

Event Generation with MadGraph 5

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with Olivier Mattelaer

Lectures and exercises found at

<https://server06.fynu.ucl.ac.be/projects/madgraph/wiki/SchoolKias>

Special thanks to Fabio Maltoni from whom I have shamelessly borrowed several of the following slides

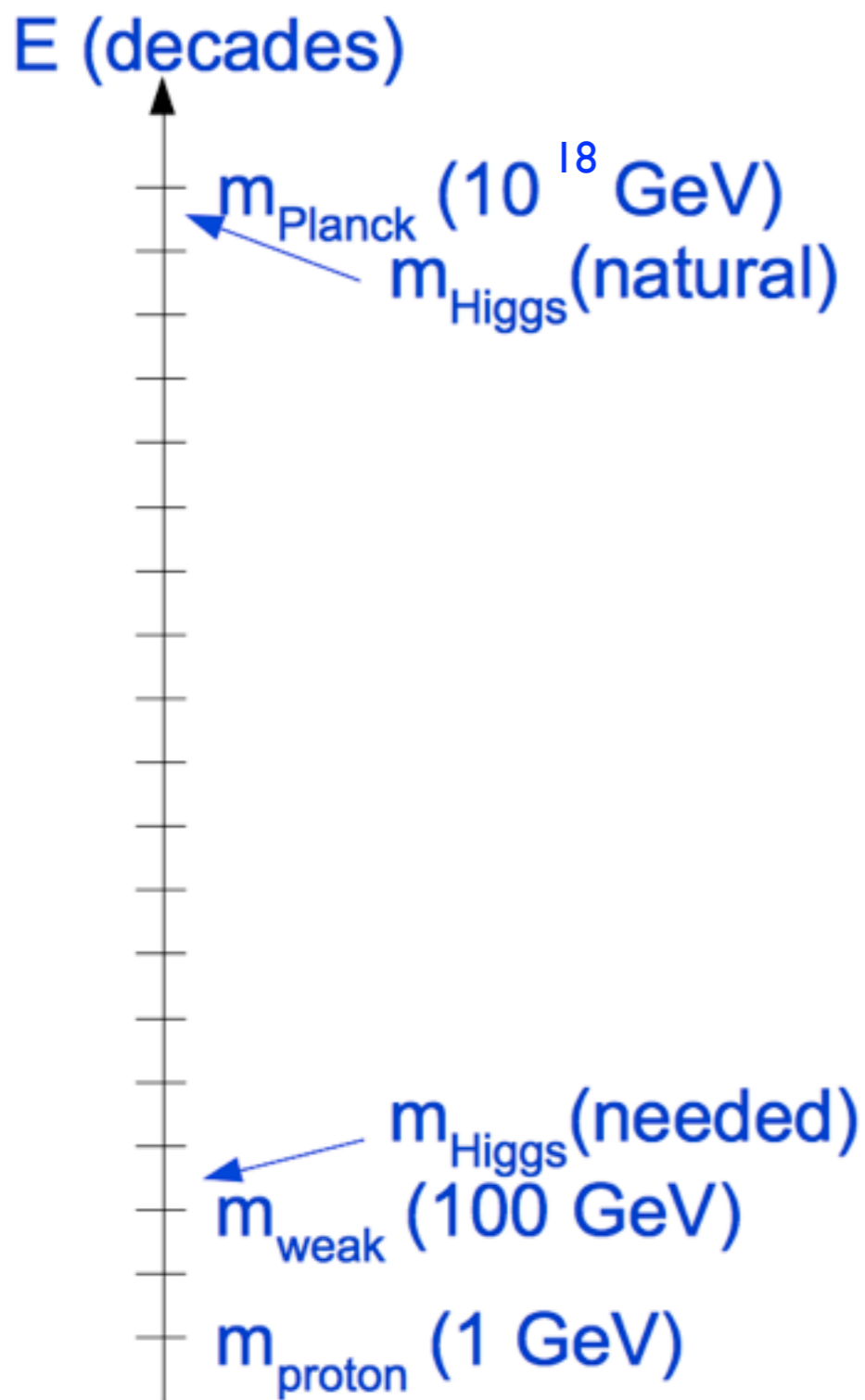
Outline of lectures

- Lecture I:
 - ➔ New Physics at hadron colliders
 - ➔ QCD basics
 - ➔ Monte Carlo integration and generation
- Lecture II:
 - ➔ Complete collider event simulation
 - ➔ Parton showering and jet matching (teaser)
 - ➔ Event simulation in practice using MadGraph
- Lecture III (by Olivier Mattelaer):
 - ➔ What is MadGraph 5?
 - ➔ Implementing new physics models in MadGraph (teaser)

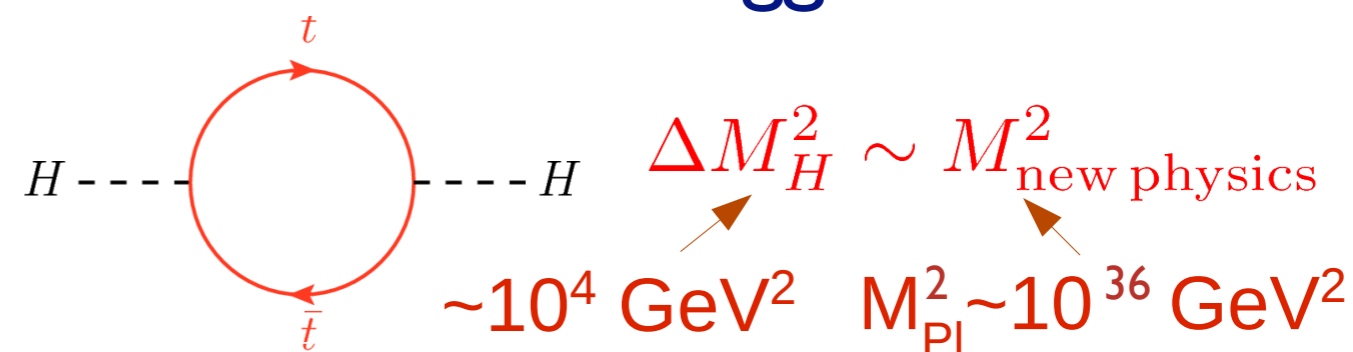
Aims for these lectures

- Get you acquainted with the concepts and techniques used in event generation
- Give you hands-on experience with matrix element generation, event generation and analysis using MadGraph
- Answer as many of your questions as I can (so please ask questions!)

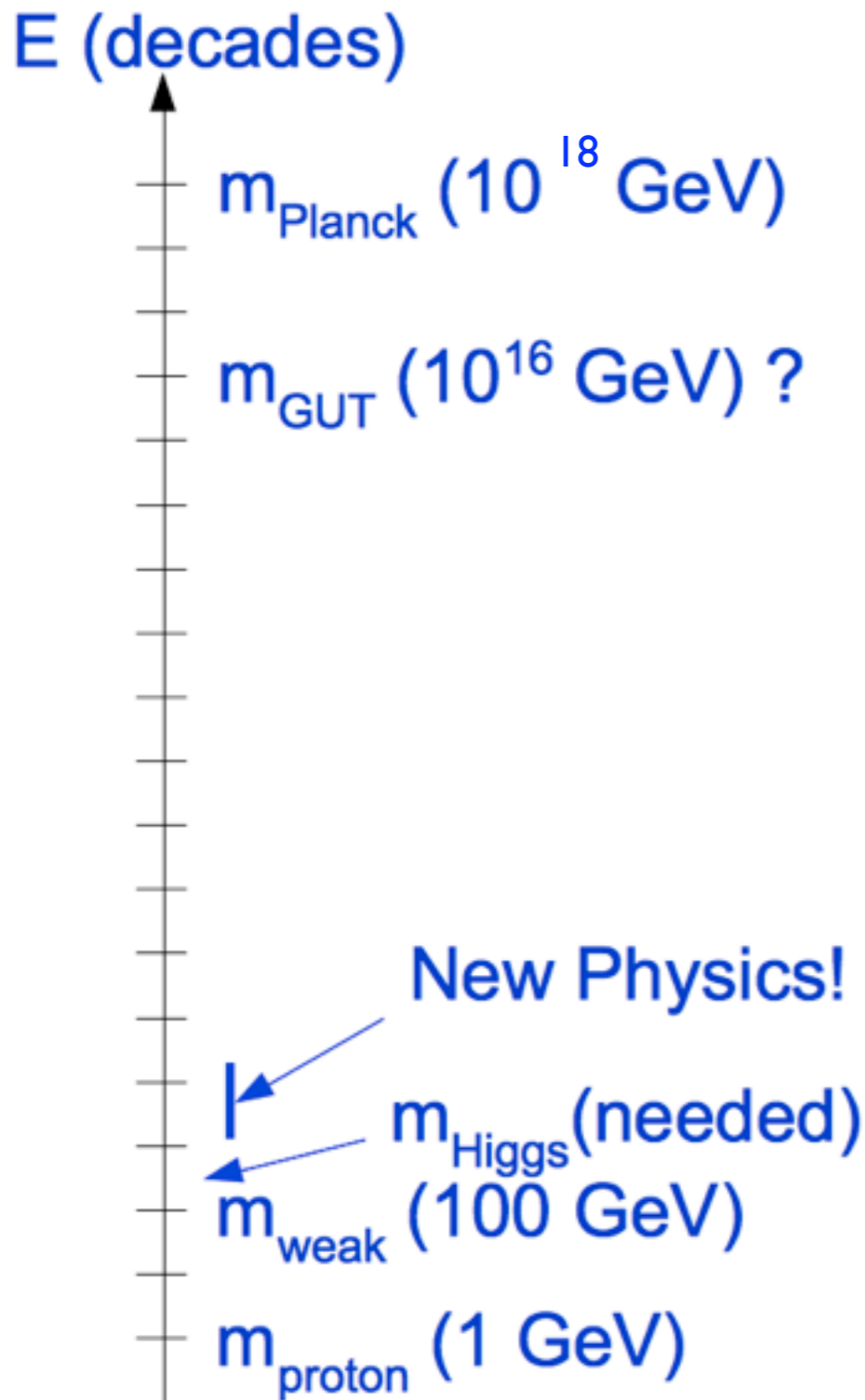
Why the LHC?



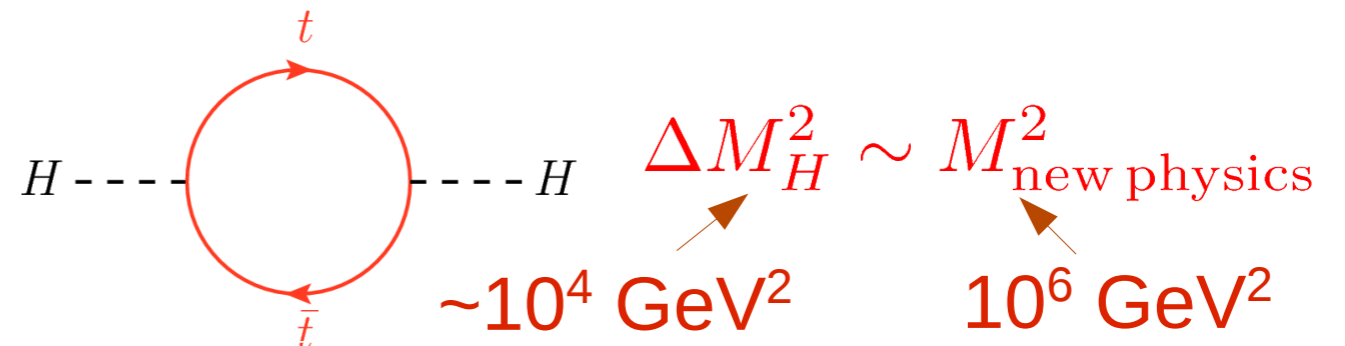
- Higgs boson mass “naturally” at mass of new physics (only known “NP scale”: Planck scale at $\sim 10^{18} \text{ GeV}$)
- Standard Model only “works” if Higgs mass below $\sim 800 \text{ GeV}$
- New Physics scale communicated through quantum loop contributions to Higgs mass



Why the LHC?



- ΔM_H contribution must be canceled by bare mass term. For fine-tuning less than 1%, need new physics which cuts off the quadratic loops at $\sim 1 \text{ TeV}$



Why the LHC?

The Hierarchy problem, together with Dark Matter (and to some extent Grand Unification) have been driving New Physics model building in past 30 years

- Supersymmetry
- Large Extra Dimensions
- Randall-Sundrum (warped extra dimensions)
- Little Higgs theories
- ... (mostly variants/combinations)

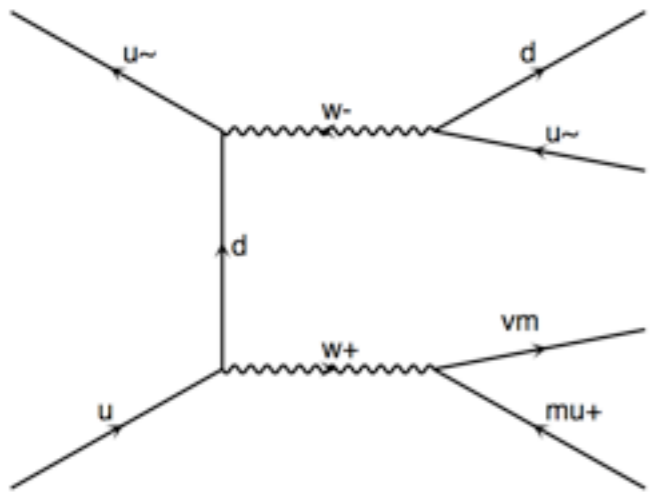
But of course, we might also find something completely unexpected!

New Physics at hadron colliders

- The LHC has taken over from the Tevatron!
- Significant luminosities
 - ➔ Tevatron collected $>10 \text{ fb}^{-1}$ in the last 10 years
 - ➔ Fantastic legacy, including several interesting excesses!
 - ➔ LHC already has a spectacular 5 fb^{-1} !
(perhaps as much as 20 fb^{-1} by end of 2012!)
 - ➔ Allows ever-more stringent tests of the SM!
- How interpret excesses? How determine Standard Model backgrounds?
 - ➔ **Monte Carlo simulation!**
(combined with data-driven methods)

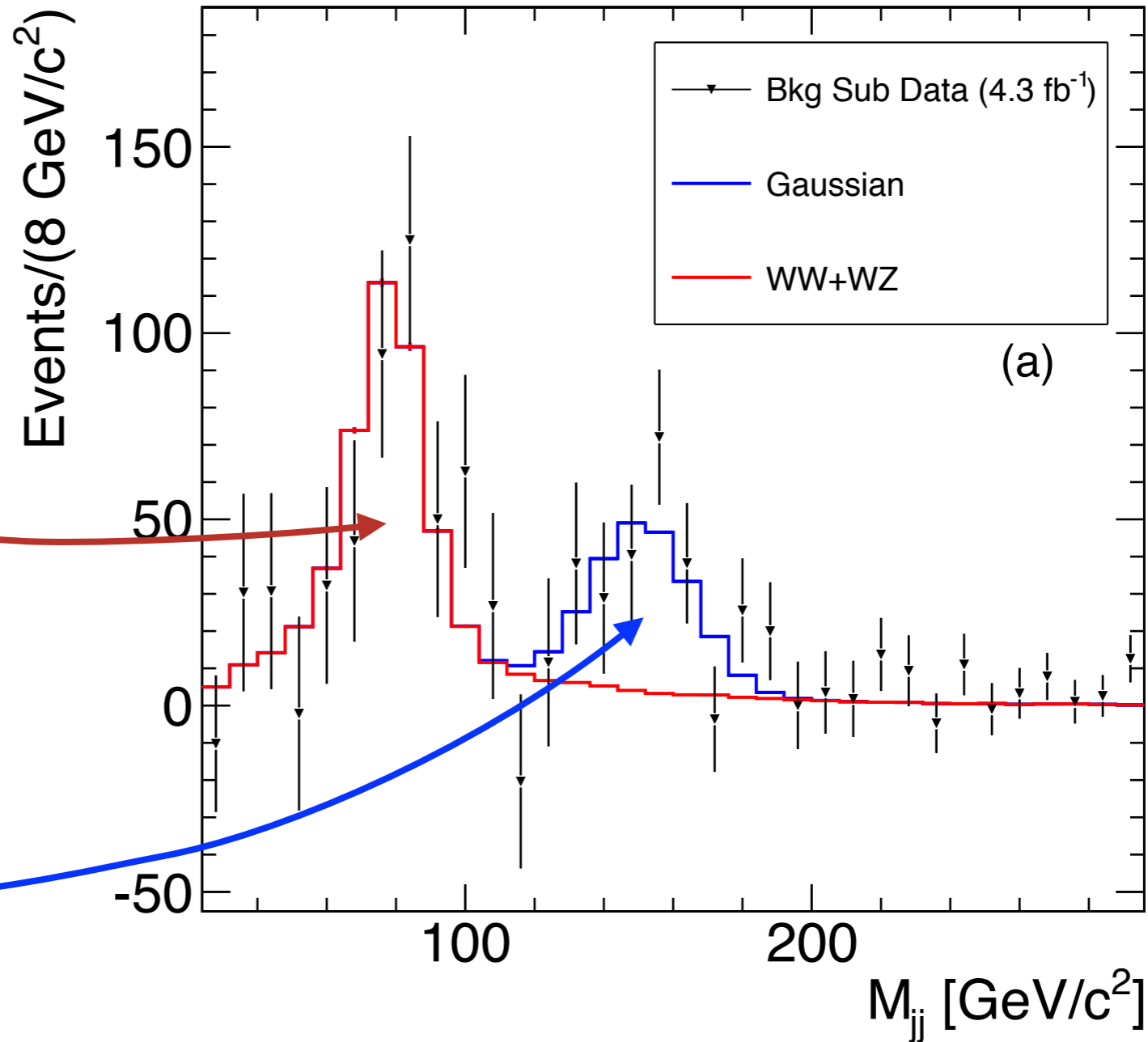
Example: CDF excess in W + 2 jets

CDF collaboration, arXiv:1104.0699



WW, WZ

W + Nobody knows what?

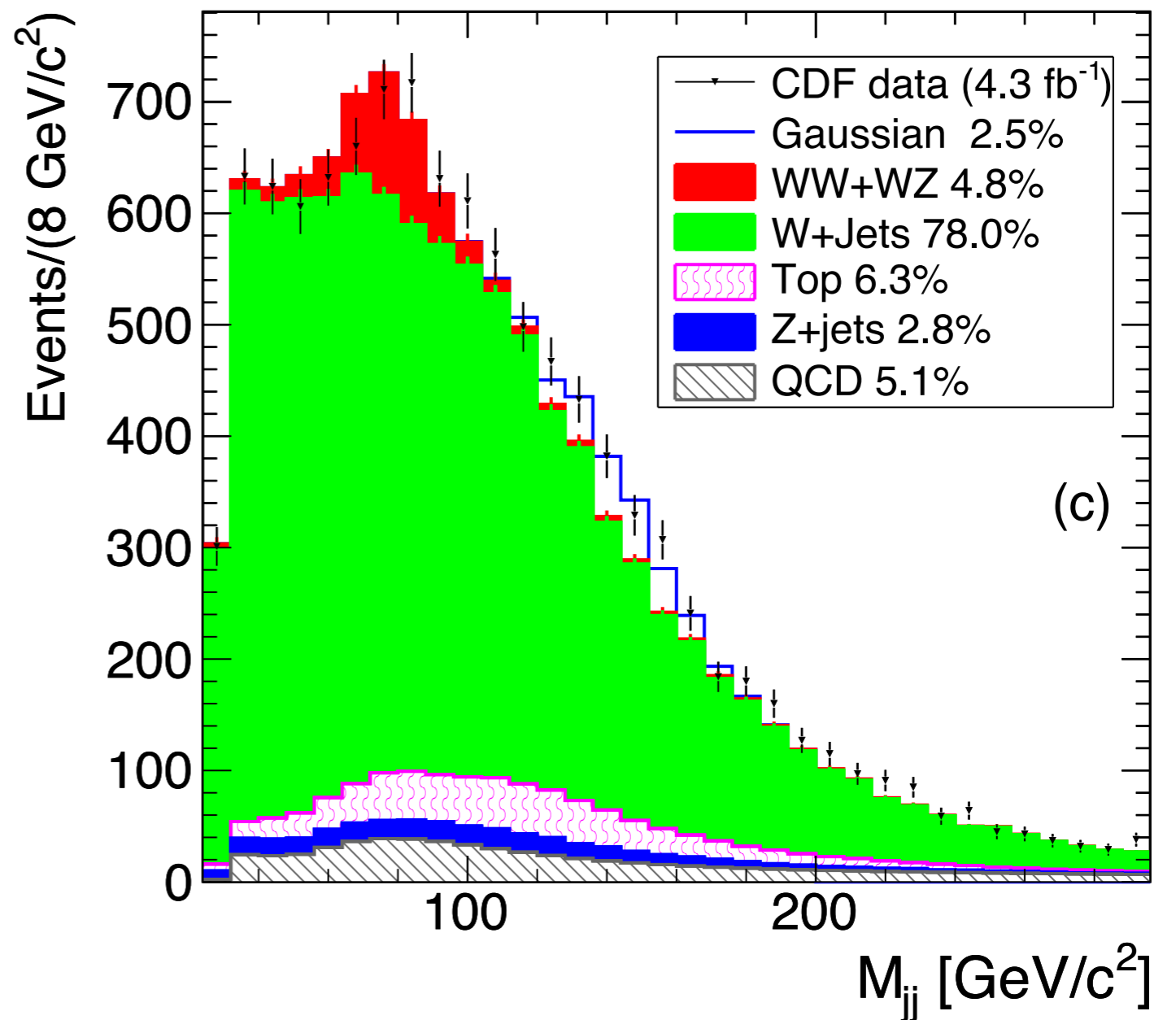


Background subtracted data (except WW/WZ)

Example: CDF excess in W + 2 jets

A more complete picture

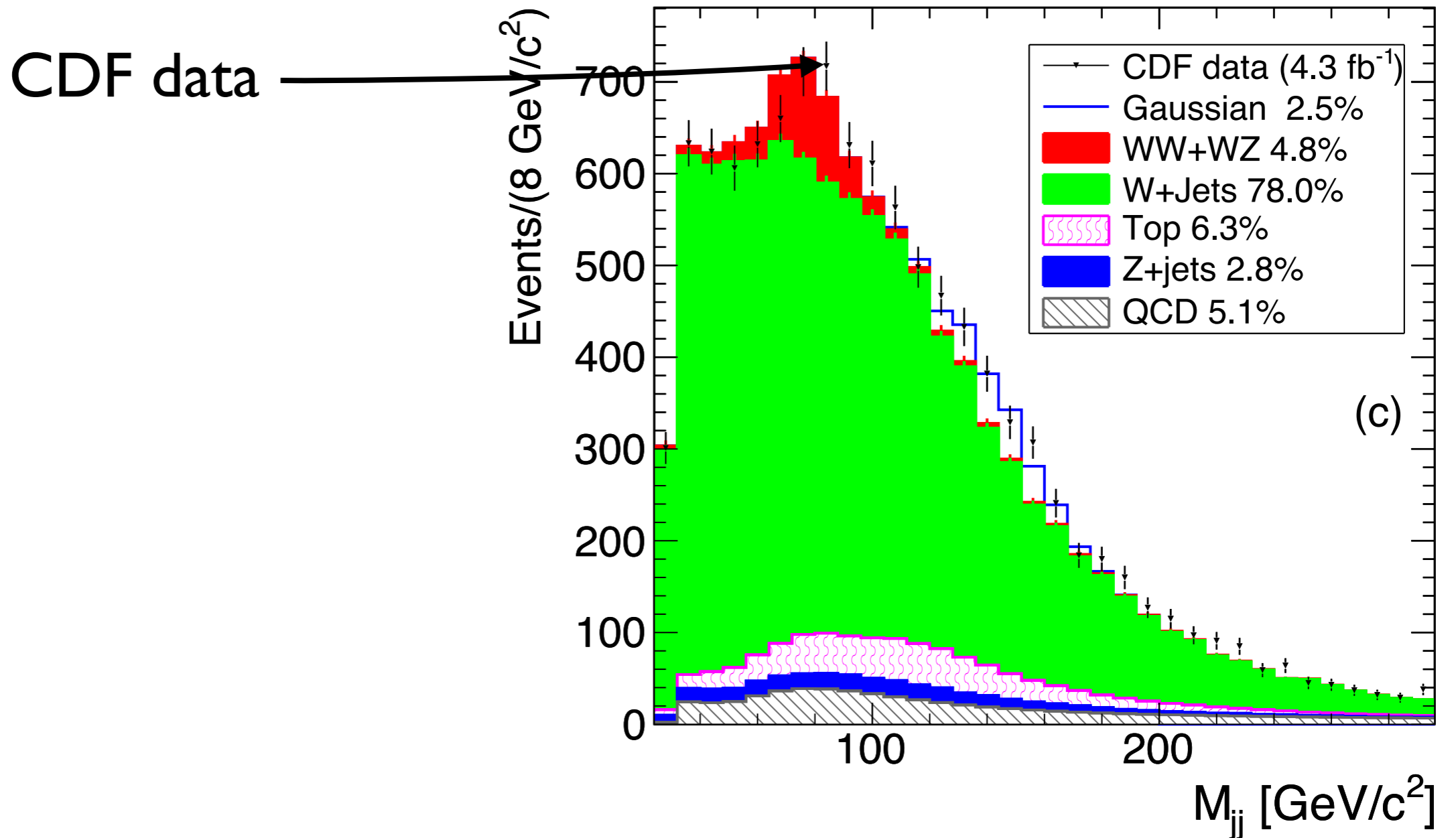
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Example: CDF excess in W + 2 jets

A more complete picture

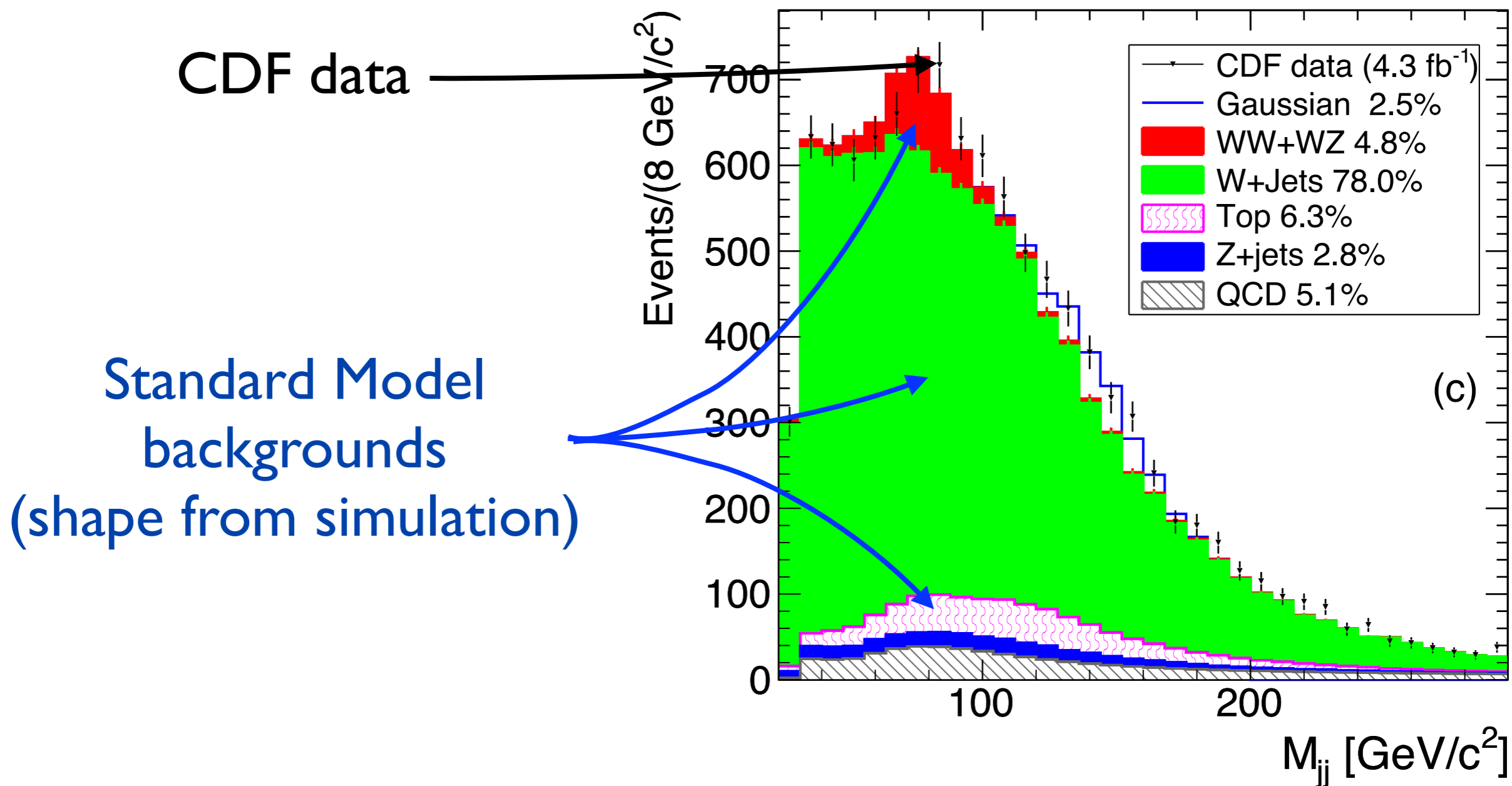
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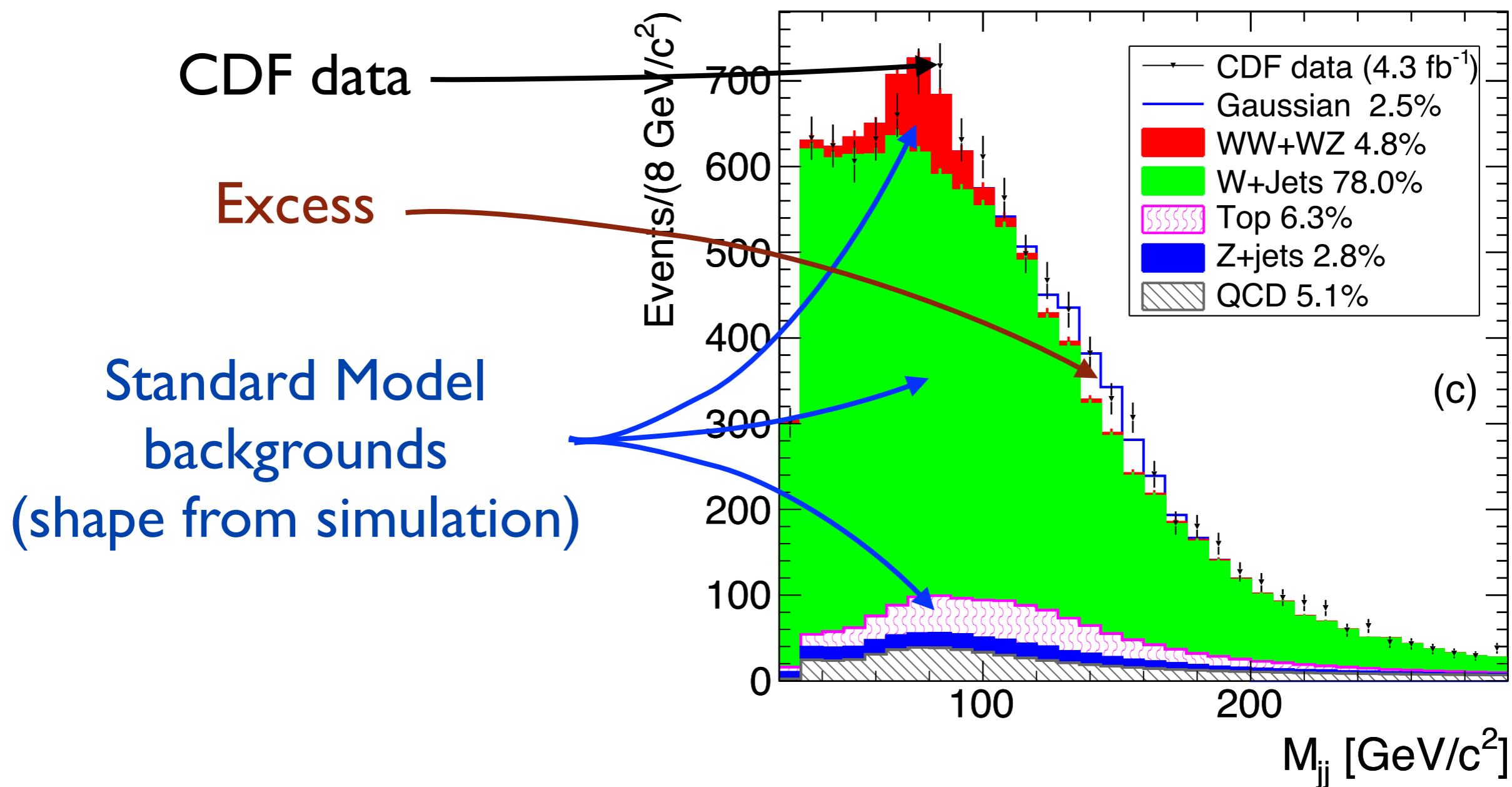
CDF collaboration, arXiv:1104.0699



Example: CDF excess in W + 2 jets

A more complete picture

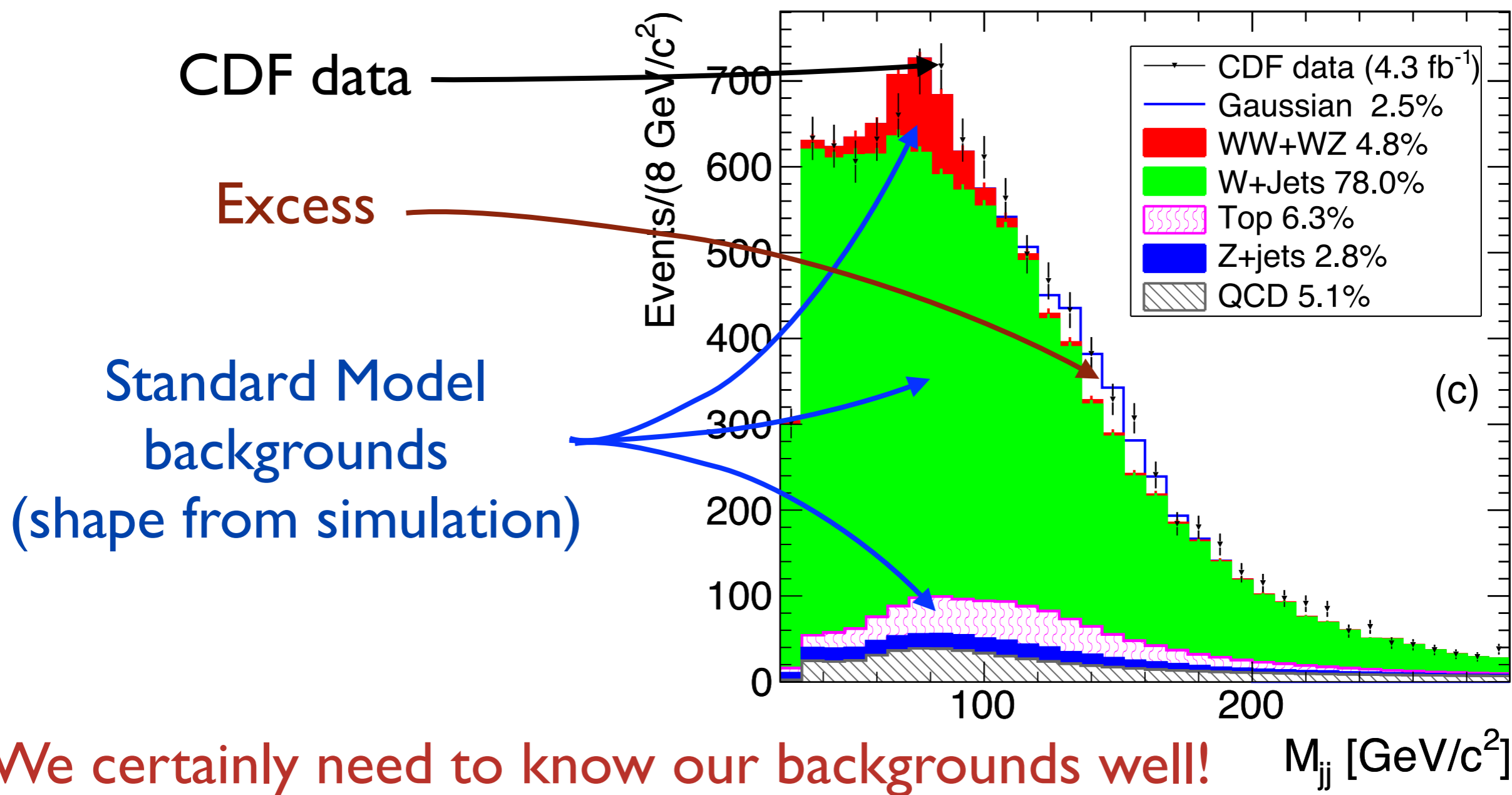
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Example: CDF excess in W + 2 jets

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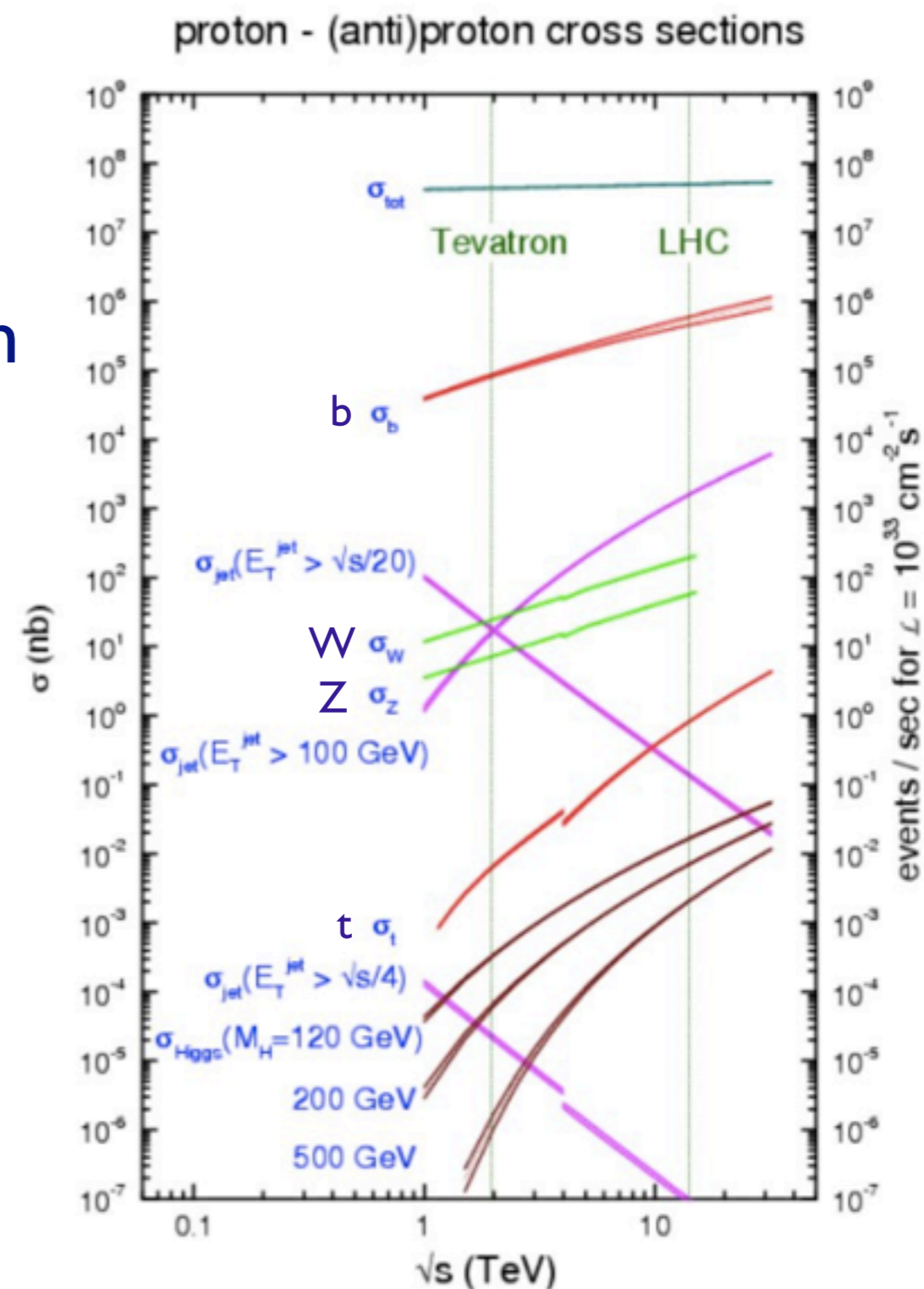


Processes at Hadron Colliders

First: Understand our processes!

Cross sections at a collider depend on

- Coupling strength
- Coupling to what?
(light quarks, gluons, heavy quarks, EW gauge bosons?)
- Mass
- Single production/pair production

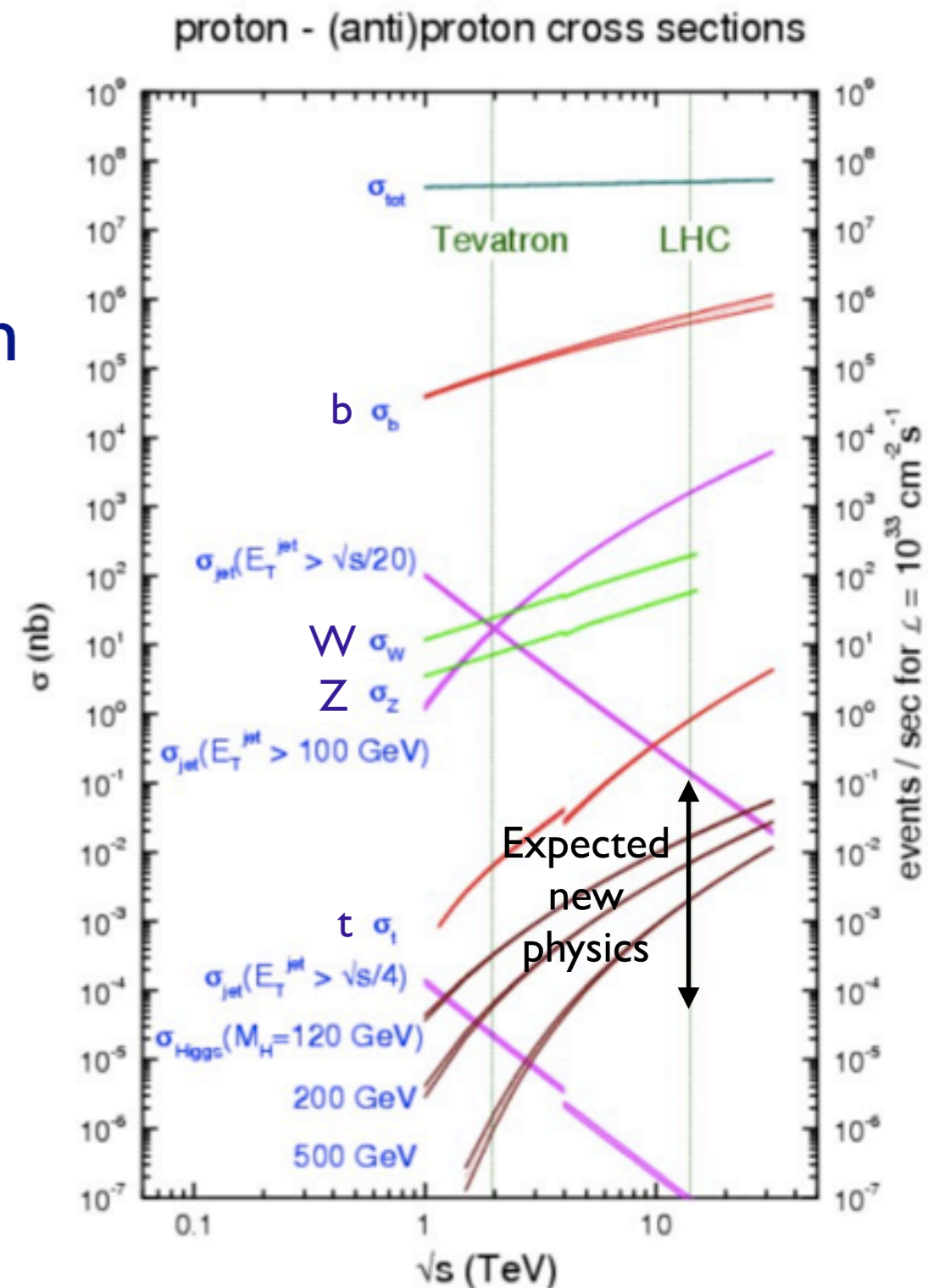


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Master formula

Master formula

$$\hat{\sigma}_{ab \rightarrow X}(\hat{s}, \dots)$$

Parton level
cross section

- Parton level cross section from matrix element

Master formula

$$\hat{\sigma}_{ab \rightarrow X}(\hat{s}, \dots) f_a(x_1) f_b(x_2)$$

Parton level
cross section

Parton density
functions

- Parton level cross section from matrix element
- Parton density (or distribution) functions:
Process independent, determined by particle type

Master formula

$$\int \hat{\sigma}_{ab \rightarrow X}(\hat{s}, \dots) f_a(x_1) f_b(x_2) dx_1 dx_2 d\Phi_{FS}$$

Parton level cross section
Parton density functions
Phase space integral

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Parton level cross section
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- Parton level cross section from matrix element
- Parton density (or distribution) functions:
Process independent, determined by particle type
- $\hat{s} = x_1 x_2 s$ (s = collision energy of the collider)
- Difference between colliders given by parton luminosities

Tevatron vs. the LHC



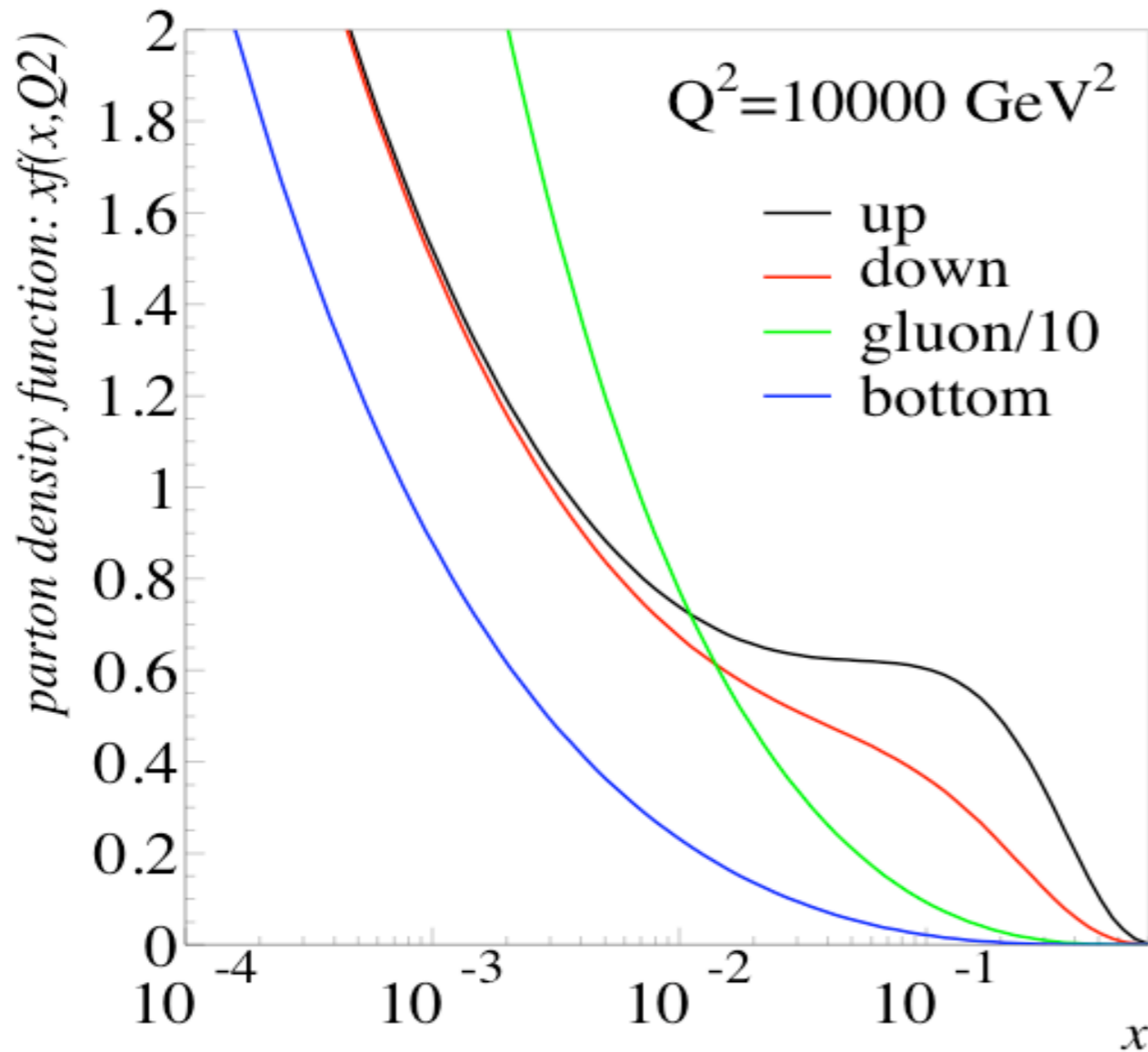
- Tevatron: 2 TeV proton-antiproton collider
 - ➔ Most important: $q\text{-}\bar{q}$ annihilation (85% of $t\bar{t}$)
- LHC: 8-14 TeV proton-proton collider
 - ➔ Most important: $g\text{-}g$ annihilation (90% of $t\bar{t}$)

Tevatron vs. the LHC

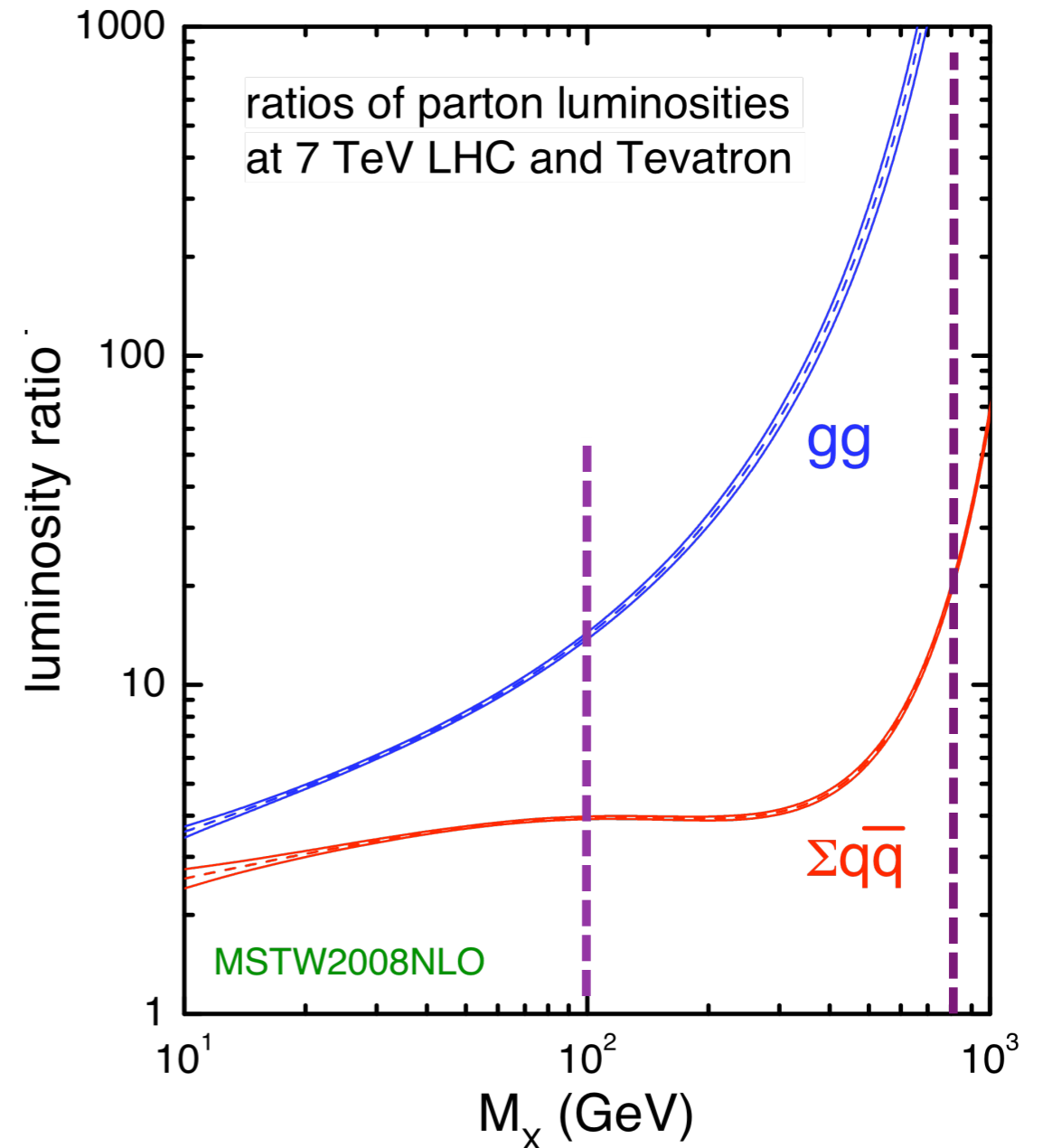


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Parton densities

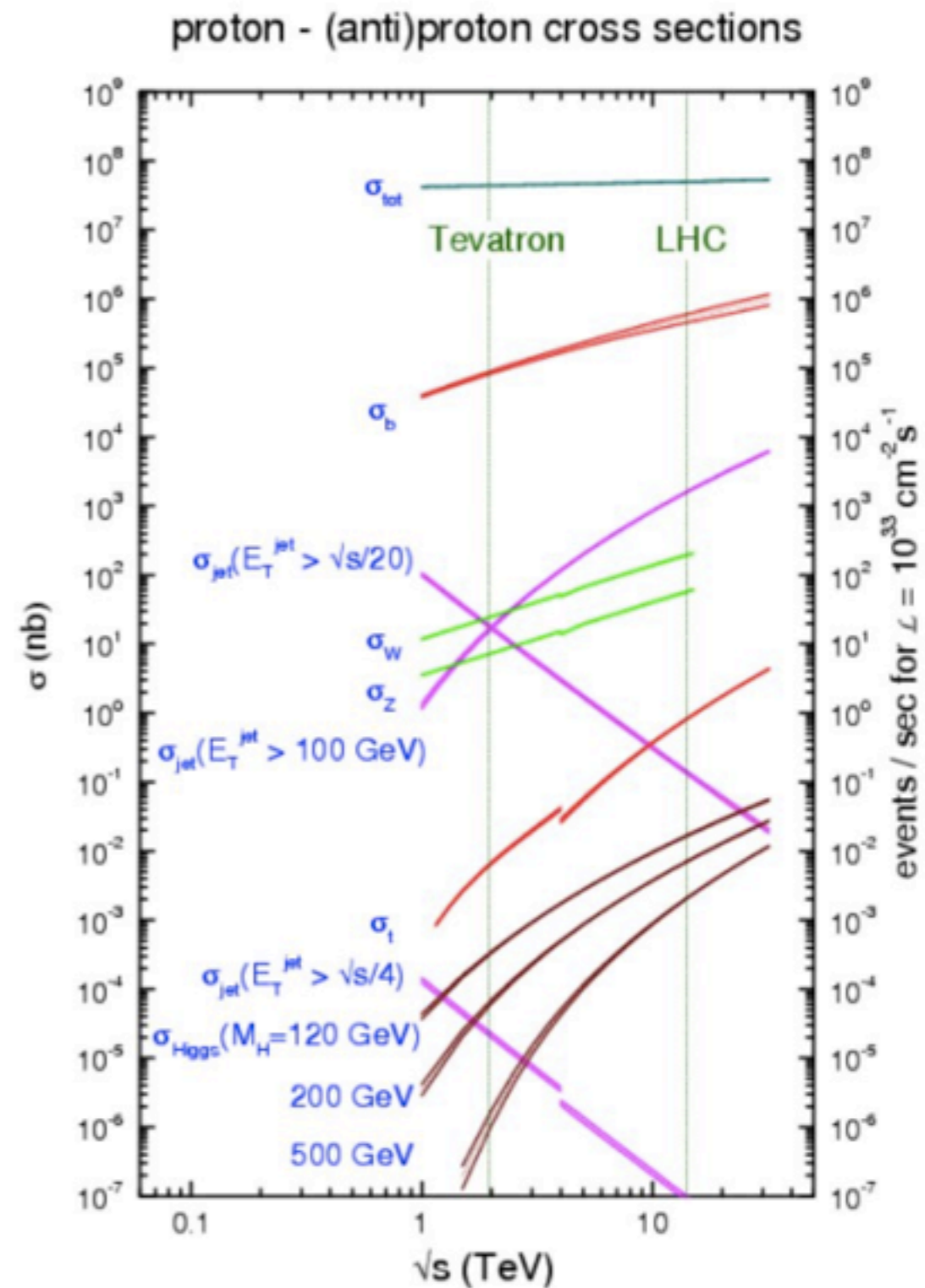
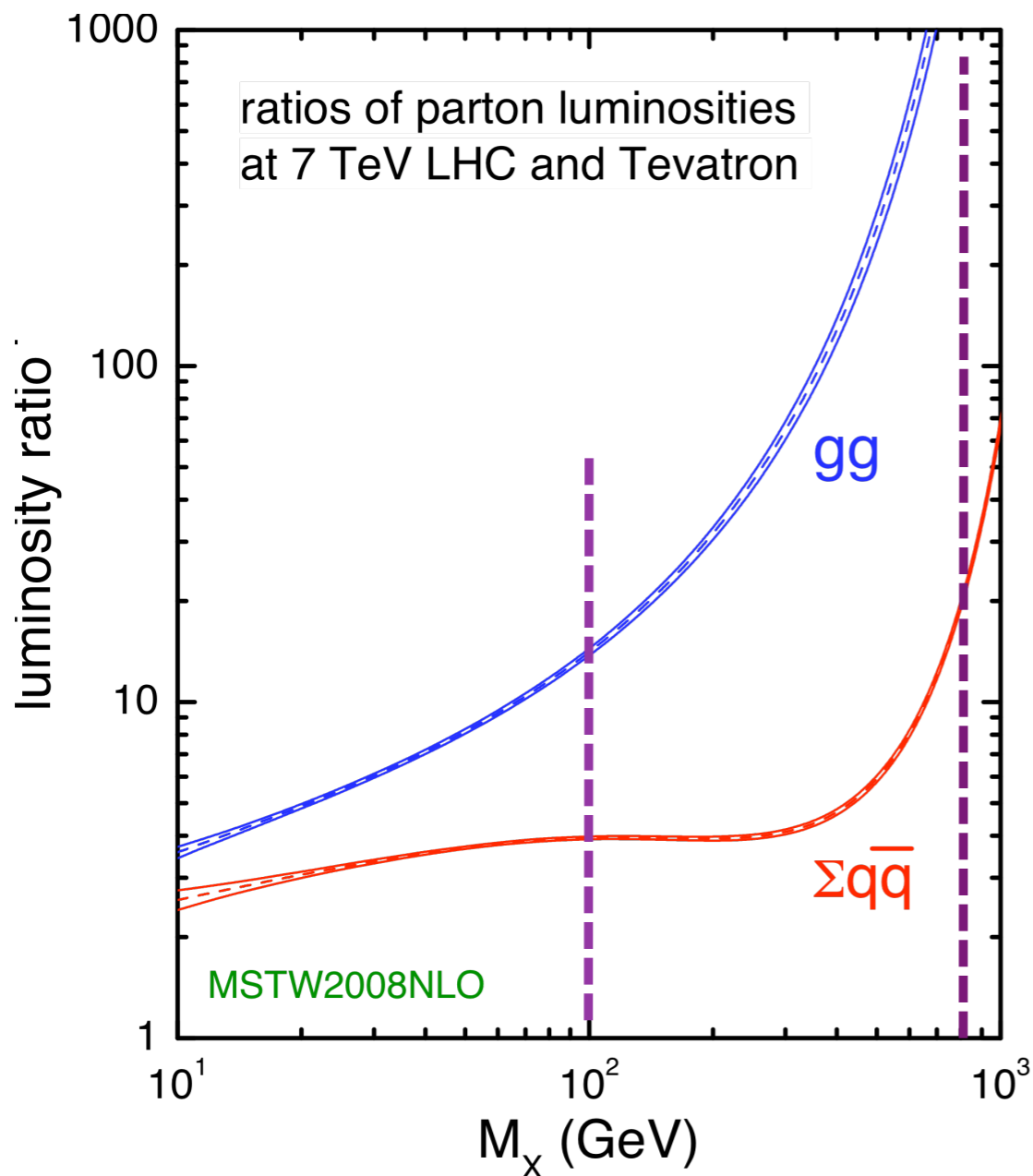


At small x (small \hat{s}), gluon domination.
At large x valence quarks

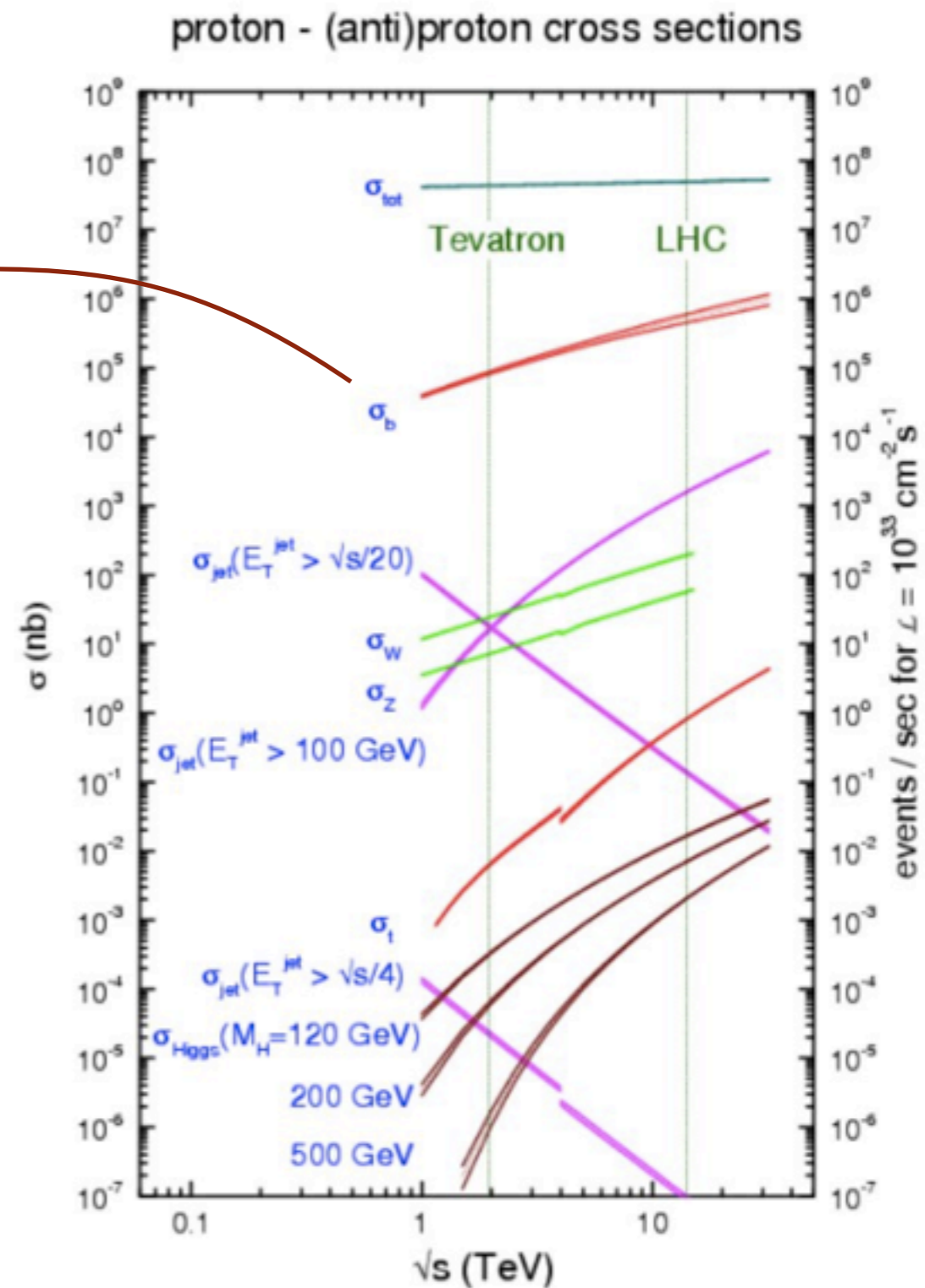
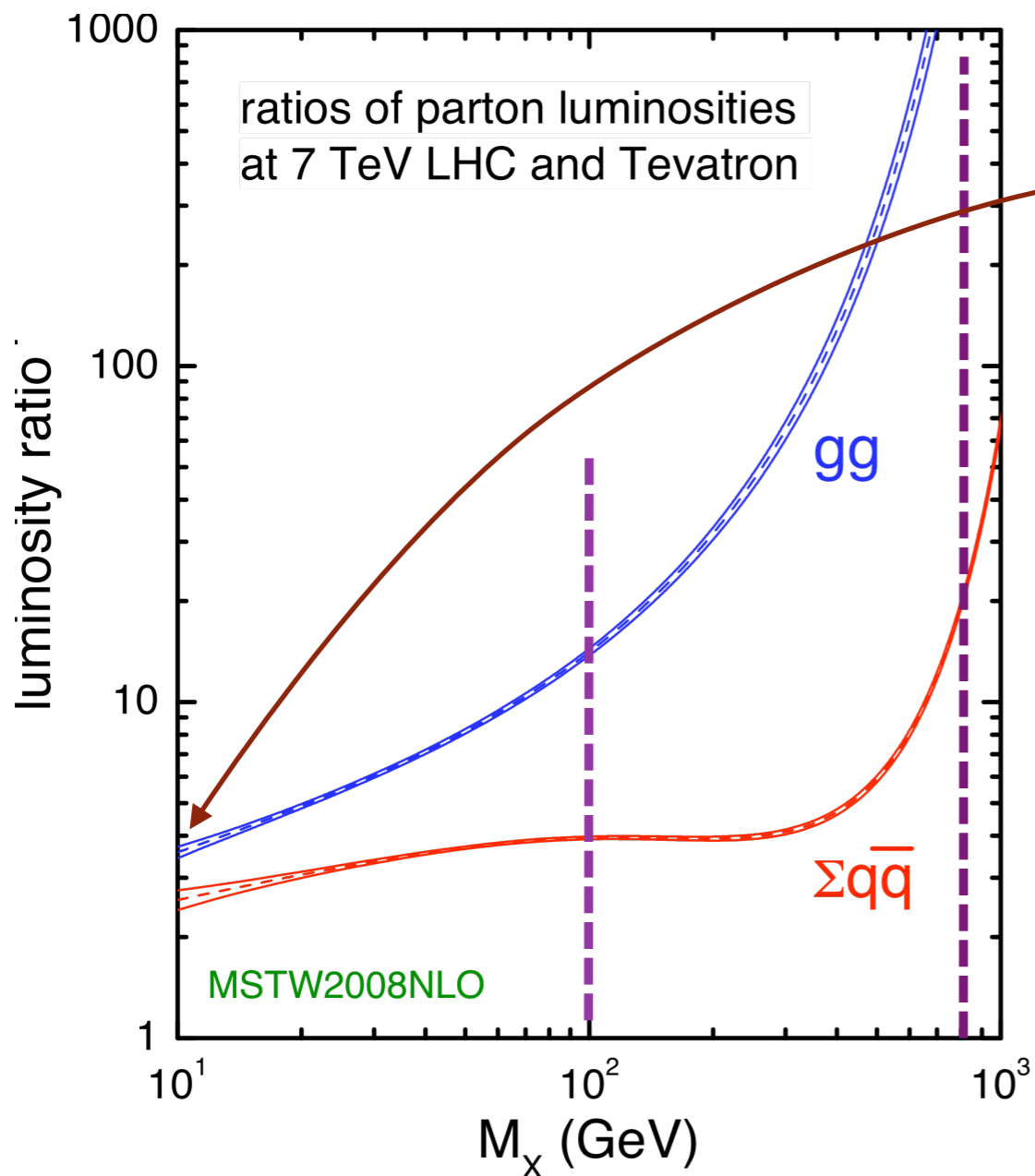


LHC formidable at large mass –
For low mass, Tevatron backgrounds smaller

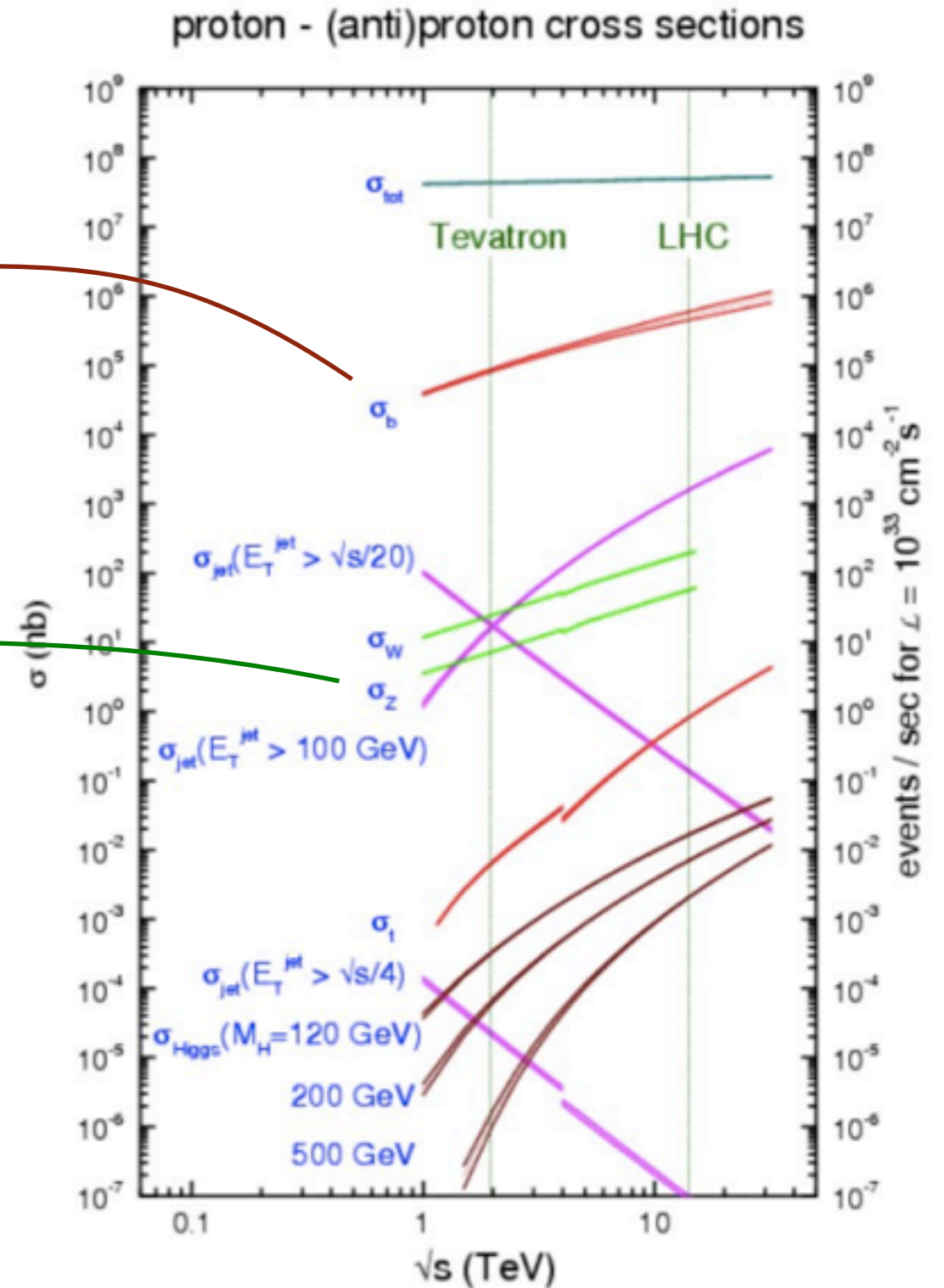
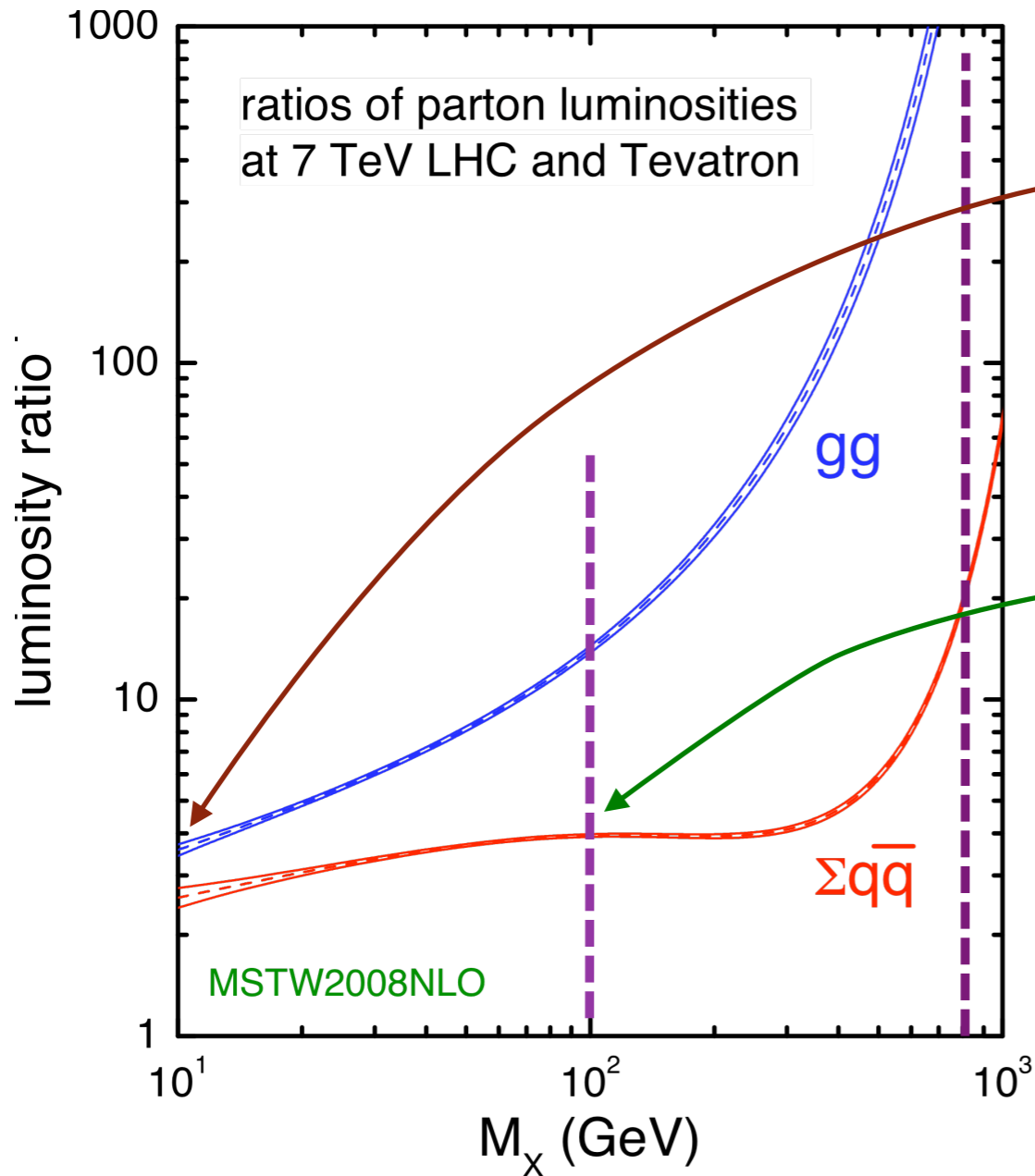
Back to the processes



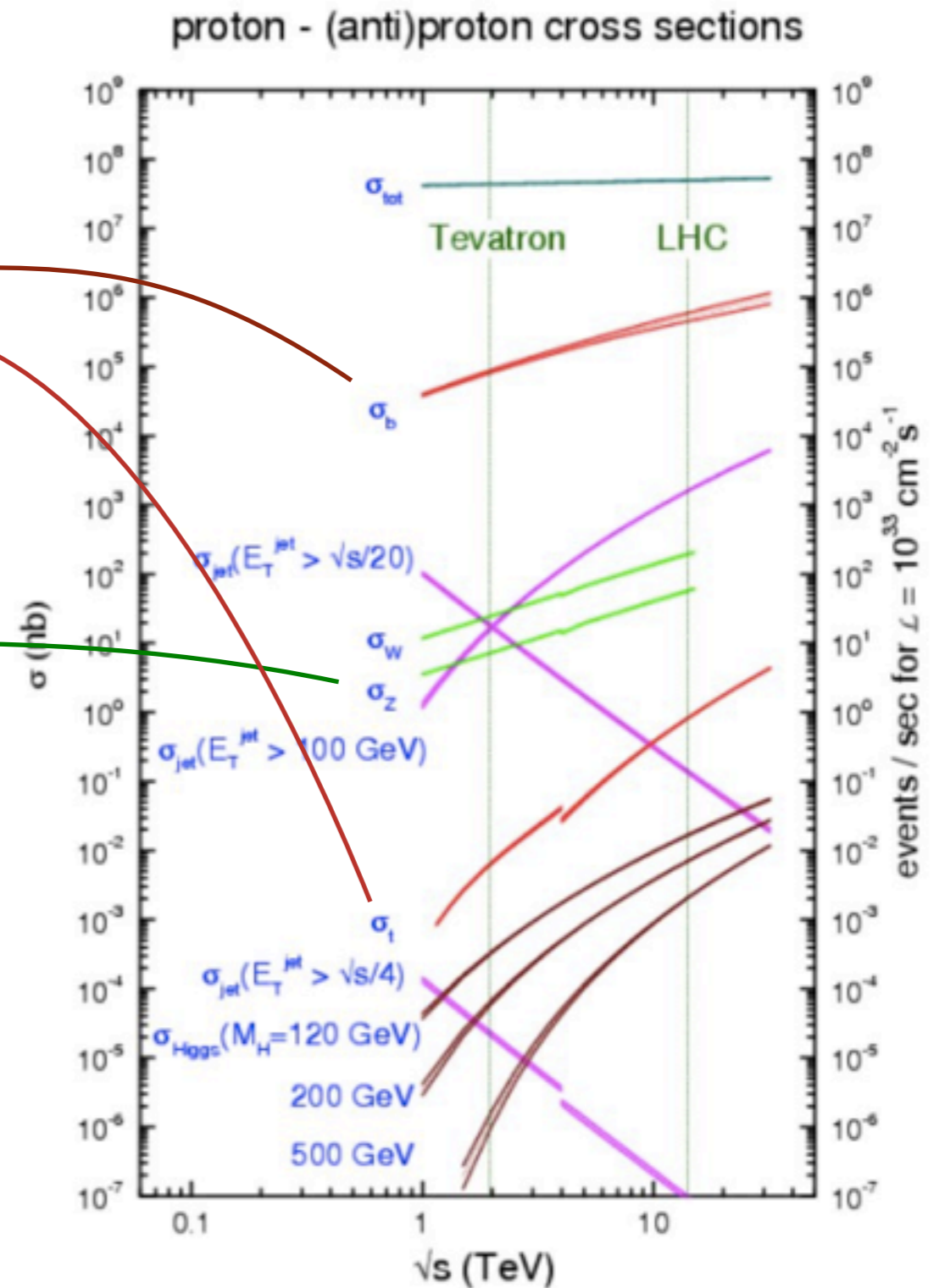
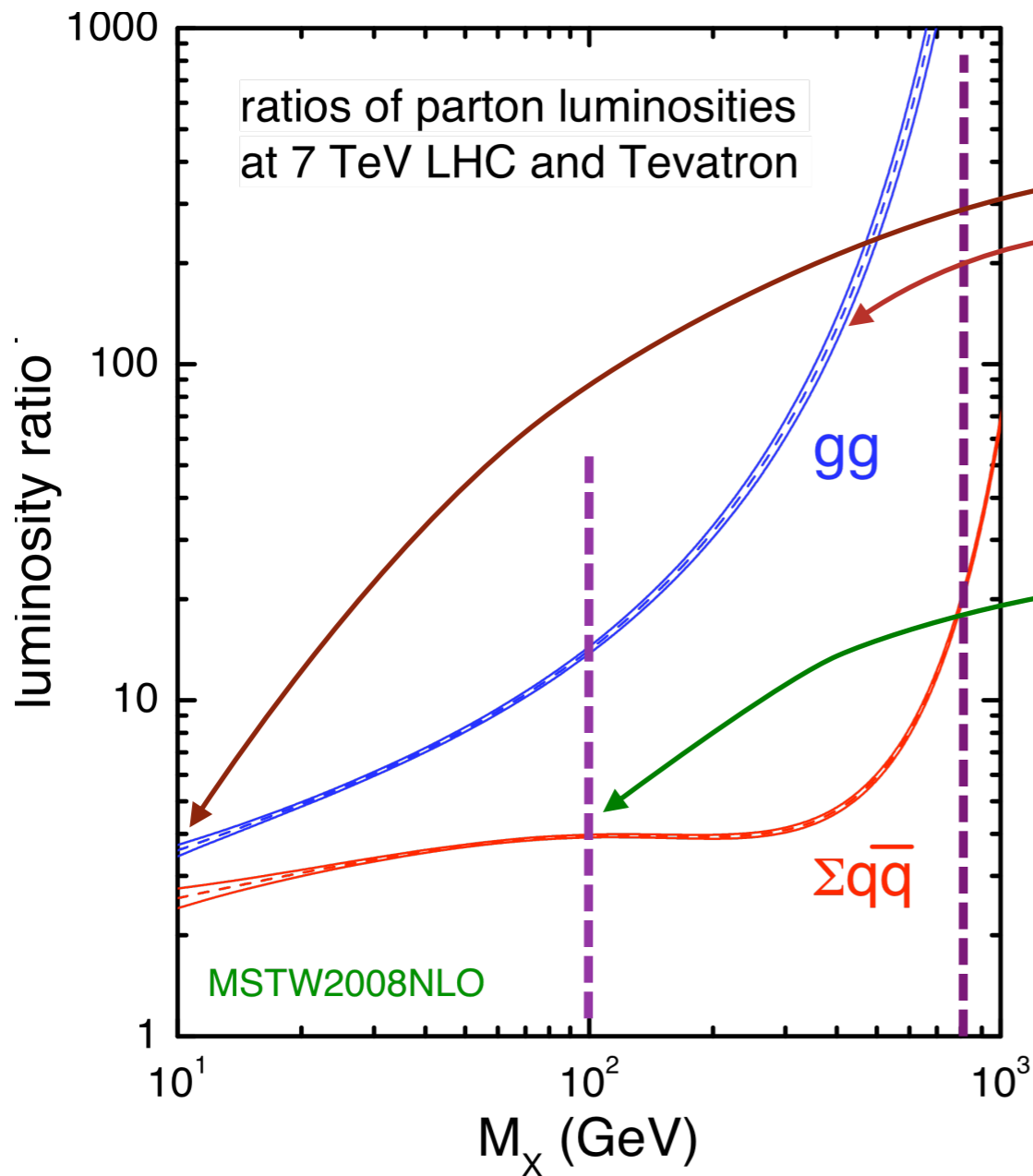
Back to the processes



Back to the processes



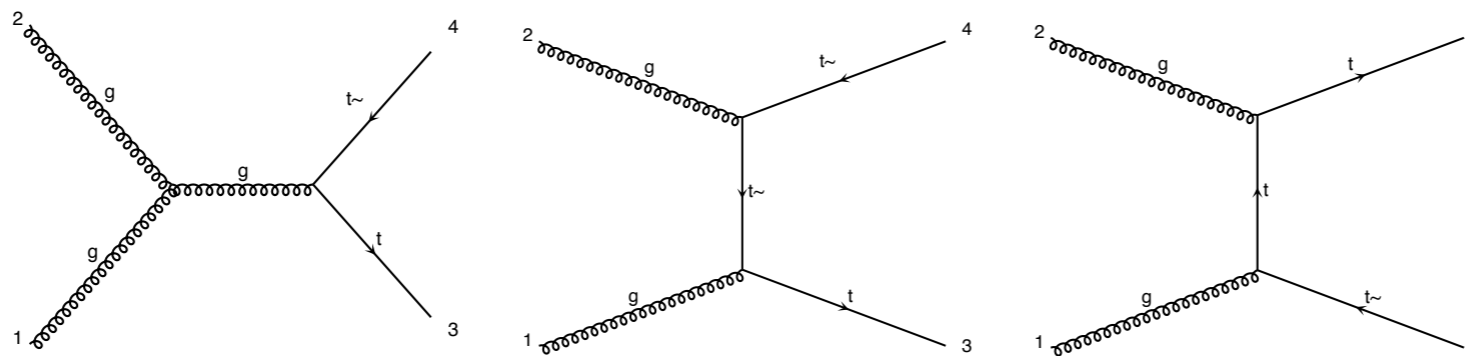
Back to the processes



Matrix Element calculation at Hadron Colliders

To calculate a given process (e.g., $p p \rightarrow t \bar{t}$)

- Determine contributing subprocesses
 $g g \rightarrow t \bar{t}$, $q \bar{q} \rightarrow t \bar{t}$, $q q \rightarrow t \bar{t}$ with $q = d, u, s, c, (\bar{b})$
- Determine matrix element for each subprocess



- Perform phase space integration for each subprocess

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Matrix Element calculation at Hadron Colliders

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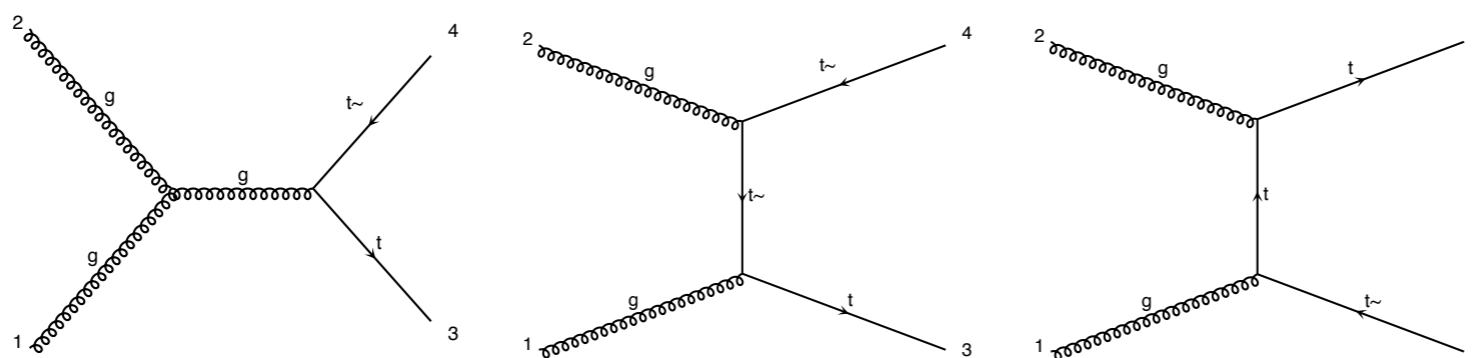
1. Determine contributing subprocesses

$g g \rightarrow t \bar{t}, q \bar{q} \rightarrow t \bar{t}, q q \rightarrow t \bar{t}$ with $q = d, u, s, c, (\bar{b})$

← Easy enough

2. Determine matrix element for each subprocess

← Hard



3. Perform phase space integration for each subprocess

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

← Very Hard (in general)

Matrix Element calculation at Hadron Colliders

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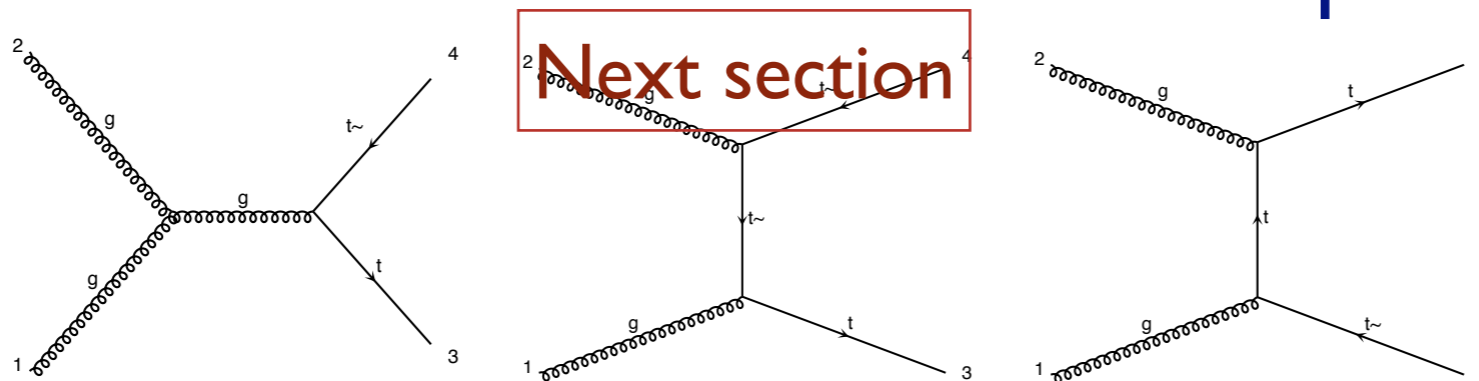
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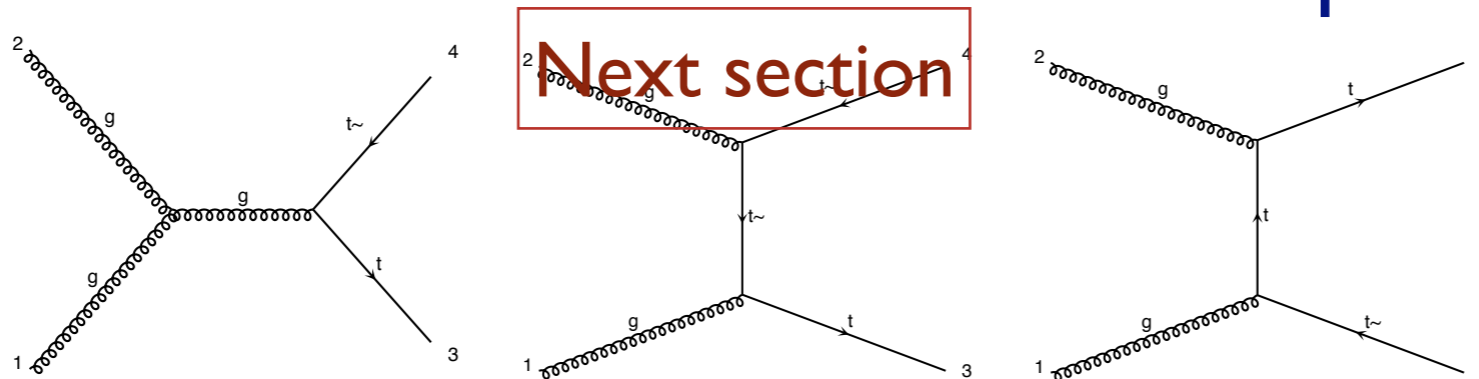
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3. Perform phase space integration for each subprocess

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← The section after

← Very Hard (in general)

Minimal QCD: Basics

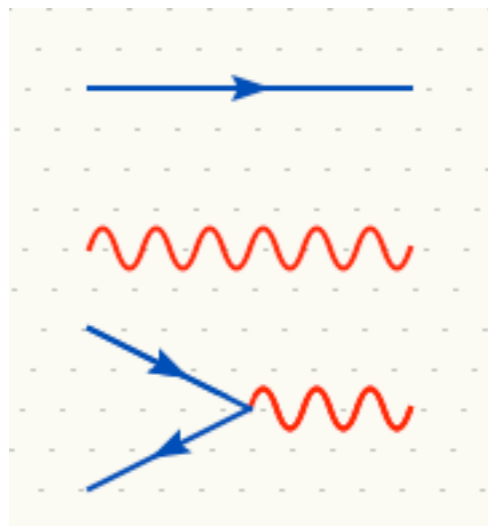
From QED to QCD: abelian vs. non-abelian

The QED Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial - m)\psi - eQ\bar{\psi}A\psi$$

where

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$$



$$= \frac{i}{\not{p} - m + i\epsilon} = i \frac{\not{p} + m}{p^2 - m^2 + i\epsilon}$$

$$= -i \frac{g_{\mu\nu}}{p^2 + i\epsilon} \text{ (Feynman gauge)}$$

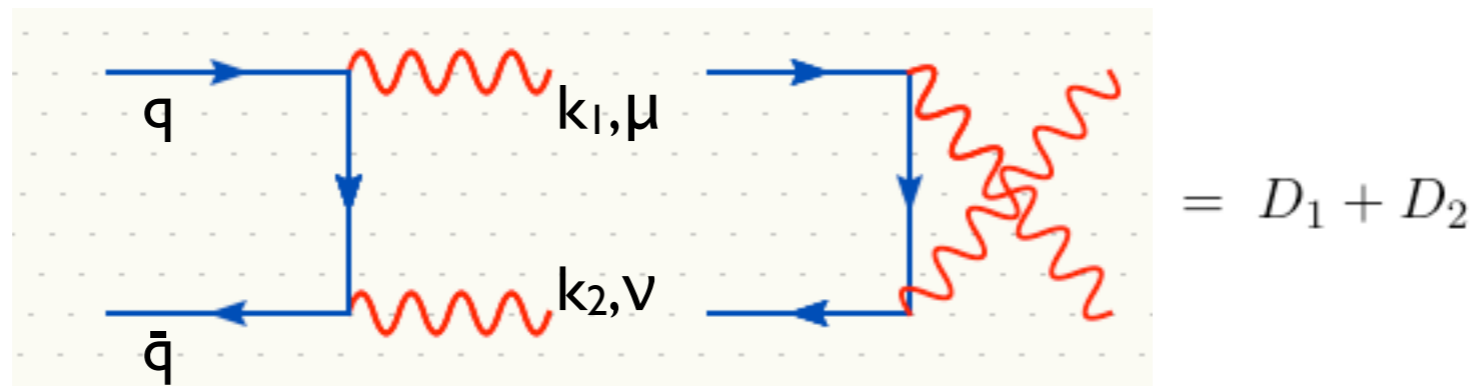
$$= -ie\gamma_{\mu}Q \quad (Q = -1 \text{ for the electron, } Q = 2/3 \text{ for the u-quark, etc})$$

From QED to QCD: abelian vs. non-abelian

We want to heuristically derive the properties of QCD using gauge invariance.

Let's start with the computation of a simple process $e^+e^- \rightarrow \gamma\gamma$.

There are two diagrams:



$$\frac{i}{e^2} M_\gamma \equiv D_1 + D_2 = \bar{v}(\bar{q}) \not{\epsilon}_2 \frac{1}{\not{q} - \not{k}_1} \not{\epsilon}_1 u(q) + \bar{v}(\bar{q}) \not{\epsilon}_1 \frac{1}{\not{q} - \not{k}_2} \not{\epsilon}_2 u(q) \equiv M_{\mu\nu} \epsilon_1^\mu \epsilon_2^\nu$$

From QED to QCD: abelian vs. non-abelian

Gauge invariance demands that:

$$\epsilon_2^\nu \partial^\mu M_{\mu\nu} = \epsilon_1^\mu \partial^\nu M_{\mu\nu} = 0$$

So let us perform the calculation:

$$\begin{aligned} k_1^\mu \epsilon_2^\nu M_{\mu\nu} &= \bar{v}(\bar{q}) \not{\epsilon}_2 \frac{1}{\not{q} - \not{k}_1} (\not{k}_1 - \not{q}) u(q) + \bar{v}(\bar{q}) (\not{k}_1 - \not{q}) \frac{1}{\not{k}_1 - \not{q}} \not{\epsilon}_2 u(q) \\ &= -\bar{v}(\bar{q}) \not{\epsilon}_2 u(q) + \bar{v}(\bar{q}) \not{\epsilon}_2 u(q) = 0 \end{aligned}$$

Only the sum of the two diagrams is gauge-invariant.

From QED to QCD: abelian vs. non-abelian

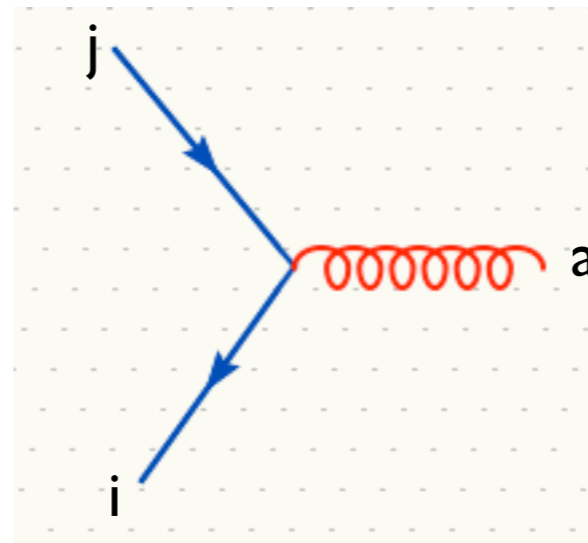
Let's now generalize what we have done for the non-abelian SU(3) of color, with

$$[t^a, t^b] = if^{abc}t^c$$

In this case we take the (anti-)quarks to be in the (anti-)fundamental representation of SU(3), 3 and 3*. The current is in a $3 \otimes 3^* = 1 \oplus 8$.

We identify the gluon with the octet and generalize the QED vertex to :

$$-ig_s t_{ij}^a \gamma^\mu$$



From QED to QCD: abelian vs. non-abelian

So now let's calculate $q\bar{q} \rightarrow gg$ and we obtain

$$\frac{i}{g_s^2} M_g \equiv (t^b t^a)_{ij} D_1 + (t^a t^b)_{ij} D_2$$

$$M_g = (t^a t^b)_{ij} M_\gamma - g^2 f^{abc} t_{ij}^c D_1$$

To satisfy gauge invariance we still need:

$$k_1^\mu \epsilon_2^\nu M_g^{\mu,\nu} = k_2^\nu \epsilon_1^\mu M_g^{\mu,\nu} = 0.$$

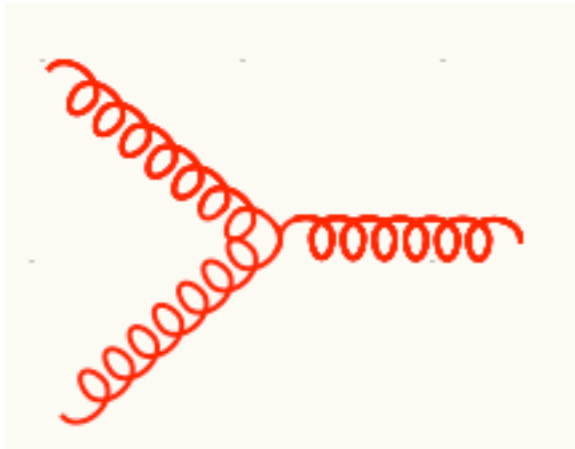
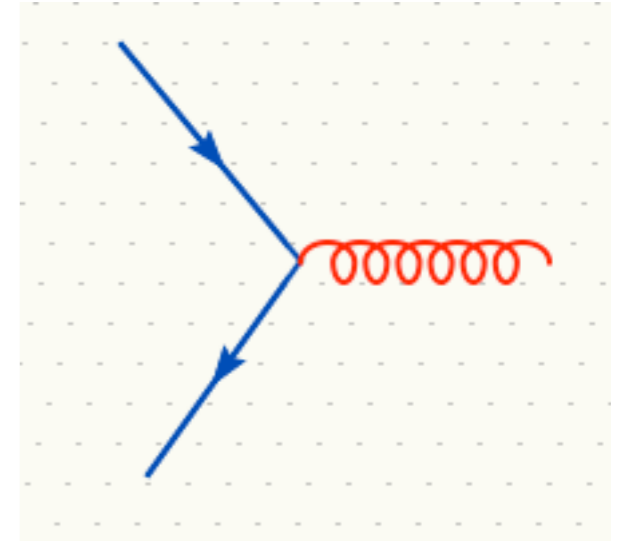
But in this case one piece is left out

$$k_{1\mu} M_g^\mu = -g_s^2 f^{abc} t_{ij}^c \bar{v}_i(\bar{q}) \not{\epsilon}_2 u_i(q)$$

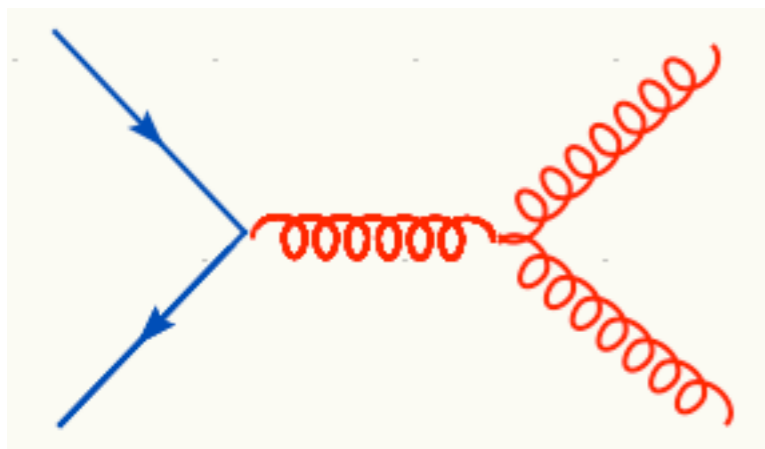
$$k_{1\mu} M_g^\mu = i(-g_s f^{abc} \epsilon_2^\mu) (-ig_s t_{ij}^c \bar{v}_i(\bar{q}) \gamma_\mu u_i(q))$$

From QED to QCD: abelian vs. non-abelian

We can interpret this as the normal vertex times a new 3 gluon vertex:



$$-g_s f^{abc} V_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$



$$-ig_s^2 D_3 = \left(-ig_s t_{ij}^a \bar{v}_i(\bar{q}) \gamma^\mu u_j(q) \right) \times \left(\frac{-i}{p^2} \right) \times$$

$$\left(-g f^{abc} V_{\mu\nu\rho}(-p, k_1, k_2) \epsilon_1^\nu(k_1) \epsilon_2^\rho(k_2) \right)$$

From QED to QCD: abelian vs. non-abelian

Can we guess the Lorentz part for this new interaction? If we assume

1. Lorentz invariance : only structure of the type $g_{\mu\nu} p_\rho$ are allowed
2. Fully anti-symmetry : only structure of the type remain $g_{\mu_1\mu_2}(k_1)_{\mu_3}$ are allowed...
3. Dimensional analysis : only one power of the momentum.

Then we get a unique form of the vertex:

$$V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = V_0 [(p_1 - p_2)_{\mu_3} g_{\mu_1\mu_2} + (p_2 - p_3)_{\mu_1} g_{\mu_2\mu_3} + (p_3 - p_1)_{\mu_2} g_{\mu_3\mu_1}]$$

With this expression we obtain the contribution to the gauge variation:

$$k_1 \cdot D_3 = g^2 f^{abc} t^c V_0 \left[\bar{v}(\bar{q}) \not{\epsilon}_2 u(q) - \frac{k_2 \cdot \epsilon_2}{2k_1 \cdot k_2} \bar{v}(\bar{q}) \not{k}_1 u(q) \right]$$

The first term cancels the gauge variation of $D_1 + D_2$ if $V_0=1$, the second term is zero IFF the other gluon is physical ($k_2 \cdot \epsilon_2 = 0$).

The QCD Lagrangian

This means that the gluon is itself charged under QCD!

For full gauge invariance, also a 4-gluon vertex is necessary.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \sum_f \bar{\psi}_i^{(f)} (i\partial - m_f) \psi_i^{(f)} - \bar{\psi}_i^{(f)} (g_s t_{ij}^a A_a) \psi_j^{(f)}$$

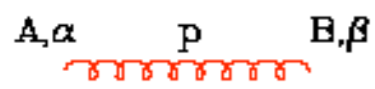
Gauge
Fields and
their
interact.

→

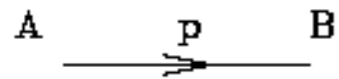
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

Matter

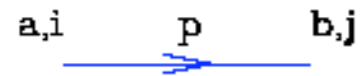
Interaction



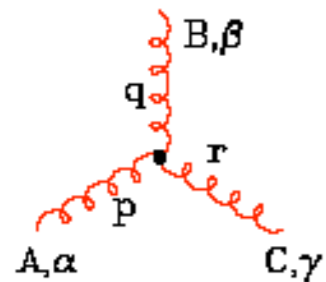
$$\delta^{AB} \left[-g^{\alpha\beta} + (1-\lambda) \frac{p^\alpha p^\beta}{p^2 + i\epsilon} \right] \frac{i}{p^2 + i\epsilon}$$



$$\delta^{AB} \frac{i}{(p^2 + i\epsilon)}$$

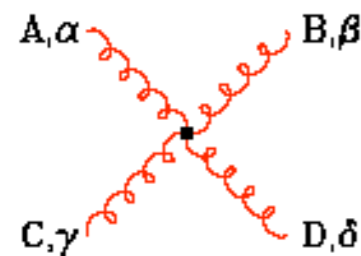


$$\delta^{ab} \frac{i}{(\not{p} - m + i\epsilon)_\mu}$$

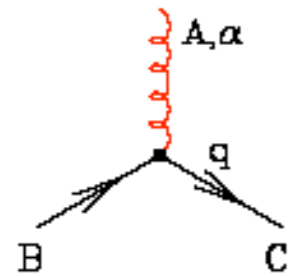
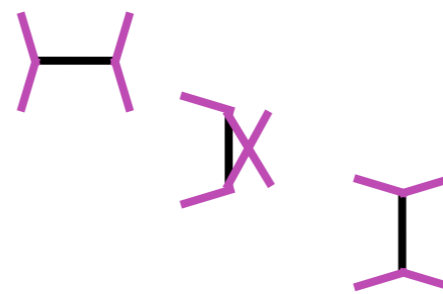


$$-g f^{ABC} [(p-q)^\gamma g^{\alpha\beta} + (q-r)^\alpha g^{\beta\gamma} + (r-p)^\beta g^{\gamma\alpha}]$$

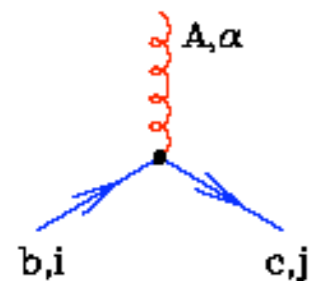
(all momenta incoming)



$$\begin{aligned} & -ig^2 f^{XAC} f^{XBD} [g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\delta} g^{\beta\gamma}] \\ & -ig^2 f^{XAD} f^{XBC} [g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\gamma} g^{\beta\delta}] \\ & -ig^2 f^{XAB} f^{XCD} [g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\gamma}] \end{aligned}$$



$$g f^{ABC} q^\alpha$$



$$-ig (t^A)_{cb} (\gamma^\alpha)_\mu$$

Monte Carlo Integration and Generation

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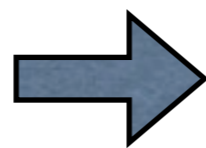
$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n) \quad \swarrow \text{Dim}[\Phi(n)] \sim 3n$$

General and flexible method is needed

Integrals as averages

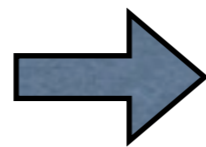


$$I = \int_{x_1}^{x_2} f(x) dx$$



$$I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x)$$

$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2$$



$$V_N = (x_2 - x_1)^2 \frac{1}{N} \sum_{i=1}^N [f(x)]^2 - I_N^2$$

Integrals as averages



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$$I = I_N \pm \sqrt{V_N/N}$$

Integrals as averages



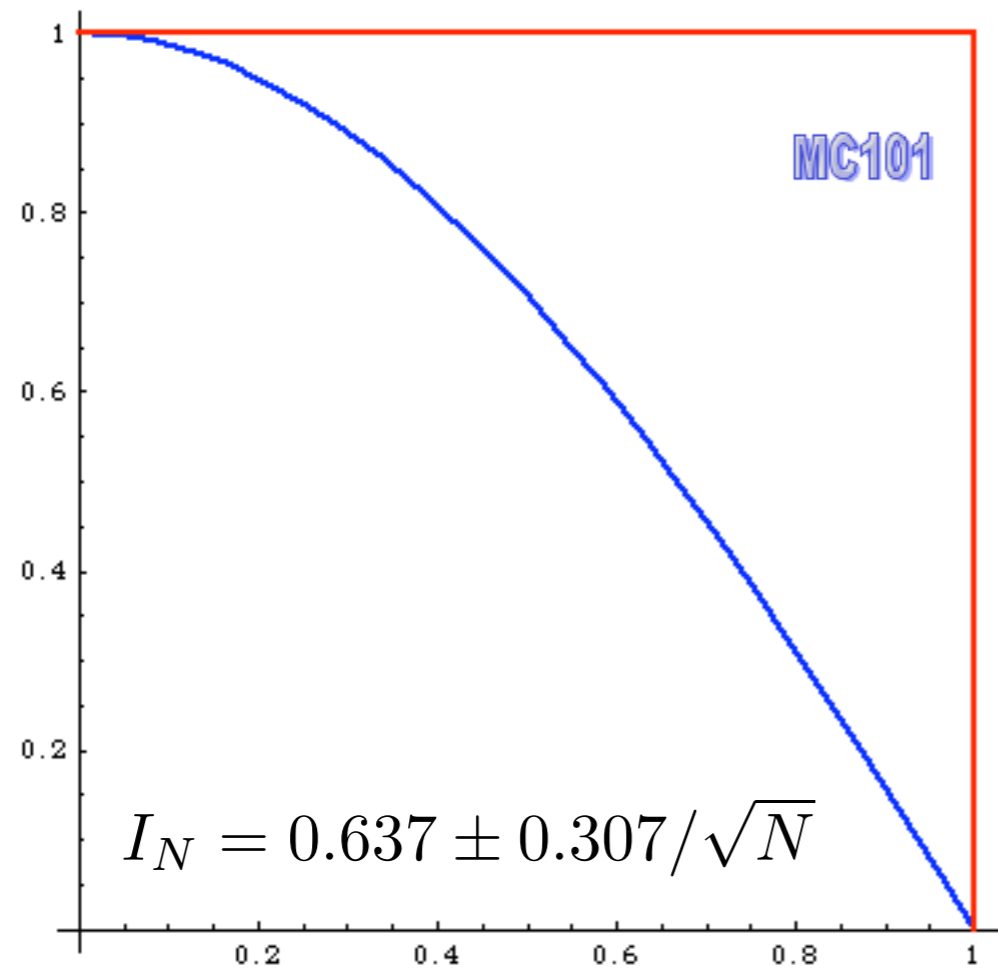
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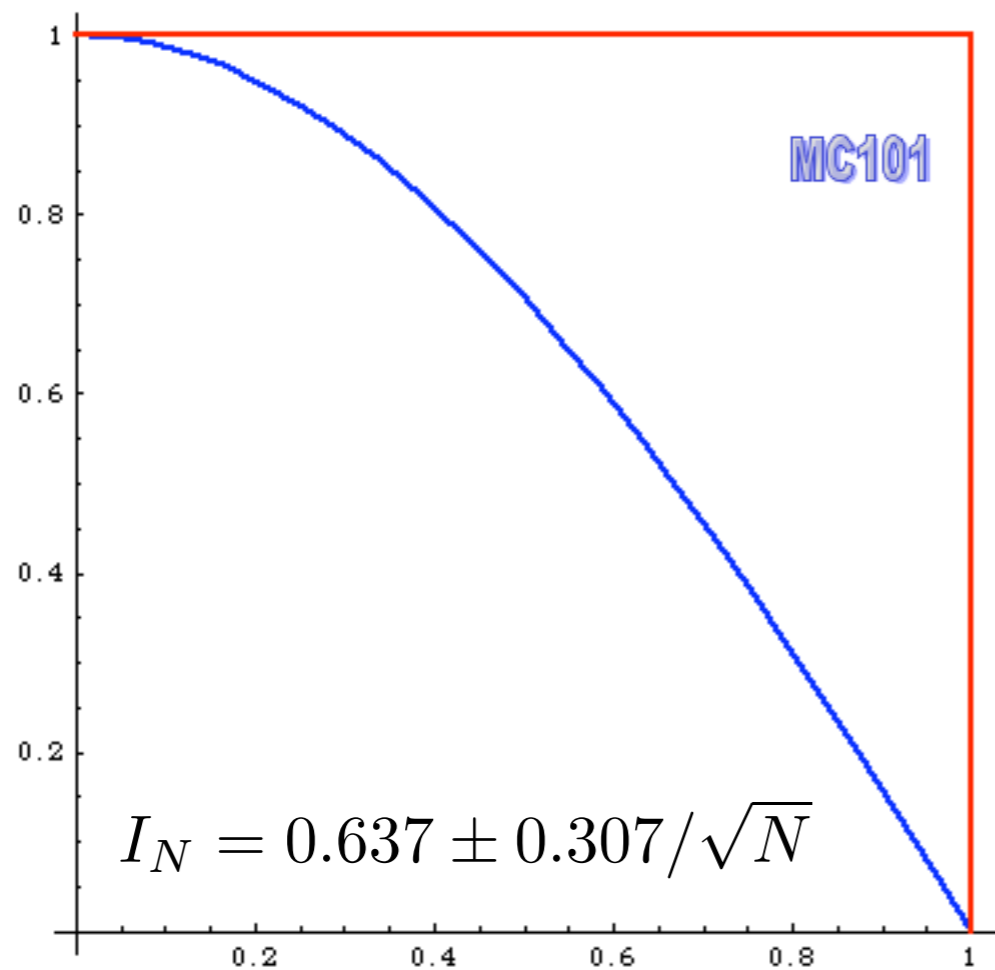
- ☞ Convergence is slow but it can be easily estimated
- ☞ Error does not depend on # of dimensions!
- ☞ Improvement by minimizing V_N .
- ☞ Optimal/Ideal case: $f(x)=C \Rightarrow V_N=0$

Importance Sampling

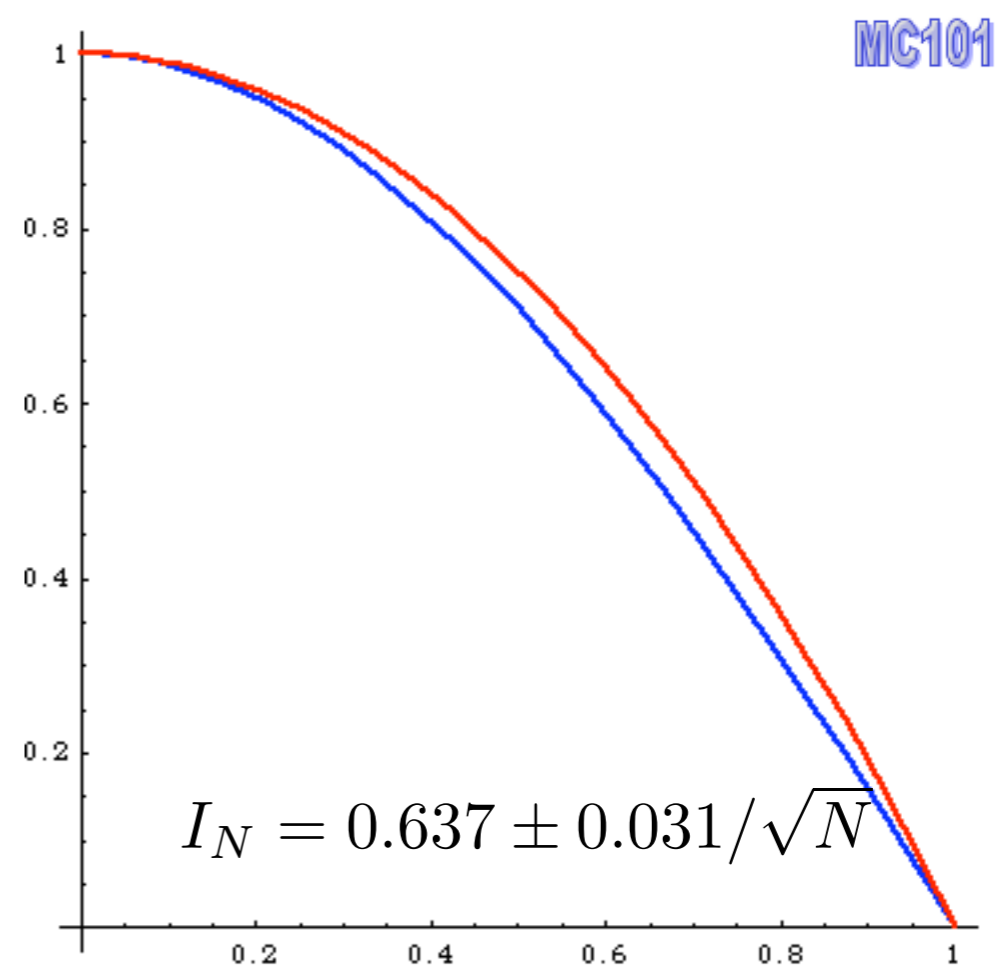


$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

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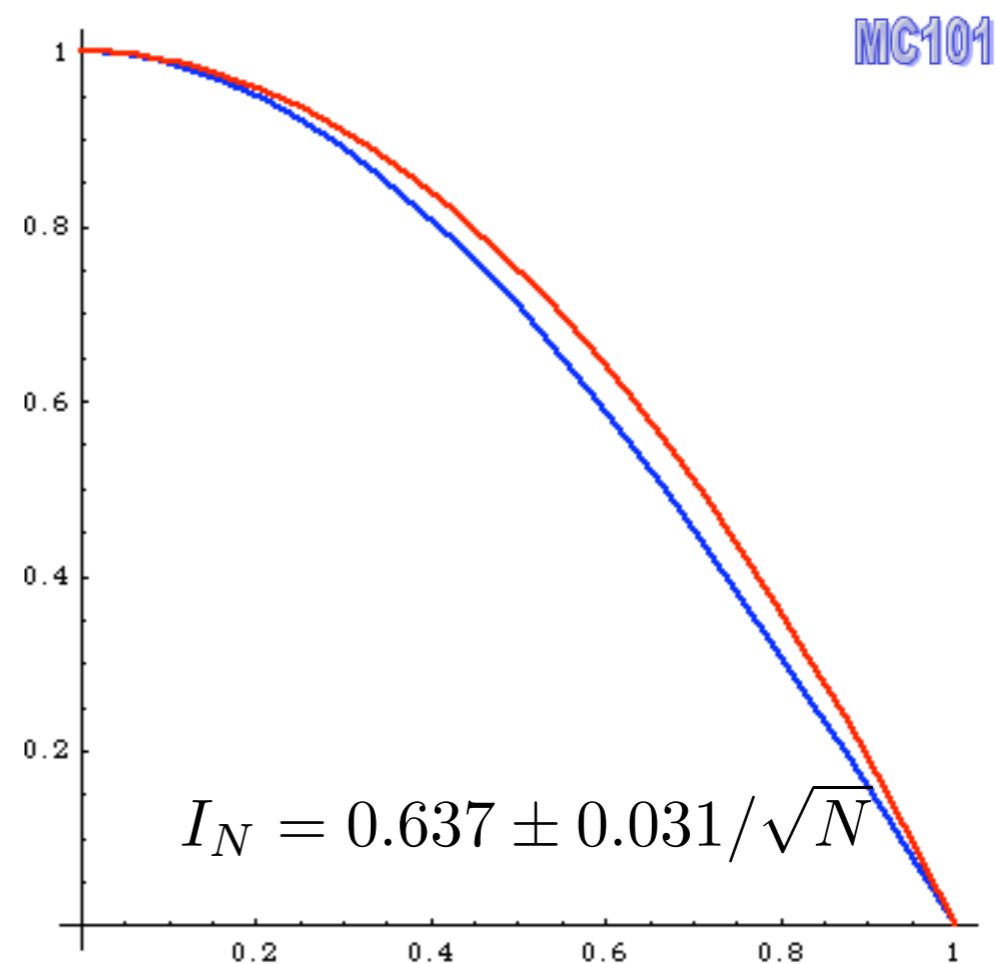
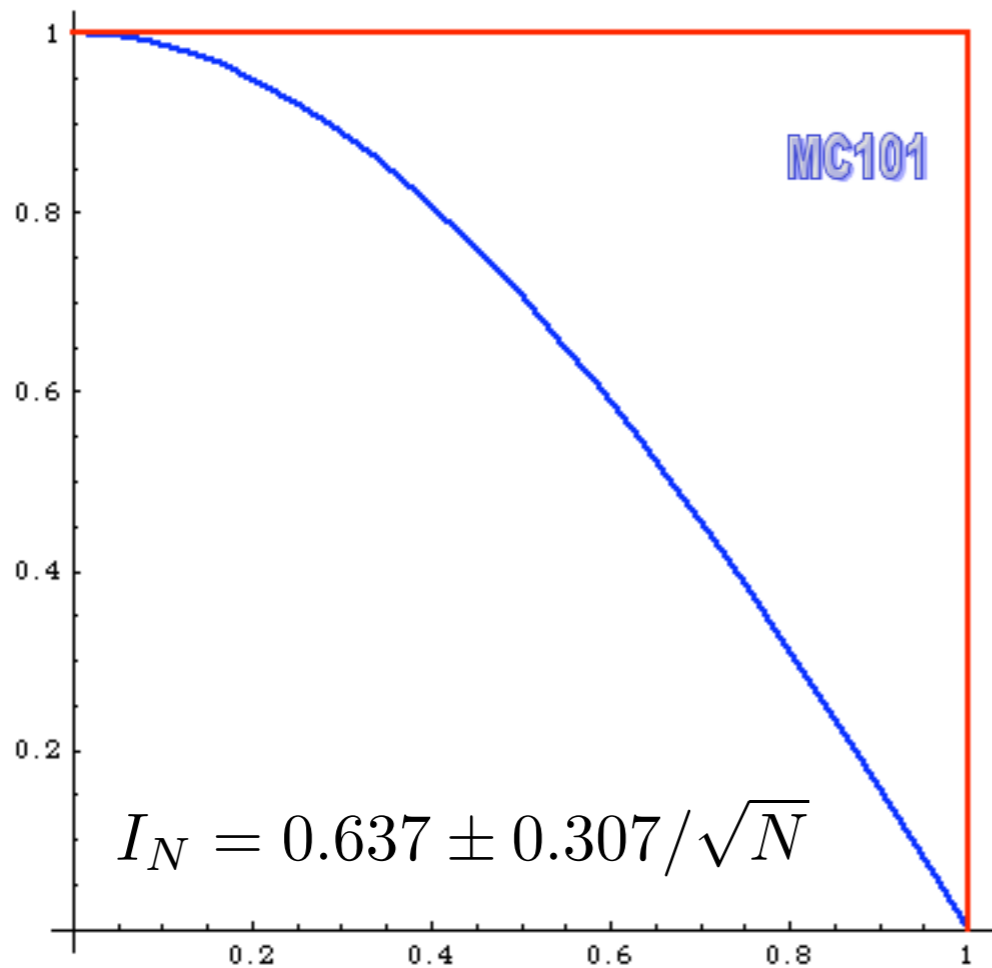


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$$I = \int_0^1 dx (1 - x^2) \frac{\cos \frac{\pi}{2} x}{1 - x^2}$$

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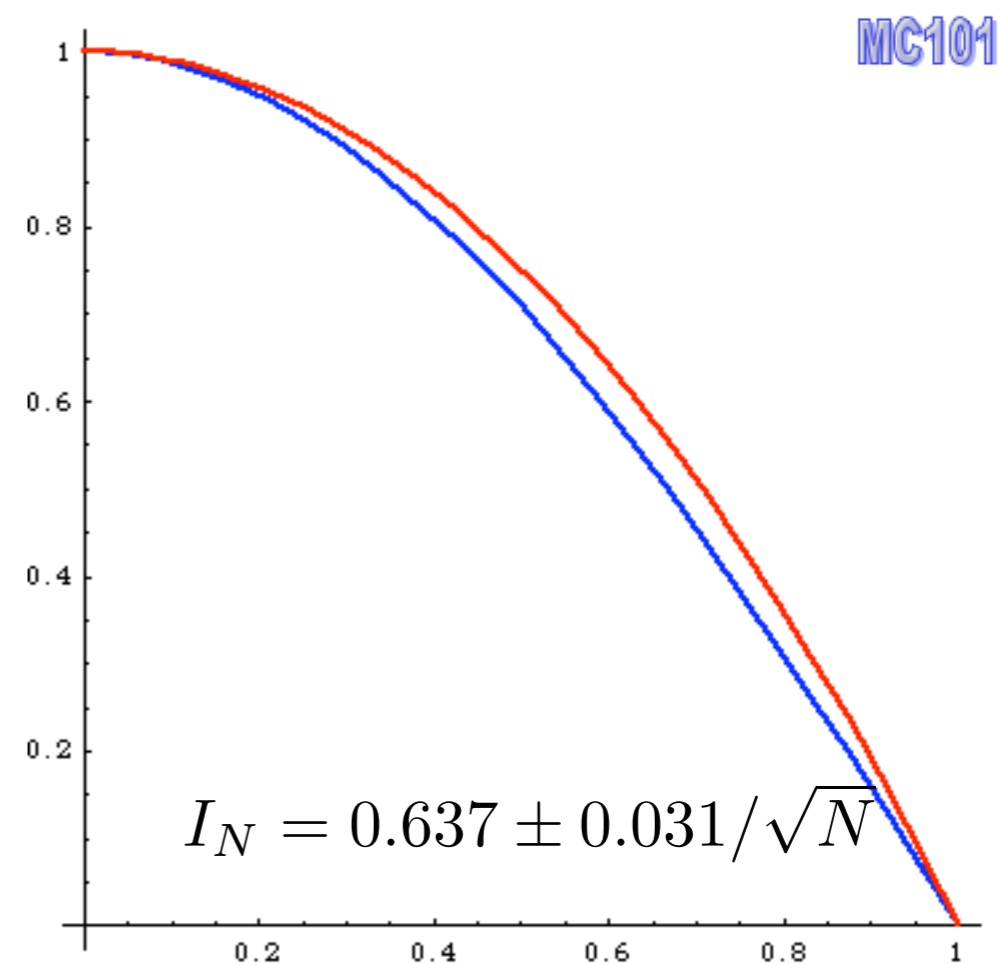
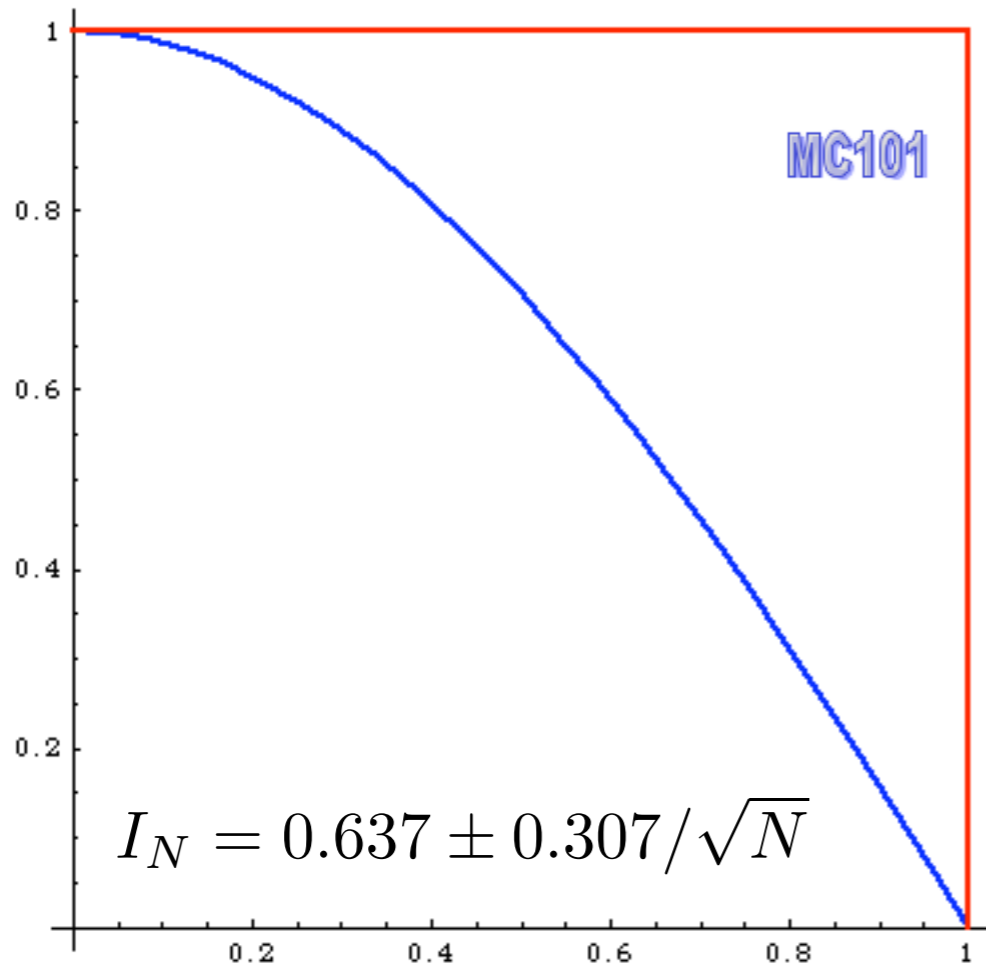


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$$= \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1-x[\xi]^2}$$

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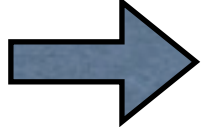
$$= \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2} \rightarrow \simeq 1$$

Importance Sampling

but... you need to know much about $f(x)$!

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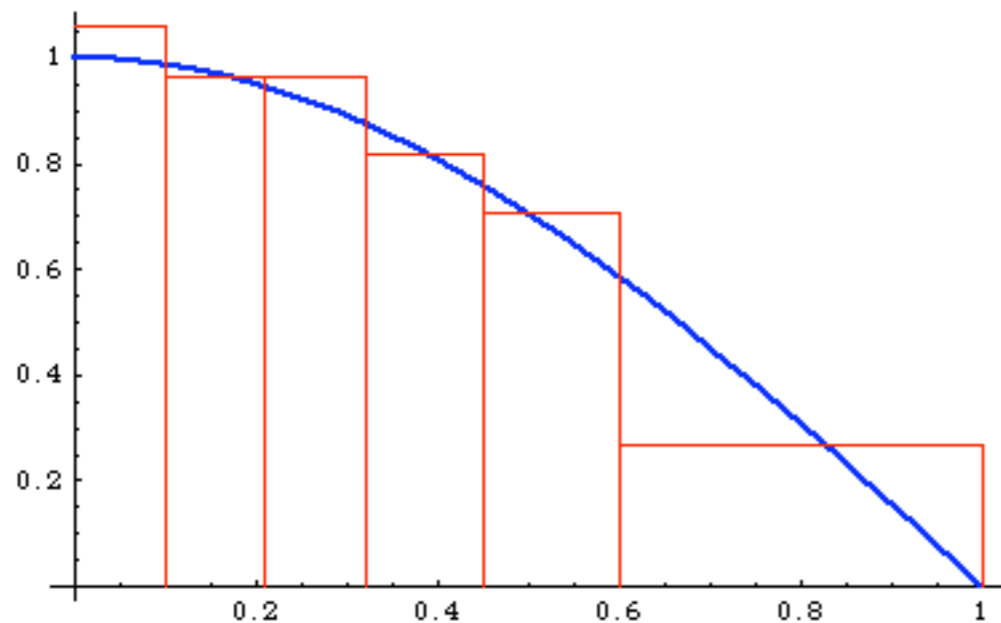
Alternative: learn during the run and build a step-function approximation $p(x)$ of $f(x)$  VEGAS

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MC101

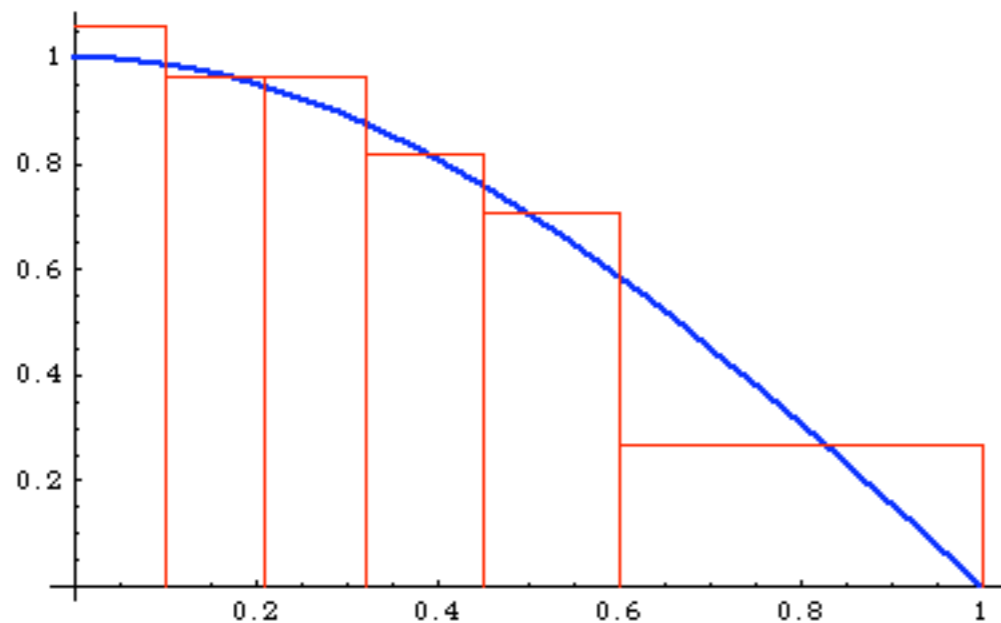


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MC101



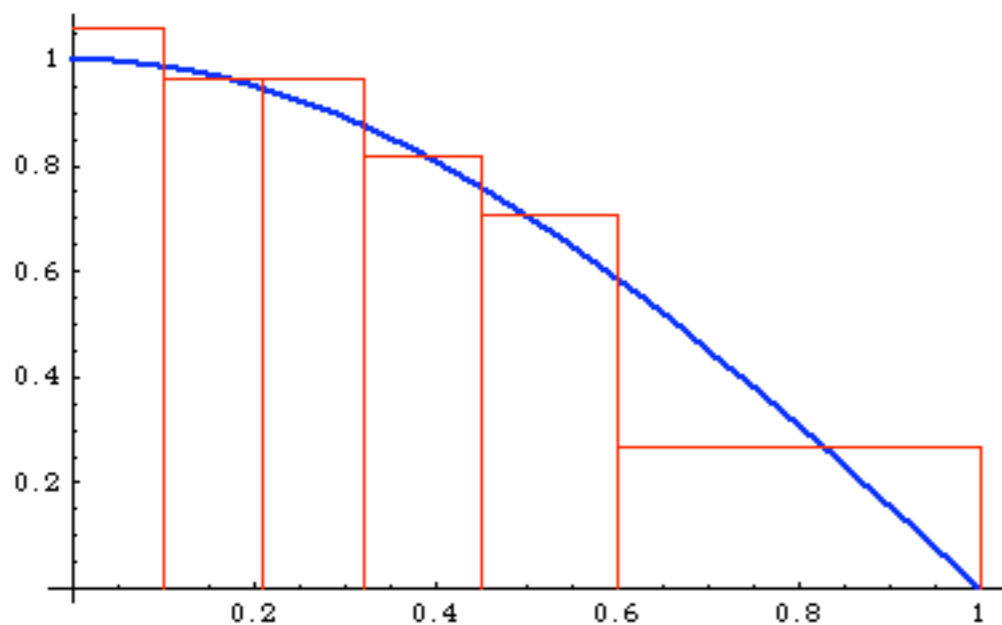
many bins where $f(x)$ is large

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but... you need to know much about $f(x)$!

Alternative: learn during the run and build a step-function approximation $p(x)$ of $f(x)$ \rightarrow VEGAS

MC101



many bins where $f(x)$ is large

$$p(x) = \frac{1}{N_b \Delta x_i}, \quad x_i - \Delta x_i < x < x_i$$

Importance Sampling

can be generalized to n dimensions:

$$p(\vec{x}) = p(x) \cdot p(y) \cdot p(z) \dots$$

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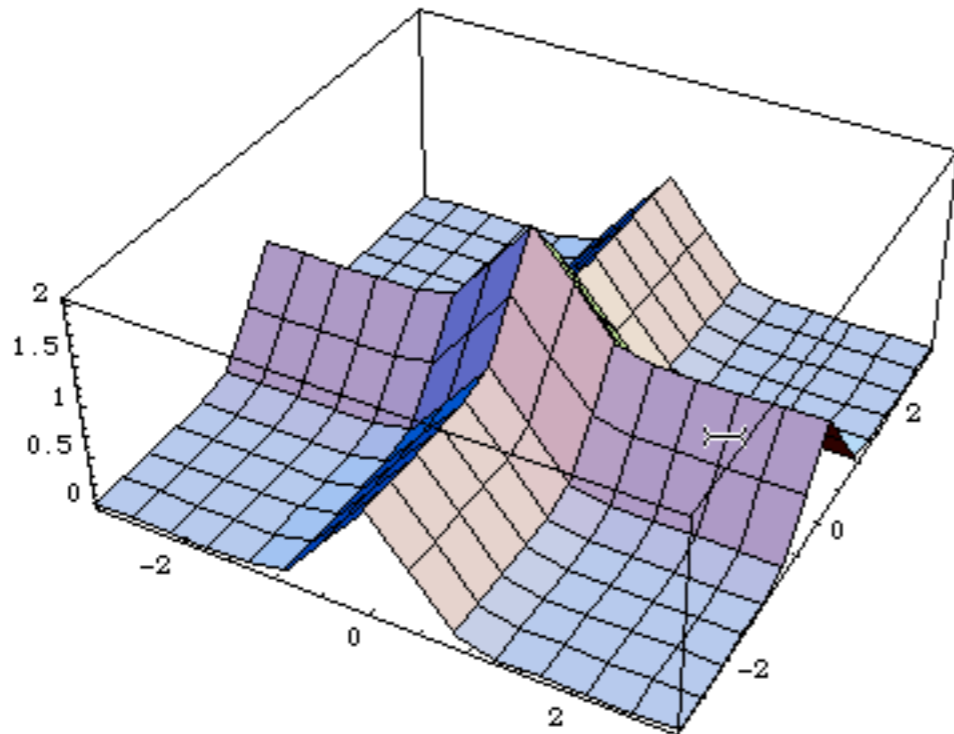
but the peaks of $f(\vec{x})$ need to be “aligned” to the axis!

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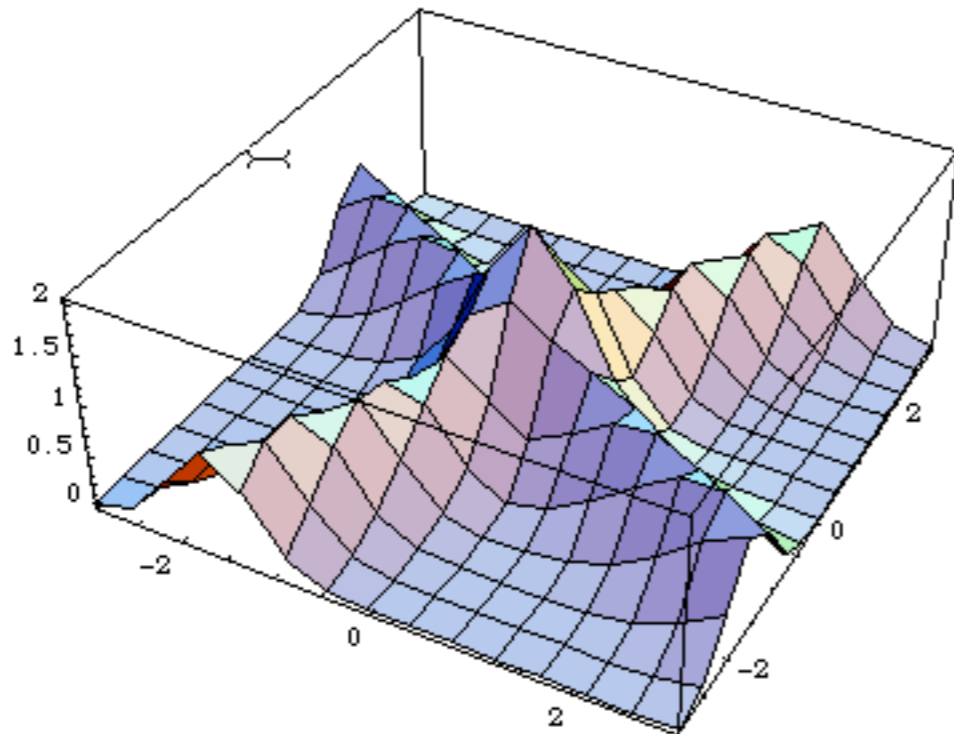
This is ok...

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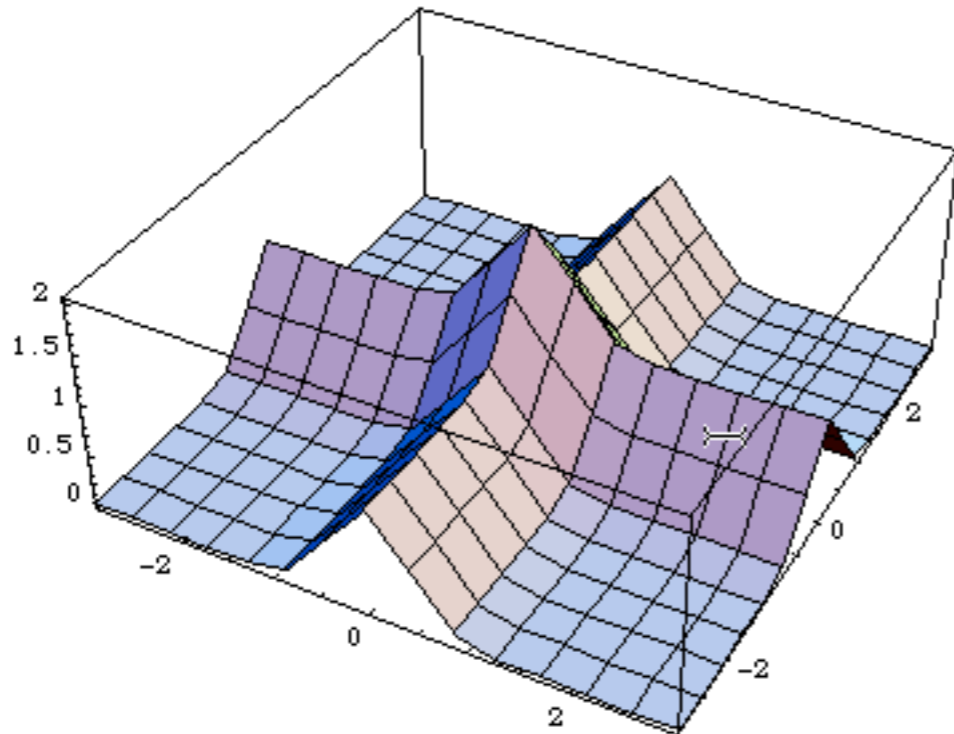
This is not ok...

Importance Sampling

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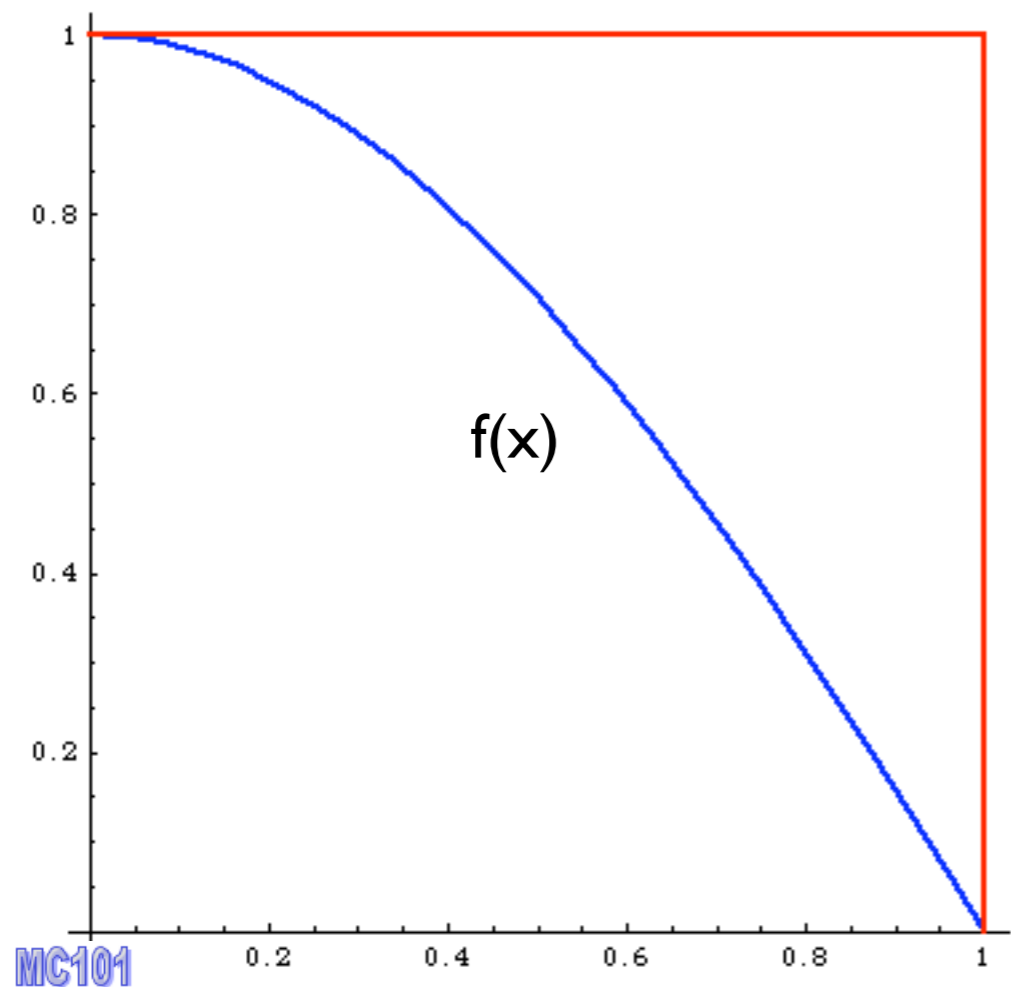
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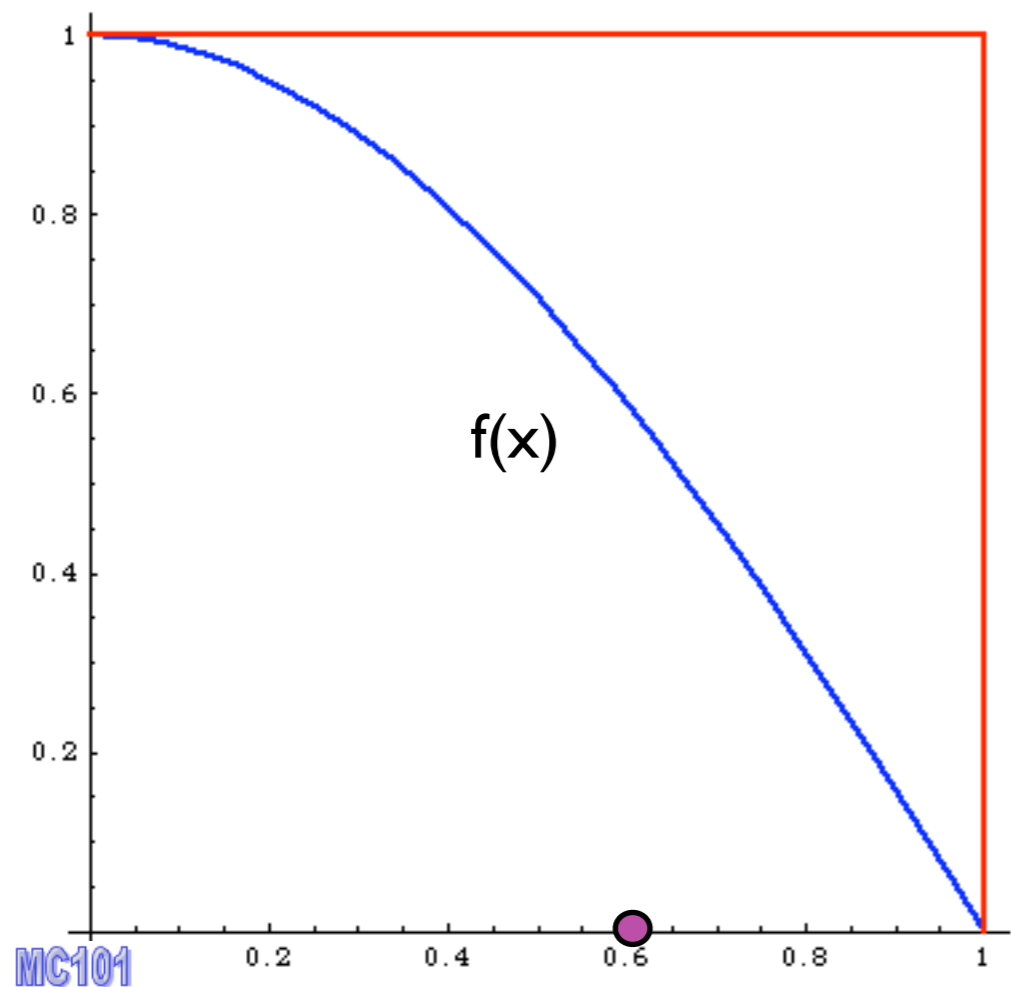
but it is sufficient to make
a change of variables!

Event generation



Alternative way

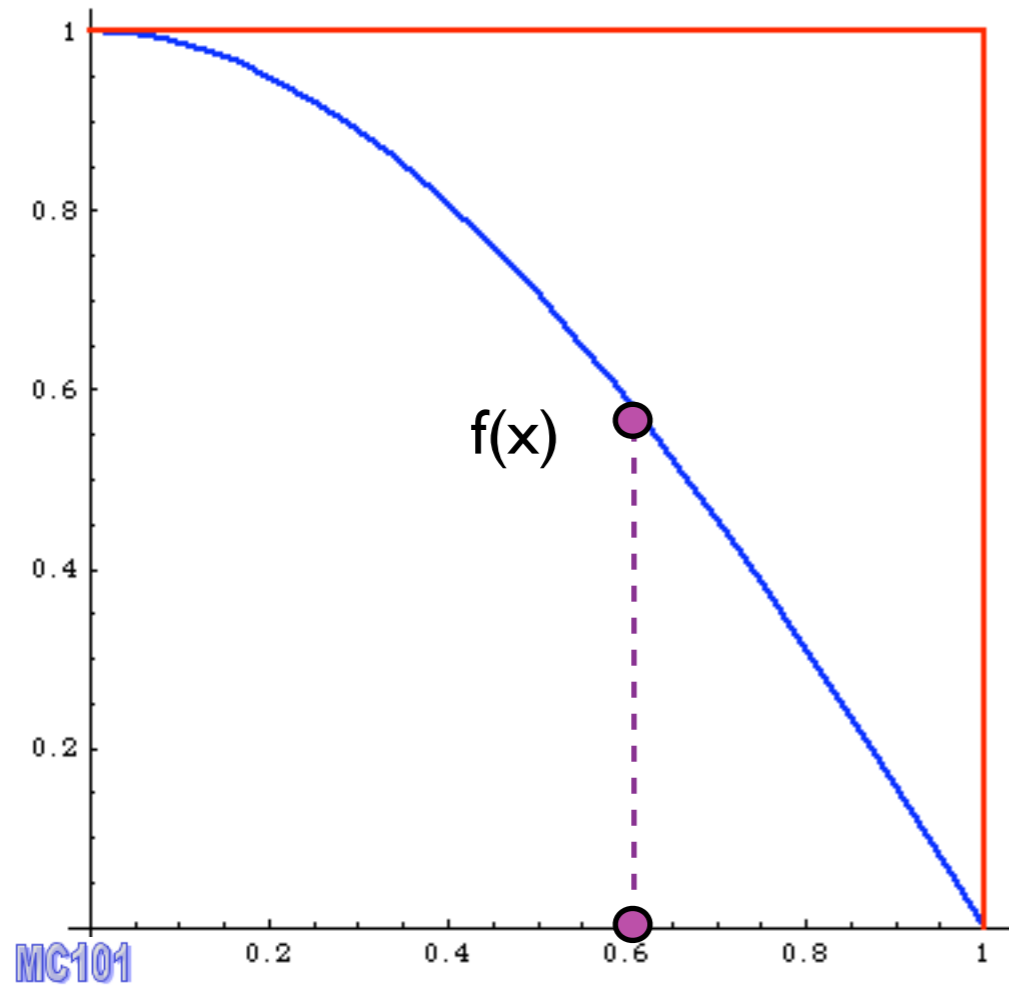
Event generation



Alternative way

1. pick x

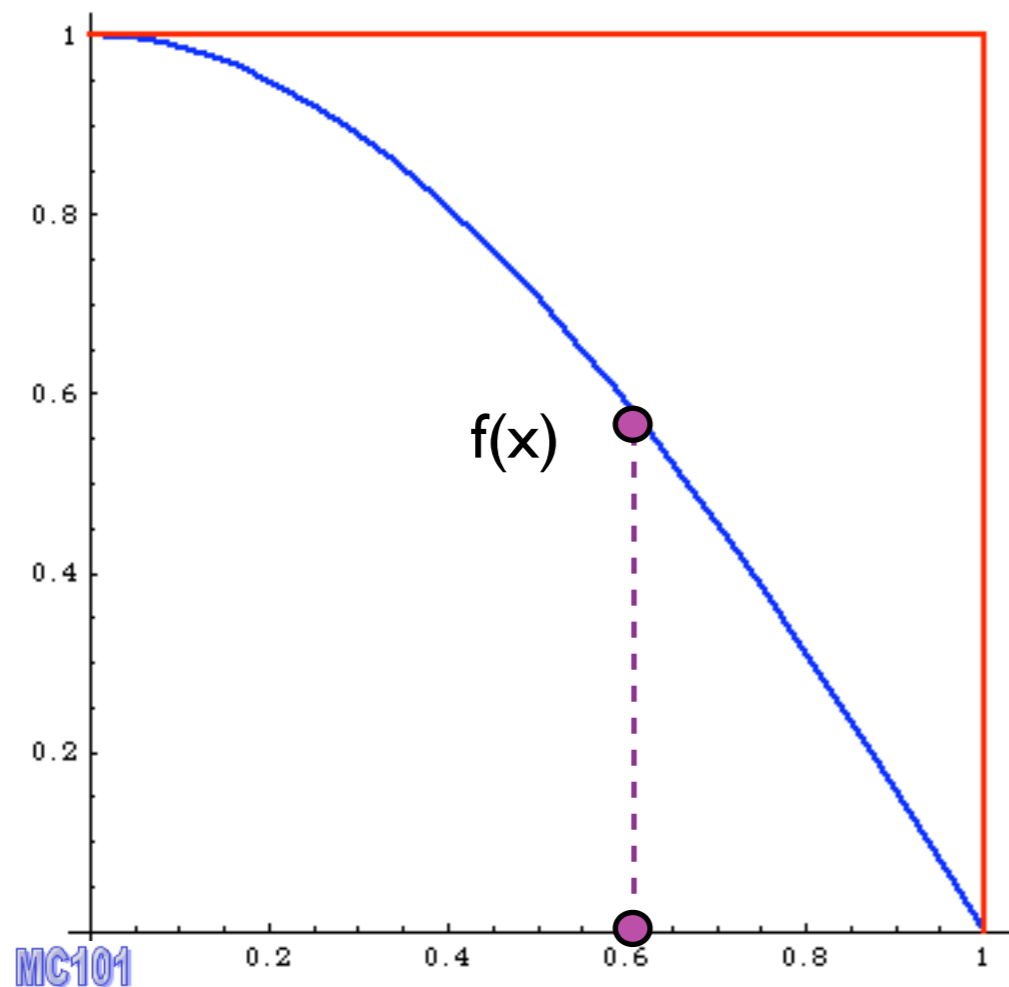
Event generation



Alternative way

1. pick x
2. calculate $f(x)$

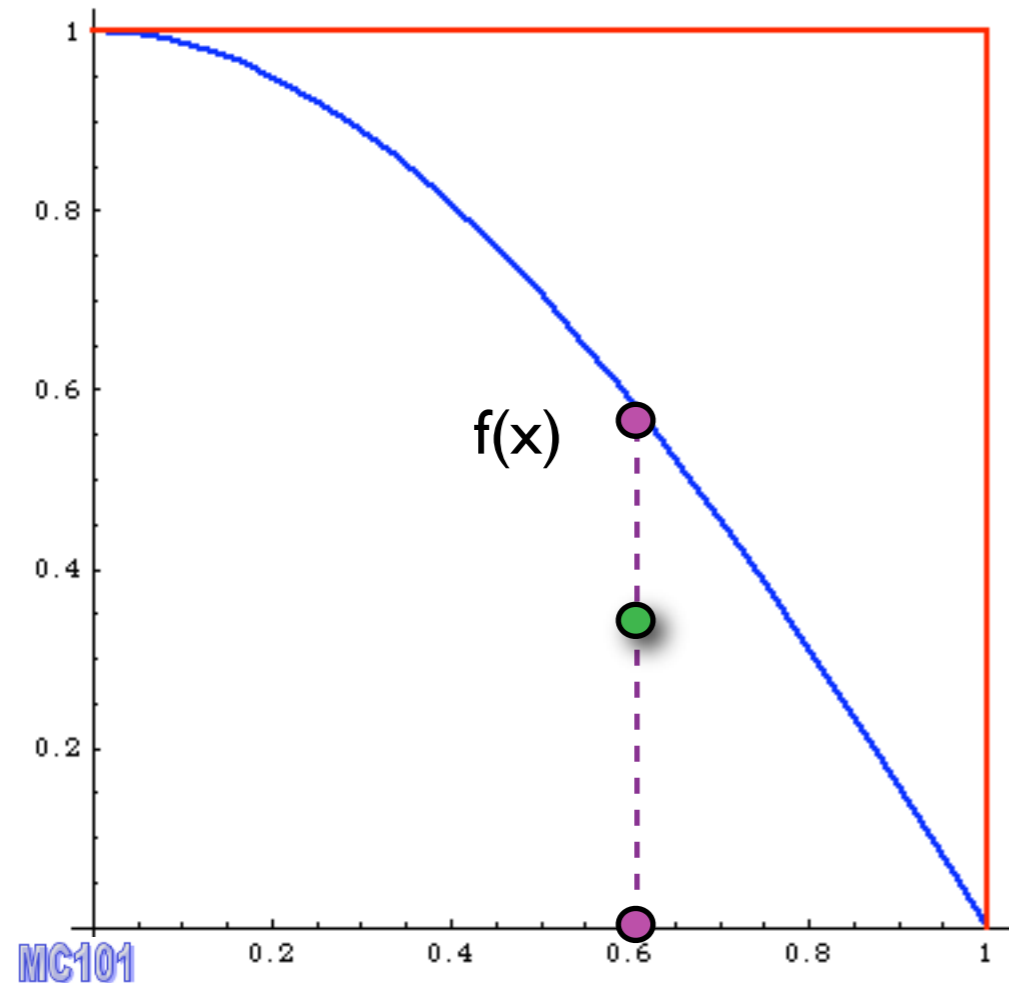
Event generation



Alternative way

1. pick x
2. calculate $f(x)$
3. pick $0 < y < f_{\max}$

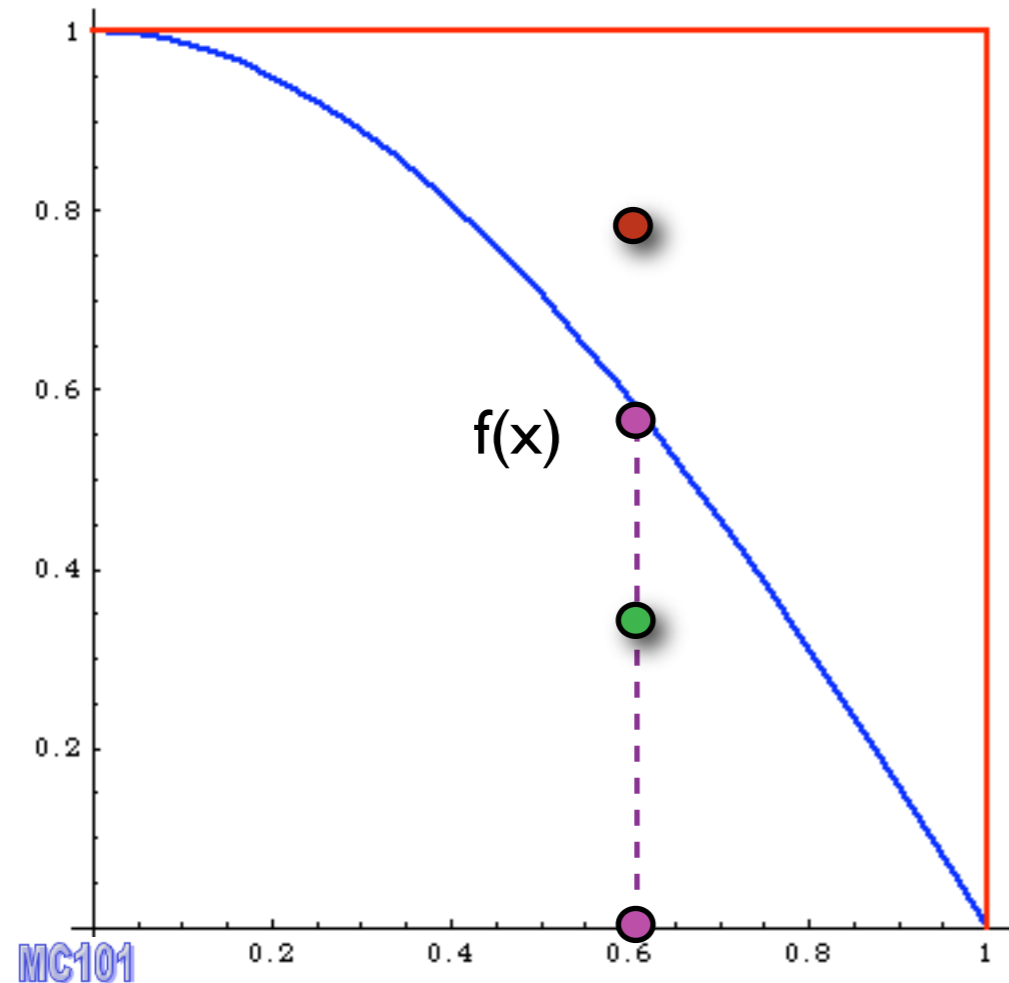
Event generation



Alternative way

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4. Compare:
if $f(x) > y$ accept event,

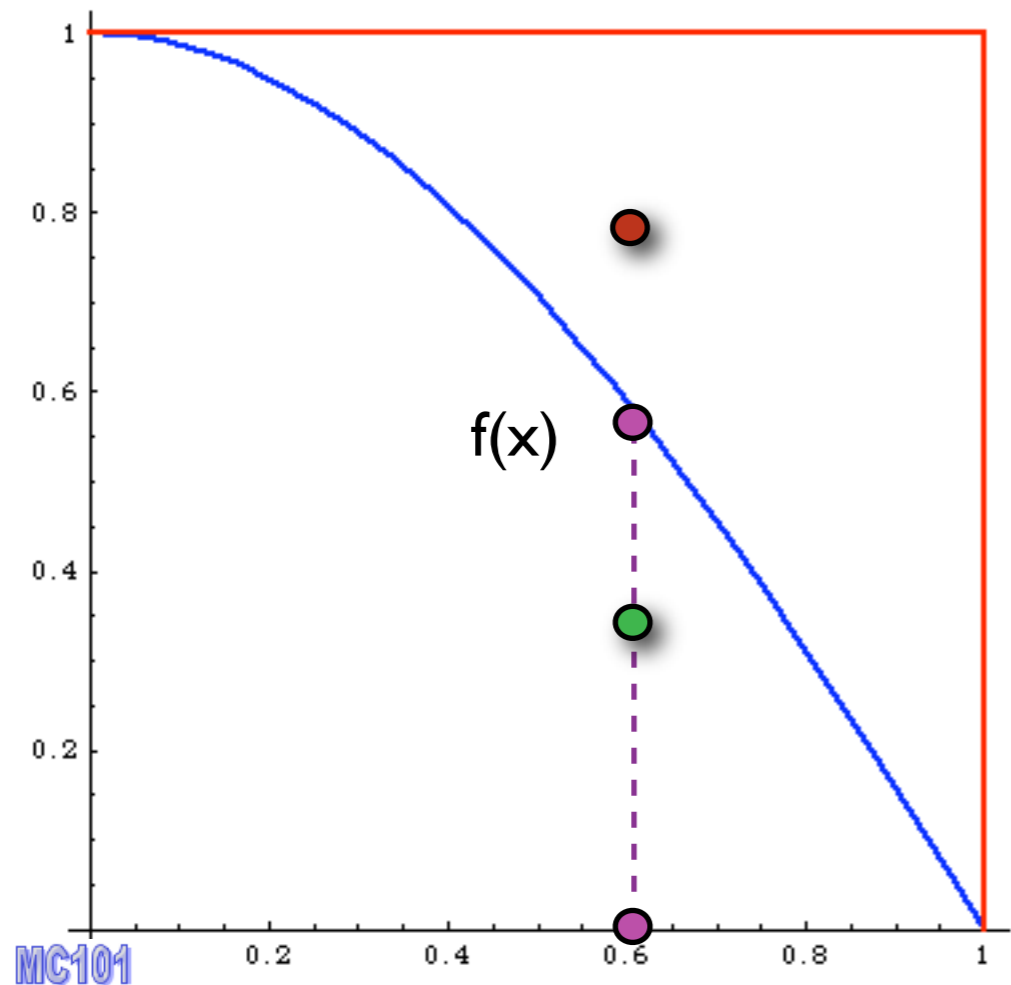
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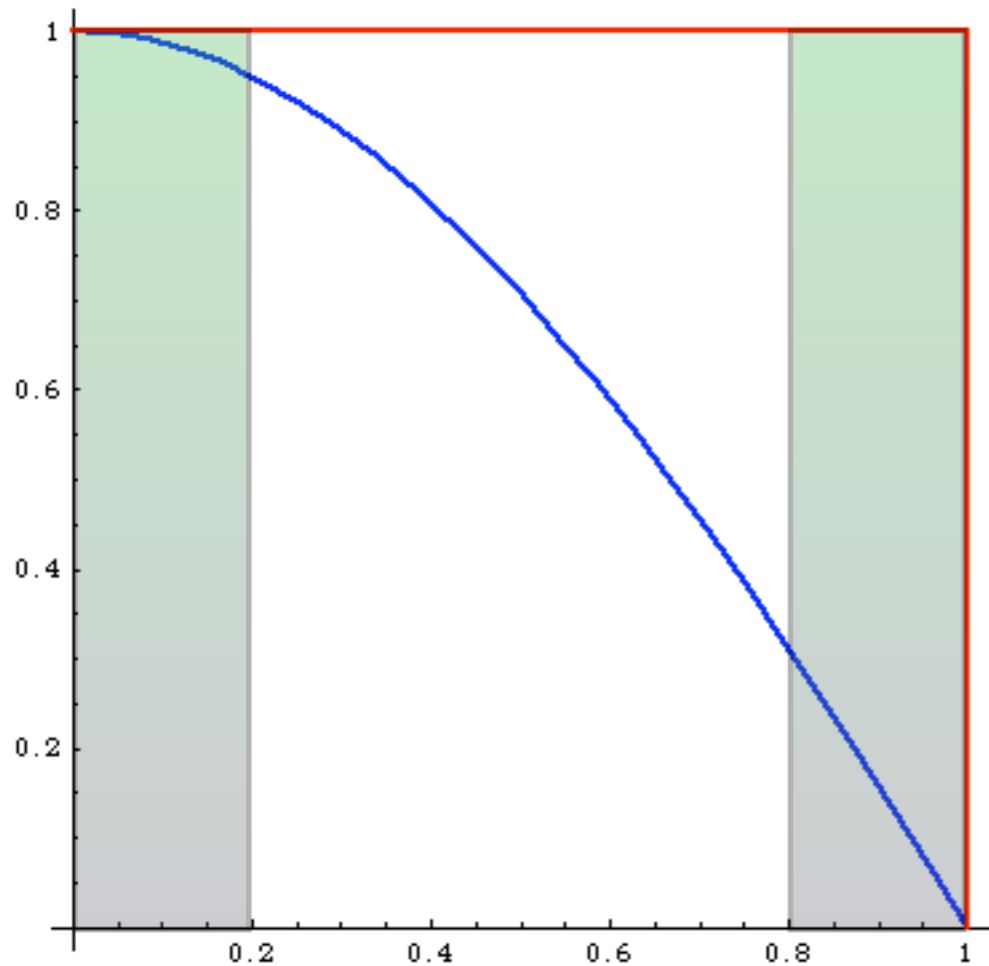


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$$| = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

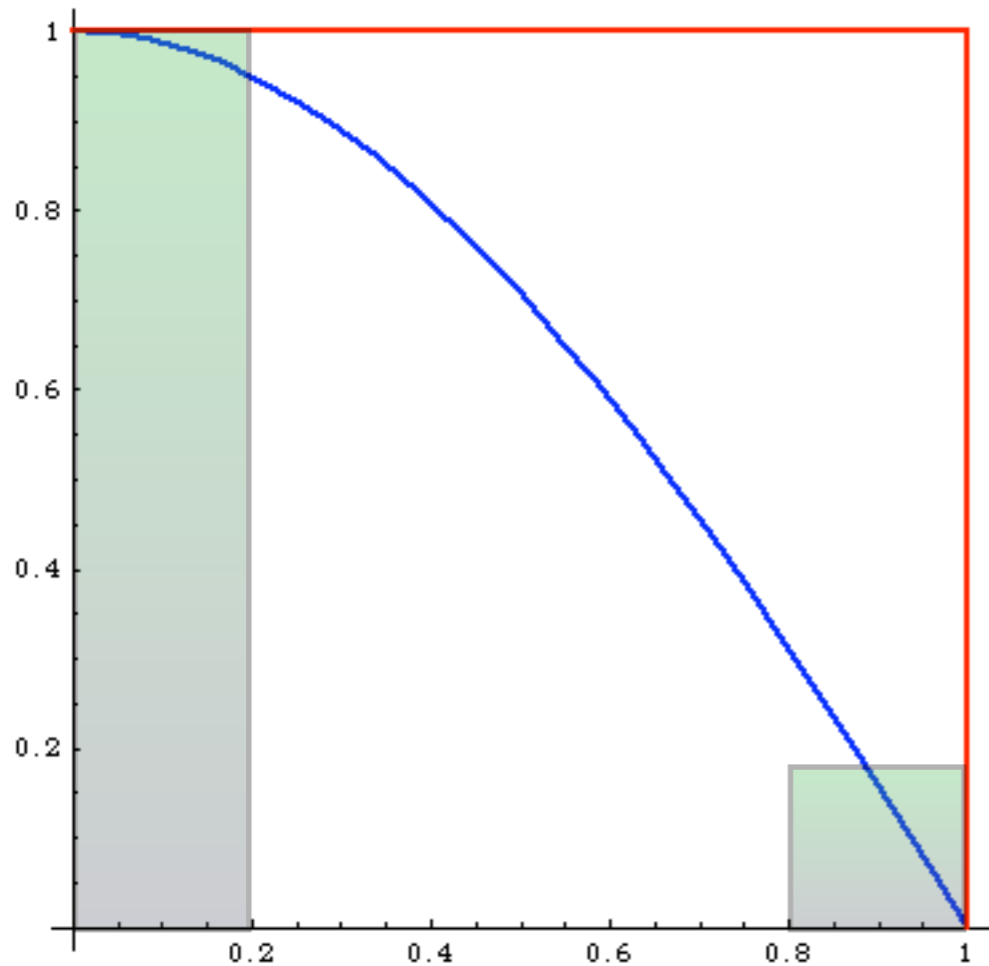


What's the difference?

Before:

Same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



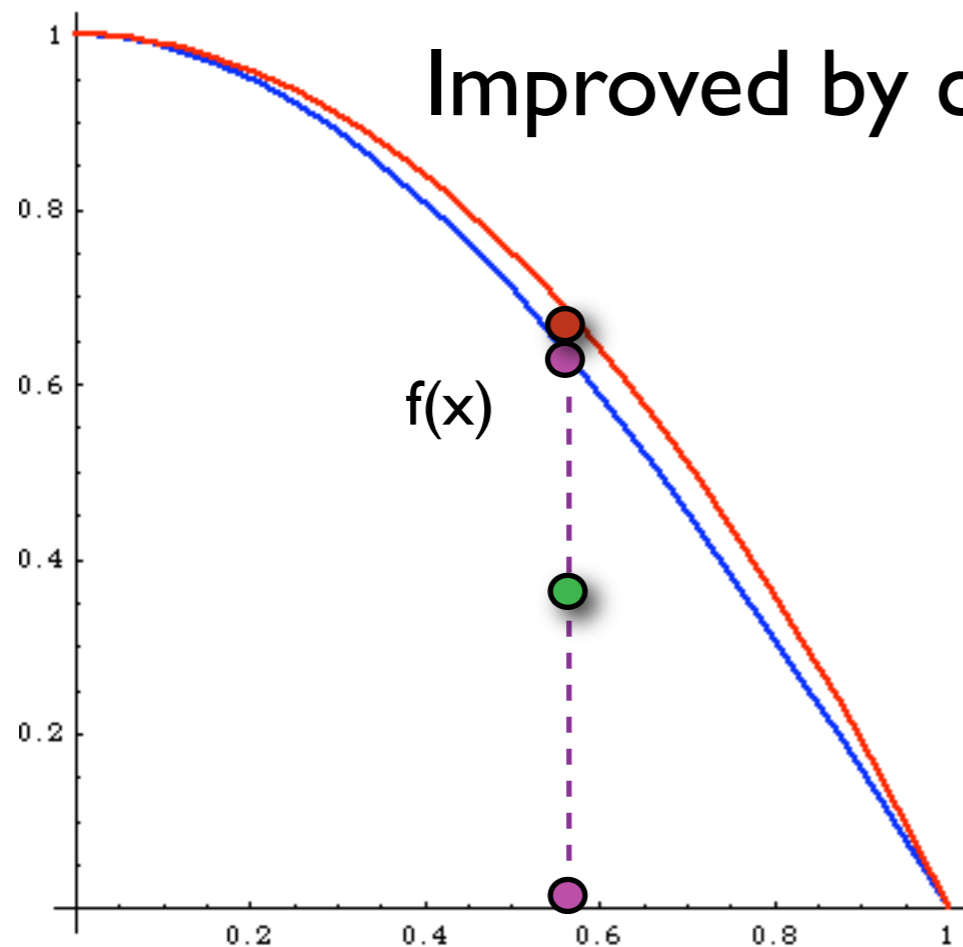
What's the difference?

After:

events is proportional to the probability of areas of phase space:
 events have all the same weight ("unweighted")

Events distributed as in Nature

Event generation



Improved by combining with importance sampling:

1. pick x distributed as $p(x)$
2. calculate $f(x)$ and $p(x)$
3. pick $0 < y < 1$
4. Compare:
if $f(x) > y p(x)$ accept event,
else reject it.

much better efficiency!!!

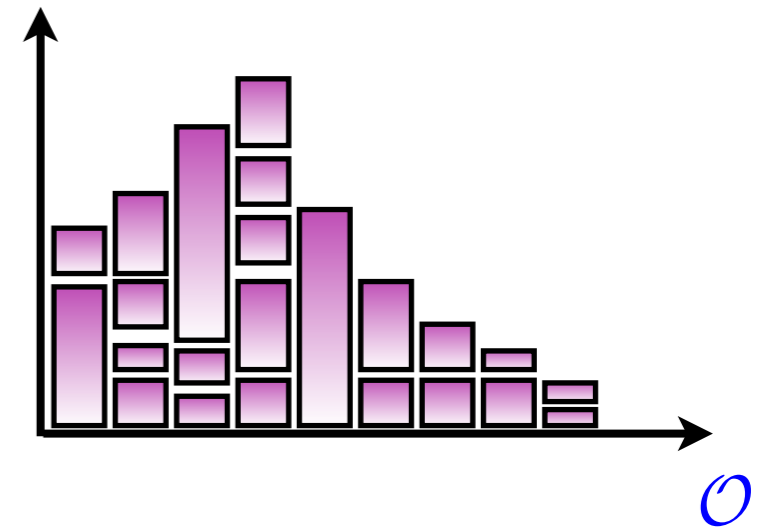
Event generation

MC integrator

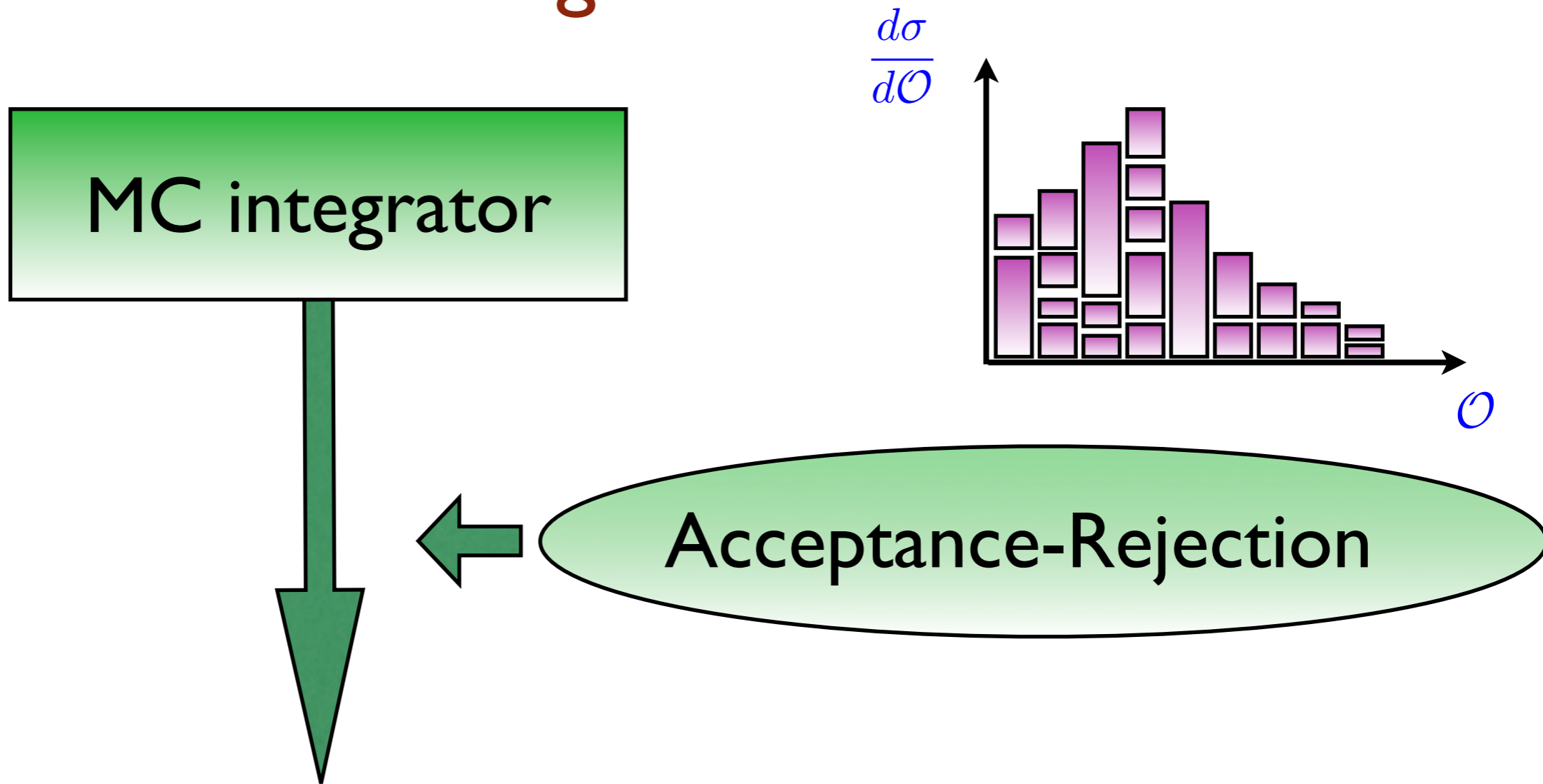
Event generation

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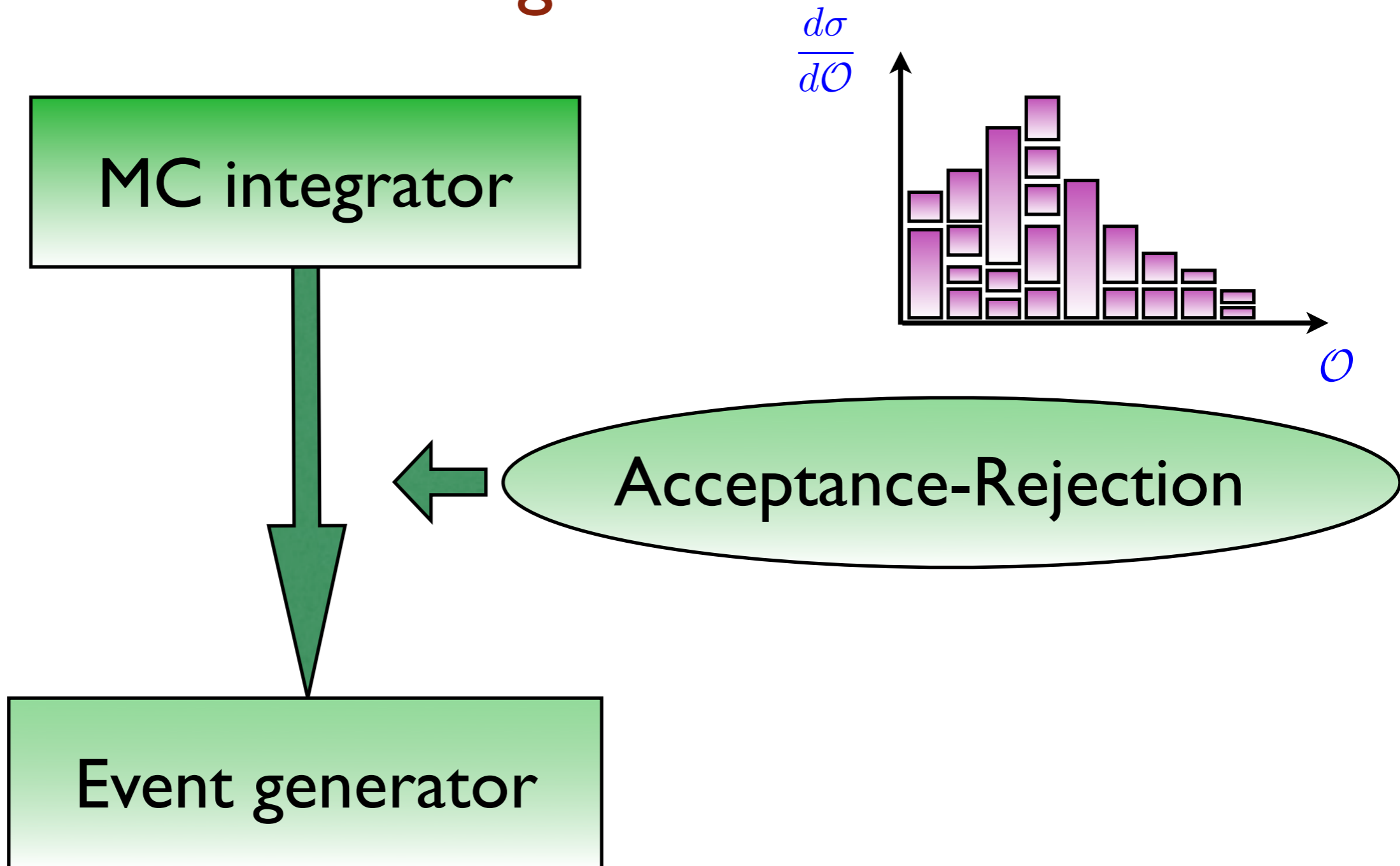
$$\frac{d\sigma}{d\mathcal{O}}$$



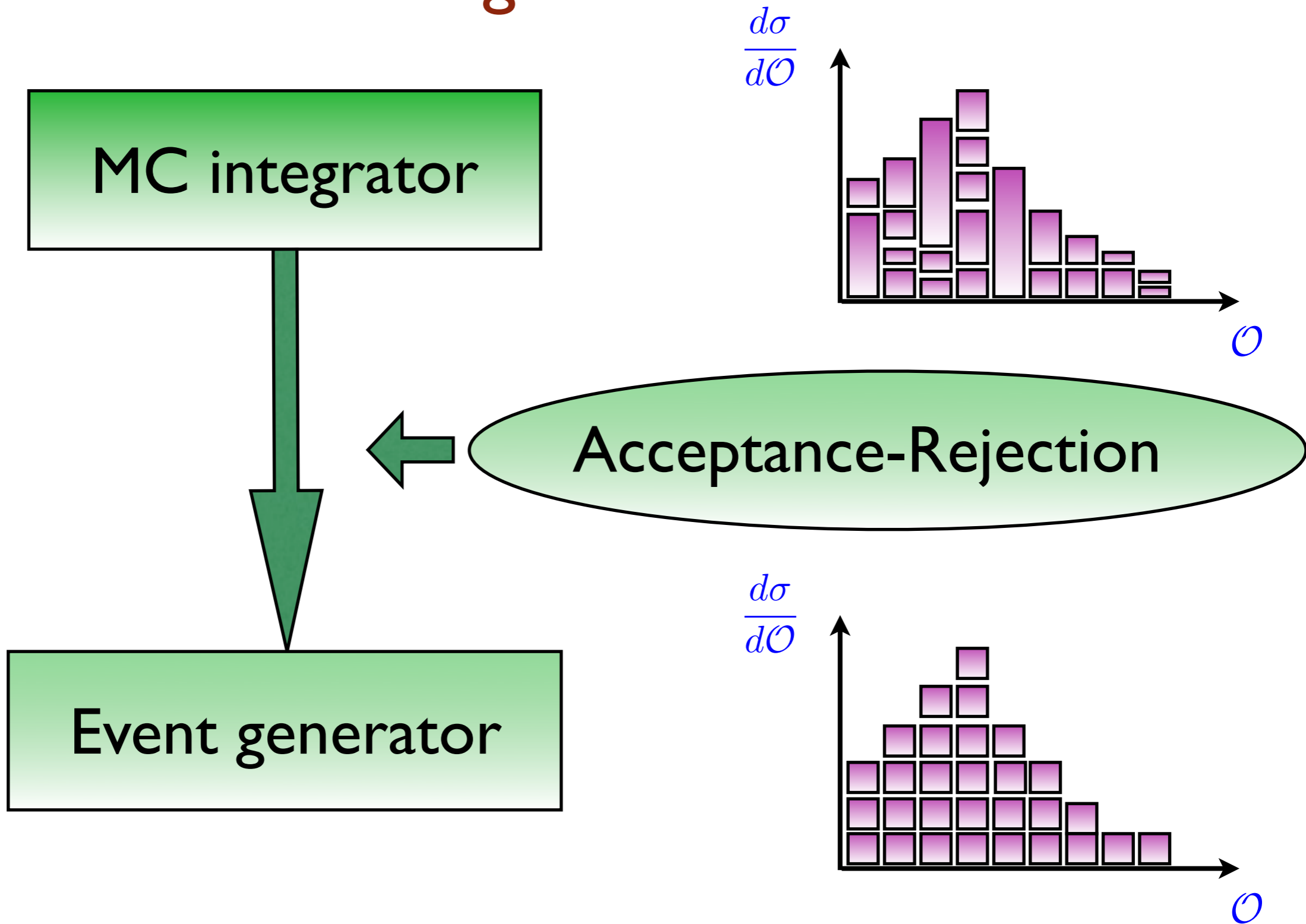
Event generation



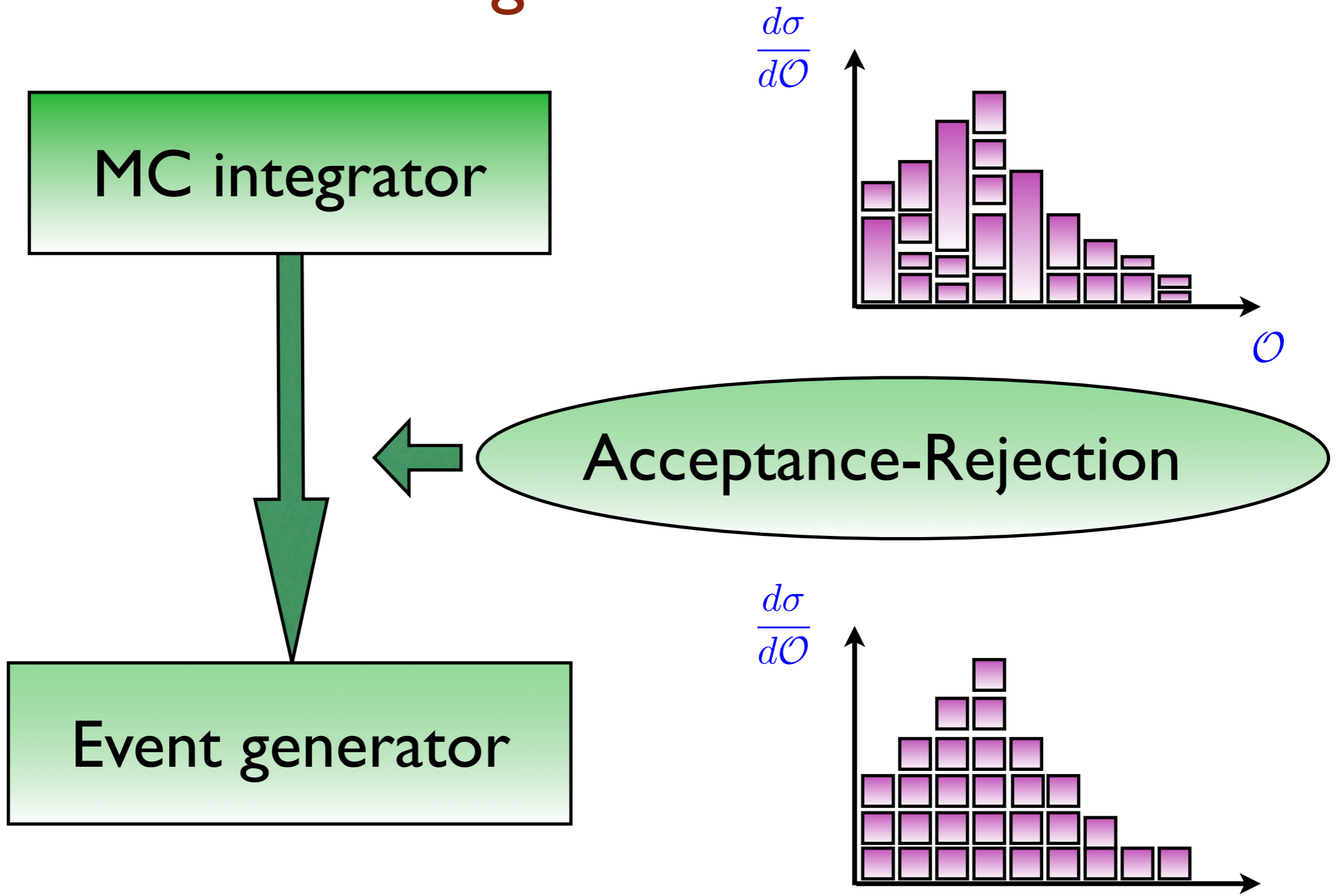
Event generation



Event generation



Event generation



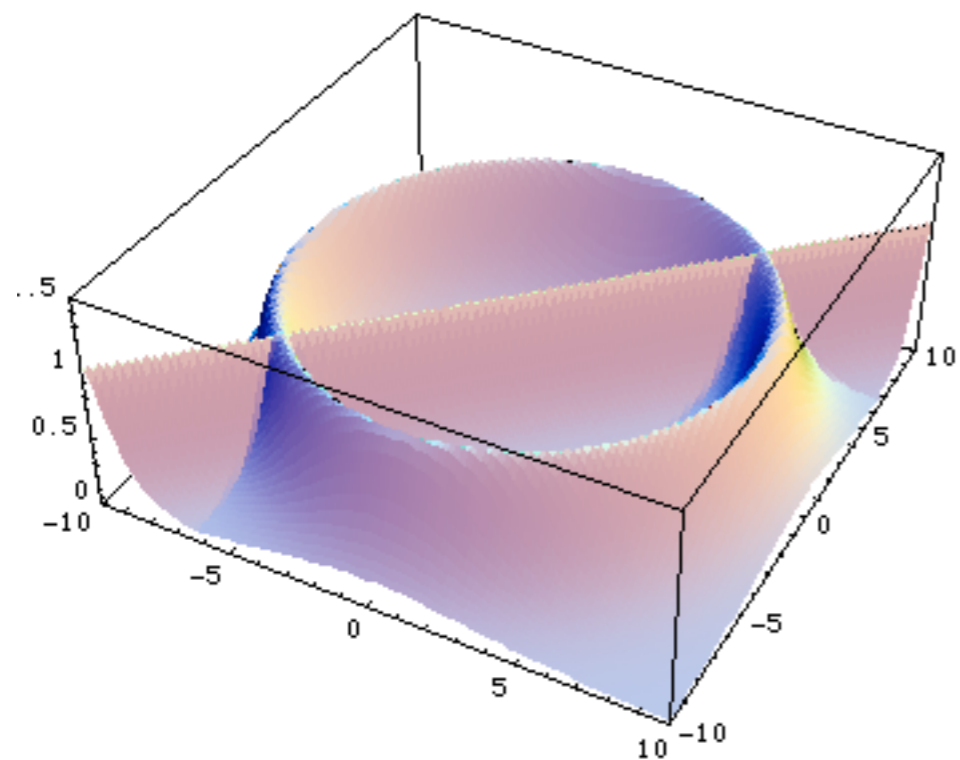
👉 This is possible only if $f(x) < \infty$ AND has definite sign!

Monte Carlo Event Generator: Definiton

At the most basic level a Monte Carlo event generator is a program which produces particle physics events with the same probability as they occur in nature (virtual collider).

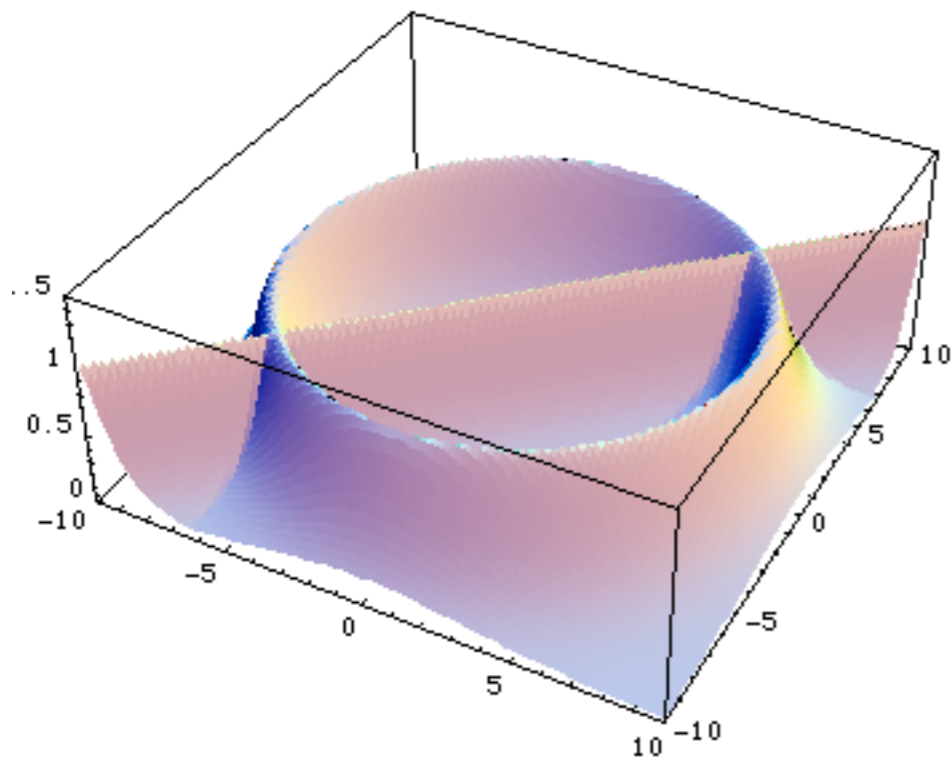
In practice it performs a large number of (sometimes very difficult) integrals and then unweight to give the four momenta of the particles that interact with the detector (simulation).

Multi-channel



What do we do if there is no transformation that aligns all integrand peaks to the chosen axes?
Vegas is bound to fail!

Multi-channel



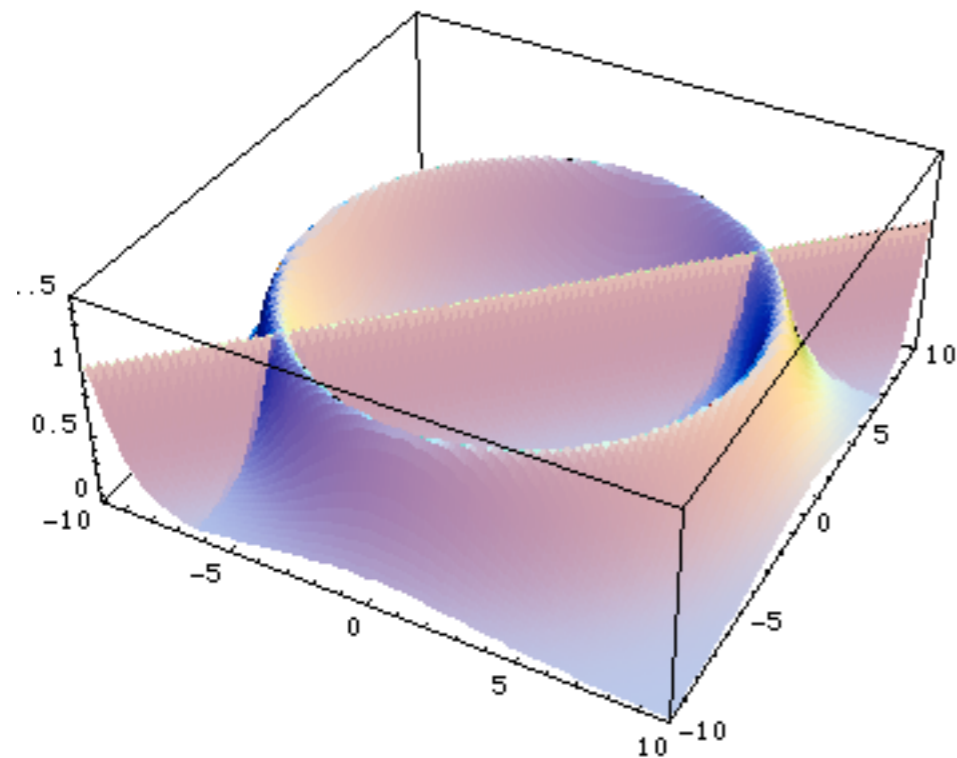
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Solution: use different transformations = channels

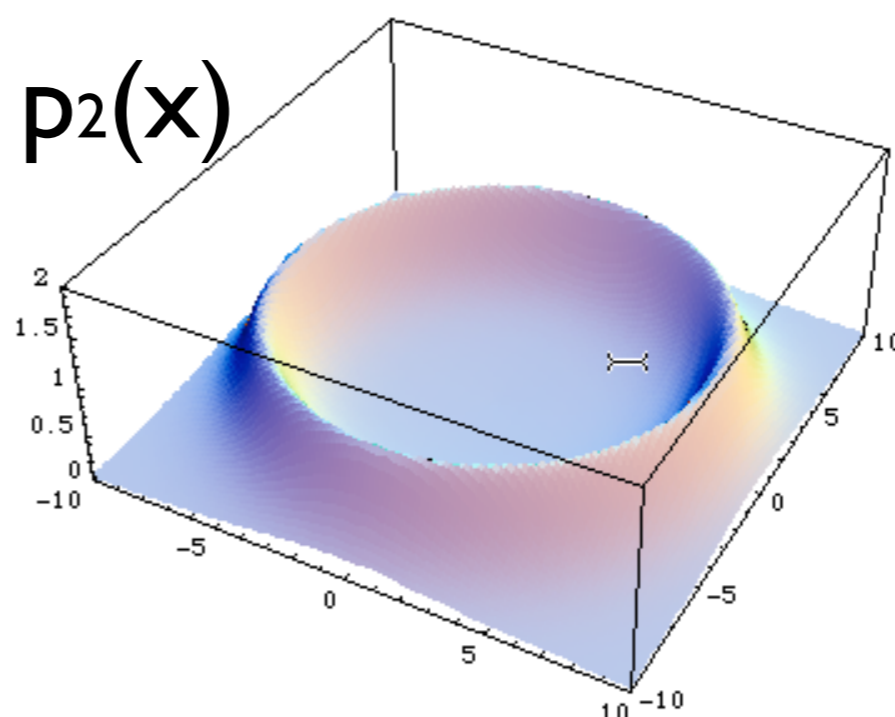
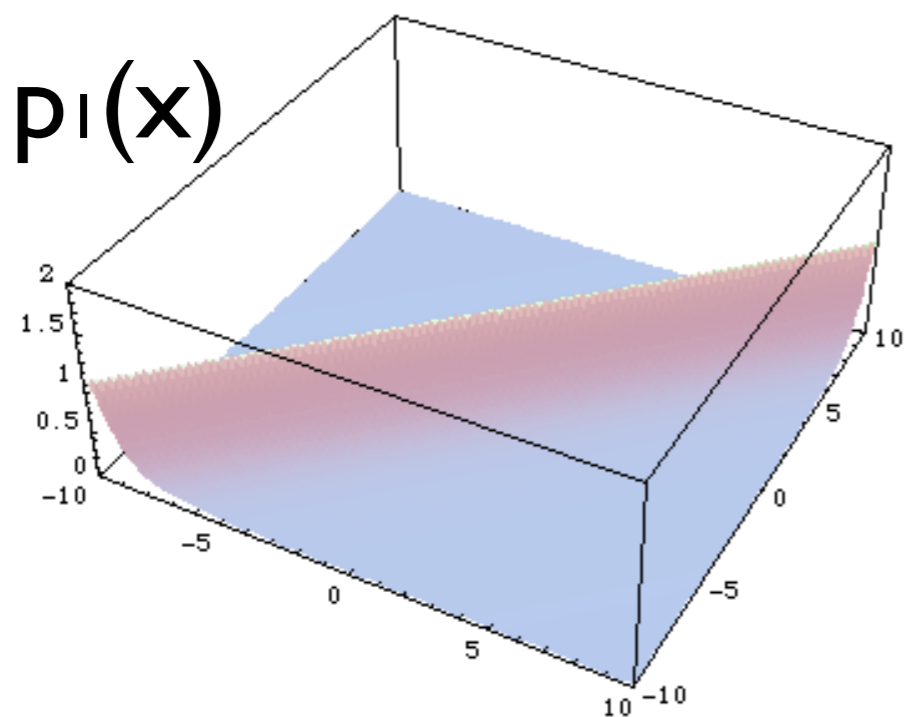
$$p(x) = \sum_{i=1}^n \alpha_i p_i(x) \quad \text{with} \quad \sum_{i=1}^n \alpha_i = 1$$

with each $p_i(x)$ taking care of one “peak” at the time

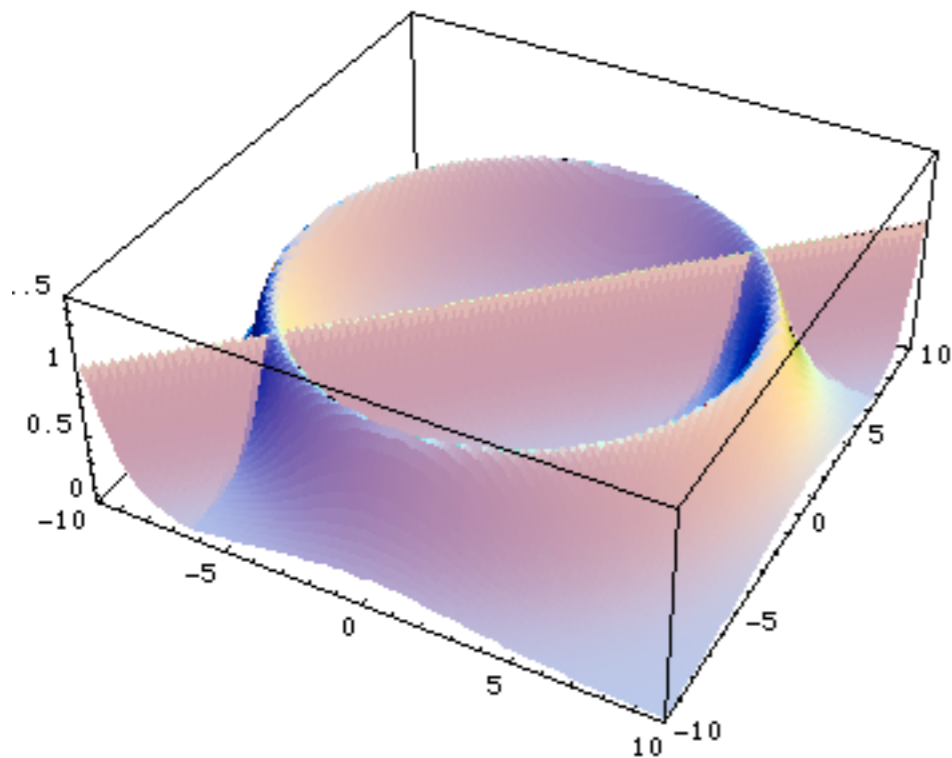
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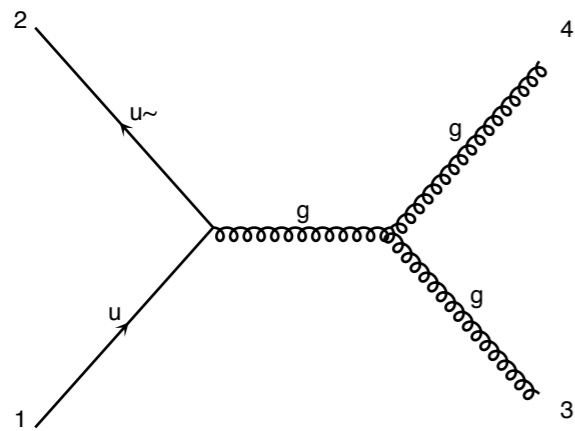
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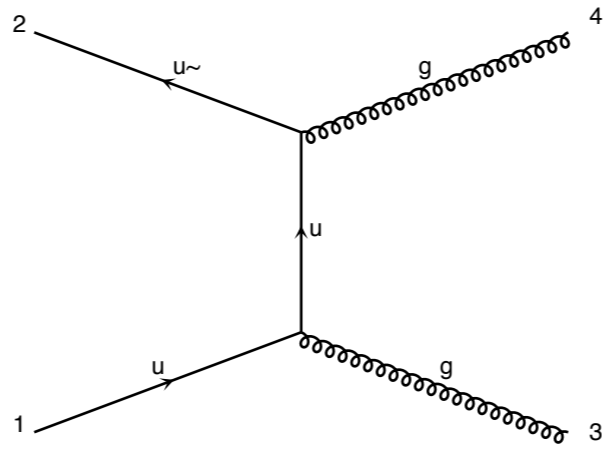
Multi-channel

- **Advantages**
 - The integral does not depend on the α_i but the variance does and can be minimized by a careful choice
- **Drawbacks**
 - Need to calculate all g_i values for each point
 - Each phase space channel must be invertible
 - N coupled equations for α_i so it might only work for small number of channels

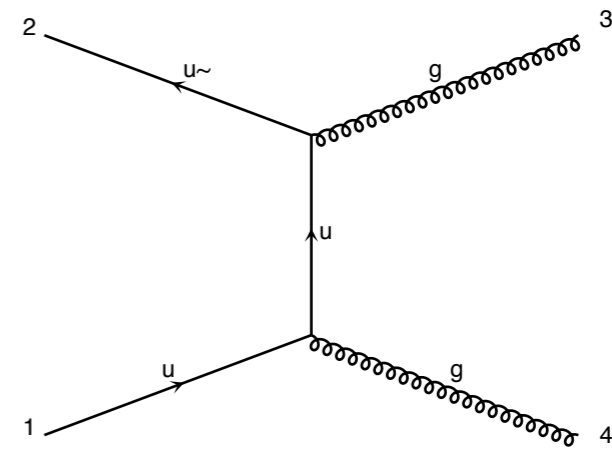
Example: QCD $2 \rightarrow 2$ production



$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2}$$



$$\propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2}$$



$$\propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

Multi-channel based on single diagrams

Consider the integration of an amplitude $|M|^2$ at tree level which many contributing diagrams. If there were a basis of functions,

$$f = \sum_{i=1}^n f_i \quad \text{with} \quad f_i \geq 0, \quad \forall i,$$

such that:

1. we know how to integrate each one of them,
2. they describe all possible peaks,

then the problem would be solved:

$$I = \int d\vec{\Phi} f(\vec{\Phi}) = \sum_{i=1}^n \int d\vec{\Phi} g_i(\vec{\Phi}) \frac{f_i(\vec{\Phi})}{g_i(\vec{\Phi})} = \sum_{i=1}^n I_i,$$

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Does such a basis exist?

Multi-channel based on single diagrams*

YES! $f_i = \frac{|A_i|^2}{\sum_i |A_i|^2} |A_{\text{tot}}|^2$

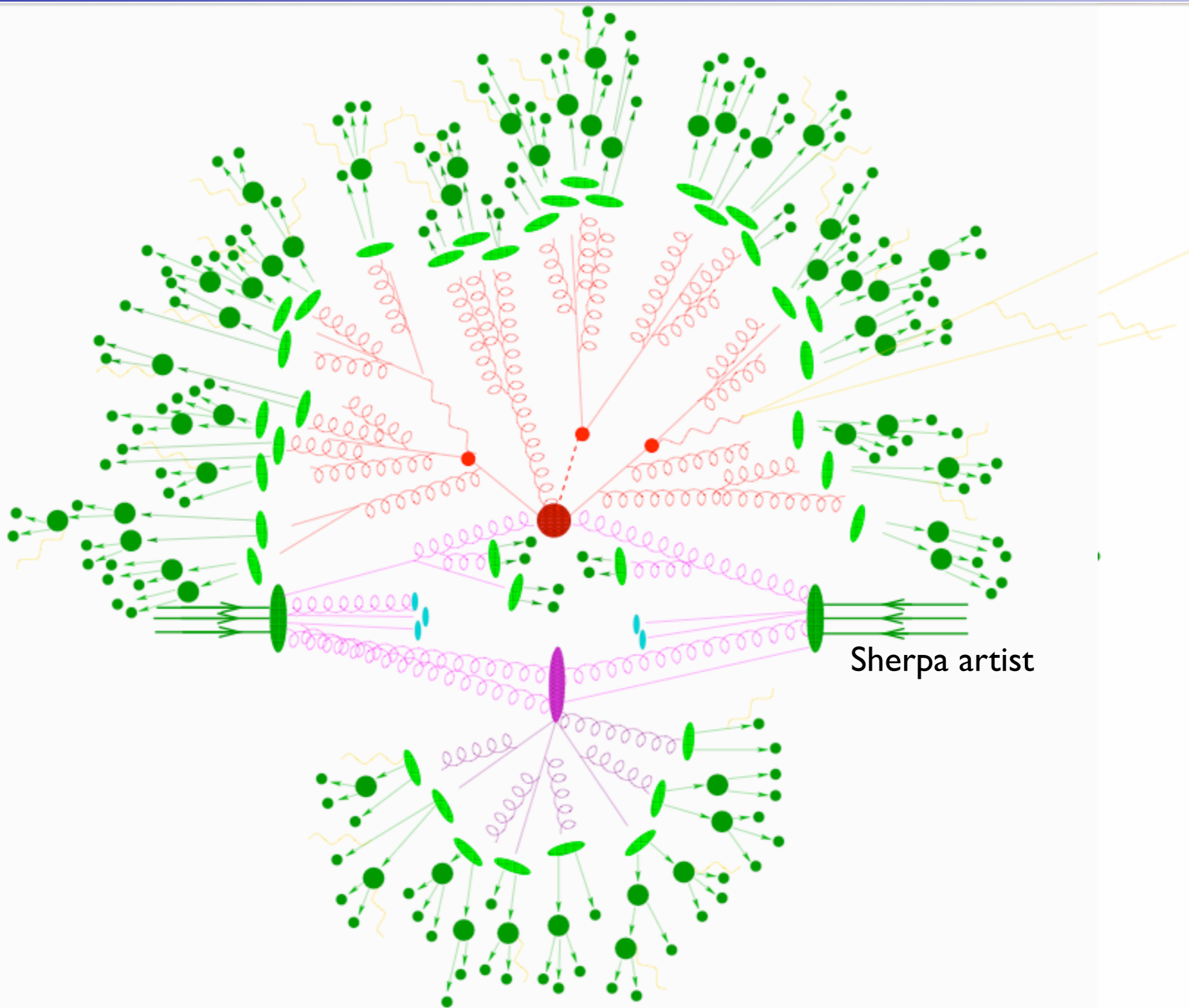
Multi-channel based on single diagrams*

YES!
$$f_i = \frac{|A_i|^2}{\sum_i |A_i|^2} |A_{\text{tot}}|^2$$

- Key Idea
 - Any single diagram is “easy” to integrate (pole structures known based on propagators)
 - Divide integration into pieces, based on diagrams
- Get N independent integrals
 - Errors add in quadrature so no extra cost
 - No need to calculate “weight” function from other channels.
 - Can optimize # of points for each one independently
 - Parallel in nature
- What about interference?
 - Never creates “new” peaks, so we’re OK!

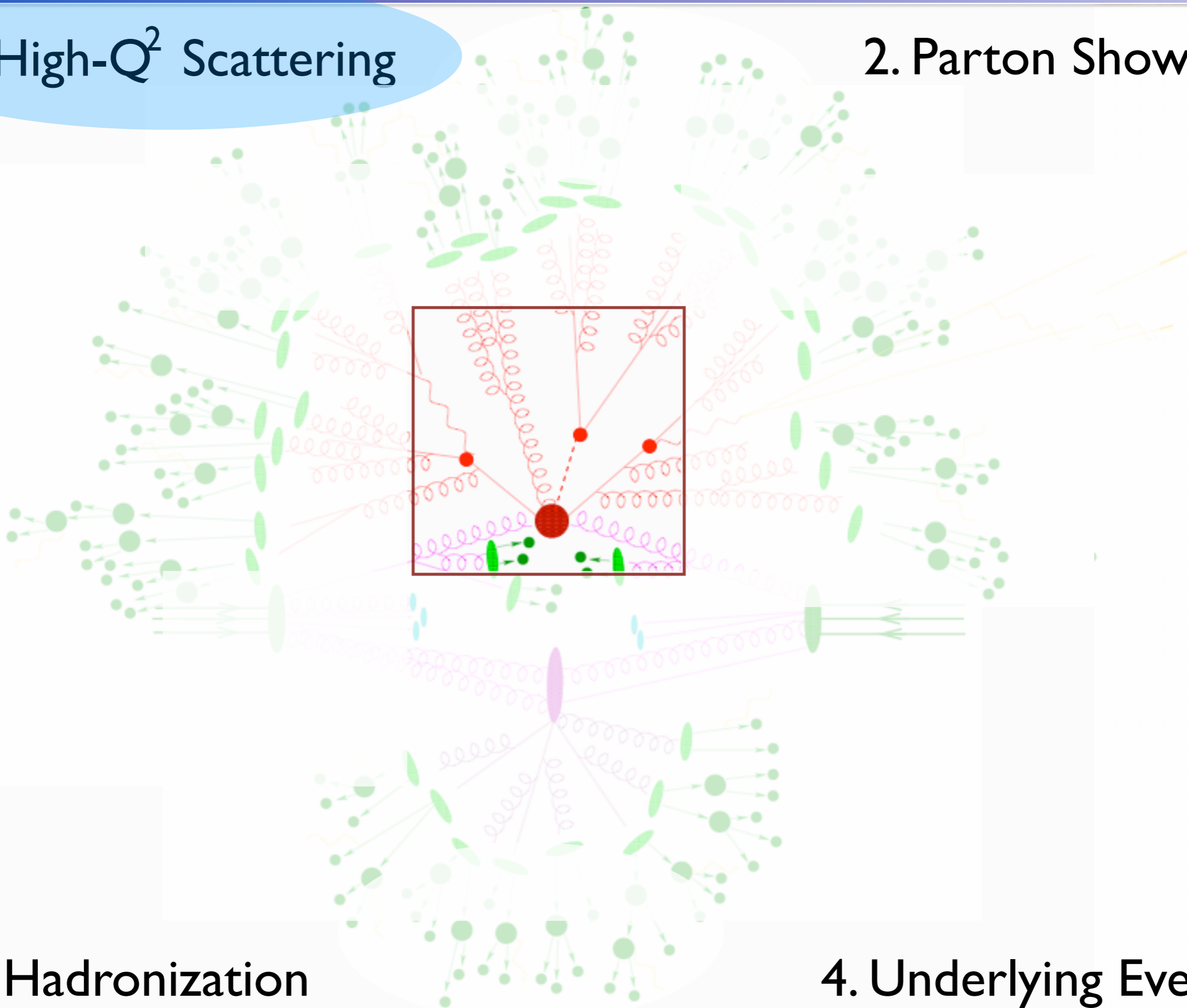
*Method used in MadGraph

Complete simulation of collider events



I. High- Q^2 Scattering

2. Parton Shower



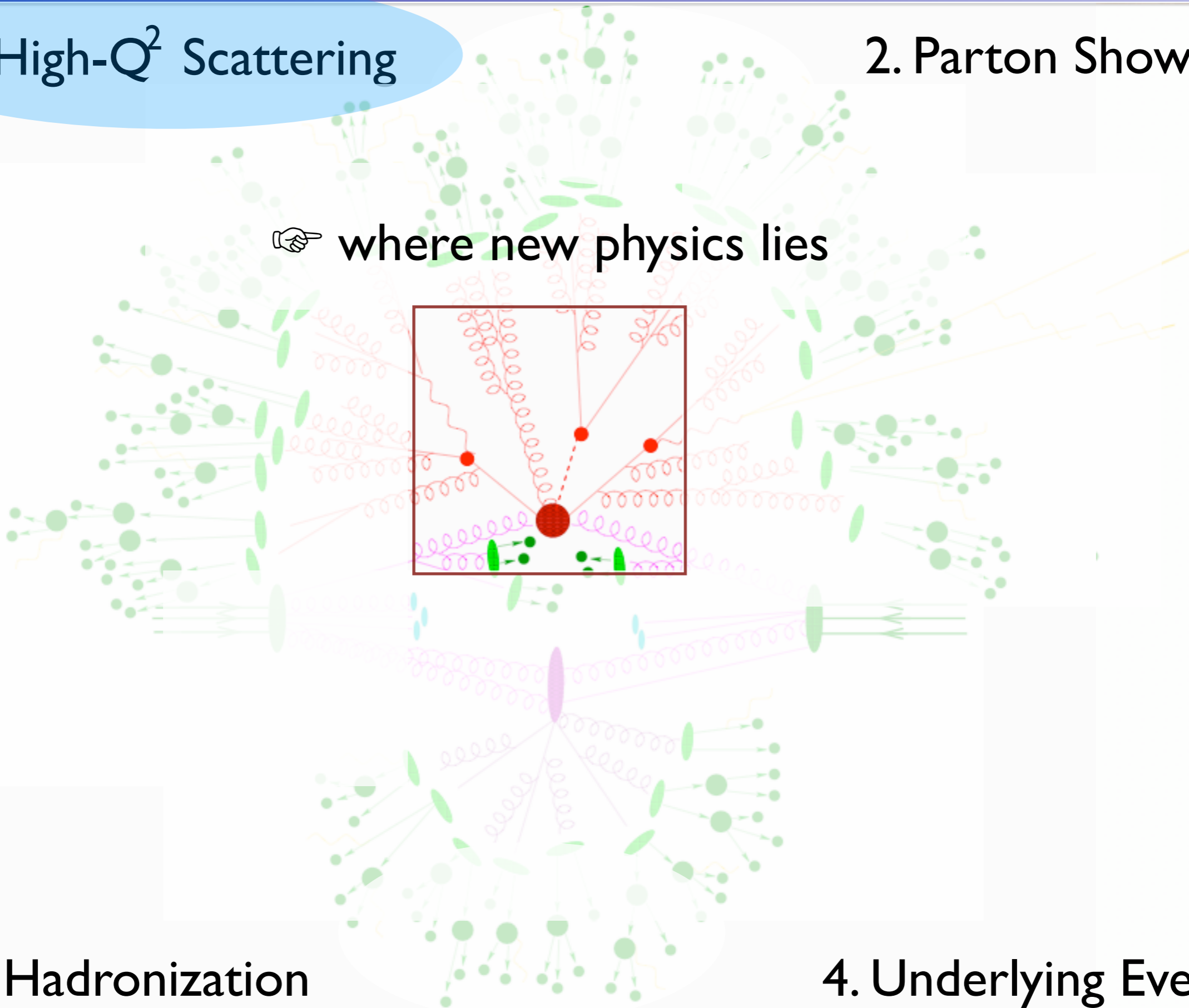
3. Hadronization

4. Underlying Event

I. High- Q^2 Scattering

2. Parton Shower

👉 where new physics lies

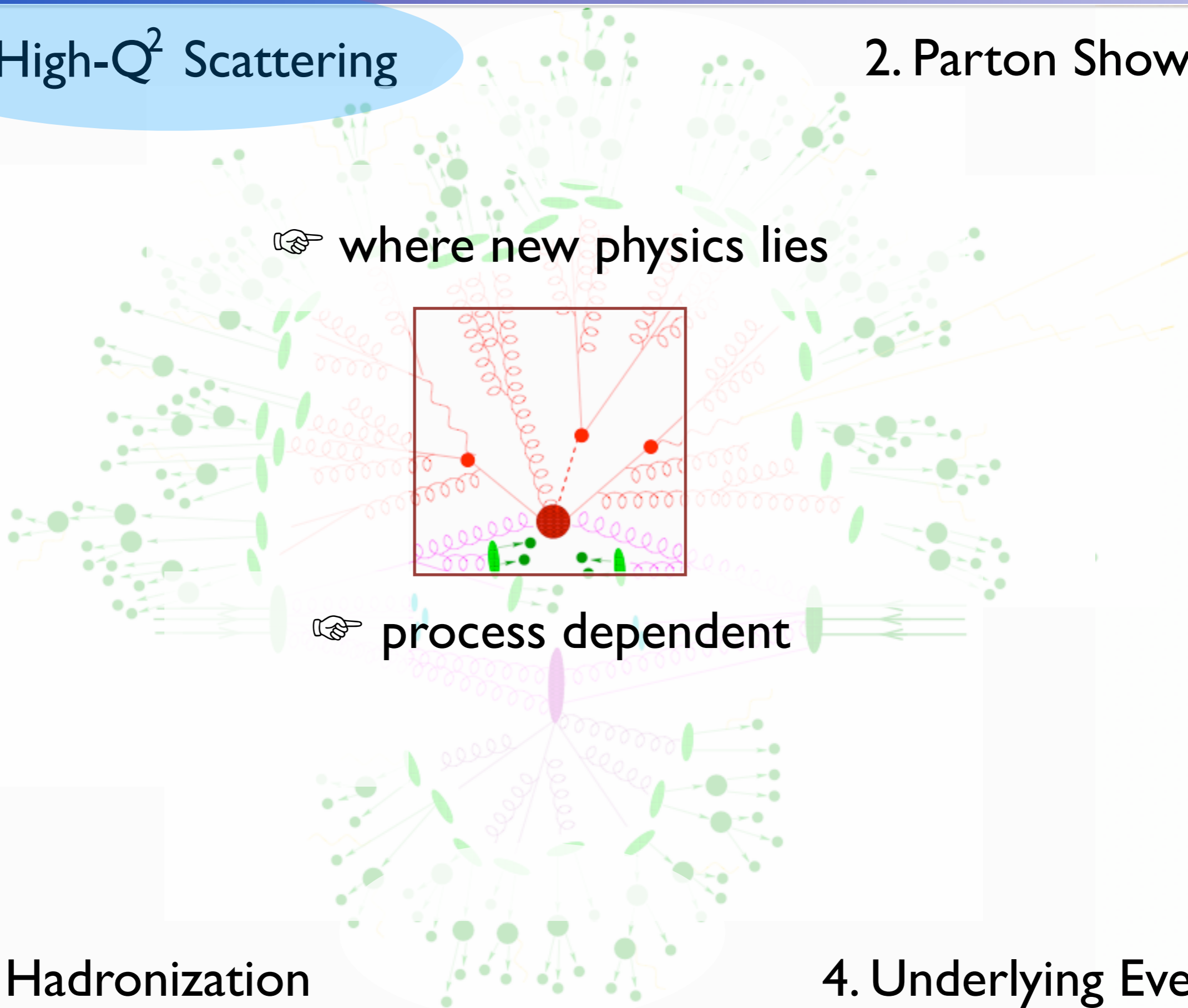


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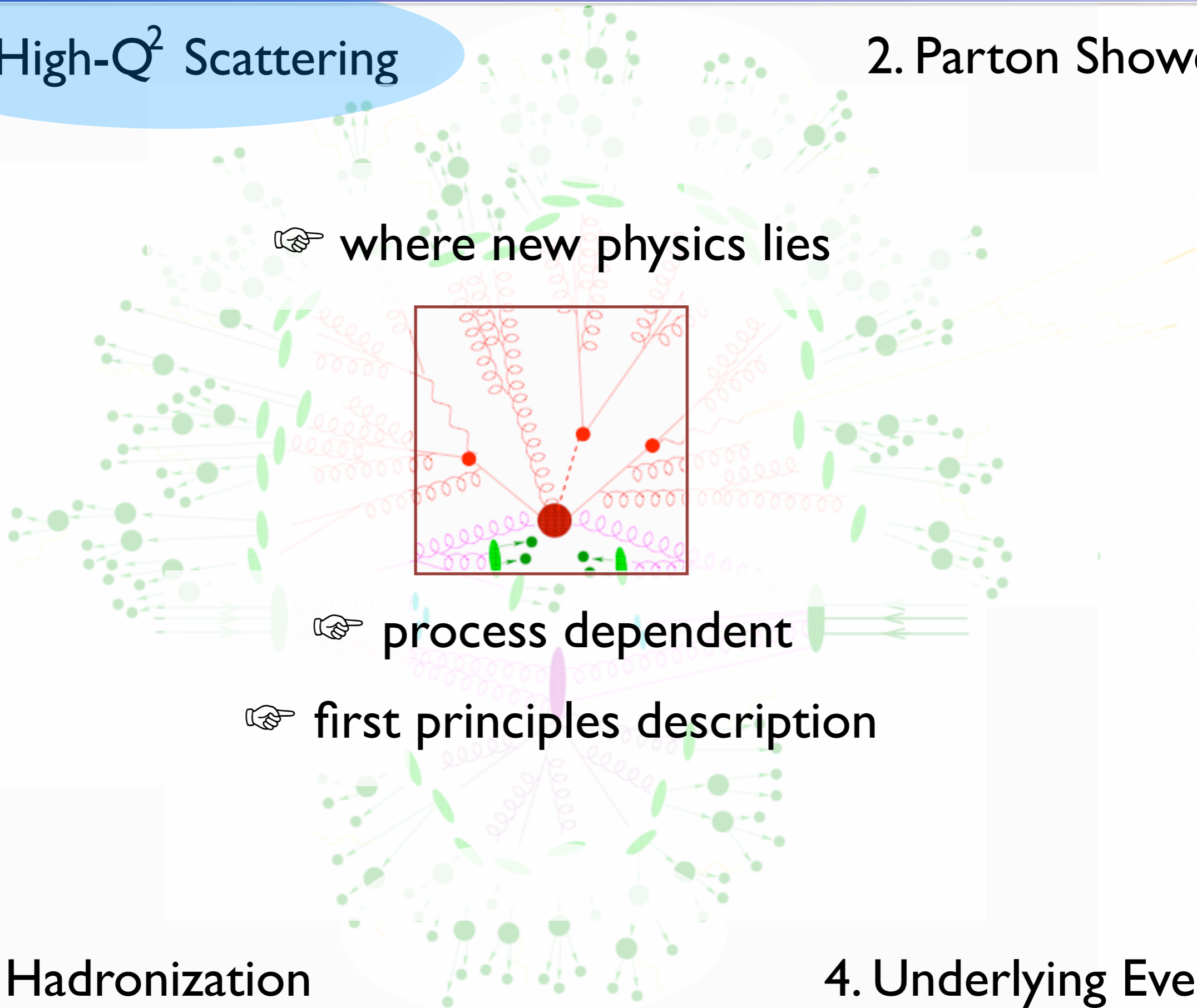


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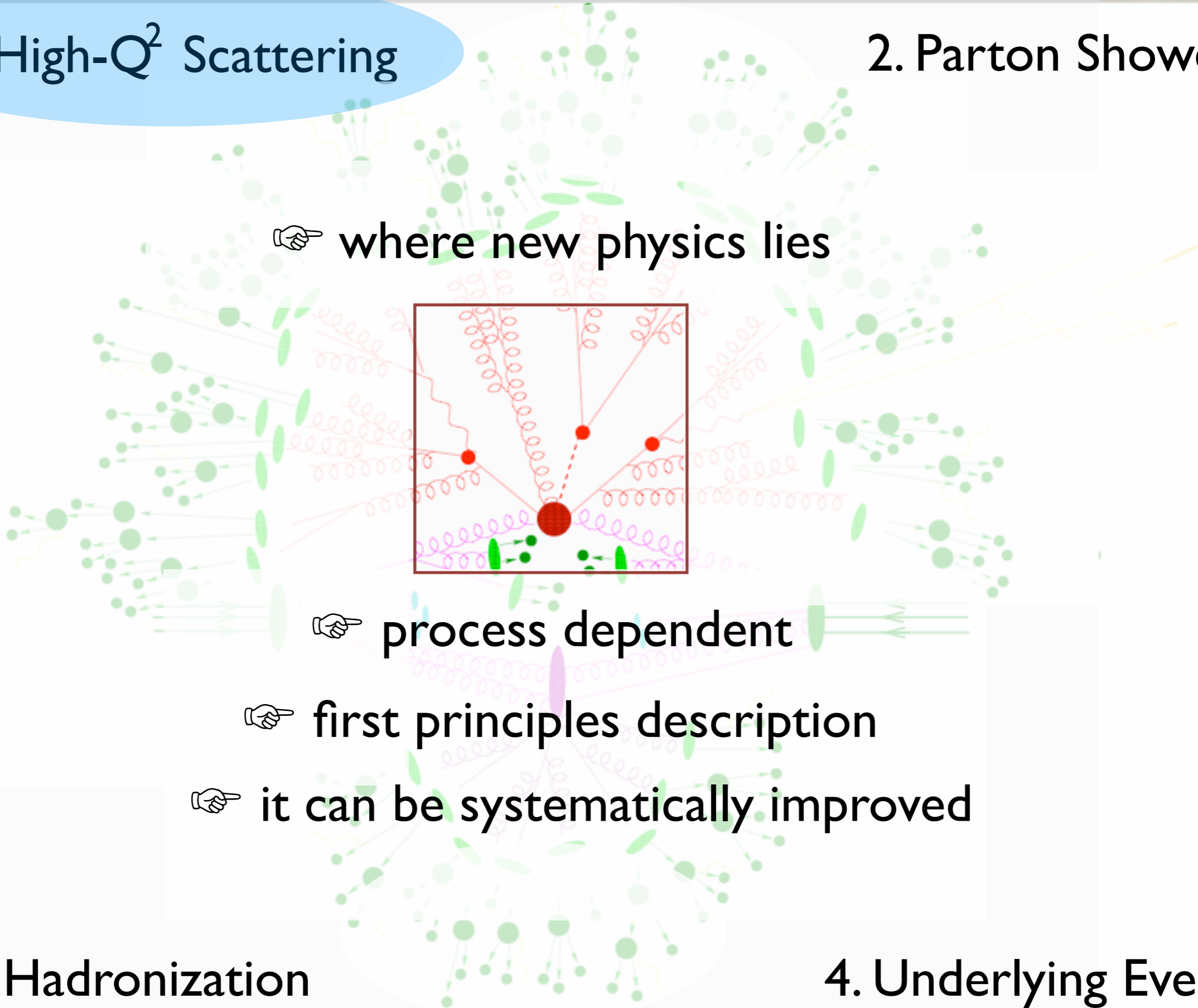


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where new physics lies

process dependent

first principles description

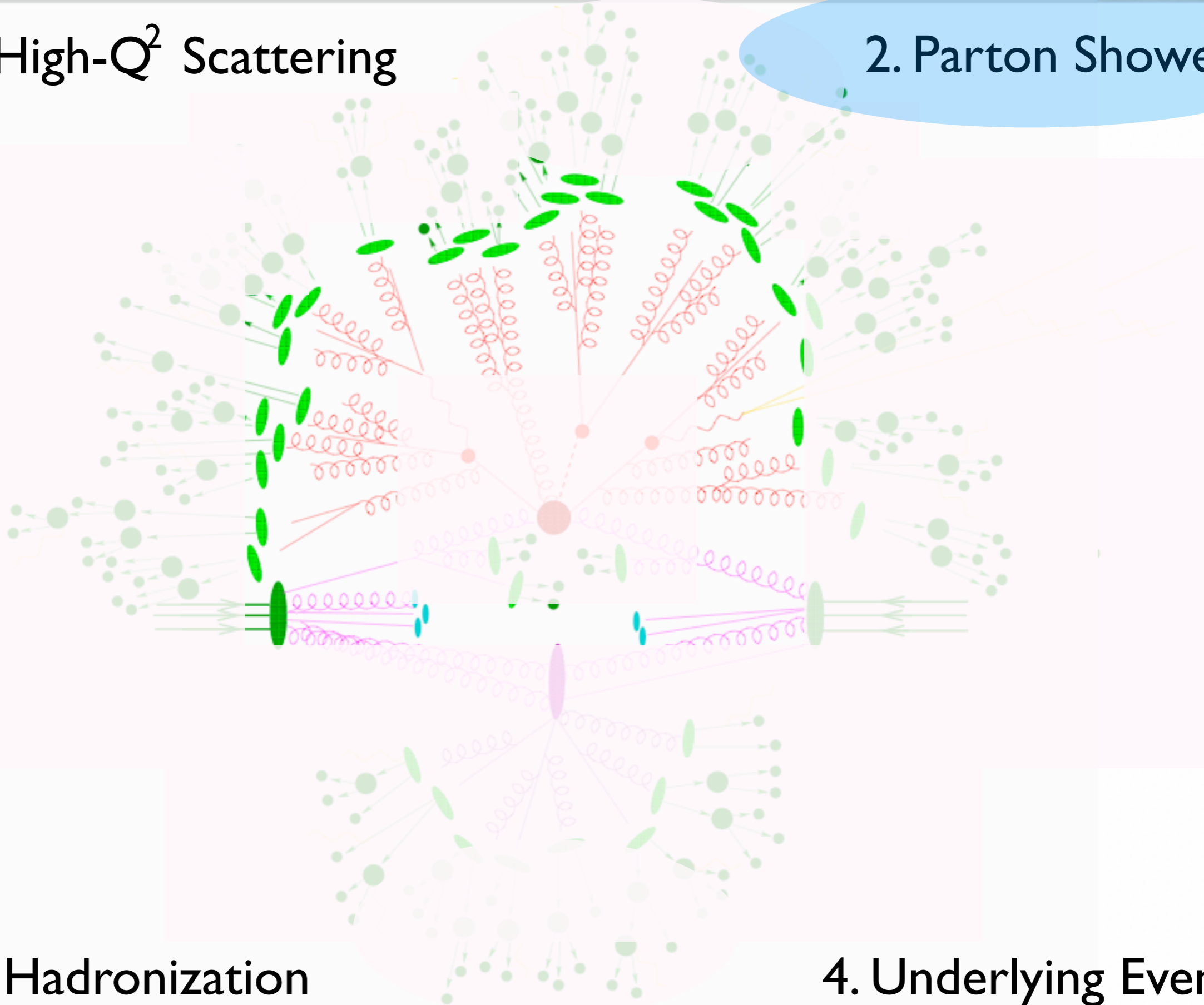
it can be systematically improved

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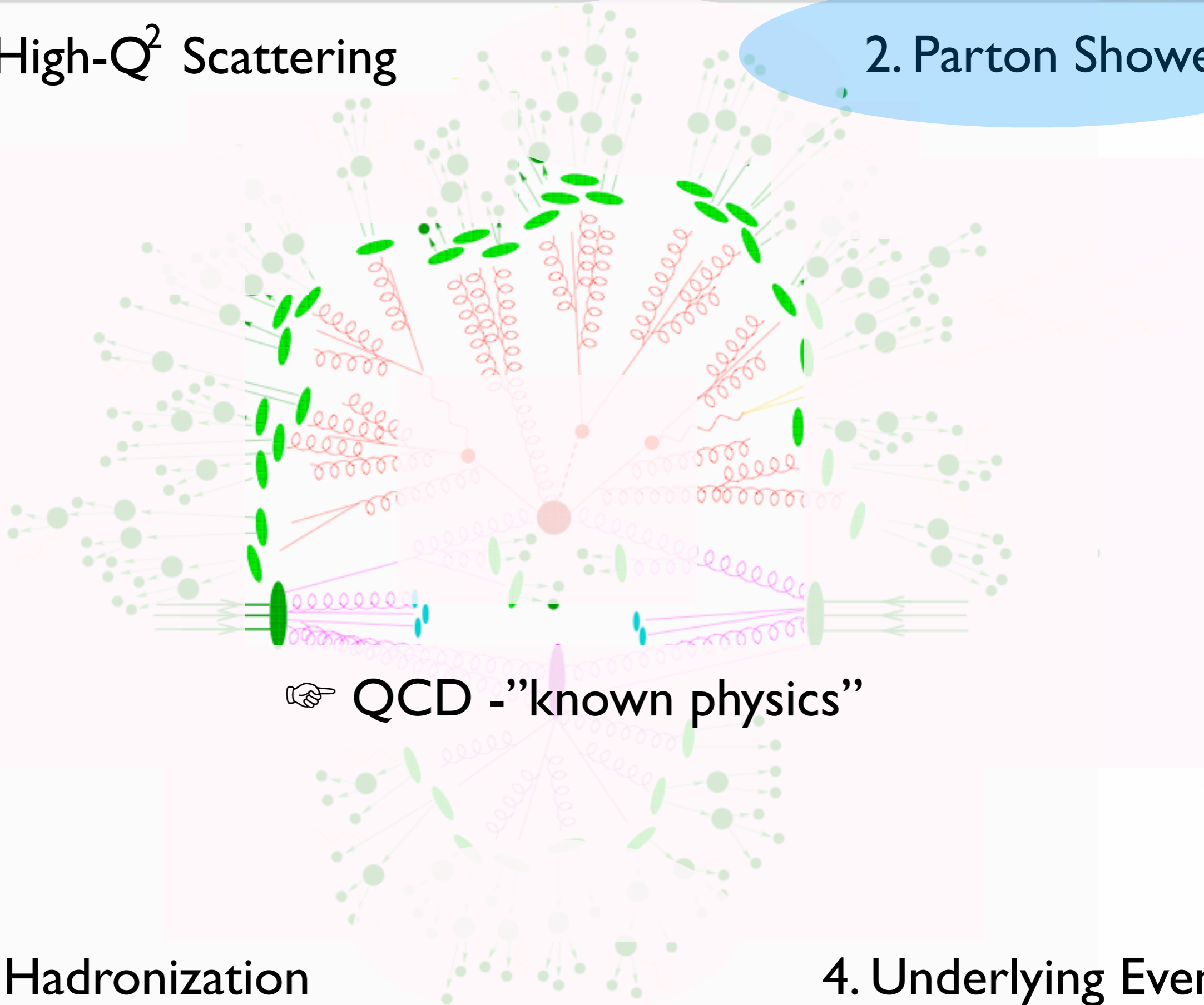


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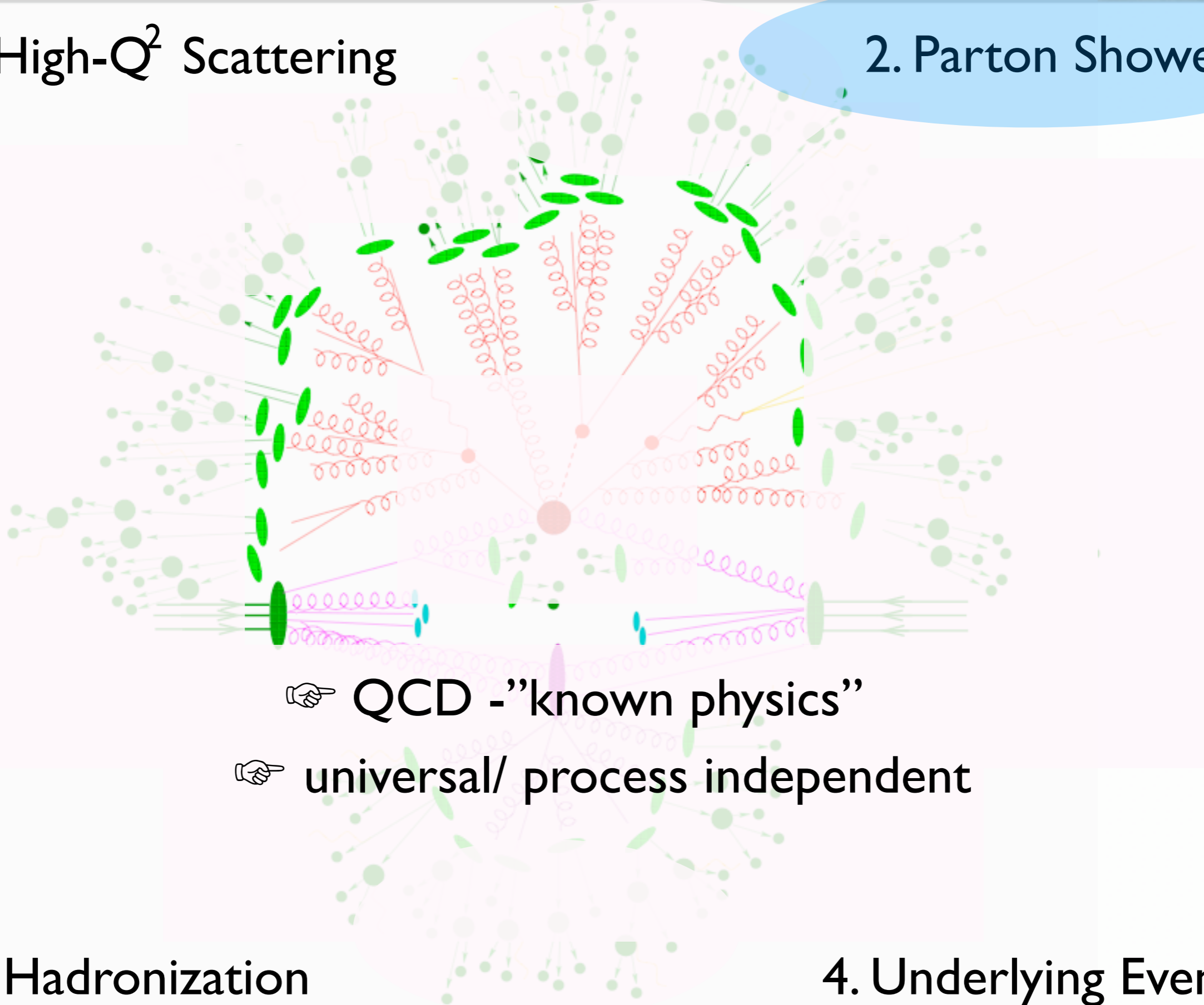
☞ QCD - "known physics"

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4. Underlying Event

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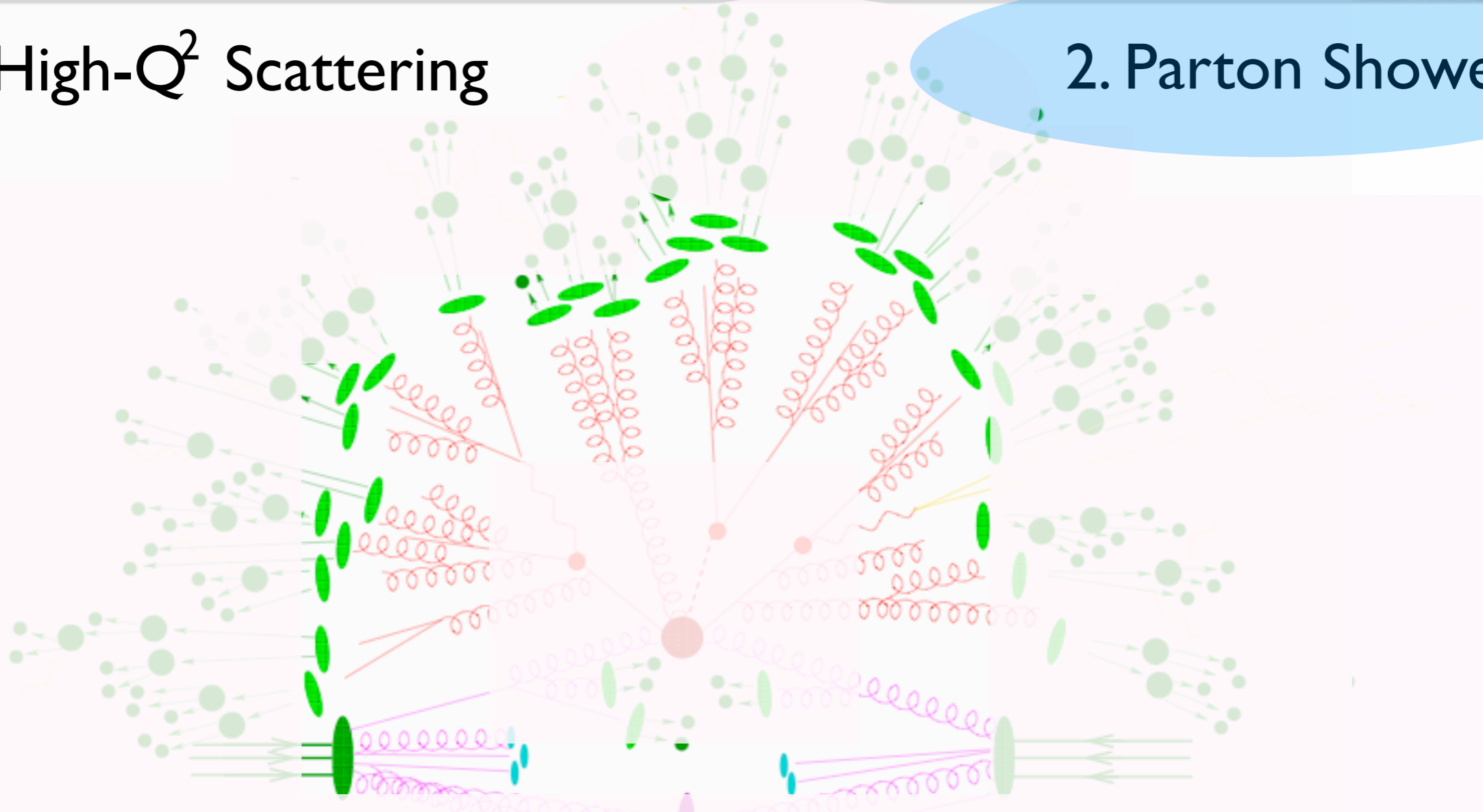
☞ universal/ process independent

3. Hadronization

4. Underlying Event

I. High- Q^2 Scattering

2. Parton Shower



☞ QCD - "known physics"

☞ universal/ process independent

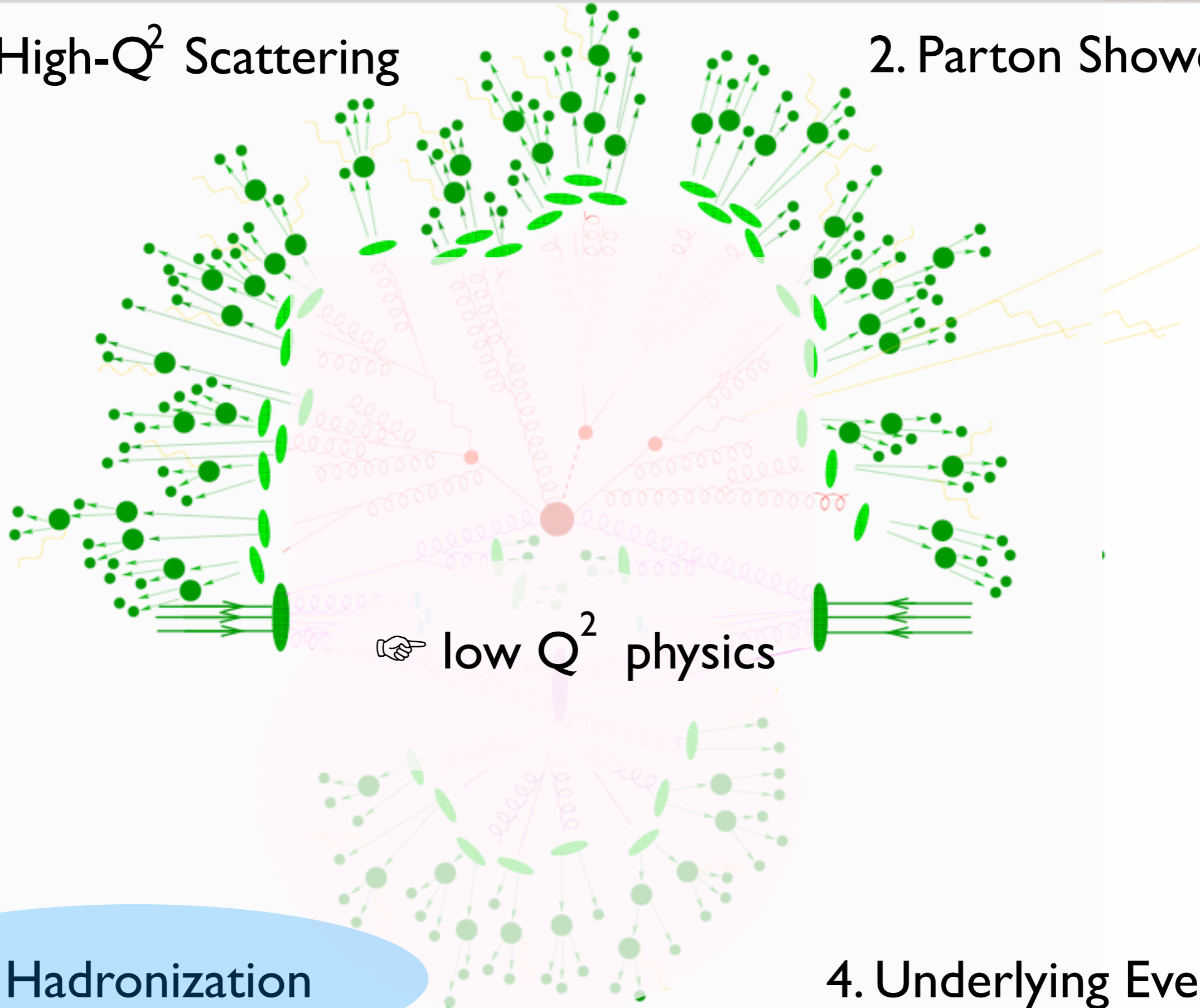
☞ first principles description

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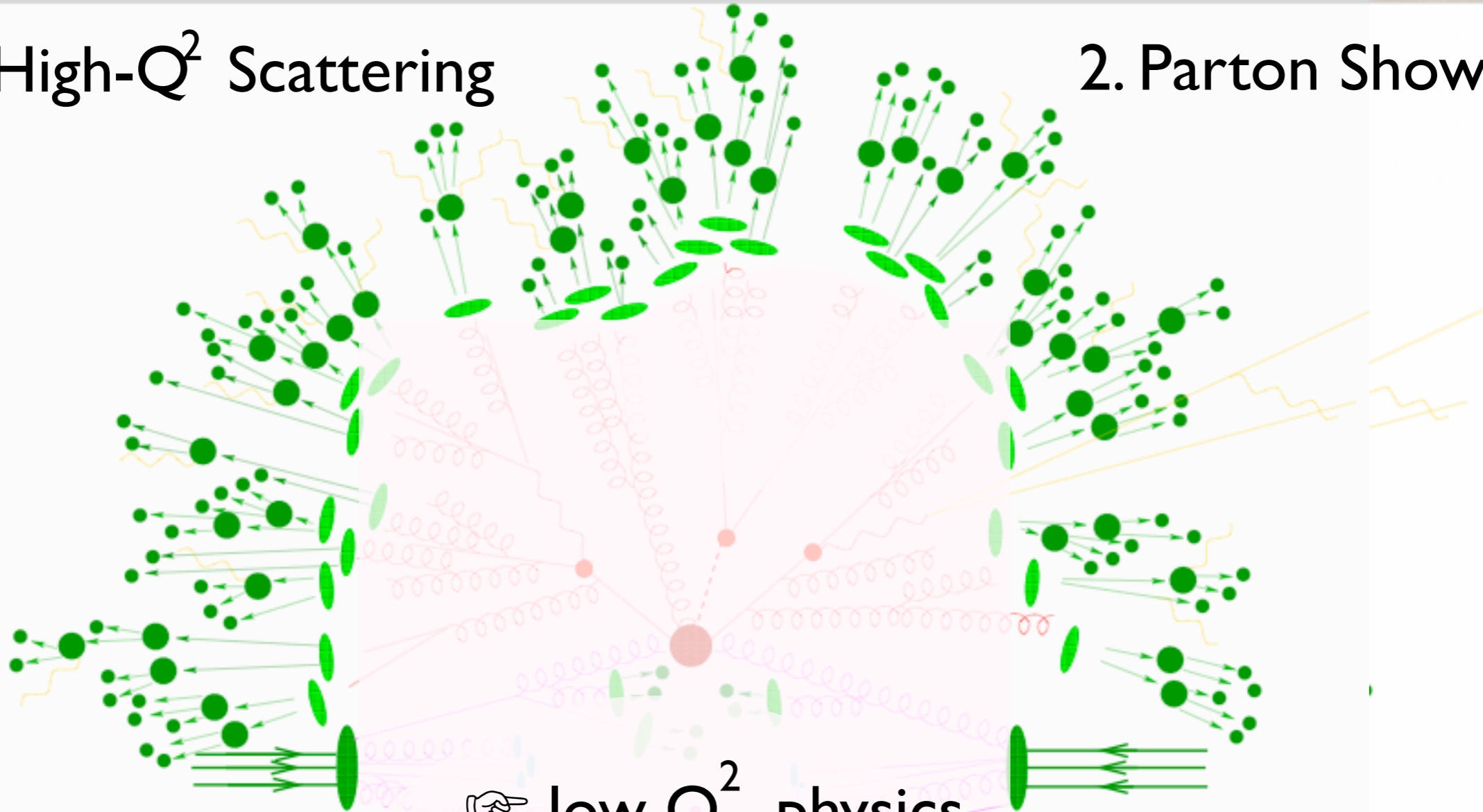
low Q^2 physics

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low Q^2 physics

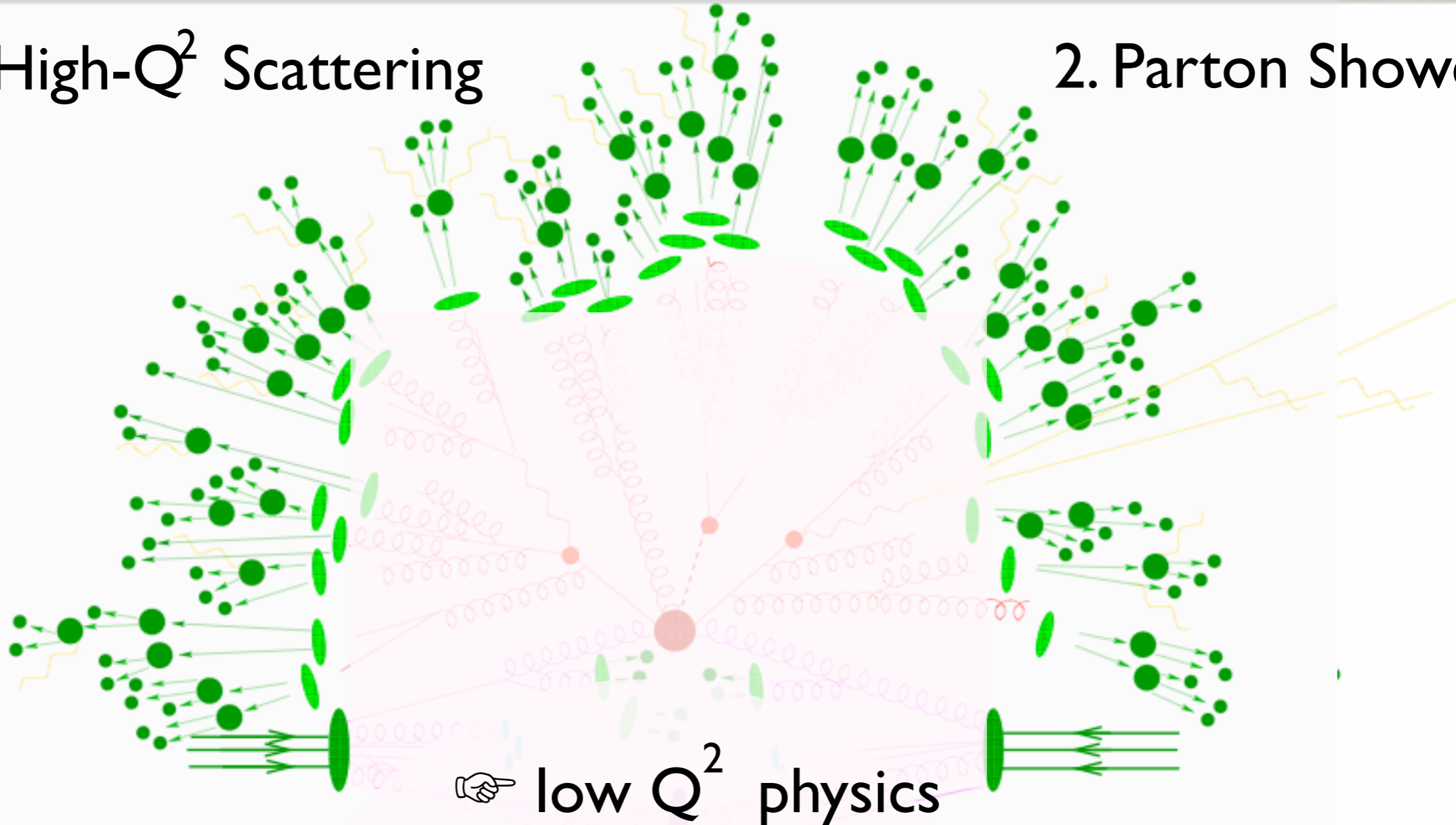
universal/ process independent

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👉 low Q^2 physics

👉 universal/ process independent

👉 model-based description

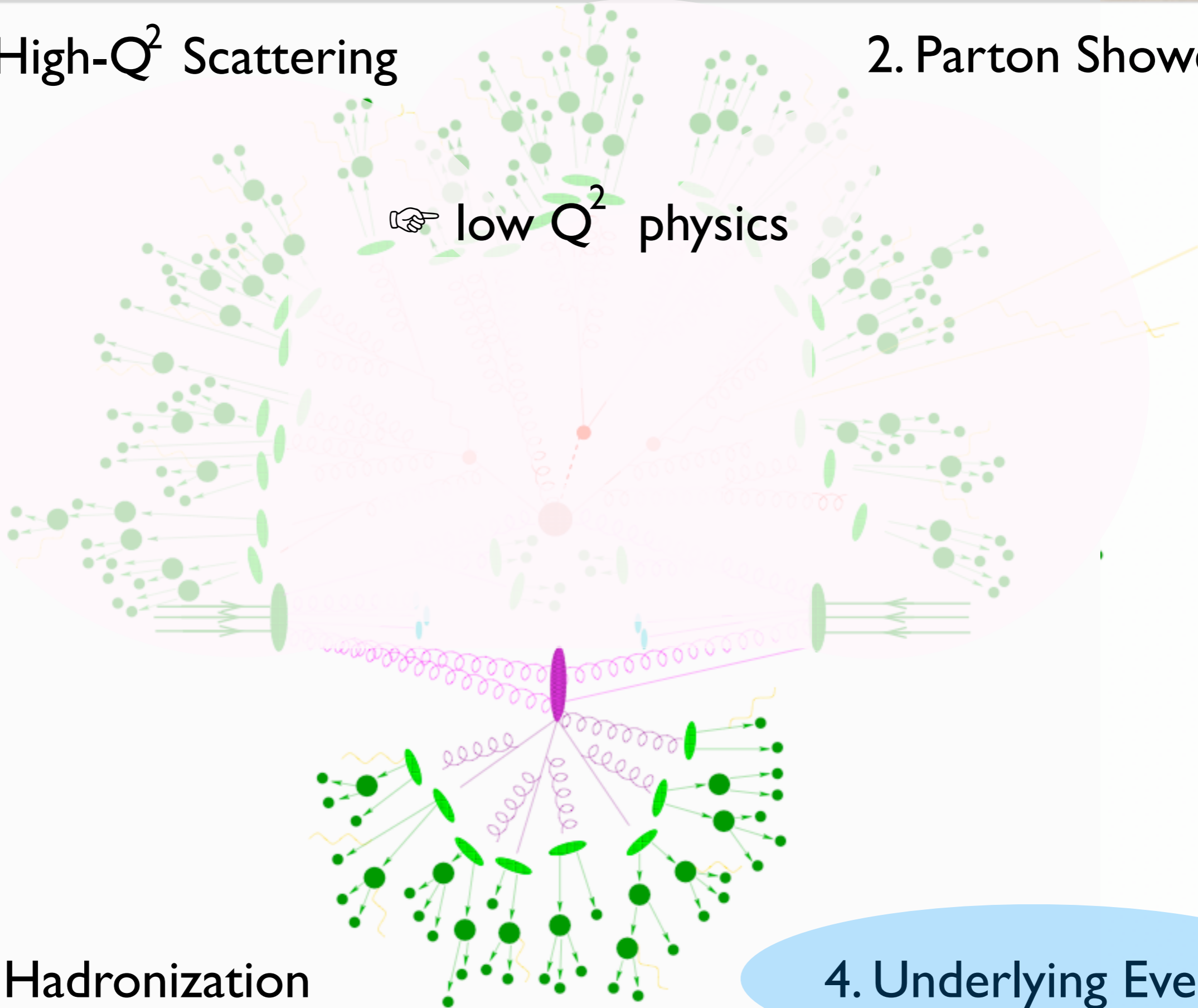
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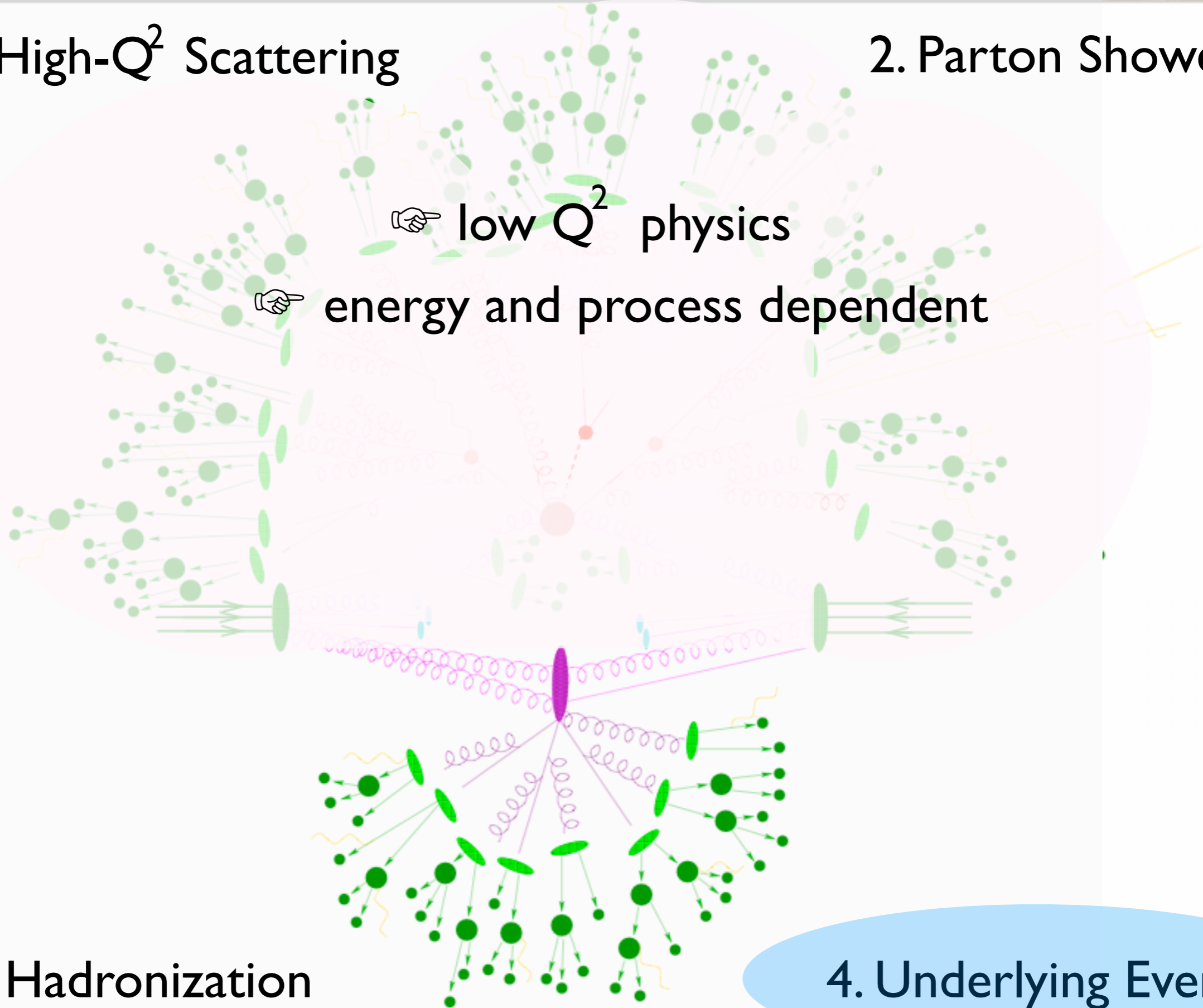


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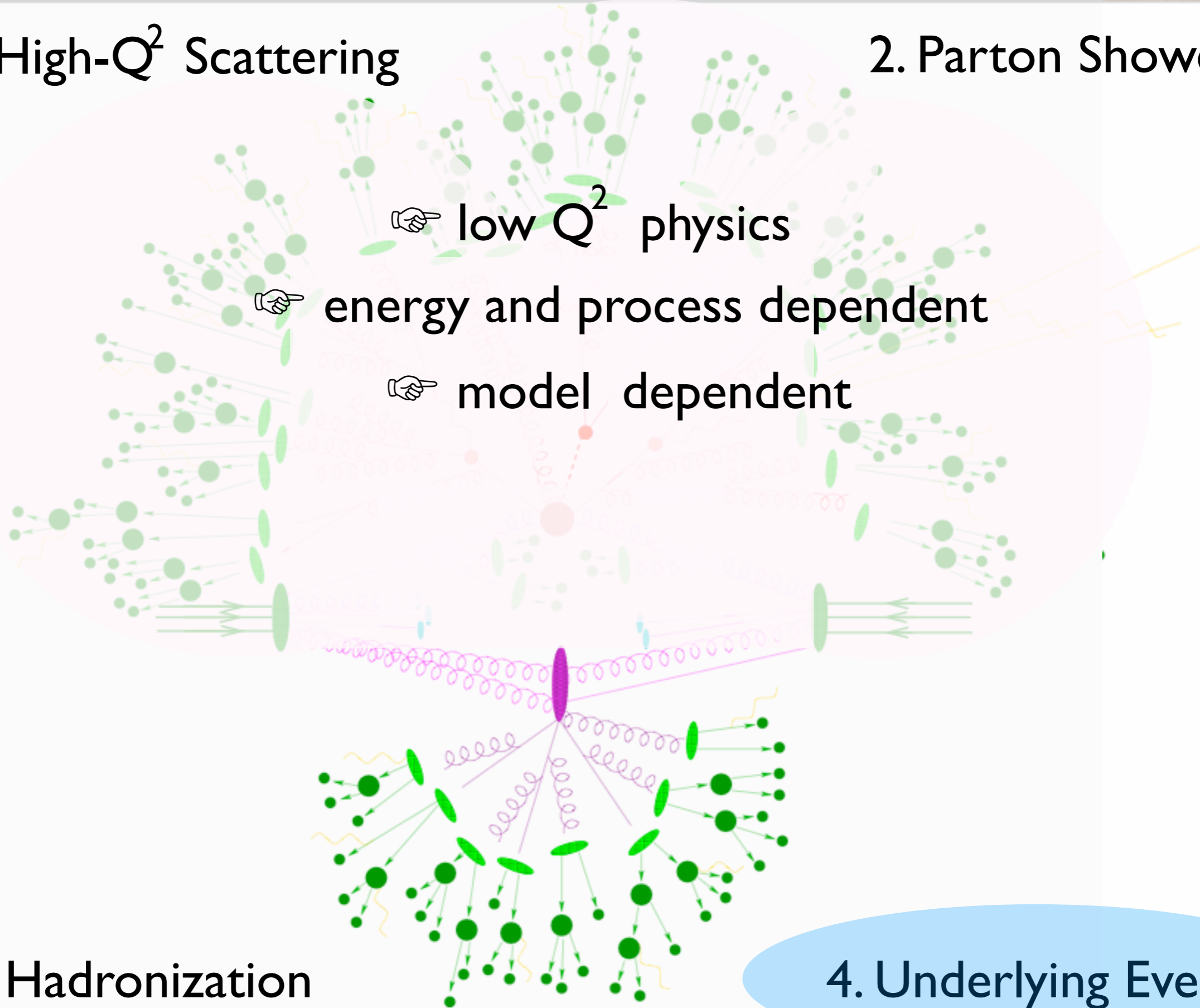


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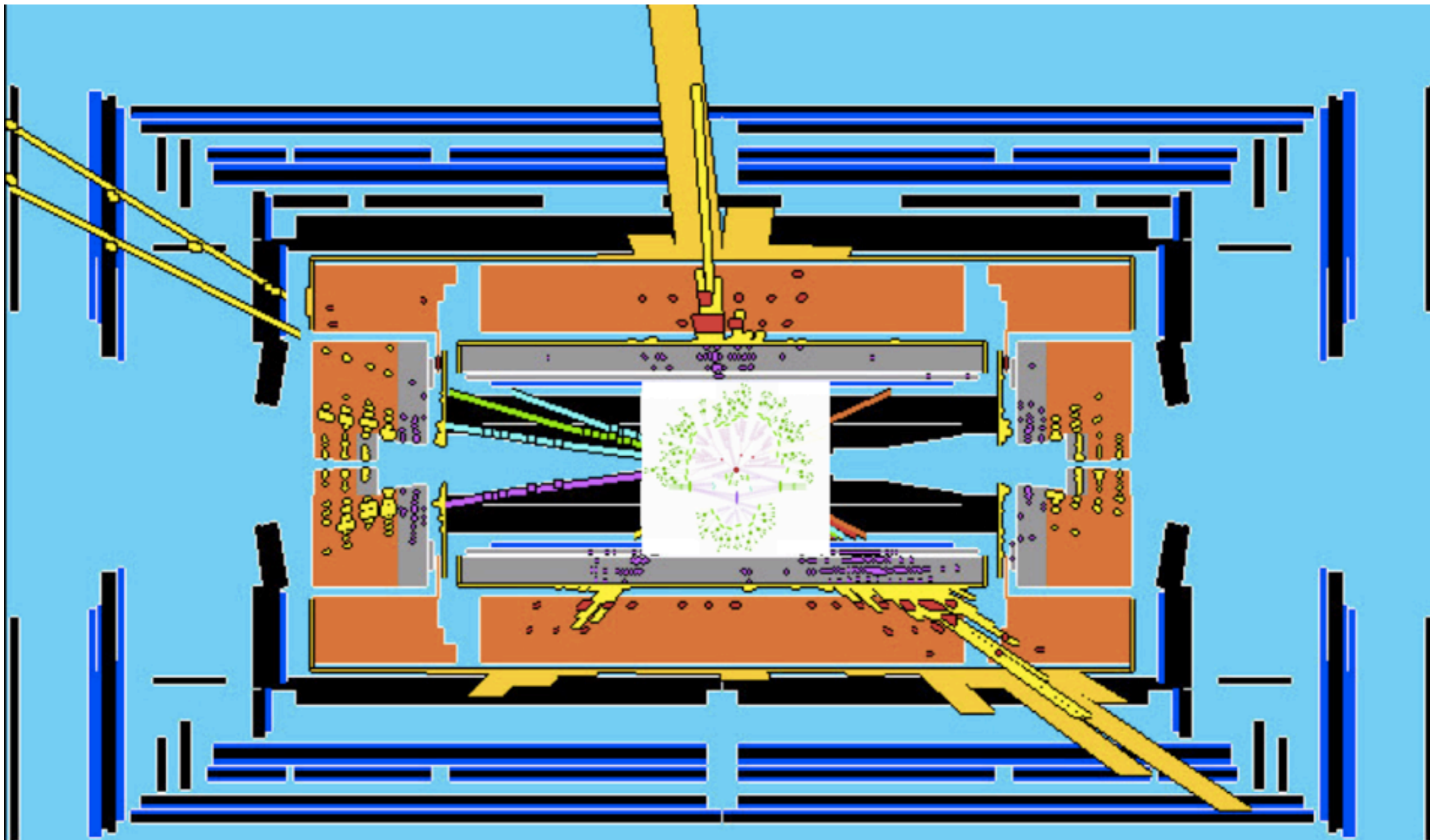
low Q^2 physics

energy and process dependent

model dependent

3. Hadronization

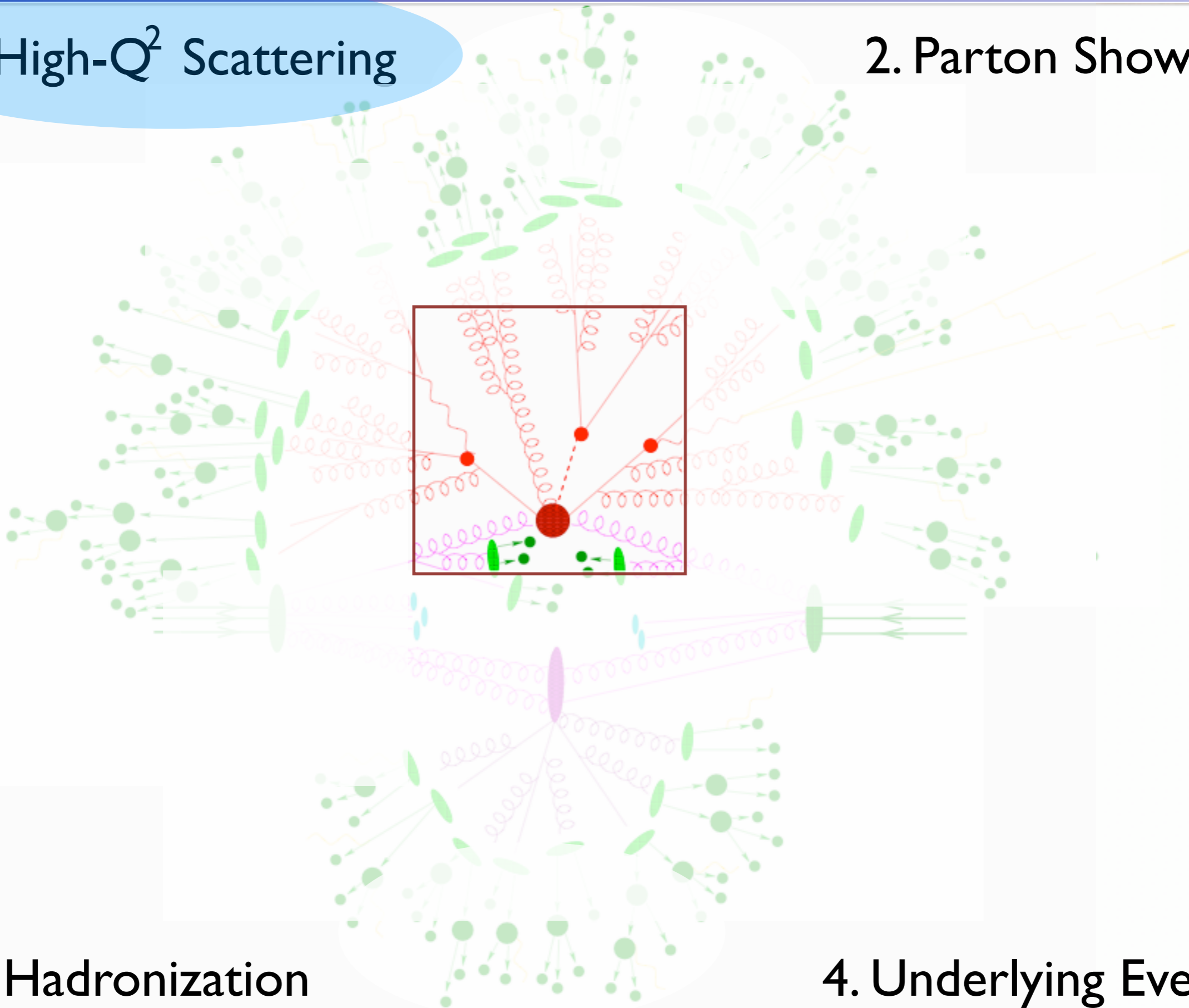
4. Underlying Event



5. Detector simulation

I. High- Q^2 Scattering

2. Parton Shower



3. Hadronization

4. Underlying Event

Automatized Matrix Element Generators

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- High- Q^2 scattering processes: In principle infinite number of processes for innumerable models

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- Implementation by hand time-consuming, labor intensive and error prone (bad use of PhD student time!)

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- High- Q^2 scattering processes: In principle infinite number of processes for innumerable models
- Implementation by hand time-consuming, labor intensive and error prone (bad use of PhD student time!)
- Instead: Automatized matrix element generators
 - ➔ Use Feynman rules to build diagrams

Automatized Matrix Element Generators

- Automatic matrix element generators:
 - ➔ CalcHep / CompHep
 - ➔ MadGraph
 - ➔ AMEGIC++ (Sherpa)
 - ➔ Whizard
- Standard Model only, with faster matrix elements:
 - ➔ AlpGen
 - ➔ HELAC
 - ➔ COMIX (Sherpa)

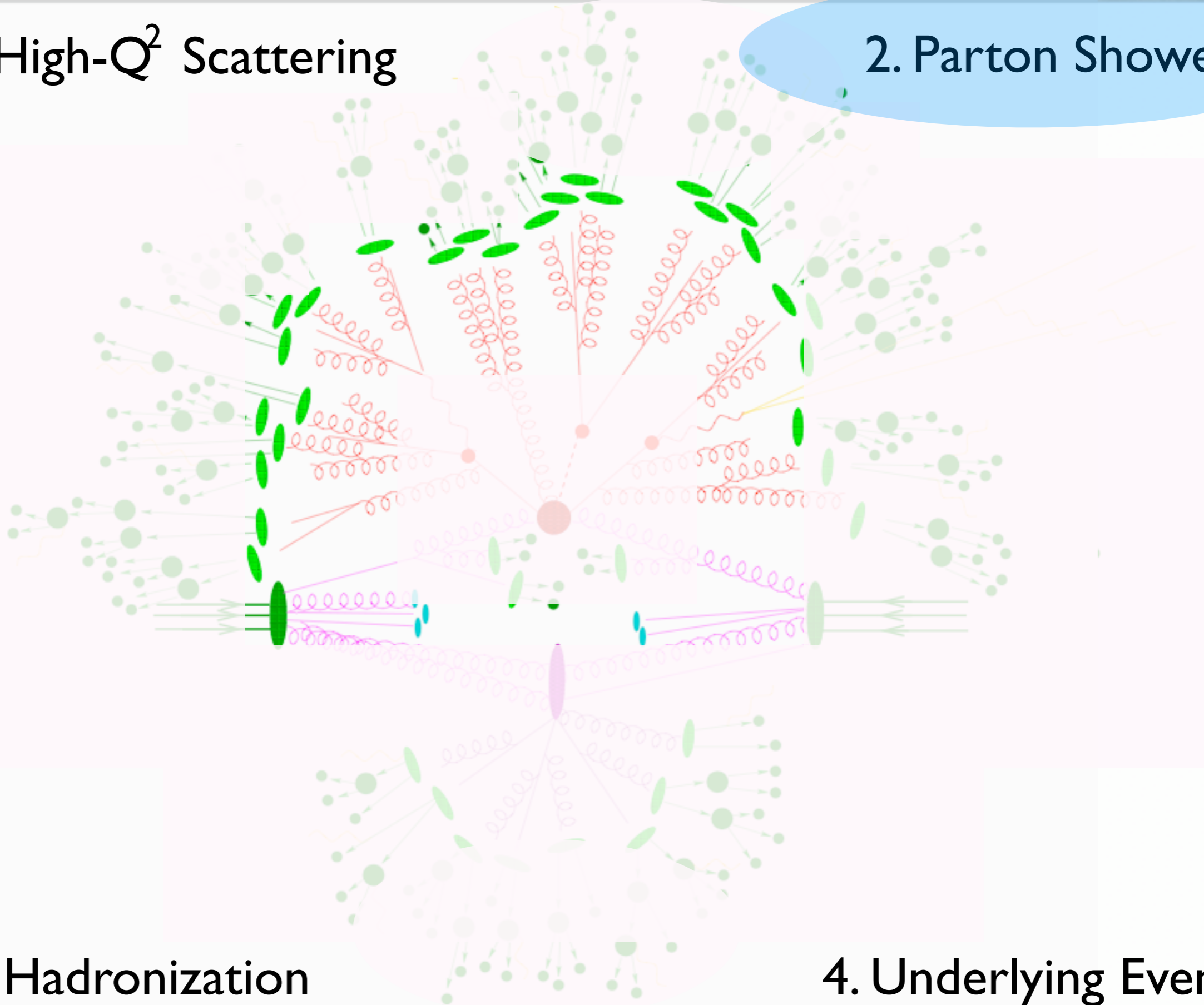
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See more later!

I. High- Q^2 Scattering

2. Parton Shower



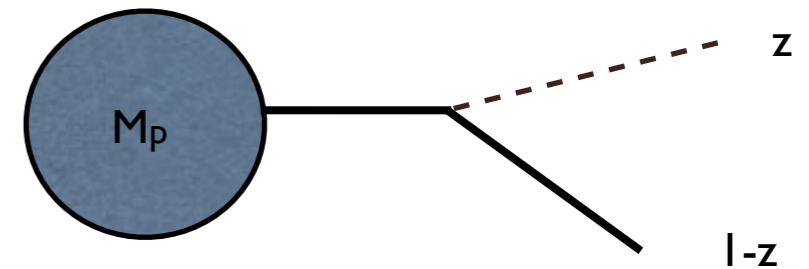
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Parton Shower MC event generators

Matrix elements involving $q \rightarrow q g$ (or $g \rightarrow gg$) are strongly enhanced when the final state particles are close in the phase space:

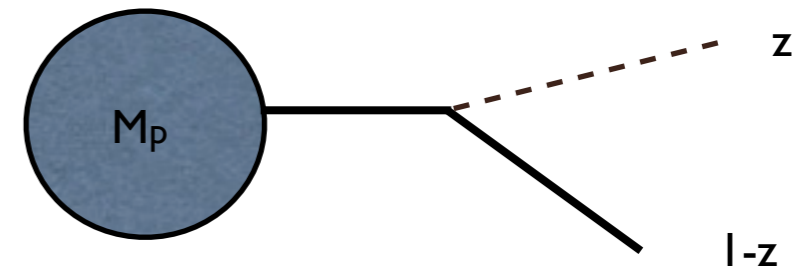
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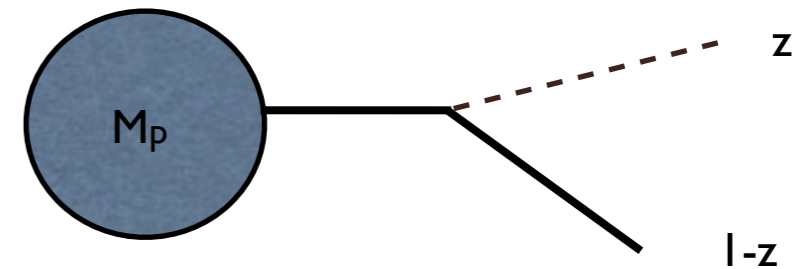


Soft

Parton Shower MC event generators

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Soft and collinear divergences!

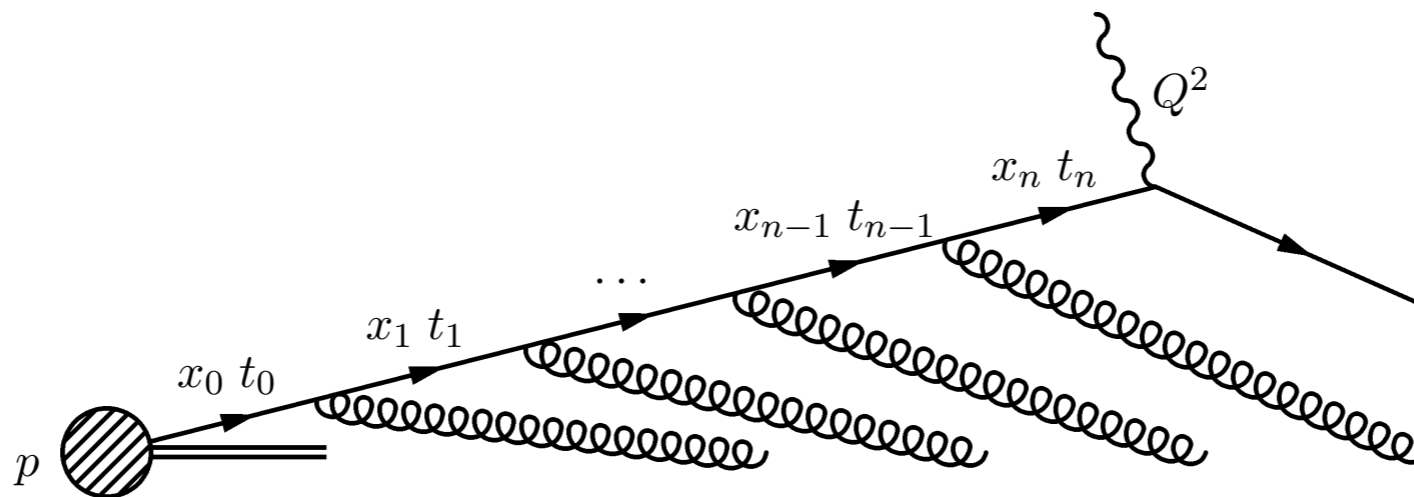
Parton Shower MC event generators

Collinear factorization:

$$|M_{p+1}|^2 d\Phi_{p+1} \simeq |M_p|^2 d\Phi_p \frac{dt}{t} \frac{\alpha_S}{2\pi} P(z) dz d\phi$$

Allows for a step-by-step (Markov process) evolution:

“Parton shower”



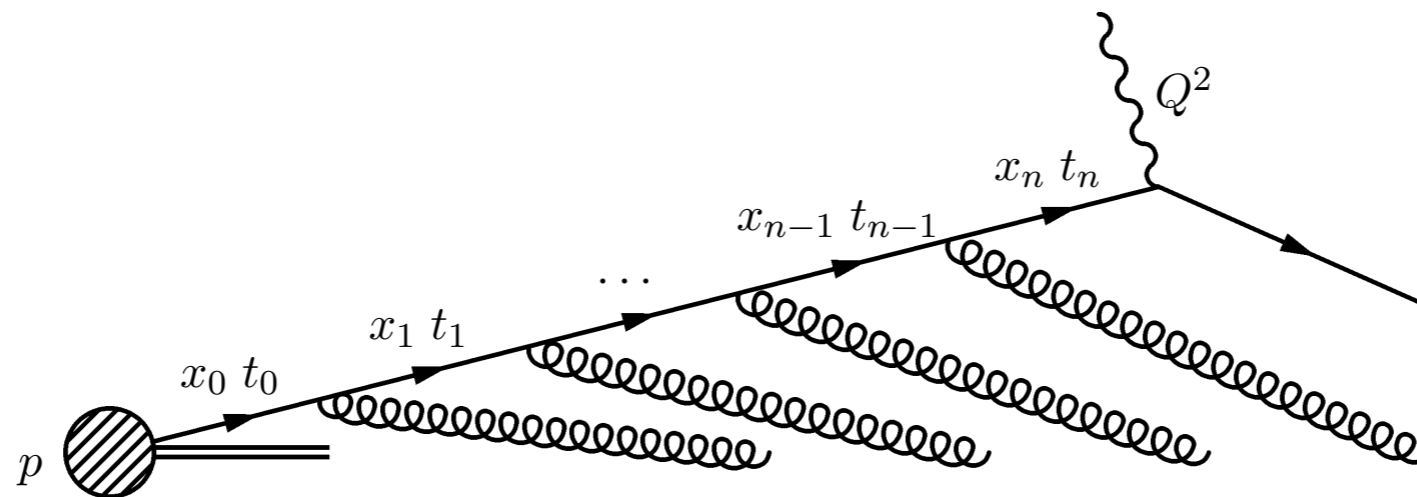
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See lectures on PS by me and Grigory for (much) more details!

Parton Shower MC event generators

- General-purpose tools
- Complete exclusive description of the events: hard scattering, showering, hadronization, underlying event
- Reliable and well tuned to experimental data.

most famous: PYTHIA, HERWIG, SHERPA

- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD [Nagy, Soper, 2005; Giele, Kosower, Skands, 2007; Krauss, Schumman, 2007]

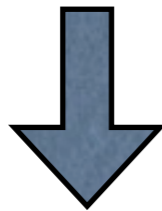
Detector simulation

- Detector simulation
 - ➔ Fast general-purpose detector simulators: Delphes, PGS (“Pretty good simulations”), AcerDet
 - ➔ Specify parameters to simulate different experiments
- Experiment-specific fast simulation
 - ➔ Detector response parameterized
 - ➔ Run time ms-s/event
- Experiment-specific full simulation
 - ➔ Full tracking of particles through detector using GEANT
 - ➔ Run time several minutes/event

Matrix Elements vs. Parton Showers (teaser)

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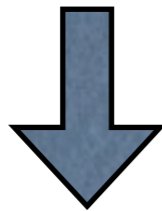
ME



1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

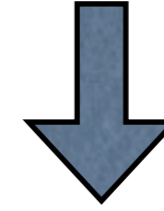
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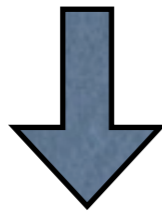
Shower MC



1. Resums large logs to all orders
2. Computationally cheap
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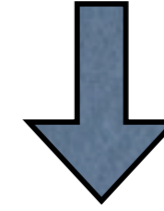
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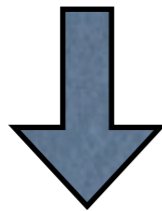


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Approaches are complementary: merge them!

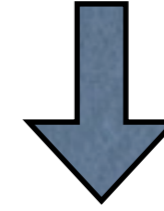
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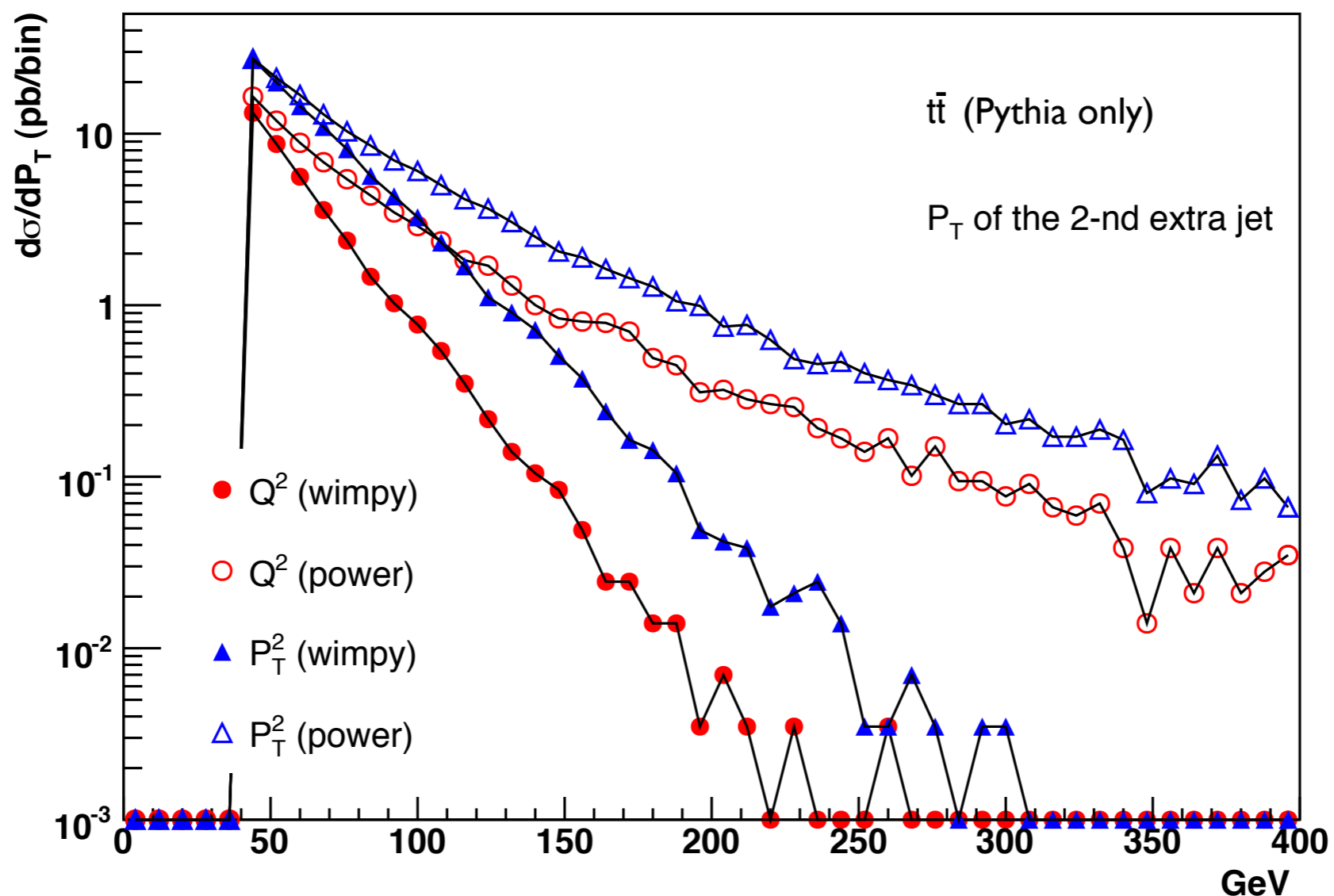
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Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

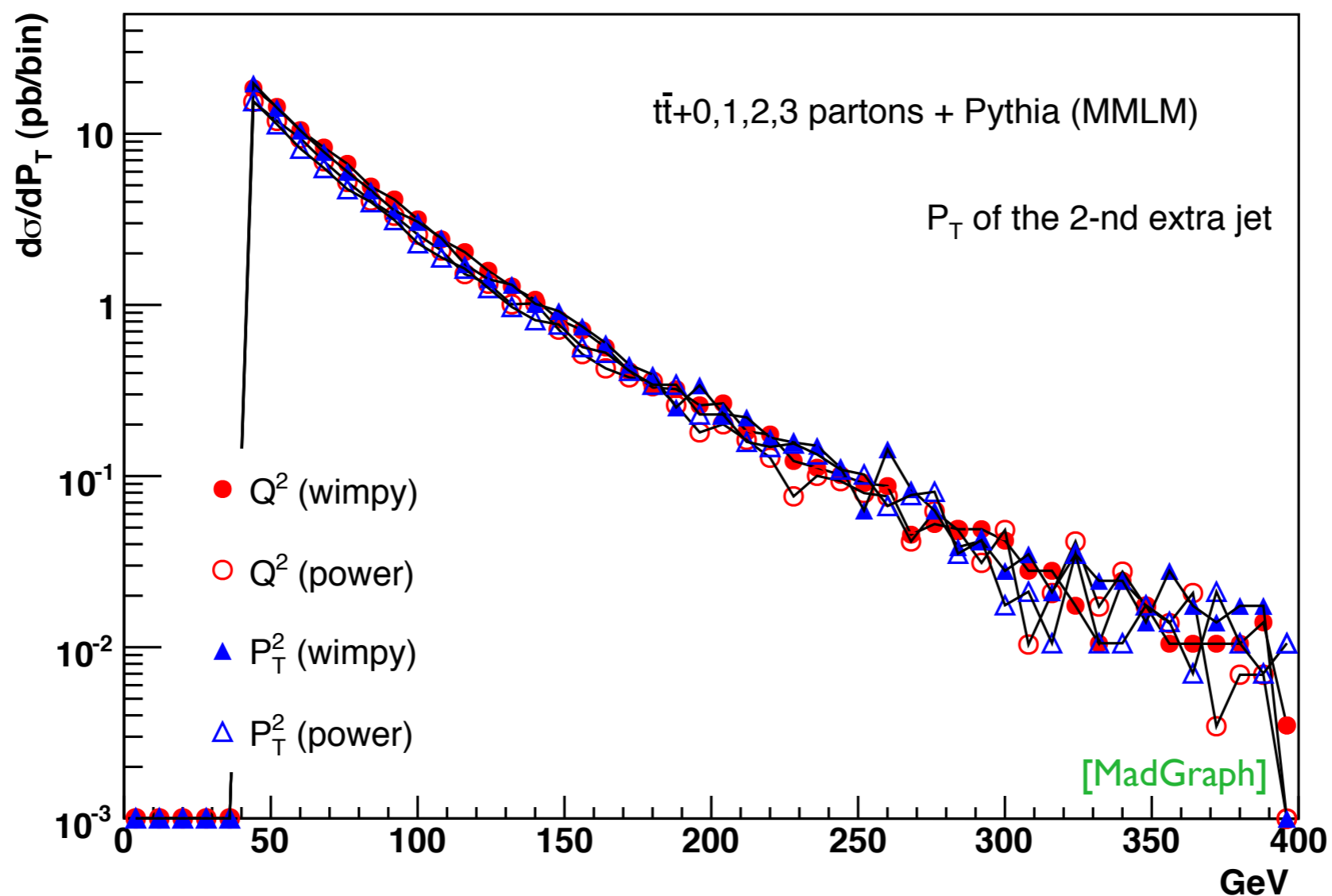
PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result \Rightarrow Large variation in results (small prediction power)



PS alone vs matched samples

In a matched sample these differences are irrelevant since the behavior at high p_T is dominated by the matrix element.

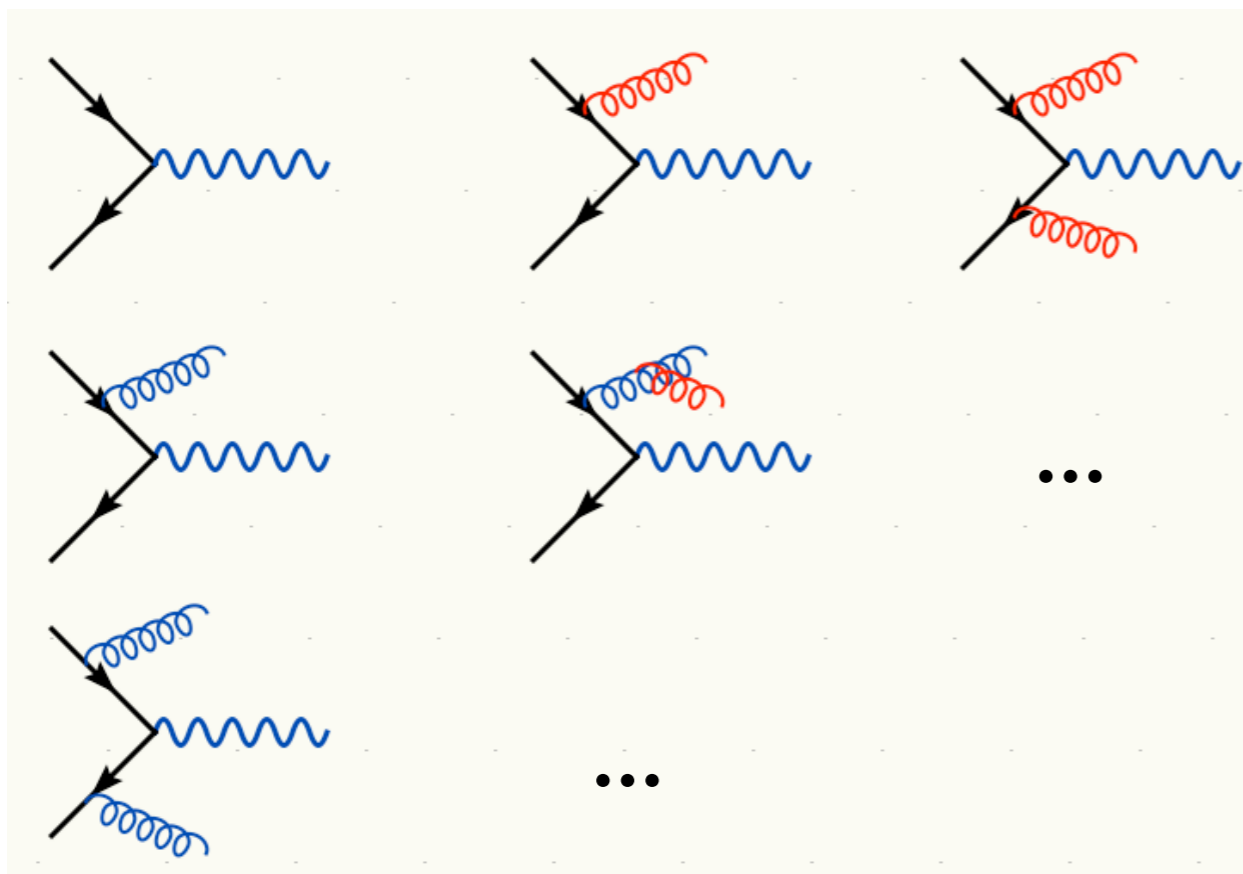


Merging ME with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]

PS →

ME
↓

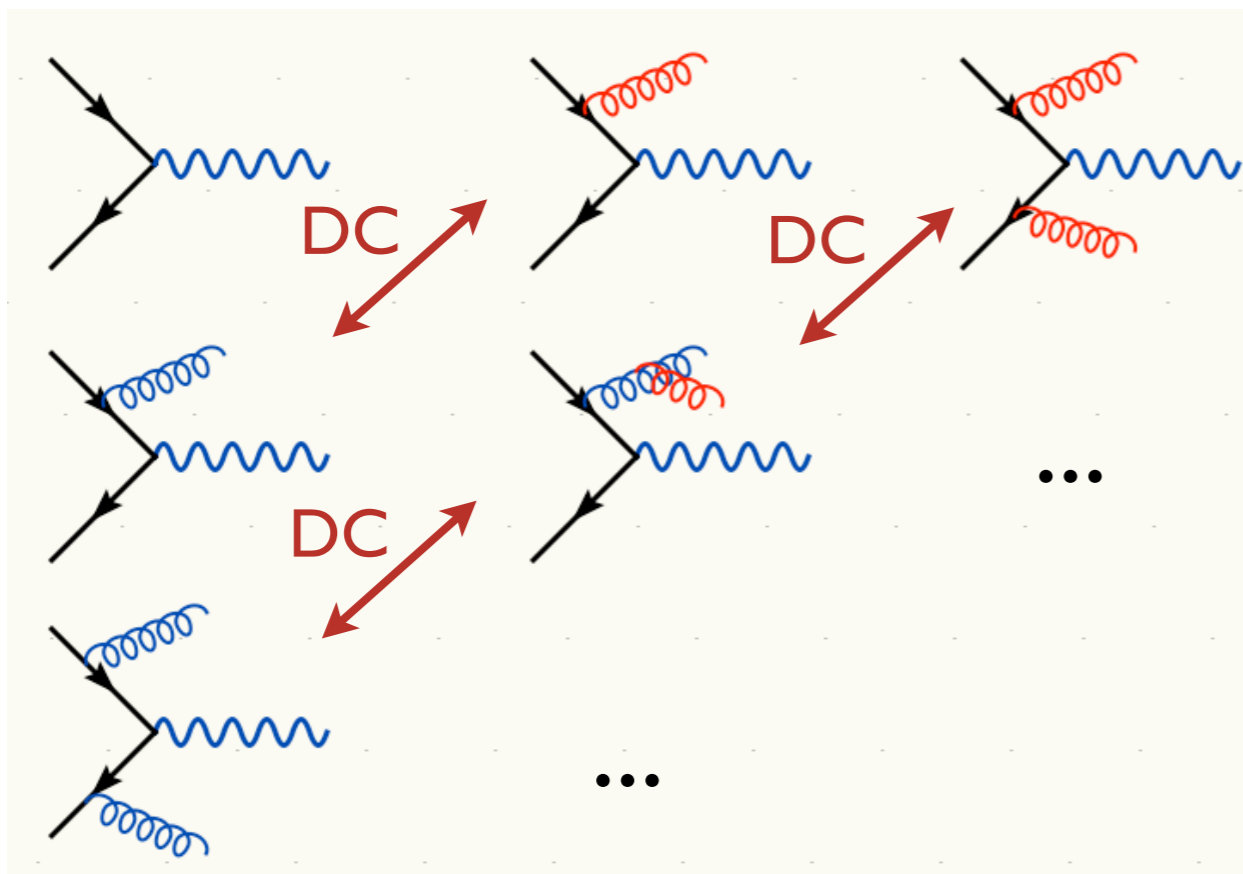


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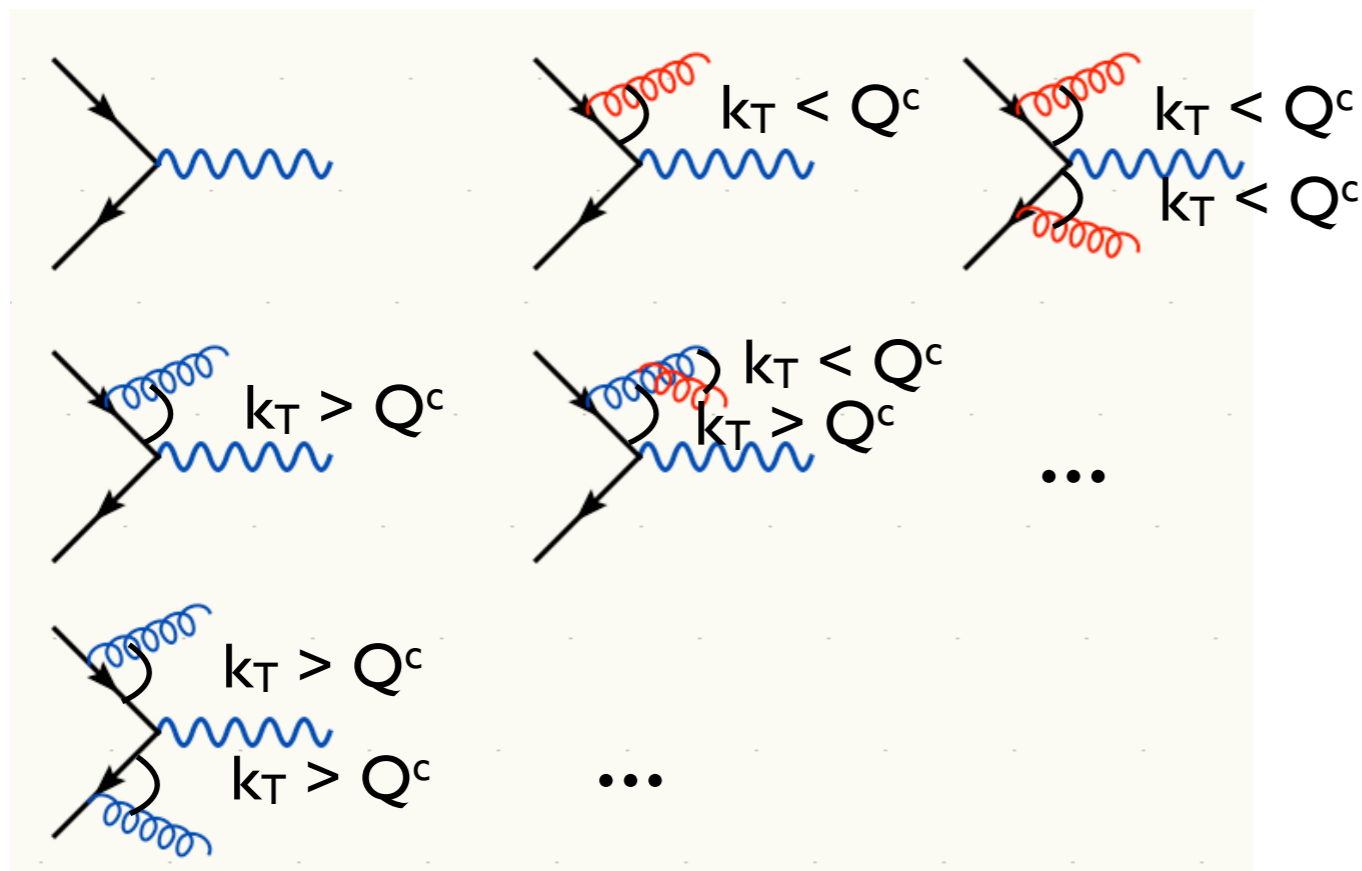


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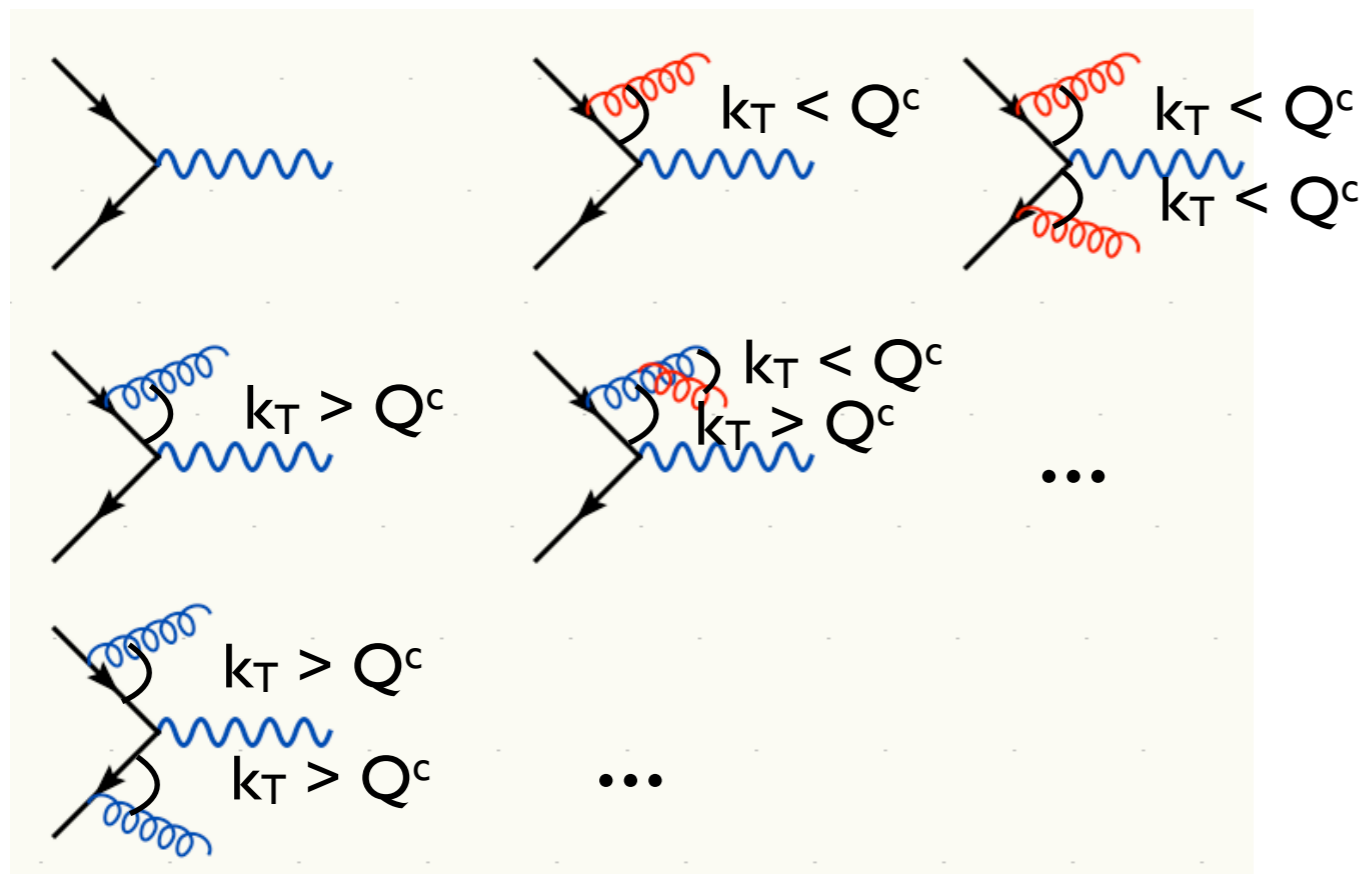


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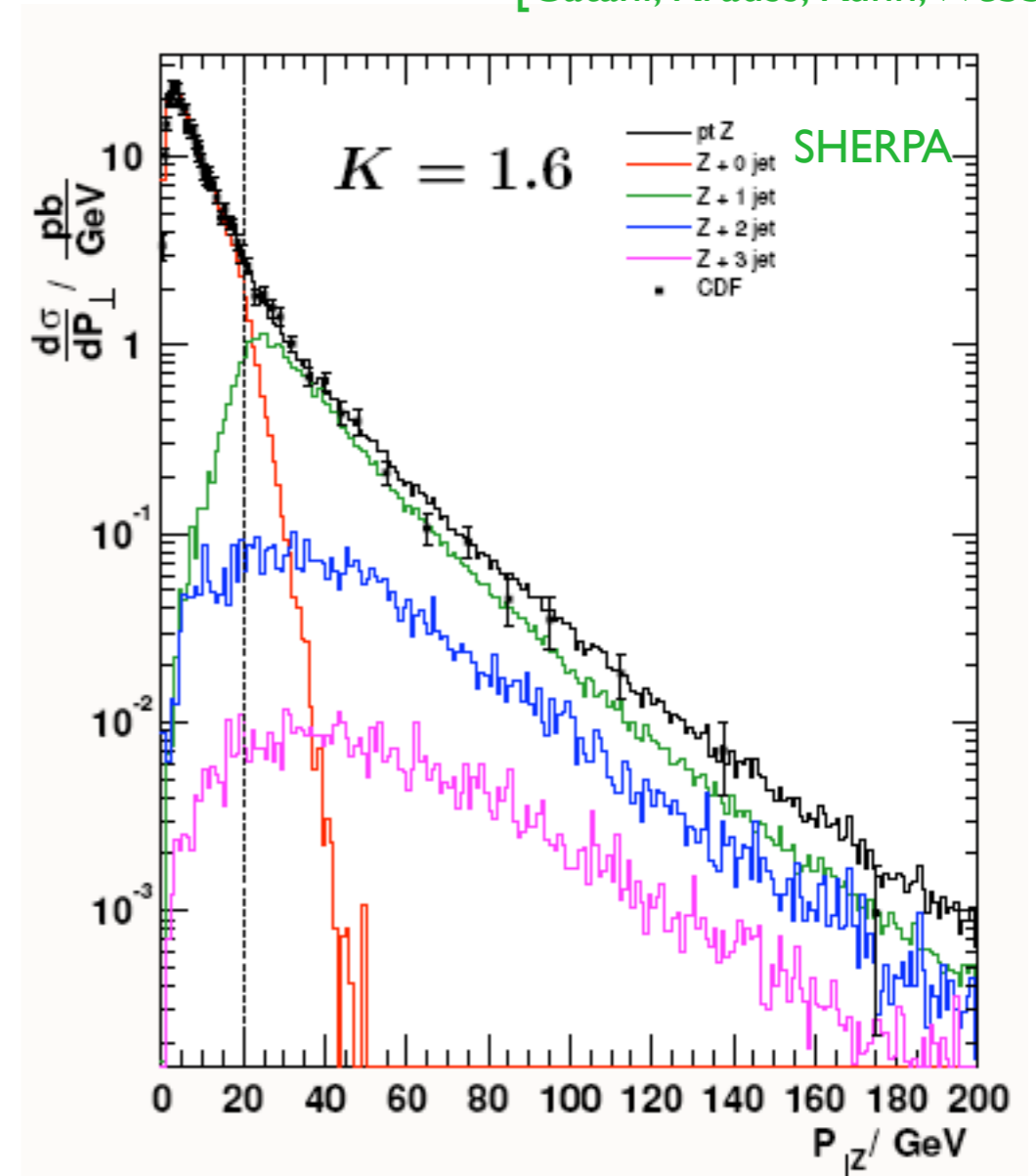
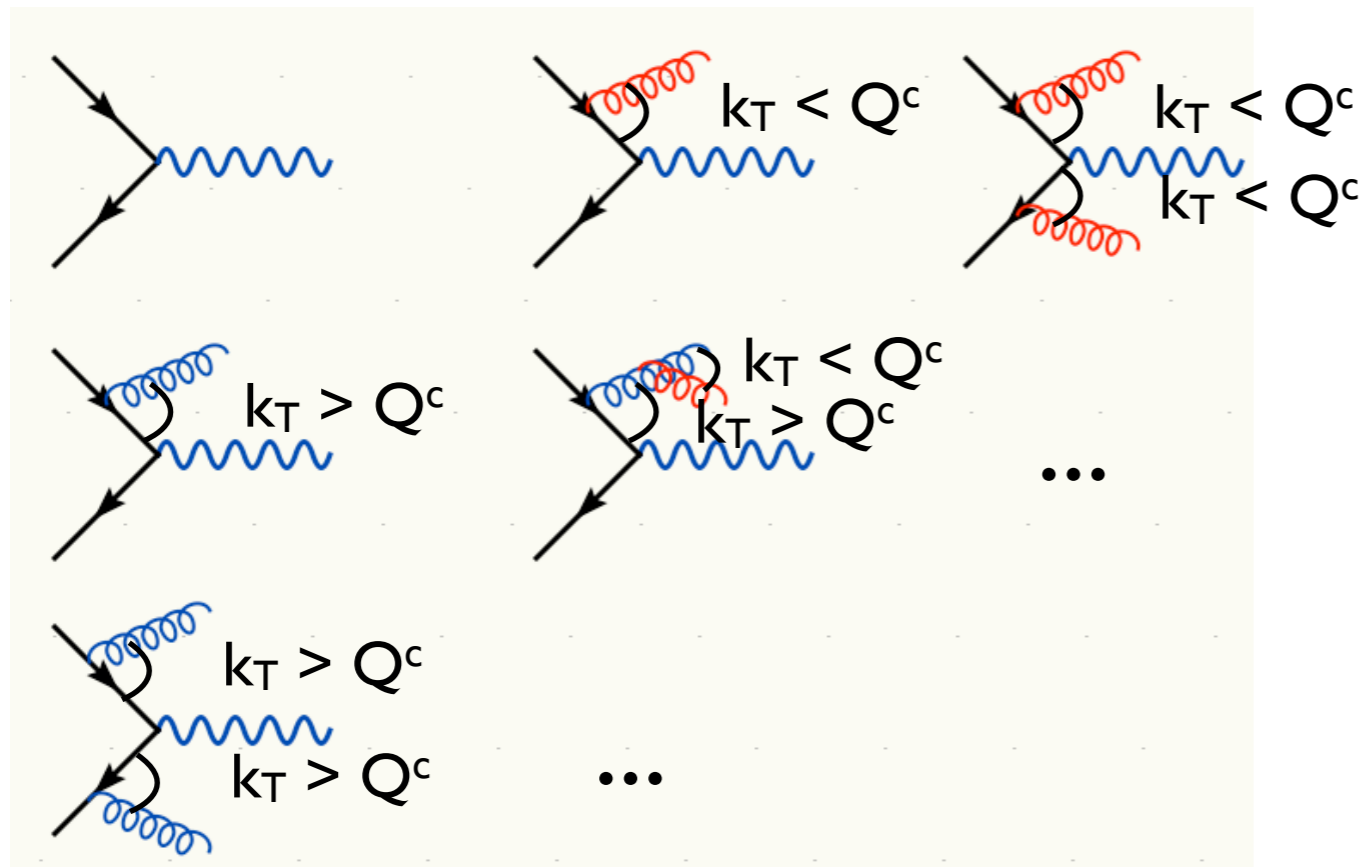
Double counting between ME and PS avoided using phase space cut between the two: PS below cutoff, ME above cutoff. Resulting events **exclusive** and can be added together into an inclusive sample. Smoothness of distributions achieved by careful treatment of ME samples to match PS.

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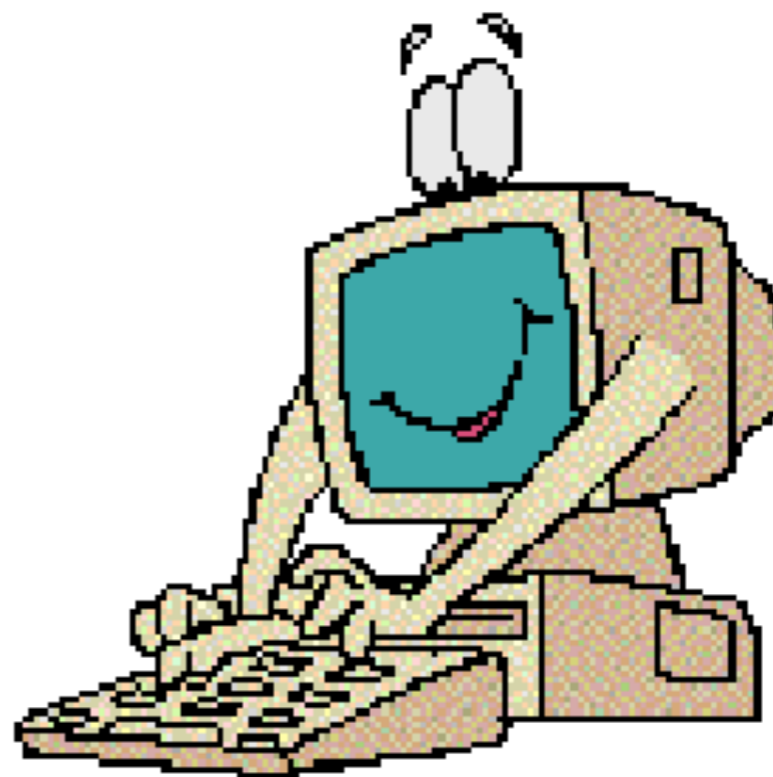
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Event simulation in practice Using MadGraph

Using MadGraph on the Web!

To generate matrix elements using MadGraph:

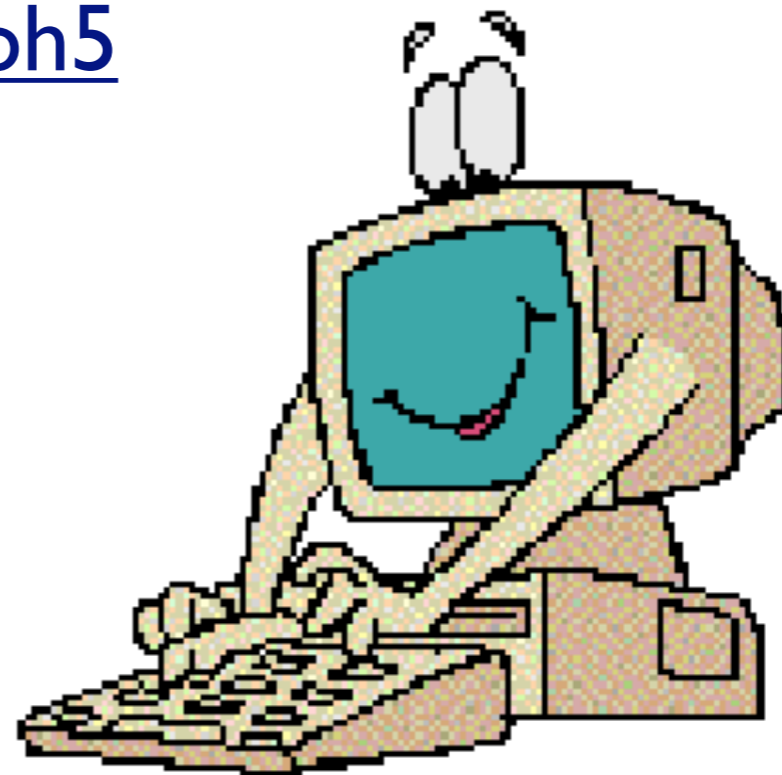
- Go to <http://madgraph.hep.uiuc.edu/>
(or google for MadGraph)
- Register
- Write your process
- Press
- Download the tar file or
generate events directly online on our clusters!



Using MadGraph on your computer!

To generate matrix elements and events:

- Download MadGraph 5 from <https://launchpad.net/madgraph5>
- Untar and run bin/mg5
- Write “generate *process*”
- Write “output”
- Write “launch”



Sounds easy? It is! Let me show you!

Examples shown

- $p p \rightarrow t \bar{t}$
This gives only (the dominant) QCD vertices, and ignores (the negligible) QED vertices.
- $p p \rightarrow t \bar{t}$ QED=2
This gives both QED and QCD vertices.
- $p p \rightarrow w^+ j j, w^+ \rightarrow l^+ \nu_l$
More complicated example.

More syntax examples

- $p p > t t^{\sim} j$ QED=2: Generate all combinations of processes for particles defined in multiparticle labels p / j , including up to two QED vertices (and unlimited QCD vertices)
- $p p > t t^{\sim}, (t > b w^+, w^+ > l^+ \nu_l), t^{\sim} > b^{\sim} j j$:
 - Only diagrams compatible with given decay
 - Only t / t^{\sim} and W^+ close to mass shell in event generation
- $p p > w^+ w^- / h$: Exclude any diagrams with h
- $p p > w^+ w^- \$ h$: Exclude on-shell h in event generation (but retain interference effects)

Thanks for listening!

And now over to Olivier...