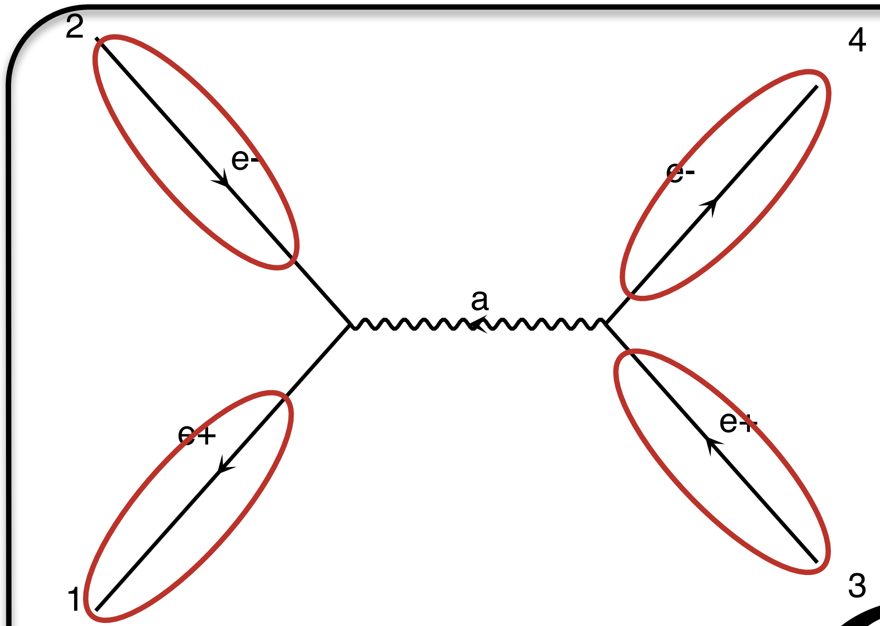

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Numbers for given helicity and momenta

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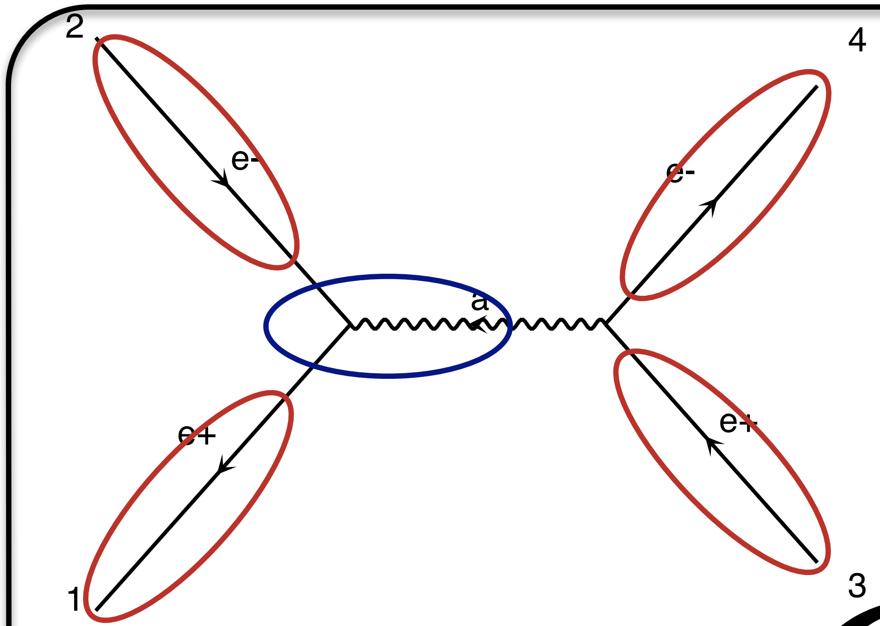

$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Lines present in the code.

$$\left\{ \begin{aligned} \bar{v}_1 &= fct(\vec{p}_1, m_1) \\ u_2 &= fct(\vec{p}_2, m_2) \\ v_3 &= fct(\vec{p}_3, m_3) \\ \bar{u}_4 &= fct(\vec{p}_4, m_4) \\ W_a &= fct(\bar{v}_1, u_2, m_a, \Gamma_a) \end{aligned} \right.$$

Idea

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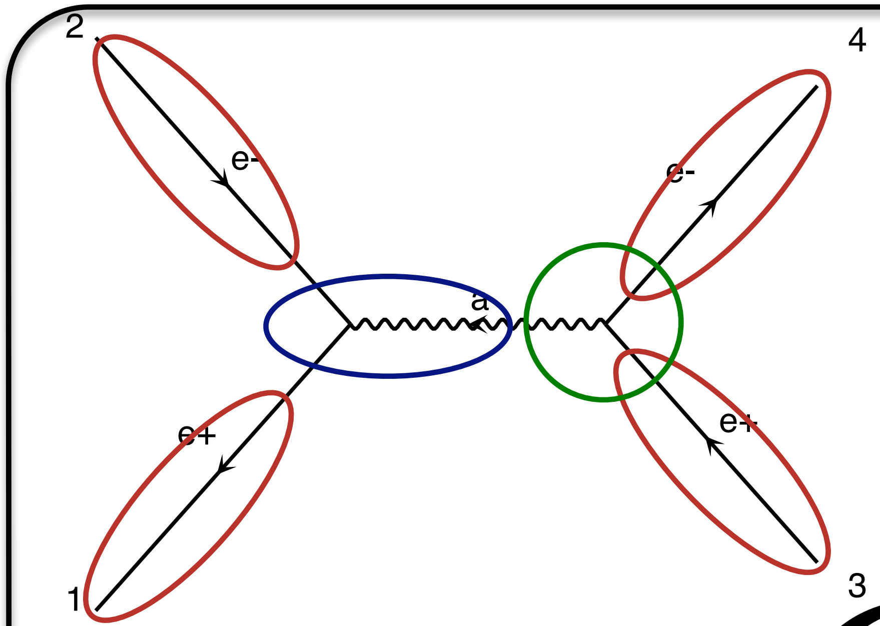


$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

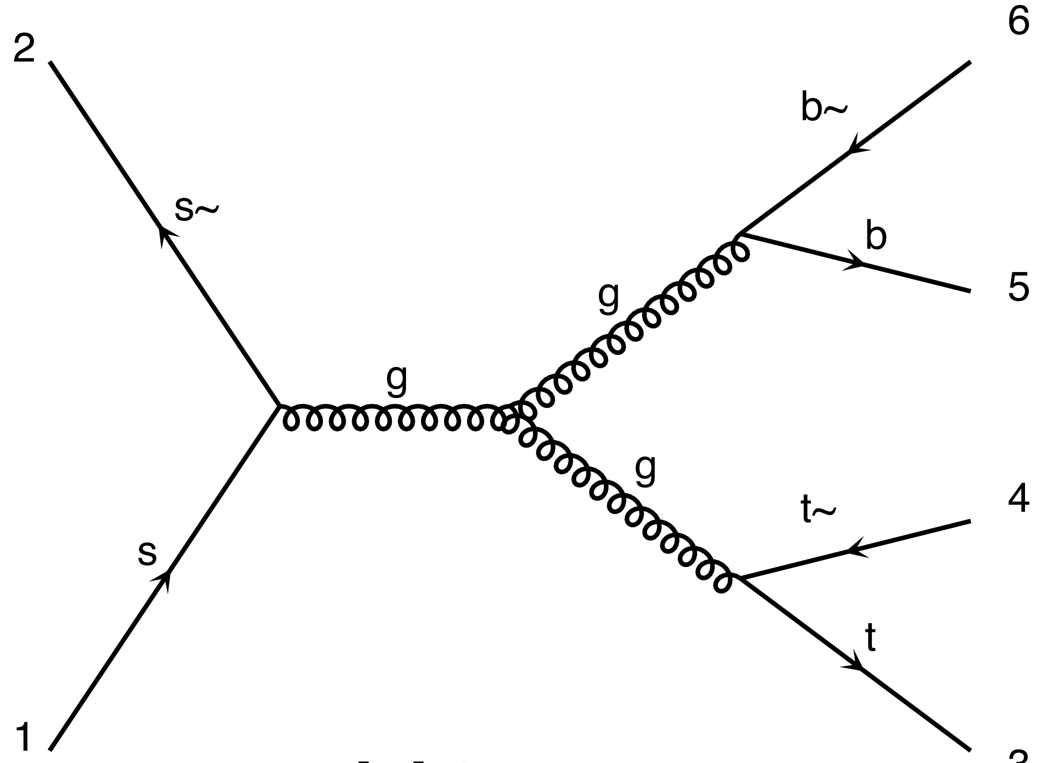
Finally evaluate amplitude (c-number)

Lines present in the code.

$$\left\{ \begin{array}{l} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a) \\ \mathcal{M} = fct(v_3, \bar{u}_4, W_a) \end{array} \right.$$

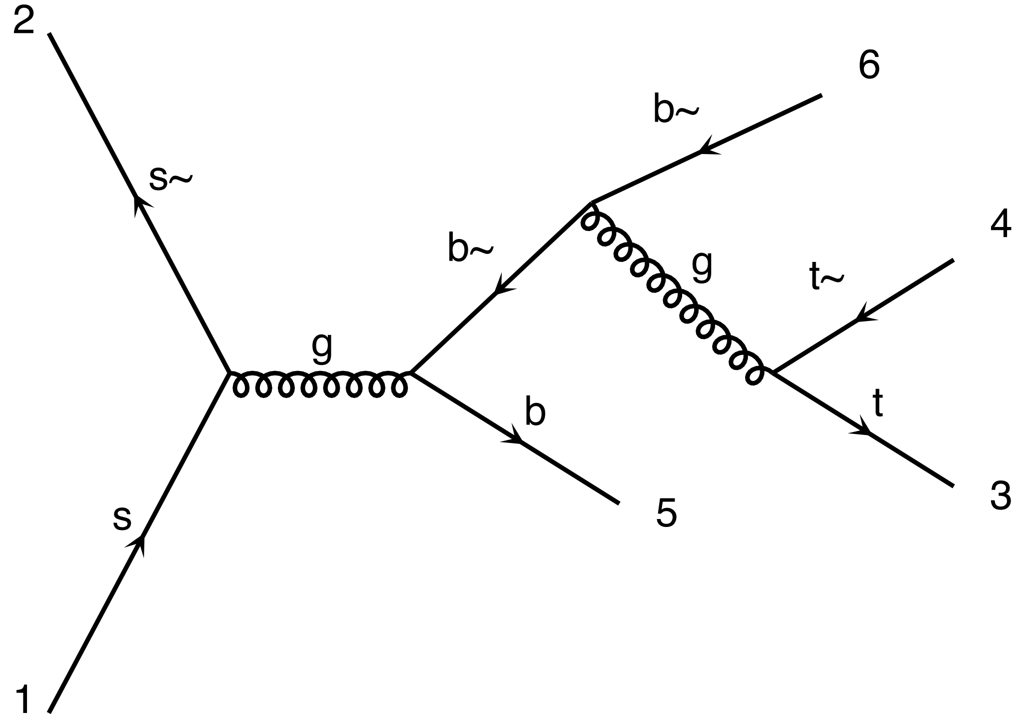
Real case

🟩 Known



M1

Number of routines: 0



M2

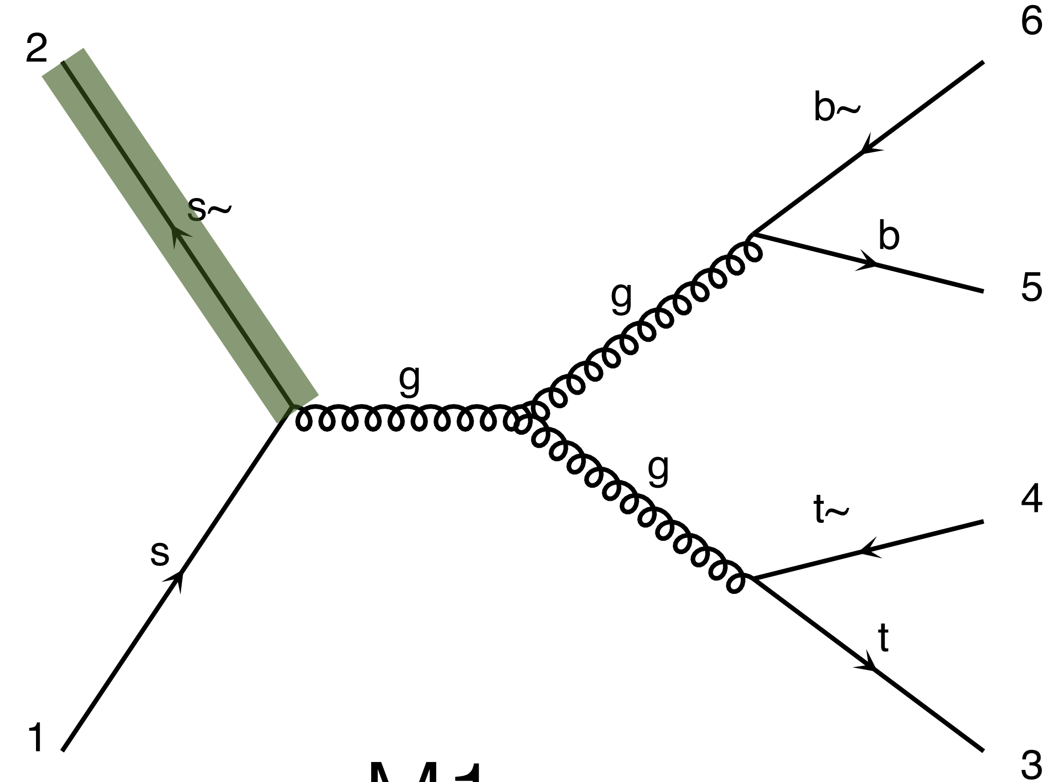
Number of routines: 0

Number of routines for both:0

$$|M|^2 = |M_1 + M_2|^2$$

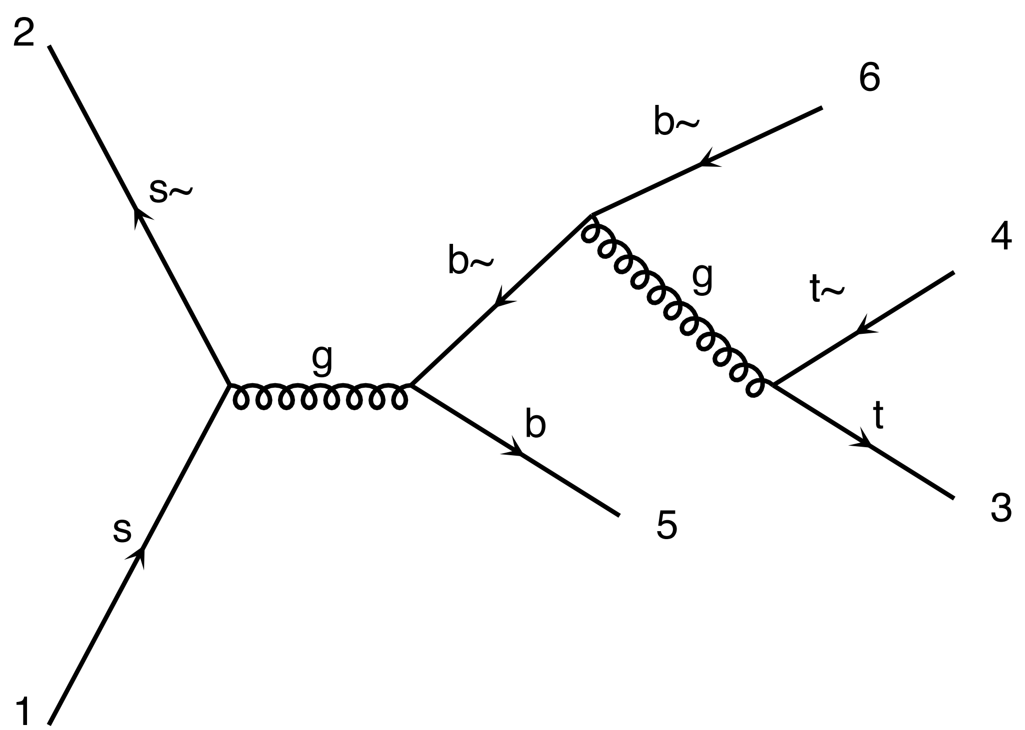
Real case

🟩 Known



M1

Number of routines: 1



M2

Number of routines: 0

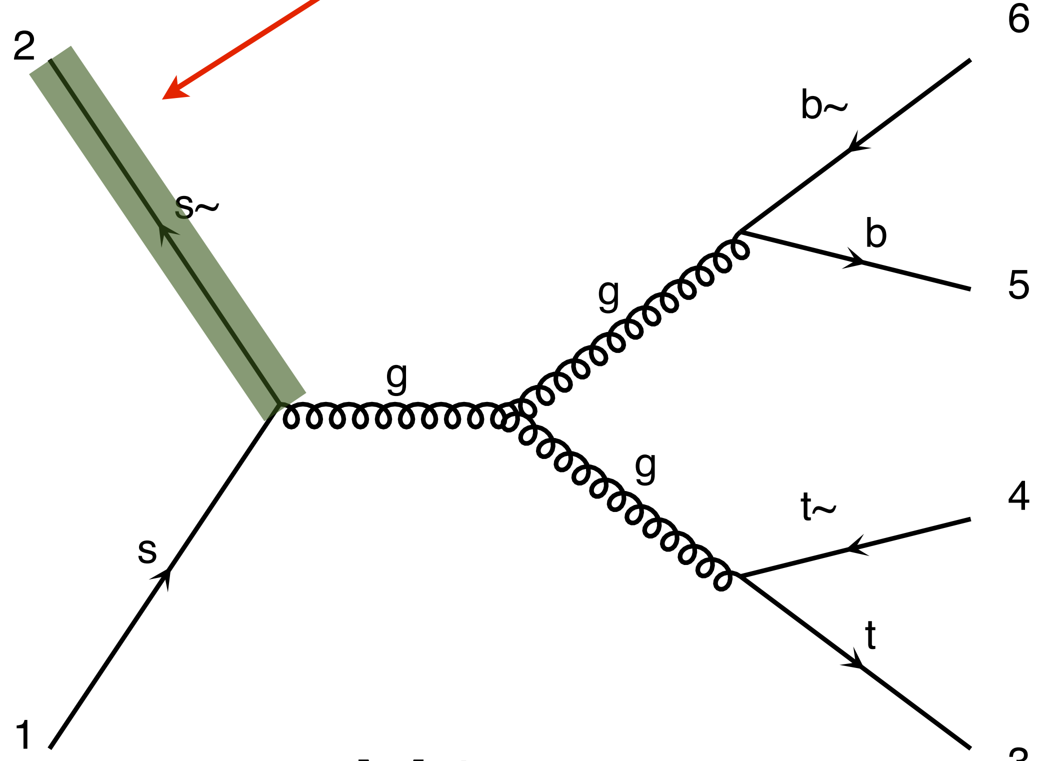
Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

Real case

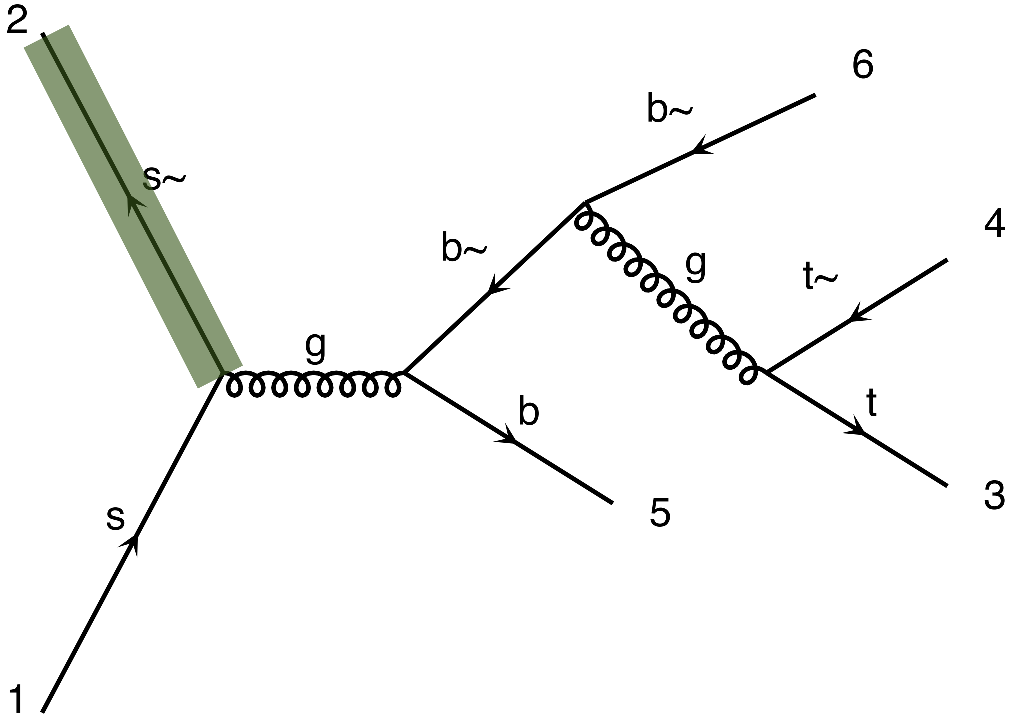
Identical

🟩 Known



M1

Number of routines: 1



M2

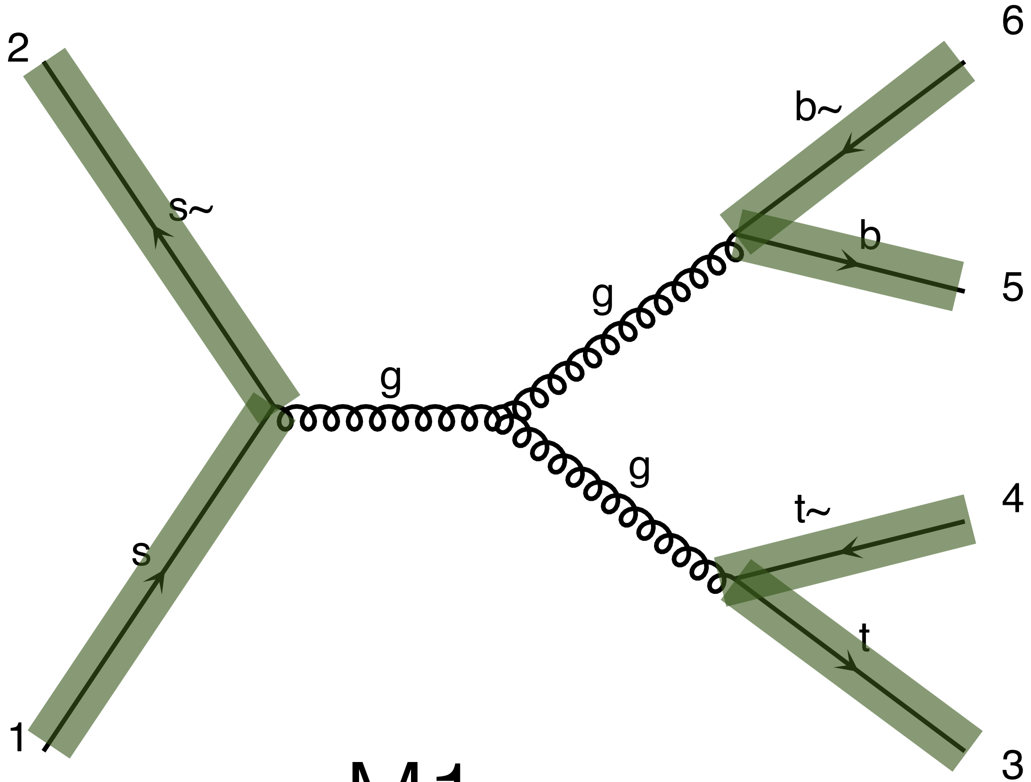
Number of routines: 1

Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

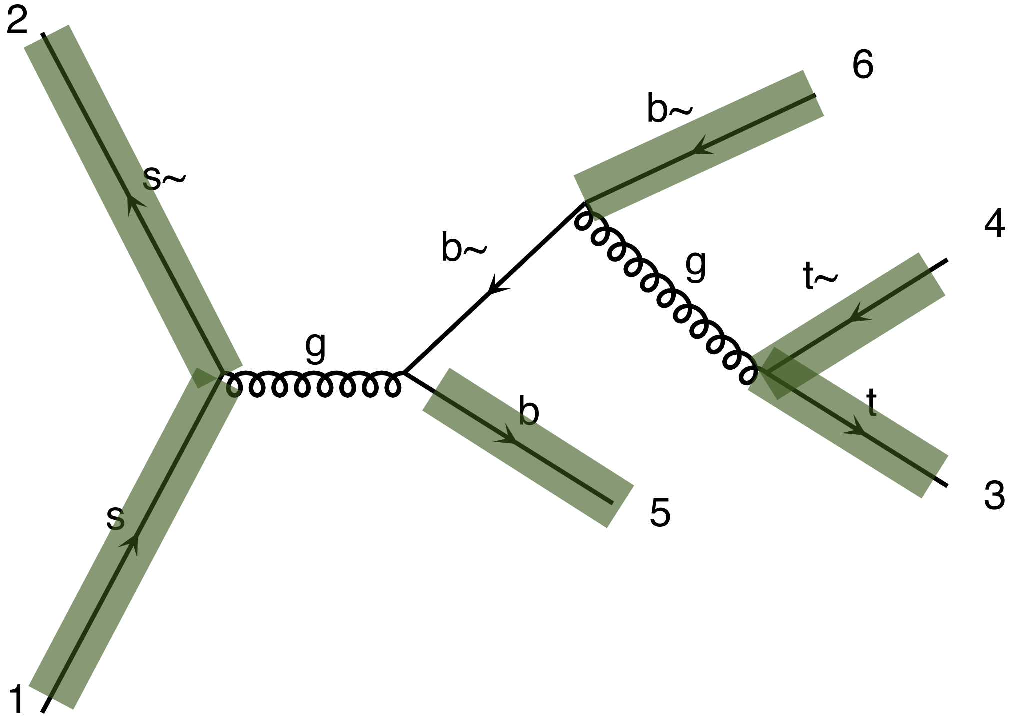
Real case

🟩 Known



M1

Number of routines: 6



M2

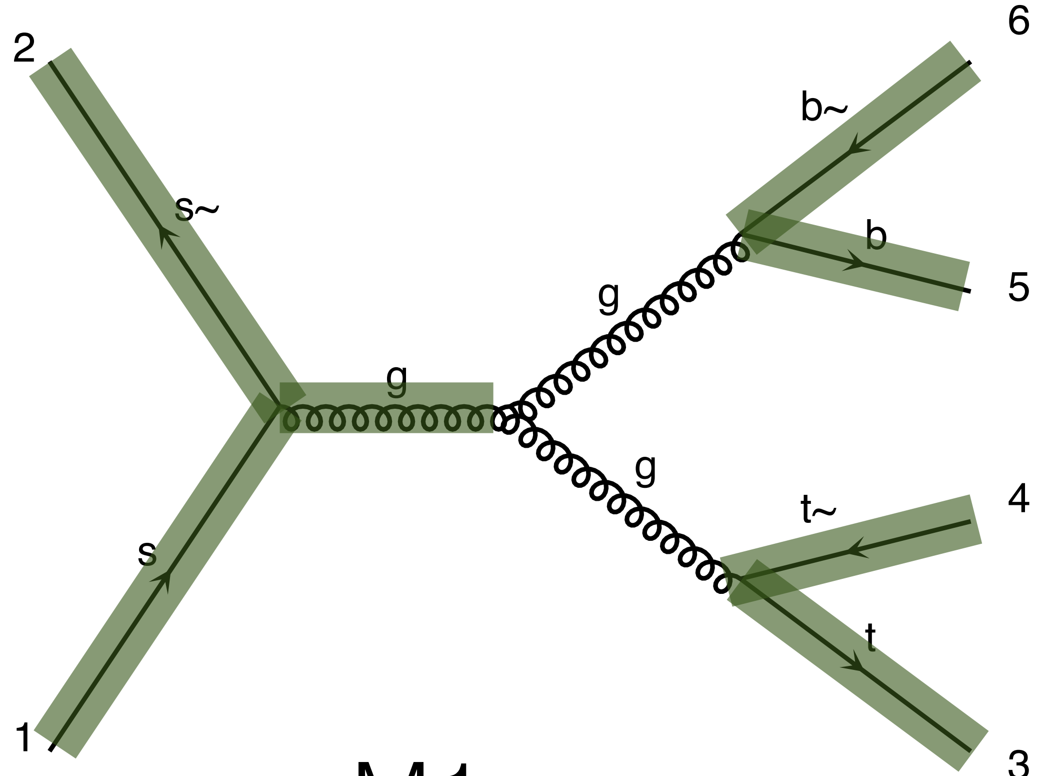
Number of routines: 6

Number of routines for both: 6

$$|M|^2 = |M_1 + M_2|^2$$

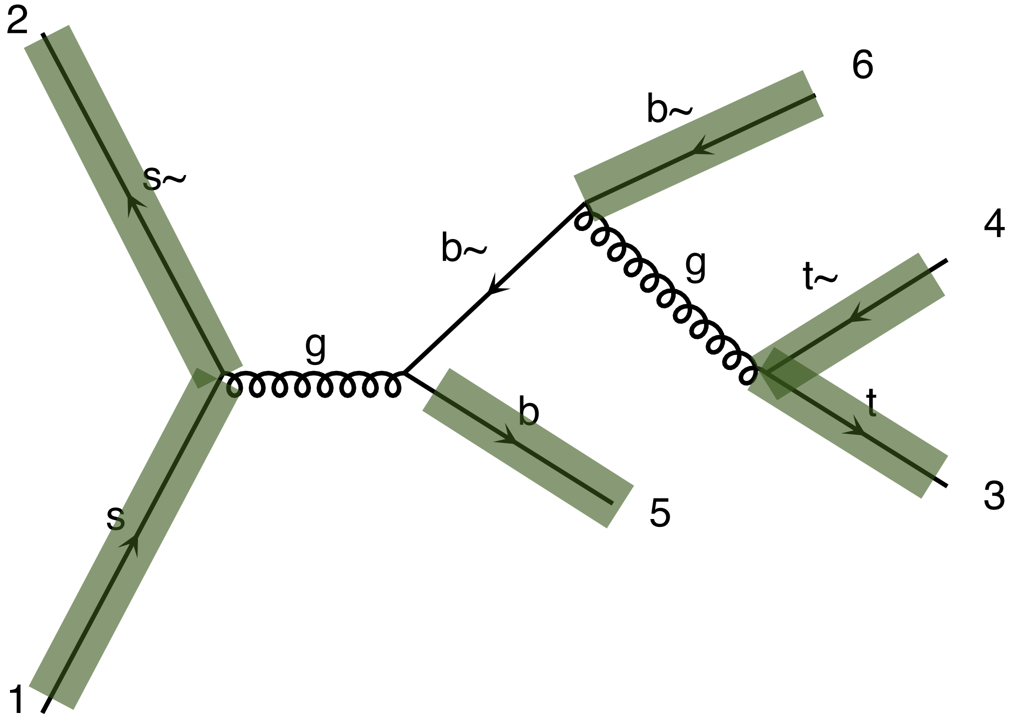
Real case

🟩 Known



M1

Number of routines: 7



M2

Number of routines: 6

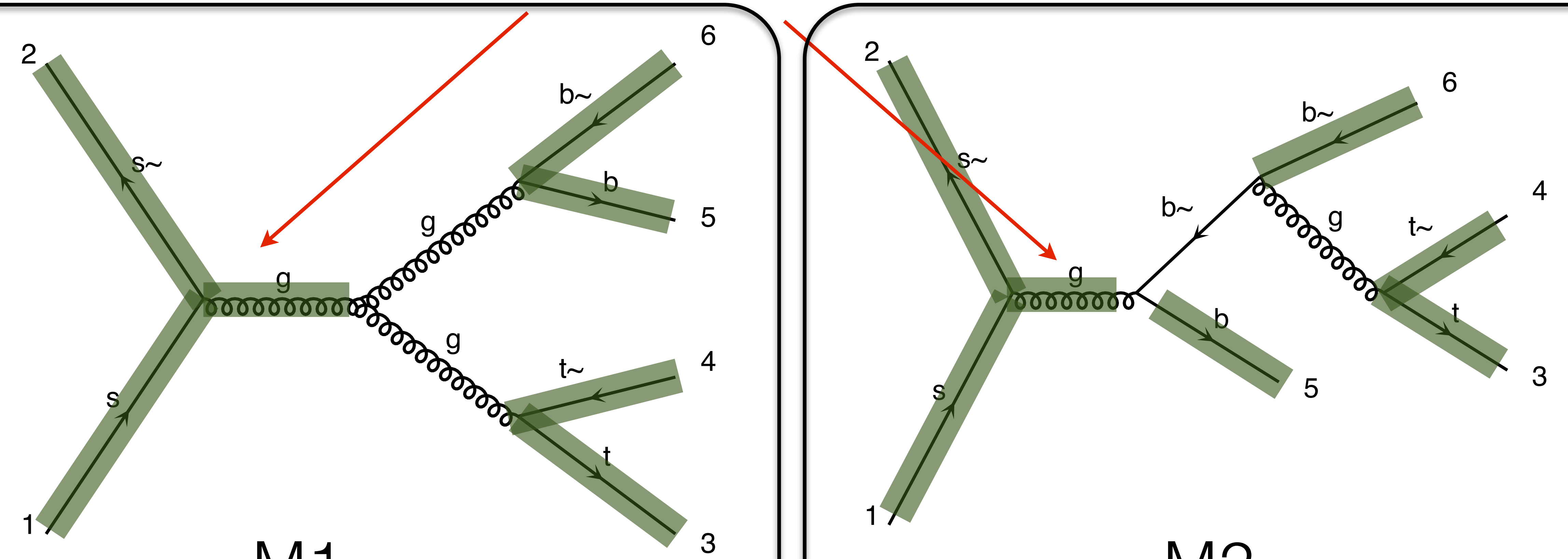
Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

Real case

🟩 Known

Identical



M1

Number of routines: 7

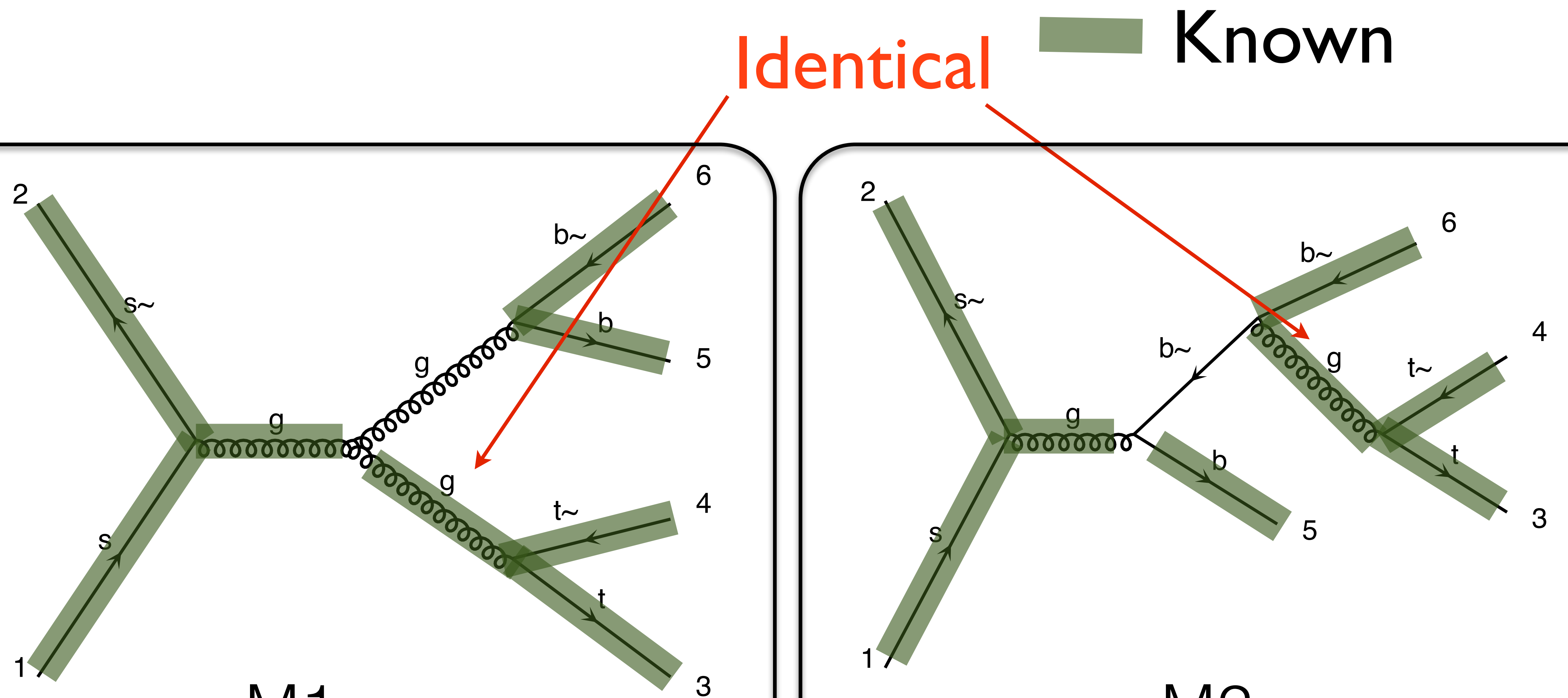
M2

Number of routines: 7

Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

Real case



M1

Number of routines: 8

M2

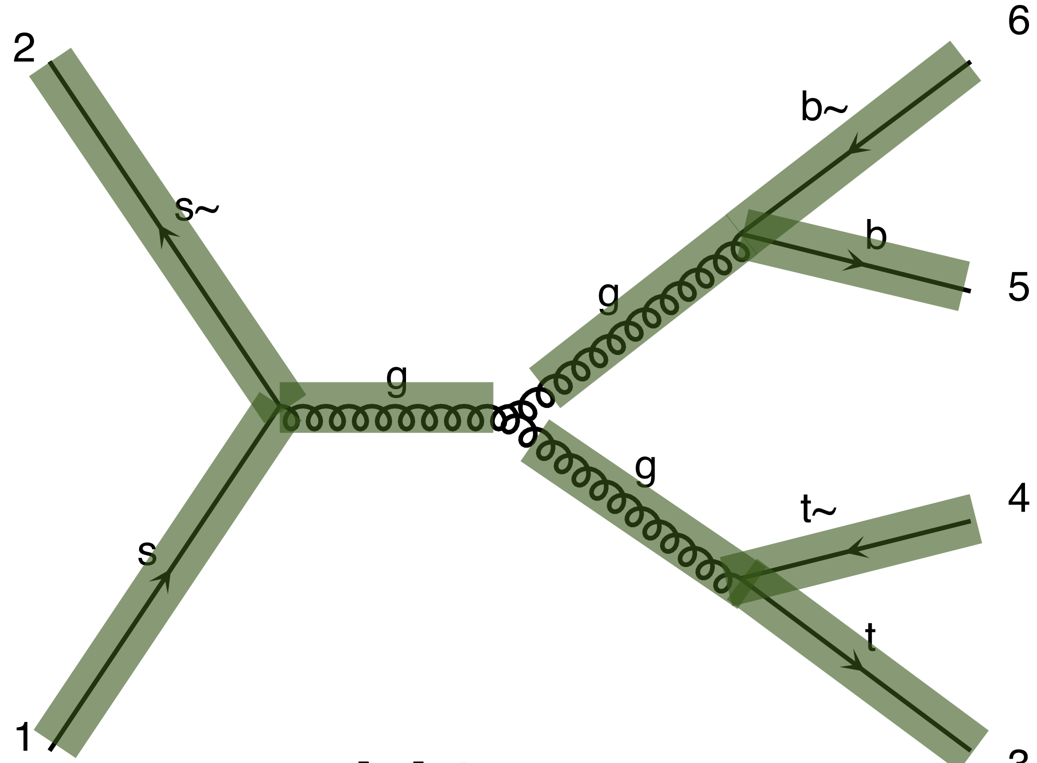
Number of routines: 8

Number of routines for both: 8

$$|M|^2 = |M_1 + M_2|^2$$

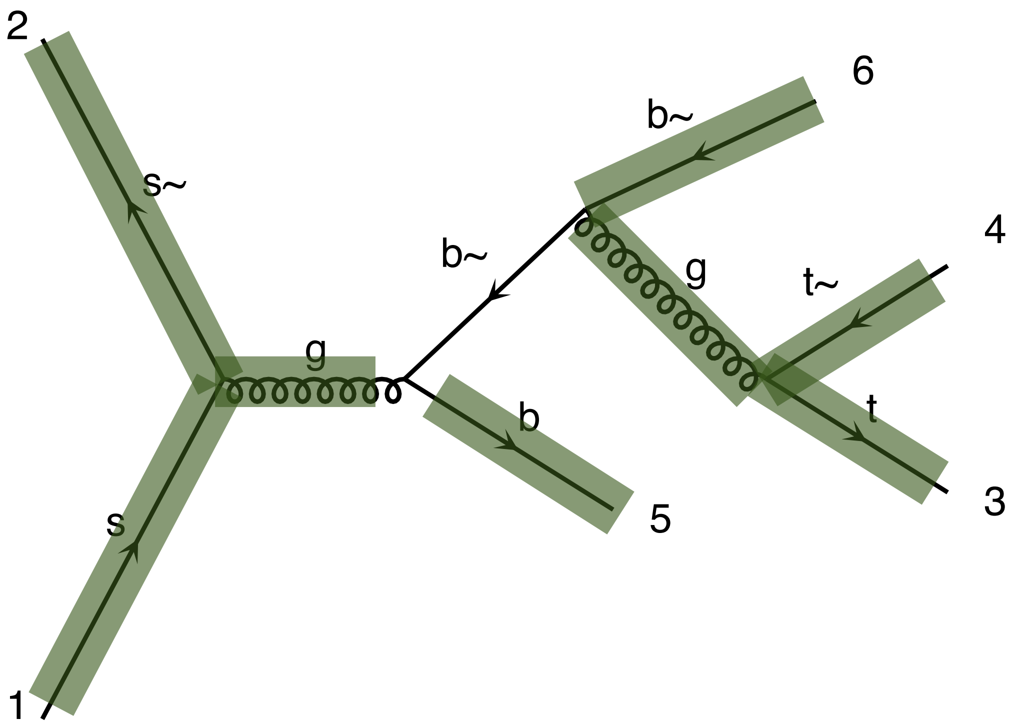
Real case

🟩 Known



M1

Number of routines: 9



M2

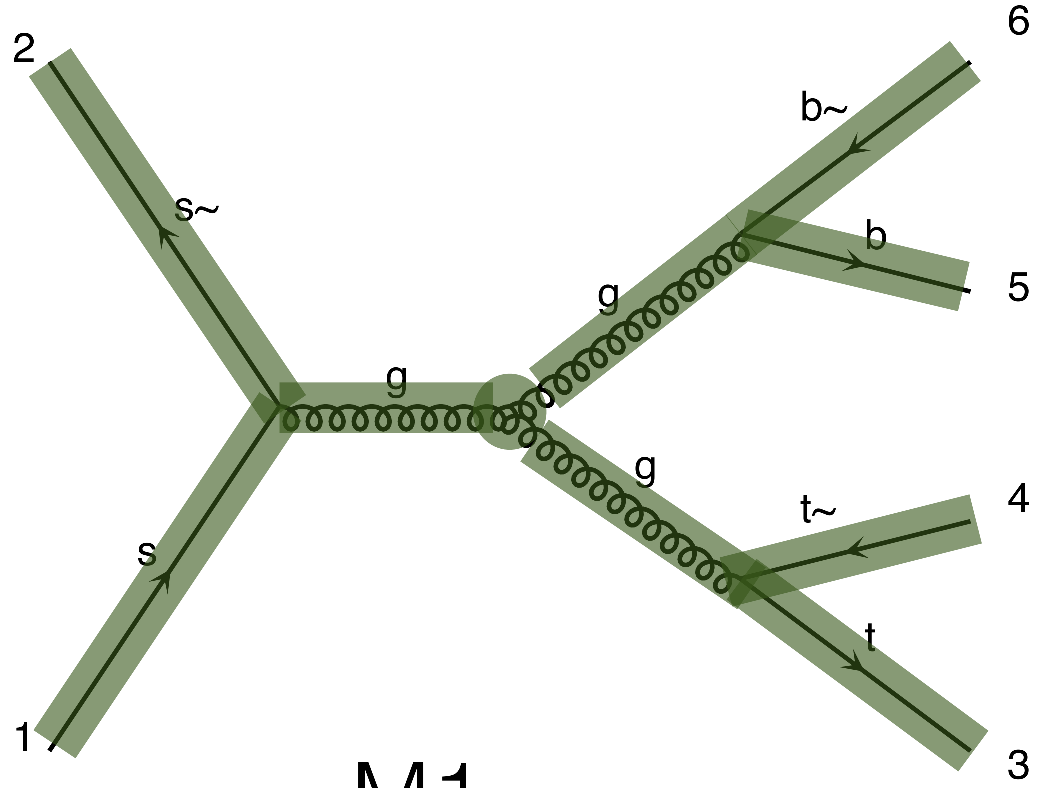
Number of routines: 8

Number of routines for both: 9

$$|M|^2 = |M_1 + M_2|^2$$

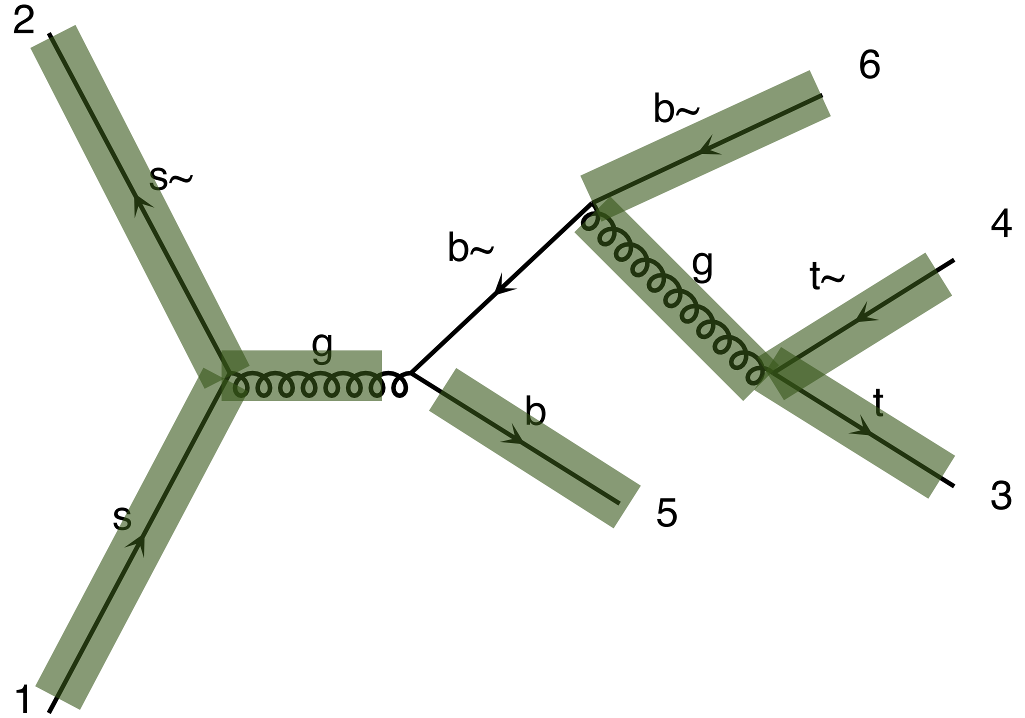
Real case

🟩 Known



M1

Number of routines: 10



M2

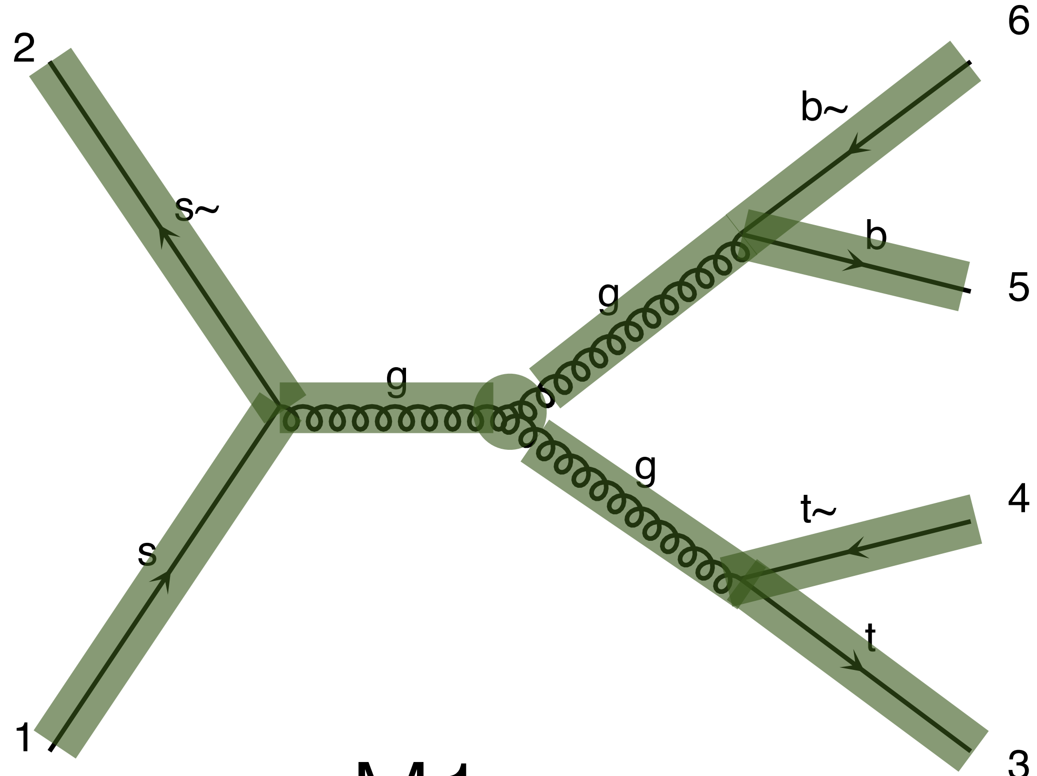
Number of routines: 8

Number of routines for both: 10

$$|M|^2 = |M_1 + M_2|^2$$

Real case

🟩 Known



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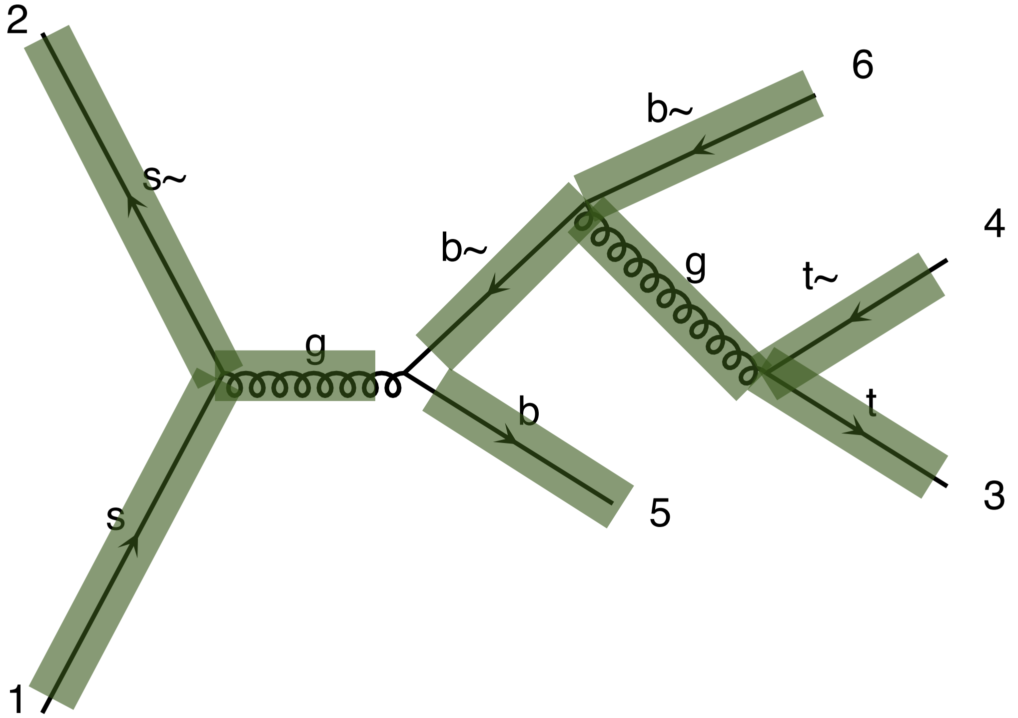
Number of routines: 10



M2

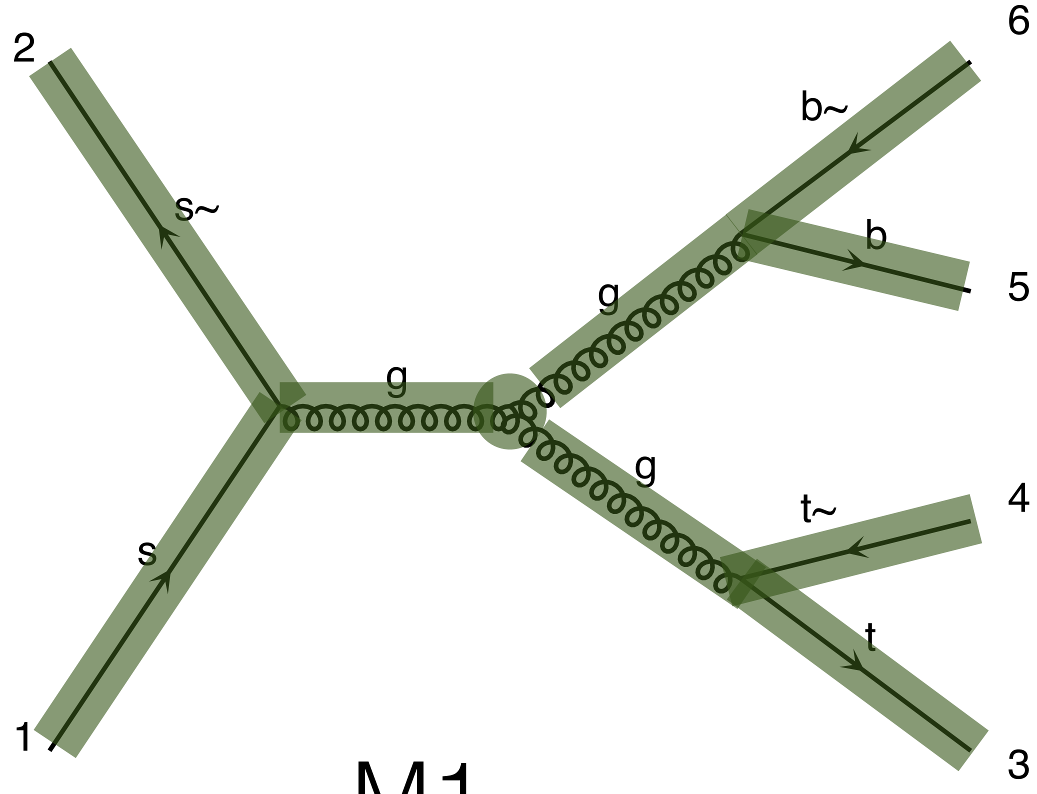
Number of routines: 9

Number of routines for both: 11

$$|M|^2 = |M_1 + M_2|^2$$

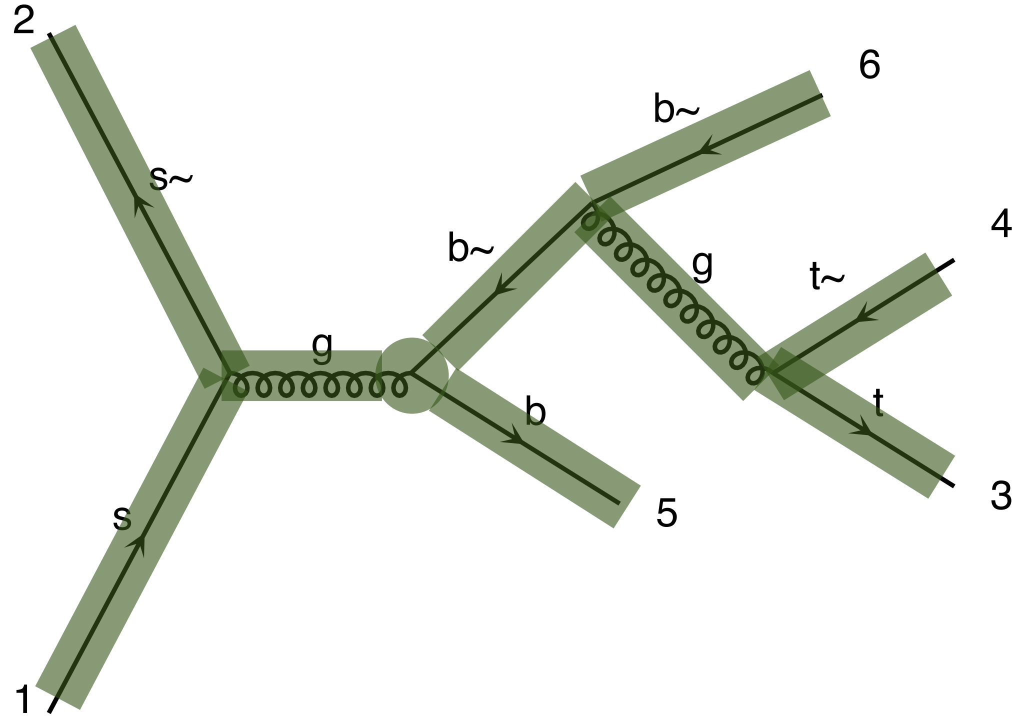
Real case

🟩 Known



M1

Number of routines: 10



M2

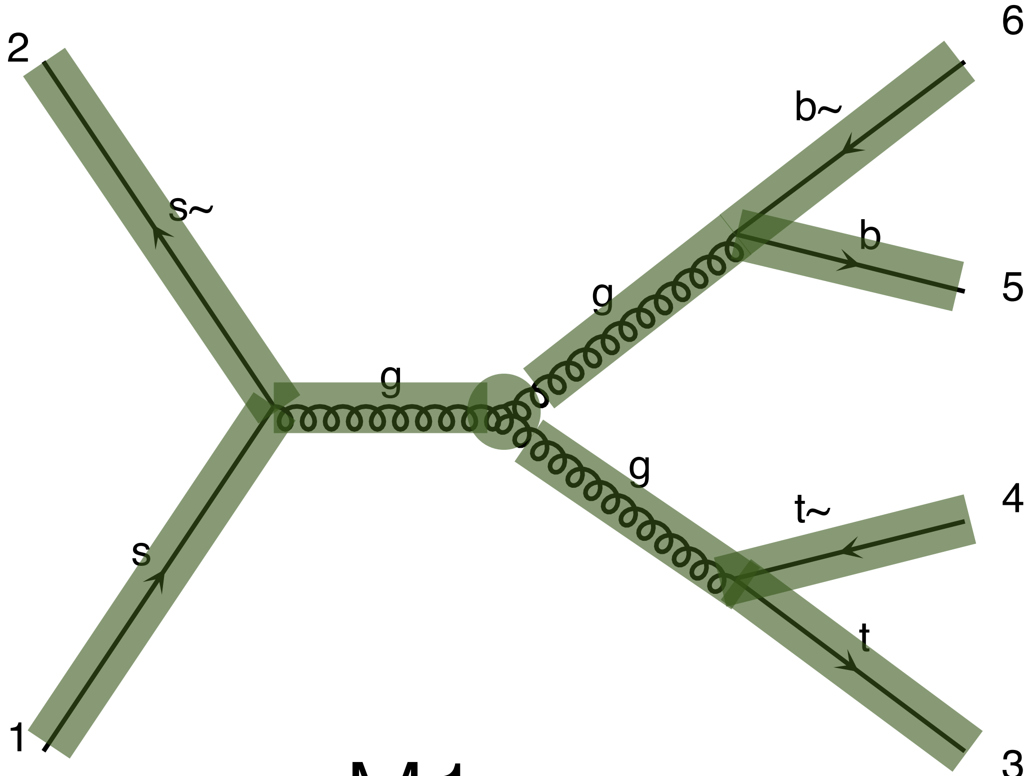
Number of routines: 10

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

Real case

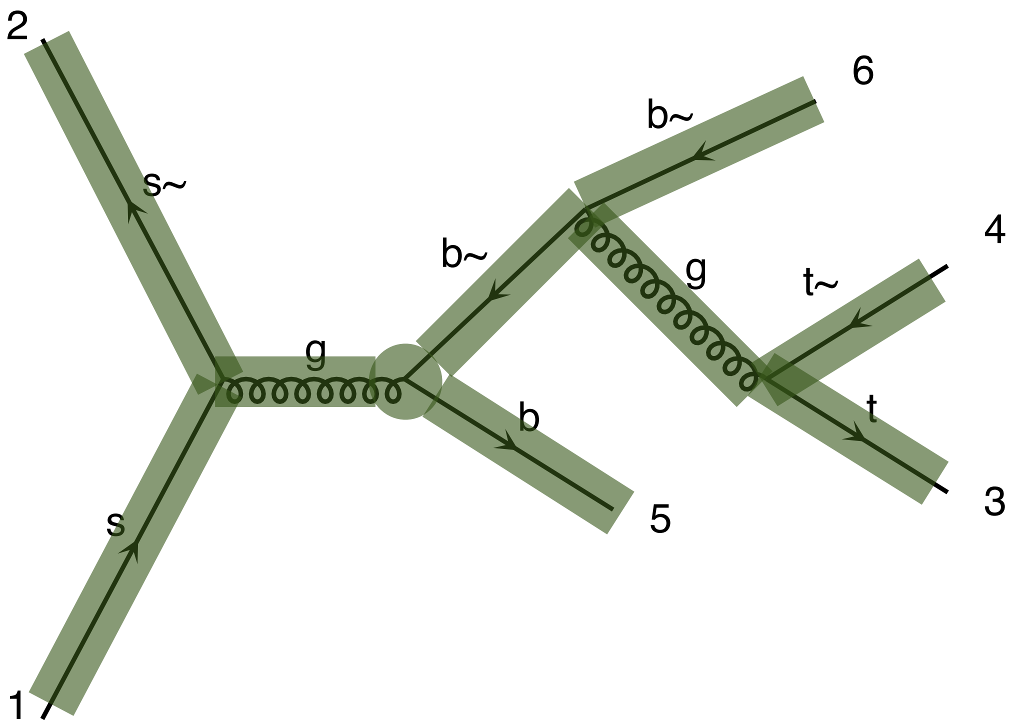
🟩 Known



M1

Number of routines: 10

2(N+1)



M2

Number of routines: 10

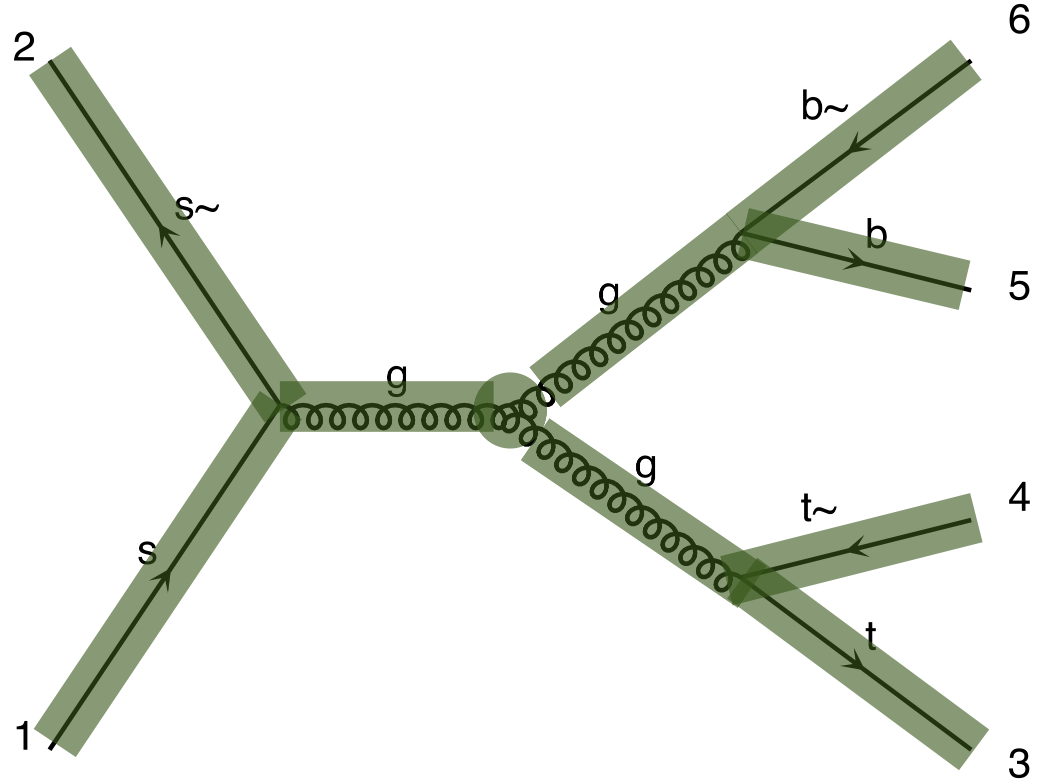
2(N+1)

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

Real case

🟩 Known



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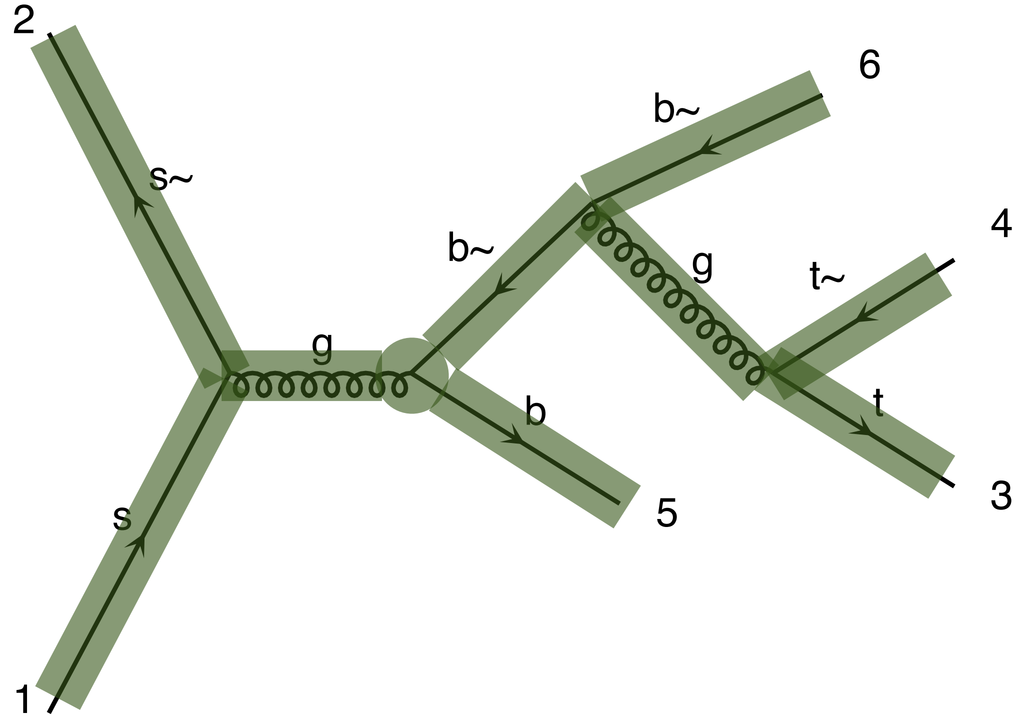

Number of routines: 10

2(N+1)

Number of routines for both: 12

$N!*2(N+1) \longrightarrow N!$

HELAS

- **Original HELicity Amplitude Subroutine library**

  [Murayama, Watanabe, Hagiwara]

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  - ➡**MSSM** [cho, al] hep-ph/0601063 (2006)
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Chiral Perturbation

BNV Model

SLIH

Effective Field Theory

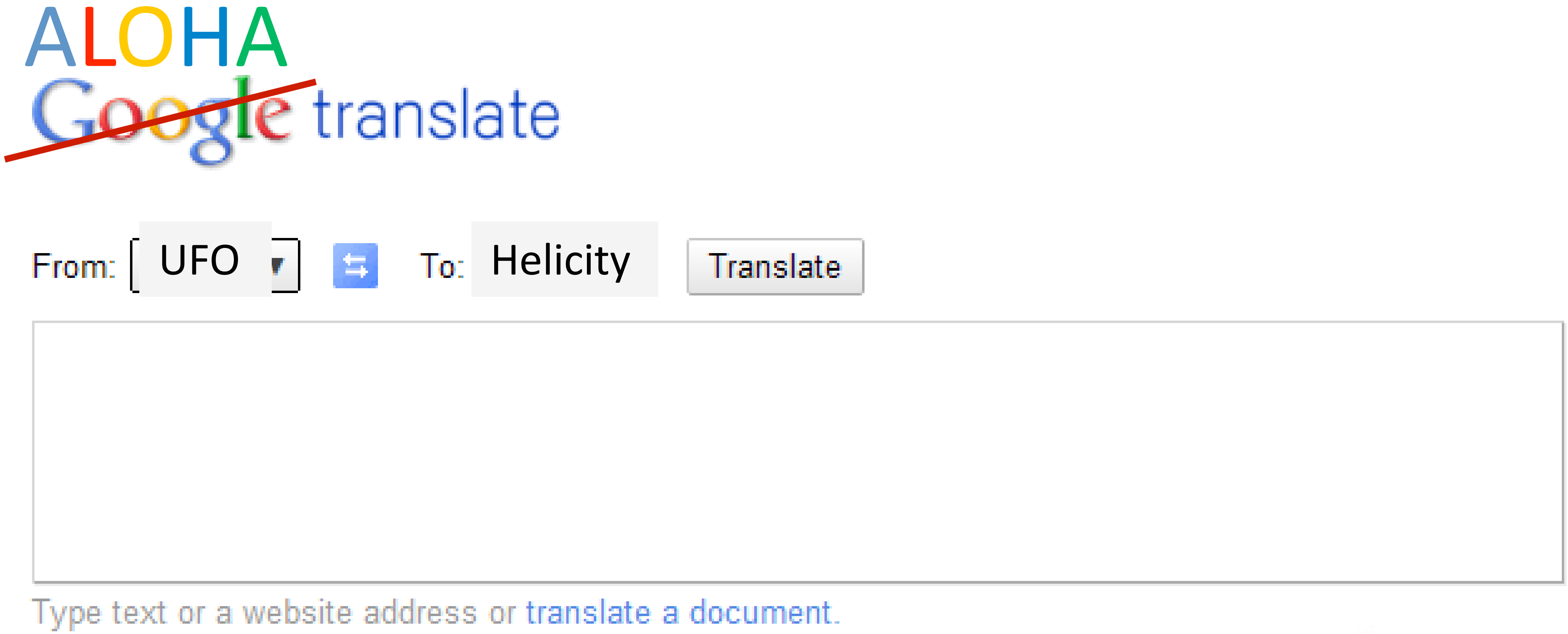
NMSSM

Chromo-magnetic operator

Full HEFT

Black Holes

ALOHA



ALOHA

~~Google~~ translate

From: UFO ▾ ⇆ To: Helicity Translate

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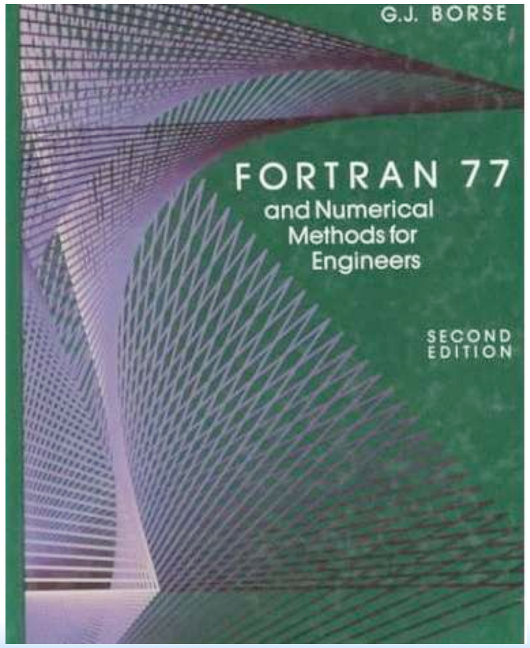
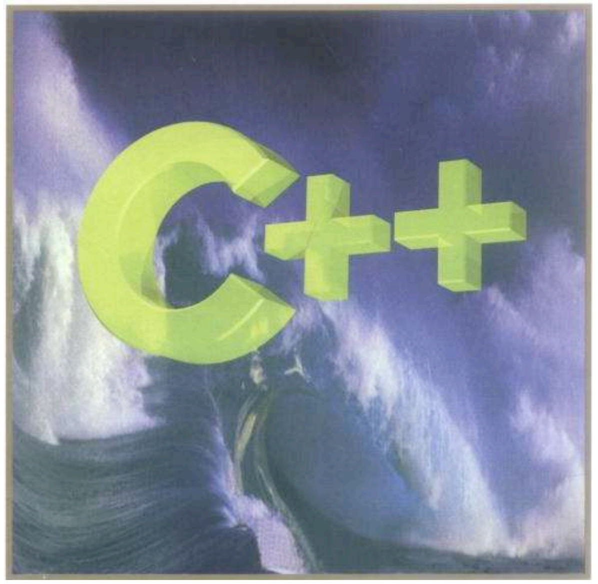
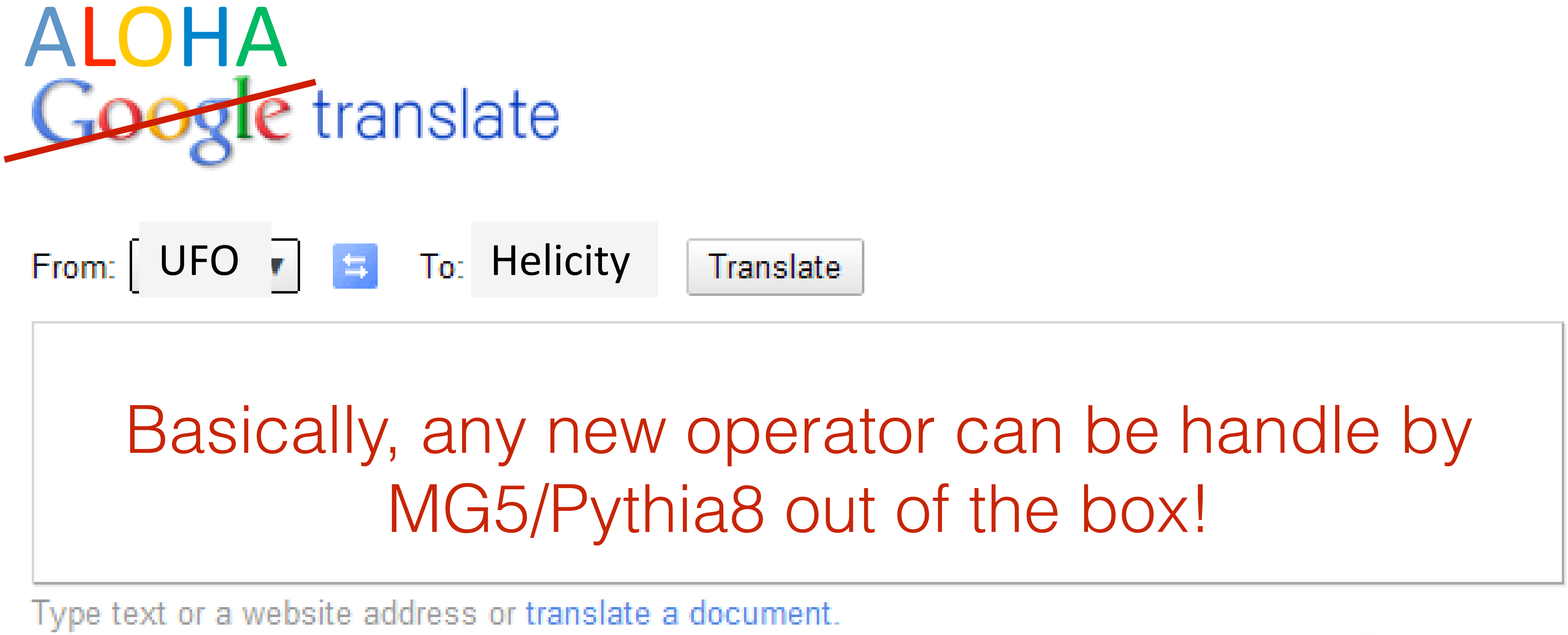




ALOHA



ALOHA

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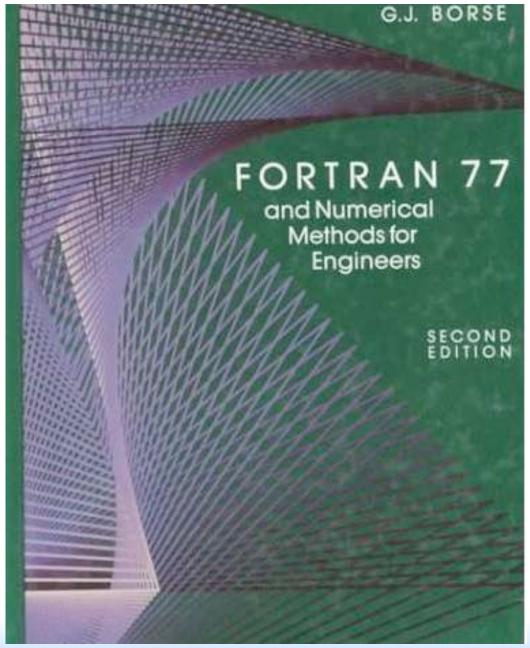
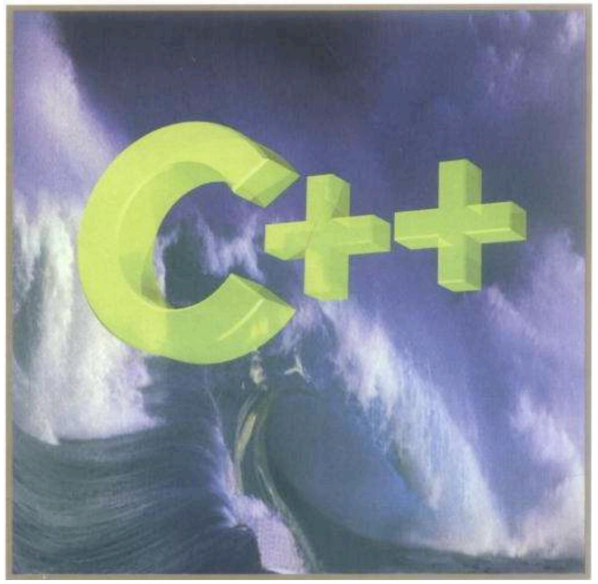
Basically, any new operator can be handle by MG5/Pythia8 out of the box!

Type text or a website address or translate a document.





To Remember

- Numerical computation faster than analytical computation
- We are able to compute matrix-element
  - ➡ for large number of final state
  - ➡ for any BSM theory

Plan

- Details of the computation
  - Evaluation of matrix-element
  - Phase-Space integration
- What is MG5_aMC?

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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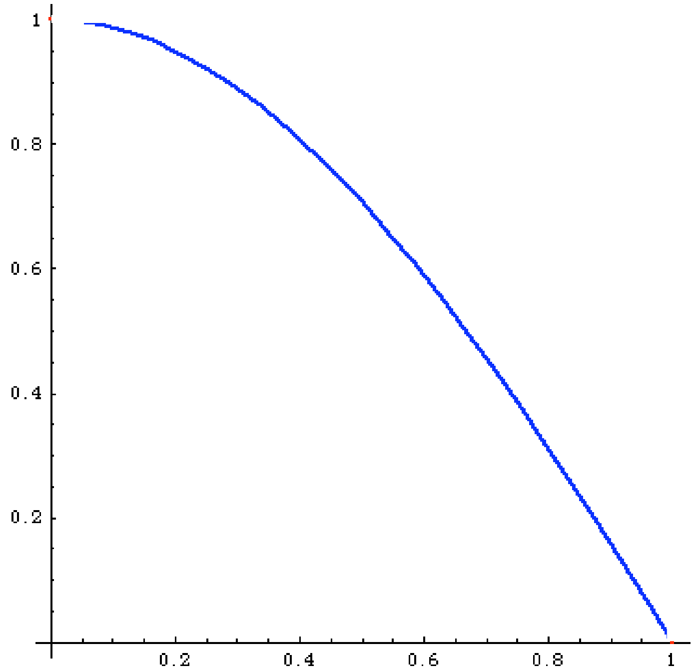
$$Dim[\Phi(n)] \sim 3n$$

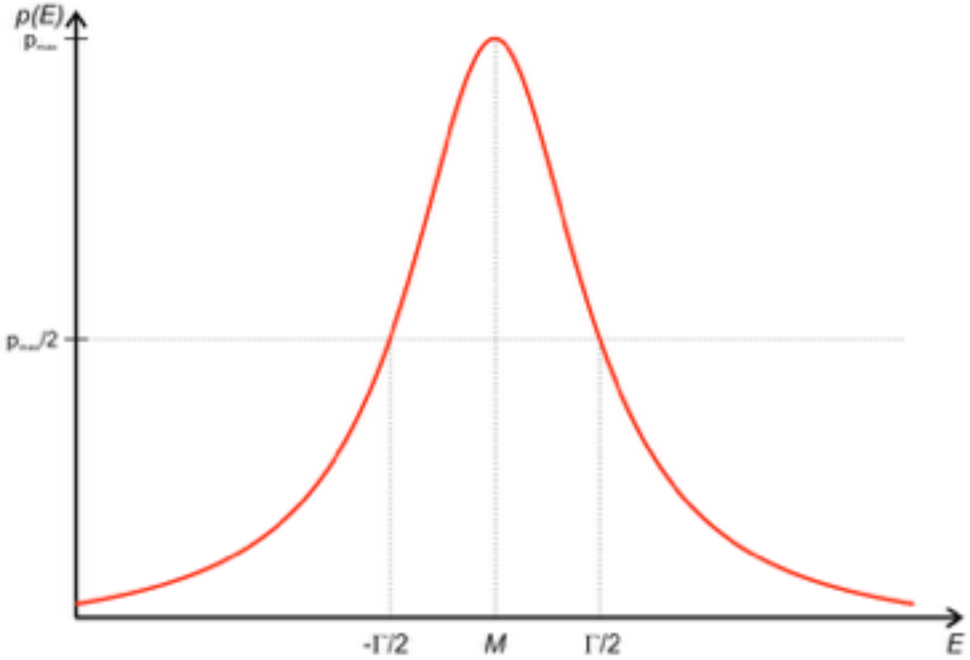
General and flexible method is needed

Integration

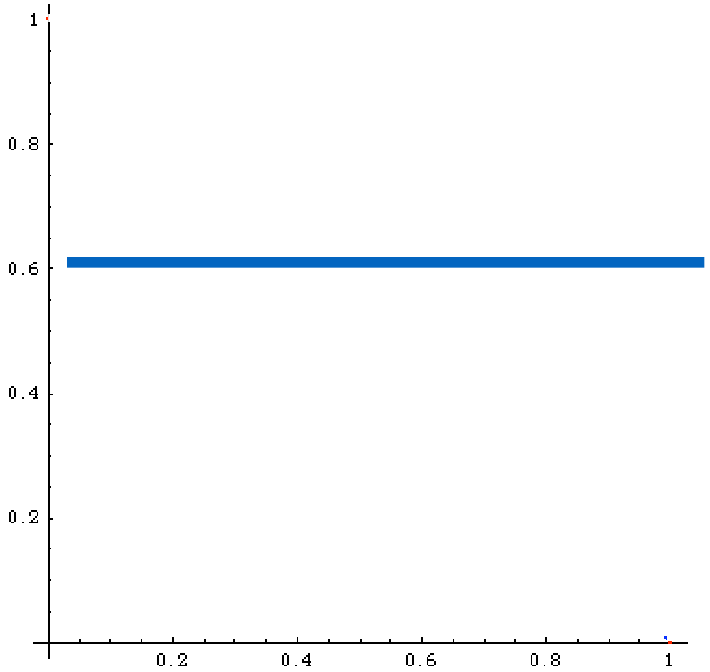
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$



$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



$$\int dx\, C$$



Integration

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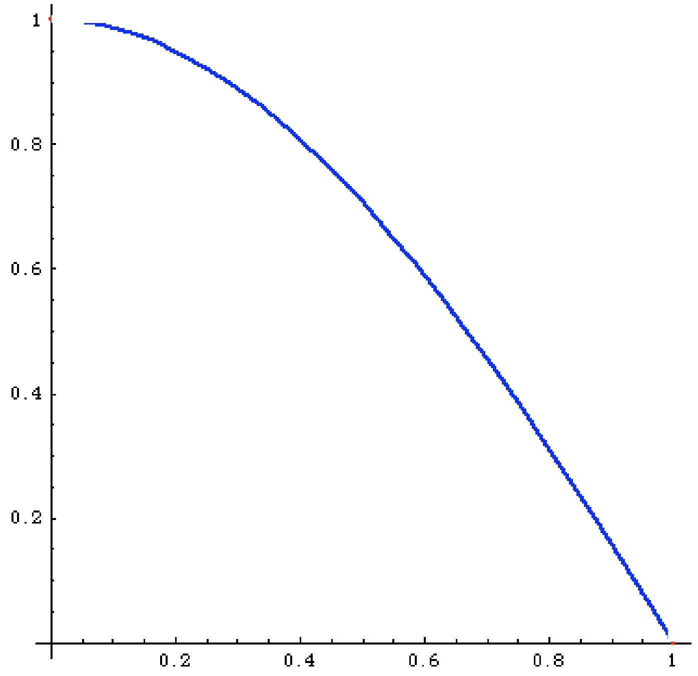

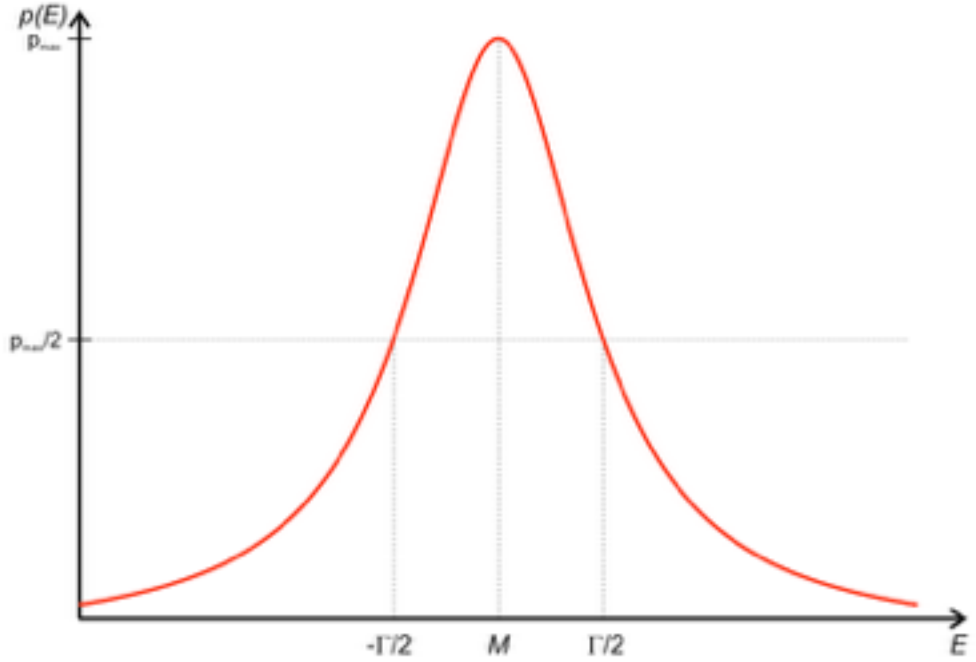

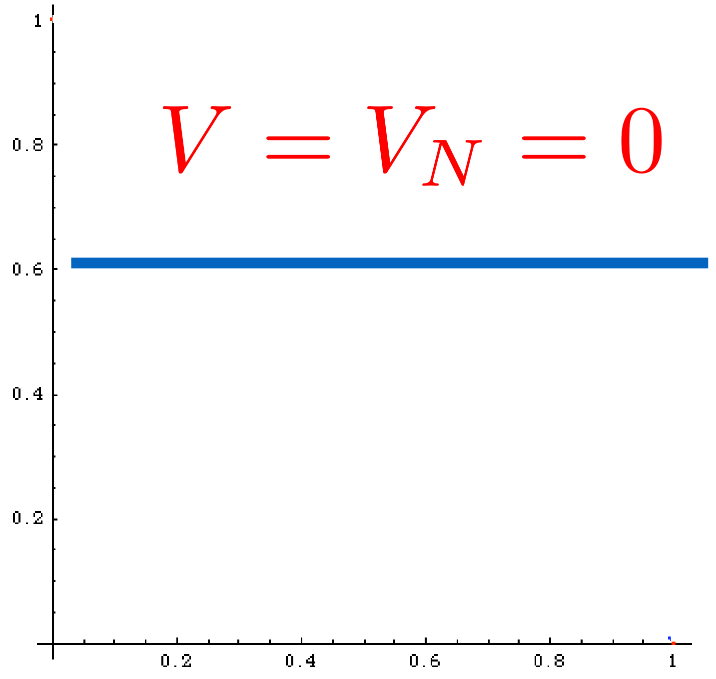

$V = V_N = 0$

Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

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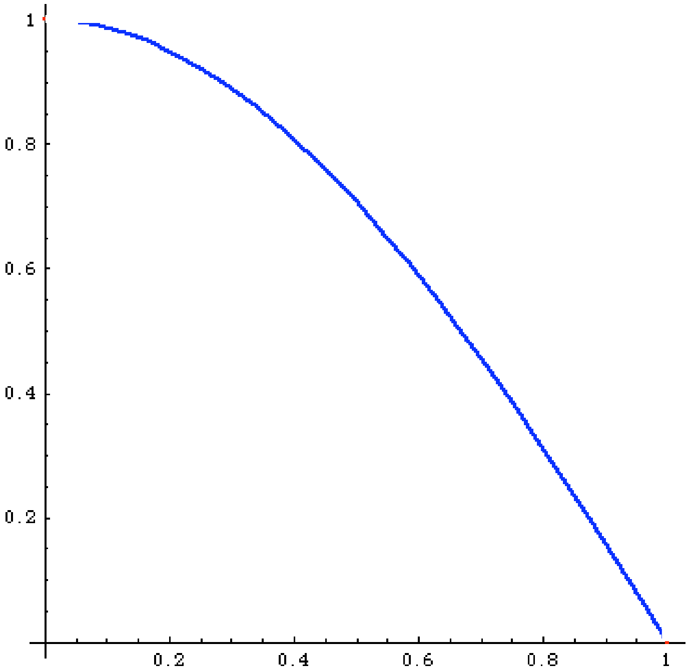

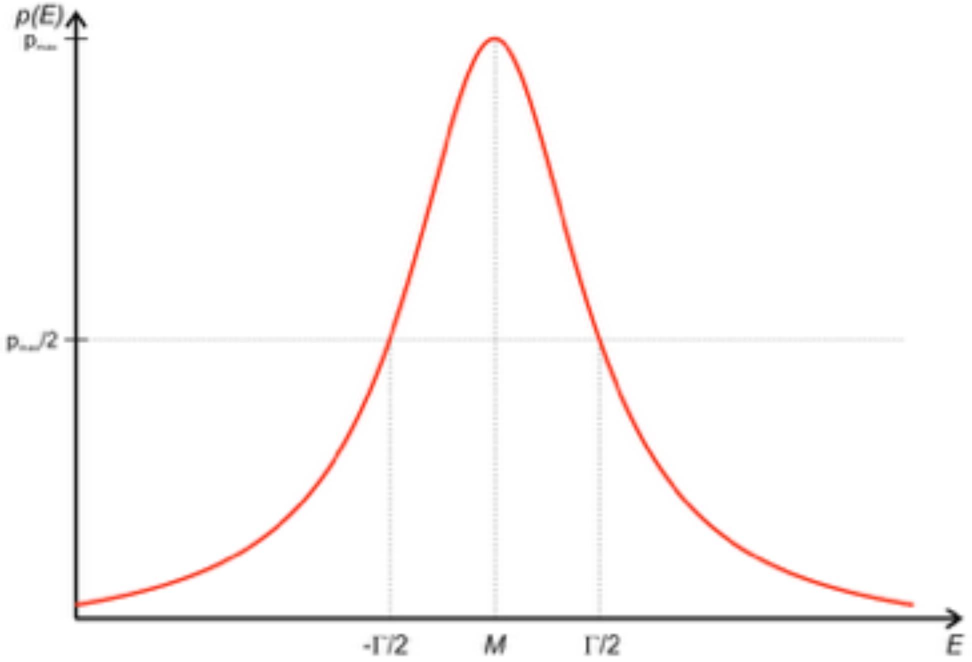

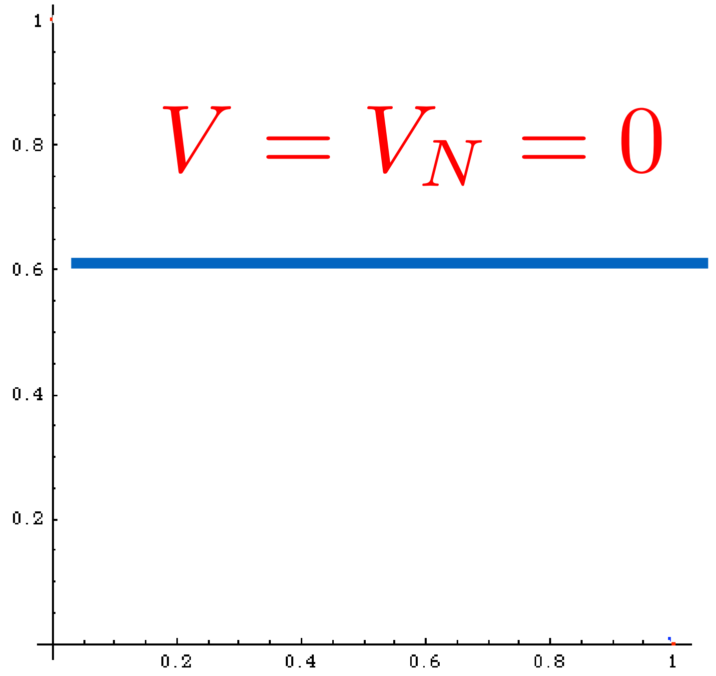

$$V = V_N = 0$$

	simpson	MC
3	0.638	0.3
5	0.6367	0.8
20	0.63662	0.6
100	0.636619	0.65
1000	0.636619	0.636

Method of evaluation

- MonteCarlo $1/\sqrt{N}$
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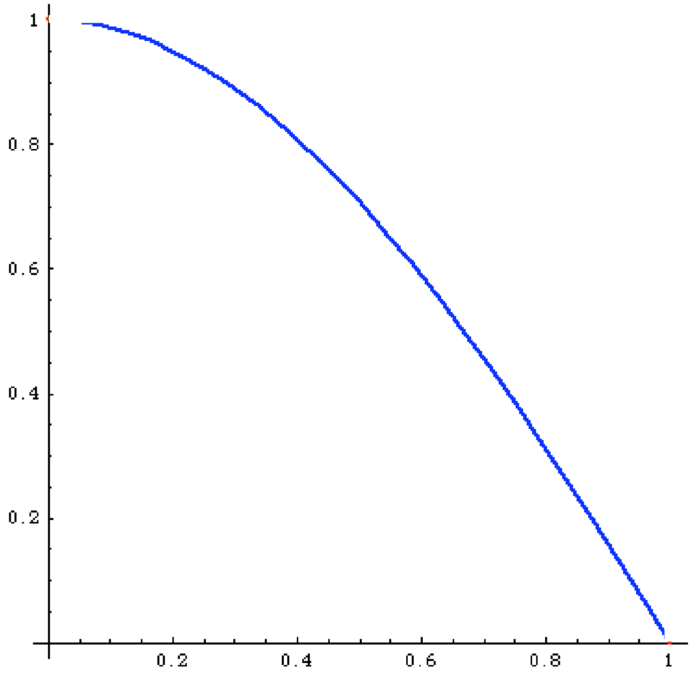
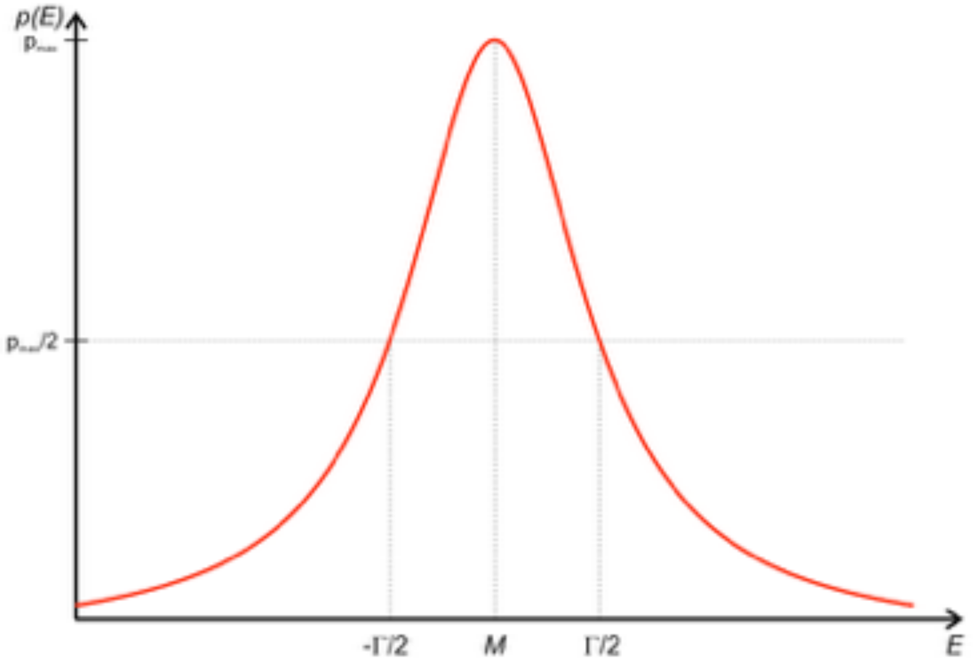
Integration

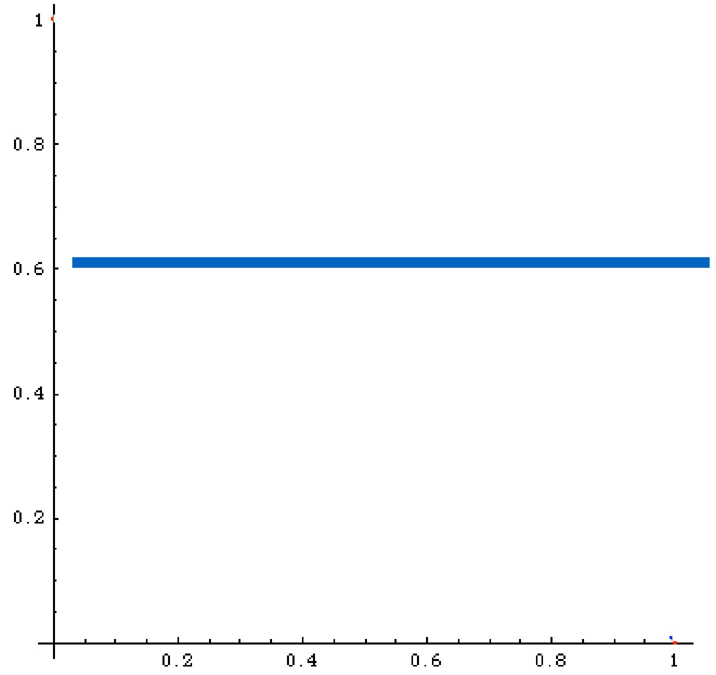
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



$$\int dx\, C$$



Method of evaluation

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More Dimension

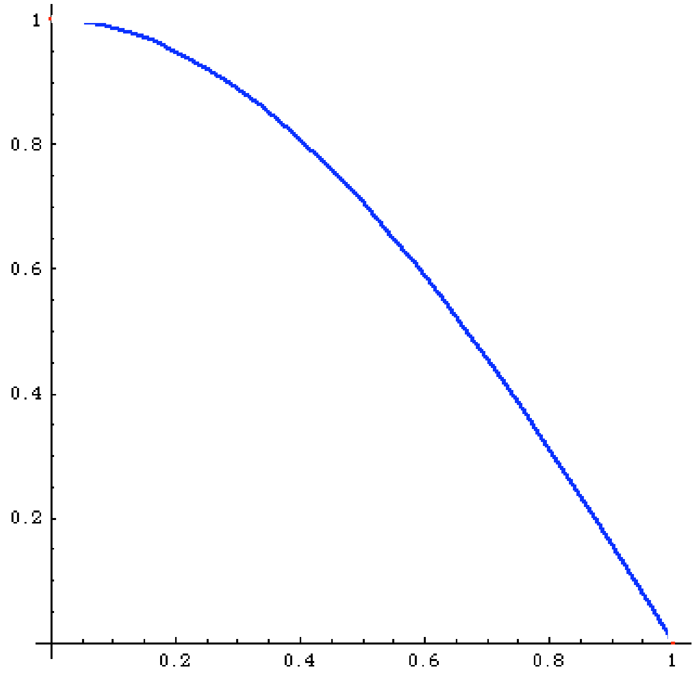
$1/\sqrt{N}$

$1/N^{2/d}$

$1/N^{4/d}$

Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$



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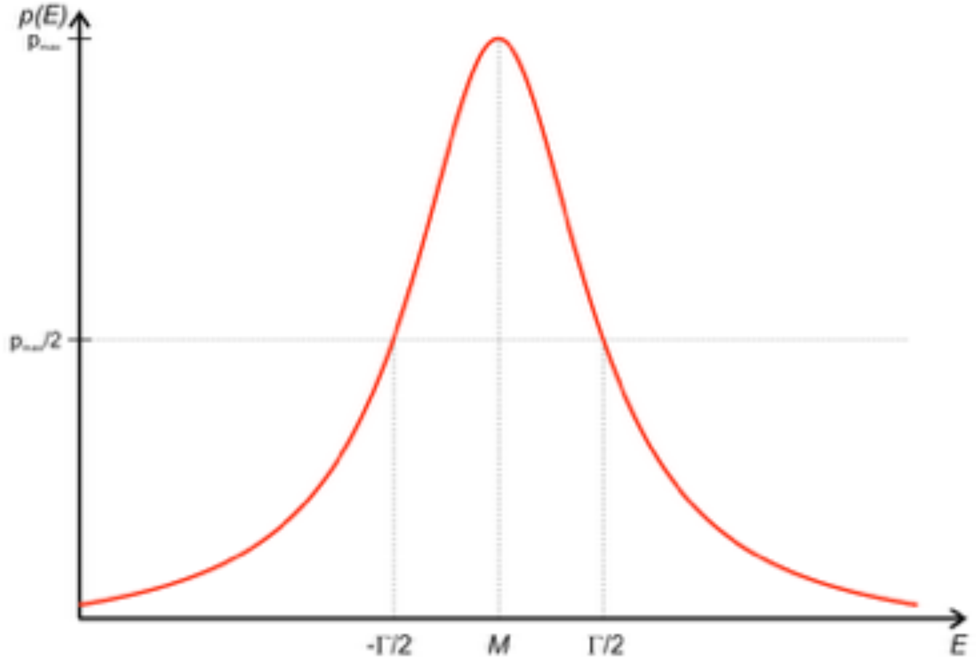

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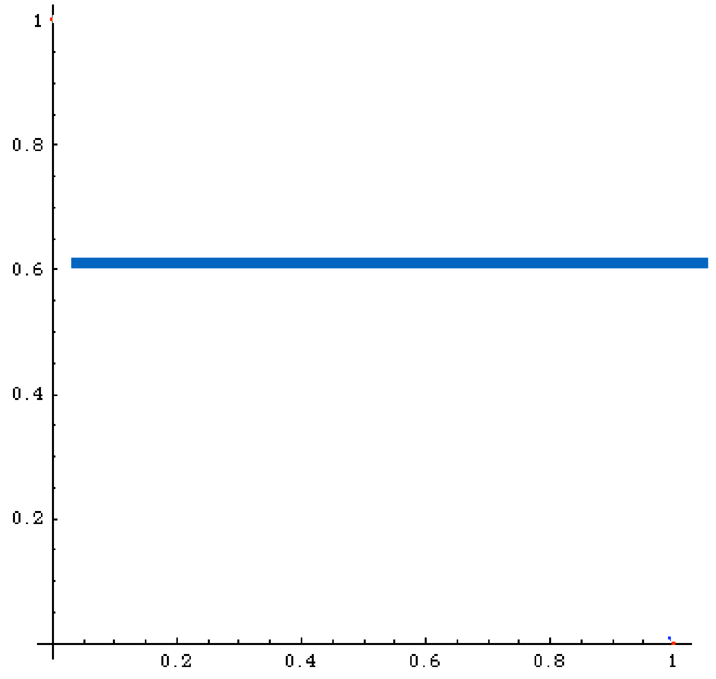

$$I = \int_{x_1}^{x_2} f(x) dx \quad \Rightarrow \quad I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N} \sum_{i=1}^{N} [f(x)]^2 - I_N^2$$

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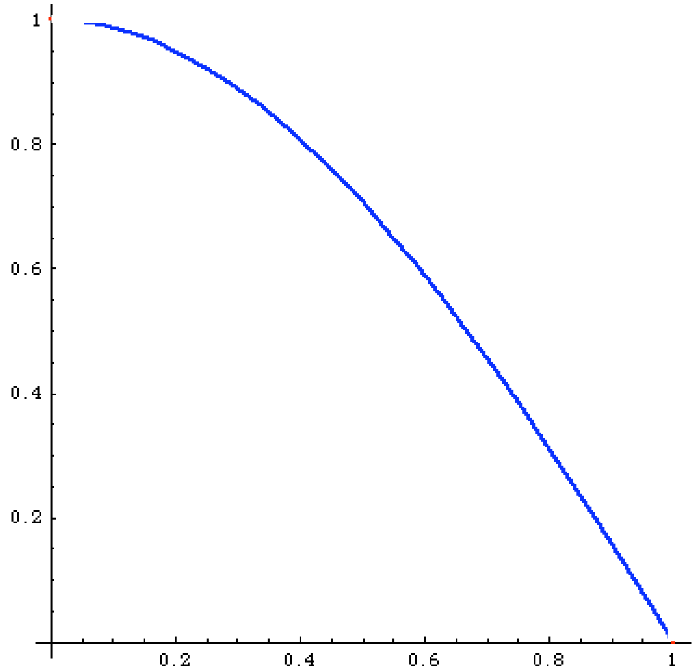

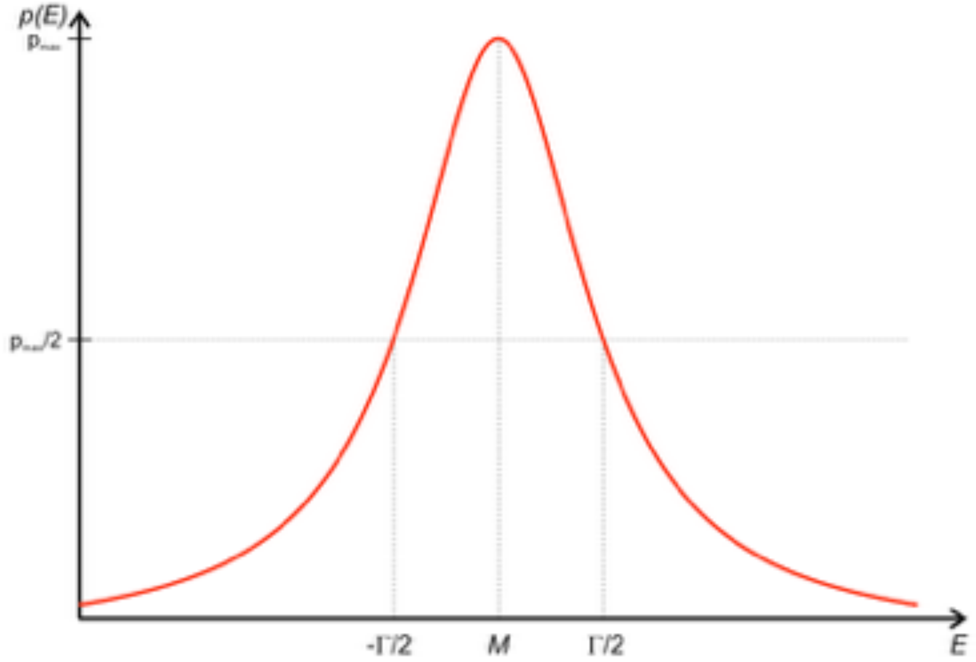

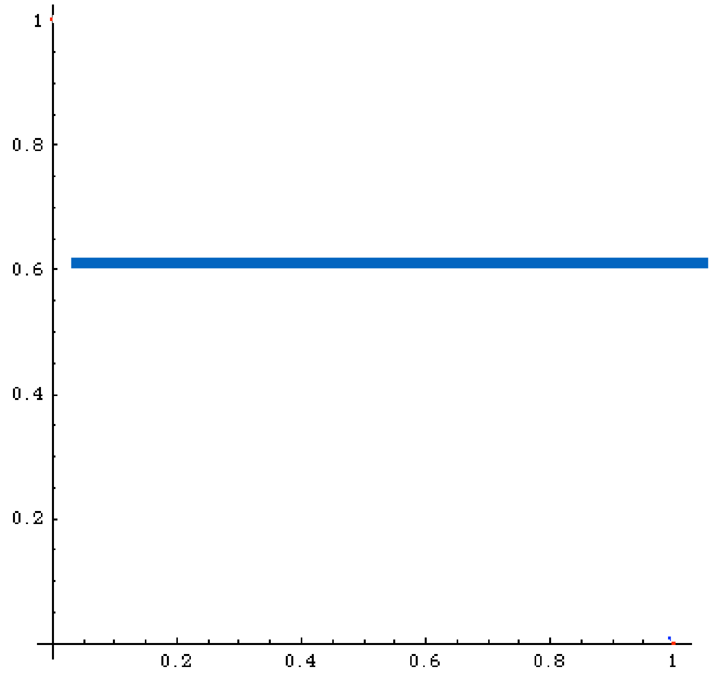

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$$I = I_N \pm \sqrt{V_N/N}$$

Integration

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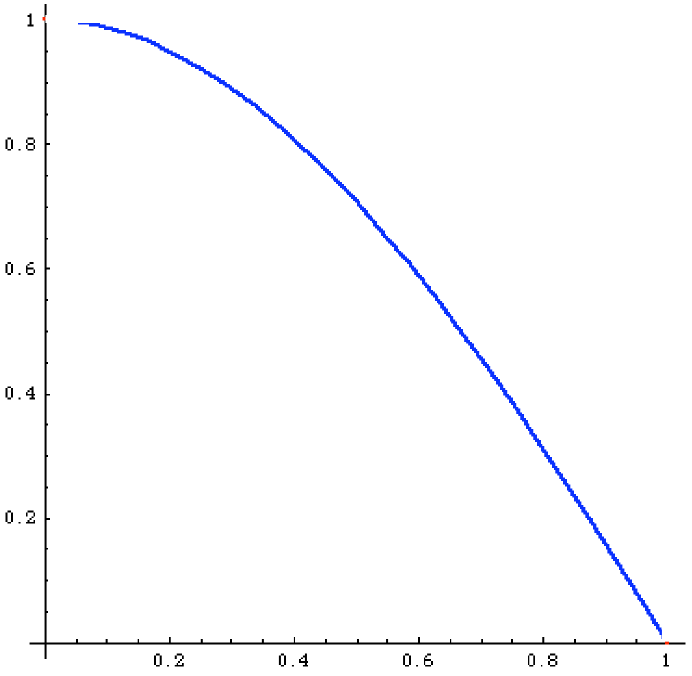

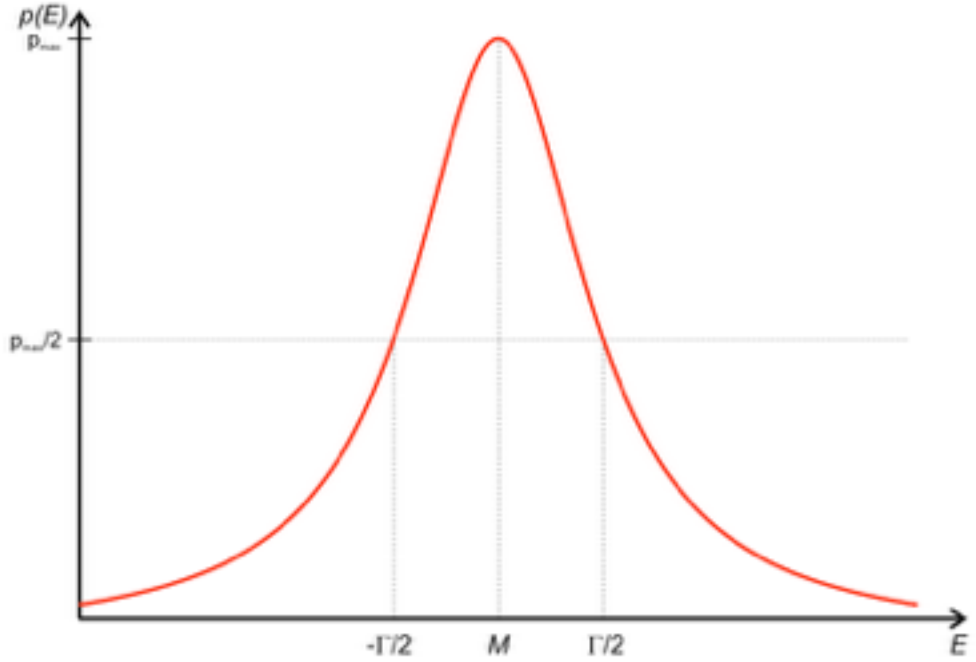

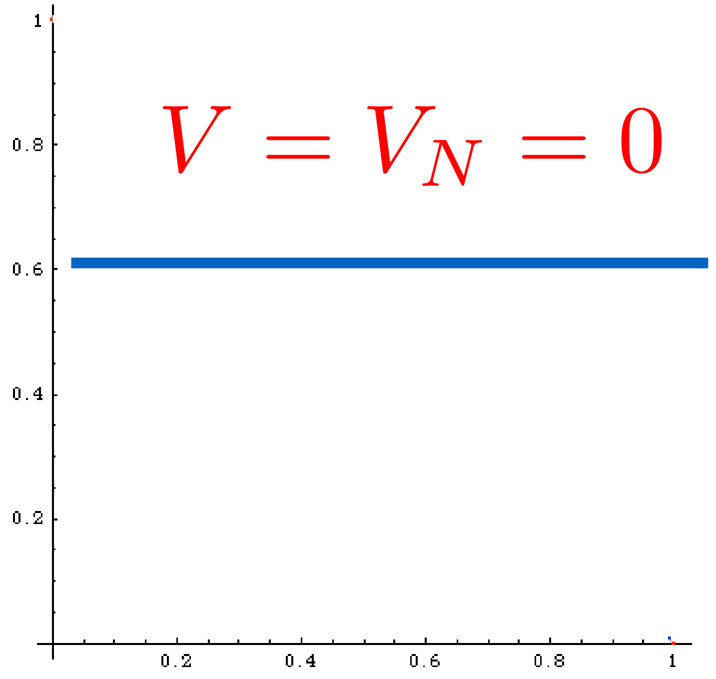

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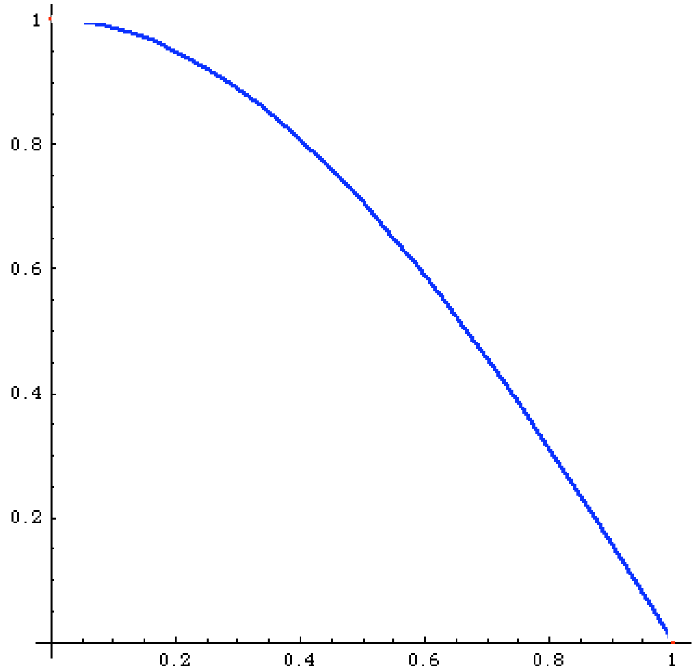

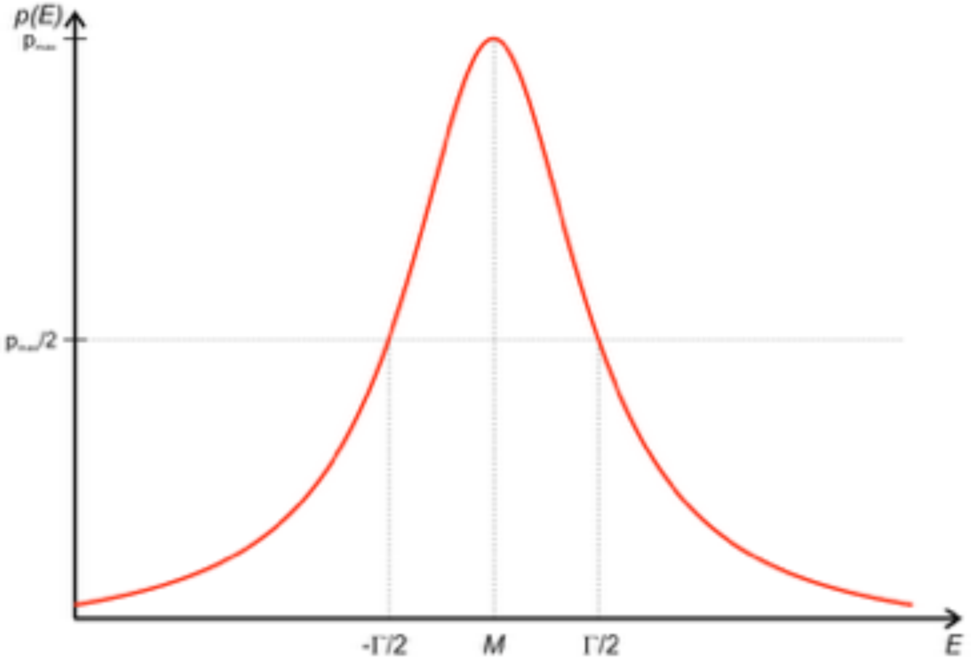

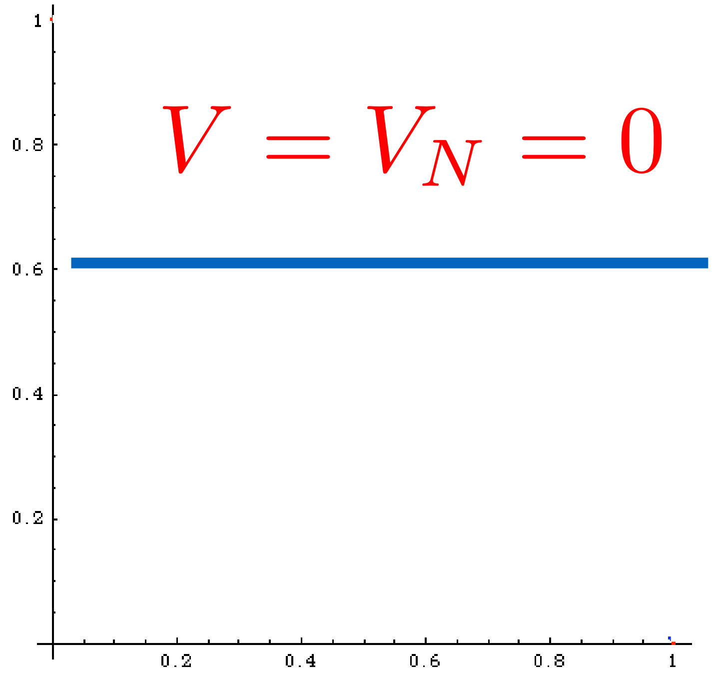

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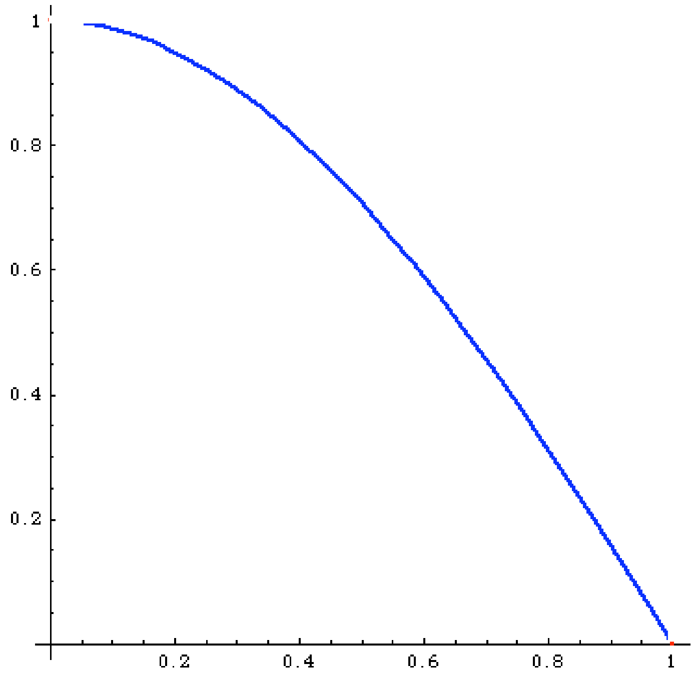
Can be minimized!

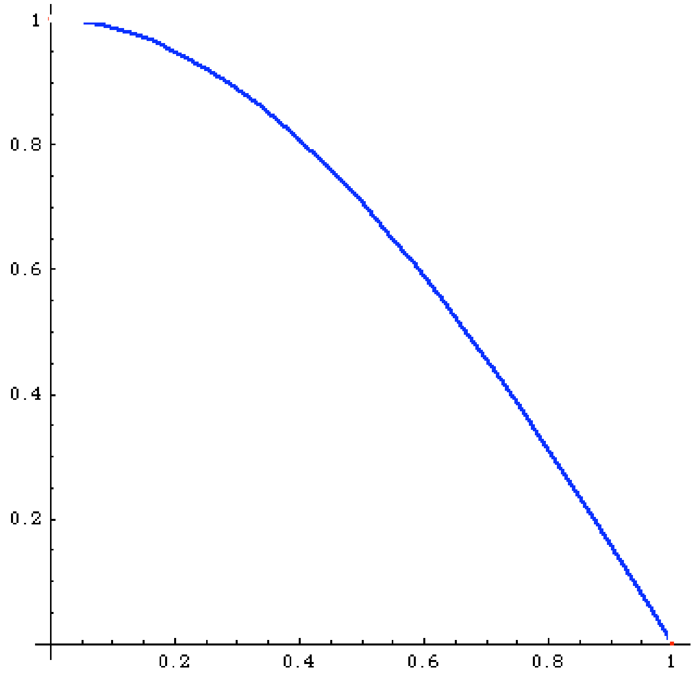
Importance Sampling



$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

Importance Sampling



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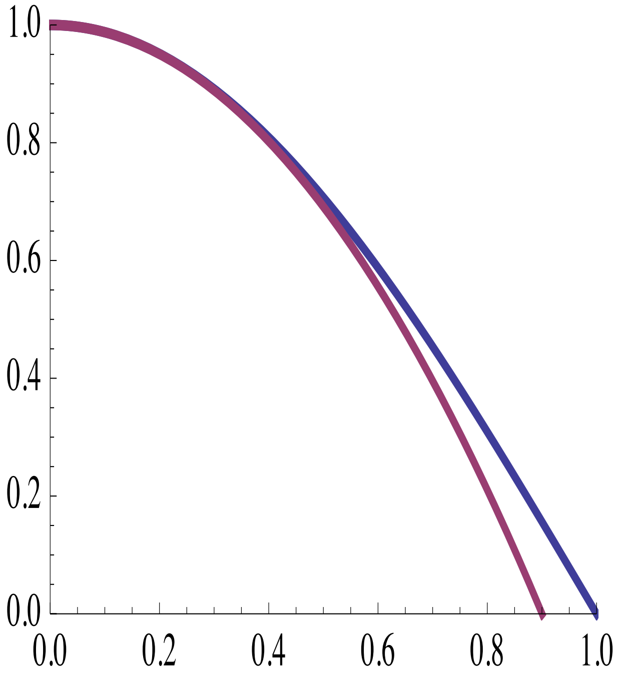
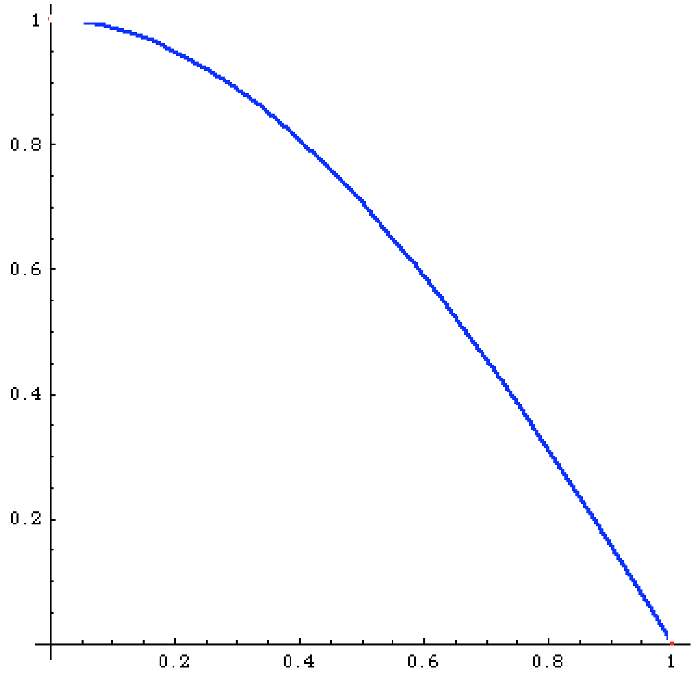


$$I = \int_0^1 dx(1-cx^2)\frac{\cos\left(\frac{\pi}{2}x\right)}{(1-cx^2)}$$

Importance Sampling



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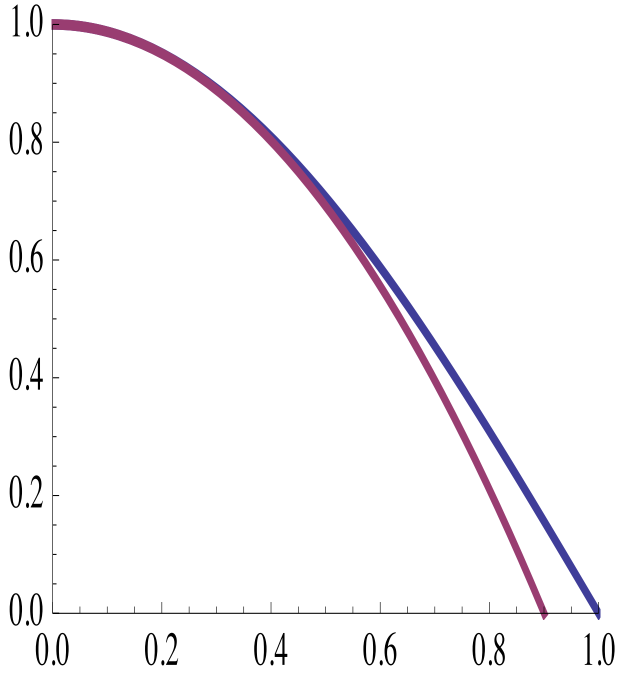

$$I = \int_0^1 dx(1-cx^2)\frac{\cos\left(\frac{\pi}{2}x\right)}{(1-cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2c}$$

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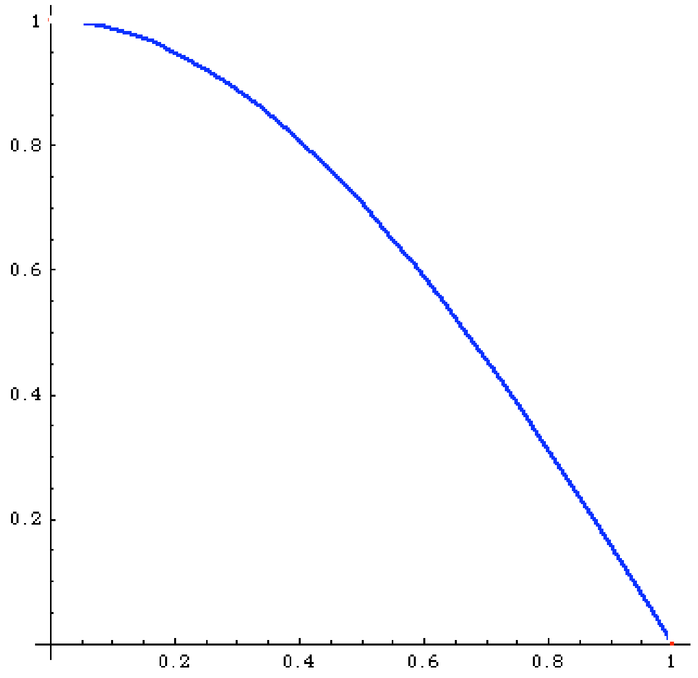

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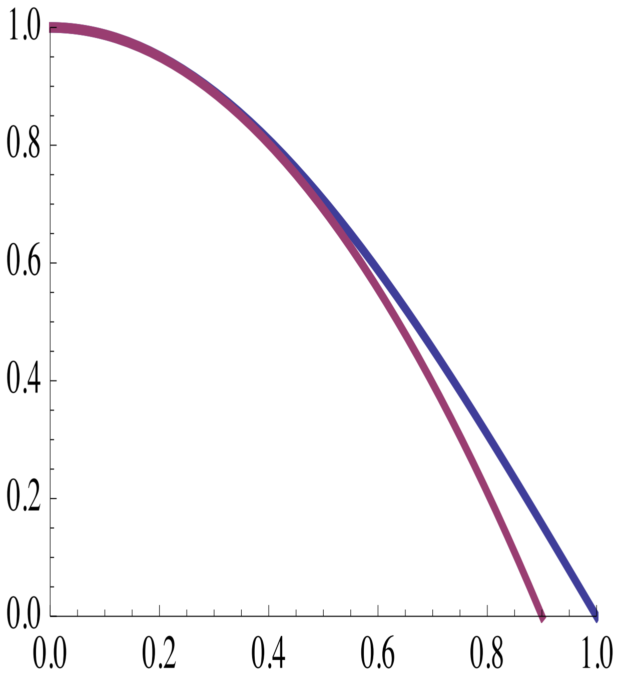

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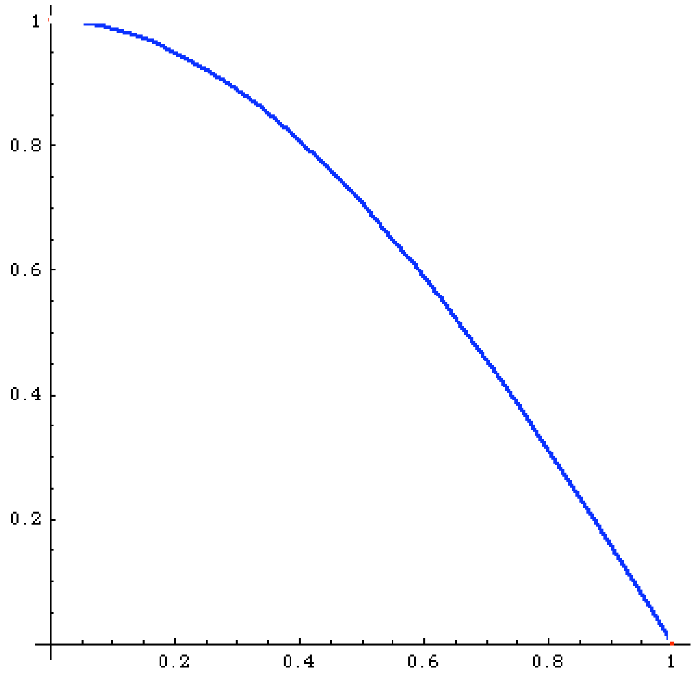
$$\Rightarrow \simeq 1$$

Importance Sampling



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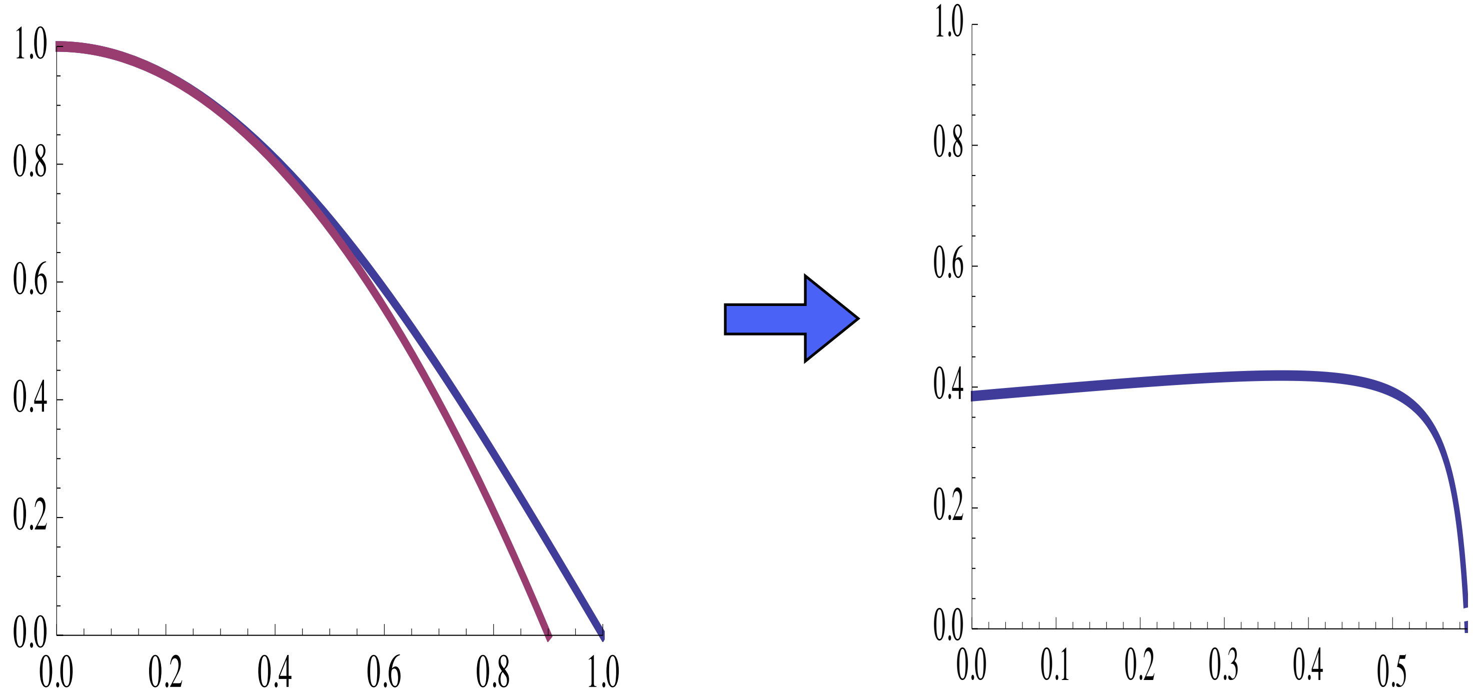

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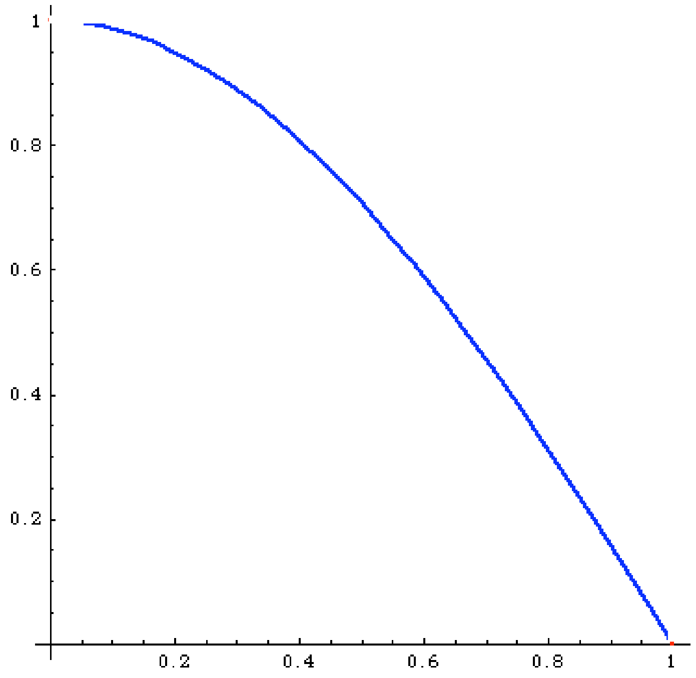
Importance Sampling



$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

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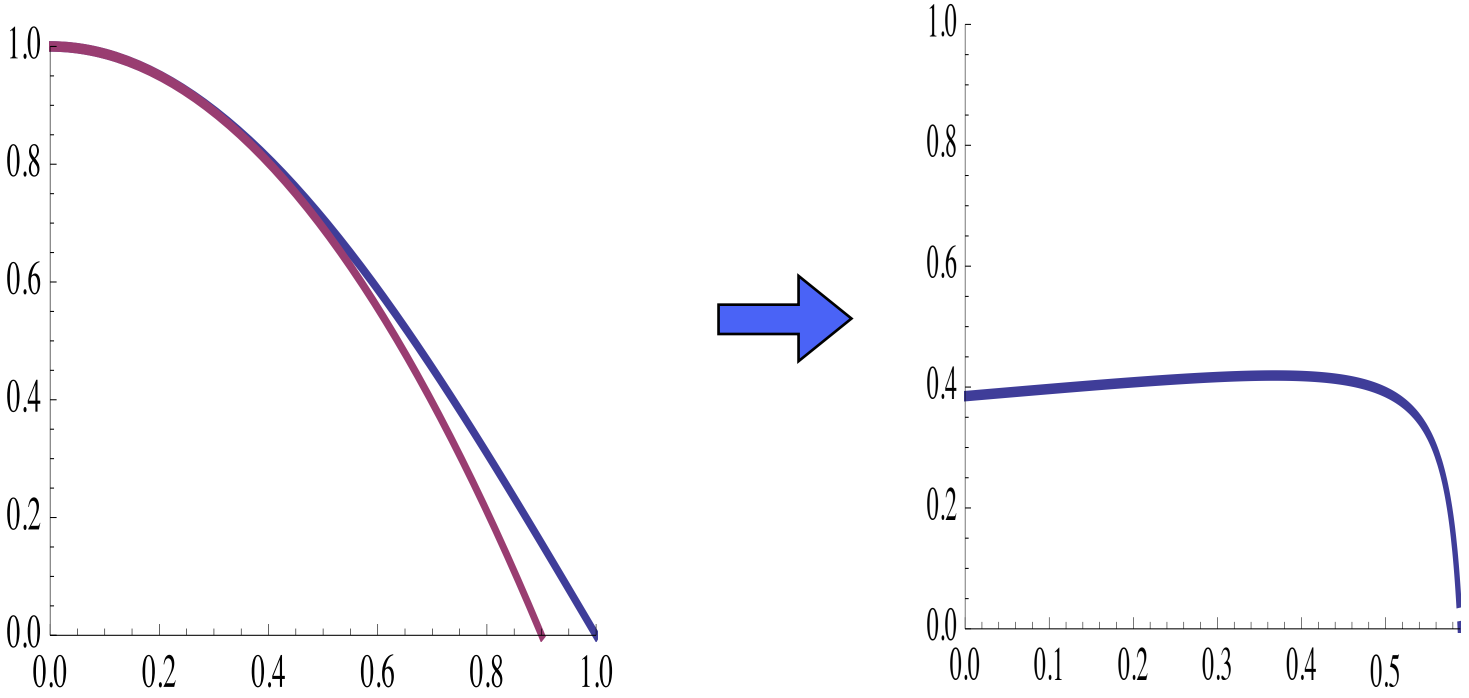

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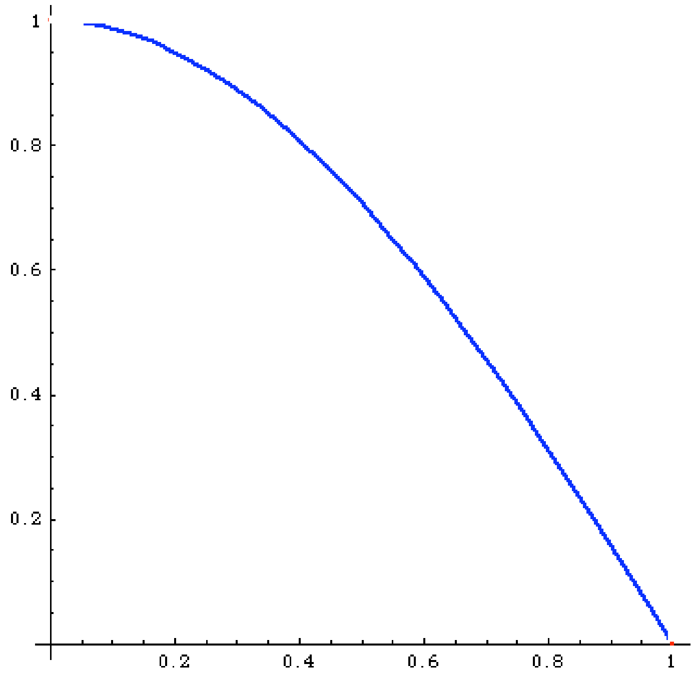
$$\rightarrow \simeq 1$$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

Importance Sampling



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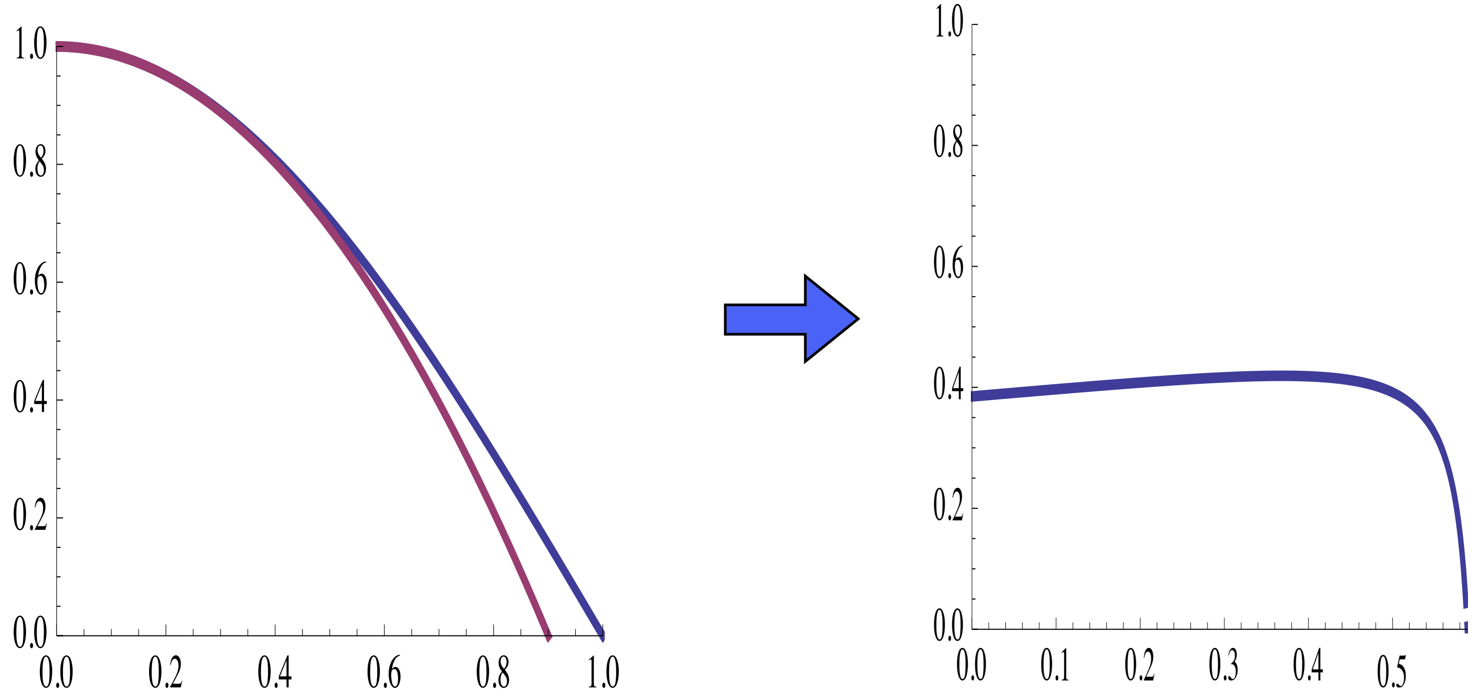

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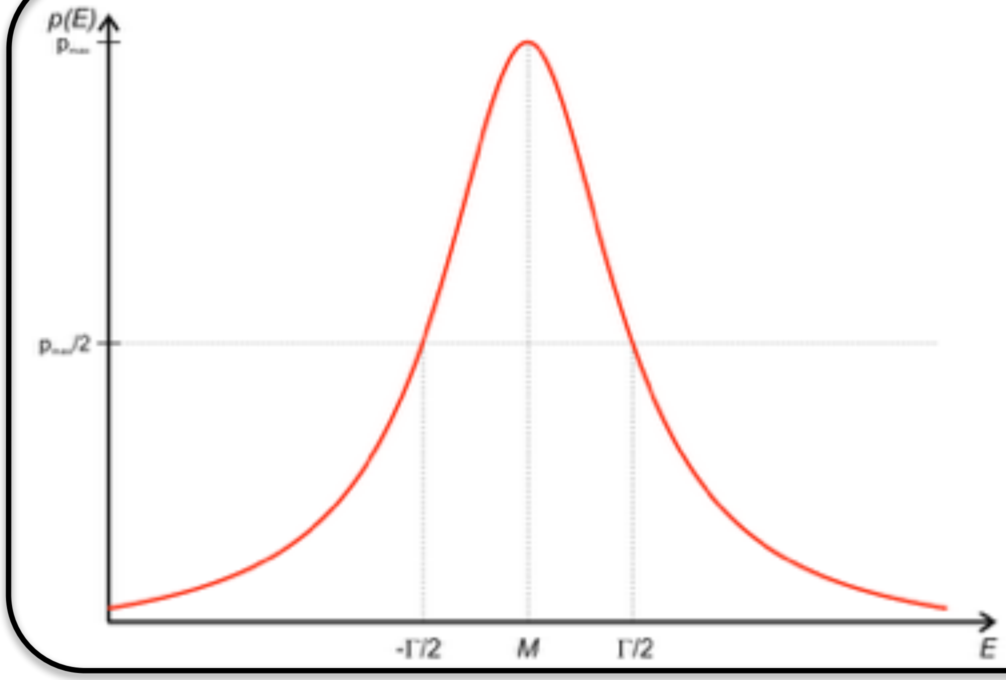
$$\Rightarrow \simeq 1$$

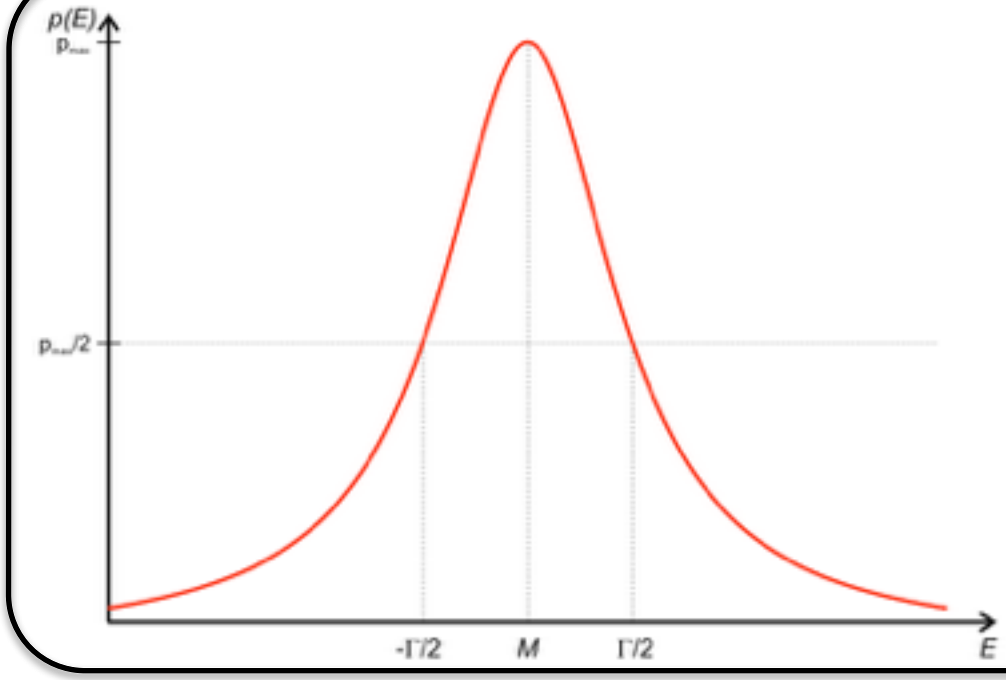
$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

The Phase-Space parametrization is important to have an efficient computation!

Importance Sampling



Importance Sampling



$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\xi = arctan\left(\frac{q^2 - M^2}{\Gamma\, M}\right)$$

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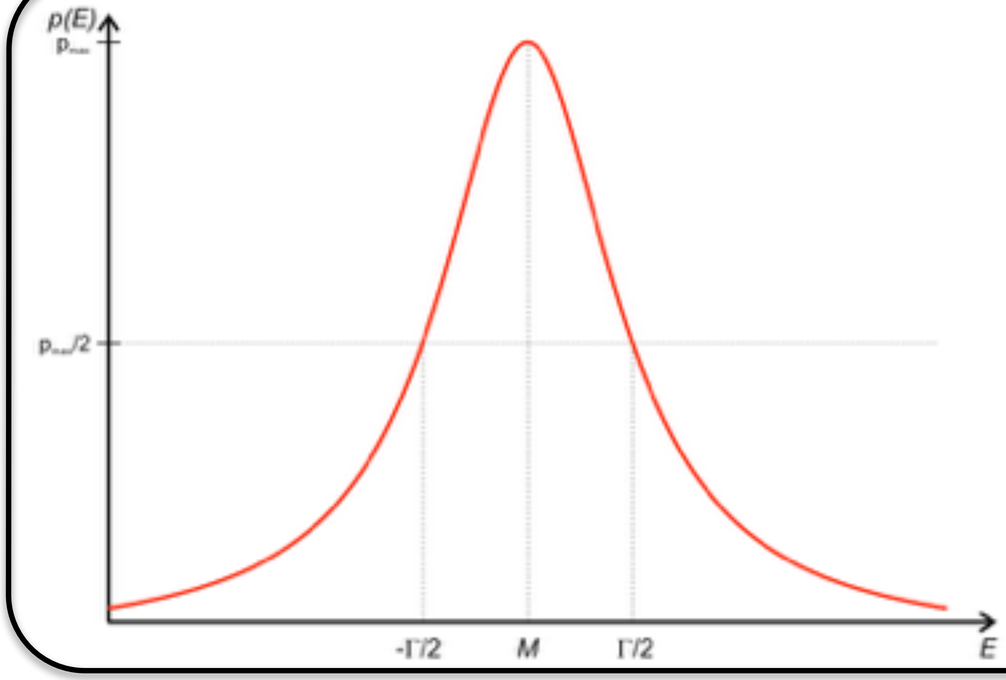

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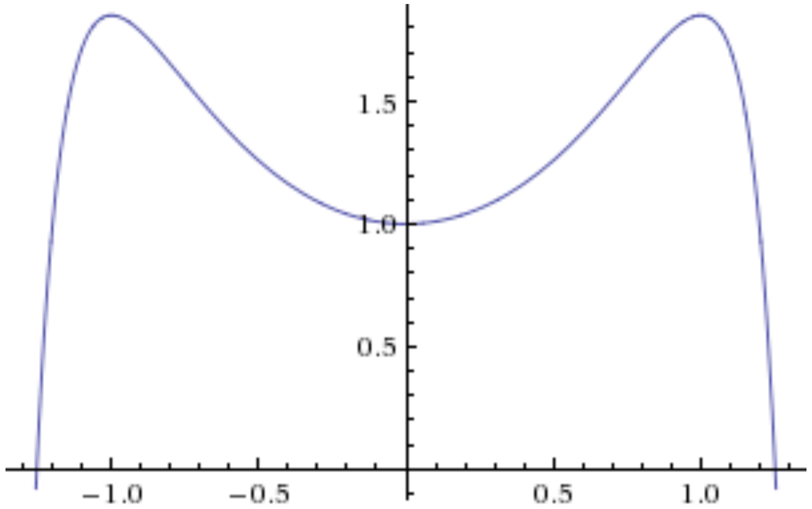

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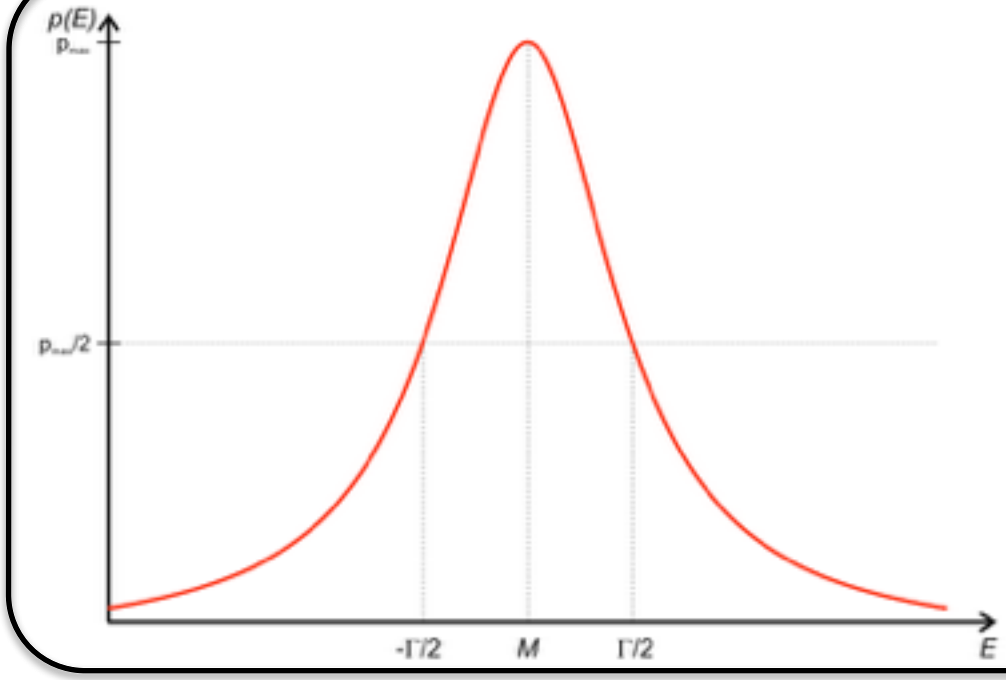

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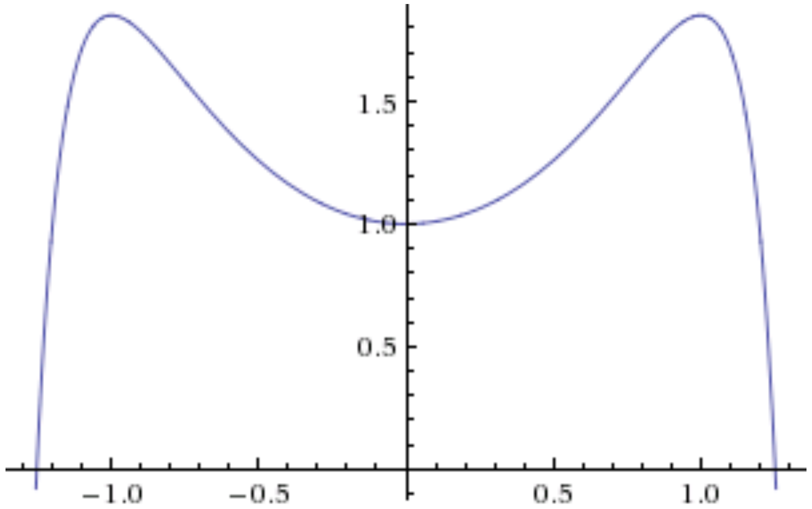
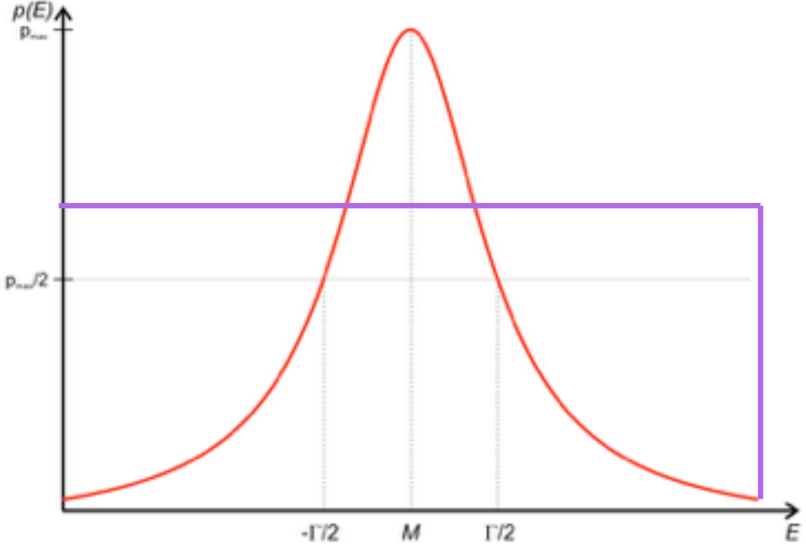
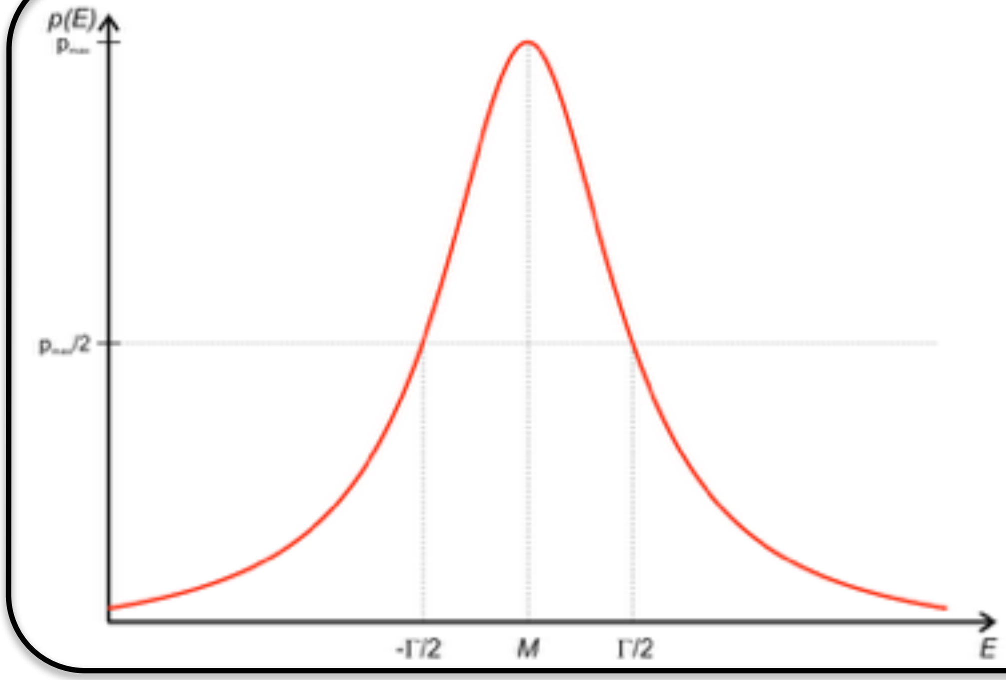


Why Importance Sampling?



Probability of using
that point p(x)

Importance Sampling



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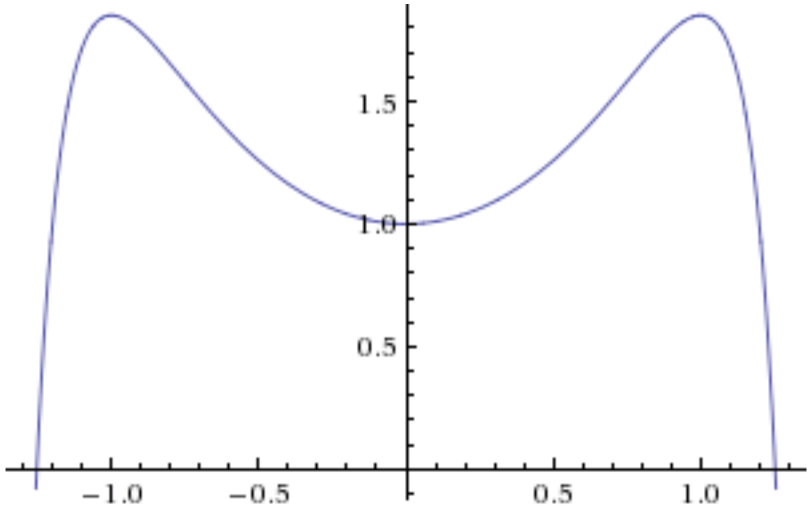

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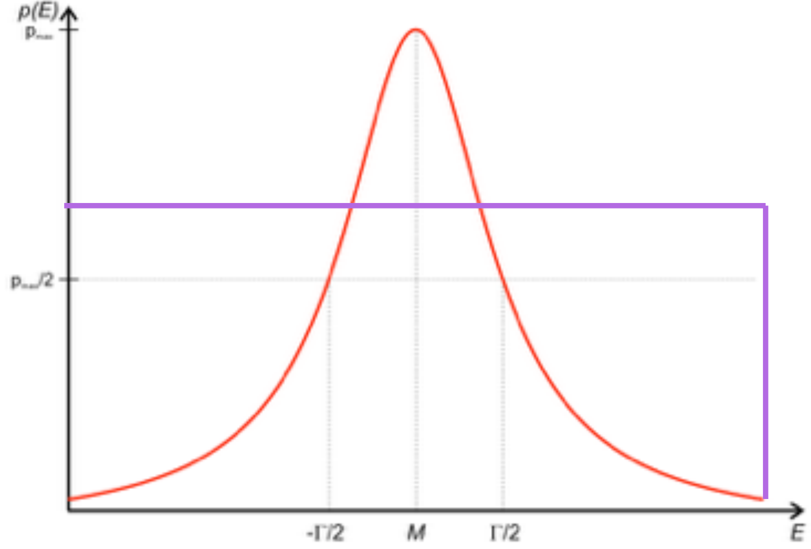

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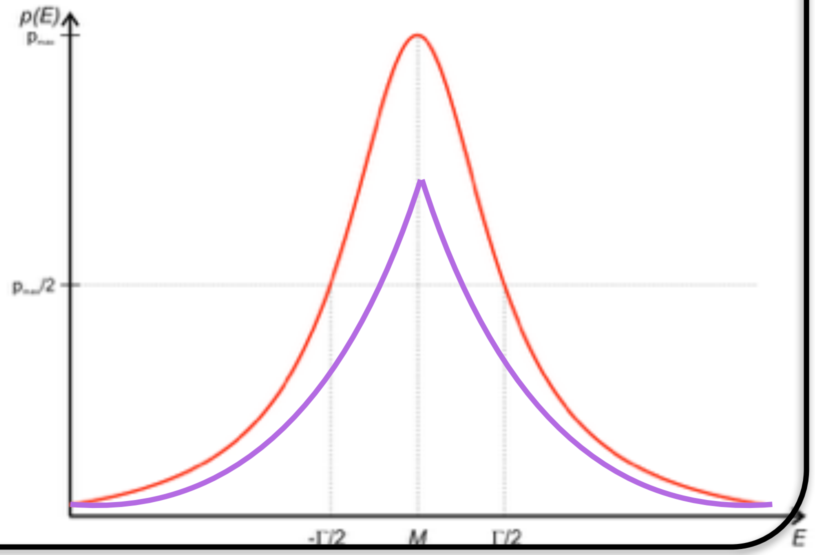

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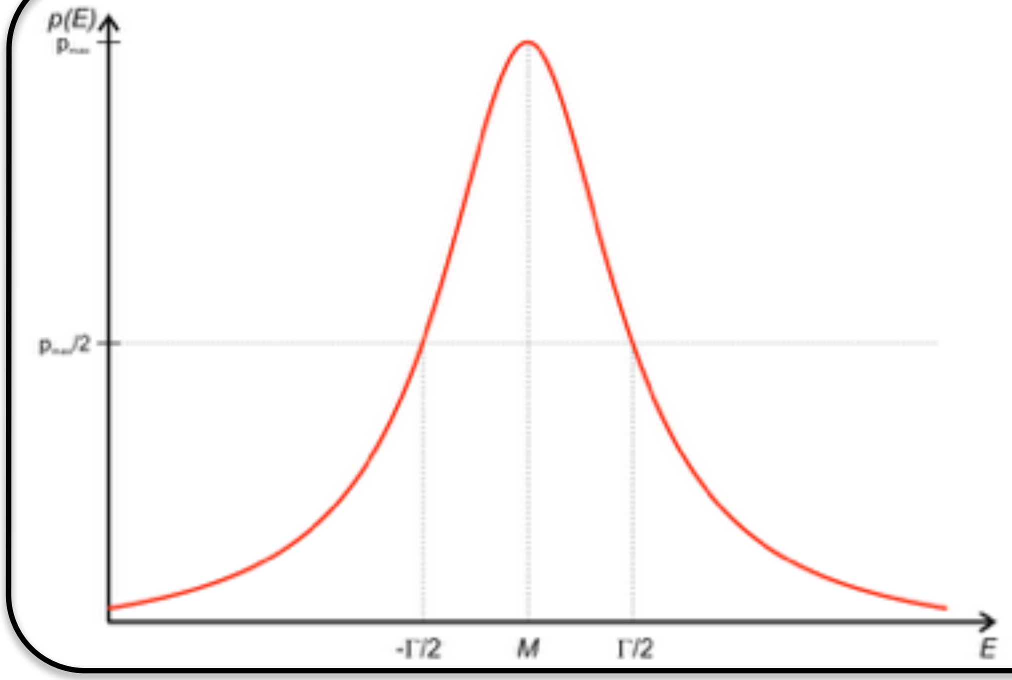

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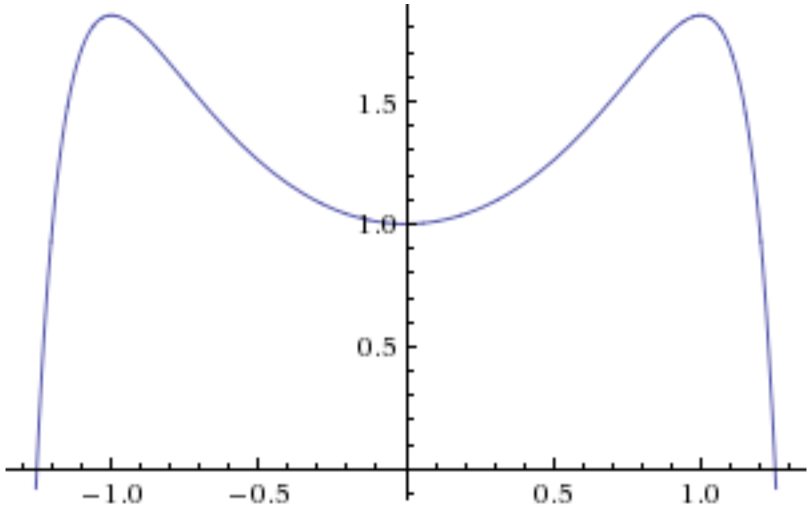

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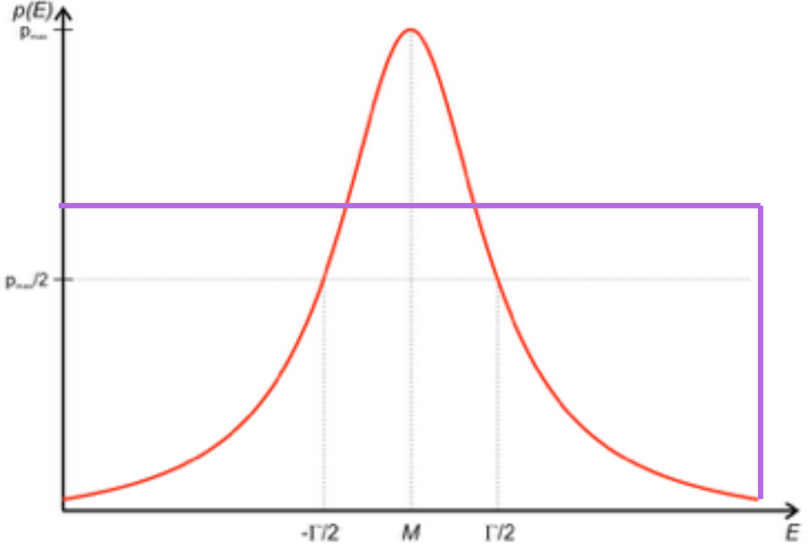

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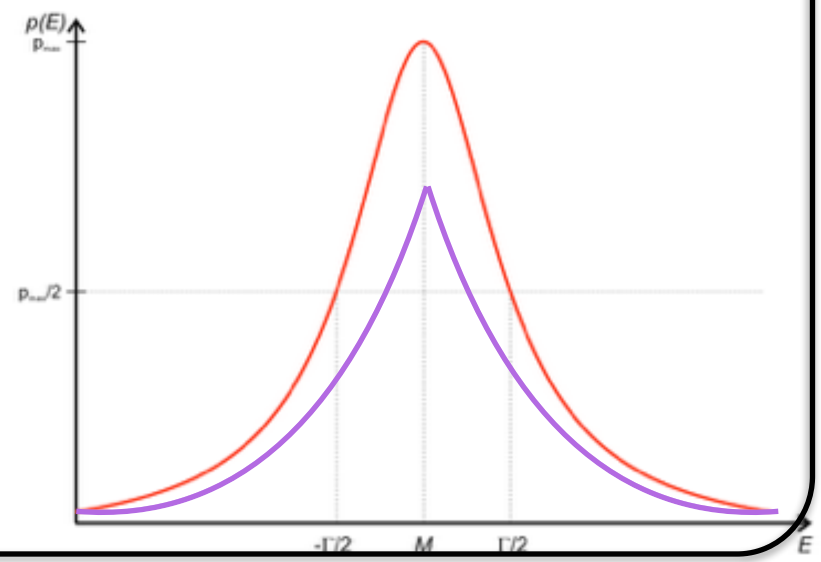

The change of variable ensure that the evaluation of the function is done where the function is the largest!

Importance Sampling

Key Point

- Generate the random point in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is flatter in this new variable.
- Needs to know an approximate function.

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

Adaptative Monte-Carlo

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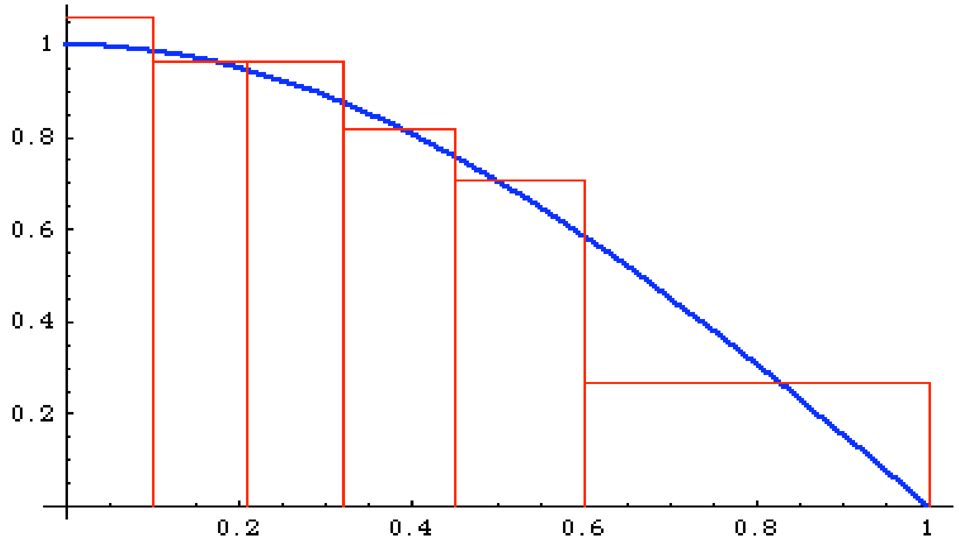

Algorithm

1. Creates bin such that each of them have the same contribution.

   ➡Many bins where the function is large

2. Use the approximate for the importance sampling method.

VEGAS

More than one Dimension

- VEGAS works only with 1(few) dimension

  ➡memory problem

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Solution

- Use projection on the axis

$$\vec{p}(x)= p(x)\bullet p(y)\bullet p(z)\ldots$$

VEGAS

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Solution

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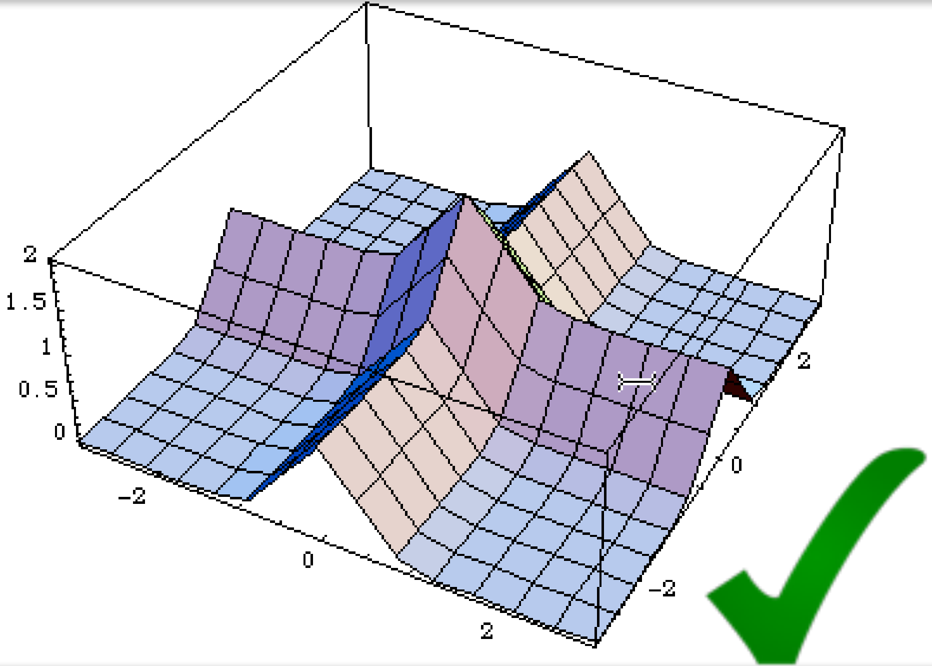
$$\vec{p}(x) = p(x) \cdot p(y) \cdot p(z) \ldots$$



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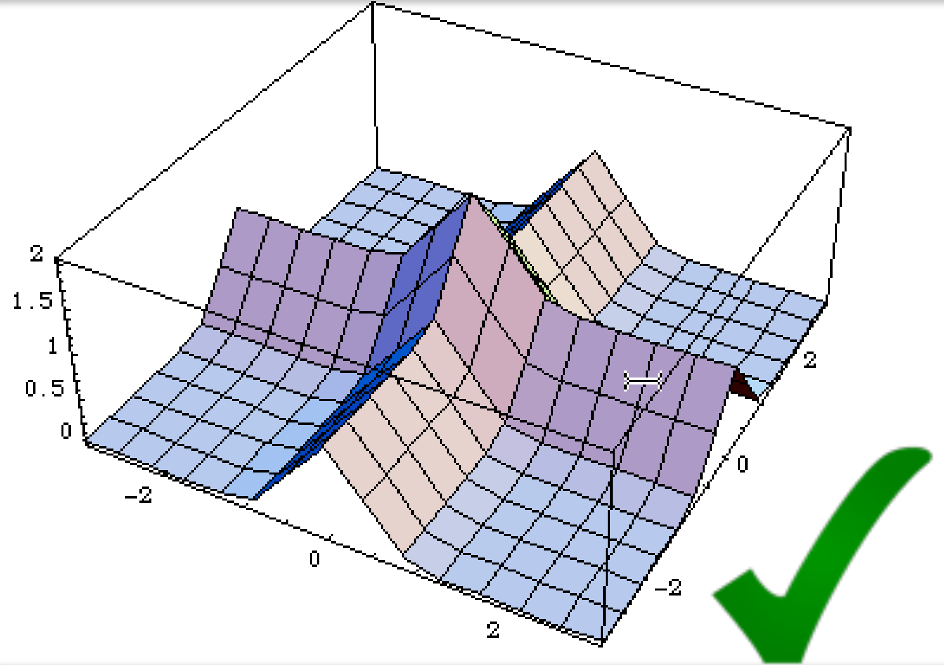
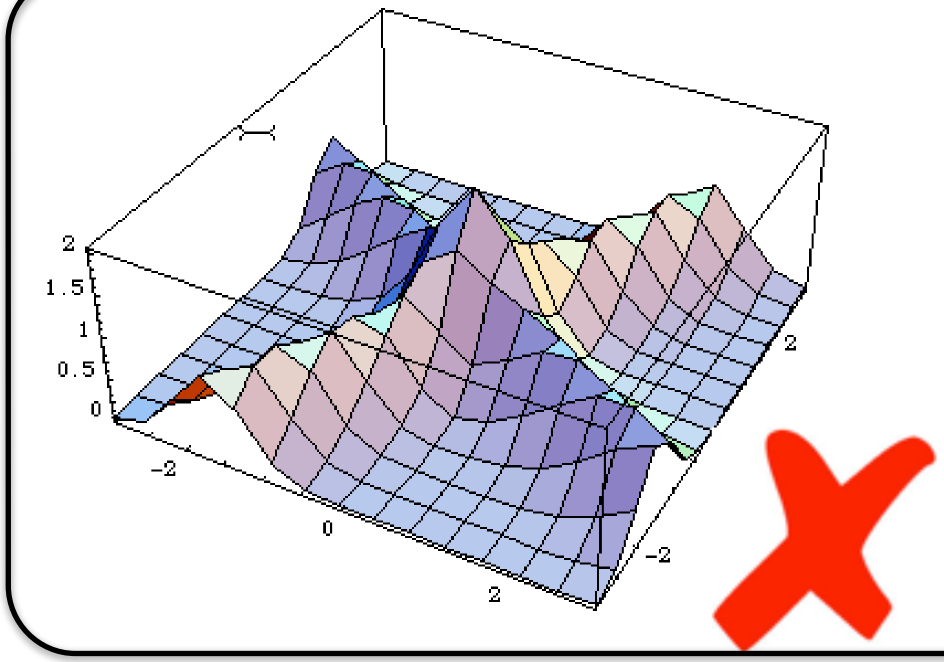
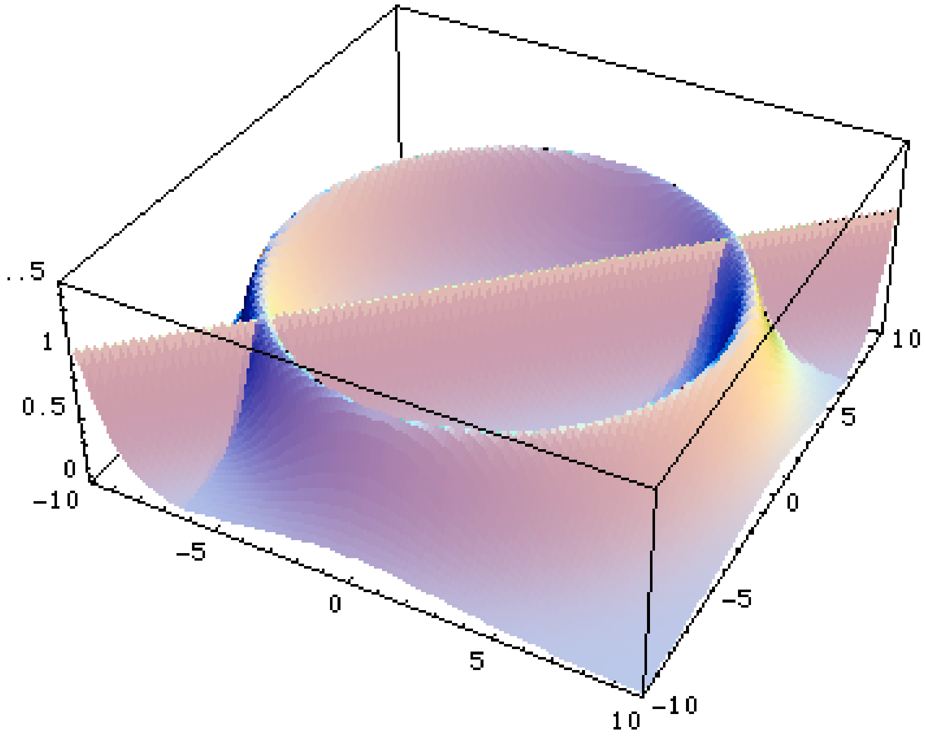




- We need to ensure the factorization !

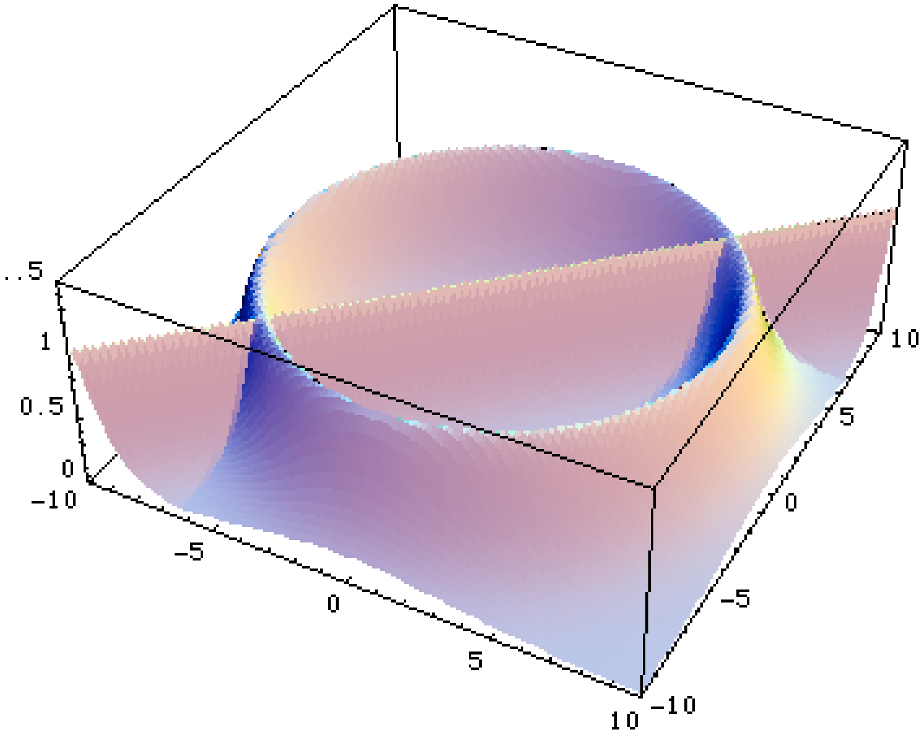
➡Additional change of variable

Multi-channel



What do we do if there is no transformation that aligns all integrand peaks to the chosen axes?
Vegas is bound to fail!

Multi-channel



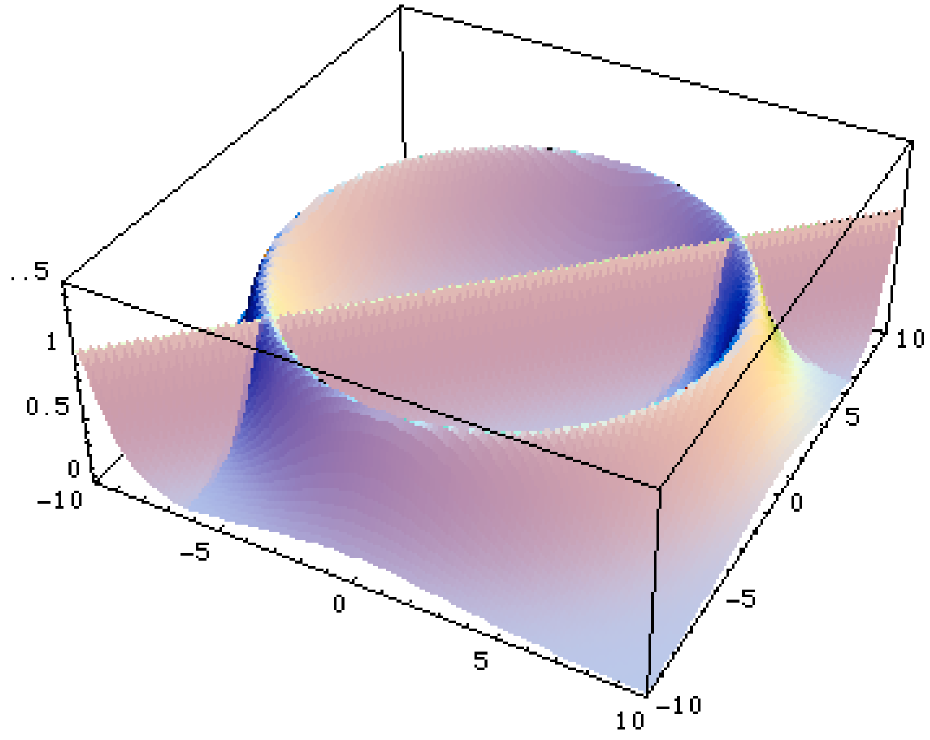
What do we do if there is no transformation that aligns all integrand peaks to the chosen axes?
Vegas is bound to fail!

Solution: use different transformations = channels

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

with each $p_i(x)$ taking care of one “peak” at the time

Multi-channel



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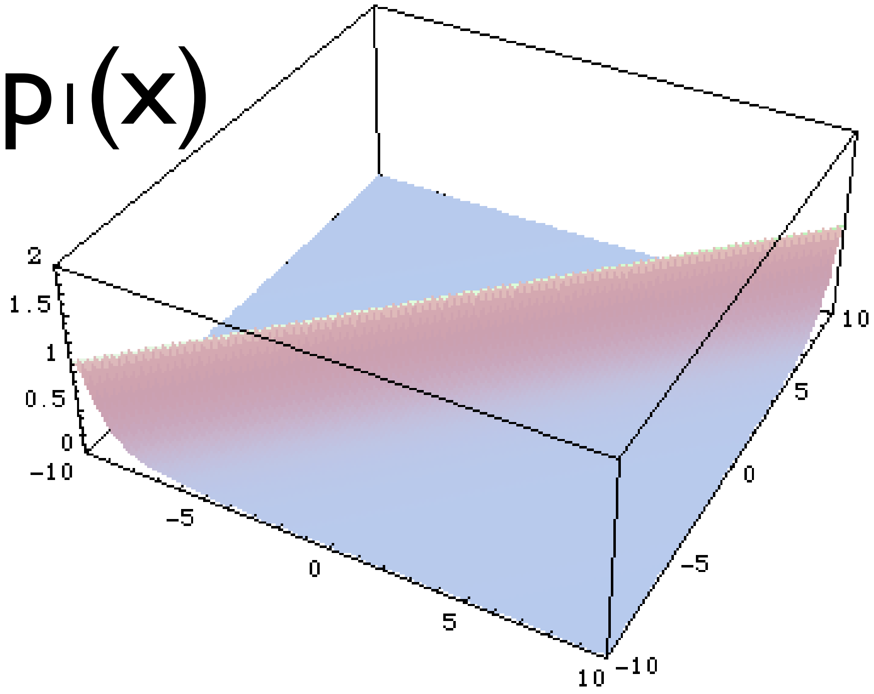
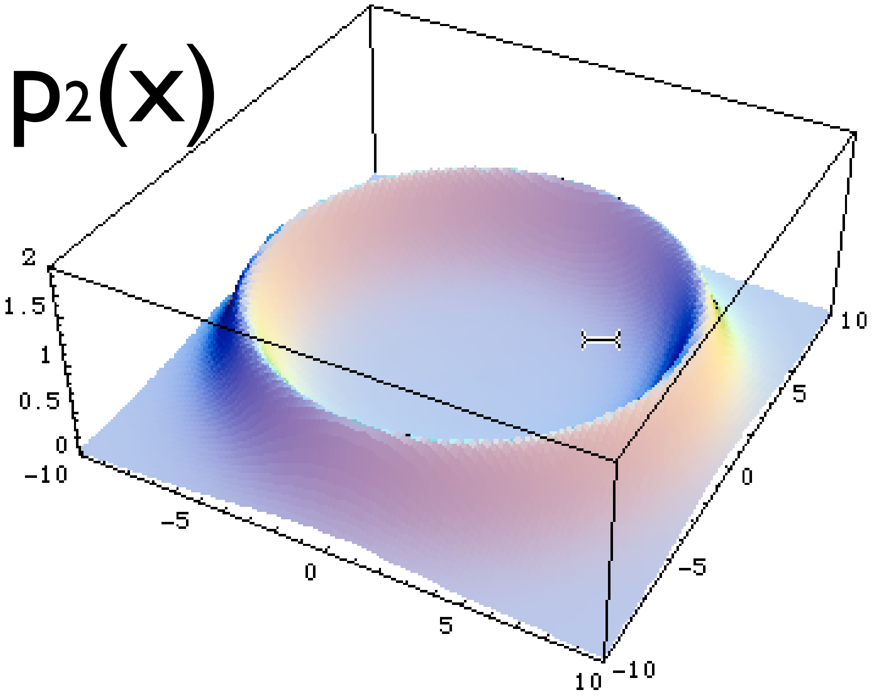
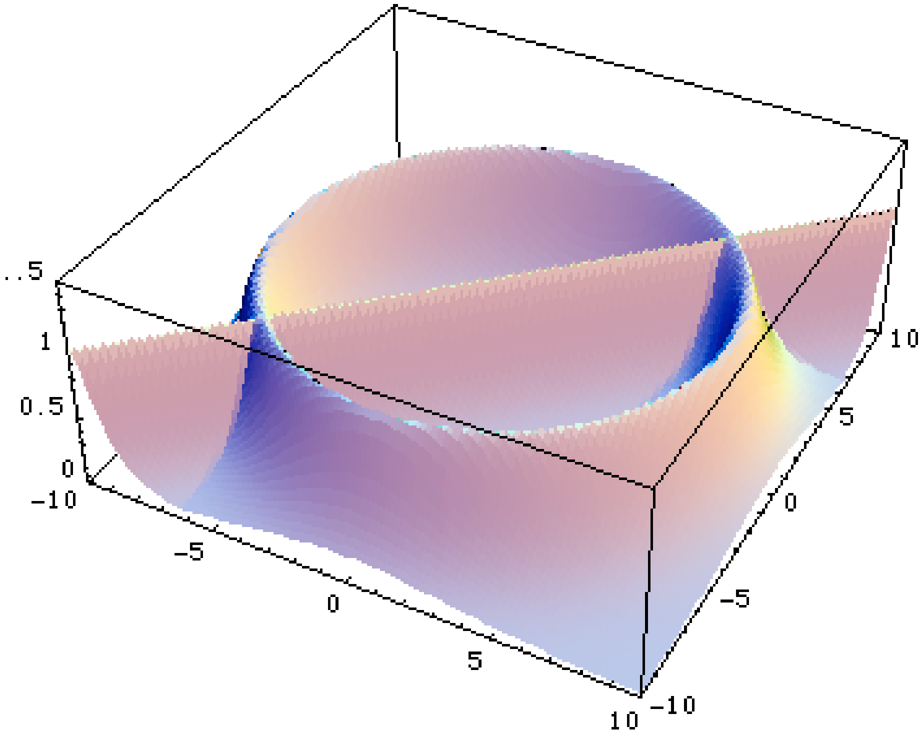
$p_1(x)$



$p_2(x)$



Multi-channel



$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x)$$

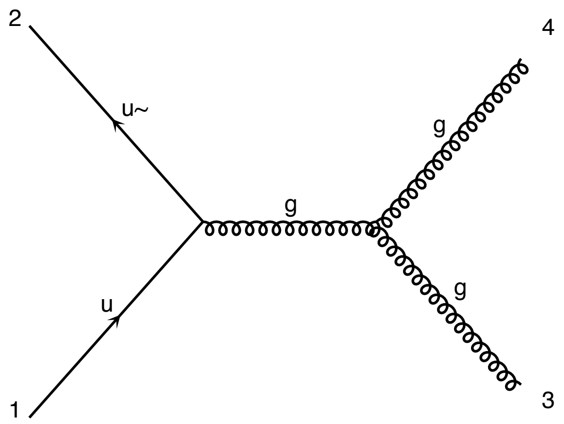
with

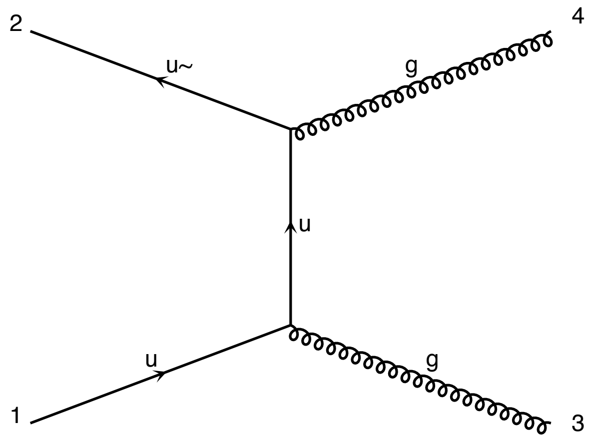
$$\sum_{i=1}^{n} \alpha_i = 1$$

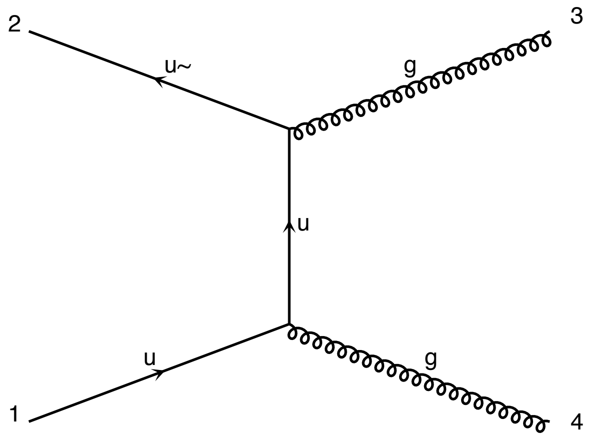
Then,

$$I = \int f(x)dx = \sum_{i=1}^{n} \alpha_i \int \frac{f(x)}{p(x)} p_i(x)dx$$

Example: QCD 2 → 2







$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2} \qquad \propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2} \qquad \propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

Multi-channel based on single diagrams*

*Method used in MadGraph

Does a basis exist?

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

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$$\approx 1$$

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Key Idea

- Any single diagram is “easy” to integrate (pole structures/ suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

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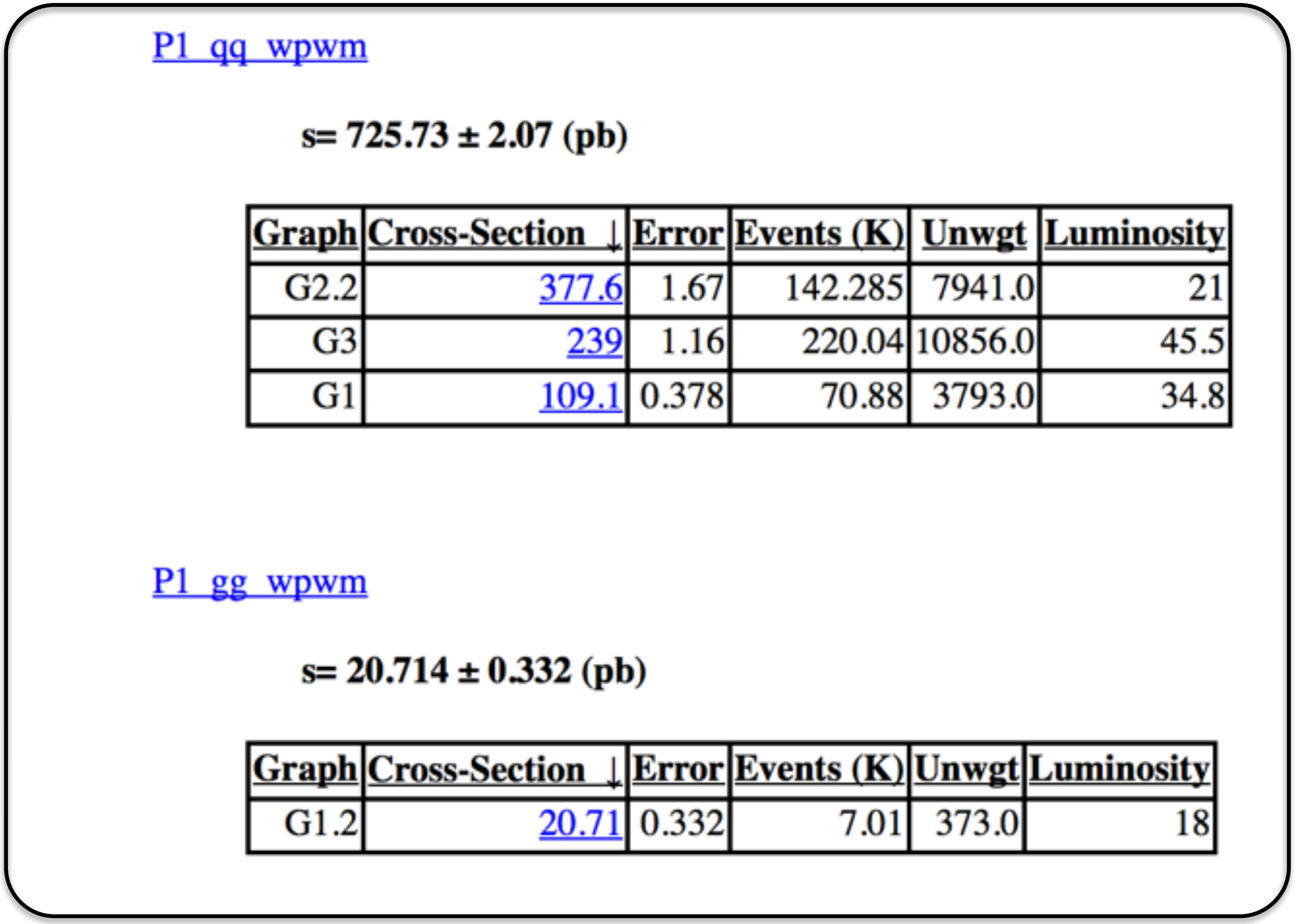
N Integral

- Errors add in quadrature so no extra cost
- "Weight" functions already calculated during $|M|^2$ calculation
- Parallel in nature

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$



P1_qq_wpwm

s= 725.73 ± 2.07 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G2.2	377.6	1.67	142.285	7941.0	21
G3	239	1.16	220.04	10856.0	45.5
G1	109.1	0.378	70.88	3793.0	34.8

P1_gg_wpwm

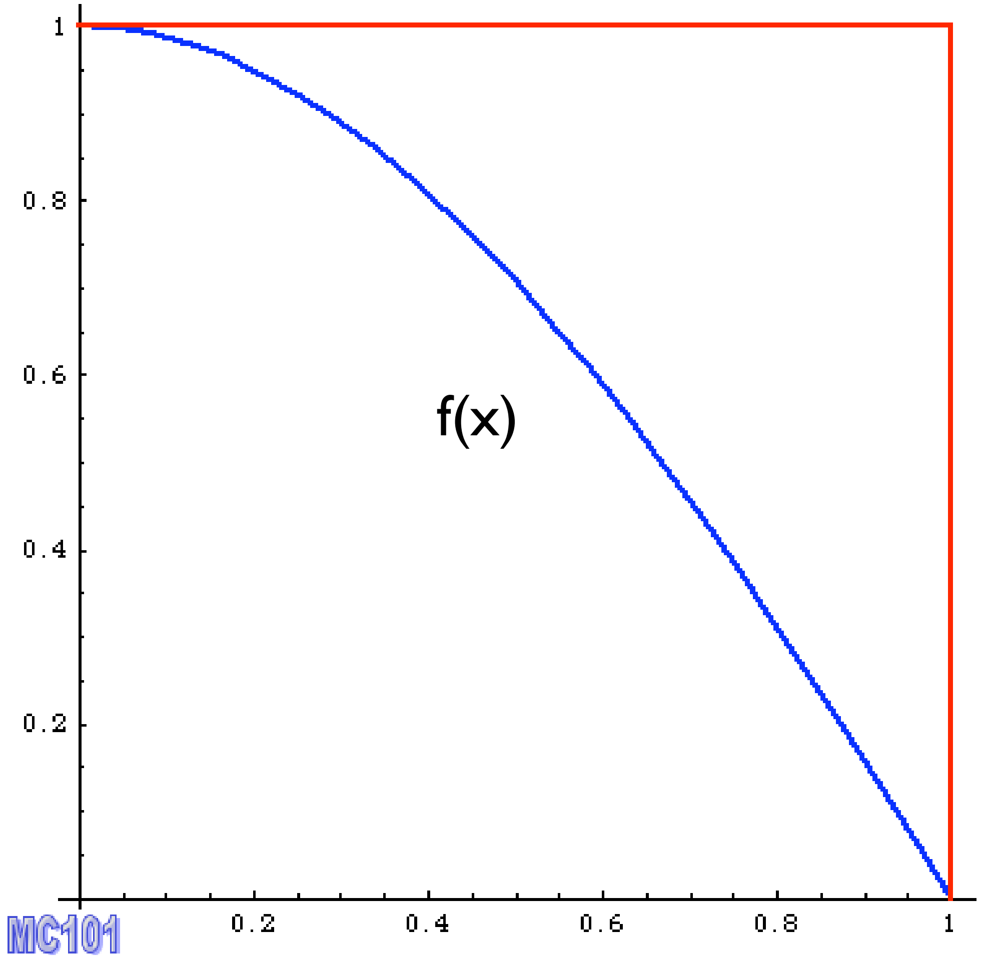
s= 20.714 ± 0.332 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.2	20.71	0.332	7.01	373.0	18

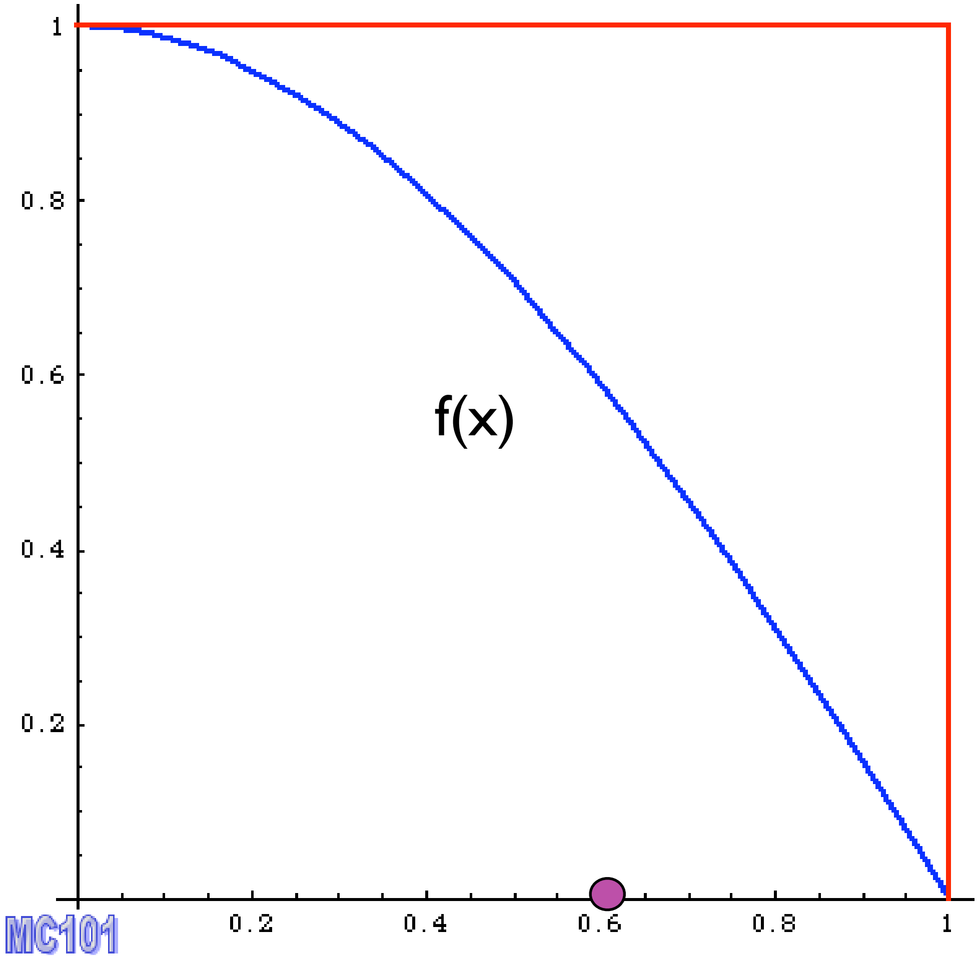
term of the above sum.

each term might not be gauge invariant

Event generation

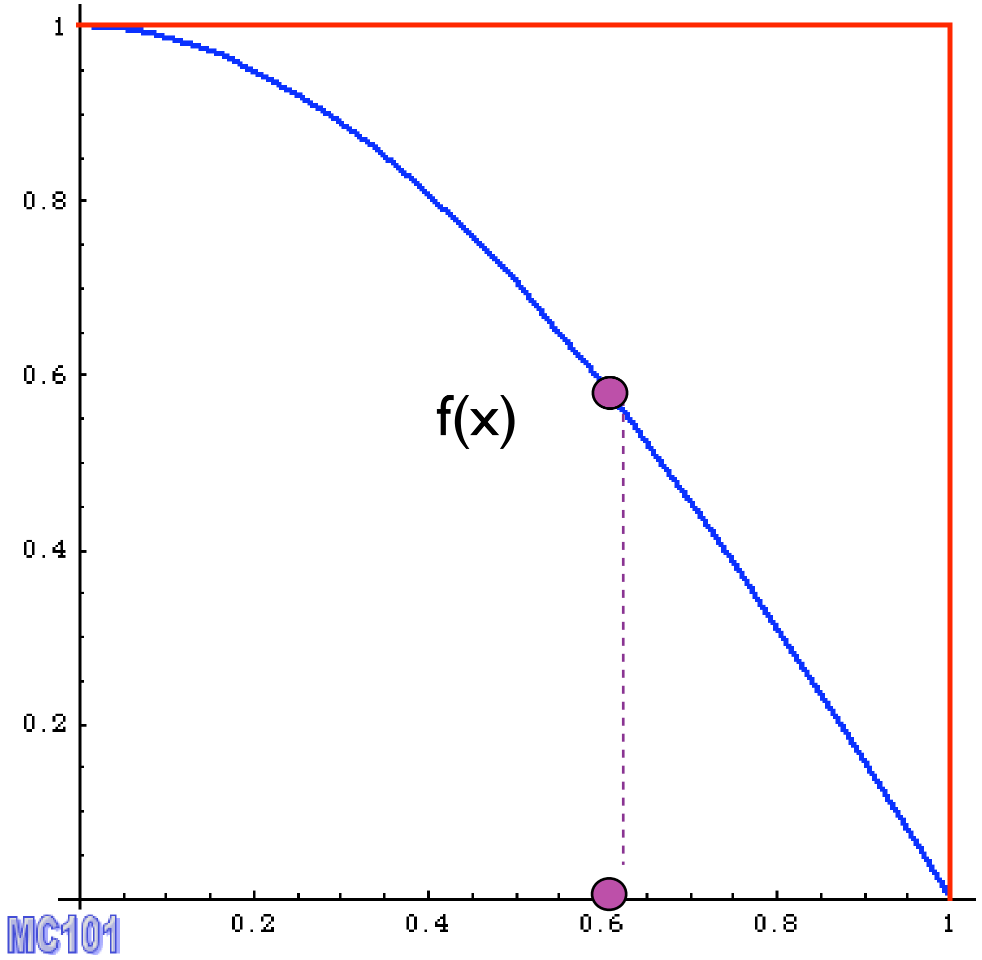


Event generation



1. pick x

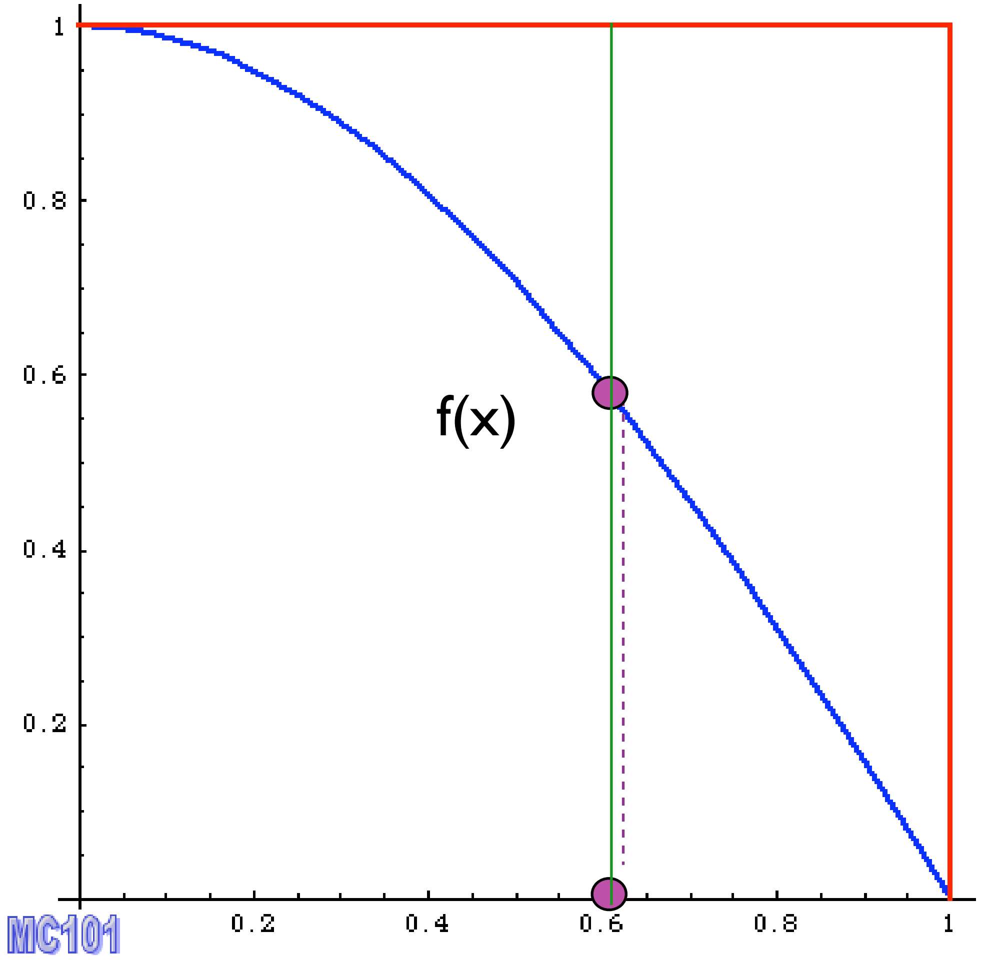
Event generation



1. pick x

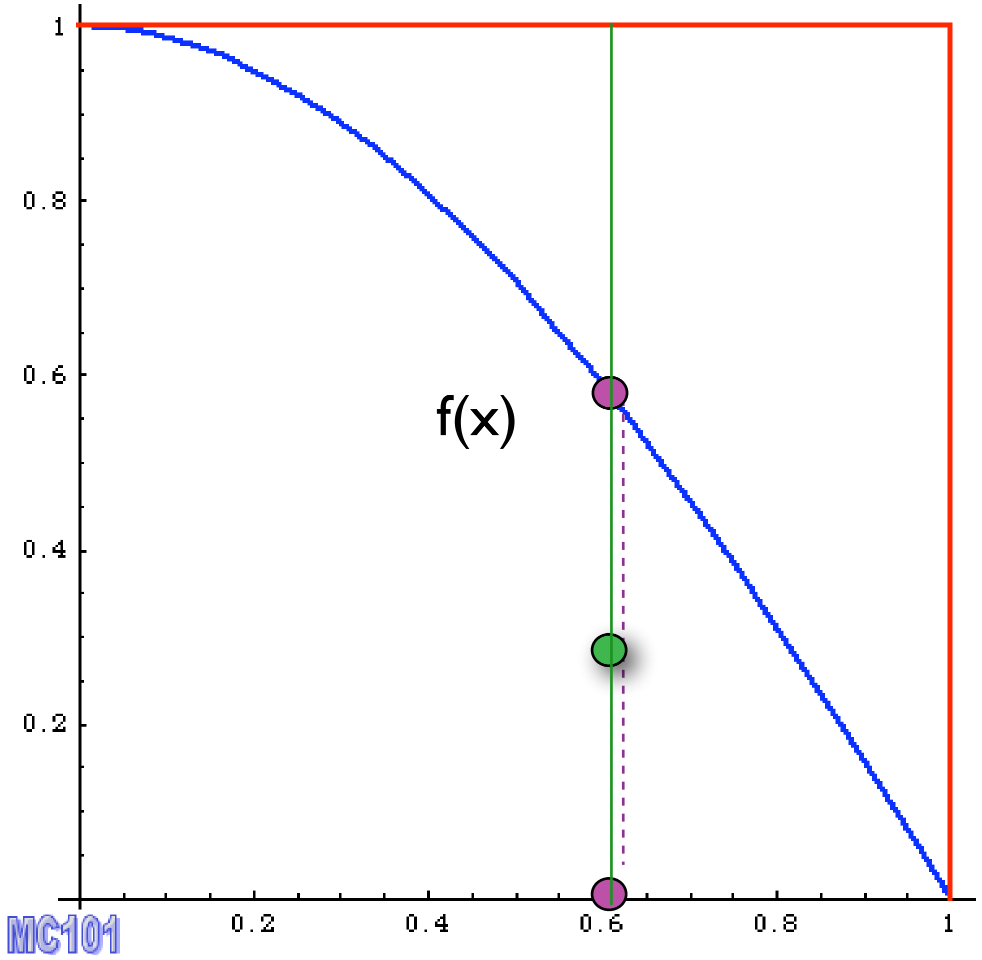
2. calculate f(x)

Event generation



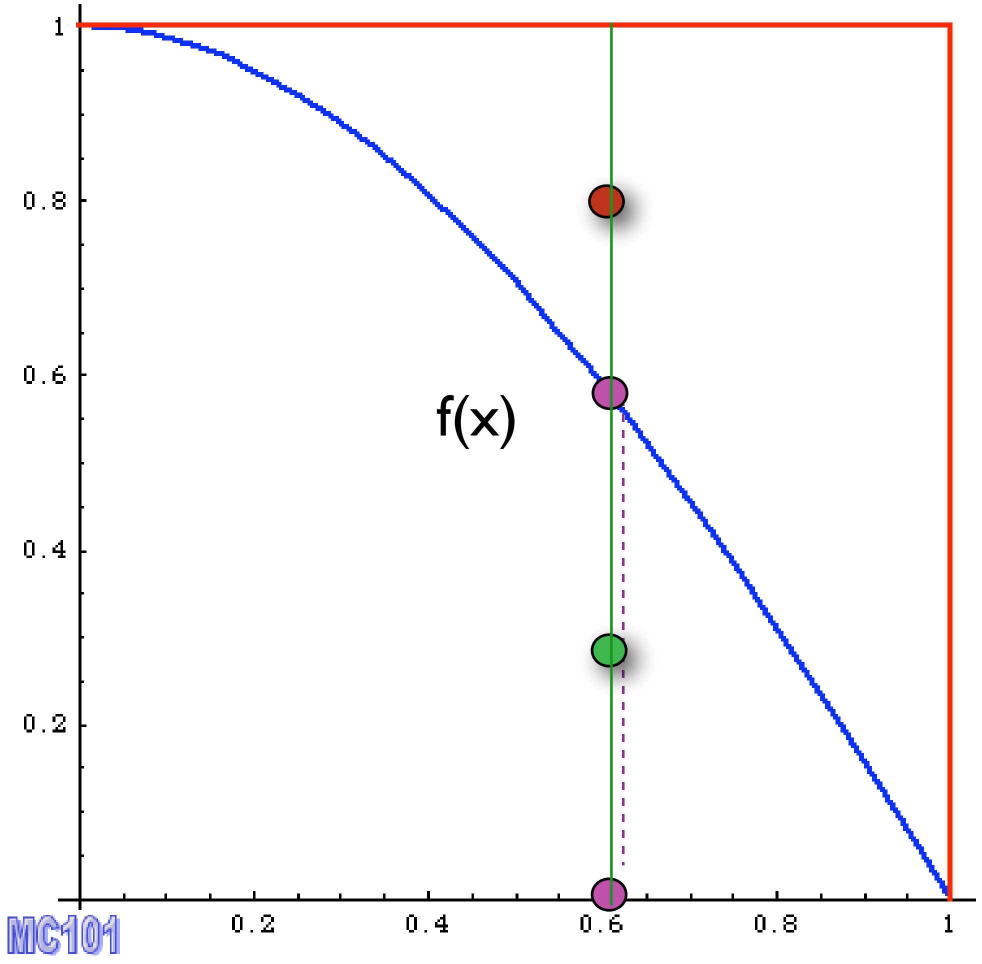
1. pick x
2. calculate f(x)
3. pick 0<y<fmax

Event generation



1. pick x
2. calculate f(x)
3. pick 0<y<fmax
4. Compare:
   if f(x)>y accept event,

Event generation



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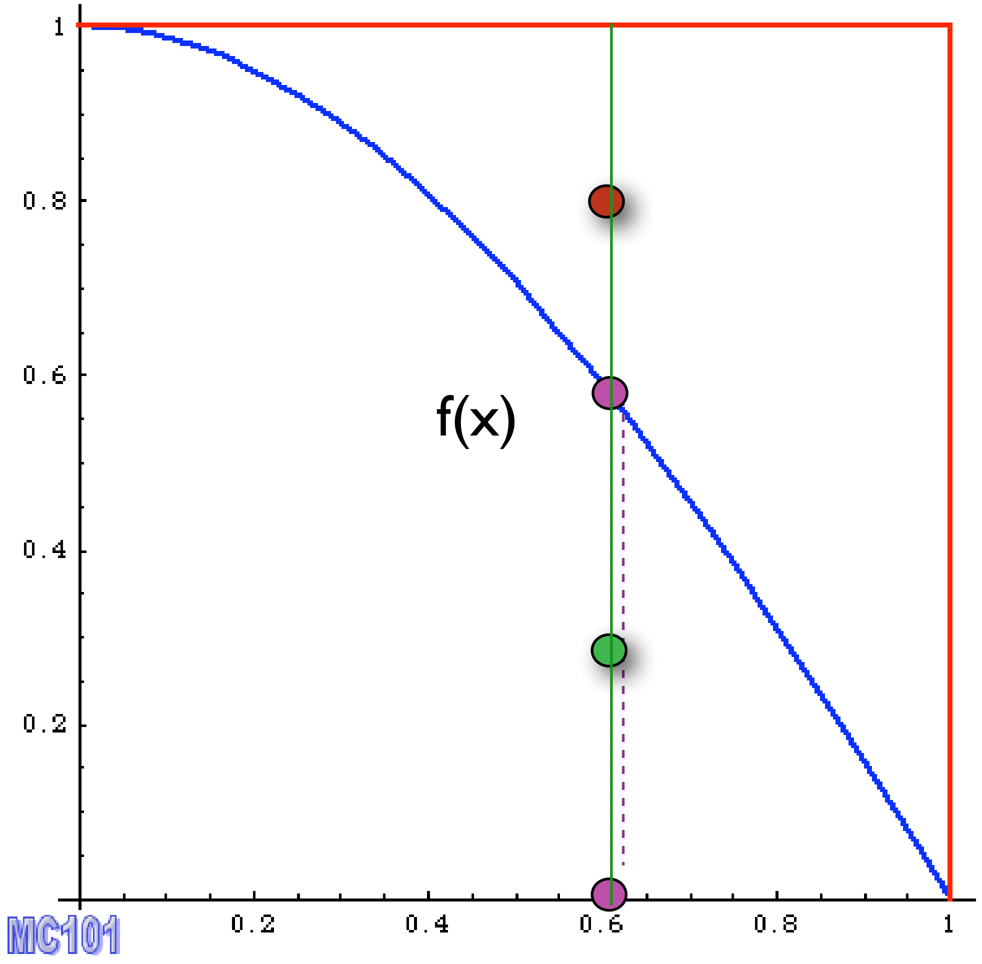


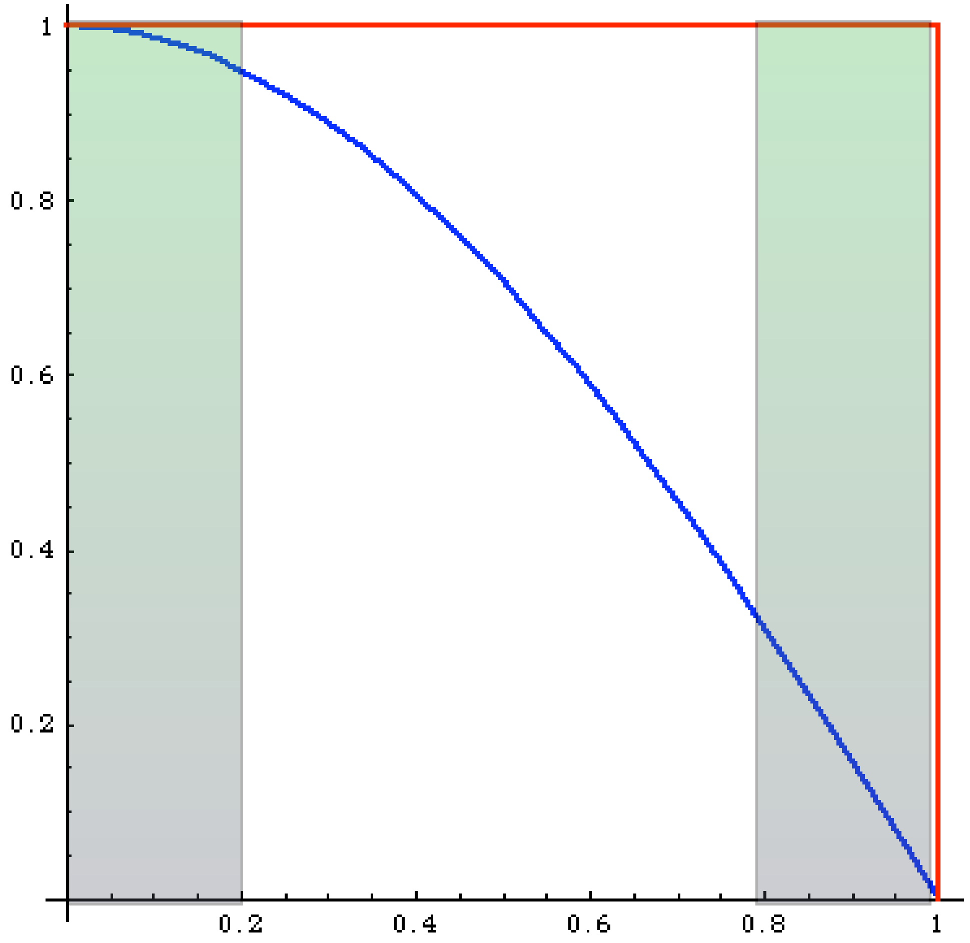
1. pick x
2. calculate f(x)
3. pick 0<y<fmax
4. Compare:

   if f(x)>y accept event,

   else reject it.

$$I= \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

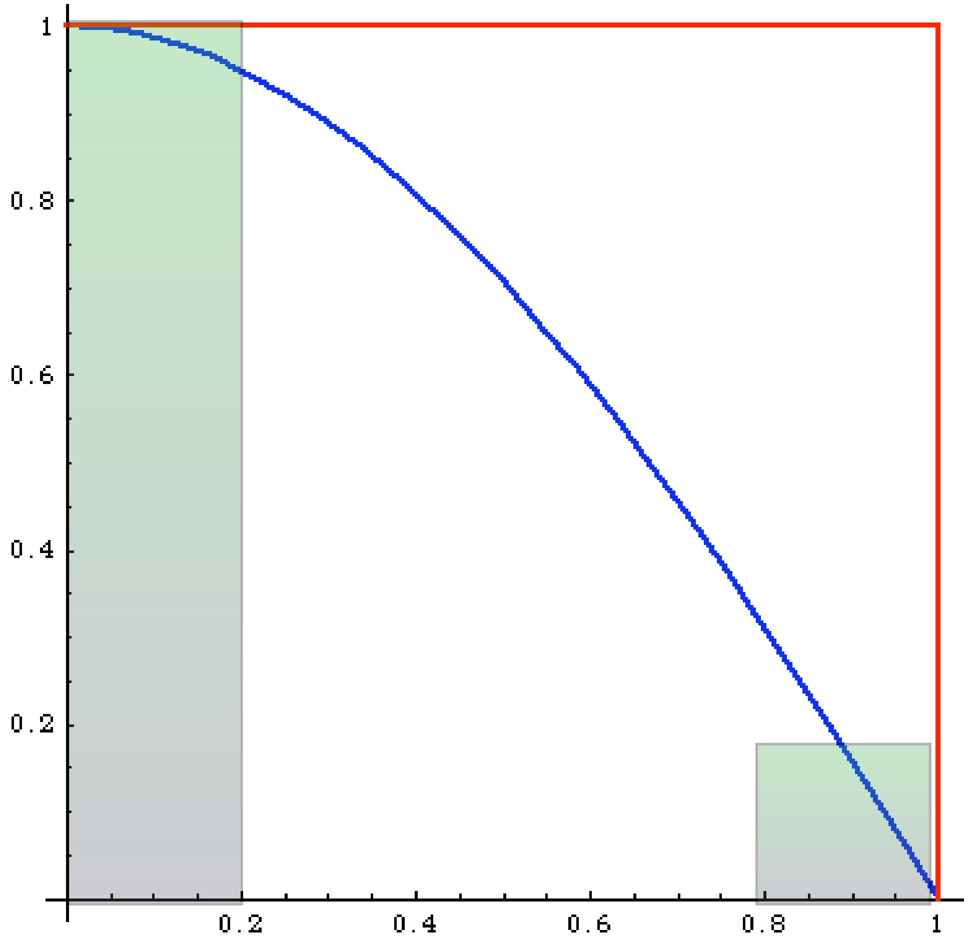


What's the difference between weighted and unweighted?

Weighted:

Same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



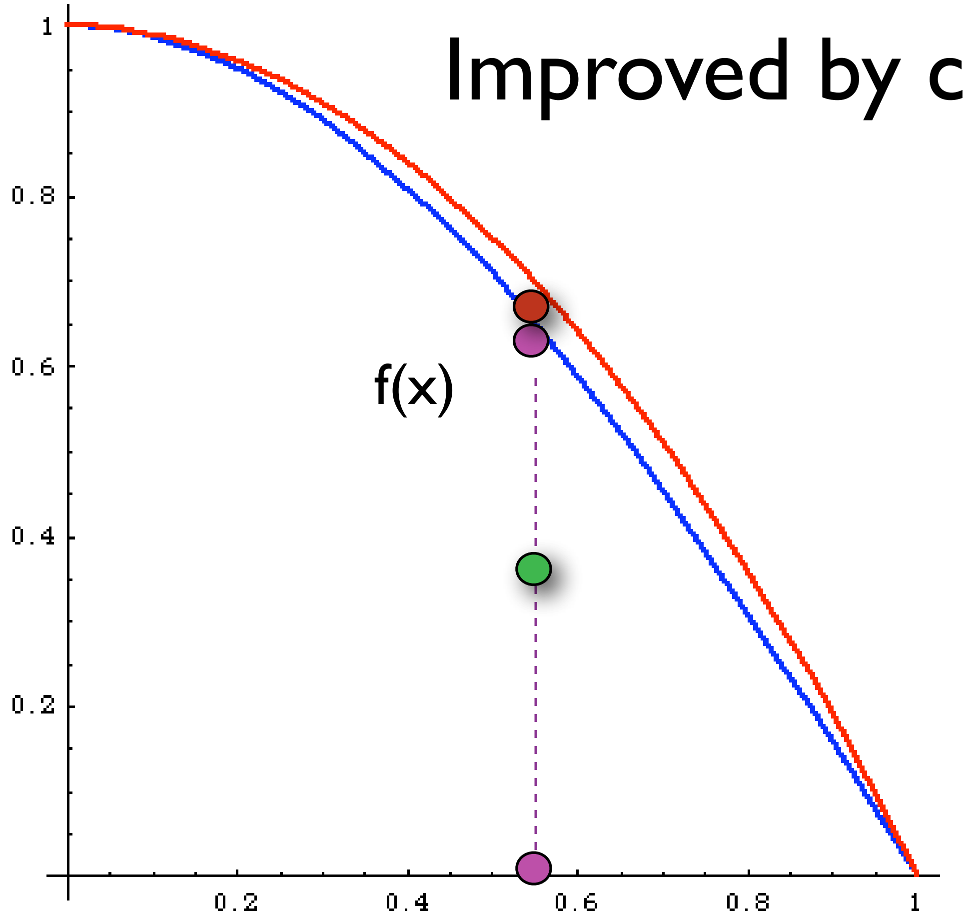
What's the difference between weighted and unweighted?

Unweighted:

events is proportional to the probability of areas of phase space:
events have all the same weight (”unweighted”)

Events distributed as in nature

Event generation



Improved by combining with importance sampling:

1. pick x distributed as p(x)
2. calculate f(x) and p(x)
3. pick 0<y<1
4. Compare:
   if f(x)>y p(x) accept event,

else reject it.

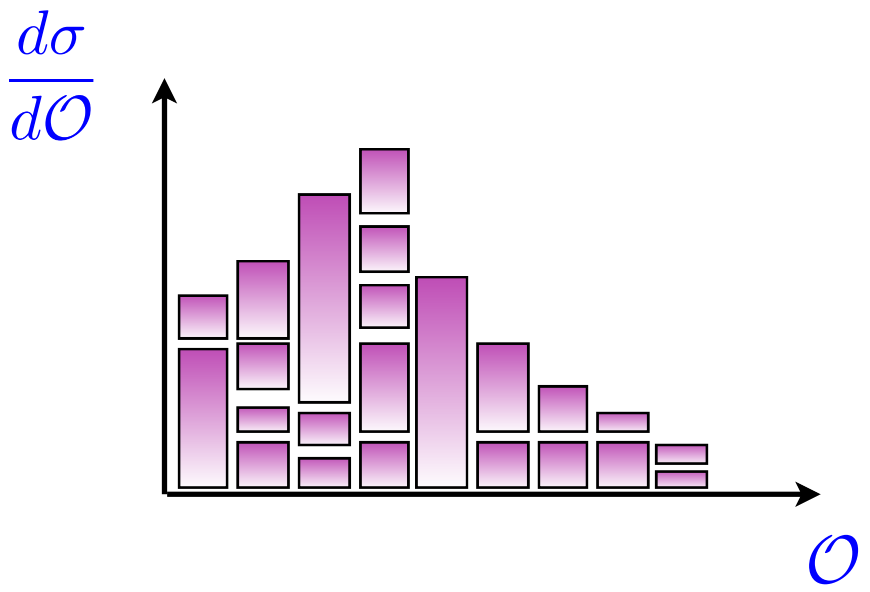
much better efficiency!!!

Event generation

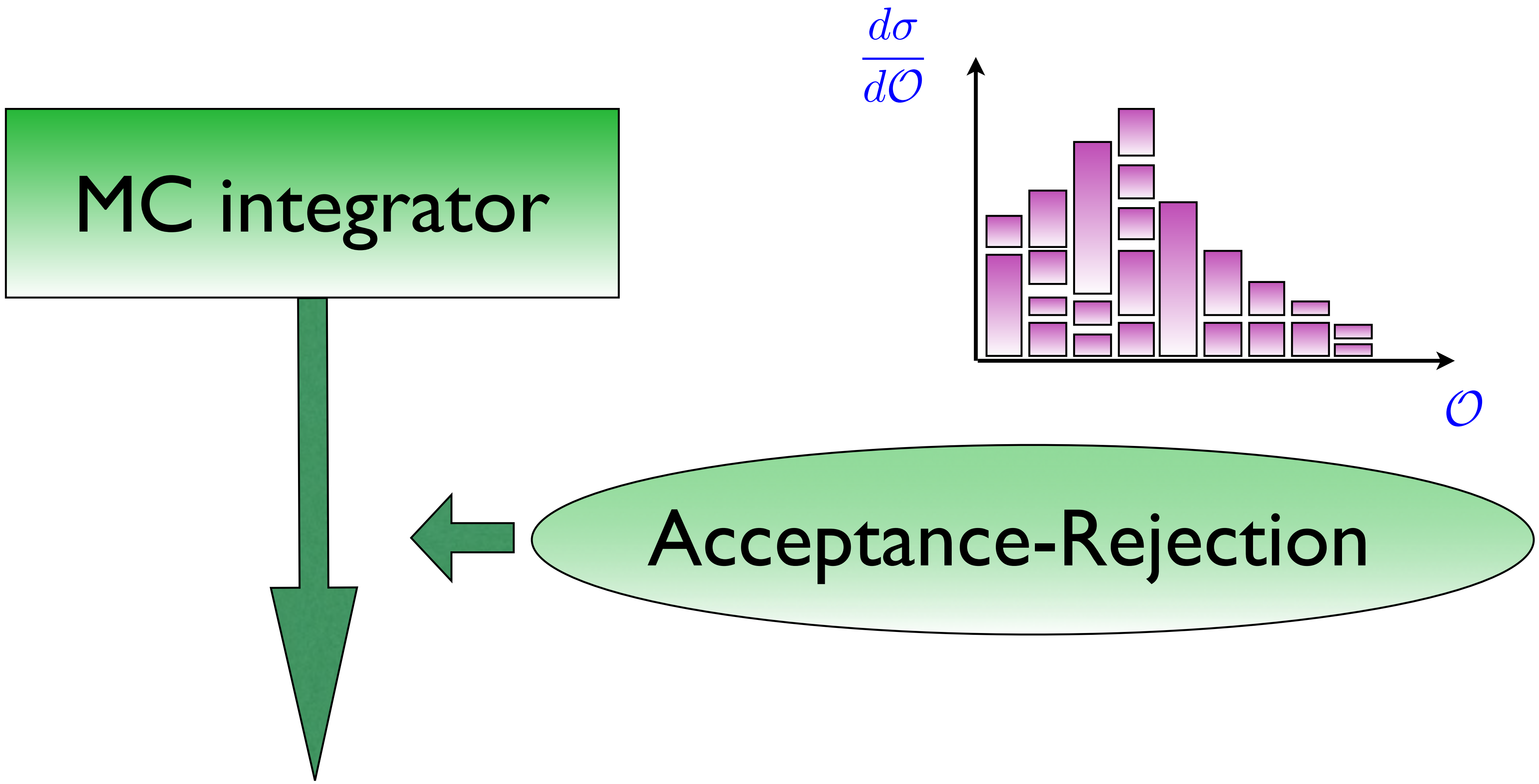
MC integrator

Event generation

MC integrator



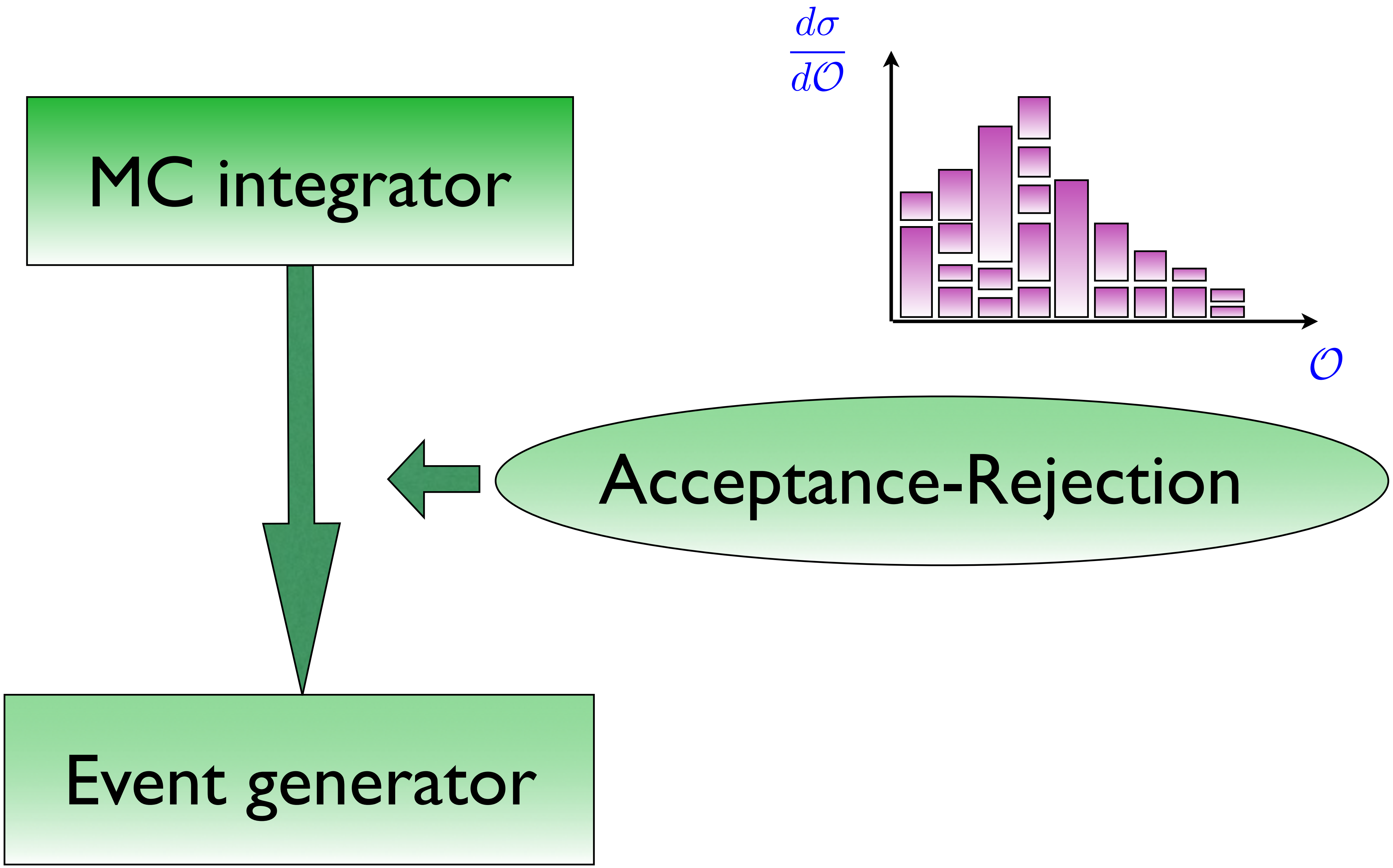
Event generation



MC integrator

Acceptance-Rejection

Event generation

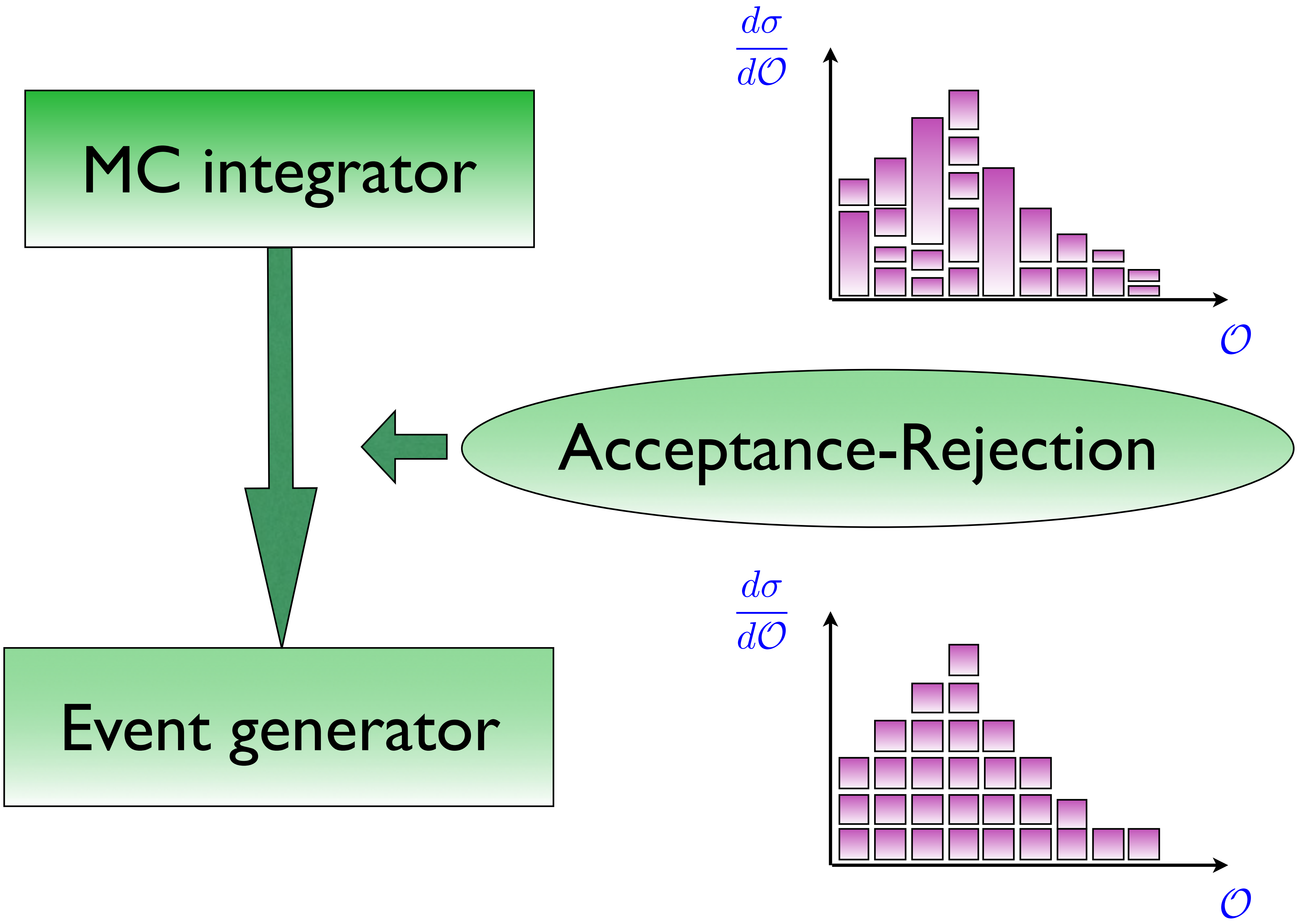


MC integrator

Acceptance-Rejection

Event generator

Event generation

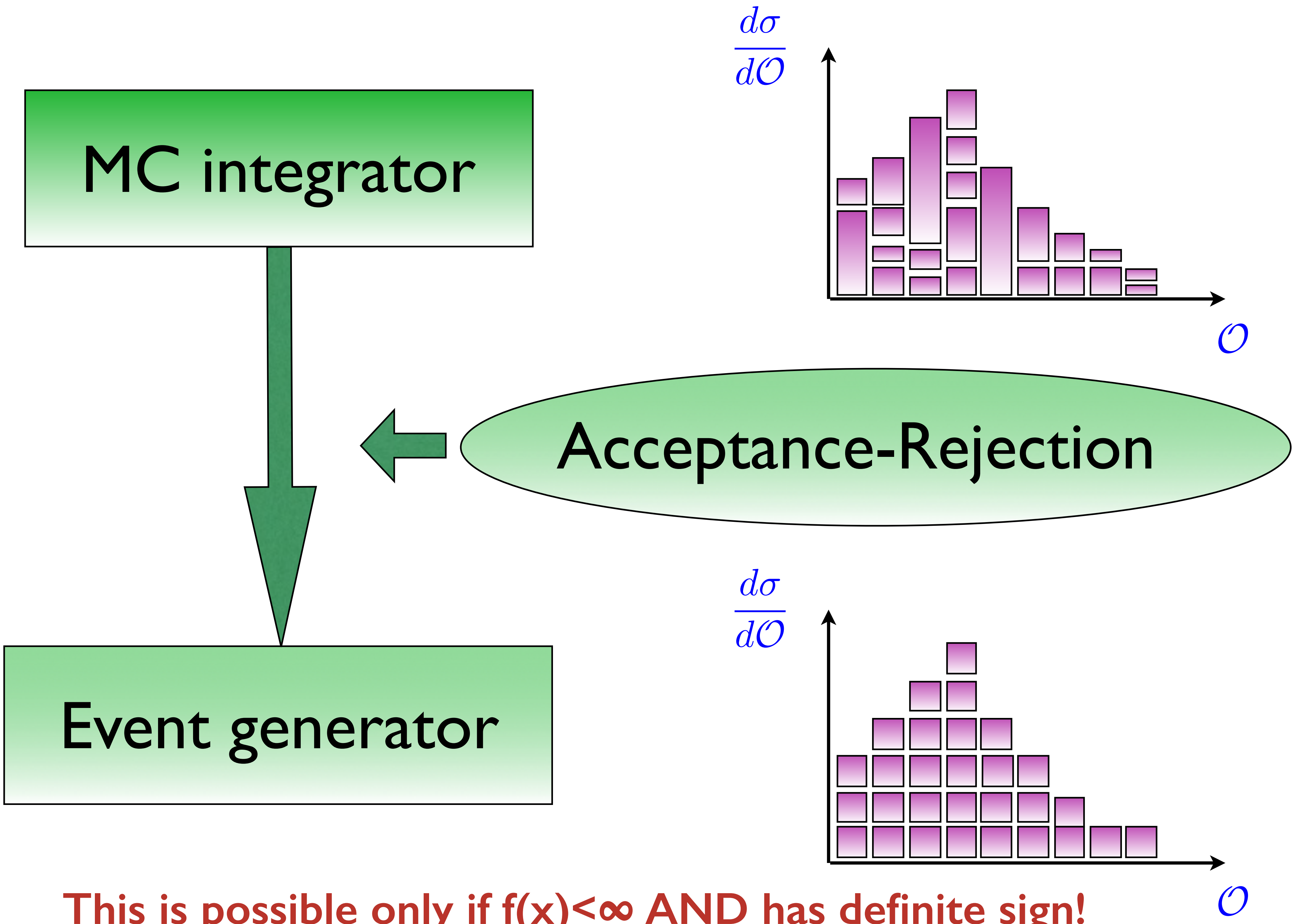


MC integrator

Acceptance-Rejection

Event generator

Event generation


MC integrator

Acceptance-Rejection

Event generator

☞ **This is possible only if f(x)<∞ AND has definite sign!**

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension
- Need to know the function
  - Impact on cut

Monte-Carlo Summary

Bad Point

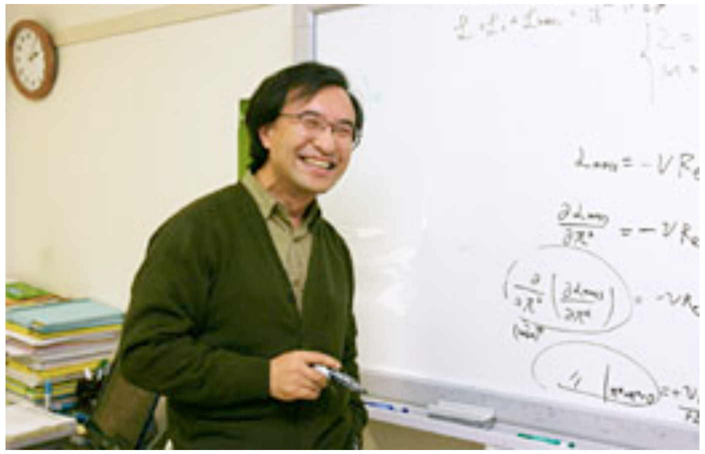
- Slow Convergence (especially in low number of Dimension
- Need to know the function
  - Impact on cut

Good Point

- Complex area of Integration
- Easy Error estimate
- quick estimation of the integral
- Possibility to have unweighted events

MadGraph

1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO



- Computing Matrix Element for a fixed Helicity and sum over the helicities.

- Suite of Routine, which allow to write the matrix element for any (SM) process

MadGraph

1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
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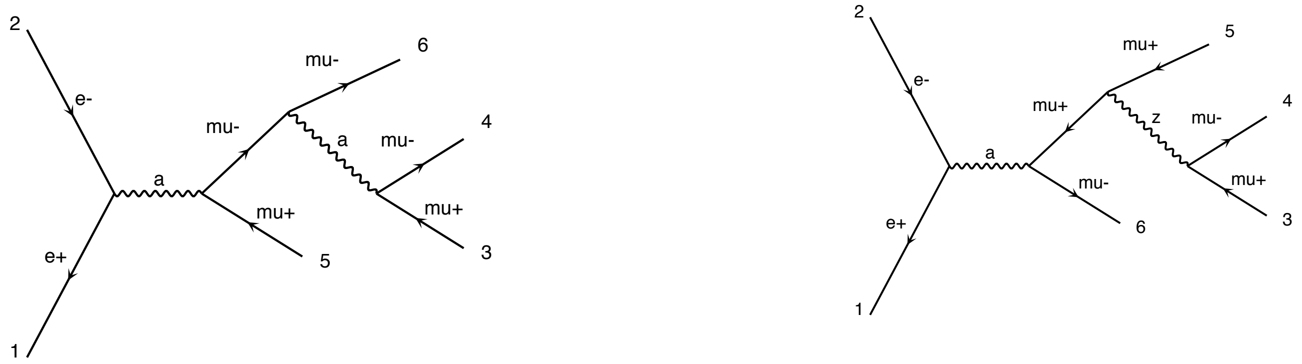
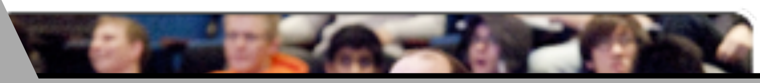
2014	MadGraph5_aMC@NLO

- Automate the creation of the diagram generation and the writing of the HELAS routine



MadGraph

1991 HELAS



MAD stands for Madison

1994

2002

am

2006

2011 MadGraph5

2014 MadGraph5_aMC@NLO

MadGraph

1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO

- Multi-Channel Method!
- Automatic phase-space Integration
- Generation of Events

- Support for the MSSM (SMADGRAPH)

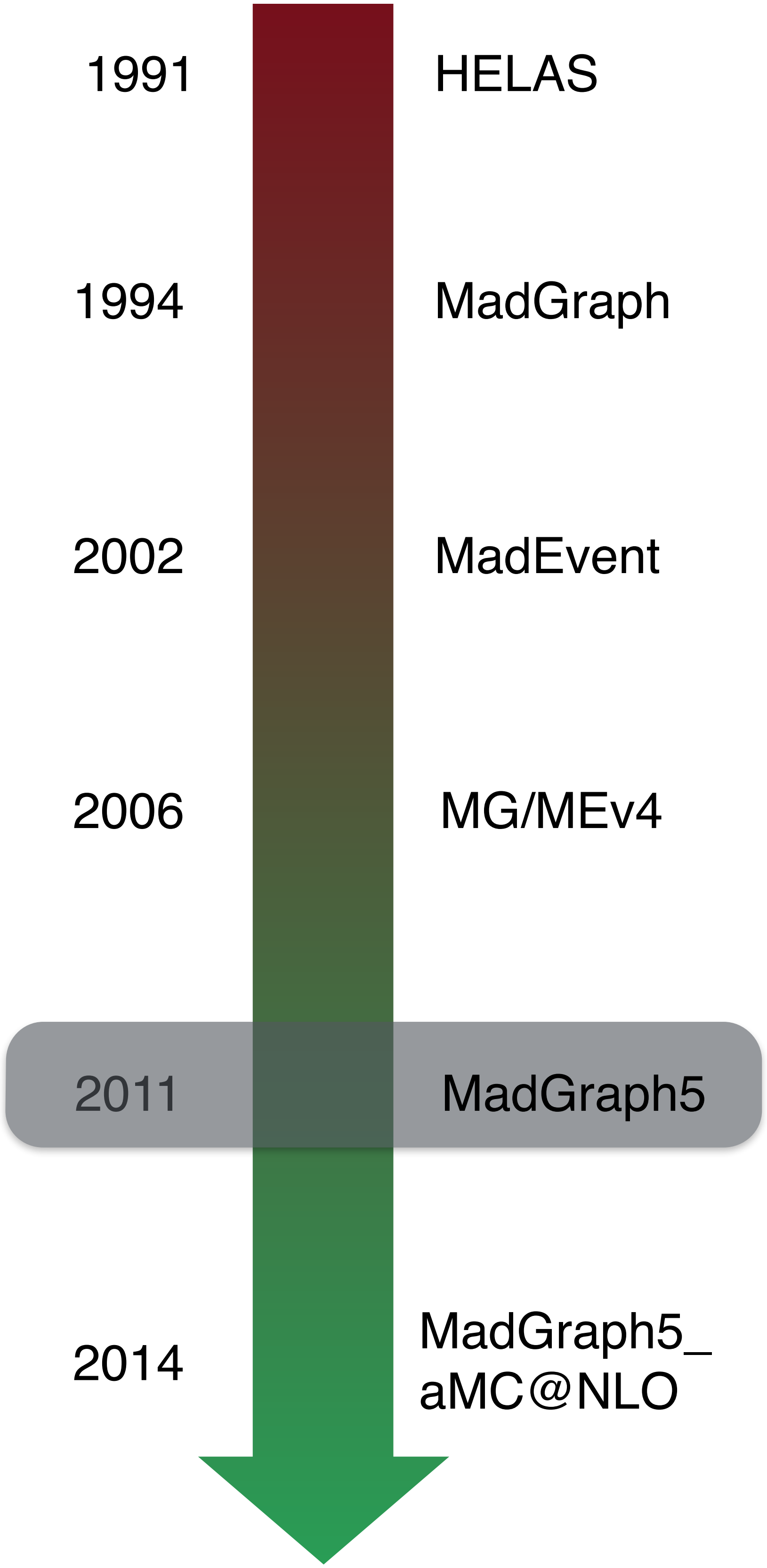
MadGraph

1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO

- Support for BSM
- Decay Chain
- Pass to a platform (MadOnia/MadWeight/…)
- Link to Pythia/PGS
- Matching/Merging

- Official/Main SM generator for CMS

MadGraph



1991 HELAS

1994 MadGraph

2002 MadEvent

2006 MG/MEv4

2011 MadGraph5

2014 MadGraph5_ aMC@NLO

- Full restart of the MadGraph part in Python
- Fully Automatic BSM
- Various Output Format
- Huge Improvement

MadGraph

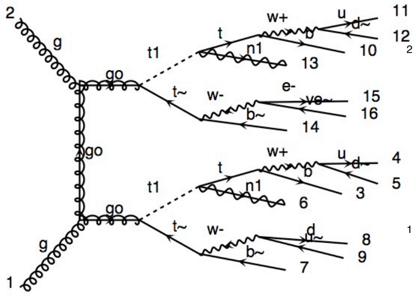
1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO

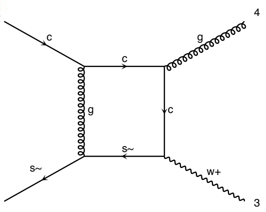
- Fully Automatic computation at
  - NLO* (cross-section)
  - NLO* matched to PS

*NLO= NLO in QCD

Type of generation

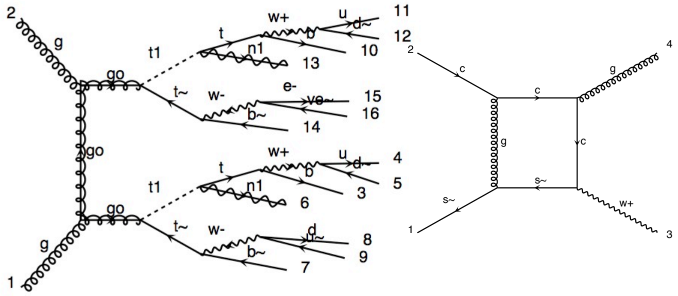
	Tree (B)SM	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	Loop Induced (B)SM
Fix Order	✓	✓	✓	✓	✓

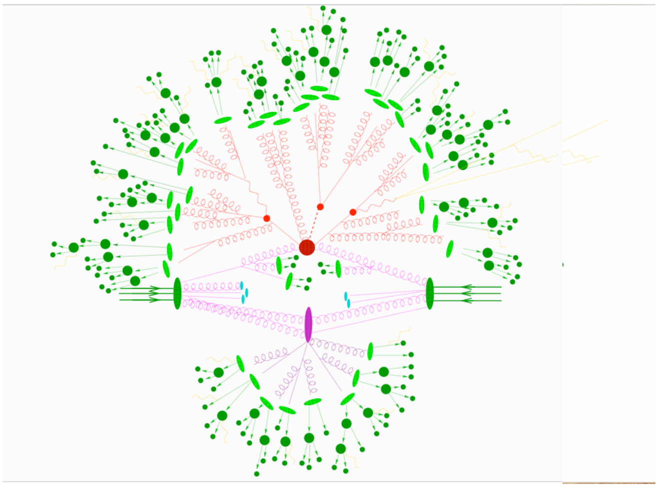




Type of generation

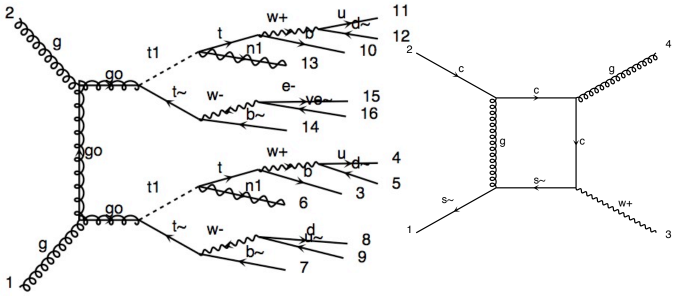
	Tree (B)SM	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	Loop Induced (B)SM
Fix Order	✓	✓	✓	✓	✓
+Parton Shower	✓	✓	✓	✗	✓

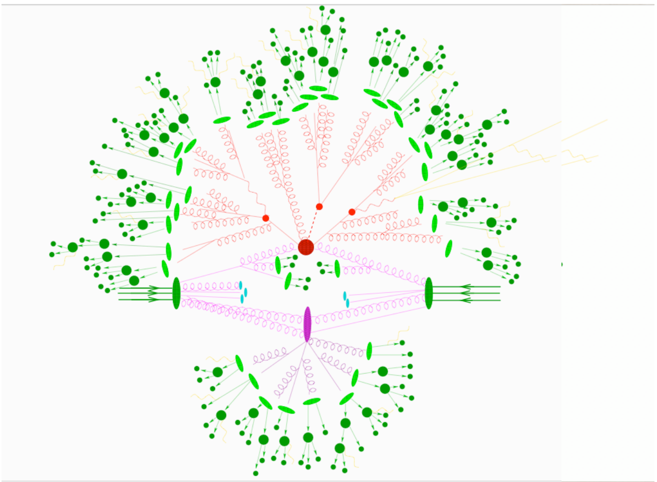


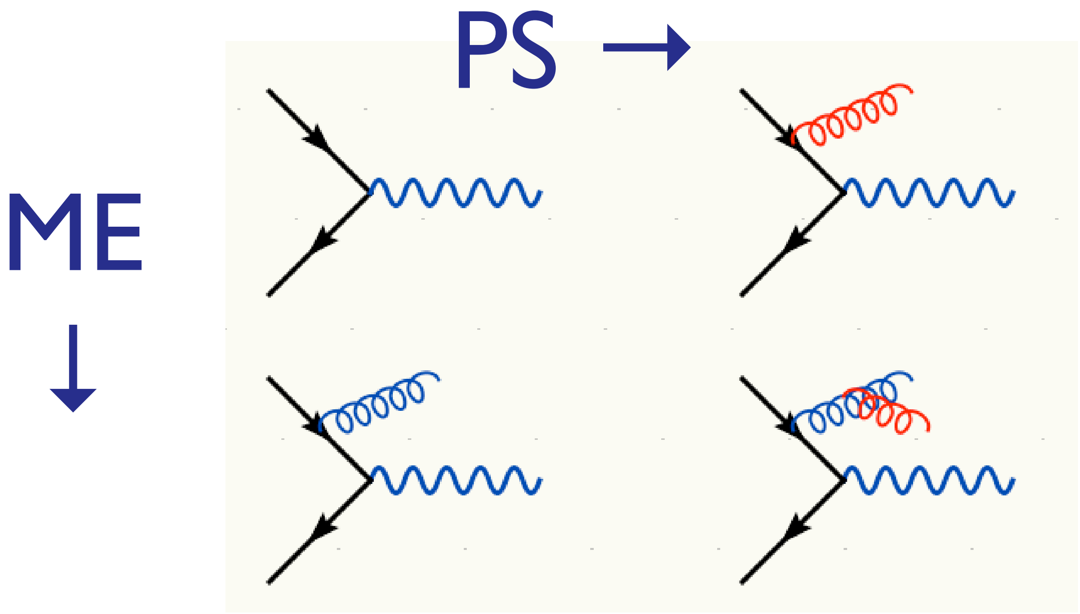


Type of generation

	Tree (B)SM	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	Loop Induced (B)SM
Fix Order	✓	✓	✓	✓	✓
+Parton Shower	✓	✓	✓	✗	✓
Merged Sample	✓	✓	?	✗	✓







List of package

- SysCalc (computation of systematics)
- MadWidth (computation of width in NWA)
- MadSpin (decay with full spin-correlation)
- Re-Weighting (change of the weight of an event)
- Shower / Detector Interface
- MadWeight (Matrix-Element Method)
- Interference
- MadAnalysis5
- Tau Decay
- MadDM
- GPU

What to remember

- Analytical computation can be slower than numerical method
- Any BSM model are supported (at LO)
- Phase Space integration are slow
  - need knowledge of the function
  - cuts can be problematic
- Event generation are from free.
- All this are automated in MG5_aMC@NLO
- Important to know the physical hypothesis