MadGraph5 (II)

Olivier Mattelaer

Aim of the Lecture

- Get you acquainted with the concepts and techniques used in event generation
- Give you hands-on experience
- Answer as many of your questions as I can

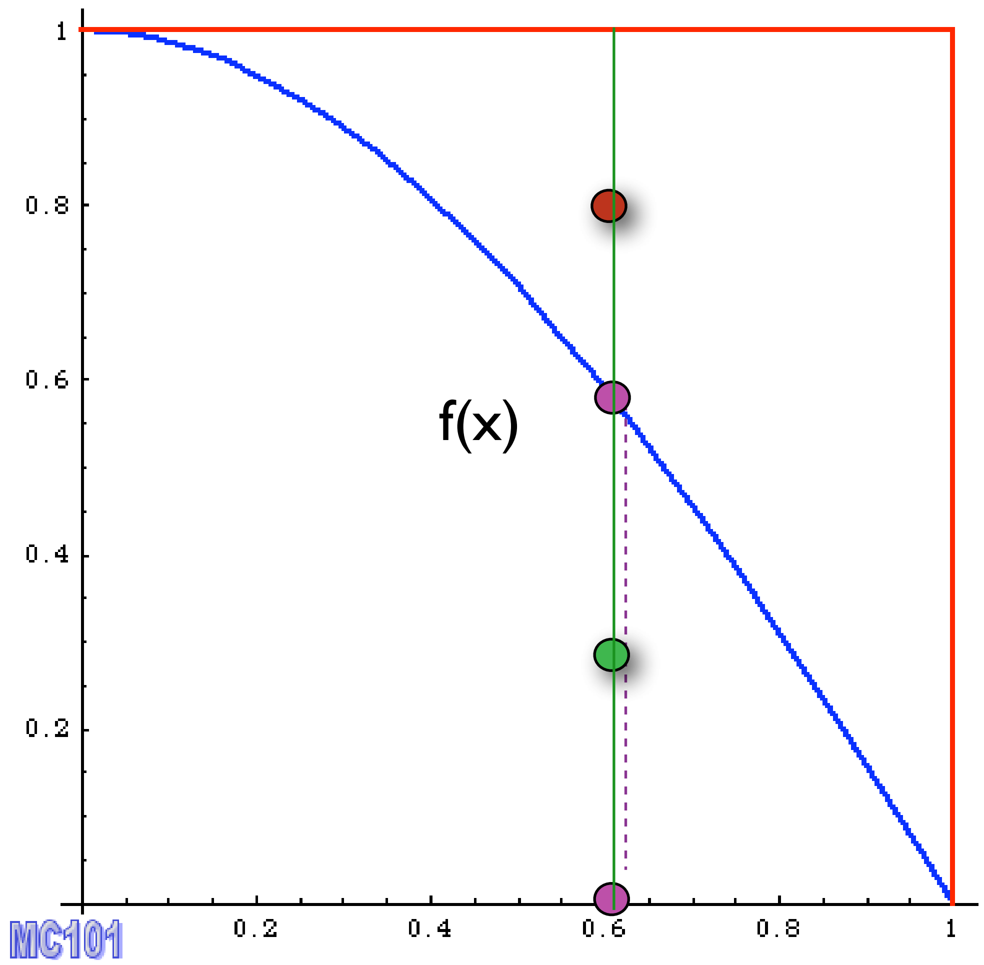
Lecture I

- Evaluation of Matrix Element
- Integration of the cross-section/ events generation

Lecture II

- Shower Monte-Carlo
- Matching/Merging
- NLO

Event generation



1. pick x
2. calculate f(x)
3. pick 0<y<fmax
4. Compare:
   if f(x)>y accept event,
   else reject it.

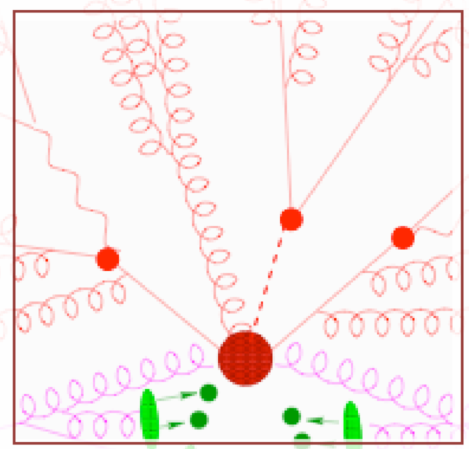
$$I = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

What are the MC for?

1. High-Q^2 Scattering

2. Parton Shower

☞ where new physics lies



☞ process dependent

☞ first principles description

☞ it can be systematically improved

3. Hadronization

4. Underlying Event

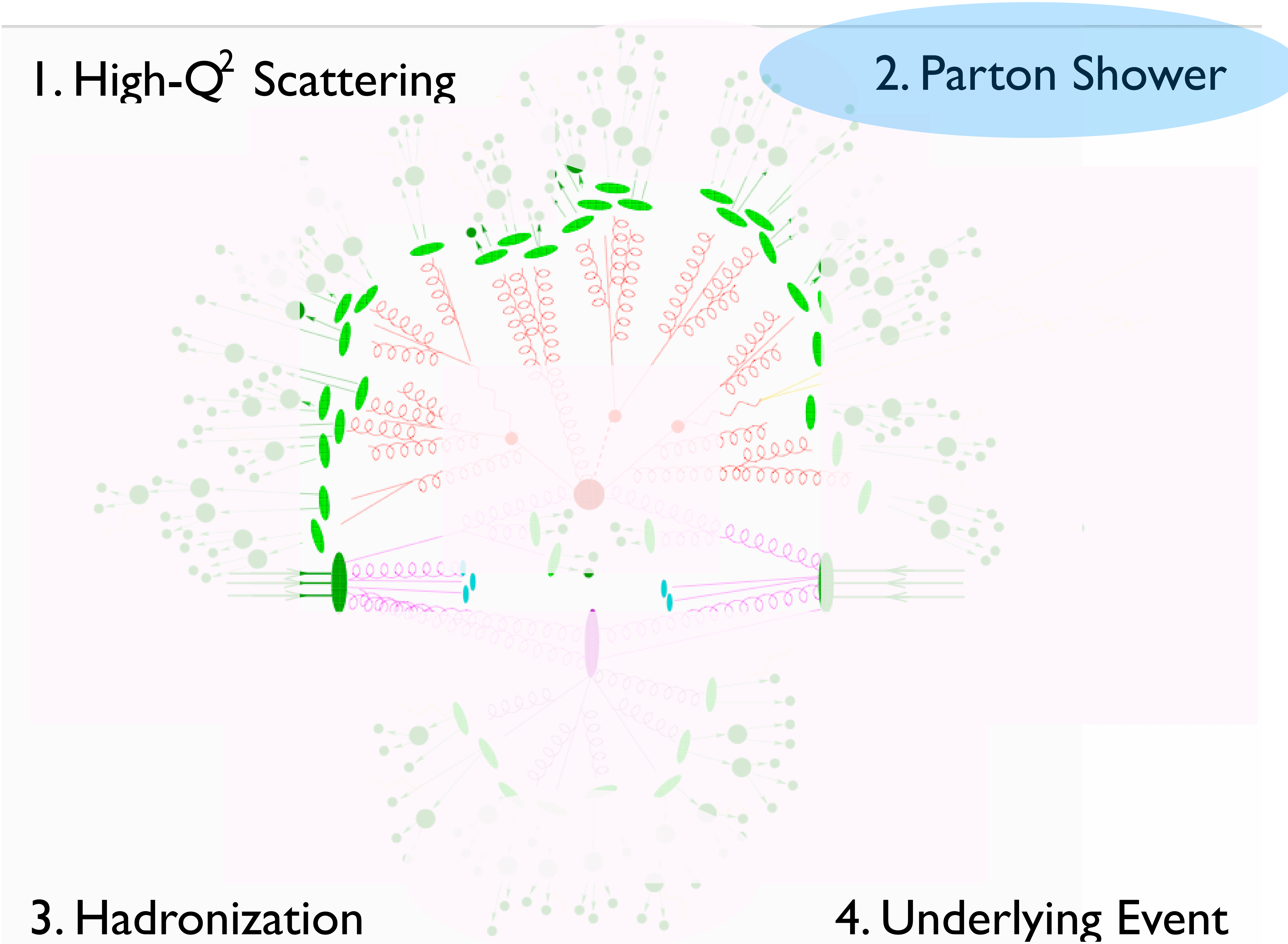
Summary

$$\int \hat{\sigma}_{ab\to X}(\hat{s},\ldots)\, f_a(x_1) f_b(x_2)\, dx_1 dx_2 d\Phi_{FS}$$

Parton level cross section | Parton density functions | Phase space integral

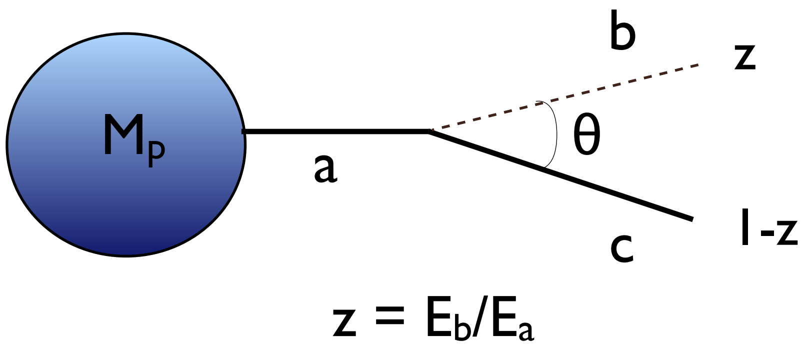
- MadGraph use Numerical method for the matrix element
  - ➡ Faster than analytical formula
  - ➡ Available For ANY BSM (thanks to UFO/ALOHA)
- Numerical integration is not trivial
  - ➡ We use Monte-Carlo integration
  - ➡ Return physical sample of events!
- MG5
  - ➡ decay chains
  - ➡ nice interface
  - ➡ several output formats

What are the MC for?



Matrix elements involving q →q g or g → gg are strongly enhanced when the final state particles are close in the phase space:

$$\frac{1}{(p_b+p_c)^2} \simeq \frac{1}{2E_bE_c(1-\cos\theta)} = \frac{1}{t}$$



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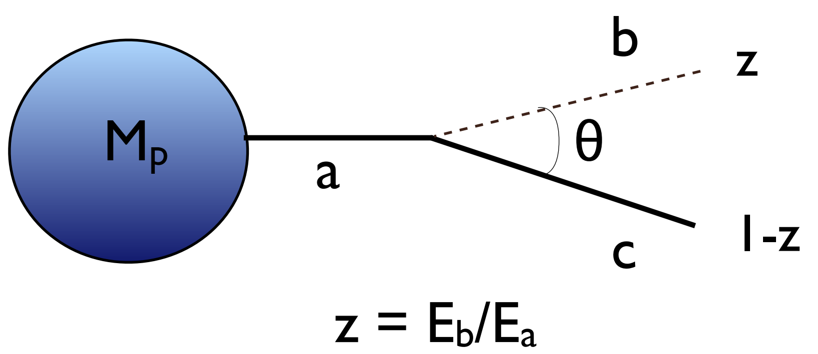
soft



Parton Shower basics

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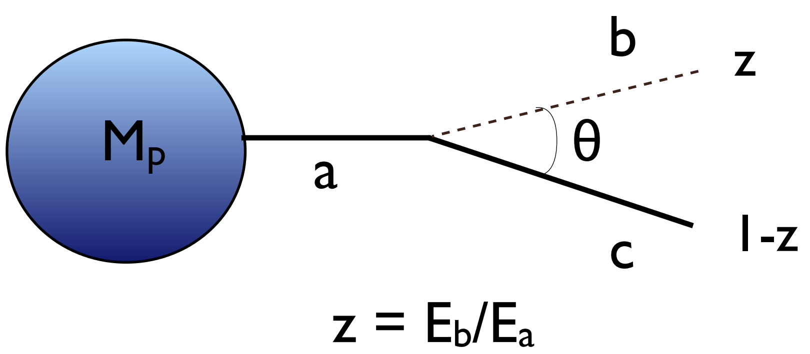

soft and collinear

divergencies

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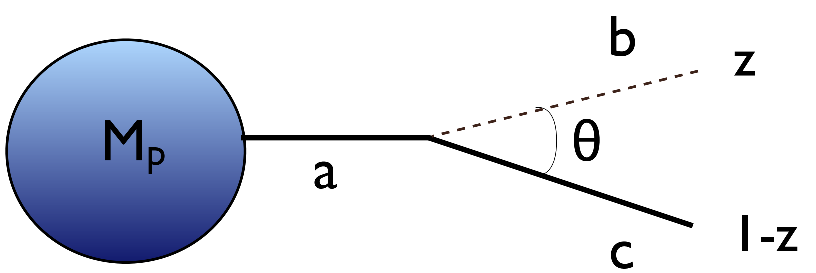

z = E_b/E_a

soft and collinear

divergencies

Collinear factorization:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

when θ is small.

Parton Shower basics

The spin averaged (unregulated) splitting functions for the various types of branching are:

$$\hat{P}_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)} \right],$$

$$\hat{P}_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right],$$

$$\hat{P}_{qg}(z) = T_R \left[z^2 + (1-z)^2 \right],$$

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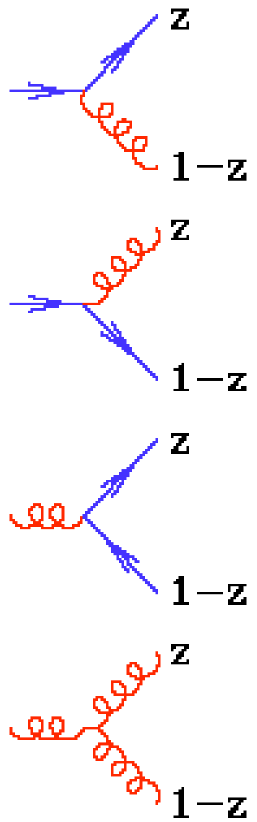

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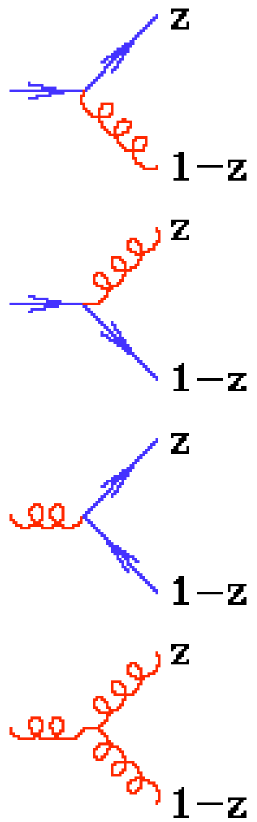

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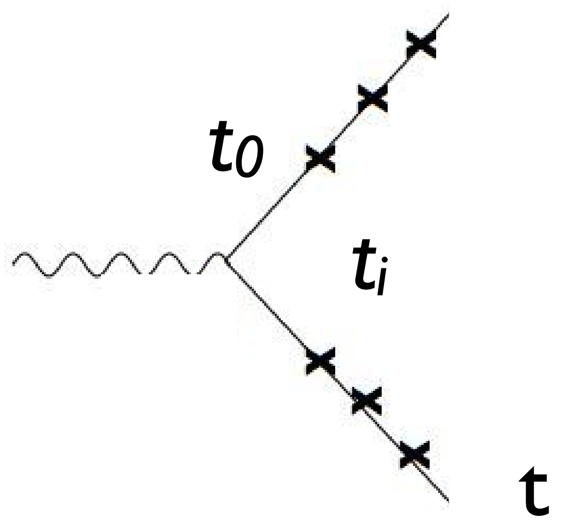
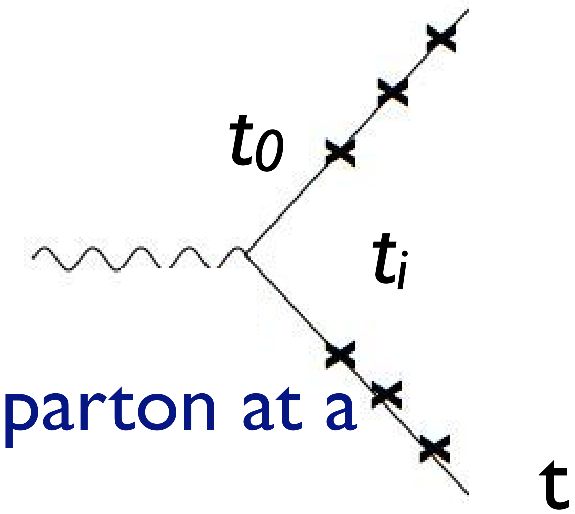
Comments:
* Gluons radiate the most
* There are soft divergences in z=1 and z=0.
* P_{qg} has no soft divergences.

Parton Shower basics





- Now, consider the non-branching probability for a parton at a given virtuality t_i:

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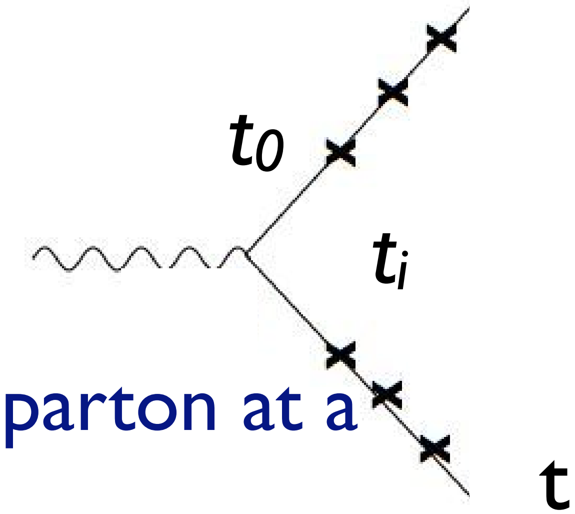
Parton Shower basics



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$$\simeq e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz \hat{P}(z)\right)}$$

$$\simeq e^{-\int_t^{t_0}\frac{dt'}{t'}\frac{\alpha_s}{2\pi}\int dz \hat{P}(z)} = \Delta(t, t_0)$$

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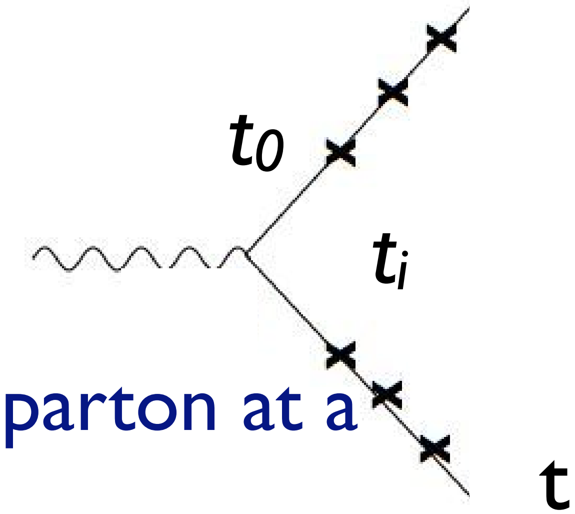

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- This is the famous "Sudakov form factor"

Final-state parton showers

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$\Delta(t_{i+1},t_i) = R$
where R is a random number (uniform on [0, 1]).

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5. For each emitted particle, iterate steps 2-4 until branching stops.

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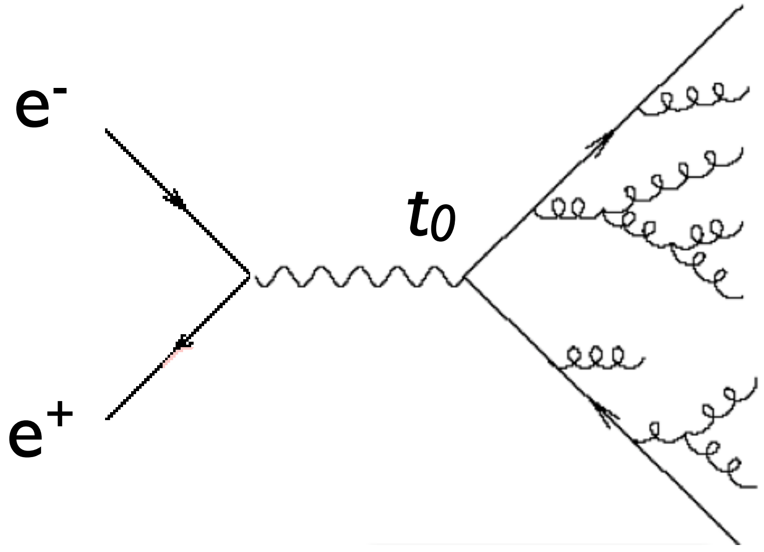
- The result is a "cascade" or "shower" of partons with ever smaller virtualities.



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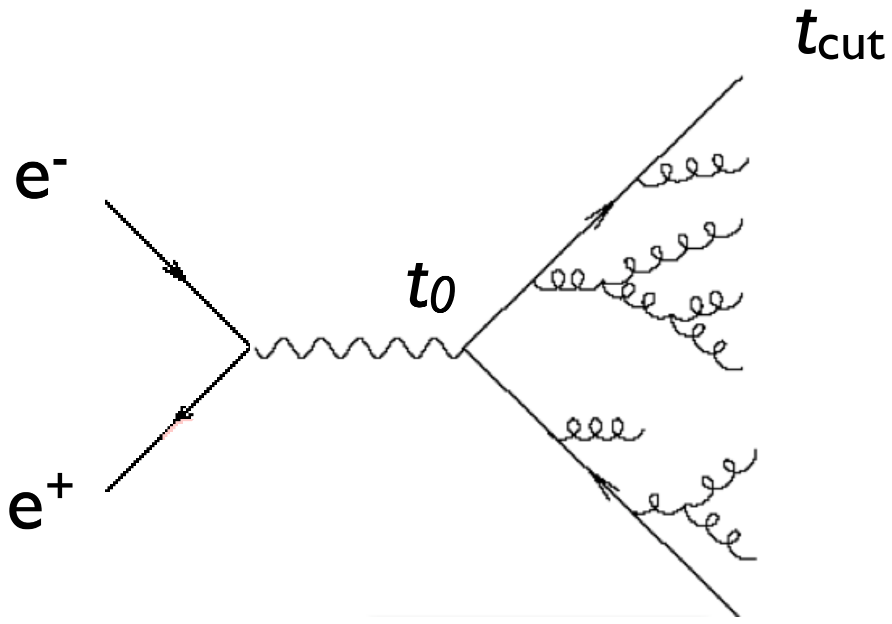

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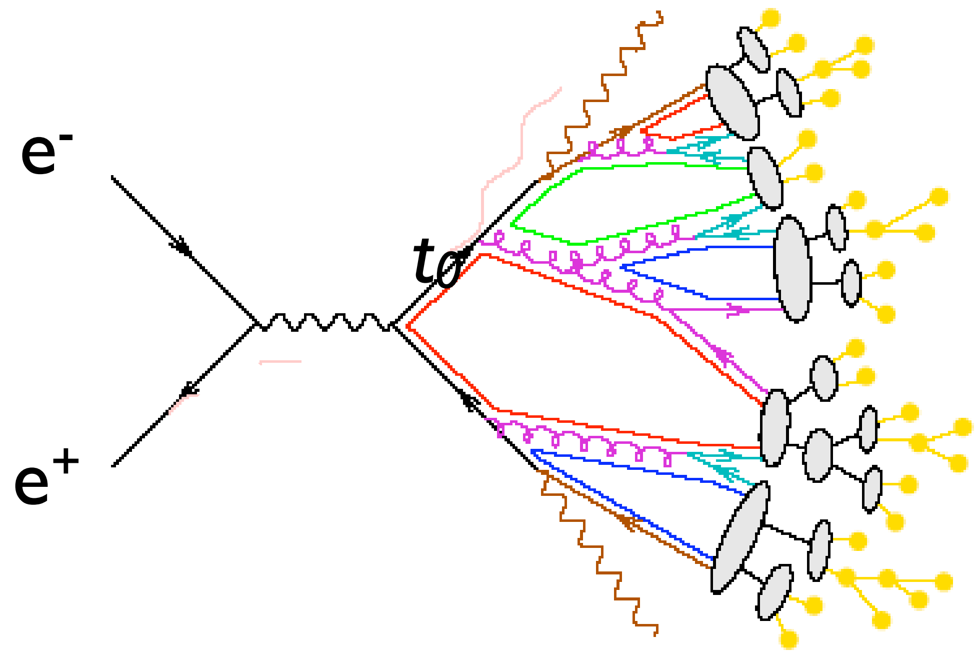
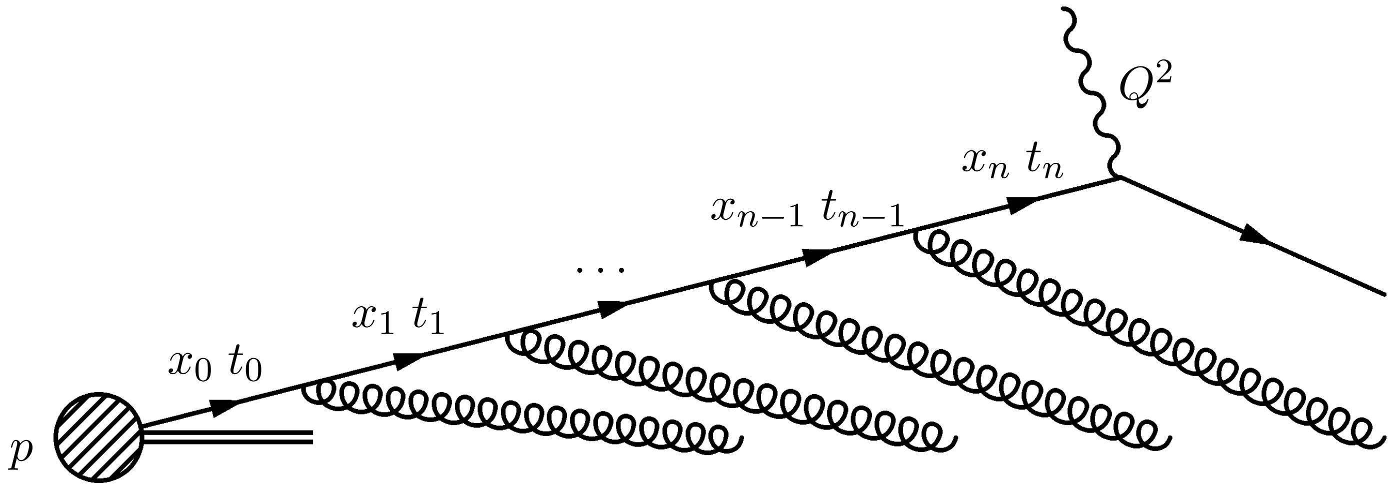
- The cutoff scale t_{cut} is usually set close to 1 GeV, the scale where non-perturbative effects start dominating over the perturbative parton shower.
- At this point, phenomenological models are used to simulate how the partons turn into color-neutral hadrons. Hadronization not sensitive to the physics at scale t_0, but only t_{cut}! (can be tuned once and for all!)



Initial-state parton splittings

- So far, we have looked at final-state (time-like) splittings
- For initial state, the splitting functions are the same
- However, there is another ingredient - the parton density (or distribution) functions (PDFs)
  - ➡ Probability to find a given parton in a hadron at a given momentum fraction $x = p_z/P_z$ and scale t
- How do the PDFs evolve with increasing t?

Initial-state parton splittings



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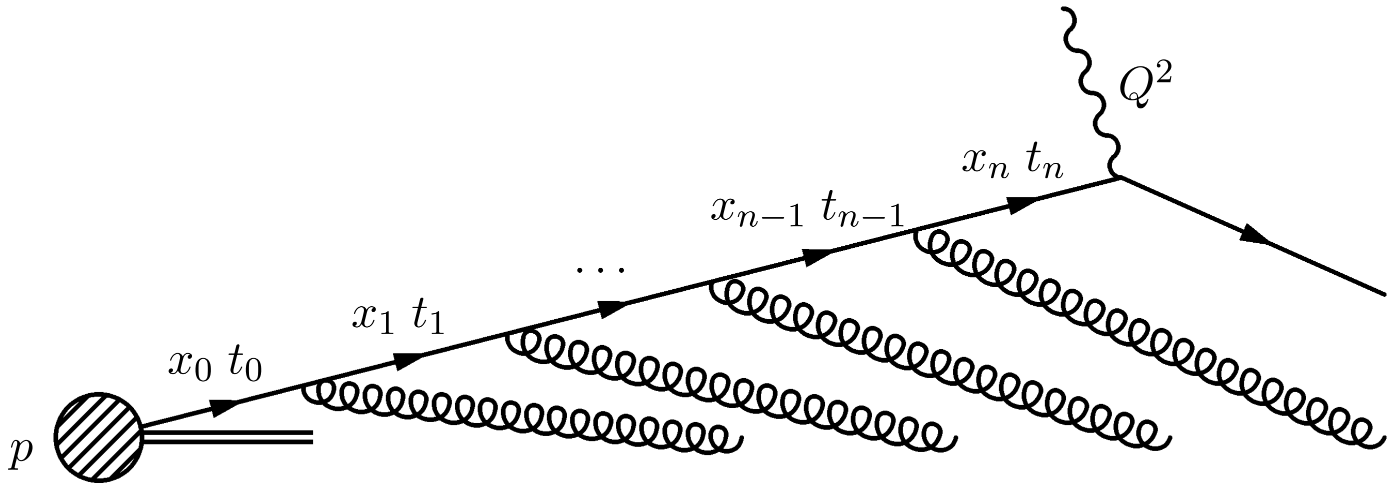

- Start with a quark PDF $f_0(x)$ at scale t_0. After a single parton emission, the probability to find the quark at virtuality $t > t_0$ is

$$f(x,t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) f_0\left(\frac{x}{z}\right)$$

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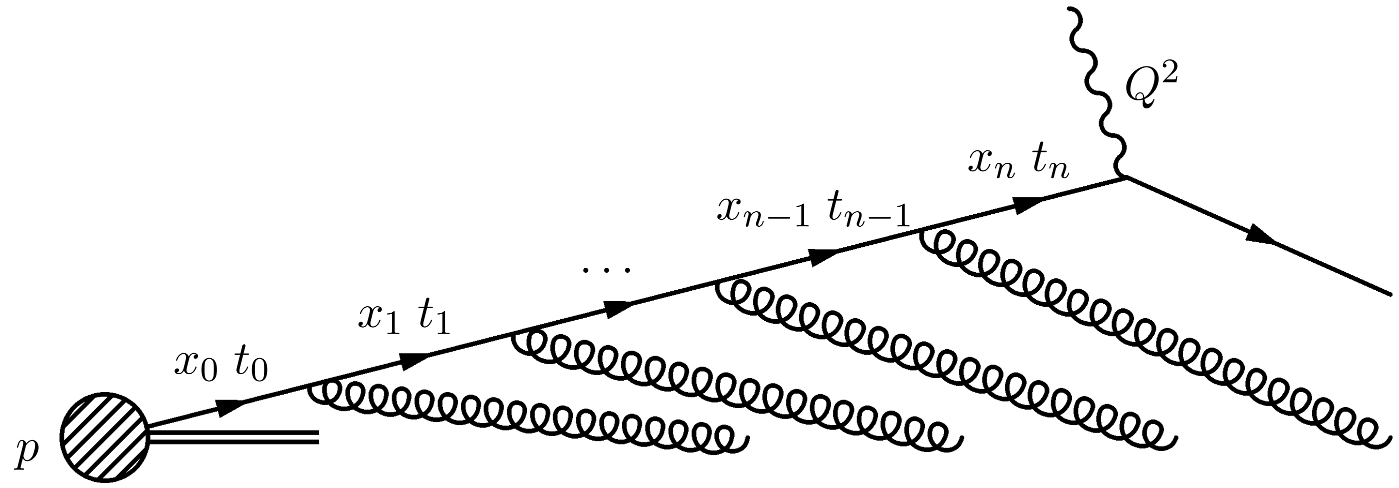

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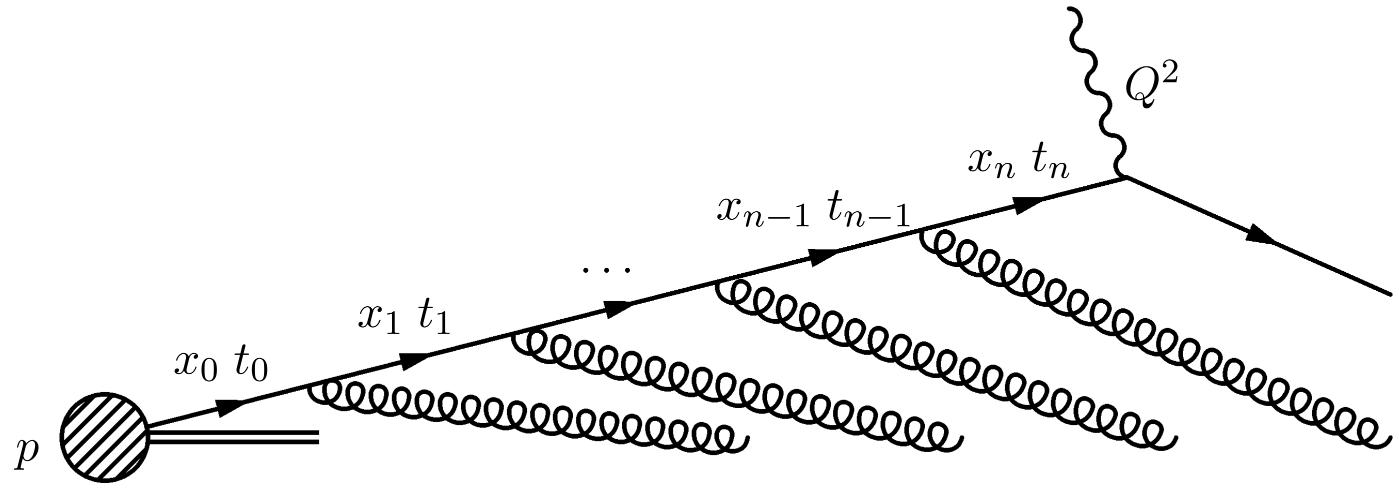

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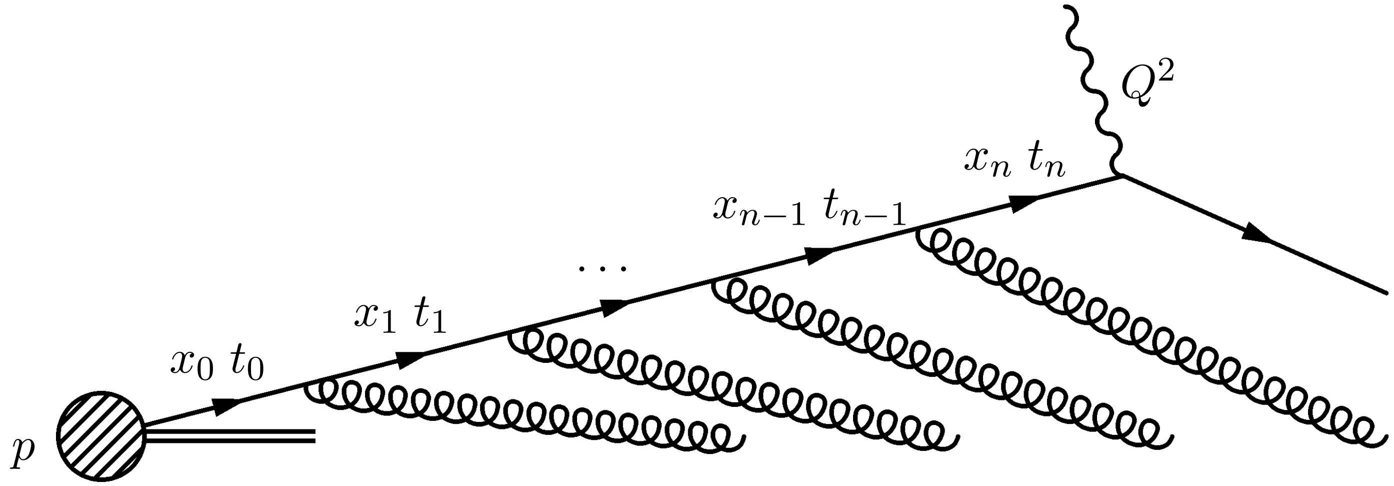
f(x/z, t')

The DGLAP equation



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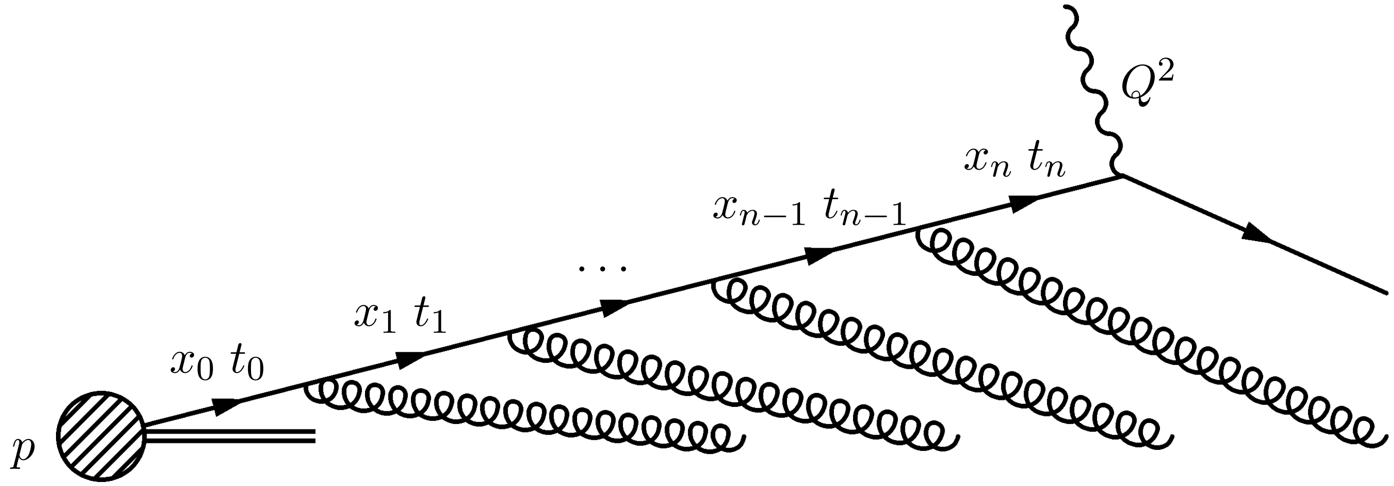

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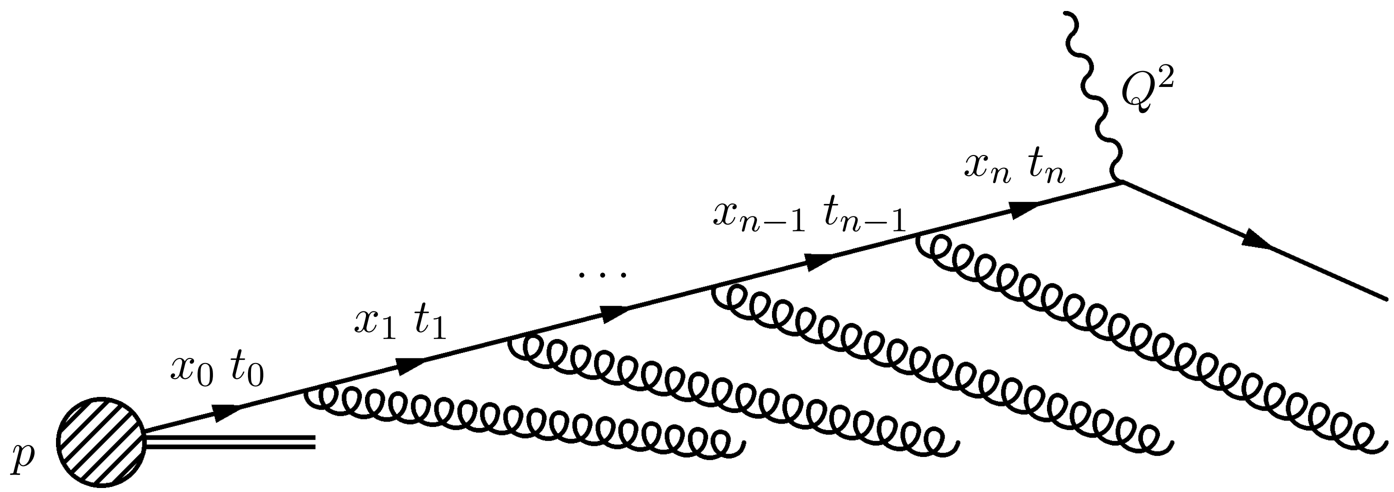
The DGLAP equation



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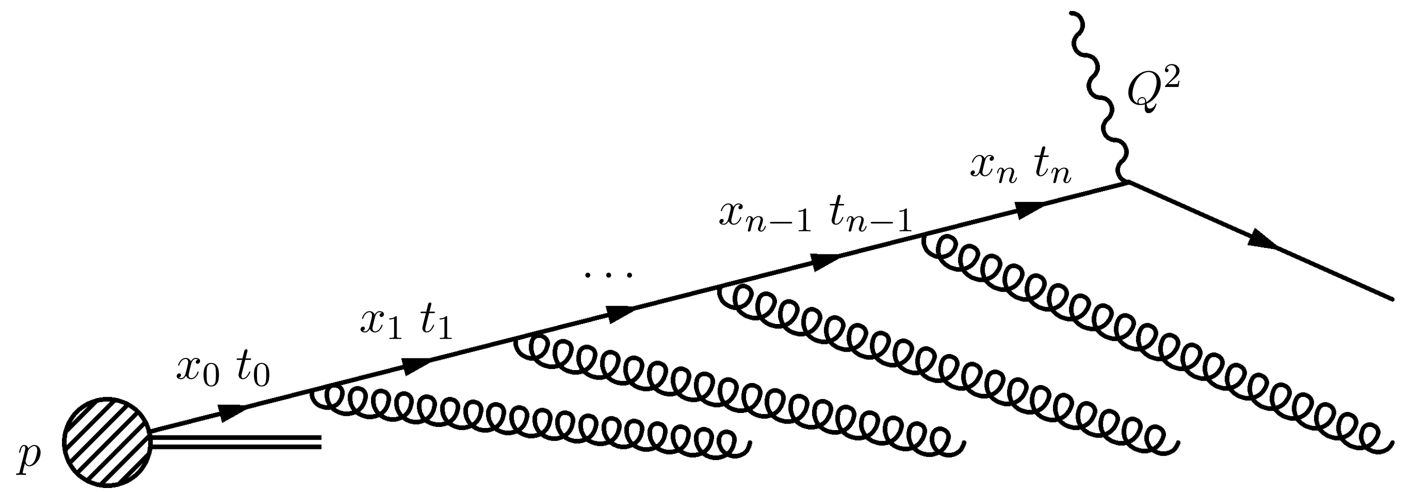

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- This is the famous DGLAP equation (where we have taken into account the multiple parton species *i*, *j*). The boundary condition for the equation is the initial PDFs $f_{i0}(x)$ at a starting scale t_0 (again around 1 GeV).

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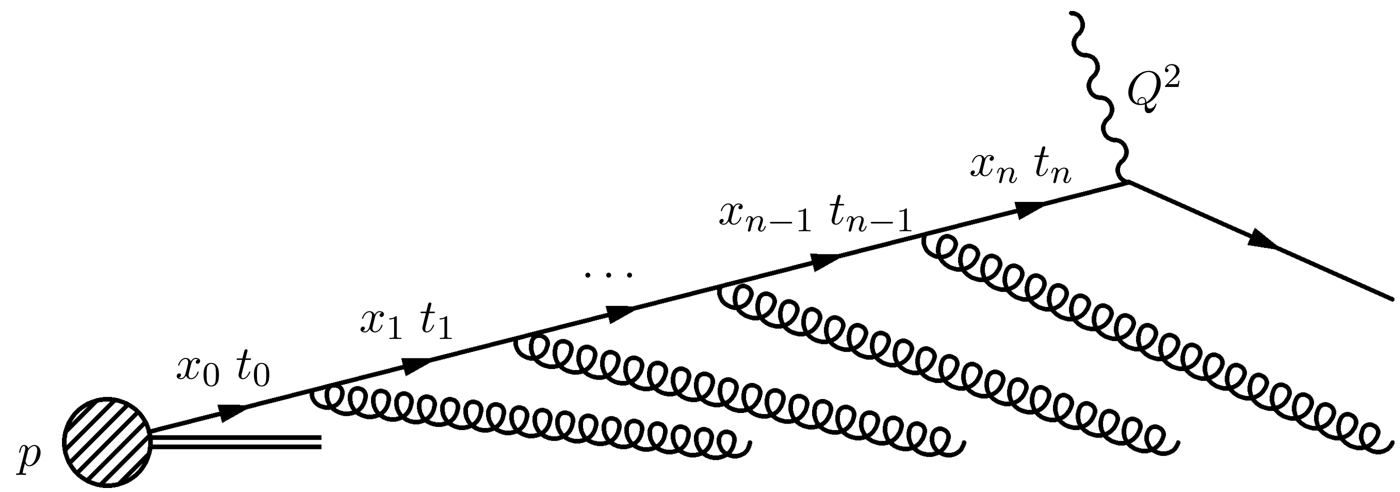

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- These starting PDFs are fitted to experimental data.

Initial-state parton showers

- To simulate parton radiation from the initial state, we start with the hard scattering, and then "devolve" the DGLAP evolution to get back to the original hadron: Backwards evolution!

- In backwards evolution, the Sudakovs include also the PDFs - this follows from the DGLAP equation and ensures conservation of probability:

$$\Delta_{Ii}(x,t_1,t_2) = \exp\left\{ -\int_{t_1}^{t_2} dt' \sum_j \int_x^1 \frac{dx'}{x'} \frac{\alpha_s(t')}{2\pi} P_{ij}\left(\frac{x}{x'}\right) \frac{f_i(x',t')}{f_j(x,t')} \right\}$$

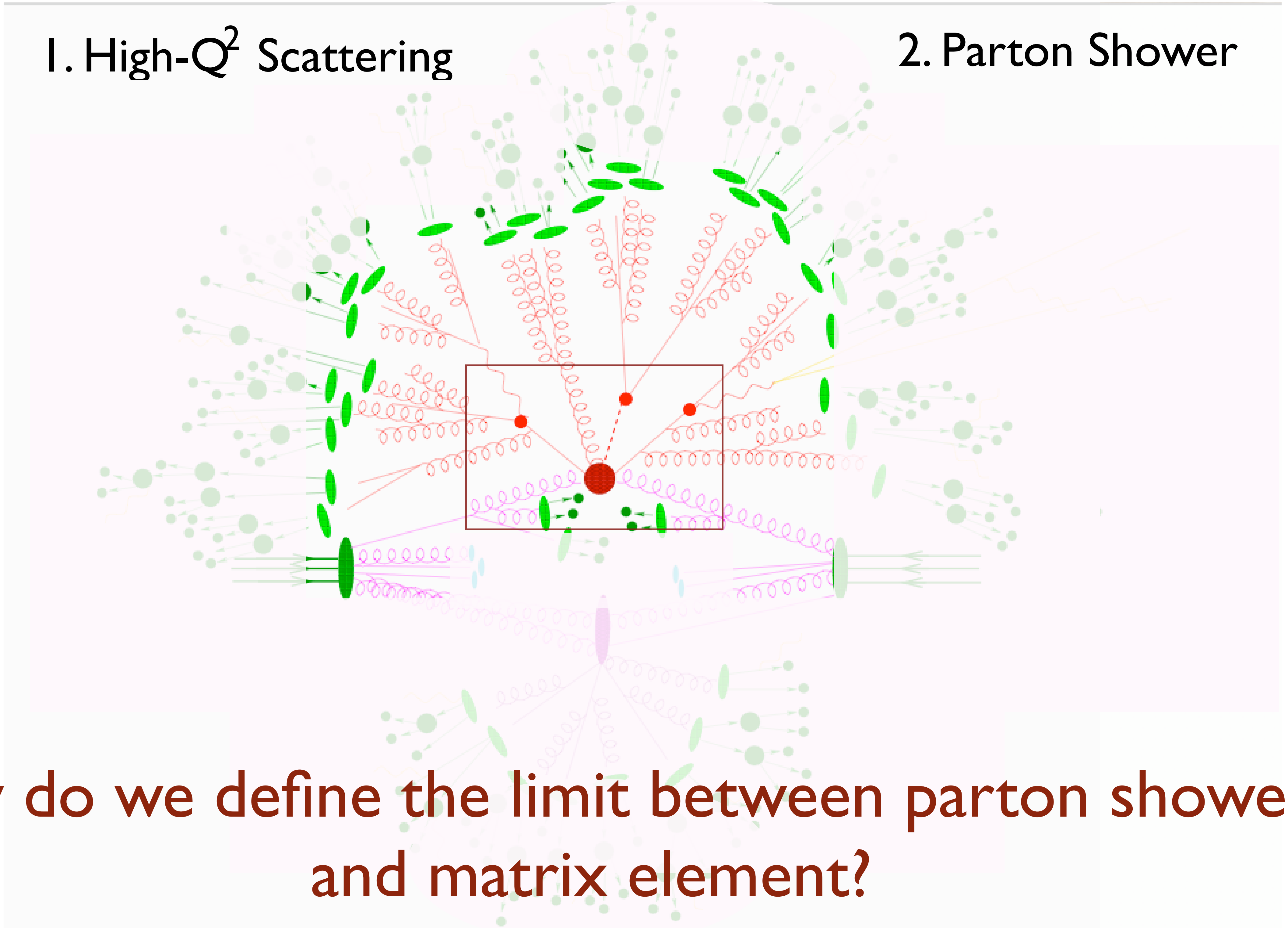
  This represents the probability that parton *i* will stay at the same *x* (no splittings) when evolving from t_1 to t_2.

- The shower simulation is now done as in FS shower!

Parton Shower MC event generators

- In both initial-state and final-state showers, the definition of t is not unique, as long as it has the dimension of scale:
- Different parton shower generators have made different choices:
  - ➡ Ariadne: "dipole p_T"
  - ➡ Herwig: $E \cdot \theta$
  - ➡ Pythia (old): virtuality q^2
  - ➡ Pythia 6.4 and Pythia 8: p_T
  - ➡ Sherpa: v. 1.1 virtuality q^2, v. 1.2 "dipole p_T"
- Note that all of the above are complete MC event generators with matrix elements, parton showers, hadronization, decay, and underlying event simulation.

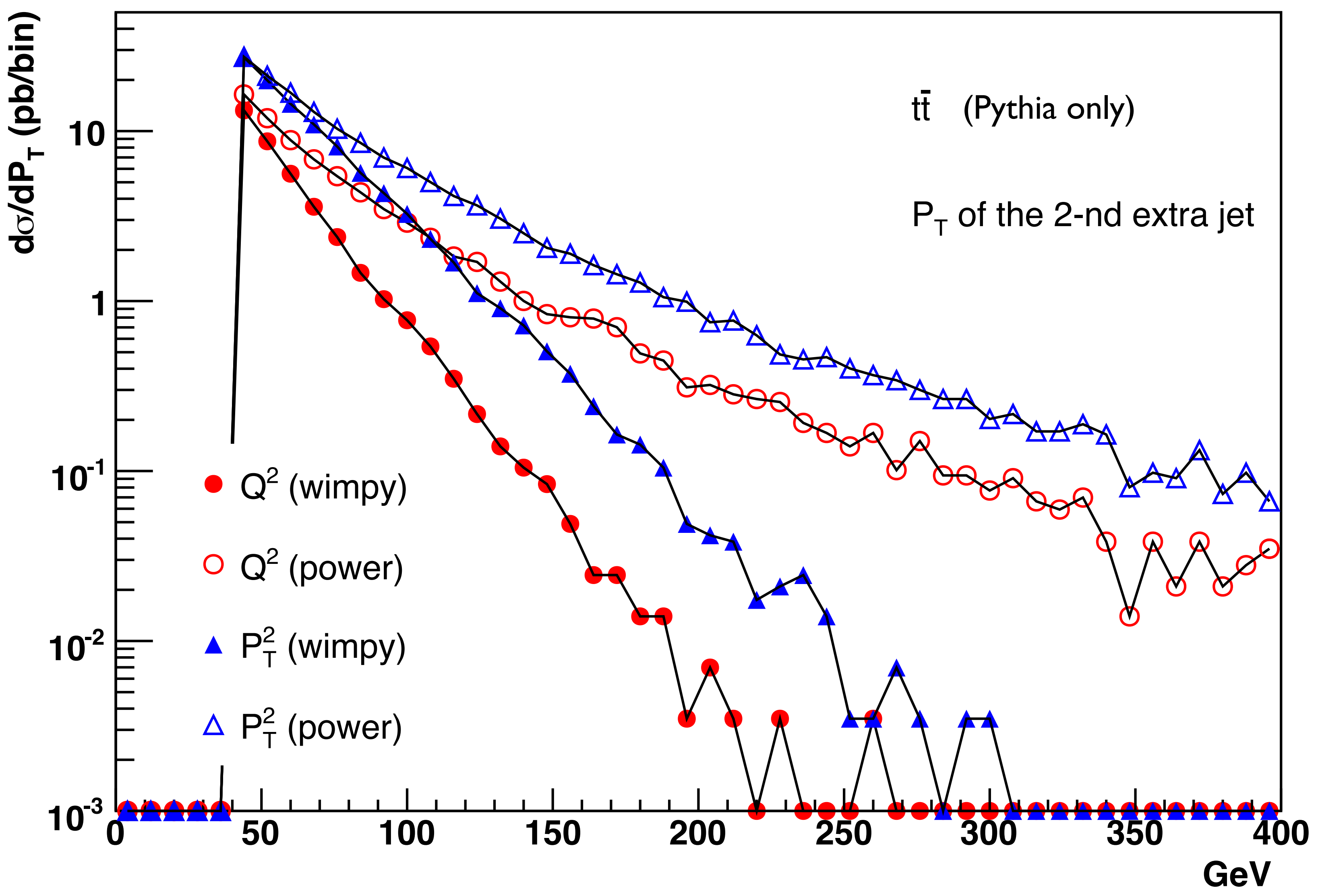
Back to our favorite piece of art!



How do we define the limit between parton shower and matrix element?

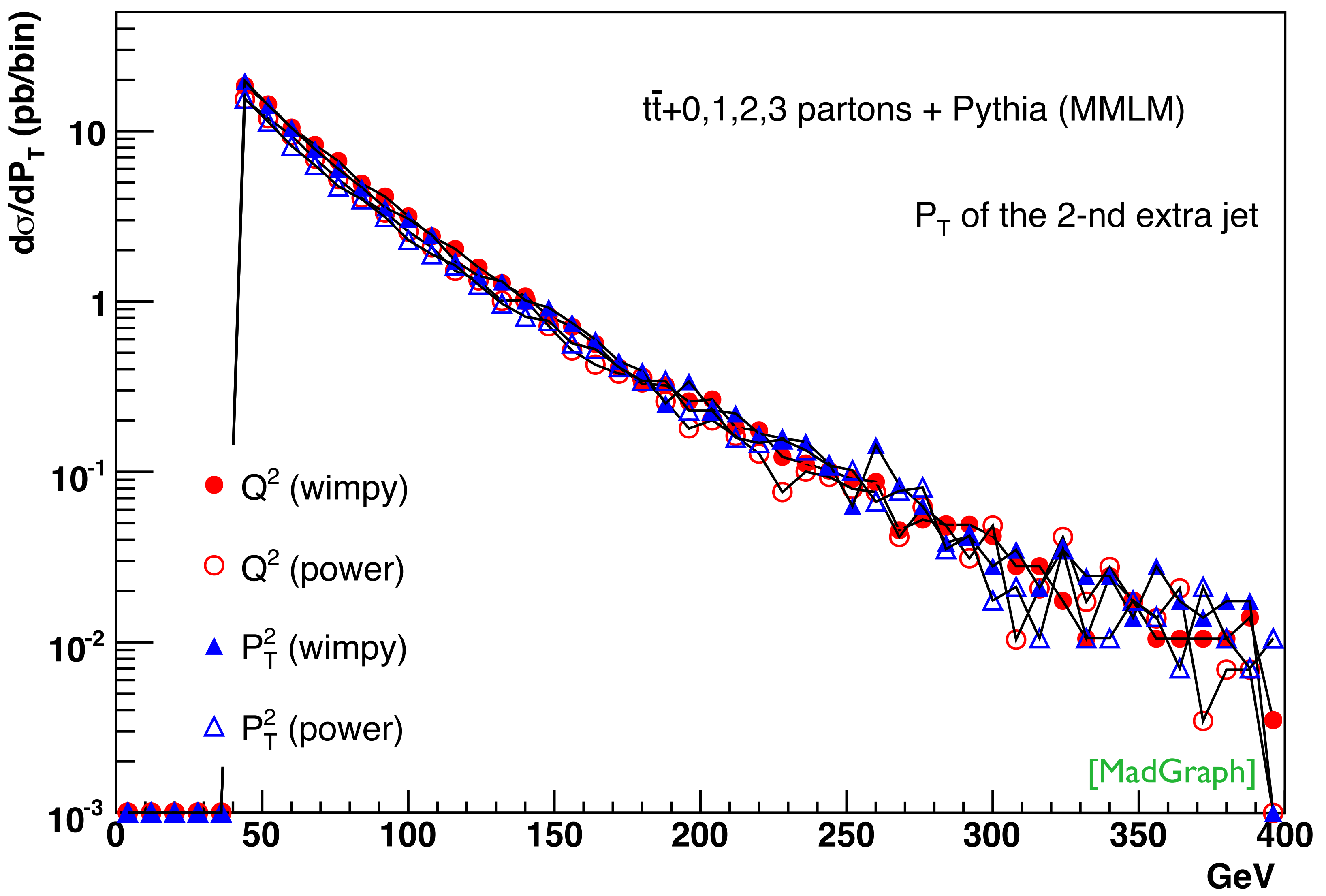
PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)



PS alone vs ME matching

In a matched sample these differences are irrelevant since the behavior at high pt is dominated by the matrix element.



Matrix Elements vs. Parton Showers

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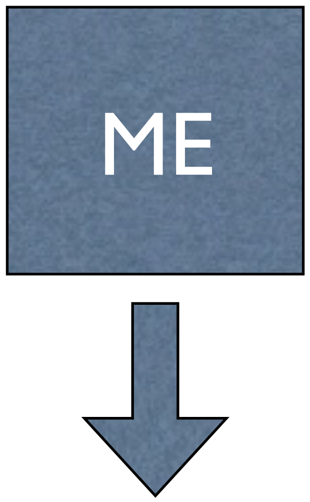

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Matrix Elements vs. Parton Showers

ME

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4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Shower MC

1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are collinear and/or soft
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Matrix Elements vs. Parton Showers

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Approaches are complementary: merge them!

Matrix Elements vs. Parton Showers

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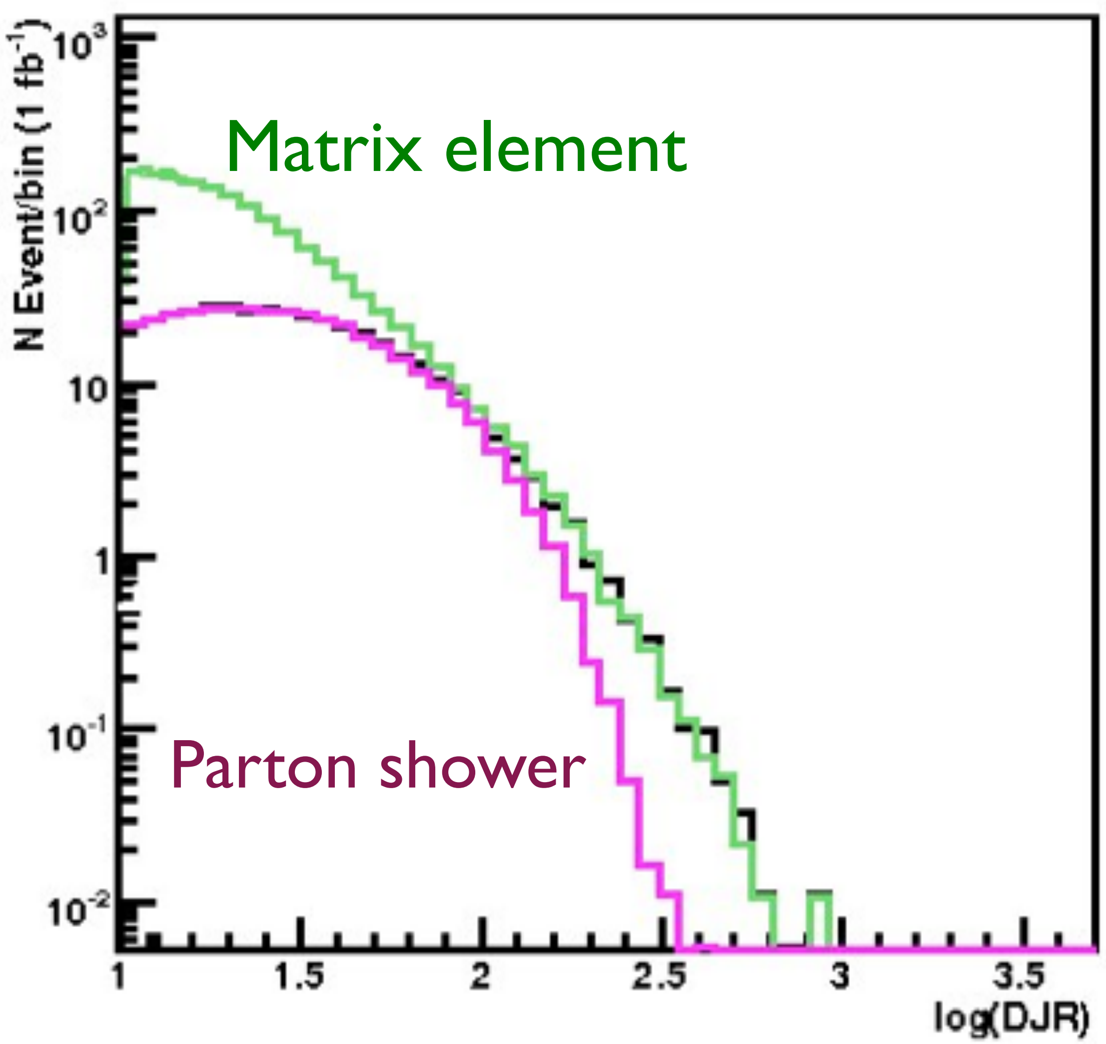
Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

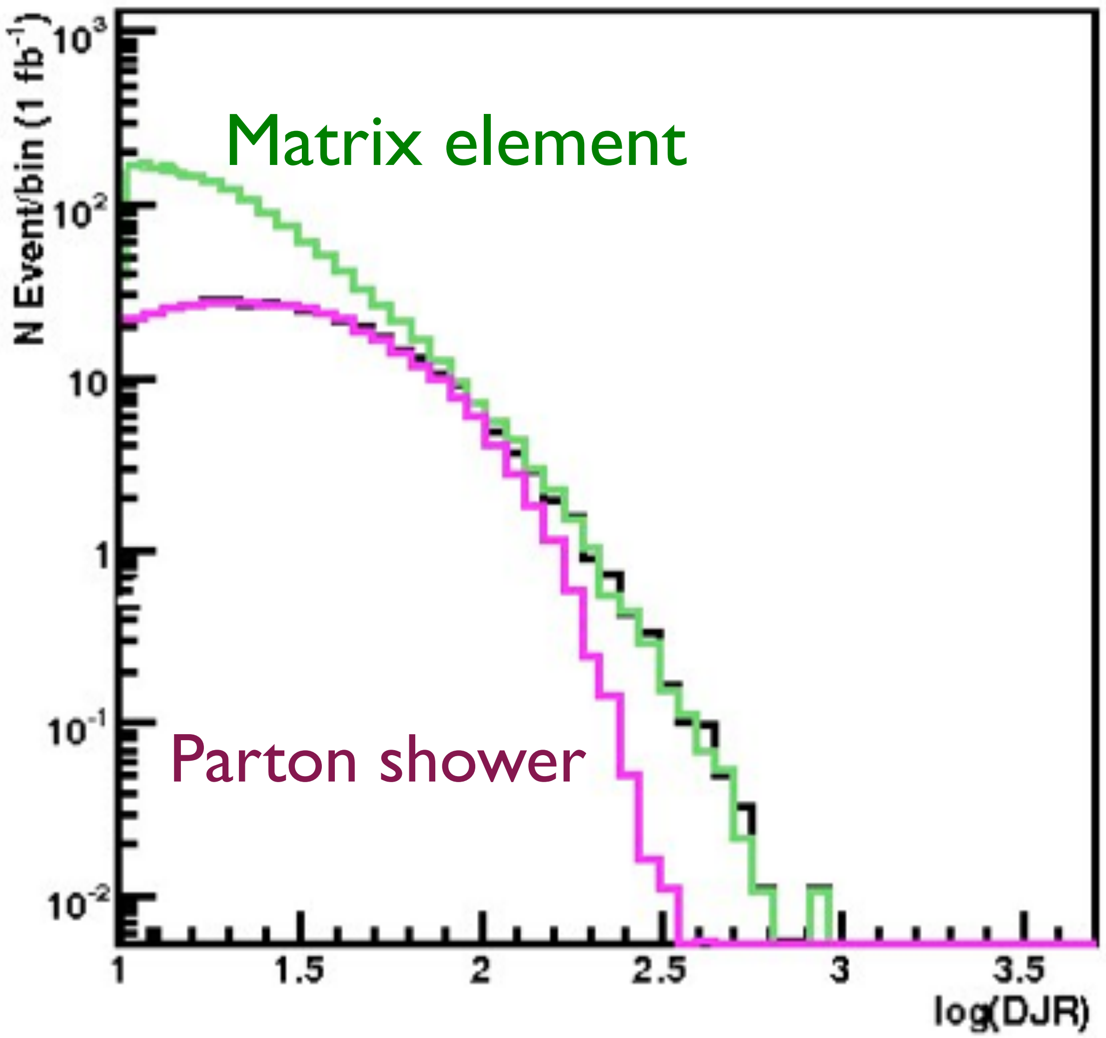
Goal for ME-PS merging/matching



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Goal for ME-PS merging/matching

- Regularization of matrix element divergence



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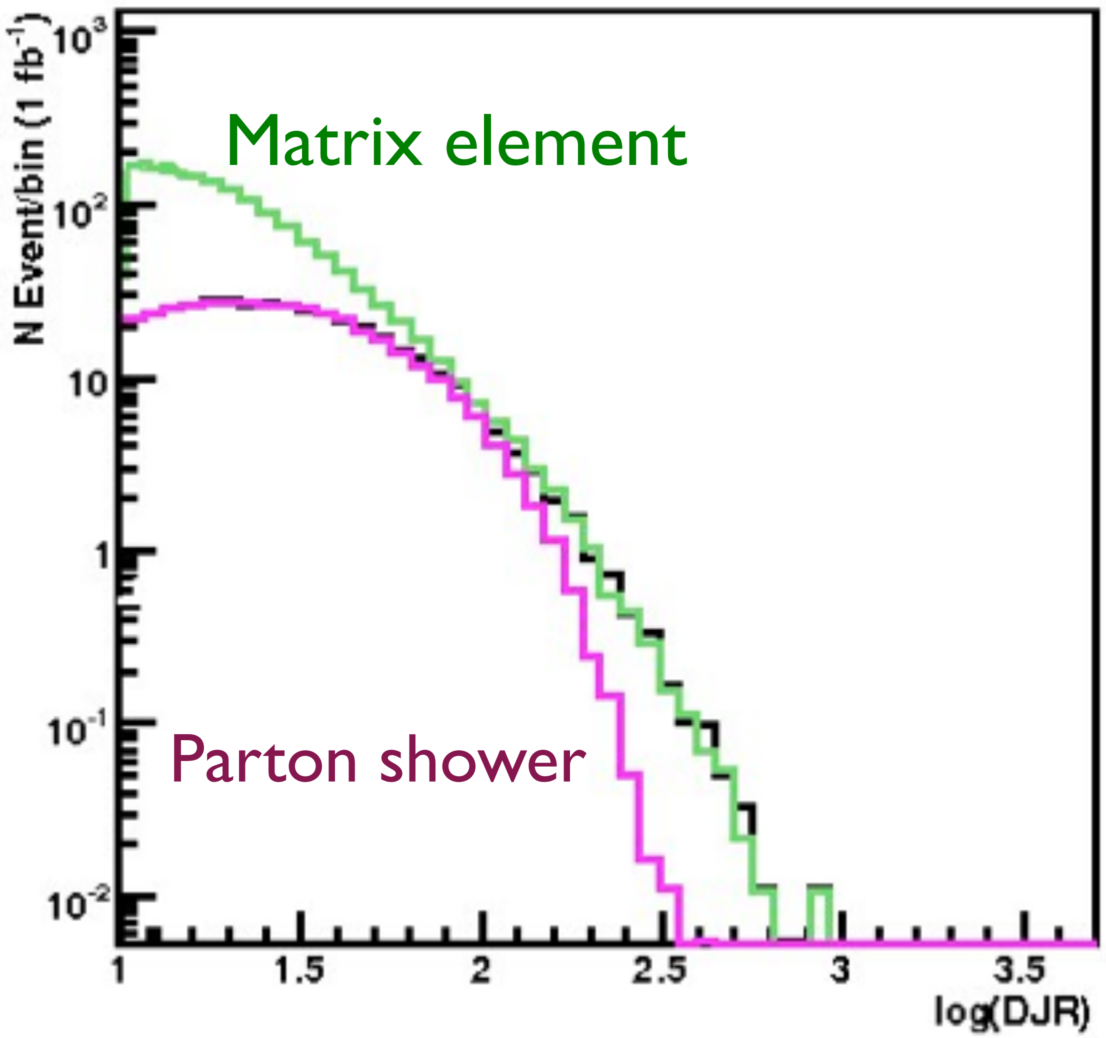

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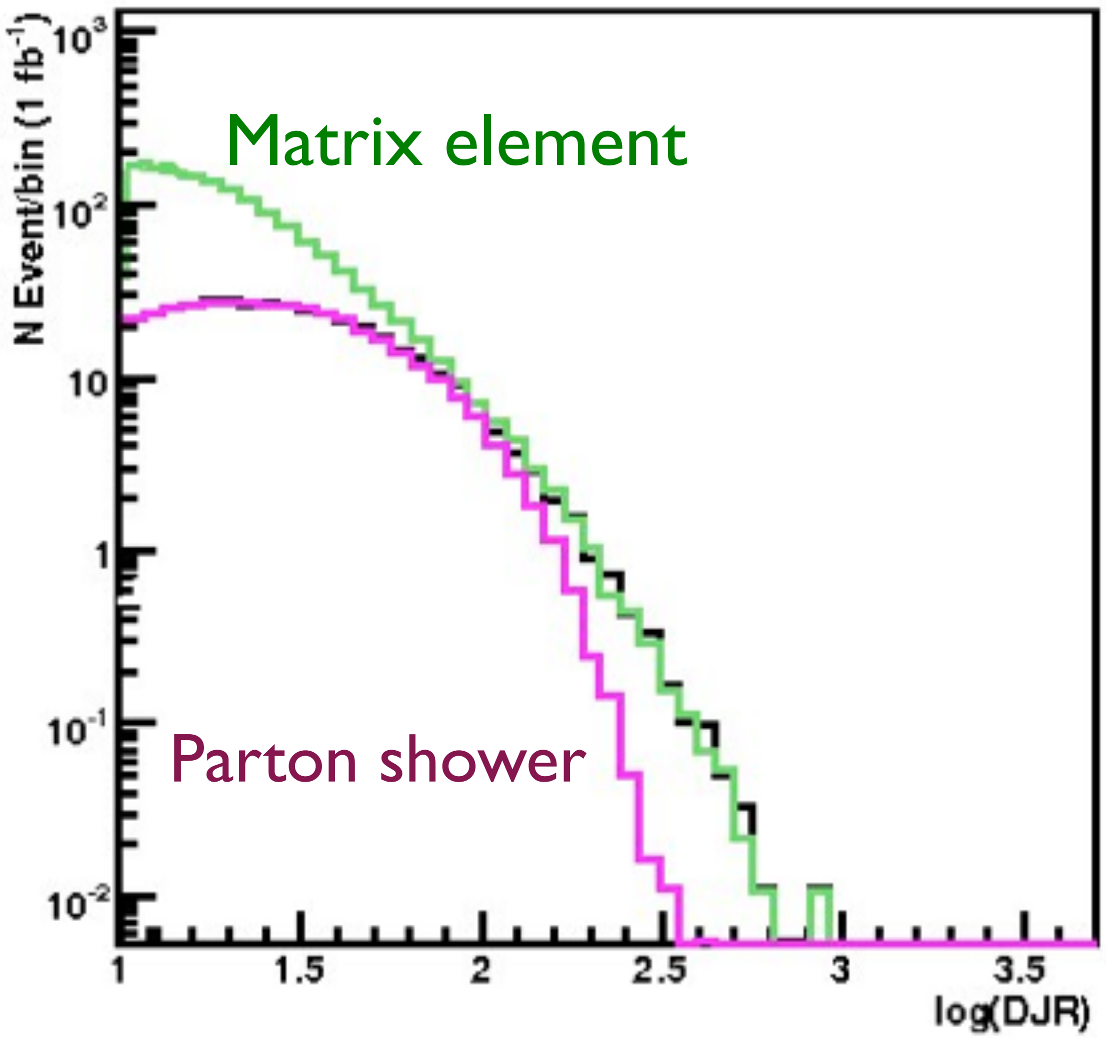

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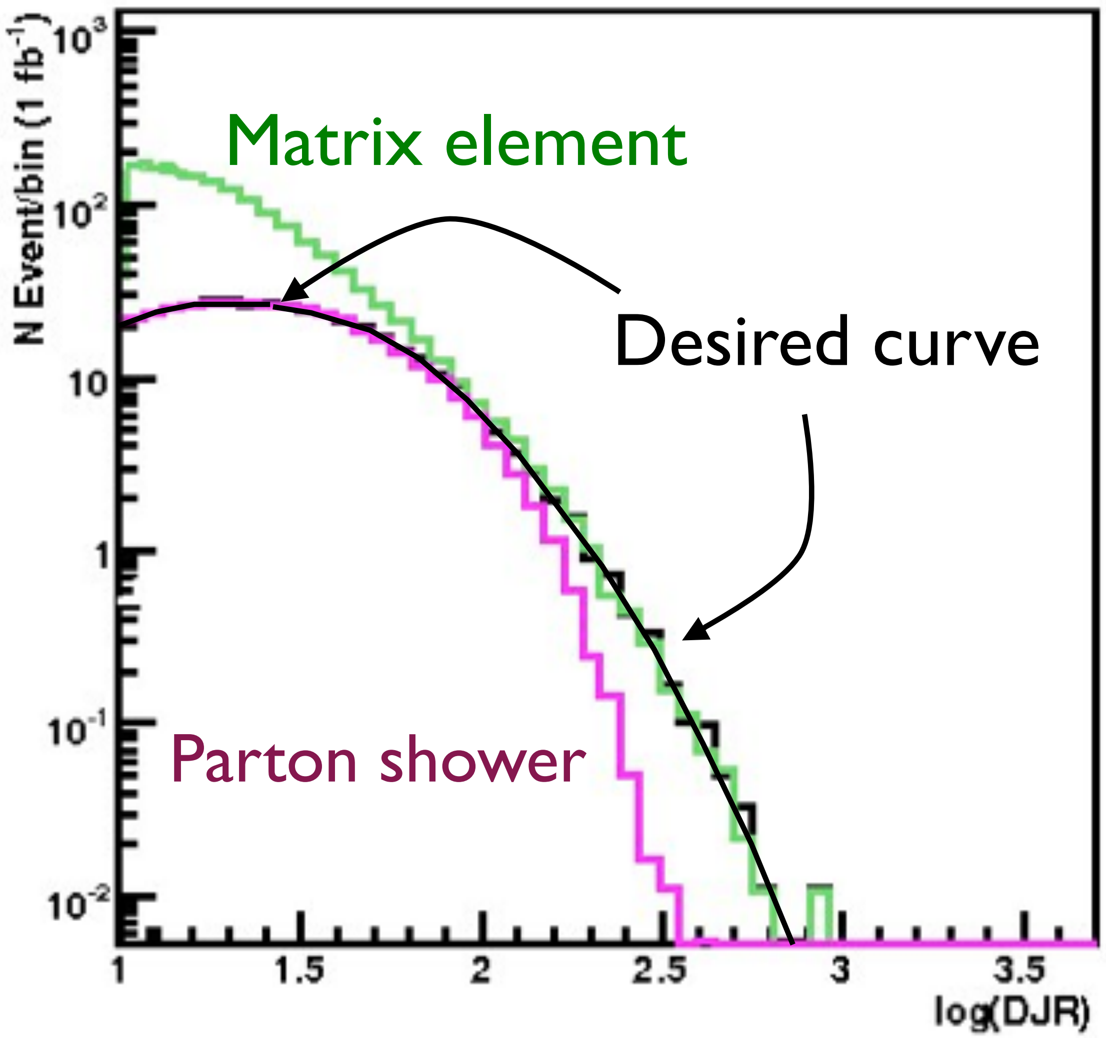

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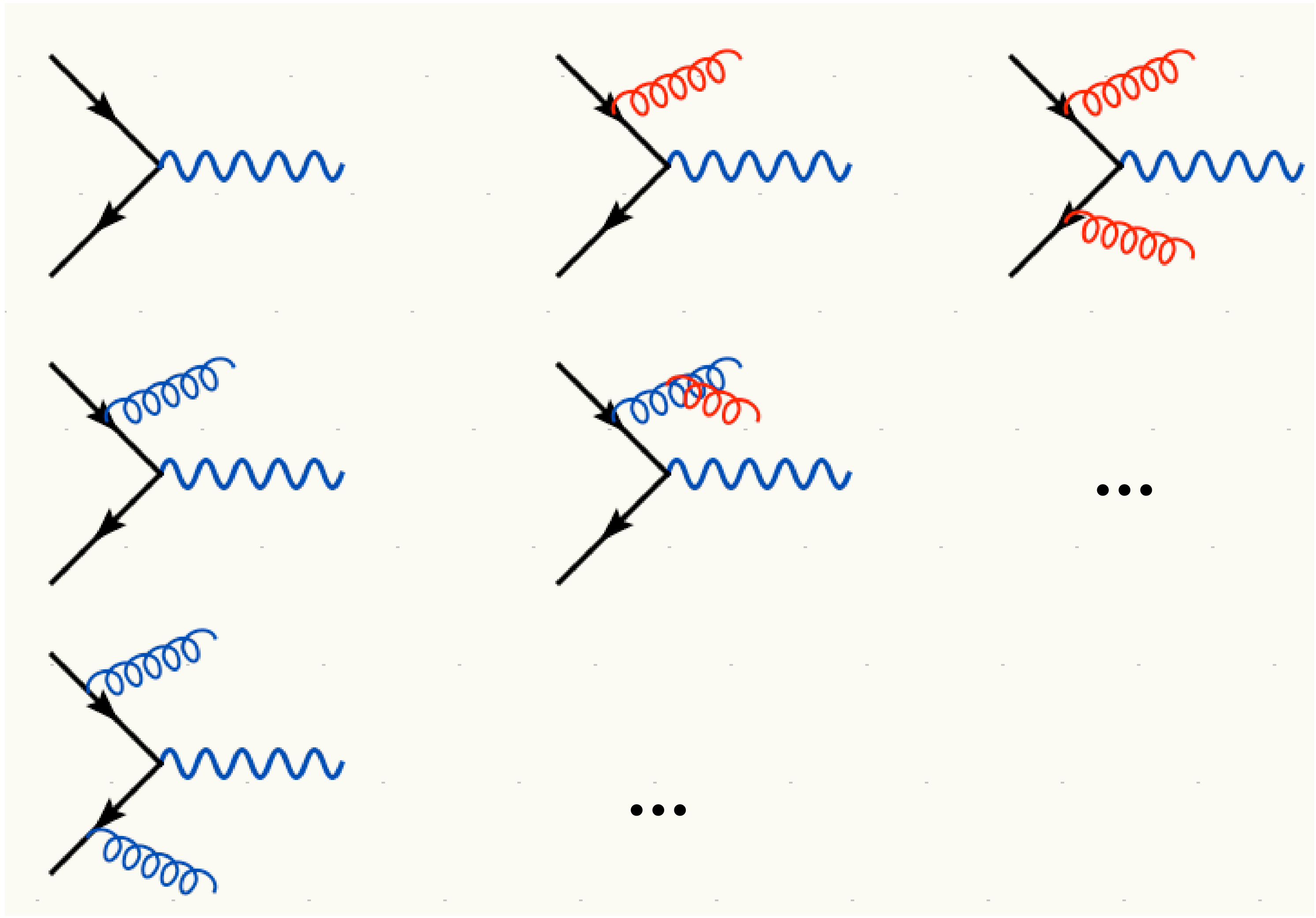
Merging ME with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]

PS →

ME ↓



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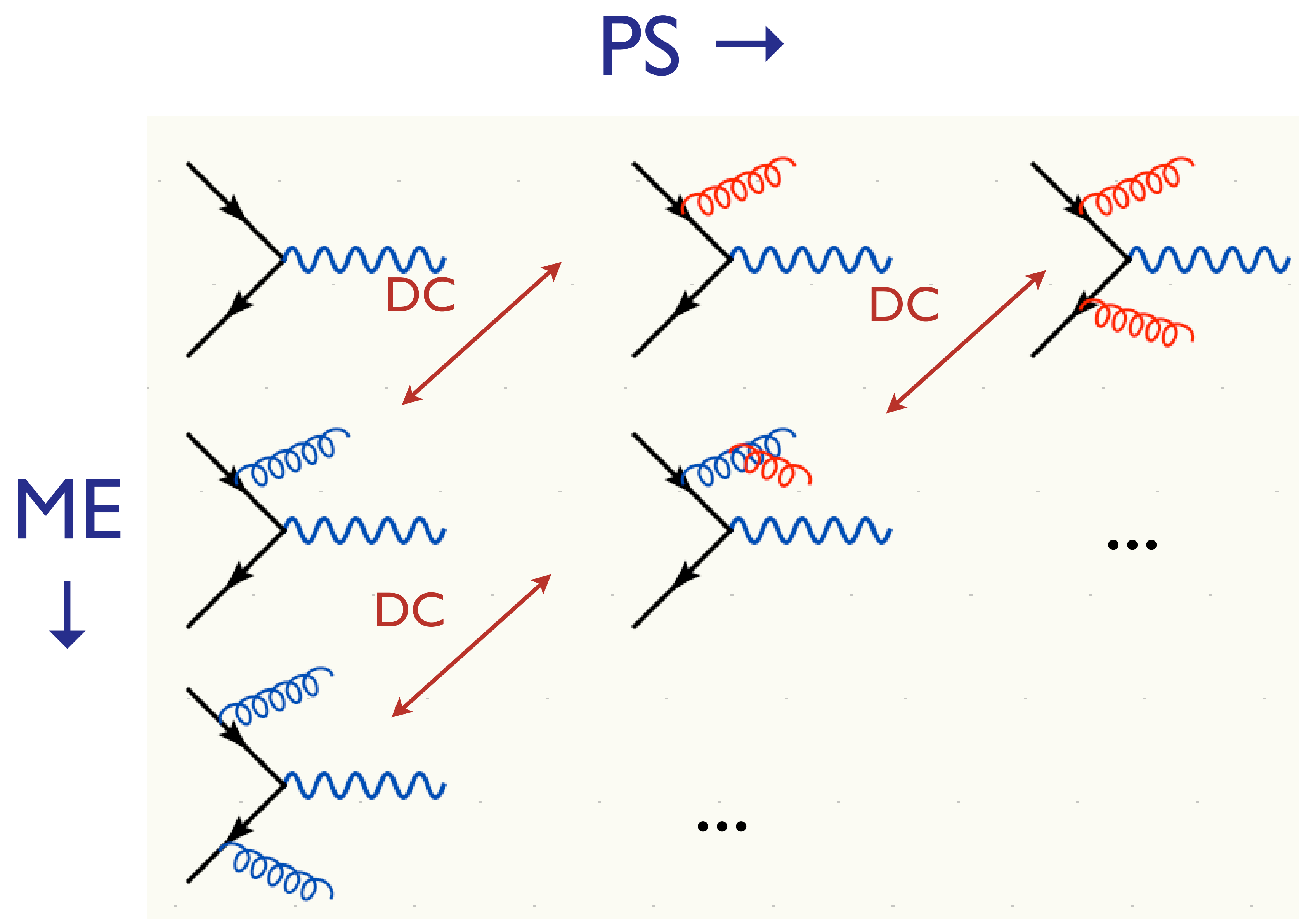

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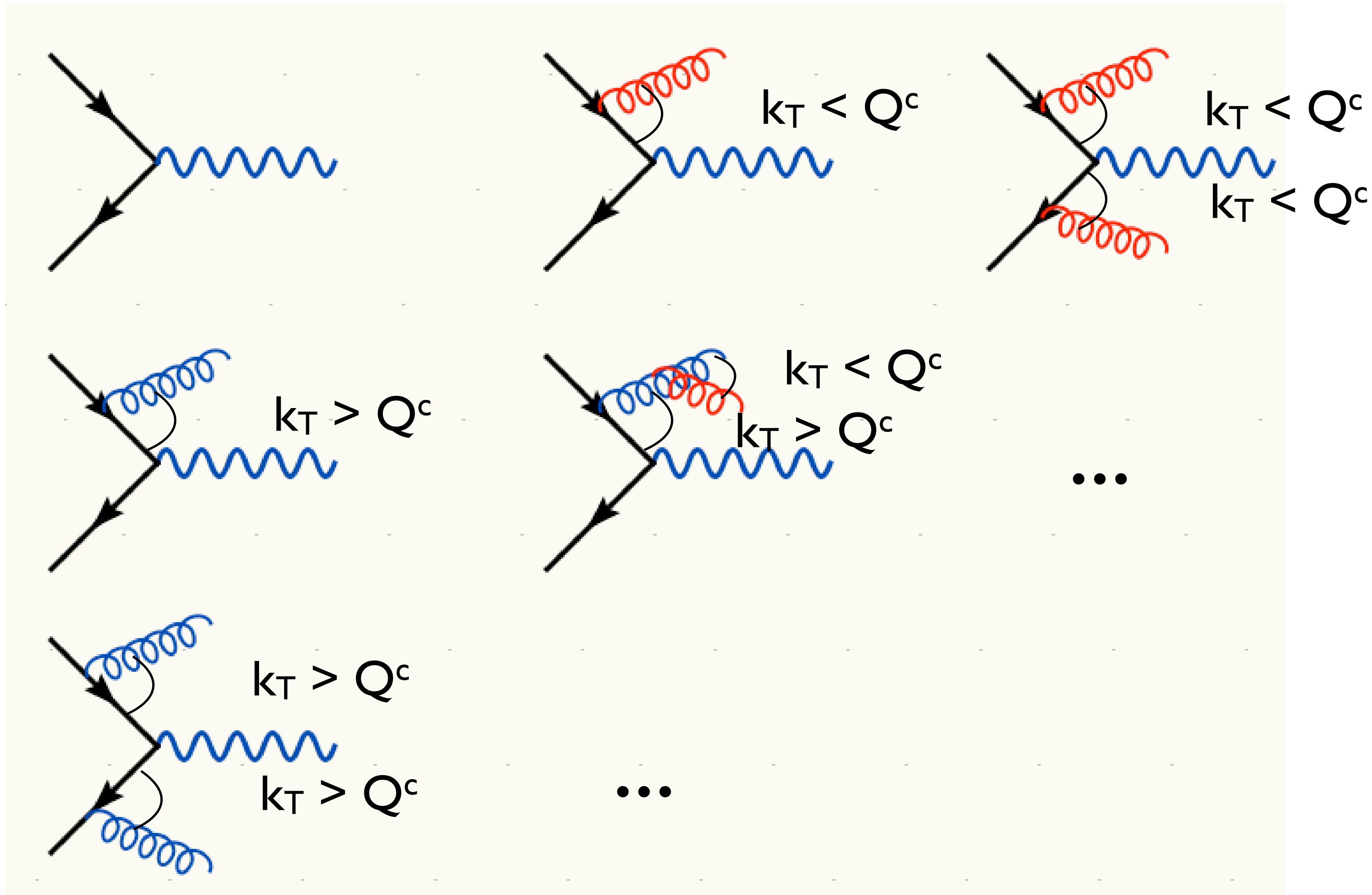

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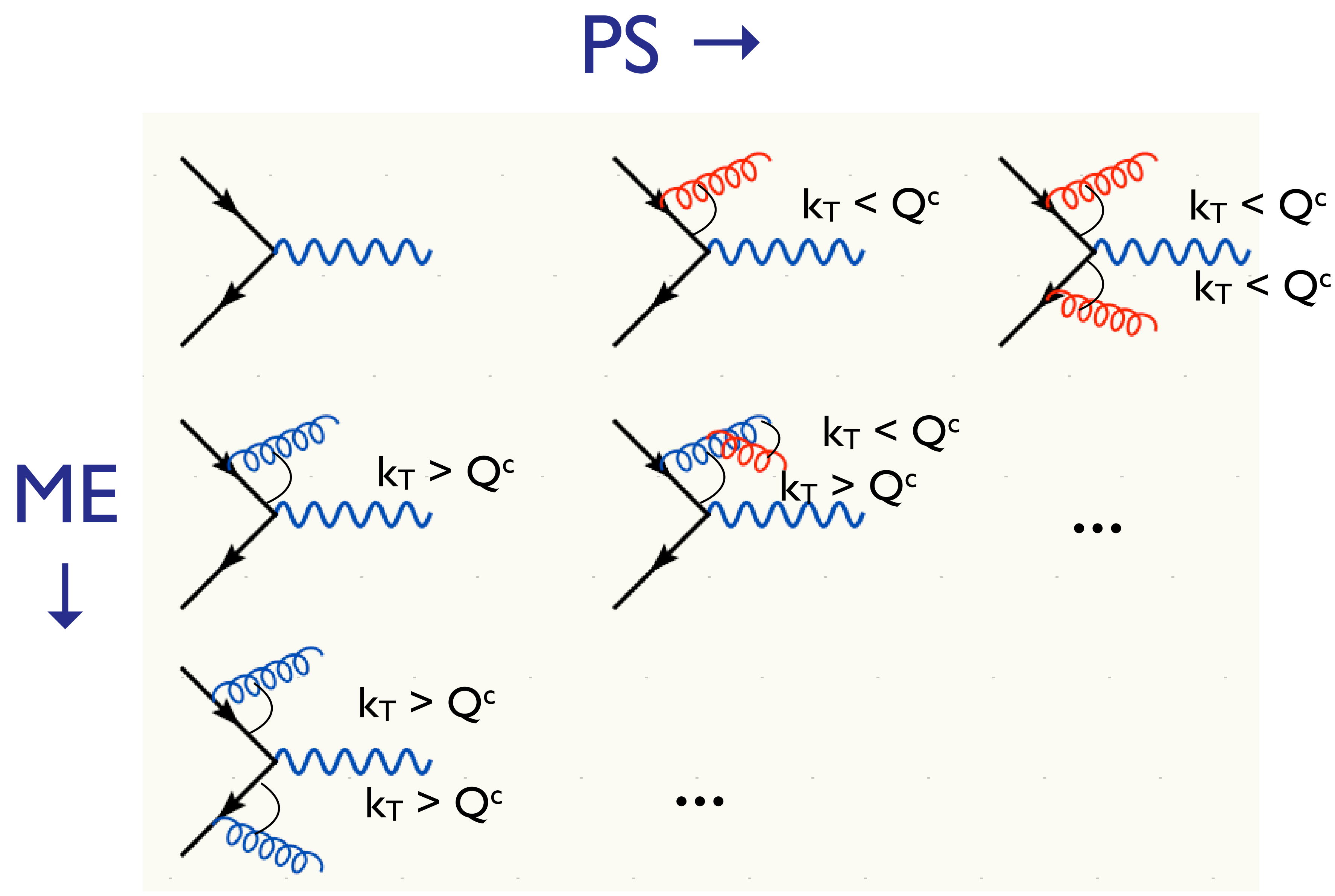
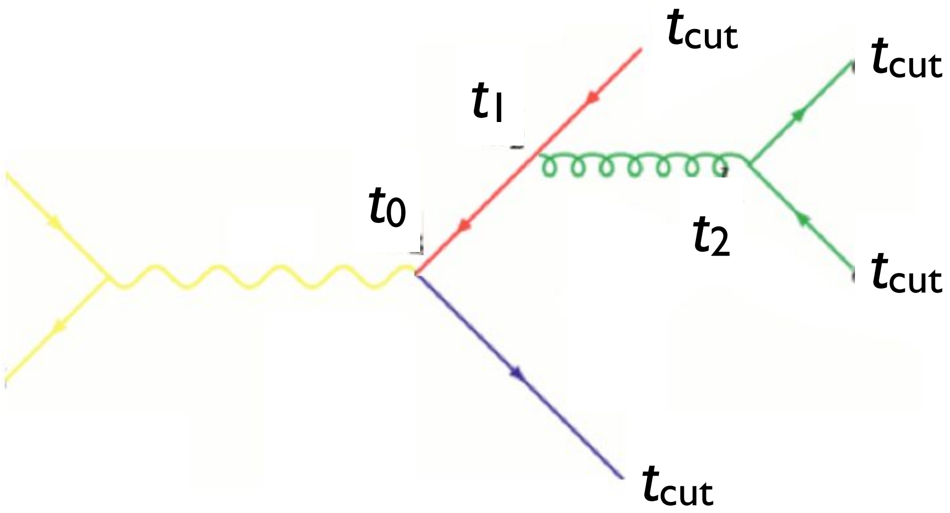


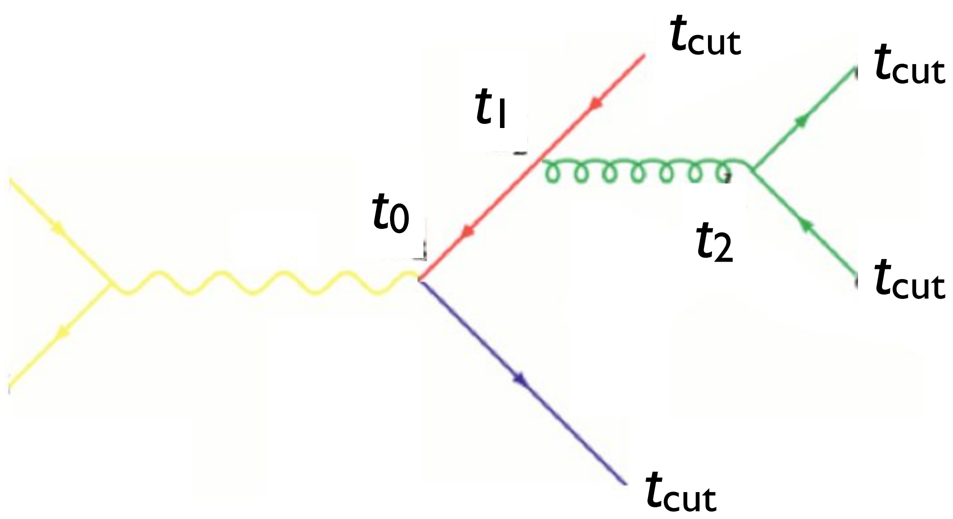
Double counting between ME and PS easily avoided using phase space cut between the two: PS below cutoff, ME above cutoff.

Merging ME with PS

- So double counting problem easily solved, but what about getting smooth distributions that are independent of the precise value of Q^c?
- Below cutoff, distribution is given by PS
  - need to make ME look like PS near cutoff
- Let's take another look at the PS!

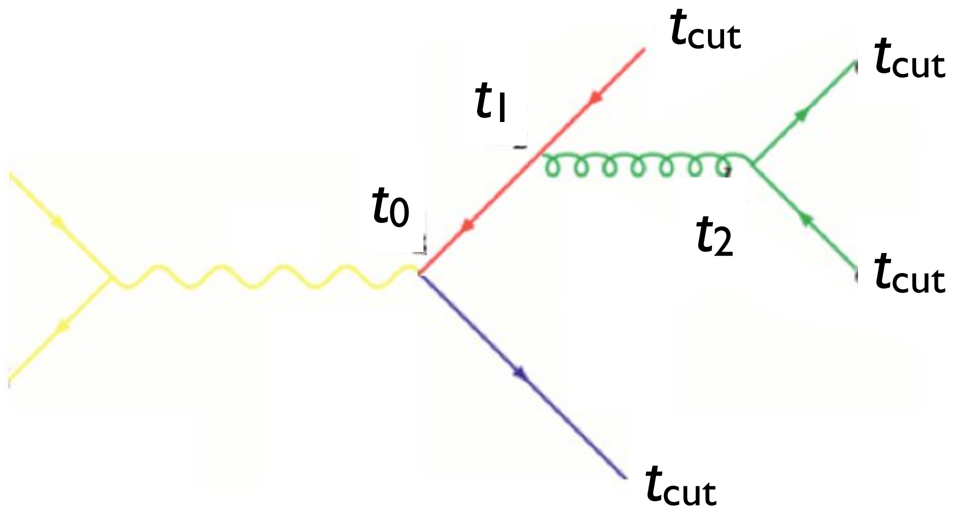
Merging ME with PS





- How does the PS generate the configuration above?

Merging ME with PS



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- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

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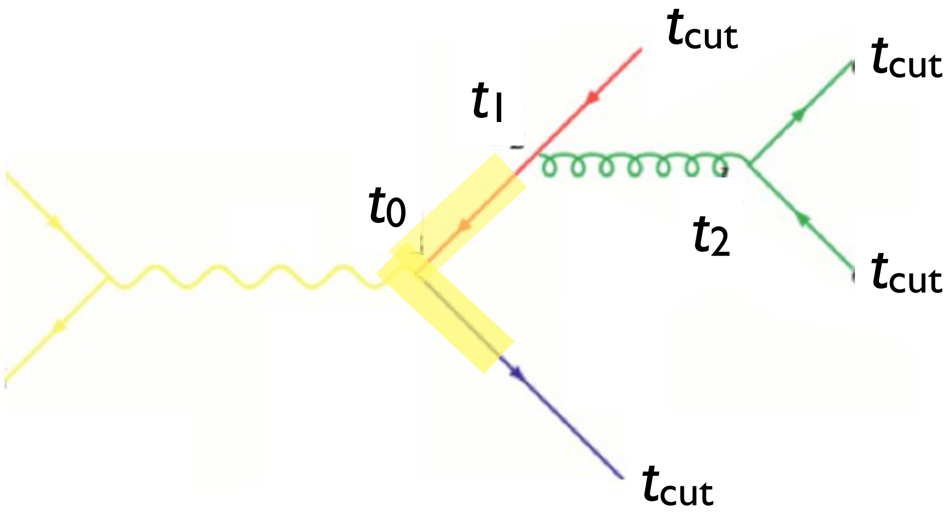

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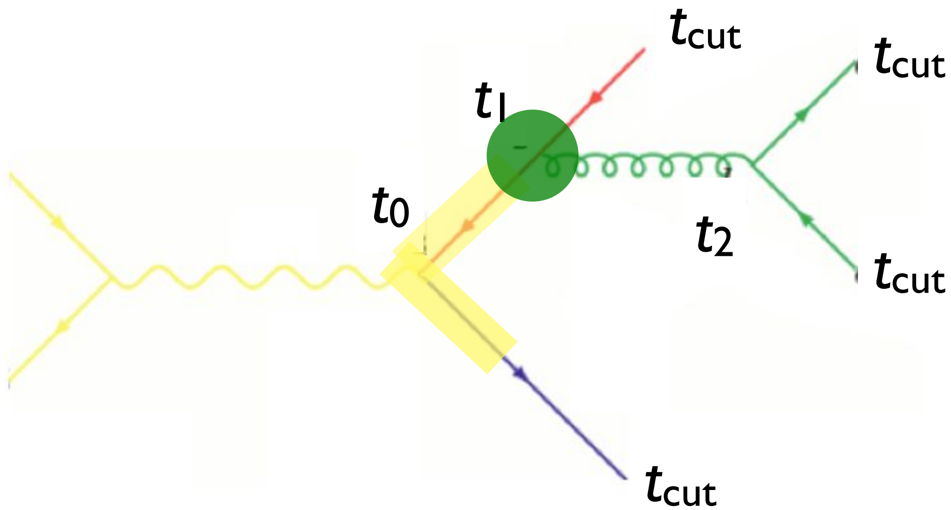

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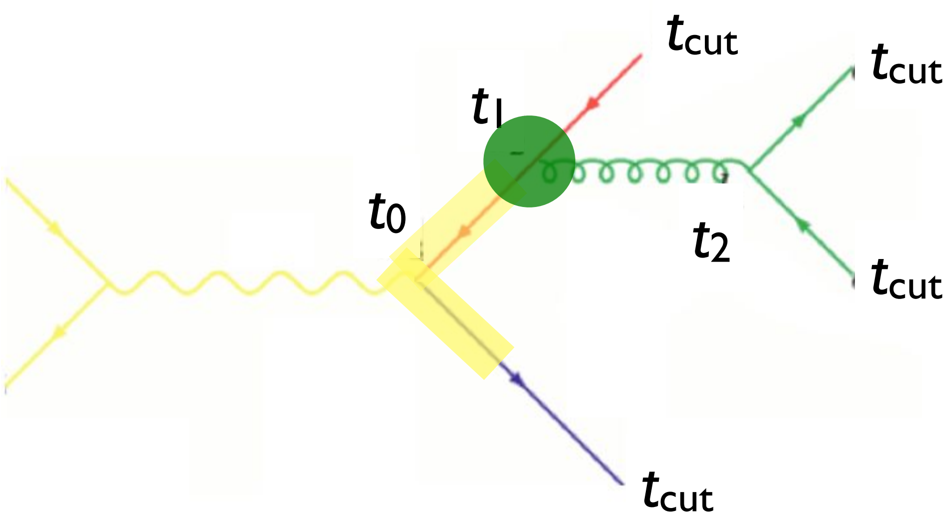

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and for the whole tree

$$(\Delta_q(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

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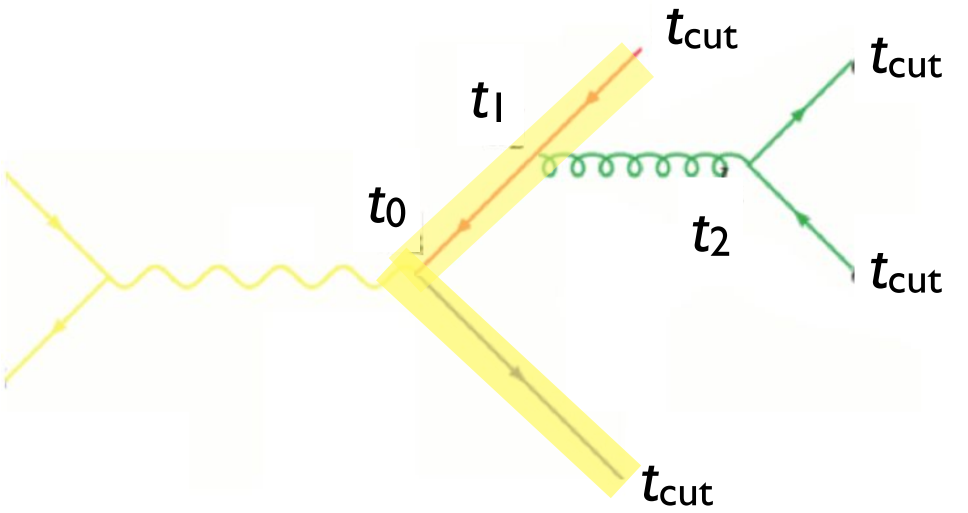

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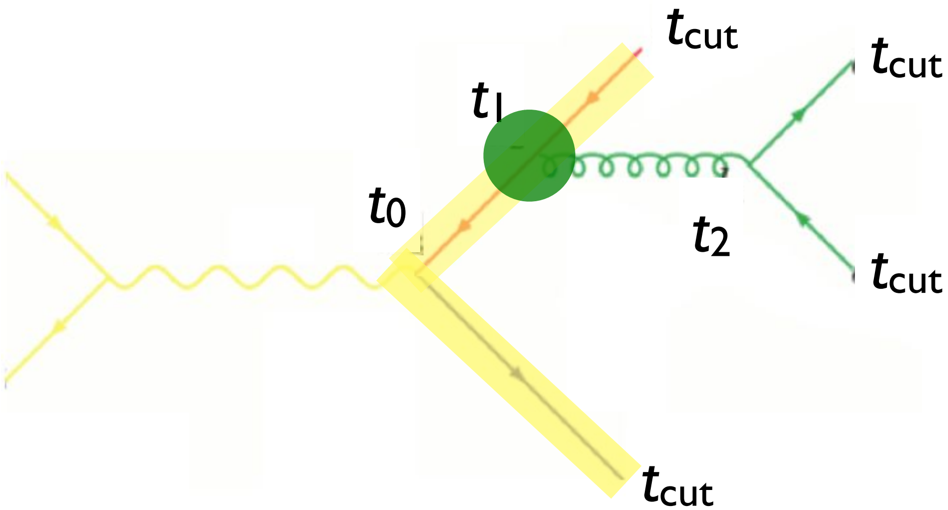

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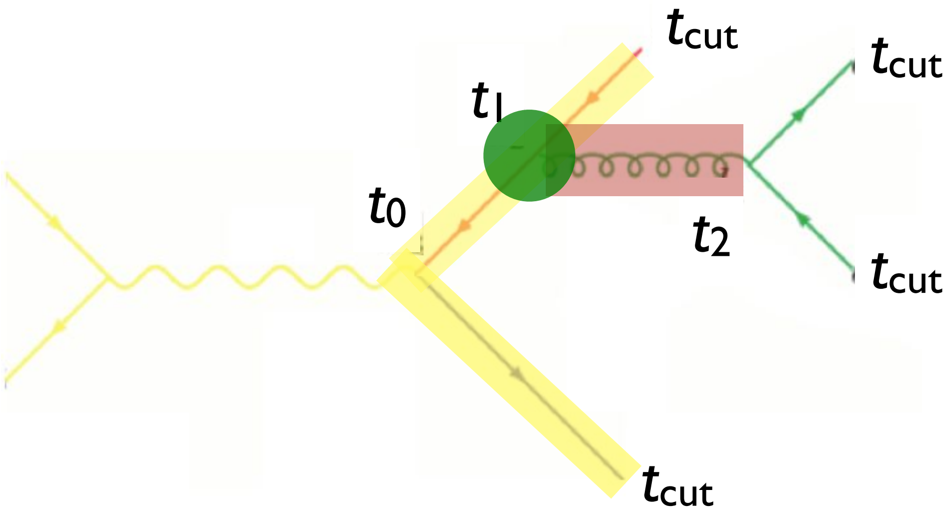

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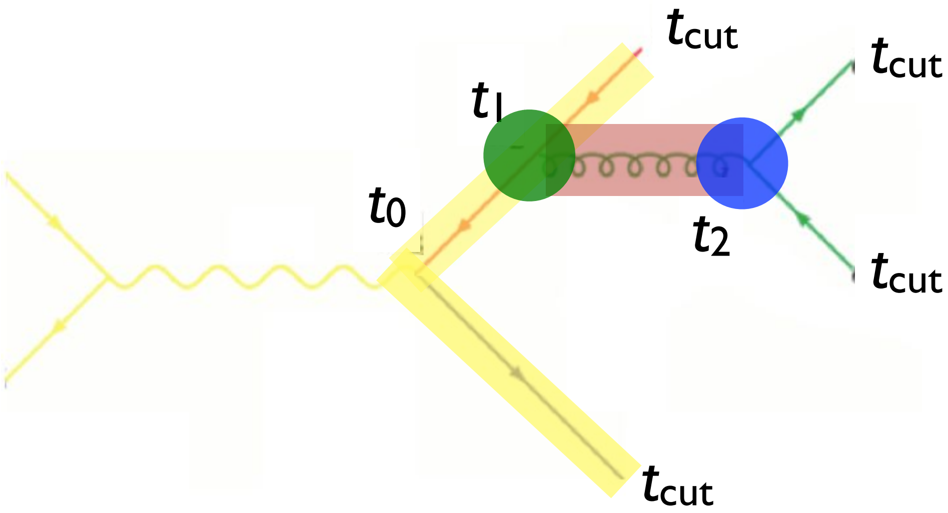

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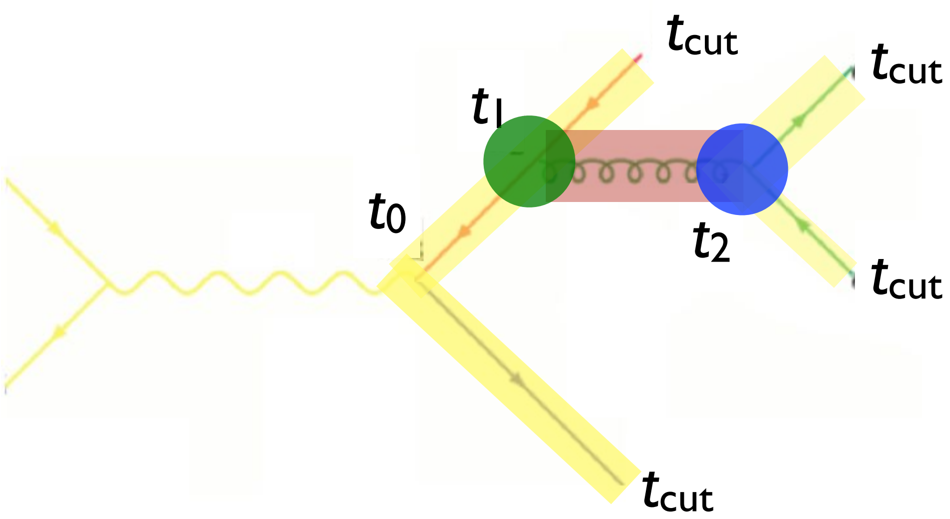

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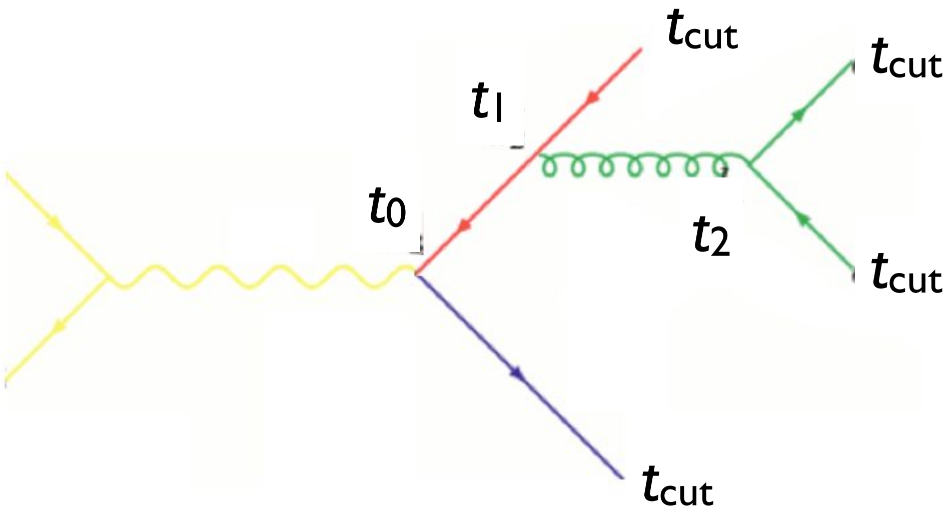

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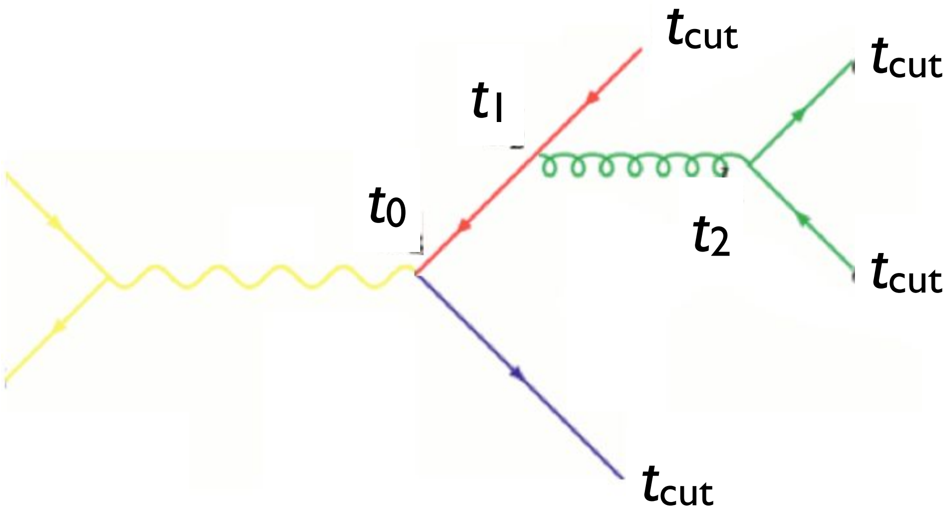

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Corresponds to the matrix element
BUT with α_s evaluated at the scale of each splitting

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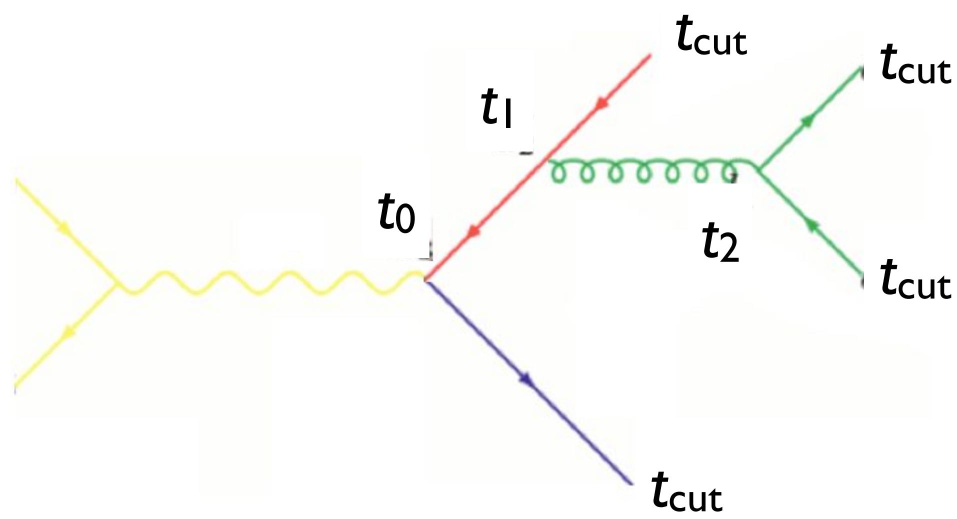

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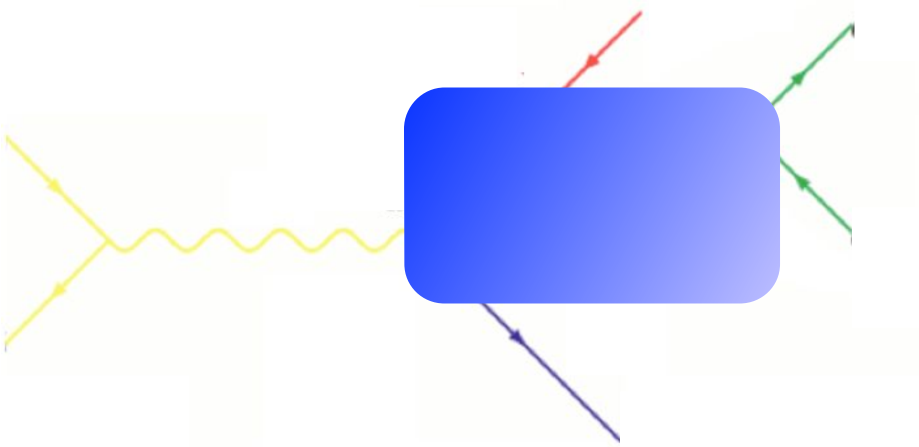
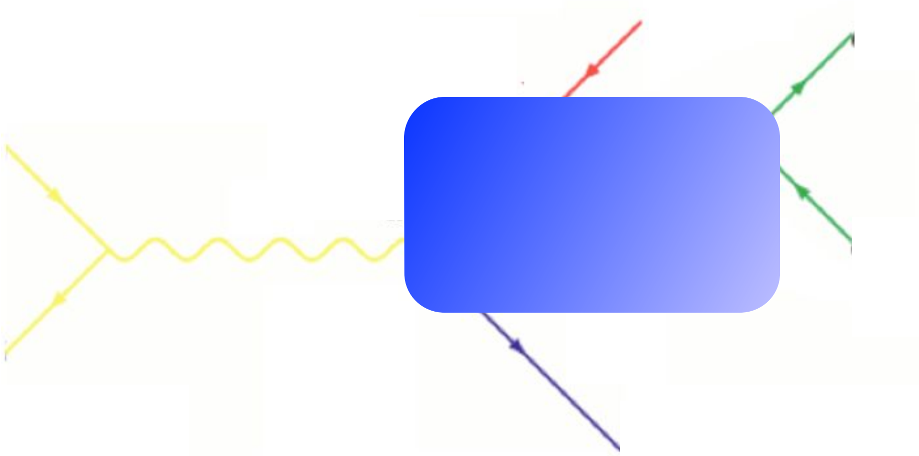
Sudakov suppression due to disallowing additional radiation above the scale t_{cut}



$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$$

Merging ME with PS



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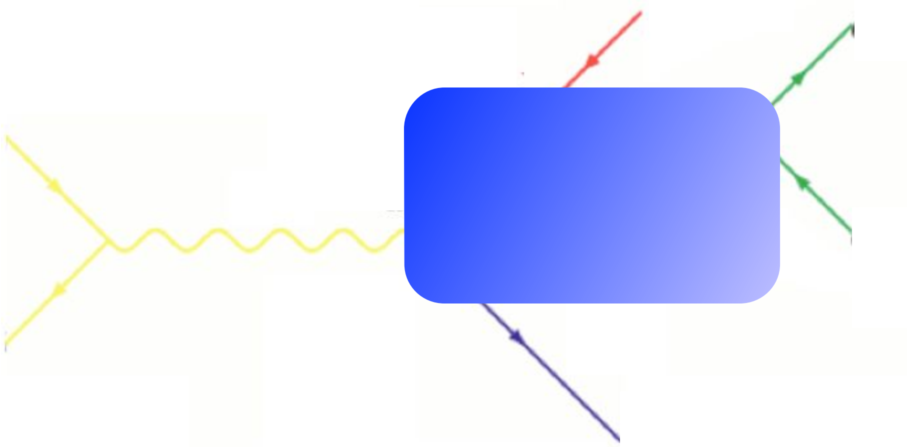

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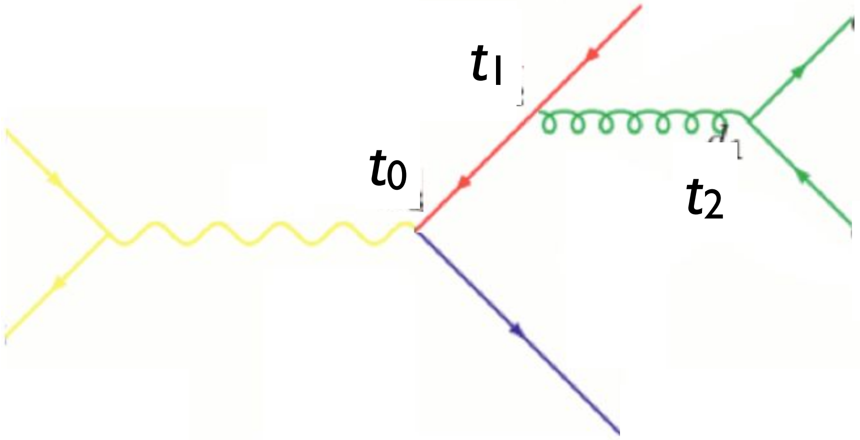

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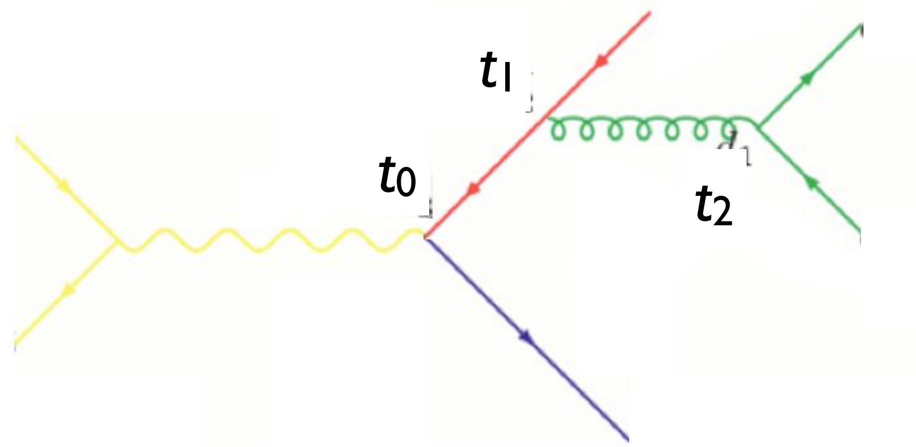

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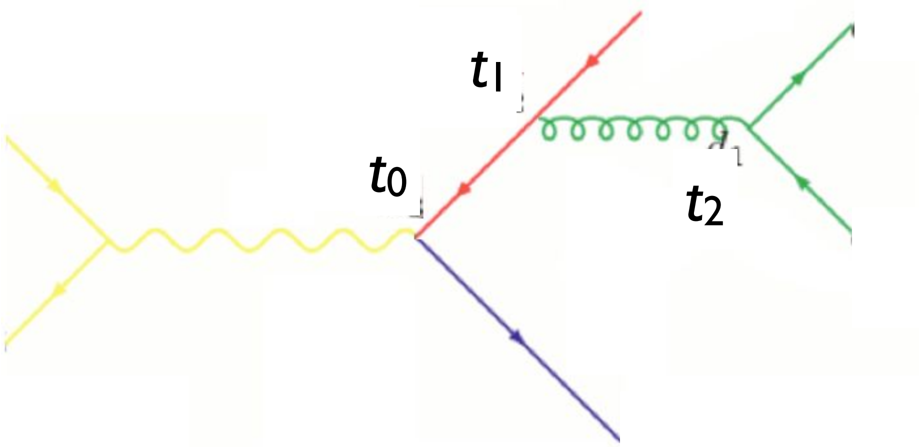
Merging ME with PS



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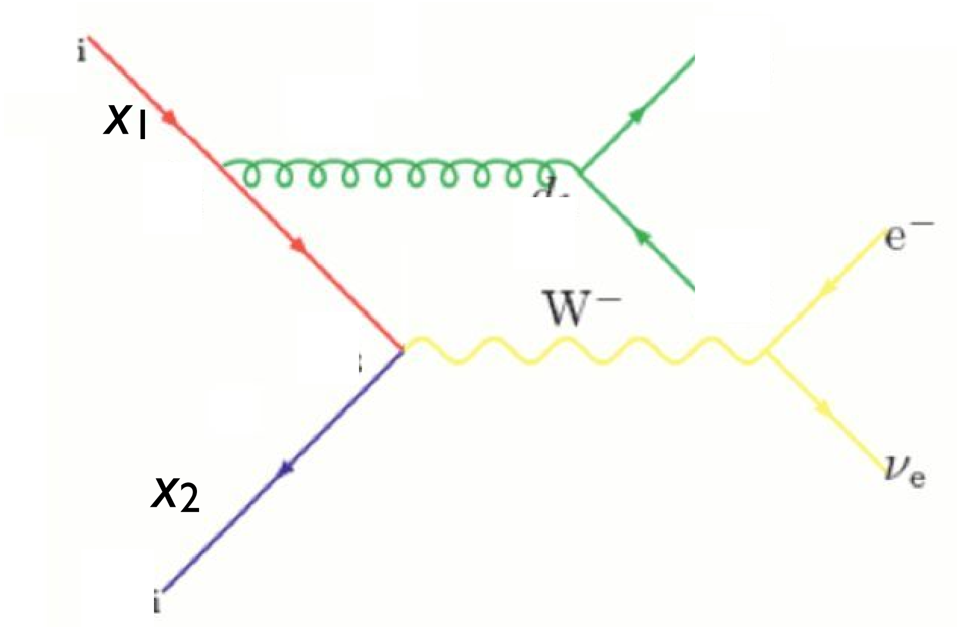
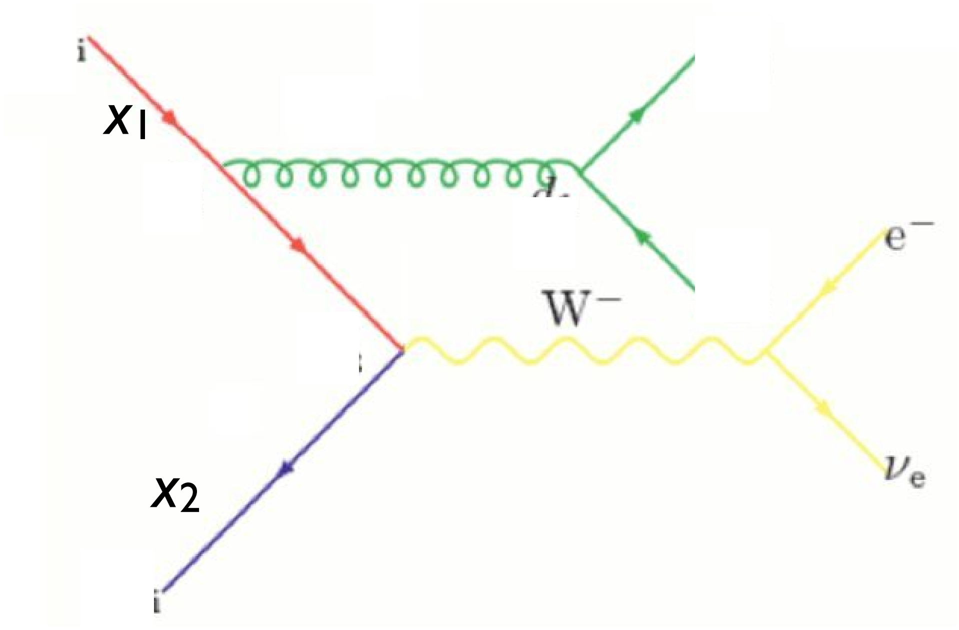
3. Use some algorithm to apply the equivalent Sudakov suppression $(\Delta_q(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q({\rm cut}, t_2))^2$

Matching for initial state radiation



- We are of course not interested in e^+e^- but p-p(bar) - what happens for initial state radiation?



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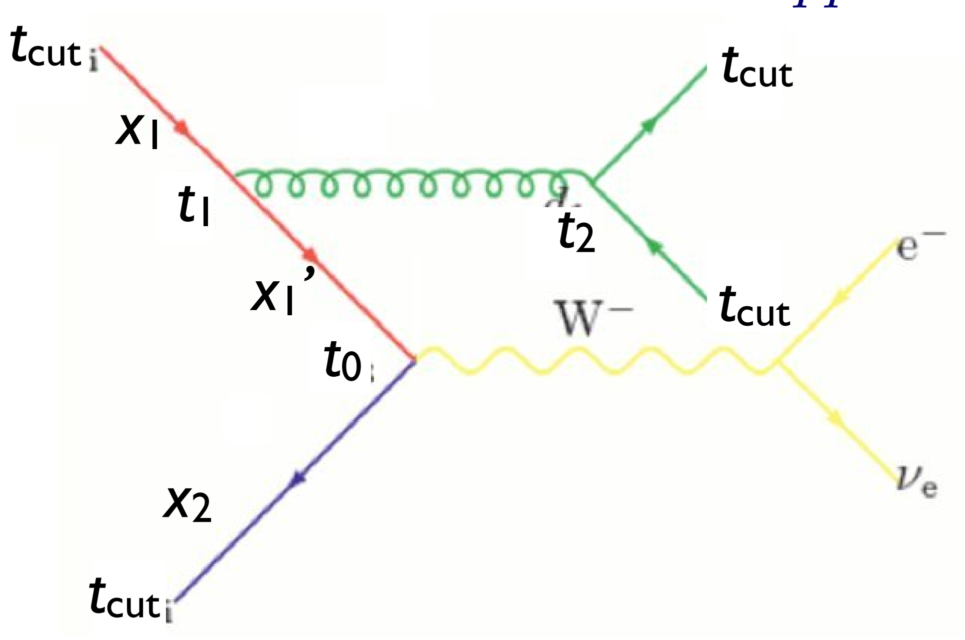

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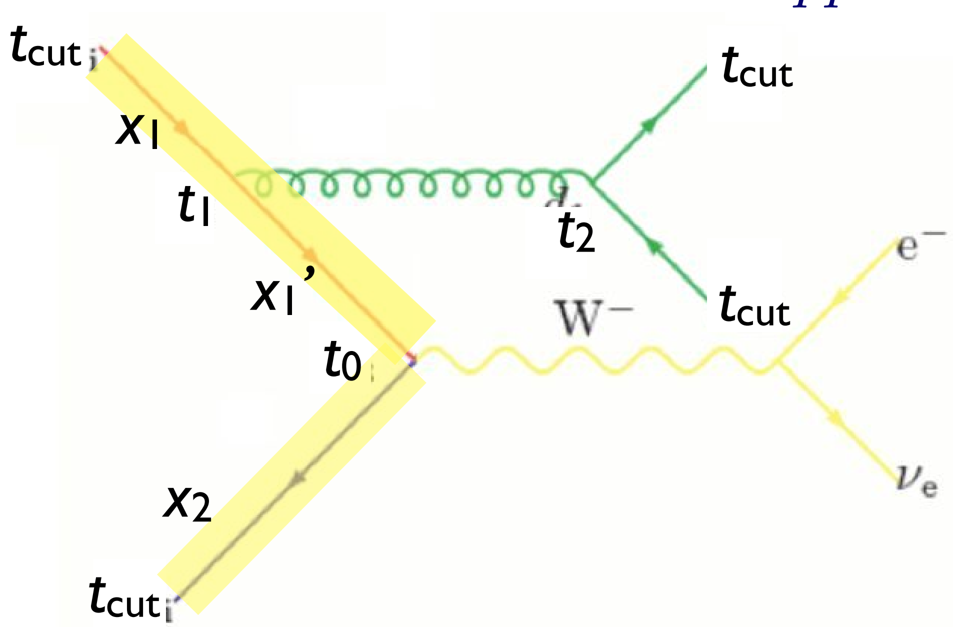

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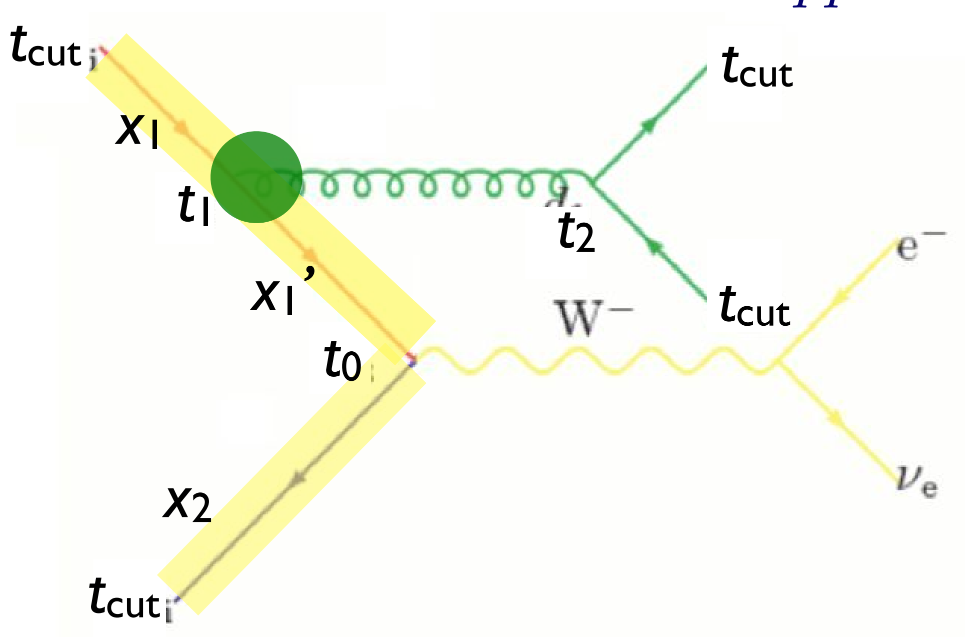

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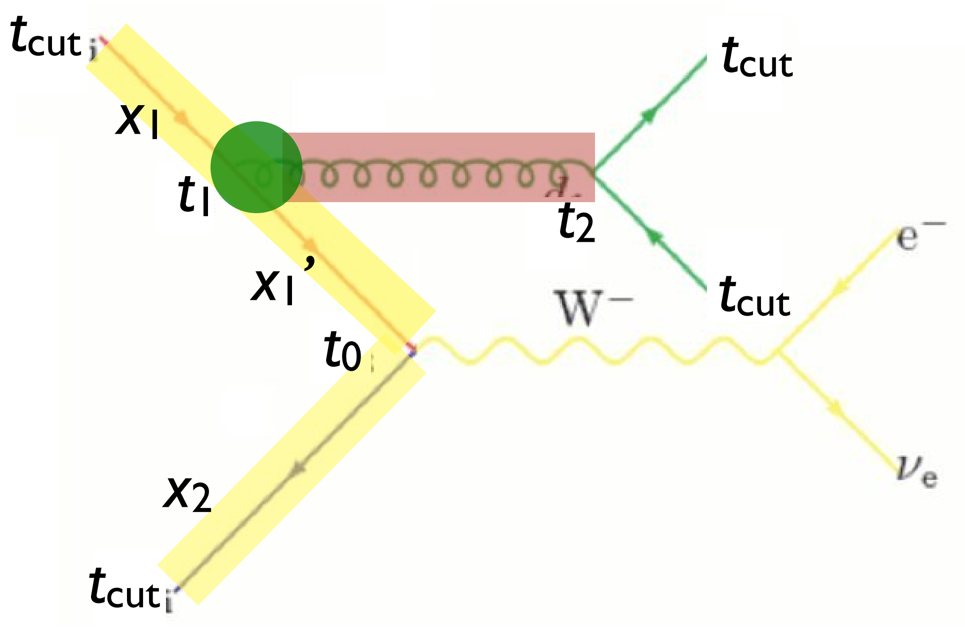

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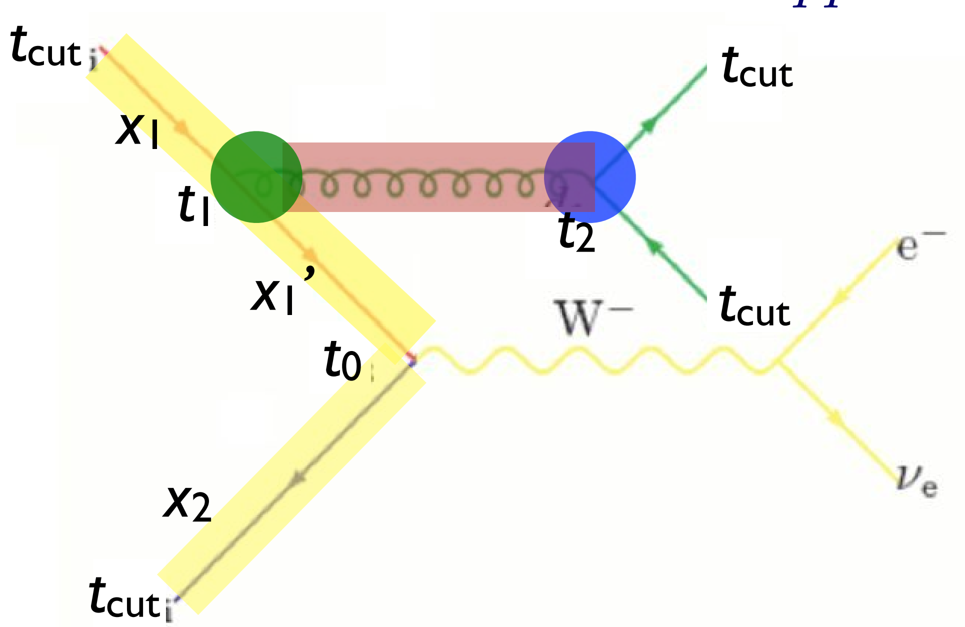

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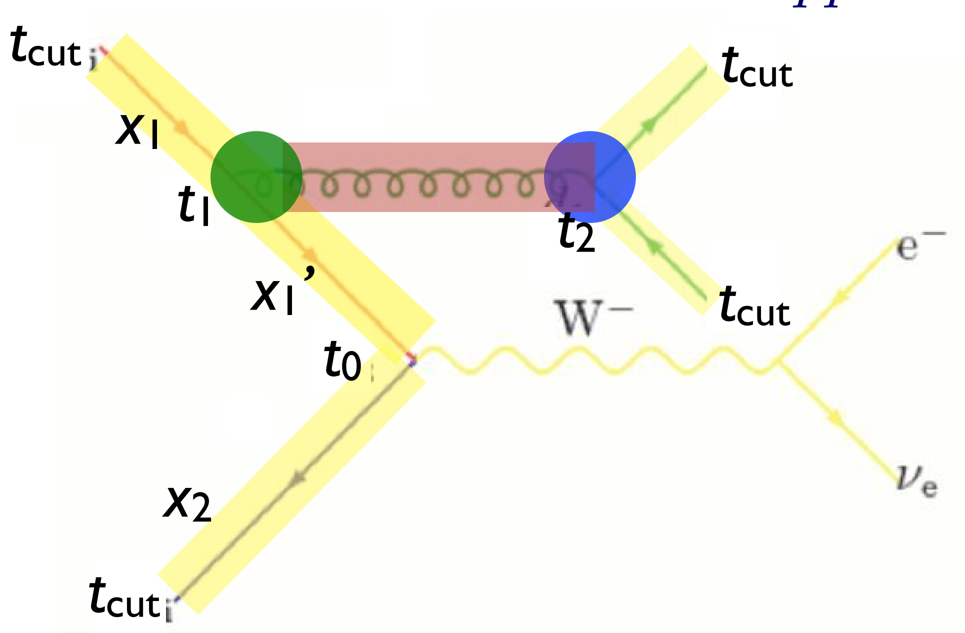
- We are of course not interested in e^+e^- but p-p(bar) - what happens for initial state radiation?
- Let's do the same exercise as before:

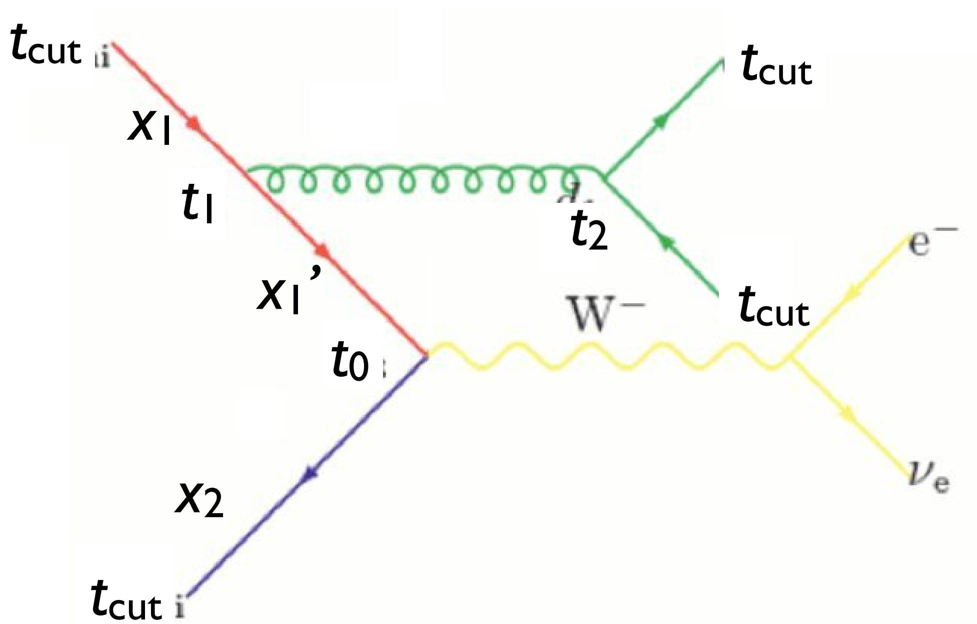
$$\mathcal{P} = (\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x_1', t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

$$\times \hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s}, \ldots) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0)$$



Matching for initial state radiation

$$(\Delta_{Iq}(t_{\rm cut},t_0))^2\Delta_g(t_2,t_1)(\Delta_q(t_{\rm cut},t_2))^2\frac{\alpha_s(t_1)}{2\pi}\frac{P_{gq}(z)}{z}\frac{f_q(x_1,t_1)}{f_q(x'_1,t_1)}\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$
$$\times\hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s},\ldots)f_q(x'_1,t_0)f_{\bar{q}}(x_2,t_0)$$

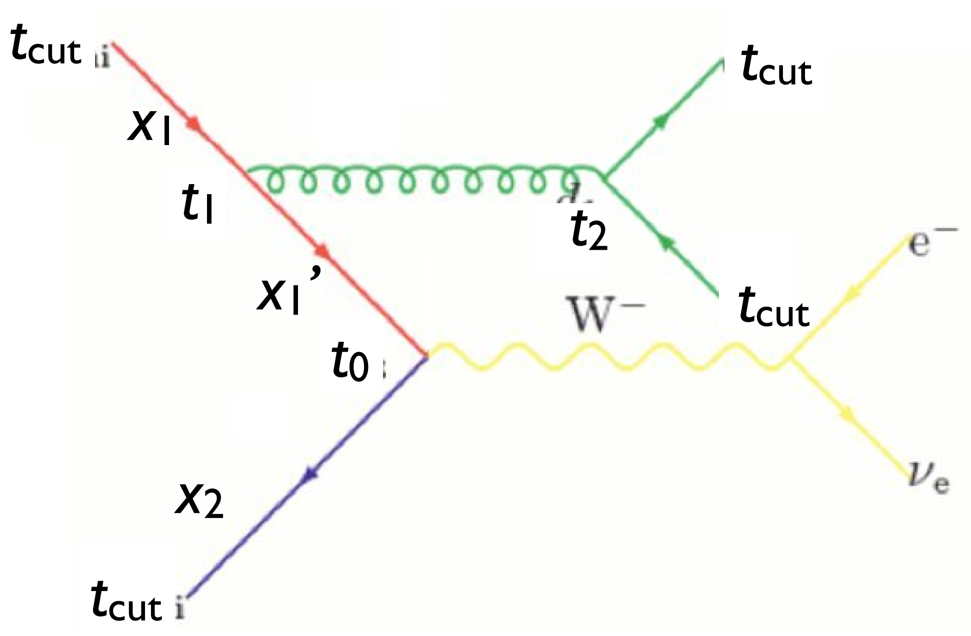


Matching for initial state radiation

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x_1', t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

$$\times \hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s}, \ldots) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0)$$

ME with α_s evaluated at the scale of each splitting



Matching for initial state radiation

$$(\Delta_{Iq}(t_{\rm cut},t_0))^2\Delta_g(t_2,t_1)(\Delta_q(t_{\rm cut},t_2))^2\frac{\alpha_s(t_1)}{2\pi}\frac{P_{gq}(z)}{z}\frac{f_q(x_1,t_1)}{f_q(x_1',t_1)}\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$

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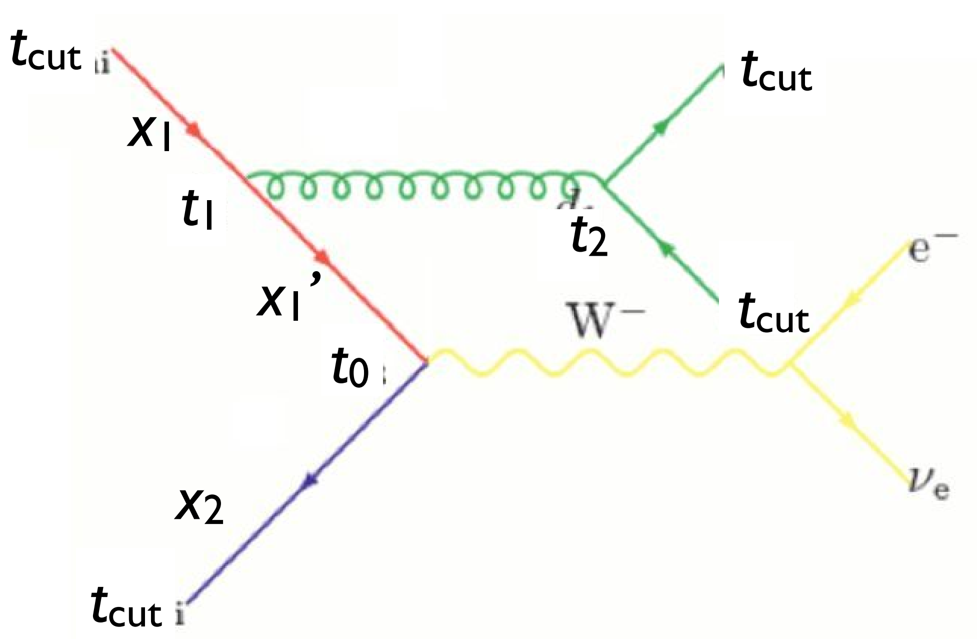
ME with α_s evaluated at the scale of each splitting

PDF reweighting



Matching for initial state radiation

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x'_1, t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

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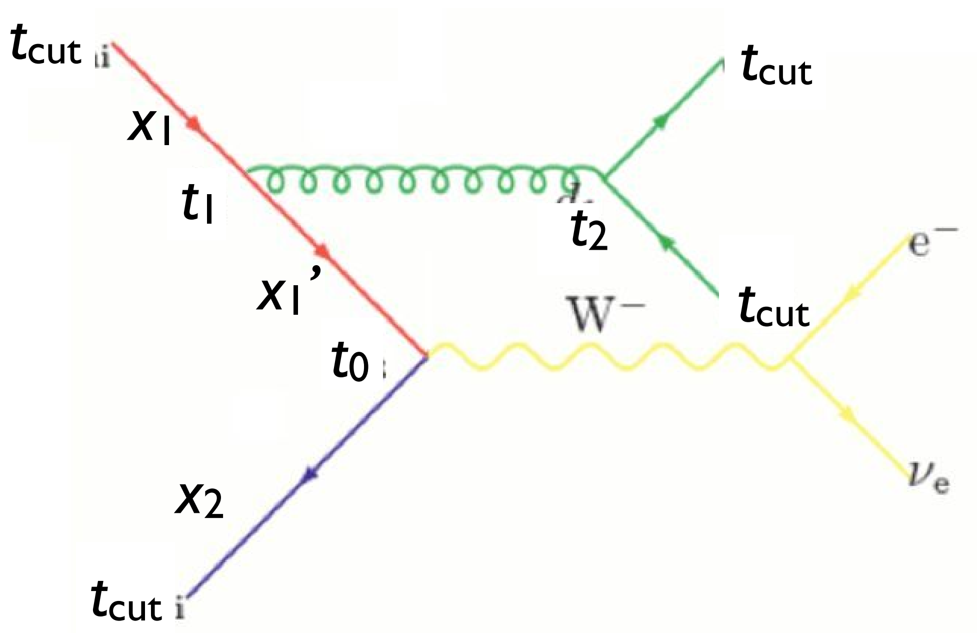
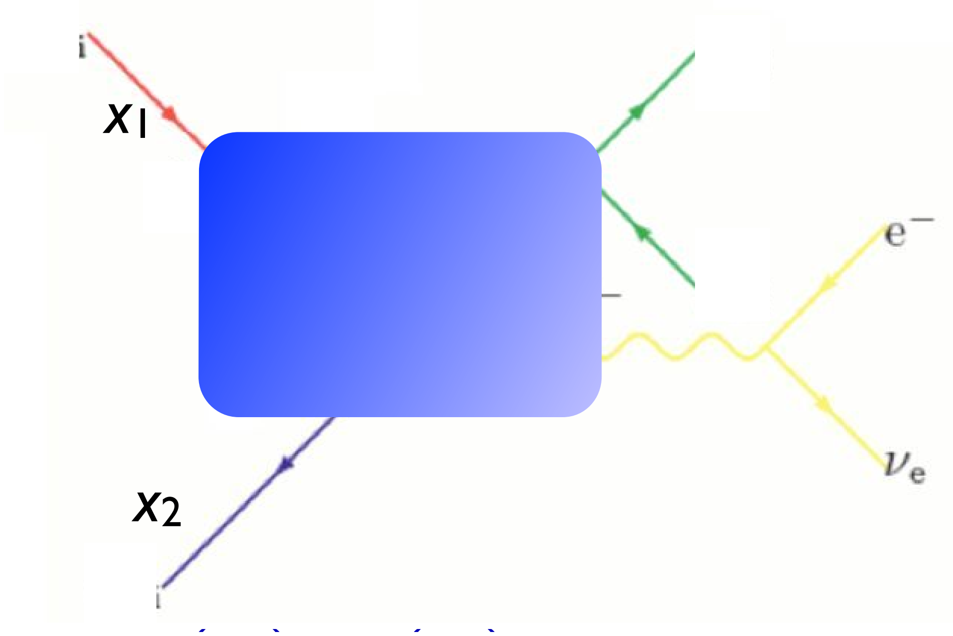
ME with α_s evaluated at the scale of each splitting

PDF reweighting

Sudakov suppression due to non-branching above scale $t_{\rm cut}$

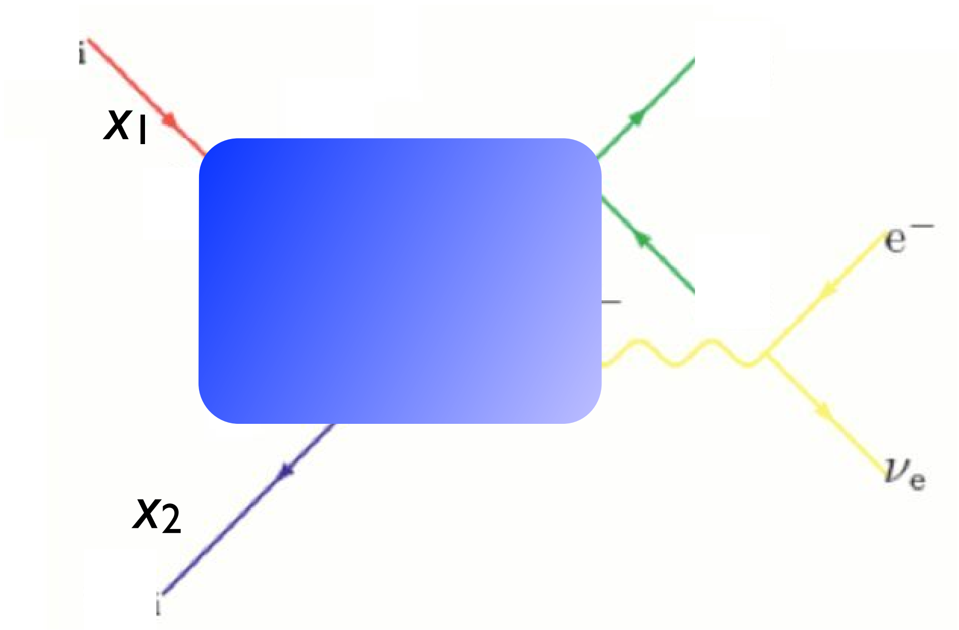


Matching for initial state radiation



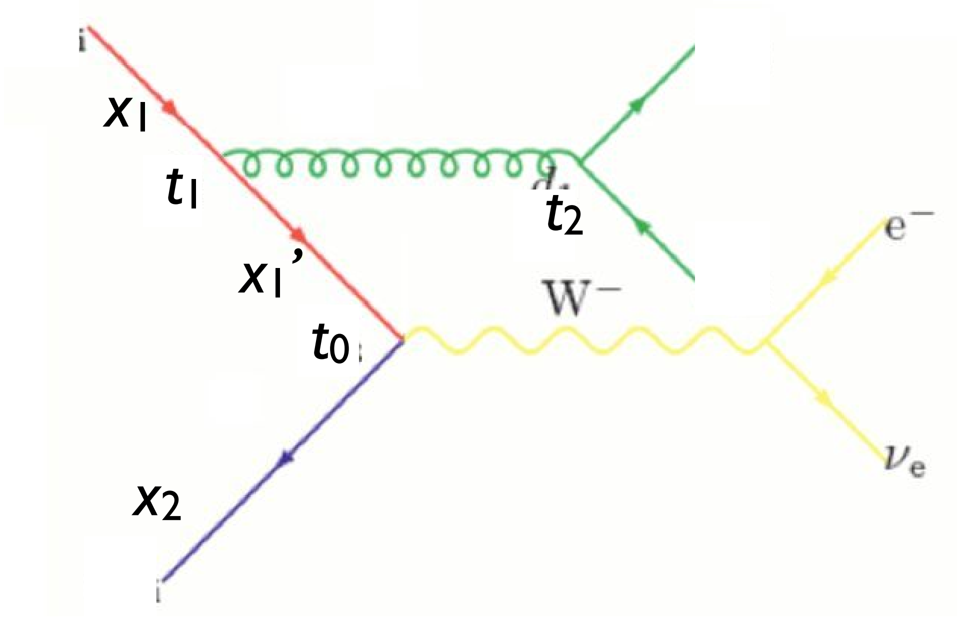
Matching for initial state radiation

- Again, use a clustering scheme to get a parton shower history



Matching for initial state radiation

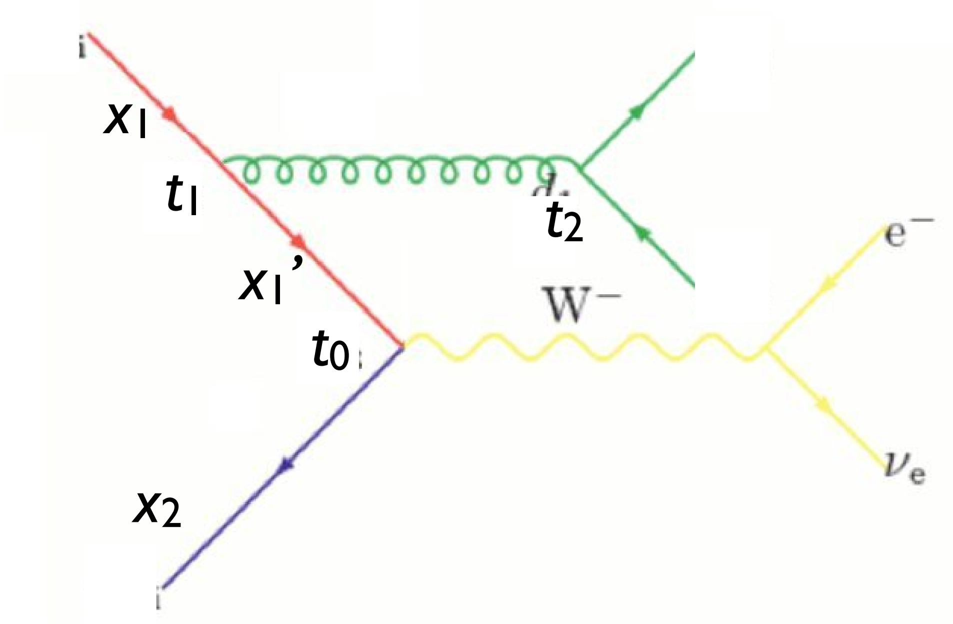
- Again, use a clustering scheme to get a parton shower history



Matching for initial state radiation

- Again, use a clustering scheme to get a parton shower history
- Now, reweight both due to α_s and PDF

$$|\mathcal{M}|^2 \rightarrow |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)} \frac{f_q(x_1', t_0)}{f_q(x_1', t_1)}$$



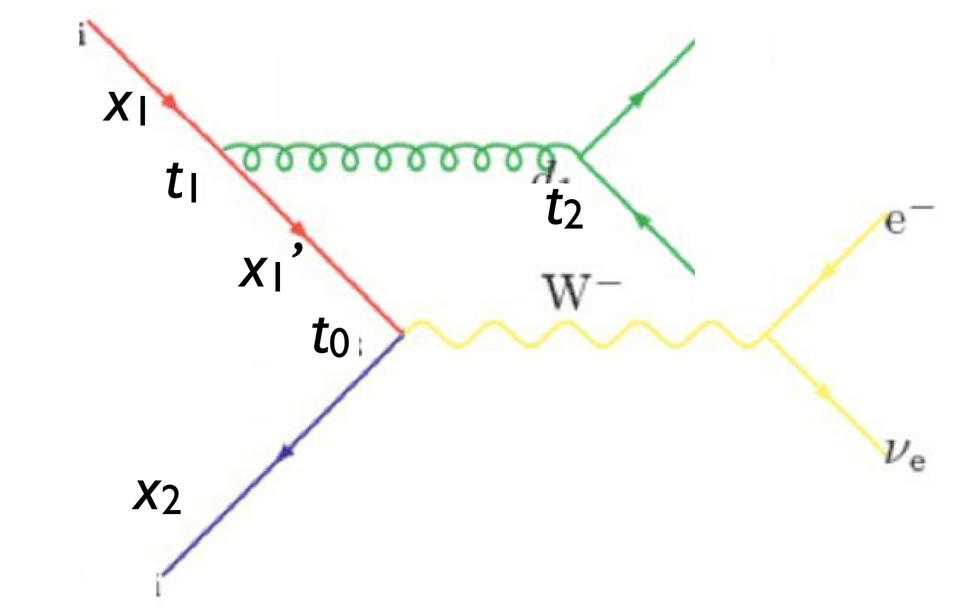
Matching for initial state radiation

- Again, use a clustering scheme to get a parton shower history
- Now, reweight both due to α_s and PDF

$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)} \frac{f_q(x_1', t_0)}{f_q(x_1', t_1)}$$

- Remember to use first clustering scale on each side for PDF scale:

$$\mathcal{P}_{\text{event}} = \hat{\sigma}(x_1, x_2, p_3, p_4, \ldots) f_q(x_1, t_1) f_{\bar{q}}(x_2, t_0)$$



K_T clustering schemes

The default clustering scheme used (in MG/Sherpa/AlpGen)to determine the parton shower history is the Durham k_T scheme. For e^+e^-:

$$k_{Tij}^2 = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$$

and for hadron collisions, the minimum of:

$$k_{Tibeam}^2 = m_i^2 + p_{Ti}^2 = (E_i + p_{zi})(E_i - p_{zi})$$

and

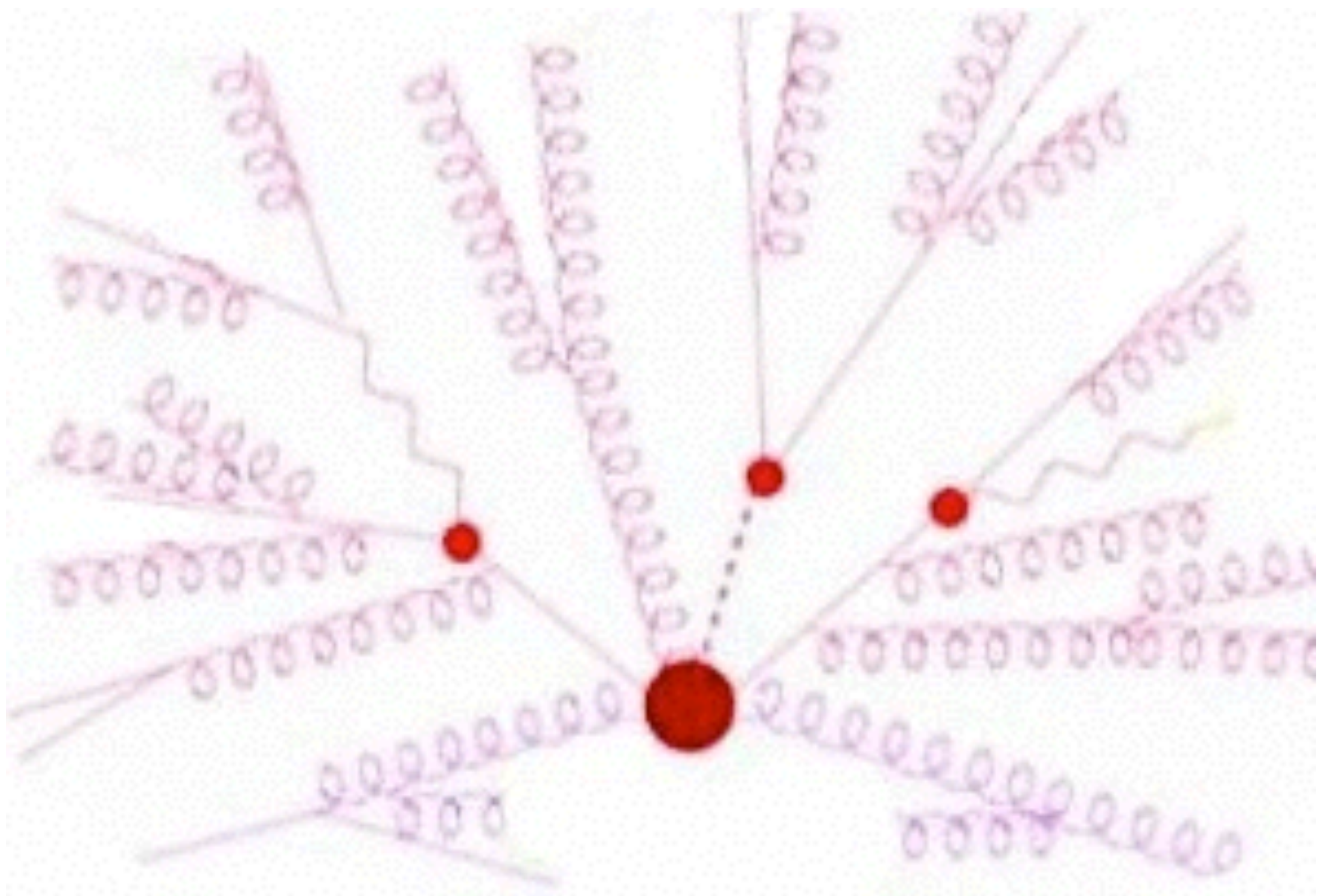
$$k_{Tij}^2 = \max(m_i^2, m_2^2) + \min(p_{Ti}^2, p_{Tj}^2)R_{ij}$$

with

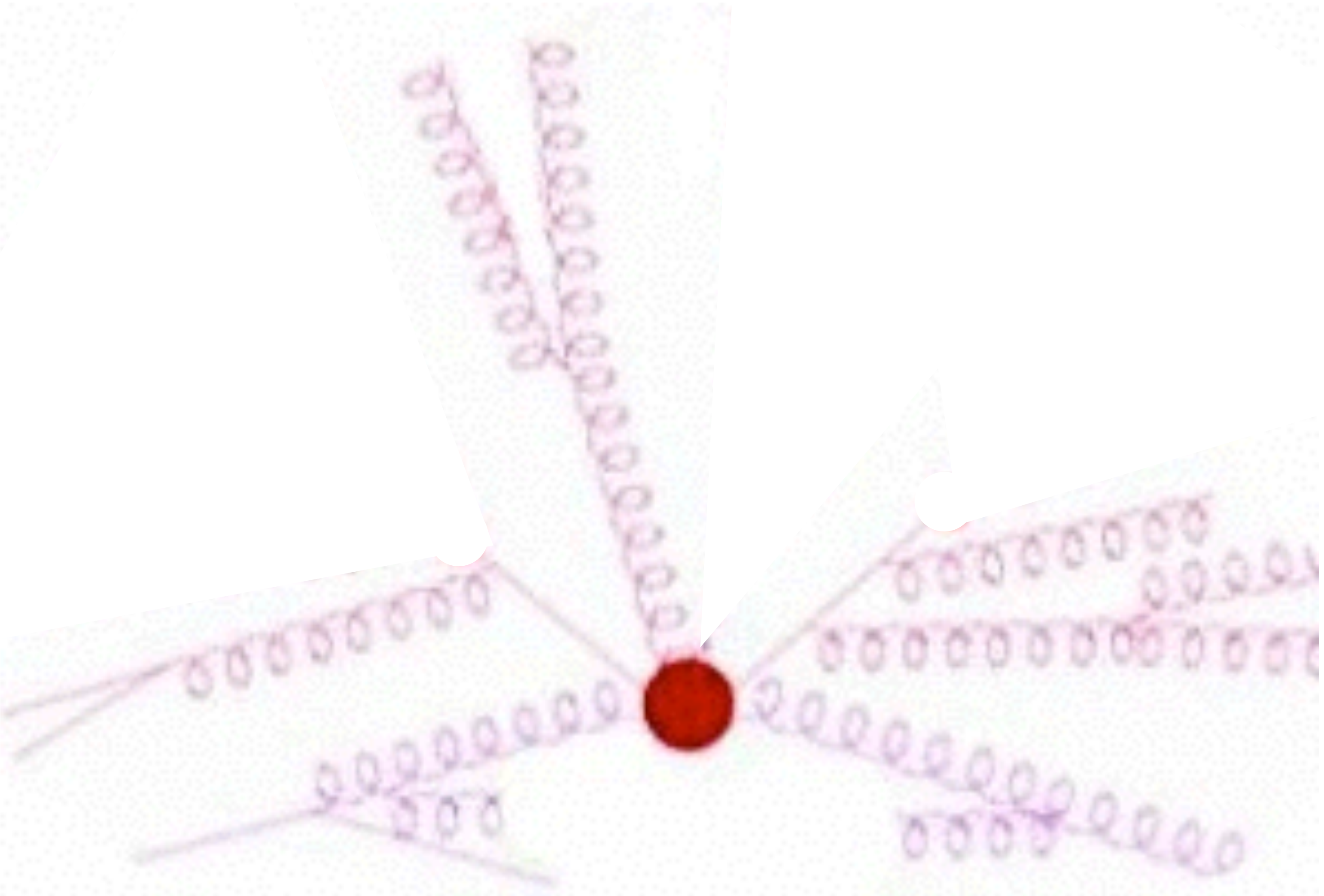
$$R_{ij} = 2[\cosh(y_i - y_j) - \cos(\phi_i - \phi_j)] \simeq (\Delta y)^2 + (\Delta\phi)^2$$

Find the smallest k_{Tij} (or k_{Tibeam}), combine partons i and j (or i and the beam), and continue until you reach a 2 → 2 (or 2 → 1) scattering.

Clustering example



Clustering example



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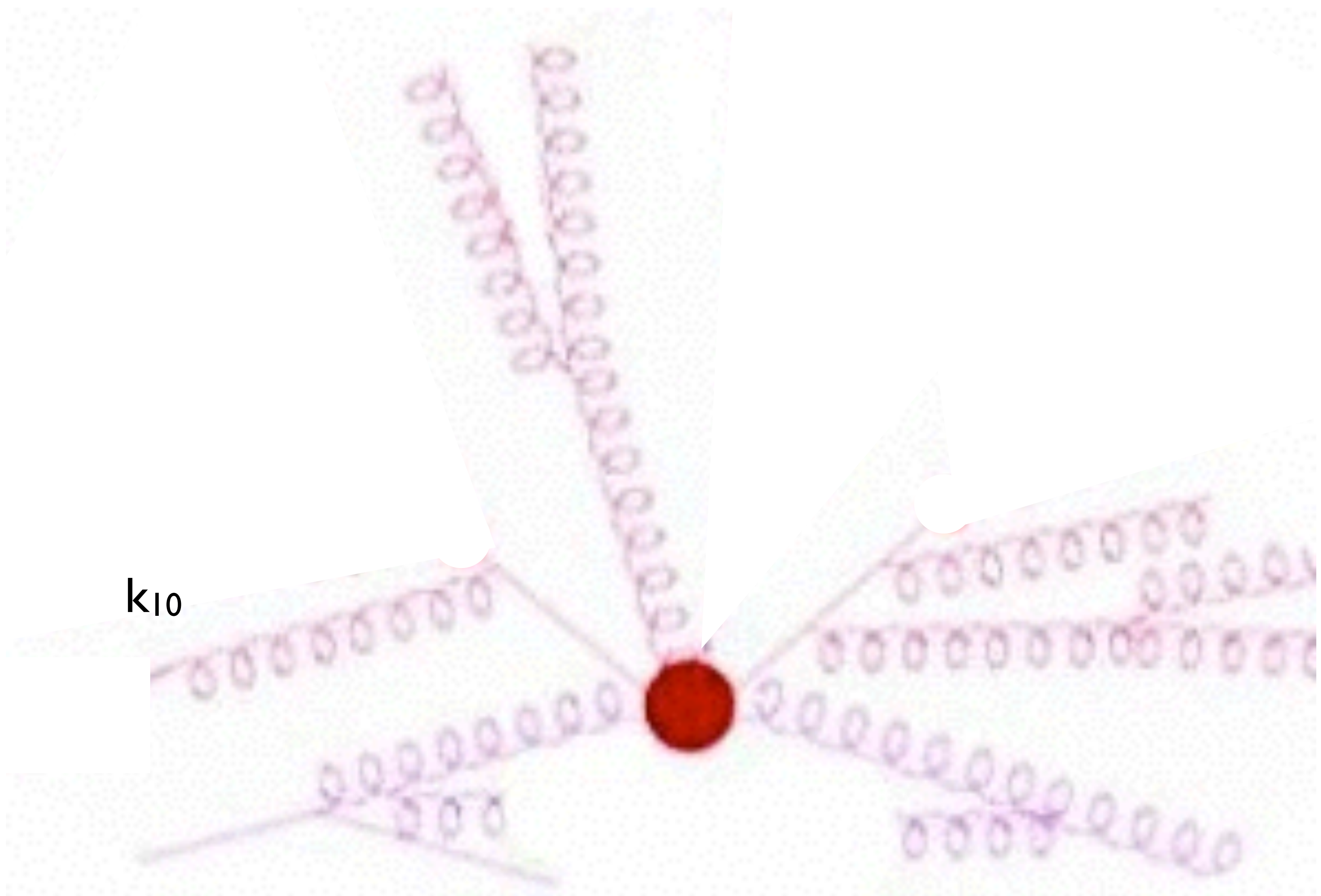
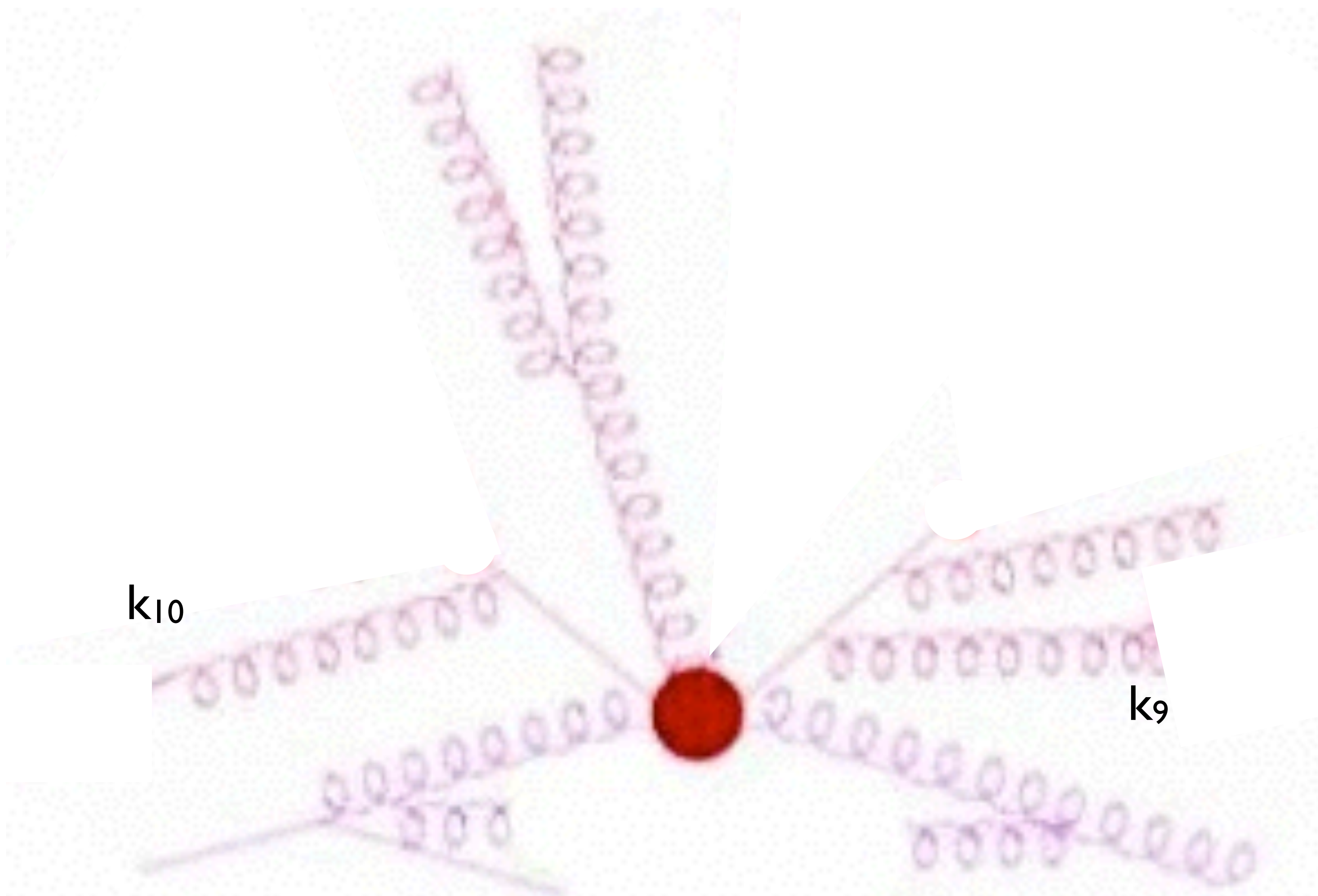
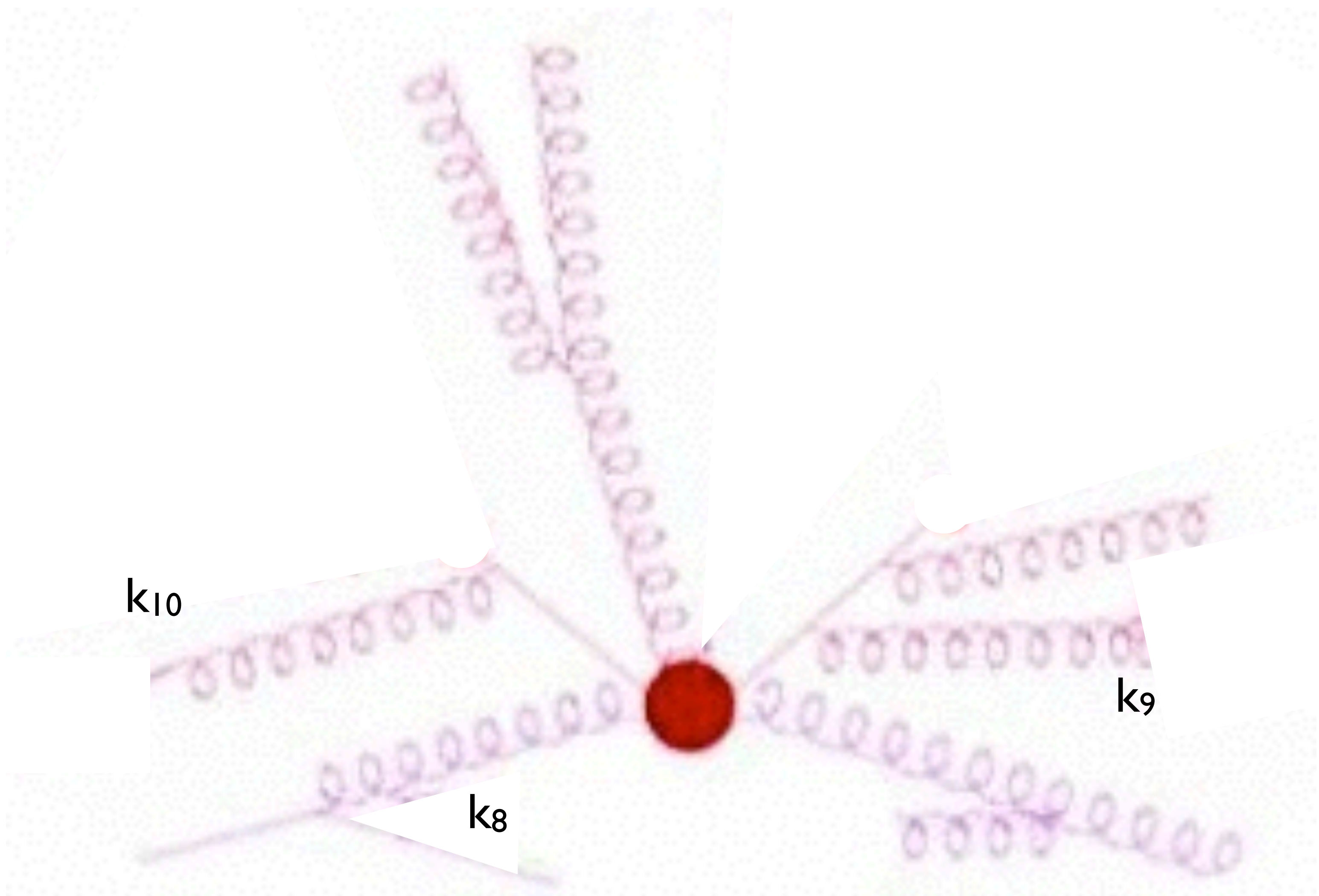


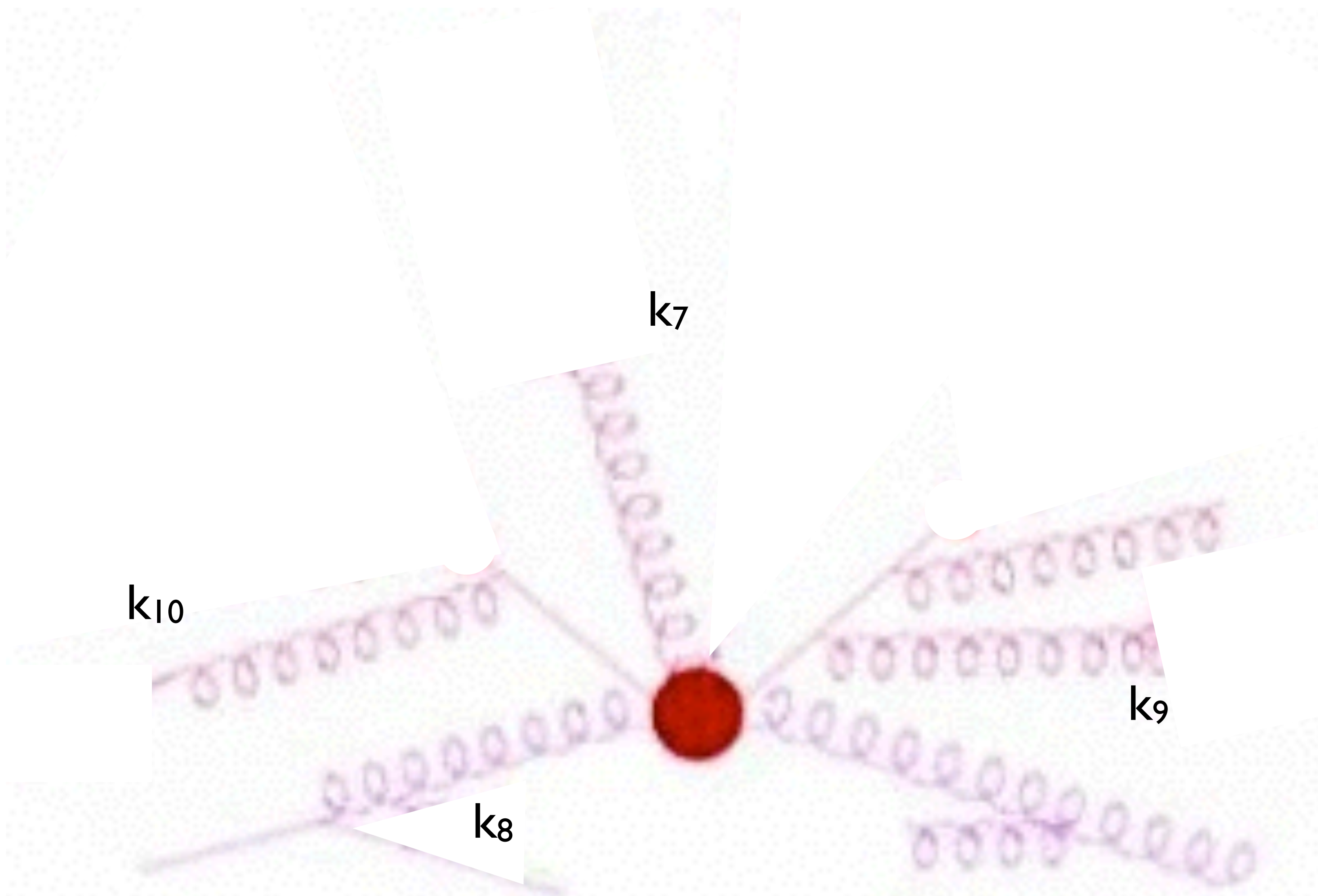
Clustering example



Clustering example



Clustering example



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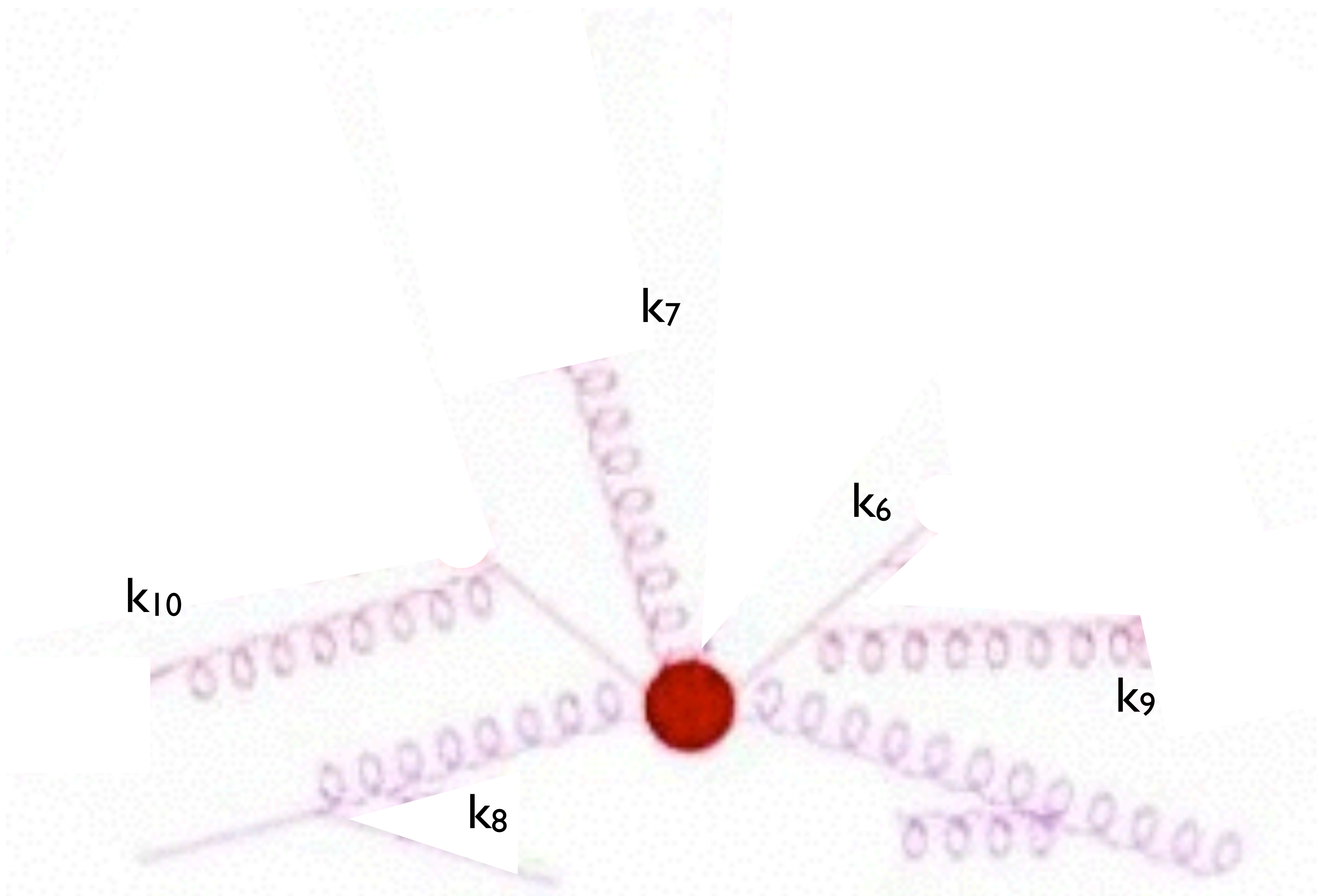

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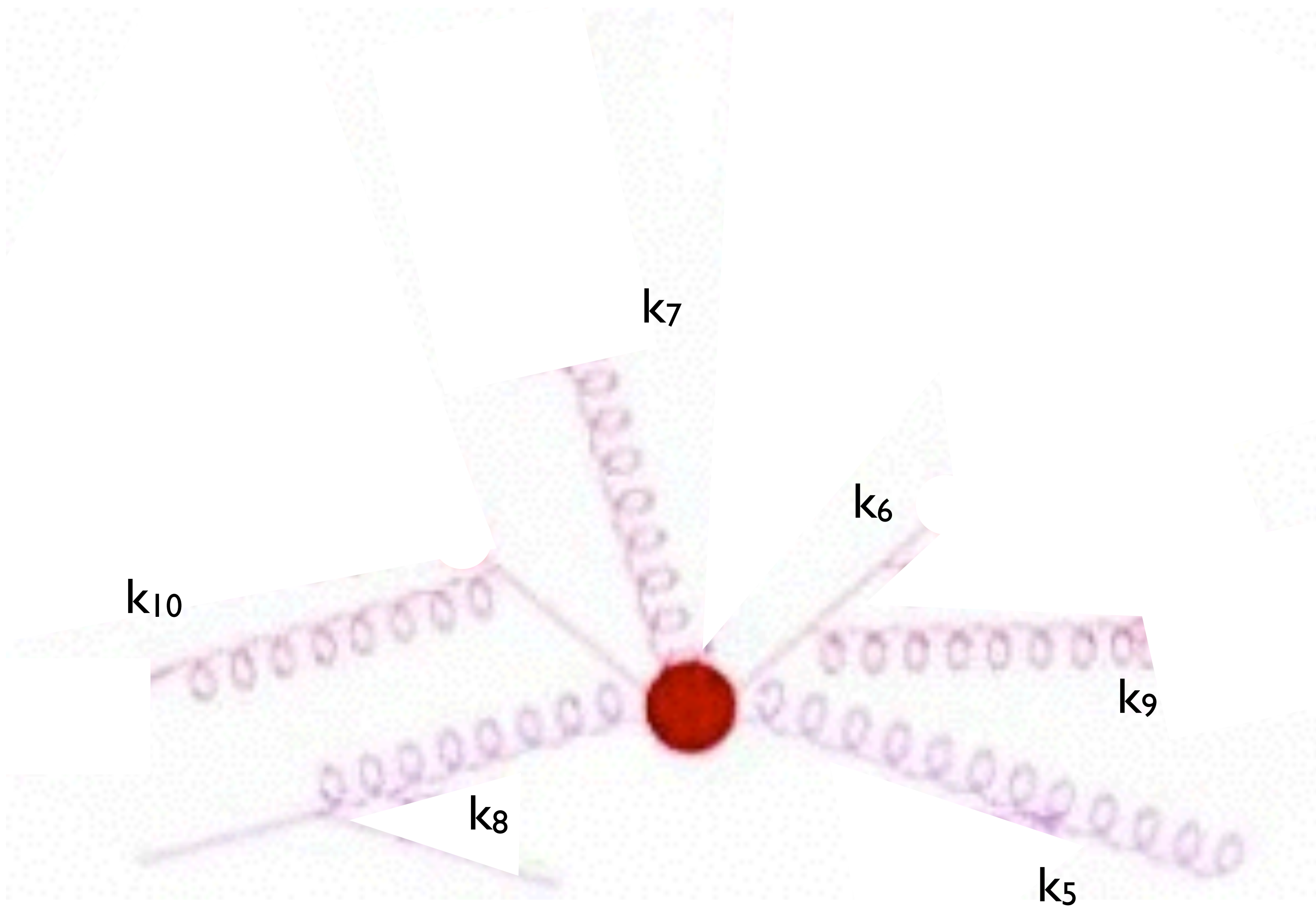
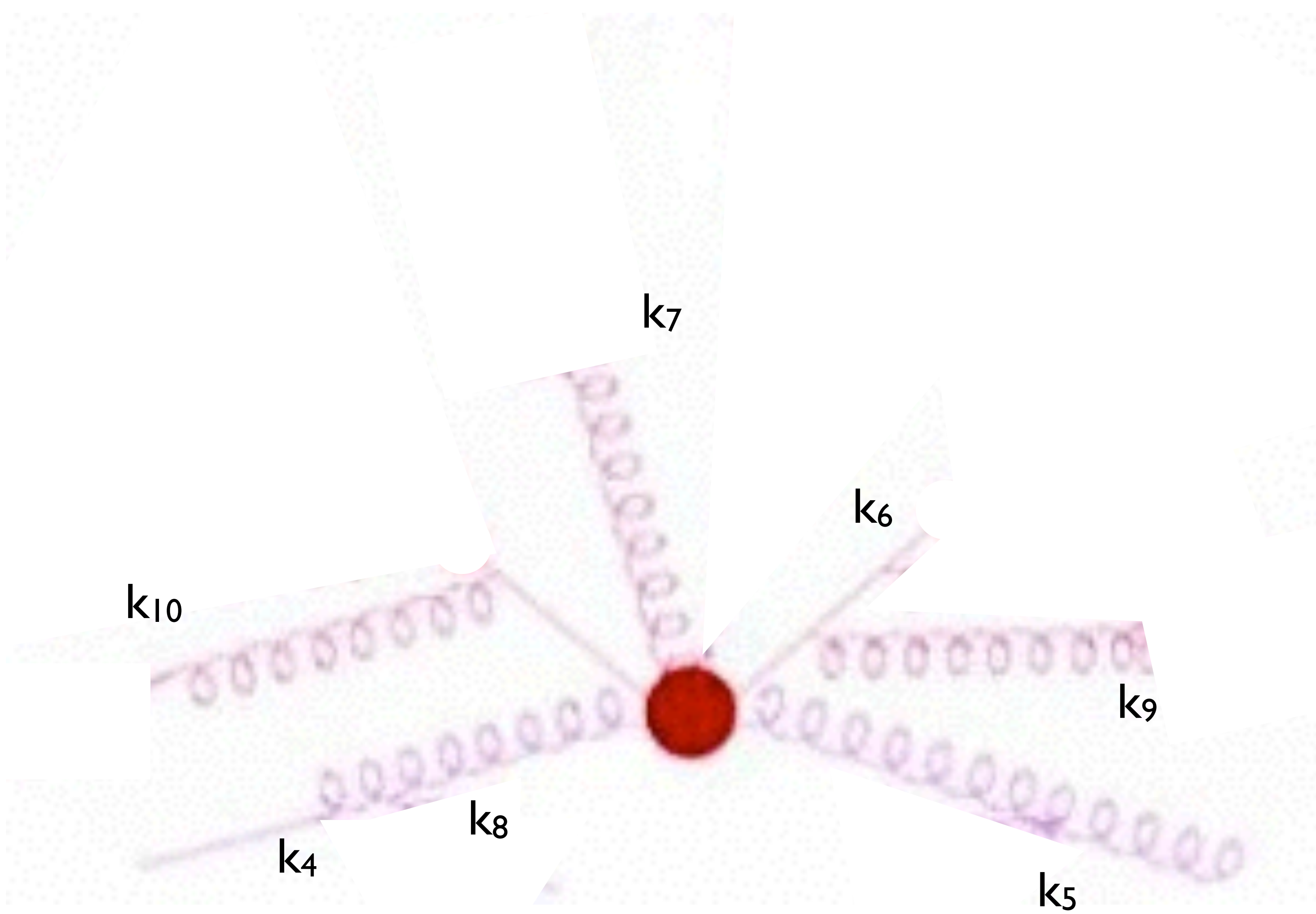


Clustering example



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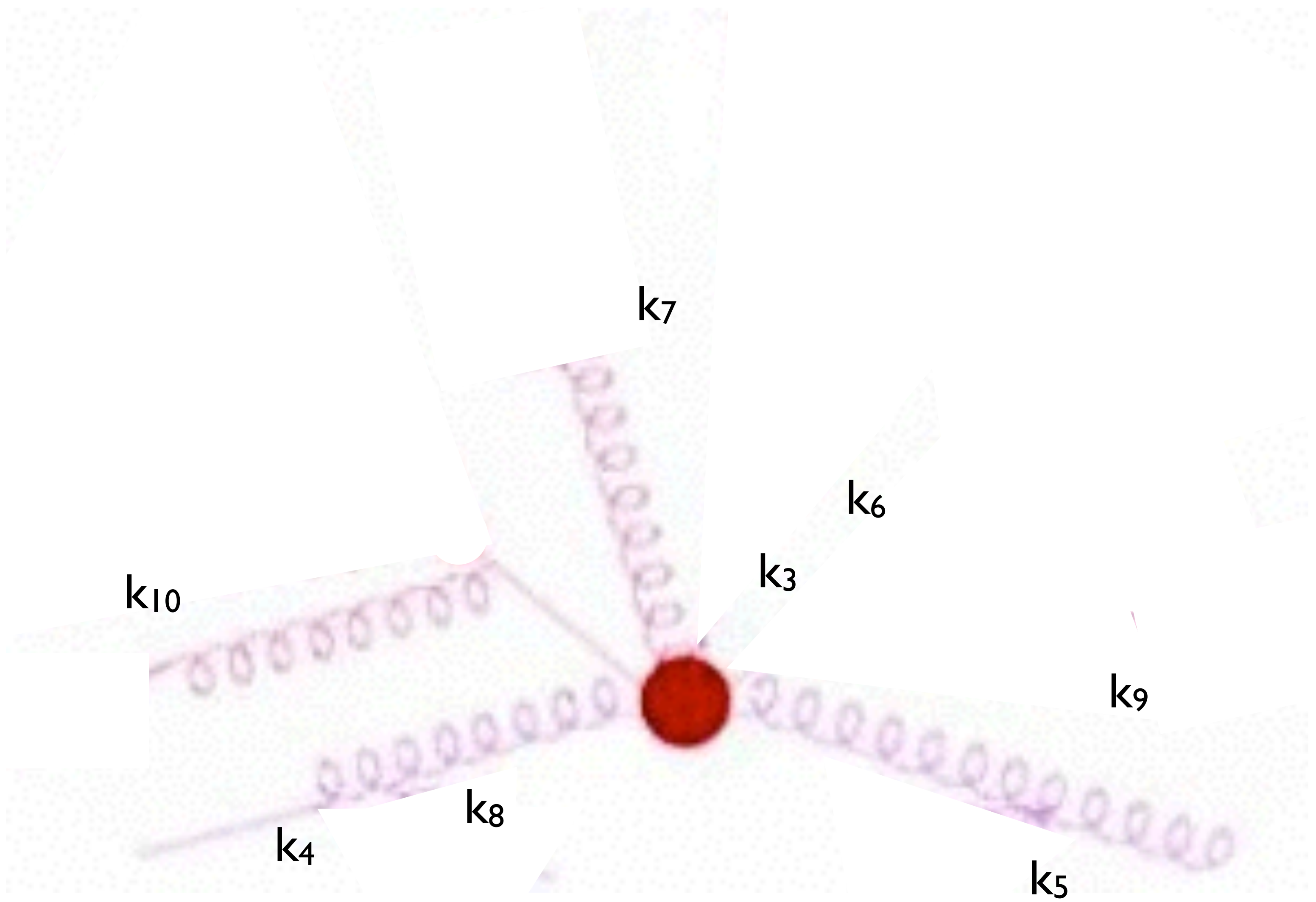

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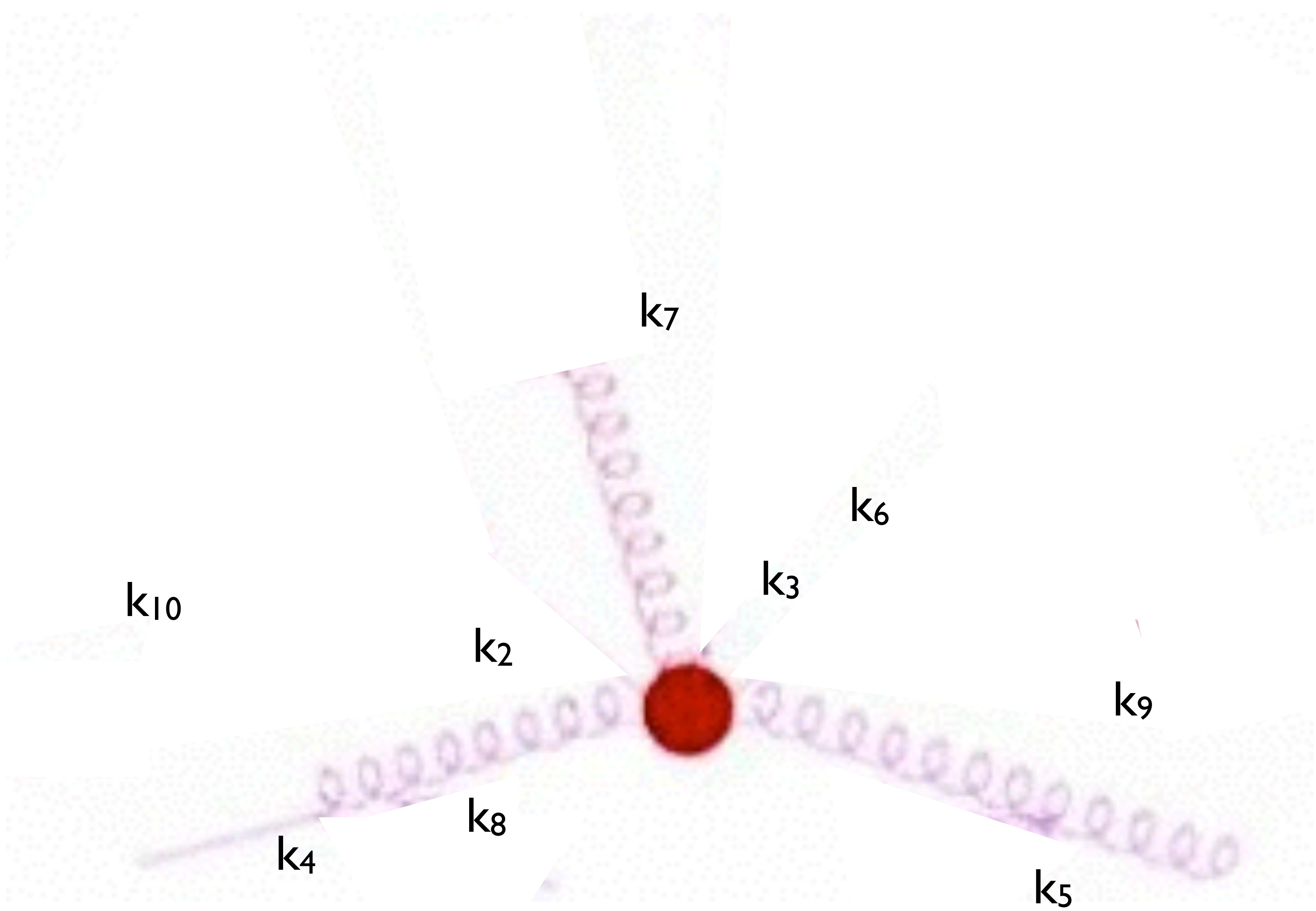

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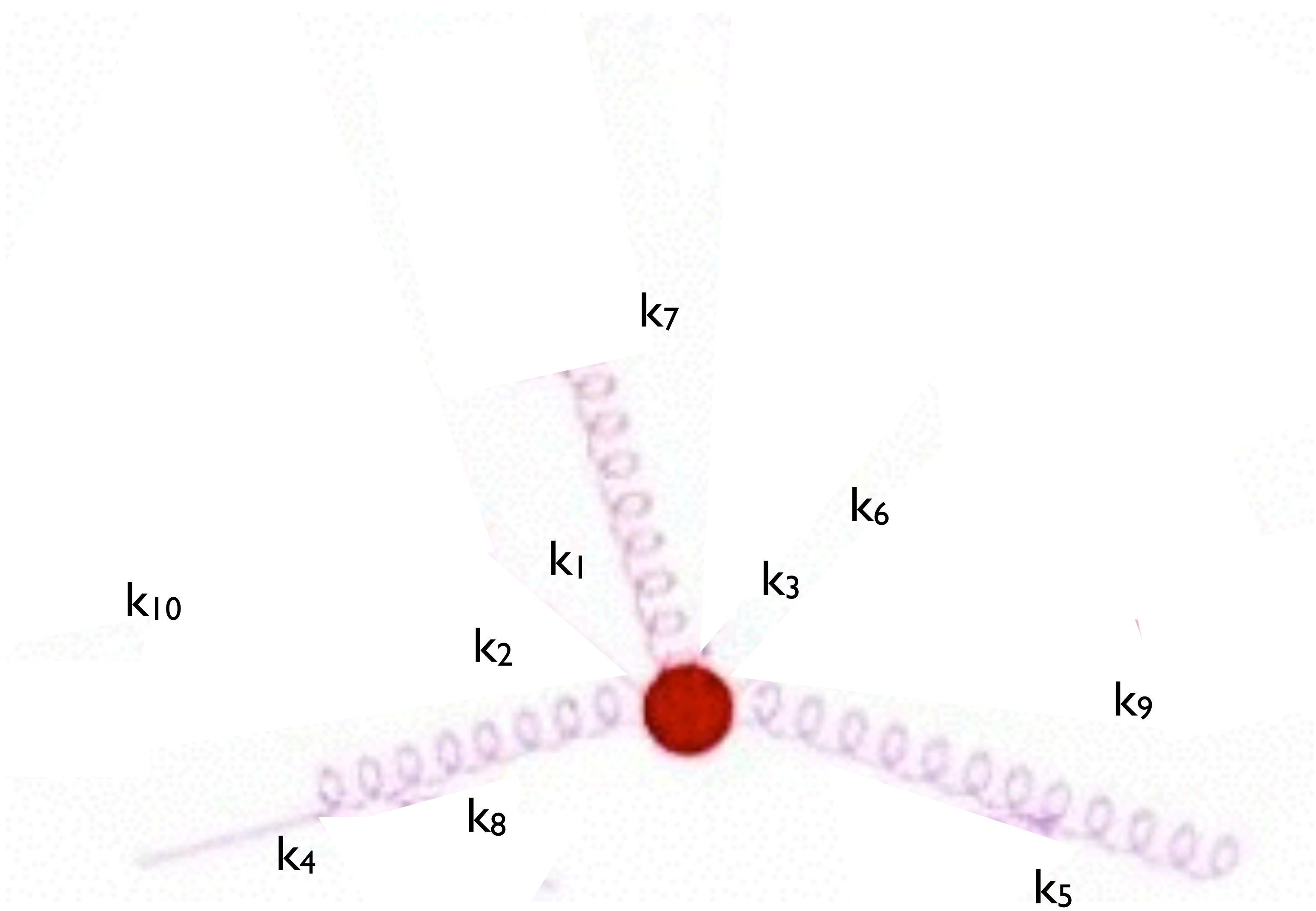

Matching schemes

- We still haven't specified how to apply the Sudakov reweighting to the matrix element
- Three general schemes available in the literature:
  - ➡ CKKW scheme [Catani,Krauss,Kuhn,Webber 2001; Krauss 2002]
  - ➡ Lönnblad scheme (or CKKW-L) [Lönnblad 2002]
  - ➡ MLM scheme [Mangano *unpublished* 2002; Mangano et al. 2007]

CKKW matching

[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]

CKKW matching

[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]

- Apply the required Sudakov suppression

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2$$

analytically, using the best available (NLL) Sudakovs.

CKKW matching

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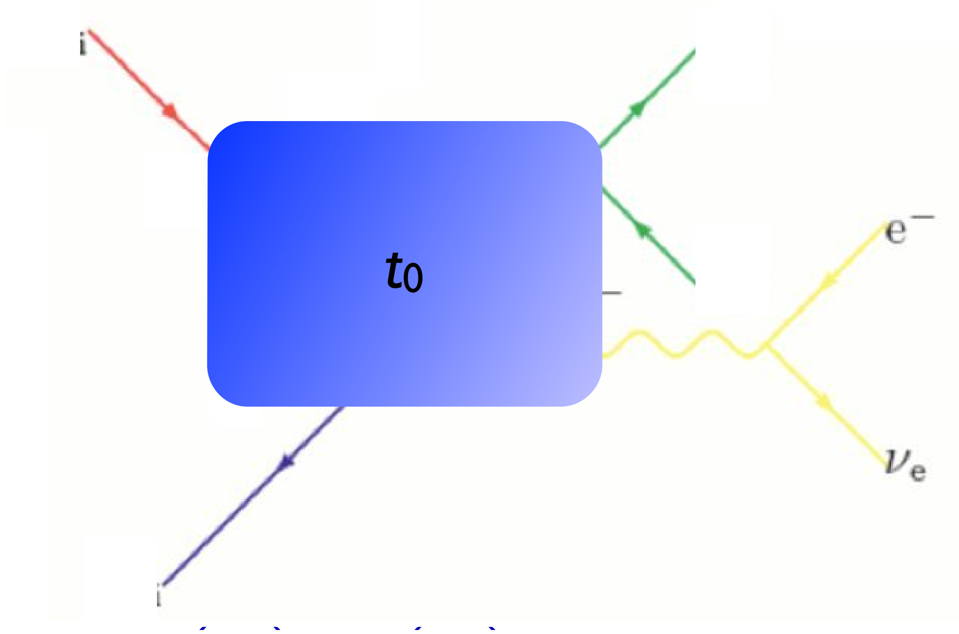
[Krauss 2002]

- Apply the required Sudakov suppression

$$(\Delta_{Iq}(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2$$

analytically, using the best available (NLL) Sudakovs.

- Perform "truncated showering": Run the parton shower starting at t_0, but forbid any showers above the cutoff scale t_{cut}.



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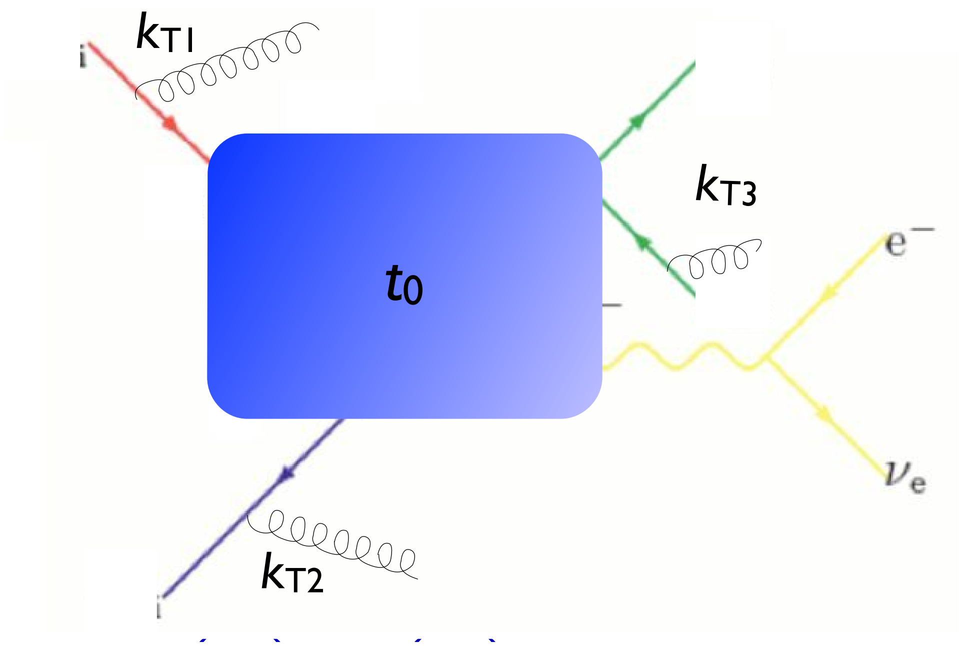

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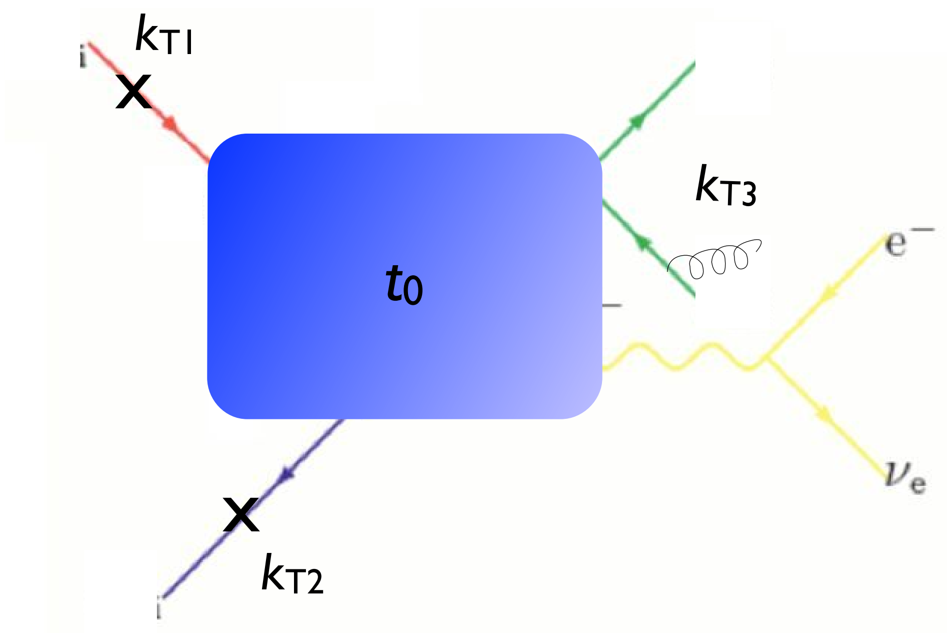

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analytically, using the best available (NLL) Sudakovs.

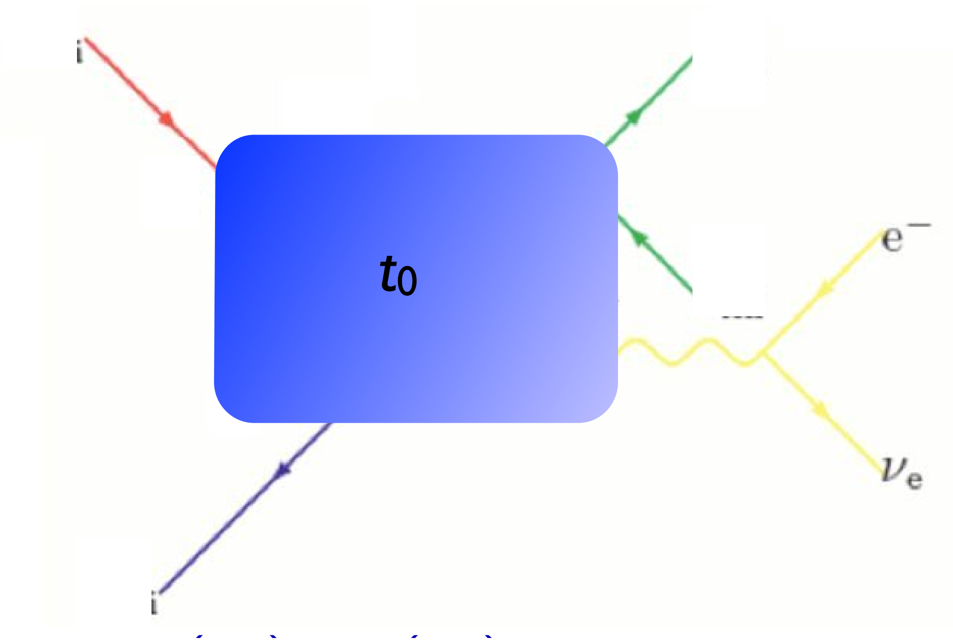
- Perform "truncated showering": Run the parton shower starting at t_0, but forbid any showers above the cutoff scale $t_{\rm cut}$.

✓ Best theoretical treatment of matrix element

- Requires dedicated PS implementation
- Mismatch between analytical Sudakov and (non-NLL) shower
- Implemented in Sherpa (v. 1.1)

CKKW-L matching

[Lönnblad 2002]
[Hoeche et al. 2009]



CKKW-L matching

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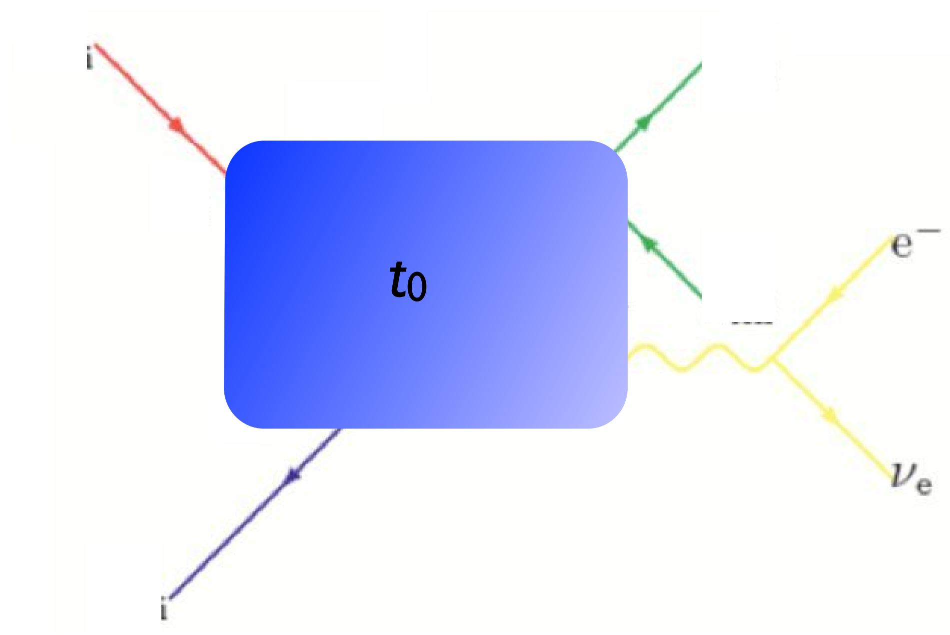
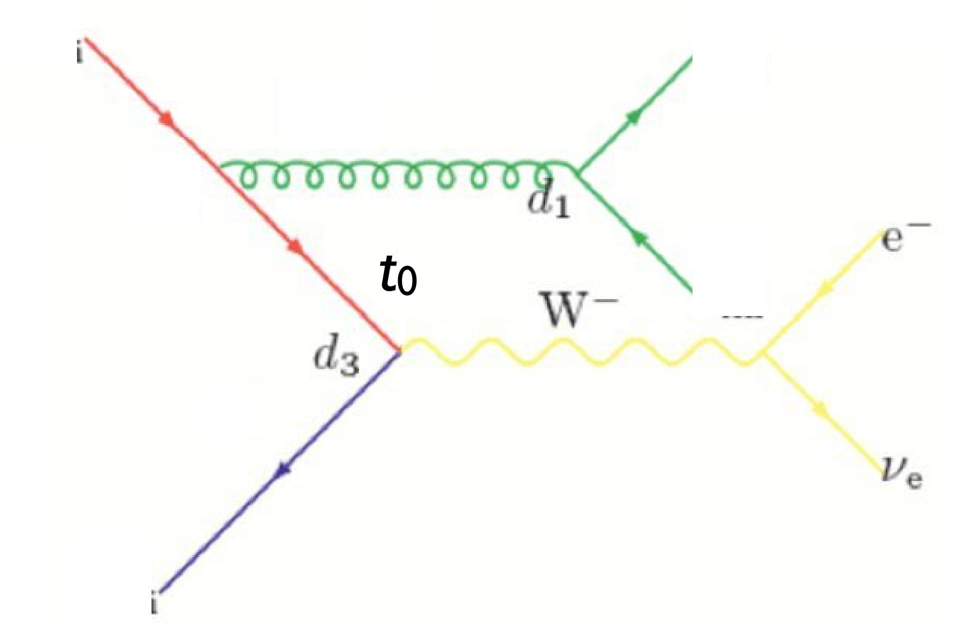
[Hoeche et al. 2009]



- Cluster back to "parton shower history"

CKKW-L matching

[Lönnblad 2002]
[Hoeche et al. 2009]



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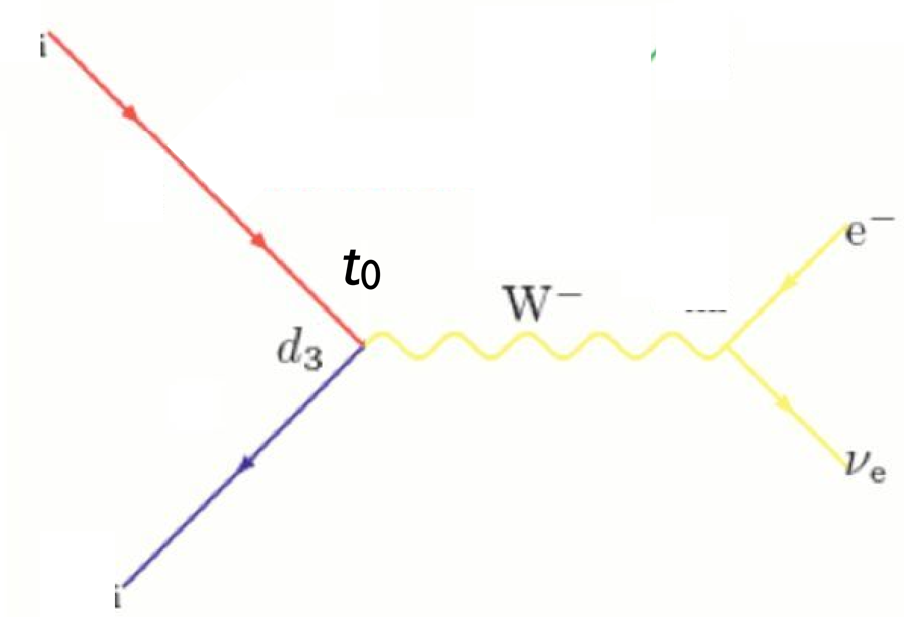
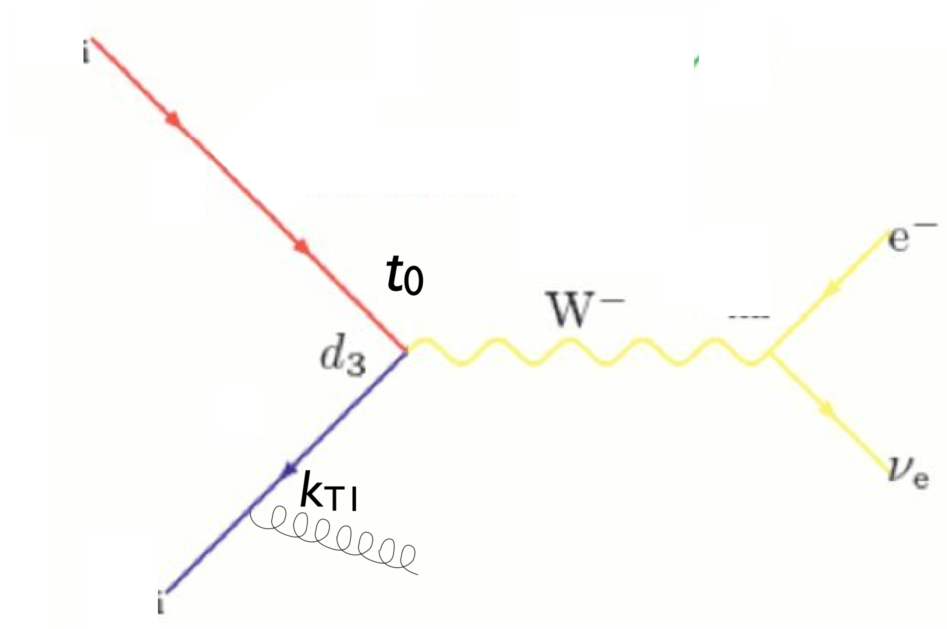


- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step

[Lönnblad 2002]
[Hoeche et al. 2009]



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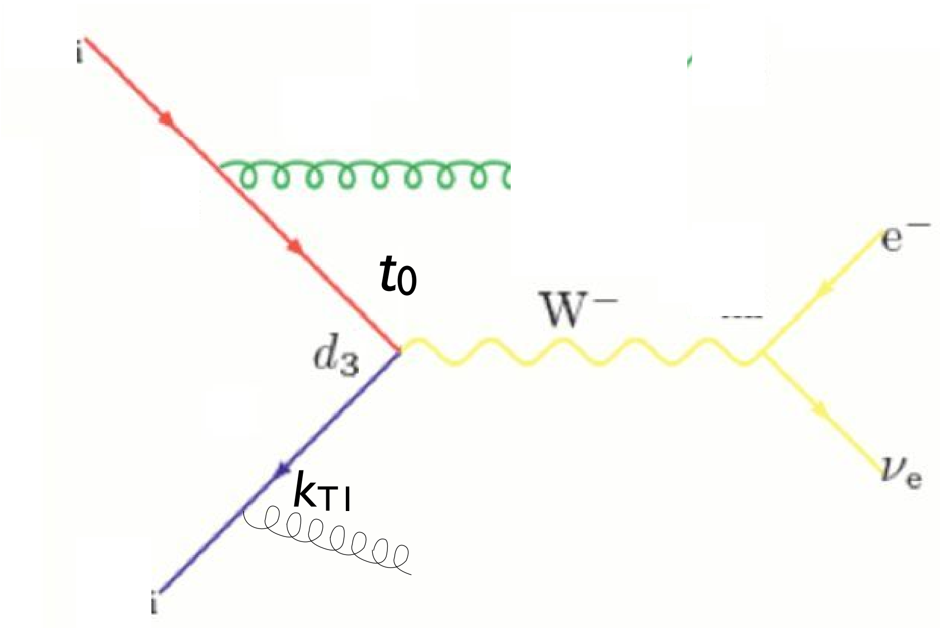

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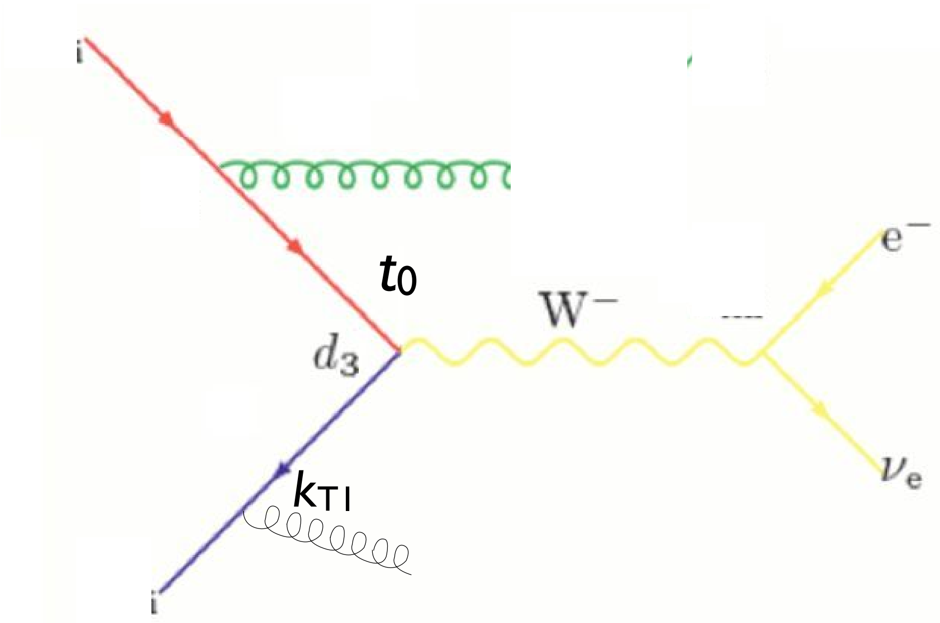

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- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
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CKKW-L matching

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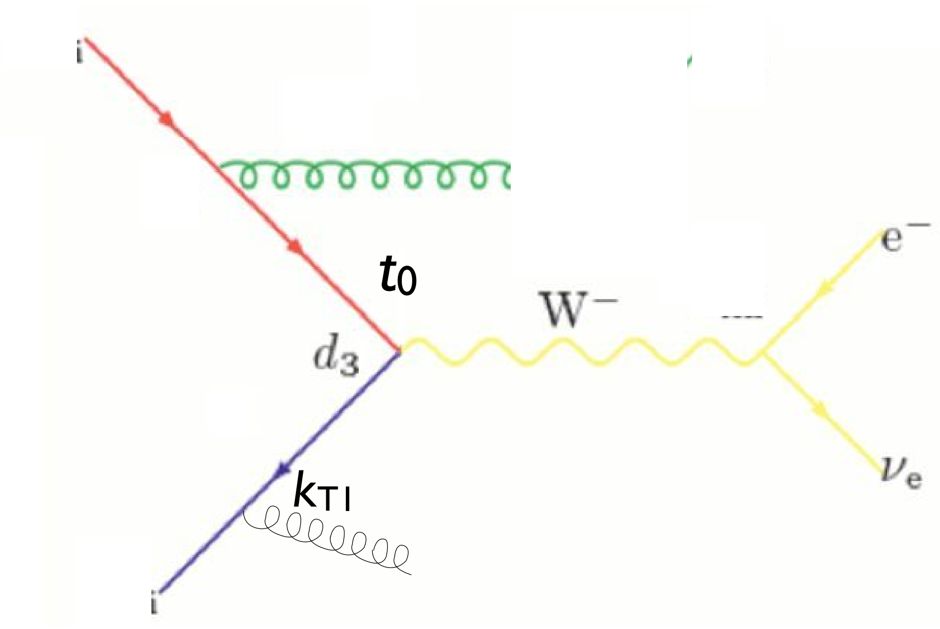
[Hoeche et al. 2009]



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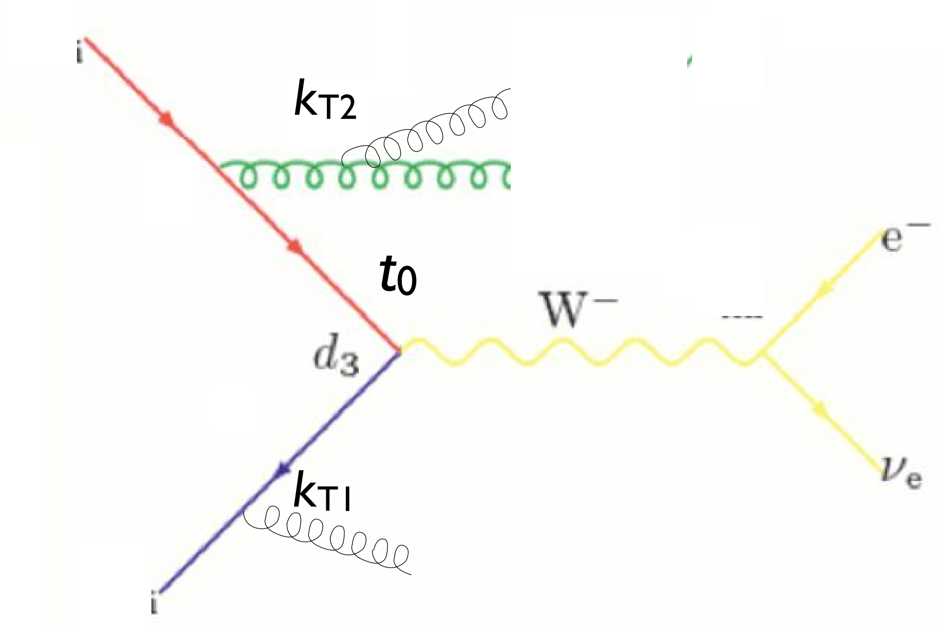
- Keep any shower emissions that are softer than the clustering scale for the next step

CKKW-L matching

[Lönnblad 2002]
[Hoeche et al. 2009]



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CKKW-L matching

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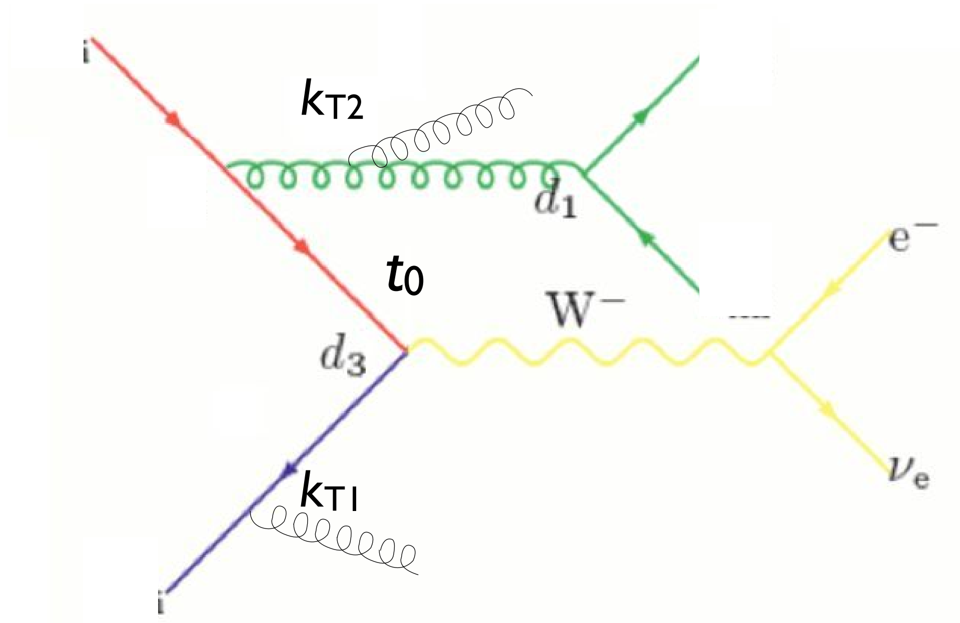

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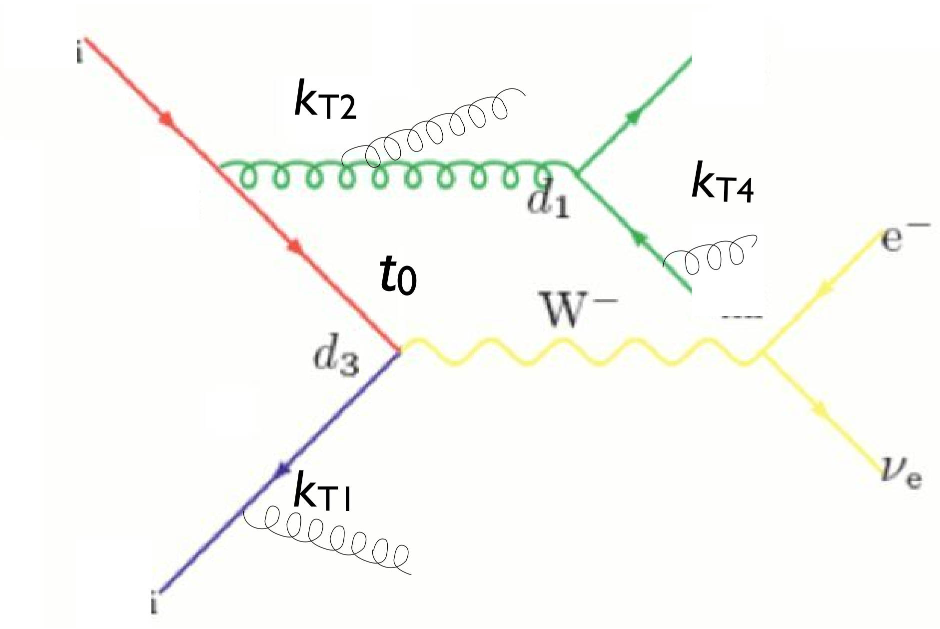

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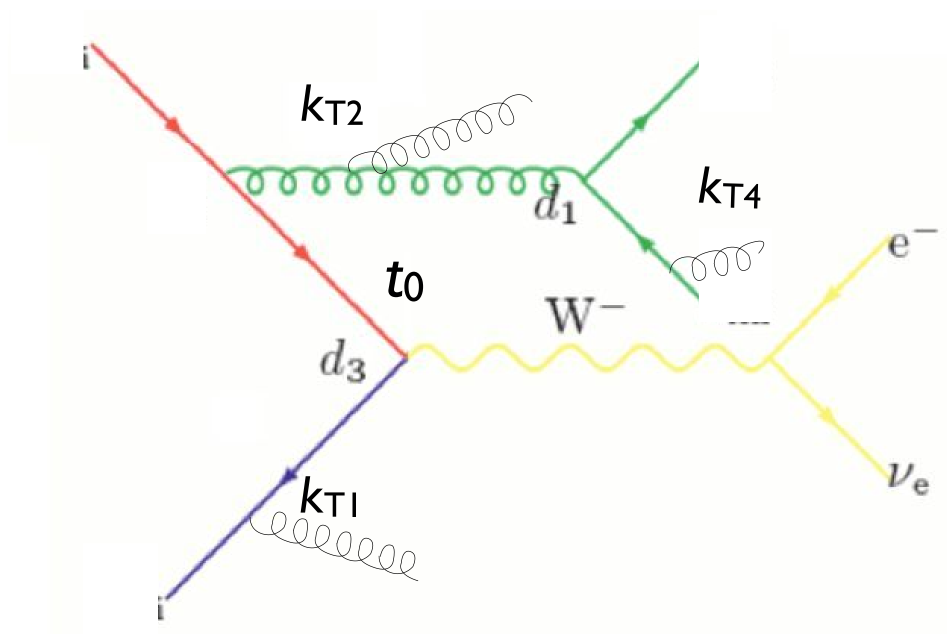


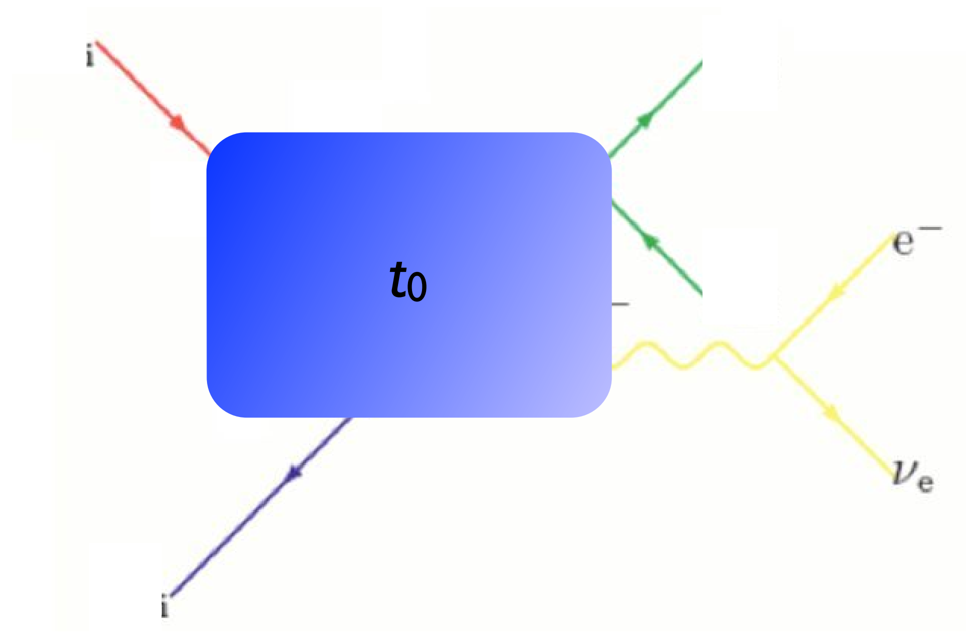
- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step

✓ Automatic agreement between Sudakov and shower

- Requires dedicated PS implementation
  - ➡ Need multiple implementations to compare between showers

- Implemented in Ariadne, Sherpa (v. 1.2), and Pythia 8

[M.L. Mangano, ~2002, 2007]
[J.A. et al 2007, 2008]



[M.L. Mangano, ~2002, 2007]
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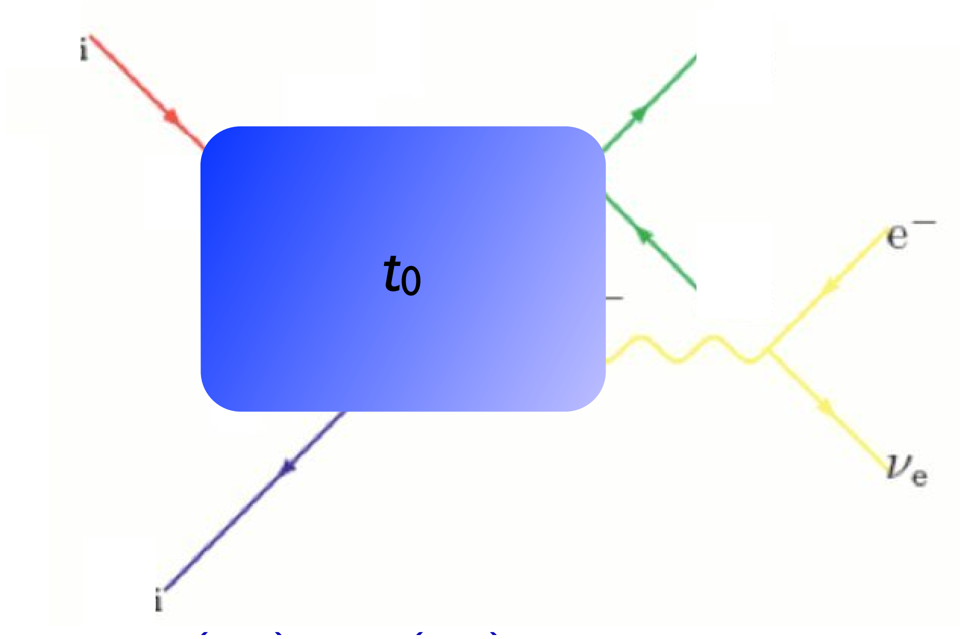
- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t_0!



MLM matching

[M.L. Mangano, ~2002, 2007]
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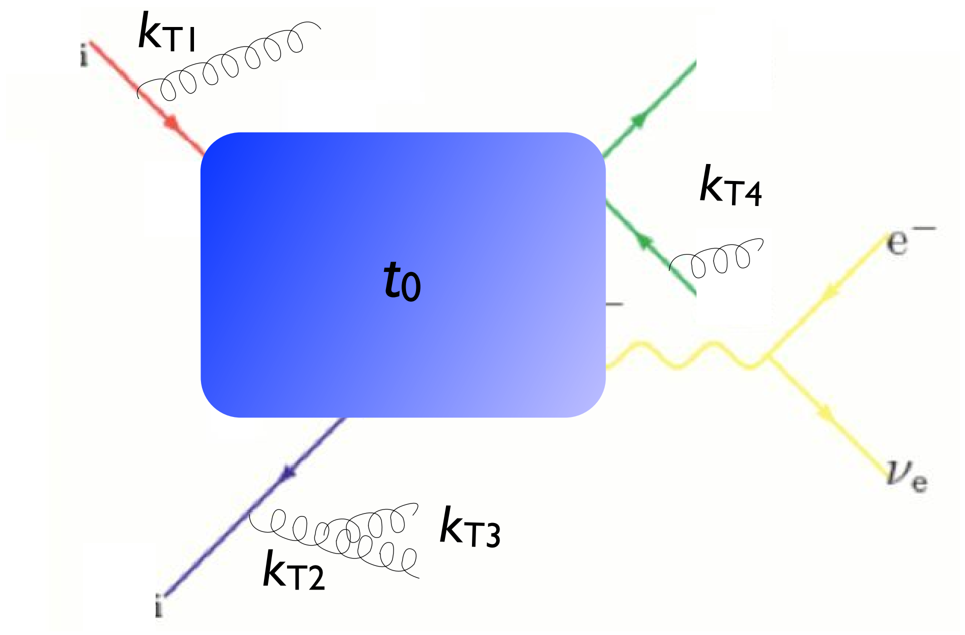

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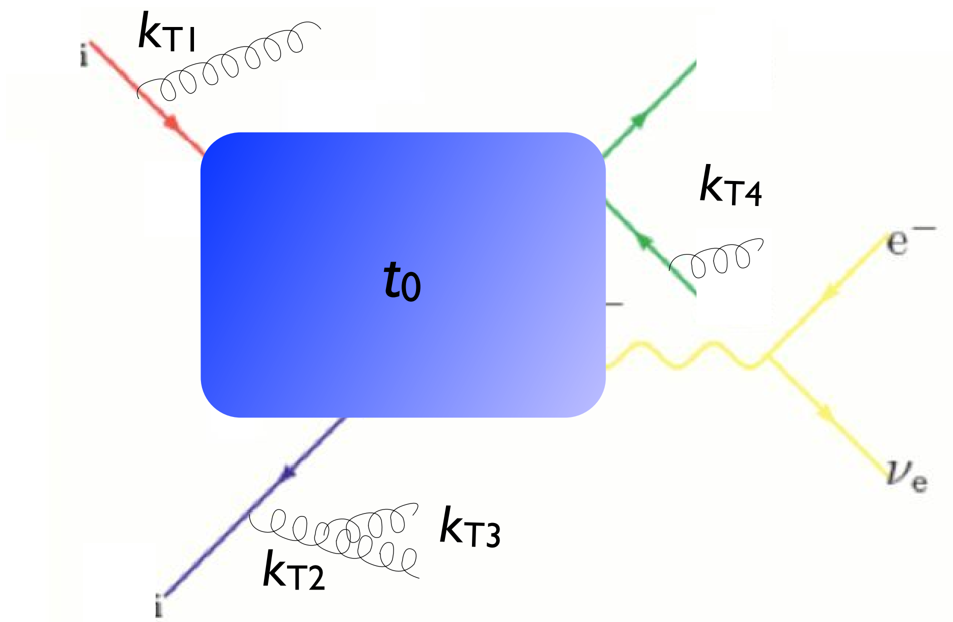
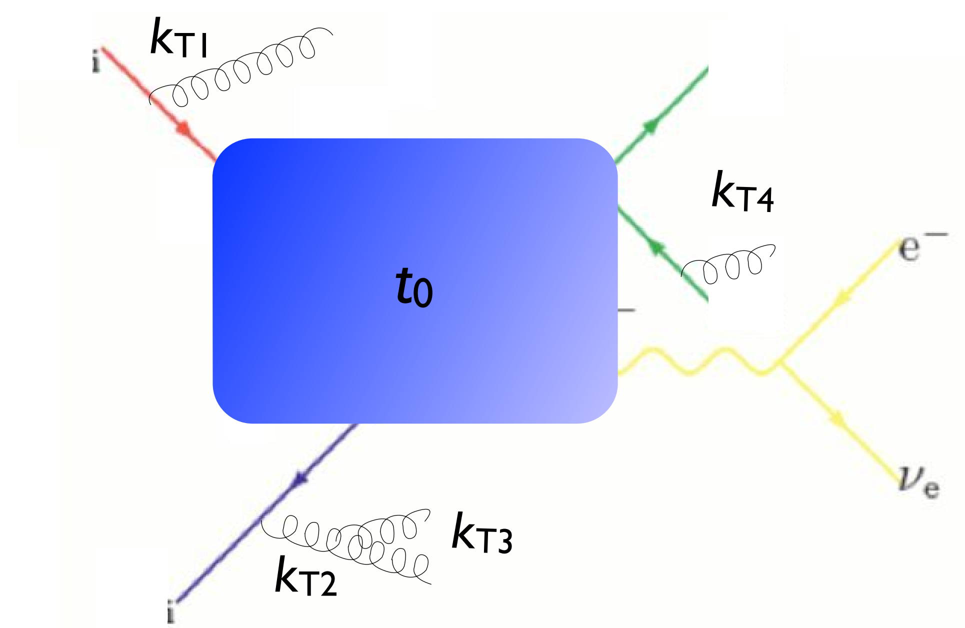


- Perform jet clustering after PS - if hardest jet $k_{T1} > t_{cut}$ or there are jets not matched to partons, reject the event

MLM matching

[M.L. Mangano, ~2002, 2007]
[J.A. et al 2007, 2008]

- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t_0!



- Perform jet clustering after PS - if hardest jet $k_{T1} > t_{cut}$ or there are jets not matched to partons, reject the event
- The resulting Sudakov suppression from the procedure is

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 (\Delta_q(t_{\rm cut}, t_0))^2$$

which turns out to be a good enough approximation of the correct expression $$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\rm cut}, t_2))^2$$

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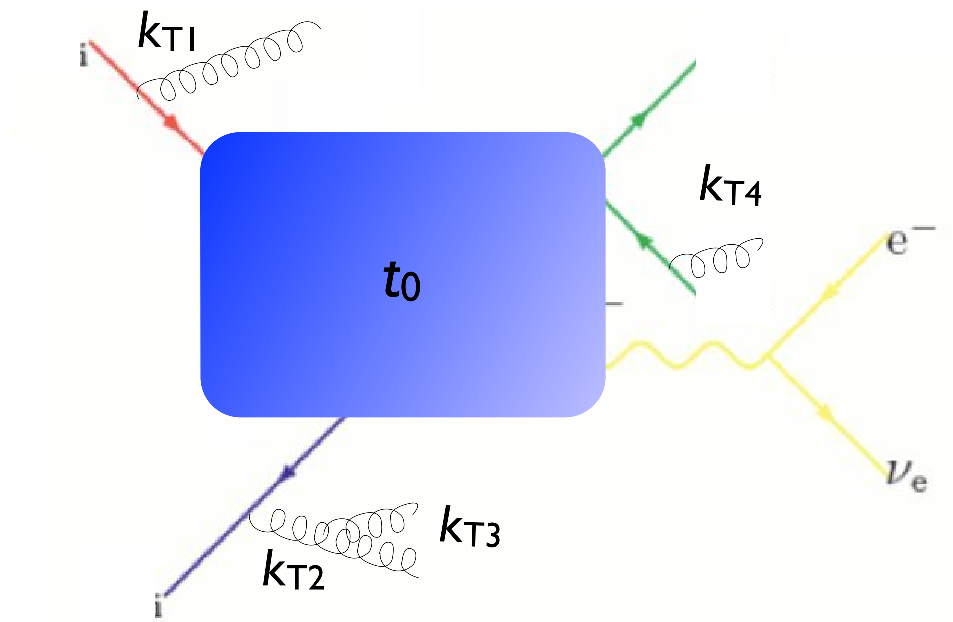
- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t_0!



- Perform jet clustering after PS - if hardest jet $k_{T1} > t_{cut}$ or there are jets not matched to partons, reject the event
  - ✓ Simplest available scheme
  - ✓ Allows matching with any shower, without modification
  - ➡ Sudakov suppression not exact, minor mismatch with shower
  - Implemented in AlpGen, HELAC, MadGraph+Pythia 6

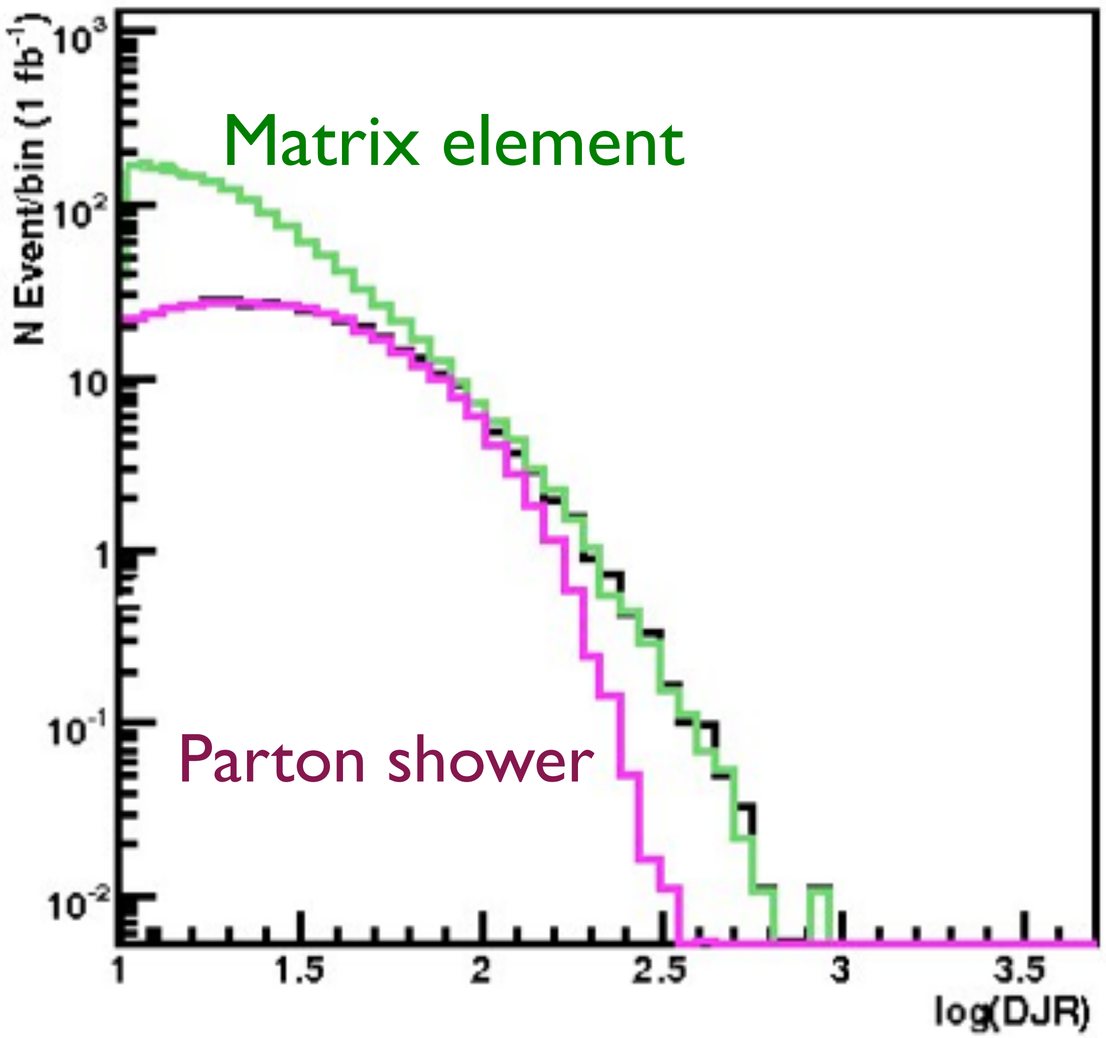
Highest multiplicity sample

- In the previous, assumed we can simulate all parton multiplicities by the ME
- In practice, we can only do limited number of final-state partons with matrix element (up to 4-5 or so)
- For the highest jet multiplicity that we generate with the matrix element, we need to allow additional jets above the matching scale t_{cut}, since we will otherwise not get a jet-inclusive description – but still can't allow PS radiation harder than the ME partons

➡ Need to replace t_{cut} by the clustering scale for the softest ME parton for the highest multiplicity

Back to the "matching goal"

- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

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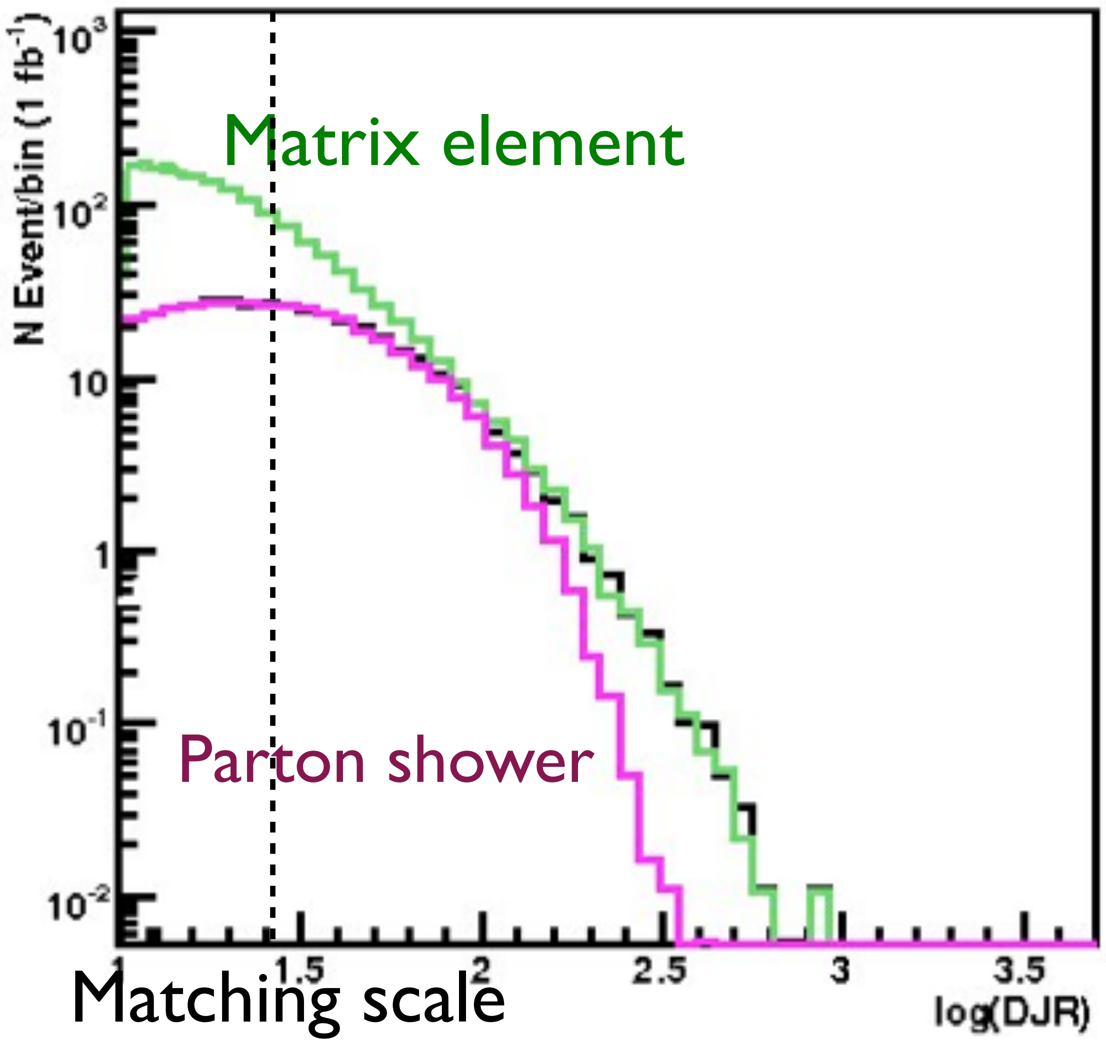

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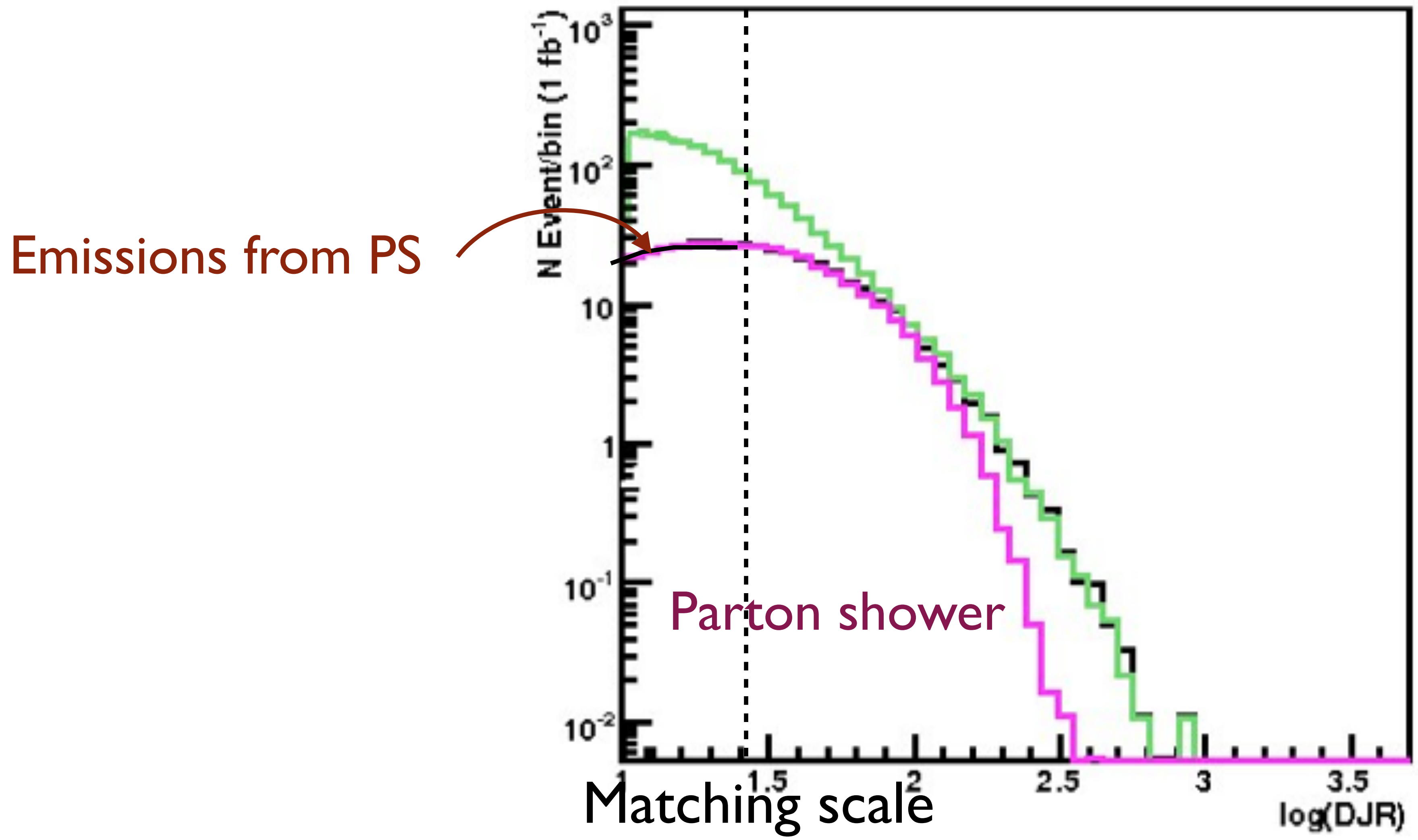

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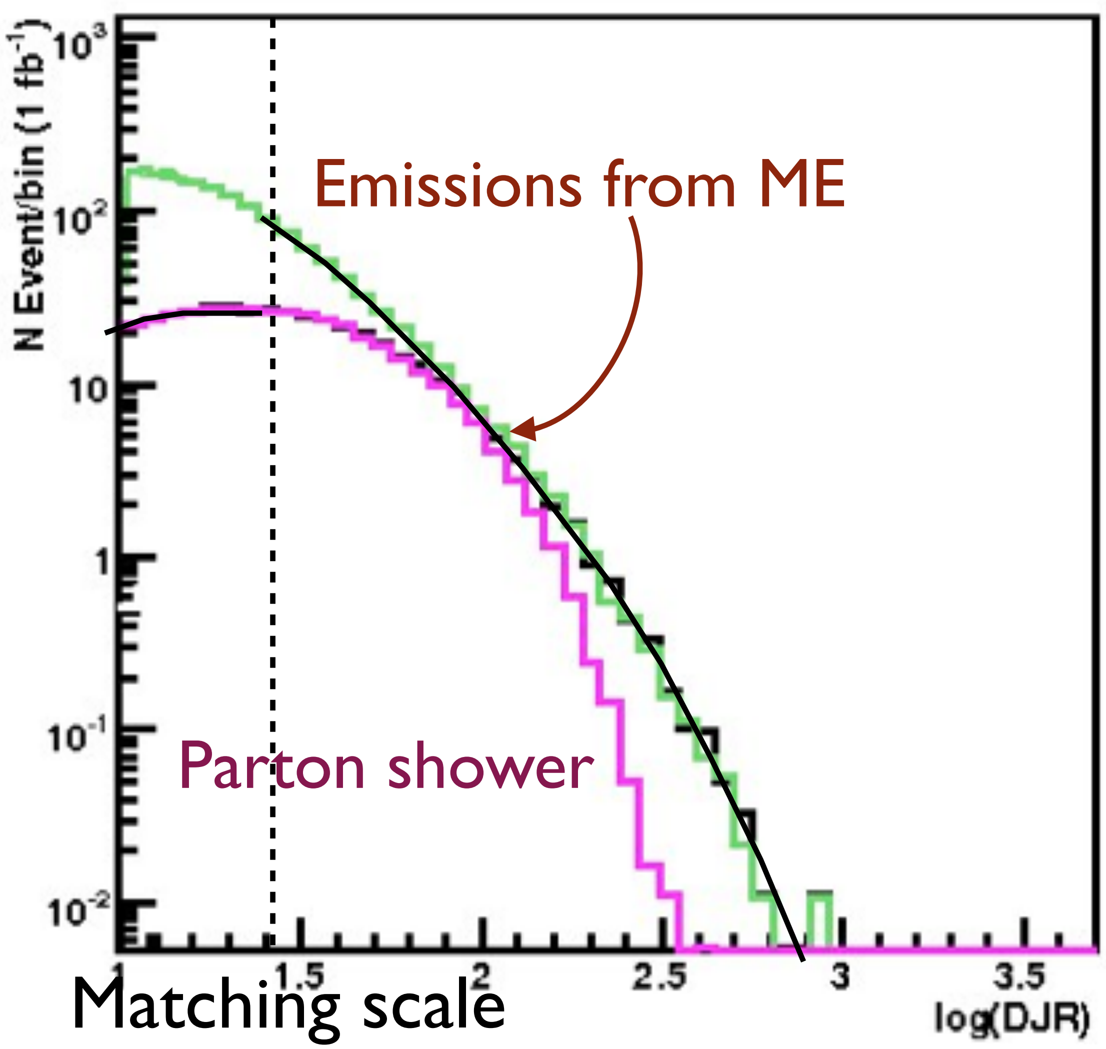

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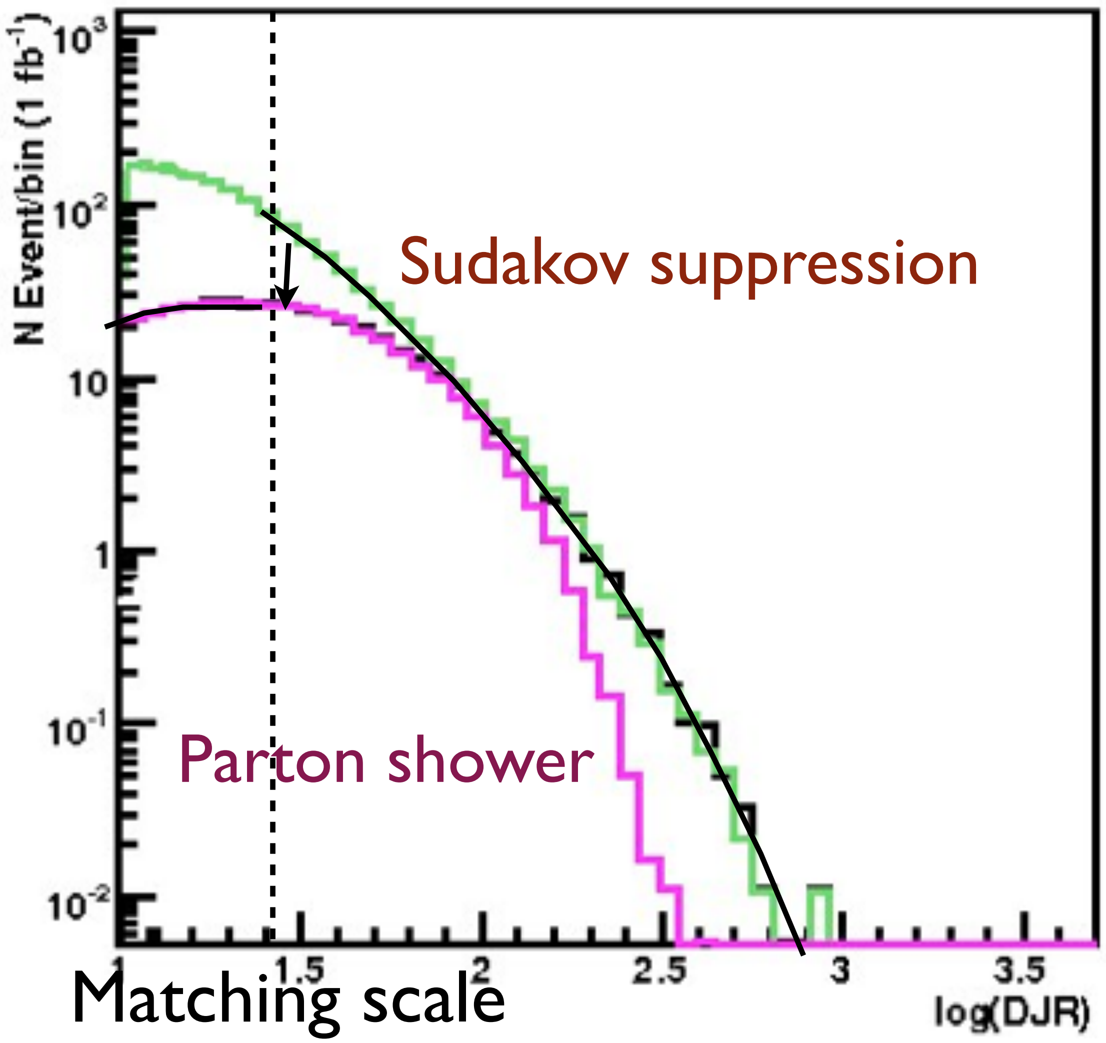

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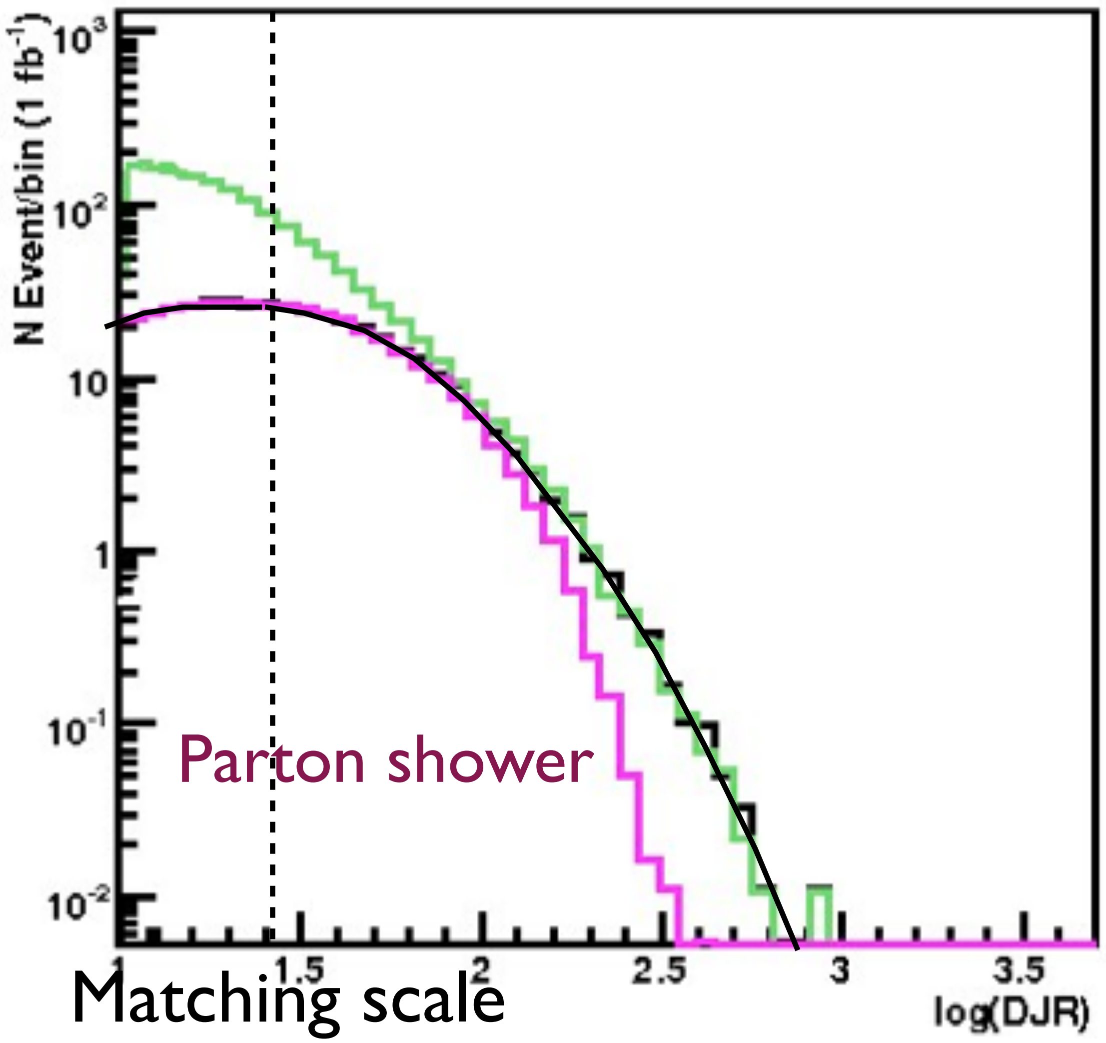

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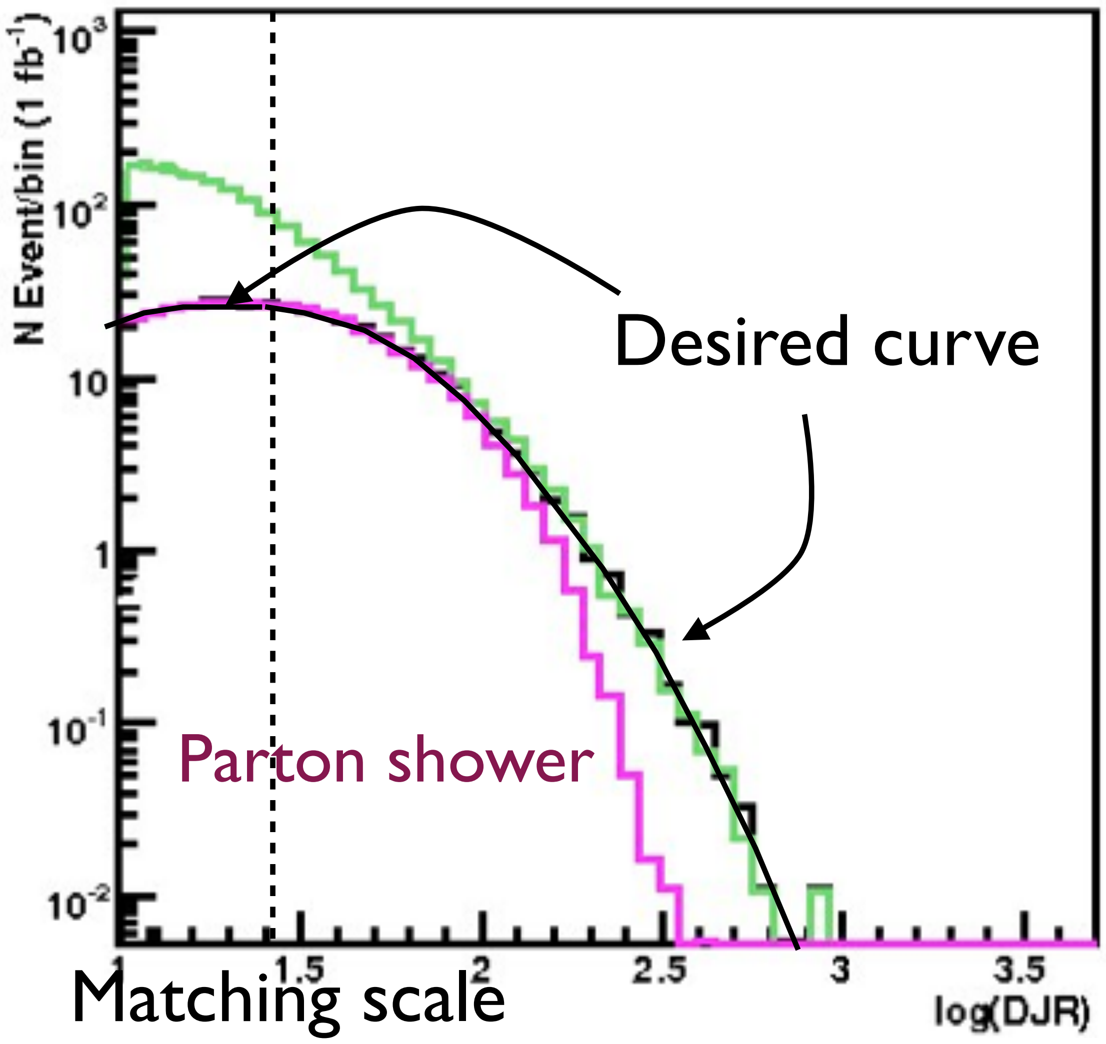
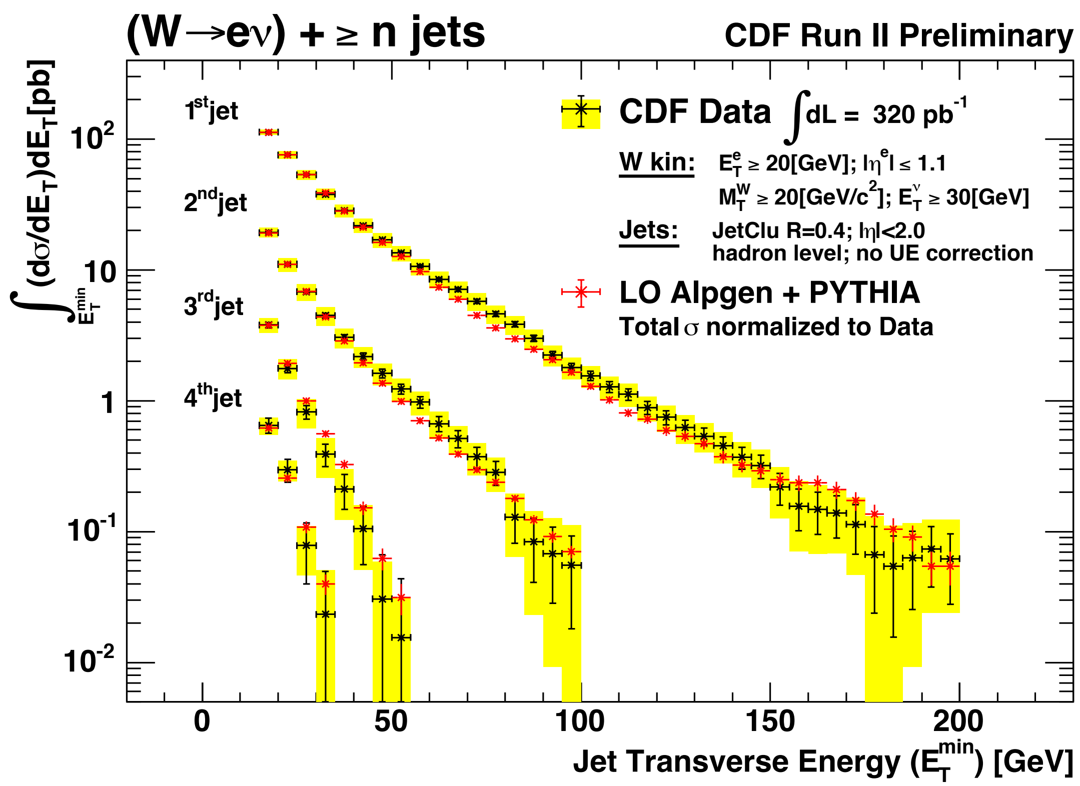


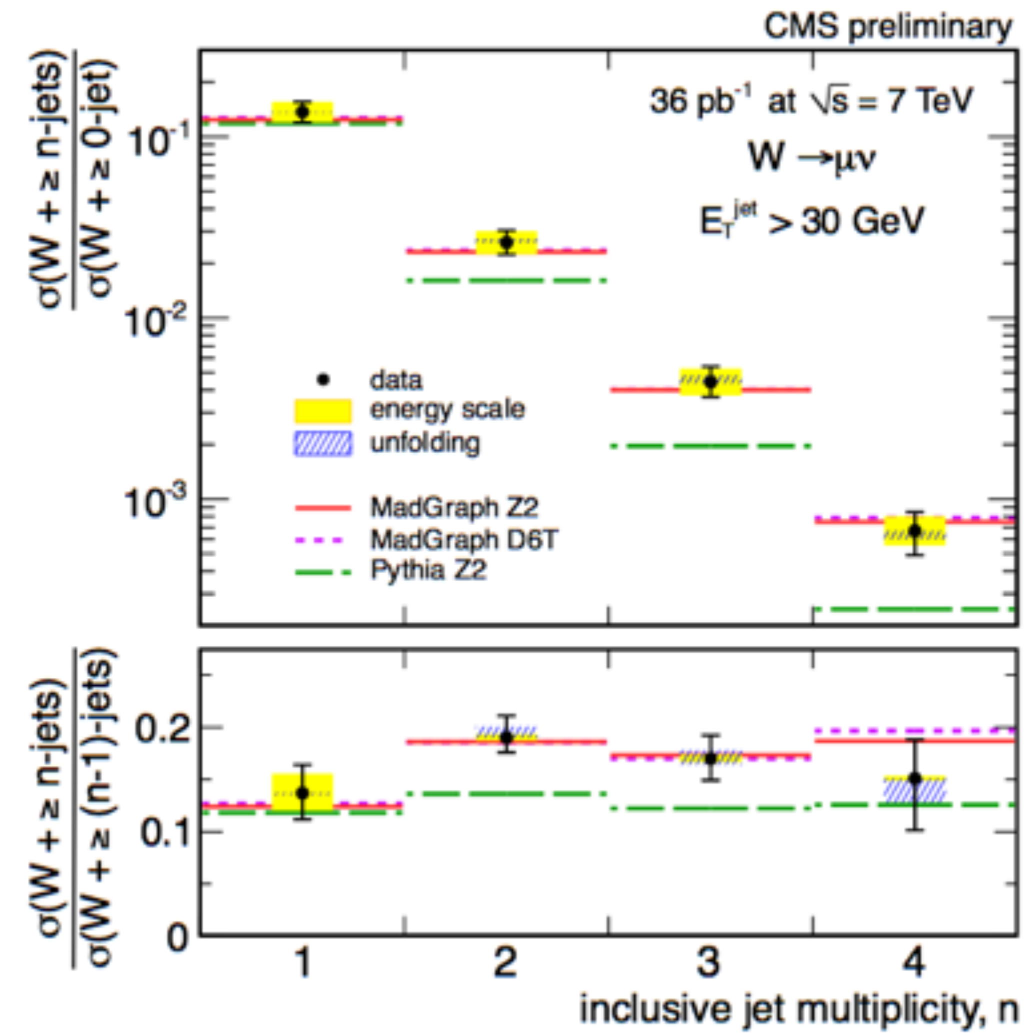
2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Summary of Matching Procedure

1. Generate ME events (with different parton multiplicities) using parton-level cuts ($p_T^{ME}/\Delta R$ or k_T^{ME})
2. Cluster each event and reweight α_s and PDFs based on the scales in the clustering vertices
3. Apply Sudakov factors to account for the required non-radiation above clustering cutoff scale and generate parton shower emissions below clustering cutoff:
   a. (CKKW) Analytical Sudakovs + truncated showers
   b. (CKKW-L) Sudakovs from truncated showers
   c. (MLM) Sudakovs from reclustered shower emissions

Comparing to experiment: W+jets





- Very good agreement at Tevatron (left) and LHC (right)

- Matched samples obtained via different matching schemes (MLM and CKKW) consistent within the expected uncertaintes.

- Pure parton shower (Pythia) doesn't describe the data beyond 1st jet.

Example: Simulation of pp→W with 0, 1, 2 jets (comfortable on a laptop)

```
mg5> generate p p > w+, w+ > l+ vl @0
mg5> add process p p > w+ j, w+ > l+ vl @1
mg5> add process p p > w+ j j, w+ > l+ vl @2
mg5> output
```

```
In run_card.dat:
…
  1 = ickkw
…
  0 = ptj
…
 15 = xqcut
```

- ickkw: Matching on
- ptj: No cone matching
- xqcut: k_T matching scale

Matching automatically done when run through MadEvent and Pythia!

matching in MadGraph+Pythia

- By default, k_T-MLM matching is run if xqcut > 0, with the matching scale QCUT = max(xqcut*1.4, xqcut+10)
- For shower-kT, by default QCUT = xqcut
- If you want to change the Pythia setting for matching scale or switch to shower-k_T matching:

```
In pythia_card.dat:
…
! This sets the matching scale, needs to be > xqcut
QCUT = 30
! This switches from kT-MLM to shower-kT matching
! Note that MSTP(81)>=20 needed (pT-ordered shower)
SHOWERKT = T
```

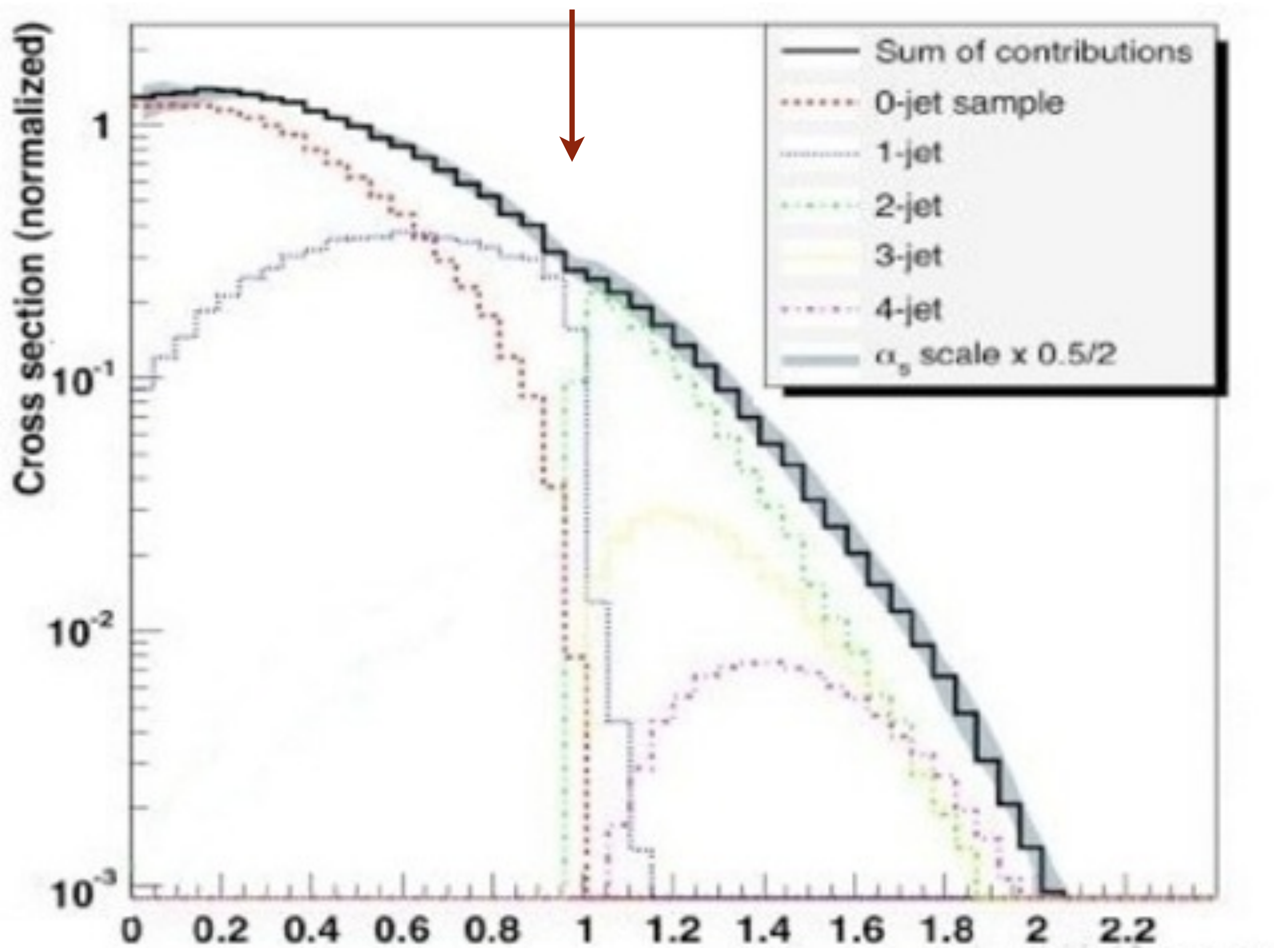
How to do validate the matching

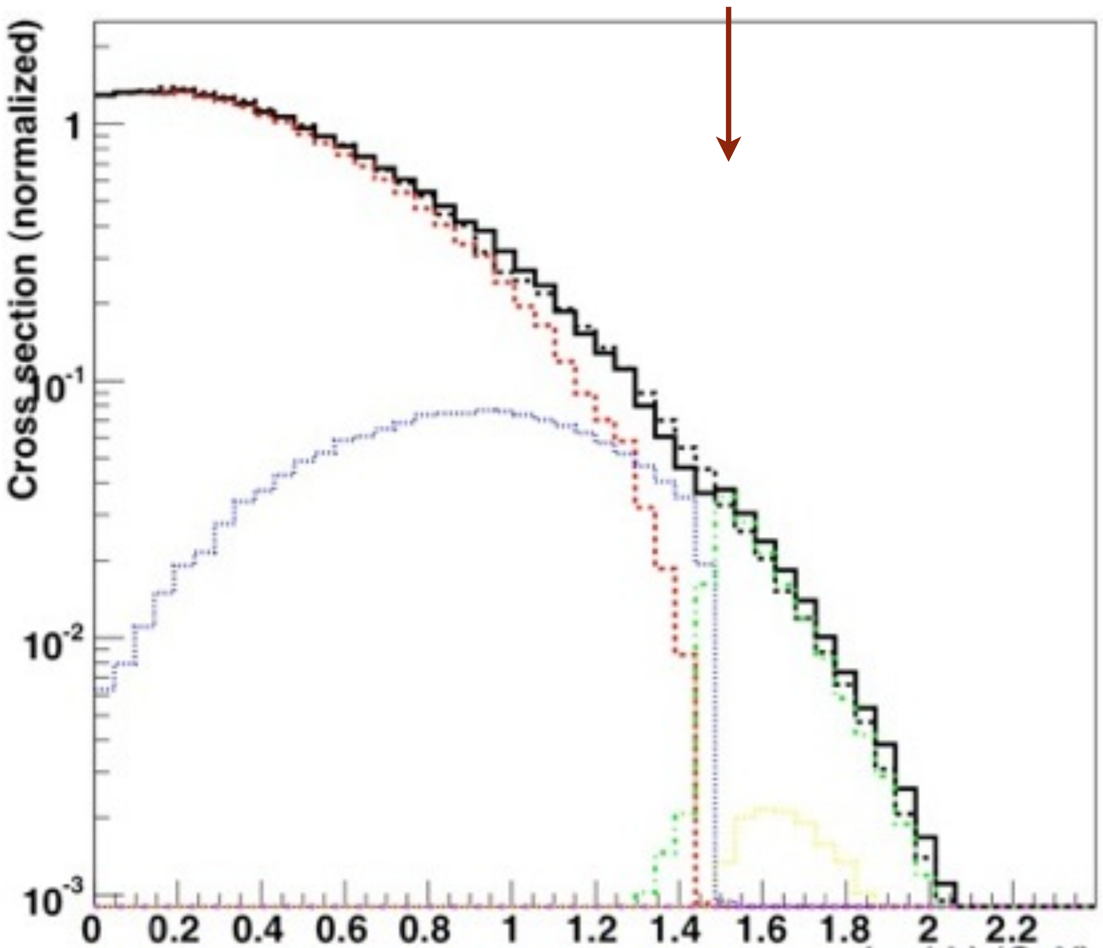
- The matching scale (QCUT) should typically be chosen around 1/6-1/2 x hard scale (so xqcut correspondingly lower)
- The matched cross section (for X+0,1,... jets) should be close to the unmatched cross section for the 0-jet sample (found on the process HTML page)
- The differential jet rate plots should be smooth
- When QCUT is varied (within the region of validity), the matched cross section or differential jet rates should not vary significantly

Matching validation

W+jets production at the Tevatron for MadGraph+Pythia
(k_T-jet MLM scheme, q^2-ordered Pythia showers)

Q^{match} = 10 GeV Q^{match} = 30 GeV





log(Differential jet rate for 1 → 2 radiated jets ~ pT(2nd jet))

Jet distributions smooth, and stable when we vary the matching scale!

Matching Summary

- Despite the apparent enormous complexity of simulation of complete collider events, nature has kindly allowed us to factorize the simulation into separate steps
- The Monte Carlo method allows us to step-by-step simulate hard scattering, parton shower, particle decays, hadronization, and underlying event
- Jet matching between matrix elements and parton showers gives crucial improvement of simulation of background as well as signal processes
- Running matching with MadGraph + Pythia is very easy, but the results should always be checked for consistency