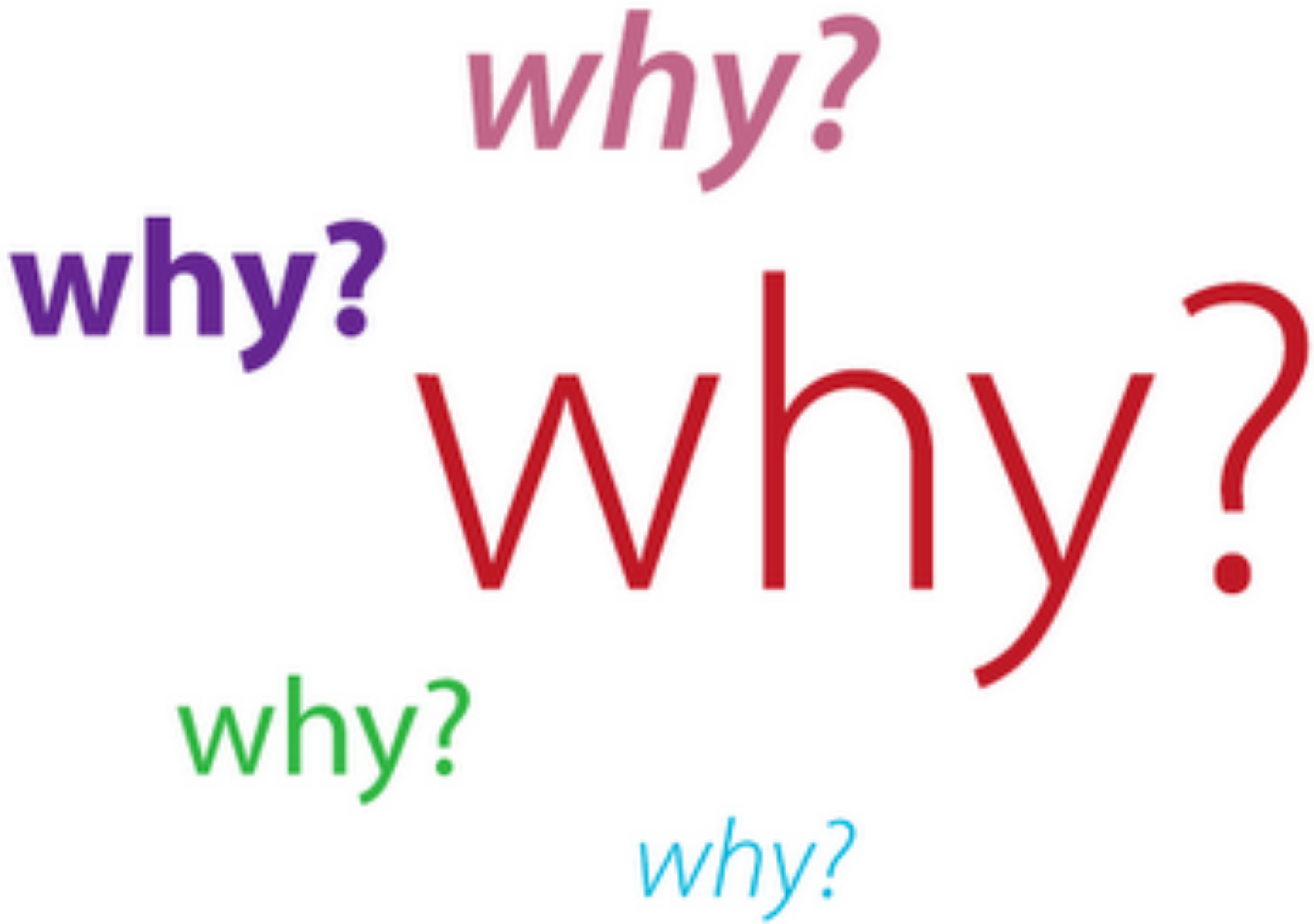
NLO: How to?

Marco Zaro
marco.zaro@mi.infn.it

Milan PhD School

Introduction:
Why do we need N$^{(k)}$LO?



Discoveries at hadron colliders

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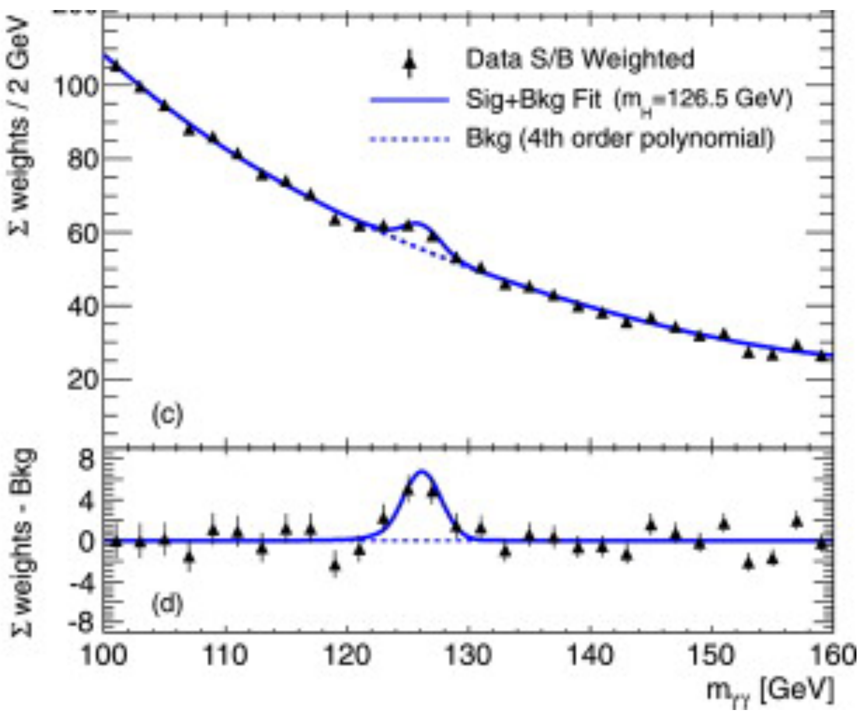
Peak

$H \to \gamma\gamma$



EASY

Background directly measured from **data**.
Theory needed only for parameter extraction

Discoveries at hadron colliders

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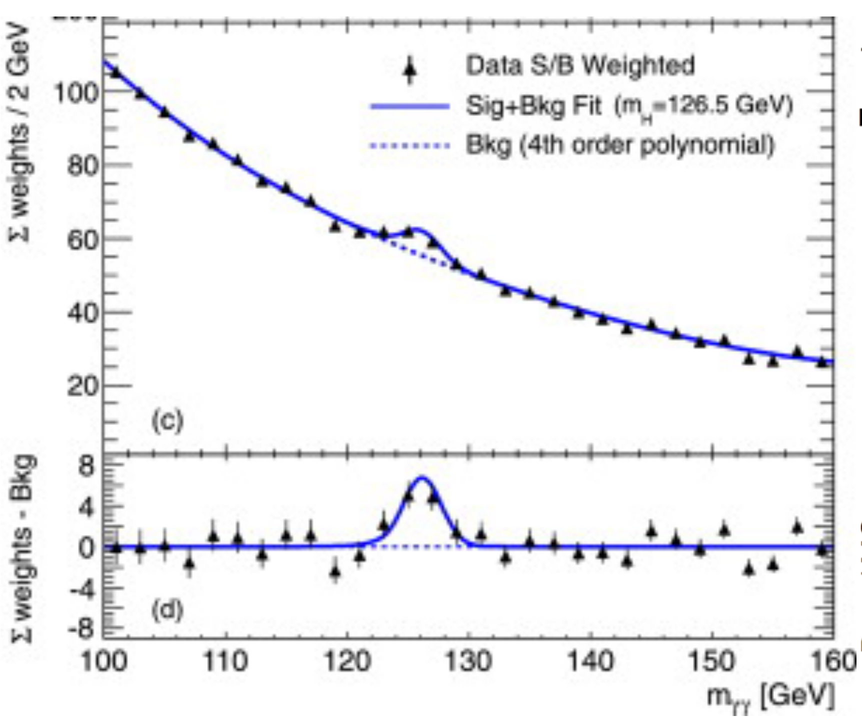

EASY

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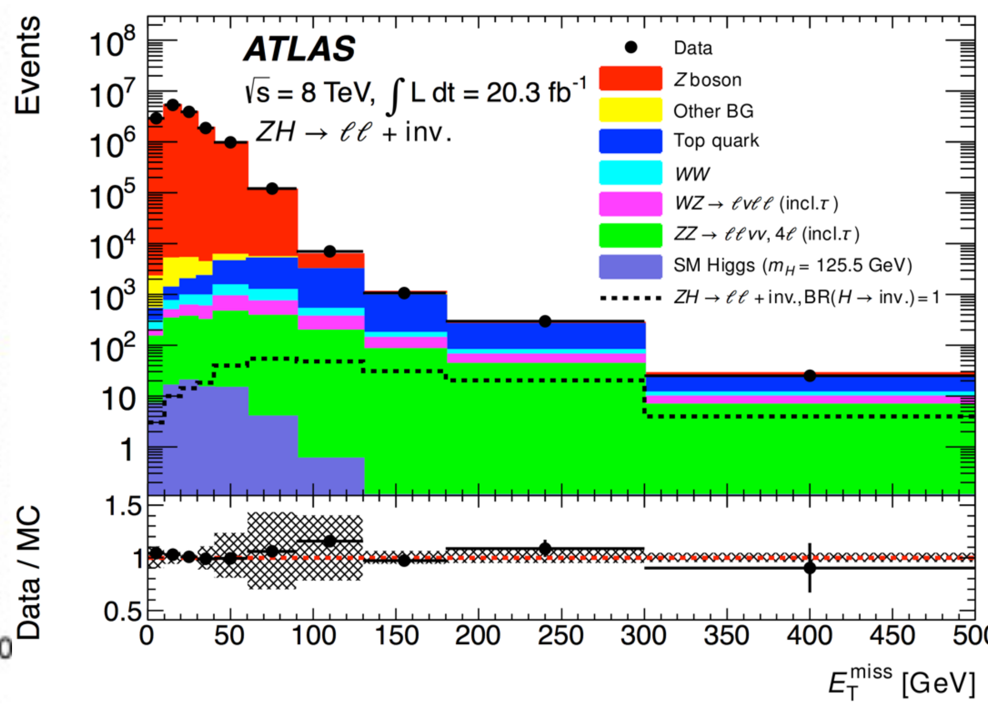
Shape

$Z\,H \to l^+l^- + inv.$

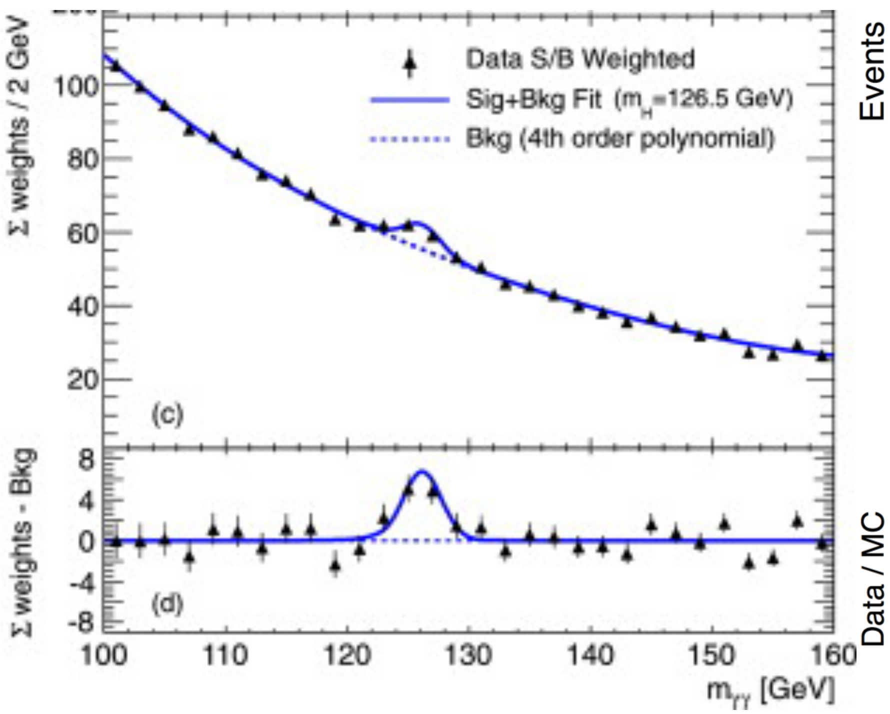


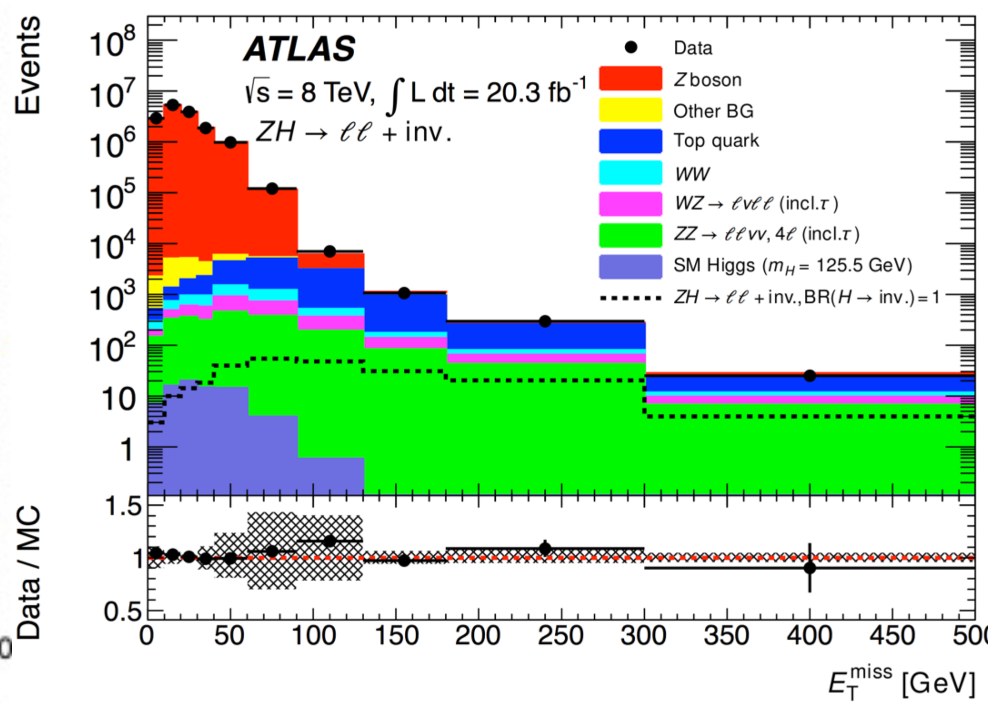
HARD

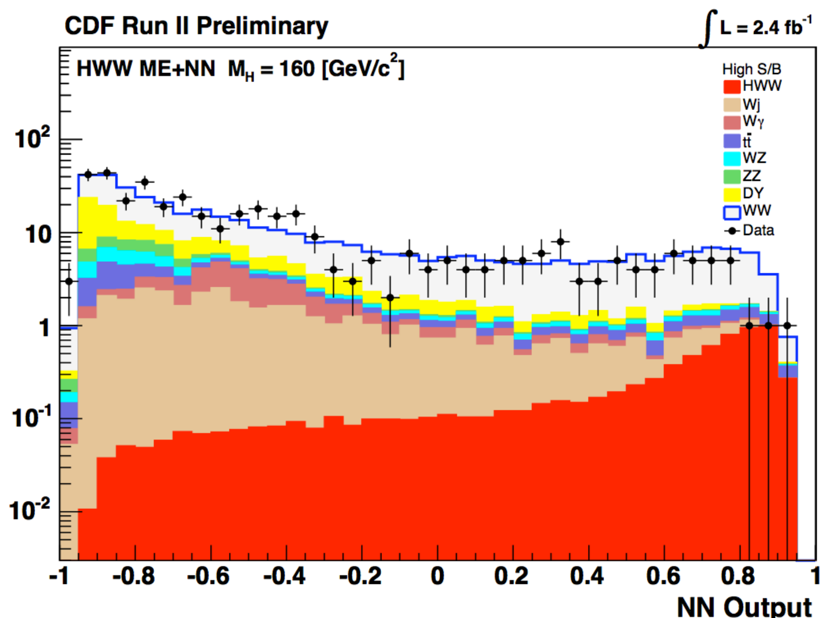
Background **SHAPE** needed.
Flexible MC for both signal and background validated and tuned to data

Discoveries at hadron colliders

Peak	Shape	Rate
$H \to \gamma\gamma$	$Z\,H \to l^+ l^- + inv.$	$H \to W^+ W^-$



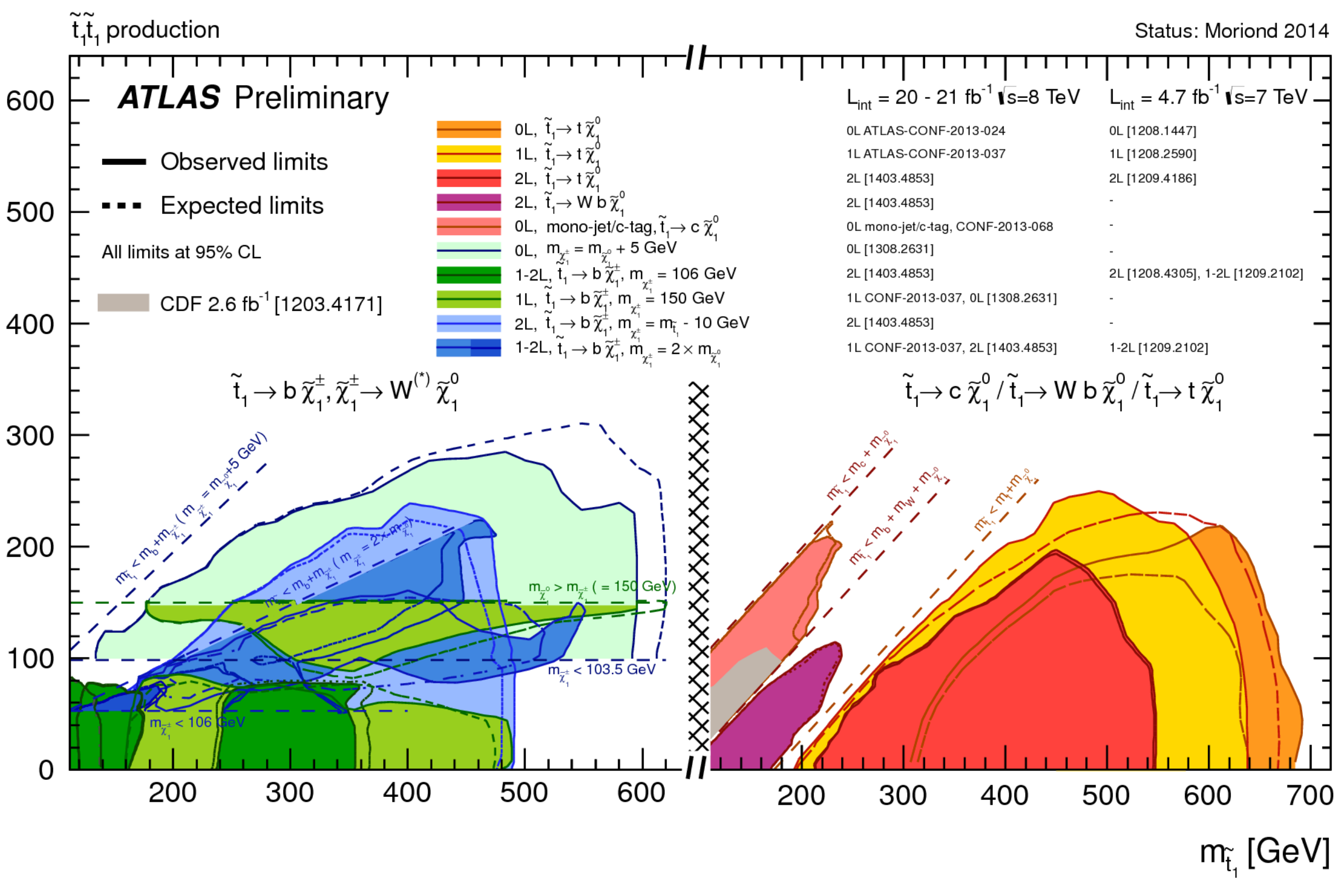


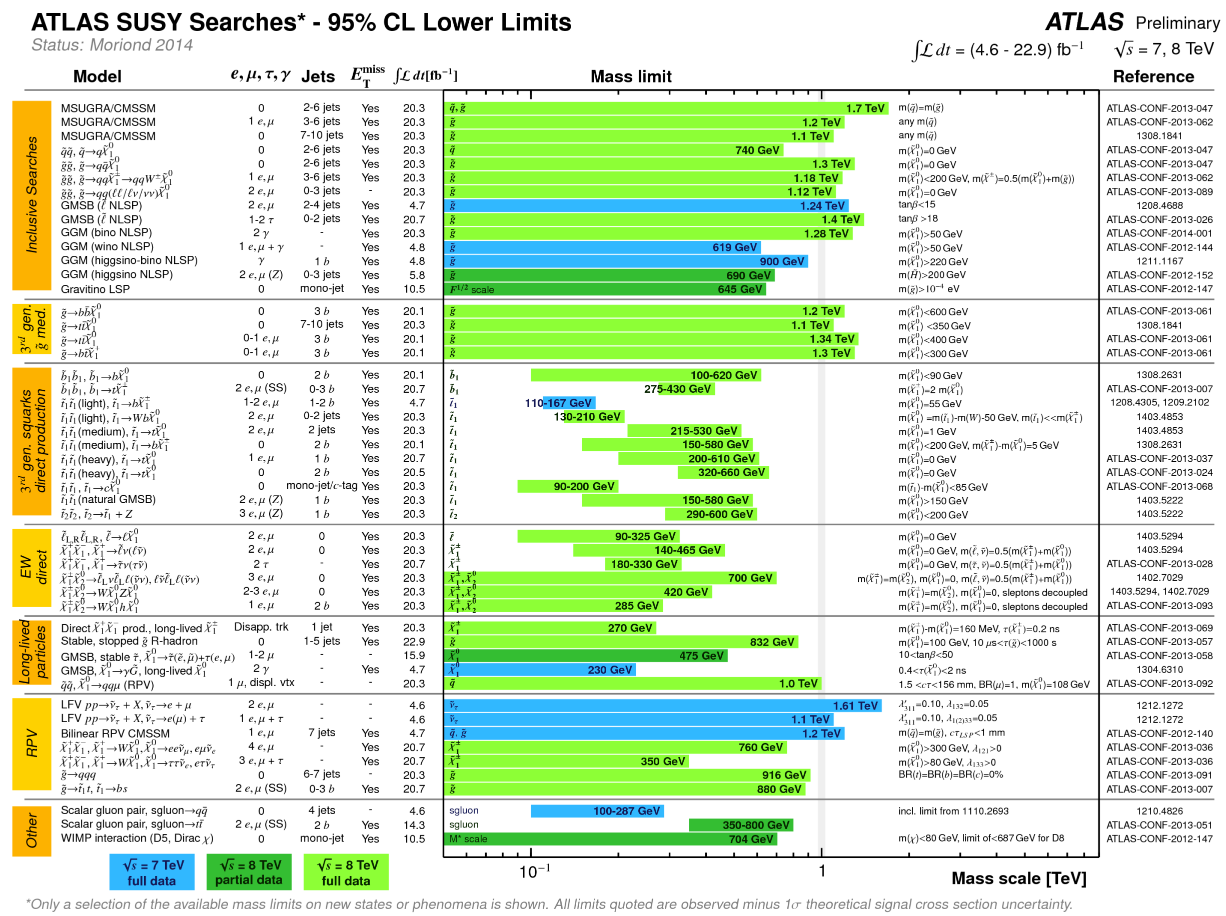


EASY	HARD	VERY HARD
Background directly measured from **data**. Theory needed only for parameter extraction	Background **SHAPE** needed. Flexible MC for both signal and background validated and tuned to data	Relies on prediction for both **shape** and **normalization**. Complicated interplay of best simulations and data

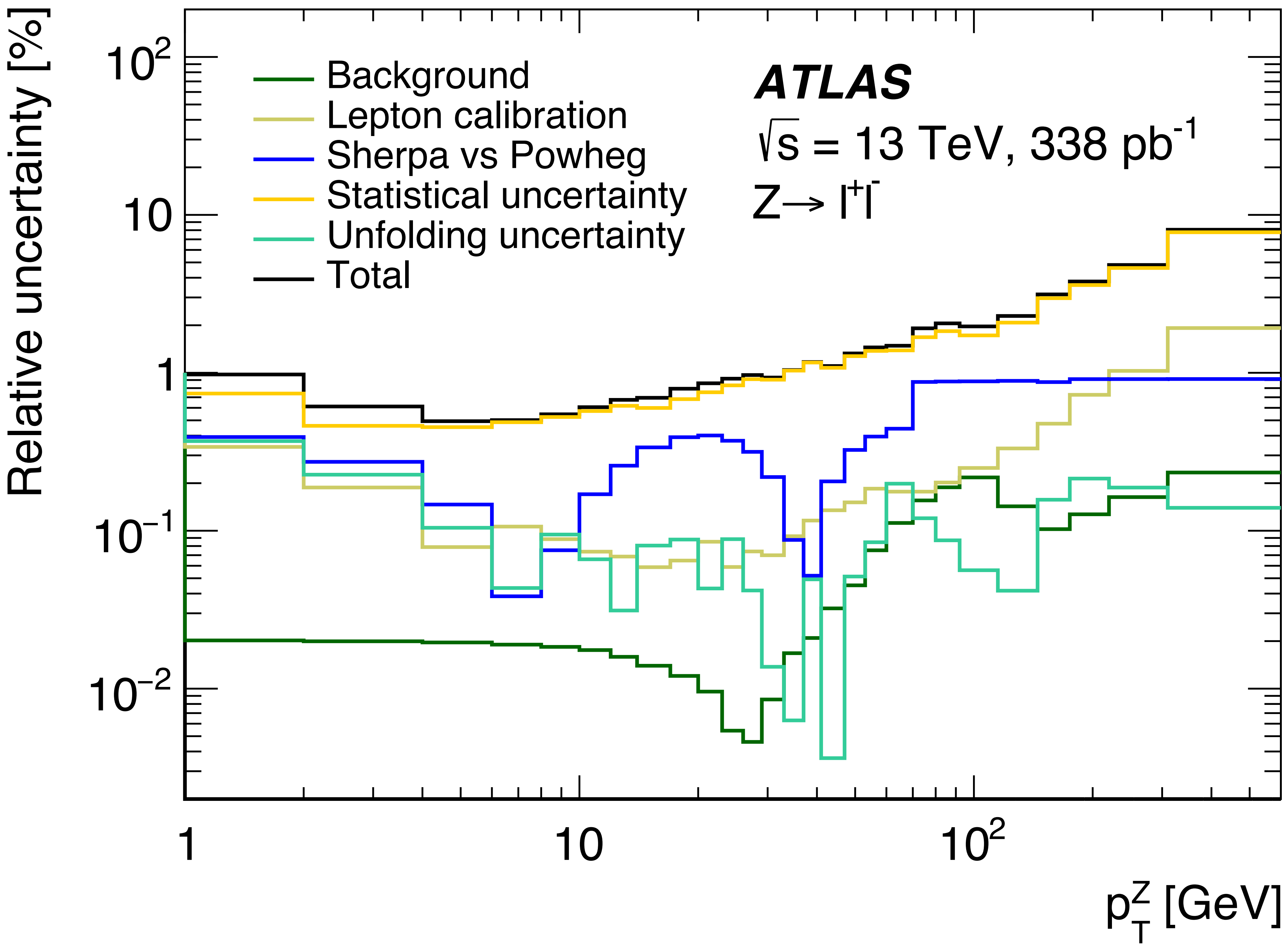
New physics?

- No NP has been discovered yet
- Either there is no NP, or it is hiding very well
- If it is there, it will be a ‘Hard’ or ‘very Hard’ discovery
  - Need for accurate predictions for signal and background



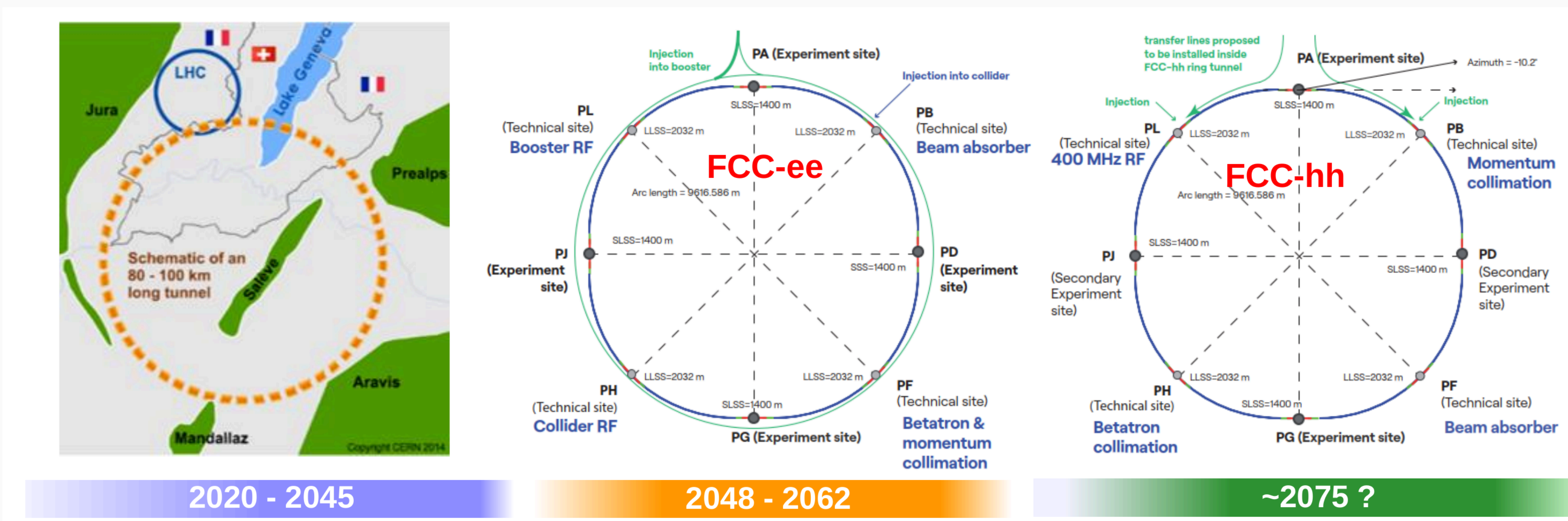


Precision at the LHC



Where this fits: the integrated FCC programme

- **FCC** = one 90.7 km tunnel, two machines staged.
  - **FCC-ee** — e^+e^- precision / Higgs / EW / flavour / top factory
  - **FCC-hh** — a $\sim$ 100 TeV pp energy frontier machine reusing the same tunnel.



M. Benedikt & F. Zimmermann, FCC Status & Plans for Reference Design Phase, *FCC Week 2026, Helsinki, 8 June 2026*

Takeaway. **FCC-ee** would run for $\sim$ 15 yr, starting a few years after HL-LHC ends, project decision in the next few years. **FCC-hh** as a possible future next step.

Future colliders

A staged run plan across four energies (Table 1)

Working point	Z pole	WW thr.	ZH	$t\bar{t}$ thr.	$t\bar{t}$
$\sqrt{s}$ (GeV)	88 / 91 / 94	157 / 163	240	340–350	365
Lumi / IP (10^{34})	140	20	7.5	1.8	1.4
Lumi / yr (ab^{-1})	68	9.6	3.6	0.83	0.67
Run time (yr)	4	2	3	1	4
$\int \mathcal{L}$ (ab^{-1})	205	19.2	10.8	0.42	2.70
Events	$6 \times 10^{12} Z$	$2.4 \times 10^{8} WW$	$2.2 \times 10^{6} ZH$	$2 \times 10^{6} t\bar{t}$	

Four heaviest SM particles, one machine: Z (91) · W (80) · H (125) · t (173) GeV.
Four interaction points — diversity, redundancy, $\frac{1}{\sqrt{N_{\text{exp}}}}$ statistical uncertainty (and some reduction/ complementarity in experimental systematics).

Implication. $10^5 \times$ more Z than LEP $\rightarrow$ statistical errors shrink $\sim$ 100–1000 $\times$. Every interpretation or theoretical input to the measurements now needs SM theory at that level.

Primary reference: 2025 FCC *Feasibility Study Report* Vol. 1 (arXiv:2505.00272) *Physics, Experiments, Detectors.*

Future colliders

The five physics pillars (and a discovery engine)

Electroweak — Z lineshape, $\sin^2\theta_{\text{eff}}$, m_W: ppm EWPOs, the precision backbone.

Higgs — recoil method, mass to a few MeV, per-mil κ_Z, model-independent Γ_H and couplings.

QCD — α_S to 0.1%, jets, fragmentation, hadronisation as a calibrated science.

Flavour — 10^{12} b/c/τ: rare decays, CKM, lepton universality, deep CP.

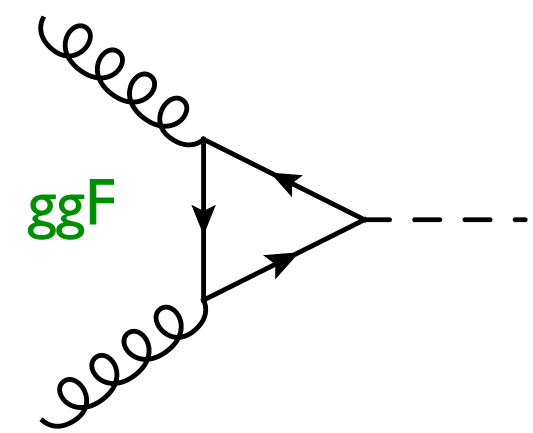
Top — threshold scan: m_t to 10 MeV, EW couplings, the EW-fit anchor.

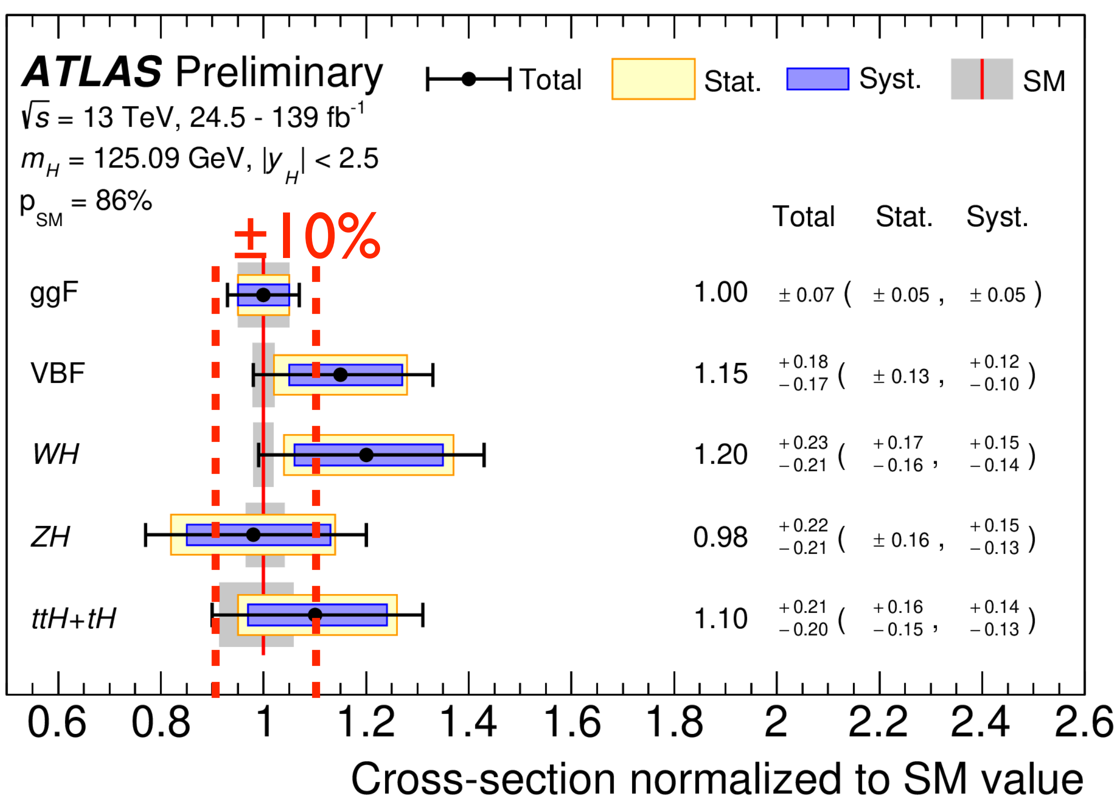
Discovery — indirect reach to 100 TeV (SMEFT) + direct: HNLs, ALPs, dark sectors.

Takeaway. Every pillar is **precision-driven** — and each one converts, at FCC-ee statistics, into a demand on the **theory** calculation that interprets it (or any corresponding theory inputs to the measurements).

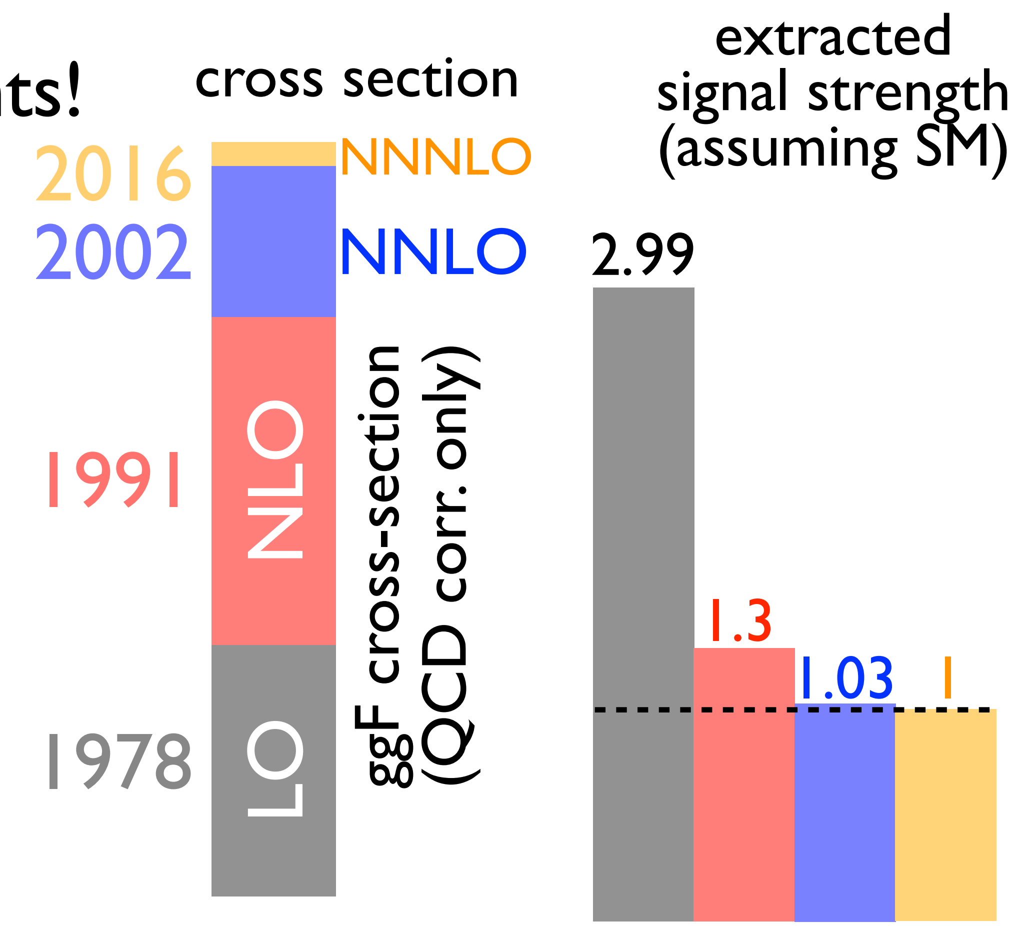
Cross-section measurements

- The discovery of the Higgs boson is an emblematic example of the need for precision
- Large perturbative corrections for the dominant channel (gluon fusion)
- Without higher-order corrections, measured signal strength ~3 * SM
- Very competitive experimental measurements!



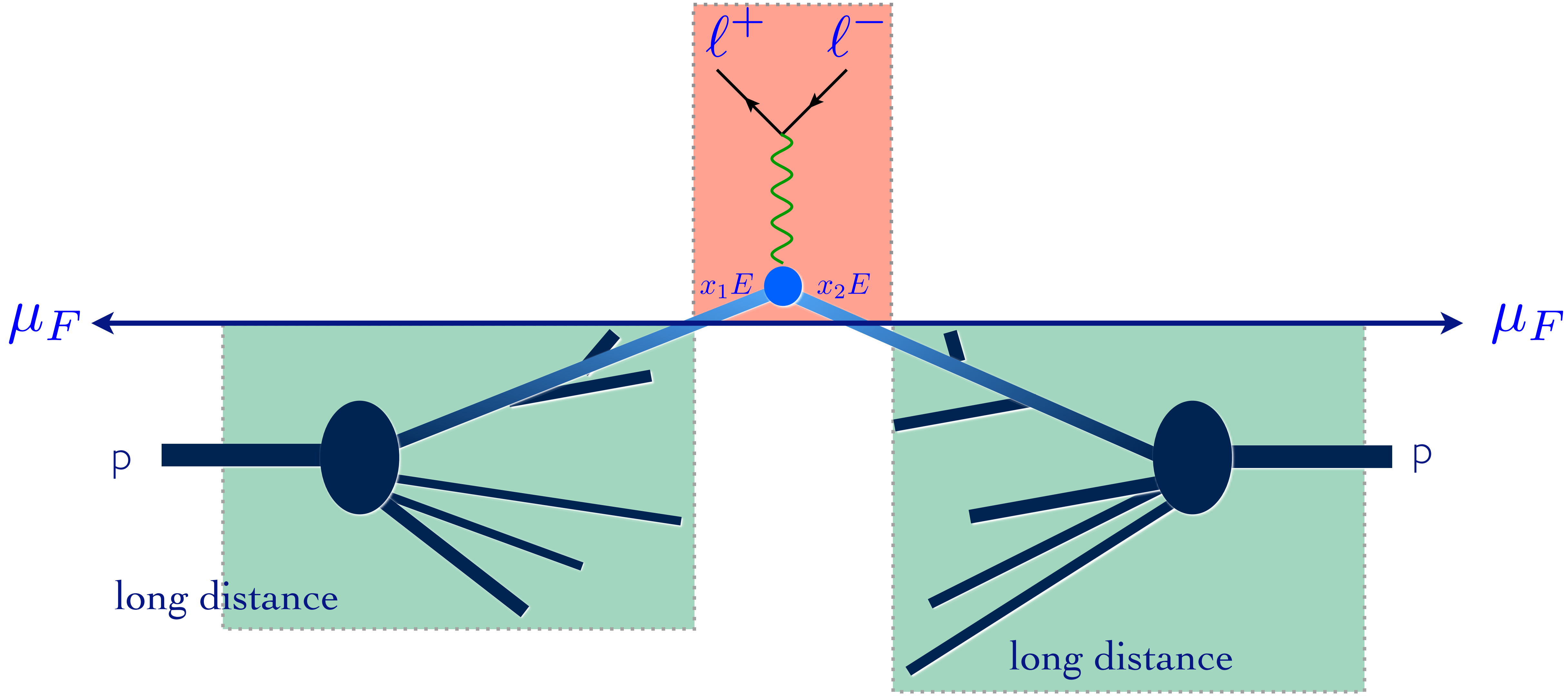


$$\mu = \frac{\sigma_{\mathrm{EXP}}}{\sigma_{\mathrm{TH}}}$$



Anastasiou et al, arXiv:1602.00695

How to compute a cross-section



$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat\sigma_{ab\to X}(\hat s, \mu_F, \mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

Perturbation theory at work

$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ Parton-level cross section

The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter

$$\hat{\sigma} = \alpha_s^b \sigma_0 + \alpha_s^{b+1}\sigma_1 + \alpha_s^{b+2}\sigma_2 + \alpha_s^{b+3}\sigma_3 + \dots$$

Remember:

$$\alpha_s = \alpha_s(\mu_R) \qquad \sigma_i = \sigma_i(\mu_R, \mu_F)$$

Coupling and cross section depend on *unphysical* scales

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LO

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LO

NLO

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LO NLO NNLO NNNLO

Remember:

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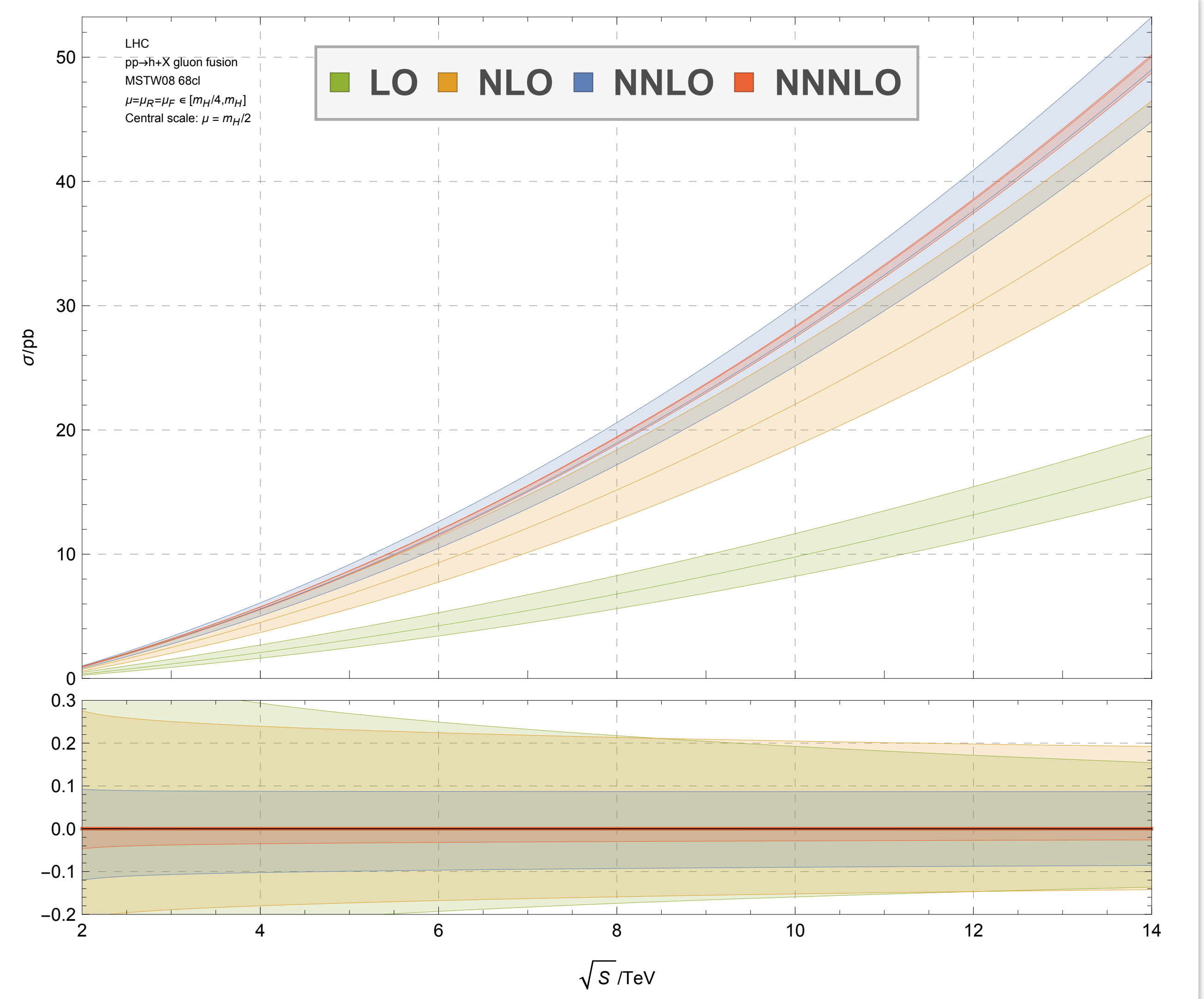
Coupling and cross section depend on *unphysical* scales

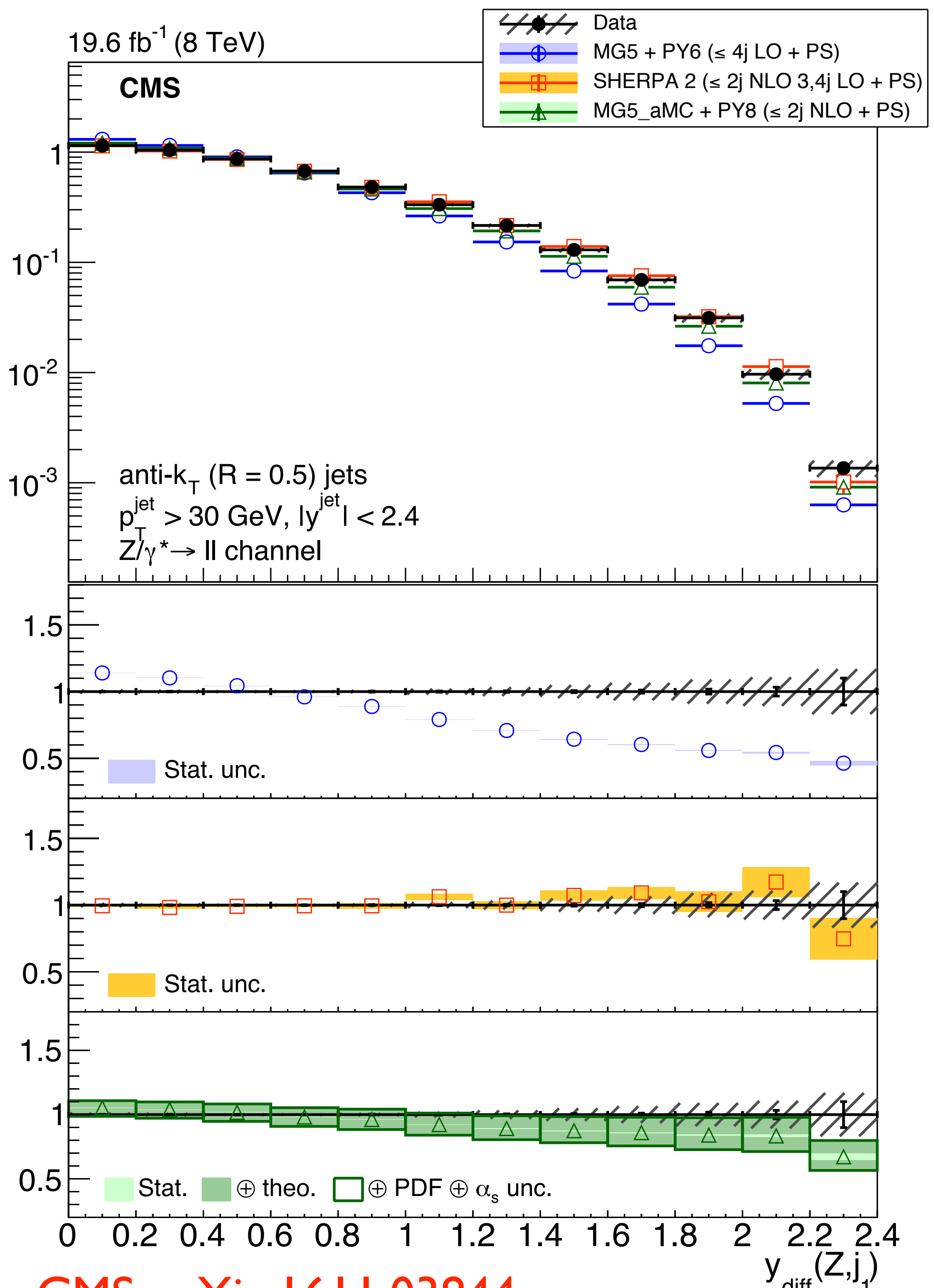
Perturbation theory at work

- The inclusion of higher orders improves the reliability of a given computation
  - More reliable description of total rates and shapes
  - Residual uncertainties related to the arbitrary scales in the process decrease
  - The computational complexity grows exponentially
  - NLO is mandatory for LHC physics!



Anastasiou, et al, arXiv:1503.06056

Perturbation theory at work



CMS, arXiv:1611.03844

- In order to describe data, LO predictions must be rescaled to match the cross section including higher orders (typically NNLO)
- NLO predictions are generally not rescaled →More predictive power
- NLO effects can be important even if merged samples are used at LO

In these lectures:

- How to compute effectively a NLO cross section?
  - How to deal with infrared divergences?
  - ~~How to compute loops?~~
  - How about EW corrections?

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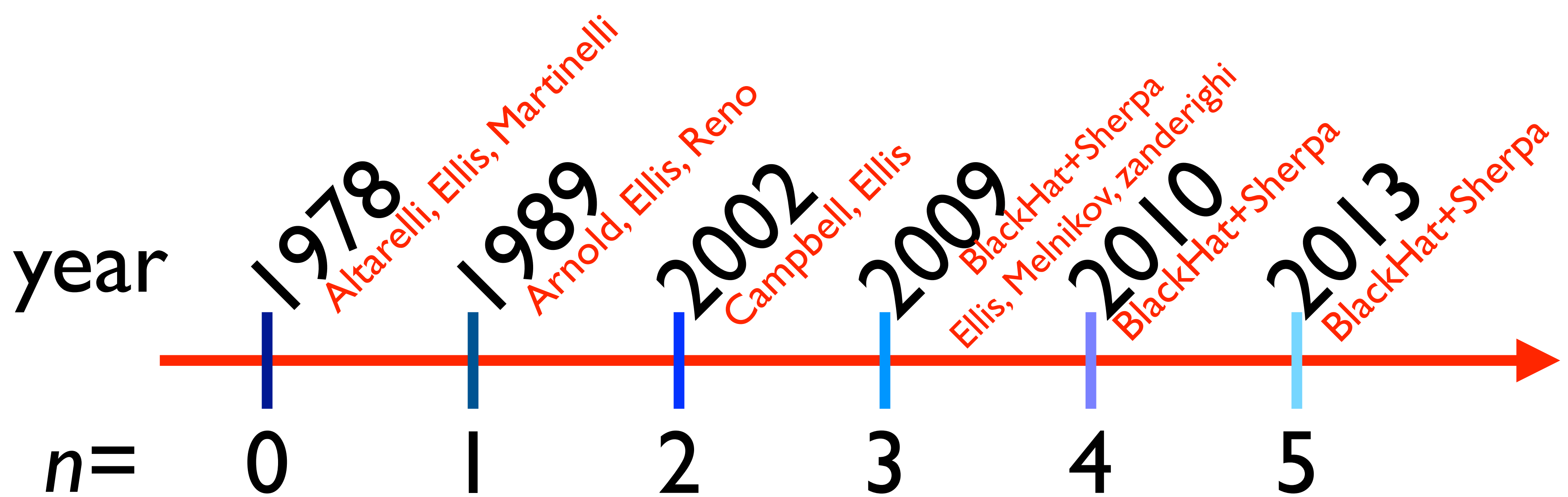
NLO (pre)history

- NLO evolution:
  - e.g. pp→W+n jets



#virt diag $u\bar{d} \to W^+$ ng	2	43	416	4489	57026	…

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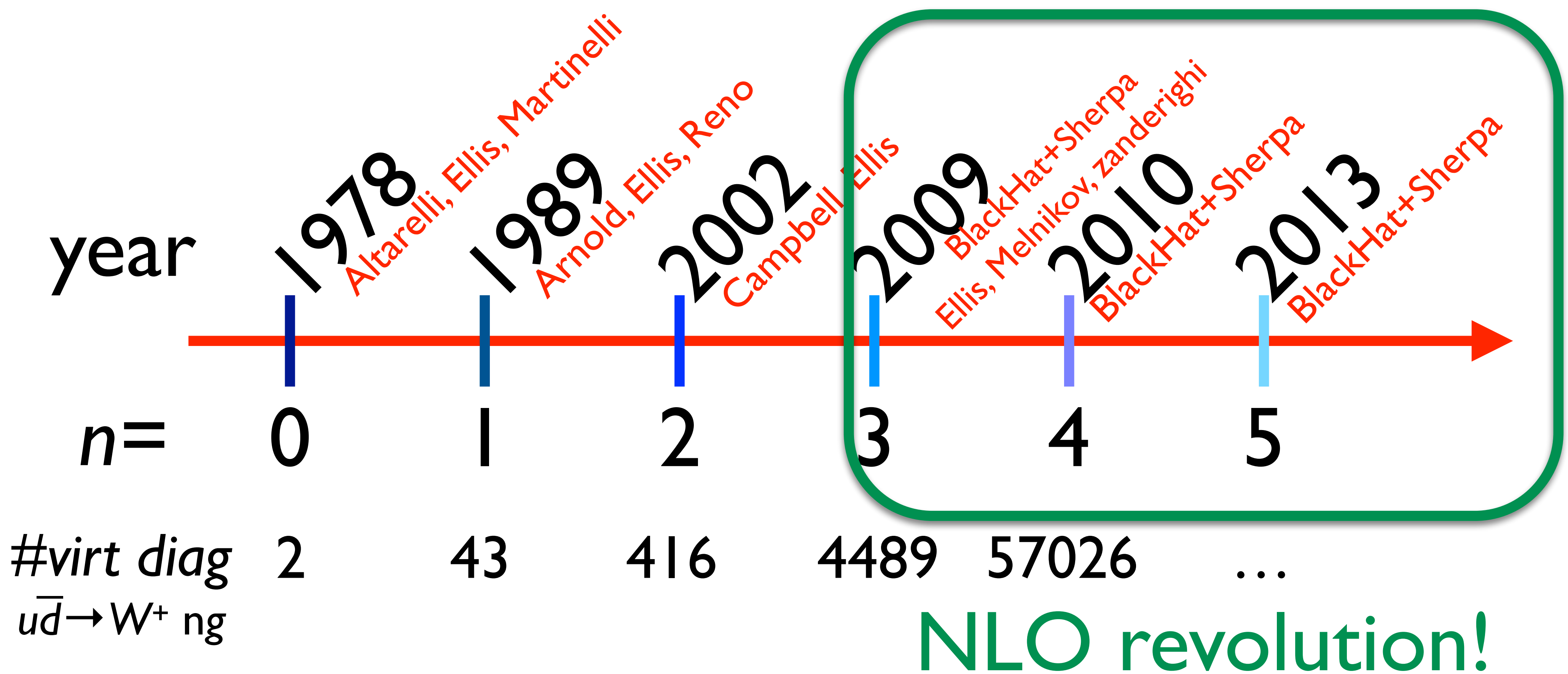

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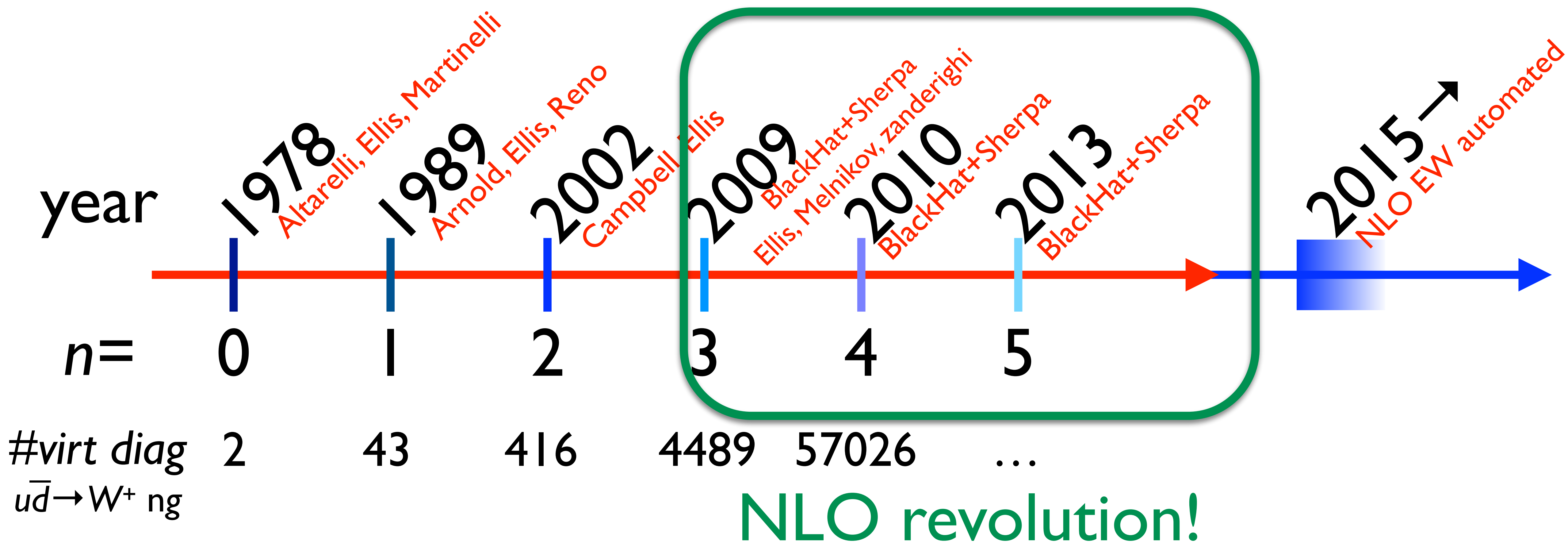


NLO revolution

- Amazing development of computational techniques to tackle *any* process at NLO
  - Local subtraction
- Computation of loop MEs
  - Tensor reduction
  - Generalized unitarity
  - Integrand reduction

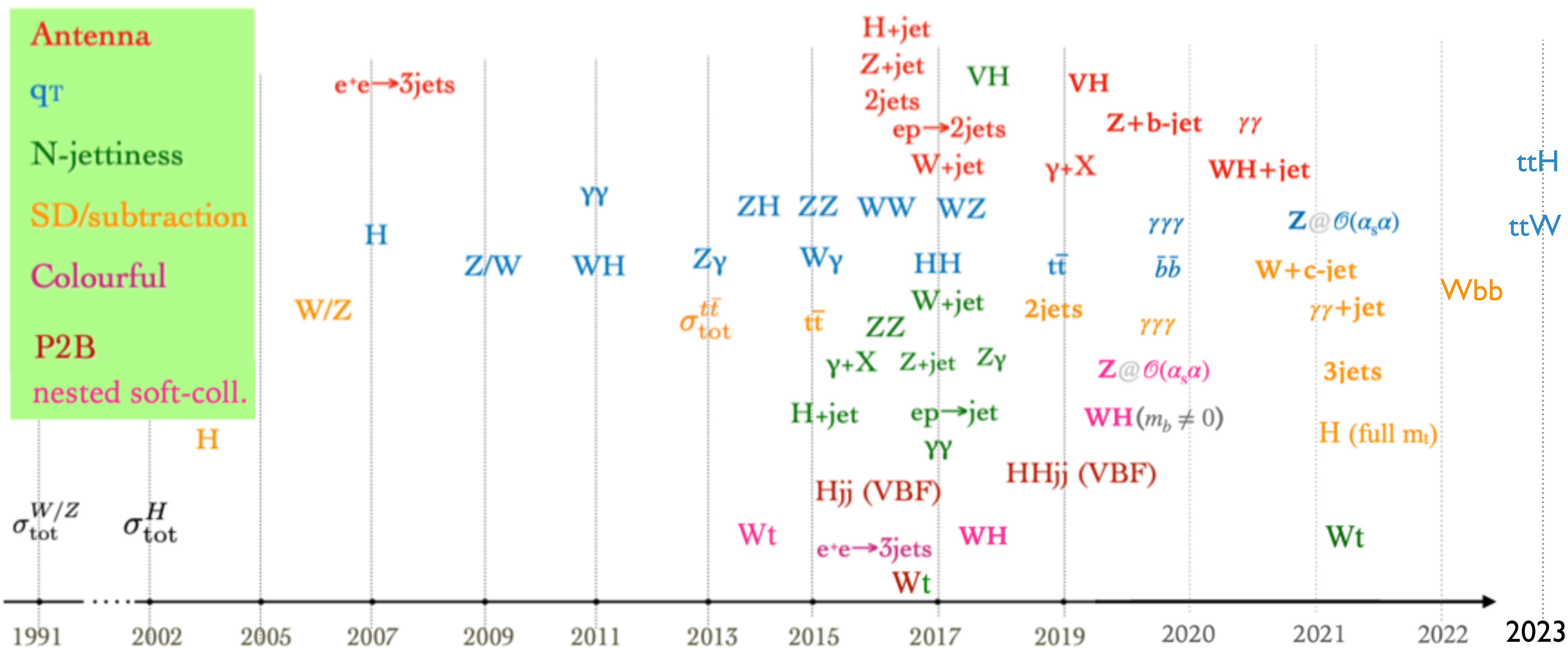
Frixione, Kunszt, Signer, hep-ph/9512328
Catani, Seymour, hep-ph/9605323

Passarino, Veltman, 1979
Denner, Dittmaier, hep-ph/509141
Binoth, Guillet, Heinrich, Pilon, Reiter, arXiv:0810.0992

Bern, Dixon, Dunbar, Kosower, hep-ph/9403226 + …
Ellis, Giele, Kunszt, arXiv:0708.2398
\+ Melnikov, arXiv:0806.3467

Ossola, Papadopoulos, Pittau, hep-ph/0609007
Del Aguila, Pittau, hep-ph/0404120
Mastrolia, Ossola, Reiter, Tramontano, arXiv:1006.0710

The NNLO revolution is happening now!



Adapted from G. Zanderighi @LHCP23

Going NLO

$$\hat{\sigma} = \alpha_s^b \sigma_0 + \alpha_s^{b+1} \sigma_1 + \alpha_s^{b+2} \sigma_2 + \alpha_s^{b+3} \sigma_3 + \ldots$$

- NLO is the first order where the scale dependence in α_s and PDFs is compensated by loop corrections
  - First reliable predictions for rates and uncertainties
- Better description of final state (inclusion of extra radiation)
- Opening of new partonic channels from real emissions
- Learning NLO technicalities will set the basis for us (you!) to tackle NNLO or beyond

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NLO: how to?

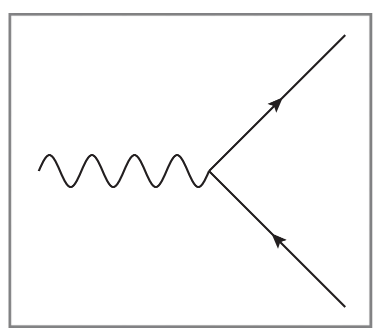
- Three ingredients need to be computed at NLO

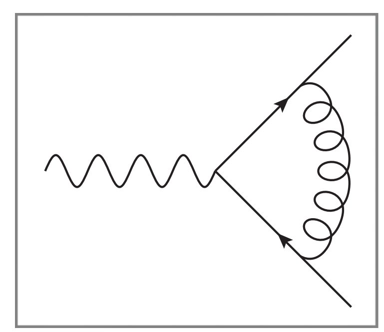
$$\sigma_{NLO} = \int_n \alpha_s^b d\sigma_0 + \int_n \alpha_s^{b+1} d\sigma_V + \int_{n+1} \alpha_s^{b+1} d\sigma_R$$

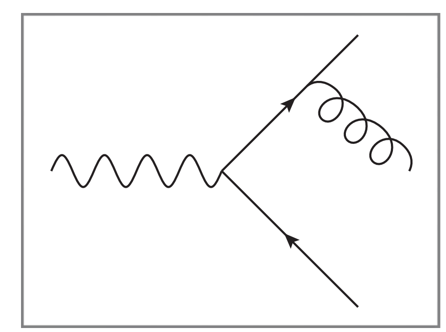
Born cross section

Virtual corrections

Real-emission corrections

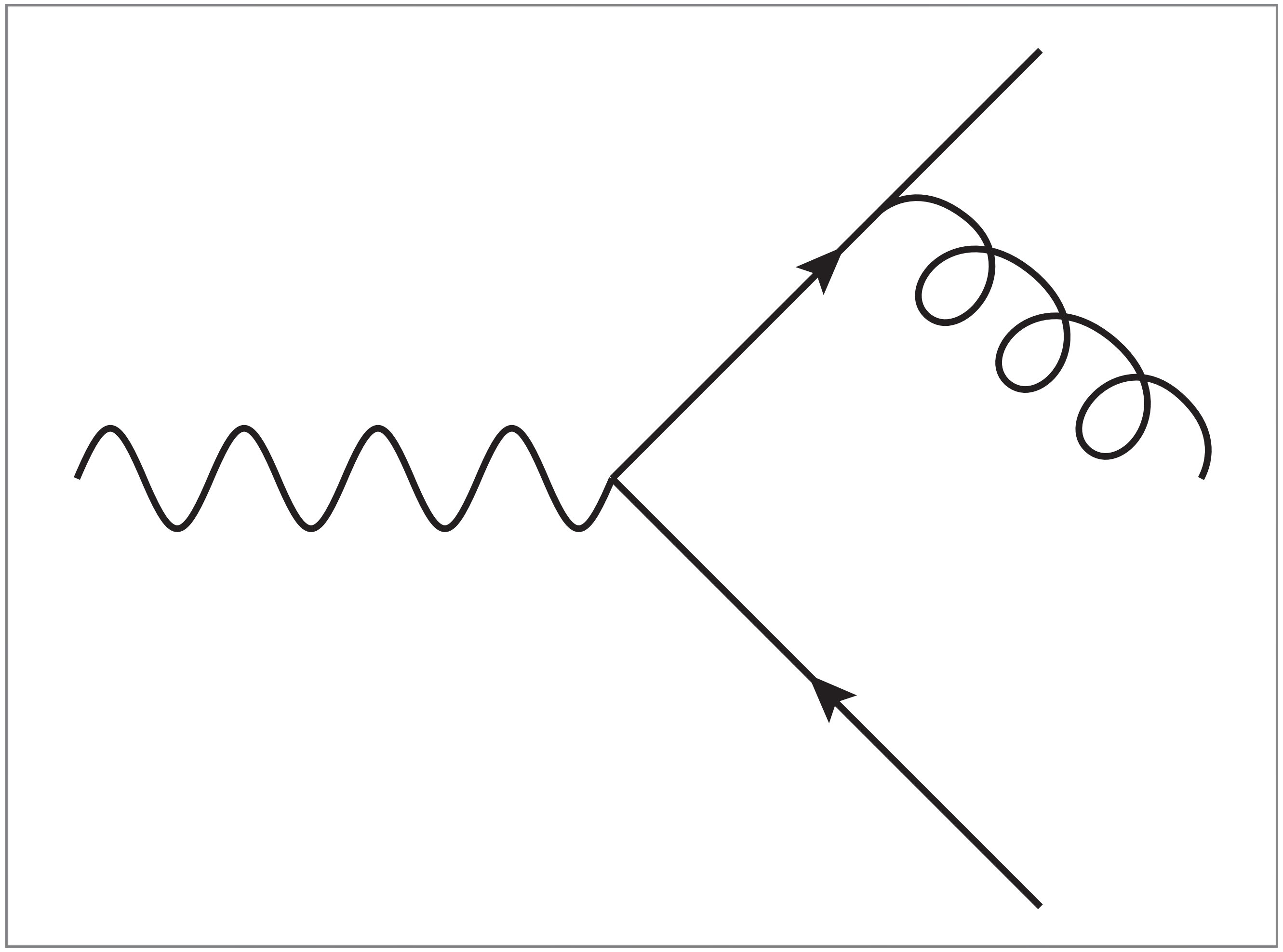






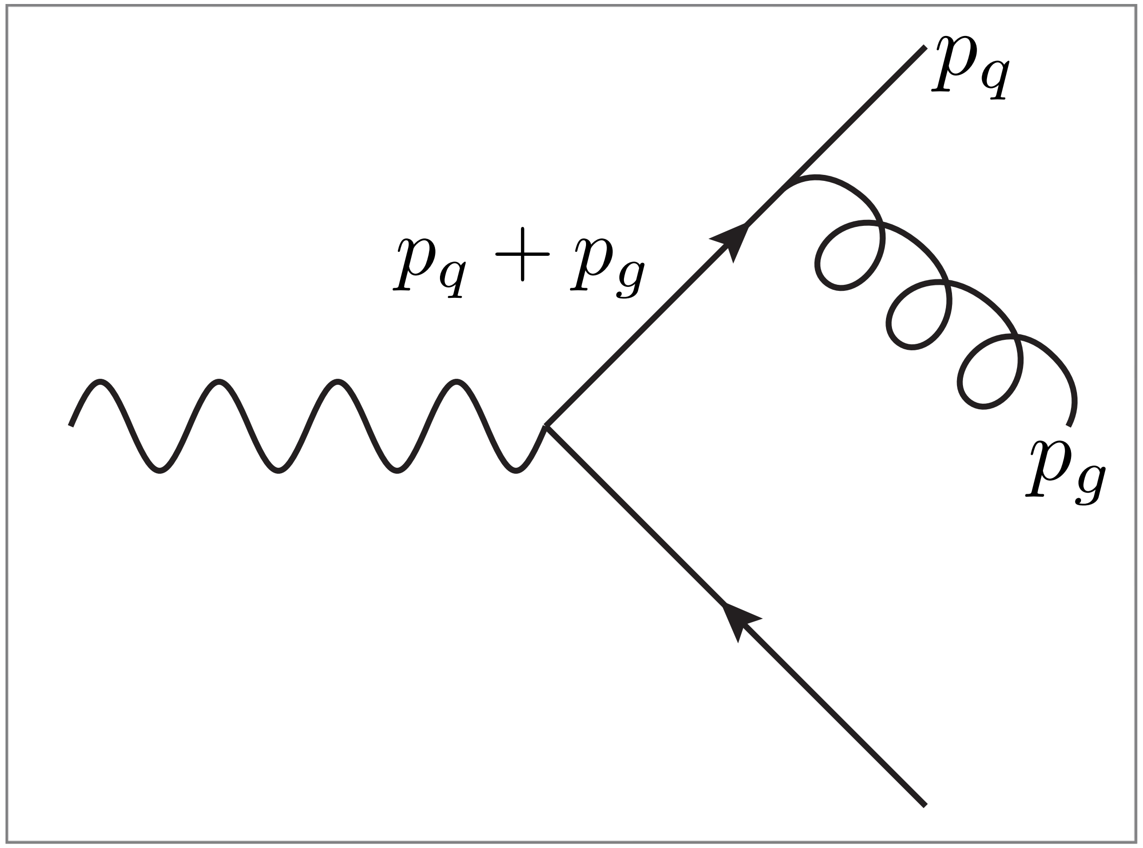
- Remember: virtual and reals are not separately finite, but their sum is (KLN theorem). Divergences have to be subtracted before numerical integration. We will shortly see how

Infrared divergences



$$\sigma_{NLO} = \int_n \alpha_s^b d\sigma_0 + \int_n \alpha_s^{b+1} d\sigma_V + \int_{n+1} \alpha_s^{b+1} d\sigma_R$$

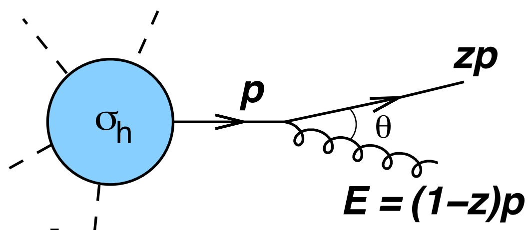
Branching



$$\int_{n+1} \alpha_s^{b+1} d\sigma_R$$

Singularities

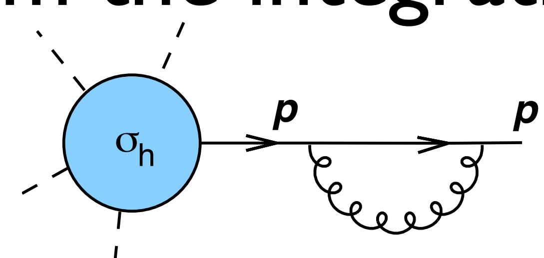
- Let us rewrite the branching of a gluon from a quark as



$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

Where k_t is the transverse momentum of the gluon $k_t = E\sin\theta$. It diverges in the soft ($z \to 1$) and collinear ($k_t \to 0$) region

- These singularities cancel with the virtual contribution, which comes from the integration of the loop momentum



$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

- The cancelation happens if we cannot distinguish between the case of no branching, and that of a soft/collinear branching

Cancellation of divergences

- The KLN theorem tells us that divergences from the virtual and real emission cancel in the sum *if observables are insensitive to soft and collinear branchings* (IR-safety)
- When doing an analytic computation in dimensional regularisation, divergences appear as poles in the regularisation parameter ε
- In the real emissions, poles appear *after* the phase space integration in d dimension

Infrared safety

- In order to have meaningful predictions in fixed-order perturbation theory, observables must be IR-safe, *i.e.* not sensitive to the emission of soft or collinear partons.
- In particular, if an observable depends on the momentum p_i, it must not be sensitive on the branching $p_i \to p_j + p_k$, where either p_j is soft or p_j and p_k are collinear
- For example
  - The number of gluons in an event
  - The number of jets with $p_T > p_T^{min}$
  - The hardest parton in an event
  - The hardest jet

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- For example
  - The number of gluons in an event is not IR-safe ❌
  - The number of jets with $p_T > p_T^{min}$ is IR-safe ✅
  - The hardest parton in an event is not IR-safe ❌
  - The hardest jet is IR-safe ✅

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Phase space integration

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} + \int d^4\Phi_n \mathcal{V} + \int d^4\Phi_{n+1} \mathcal{R}$$

contains $\int d^d l$

- For complicated processes the integrations have to be done via MonteCarlo techniques, in an integer number of dimensions
- Divergences have to be canceled explicitly
- Slicing/Subtraction methods have been developed to extract divergences from the phase-space integrals

Example

- Suppose that we can cast the phase space integral in the form

$$\int_0^1 dx f(x) \quad \text{with} \quad f(x) = \frac{g(x)}{x} \quad \text{and } g(x) \text{ a regular function}$$

- We introduce a regulator which renders the integral finite

$$\int_0^1 dx x^{\varepsilon} f(x) = \int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- The divergence will turn into a pole in ε. How can we extract the pole?

Phase space slicing

$$\lim_{\varepsilon\to 0}\int_0^1 dx x^{\varepsilon} f(x) = \lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- We introduce a small parameter $\delta \ll 1$:

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Phase space slicing

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&\simeq \lim_{\varepsilon\to 0}\left(\int_0^{\delta} dx \frac{g(0)}{x^{1-\varepsilon}} + \int_{\delta}^1 dx \frac{g(x)}{x^{1-\varepsilon}}\right) \\
&= \lim_{\varepsilon\to 0}\frac{\delta^{\varepsilon}}{\varepsilon} g(0) + \int_{\delta}^1 dx \frac{g(x)}{x}
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pole in ε

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pole in ε

finite integral
(can be computed numerically)

Subtraction method

$$\lim_{\varepsilon\to 0}\int_0^1 dx x^{\varepsilon} f(x) = \lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- Add and subtract $g(0)/x$

$$\lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}} = \lim_{\varepsilon\to 0}\int_0^1 dx x^{\varepsilon}\left(\frac{g(0)}{x} + \frac{g(x)}{x} - \frac{g(0)}{x}\right)$$

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finite integral
(can be computed numerically)

pole in ε

Slicing vs Subtraction

- In both cases the pole is extracted and we end up with a finite remainder:

$$g(0)\log\delta + \int_\delta^1 dx \frac{g(x)}{x} \qquad \int_0^1 dx \frac{g(x)-g(0)}{x}$$

- Subtraction acts like a plus distribution
- Slicing works only for small δ: δ-independence of cross section and distributions must be proven; subtraction is exact
- Both methods have cancelations between large numbers. If for a given observable $\lim_{x\to 0} O(x) \neq O(0)$ or we choose a too small bin size, instabilities will arise (we cannot ask for an infinite resolution)
- Subtraction is in general more flexible: good for automation

NLO with subtraction

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} + \int d^4\Phi_n \mathcal{V} + \int d^4\Phi_{n+1} \mathcal{R}$$

- With the subtraction terms the expression becomes

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B}$$

Integrating C is much simpler than than R
Can be done in D=D dimension (once and for all)

$$+ \int d^4\Phi_n \left(\mathcal{V} + \int d^d\Phi_1 \mathcal{C} \right)_{\varepsilon\to 0}$$

$$+ \int d^4\Phi_{n+1} \left(\mathcal{R} - \mathcal{C} \right)$$

- Terms in brackets are finite and can be integrated numerically in *d*=4 and independently one from another

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The subtraction term

- The subtraction term C should be chosen such that:
  - It exactly matches the singular behaviour of R
  - It can be integrated numerically in a convenient way
  - It can be integrated exactly in d dimension, leading to the soft and/or collinear poles in the dimensional regulator
  - It is process independent (overall factor times Born)

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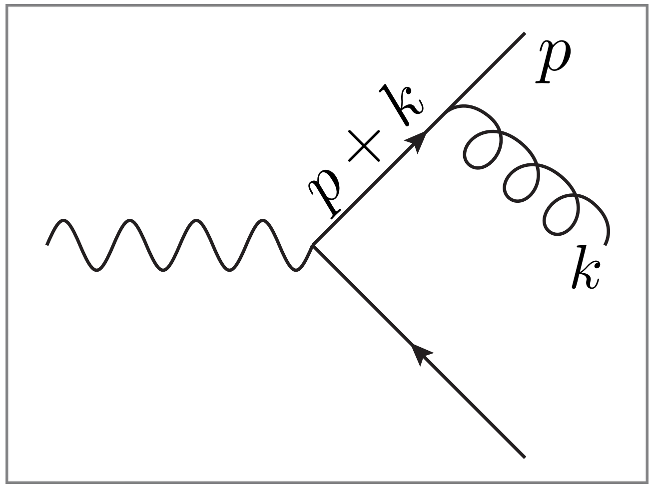
  - It can be integrated exactly in d dimension, leading to the soft and/or collinear poles in the dimensional regulator
  - It is process independent (overall factor times Born)
- QCD comes to help: structure of divergences is universal:



$$(p+k)^2 = 2E_p E_k (1-\cos\theta_{pk})$$

- Collinear singularity:

$$\lim_{p//k} |M_{n+1}|^2 \simeq |M_n|^2 \; P^{AP}(z)$$

- Soft singularity:

$$\lim_{k\to 0} |M_{n+1}|^2 \simeq \sum_{ij} |M_n^{ij}|^2 \frac{p_i p_j}{p_i k \; p_j k}$$

Two subtraction methods

Dipole subtraction

Catani, Seymour, hep-ph/9602277 & hep-ph/9605323

- Recoil taken by one parton $\rightarrow N^3$ scaling
- Method evolves from cancelation of soft divergences
- Proven to work for simple and complicated processes
- Automated in MadDipole, AutoDipole, Sherpa, Helac-NLO, …

FKS subtraction

Frixione, Kunszt, Signer, hep-ph/9512328

- Recoil distributed among all particles $\rightarrow N^2$ scaling
- Method evolves from cancelation of collinear divergences
- Proven to work for simple and complicated processes
- Automated in MadGraph5_aMC@NLO and in the Powheg box/Powhel

FKS subtraction #1
Phase space partition

- Let us consider the real emission

$$d\sigma_R = \left|M^{n+1}\right|^2 d\Phi_{n+1}$$

- The matrix element $|M^{n+1}|^2$ diverges as

$$\left|M^{n+1}\right| \sim \frac{1}{\xi_i^2}\frac{1}{1-y_{ij}}$$

$$\xi_i = \frac{2E_i}{\sqrt{s}}$$

$$y_{ij} = \cos\theta_{ij}$$

- Partition the phase space in order to have at most one soft and one collinear singularity

$$d\sigma_R = \sum_{ij} S_{ij}\left|M^{n+1}\right|^2 d\Phi_{n+1} \qquad \sum_{ij} S_{ij} = 1$$

$$S_{ij} \to 1 \text{ if } k_i \cdot k_j \to 0 \qquad S_{ij} \to 0 \text{ if } k_{m\neq i} \cdot k_{n\neq j} \to 0$$

Divide et impera

- Real emissions can have singularities in different regions of the phase-space:
- E.g. $gg \to t\bar{t}g$:
  - g collinear to g or g
  - g soft
- Numeric integrators (VEGAS) are quite dumb (still, that is the best one can do): peaks need to be well aligned with the integration variables
- *"Divide et impera"* solution: integrate one singularity at the time, with the most suitable phase-space parameterisation:

$$|M|^2 = \sum_{ij} S_{ij} |M|^2 = \sum_{ij} |M|^2_{ij} \qquad \sum S_{ij} = 1$$

$$S_{ij} \to 1 \text{ if } k_i \cdot k_j \to 0 \qquad S_{ij} \to 0 \text{ if } k_{m\neq i} \cdot k_{n\neq j} \to 0$$

FKS subtraction #2
Plus prescriptions

- Use plus prescriptions in y_{ij} and ξ_i to subtract the divergences

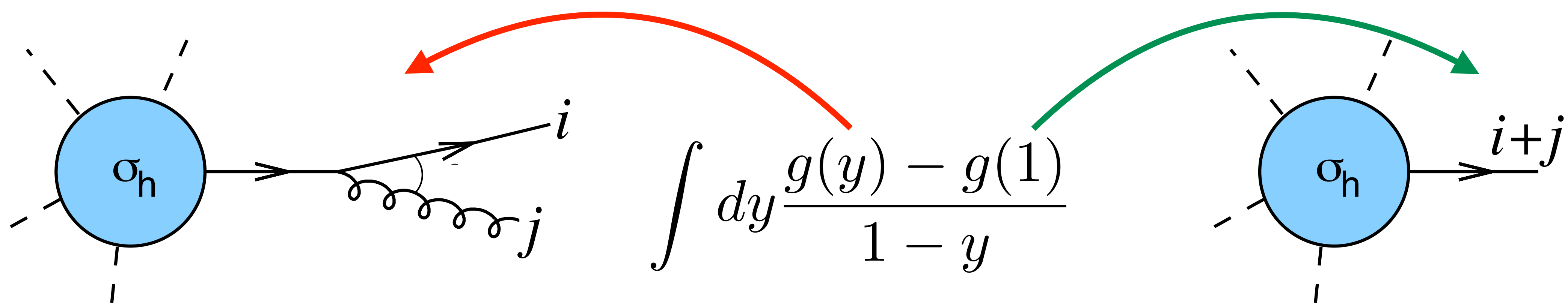
$$d\sigma_{\tilde{R}} = \sum_{ij} \left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i (1-y_{ij}) S_{ij} \left|M^{n+1}\right|^2 d\Phi_{n+1}$$

- Plus prescriptions are defined as

$$\int d\xi \left(\frac{1}{\xi}\right)_+ f(\xi) = \int d\xi \frac{f(\xi)-f(0)}{\xi} \qquad \int dy \left(\frac{1}{1-y}\right)_+ g(y) = \int dy \frac{g(y)-g(1)}{1-y}$$

- Maximally three counterevents are needed
  - Soft counterevent ($\xi_i \longrightarrow 0$)
  - Collinear counterevents ($y_{ij} \longrightarrow 1$)
  - Soft-collinear counterevents ($\xi_i \longrightarrow 0$ and $y_{ij} \longrightarrow 1$)
- The counterevents will feature the *same* kinematics

Kinematics of counterevents



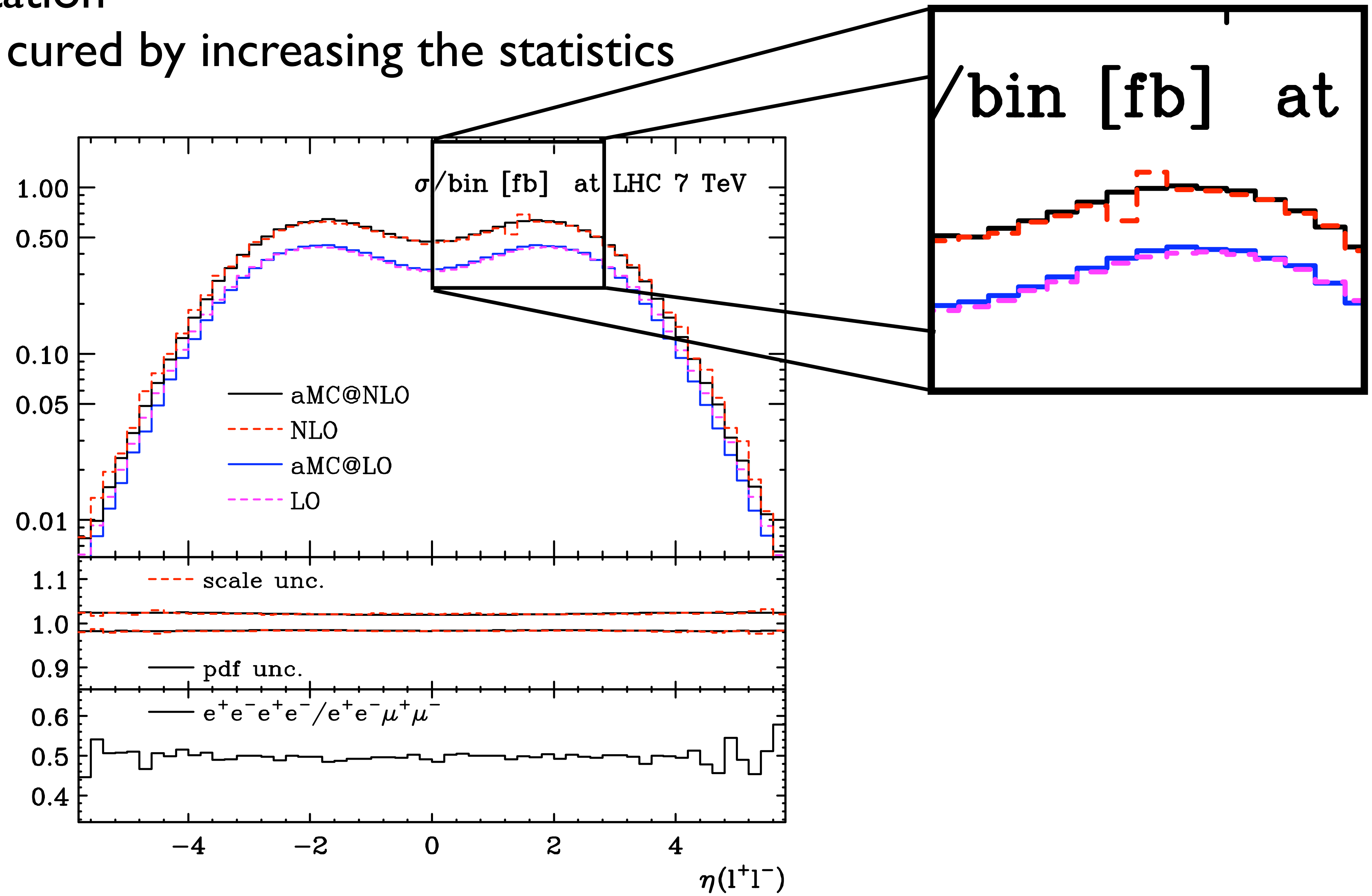
$$\int dy \frac{g(y) - g(1)}{1 - y}$$

Real emission Subtraction term

- If i and j are on-shell in the event, for the counterevent the combined particle $i+j$ must be on shell
- $i+j$ can be put on shell only be reshuffling the momenta of the other particles
- It can happen that event and counterevent end up in different histogram bins
  - Use IR-safe observables and don't ask for infinite resolution!
  - Still, these precautions do not eliminate the problem…

An example in 4-lepton production

- The NLO result shows the typical peak-dip structure that hampers fixed-order computation
- Can be cured by increasing the statistics



Can we generate unweighted events at NLO?

- Another consequence of the kinematic mismatch is that we cannot generate events at NLO
- $n+1$-body contribution and n-body contribution are not bounded from above → unweighting not possible
- Further ambiguity on which kinematics to use for the unweighted events

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More tomorrow

Filling histograms on-the-fly

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B}$$
$$+ \int d^4\Phi_n \left(\mathcal{V} + \int d^d\Phi_1 \mathcal{C}\right)_{\varepsilon\to 0}$$
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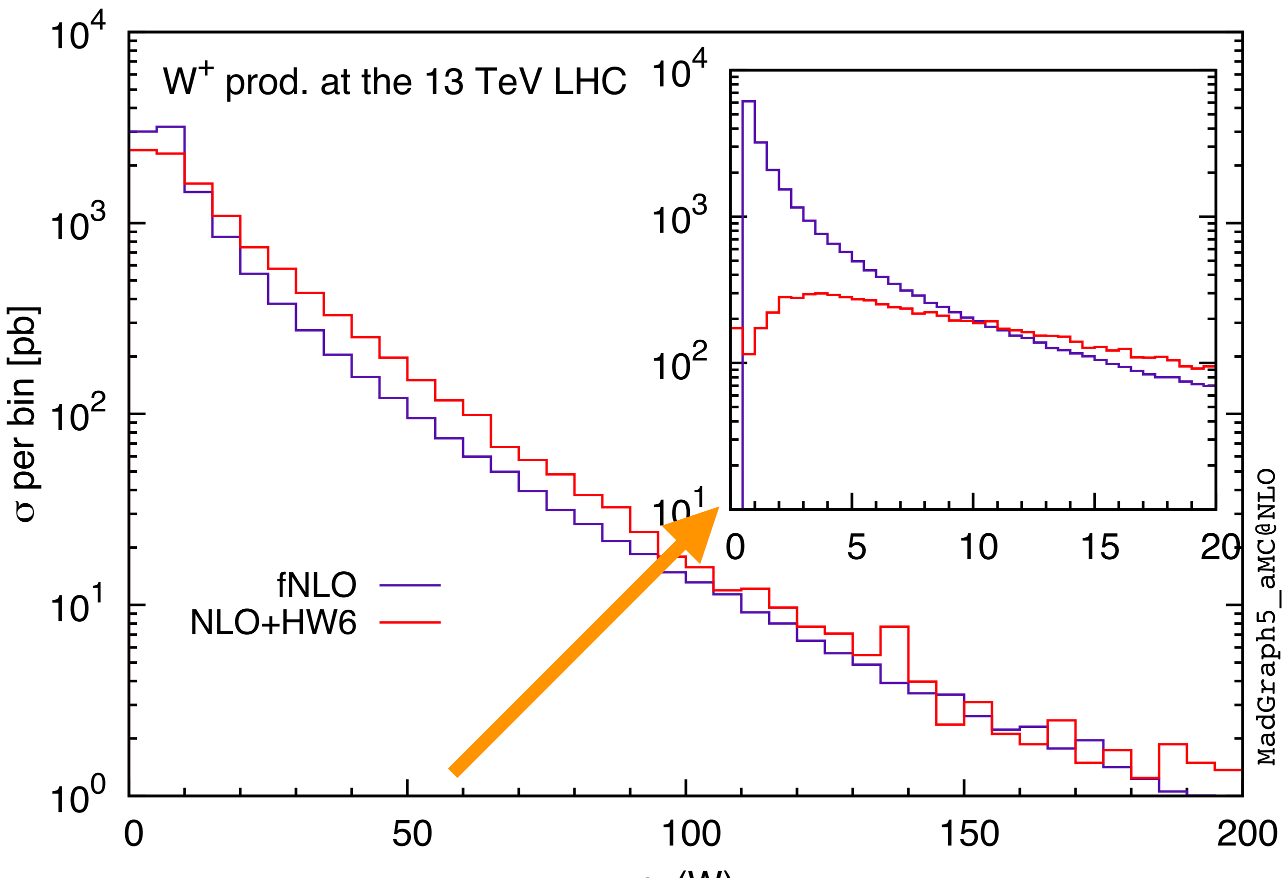
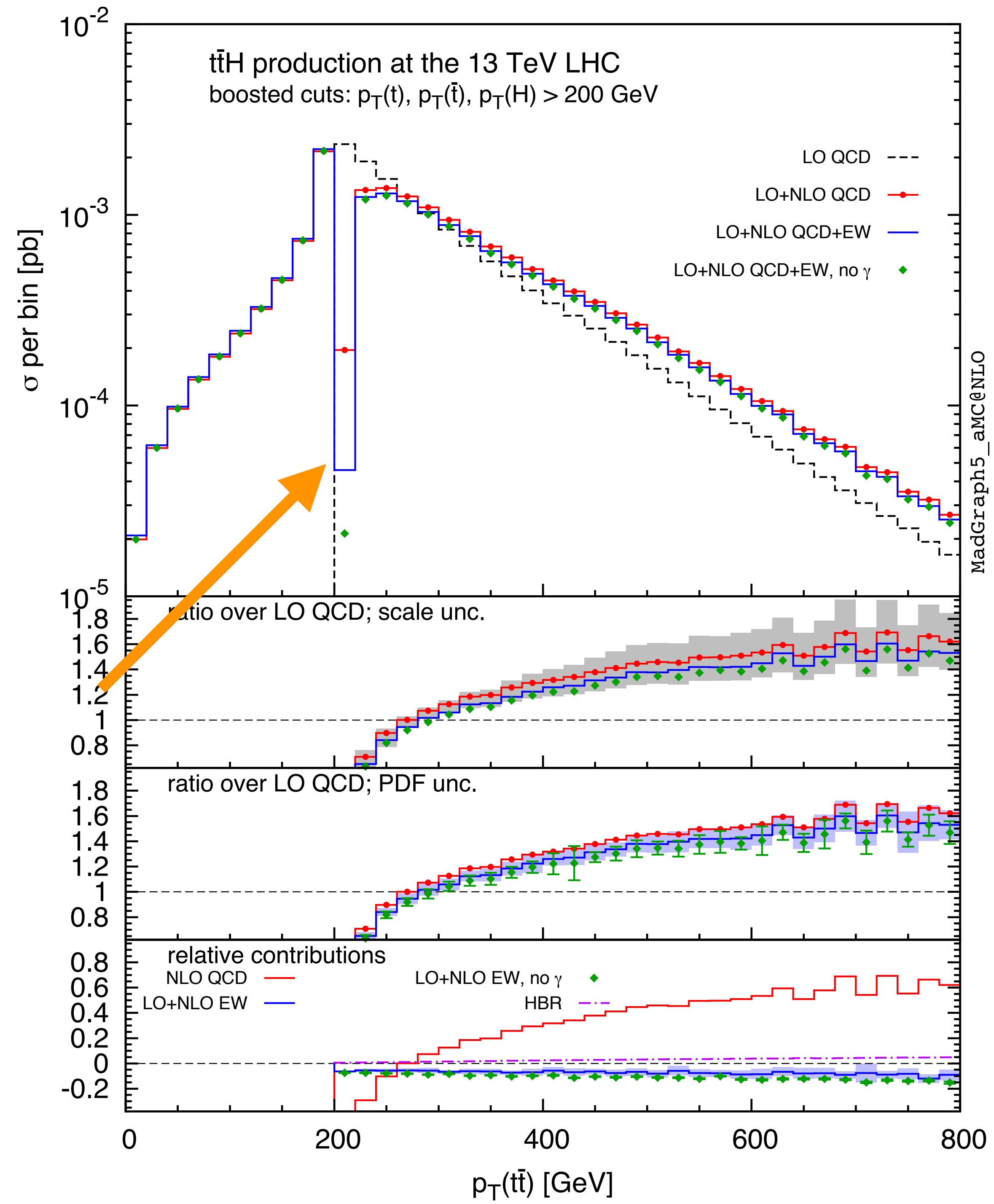
- In practice, two set of momenta are generated during the MC integration
  - One (or more) n-body set(s), for Born, virtuals and counterterms
  - One n+1-body set, for the real emission
- The various terms are computed. Cuts are applied on the corresponding momenta and histograms are filled with the weight and kinematics of each term

Instabilities at fixed order

- Besides the mis-binning problem, the kinematics mismatch can lead to odd behaviours of certain observables, in particular when some constraint coming from the n-body kinematics is relaxed in the $n+1$-body one





Subtracting IR divergences: Summary

- Virtual and real matrix element are not finite, but their sum is. Subtraction methods can be used to extract divergences for real-emission matrix elements and cancel explicitly the poles from the virtuals
- Event and counterevents have different kinematics. Unweighting is not possible, we need to fill plots on-the-fly with weighted events
- For plots, only IR-safe observable with finite resolution must be used!

Intermezzo: Is everything NLO accurate?

- Suppose we have a code for pp→$t\bar{t}$ @NLO. Are all the following (IR-safe) variables described at NLO?
  - top p_T
  - $t\bar{t}$ pair p_T
  - $t\bar{t}$ pair invariant mass
  - jet (extra parton) p_T
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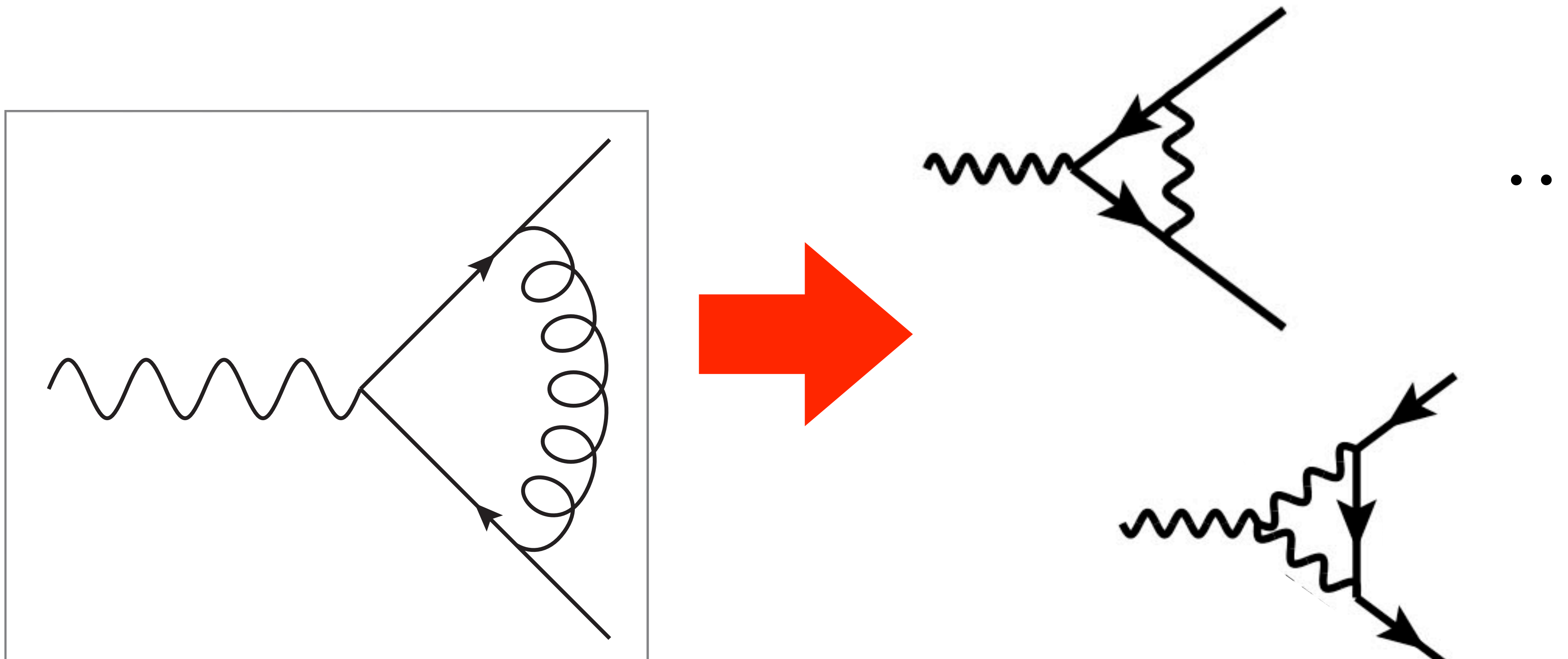
From QCD to EW corrections

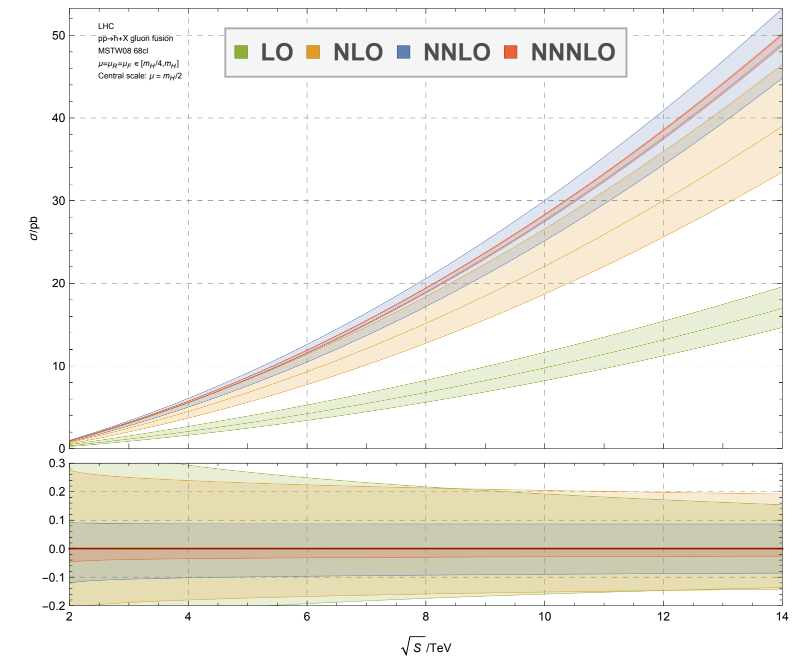
a brief overview



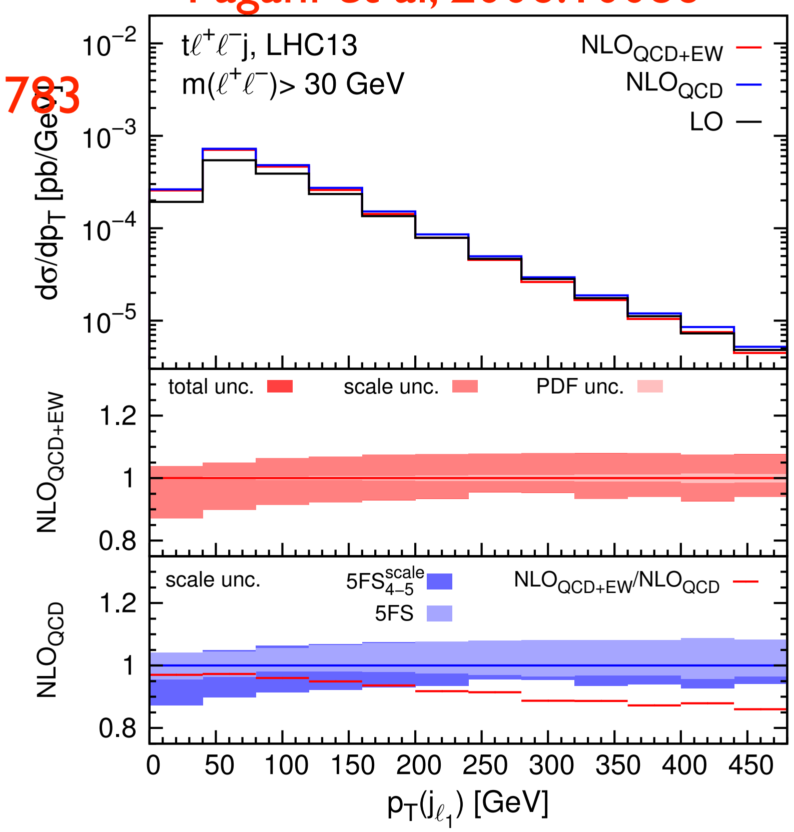
Why bothering?

- QCD corrections generally improve precision of computations (shrink theoretical errors)
- EW corrections necessary to improve accuracy of predictions, specially in the tails of distributions (Sudakov enhancement)
- EW corrections are crucial at lepton colliders
- EW and complete-NLO corrections automated! Sherpa+Openloops: 1412.5157; Sherpa+Recola: 1704.05783 MG5_aMC: 1804.10017
- In some cases, EW corrections do not behave as expected: can give effects as large as QCD!

Anastasiou et al, 1503.06056



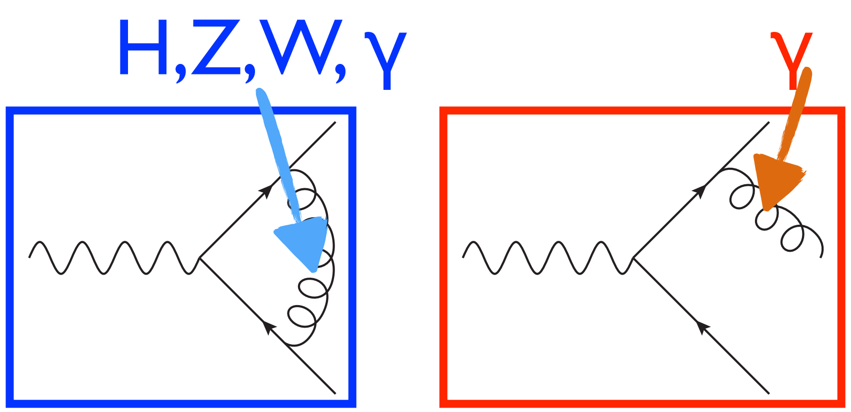
Pagani et al, 2006.10086

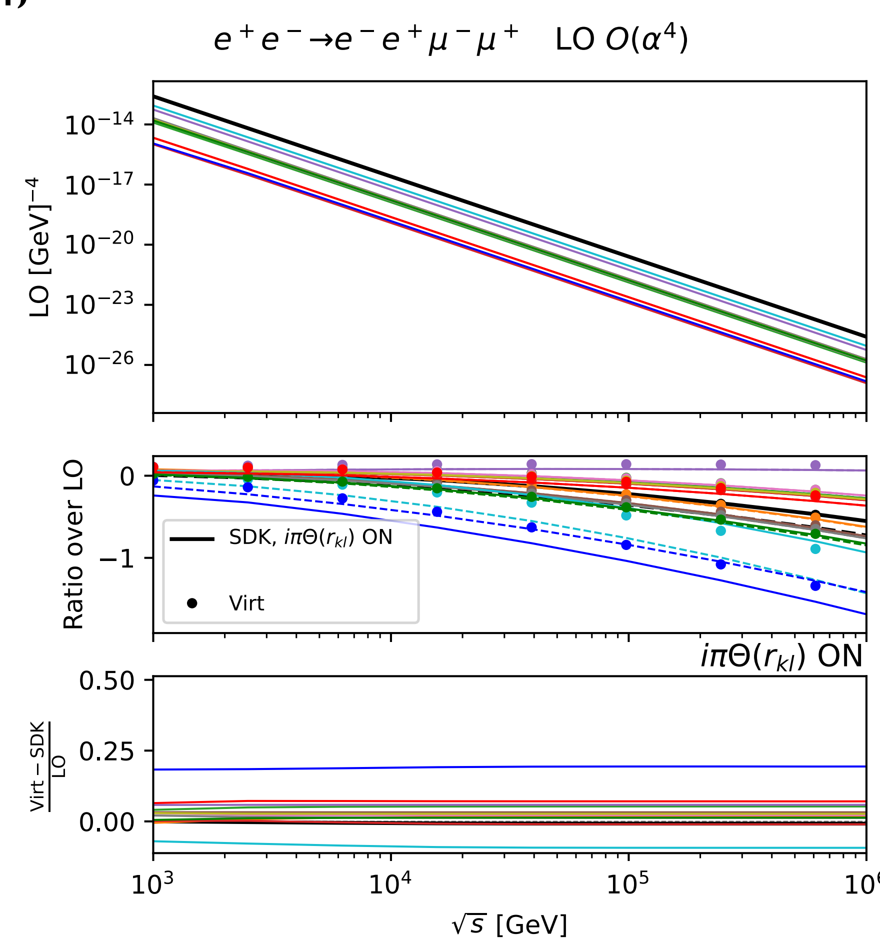


Sudakov enhancement

Denner, Pozzorini, hep-ph/0010201 & hep-ph/0104127
Pagani, MZ, arXiv:2110.03714

- EW bosons are massive: a real W/Z/Higgs emission is detectable (at least in principle)
- Radiation of W/Z/Higgs bosons is in general not included in EW corrections, which remain finite
- When the process scale Q is large, Q≫M~m_W,m_Z,m_H, the would-be IR divergence associated to the heavy boson shows up with double and single log(Q/M)
- In the regime where all invariants are ≫M, these logs are universal, and exponentiate at all orders (resummation possible)
- Sudakov approximation is excellent at high-energy (only a constant part is missing)

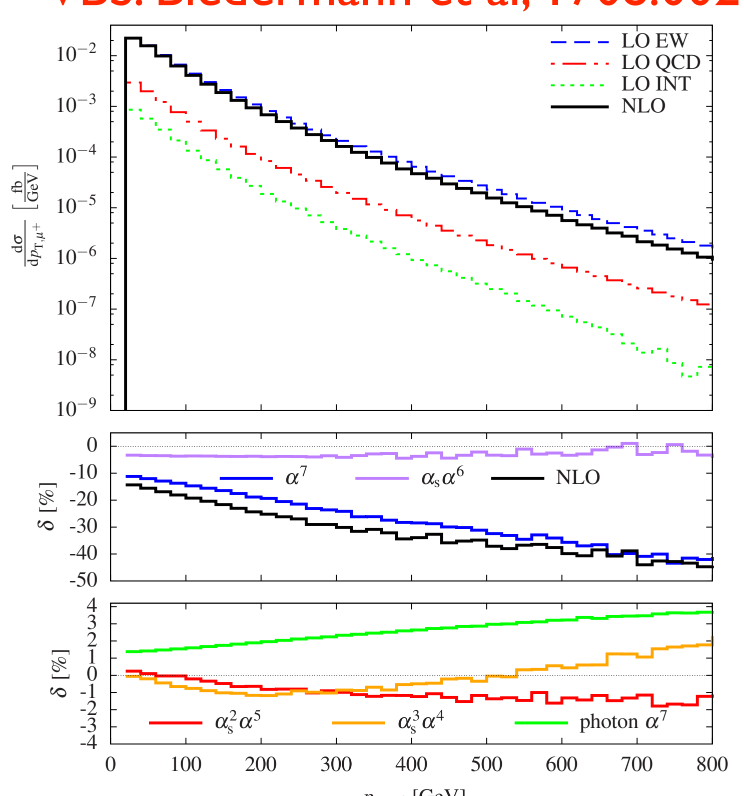




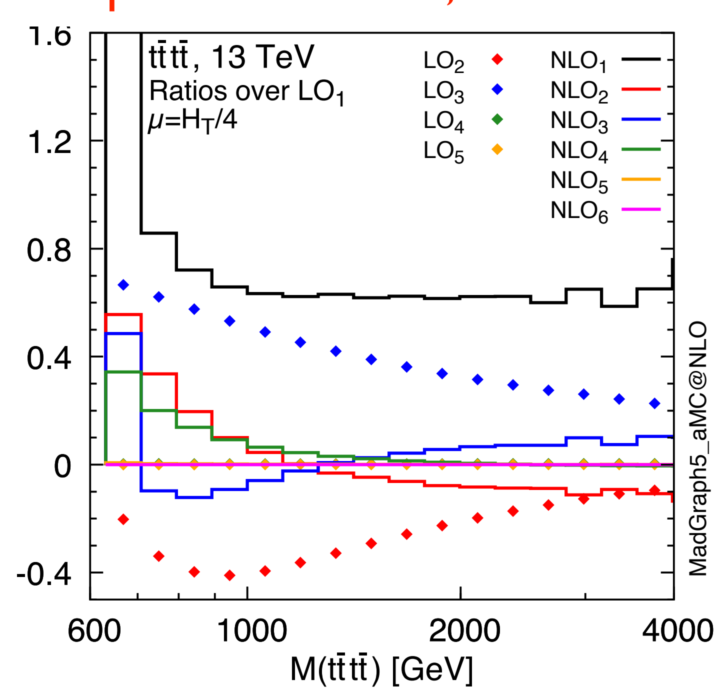
Large EW corrections: not only Sudakov logs

- Despite the naive estimate $\alpha \sim \alpha_s^2$, there are cases when EW corrections comparable to NLO QCD or larger. It happens when:
  - Large scales are probed (VBS) feature of all VBS channels, see also Denner et al, 1904.00882, 2009.00411
  - Power counting is altered (4 top: y_t vs α)
  - New production mechanisms, different than those at the "dominant" LO, enter (ttW, bbH)

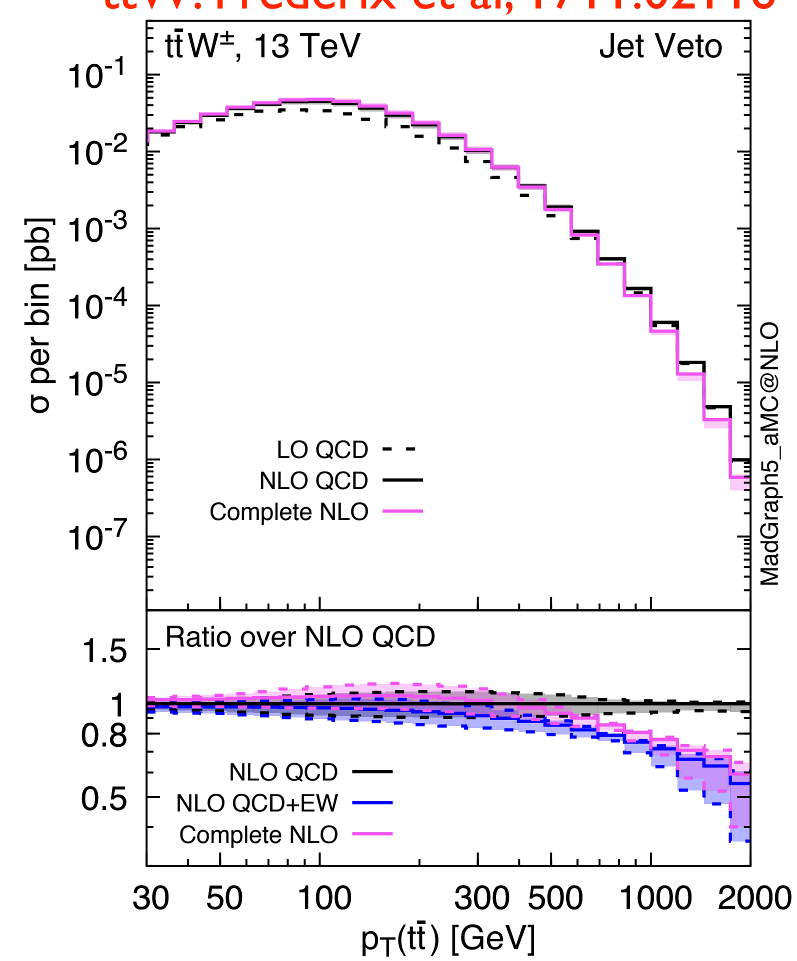
VBS: Biedermann et al, 1708.00268



4 top: Frederix et al, 1711.02116

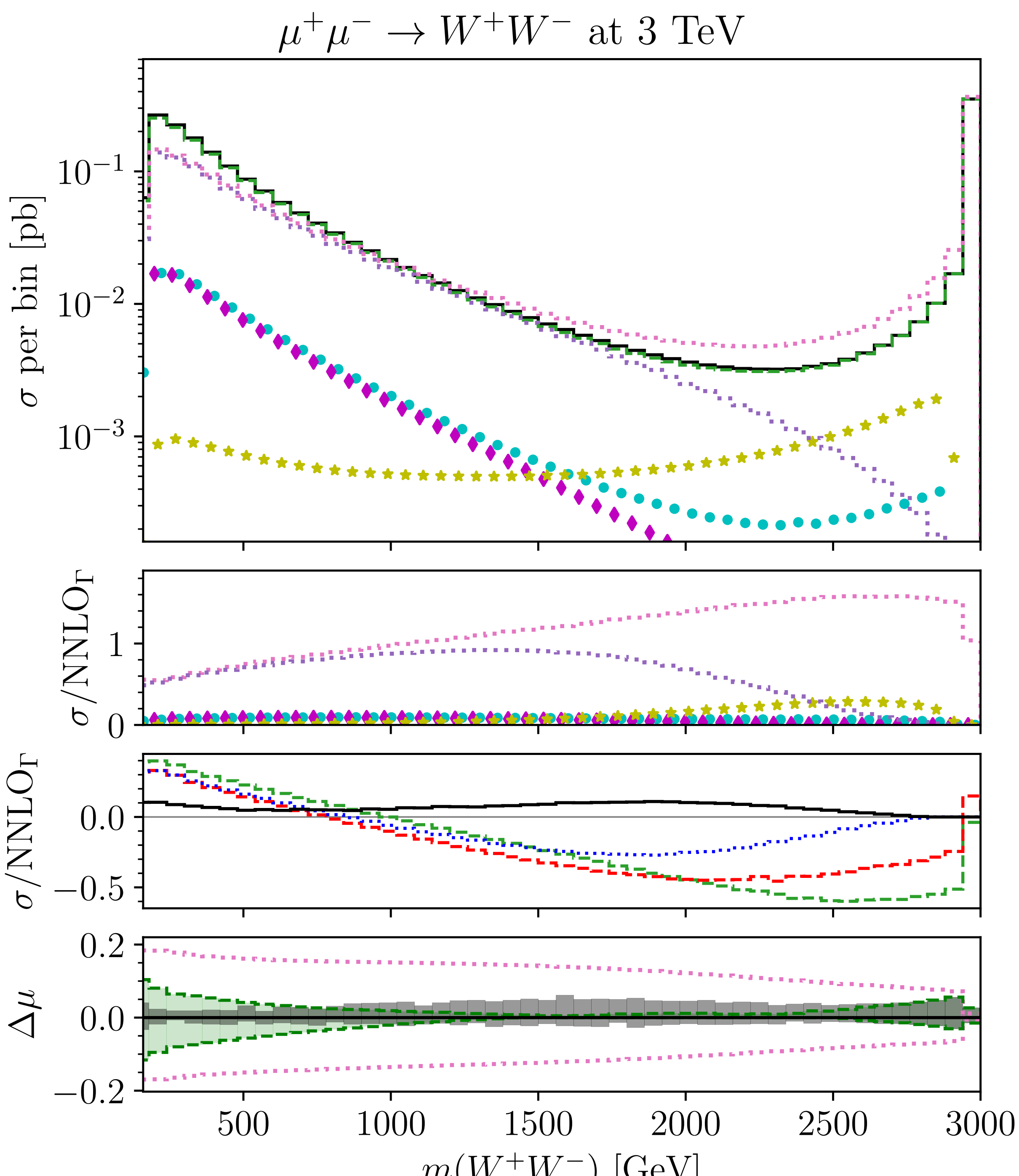


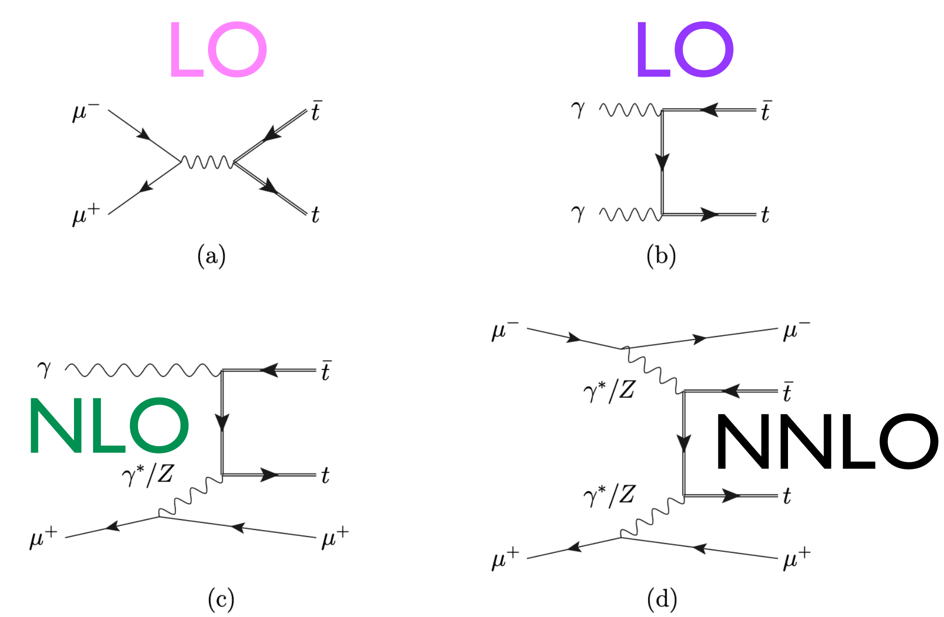
ttW: Frederix et al, 1711.02116

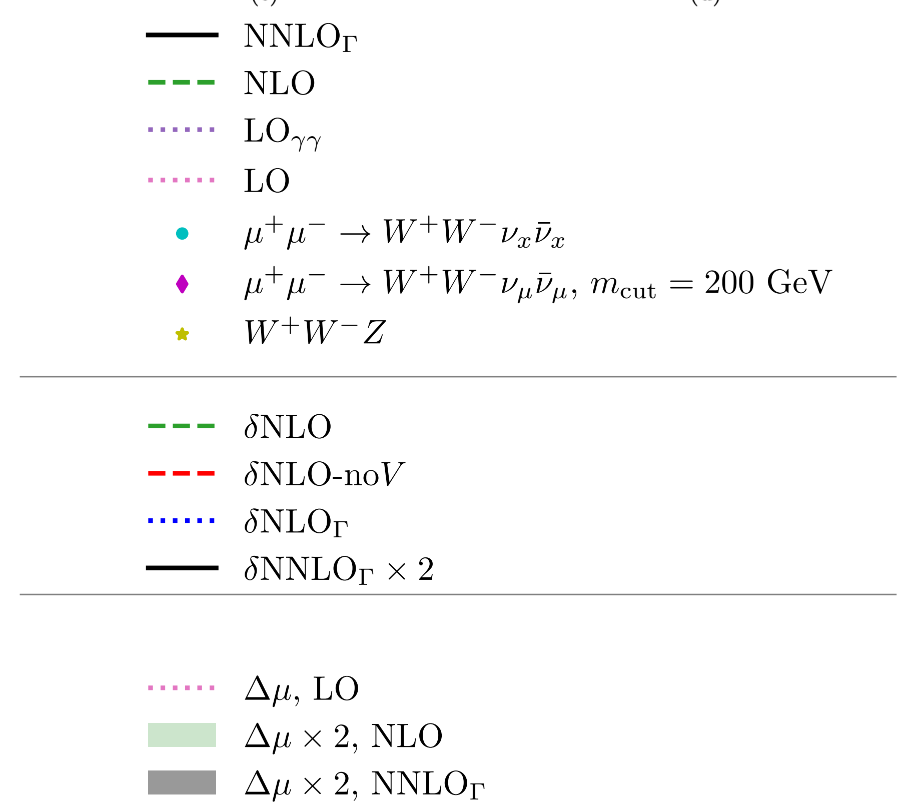


EW corrections at a Muon collider

Frixione, Maltoni, Pagani, MZ, 2506.10732 & 2506.10733

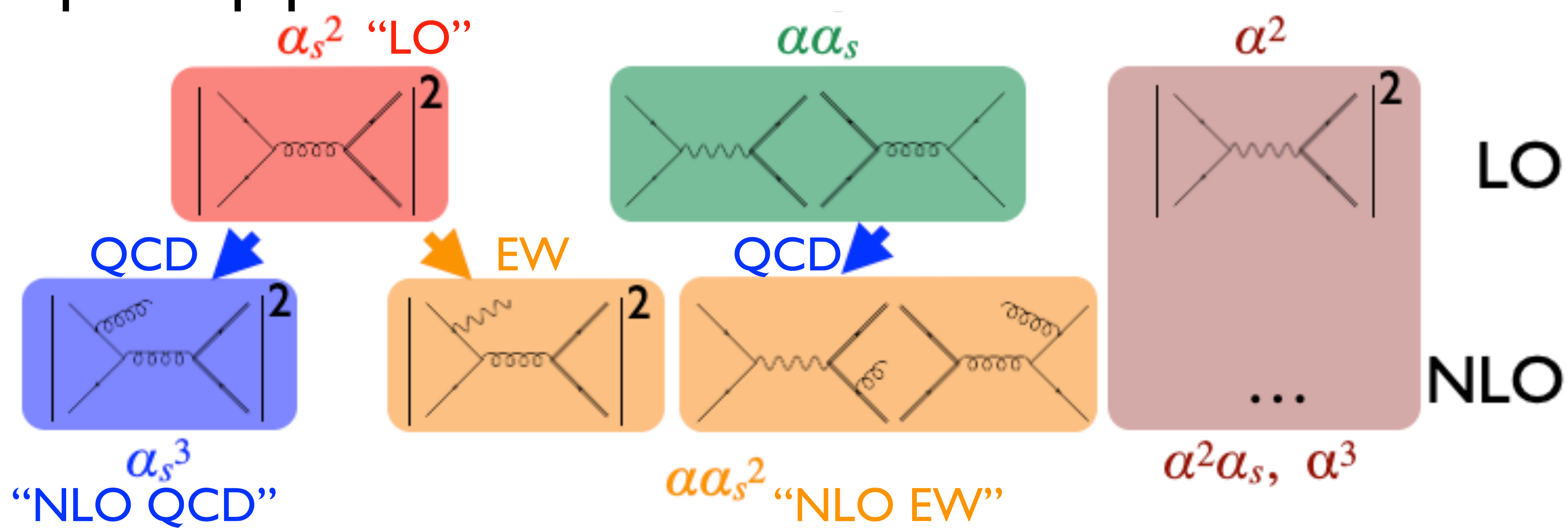






Anatomy of EW corrections: EW corrections vs EW effects

- A general process has more contributions at LO, NLO, …
- Example: top pair



- The LO is often identified with the contribution with most α_s
- At NLO the first two contributions are identified with the NLO QCD and NLO EW corrections
- This structures induces mixed QCD-EW effects at NLO: $\mathrm{NLO}_i = \mathrm{LO}_{i-1} \otimes \mathrm{EW} + \mathrm{LO}_i \otimes \mathrm{QCD}$

Multi-coupling expansion

Single coupling $$\hat{\sigma} = \alpha_s^b \sigma_0 + \alpha_s^{b+1} \sigma_1 + \alpha_s^{b+2} \sigma_2 + \alpha_s^{b+3} \sigma_3 + \ldots$$

LO NLO NNLO NNNLO

Multi-coupling

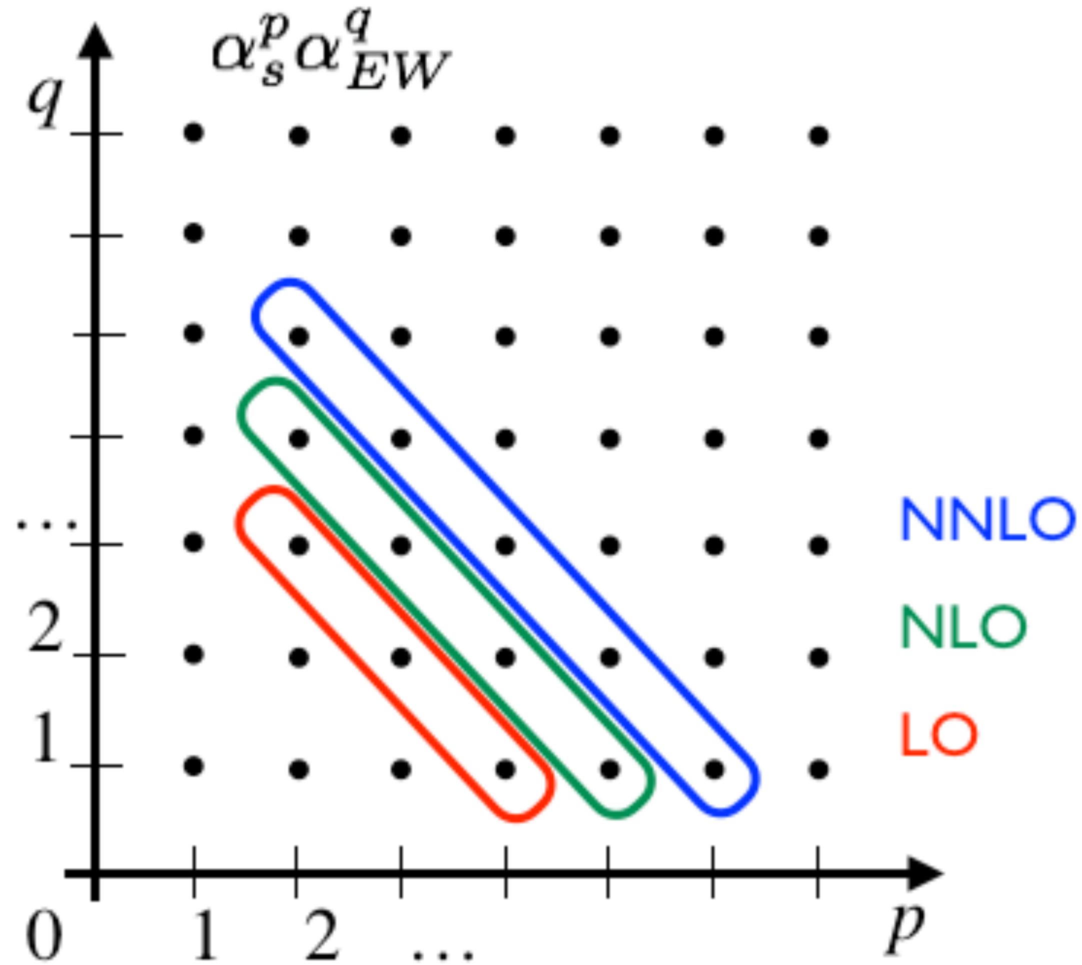
Multi-coupling expansion

Single coupling

$$\hat{\sigma} = \alpha_s^b \sigma_0 + \alpha_s^{b+1}\sigma_1 + \alpha_s^{b+2}\sigma_2 + \alpha_s^{b+3}\sigma_3 + \ldots$$

LO NLO NNLO NNNLO

Multi-coupling



Steps towards the automation of EW corrections

- Apart for the (much) more complex book-keeping, automation of NLO EW corrections largely builds on techniques for NLO QCD (modulo bookkeeping)
- IR subtraction: techniques established for QCD corrections can be extended to EW ones
- Replace color factors with charges ($C_F \rightarrow q_i^2$, $C_A \rightarrow 0$, $T_F \rightarrow N_{C,i}\, q_i^2$) Replace color-linked Borns with charge-links
- Loop amplitudes: one-loop techniques can be exploited for EW loops.
- UV/R2 counterterms for the EW interactions are needed
- Higher ranks appear, integrand-reduction may lead to unstable results Switch to other techniques (Tensor-integral reduction, Laurent-series expansion,…)
- Use scalar-integral libraries that support complex masses

EW renormalisation schemes in a nutshell

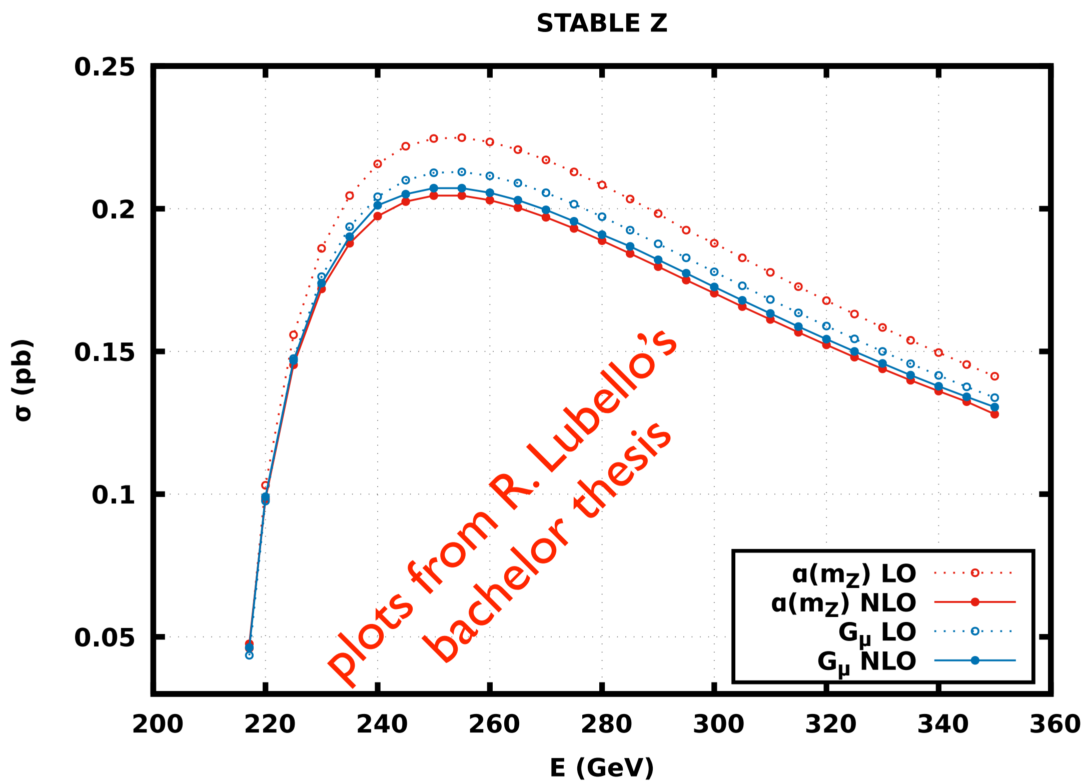
The renormalisation of α can be performed in different schemes:

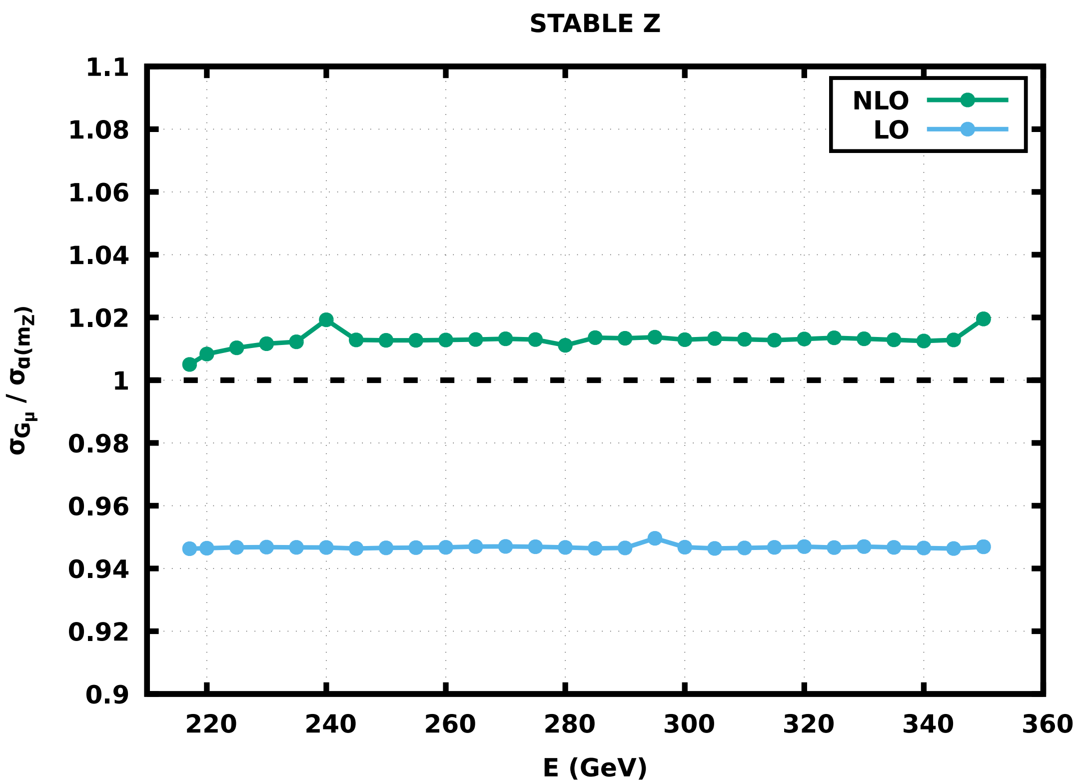
- $\alpha(0)$: α is measured in the Thompson scattering, in the zero-momentum limit. Terms ~$\log(Q/m_f)$ appear in the cross section, except for external photons. Fermion masses must be retained.
- $\alpha(M_Z)$: α is measured at the Z peak (e.g. at LEP). It removes the dependence on the fermion masses, which can be set to zero.
- G_μ scheme: the Fermi constant is measured from the muon lifetime, then α is extracted. W.r.t. the $\alpha(M_Z)$ scheme, also contributions of weak origin ($\Delta\rho$) are resummed

The G_μ scheme is generally preferred for processes without final-state photons at the LO.

Renormalisation-scheme dependence of cross sections

- Consider $e^+e^- \to ZH$

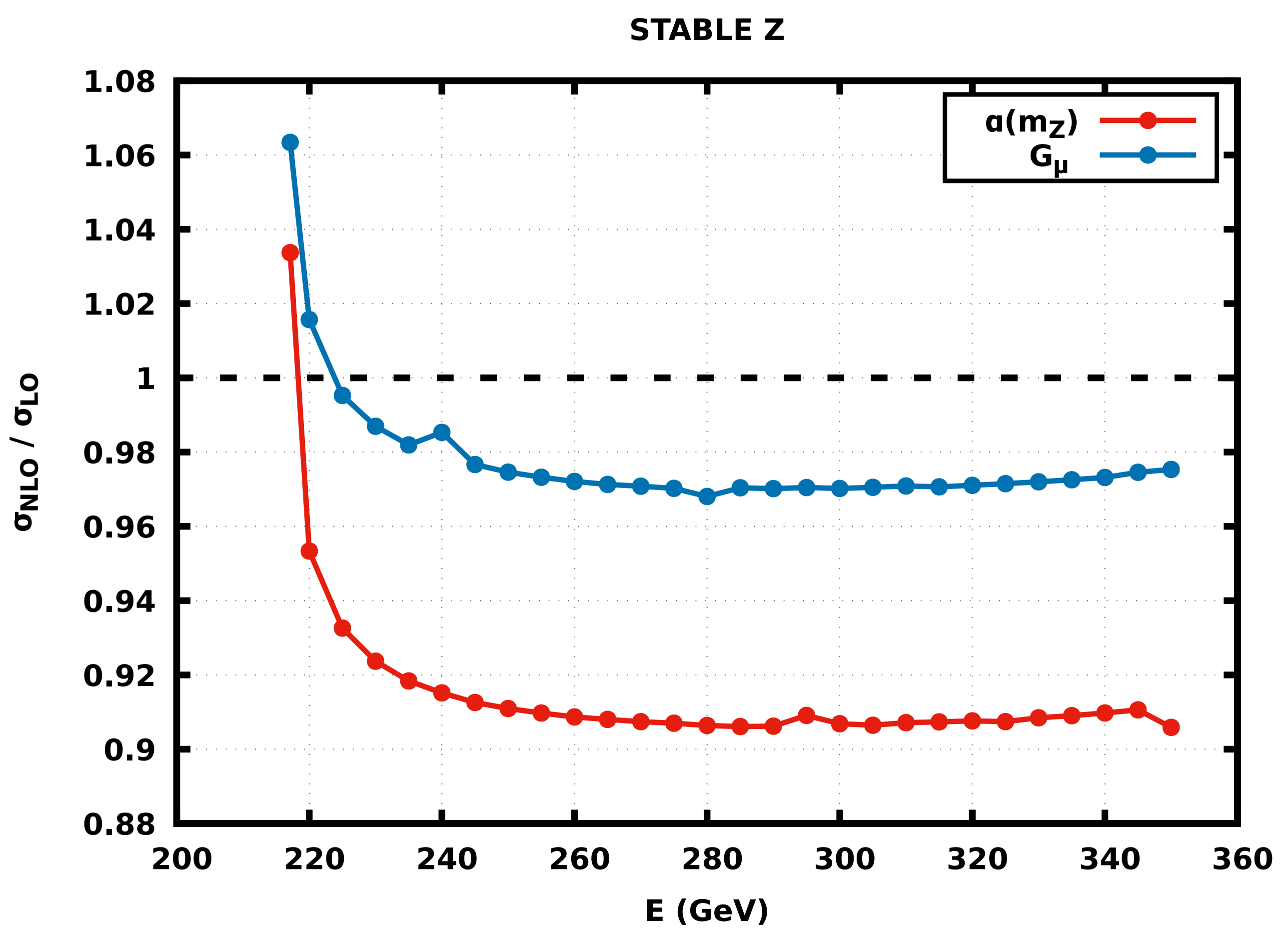




- Ren.Scheme dependence reduces from LO to NLO: at N^{p}LO is always O(N^{p+1}LO)

Renormalisation-scheme dependence of cross sections

- NLO corrections are smaller (K-factor closer to 1) in the G_μ scheme: the coupling already incorporates higher-order weak effects

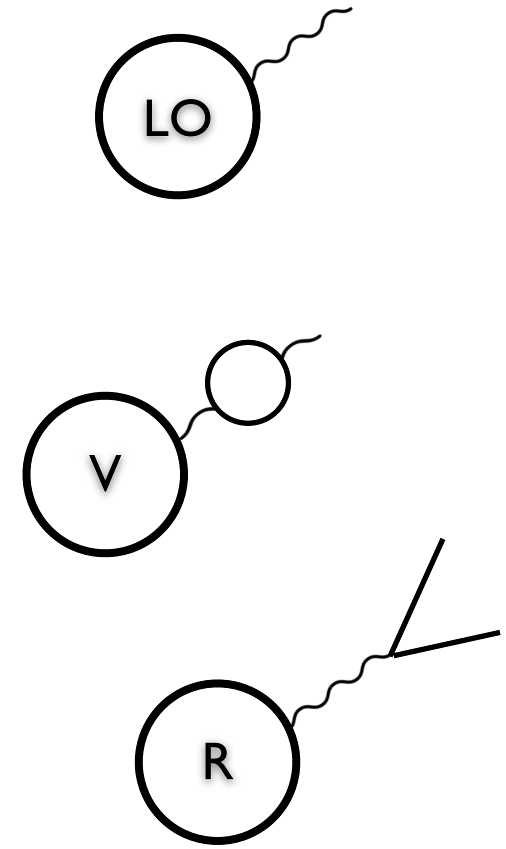


plots from R. Lubello's bachelor thesis

Processes with tagged photons

Pagani, Tsinikos, MZ arXiv:2106.02059

- The definition of a "photon" in the presence of EW corrections is not IR-safe (in a scheme with massless quarks/leptons)
- This is why democratic jets are usually employed
- In order to define photons as physical objects, a renormalisation scheme which takes into account fermion masses must be employed (only for the vertices related to tagged photons). Such a scheme exists: $\alpha(0)$
- Renormalisation conditions define α from the low-energy Thomson scattering. IR-poles differ from a high-energy scheme such as G_μ or $\alpha(m_Z)$
- The difference of IR poles accounts for the fact that real emissions with $\gamma \to 2f$ splittings are not included
- Alternative: use fragmentation functions (more involved)



NLO: Summary

- Precise predictions crucial for success of LHC programme
- They entail a lot of complexity: NLO is just the first bite!
- 10 years ago: NLO revolution. We have harvested many fruits
  - Automation: complexity hidden to the user!
  - NLO event generators ubiquitous in exp. analyses
  - Techniques proved successful also beyond QCD: automation of electroweak corrections (see backup slides for extra informations)

Next?

- Beyond NLO: NNLO is the new Holy Graal:
  - Several subtraction techniques are being studied at NNLO. They all work on paper, need for numeric implementation and testing
  - No general algorithm to compute 2-loop amplitudes, but huge progress (first results for massless $2\to3$ processes available)
  - In general, huge amount of complexity and of running time (~1M CPU hours for $2\to2$ with coloured FS)
- Is the NNLO revolution approaching?

Matching NLO computations with parton showers

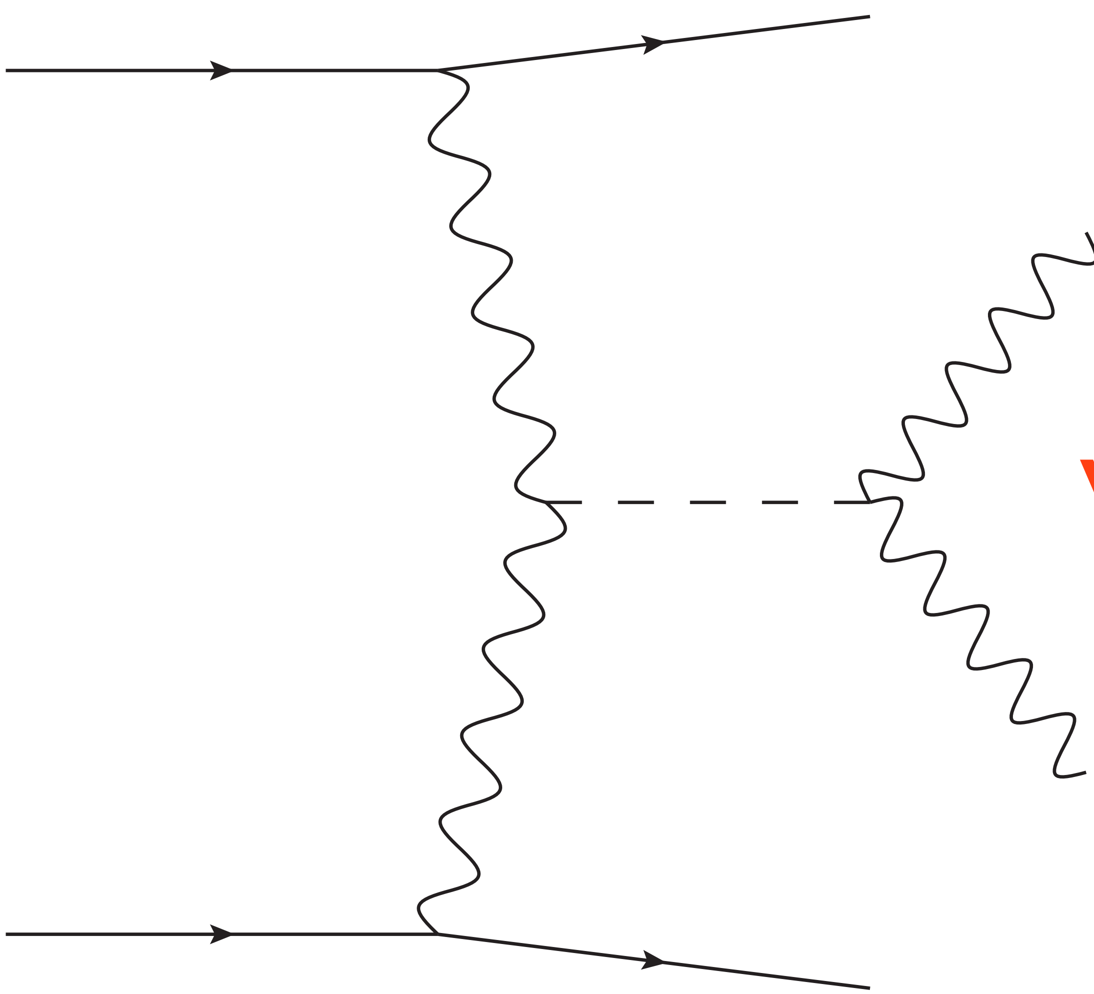
Marco Zaro
marco.zaro@mi.infn.it

Milan PhD School

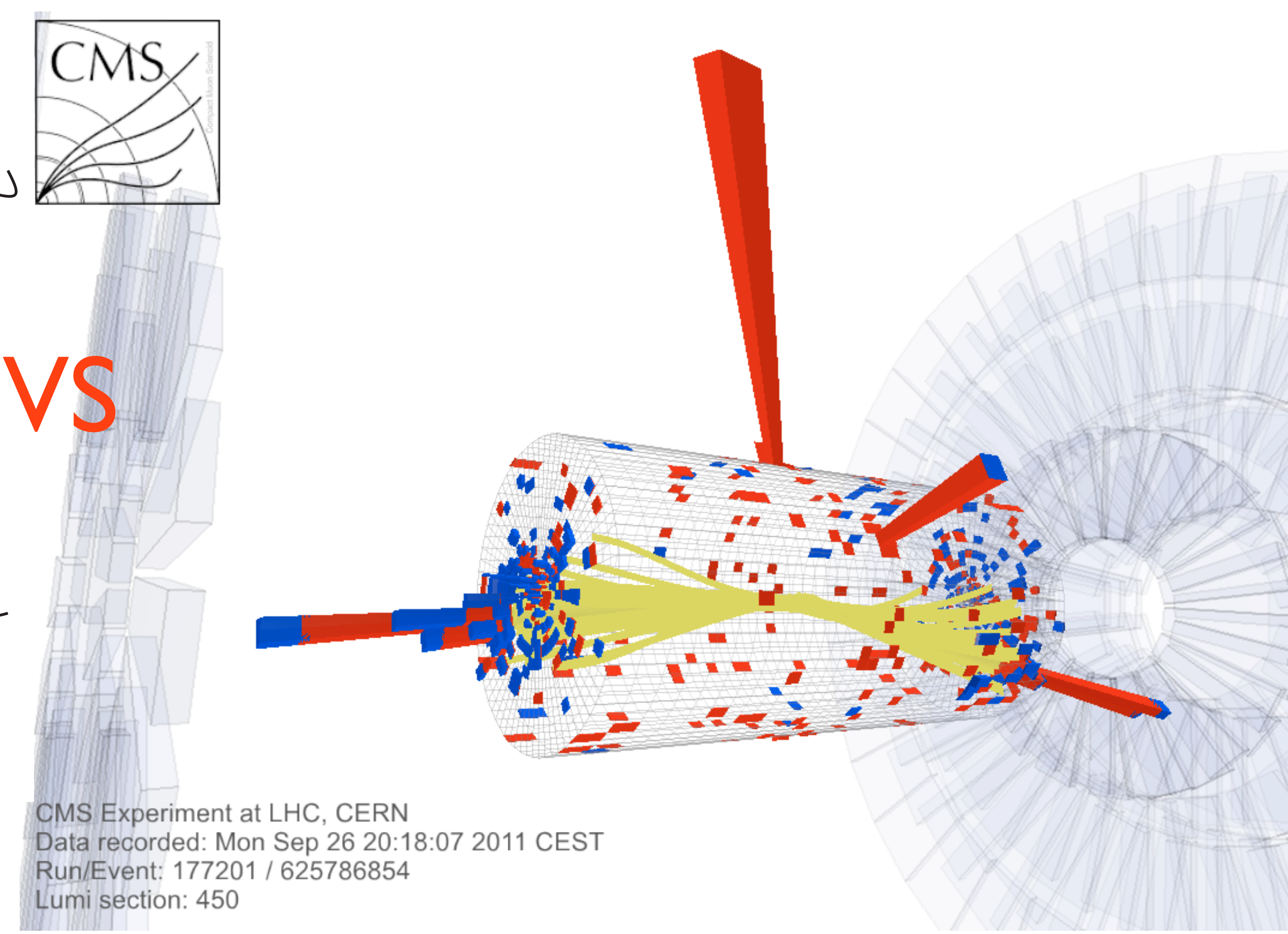
UNIVERSITAS STUDIORUM MEDIOLANENSIS

INFN

Matching NLO computations with parton showers



VS



Recap from yesterday

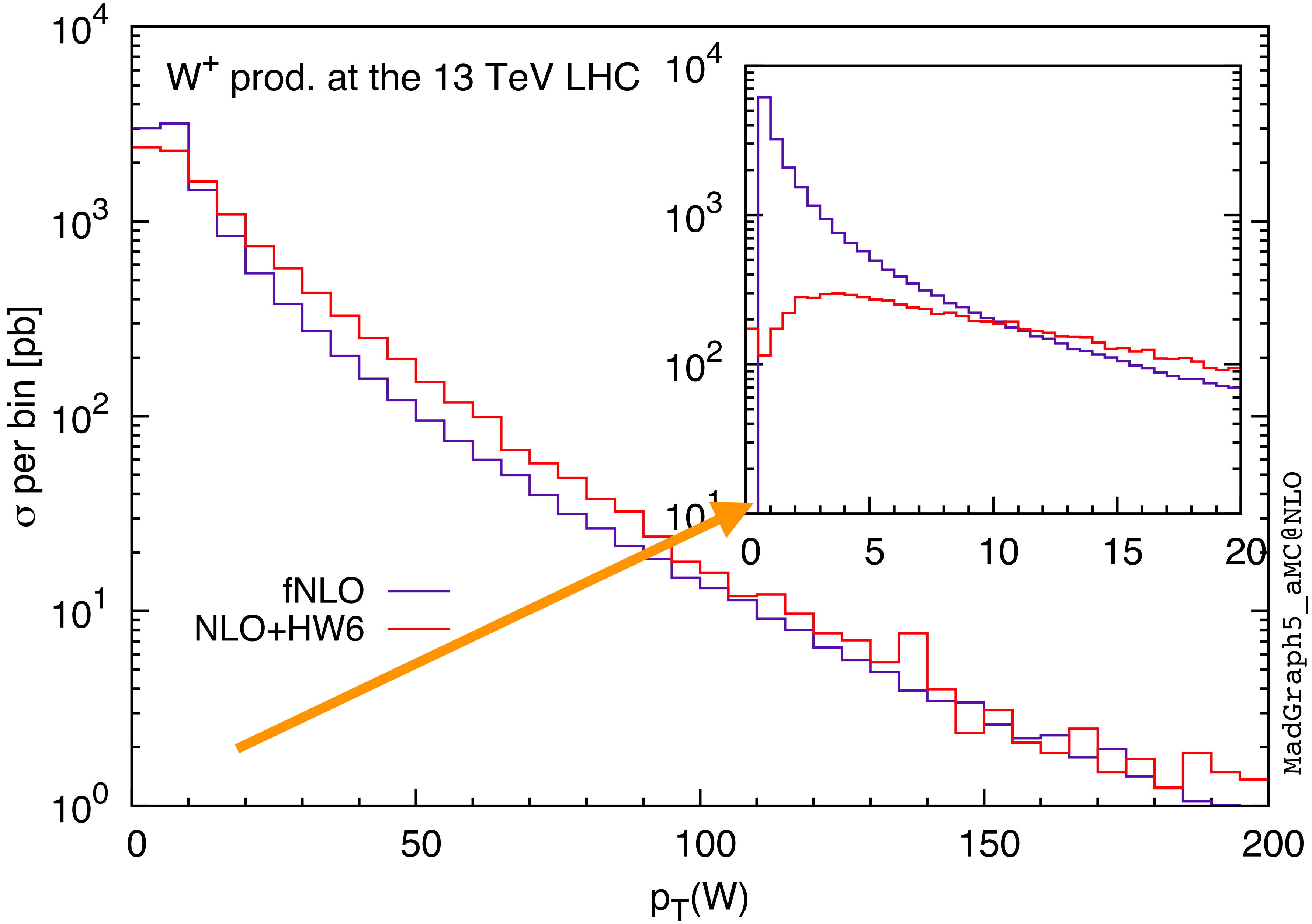
- Virtual and real matrix element are not finite, but their sum is. Subtraction methods can be used to extract divergences for real-emission matrix elements and cancel explicitly the poles from the virtuals
- Event and counterevents have different kinematics. Unweighting and event generation not possible, plots are filled on-the-fly with weighted events
- For plots, only IR-safe observable with finite resolution must be used!

Matching NLO predictions and parton showers

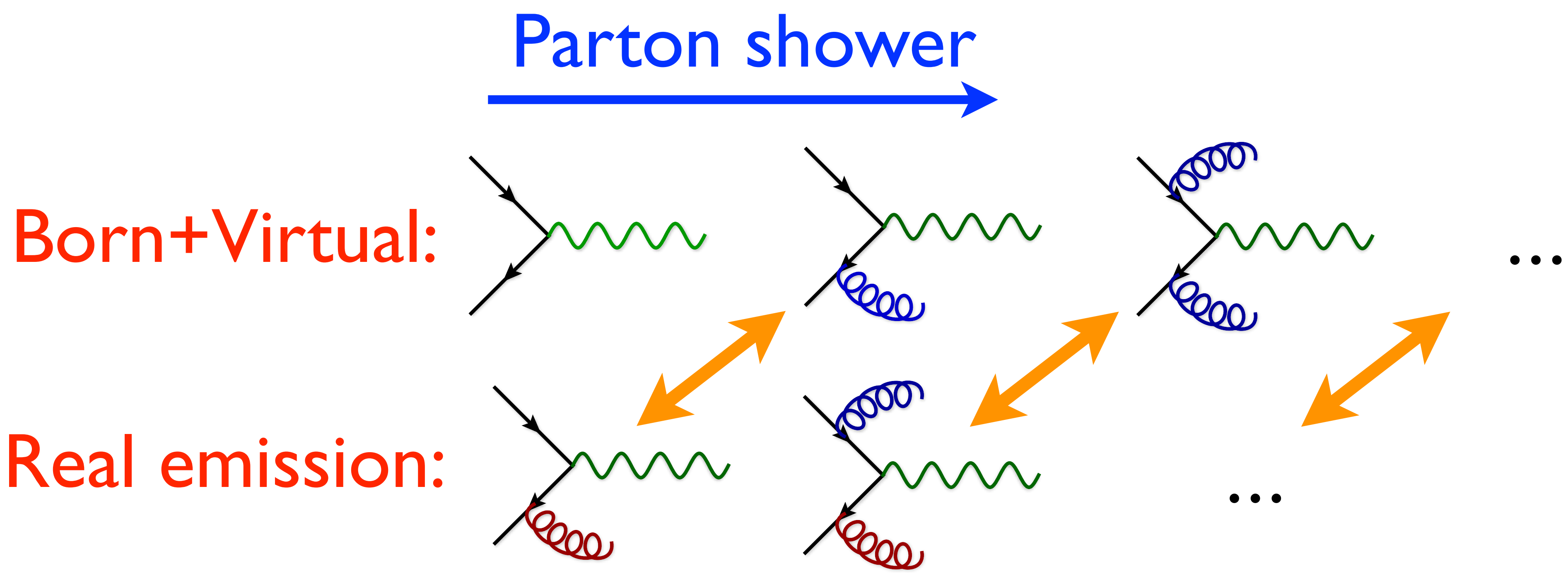
- Parton showers evolve hard partons by emitting extra QCD radiation down to a more realistic final state made of hadrons
- This resums the effect of soft gluon radiations, and cures fixed-order instabilities
- After the parton shower, a fully exclusive description of the event is available
- NLO corrections are inclusive by definition, but they provide the first reliable estimate of rates and uncertainties

Can we attach a parton shower to NLO simulations?

Fixed order instabilities, again



Warning: double counting!



- There is a double counting between real emission and the parton shower
- There is also double counting between the virtuals and the non-emission probability from the Sudakov factor

Double counting the virtuals

- The Sudakov factor Δ, responsible for the resummation performed by the shower, gives the no-emission probability 1-P, P being the emission probability)

shower variables

$$\Delta(Q,Q_0) = \exp\left[-\int_{Q_0}^{Q} d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}\right] \qquad d\Phi_1 = \frac{dt}{t}\, dz \frac{d\phi}{2\pi}$$

- Δ contains implicitly contributions from the virtual and real corrections
- We should therefore avoid to double counting the contribution from the virtuals in the matrix element and in the Sudakov
- Because of unitarity, what is double counted in the virtuals is exactly opposite to what is double counted by the reals

How to avoid double counting at NLO?

- Two methods exist:
  - MC@NLO Frixione, Webber hep-ph/0204244
  - Powheg Nason, hep-ph/0409146

Naive (wrong) matching

- Let us *assume* we can generate events separately for Born, virtuals and real emissions, and that we pass them to a parton shower

$$\frac{d\sigma_{\text{``}NLO+PS\text{''}}}{dO} = [\mathcal{B}+\mathcal{V}]\, d\Phi_n\, I^n_{MC}(O) + d\Phi_{n+1}\mathcal{R}\, I^{n+1}_{MC}(O)$$

- Do we get the NLO cross section?
  - Let us expand the shower operator at order α_S (0 or 1 emission)

$$I_{MC} = \Delta_a\left(Q, Q_0\right) + \Delta_a\left(Q, Q_0\right) d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

$$\Delta_a\left(Q, Q_0\right) = \exp\left[-\int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}\right] \simeq 1 - \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

$$I_{MC} \simeq 1 - \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc} + d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

Naive (wrong) matching

- At order α_S we get

$$\frac{d\sigma_{\text{“}NLO+PS\text{”}}}{dO} = [\mathcal{B} + \mathcal{V}]\, d\Phi_n + d\Phi_{n+1}\mathcal{R}$$

$$- \mathcal{B}\, d\Phi_n \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc} + \mathcal{B}\, d\Phi_n d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

- Which is not the NLO

MC@NLO matching

- In the MC@NLO formalism, double counting can be cured by the so-called Monte Carlo counterterms, defined as

$$\Delta(Q,Q_0) = \exp\left(-\int d\Phi_1 MC\right) \qquad MC = \left|\frac{\partial \Phi_1^{MC}}{\partial \Phi_1}\right| \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

- The MC@NLO cross section is defined as

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n\, I_{MC}^n(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I_{MC}^{n+1}(O)$$

- Again, if we expand up to α_S we recover the NLO

$$I_{MC} = 1 - \int d\Phi_1 MC + d\Phi_1 MC$$

$$\frac{d\sigma_{\text{“}MC@NLO\text{”}}}{dO} = \left[\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right] d\Phi_n + d\Phi_{n+1}\left[\mathcal{R} - MC\right]$$

$$+\,\mathcal{B}\left[-\int d\Phi_1 MC + d\Phi_1 MC\right] d\Phi_n$$

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- Again, if we expand up to α_S we recover the NLO

$$I_{MC} = 1 - \int d\Phi_1 MC + d\Phi_1 MC$$

$$\frac{d\sigma_{\text{“}MC@NLO\text{”}}}{dO} = \left[\mathcal{B}+\mathcal{V}+\cancel{\int d\Phi_1 MC}\right] d\Phi_n + d\Phi_{n+1}\left[\mathcal{R}-MC\right]$$

$$+\,\mathcal{B}\left[\cancel{-\int d\Phi_1 MC} + d\Phi_1 MC\right] d\Phi_n$$

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- Again, if we expand up to α_S we recover the NLO

$$I_{MC} = 1 - \int d\Phi_1 MC + d\Phi_1 MC$$

$$\frac{d\sigma_{\text{“}MC@NLO\text{”}}}{dO} = \left[\mathcal{B} + \mathcal{V} + \cancel{\int d\Phi_1 MC}\right] d\Phi_n + d\Phi_{n+1}\left[\mathcal{R} - \cancel{MC}\right]$$

$$+\,\mathcal{B}\left[\cancel{-\int d\Phi_1 MC} + \cancel{d\Phi_1 MC}\right] d\Phi_n$$

The *MC* counterterm

- *MC* has some remarkable properties:
  - It avoids double counting when matching to PS ←Just shown
  - It matches the singular behaviour of the real-emission ME, making it possible to unweight events (some special cares are needed for the soft region)
  - It ensures a smooth matching: NLO+PS has the same shape of the shower in the soft/collinear region; in the hard region, it approaches the NLO
  - It is PS dependent, as it depends on the PS details. For each PS, we need its own *MC* counterterms

Unweighting

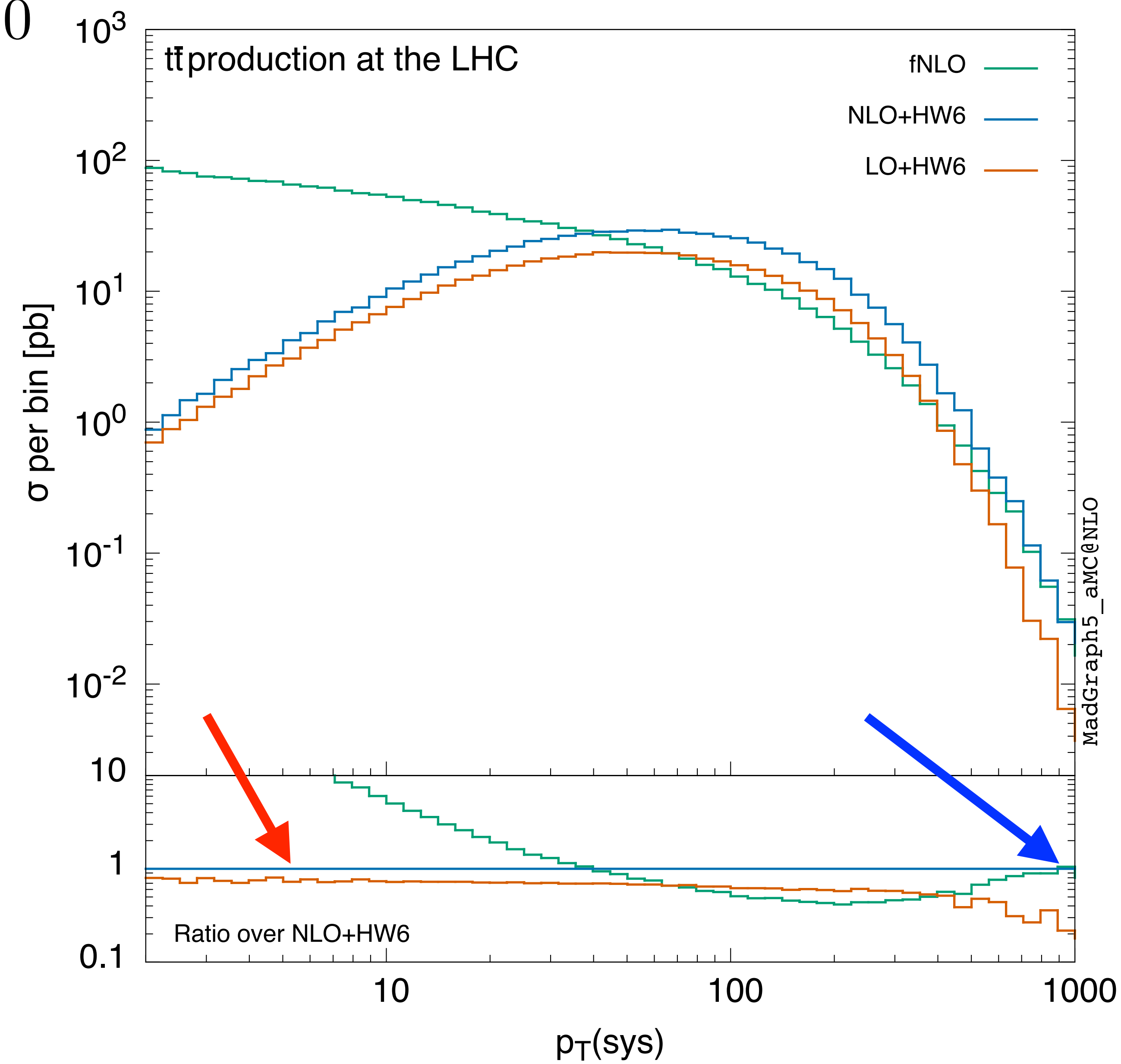
$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n \, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- MC is by construction what the shower does to go from n to $n+1$. It matches exactly R in the soft-collinear region. Furthermore, it has the same kinematics as R, therefore there is no reshuffling needed. The n and $n+1$ body contributions are separately finite and bounded. Unweighted events can be generated!
  - S-events, with n-body kinematics
  - H-events, with $n+1$-body kinematics

Smooth matching

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n\, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- In the soft/collinear region, $\mathcal{R} - MC \sim 0$ so that
$$\frac{d\sigma_{MC@NLO}}{dO} \simeq I^n_{MC}(O)$$
- In the hard region, MC=0 (it must be zero far from singular regions). The only contribution comes from the real-emission ME



The MC counterterms and the FKS subtraction

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n \, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- The MC counterterms already make the cross-section finite. Are the local counterterms still needed?
- Yes, because we cannot integrate MC analytically to extract the poles
- In practice, we have

$$\frac{d\sigma_{MC@NLO}}{dO} = \left[\mathcal{B} + \left(\mathcal{V} + \int d\Phi_1 \mathcal{C}\right) + \int d\Phi_1 \left(MC - \mathcal{C}\right)\right] d\Phi_n \, I^n_{MC}(O)$$
$$+ (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

Negative weights

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n \, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- Events are generated for n- and n+1-body kinematics separately
- Nothing guarantees that the two contributions are separately positive
- The unweighting has to be done up to a sign, and the sign should be taken into account when filling plots
- Remember: results are physical only after having showered the events!

Powheg

- Let us consider the LO+PS cross-section expanded up to the first emission:

$$d\sigma_{LO+PS} = \mathcal{B}\, d\Phi_n \left[\Delta(Q,Q_0) + \Delta(Q,Q_0) d\Phi_1 \frac{\alpha_s(t)}{2\pi} P\right]$$

- We could think of going NLO by replacing the Born with the NLO cross section

$$d\sigma_{\text{“}NLO+PS\text{”}} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[\Delta(Q,Q_0) + \Delta(Q,Q_0) d\Phi_1 \frac{\alpha_s(t)}{2\pi} P\right]$$

$$= \mathcal{B}\left(1 - \int \underline{d\Phi_1 MC} - d\Phi_1 MC\right) + \mathcal{V} + \int \underline{d\Phi_1 \mathcal{R}}$$

- Of course, there is double counting. This is in particular due by the fact that the integral in the Sudakov does not contain R

A modified Sudakov

- In order to avoid double counting one could use a modified Sudakov

$$\tilde{\Delta}(Q, Q_0) = \exp\left(- \int_{Q_0^2}^{Q^2} d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}} \right)$$

- Such that

$$d\sigma_{\text{“}NLO+PS\text{”}} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R} \right) d\Phi_n \left[\tilde{\Delta}(Q, Q_0) + \tilde{\Delta}(Q, Q_0) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}} \right]$$

- But the total rate is not the NLO! The second parentheses does not integrate to 1 (see next slide). It has to be modified to

$$d\sigma_{\text{“}NLO+PS\text{”}} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R} \right) d\Phi_n \left[\tilde{\Delta}(Q, Q_0) + \tilde{\Delta}(Q, t) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}} \right]$$

- Where t is the scale at which R/B is evaluated

- Note that

$$d\sigma_{Powheg} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[\tilde{\Delta}(Q, Q_0) + \tilde{\Delta}(Q, t) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right]$$

- Therefore

$$\tilde{\Delta}(Q, t)\frac{\mathcal{R}}{\mathcal{B}} = \frac{d\tilde{\Delta}(Q, t)}{dt}$$

Lower integration bound

- So the [] integrates to 1.

$$\int_{Q_0}^{Q} dt \tilde{\Delta}(Q, t)\frac{\mathcal{R}}{\mathcal{B}} = \tilde{\Delta}(Q, Q) - \tilde{\Delta}(Q, Q_0) = 1 - \tilde{\Delta}(Q, Q_0)$$

- The NLO normalisation is kept
- Double counting is avoided

Comments

$$d\sigma_{Powheg} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[\tilde{\Delta}(Q, Q_0) + \tilde{\Delta}(Q, t) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right]$$

- The Powheg cross section has the same structure as an ordinary shower, with a global K-factor correction and a different Sudakov for the first emission
- Note that when matching to PS one has to veto emissions harder than t (in the Powheg formalism, is has to be interpreted as transverse momentum), even for showers with a different ordering variable
  - Formula to be modified for angular-ordered PS in order to keep color coherence
- MC@NLO and Powheg are formally equivalent at NLO level. In practice, their predictions may visibly differ

MC@NLO vs Powheg

- The two matching procedure can be cast in a single formula

$$d\sigma_{NLO+PS} = d\Phi_n \bar{\mathcal{B}}^s \left[\Delta^s(Q, Q^0) + d\Phi_1 \frac{\mathcal{R}^s}{\mathcal{B}} \Delta^s(Q,t) \right] + d\Phi_{n+1} \mathcal{R}^f$$

- With

$$\bar{\mathcal{B}}^s = \mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}^s$$

- And the real-emission ME has been split in a singular and non-singular (finite) part

$$\mathcal{R} = \mathcal{R}^s + \mathcal{R}^f$$

- The difference between the two methods is in R^s:

MC@NLO

$$\mathcal{R}^s = \frac{\alpha_s}{2\pi} P\mathcal{B} = MC$$

Powheg

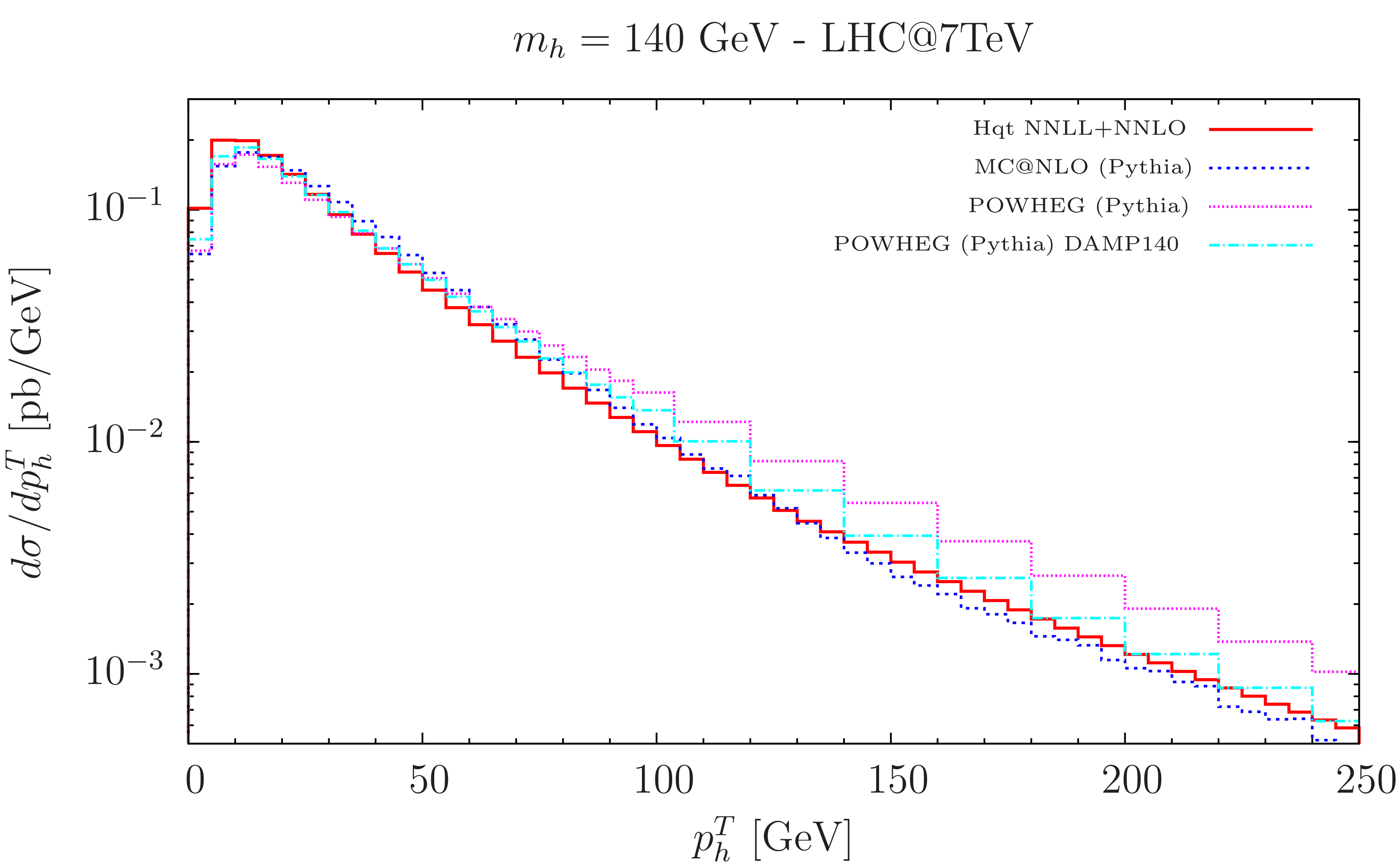
$$\mathcal{R}^s = F\mathcal{R} \quad \mathcal{R}^f = (1-F)\mathcal{R}$$

default F=1,
but can be tuned in order to suppress non-singular part of R

Effect of F

$$F = \frac{h^2}{h^2 + p_T^2}$$

$p_T \gg h$ are suppressed

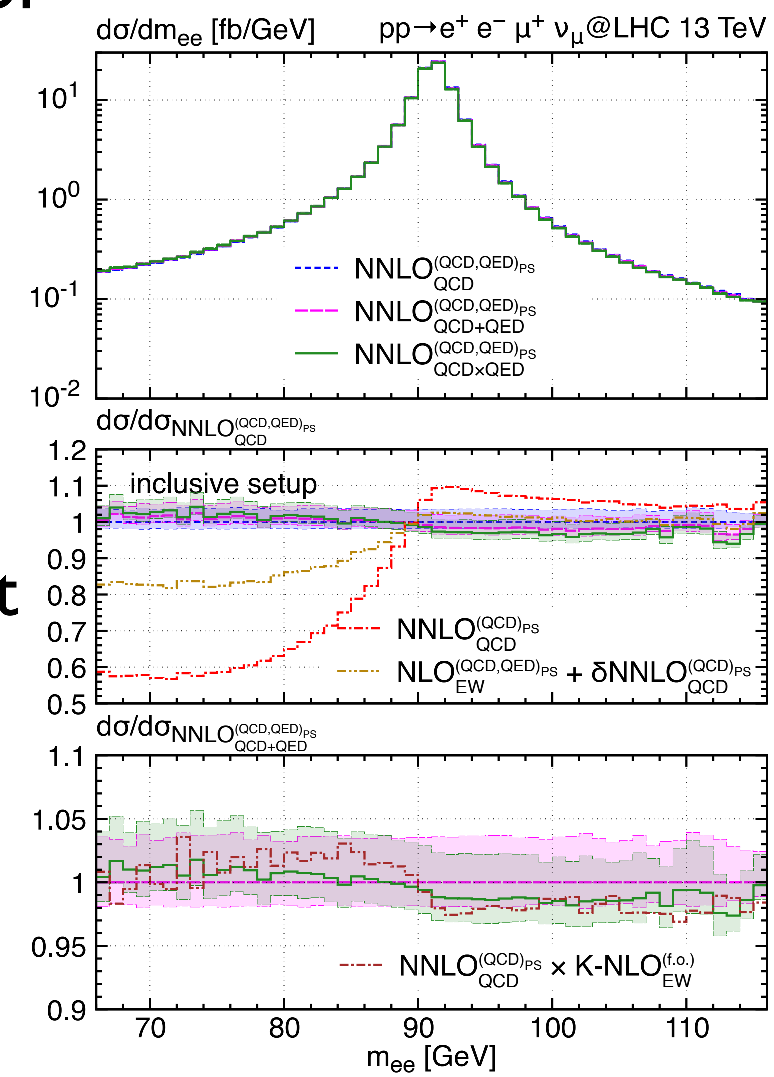


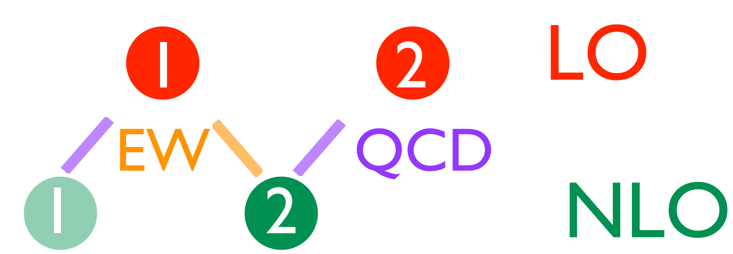
MC@NLO naturally matches analytic resummation+FO curve at large p_T
Powheg (without damping) overshoots the FO
Damping recovers matching at large p_T

Matching EW corrections with PS?

- EW corrections matched with PS still not available for general processes
  - Approximate approaches exist, only including n-body contribution (“EWvirt” or EWSL). Accuracy depends on kinematics region
    EWVirt: VV(J): Brauer et al, 2005.12128; top: Gutschov et al, 1803.00950; V+jets: Kallweit et al, 1511.08692, …
  - Exact matching performed only for processes with just LO_1 (in the Powheg scheme)
    DY: Barzè et al,1302.4606; HV(J): Granata et al, 1706.03522; VBS: Chiesa et al, 1906.01863,VV: Chiesa et al, 2005.12146; WZ@NNLO+PS: Lindert et al, 2208.12660
- Main issue: how to assign colour-flows to interferences (LO_2 is mostly an interference contribution) Some ideas: Frixione et al, 2106.13471

Lindert et al, 2208.12660





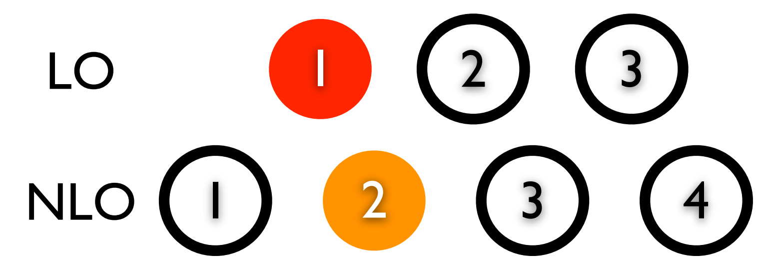
NLO event generation and matching: Summary

- Generating and showering events at NLO is much trickier than at LO: $n(+1)$ body contributions are not separately finite +need not to double count contributions
- Methods have been developed to overcome this
  - MC@NLO achieves this by means of the so-called MC counterterm
  - Powheg generates the hardest emission with its own Sudakov
- Both have proven very successful, and today NLO+PS simulations are the workhorses of all experimental analyses

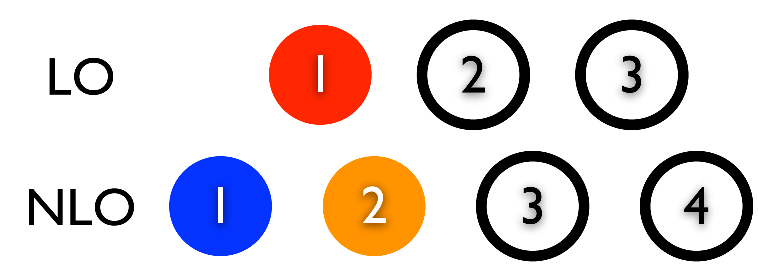
Backup

MG5_aMC Syntax (I)

- The syntax to generate NLO EW corrections is very similar to the one for QCD:
  - e.g.: ttbar@NLO EW: generate p p > t t~ [QED]
  - Since no orders are specified, it will take the LO contribution with the largest power of $\alpha_s{}^2$, $O(\alpha_s{}^2)$, and generate NLO corrections with one extra power of α, $O(\alpha_s{}^2\alpha)$

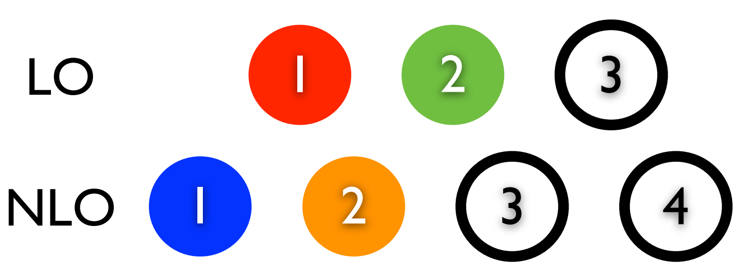


  - If one wants to also generate NLO QCD corrections, the syntax is
    generate p p > t t~ [QED QCD]
    In this case NLO contributions with both one extra power of α and of α_s will be generated



MG5_aMC Syntax (II)

- In the previous slide, the syntax would have been equivalent had we explicitly selected the dominant LO contribution.
  - This could be done by adding QED^2=0 QCD^2=4 to the generate command (note the squared-order constraints, applied at the amplitude level)
- Now, suppose you want to include also the first subleasing LO term (LO_2), together with NLO QCD and EW corrections.
  The syntax is: generate p p > t t~ QED^2=2 QCD^2=4 [QCD].
  While counterintuitive, this is interpreted as in the previous slide:
  - Generate LO contributions which satisfy the squared-order constraints ($O(\alpha_s^2)$ and $O(\alpha_s\alpha)$)
  - For the NLO corrections, add a power of α_s on top of both. This will give ($O(\alpha_s^3)$ and $O(\alpha_s^2\alpha)$)



MG5_aMC Syntax (III)

- Can I use diagram-order constraints?
- While this will give inconsistencies when NLO EW corrections are computed, it may be useful e.g. in EFT studies
- If the user asks for diagram constraints together with NLO corrections, the code will issue a clear warning, asking the user to acknowledge what he/she wants to do
- More info on http://amcatnlo.cern.ch/co.htm

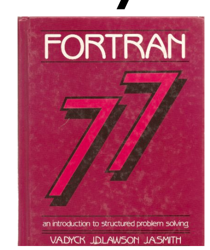
Processes with tagged photons: how to

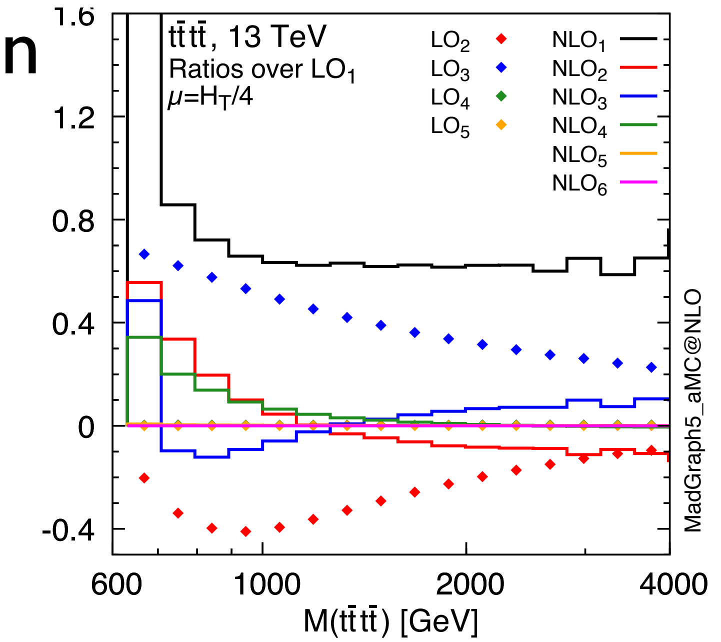
- In practice: a new model with both the HE renormalisation scheme (G_μ) and the $\alpha(0)$ is available: loop_qcd_qed_sm_Gmu-a0
- Once loaded, tagged photons can be specified via the generate syntax: generate t t~ !a! [QED]
- Photons marked as tagged will not originate real emissions where $\gamma \to 2f$ and the corresponding (local and integrated) FKS counterterms will not be included
- For each tagged photon, a term proportional to the difference between $\alpha(0)$ and $\alpha_{G\mu}$ is added (it has IR poles)
- The final result is rescaled by $(\alpha(0)/\alpha_{G\mu})^{\mathrm{NTagPhotons}}$
- Result presented for top-pair and single-top production + photons
  Pagani, Shao, Tsinikos, MZ 2106.02059
- Available in v3.3.0

Accessing the various coupling combinations

- The different coupling combinations to the cross section are evaluated in the same run
- Histograms can be booked for each of them in the analysis
- The coupling combination can be detected by using the `orders_tag_plot` variable

```
integer orders_tag_plot
common /corderstagplot/ orders_tag_plot
```





- It is typically computed as 100*QED + 1*QCD (may change if more coupling types are around)
- In any case, the specific values are printed inside the log file

```
INFO: orders_tag_plot is computed as:         + QCD *        1          + QED *      100
 orders_tag_plot=            4   for QCD,QED, =           4 ,           0 ,
 orders_tag_plot=          202   for QCD,QED, =           2 ,           2 ,
 orders_tag_plot=          400   for QCD,QED, =           0 ,           4 ,
 orders_tag_plot=            6   for QCD,QED, =           6 ,           0 ,
 orders_tag_plot=          204   for QCD,QED, =           4 ,           2 ,
 orders_tag_plot=          402   for QCD,QED, =           2 ,           4 ,
```

Accessing the various coupling combinations in LHE events

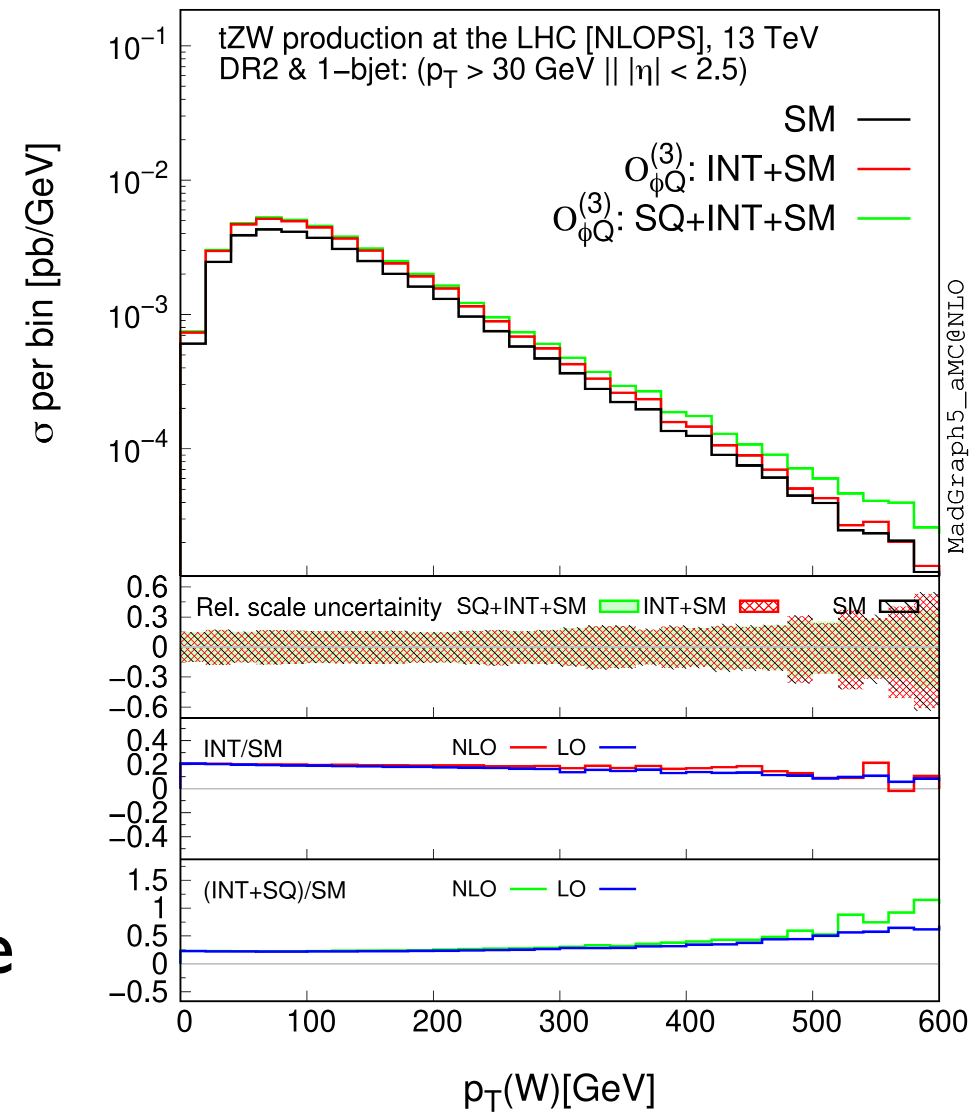
- The same coupling structure can be accessed inside the LHE event file (when PS-matching is possible)
- Weights are stored in the same format as the scale/PDF variations

```
<initrwgt>
  <weightgroup name='scale_variation              0     0' combine='envelope'>
    <weight id='1001'> tag=           0 dyn=    0 muR=0.10000E+01 muF=0.10000E+01 </weight>
    <weight id='1002'> tag=           0 dyn=    0 muR=0.20000E+01 muF=0.10000E+01 </weight>
    <weight id='1003'> tag=           0 dyn=    0 muR=0.50000E+00 muF=0.10000E+01 </weight>
    <weight id='1004'> tag=           0 dyn=    0 muR=0.10000E+01 muF=0.20000E+01 </weight>
    <weight id='1005'> tag=           0 dyn=    0 muR=0.20000E+01 muF=0.20000E+01 </weight>
    <weight id='1006'> tag=           0 dyn=    0 muR=0.50000E+00 muF=0.20000E+01 </weight>
    <weight id='1007'> tag=           0 dyn=    0 muR=0.10000E+01 muF=0.50000E+00 </weight>
    <weight id='1008'> tag=           0 dyn=    0 muR=0.20000E+01 muF=0.50000E+00 </weight>
    <weight id='1009'> tag=           0 dyn=    0 muR=0.50000E+00 muF=0.50000E+00 </weight>
  </weightgroup>
  <weightgroup name='scale_variation          40200     0' combine='envelope'>
    <weight id='1010'> tag=       40200 dyn=    0 muR=0.10000E+01 muF=0.10000E+01 </weight>
    <weight id='1011'> tag=       40200 dyn=    0 muR=0.20000E+01 muF=0.10000E+01 </weight>
    <weight id='1012'> tag=       40200 dyn=    0 muR=0.50000E+00 muF=0.10000E+01 </weight>
    <weight id='1013'> tag=       40200 dyn=    0 muR=0.10000E+01 muF=0.20000E+01 </weight>
    <weight id='1014'> tag=       40200 dyn=    0 muR=0.20000E+01 muF=0.20000E+01 </weight>
    <weight id='1015'> tag=       40200 dyn=    0 muR=0.50000E+00 muF=0.20000E+01 </weight>
    <weight id='1016'> tag=       40200 dyn=    0 muR=0.10000E+01 muF=0.50000E+00 </weight>
    <weight id='1017'> tag=       40200 dyn=    0 muR=0.20000E+01 muF=0.50000E+00 </weight>
    <weight id='1018'> tag=       40200 dyn=    0 muR=0.50000E+00 muF=0.50000E+00 </weight>
  </weightgroup>
  <weightgroup name='scale_variation          40202     0' combine='envelope'>
    <weight id='1019'> tag=       40202 dyn=    0 muR=0.10000E+01 muF=0.10000E+01 </weight>
 ...
```

```
<event>
 5      0 0.15776264E+00 0.21383348
      -5  -1     0     0     0   501 0.0
      21  -1     0     0   501   502 0.0
      -6   1     1     2     0   502 -.4
      24   1     1     2     0     0 0.7
      23   1     1     2     0     0 -.3
#aMCatNLO 1   0   0   1   2 0.91081533E-
0.00000000E+00
  <rwgt>
   <wgt id='1001'> 0.15776E+00 </wgt
   <wgt id='1002'> 0.15496E+00 </wgt
   <wgt id='1003'> 0.15846E+00 </wgt
   <wgt id='1004'> 0.16498E+00 </wgt
   <wgt id='1005'> 0.16195E+00 </wgt
   <wgt id='1006'> 0.16585E+00 </wgt
   <wgt id='1007'> 0.14640E+00 </wgt
   <wgt id='1008'> 0.14389E+00 </wgt
   <wgt id='1009'> 0.14693E+00 </wgt
   <wgt id='1010'> 0.13388E+00 </wgt
   <wgt id='1011'> 0.12227E+00 </wgt
   <wgt id='1012'> 0.14798E+00 </wgt
   <wgt id='1013'> 0.13946E+00 </wgt
   <wgt id='1014'> 0.12736E+00 </wgt
   <wgt id='1015'> 0.15414E+00 </wgt
...
```

Accessing the various coupling combinations

- In either case, having all the couplings available from the same run makes them all statistically-correlated
- It is specially useful in the context of EFT studies, where different admixtures of new-physics can be morphed starting from the event weights
- Careful when matching to PS!
  If the statistical distribution of colour-flows is very different from one coupling combination to another (e.g. EFT vs SM), morphing could be dangerous!



El Faham, Maltoni, Mimasu, MZ
arXiv:2111.03080