NLO: How to?

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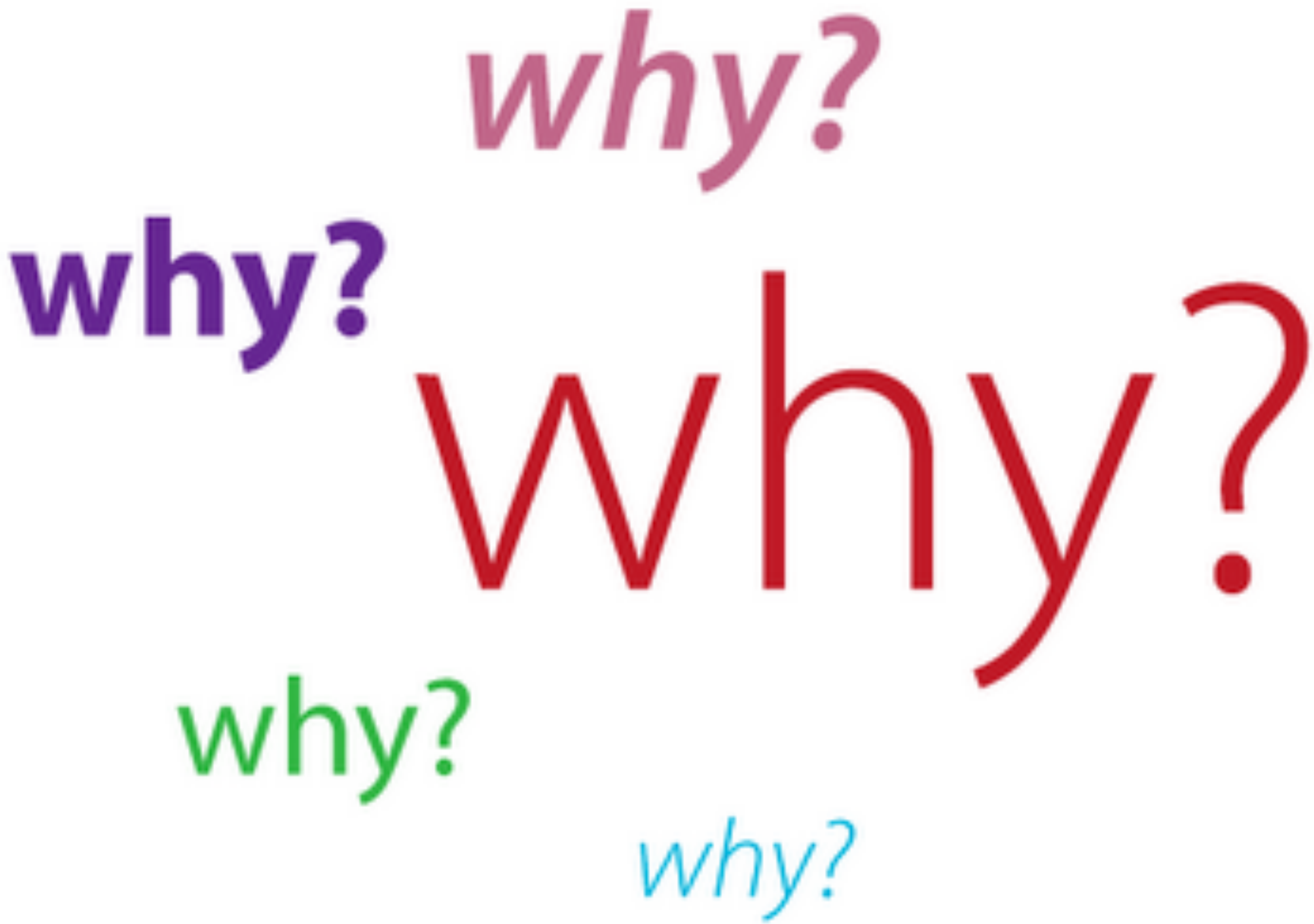
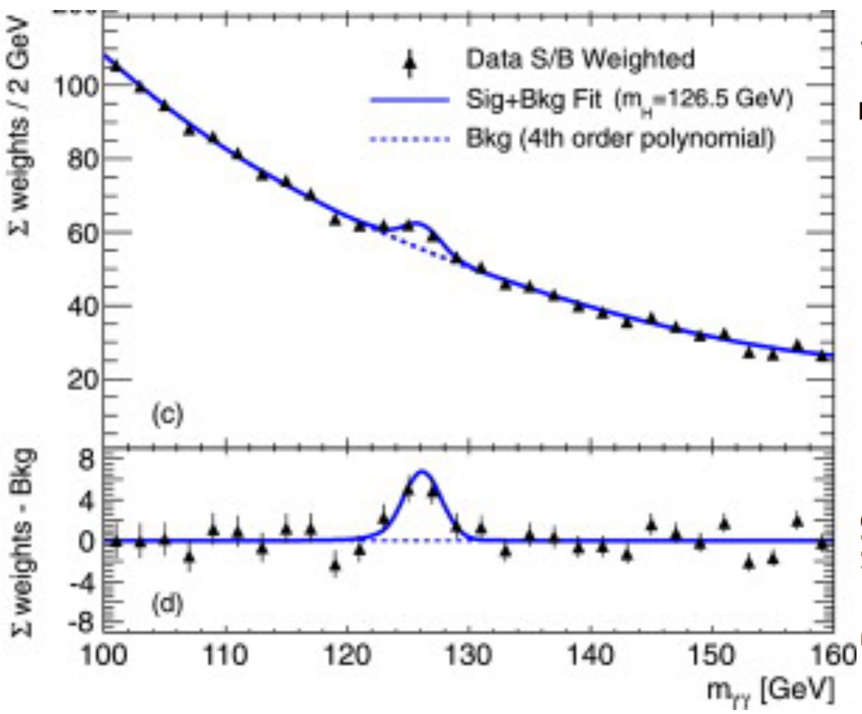
Introduction: Why do we need N$^{(k)}$LO?



Discoveries at hadron colliders

Peak

$H \to \gamma\gamma$

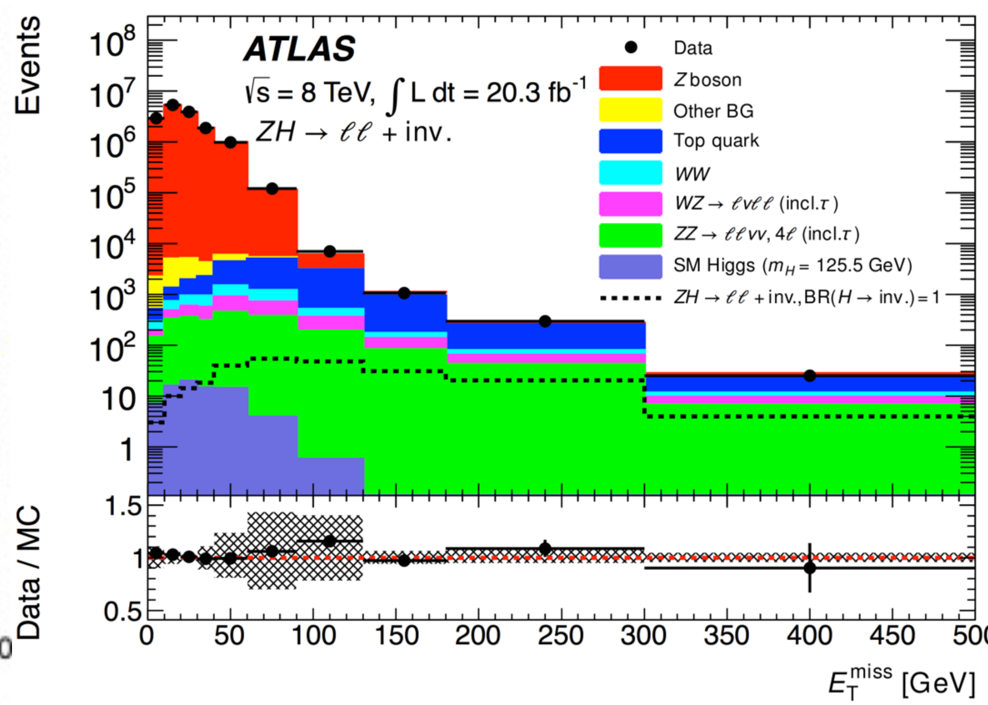


EASY

Background directly measured from **data**.
Theory needed only for parameter extraction

Shape

$Z\,H \to l^+l^- + inv.$

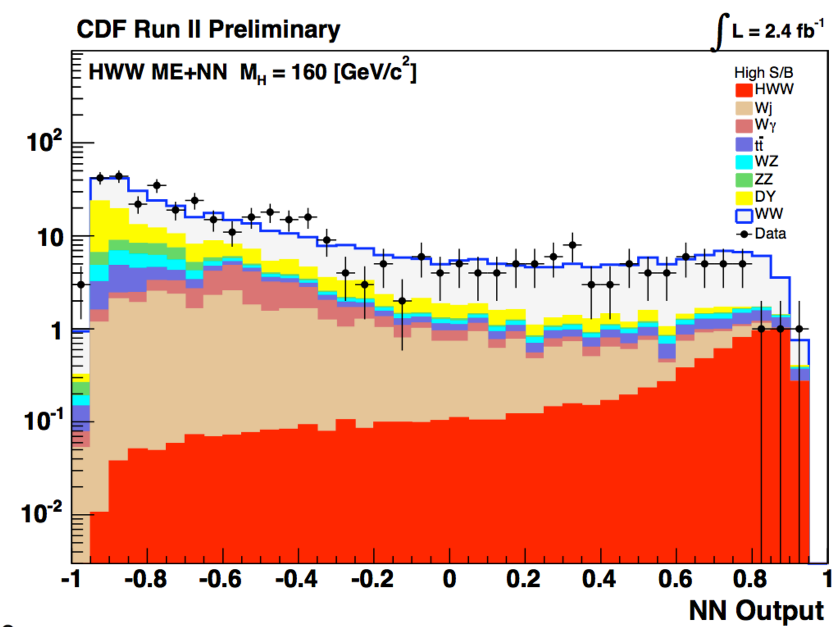


HARD

Background **SHAPE** needed.
Flexible MC for both signal and background validated and tuned to data

Rate

$H \to W^+\,W^-$

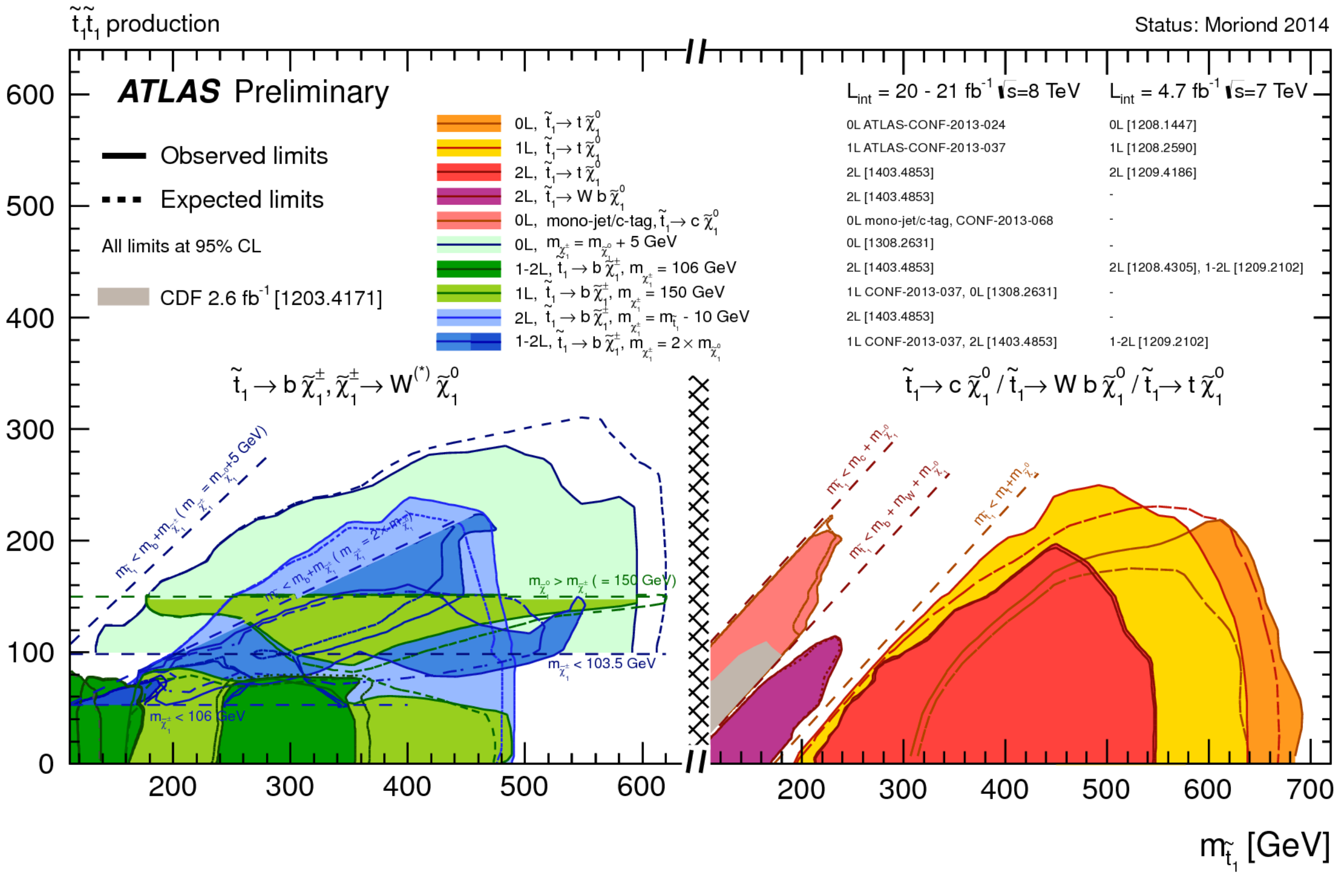


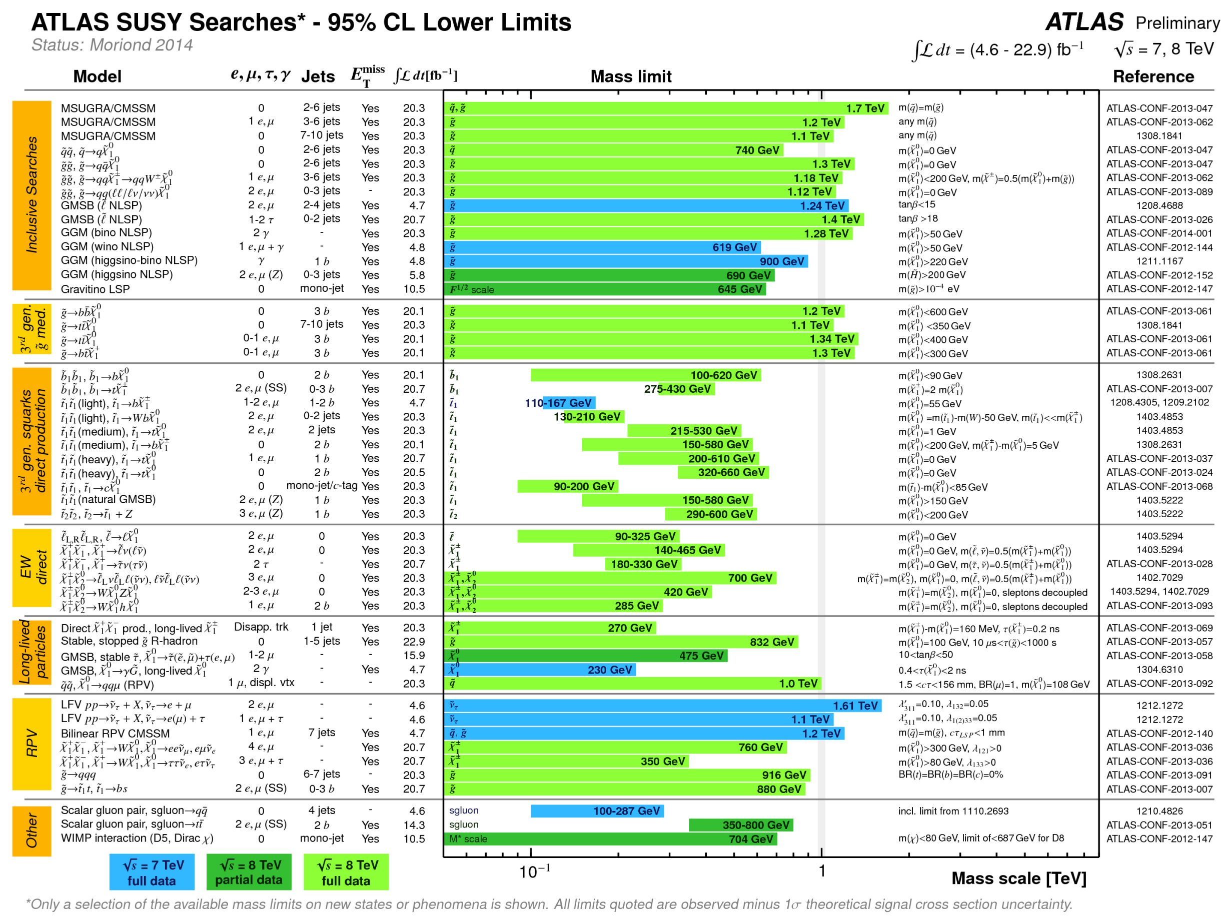
VERY HARD

Relies on prediction for both **shape** and **normalization**.
Complicated interplay of best simulations and data

New physics?

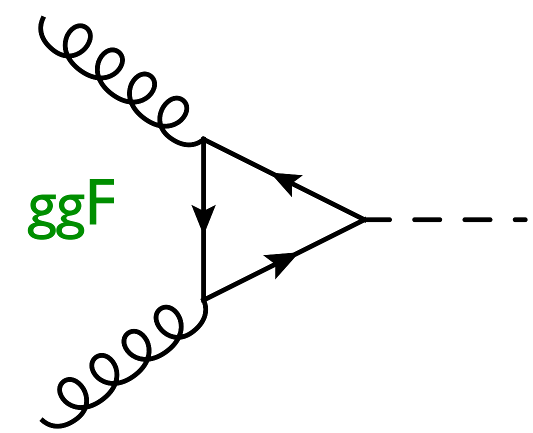
- No NP has been discovered yet
- Either there is no NP, or it is hiding very well
- If it is there, it will be a 'Hard' or 'very Hard' discovery
  - Need for accurate predictions for signal and background

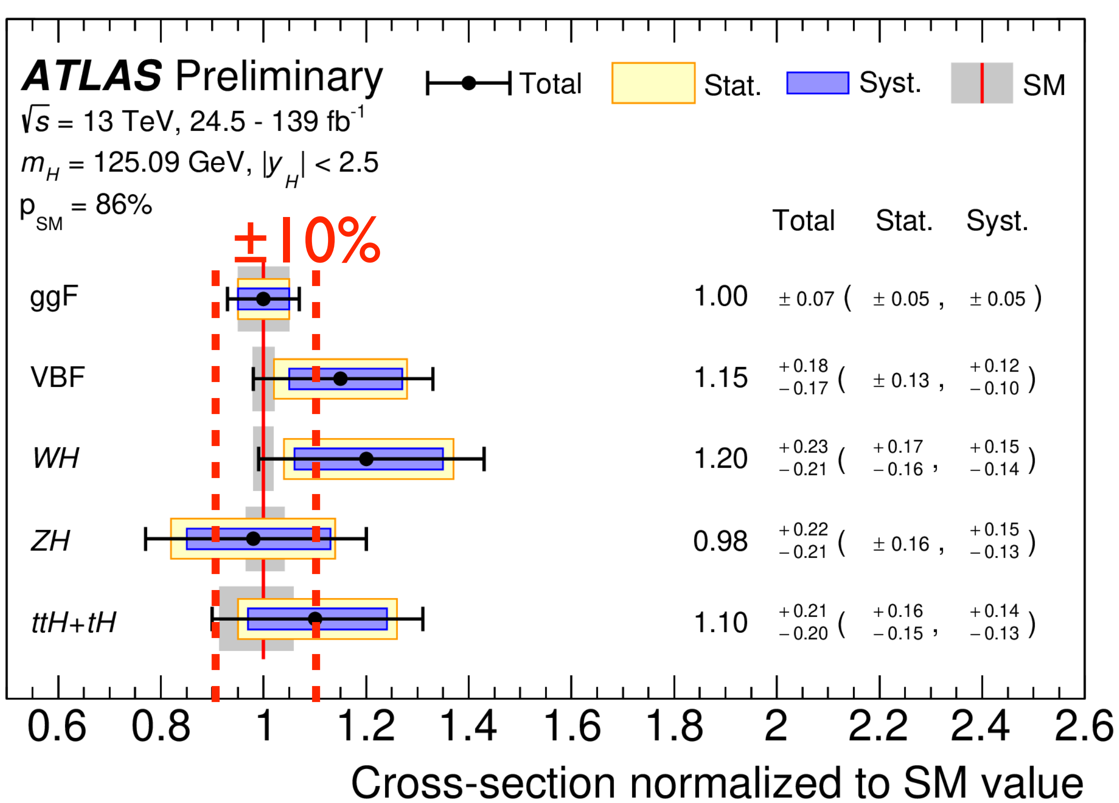




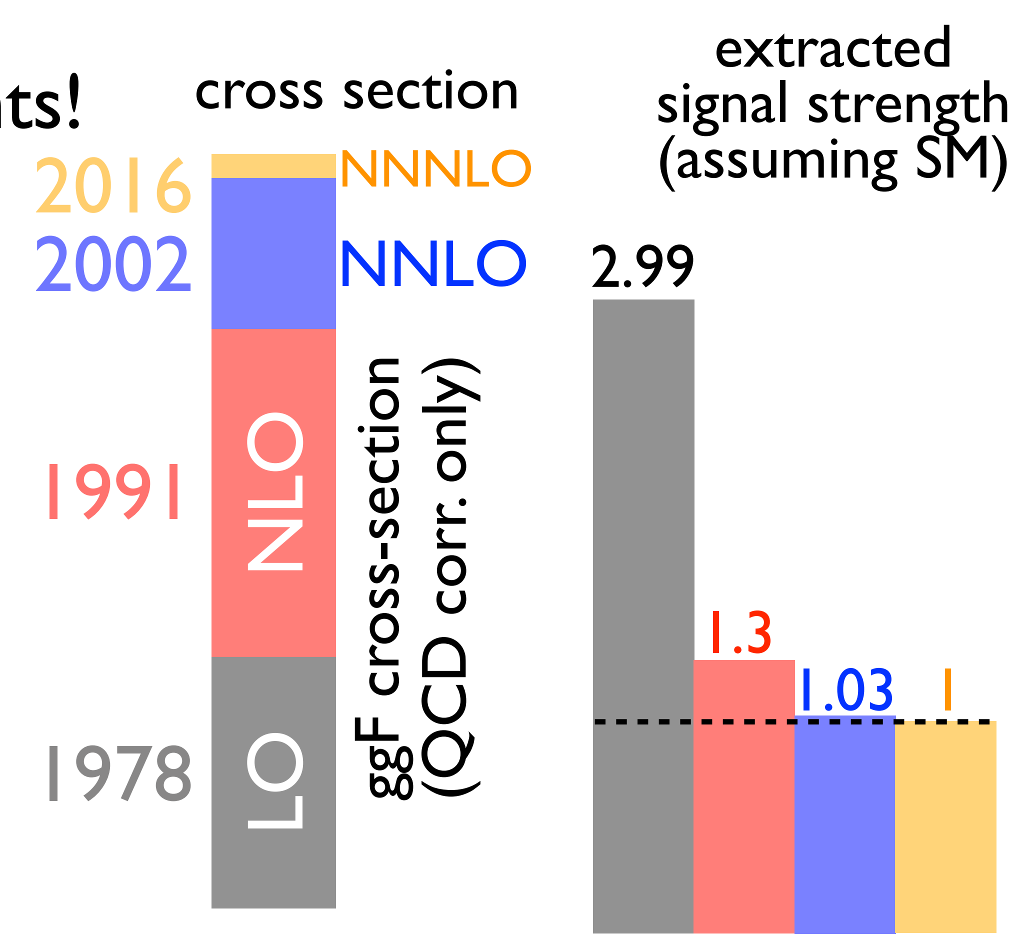
Cross-section measurements

- The discovery of the Higgs boson is an emblematic example of the need for precision
- Large perturbative corrections for the dominant channel (gluon fusion)
- Without higher-order corrections, measured signal strength ~3 * SM
- Very competitive experimental measurements!



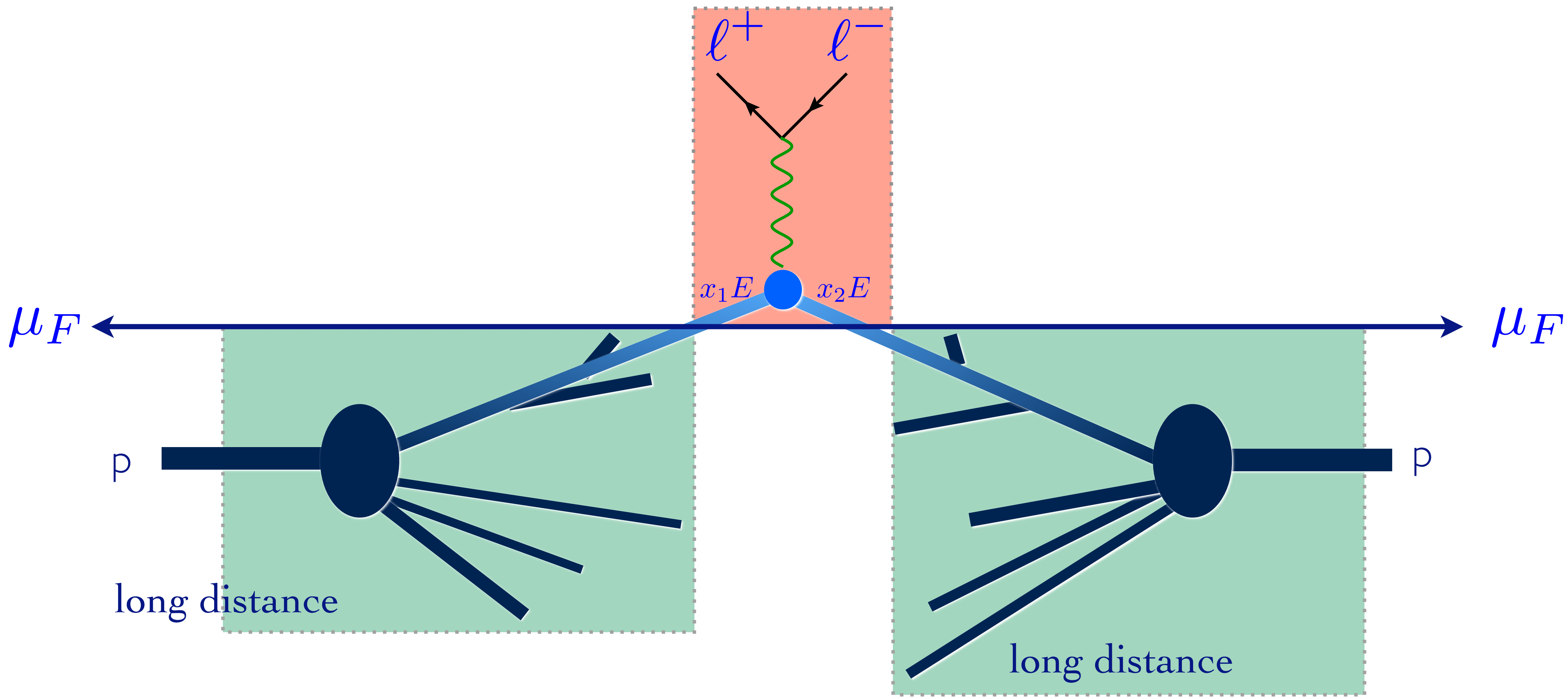


$$\mu = \frac{\sigma_{\mathrm{EXP}}}{\sigma_{\mathrm{TH}}}$$



Anastasiou et al, arXiv:1602.00695

How to compute a cross-section



$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

Perturbation theory at work

$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ Parton-level cross section

The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter

$$\hat{\sigma} = \alpha_s^b \sigma_0 + \alpha_s^{b+1}\sigma_1 + \alpha_s^{b+2}\sigma_2 + \alpha_s^{b+3}\sigma_3 + \dots$$

LO | NLO | NNLO | NNNLO

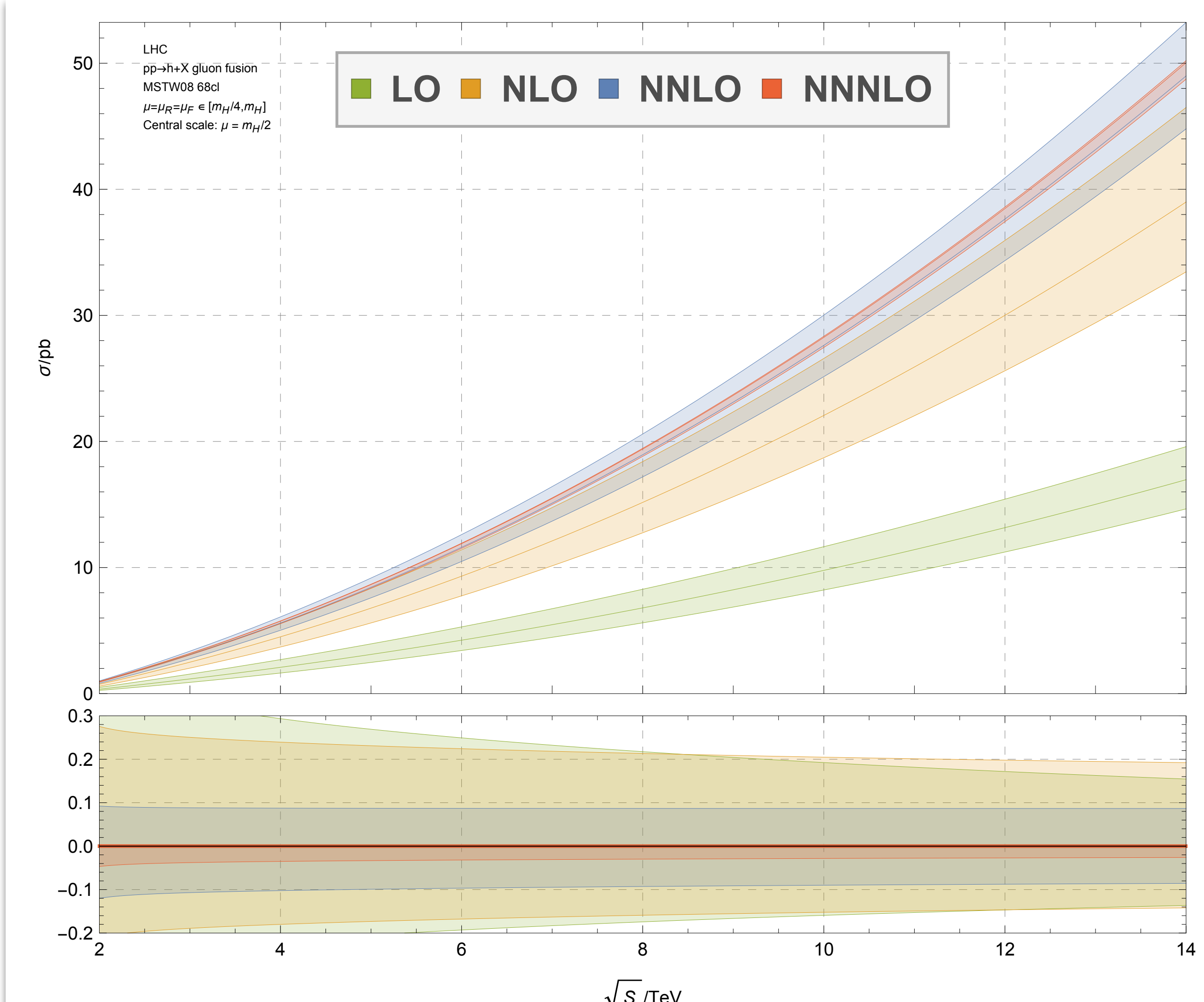
Remember:

$$\alpha_s = \alpha_s(\mu_R) \qquad \sigma_i = \sigma_i(\mu_R, \mu_F)$$

Coupling and cross section depend on *unphysical* scales

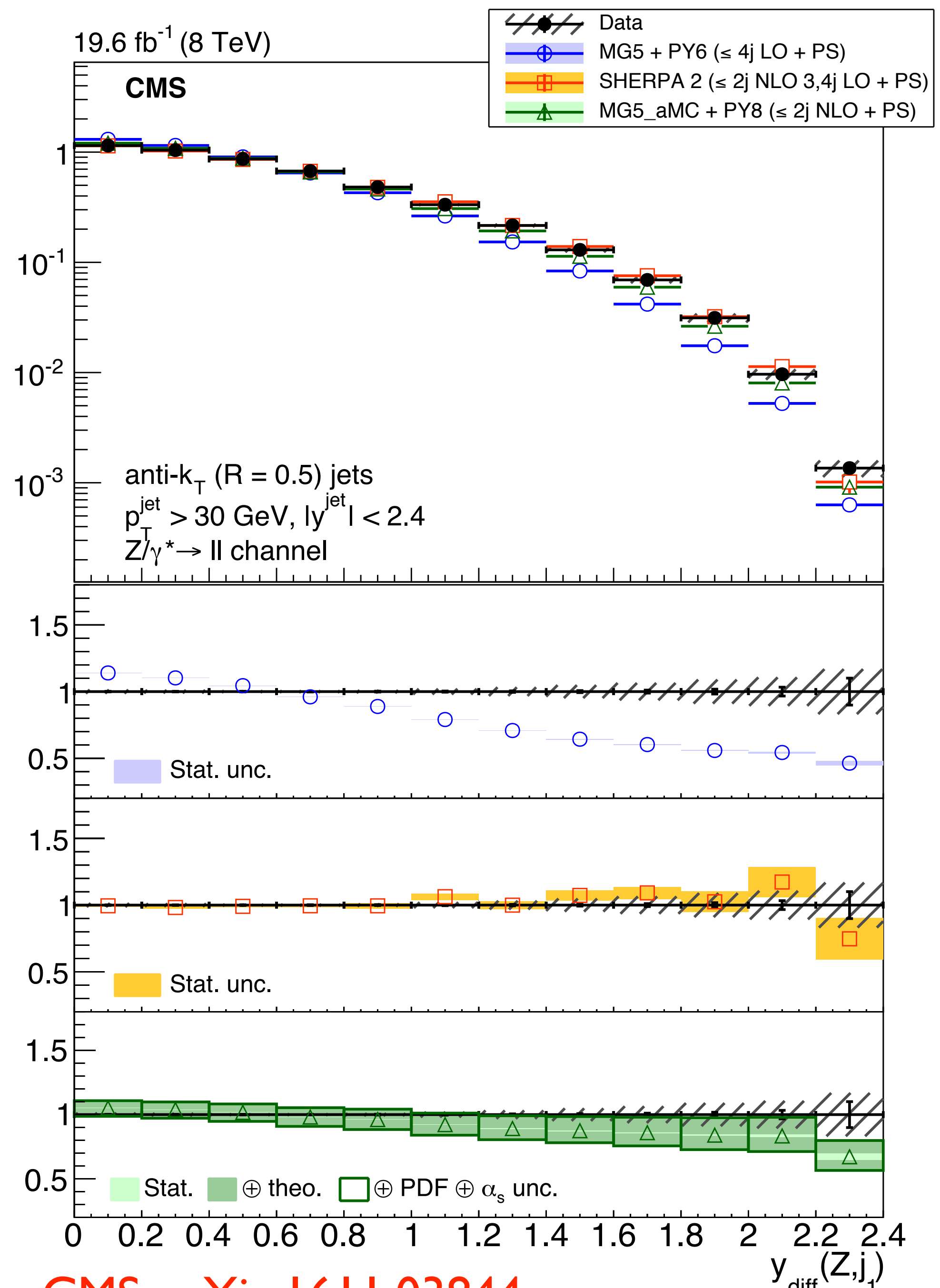
Perturbation theory at work

- The inclusion of higher orders improves the reliability of a given computation
  - More reliable description of total rates and shapes
  - Residual uncertainties related to the arbitrary scales in the process decrease
  - The computational complexity grows exponentially
  - NLO is mandatory for LHC physics!



Anastasiou, et al, arXiv:1503.06056

Perturbation theory at work



CMS, arXiv:1611.03844

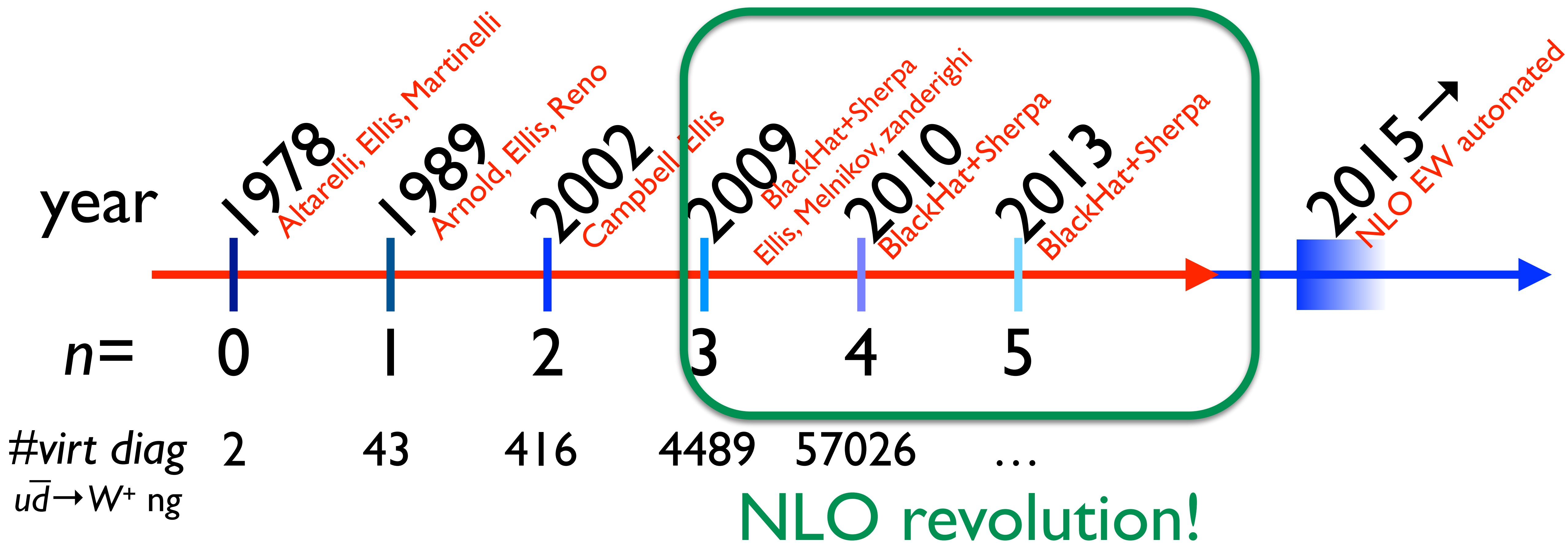
- In order to describe data, LO predictions must be rescaled to match the cross section including higher orders (typically NNLO)
- NLO predictions are generally not rescaled →More predictive power
- NLO effects can be important even if merged samples are used at LO

In these lectures:

- How to compute effectively a NLO cross section?
  - How to deal with infrared divergences?
  - How to compute loops?
  - How about EW corrections?

NLO (pre)history

- NLO evolution:
  - e.g. pp→W+*n* jets



NLO revolution

- Amazing development of computational techniques to tackle *any* process at NLO
  - Local subtraction
- Computation of loop MEs
  - Tensor reduction
  - Generalized unitarity
  - Integrand reduction

Frixione, Kunszt, Signer, hep-ph/9512328
Catani, Seymour, hep-ph/9605323

Passarino, Veltman, 1979
Denner, Dittmaier, hep-ph/509141
Binoth, Guillet, Heinrich, Pilon, Reiter, arXiv:0810.0992

Bern, Dixon, Dunbar, Kosower, hep-ph/9403226 + …
Ellis, Giele, Kunszt, arXiv:0708.2398
+ Melnikov, arXiv:0806.3467

Ossola, Papadopoulos, Pittau, hep-ph/0609007
Del Aguila, Pittau, hep-ph/0404120
Mastrolia, Ossola, Reiter, Tramontano, arXiv:1006.0710

Going NLO

$$\hat{\sigma} = \alpha_s^b \sigma_0 + \alpha_s^{b+1} \sigma_1 + \alpha_s^{b+2} \sigma_2 + \alpha_s^{b+3} \sigma_3 + \dots$$

LO NLO NNLO NNNLO

- NLO is the first order where the scale dependence in α_s and PDFs is compensated by loop corrections
  - First reliable predictions for rates and uncertainties
- Better description of final state (inclusion of extra radiation)
- Opening of new partonic channels from real emissions

NLO: how to?

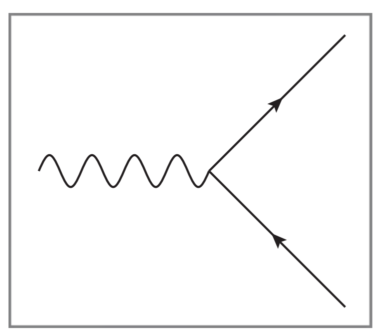
- Three ingredients need to be computed at NLO

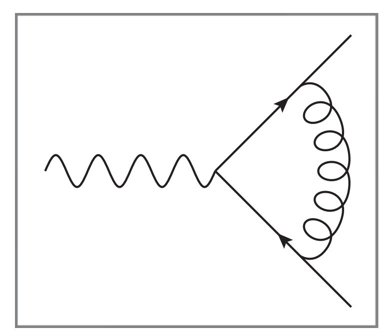
$$\sigma_{NLO} = \int_n \alpha_s^b d\sigma_0 + \int_n \alpha_s^{b+1} d\sigma_V + \int_{n+1} \alpha_s^{b+1} d\sigma_R$$

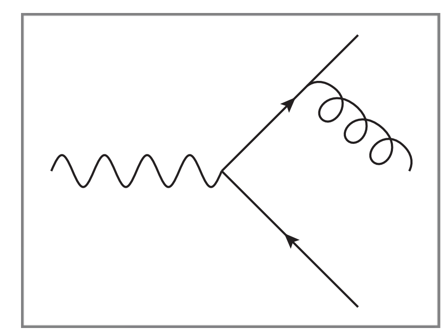
Born cross section

Virtual corrections

Real-emission corrections

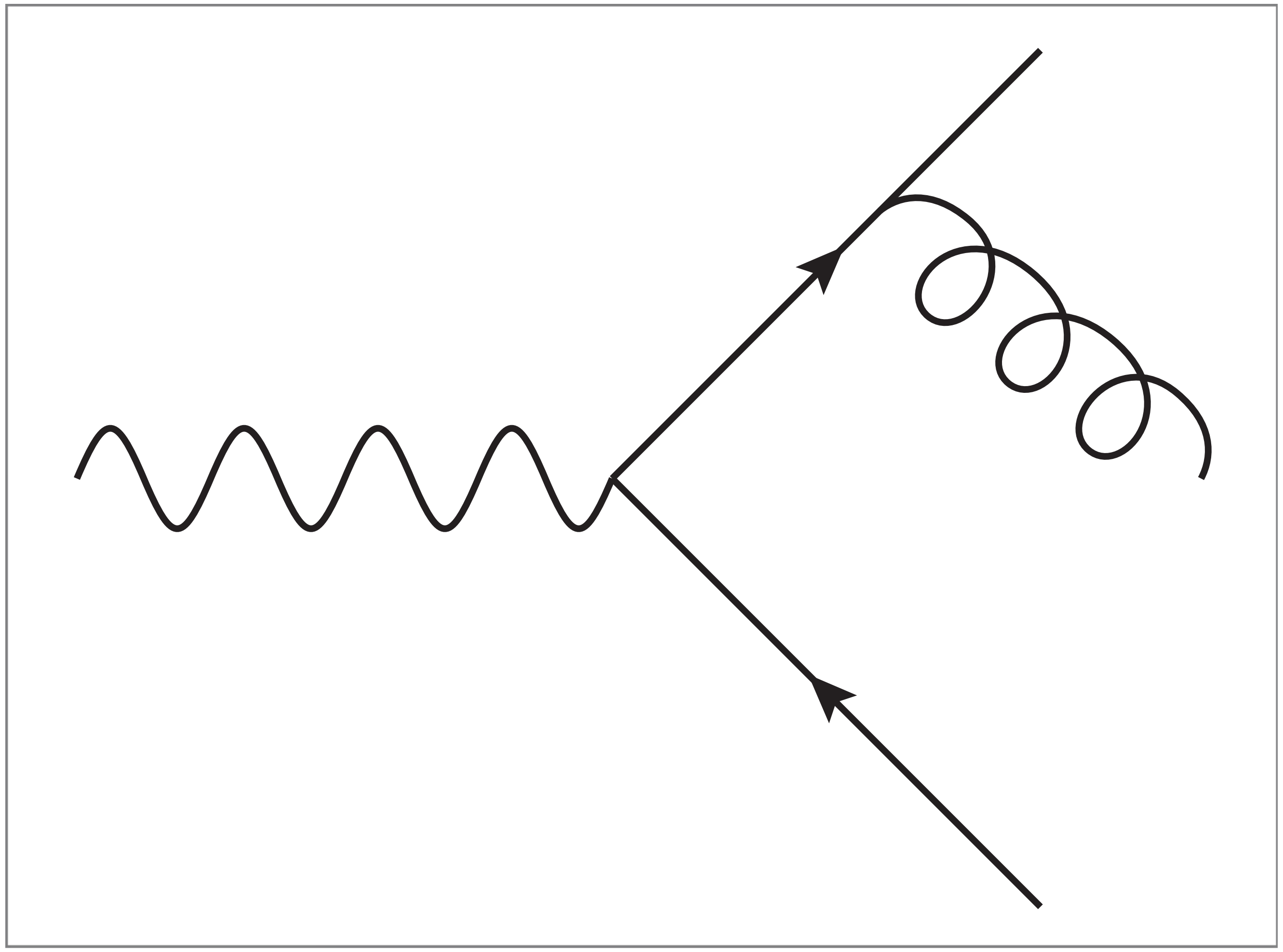






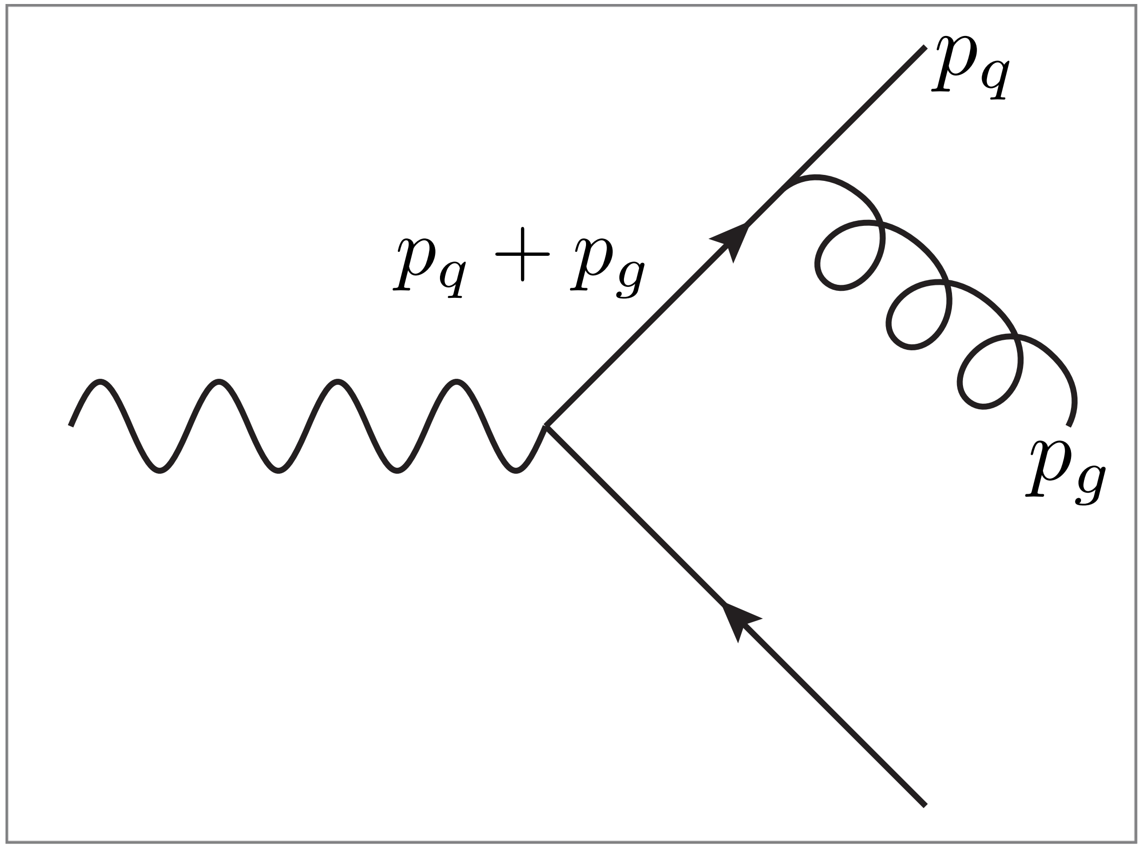
- Remember: virtual and reals are not separately finite, but their sum is (KLN theorem). Divergences have to be subtracted before numerical integration. We will shortly see how

Infrared divergences



$$\sigma_{NLO} = \int_n \alpha_s^b d\sigma_0 + \int_n \alpha_s^{b+1} d\sigma_V + \int_{n+1} \alpha_s^{b+1} d\sigma_R$$

Branching

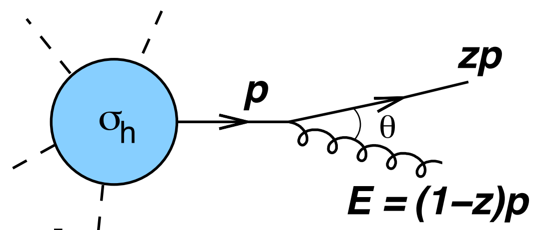


$$\int_{n+1} \alpha_s^{b+1} d\sigma_R$$

- When the integral over the phase-space of the gluon is performed, one can have $(p_q+p_g)^2=0$
- Since $(p_q+p_g)^2=2E_qE_g(1\text{-}\cos\theta)$ it happens when the gluon is soft (E_g=0) or collinear to the quark (θ=0)
- In both cases, the propagator leads to a divergent cross section

Singularities

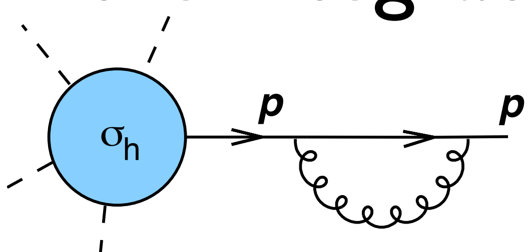
- Let us rewrite the branching of a gluon from a quark as



$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

Where k_t is the transverse momentum of the gluon $k_t = E\sin\theta$. It diverges in the soft ($z \to 1$) and collinear ($k_t \to 0$) region

- These singularities cancel with the virtual contribution, which comes from the integration of the loop momentum



$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

- The cancelation happens if we cannot distinguish between the case of no branching, and that of a soft/collinear branching

Cancellation of divergences

- The KLN theorem tells us that divergences from the virtual and real emission cancel in the sum *if observables are insensitive to soft and collinear branchings* (IR-safety)
- When doing an analytic computation in dimensional regularisation, divergences appear as poles in the regularisation parameter ε
- In the real emissions, poles appear *after* the phase space integration in d dimension

Infrared safety

- In order to have meaningful predictions in fixed-order perturbation theory, observables must be IR-safe, *i.e.* not sensitive to the emission of soft or collinear partons.
- In particular, if an observable depends on the momentum p_i, it must not be sensitive on the branching $p_i \to p_j + p_k$, where either p_j is soft or p_j and p_k are collinear
- For example
  - The number of gluons in an event ❌
  - The number of jets with $p_T > p_T^{min}$ ✅
  - The hardest parton in an event ❌
  - The hardest jet ✅

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Phase space integration

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} + \int d^4\Phi_n \mathcal{V} + \int d^4\Phi_{n+1} \mathcal{R}$$

contains $\int d^d l$

- For complicated processes the integrations have to be done via MonteCarlo techniques, in an integer number of dimensions
- Divergences have to be canceled explicitly
- Slicing/Subtraction methods have been developed to extract divergences from the phase-space integrals

Example

- Suppose that we can cast the phase space integral in the form

$$\int_0^1 dx f(x) \quad \text{with} \quad f(x) = \frac{g(x)}{x} \quad \text{and } g(x) \text{ a regular function}$$

- We introduce a regulator which renders the integral finite

$$\int_0^1 dx x^{\varepsilon} f(x) = \int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- The divergence will turn into a pole in ε. How can we extract the pole?

Phase space slicing

$$\lim_{\varepsilon\to 0}\int_0^1 dx x^\varepsilon f(x) = \lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- We introduce a small parameter δ≪1:

$$\lim_{\varepsilon\to 0}\int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}} = \lim_{\varepsilon\to 0}\left(\int_0^\delta dx \frac{g(x)}{x^{1-\varepsilon}} + \int_\delta^1 dx \frac{g(x)}{x^{1-\varepsilon}}\right)$$

finite integral
(can be computed numerically)

pole in ε

Subtraction method

$$\lim_{\varepsilon \to 0} \int_0^1 dx x^\varepsilon f(x) = \lim_{\varepsilon \to 0} \int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}}$$

- Add and subtract $g(0)/x$

$$\lim_{\varepsilon \to 0} \int_0^1 dx \frac{g(x)}{x^{1-\varepsilon}} = \lim_{\varepsilon \to 0} \int_0^1 dx x^\varepsilon \left(\frac{g(0)}{x} + \frac{g(x)}{x} - \frac{g(0)}{x} \right)$$

nite integral
(can be computed numerically)

pole in ε

Slicing vs Subtraction

- In both cases the pole is extracted and we end up with a finite remainder:

$$g(0) \log \delta + \int_\delta^1 dx \frac{g(x)}{x} \qquad \int_0^1 dx \frac{g(x) - g(0)}{x}$$

- Subtraction acts like a plus distribution
- Slicing works only for small δ: δ-independence of cross section and distributions must be proven; subtraction is exact
- Both methods have cancelations between large numbers. If for a given observable $\lim_{x \to 0} O(x) \neq O(0)$ or we choose a too small bin size, instabilities will arise (we cannot ask for an infinite resolution)
- Subtraction is in general more flexible: good for automation

NLO with subtraction

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} + \int d^4\Phi_n \mathcal{V} + \int d^4\Phi_{n+1} \mathcal{R}$$

- With the subtraction terms the expression becomes

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B}$$

$$+ \int d^4\Phi_n \left(\mathcal{V} + \int d^d\Phi_1 \mathcal{C}\right)_{\varepsilon\to 0}$$

Poles cancel from *d*-dim integration

$$+ \int d^4\Phi_{n+1} \left(\mathcal{R} - \mathcal{C}\right)$$

Integrand is finite in 4 dimension

- Terms in brackets are finite and can be integrated numerically in d=4 and independently one from another

The subtraction term

- The subtraction term C should be chosen such that:
  - It exactly matches the singular behaviour of R
  - It can be integrated numerically in a convenient way
  - It can be integrated exactly in d dimension, leading to the soft and/or collinear poles in the dimensional regulator
  - It is process independent (overall factor times Born)

Two subtraction methods

Dipole subtraction

Catani, Seymour, hep-ph/9602277 & hep-ph/9605323

- Recoil taken by one parton $\rightarrow N^3$ scaling
- Method evolves from cancelation of soft divergences
- Proven to work for simple and complicated processes
- Automated in MadDipole, AutoDipole, Sherpa, Helac-NLO, …

FKS subtraction

Frixione, Kunszt, Signer, hep-ph/9512328

- Recoil distributed among all particles $\rightarrow N^2$ scaling
- Method evolves from cancelation of collinear divergences
- Proven to work for simple and complicated processes
- Automated in MadGraph5_aMC@NLO and in the Powheg box/Powhel

FKS subtraction #1
Phase space partition

- Let us consider the real emission

$$d\sigma_R = \left|M^{n+1}\right|^2 d\Phi_{n+1}$$

- The matrix element $|M^{n+1}|^2$ diverges as

$$\left|M^{n+1}\right| \sim \frac{1}{\xi_i^2}\frac{1}{1-y_{ij}}$$

$$\xi_i = E_i\sqrt{\hat{s}}$$

$$y_{ij} = \cos\theta_{ij}$$

- Partition the phase space in order to have at most one soft and one collinear singularity

$$d\sigma_R = \sum_{ij} S_{ij}\left|M^{n+1}\right|^2 d\Phi_{n+1} \qquad \sum_{ij} S_{ij} = 1$$

$$S_{ij} \to 1 \text{ if } k_i \cdot k_j \to 0 \qquad S_{ij} \to 0 \text{ if } k_{m\neq i} \cdot k_{n\neq j} \to 0$$

FKS subtraction #2
Plus prescriptions

- Use plus prescriptions in y_{ij} and ξ_i to subtract the divergences

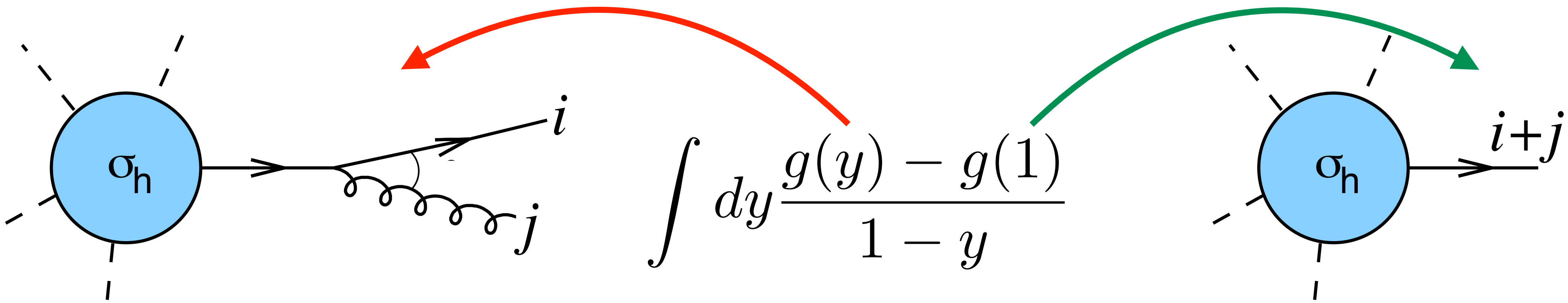
$$d\sigma_{\tilde{R}} = \sum_{ij} \left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i (1-y_{ij}) S_{ij} \left|M^{n+1}\right|^2 d\Phi_{n+1}$$

- Plus prescriptions are defined as

$$\int d\xi \left(\frac{1}{\xi}\right)_+ f(\xi) = \int d\xi \frac{f(\xi)-f(0)}{\xi} \qquad \int dy \left(\frac{1}{1-y}\right)_+ g(y) = \int dy \frac{g(y)-g(1)}{1-y}$$

- Maximally three counterevents are needed
  - Soft counterevent ($\xi_i \rightarrow 0$)
  - Collinear counterevents ($y_{ij} \rightarrow 1$)
  - Soft-collinear counterevents ($\xi_i \rightarrow 0$ and $y_{ij} \rightarrow 1$)
- The counterevents will feature the *same* kinematics

Kinematics of counterevents



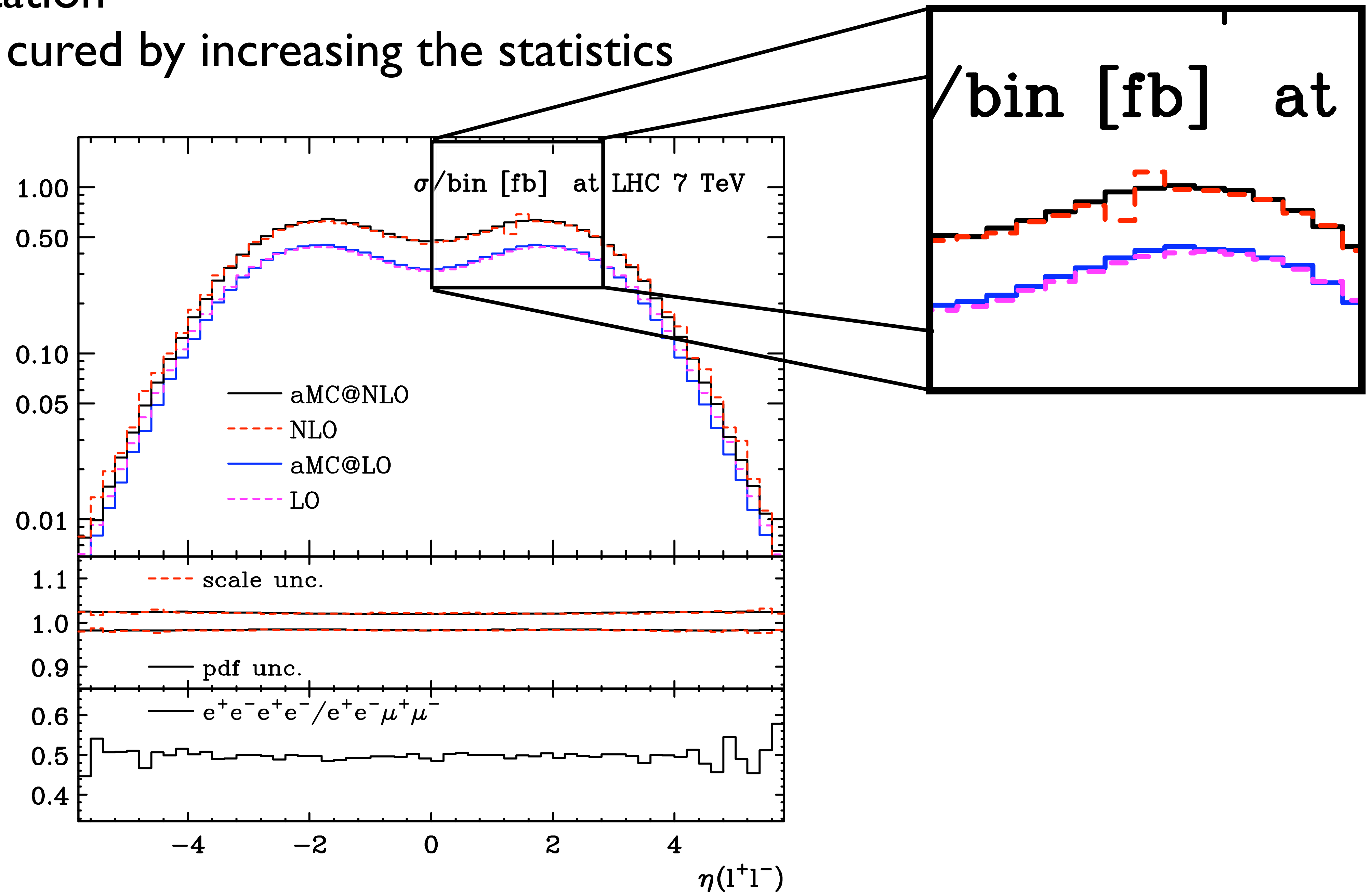
$$\int dy \frac{g(y) - g(1)}{1 - y}$$

Real emission | Subtraction term

- If i and j are on-shell in the event, for the counterevent the combined particle $i+j$ must be on shell
- $i+j$ can be put on shell only be reshuffling the momenta of the other particles
- It can happen that event and counterevent end up in different histogram bins
  - Use IR-safe observables and don't ask for infinite resolution!
  - Still, these precautions do not eliminate the problem…

An example in 4-lepton production

- The NLO result shows the typical peak-dip structure that hampers fixed-order computation
- Can be cured by increasing the statistics



Can we generate unweighted events at NLO?

- Another consequence of the kinematic mismatch is that we cannot generate events at NLO
- n+1-body contribution and n-body contribution are not bounded from above → unweighting not possible
- Further ambiguity on which kinematics to use for the unweighted events

More tomorrow

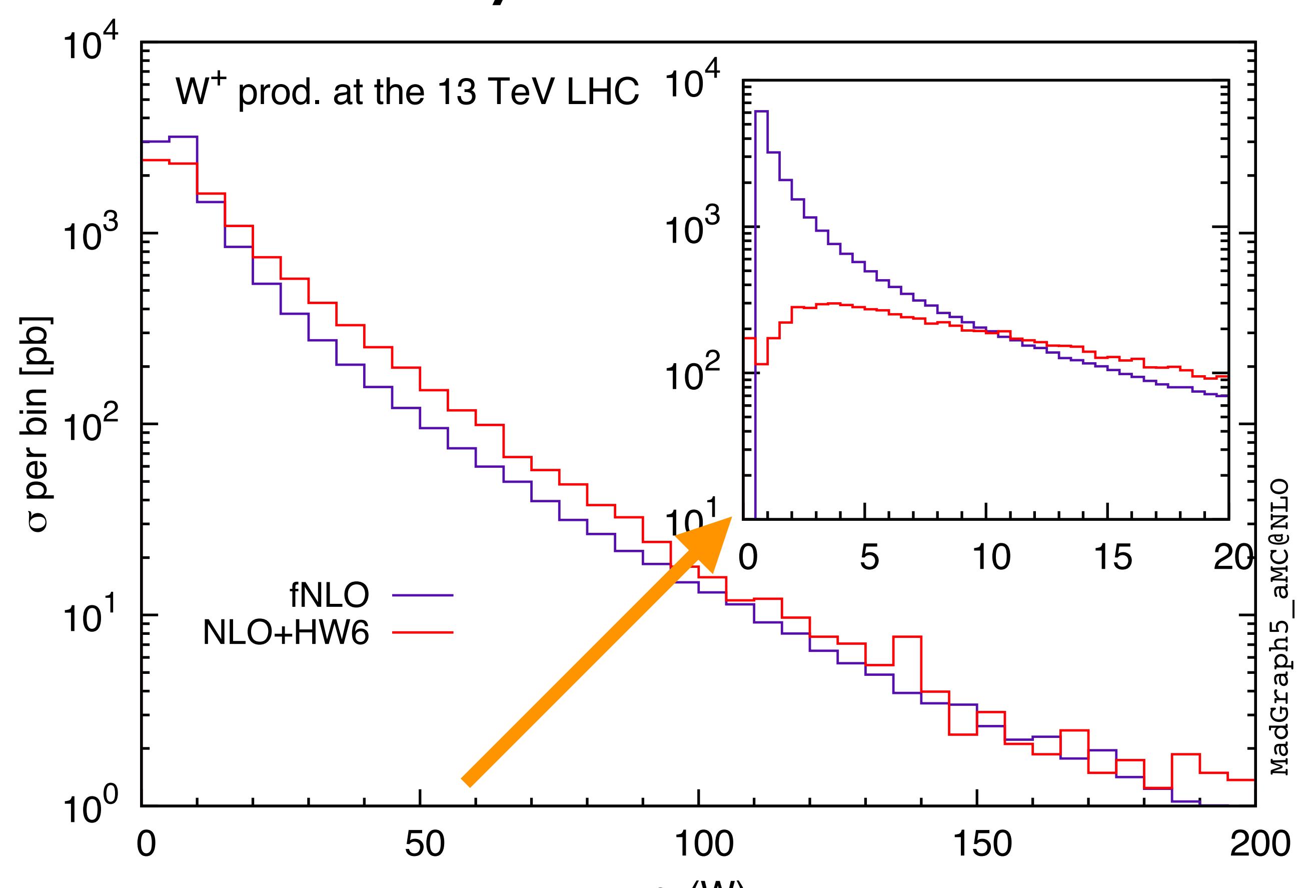
Filling histograms on-the-fly

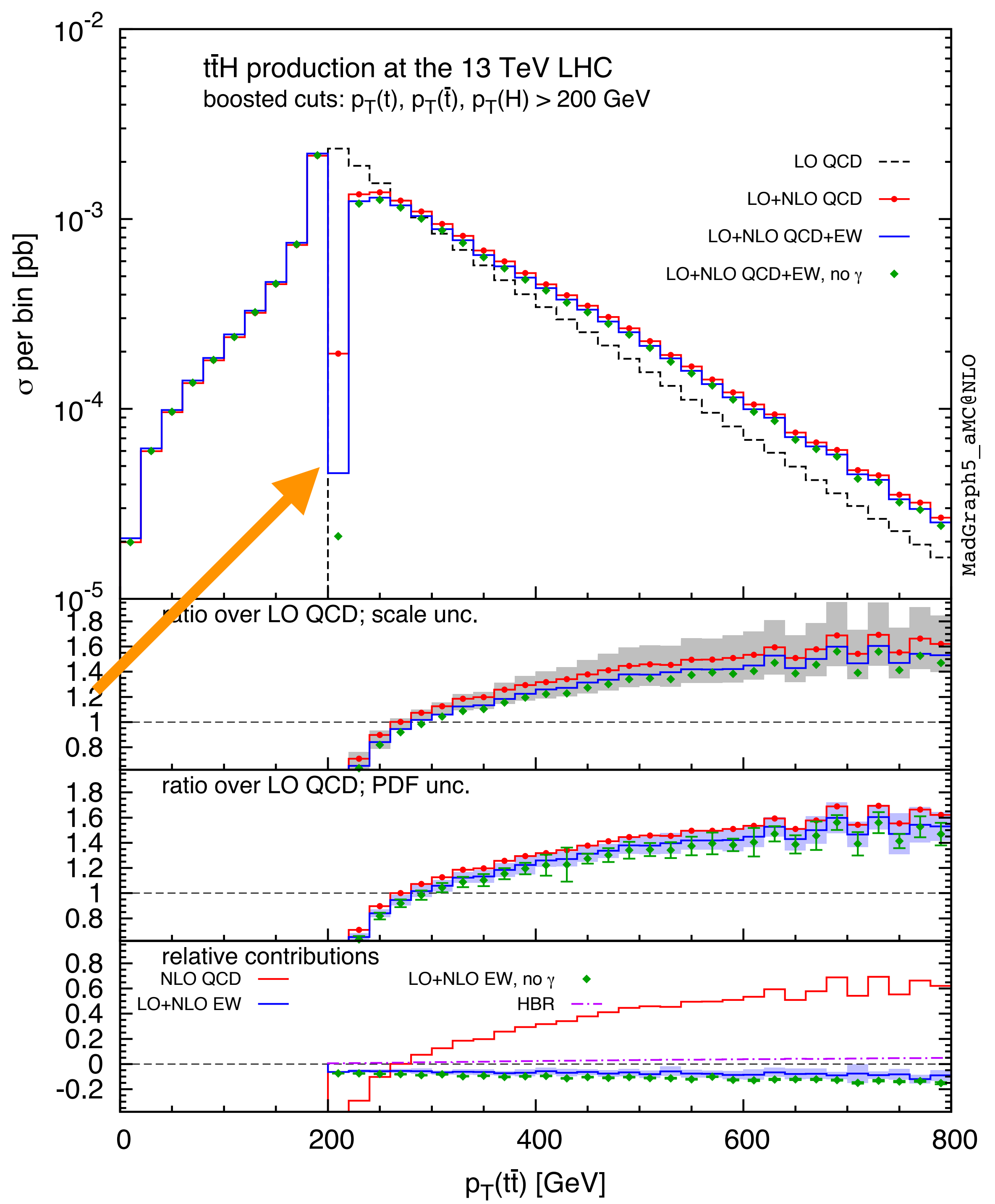
$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} + \int d^4\Phi_n \left(\mathcal{V} + \int d^d\Phi_1 \mathcal{C}\right)_{\varepsilon\to 0} + \int d^4\Phi_{n+1} \left(\mathcal{R} - \mathcal{C}\right)$$

- In practice, two set of momenta are generated during the MC integration
  - One (or more) n-body set(s), for Born, virtuals and counterterms
  - One n+1-body set, for the real emission
- The various terms are computed. Cuts are applied on the corresponding momenta and histograms are filled with the weight and kinematics of each term

Instabilities at fixed order

- Besides the mis-binning problem, the kinematics mismatch can lead to odd behaviours of certain observables, in particular when some constraint coming from the n-body kinematics is relaxed in the $n+1$-body one





Subtracting IR divergences: Summary

- Virtual and real matrix element are not finite, but their sum is. Subtraction methods can be used to extract divergences for real-emission matrix elements and cancel explicitly the poles from the virtuals
- Event and counterevents have different kinematics. Unweighting is not possible, we need to fill plots on-the-fly with weighted events
- For plots, only IR-safe observable with finite resolution must be used!

Intermezzo:
Is it all at NLO?

- Suppose we have a code for $pp \to t\bar{t}$ @NLO. Are all the following (IR-safe) variables described at NLO?
  - top p_T — YES
  - $t\bar{t}$ pair p_T — NO
  - $t\bar{t}$ pair invariant mass — YES
  - jet (extra parton) p_T — NO
  - $t\bar{t}$ azimuthal distance — NO

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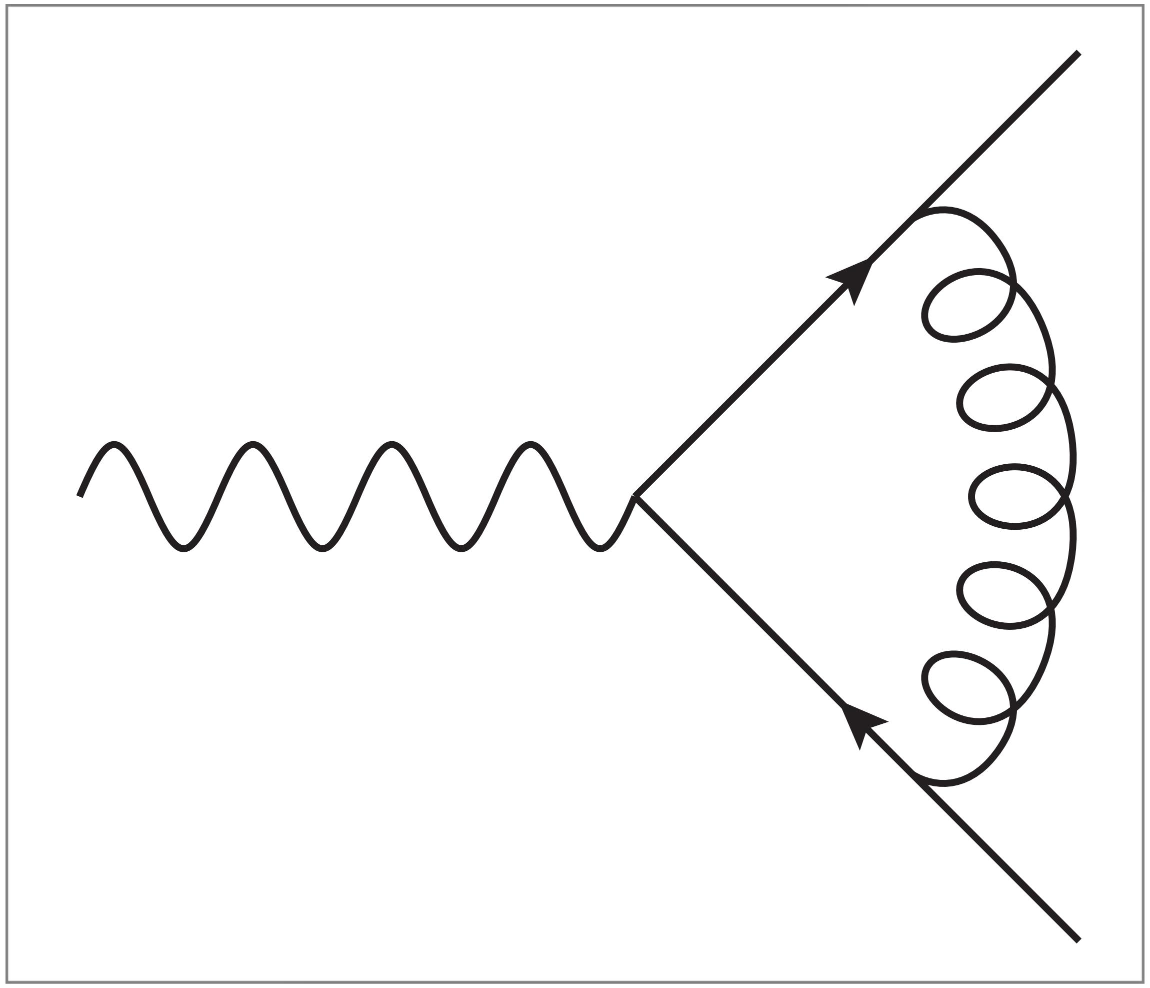
How to compute loops?

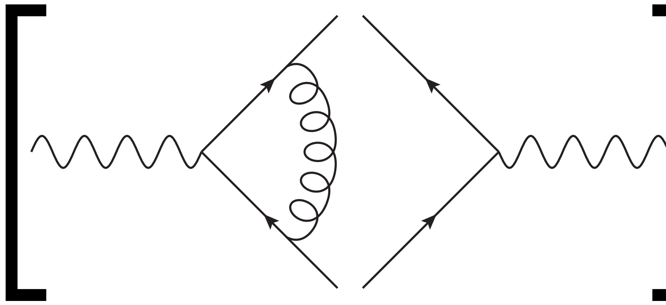


$$\sigma_{NLO} = \int_n \alpha_s^b d\sigma_0 + \int_n \alpha_s^{b+1} d\sigma_V + \int_{n+1} \alpha_s^{b+1} d\sigma_R$$

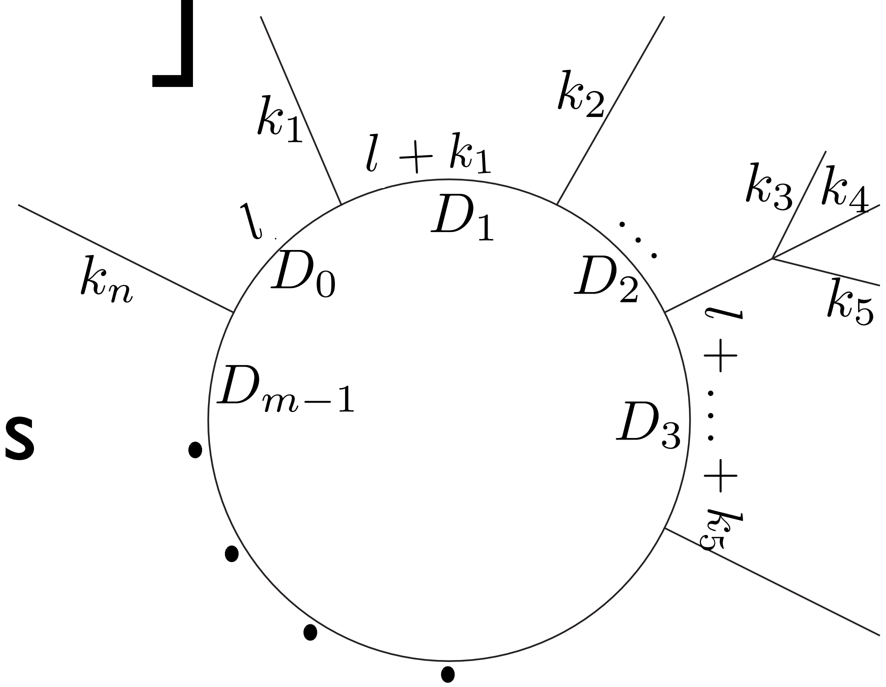
Computing loops numerically

- Consider a m-point one-loop diagram with n external momenta

$$d\sigma_V = 2\Re\left[\quad\right]$$



- The integral to compute is



$$p_1 = k_1$$
$$p_2 = k_2$$
$$p_3 = k_3 + k_4 + k_5$$

$$\int d^d l \frac{N(l)}{D_0 D_1 \ldots D_{m-1}} \qquad D_i = (l + p_i)^2 - m_i^2$$

A hint…

- Any one-loop integral can be cast in the form

$$\int d^d l \frac{N(l)}{D_0 D_1 \dots D_{m-1}} = \sum \mathrm{coeff}_i \int d^d l \frac{1}{D_{i_0} D_{i_1} \dots}$$

- It is a linear combination of scalar integrals
- If d=4+ε, only scalar integrals with up to 4 denominators are needed → the basis is finite!
- The coefficients depend only on external momenta and parameters

Scalar integrals

- Scalar integrals are known and available as libraries

FF (van Oldenborgh, CPC 66,1991)
QCDLoop (Ellis, Zanderighi, arXiv:0712.1851)
OneLOop (Van Hameren, arXiv:1007.4716)

$$\mathcal{M}^{1\text{loop}} = \sum_{i_0,i_1,i_2,i_3} d_{i_0 i_1 i_2 i_3} \mathcal{D}_{i_0 i_1 i_2 i_3} + \sum_{i_0,i_1,i_2} c_{i_0 i_1 i_2} \mathcal{C}_{i_0 i_1 i_2} + \sum_{i_0,i_1} b_{i_0 i_1} \mathcal{B}_{i_0 i_1} + \sum_{i_0} a_{i_0} \mathcal{A}_{i_0}$$

Box $\mathcal{D}_{i_0 i_1 i_2 i_3} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2} D_{i_3}}$

Triangle $\mathcal{C}_{i_0 i_1 i_2} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2}}$

Bubble $\mathcal{B}_{i_0 i_1} = \int d^d l \frac{1}{D_{i_0} D_{i_1}}$

Tadpole $\mathcal{A}_{i_0} = \int d^d l \frac{1}{D_{i_0}}$

How to compute the coefficients?

- Several techniques exist
  - Computation of loop MEs
    - Tensor reduction
    - Generalized unitarity
    - Integrand reduction

Passarino, Veltman, 1979
Denner, Dittmaier, hep-ph/509141
Binoth, Guillet, Heinrich, Pilon, Reiter, arXiv:0810.0992

Bern, Dixon, Dunbar, Kosower, hep-ph/9403226 + …
Ellis, Giele, Kunszt, arXiv:0708.2398
+ Melnikov, arXiv:0806.3467

Ossola, Papadopoulos, Pittau, hep-ph/0609007
Del Aguila, Pittau, hep-ph/0404120
Mastrolia, Ossola, Reiter, Tramontano, arXiv:1006.0710

Integrand reduction

- Can we take away the integral?

$$\int d^d l \frac{N(l)}{D_0 D_1 \ldots D_{m-1}} = \sum \text{coeff}_i \int d^d l \frac{1}{D_{i_0} D_{i_1} \ldots}$$

- Of course not, we must take into account for terms which integrate to 0, the so-called spurious terms:

$$\frac{N(l)}{D_0 D_1 \ldots D_{m-1}} \neq \sum \text{coeff}_i \frac{1}{D_{i_0} D_{i_1} \ldots}$$

$$\frac{N(l)}{D_0 D_1 \ldots D_{m-1}} = \sum \left(\text{coeff}_i + \text{spurious}_i(l)\right) \frac{1}{D_{i_0} D_{i_1} \ldots}$$

Spurious terms

- The functional form of the spurious terms is known and depends on the rank (powers of l in the numerator) and on the number of denominators Del Aguila, Pittau, hep-ph/0404120
  - E.g. a rank-1 box

$$\tilde{d}_{i_0 i_1 i_2 i_3}(l) = \tilde{d}_{i_0 i_1 i_2 i_3}\, \epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma$$

  - The integral is 0

$$\int d^d l \frac{\tilde{d}_{i_0 i_1 i_2 i_3}(l)}{D_0 D_1 D_2 D_3} = \tilde{d}_{i_0 i_1 i_2 i_3} \int d^d l \frac{\epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma}{D_0 D_1 D_2 D_3} = 0$$

OPP decomposition

Ossola, Papadopoulos, Pittau, hep-ph/0609007

$$\frac{N(l)}{D_0 D_1 \dots D_{m-1}} = \sum \left(\text{coeff}_i + \text{spurious}_i(l)\right) \frac{1}{D_{i_0} D_{i_1} \dots}$$

- If we multiply both sides times $D_0D_1\dots D_{m\text{-}1}$ we get

$$\begin{aligned} N(l) = & \sum_{i_0,i_1,i_2,i_3} \left(d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}\right) \prod_{i \neq i_0,i_1,i_2,i_3} D_i \\ & + \sum_{i_0,i_1,i_2} \left(c_{i_0 i_1 i_2} + \tilde{c}_{i_0 i_1 i_2}\right) \prod_{i \neq i_0,i_1,i_2} D_i \\ & + \sum_{i_0,i_1} \left(b_{i_0 i_1} + \tilde{b}_{i_0 i_1}\right) \prod_{i \neq i_0,i_1} D_i \\ & + \sum_{i_0} \left(a_{i_0} + \tilde{a}_{i_0}\right) \prod_{i \neq i_0} D_i \\ & + \tilde{P}(l) \prod_i D_i + \mathcal{O}(\varepsilon) \end{aligned}$$

Getting the coefficients

- *N(l)* is known from the diagrams and the functional form of spurious terms is known too
  - We can sample *N(l)* at various values of the loop momentum, and get a system of linear equations
  - The sampling can be done numerically
  - By choosing smart values of *l* (in the complex plane), the system can be greatly simplified
  - E.g. we can choose l such that

$$D_1(l^\pm) = D_2(l^\pm) = D_3(l^\pm) = D_4(l^\pm) = 0$$

$$\Downarrow$$

$$N(l^\pm) = (d_{1234} + \tilde{d}_{1234}(l^\pm)) \prod_{i \neq 1,2,3,4} D_i(l^\pm)$$

Getting the coefficients

- Two values of l and the knowledge of the spurious terms functional form are enough to extract the box coefficient
$$d_{1234} = \frac{1}{2}\left(\frac{N(l^+)}{\prod_{i\neq 1,2,3,4} D_i(l^+)} + \frac{N(l^-)}{\prod_{i\neq 1,2,3,4} D_i(l^-)}\right)$$
- Similarly, all the box coefficients can be determined
- Then one can move on to the triangles (choosing l such that 3 denominators vanish)
- Then to the bubbles, and finally to the tadpoles

Getting the coefficient: recap

- For each PS point, we have to solve a system of equations numerically
- The system reduces when special values of the loop momentum are chosen
- $N(l)$ can be the numerator of the full matrix element, of a single diagram or anything in between
- For a given PS point, the numerator has to be sampled several times (~50 for a 4-point diagrams)

The evil is in the *d*etails: Complications in *d* dimensions

- So far, we did not care much about the number of dimensions we were using
- In general, external momenta and polarisations are in 4 dimensions; only the loop momentum is in d
- To be more rigorous, we compute the integral

$$\int d^d l \frac{N(l,\tilde{l})}{\bar{D}_0 \bar{D}_1 \dots \bar{D}_{m-1}}$$

$$\bar{l} = l + \tilde{l}$$

$\bar{l}$: d-dim; l: 4-dim; $\tilde{l}$: ε-dim

$$\bar{D}_i = (\bar{l} + p_i)^2 - m_i = (l + p_i)^2 - m_i^2 + \tilde{l}^2 = D_i + \tilde{l}^2$$

$$l \cdot \tilde{l} = 0 \qquad \bar{l} \cdot p_i = l \cdot p_i \qquad \bar{l} \cdot \bar{l} = l \cdot l + \tilde{l} \cdot \tilde{l}$$

Implications

- The reduction should be consistently done in d dimensions

$$\mathcal{M}^{1\text{loop}} = \sum_{i_0,i_1,i_2,i_3} d_{i_0 i_1 i_2 i_3} \bar{\mathcal{D}}_{i_0 i_1 i_2 i_3} + \sum_{i_0,i_1,i_2} c_{i_0 i_1 i_2} \bar{\mathcal{C}}_{i_0 i_1 i_2} + \sum_{i_0,i_1} b_{i_0 i_1} \bar{\mathcal{B}}_{i_0 i_1} + \sum_{i_0} a_{i_0} \bar{\mathcal{A}}_{i_0} + \mathcal{O}(\varepsilon)$$

$\Rightarrow$

$$\mathcal{M}^{1\text{loop}} = \sum_{i_0,i_1,i_2,i_3} d_{i_0 i_1 i_2 i_3} \mathcal{D}_{i_0 i_1 i_2 i_3} + \sum_{i_0,i_1,i_2} c_{i_0 i_1 i_2} \mathcal{C}_{i_0 i_1 i_2} + \sum_{i_0,i_1} b_{i_0 i_1} \mathcal{B}_{i_0 i_1} + \sum_{i_0} a_{i_0} \mathcal{A}_{i_0} + R + \mathcal{O}(\varepsilon)$$

That is why the *rational terms* are needed

The rational terms

OPP, arXiv:0802.1876

- In the OPP method, two types of rational terms are there: $R=R_1+R_2$
- Both originate from the UV part of the model, but only R_1 can be computed in the OPP decomposition
- R_1 originates from the *denominators* (propagators) in the loops

$$\frac{1}{\bar{D}_i} = \frac{1}{D_i}\left(1 - \frac{\tilde{l}^2}{D_i}\right)$$

- The denominator structure is known, so these terms can be directly included in the OPP reduction
- R_1 contributions are proportional to

$$\int d^d l \frac{\tilde{l}^2}{\bar{D}_i\bar{D}_j} = -\frac{i\pi^2}{2}\left[m_i^2 + m_j^2 - \frac{(p_i-p_j)^2}{2}\right] + \mathcal{O}(\varepsilon)$$

$$\int d^d l \frac{\tilde{l}^2}{\bar{D}_i\bar{D}_j\bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\varepsilon)$$

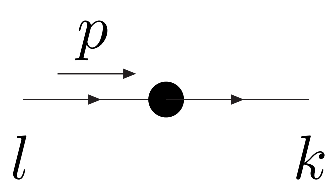
$$\int d^d l \frac{\tilde{l}^2}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\varepsilon)$$

R_2

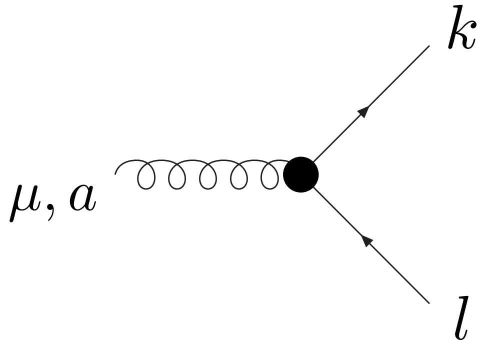
- R_2 terms originate from the numerator.
  Integrals with rank ≥2 can have terms in the numerator ~ to $\tilde{l}^2$
- This dependence can be quite hidden and become explicit only after having done the Clifford algebra
- Since we want a fully numerical approach, these terms cannot be obtained directly with the OPP reduction
- Within a given (renormalizable) model, only a finite set of terms that can give rise to these terms exists. They can be identified and computed as the "R_2 counterterms"

R_2 Feynman rules

- In a renormalizable theory, only up to 4-point integrals contribute to the R_2 terms
- They can be included in the computation using special Feynman rules (as it is done for the UV renormalisation). For example:



$$= \frac{ig^2}{16\pi^2} \frac{N_{col}^2 - 1}{2N_{col}} \delta_{kl} (-\not{p} + 2m_q) \, \lambda_{HV}$$



$$= \frac{ig^3}{16\pi^2} \frac{N_{col}^2 - 1}{2N_{col}} \, t^a_{kl} \gamma_\mu \, (1 + \lambda_{HV})$$

Draggiotis, Garzelli, Papadopoulos, Pittau, arXiv:0903.0356

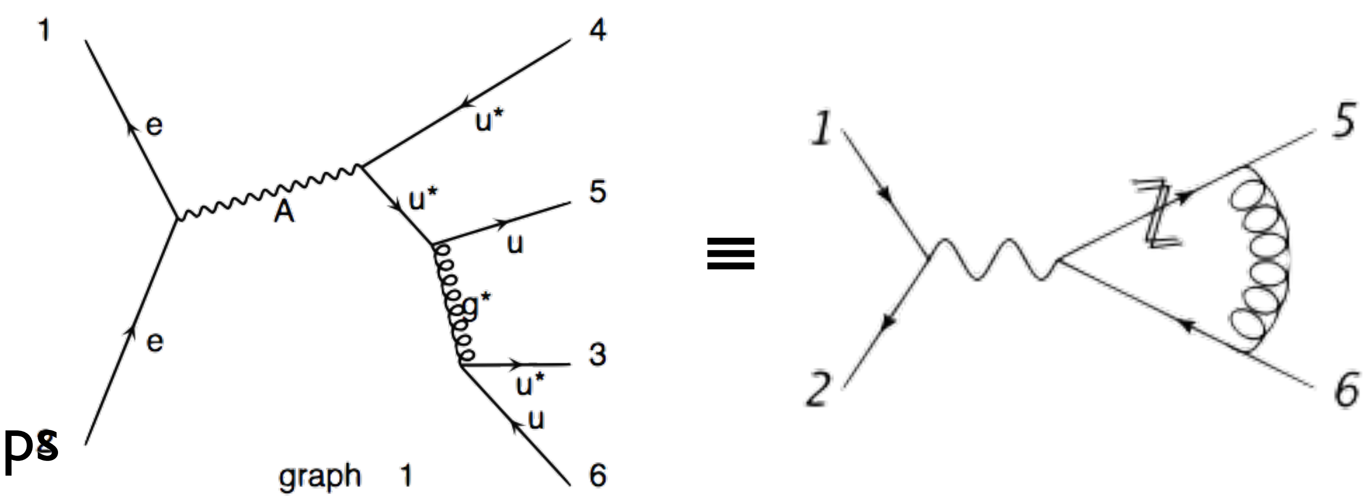
- Similarly to the UV counterterms, the R_2 terms are model dependent and need to be explicitly computed for BSM models This is now automated for renormalizable theories

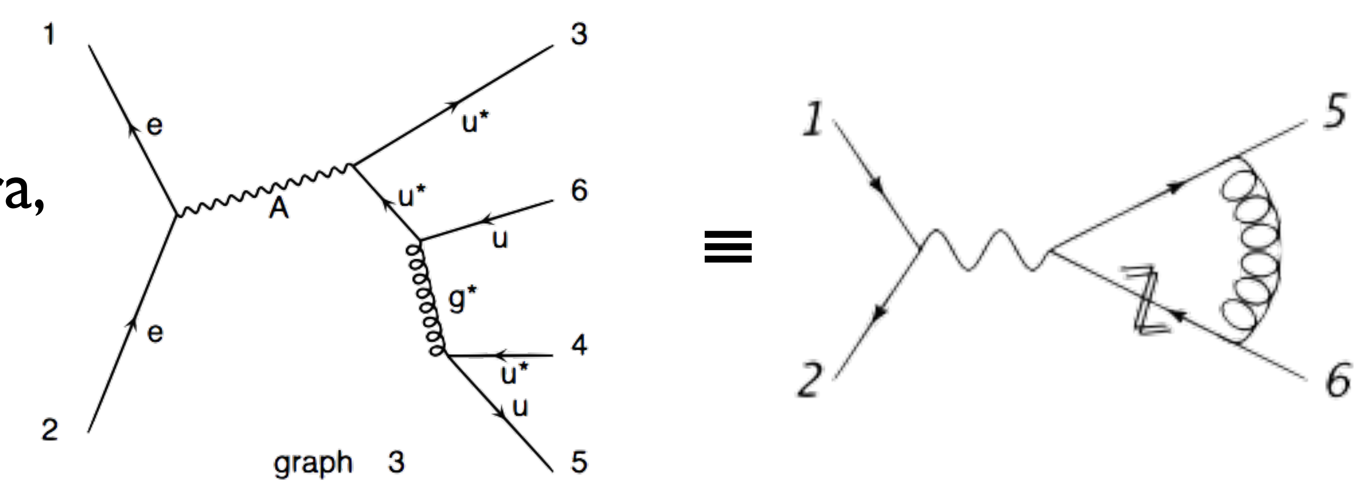
Degrande, arXiv:1406.3030

MadLoop

Hirschi et al, arXiv:1103.0621

- How to automate loop computation?
- Exploit MadGraph's capabilities to generate tree-level diagrams
- Loop diagrams with n external legs can be cut, leading to tree diagrams with n+2 legs





- All diagrams with 2 extra particles are generated, those which are needed are filtered out
- Each diagram is assigned a tag, which helps removing mirror/cyclic configurations
- Additional filters to remove tadpole/ bubbles on external legs
- Contract with Born, do the color algebra, re-glue the cut particle, etc…
- Add UV and R2 counterterms as extra vertices

Lot of results!

	Process	Syntax	LO 13 TeV	LO scale unc.	LO PDF unc.	NLO 13 TeV	NLO scale unc.	NLO PDF unc.
Vector boson +jets								
a.1	$pp \to W^{\pm}$	`p p > wpm`	$1.375 \pm 0.002 \cdot 10^{5}$	+15.4% −16.6%	+2.0% −1.6%	$1.773 \pm 0.007 \cdot 10^{5}$	+5.2% −9.4%	+1.9% −1.6%
a.2	$pp \to W^{\pm} j$	`p p > wpm j`	$2.045 \pm 0.001 \cdot 10^{4}$	+19.7% −17.2%	+1.4% −1.1%	$2.843 \pm 0.010 \cdot 10^{4}$	+5.9% −8.0%	+1.3% −1.1%
a.3	$pp \to W^{\pm} jj$	`p p > wpm j j`	$6.805 \pm 0.015 \cdot 10^{3}$	+24.5% −18.6%	+0.8% −0.7%	$7.786 \pm 0.030 \cdot 10^{3}$	+2.4% −6.0%	+0.9% −0.8%
a.4	$pp \to W^{\pm} jjj$	`p p > wpm j j j`	$1.821 \pm 0.002 \cdot 10^{3}$	+41.0% −27.1%	+0.5% −0.5%	$2.005 \pm 0.008 \cdot 10^{3}$	+0.9% −6.7%	+0.6% −0.5%
a.5	$pp \to Z$	`p p > z`	$4.248 \pm 0.005 \cdot 10^{4}$	+14.6% −15.8%	+2.0% −1.6%	$5.410 \pm 0.022 \cdot 10^{4}$	+4.6% −8.6%	[illegible]
a.6	$pp \to Zj$	`p p > z j`	$7.209 \pm 0.005 \cdot 10^{3}$	+19.3% −17.0%	+1.2% −1.0%	$9.742 \pm$ [illegible] $\cdot 10^{3}$	[illegible] −7.8%	[illegible]
a.7	$pp \to Zjj$	`p p > z j j`	$2.348 \pm 0.006 \cdot 10^{3}$	+24.3% −18.5%	+0.6% −0.6%	[illegible] $\cdot 10^{3}$	+2.5% [illegible]	[illegible] −0.7%
a.8	$pp \to Zjjj$	`p p > z j j j`	$6.314 \pm 0.008 \cdot 10^{2}$	+40.8% −27.0%	+0.5% −0.5%	$6.996 \pm 0.028 \cdot 10^{2}$	+1.1% −6.8%	+0.5% −0.5%
a.9	$pp \to \gamma j$	`p p > a j`	$1.964 \pm 0.001 \cdot 10^{4}$	+31.2% −26.0%	+1.7% −1.8%	$5.218 \pm 0.025 \cdot 10^{4}$	+24.5% −21.4%	+1.4% −1.6%
a.10	$pp \to \gamma jj$	`p p > a j j`	$7.815 \pm 0.008 \cdot 10^{3}$	+32.8% −24.2%	+0.9% −1.2%	$1.004 \pm 0.004 \cdot 10^{4}$	+5.9% −10.9%	+0.8% −1.2%

	Process	Syntax	LO 13 TeV	LO scale unc.	LO PDF unc.	NLO 13 TeV	NLO scale unc.	NLO PDF unc.
Vector-boson pair +jets								
b.1	$pp \to W^{+}W^{-}$ (4f)	`p p > w+ w-`	$7.355 \pm 0.005 \cdot 10^{1}$	+5.0% −6.1%	+2.0% −1.5%	$1.028 \pm 0.003 \cdot 10^{2}$	+4.0% −4.5%	+1.9% −1.4%
b.2	$pp \to ZZ$	`p p > z z`	$1.097 \pm 0.002 \cdot 10^{1}$	+4.5% −5.6%	+1.9% −1.5%	$1.415 \pm 0.005 \cdot 10^{1}$	+3.1% −3.7%	+1.8% −1.4%
b.3	$pp \to ZW^{\pm}$	`p p > z wpm`	$2.777 \pm 0.003 \cdot 10^{1}$	+3.6% −4.7%	+2.0% −1.5%	$4.487 \pm 0.013 \cdot 10^{1}$	+4.4% −4.4%	+1.7% −1.3%
b.4	$pp \to \gamma\gamma$	`p p > a a`	$2.510 \pm 0.002 \cdot 10^{1}$	+22.1% −22.4%	+2.4% −2.1%	$6.593 \pm 0.021 \cdot 10^{1}$	+17.6% −18.8%	+2.0% −1.9%
b.5	$pp \to \gamma Z$	`p p > a z`	$2.523 \pm 0.004 \cdot 10^{1}$	+9.9% −11.2%	+2.0% −1.6%	$3.695 \pm 0.013 \cdot 10^{1}$	+5.4% −7.1%	+1.8% −1.4%
b.6	$pp \to \gamma W^{\pm}$	`p p > a wpm`	$2.954 \pm 0.005 \cdot 10^{1}$	+9.5% −11.0%	+2.0% −1.7%	$7.124 \pm 0.026 \cdot 10^{1}$	+9.7% −9.9%	+1.5% −1.3%
b.7	$pp \to W^{+}W^{-}j$ (4f)	`p p > w+ w- j`	$2.865 \pm 0.003 \cdot 10^{1}$	+11.6% −10.0%	+1.0% −0.8%	$3.730 \pm 0.013 \cdot 10^{1}$	+4.9% −4.9%	+1.1% −0.8%
b.8	$pp \to ZZj$	`p p > z z j`	$3.662 \pm 0.003 \cdot 10^{0}$	+10.9% −9.3%	+1.0% −0.8%	$4.830 \pm 0.016 \cdot 10^{0}$	+5.0% −4.8%	+1.1% −0.9%
b.9	$pp \to ZW^{\pm}j$	`p p > z wpm j`	$1.605 \pm 0.005 \cdot 10^{1}$	+11.6% −10.0%	+0.9% −0.7%	$2.086 \pm 0.007 \cdot 10^{1}$	+4.9% −4.8%	+0.9% −0.7%
b.10	$pp \to \gamma\gamma j$	`p p > a a j`	$1.022 \pm 0.001 \cdot 10^{1}$	+20.3% −17.7%	+1.2% −1.5%	$2.292 \pm 0.010 \cdot 10^{1}$	+17.2% −15.1%	+1.0% −1.4%
b.11*	$pp \to \gamma Zj$	`p p > a z j`	$8.310 \pm 0.017 \cdot 10^{0}$	+14.5% −12.8%	+1.0% −1.0%	$1.220 \pm 0.005 \cdot 10^{1}$	+7.3% −7.4%	+0.9% −0.9%
b.12*	$pp \to \gamma W^{\pm}j$	`p p > a wpm j`	$2.546 \pm 0.010 \cdot 10^{1}$	+13.7% −12.1%	+0.9% −1.0%	$3.713 \pm 0.015 \cdot 10^{1}$	+7.2% −7.1%	+0.9% −1.0%
b.13	$pp \to W^{+}W^{+}jj$	`p p > w+ w+ j j`	$1.484 \pm$ [illegible] $\cdot 10^{-1}$	+25.4% [illegible]	[illegible]	$2.251 \pm$ [illegible] $\cdot 10^{-1}$	+10.5% −10.6%	+2.2% −1.6%
b.14	$pp \to W^{-}W^{-}jj$	`p p > w- w- j j`	6.752 [illegible]	[illegible]	+2.4% −1.7%	$1.003 \pm$ [illegible]	[illegible] 10.1% [illegible] 10.4%	[illegible]
b.15	$pp \to W^{+}W^{-}jj$ (4f)	`p p > w+ w- j j`	1.14[illegible] $\pm 0.002 \cdot 10^{1}$	+27[illegible] −19[illegible]	[illegible]	$1.396 \pm$ [illegible] $\cdot 10^{1}$	[illegible]	[illegible]
b.16	$pp \to ZZjj$	`p p > z z j j`	$1.344 \pm 0.002 \cdot 10^{0}$	+26.6% −19.6%	+0.7% −0.6%	$1.706 \pm 0.011 \cdot 10^{0}$	+5.8% −7.2%	+0.8% −0.6%
b.17	$pp \to ZW^{\pm}jj$	`p p > z wpm j j`	$8.038 \pm 0.009 \cdot 10^{0}$	+26.7% −19.7%	+0.7% −0.5%	$9.139 \pm 0.031 \cdot 10^{0}$	+3.1% −5.1%	+0.7% −0.5%
b.18	$pp \to \gamma\gamma jj$	`p p > a a j j`	$5.377 \pm 0.029 \cdot 10^{0}$	+26.2% −19.8%	+0.6% −1.0%	$7.501 \pm 0.032 \cdot 10^{0}$	+8.8% −10.1%	+0.6% −1.0%
b.19*	$pp \to \gamma Zjj$	`p p > a z j j`	$3.260 \pm 0.009 \cdot 10^{0}$	+24.[illegible] −18.[illegible]	[illegible] −0.6%	$4.242 \pm 0.016 \cdot 10^{0}$	+6.5[illegible] −7.3[illegible]	[illegible] −0.6[illegible]
b.20*	$pp \to \gamma W^{\pm}jj$	`p p > a wpm j j`	$1.233 \pm 0.002 \cdot 10^{1}$	+24.[illegible] −18.[illegible]	+0.6% [illegible]	1.44[illegible] $\pm 0.005 \cdot 10^{1}$	+3.6[illegible] −5.4[illegible]	+0.6[illegible]

	Process	Syntax	LO 13 TeV	LO scale unc.	LO PDF unc.	NLO 13 TeV	NLO scale unc.	NLO PDF unc.
Three vector bosons +jet								
c.1	$pp \to W^{+}W^{-}W^{\pm}$ (4f)	`p p > w+ w- wpm`	$1.307 \pm 0.003 \cdot 10$[illegible]	+0.0% −0.3%	+2.0% −1.5%	$2.109 \pm 0.006 \cdot 10^{-1}$	+5.1% −4.1%	+1.6% −1.2%
c.2	$pp \to ZW^{+}W^{-}$ (4f)	`p p > z w+ w-`	$9.658 \pm 0.065 \cdot 10^{-2}$	+0.8% −1.1%	+2.1% −1.6%	$1.679 \pm 0.005 \cdot 10^{-1}$	+6.3% −5.1%	+1.6% −1.2%
c.3	$pp \to ZZW^{\pm}$	`p p > z z wpm`	$2.996 \pm 0.016 \cdot 10^{-2}$	+1.0% −1.4%	+2.0% −1.6%	$5.550 \pm 0.020 \cdot 10^{-2}$	+6.8% −5.5%	+1.5% −1.1%
c.4	$pp \to ZZZ$	`p p > z z z`	$1.085 \pm 0.002 \cdot 10^{-2}$	+0.0% −0.5%	+1.9% −1.5%	$1.417 \pm 0.005 \cdot 10^{-2}$	+2.7% −2.1%	+1.9% −1.5%
c.5	$pp \to \gamma W^{+}W^{-}$ (4f)	`p p > a w+ w-`	$1.427 \pm 0.011 \cdot 10^{-1}$	+1.9% −2.6%	+2.0% −1.5%	$2.581 \pm 0.008 \cdot 10^{-1}$	+5.4% −4.3%	+1.4% −1.1%
c.6	$pp \to \gamma\gamma W^{\pm}$	`p p > a a wpm`	$2.681 \pm 0.007 \cdot 10^{-2}$	+4.4% −5.6%	+1.9% −1.6%	$8.251 \pm 0.032 \cdot 10^{-2}$	+7.6% −7.0%	+1.0% −1.0%
c.7	$pp \to \gamma ZW^{\pm}$	`p p > a z wpm`	$4.994 \pm 0.011 \cdot 10^{-2}$	+0.8% −1.4%	+1.9% −1.6%	$1.117 \pm 0.004 \cdot 10^{-1}$	+7.2% −5.9%	+1.2% −0.9%
c.8	$pp \to \gamma ZZ$	`p p > a z z`	$2.320 \pm 0.005 \cdot 10^{-2}$	+2.0% −2.9%	+1.9% −1.5%	$3.118 \pm 0.012 \cdot 10^{-2}$	+2.8% −2.7%	+1.8% −1.4%
c.9	$pp \to \gamma\gamma Z$	`p p > a a z`	$3.078 \pm 0.007 \cdot 10^{-2}$	+5.6% −6.8%	+1.9% −1.6%	$4.634 \pm 0.020 \cdot 10^{-2}$	+4.5% −5.0%	+1.7% −1.3%
c.10	$pp \to \gamma\gamma\gamma$	`p p > a a a`	$1.269 \pm 0.003 \cdot 10^{-2}$	+9.8% −11.0%	+2.0% −1.8%	$3.441 \pm 0.012 \cdot 10^{-2}$	+11.8% −11.6%	+1.4% −1.5%
c.11	$pp \to W^{+}W^{-}W^{\pm}j$ (4f)	`p p > w+ w- wpm j`	$9.167 \pm 0.010 \cdot 10^{-2}$	+15.0% −12.2%	+1.0% −0.7%	$1.197 \pm 0.004 \cdot 10^{-1}$	+5.2% −5.6%	+1.0% −0.8%
c.12*	$pp \to ZW^{+}W^{-}j$ (4f)	`p p > z w+ w- j`	$8.340 \pm 0.010 \cdot 10^{-2}$	+15.6% −12.6%	+1.0% −0.7%	$1.066 \pm 0.003 \cdot 10^{-1}$	+4.5% −5.3%	+1.0% −0.7%
c.13*	$pp \to ZZW^{\pm}j$	`p p > z z wpm j`	$2.810 \pm 0.004 \cdot 10^{-2}$	+16.1% −13.0%	+1.0% −0.7%	$3.660 \pm 0.013 \cdot 10^{-2}$	+4.8% −5.6%	+1.0% −0.7%
c.14*	$pp \to ZZZj$	`p p > z z z j`	$4.823 \pm 0.011 \cdot 10^{-3}$	+14.3% −11.8%	+1.4% −1.0%	$6.341 \pm 0.025 \cdot 10^{-3}$	+4.9% −5.4%	+1.4% −1.0%
c.15*	$pp \to \gamma W^{+}W^{-}j$ (4f)	`p p > a w+ w- j`	$1.182 \pm 0.004 \cdot 10^{-1}$	+13.4% −11.2%	+0.8% −0.7%	$1.233 \pm 0.004 \cdot 10^{3}$	+18.9% −19.9%	+1.0% −1.5%
c.16	$pp \to \gamma\gamma W^{\pm}j$	`p p > a a wpm j`	$4.107 \pm 0.015 \cdot 10^{-2}$	+11.8% −10.2%	+0.6% −0.8%	$5.807 \pm 0.023 \cdot 10^{-2}$	+5.8% −5.5%	+0.7% −0.7%
c.17*	$pp \to \gamma ZW^{\pm}j$	`p p > a z wpm j`	$5.833 \pm 0.023 \cdot 10^{-2}$	+14.4% −12.0%	+0.7% −0.6%	$7.764 \pm 0.025 \cdot 10^{-2}$	+5.1% −5.5%	+0.8% −0.6%
c.18*	[illegible]	`p p > a z z j`	$9.995 \pm 0.013 \cdot 10^{-3}$	+12.5% −10.6%	+1.2% −0.9%	$1.371 \pm 0.005 \cdot 10^{-2}$	+5.6% −5.5%	+1.2% −0.9%
c.19*	$pp \to \gamma\gamma Zj$	`p p > a a z j`	$1.372 \pm 0.003 \cdot 10^{-2}$	+10.9% −9.4%	+1.0% −0.9%	$2.051 \pm 0.011 \cdot 10^{-2}$	+7.0% −6.3%	+1.0% −0.9%

	Process	Syntax	LO 13 TeV	LO scale unc.	LO PDF unc.	NLO 13 TeV	NLO scale unc.	NLO PDF unc.
Four vector bosons								
c.21*	$pp \to W^{+}W^{-}W^{+}W^{-}$ (4f)	`p p > w+ w- w+ w-`	$5.721 \pm 0.014 \cdot 10^{-4}$	+3.7% −3.5%	+2.3% −1.7%	$9.959 \pm 0.035 \cdot 10^{-4}$	+7.4% −6.0%	+1.7% −1.2%
c.22*	$pp \to W^{+}W^{-}W^{\pm}Z$ (4f)	`p p > w+ w- wpm z`	$6.391 \pm 0.076 \cdot 10^{-4}$	+4.4% −4.1%	+2.4% −1.8%	$1.188 \pm 0.004 \cdot 10^{-3}$	+8.4% −6.8%	+1.7% −1.2%
c.23*	$pp \to W^{+}W^{-}W^{\pm}\gamma$ (4f)	`p p > w+ w- wpm a`	$8.115 \pm 0.064 \cdot 10^{-4}$	+2.5% −2.5%	+2.2% −1.7%	$1.546 \pm 0.005 \cdot 10^{-3}$	+7.9% −6.3%	+1.5% −1.1%
c.24*	$pp \to W^{+}W^{-}ZZ$ (4f)	`p p > w+ w- z z`	$4.320 \pm 0.013 \cdot 10^{-4}$	+4.4% −4.1%	+2.4% −1.7%	$7.107 \pm 0.020 \cdot 10^{-4}$	+7.0% −5.7%	+1.8% −1.3%
c.25*	$pp \to W^{+}W^{-}Z\gamma$ (4f)	`p p > w+ w- z a`	$8.403 \pm 0.016 \cdot 10^{-4}$	+3.0% −2.9%	+2.3% −1.7%	$1.483 \pm 0.004 \cdot 10^{-3}$	+7.2% −5.8%	+1.6% −1.2%
c.26*	[illegible]	`p p > w+ w- a a`	$5.198 \pm 0.012 \cdot 10^{-4}$	+0.6% −0.9%	+2.1% −1.6%	$9.381 \pm 0.032 \cdot 10^{-4}$	+6.7% −5.3%	+1.4% −1.1%
c.27*	[illegible]	`p p > wpm z z z`	$5.862 \pm 0.010 \cdot 10^{-5}$	+5.1% −4.7%	+2.4% −1.8%	$1.240 \pm 0.004 \cdot 10^{-4}$	+9.9% −8.0%	+1.7% −1.2%
c.28*	$pp \to W^{\pm}ZZ\gamma$	`p p > wpm z z a`	$1.148 \pm 0.003 \cdot 10^{-4}$	+3.6% −3.5%	+2.2% −1.7%	$2.945 \pm 0.008 \cdot 10^{-4}$	+10.8% −8.7%	+1.3% −1.0%
c.29*	$pp \to W^{\pm}Z\gamma\gamma$	`p p > wpm z a a`	$1.054 \pm 0.004 \cdot 10^{-4}$	+1.7% −1.9%	+2.1% −1.7%	$3.033 \pm 0.010 \cdot 10^{-4}$	+10.6% −8.6%	+1.1% −0.8%
c.30*	$pp \to W^{\pm}\gamma\gamma\gamma$	`p p > wpm a a a`	$3.600 \pm 0.013 \cdot 10^{-5}$	+0.4% −1.0%	+2.0% −1.6%	$1.246 \pm 0.005 \cdot 10^{-4}$	+9.8% −8.1%	+0.9% −0.8%
c.31*	$pp \to ZZZZ$	`p p > z z z z`	$1.989 \pm 0.002 \cdot 10^{-5}$	+3.8% −3.6%	+2.2% −1.7%	$2.629 \pm 0.008 \cdot 10^{-5}$	+3.5% −3.0%	+2.2% −1.7%
c.32*	$pp \to ZZZ\gamma$	`p p > z z z a`	$3.945 \pm 0.007 \cdot 10^{-5}$	+1.9% −2.1%	+2.1% −1.6%	$5.224 \pm 0.016 \cdot 10^{-5}$	+3.3% −2.7%	+2.1% −1.6%
c.33*	$pp \to ZZ\gamma\gamma$	`p p > z z a a`	$5.513 \pm 0.017 \cdot 10^{-5}$	+0.0% −0.3%	+2.1% −1.6%	$7.518 \pm 0.032 \cdot 10^{-5}$	+3.4% −2.6%	+2.0% −1.5%
c.34*	$pp \to Z\gamma\gamma\gamma$	`p p > z a a a`	$4.790 \pm 0.012 \cdot 10^{-5}$	+2.3% −3.1%	+2.0% −1.6%	$7.103 \pm 0.026 \cdot 10^{-5}$	+3.4% −3.2%	+1.6% −1.5%
c.35*	$pp \to \gamma\gamma\gamma\gamma$	`p p > a a a a`	$1.594 \pm 0.004 \cdot 10^{-5}$	+4.7% −5.7%	+1.9% −1.7%	$3.389 \pm 0.012 \cdot 10^{-5}$	+7.0% −6.7%	+1.3% −1.3%

	Process	Syntax	LO 13 TeV	LO scale unc.	LO PDF unc.	NLO 13 TeV	NLO scale unc.	NLO PDF unc.
Heavy quarks and jets								
d.1	$pp \to jj$	`p p > j j`	$1.162 \pm 0.001 \cdot 10^{6}$	+24.9% −18.8%	+0.8% −0.9%	$1.580 \pm 0.007 \cdot 10^{6}$	+8.4% −9.0%	+0.7% −0.9%
d.2	$pp \to jjj$	`p p > j j j`	$8.940 \pm 0.021 \cdot 10^{4}$	+43.8% −28.4%	+1.2% −1.4%	$7.791 \pm 0.037 \cdot 10^{4}$	+2.1% −23.2%	+1.1% −1.3%
d.3	$pp \to b\bar{b}$ (4f)	`p p > b b~`	$3.743 \pm 0.004 \cdot 10^{3}$	+25.2% −18.9%	+1.5% −1.8%	$6.438 \pm 0.028 \cdot 10^{3}$	+15.9% −13.3%	+1.5% −1.7%
d.4*	$pp \to b\bar{b}j$ (4f)	`p p > b b~ j`	$1.050 \pm 0.002 \cdot 10^{3}$	+44.1% −28.5%	+1.6% −1.8%	$1.327 \pm 0.007 \cdot 10^{3}$	+6.8% −11.6%	+1.5% −1.8%
d.5*	$pp \to b\bar{b}jj$ (4f)	`p p > b b~ j j`	$1.852 \pm 0.006 \cdot 10^{2}$	+61.8% −35.6%	+2.1% −2.4%	$2.471 \pm 0.012 \cdot 10^{2}$	+8.2% −16.4%	+2.0% −2.3%
d.6	$pp \to b\bar{b}b\bar{b}$ (4f)	`p p > b b~ b b~`	$5.050 \pm 0.007 \cdot 10^{-1}$	+61.7% −35.6%	+2.9% −3.4%	$8.736 \pm 0.034 \cdot 10^{-1}$	+20.9% −22.0%	+2.9% −3.4%
d.7	$pp \to t\bar{t}$	`p p > t t~`	$4.584 \pm$ [illegible] $\cdot 10^{2}$	+29.0% −21.1%	+1.8% −2.0%	$6.741 \pm 0.023 \cdot 10^{2}$	+9.8% −10.9%	+1.8% −2.1%
d.8	$pp \to t\bar{t}j$	[illegible] `j`	[illegible]$.135 \pm$ [illegible] $\cdot 10^{2}$	+45.1% −29.0%	+2.2% −2.5%	$4.106 \pm 0.015 \cdot 10^{2}$	+8.1% −12.2%	+2.1% −2.5%
d.9	$pp \to t\bar{t}jj$	[illegible] `j j`	[illegible]$.361 \pm 0.001 \cdot 10^{2}$	+61.4% −35.6%	+2.6% −3.0%	$1.795 \pm 0.006 \cdot 10^{2}$	+9.3% −16.1%	+2.4% −2.9%
d.10	$pp \to t\bar{t}t\bar{t}$	[illegible]	[illegible] $\cdot 10^{-3}$	+63.8% −36.5%	+5.4% −5.7%	$9.201 \pm 0.028 \cdot 10^{-3}$	+30.8% −25.6%	+5.5% −5.9%
d.11	$pp \to t\bar{t}b\bar{b}$ (4f)	`p p > t t~ b b~`	$6.119 \pm 0.004 \cdot 10^{0}$	+62.1% −35.7%	+2.9% −3.5%	$1.452 \pm 0.005 \cdot 10^{1}$	+37.6% −27.5%	+2.9% −3.5%

	Process	Syntax	LO 13 TeV	LO scale unc.	LO PDF unc.	NLO 13 TeV	NLO scale unc.	NLO PDF unc.
Heavy quarks+vector bosons								
e.1	$pp \to W^{\pm} b\bar{b}$ (4f)	`p p > wpm b b~`	$3.074 \pm 0.002 \cdot 10^{2}$	+42.3% −29.2%	+2.0% −1.6%	$8.162 \pm 0.034 \cdot 10^{2}$	+29.8% −23.6%	+1.5% −1.2%
e.2	$pp \to Z b\bar{b}$ (4f)	`p p > z b b~`	$6.993 \pm 0.003 \cdot 10^{2}$	+33.5% −24.4%	+1.0% −1.4%	$1.235 \pm 0.004 \cdot 10^{3}$	+19.9% −17.4%	+1.0% −1.4%
e.3	$pp \to \gamma b\bar{b}$ (4f)	`p p > a b b~`	$1.731 \pm 0.001 \cdot 10^{3}$	+51.9% −34.8%	+1.6% −2.1%	$4.171 \pm 0.015 \cdot 10^{3}$	+33.7% −27.1%	+1.4% −1.9%
e.4*	$pp \to W^{\pm} b\bar{b} j$ (4f)	`p p > wpm b b~ j`	$1.861 \pm 0.003 \cdot 10^{2}$	+42.5% −27.7%	+0.7% −0.7%	$3.957 \pm 0.013 \cdot 10^{2}$	+27.0% −21.0%	+0.7% −0.6%
e.5*	$pp \to Z b\bar{b} j$ (4f)	`p p > z b b~ j`	$1.604 \pm 0.001 \cdot 10^{2}$	+42.4% −27.6%	+0.9% −1.1%	$2.805 \pm 0.009 \cdot 10^{2}$	+21.0% −17.6%	+0.8% −1.0%
e.6*	$pp \to \gamma b\bar{b} j$ (4f)	`p p > a b b~ j`	$7.812 \pm 0.017 \cdot 10^{2}$	+51.2% −32.0%	+1.0% −1.5%	$1.233 \pm 0.004 \cdot 10^{3}$	+18.9% −19.9%	+1.0% −1.5%
e.7	$pp \to t\bar{t}W^{\pm}$	`p p > t t~ wpm`	$3.777 \pm 0.003 \cdot 10^{-1}$	+23.9% −18.0%	+2.1% −1.6%	$5.662 \pm 0.021 \cdot 10^{-1}$	+11.2% −10.6%	+1.7% −1.3%
e.8	$pp \to t\bar{t}Z$	`p p > t t~ z`	$5.273 \pm 0.004 \cdot 10^{-1}$	+30.5% −21.8%	+1.8% −2.1%	$7.598 \pm 0.026 \cdot 10^{-1}$	+9.7% −11.1%	+1.9% −2.2%
e.9	$pp \to t\bar{t}\gamma$	`p p > t t~ a`	$1.204 \pm 0.001 \cdot 10^{0}$	+29.6% −21.3%	+1.6% −1.8%	$1.744 \pm 0.005 \cdot 10^{0}$	+9.8% −11.0%	+1.7% −2.0%
e.10*	$pp \to t\bar{t}W^{\pm}j$	`p p > t t~ wpm j`	$2.352 \pm 0.002 \cdot 10^{-1}$	+40.9% −27.1%	+1.3% −1.0%	$3.404 \pm 0.011 \cdot 10^{-1}$	+11.2% −14.0%	+1.2% −0.9%
e.11*	$pp \to t\bar{t}Zj$	`p p > t t~ z j`	$3.953 \pm 0.004 \cdot 10^{-1}$	+46.2% −29.5%	+2.7% −3.0%	$5.074 \pm 0.016 \cdot 10^{-1}$	+7.0% −12.3%	+2.5% −2.9%
e.12*	$pp \to t\bar{t}\gamma j$	`p p > t t~ a j`	$8.726 \pm 0.010 \cdot 10^{-1}$	+45.4% −29.1%	+2.3% −2.6%	$1.135 \pm 0.004 \cdot 10^{0}$	+7.5% −12.2%	+2.2% −2.5%
e.13*	$pp \to t\bar{t}W^{-}W^{+}$ (4f)	`p p > t t~ w+ w-`	$6.675 \pm 0.006 \cdot 10^{-3}$	+30.9% −21.9%	+2.1% −2.0%	$9.904 \pm 0.026 \cdot 10^{-3}$	+10.9% −11.8%	+2.1% −2.1%
e.14*	$pp \to t\bar{t}W^{\pm}Z$	`p p > t t~ wpm z`	$2.404 \pm 0.002 \cdot 10^{-3}$	+26.6% −19.6%	+2.5% −1.8%	$3.525 \pm 0.010 \cdot 10^{-3}$	+10.6% −10.8%	+2.3% −1.6%
e.15*	$pp \to t\bar{t}W^{\pm}\gamma$	`p p > t t~ wpm a`	$2.718 \pm 0.003 \cdot 10^{-3}$	+25.4% −18.9%	+2.3% −1.8%	$3.927 \pm 0.013 \cdot 10^{-3}$	+10.3% −10.4%	+2.0% −1.5%
e.16*	$pp \to t\bar{t}ZZ$	`p p > t t~ z z`	$1.349 \pm 0.014 \cdot 10^{-3}$	+29.3% −21.1%	+1.7% −1.5%	$1.840 \pm 0.007 \cdot 10^{-3}$	+7.9% −9.9%	+1.7% −1.5%
e.17*	$pp \to t\bar{t}Z\gamma$	`p p > t t~ z a`	$2.548 \pm 0.003 \cdot 10^{-3}$	+30.1% −21.5%	+1.7% −1.6%	$3.656 \pm 0.012 \cdot 10^{-3}$	+9.7% −11.0%	+1.8% −1.9%
e.18*	$pp \to t\bar{t}\gamma\gamma$	`p p > t t~ a a`	$3.272 \pm 0.006 \cdot 10^{-3}$	+28.4% −20.6%	+1.3% −1.1%	$4.402 \pm 0.015 \cdot 10^{-3}$	+7.8% −9.7%	+1.4% −1.4%

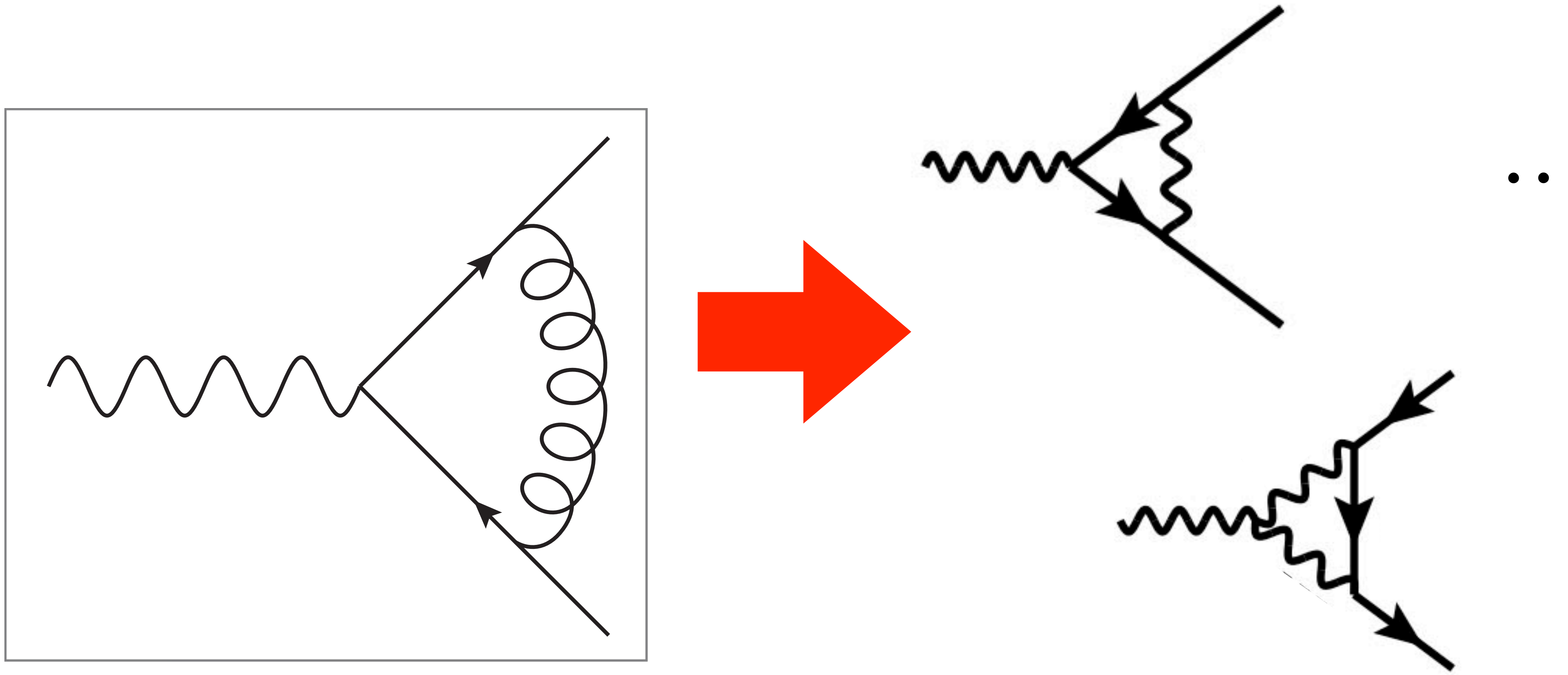
And much more! (EW/BSM/...)

How to compute loops: Summary

- There has been an enormous progress in loop computation techniques in the recent years
- For one-loop computation, we need to find the coefficient which multiply the scalar integrals
- OPP is a powerful method to compute the coefficients numerically. Some cares need to be taken because of dimensional regularisation

From QCD to EW corrections

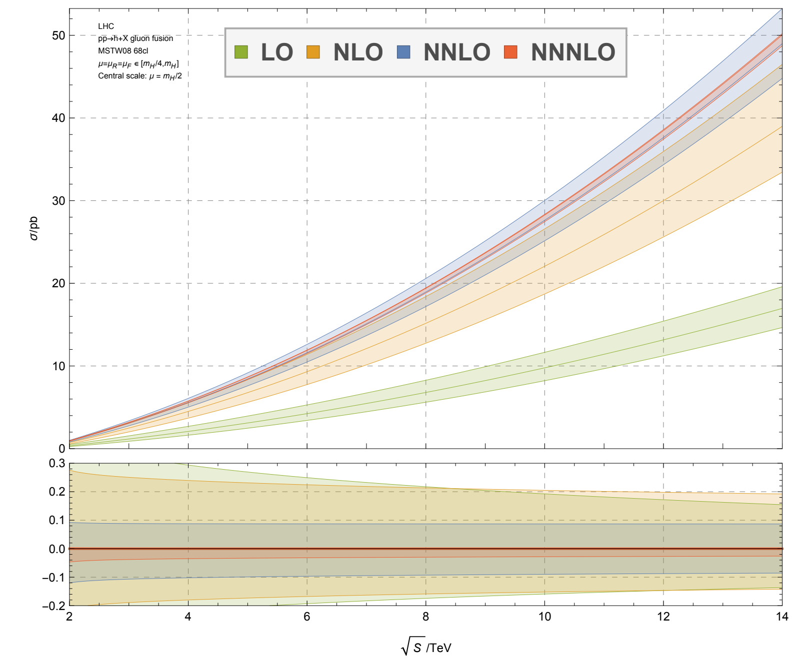
a brief overview



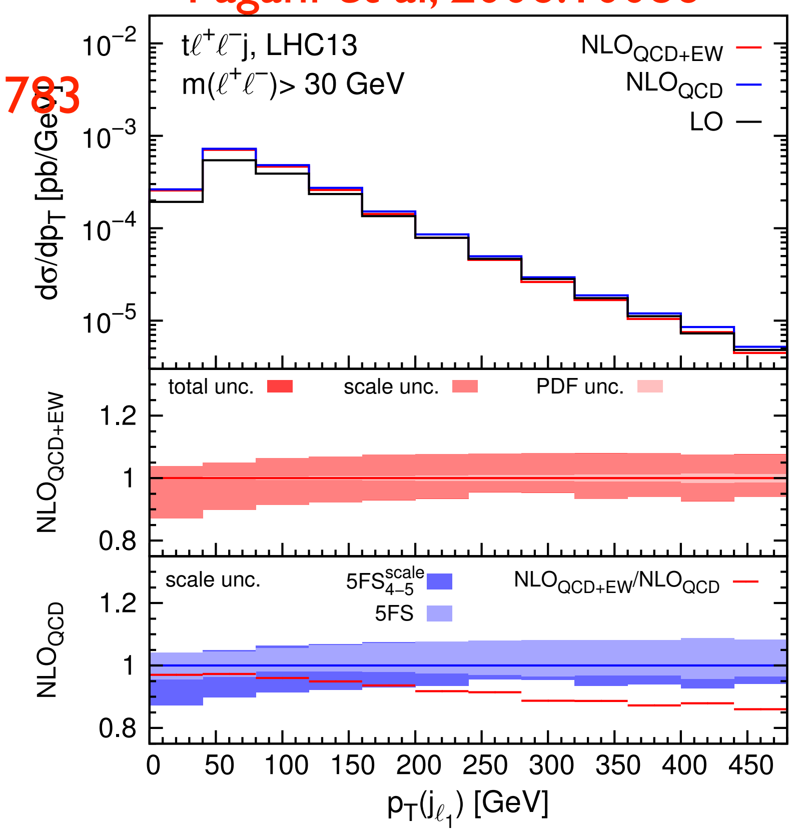
Why bothering?

- QCD corrections generally improve precision of computations (shrink theoretical errors)
- EW corrections necessary to improve accuracy of predictions, specially in the tails of distributions (Sudakov enhancement)
- EW corrections are crucial at lepton colliders
- EW and complete-NLO corrections automated! Sherpa+Openloops: 1412.5157; Sherpa+Recola: 1704.05783 MG5_aMC: 1804.10017
- In some cases, EW corrections do not behave as expected: can give effects as large as QCD!

Anastasiou et al, 1503.06056



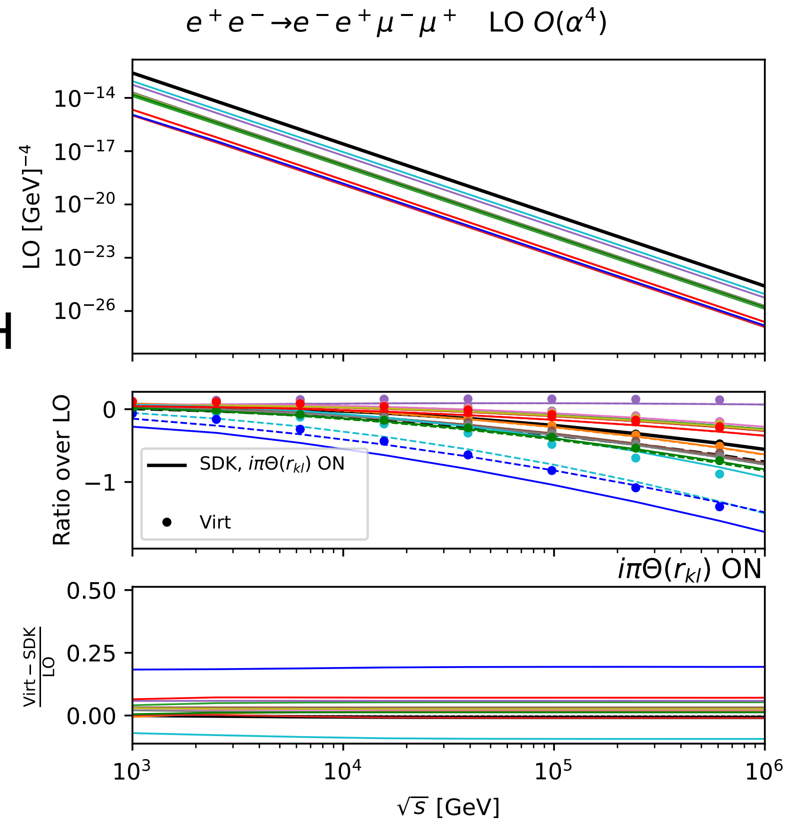
Pagani et al, 2006.10086



Sudakov enhancement

Denner, Pozzorini, hep-ph/0010201 & hep-ph/0104127
Pagani, MZ, arXiv:2110.03714

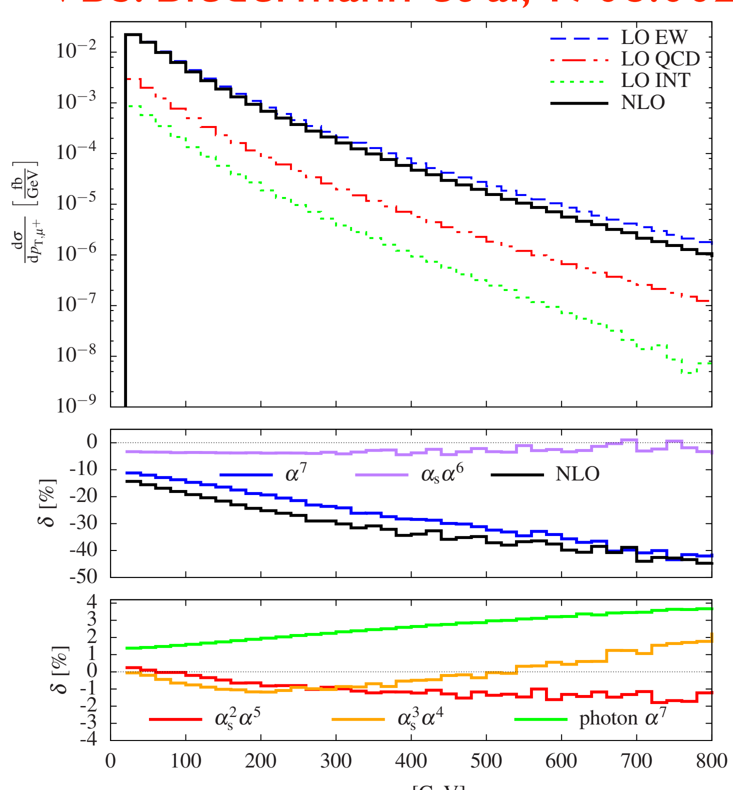
- EW bosons are massive: a real W/Z/Higgs emission is detectable
  (at least in principle)
- Radiation of W/Z/Higgs bosons is in general not included in EW corrections, which remain finite
- When the process scale Q is large, Q≫M~m_W,m_Z,m_H the would-be IR divergence associated to the heavy boson shows up with double and single log(Q/M)
- In the regime where all invariants are ≫M, these logs are universal, and exponentiate at all orders (resummation possible)
- Sudakov approximation is excellent at high-energy (only a constant part is missing)



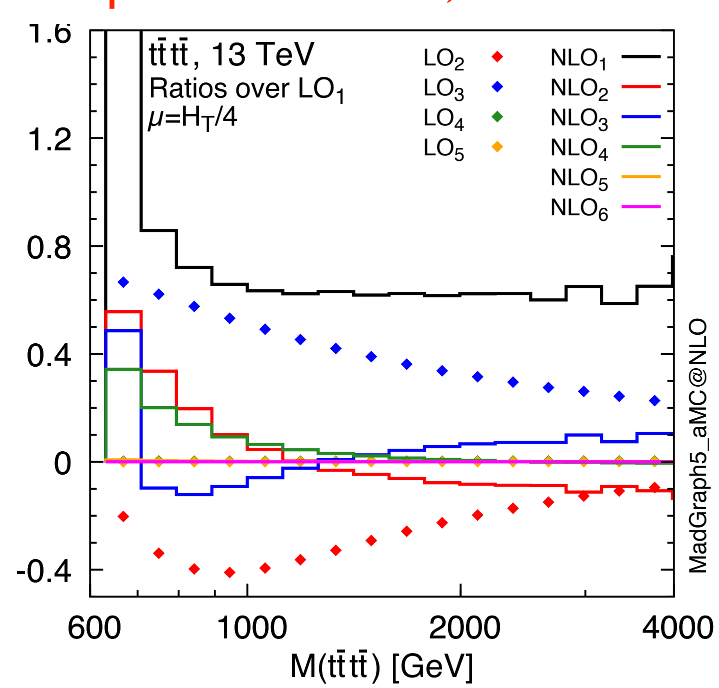
Large EW corrections: not only Sudakov logs

- Despite the naive estimate $\alpha \sim \alpha_s^2$, there are cases when EW corrections comparable to NLO QCD or larger. It happens when:
  - Large scales are probed (VBS) — feature of all VBS channels, see also Denner et al, 1904.00882, 2009.00411
  - Power counting is altered (4 top: y_t vs α)
  - New production mechanisms, different than those at the "dominant" LO, enter (ttW, bbH)

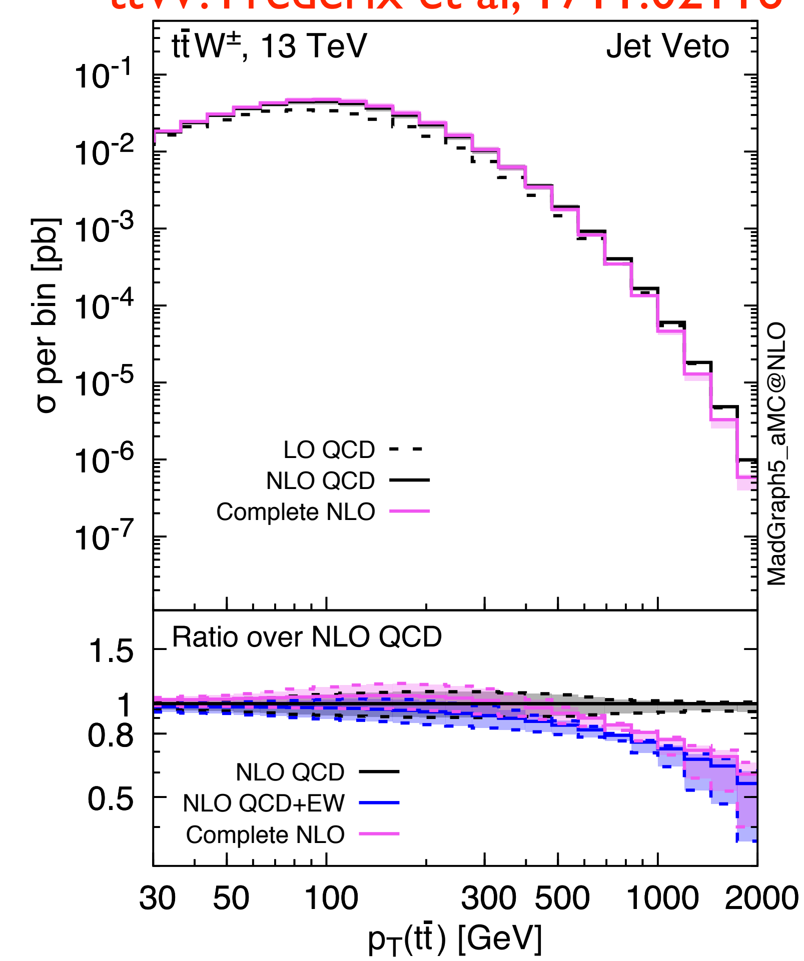
VBS: Biedermann et al, 1708.00268



4 top: Frederix et al, 1711.02116

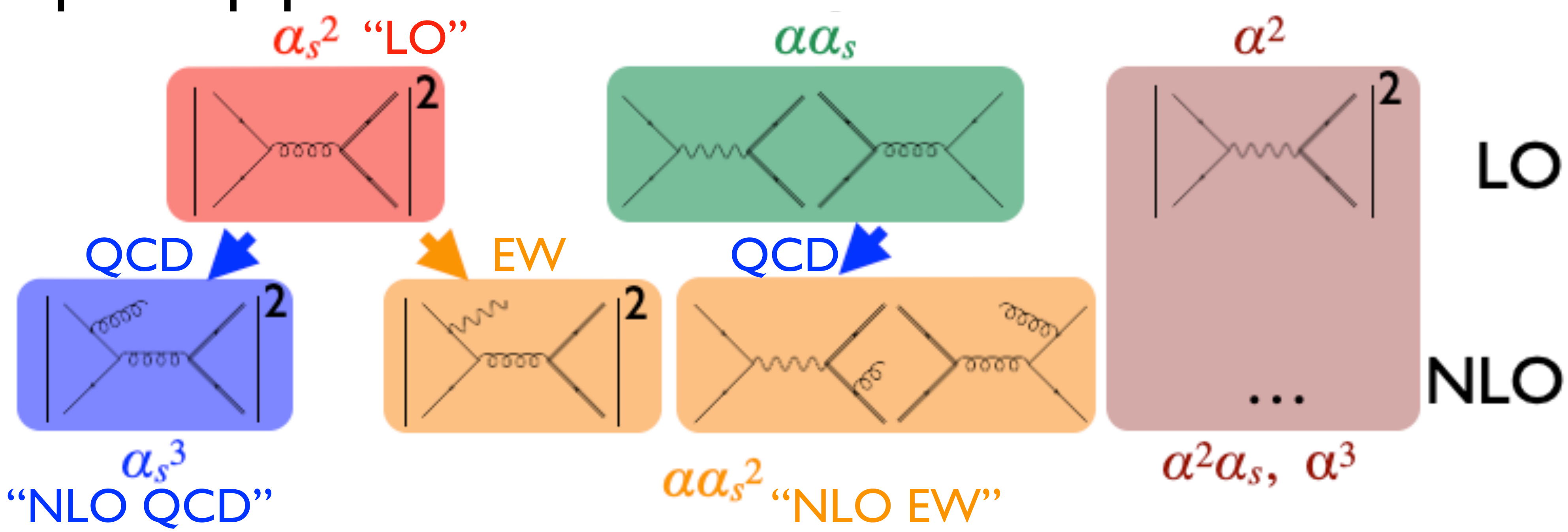


ttW: Frederix et al, 1711.02116



Anatomy of EW corrections: EW corrections vs EW effects

- A general process has more contributions at LO, NLO, …
- Example: top pair



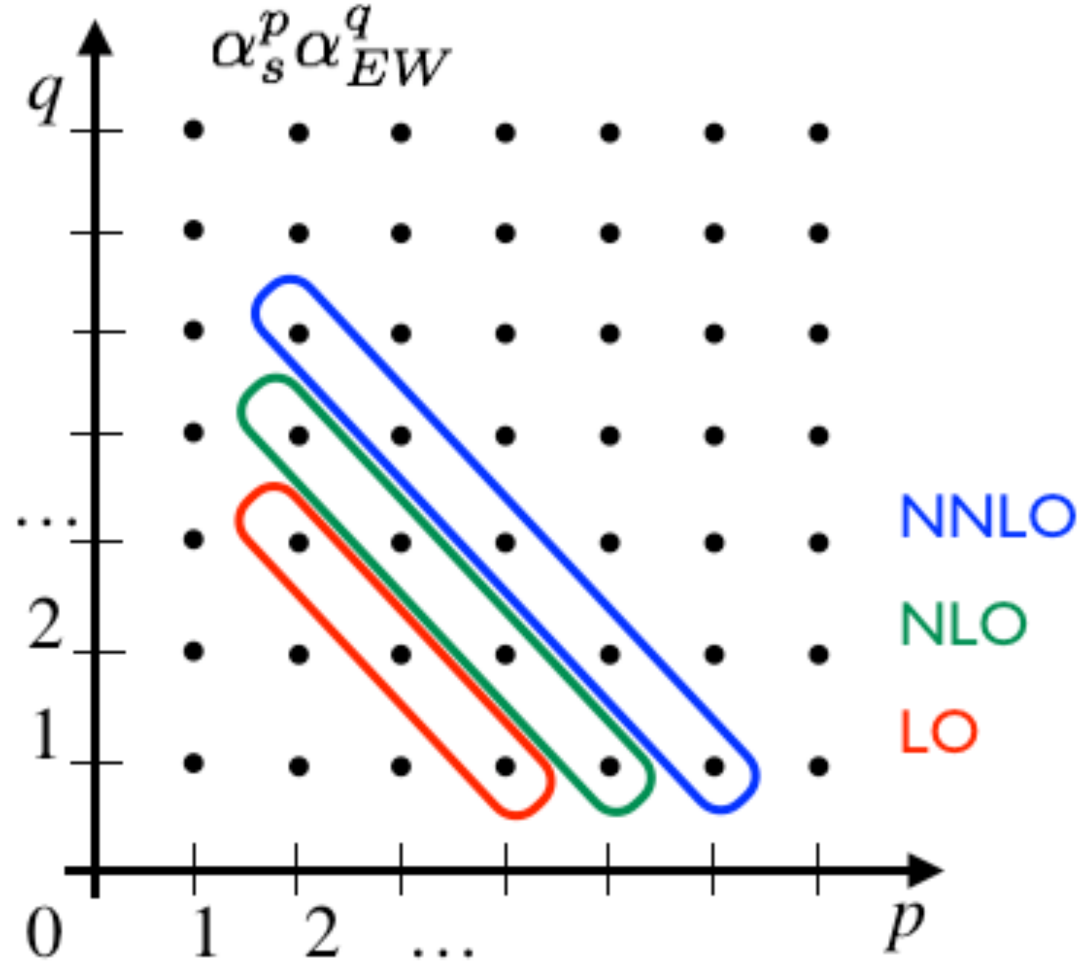
- The LO is often identified with the contribution with most α_s
- At NLO the first two contributions are identified with the NLO QCD and NLO EW corrections
- This structures induces mixed QCD-EW effects at NLO: $NLO_i = LO_{i-1} \otimes EW + LO_i \otimes QCD$

Multi-coupling expansion

Single coupling $\hat{\sigma} = \alpha_s^b \sigma_0 + \alpha_s^{b+1} \sigma_1 + \alpha_s^{b+2} \sigma_2 + \alpha_s^{b+3} \sigma_3 + \ldots$

LO NLO NNLO NNNLO

Multi-coupling



Steps towards the automation of EW corrections

- Apart for the (much) more complex book-keeping, automation of NLO EW corrections largely builds on techniques for NLO QCD (modulo bookkeeping)
- IR subtraction: techniques established for QCD corrections can be extended to EW ones
- Replace color factors with charges ($C_F \to q_i^2, C_A \to 0, T_F \to N_{C,i}\, q_i^2$) Replace color-linked Borns with charge-links
- Loop amplitudes: one-loop techniques can be exploited for EW loops.
- UV/R2 counterterms for the EW interactions are needed
- Higher ranks appear, integrand-reduction may lead to unstable results Switch to other techniques (Tensor-integral reduction, Laurent-series expansion,…)
- Use scalar-integral libraries that support complex masses

EW renormalisation schemes in a nutshell

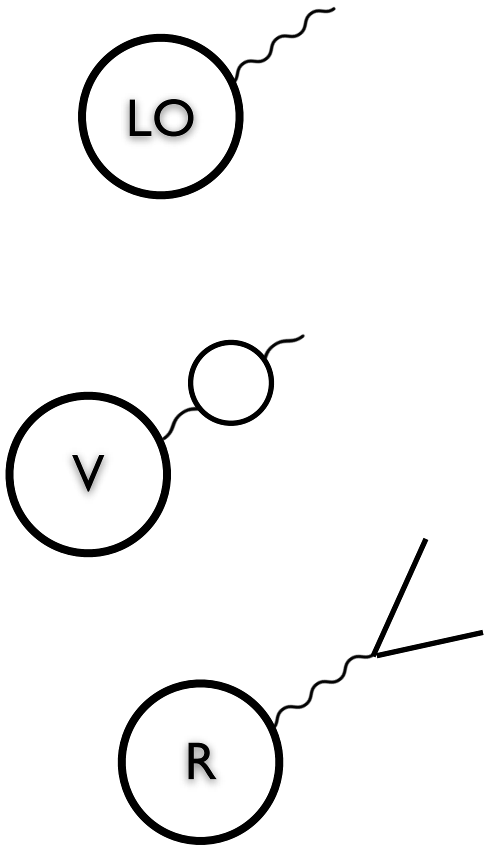
The renormalisation of α can be performed in different schemes:

- $\alpha(0)$: α is measured in the Thompson scattering, in the zero-momentum limit. Terms ~$\log(Q/m_f)$ appear in the cross section, except for external photons. Fermion masses must be retained.
- $\alpha(M_Z)$: α is measured at the Z peak (e.g. at LEP). It removes the dependence on the fermion masses, which can be set to zero.
- G_μ scheme: the Fermi constant is measured from the muon lifetime, then α is extracted. W.r.t. the $\alpha(M_Z)$ scheme, also contributions of weak origin ($\Delta\rho$) are resummed

The G_μ scheme is generally preferred for processes without final-state photons at the LO.

Processes with tagged photons

Pagani, Tsinikos, MZ arXiv:2106.02059



- The definition of a "photon" in the presence of EW corrections is not IR-safe (in a scheme with massless quarks/leptons)
- This is why democratic jets are usually employed
- In order to define photons as physical objects, a renormalisation scheme which takes into account fermion masses must be employed (only for the vertices related to tagged photons). Such a scheme exists: $\alpha(0)$
- Renormalisation conditions define α from the low-energy Thomson scattering. IR-poles differ from a high-energy scheme such as G_μ or $\alpha(m_Z)$
- The difference of IR poles accounts for the fact that real emissions with $\gamma \to 2f$ splittings are not included
- Alternative: use fragmentation functions (more involved)

NLO: Summary

- Precise predictions crucial for success of LHC programme
- They entail a lot of complexity: NLO is just the first bite!
- 10 years ago: NLO revolution. We have harvested many fruits
  - Automation: complexity hidden to the user!
  - NLO event generators ubiquitous in exp. analyses
  - Techniques proved successful also beyond QCD: automation of electroweak corrections (see backup slides for extra informations)

Matching NLO computations with parton showers

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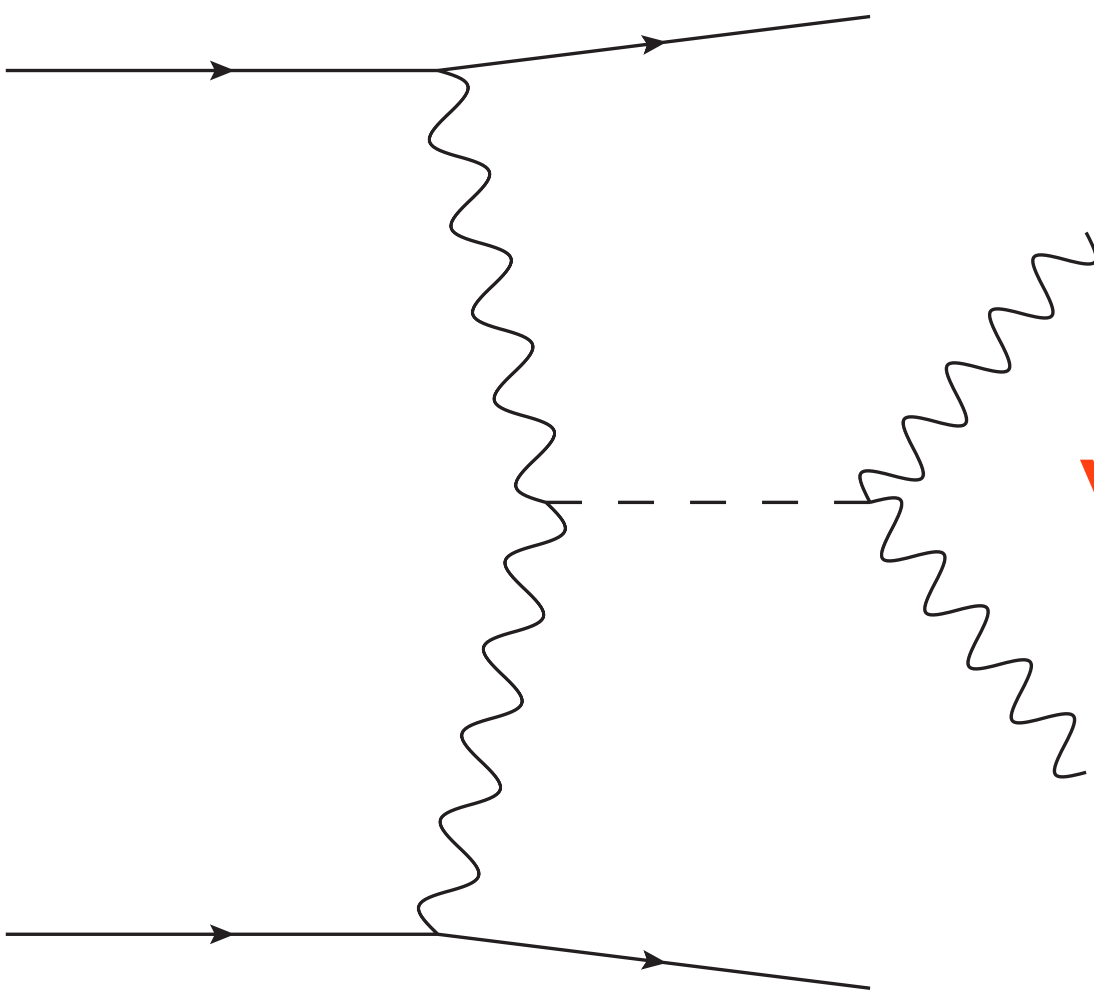
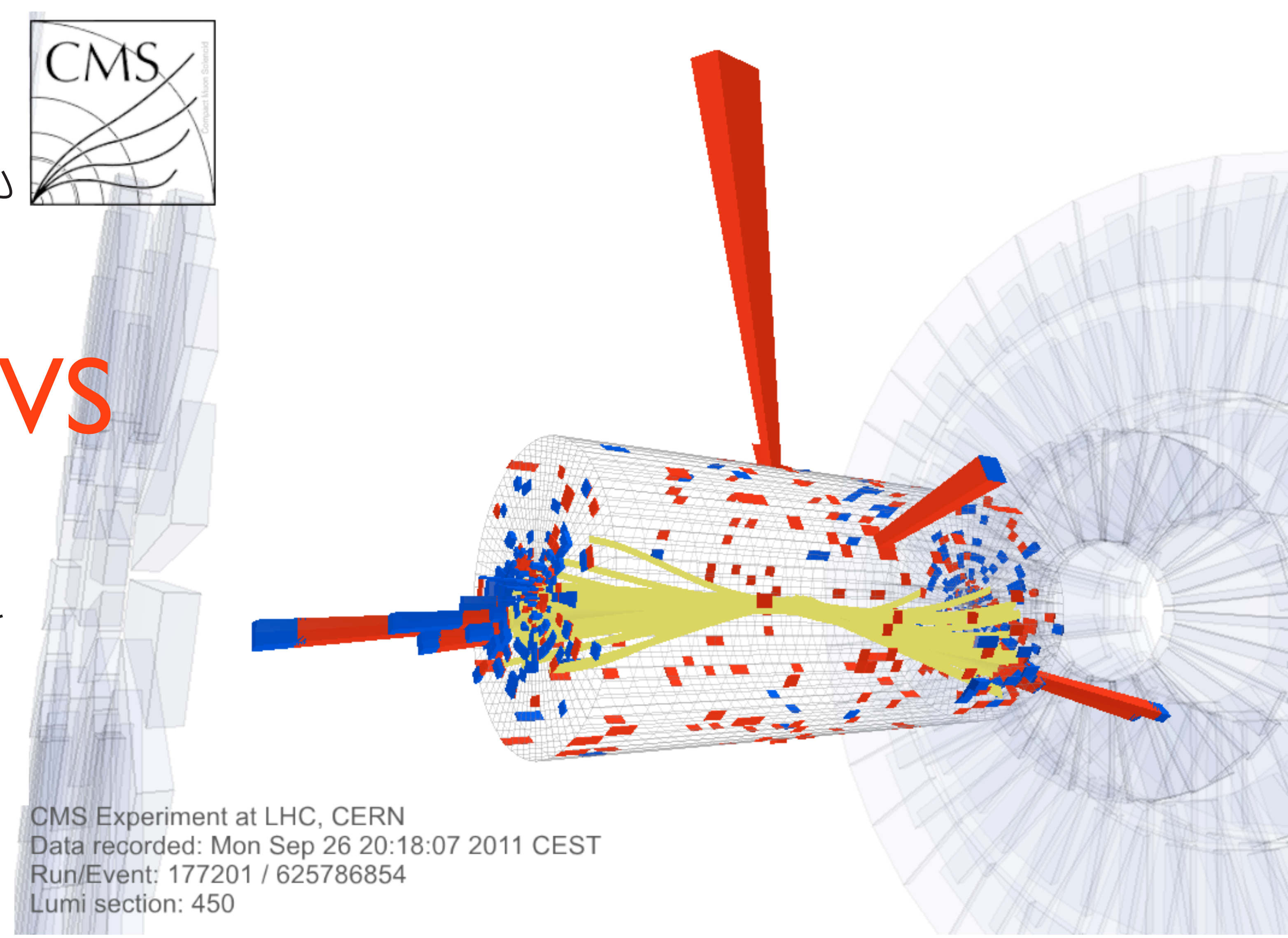
Matching NLO computations with parton showers



VS



Recap from yesterday

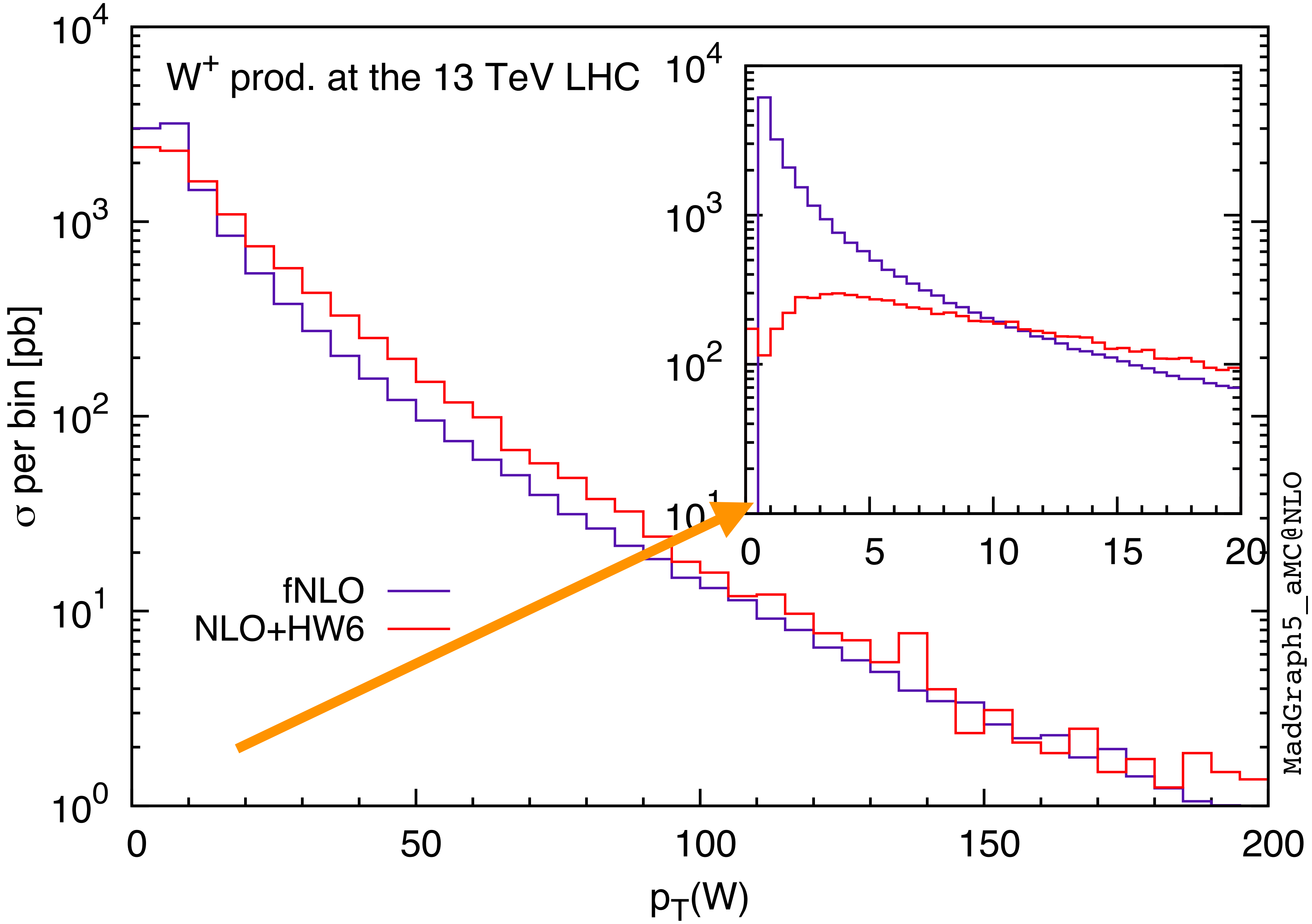
- Virtual and real matrix element are not finite, but their sum is. Subtraction methods can be used to extract divergences for real-emission matrix elements and cancel explicitly the poles from the virtuals
- Event and counterevents have different kinematics. Unweighting and event generation not possible, plots are filled on-the-fly with weighted events
- For plots, only IR-safe observable with finite resolution must be used!

Matching NLO predictions and parton showers

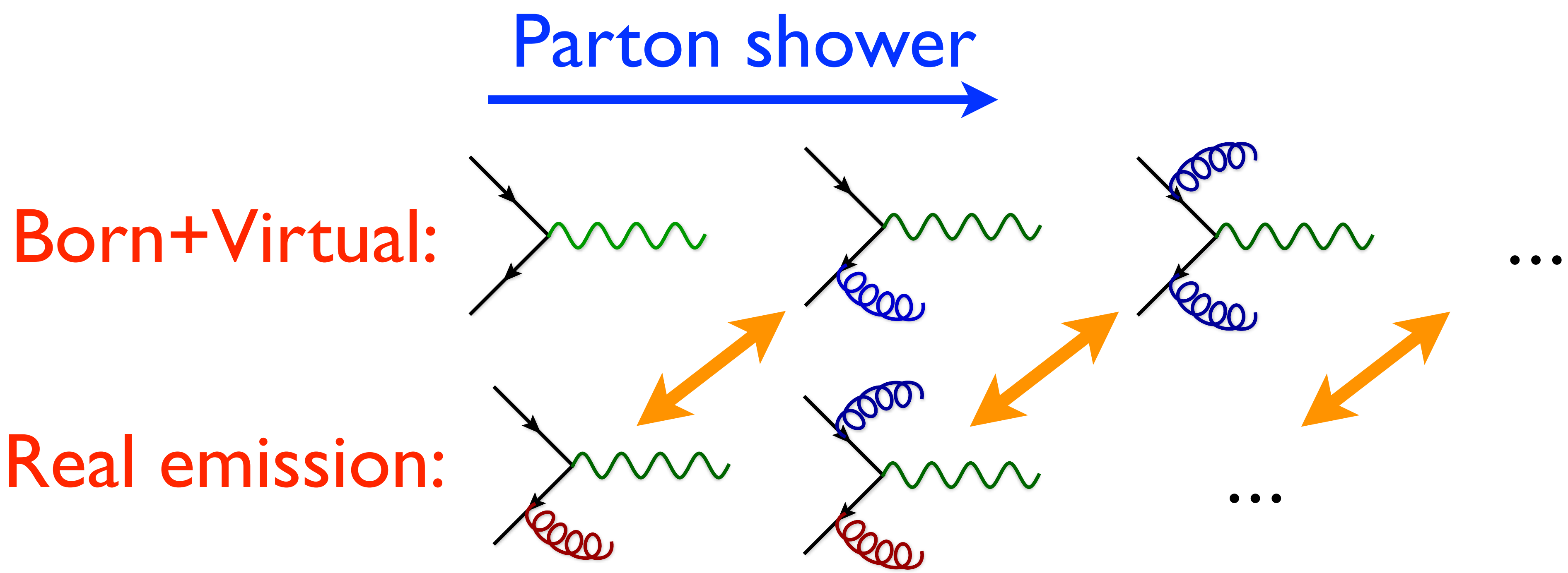
- Parton showers evolve hard partons by emitting extra QCD radiation down to a more realistic final state made of hadrons
- This resums the effect of soft gluon radiations, and cures fixed-order instabilities
- After the parton shower, a fully exclusive description of the event is available
- NLO corrections are inclusive by definition, but they provide the first reliable estimate of rates and uncertainties

Can we attach a parton shower to NLO simulations?

Fixed order instabilities, again



Warning: double counting!



- There is a double counting between real emission and the parton shower
- There is also double counting between the virtuals and the non-emission probability from the Sudakov factor

Double counting the virtuals

- The Sudakov factor Δ, responsible for the resummation performed by the shower, gives the no-emission probability 1-P, P being the emission probability)

$$\Delta(Q, Q_0) = \exp\left[-\int_{Q_0}^{Q} d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}\right]$$

shower variables

$$d\Phi_1 = \frac{dt}{t}\, dz \frac{d\phi}{2\pi}$$

- Δ contains implicitly contributions from the virtual and real corrections
- We should therefore avoid to double counting the contribution from the virtuals in the matrix element and in the Sudakov
- Because of unitarity, what is double counted in the virtuals is exactly opposite to what is double counted by the reals

How to avoid double counting at NLO?

- Two methods exist:
  - MC@NLO Frixione, Webber hep-ph/0204244
  - Powheg Nason, hep-ph/0409146

Naive (wrong) matching

- Let us *assume* we can generate events separately for Born, virtuals and real emissions, and that we pass them to a parton shower

$$\frac{d\sigma_{\text{“}NLO+PS\text{”}}}{dO} = [\mathcal{B}+\mathcal{V}]\, d\Phi_n\, I^n_{MC}(O) + d\Phi_{n+1}\mathcal{R}\, I^{n+1}_{MC}(O)$$

- Do we get the NLO cross section?
  - Let us expand the shower operator at order α_S (0 or 1 emission)

$$I_{MC} = \Delta_a\left(Q,Q_0\right) + \Delta_a\left(Q,Q_0\right) d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

$$\Delta_a\left(Q,Q_0\right) = \exp\left[-\int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}\right] \simeq 1 - \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

$$I_{MC} \simeq 1 - \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc} + d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

Naive (wrong) matching

- At order α_S we get

$$\frac{d\sigma_{\text{“}NLO+PS\text{”}}}{dO} = [\mathcal{B}+\mathcal{V}]\, d\Phi_n + d\Phi_{n+1}\mathcal{R}$$
$$- \mathcal{B}\, d\Phi_n \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc} + \mathcal{B}\, d\Phi_n d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

- Which is not the NLO

MC@NLO matching

- In the MC@NLO formalism, double counting can be cured by the so-called Monte Carlo counterterms, defined as

$$\Delta(Q,Q_0) = \exp\left(-\int d\Phi_1 MC\right) \qquad MC = \left|\frac{\partial \Phi_1^{MC}}{\partial \Phi_1}\right| \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

- The MC@NLO cross section is defined as

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n\, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- Again, if we expand up to α_S we recover the NLO

$$I_{MC} = 1 - \int d\Phi_1 MC + d\Phi_1 MC$$

$$\frac{d\sigma_{\text{“}MC@NLO\text{”}}}{dO} = \left[\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right] d\Phi_n + d\Phi_{n+1}\left[\mathcal{R} - MC\right]$$

$$+\mathcal{B}\left[-\int d\Phi_1 MC + d\Phi_1 MC\right] d\Phi_n$$

The *MC* counterterm

- *MC* has some remarkable properties:
  - It avoids double counting when matching to PS ←Just shown
  - It matches the singular behaviour of the real-emission ME, making it possible to unweight events (some special cares are needed for the soft region)
  - It ensures a smooth matching: NLO+PS has the same shape of the shower in the soft/collinear region; in the hard region, it approaches the NLO
  - It is PS dependent, as it depends on the PS details. For each PS, we need its own *MC* counterterms

Unweighting

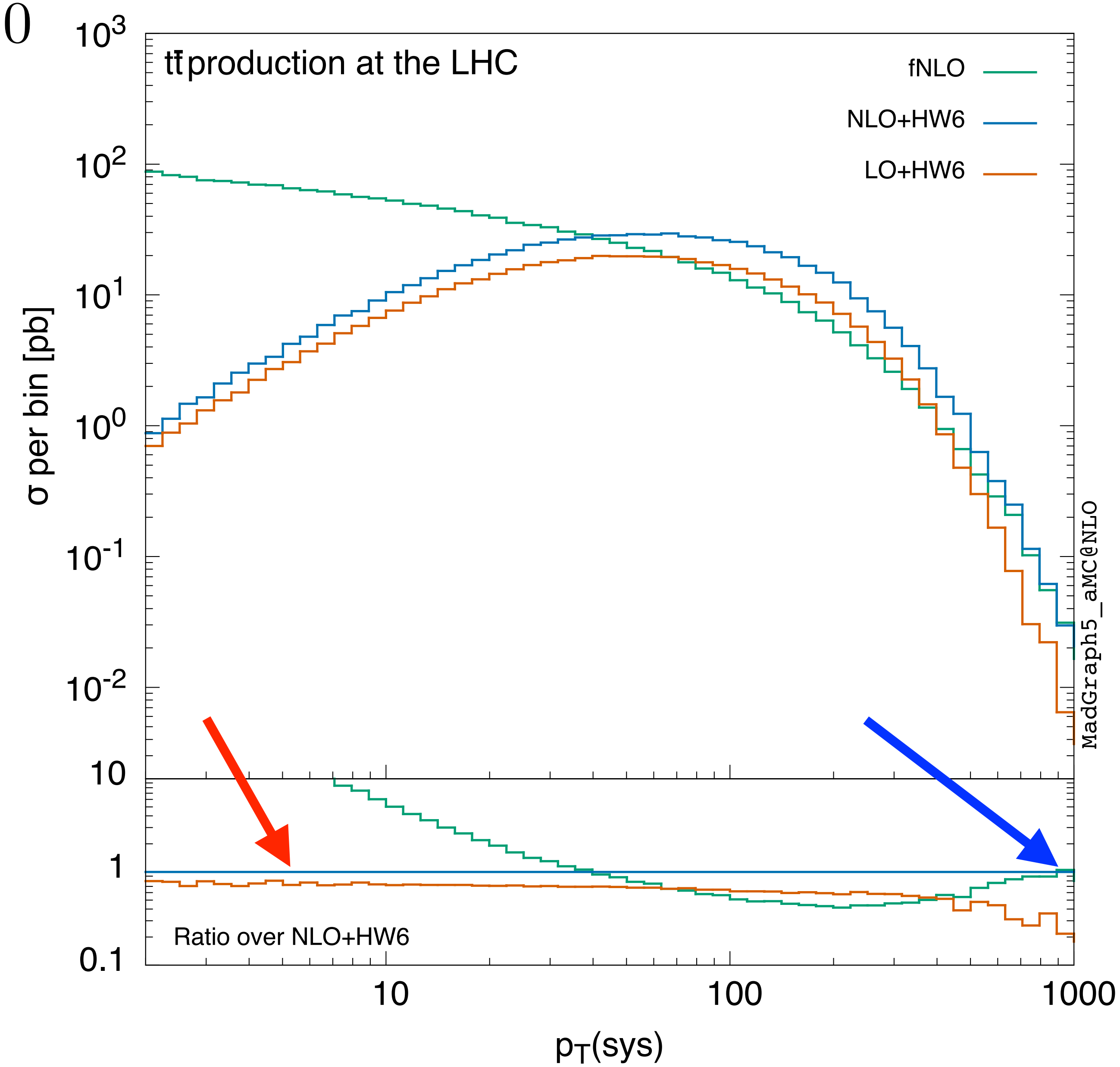
$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n \, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- MC is by construction what the shower does to go from n to n+1. It matches exactly R in the soft-collinear region. Furthermore, it has the same kinematics as R, therefore there is no reshuffling needed. The n and n+1 body contributions are separately finite and bounded. Unweighted events can be generated!
  - S-events, with n-body kinematics
  - H-events, with n+1-body kinematics

Smooth matching

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n \, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- In the soft/collinear region, $\mathcal{R} - MC \sim 0$ so that
$$\frac{d\sigma_{MC@NLO}}{dO} \simeq I^n_{MC}(O)$$
- In the hard region, MC=0 (it must be zero far from singular regions). The only contribution comes from the real-emission ME



The MC counterterms and the FKS subtraction

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n\, I^n_{MC}(O) + (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- The MC counterterms already make the cross-section finite. Are the local counterterms still needed?
- Yes, because we cannot integrate MC analytically to extract the poles
- In practice, we have

$$\frac{d\sigma_{MC@NLO}}{dO} = \left[\mathcal{B} + \left(\mathcal{V} + \int d\Phi_1 \mathcal{C}\right) + \int d\Phi_1\,(MC - \mathcal{C})\right] d\Phi_n\, I^n_{MC}(O)$$
$$+ (\mathcal{R} - MC)\, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

Negative weights

$$\frac{d\sigma_{MC@NLO}}{dO} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right) d\Phi_n \, I^n_{MC}(O) + (\mathcal{R} - MC) \, d\Phi_{n+1} I^{n+1}_{MC}(O)$$

- Events are generated for n- and n+1-body kinematics separately
- Nothing guarantees that the two contributions are separately positive
- The unweighting has to be done up to a sign, and the sign should be taken into account when filling plots
- Remember: results are physical only after having showered the events!

Powheg

- Let us consider the LO+PS cross-section expanded up to the first emission:

$$d\sigma_{LO+PS} = \mathcal{B}\, d\Phi_n \left[\Delta(Q,Q_0) + \Delta(Q,Q_0) d\Phi_1 \frac{\alpha_s(t)}{2\pi} P \right]$$

- We could think of going NLO by replacing the Born with the NLO cross section

$$d\sigma_{\text{``}NLO+PS\text{''}} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R} \right) d\Phi_n \left[\Delta(Q,Q_0) + \Delta(Q,Q_0) d\Phi_1 \frac{\alpha_s(t)}{2\pi} P \right]$$

$$= \mathcal{B} \left(1 - \int \underline{d\Phi_1 MC} - d\Phi_1 MC \right) + \mathcal{V} + \int \underline{d\Phi_1 \mathcal{R}}$$

- Of course, there is double counting. This is in particular due by the fact that the integral in the Sudakov does not contain R

A modified Sudakov

- In order to avoid double counting one could use a modified Sudakov

$$\tilde{\Delta}(Q, Q_0) = \exp\left(-\int_{Q_0^2}^{Q^2} d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right)$$

- Such that

$$d\sigma_{\text{“}NLO+PS\text{”}} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[\tilde{\Delta}(Q, Q_0) + \tilde{\Delta}(Q, Q_0) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right]$$

- But the total rate is not the NLO! The second parentheses does not integrate to 1 (see next slide). It has to be modified to

$$d\sigma_{\text{“}NLO+PS\text{”}} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[\tilde{\Delta}(Q, Q_0) + \tilde{\Delta}(Q, t) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right]$$

- Where t is the scale at which R/B is evaluated

- Note that

$$d\sigma_{Powheg} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[\tilde{\Delta}(Q, Q_0) + \tilde{\Delta}(Q, t) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right]$$

- Therefore

Lower integration bound

$$\tilde{\Delta}(Q, t)\frac{\mathcal{R}}{\mathcal{B}} = \frac{d\tilde{\Delta}(Q, t)}{dt}$$

- So the [] integrates to 1. The NLO normalisation is kept
- Indeed one expands at order α_S:

$$\int_{Q_0}^{Q} dt \tilde{\Delta}(Q, t)\frac{\mathcal{R}}{\mathcal{B}} = \tilde{\Delta}(Q, Q) - \tilde{\Delta}(Q, Q_0) = 1 - \tilde{\Delta}(Q, Q_0)$$

- Double counting is avoided

$$d\sigma_{Powheg} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[1 - \int d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}} + d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right] = d\sigma_{NLO}$$

Comments

$$d\sigma_{Powheg} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}\right) d\Phi_n \left[\tilde{\Delta}(Q, Q_0) + \tilde{\Delta}(Q, t) d\Phi_1 \frac{\mathcal{R}}{\mathcal{B}}\right]$$

- The Powheg cross section has the same structure as an ordinary shower, with a global K-factor correction and a different Sudakov for the first emission
- Note that when matching to PS one has to veto emissions harder than t (in the Powheg formalism, is has to be interpreted as transverse momentum), even for showers with a different ordering variable
  - Formula to be modified for angular-ordered PS in order to keep color coherence
- MC@NLO and Powheg are formally equivalent at NLO level. In practice, their predictions may visibly differ

MC@NLO vs Powheg

- The two matching procedure can be cast in a single formula

$$d\sigma_{NLO+PS} = d\Phi_n \bar{\mathcal{B}}^s \left[\Delta^s(Q,Q^0) + d\Phi_1 \frac{\mathcal{R}^s}{\mathcal{B}} \Delta^s(Q,t)\right] + d\Phi_{n+1}\mathcal{R}^f$$

- With

$$\bar{\mathcal{B}}^s = \mathcal{B} + \mathcal{V} + \int d\Phi_1 \mathcal{R}^s$$

- And the real-emission ME has been split in a singular and non-singular (finite) part

$$\mathcal{R} = \mathcal{R}^s + \mathcal{R}^f$$

- The difference between the two methods is in R^s:

MC@NLO

$$\mathcal{R}^s = \frac{\alpha_s}{2\pi} P\mathcal{B} = MC$$

Powheg

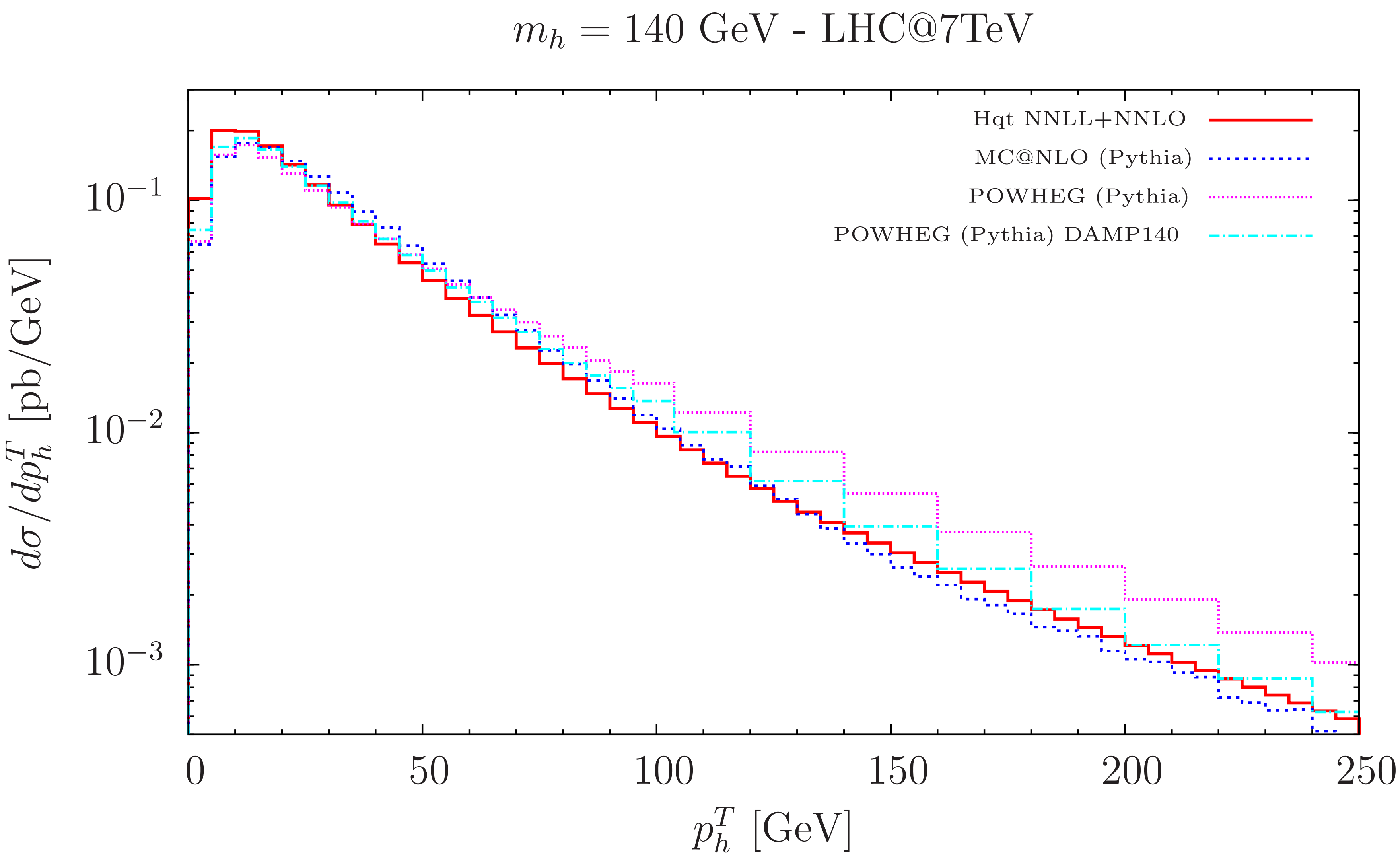
$$\mathcal{R}^s = F\mathcal{R} \quad \mathcal{R}^f = (1-F)\mathcal{R}$$

default F=1,
but can be tuned in order to
suppress non-singular part of R

Effect of F

$$F = \frac{h^2}{h^2 + p_T^2}$$

$p_T \gg h$ are suppressed



MC@NLO naturally matches analytic resummation+FO curve at large p_T

Powheg (without damping) overshoots the FO

Damping recovers matching at large p_T

NLO event generation and matching: Summary

- Generating and showering events at NLO is much trickier than at LO: $n(+1)$ body contributions are not separately finite +need not to double count contributions
- Methods have been developed to overcome this
  - MC@NLO achieves this by means of the so-called MC counterterm
  - Powheg generates the hardest emission with its own Sudakov
- Both have proven very successful, and today NLO+PS simulations are the workhorses of all experimental analyses

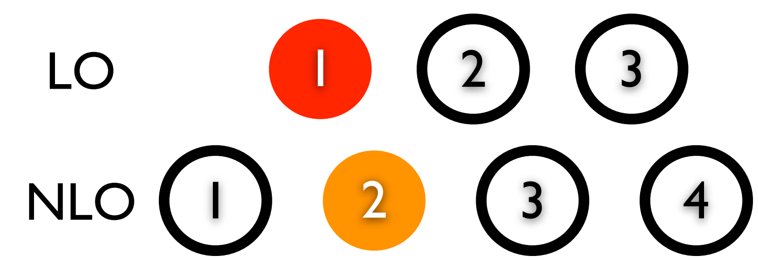
Next?

- Beyond NLO: NNLO is the new Holy Graal:
  - Several subtraction techniques are being studied at NNLO. They all work on paper, need for numeric implementation and testing
  - No general algorithm to compute 2-loop amplitudes, but huge progress (first results for massless $2\to3$ processes available)
  - In general, huge amount of complexity and of running time (~1M CPU hours for $2\to2$ with coloured FS)
- Is the NNLO revolution approaching?

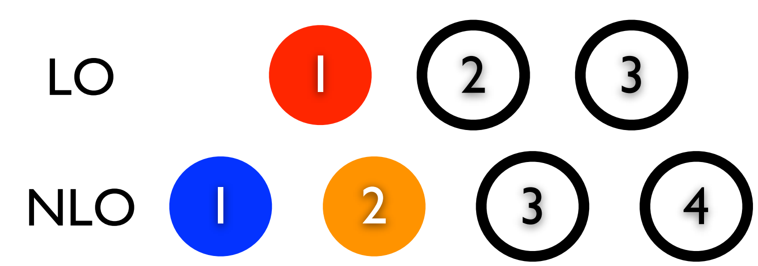
Backup

MG5_aMC Syntax (I)

- The syntax to generate NLO EW corrections is very similar to the one for QCD:
  - e.g.: ttbar@NLO EW: generate p p > t t~ [QED]
  - Since no orders are specified, it will take the LO contribution with the largest power of α_s^2, $O(\alpha_s^2)$, and generate NLO corrections with one extra power of α, $O(\alpha_s^2\alpha)$

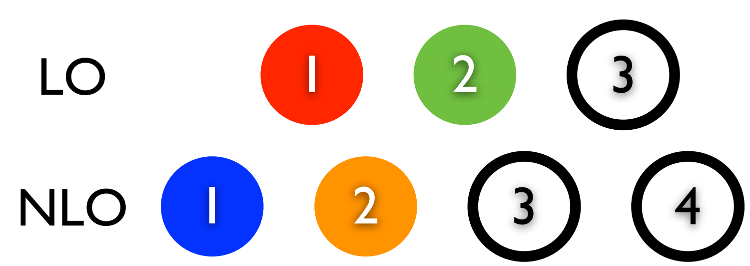


  - If one wants to also generate NLO QCD corrections, the syntax is
    generate p p > t t~ [QED QCD]
    In this case NLO contributions with both one extra power of α and of α_s will be generated



MG5_aMC Syntax (II)

- In the previous slide, the syntax would have been equivalent had we explicitly selected the dominant LO contribution.
  - This could be done by adding QED^2=0 QCD^2=4 to the generate command (note the squared-order constraints, applied at the amplitude level)
- Now, suppose you want to include also the first subleasing LO term (LO_2), together with NLO QCD and EW corrections.
  The syntax is: generate p p > t t~ QED^2=2 QCD^2=4 [QCD].
  While counterintuitive, this is interpreted as in the previous slide:
  - Generate LO contributions which satisfy the squared-order constraints ($O(\alpha_s^2)$ and $O(\alpha_s\alpha)$)
  - For the NLO corrections, add a power of α_s on top of both. This will give ($O(\alpha_s^3)$ and $O(\alpha_s^2\alpha)$)



MG5_aMC Syntax (III)

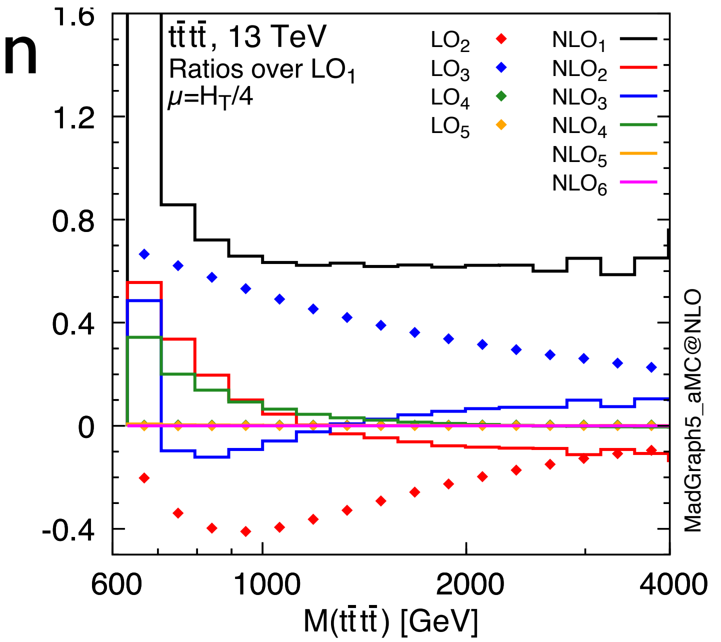
- Can I use diagram-order constraints?
- While this will give inconsistencies when NLO EW corrections are computed, it may be useful e.g. in EFT studies
- If the user asks for diagram constraints together with NLO corrections, the code will issue a clear warning, asking the user to acknowledge what he/she wants to do
- More info on http://amcatnlo.cern.ch/co.htm

Processes with tagged photons: how to

- In practice: a new model with both the HE renormalisation scheme (G_μ) and the $\alpha(0)$ is available: loop_qcd_qed_sm_Gmu-a0
- Once loaded, tagged photons can be specified via the generate syntax: generate t t~ !a! [QED]
- Photons marked as tagged will not originate real emissions where $\gamma \to 2f$ and the corresponding (local and integrated) FKS counterterms will not be included
- For each tagged photon, a term proportional to the difference between $\alpha(0)$ and $\alpha_{G\mu}$ is added (it has IR poles)
- The final result is rescaled by $(\alpha(0)/\alpha_{G\mu})^{\mathrm{NTagPhotons}}$
- Result presented for top-pair and single-top production + photons
  Pagani, Shao, Tsinikos, MZ 2106.02059
- Available in v3.3.0

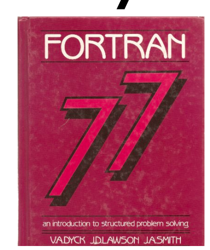
Accessing the various coupling combinations

- The different coupling combinations to the cross section are evaluated in the same run



- Histograms can be booked for each of them in the analysis
- The coupling combination can be detected by using the `orders_tag_plot` variable

```
integer orders_tag_plot
common /corderstagplot/ orders_tag_plot
```



- It is typically computed as 100*QED + 1*QCD (may change if more coupling types are around)
- In any case, the specific values are printed inside the log file

```
INFO: orders_tag_plot is computed as:         + QCD *        1          + QED *      100
 orders_tag_plot=           4   for QCD,QED, =         4 ,          0 ,
 orders_tag_plot=         202   for QCD,QED, =         2 ,          2 ,
 orders_tag_plot=         400   for QCD,QED, =         0 ,          4 ,
 orders_tag_plot=           6   for QCD,QED, =         6 ,          0 ,
 orders_tag_plot=         204   for QCD,QED, =         4 ,          2 ,
 orders_tag_plot=         402   for QCD,QED, =         2 ,          4 ,
```

Accessing the various coupling combinations in LHE events

- The same coupling structure can be accessed inside the LHE event file (when PS-matching is possible)
- Weights are stored in the same format as the scale/PDF variations

```
<initrwgt>
  <weightgroup name='scale_variation                 0    0' combine='envelope'>
    <weight id='1001'> tag=           0 dyn=    0 muR=0.10000E+01 muF=0.10000E+01 </weight>
    <weight id='1002'> tag=           0 dyn=    0 muR=0.20000E+01 muF=0.10000E+01 </weight>
    <weight id='1003'> tag=           0 dyn=    0 muR=0.50000E+00 muF=0.10000E+01 </weight>
    <weight id='1004'> tag=           0 dyn=    0 muR=0.10000E+01 muF=0.20000E+01 </weight>
    <weight id='1005'> tag=           0 dyn=    0 muR=0.20000E+01 muF=0.20000E+01 </weight>
    <weight id='1006'> tag=           0 dyn=    0 muR=0.50000E+00 muF=0.20000E+01 </weight>
    <weight id='1007'> tag=           0 dyn=    0 muR=0.10000E+01 muF=0.50000E+00 </weight>
    <weight id='1008'> tag=           0 dyn=    0 muR=0.20000E+01 muF=0.50000E+00 </weight>
    <weight id='1009'> tag=           0 dyn=    0 muR=0.50000E+00 muF=0.50000E+00 </weight>
  </weightgroup>
  <weightgroup name='scale_variation             40200    0' combine='envelope'>
    <weight id='1010'> tag=       40200 dyn=    0 muR=0.10000E+01 muF=0.10000E+01 </weight>
    <weight id='1011'> tag=       40200 dyn=    0 muR=0.20000E+01 muF=0.10000E+01 </weight>
    <weight id='1012'> tag=       40200 dyn=    0 muR=0.50000E+00 muF=0.10000E+01 </weight>
    <weight id='1013'> tag=       40200 dyn=    0 muR=0.10000E+01 muF=0.20000E+01 </weight>
    <weight id='1014'> tag=       40200 dyn=    0 muR=0.20000E+01 muF=0.20000E+01 </weight>
    <weight id='1015'> tag=       40200 dyn=    0 muR=0.50000E+00 muF=0.20000E+01 </weight>
    <weight id='1016'> tag=       40200 dyn=    0 muR=0.10000E+01 muF=0.50000E+00 </weight>
    <weight id='1017'> tag=       40200 dyn=    0 muR=0.20000E+01 muF=0.50000E+00 </weight>
    <weight id='1018'> tag=       40200 dyn=    0 muR=0.50000E+00 muF=0.50000E+00 </weight>
  </weightgroup>
  <weightgroup name='scale_variation             40202    0' combine='envelope'>
    <weight id='1019'> tag=       40202 dyn=    0 muR=0.10000E+01 muF=0.10000E+01 </weight>
 ...
```

```
 <event>
  5        0 0.15776264E+00 0.21383348
       -5 -1     0     0     0  501 0.0
       21 -1     0     0   501  502 0.0
       -6  1     1     2     0  502 -.4
       24  1     1     2     0    0 0.7
       23  1     1     2     0    0 -.3
#aMCatNLO 1   0   0   1   2 0.91081533E-
0.00000000E+00
  <rwgt>
   <wgt id='1001'> 0.15776E+00 </wgt
   <wgt id='1002'> 0.15496E+00 </wgt
   <wgt id='1003'> 0.15846E+00 </wgt
   <wgt id='1004'> 0.16498E+00 </wgt
   <wgt id='1005'> 0.16195E+00 </wgt
   <wgt id='1006'> 0.16585E+00 </wgt
   <wgt id='1007'> 0.14640E+00 </wgt
   <wgt id='1008'> 0.14389E+00 </wgt
   <wgt id='1009'> 0.14693E+00 </wgt
   <wgt id='1010'> 0.13388E+00 </wgt
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   <wgt id='1012'> 0.14798E+00 </wgt
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   <wgt id='1015'> 0.15414E+00 </wgt
...
```
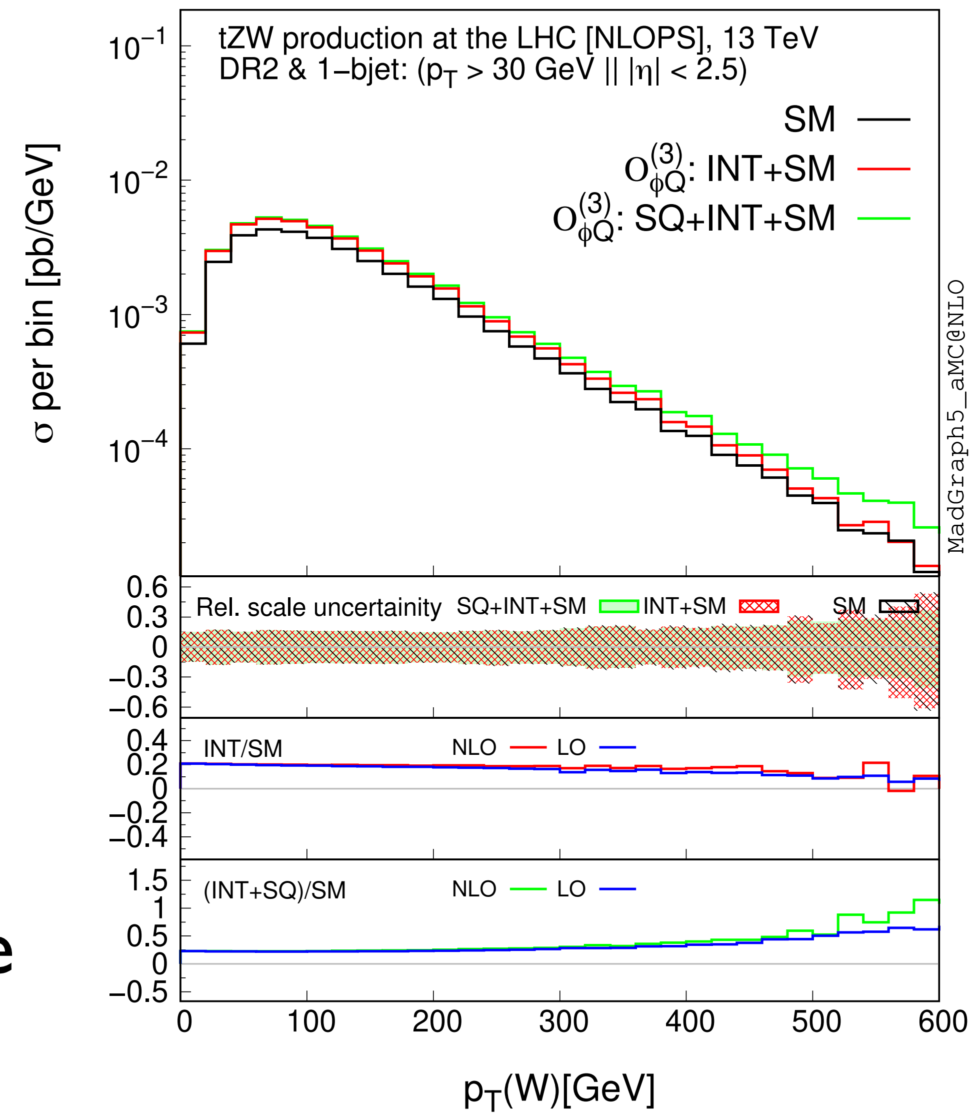

Accessing the various coupling combinations

- In either case, having all the couplings available from the same run makes them all statistically-correlated
- It is specially useful in the context of EFT studies, where different admixtures of new-physics can be morphed starting from the event weights
- Careful when matching to PS!
  If the statistical distribution of colour-flows is very different from one coupling combination to another (e.g. EFT vs SM), morphing could be dangerous!



El Faham, Maltoni, Mimasu, MZ
arXiv:2111.03080