Monte-Carlo Generation

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Plan

Lectures

- Matrix-element, phase-space generation
- Parton-Shower matching/merging
- Loop Computation
- NLO

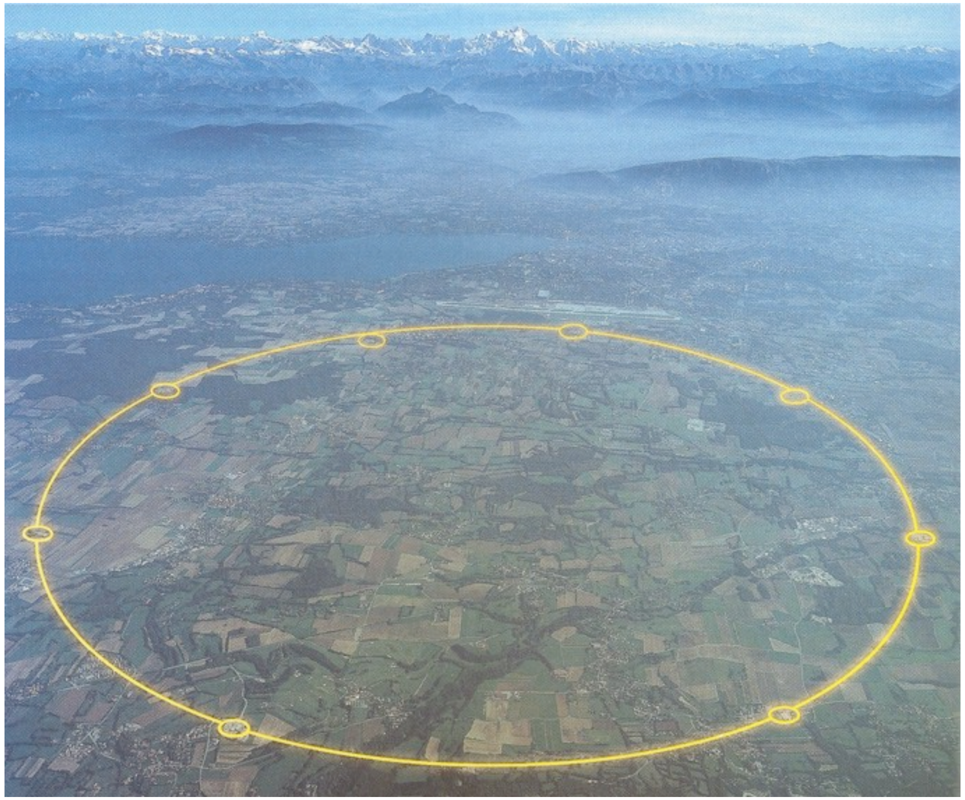
Tutorial

- MG5aMC
- Parton-Shower + MLM matching/merging
- Loop induced processes
- NLO

Introduction

Topic

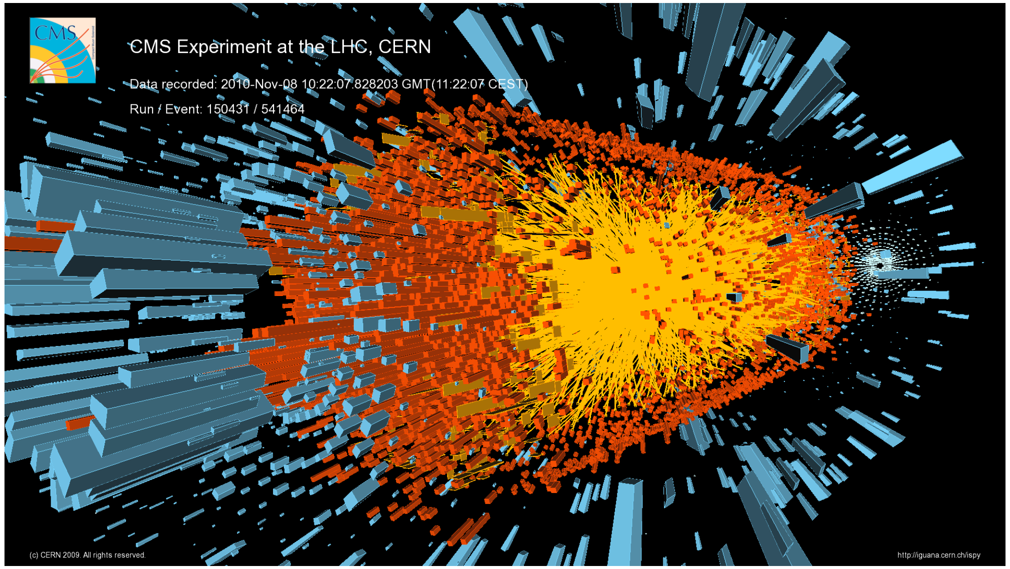
- Collider Physics
  - accelerating particle -> High Energy collision

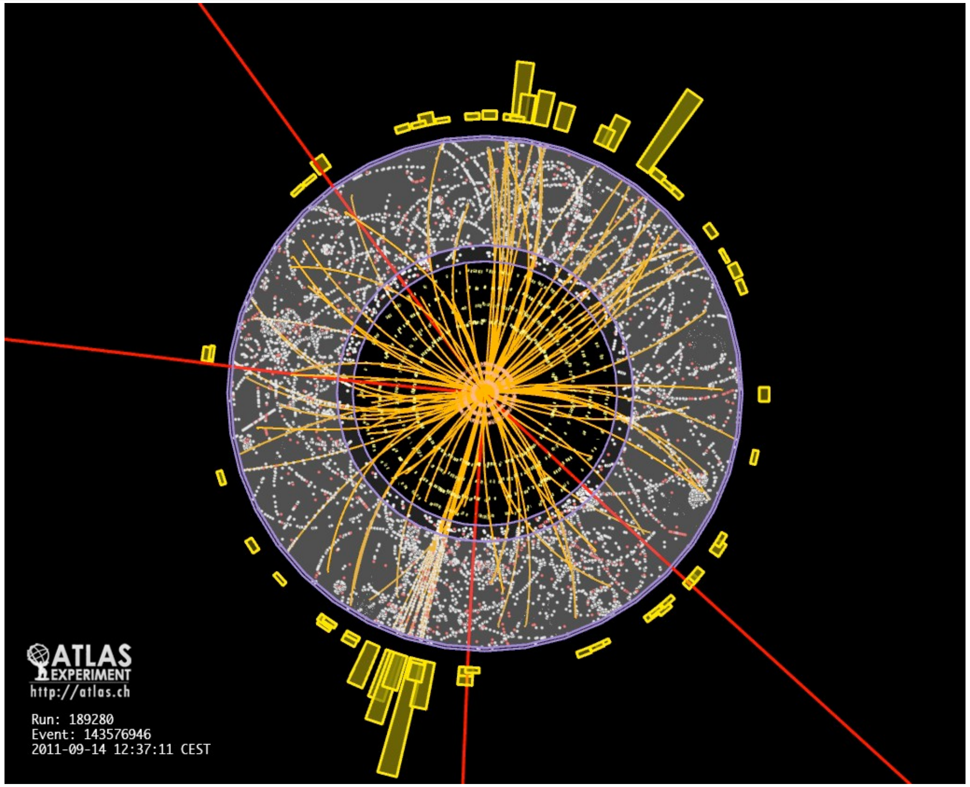


Introduction

Topic

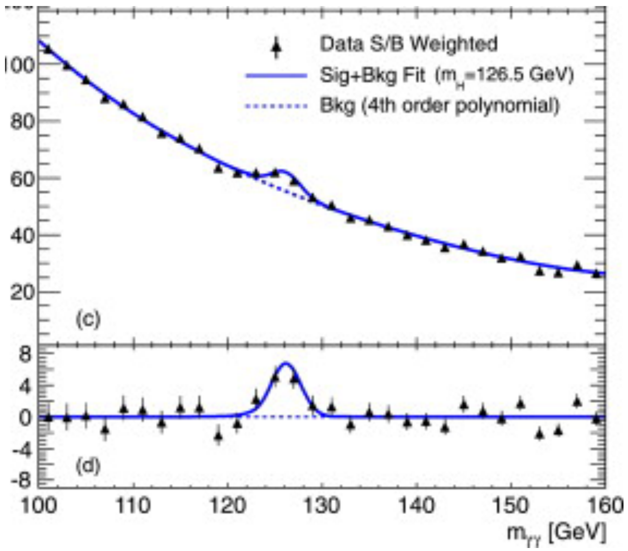
- Collider Physics
  - accelerating particle -> High Energy collision
- What do we need to predict/understand such collision?





Kind of measurement

Peak



Background directly measured from **data**. Theory needed only for parameter extraction

Kind of measurement

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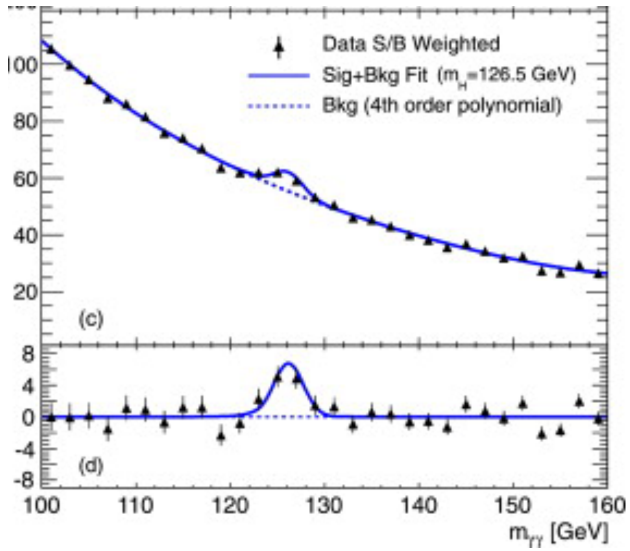
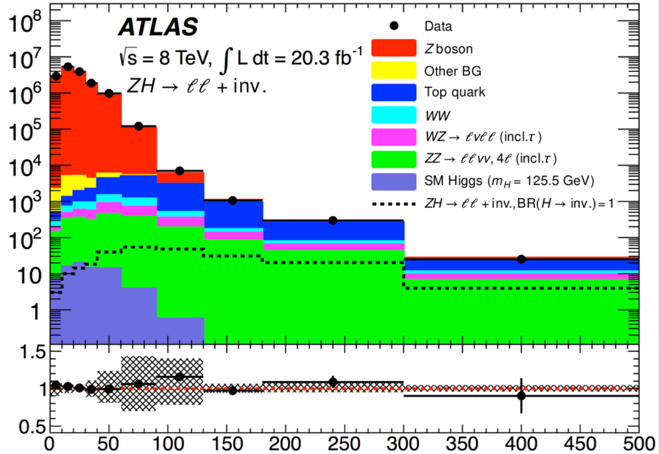


Background directly measured from **data**. Theory needed only for parameter extraction

Shape



Background **SHAPE** needed. Flexible MC for both signal and background validated and tuned to data

Kind of measurement

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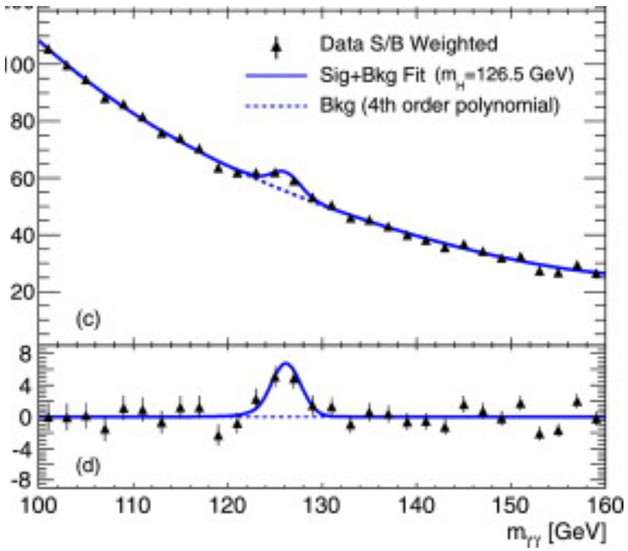

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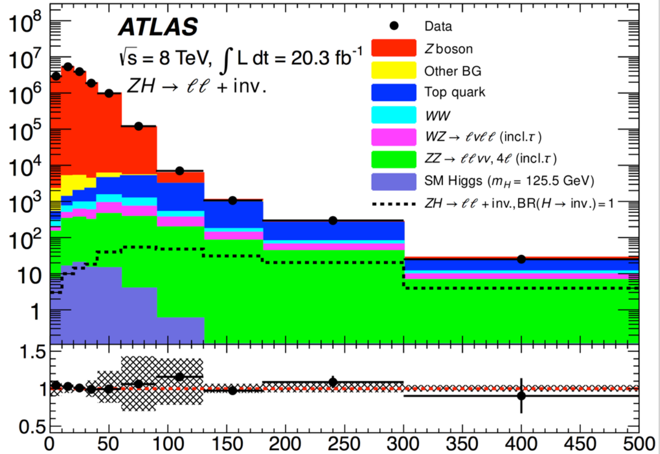
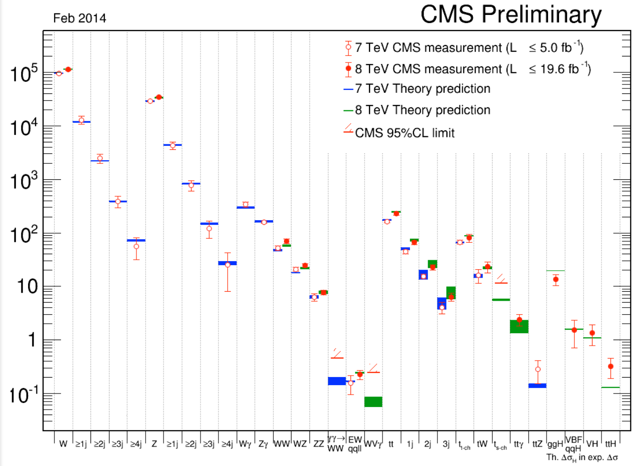


Background **SHAPE** needed. Flexible MC for both signal and background validated and tuned to data

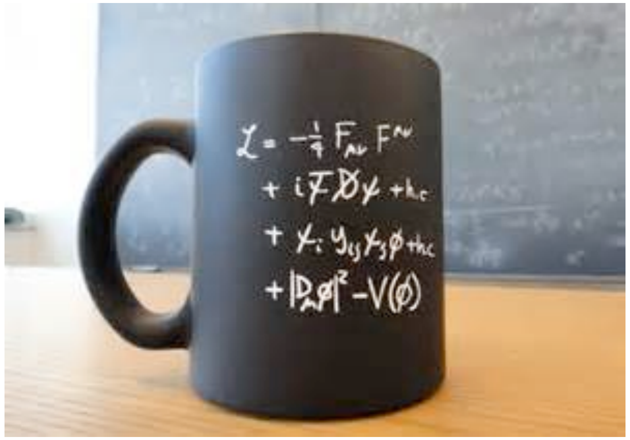
Rate



Relies on prediction for both **shape** and **normalization**. Complicated interplay of best simulations and data

Theory side

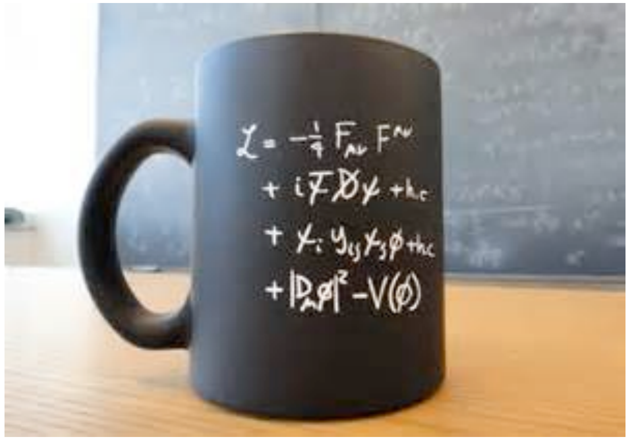
Lagrangian



- This is Where the new idea are expressed

Theory side

Lagrangian



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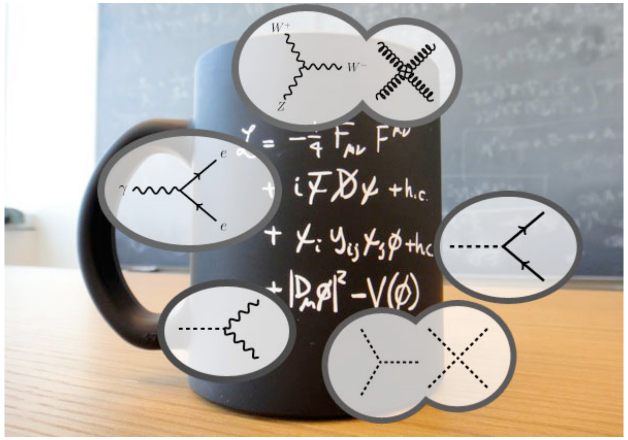
Feynman Rule



- Same information as the Lagrangian

FeynRules

Theory side

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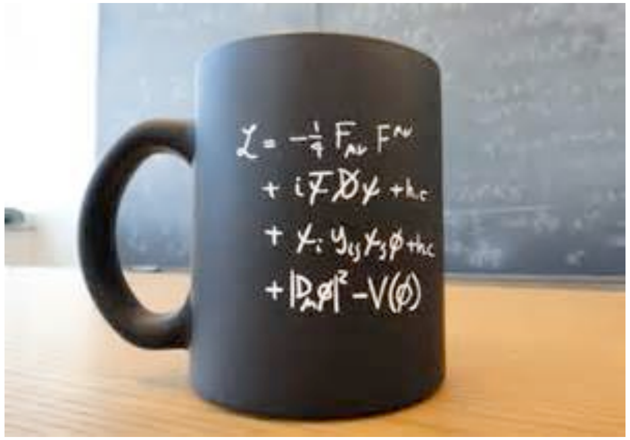

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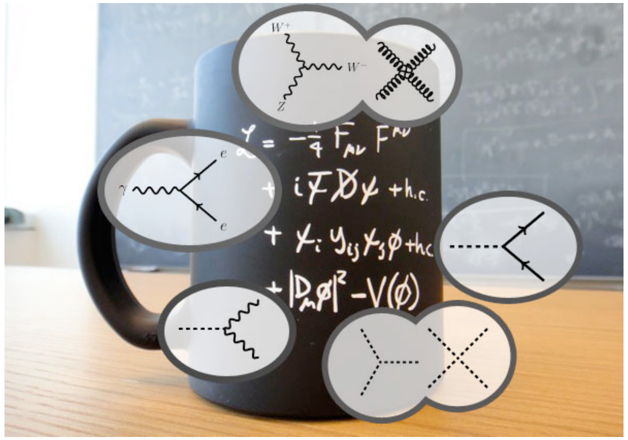
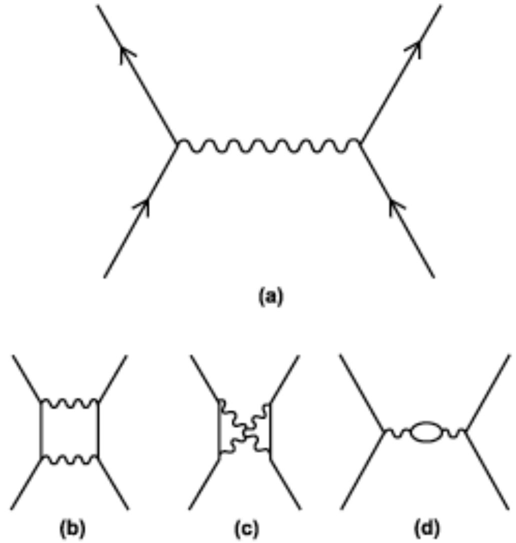


- Same information as the Lagrangian

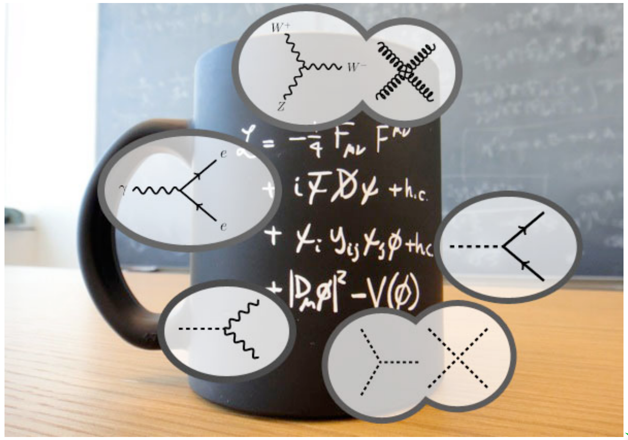
FeynRules

Cross-section

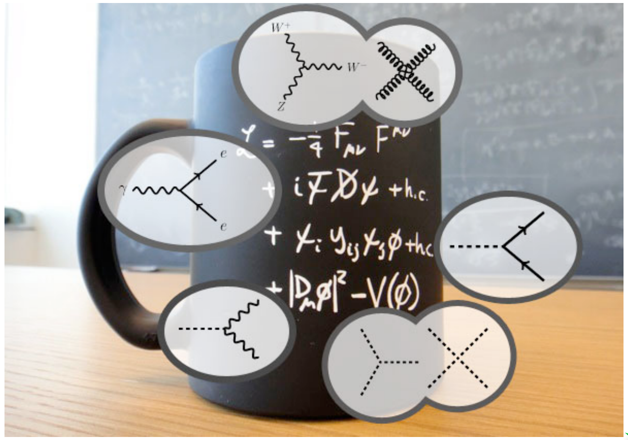
$$\frac{d\sigma}{d\ \cos\theta} = \left(\frac{d\sigma}{d\ \cos\theta}\right)_R \frac{(1+\cos\theta)/2}{\left[1 + \frac{(1-\cos\theta)KE}{Mc^2}\right]}$$



- What is the precision?



Monte-Carlo Physics



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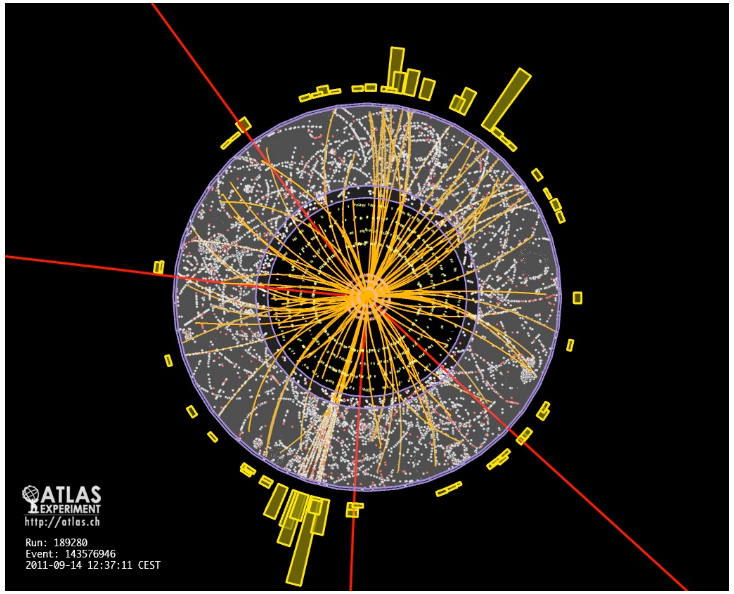
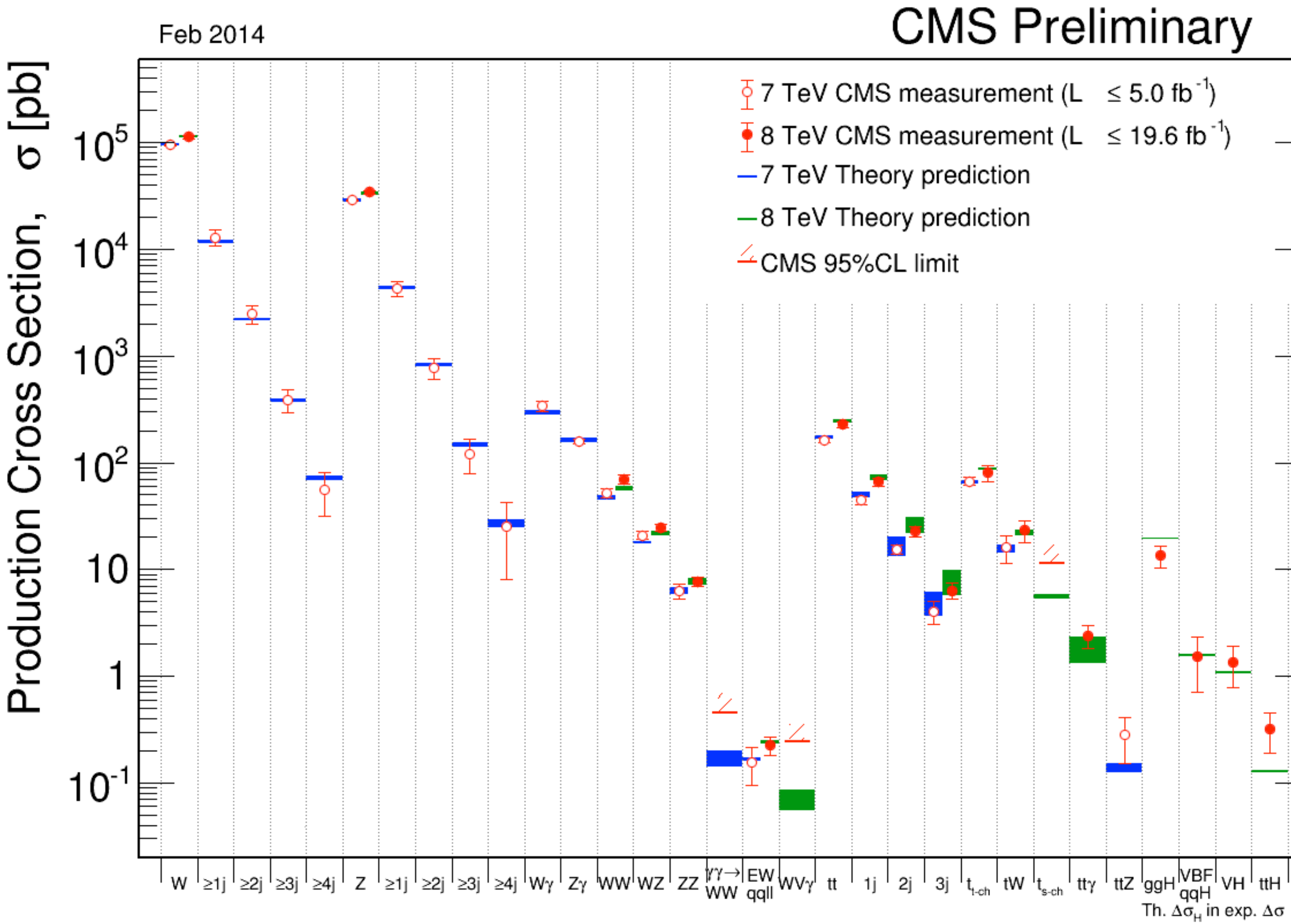


Standard Model



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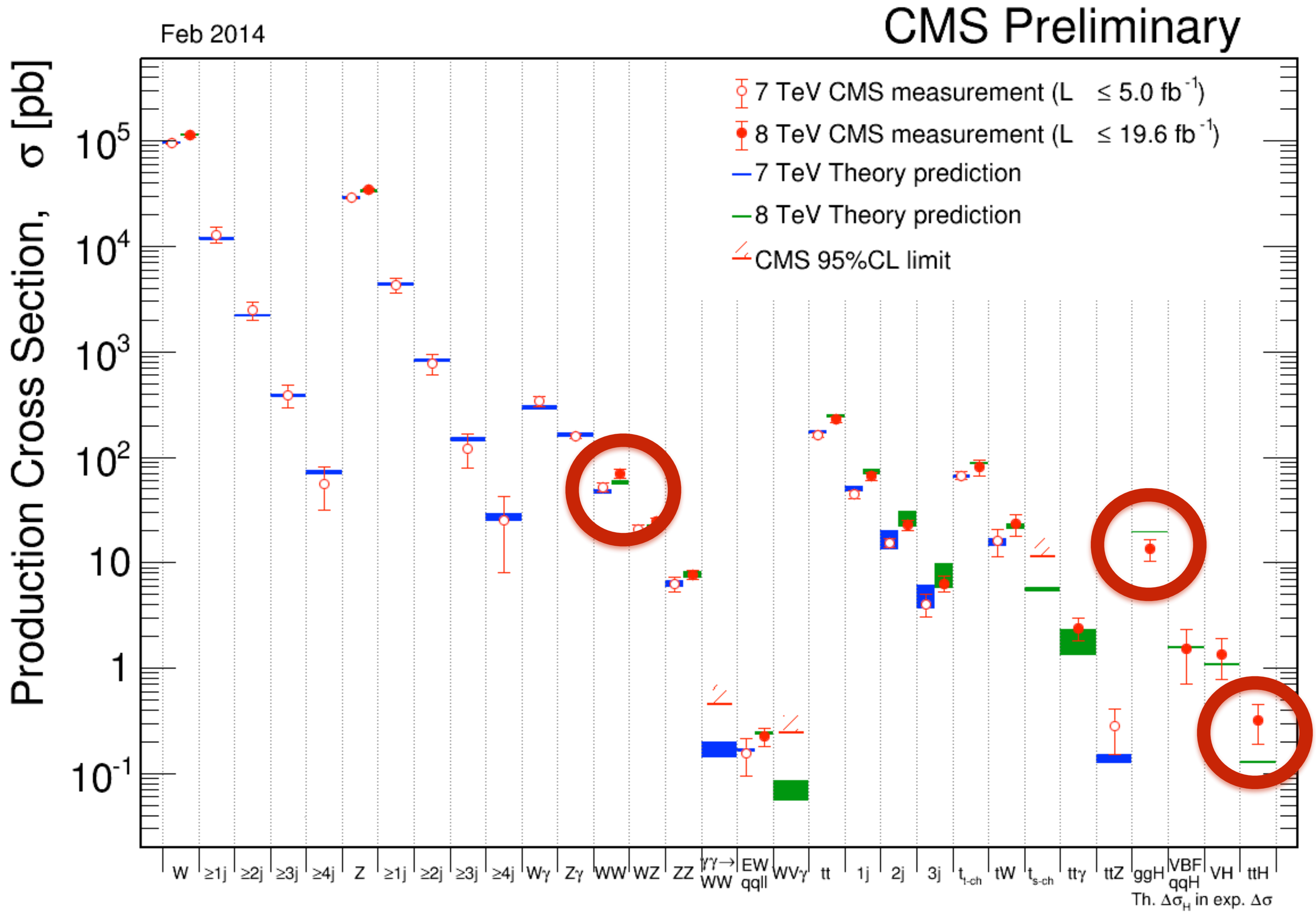
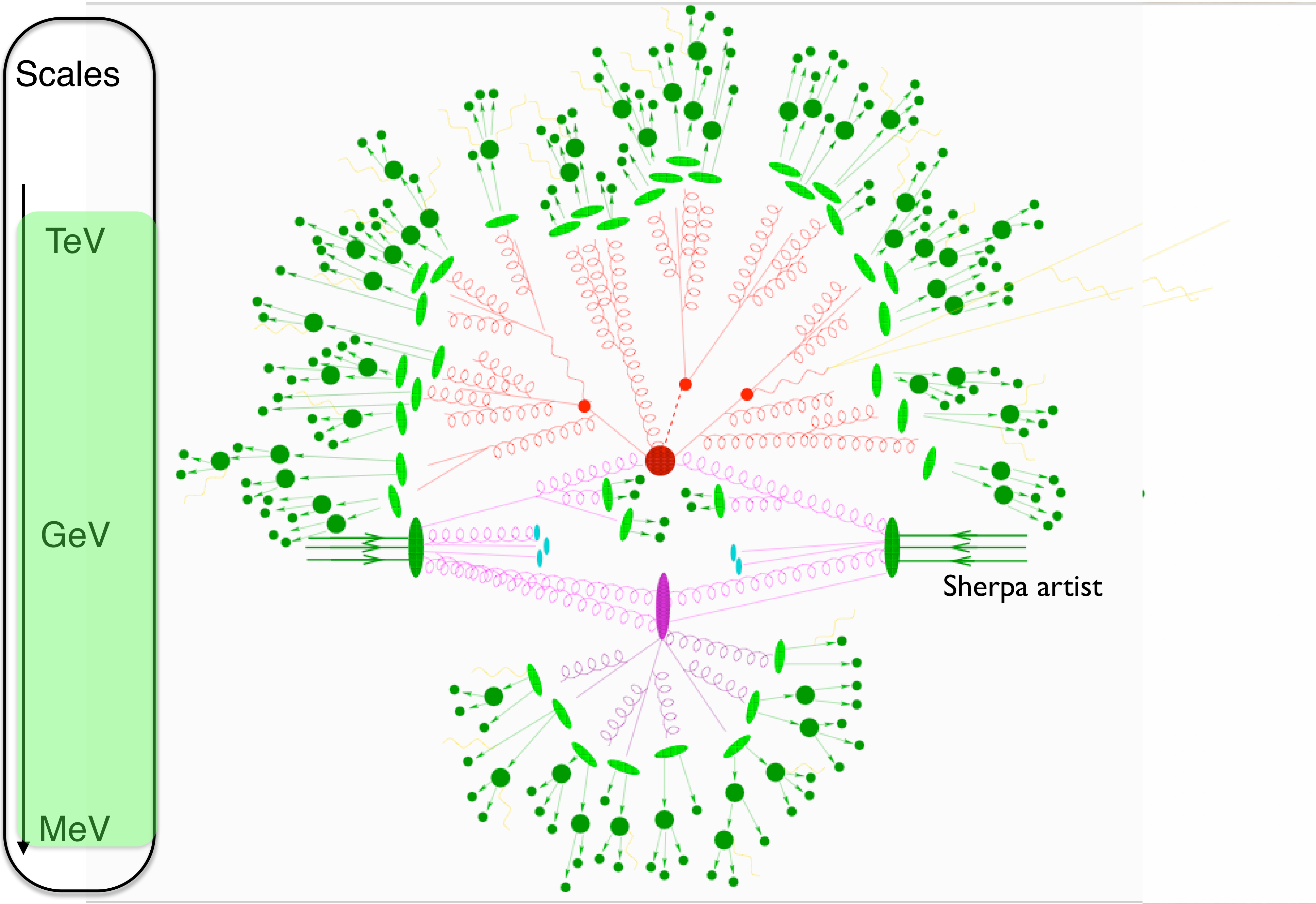


Simulation of collider events

What are the MC for?



What are the MC for?

Scales

TeV

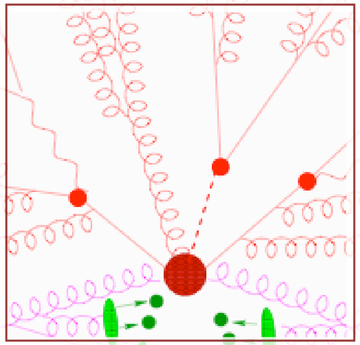
GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ where BSM physics lies



☞ process dependent

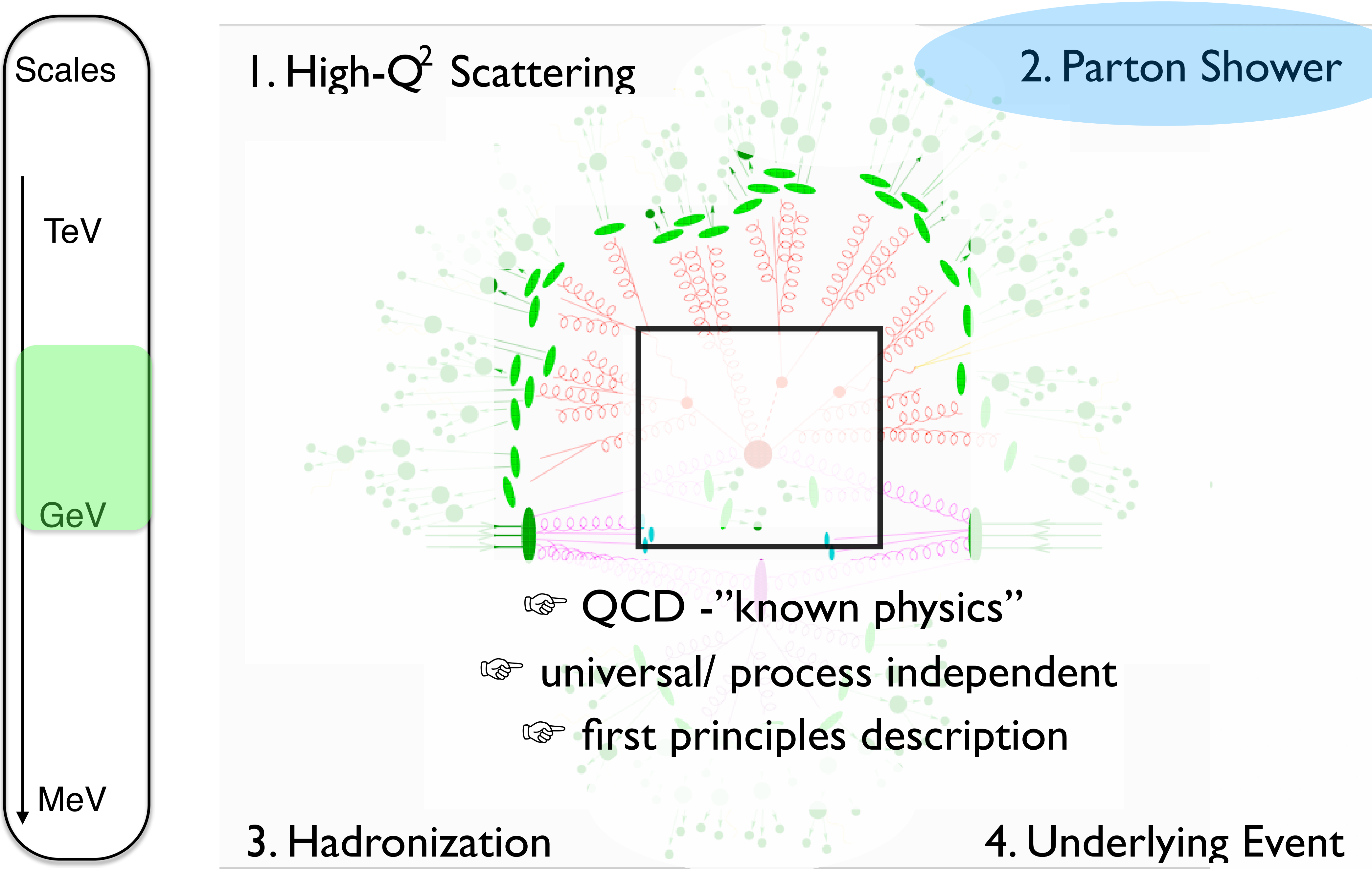
☞ first principles description

☞ it can be systematically improved

3. Hadronization

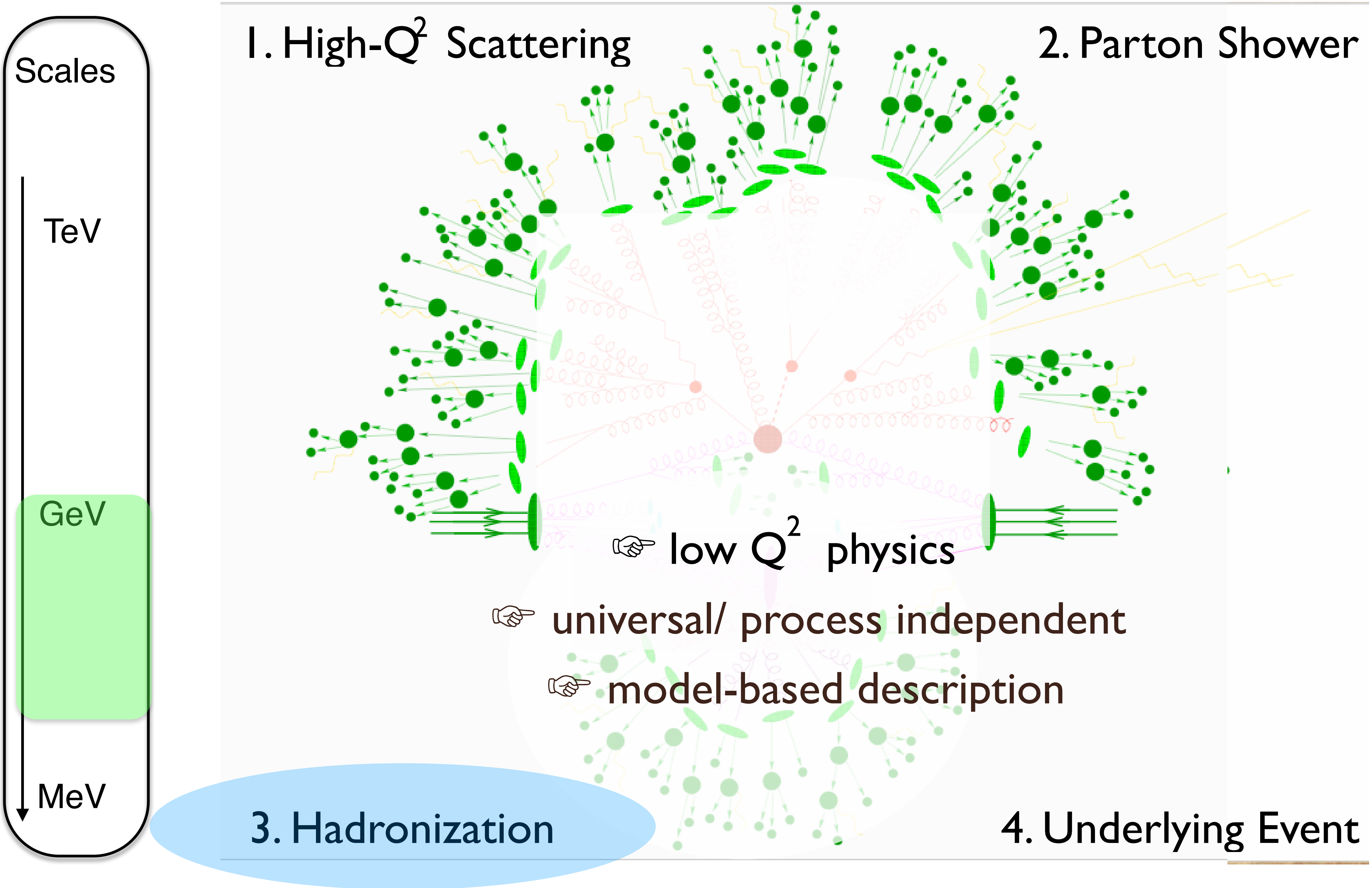
4. Underlying Event

What are the MC for?



- ☞ QCD -"known physics"
- ☞ universal/ process independent
- ☞ first principles description

What are the MC for?



☞ low Q^2 physics

☞ universal/ process independent

☞ model-based description

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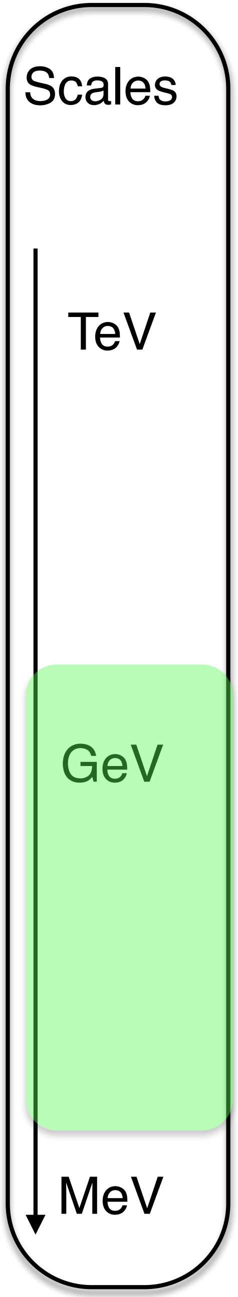
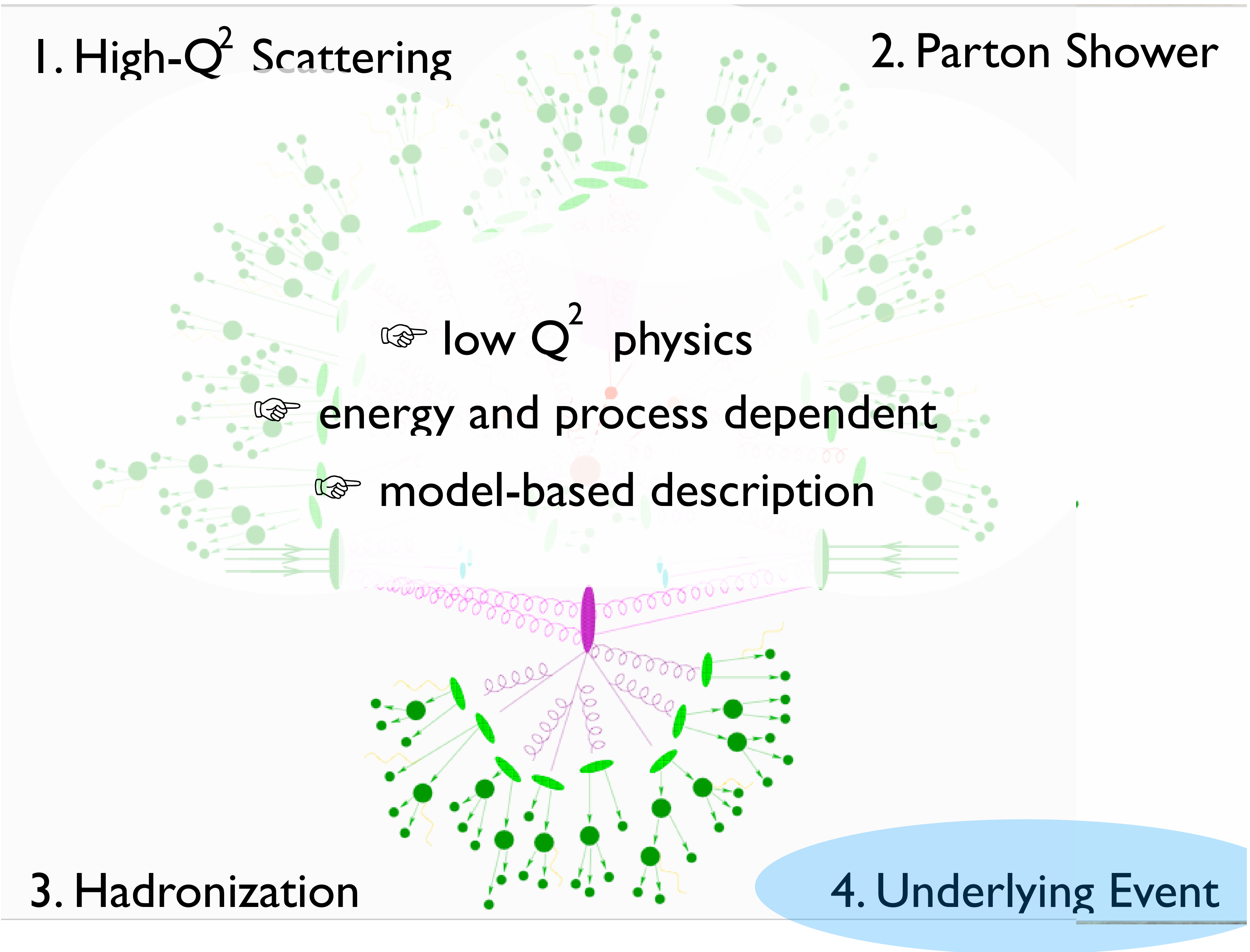
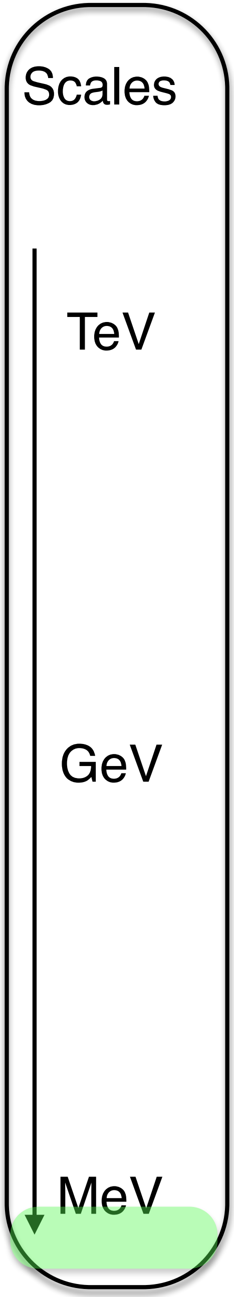
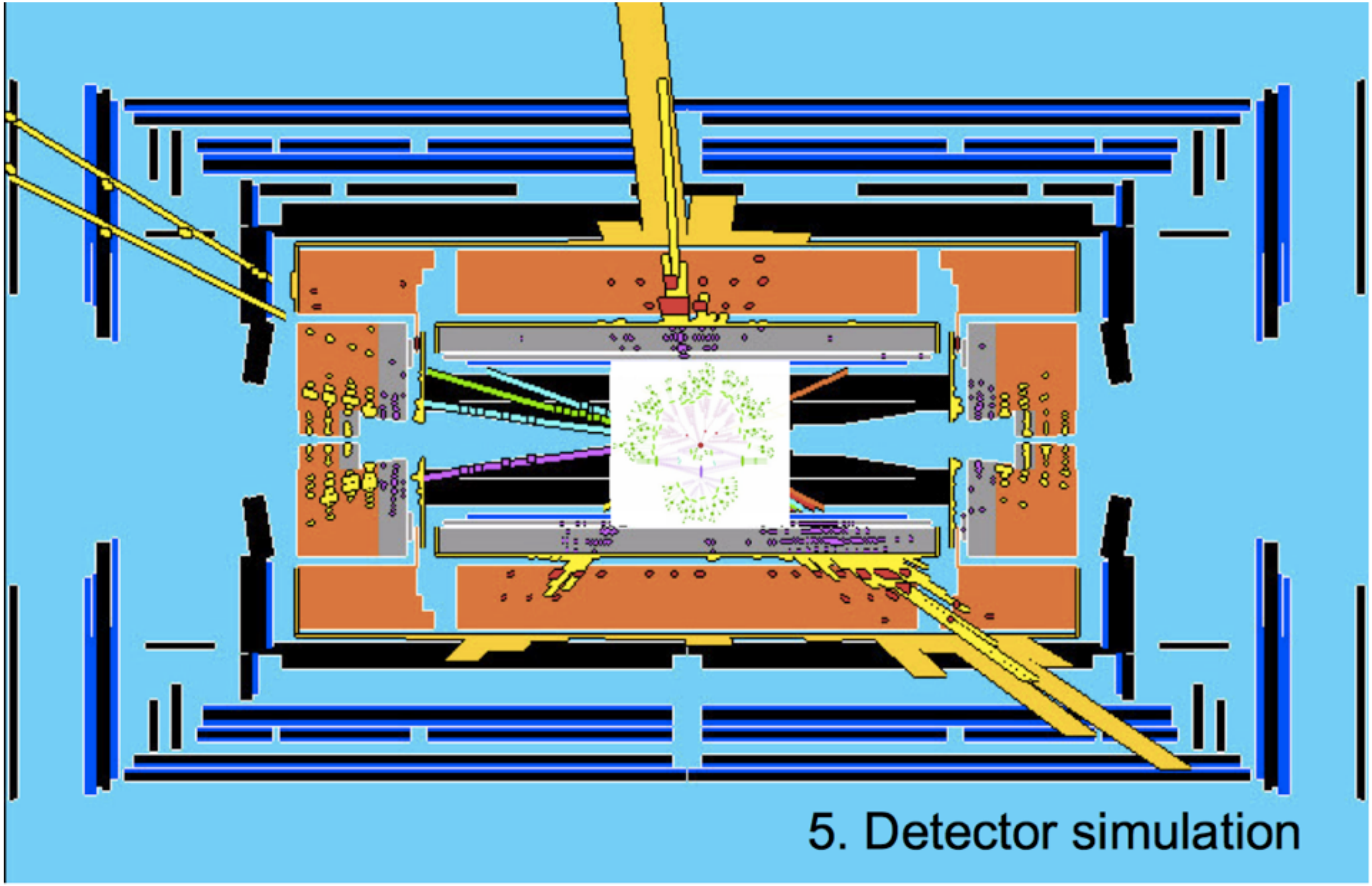




☞ low Q^2 physics
☞ energy and process dependent
☞ model-based description

What are the MC for?

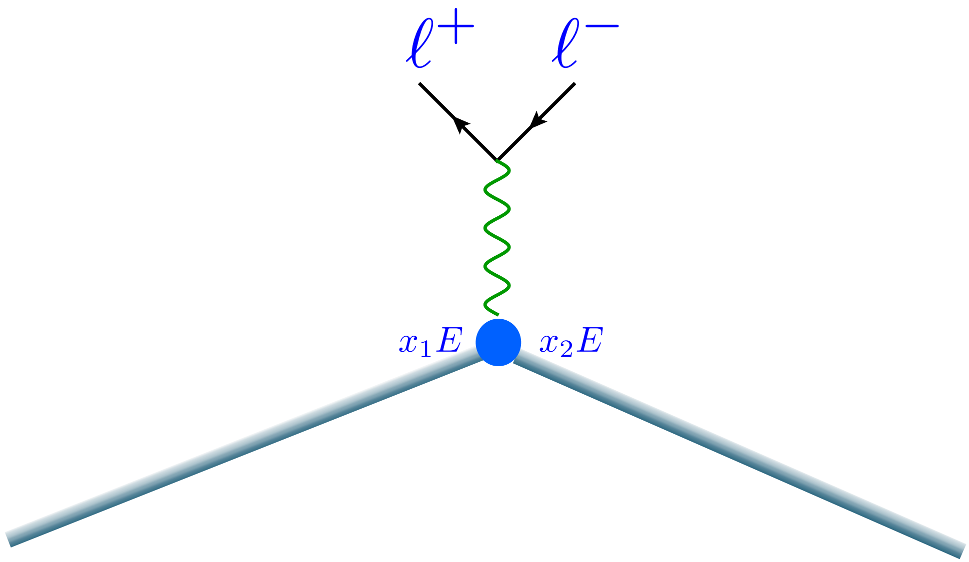




To Remember

- Multi-scale problem
  - ➡ New physics visible only at High scale
  - ➡ Problem split in different scale

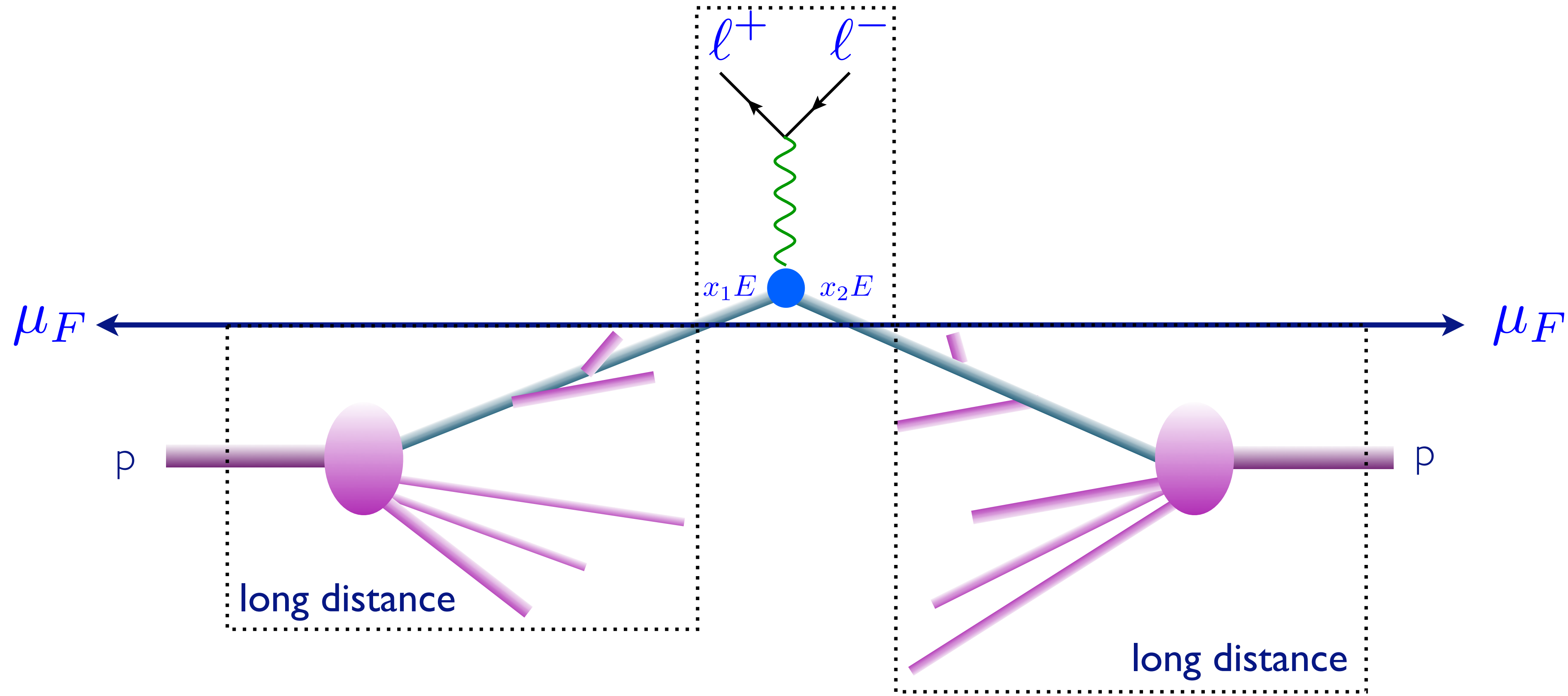
Master formula for the LHC



$$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Parton-level cross section

Master formula for the LHC



$$f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

Parton density functions

Parton-level cross section

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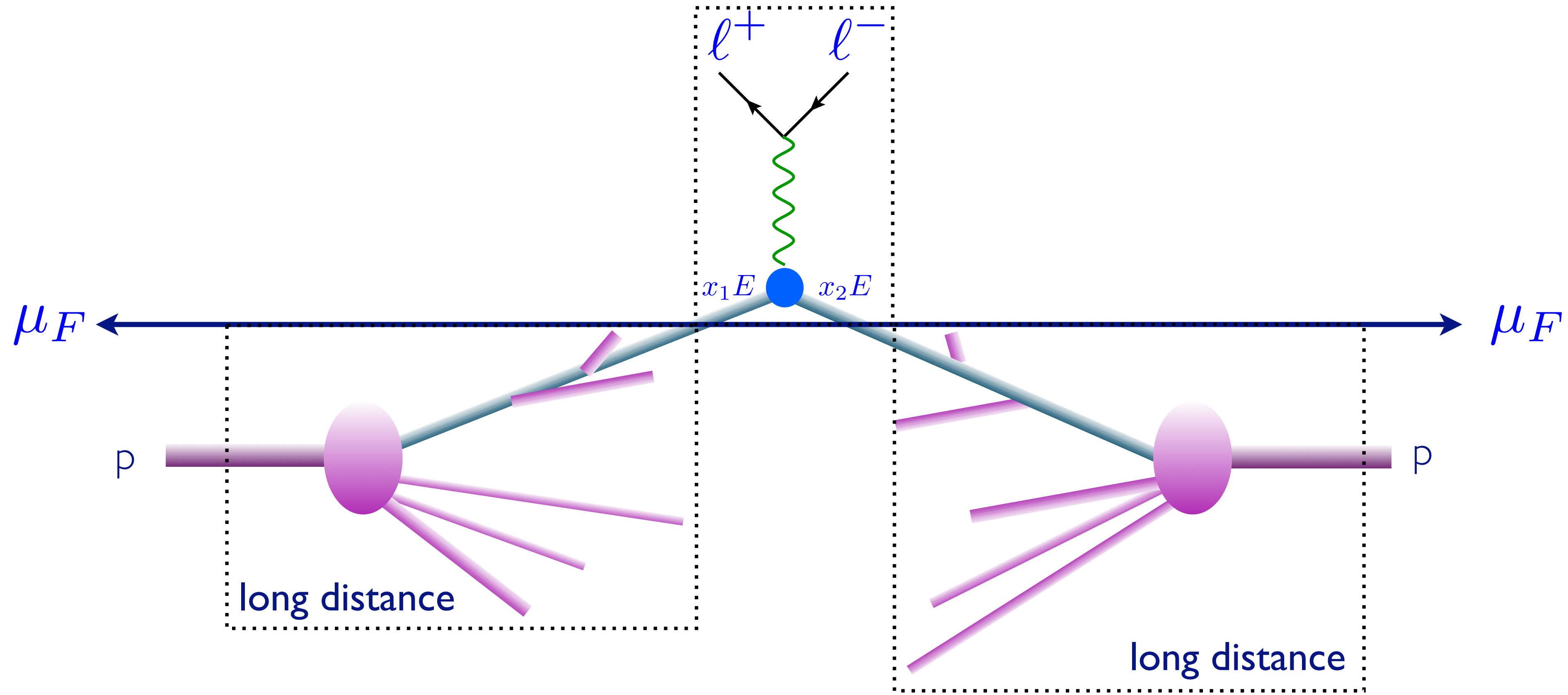

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\mathrm{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

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LO predictions

Perturbative expansion

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LO predictions

NLO corrections

NNLO corrections

N3LO or NNNLO corrections

- Including higher corrections improves predictions and reduces theoretical uncertainties

Improved predictions

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \;\; f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, d\hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

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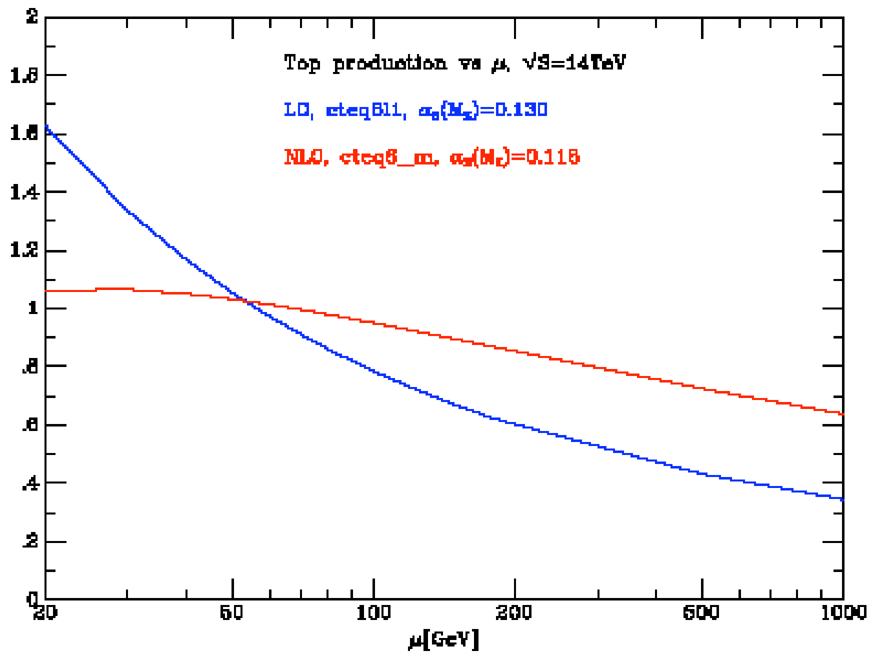
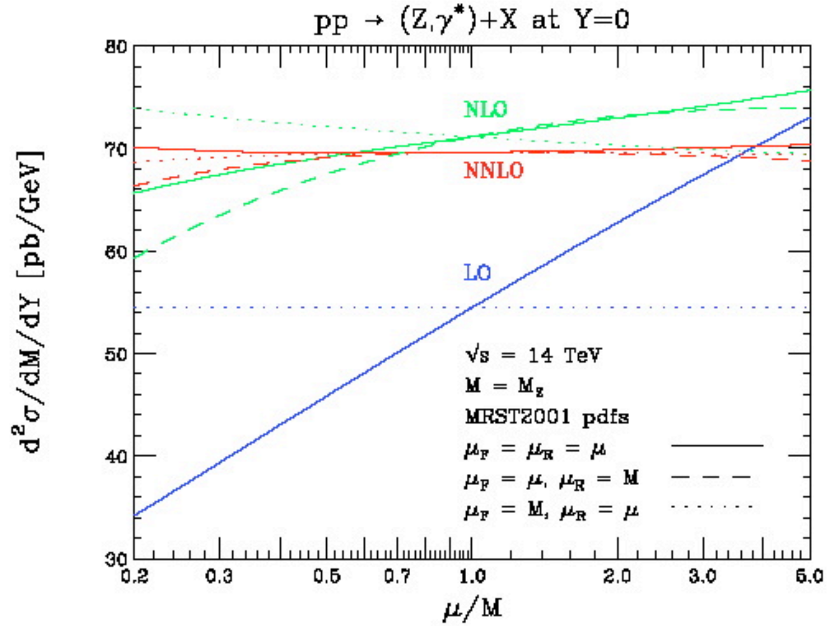
- Leading Order predictions can depend strongly on the renormalization and factorization scales
- Including higher order corrections reduces the dependence on these scales



Going NNLO...?

- **NNLO is the current state-of-the-art. There are only a few results available: Higgs, Drell-Yan, ttbar**
- **Why do we need it?**
  - control of the uncertainties in a calculation
  - It is "mandatory" if NLO corrections are very large to check the behavior of the perturbative series
  - It is needed for Standard Candles and very precise tests of perturbation theory, exploiting all the available information, e.g. for determining NNLO PDF sets



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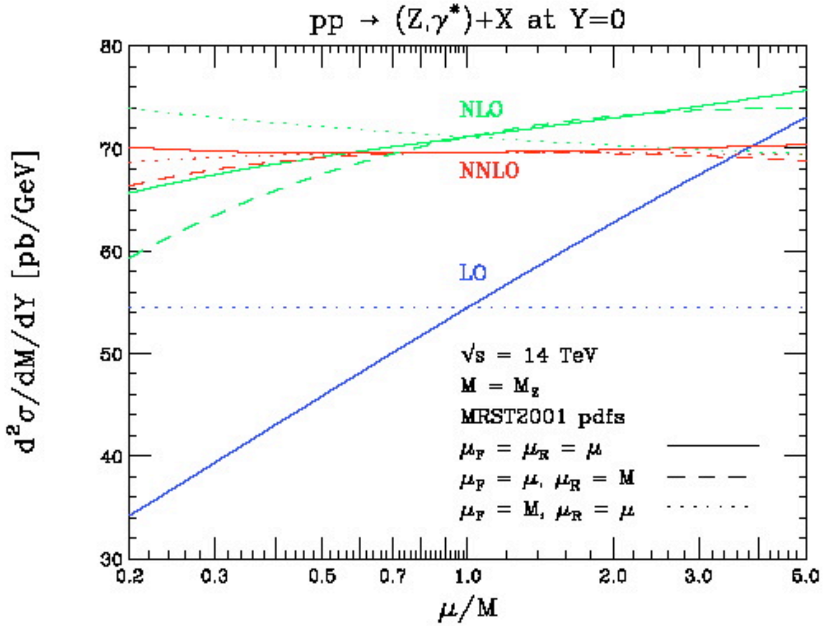

Let's focus on LO

Tevatron vs. the LHC

- Tevatron: 2 TeV proton-antiproton collider
  - ➡ Most important: q-$\bar{q}$ annihilation (85% of $t\bar{t}$)
- LHC: 7-14 TeV proton-proton collider
  - ➡ Most important: g-g annihilation (90% of $t\bar{t}$)

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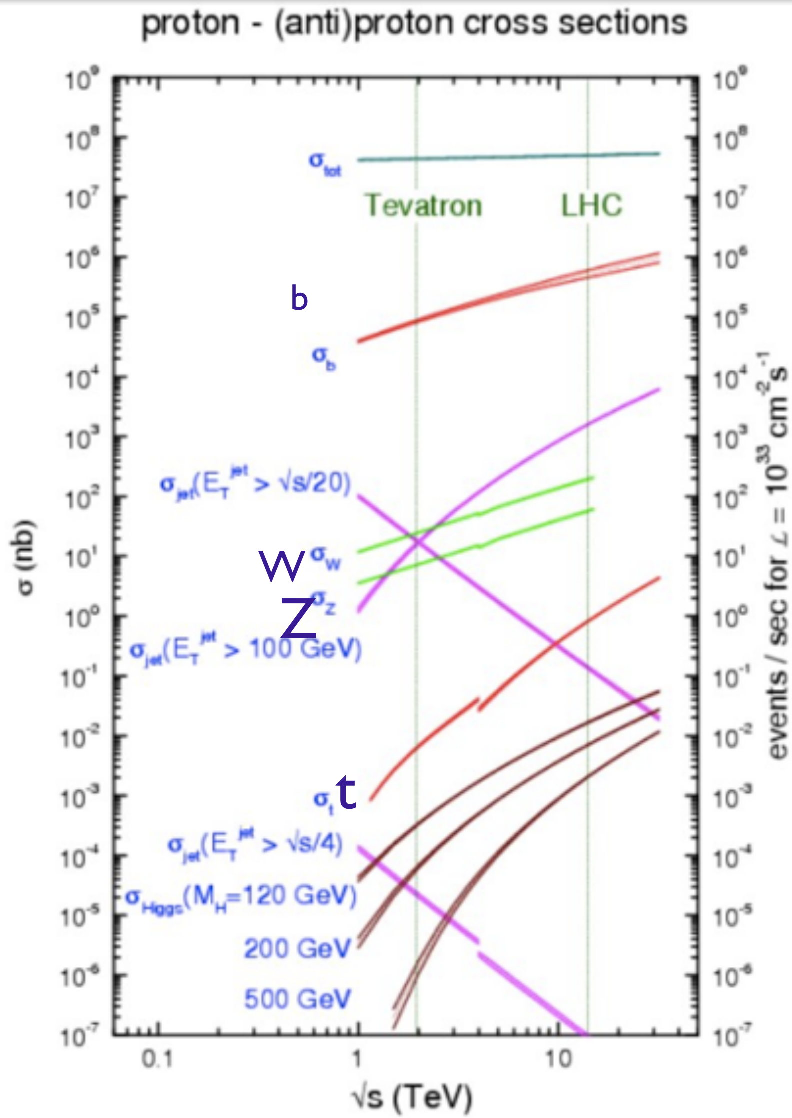
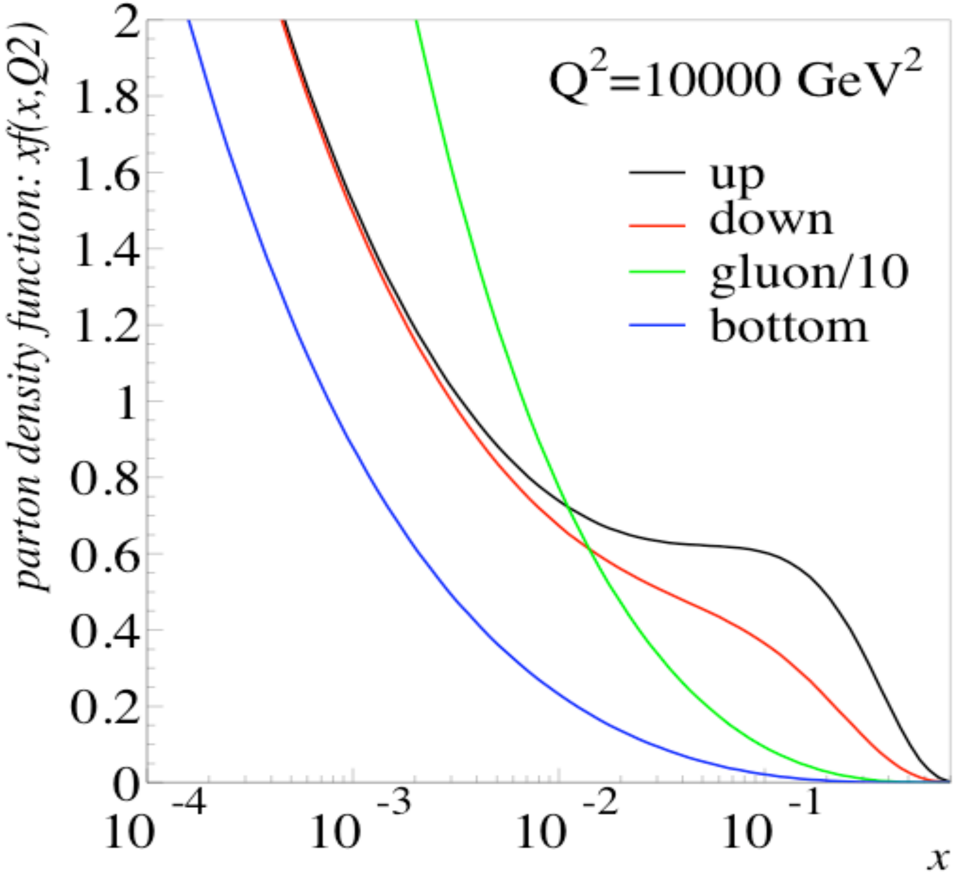
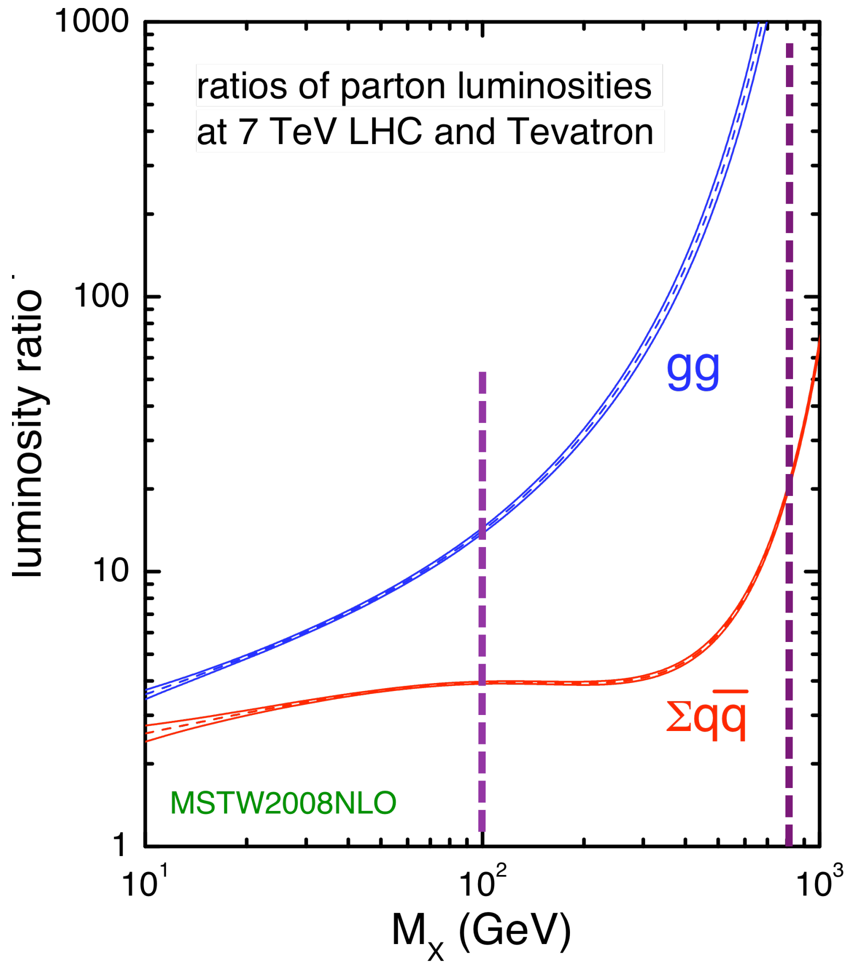
Hadron Colliders

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$



Parton densities

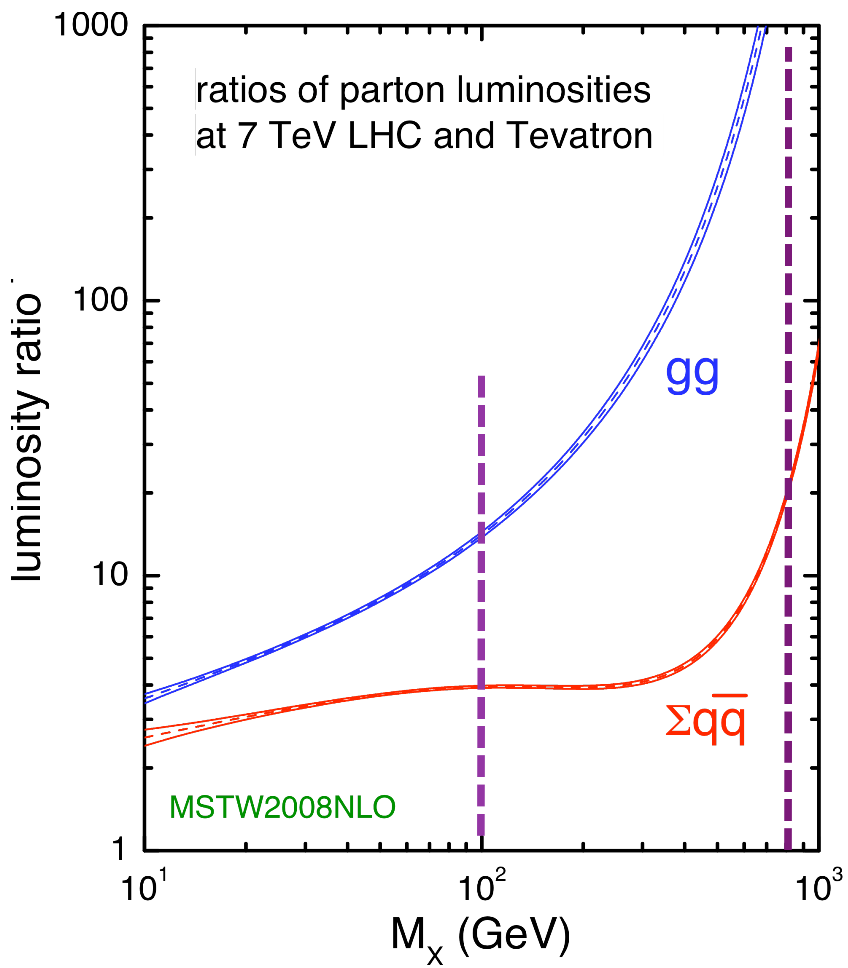


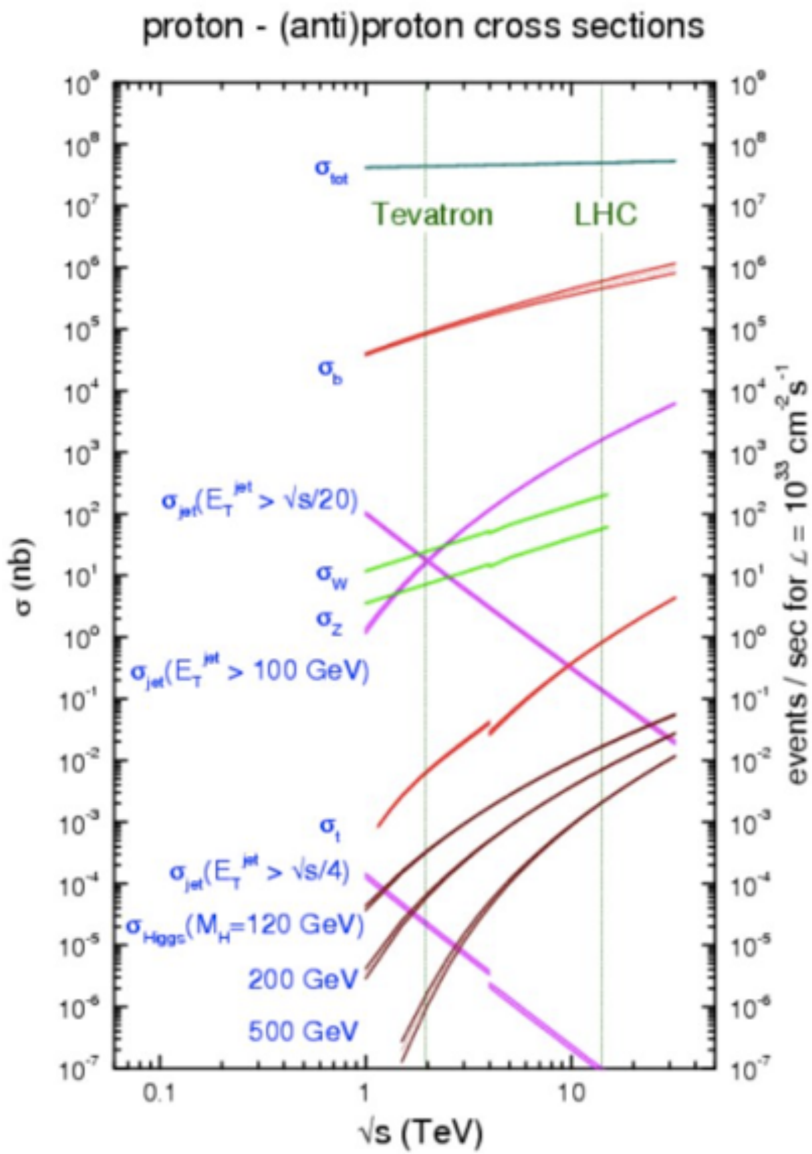


At small x (small $\hat{s}$), gluon domination.
At large x valence quarks

LHC formidable at large mass –
For low mass, Tevatron backgrounds smaller

Back to the processes





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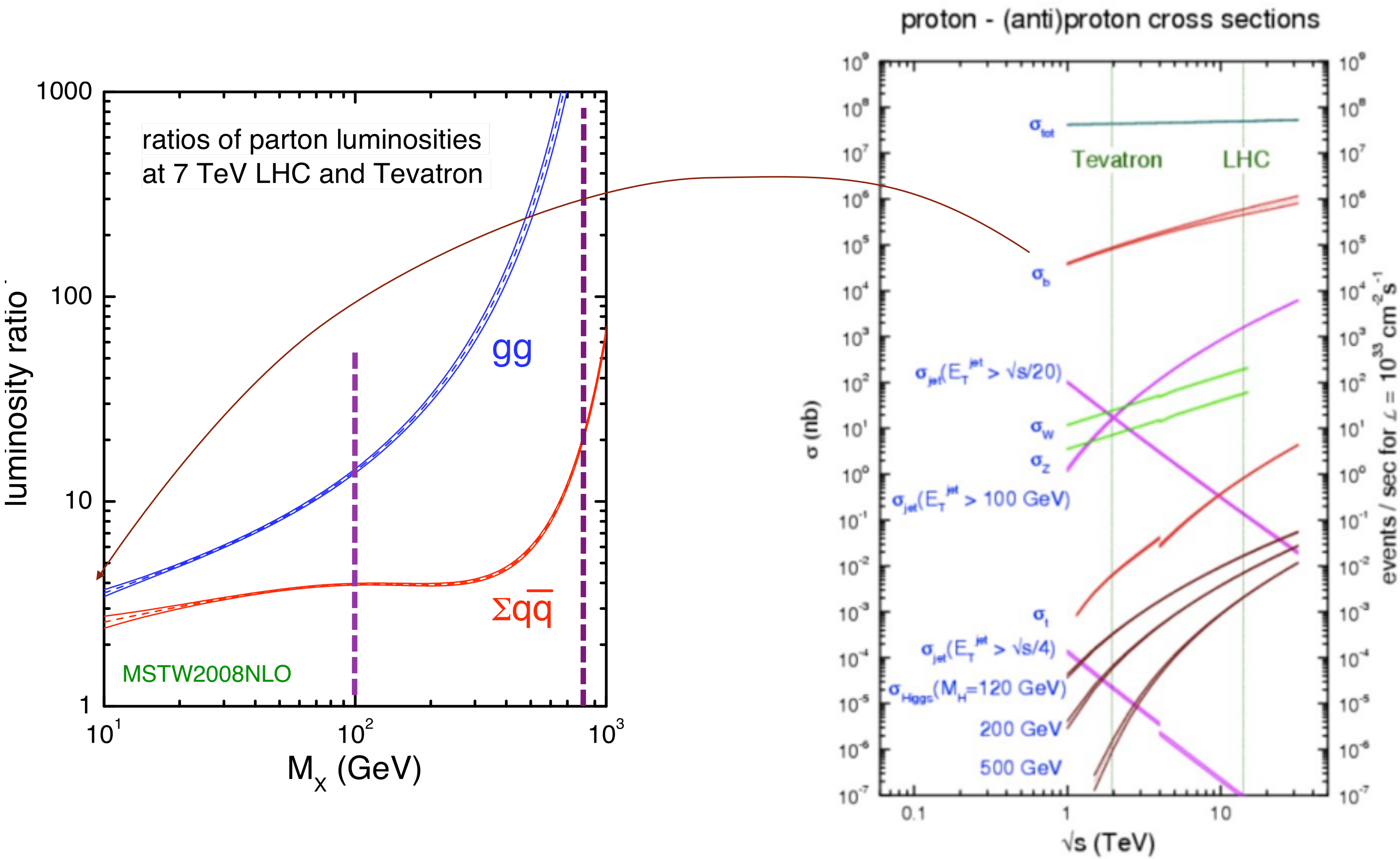

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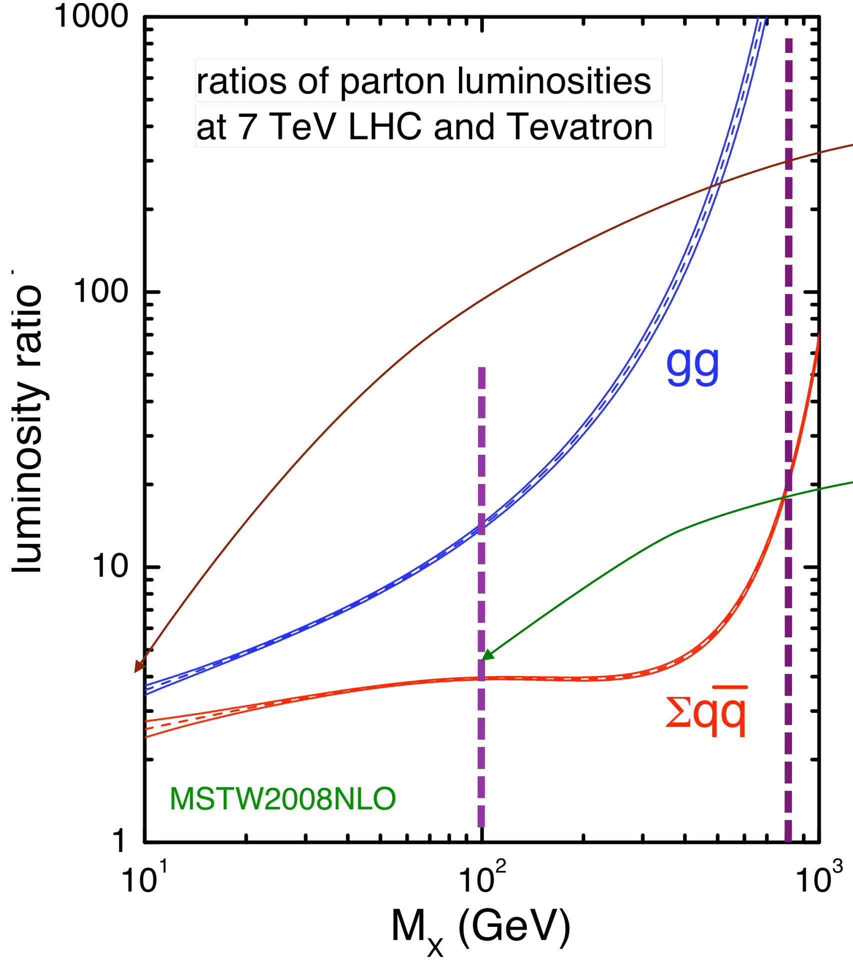

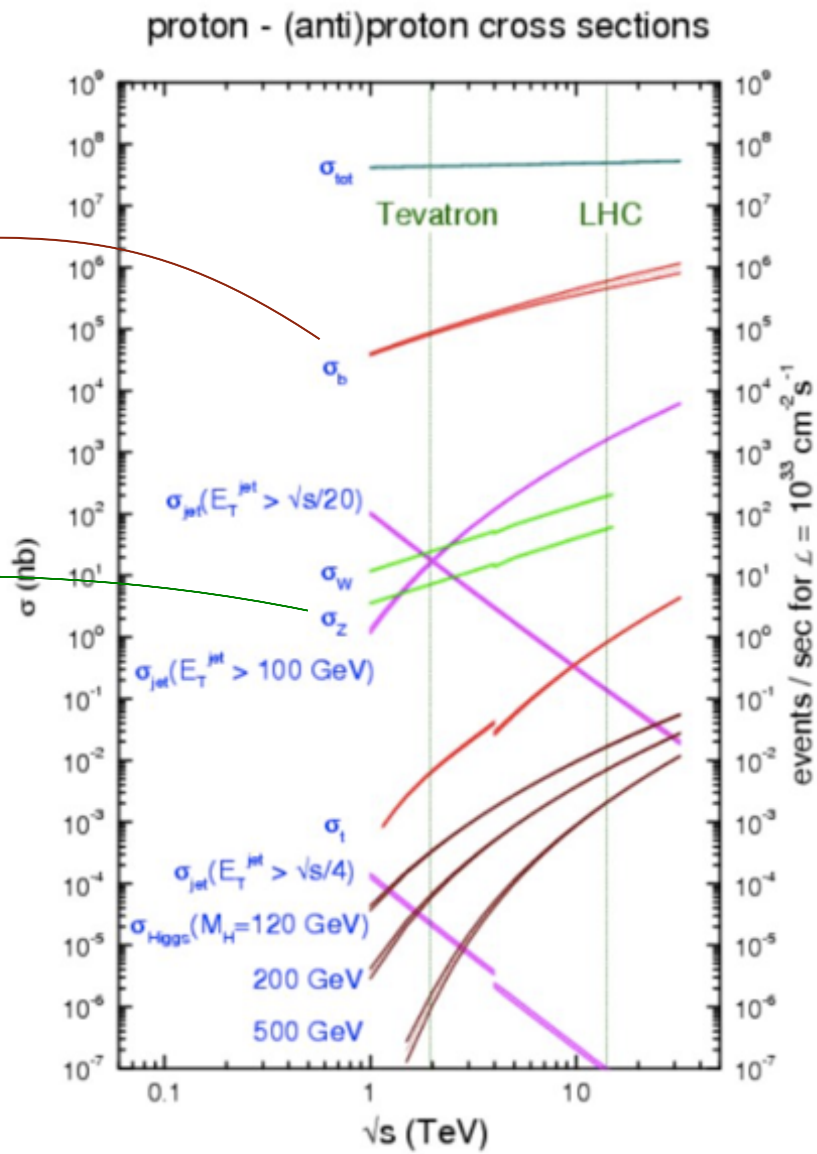

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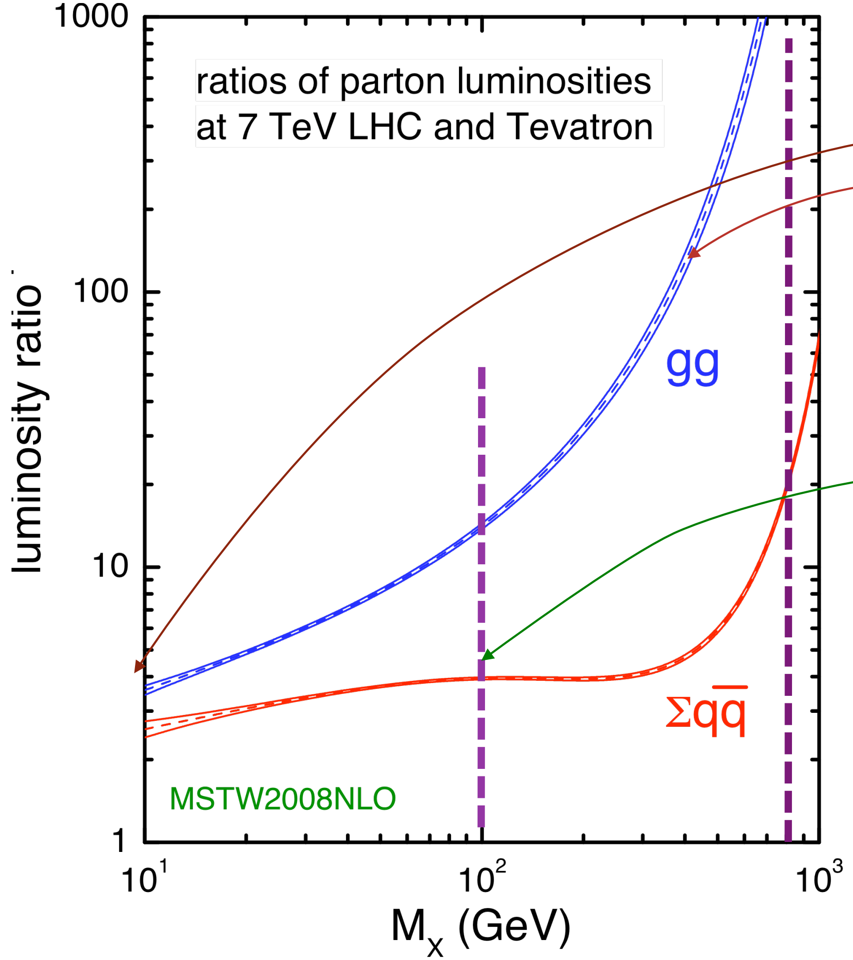

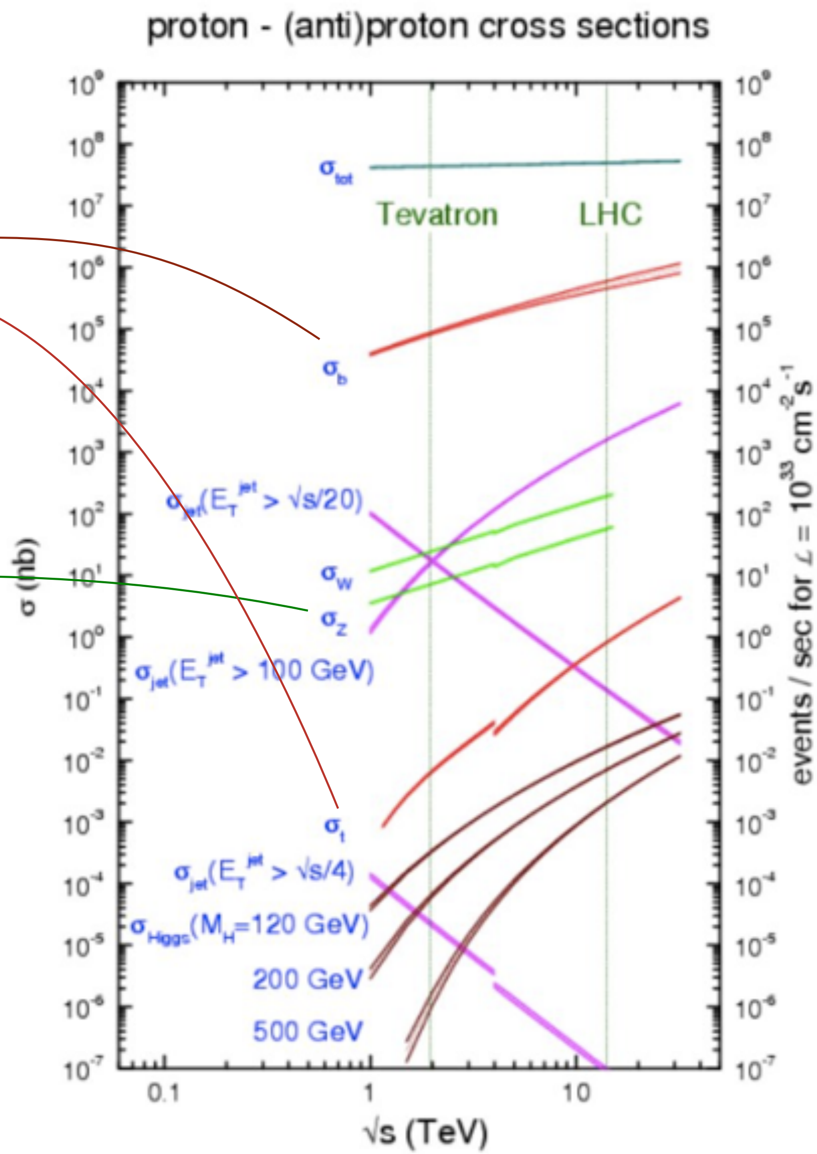

To Remember

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Phase-space integral

Parton density functions

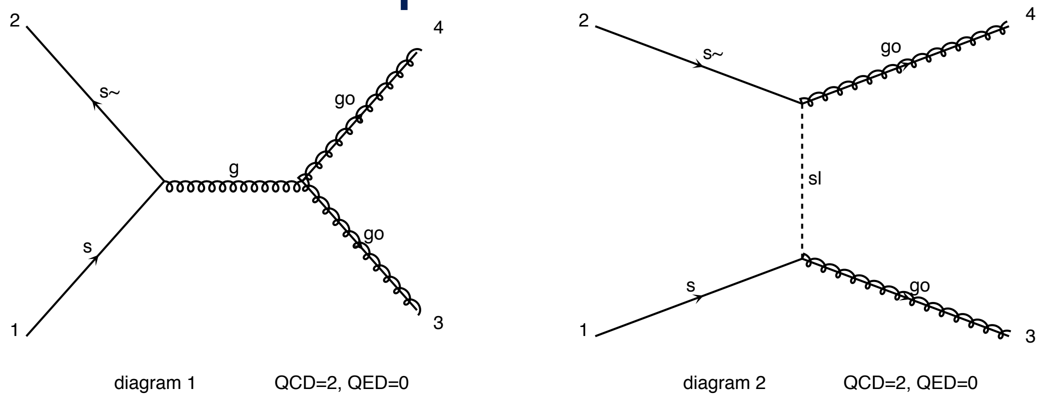
Parton-level cross section

- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- NLO/NNLO: Reduce scale uncertainty linked to your division of your multi-scale problem

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$$|\mathcal{M}|^2$$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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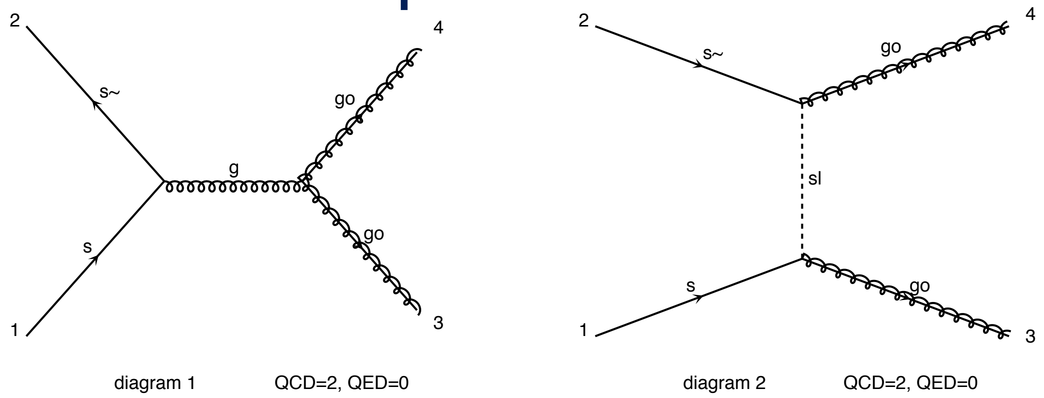

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Easy enough

Hard

Very Hard (in general)

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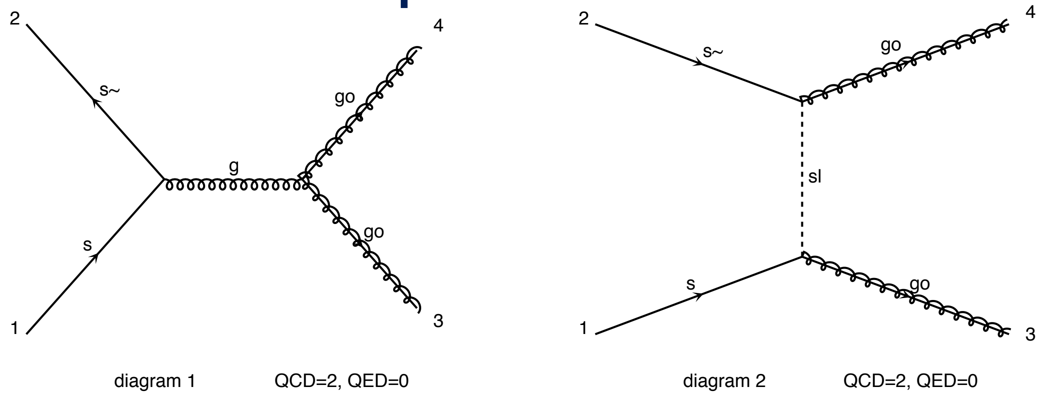

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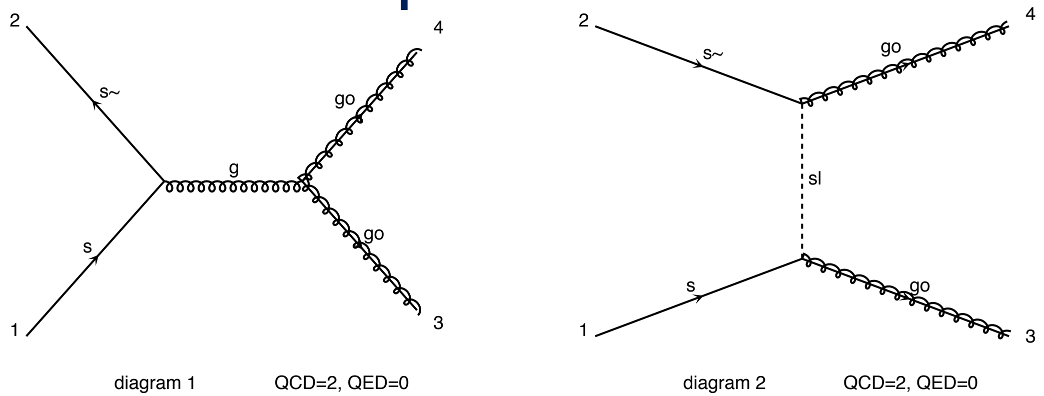

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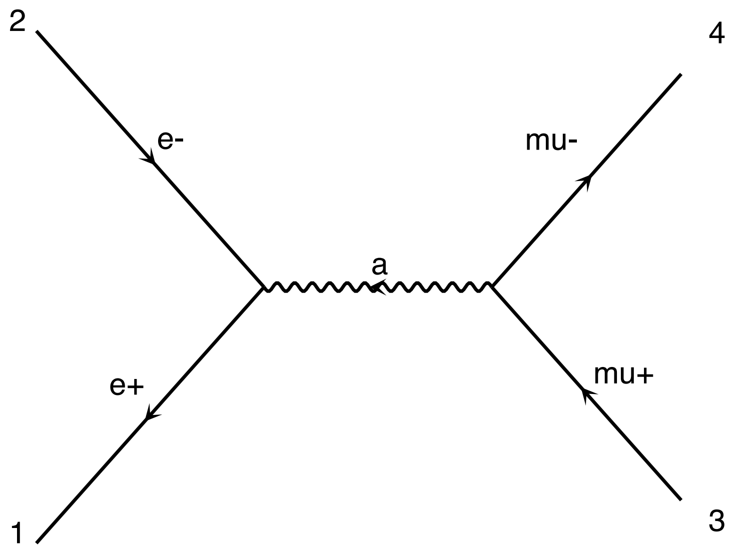
Very Hard (in general)

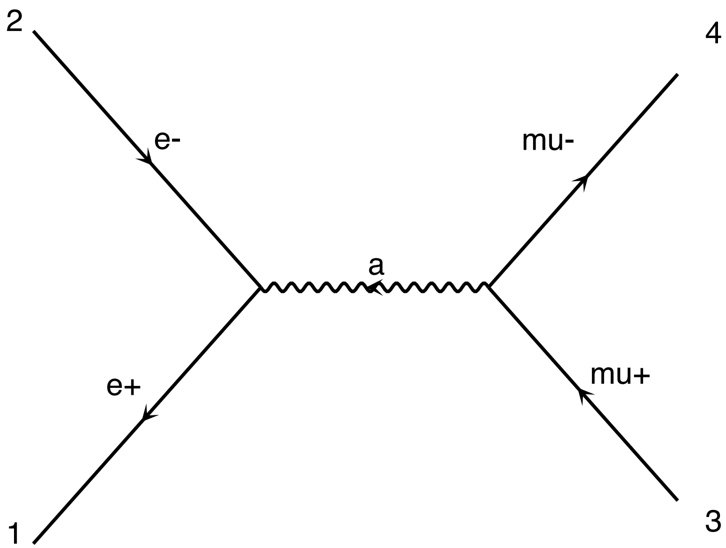
After

Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

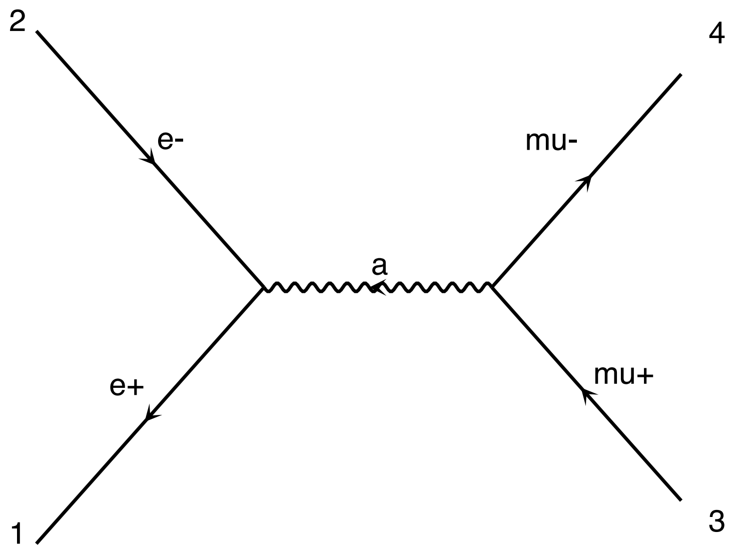
Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol} |\mathcal{M}|^2 = \frac{1}{4}\sum_{pol} \mathcal{M}^* \mathcal{M}$$

Matrix Element

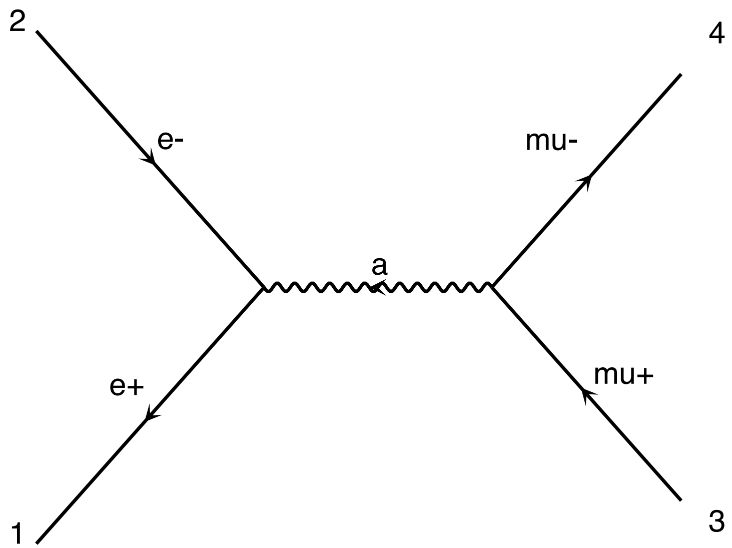


$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

$$\frac{1}{4} \sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} \mathcal{M}^* \mathcal{M}$$

$$\sum_{pol} \bar{u}u = \not{p} + m$$

Matrix Element



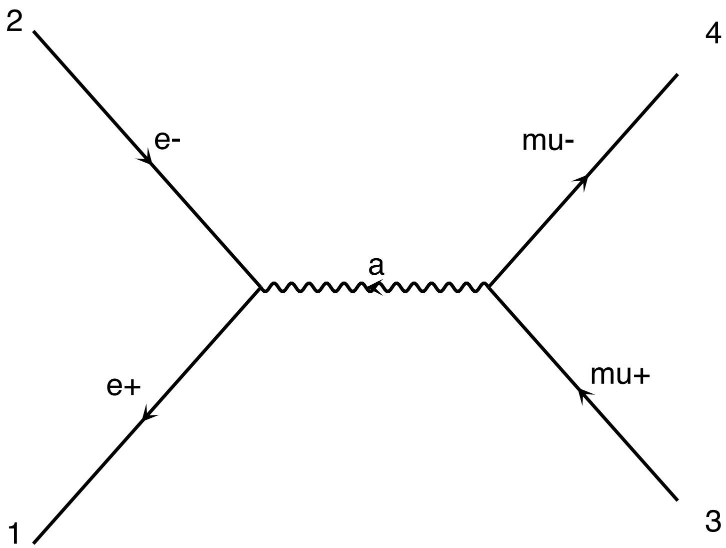
$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

$$\sum_{pol}\bar{u}u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4}Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

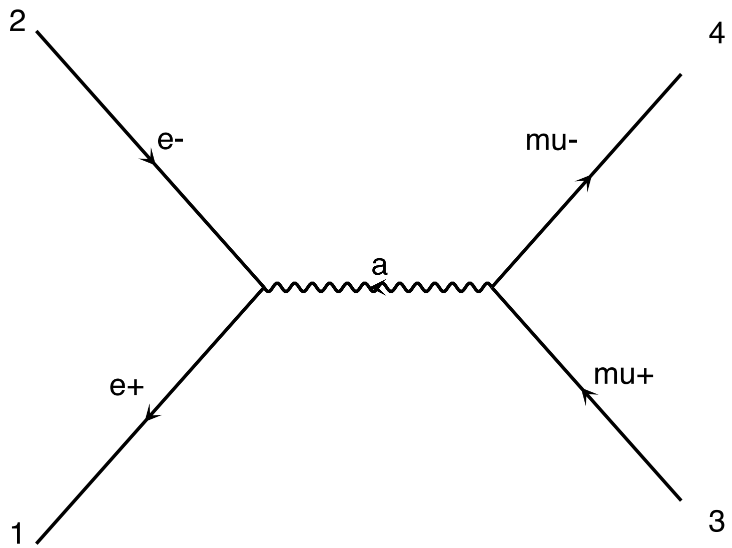
$$\frac{1}{4}\sum_{pol} |\mathcal{M}|^2 = \frac{1}{4}\sum_{pol} \mathcal{M}^* \mathcal{M}$$

$$\sum_{pol} \bar{u}u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4} Tr[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu] Tr[\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu]$$

$$\rightarrow \frac{8e^4}{q^4} [(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

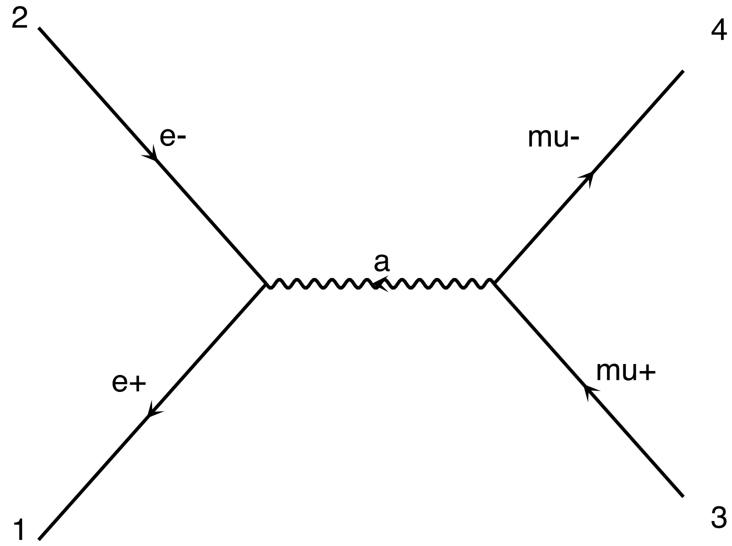
$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

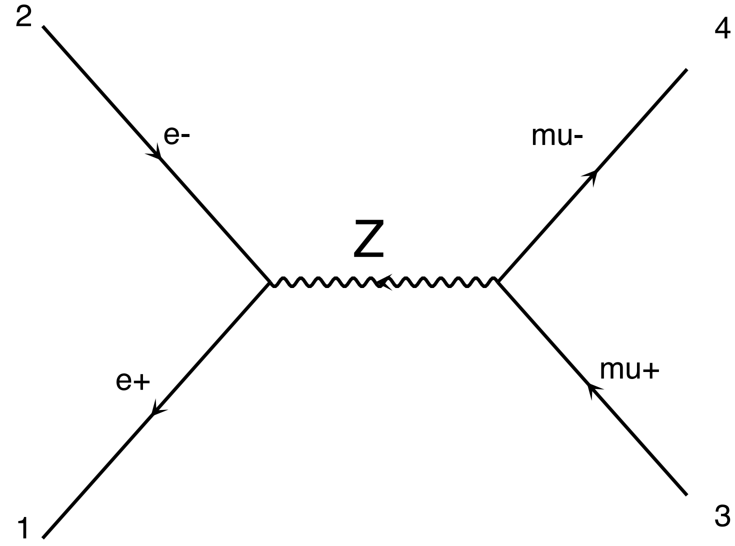
$$\sum_{pol}\bar{u}u = \not{p} + m$$

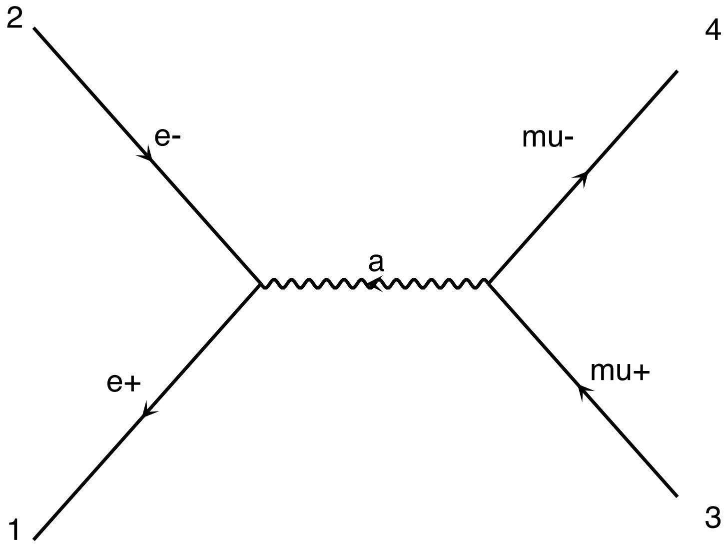
→ $$\frac{e^4}{4q^4}Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

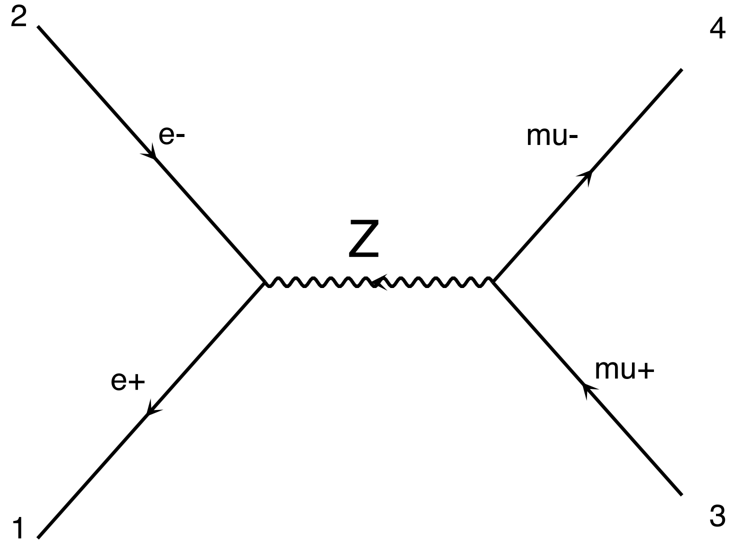
→ $$\frac{8e^4}{q^4}[(p_1.p_3)(p_2.p_4)+(p_1.p_4)(p_2.p_3)]$$

Very Efficient !!!

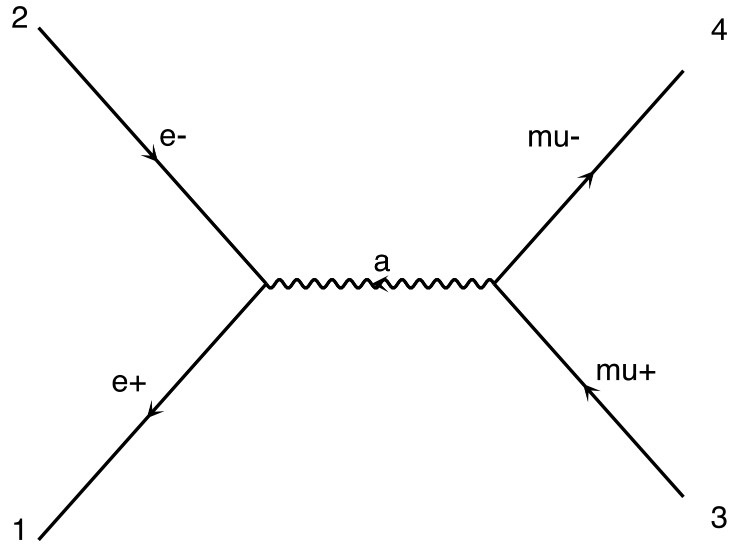

2
e-
mu-
4
a
e+
mu+
1
3

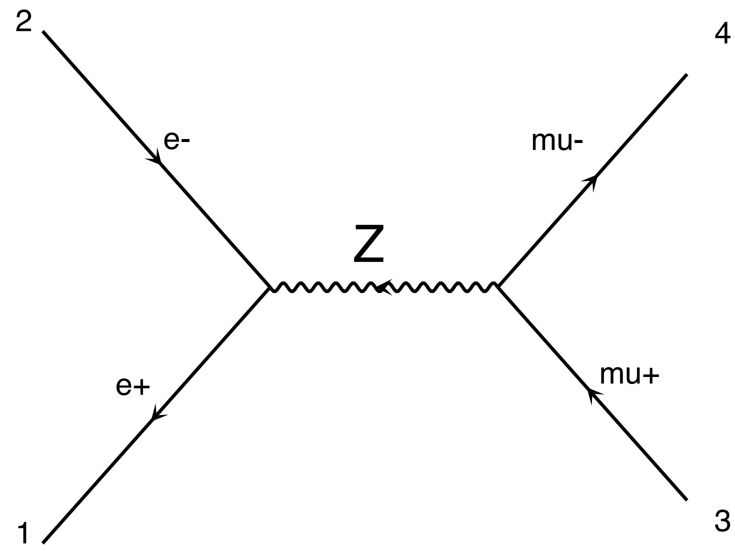

2
e-
mu-
4
Z
e+
mu+
1
3





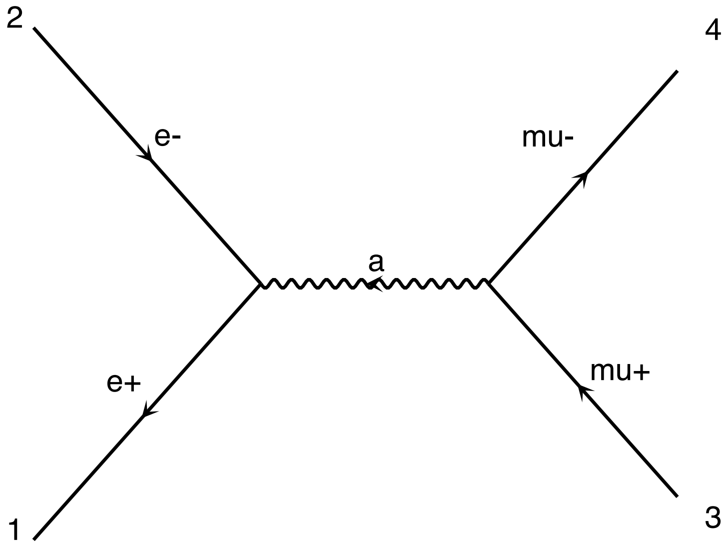
Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$

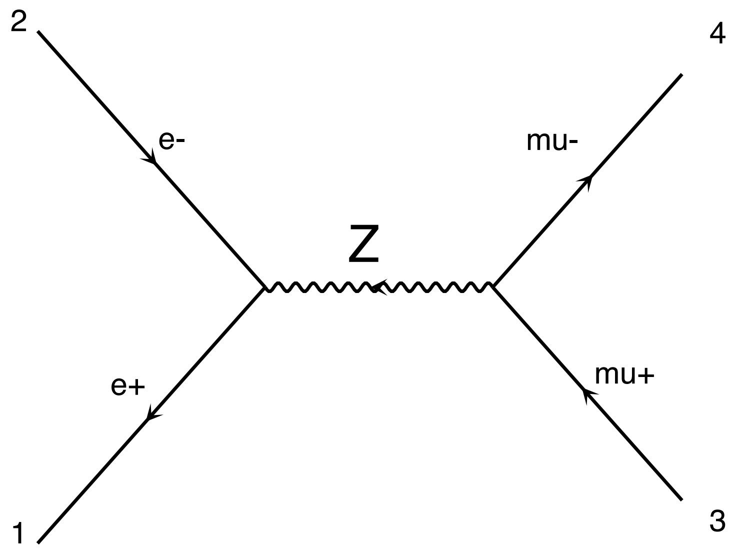




Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$

So for M Feynman diagram we need to compute M^2 different term

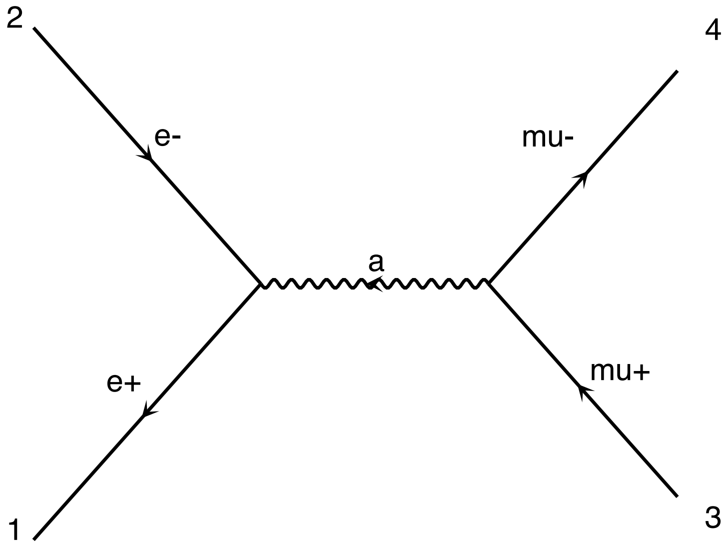


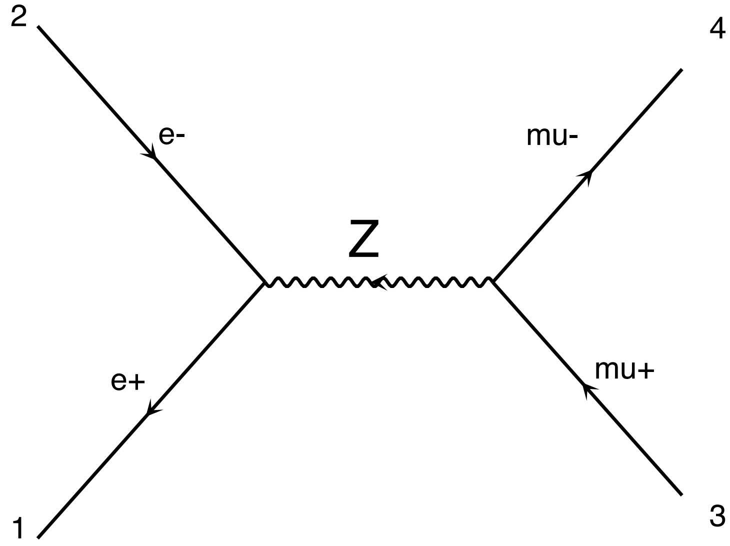


Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$

So for M Feynman diagram we need to compute M^2 different term

The number of diagram scales factorially with the number of particle





Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$

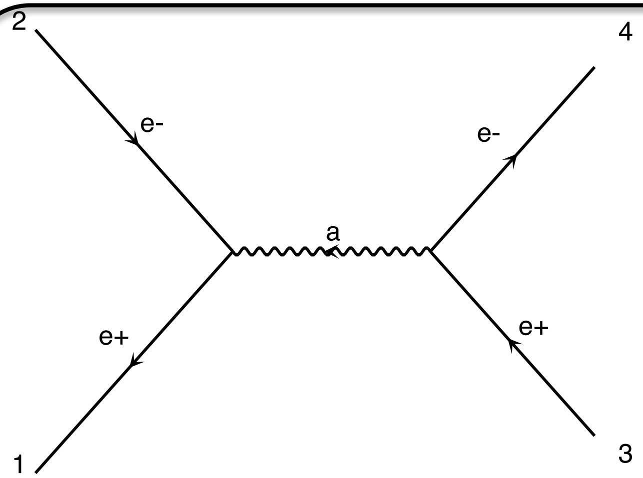
So for M Feynman diagram we need to compute M^2 different term

The number of diagram scales factorially with the number of particle

In practise possible up to 2>4

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\, e\, \gamma^\nu u)$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
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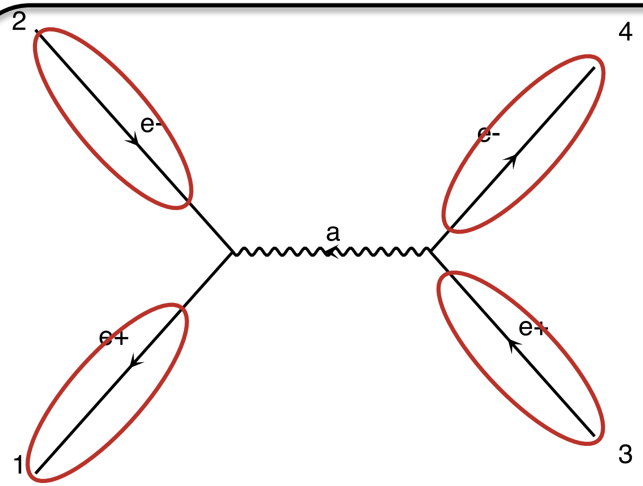
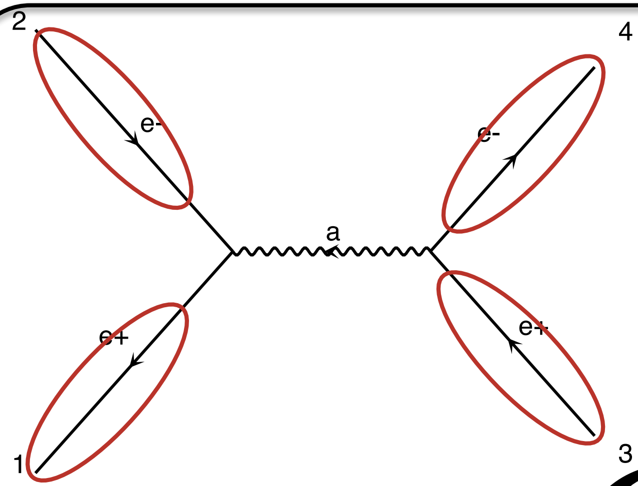


$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v} e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

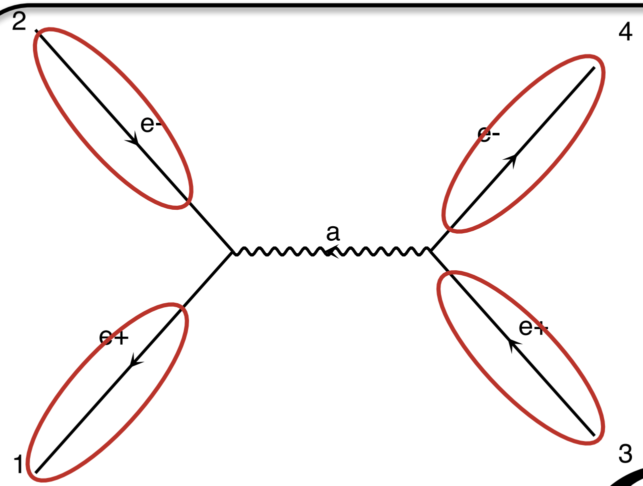
Numbers for given helicity and momenta

Lines present in the code.

$$\left\{ \begin{array}{l} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{array} \right.$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

Lines present in the code.

$$\begin{aligned} \bar{v}_1 &= fct(\vec{p}_1, m_1) \\ u_2 &= fct(\vec{p}_2, m_2) \\ v_3 &= fct(\vec{p}_3, m_3) \\ \bar{u}_4 &= fct(\vec{p}_4, m_4) \end{aligned}$$

$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_\lambda(\vec{p}) \\ \omega_\lambda(p)\chi_\lambda(\vec{p}) \end{pmatrix}$$

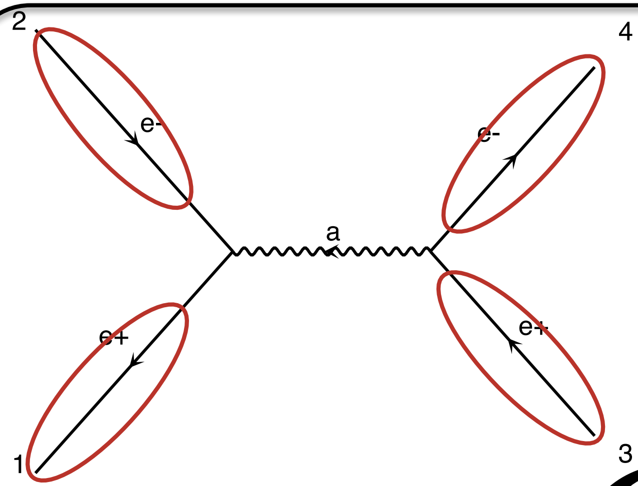
$$\omega_\pm(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_z)}} \begin{pmatrix} |\vec{p}| + p_z \\ p_x + ip_y \end{pmatrix},$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_z)}} \begin{pmatrix} -p_x + ip_y \\ |\vec{p}| + p_z \end{pmatrix}.$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v) \frac{g_{\mu\nu}}{q^2} (\bar{v} e\, \gamma^{\nu} u)$$

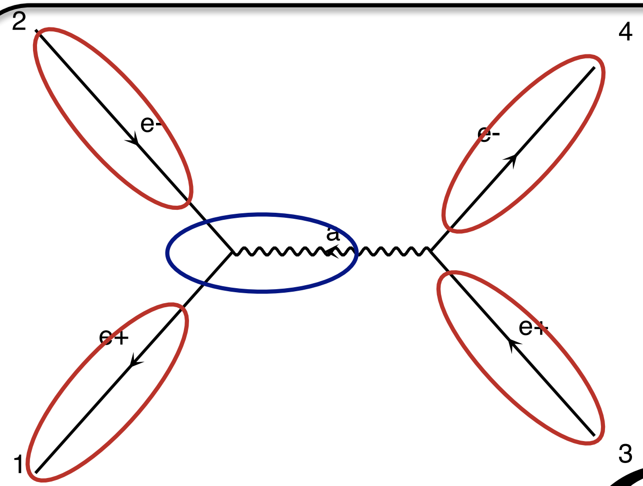
Numbers for given helicity and momenta

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{cases}$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v)\, \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

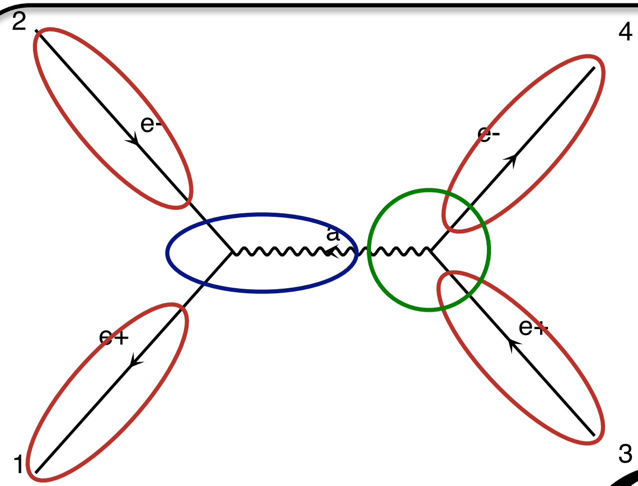
Calculate propagator wavefunctions

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a) \end{cases}$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|^2$
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

Lines present in the code.

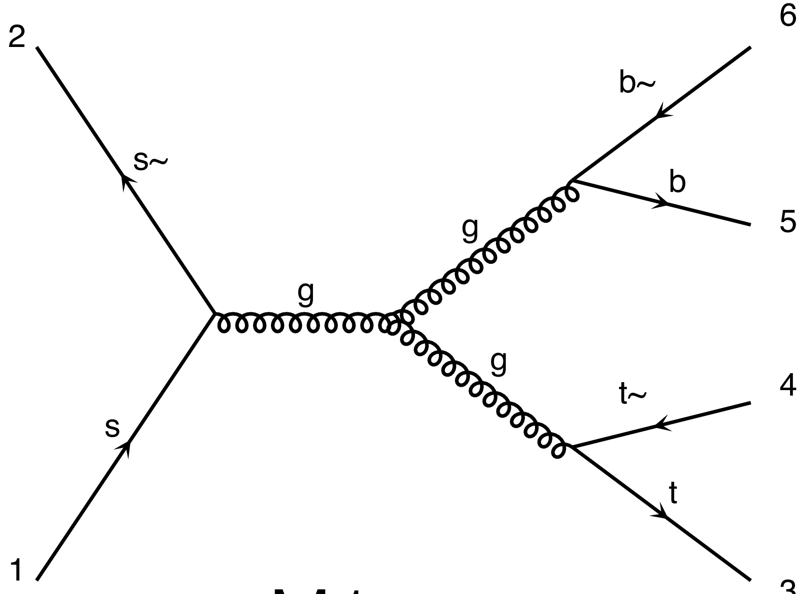
$$\left\{ \begin{aligned} \bar{v}_1 &= fct(\vec{p}_1, m_1) \\ u_2 &= fct(\vec{p}_2, m_2) \\ v_3 &= fct(\vec{p}_3, m_3) \\ \bar{u}_4 &= fct(\vec{p}_4, m_4) \\ W_a &= fct(\bar{v}_1, u_2, m_a, \Gamma_a) \\ \mathcal{M} &= fct(v_3, \bar{u}_4, W_a) \end{aligned} \right.$$

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$

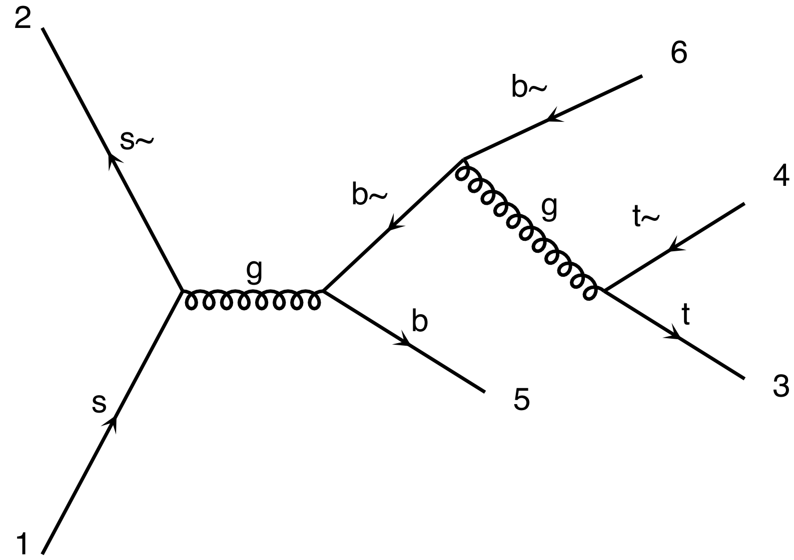
Real case

Known



M1

Number of routines: 0



M2

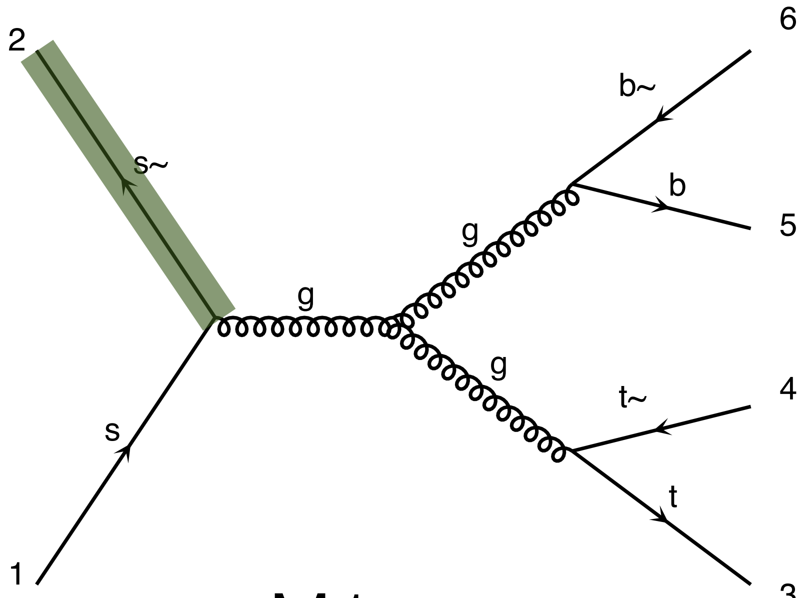
Number of routines: 0

Number of routines for both: 0

$$|M|^2 = |M_1 + M_2|^2$$

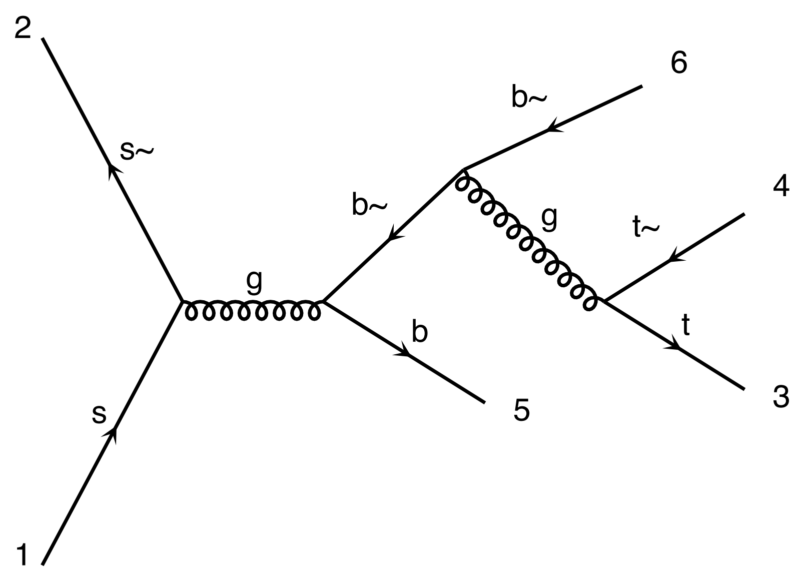
Real case

Known



M1

Number of routines: 1



M2

Number of routines: 0

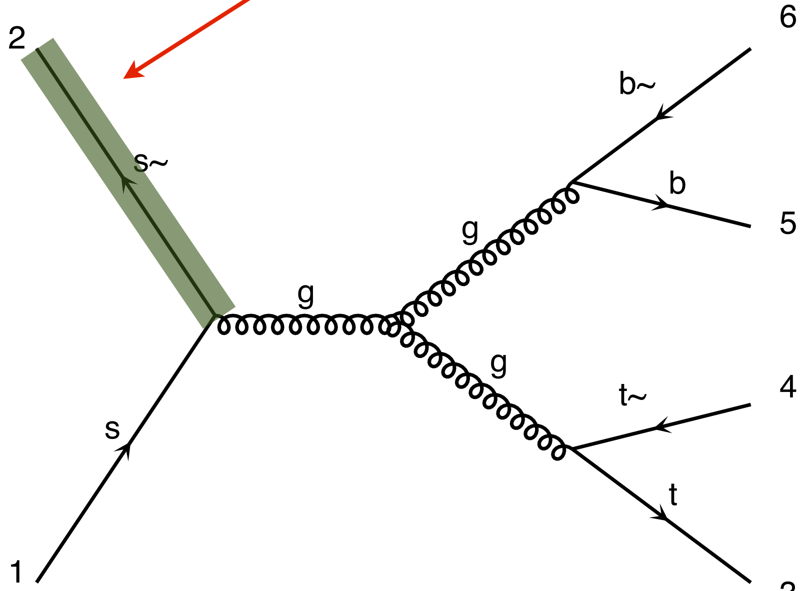
Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

Real case

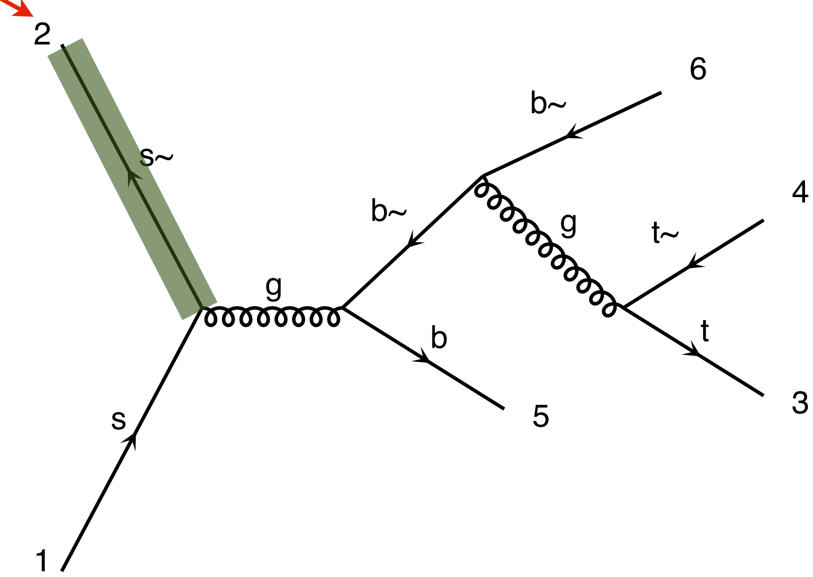
Identical

Known



M1

Number of routines: 1



M2

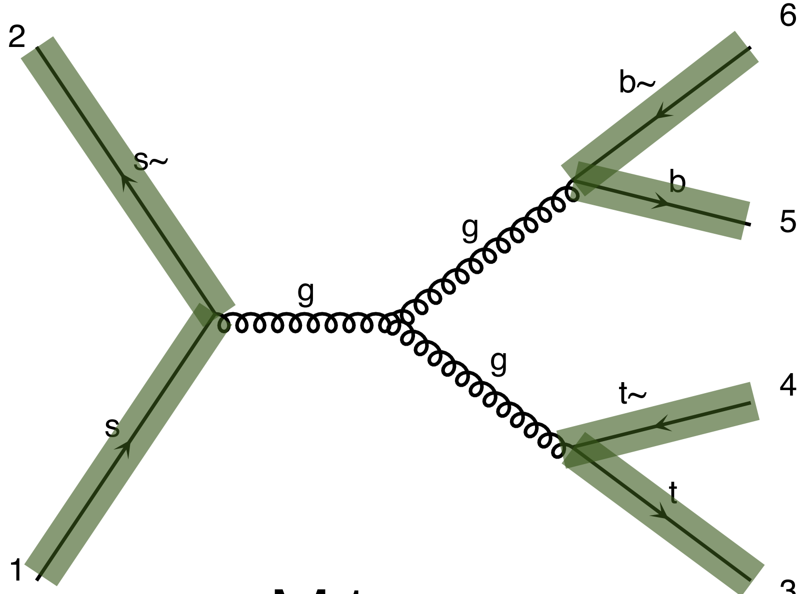
Number of routines: 1

Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

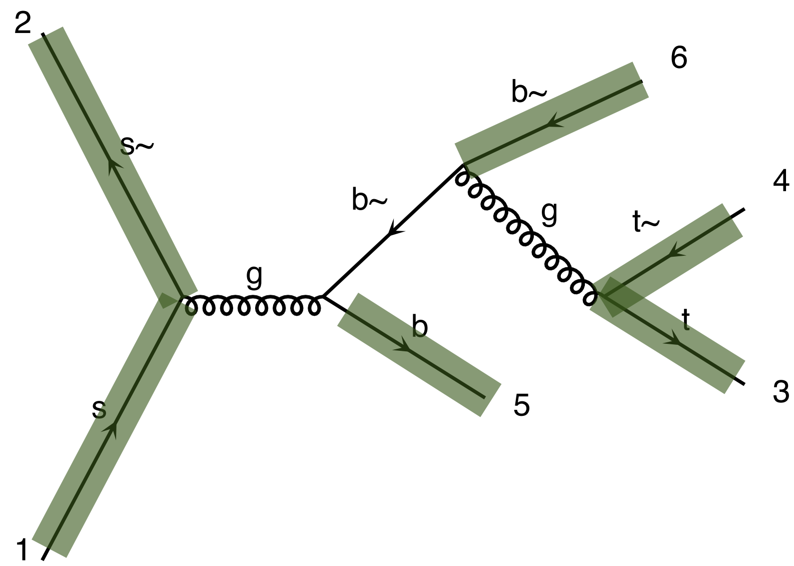
Real case

Known



M1

Number of routines: 6



M2

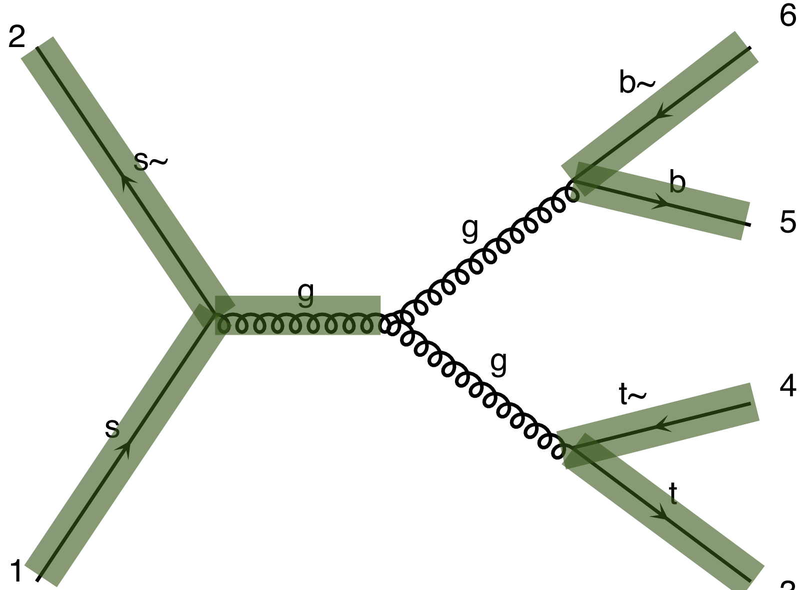
Number of routines: 6

Number of routines for both: 6

$$|M|^2 = |M_1 + M_2|^2$$

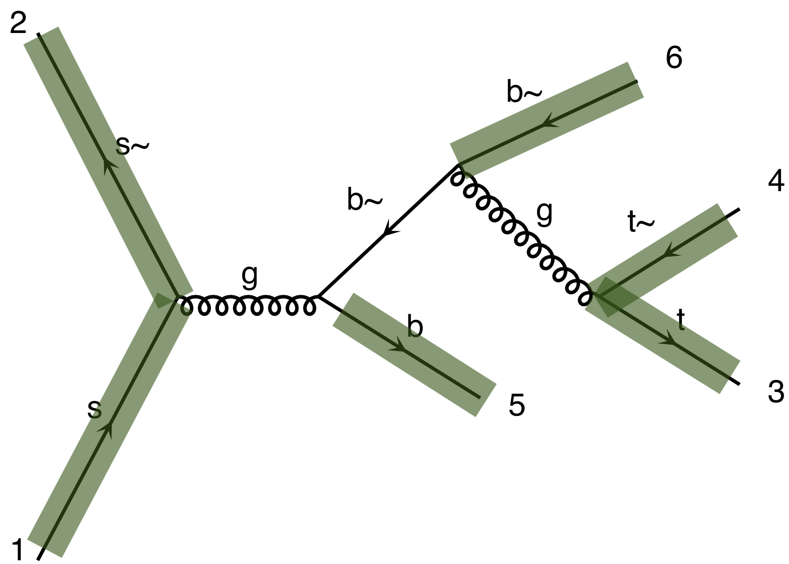
Real case

Known



M1

Number of routines: 7



M2

Number of routines: 6

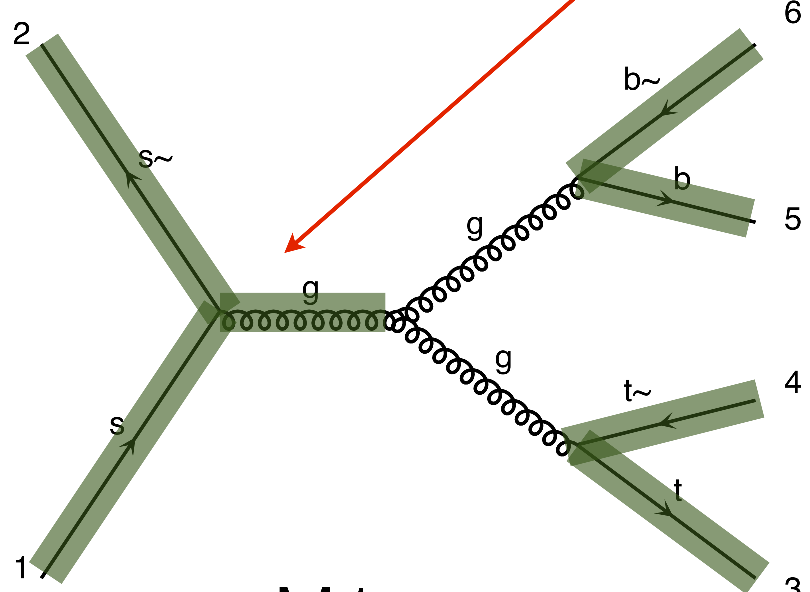
Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

Real case

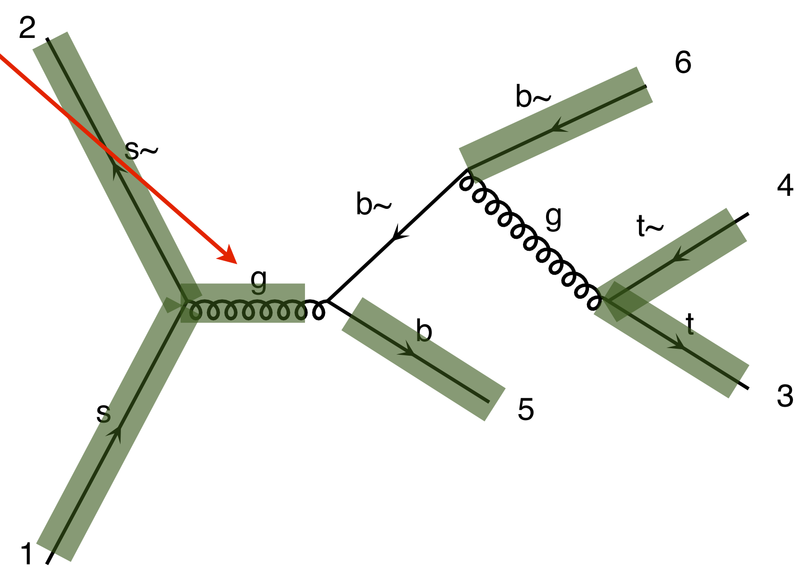
Known

Identical



M1

Number of routines: 7



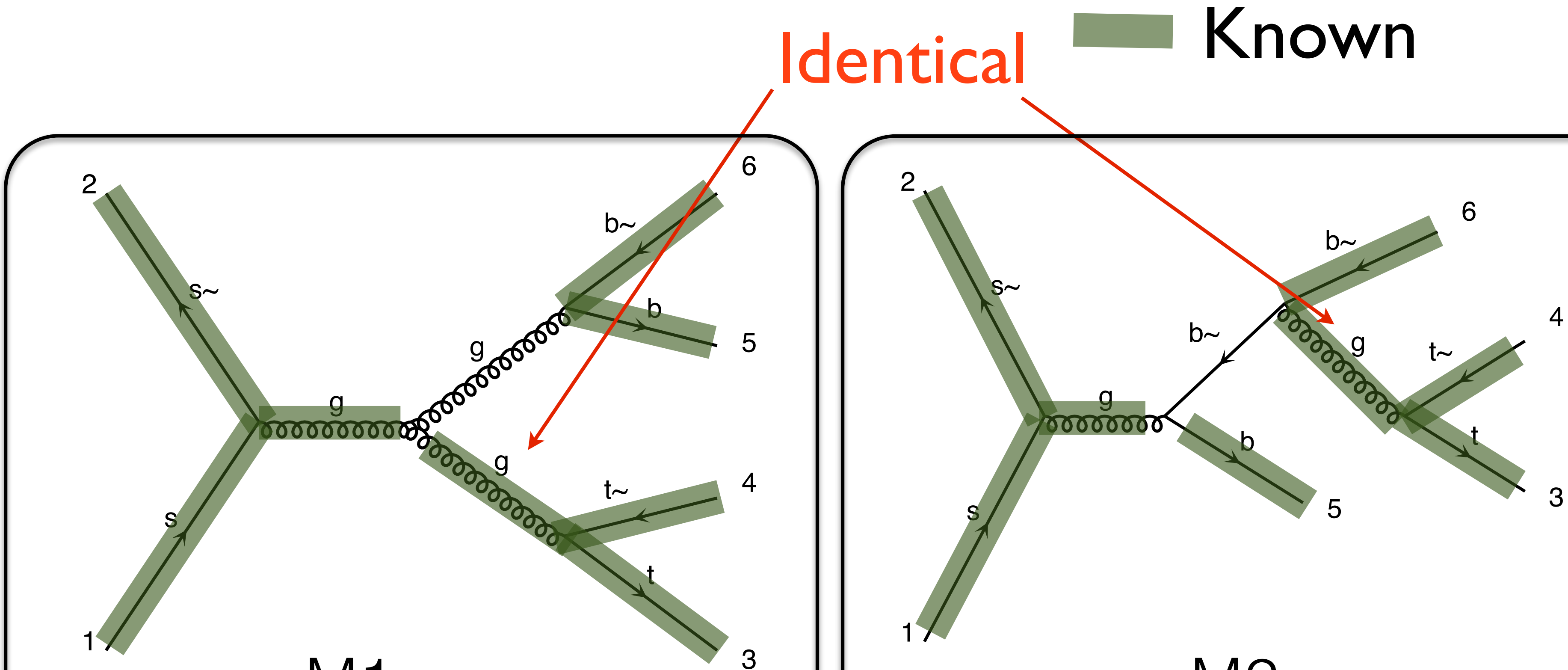
M2

Number of routines: 7

Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

Real case



M1

Number of routines: 8

M2

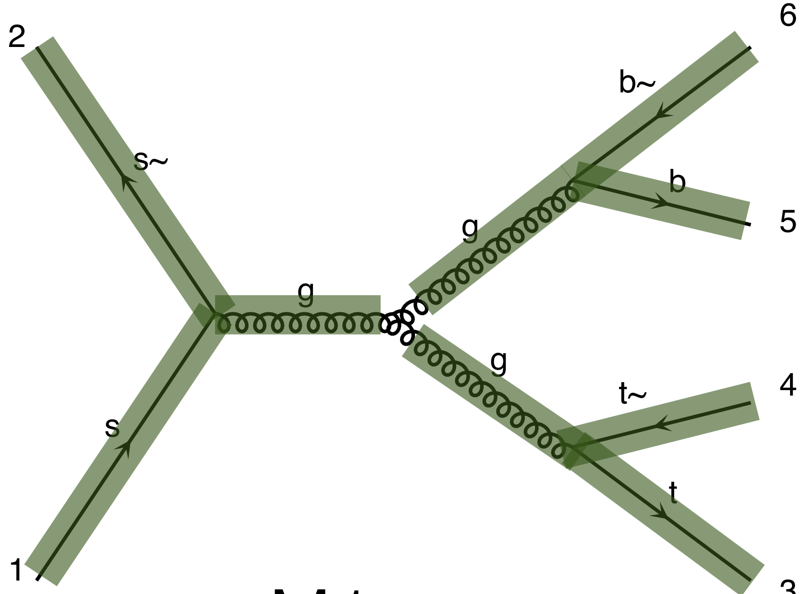
Number of routines: 8

Number of routines for both: 8

$$|M|^2 = |M_1 + M_2|^2$$

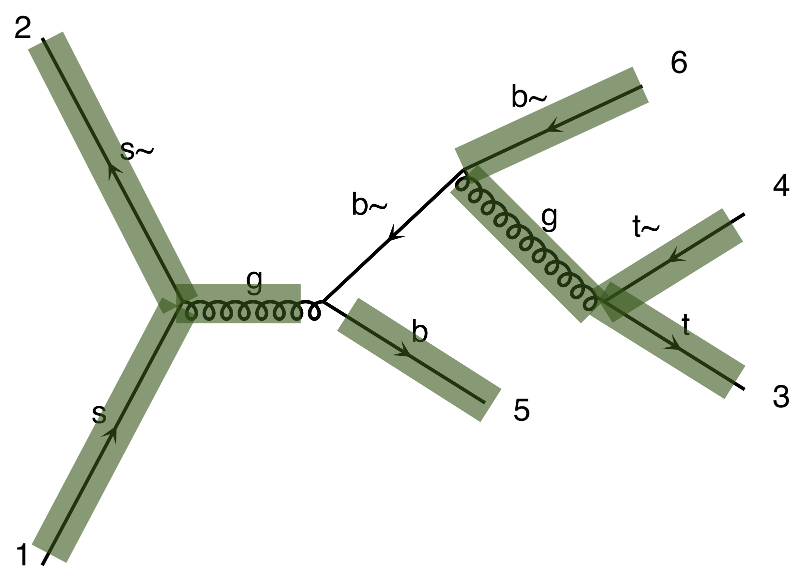
Real case

Known



M1

Number of routines: 9



M2

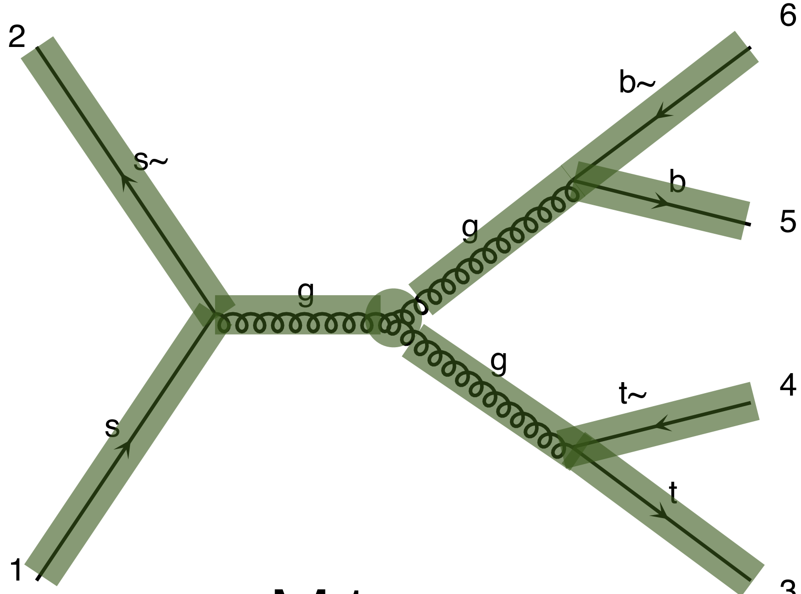
Number of routines: 8

Number of routines for both: 9

$$|M|^2 = |M_1 + M_2|^2$$

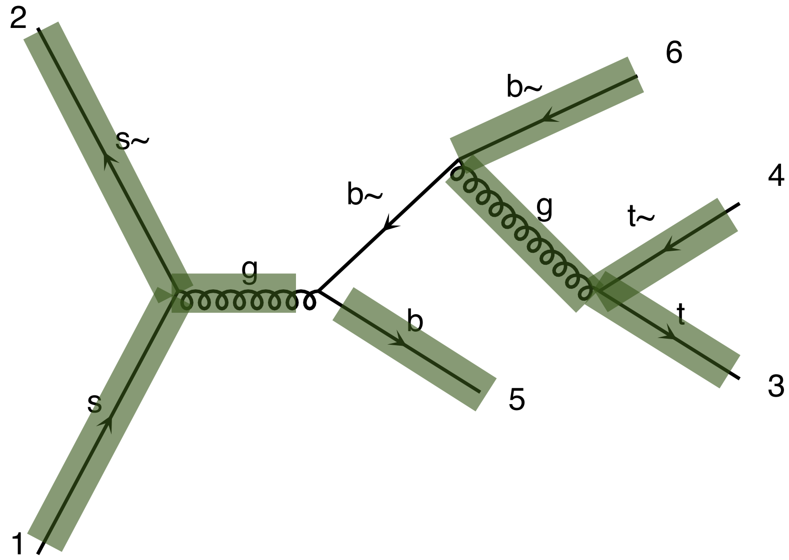
Real case

Known



M1

Number of routines: 10



M2

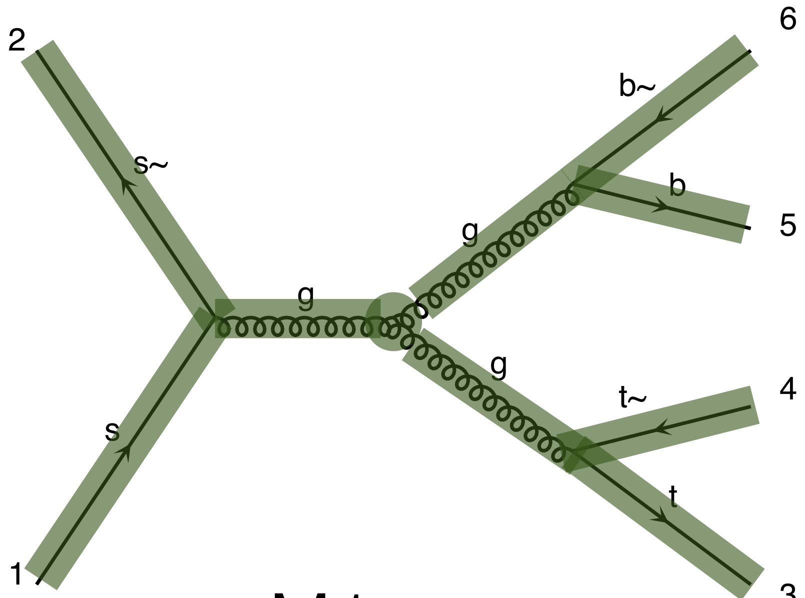
Number of routines: 8

Number of routines for both: 10

$$|M|^2 = |M_1 + M_2|^2$$

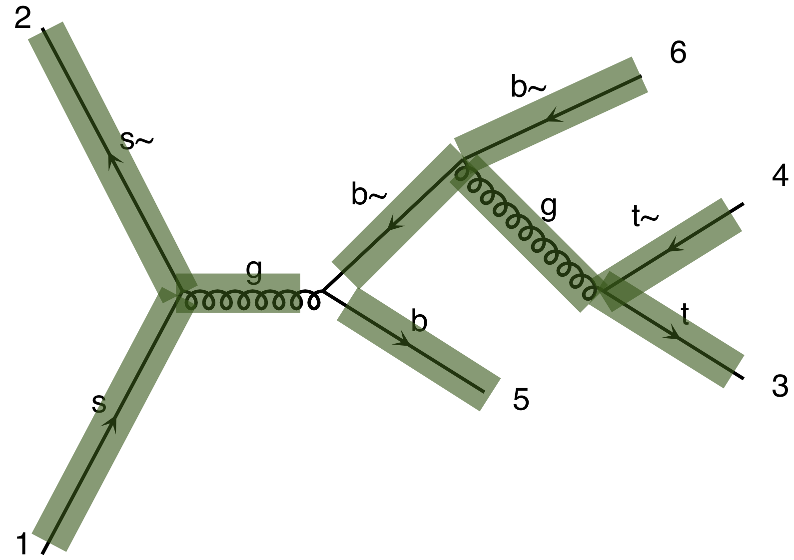
Real case

Known



M1

Number of routines: 10



M2

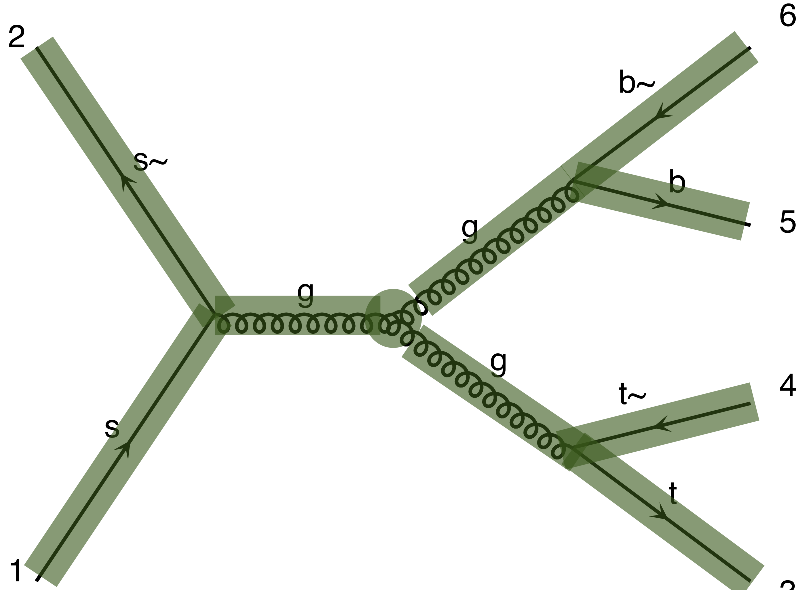
Number of routines: 9

Number of routines for both: 11

$$|M|^2 = |M_1 + M_2|^2$$

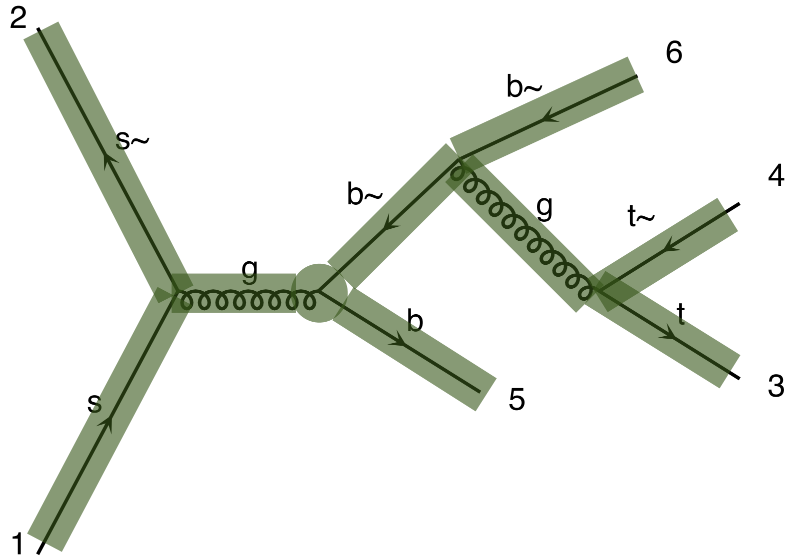
Real case

Known



M1

Number of routines: 10



M2

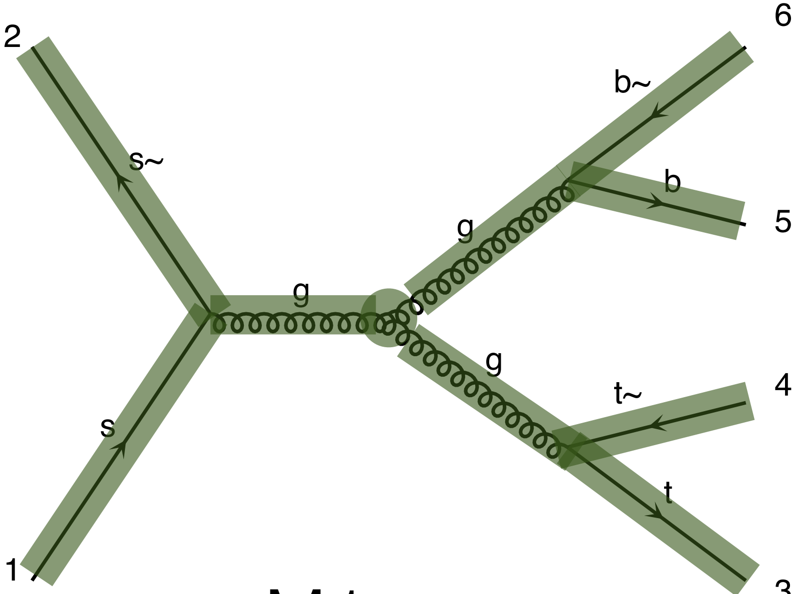
Number of routines: 10

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

Real case

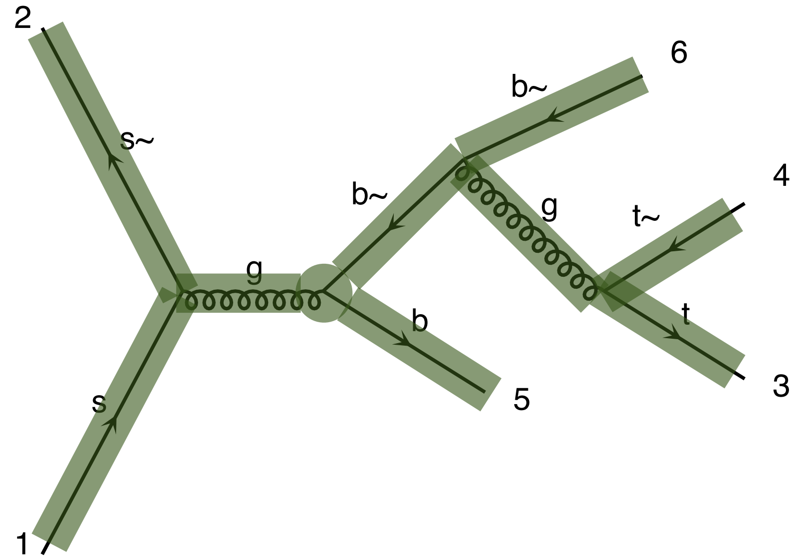
Known



M1

Number of routines: 10

2(N+1)



M2

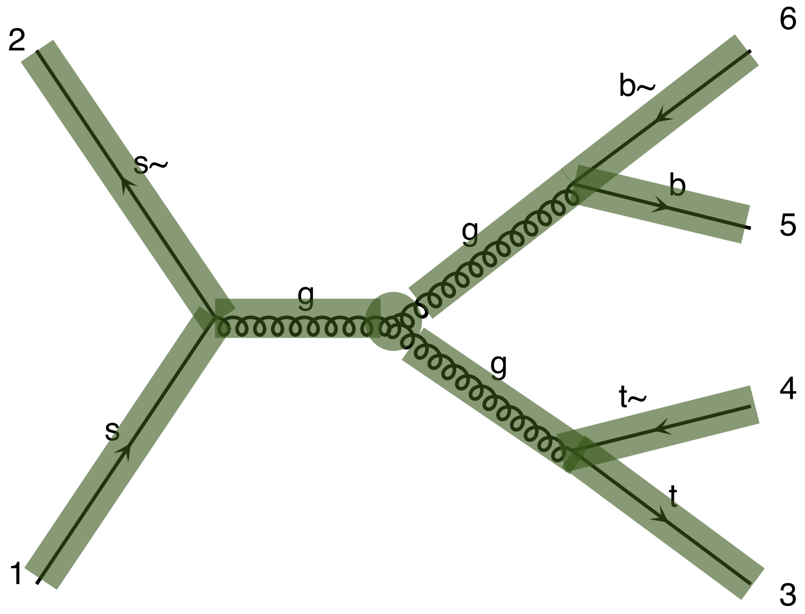
Number of routines: 10

2(N+1)

Number of routines for both: 12

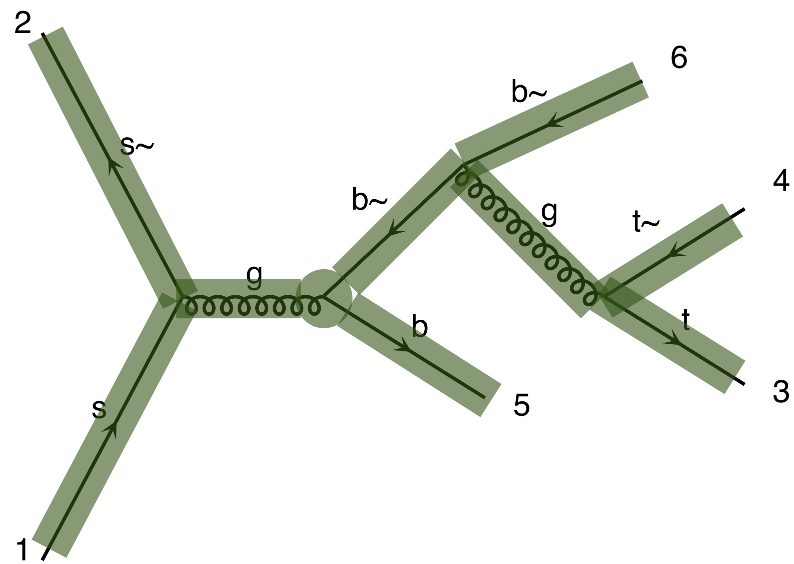
$$|M|^2 = |M_1 + M_2|^2$$

Real case



Number of routines: 10

2(N+1)



Number of routines: 10

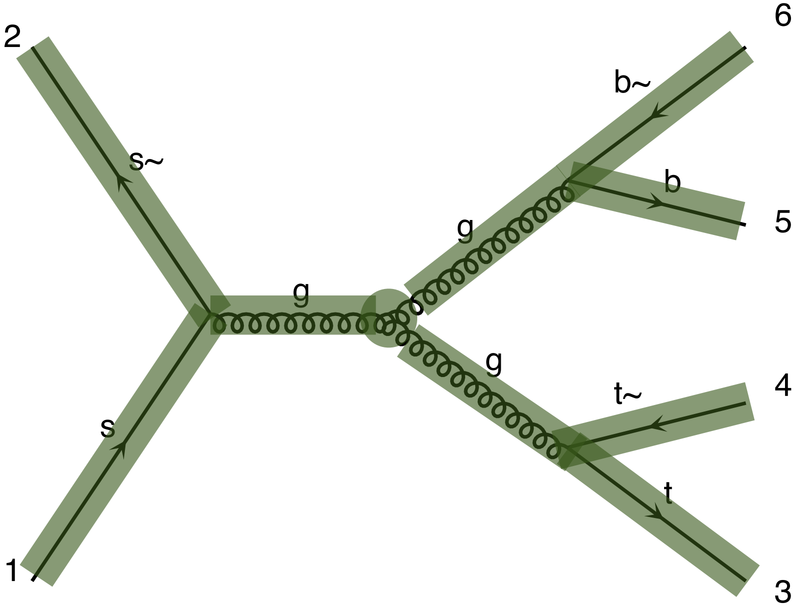
2(N+1)

Number of routines for both: 12

N!*2(N+1) ⟶ N!

Real case

Known



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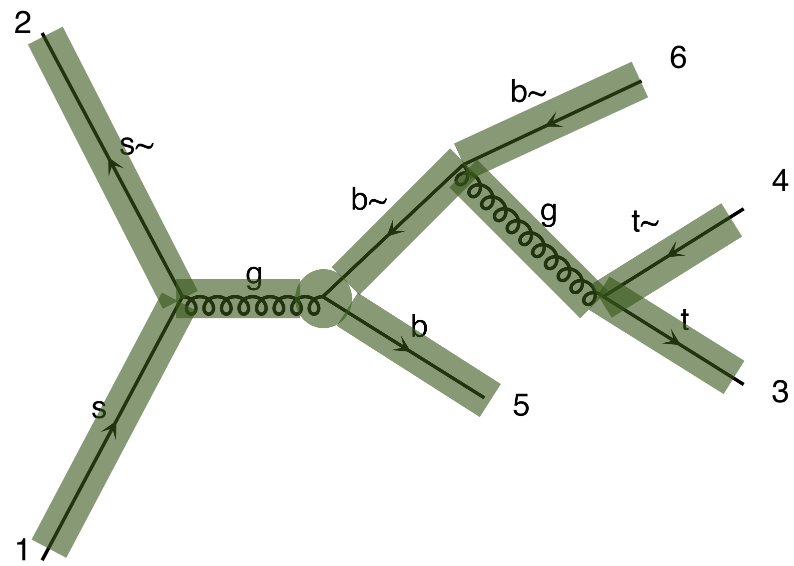

Number of routines: 10

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Number of routines for both: 12

N!*2(N+1) ⟶ N! $\xrightarrow{\text{recursion}}$ 2^N

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$
Recycling	M	$(N-1)!\,2^{(N-1)}$

Comparison

	M diag	N particle	2 > 6
Analytical	M^2	$(N!)^2$	1.6e9
Helicity	M	$(N!)\,2^N$	1.0e7
Recycling	M	$(N-1)!\,2^{(N-1)}$	6.5e5
Recursion Relation	$log(M)$	$2^N\,2^{(N-1)}$	3.2e4

Helicity amplitudes

- Thanks to new diagram generation algorithm, wf recycling much more efficient in MG5 than MG4

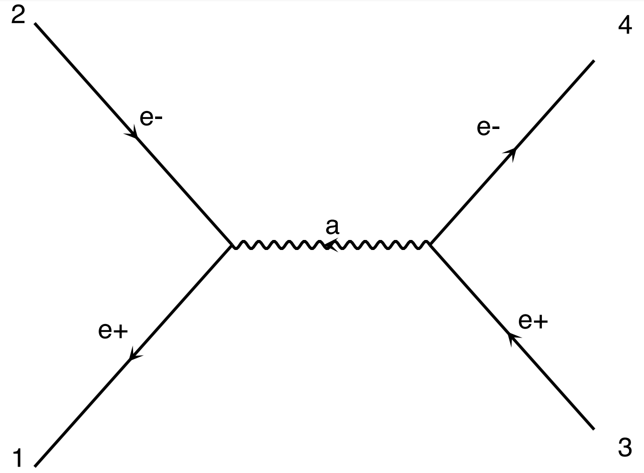
Process	Amplitudes	Wavefunctions		Run time	
		MG 4	MG 5	MG 4	MG 5
$u\bar{u} \to e^+e^-$	2	6	6	$< 6\mu s$	$< 6\mu s$
$u\bar{u} \to e^+e^-e^+e^-$	48	62	32	0.22 ms	0.14 ms
$u\bar{u} \to e^+e^-e^+e^-e^+e^-$	3474	3194	301	46.5 ms	19.0 ms
$u\bar{u} \to d\bar{d}$	1	5	5	$< 4\mu s$	$< 4\mu s$
$u\bar{u} \to d\bar{d}g$	5	11	11	27 μs	27 μs
$u\bar{u} \to d\bar{d}gg$	38	47	29	0.42 ms	0.31 ms
$u\bar{u} \to d\bar{d}ggg$	393	355	122	10.8 ms	6.75 ms
$u\bar{u} \to u\bar{u}gg$	76	84	40	1.24 ms	0.80 ms
$u\bar{u} \to u\bar{u}ggg$	786	682	174	35.7 ms	17.2 ms
$u\bar{u} \to d\bar{d}d\bar{d}$	14	28	19	84 μs	83 μs
$u\bar{u} \to d\bar{d}d\bar{d}g$	132	178	65	1.88 ms	1.15 ms
$u\bar{u} \to d\bar{d}d\bar{d}gg$	1590	1782	286	141 ms	34.4 ms
$u\bar{u} \to d\bar{d}d\bar{d}d\bar{d}$	612	758	141	42.5 ms	6.6 ms

no recycling

300,000

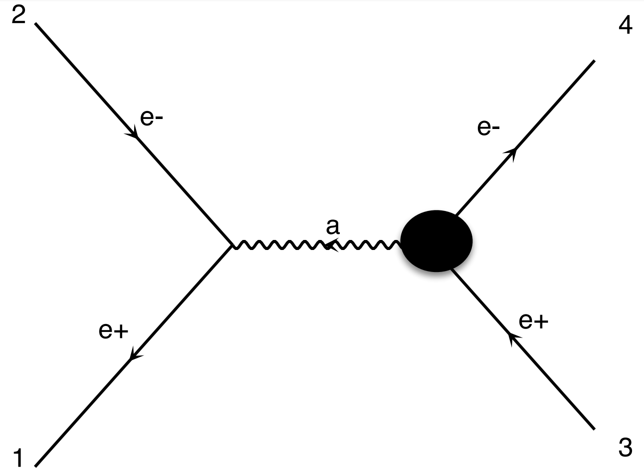
5500

Time for matrix element evaluation on a Sony Vaio TZ laptop



$$\mathcal{M} = \bar{u}\gamma^\mu v P_{\mu\nu} \bar{u}\gamma^\nu v$$

$$\bar{u}_1 = fct(\vec{p}_1, m)$$
$$v_2 = fct(\vec{p}_2, m)$$
$$\bar{u}_3 = fct(\vec{p}_3, m)$$
$$v_4 = fct(\vec{p}_4, m)$$
$$W_a = fct(\bar{u}_1, v_2, M_a, \Gamma_a)$$
$$\mathcal{M} = fct(\bar{u}_3, v_4, W_a)$$



$$\mathcal{M} = \bar{u}\gamma^\mu v P_{\mu\nu} \bar{u}\Gamma^\mu v$$

$$\bar{u}_1 = fct(\vec{p}_1, m)$$
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$$\mathcal{M} = fct(\bar{u}_3, v_4, W_a)$$

HELAS

- **Original HELicity Amplitude Subroutine library**

[Murayama, Watanabe, Hagiwara]

HELAS

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- **One routine by Lorentz structure**
  - ➡**MSSM** [cho, al] hep-ph/0601063 (2006)
  - ➡**HEFT** [Frederix] (2007)
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  - ➡**Spin 3/2** [Mawatari, al] 1101.1289 (2011)

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Chiral Perturbation

BNV Model

SLIH

Effective Field Theory

NMSSM

Chromo-magnetic operator

Full HEFT

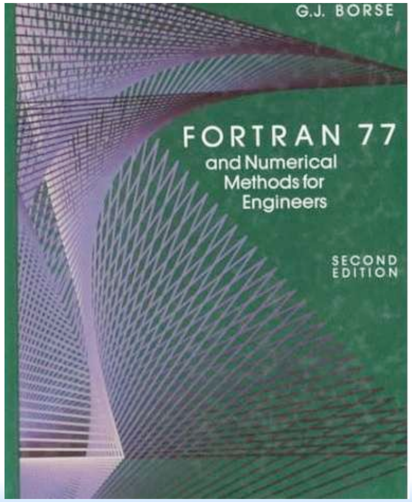
Black Holes

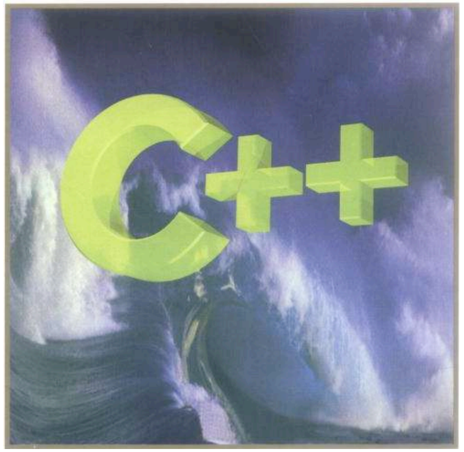
ALOHA

ALOHA
~~Google~~ translate

From: UFO ▾ To: Helicity [Translate]

Type text or a website address or translate a document.





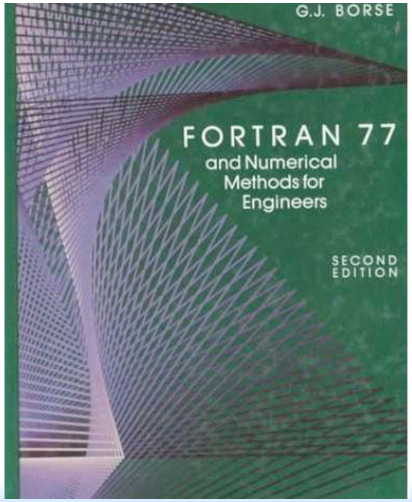
ALOHA

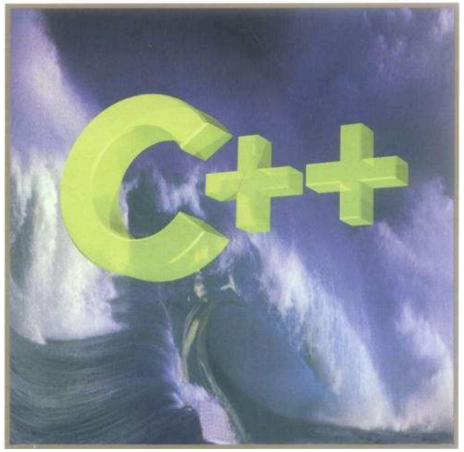
ALOHA
~~Google~~ translate

From: UFO To: Helicity Translate

Basically, any new operator can be handle by MG5/Pythia8 out of the box!

Type text or a website address or translate a document.





ALOHA DETAIL

Input

```
FFV1 = Lorentz(name = 'FFV1',
               spins = [ 2, 2, 3 ],
               structure = 'Gamma(3,2,1)')
```

Output

```
C     This File is Automatically generated by ALOHA
C     The process calculated in this file is:
C     Gamma(3,2,1)
C
      SUBROUTINE FFV1_0(F1,F2,V3,C,VERTEX)
      IMPLICIT NONE
      DOUBLE COMPLEX F1(6)
      DOUBLE COMPLEX F2(6)
      DOUBLE COMPLEX V3(6)
      DOUBLE COMPLEX C
      DOUBLE COMPLEX VERTEX


      VERTEX = C*( (F2(1)*( (F1(3)*( (0, -1)*V3(1)+(0, 1)*V3(4)))
     $ +(F1(4)*( (0, 1)*V3(2)+V3(3)))))+( (F2(2)*( (F1(3)*( (0, 1)
     $ *V3(2)-V3(3)))+(F1(4)*( (0, -1)*V3(1)+(0, -1)*V3(4)))))
     $ +( (F2(3)*( (F1(1)*( (0, -1)*V3(1)+(0, -1)*V3(4)))+(F1(2)
     $ *( (0, -1)*V3(2)-V3(3)))))+(F2(4)*( (F1(1)*( (0, -1)*V3(2)
     $ +V3(3)))+(F1(2)*( (0, -1)*V3(1)+(0, 1)*V3(4)))))))

      END
```

ALOHA

- Compute those Function Analytically
- Code in Python
- Can handle
  - ➡ all spin up to 2
  - ➡ custom propagator
  - ➡ majorana (but in 4 fermion operator)
  - ➡ Any dimensional operator
- Only use in MadGraph5_aMC@NLO
- Plan to have similar tools for the other generator

To Remember

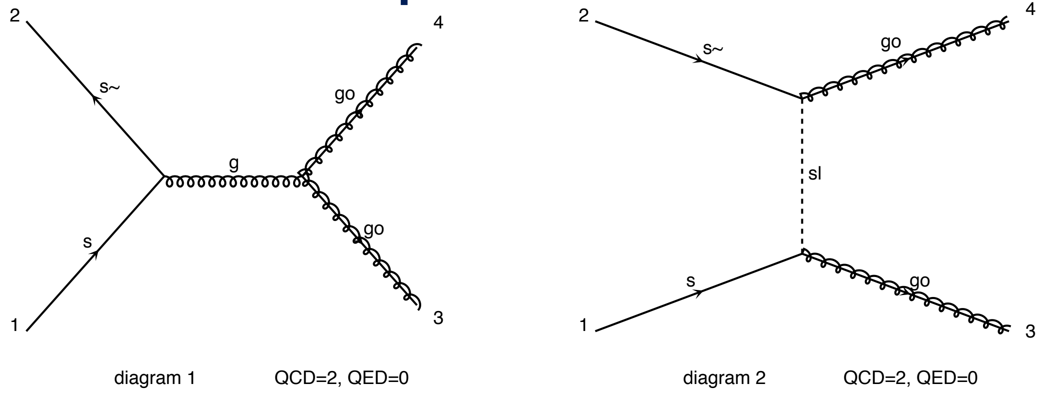
- Numerical computation faster than analytical computation
- We are able to compute matrix-elment
    - ➡ for large number of final state
    - ➡ for any BSM theory
    - ➡ actually also for loop

Monte Carlo Integration and Generation

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$$|\mathcal{M}|^2$$

➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Matrix-Element

Calculate a given process (e.g. gluino pair)

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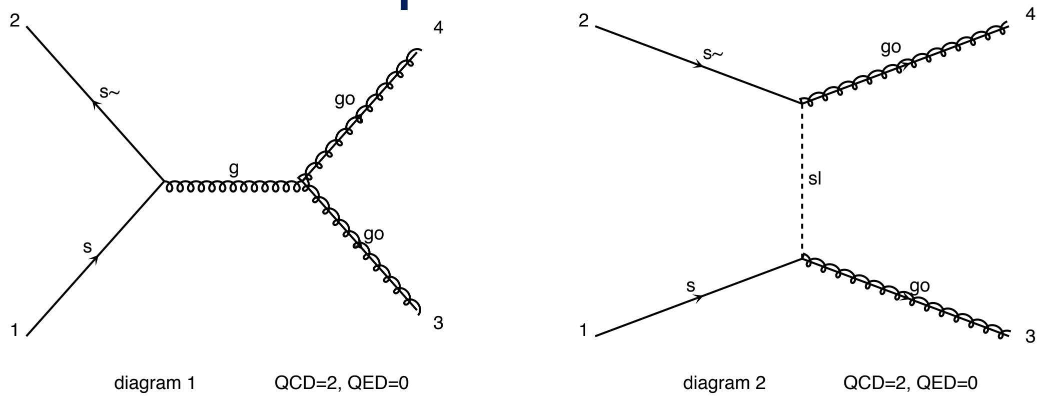

- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

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$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

Very Hard (in general)

Matrix-Element

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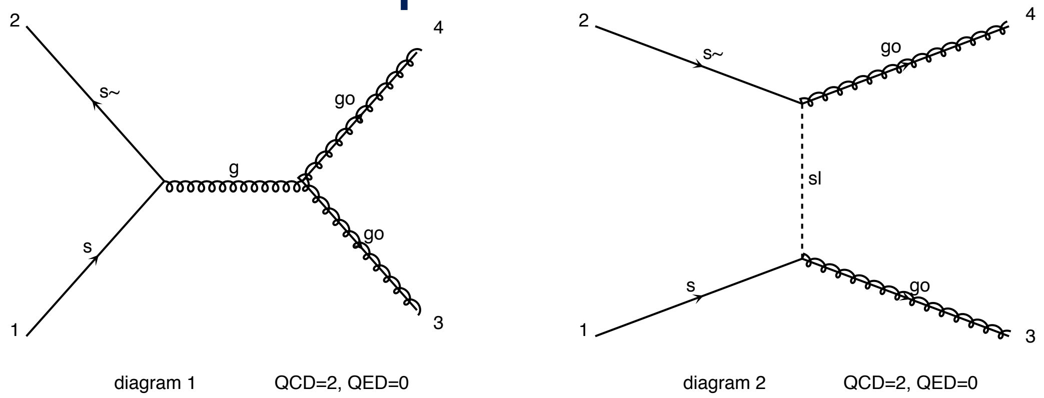
- Determine the production mechanism



- Evaluate the matrix-element

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$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

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Very Hard (in general)

Now

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$Dim[\Phi(n)] \sim 3n$$

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Monte Carlo Integration

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General and flexible method is needed

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$Dim[\Phi(n)] \sim 3n$$

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

General and flexible method is needed

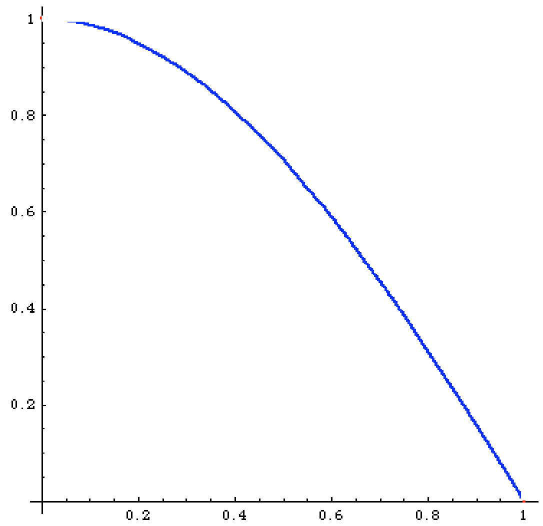
Not only integrating but also **generates events**

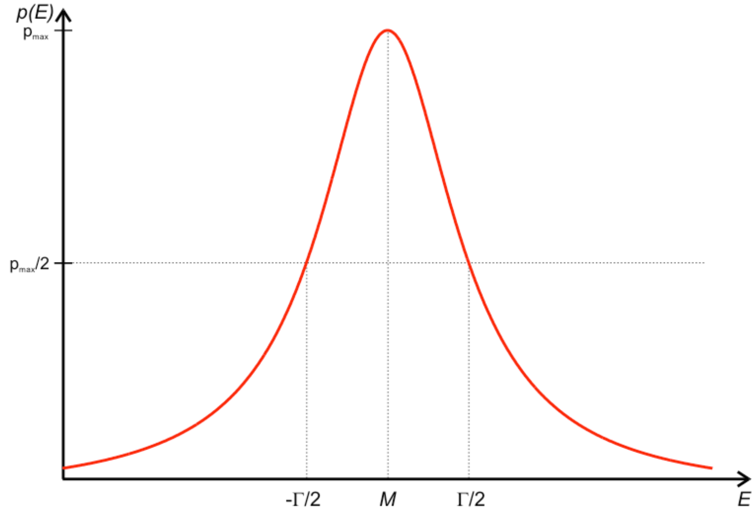
Integration

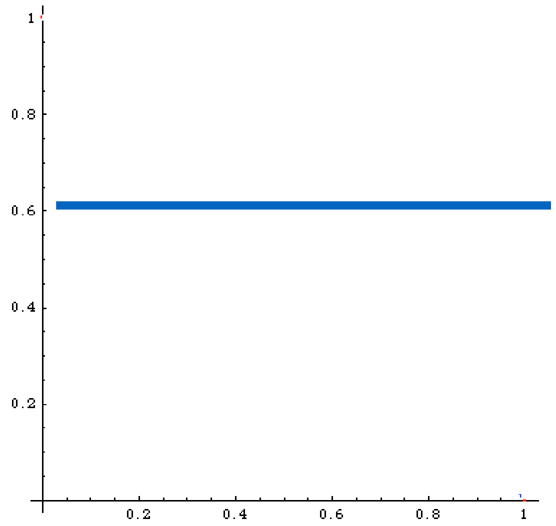
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$







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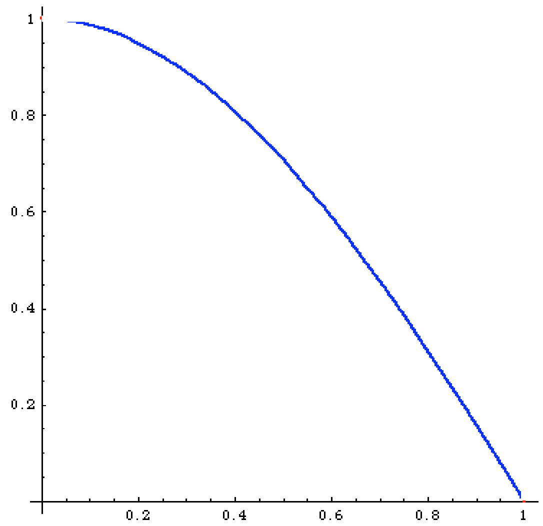
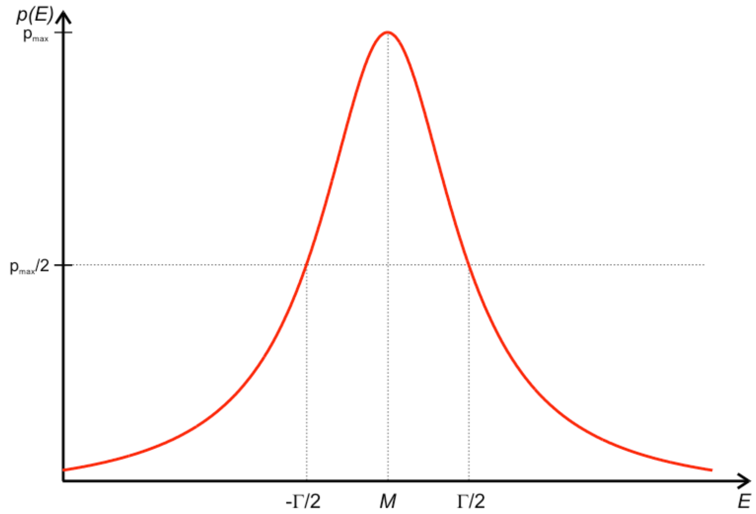


$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



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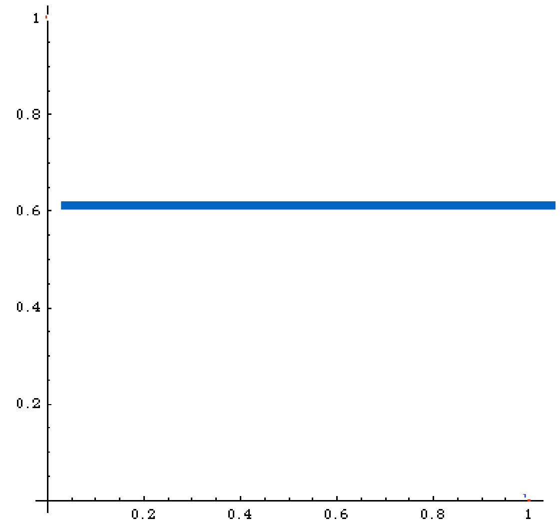

Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

Integration

$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

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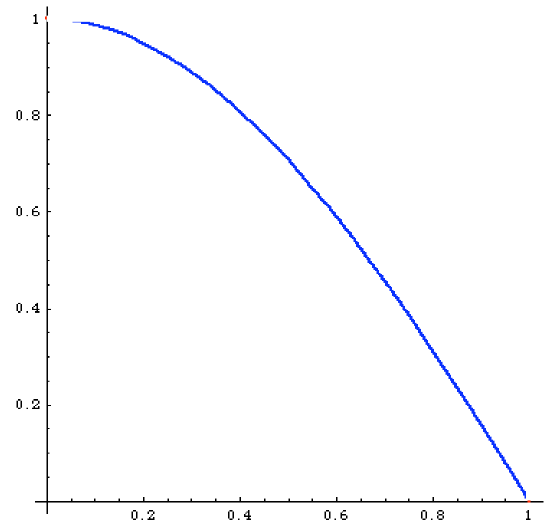

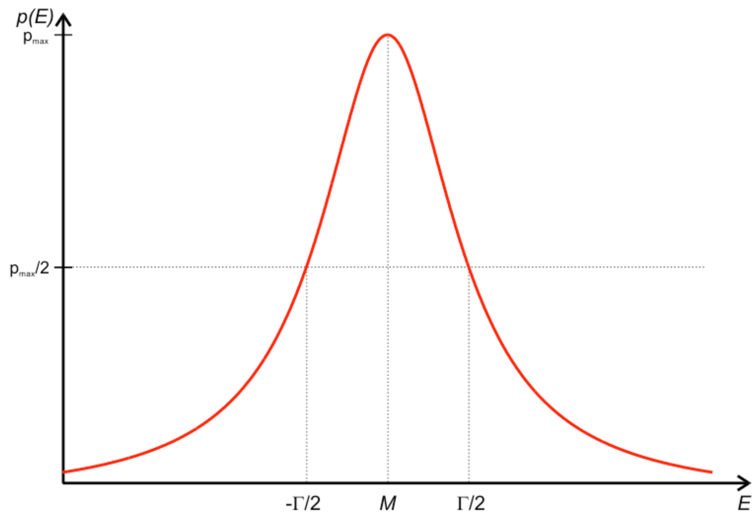

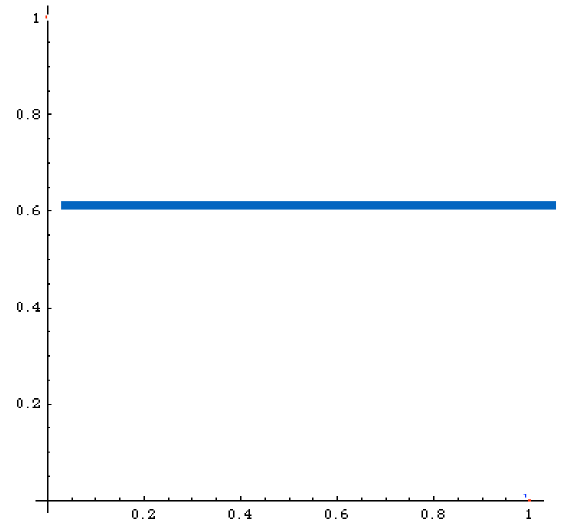

	simpson	MC
3	0,638	0,3
5	0,6367	0,8
20	0,63662	0,6
100	0,636619	0,65
1000	0,636619	0,636

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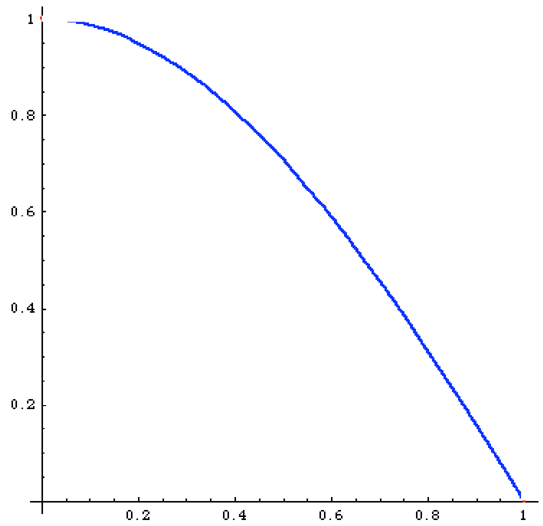
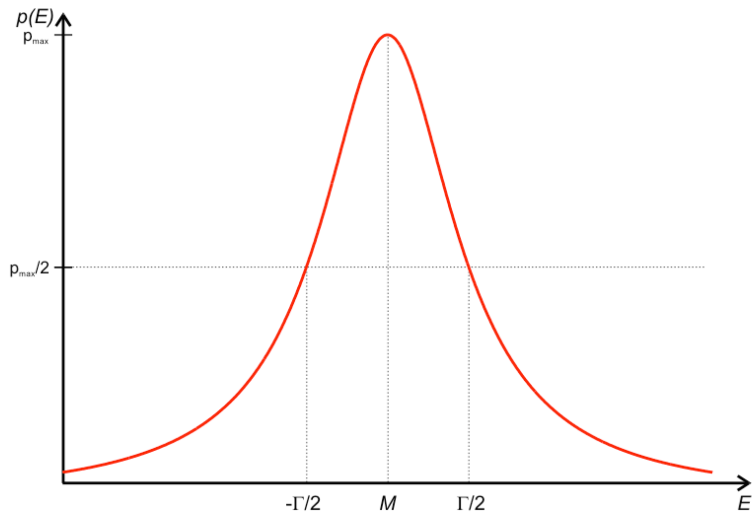
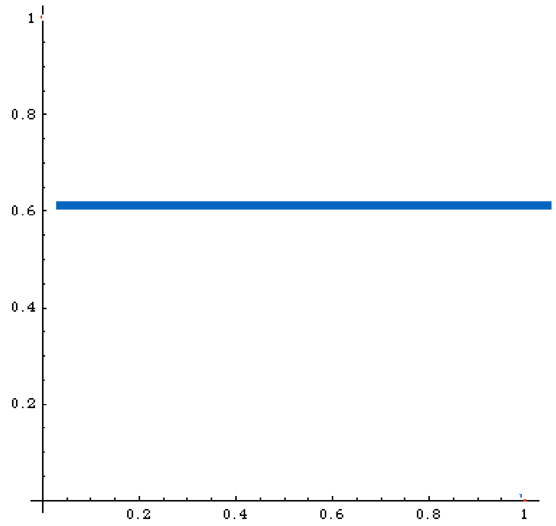


$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



$$\int dx\, C$$



Method of evaluation

- MonteCarlo $1/\sqrt{N}$
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- Simpson $1/N^4$

More Dimension →

$1/\sqrt{N}$

$1/N^{2/d}$

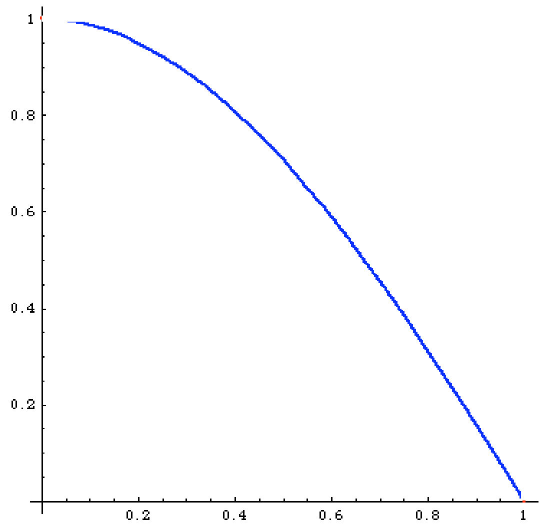
$1/N^{4/d}$

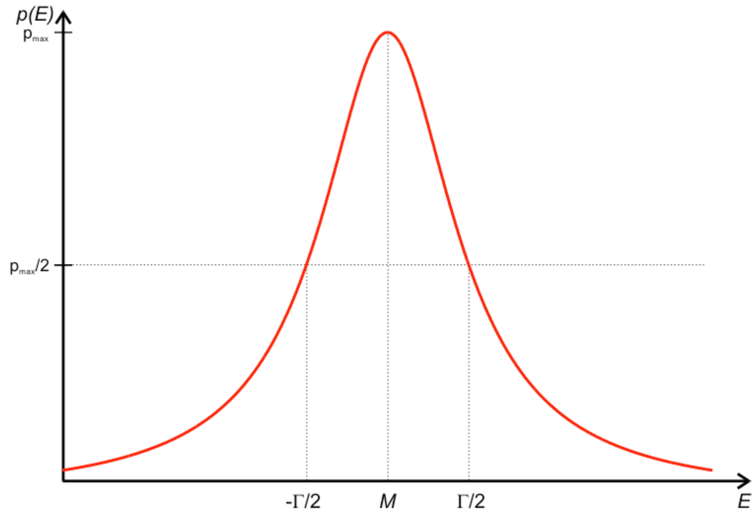
Integration

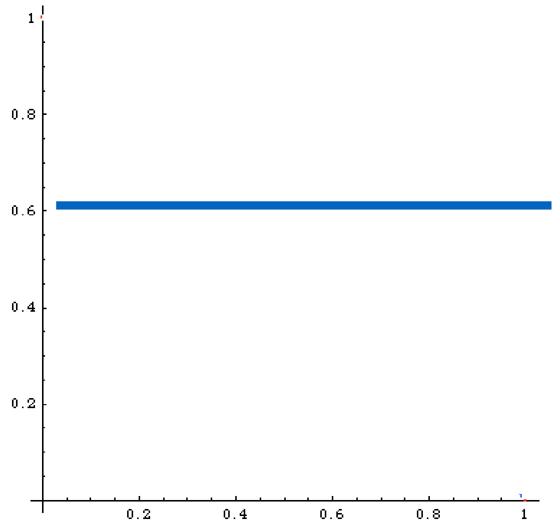
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$







$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

Integration

$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

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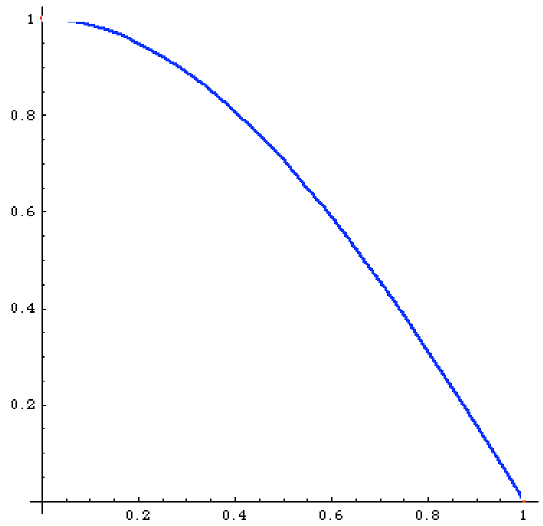
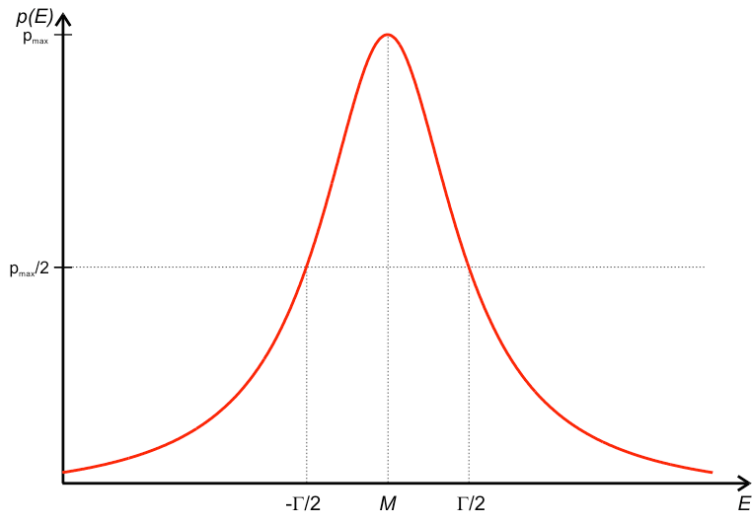
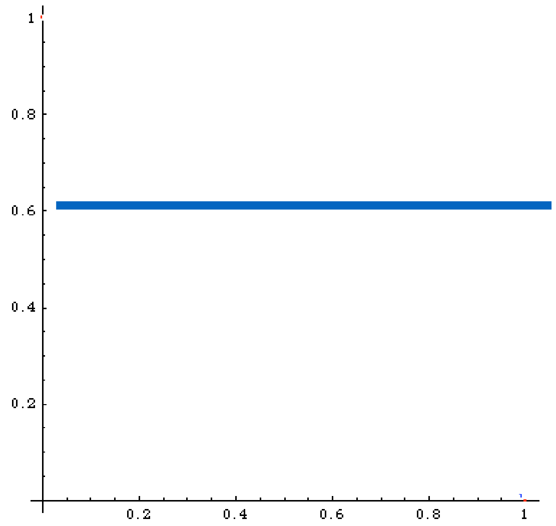






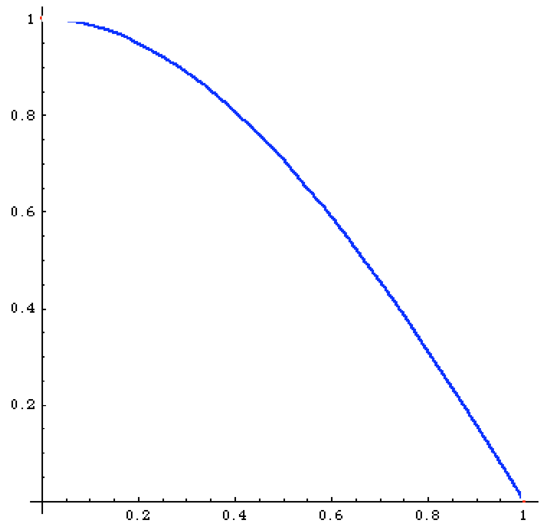
$$I = \int_{x_1}^{x_2} f(x)dx \Rightarrow I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \Rightarrow V_N = (x_2 - x_1)^2\frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

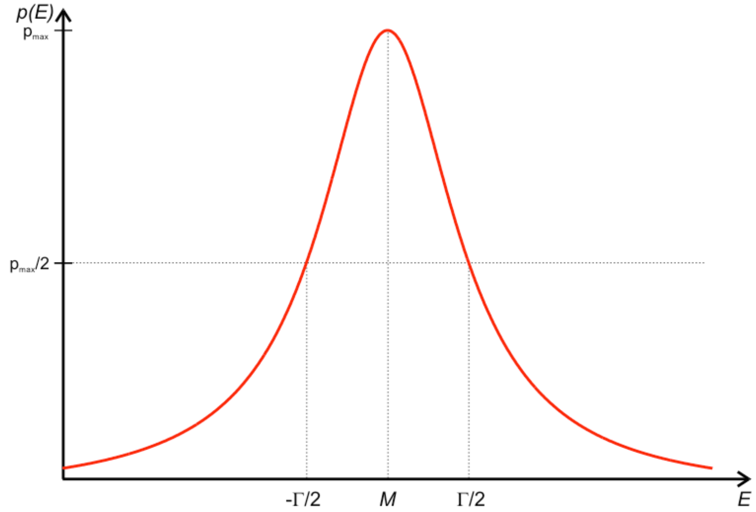
$$I = I_N \pm \sqrt{V_N/N}$$

Integration

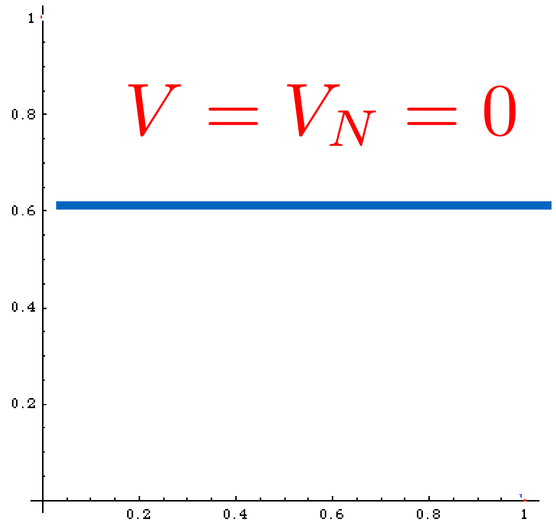
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



$$\int dx\, C$$



$$V = V_N = 0$$

$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2}[f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

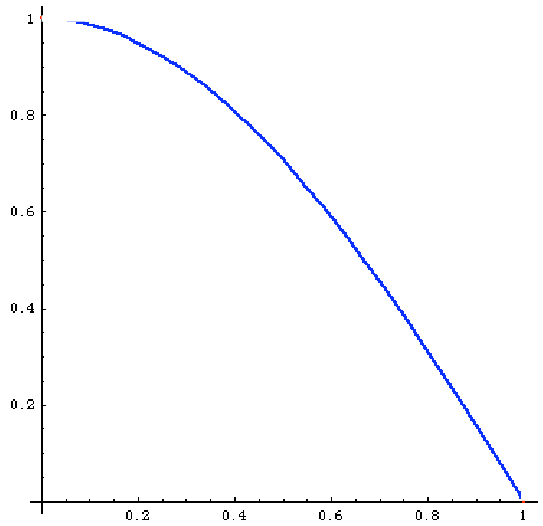
$$I = I_N \pm \sqrt{V_N/N}$$

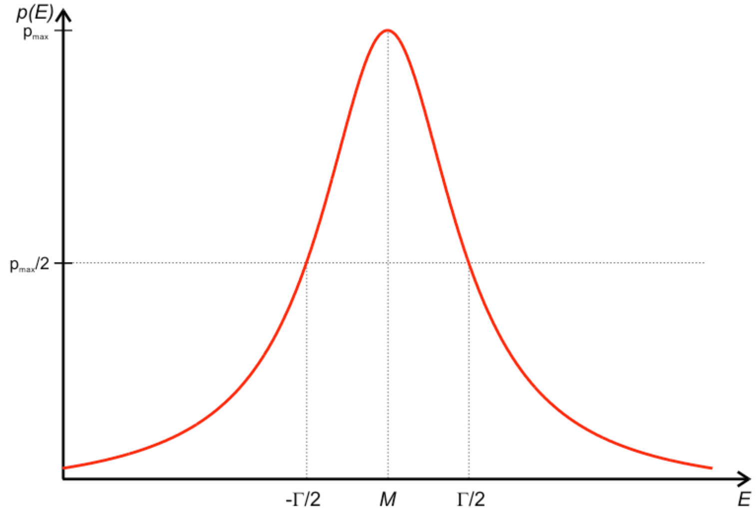
Integration

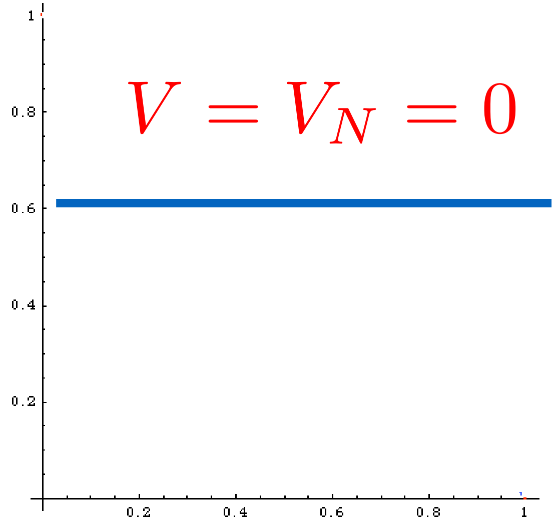
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$







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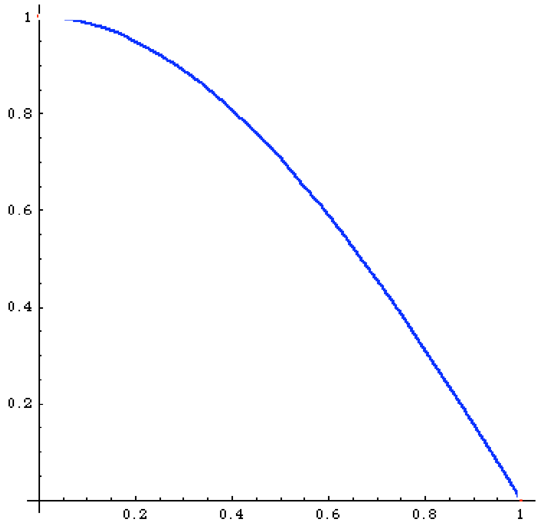
$$V = (x_2 - x_1)\int_{x_1}^{x_2}[f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2\frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

$$I = I_N \pm \sqrt{V_N/N}$$

Can be minimized!

Importance Sampling



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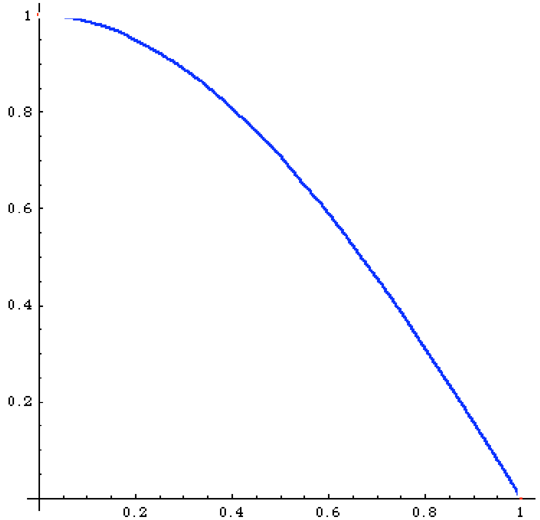

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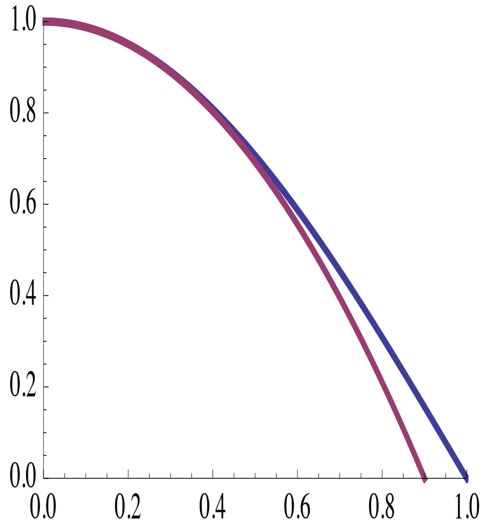

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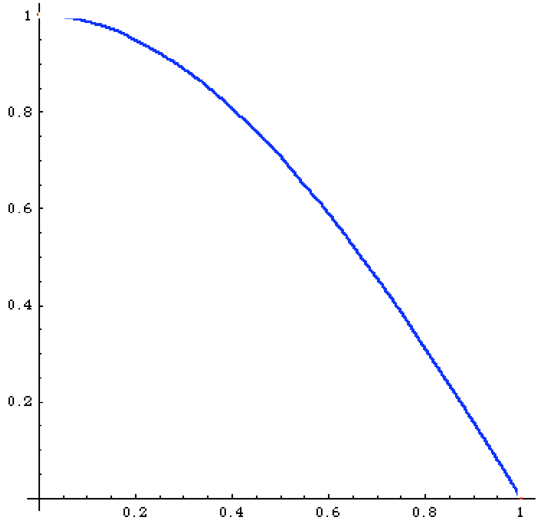

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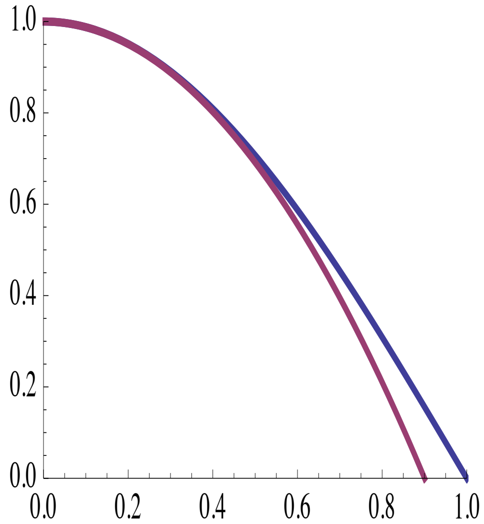

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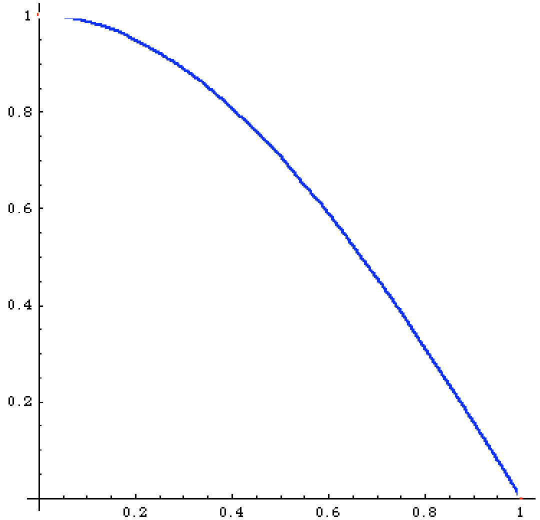

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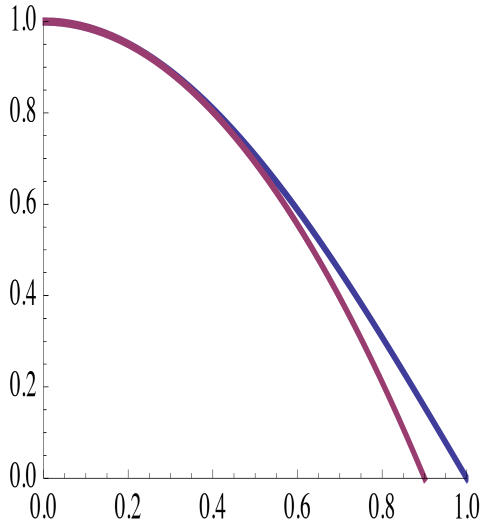

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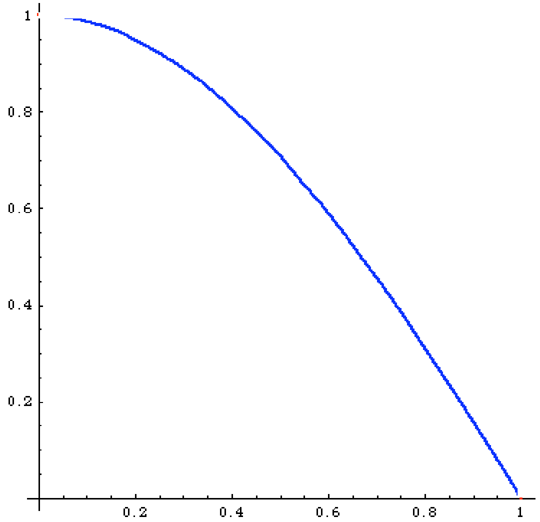

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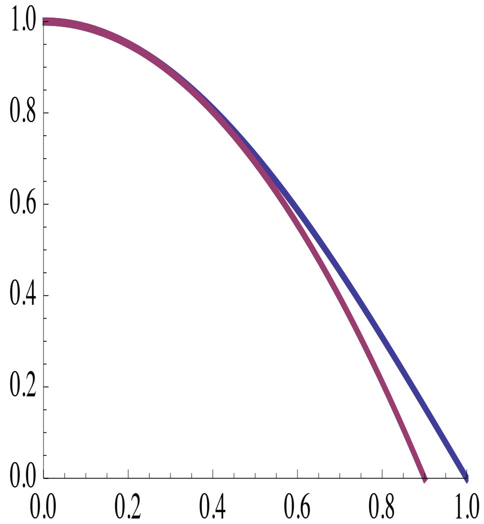

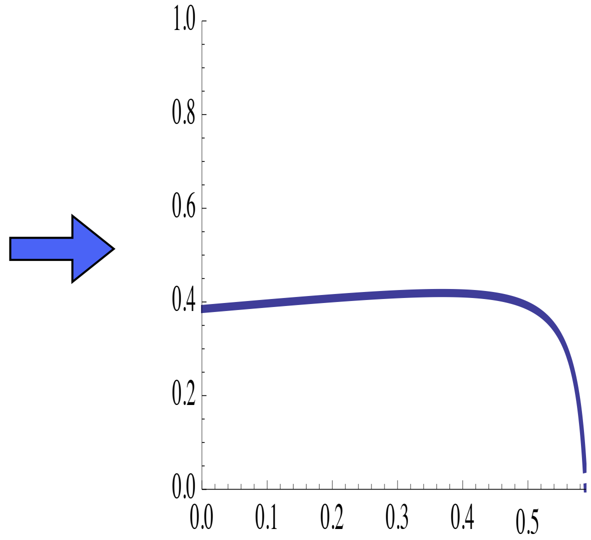

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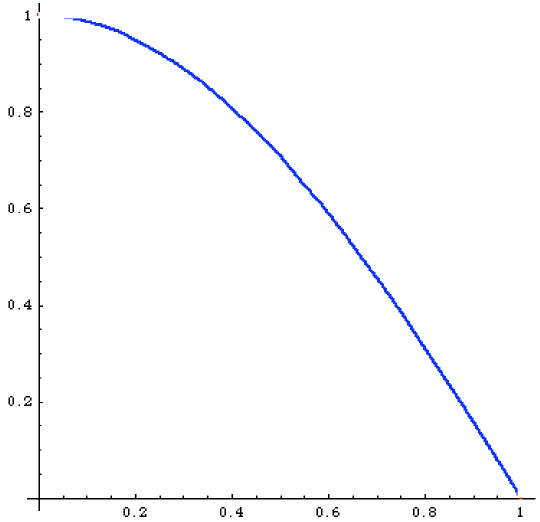

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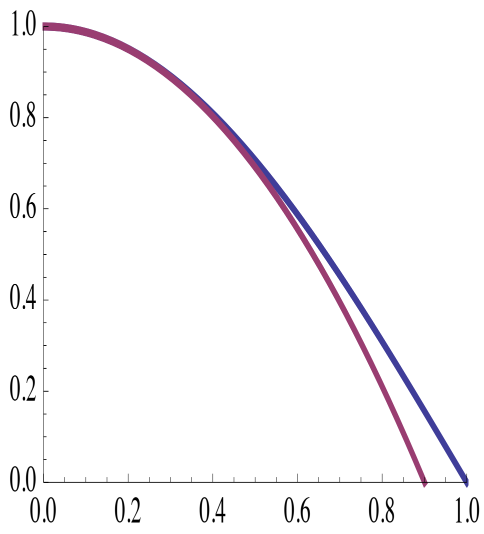

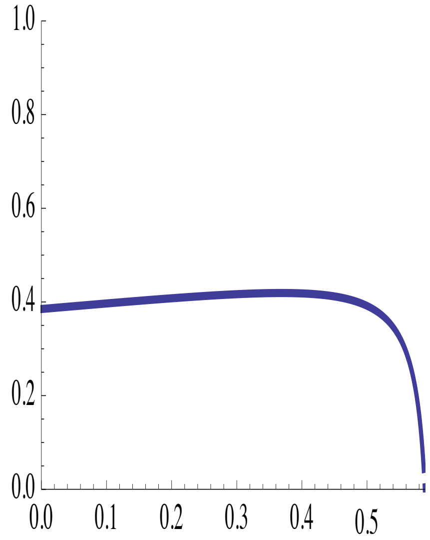

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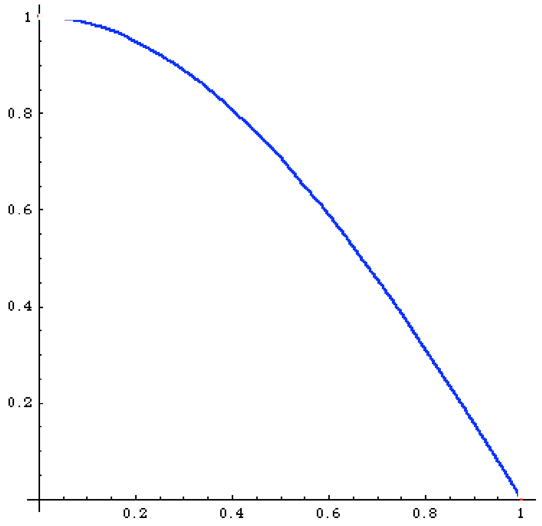
Importance Sampling



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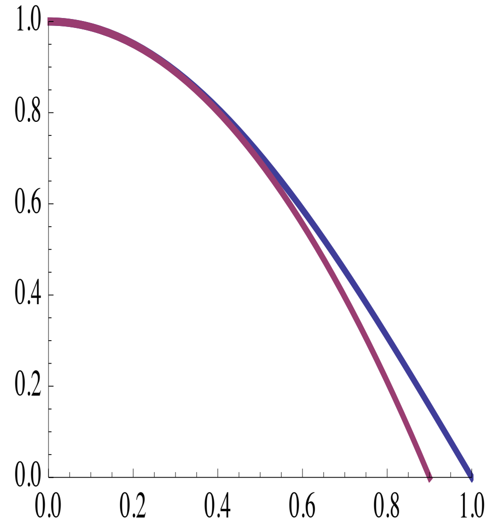

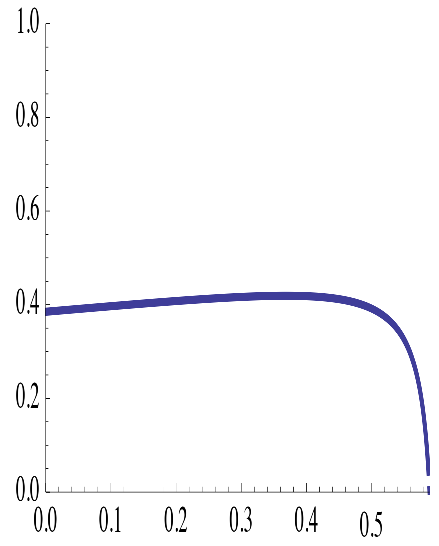

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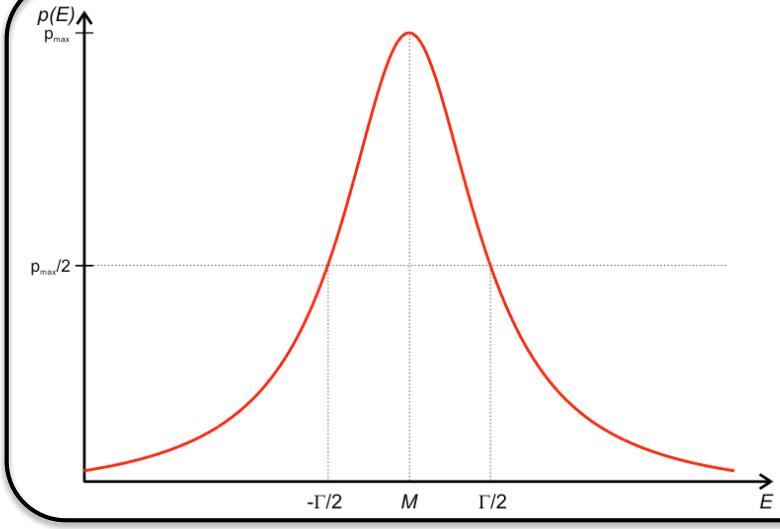
The Phase-Space parametrization is important to have an efficient computation!

Importance Sampling



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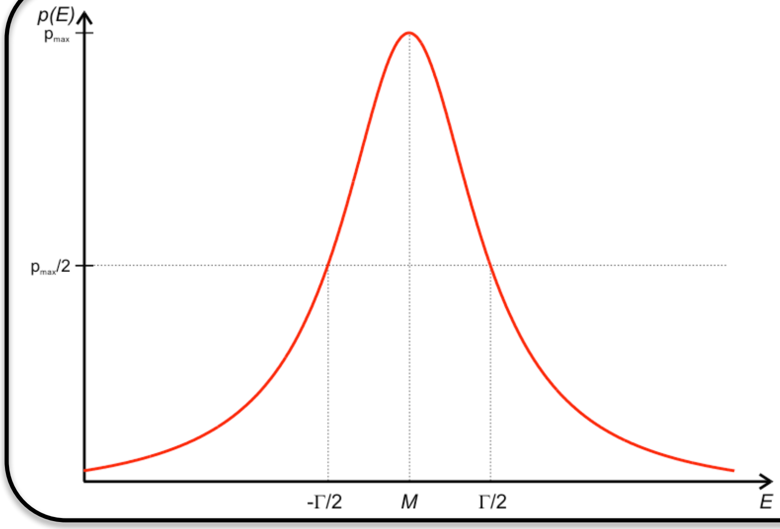

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

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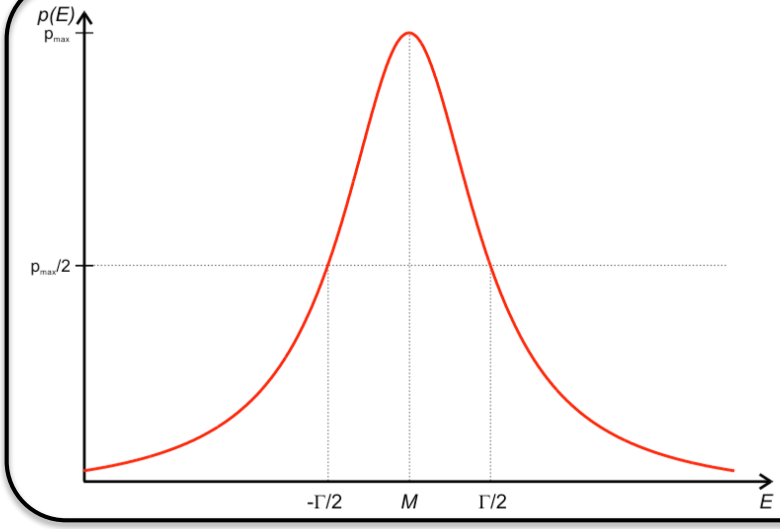

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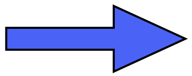

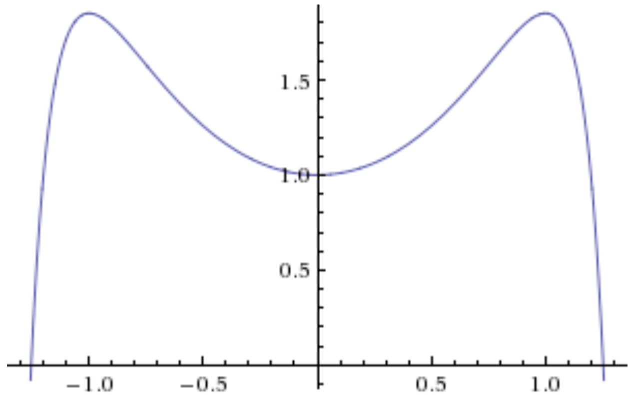

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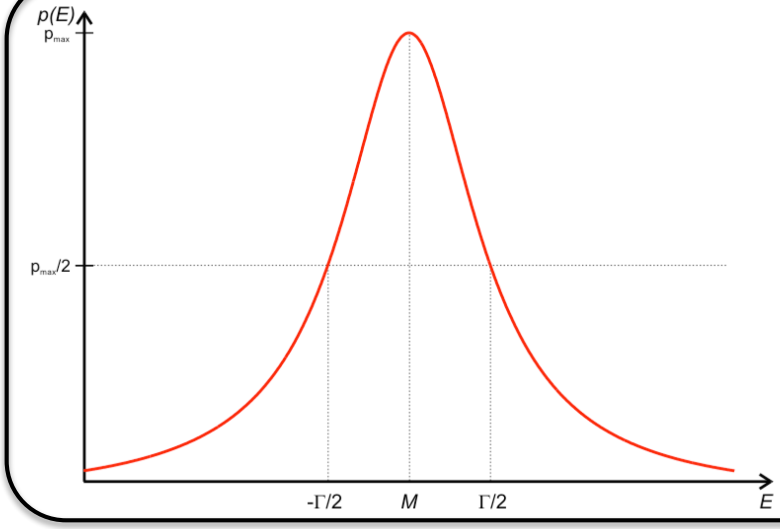

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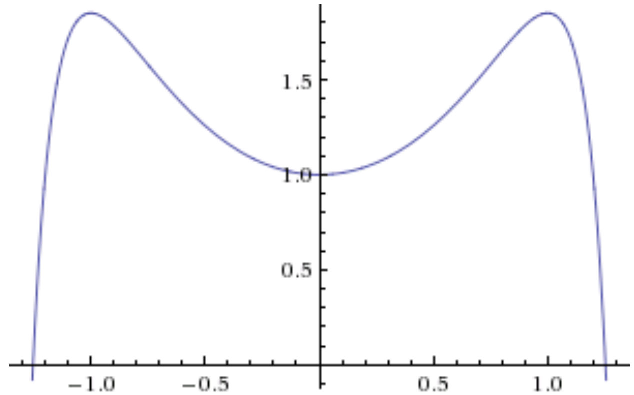
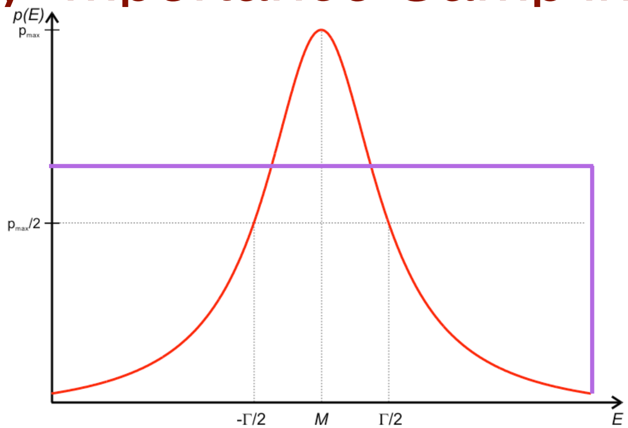


Why Importance Sampling?



Probability of using that point p(x)

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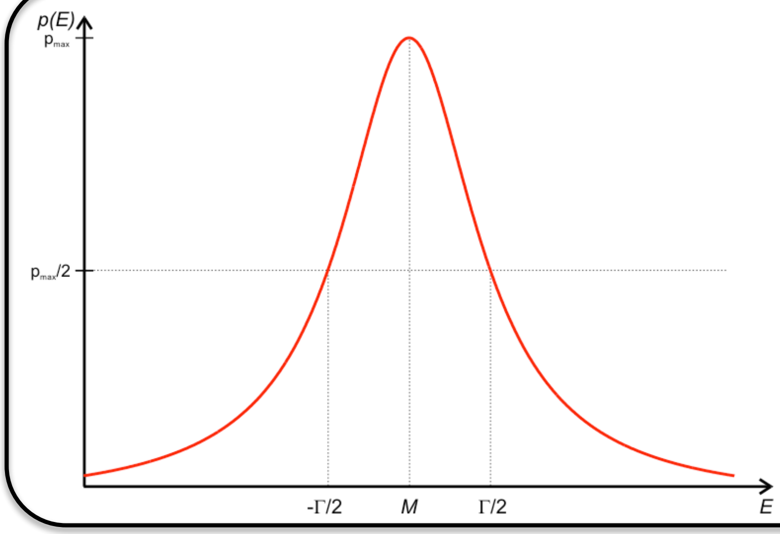

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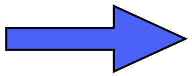

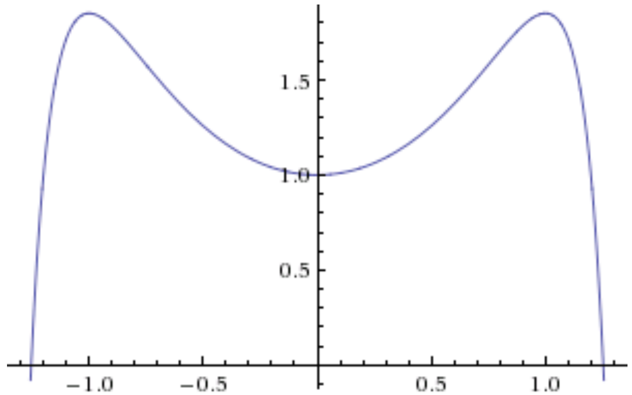

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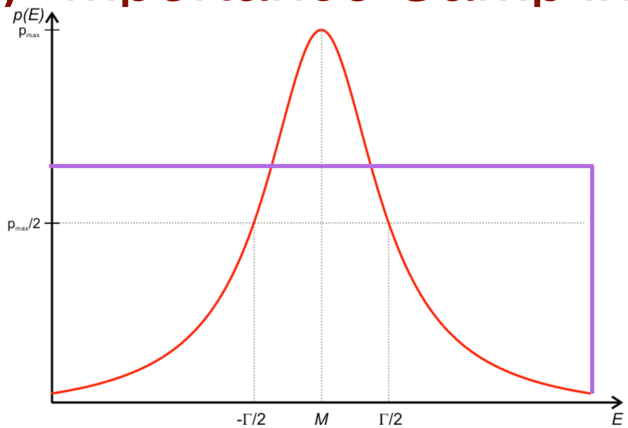

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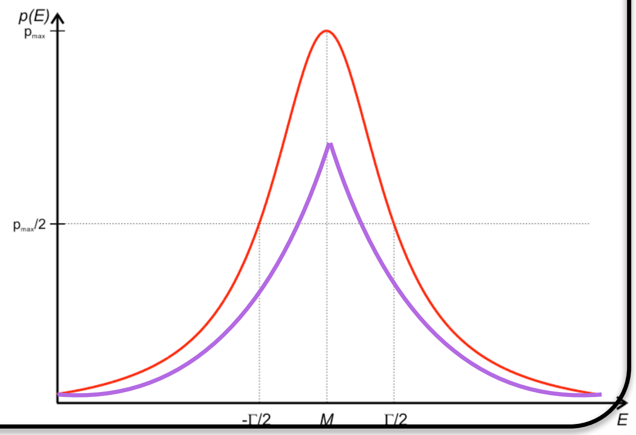

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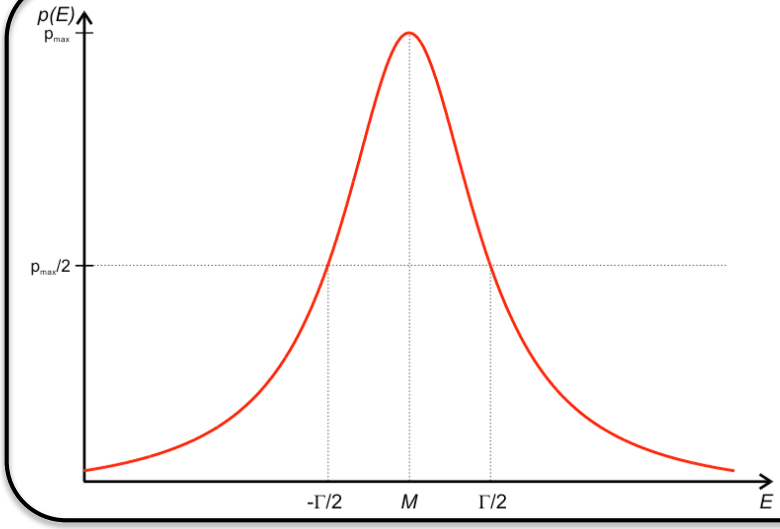

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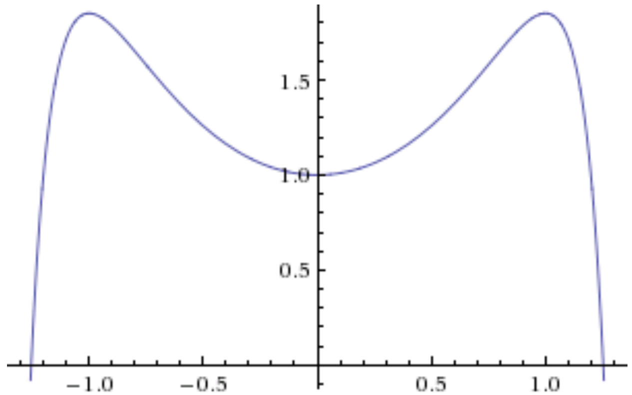

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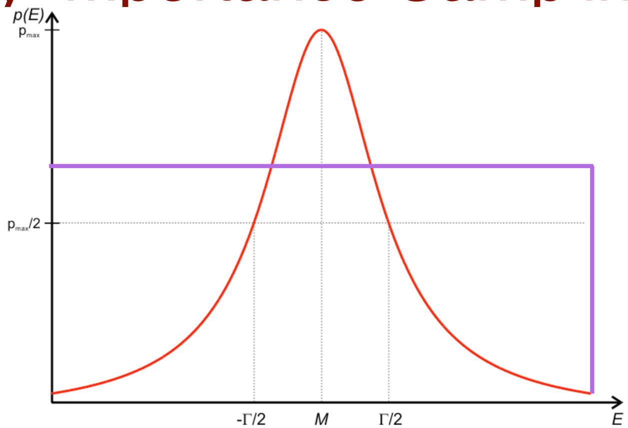

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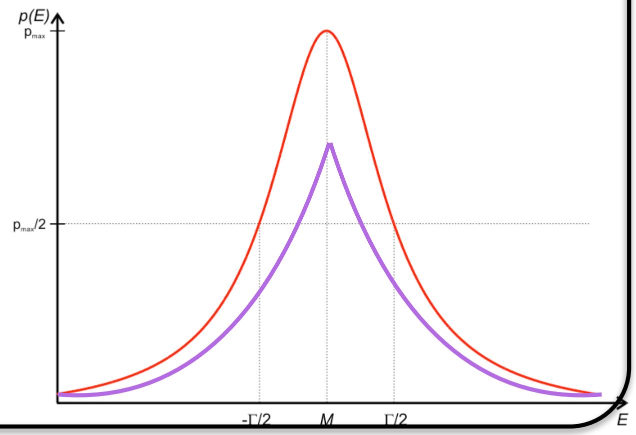
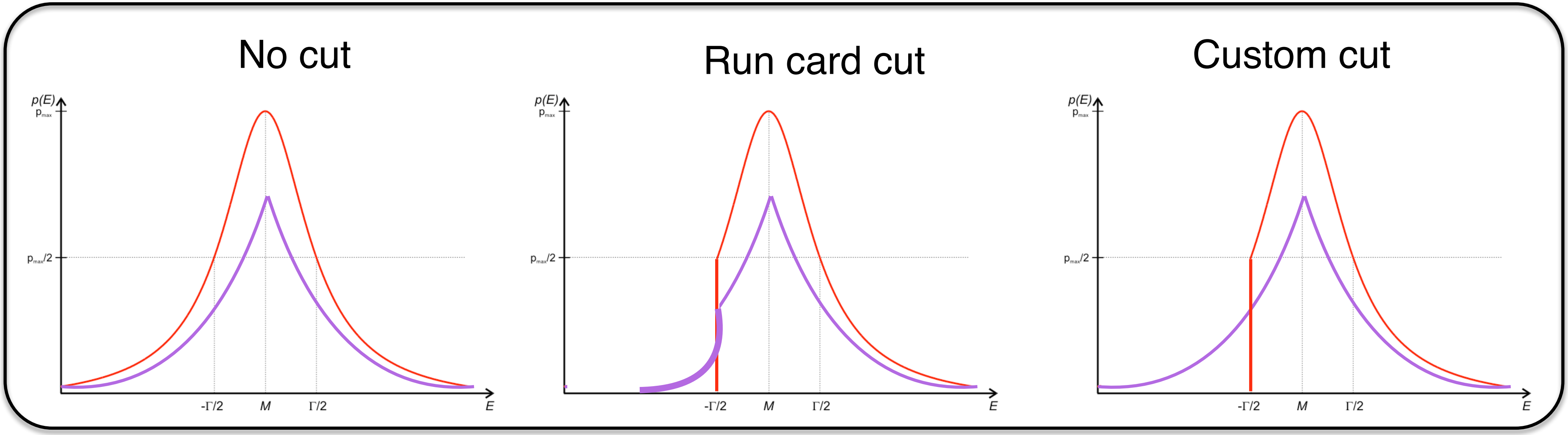


The change of variable ensure that the evaluation of the function is done where the function is the largest!

Cut Impact

- Events are generated according to our best knowledge of the function
  - ➡Basic cut include in this “best knowledge”
  - ➡Custom cut are ignored



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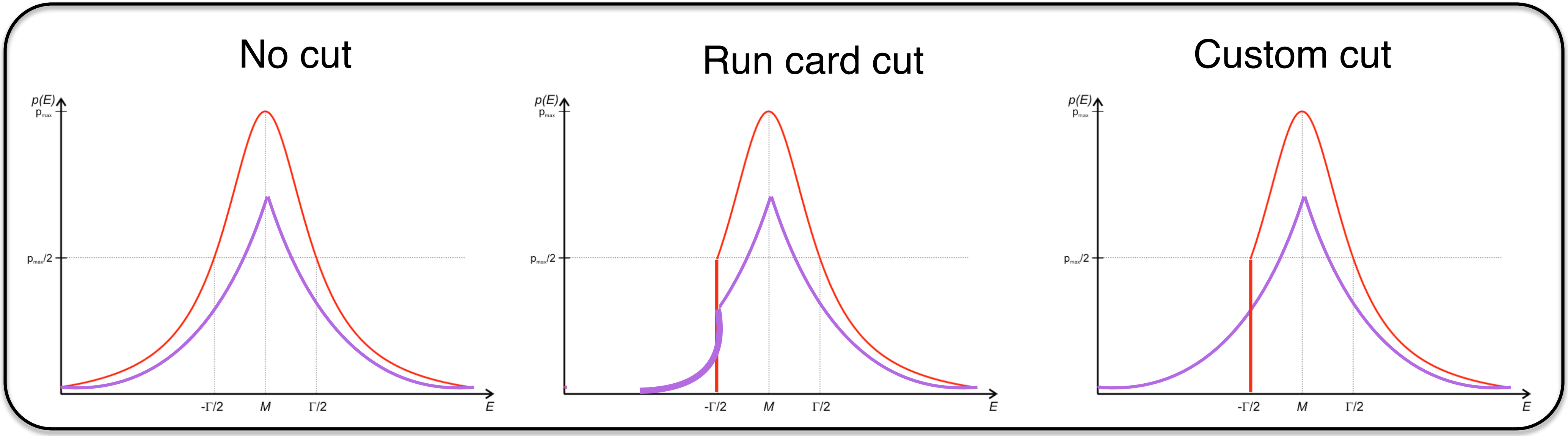

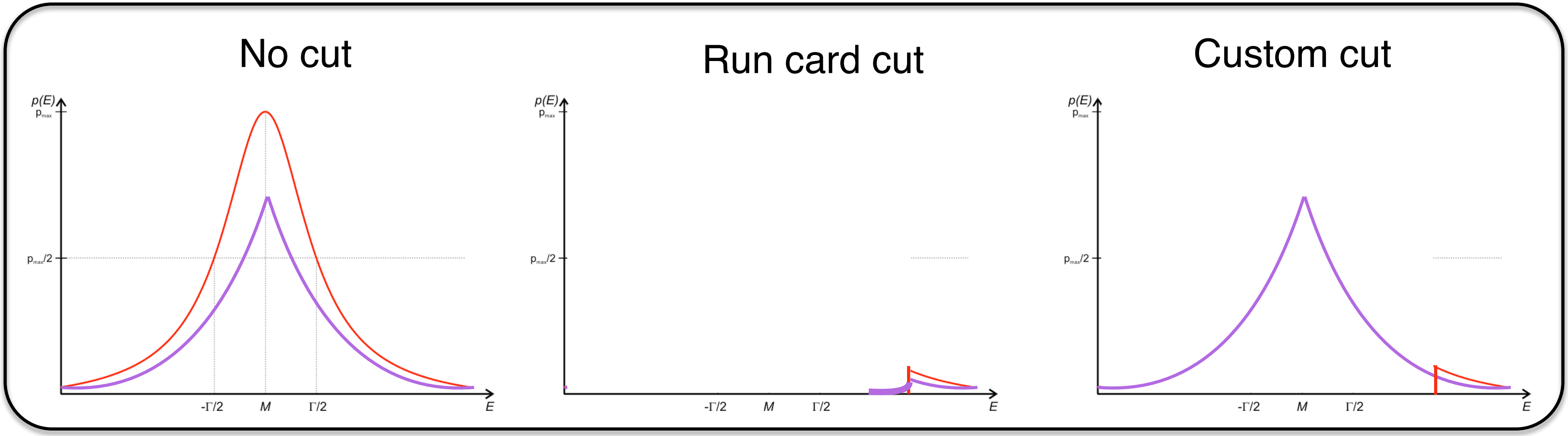

Might miss the contribution and think it is just zero.

Importance Sampling

Key Point

- Generate the random point in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is flatter in this new variable.
- Needs to know an approximate function.

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

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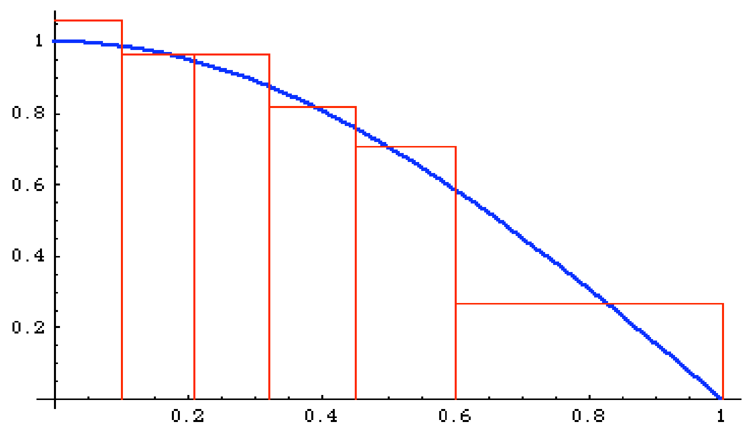

Algorithm

1. Creates bin such that each of them have the same contribution.

   ➡Many bins where the function is large

2. Use the approximate for the importance sampling method.

VEGAS

More than one Dimension

- VEGAS works only with 1(few) dimension
  - ➡memory problem

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- Use projection on the axis

$$\vec{p}(x) = p(x) \cdot p(y) \cdot p(z) \ldots$$

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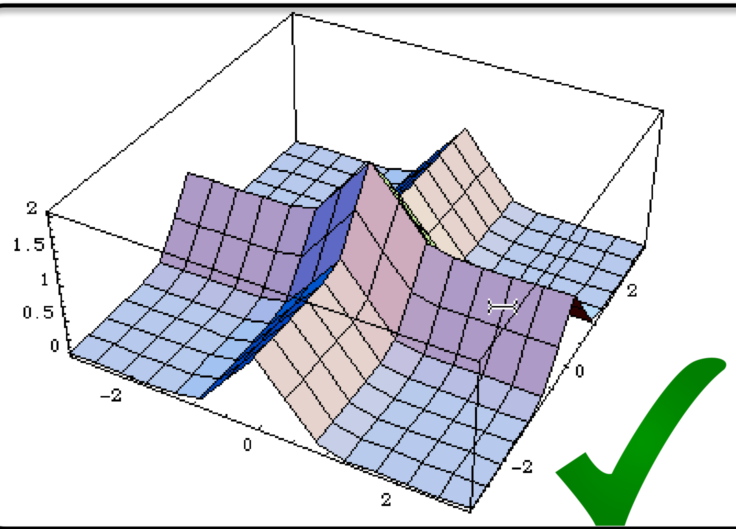

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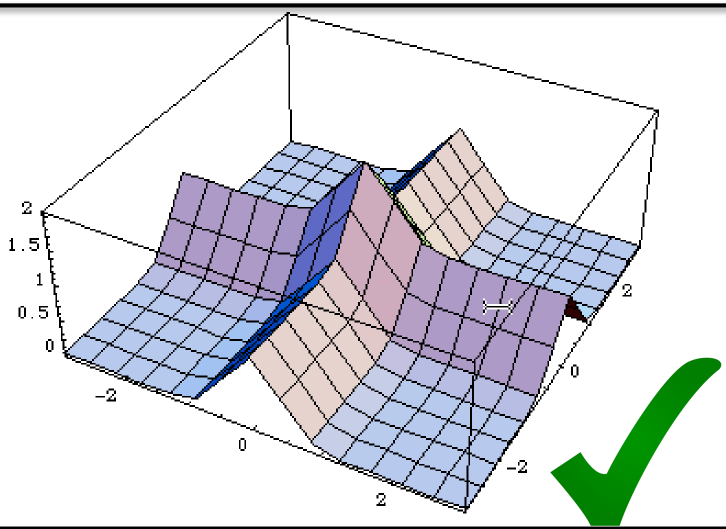
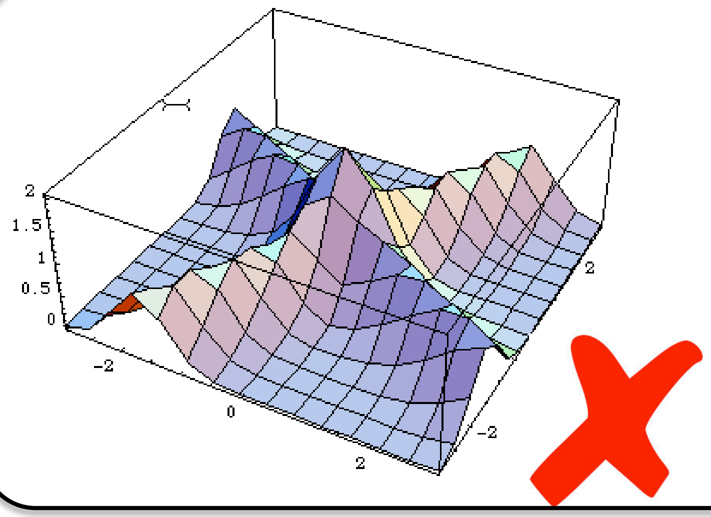


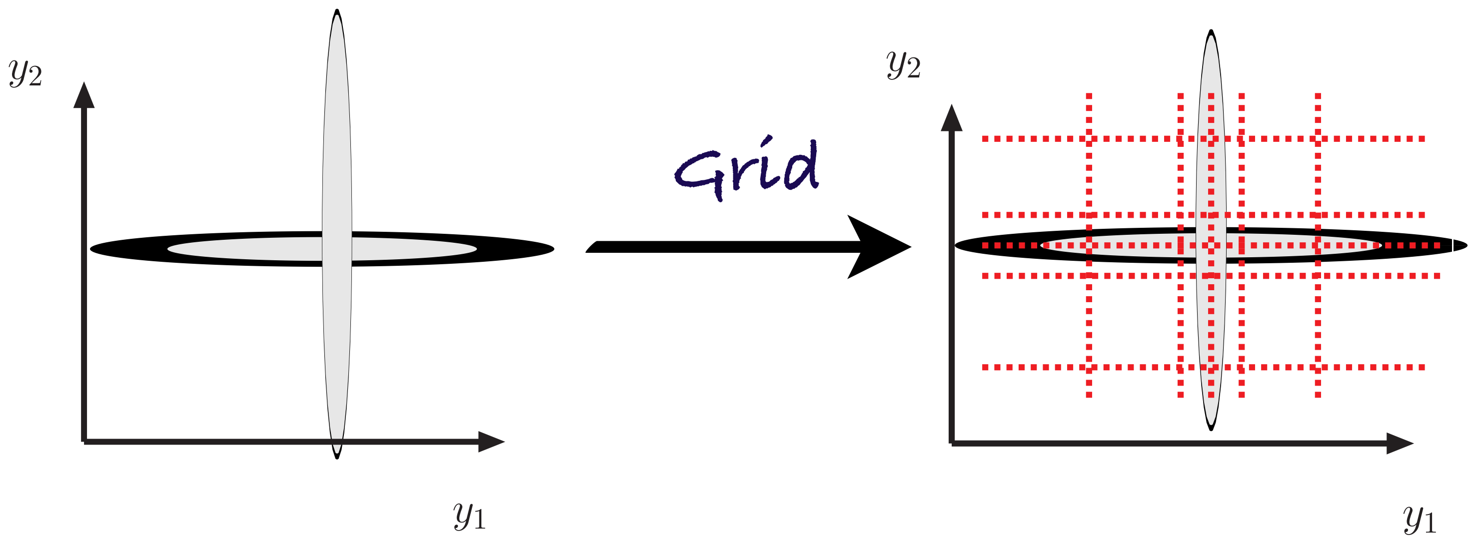


- We need to ensure the factorization !

➡Additional change of variable

Monte-Carlo Integration

- The choice of the parameterisation has a strong impact on the efficiency



- ☐ The adaptive Monte-Carlo Technique picks point in interesting areas

→ The technique is efficient

Monte-Carlo Integration

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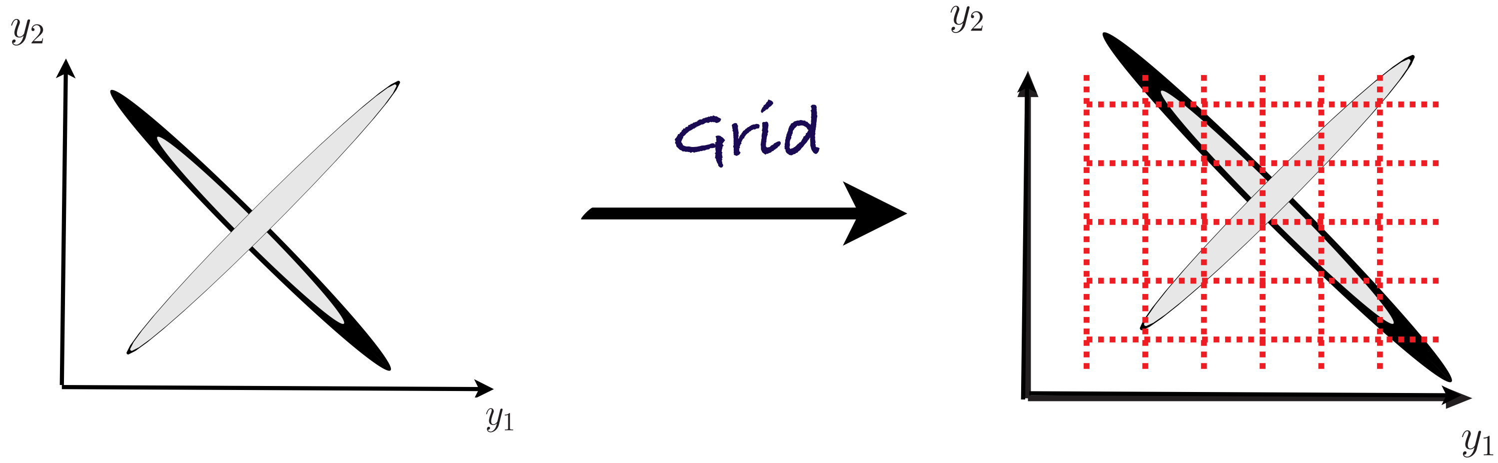

- ☐ The adaptive Monte-Carlo Techniques picks points everywhere
  → The integral converges slowly

Monte-Carlo Integration

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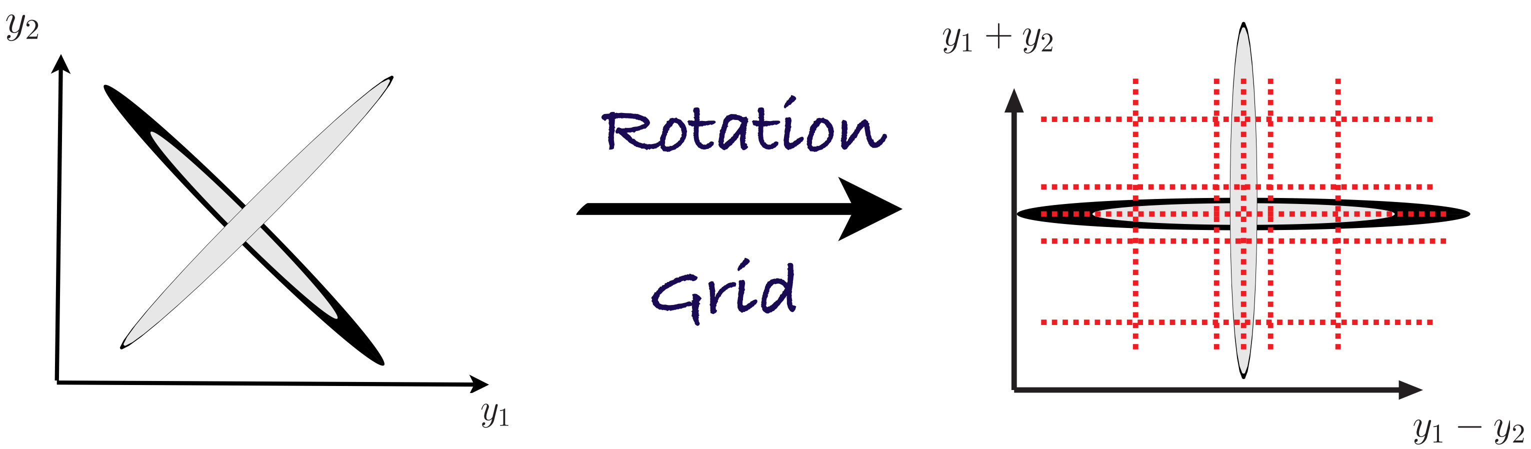

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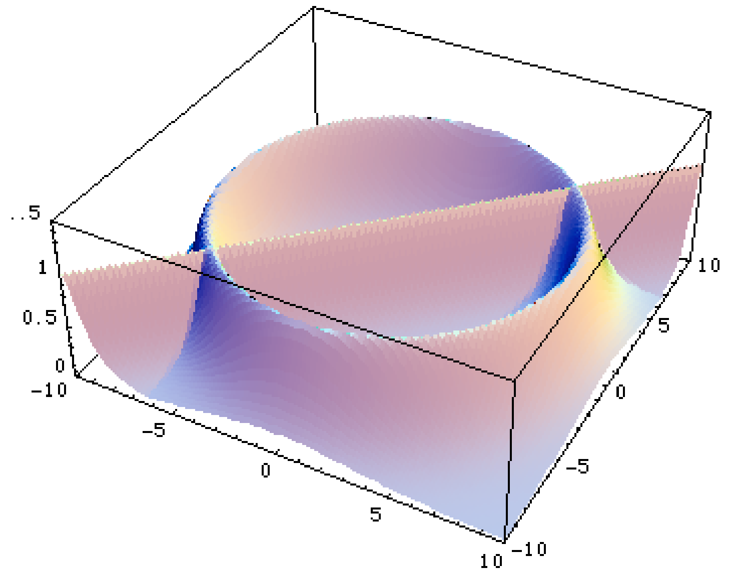
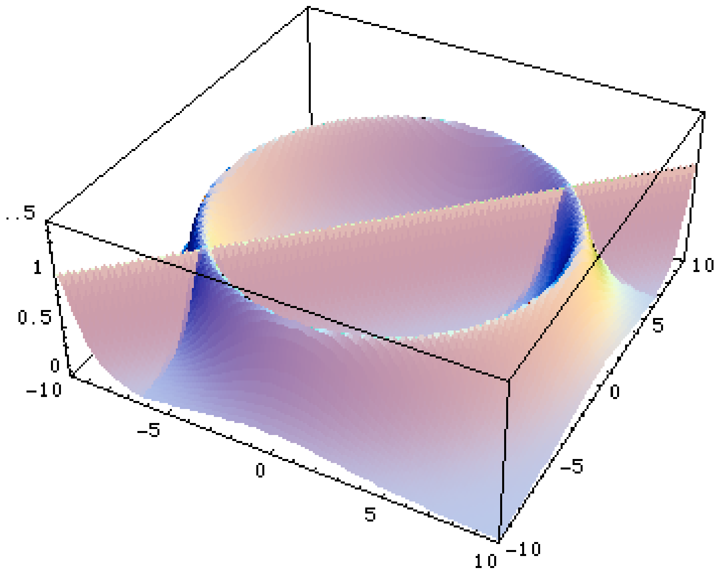
Multi-channel



What do we do if there is no transformation that aligns all integrand peaks to the chosen axes?
Vegas is bound to fail!

Multi-channel



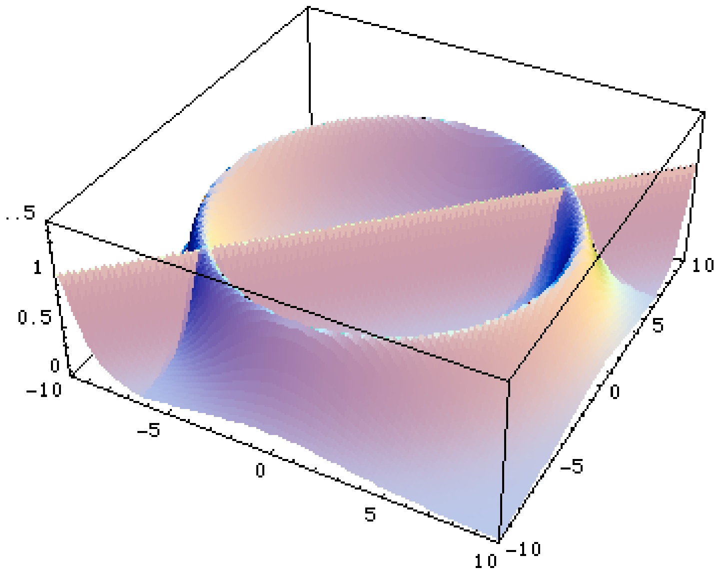
What do we do if there is no transformation that aligns all integrand peaks to the chosen axes?
Vegas is bound to fail!

Solution: use different transformations = channels

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

with each $p_i(x)$ taking care of one "peak" at the time

Multi-channel



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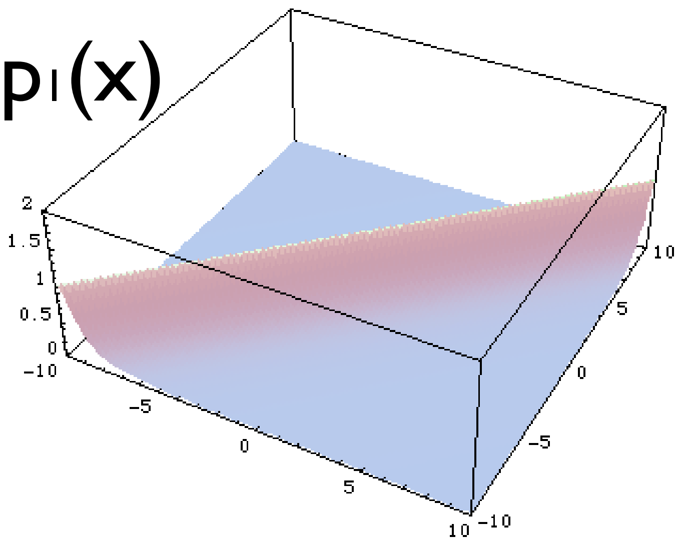
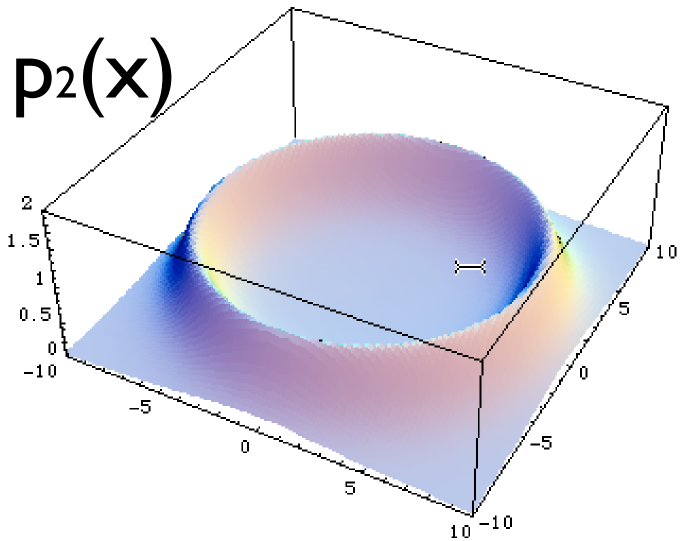
$p_1(x)$



$p_2(x)$



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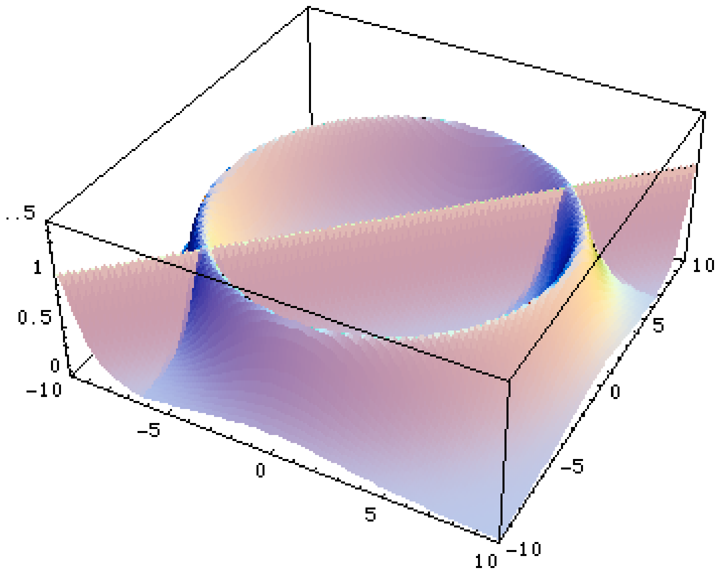


$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x)$$

with

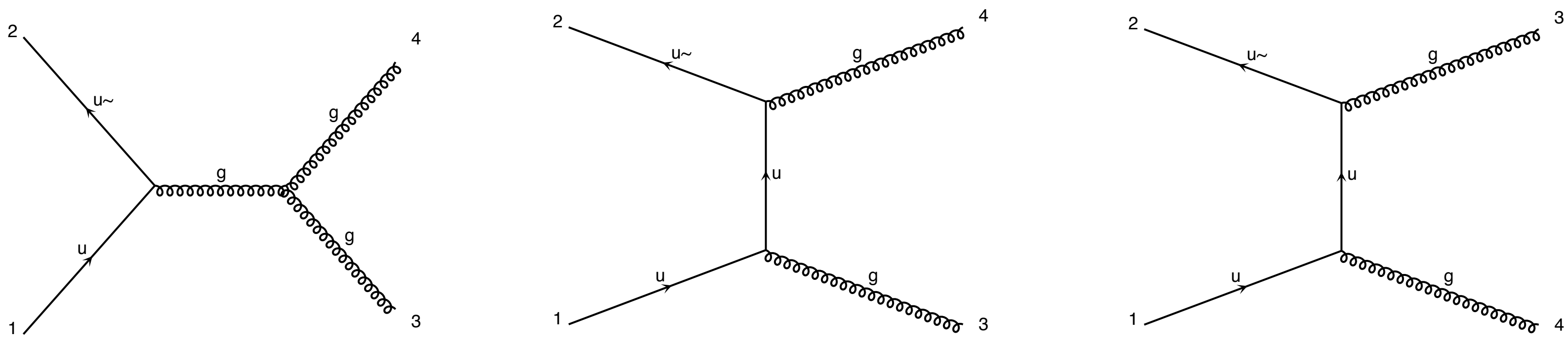
$$\sum_{i=1}^{n} \alpha_i = 1$$

Then,

$$I = \int f(x)dx = \sum_{i=1}^{n} \alpha_i \int \frac{f(x)}{p(x)} p_i(x) dx$$

$$\approx 1$$

Example: QCD 2 → 2



$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2} \qquad \propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2} \qquad \propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

Multi-channel

Consider the integration of an amplitude |M|^2 at tree level which many contributing diagrams. We would like to have a basis of functions,

$$f = \sum_{i=1}^{n} f_i \quad \text{with} \quad f_i \geq 0\,, \quad \forall \;\; i\,,$$

such that:

1. we know how to integrate each one of them,
2. they describe all possible peaks,

giving us the combined integral

$$I = \int d\vec{\Phi} f(\vec{\Phi}) = \sum_{i=1}^{n} \int d\vec{\Phi}\, g_i(\vec{\Phi})\, \frac{f_i(\vec{\Phi})}{g_i(\vec{\Phi})} = \sum_{i=1}^{n} I_i\,,$$

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Does such a basis exist?

Multi-channel based on single diagrams*

*Method used in MadGraph

Trick in MadEvent: Split the complexity

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

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≈ 1

Multi-channel based on single diagrams*

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Trick in MadEvent: Split the complexity

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

$$\approx 1$$

Key Idea

- Any single diagram is "easy" to integrate (pole structures/ suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

Multi-channel based on single diagrams*

*Method used in MadGraph

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≈ 1

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N Integral

- Errors add in quadrature so no extra cost
- "Weight" functions already calculated during $|M|^2$ calculation
- Parallel in nature

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P1_qq_wpwm

s= 725.73 ± 2.07 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G2.2	377.6	1.67	142.285	7941.0	21
G3	239	1.16	220.04	10856.0	45.5
G1	109.1	0.378	70.88	3793.0	34.8

P1_gg_wpwm

s= 20.714 ± 0.332 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.2	20.71	0.332	7.01	373.0	18

term of the above sum.

each term might not be gauge invariant

To Remember

- Phase-Space integration are difficult
- We need to know the function
  - ➡ Be careful with cuts
- MadGraph split the integral in different contribution linked to the Feynman Diagram
  - ➡Those are not the contribution of a given diagram

To Remember

- Phase-Space integration are difficult
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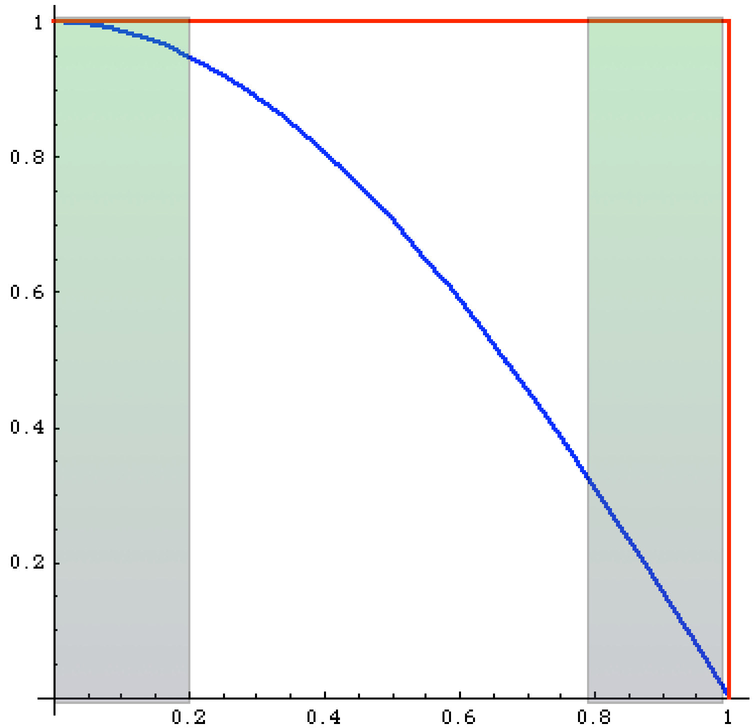
Event generation

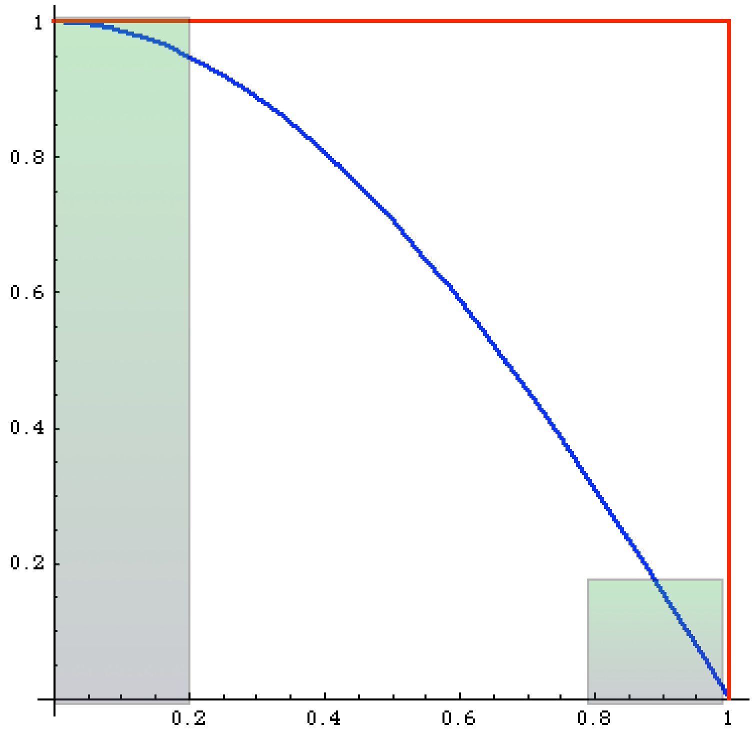


What's the difference between weighted and unweighted?

Weighted:

Same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



What's the difference between weighted and unweighted?

Unweighted:

events is proportional to the probability of areas of phase space:
events have all the same weight (”unweighted”)

Events distributed as in nature

Event generation

$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

Event generation

$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i) = \frac{1}{N}\sum_{i=1}^{N} \frac{f(x_i)}{max(f)} max(f)$$

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Number between 0 and 1 (assuming positive function)
-> re-interpret as the probability to keep the events

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For each events throw the dice and see if we keep or reject the events

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$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i) = \frac{1}{N}\sum_{i=1}^{N} P(x_i) max(f) \simeq \frac{max(f)}{N}\sum_{i=1}^{n} 1$$

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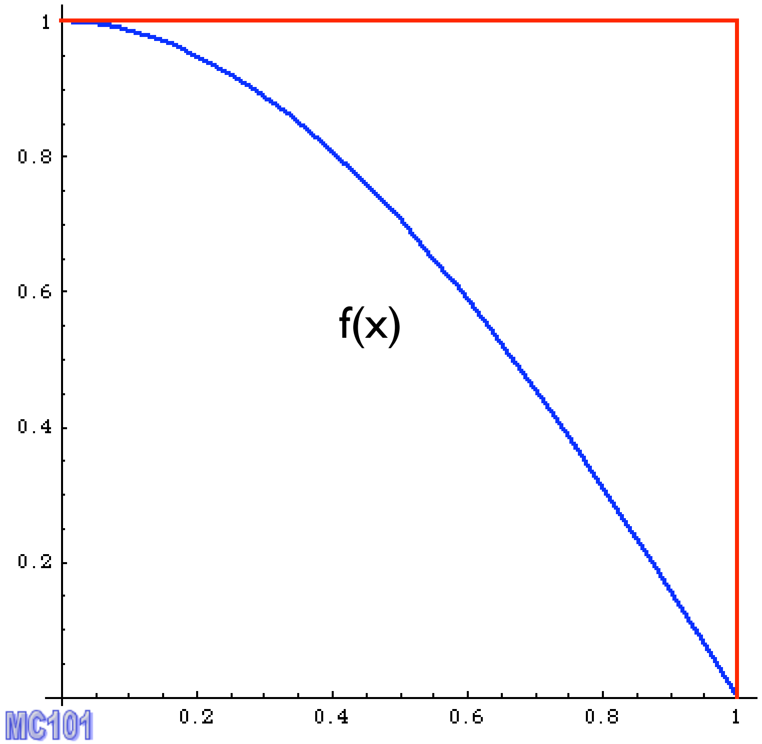

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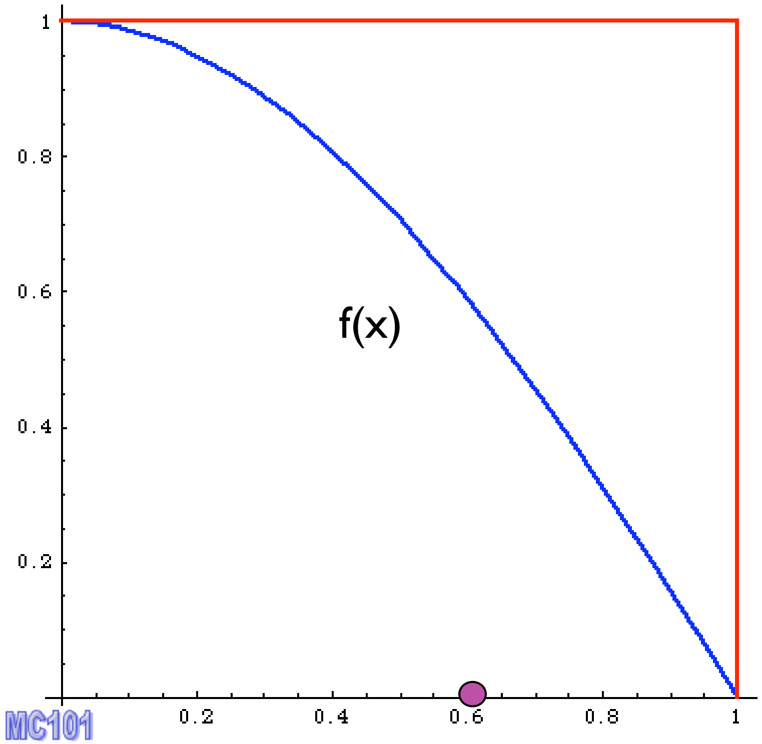

1. pick x_i

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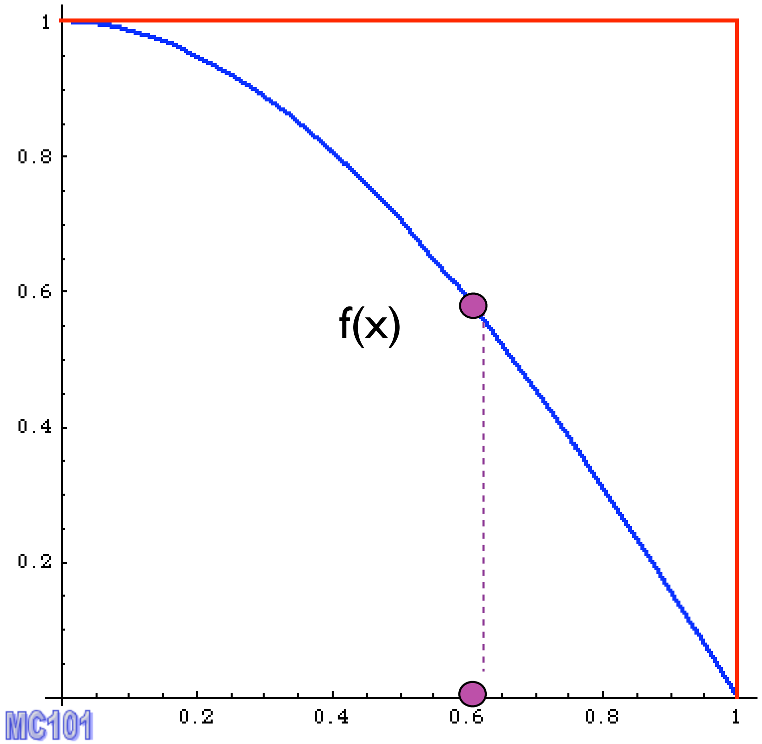

1. pick x_i
2. calculate $f(x_i)$

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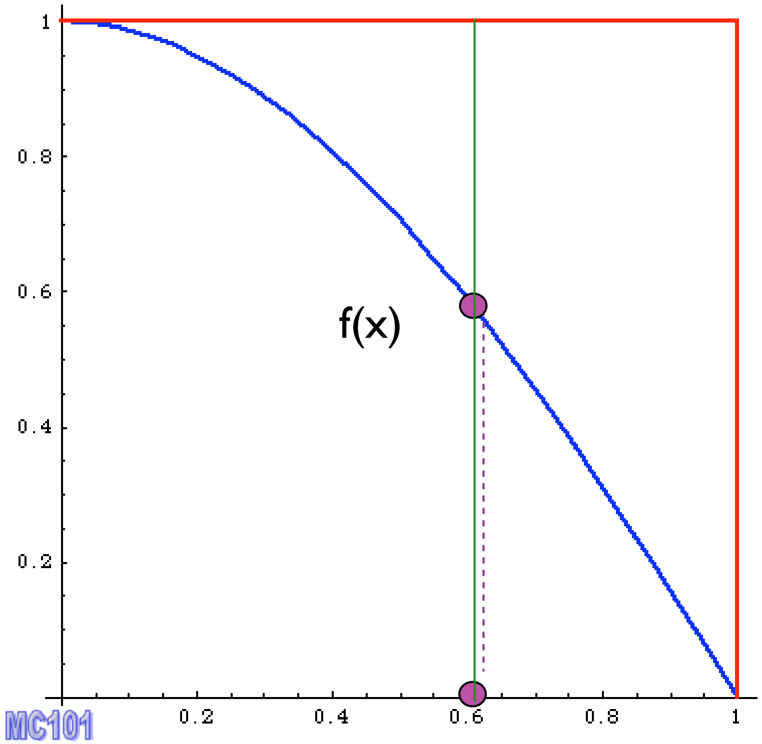

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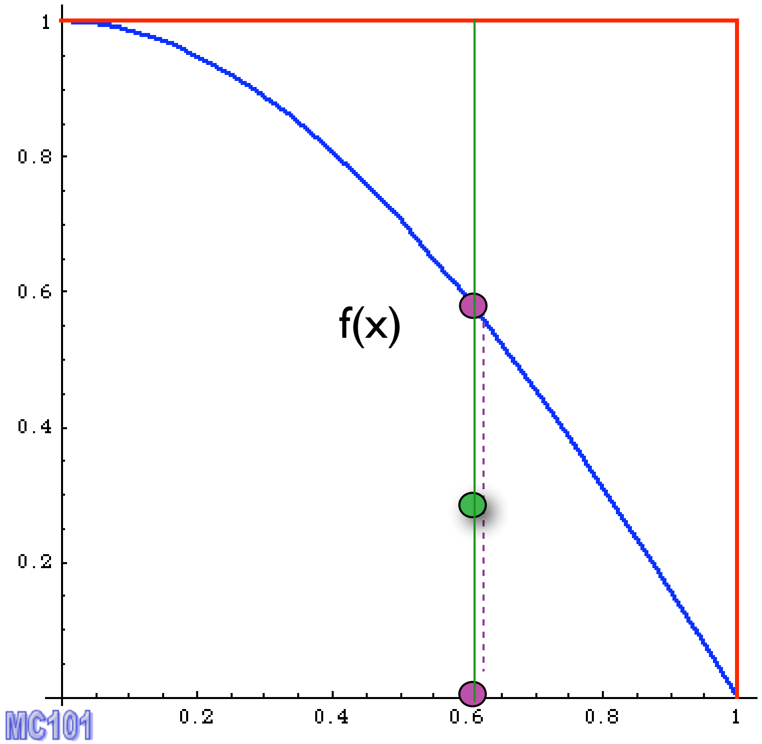

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   if $y < f(x_i)$ accept event,

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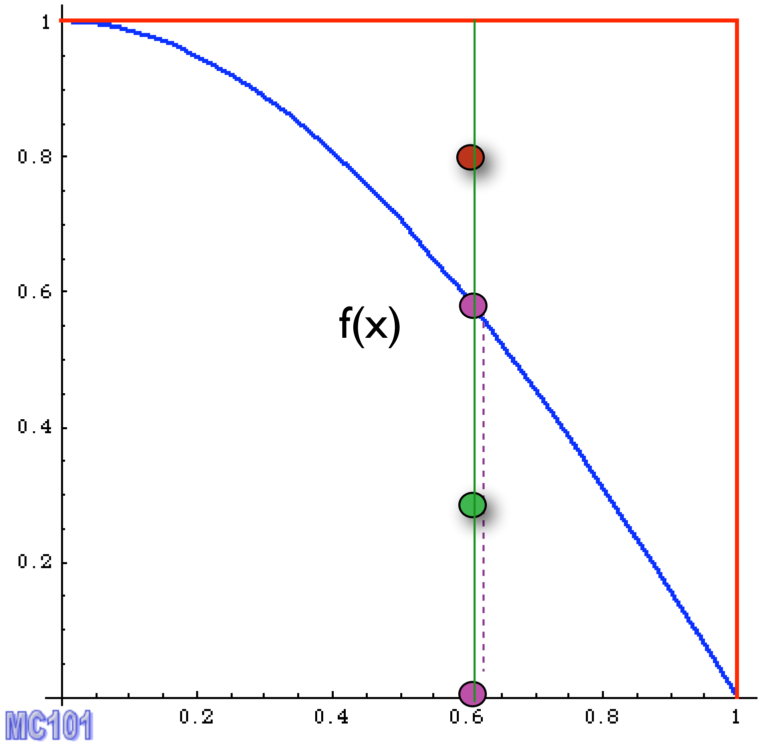

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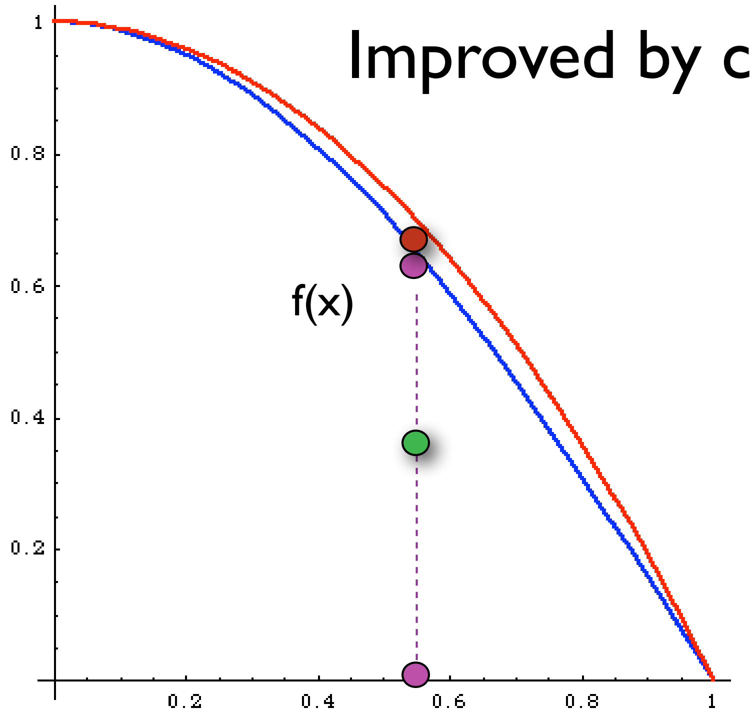
Event generation



Improved by combining with importance sampling:

1. pick x distributed as p(x)
2. calculate f(x) and p(x)
3. pick 0<y<1
4. Compare:
   if f(x)>y p(x) accept event,

else reject it.

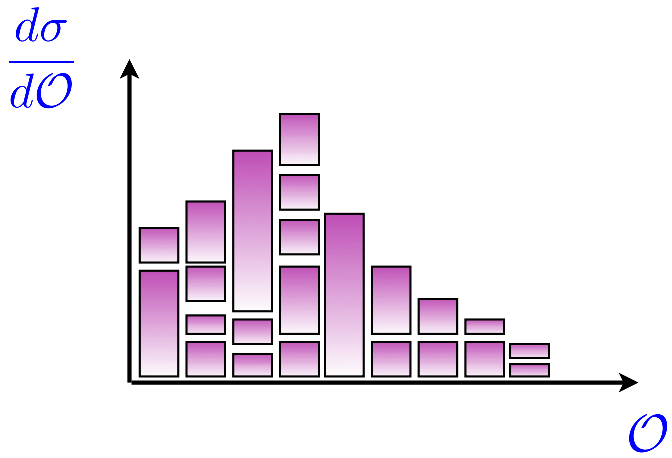
much better efficiency!!!

Event generation

MC integrator

Event generation

MC integrator



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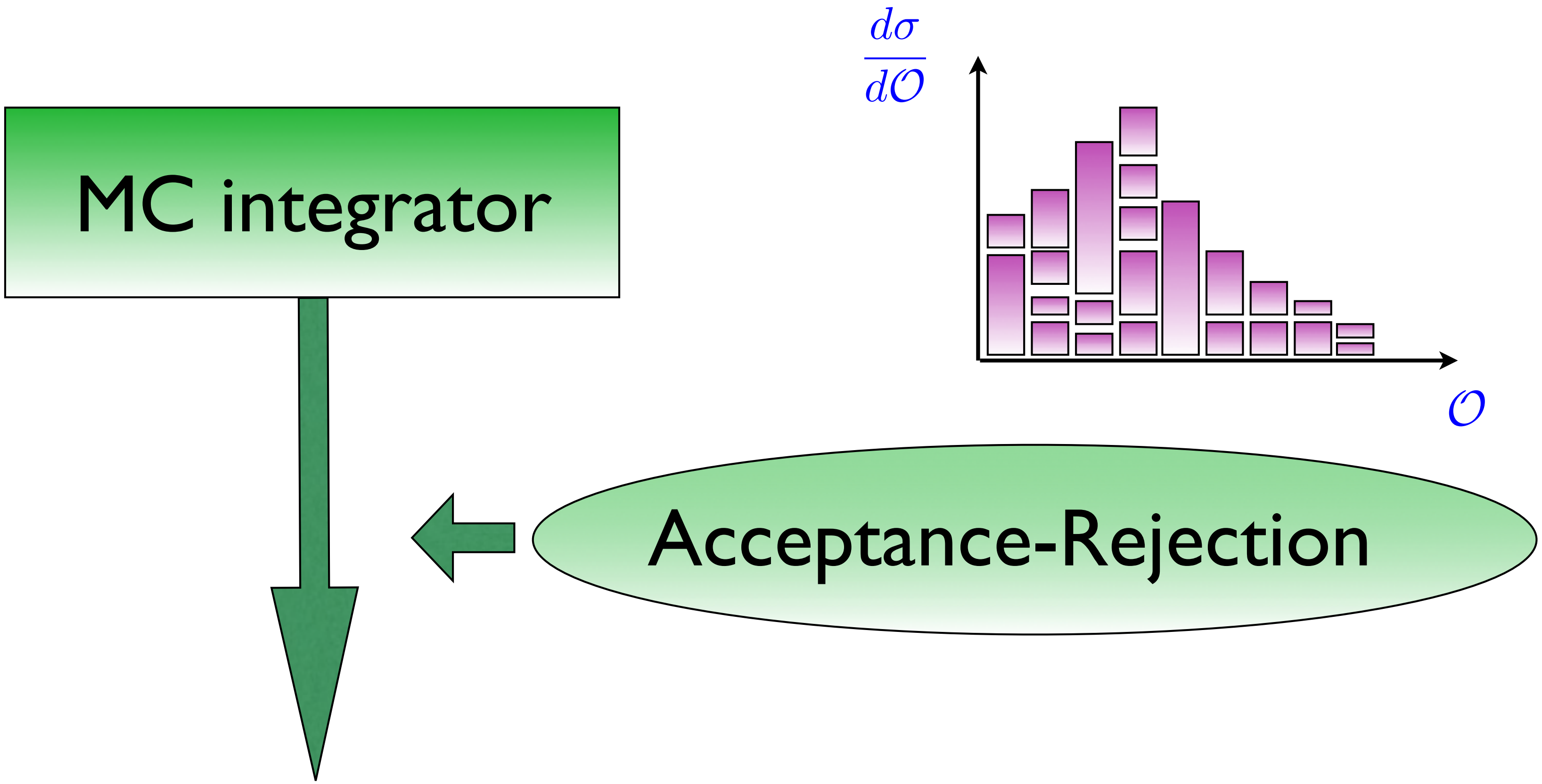

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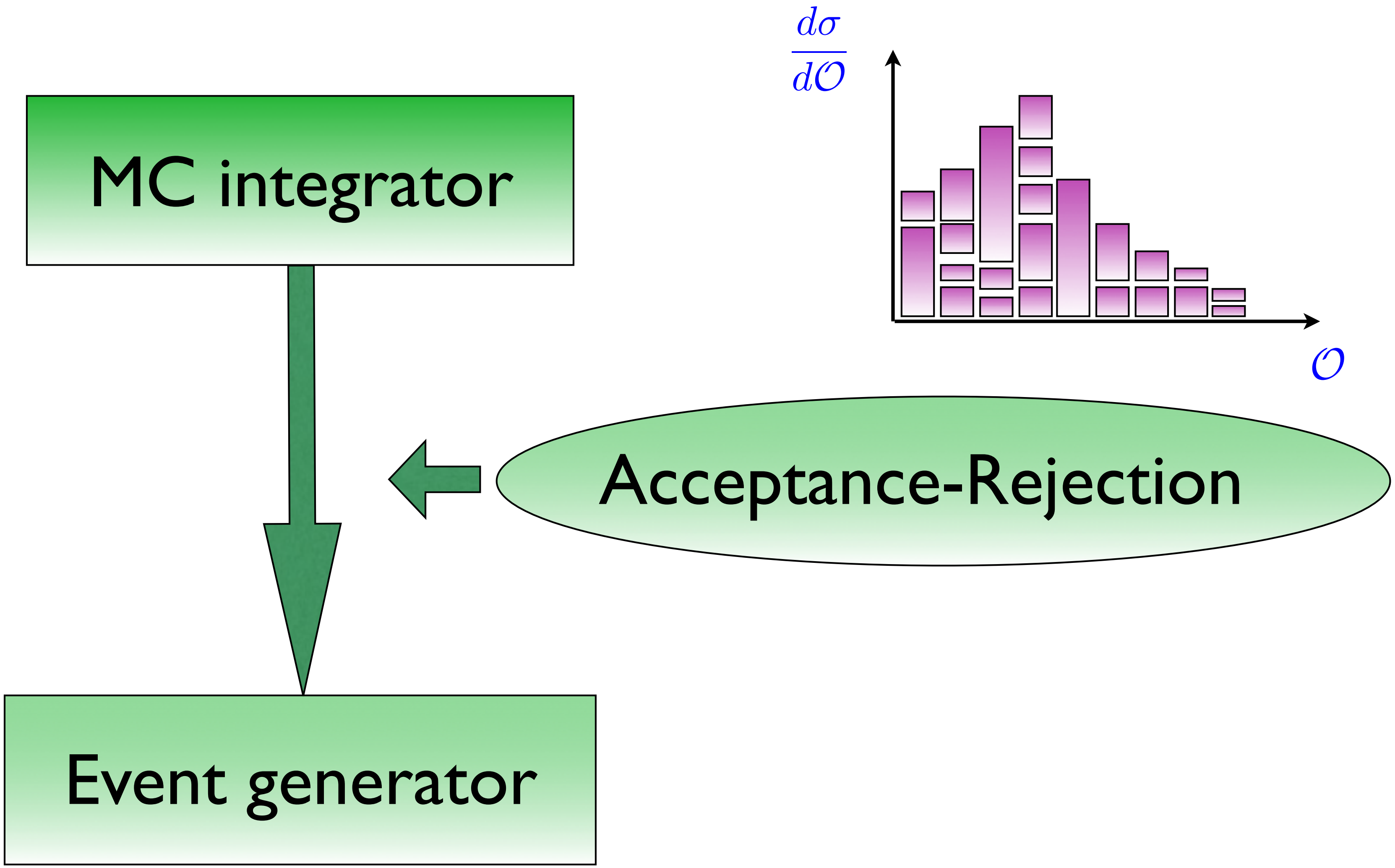

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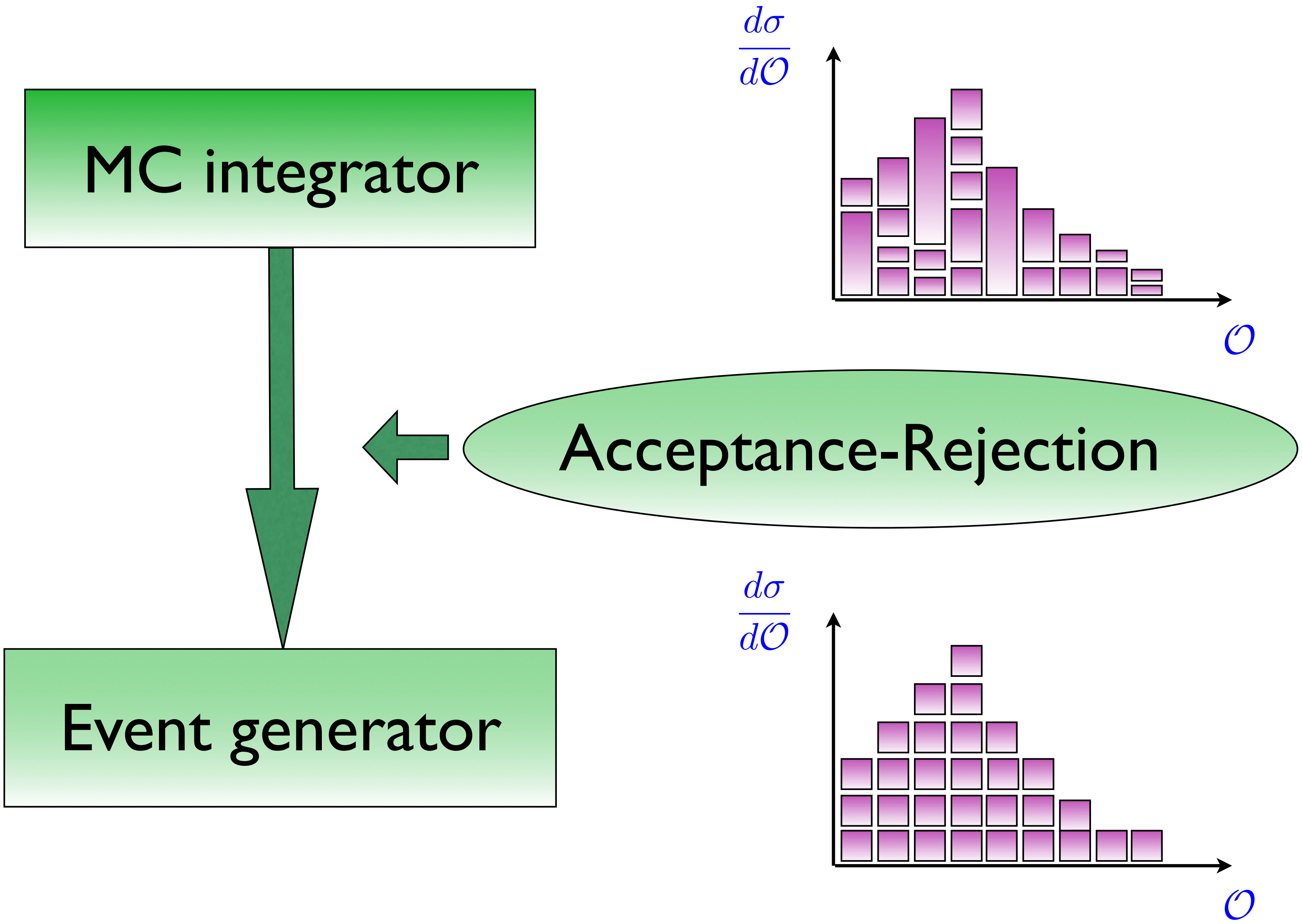

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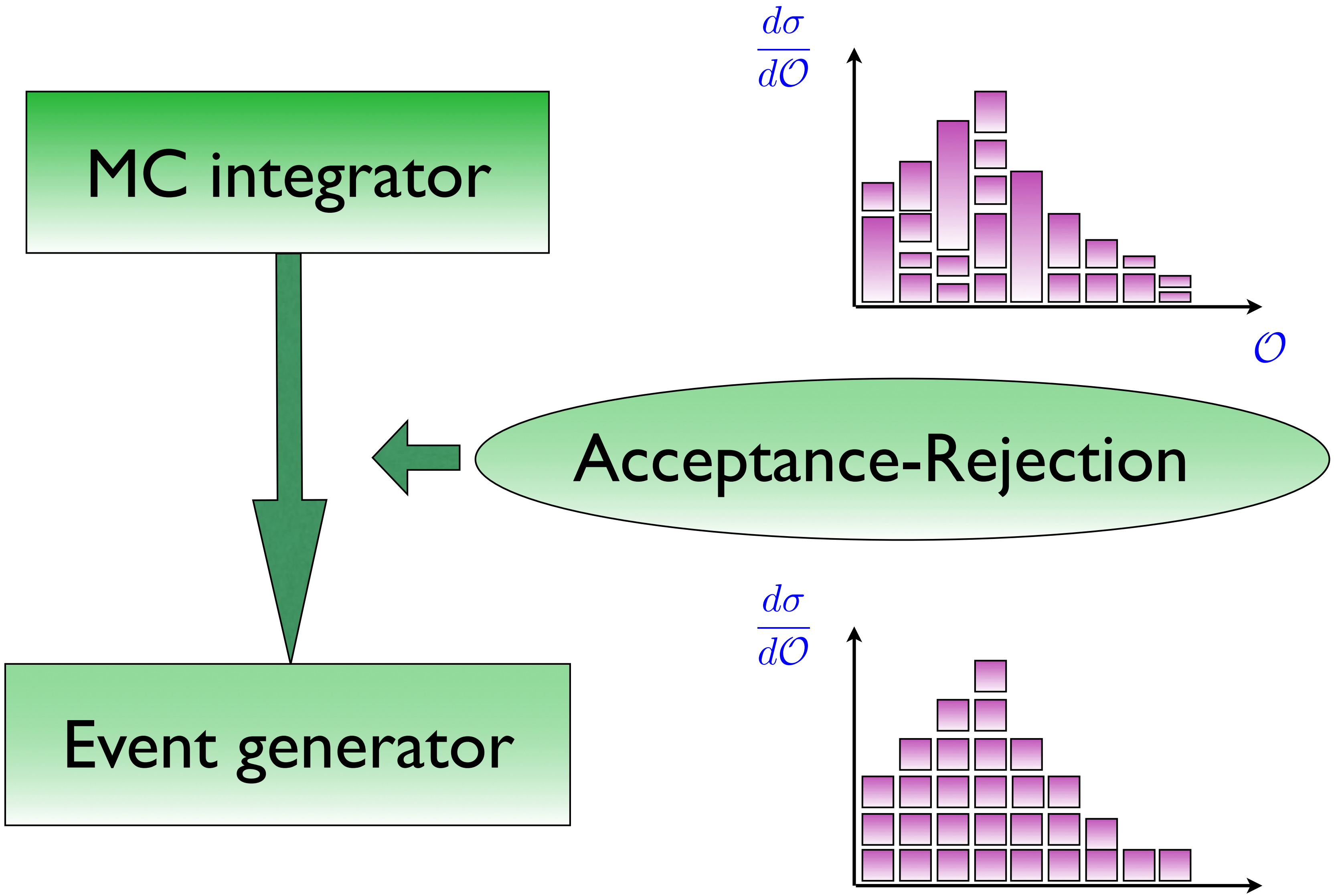

☞ **This is possible only if f(x)<∞ AND has definite sign!** $\mathcal{O}$

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension)
- Need to know the function
    - Impact on cut

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension)
- Need to know the function
  - Impact on cut

Good Point

- Complex area of Integration
- Easy error estimate
- quick estimation of the integral
- Possibility to have unweighted events

What have we learned!

$$\sum_{a,b}\int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat\sigma_{ab\to X}(\hat s,\mu_F,\mu_R)$$

Phase-space integral | Parton density functions | Parton-level cross section

- The Importance of PDF
  - ➡Defines the physics
- Evaluation of Matrix Element
  - ➡ Numerical method faster than analytical formula
  - ➡ cross-section prediction needs NLO
- Phase Space Integration
  - ➡ Need to know in advance what we integrate. Be careful with strong cuts!