Contents

☀ Before the coffee break

  ☀ Ingredients to a NLO calculation

  ☀ Real-emission and Virtual corrections

☀ After the coffee break

  ☀ NLO event generation and aMC@NLO

  ☀ FxFx merging

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Master equation for hadron colliders

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \ f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, d\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

Parton density functions

Parton-level (differential) cross section

* Parton-level cross section from matrix elements: model and process dependent

* Parton density (or distribution) functions: process independent

* Differences between colliders given by parton luminosities

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Perturbative expansion

$d\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$ Parton-level cross section

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \ldots\right)$$

Perturbative expansion

$$d\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R) \quad \text{Parton-level cross section}$$

✸ The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

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✳ Including higher corrections improves predictions and reduces theoretical uncertainties

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LO predictions

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PERTURBATIVE EXPANSION

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LO predictions

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NNLO corrections

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LO predictions	NLO corrections	NNLO corrections	NNNLO corrections

※ Including higher corrections improves predictions and reduces theoretical uncertainties

Improved predictions

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \; f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, d\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \dots \right)$$

* Remember, **predictions are inclusive**: also at LO initial state radiation is included via the PDF; final state radiation by the definition of the parton, which represents all final state evolutions

    * Can be made explicit by using a parton shower (which is unitary)

* Due to these approximations, Leading Order predictions can depend strongly on the renormalization and factorization scales

* Including higher order corrections reduces the dependence on these scales

Going NLO

※ At NLO the dependence on the renormalization and factorization scales is reduced

 ※ First order where scale dependence in the running coupling and the PDFs is compensated for via the loop corrections: **first reliable estimate of the total cross section**

 ※ Better description of final state: impact of extra radiation included (e.g. jets can have substructure)

 ※ Opening of additional initial state partonic channels
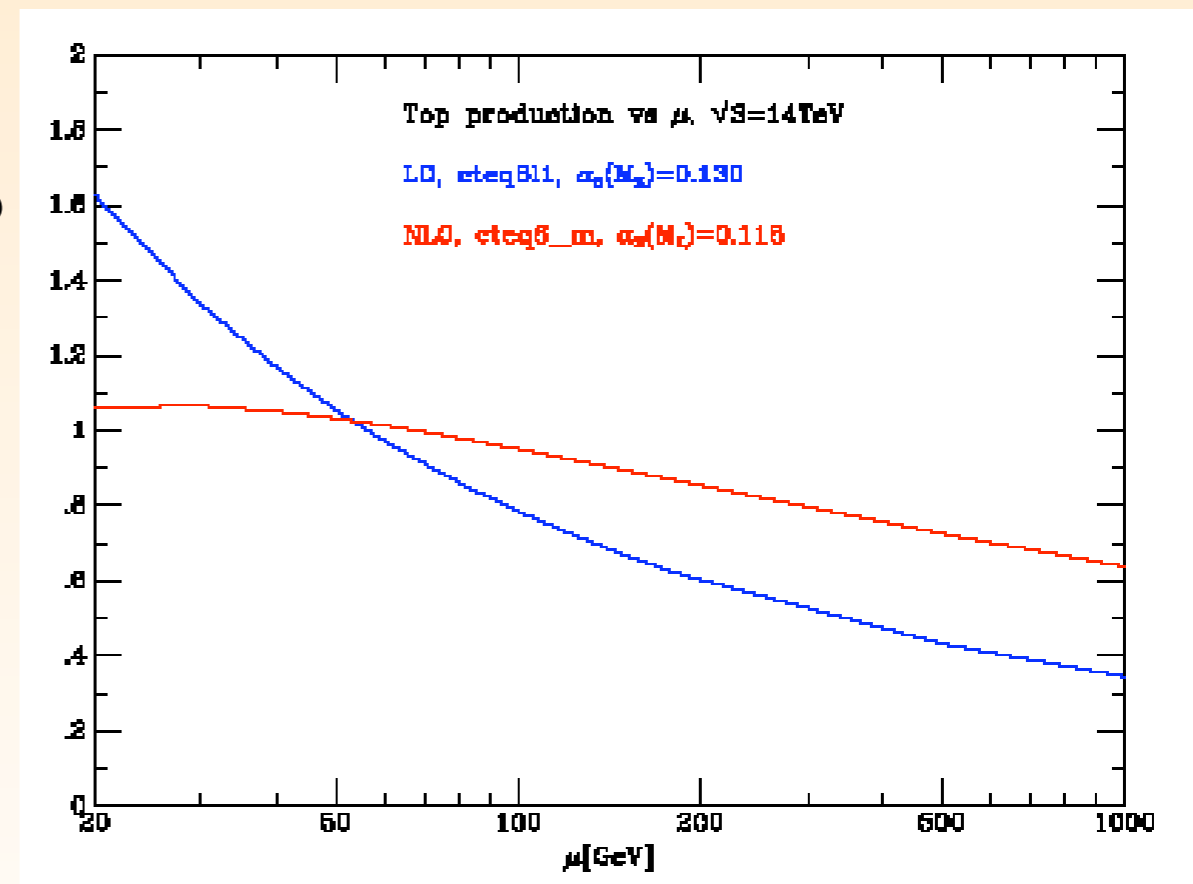
NLO CORRECTIONS

* NLO corrections have three parts:

    * The Born contribution, i.e. the Leading order.

    * Virtual (or Loop) corrections: formed by an amplitude with a closed loop of particles interfered with the Born amplitudes

    * Real emission corrections: formed by amplitudes with one extra parton compared to the Born process

* Both Virtual and Real emission have one power of α_s extra compared to the Born process

$$\sigma^{\mathrm{NLO}} = \int_m d\sigma^B + \int_m d\sigma^V + \int_{m+1} d\sigma^R$$

NLO PREDICTIONS

* As an example, consider Drell-Yan Z/γ^* production

ℓ^+ ℓ^-

$x_1 E$ $x_2 E$
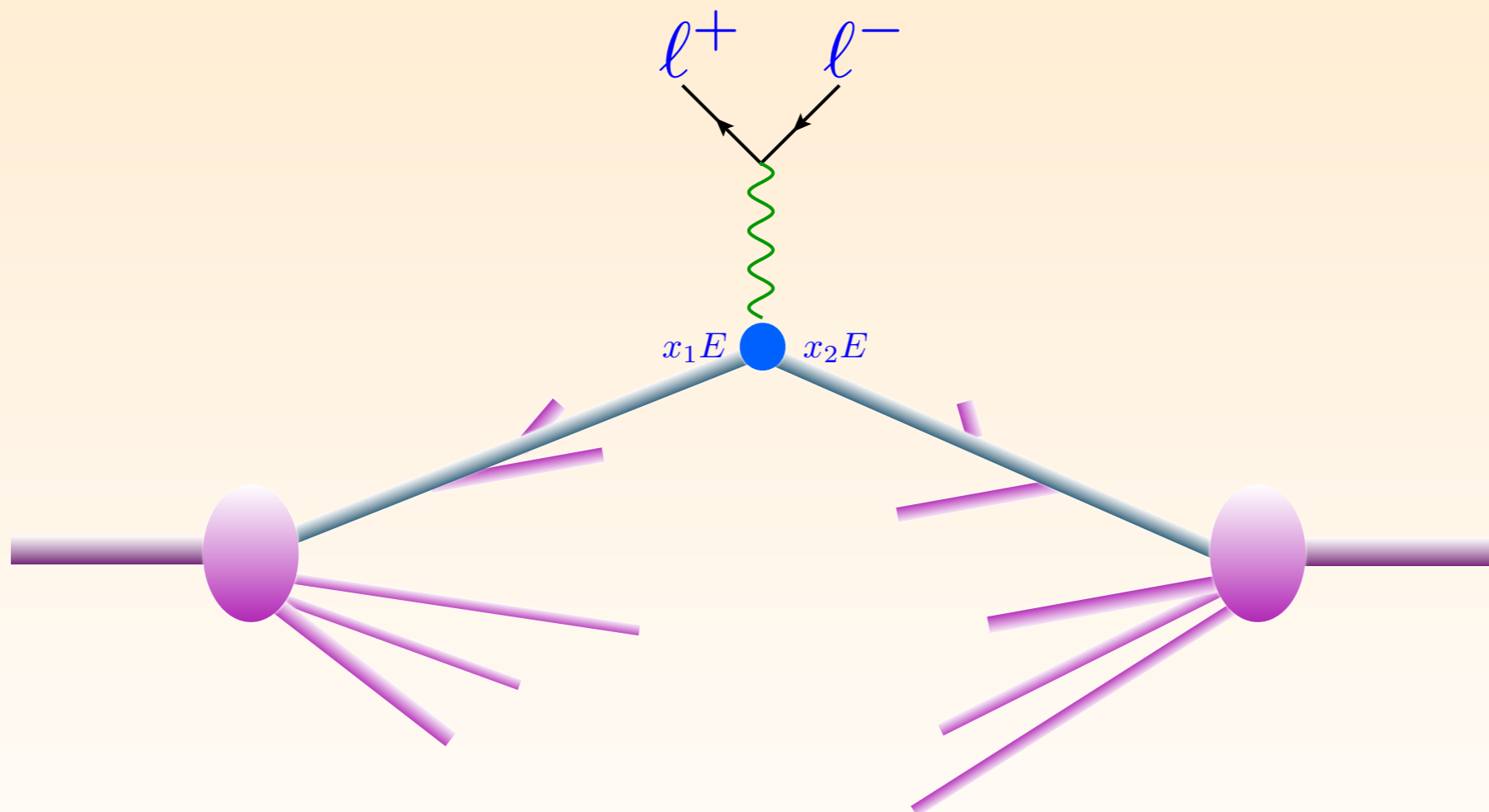
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NLO PREDICTIONS

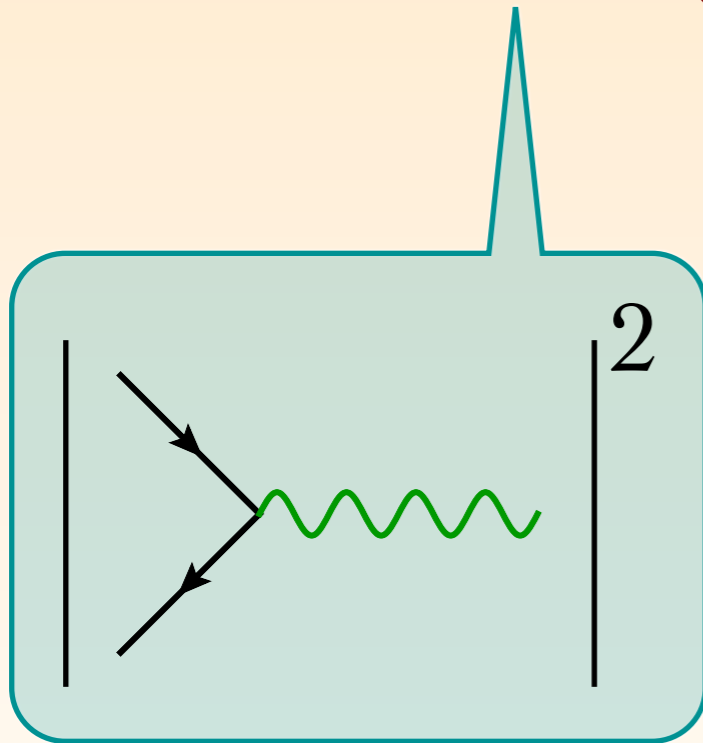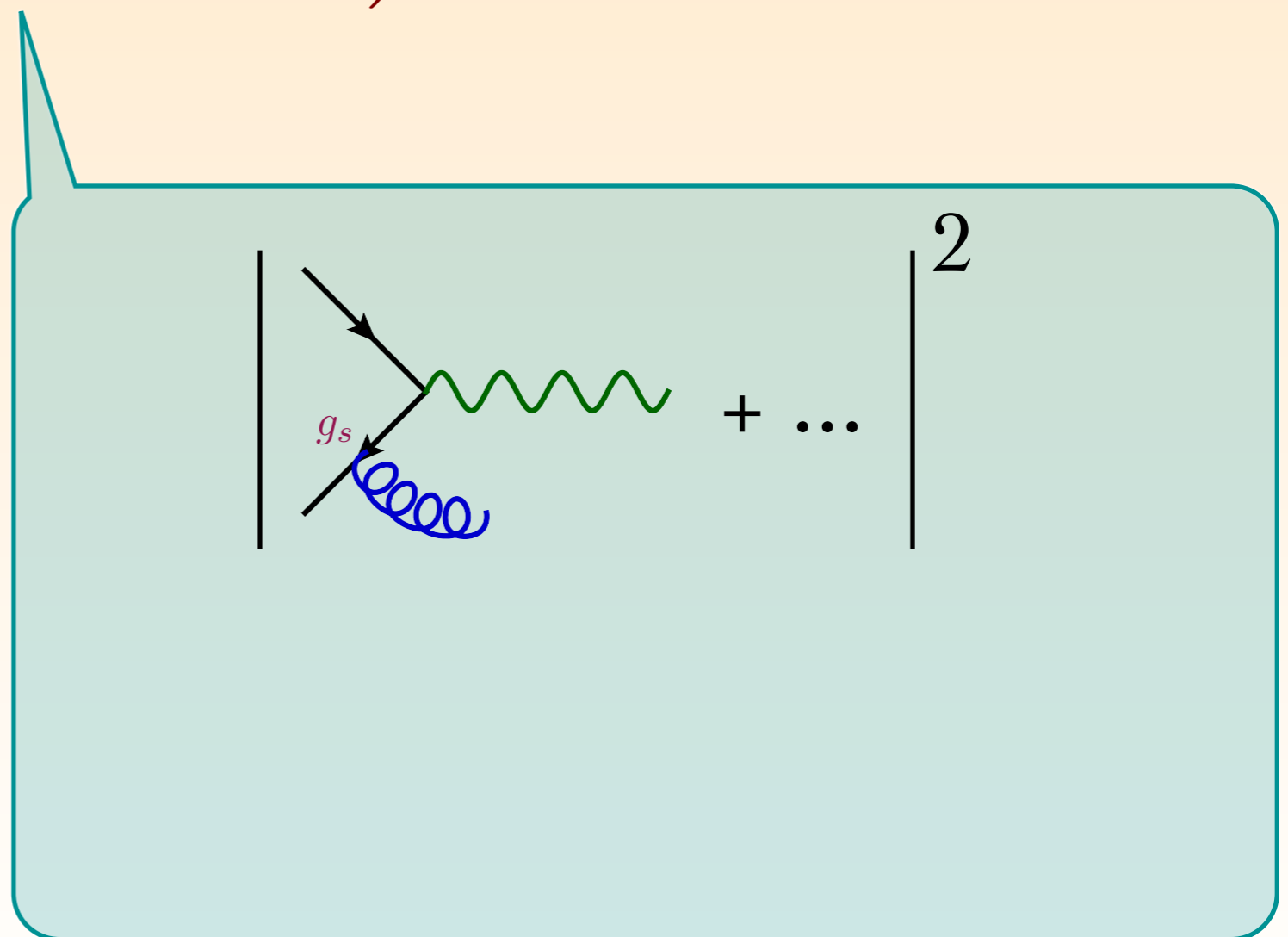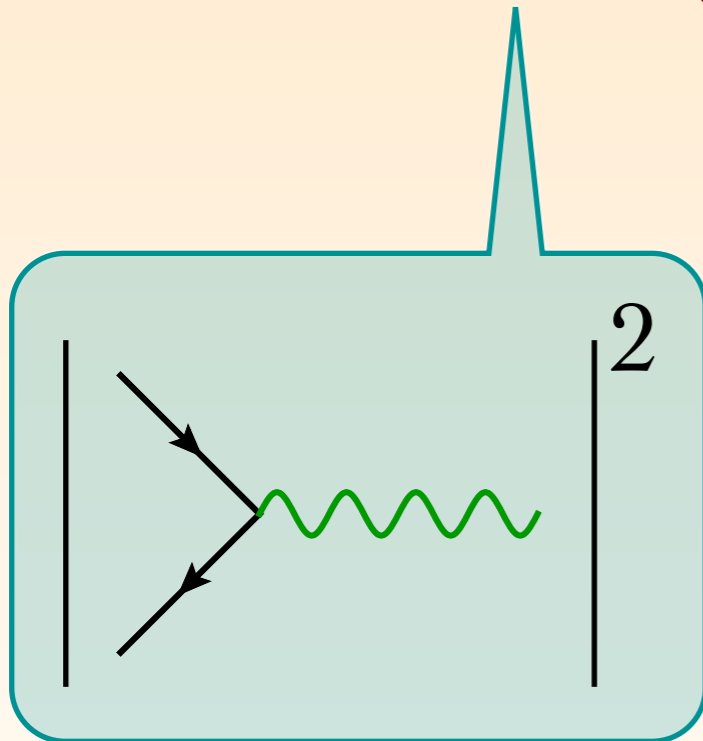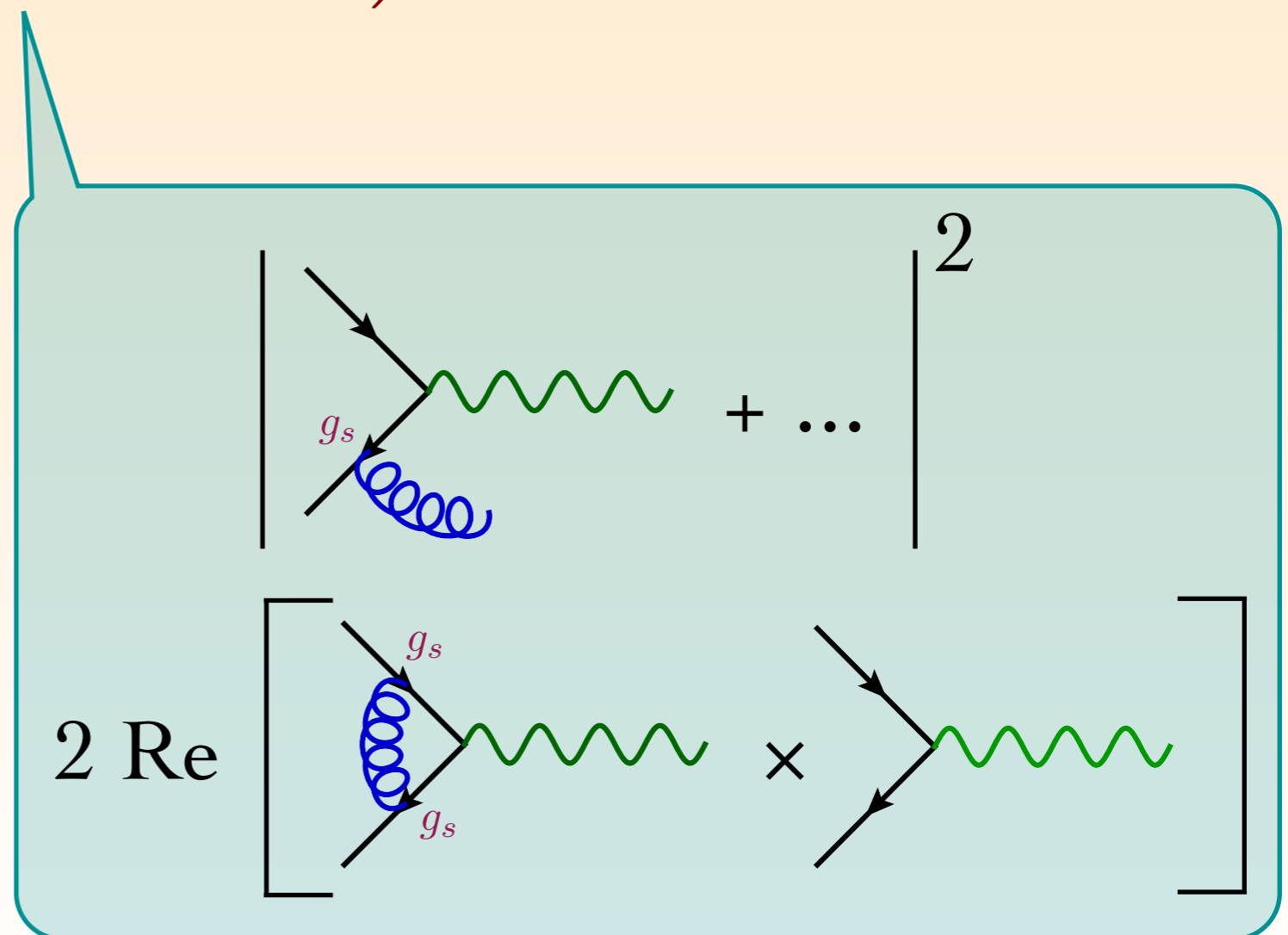* As an example, consider Drell-Yan Z/γ^* production

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \dots \right)$$

NLO PREDICTIONS

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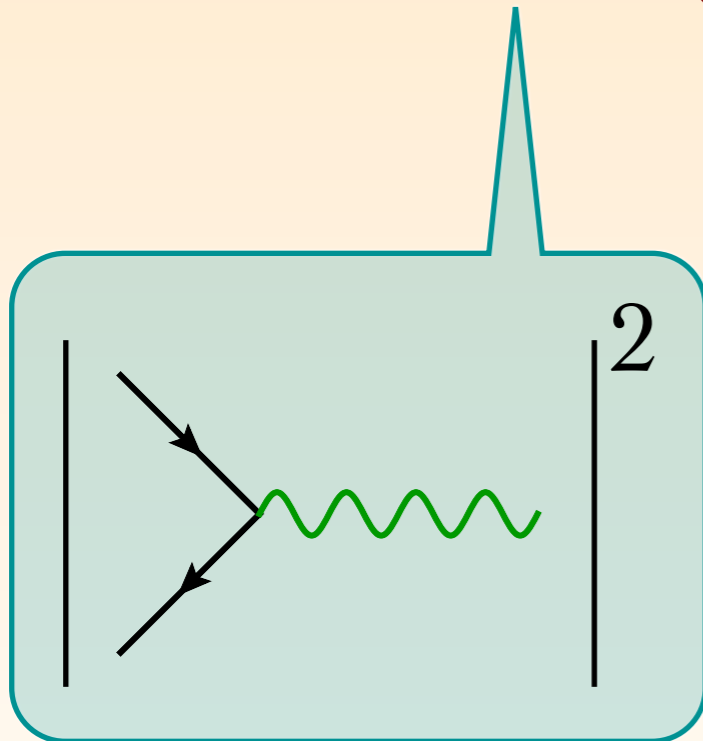$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \dots\right)$$

NLO PREDICTIONS

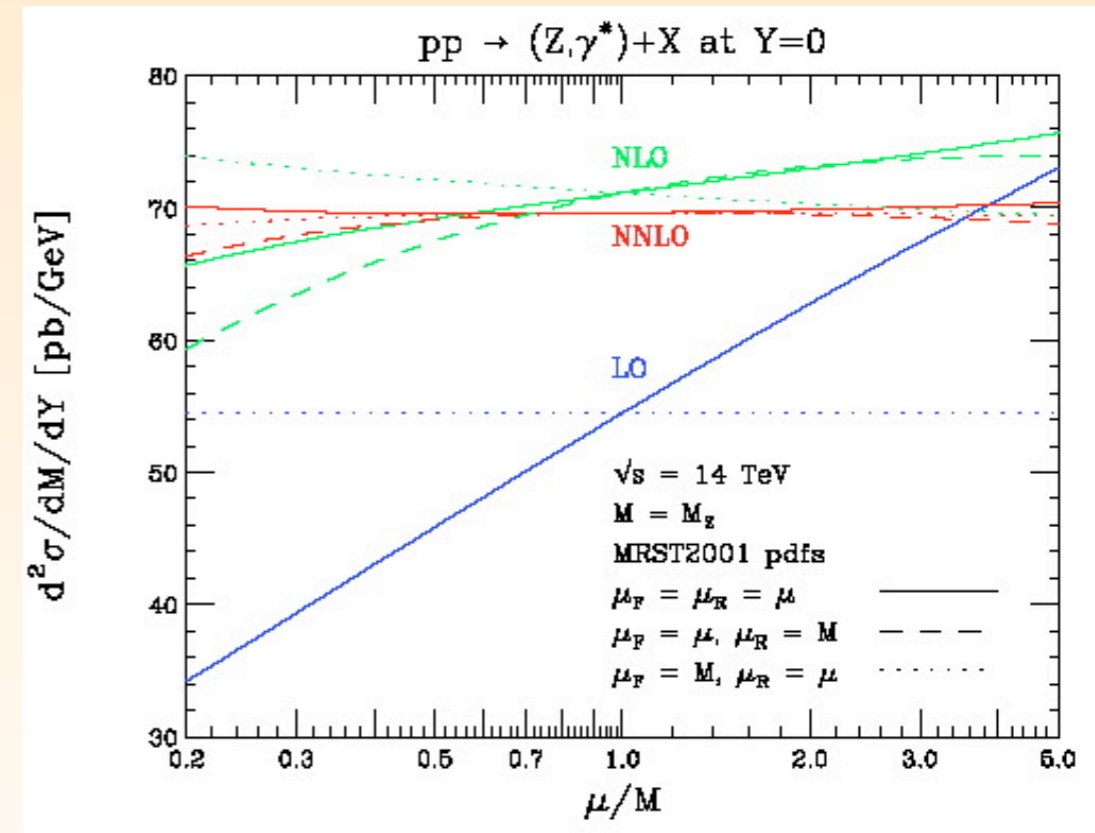✳ As an example, consider Drell-Yan Z/γ^* production

$$\hat{\sigma} = \sigma^{\mathrm{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \dots\right)$$

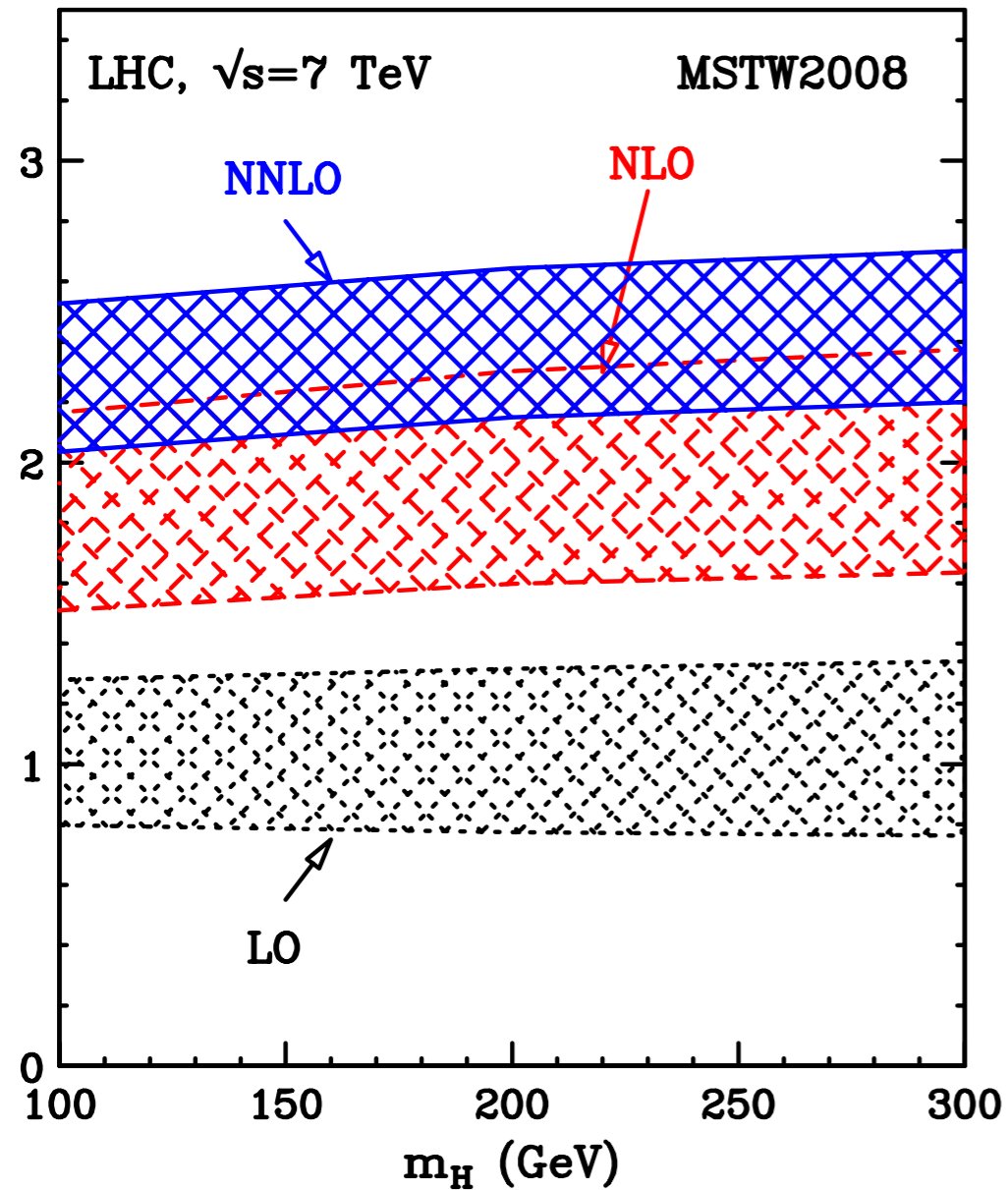# NLO PREDICTIONS

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GOING NNLO...?

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

- NNLO is the current state-of-the-art. There are only a few complete results available, but this year great progress has been made and NNLO results for ttbar, H+1j, dijet appeared

$$\sigma_{ab \to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

- Why do we need it?

  - An NNLO calculation gives control of the uncertainties in a calculation

  - It is "mandatory" if NLO corrections are very large to check the behavior of the perturbative series

  - It is the best we have! It is needed for Standard Candles and very precise tests of perturbation theory, exploiting all the available information, e.g. for determining NNLO PDF sets
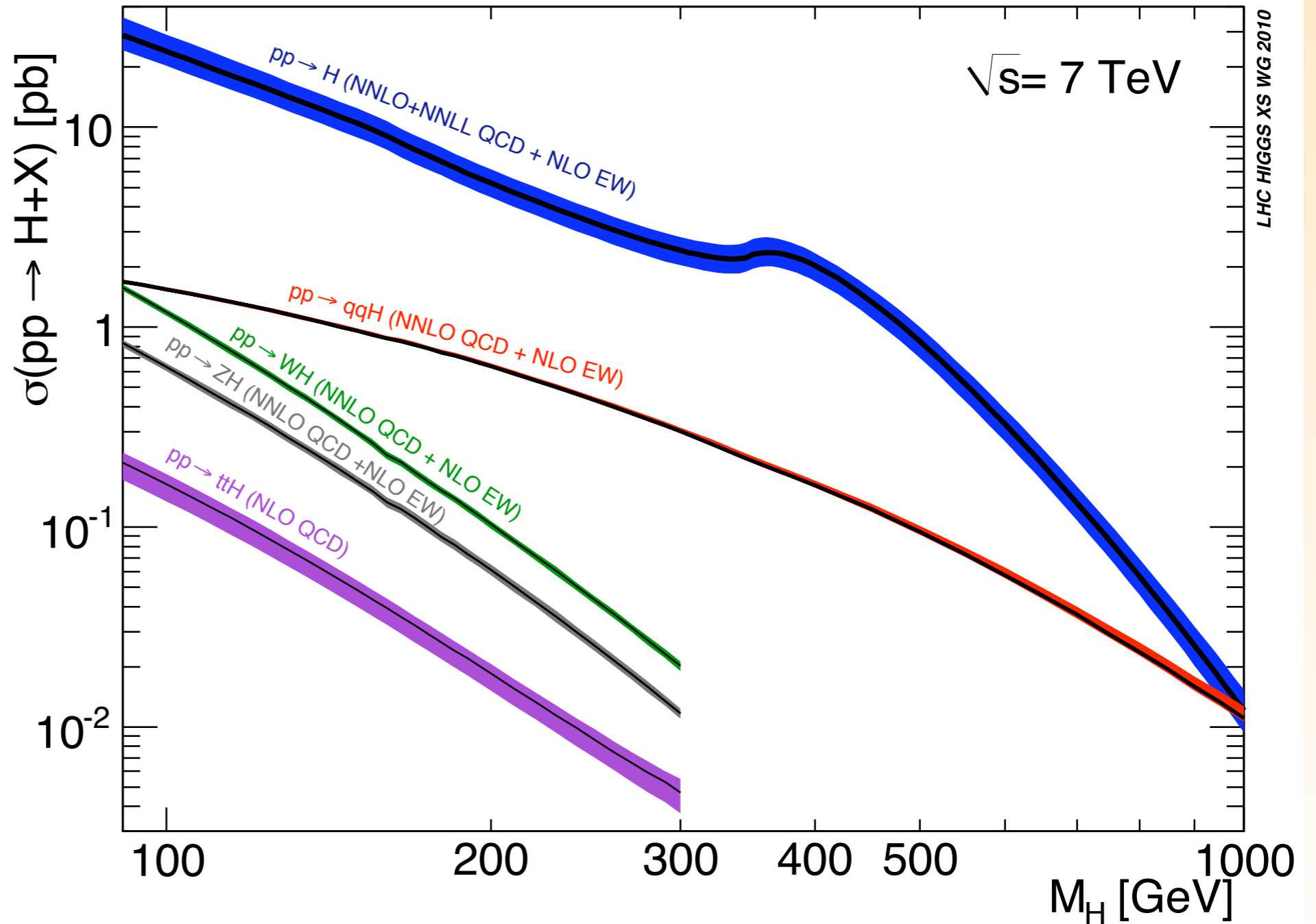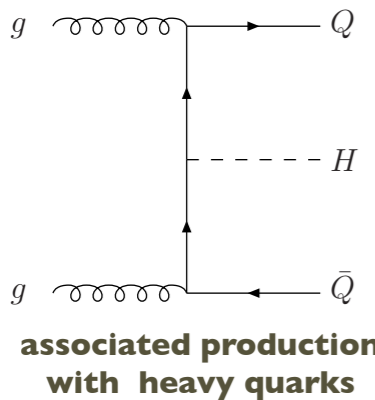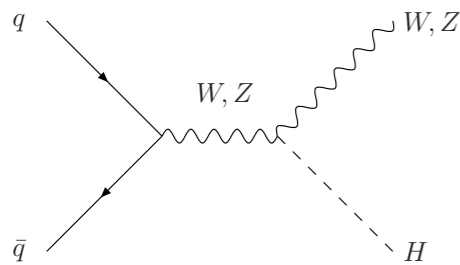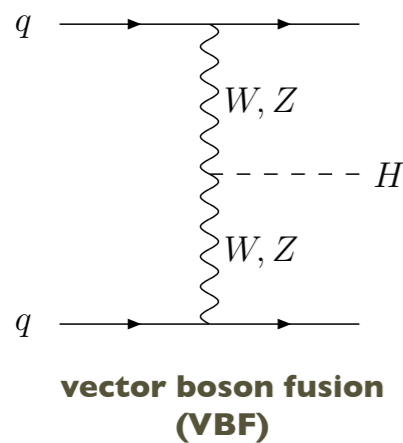

pp → (Z,γ*)+X at Y=0

NLO
NNLO
LO

$\sqrt{s}$ = 14 TeV
M = M_z
MRST2001 pdfs
$\mu_F = \mu_R = \mu$ ———
$\mu_F = \mu, \mu_R = M$ — — —
$\mu_F = M, \mu_R = \mu$ ·········

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Higgs predictions at NNLO

LHC, √s=7 TeV MSTW2008

NNLO NLO

LO

m_H (GeV)
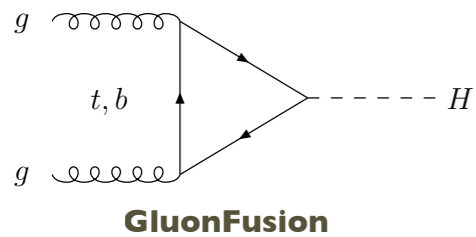
- ❋ LO calculation is not reliable,

- ❋ but the perturbative series stabilizes at NNLO

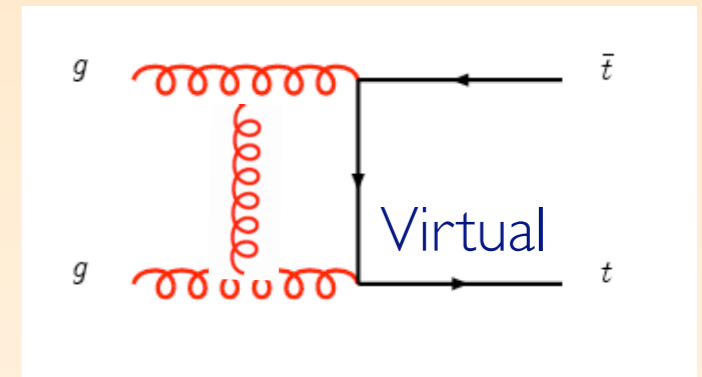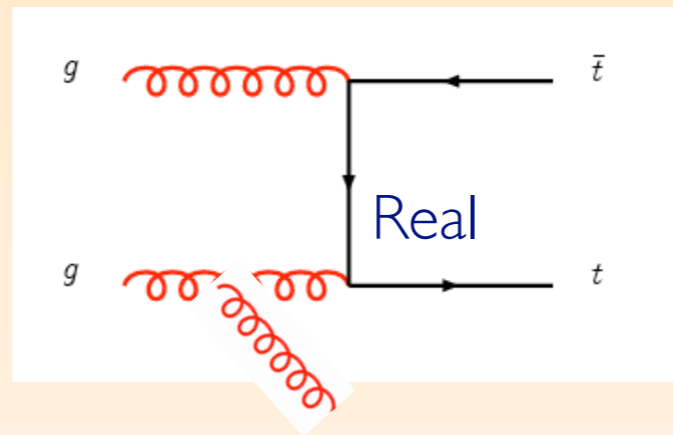- ❋ NLO estimation of the uncertainties (by scale variation) works reasonably well

NNNLO?

NNNLO?

※ Nothing known here... Too complicated!

NLO...?

✳ Are all (IR-safe) observables that we can compute using a NLO code correctly described at NLO? Suppose we have a NLO code for pp → ttbar

✳ Total cross section

✳ Transverse momentum of the top quark

✳ Transverse momentum of the top-antitop pair

✳ Transverse momentum of the jet

✳ Top-antitop invariant mass

✳ Azimuthal distance between the top and anti-top

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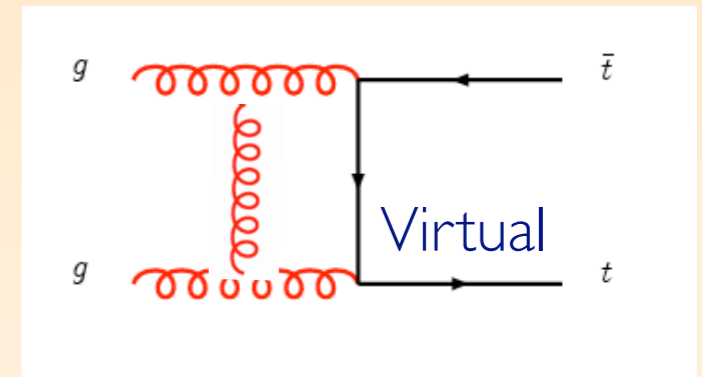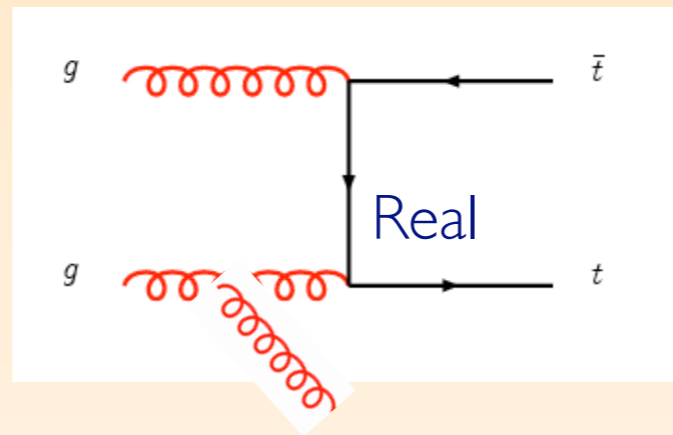
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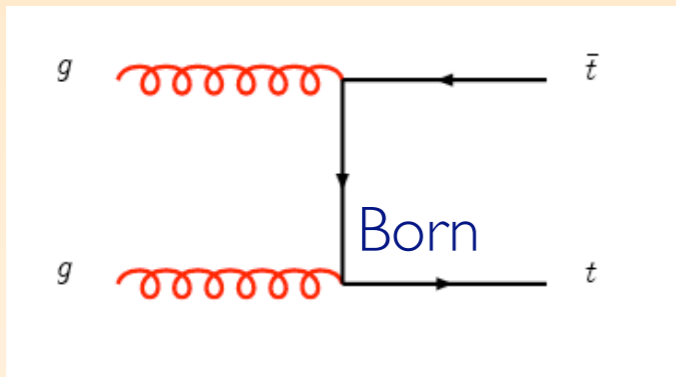
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| | Born | Real | Virtual |

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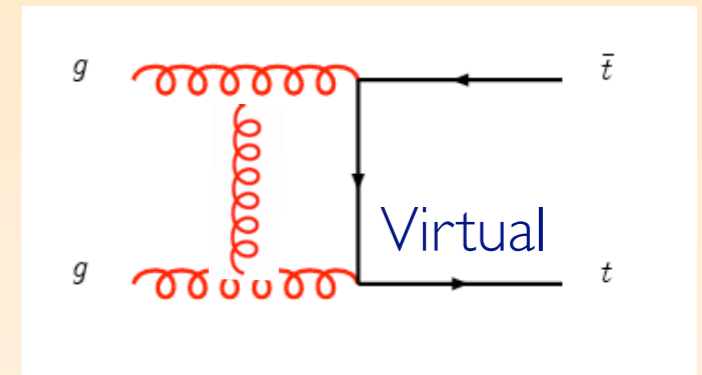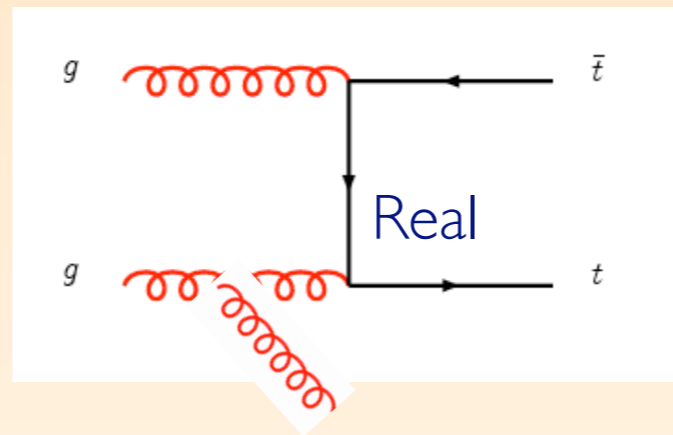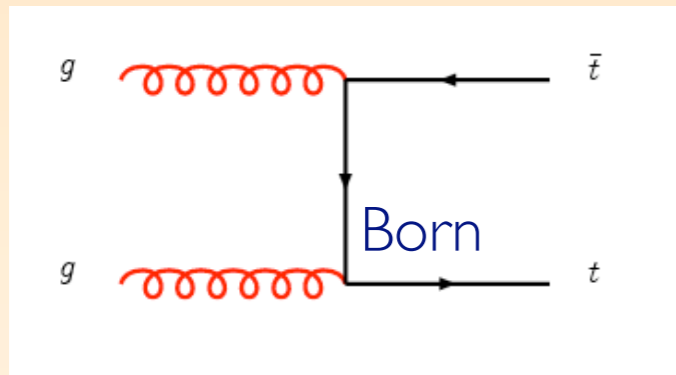

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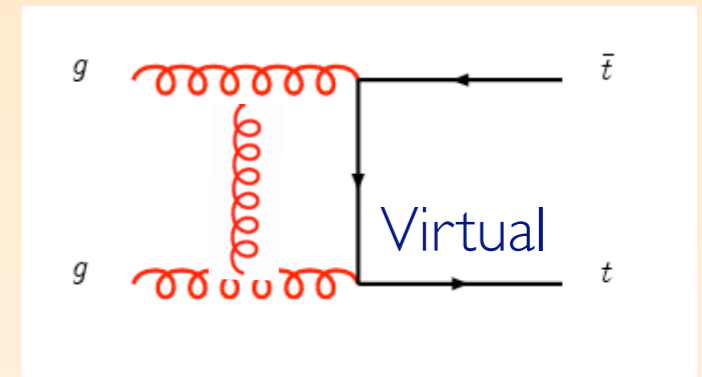

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NLO...?

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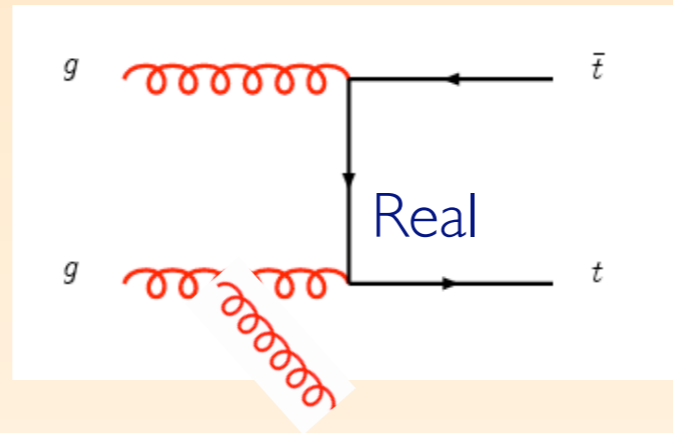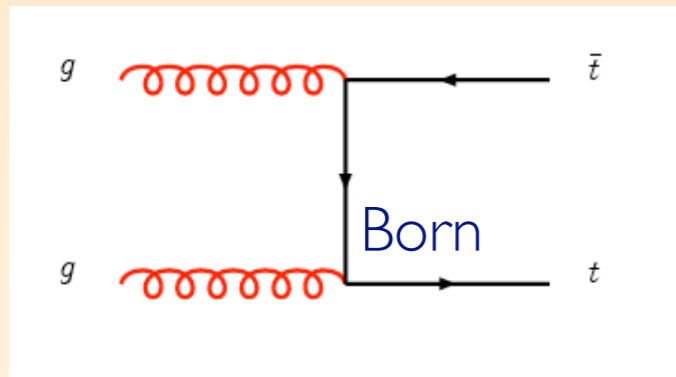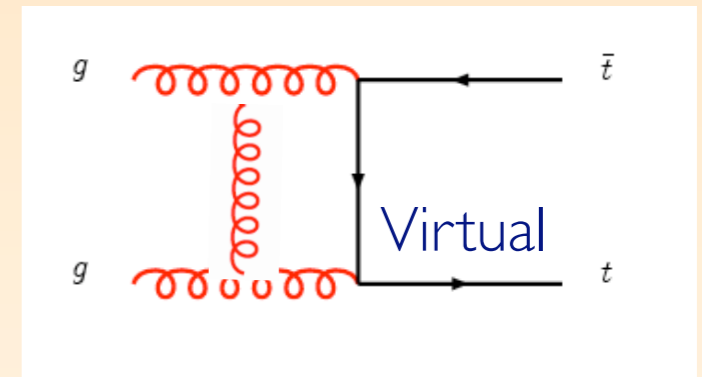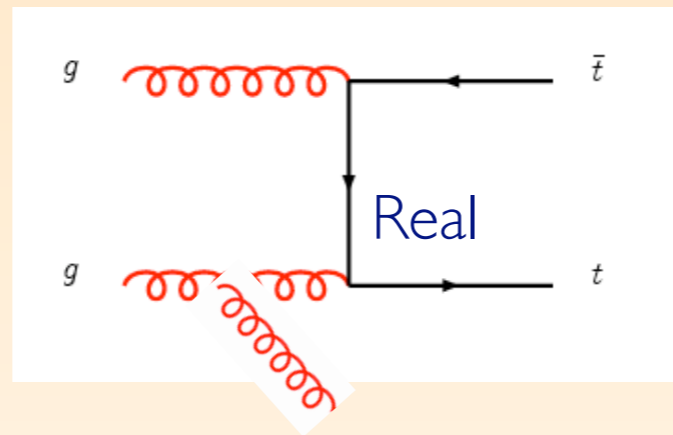

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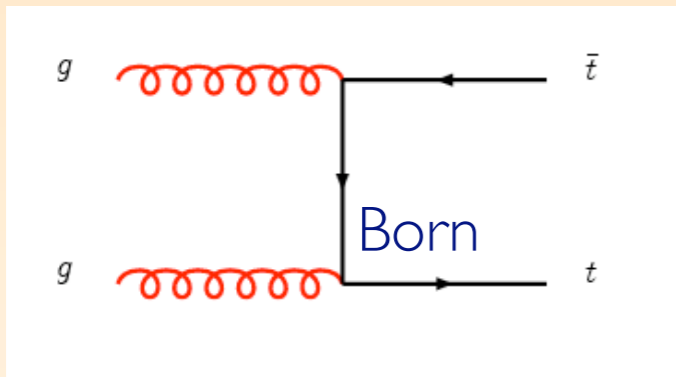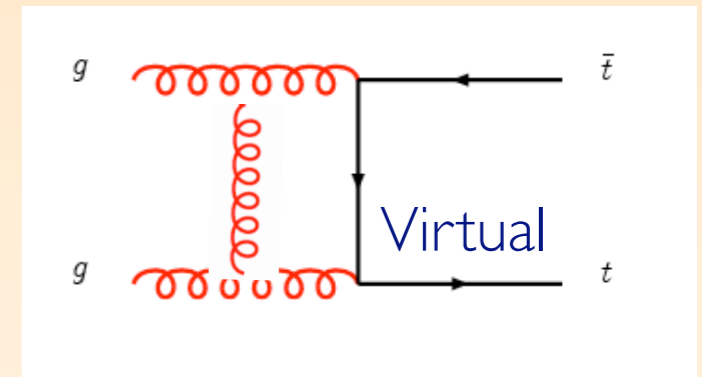

	NLO?
Total cross section	✔
Transverse momentum of the top quark	✔
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Transverse momentum of the jet	✘
Top-antitop invariant mass	✔
Azimuthal distance between the top and anti-top	✘

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Obstacles

$$= \quad + \quad + \quad + \; O(\alpha_s^2)$$

+ anything

* Let us focus on NLO... there are already enough steps to be taken:

  * Virtual amplitudes: how to compute the loops automatically in a reasonable amount of time

  * How to deal with infra-red divergences: virtual corrections and real-emission corrections are separately divergent and only their sum is finite (for IR-safe observables) according to the KLN theorem

  * How to match these processes to a parton shower without double counting

Why an automatic tool?

☀ To save time

Trade human time and expertise on computing one process at the time with time on physics and phenomenology.

☀ Robustness

Modular code structure means that elements can be checked systematically and extensively once and for all. Trust can easily be build.

☀ Wide accessibility

One framework for all. Available to everybody for an unlimited set of applications. Suitable for Experimental collaborations.

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Canceling infrared divergences

NLO PREDICTIONS

❋ As an example, consider Drell-Yan production

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \dots\right)$$
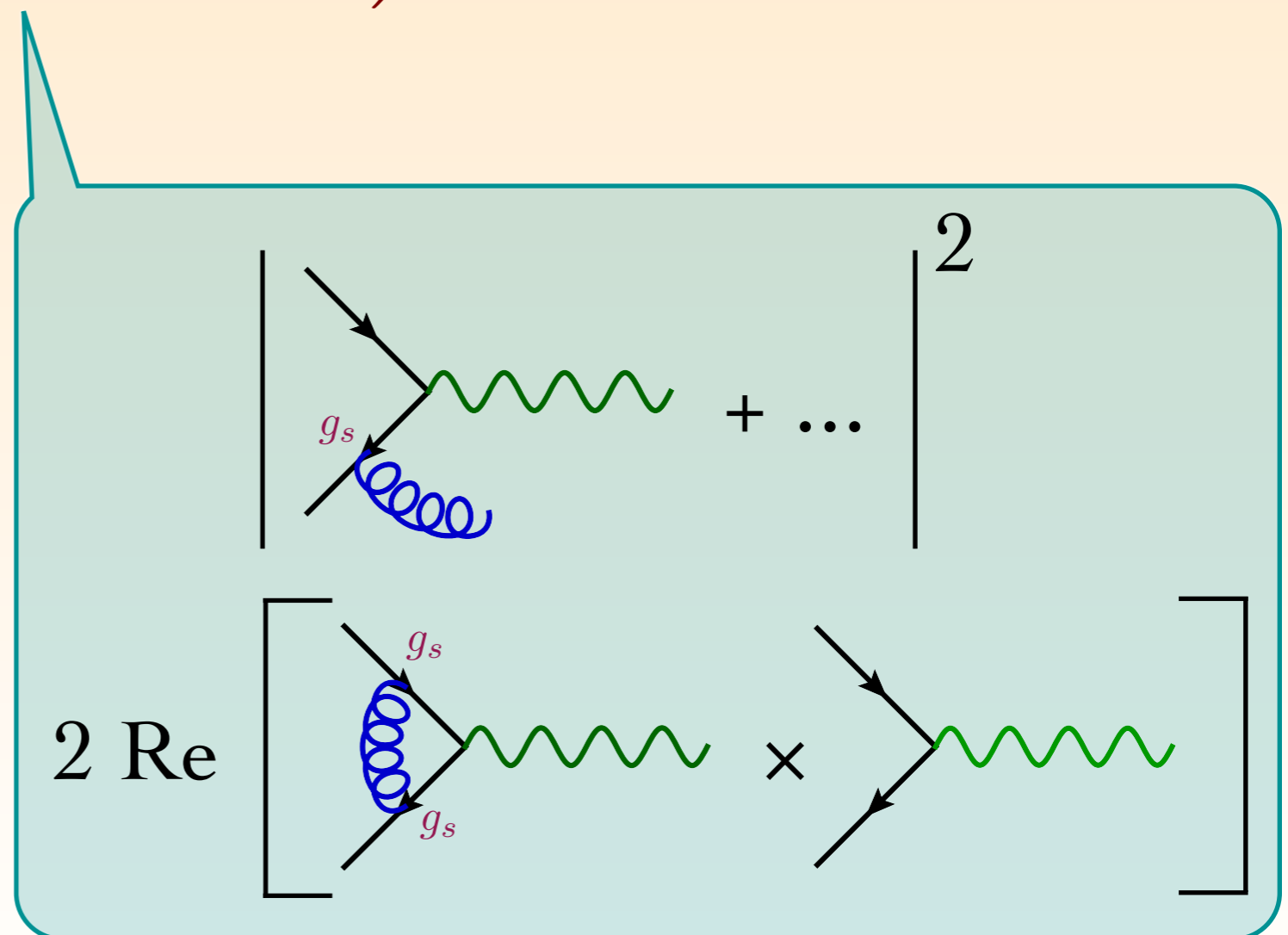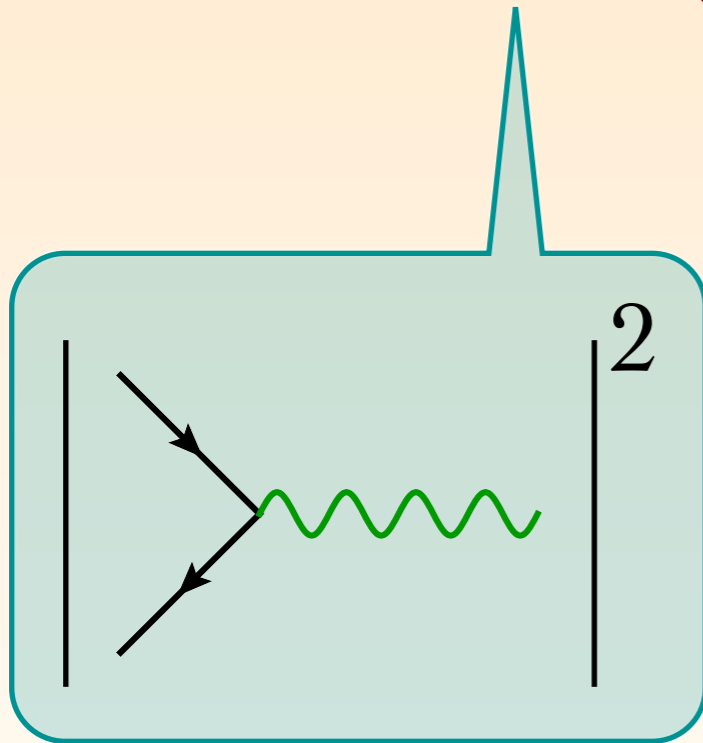
Branching

※ In the soft and collinear region, the branching of a gluon from a quark can be written as
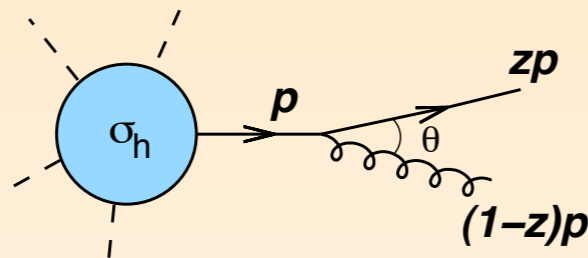
$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

where k_t is the transverse momentum of the gluon, $k_t = E \sin\theta$.

※ The singularities cancel against the singularities in the virtual corrections, which result from the integral over the loop momentum of the function
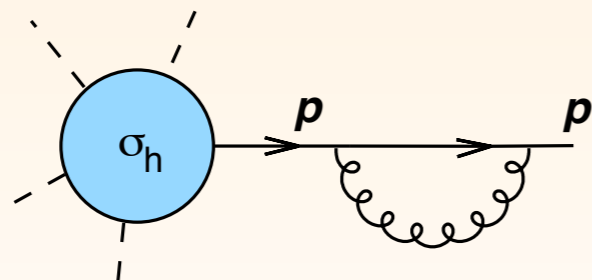
$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

※ The sum is finite for observables that cannot distinguish between two collinear partons ($k_t \longrightarrow 0$); a hard and a soft parton ($z \longrightarrow 1$); and a single parton (in the virtual contributions)

Infrared cancellation

$$\sigma^{\mathrm{NLO}} \sim \int d^4\Phi_m \, B(\Phi_m) + \int d^4\Phi_m \int_{\mathrm{loop}} d^d l \, V(\Phi_m) + \int d^d\Phi_{m+1} \, R(\Phi_{m+1})$$

* The KLN theorem tells us that divergences from virtual and real-emission corrections cancel in the sum for observables insensitive to soft and collinear radiation ("IR-safe observables")

* When doing an analytic calculation in dimensional regularization this can be explicitly seen in the cancellation of the $1/\epsilon$ and $1/\epsilon^2$ terms (with ϵ the regulator, $\epsilon \rightarrow 0$)

* In the real emission corrections, the explicit poles enter after the phase-space integration (in d dimensions)

Infrared safe observables

✤ For an observable to be calculable in fixed-order perturbation theory, the observable should be infrared safe, i.e., it should be insensitive to the emission of soft or collinear partons.

✤ In particular, if p_i is a momentum occurring in the definition of an observable, it most be invariant under the branching

$$p_i \longrightarrow p_j + p_k,$$

whenever p_j and p_k are collinear or one of them is soft.

✤ Examples

   ✤ "The number of gluons" produced in a collision is not an infrared safe observable

   ✤ "The number of hard jets defined using the k_T algorithm with a transverse momentum above 40 GeV," produced in a collision is an infrared safe observable

PHASE-SPACE INTEGRATION

$$\sigma^{\mathrm{NLO}} \sim \int d^4\Phi_m \, B(\Phi_m) + \int d^4\Phi_m \int_{\mathrm{loop}} d^d l \, V(\Phi_m) + \int d^d\Phi_{m+1} \, R(\Phi_{m+1})$$

- ✳ For complicated processes we have to result to numerical phase-space integration techniques ("Monte Carlo integration"), which can only be performed in an integer number of dimensions

    - ✳ Cannot use a finite value for the dimensional regulator and take the limit to zero in a numerical code

- ✳ But we still have to cancel the divergences explicitly

- ✳ Use a subtraction method to explicitly factor out the divergences from the phase-space integrals

Example

* Suppose we want to compute the integral ("real emission radiation", where the 1-particle phase-space is referred to as the 1-dimensional x)

$$\int_0^1 dx\, f(x) \qquad \left| \begin{array}{c} \end{array} + ... \right|^2$$
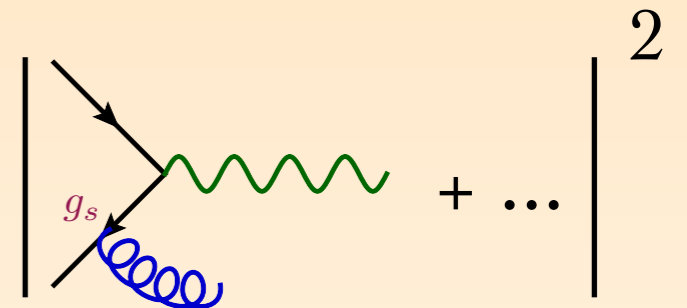
where $f(x) = \dfrac{g(x)}{x}$ and $g(x)$ is finite everywhere

* Let's introduce a regulator

$$\lim_{\epsilon \to 0} \int_0^1 dx\, \frac{g(x)}{x^{1+\epsilon}} = \lim_{\epsilon \to 0} \int_0^1 dx\, x^{-\epsilon} f(x)$$

for any non-integer non-zero value for ϵ this integral is finite

* We would like to factor out the explicit poles in ϵ so that they can be canceled explicitly against the virtual corrections

Subtraction method

$$\lim_{\epsilon \to 0} \int_0^1 dx \, x^{-\epsilon} f(x) \qquad\qquad f(x) = \frac{g(x)}{x}$$

※ Add and subtract the same term

$$\lim_{\epsilon \to 0} \int_0^1 dx \, x^{-\epsilon} f(x) = \lim_{\epsilon \to 0} \int_0^1 dx \, x^{-\epsilon} \left[\frac{g(0)}{x} + f(x) - \frac{g(0)}{x} \right]$$

$$= \lim_{\epsilon \to 0} \int_0^1 dx \left[g(0) \frac{x^{-\epsilon}}{x} + \frac{g(x) - g(0)}{x^{1+\epsilon}} \right]$$

$$= \lim_{\epsilon \to 0} \frac{-1}{\epsilon} g(0) + \int_0^1 dx \, \frac{g(x) - g(0)}{x}$$

※ We have factored out the $1/\epsilon$ divergence and are left with a finite integral

※ According to the KLN theorem the divergence cancels against the virtual corrections

Limitations

Subtraction: $$\int_0^1 dx \, \frac{g(x) - g(0)}{x}$$

"Plus distribution"

✸ Even though the divergence is factored, there are cancellations between large numbers: if for an observable O, if $\lim_{x \to 0} O(x) \neq O(0)$ or we choose the bin-size too small, instabilities render the computation useless

✸ We already knew that! KLN is sufficient; one must have infra-red safe observables and cannot ask for infinite resolution (need a finite bin-size)

NLO with Subtraction

$$\sigma^{\mathrm{NLO}} \sim \int d^4 \Phi_m \, B(\Phi_m) + \int d^4 \Phi_m \int_{\mathrm{loop}} d^d l \, V(\Phi_m) + \int d^d \Phi_{m+1} \, R(\Phi_{m+1})$$

✳ With the subtraction method this is replace by

$$\sigma^{\mathrm{NLO}} \sim \int d^4 \Phi_m \, B(\Phi_m)$$

$$+ \int d^4 \Phi_m \left[\int_{\mathrm{loop}} d^d l \, V(\Phi_m) + \int d^d \Phi_1 G(\overline{\Phi}_{m+1}) \right]_{\epsilon \to 0}$$

$$+ \int d^4 \Phi_{m+1} \left[R(\Phi_{m+1}) - G(\overline{\Phi}_{m+1}) \right]$$

✳ Terms between the brackets are finite. Can integrate them numerically and independent from one another in 4 dimensions

Subtraction methods

※ $G(\overline{\Phi}_{m+1})$ should be defined such that

1) it exactly matches the singular behavior of $R(\Phi_{m+1})$

2) its form is convenient for numerical integration techniques

3) it is exactly integrable in d dimensions over the one-particle subspace $\int d^d\Phi_1 G(\overline{\Phi}_{m+1})$, leading to soft and/or collinear divergences as explicit poles in the dimensional regulator

4) it is universal, i.e. process independent
   $\rightarrow$ overall factor times the Born process

Two methods

* **Catani-Seymour (CS) dipole subtraction**

  ☑ Most used method

  ☑ Clear written paper on how to use this method in practice

  ☑ Recoil taken by one (color-connected) parton: N^3 scaling

  ☑ Method evolved from cancellation of the soft divergence

  ☑ Proven to work for simple as well as complicated processes

  ☑ Automation in publicly available packages: MadDipole, AutoDipole, Helac-Dipoles, Sherpa

* **Frixione-Kunszt-Signer (FKS) subtraction**

  ☑ Not so well-known

  ☑ (Probably) more efficient, because less subtraction terms are needed

  ☑ Recoil evenly distributed by all particles: N^2 scaling

  ☑ Collinear divergences as a starting point

  ☑ Proven to work for simple as well as complicated processes

  ☑ Automated in aMC@NLO & POWHEG BOX

FKS SUBTRACTION

※ FKS subtraction: Frixione, Kunszt & Signer 1996.
Standard subtraction method in MC@NLO and POWHEG, but
can also be used for 'normal' NLO computations

※ Also known as "residue subtraction"

※ Based on using plus-distributions to regulate the infrared
divergences of the real emission matrix elements

FKS subtraction: Phase-space partitions

* Easiest to understand by starting from real emission:

$$d\sigma^R = |M^{n+1}|^2 d\phi_{n+1}$$

* $|M^{n+1}|^2$ blows up like $\quad \dfrac{1}{\xi_i^2}\dfrac{1}{1-y_{ij}} \quad$ with $\qquad \xi_i = E_i/\sqrt{\hat{s}}$

$$y_{ij} = \cos\theta_{ij}$$

* Partition the phase space in such a way that each partition has at most one soft and one collinear singularity

$$d\sigma^R = \sum_{ij} S_{ij}|M^{n+1}|^2 d\phi_{n+1} \qquad\qquad \sum_{ij} S_{ij} = 1$$

* Use plus distributions to regulate the singularities

$$d\tilde{\sigma}^R = \sum_{ij} \left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i(1-y_{ij})S_{ij}|M^{n+1}|^2 d\phi_{n+1}$$

FKS subtraction: Regularized by plus prescription

$$d\tilde{\sigma}^R = \sum_{ij} \left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i(1-y_{ij})S_{ij}|M^{n+1}|^2 d\phi_{n+1}$$

* Definition plus distribution

$$\int d\xi \left(\frac{1}{\xi}\right)_+ g(\xi) = \int d\xi \frac{g(\xi) - g(0)}{\xi}$$

* One event has maximally three counter events:

  * Soft: $\qquad \xi_i \rightarrow 0$

  * Collinear: $\qquad y_{ij} \rightarrow 1$

  * Soft-collinear: $\qquad \xi_i \rightarrow 0 \qquad y_{ij} \rightarrow 1$

KINEMATICS OF COUNTER EVENTS

Real emission

Subtraction term

i

j

i+j
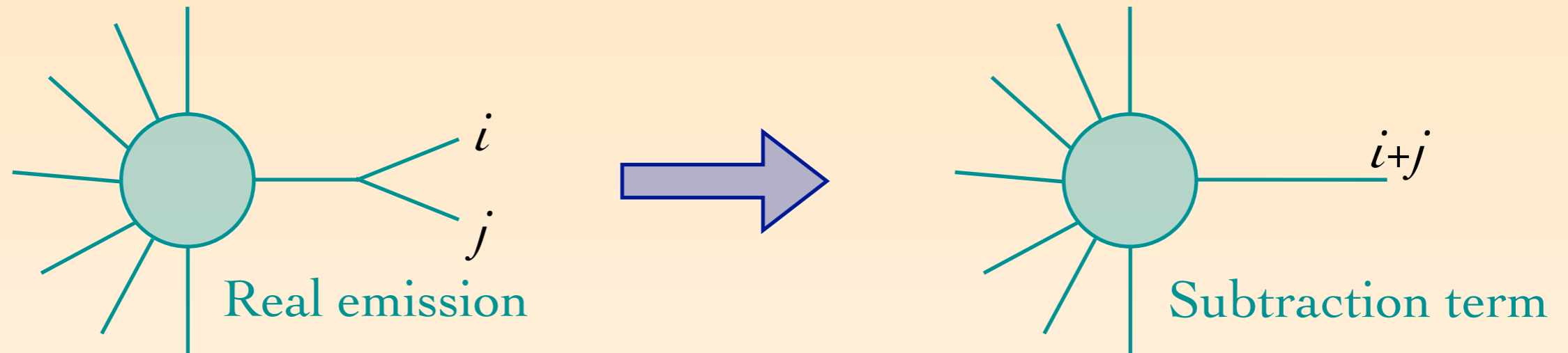
* If *i* and *j* are two on-shell particles that are present in a splitting that leads to an singularity, for the counter events we need to combine their momenta to a new on-shell parton that's the sum of *i+j*

* This is not possible without changing any of the other momenta in the process

* When applying cuts or making plots, events and counter events might end-up in different bins

  * Use IR-safe observables and don't ask for infinite resolution! (KLN theorem)

Example in 4 charged lepton production
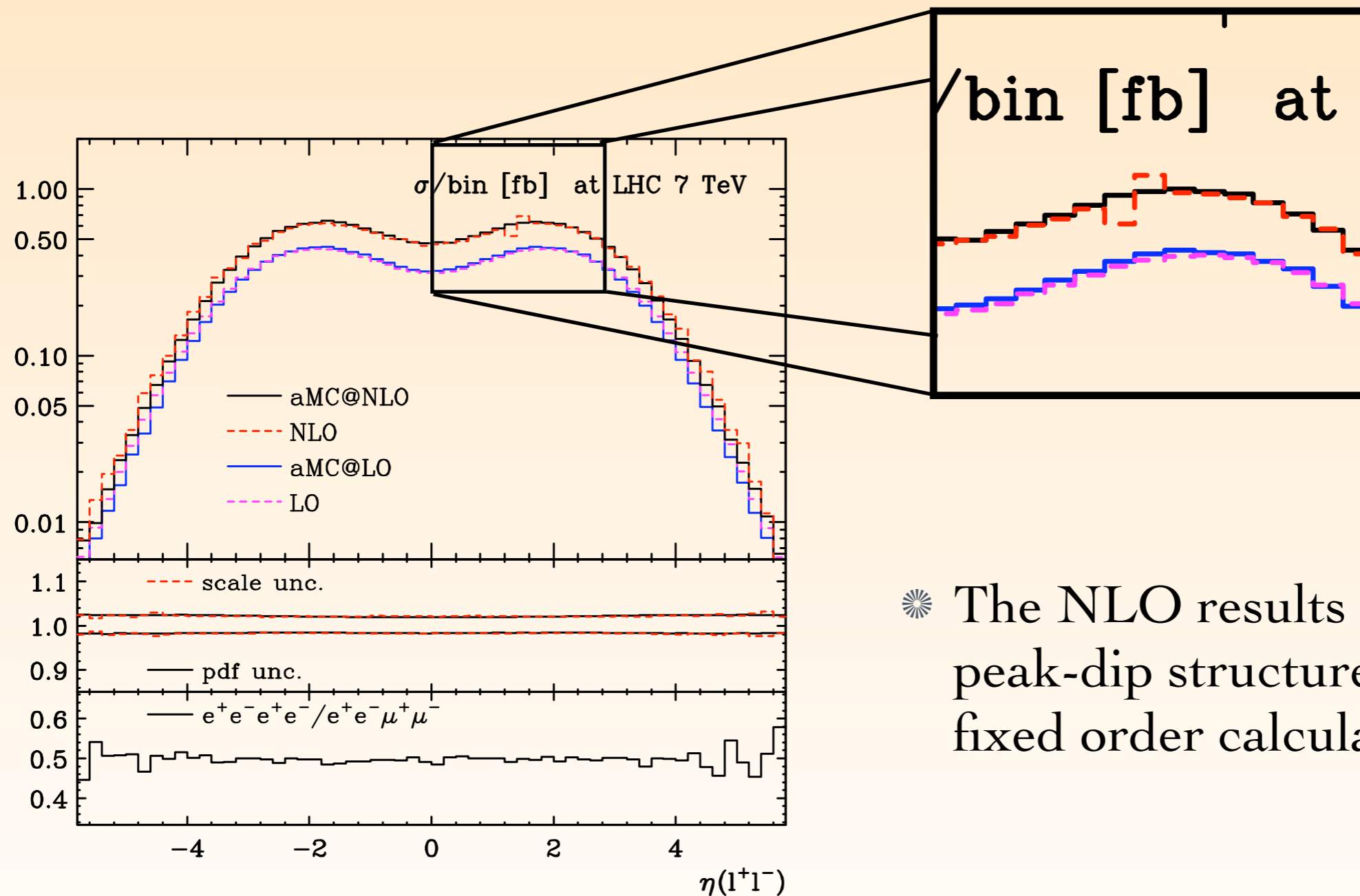
- The NLO results shows a typical peak-dip structure that hampers fixed order calculations

Event unweighting?

※ Another consequence of this kinematic mismatch is that we cannot generate events at fixed order NLO

  ※ Even though the integrals are finite, they are not bounded (compare with $\int_0^1 dx \, \frac{1}{\sqrt{x}}$), so there is no maximum to unweight against: a single event can have an arbitrarily large weight!

※ Furthermore, event and counter event have different kinematics: which one to use for the unweighted event?

Event generation

MC integrator

$$\frac{d\sigma}{d\mathcal{O}}$$

$\mathcal{O}$

Acceptance-Rejection

Event generator

$$\frac{d\sigma}{d\mathcal{O}}$$

$\mathcal{O}$
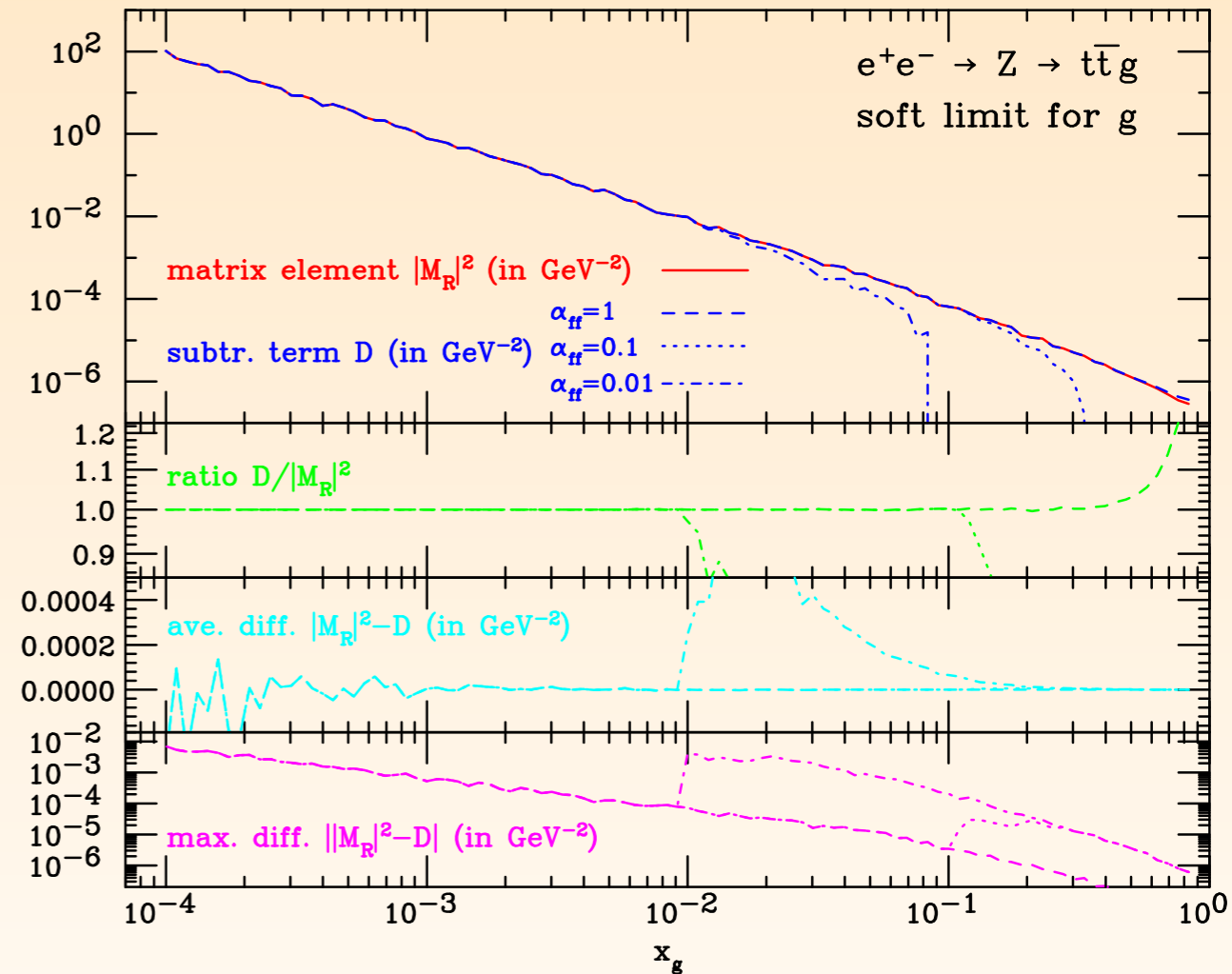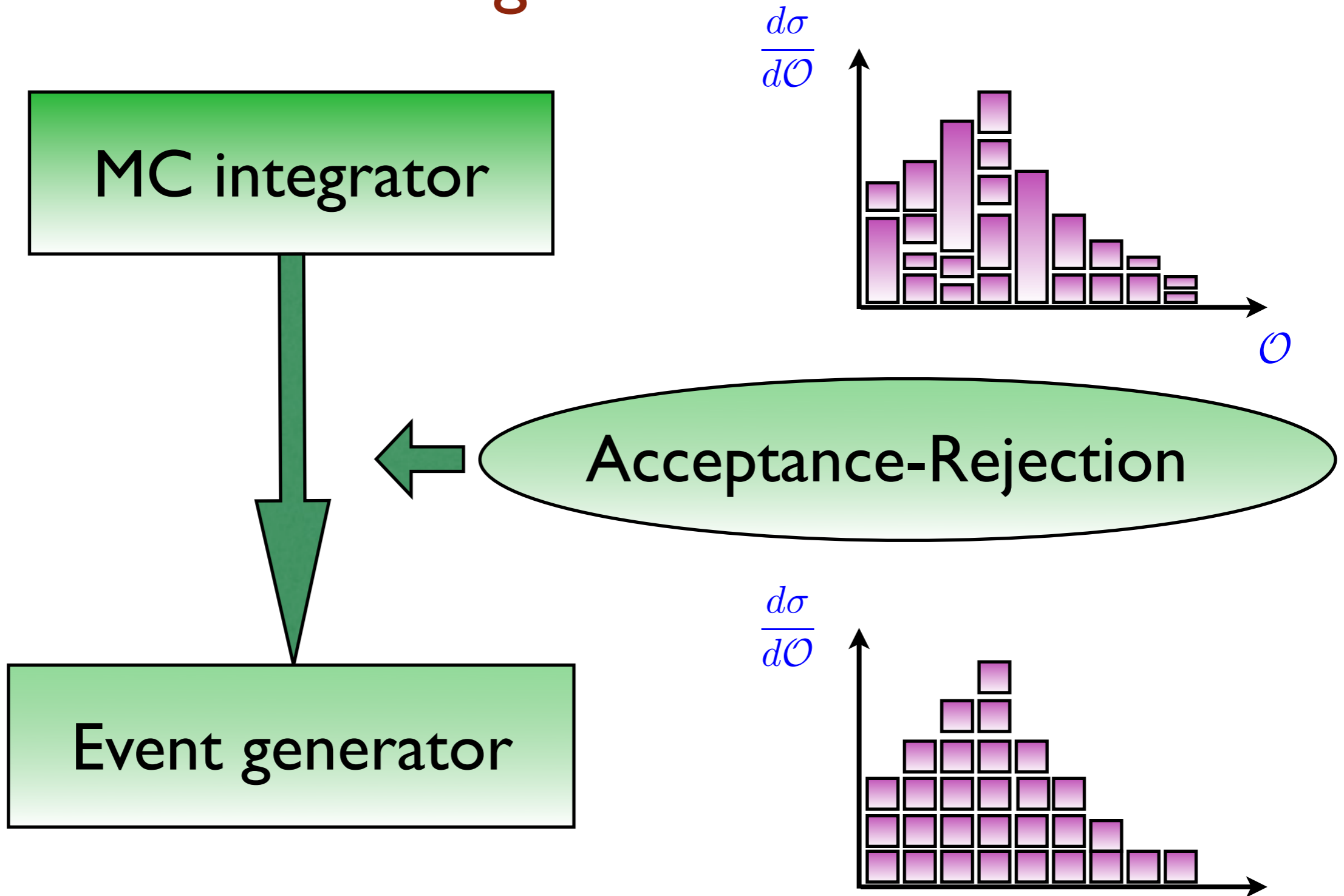
☞ **This is possible only if f(x)<∞ AND has definite sign!**

Event unweighting?

- Another consequence of this kinematic mismatch is that we cannot generate events at fixed order NLO

  - Even though the integrals are finite, they are not bounded (compare with $\int_0^1 dx \frac{1}{\sqrt{x}}$), so there is no maximum to unweight against: a single event can have an arbitrarily large weight!

  - Furthermore, event and counter event have different kinematics: which one to use for the unweighted event?

Filling histograms on-the-fly

$$\sigma^{\mathrm{NLO}} \sim \int d^4\Phi_m \, B(\Phi_m)$$

$$+ \int d^4\Phi_m \left[\int_{\mathrm{loop}} d^d l \, V(\Phi_m) + \int d^d\Phi_1 \, G(\overline{\Phi}_{m+1}) \right]_{\epsilon \to 0}$$

$$+ \int d^4\Phi_{m+1} \left[R(\Phi_{m+1}) - G(\overline{\Phi}_{m+1}) \right]$$

✳ In practice, when we do the MC integration we generate 2 sets of momenta

  1. An m-body set (for the Born, virtual and integrated counter terms)

  2. An m+1-body (for the NLO) which we map to the counter term momenta (for the counter terms)

✳ We compute the above formula; and apply cuts and fill histograms using the momenta corresponding to each term with the weight of that corresponding term

Summary

- Both the virtual and real-emission corrections are IR divergent, but their sum is finite: We can use a subtraction methods to factor the divergences in the real-emission phase-space integration and cancel them explicitly against the terms in the virtual corrections

- This generates events and counter events with slightly different kinematics. This means we cannot generate unweighed events (integrals are not bounded), but we can fill plots with weighted events: MC integrator (not an MC event generator)

- When making plots or applying cuts, use only IR safe observables with finite resolution

VIRTUAL CORRECTIONS

ONE-LOOP INTEGRAL

k_1

k_2

k_n

l

$l + k_1 = l + p_1$

k_3

D_0　D_1

D_{m-1}

D_2　$l + k_1 + k_2 + k_3 = l + p_2$

k_4

D_3

$l + k_1 + \ldots + k_6 = l + p_3$

k_5

k_6
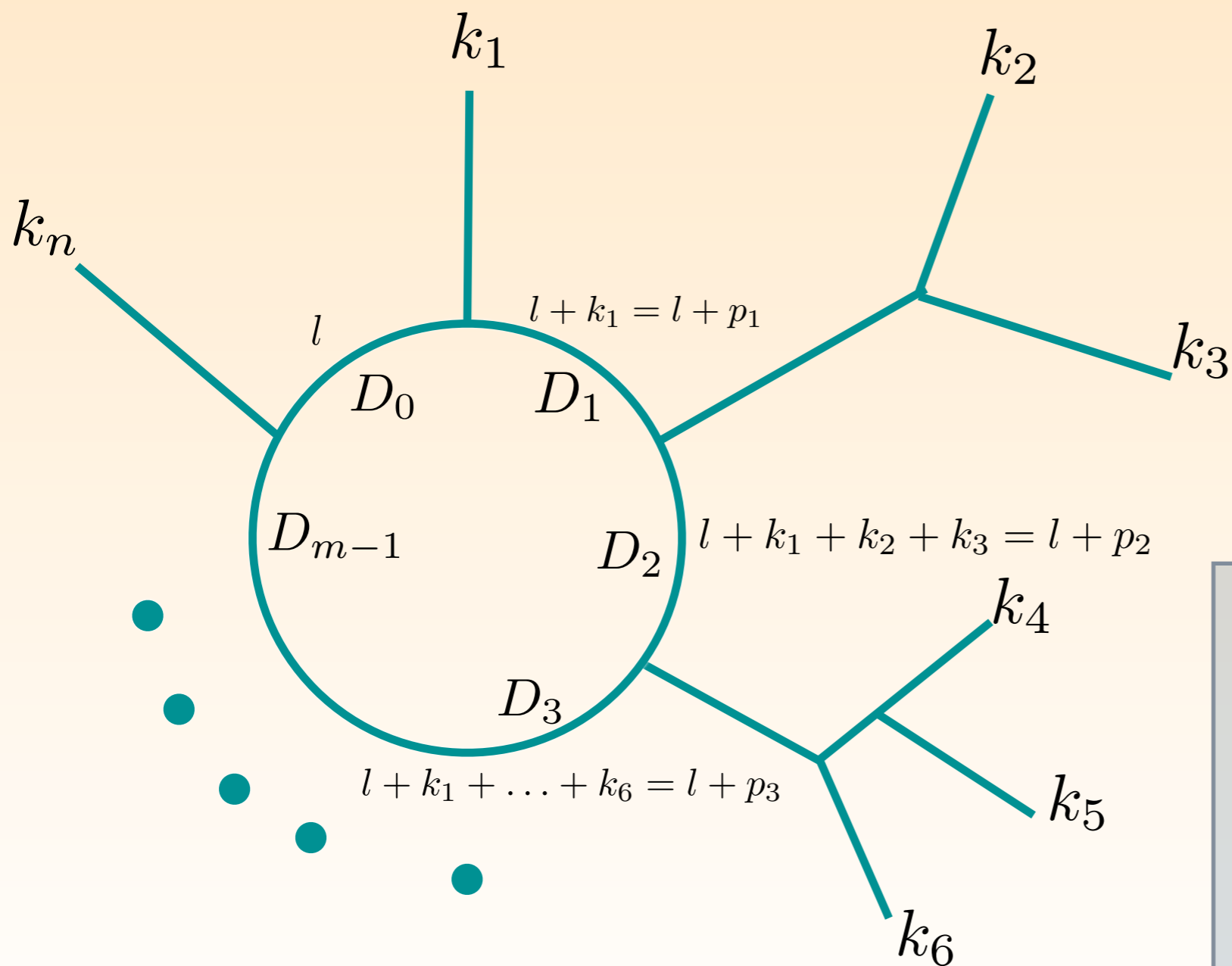
✳ Consider this *m*-point loop diagram with *n* external momenta

✳ The integral to compute is

$$\int d^d l \, \frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}}$$

$$D_i = (l + p_i)^2 - m_i^2$$

What is the goal

※ Any one-loop integral can be written as:

$$\int d^d l \, \frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}} = \sum_i \text{coeff}_i \int d^d l \, \frac{1}{D_{i_0} D_{i_1} \cdots}$$

※ Reduce a general integral to "scalar integrals", with at most 4 denominator factors D_i

※ To compute the virtual corrections, we "only" need to find the values of the coefficients multiplying the scalar integrals

Basis of scalar integrals

$$\mathcal{M}^{\text{1-loop}} = \sum_{i_0 < i_1 < i_2 < i_3} d_{i_0 i_1 i_2 i_3} \text{Box}_{i_0 i_1 i_2 i_3}$$

$$+ \sum_{i_0 < i_1 < i_2} c_{i_0 i_1 i_2} \text{Triangle}_{i_0 i_1 i_2}$$

$$+ \sum_{i_0 < i_1} b_{i_0 i_1} \text{Bubble}_{i_0 i_1}$$

$$+ \sum_{i_0} a_{i_0} \text{Tadpole}_{i_0}$$

$$+ R + \mathcal{O}(\epsilon)$$

✳ The a, b, c, d and R coefficients depend only on external parameters and momenta

$$D_i = (l + p_i)^2 - m_i^2$$

$$\text{Tadpole}_{i_0} = \int d^d l \frac{1}{D_{i_0}}$$

$$\text{Bubble}_{i_0 i_1} = \int d^d l \frac{1}{D_{i_0} D_{i_1}}$$

$$\text{Triangle}_{i_0 i_1 i_2} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2}}$$

$$\text{Box}_{i_0 i_1 i_2 i_3} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2} D_{i_3}}$$

- All these scalar integrals are known and available in computer libraries (FF [v. Oldenborgh], QCDLoop [Ellis, Zanderighi], OneLOop [v. Hameren])

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New loop techniques

* The "loop revolution": new techniques for computing one-loop matrix elements are now established:

  * Generalized unitarity (e.g. BlackHat, Njet, ...)

    [Bern, Dixon, Dunbar, Kosower, 1994...; Ellis Giele Kunst 2007 + Melnikov 2008; Badger...]

  * Integrand reduction (OPP method) (e.g. MadLoop (aMC@NLO), GoSam)

    [Ossola, Papadopoulos, Pittau 2006; del Aguila, Pittau 2004; Mastrolia, Ossola, Reiter, Tramontano 2010;...]

  * Tensor reduction (e.g. Golem, Openloops)

    [Passarino, Veltman 1979; Denner, Dittmaier 2005; Binoth Guillet, Heinrich, Pilon, Reiter 2008; Cascioli, Maierhofer, Pozzorini 2011;...]

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Integrand Reduction

$$\int d^d l \, \frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}} = \sum_i \text{coeff}_i \int d^d l \, \frac{1}{D_{i_0} D_{i_1} \ldots}$$

※ Of course the above equation does no longer hold when we take away the integral

$$\frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}} \neq \sum_i \frac{\text{coeff}_i}{D_{i_0} D_{i_1} \ldots}$$

※ But we can fix it by introducing "spurious terms" (that depend on the loop momenta)

$$\frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}} = \sum_i \frac{\text{coeff}_i + \text{spurious}_i(l)}{D_{i_0} D_{i_1} \ldots}$$

Functional form of the spurious terms

⁂ The functional form of the spurious terms is known (it depends on the rank of the integral and the number of propagators in the loop) [del Aguila, Pittau 2004]

  ⁂ for example, a box coefficient from a rank 1 numerator is

  $$\tilde{d}_{i_0 i_1 i_2 i_3}(l) = \tilde{d}_{i_0 i_1 i_2 i_3}\, \epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma$$

  (remember that p_i is the sum of the momentum that has entered the loop so far, so we always have $p_0 = 0$)

  ⁂ The integral is zero

  $$\int d^d l \frac{\tilde{d}_{i_0 i_1 i_2 i_3}(l)}{D_0 D_1 D_2 D_3} = \tilde{d}_{i_0 i_1 i_2 i_3}\int d^d l \frac{\epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma}{D_0 D_1 D_2 D_3} = 0$$

$$\frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}} = \sum_i \frac{\text{coeff}_i + \text{spurious}_i(l)}{D_{i_0} D_{i_1} \ldots}$$

✳ Multiplying both sides by the product of all the **D_i**'s leaves:

$$
\begin{aligned}
N(l) = &\sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}(l) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
&+ \sum_{i_0 < i_1 < i_2}^{m-1} \left[c_{i_0 i_1 i_2} + \tilde{c}_{i_0 i_1 i_2}(l) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0 < i_1} \left[b_{i_0 i_1} + \tilde{b}_{i_0 i_1}(l) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
&+ \sum_{i_0} \left[a_{i_0} + \tilde{a}_{i_0}(l) \right] \prod_{i \neq i_0}^{m-1} D_i \\
&+ \tilde{P}(l) \prod_i^{m-1} D_i
\end{aligned}
$$

Numerical evaluation

❊ *N(l)* is known from the diagrams; also the function form of the spurious terms is known, so:

❊ **By choosing specific values for the loop momentum *l*, we end up with a system of linear equations that we can solve easily**

   ❊ In a renormalizable theory, the rank of the integrand is always smaller (or equal) to the number of particles in the loop (with a conveniently chosen gauge)

   ❊ We can straight-forwardly set the it up by sampling the numerator numerically for various values of the loop momentum *l*

   ❊ By choosing *l* smartly, the system greatly reduces

      ❊ In particular when we chose *l* to be a complex 4-vector

$$N(l) = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}(l) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i$$

$$+ \sum_{i_0 < i_1 < i_2}^{m-1} \left[c_{i_0 i_1 i_2} + \tilde{c}_{i_0 i_1 i_2}(l) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i$$

$$+ \sum_{i_0 < i_1}^{m-1} \left[b_{i_0 i_1} + \tilde{b}_{i_0 i_1}(l) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i$$

$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l) \right] \prod_{i \neq i_0}^{m-1} D_i$$

$$+ \tilde{P}(l) \prod_{i}^{m-1} D_i$$

To solve the OPP reduction, choosing special values for the loop momenta helps a lot

For example, choosing l such that

$$D_0(l^{\pm}) = D_1(l^{\pm}) = D_2(l^{\pm}) = D_3(l^{\pm}) = 0$$

sets all the terms in this equation to zero except the first line

There are two (complex) solutions to this equation due to the quadratic nature of the propagators

How it works...

$$N(l^{\pm}) = d_{0123} + \tilde{d}_{0123}(l^{\pm}) \prod_{i \neq 0,1,2,3}^{m-1} D_i(l^{\pm})$$

✳ Two values are enough given the functional form for the spurious term. We can immediately determine the Box coefficient

$$d_{0123} = \frac{1}{2} \left[\frac{N(l^+)}{\prod_{i \neq 0,1,2,3}^{m-1} D_i(l^+)} + \frac{N(l^-)}{\prod_{i \neq 0,1,2,3}^{m-1} D_i(l^-)} \right]$$

✳ By choosing other values for l, that set other combinations of 4 "denominators" to zero, we can get all the Box coefficients

How it works...

* Now that we have all the Box coefficients we can start choosing values for l that set 3 "denominators" to zero to get the Triangle coefficients. Of course, now both the first and the second lines contribute.

$$N(l) = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}(l) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i$$

$$+ \sum_{i_0 < i_1 < i_2}^{m-1} \left[c_{i_0 i_1 i_2} + \tilde{c}_{i_0 i_1 i_2}(l) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i$$

* We already have solved the coefficients of the first line in the previous iteration, so also here there is only a simple system of equations to solve

* Once we have all the Triangle coefficients, we can continue to determine the Bubble coefficients; and finally the Tadpole coefficients

How it works…

※ For each phase-space point we have to solve the system of equations numerically

※ Due to the fact that the system reduces when picking special values for the loop momentum, the system greatly reduces

※ We can decompose the system at the level of the squared matrix element, amplitude, diagram or anywhere in between. As long as we provide the corresponding numerator function

※ For a given phase-space point, we have to compute the numerator function several times (~50 or so for a 4-point loop diagram)

Complications in d dimensions

* In the previous consideration I was very sloppy in considering if we are working in 4 or d dimensions

* In general, external momenta and polarization vectors are in 4 dimensions; only the loop momentum is in d dimensions

* To be more correct, we compute the integral

$$\int d^d l \, \frac{N(l, \tilde{l})}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \cdots \bar{D}_{m-1}}$$

$$\bar{l} = l + \tilde{l}$$

d dim   4 dim   epsilon dim

$$\bar{D}_i = (\bar{l} + p_i)^2 - m_i^2 = (l + p_i)^2 - m_i^2 + \tilde{l}^2 = D_i + \tilde{l}^2$$

$$l \cdot \tilde{l} = 0 \qquad\qquad \bar{l} \cdot p_i = l \cdot p_i \qquad\qquad \bar{l} \cdot \bar{l} = l \cdot l + \tilde{l} \cdot \tilde{l}$$

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IMPLICATIONS

$$\sum_{0 \leq i_0 < i_1 < i_2 < i_3}^{m-1} d(i_0 i_1 i_2 i_3) \int d^d \bar{\ell} \, \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2} \bar{D}_{i_3}}$$

$$+ \sum_{0 \leq i_0 < i_1 < i_2}^{m-1} c(i_0 i_1 i_2) \int d^d \bar{\ell} \, \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2}}$$

$$+ \sum_{0 \leq i_0 < i_1}^{m-1} b(i_0 i_1) \int d^d \bar{\ell} \, \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1}}$$

$$+ \sum_{i_0 = 0}^{m-1} a(i_0) \int d^d \bar{\ell} \, \frac{1}{\bar{D}_{i_0}}$$

$$+ R \, .$$

✳ The decomposition in terms of scalar integrals has to be done in d dimensions

✳ This is why the rational part R is needed

Rational terms

✳ The main difference is how we get the rational terms (we already saw them in the Passarino-Veltman reduction)

✳ In the OPP method, they are split into two contributions, generally called
$$R = R_1 + R_2$$

✳ Both have their origin in the UV part of the model, but only R_1 can be directly computed in the OPP reduction

R₁

- ✳ The origin of R_1 is coming is the denominators of the propagators in the loop

$$\frac{1}{D_i} \rightarrow \frac{1}{\bar{D}_i} = \frac{1}{D}\left(1 - \frac{\tilde{l}^2}{D_i}\right)$$

- ✳ Of course, the propagator structure is known, so these contributions can be included in the OPP reduction

- ✳ They give contributions proportional to

$$\int d^d\bar{l} \frac{\tilde{l}^2}{\bar{D}_i \bar{D}_j} = -\frac{i\pi^2}{2}\left[m_i^2 + m_j^2 - \frac{(p_i - p_j)^2}{3}\right] + \mathcal{O}(\epsilon)$$

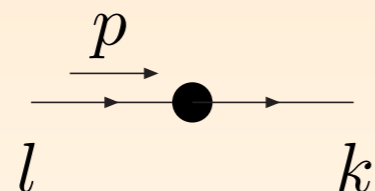$$\int d^d\bar{l} \frac{\tilde{l}^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon)$$

$$\int d^d\bar{l} \frac{\tilde{l}^4}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\epsilon)$$
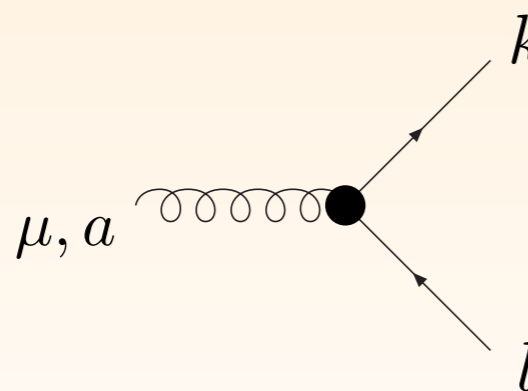
R₂

☀ The other origin of rational terms is the numerator itself. For integrals with rank > 2 we can have dependence in the numerator that is proportional to $\bar{l}^2$

☀ Unfortunately, this dependence can be quite hidden; maybe it is only explicitly there after doing the Clifford algebra

☀ Because we want to solve the system without doing this algebra analytically (we want to solve it numerically) we cannot get these contributions directly within the OPP reduction

☀ Within a given model, there is only a finite number of sources that can give these contributions; They have all been identified within the SM, and can be computed with the "R₂ counter terms"

R₂ Feynman rules

* Given that the R_2 contributions are of UV origin, only up to 4-point functions contribute to it (in a renormalizable theory)

* They can be computed using special Feynman rules, similarly to the UV counter term Feynman rules needed for the UV renormalization, e.g.

$$= \frac{ig^2}{16\pi^2} \frac{N_{col}^2 - 1}{2N_{col}} \delta_{kl}(-\not{p} + 2m_q)\lambda_{HV}$$

$$= \frac{ig^3}{16\pi^2} \frac{N_{col}^2 - 1}{2N_{col}} t_{kl}^a \gamma_\mu (1 + \lambda_{HV})$$

[Draggiotis, Garzelli, Papadopoulos, Pittau]

* Unfortunately these Feynman rules are model dependent, which means the need to be explicitly computed when going to BSM (just like the UV renormalisation)

Summary

※ There has been an enormous amount of progress in computing loops during the last 5 years

※ For the one-loop corrections, we need to find the coefficients multiplying the scalar integrals

※ OPP or integrand reduction is an efficient numerical method to get those. However, need to be careful due to the need of dimensional regularization.

※ OPP method implemented in MadLoop, which can generate QCD virtual corrections for any process within the SM

  ※ For BSM, a bit of work is needed in the model file, because the renormalisation (and R2 contribution) is not completely automated

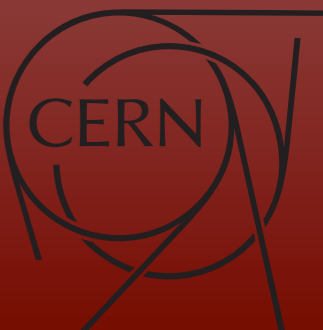# Master equation for hadron colliders

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \; f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, d\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$
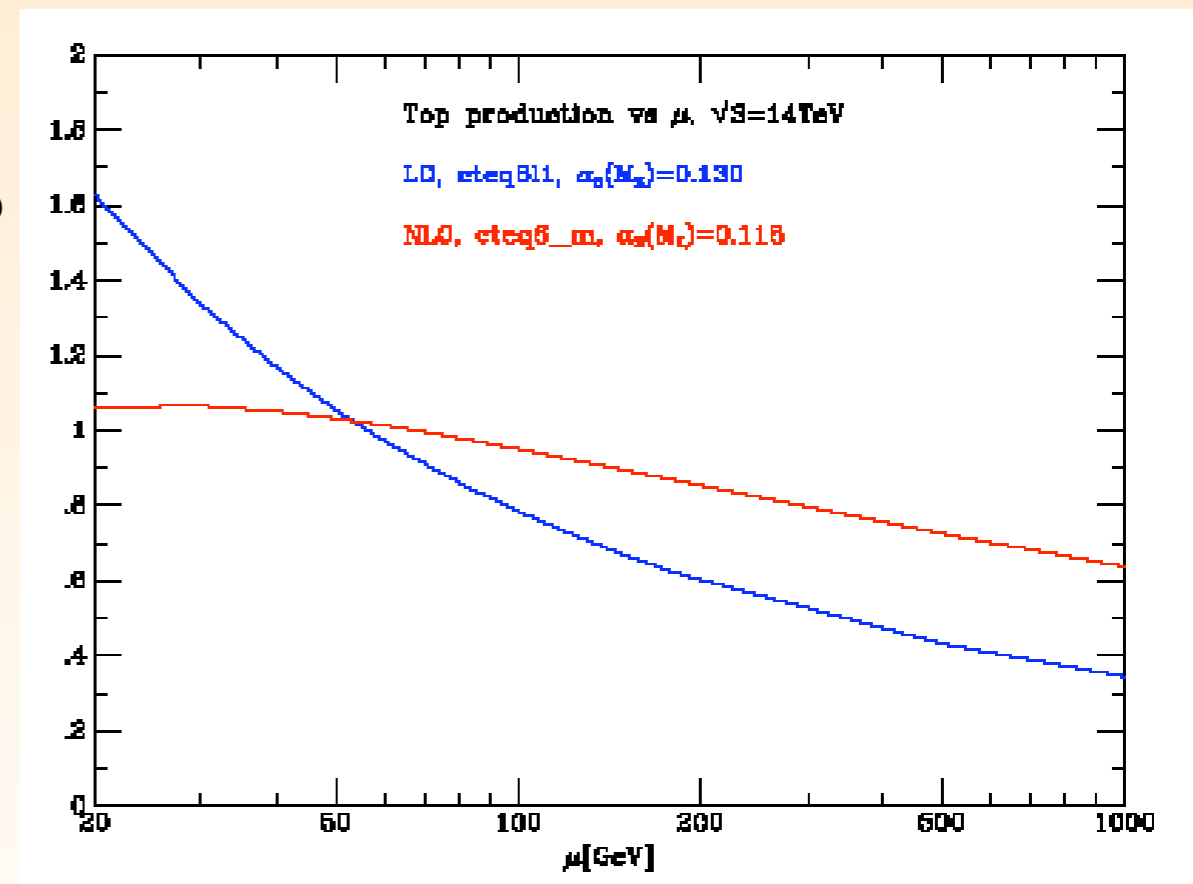
Parton density functions

Parton-level (differential) cross section

✹ Parton-level cross section from matrix elements: model and process dependent

✹ Parton density (or distribution) functions: process independent

✹ Differences between colliders given by parton luminosities
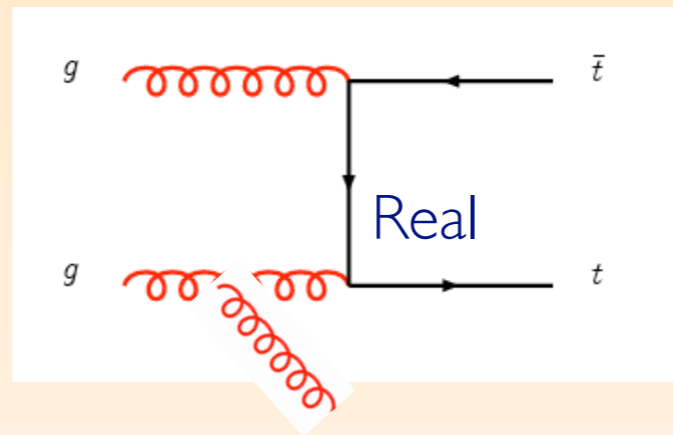
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GOING NLO

* At NLO the dependence on the renormalization and factorization scales is reduced

  * First order where scale dependence in the running coupling and the PDFs is compensated for via the loop corrections: **first reliable estimate of the total cross section**

  * Better description of final state: impact of extra radiation included (e.g. jets can have substructure)

  * Opening of additional initial state partonic channels

NLO...?

* Are all (IR-safe) observables that we can compute using a NLO code correctly described at NLO? Suppose we have a NLO code for pp → ttbar

LO

Real

Virt

NLO?
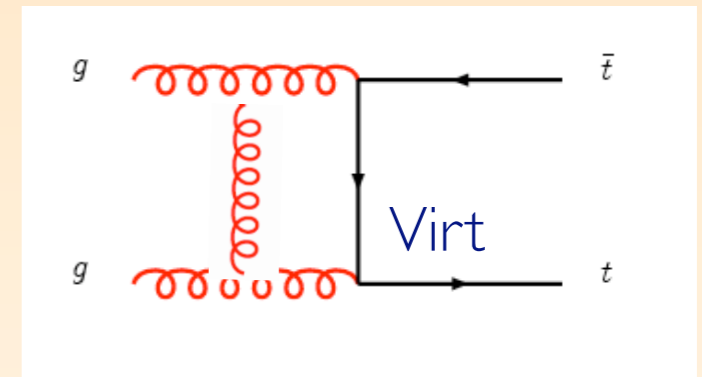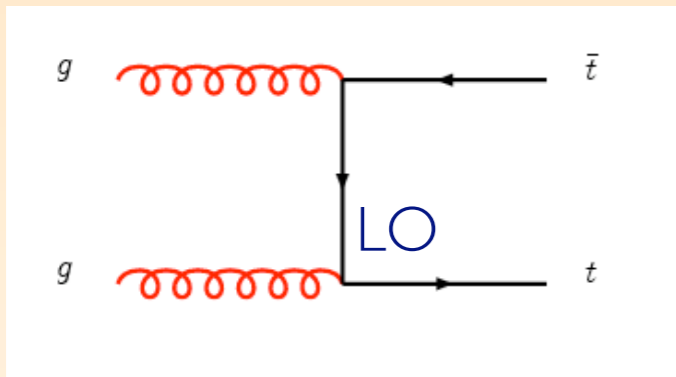
* Total cross section ✔

* Transverse momentum of the top quark ✔

* Transverse momentum of the top-antitop pair ✘

* Transverse momentum of the jet ✘

* Top-antitop invariant mass ✔

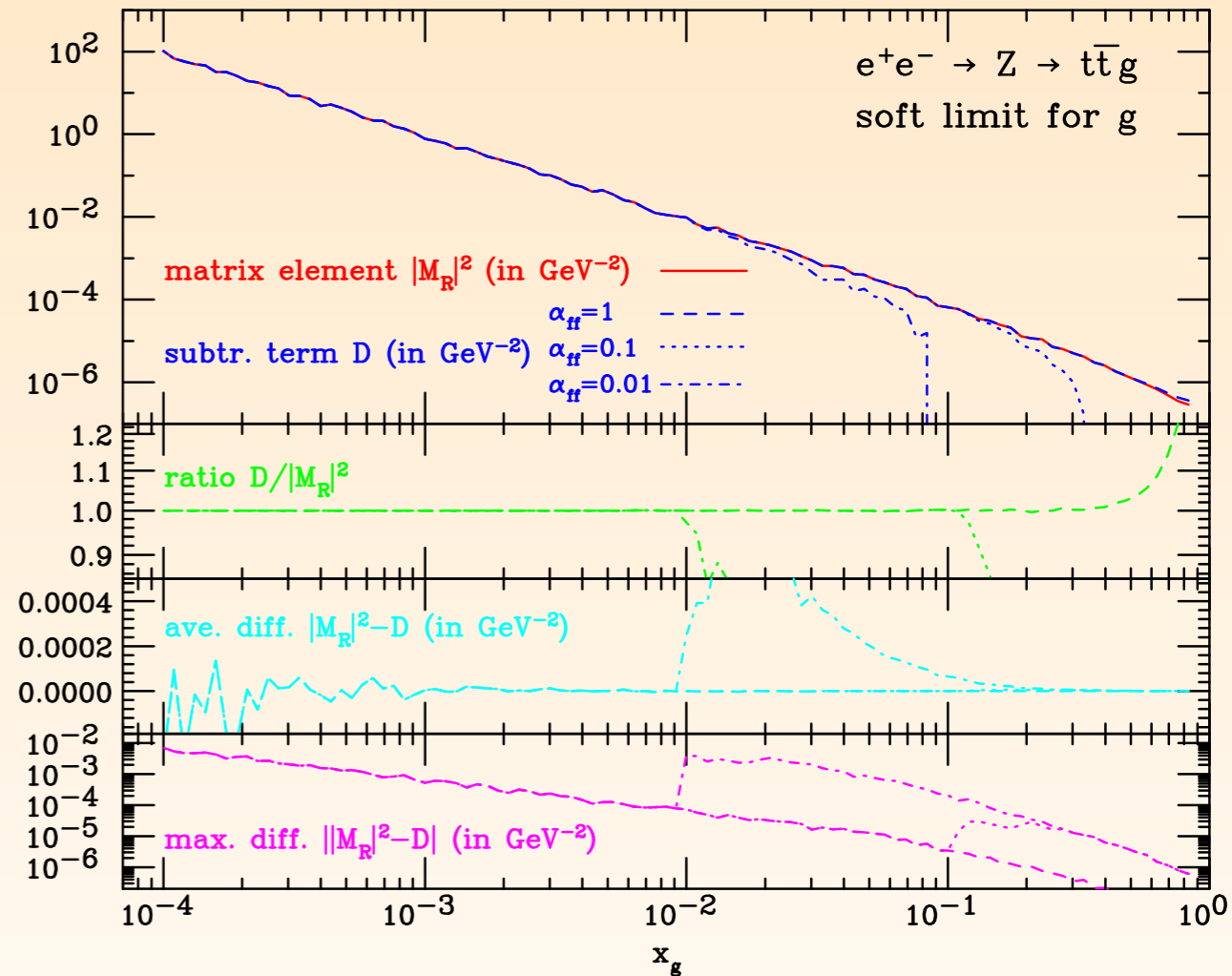* Azimuthal distance between the top and anti-top ✘

Event unweighting?

* Another consequence of this kinematic mismatch is that we cannot generate events at fixed order NLO

  * Even though the integrals are finite, they are not bounded (compare with $\int_0^1 dx \, \frac{1}{\sqrt{x}}$), so there is no maximum to unweight against: a single event can have an arbitrarily large weight!

  * Furthermore, event and counter event have different kinematics: which one to use for the unweighted event?

$e^+e^- \to Z \to t\bar{t}g$
soft limit for g

matrix element $|M_R|^2$ (in GeV^{-2})
$\alpha_{ff}=1$
subtr. term D (in GeV^{-2}) $\alpha_{ff}=0.1$
$\alpha_{ff}=0.01$

ratio $D/|M_R|^2$

ave. diff. $|M_R|^2-D$ (in GeV^{-2})

max. diff. $||M_R|^2-D|$ (in GeV^{-2})

x_g

$\frac{d\sigma}{d\mathcal{O}}$

not possible at NLO

$\frac{d\sigma}{d\mathcal{O}}$

$\mathcal{O}$

$\mathcal{O}$

Filling histograms on-the-fly

$$\sigma^{\text{NLO}} \sim \int d^4\Phi_m \, B(\Phi_m)$$

$$+ \int d^4\Phi_m \left[\int_{\text{loop}} d^d l \, V(\Phi_m) + \int d^d\Phi_1 \, G(\overline{\Phi}_{m+1}) \right]_{\epsilon \to 0}$$

$$+ \int d^4\Phi_{m+1} \left[R(\Phi_{m+1}) - G(\overline{\Phi}_{m+1}) \right]$$

✺ In practice, when we do the MC integration we generate 2 sets of momenta

  1. An *m*-body set (for the Born, virtual and integrated counter terms)

  2. An *m*+1-body (for the NLO) which we map to the counter term momenta (for the counter terms)

✺ We compute the above formula; and apply cuts and fill histograms using the momenta corresponding to each term with the weight of that corresponding term

Obstacles
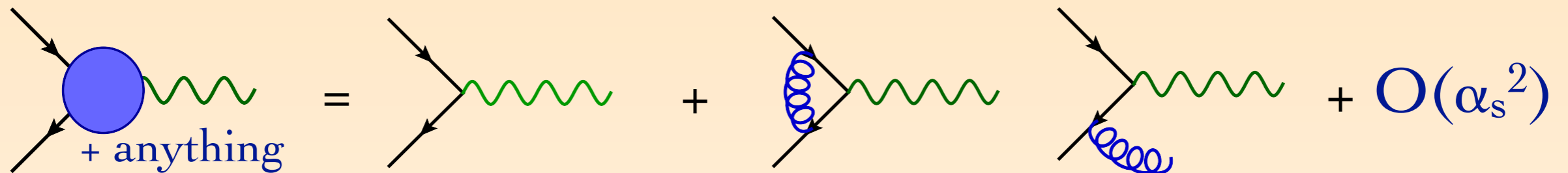
- Let us focus on NLO... there are already enough steps to be taken:

  - Virtual amplitudes: how to compute the loops automatically in a reasonable amount of time

  - How to deal with infra-red divergences: virtual corrections and real-emission corrections are separately divergent and only their sum is finite (for IR-safe observables) according to the KLN theorem

 - How to match these processes to a parton shower without double counting

NLO+PS MATCHING

IMPROVING MC'S

✳ There are two ways to improve a Parton Shower Monte Carlo event generator with matrix elements:

  ✳ ME+PS merging: Include matrix elements with more final state partons to describe hard, well-separated radiation better (already discussed by Johan)

  ✳ NLO+PS matching: Include full NLO corrections to the matrix elements to reduce theoretical uncertainties in the matrix elements. The real-emission matrix elements will describe the hard radiation
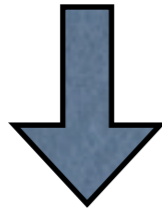
LIMITATIONS OF FIXED ORDER CALCULATIONS

⁂ In fact, for the observables that are not described at NLO accuracy, the situation is actually a bit worse:

⁂ In the small transverse momentum region, this calculation breaks down (it's even negative in the first bin!), and anywhere else it is purely a LO calculation for V+1j
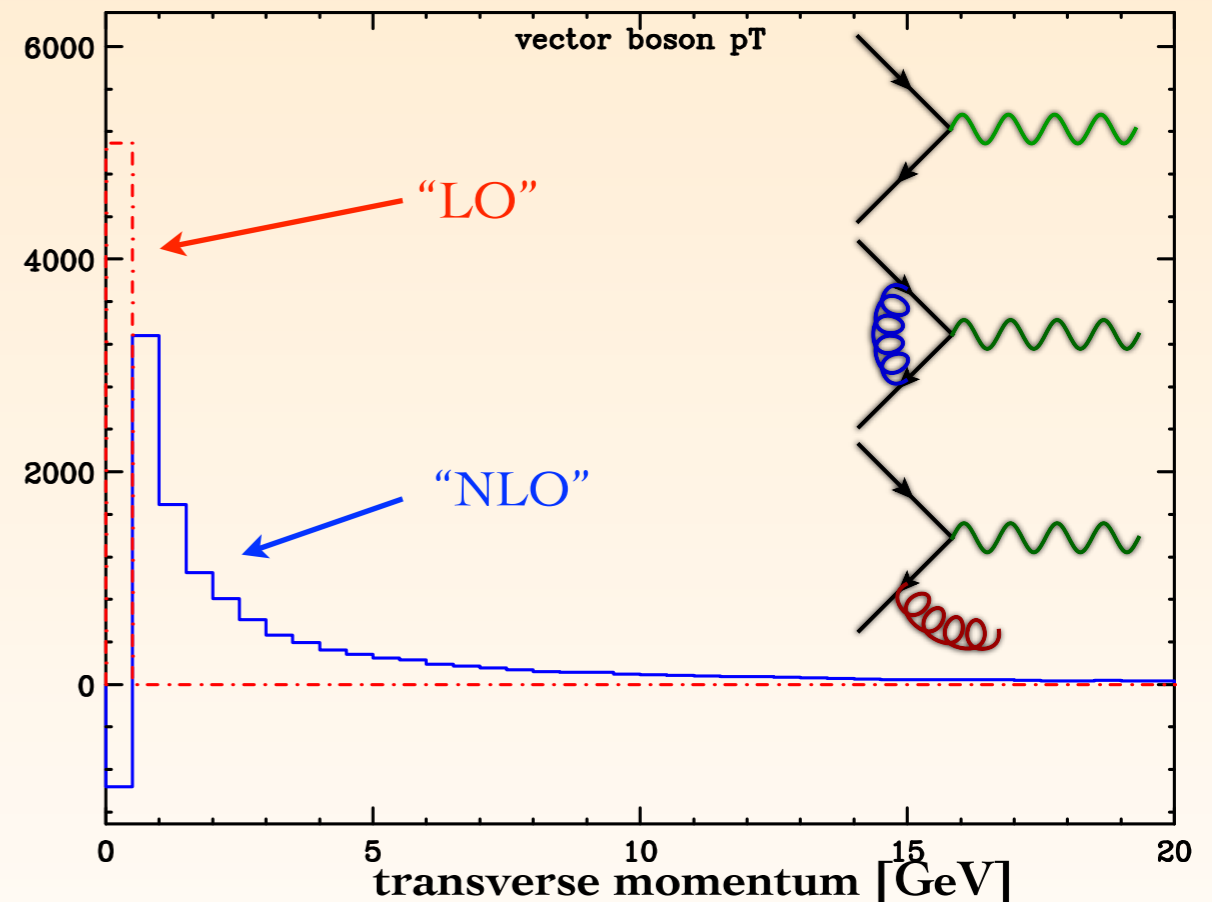
Matrix Elements vs. Parton Showers

ME

Shower MC

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are collinear and/or soft
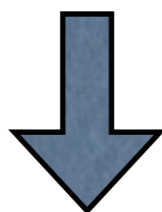5. Partial interference through angular ordering
6. Needed for hadronization

Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

Matrix Elements vs. Parton Showers

ME

Shower MC

1. Fixed order calculation
2. Computationally expensive
3. ~~Limited number of particles~~
4. Valid when partons are hard and well separated
5. ~~Quantum interference correct~~
6. Needed for multi-jet description

1. Resums logs to all orders
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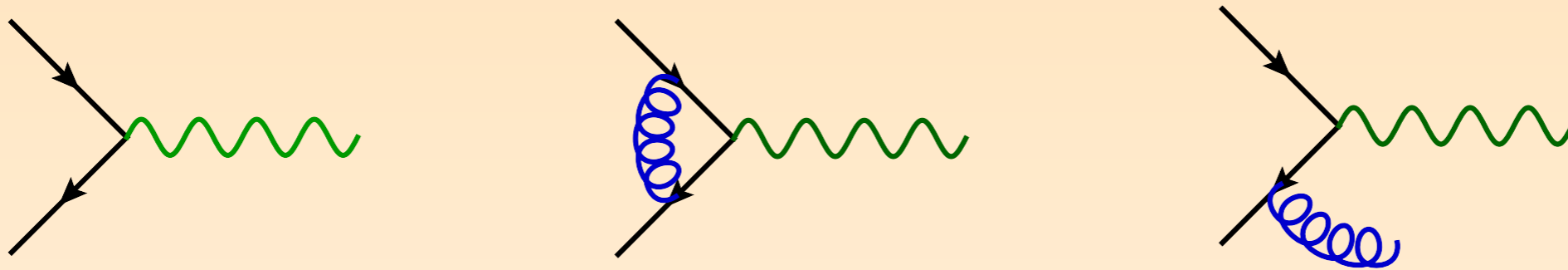**No longer true at NLO!**

Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

AT NLO

$$\left| \quad \substack{g_s} \quad + \ldots \right|^2$$
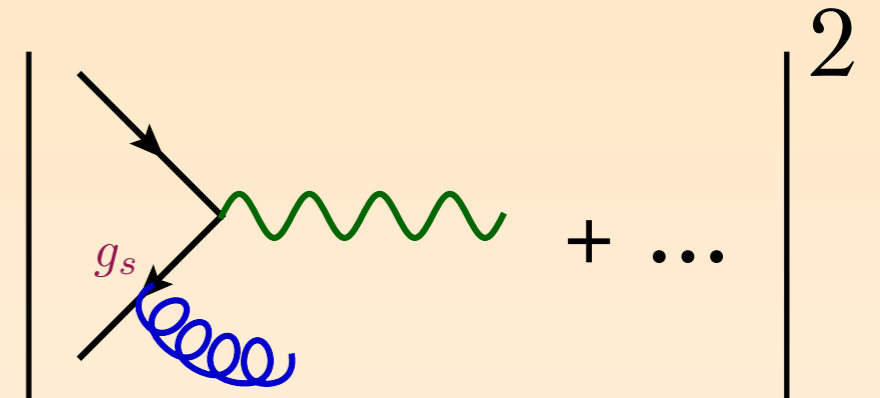
※ We have to integrate the real emission over the **complete** phase-space of the one particle that can go soft or collinear to obtain the infra-red poles that will cancel against the virtual corrections

※ We cannot use the same matching procedure: requiring that all partons should produce separate jets is not infrared safe

※ We have to invent a new procedure to match NLO matrix elements with parton showers
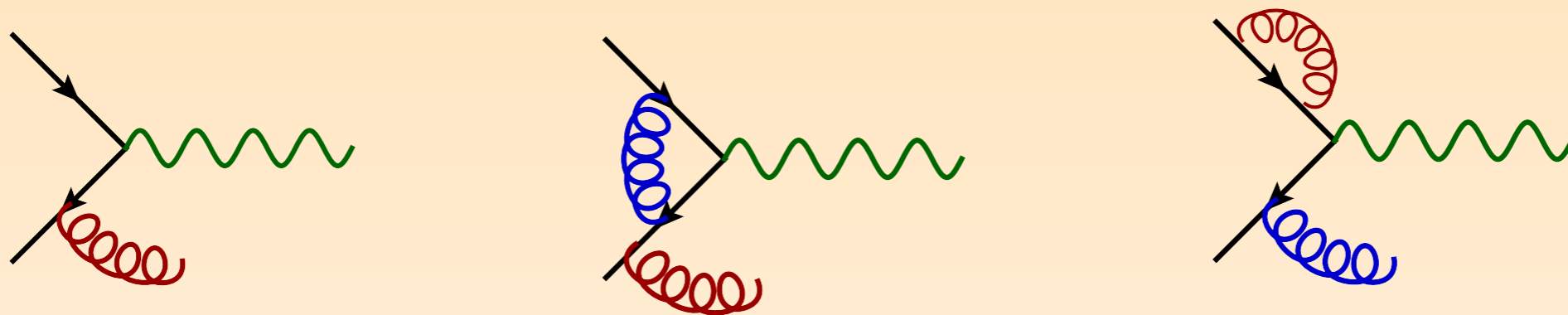
Naive (wrong) approach

- In a fixed order calculation we have contributions with m final state particles and with $m+1$ final state particles

$$\sigma^{\mathrm{NLO}} \sim \int d^4\Phi_m \, B(\Phi_m) + \int d^4\Phi_m \int_{\mathrm{loop}} d^d l \, V(\Phi_m) + \int d^d\Phi_{m+1} \, R(\Phi_{m+1})$$

- We could try to shower them independently

- Let $I_{\mathrm{MC}}^{(k)}(O)$ be the parton shower spectrum for an observable O, showering from a k-body initial condition

- We can then try to shower the m and $m+1$ final states independently

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \Big(B + \int_{\mathrm{loop}} V \Big) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} R \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

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Naive (wrong) approach

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$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V \right) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} R \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

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Double counting

$$\frac{d\sigma_{\text{NLOwPS}}}{dO} = \left[d\Phi_m(B + \int_{\text{loop}} V)\right] I_{\text{MC}}^{(m)}(O) + \left[d\Phi_{m+1} R\right] I_{\text{MC}}^{(m+1)}(O)$$
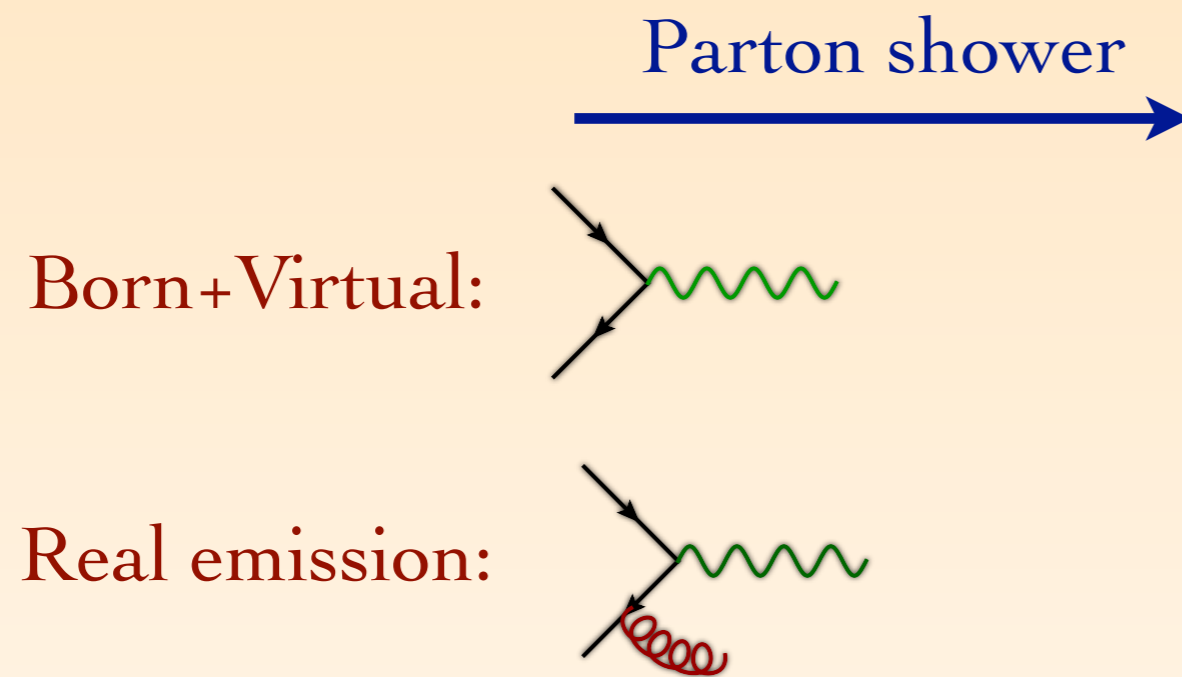
✴ But this is wrong!

✴ If you expand this equation out up to NLO, there are more terms then there should be and the total rate does not come out correctly

✴ Schematically $I_{\text{MC}}^{(k)}(O)$ for 0 and 1 emission is given by

$$I_{\text{MC}}^{(k)}(O) \sim \Delta_a(Q^2, Q_0^2)$$

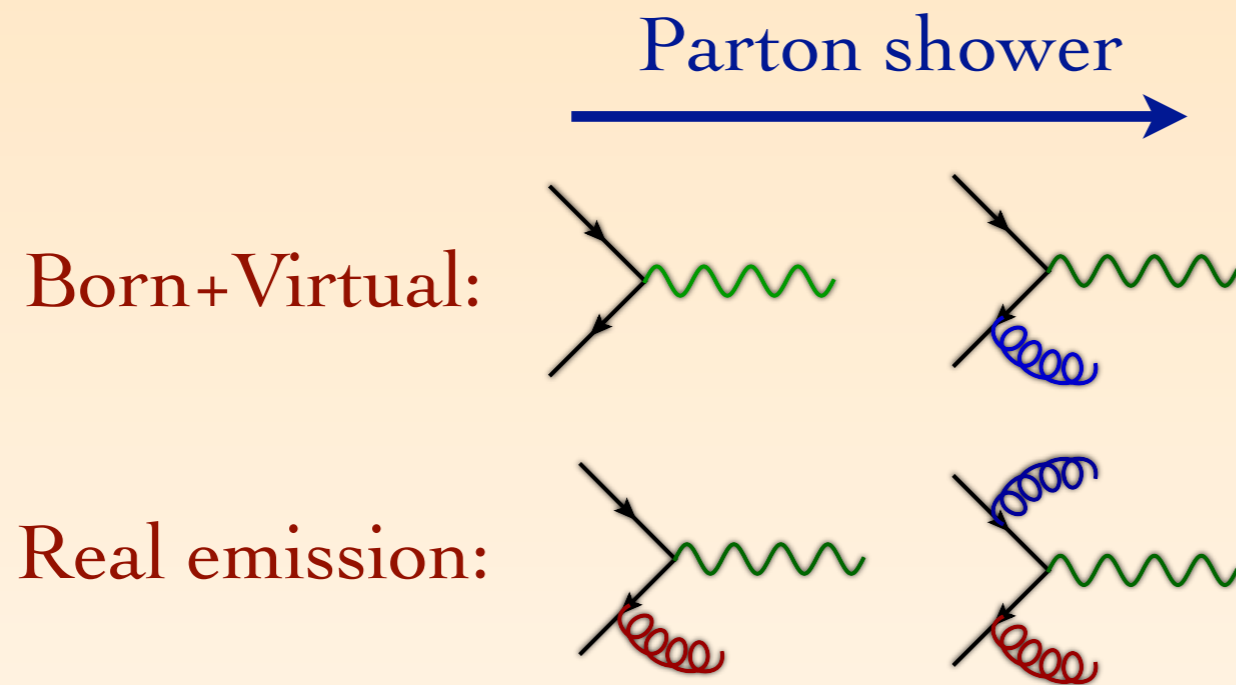$$+ \Delta_a(Q^2, t) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s(t)}{2\pi} P_{a \to bc}(z)$$

✴ And Δ is the Sudakov factor

$$\Delta_a(Q^2, t) = \exp\left[-\sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_s(t')}{2\pi} P_{a \to bc}\right]$$
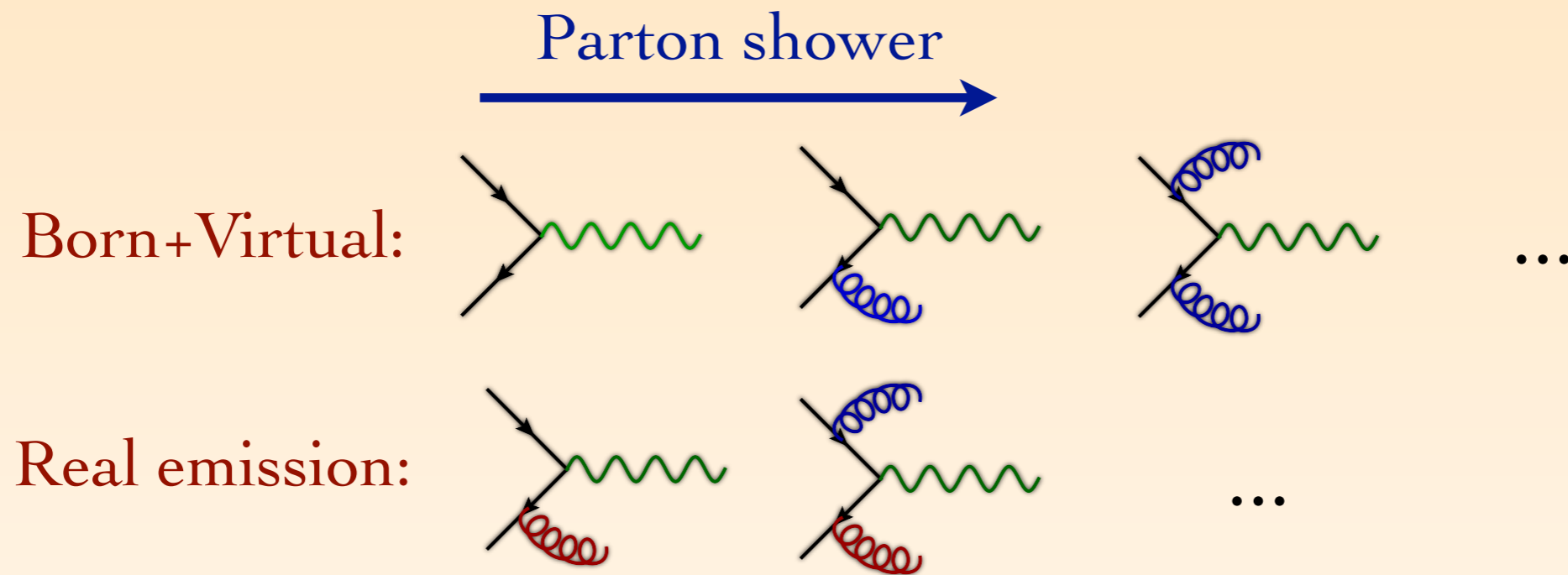
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Sources of double counting

Parton shower →

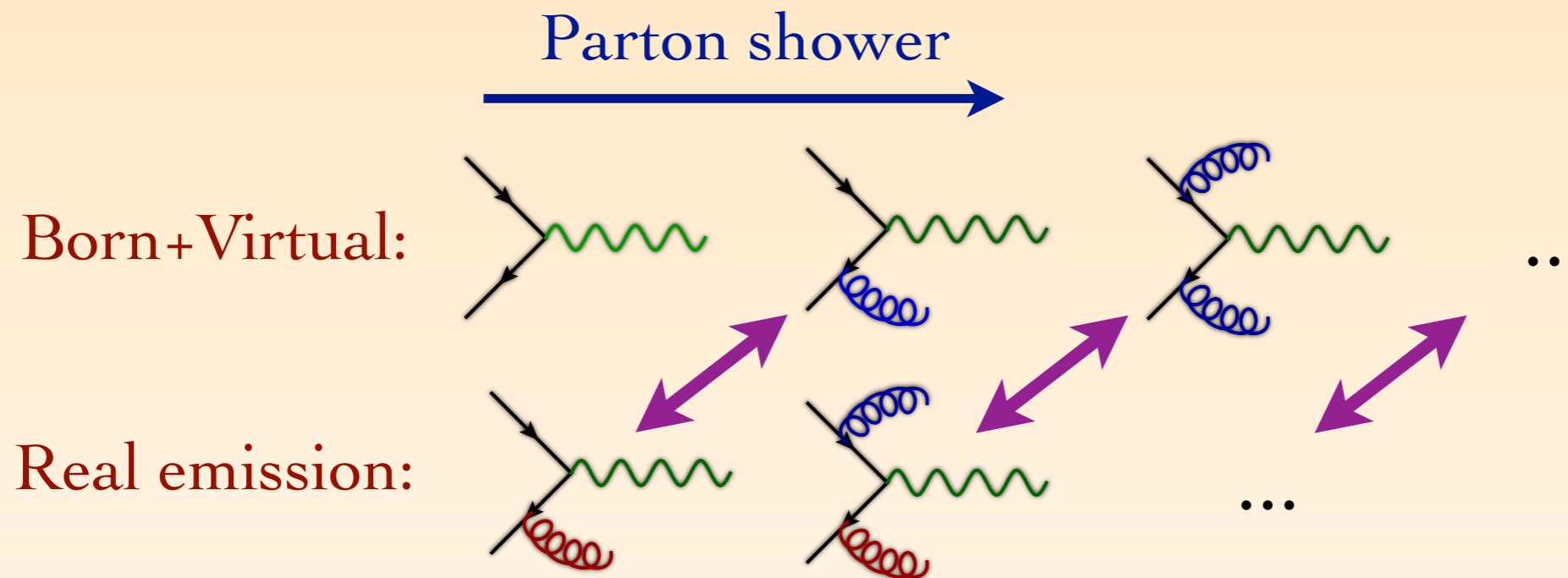Born+Virtual:

Real emission:

Sources of double counting

Parton shower

Born+Virtual:

Real emission:

Sources of double counting

Parton shower →

Born+Virtual:

Real emission:

Sources of double counting

Parton shower

Born+Virtual:

Real emission:

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Sources of double counting

Parton shower

Born+Virtual:

Real emission:

...
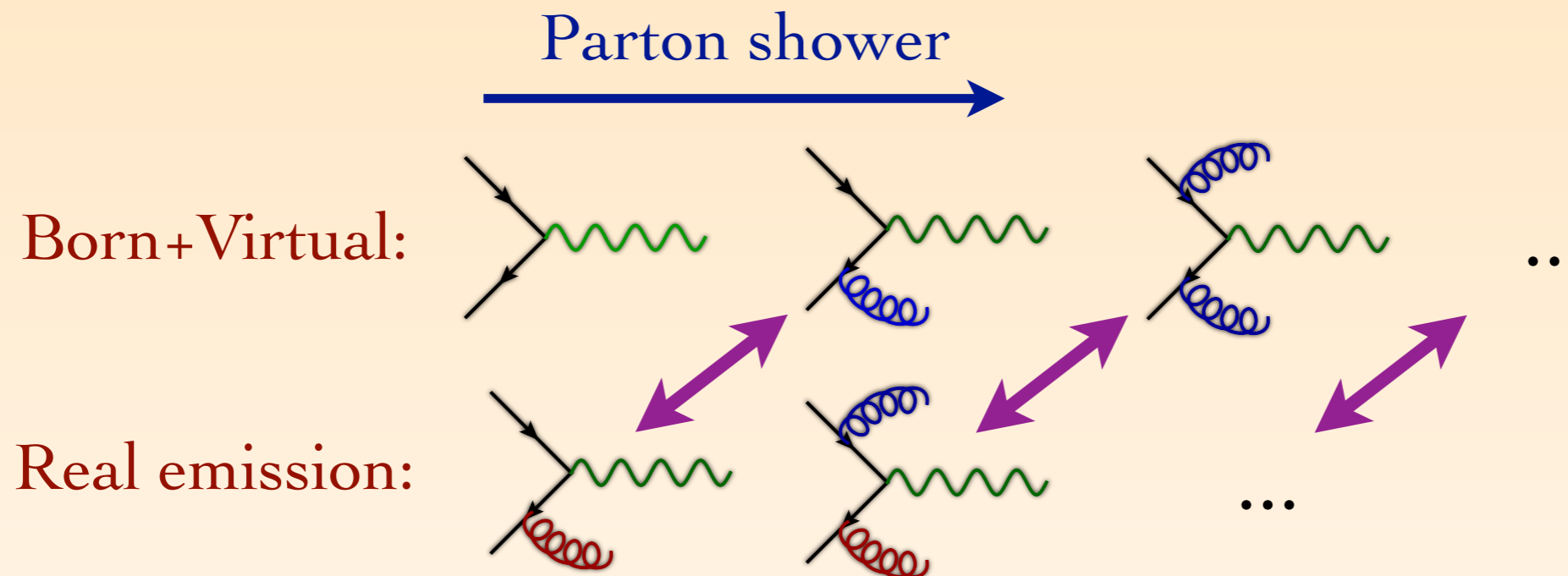
* There is double counting between the real emission matrix elements and the parton shower: the extra radiation can come from the matrix elements or the parton shower

* There is also an overlap between the virtual corrections and the Sudakov suppression in the zero-emission probability

Double counting in virtual/ Sudakov

* The Sudakov factor Δ (which is responsible for the resummation of all the radiation in the shower) is the no-emission probability

* It's defined to be $\Delta = 1 - P$, where P is the probability for a branching to occur

* By using this conservation of probability in this way, Δ contains contributions from the virtual corrections implicitly

* Because at NLO the virtual corrections are already included via explicit matrix elements, Δ is double counting with the virtual corrections

* In fact, because the shower is unitary, what we are double counting in the real emission corrections is exactly equal to what we are double counting in the virtual corrections (but with opposite sign)!

Avoiding double counting

✳ There are two methods to circumvent this double counting

   ✳ MC@NLO (Frixione & Webber)

   ✳ POWHEG (Nason)

MC@NLO PROCEDURE

❋ To remove the double counting, we can add and subtract the same term to the m and m+1 body configurations

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC \right) \right] I_{\mathrm{MC}}^{(m)}(O)$$
$$+ \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

Where the MC are defined to be the contribution of the parton shower to get from the m body Born final state to the m+1 body real emission final state

MC@NLO PROCEDURE

Parton shower →

Born+Virtual:

Real emission:
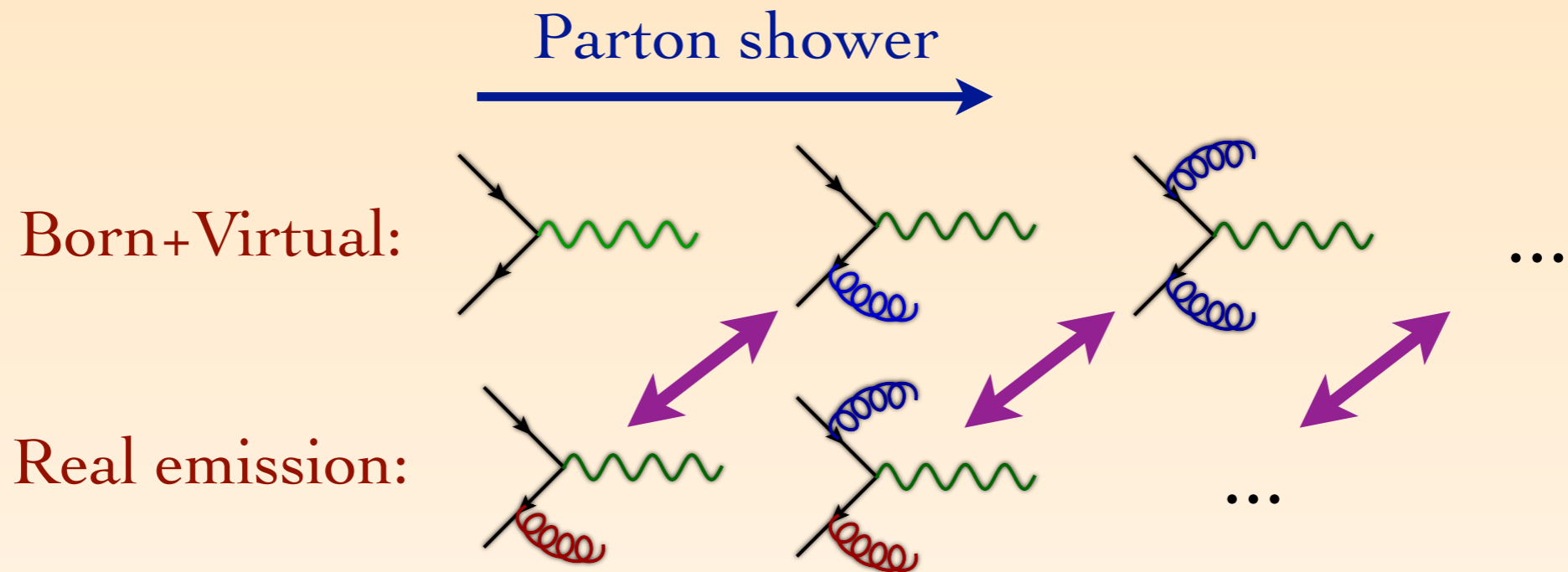
$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC \right) \right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

✳ Double counting is explicitly removed by including the "shower subtraction terms"

MC@NLO PROPERTIES

☀ Good features of including the subtraction counter terms

1. **Double counting avoided**: The rate expanded at NLO coincides with the total NLO cross section

2. **Smooth matching**: MC@NLO coincides (in shape) with the parton shower in the soft/collinear region, while it agrees with the NLO in the hard region

3. **Stability**: weights associated to different multiplicities are separately finite. The *MC* term has the same infrared behavior as the real emission (there is a subtlety for the soft divergence)

☀ Not so nice feature (for the developer):

1. **Parton shower dependence**: the form of the *MC* terms depends on what the parton shower does exactly. Need special subtraction terms for each parton shower to which we want to match

Double counting avoided

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC)\right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC)\right] I_{\mathrm{MC}}^{(m+1)}(O)$$

✳ Expanded at NLO

$$I_{\mathrm{MC}}^{(m)}(O)dO = 1 - \int d\Phi_1 \frac{MC}{B} + d\Phi_1 \frac{MC}{B} + \dots$$

$$d\sigma_{\mathrm{NLOwPS}} = \left[d\Phi_m(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC)\right] I_{\mathrm{MC}}^{(m)}(O)dO$$

$$+ \left[d\Phi_{m+1}(R - MC)\right]$$

$$\simeq d\Phi_m(B + \int_{\mathrm{loop}} V) + d\Phi_{m+1} R = d\sigma_{\mathrm{NLO}}$$

Smooth matching

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

✳ Smooth matching:

✳ Soft/collinear region: $R \simeq MC \implies d\sigma_{\mathrm{MC@NLO}} \sim I_{\mathrm{MC}}^{(m)}(O) dO$

✳ Hard region (shower effects suppressed), ie.

$$MC \simeq 0 \quad I_{\mathrm{MC}}^{(m)}(O) \simeq 0 \quad I_{\mathrm{MC}}^{(m+1)}(O) \simeq 1$$

$$\implies d\sigma_{\mathrm{MC@NLO}} \sim d\Phi_{m+1} R$$

Stability & unweighting

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC \right) \right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

✳ The *MC* subtraction terms are defined to be what the shower does to get from the *m* to the *m+1* body matrix elements. Therefore the cancellation of singularities is exact in the (*R - MC*) term: there is no mapping of the phase-space in going from events to counter events as we saw in the FKS subtraction

✳ The integral is bounded all over phase-space; we can therefore generate **unweighted events**!

✳ "S-events" (which have *m* body kinematics)

✳ "H-events" (which have *m+1* body kinematics)

FKS SUBTRACTION

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

✳ The MC counter terms render the real emission finite

✳ So, do we still need the FKS subtraction terms?

FKS SUBTRACTION

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

❋ The *MC* counter terms render the real emission finite

❋ So, do we still need the FKS subtraction terms?

YES!

FKS SUBTRACTION

$$\frac{d\sigma_{\text{NLOwPS}}}{dO} = \left[d\Phi_m (B + \int_{\text{loop}} V + \int d\Phi_1 MC) \right] I_{\text{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC) \right] I_{\text{MC}}^{(m+1)}(O)$$

✳ We cannot do the one-particle integral over the MC terms analytically: we do not get the explicit poles in $1/\epsilon$ and $1/\epsilon^2$ to cancel the poles in the virtual corrections. So we need to extract them using a subtraction method G

$$\frac{d\sigma_{\text{NLOwPS}}}{dO} = \left[d\Phi_m (B + (\int_{\text{loop}} V + \int d\Phi_1 G) + \int d\Phi_1 (MC - G) \right] I_{\text{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC) \right] I_{\text{MC}}^{(m+1)}(O)$$

Negative weights

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC \right) \right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

* We generate events for the two terms between the square brackets (S- and H-events) separately

* There is no guarantee that these contributions are separately positive (even though predictions for infra-red safe observables should always be positive!)

* Therefore, when we do event unweighting we can only unweight the events **up to a sign**. These signs should be taken into account when doing a physics analysis (i.e. making plots etc.)

* The events are only physical when they are showered

POWHEG

❋ Consider the probability of the first emission of a leg (inclusive over later emissions)

$$d\sigma = d\Phi_m \, B\left[\Delta(Q^2, Q_0^2) + \Delta(Q^2, t)d\Phi_{(+1)}\frac{MC}{B}\right]$$

❋ One could try to get NLO accuracy by replacing B with the NLO rate (integrated over the extra phase-space)

$$B \to B + V + \int d\Phi_{(+1)} \, R$$

❋ This naive definition is not correct: the radiation is still described only at leading logarithmic accuracy, which is not correct for hard emissions.

POWHEG

※ This is double counting.
To see this, expand the equation up to the first emission

$$d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[1 - \int d\Phi_{(+1)} \frac{MC}{B} + d\Phi_{(+1)} \frac{MC}{B} \right]$$

which is not equal to the NLO

※ In order to avoid double counting, one should replace the definition of the Sudakov form factor with the following:

$$\Delta(Q^2, Q_0^2) = \exp \left[-\int_{Q_0^2}^{Q^2} d\Phi_{(+1)} \frac{MC}{B} \right] \to \tilde{\Delta}(Q^2, Q_0^2) = \exp \left[-\int_{Q_0^2}^{Q^2} d\Phi_{(+1)} \frac{R}{B} \right]$$

corresponding to a modified differential branching probability

$$d\tilde{p} = d\Phi_{(+1)} R/B$$

※ Therefore we find for the POWHEG differential cross section

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[\tilde{\Delta}(Q^2, Q_0^2) + \tilde{\Delta}(Q^2, t) \, d\Phi_{(+1)} \frac{R}{B} \right]$$

Properties

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[\tilde{\Delta}(Q^2, Q_0^2) + \tilde{\Delta}(Q^2, t)\, d\Phi_{(+1)} \frac{R}{B} \right]$$

* The term in the square brackets integrates to one (integrated over the extra parton phase-space between scales $Q_0{}^2$ and Q^2)
  (this can also be understood as unitarity of the shower below scale t)
  POWHEG cross section is normalized to the NLO

* Expand up to the first-emission level:

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[1 - \int d\Phi_{(+1)} \frac{R}{B} + d\Phi_{(+1)} \frac{R}{B} \right] = d\sigma_{\text{NLO}}$$

  so double counting is avoided

* Its structure is identical an ordinary shower, with normalization rescaled by a global K-factor and a different Sudakov for the first emission: no negative weights are involved.

MC@NLO/POWHEG

The MC@NLO and POWHEG procedures can be cast in a single formula:

$$d\sigma_{\text{NLO+PS}} = d\Phi_B \bar{B}^s(\Phi_B) \left[\Delta^s(p_\perp^{\min}) + d\Phi_{R|B} \frac{R^s(\Phi_R)}{B(\Phi_B)} \Delta^s(p_T(\Phi)) \right] + d\Phi_R R^f(\Phi_R)$$

where

$$\bar{B}^s(\Phi_B) = B(\Phi_B) + \left[V(\Phi_B) + \int d\Phi_{R|B} R^s(\Phi_{R|B}) \right]$$

and we have split the Real emission matrix elements in a singular and finite part:

$$R(\Phi_R) = R^s(\Phi_R) + R^f(\Phi_R)$$

The difference between MC@NLO and POWHEG is in the way the real matrix elements are split:

MC@NLO $R^s(\Phi) = P(\Phi_{R|B}) B(\Phi_B) = MC$

POWHEG $R^s(\Phi) = F R(\Phi), \quad R^f(\Phi) = (1-F)R(\Phi)$

Need exact mapping $(\Phi_R, \Phi_B) \Rightarrow \Phi$ in MC subtraction term R^s

Default is $F = 1$: exponentiate the full real; it can be damped by hand

MC@NLO vs POWHEG

|	MC@NLO	POWHEG
Parton showers are (usually) not exact in the soft limit: MC@NLO needs an artificial smoothing	☹	☺
MC@NLO does not exponentiate the non-singular part of the real emission amplitudes	☺	☹
MC@NLO does not require any tricks for treating Born zeros	☺	☹
POWHEG is independent from the parton shower (although, in general the shower should be a truncated vetoed)	☹	☺
POWHEG has (almost) no negatively weighted events	☹	☺
Automation of the methods: http://amcatnlo.cern.ch, http://powhegbox.mib.infn.it, http://www.sherpa-mc.de	☺	☺

Four-lepton production
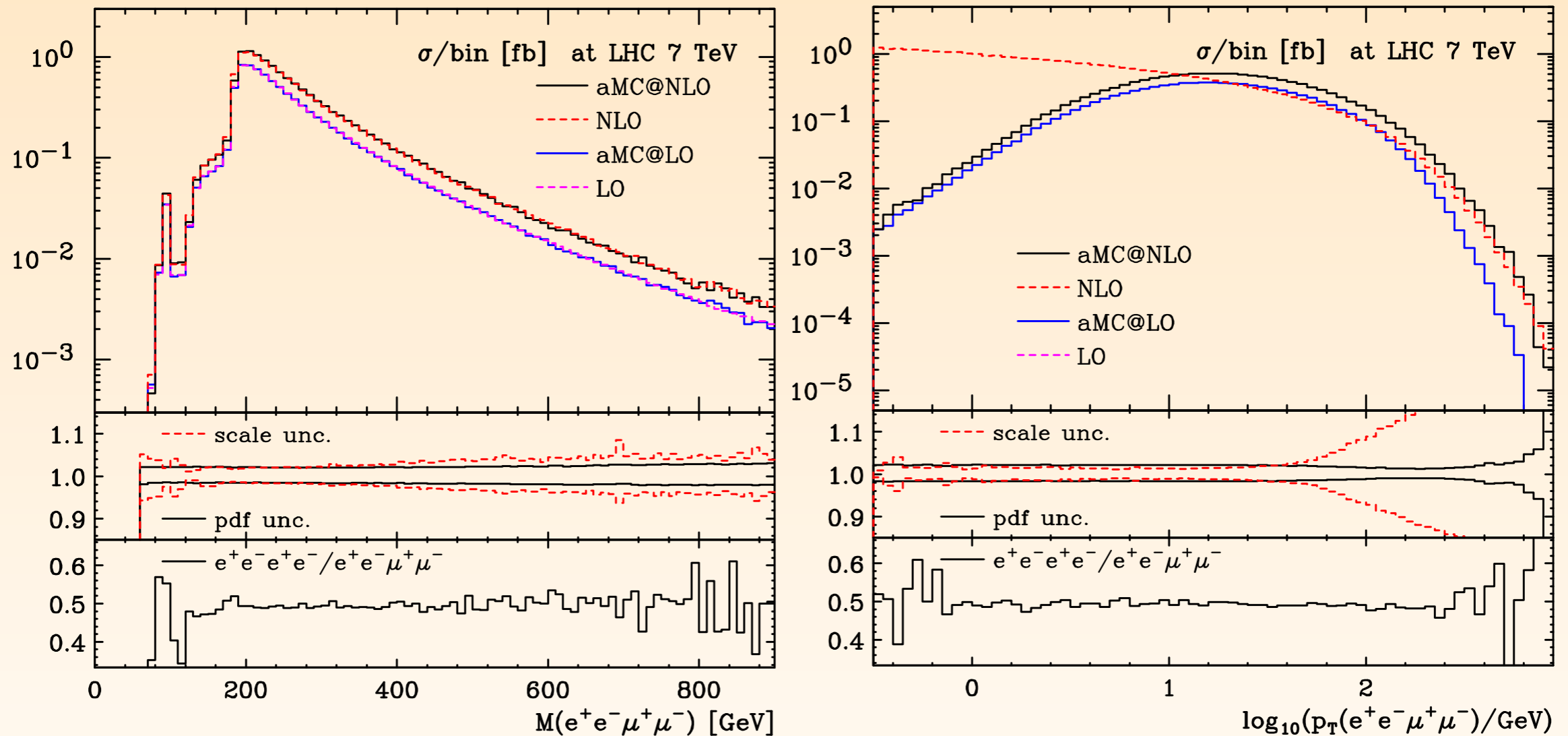
- 4-lepton invariant mass is almost insensitive to parton shower effects. 4-lepton transverse moment is extremely sensitive

- Including scale uncertainties

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Four-lepton production
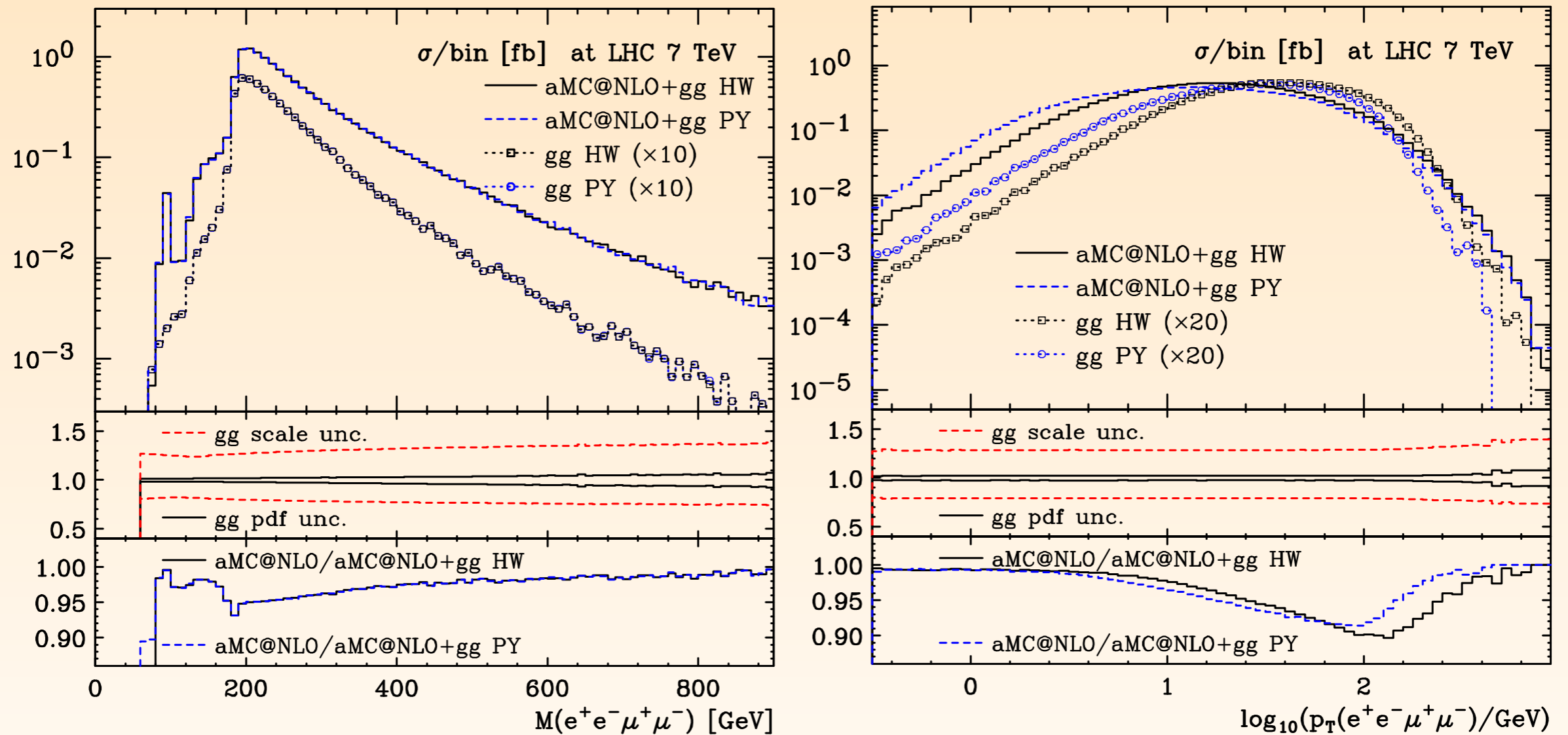
* Differences between Herwig (black) and Pythia (blue) showers large in the Sudakov suppressed region (much larger than the scale uncertainties)

* Contributions from gg initial state (formally NNLO) are of 5-10%

AMC@NLO JOINT VENTURE

AMC@NLO JOINT VENTURE

MadGraph 5

MadGraph 5

aMC@NLO

AMC@NLO JOINT VENTURE

MadGraph 5

aMC@NLO

MC@NLO method
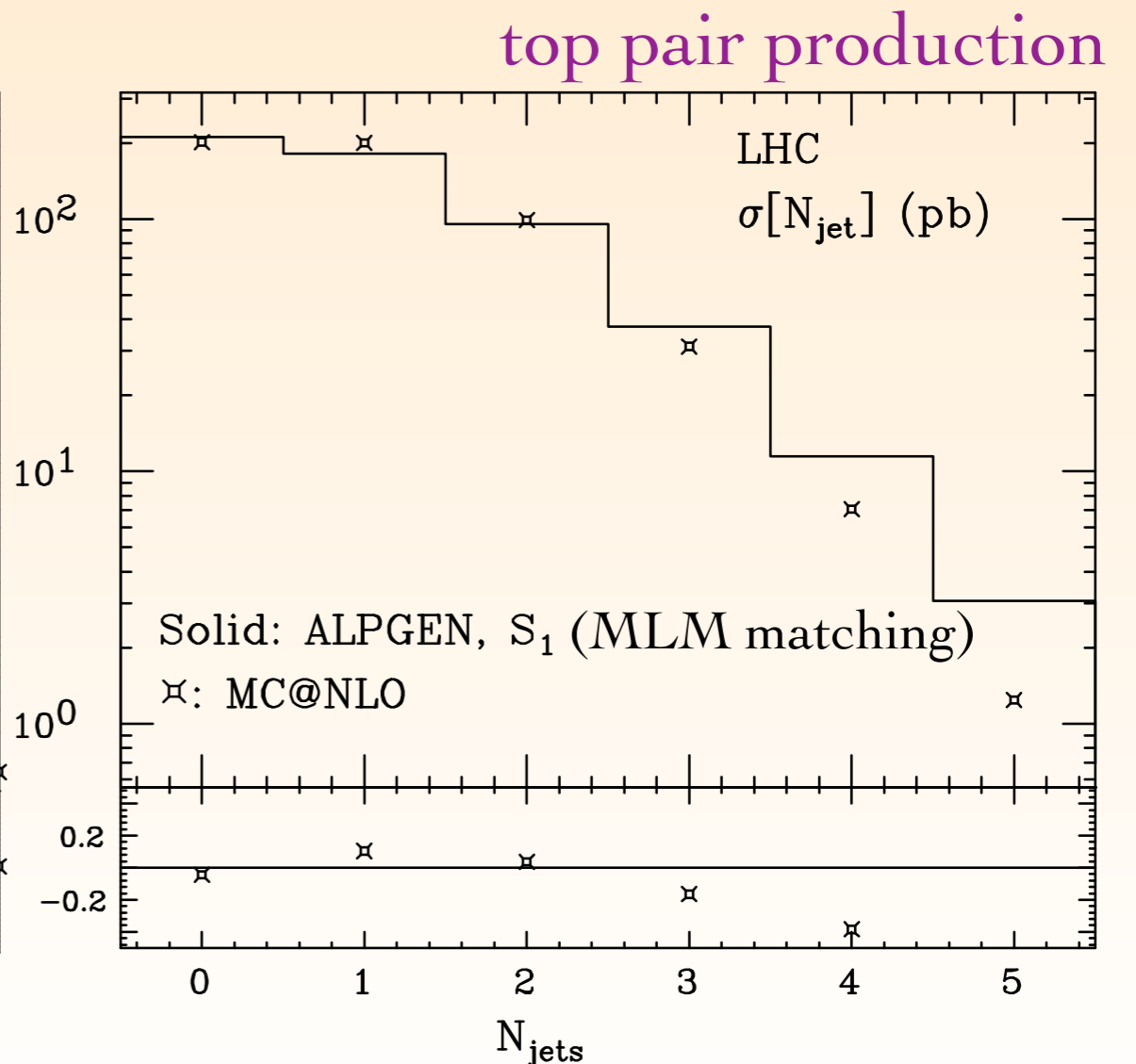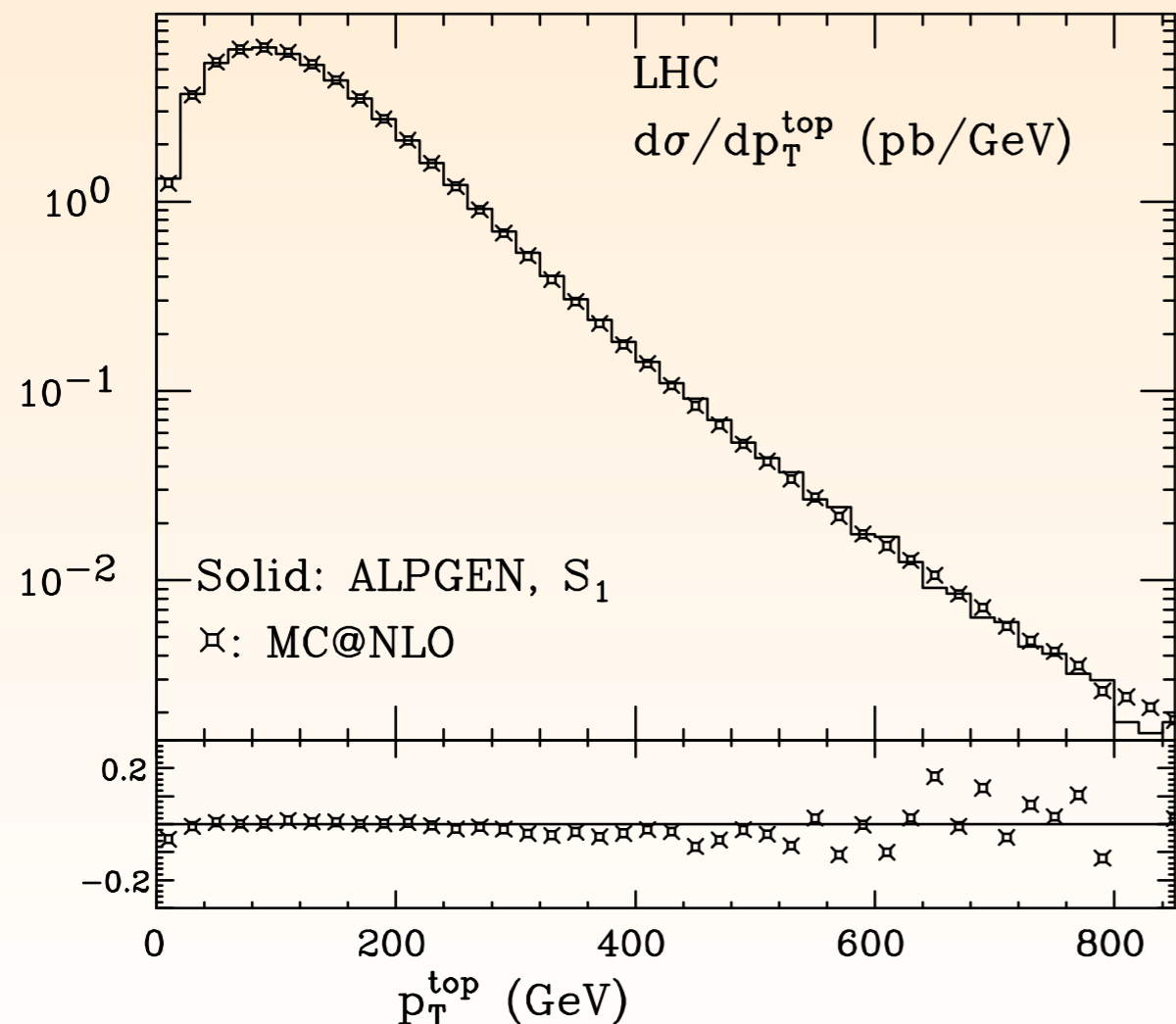to match NLO to parton shower
(Herwig(++) & Pythia6/8)

MadLoop (+CutTools)
for the one-loop virtual corrections
-- also possible to use external tools via
Binoth-LHA

MadFKS
to factor out IR divergences in
phase-space integrals

MadSpin
to keep spin-correlations in
particle decays

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Is NLO+PS always the preferred method?

- It is the preferred method if the observable is described at NLO accuracy

- But there are many observables for which a given NLO+PS code has only zeroth order accuracy.

top pair production

Left plot:
LHC
$d\sigma/dp_T^{top}$ (pb/GeV)
10^0
10^{-1}
10^{-2}
Solid: ALPGEN, S_1
⋈: MC@NLO
0.2
−0.2
0 200 400 600 800
p_T^{top} (GeV)

Right plot:
LHC
$\sigma[N_{jet}]$ (pb)
10^2
10^1
10^0
Solid: ALPGEN, S_1 (MLM matching)
⋈: MC@NLO
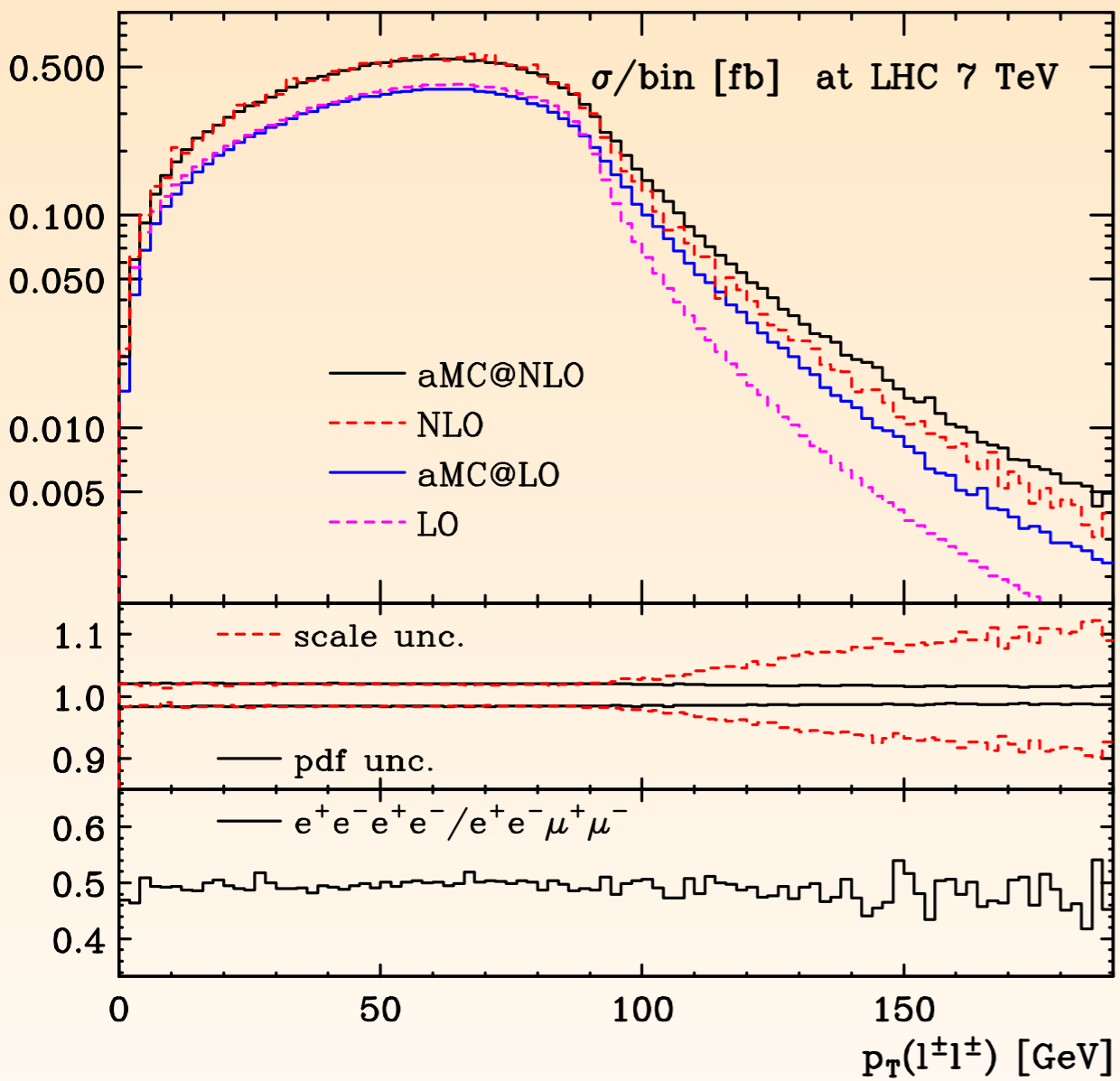0.2
−0.2
0 1 2 3 4 5
N_{jets}

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Summary

* We want to match NLO computations to parton showers to keep the good features of both approximations

    * In the MC@NLO method:
      by including the shower subtraction terms in our process we avoid double counting between NLO processes and parton showers

    * In the POWHEG method:
      apply an overall K-factor, and modify the (Sudakov of the) first emission to fill the hard region of phase-space according to the real-emission matrix elements

* First studies to combine NLO+PS matching with ME+PS merging have been made and result look very promising..., see next slides

Merging ME+PS at NLO accuracy
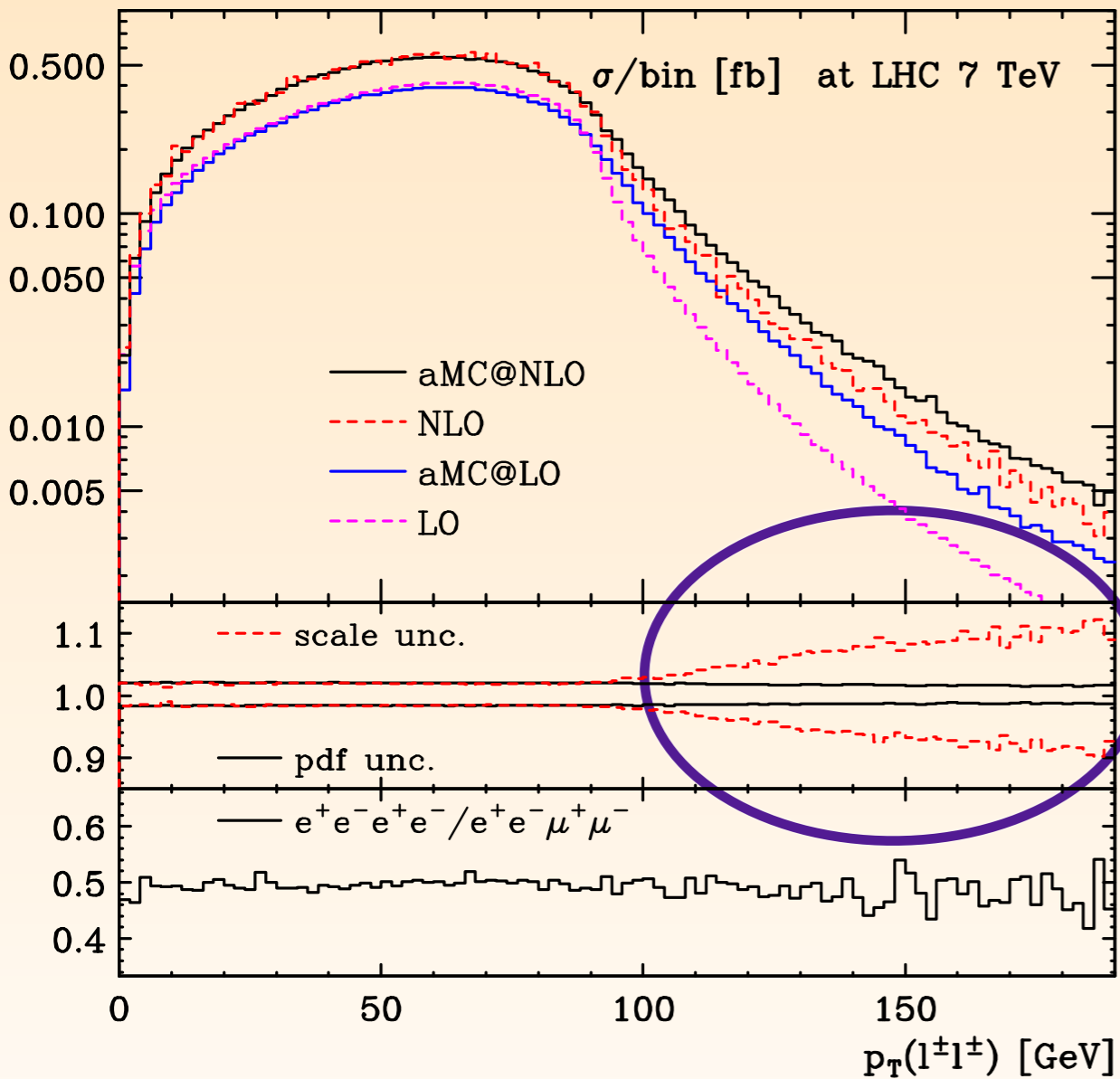### (will become available in a couple of weeks)

Four-lepton production

σ/bin [fb] at LHC 7 TeV

- aMC@NLO
- NLO
- aMC@LO
- LO

scale unc.

pdf unc.

$e^+e^-e^+e^-/e^+e^-\mu^+\mu^-$

$p_T(l^\pm l^\pm)$ [GeV]

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Four-lepton production

In the tail of the pT spectrum, there are large theoretical uncertainties. This is no surprise! Here the NLO calculation has actually only LO accuracy, because there must be a hard parton/jet recoiling against the 4-lepton system.
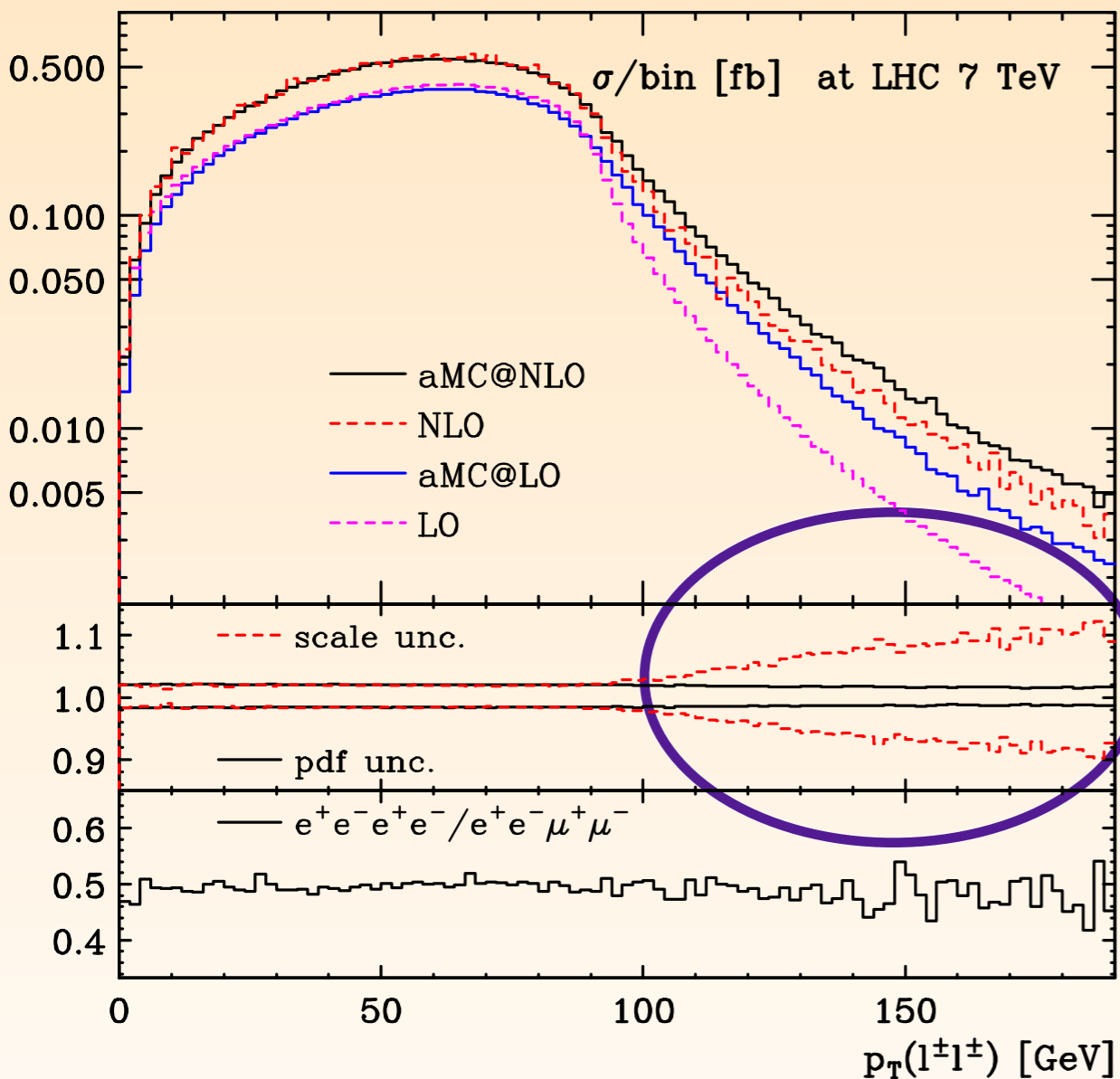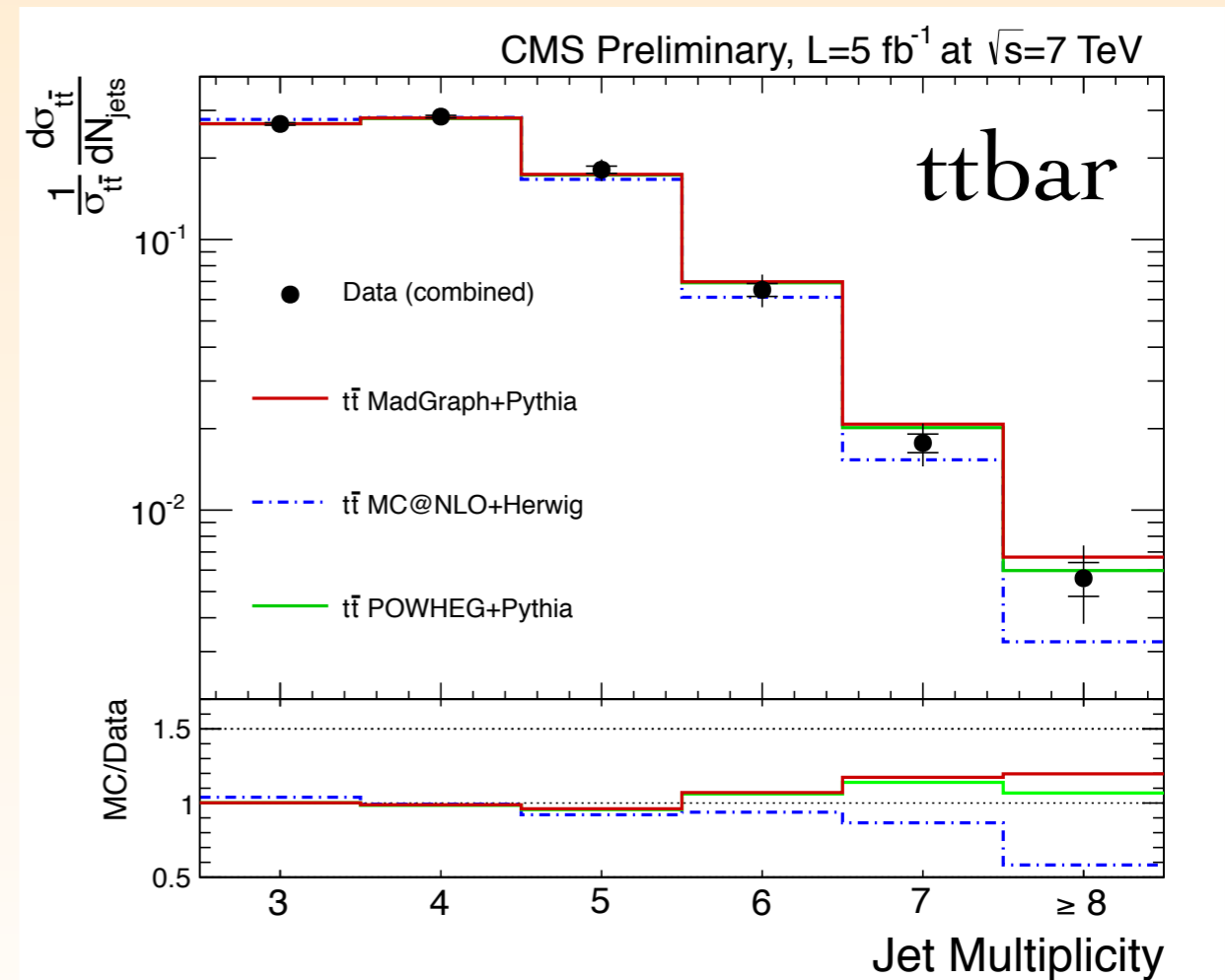
Four-lepton production

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Can we include the NLO corrections to
4 leptons + 1 (hard) jet here?

Limitations

There are more observables very sensitive to theory uncertainties -- all related to **hard emissions** in the real-emission matrix elements and even stronger if they are emitted by the shower.

Even though our NLO computation is "inclusive in all extra radiation" (which is made explicit by the parton shower), the shower is only correct in the strict collinear approximation. It cannot generate hard extra jets correctly (i.e. jets beyond the first, which is included in the real emission corrections of the NLO computation and therefore already has a large uncertainty associated with it)

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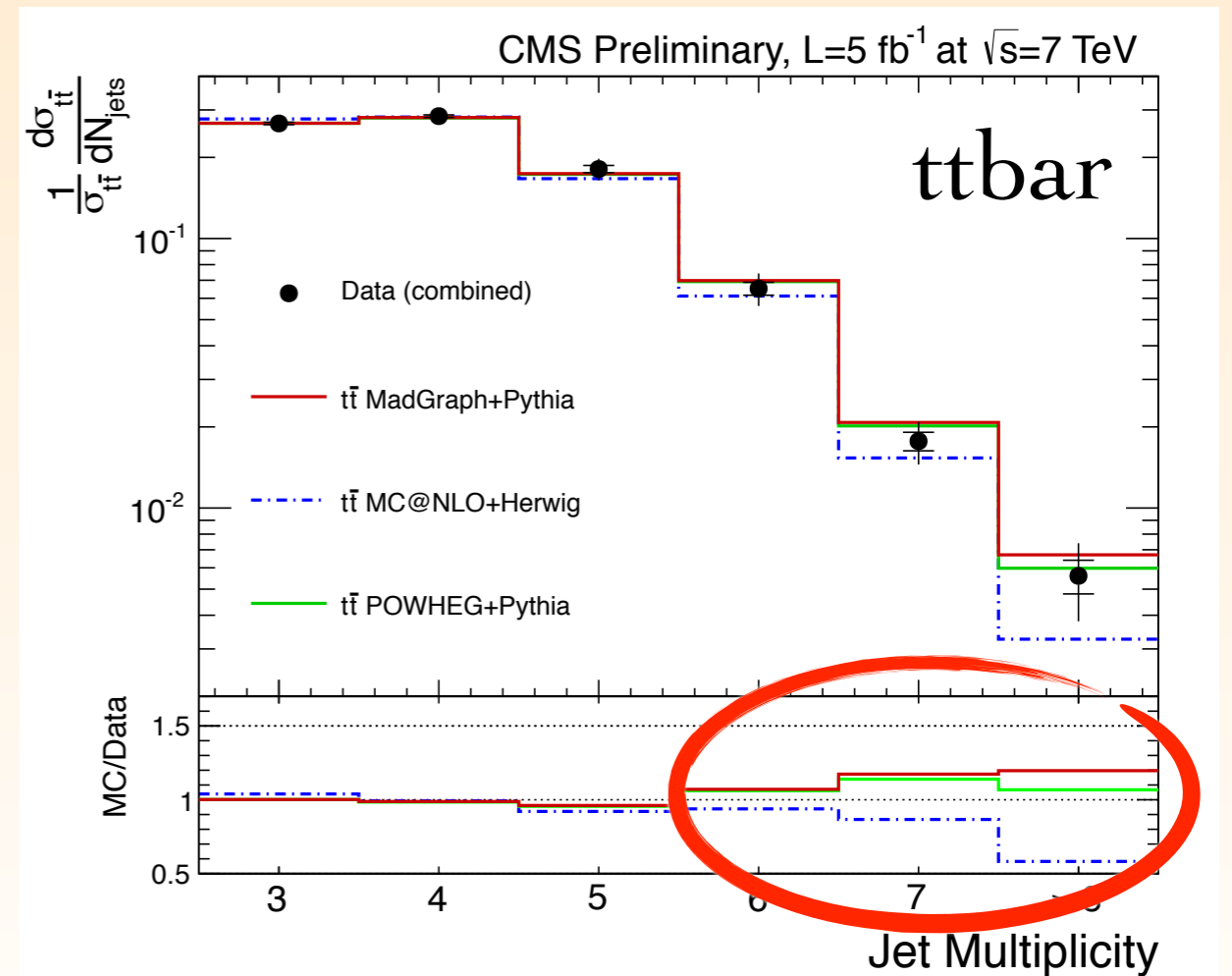

Large dependence on the shower/scales

Merging ME with PS

❋ At LO this has been solved ~10 years ago: use tree-level matrix elements of various multiplicities to generate hard radiation, and the parton shower for the collinear and soft

❋ Double counting no problem: we simply throw events away when the matrix-element partons are too soft, or when the parton shower generates too hard radiation

❋ Applying the matrix-element cut is easy: during phase-space integration, we only generate events with partons above the matching scale

$k_T < Q^c$

$k_T > Q^c$
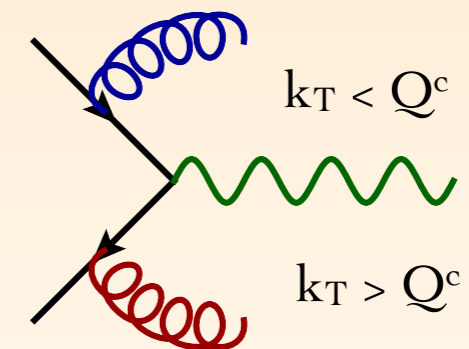
❋ For the cut on the shower, there are two methods. Throwing events away after showering is not very efficient, although it is working ("MLM method")

❋ Instead we can also multiply the Born matrix elements by suitable product of Sudakov factors (i.e. the no-emission probabilities) $\Delta(Q^{max}, Q^c)$ and start the shower at the scale Q^c ("CKKW method"):

❋ For a given multiplicity we have $\sigma_{n,\text{excl}}^{\text{LO}} = B_n \Theta(k_{T,n} - Q^c)\Delta_n(Q_{\max}, Q^c)$

Merging at NLO

※ To make a LO prediction exclusive in the number of jets, we need to multiply it by a Sudakov damping factor; this is CKKW method:

$$\sigma_{n,\text{excl}}^{\text{LO}} = B_n \Theta(k_{T,n} - Q^c) \Delta_n(Q_{\text{max}}, Q^c)$$

This makes the prediction exclusive at leading logarithmic accuracy

※ Similarly we can make an NLO prediction exclusive at leading logarithm

$$\sigma_{n,\text{excl, LL}}^{\text{NLO}} = \left\{ B_n + V_n + \int d\Phi_1 \, R_{n+1} \right\} \Theta(k_{T,n} - Q^c) \Delta_n(Q_{\text{max}}, Q^c)$$

※ We can improve here and use the real-emission matrix elements instead of just the Sudakov:

$$\sigma_{n,\text{excl, LL}}^{\text{NLO}} = \left\{ B_n + V_n + \int_0^{Q^c} d\Phi_1 \, R_{n+1} - B_n \Delta_n^{(1)}(Q_{\text{max}}, Q^c) \right\}$$
$$\Theta(k_{T,n} - Q^c) \Delta_n(Q_{\text{max}}, Q^c)$$

Exclusive MC@NLO: FxFx merging

⁂ Converting the NLO exclusive predictions in the number of jets to the MC@NLO event generation is straight-forward:

$\mathbb{S}$-events:
$$\left\{ B_n + V_n + \int_0^{Q^c} d\Phi_1 \, \text{MC} - B_n \Delta_n^{(1)}(Q_{\max}, Q^c) \right\}$$
$$\Theta(k_{T,n}^B - Q^c) \Delta_n(Q_{\max}^B, Q^c)$$

$\mathbb{H}$-events:
$$\left\{ R_{n+1} \Theta(k_{T,n}^R - Q^c) - \text{MC} \, \Theta(k_{T,n}^B - Q^c) \right\}$$
$$\Theta(Q^c - k_{T,n+1}^R) \Delta_n(Q_{\max}^R, Q^c)$$

⁂ Indeed, that doesn't look very hard...
   It's a simple extension of the LO meging method, isn't it?

The devil is in the details...

✳ What to choose for the renormalization scale (it does not only enter as argument of the strong coupling at NLO)?

✳ What to choose for the factorization scale (it does not only enter in the PDFs at NLO)?

✳ What to do for the PDF reweighting (NLO PDF counter terms)?

✳ What to choose for the starting scales of the parton shower?

✳ How to apply the Sudakov suppression (MLM or CKKW)?

✳ How to treat the extra parton in the real-emission? Do we need a Sudakov?

✳ What to do with the matching scale (fixed or a smooth function)?
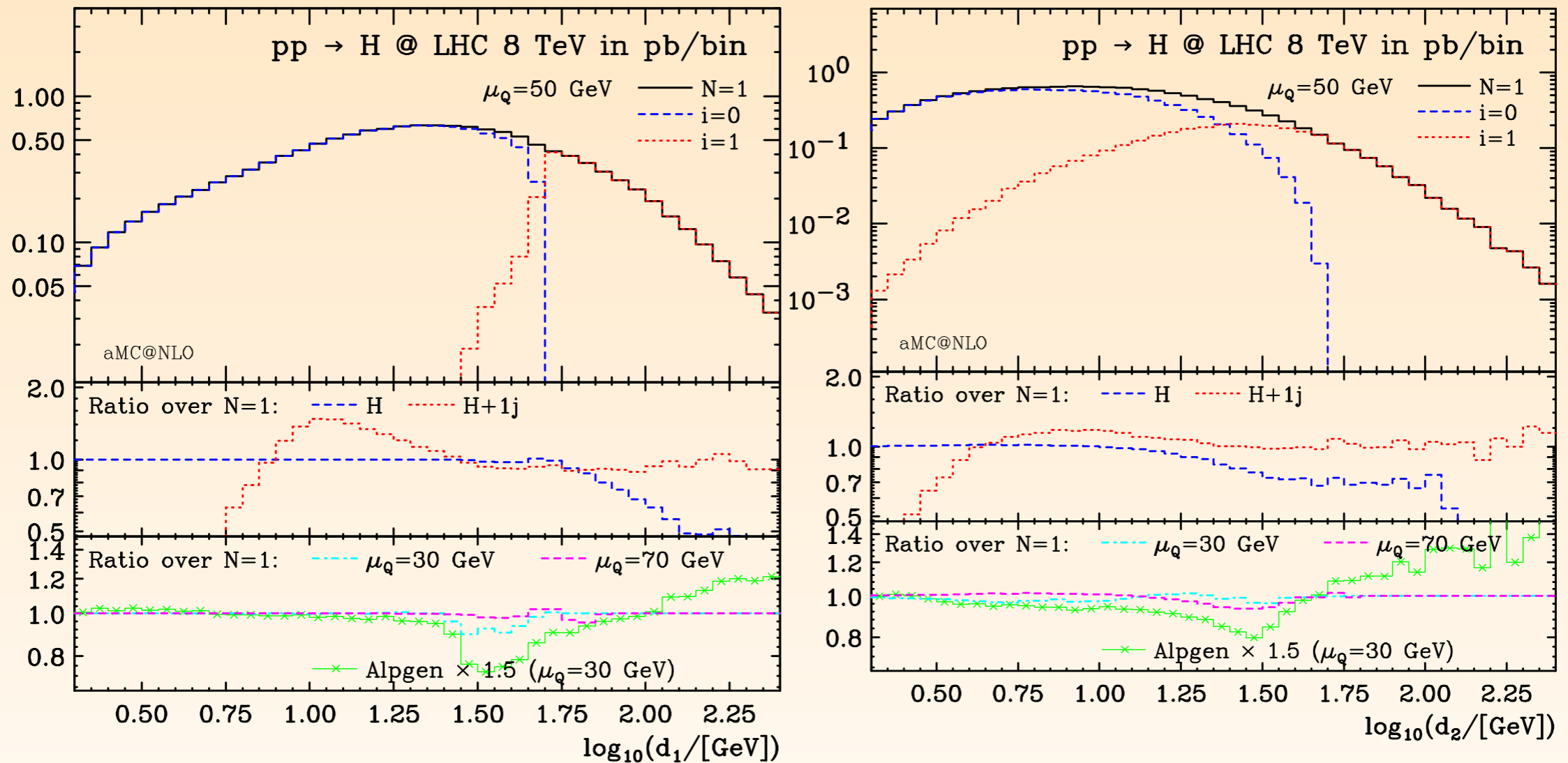
✳ ...

FxFx merging: Higgs boson production
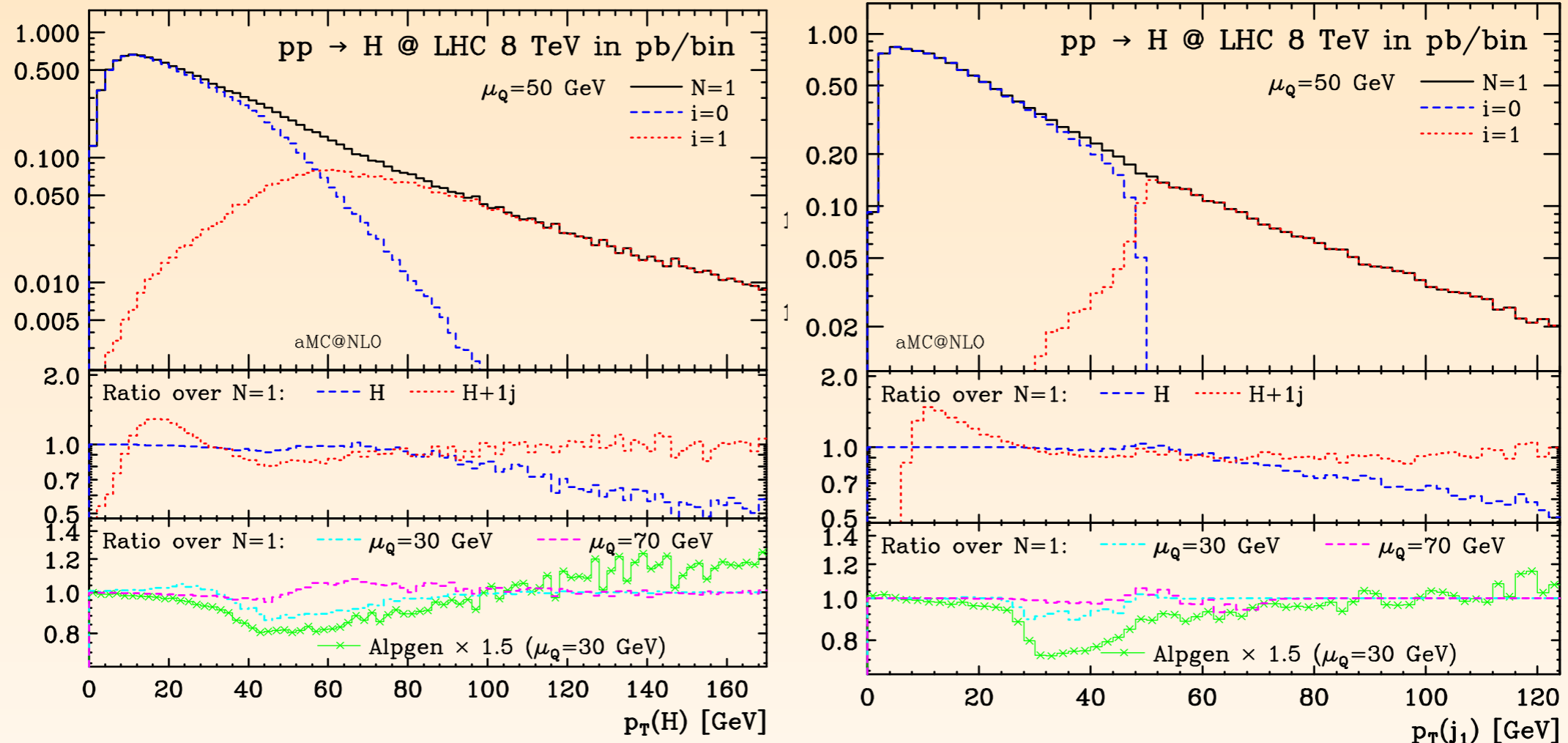
- Transverse momentum of the Higgs and of the 1st jet.

- Agreement with H+0j at MC@NLO and H+1j at MC@NLO in their respective regions of phase-space; Smooth matching in between; Small dependence on matching scale

- Alpgen (LO matching) shows larger kinks
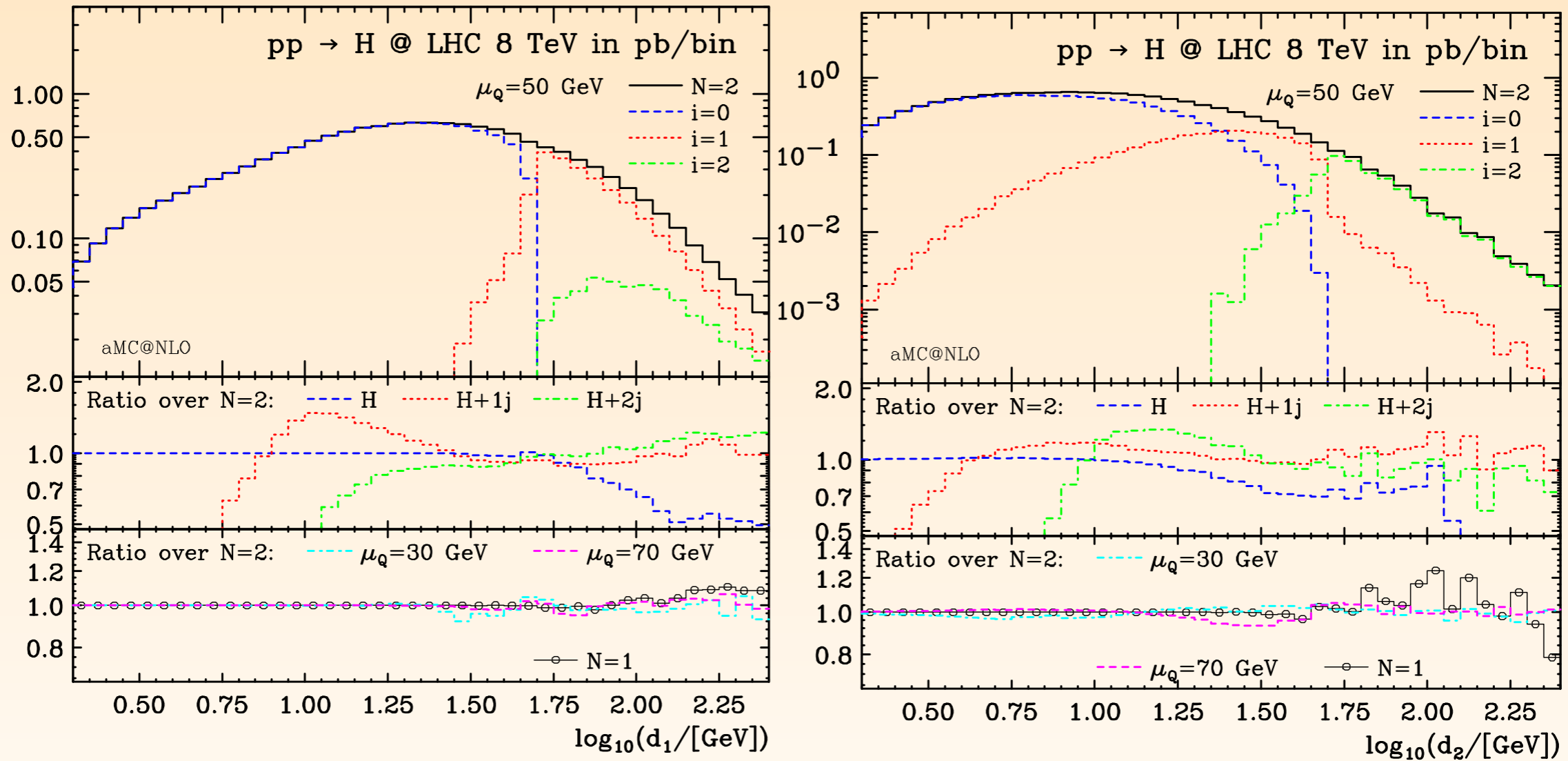
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FxFx merging: Higgs boson production

pp → H @ LHC 8 TeV in pb/bin
μ_Q=50 GeV
N=1, i=0, i=1
aMC@NLO

Ratio over N=1: H, H+1j

Ratio over N=1: μ_Q=30 GeV, μ_Q=70 GeV
Alpgen × 1.5 (μ_Q=30 GeV)

$\log_{10}(d_1/[GeV])$

$\log_{10}(d_2/[GeV])$

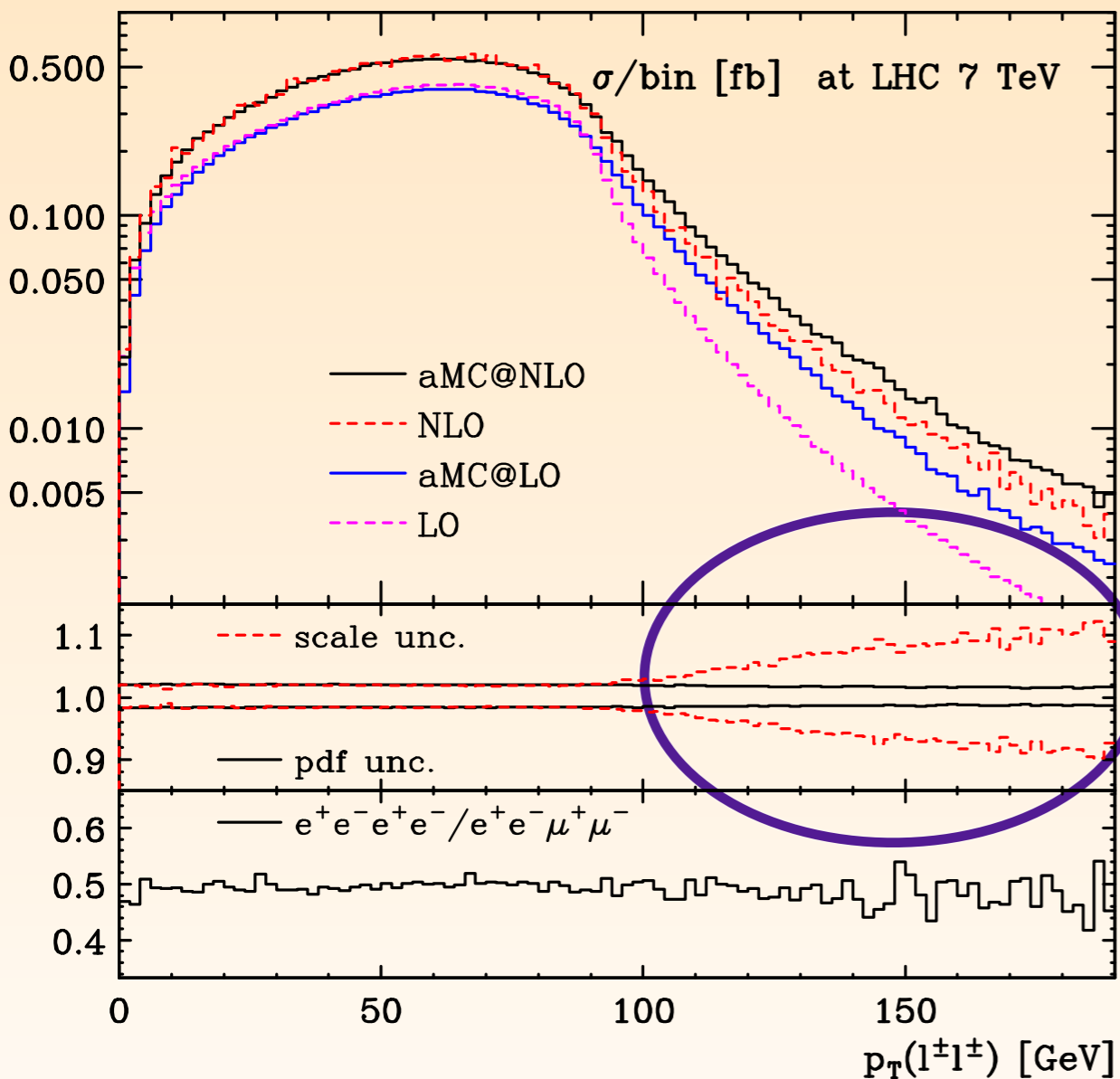✳ Differential jet rates for 1->0 and 2->1

FxFx merging: Higgs boson production

* Differential jet rates

* Matching up to 2 jets at NLO

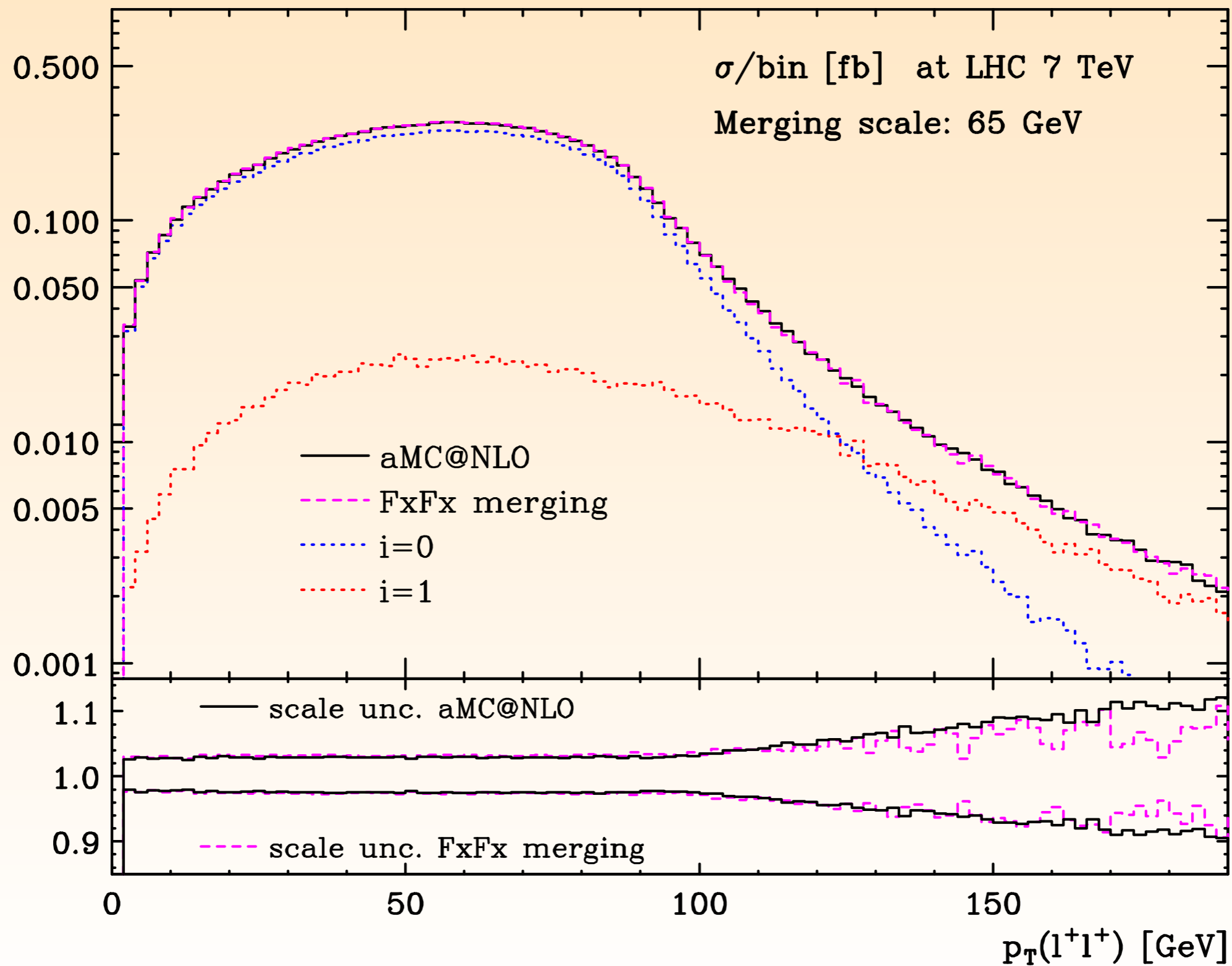* Results very much consistent with matching up to 1 jet at NLO

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Can we include the NLO corrections to
4 leptons + 1 (hard) jet here?

Four-lepton production

σ/bin [fb] at LHC 7 TeV

Merging scale: 65 GeV

aMC@NLO
FxFx merging
i=0
i=1

scale unc. aMC@NLO

scale unc. FxFx merging

$p_T(l^+l^+)$ [GeV]

Conclusions

- In the last couple of years the accuracy of event generation has greatly improved, and full automation has been achieved at NLO accuracy

- A lot of freedom in tuning has been replaced by accurate theory descriptions:

    - More predictive power

    - Better control on uncertainties

    - Greater trust in the measurements

- The only public tool that can generate events at NLO accuracy (in QCD) for any process in the SM (or simple extensions) is the `aMC@NLO` project. It is only limited in CPU time available

- Latest developments include the FxFx merging, which allows for the combination of various multiplicity matrix elements at NLO into a single inclusive sample