

Milan, June 2026

Automated computational tools for LHC simulations

Abstract

A set of exercises is proposed to train on the concepts and applications illustrated in the course as well as a take-home exam.

Please provide a report where all the exercises in section 1 are solved, and one on choice in section 2 and 3.

1 Compulsory exercises

1. Calculate the partial widths of the Z and W bosons and the t quark and compare with the analytic formulas you find in preferred QFT book.
2. Calculate the total width of a Higgs boson of 200 GeV mass.
3. Calculate the width of the muon. Plot the energy spectrum of the electron. Note that you need to choose a MG model where the muon has a mass (`sm-full`).
4. Calculate $\sigma(e^+e^- \rightarrow q\bar{q})$ @ NLO with **MG5aMC**. What is the K -factor?
5. Calculate $\sigma(pp \rightarrow e^+e^-)$ @ NLO with **MG5aMC**. What is the K -factor? Using the analysis `analysis_HwU_pp_lplm`, comment and explain the differences between the LO and NLO distributions.
6. Generate and output the folder for $pp \rightarrow t\bar{t}$ at NLO with **MG5aMC**. Using that directory:
 - (a) Calculate the cross section at LO and LO+PS (using `Pythia8` as shower).
Hint: use the analyses `analysis_HwU_pp_ttx_v2` and `py8an_HwU_pp_ttx_v2` for the two runs, respectively, and note that they have to be set inside different cards.
What is the K -factor? Can you explain the differences between the LO and LO+PS distributions?
 - (b) Calculate $\sigma(pp \rightarrow t\bar{t})$ @ NLO with **MG5aMC**. What is the K -factor? Comment and explain the differences between the LO and NLO distributions.

2 One exercise in this list

1. (a) Derive the expression for the amplitude squared $e^+e^- \rightarrow q\bar{q}$, in terms of the invariants, s, t, u , for massless quarks. Include only

photon exchange. Express it in terms of the c.m.s. variables $\cos \theta, \phi$ and write the differential cross section:

$$\frac{d\sigma}{d\cos\theta} = N_c \left(\sum_f Q_f^2 \right) \frac{\pi\alpha^2}{2s} (1 + \cos\theta^2). \quad (1)$$

Which quarks should be included in the sum over flavors f ?

- (b) Include the diagram where a Z is exchanged and recall that the interaction vertex $q\bar{q}Z$ is given by:

$$\frac{-ig_w}{2\sqrt{2}} \gamma_\mu (V_f - A_f \gamma_5), \quad (2)$$

and the axial and vector couplings of the fermions to the Z are

$$V_f = T_f^3 - 2Q_f \sin^2 \theta_W, \quad A_f = T_f^3, \quad (3)$$

with $T_f^3 = 1/2$ for $f = \nu, u, \dots$ and $T_f^3 = -1/2$ for $f = e, d, \dots$

What happens to the $\cos \theta$ distribution?

2. $t\bar{t}$ production at hadron collider come from both $q\bar{q}$ annihilation and gg fusion.

- (a) Find the LO cross sections for $t\bar{t}$ production at Tevatron and LHC. Which initial parton contributions are dominating in the two cases?
- (b) Find the cross sections for $t\bar{t} + 1j$ production at the LHC. Select events for which the jet has $p_T > 20$ GeV and $|\eta| < 4$ (Is a ΔR cut needed to have a finite cross section?). Estimate the cross section and compare it with the LO result for $t\bar{t}$. Is the result reasonable? What's going on? Explain.

3. (a) The differential cross section for the case of massive final state for $e^+e^- \rightarrow Q\bar{Q}g$ is

$$\begin{aligned} \frac{1}{\sigma^{LO}} \frac{d^2\sigma}{dx_1 dx_2} = & \frac{1}{\beta} C_F \frac{\alpha_S}{2\pi} \left[\frac{2(x_1 + x_2 - 1 - \rho/2)}{(1-x_1)(1-x_2)} \right. \\ & - \frac{\rho}{2} \left(\frac{1}{(1-x_1)^2} + \frac{1}{(1-x_2)^2} \right) \\ & \left. + \frac{1}{1+\rho/2} \frac{(1-x_1)^2 + (1-x_2)^2}{(1-x_1)(1-x_2)} \right], \quad (4) \end{aligned}$$

where

$$\rho = \frac{4m^2}{s} \leq 1, \quad \beta = \sqrt{1 - \rho}. \quad (5)$$

and σ^{LO} is the leading order cross section for $e^+e^- \rightarrow Q\bar{Q}$. Write the soft and collinear approximation of the amplitude in the case the gluon is close to the quark:

$$\frac{1}{\sigma^{LO}} \frac{d^2\sigma}{dzd\theta^2} = C_F \frac{\alpha_S}{\pi} \frac{1}{z} \frac{\theta^2}{(\theta^2 + \rho)^2} \quad (6)$$

where $z = 2E_g/\sqrt{s}$ is the energy fraction of the gluon and θ the angle between the gluon and the quark. Plot the behaviour of the matrix element in the massless and massive cases and compare with Fig. 1. Explain this behaviour in terms of angular momentum conservation.

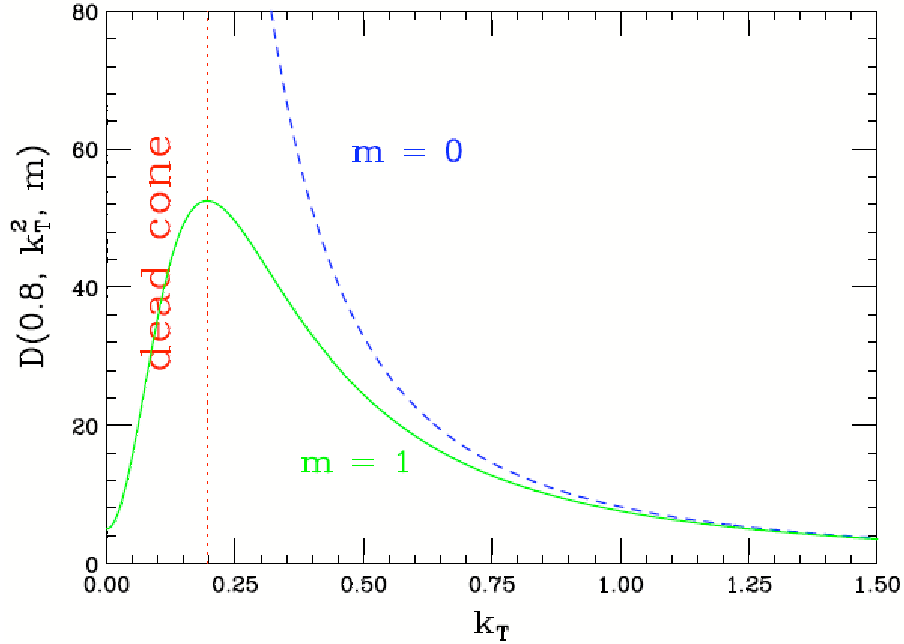


Figure 1: Dead cone: emission of collinear (soft) gluons from a massive quark is suppressed by angular momentum conservation [Courtesy of C. Oleari].

- (c) Use **MG5aMC** and verify that there are no collinear divergences to be regulated and the cross section is finite with just a minimum

cut on the energy of the gluon. Plot the behaviour of the cross sections as a function of the quark mass and verify that it has a logarithmic behaviour.

4. Use **MG5aMC** to obtain the cross sections at LO and NLO, when possible, for the main Higgs production ($m_H = 125$ GeV) channels at the LHC. Beware that you have to choose what to do for the Higgs from gluon fusion. This can be calculated at NLO in the HEFT using the **HC_NLO_X0_UFO-heft** model.

