Combining parton showers with fixed-order calculations

Andreas Papaefstathiou

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content

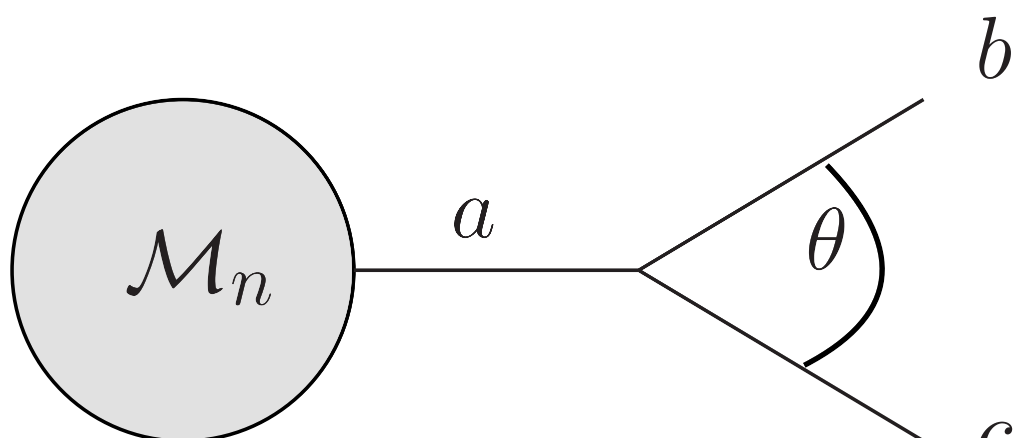
- **(0) brief recap of parton shower.**
- **(I) matching the parton shower to fixed-order calculations.**
- **(II) multi-jet merging at LO and NLO.**
- **(III) future directions: NNLO matching.**

(0) brief recap of elements of the PS formalism.

parton shower recap

- QCD scattering cross sections factorise, e.g.:

$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \frac{\mathrm{d}t}{t} \frac{\alpha_S}{2\pi} \hat{P}_{ba}(z)$$



$$z = E_b / E_a \qquad t \equiv p_a^2$$

- this factorisation is suitable for numerical implementation.

parton shower recap

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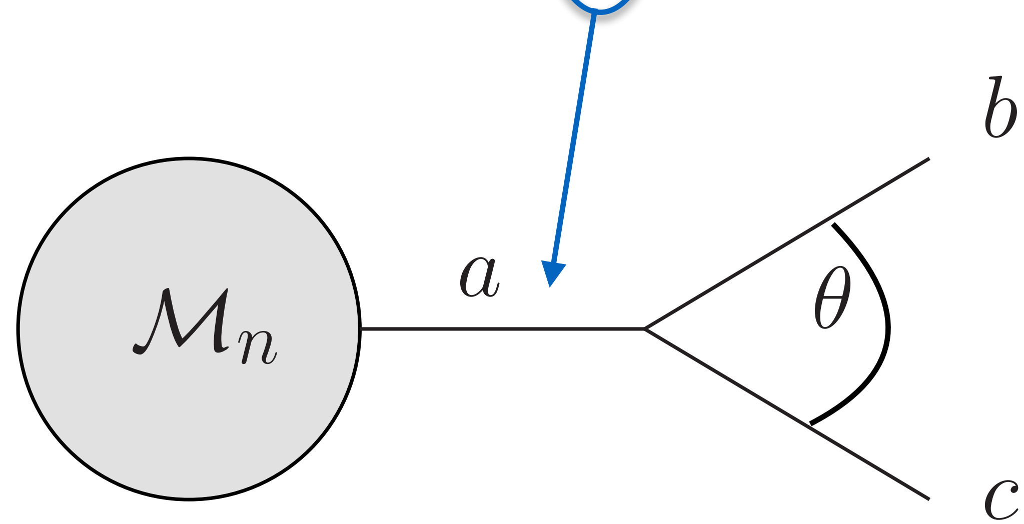

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parton shower recap

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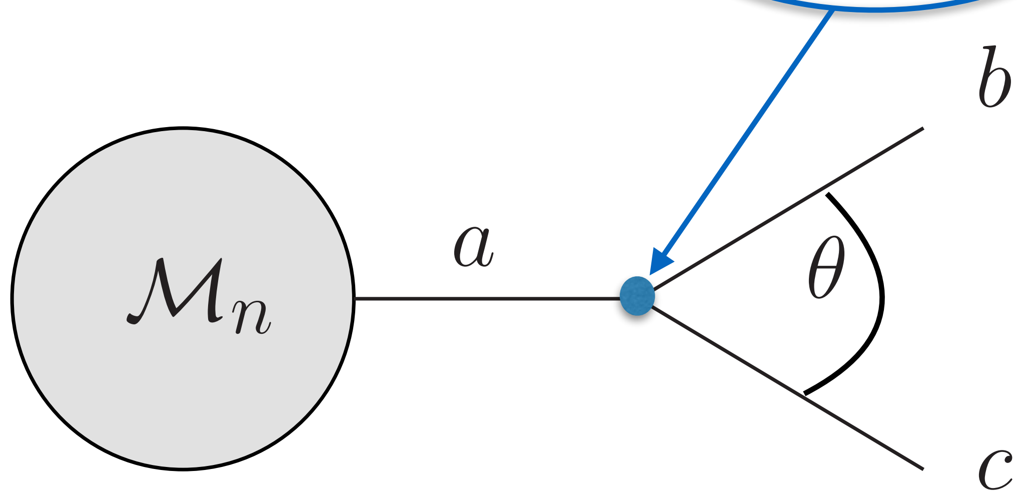

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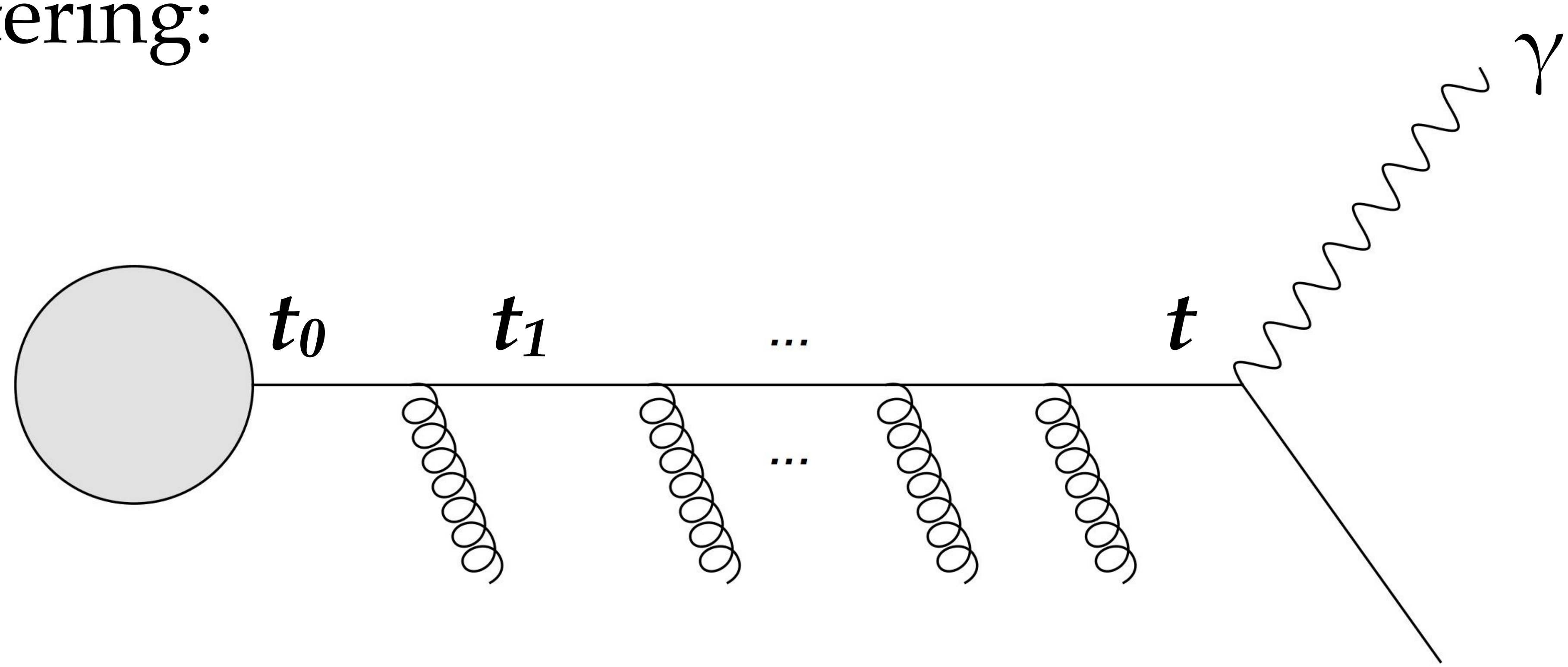
- this factorisation is suitable for numerical implementation.

parton shower recap

- e.g. initial-state branching in deep inelastic lepton scattering:



- result: parton shower (**PS** henceforth) produces "exclusive" final-states:

- i.e. tells us how the inclusive cross section splits into exclusive pieces.

parton shower recap

- exclusive cross sections are defined through **no-emission probabilities**,

- also known as "Sudakov form factors":

$$\Delta_i(t, t_0) = \exp\left[-\sum_j \int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \int \mathrm{d}z \frac{\alpha_S}{2\pi} \hat{P}_{ij}(z)\right]$$

- = the probability of evolution from $\boldsymbol{t_0}$ to $\boldsymbol{t}$ without branching.

PS versus fixed order

- **PS** give an approximate multi-parton cross section which:
  + is always finite,
  + can produce any number of emissions,
  - is only valid in certain regions (soft/collinear).

- **what if we wish to describe a process with many hard jets?**

- Fixed-order calculations (henceforth **FO**):
  + contain all terms at one order of α_S,
  + valid also for high relative p_T,
  - only feasible for few emissions.

the best of both worlds through:

- **Matrix element corrections:**
  - ✦ oldest scheme: correct according to full real emission calculations,
  - ✦ hard to iterate.
- **FO-PS Matching:**
  - ✦ combine an NLO calculation with the parton shower consistently.
  - ✦ hard to iterate.
- **FO-PS Merging:**
  - ✦ divide phase space: use **FO** for hard jets, **PS** for soft jets.
  - ✦ easy to iterate.

Matrix element corrections

- **basic idea: modify** the **PS** probabilities so that they add up to the full **real emission** matrix element:

  - ✳ choose a branching according to the **PS** probability, $P_{\mathrm{PS},i}$.

  - ✳ but accept according to a "corrective" probability, $P_{\mathrm{MECorr},i}$,

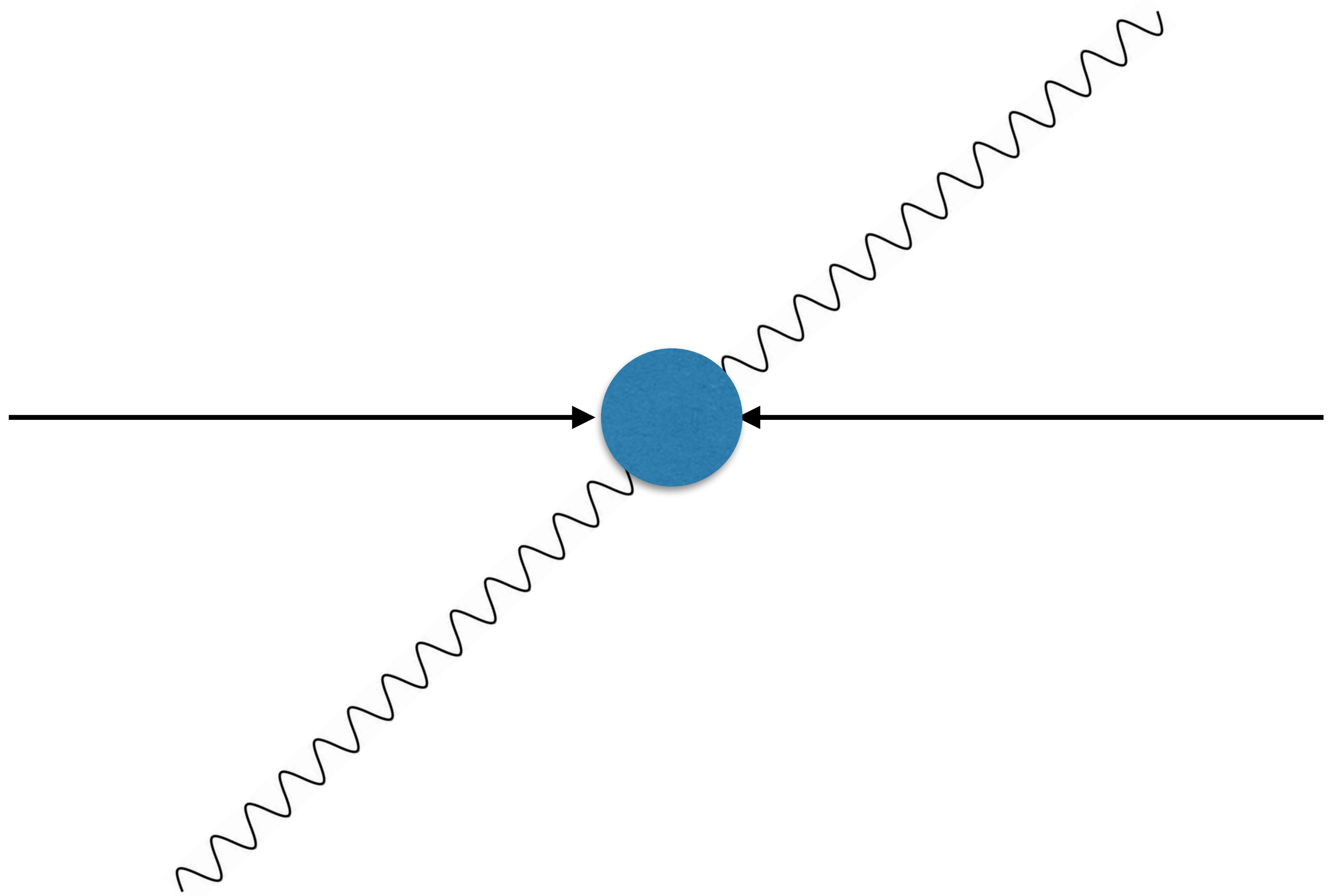
  - ✳ such that: $$\sum P_{\mathrm{PS},i} \times P_{\mathrm{MECorr},i} = P_{\mathrm{full-ME}}$$

+ natural within **PS** formalism & efficient,
- technically fiddly,
- difficult to iterate.

(I) matching the parton shower to fixed-order calculations.

combining **PS & FO** is *essential*

- consider $\Delta\phi(W^+W^-)$ in: $pp \to W^+ + W^-$



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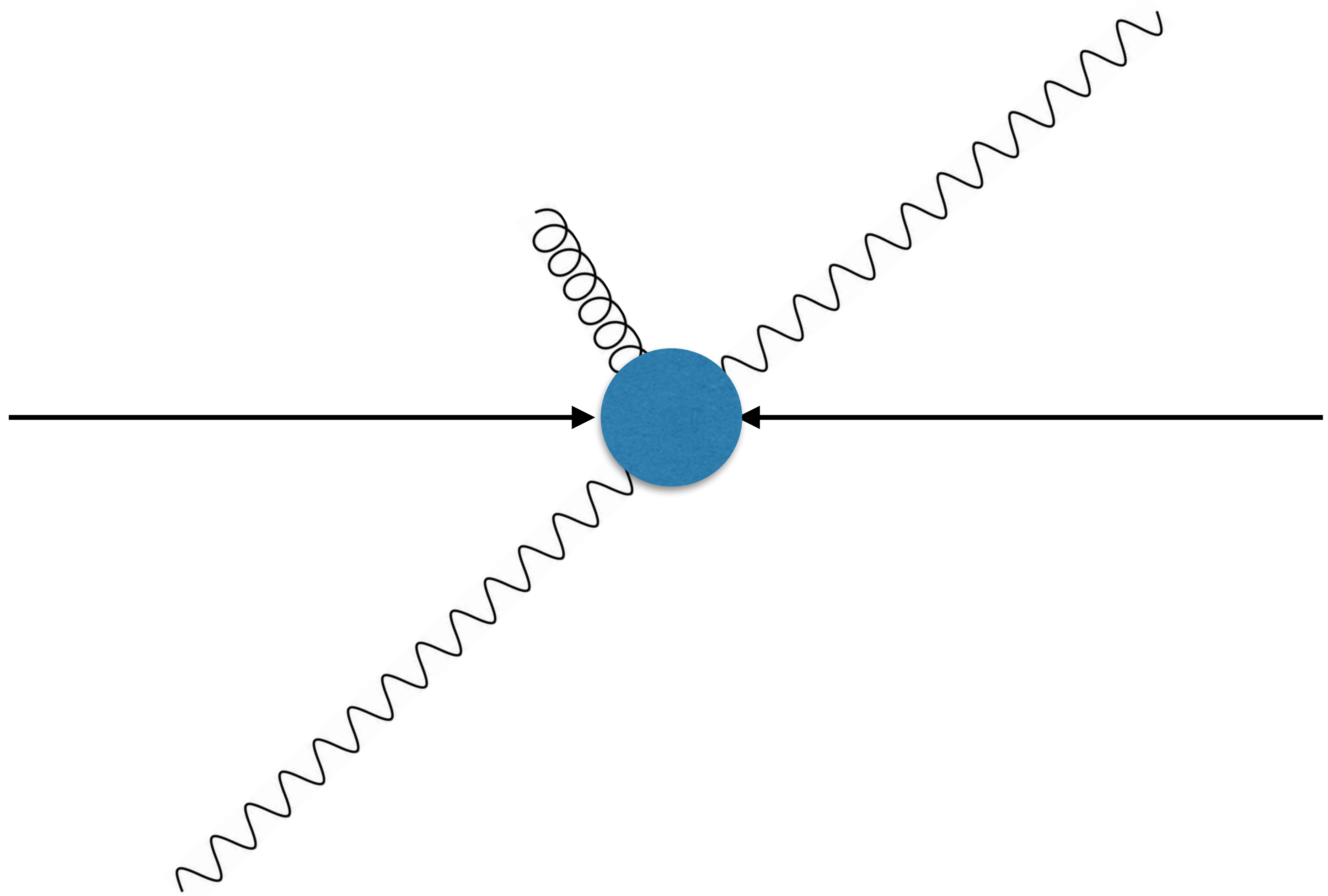

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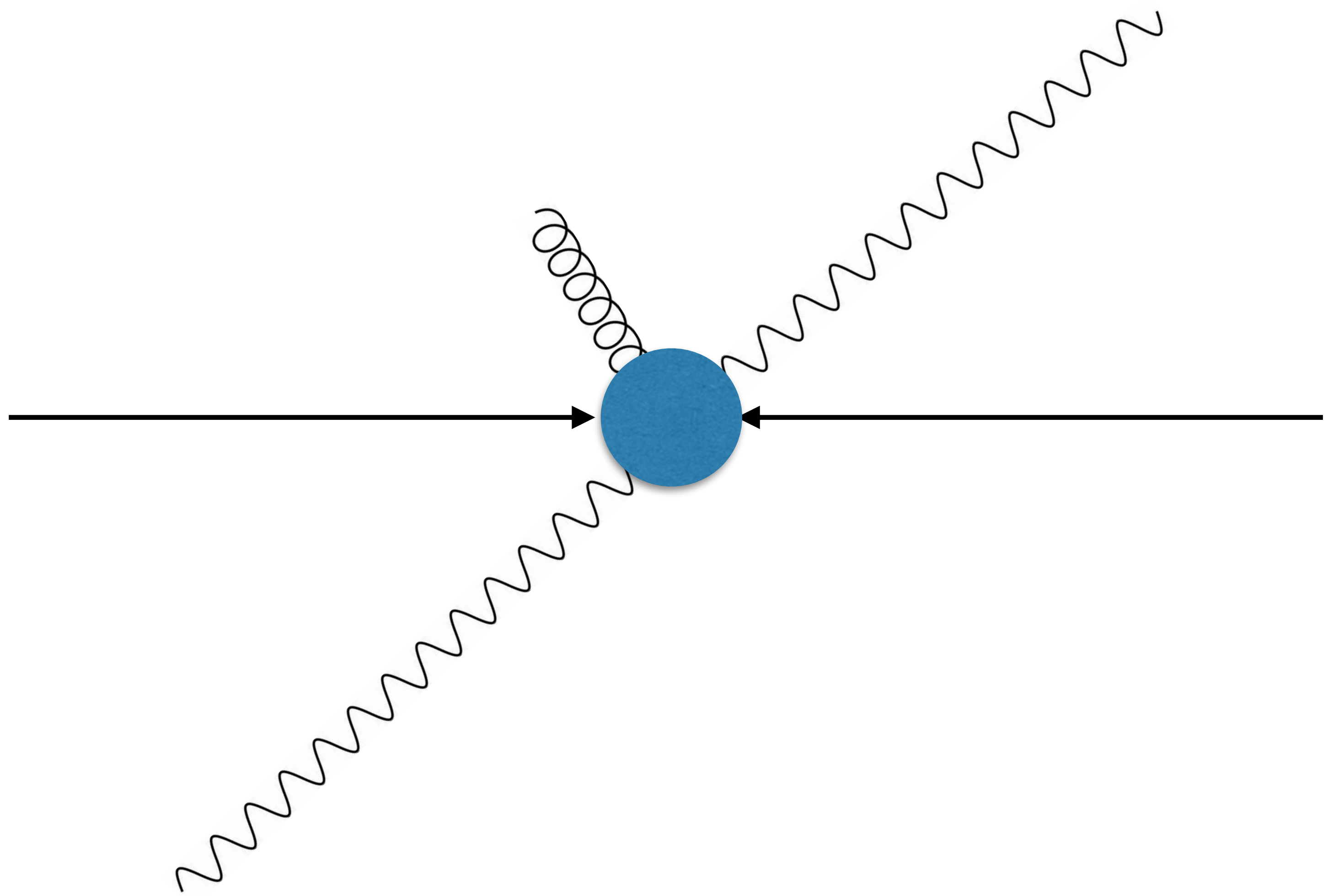

at $\boldsymbol{\Delta\phi(W^+W^-)} \sim \pi$, the **FO (NLO)** diverges logarithmically.

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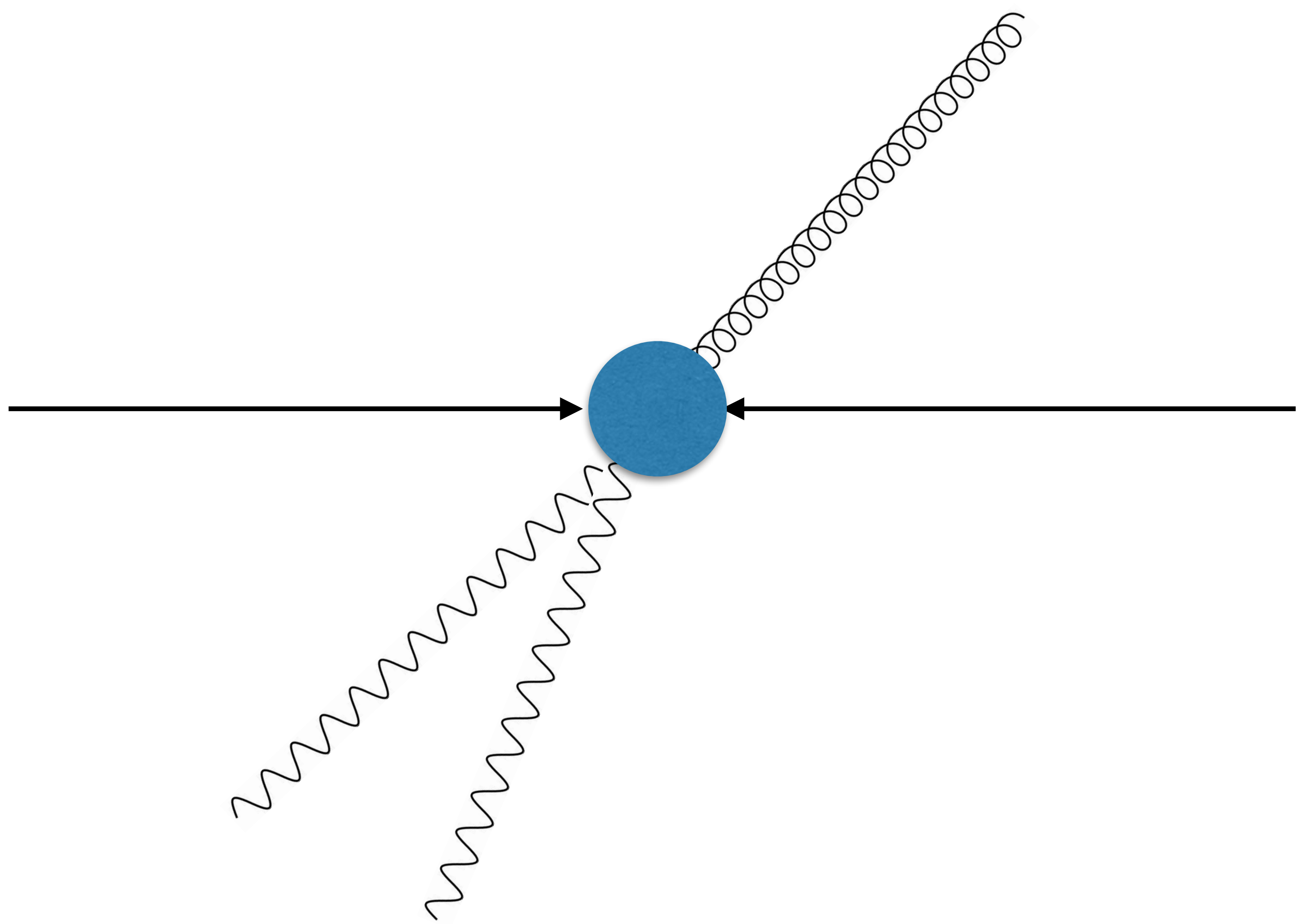
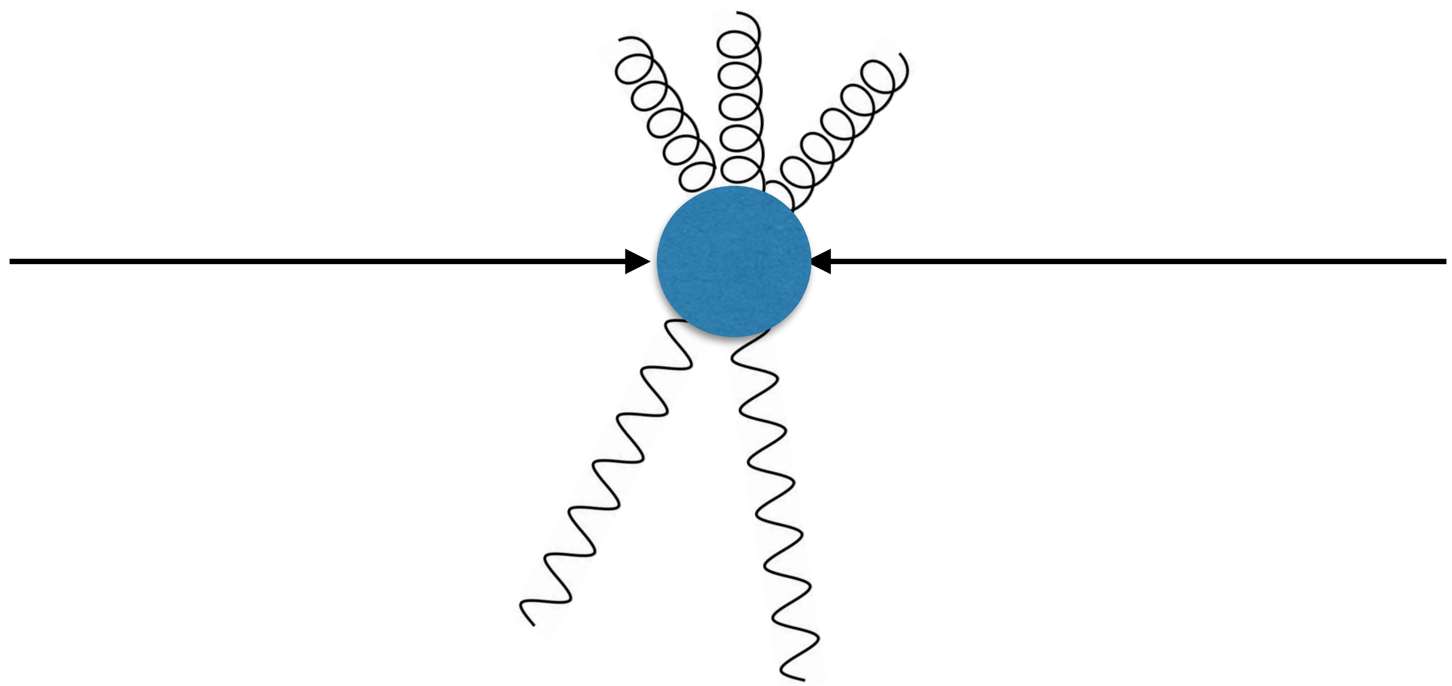


at $\boldsymbol{\Delta\phi(W^+W^-)}$ ~ 0 NLO contributes: recoil against hard jet.

combining **PS & FO** is *essential*

- consider $\Delta\phi(W^+W^-)$ in: $pp \to W^+ + W^-$



but also, at $\boldsymbol{\Delta\phi(W^+W^-)} \sim 0$, **PS** contributes.

combining **PS** & **FO** is *essential*

- some observables: require both FO & PS to be described in whole of phase space:
  - $\boldsymbol{\Delta\phi(W^+W^-) \sim \pi}$, NLO prediction diverges logarithmically.
  - $\boldsymbol{\Delta\phi(W^+W^-) \sim 0}$, receives contributions from both NLO and **PS**.

➡ desirable: **FO** + **PS**.

➡ combine PS & FO to get:

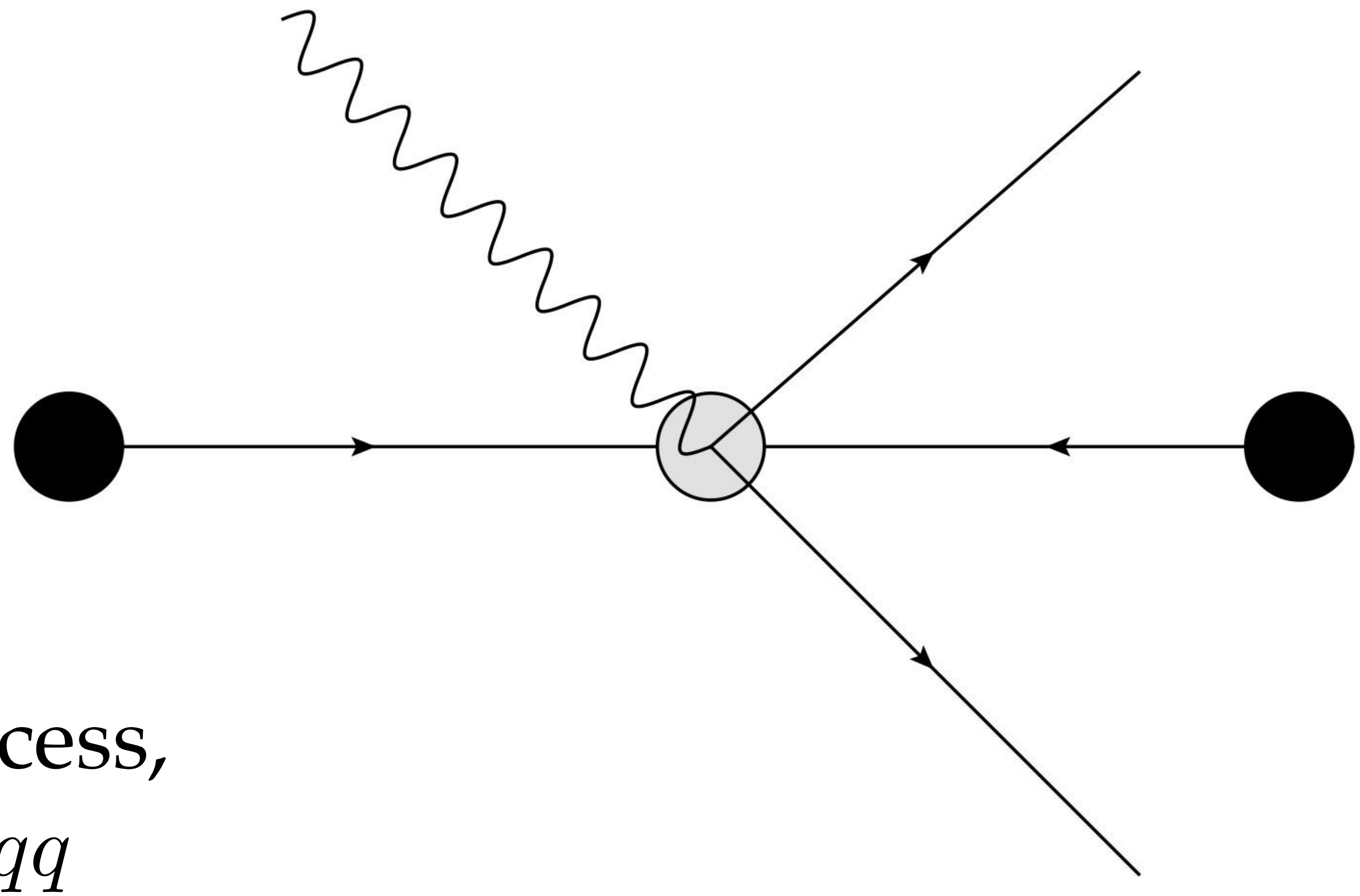
PS Monte Carlo	**FO**
fully exclusive, hadronic final states	total rates
multiple soft/ collinear emissions	some hard/ wide angle emissions

& smooth matching between soft/hard regions

for NLO + PS:

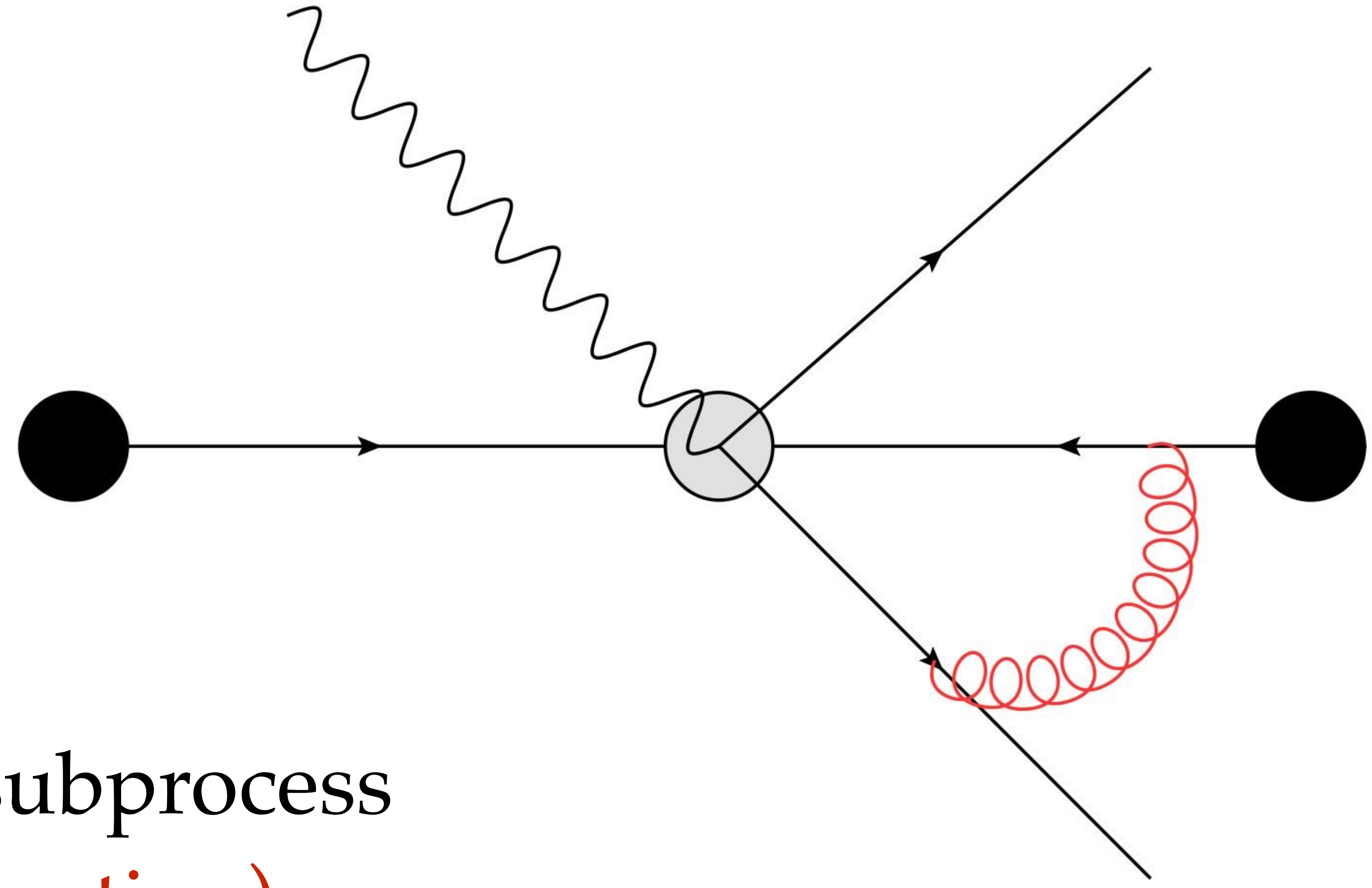
- MC@NLO and POWHEG methods. [Frixione, Webber, hep-ph/0204244] [Nason, hep-ph/0409146]
- [see also krkNLO: [Jadach, Płaczek, Sapeta, Siódmok, Skrzypek, 1503.06849]]
- they construct an MC that works *like* ‘LO+PS’ but knows how to treat hard radiation,
- also remove double-counting between the PS and FO.

matching the PS with FO calculations: double counting



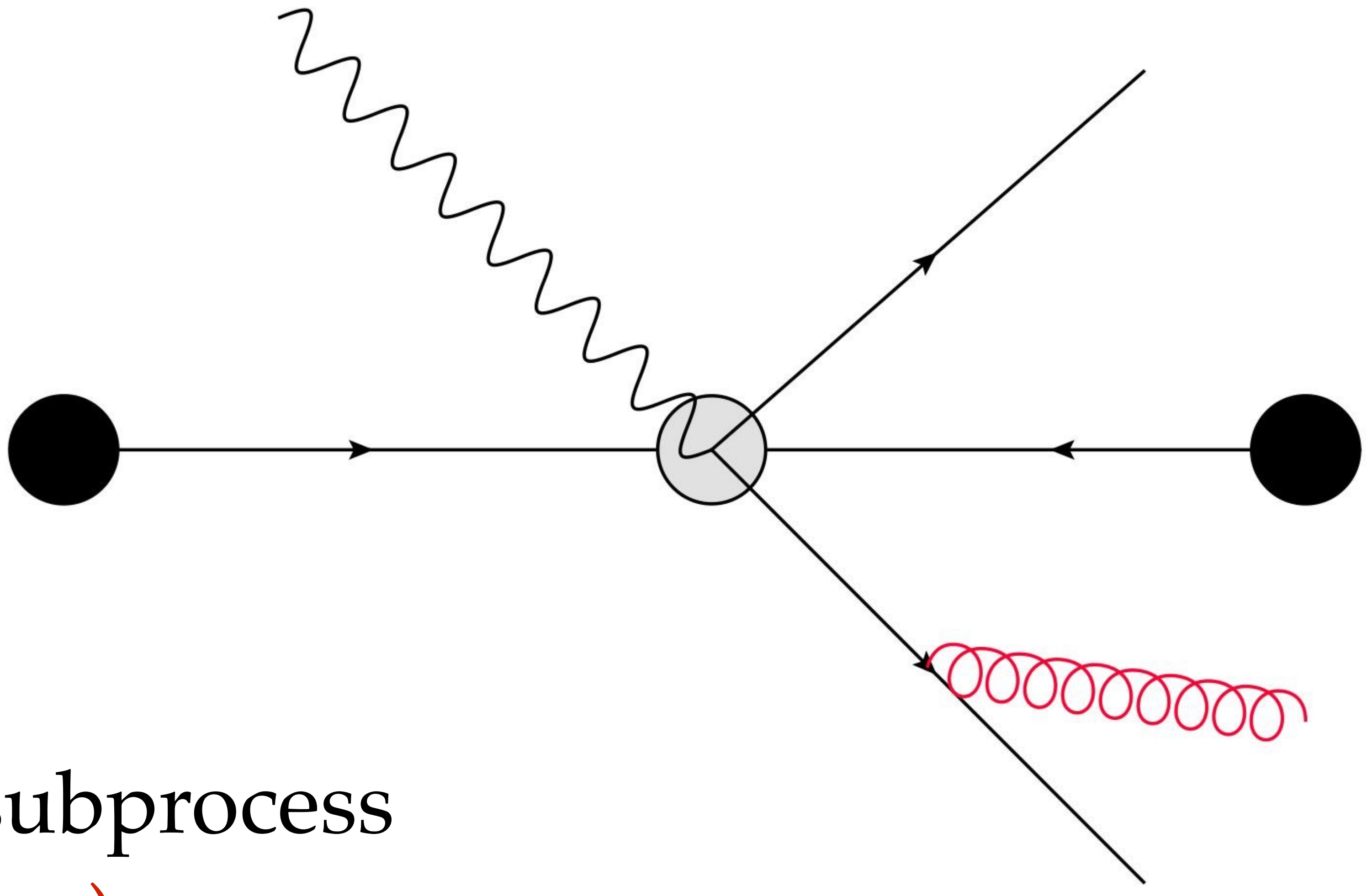
hard subprocess,
e.g. $qq \to Zqq$

matching the PS with FO calculations: double counting



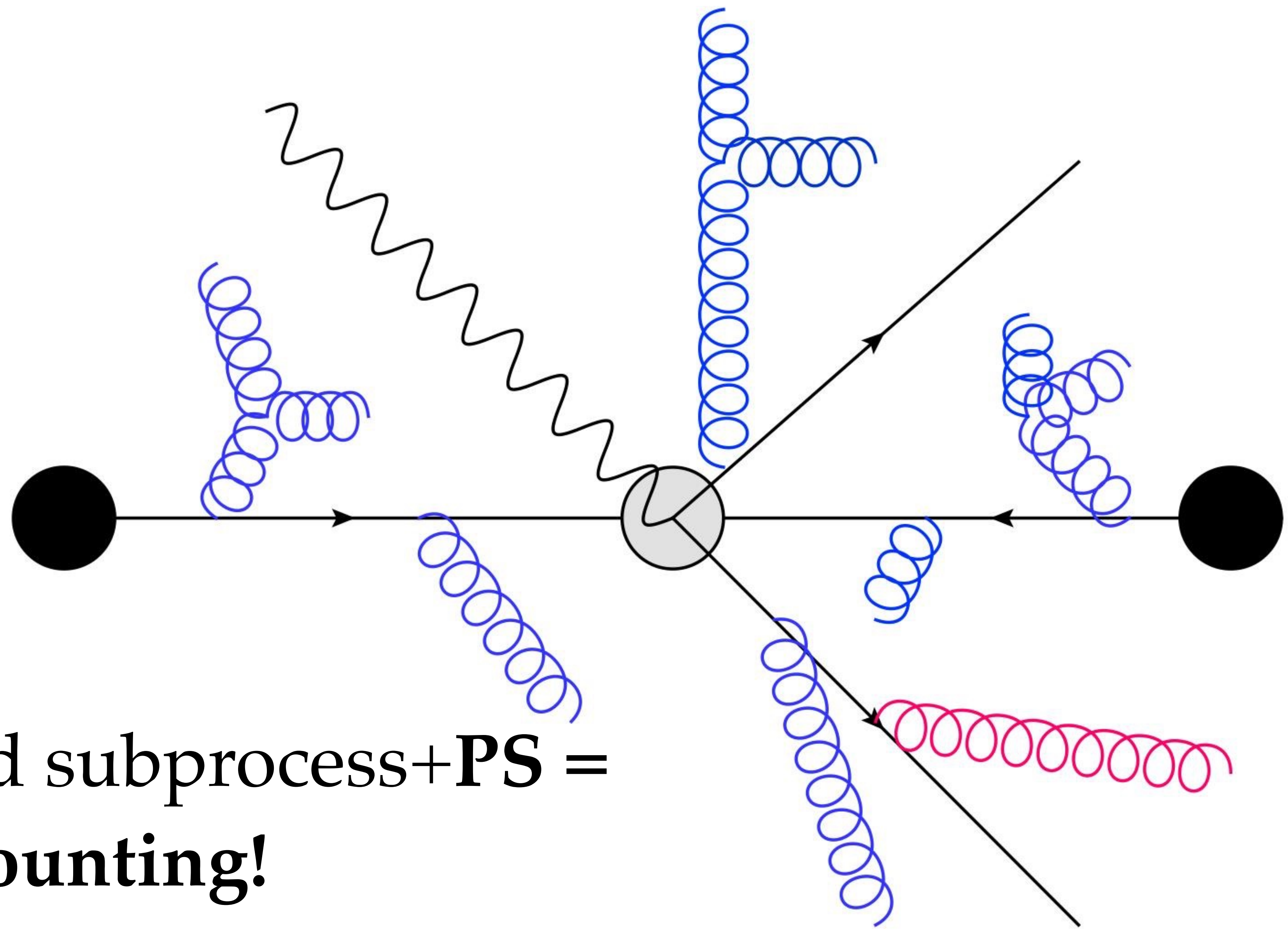
NLO hard subprocess
(virtual correction)

matching the PS with FO calculations: double counting



NLO hard subprocess
(real emission)

matching the PS with FO calculations: double counting



NLO hard subprocess+**PS =**
double counting!

MC@NLO for "toy model"

- the MC@NLO method removes the double counting by **modifying the NLO subtraction.** [Frixione, Webber, hep-ph/0204244]

- **MC@NLO subtraction,** emission of "photons" of energy ***x*:**

$$\langle O \rangle_{\mathrm{mod}} = \int_0^1 \mathrm{d}x \left[I_{\mathrm{MC}}(O, x_M(x)) \frac{a[R(x) - BQ(x)]}{x} + I_{\mathrm{MC}}(O,1) \left(B + aV - \frac{a[BQ(x) - 1]}{x} \right) \right]$$

add & subtract

- I_{MC} is the effect of the **PS** on a given "seed" configuration.

- the function $Q(x)/x$ is the **splitting function**!

[a is equivalent to QCD α_S]

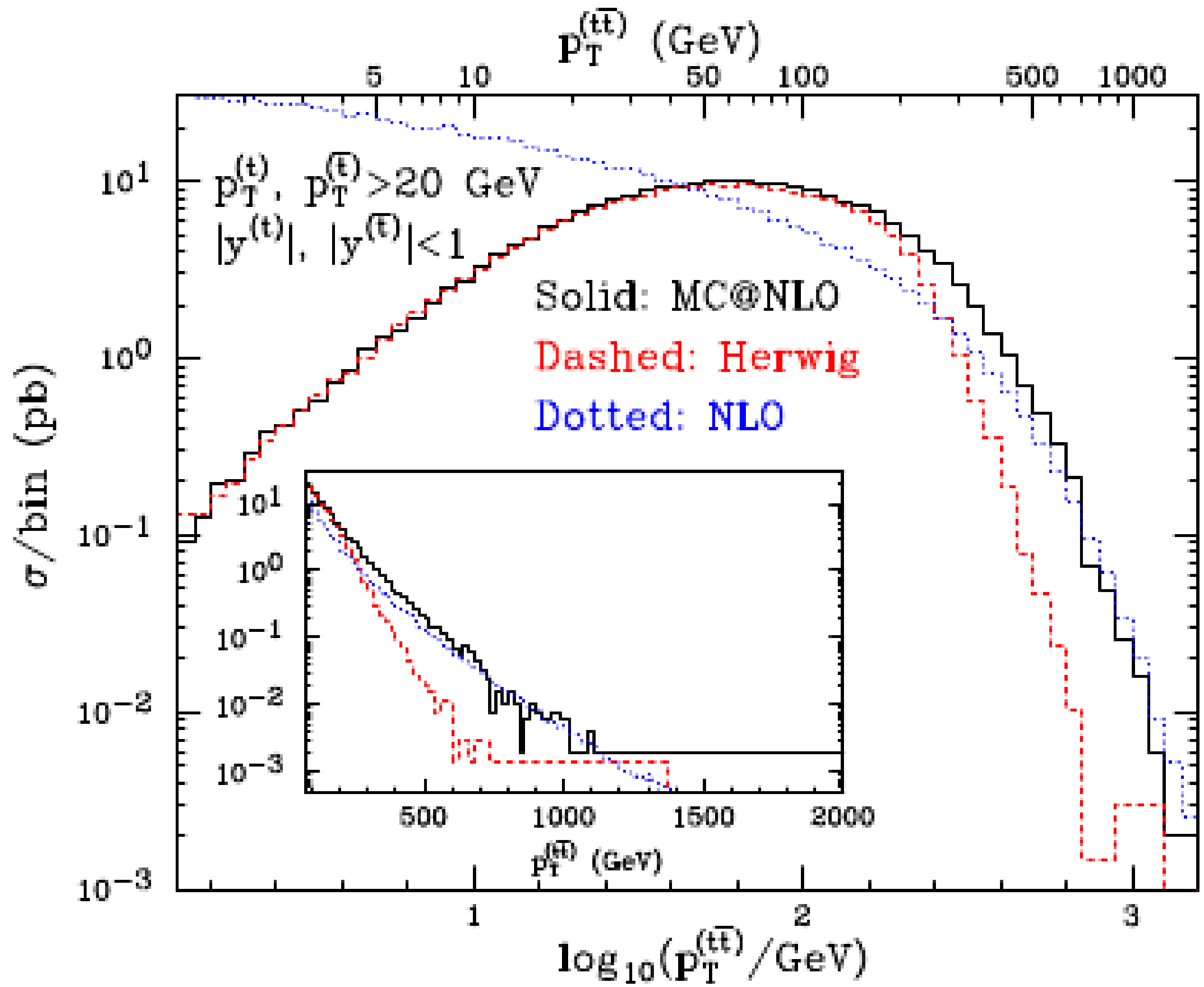
the POWHEG method

- **PO**sitive **W**eight **H**ardest **E**mission **G**enerator
- aim: distribute the **hardest emission** according to the **exact** NLO matrix element.
  - Sudakov form factor modified with real emission matrix elements,
  - almost eliminates negative weights
  - but some uncontrolled terms beyond NLO.

POWHEG vs MC@NLO

MC@NLO	POWHEG
+ controlled modifications to PS resummation	+ mostly positive weights
- negative weights	- interface can be subtle (e.g. p_T veto and truncated showers in angular-ordered showers)
- difficult to iterate	- difficult to iterate

some results

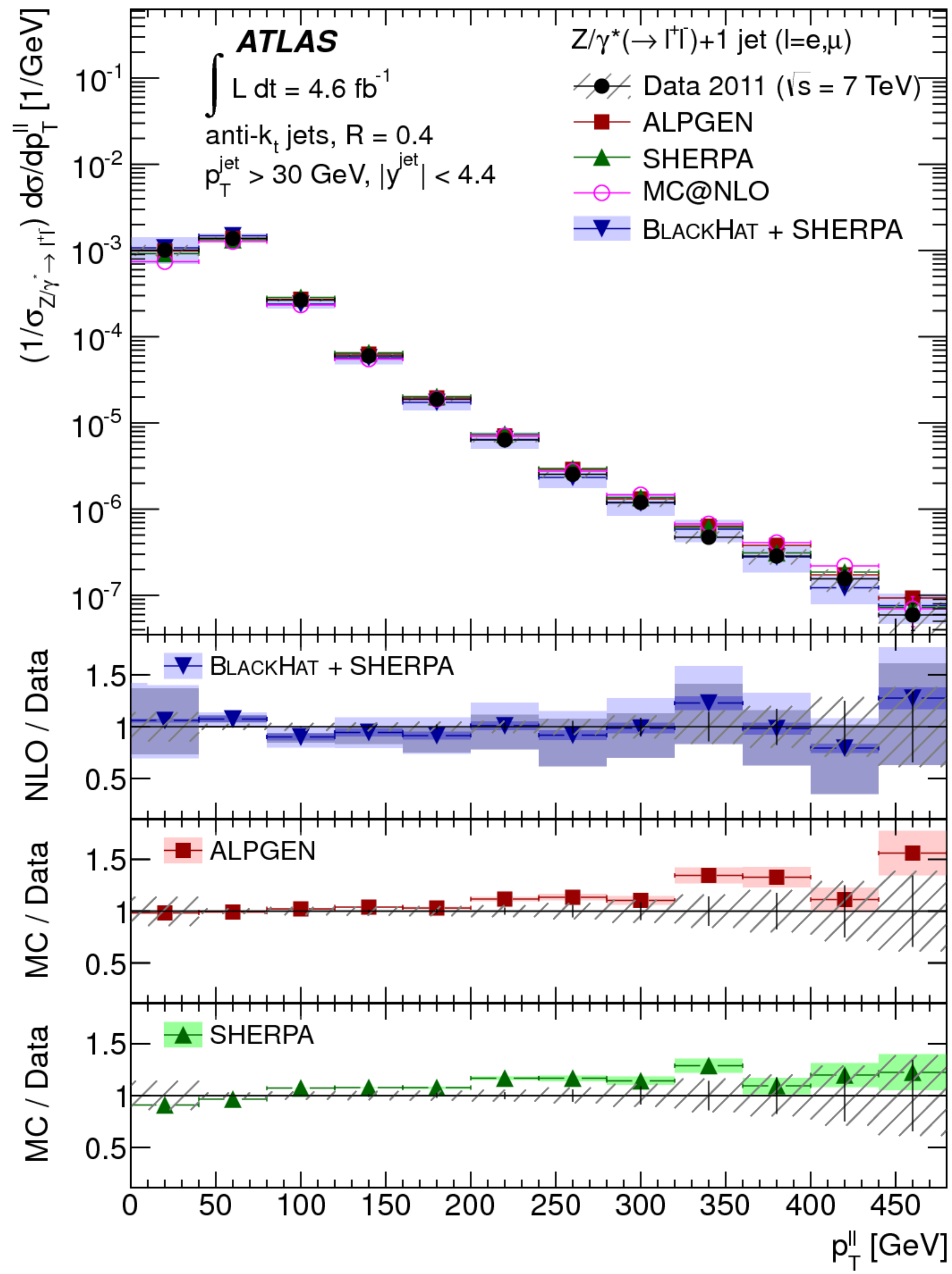


e.g. transverse momentum of top-anti-top system in top quark pair production at 14 TeV.

some results

e.g. transverse momentum of lepton pair in Z+jets vs **ATLAS 7 TeV Data.**

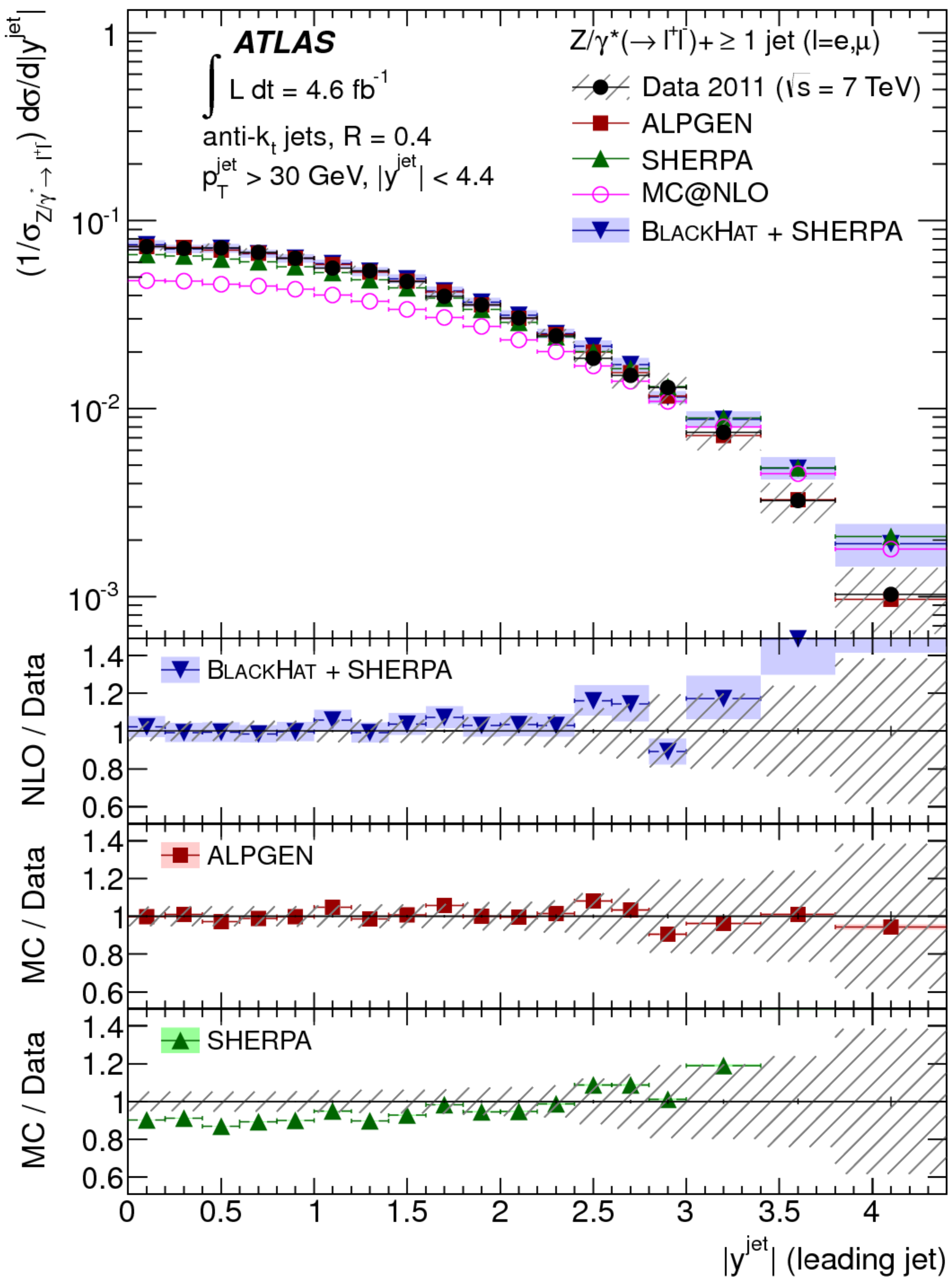
NLO for MC@NLO!



some results

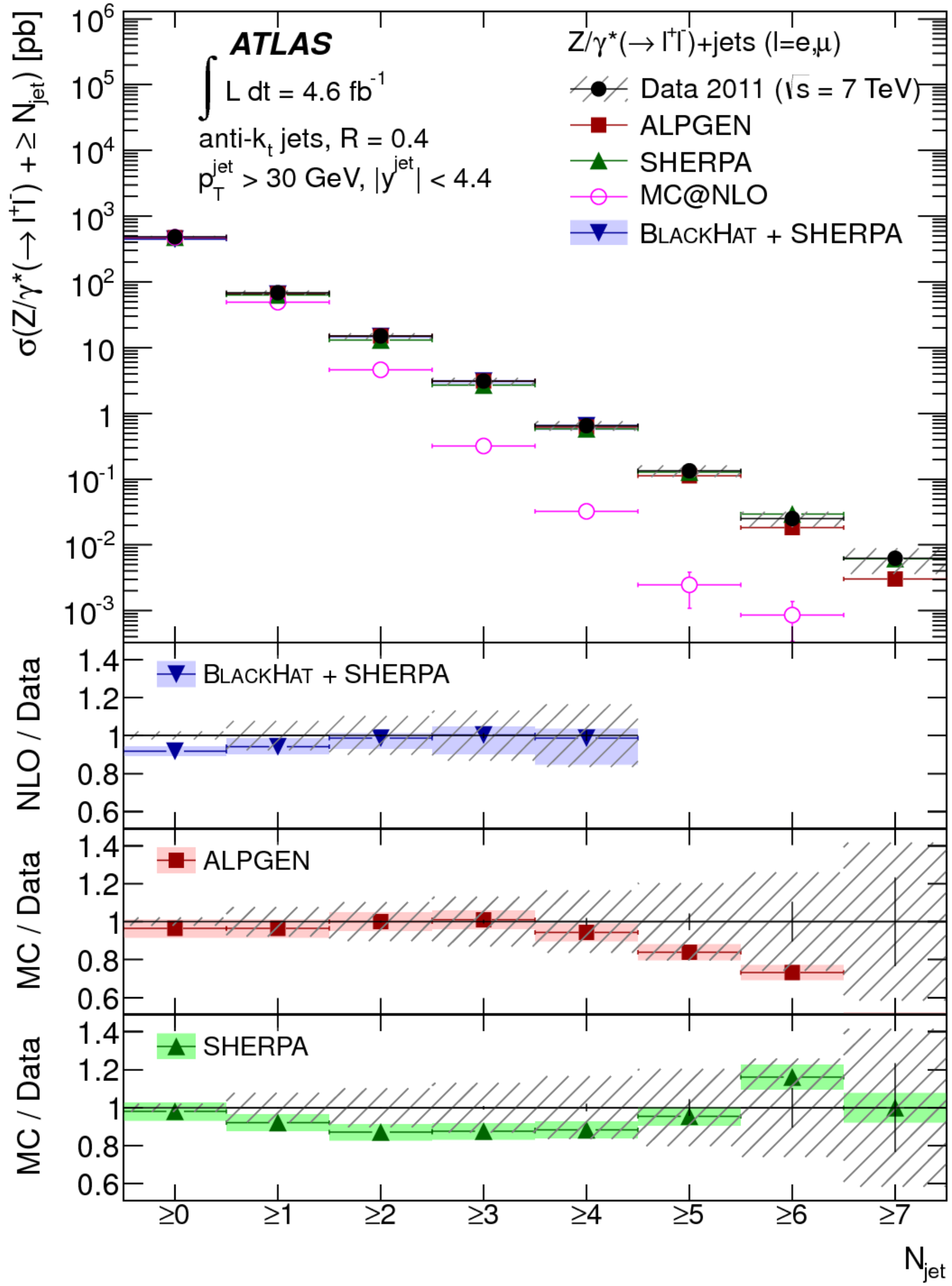
e.g. rapidity of leading jet in Z+jets vs **ATLAS 7 TeV Data.**

LO for MC@NLO!



some results

e.g. inclusive jet multiplicity distribution in Z+jets vs **ATLAS 7 TeV Data.**

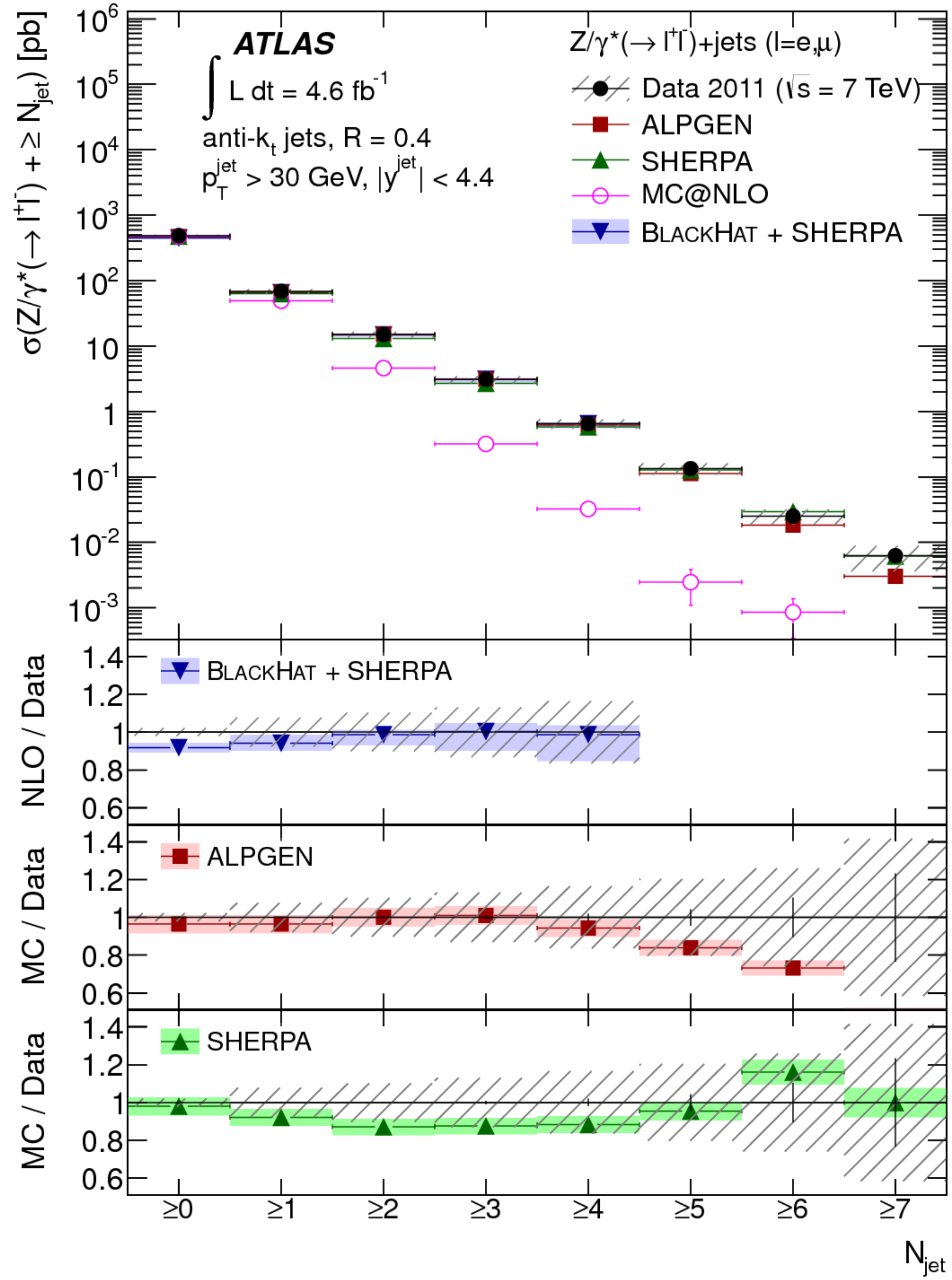


some results

e.g. inclusive jet multiplicity distribution in Z+jets vs **ATLAS 7 TeV Data.**

"ALPGEN" and "SHERPA": higher-multiplicity MEs merged to the **PS**.

➡ focus on this.



(II) multi-jet merging at LO and NLO

the ME+PS merging problem

- **goal**: get an accurate prediction of multi-jet observables.
- **approach**: combine predictions for multiple jets to a single calculation.
- **the challenges**:
  - avoid double counting between the different calculations.
  - **FO** predictions may break down for collinear or soft partons (e.g. tree-level).

tree-level merging

- **what we want to achieve:**
  - ***n* hardest jets** described by **FO** calculation: good description of high-p_T multi-jet data.
  - **any other emission** described by the **PS**: since it gets soft/collinear patrons right.
- start with **tree-level** calculations:
  - ✳ remove their singularities with a phase-space cut t_{MS}: the **merging scale**.
  - ✳ ***n* hardest partons** (above t_{MS}) described with tree-level accuracy.
  - ✳ softer partons (below t_{MS}) described by the **PS**.
  - ✳ whatever the algorithm should be, dependence on arbitrary t_{MS} should be **small**.

tree-level merging methods

- **MLM**: approximate no-emission probabilities by veto on jets.
- **CKKW:** analytic Sudakov factors as no-emission probabilities.
- **CKKW-L:** PS no-emission probabilities directly from PS trial showers.
- **Unitarised merging**: CKKW-L-inspired, does not change the inclusive cross sections.

[Lonnblad, Prestel, 1211.7278, Plätzer, 1211.5467]

CKKW merging

[Catani, Krauss, Kuhn, Webber, hep-ph/0109231]

- **the algorithm:**

  ✳ generate tree-level n-jet configurations, **defined** by the $\boldsymbol{k}_{\rm T}$ jet algorithm, with a resolution parameter $\boldsymbol{k}_0$.

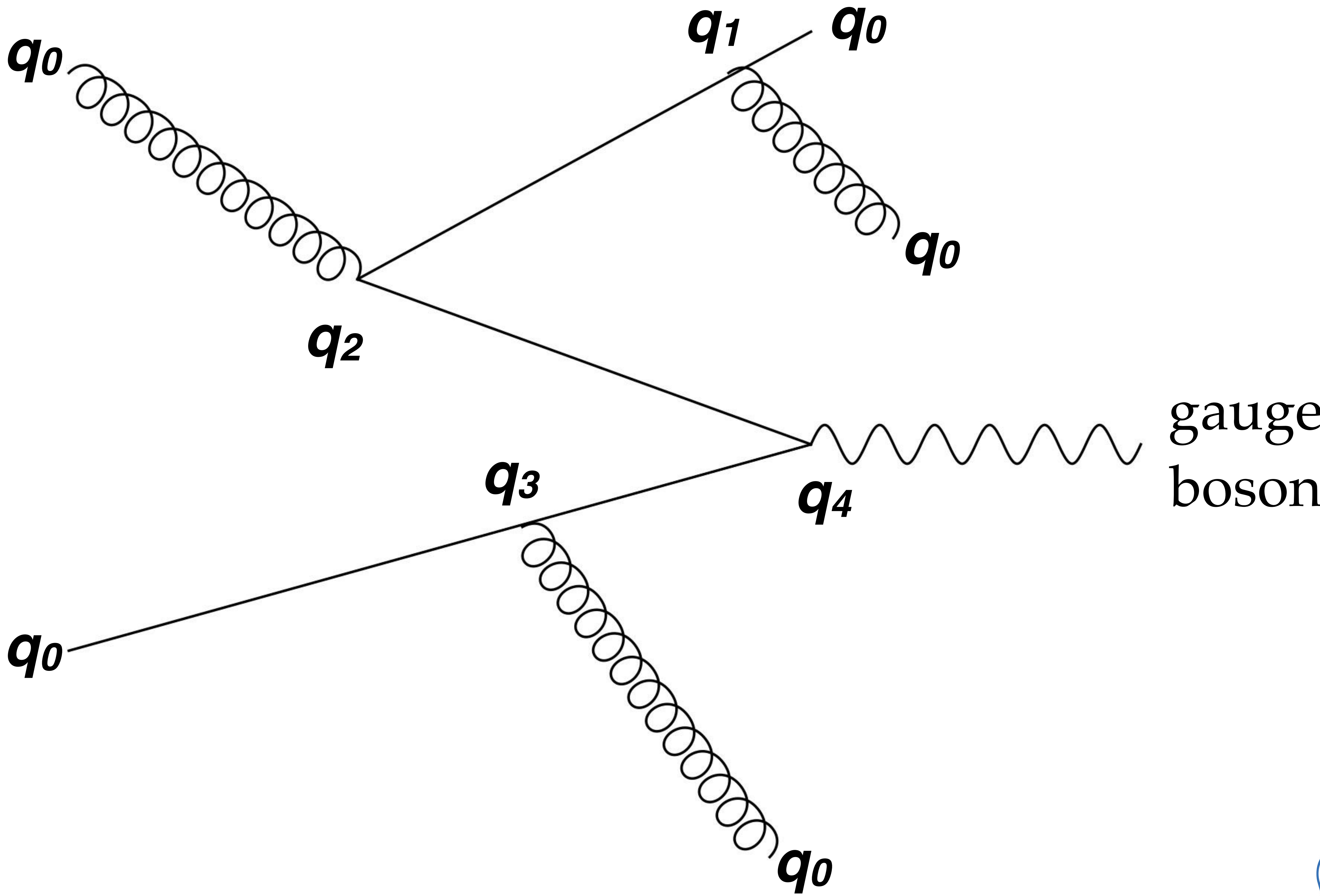
  ✳ for each line in the tree, associate a **Sudakov weight** giving the probability that no emission has taken place along this line.

  ✳ run the **PS** and put the samples together,

  ✳ a vetoed **PS** algorithm is used to guarantee that no unwanted hard jets are produced during jet evolution.

CKKW merging, schematically

$$R(q_i, q_j) = \Delta(q_i, q_0)/\Delta(q_j, q_0)$$



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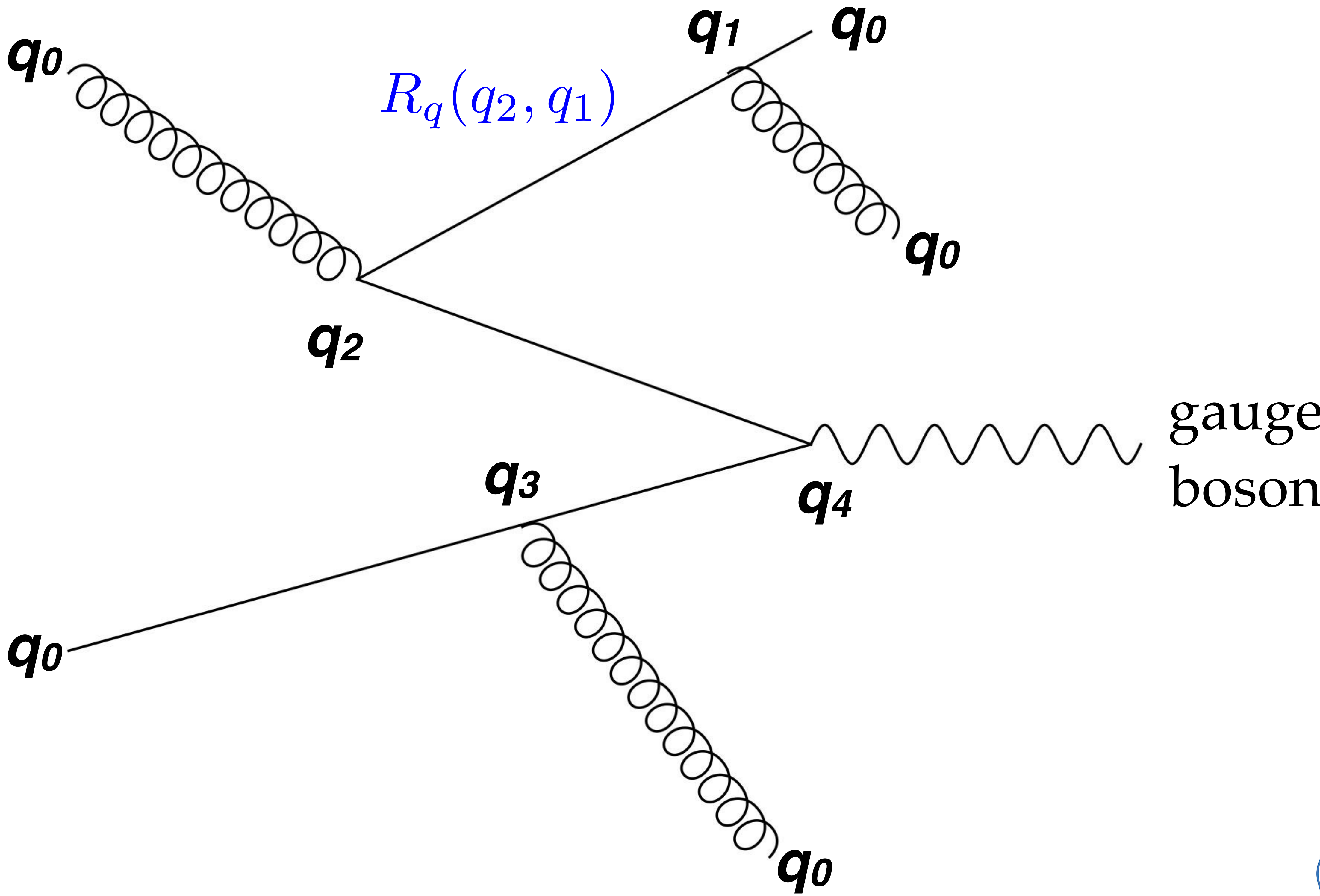

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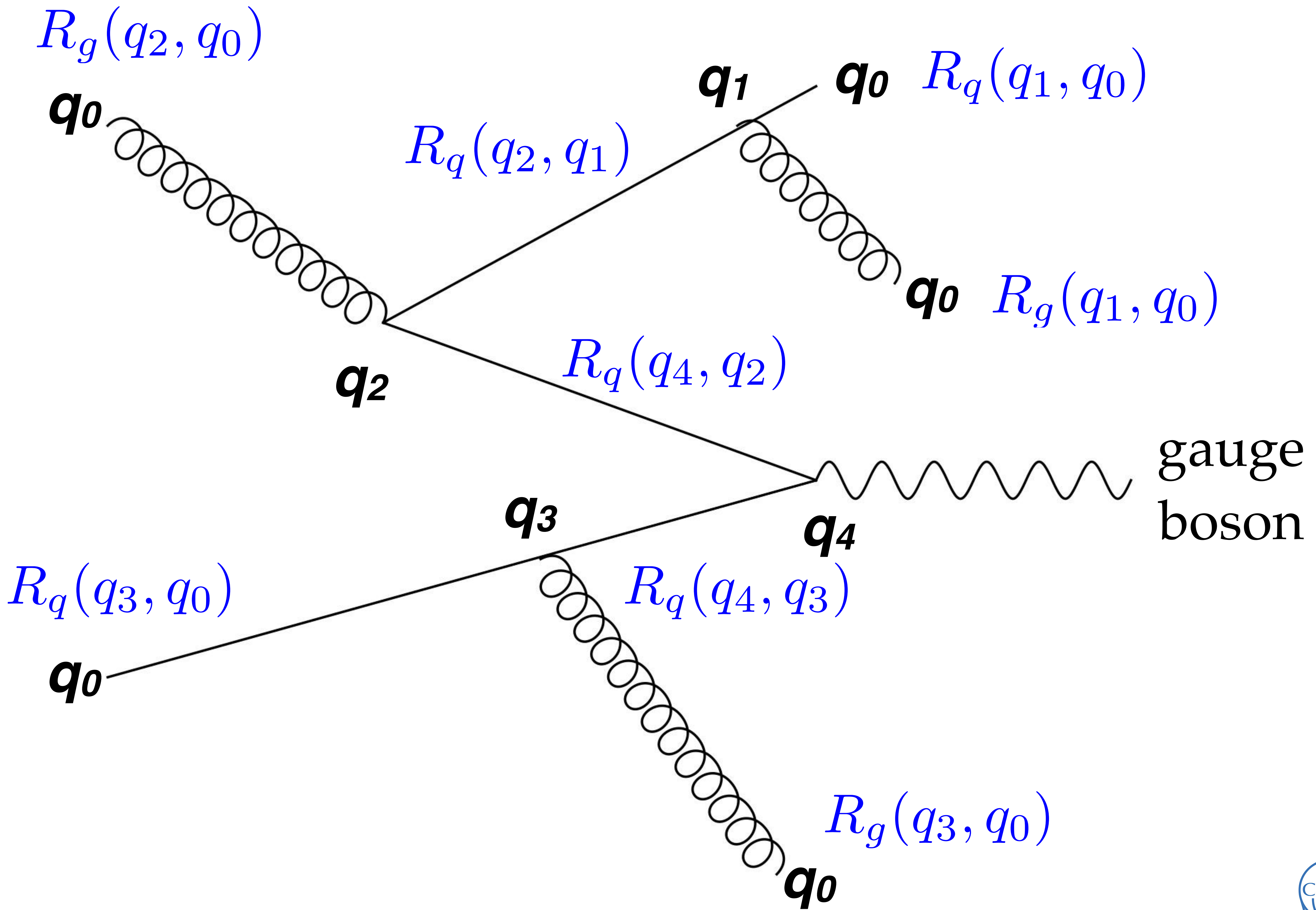

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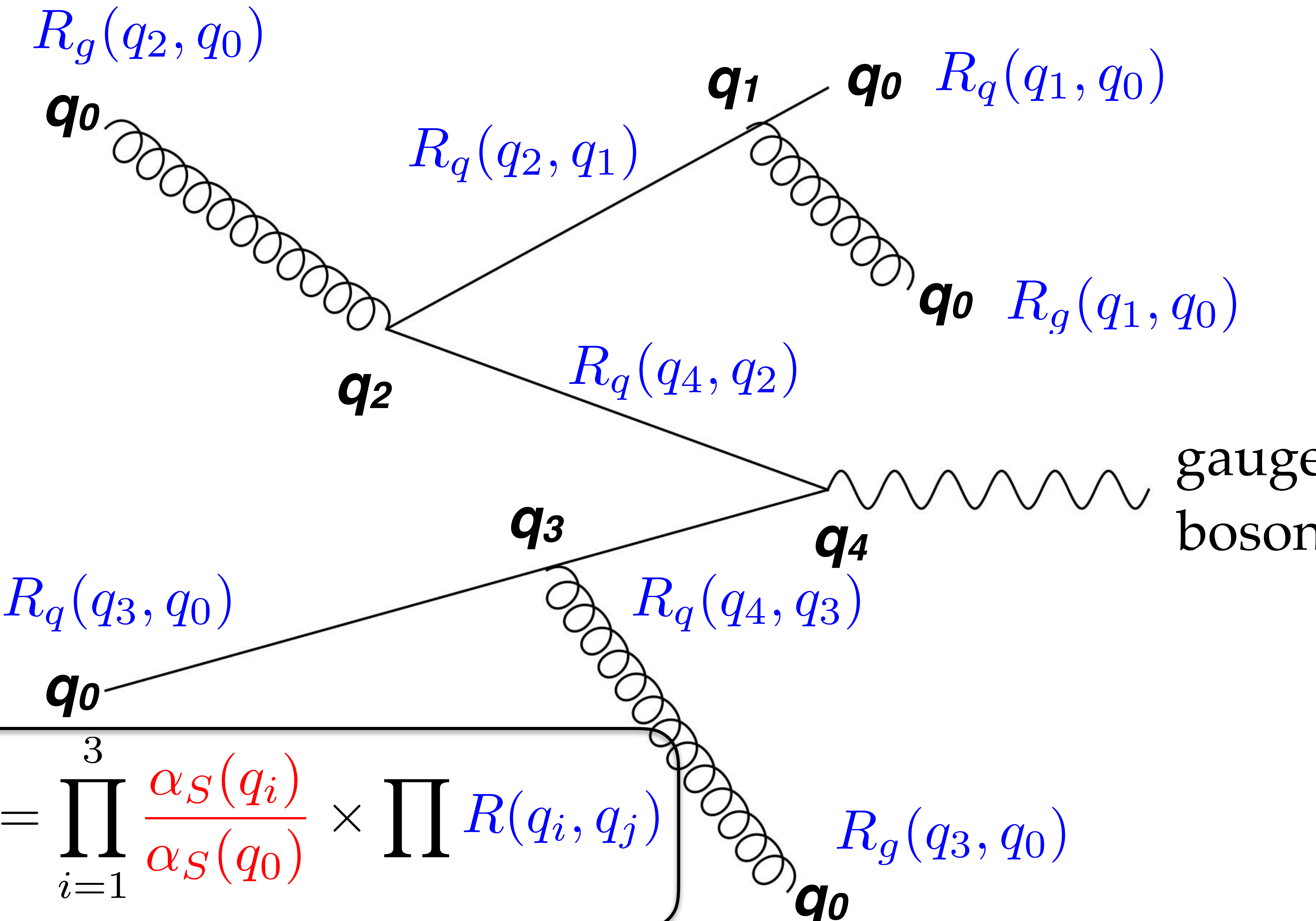


$$w = \prod_{i=1}^{3} \frac{\alpha_S(q_i)}{\alpha_S(q_0)} \times \prod R(q_i, q_j)$$

CKKW merging

$$w = \prod_{i=1}^{3} \frac{\alpha_S(q_i)}{\alpha_S(q_0)} \times \prod R(q_i, q_j)$$

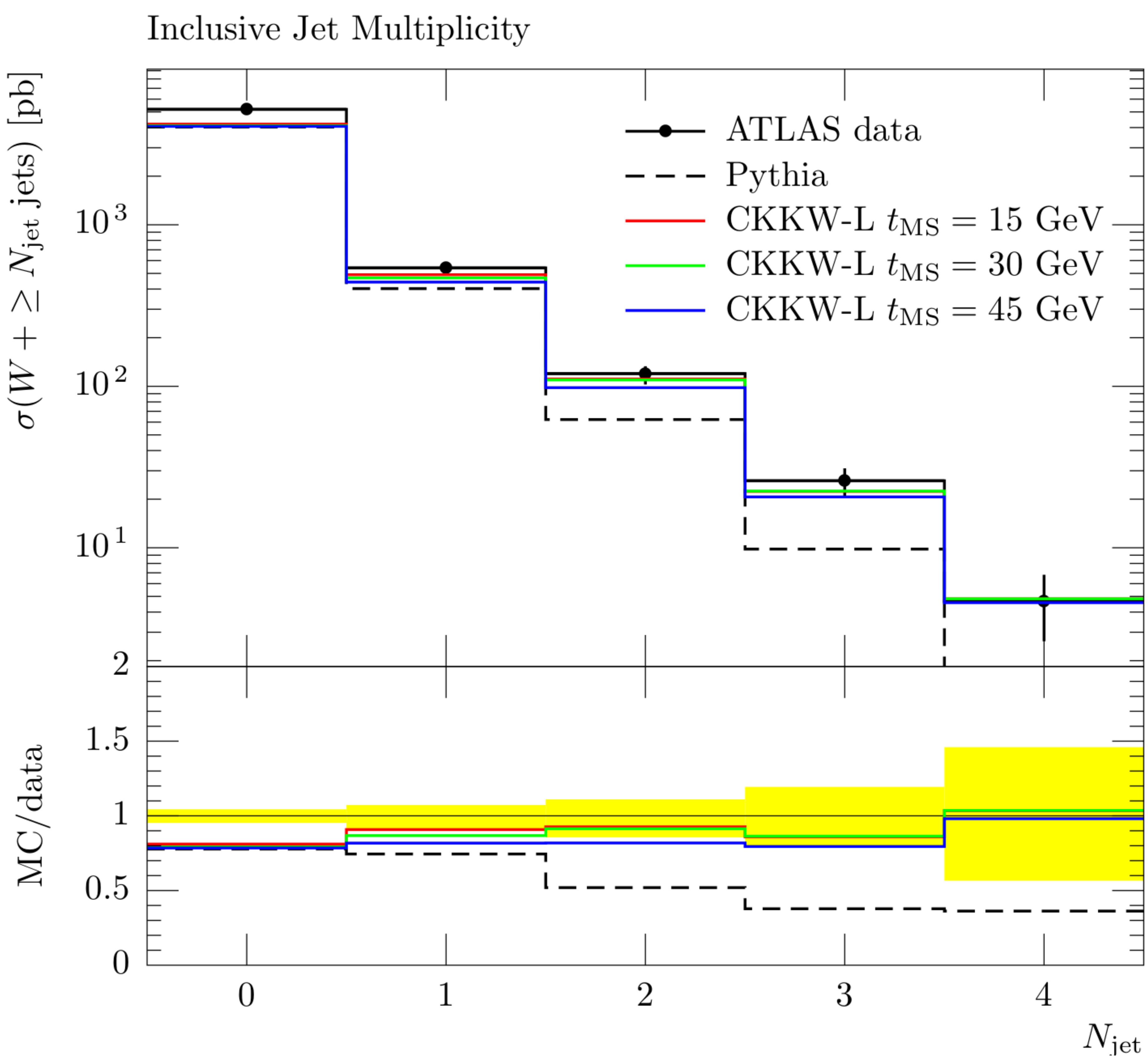
take into account running of α_S in shower.

no-emission probabilities.
note: for initial-state radiation need to amend with PDF reweighting.

CKKW-L

- main source of uncertainty of CKKW:
- **mismatch** between the k_{T} scale used to define the Sudakov reweighting and the evolution scale used in the **PS**.
- problem evaded in CKKW-L through construction of **PS** "histories" and the use of a veto algorithm.
- this reproduces the no-emission probabilities present in the **PS**. [L.Lonnblad **JHEP 0205:046,2002**]

CKKW-L example



Data: ATLAS W+jets, inclusive jet multiplicity.

MLM merging

- **the algorithm:**

  - ✳ generate tree-level configurations up to the desired multiplicity: e.g. Z+0, 1, 2, …, $\boldsymbol{n}$ partons with phase space cuts.

  - ✳ run the **PS** on the events and

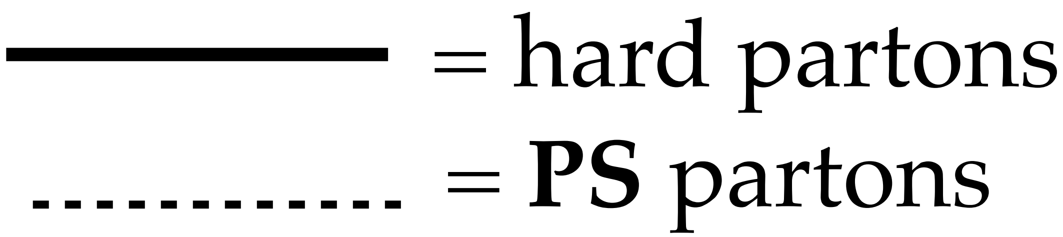
  - ✳ run a jet-cone algorithm defined by a cone size $\boldsymbol{R}_{\mathrm{clus}}$ and minimum transverse energy $\boldsymbol{E}_{\mathrm{Tclus}}$.

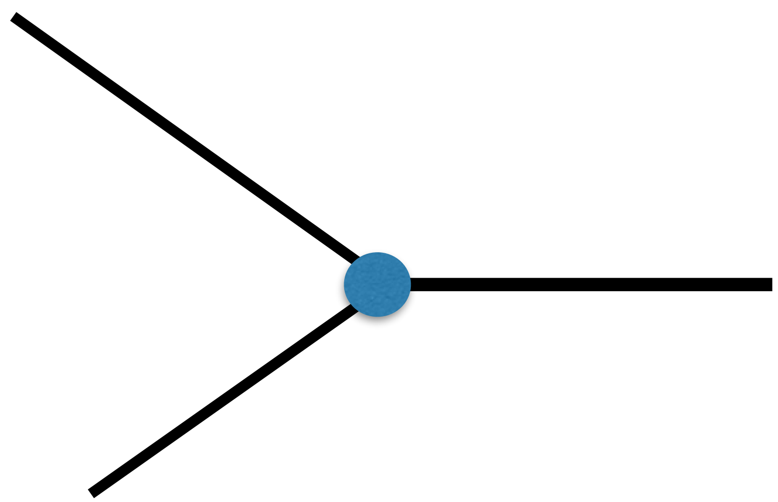
  - ✳ compare the resulting jets with the partons before the shower: if the parton-jet distance is less than 1.5x$\boldsymbol{R}_{\mathrm{clus}}$ they "match", remove the jet from list of jets and continue.

  - ✳ if there are partons that have not been matched to jets: **VETO** event.

  - ✳ if the parton sample contains **extra** jets and is **not** the highest-multiplicity: **VETO** event: removes double-counting. (for the highest-mult. they are allowed but they should be softer than the matched jets).

MLM merging, examples

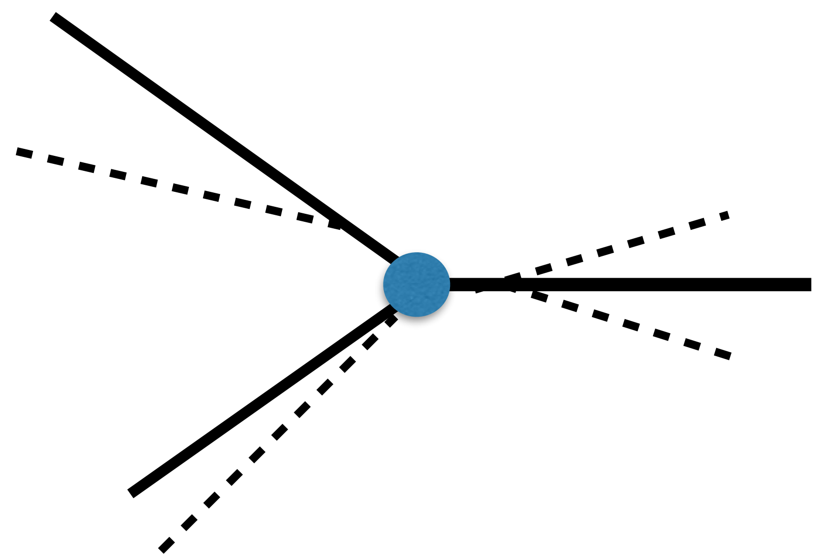




MLM merging, examples

= hard partons

= **PS** partons



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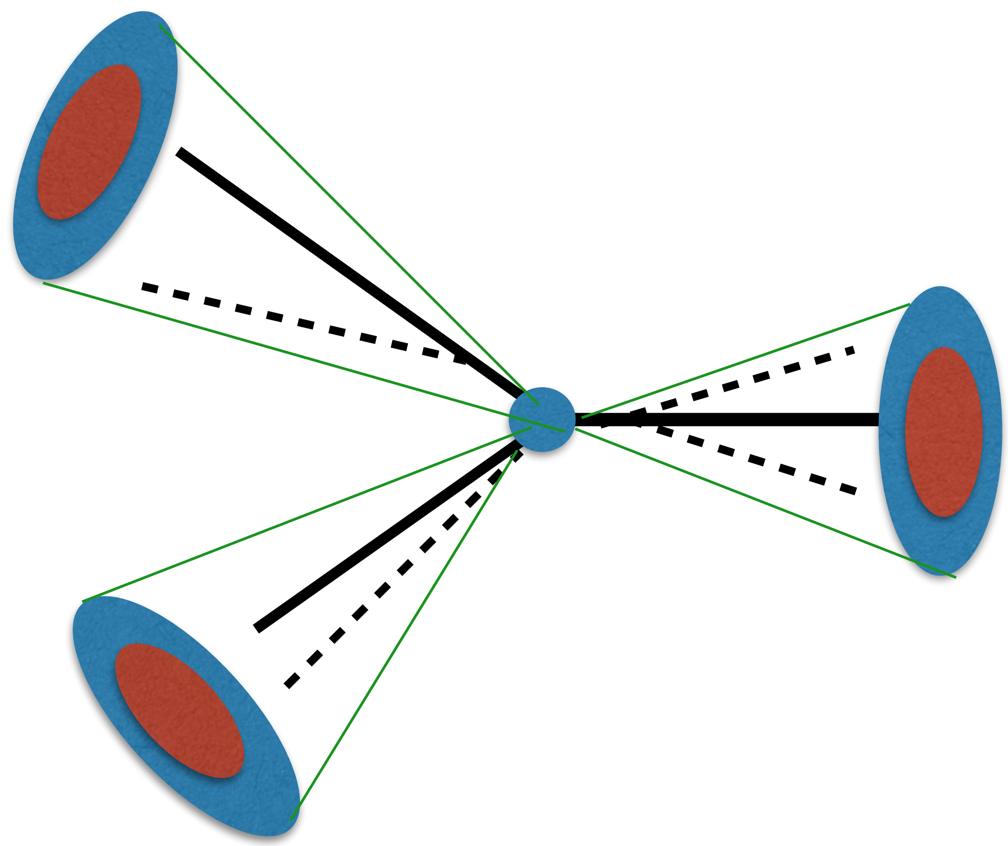
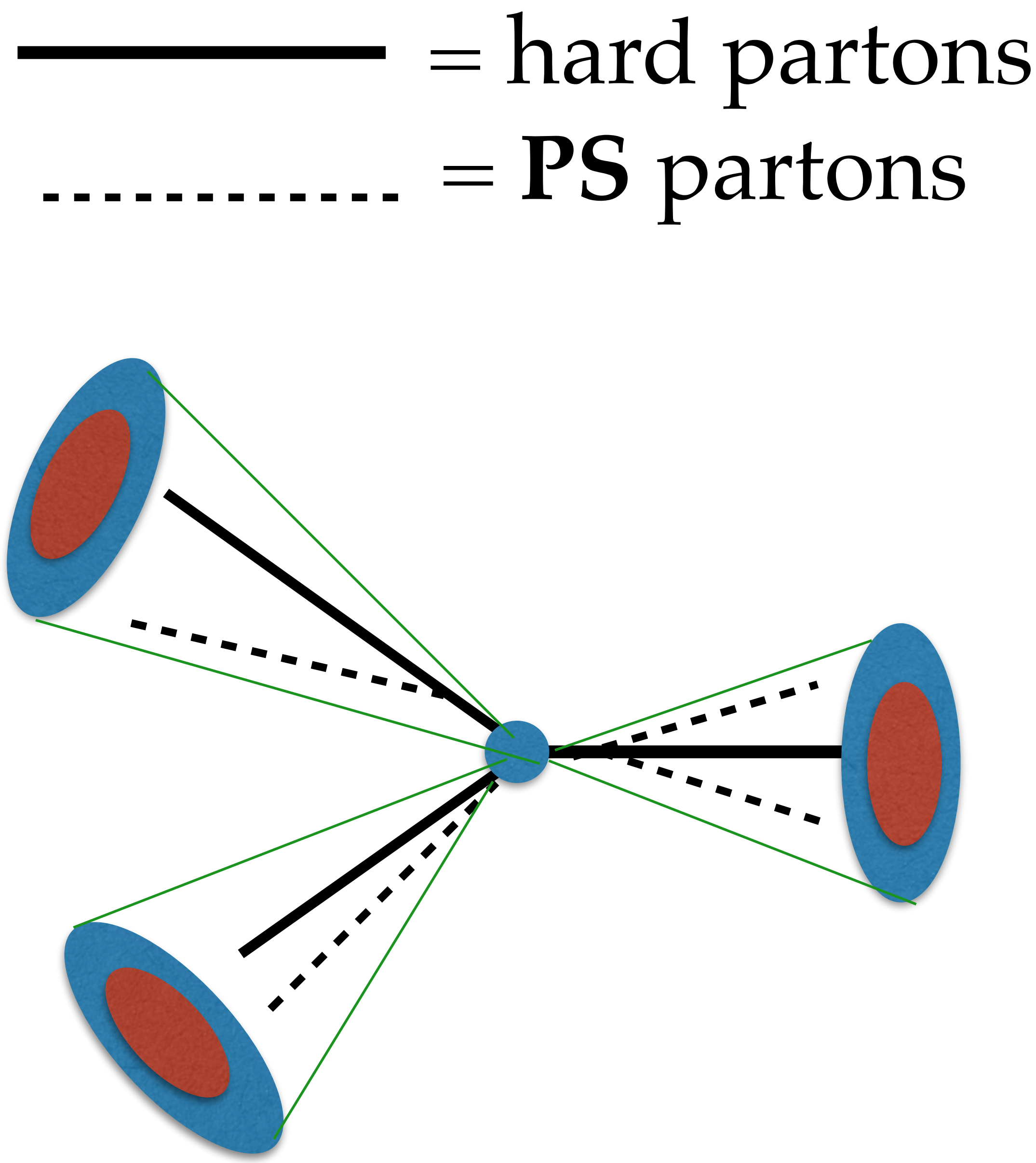
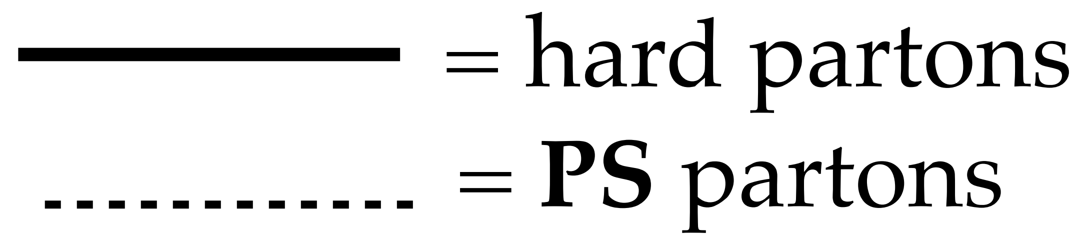


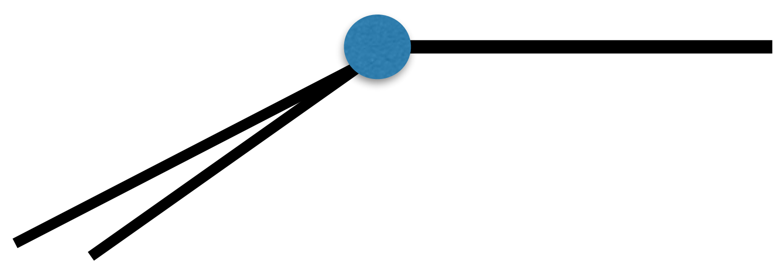
MLM merging, examples



all partons matched: **keep** event.

MLM merging, examples





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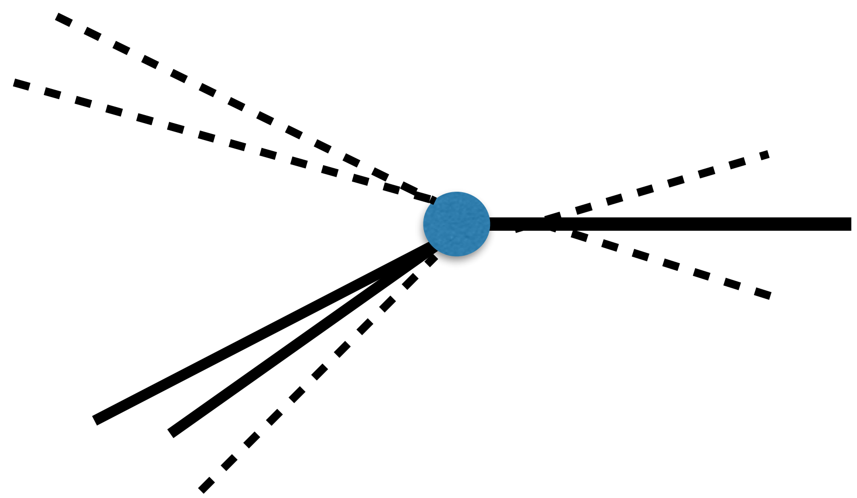
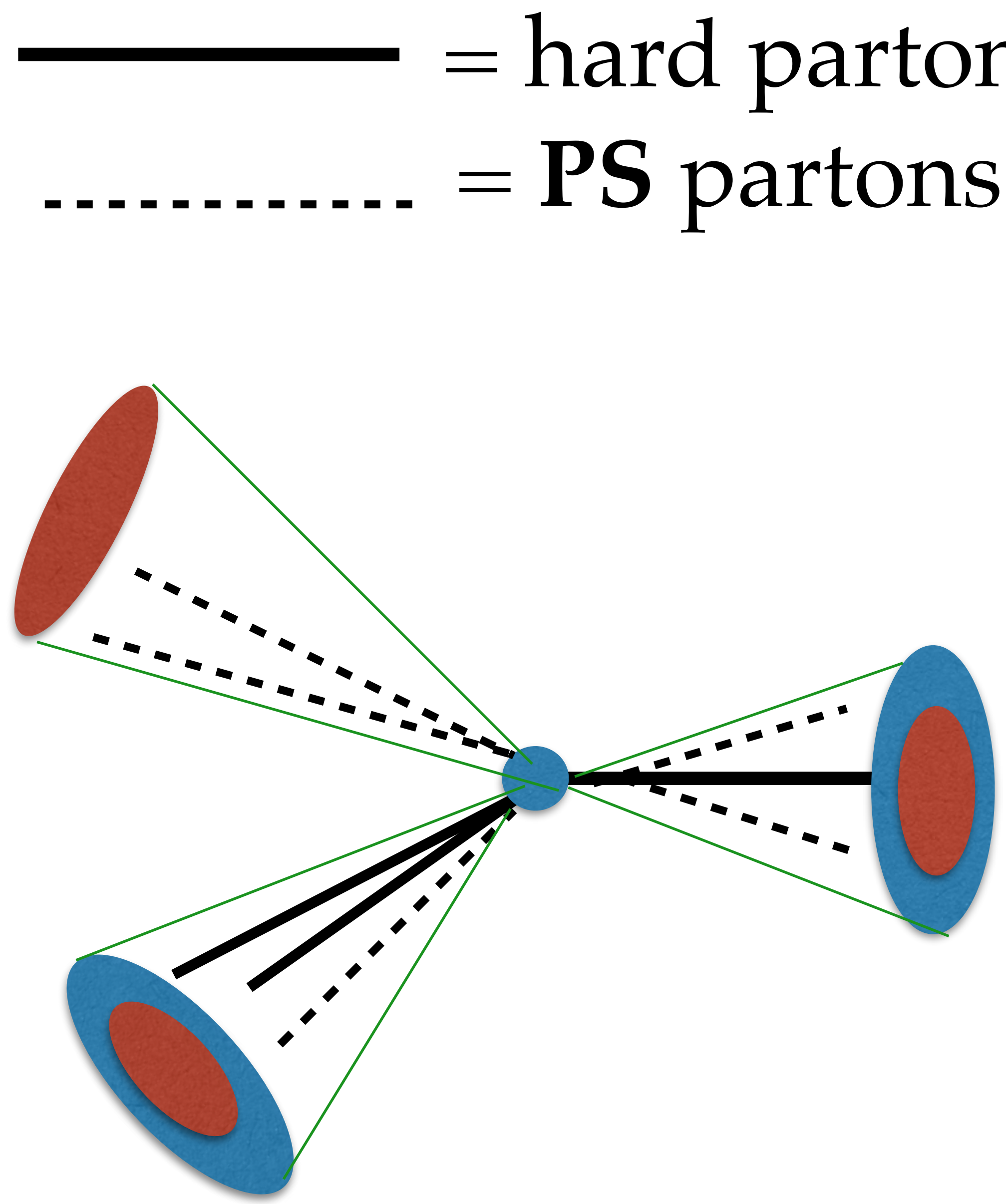
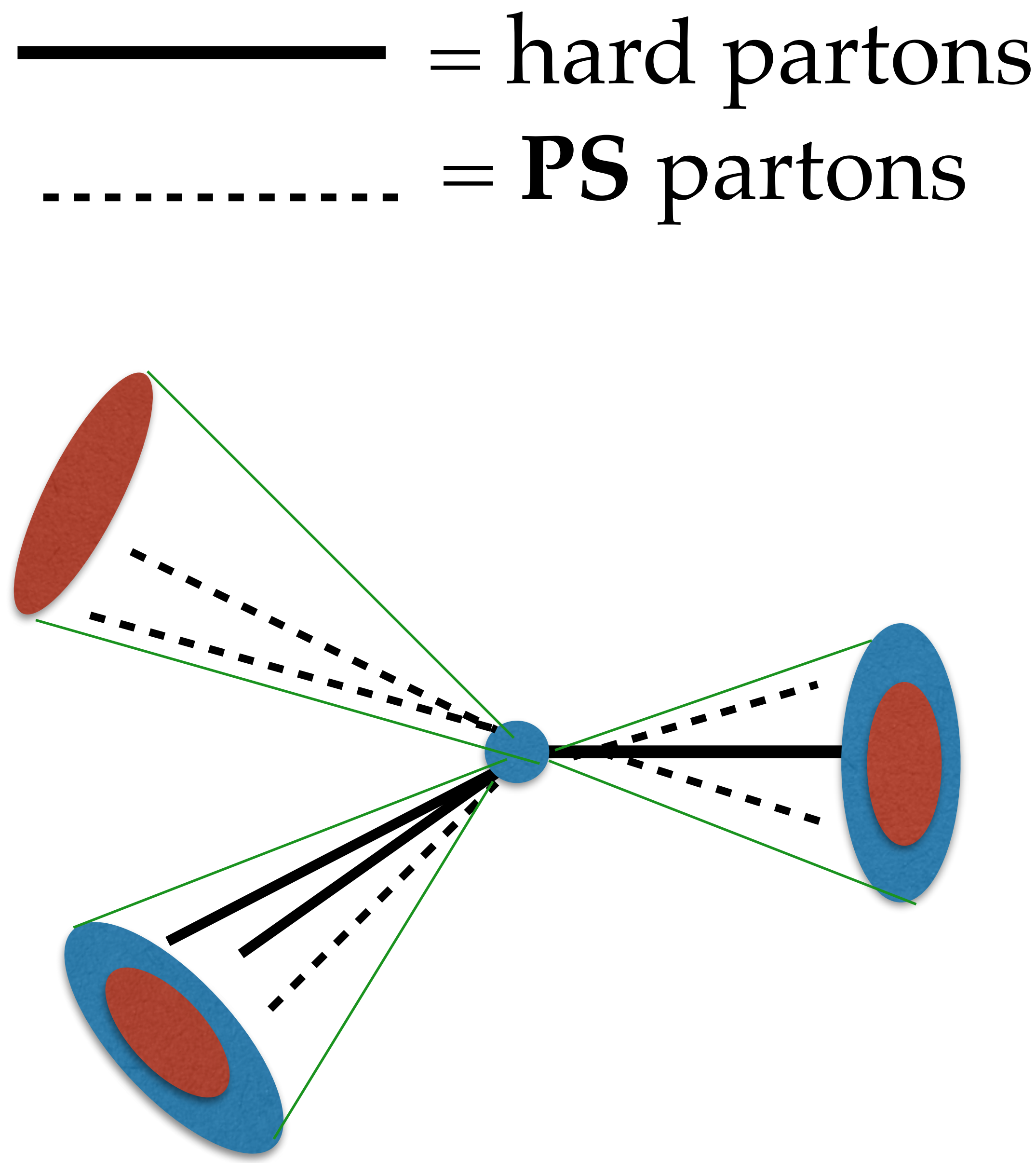


MLM merging, examples



MLM merging, examples

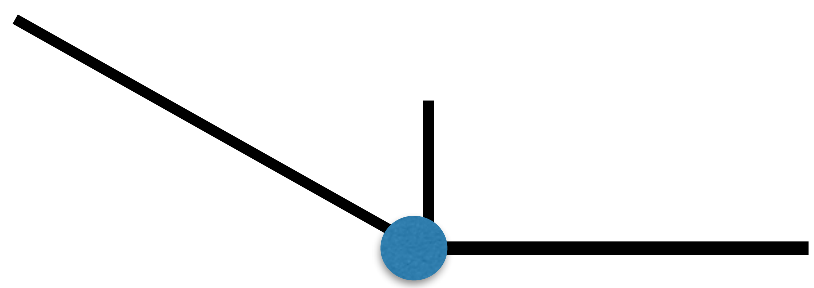


not all partons match: **veto** event.
(collinear double-log double-counting)

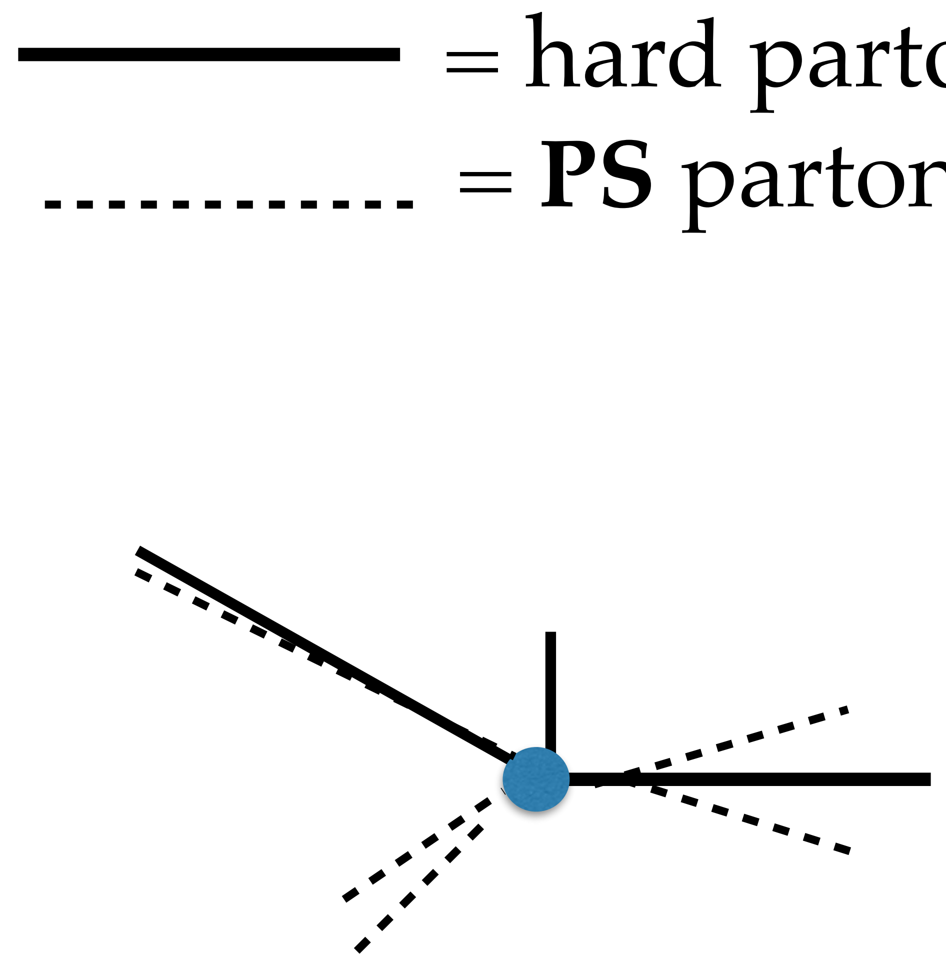
MLM merging, examples

(solid line) = hard partons

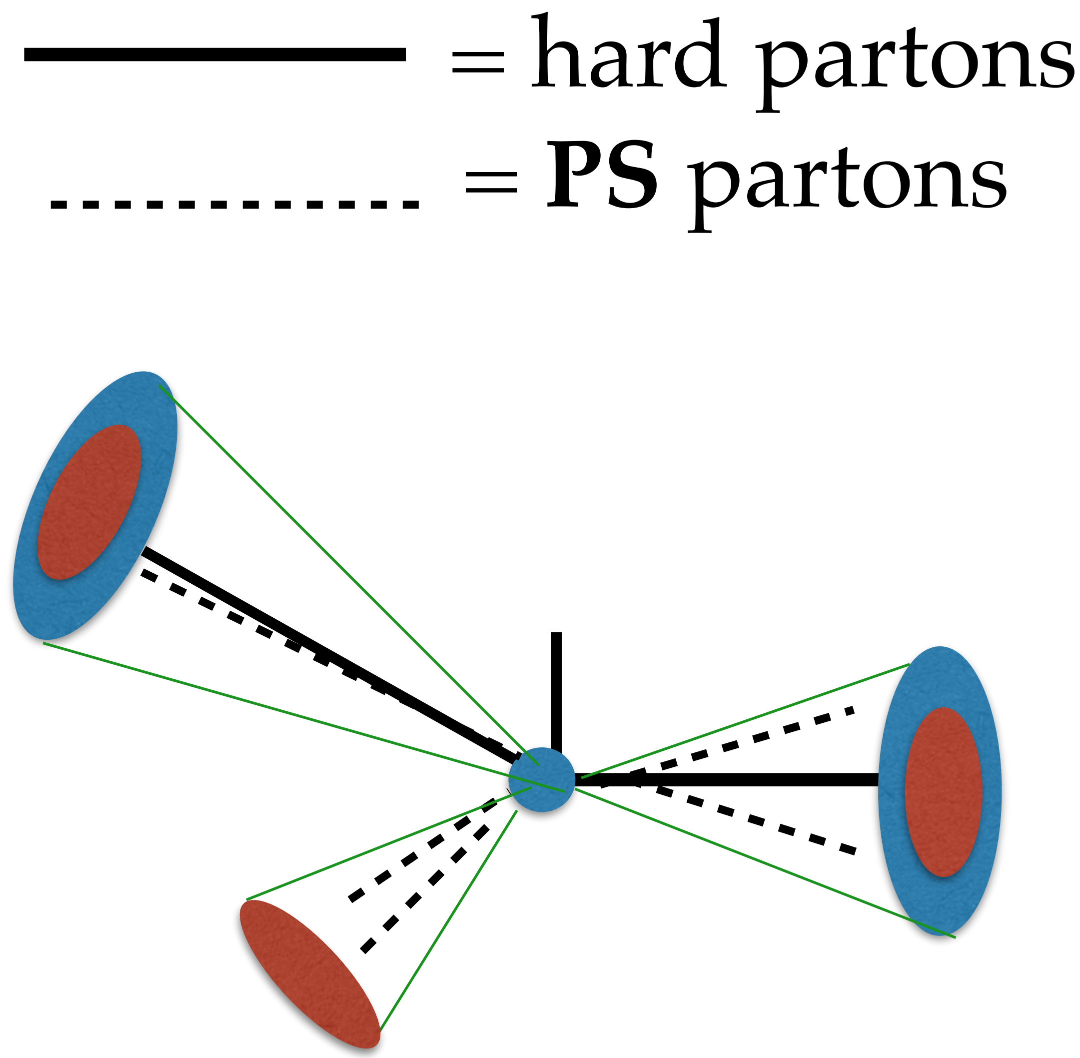
(dotted line) = **PS** partons



MLM merging, examples



MLM merging, examples



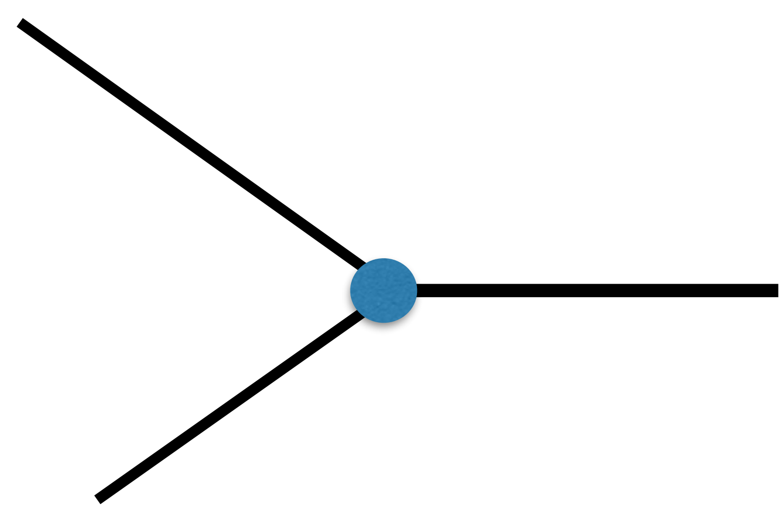
not all partons match: **veto** event.

(soft single-log double-counting)

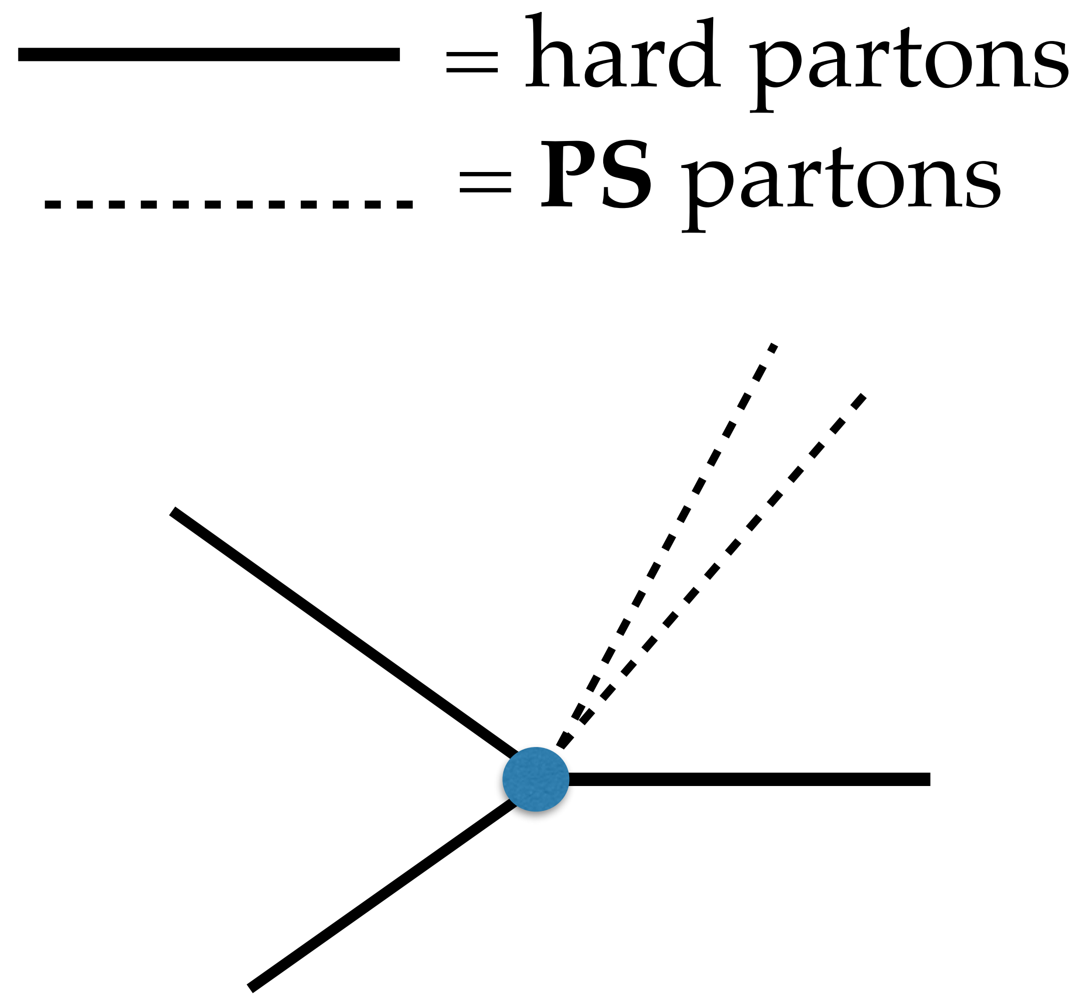
MLM merging, examples

= hard partons

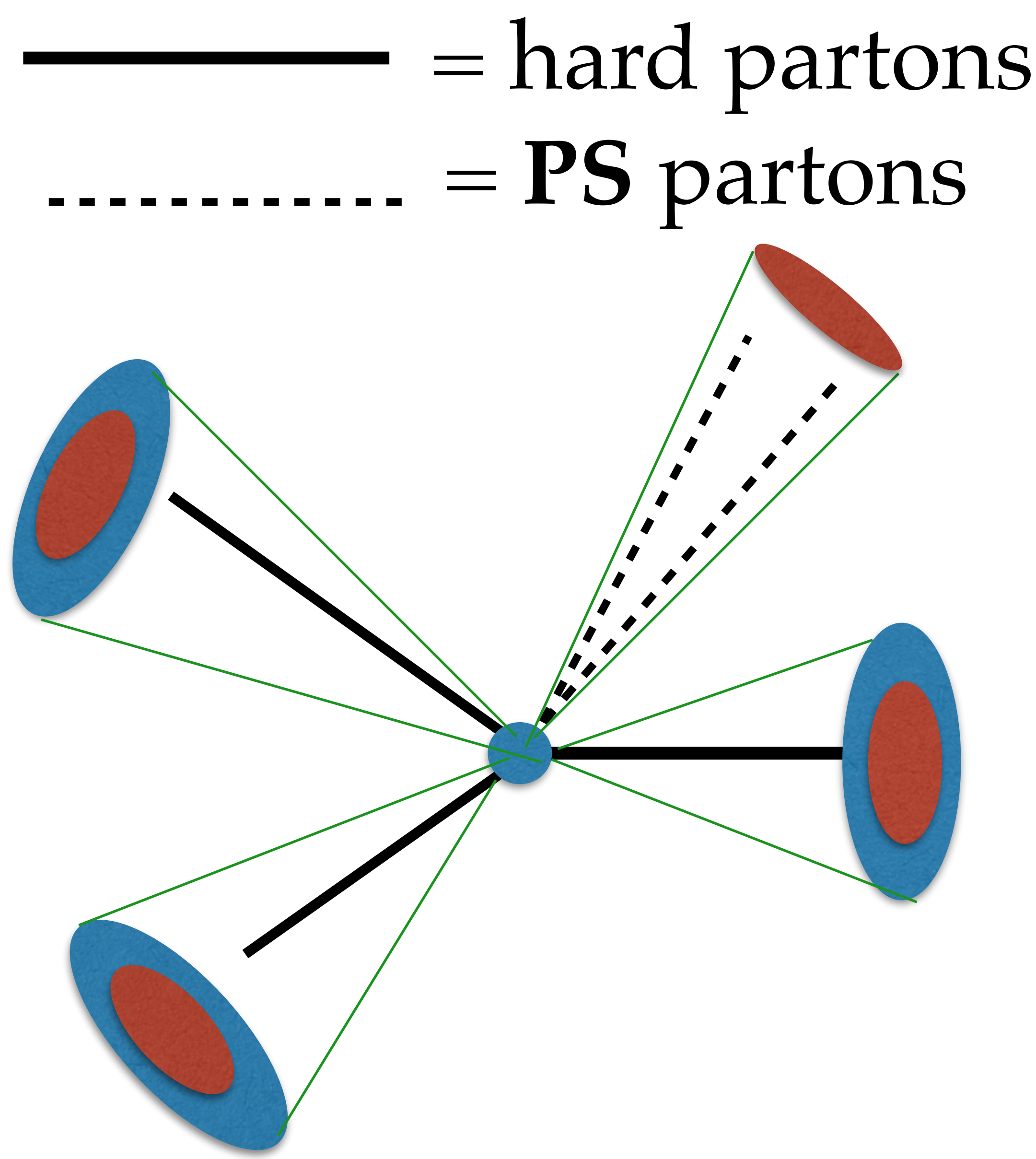
= **PS** partons



MLM merging, examples



MLM merging, examples



MLM merging, examples

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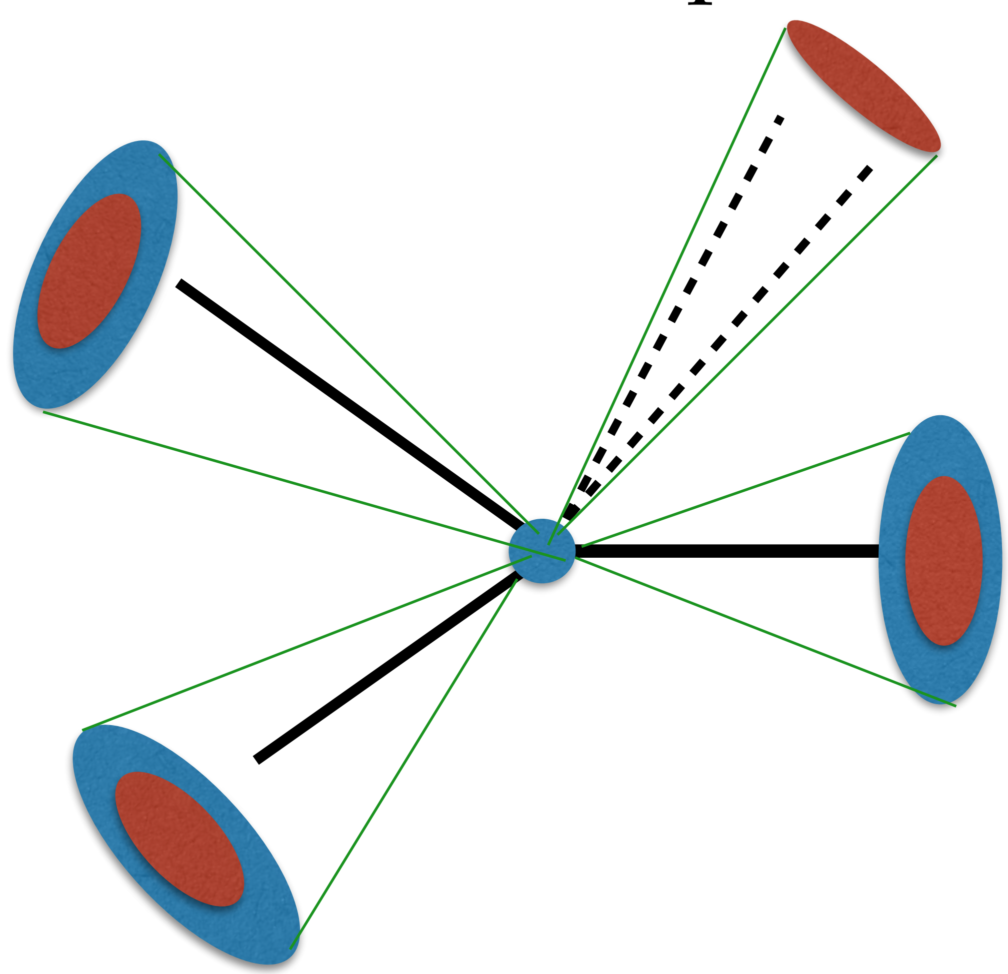


all partons match:

keep for the inclusive sample (maximum ME multiplicity),

but **veto** for exclusive samples.

merging of NLO+PS

- **NLO matching:** NLO-correct for inclusive observables: reliable uncertainty estimates, but limited applicability.
- **Multi-jet merging**: uncertainty estimates not reliable but broad applicability.
- **combine both strategies for an improved result!**
- we want to use the full NLO whenever possible, i.e., have:
  - NLO accuracy for inclusive W+0 jet observables,
  - NLO accuracy for inclusive W+1 jet observables,
  - NLO accuracy for inclusive W+2 jet observables
  - […]

merging of NLO+PS

- **to achieve multi-jet merging at NLO:**
  - ✳ add multiple NLO calculations,
  - ✳ ensure that real emission parts of NLO calculations do not overlap.

merging @ NLO+PS methods

- **FxFx**: combine multiple MC@NLOs by MLM-inspired jet matching at NLO.
- **MEPS@NLO**: combine MC@NLOs.
- **UNLOPS**: combine MC@NLOs or POWHEGs by Unitarised merging@NLO.
- **MiNLO**: get the zero-jet NLO by CKKW-reweighed 1-jet POWHEG after integration.

FxFx

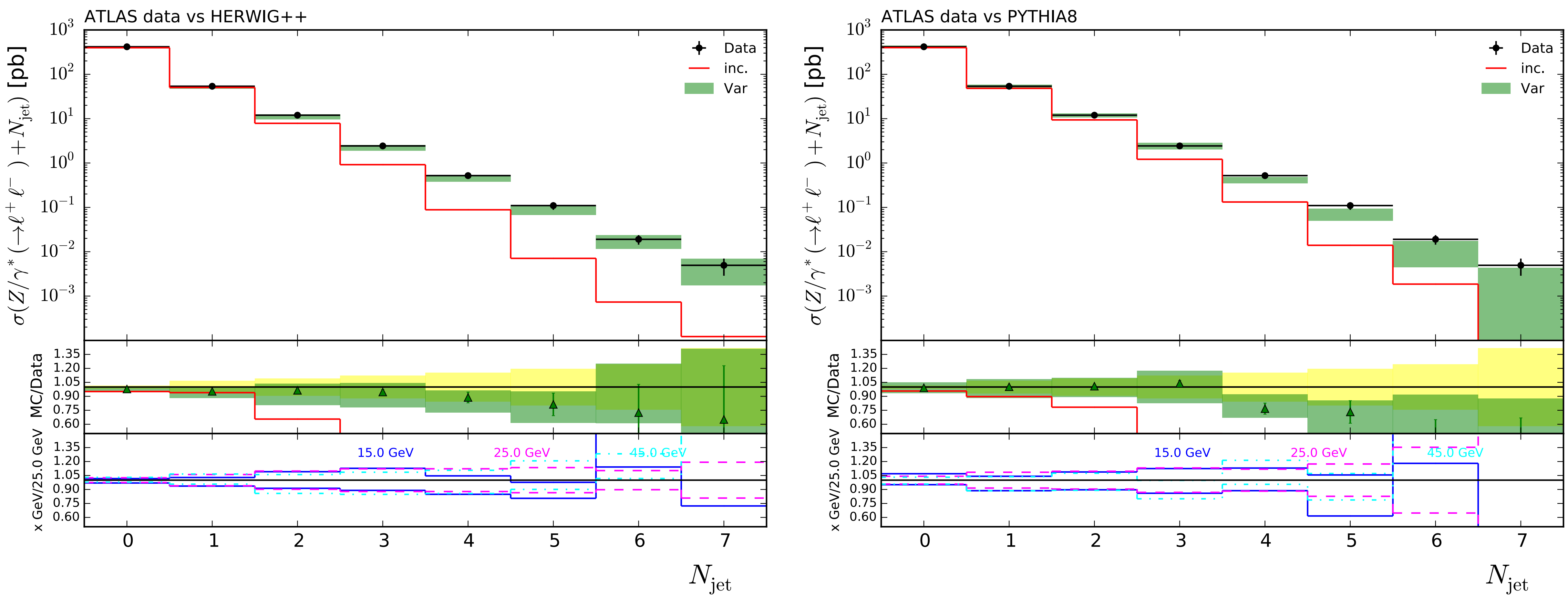
[Frederix, Frixione, 1209.6215]

- **the algorithm**:
  - ✳ construct MC@NLO for X+0, 1, 2,…, ***n*** jets,
  - ✳ multiply the matrix elements by appropriate Sudakov factors,
  - ✳ shower the events and apply an MLM-type rejection, but for jet-to-jet matching instead of parton-to-jet.

FxFx results

[Frederix, Frixione, AP, Prestel, Torrielli, 1511.00847]

e.g. ATLAS@7 TeV **exclusive jet multiplicity** in Z+jets **VS** aMC@NLO FxFx with Herwig++ or Pythia8:



NLO-Merged (FxFx): Z+0/1/2j.

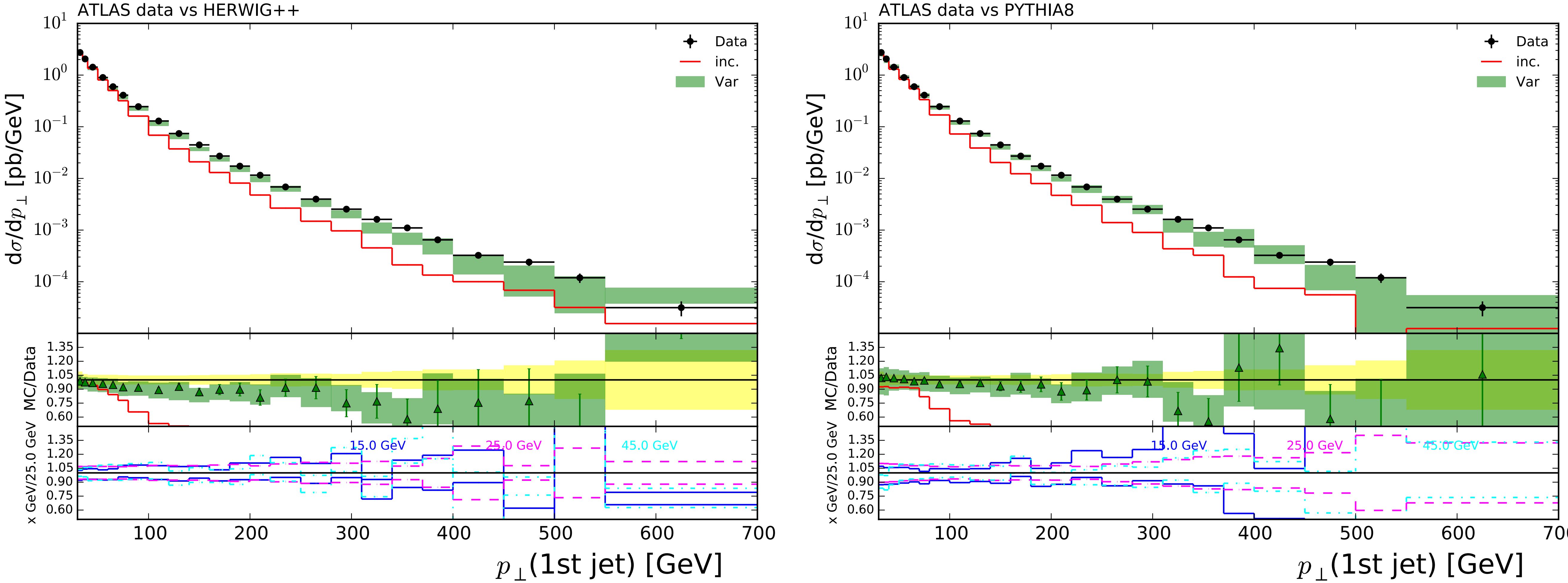
MC@NLO: Z+0j.

improved description of higher-multiplicities!

FxFx results

[Frederix, Frixione, AP, Prestel, Torrielli, 1511.00847]

e.g. ATLAS@7 TeV **1st jet p_T** in Z+jets **VS** aMC@NLO FxFx with Herwig++ or Pythia8:



NLO-Merged (FxFx): Z+0/1/2j.

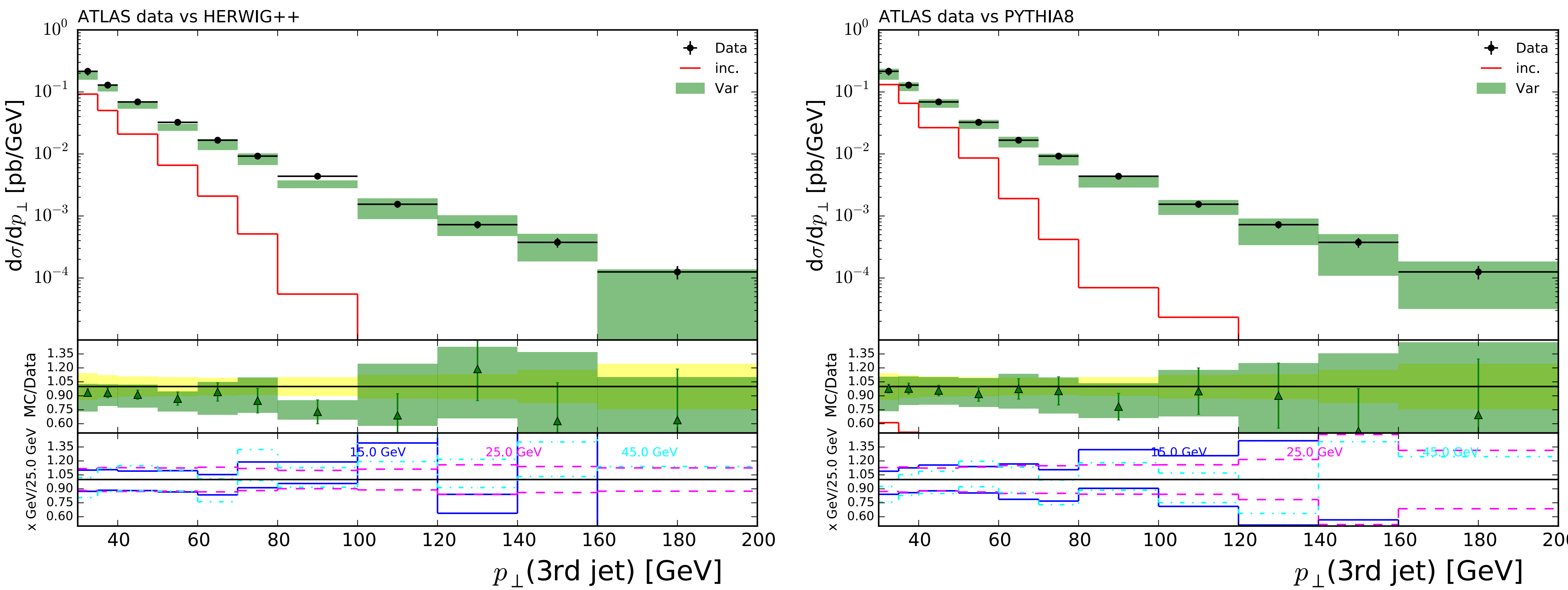
MC@NLO: Z+0j.

NLO-accurate observable.

FxFx results

[Frederix, Frixione, AP, Prestel, Torrielli, 1511.00847]

e.g. ATLAS@7 TeV **3rd jet p_T** in Z+jets <u>**VS**</u> aMC@NLO FxFx with Herwig++ or Pythia8:



NLO-Merged (FxFx): Z+0/1/2j.

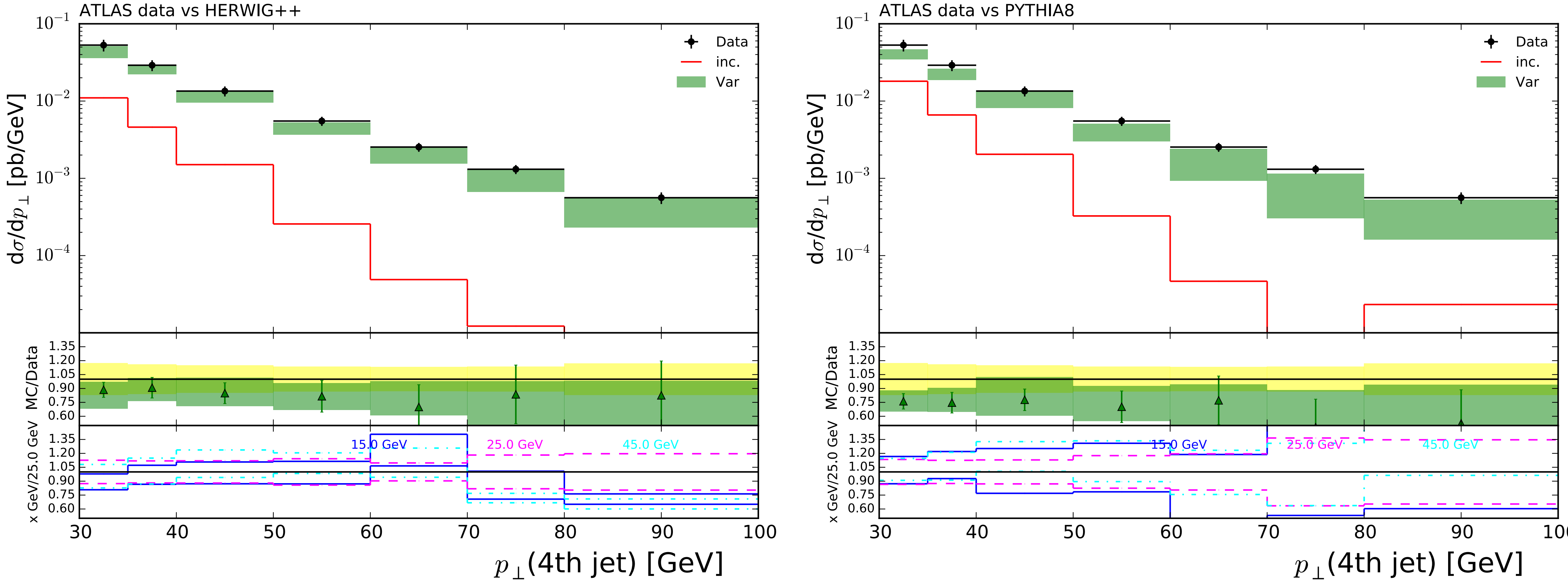
MC@NLO: Z+0j.

LO-accurate observable.

FxFx results

[Frederix, Frixione, AP, Prestel, Torrielli, 1511.00847]

e.g. ATLAS@7 TeV **4th jet p_T** in Z+jets **VS** aMC@NLO FxFx with Herwig++ or Pythia8:



NLO-Merged (FxFx): Z+0/1/2j.

MC@NLO: Z+0j.

PS-accurate observable.

(III) future directions: NNLO matching

NNLO + parton shower

- some approaches already exist, most exploiting NLO merged calculations.
- e.g. consider H and H+jet merged (both NLO +parton shower):

sample:	total σ	jet 1	jet 2	jet ≥ 3
H NLO+PS, H+jet NLO+PS, merged.	$\sim \sigma_{NLO}$	**NLO**	**LO**	**PS**
H NNLO+PS.	$\sim \sigma_{NNLO}$	**NLO**	**LO**	**PS**

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H NNLO+PS.	**~ σ_{NNLO}**	**NLO**	**LO**	**PS**

['MiNLO': Hamilton, Nason, Re, Zanderighi, 1309.0017, Hamilton, Nason, Zanderighi, 1501.04637]

achieve by reweighing

+ [Höche, Prestel, 1506.05057] [Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi, 1311.0286, Alioli, Bauer, Bergrren, Tackmann, Walsh, 1508.01475]

open questions & challenges

- accuracy and uncertainties of NLO+PS merging a debated matter.

- no NNLO matching for QCD final states: multi-jets, top pairs, etc.

- do we need to develop next-to-leading-log (or even NNLL) showers for 'proper' NNLO matching?

- technically challenging, computationally intensive.

➡ we are a long way from N^nLO+PS generalized/ 'automated' matching for $n \geq 1$, but certainly *on* the way.

summary

- **PS** can be systematically improved with **FO** calculations.
- three major methods with some overlap in philosophy:
  - Matrix-element corrections: oldest scheme for simple processes in PS,
  - Matching: MC@NLO, POWHEG,
  - Merging: tree-level, or at NLO.

- **Matrix element corrections:**
  - Pythia (PLB 185 (1987) 435, NPB 289 (1987) 810, PLB 449 (1999) 313, NPB 603 (2001) 297)\ \
  - Herwig (CPC 90 (1995) 95)
  - Vincia (Phys.Rev. D78 (2008) 014026, Phys.Rev. D84 (2011) 054003, Phys.Rev. D85 (2012) 014013, Phys.Lett. B718 (2013) 1345-1350, Phys.Rev. D87 (2013) 5, 054033, JHEP 1310 (2013) 127)
- **NLO matching:**
  - POWHEG: JHEP 0411 (2004) 040, JHEP 0711 (2007) 070, POWHEG-BOX (JHEP 1006 (2010) 043)
  - MC@NLO:
  - Original (JHEP 0206 (2002) 029), Herwig++ (Eur.Phys.J. C72 (2012) 2187)\ \
  - Sherpa (JHEP 1209 (2012) 049), MC@NLO (arXiv:1405.0301)\ \
- **Tree-level merging:**
  - MLM (Mangano, http://www-cpd.fnal.gov/personal/mrenna/tuning/nov2002/mlm.pdf. Talk presented at the Fermilab ME/MC Tuning Workshop, Oct 4, 2002, Mangano et al. JHEP 0701 (2007) 013)
  - Pseudoshower (JHEP 0405 (2004) 040)
- CKKW (JHEP 0111 (2001) 063, JHEP 0208 (2002) 015)
- CKKW-L (JHEP 0205 (2002) 046, JHEP 0507 (2005) 054, JHEP 1203 (2012) 019)

[based on lectures given by S. Prestel at "School on QCD and LHC Physics", Sao Paolo, July 2015]

(IV) appendices

MC@NLO for "toy model"

- the MC@NLO method removes the double counting by **modifying the NLO subtraction.** [Frixione, Webber, hep-ph/0204244]

- start with a toy model for radiation of a particle of energy $\mathbf{0 \leq x \leq 1}$:

  [a is equivalent to QCD α_S]

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_B = B\delta(x) \longrightarrow \text{"Born"} = \text{LO}$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_V = a\left(\frac{B}{2\epsilon} + V\right)\delta(x) \longrightarrow$$

virtual correction [ϵ: parameter entering dimensional regularization]

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_R = a\frac{R(x)}{x} \longrightarrow$$

real emission: $\lim_{x\to 0} R(x) = B$

MC@NLO for "toy model"

- an **NLO (fixed-order)** observable ***O(x)*** is then given by:

$$\langle O \rangle = \lim_{\epsilon \to 0} \int_0^1 \mathrm{d}x \; x^{-2\epsilon} O(x) \left[\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x} \right)_B + \left(\frac{\mathrm{d}\sigma}{\mathrm{d}x} \right)_V + \left(\frac{\mathrm{d}\sigma}{\mathrm{d}x} \right)_R \right]$$

- main technical problem: due to regularising parameter, ***ε***.

- extract the pole in *ε* from real in order to cancel the one from virtual: to have an efficient numerical procedure.

- i.e. we must make the integrands **separately** finite.

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[phase-space factor from dimensional regularisation]

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“toy model” NLO subtraction

- rewrite **real contribution as**:

$$\langle O \rangle_R = \lim_{\epsilon \to 0} \int_0^1 \mathrm{d}x \; x^{-2\epsilon} O(x) \left[a \frac{BO(0)}{x} + a \frac{R(x)O(x) - BO(0)}{x} \right]$$

add & subtract

$$= \lim_{\epsilon \to 0} \left[-a \frac{BO(0)}{2\epsilon} + \int_0^1 \mathrm{d}x \; x^{-2\epsilon} a \frac{R(x)O(x) - BO(0)}{x} \right]$$

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no singularities! set $\varepsilon = 0$

CERN

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cancels pole in virtual! no singularities! set ***ε = 0***

$$\Rightarrow \ \langle O \rangle_{\mathrm{sub}} = BO(0) + a \left[VO(0) + \int_0^1 \mathrm{d}x \frac{R(x)O(x) - BO(0)}{x} \right]$$

effect of the PS

- for ease of use in Monte Carlo, rewrite the **NLO-subtracted** observable as:

$$\langle O \rangle_{\text{sub}} = \int_0^1 \mathrm{d}x \left[O(x) \frac{aR(x)}{x} + O(0) \left(B + aV - \frac{aB}{x} \right) \right]$$

- given a LO configuration: $\langle O \rangle_{\text{LO}} = BO(0)$

- the **PS** produces a configuration:

$$\langle O \rangle_{\text{MC@LO}} = BI_{\text{MC}}(O, x_M = 1)$$

total energy of the system (= 1 for LO)

- i.e. the PS performs substitution: $O(x) \longrightarrow I_{\text{MC}}(O, x_M(x))$

NLO matching: naive subtraction

- "naive" subtraction: substitution $O(x) \longrightarrow I_{\rm MC}(O, x_M(x))$ in NLO-subtracted expectation value:

$$\langle O \rangle_{\rm naive} = \int_0^1 {\rm d}x \left[I_{\rm MC}(O, x_M(x)) \frac{aR(x)}{x} + I_{\rm MC}(O, 1) \left(B + aV - \frac{aB}{x} \right) \right]$$

- suggests the following algorithm:

✳ pick at random $\mathbf{0 \leq x \leq 1}$,

✳ generate MC "event" with $x_M(x)$ available to the 1st branching, has weight according to the 1st term: $aR(x)/x$,

✳ generate MC "counter-event" with $x_M=1$ and weight according to 2nd term: $B + aV - aB/x$

NLO matching: naive subtraction

- "naive" subtraction (as the name suggests) fails because:
  - ✳ individual weights diverge,
  - ✳ issue of double counting: equivalent to the schematic diagrams drawn earlier: I_{MC} contains terms included in the real radiation.
- **modified subtraction** amends the above:

$$\langle O \rangle_{\text{mod}} = \int_0^1 \mathrm{d}x \left[I_{\text{MC}}(O, x_M(x)) \frac{a[R(x) - BQ(x)]}{x} + I_{\text{MC}}(O,1) \left(B + aV - \frac{a[BQ(x) - 1]}{x} \right) \right]$$

add & subtract

- the function ***Q(x)/x*** is the **splitting function**! corresponding to PS Sudakov form factor:

$$\Delta(x_1, x_2) = \exp\left[-a \int_{x_1}^{x_2} \mathrm{d}z \frac{Q(z)}{z} \right]$$

NLO matching: naive subtraction

- **modified subtraction:**

$$\langle O\rangle_{\mathrm{mod}} = \int_0^1 \mathrm{d}x \left[I_{\mathrm{MC}}(O, x_M(x)) \frac{a[R(x) - BQ(x)]}{x} \right.$$
$$\left. + I_{\mathrm{MC}}(O,1)\left(B + aV - \frac{a[BQ(x) - 1]}{x} \right) \right]$$

- a property of ***Q***: $\lim_{z\to 0} Q(z) = 1$

- integrands for "events" and "counter-events" are now separately finite.

- left as an exercise: show that that the double counting vanishes.

[Frixione, Webber, hep-ph/0204244]

QCD MC@NLO

- **truth is stranger than fiction**: (i.e. QCD is more complicated than toy model)
  - initial-state collinear divergences need to be subtracted as well (related to the parton densities).
  - colour structure of the emissions needs to be taken into account.
  - subtleties with phase-space mapping between the different configurations.

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see NLO+PS matching lectures for more details!

$\Delta\phi(W^+W^-)$

[Frixione, Webber, hep-ph/0204244]

$$\frac{\pi - \Delta\phi(W^+W^-)}{\pi}$$