UNIVERSITÀ DEGLI STUDI DI MILANO

LHC Phenomenology with MadGraph

Three introductory lectures

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A simple plan

- Intro: the LHC challenge
- Tree-level matrix elements
- Parton-level cross sections and events
- Events at the LHC

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- Tree-level matrix elements
- Parton-level cross sections and events — now
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Master QCD formula

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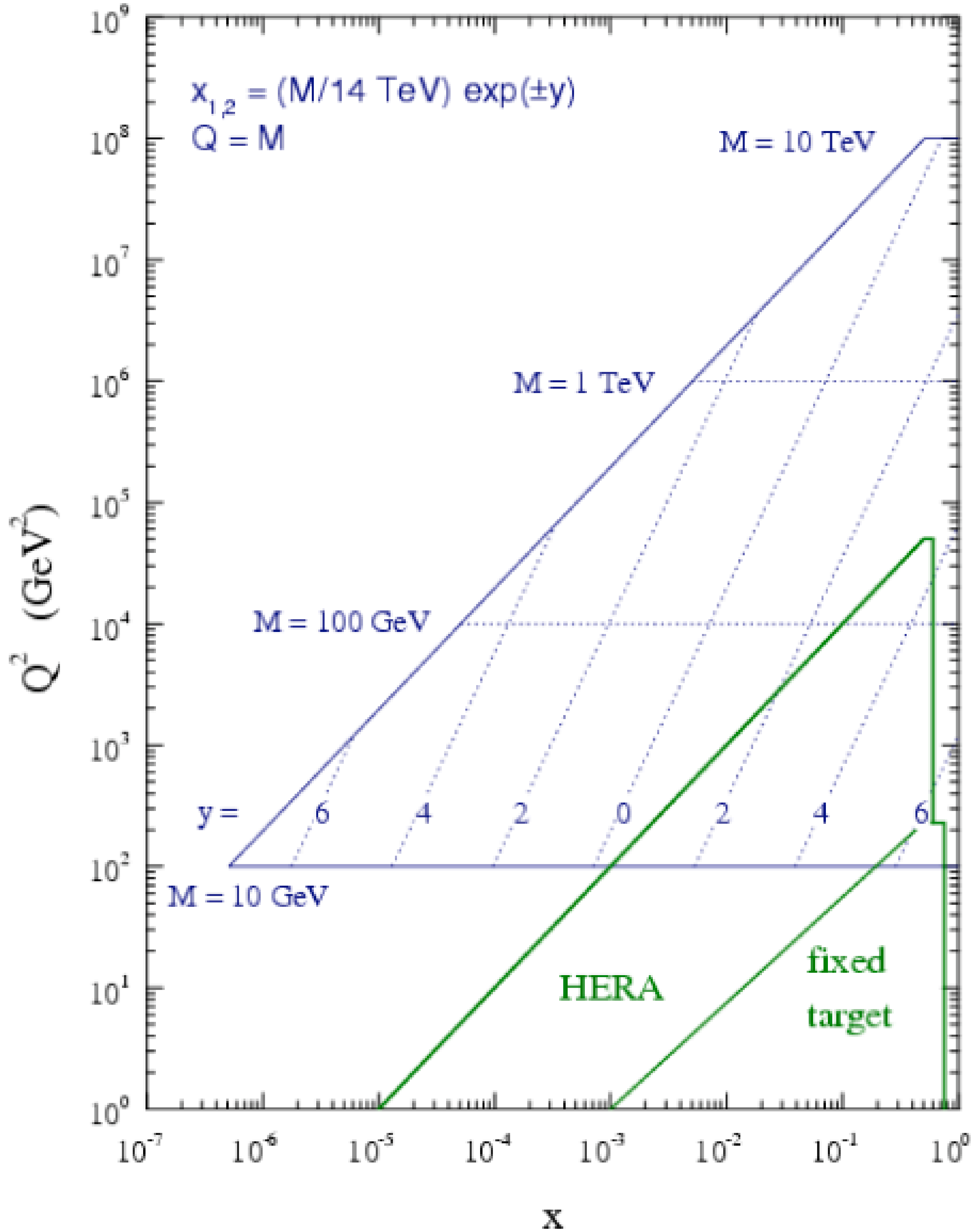
Two ingredients necessary:

1. Parton Distribution functions (from exp, but evolution from th).

Progress in the PDF

PDF measured at HERA and fixed-target experiments. x dependence from data.
Q^2 dependence from DGLAP evolution.



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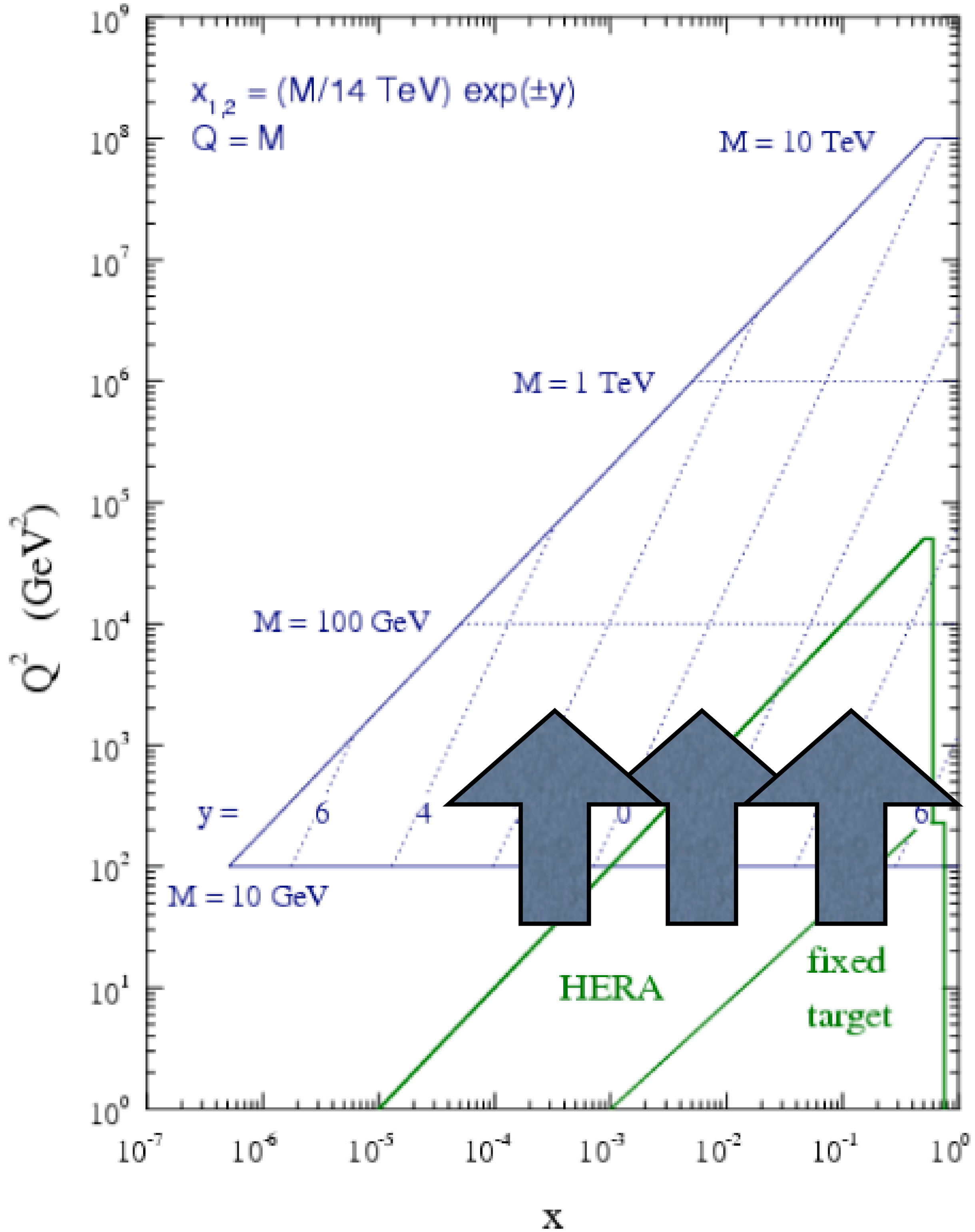

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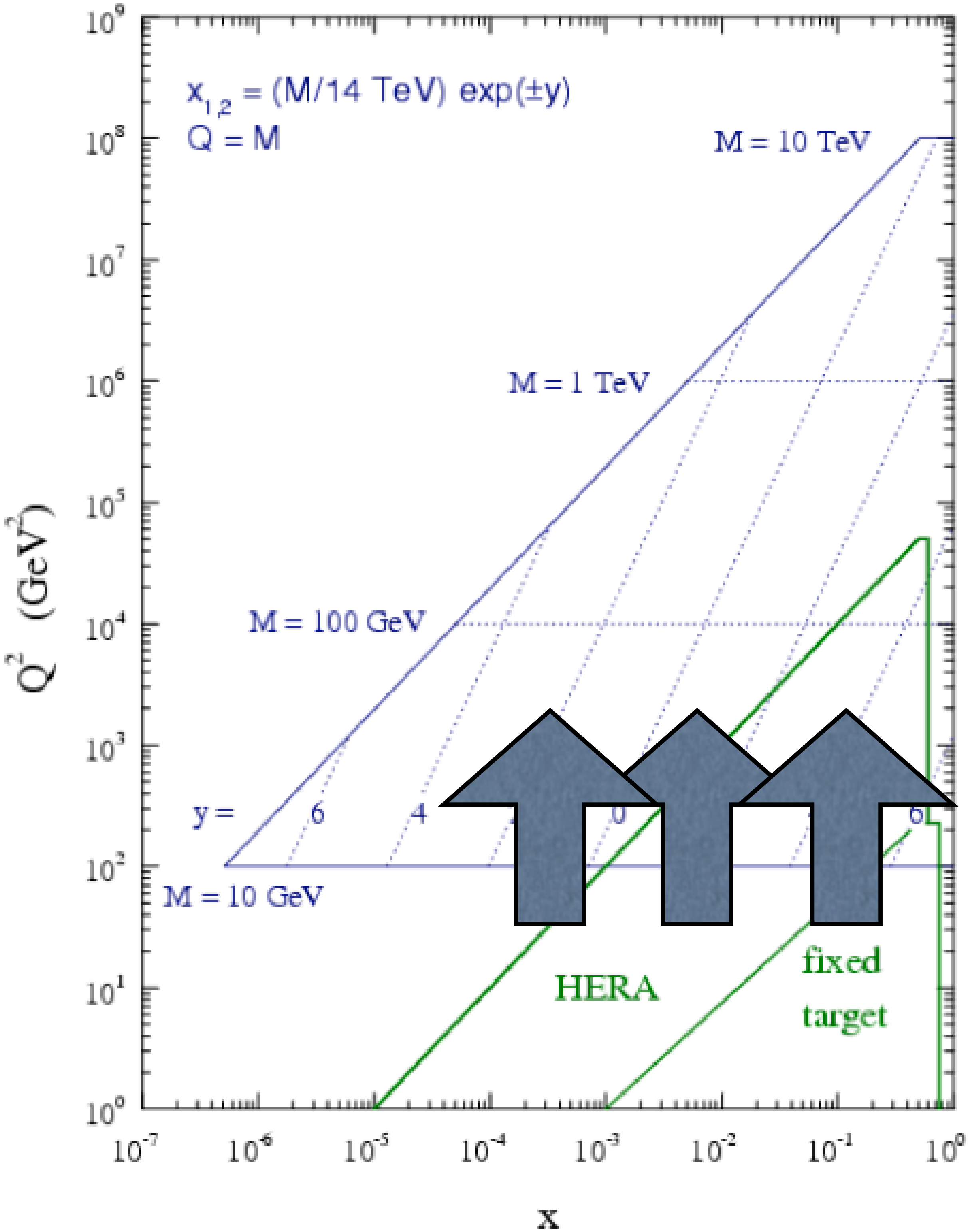
Recently:

NNLO calculation of the 3-loop splitting kernels ("the hardest calculation in QCD") [Moch, Vermaseren, Vogt. 2004]

Together with short distance NNLO calculation first sets of NNLO PDF sets. [MRST and Alekhin, 2004]

PDF's with errors: Various "traditional methods", [CTEQ and MRST, 2003]. Also new approaches, the functional space [Giele, Keller, Kosower.2001] and the Neural Network (NNPDF) approach [Del Debbio, Forte, La Torre, Piccione, Rojo. 2002,2005].



Issues:
1. small-x effects
2. Heavy flavors pdf

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Two ingredients necessary:

1. Parton Distribution functions (from exp, but evolution from th).
2. Short distance coefficients as an expansion in α_S (from th).

$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Leading order

Next-to-leading order

Next-to-next-to-leading order

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What's a matrix-element based generator?

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- Matrix element calculators provide our first estimation of rates for inclusive final states.

- Extra radiation is included: it is described by the PDF's in the initial state and by the definition of a final state parton, which at LO represents all possible final state evolutions.

- Due to the above approximations a cross section at LO can strongly depend on the factorization and renormalization scales.

- Any tree-level calculation for a final state F can be promoted to the exclusive F + X through a shower. However, a naive sum of final states with different jet multiplicities would lead to double counting.

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General and flexible method is needed

Phase Space

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$$d\Phi_n = \left[\Pi_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 (2E_i)}\right] (2\pi)^4 \delta^{(4)}(p_0 - \sum_{i=1}^{n} p_i)$$

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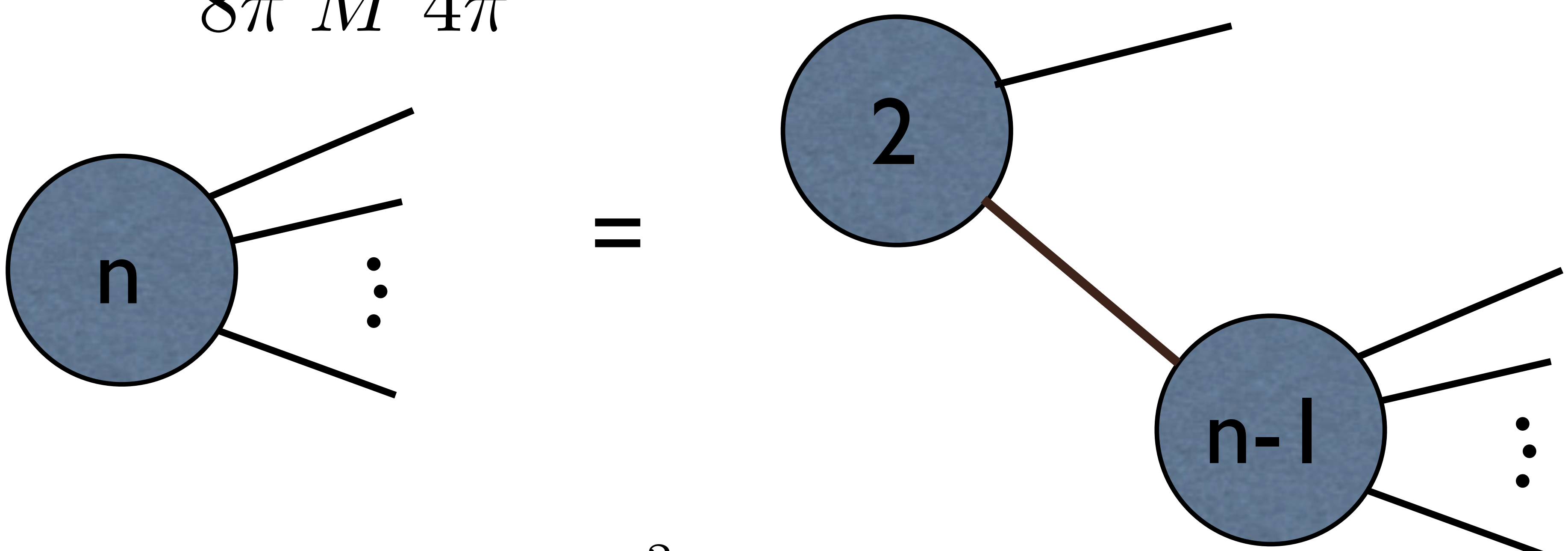

$$d\Phi_n(M) = \frac{1}{2\pi}\int_0^{(M-\mu)^2} d\mu^2 d\Phi_2(M) d\Phi_{n-1}(\mu)$$

Integrals as averages

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$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

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- ☞ Convergence is slow but it can be estimated easily
- ☞ Error does not depend on # of dimensions!
- ☞ Improvement by minimizing V_N.
- ☞ Optimal/Ideal case: $f(x)=C \Rightarrow V_N=0$

Importance Sampling

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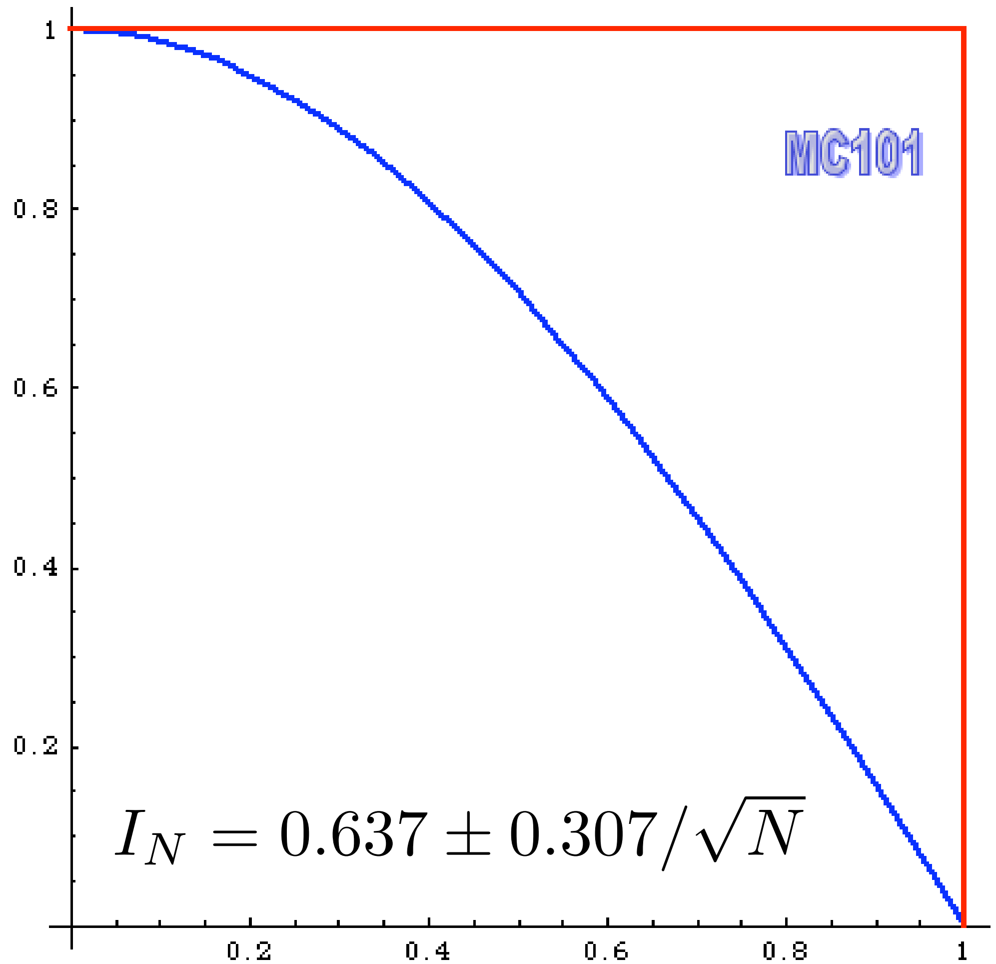

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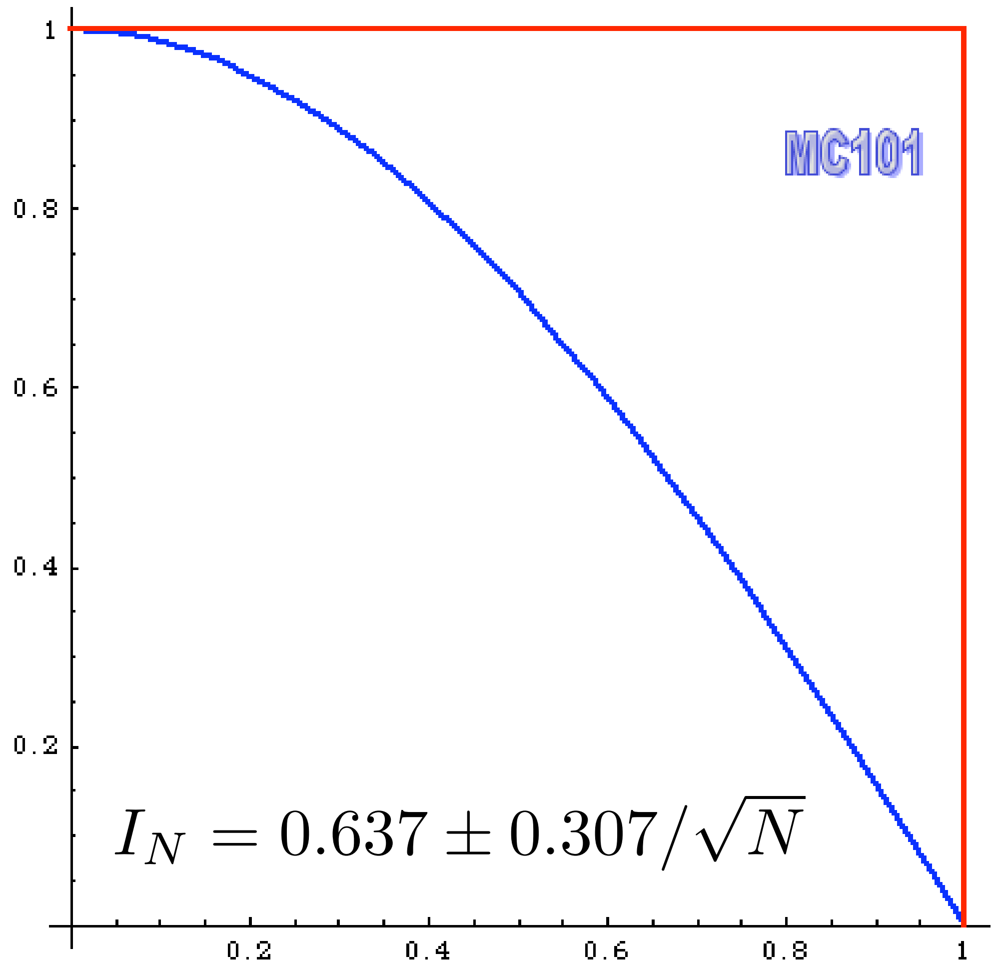

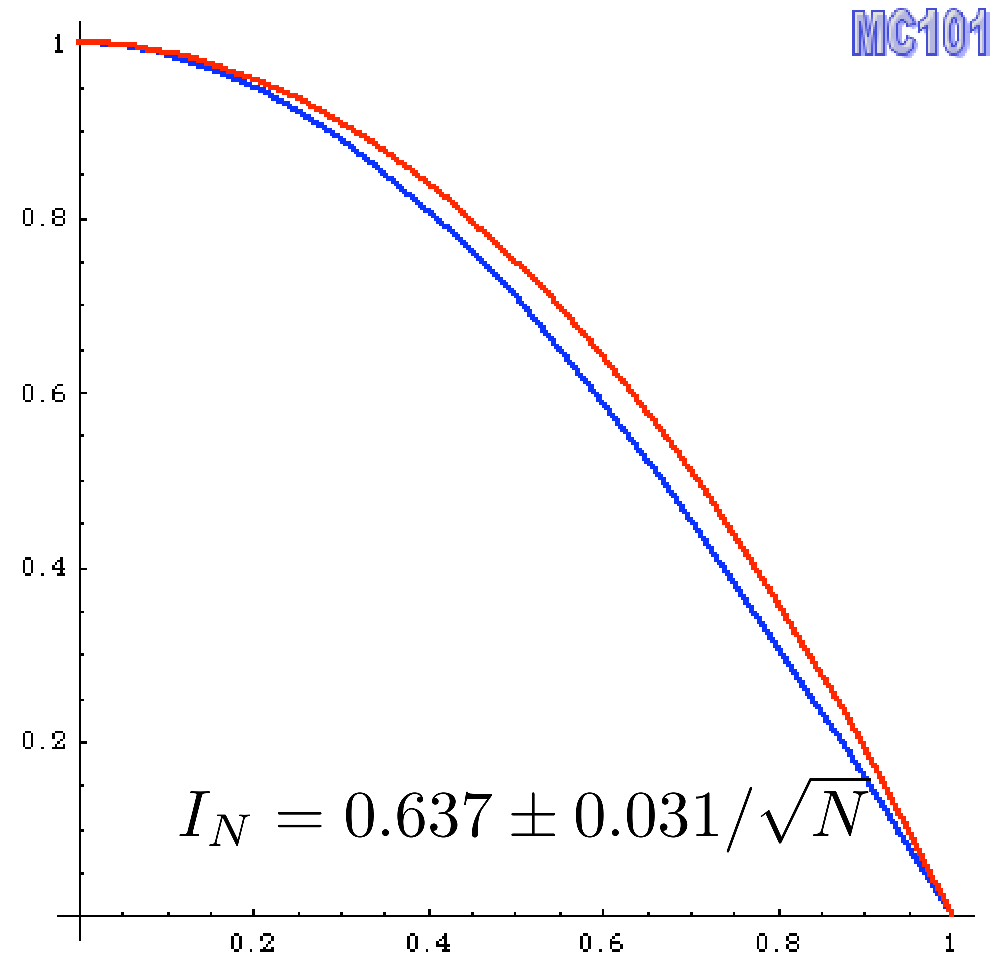

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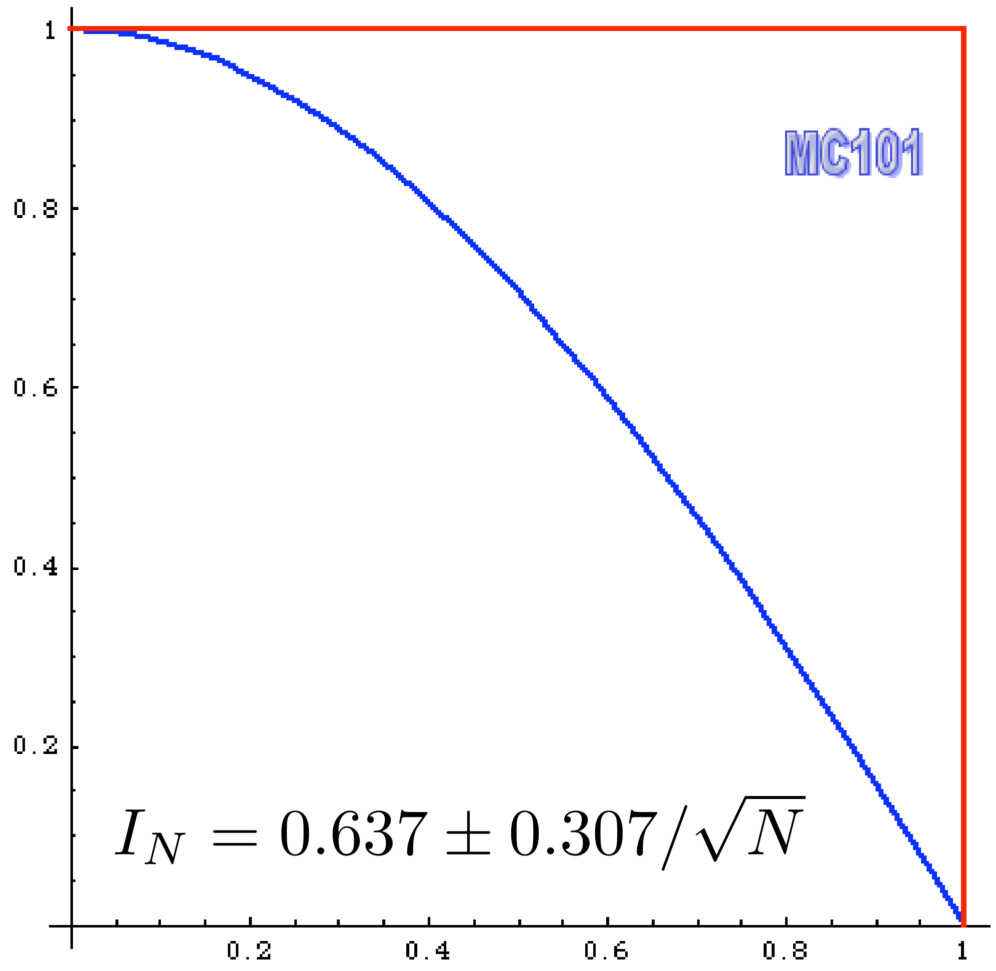

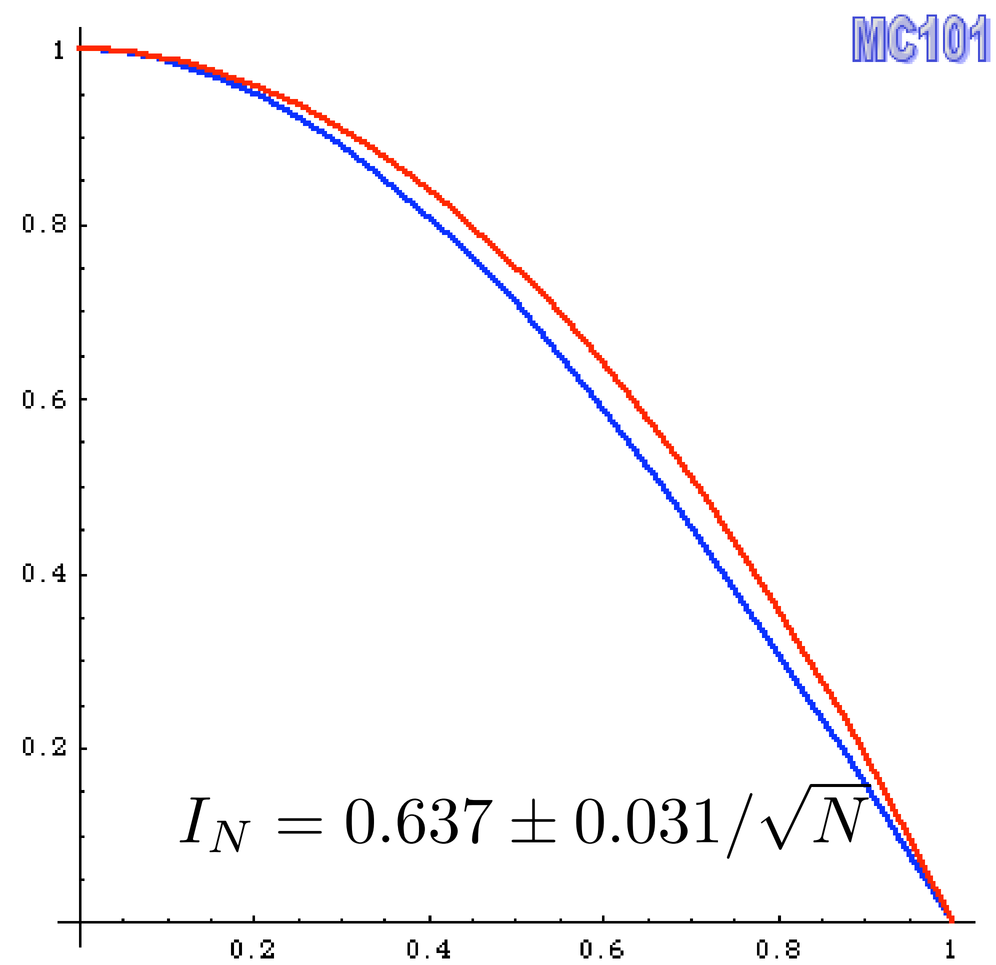

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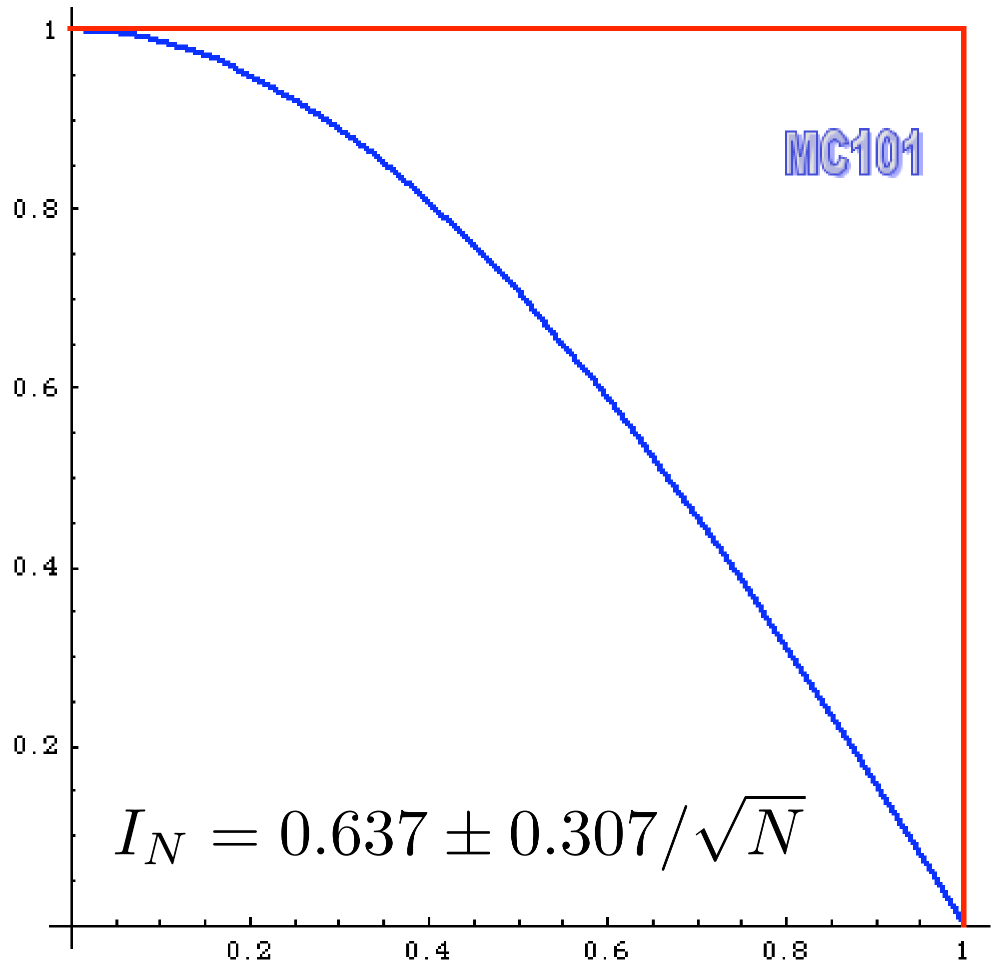

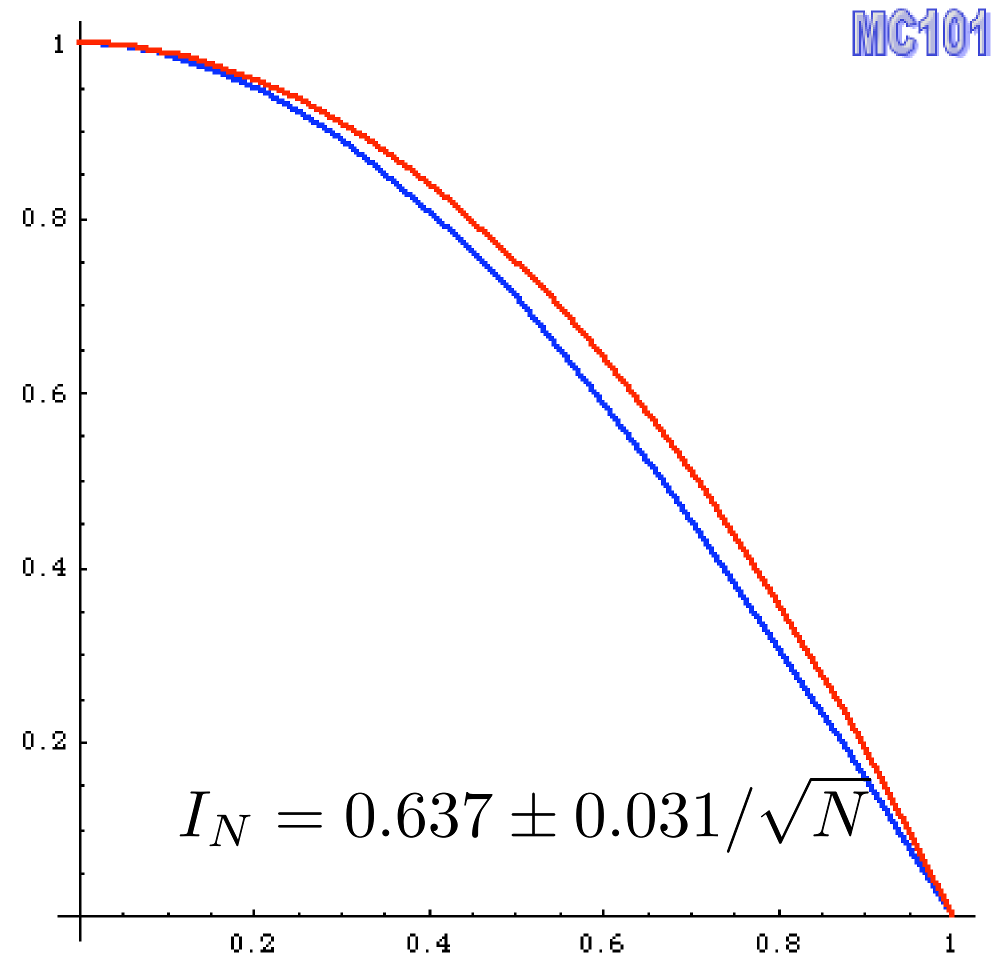

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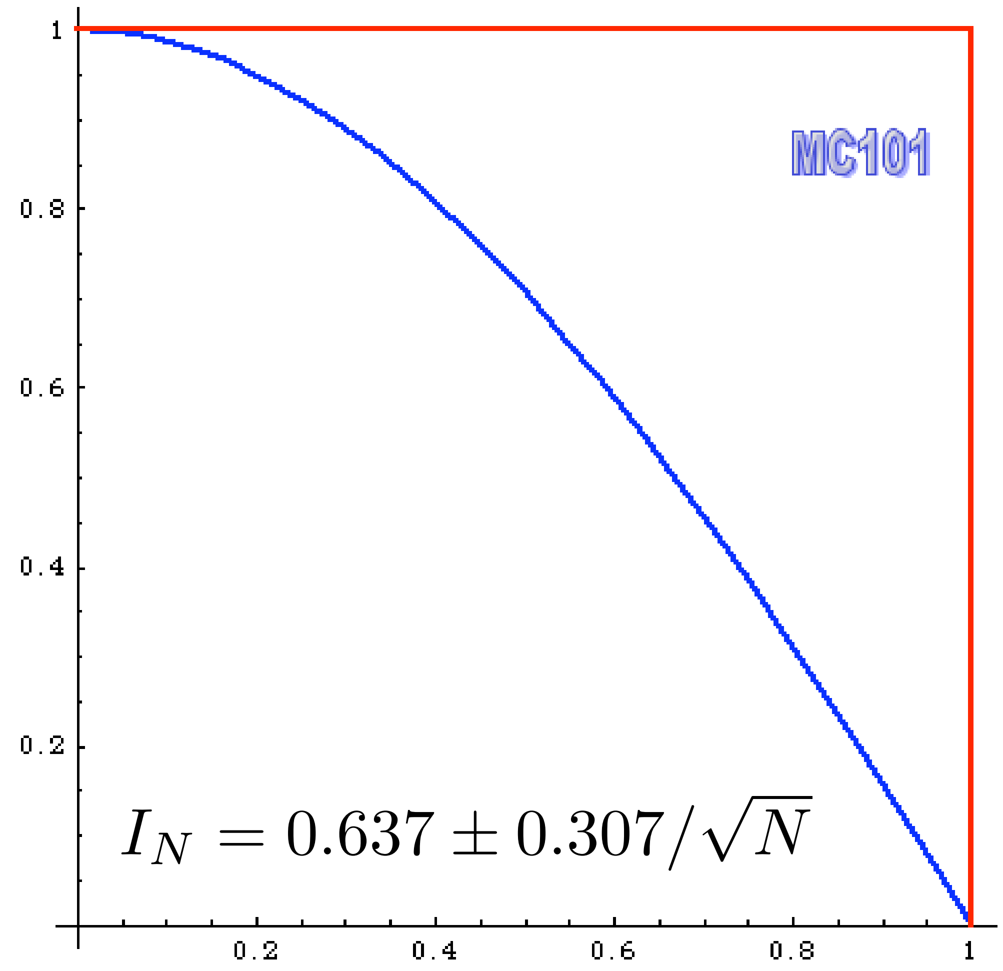

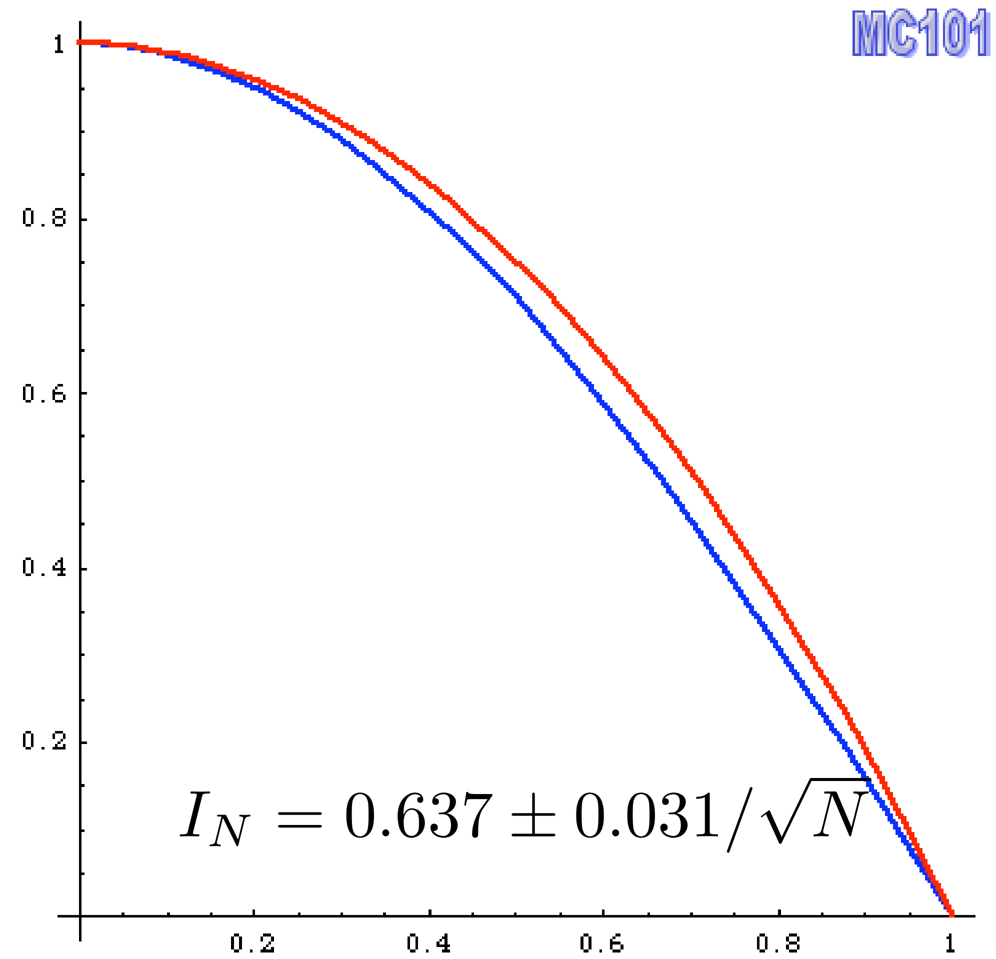

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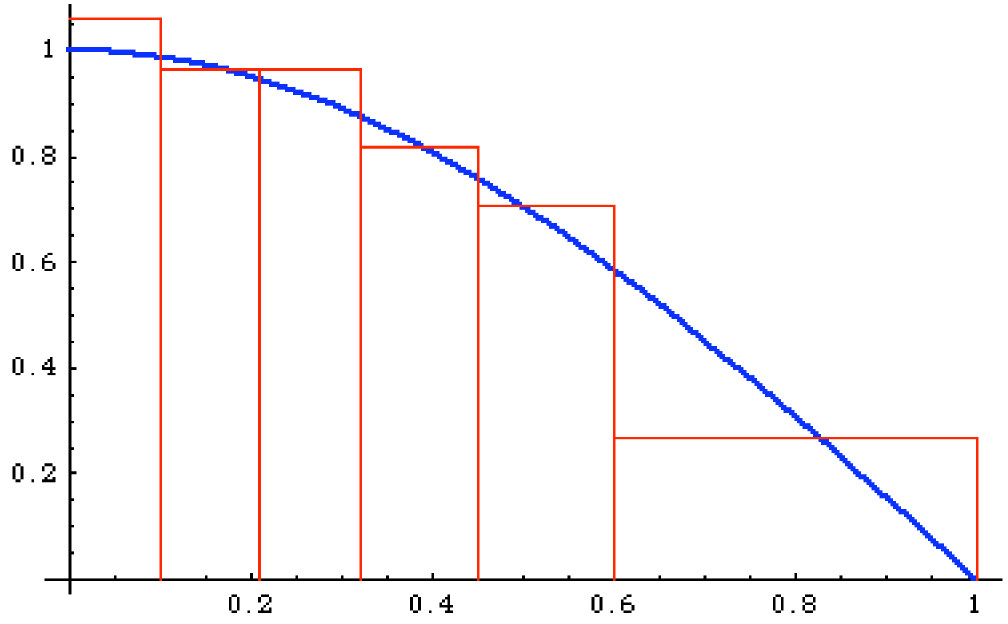

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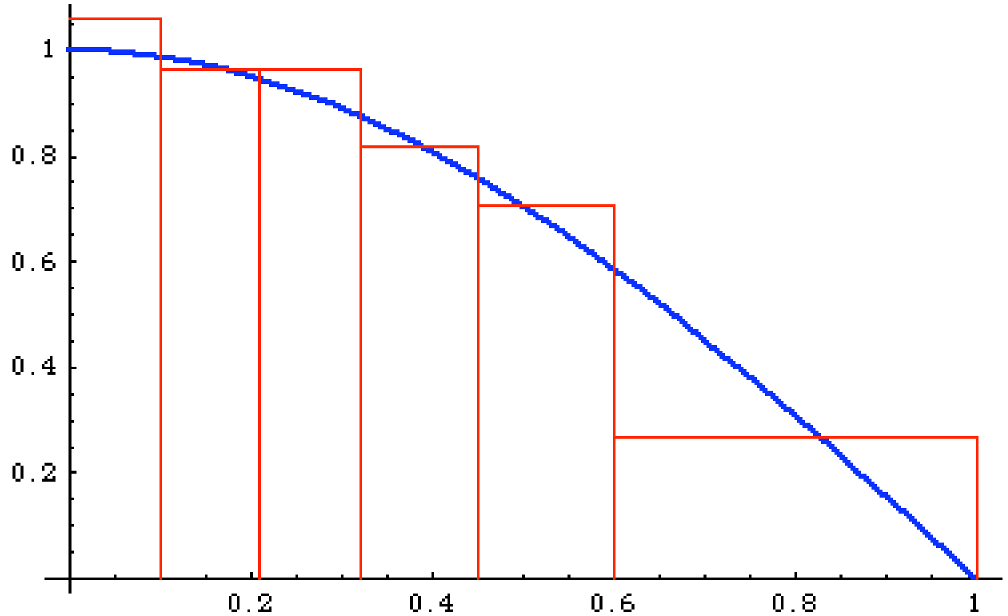

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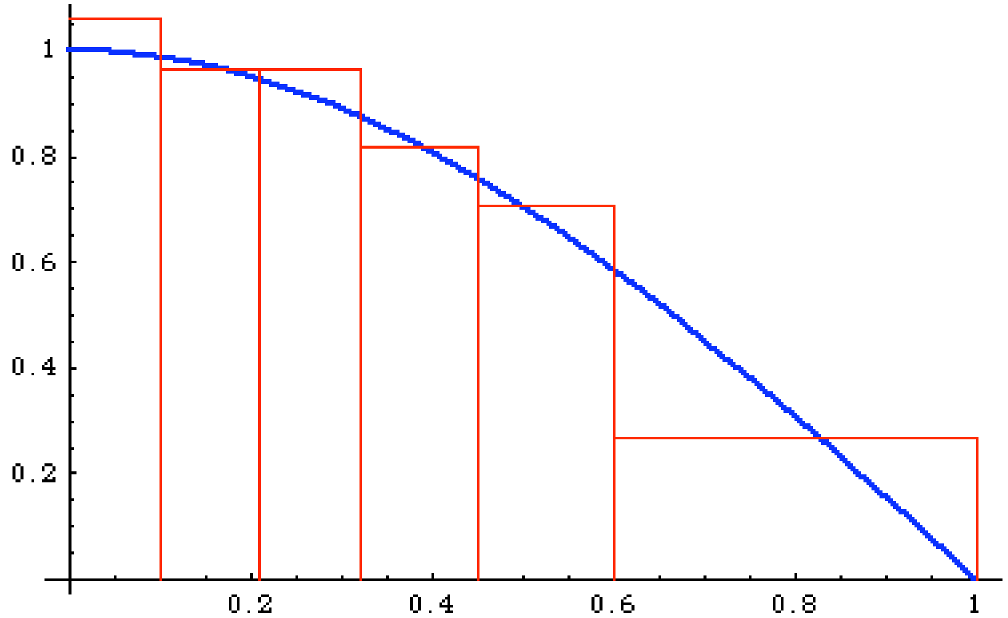

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$$p(x) = \frac{1}{N_b \Delta x_i}, \quad x_i - \Delta x_i < x < x_i$$

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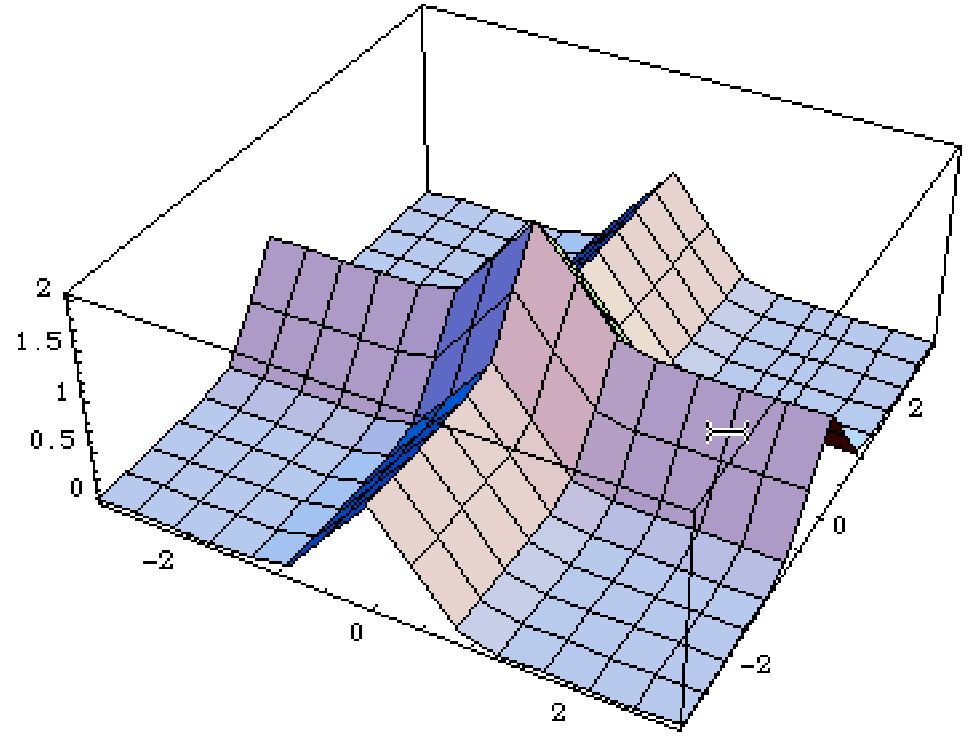

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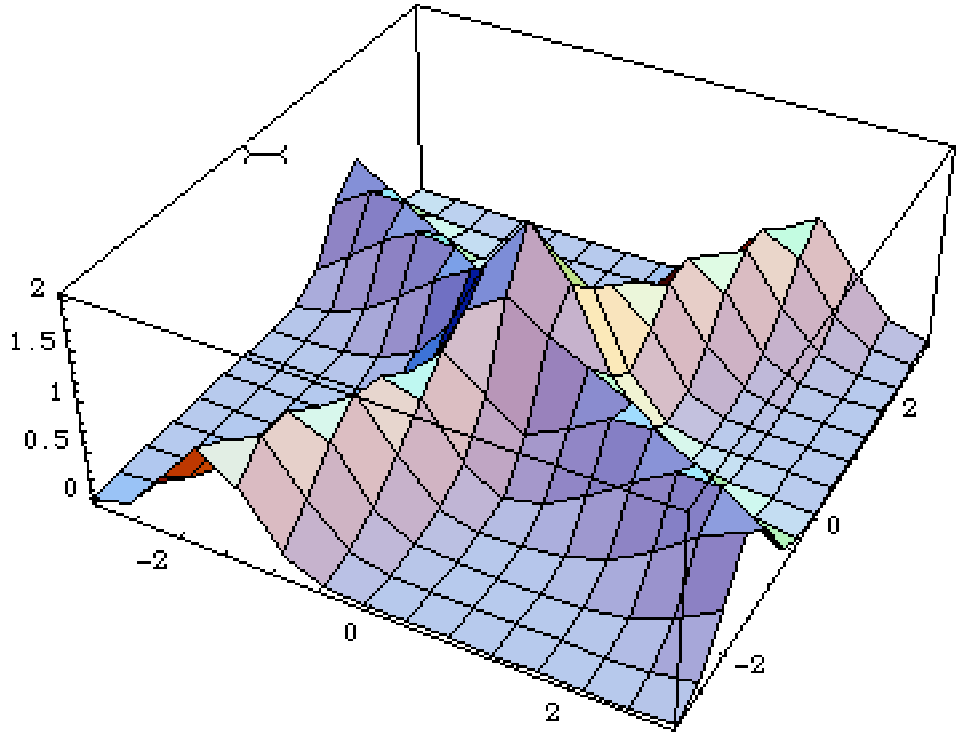

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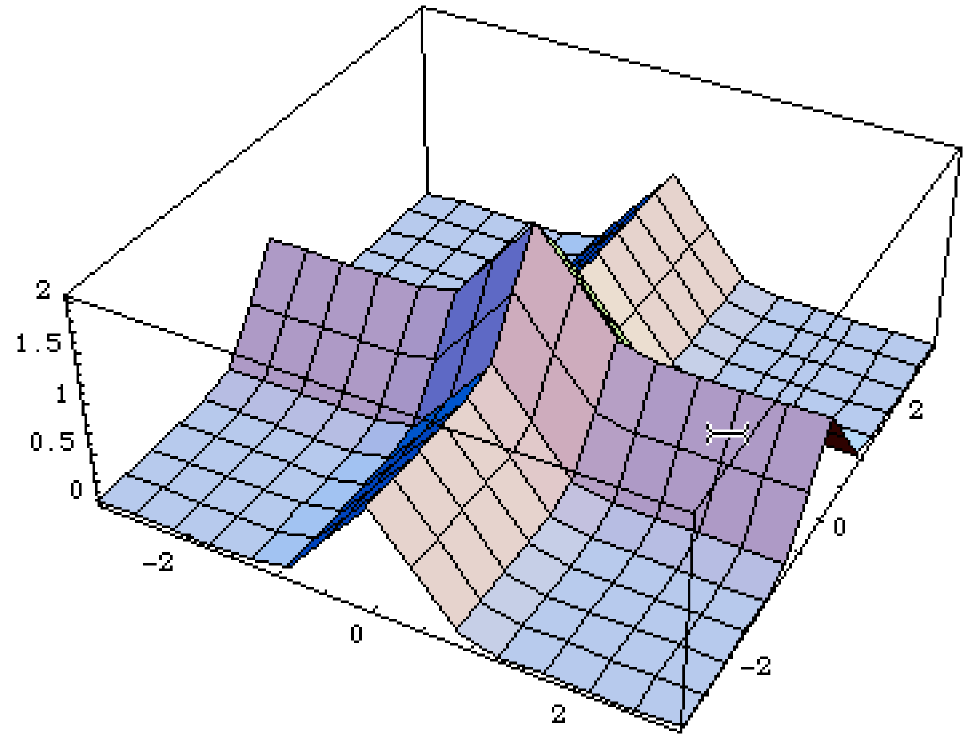

but it is sufficient to make a change of variables!

Multi-channel

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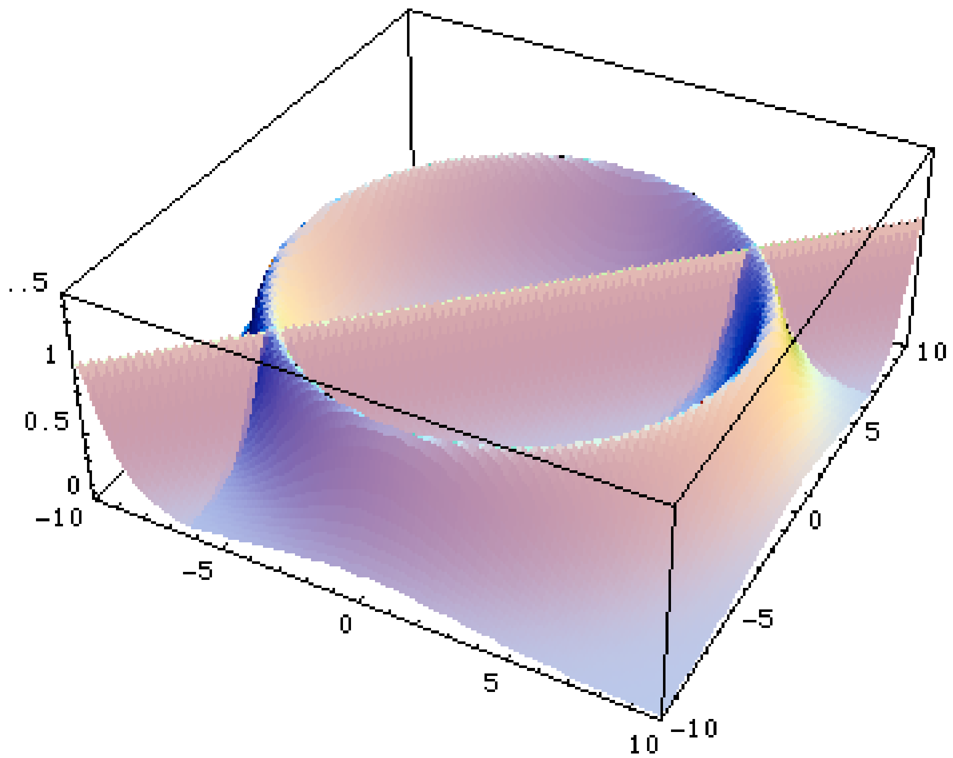

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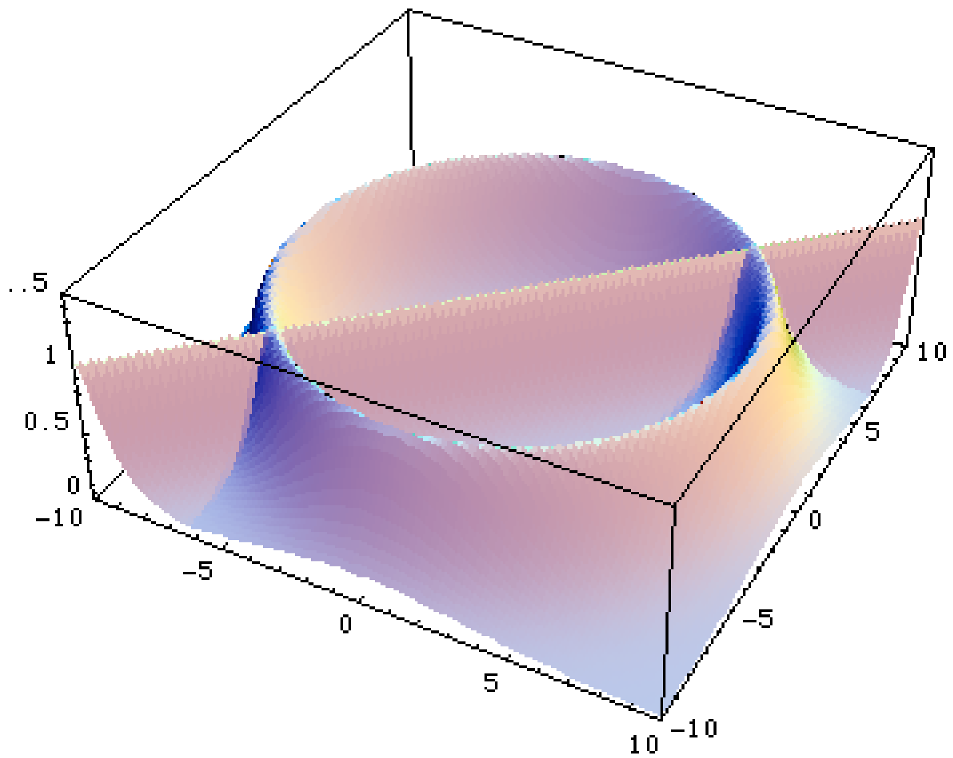

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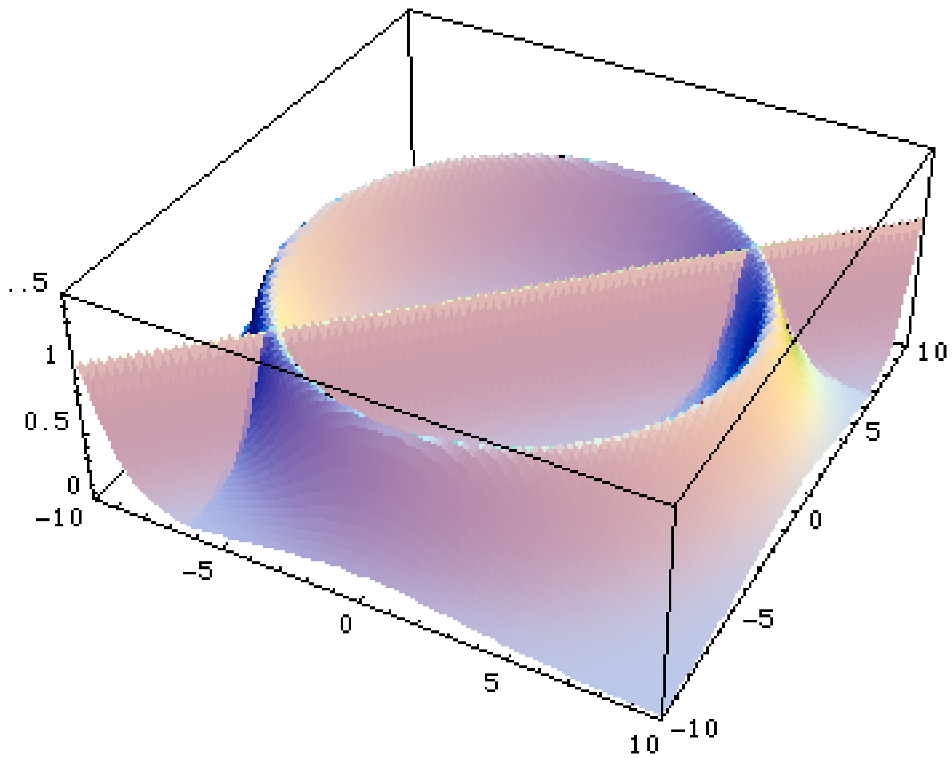

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Solution: use different transformations= channels

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

with each $p_i(x)$ taking care of one "peak" at the time

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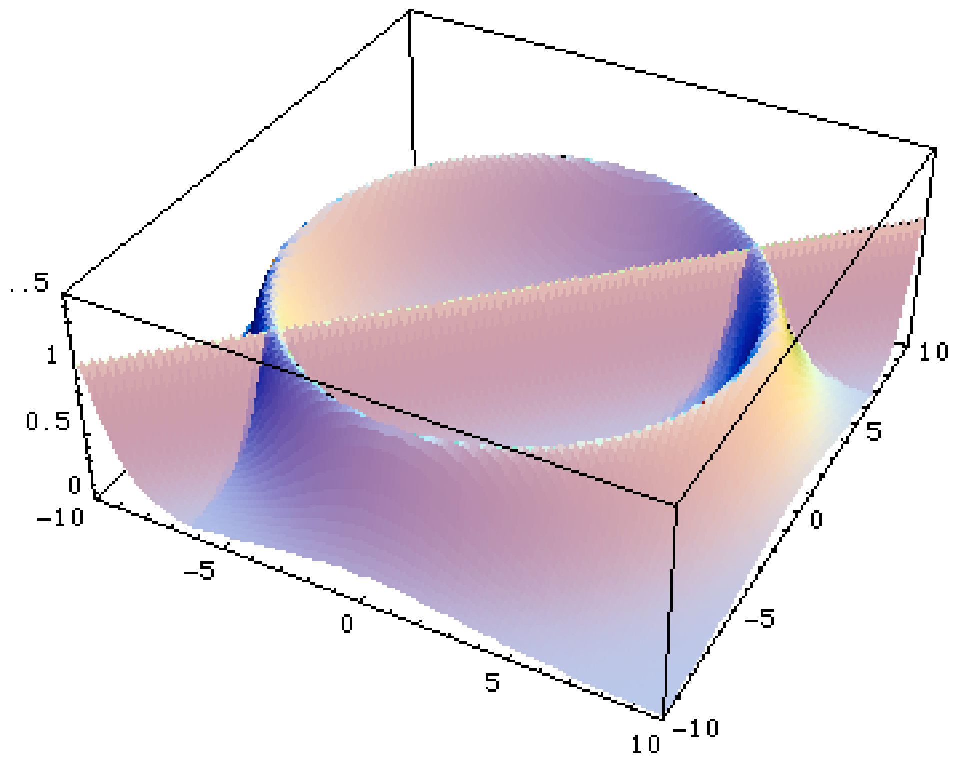

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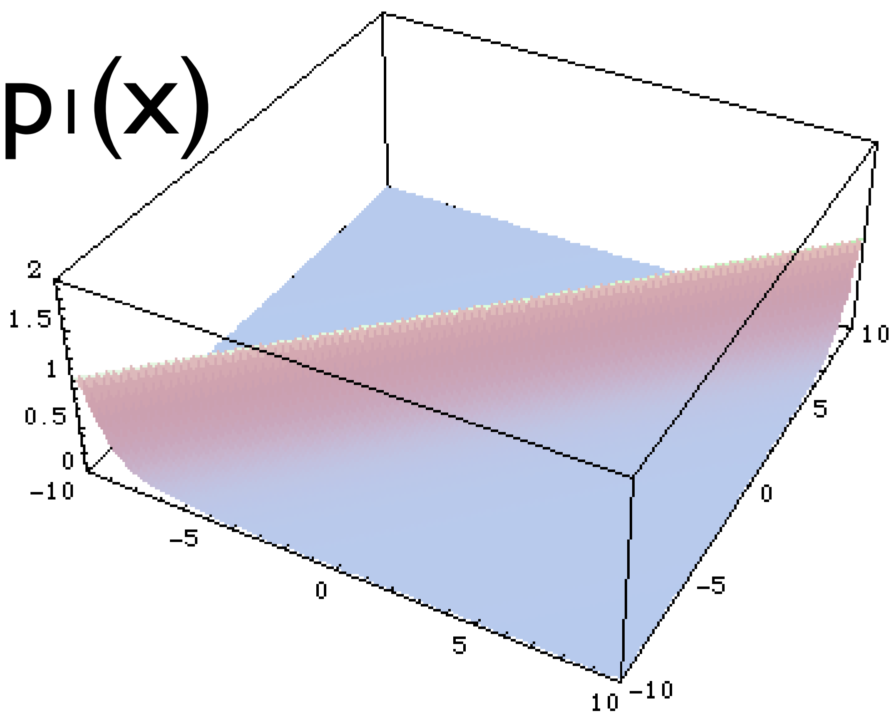
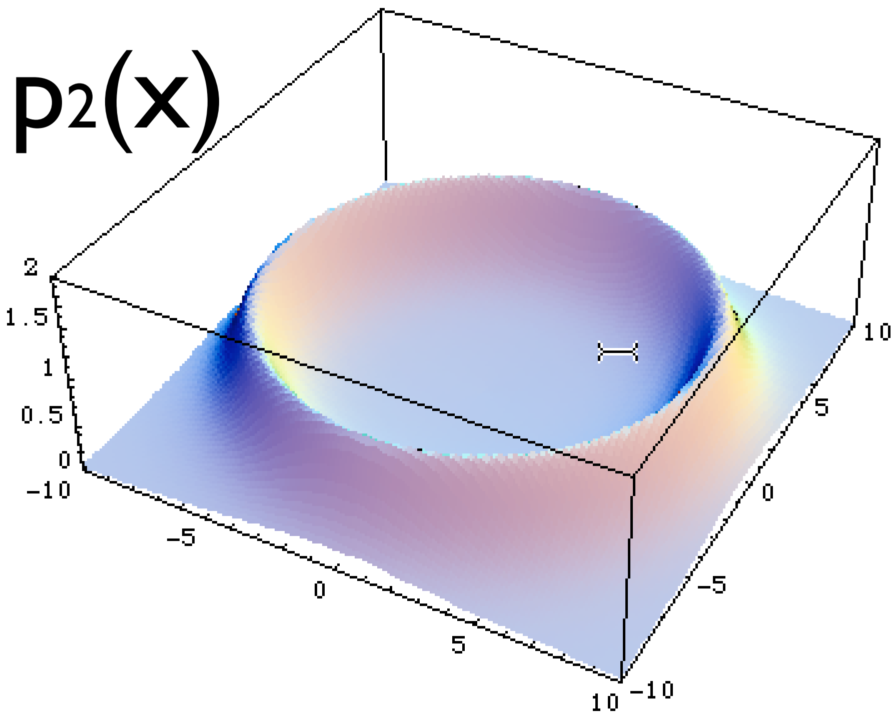


$p_2(x)$



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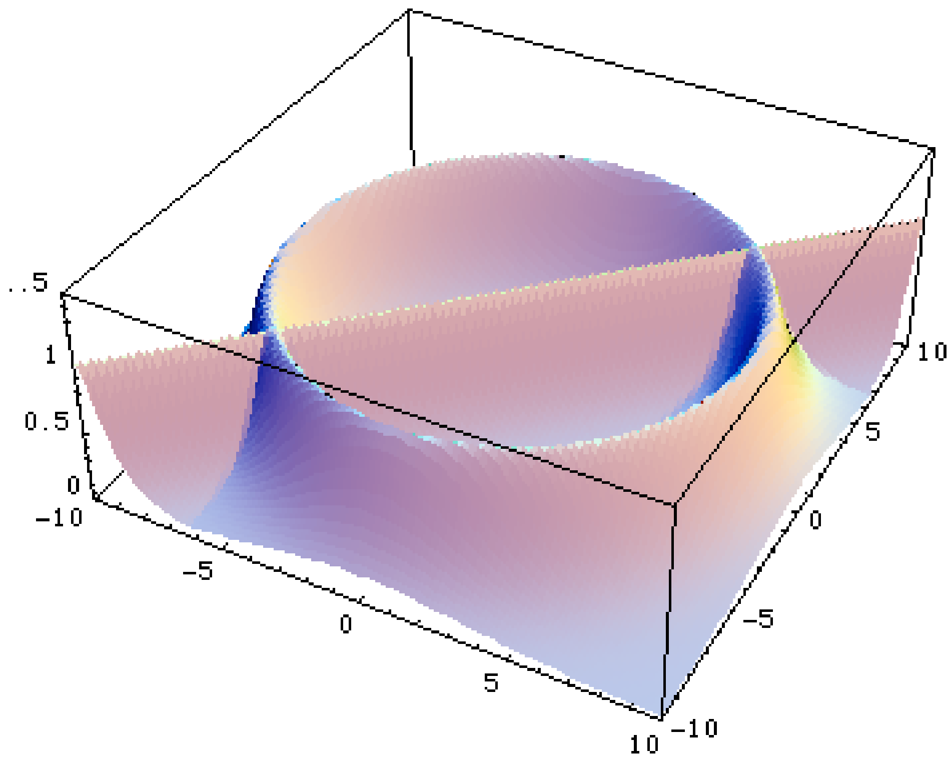

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But if you know where the peaks are (=in which variables) we can use different transformations= channels:

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- Advantages
  - The integral does not depend on the α_i but the variance does and can be minimised by a careful choice
- Limitations
  - Need to calculate all g_i values for each point
  - Each phase space channel must be invertible
  - N coupled equations for α_i so it might only work for small number of channels

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Very commonly used method!

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$$I = \int d\vec{\Phi} f(\vec{\Phi}) = \sum_{i=1}^{n} \int d\vec{\Phi}\, g_i(\vec{\Phi})\, \frac{f_i(\vec{\Phi})}{g_i(\vec{\Phi})} = \sum_{i=1}^{n} I_i$$

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The MadEvent method

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$$f = \sum_{i=1}^{n} f_i \qquad \text{with} \qquad f_i \geq 0\,, \qquad \forall\, i$$

such that:

1. we know how to integrate each one of them
2. they describe all possible peaks

$$I = \int d\vec{\Phi} f(\vec{\Phi}) = \sum_{i=1}^{n} \int d\vec{\Phi}\, g_i(\vec{\Phi})\, \frac{f_i(\vec{\Phi})}{g_i(\vec{\Phi})} = \sum_{i=1}^{n} I_i$$

Then the problem would be solved!

Does such a basis exist?

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YES!

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Then the problem would be solved!

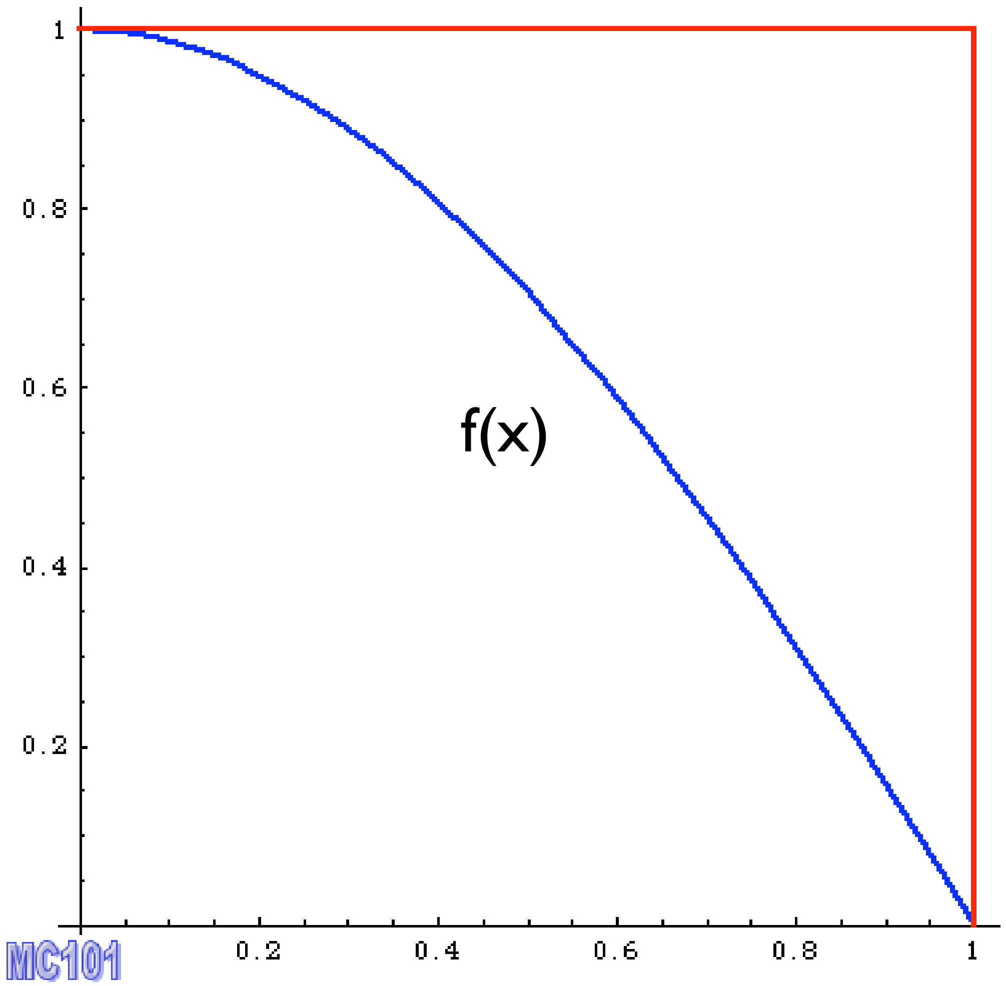
$$f_i = \frac{|A_i|^2}{\sum_i |A_i|^2} |A_{\rm tot}|^2$$

Does such a basis exist?

Single-Diagram-Enhanced technique

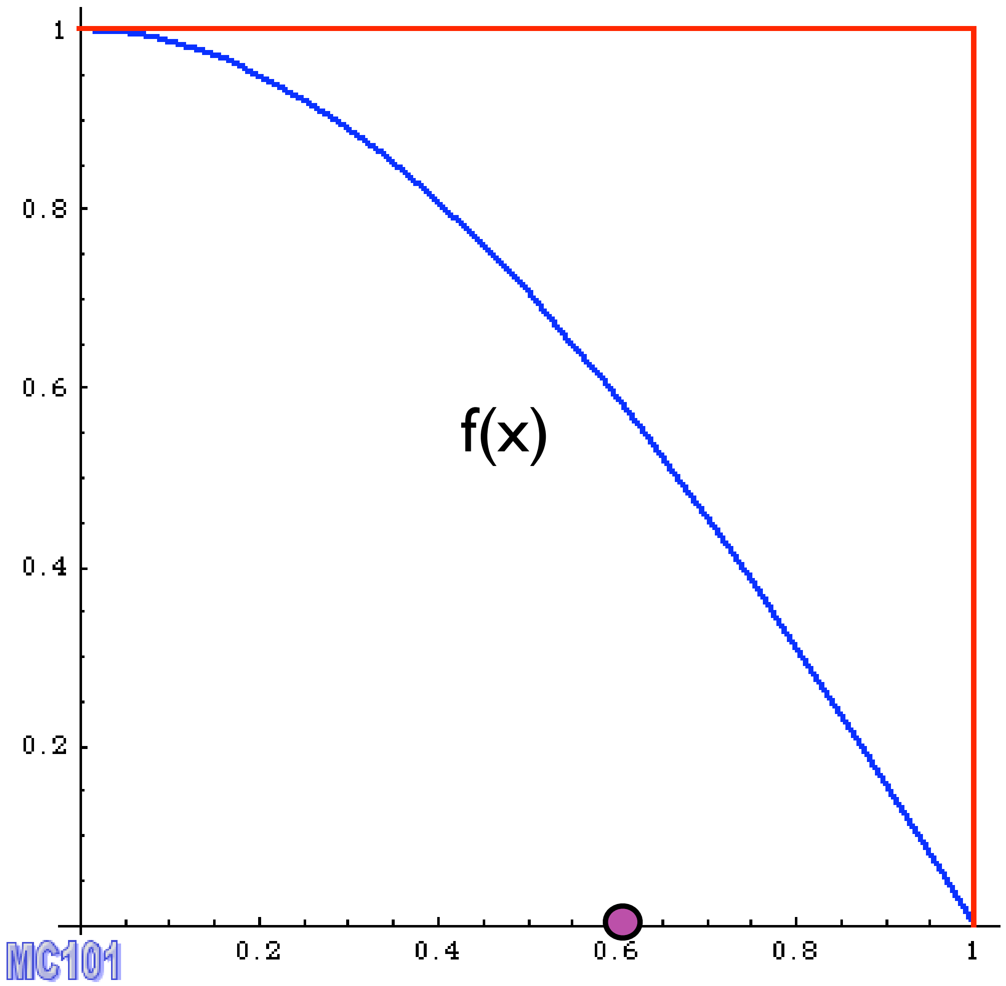
- Key Idea
  - Any single diagram is "easy" to integrate
  - Divide integration into pieces, based on diagrams
- Get N independent integrals
  - Errors add in quadrature so no extra cost
  - No need to calculate "weight" function from other channels.
  - Can optimize # of points for each one independently
  - Parallel in nature
- What about interference?
  - Never creates "new" peaks, so we're OK!

Event generation



Alternative way

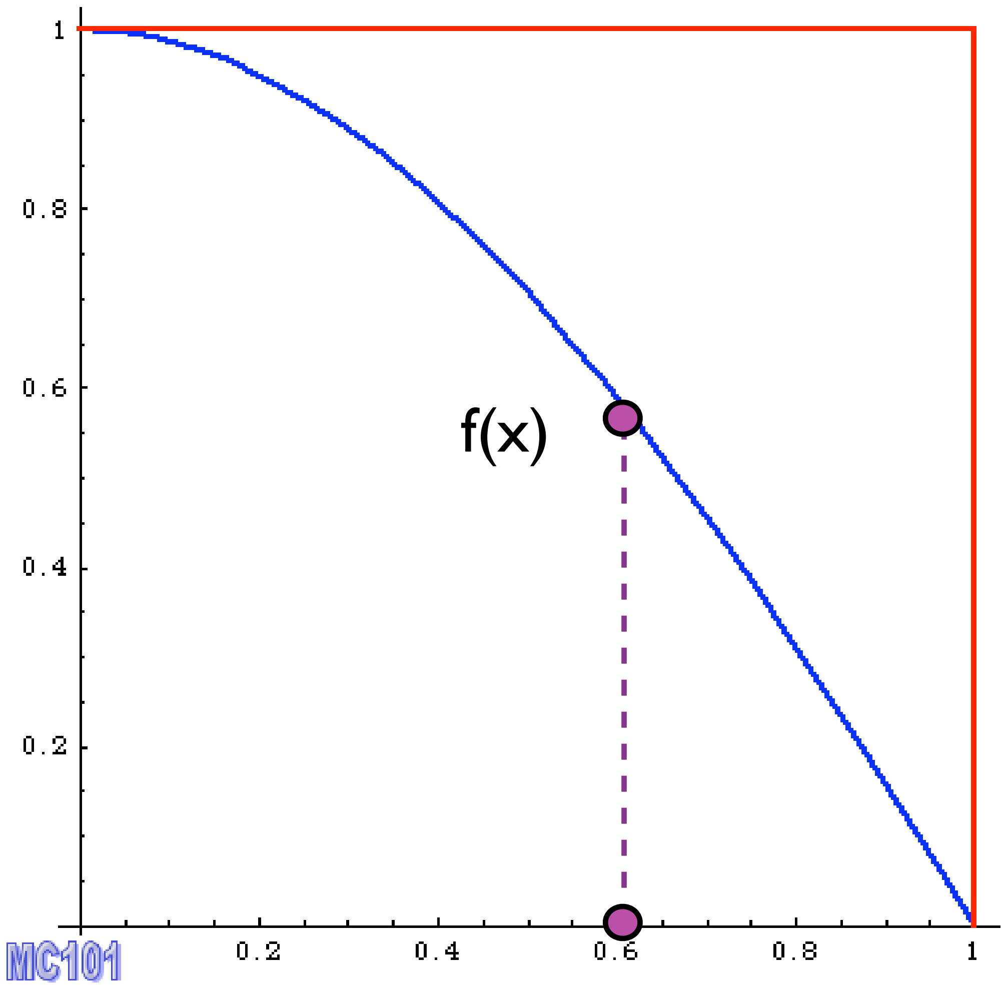
Event generation



Alternative way

1. pick x

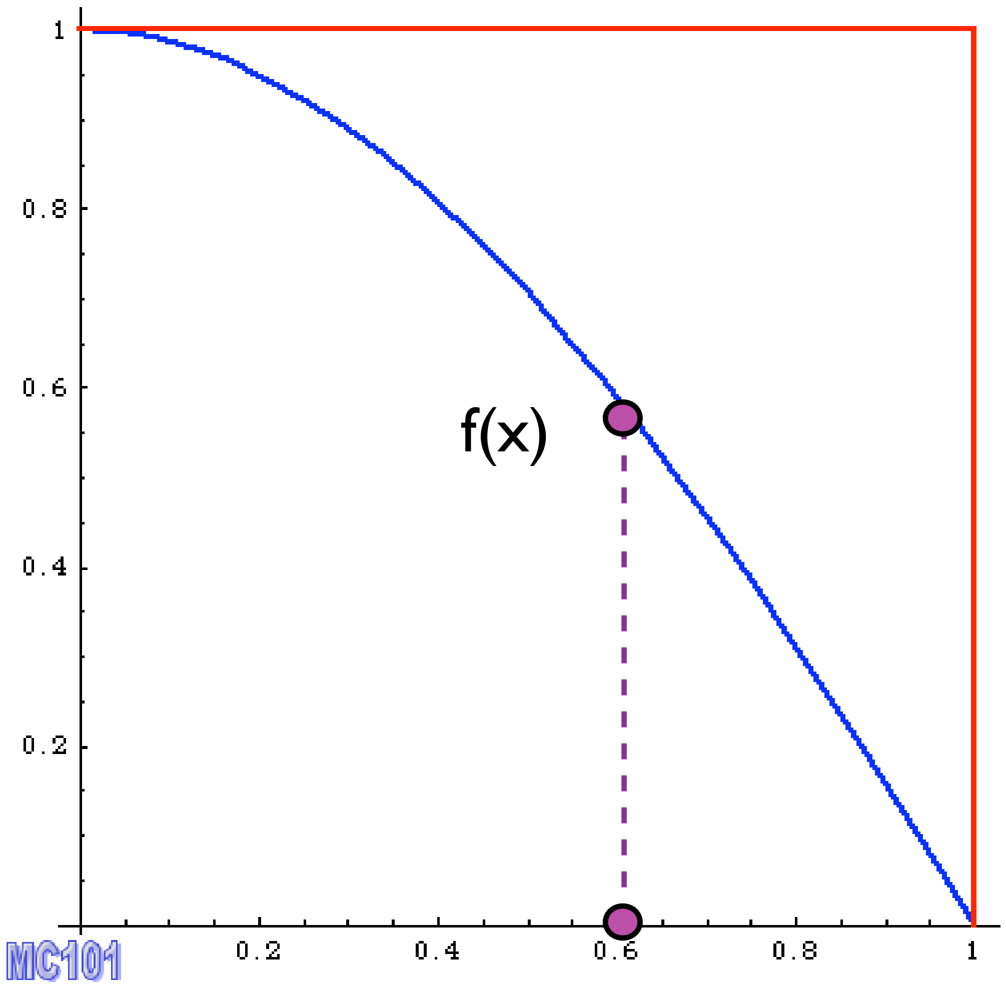
Event generation



Alternative way

1. pick x
2. calculate f(x)

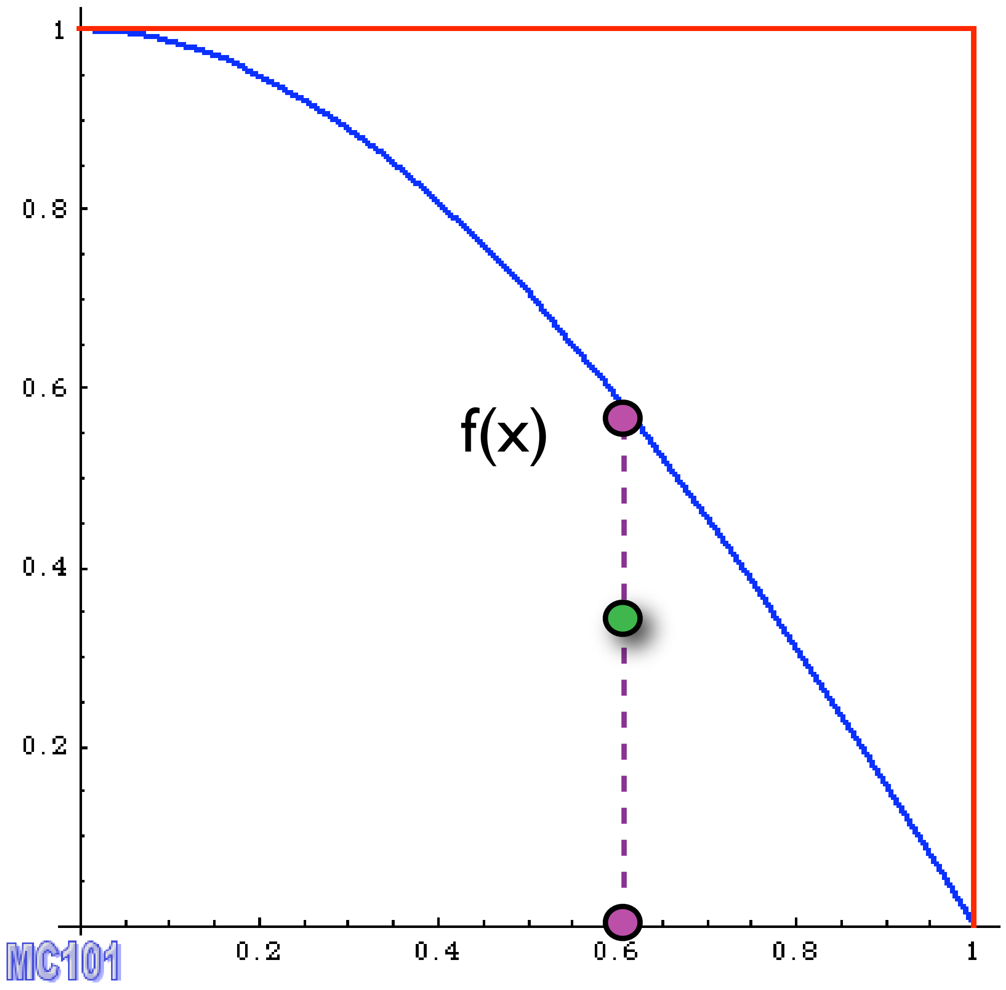
Event generation



Alternative way

1. pick x
2. calculate f(x)
3. pick 0<y<fmax

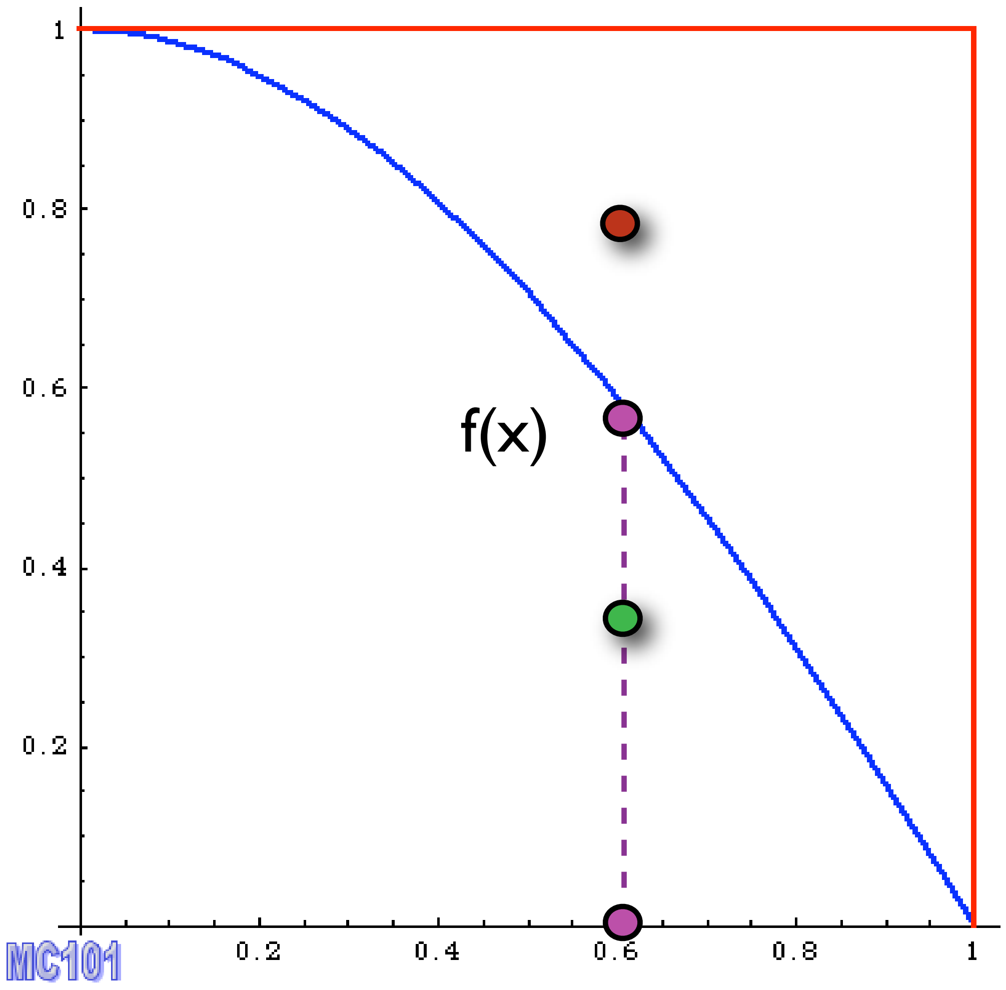
Event generation



Alternative way

1. pick x
2. calculate f(x)
3. pick 0<y<fmax
4. Compare:
   if f(x)>y accept event,

Event generation



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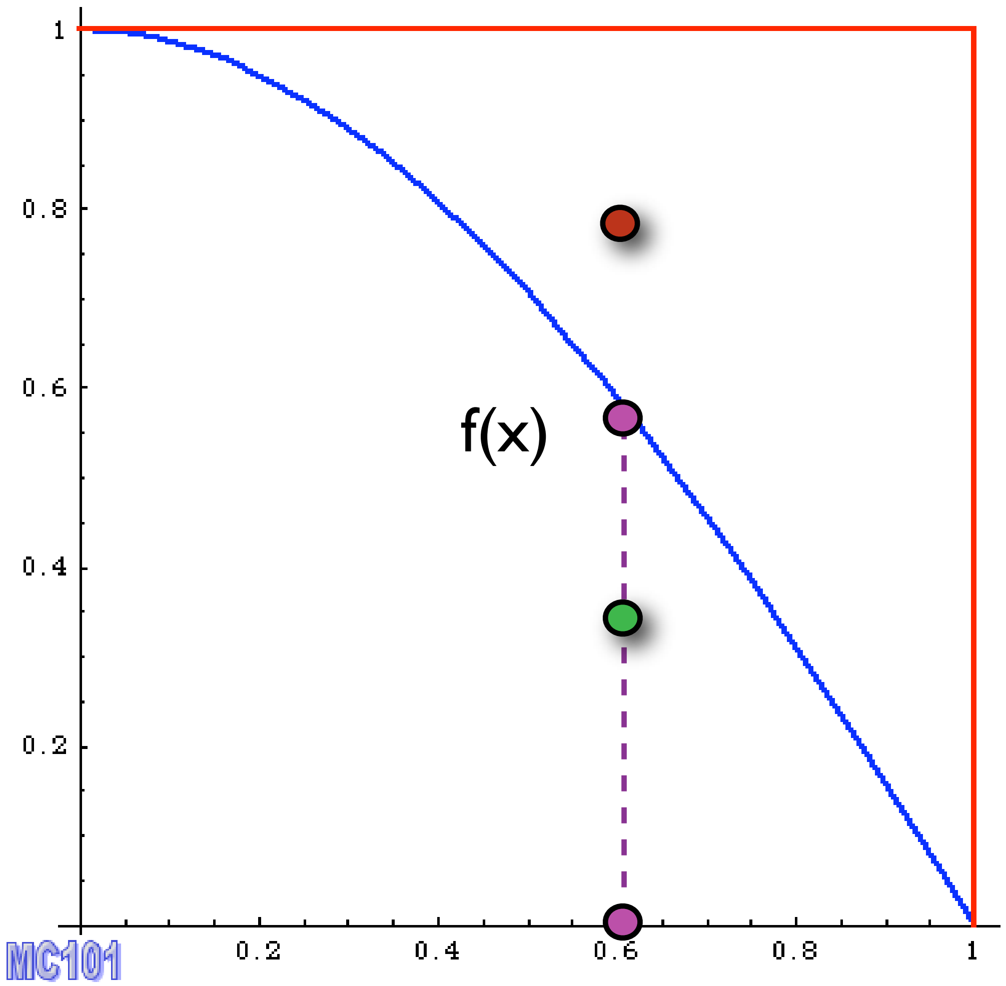
Event generation

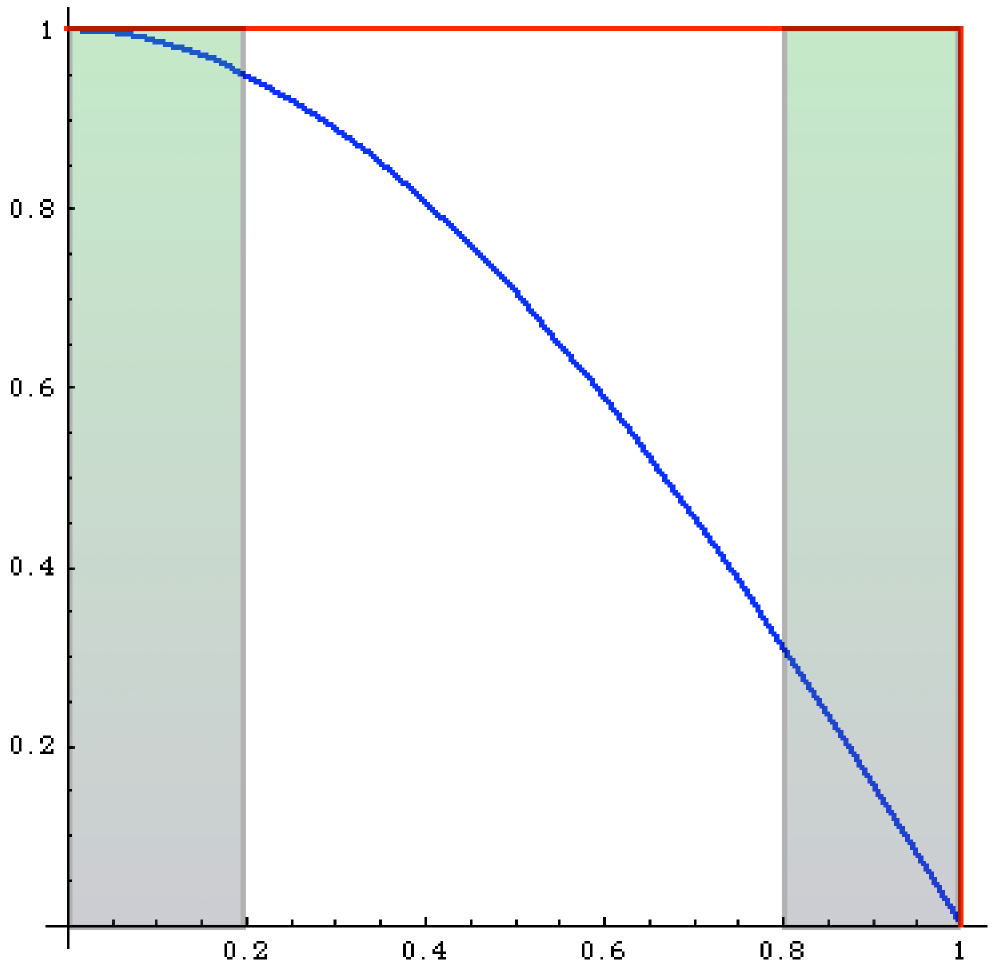


Alternative way

1. pick x
2. calculate f(x)
3. pick 0<y<fmax
4. Compare:
   if f(x)>y accept event,
   else reject it.

$$I = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

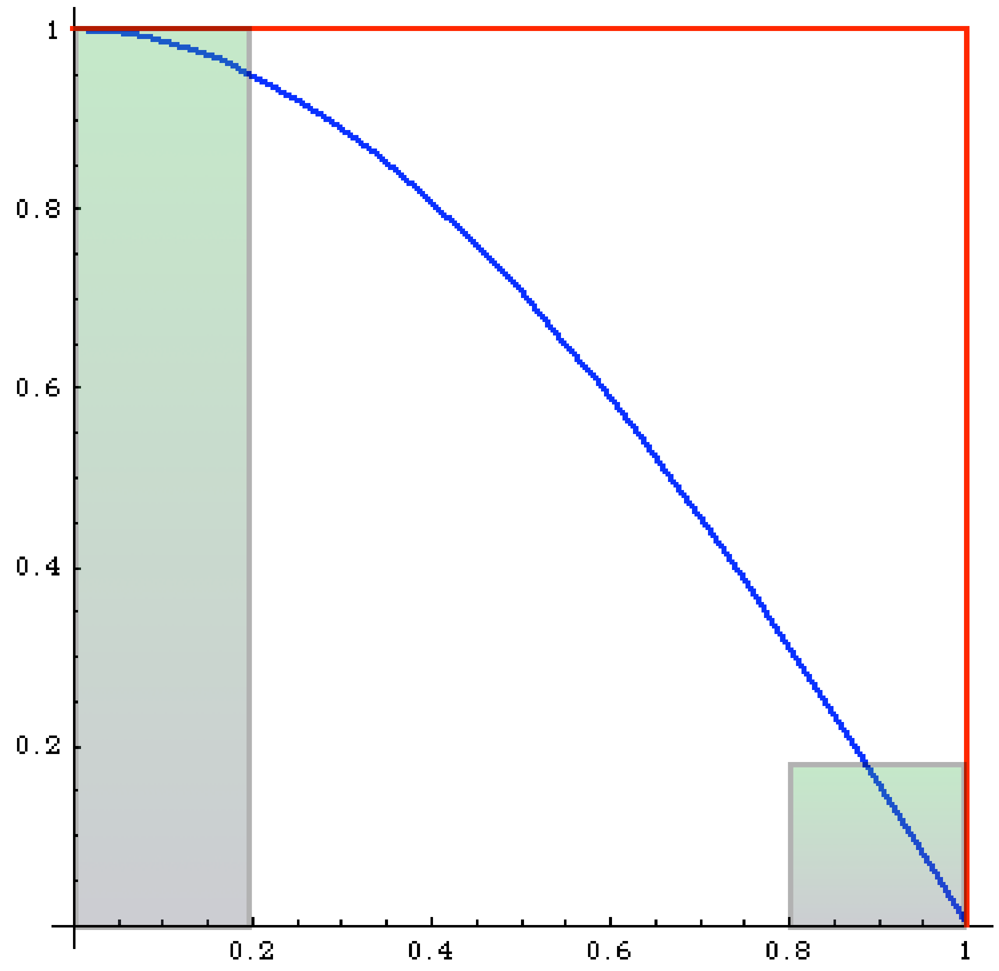


What's the difference?

before:

same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



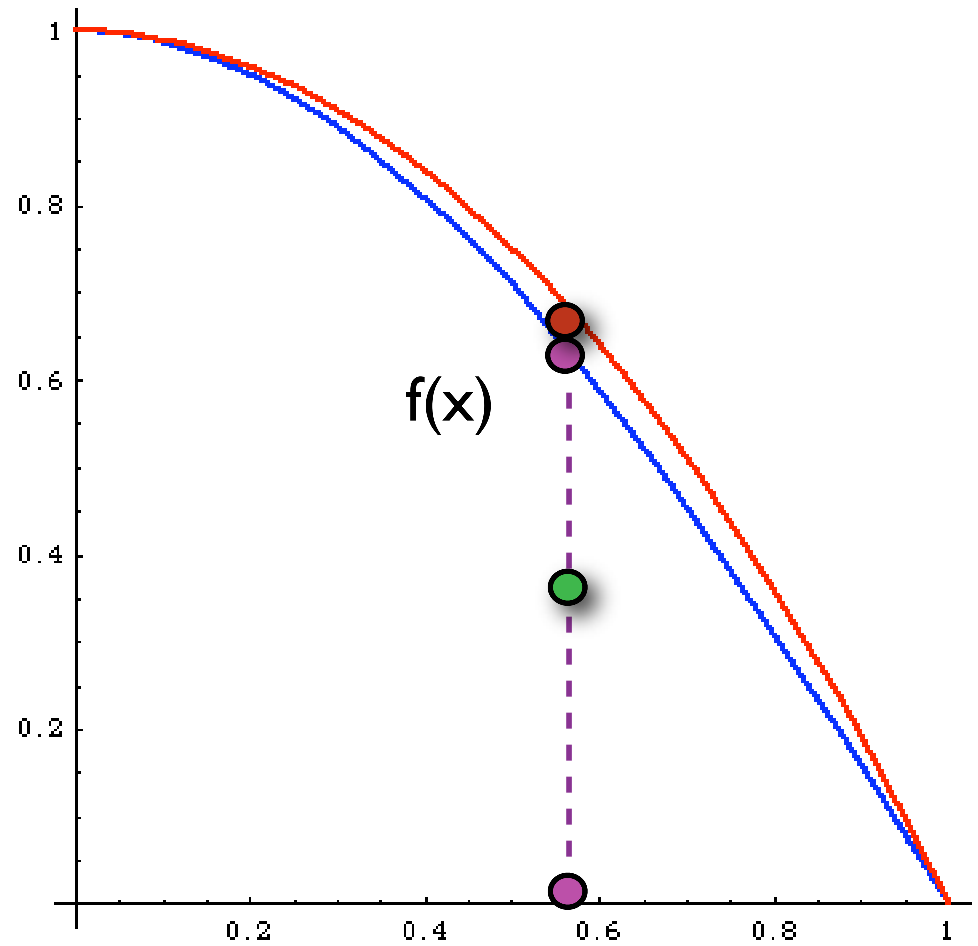
What's the difference?

after:

events is proportional to the probability of areas of phase space:
events have all the same weight (”unweighted”)

Events distributed as in Nature

Event generation



Improved

1. pick x distributed as p(x)
2. calculate f(x) and p(x)
3. pick 0<y<1
4. Compare:
   if f(x)>y p(x) accept event,

else reject it.

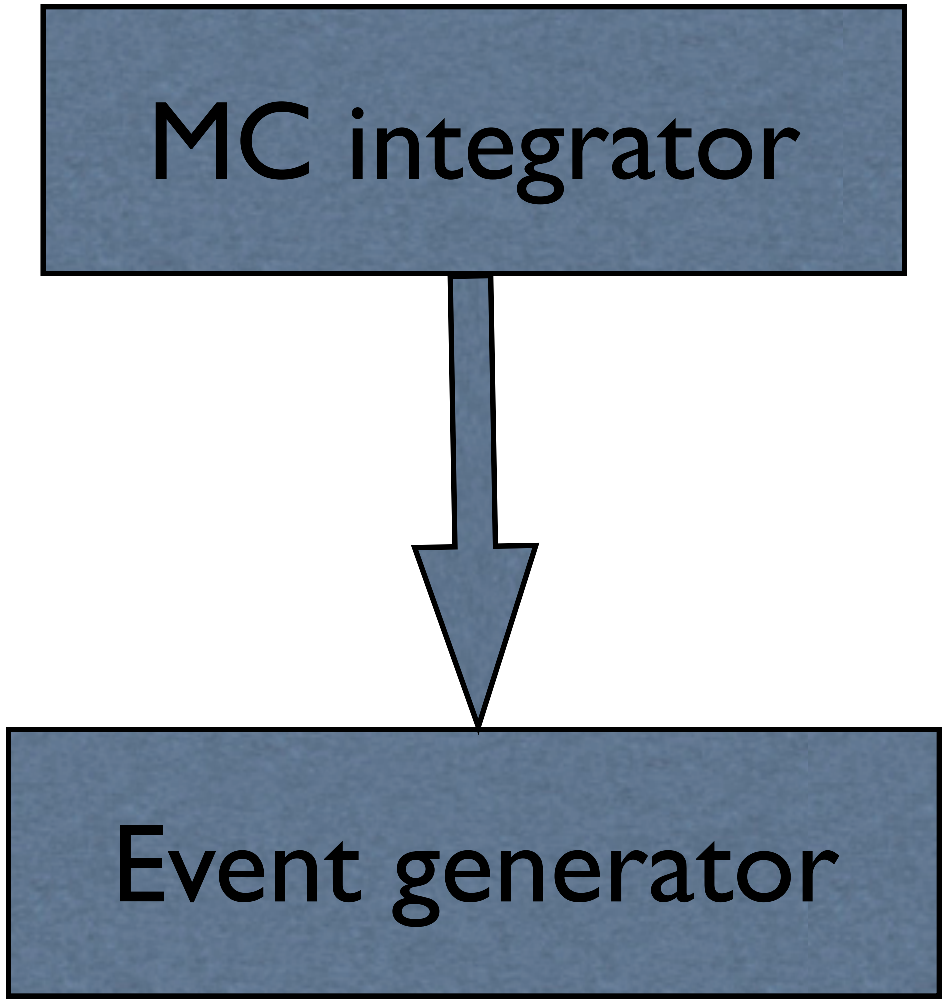
much better efficiency!!!

Event generation

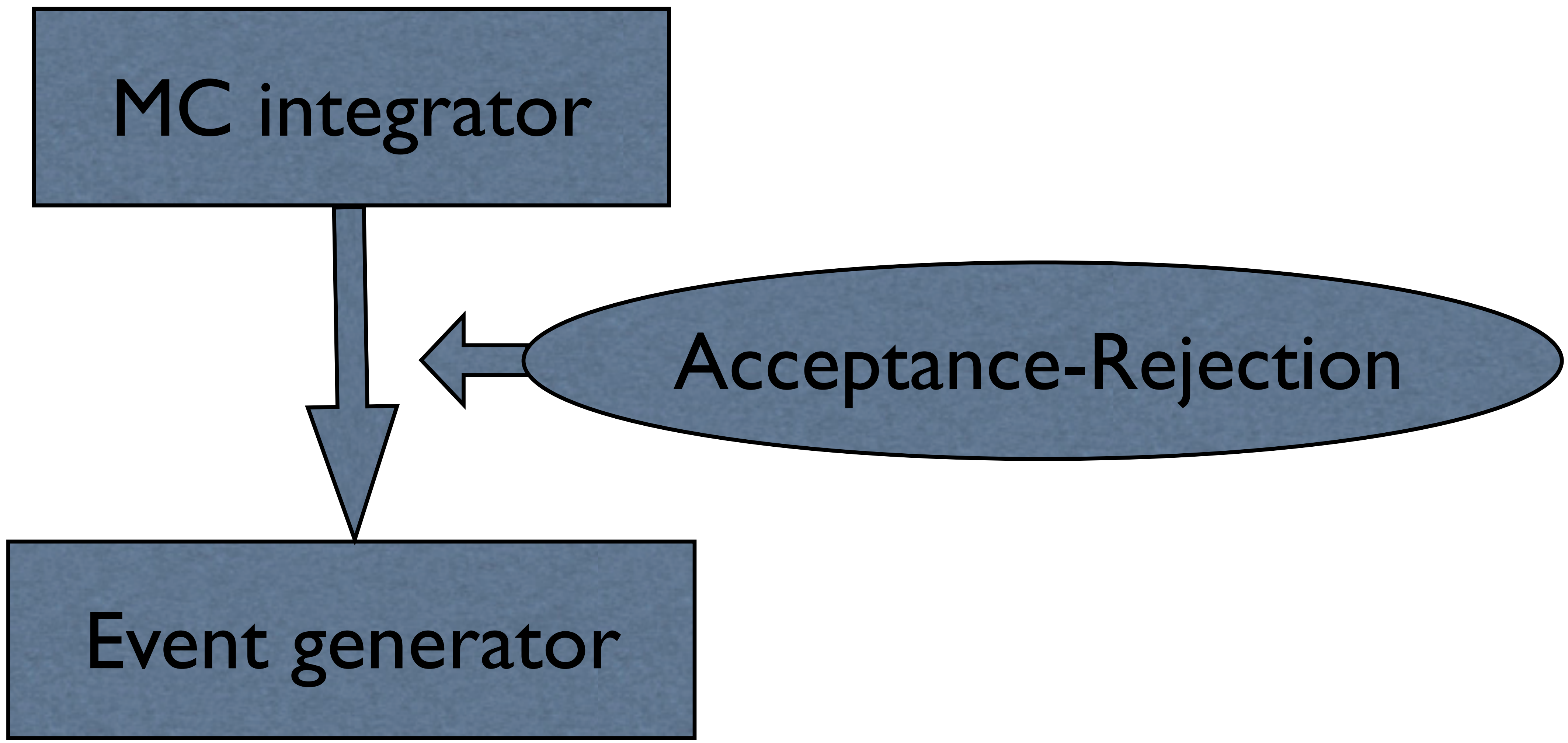
Event generation

MC integrator

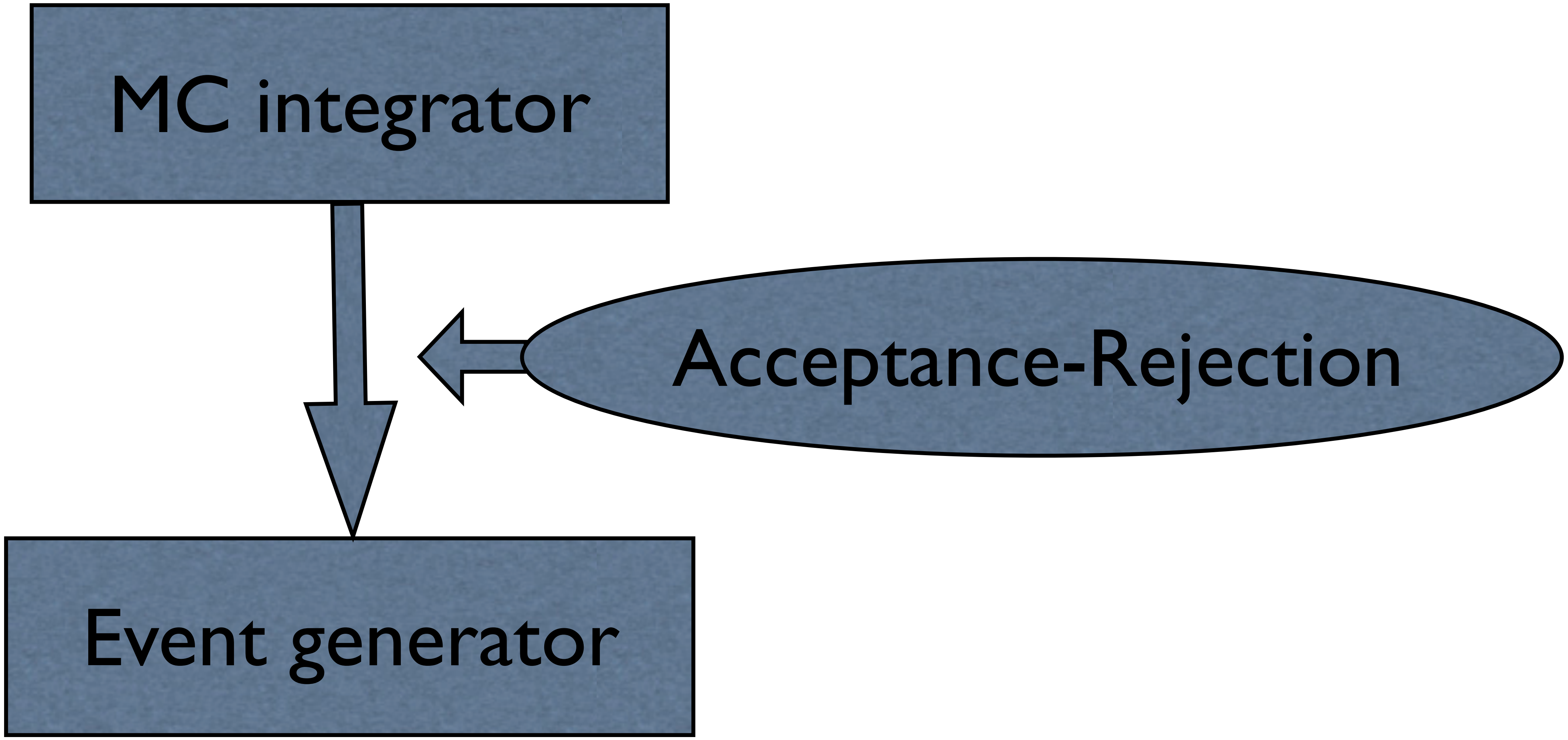
Event generation



Event generation



Event generation



☞ This is possible only if $f(x)<\infty$ AND has definite sign!

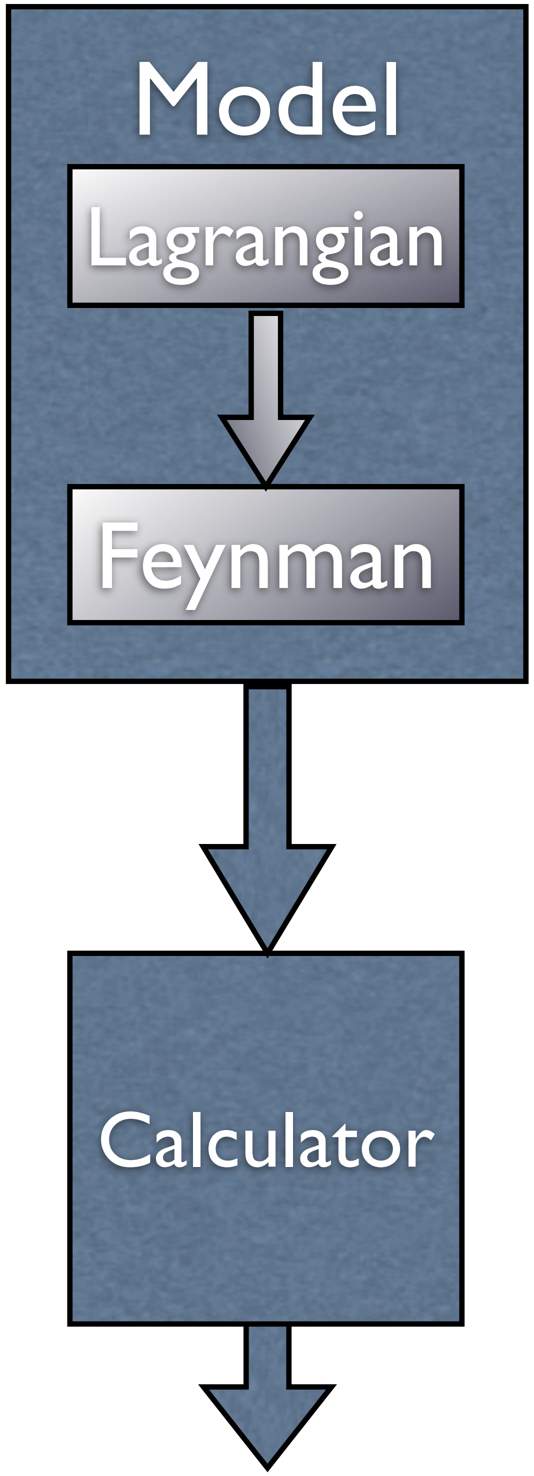
Monte Carlo Event Generator: definiton

At the most basic level a Monte Carlo event generator is a program which produces particle physics events with the same probability as they occur in nature (virtual collider).

In practice it performs a large number of (sometimes very difficult) integrals and then unweights to give the four momenta of the particles that interact with the detector (simulation).

Note that, at least among theorists, the definition of a "Monte Carlo program" also includes codes which don't provide a fully exclusive information on the final state but only cross sections or distributions at the parton level, even when no unweighting can be performed. I will refer to these kind of codes as "MC integrators".

MadGraph



Les Houches interface

Invent a model, renormalizable or not, with new physics. Write the Lagrangian and get the Feynman Rules.

The particles content, the type of interactions and the analytic form of the couplings in the Feynman rules define the model at tree level.

Interfaced to FeynRules

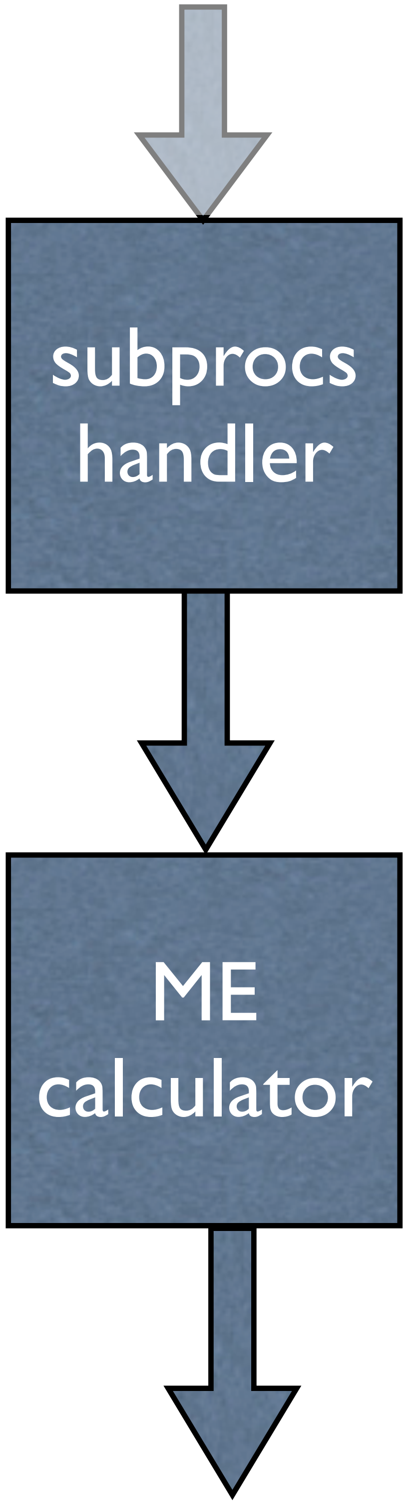
SUSY, Little Higgs, Higgsless, GUT, Extra dimensions (flat, warped, universal,...)

Parameters Calculator.
Given the "primary" couplings, all relevant quantities are calculated: masses, widths and the values of the couplings in the Feynman rules.

Caution: tree-level relations have to be satisfied to avoid gauge violations and/or wrong branching ratios.

FeynHiggs, ISAJET, NMHDecay, SOFTSUSY, SPHENO, SUSPECT, SDECAY...

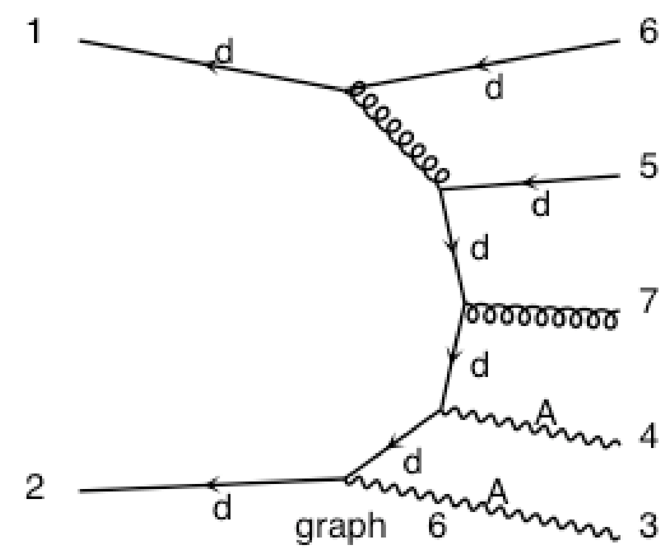
MadGraph



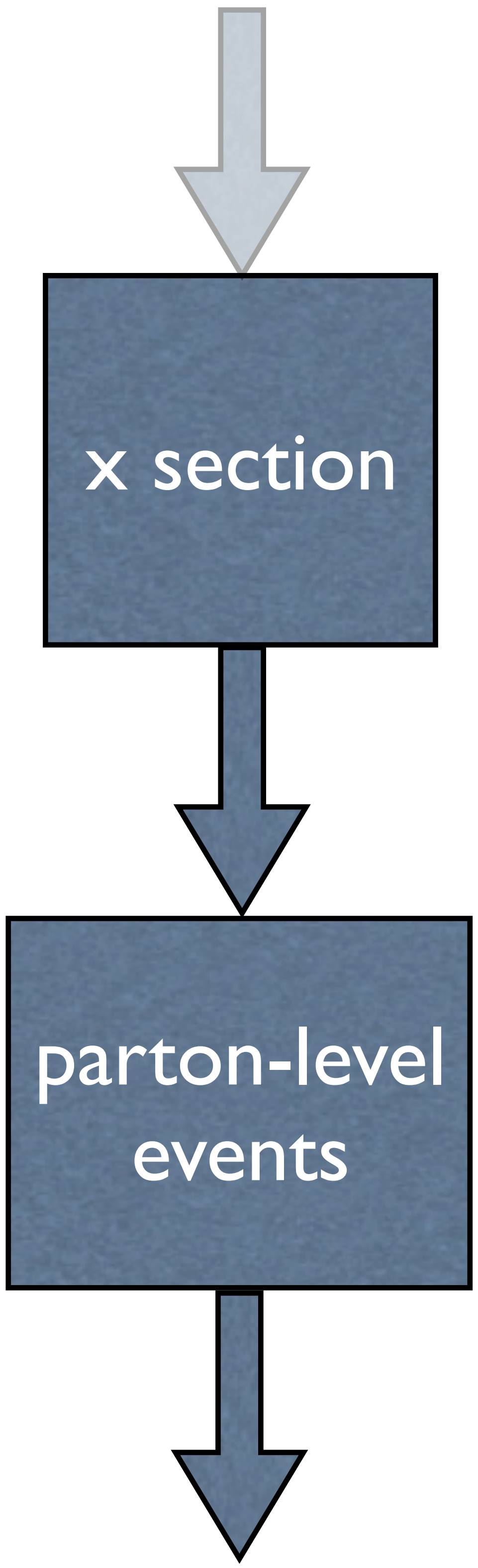
Includes all possible subprocess leading to a given multi-jet final state automatically

d~ d -> a a u u~ g
d~ d -> a a c c~ g
s~ s -> a a u u~ g
s~ s -> a a c c~ g

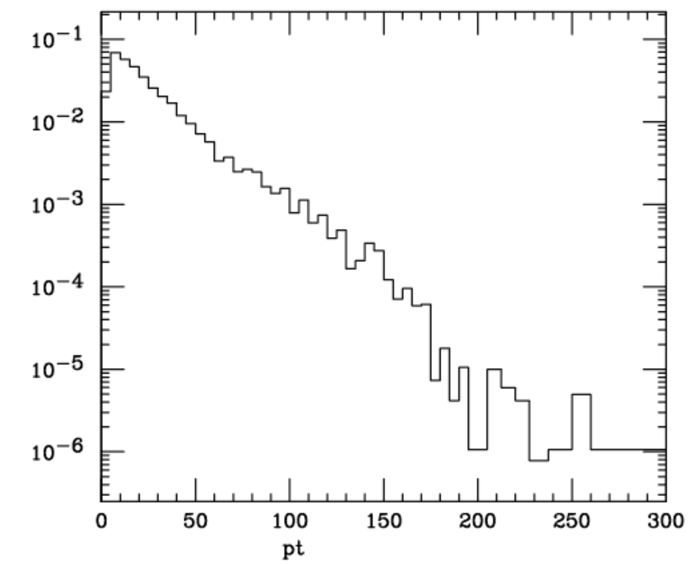
Automatically generates a code to calculate |M|^2 for arbitrary processes. Most use Feynman diagrams w/ tricks to reduce the factorial growth [MadGraph, SHERPA], others have recursive relations to reduce the complexity to exponential [Alpgen, HELAC, Comix].



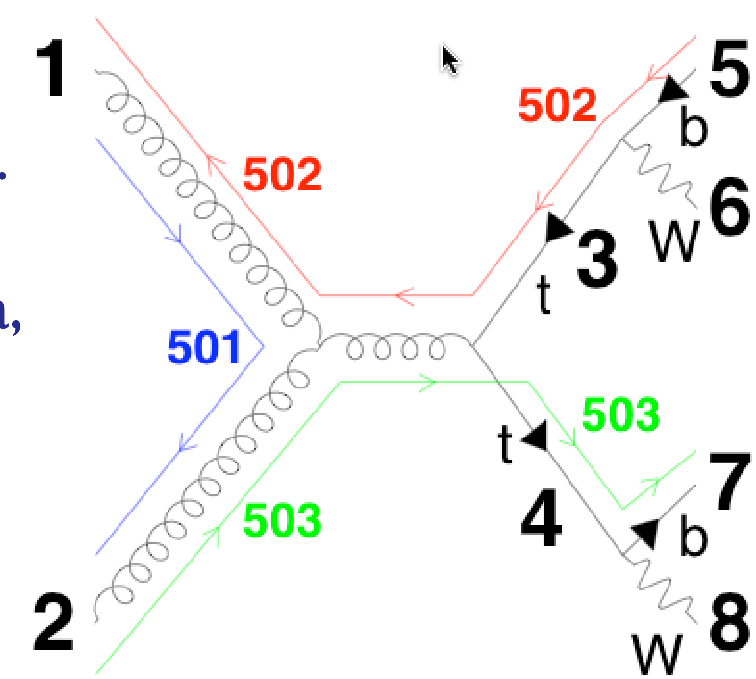
MadGraph



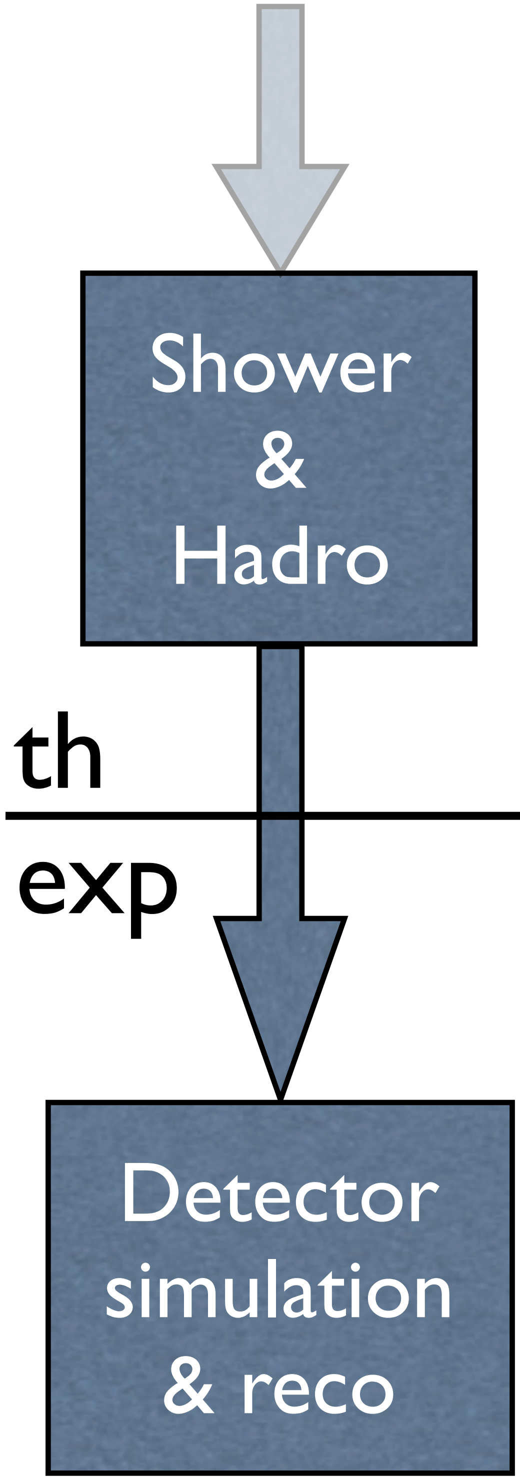
Integrate the matrix element over the phase space using a multi-channel technique and using parton-level cuts.



Events are obtained by unweighting. These are at the parton-level. Information on particle id, momenta, spin, color and mother-daugther is given in the Les Houches format.



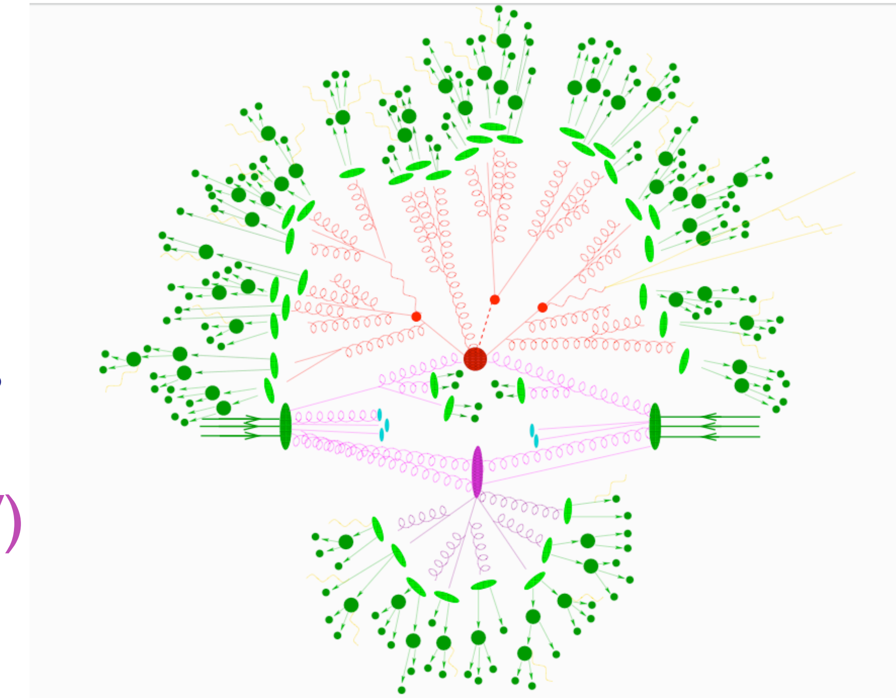
MadGraph



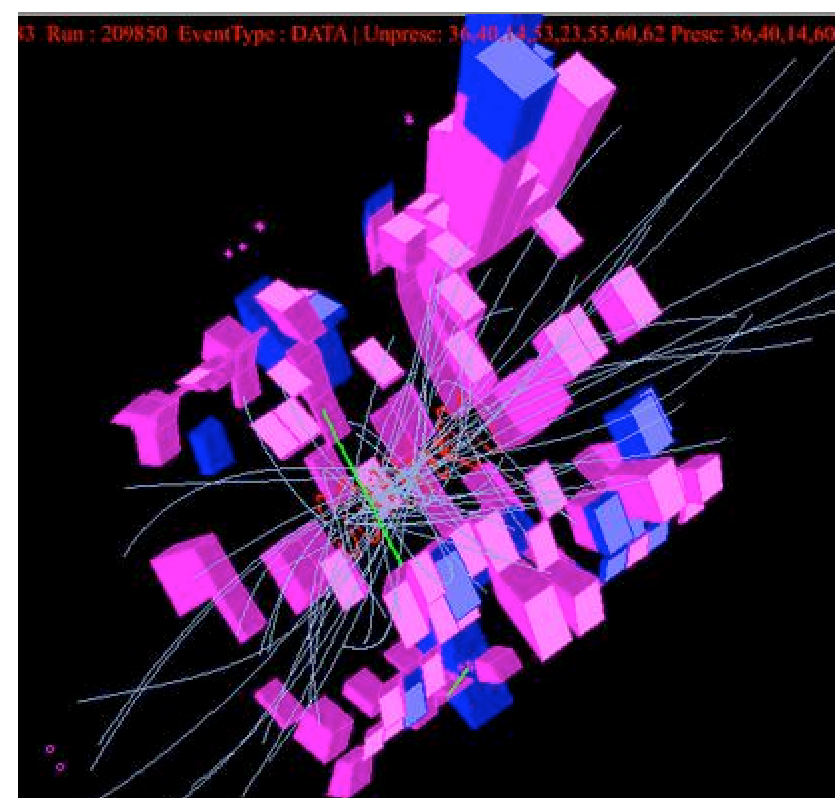
Events in the LH format are passed to the showering and hadronization⇒

high multiplicity hadron-level events

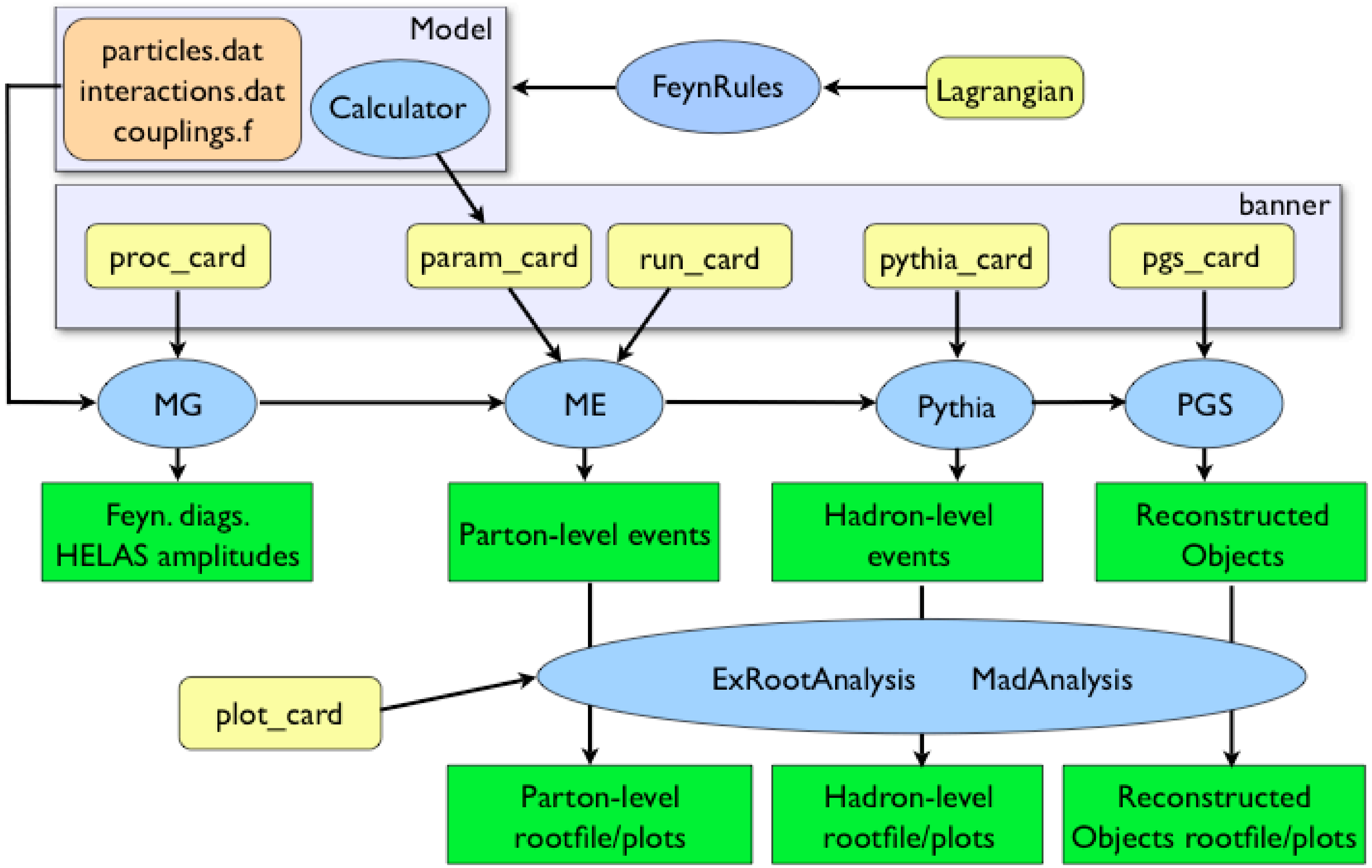
Parton-Jet merging (MLM or CKKW) happens here!



Events in stdhep format are passed through fast or full simulation, and physical objects (leptons, photons, jet, b-jets, taus) are reconstructed.



FlowChart


Model
particles.dat
interactions.dat
couplings.f
Calculator
FeynRules
Lagrangian
banner
proc_card
param_card
run_card
pythia_card
pgs_card
MG
ME
Pythia
PGS
Feyn. diags.
HELAS amplitudes
Parton-level events
Hadron-level
events
Reconstructed
Objects
ExRootAnalysis
MadAnalysis
plot_card
Parton-level
rootfile/plots
Hadron-level
rootfile/plots
Reconstructed
Objects rootfile/plots

Let’s plug ... & play!

1. t tbar production: pp>tt~>bb~mu+ e- ve~ vm (or fully hadronic:pp>tt~>bb~jjjj).
2. t tbar + Higgs : pp>h>tt~bb~ (QCD=2,QED=2). Generate the background pp>tt~bb~ (QCD=99,QED=0) and put a min cut on the m(bb)=100 GeV.
3. Single top + Higgs: pp>tHj (QCD=0, QED=3,j=gudsc, p=gudscb). Show that there is a large negative interference between the diagrams.
4. gg>h: pp>h>mu+ e- ve~ vm (HEFT,QED). Generate the background, pp>W+W-> mu+ e- ve~ vm/h (QCD=0,QED=4). Use different Higgs masses (mh=120,mh=170). Identify a smart discriminating variable among those plotted automatically.

Installing the MG/ME & analysis routines:

1. Get the full thing:

wget http://madgraph.phys.ucl.ac.be/Downloads/MG_ME_V4.4.38.tar.gz;
tar zxvf MG_ME_V4.4.38.tar.gz;
cd MG_ME_V4.4.38

2. Get a very simple LHE and LHCO event analyzer:

wget http://madgraph.phys.ucl.ac.be/Downloads/MadAnalysis_V1.1.2.tar.gz;
tar zxvf MadAnalysis_V1.1.2.tar.gz

3. make

4. Install topdrawer :
cd MadAnalysis; wget http://madgraph.phys.ucl.ac.be/Downloads/td.tgz

Take-home project : pp→Higgs

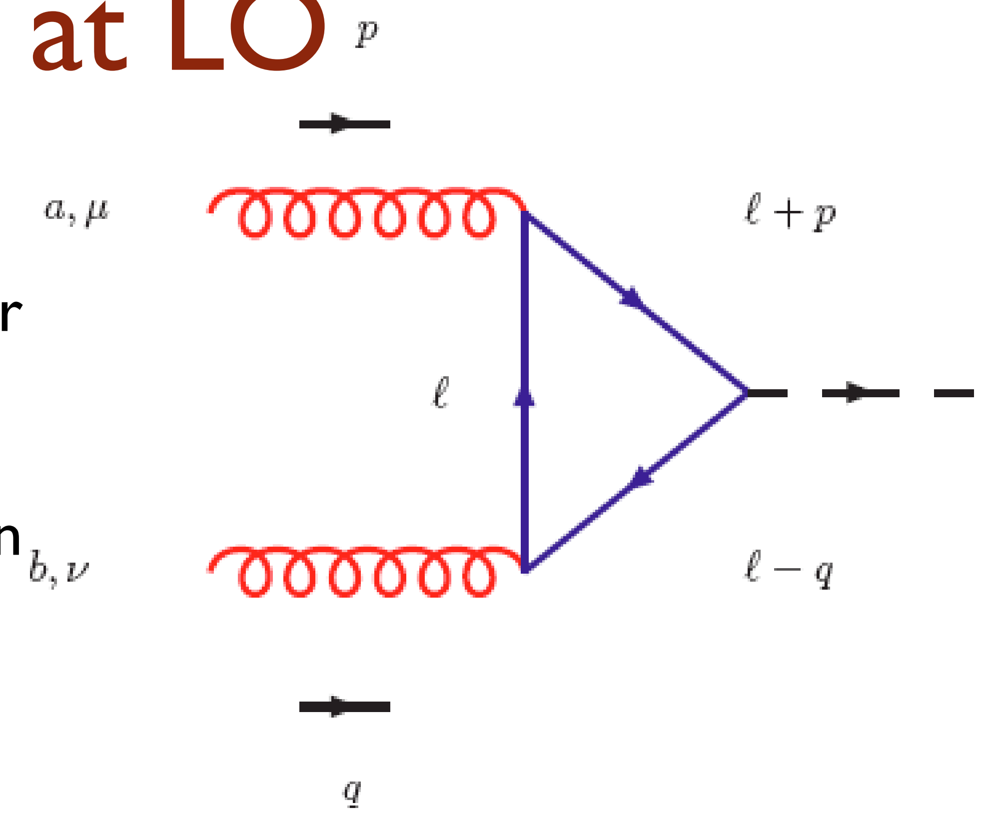
- LO : 1-loop calculation and HEFT
- Cross sections at the LHC

pp→H at LO

This is a "simple" 2→1 process.

However, at variance with pp→W, the LO order process already proceeds through a loop.

In this case, this means that the loop calculation has to give a finite result!



pp→H at LO

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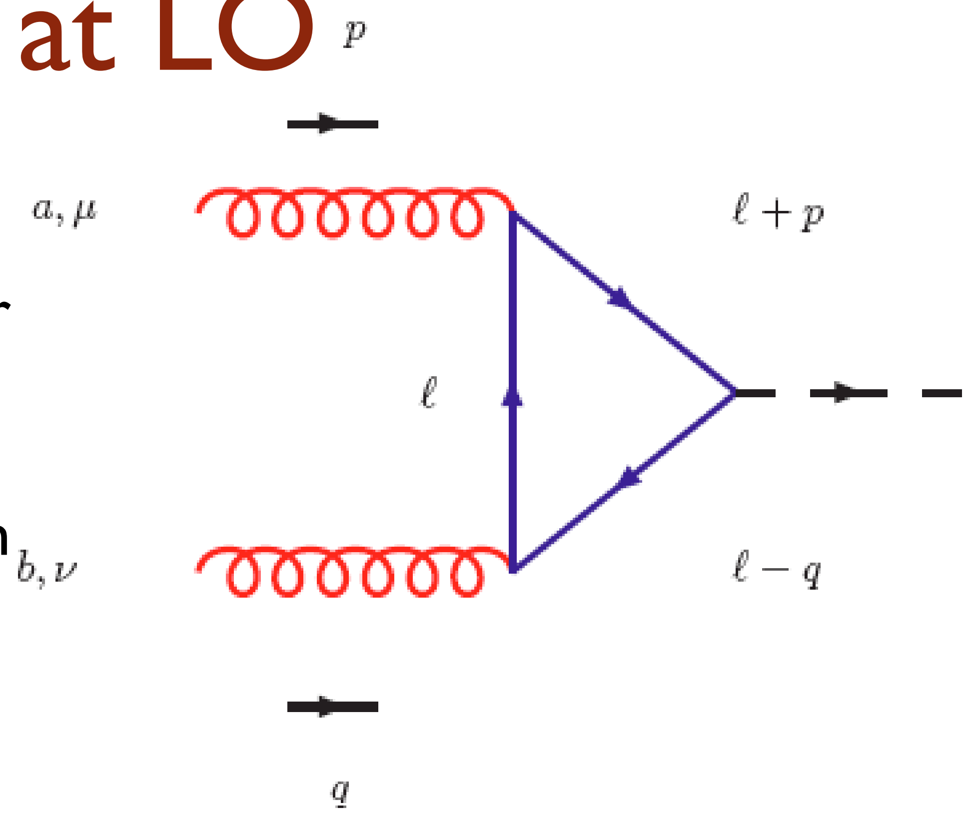
Let's do the calculation!



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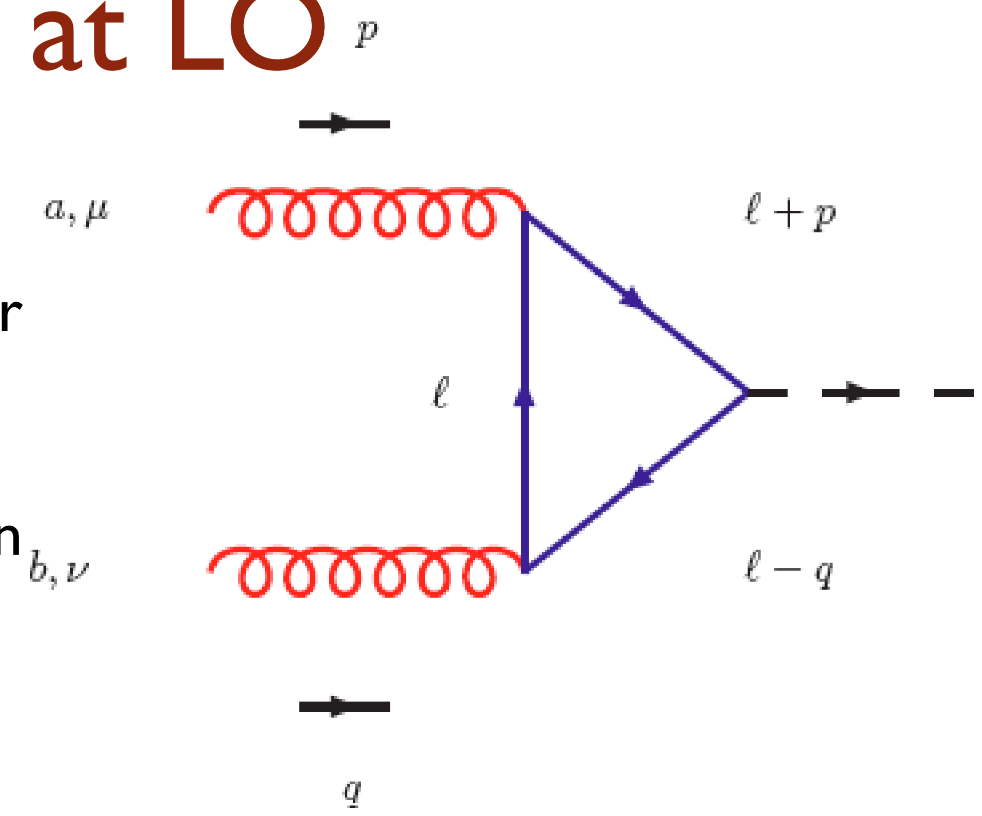

$$iA = -(-ig_s)^2 \mathrm{Tr}(t^a t^b)\left(\frac{-im_t}{v}\right)\int \frac{d^d\ell}{(2\pi)^n}\frac{T^{\mu\nu}}{\mathrm{Den}}(i)^3 \epsilon_\mu(p)\epsilon_\nu(q)$$

where

$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$$

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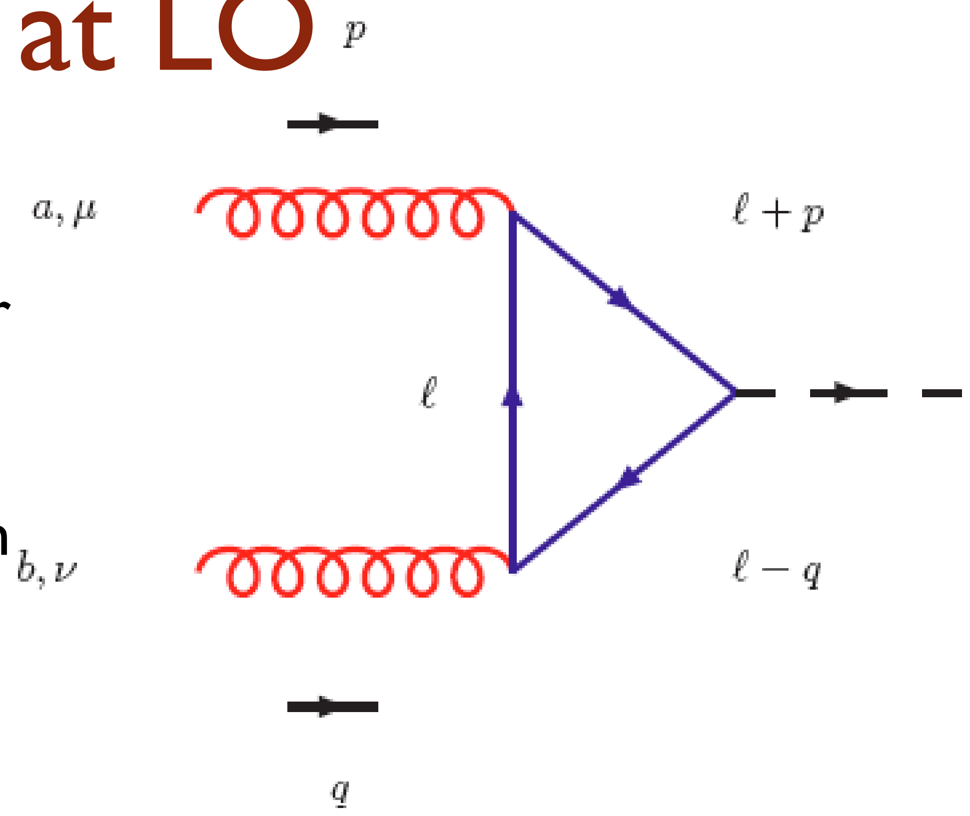

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where

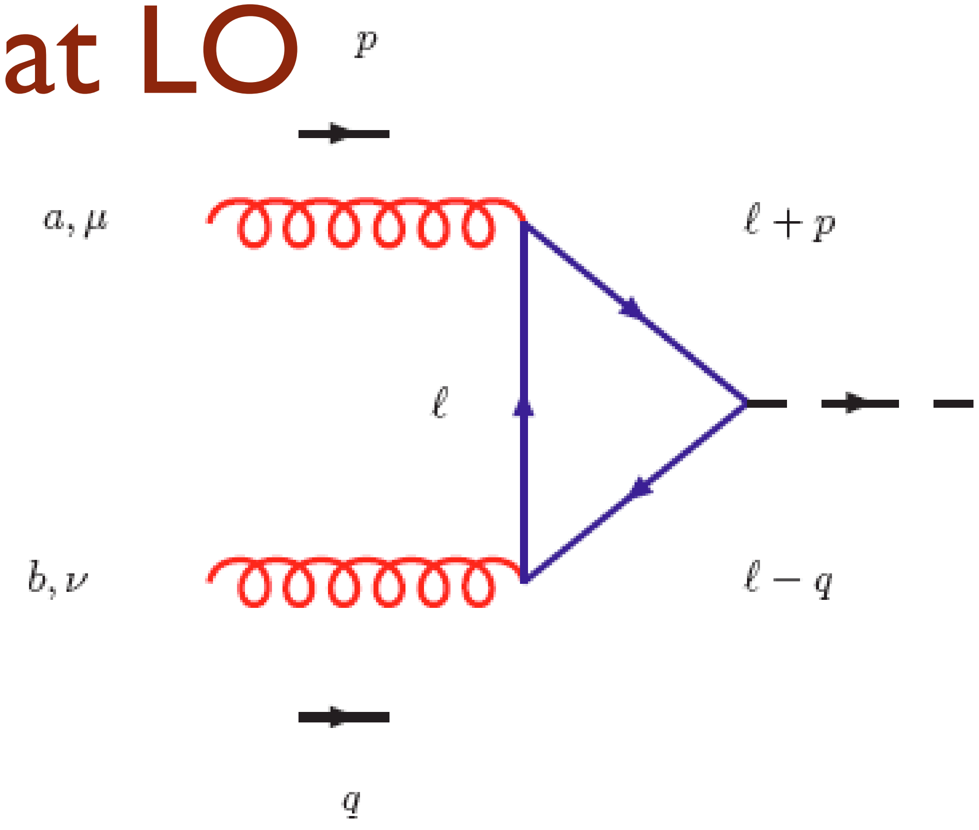
$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$$

We combine the denominators into one by using $\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1-x-y)]^3}$

$$\frac{1}{\mathrm{Den}} = 2\int dx\, dy \frac{1}{[\ell^2 - m_t^2 + 2\ell\cdot(px - qy)]^3}.$$

pp→H at LO

$$\int \frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^2-C)^3} = \frac{i}{32\pi^2}(4\pi)^\epsilon\frac{\Gamma(1+\epsilon)}{\epsilon}(2-\epsilon)C^{-\epsilon}$$
$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-C)^3} = -\frac{i}{32\pi^2}(4\pi)^\epsilon\Gamma(1+\epsilon)C^{-1-\epsilon}.$$



where d=4-2eps. By substituting we arrive at a very simple final result!!

$$\mathcal{A}(gg\to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2}-p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2-m_H^2xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

Comments:

* The final dependence of the result is m_t^2 : one from the Yukawa coupling, one from the spin flip.
* The tensor structure could have been guessed by gauge invariance.
* The integral depends on mt and mh.

LO cross section

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \, g(x_1,\mu_f) g(x_2,\mu_f) \, \hat{\sigma}(gg \to H)$$

$$x_1 \equiv \sqrt{\tau} e^{y} \quad x_2 \equiv \sqrt{\tau} e^{-y} \quad \tau = x_1 x_2 \qquad \tau_0 = M_H^2/S \quad z = \tau_0/\tau$$

$$= \frac{\alpha_S^2}{64\pi v^2} \mid I\left(\frac{M_H^2}{m^2}\right) \mid^2 \tau_0 \int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy g(\sqrt{\tau_0}e^{y}) g(\sqrt{\tau_0}e^{-y})$$

The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

LO cross section

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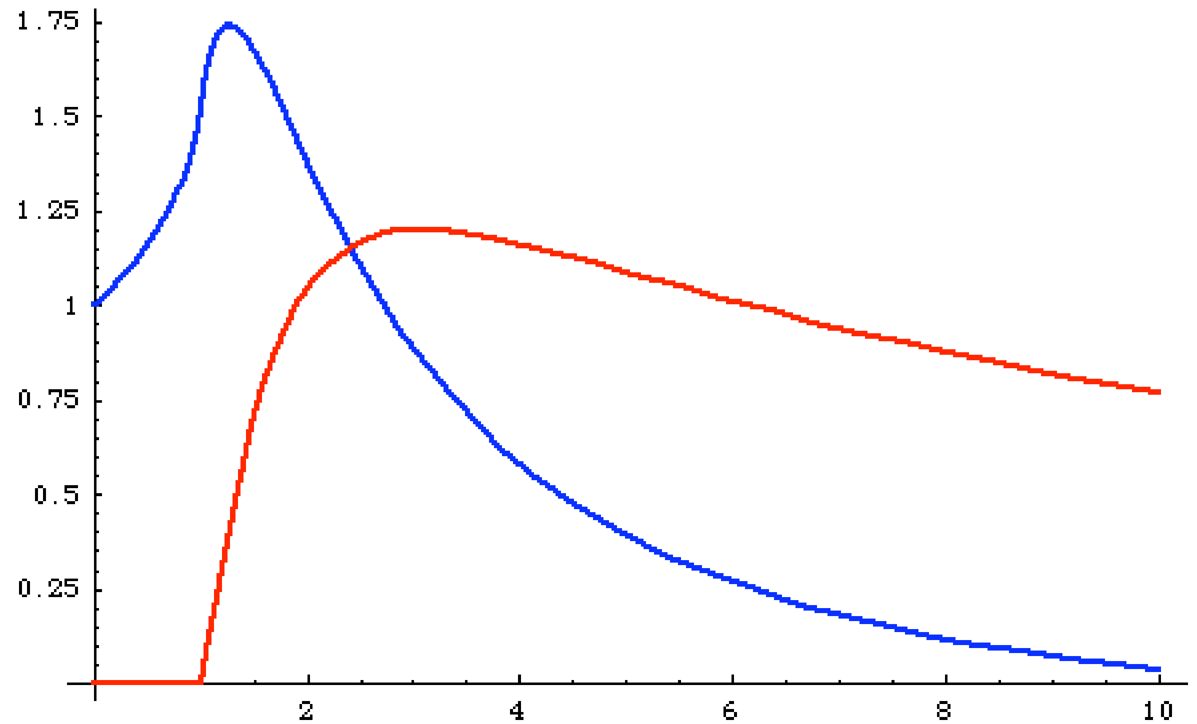
$$= \frac{\alpha_S^2}{64\pi v^2} \left| I\left(\frac{M_H^2}{m^2}\right) \right|^2 \tau_0 \int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy\, g(\sqrt{\tau_0} e^{y}) g(\sqrt{\tau_0} e^{-y})$$

The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

I(x) has both a real and imaginary part, which develops at mh=2mt.



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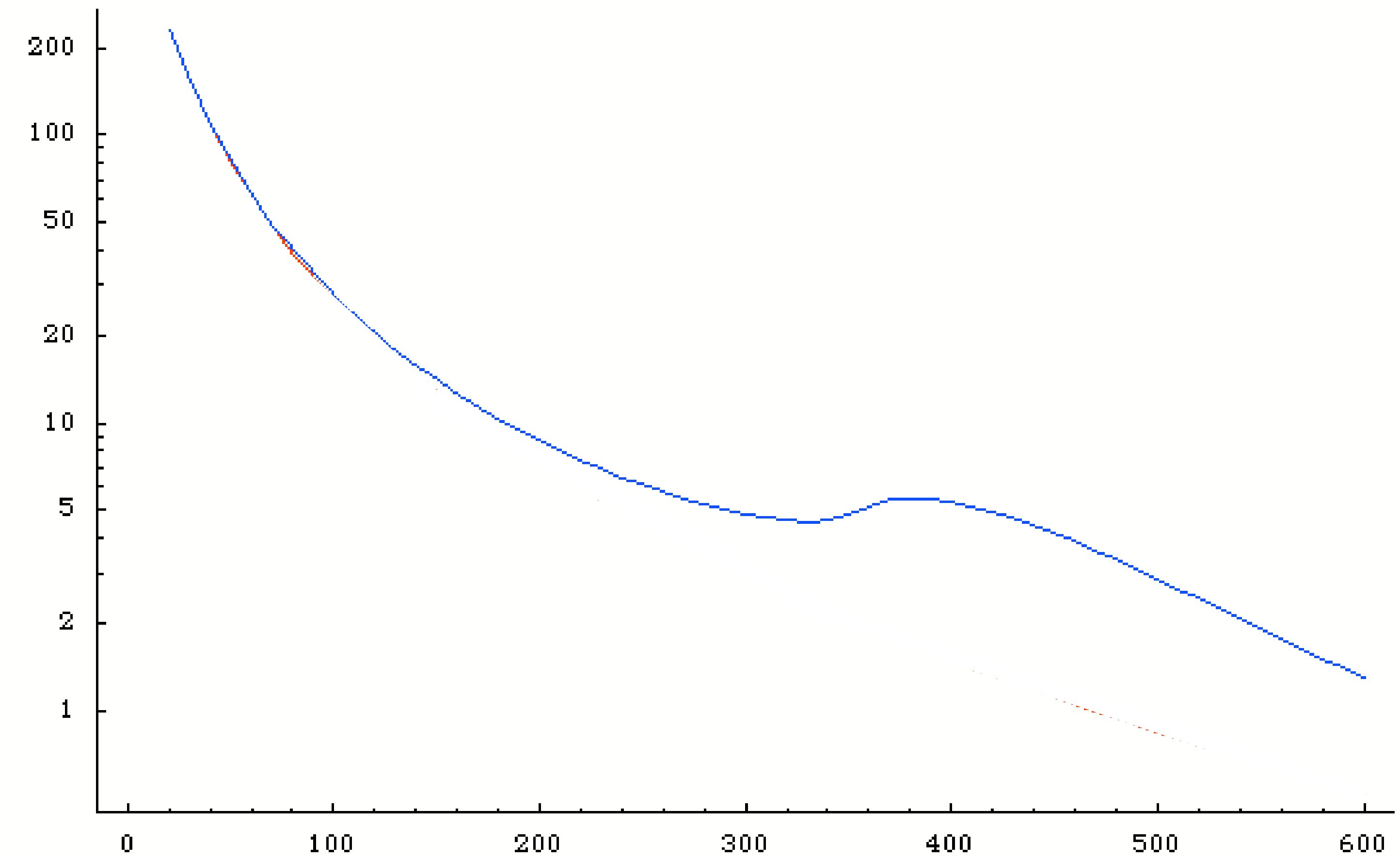
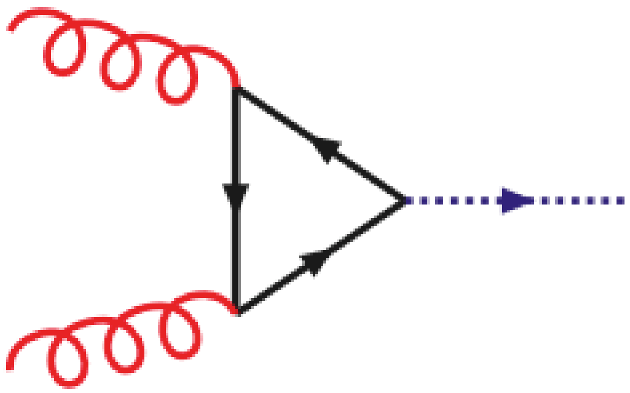
The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

I(x) has both a real and imaginary part, which develops at mh=2mt.

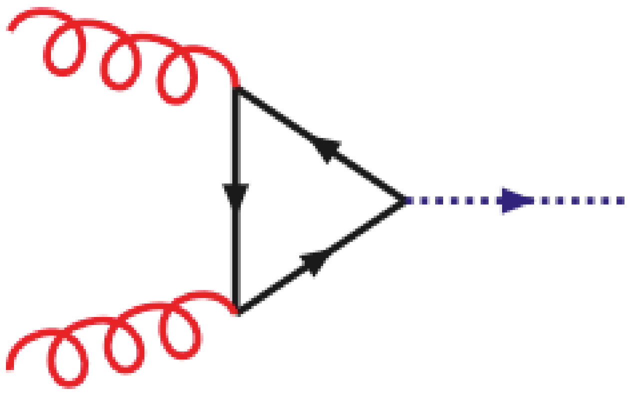
This causes a bump in the cross section.



pp →H in the EFT



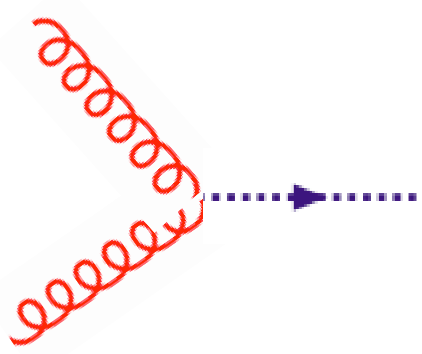
pp →H in the EFT



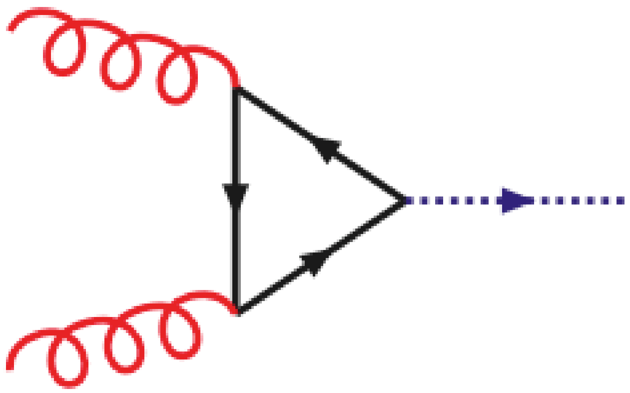
Let's consider the case where the Higgs is light:

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

$$\xrightarrow{m \gg M_H} -\frac{\alpha_S}{3\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\epsilon_\mu(p)\epsilon_\nu(q).$$



pp →H in the EFT



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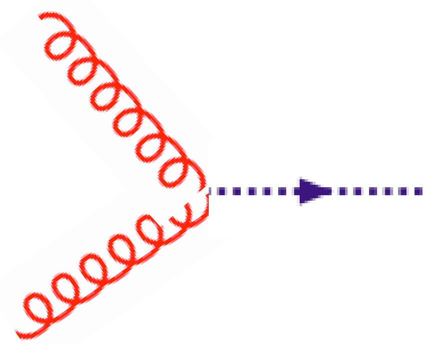
$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

$$\xrightarrow{m \gg M_H} -\frac{\alpha_S}{3\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\epsilon_\mu(p)\epsilon_\nu(q).$$



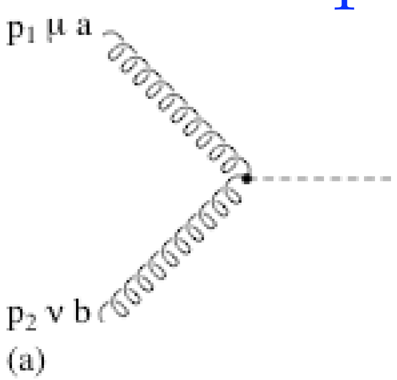
This looks like a local vertex, ggH.

The top quark has disappeared from the low energy theory but it has left something behind (non-decoupling).

Higgs effective field theory

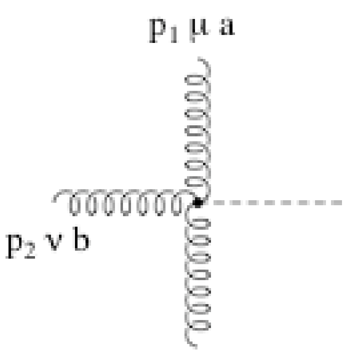
$$\mathcal{L}_{\rm eff} = -\frac{1}{4}\left(1 - \frac{\alpha_S}{3\pi}\frac{H}{v}\right) G^{\mu\nu}G_{\mu\nu}$$

This is an effective non-renormalizable theory (no top) which describes the Higgs couplings to QCD.



$iA\delta^{ab}H^{\mu\nu}(p_1,p_2)$

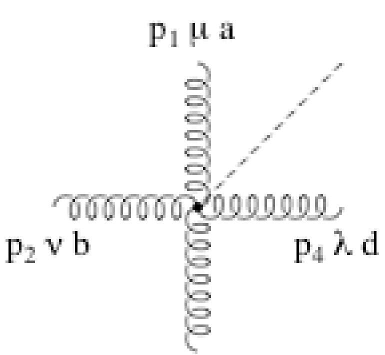
$$H^{\mu\nu}(p_1,p_2) = g^{\mu\nu} p_1\cdot p_2 - p_1^\nu p_2^\mu .$$



(b)

$-Agf^{abc}V^{\mu\nu\sigma}(p_1,p_2,p_3)$

$$V^{\mu\nu\rho}(p_1,p_2,p_3) = (p_1-p_2)^\rho g^{\mu\nu} + (p_2-p_3)^\mu g^{\nu\rho} + (p_3-p_1)^\nu g^{\rho\mu},$$



(c)

$-iAg^2X^{abcd}_{\mu\nu\sigma\lambda}$

$$X^{\mu\nu\rho\sigma}_{abcd} = f_{abe}f_{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f_{ace}f_{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f_{ade}f_{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma}).$$

LO cross section: full vs HEFT

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, g(x_1, \mu_f) g(x_2, \mu_f)\, \hat{\sigma}(gg \to H)$$

The accuracy of the calculation in the HEFT calculation can be directly assessed by taking the limit m→∞.

For light Higgs is better than 10%.