KIAS School on MadGraph for LHC Physics (Oct/24-29 2011)

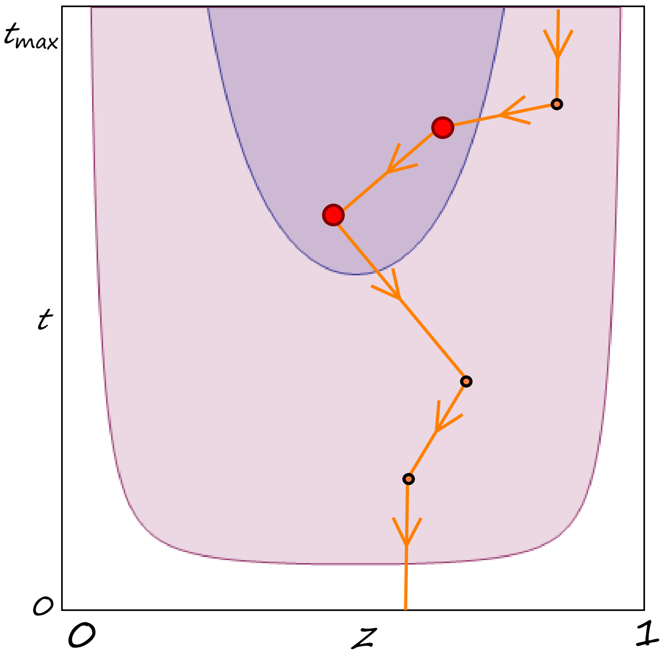
Matrix Elements + Parton Showers

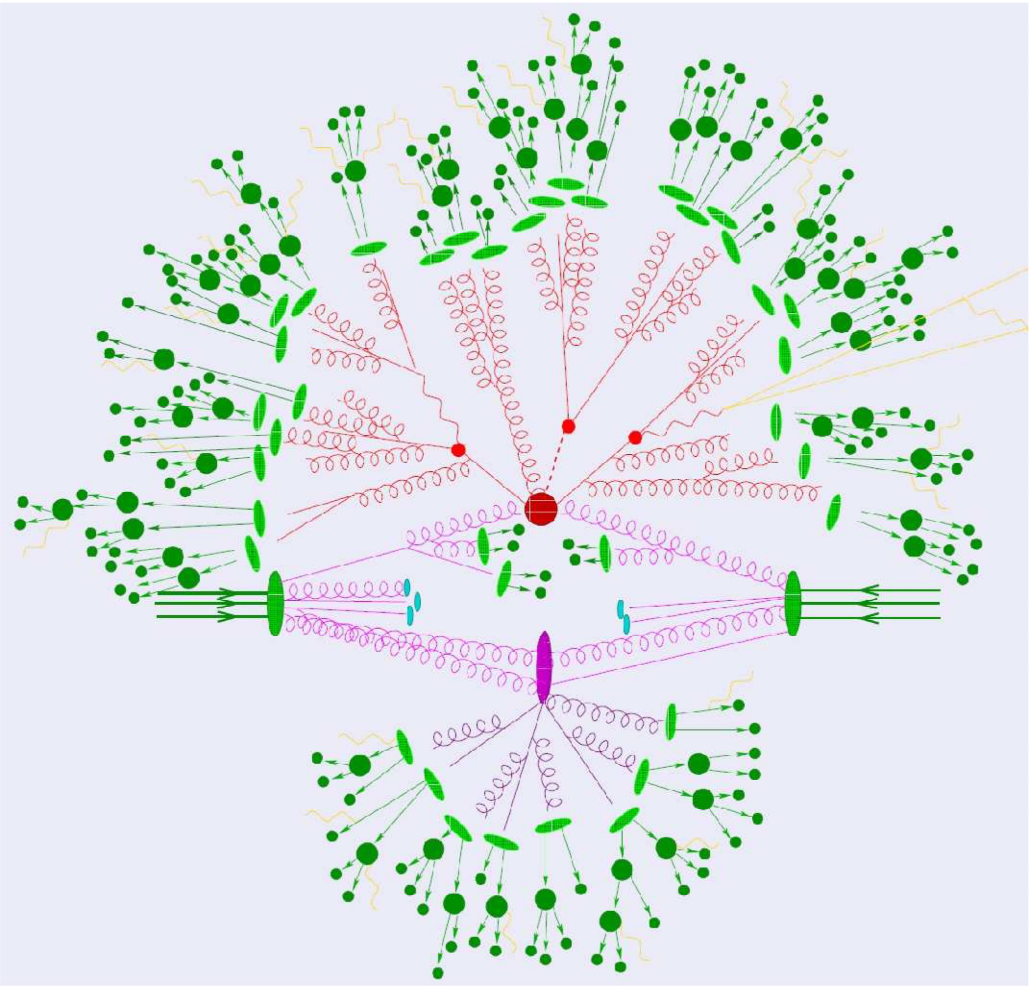
Grisha Kirilin/KEK

Literature for the third lecture:

- S. Catani, F. Krauss, B.R. Webber, R. Kuhn, QCD matrix elements + parton showers, JHEP 11 (2001) 063

Jets





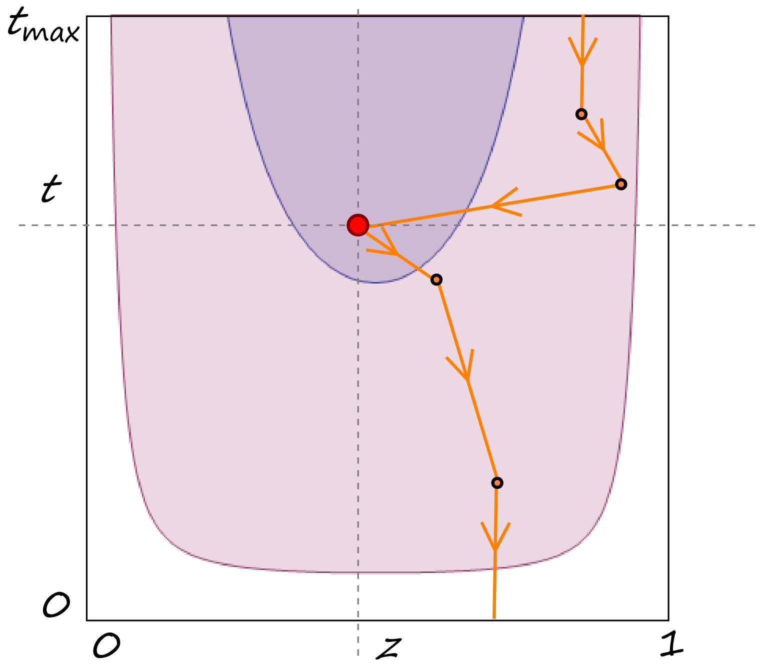
if we stop custering algorithm $y_{ij} > \tau_{ini}/Q^2$,

i.e., all distances between pseudoparticles

are grater than τ_{ini}. A bunch of particles

already clustered into a single preudoparticles is called a **jet**.

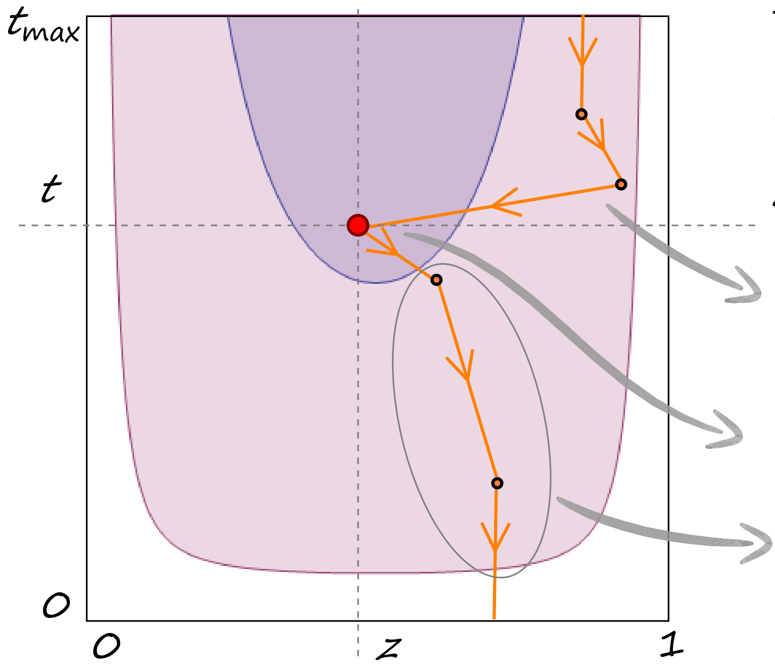
Single resolved splitting



The following statements are equivalent:

1. There is only one (the hardest) splitting with $(k_\perp^2)_2 > \tau_{ini}$
2. The second hardest splitting has $(k_\perp^2)_3 < \tau_{ini}$

Single resolved splitting



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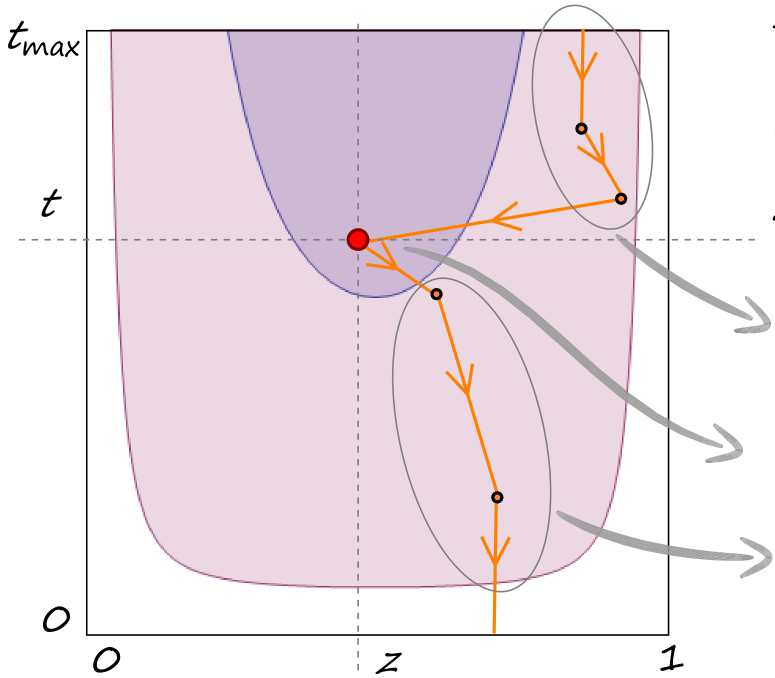
???

$$\times \frac{dt}{t}\, dz\, P(\alpha_s, z)\, \Theta\left(k_\perp^2 - \tau_{ini}\right) \qquad k_\perp^2 = \min\{z^2, \bar{z}^2\}\, t$$

$$\times\, \Delta_q\left(z^2\, t \,\middle|\, \tau_{ini}\right)$$

Single resolved splitting



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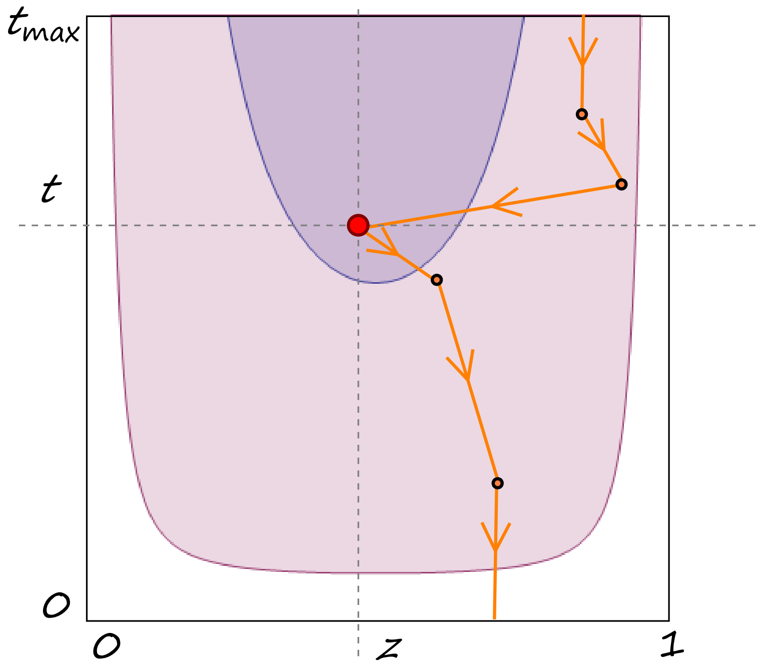
$$\frac{\Delta_q(t_{max}\,|\,\tau_{ini})}{\Delta_q(t\,|\,\tau_{ini})}$$

$$\times \frac{dt}{t}\, dz\, P(\alpha_s, z)\, \Theta\left(k_\perp^2 - \tau_{ini}\right) \qquad k_\perp^2 = \min\{z^2, \bar{z}^2\}\, t$$

$$\times\, \Delta_q\left(z^2\, t\,|\,\tau_{ini}\right)$$

Single resolved splitting



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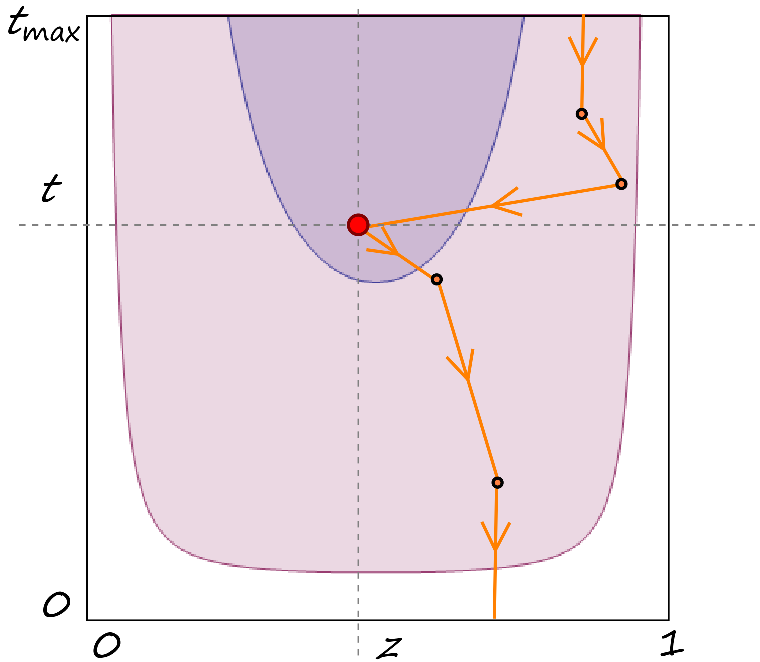
$$\frac{\Delta_q(t_{max}\,|\,\tau_{ini})}{\Delta_q(t\,|\,\tau_{ini})} \times \frac{d\,t}{t}\, d\,z\, P(\alpha_s, z)\, \Theta(k_\perp^2 - \tau_{ini}) \times \Delta_q(z^2\, t\,|\,\tau_{ini})$$

$$\approx \Delta_q(t_{max}\,|\,\tau_{ini}) \times \frac{d\,t}{t}\, d\,z\, P(\alpha_s, z)\, \Theta(k_\perp^2 - \tau_{ini})$$

What is about total probability?

Single resolved splitting



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$$\frac{\Delta_q(t_{max} \mid \tau_{ini})}{\Delta_q(t \mid \tau_{ini})} \times \frac{d\,t}{t}\, d\,z\, P(\alpha_s, z)\, \Theta(k_\perp^2 - \tau_{ini}) \times \Delta_q(z^2\, t \mid \tau_{ini})$$

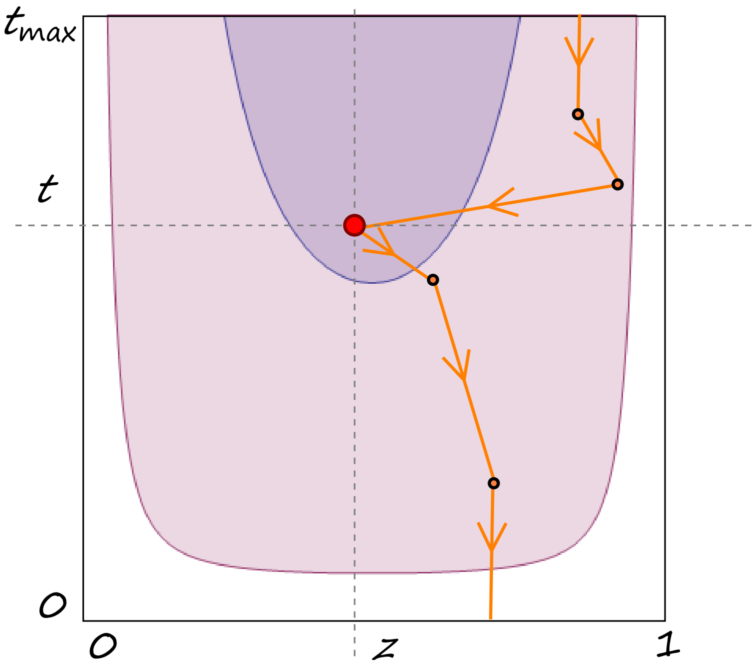
$$\approx \Delta_q(t_{max} \mid \tau_{ini}) \times \frac{d\,t}{t}\, d\,z\, P(\alpha_s, z)\, \Theta(k_\perp^2 - \tau_{ini})$$

$$R_3(\tau_{ini}) = \Delta_q(t_{max} \mid \tau_{ini}) \times \int_0^{t_{max}} \frac{d\,t}{t} \int_0^1 d\,z\, P(\alpha_s, z)\, \Theta(k_\perp^2 - \tau_{ini}) \approx \Delta_q(t_{max} \mid \tau_{ini}) \times \int_{\tau_{ini}}^{t_{max}} d\,\tau\, \Gamma_q(t_{max} \mid \tau)$$

$$\Gamma_q(T \mid \tau) = \frac{\alpha_s(\tau)\, C_F}{2\pi} \frac{1}{\tau} \left(\ln \frac{T}{\tau} - \frac{3}{2} \right)$$

$$f(\tau_3) = \frac{d\, R_3(\tau_3)}{d\,\tau_3}$$

Single resolved splitting



The following statements are equivalent:

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$$\frac{\Delta_q(t_{max}\,|\,\tau_{ini})}{\Delta_q(t\,|\,\tau_{ini})} \times \frac{dt}{t}\, dz\, P(\alpha_s, z)\, \Theta\big(k_\perp^2 - \tau_{ini}\big) \times \Delta_q\big(z^2 t\,|\,\tau_{ini}\big)$$

$$\approx \Delta_q(t_{max}\,|\,\tau_{ini}) \times \frac{dt}{t}\, dz\, P(\alpha_s, z)\, \Theta\big(k_\perp^2 - \tau_{ini}\big)$$

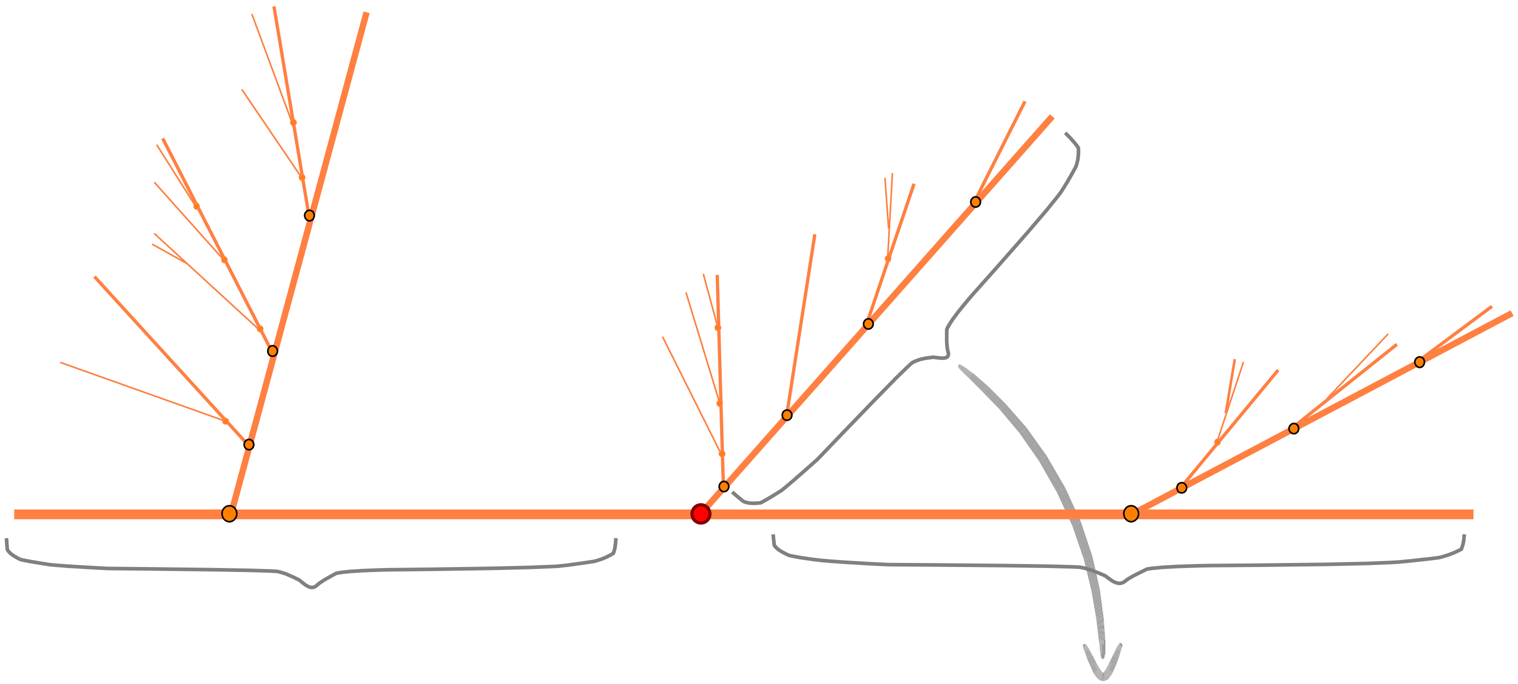
$$R_3(\tau_{ini}) = \Delta_q(t_{max}\,|\,\tau_{ini}) \times \int_0^{t_{max}} \frac{dt}{t} \int_0^1 dz\, P(\alpha_s, z)\, \Theta\big(k_\perp^2 - \tau_{ini}\big) \approx \Delta_q(t_{max}\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{t_{max}} d\tau\, \Gamma_q(t_{max}\,|\,\tau)$$

$$\Gamma_q(T\,|\,\tau) = \frac{\alpha_s(\tau)\, C_F}{2\pi} \frac{1}{\tau} \left(\ln \frac{T}{\tau} - \frac{3}{2} \right)$$

$$f(\tau_3) = \frac{d R_3(\tau_3)}{d\tau_3}$$

Actually it is not correct because we considered only one rod.

Single resolved splitting

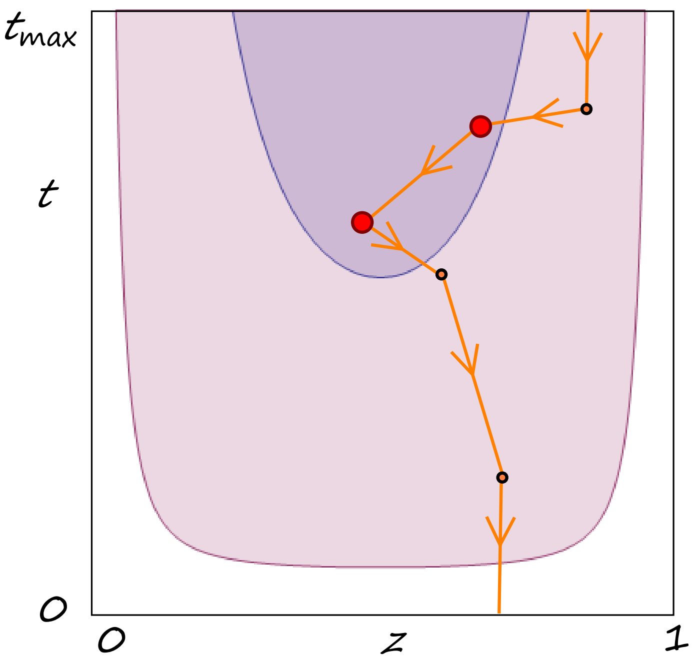


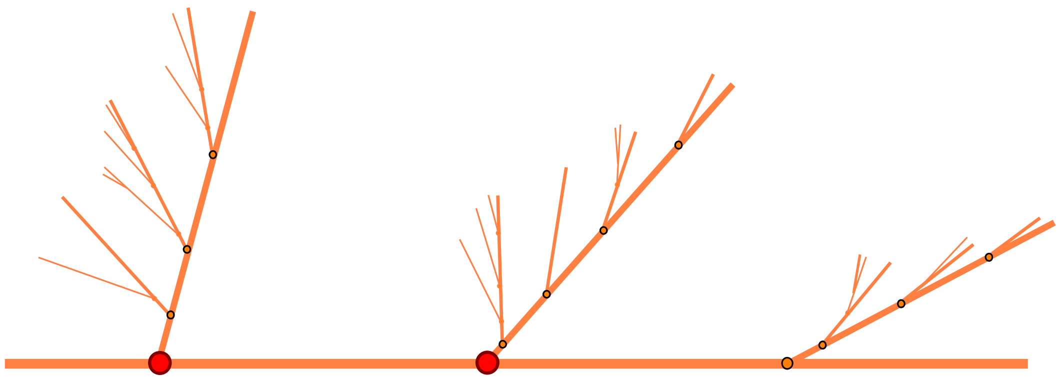
$$\frac{\Delta_q(t_{max} \mid \tau_{ini})}{\Delta_q(t \mid \tau_{ini})} \times \frac{dt}{t}\, dz\, P(\alpha_s, z)\, \Theta\left(k_\perp^2 - \tau_{ini}\right) \times \Delta_q\left(z^2\, t \mid \tau_{ini}\right) \times \Delta_g\left(\bar{z}^2\, t \mid \tau_{ini}\right)$$

$$\approx \Delta_q(t_{max} \mid \tau_{ini}) \times \frac{dt}{t}\, dz\, P(\alpha_s, z)\, \Theta\left(k_\perp^2 - \tau_{ini}\right) \times \Delta_g\left(k_\perp^2 \mid \tau_{ini}\right)$$

$$R_3(\tau_3) \approx \Delta_q(t_{max} \mid \tau_3) \times \int_{\tau_3}^{t_{max}} d\tau\, \Gamma_q(t_{max} \mid \tau)\, \Delta_g(\tau \mid \tau_3)$$

Double resolved splitting





$$1 = \frac{\Delta_q(t_{max}\,|\,\tau_{ini})}{\Delta_q(t_{max}\,|\,\tau_{ini})} = \Delta_q(t_{max}\,|\,\tau_{ini})\, exp\left[\int_{\tau_{ini}}^{t_{max}} d\tau\, \Gamma_q(t_{max}\,|\,\tau)\right]$$

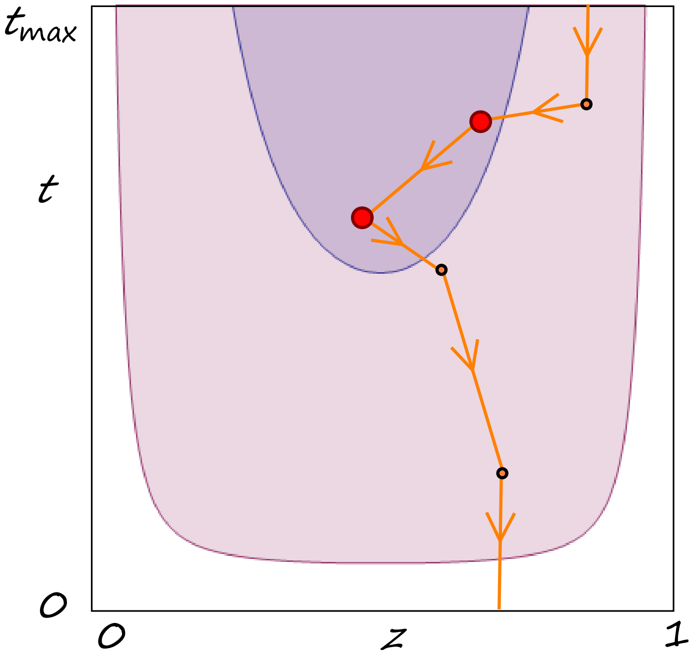
No resolved splittings → $= \Delta_q(t_{max}\,|\,\tau_{ini})$

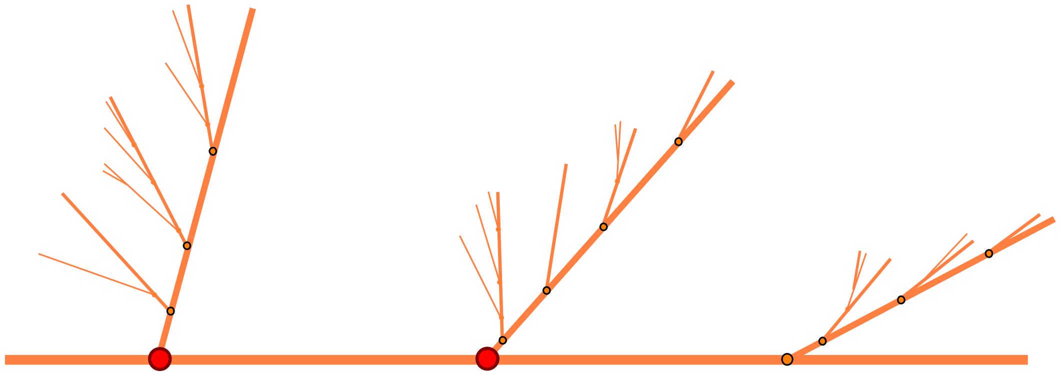
One resolved splitting → $+\Delta_q(t_{max}\,|\,\tau_{ini}) \int_{\tau_{ini}}^{t_{max}} d\tau\, \Gamma_q(t_{max}\,|\,\tau)$

Two resolved splittings → $+\Delta_q(t_{max}\,|\,\tau_{ini})\, \frac{1}{2}\left[\int_{\tau_{ini}}^{t_{max}} d\tau\, \Gamma_q(t_{max}\,|\,\tau)\right]^2$

Three resolved splittings → $+\Delta_q(t_{max}\,|\,\tau_{ini})\, \frac{1}{3!}\left[\int_{\tau_{ini}}^{t_{max}} d\tau\, \Gamma_q(t_{max}\,|\,\tau)\right]^3 + \ldots$

Double resolved splitting





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No resolved splittings ⟶ $= \Delta_q(t_{max}\,|\,\tau_{ini})$

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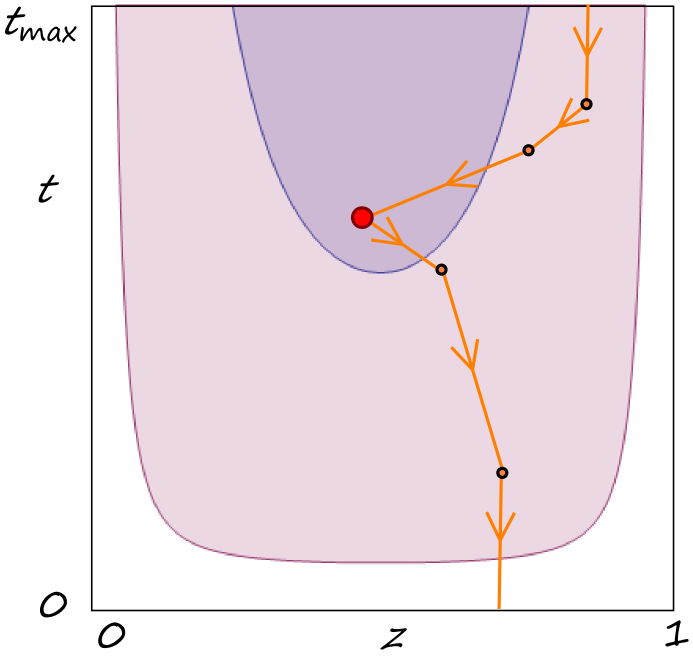
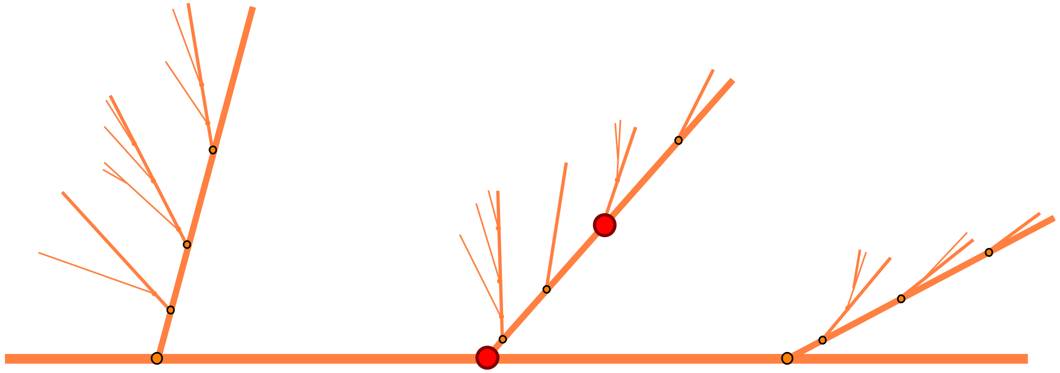
Two resolved splittings ⟶ $+\Delta_q(t_{max}\,|\,\tau_{ini})\, \frac{1}{2}\left[\int_{\tau_{ini}}^{t_{max}} d\tau\, \Gamma_q(t_{max}\,|\,\tau)\right]^2$

Three resolved splittings ⟶ $+\Delta_q(t_{max}\,|\,\tau_{ini})\, \frac{1}{3!}\left[\int_{\tau_{ini}}^{t_{max}} d\tau\, \Gamma_q(t_{max}\,|\,\tau)\right]^3 + \ldots$

Double resolved splitting





$$R_4(\tau_{ini}) = \Delta_q(t_{max}\,|\,\tau_{ini})\,\frac{1}{2}\int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini})$$

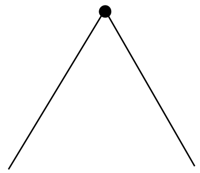
$$+\Delta_q(t_{max}\,|\,\tau_{ini})\int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\times\Delta_g(\tau\,|\,\tau_{ini})\int_{\tau_{ini}}^{\tau} d\tau'\,\Gamma_g(\tau\,|\,\tau')\times\Delta_g(\tau'\,|\,\tau_{ini})$$

$$+\Delta_q(t_{max}\,|\,\tau_{ini})\int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\times\Delta_g(\tau\,|\,\tau_{ini})\int_{\tau_{ini}}^{\tau} d\tau'\,\Gamma_f(\tau\,|\,\tau')\times\Delta_f(\tau'\,|\,\tau_{ini})$$

$Z \to q\bar{q}$

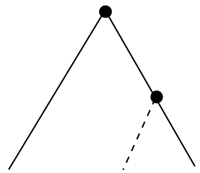
Two-jet rate

$$R_2(\tau_{ini}) = \Delta_q(t_{max} \,|\, \tau_{ini}) \times \Delta_q(t_{max} \,|\, \tau_{ini})$$



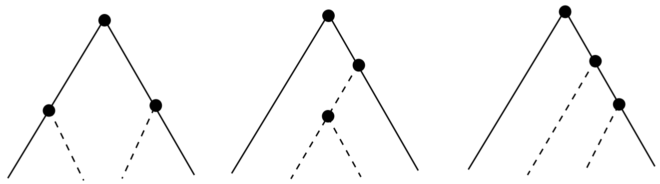
Three-jet rate

$$R_3(\tau_{ini}) \approx 2\left[\Delta_q(t_{max} \,|\, \tau_{ini})\right]^2 \times \int_{\tau_{ini}}^{t_{max}} d\tau \, \Gamma_q(t_{max} \,|\, \tau)\, \Delta_g(\tau \,|\, \tau_{ini})$$

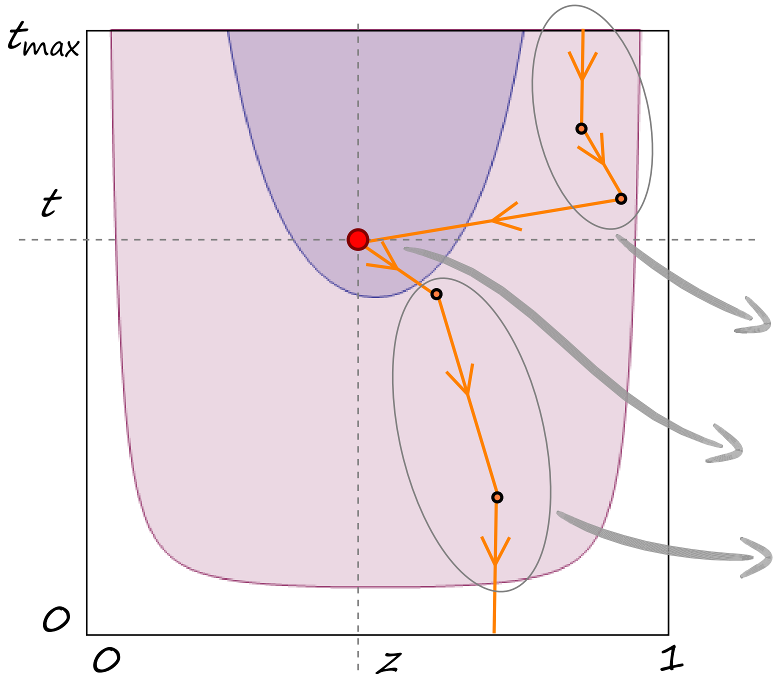


Four-jet rate

$$R_4(\tau_{ini}) = 2\, \Delta_q(t_{max} \,|\, \tau_{ini}) \times \Delta_q(t_{max} \,|\, \tau_{ini})$$
$$\times \Bigg[\int_{\tau_{ini}}^{t_{max}} d\tau \, \Gamma_q(t_{max} \,|\, \tau)\, \Delta_g(\tau \,|\, \tau_{ini}) \times \int_{\tau_{ini}}^{t_{max}} d\tau' \, \Gamma_q(t_{max} \,|\, \tau')\, \Delta_g(\tau' \,|\, \tau_{ini})$$
$$+ \int_{\tau_{ini}}^{t_{max}} d\tau \, \Gamma_q(t_{max} \,|\, \tau)\, \Delta_g(\tau \,|\, \tau_{ini}) \times \int_{\tau_{ini}}^{\tau} d\tau' \, \Gamma_g(\tau \,|\, \tau')\, \Delta_g(\tau' \,|\, \tau_{ini})$$
$$+ \int_{\tau_{ini}}^{t_{max}} d\tau \, \Gamma_q(t_{max} \,|\, \tau)\, \Delta_g(\tau \,|\, \tau_{ini}) \times \int_{\tau_{ini}}^{\tau} d\tau' \, \Gamma_f(\tau \,|\, \tau')\, \Delta_f(\tau' \,|\, \tau_{ini}) \Bigg]$$



Renormalization



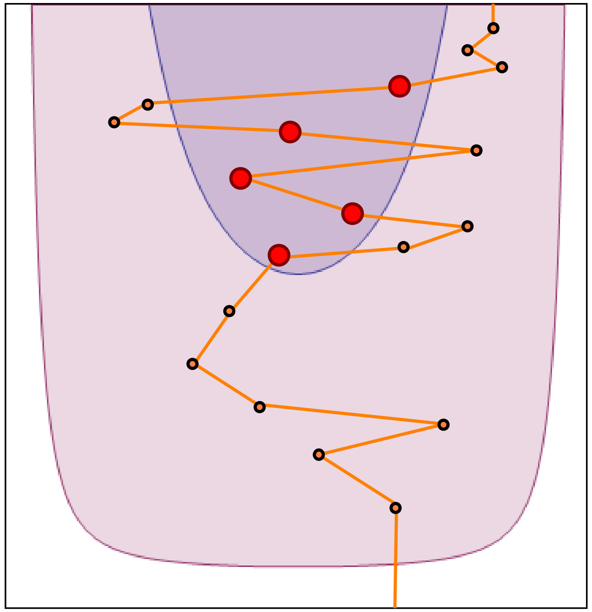
$$\frac{\Delta_q(t_{max} \,|\, \tau_{ini})}{\Delta_q(t \,|\, \tau_{ini})}$$

$$\times \frac{dt}{t} \, dz \, P(\alpha_s, z) \, \Theta\left(k_\perp^2 - \tau_{ini}\right) \qquad k_\perp^2 = \min\{z^2, \bar{z}^2\}\, t$$

$$\times \, \Delta_q\left(z^2 \, t \,\middle|\, \tau_{ini}\right)$$

Actually, it is the expression for the single splitting probability for a PS generator with cutoff τ_{ini}

Renormalization



PS Gen. 1:

cutoff is τ_0,

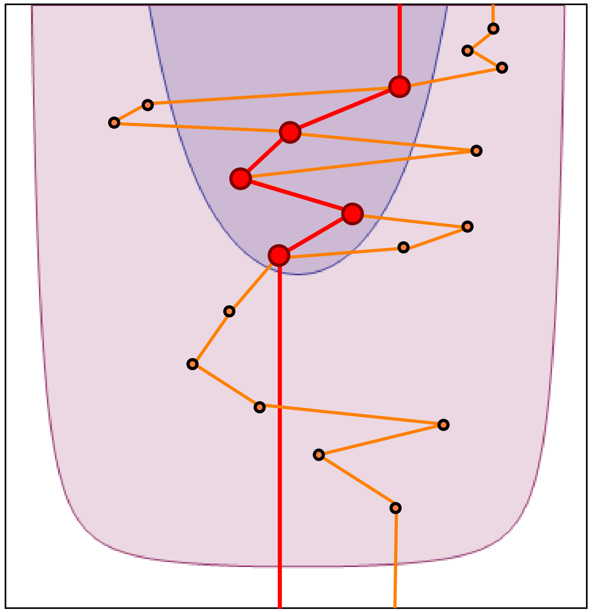
$\Delta(t \mid \tau_0)$

Renormalization

PS Gen. 2:

cutoff is $\tau_{ini} > \tau_O$,

$\Delta(t\,|\,\tau_{ini})$



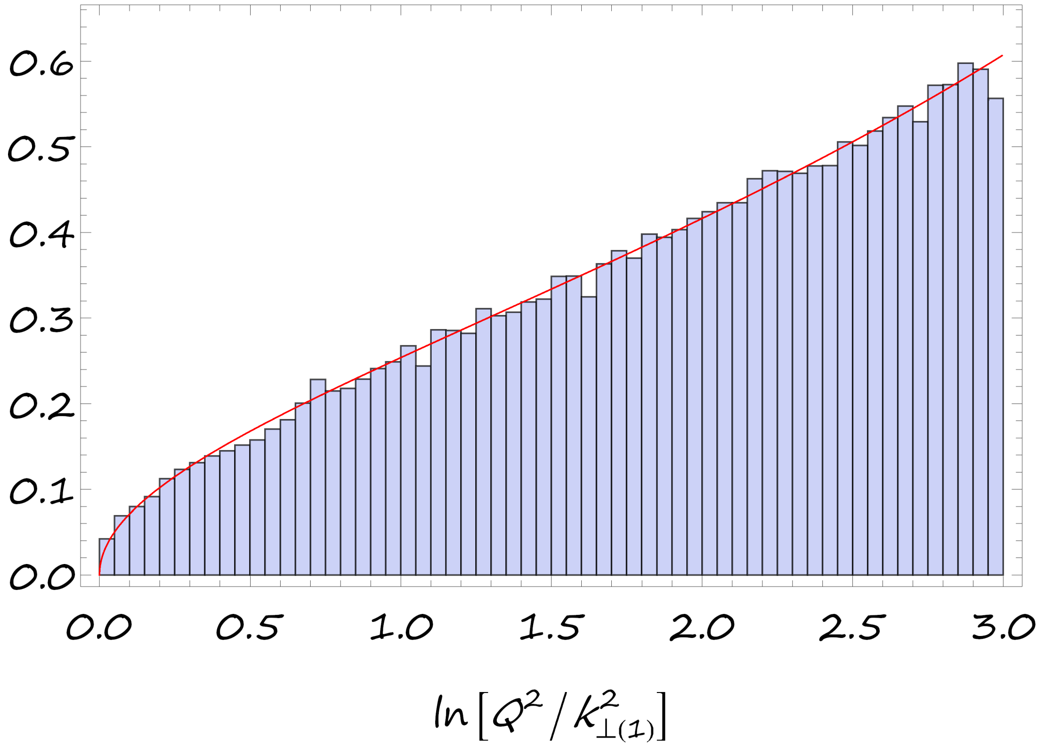
PS Gen. 1:

cutoff is τ_O,

$\Delta(t\,|\,\tau_O)$

- If we select the nodes with $k_\perp^2 > \tau_{ini}$ which were generated by Gen 1, then we obtain the same distributions as those generated by Gen 2.
- The objects generated by Gen 2 are exclusive in the sense they are separated at least by the distance τ_{ini}, but completely inclusive with respect to all partons closer than τ_{ini}.
- Gen 1 can help us to reveal the exclusive structure of radiation inside the region $\tau_O < k_\perp^2 < \tau_{ini}$.

Renormalization



$k^2_{\perp(1)}$ is the transverse moment of the node which is the first on the (angular ordered) rod among resolved ones ($k^2_\perp > \tau_{ini}$)

Histogram is the result of Gen. 1

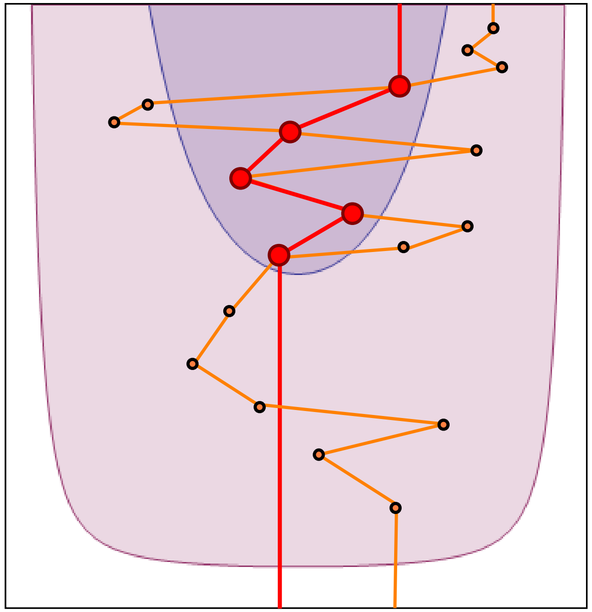
Red curve is the result of Gen. 2

Renormalization

PS Gen. 2:

cutoff is $\tau_{ini} > \tau_O$,

$\Delta(t \mid \tau_{ini})$



PS Gen. 1:

cutoff is τ_O,

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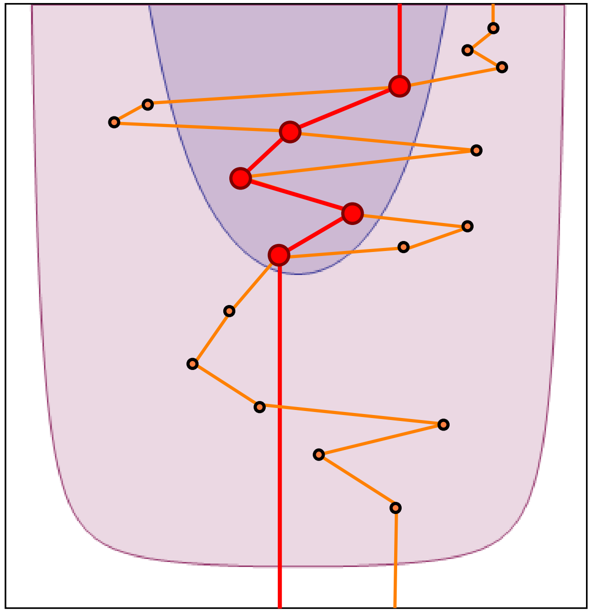
By the way, why do we have to use Gen 2?

Renormalization

PS Gen. 2:

cutoff is $\tau_{ini} > \tau_O$,

$\Delta(t\,|\,\tau_{ini})$



PS Gen. 1:

cutoff is τ_O,

$\Delta(t\,|\,\tau_O)$

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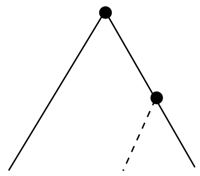
- Gen 1 can help us to reveal the exclusive structure of radiation inside the region $\tau_O < k_\perp^2 < \tau_{ini}$.

By the way, why do we have to use Gen 2? We don't. Let's replace it by e.g. MadGraph

Bare particles and Sudakov factor

- Three-jet rate

$$R_3(\tau_{ini}) = \left[\Delta_q(t_{max}\,|\,\tau_{ini})\right]^2 \times \int_{\tau_{ini}}^{t_{max}} d\tau\, 2\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini})$$



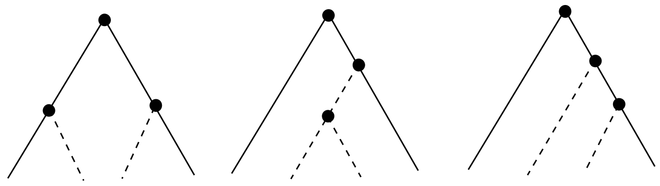
- Four-jet rate

$$R_4(\tau_{ini}) = 2\,\Delta_q(t_{max}\,|\,\tau_{ini}) \times \Delta_q(t_{max}\,|\,\tau_{ini})$$

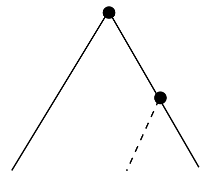
$$\times \Bigg[\int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{t_{max}} d\tau'\,\Gamma_q(t_{max}\,|\,\tau')\,\Delta_g(\tau'\,|\,\tau_{ini})$$

$$+ \int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{\tau} d\tau'\,\Gamma_g(\tau\,|\,\tau')\,\Delta_g(\tau'\,|\,\tau_{ini})$$

$$+ \int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{\tau} d\tau'\,\Gamma_f(\tau\,|\,\tau')\,\Delta_f(\tau'\,|\,\tau_{ini}) \Bigg]$$

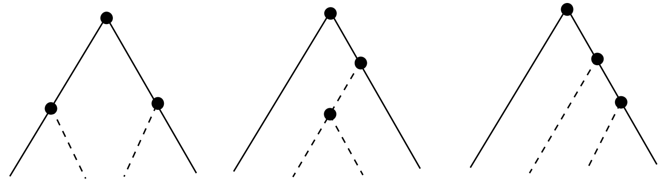


Bare particles and Sudakov factor



Three-jet rate

$$R_3(\tau_{ini}) = \left[\Delta_q(t_{max}\,|\,\tau_{ini})\right]^2 \times \int_{\tau_{ini}}^{t_{max}} d\tau\, 2\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini})$$



Four-jet rate

$$R_4(\tau_{ini}) = 2\,\Delta_q(t_{max}\,|\,\tau_{ini}) \times \Delta_q(t_{max}\,|\,\tau_{ini})$$

$$\times\Bigg[\int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{t_{max}} d\tau'\,\Gamma_q(t_{max}\,|\,\tau')\,\Delta_g(\tau'\,|\,\tau_{ini})$$

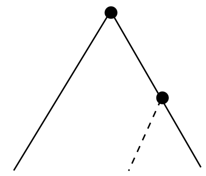
$$+ \int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{\tau} d\tau'\,\Gamma_g(\tau\,|\,\tau')\,\Delta_g(\tau'\,|\,\tau_{ini})$$

$$+ \int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{\tau} d\tau'\,\Gamma_f(\tau\,|\,\tau')\,\Delta_f(\tau\,|\,\tau_{ini})\Bigg]$$

PS approximation of the matrix element squared should be replaced by the exact ME.

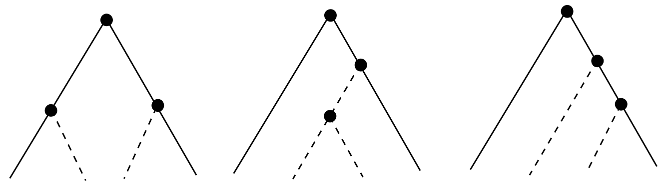
$$\Gamma_q(T\,|\,\tau) = \frac{\alpha_s(\tau)\,C_F}{2\pi}\,\frac{1}{\tau}\left(\ln\frac{T}{\tau} - \frac{3}{2}\right) \qquad \Gamma_g(T\,|\,\tau) = \frac{\alpha_s(\tau)\,C_g}{2\pi}\,\frac{1}{\tau}\left(\ln\frac{T}{\tau} - \frac{11}{6}\right) \qquad \Gamma_f(T,\tau) = \frac{\alpha_s(\tau)}{2\pi}\,\frac{N_f}{3}\,\frac{1}{\tau}$$

Bare particles and Sudakov factor



Three-jet rate

$$R_3(\tau_{ini}) = \left[\Delta_q(t_{max}\,|\,\tau_{ini})\right]^2 \times \int_{\tau_{ini}}^{t_{max}} d\tau\, 2\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini})$$



Four-jet rate

$$R_4(\tau_{ini}) = 2\,\Delta_q(t_{max}\,|\,\tau_{ini}) \times \Delta_q(t_{max}\,|\,\tau_{ini})$$

$$\times\Bigg[\int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{t_{max}} d\tau'\,\Gamma_q(t_{max}\,|\,\tau')\,\Delta_g(\tau'\,|\,\tau_{ini})$$

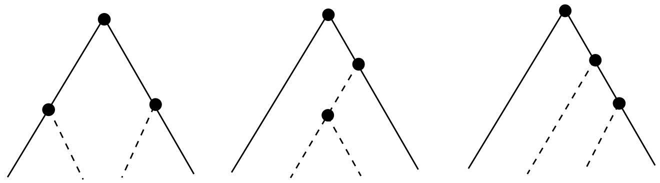
$$+ \int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{\tau} d\tau'\,\Gamma_g(\tau\,|\,\tau')\,\Delta_g(\tau'\,|\,\tau_{ini})$$

$$+ \int_{\tau_{ini}}^{t_{max}} d\tau\,\Gamma_q(t_{max}\,|\,\tau)\,\Delta_g(\tau\,|\,\tau_{ini}) \times \int_{\tau_{ini}}^{\tau} d\tau'\,\Gamma_f(\tau\,|\,\tau')\,\Delta_f(\tau\,|\,\tau_{ini})\Bigg]$$

Exclusive nature of partons generated is taken into account by the Sudakov form factor.
Sudakov form factor is just the product of the Sudakov factors for each parton: $\Delta_i(\tau_i, \tau_{ini})$, where τ_i is (approx.) $k_\perp^2$ in the "place of birth" of the parton.

$$F_{qqgg}(t_{max}, \tau, \tau'\,|\,\tau_{ini}) = \Delta_q(t_{max}\,|\,\tau_{ini}) \times \Delta_q(t_{max}\,|\,\tau_{ini}) \times \Delta_g(\tau\,|\,\tau_{ini}) \times \Delta_g(\tau'\,|\,\tau_{ini})$$

Sudakov form factor



- Sudakov form factors depend on parton identities
- Sudakov form factors do not depend on topology of branching history — à la gauge invariance
- Sudakov form factors depend on the set of nodal values: $\{\tau_i\}$
- Each splitting can be presented as a "parent $\longrightarrow$ parent+child" processes. The scale τ_i is $(t_i)_{max}$ for the child partion, $(t_i)_{max} \approx \left(k_T^2\right)_{i-1}$ — the value of k_T at the node where the child was born.
- Each final-state parton of type "a" yields $\Delta_a(\tau_i \,|\, \tau_{ini})$ contribution to the Sudakov form factor (except partons which were born in $g \to q\,\overline{q}$ splitting)
- The factor $\Delta_a(\tau \,|\, \tau_{ini})$ accounts for the excluding radiation in the cone of size τ and $k_{Tj}^2 > \tau_{ini}$ in branchings.

ME reweighting

1. Select parton identities id = $(q, \bar{q}, g, \ldots)$ and multiplicity n:

$$P^{(0)}(n, id) = \frac{\sigma^{(0)}_{n,id}}{\sum_{k,id'}^{k=N} \sigma^{(0)}_{k,id'}}.$$

2. Generate these partons using ME generator, using fixed $\alpha_s(\tau_{ini})$ – the biggest one.

3. Run k_T-clustering algorithm to find τ-s: $\{\tau_1 = Q^2, \tau_2, \tau_3, \ldots, \tau_{n-1}\} > \tau_{ini}$

4. Identify each parton j with its "place of birth" τ_j

5. Calculate the Sudakov form factor as a product of Sudakov factors:

$$F_{id}(\tau_2, \ldots, \tau_{n-1}) = \prod_j \Delta_j(\tau_j \mid \tau_{ini}).$$

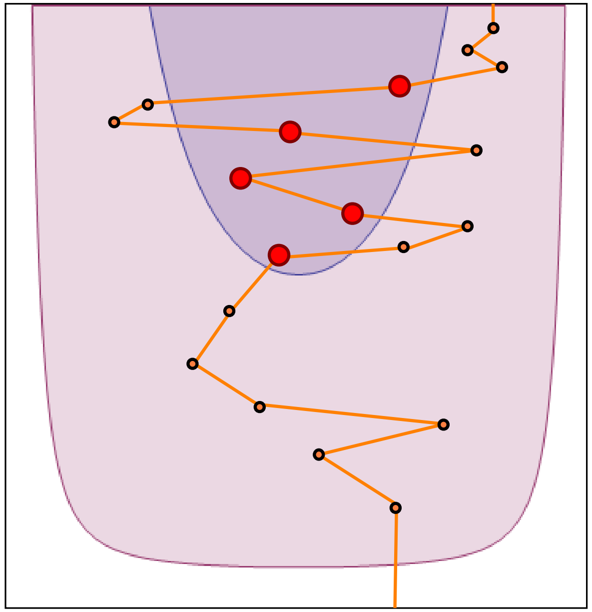
6. Calculate the total weight:

$$W_{id}(\tau_2, \ldots, \tau_{n-1}) = F_{id}(\tau_2, \ldots, \tau_{n-1}) \frac{\alpha_s(\tau_2)\,\alpha_s(\tau_3) \ldots \alpha_s(\tau_{n-1})}{[\alpha_s(\tau_{ini})]^{n-2}}.$$

7. Generate random number $r \in [0, 1]$. If $r < W_{id}$ then accept the configuration, else goto step 1

Vetoed (truncated) Parton Showers

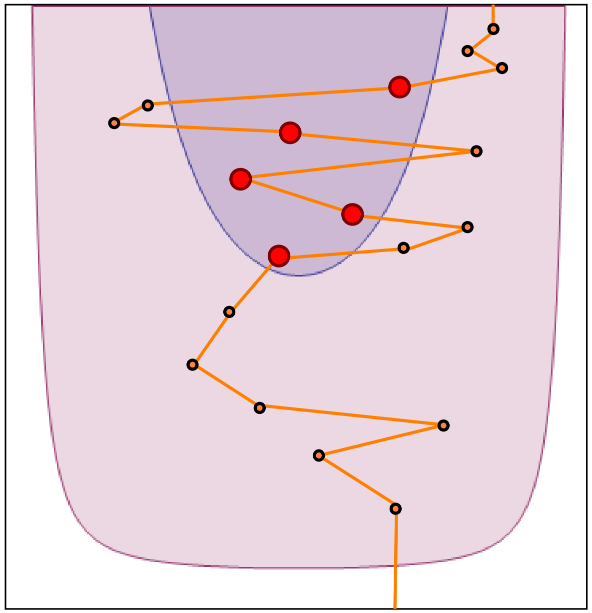
After generating partons with ME-generator, one should "dress" these partons with PS.



- For each parton j we know $\tau_j = k_\perp^2$ in its "place of birth".
- Generate "vetoed" PS for each parton with initial scale τ_j
- "Vetoed" means that if we generate splitting (t, z) and $\min\{z^2, \bar{z}^2\}\, t > \tau_{ini}$ then reload your

PS gun in this point, but don't append the splitting (t, z) to your branching tree.

Final comments



- Parton shower approximation (for the red points) is the singular part of the total ME squared.
- This singular part is smoothly merged with the vetoed parton shower.
- Only the nonsingular (or singular but beyond the NLL level) parts have discontinuities.
- Inclusive mode helps to overcome difficulties with the finite multiplicity available for ME generator

Summary of the third lecture

- The total phase space should be clearly separated by the scale τ_{ini} into domains of applicability of ME and PS generators.

- Such separation also helps to avoid double counting.

- Smoothness of the merging is guaranteed by: on the one side ME modification by Sudakov form factors, on the other side, by implementing "vetoed" PS.

- ME gen. tends to generate more configurations near ME singularities (although they are isolated by τ_{ini}), but these configurations are suppressed by Sudakov factors. Thus CKKW procedure is rather effective. MLM procedure looks less effective (because Sudakov f.f. are calculated on the fly) but more universal.