QCD Basics for accurate LHC Physics

Lecture II

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Lectures

1. Intro and QCD fundamentals
2. QCD in the final state
3. From accurate QCD to useful QCD
4. Advanced QCD with applications at the LHC

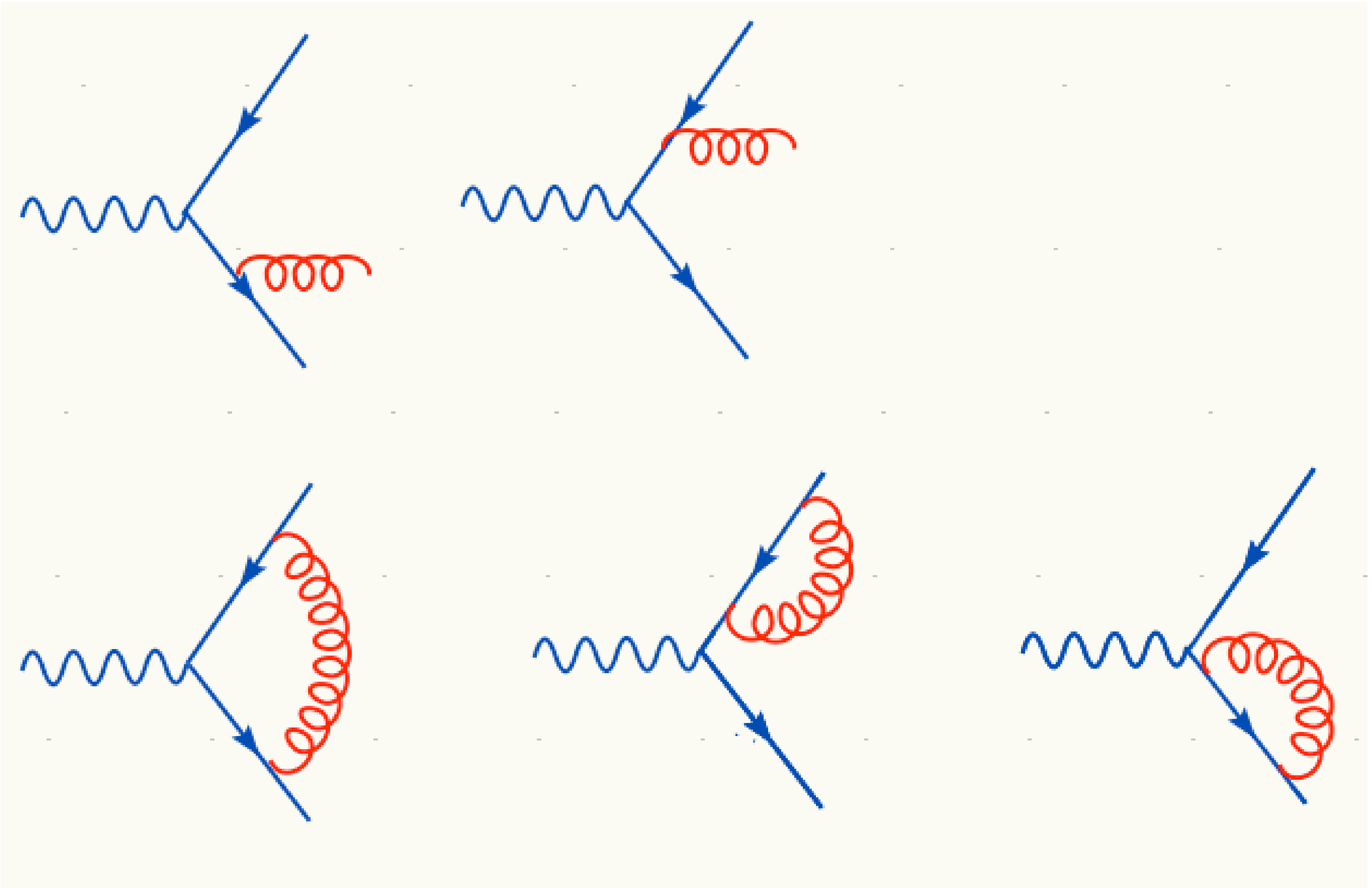
New set of questions

1. How can we identify a cross sections for producing quarks and gluons with a cross section for producing hadrons?

2. Given the fact that free quarks are not observed, why is the computed Born cross section so good?

3. Are there other calculable, i.e., that do not depend on the non-perturbative dynamics (like hadronization), quantities besides the total cross section?

Anatomy of a NLO calculation



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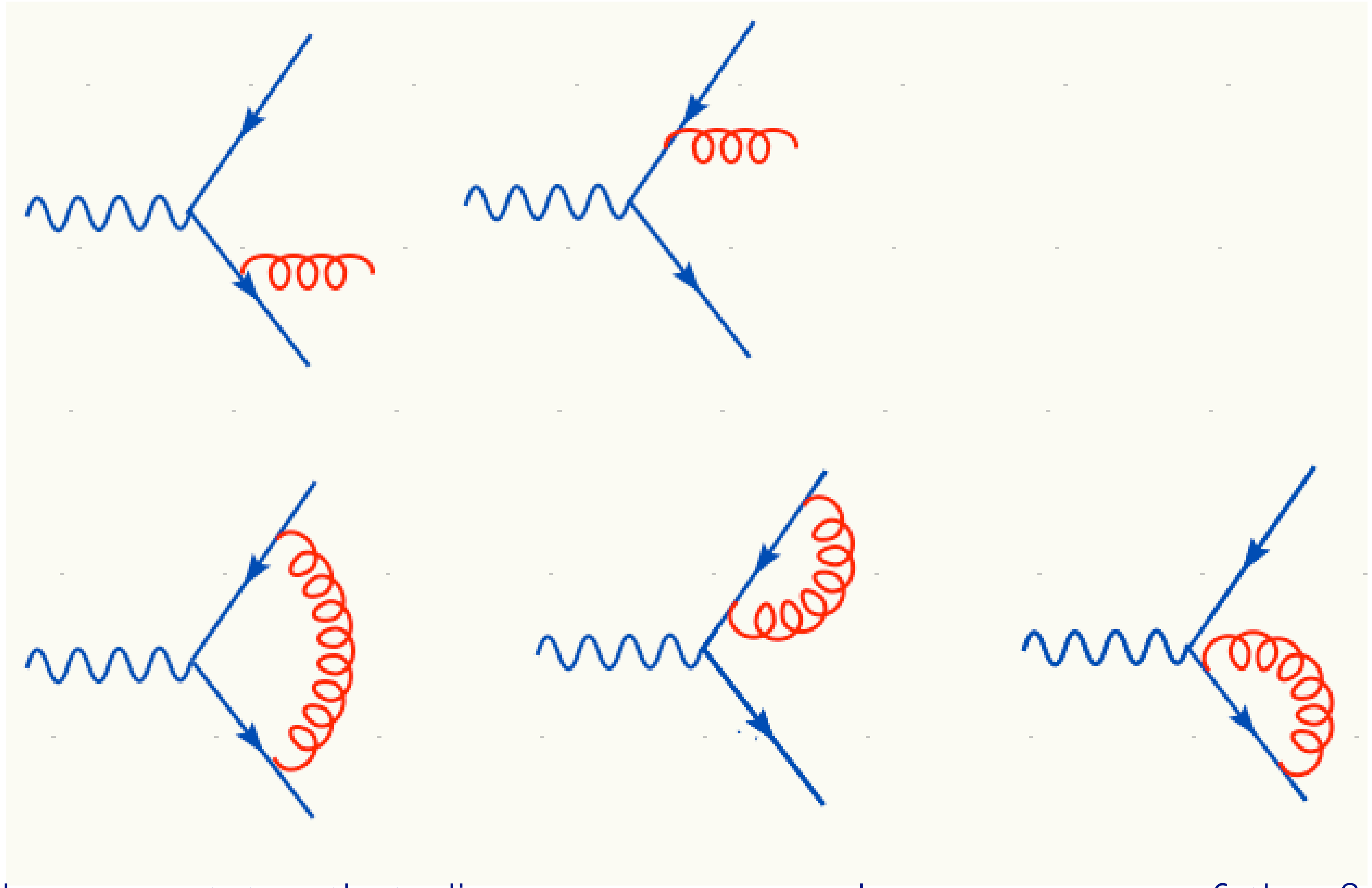

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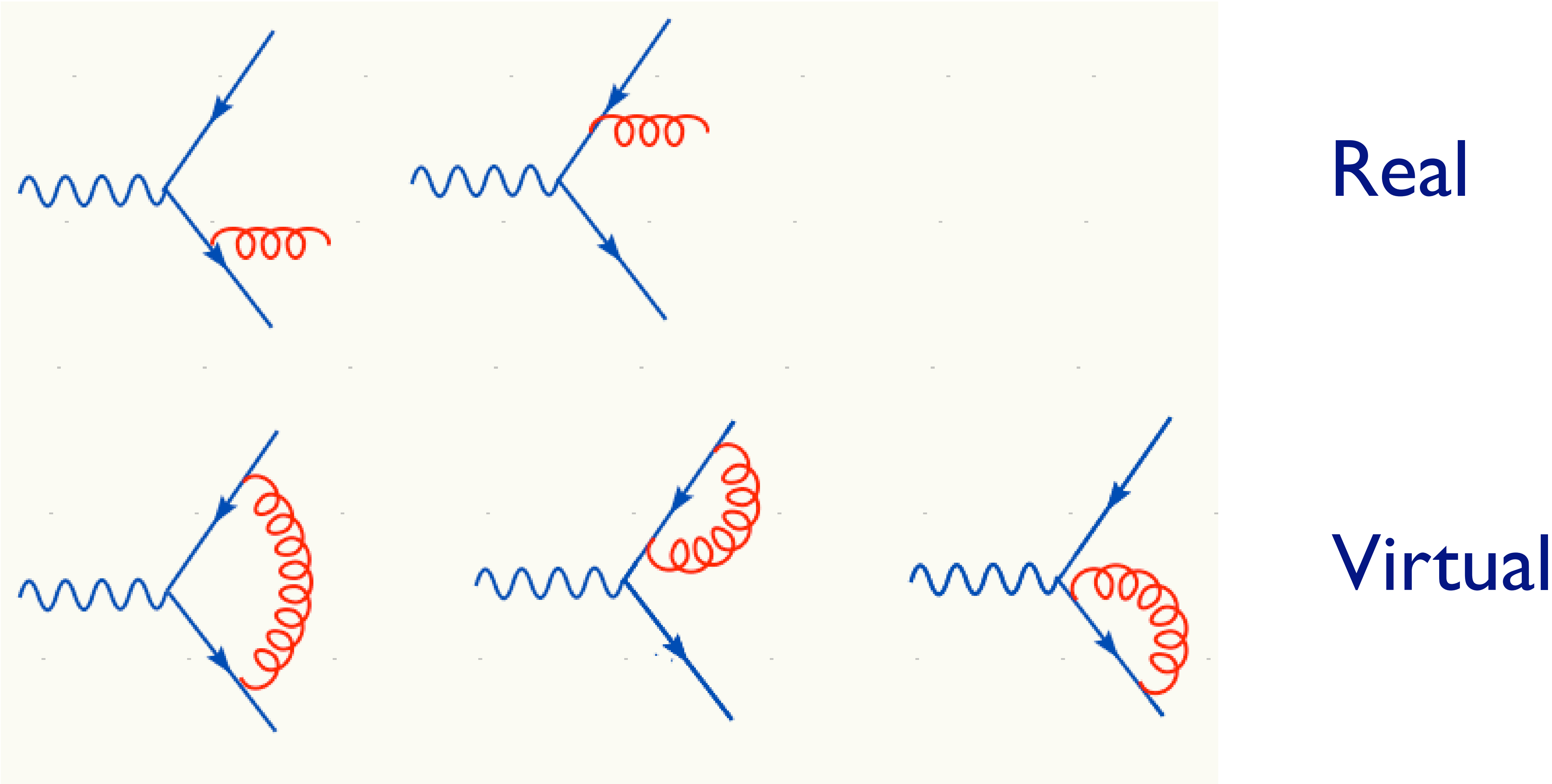

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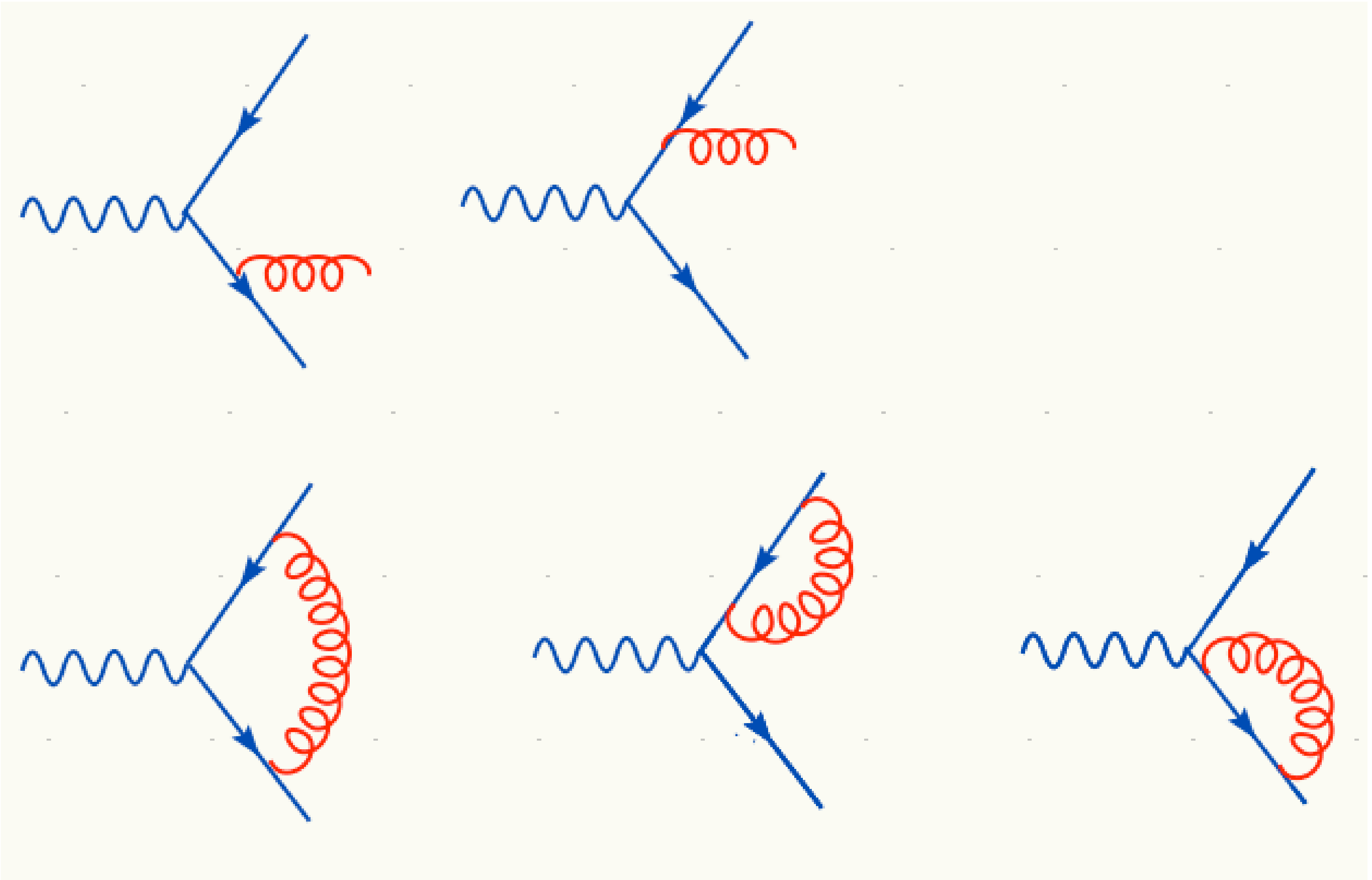

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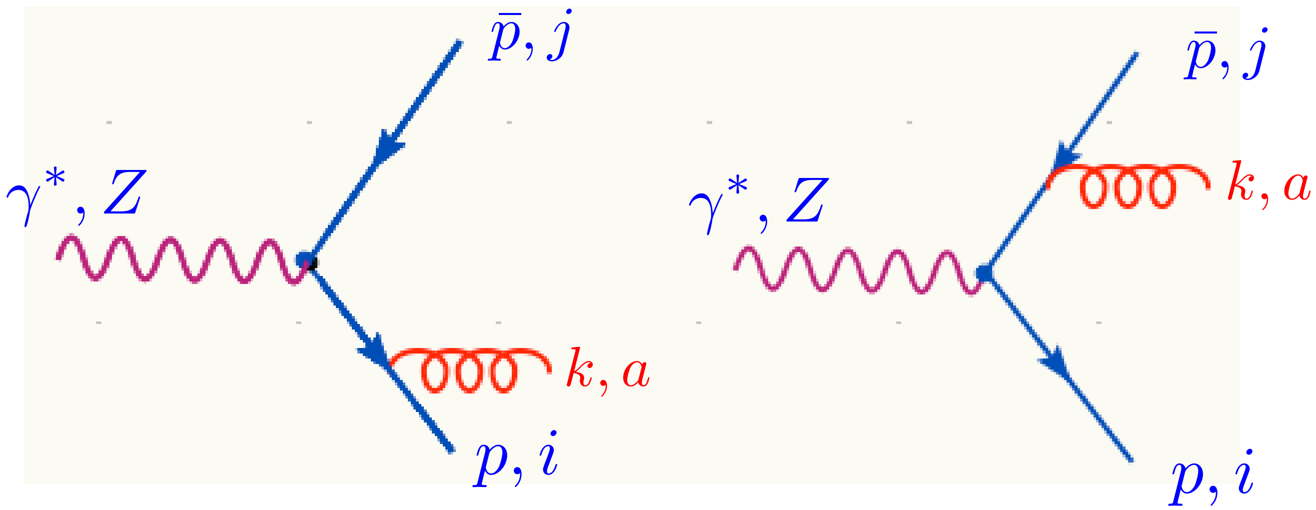

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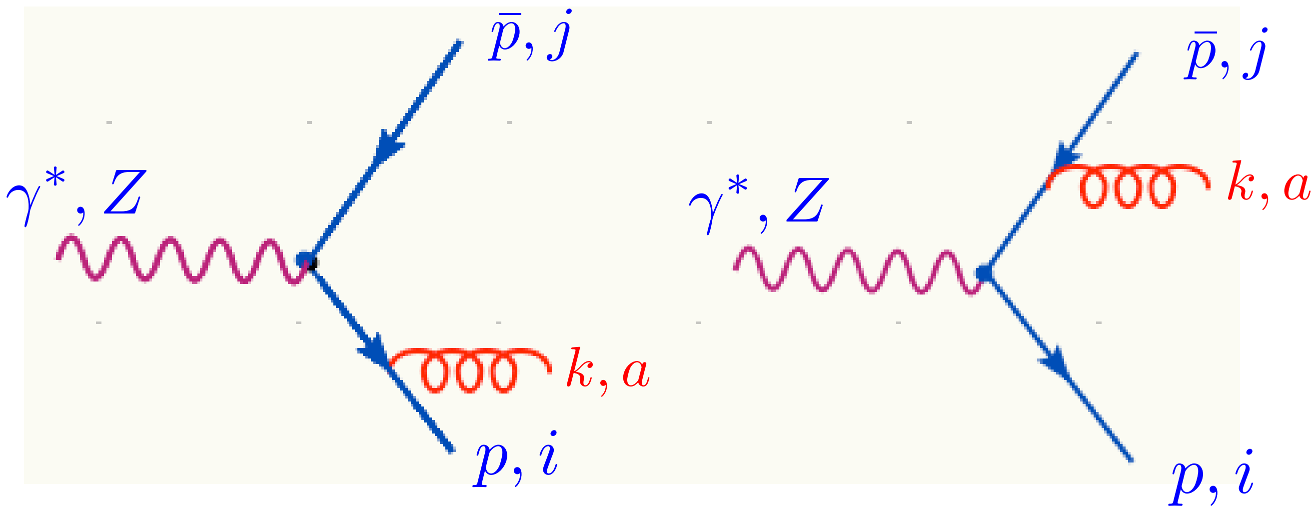

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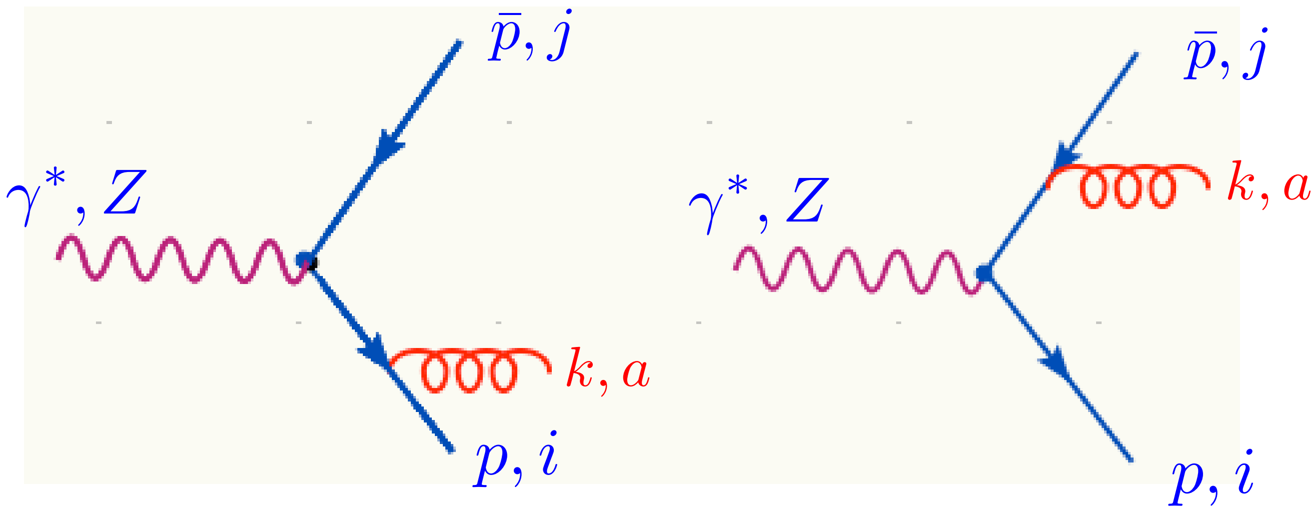

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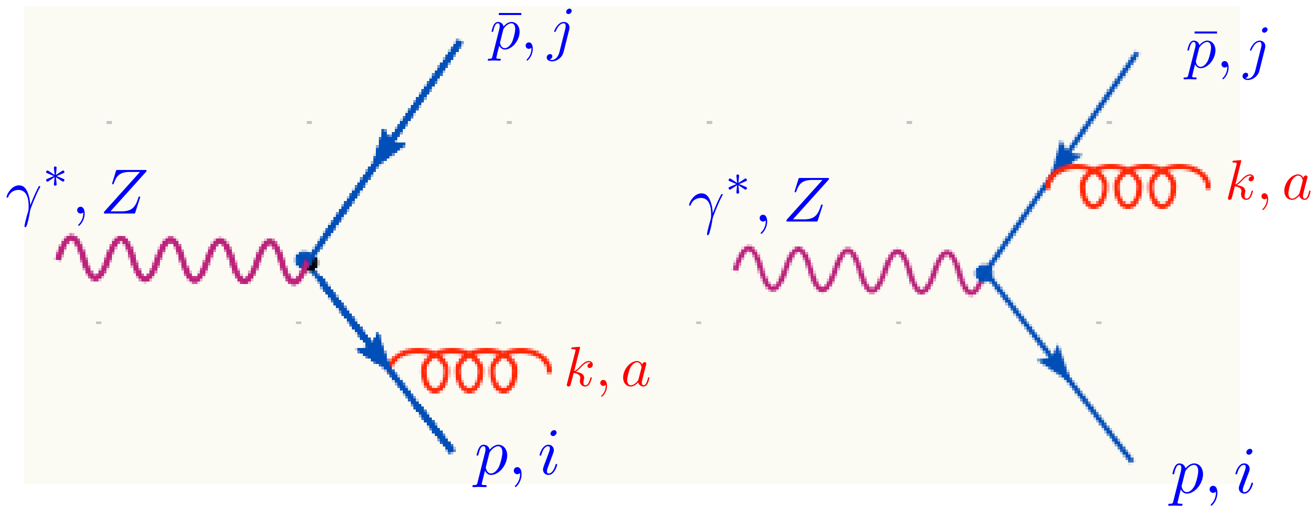

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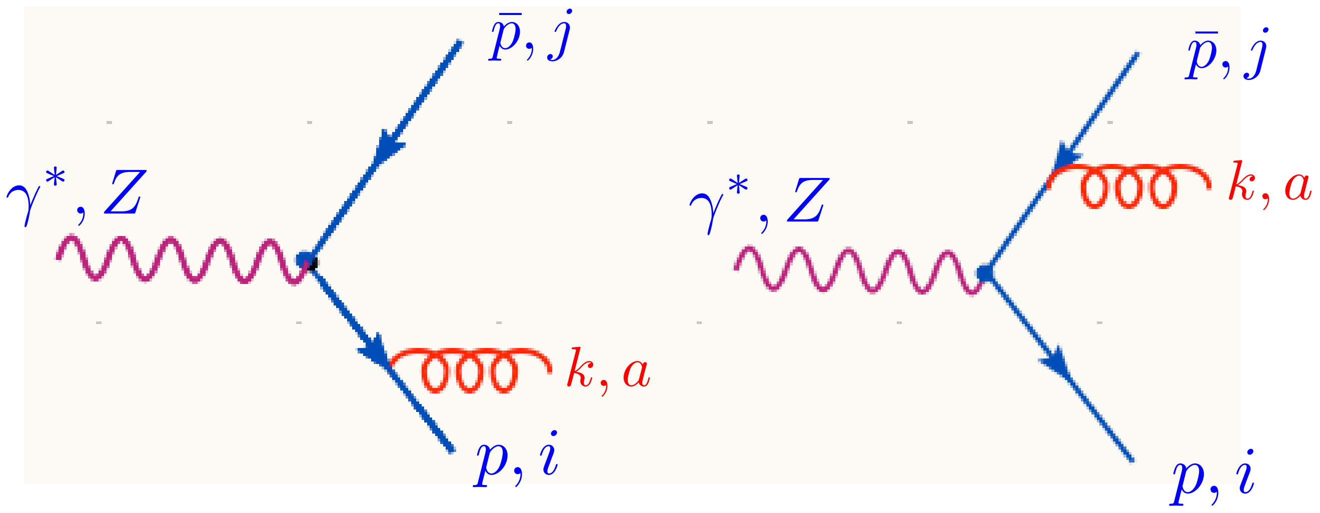

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The denominators $2p\cdot k = p_0 k_0(1-\cos\theta)$ give singularities for collinear ($\cos\theta \to 1$) or soft ($k_0 \to 0$) emission. By neglecting k in the numerators and using the Dirac equation, the amplitude simplifies and factorizes over the Born amplitude:

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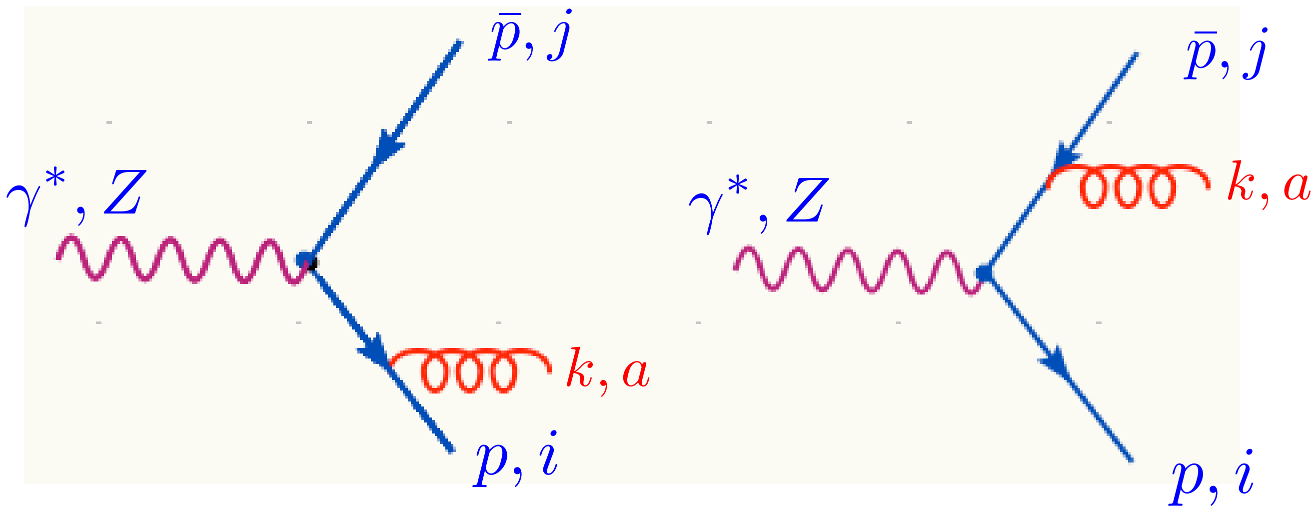

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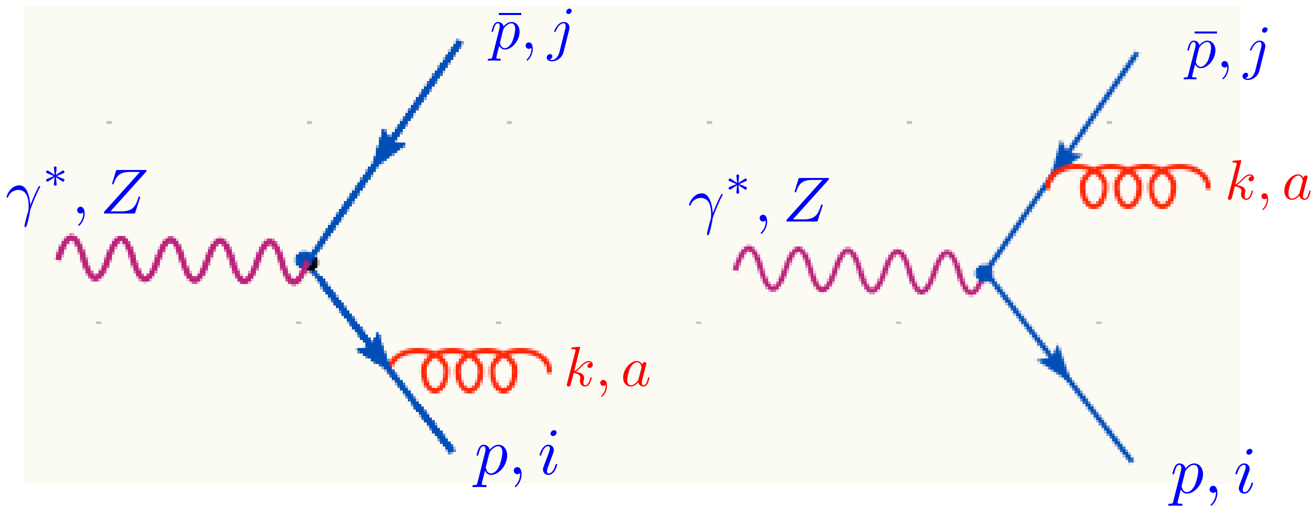

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Factorization: Independence of long-wavelength (soft) emission form the hard (short-distance) process. Soft emission is universal!!

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Two collinear divergences and a soft one. Very often you find the integration over phase space expressed in terms of x_1 and x_2, the fraction of energies of the quark and anti-quark:

$$x_1 = 1 - x_2 x_3 (1-\cos\theta_{23})/2$$

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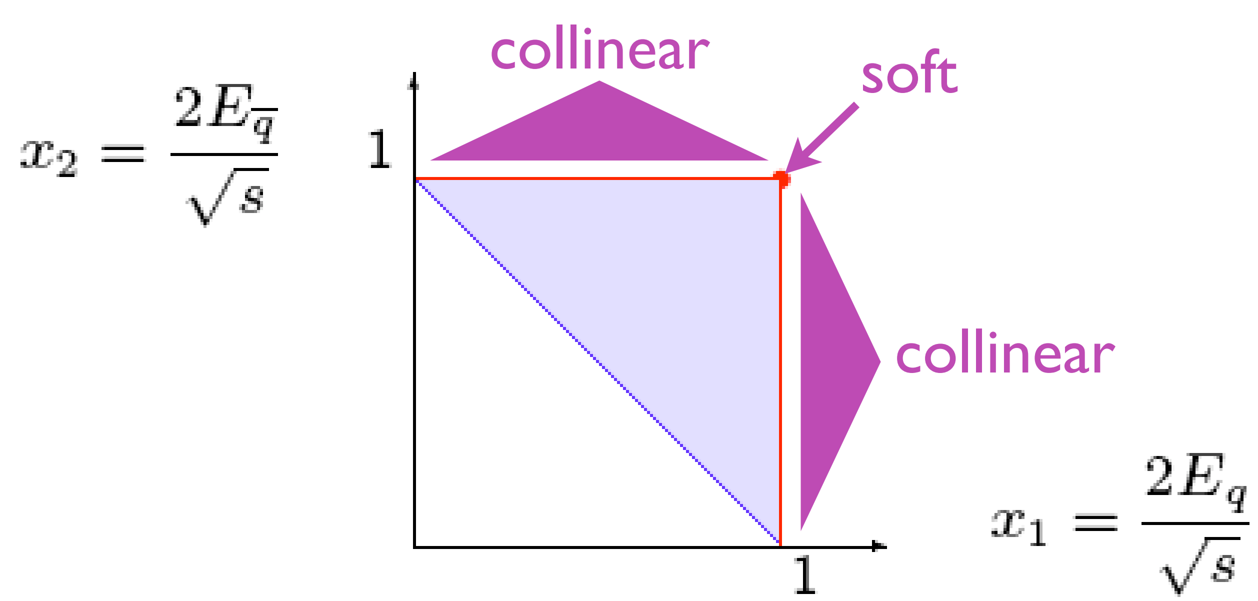

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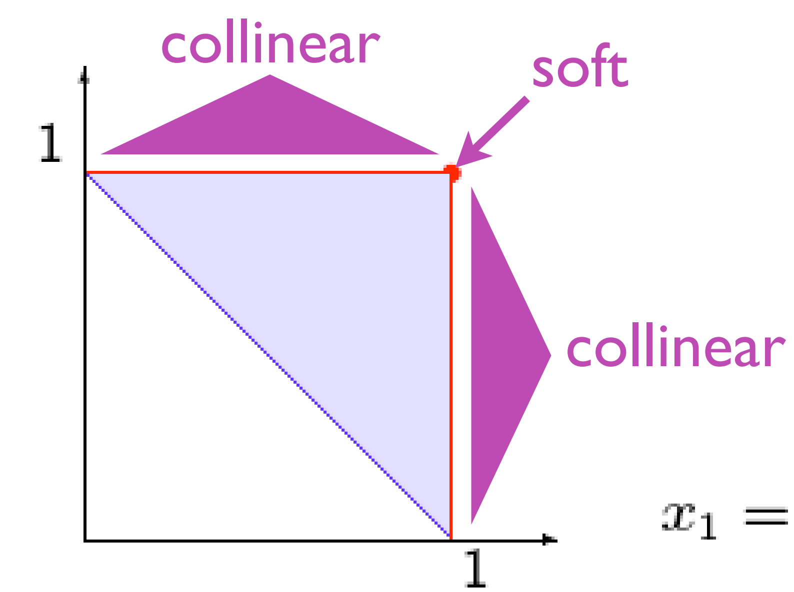

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So we can now predict the divergent part of the virtual contribution, while for the finite part an explicit calculation is necessary:

$$\sigma_{q\bar{q}}^{\text{VIRT}} = -\sigma_{q\bar{q}}^{\text{Born}} C_F \frac{\alpha_S}{2\pi} \int d\cos\theta' \frac{dk_0'}{k_0'} \frac{1}{1-\cos^2\theta} 2\delta(k_0')[\delta(1-\cos\theta')+\delta(1+\cos\theta')]+\ldots$$

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A further insight can be gained by thinking of what happens in QED and what is different there. For instance soft and collinear divergence are also there. In QED one can prove that cross section for producing "only two muons" is zero...

Infrared divergences

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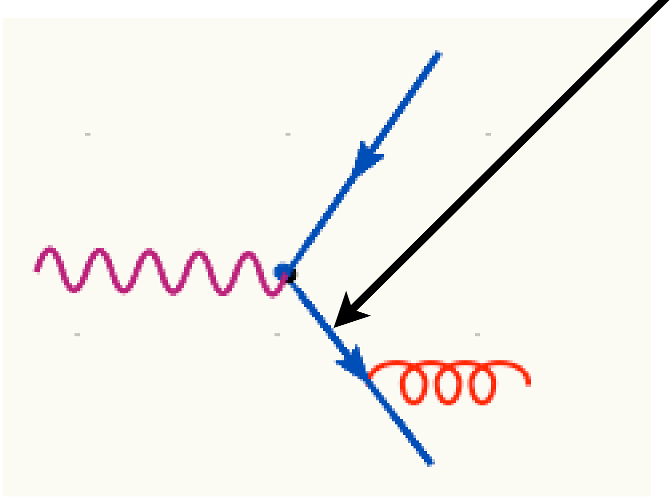
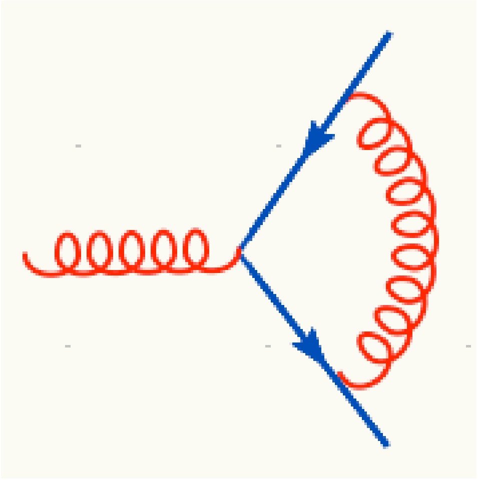


Even in high-energy, short-distance regime, long-distance aspects of QCD cannot be ignored.

This is because there are configurations in phase space for gluons and quarks, i.e. when gluons are soft and/or when are pairs of partons are collinear.

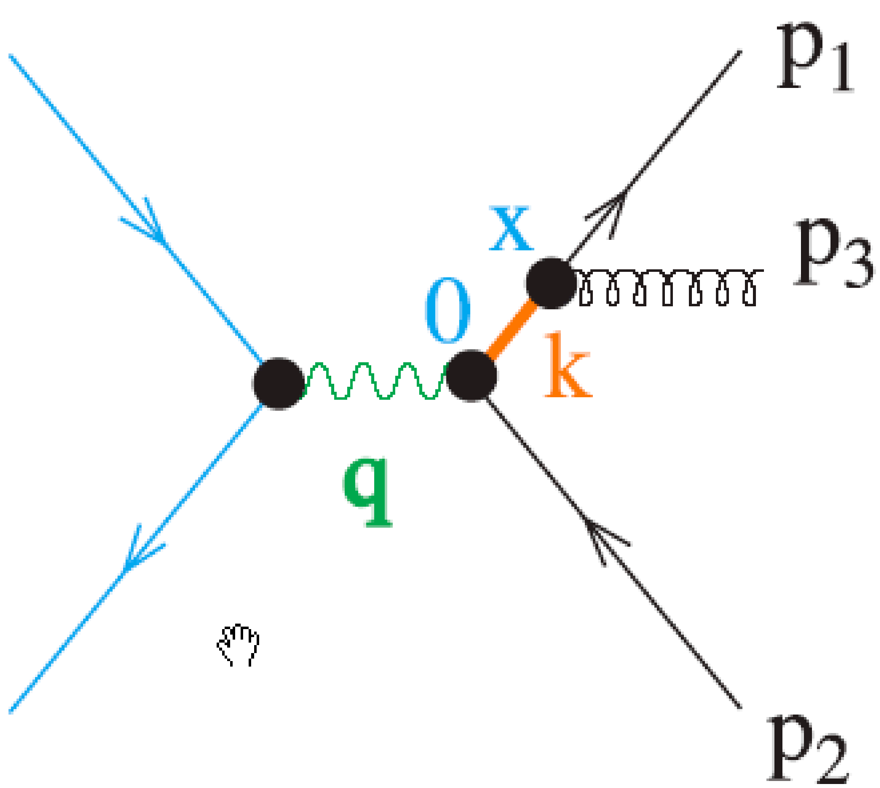


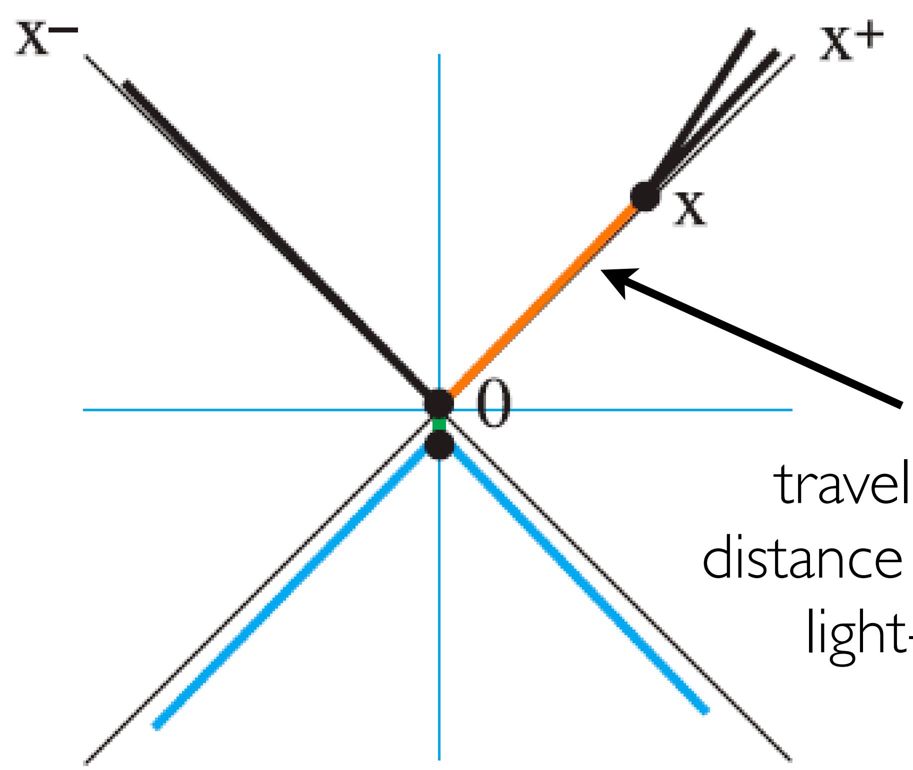
$$\Rightarrow \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k+p)^2 (k-\bar{p})^2}$$

also for soft and collinear or collinear configurations associated to the virtual partons with the region of integration of the loop momenta.

Space-time picture of IR singularities

The singularities can be understood in terms of light-cone coordinates. Take $p^\mu=(p^0, p^1, p^2, p^3)$ and define $p^\pm=(p^0 \pm p^3)/\sqrt{2}$. Then choose the direction of the + axis as the one of the largest between + and - . A particle with small virtuality travels for a long time along the x^+ direction.





$$k^+ \simeq \sqrt{s}/2$$ large

$$k^- \simeq (k^T + 2k^+k^-)\sqrt{s}/2$$ small

$$x^+ \simeq 1/k^-$$ large

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YES! It is called INFRARED SAFETY

Infrared-safe quantities

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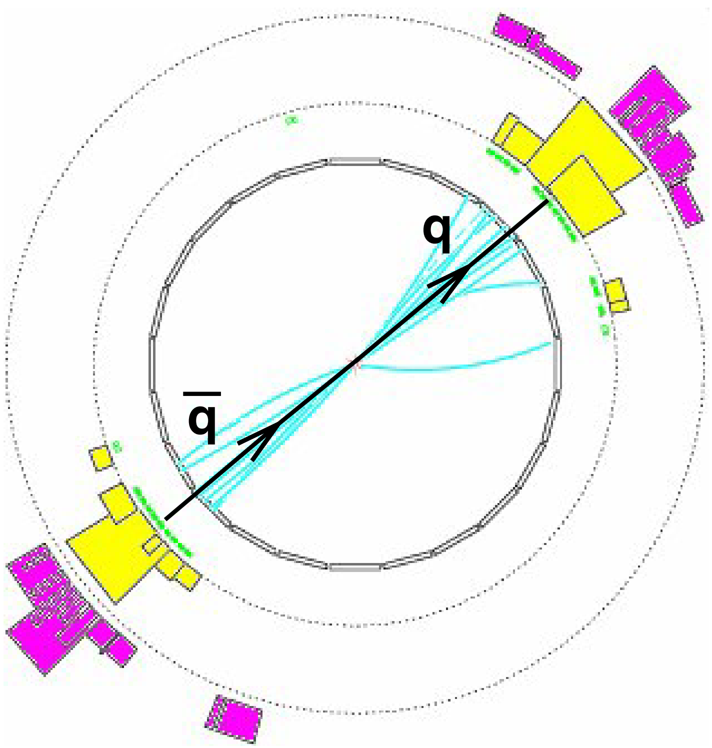
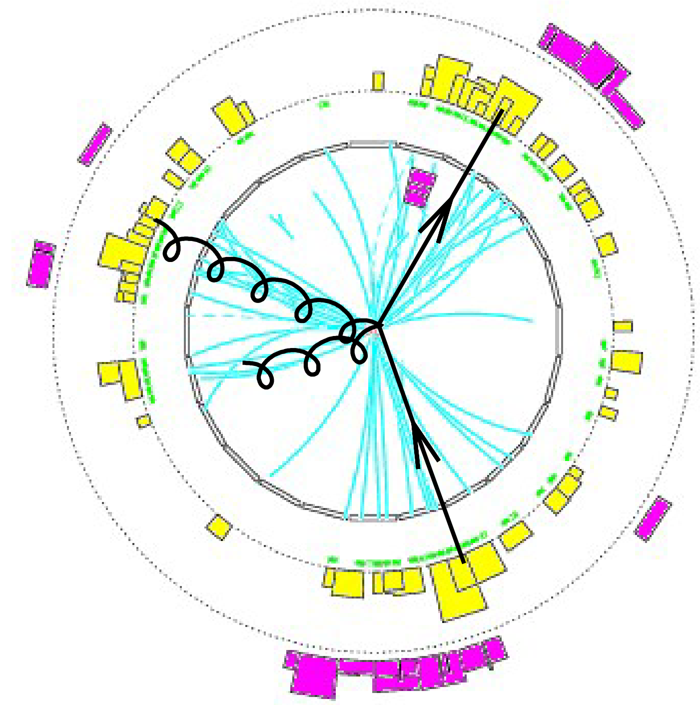
Examples:

1. Multiplicity of gluons is not IRC safe
2. Energy of hardest particle is not IRC safe
3. Energy flow into a cone is IRC safe

Event shape variables





pencil-like → spherical

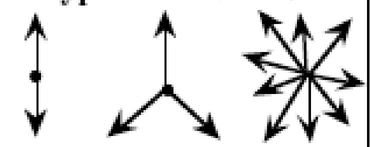
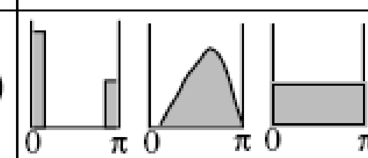
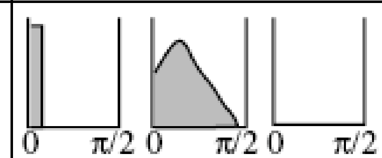
Event shape variables

The idea is to give more information than just total cross section by defining "shapes" of an hadronic event (pencil-like, planar, spherical, etc..)

In order to be comparable with theory it MUST be IR-safe, that means that the quantity should not change if one of the parton "branches" $p_k \to p_i + p_j$

Examples are: Thrust, Spherocity, C-parameters,...

Similar quantities exist for hadron collider too, but they much less used.

Name of Observable	Definition	Typical Value for: 			QCD calculation
Thrust	$T = \max_{\vec{n}} \left(\frac{\Sigma_i \lvert \vec{p}_i \vec{n} \rvert}{\Sigma_i \lvert \vec{p}_i \rvert} \right)$	1	≥2/3	≥1/2	(resummed) $O(\alpha_s^2)$
Thrust major	Like T, however T_{maj} and $\vec{n}_{maj}$ in plane $\perp \vec{n}_T$	0	≤1/3	≤1/√2	$O(\alpha_s^2)$
Thrust minor	Like T, however T_{min} and $\vec{n}_{min}$ in direction $\perp$ to $\vec{n}_T$ and $\vec{n}_{maj}$	0	0	≤1/2	$O(\alpha_s^2)$
Oblateness	$O = T_{maj} - T_{min}$	0	≤1/3	0	$O(\alpha_s^2)$
Sphericity	$S = 1.5\,(Q_1 + Q_2)$; $Q_1 \leq ... \leq Q_3$ are Eigenvalues of $S^{\alpha\beta} = \frac{\Sigma_i p_i^\alpha p_i^\beta}{\Sigma_i p_i^2}$	0	≤3/4	≤1	none (not infrared safe)
Aplanarity	$A = 1.5\, Q_1$	0	0	≤1/2	none (not infrared safe)
Jet (Hemisphere) masses	$M_\pm^2 = \left(\Sigma_i E_i^2 - \Sigma_i \vec{p}_i^{\,2}\right)_{i \in S_\pm}$ ($S_\pm$: Hemispheres $\perp$ to $\vec{n}_T$) $M_H^2 = \max(M_+^2, M_-^2)$ $M_D^2 = \lvert M_+^2 - M_-^2 \rvert$	 0 0	 ≤1/3 ≤1/3	 ≤1/2 0	(resummed) $O(\alpha_s^2)$
Jet broadening	$B_\pm = \frac{\Sigma_{i \in S_\pm} \lvert \vec{p}_i \times \vec{n}_T \rvert}{2\, \Sigma_i \lvert \vec{p}_i \rvert}$; $B_T = B_+ + B_-$ $B_w = \max(B_+, B_-)$	0 0	≤1/(2√3) ≤1/(2√3)	≤1/(2√2) ≤1/(2√3)	(resummed) $O(\alpha_s^2)$
Energy-Energy Correlations	$EEC(\chi) = \sum_{events} \sum_{i,j} \frac{E_i E_j}{E_{vis}^2} \int_{\chi - \frac{\Delta\chi}{2}}^{\chi + \frac{\Delta\chi}{2}} \delta(\chi - \chi_{ij})$				(resummed) $O(\alpha_s^2)$
Asymmetry of EEC	$AEEC(\chi) = EEC(\pi - \chi) - EEC(\chi)$				$O(\alpha_s^2)$
Differential 2-jet rate	$D_2(y) = \frac{R_2(y - \Delta y) - R_2(y)}{\Delta y}$				(resummed) $O(\alpha_s^2)$

Is the thrust IR safe?

$$T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i \vec{p}_i}$$

Contribution from a particle with momentum going to zero drops out.

Replacing one particle with two collinear ones does not change the thrust:

$$|(1-\lambda)\vec{p}_k \cdot \vec{u}| + |\lambda \vec{p}_k \cdot \vec{u}| = |\vec{p}_k \cdot \vec{u}|$$

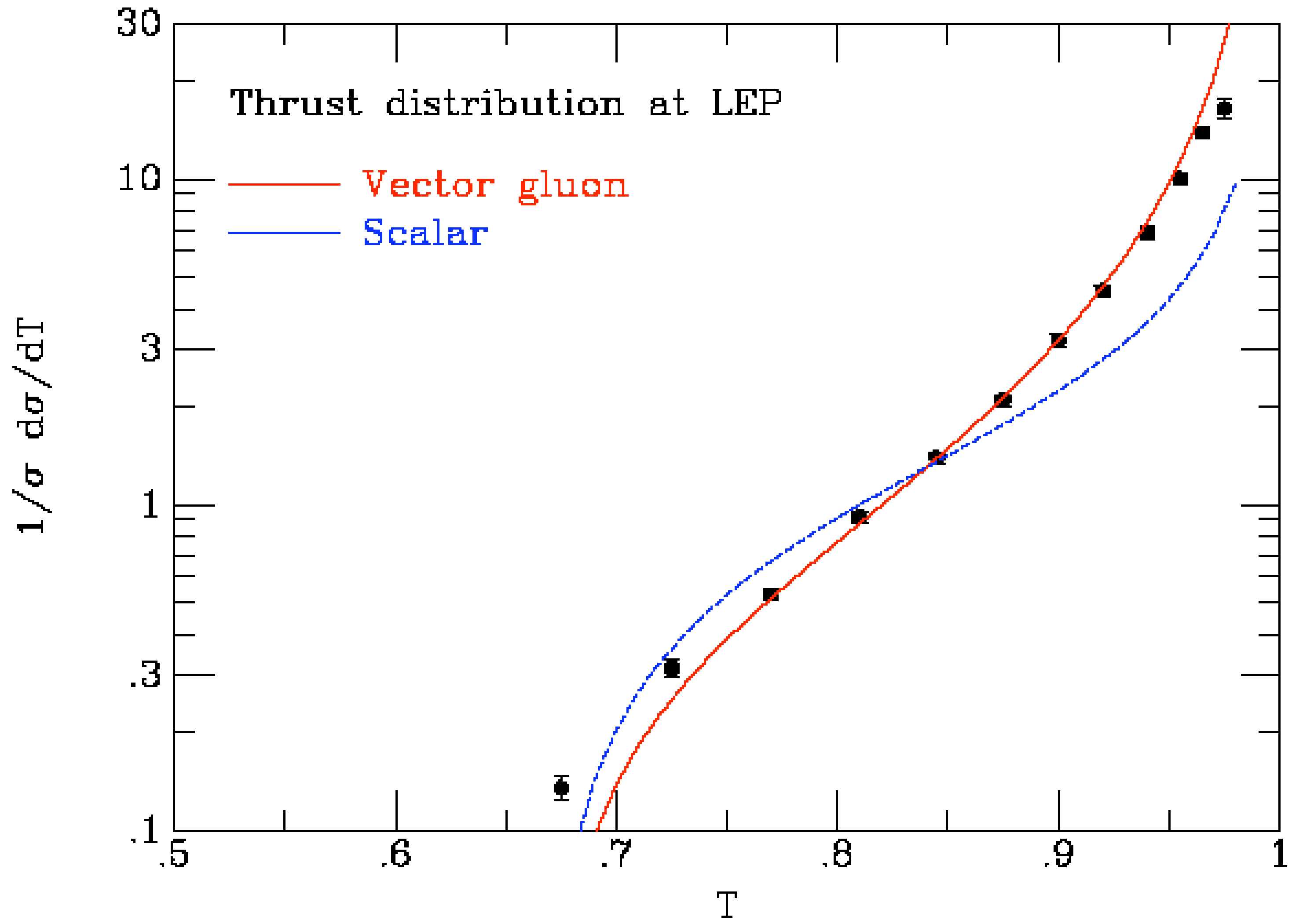
and

$$|(1-\lambda)\vec{p}_k| + |\lambda \vec{p}_k| = |\vec{p}_k|$$

Calculation of event shape variables: Thrust

The values of the different event-shape variables for different topologies are

$$\frac{1}{\sigma}\frac{d\sigma}{dT} = C_F \frac{\alpha_S}{2\pi}\left[\frac{2(3T^2-3T+2)}{T(1-T)}\log\left(\frac{2T-1}{1-T}\right) - \frac{3(3T-2)(2-T)}{1-T}\right].$$



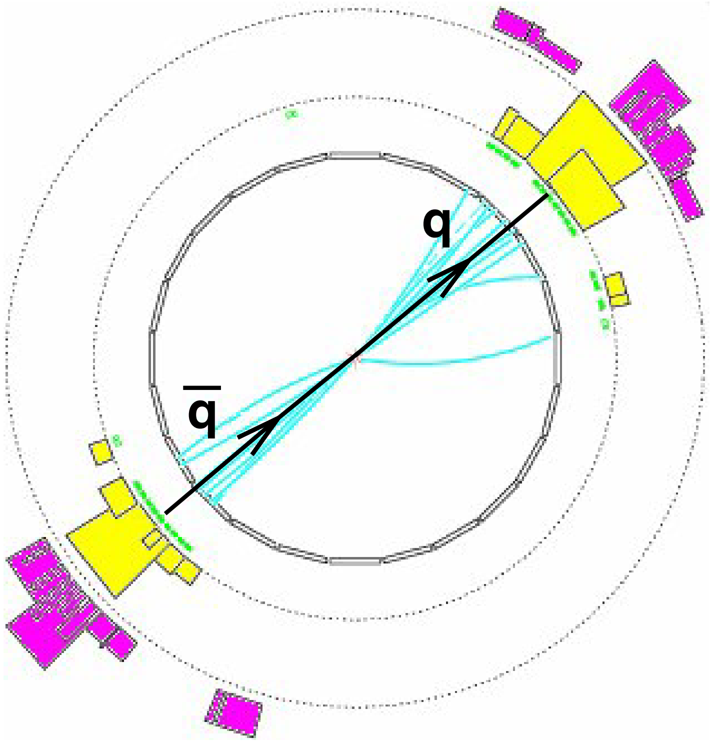
$O(\alpha_S^2)$ corrections (NLO) are also known. Comparison with data provide test of QCD matrix elements, through shape distribution and measurement of α_S from overall rate. Care must be taken around T=1 where

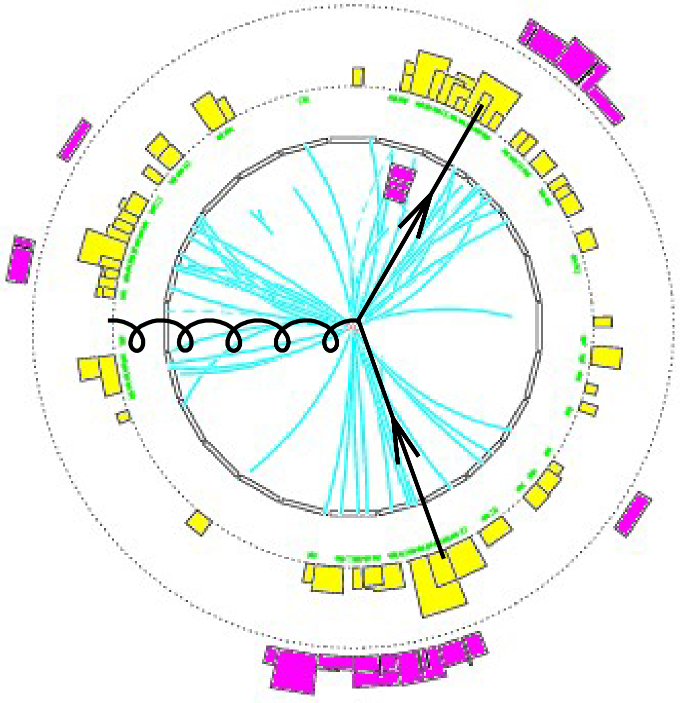
(a) hadronization effects become large and

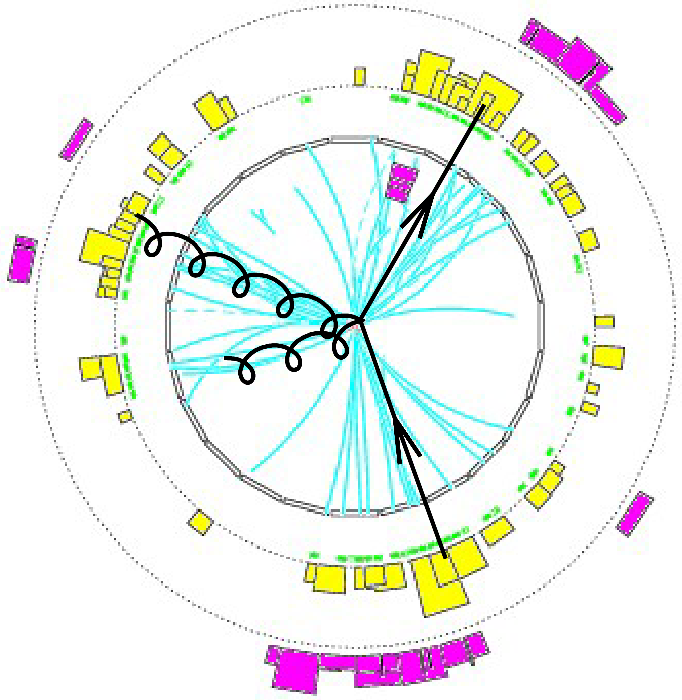
(b) large higher order terms of the form $\alpha_S^N\,[\log^{2N-1}(1-T)]/(1-T)$ need to be resummed.

At lower T multi-jet matrix element become important.

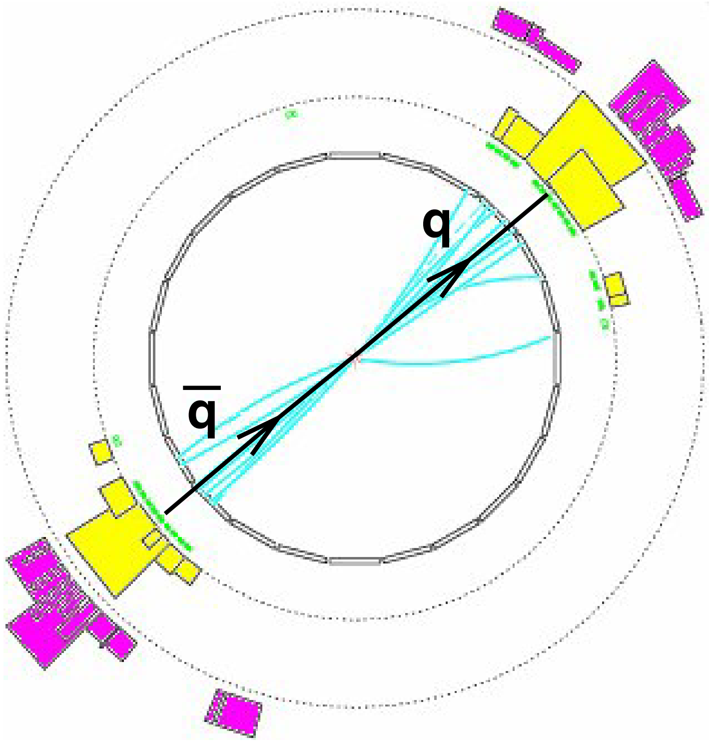
Jet algorithms

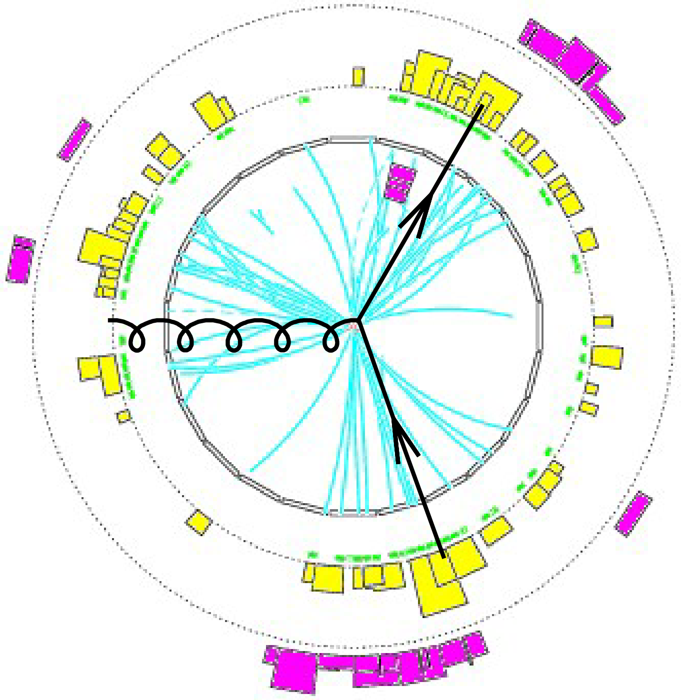


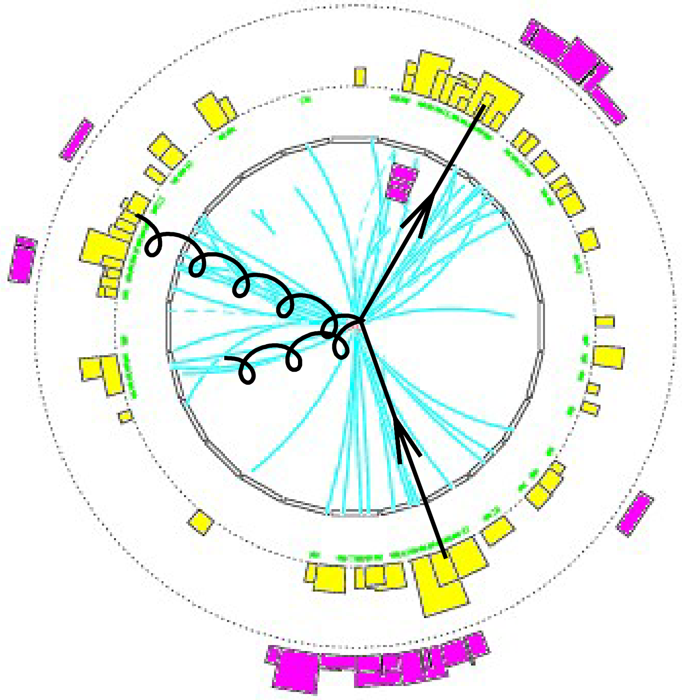




JET ALGORITHMS

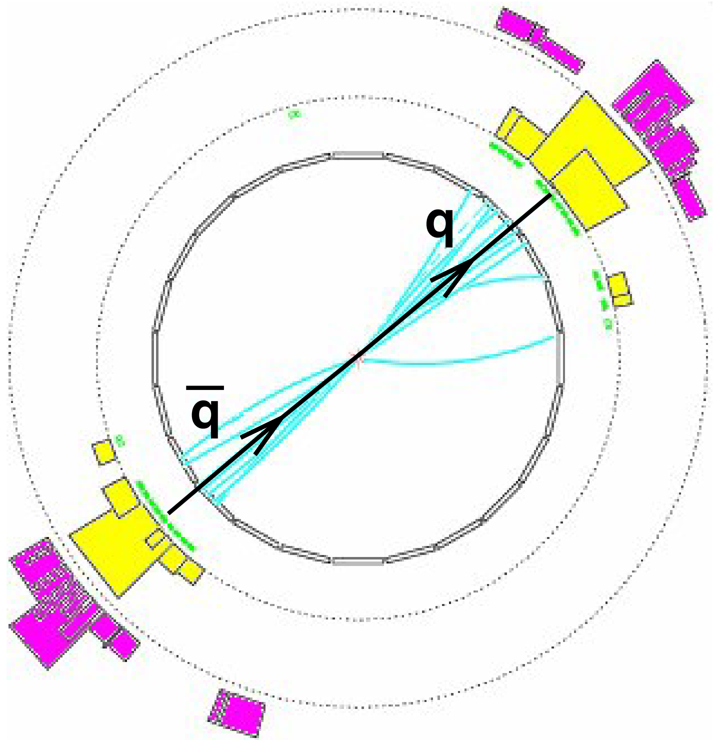


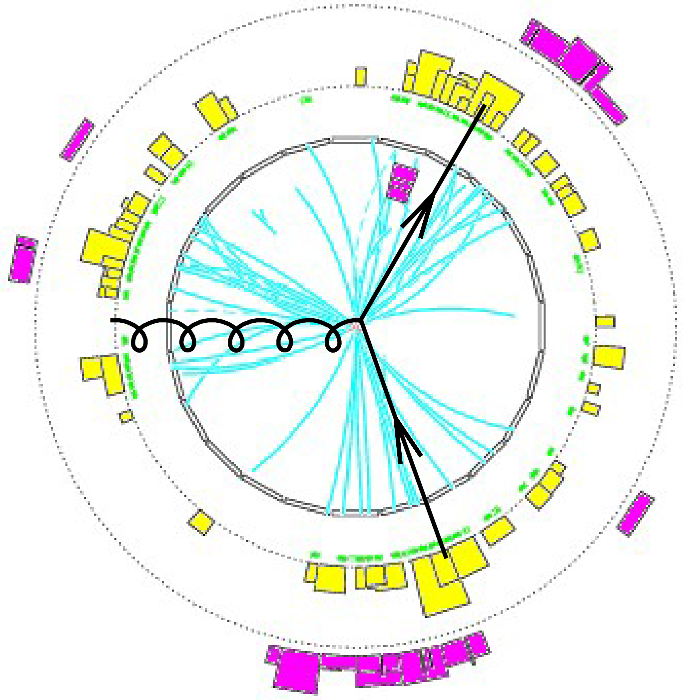


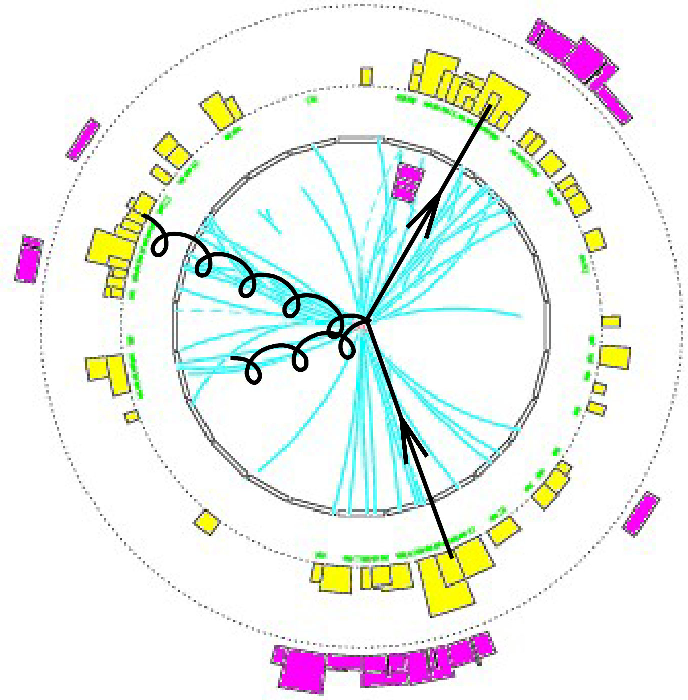


2-jets

Jet algorithms



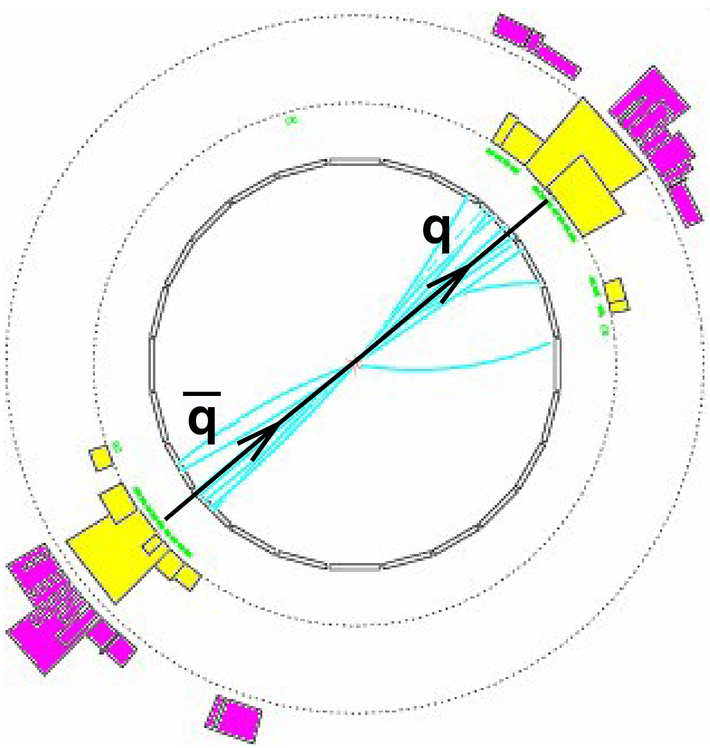




2-jets

3-jets

Jet algorithms



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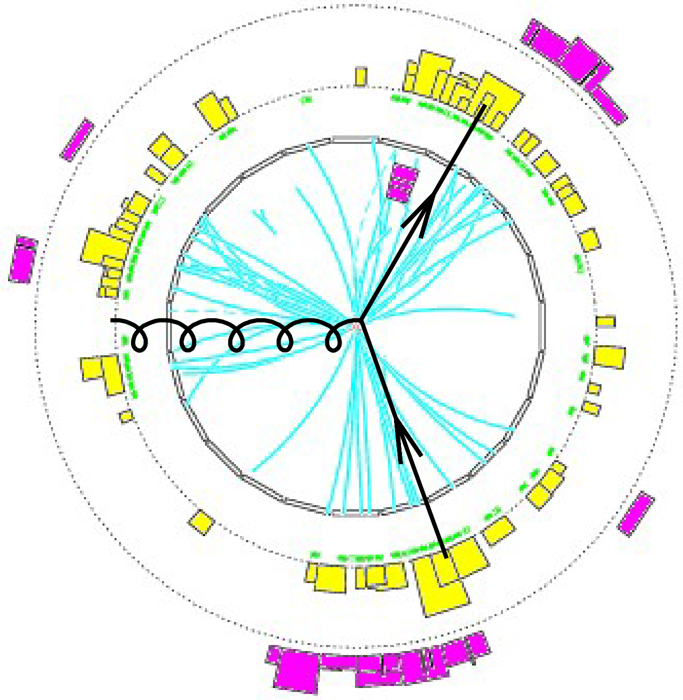
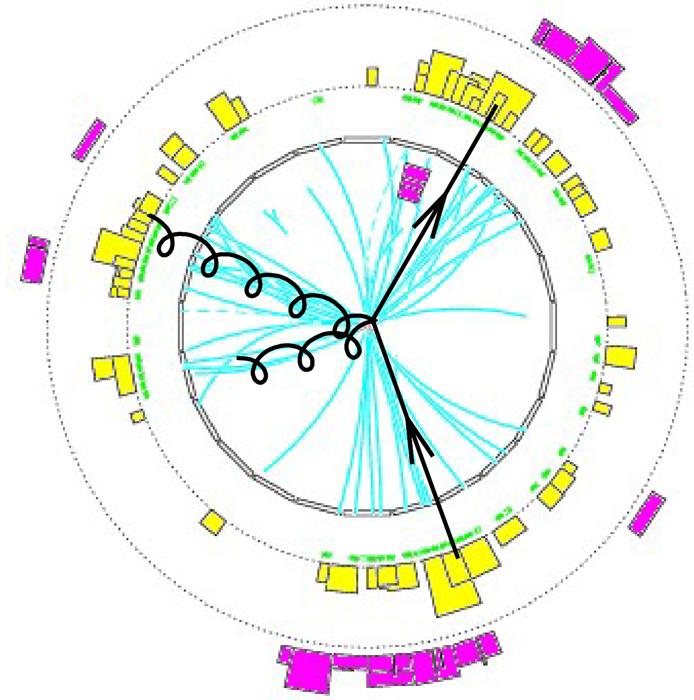
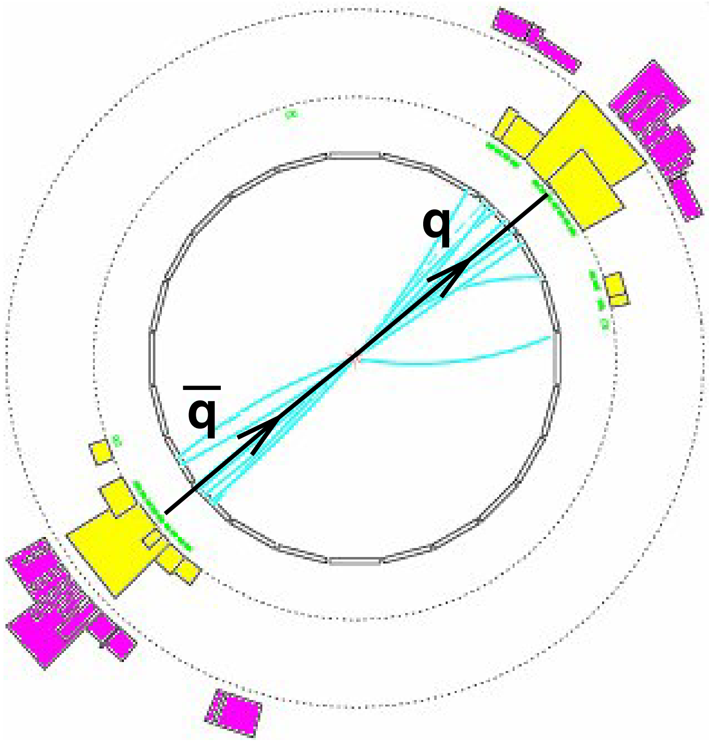


3-jets

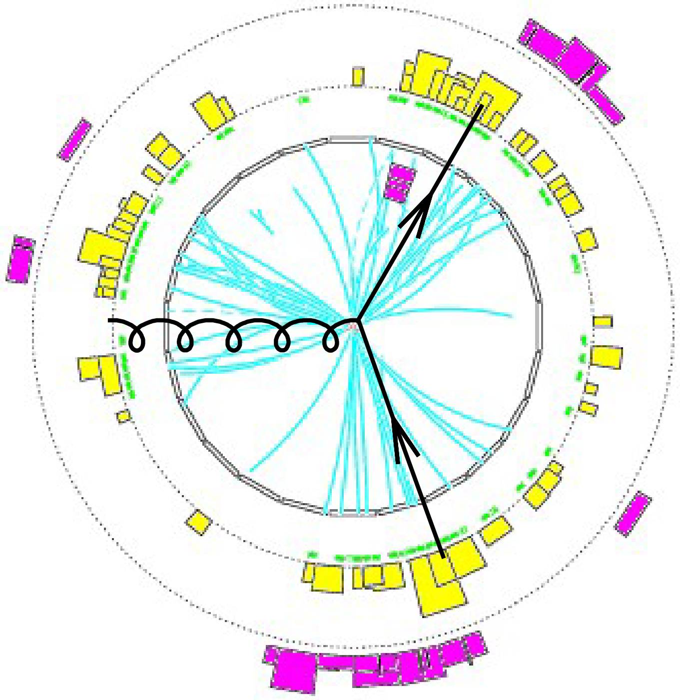


4-jets

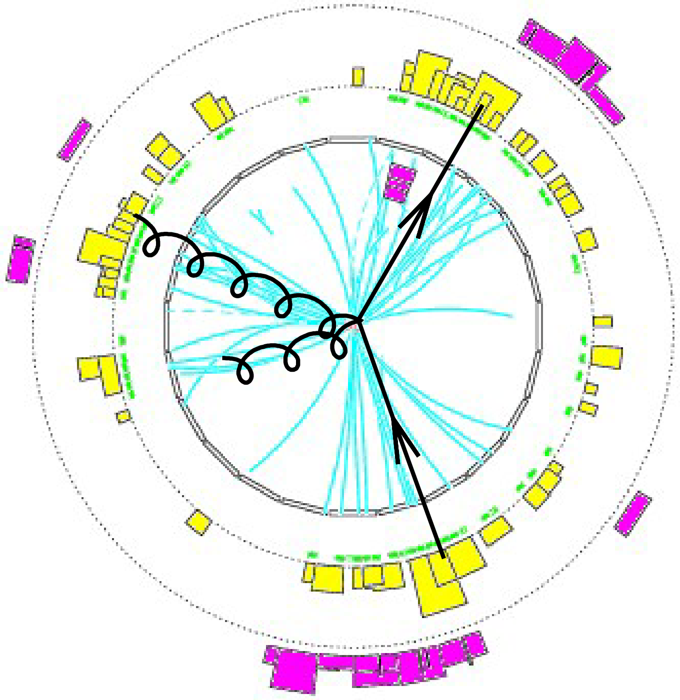
Jet algorithms



2-jets



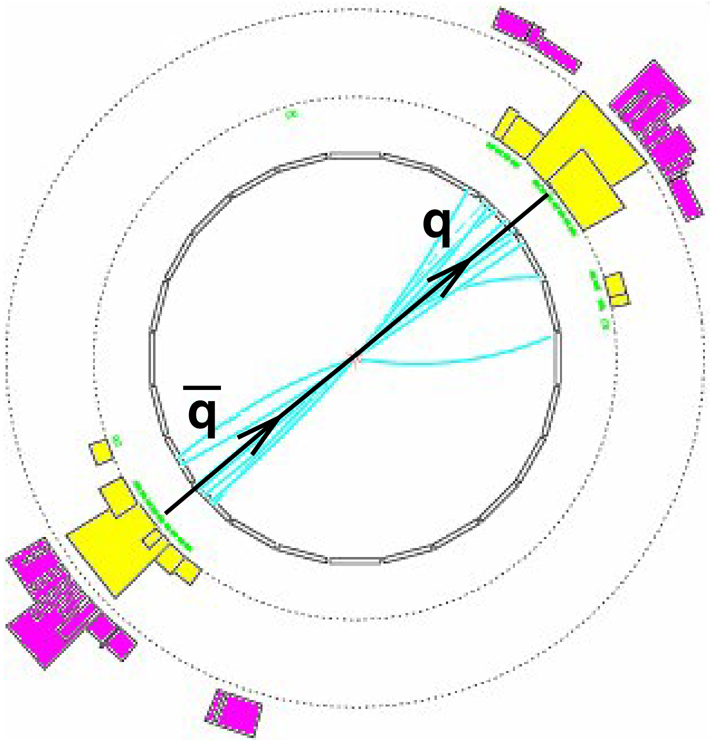
3-jets

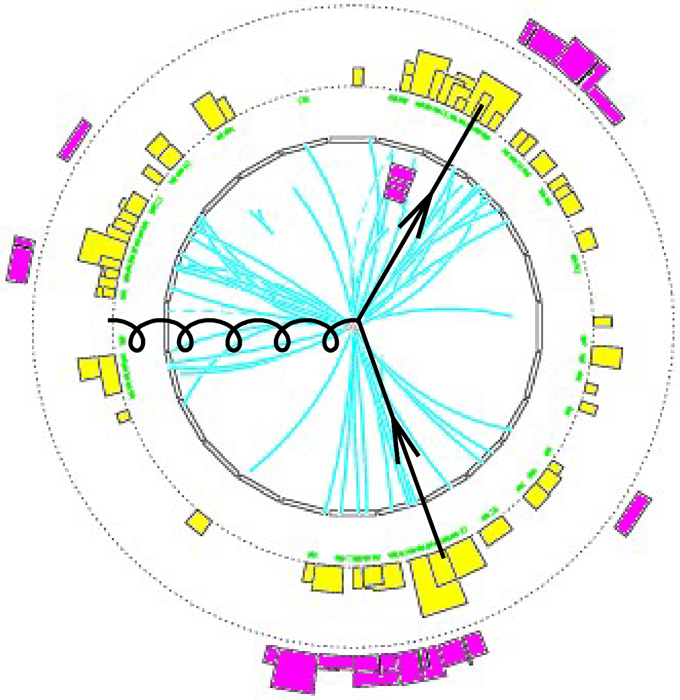


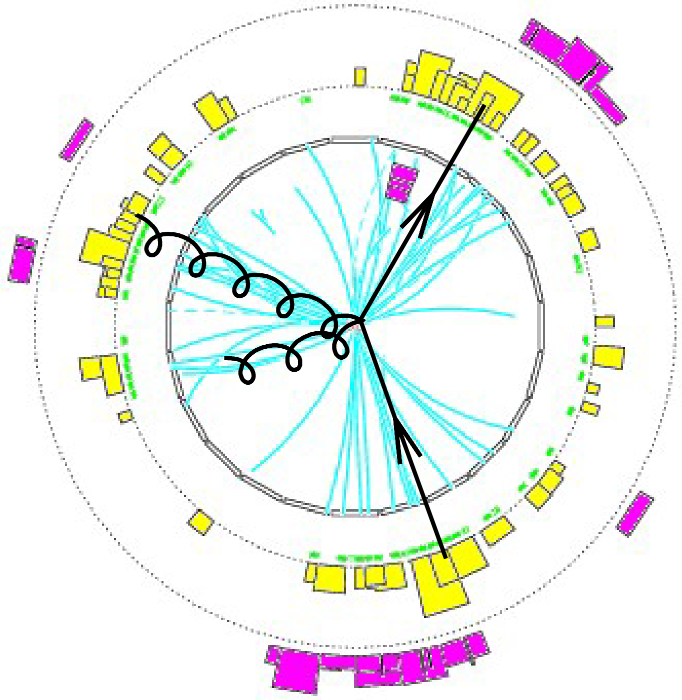
4-jets

Want more?

Jet algorithms







2-jets

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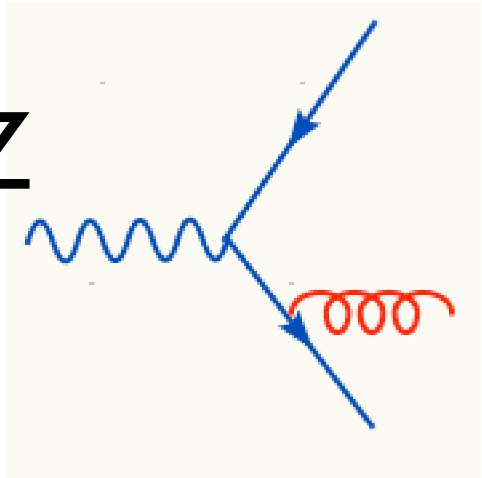
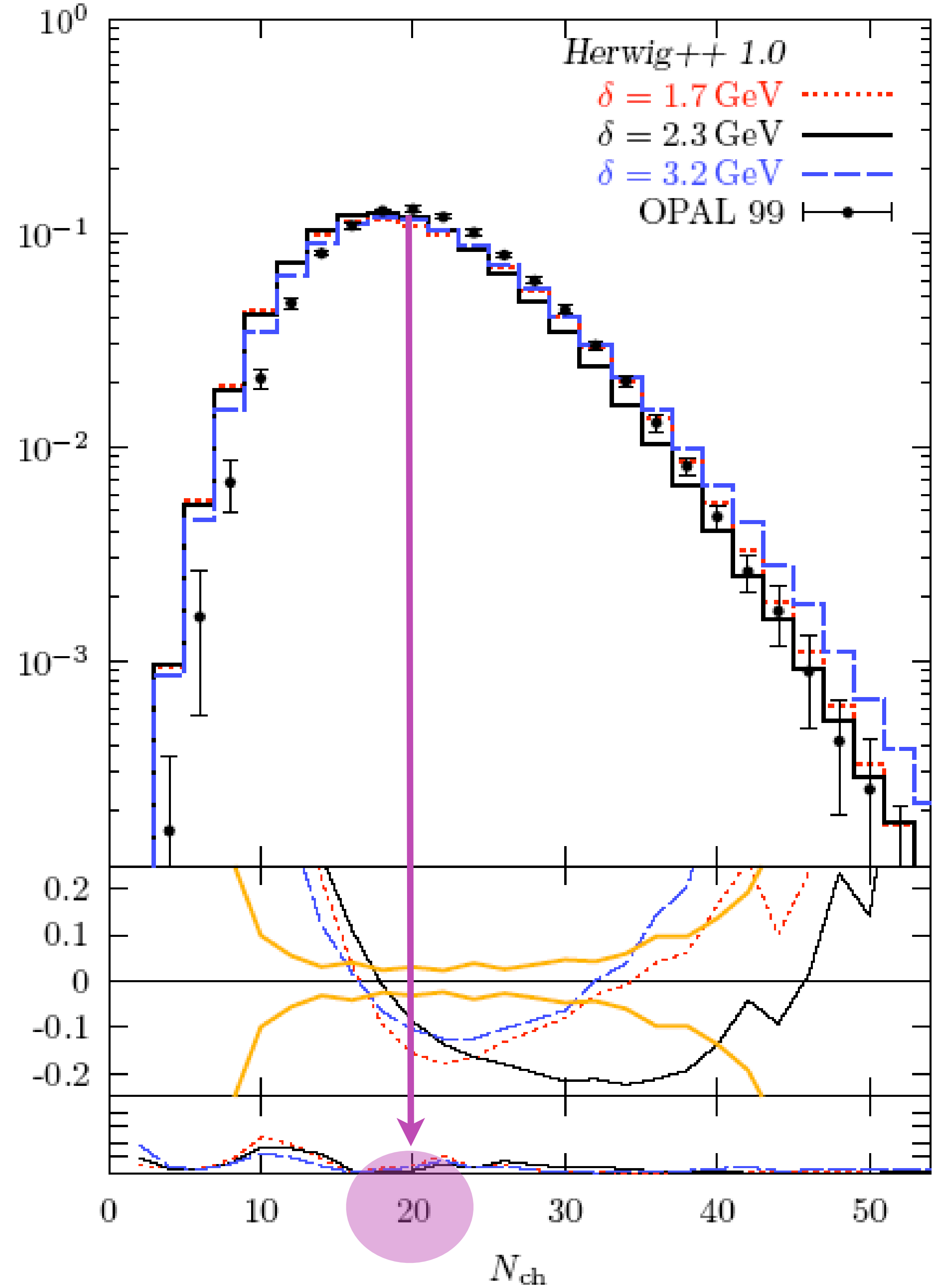
Want more?

γ*,Z



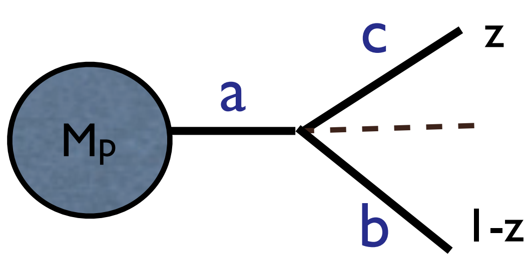
?



Herwig++ 1.0
δ = 1.7 GeV
δ = 2.3 GeV
δ = 3.2 GeV
OPAL 99
10^0
10^{-1}
10^{-2}
10^{-3}
0.2
0.1
0
-0.1
-0.2
0
10
20
30
40
50
N_{ch}

Parton showers

ME involving q →q g (or g → gg) are strongly enhanced when they are close in the phase space:



$$z = E_b/E_a\,, t = k_a^2$$

$$\theta = \theta_b + \theta_c = \frac{\theta_b}{1-z} = \frac{\theta_c}{z} = \frac{1}{E_a}\sqrt{\frac{t}{z(1-z)}}$$

$$\frac{1}{(p_q + p_g)^2} \simeq \frac{1}{2E_q E_g (1 - \cos\theta)}$$

$$d\sigma_{N+1} = d\sigma_N \frac{dt}{t}\frac{d\phi}{2\pi} dz \frac{\alpha_S}{2\pi} |K_{ba}(z)|^2$$

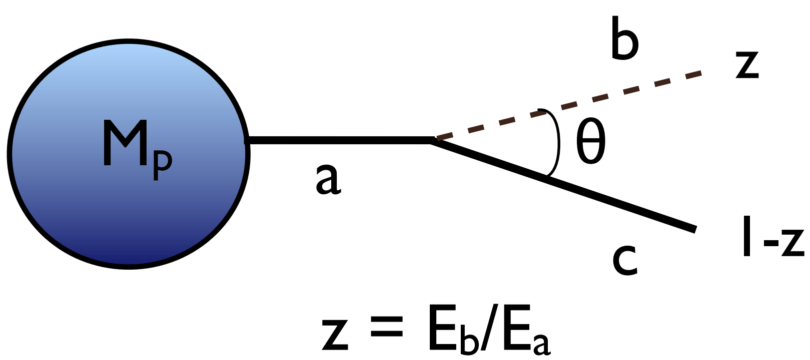
$$d\bar{\sigma}_{N+1} = d\bar{\sigma}_N \frac{dt}{t} dz \frac{\alpha_S}{2\pi} P_{ba}(z)$$

In the collinear limit the cross section factorizes. The splitting can be iterated.

Parton Shower basics

Matrix elements involving q →q g (or g → gg) are strongly enhanced when the final state particles are close in the phase space:

$$\frac{1}{(p_b+p_c)^2} \simeq \frac{1}{2E_bE_c(1-\cos\theta)} = \frac{1}{t}$$



soft and collinear

divergencies

Collinear factorization:

$$|M_{p+1}|^2 d\Phi_{p+1} \simeq |M_p|^2 d\Phi_p \frac{dt}{t}\frac{\alpha_S}{2\pi}P(z)dzd\phi$$

when θ is small.

Parton Shower basics

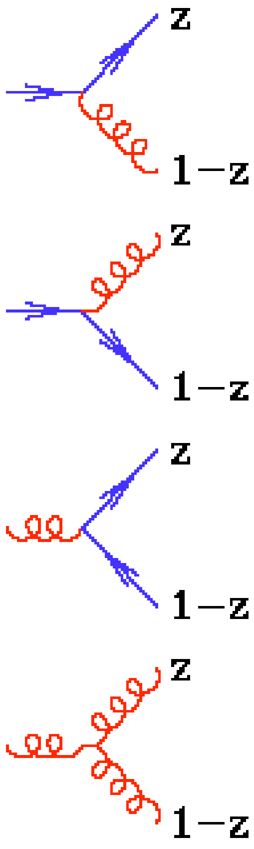
The spin averaged (unregulated) splitting functions for the various types of branching are:

$$\hat{P}_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)}\right],$$

$$\hat{P}_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z}\right],$$

$$\hat{P}_{qg}(z) = T_R \left[z^2+(1-z)^2\right],$$

$$\hat{P}_{gg}(z) = C_A \left[\frac{z}{(1-z)} + \frac{1-z}{z} + z\,(1-z)\right].$$



$C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}.$

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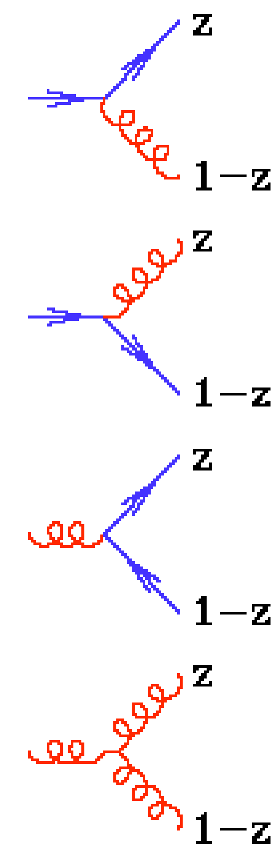

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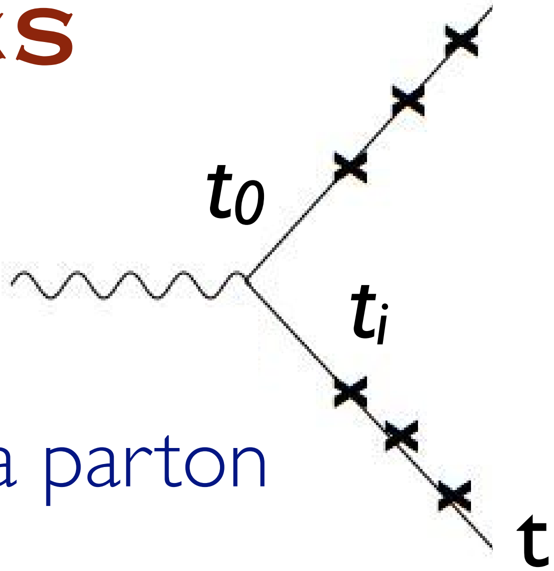
Comments:

* Gluons radiate the most

* There soft divergences in z=1 and z=0.

* P_{qg} has no soft divergences.

PARTON SHOWER BASICS



- Now, consider the non-branching probability for a parton at a given virtuality t_i:

$$\mathcal{P}_{\mathrm{non-branching}}(t_i) = 1 - \mathcal{P}_{\mathrm{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int_z^1 \frac{dz}{z}\hat{P}(z)$$

- The total non-branching probability between virtualities t and t_0:

$$\mathcal{P}_{\mathrm{non-branching}}(t, t_0) \simeq \prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int_z^1 \frac{dz}{z}\hat{P}(z)\right)$$

$$= e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int_z^1 \frac{dz}{z}\hat{P}(z)\right)}$$

$$\simeq e^{-\int_t^{t_0}\frac{dt'}{t'}\frac{\alpha_s}{2\pi}\int_z^1 \frac{dz}{z}\hat{P}(z)} = \Delta(t, t_0)$$

- This is the famous "Sudakov form factor"

Final-state parton showers

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2. Given a virtual mass scale t_i and momentum fraction x_i at some stage in the evolution, generate the scale of the next emission t_{i+1} according to the Sudakov probability $\Delta(t_i,t_{i+1})$ by solving
   $\Delta(t_{i+1},t_i) = R$
   where R is a random number (uniform on [0, 1]).

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where R is a random number (uniform on [0, 1]).
3. If $t_{i+1} < t_{cut}$ it means that the shower has finished.
4. Otherwise, generate $z = z_i/z_{i+1}$ with a distribution proportional to $(\alpha_s/2\pi)P(z)$, where P(z) is the appropriate splitting function.

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With the Sudakov form factor, we can now implement a final-state parton shower in a Monte Carlo event generator!

1. Start the evolution at the virtual mass scale t_0 (e.g. the mass of the decaying particle) and momentum fraction $z_0 = 1$
2. Given a virtual mass scale t_i and momentum fraction x_i at some stage in the evolution, generate the scale of the next emission t_{i+1} according to the Sudakov probability $\Delta(t_i,t_{i+1})$ by solving
   $\Delta(t_{i+1},t_i) = R$
   where R is a random number (uniform on [0, 1]).
3. If $t_{i+1} < t_{cut}$ it means that the shower has finished.
4. Otherwise, generate $z = z_i/z_{i+1}$ with a distribution proportional to $(\alpha_s/2\pi)P(z)$, where $P(z)$ is the appropriate splitting function.
5. For each emitted particle, iterate steps 2-4 until branching stops.

Parton showers

Formulation in terms of Sudakov form factor is well suited to computer implementation, and is the basis of parton shower Monte Carlo programs. Let's rewrite the formula using p_T and a parton-level event at the Born level:

$$d\sigma^{\rm PS} = d\Phi_B B(\Phi_B)\left[\Delta(p_\perp^{\rm min}) + d\Phi_{R|B}\Delta(p_T(\Phi_{R|B}))\frac{R^{\rm PS}(\Phi_R)}{B(\Phi_B)}\right]$$

$$\Delta(p_T) = \exp\left[-\int d\Phi_{R|B}\frac{R^{\rm PS}(\Phi_R)}{B(\Phi_B)}\Theta(p_T(\Phi_R)-p_T)\right] \;.\; R^{\rm PS}(\Phi) = P(\Phi_{R|B})\,B(\Phi_B).$$

Monte Carlo branching algorithm operates as follows. Given an initial configuration (parton-level event at the Born level), a parton is chosen, a rnd value of p_T is chosen accordingly to the probability of non-emission down to p_T . If it is larger than a p_T^{min}, than a branching occurs at p_T , and x is generated according to the splitting function $P(\Phi_{R|B})$ (as well as a flat azimuthal angle). An extra parton is now included and the process starts from there.

Due to successive branching, a parton cascade or shower develops. Each outgoing line is source of a new cascade, until all lines have stopped branching. At this stage, which depends on p_T^{min}, outgoing partons have to be converted into hadrons.

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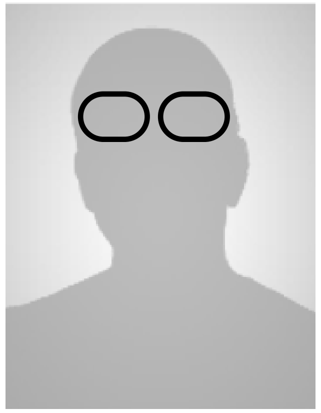
Want more?

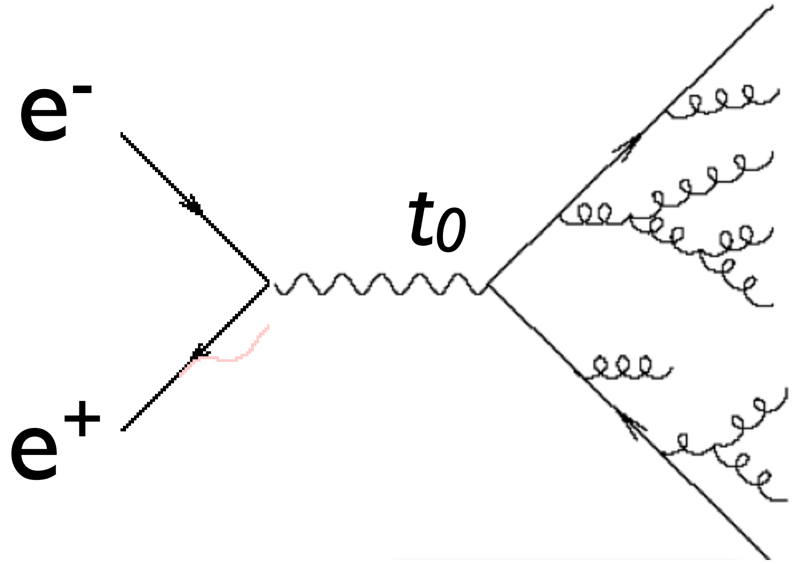


Ask Paolo!

Final-state parton showers

Final-state parton showers

- The result is a "cascade" or "shower" of partons with ever smaller virtualities.



Final-state parton showers

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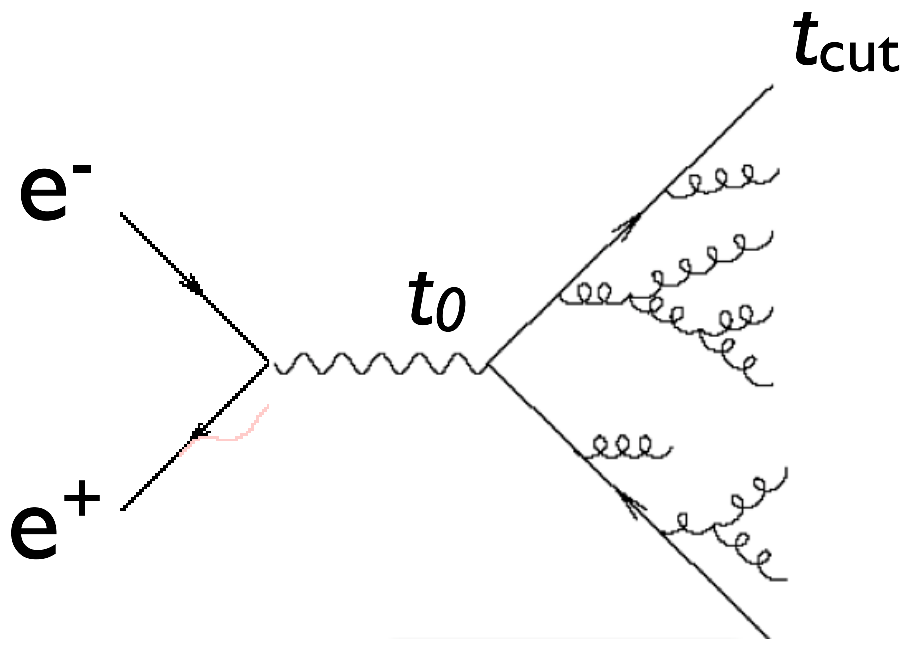
- The cutoff scale t_{cut} is usually set close to 1 GeV, and is the scale where non-perturbative effects start dominating over the perturbative parton shower.



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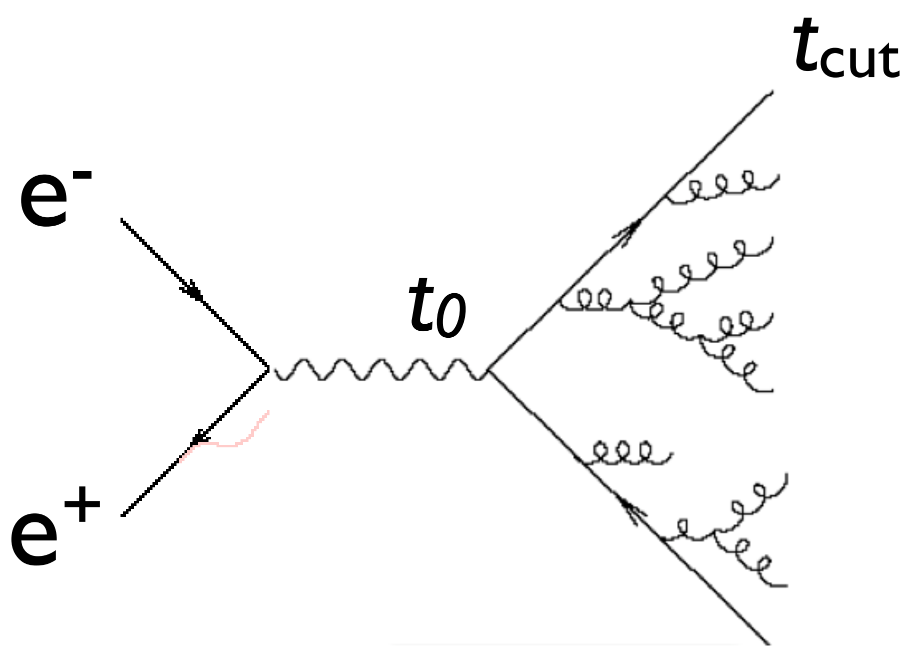
  Main point: Hadronization not sensitive to the physics at scale t_0, but only t_{cut}!



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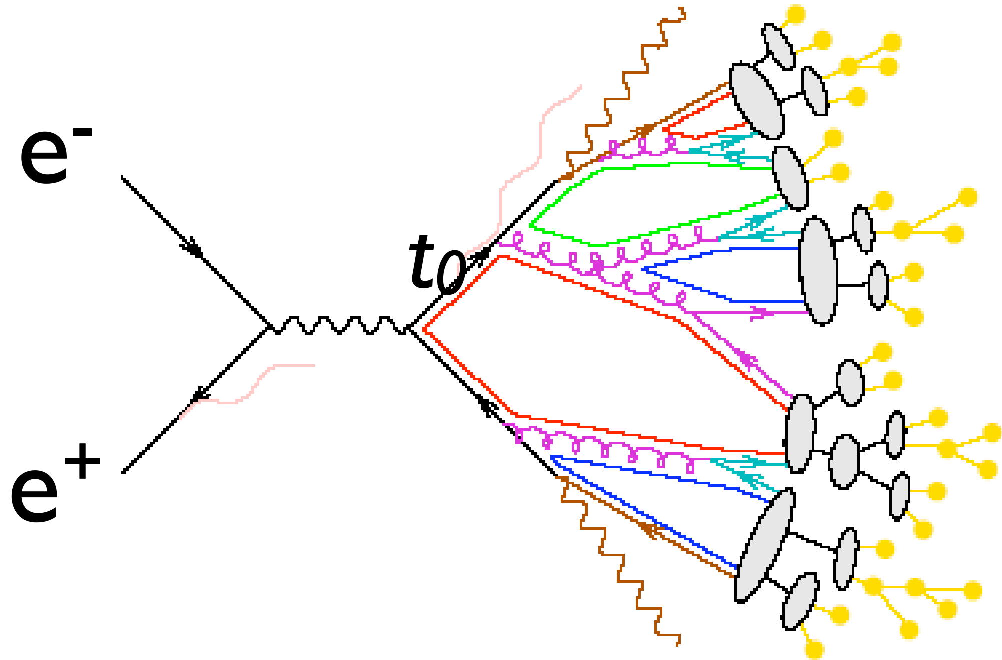
- (can be tuned once and for all)



Parton showers

Note that we can define the following quantities with mass squared dimensions

and obtain

$$
\begin{aligned}
Q^2 &= z(1-z)\theta^2 E^2 \\
p_T^2 &= z^2(1-z)^2\theta^2 E^2 \\
\tilde{t} &= \theta^2 E^2
\end{aligned}
$$

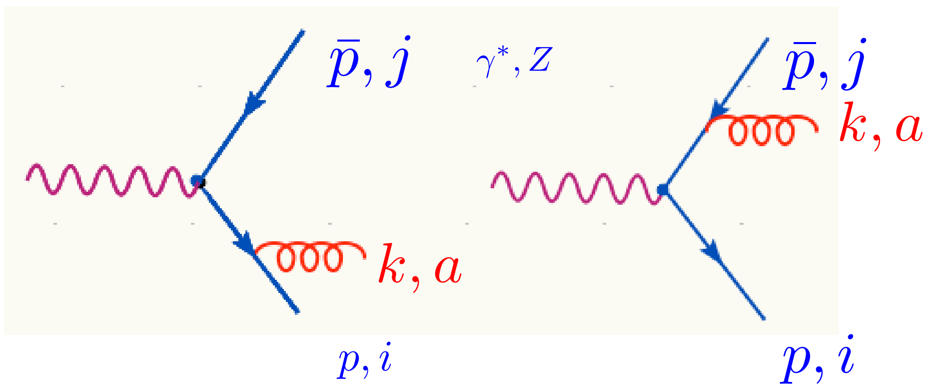
$$\frac{d\theta^2}{\theta^2} = \frac{dQ^2}{Q^2} = \frac{dp_T^2}{p_T^2} = \frac{d\tilde{t}}{\tilde{t}}$$

Different MC programs make different choices for the variable. HERWIG uses θ, while Pythia uses p_T.

This fact has an important consequence: the evolution parameter of the shower is not uniquely defined. This is because the scales chosen above have all the same angular behavior, provided that z is not too close to 0 or 1.

Differences stem from the SOFT region. It is therefore necessary to study what happens for soft emissions to find the optimal choice.

ANGULAR ORDERING



$$d\sigma_{qqg} = C_F \frac{\alpha_S}{2\pi} \sigma^{\text{Born}} d\cos\theta \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} d\cos\theta$$

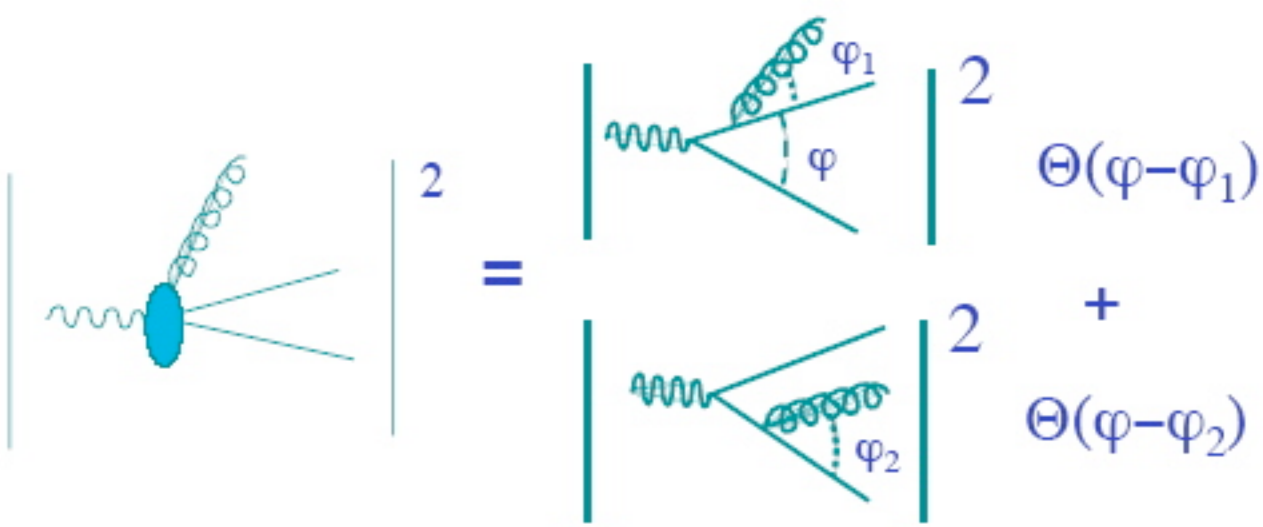
You can easily prove that:

$$\frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} = \frac{1}{2}\left[\frac{\cos\theta_{jk}-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} + \frac{1}{(1-\cos\theta_{jk})}\right] + \frac{1}{2}[i \to j]$$

The probabilistic interpretation of W_i and W_j is achieved simply by azimuthal averaging:

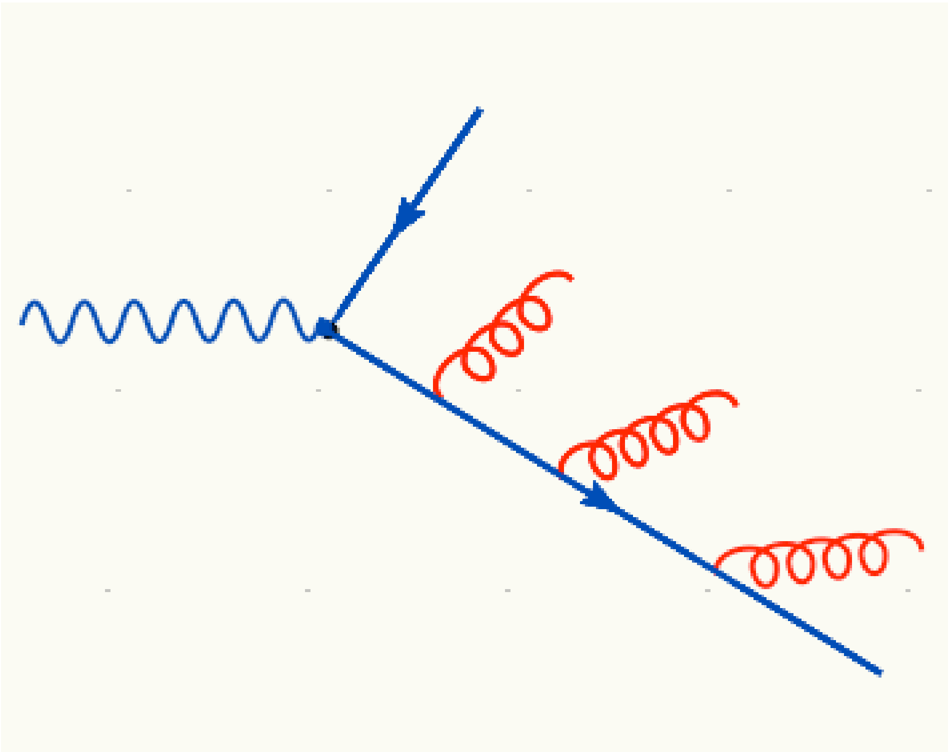
$$\int \frac{d\phi}{2\pi} W_i = \frac{1}{1-\cos\theta_{ik}} \quad \text{if} \quad \theta_{ik} < \theta_{ij}\ , 0\,\text{otherwise}$$

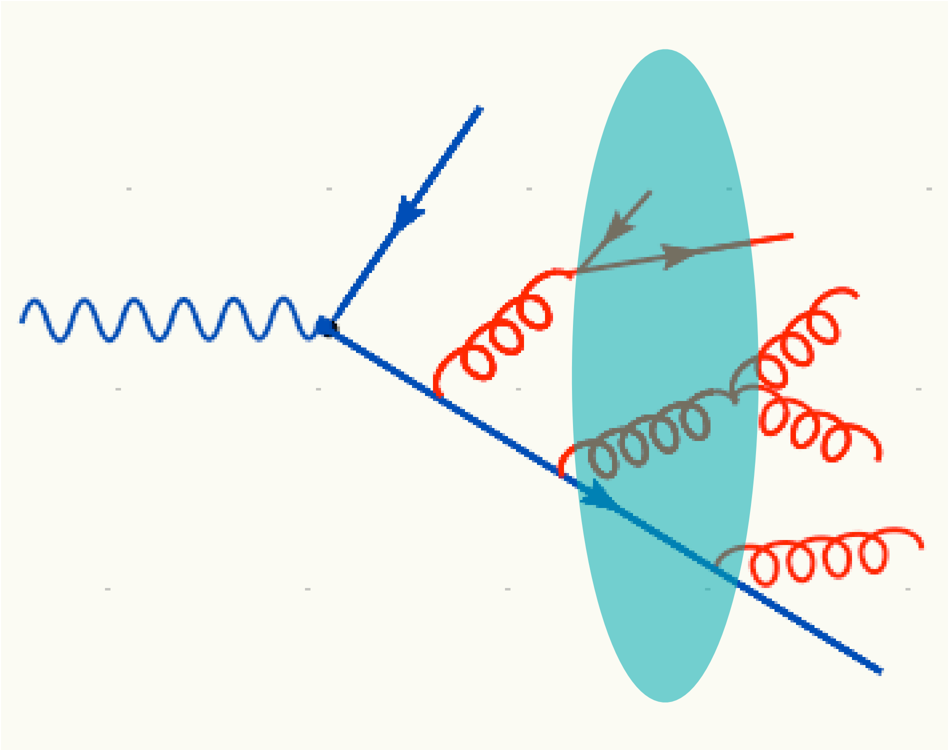
And the same for W_j



Radiation happens only for angles smaller than the color connected (antenna) opening angle!

Angular ordering





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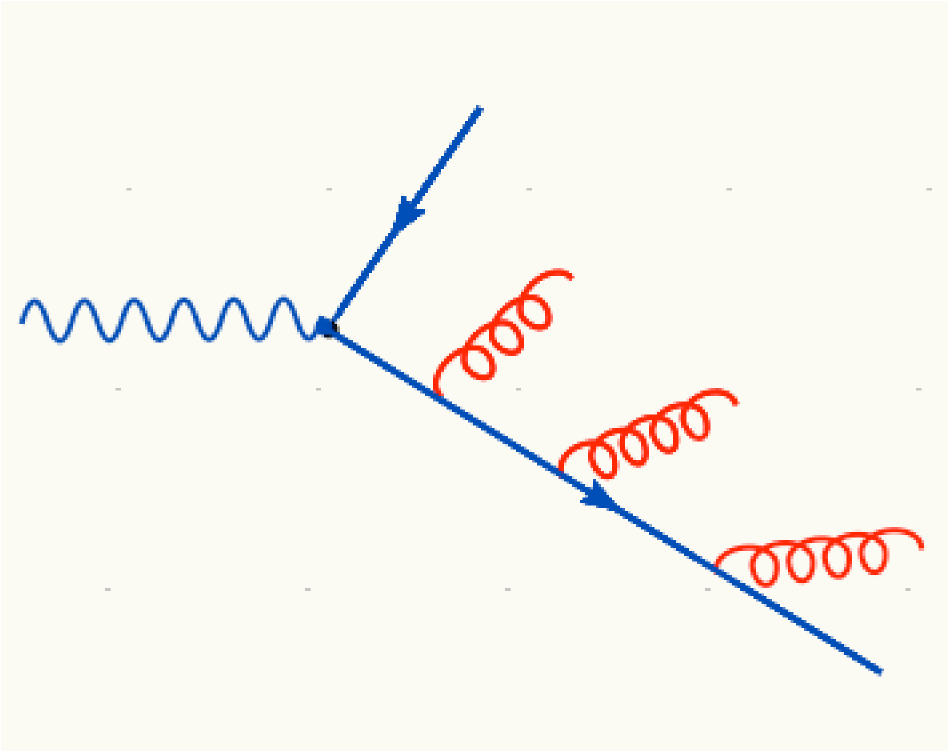
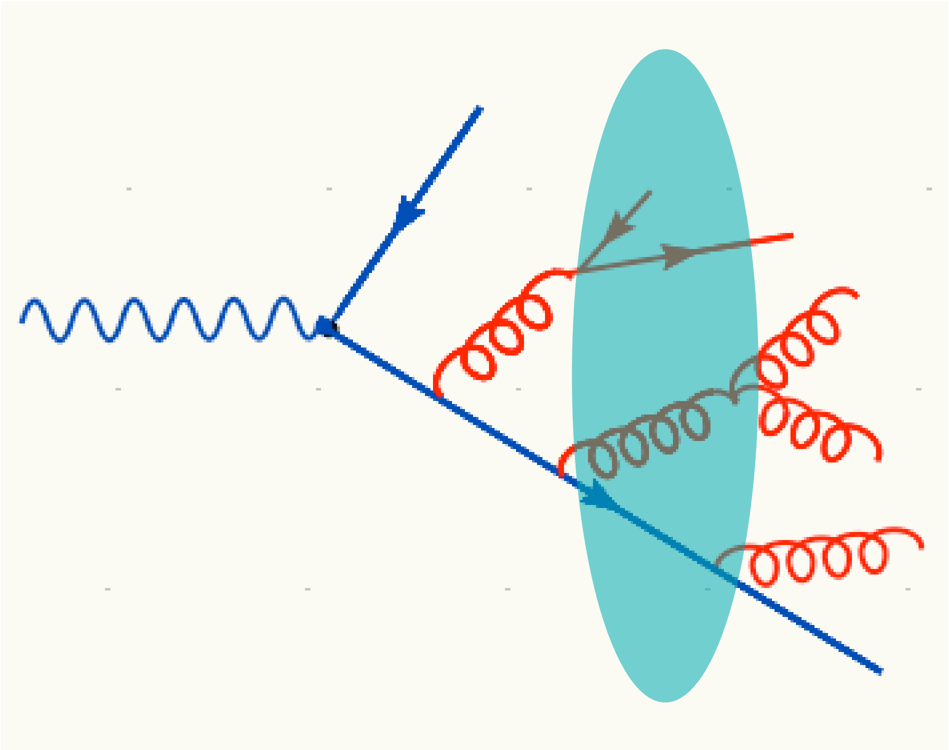
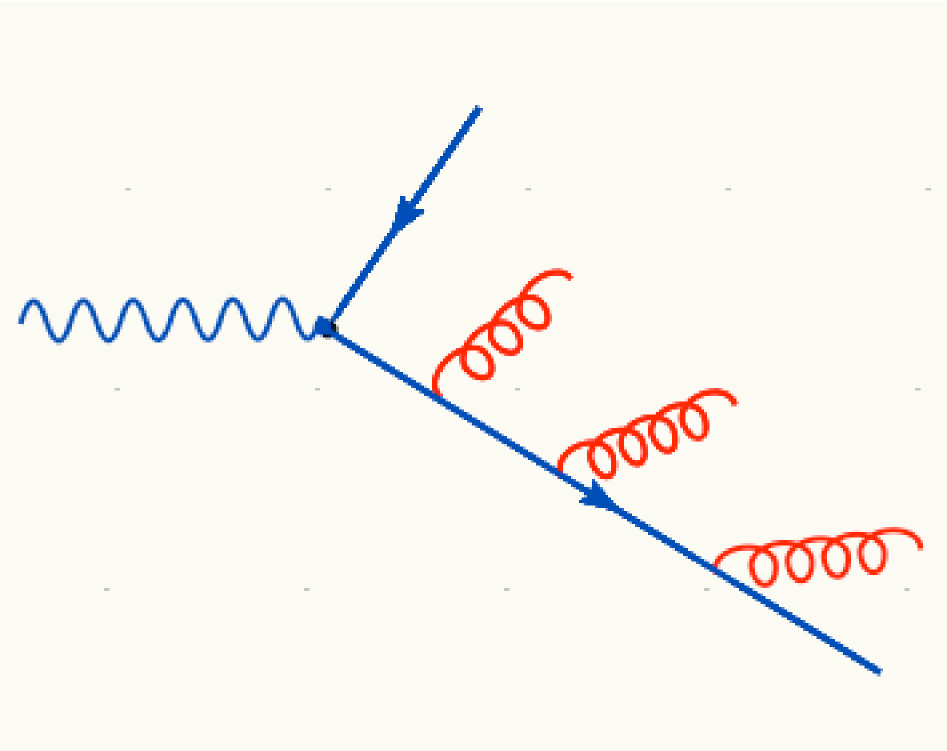
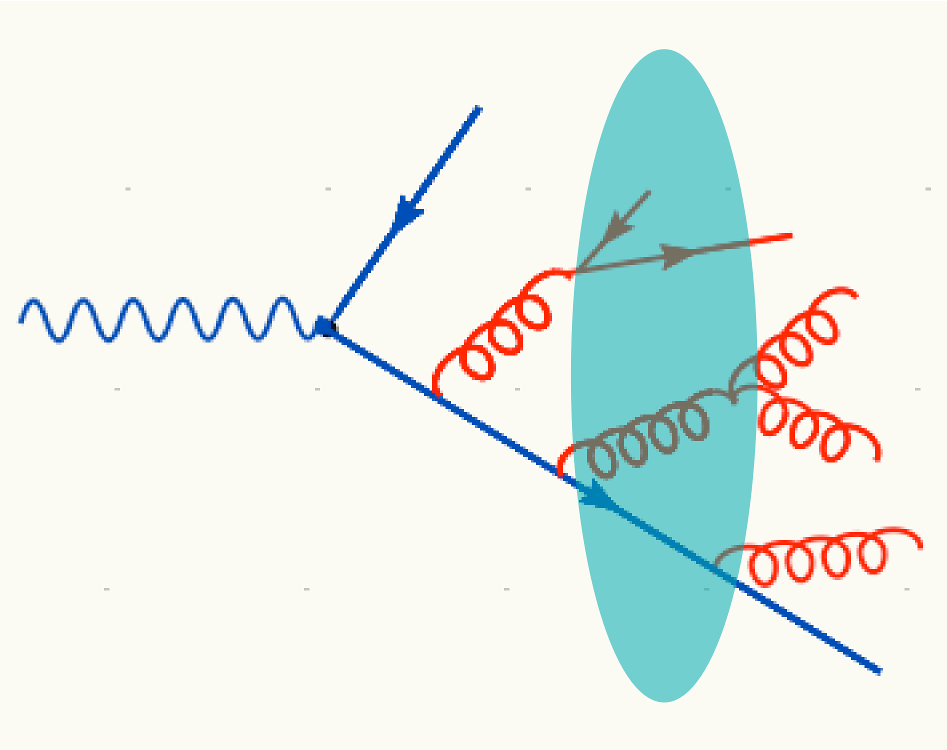




The construction can be iterated to the next emission, with the result that the emission angles keep getting smaller and smaller.

Angular ordering





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One can generalize it to a generic parton of color charge Qk splitting into two partons i and j , Qk=Qi +Qj. The result is that inside the cones i and j emit as independent charges, and outside their angular-order cones the emission is coherent and can be treated as if it was directly from color charge Qk.

KEY POINT FOR THE MC!

Angular ordering is automatically satisfied in p_T and θ ordered showers!

Angular ordering

Angular ordering is:

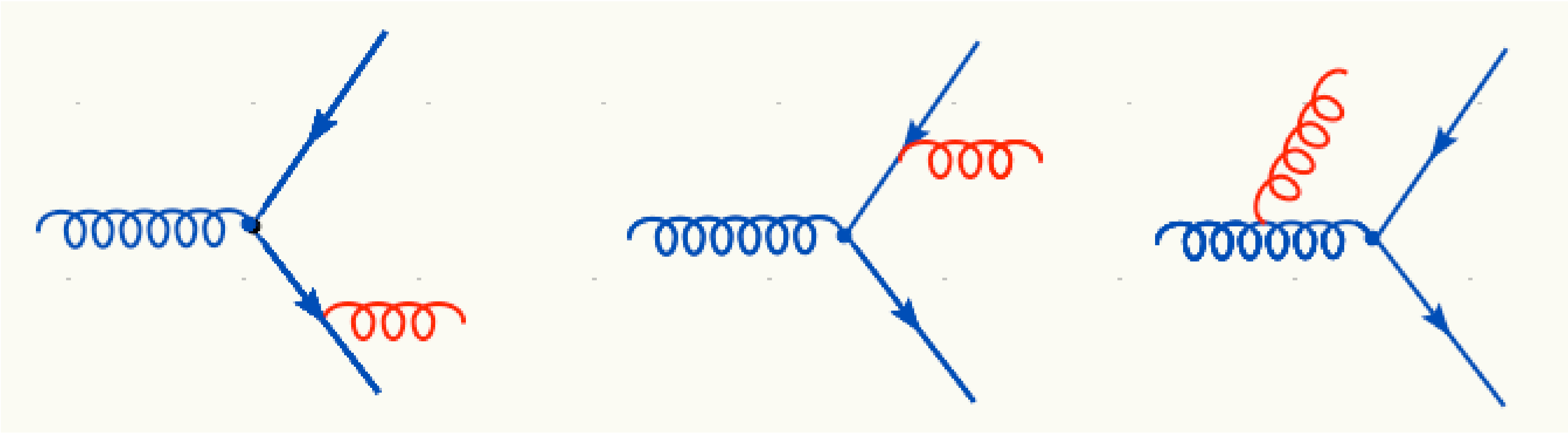
1. A quantum effect coming from the interference of different Feynman diagrams.

2. Nevertheless it can be expressed in "a classical fashion" (square of a amplitude is equal to the sum of the squares of two special "amplitudes"). The classical limit is the dipole-radiation.

3. It is not an exclusive property of QCD (i.e., it is also present in QED) but in QCD produces very non-trivial effects, depending on how particles are color connected.

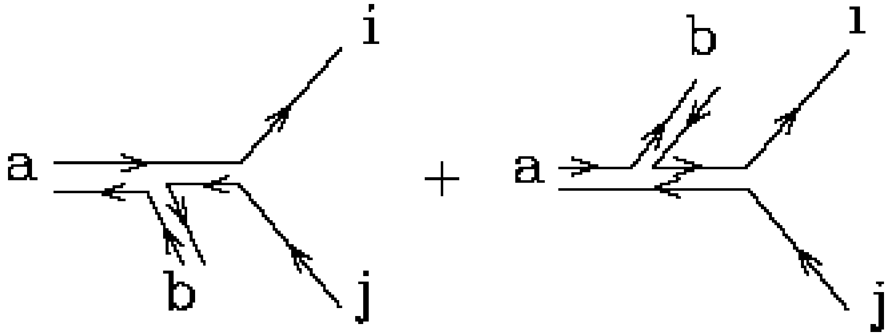
Angular ordering

How does look the amplitude for a soft-emission in a qqg system? (Virtual photon not shown, coming out of the screen)



$$A_{soft} = -g_s \left\{ (t^a t^b)_{ij} \left[\frac{Q\cdot\epsilon}{Q\cdot k} - \frac{\bar{p}\cdot\epsilon}{\bar{p}\cdot k} \right] - (t^b t^a)_{ij} \left[\frac{p\cdot\epsilon}{p\cdot k} - \frac{Q\cdot\epsilon}{Q\cdot k} \right] \right\} A_{Born}$$

The two terms correspond to the two possible ways colour can flow in these diagrams:



The interference between the two color structures is suppressed by 1/Nc2:

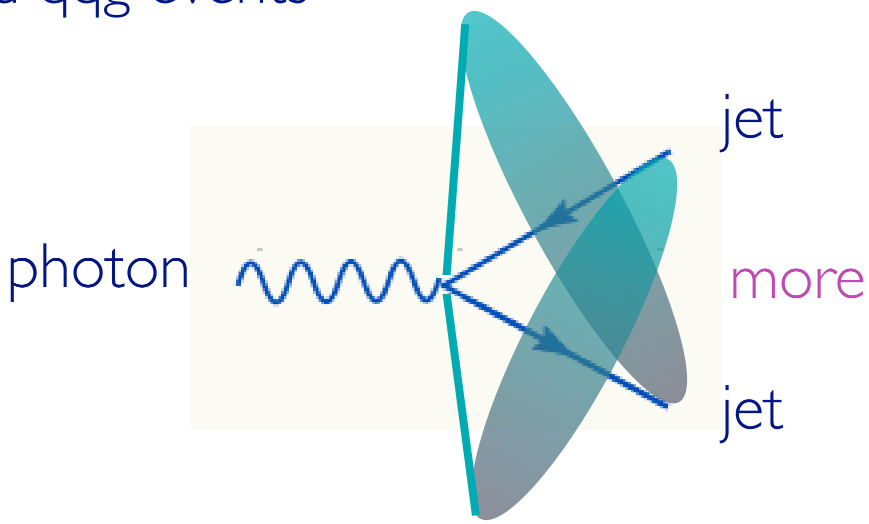
$$\sum_{a,b,i,j} |(t^a t^b)|^2 = \frac{N_c^2-1}{2}\frac{N_c^2-1}{2N_c} = O(N_c^3) \qquad \sum_{a,b,i,j} (t^a t^b)(t^b t^a)^\dagger = \frac{N_c^2-1}{2}(-\frac{1}{2N_c}) = O(N_c)$$

In the large Nc limit, this is equivalent to the incoherent sum of the emission from the two currents.

Angular ordering

(a) amount of radiation between two quark-jets in qqγ and qqg events

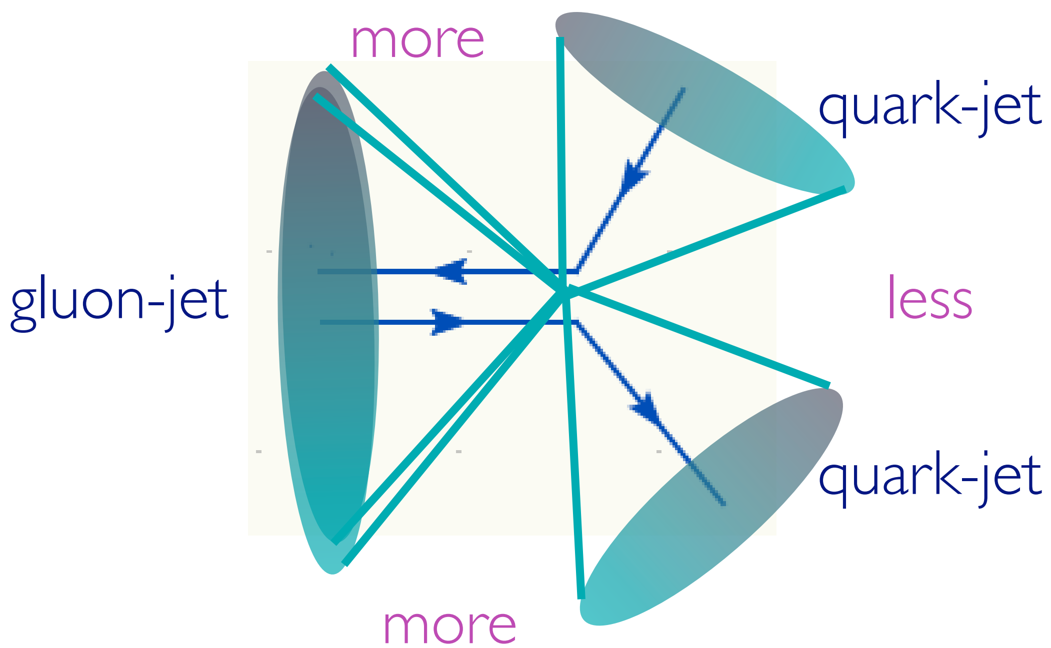
$$\frac{dN_{q\bar{q}}^{(q\bar{q}\gamma)}}{dN_{q\bar{q}}^{(q\bar{q}g)}} \simeq \frac{2(N_c^2-1)}{N_c^2-2} = \frac{16}{7}$$



$$(\text{experiment}: 2.3 \pm 0.2)$$

(b) radiation between the qg and qq

$$\frac{dN_{qg}^{(q\bar{q}g)}}{dN_{q\bar{q}}^{(q\bar{q}g)}} \simeq \frac{5(N_c^2-1)}{2N_c^2-4} = \frac{22}{7}$$



Parton Shower MC event generators

A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

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Complete MC Generators: PYTHIA, HERWIG, SHERPA

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