QCD Basics for accurate LHC Physics

Lecture I

Fabio Maltoni
Center for Particle Physics and Phenomenology (CP3)
Université Catholique de Louvain

Claims and Aims

LHC is live and kicking!!!!

There has been a number of key theoretical results recently in the quest of achieving the best possible predictions and description of events at the LHC.

Perturbative QCD applications to LHC physics in conjunction with Monte Carlo developments are VERY active lines of theoretical research in particle phenomenology.

In fact, new dimensions have been added to
Theory ⇔ Experiment interactions

Claims and Aims

Four lectures:

1. Intro and QCD fundamentals
2. QCD in the final state

basics

3. From accurate QCD to useful QCD
4. Advanced QCD with applications at the LHC

apps

Claims and Aims

- perspective: **the big picture**
- concepts: QCD from high-Q^2 to low-Q^2, asymptotic freedom, infrared safety, factorization
- tools & techniques: Fixed Order (FO) computations, Parton showers, Monte Carlo's (MC)
- recent progress: merging MC's with FO, new jet algorithms
- sample applications at the LHC: Drell-Yan, Higgs, Jets, BSM,...

Test : How much do I know about MC's?

	Statements	TRUE	FALSE	IT DEPENDS	I have no clue
0	MC's are black boxes, I don't need to know the details as long as the are no bugs.				
1	A MC generator produces "unweighted" events , i.e., events distributed as in Nature.				
2	MC's are based on a classical approximation (Markov Chain), QM effects are not included.				
3	The "Sudakov form factor" directly quantifies how likely is for a parton to undergo branching.				
4	A calculations/code at NLO for a process provides NLO predictions for any IR safe observable.				
5	Tree-level based MC's are less accurate than those at NLO.				

Test : How much do I know about MC's?

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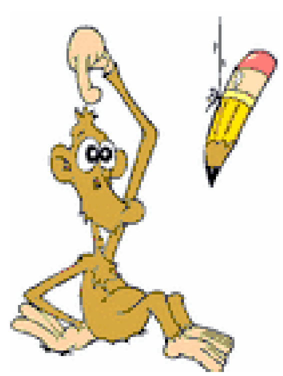
1	A MC generator produces "unweighted" events , i.e., events distributed as in Nature.	✓			
2	MC's are based on a classical approximation (Markov Chain), QM effects are not included.		✓		
3	The "Sudakov form factor" directly quantifies how likely is for a parton to undergo branching.		✓		
4	A calculations/code at NLO for a process provides NLO predictions for any IR safe observable.		✓		
5	Tree-level based MC's are less accurate than those at NLO.			✓	

Test : How much do I know about MC's?

Score	Result	Comment
5	Addict	Always keep in mind that there are also other interesting activities in the field.
4	Excellent	No problem in following these lectures.
3	Fair	Check out carefully the missed topics.
≤2	Room for improvement (not passing the Turing test)	Enroll in a MC crash course at your home institution.
5 No clue	No clue	

Claims and (your) Aims



Think

Ask

Work

Mathematica notebooks on a "simple" NLO calculation and other exercises on QCD applications to LHC phenomenology available on the MadGraph Wiki.

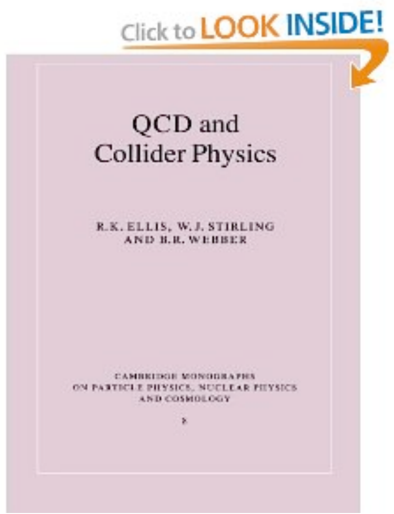
https://server06.fynu.ucl.ac.be/projects/madgraph/wiki/SchoolIndia

Schedule

	MON	TUE	WED	THU	FRI
9:00	Fabio Maltoni	Fabio Maltoni	Safari since 7:00	Rikkert Frederix	Paolo Torrielli
10:00	Fabio Maltoni	Matteo Cacciari	Back-from Safari	Matteo Cacciari	Paolo Torrielli
11:00	Break	Break	Break	Break	Break
11:30	Matteo Cacciari	Rikkert Frederix	Fabio Maltoni	Paolo Torrielli	Matteo Cacciari
12:30	Matteo Cacciari	Rikkert Frederix	Paolo Torrielli	Paolo Torrielli	Matteo Cacciari
13:30	Lunch	Lunch	Lunch	Lunch	Lunch
15:00	Fabio Maltoni Matteo Cacciari	Fabio Maltoni Matteo Cacciari	Rikkert Frederix	Paolo Torrielli	Matteo Cacciari
17:00	Break	Break	Break	Break	Break
17:30	Work	Work	Rikkert Frederix	Work	Work
19:30	End/Start	End/Start	End/Start	End/Start	End/START!

Minimal References

- Ellis, Stirling and Webber: The Pink Book



- Excellent lectures on the archive
  by M. Mangano, P. Nason,
  and more recently
  by G. Salam, P. Skands.

QCD : THE FUNDAMENTALS

1. QCD is a good theory for strong interactions: facts
2. From QED to QCD: the importance of color
3. Renormalization group and asymptotic freedom

Strong interactions

Strong interactions are characterized at moderate energies by a single* dimensionful scale, Λ_S , of few hundreds of MeV:

$$\sigma_h \cong 1/\Lambda_s^2 \cong 10 \text{ mb}$$

$$\Gamma_h \cong \Lambda_s$$

$$R \cong 1/\Lambda_s \cong 1 \text{ fm}$$

No hint to the presence of a small parameter! Very hard to understand and many attempts...

*neglecting quark masses..!!!

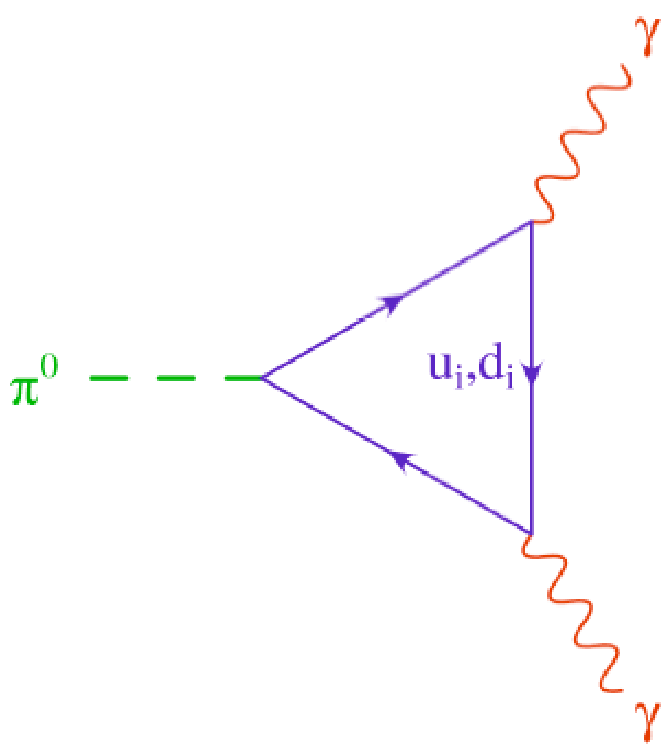
Strong interactions

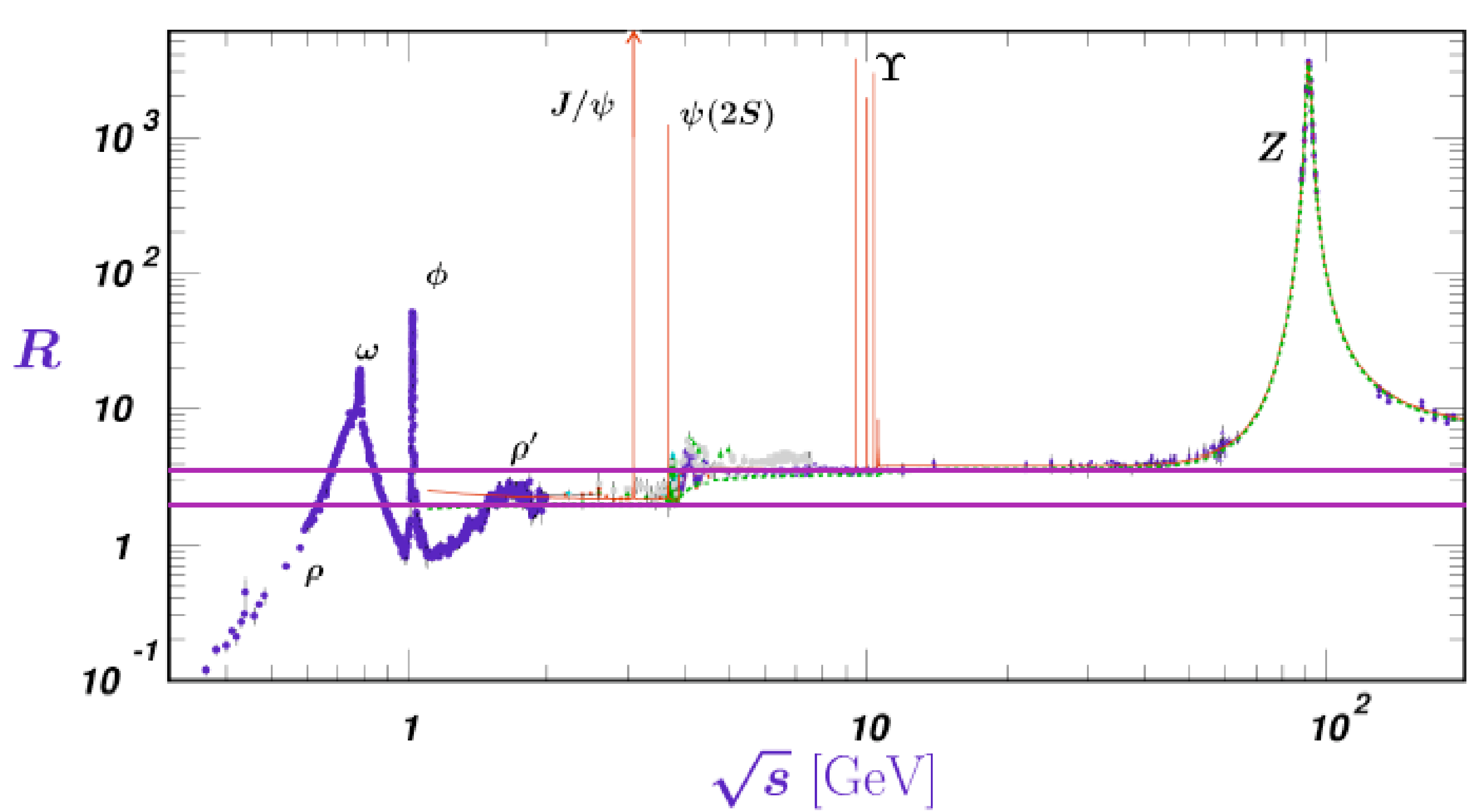
Nowadays we have a satisfactory model of strong interactions based on a non-abelian gauge theory, QCD.

Why is QCD a good theory?

1. Hadron spectrum
2. Scaling
3. QCD: a consistent QFT
4. Low energy symmetries
5. MUCH more....

How many colors?





$$\Gamma \sim N_c^2 \left[Q_u^2 - Q_d^2\right]^2 \frac{m_\pi^3}{f_\pi^2}$$

$$\Gamma_{TH} = \left(\frac{N_c}{3}\right)^2 7.6\,\mathrm{eV}$$

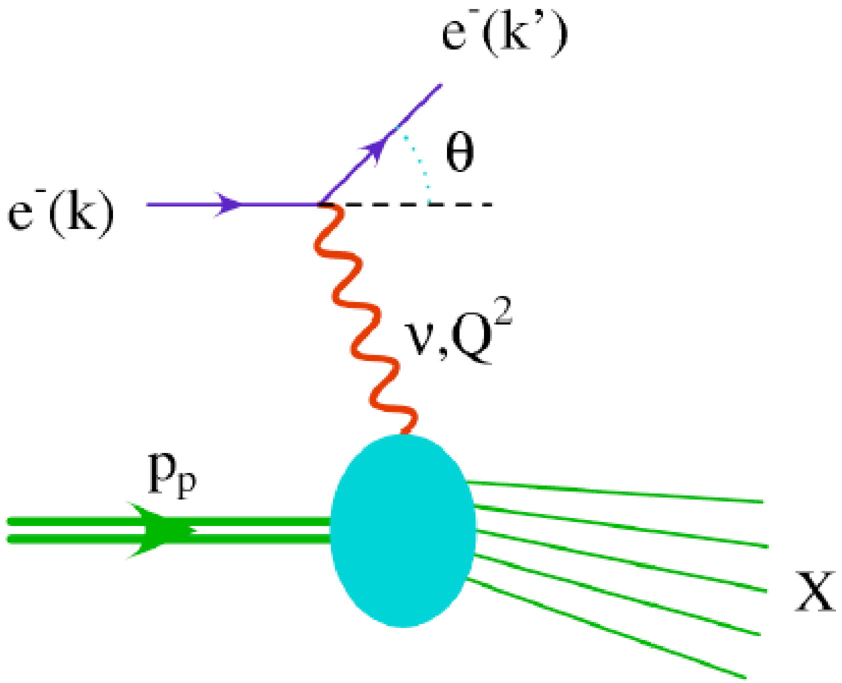
$$\Gamma_{EXP} = 7.7 \pm 0.6 \quad \mathrm{eV}$$

$$R = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \sim N_c \sum_q e_q^2$$

$$= 2(N_c/3) \quad q = u, d, s$$

$$= 3.7(N_c/3) \quad q = u, d, s, c, b$$

Scaling



$$s = (P+k)^2$$ cms energy²

$$Q^2 = -(k-k')^2$$ momentum transfer²

$$x = Q^2/2(P\cdot q)$$ scaling variable

$$\nu = (P\cdot q)/M = E - E'$$ energy loss

$$y = (P\cdot q)/(P\cdot k) = 1 - E'/E$$ rel. energy loss

$$W^2 = (P+q)^2 = M^2 + \frac{1-x}{x}Q^2$$ recoil mass

$$\frac{d\sigma_{\text{elastic}}}{dq^2} = \left(\frac{d\sigma}{dq^2}\right)_{\text{point}} \cdot F^2_{\text{elastic}}(q^2)\,\delta(1-x)\,dx$$

$$\frac{d\sigma_{\text{inelastic}}}{dq^2} = \left(\frac{d\sigma}{dq^2}\right)_{\text{point}} \cdot F^2_{\text{inelastic}}(q^2,x)\,dx$$

What should we expect for $F(q^2,x)$?

Scaling

Two plausible and one crazy scenarios for the $|q^2| \to \infty$ (Bjorken) limit:

1. Smooth electric charge distribution: (classical picture)

$$F^2_{elastic}(q^2) \sim F^2_{inelastic}(q^2) \ll 1$$

i.e., external probe penetrates the proton as knife through the butter!

2. Tightly bound point charges inside the proton: (bound quarks)

$$F^2_{elastic}(q^2) \sim 1 \text{ and } F^2_{inelastic}(q^2) \ll 1$$

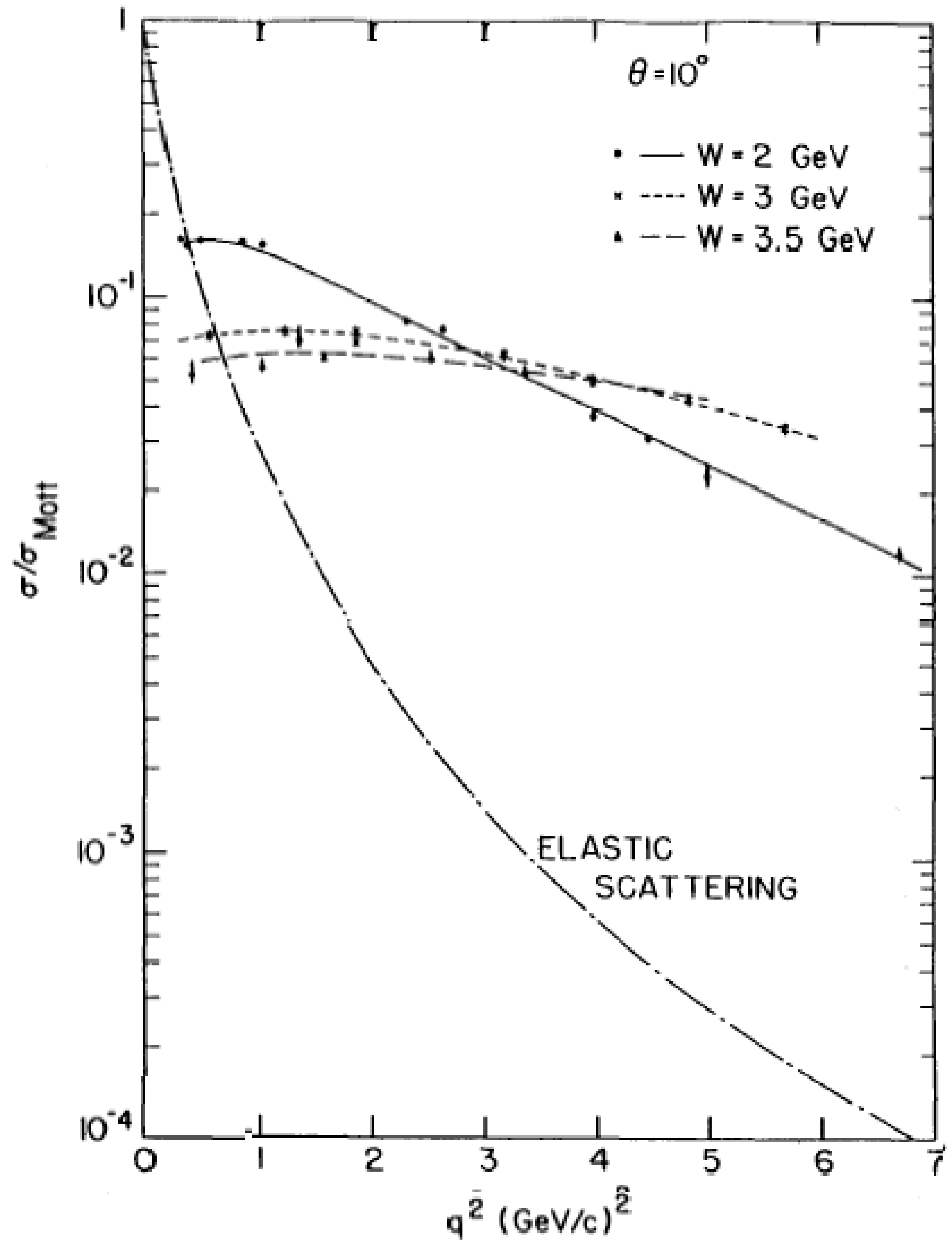
i.e., quarks get hit as single particles, but momentum is immediately redistributed as they are tightly bound together (confinement) and cannot fly away.

3. And now the crazy one: (free quarks)

$$F^2_{elastic}(q^2) \ll 1 \text{ and } F^2_{inelastic}(q^2) \sim 1$$

i.e., there are points (quarks!) inside the protons, however the hit quark behaves as a free particle that flies away without feeling or caring about confinement!!!

Scaling



$$\frac{d^2\sigma^{\mathrm{EXP}}}{dxdy} \sim \frac{1}{Q^2}$$

Remarkable!!! Pure dimensional analysis!

The right hand side does not depend on Λ_S !

This is the same behaviour one may find in a renormalizable theory like in QED.

Other stunning example is again $e^+e^- \rightarrow$ hadrons.

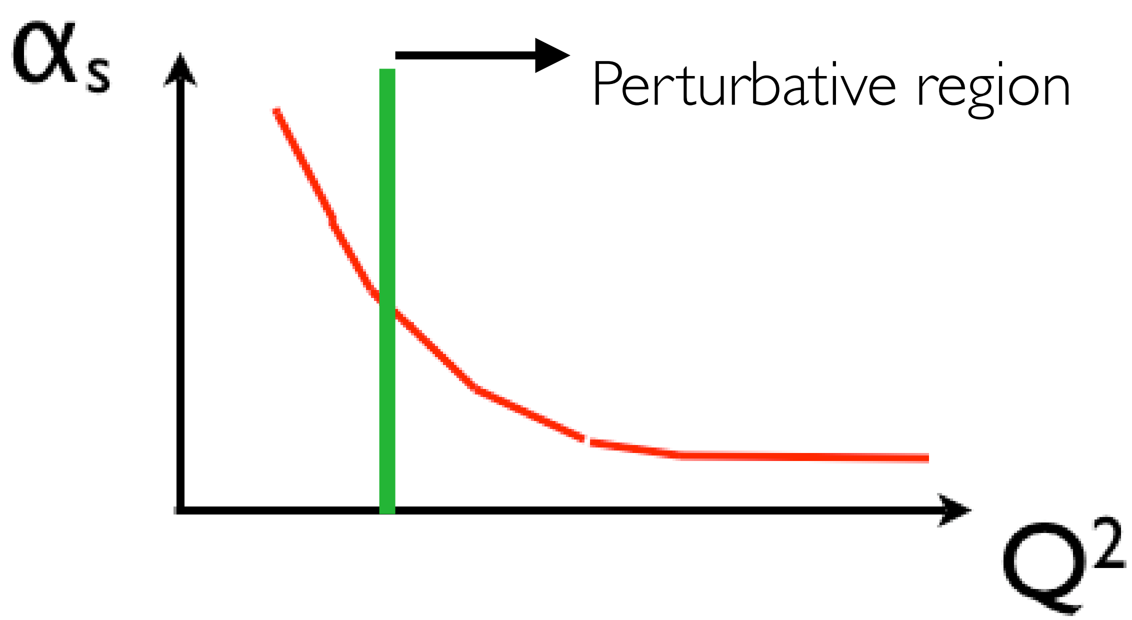
This motivated the search for a weakly-coupled theory at high energy!

Asymptotic freedom

Among QFT theories in 4 dimension only the non-Abelian gauge theories are "asymptotically free".

It becomes then natural to promote the global color SU(3) symmetry into a local symmetry where color is a charge.

This also hints to the possibility that the color neutrality of the hadrons could have a dynamical origin



In renormalizable QFT's scale invariance is broken by the renormalization procedure and couplings depend logarithmically on scales.

The QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \sum_f \bar{\psi}_i^{(f)}(i\partial\!\!\!/ - m_f)\psi_i^{(f)} - \bar{\psi}_i^{(f)}(g_s t^a_{ij} A\!\!\!/_a)\psi_j^{(f)}$$

Gauge Fields — Matter — Interaction

$$[t^a, t^b] = i f^{abc} t^c$$ → Algebra of SU(N)

$$\mathrm{tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$$ → Normalization

Very similar to the QED Lagrangian.. we'll see in a moment where the differences come from!

Why do we care about QCD?

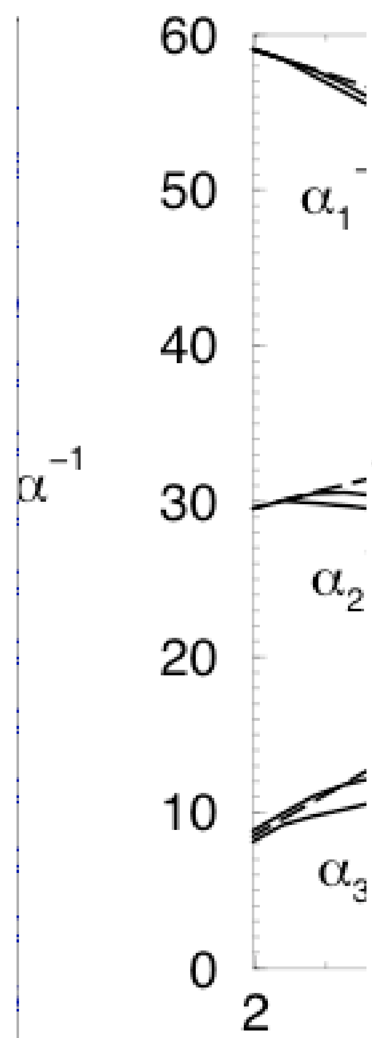
At high energy:

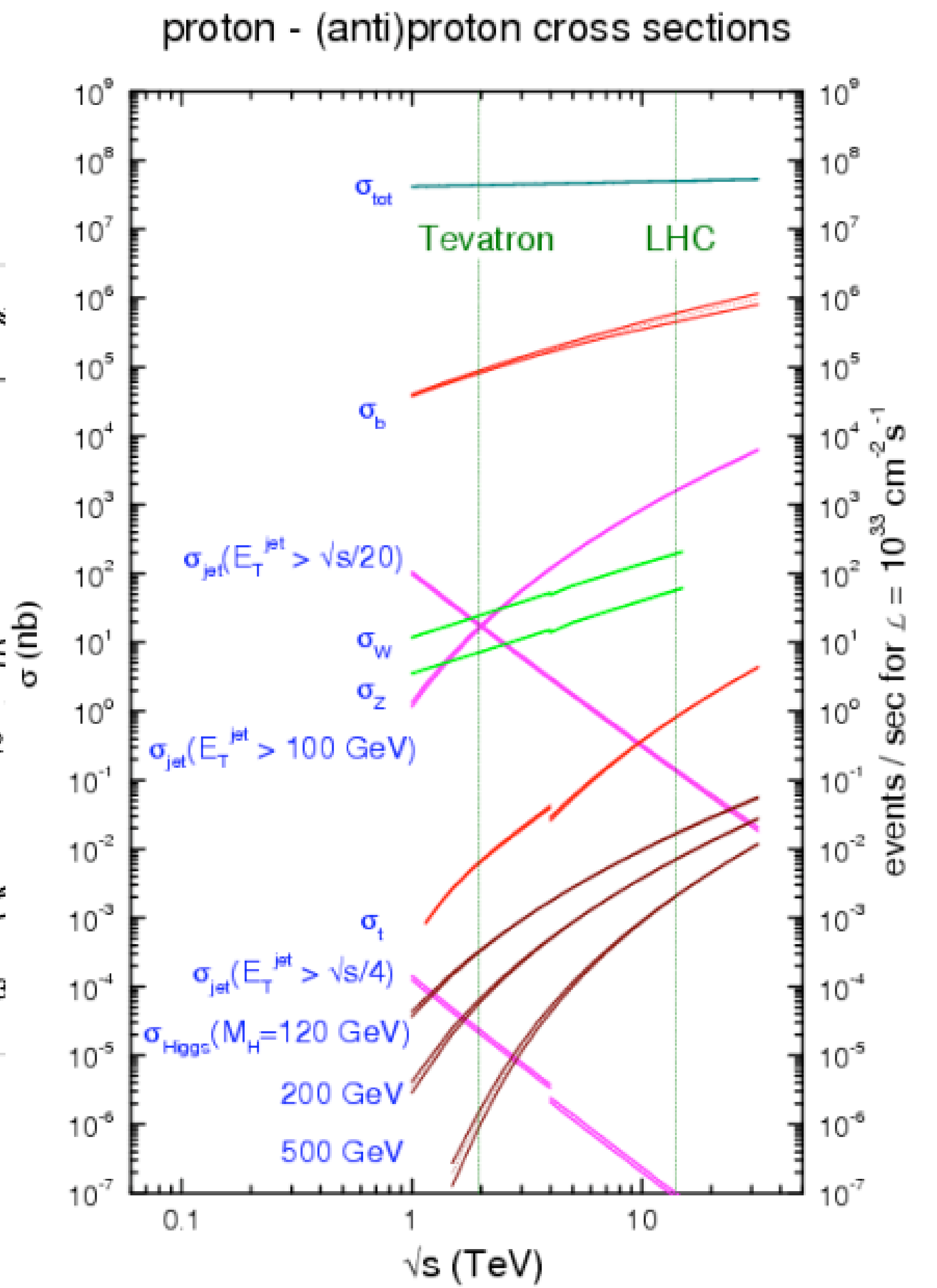
QCD is a necessary tool to decode most hints that Nature is giving us on the fundamental issues!

*Measurement of α_S, $\sin^2\theta_W$ give information on possible patterns of unification.

*Measurements and discoveries at hadron colliders need accurate predictions for QCD backgrounds!

BTW, is this really true?

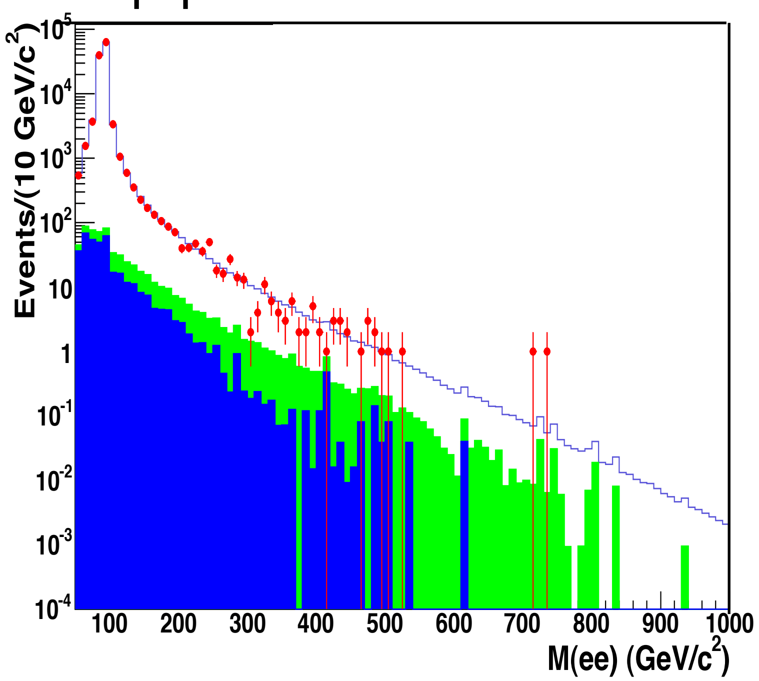




Discoveries at hadron colliders

peak

$pp \to Z' \to e^+e^-$

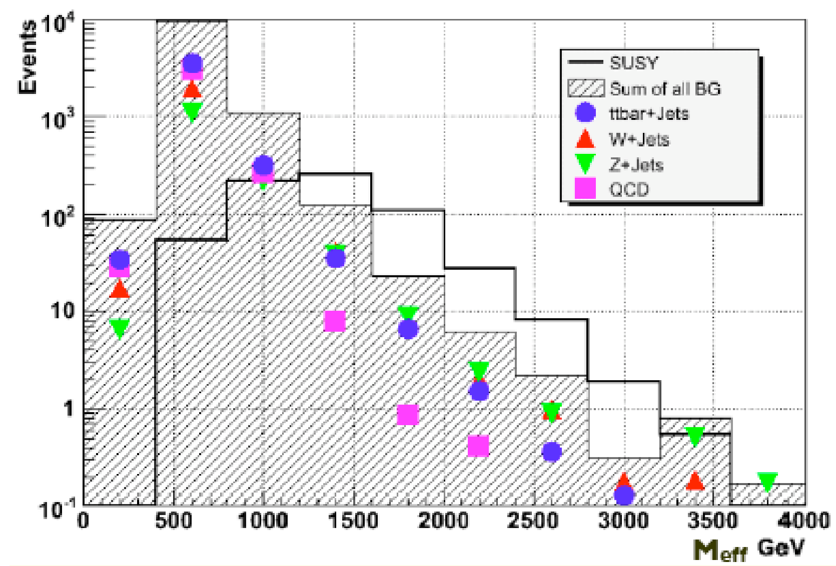


"easy"

Background directly measured from data. TH needed only for parameter extraction (Normalization, acceptance,...)

shape

$pp \to \tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q} \to jets + \not{E}_T$

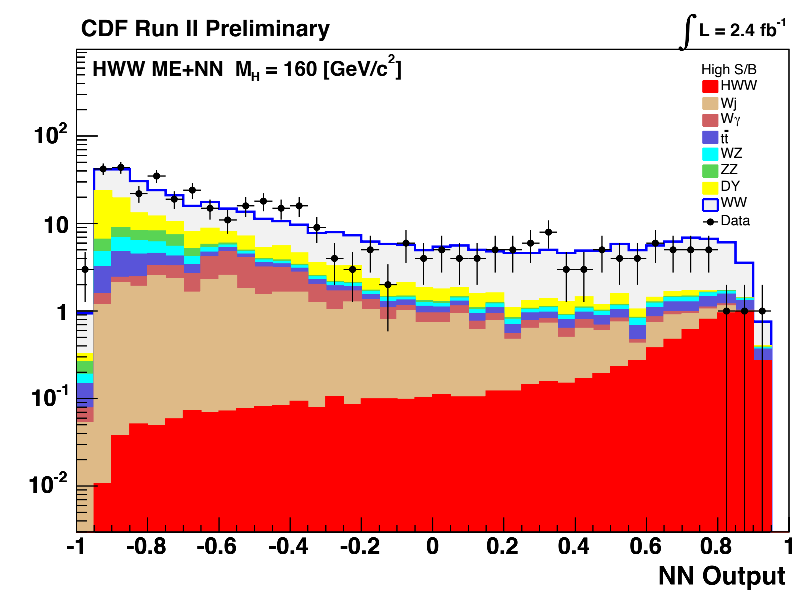


hard

Background shapes needed. Flexible MC for both signal and backgroud tuned and validated with data.

rate

$pp \to H \to W^+W^-$



very hard

Background normalization and shapes known very well. Interplay with the best theoretical predictions (via MC) and data.

Motivations for QCD predictions

- Accurate and experimental friendly predictions for collider physics range from being very useful to strictly necessary.
- Confidence on possible excesses, evidences and eventually discoveries builds upon an intense (and often non-linear) process of description/prediction of data via MC's.
- Measurements and exclusions always rely on accurate predictions.
- Predictions for both SM and BSM on the same ground.

no QCD $\Rightarrow$ no PARTY !

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2. From QED to QCD: the importance of color
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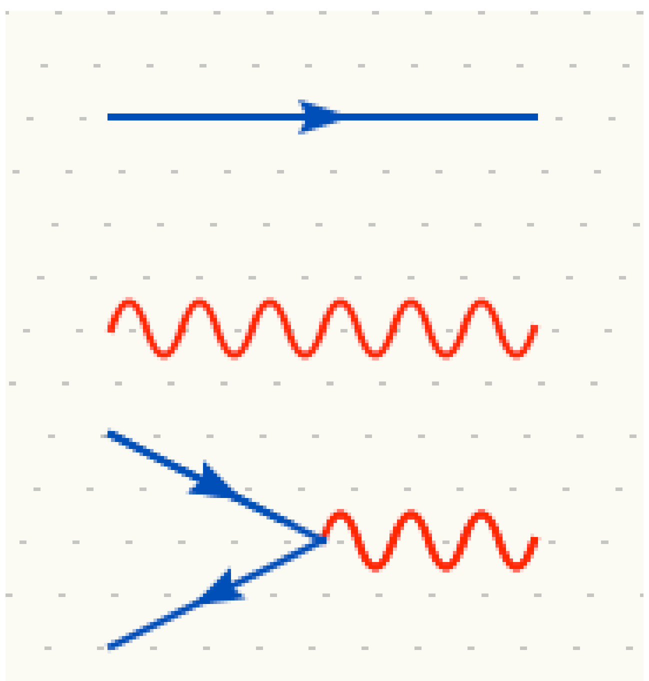
From QED to QCD

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi - eQ\bar{\psi}\not{A}\psi$$

where $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$



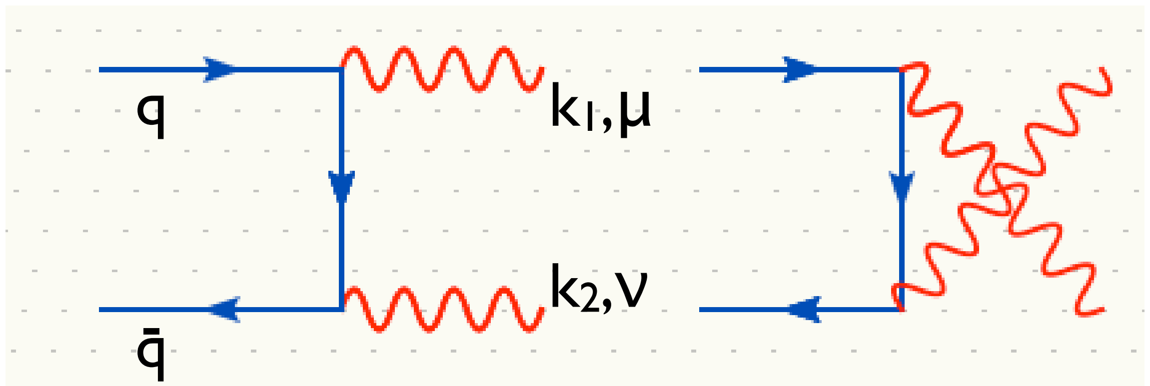
$$= \frac{i}{\not{p} - m + i\epsilon}$$

$$= \frac{-ig_{\mu\nu}}{p^2 + i\epsilon}$$

$$= -ie\gamma_\mu Q$$

From QED to QCD

We want to focus on how gauge invariance is realized in practice.
Let's start with the computation of a simple proces $e^+e^- \to \gamma\gamma$. There are two diagrams:



$= D_1 + D_2$

$$i\mathcal{M} = \mathcal{M}_{\mu\nu}\epsilon_1^{*\mu}\epsilon_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q})\not{\epsilon}_2 \frac{1}{\not{q} - \not{k}_1}\not{\epsilon}_1 u(q) + \bar{v}(\bar{q})\not{\epsilon}_1 \frac{1}{\not{q} - \not{k}_2}\not{\epsilon}_2 u(q) \right)$$

Gauge invariance requires that:

$$\epsilon_1^{*\mu} k_2^{\nu} \mathcal{M}_{\mu\nu} = \epsilon_2^{*\nu} k_1^{\mu} \mathcal{M}_{\mu\nu} = 0$$

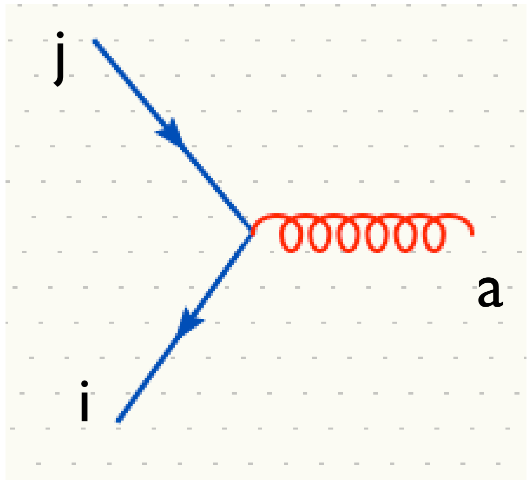
From QED to QCD

$$\mathcal{M}_{\mu\nu} k_1^{*\mu} \epsilon_2^{*\nu} = D_1 + D_2 = e^2 \left(\bar{v}(\bar{q}) \not\epsilon_2 \frac{1}{\not q - \not k_1} (\not k_1 - \not q) u(q) + \bar{v}(\bar{q}) (\not k_1 - \not{\bar{q}}) \frac{1}{\not k_1 - \not q} \not\epsilon_2 u(q) \right)$$
$$= -\bar{v}(\bar{q}) \not\epsilon_2 u(q) + \bar{v}(\bar{q}) \not\epsilon_2 u(q) = 0$$

Only the sum of the two diagrams is gauge invariant. For the amplitude to be gauge invariant it is enough that one of the polarizations is longitudinal. The state of the other gauge boson is irrelevant.

Let's try now to generalize what we have done for SU(3). In this case we take the (anti-) quarks to be in the (anti-)fundamental representation of SU(3), 3 and 3^*. Then the current is in a $3 \otimes 3^* = 1 \oplus 8$. The singlet is like a photon, so we identify the gluon with the octet and generalize the QED vertex to :

$$-i g_s t^a_{ij} \gamma^\mu$$



with $[t^a, t^b] = i f^{abc} t^c$

So now let's calculate qq → gg and we obtain

$$\frac{i}{g_s^2} M_g \equiv (t^b t^a)_{ij} D_1 + (t^a t^b)_{ij} D_2$$

$$M_g = (t^a t^b)_{ij} M_\gamma - g^2 f^{abc} t^c_{ij} D_1$$

From QED to QCD

To satisfy gauge invariance we still need:

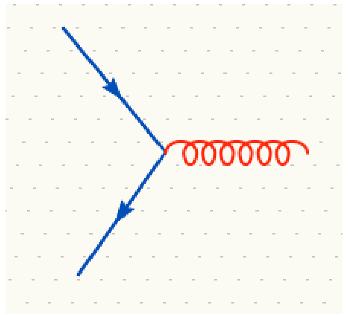
$$k_1^{\mu}{\epsilon_2}^{\nu} M_g^{\mu,\nu} = k_2^{\nu}\epsilon_1^{\mu} M_g^{\mu,\nu} = 0.$$

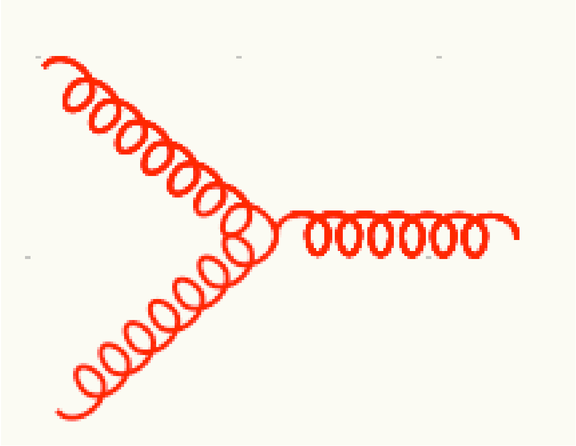
But in this case one piece is left out

$$k_{1\mu} M_g^{\mu} = -g_s^2 f^{abc} t^c_{ij} \bar{v}_i(\bar{q}) \not{\epsilon}_2 u_i(q)$$

$$k_{1\mu} M_g^{\mu} = i(-g_s f^{abc} \epsilon_2^{\mu})(-i g_s t^c_{ij} \bar{v}_i(\bar{q}) \gamma_\mu u_i(q))$$

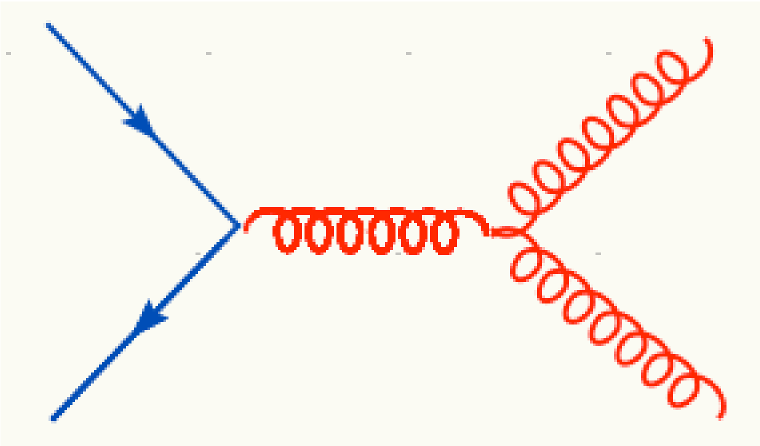
We indeed see that we interpret as the normal vertex times a new 3 gluon vertex:





$$-g_s f^{abc} V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$$

From QED to QCD



$$-ig_s^2 D_3 = (-ig_s t^a_{ij} \bar{v}_i(\bar{q})\gamma^\mu u_j(q)) \times \left(\frac{-i}{p^2}\right) \times$$

$$(-g f^{abc} V_{\mu\nu\rho}(-p, k_1, k_2)\epsilon_1^\nu(k_1)\epsilon_2^\rho(k_2))$$

How do we write down the Lorentz part for this new interaction? We can impose

1. Lorentz invariance : only structure of the type $g_{\mu\nu}\, p_\rho$ are allowed
2. fully anti-symmetry : only structure of the type remain $g_{\mu_1\mu_2}$ $(k_1)_{\mu_3}$ are allowed...
3. dimensional analysis : only one power of the momentum.

that uniquely constrain the form of the vertex:

$$V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = V_0 \left[(p_1 - p_2)_{\mu_3} g_{\mu_1\mu_2} + (p_2 - p_3)_{\mu_1} g_{\mu_2\mu_3} + (p_3 - p_1)_{\mu_2} g_{\mu_3\mu_1}\right]$$

With the above expression we obtain a contribution to the gauge variation:

$$k_1 \cdot D_3 = g^2 f^{abc} t^c V_0 \left[\bar{v}(\bar{q}) \not{\epsilon}_2 u(q) - \frac{k_2 \cdot \epsilon_2}{2k_1 \cdot k_2} \bar{v}(\bar{q}) \not{k}_1 u(q)\right]$$

The first term cancels the gauge variation of $D_1 + D_2$ if $V_0 = 1$, the second term is zero IFF the other gluon is physical!!

One can derive the form of the four-gluon vertex using the same heuristic method.

THE QCD LAGRANGIAN

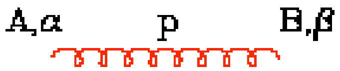
By direct inspection and by using the form non-abelian covariant derivation, we can check that indeed non-abelian gauge symmetry implies self-interactions. This is not surprising since the gluon itself is charged (In QED the photon is not!)

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_f \bar{\psi}_i^{(f)} (i\partial\!\!\!/ - m_f)\psi_i^{(f)} - \bar{\psi}_i^{(f)} (g_s t^a_{ij} A\!\!\!/_a)\psi_j^{(f)}$$

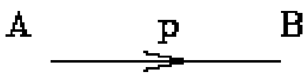
Gauge Fields and their interact. — Matter — Interaction

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

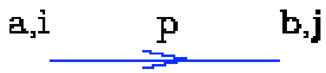
The Feynman Rules of QCD



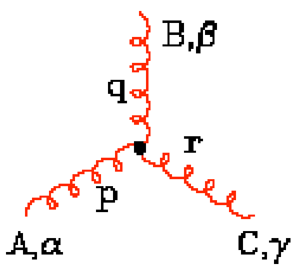
$$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^{\alpha}p^{\beta}}{p^{2}+i\epsilon}\right]\frac{i}{p^{2}+i\epsilon}$$



$$\delta^{AB}\frac{i}{(p^{2}+i\epsilon)}$$

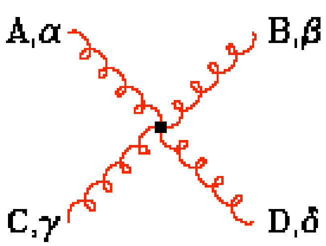


$$\delta^{ab}\frac{i}{(\not{p}-m+i\epsilon)_{ji}}$$

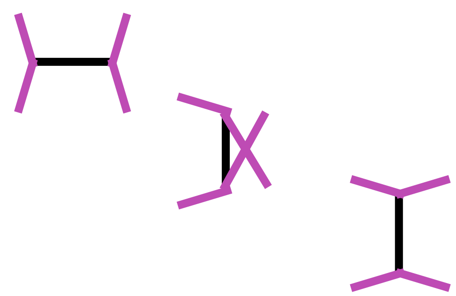


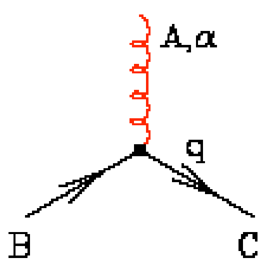
$$-g\ f^{ABC}[(p-q)^{\gamma}g^{\alpha\beta}+(q-r)^{\alpha}g^{\beta\gamma}+(r-p)^{\beta}g^{\gamma\alpha}]$$

(all momenta incoming)



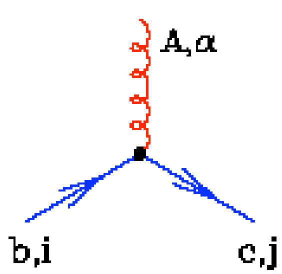
$$-ig^{2}\ f^{XAC}f^{XBD}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$
$$-ig^{2}\ f^{XAD}f^{XBC}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$$
$$-ig^{2}\ f^{XAB}f^{XCD}\ [g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$





$$g\ f^{ABC}q^{\alpha}$$

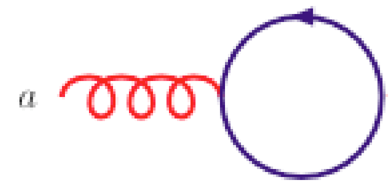
←what is this?



$$-ig\ (t^{A})_{cb}\ (\gamma^{\alpha})_{ji}$$

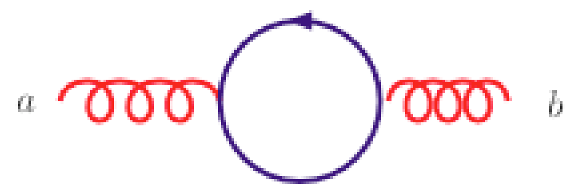
The color algebra

$$\mathrm{Tr}(t^a) = 0$$



= 0

$$\mathrm{Tr}(t^a t^b) = T_R \delta^{ab}$$



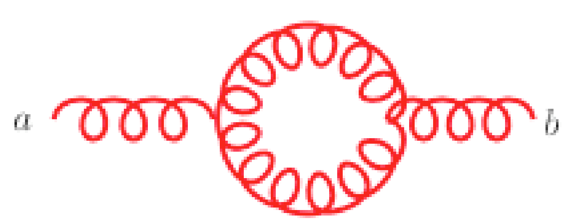
= T_R *

$$(t^a t^a)_{ij} = C_F \delta_{ij}$$

= C_F *

$$\sum_{cd} f^{acd} f^{bcd}$$

$$= (F^c F^c)_{ab} = C_A \delta_{ab}$$

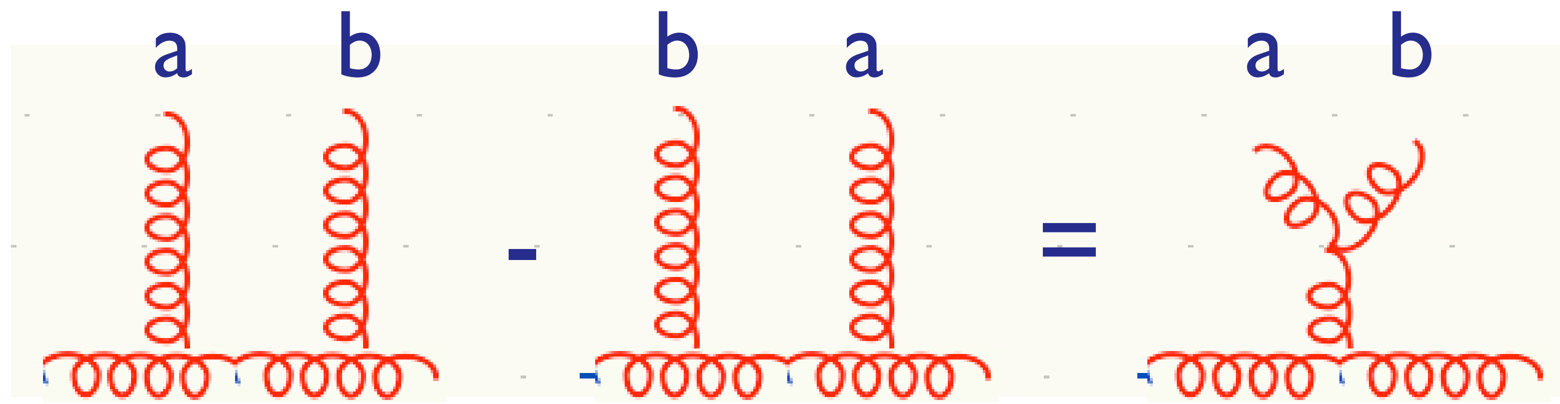


= C_A*

The color algebra

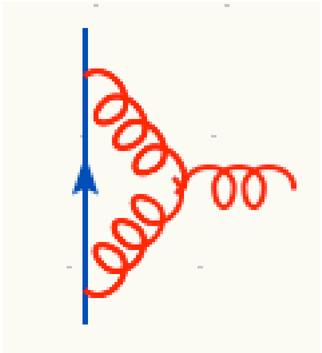
$$[t^a, t^b] = i f^{abc} t^c$$

$$[F^a, F^b] = i f^{abc} F^c$$

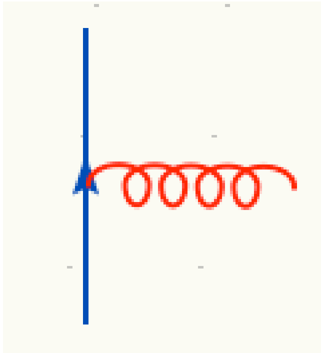


1-loop vertices

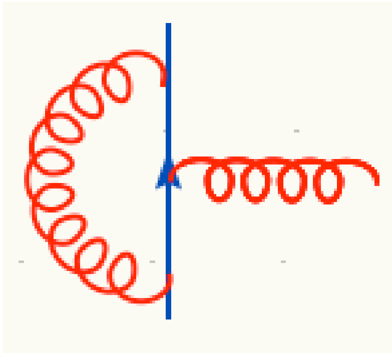
$$i f^{abc} (t^b t^c)_{ij} = \frac{C_A}{2} t^a_{ij}$$



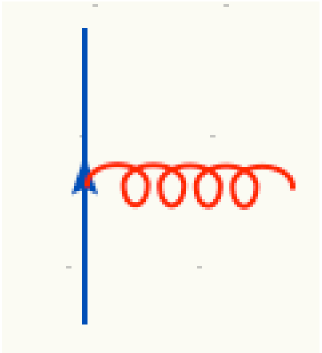
= $C_A/2$ *



$$(t^b t^a t^b)_{ij} = (C_F - \frac{C_A}{2}) t^a_{ij}$$

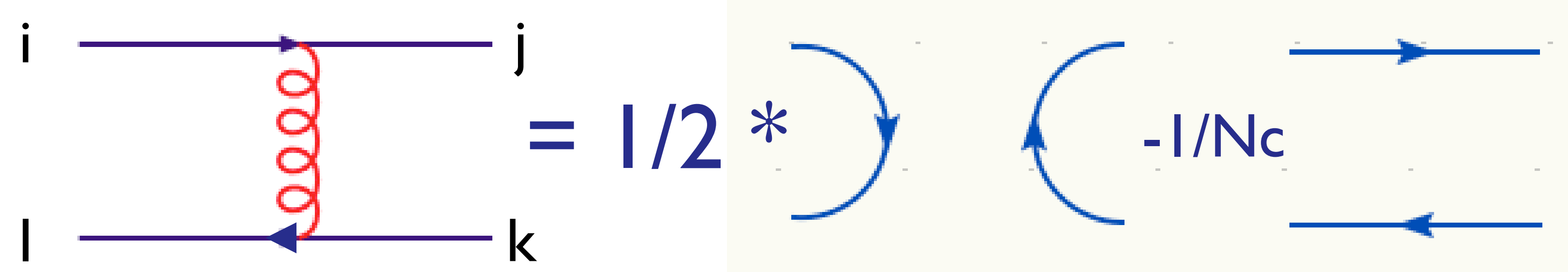


= -1/2/Nc *



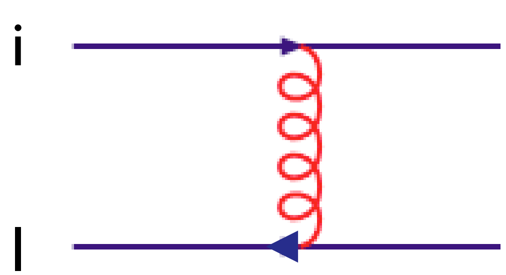
THE COLOR ALGEBRA

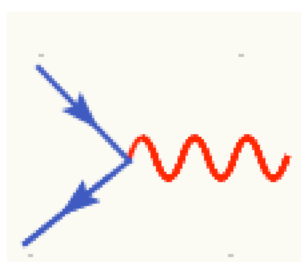
$$t^a_{ij}t^a_{kl} = \frac{1}{2}(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl})$$



Problem: Show that the one-gluon exchange between quark-antiquark pair can be attractive or repulsive. Calculate the relative strength.

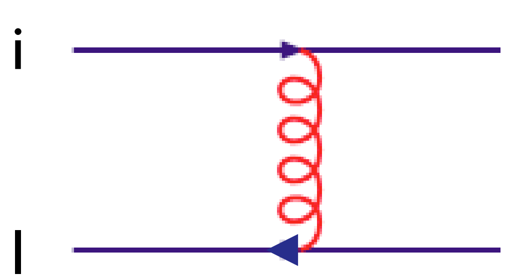
Solution: a q qb pair can be in a singlet state (photon) or in octet (gluon) : $3 \otimes \bar{3} = 1 \oplus 8$

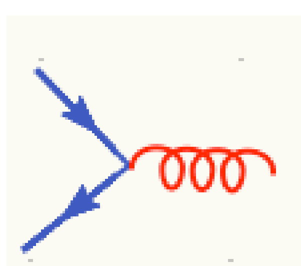




$$\frac{1}{2}(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{lk})\delta_{ki} = \frac{1}{2}\delta_{lj}(N_c - \frac{1}{N_c}) = C_F\delta_{lj}$$

>0, attractive

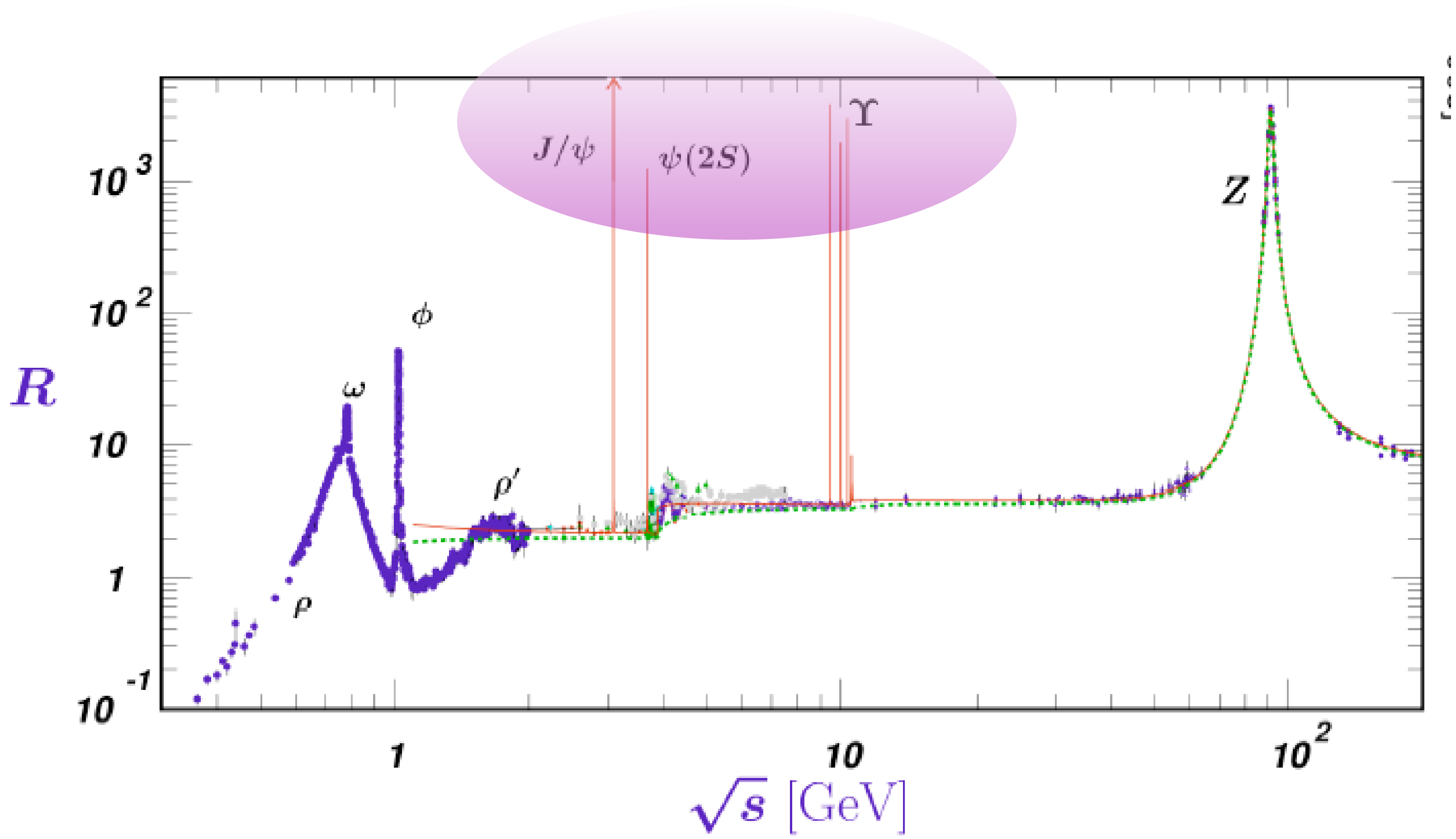


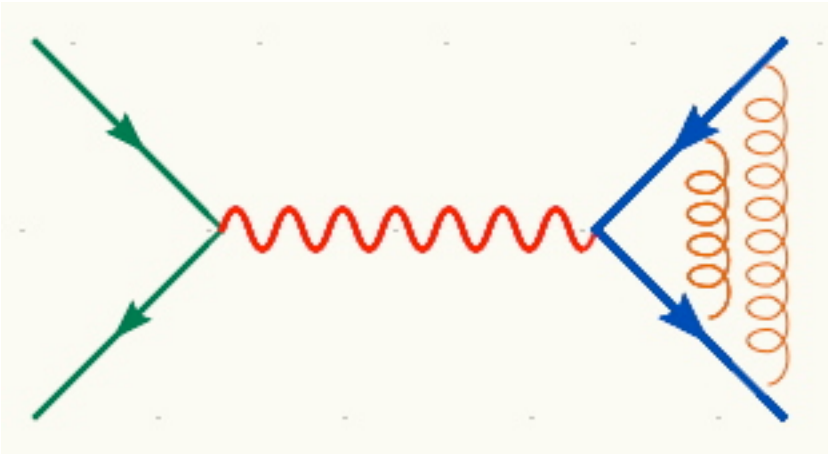


$$\frac{1}{2}(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{lk})t^a_{ki} = -\frac{1}{2N_c}t^a_{lj}$$

<0, repulsive

Quarkonium states

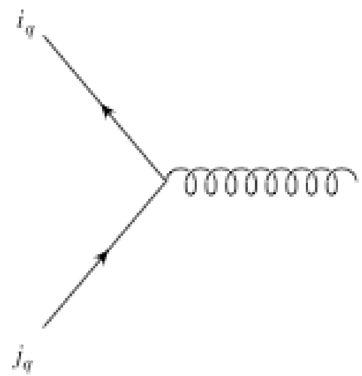


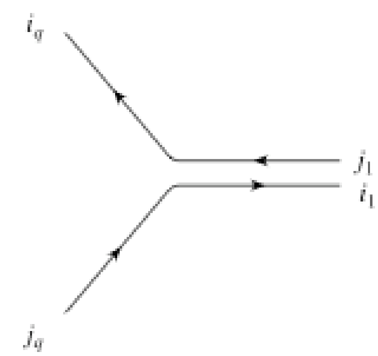


Very sharp peaks => small widths (~ 100 KeV) compared to hadronic resonances (100 MeV) => very long lived states. QCD is "weak" at scales >> ΛQCD (asymptotic freedom), non-relativistic bound states are formed like positronium!

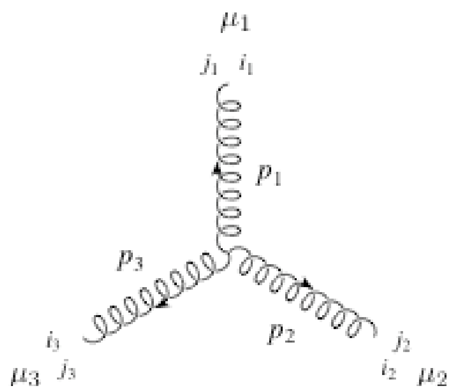
The QCD-Coulomb attractive potential is like: $V(r) \simeq -C_F \frac{\alpha_S(1/r)}{r}$

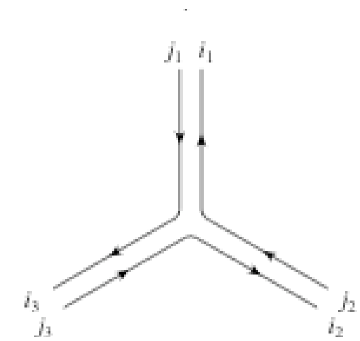
COLOR ALGEBRA: ‘T HOOFT DOUBLE LINE





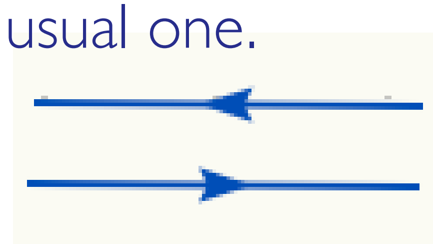
$$i\frac{g}{\sqrt{2}}\gamma_1^{\mu}\delta_{j_1}^{i_q}\delta_{j_q}^{i_1}$$





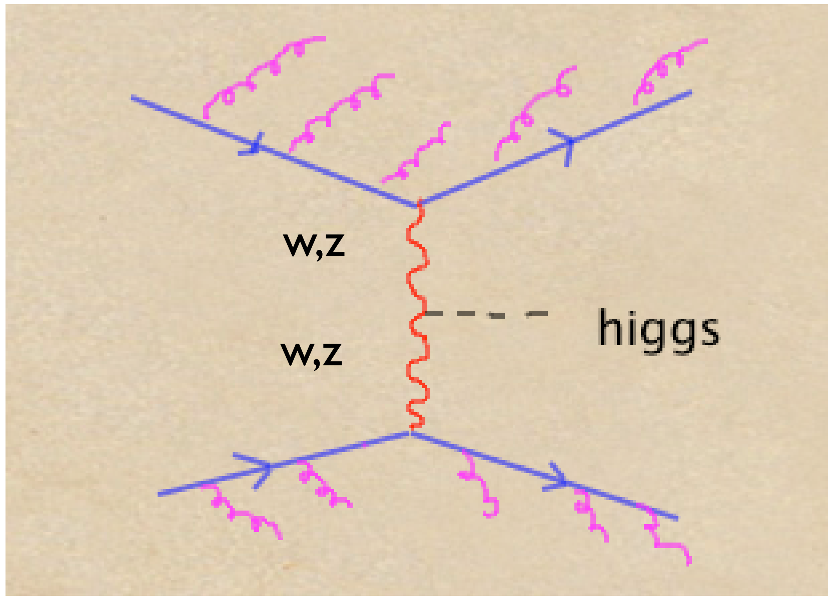
$$i\frac{g}{\sqrt{2}}\sum K^{\mu_1\mu_2\mu_3}\delta_{j_1}^{i_3}\delta_{j_2}^{i_1}\delta_{j_3}^{i_2}$$

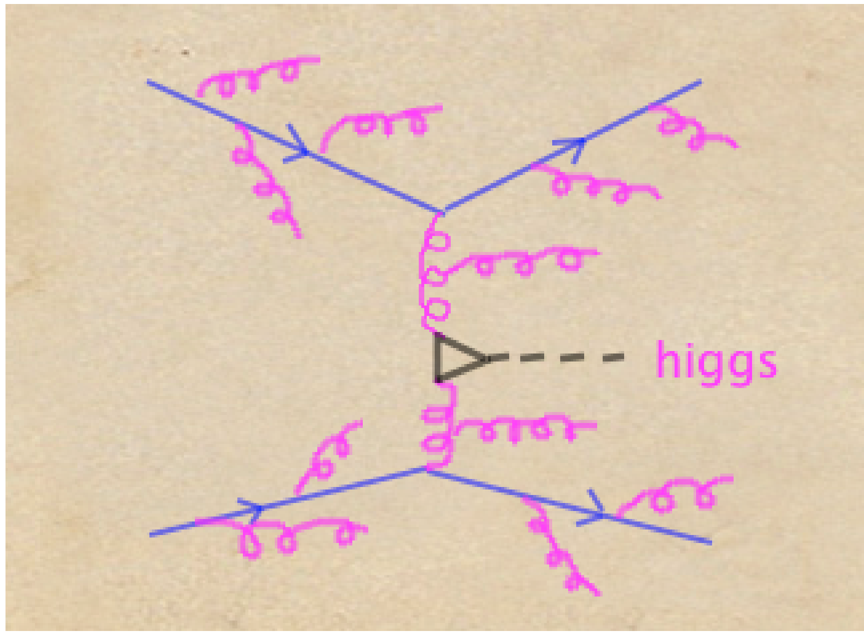
This formulation leads to a graphical representation of the simplifications occuring in the large Nc limit, even though it is exactly equivalent to the usual one.

 ≈ 1/2 

In the large Nc limit, a gluon behaves as a quark-antiquark pair. In addition it behaves classically, in the sense that quantum interference, which are effects of order $1/Nc^2$ are neglected. Many QCD algorithms and codes (such a the parton showers) are based on this picture.

Example: VBF fusion





Facts:

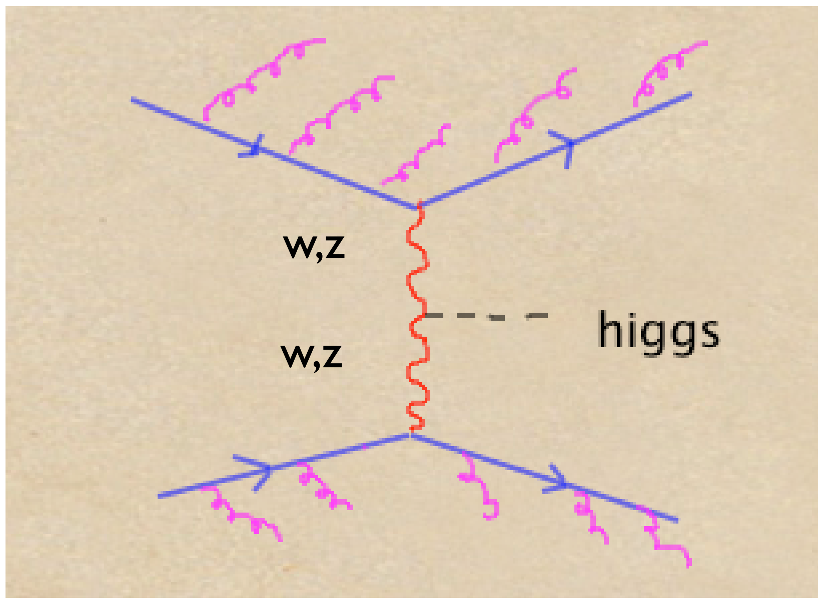
1. Important channel for light Higgs both for discovery and measurement

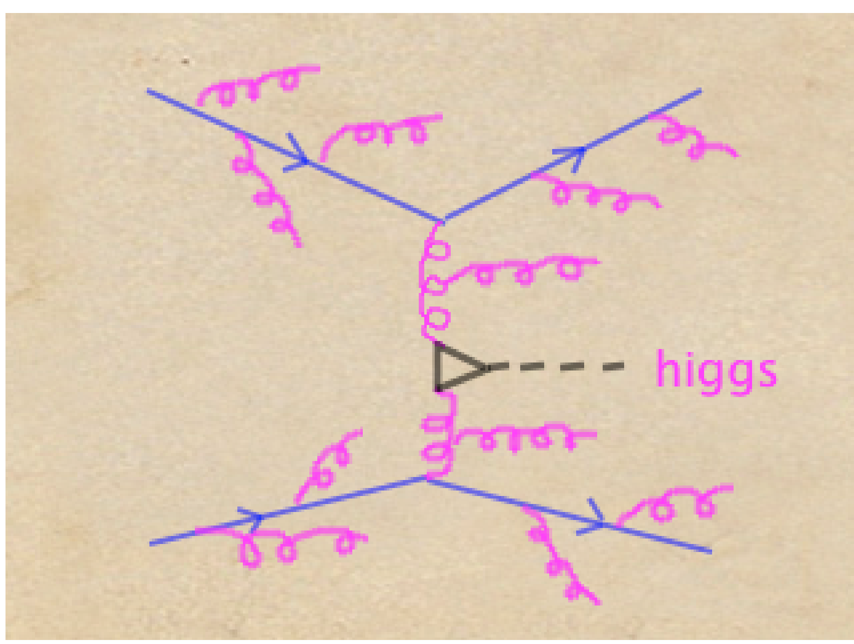
2. Color singlet exchange in the t-channel

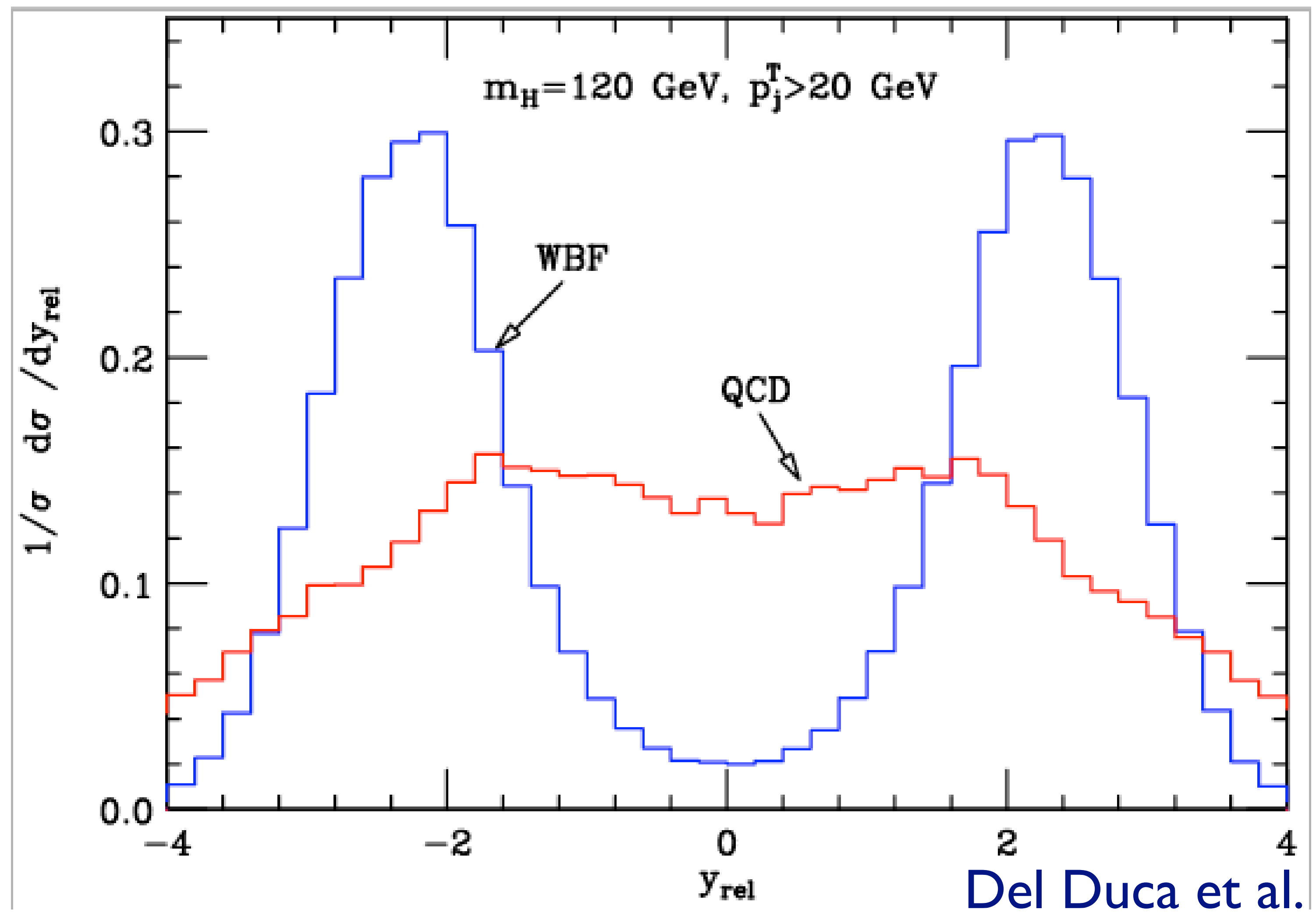
3. Characteristic signature: forward-backward jets + RAPIDITY GAP

4. QCD production is a background to precise measurements of couplings

Example: VBF fusion





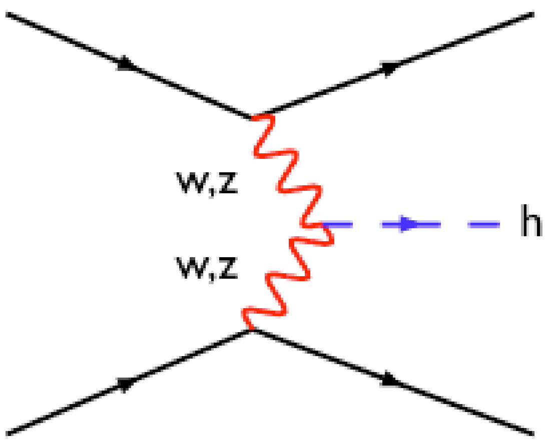


Del Duca et al.

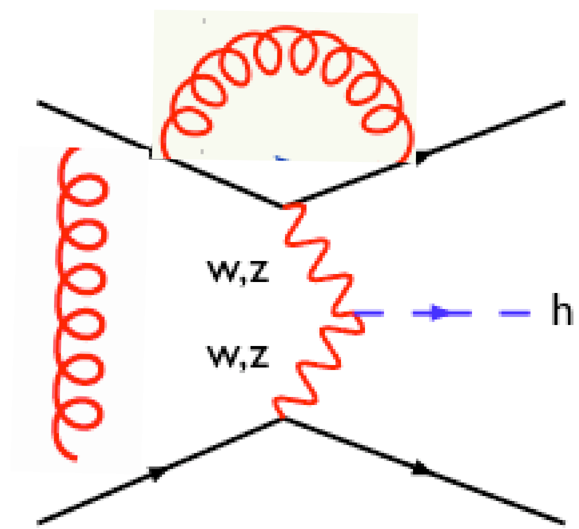
Third jet distribution

EXAMPLE: VBF FUSION

Consider VBF: at LO there is no exchange of color between the quark lines:



$$\delta_{ij}\delta_{kl}$$



$$C_F \delta_{ij}\delta_{kl} \Rightarrow$$

$$M_{\rm tree} M^*_{1-{\rm loop}} = C_F N_c^2 \simeq N_c^3$$

$$\frac{1}{2}\left(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right) \Rightarrow$$

$$M_{\rm tree} M^*_{1-{\rm loop}} = 0$$

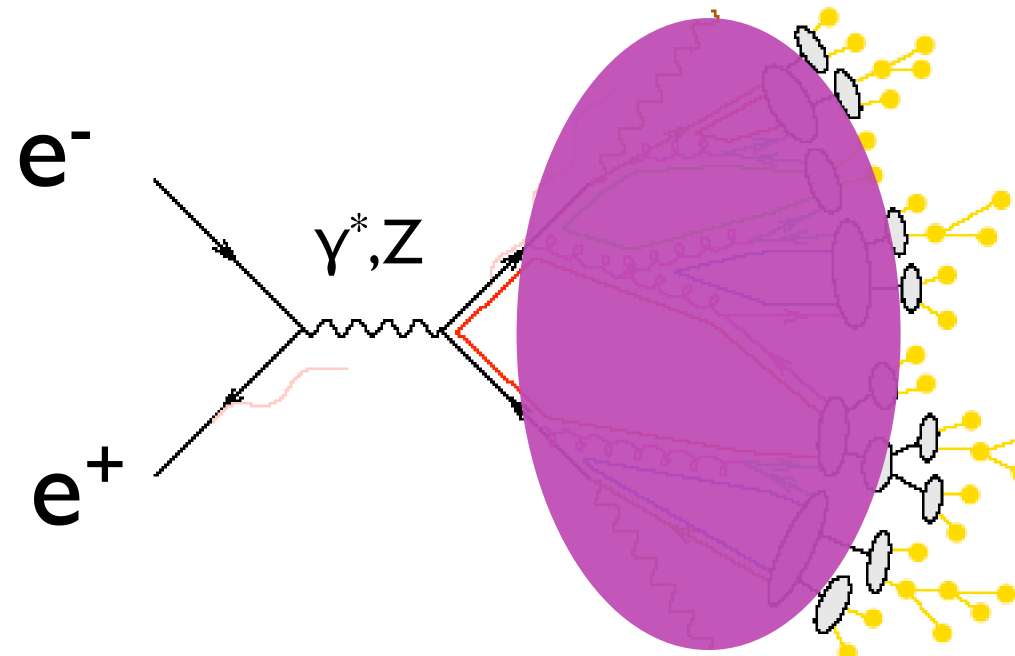
Also at NLO there is no color exchange! With one little exception....
At NNLO exchange is possible but it suppressed by $1/Nc^2$

QCD : THE FUNDAMENTALS

1. QCD is a good theory for strong interactions: facts
2. From QED to QCD: the importance of color
3. Renormalization group and asymptotic freedom

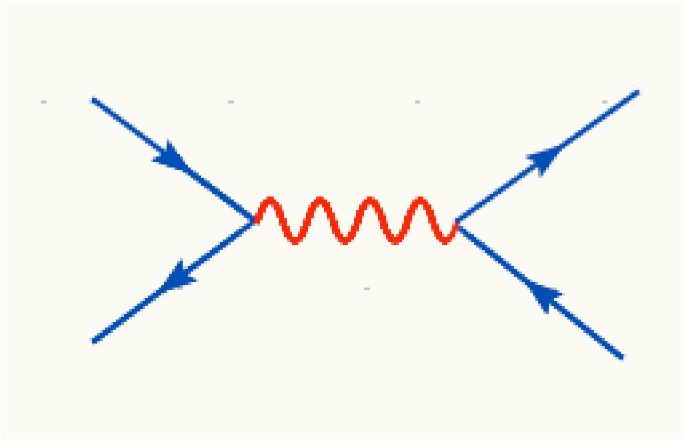
Ren. group and asymptotic freedom

Let us consider the process:
$e^- e^+ \rightarrow$ hadrons and for a $Q^2 >> \Lambda_S$.
At this pont (though we will!) we don't have an idea how to calculate the details of such a process.
So let's take the most inclusive approach ever: we just want to count how many events with hadrons in the final state there are wrt to a pair of muons.



Zeroth Level: e+ e- → qq

$$R_0 = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_f Q_f^2$$



Very simple exercise. The calculation is exactly the same as for the $\mu^+\mu^-$.

Ren. group and asymptotic freedom

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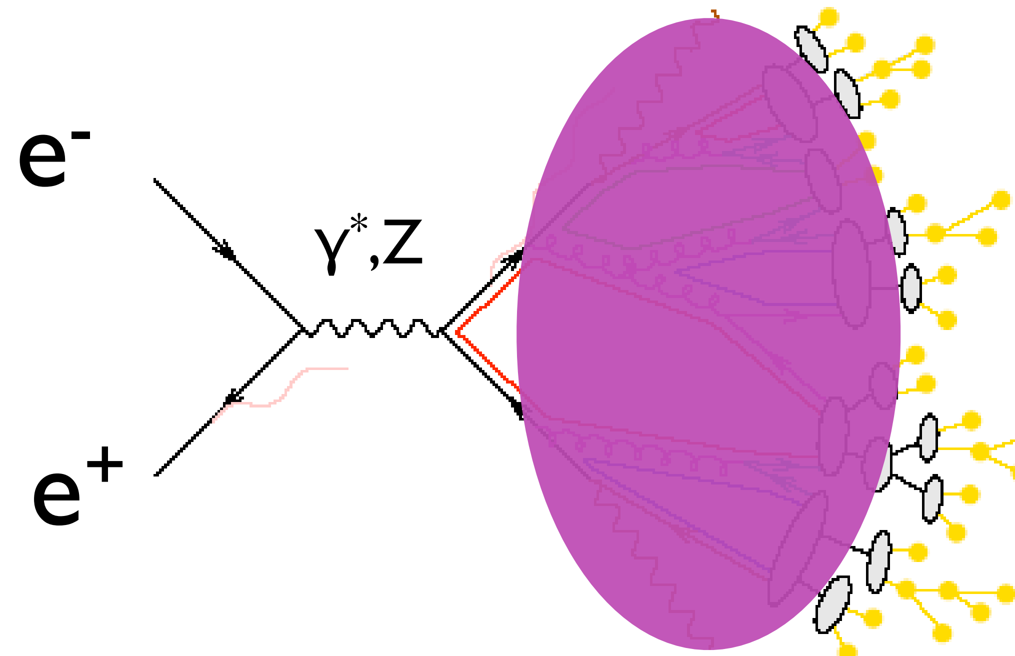
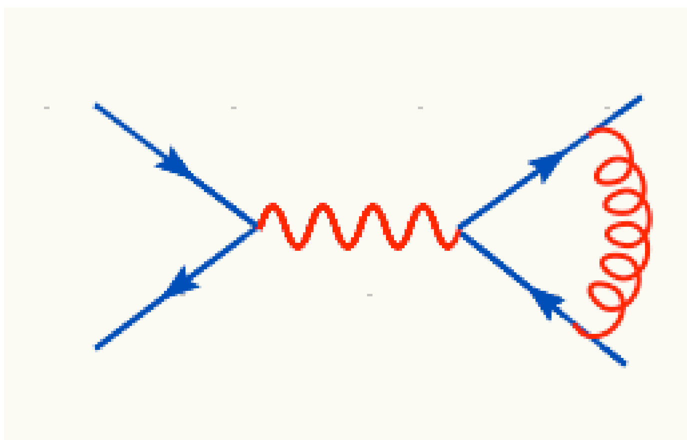


First improvement: e+ e- → qq at NLO
Already a much more difficult calculation!
There are real and virtual contributions. There are:
* UV divergences coming from loops
* IR divergences coming from loops and real diagrams. Ignore the IR for the moment (they cancel anyway) We need some kind of trick to regulate the divergences. Like dimensional regularization or a cutoff M. At the end the result is VERY SIMPLE:

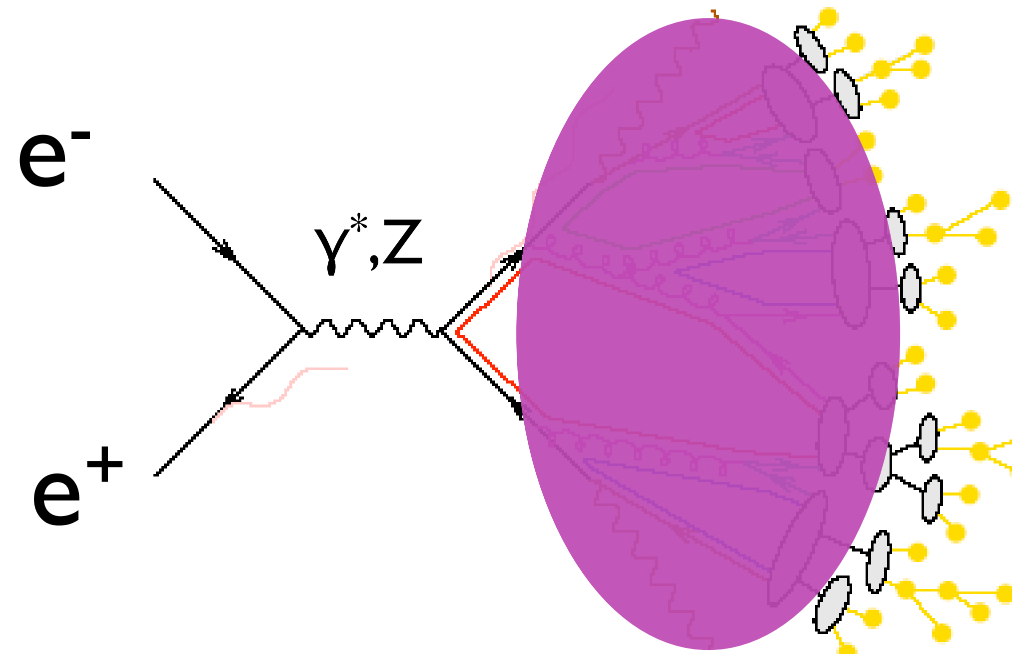


$$R_1 = R_0 \left(1 + \frac{\alpha_S}{\pi}\right)$$

No renormalization is needed! Electric charge is left untouched by strong interactions!

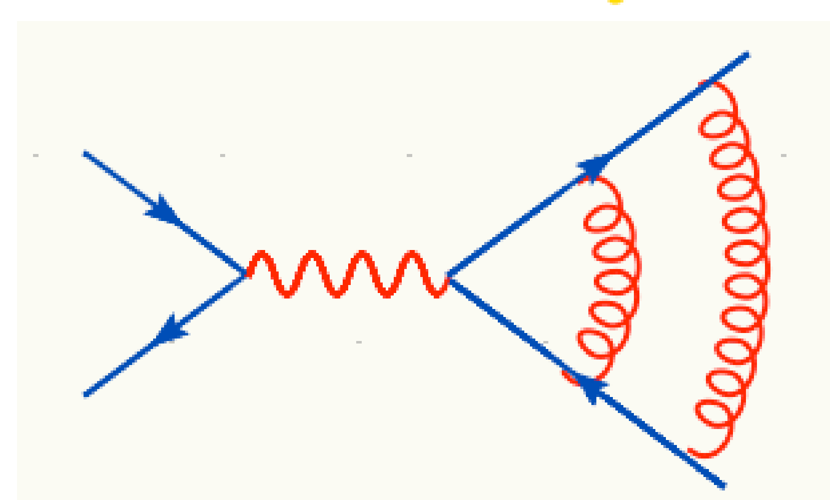
Ren. group and asymptotic freedom

Let us consider the process:
$e^-e^+ \rightarrow$ hadrons and for a $Q^2 >> \Lambda_S$.
At this pont (though we will!) we don't have an idea how to calculate the details of such a process.
So let's take the most inclusive approach ever: we just want to count how many events with hadrons in the final state there are wrt to a pair of muons.



Second improvement: e+ e- → qq at NNLO
Extremely difficult calculation!
Something new happens:

$$R_2 = R_0 \left(1 + \frac{\alpha_S}{\pi} + \left[c + \pi b_0 \log \frac{M^2}{Q^2}\right] \left(\frac{\alpha_S}{\pi}\right)^2\right)$$



The result is explicitly dependent on the arbitrary cutoff scale. We need to perform normalization of the coupling and since QCD is renormalizable we are garanteed that this fixes all the UV problems at this order.

$$\alpha_S(\mu) = \alpha_S + b_0 \log \frac{M^2}{\mu^2} \alpha_S^2$$

Ren. group and asymptotic freedom

(1) $$R_2^{\rm ren}(\alpha_S(\mu), \frac{\mu^2}{Q^2}) = R_0 \left(1 + \frac{\alpha_S(\mu)}{\pi} + \left[c + \pi b_0 \log\frac{\mu^2}{Q^2}\right]\left(\frac{\alpha_S(\mu)}{\pi}\right)^2\right)$$

(2) $$\alpha_S(\mu) = \alpha_S + b_0 \log\frac{M^2}{\mu^2}\alpha_S^2 \qquad b_0 = \frac{11N_c - 2n_f}{12\pi} > 0$$

Comments:

1. Now R_2 is finite but depends on an arbitrary scale **μ**, directly and through **α_s**. We had to introduce **μ** because of the presence of M.

2. Renormalizability garantes than any physical quantity can be made finite with the SAME substitution. If a quantity at LO is $A\alpha_s^N$ then the UV divergence will be N A b_0 log M^2 α_s^{N+1}.

3. R is a physical quantity and therefore cannot depend on the arbitrary scale **μ**!! One can show that at order by order:

$$\mu^2 \frac{d}{d\mu^2} R^{\rm ren} = 0 \Rightarrow R^{\rm ren}(\alpha_S(\mu), \frac{\mu^2}{Q^2}) = R^{\rm ren}(\alpha_S(Q), 1)$$

which is obviously verified by Eq. (1). Choosing **μ** ≈ Q the logs ...are resummed!

Ren. group and asymptotic freedom

(2) $\alpha_S(\mu) = \alpha_S + b_0 \log \frac{M^2}{\mu^2} \alpha_S^2$ $\qquad b_0 = \frac{11N_c - 2n_f}{12\pi}$ >0

4. From (b) one finds that:

$$\beta(\alpha_S) \equiv \mu^2 \frac{\partial \alpha_S}{\partial \mu^2} = -b_0 \alpha_S^2 \quad \Rightarrow \quad \alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}}$$

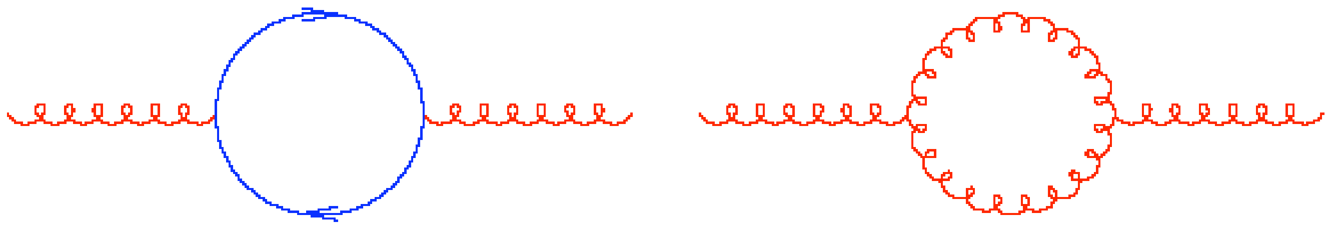
This gives the running of α_S. Since $b_0 > 0$, this expression make sense for all scale $\mu > \Lambda$.
In general one has:

$$\frac{d\alpha_S(\mu)}{d \log \mu^2} = -b_0 \alpha_S^2(\mu) - b_1 \alpha_S^3(\mu) - b_2 \alpha_S^4(\mu) + \ldots$$

where all b_i are finite (renormalization!). At present we know the b_i up to b_3 (4 loop calculation!!). b_1 and b_2 are renormalization scheme independent. Note that the expression for $\alpha_S(\mu)$ changes accordingly to the loop order. At two loops we have:

$$\alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}} \left[1 - \frac{b_1}{b_0^2} \frac{\log \log \mu^2/\Lambda^2}{\log \mu^2/\Lambda^2} \right]$$

WHY IS THE BETA FUNCTION NEGATIVE IN QCD?



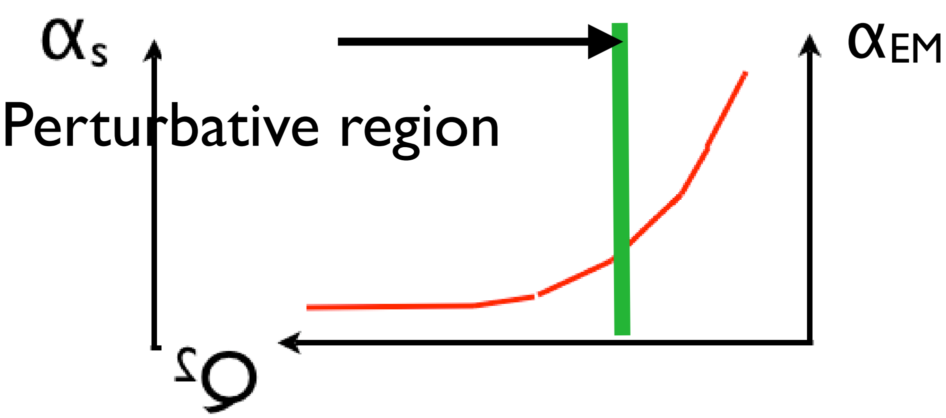
(a) (b)

Roughly speaking, quark loop diagram (a) contributes a negative N_f term in b_0, while the gluon loop, diagram (b) gives a positive contribution proportional to the number of colors N_c, which is dominant and make the overall beta function negative.

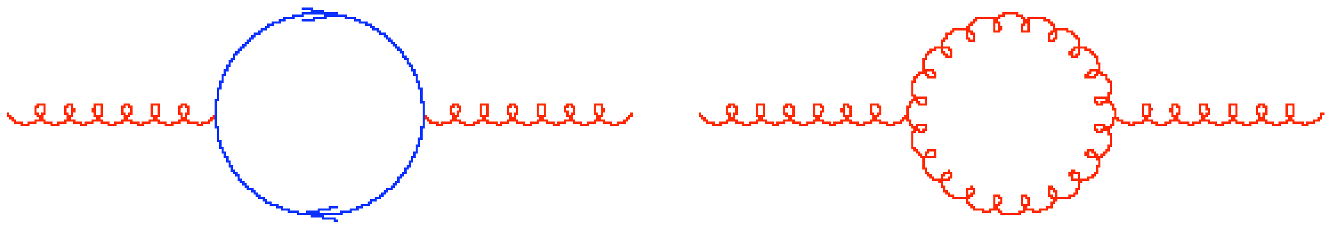
$$b_0 = \frac{11N_c - 2n_f}{12\pi} > 0 \quad \Rightarrow \quad \beta(\alpha_S) < 0 \text{ in QCD}$$

$$b_0 = -\frac{n_f}{3\pi} < 0 \quad \Rightarrow \quad \beta(\alpha_S) > 0 \text{ in QED}$$

$$\alpha_{EM}(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2_{\rm QED}}}$$



Why is the beta function negative in QCD?



(a)

Roughly speaking, quark loop diagram (a) contributes
loop, diagram (b) gives a positive contribution propor
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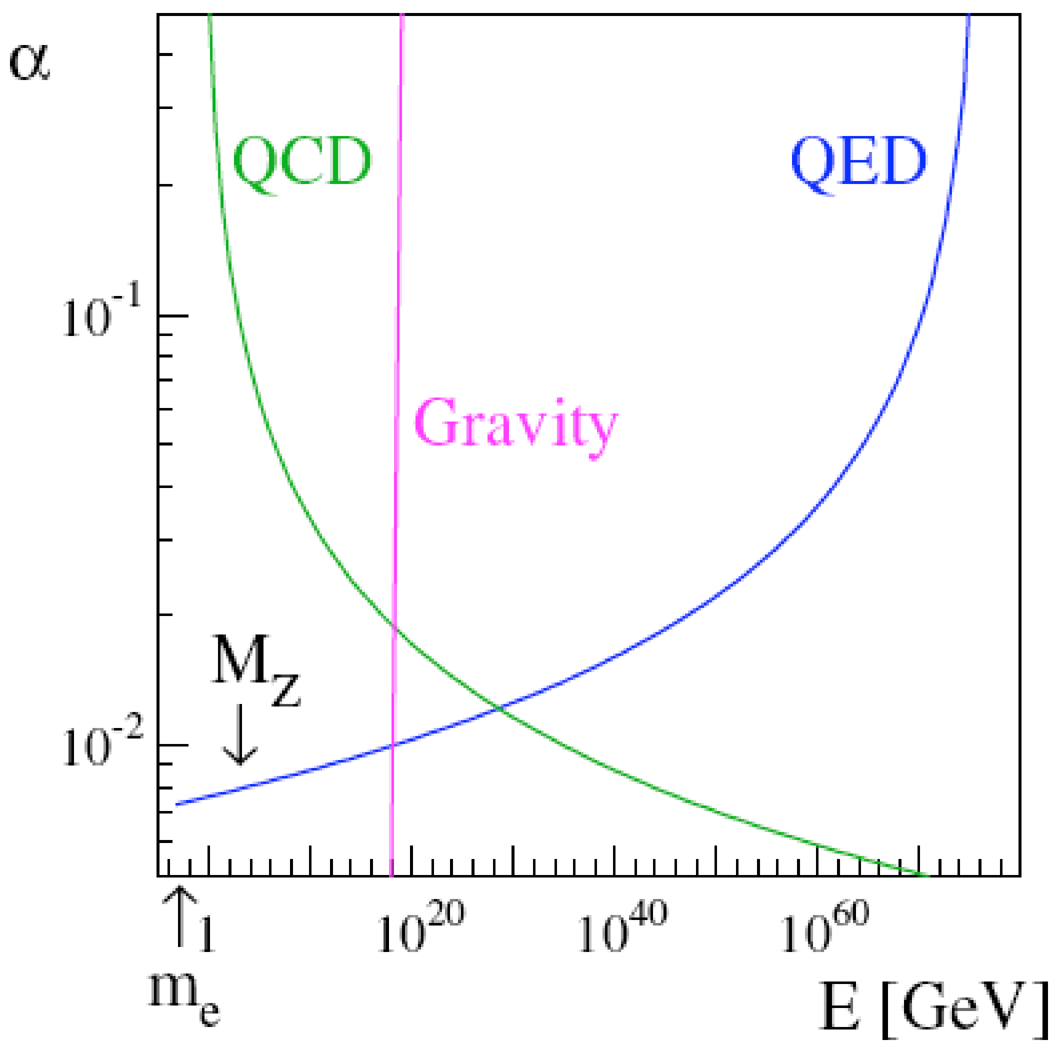
$$b_0 = \frac{11N_c - 2n_f}{12\pi} \quad >0 \quad \Rightarrow$$

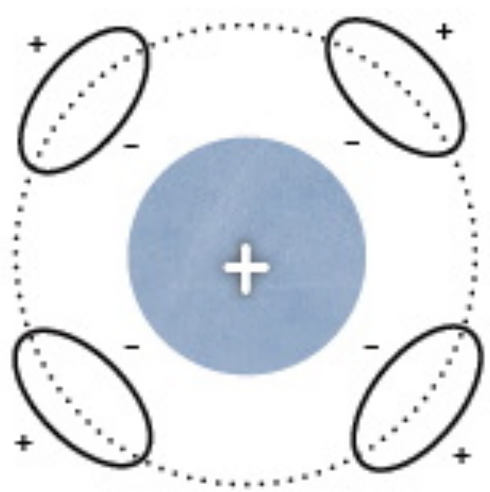
$$b_0 = -\frac{n_f}{3\pi} \quad <0 \quad \Rightarrow$$

$$\alpha_{EM}(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2_{\rm QED}}}$$

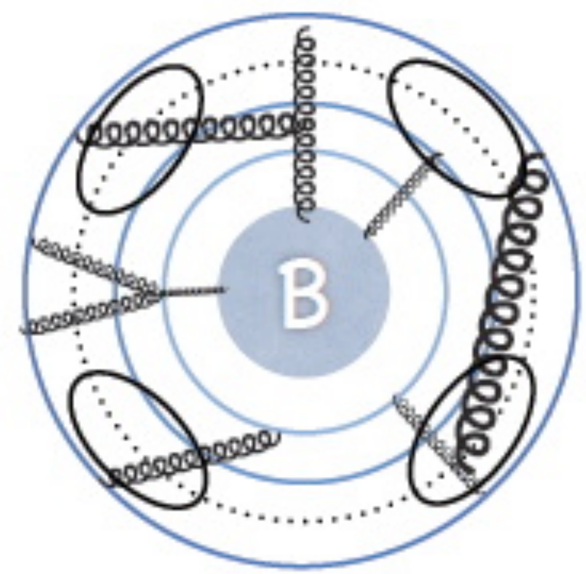


Why is the beta function negative in QCD?

QED



QCD



Ren. group and asymptotic freedom

Given

$$\alpha_S(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}} \qquad b_0 = \frac{11N_c - 2n_f}{12\pi}$$

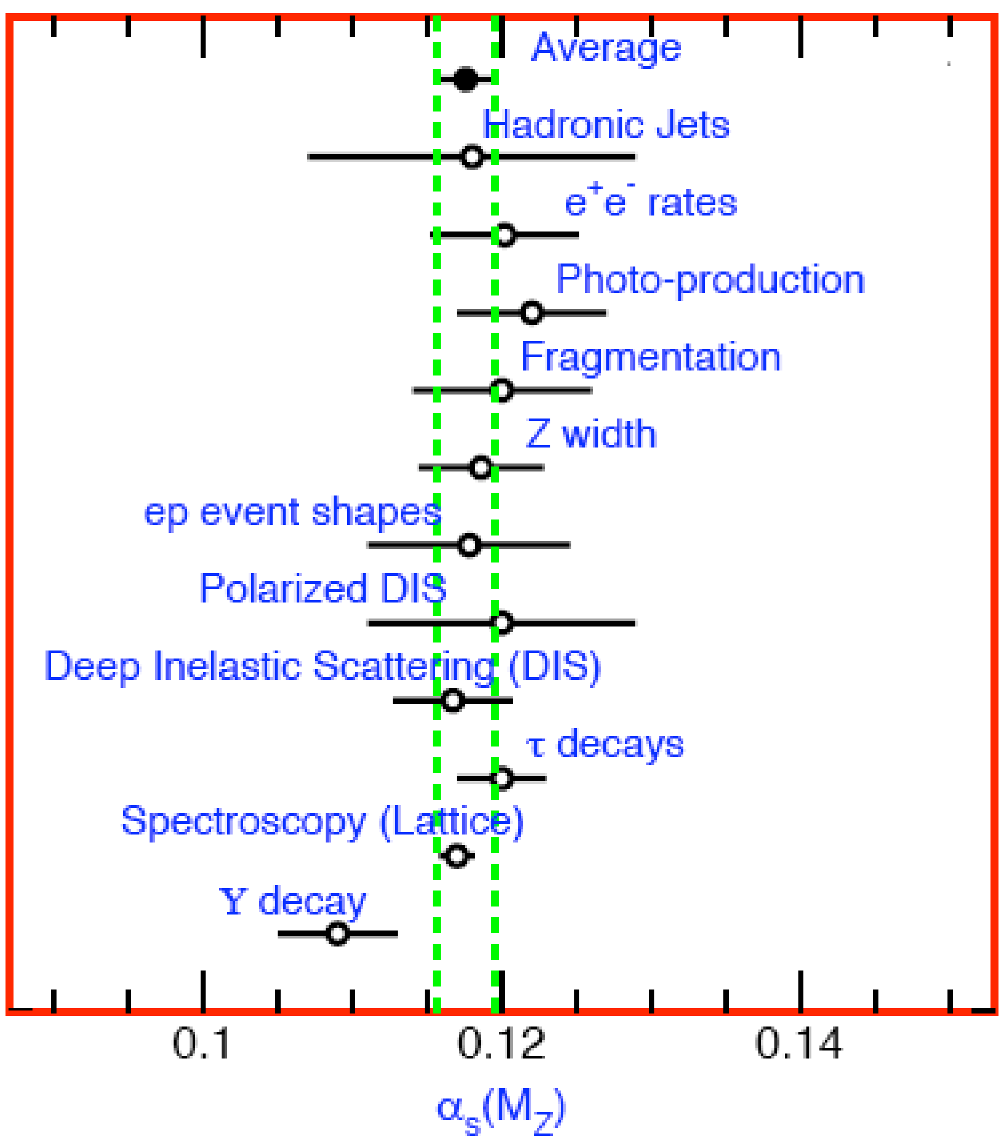
It is tempting to use identify Λ with Λ_S=300 MeV and see what we get for LEP I

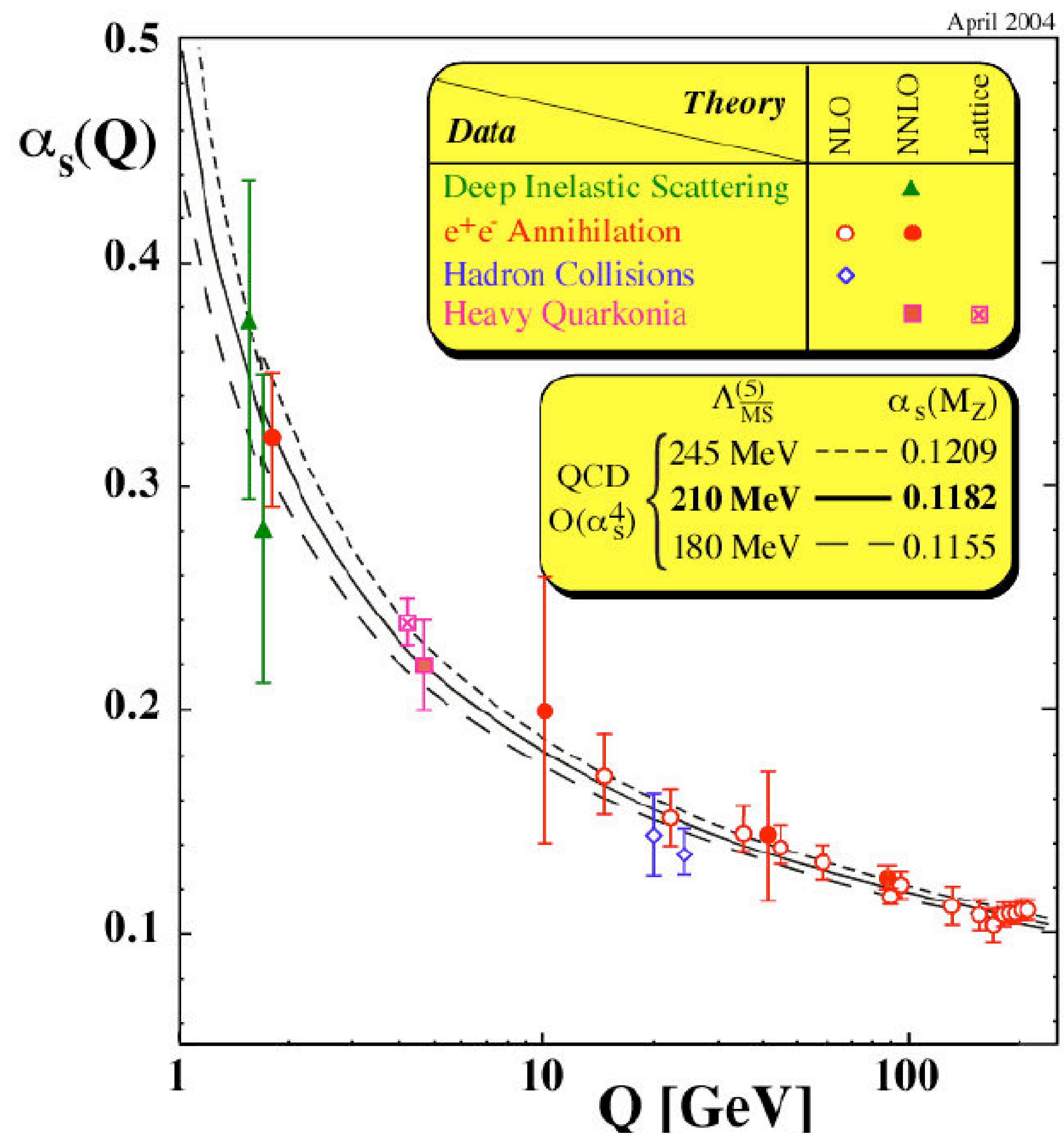
$$R(M_Z) = R_0 \left(1 + \frac{\alpha_S(M_Z)}{\pi}\right) = R_0(1 + 0.046)$$

which is in very reasonable agreement with LEP.

This example is very sloppy since it does not take into account heavy flavor thresholds, higher order effects, and so on. However it is important to stress that had we measured 8% effect at LEP I we would have extracted Λ= 5 GeV, a totally unacceptable value...

α_S: Experimental results





Many measurements at different scales all leading to very consistent results once evolved to the same reference scale, M_Z.

Scale dependence

$$R_2^{\rm ren}(\alpha_S(\mu),\frac{\mu^2}{Q^2}) = R_0\left(1+\frac{\alpha_S(\mu)}{\pi}+\left[c+\pi b_0\log\frac{\mu^2}{Q^2}\right]\left(\frac{\alpha_S(\mu)}{\pi}\right)^2\right)$$

As we said, at all orders physical quantities do not depend on the choice of the renormalization scale. At fixed order, however, there is a residual dependence due to the non-cancellation of the higher order logs:

$$\frac{d}{d\log\mu}\sum_{n=1}^{N} c_n(\mu)\alpha_S^n(\mu) \sim \mathcal{O}\left(\alpha_S(\mu)^{N+1}(\mu)\right)$$

So possible (related) questions are:

* Is there a systematic procedure to estimate the residual uncertainty in the theoretical prediction?

* Is it possible to identify a scale corresponding to our best guess for the theoretical prediction?

BTW: The above argument proves that the more we work the better a prediction becomes!

CHOOSING THE SCALE IN E+E- → HADRONS

Cross section for e+e- → hadrons:

$$\sigma_{tot} = \frac{12\pi\alpha^2}{s}\left(\sum_q q_f^2\right)(1+\Delta)$$

Let's take our best TH prediction

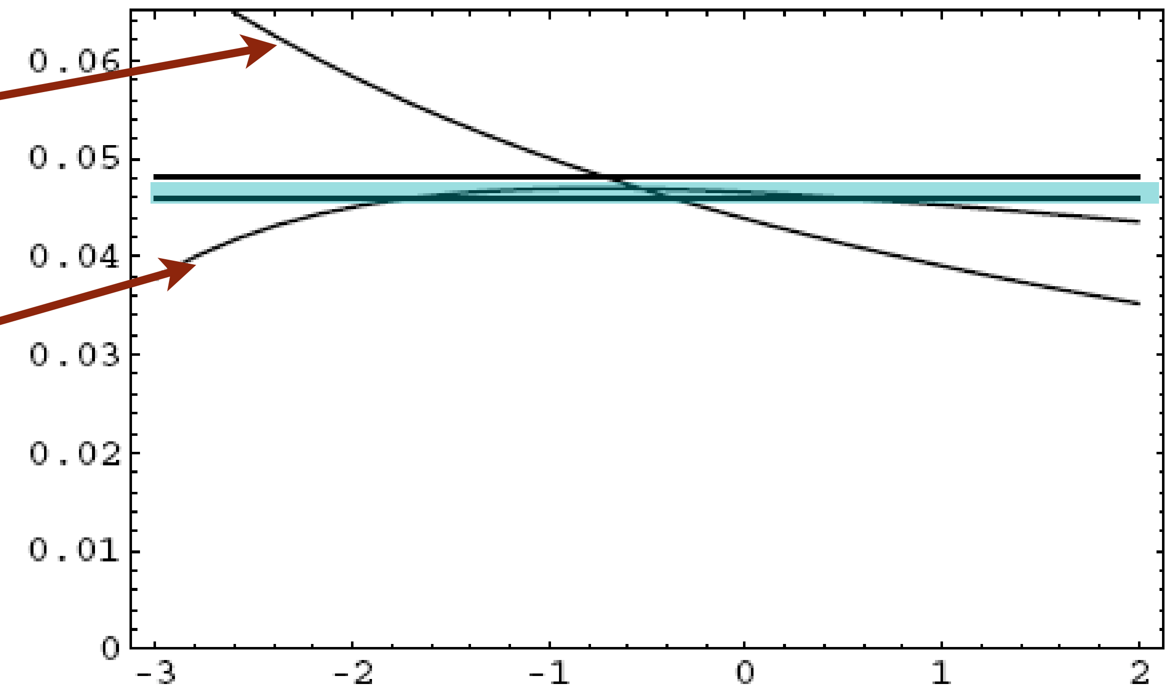
$$\Delta(\mu) = \frac{\alpha_S(\mu)}{\pi} + [1.41 + 1.92\log(\mu^2/s)]\left(\frac{\alpha_S(\mu)}{\pi}\right)^2$$
$$= [-12.8 + 7.82\log(\mu^2/s) + 3.67\log^2(\mu^2/s)]\left(\frac{\alpha_S(\mu)}{\pi}\right)^3$$

CHOOSING THE SCALE IN E+E- → HADRONS

Take $\alpha_s(M_Z)$ = 0.117, $\sqrt{s}$ = 34 GeV, 5 flavors and let's plot $\Delta(\mu)$ as function of p where $\mu=2^p\sqrt{s}$.

First curve Δ_1

Second curve Δ_2



Possible choice:

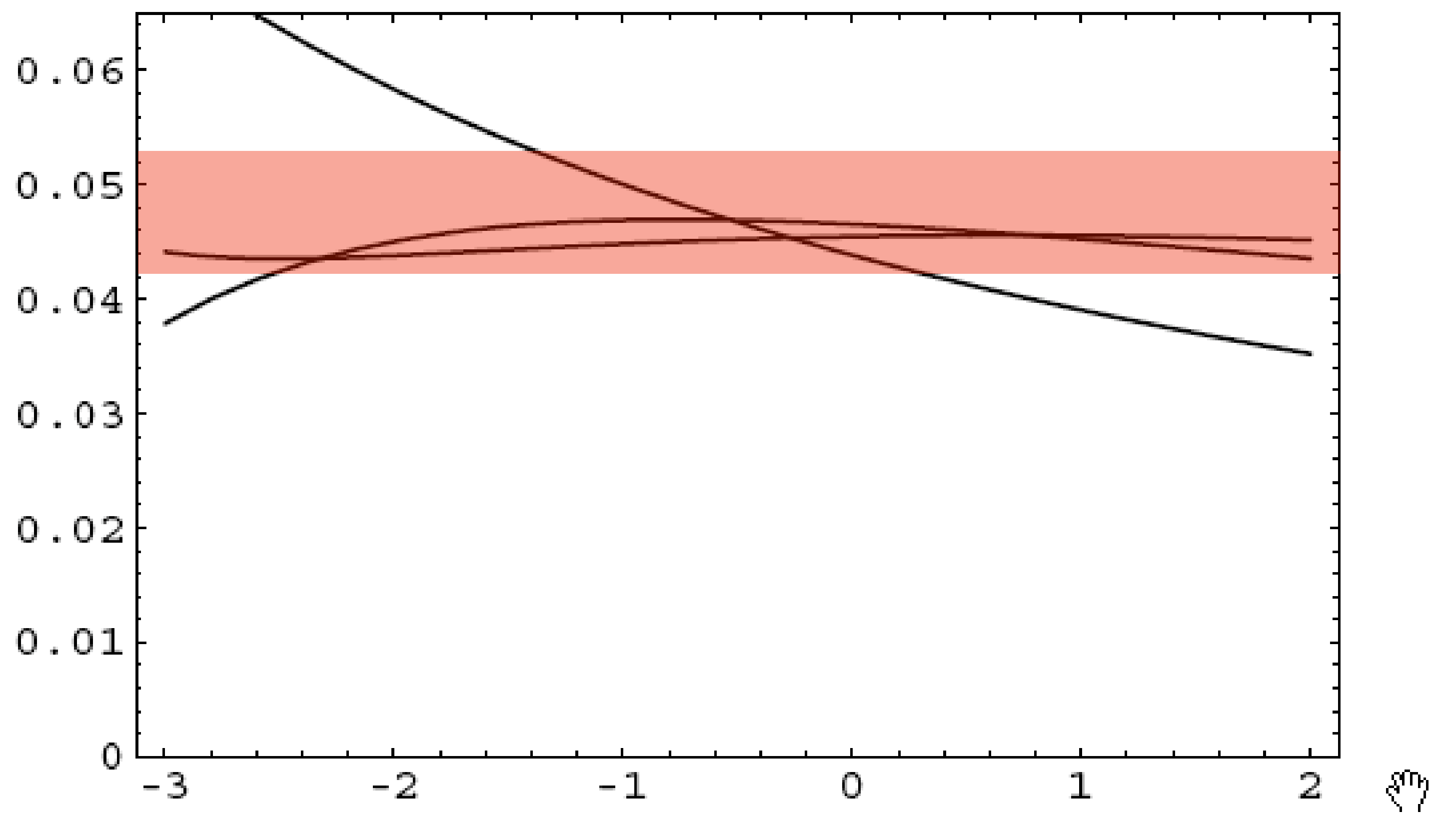
$\Delta_{PMS} = \Delta(\mu_0)$ where at μ_0 $d\Delta/d\mu=0$ and error band $p\in[1/2,2]$

Principle of mimimal sensitivity!

Improvement of a factor of two from LO to NLO!
How good is our error estimate?

CHOOSING THE SCALE IN E+E- → HADRONS

What happens at α_s^3?



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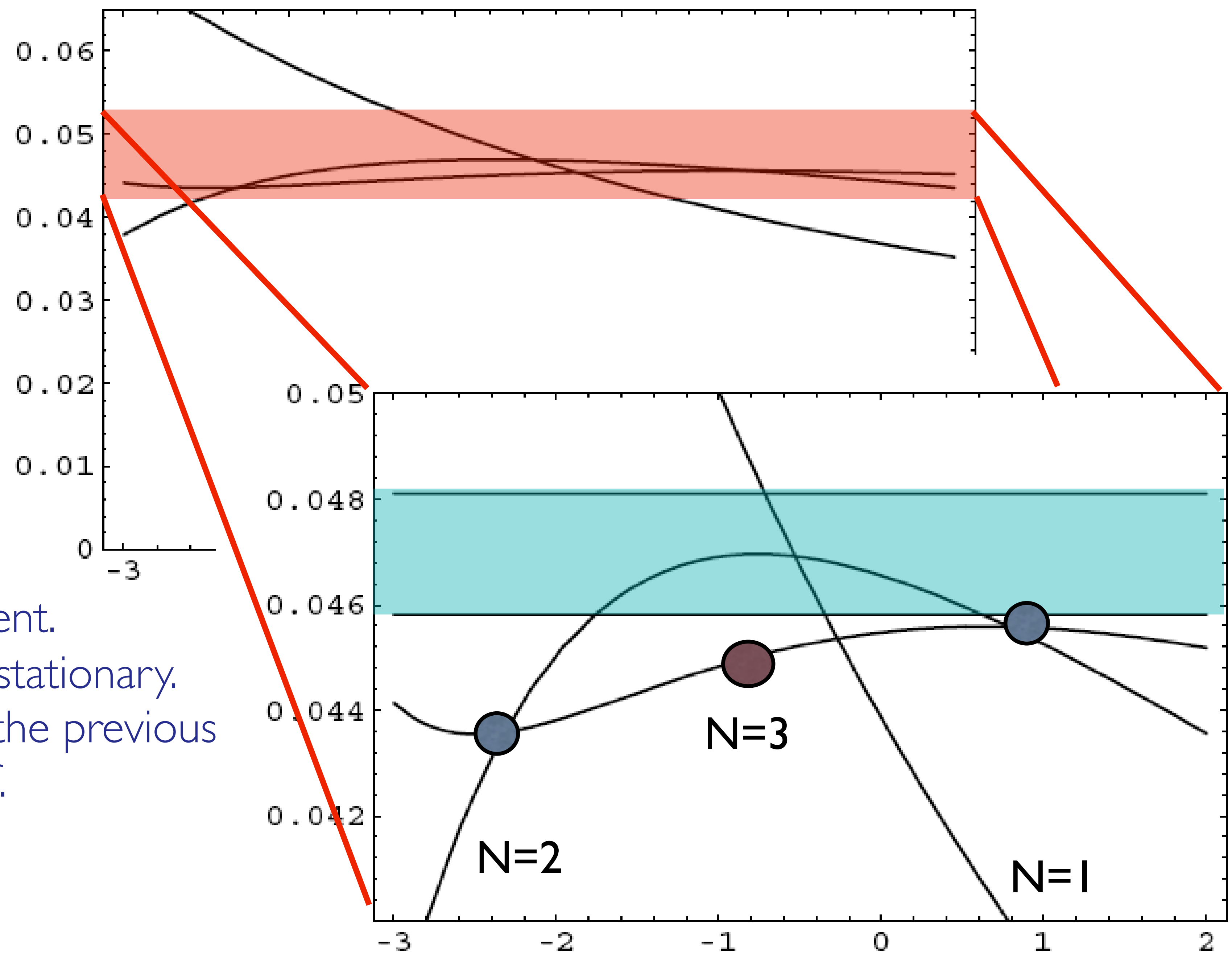

N=3 less scale dependent.
Two places where μ is stationary.
Take the average, then the previous estimate was sligthly off.

CHOOSING THE SCALE IN E+E- → HADRONS

Bottom line

There is no theorem that states the right 95% confidence interval for the uncertainty associated to the scale dependence of a theoretical predictions.

There are however many recipes available, where educated guesses (meaning physical). For example the so-called BLM choice.

In hadron-hadron collisions things are even more complicated due to the presence of another scale, the factorization scale, and in general also on a multi-scale processes...

Summary

1. We have given evidence of why we think QCD is a good theory: scaling, QCD is a renormalizable and asymptotically free QFT. We have seen how gauge invariance is realized in QCD starting from QED.

2. We have illustrated with an example the use of the renormalization group and the appearance of asymptotic freedom.