Monte-Carlo Generation

Olivier Mattelaer
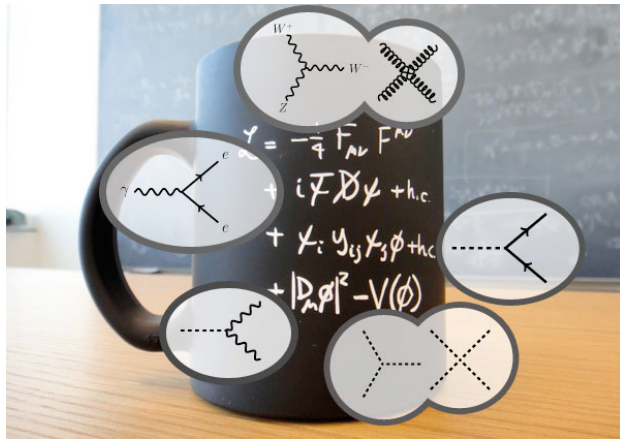**CP3/UCLouvain**
Marco Zaro
Milan

Plan

Lectures

- Matrix-element, phase-space generation
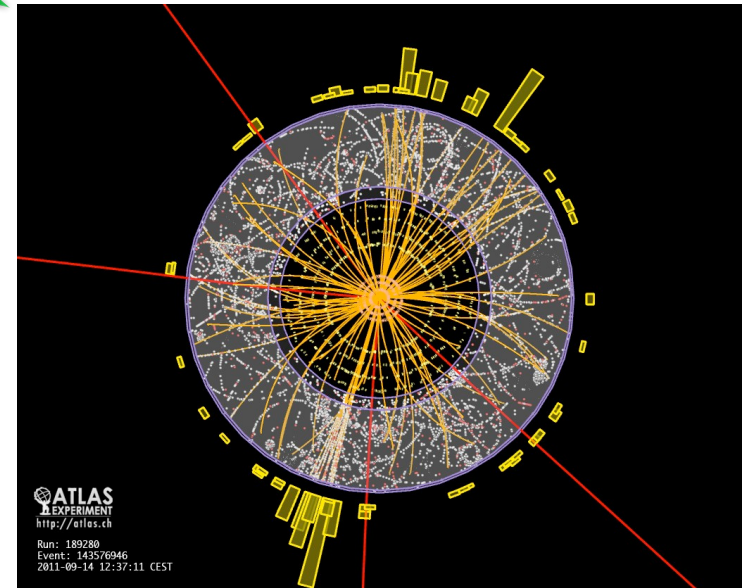- NLO and Loop
- NLO+PS and merging

Tutorial

- Basic of MG5aMC
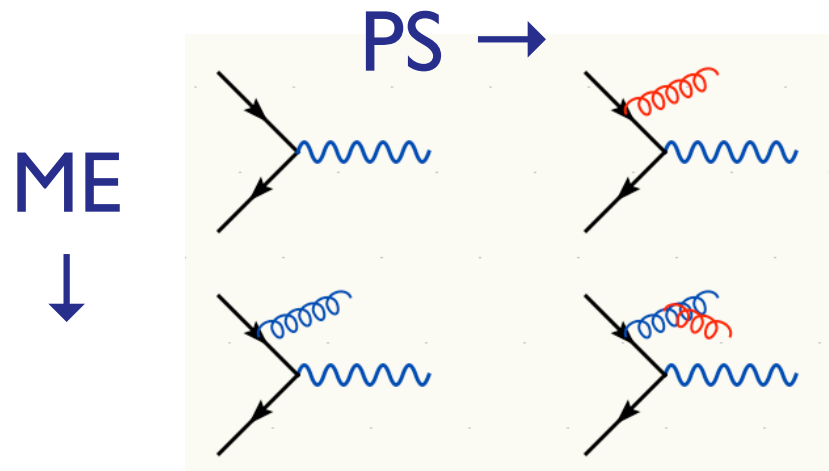- NLO and loop-induced
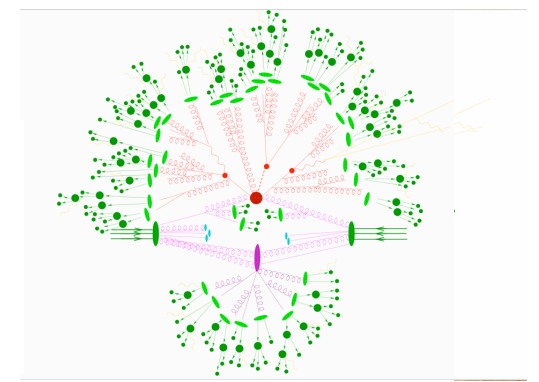- NLO+PS and MLM merging
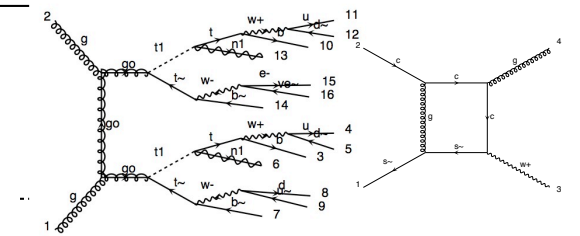
Monte-Carlo Physics

Our goal

- Cross-section
- Differential cross-section
- Un-weighted events

Type of generation

	Tree (B)SM	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	Loop Induced (B)SM
Fix Order	✔	✔	✔	✔	✔

PS →

ME ↓

Simulation of collider events

What are the MC for?
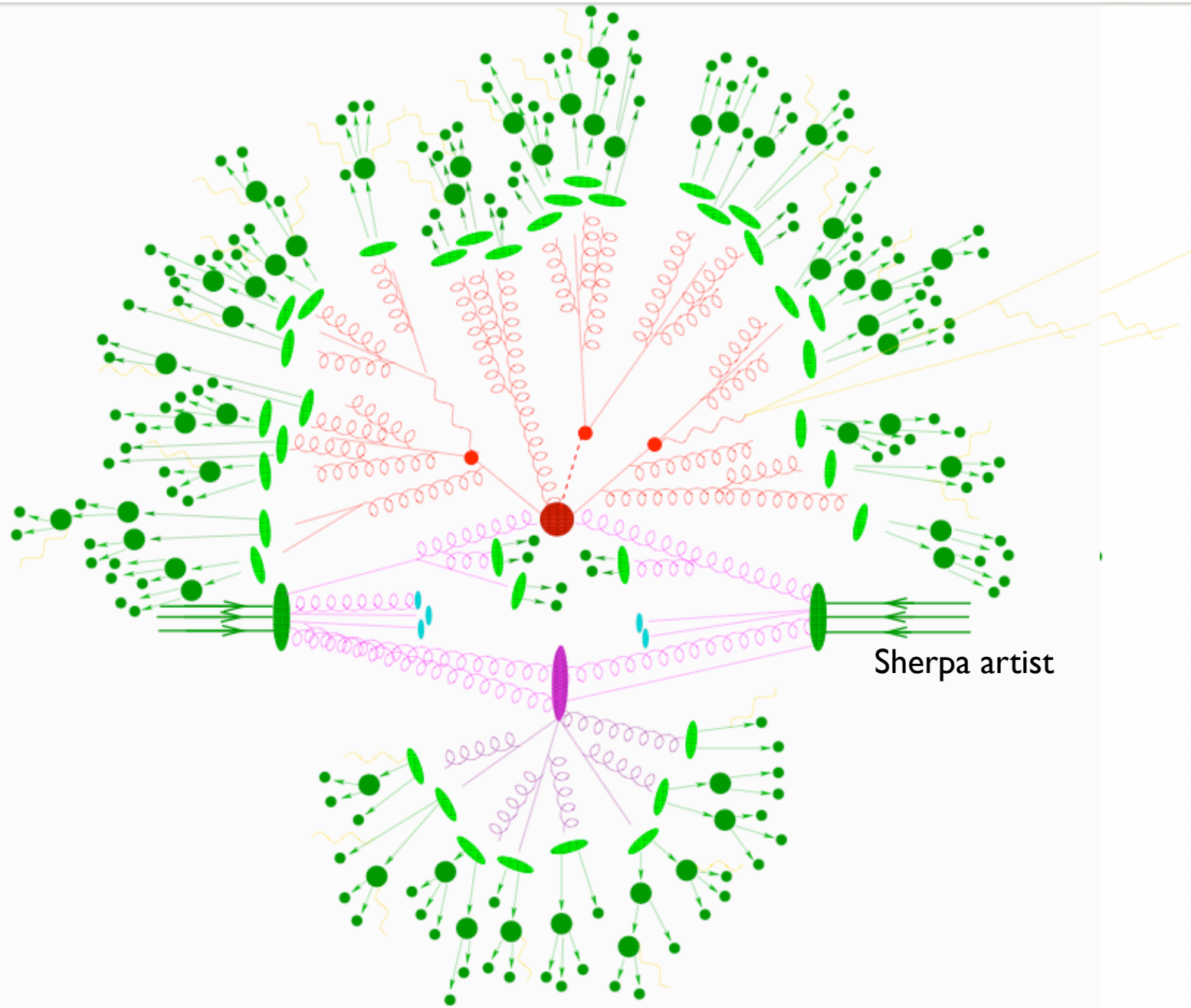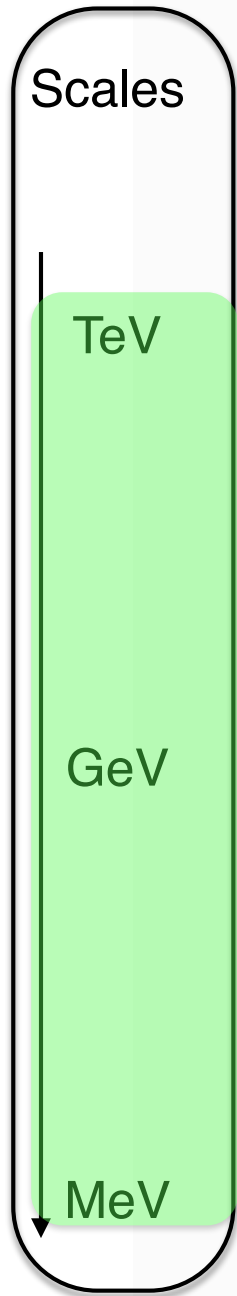
Scales

TeV

GeV

MeV

Sherpa artist

What are the MC for?

Scales

TeV

GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ where BSM physics lies

☞ process dependent

☞ first principles description

☞ it can be systematically improved

3. Hadronization

4. Underlying Event

What are the MC for?

TeV

GeV

MeV
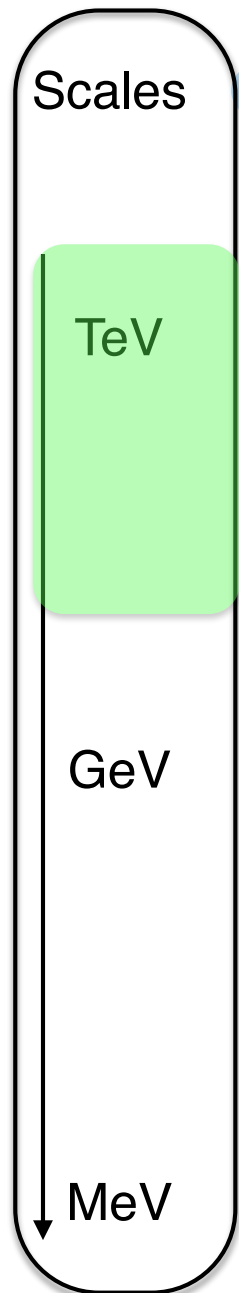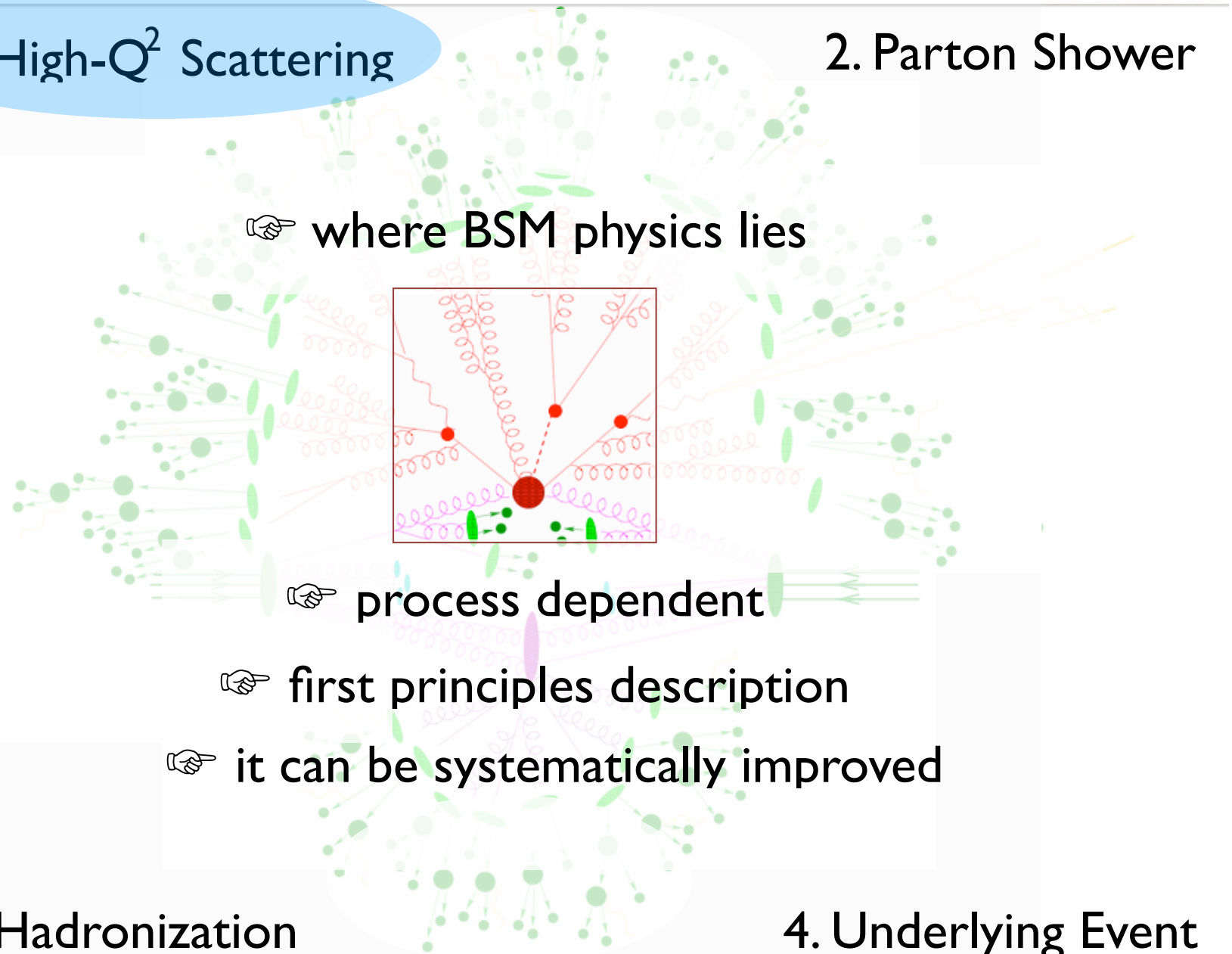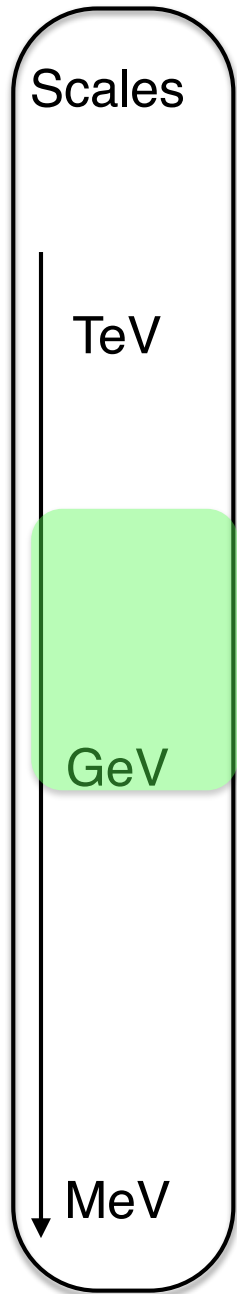
1. High-Q^2 Scattering
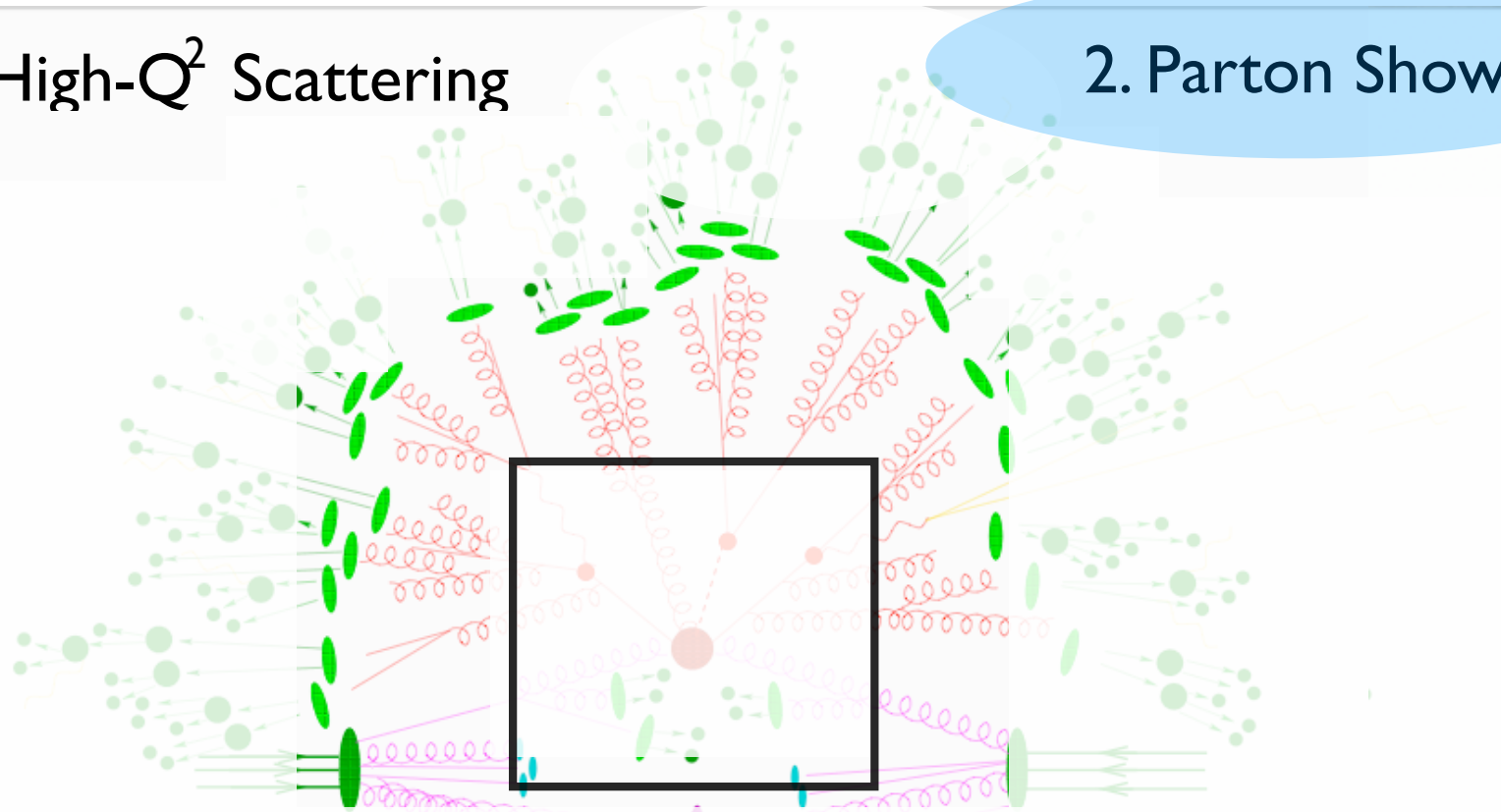
2. Parton Shower

☞ QCD -"known physics"

☞ universal/ process independent

☞ first principles description

3. Hadronization

4. Underlying Event

Parton shower

Goal

- We want to an **explicit** description of the SOFT radiation that are **ALREADY** included **implicitly in the LO events** (via the scale)

Important

- Parton-Shower is **not ADDING radiation**

  - Such radiations are already included within the event-generator

- This effect should be **unitary:** the inclusive cross section shouldn't change when extra radiation is added

What are the MC for?
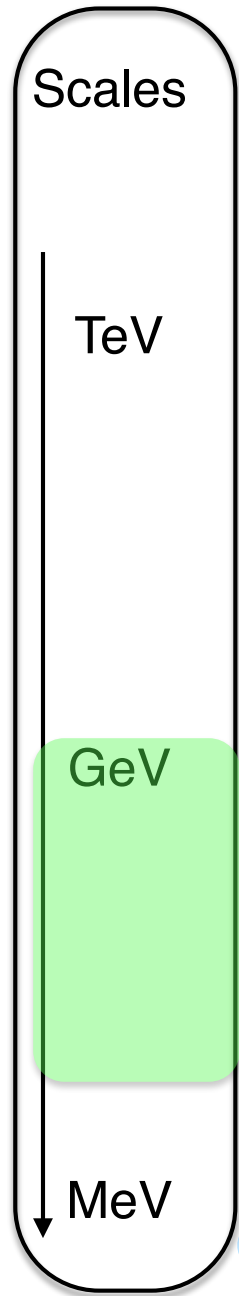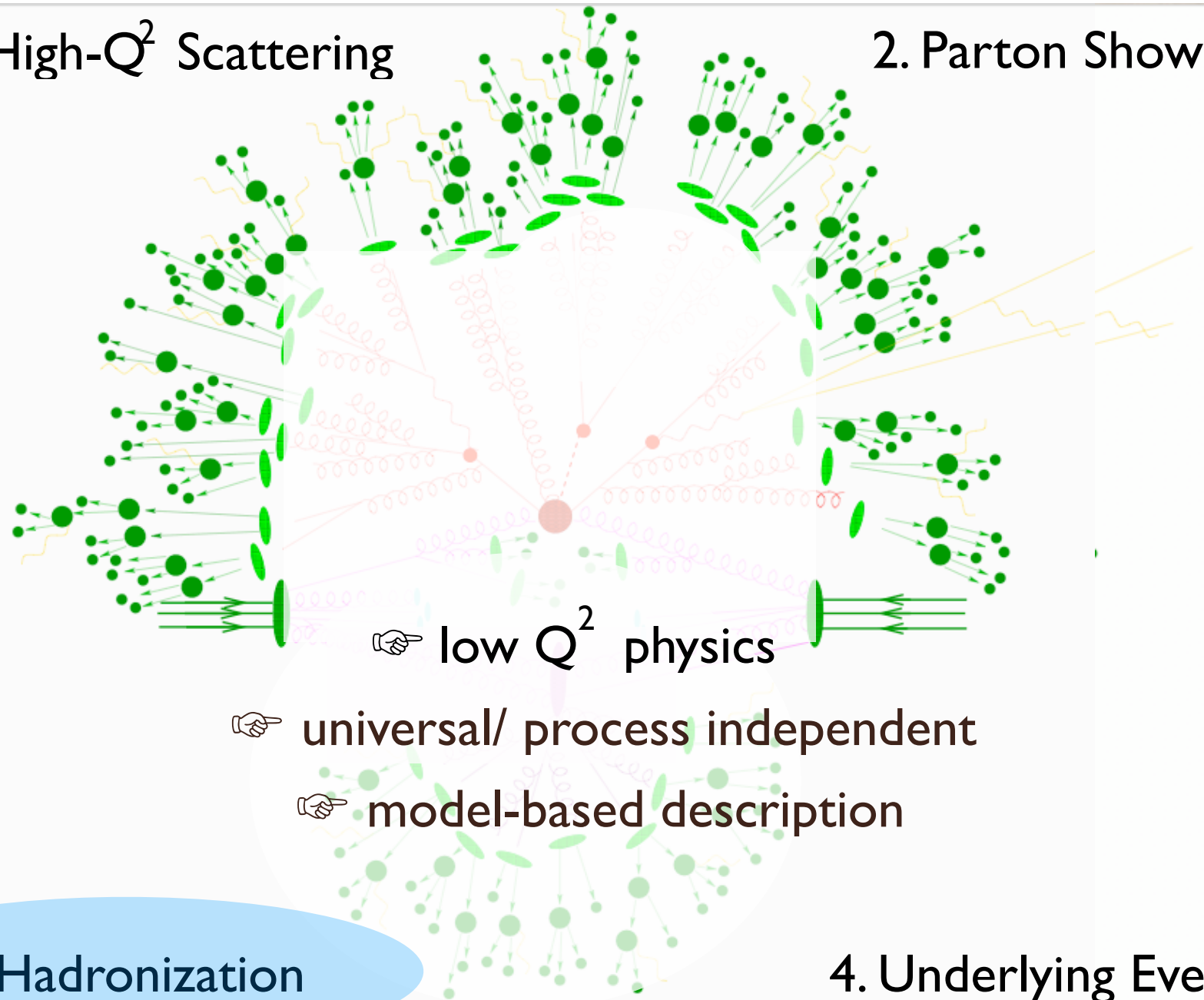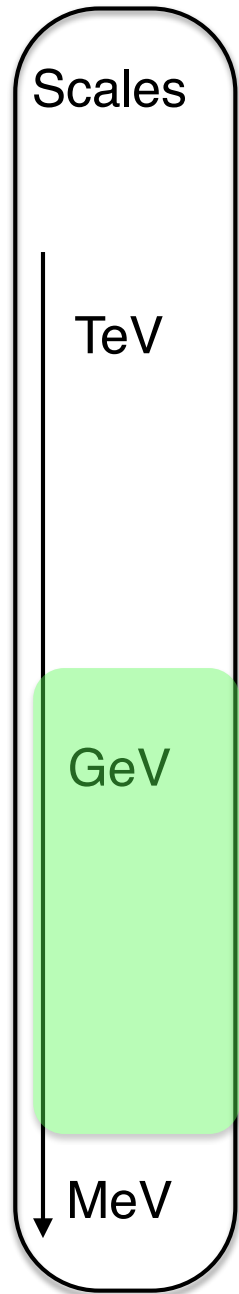
Scales

TeV

GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ low Q^2 physics

☞ universal/ process independent

☞ model-based description

3. Hadronization

4. Underlying Event

What are the MC for?

Scales

TeV

GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ low Q^2 physics

☞ energy and process dependent

☞ model-based description

3. Hadronization

4. Underlying Event

What are the MC for?

Scales

TeV

GeV

MeV

5. Detector simulation
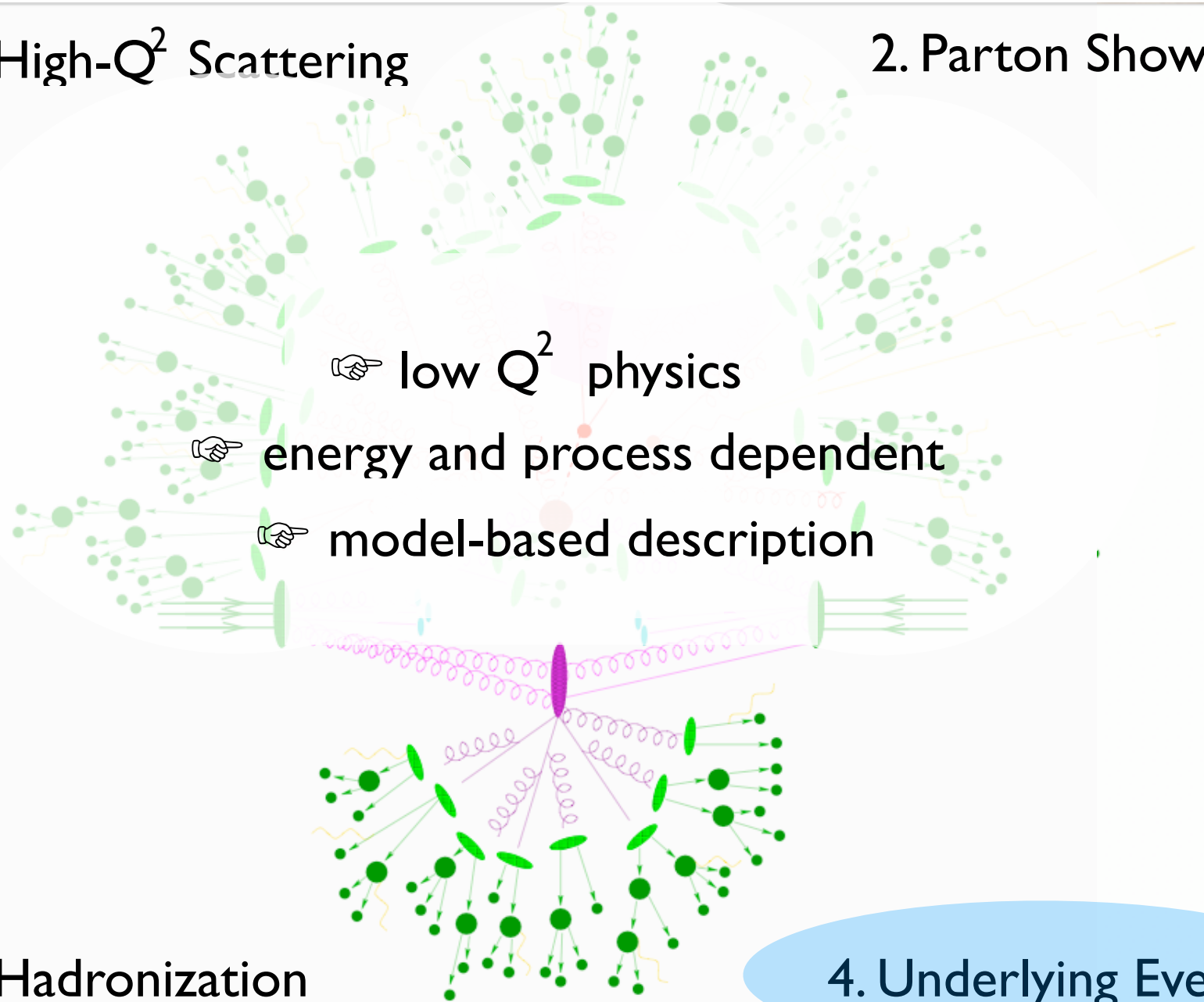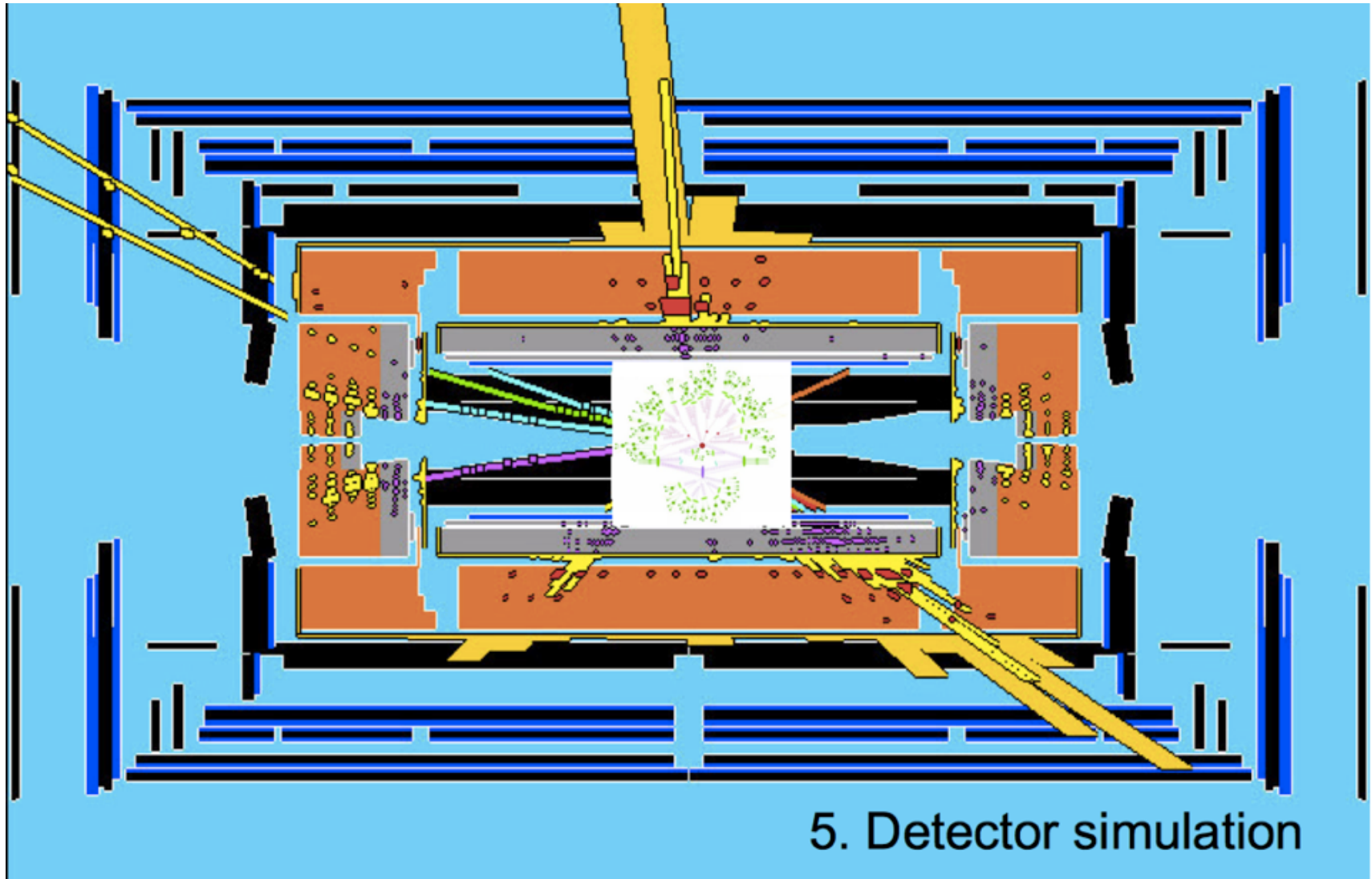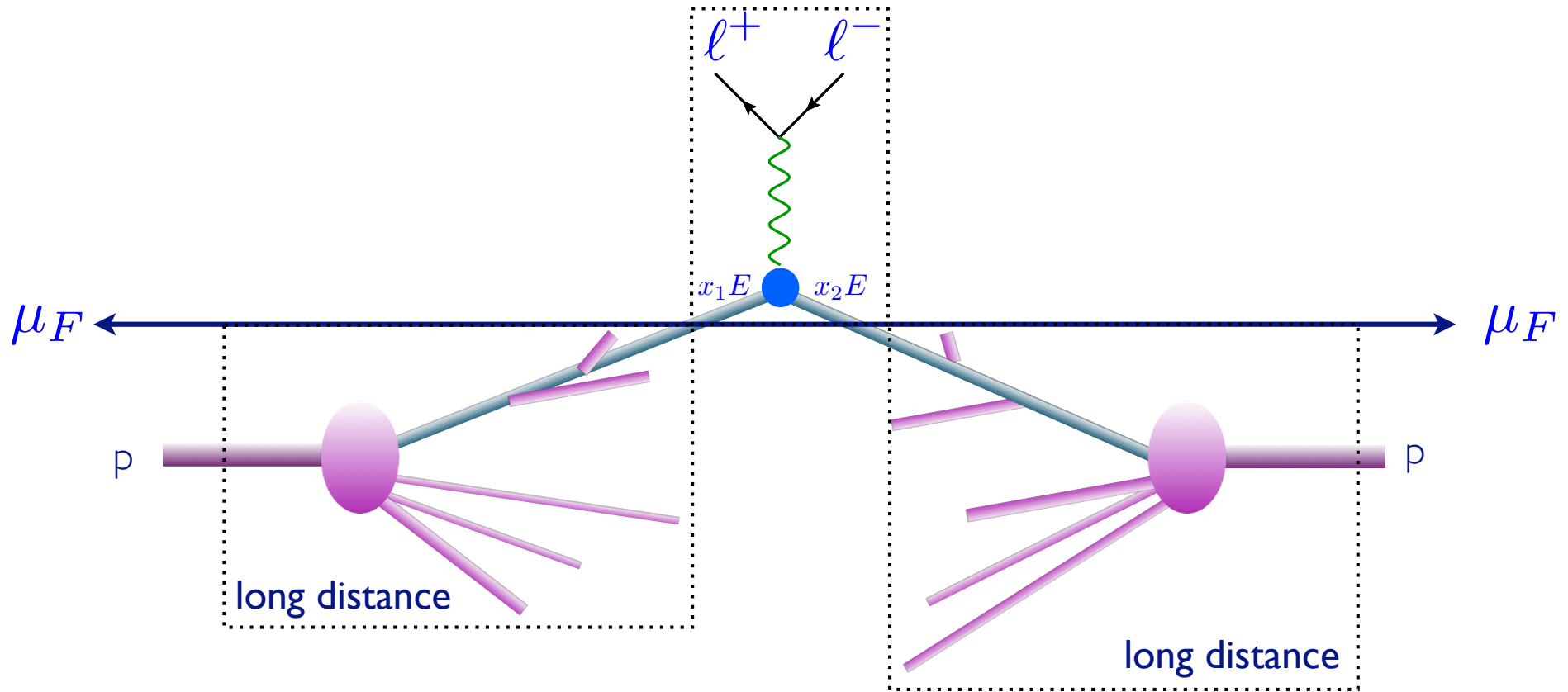
To Remember

- Multi-scale problem

  ➡ New physics visible only at High scale

  ➡ Problem split in different scale

- Computation used renormalised object

  ➡ Soft radiation below scale are hidden in the pdf/strong coupling

- Hard process gives the cross-section

- Parton-Shower does not ADD radiation (unitary process)

$$\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

Parton-level cross section

Perturbative expansion

$d\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

| LO predictions | NLO corrections | NNLO corrections | N3LO or NNNLO corrections |

- Including higher corrections improves predictions and reduces theoretical uncertainties

Improved predictions

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \ f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, d\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \dots\right)$$

- Leading Order predictions can depend strongly on the renormalization and factorization scales

- Including higher order corrections reduces the dependence on these scales

Top production vs μ, $\sqrt{S}$=14TeV

LO, cteq6l1, $\alpha_s(M_z)$=0.130

NLO, cteq6_m, $\alpha_s(M_z)$=0.118

μ[GeV]
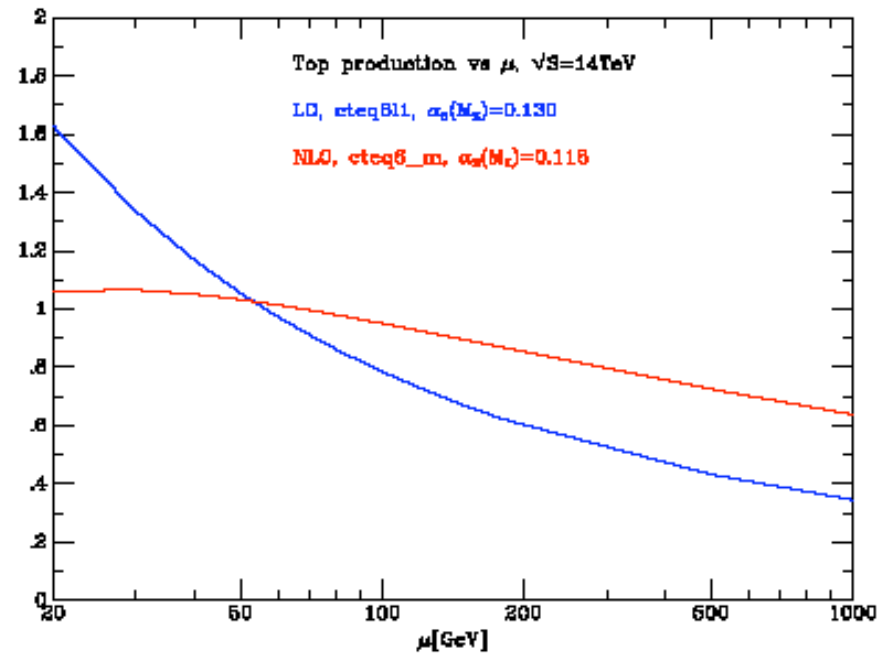
LO

LO computation (top quark pair)

normalized to one

Same variable
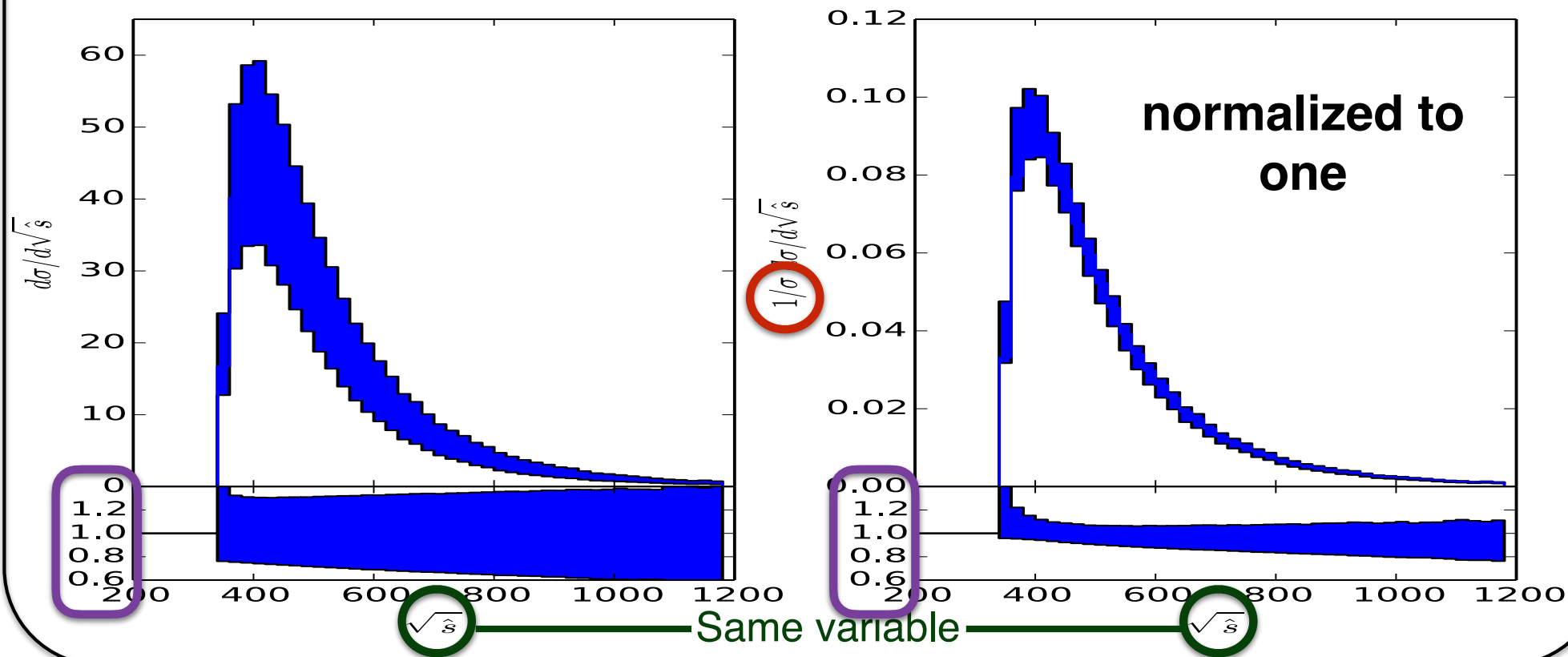
At LO:
- Large scale uncertainty
- but mainly in the Normalisation (in the cross-section)
- LO is good for shape

Hadron Colliders

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\text{FS}} \, f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

proton - (anti)proton cross sections
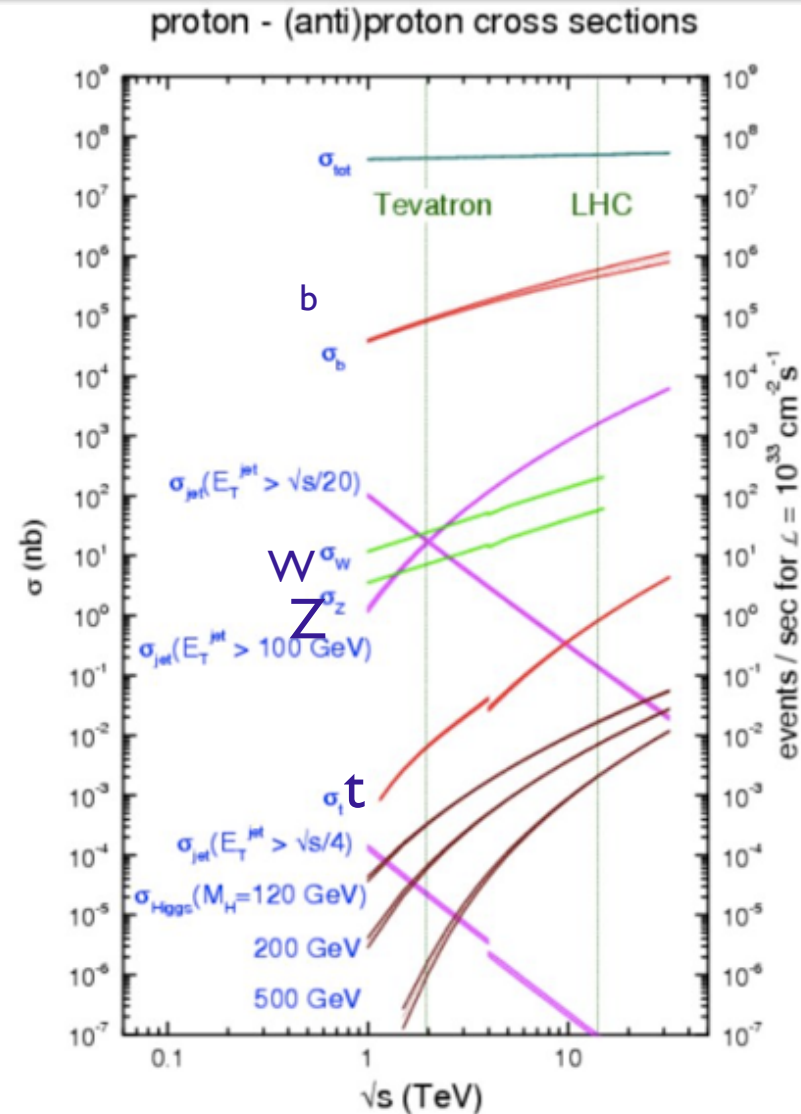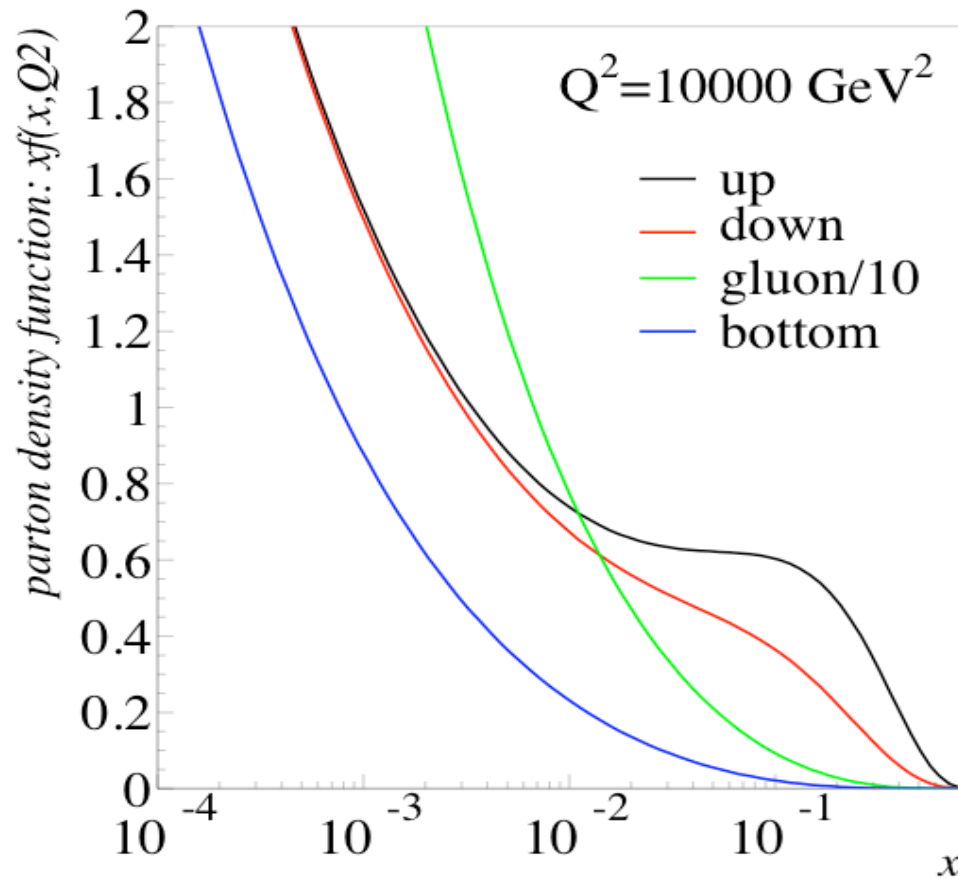
Parton densities

At small x (small ŝ), gluon domination.
At large x valence quarks

LHC formidable at large mass –
For low mass, Tevatron backgrounds smaller

Back to the processes

ratios of parton luminosities at 7 TeV LHC and Tevatron

luminosity ratio

gg

$\Sigma q\bar{q}$

MSTW2008NLO

M_X (GeV)

proton - (anti)proton cross sections

Tevatron   LHC

σ_{tot}

σ_b

$\sigma_{jet}(E_T^{jet} > \sqrt{s}/20)$

σ_W

σ_Z

$\sigma_{jet}(E_T^{jet} > 100\ GeV)$

σ_t

$\sigma_{jet}(E_T^{jet} > \sqrt{s}/4)$

$\sigma_{Higgs}(M_H=120\ GeV)$

200 GeV

500 GeV

σ (nb)

events / sec for $\mathcal{L} = 10^{33}$ cm^{-2} s^{-1}

$\sqrt{s}$ (TeV)
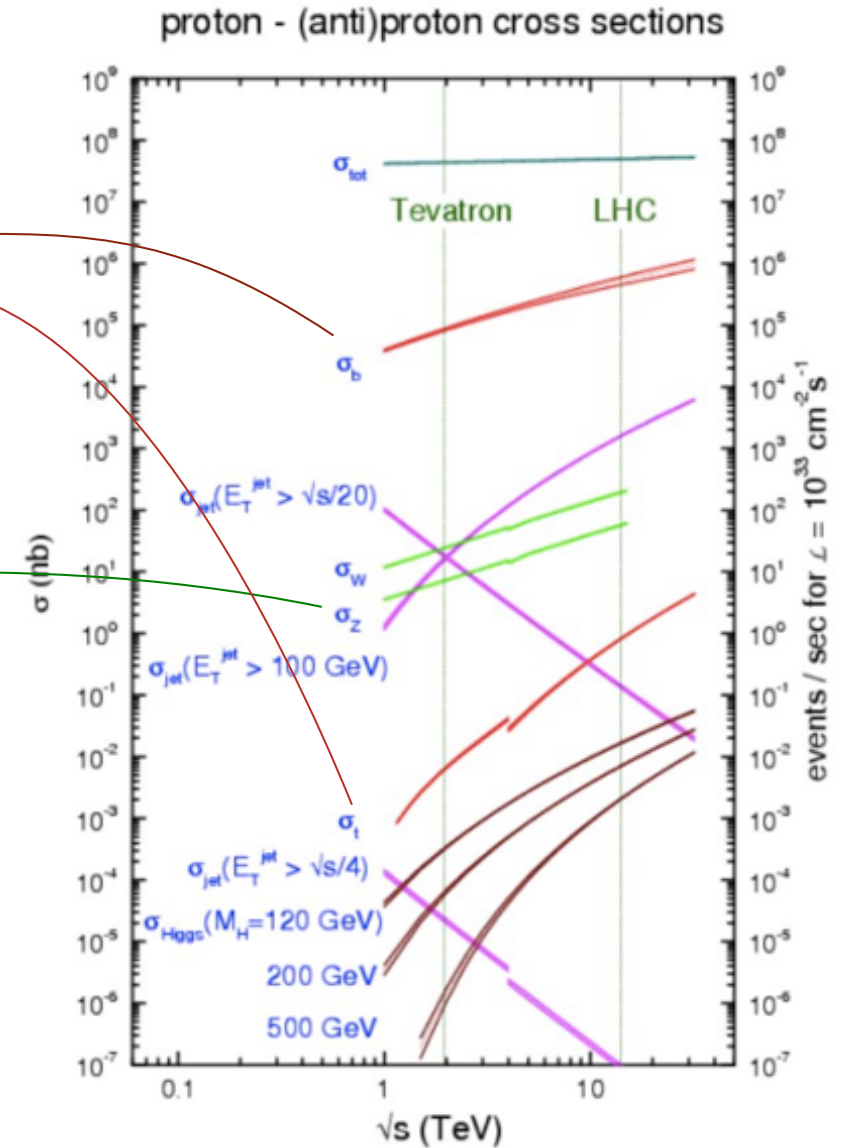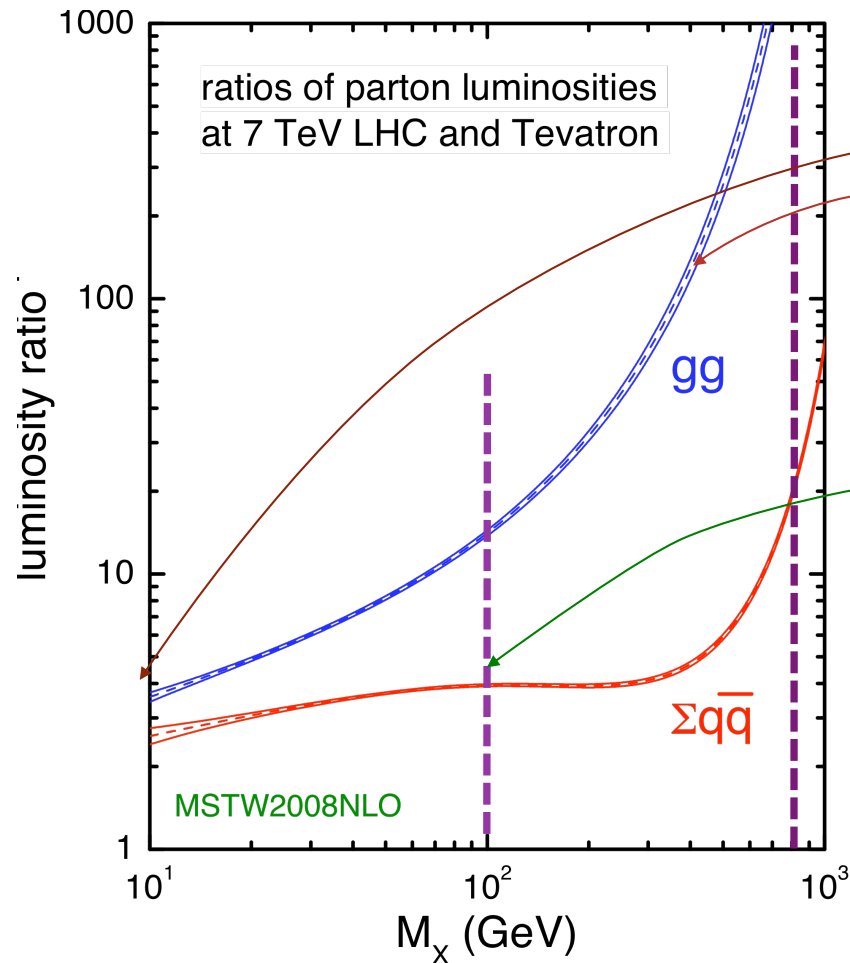
To Remember

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}} \, f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$
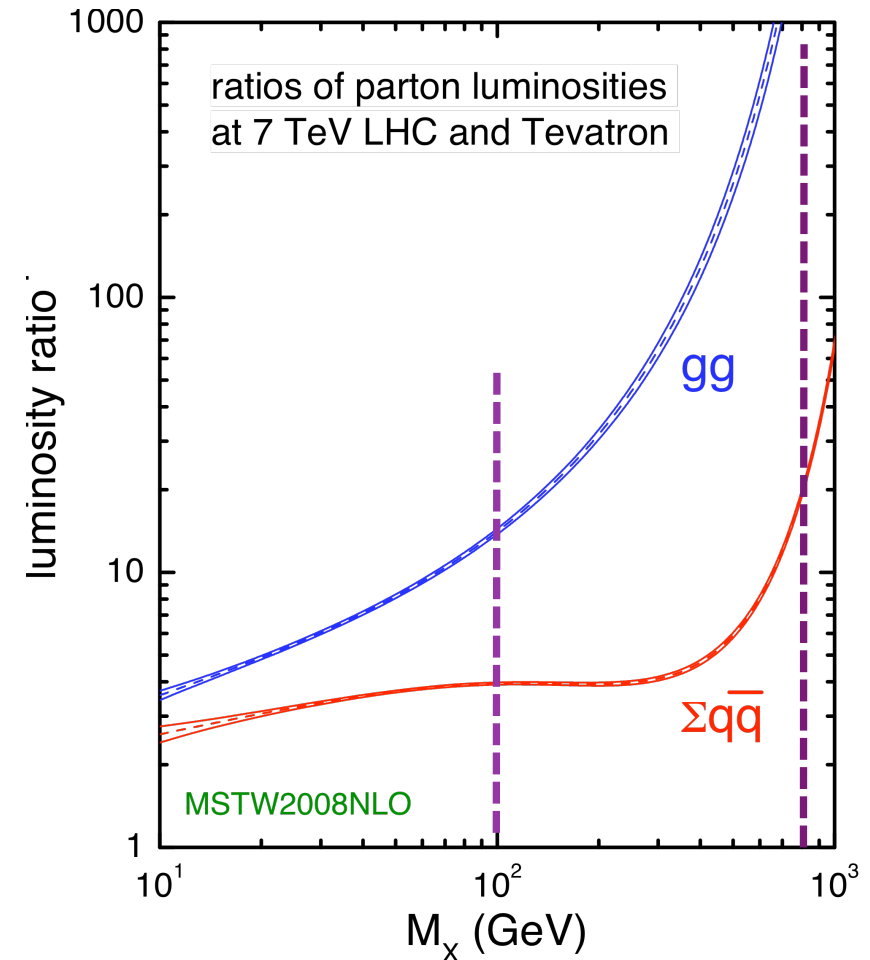
Phase-space integral   Parton density functions   Parton-level cross section

- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- NLO/NNLO: Reduce scale uncertainty linked to your division of your multi-scale problem
- Computation are inclusive (+ any jet) due to renormalization/factorization scale

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism

diagram 1   QCD=2, QED=0   diagram 2   QCD=2, QED=0
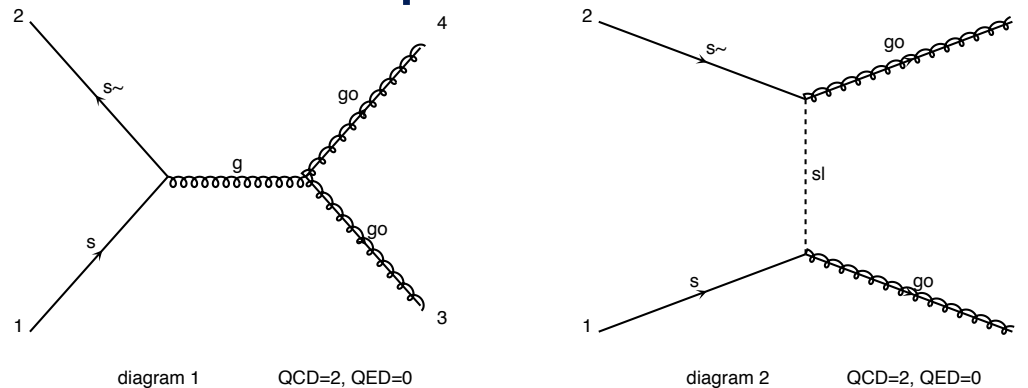
Easy enough

- Evaluate the matrix-element

$$|\mathcal{M}|^2$$ ➡ Need Feynman Rules!

Hard

Next

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Very Hard (in general)

After

Matrix Element
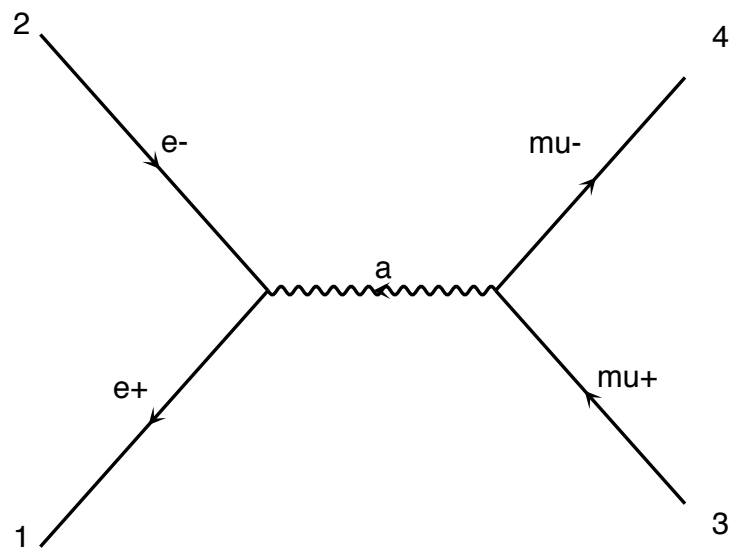
$$\mathcal{M} = e^2 (\bar{u} \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v} \gamma^\nu u)$$

$$\frac{1}{4} \sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} \mathcal{M}^* \mathcal{M}$$

$$\sum_{pol} \bar{u} u = \slashed{p} + m$$

➡ $$\frac{e^4}{4q^4} Tr[\slashed{p}_1 \gamma^\mu \slashed{p}_2 \gamma^\nu] Tr[\slashed{p}_3 \gamma_\mu \slashed{p}_4 \gamma_\nu]$$

➡ $$\frac{8e^4}{q^4} [(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Very Efficient !!!
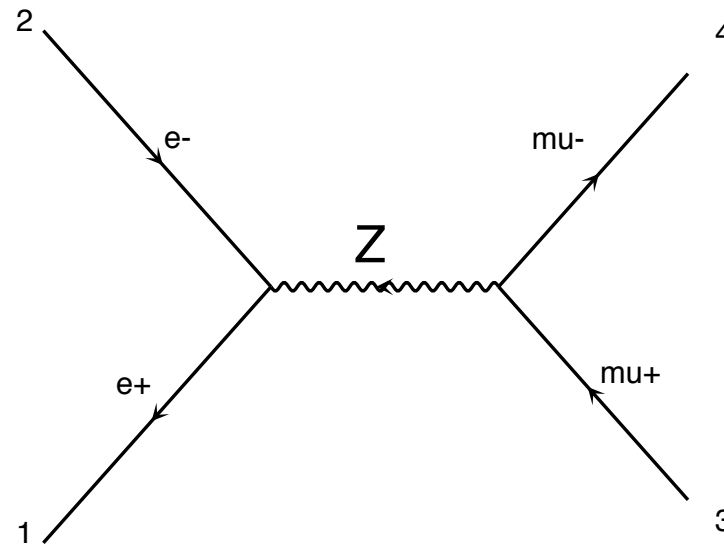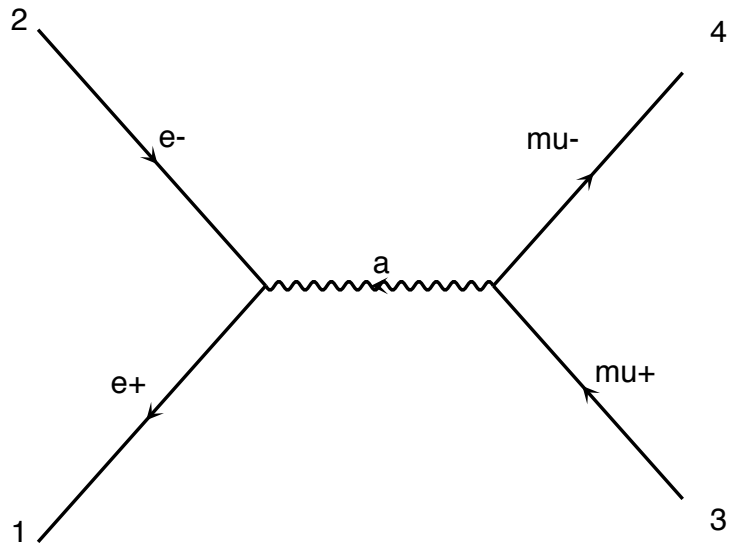
Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$

So for M Feynman diagram we need to compute M^2 different term

The number of diagram scales factorially with the number of particle

In practise possible up to 2>4

Helicity Amplitude

Idea • Evaluate $\mathcal{M}$ for fixed helicity of external particles

➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|^{\wedge}2$

➡ Loop on Helicity and average the results

Lines present in the code.

$$\mathcal{M} = ((\bar{u}e\gamma^\mu v)\frac{g_{\mu\nu}}{q^2})(\bar{v}e\gamma^\nu u))$$

Numbers for given helicity and momenta

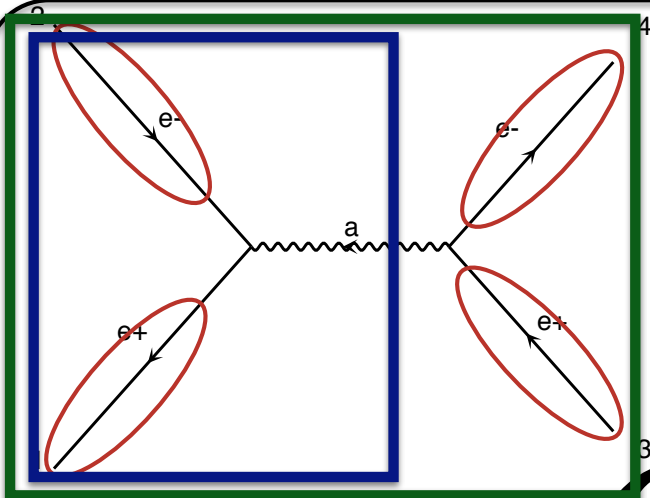*Calculate propagator wavefunctions*

Finally evaluate amplitude (c-number)

$\bar{v}_1 = fct(\vec{p}_1, m_1)$

$u_2 = fct(\vec{p}_2, m_2)$

$v_3 = fct(\vec{p}_3, m_3)$

$\bar{u}_4 = fct(\vec{p}_4, m_4)$

$W_a = fct(\bar{v}_1, u_2, e, m_a, \Gamma_a) = e\bar{v}_1\gamma^\mu u_2 \dfrac{g_{\mu\nu}}{q^2 - m_a^2 + im_a\Gamma_a}$

$\mathcal{M} = fct(\bar{v}_3, u_4, W_\nu^a, e) = e\bar{v}_3\gamma_\nu \bar{u}_4 W^a$

$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_\lambda(\vec{p}) \\ \omega_\lambda(p)\chi_\lambda(\vec{p}) \end{pmatrix}$$

$$\omega_\pm(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}}\begin{pmatrix} |\vec{p}| + p_z \\ p_x + ip_y \end{pmatrix},$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}}\begin{pmatrix} -p_x + ip_y \\ |\vec{p}| + p_z \end{pmatrix}.$$

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\, 2^N$

Real case

Identical Identical Identical Identical ▬ Known

M1

Number of routines: 10

2(N+1)

M2

Number of routines: 10

2(N+1)
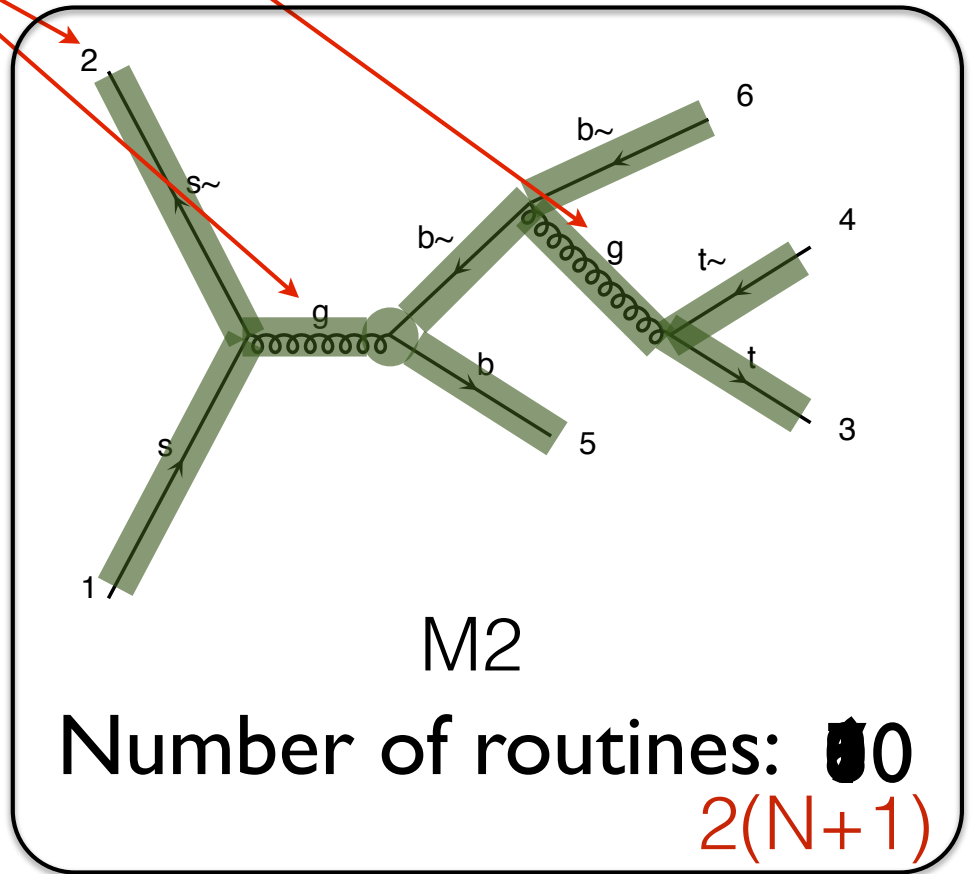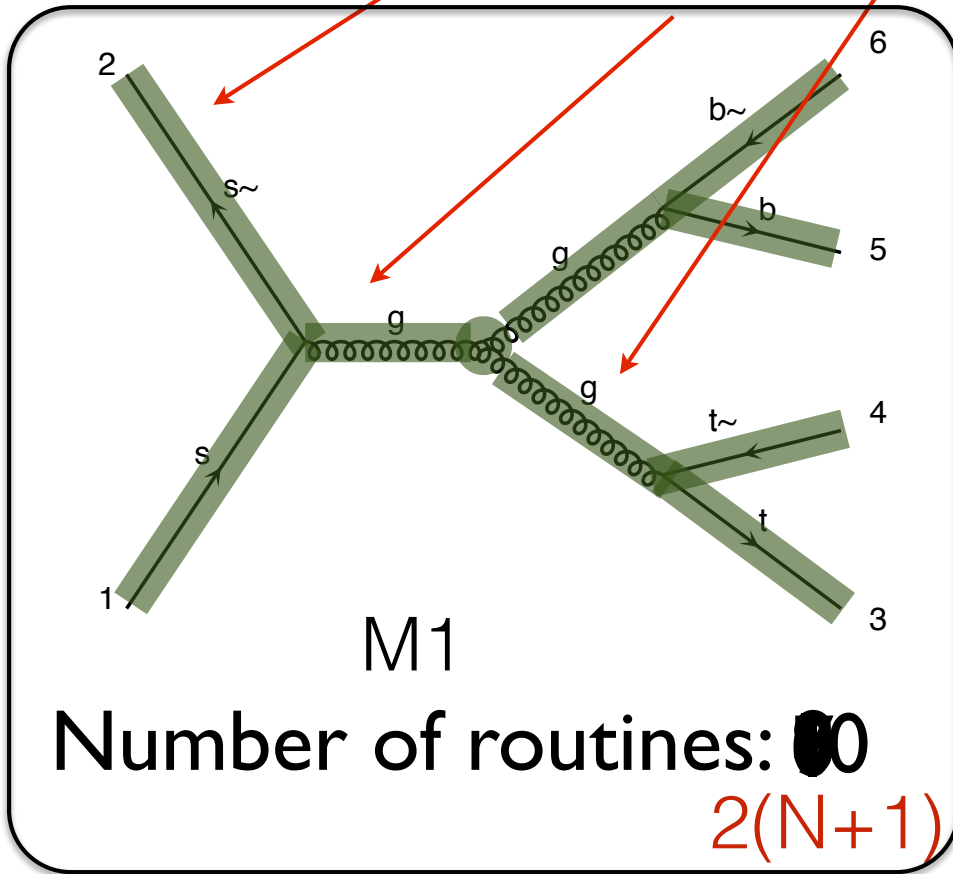
Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

Real case

Known

Number of routines: 10
2(N+1)

Number of routines: 10
2(N+1)
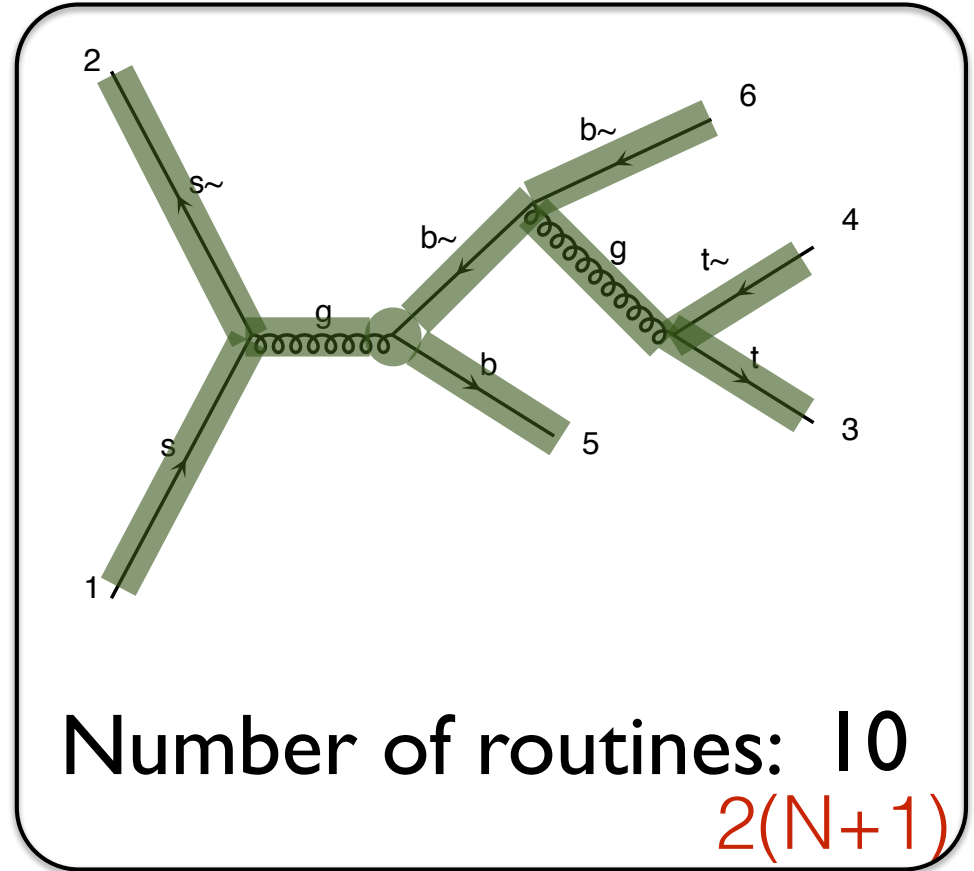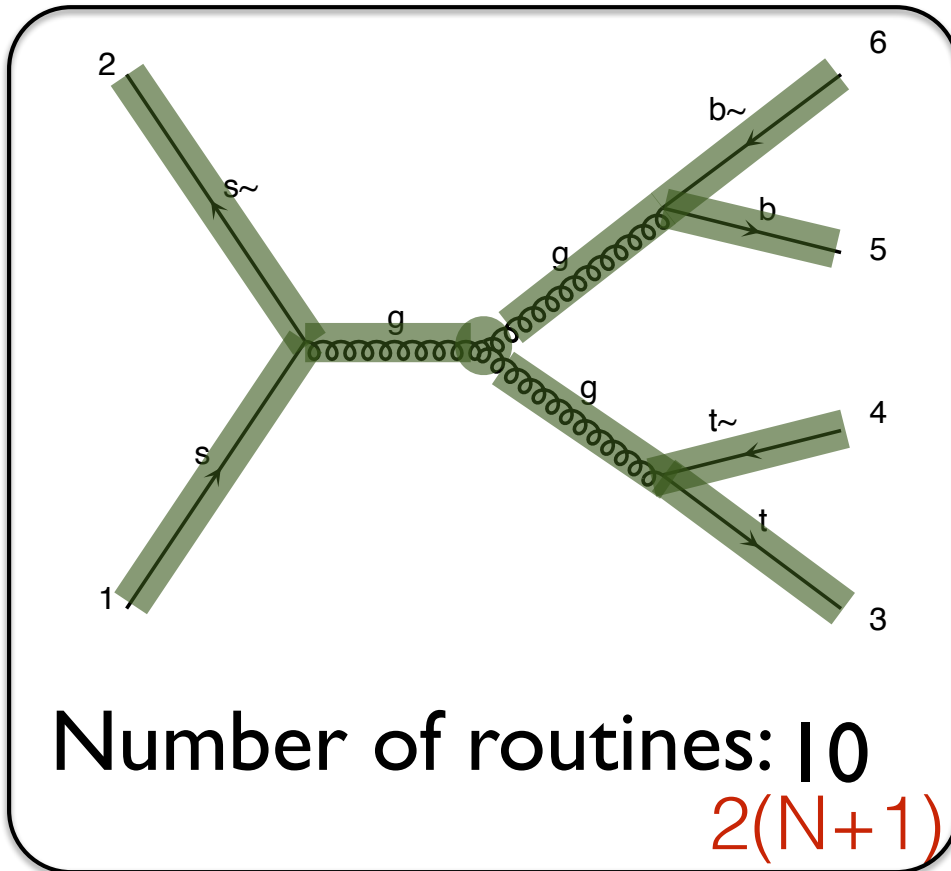
Number of routines for both: 12

N!*2(N+1) ⟶ N!

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$
Recycling	M	$(N-1)!\,2^{(N-1)}$

Can we do better? YES

- Recursion relation (used in Sherpa) [WIP]
- New in MG5aMC: Helicity Recycling 2102.00773
- 5 Dimensional helicity wave function 2203.10440
- Not full color computation [WIP]

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$
Recycling	M	$(N-1)!\,2^{(N-1)}$
Hel Recycling	M	$\approx (N-1)!\,2^{N/2}$

- 2102.00773 [hep-ph]

Can we go faster? YES

GPU

➡ Was first done a while ago (cuda)

arXiv:0908.4403,arXiv:1305.0708v2

➡ New recent focus in this direction

➡ Not only cuda:

➡ Kokkos, syCL, tensorflow

➡ Good performance but not yet integrated with the phase-space

SIMD
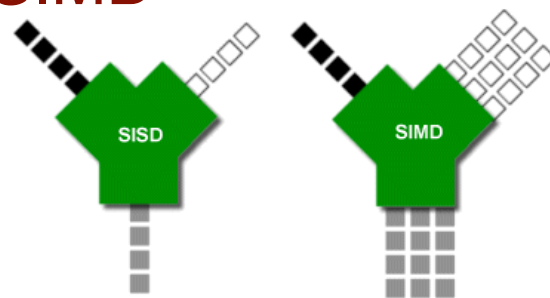
➡ Modern CPU can act as a baby GPU

➡ They can perform N identical operation as fast as one

➡ First cross-section computation with SIMD: last week

BSM and HELAS

- **Original HELicity Amplitude Subroutine library**
  [Murayama, Watanabe, Hagiwara]

- **One routine by Lorentz structure**
  - ➡ **MSSM** [cho, al] hep-ph/0601063 (2006)
  - ➡ **HEFT** [Frederix] (2007)
  - ➡ **Spin 2** [Hagiwara, al] 0805.2554 (2008)
  - ➡ **Spin 3/2** [Mawatari, al] 1101.1289 (2011)

Chiral Perturbation

SLIH

BNV Model

Effective Field Theory

NMSSM

Full HEFT

Chromo-magnetic operator

Black Holes

ALOHA

ALOHA
~~Google~~ translate

From: [UFO ▾] ⇄ To: Helicity   [Translate]

Basically, any new operator can be handle by
MG5/Pythia8 out of the box!

Type text or a website address or translate a document.
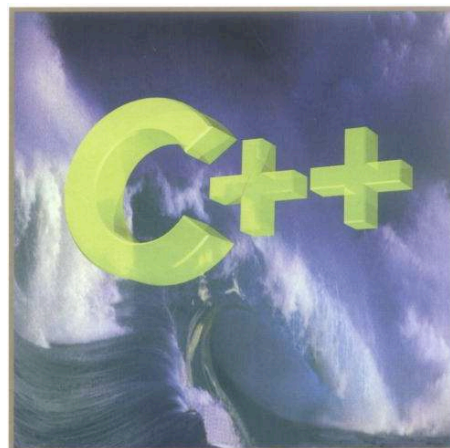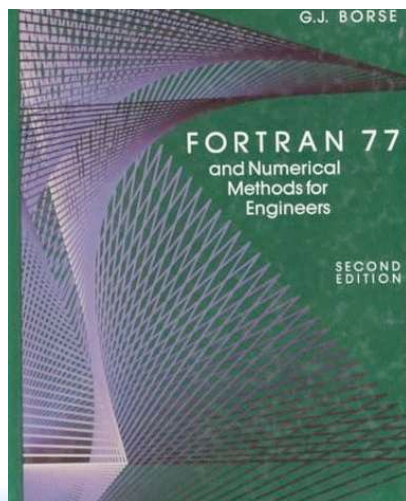
To Remember

- Numerical computation faster than analytical computation

- We are able to compute matrix-elment

  ➡ for large number of final state

  ➡ for any BSM theory

  ➡ actually also for loop (tomorrow)

Monte Carlo Integration and Generation

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism

diagram 1   QCD=2, QED=0   diagram 2   QCD=2, QED=0
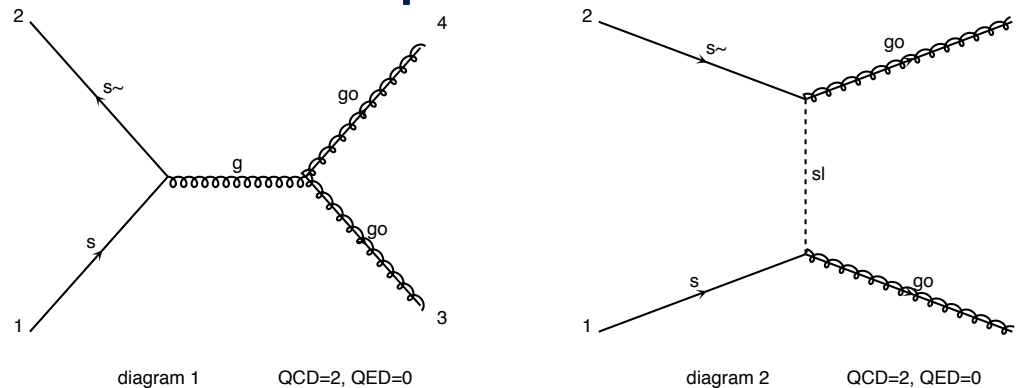
- Evaluate the matrix-element

$$|\mathcal{M}|^2$$ ➡ Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

Very Hard (in general)

Now

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

$$Dim[\Phi(n)] \sim 3n$$

General and flexible method is needed

Not only integrating but also **generates events**

Integration
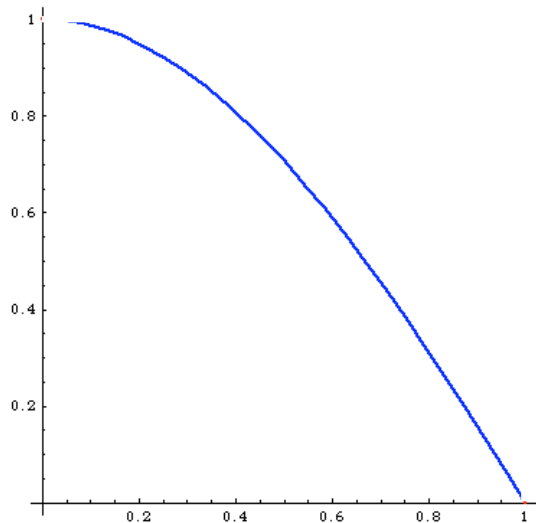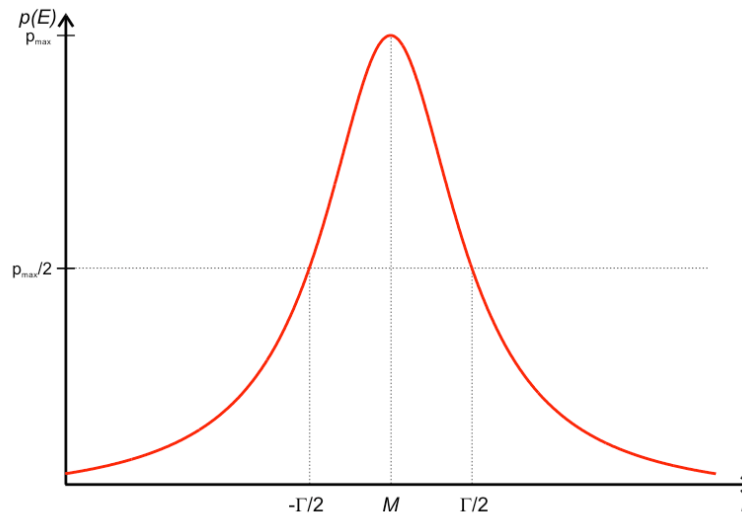
$$I = \int_0^1 dx \cos \frac{\pi}{2} x \qquad \int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2} \qquad \int dx\, C$$
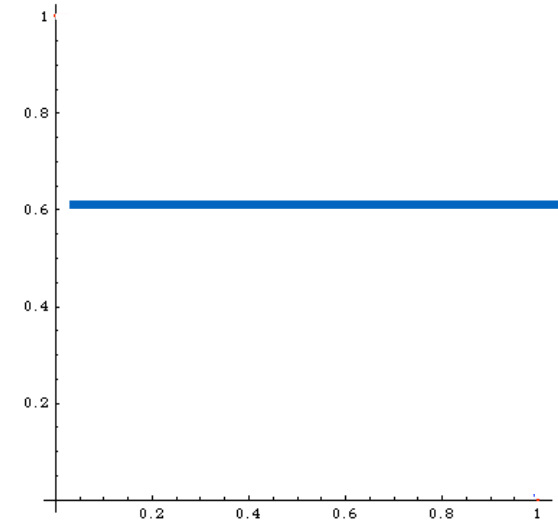
	simpson	MC
3	0,638	0,3
5	0,6367	0,8
20	0,63662	0,6
100	0,636619	0,65
1000	0,636619	0,636

Method of evaluation

- MonteCarlo $\quad 1/\sqrt{N}$
- Trapezium $\quad 1/N^2$
- Simpson $\quad 1/N^4$

Integration
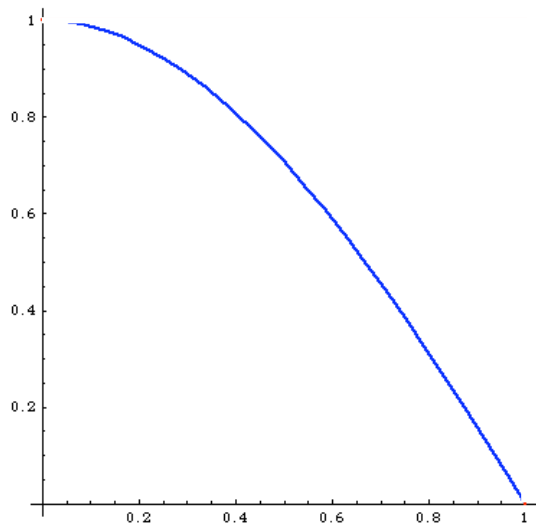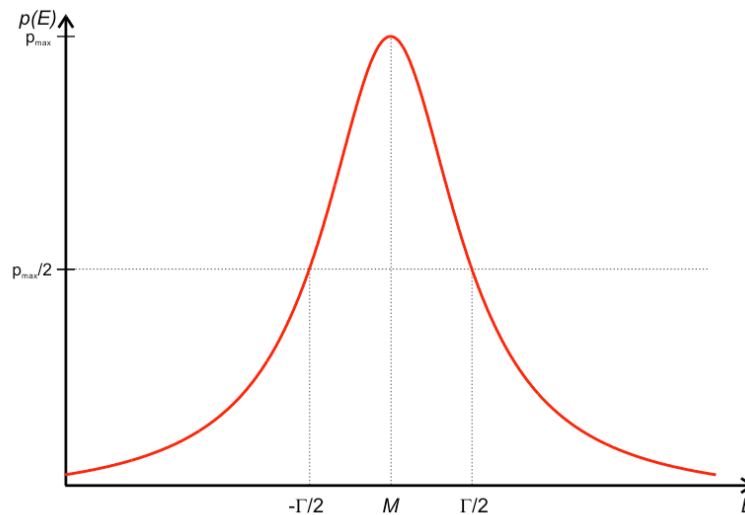
$$I = \int_0^1 dx \cos \frac{\pi}{2} x \qquad \int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2} \qquad \int dx\, C$$
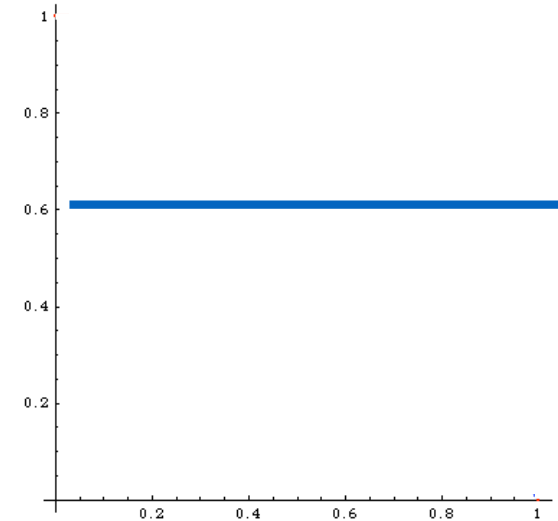
Method of evaluation

- MonteCarlo $\quad 1/\sqrt{N}$
- Trapezium $\quad 1/N^2$
- Simpson $\quad \boxed{1/N^4}$

More Dimension →

$1/\sqrt{N}$

$1/N^{2/d}$

$1/N^{4/d}$

Integration

$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$

$$V = V_N = 0$$
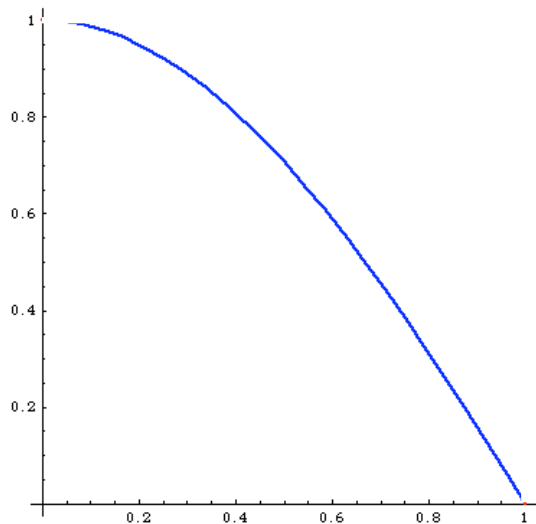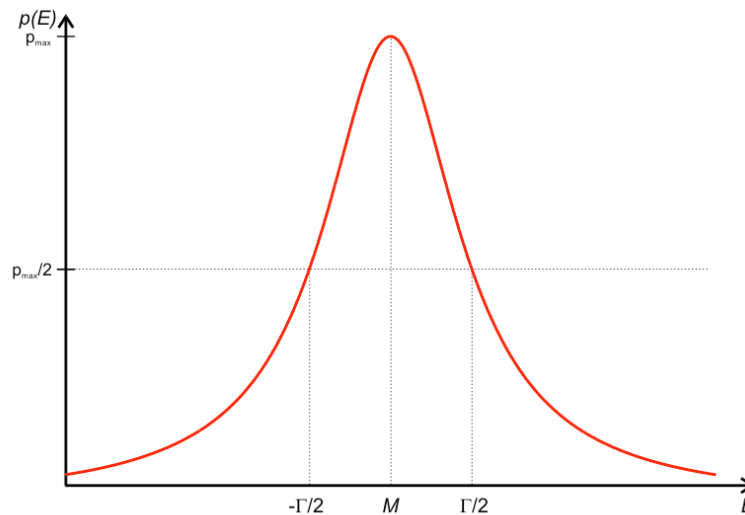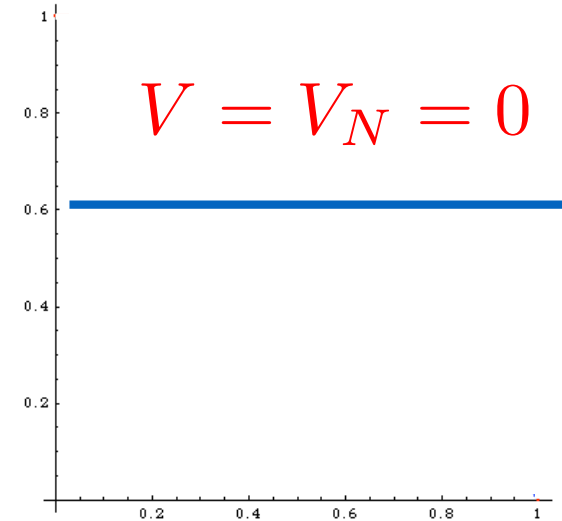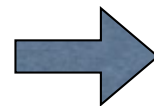
$$I = \int_{x_1}^{x_2} f(x)dx \qquad \Longrightarrow \qquad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$
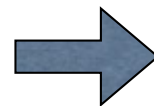
$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \qquad \Longrightarrow \qquad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

$$I = I_N \pm \sqrt{V_N/N}$$

Can be minimized!

Importance Sampling
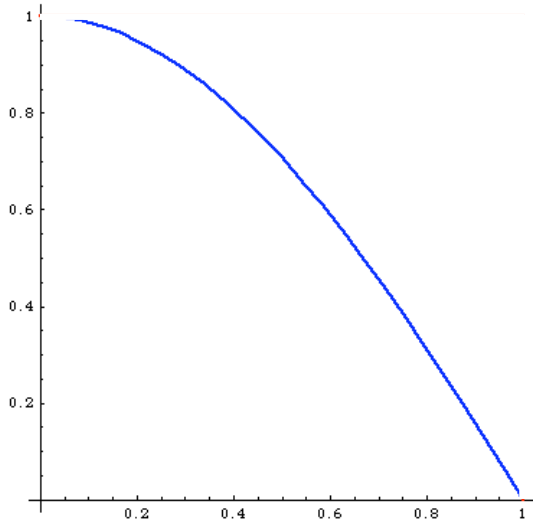
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$
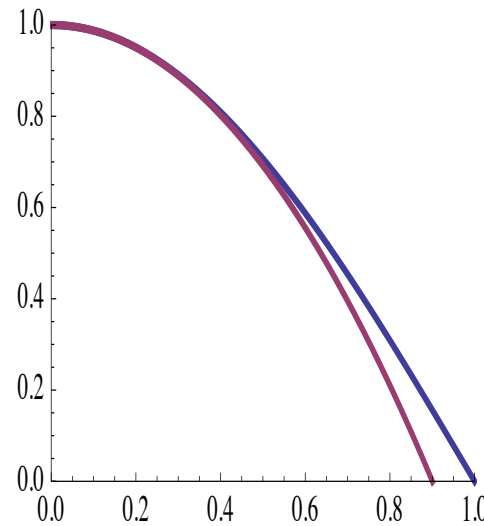
$$I = \int_0^1 dx (1 - cx^2) \frac{\cos\left(\frac{\pi}{2}x\right)}{(1 - cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2 c} \simeq 1$$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

The Phase-Space parametrization is important to have an efficient computation!

Importance Sampling

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\xi = arctan\left(\frac{q^2 - M^2}{\Gamma M}\right)$$

Why Importance Sampling?

Probability of using that point p(x)
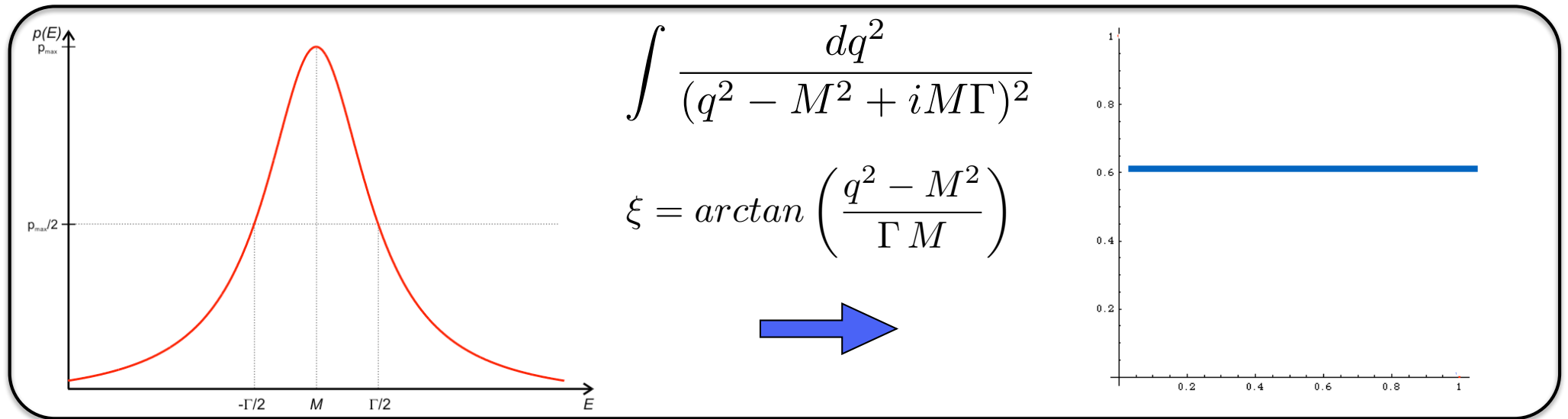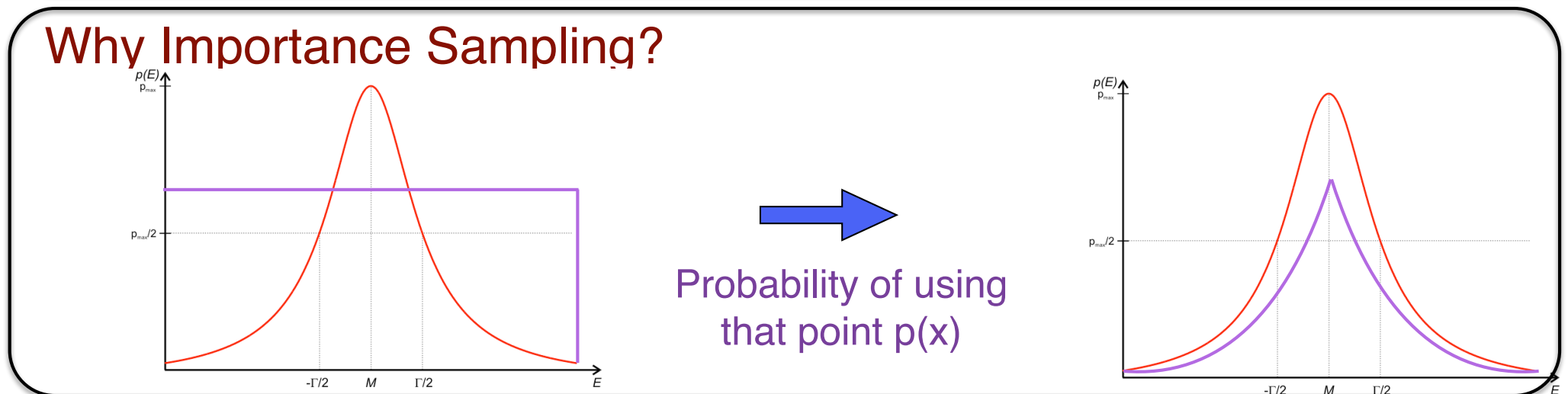
The change of variable ensure that the evaluation of the function is done where the function is the largest!

Cut Impact

- Events are generated according to our best knowledge of the function

  ➡ Basic cut include in this "best knowledge"

  ➡ Custom cut are ignored

No cut | Run card cut | Custom cut
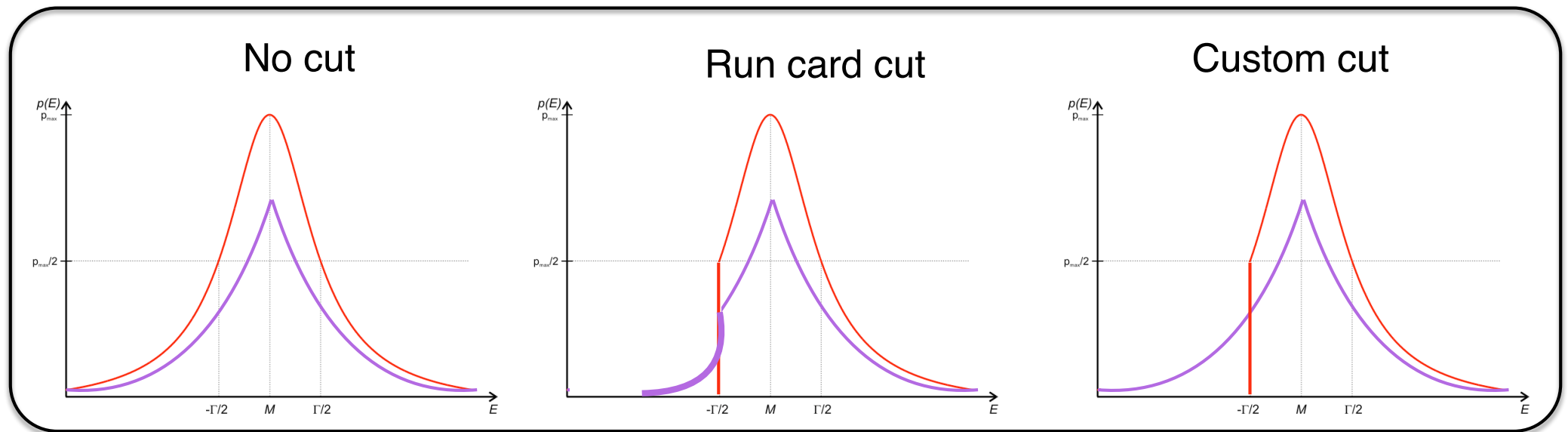
Cut Impact
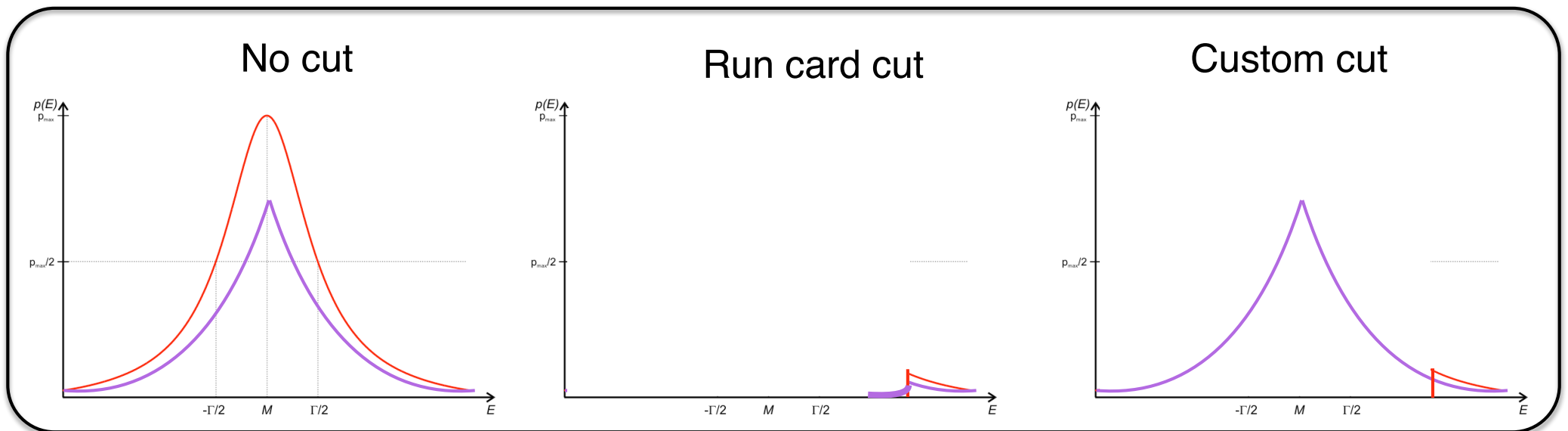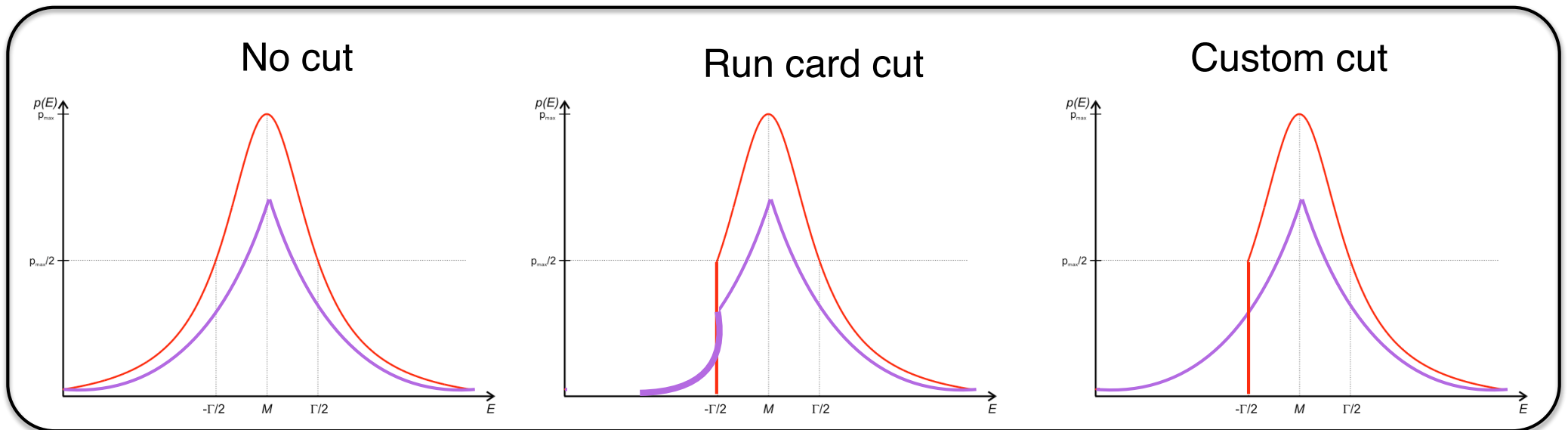
Might miss the contribution and think it is just zero.

Importance Sampling

Key Point

- Generate the random point in a distribution which is close to the function to integrate.

- This is a change of variable, such that the function is flatter in this new variable.

- Needs to know an approximate function.

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!
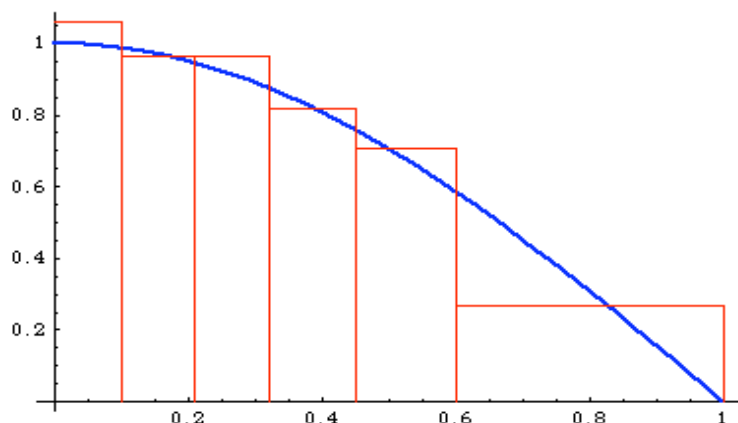
Algorithm

1. Creates bin such that each of them have the same contribution.

➡ Many bins where the function is large

2. Use the approximate for the importance sampling method.

Example: QCD 2 → 2
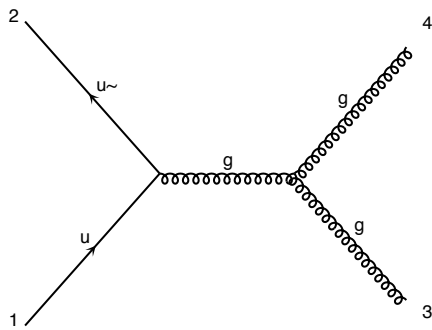
$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2}$$
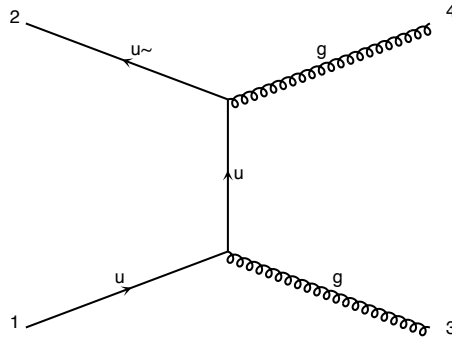
$$\propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2}$$
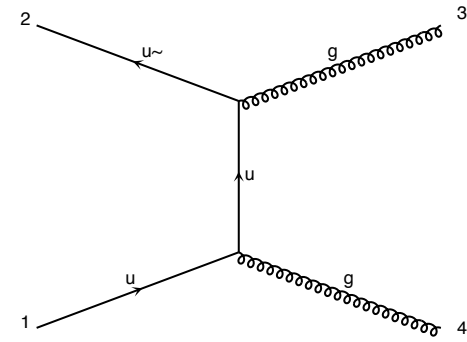
$$\propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

Multi-channel based on single diagrams*

Trick in MadEvent: Split the complexity

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

$$\approx 1$$

Key Idea

- Any single diagram is "easy" to integrate (pole structures/ suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

N Integral

- Errors add in quadrature so no extra cost
- "Weight" functions already calculated during $|M|^2$ calculation
- Parallel in nature

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

P1_qq_wpwm

s= 725.73 ± 2.07 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G2.2	377.6	1.67	142.285	7941.0	21
G3	239	1.16	220.04	10856.0	45.5
G1	109.1	0.378	70.88	3793.0	34.8

term of the above sum.

each term might not be gauge invariant

P1_gg_wpwm

s= 20.714 ± 0.332 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.2	20.71	0.332	7.01	373.0	18

To Remember

- Phase-Space integration are difficult

- We need to know the function

  ➡ Be careful with cuts

- MadGraph split the integral in different contribution linked to the Feynman Diagram

  ➡ Those are not the contribution of a given diagram

- Here speed-up can be massive

  ➡ Modern machine learning are (or will be) investigated

Event generation
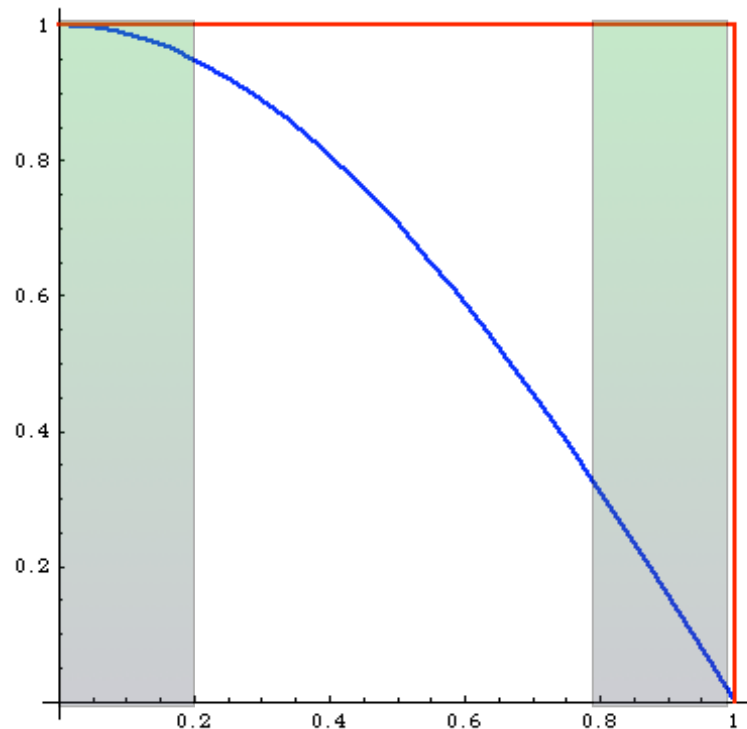
What's the difference between weighted and unweighted?

Weighted:

Same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation
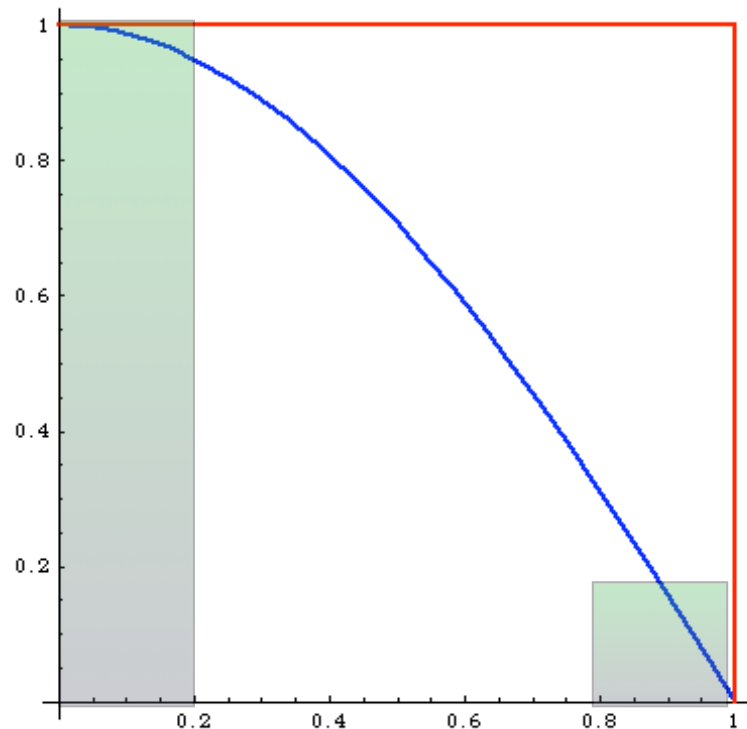
What's the difference between weighted and unweighted?

Unweighted:

events is proportional to the probability of areas of phase space:
events have all the same weight ("unweighted")

Events distributed as in nature

Event generation
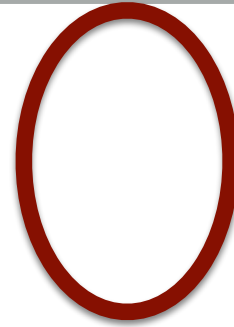
$$\int f(x)dx = \frac{1}{N} \sum_{i=1}^{N} f(x_i)$$

Number between 0 and 1 (assuming positive function)
-> re-interpret as the probability to keep the events

$$\int f(x)dx = \frac{1}{N} \sum_{i=1}^{N} f(x_i) = \frac{1}{N} \sum_{i=1}^{N} P(x_i) max(f)$$

Let's reduce the sample size by playing the lottery.
For each events throw the dice and see if we keep or reject the events

$$\int f(x)dx = \frac{1}{N} \sum_{i=1}^{N} f(x_i) = \frac{1}{N} \sum_{i=1}^{N} P(x_i) max(f) \simeq \frac{max(f)}{N} \sum_{i=1}^{n} 1$$

Event generation

$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i) = \frac{1}{N}\sum_{i=1}^{N} \frac{f(x_i)}{max(f)} max(f)$$

f(x)

MC101
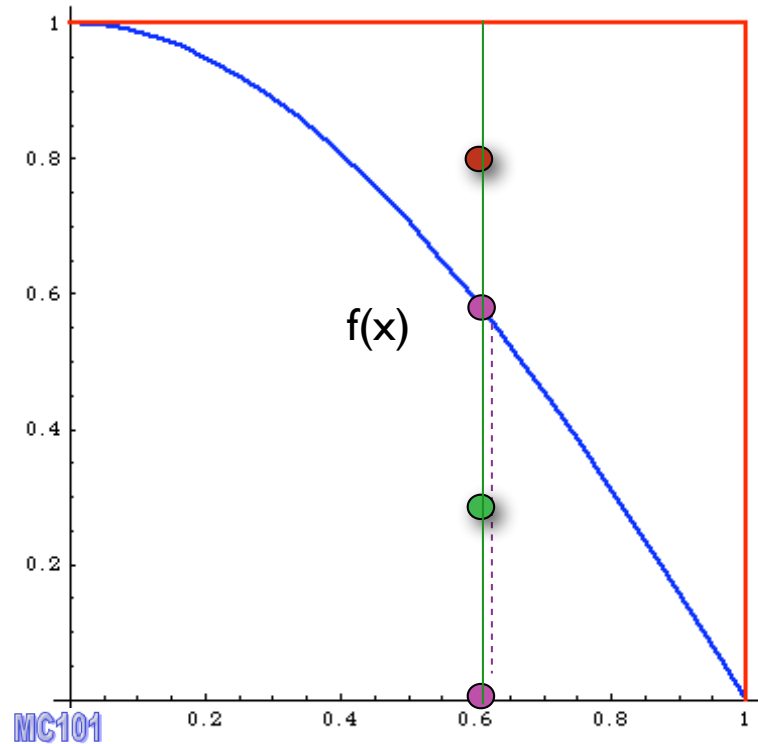
1. pick x_i

2. calculate $f(x_i)$

3. pick $y \in [0, max(f)]$

4. Compare:
   if $y < f(x_i)$ accept event,

   else reject it.

Event generation

MC integrator

Event generator

Acceptance-Rejection

$$\frac{d\sigma}{d\mathcal{O}}$$

$$\mathcal{O}$$
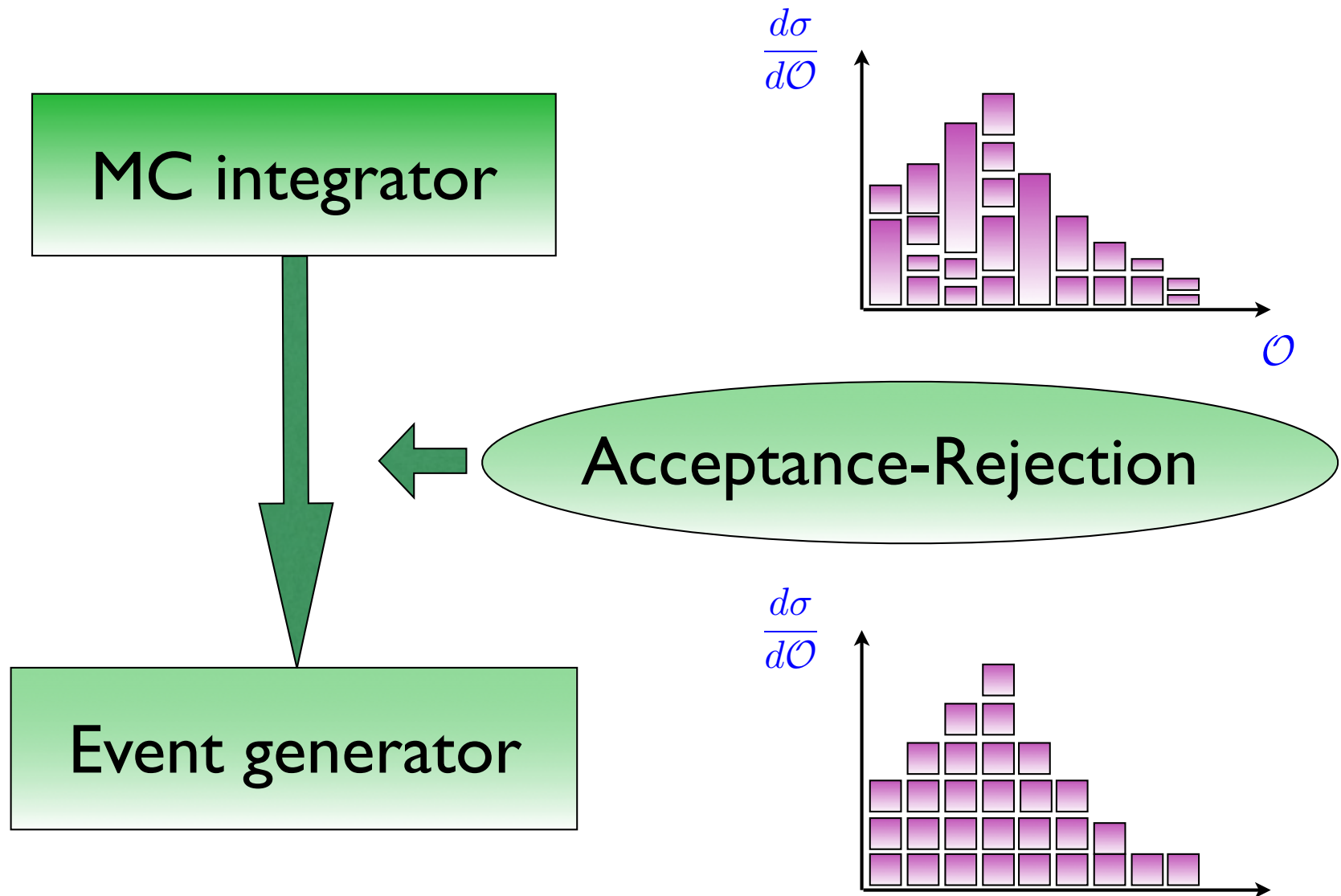
☞ **This is possible only if f(x)<∞ AND has definite sign!** $\mathcal{O}$

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension)

- Need to know the function

  - Impact on cut

Good Point

- Complex area of Integration

- Easy error estimate

- quick estimation of the integral

- Possibility to have unweighted events

What have we learned!

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\text{FS}} \, f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral   Parton density functions   Parton-level cross section

- The Importance of PDF

  ➡ Defines the physics

- Evaluation of Matrix Element

  ➡ Numerical method faster than analytical formula

  ➡ cross-section prediction needs NLO

- Phase Space Integration

  ➡ Need to know in advance what we integrate. Be careful with strong cuts!

Parton Shower

Olivier Mattelaer
CP3/UCLouvain

What are the MC for?
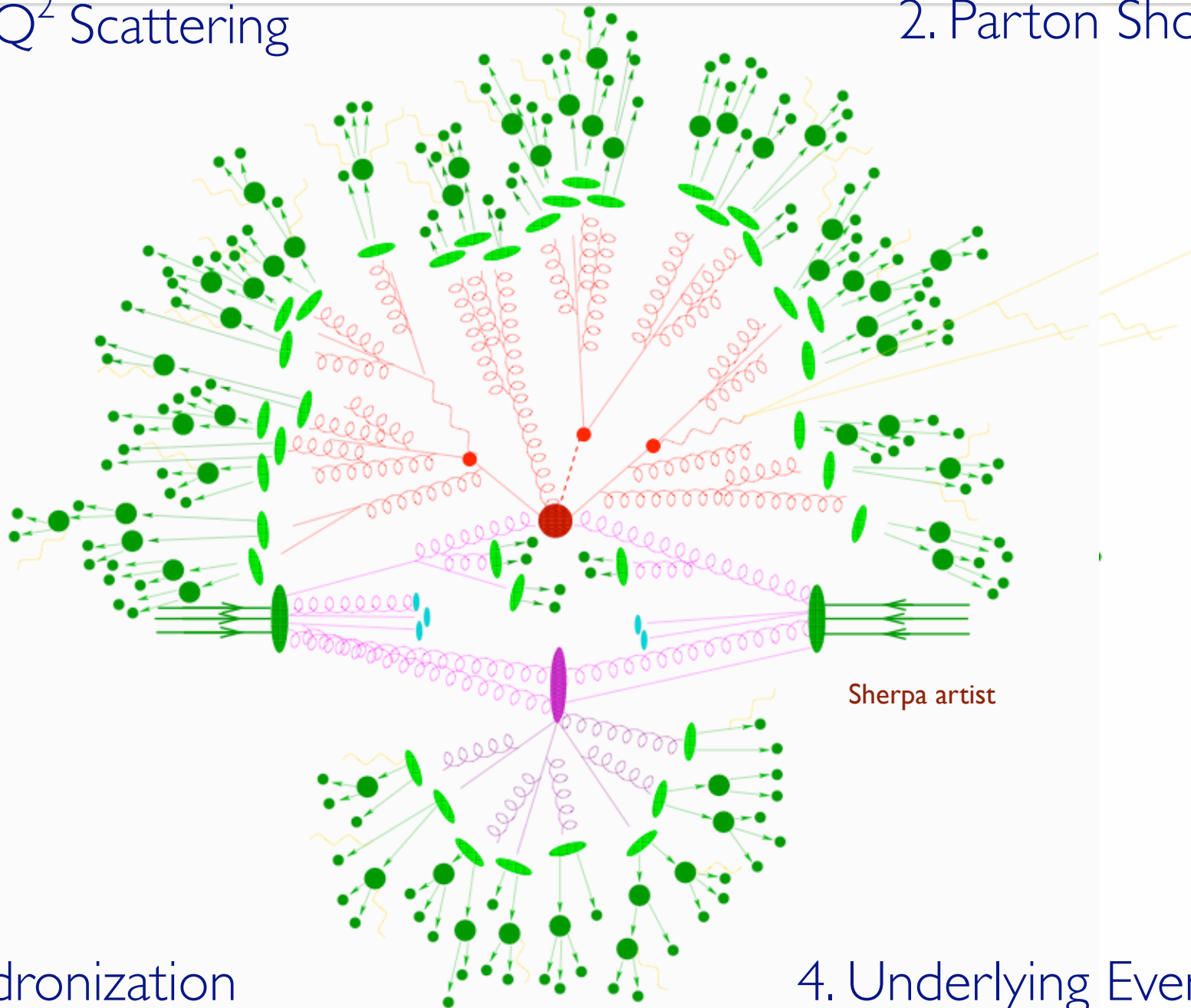
1. High-Q^2 Scattering

2. Parton Shower

Sherpa artist

3. Hadronization

4. Underlying Event

What are the MC for?

1. High-Q^2 Scattering

2. Parton Shower

☞ where new physics lies

☞ process dependent

☞ first principles description

☞ it can be systematically improved

Sherpa artist

3. Hadronization

4. Underlying Event

What are the MC for?

1. High-Q^2 Scattering

2. Parton Shower

Sherpa artist
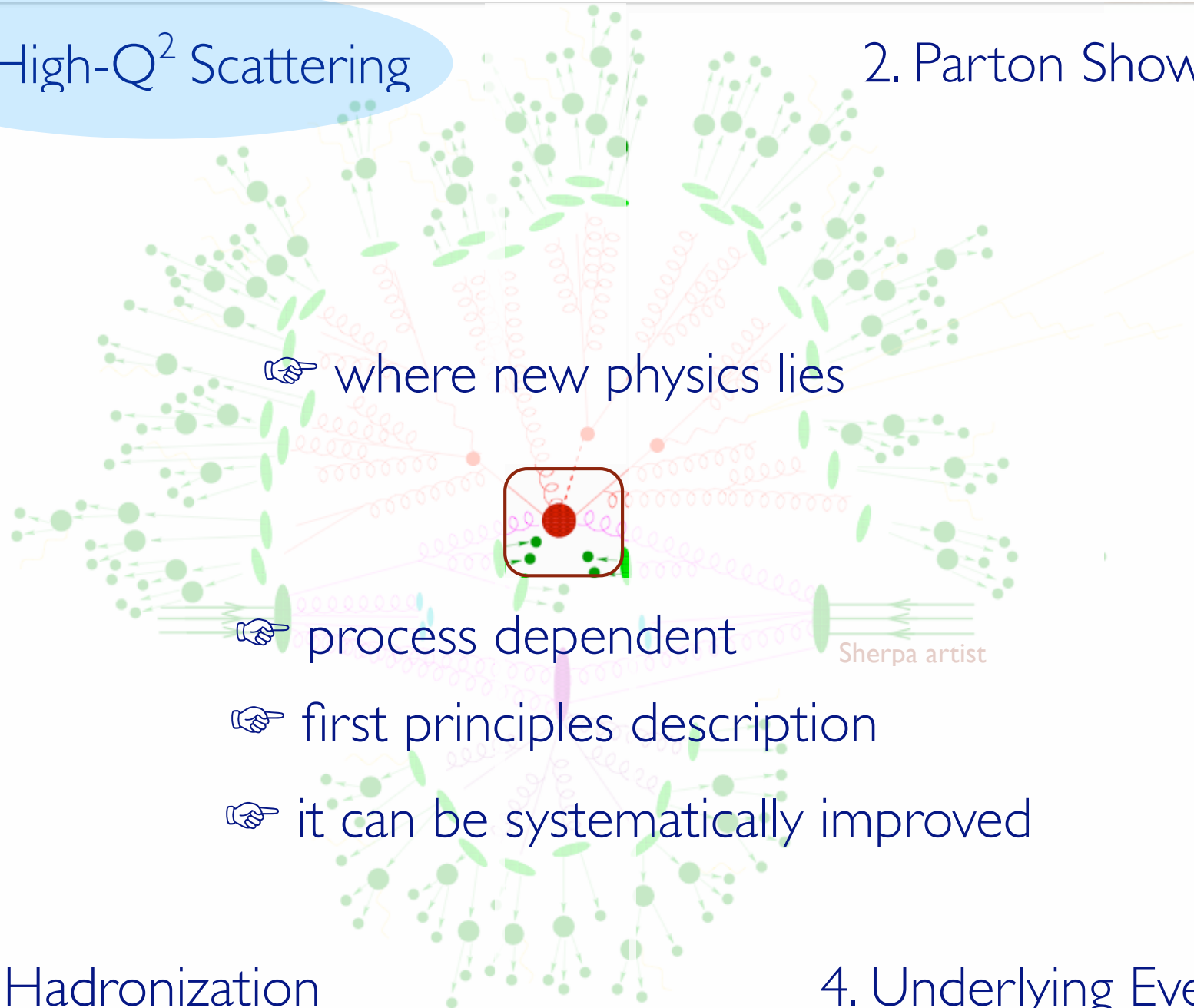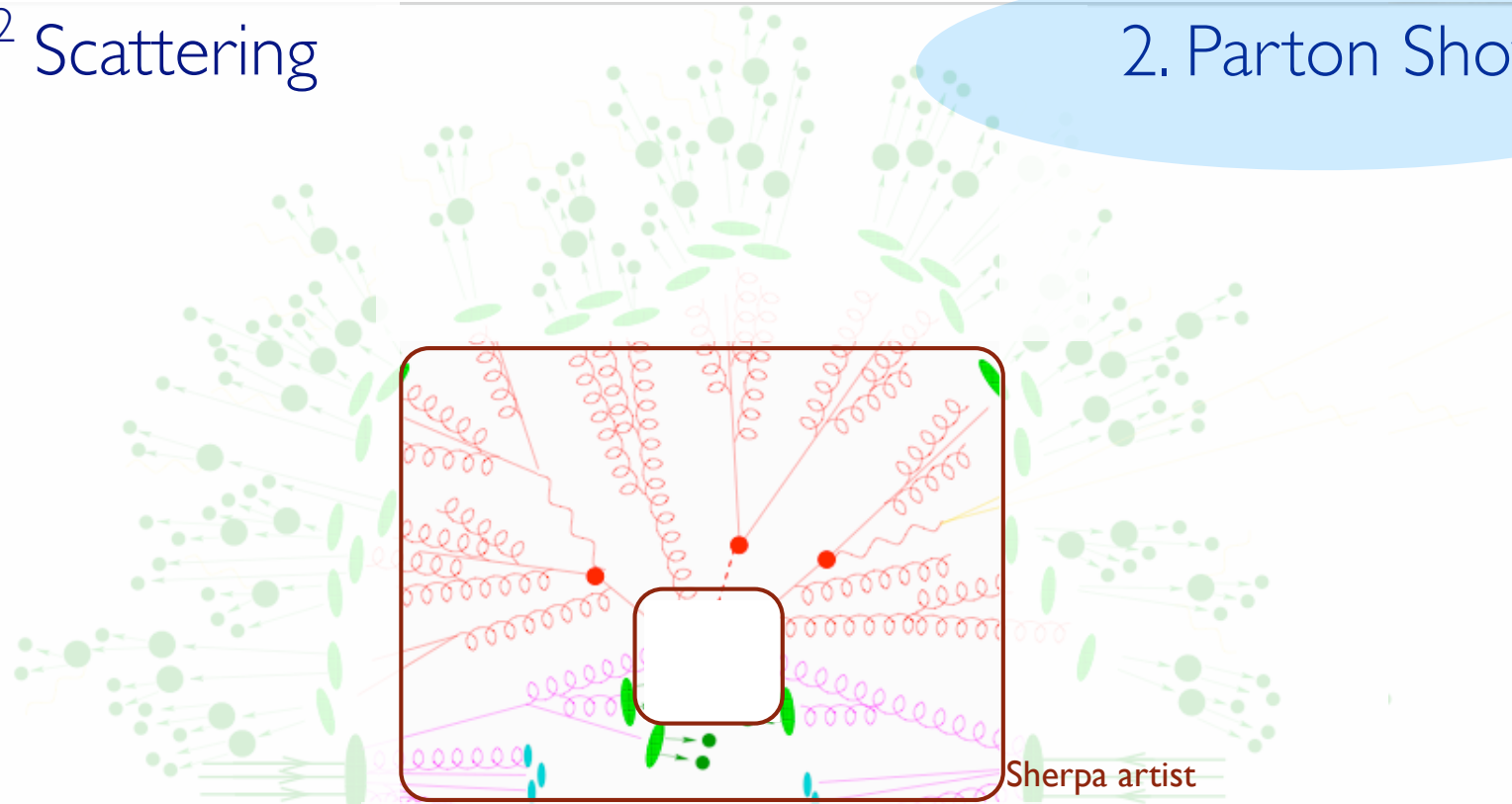
☞ QCD -"known physics"

☞ universal/ process independent

☞ first principles description

3. Hadronization

4. Underlying Event

MASTER FORMULA FOR THE LHC

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}} \, f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral   Parton density functions   Parton-level cross section

Parton shower

Goal

- We want to an **explicit** description of the SOFT radiation that are **ALREADY** included **implicitly in the LO events** (via the scale)

Important

- Parton-Shower is **not ADDING radiation**

  - Such radiations are already included within the event-generator

- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation

- This effect should be **unitary:** the inclusive cross section shouldn't change when extra radiation is added

Collinear factorization
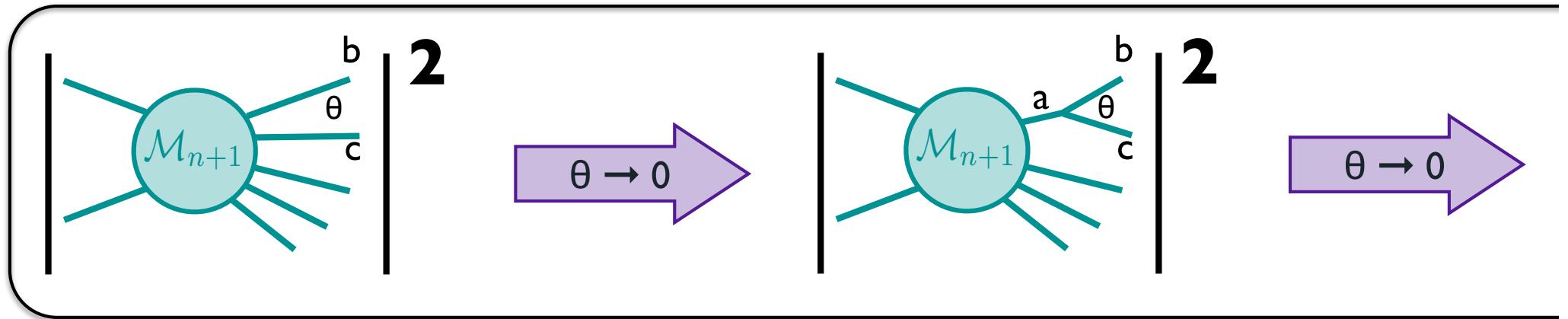
- Consider a process for which two particles are separated by a small angle θ.

- The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability.

Collinear factorization
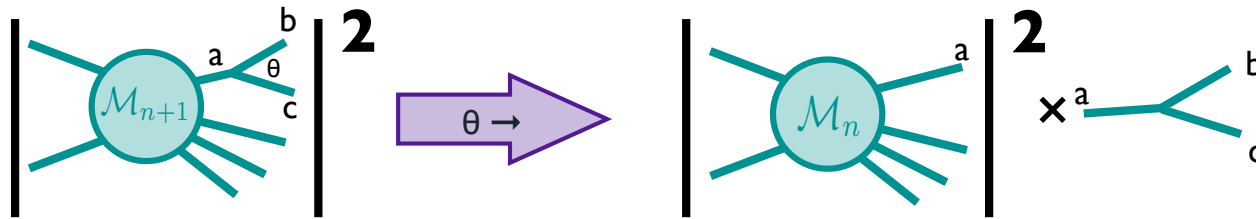
- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b + p_c)^2} \simeq \frac{1}{2 E_b E_c (1 - \cos\theta)} = \frac{1}{t}$$

$$z = E_b/E_a$$

soft and **collinear** divergencies

Collinear factorization:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \to bc}(z)$$

when θ is small.

First Example

$$e^+ e^- \rightarrow q\bar{q}g$$
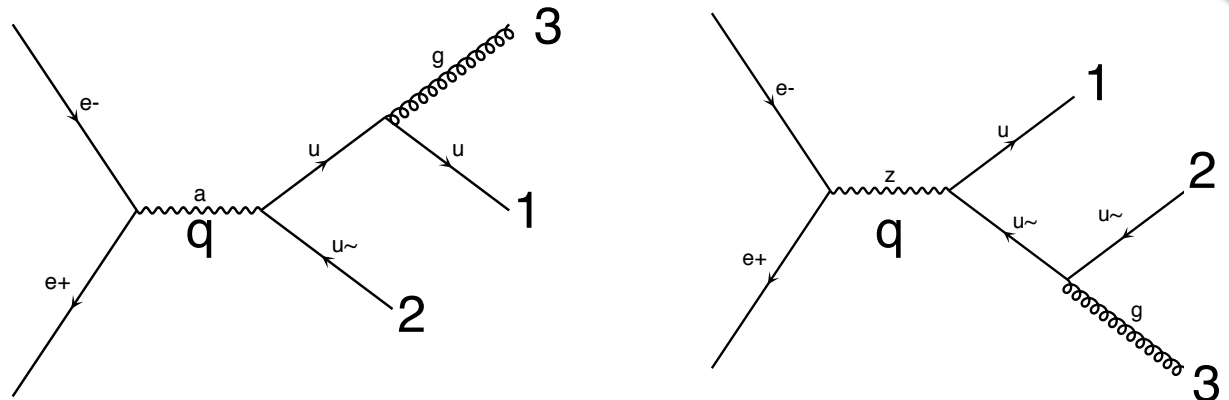
$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$

$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$

$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$

$$x_1 + x_2 + x_3 = 2$$

- Divergent at $x_1 = 1$ and $x_2 = 1$
  - Soft Divergencies
  - Collinear Divergencies

$$(1 - x_1) = \frac{x_2 x_3}{2}(1 - cos\theta_{23})$$

$$(1 - x_2) = \frac{x_1 x_3}{2}(1 - cos\theta_{13})$$

First Example

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

- Change the variable to x_3 and $\cos\theta_{13}$

$$\frac{d\sigma}{dx_3 d\cos\theta_{13}} = \sigma_0 C_F \frac{\alpha_s}{2\pi}\left(\frac{2}{\sin^2\theta_{13}} \frac{1+(1-x_3)^2}{x_3} - x_3\right)$$

- Collinear limit

  - Split our integral in two

$$\frac{2\,dcos\theta_{13}}{sin^2\theta_{13}} = \frac{dcos\theta_{13}}{1-cos\theta_{13}} + \frac{dcos\theta_{13}}{1+cos\theta_{13}}$$

$$\approx \frac{d\cos\theta_{13}}{(1-\cos\theta_{13})} + \frac{d\cos\theta_{23}}{(1-\cos\theta_{23})}$$

$$\approx \frac{d\theta_{13}^2}{\theta_{13}^2} + \frac{d\theta_{23}^2}{\theta_{23}^2}$$

First Example

- Change the variable to x_3 and $\cos\theta_{13}$

$$\frac{d\sigma}{dx_3 \, d\cos\theta_{13}} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \left(\boxed{\frac{2}{\sin^2\theta_{13}}} \frac{1+(1-x_3)^2}{\boxed{x_3}} - x_3 \right)$$

- Collinear limit

  - Split our integral in two

$$\frac{2\,d\cos\theta_{13}}{\sin^2\theta_{13}} = \frac{d\cos\theta_{13}}{1-\cos\theta_{13}} + \frac{d\cos\theta_{13}}{1+\cos\theta_{13}}$$

$$\approx \frac{d\cos\theta_{13}}{(1-\cos\theta_{13})} + \frac{d\cos\theta_{23}}{(1-\cos\theta_{23})}$$

$$\approx \frac{d\theta_{13}^2}{\theta_{13}^2} + \frac{d\theta_{23}^2}{\theta_{23}^2}$$

$$(x_3 \rightarrow z)$$

$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} \, dz \frac{1+(1-z)^2}{z}$$

☞ z fraction of energy

☞ **Generic Formula**

Parton Shower basics

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

The spin averaged (unregulated) splitting functions for the various types of branching are (Altarelli-Parisi):

$$\hat{P}_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)} \right],$$

$$\hat{P}_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right],$$

$$\hat{P}_{qg}(z) = T_R \left[z^2 + (1-z)^2 \right],$$

$$\hat{P}_{gg}(z) = C_A \left[\frac{z}{(1-z)} + \frac{1-z}{z} + z(1-z) \right].$$

$$C_F = \tfrac{4}{3}, C_A = 3, T_R = \tfrac{1}{2}.$$

Comments:
* Gluons radiate the most
* There are soft divergences in z=1 and z=0.
* P_{qg} has no soft divergences.
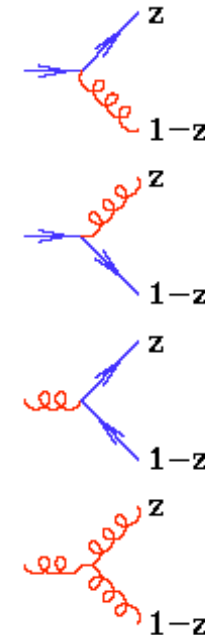
Collinear factorization

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_\mathrm{S}}{2\pi} P_{a\to bc}(z)$$

* **t** can be called the 'evolution variable' (will become clearer later): it can be the virtuality **m²** of particle a or its **p$_T$²** or **E²θ²** …

$$d\theta^2/\theta^2 = dm^2/m^2 = dp_T^2/p_T^2$$

$$m^2 \simeq z(1-z)\theta^2 E_a^2$$

$$p_T^2 \simeq zm^2$$

* It represents the hardness of the branching and tends to 0 in the collinear limit.

* Different choice of 'evolution parameter' in different Parton-shower code

Collinear factorization

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \to bc}(z)$$

* **z** is the "energy variable": it is defined to be the energy fraction taken by parton **b** from parton **a**. It represents the energy sharing between **b** and **c** and tends to 1 in the soft limit (parton c going soft)

* Φ is the azimuthal angle. It can be chosen to be the angle between the polarization of a and the plane of the branching.

Argument of α_S

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \to bc}(z)$$

- Each choice of argument for α_S is equally acceptable at the leading-logarithmic accuracy. However, there is a choice that allows one to resum certain classes of subleading logarithms.

- The more natural choices is to evaluated it at scale "t"

  - Can be proof to be a good choice since it allows to include sub-logarithm contributions.

  - Each radiation evaluates alpha_s at his own scale

    - Different from fixed order computation where all value use the renormalisation scale.

To Remember

Collinear Limit

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \to bc}(z)$$

- **t** is the evolution parameter (control the collinear behaviour)

- **z** is the energy sharing variable

- **alpha_s** need to be evaluated at the scale t

- **P** is the splitting Kernel (control the soft behaviour)

Multiple emission
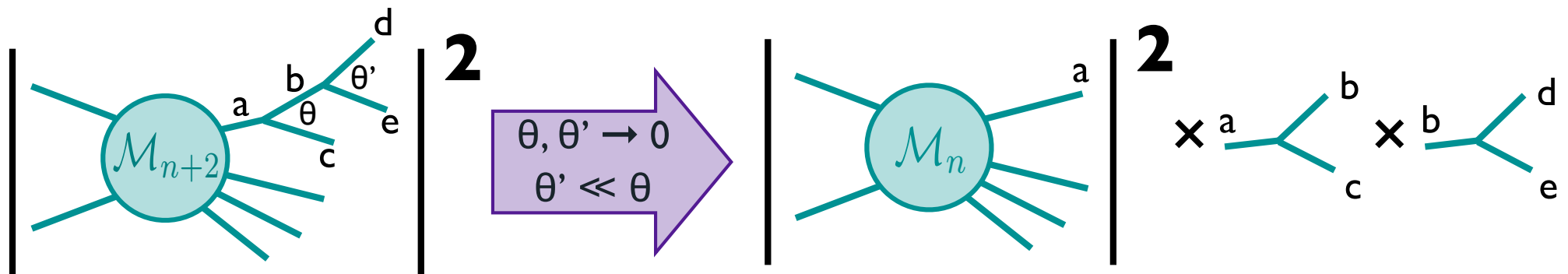
- Now consider **M_{n+1}** as the new core process and use the recipe we used for the first emission in order to get the dominant contribution to the (n+2)-body cross section: add a new branching at angle much smaller than the previous one:

$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_{\mathrm{S}}}{2\pi} P_{a\to bc}(z)$$

$$\times \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_{\mathrm{S}}}{2\pi} P_{b\to de}(z')$$

- This can be done for an arbitrary number of emissions. The recipe to get the leading collinear singularity is thus cast in the form of an iterative sequence of emissions whose probability does not depend on the past history of the system: a 'Markov chain'. No interference!!!

Multiple emission
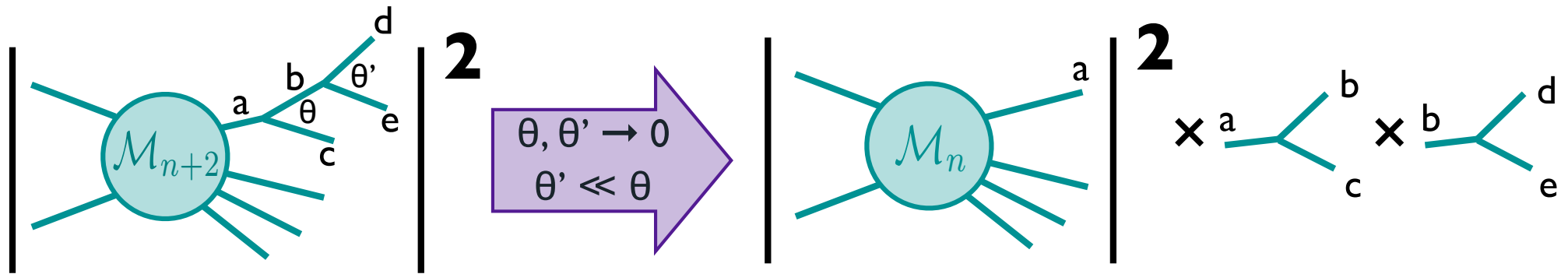
- The dominant contribution comes from the region where the subsequently emitted partons satisfy the strong ordering requirement: θ ≫ θ' ≫ θ''...
  For the rate for multiple emission we get

$$\sigma_{n+k} \propto \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \cdots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

  where Q is a typical hard scale and Q_0 is a small infrared cutoff that separates perturbative from non perturbative regimes.
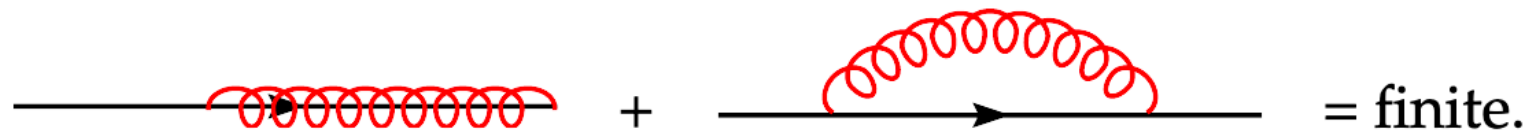
- Each power of α_s comes with a logarithm. The logarithm can be easily large, and therefore it can lead to a breakdown of perturbation theory.

Need to introduce resolution t_0, e.g. a cutoff in $p_\perp$. Prevent us from the singularity at $\theta \to 0$.

Emissions below t_0 are unresolvable.

Finite result due to virtual corrections:

$$= \text{finite}.$$

unresolvable + virtual emissions are included in Sudakov form factor via unitarity (see below!).

Sudakov Form Factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz \hat{P}(z)$$

- So the probability of no emission between two scales:

$$P_{no-branching}(Q^2, t) = \lim_{N\to\infty}\prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz \hat{P}(z)\right)$$

$$\simeq \lim_{N\to\infty} e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz \hat{P}(z)\right)}$$

Sudakov form factor

$$\Delta(Q^2, t) \qquad \simeq e^{-\int_t^{Q^2}\frac{dt'}{t'}\int dz \frac{\alpha_S}{2\pi}\hat{P}(z)} \equiv e^{-\int_t^{Q^2} dp(t')}$$

➡ Property: Δ(A,B) = Δ(A,C) Δ(C,B)

Final-state parton showers

With the Sudakov form factor, we can now implement a final-state parton shower in a Monte Carlo event generator!

1. Start the evolution at the virtual mass scale t_0 (e.g. the mass of the decaying particle) and momentum fraction $z_0 = 1$

2. Given a virtual mass scale t_i and momentum fraction x_i at some stage in the evolution, generate the scale of the next emission t_{i+1} according to the Sudakov probability $\Delta(t_i, t_{i+1})$ by solving
$$\Delta(t_{i+1}, t_i) = R$$
where R is a random number (uniform on [0, 1]).

3. If $t_{i+1} < t_{cut}$ it means that the shower has finished.

4. Otherwise, generate $z = z_i/z_{i+1}$ with a distribution proportional to $(\alpha_s/2\pi)P(z)$, where $P(z)$ is the appropriate splitting function.

5. For each emitted particle, iterate steps 2-4 until branching stops.

To Remember

- Sudakov Form-Factor: Probability of No-emission between two scale.

$$\Delta(Q^2, t) \simeq e^{-\int_t^{Q^2} \frac{dt'}{t'} dz \frac{\alpha_S}{2\pi} \hat{P}(z)} \equiv e^{-\int_t^{Q^2} dp(t')}$$

- Parton shower is unitary (and IR save)

- Parton shower is a Markov Chain

  ➡ One emission at the time

- Each interactions has its own scale for alphas

- Various choice for the evolution parameter

Matching/Merging

Olivier Mattelaer
CP3/UCLouvain

PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result $\Rightarrow$ Large variation in results (small prediction power)
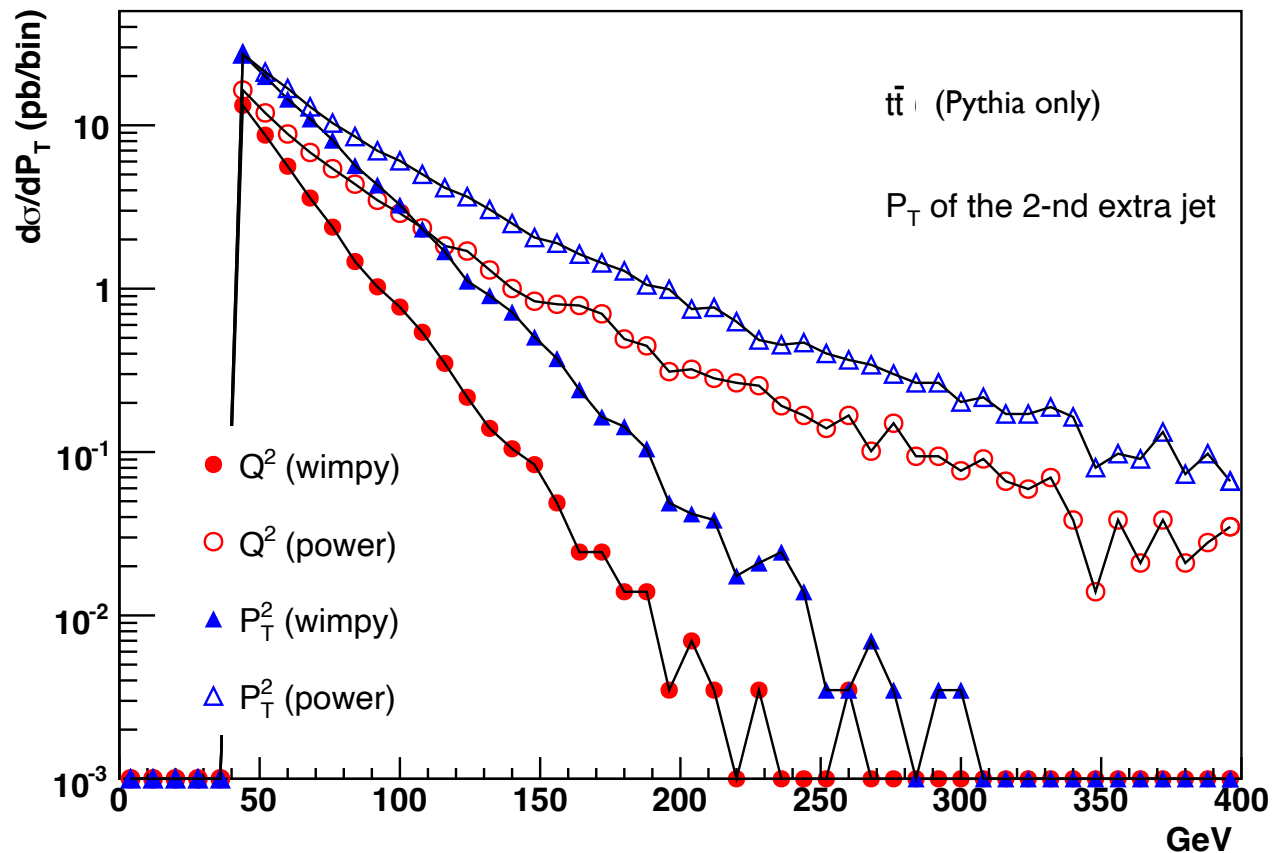
Matrix Elements vs. Parton Showers

ME
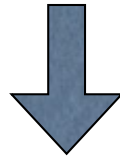
1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Shower MC
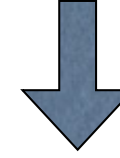
1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are collinear and/or soft
5. Partial interference through angular ordering
6. Needed for hadronization

Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

Goal for ME-PS merging/matching

- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions
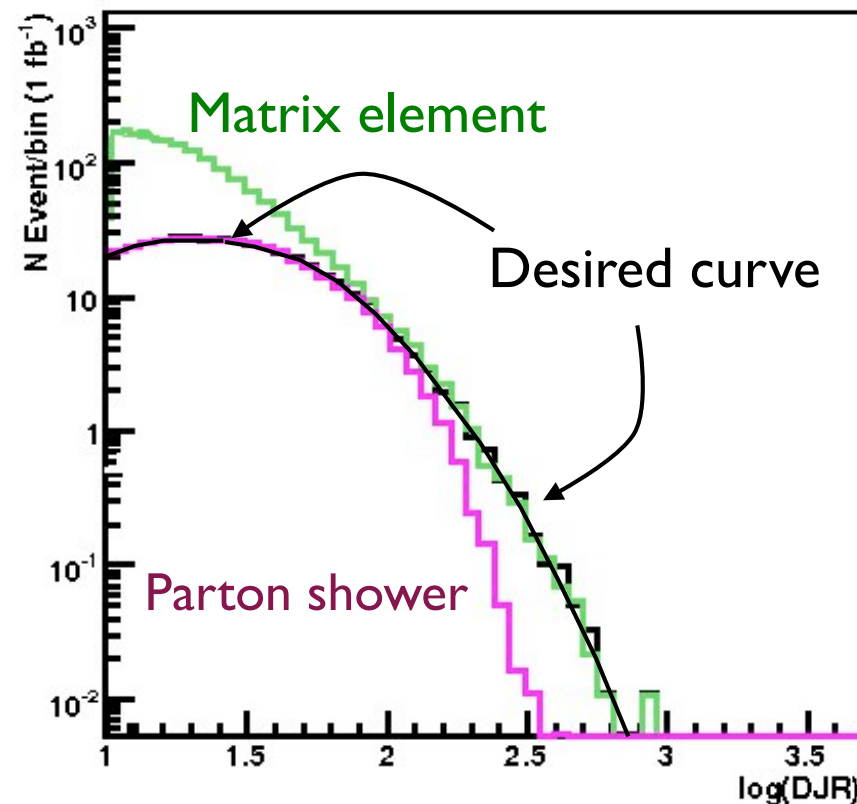
2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Merging ME with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]

PS →

ME ↓
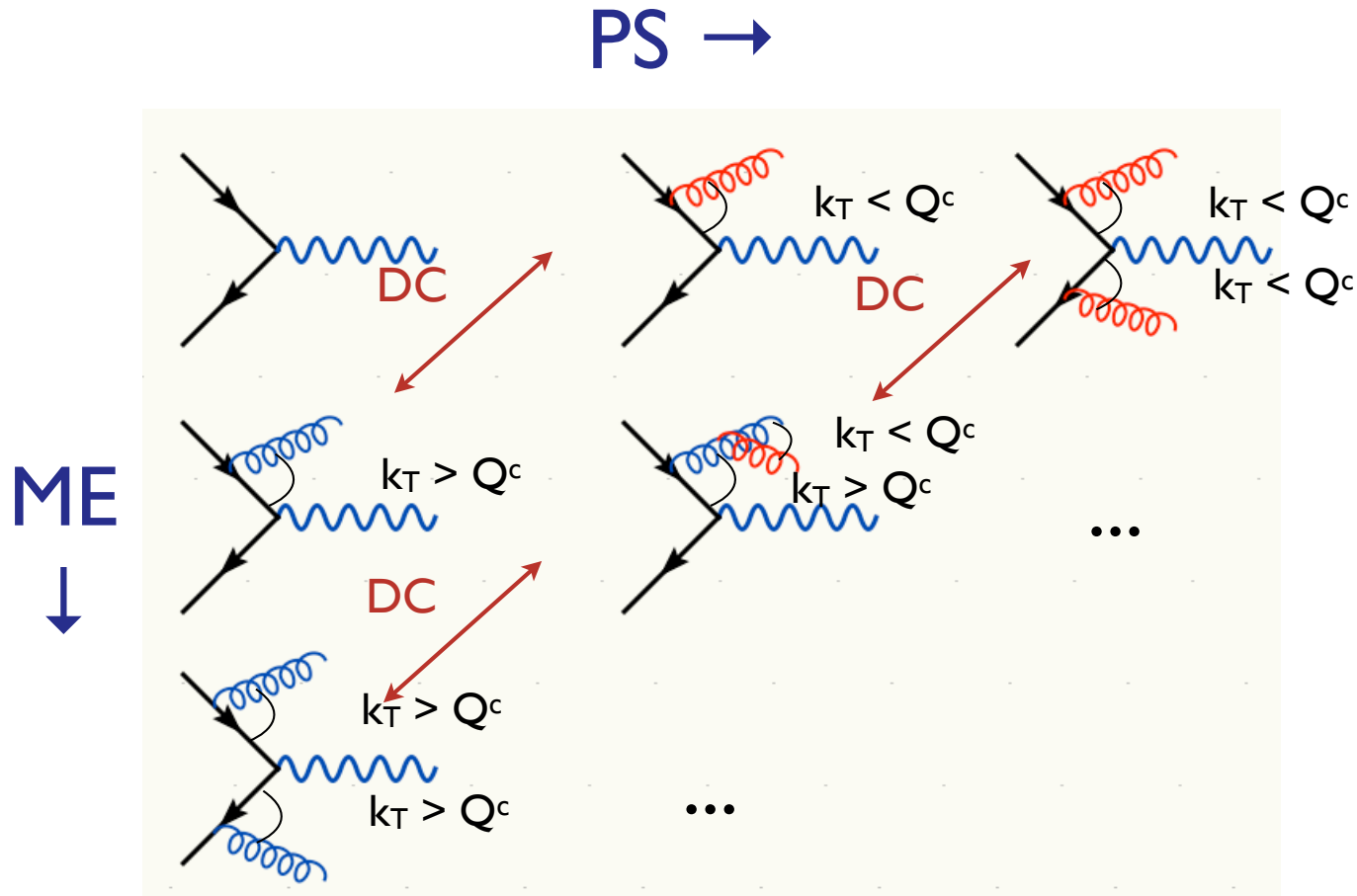
Double counting between ME and PS easily avoided using phase space cut between the two: PS below cutoff, ME above cutoff.

Merging ME with PS

- So double counting problem easily solved, but what about getting smooth distributions that are independent of the precise value of Q_c?

- Below cutoff, distribution is given by PS
  - need to make ME look like PS near cutoff

- Let's take another look at the PS!

Merging ME with PS
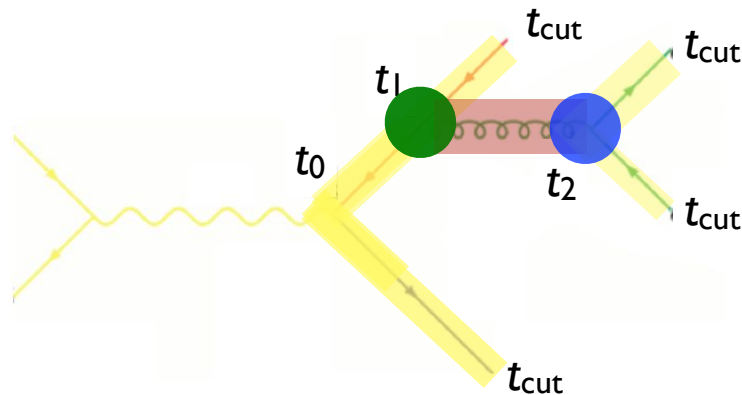
- ## How does the PS generate the configuration above?

- ## Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$(\Delta_q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

Merging ME with PS
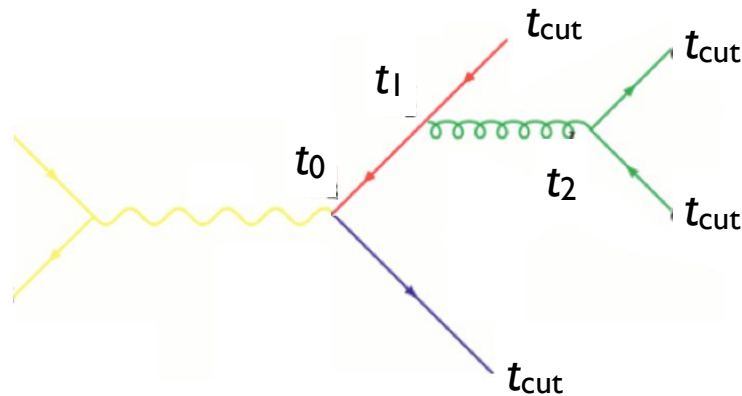
$$(\Delta_q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(\text{cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

Corresponds to the matrix element
BUT with αs evaluated at the scale of each splitting

Sudakov suppression due to not allowing additional radiation
above the scale t_{cut}

Merging ME with PS

$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, ...)$$
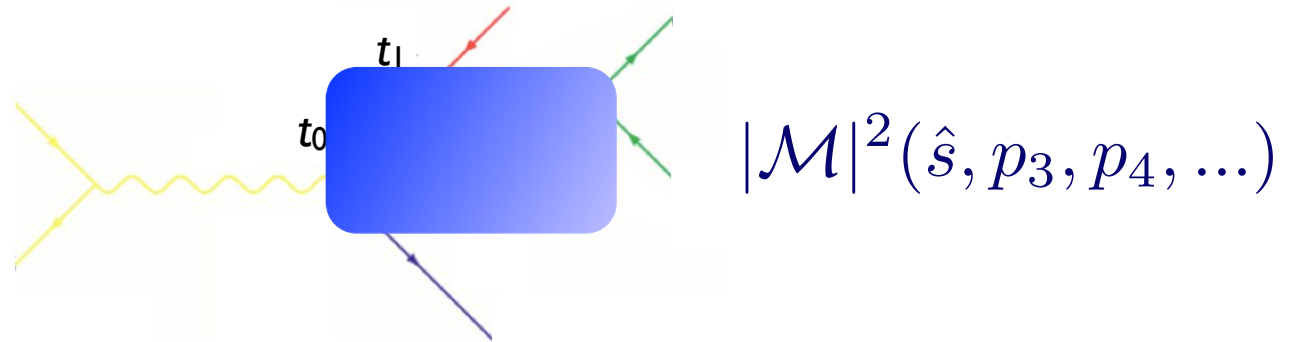
- **To get an equivalent treatment of the corresponding matrix element, do as follows:**

  1. Cluster the event using some clustering algorithm
     - this gives us a corresponding "parton shower history"

  2. Reweight α_s in each clustering vertex with the clustering scale

  $$|\mathcal{M}|^2 \rightarrow |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)}$$

  3. Use some algorithm to apply the equivalent Sudakov suppression $\quad (\Delta_q(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q({\rm cut}, t_2))^2$

Matching for initial state radiation

- We are of course not interested in e⁺e⁻ but p-p(bar) - what happens for initial state radiation?

- Let's do the same exercise as before:

$$\mathcal{P} = (\Delta_{Iq}(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x'_1, t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$
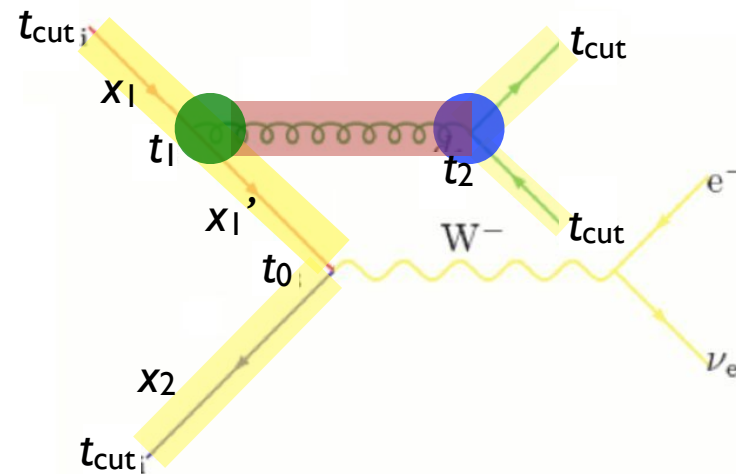
$$\times \hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s}, ...) f_q(x'_1, t_0) f_{\bar{q}}(x_2, t_0)$$

Matching for initial state radiation

$$(\Delta_{Iq}(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x_1', t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

$$\times \hat{\sigma}_{q\bar{q} \to e\nu}(\hat{s}, ...) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0)$$

ME with α_s evaluated at the scale of each splitting

PDF reweighting

Sudakov suppression due to non-branching above scale t_{cut}
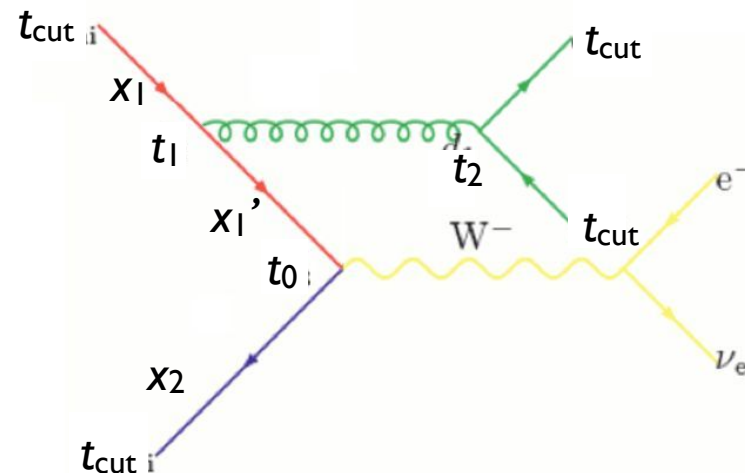
Matching for initial state radiation

- Again, use a clustering scheme to get a parton shower history

- Now, reweight both due to α_s and PDF

$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)} \frac{f_q(x_1', t_0)}{f_q(x_1', t_1)}$$
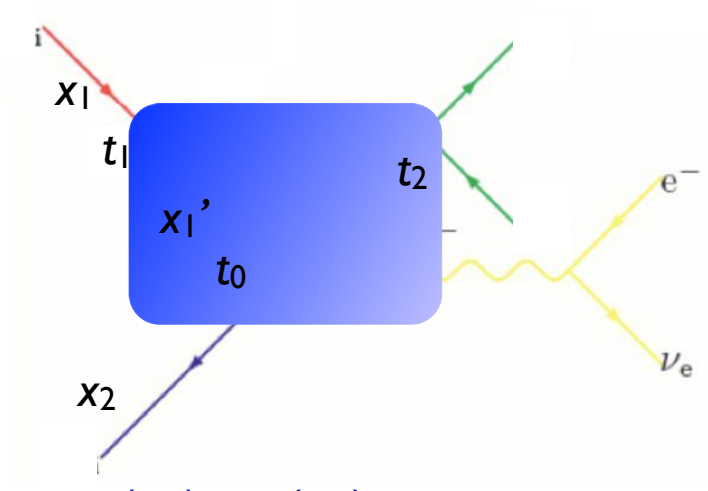
Matching schemes

- We still haven't specified how to apply the Sudakov reweighting to the matrix element

- Three general schemes available in the literature:

    ➡ CKKW scheme [Catani,Krauss,Kuhn,Webber 2001; Krauss 2002]

    ➡ Lönnblad scheme (or CKKW-L) [Lönnblad 2002]

    ➡ MLM scheme [Mangano *unpublished* 2002; Mangano et al. 2007]

CKKW matching

[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]

- Apply the required Sudakov suppression

$$(\Delta_{Iq}(t_{\mathrm{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\mathrm{cut}}, t_2))^2$$

analytically, using the best available (NLL) Sudakovs.

- Perform "truncated showering": Run the parton shower starting at t_0, but forbid any showers above the cutoff scale t_{cut}.

  - ✓ Best theoretical treatment of matrix element
  - – Requires dedicated PS implementation
  - – Mismatch between analytical Sudakov and (non-NLL) shower
- Implemented in Sherpa (v. 1.1)

CKKW-L matching

[Lönnblad 2002]
[Hoeche et al. 2009]
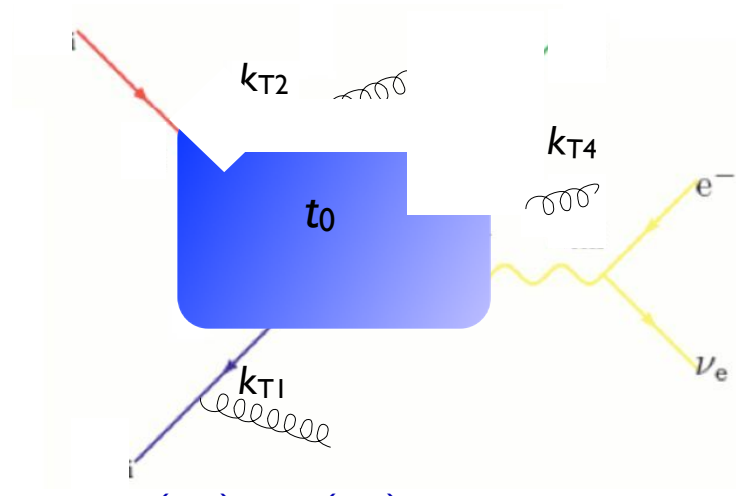
✓ Automatic agreement between Sudakov and shower

- Requires dedicated PS implementation

➡ Need multiple implementations to compare between showers

● Implemented in Ariadne, Sherpa (v. 1.2), and Pythia 8

MLM matching

- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t_0!
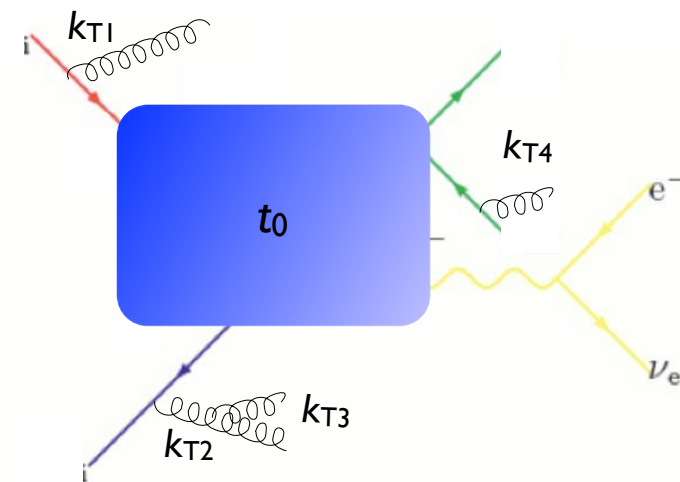
- ✓ Simplest available scheme

- ✓ Allows matching with any shower, without modification

- ➡ Sudakov suppression not exact, minor mismatch with shower

- Implemented in AlpGen, HELAC, MadGraph+Pythia 6

Highest multiplicity sample

- In the previous, assumed we can simulate all parton multiplicities by the ME

- In practice, we can only do limited number of final-state partons with matrix element (up to 4-5 or so)

- For the highest jet multiplicity that we generate with the matrix element, we need to allow additional jets above the matching scale t_{cut}, since we will otherwise not get a jet-inclusive description – but still can't allow PS radiation harder than the ME partons

➡ Need to replace t_{cut} by the clustering scale for the softest ME parton for the highest multiplicity

matching schemes

- We have a number of choices to make in the above procedure. The most important are:

1. The clustering scheme used to determine the parton shower history of the ME event

2. What to use for the scale *of hard emission*

3. How to divide the phase space between parton showers and matrix elements

Cluster schemes

1. The clustering scheme used inside MadGraph and Sherpa to determine the parton shower history is the Durham k_T scheme. For e$^+$e$^-$:

$$k_{Tij}^2 = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$$

and for hadron collisions, the minimum of:

$$k_{Ti\mathrm{beam}} = m_i^2 + p_{Ti}^2 = (E_i + p_{zi})(E_i - p_{zi})$$

and

$$k_{Tij}^2 = \min(p_{Ti}^2, p_{Tj}^2)R_{ij}$$

with $R_{ij} = 2[\cosh(y_i - y_j) - \cos(\phi_i - \phi_j)] \simeq (\Delta y)^2 + (\Delta\phi)^2$

Find the smallest k_{Tij} (or $k_{Ti\mathrm{beam}}$), combine partons i and j (or i and the beam), and continue until you reach a 2 → 2 (or 2 → 1) scattering.

2. In AlpGen a more naive cone algorithm is used.

MLM in MadGraph

- Cannot use the standard k_T clustering:

  - MadGraph and Sherpa only allow clustering according to valid diagrams in the process. This means that, e.g., two quarks or quark-antiquark of different flavor are never clustered, and the clustering always gives a physically allowed parton shower history.

  - If there is an on-shell propagator in the diagram (e.g. a top quark), only clustering according to diagrams with this propagator is allowed.

Hard scale

2. The clustering provides a convenient choice for factorization scale Q^2:
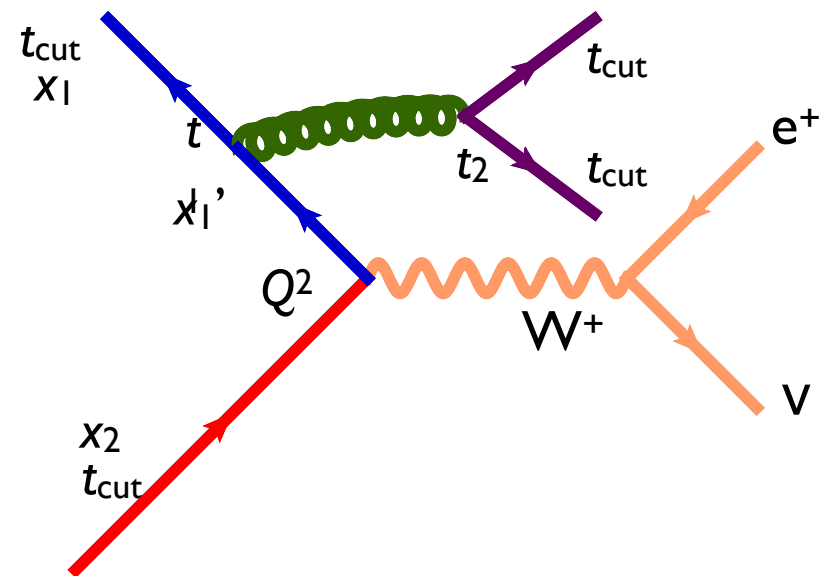
Cluster back to the 2 → 2 (here qq → W-g) system, and use the W boson transverse mass in that system.

Phase-space division

3. How to divide the phase space between PS and ME:
   This is where the schemes really differ:

   AlpGen: MLM Cone
   MadGraph: MLM Cone, k_T or shower-k_T
   Sherpa: CKKW
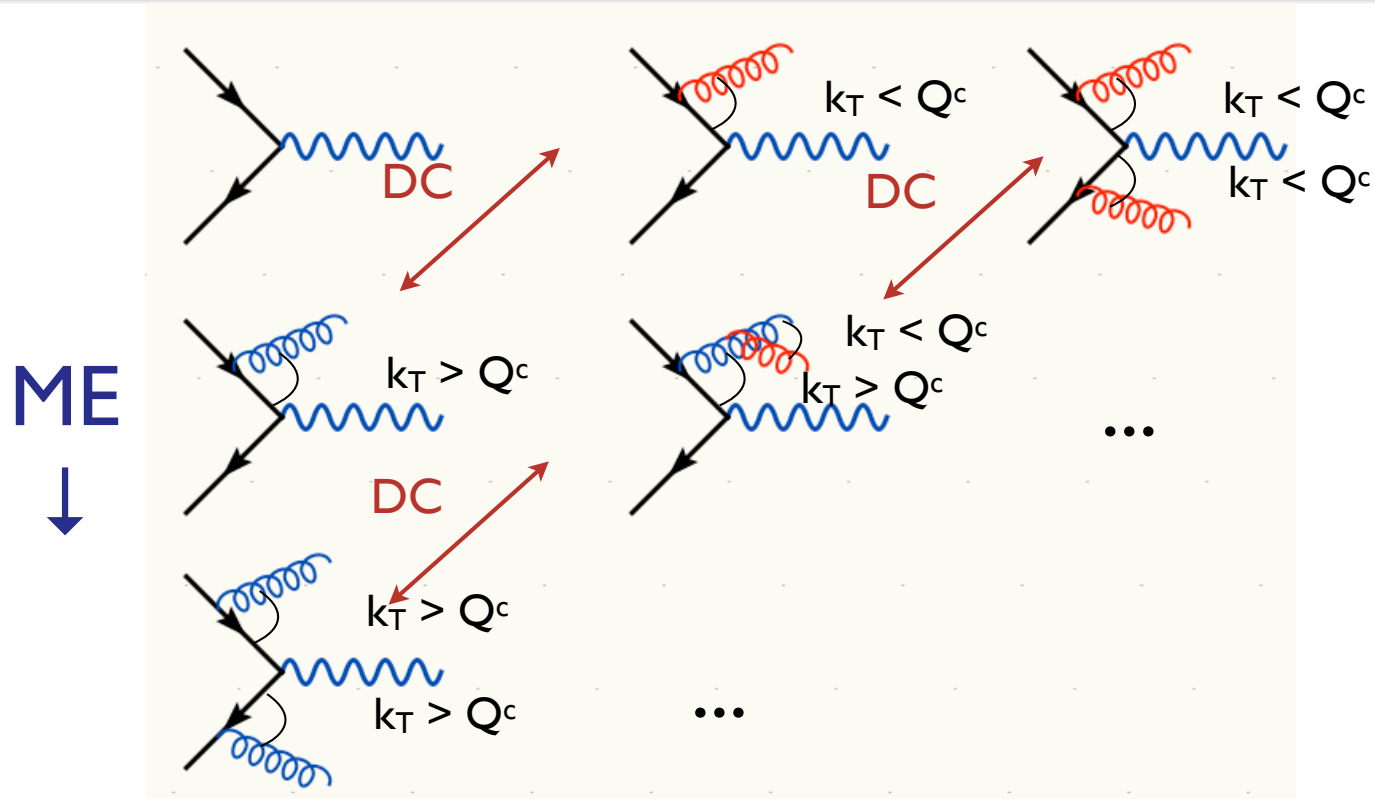
Back to the "matching goal"

- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions
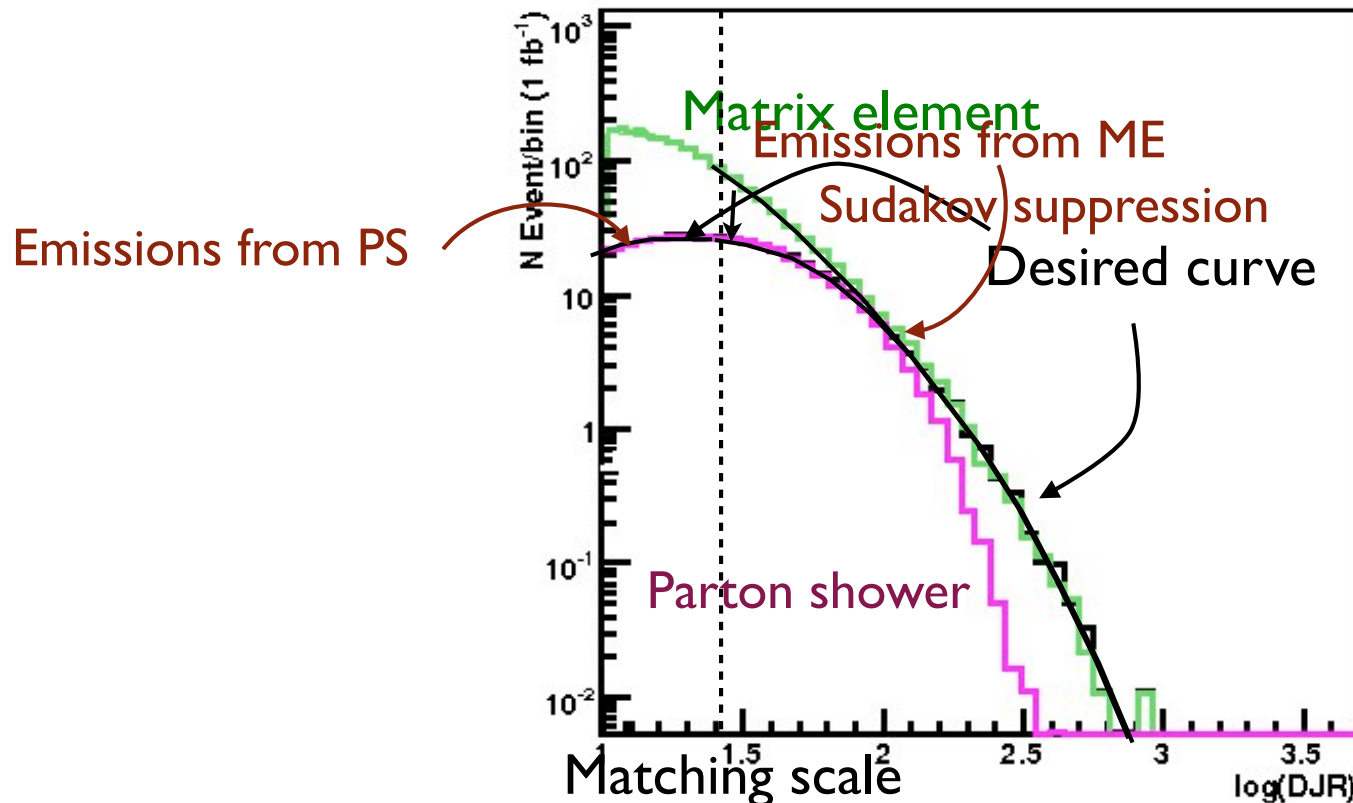
2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)
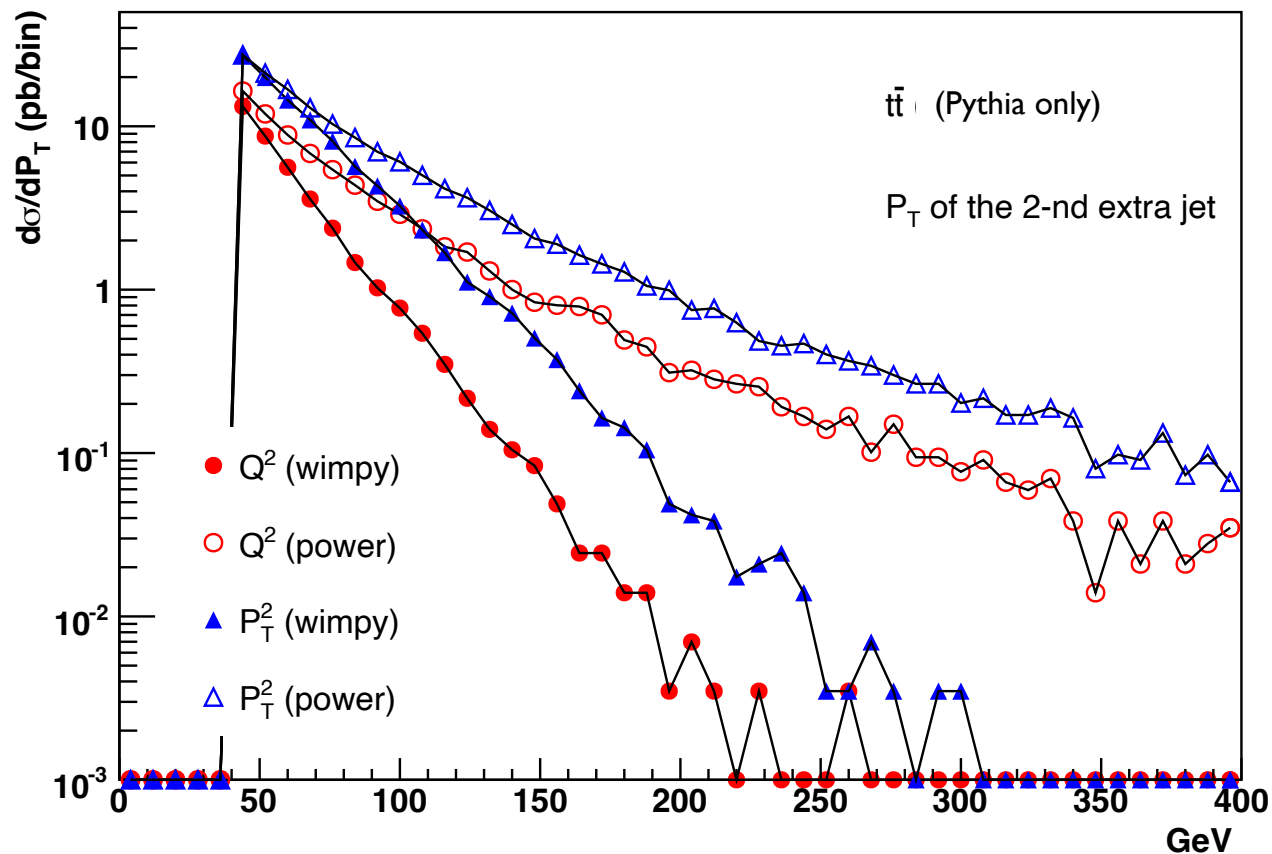
In a matched sample these differences are irrelevant since the behavior at high pt is dominated by the matrix element.
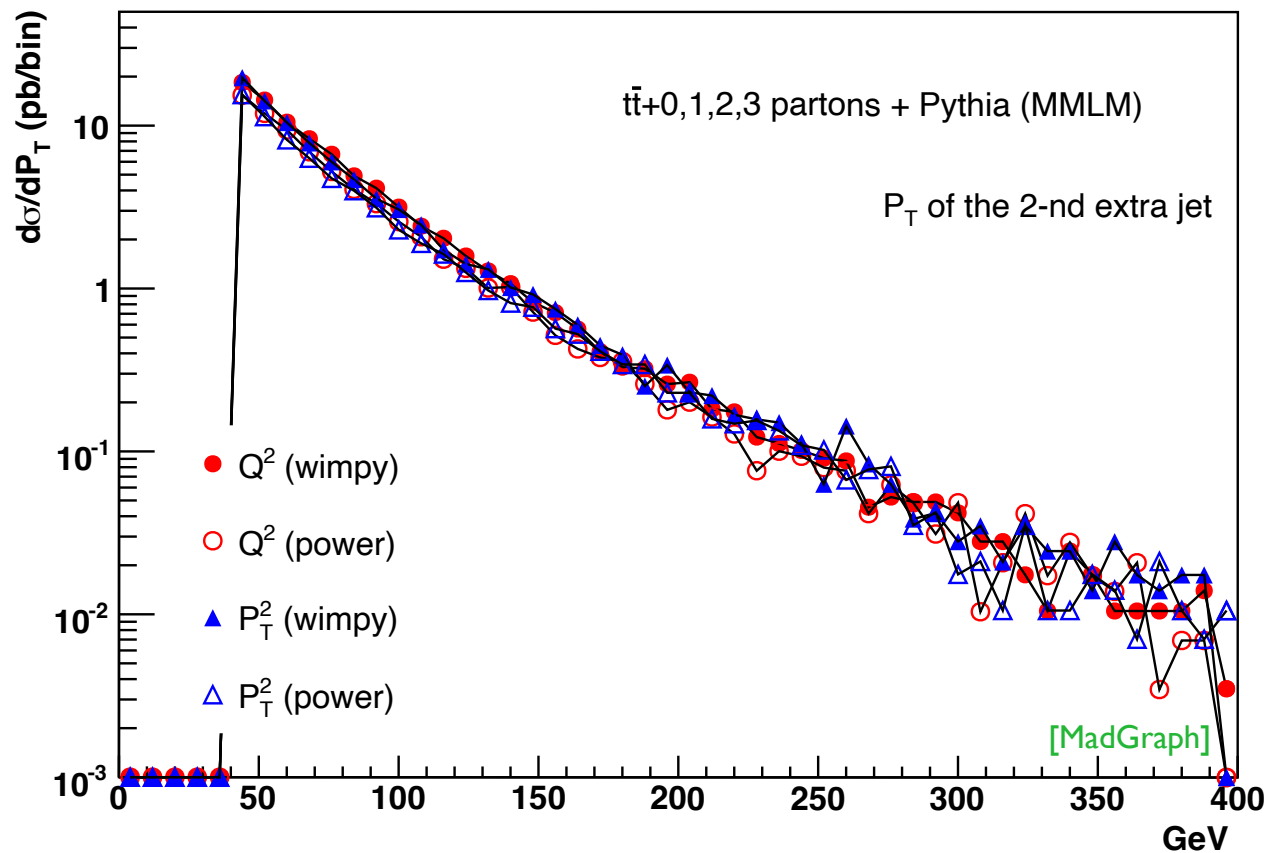
Summary of Matching Procedure

1. Generate ME events (with different parton multiplicities) using parton-level cuts ($p_T^{ME}/\Delta R$ or k_T^{ME})

2. Cluster each event and reweight α_s and PDFs based on the scales in the clustering vertices

3. Apply Sudakov factors to account for the required non-radiation above clustering cutoff scale and generate parton shower emissions below clustering cutoff:

   a. (CKKW) Analytical Sudakovs + truncated showers

   b. (CKKW-L) Sudakovs from truncated showers

   c. (MLM) Sudakovs from reclustered shower emissions

1. Apply separation cut

Tutorial

**Olivier Mattelaer
CP3/UCLouvain**

Tutorial map

Learning MG5

- follow the built-in tutorial

- cards meaning

- meaning of QCD/QED

- details of syntax ($/)

- tt~ processes

  ➡ Decay

  ➡ Scripting

Learning MG5_aMC

Where to find help?

- Ask us

- Use the command "help" / "help XXX"

  ➡ "help" tell you the next command that you need to do.

- Launchpad:

  ➡ https://answers.launchpad.net/madgraph5

  ➡ FAQ: https://answers.launchpad.net/madgraph5/+faqs

Minimal tutorial

- Launch the code
  - ➡ python2.7 ./bin/mg5_aMC
- Type tutorial
  - ➡ Follow instructions

What are those cards?

- Read the Cards and identify what they do
  - ➡ param_card: model parameters
  - ➡ run_card: beam/run parameters and cuts
    - https://answers.launchpad.net/madgraph5/+faq/2014

Exercise II: Cards Meaning

- How do you change
  - ➡ top mass
  - ➡ top width
  - ➡ W mass
  - ➡ beam energy
  - ➡ pt cut on the lepton

Exercise II : Syntax

- What's the meaning of the order QED/QCD

- What's the difference between

  ➡ p p > t t~   ➡ p p > t t~ QCD=0

  ➡ p p > t t~ QED=2   ➡ p p > t t~ QED<=2

  ➡ p p > t t~ QED=0   ➡ p p > t t~ QCD^2==2

- Compute the cross-section for each of those and check the diagram
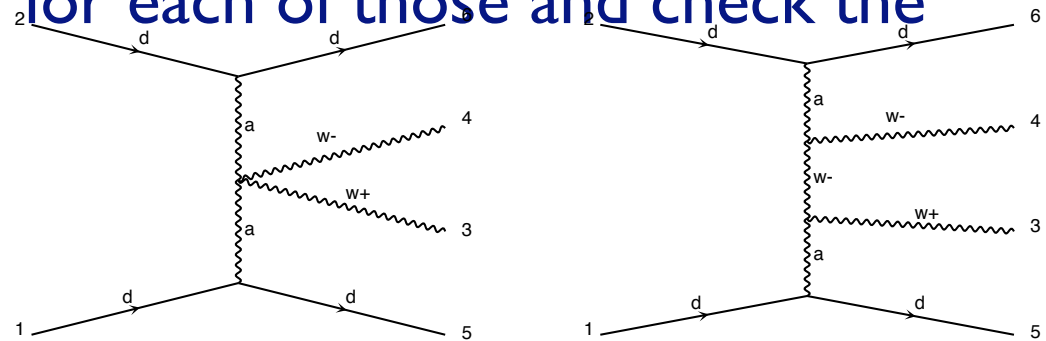
- Generate VBF process

Exercise III: Syntax

- Generate the cross-section and the distribution (invariant mass) for

  ➡ p p > e+ e-

  ➡ p p > z, z > e+ e-

  ➡ p p > e+ e- \$ z

  ➡ p p > e+ e- / z

  Hint : To plot automatically distributions:
  mg5> install MadAnalysis

- Use the invariant mass distribution to determine the

Exercise IV:
Top pair production at LO

- **Basic questions:**
  - Generate the process
  - Which partonic subprocesses contribute?
  - How many Feynman diagrams has each subprocess?
  - Output the code
  - Compute the cross-section at the LHC (8 TeV) for m_t=170 GeV
- **Extra questions:**
  - Are b-quarks included in the initial state? If not, how can I include them?
  - What does 'WEIGHTED' mean?
  - Recompute the $t\bar{t}$ cross-section for m_t=170, 172, 174 ... 180 GeV
  - Add the top decay and redo the mass scan. Anything strange?

Exercise IV: Automation/Width

- Compute the cross-section for the top pair production for 3 different mass points.

  ➡ Do NOT use the interactive interface

    - hint: you can edit the param_card/run_card via the "set" command [After the launch]

    - hint: All command [including answer to question] can be put in a file. (run ./bin/mg5 PATH_TO_FILE)

Examples

File:
```
import model EWDim6
generate p p > z z
ouput TUTO_DIM6
launch
  set nevents 5000
  set MZ 100
```

How to Run: `./bin/mg5_amc PATH`

Exercise V: Decay Chain

- Generate p p > t t~ h, fully decayed (fully leptonic decay for the top)
  - ➡ Using the decay-chain formalism
  - ➡ Using MadSpin

- Compare cross-section
  - ➡ which one is the correct one?
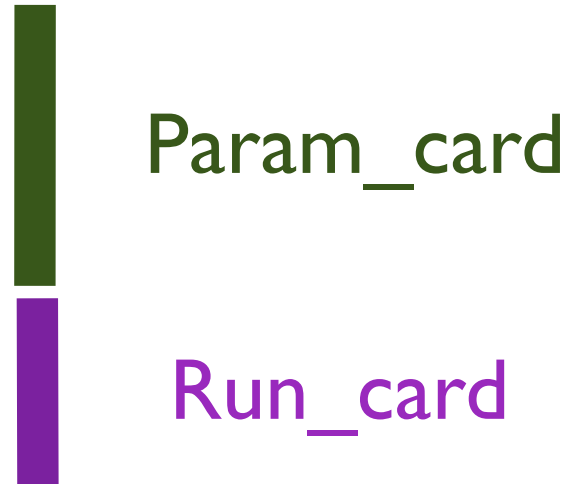 - ➡ Why are they different?

- Compare the shape.

Solution Learning MG5_aMC

Exercise II: Cards Meaning

- How do you change
  - ➡ top mass
  - ➡ top width
  - ➡ W mass
  - ➡ beam energy
  - ➡ pt cut on the lepton

Param_card

Run_card

- **top mass**

```
################################
## INFORMATION FOR MASS
################################
Block mass
      6 1.730000e+02 # MT
     23 9.118800e+01 # MZ
     25 1.200000e+02 # MH
## Dependent parameters, given by model restrictions.
## Those values should be edited following the
## analytical expression. MG5 ignores those values
## but they are important for interfacing the output of MG5
## to external program such as Pythia.
   1 0.000000 # d : 0.0
   2 0.000000 # u : 0.0
   3 0.000000 # s : 0.0
   4 0.000000 # c : 0.0
  11 0.000000 # e- : 0.0
  12 0.000000 # ve : 0.0
  13 0.000000 # mu- : 0.0
  14 0.000000 # vm : 0.0
  16 0.000000 # vt : 0.0
  21 0.000000 # g : 0.0
  22 0.000000 # a : 0.0
  24 80.419002 # w+ : cmath.sqrt(MZ__exp__2/2. + cmath.sqrt(MZ__exp__4/4. - (aEW*cmath.pi*MZ__exp__2)/(Gf*sqrt__2)))
```

```
#################################
## INFORMATION FOR MASS
#################################
Block mass
    5 4.700000e+00 # MB
    6 1.730000e+02 # MT
   15 1.777000e+00 # MTA
   23 9.118800e+01 # MZ
   25 1.200000e+02 # MH
## Dependent parameters, given by model restrictions.
## Those values should be edited following the
## analytical expression. MG5 ignores those values
## but they are important for interfacing the output of MG5
## to external program such as Pythia.
   1 0.000000 # d : 0.0
   2 0.000000 # u : 0.0
   3 0.000000 # s : 0.0
   4 0.000000 # c : 0.0
  11 0.000000 # e- : 0.0
  12 0.000000 # ve : 0.0
  13 0.000000 # mu- : 0.0
  14 0.000000 # vm : 0.0
  16 0.000000 # vt : 0.0
  21 0.000000 # g : 0.0
  22 0.000000 #
  24 80.419002 # w+ : cmath.sqrt(MZ__exp__2/2. + cmath.sqrt(MZ__exp__4/4. - (aEW*cmath.pi*MZ__exp__2)/(Gf*sqrt__2)))
```

W Mass is an internal parameter!
MG5 didn't use this value!
So you need to change MZ or Gf or alpha_EW

Exercise III: Syntax

- What's the meaning of the order QED/QCD

- What's the difference between
  - ➡ p p > t t~
  - ➡ p p > t t~ QED=2
  - ➡ p p > t t~ QED=0
  - ➡ p p > t t~ QCD^2==2

Solution 1 : Syntax

- What's the meaning of the order QED/QCD

  ➡ By default MG5 takes the lowest order in QED!

  ➡ p p > t t~ => p p > t t~ QED=0

  ➡ p p > t t~ QED=2

    - additional diagrams (photon/z exchange)

p p > t t~

Cross section (pb)
555 ± 0.84

p p > t t~ QED=2

Cross section (pb)
555.8 ± 0.91

No significant QED contribution

- QED<=2 is the SAME as QED=2

  ➡ quite often source of confusion since most of the people use the = syntax

- QCD^2==2

  ➡ returns the interference between the QCD and the QED diagram

Cross section (pb)
5.455e-17 ± 4.7e-19 ± systematics

Solution 1 Syntax

- generate p p > w+ w- j j
  - ➡ 76 processes
  - ➡ 1432 diagrams
  - ➡ None of them are VBF

- generate p p > w+ w- j j QED = 2
  - ➡ 76 processes
  - ➡ 1432 diagrams
  - ➡ None of them are VBF

- generate p p > w+ w- j j QED = 4
  - ➡ 76 processes
  - ➡ 5332 diagrams
  - ➡ VBF present! + those not VBF

- generate p p > w+ w- j j QCD = 0
  - ➡ 60 processes
  - ➡ 3900 diagrams
  - ➡ VBF present!

- generate p p > w+ w- j j QCD = 2
  - ➡ 76 processes
  - ➡ 5332 diagrams

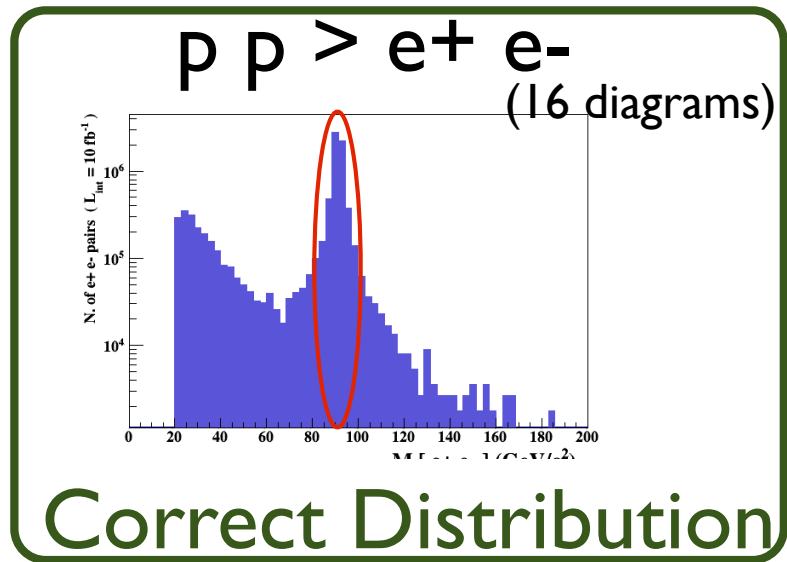- generate p p > w+ w- j j QCD = 4
  - ➡ 76 processes
  - ➡ 5332 diagrams
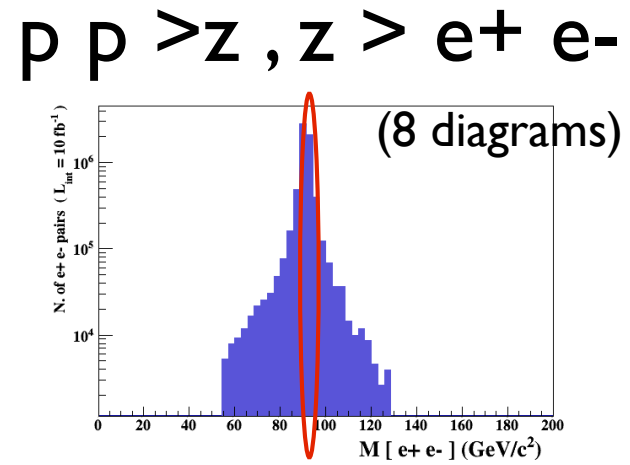
Exercise IV: Syntax

- Generate the cross-section and the distribution (invariant mass) for

  ➡ p p > e+ e-

  ➡ p p > z, z > e+ e-

  ➡ p p > e+ e- $ z

  ➡ p p > e+ e- / z

Hint : To have automatic distributions:
mg5> install MadAnalysis

p p > e+ e-
(16 diagrams)

Z Peak

p p >z , z > e+ e-
(8 diagrams)

Correct Distribution

p p > e+ e- /z

(8 diagrams)

NO Z Peak

No z/a interference

Wrong tail

No Z

p p > e+ e- $ z

(16 diagrams)

z/a interference

Correct tail
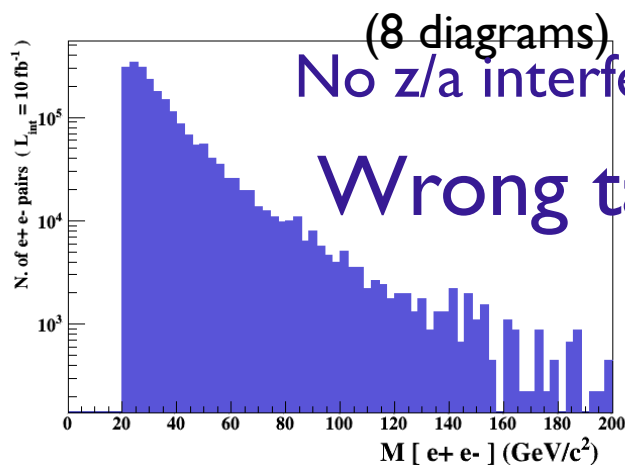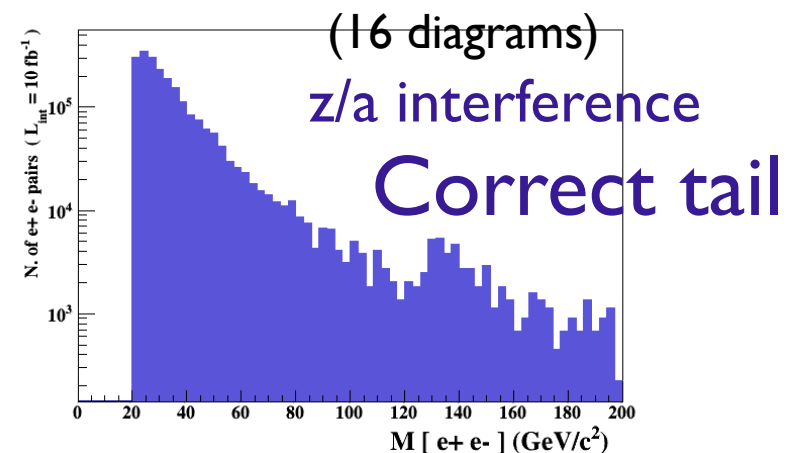
Z- onshell veto

p p > e+ e-

p p >z , z > e+ e-

p p > e+ e- $ z

(16 diagrams)

=

(8 diagrams)

+
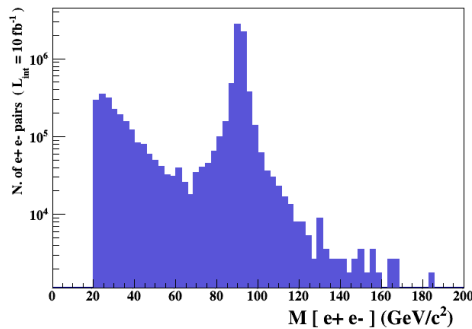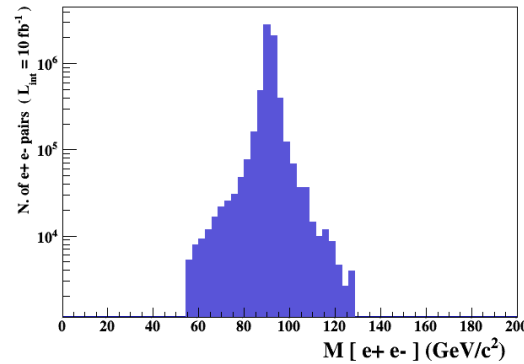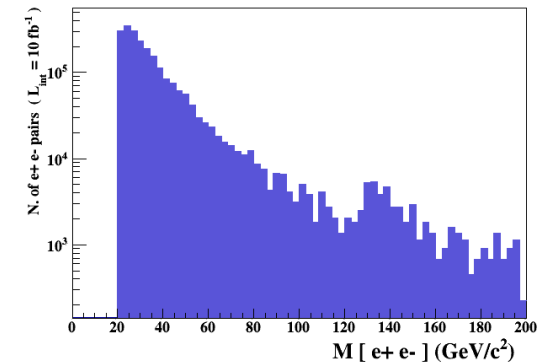
(16 diagrams)

Onshell cut: BW_cut

$$|M^* - M| < BW_{cut} * \Gamma$$

- The Physical distribution is (very close to) exact sum of the two other one.

- The "$" forbids the Z to be onshell but the photon invariant mass can be at MZ (i.e. on shell substraction).

- The "/" is to be avoid if possible since this leads to violation of gauge invariance.

WARNING

- NEXT SLIDE is generated with bw_cut =5

- This is TOO SMALL to have a physical meaning (15 the default value used in previous plot is better)

- This was done to illustrate more in detail how the "$" syntax works.

$ explanation

p p > e+ e- / Z (red curve) adding p p > e+ e- $ Z(blue curve)
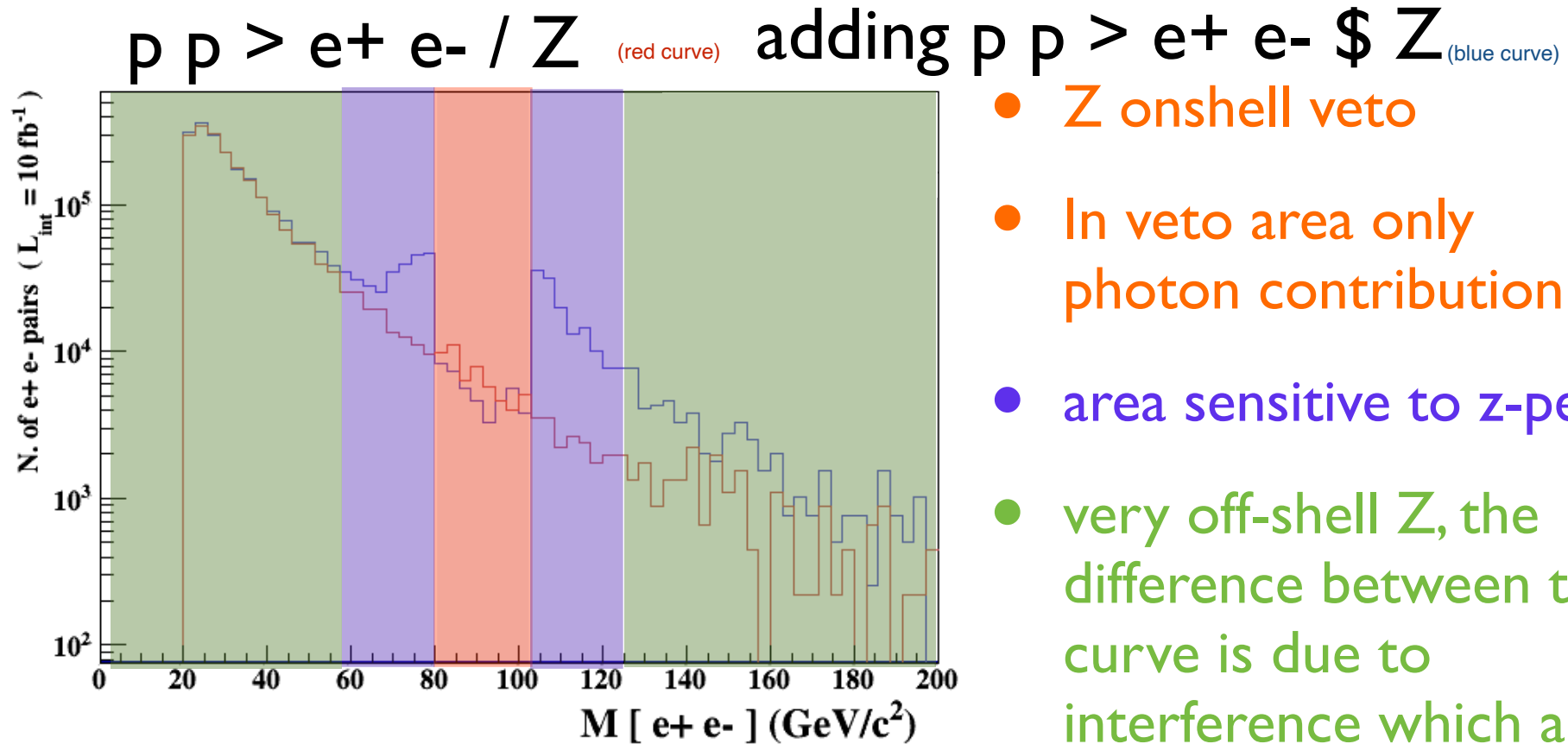
- Z onshell veto

- In veto area only photon contribution

- area sensitive to z-peak

- very off-shell Z, the difference between the curve is due to interference which are need to be KEPT in simulation.

5 times width area

15 times width area

>15 times width area

The "$" can be use to split the sample in BG/SG area

- **Syntax Like**

  ➡ p p > z > e+ e-   (ask one S-channel z)

  ➡ p p > e+ e- / z   (forbids any z)

  ➡ p p > e+ e- $$ z   (forbids any z in s-channel)

- ARE NOT GAUGE INVARIANT !

- forgets diagram interference.

- can provides un-physical distributions.

Avoid Those as much as possible!

check physical meaning and gauge/Lorentz invariance if you do.

- Syntax like

  - p p > z, z > e+ e-   (on-shell z decaying)

  - p p > e+ e- $ z   (forbids s-channel z to be on-shell)

- Are linked to cut $|M^* - M| < BW_{cut} * \Gamma$

- Are more safer to use

- Prefer those syntax to the previous slides one

Exercise 1:
Extra questions:

- Are diagrams with photons/z included? If not, how can I include them? How much does the cross-section change? What is that 'WEIGHTED'?
  - > display diagrams
- No photon/z appear.
- Are we missing anything important?

$$\left| \;\;\right|^2 \sim \alpha_s^2 \qquad \left| \;\;\right|^2 \sim \alpha_e^2 \; \frac{\alpha_e}{\alpha_s} \lesssim 0.1$$
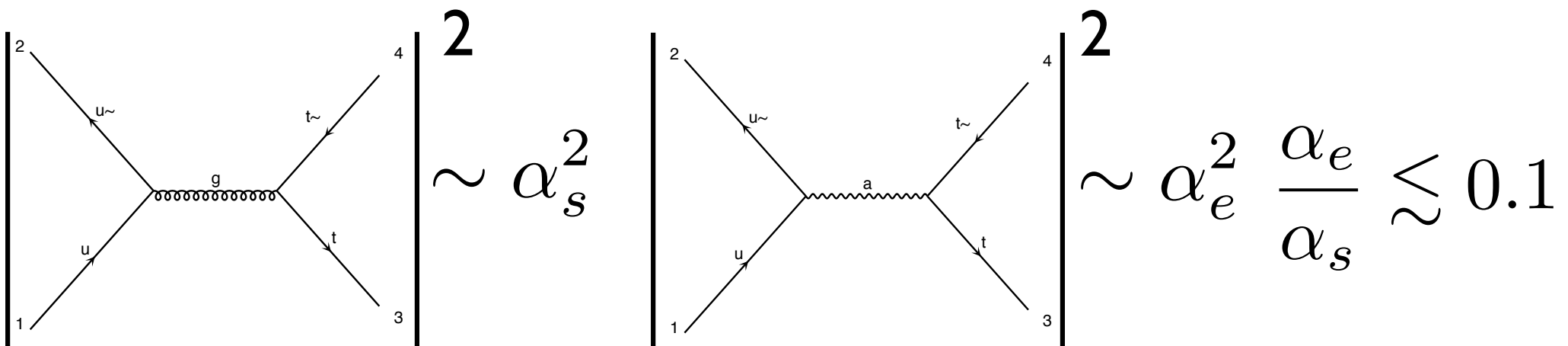
Exercise 1:
Extra questions:

- Are diagrams with photons/z included? If not, how can I include them? How much does the cross-section change? What is that 'WEIGHTED'?
  - > `display diagrams`
- No photon/z appear.
- Are we missing anything important? Does not seem the case
- How to have them anyway?
- MG5 exploits the hierarchy between QCD and QED couplings in order to give the leading (i.e. with most QCD) contribution to the cross-section by default
- It assign WEIGHTED order =1(=2) to QCD (QED) vertices and generates the process with minimum WEIGHTED order

Exercise 1:
Extra questions:

- Are diagrams with photons/z included? If not, how can I include them? How much does the cross-section change?
  - > `generate p p > t t~ WEIGHTED=4`
  - > `display diagrams`
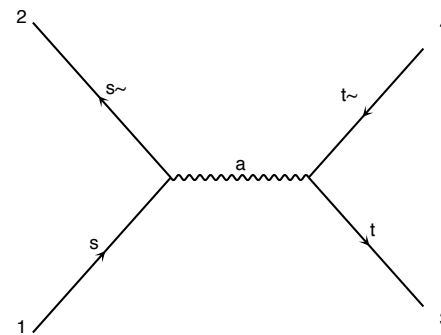 - > `output …`
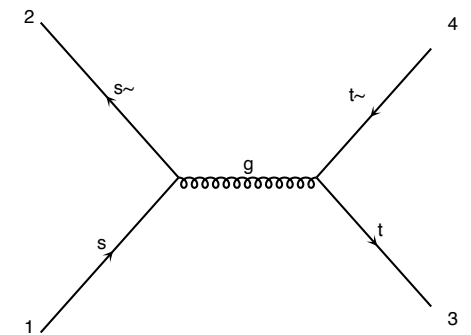 - > `launch`
  - > `…`

diagram 1   QCD=0, QED=2

diagram 2   QCD=2, QED=0

```
Cross-section :    160.8 +- 0.1999 pb
Nb of events :  10000
```

WEIGHTED=2
```
Cross-section :    160.4 +- 0.231 pb
Nb of events :  10000
```
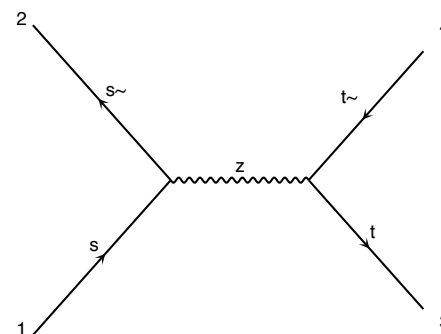

diagram 3   QCD=0, QED=2

Exercise 1:
Extra questions:

- Recompute the $t\bar{t}$ cross-section for m_t=170, 172, 174 … 180 GeV

- Be smart! Script it!

- Create a txt file `myttbar_scan.txt`

```
generate p p > t t~
output mytestdir2
launch
set ebeam1 4000
set ebeam2 4000
set MT 170
launch
set MT 172
launch
set MT 174
launch
set MT 176
launch
set MT 178
launch
set MT 180
```

- `./bin/mg5_aMC myttbar_scan.txt`

Exercise 1:
Extra questions:

- Recompute the $t\bar{t}$ cross-section for m_t=170, 172, 174 … 180 GeV
- Be smart! Script it!
- You can also launch an existing folder, without regenerating the code

```
launch mytestdir2  ⬅
set ebeam1 4000
set ebeam2 4000
set MT 170
launch
set MT 172
launch
set MT 174
launch
set MT 176
launch
set MT 178
launch
set MT 180
```

Exercise 1:
Extra questions:

- Recompute the $t\bar{t}$ cross-section for m_t=170, 172, 174 … 180 GeV

Results in the sm for p p > t t~

Available Results

Run	Collider	Banner	Cross section (pb)	Events	Data	Output	Action	
run_01	p p 4000 x 4000 GeV	tag_1	169.8 ± 0.24	10000	parton madevent	LHE	remove run	launch detector simulation
run_02	p p 4000 x 4000 GeV	tag_1	160.1 ± 0.28	10000	parton madevent	LHE	remove run	launch detector simulation
run_03	p p 4000 x 4000 GeV	tag_1	151.1 ± 0.2	10000	parton madevent	LHE	remove run	launch detector simulation
run_04	p p 4000 x 4000 GeV	tag_1	142.9 ± 0.18	10000	parton madevent	LHE	remove run	launch detector simulation
run_05	p p 4000 x 4000 GeV	tag_1	134.7 ± 0.19	10000	parton madevent	LHE	remove run	launch detector simulation
run_06	p p 4000 x 4000 GeV	tag_1	127.3 ± 0.16	10000	parton madevent	LHE	remove run	launch detector simulation

Main Page

which folder is what?

Exercise 1:
Extra questions:

- Recompute the $t\bar{t}$ cross-section for m_t=170, 172, 174 … 180 GeV
- Be smart! Script it!
- You can specify the name (instead of `run_01`…) with `-n NAME`

```
launch mytestdir2 -n run_MT170
set ebeam1 4000
set ebeam2 4000
set MT 170
launch -n run_MT172
set MT 172
launch -n run_MT174
set MT 174
launch -n run_MT176
set MT 176
launch -n run_MT178
set MT 178
launch -n run_MT180
set MT 180
```

Exercise 1:
Extra questions:

- Since recently, multiple values can be specified for parameters. Just set in the param_card, instead of the top mass
6 scan:[170,172,174,176,178]

Exercise 1:
Extra questions:

- Recompute the $t\bar{t}$ cross-section for m_t=170, 172, 174 … 180 GeV

Run	Collider	Banner	Cross section (pb)	Events	Data	Output	Action	
run_01	p p 4000 x 4000 GeV	tag_1	169.8 ± 0.24	10000	parton madevent	LHE	remove run	launch detector simulation
run_02	p p 4000 x 4000 GeV	tag_1	160.1 ± 0.28	10000	parton madevent	LHE	remove run	launch detector simulation
run_03	p p 4000 x 4000 GeV	tag_1	151.1 ± 0.2	10000	parton madevent	LHE	remove run	launch detector simulation
run_04	p p 4000 x 4000 GeV	tag_1	142.9 ± 0.18	10000	parton madevent	LHE	remove run	launch detector simulation
run_05	p p 4000 x 4000 GeV	tag_1	134.7 ± 0.19	10000	parton madevent	LHE	remove run	launch detector simulation
run_06	p p 4000 x 4000 GeV	tag_1	127.3 ± 0.16	10000	parton madevent	LHE	remove run	launch detector simulation
run_MT170	p p 4000 x 4000 GeV	tag_1	170 ± 0.22	10000	parton madevent	LHE	remove run	launch detector simulation
run_MT172	p p 4000 x 4000 GeV	tag_1	159.6 ± 0.22	10000	parton madevent	LHE	remove run	launch detector simulation
run_MT174	p p 4000 x 4000 GeV	tag_1	151.1 ± 0.22	10000	parton madevent	LHE	remove run	launch detector simulation
run_MT176	p p 4000 x 4000 GeV	tag_1	142.6 ± 0.19	10000	parton madevent	LHE	remove run	launch detector simulation
run_MT178	p p 4000 x 4000 GeV	tag_1	134.7 ± 0.18	10000	parton madevent	LHE	remove run	launch detector simulation
run_MT180	p p 4000 x 4000 GeV	tag_1	127.2 ± 0.24	10000	parton madevent	LHE	remove run	launch detector simulation

Main Page

Add the decay

MadSpin

- generate p p > t t~

MadSpin Card

➡ decay t > w+ b, w+ > e+ ve

➡ decay t~ >w- b~, w- > e- ve~

MadGraph

- generate p p > t t~ h, (t > w+ b, w+ > e+ ve), (t~ >w- b~, w- > e- ve~)

By default cut does not apply to particle originated from an on shell decay

Mass-Scan (with decay)

- Generate p p > t t~, t > w+ b, t~ > w- b~
- Output
- Launch
- Set mt scan:range(170,181,2)

Scan

Available Results

Run	Collider	Banner	Cross section (pb)	Events	Data	Output	Action
run_01	p p 6500.0 x 6500.0 GeV	tag_1	462.6 ± 0.72 ± systematics	10000	parton madevent	LHE plots	remove run launch detector simulation
run_02	p p 6500.0 x 6500.0 GeV	tag_1	476.2 ± 0.72 ± systematics	10000	parton madevent	LHE plots	remove run launch detector simulation
run_03	p p 6500.0 x 6500.0 GeV	tag_1	488.2 ± 0.86 ± systematics	10000	parton madevent	LHE plots	remove run launch detector simulation
run_04	p p 6500.0 x 6500.0 GeV	tag_1	502.5 ± 0.8 ± systematics	10000	parton madevent	LHE plots	remove run launch detector simulation
run_05	p p 6500.0 x 6500.0 GeV	tag_1	514 ± 0.78 ± systematics	10000	parton madevent	LHE plots	remove run launch detector simulation
run_06	p p 6500.0 x 6500.0 GeV	tag_1	528.6 ± 0.87 ± systematics	10000	parton madevent	LHE plots	remove run launch detector simulation

Main Page

- What's wrong?
  ➡ Why the cross-section increase?

Mass-Scan (with decay)

- The width was not updated. Let's fix it:
    - ➡Generate p p > t t~, t > w+ b, t~ > w- b~
    - ➡Output
    - ➡Launch
    - ➡Set mt scan:range(170,181,2)
    - ➡Set wt auto

• With the LO width

run_07	p p 6500.0 x 6500.0 GeV	tag_1	524.4 ± 0.89 ± systematics	10000	parton madevent	LHE plots	remove run	launch detector simulation
run_08	p p 6500.0 x 6500.0 GeV	tag_1	496.6 ± 0.9 ± systematics	10000	parton madevent	LHE plots	remove run	launch detector simulation
run_09	p p 6500.0 x 6500.0 GeV	tag_1	468.5 ± 0.79 ± systematics	10000	parton madevent	LHE plots	remove run	launch detector simulation
run_10	p p 6500.0 x 6500.0 GeV	tag_1	446 ± 0.79 ± systematics	10000	parton madevent	LHE plots	remove run	launch detector simulation
run_11	p p 6500.0 x 6500.0 GeV	tag_1	421.8 ± 0.81 ± systematics	10000	parton madevent	LHE plots	remove run	launch detector simulation
run_12	p p 6500.0 x 6500.0 GeV	tag_1	400.3 ± 0.63 ± systematics	10000	parton madevent	LHE plots	remove run	launch detector simulation

Main Page

BSM Tutorial

Exercise I: Check the model validity

- Check the model validity:
  - ➡ check p p > uv uv~
  - ➡ check p p > ev ev~
  - ➡ check p p > t t~ p1 p2

- This checks
  - ➡ gauge invariance
  - ➡ lorentz invariance
  - ➡ that various way to compute the matrix element provides the same answer

Exercise II: Width computation

- **Check with MG the width computed with FR:**
  - ➡ generate uv > all all; output; launch
  - ➡ generate ev > all all; output; launch
  - ➡ generate p1 > all all; output; launch
  - ➡ generate p2 > all all; output; launch

- **Check with MadWidth**
  - ➡ compute_widths uv ev p1 p2
  - ➡ (or Auto in the param_card)

FR Number
0.0706 GeV
0.00497 GeV
0 GeV
0.0224 GeV

- Muv = 400 GeV   Mev = 50 GeV λ=0.1
- m1 = 1GeV   m2 = 100GeV   m12 = 0.5 GeV

Exercise III:

- Compute cross-section and distribution

  ➡ uv pair production with decay in top and Φ_1/Φ_2 (semi leptonic decay for the top

- **Hint:** The width of the new physics particles has to be set correctly in the param_card.

  ➡ You can either use "Auto"   arXiv:1402.1178

  ➡ or use the value computed in exercise 1

- **Hint:** For sub-decay, you have to put parenthesis:

  ➡ example:
  p p > t t~ w+, (t > w+ b, w+ >e+ ve), (t~ > b~ w-, w- > j j), w+ > l+ vl

Too Slow?

- Use MadSpin!

  ➡ Use Narrow Width Approximation to factorize production and decay

- instead of

  ➡ p p > t t~ w+, (t > w+ b, w+ >e+ ve), (t~ > b~ w-, w- > j j), w+ > l+ vl

- Do

  ➡ p p > t t~ w+

- At the question:

```
The following switches determine which programs are run:
 1 Run the pythia shower/hadronization:                    pythia=OFF
 2 Run PGS as detector simulator:                              pgs=OFF
 3 Run Delphes as detector simulator:                      delphes=NOT INSTA
 4 Decay particles with the MadSpin module:                madspin=OFF
 5 Add weight to events based on coupling parameters:     reweight=OFF
  Either type the switch number (1 to 5) to change its default setting,
  or set any switch explicitly (e.g. type 'madspin=ON' at the prompt)
  Type '0', 'auto', 'done' or just press enter when you are done.
[0, 1, 2, 4, 5, auto, done, pythia=ON, pythia=OFF, ... ][60s to answer]
```

- At the next question edit the madspin_card and define the decay

Exercise IV: generate multiple multiplicity sample for pythia8

- We will do MLM matching

  ➡ in the run_card.dat ickkw=1

  ➡ the matching scale (Qcut) will be define in pythia

    - in madgraph we use xqcut which should be smaller than Qcut (but at least 10-20 GeV)

Exercise V: Have Fun

- Simulate Background

- Go to NLO (ask me the model)

- …

Loop Tutorial

- Continue tutorial of yesterday

  ➡ Scan on top pair production

- Compare loop-induced process with Higgs Effective Theory:

  ➡ Compare the cross-section for g g > h

  ☐ In "heft" model

  ☐ In sm (g g > h [QCD])

  ➡ Compare the jet transverse momenta for "g g > h g" in both theory

Cross-section

- HEFT:

  - ➡ Import model heft; generate g g > h; output; launch

  - ➡ 17.62 pb

- SM:
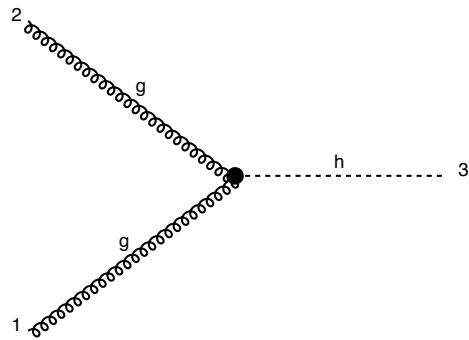
  - ➡ Import model sm; generate g g > h [QCD];output;launch

  - ➡ 15.69 +- 0.05053 pb
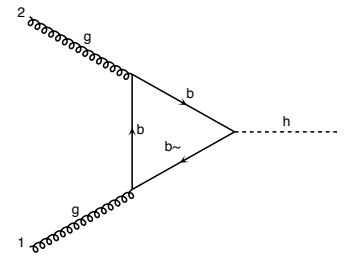
  - ➡ Why so different?

Feynman diagram

- HEFT

- SM

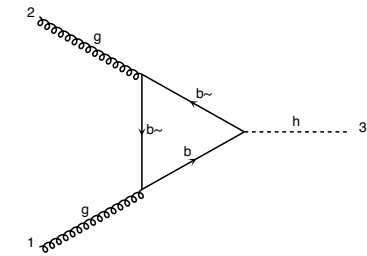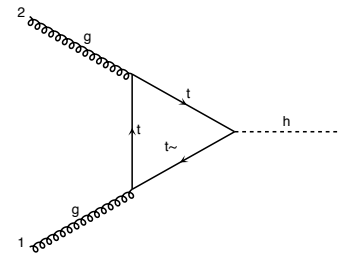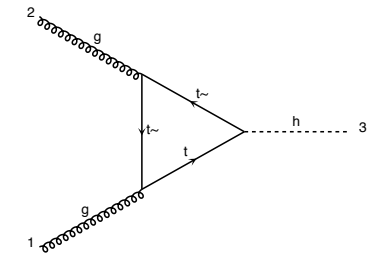- Remove the "b" loop:

 ➡ Import model sm-no_b_mass; generate g g > h [QCD];output;launch

  ➡ 17.59 pb

  ➡ The "b" loop itself is negligeable, the impact here is 100% the interference term.

  ➡ The lighter quark (mainly c) have the same effect (at the level of the percent)

process	EFT	Exact loop-Induced (LI)	Exact loop-Induced $m_b = 0$ (NoB)
$gg \to h$	19.996(4) pb	17.79(6) pb	19.94(4) pb
$pp \to hj$	13.41(2) pb	12.86(4) pb	13.24(4) pb
$pp \to hjj$	6.31(2) pb	6.18(2) pb	6.13(1) pb

PT distribution
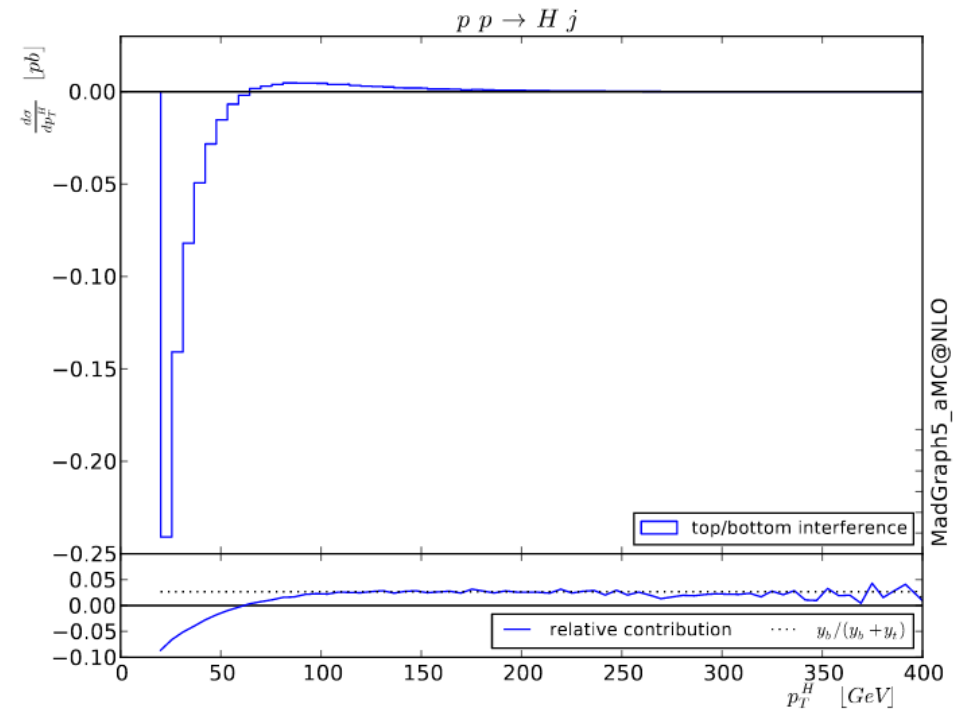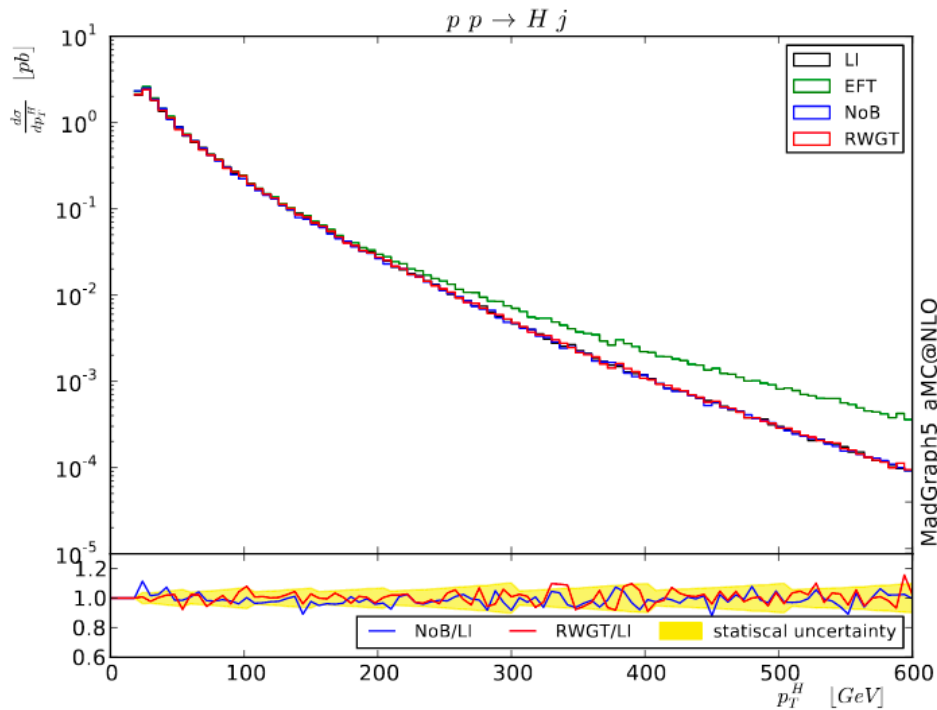
- HEFT is working fine at low energy (as expected)
- At low pt, the b diagram decrease the cross-section

Shower Tutorial

- Compare

  ➡ Top-pair production (+ parton shower)

  ➡ Top-pair plus one jet

Matching/Merging Tutorial

matching in MadGraph+Pythia

Example: Simulation of pp→W with 0, 1, 2 jets
(comfortable on a laptop)

```
mg5> generate p p > w+, w+ > l+ vl @0
mg5> add process p p > w+ j, w+ > l+ vl @1
mg5> add process p p > w+ j j, w+ > l+ vl @2
mg5> output
```

```
In run_card.dat:
…
   1 = ickkw
…
   0 = ptj
…
  15 = xqcut
```

Matching on

No cone matching

k_T matching scale

Matching automatically done when run through
MadEvent and Pythia!

Merging scale can be defined in PY8 card (or left auto)

How to do validate the matching

- The matching scale (QCUT) should typically be chosen around 1/6-1/2 x hard scale (so xqcut correspondingly lower)

- The matched cross section (for X+0,1,... jets) should be close to the unmatched cross section for the 0-jet sample (found on the process HTML page)

- The differential jet rate plots should be smooth

- When QCUT is varied (within the region of validity), the matched cross section or differential jet rates should not vary significantly
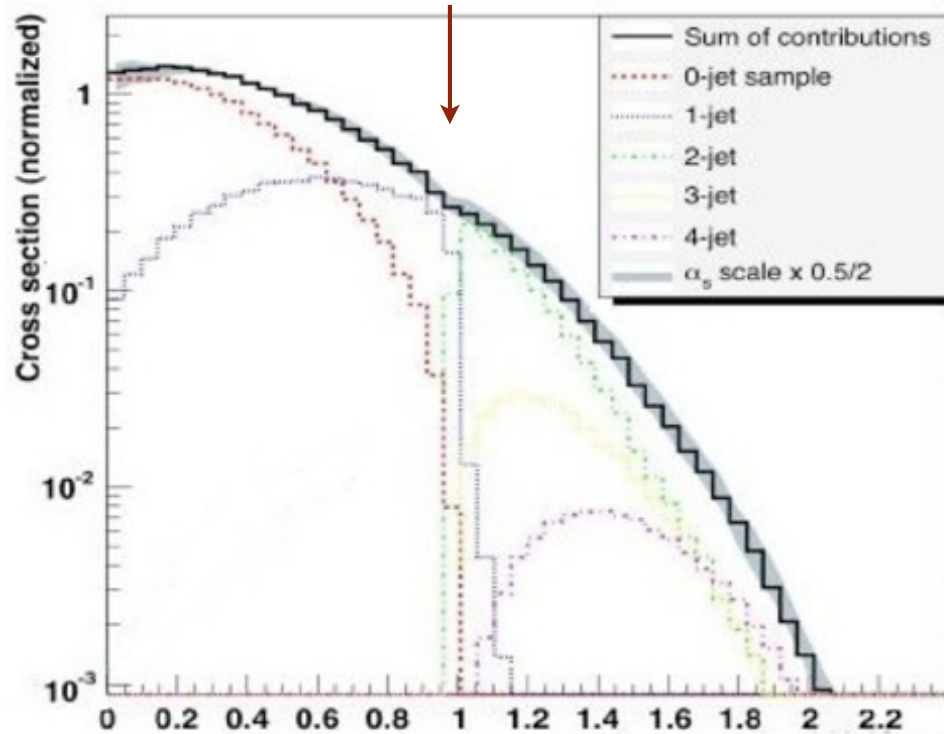
Differential Jet Rate Plot

- This are the clustering scales in the kt-jet clustering scheme

- DJR1: pT of the last remaining jet

- DJR2: The minimum between the pT of the second to last remaining jet and the kt between the last two jet.

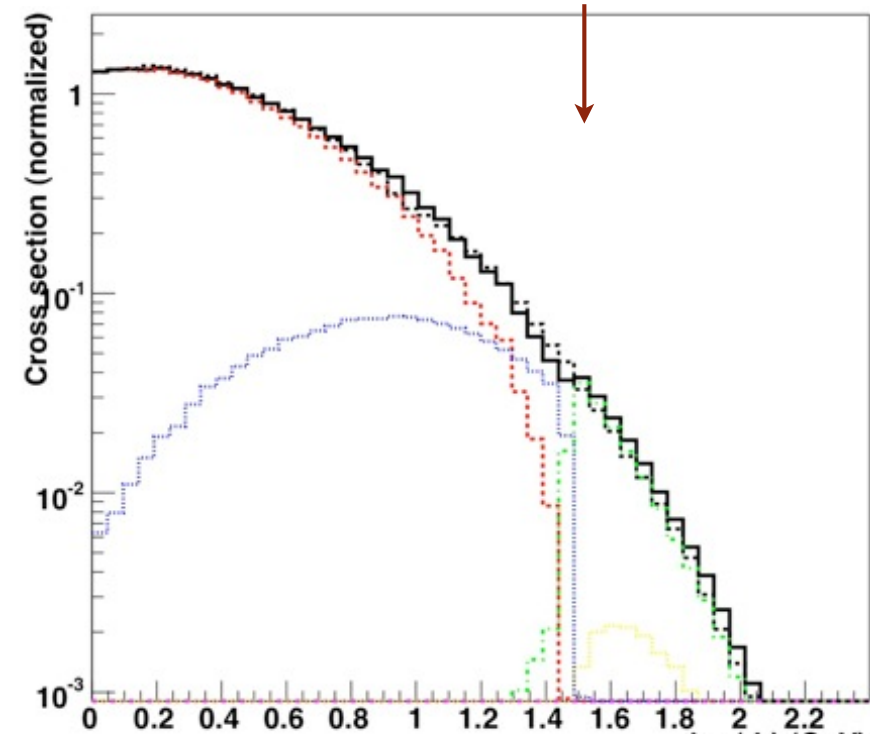- Only radiative jet (not those from decay) should enter those plot.
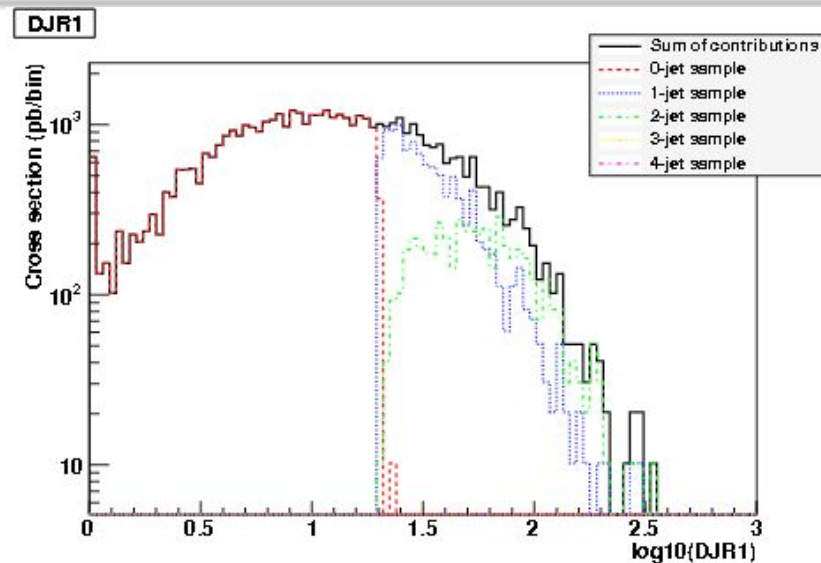
Matching Validation

W+jets production at the Tevatron for MadGraph+Pythia
(k_T-jet MLM scheme, q^2-ordered Pythia showers)

Q^{match} = 10 GeV

Q^{match} = 30 GeV

log(Differential jet rate for 1 → 2 radiated jets ~ pT(2nd jet))

Matching Validation

W+jets production at the Tevatron for MadGraph+Pythia
(k_T-jet MLM scheme, q^2-ordered Pythia showers)

Jet distributions smooth, and stable when we vary the matching scale!
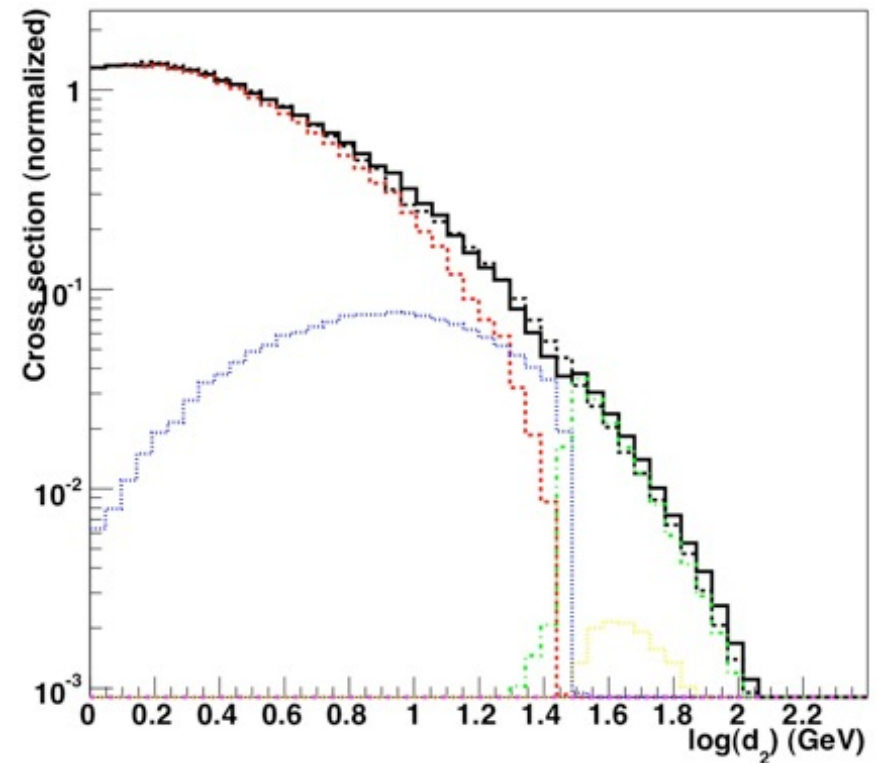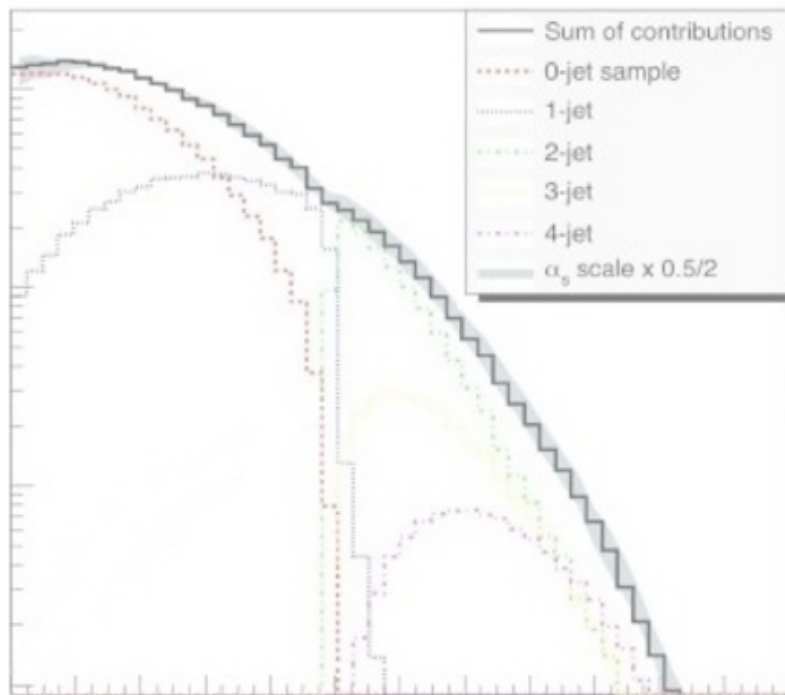
MLM

- Generate p p > t t~

- Add process p p > t t~ j

- Output; Launch
  - ➡ Ask for MadSpin and Pythia8 and MA5
  - ➡ set mpi OFF # This is for speed issue for the tuto
  - ➡ decay t > w+ b, w+ > e+ ve
  - ➡ decay t~ > w- b~, w- > e- ve~
  - ➡ set xqcut 30 #minimal distance between quark/gluon @tree-level
  - ➡ set jetmatching:Qcut 60 #the MLM matching scale

Validation of MLM
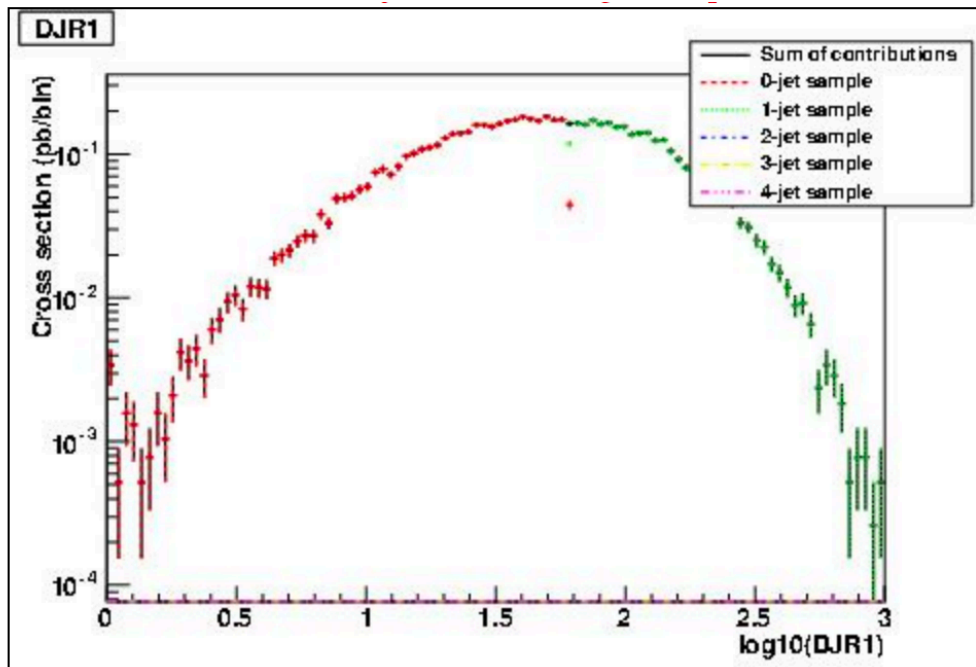
DJR1

Download PS DJR1.ps
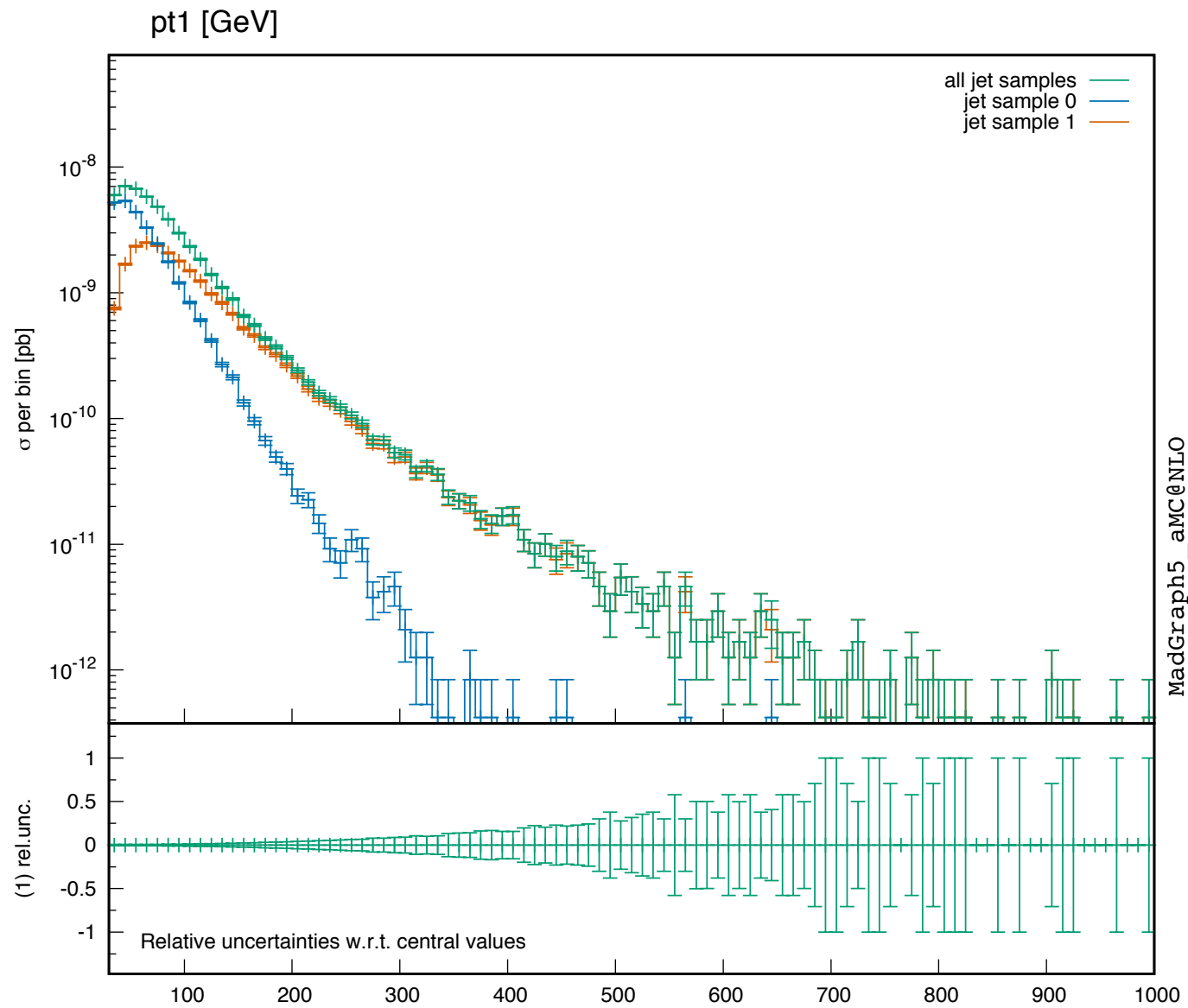
log10d01 []

PT distribution

Exercise VI: Matching+Merging

	w+0j	w+1j	w+2j	w+3j
no matching	8,35E+04	1,58E+04	8,7E+03	3,5E+03

	1GeV	10GeV	20GeV	50GeV	100GeV	500GeV
w+0	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04
0+1	1,07E+05	9,09E+04	8,91E+04	8,61E+04	8,40E+04	8.35+04
0+1+2	1,12E+05	9,29E+04	9,03E+04	8,66E+04	8,44E+04	8,35E+04
0+1+2+3	1,20E+05	9,47E+04	9,07E+04	8,68E+04	8,40E+04	8,35E+04

Slow   Fast

low efficiency   High efficiency

Exercise VI: Matching+Merging

	w+0j	w+1j	w+2j	w+3j
no matching	8,35E+04	1,58E+04	8,7E+03	3,5E+03

	1GeV	10GeV	20GeV	50GeV	100GeV	500GeV
w+0	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04
0+1	1,07E+05	9,09E+04	8,91E+04	8,61E+04	8,40E+04	8.35+04
0+1+2	1,12E+05	9,29E+04	9,03E+04	8,66E+04	8,44E+04	8,35E+04
0+1+2+3	1,20E+05	9,47E+04	9,07E+04	8,68E+04	8,40E+04	8,35E+04

Exercise VI: Matching+Merging

	w+0j	w+1j	w+2j	w+3j
no matching	8,35E+04	1,58E+04	8,7E+03	3,5E+03

	1GeV	**10GeV**	**20GeV**	**50GeV**	**100GeV**	**500GeV**
w+0	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04
0+1	1,07E+05	9,09E+04	8,91E+04	8,61E+04	8,40E+04	8.35+04
0+1+2	1,12E+05	9,29E+04	9,03E+04	8,66E+04	8,44E+04	8,35E+04
0+1+2+3	1,20E+05	9,47E+04	9,07E+04	8,68E+04	8,40E+04	8,35E+04

Exercise VI: Matching+Merging

	w+0j	w+1j	w+2j	w+3j
no matching	8,35E+04	1,58E+04	8,7E+03	3,5E+03

	1GeV	10GeV	20GeV	50GeV	100GeV	500GeV
w+0	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04
0+1	1,07E+05	9,09E+04	8,91E+04	8,61E+04	8,40E+04	8.35+04
0+1+2	1,12E+05	9,29E+04	9,03E+04	8,66E+04	8,44E+04	8,35E+04
0+1+2+3	1,20E+05	9,47E+04	9,07E+04	8,68E+04	8,40E+04	8,35E+04

Exercise VI: Matching+Merging

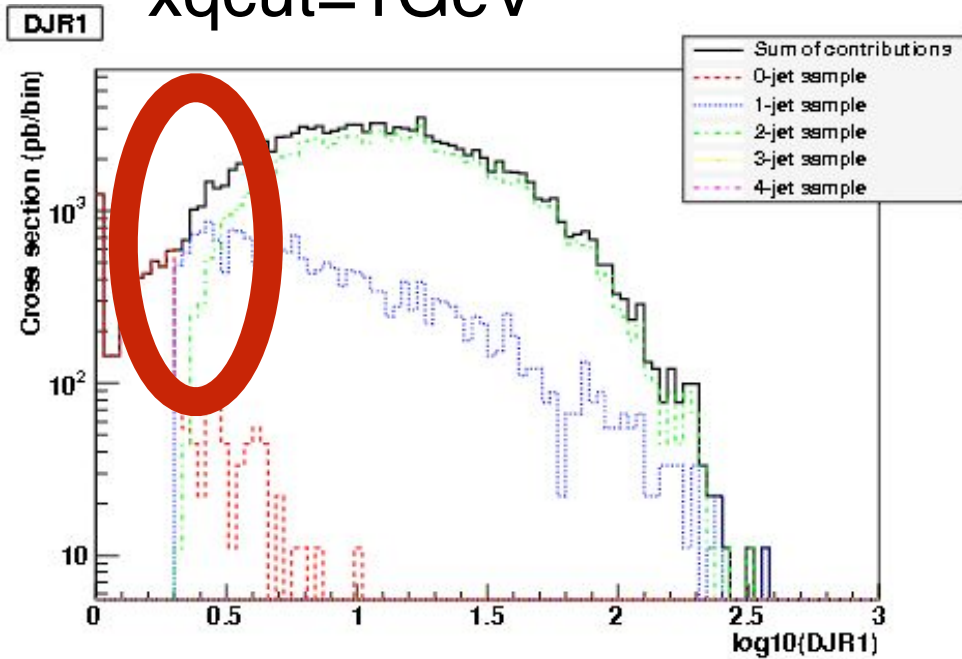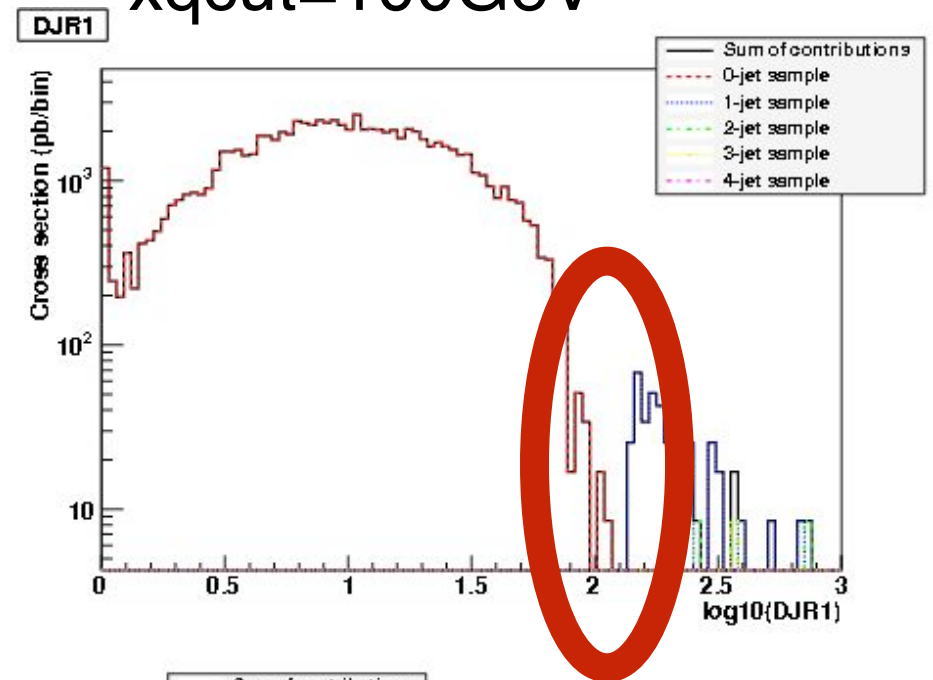|	w+0j	w+1j	w+2j	w+3j
no matching	8,35E+04	1,58E+04	8,7E+03	3,5E+03

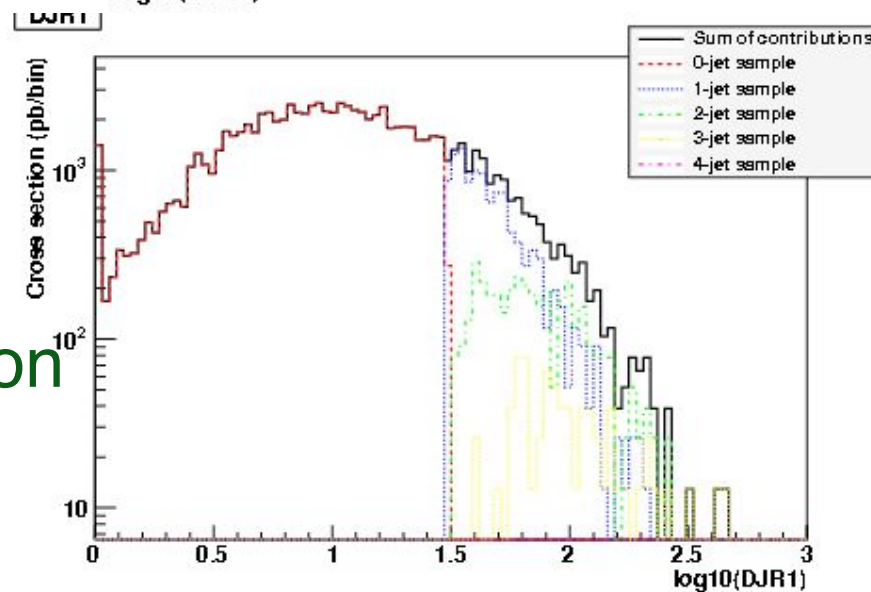|	1GeV	10GeV	20GeV	50GeV	100GeV	500GeV
w+0	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04
0+1	1,07E+05	9,09E+04	8,91E+04	8,61E+04	8,40E+04	8.35+04
0+1+2	1,12E+05	9,29E+04	9,03E+04	8,66E+04	8,44E+04	8,35E+04
0+1+2+3	1,20E+05	9,47E+04	9,07E+04	8,68E+04	8,40E+04	8,35E+04

xqcut=1GeV

xqcut=100GeV

xqcut=20GeV
smooth transition

Exercise VI: Matching+Merging

	w+0j	w+1j	w+2j	w+3j
no matching	8,35E+04	1,58E+04	8,7E+03	3,5E+03

	1GeV	10GeV	20GeV	50GeV	100GeV	500GeV
w+0	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04	8,35E+04
0+1	1,07E+05	9,99E+04	8,91E+04	8,61E+04	8,40E+04	8.35+04
0+1+2	1,12E+05	9,29E+04	9,03E+04	8,66E+04	8,44E+04	8,35E+04
0+1+2+3	1,20E+05	9,47E+04	9,07E+04	8,68E+04	8,40E+04	8,35E+04