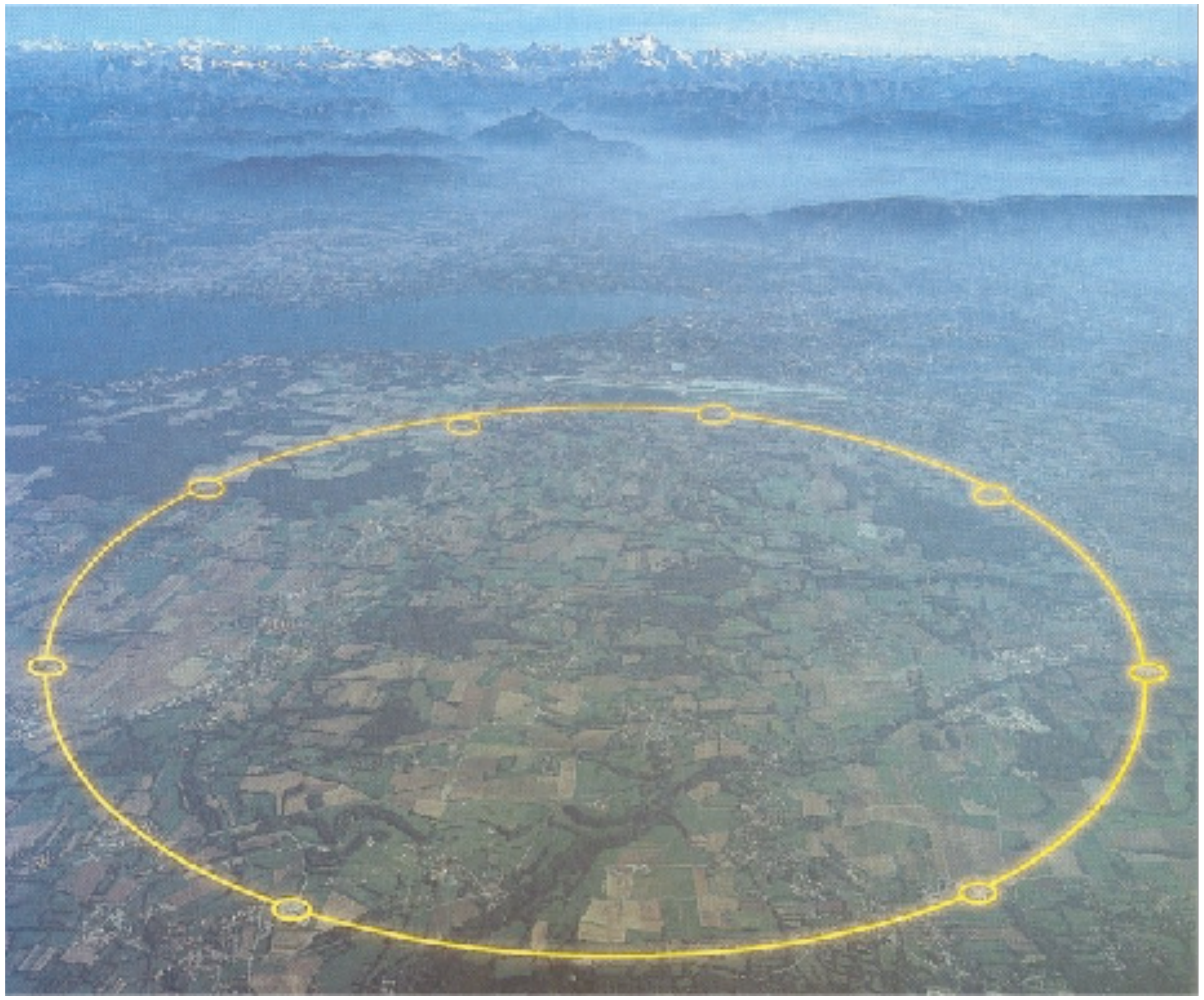
Monte-Carlo Generation

Olivier Mattelaer
IPPP/Durham

Introduction

Topic

- Collider Physics



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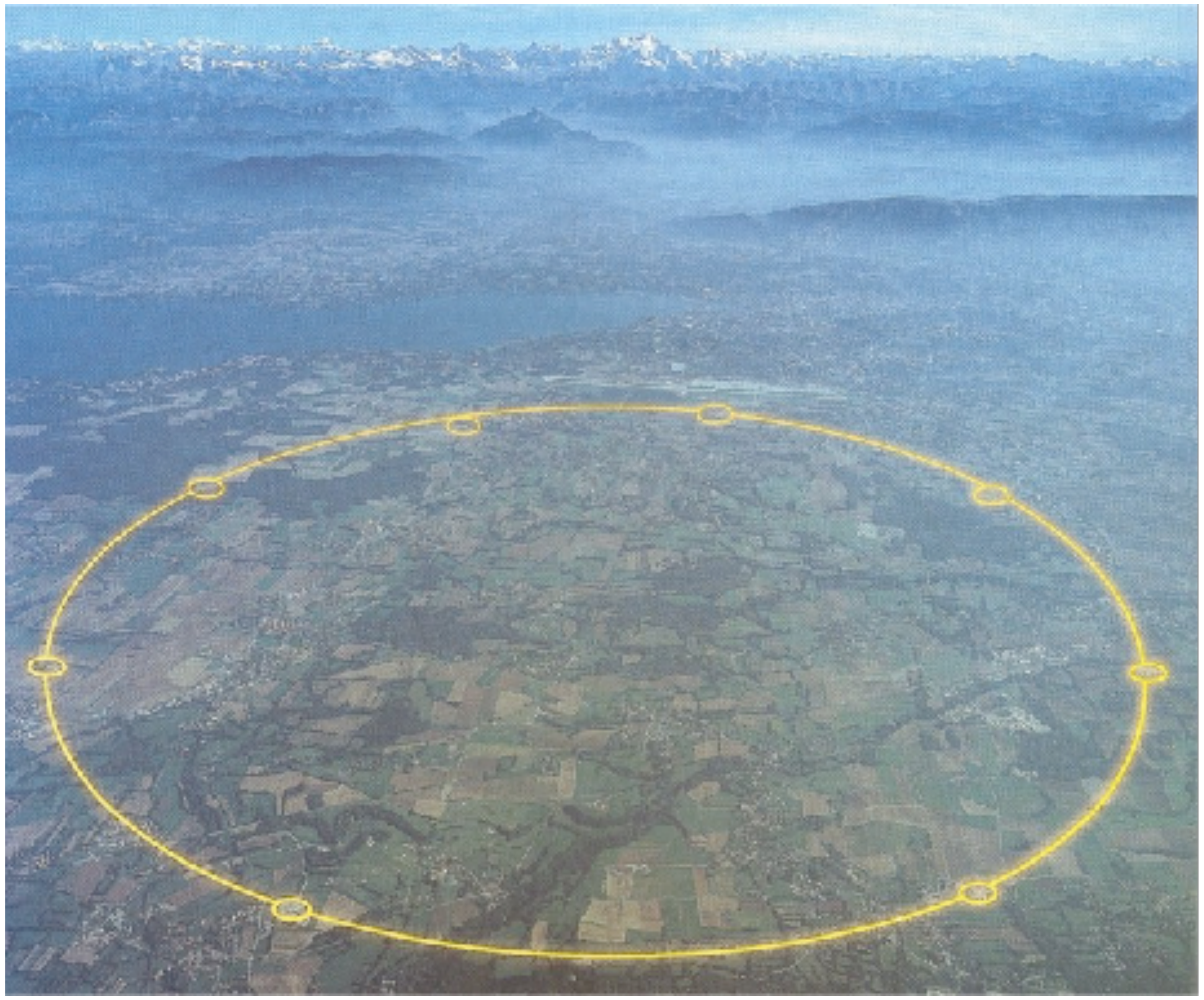
  - accelerating particle -> High Energy collision



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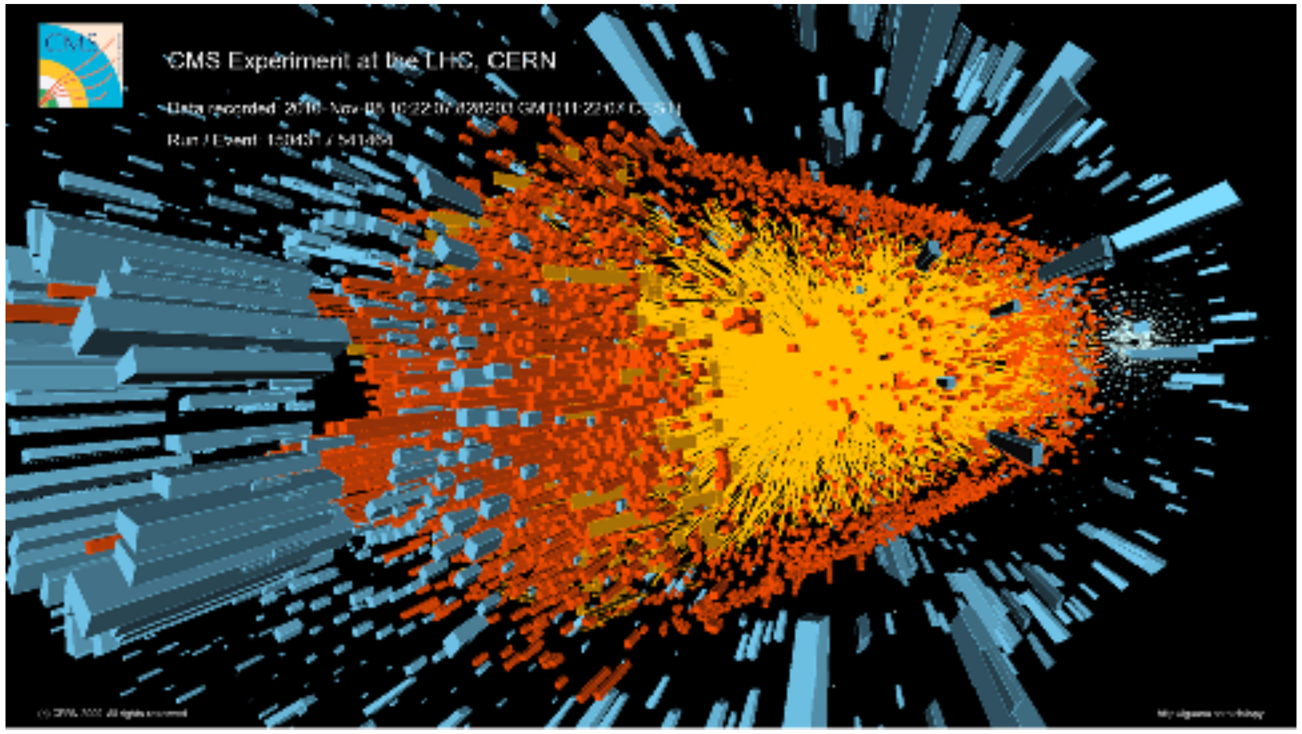
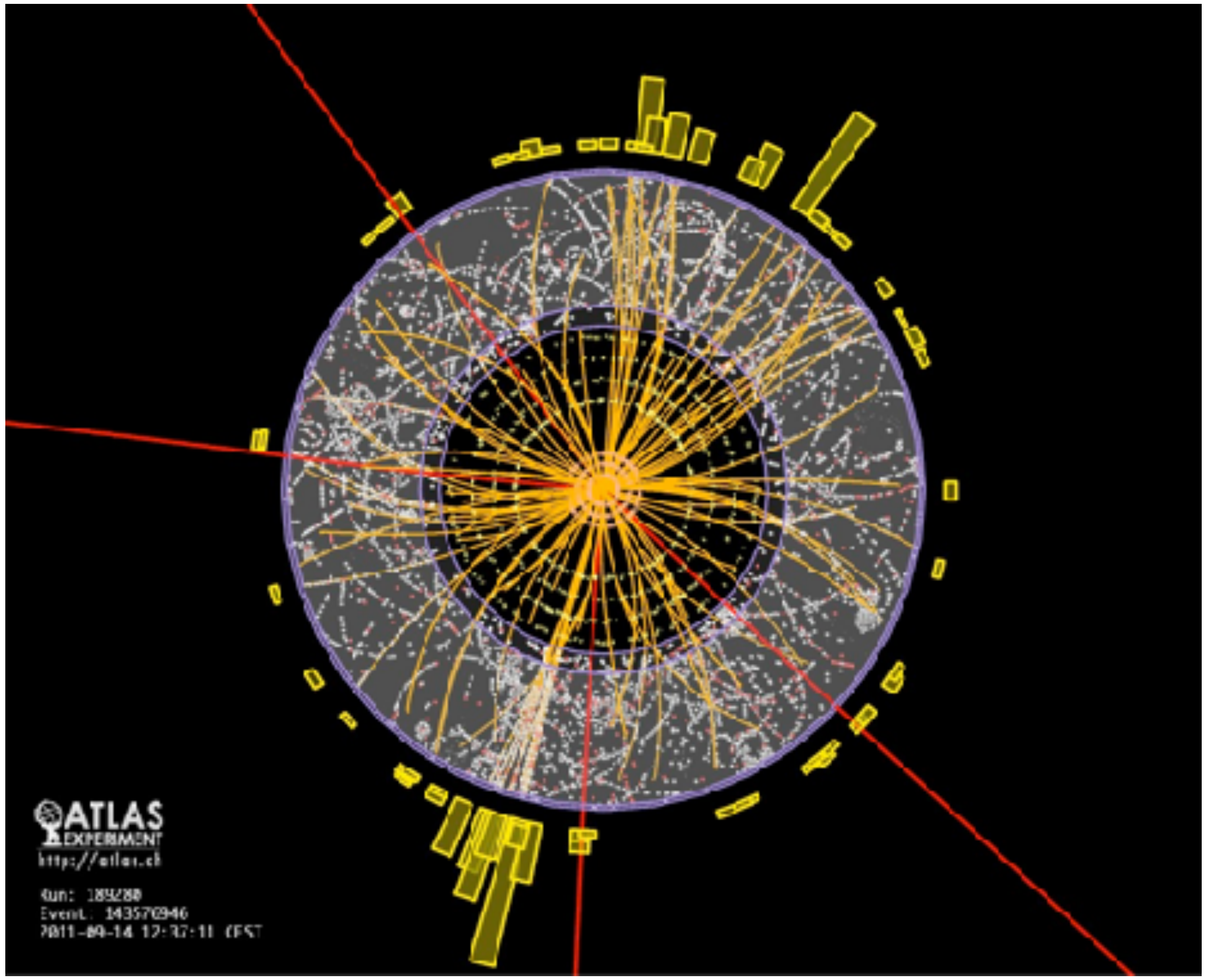
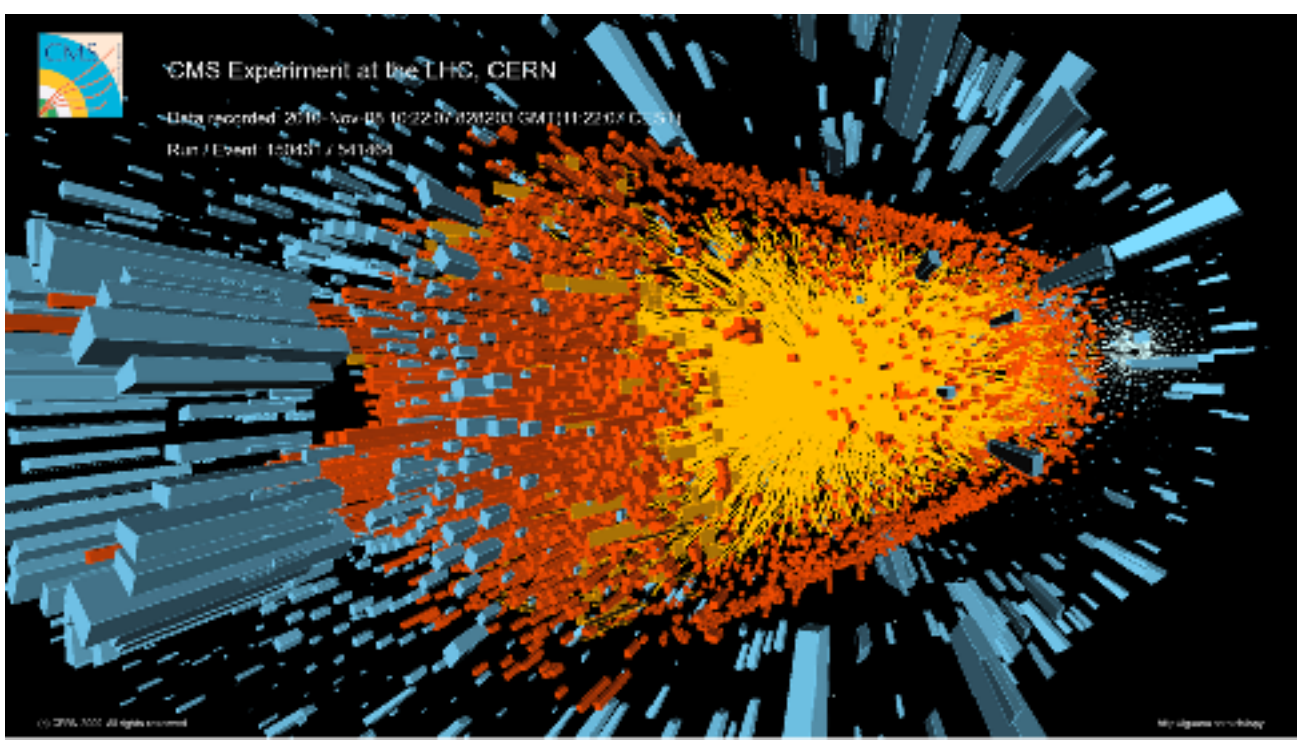


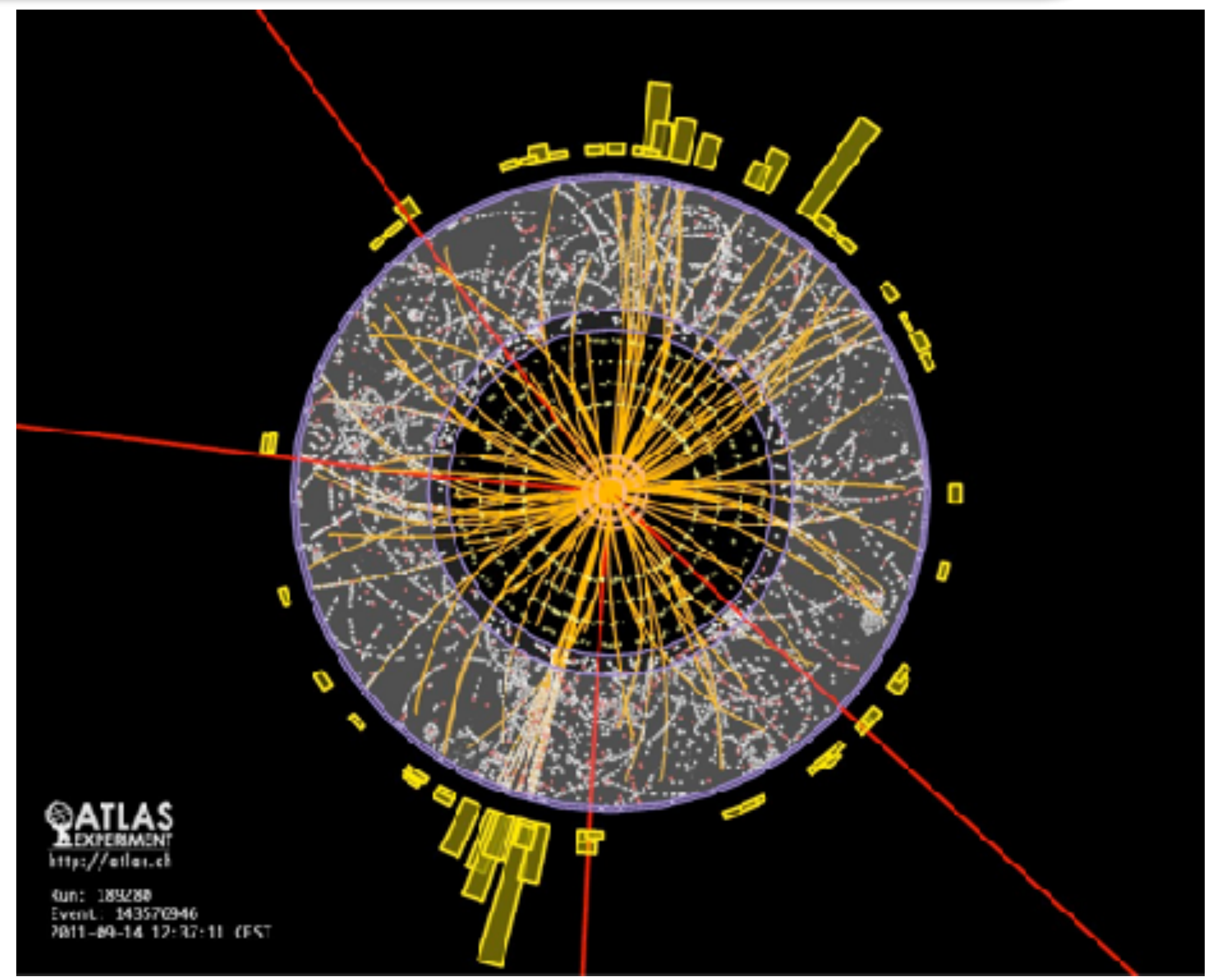


Introduction

Topic

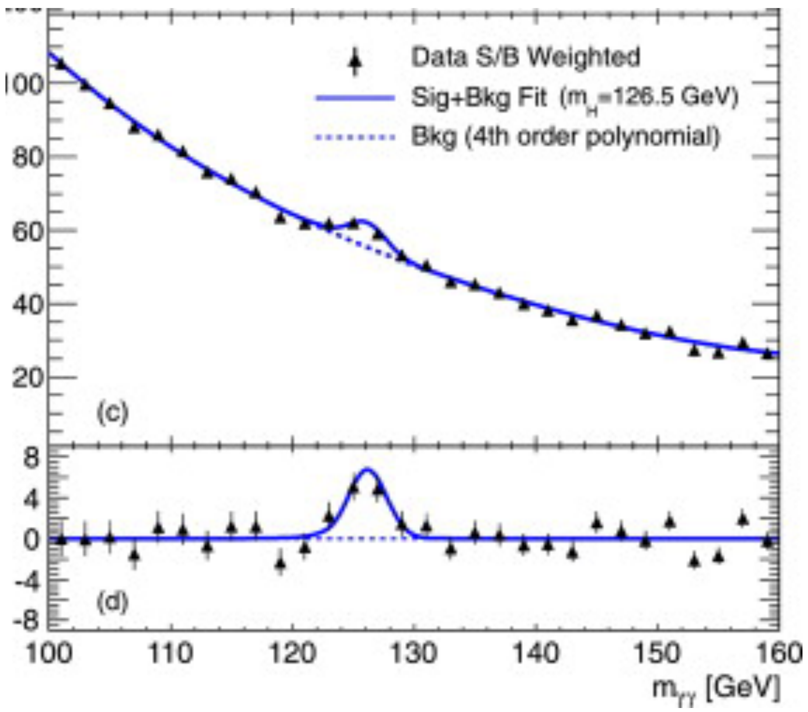
- Collider Physics
  - accelerating particle -> High Energy collision
- What do we need to predict/understand such collision?





Kind of measurement

Peak



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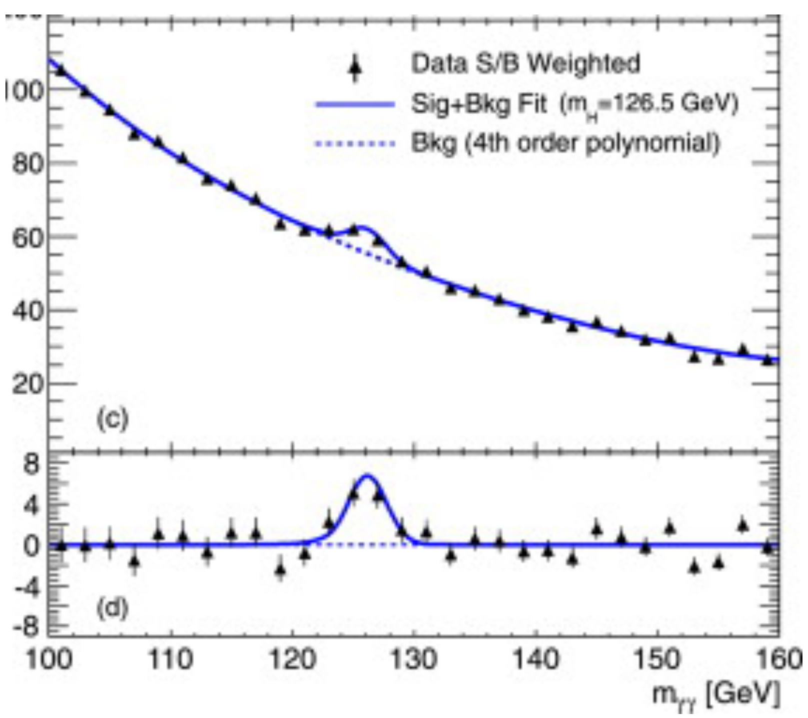
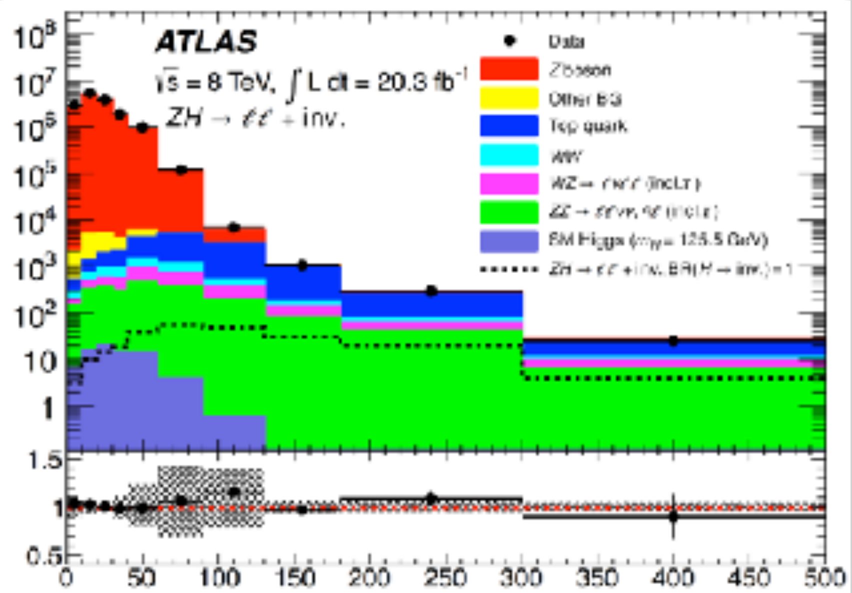


Shape



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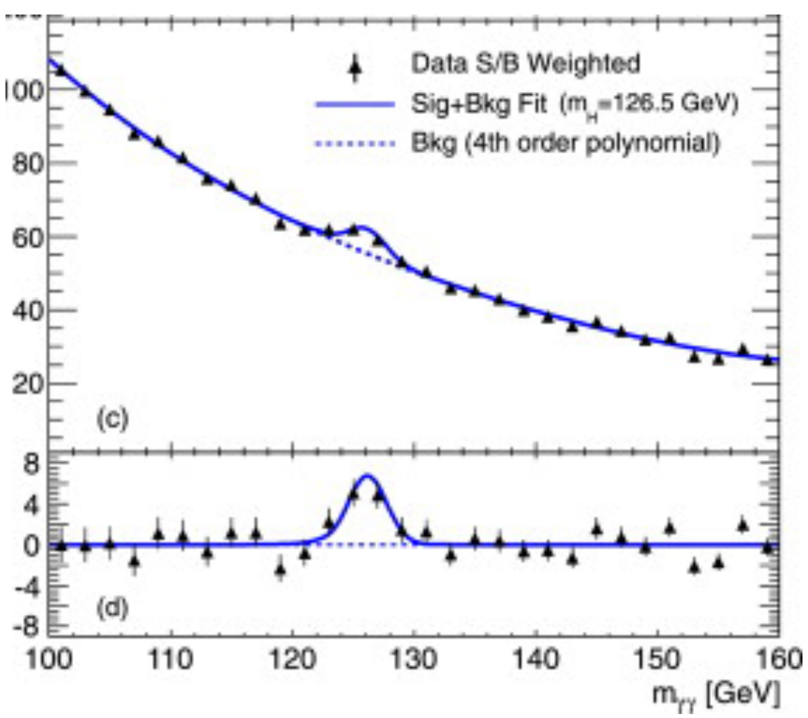

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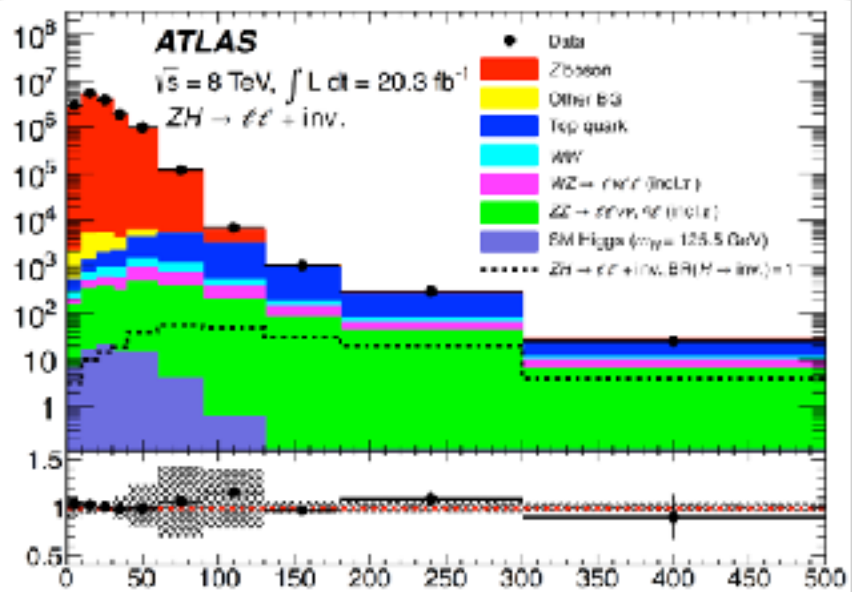
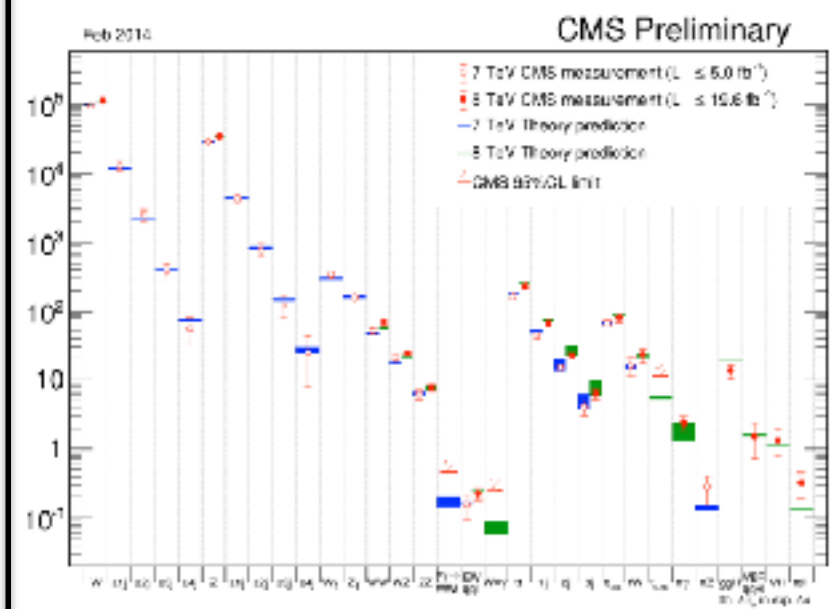


Rate



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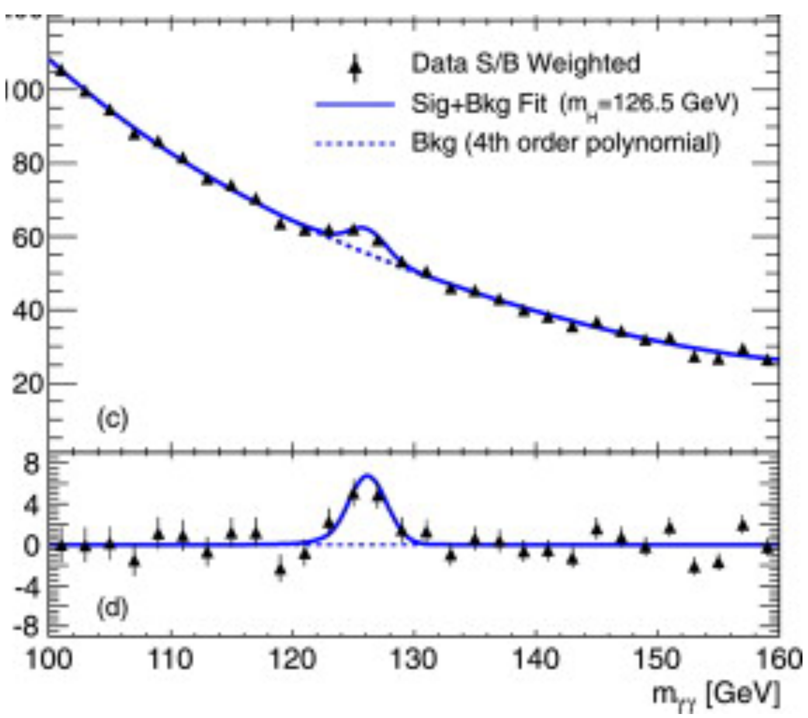
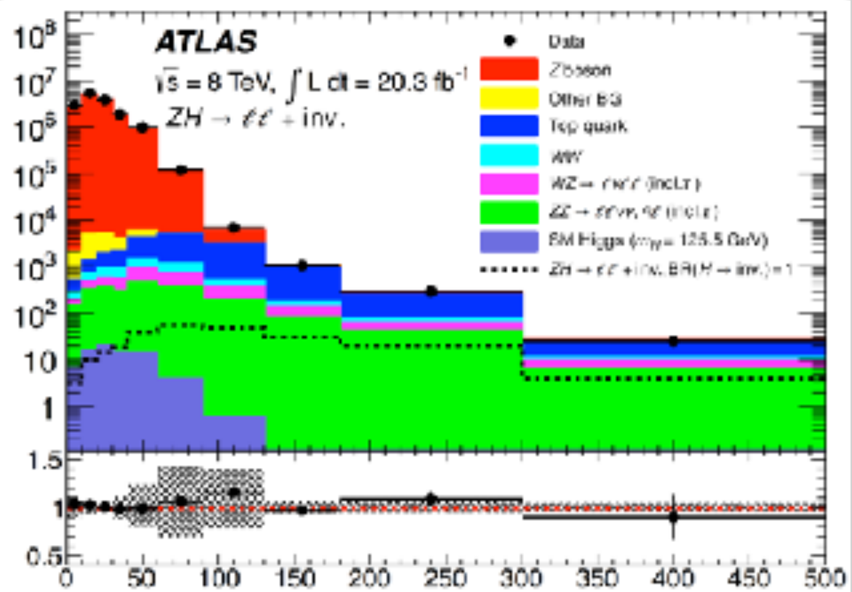


“EASY”

Shape



“HARD”

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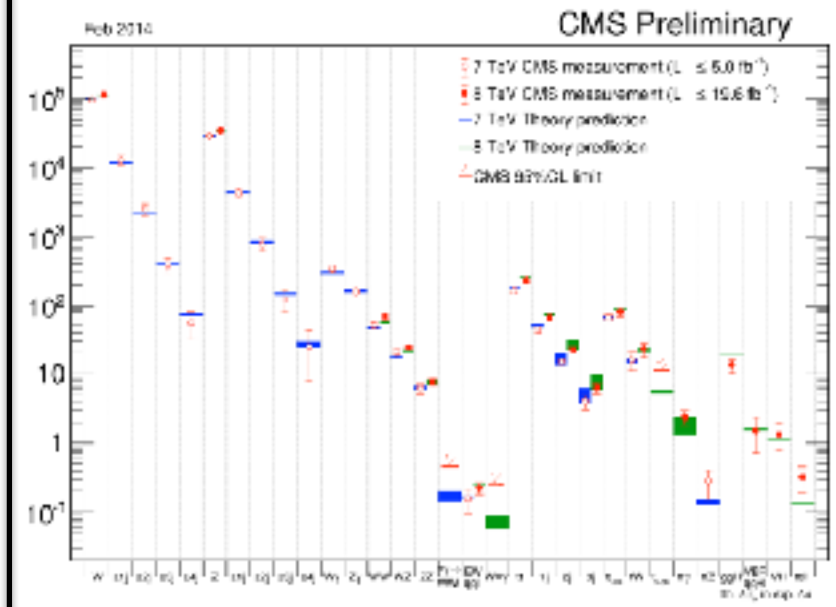

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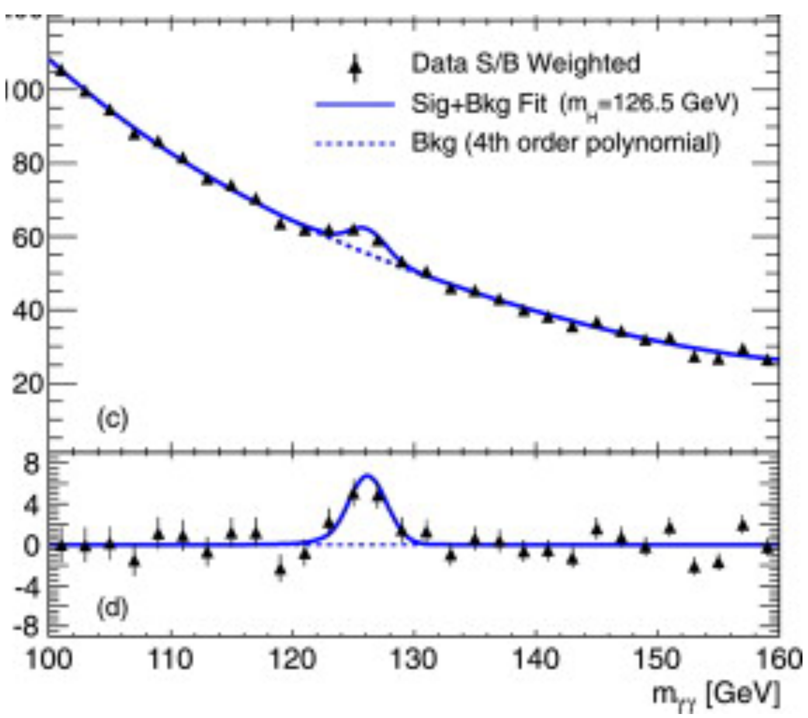
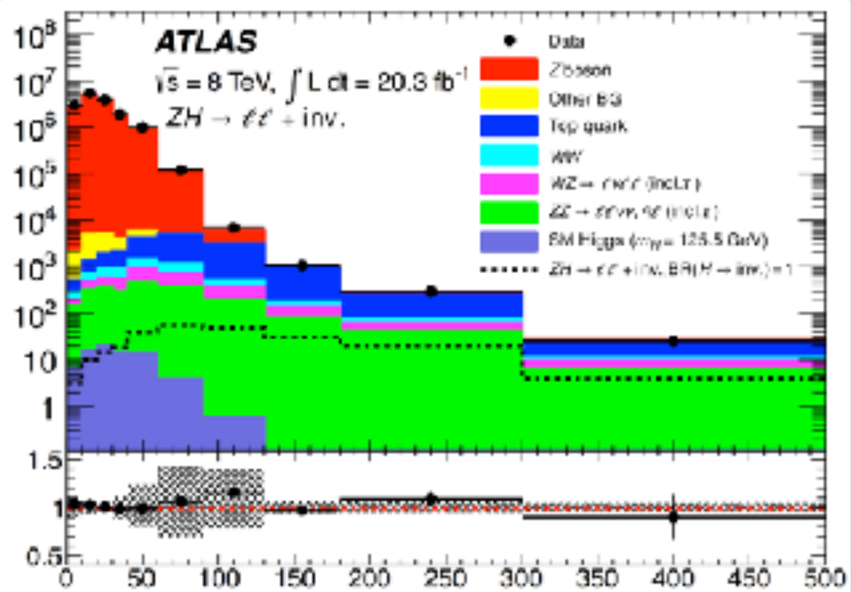


"EASY"

Background directly measured from data. Theory needed only for parameter extraction

Shape



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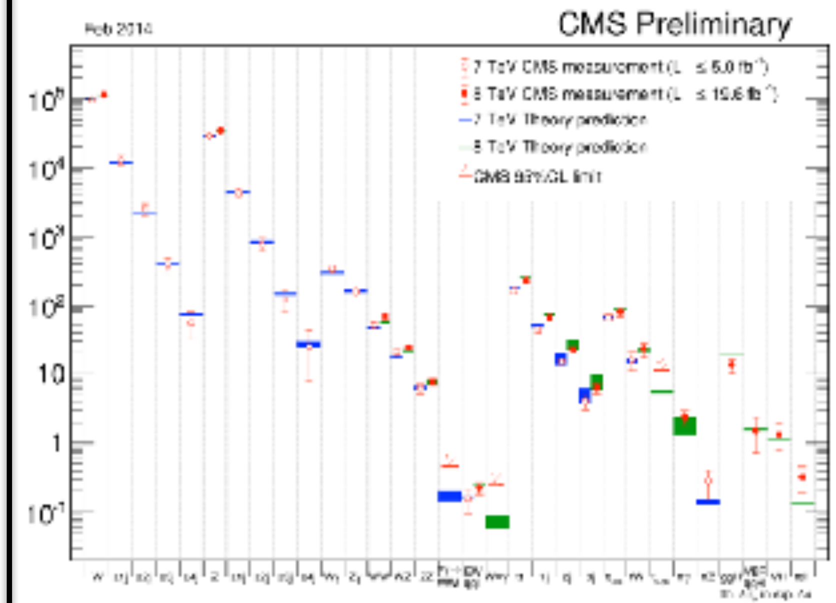

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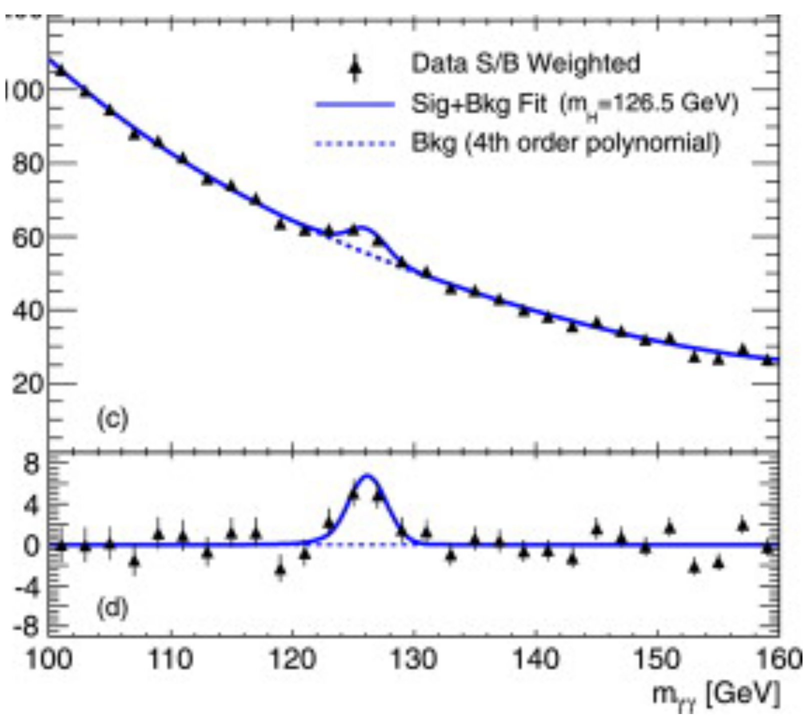

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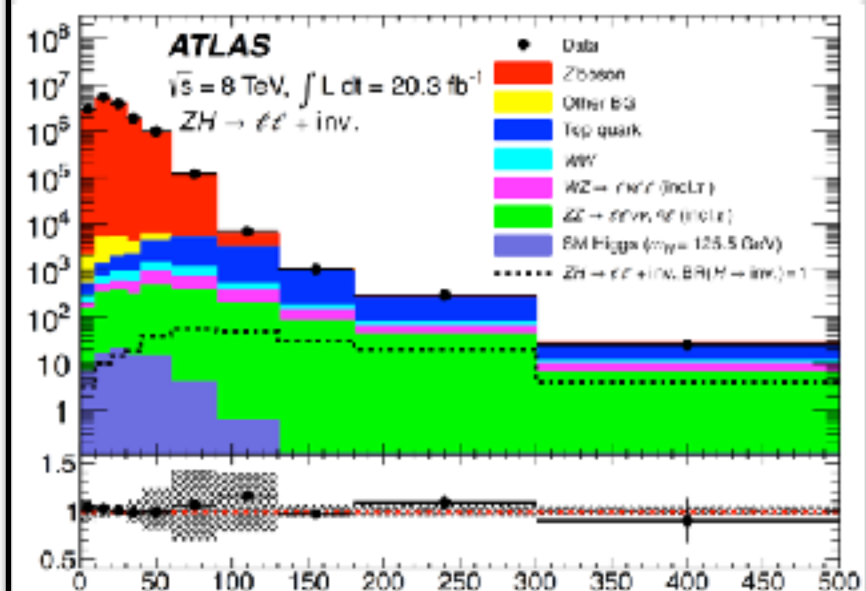
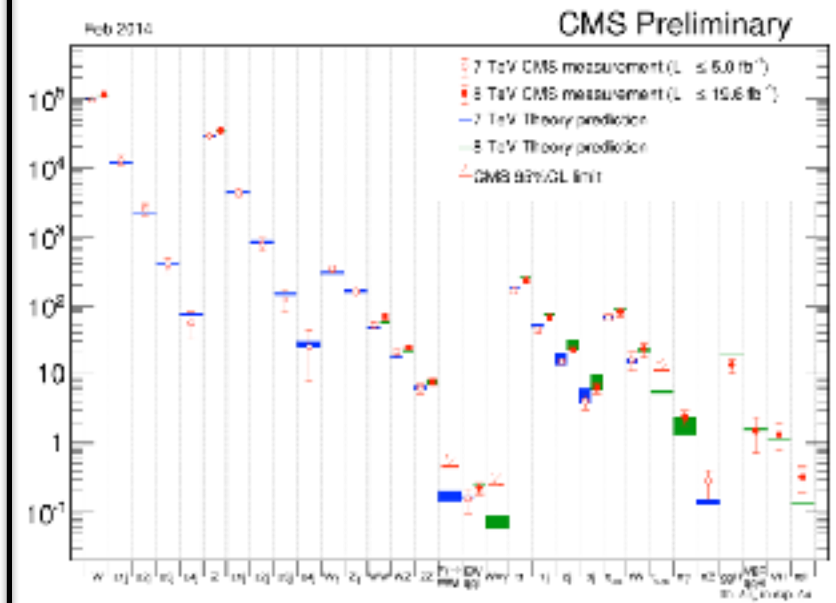


"HARD"

Background **SHAPE** needed. Flexible MC for both signal and background validated and tuned to data

Rate



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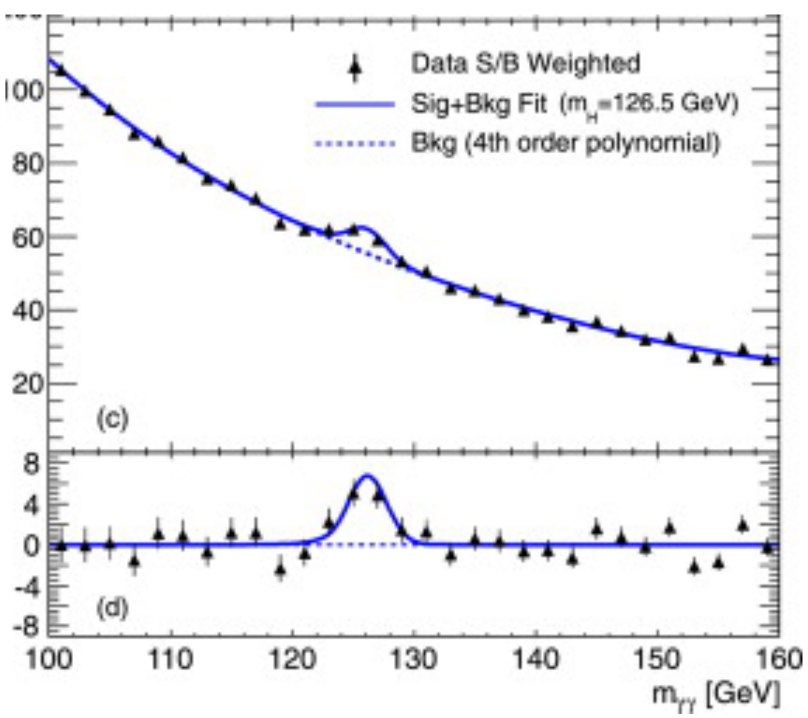

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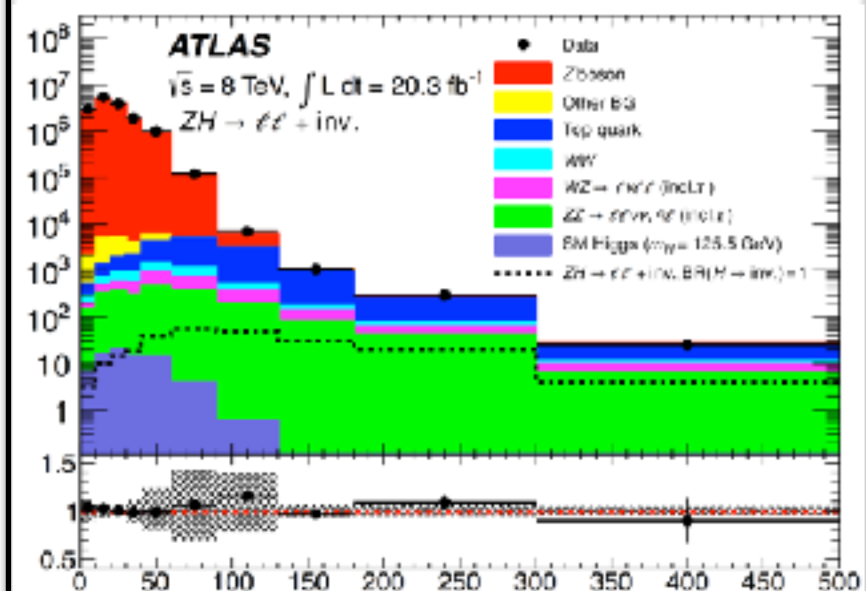

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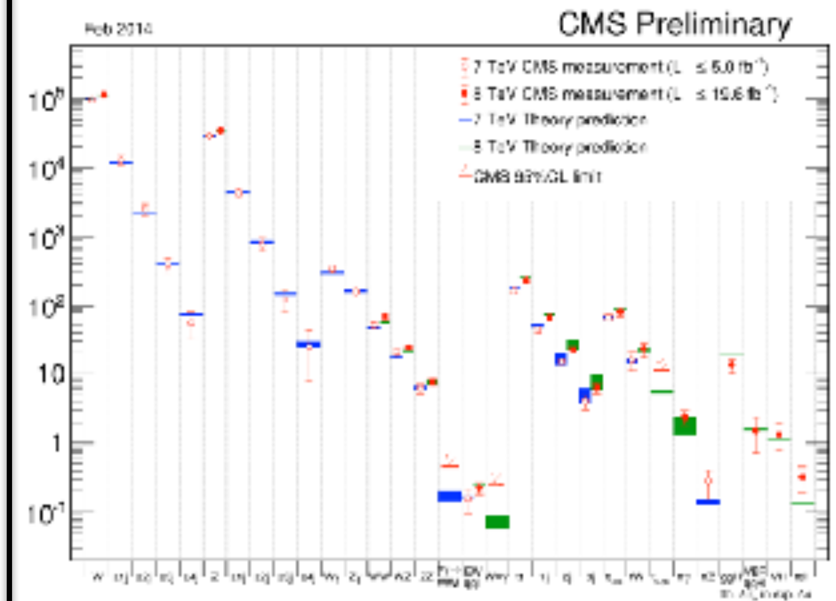
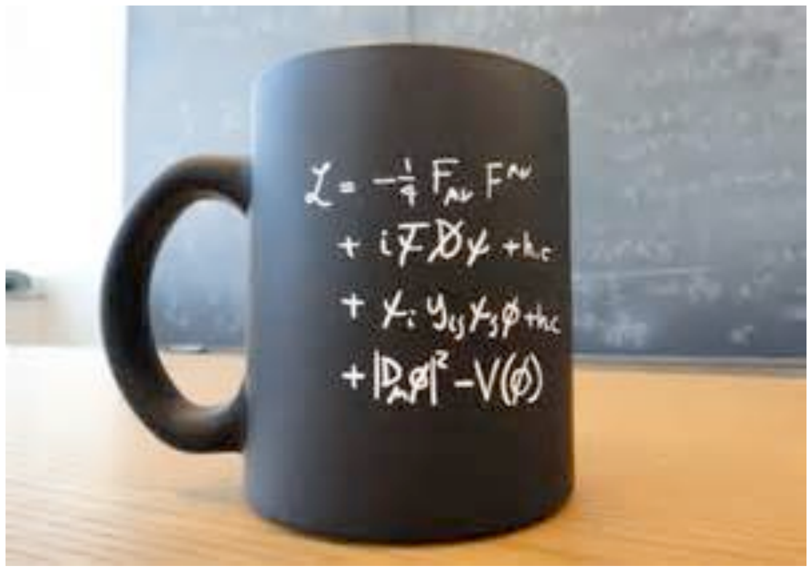


“VERY HARD”

Relies on prediction for both **shape** and **normalization**. Complicated interplay of best simulations and data

Theory side

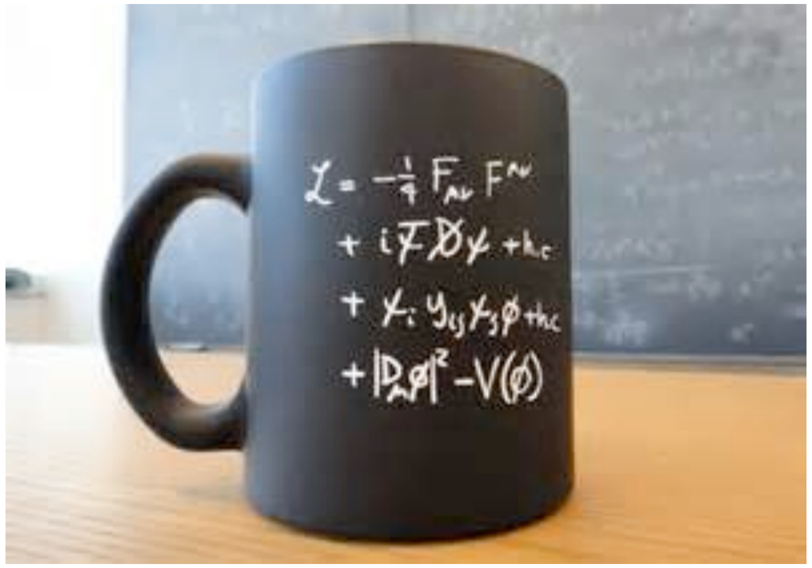
Lagrangian



- This is Where the new idea are expressed

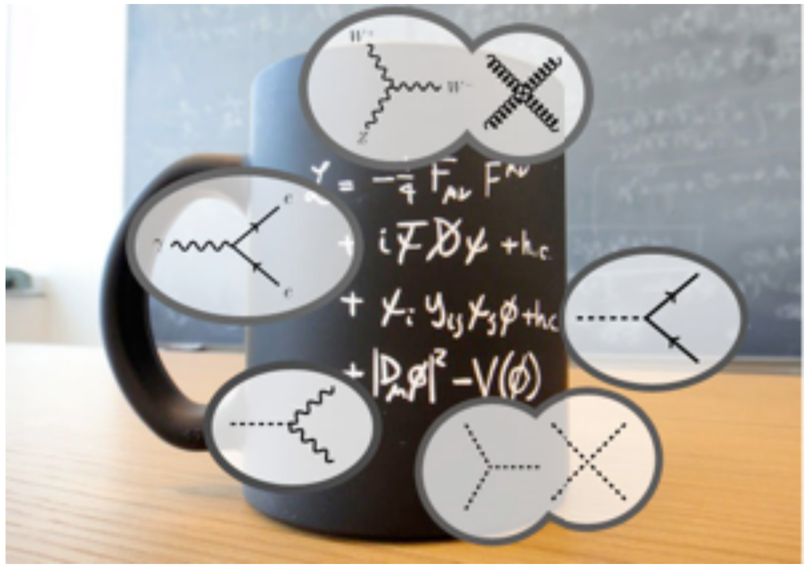
Theory side

Lagrangian



- This is Where the new idea are expressed

Feynman Rule

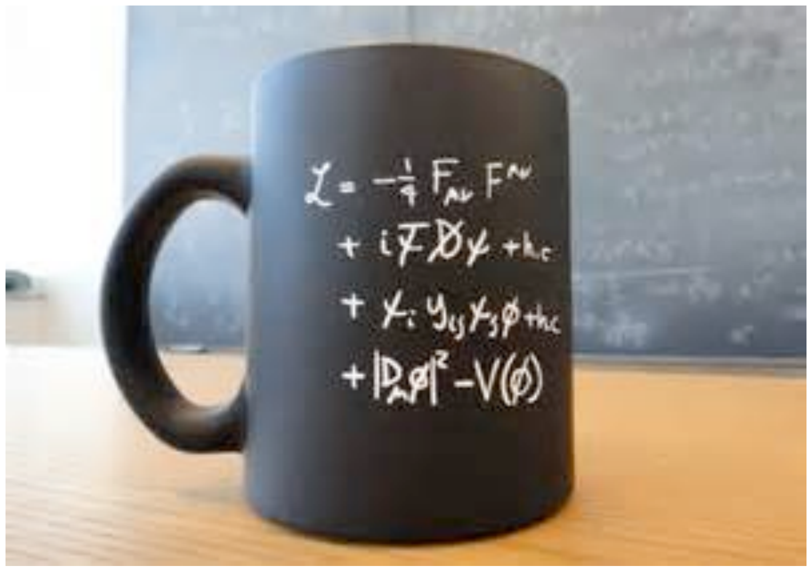


- Same information as the Lagrangian

FeynRules

Theory side

Lagrangian



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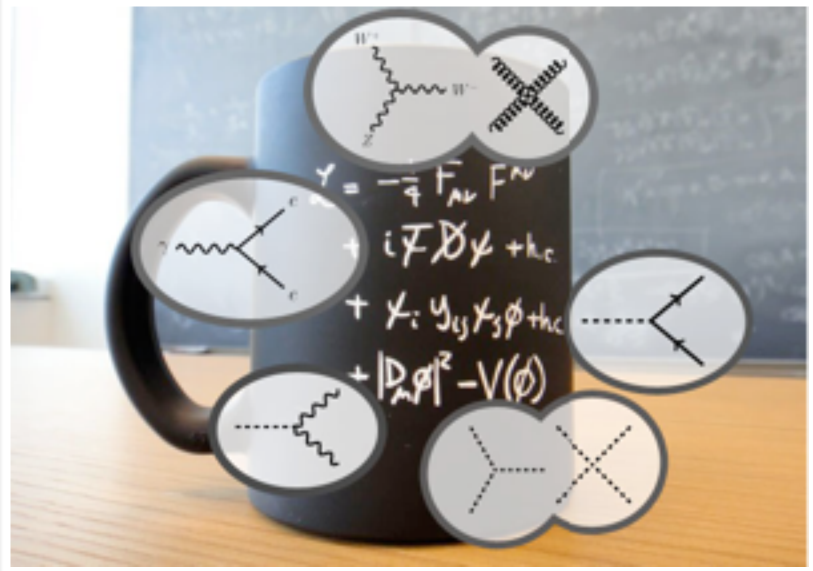
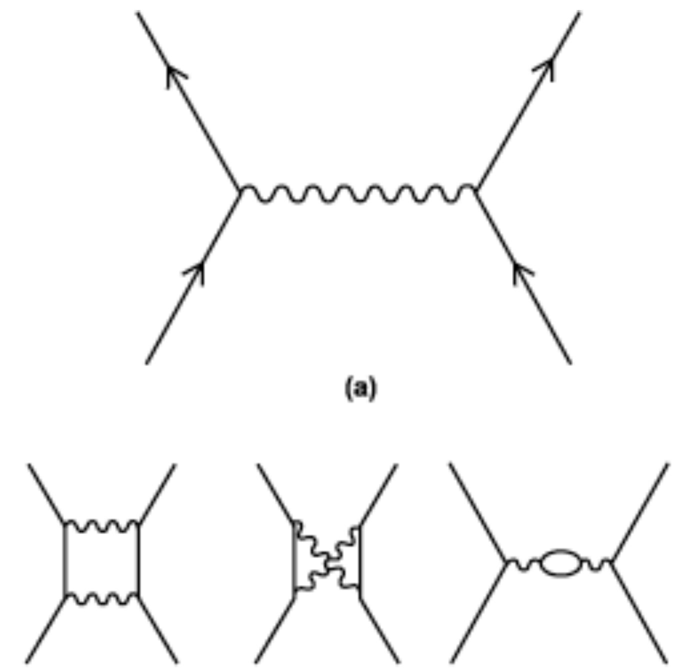


- Same information as the Lagrangian

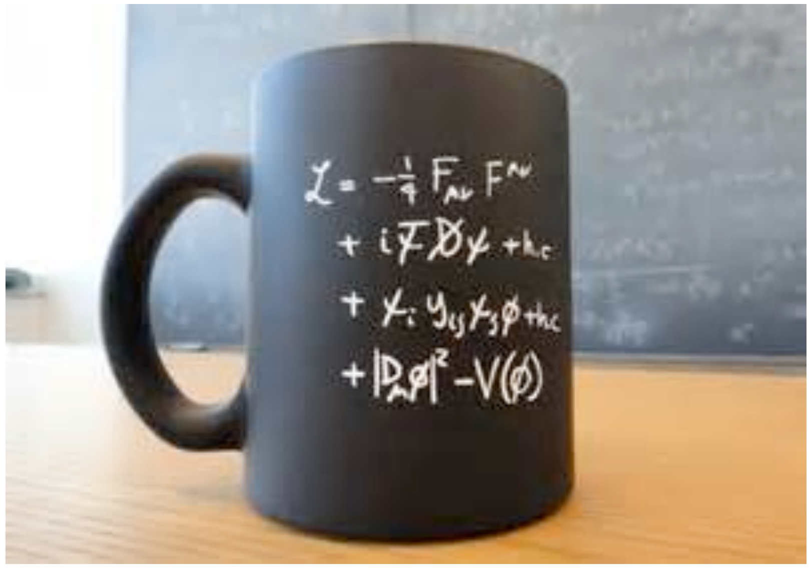
FeynRules

Cross-section

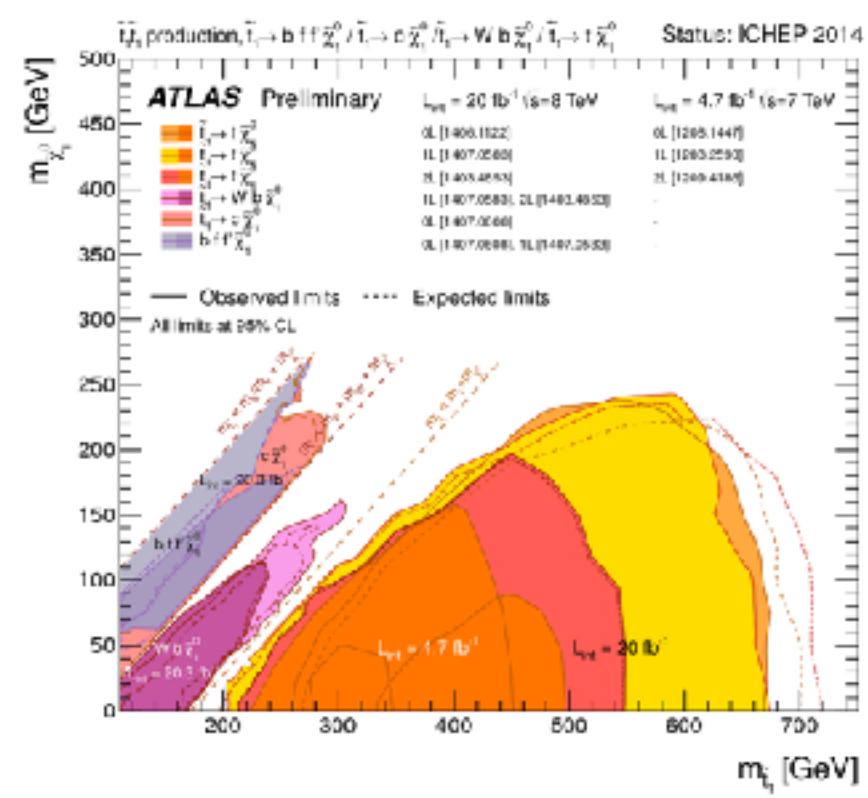
$$\frac{d\sigma}{d\ \cos\theta} = \left(\frac{d\sigma}{d\ \cos\theta}\right)_R \frac{(1+\cos\theta)/2}{\left[1+\frac{(1-\cos\theta)KE}{Mc^2}\right]}$$



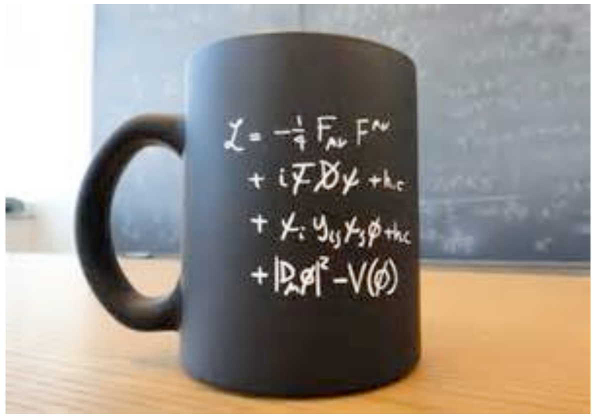
- What is the precision?

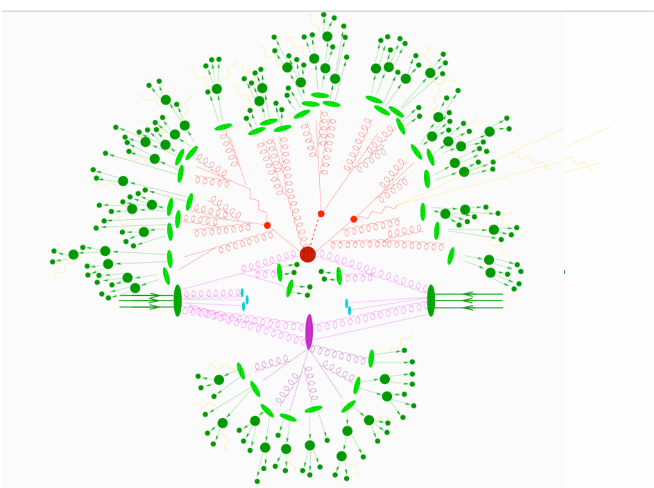


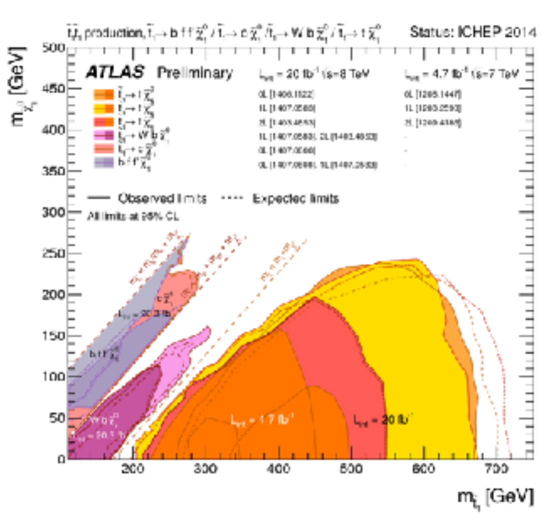
Monte-Carlo Physics



Filling the gap



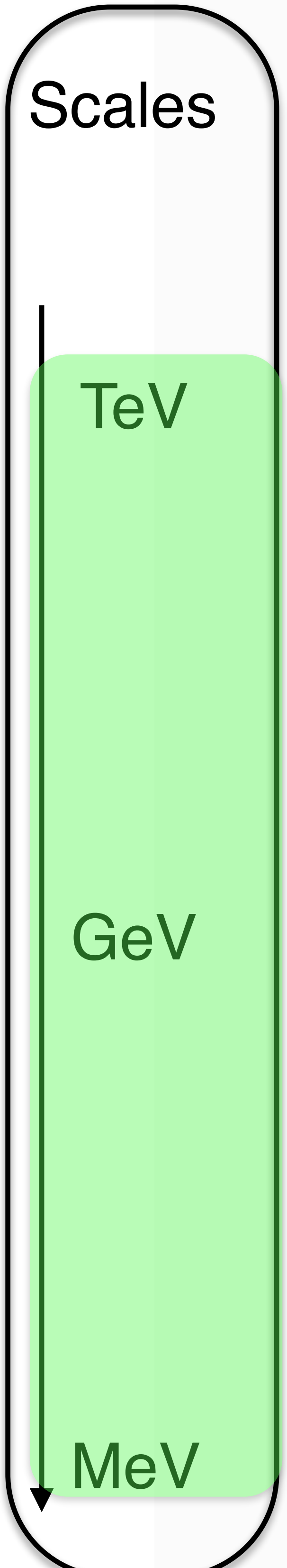


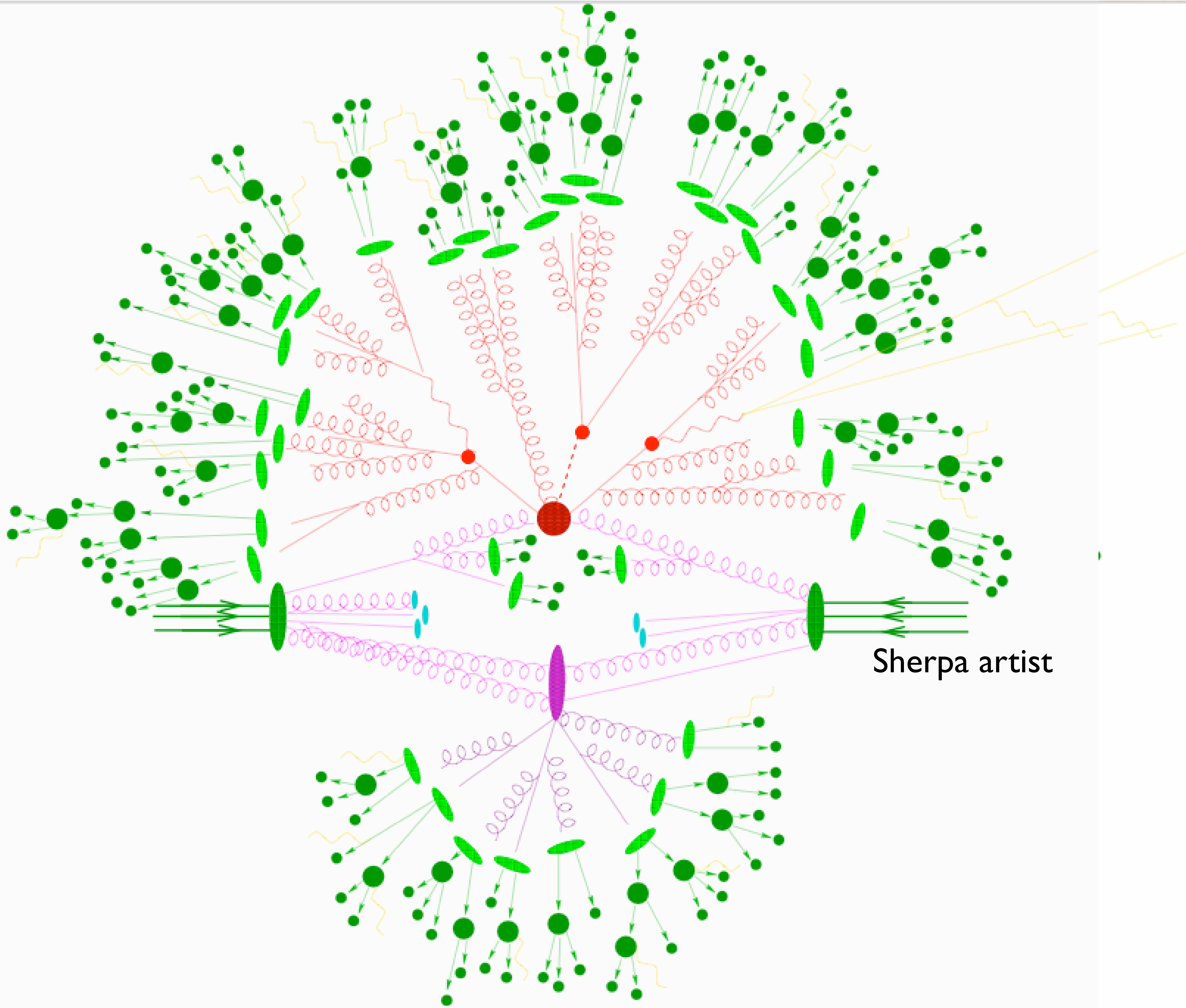


Lagrangian	FeynRules / Sarah
Feynman Rules	MadGraph / Comix
matrix element	MadEvent/Sherpa
parton events	Pythia/Herwig
shower events	Pythia/Herwig
hadronized events	Delphes/Full Sim
detector simulation	CheckMate/ MadAnalysis5
exclusion plot	

Simulation of collider events

What are the MC for?





What are the MC for?

Scales

TeV

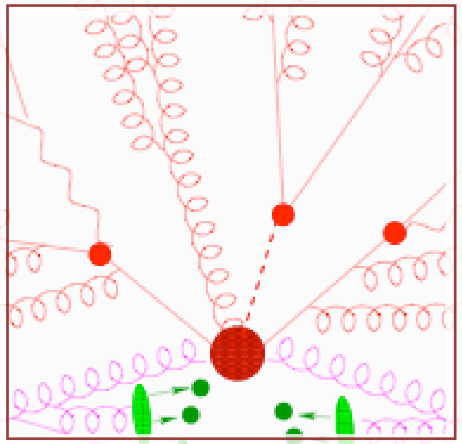
GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ where BSM physics lies



☞ process dependent

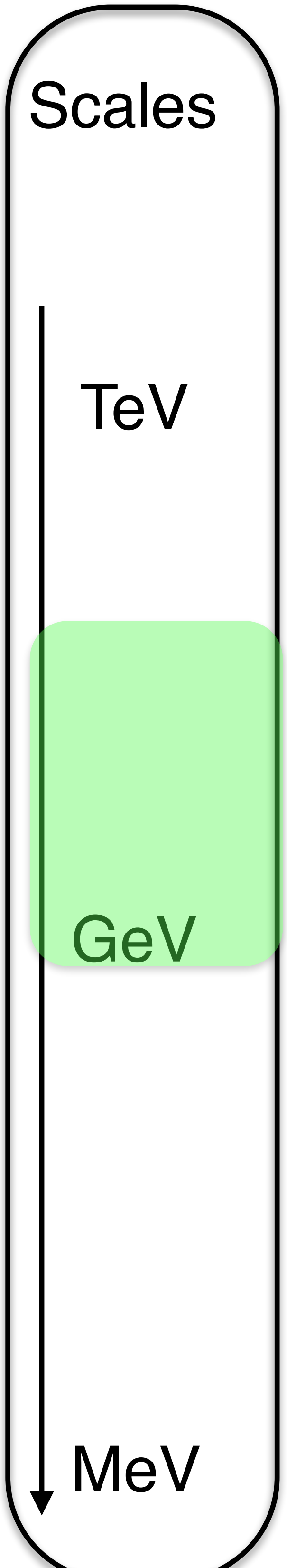
☞ first principles description

☞ it can be systematically improved

3. Hadronization

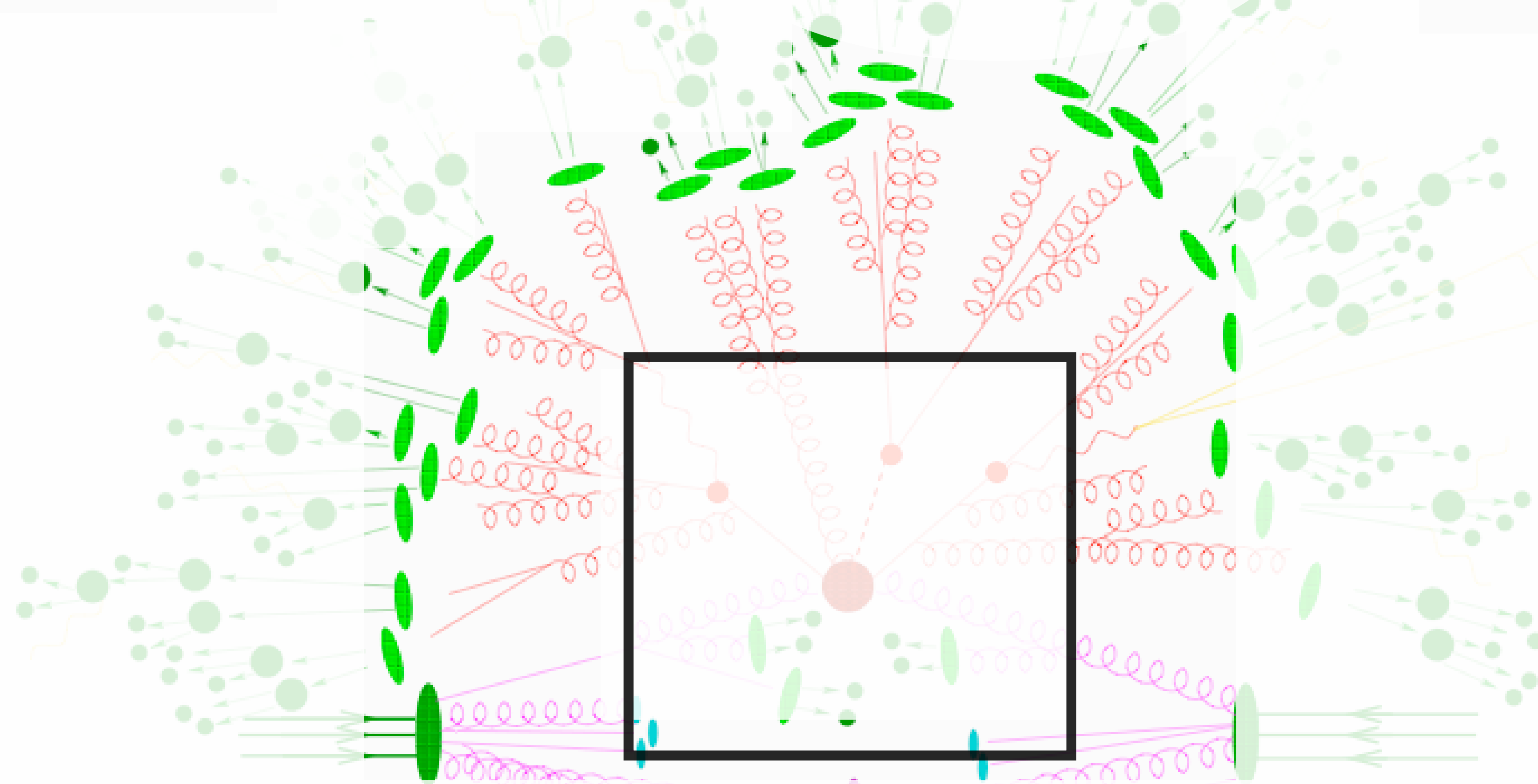
4. Underlying Event

What are the MC for?



1. High-Q^2 Scattering

2. Parton Shower



- QCD -"known physics"
- universal/ process independent
- first principles description

3. Hadronization

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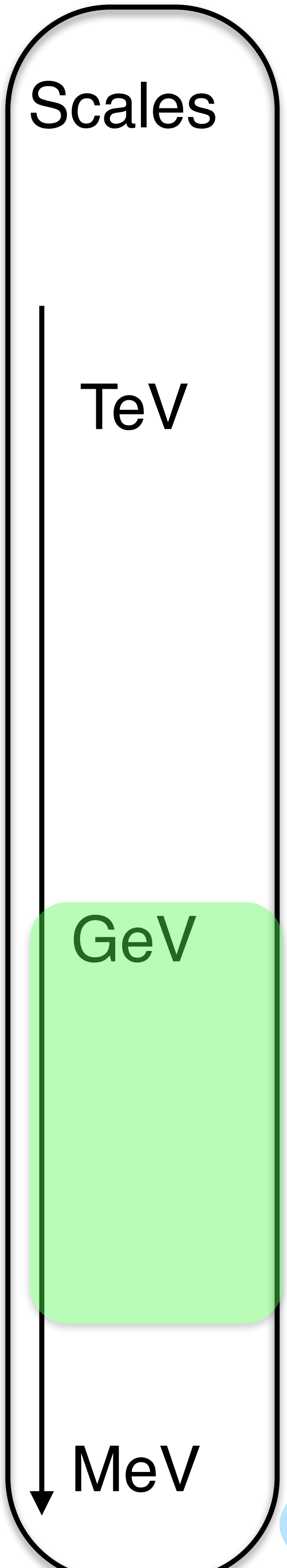

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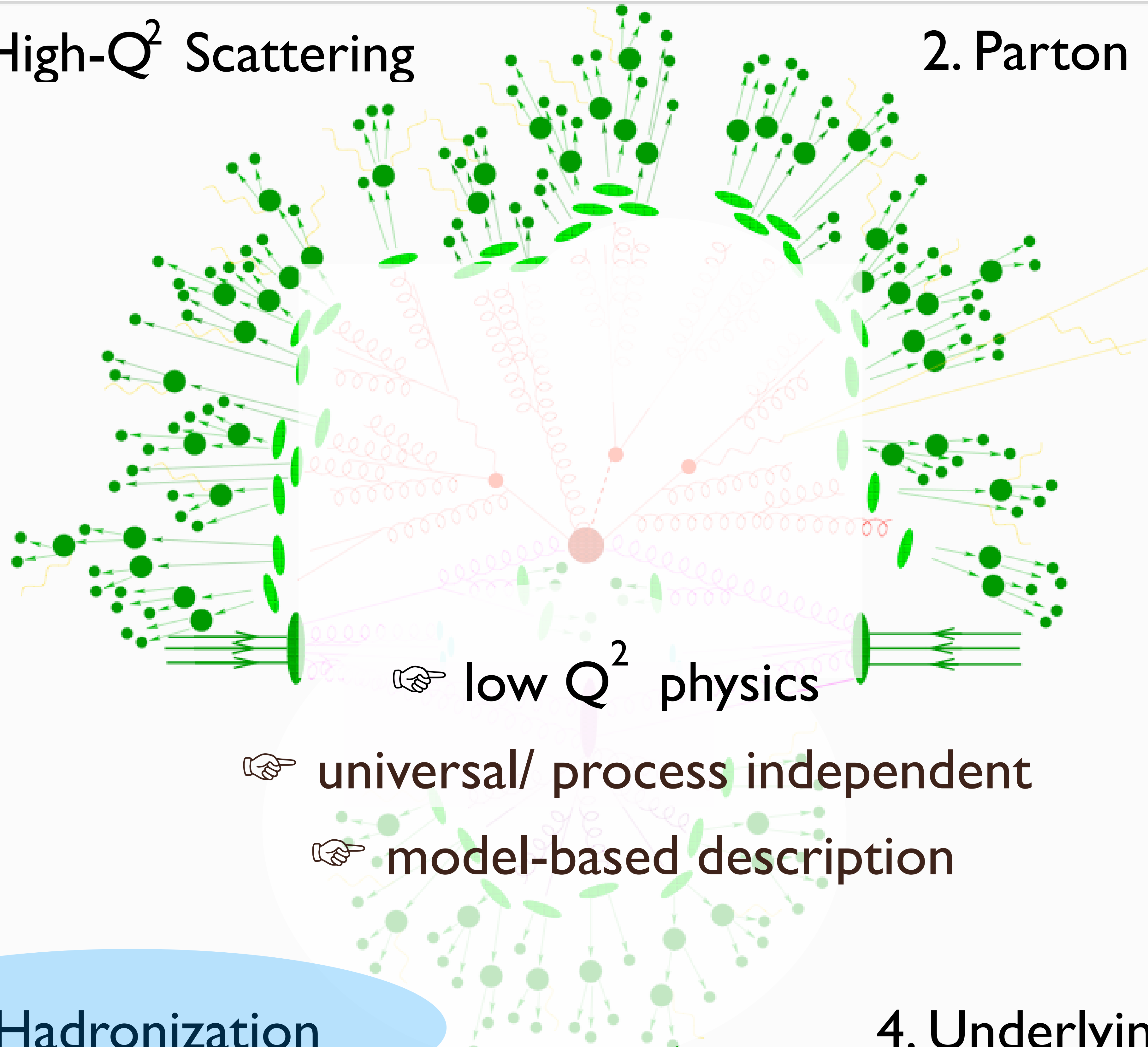

- ☞ low Q^2 physics
- ☞ universal/ process independent
- ☞ model-based description

3. Hadronization

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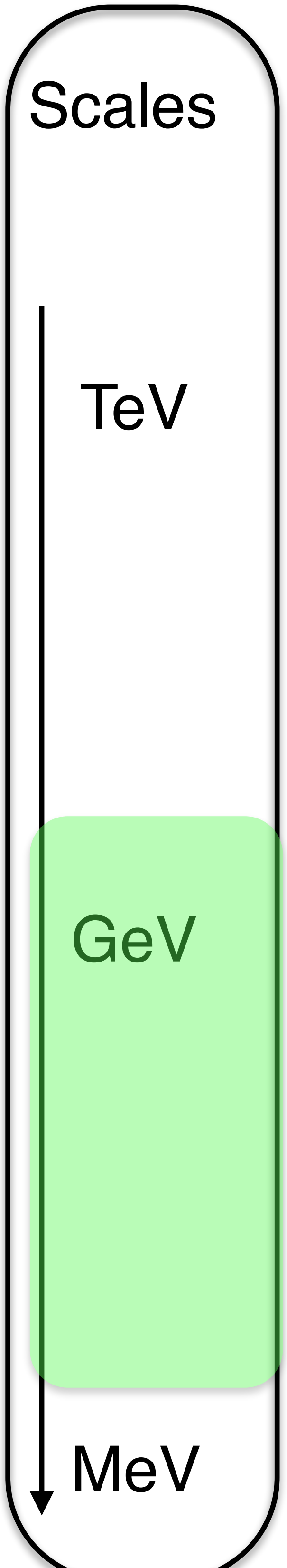
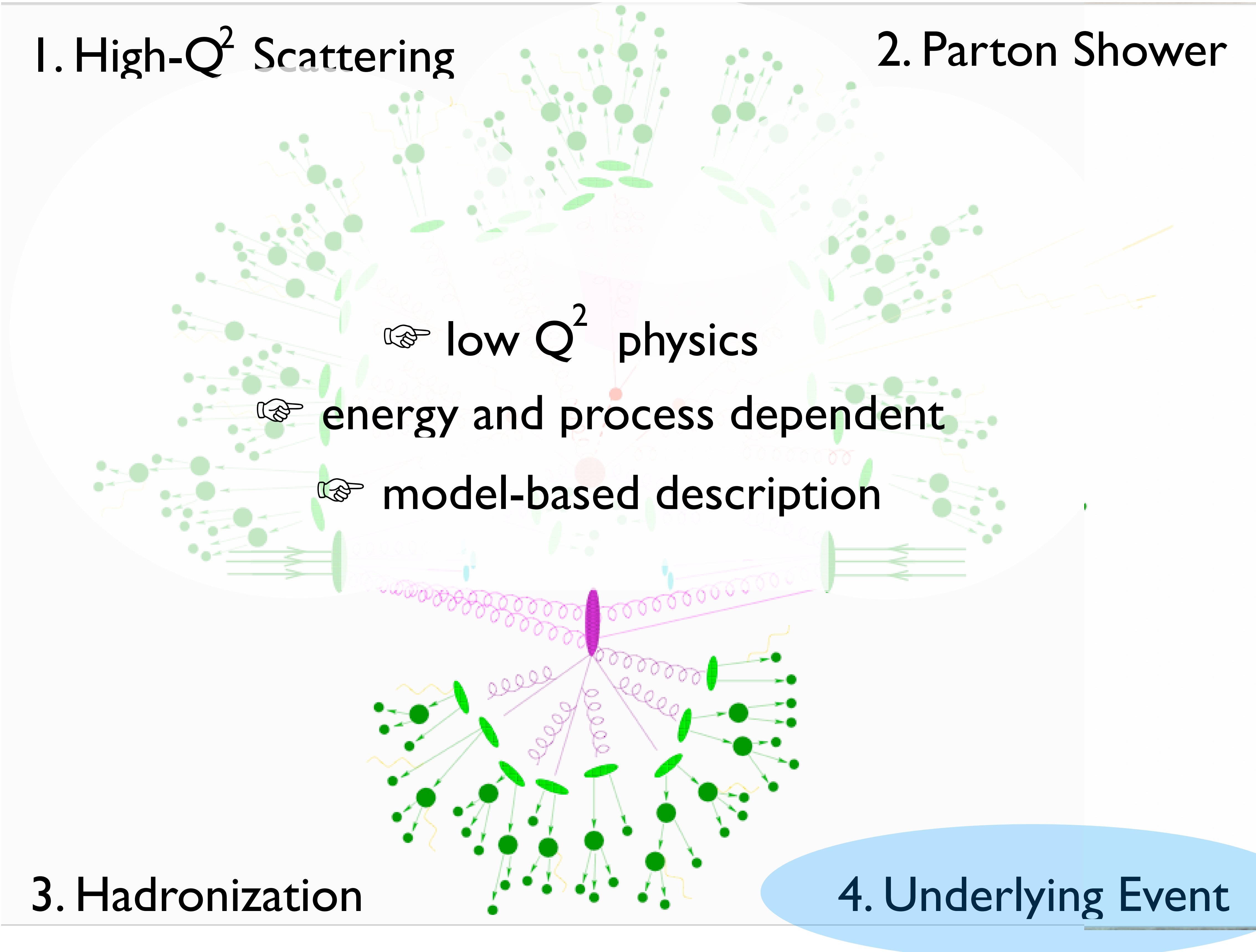
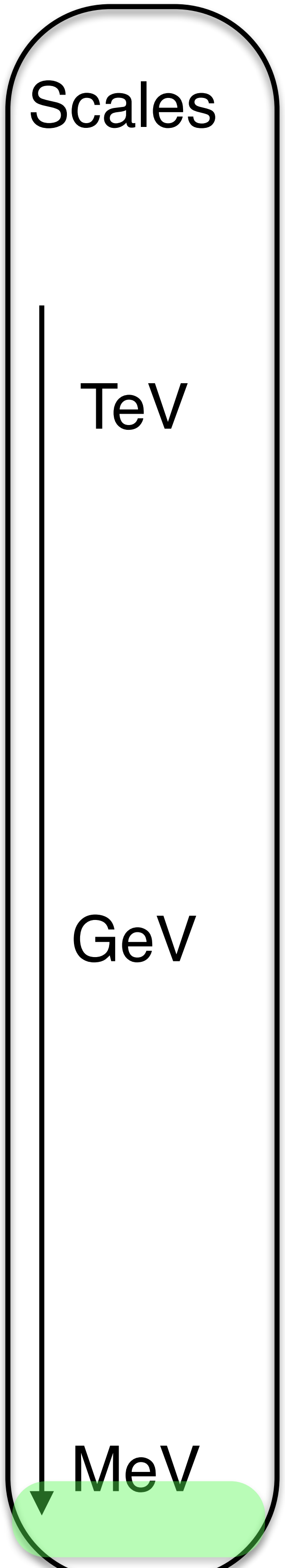
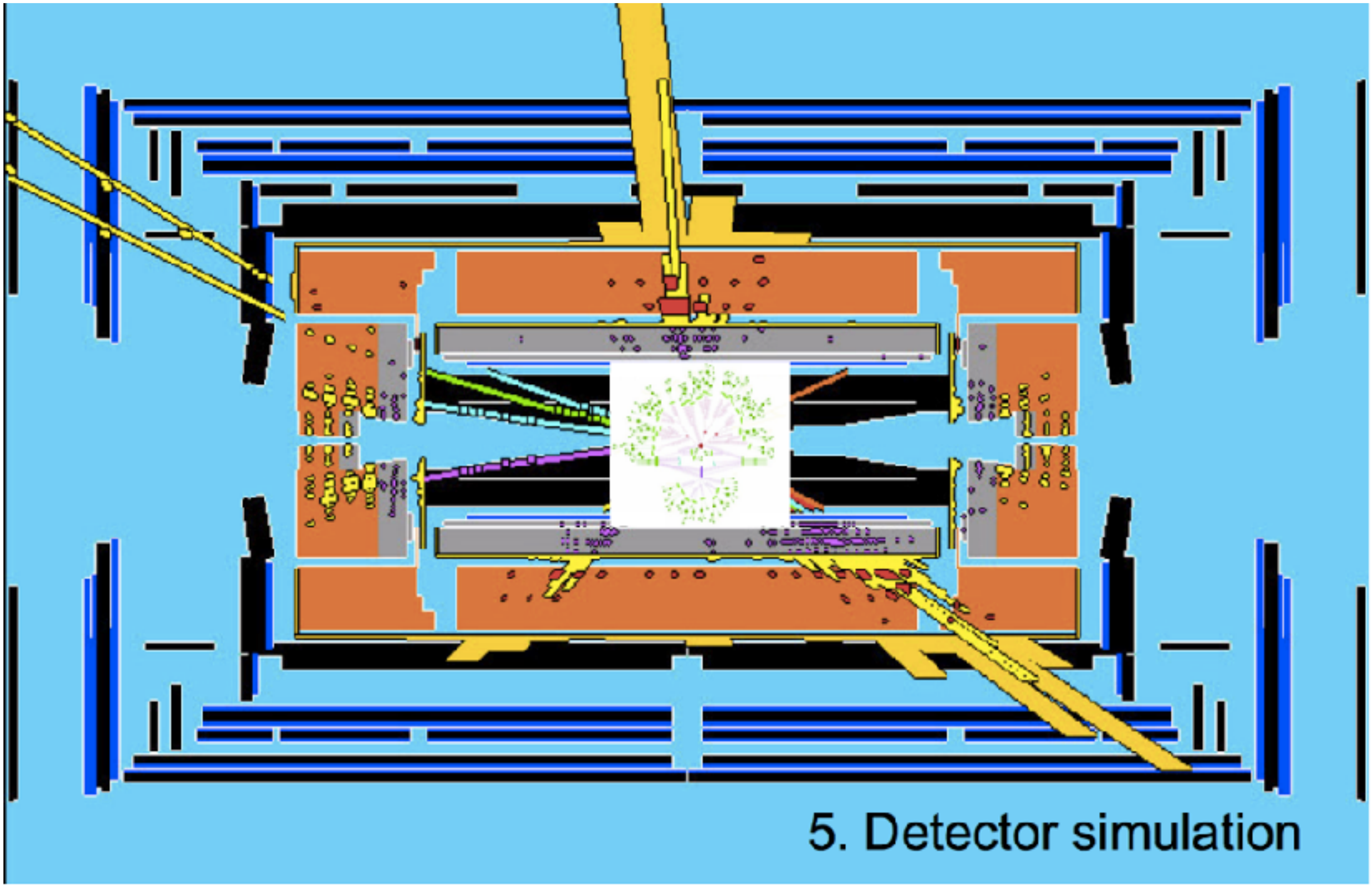




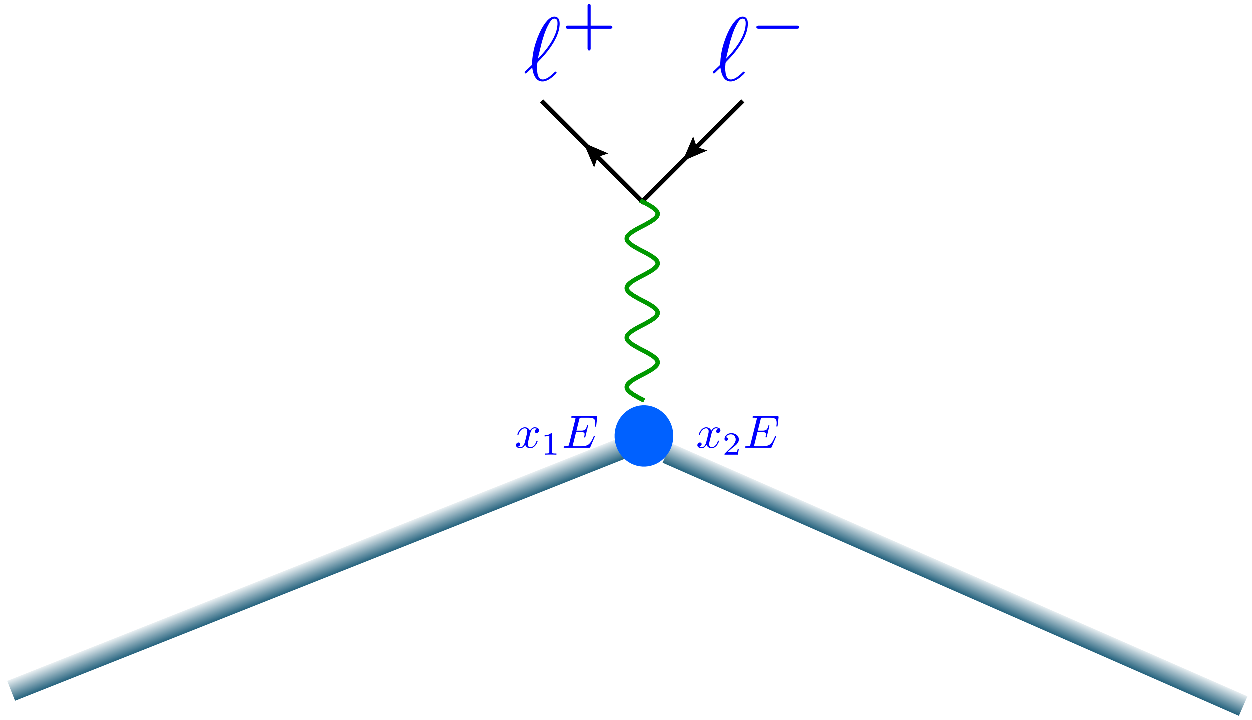
- ☞ low Q^2 physics
- ☞ energy and process dependent
- ☞ model-based description


Scales
TeV
GeV
MeV


5. Detector simulation

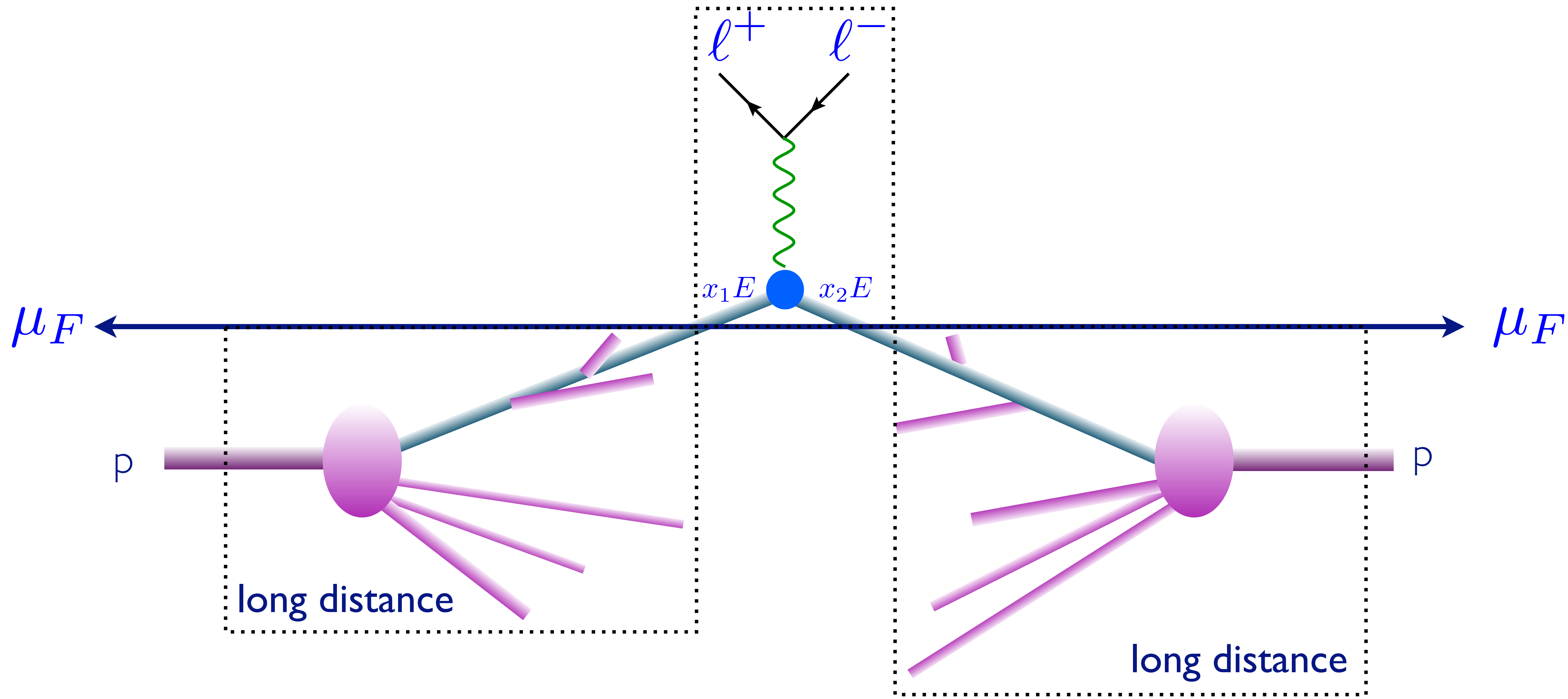
To Remember

- Multi-scale problem
  - ➡ New physics visible only at High scale
  - ➡ Problem split in different scale



$$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Parton-level cross section

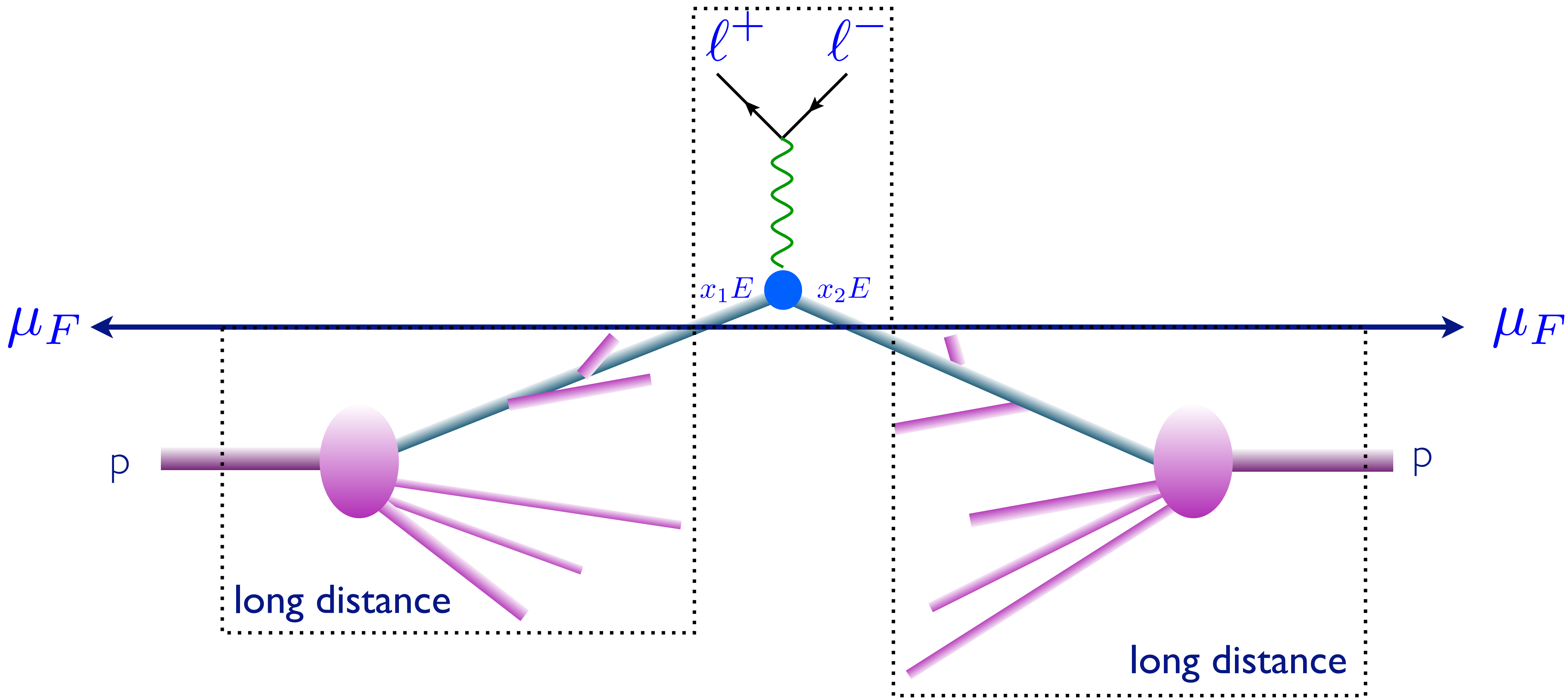


$$f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Parton density functions

Parton-level cross section

Master formula for the LHC



$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat\sigma_{ab\to X}(\hat s,\mu_F,\mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\rm Born}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

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LO predictions

Perturbative expansion

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LO predictions

NLO corrections

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LO predictions | NLO corrections | NNLO corrections

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LO predictions

NLO corrections

NNLO corrections

N3LO or NNNLO corrections

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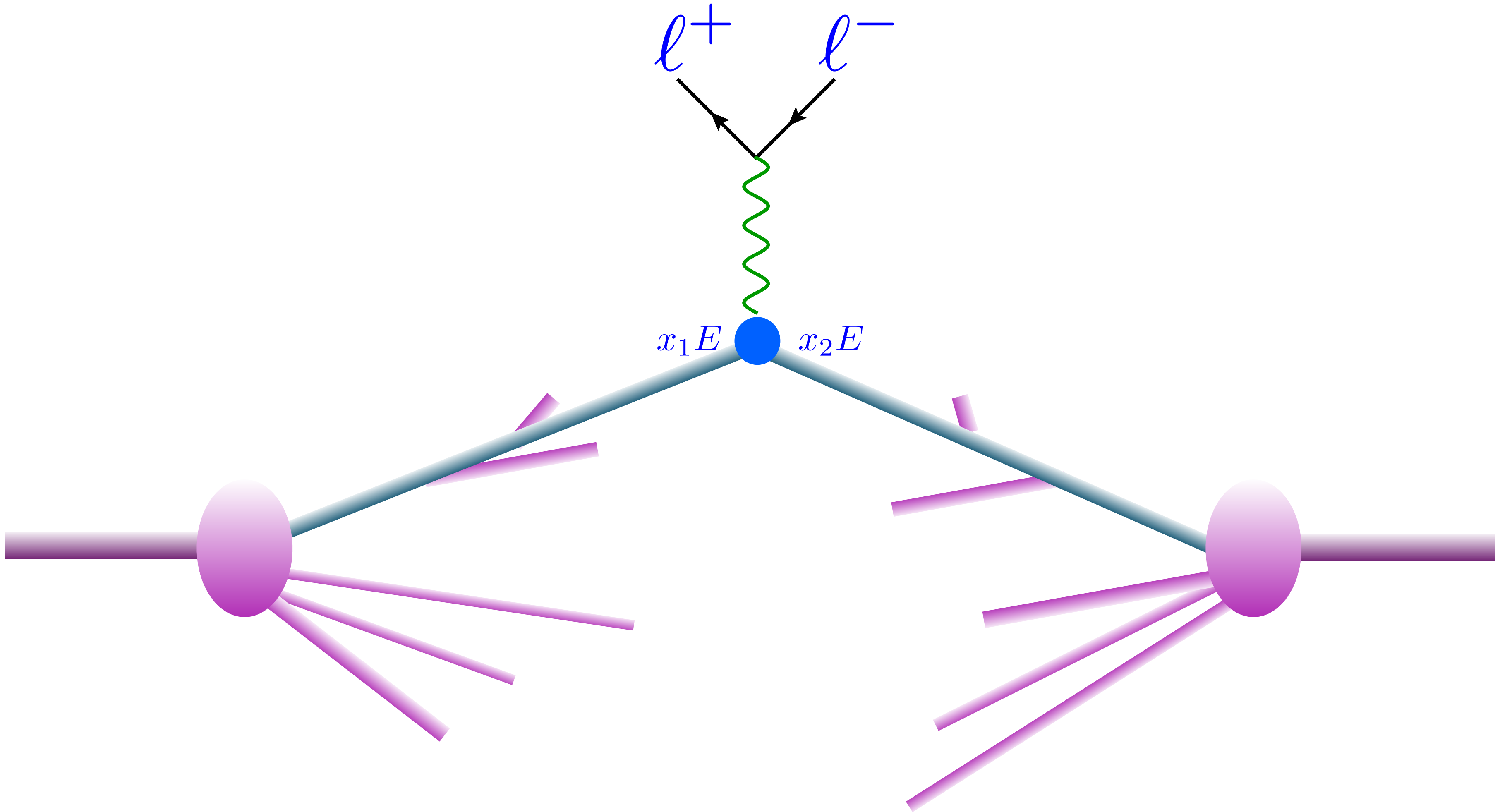
LO predictions | NLO corrections | NNLO corrections | N3LO or NNNLO corrections

- Including higher corrections improves predictions and reduces theoretical uncertainties

- As an example, consider Drell-Yan Z/γ* production



NLO predictions

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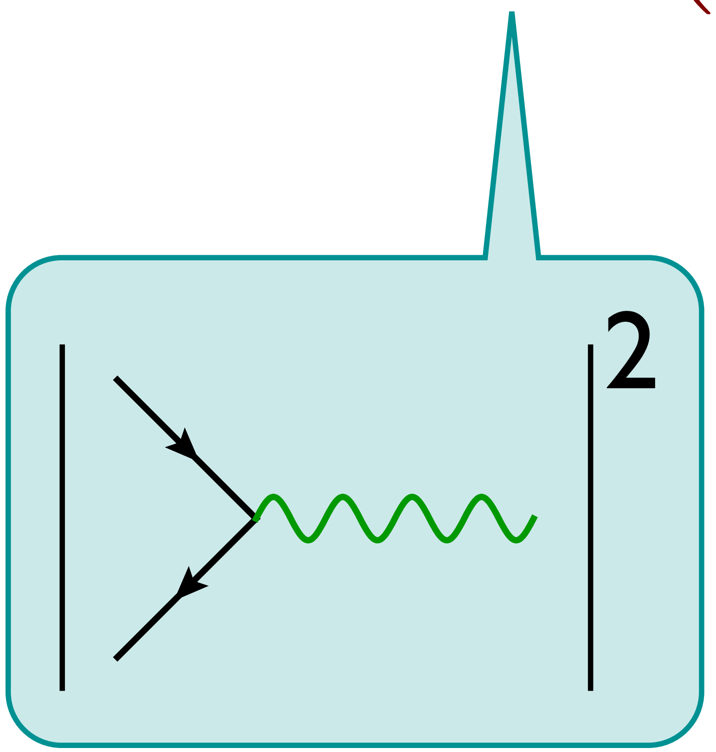

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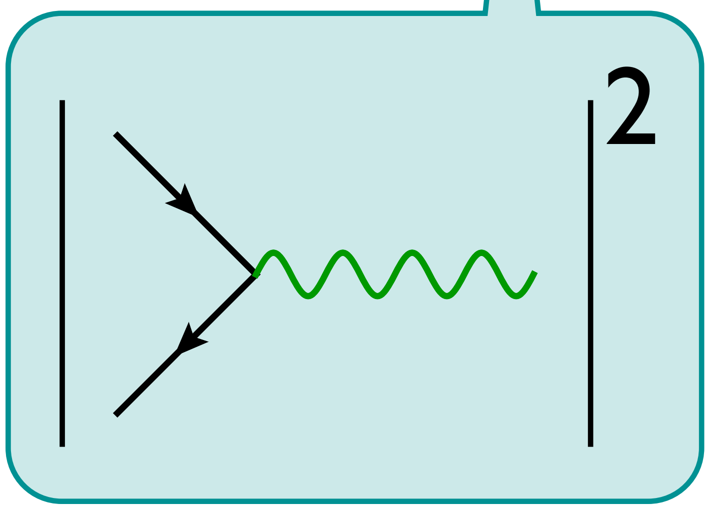

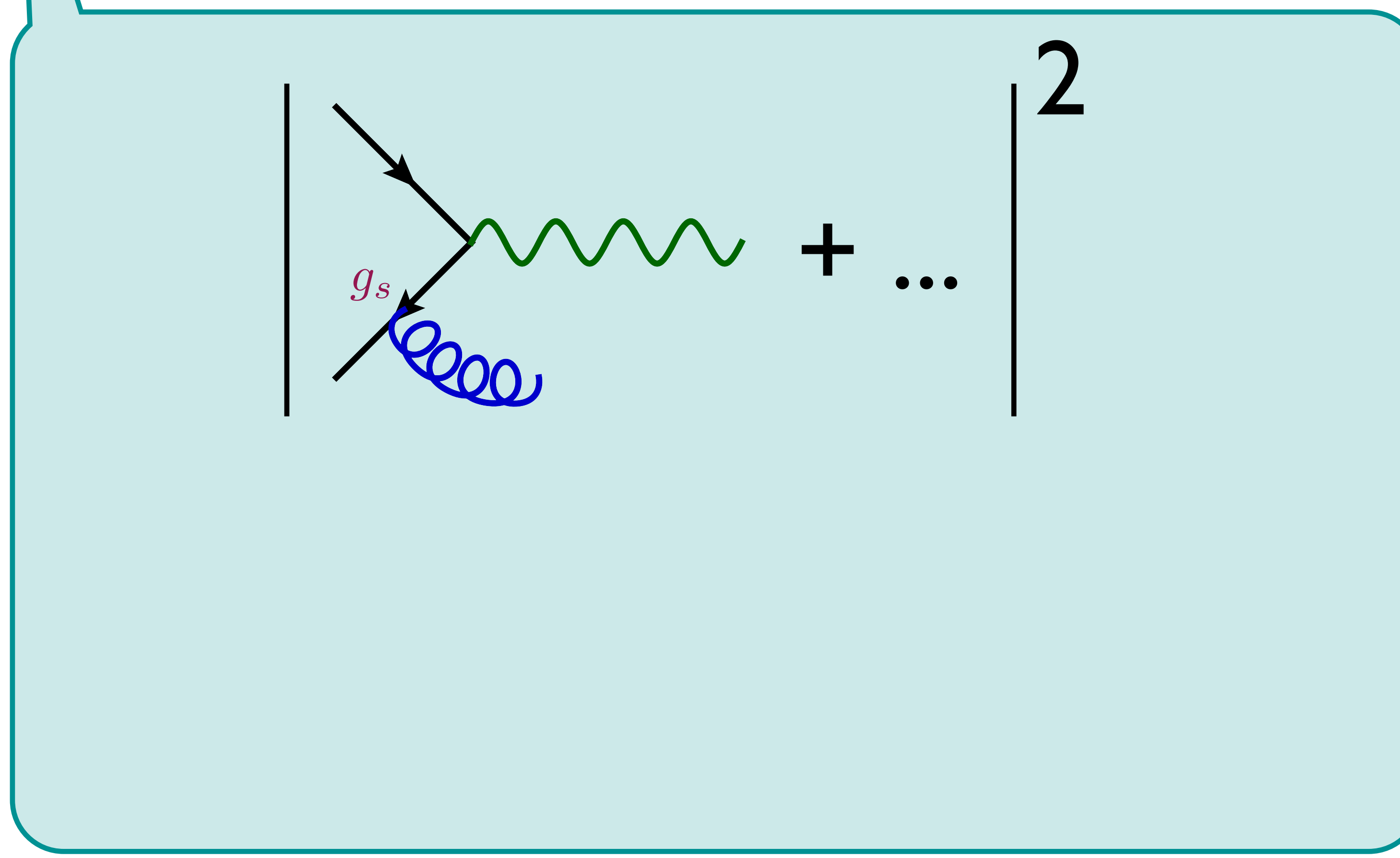

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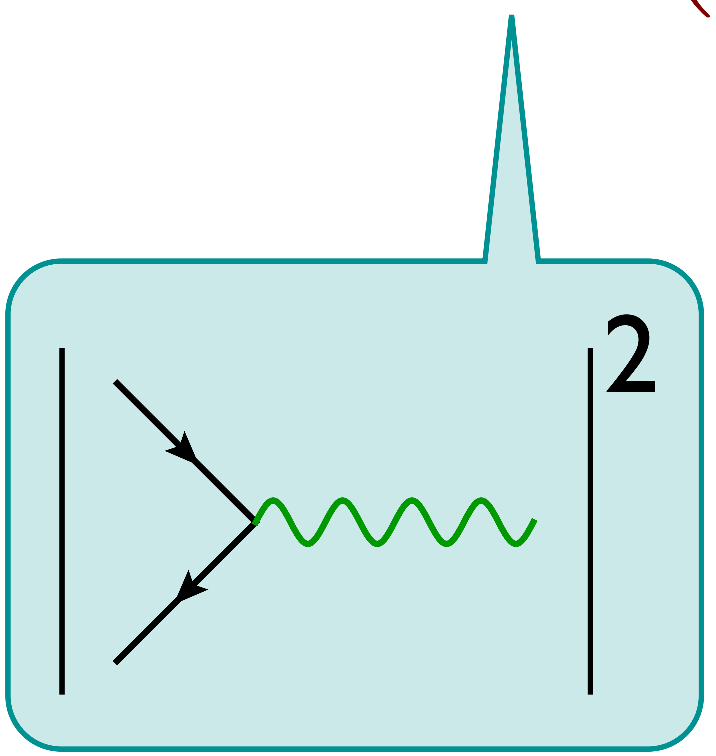
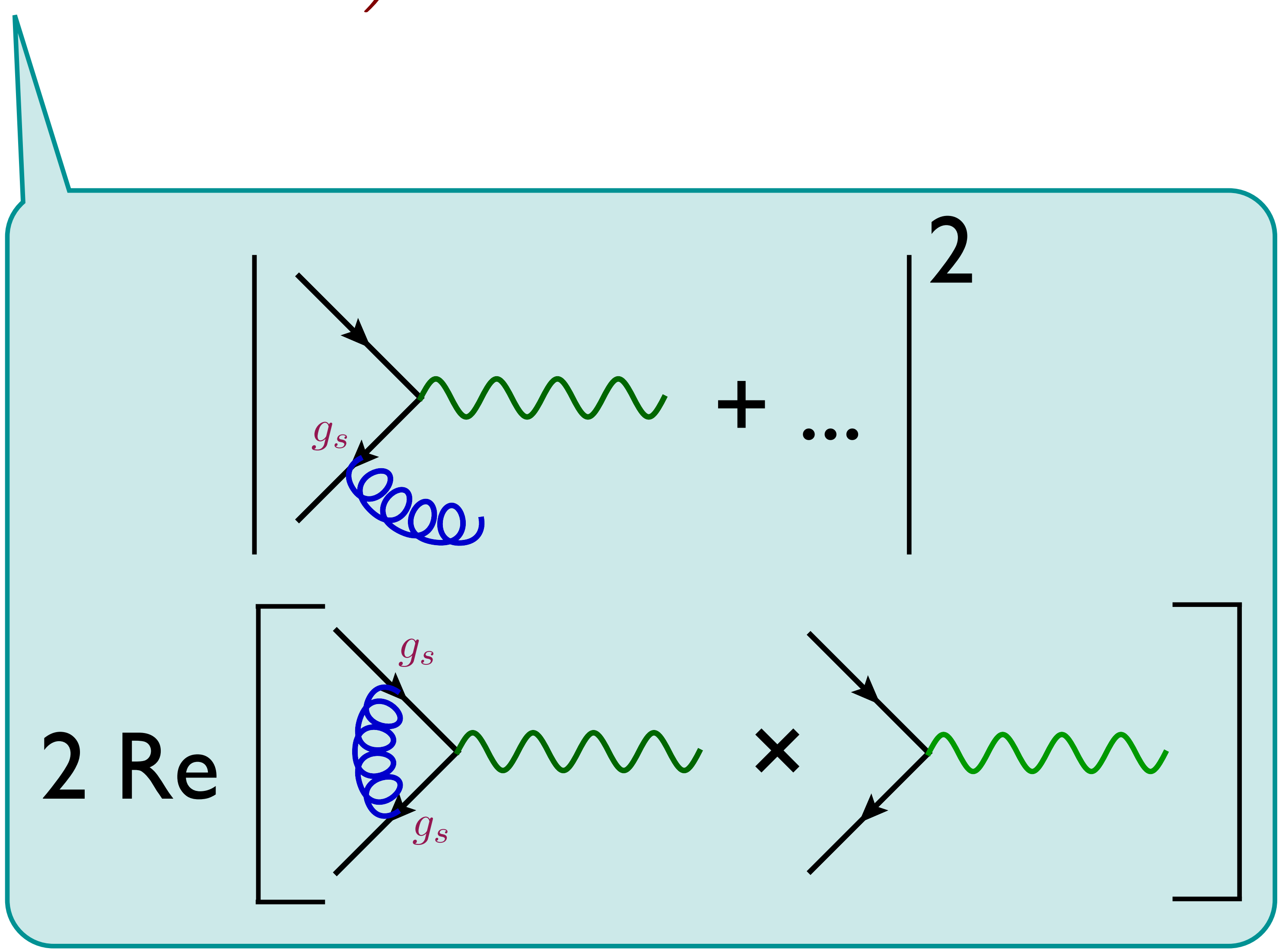
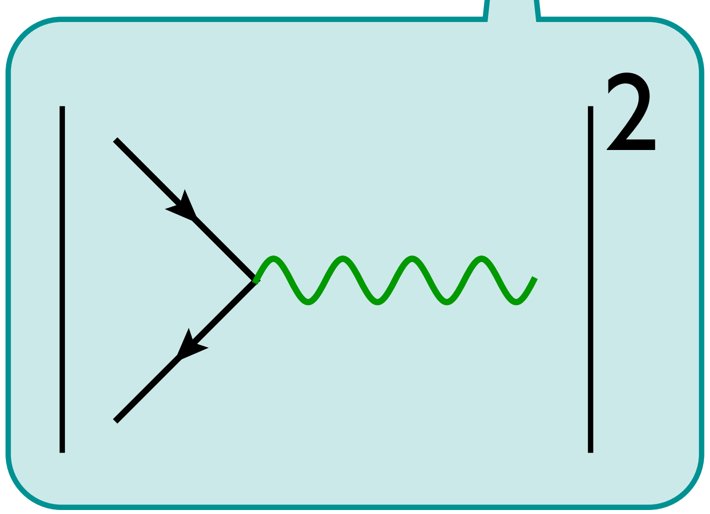


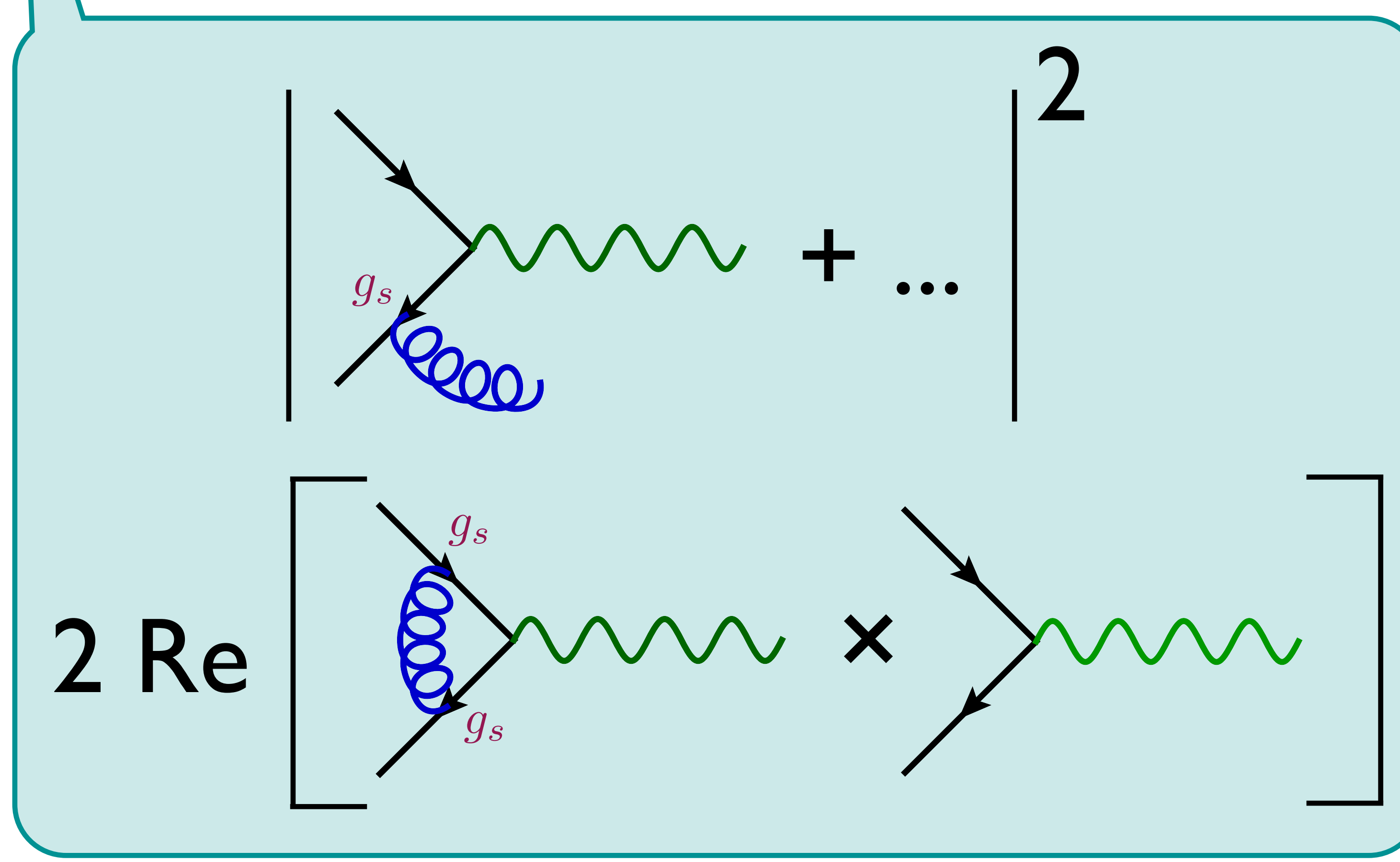


NLO predictions

- As an example, consider Drell-Yan Z/γ* production

$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_s}{2\pi} \sigma^{(1)} + \ldots \right)$$



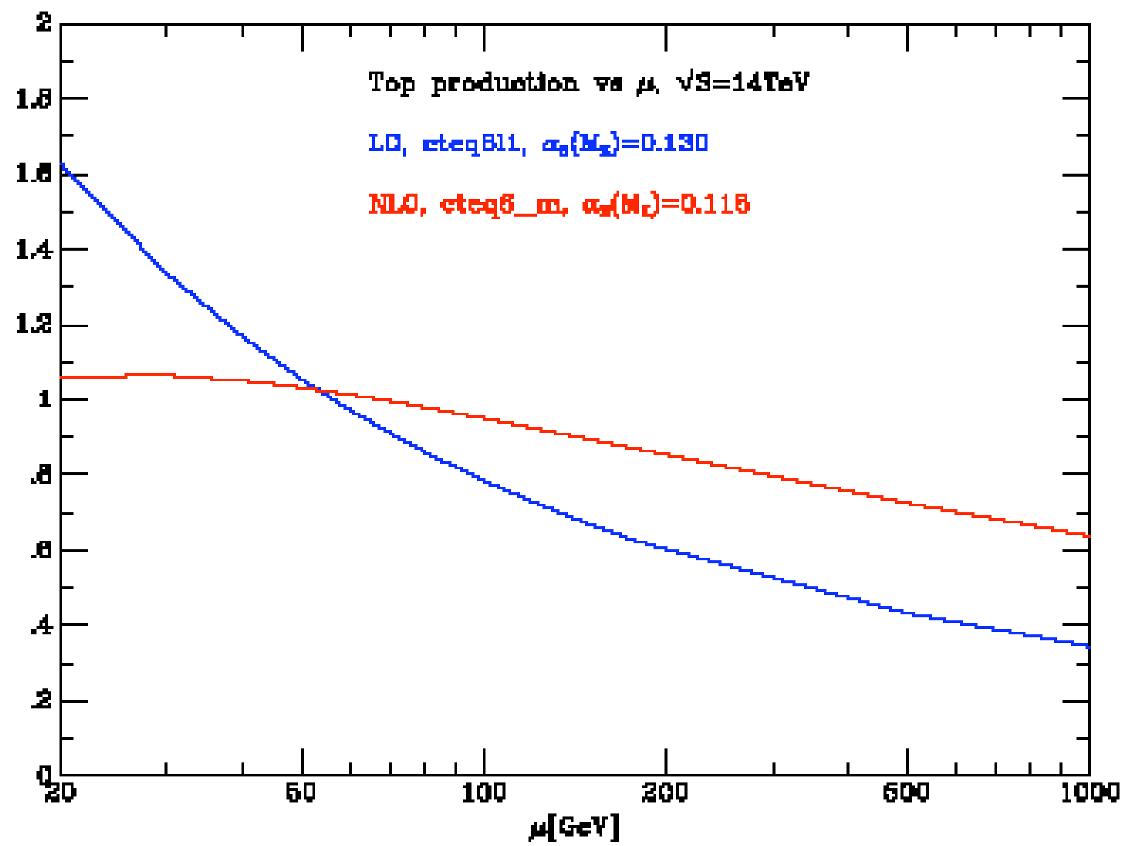


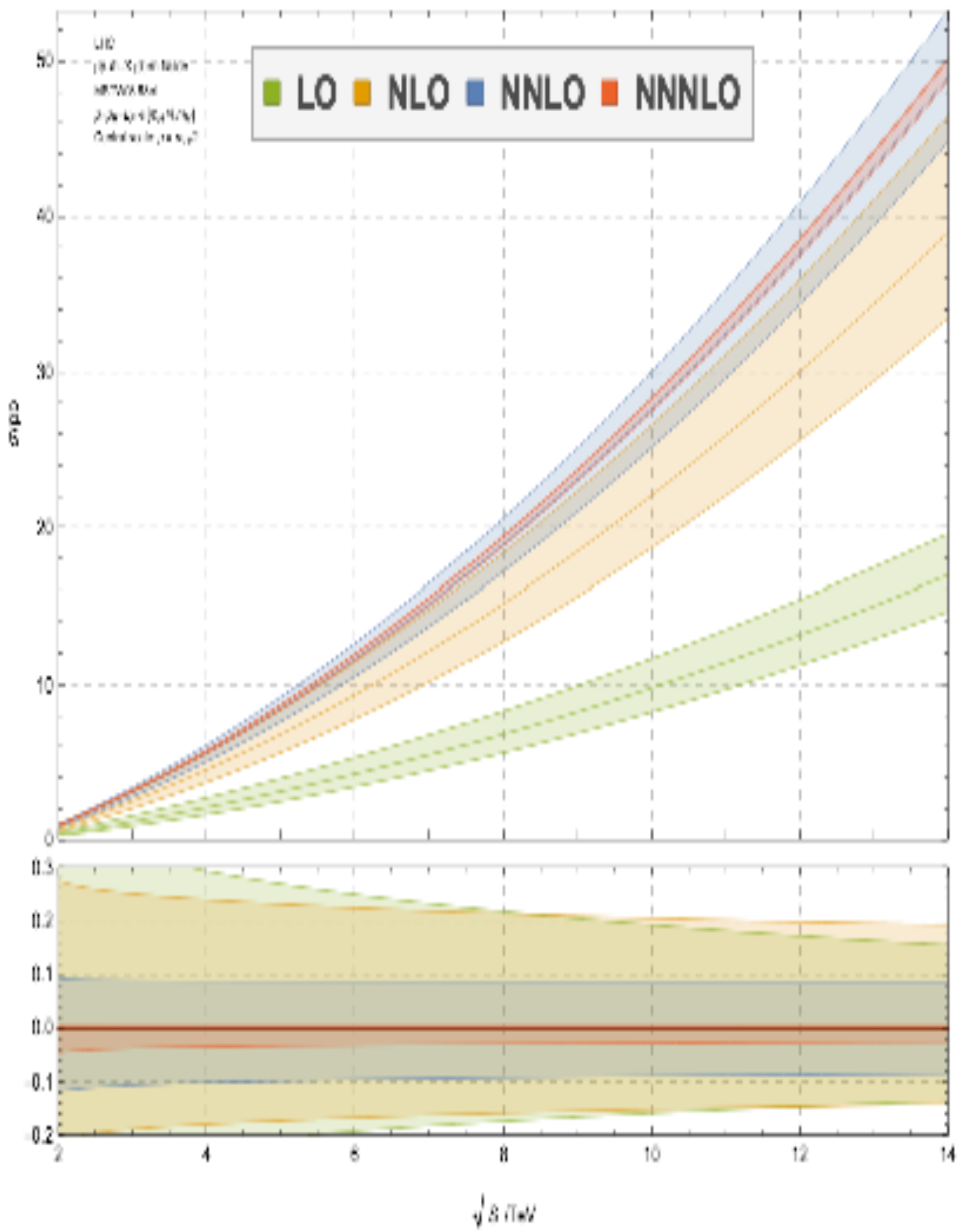
Not definite positive

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \;\; f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, d\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

$$\hat{\sigma} = \sigma^{\mathrm{Born}} \left(1 + \frac{\alpha_s}{2\pi} \sigma^{(1)} + \left(\frac{\alpha_s}{2\pi} \right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi} \right)^3 \sigma^{(3)} + \dots \right)$$

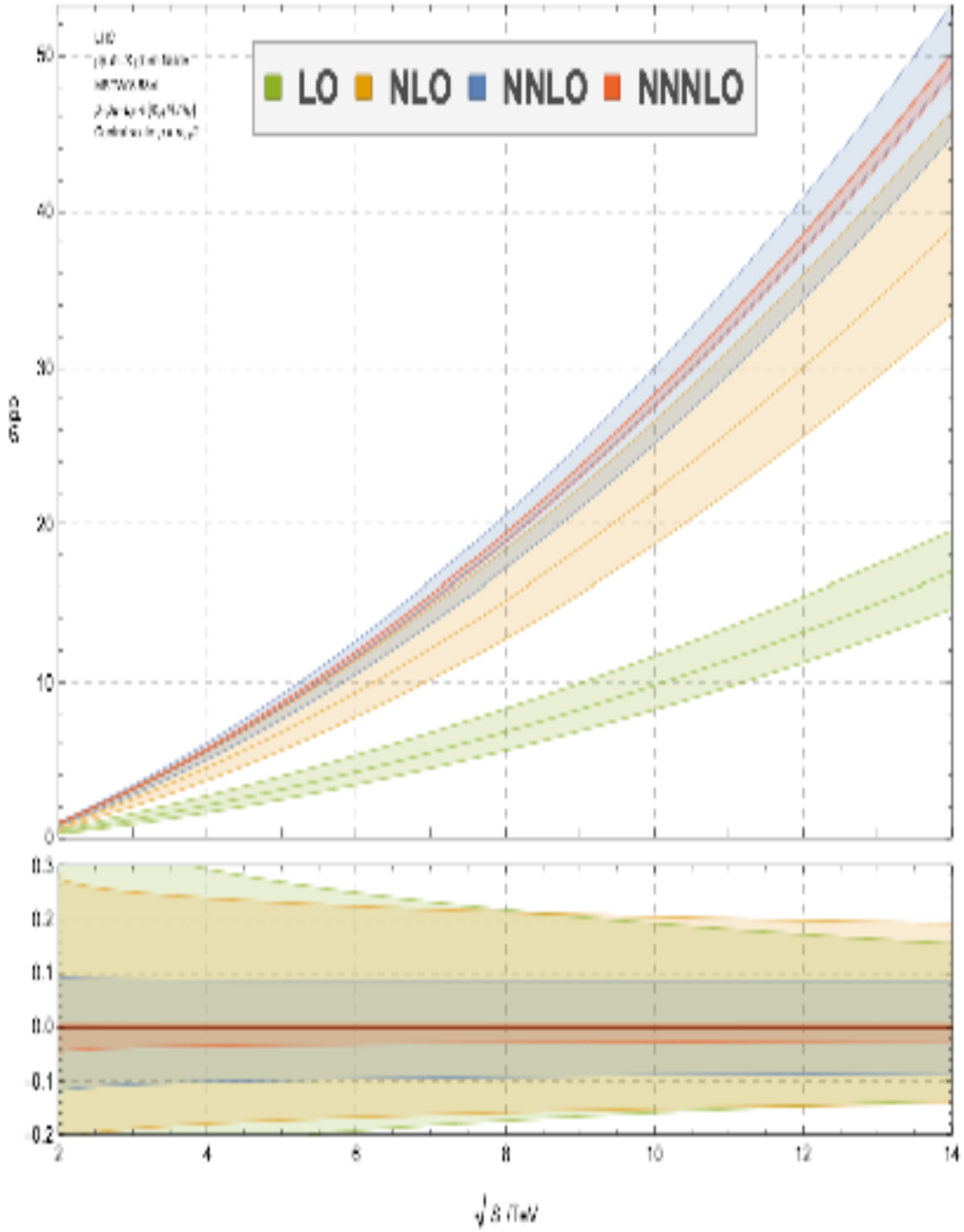
- Leading Order predictions can depend strongly on the renormalization and factorization scales
- Including higher order corrections reduces the dependence on these scales





- LO calculation is not reliable,
- but the perturbative series stabilises at NNLO/N3LO
- NLO estimation of the uncertainties (by scale variation) works reasonably well

Higgs at N3LO



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Let's focus on NLO

Infrared safe observables

- For an observable to be calculable in fixed-order perturbation theory, the observable should be infrared safe, i.e., it should be insensitive to the emission of soft or collinear partons.

- In particular, if p_i is a momentum occurring in the definition of an observable, it most be invariant under the branching

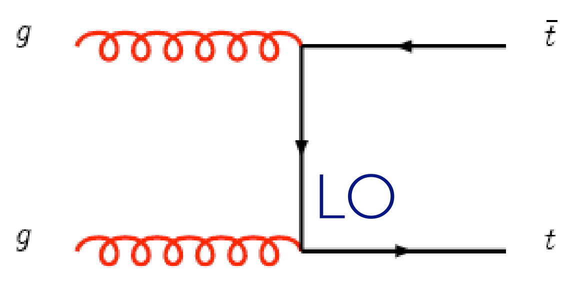
$$p_i \longrightarrow p_j + p_k,$$

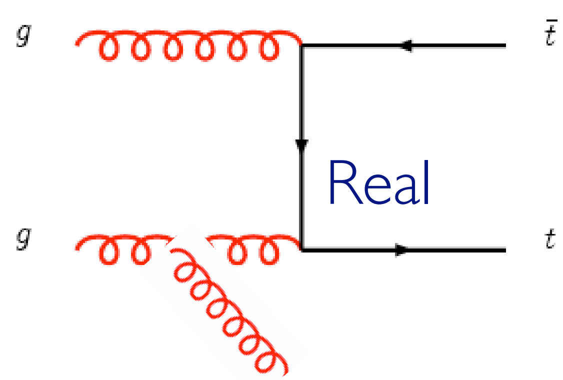
whenever p_j and p_k are collinear or one of them is soft.

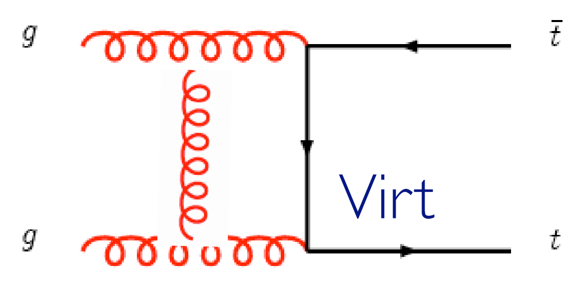
- Examples
  - "The number of gluons" produced in a collision is not an infrared safe observable
  - "The number of hard jets defined using the k_T algorithm with a transverse momentum above 40 GeV," produced in a collision is an infrared safe observable

NLO...?

- Are all (IR-safe) observables that we can compute using a NLO code correctly described at NLO? Suppose we have a NLO code for pp $\longrightarrow$ ttbar







- Total cross section
- Transverse momentum of the top quark
- Transverse momentum of the top-antitop pair
- Transverse momentum of the jet
- Top-antitop invariant mass
- Azimuthal distance between the top and anti-top

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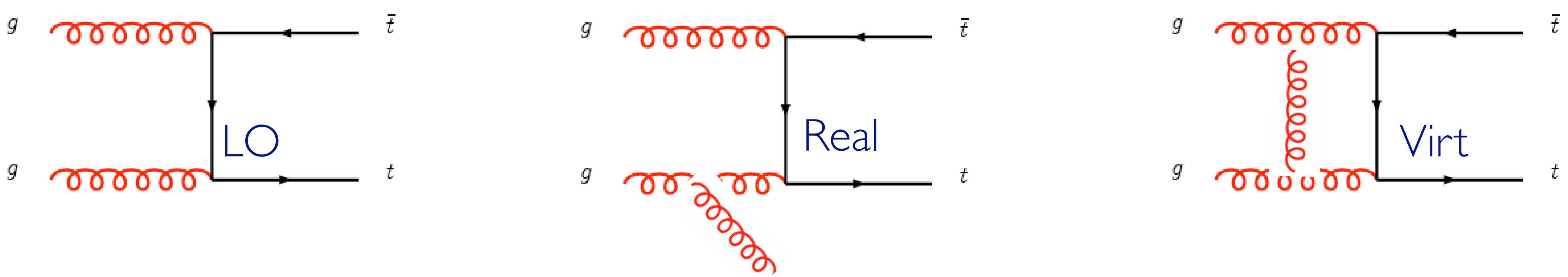

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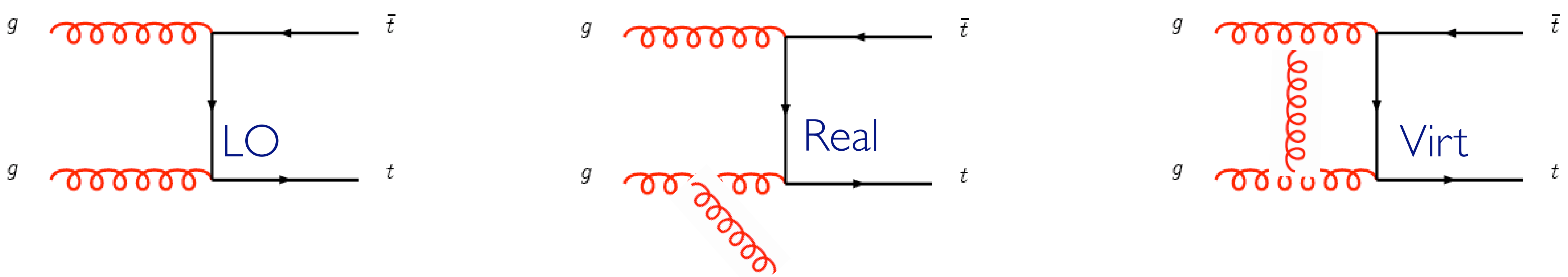

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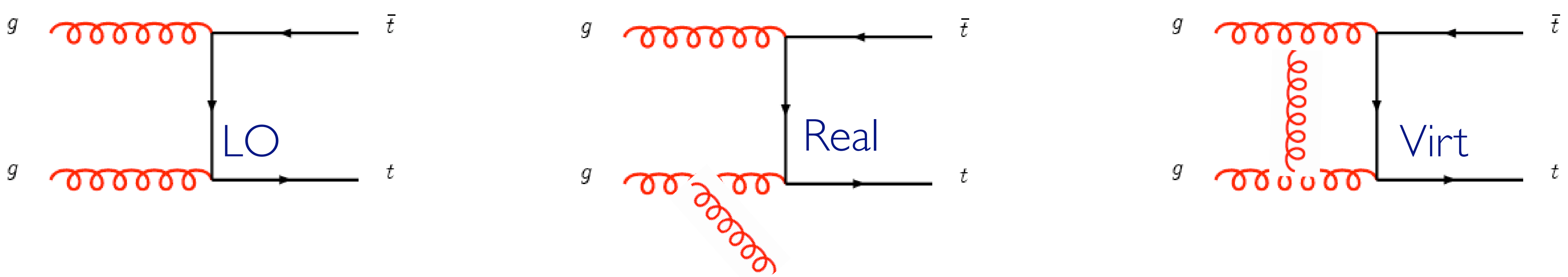

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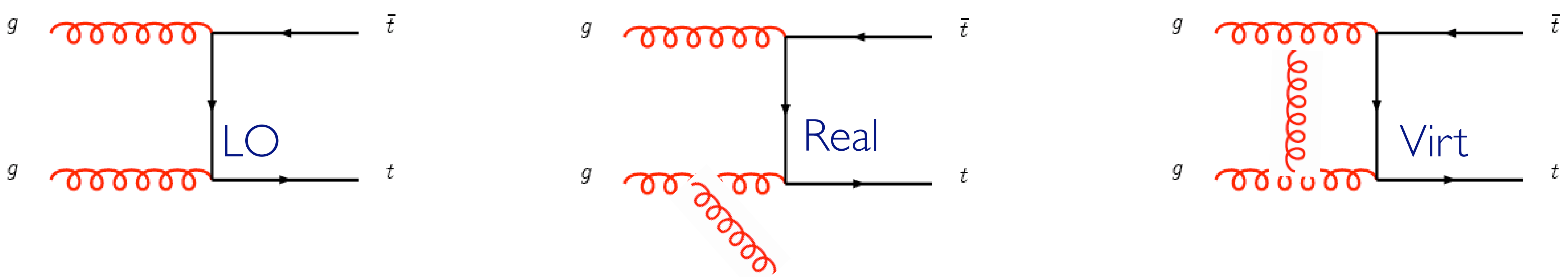

NLO?

- Total cross section ✔
- Transverse momentum of the top quark ✔
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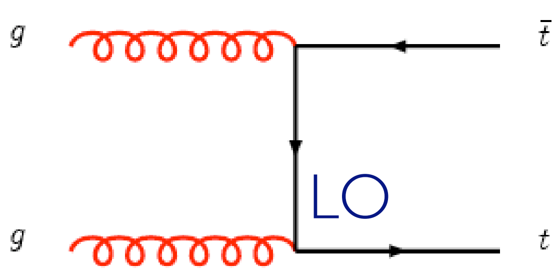

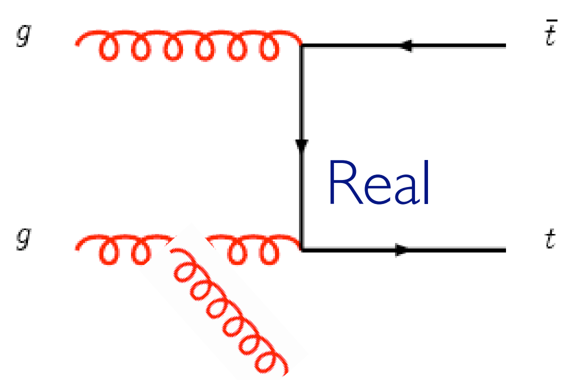

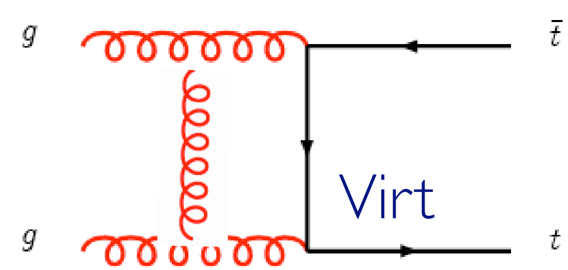

	NLO?
• Total cross section	✔
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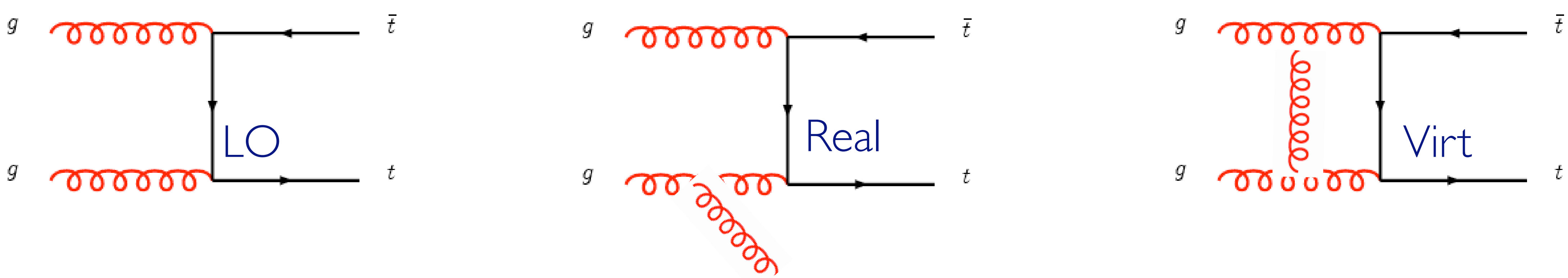

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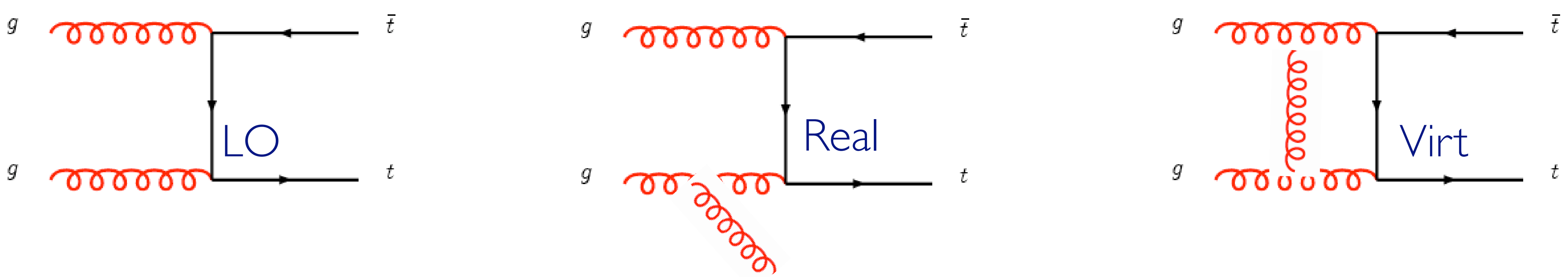

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Going NNLO...?

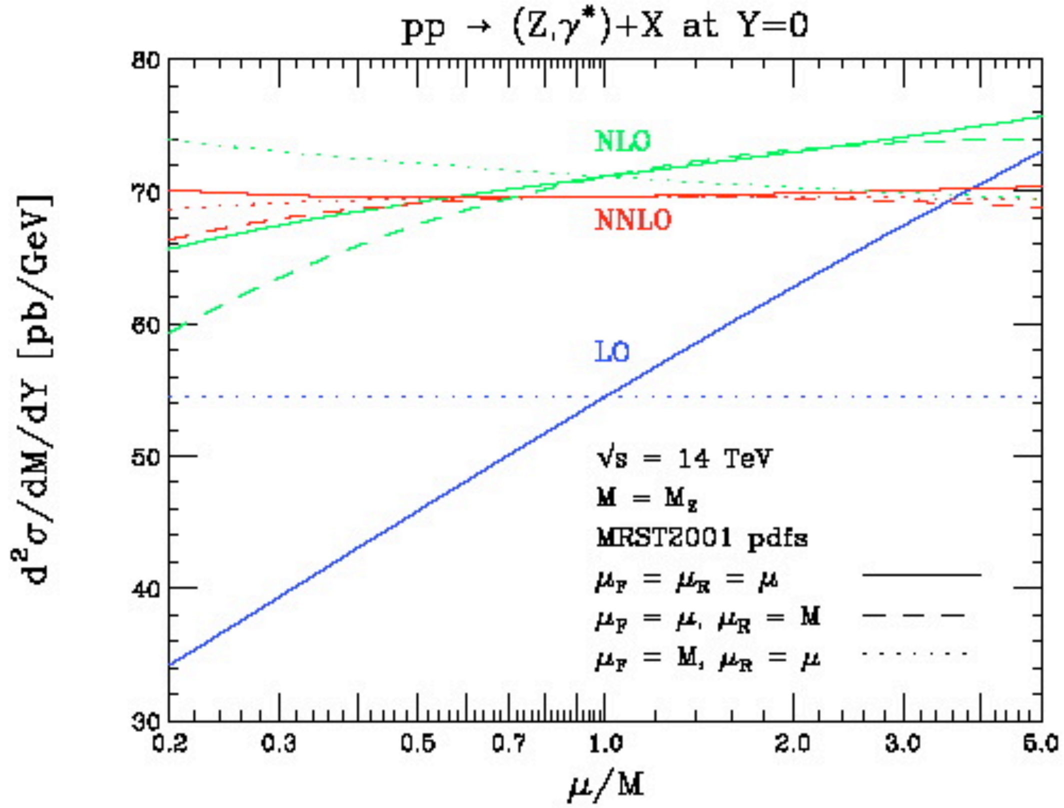
- NNLO is the current state-of-the-art. There are only a few results available: Higgs (N3LO available), Drell-Yan, ttbar

- Why do we need it?

  - control of the uncertainties in a calculation

  - It is "mandatory" if NLO corrections are very large to check the behavior of the perturbative series

  - It is needed for Standard Candles and very precise tests of perturbation theory, exploiting all the available information, e.g. for determining NNLO PDF sets



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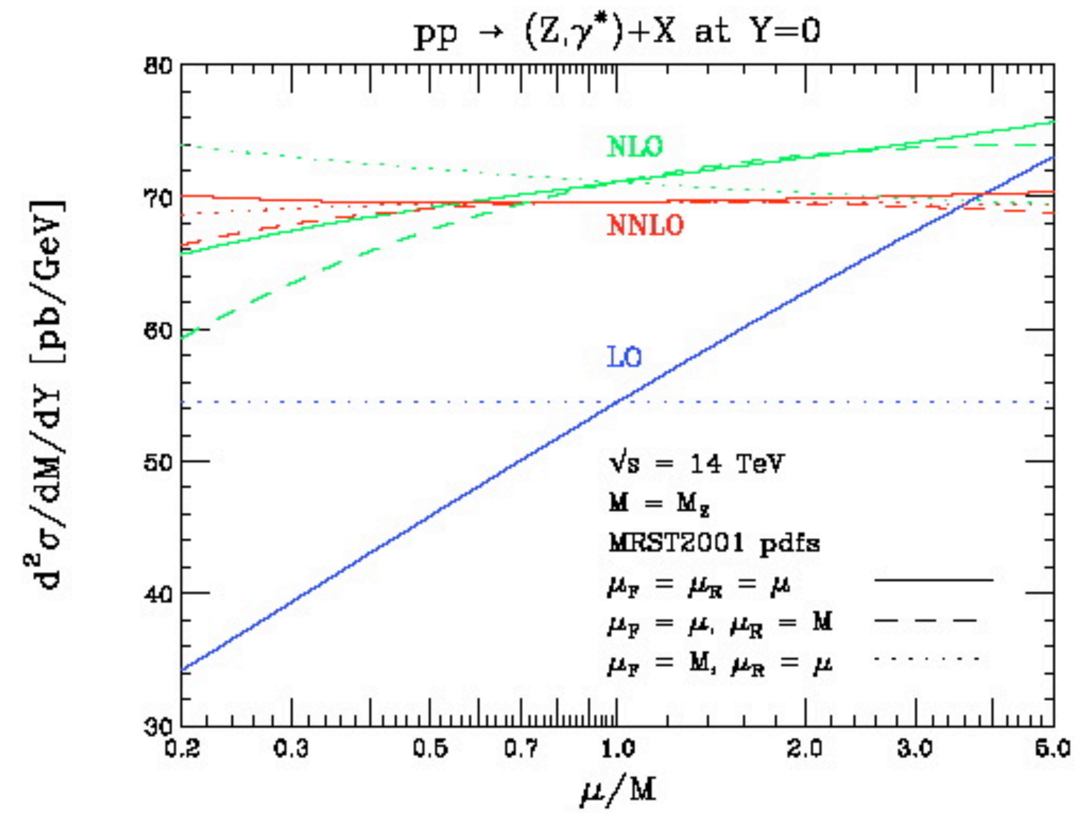
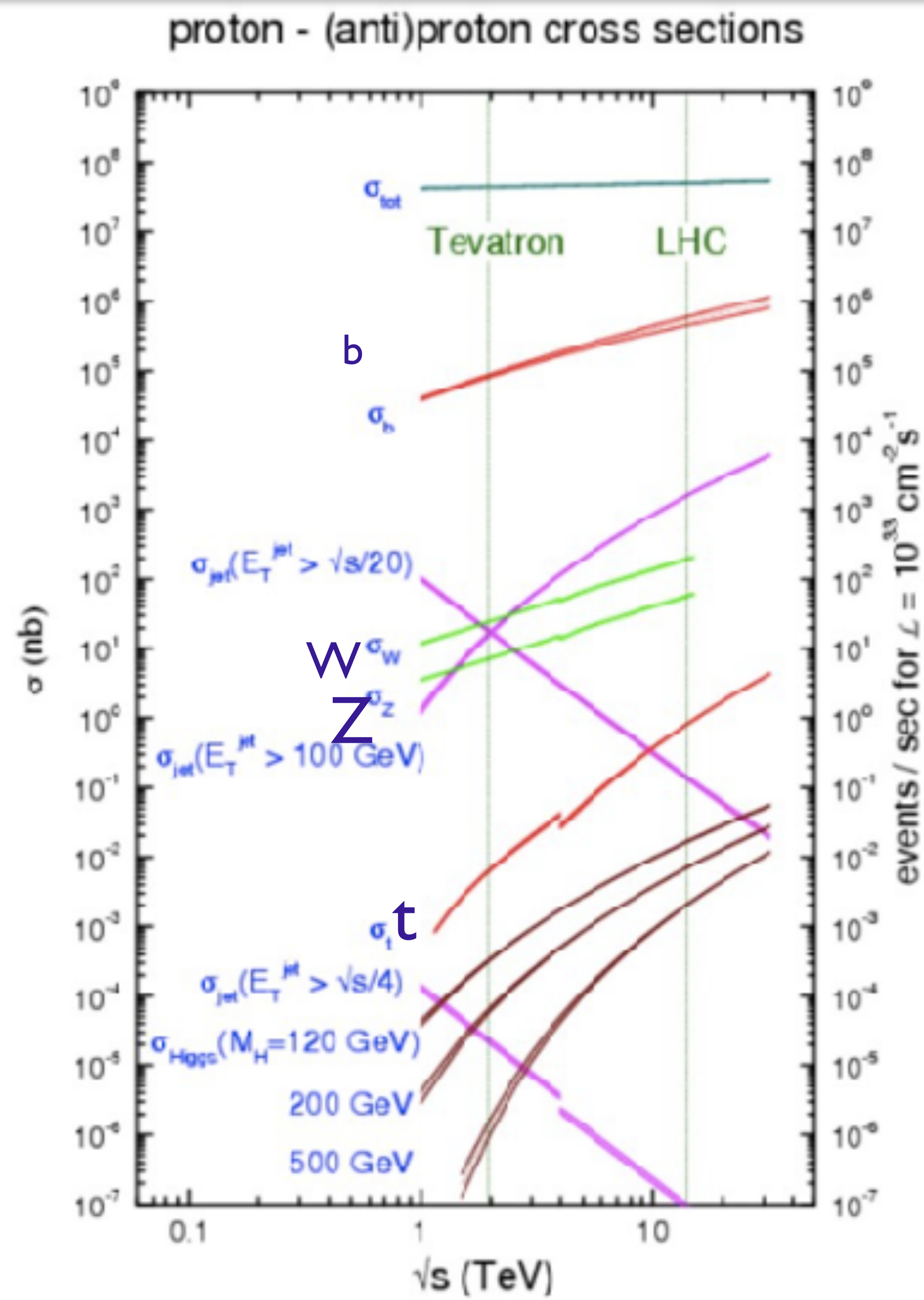


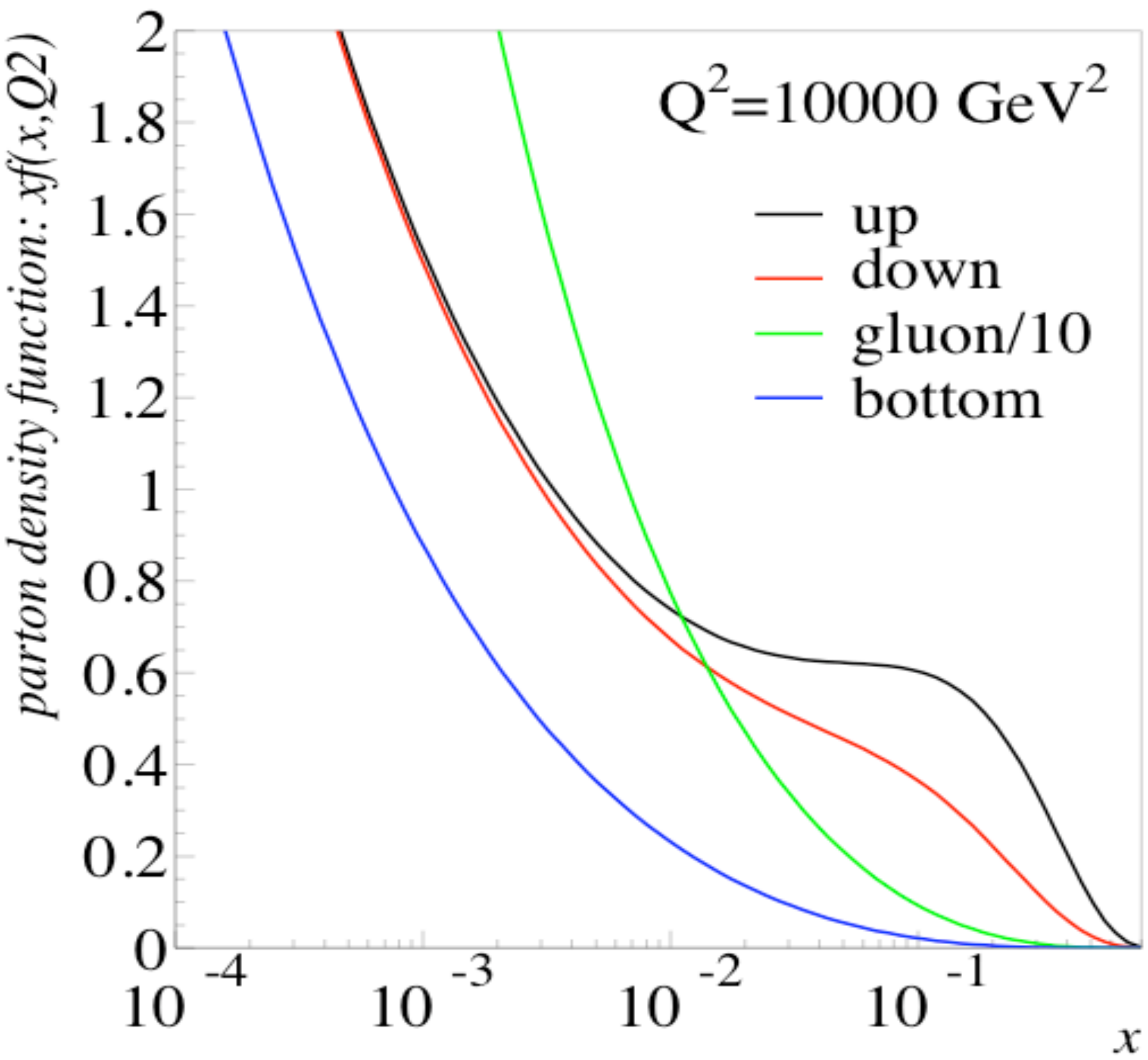
Let's focus on LO

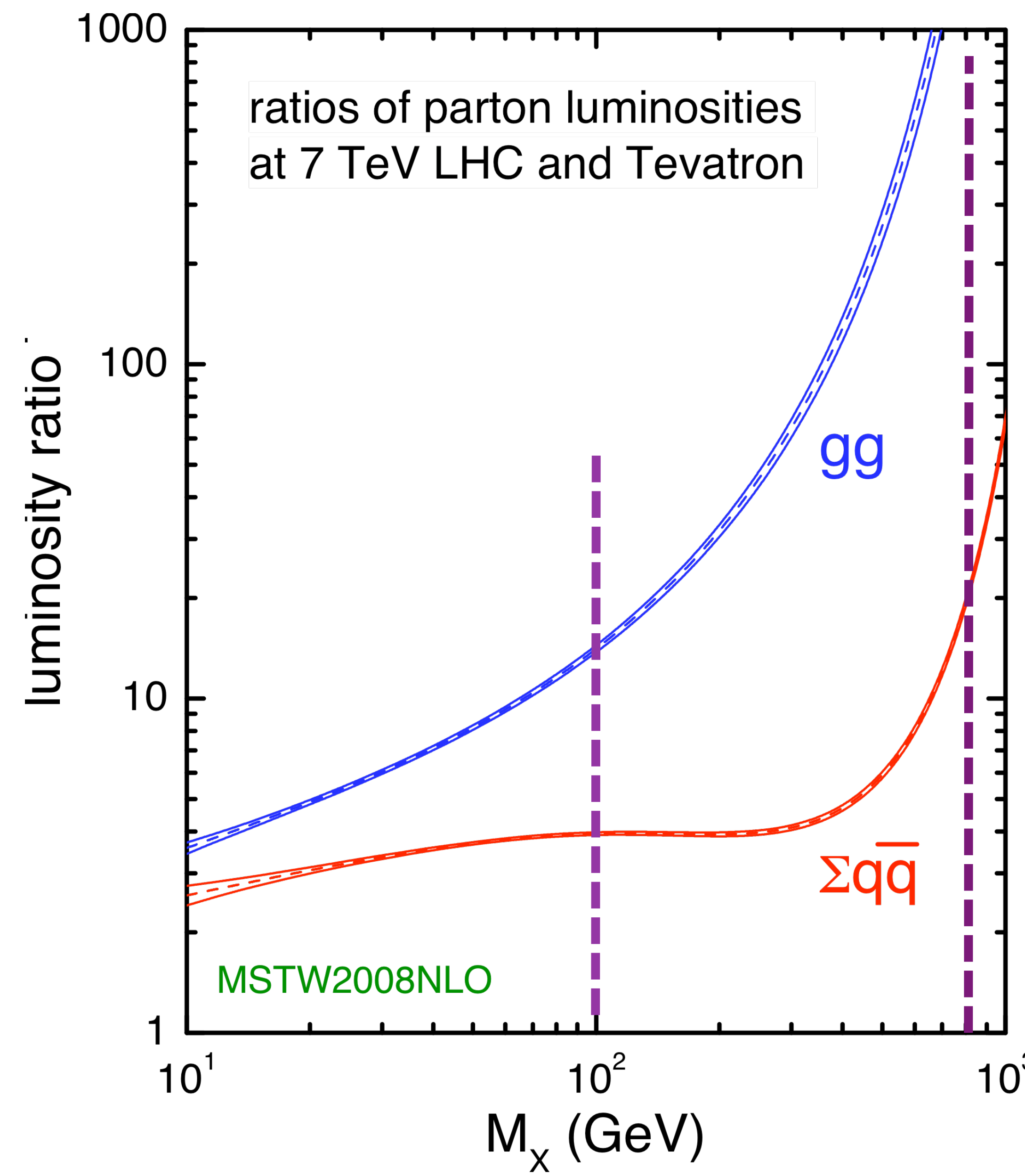
Hadron Colliders

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$



Parton densities

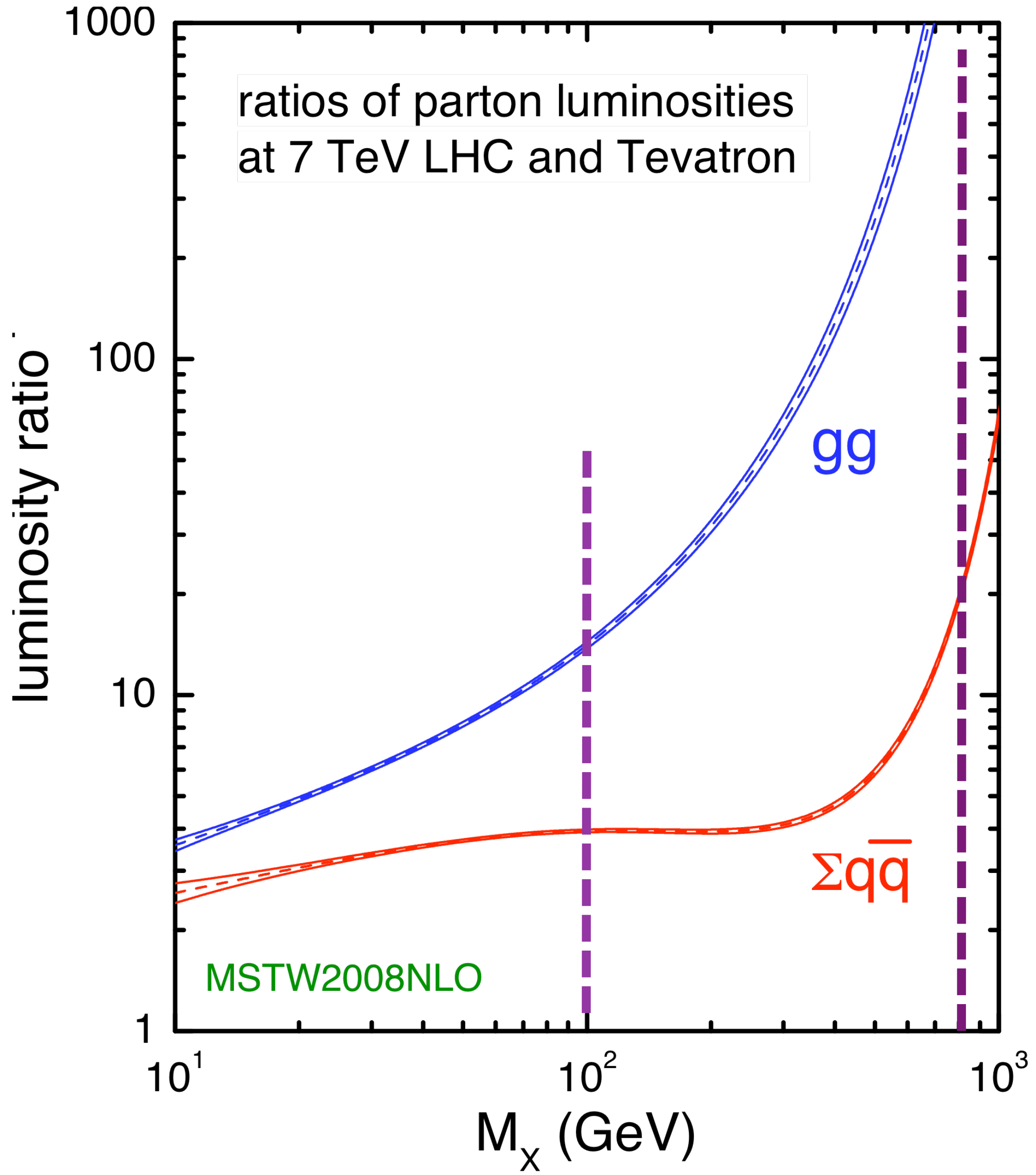


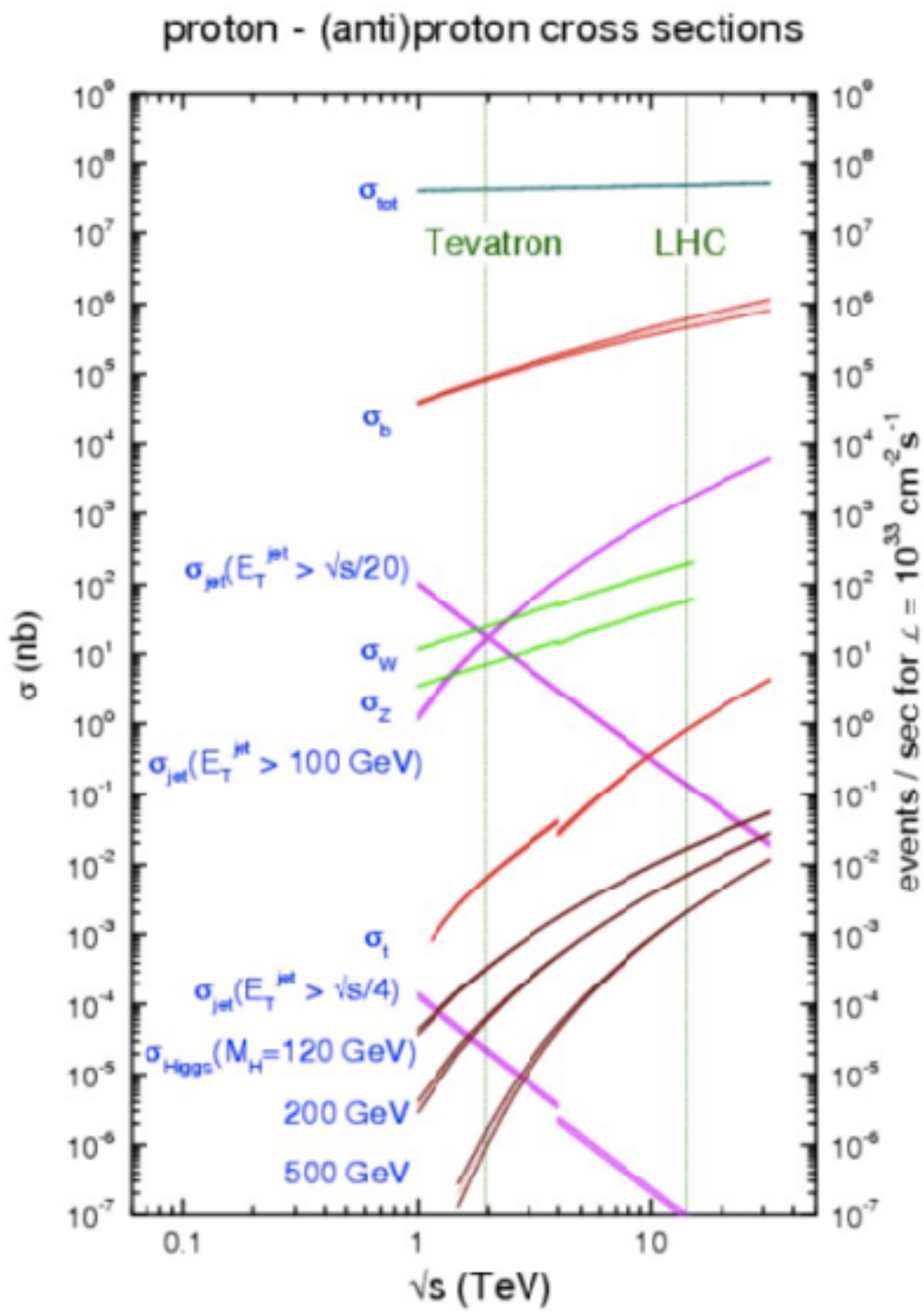


At small x (small $\hat{s}$), gluon domination.
At large x valence quarks

LHC formidable at large mass –
For low mass, Tevatron backgrounds smaller

Back to the processes





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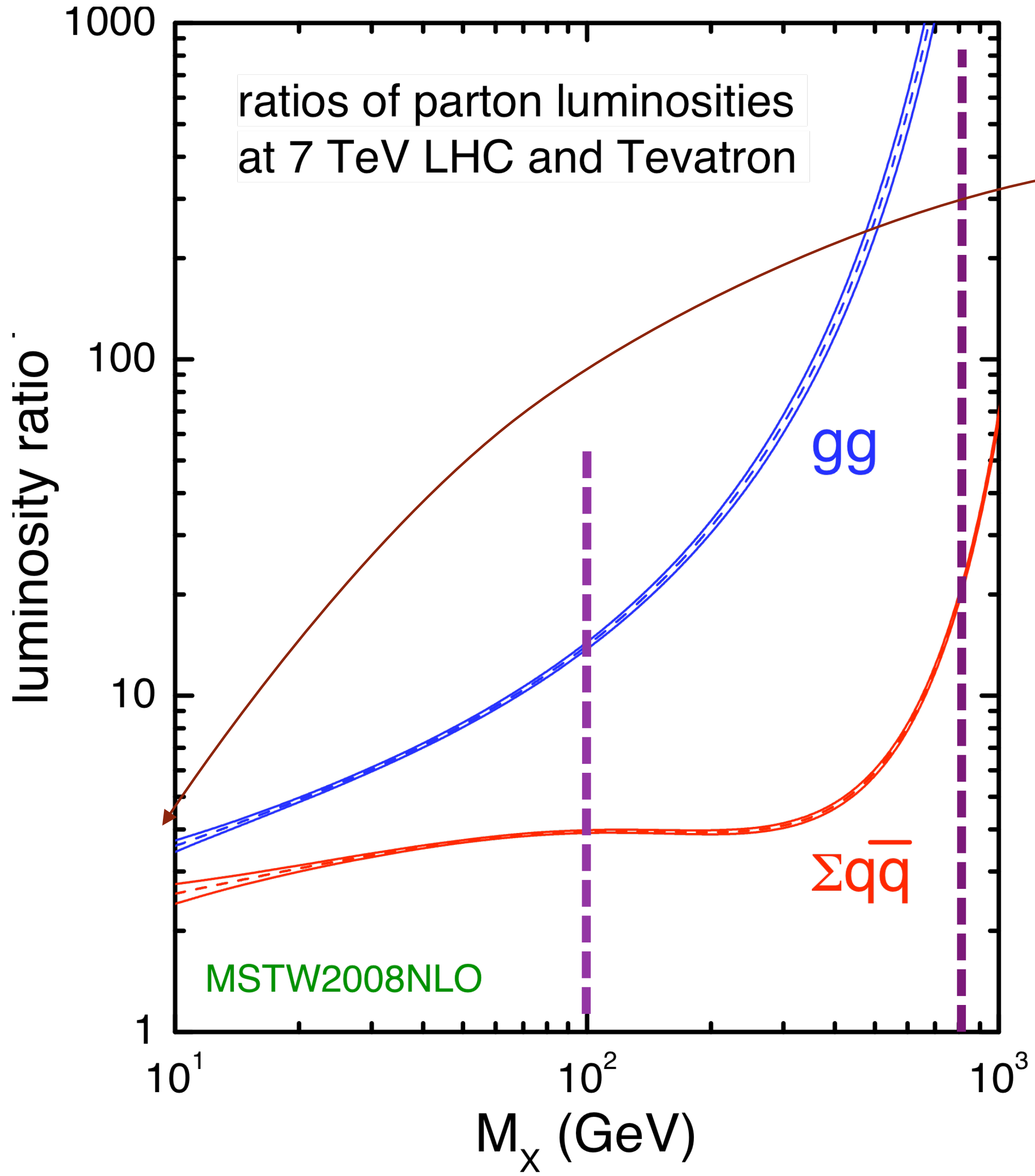

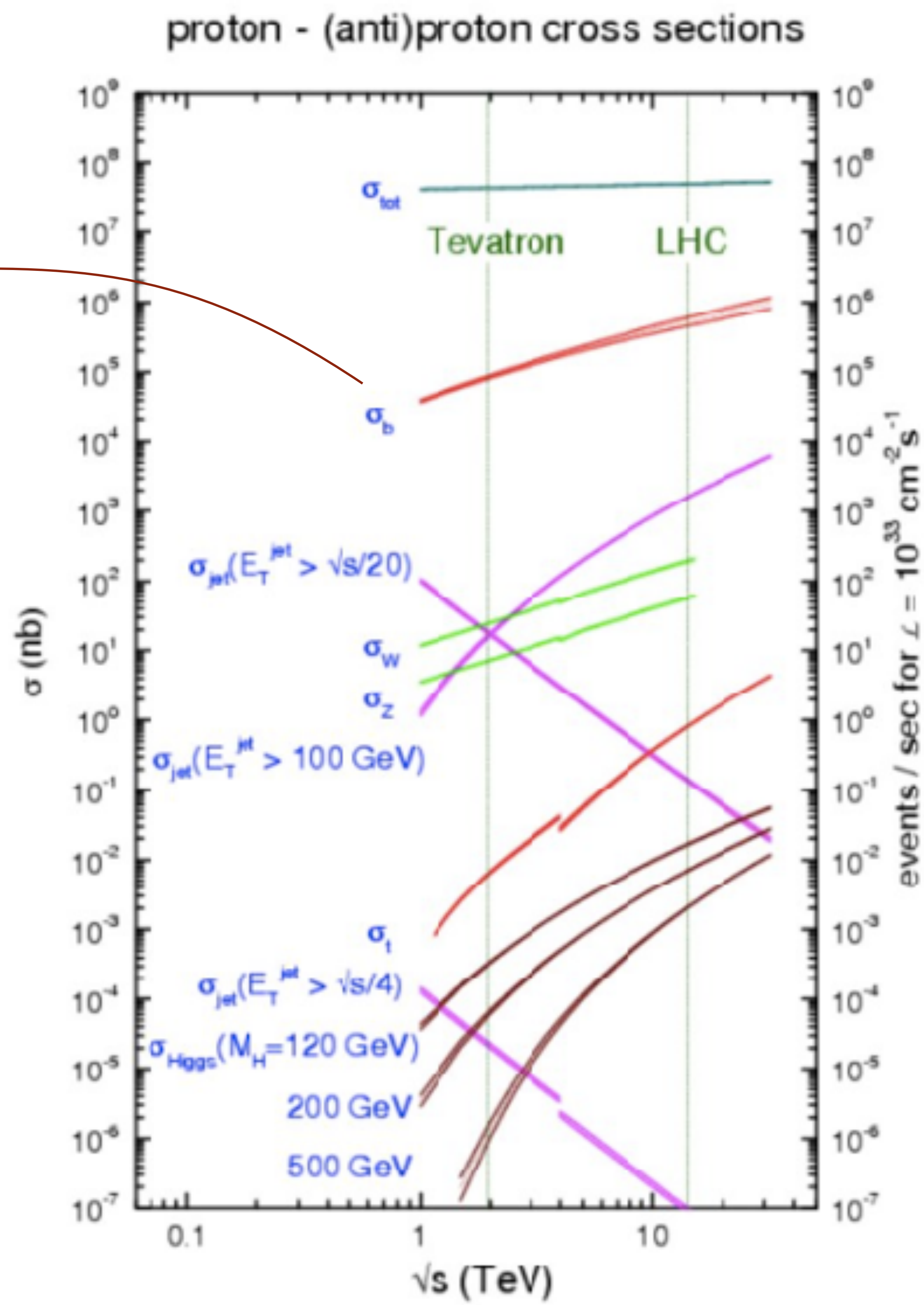

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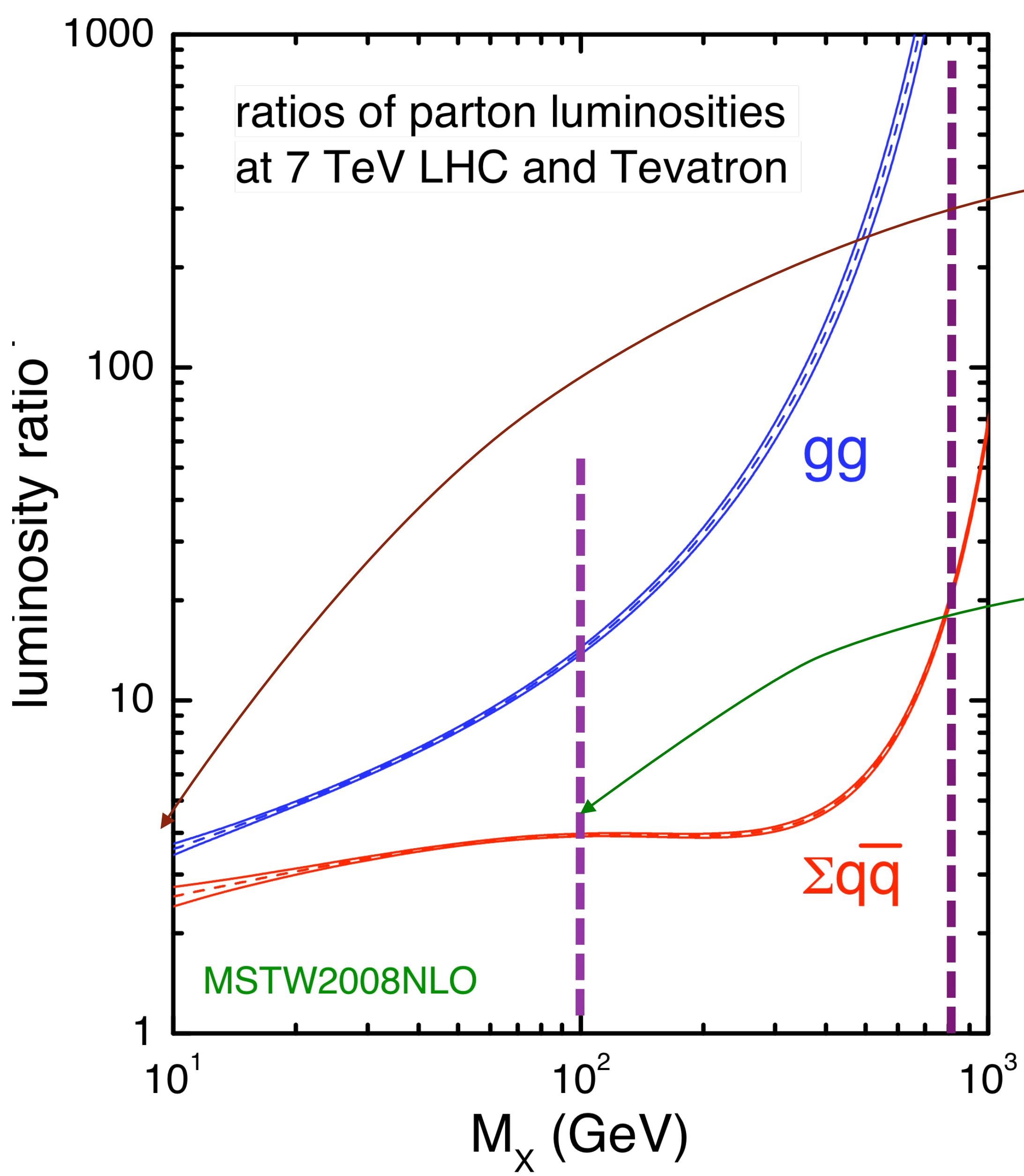

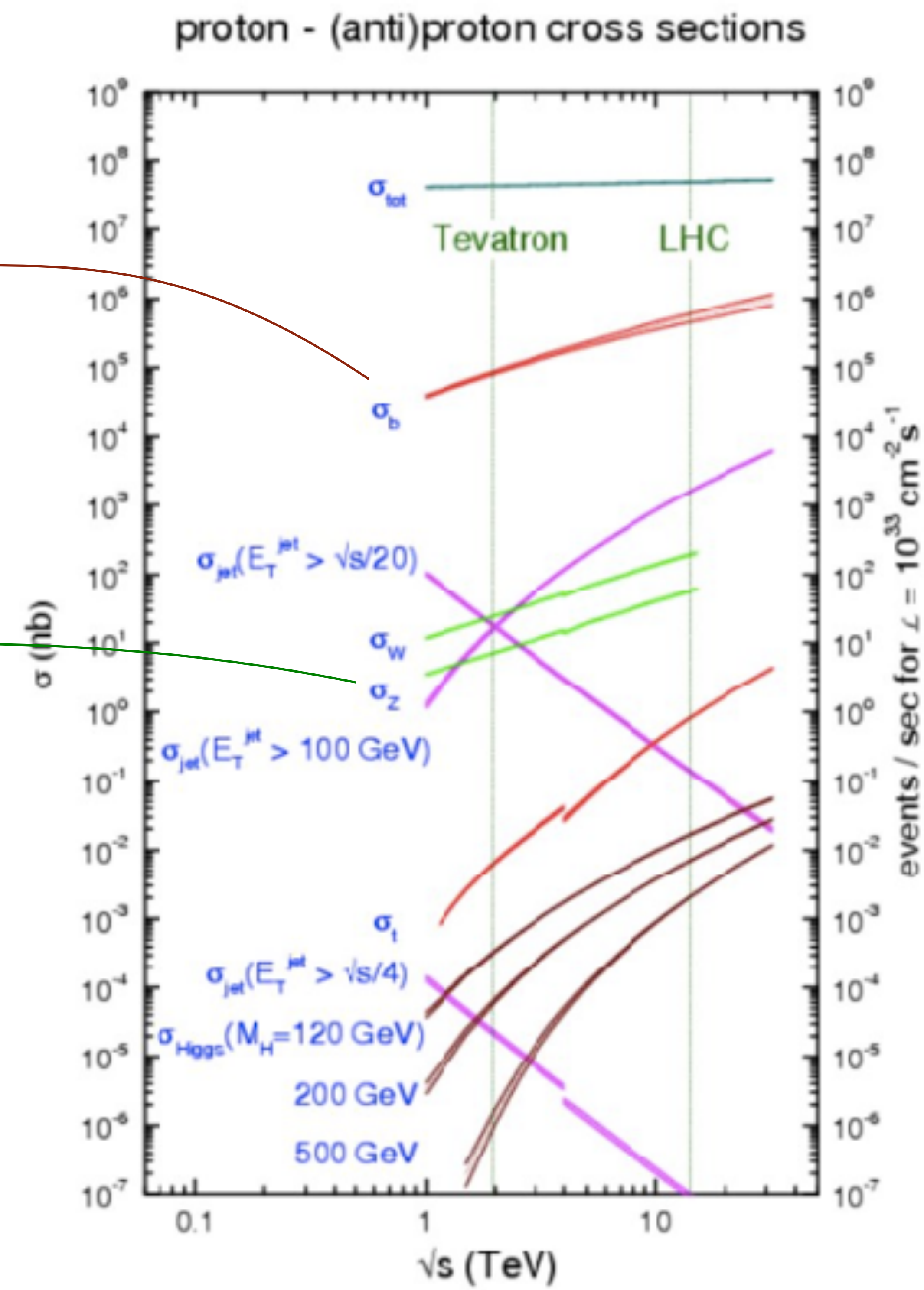

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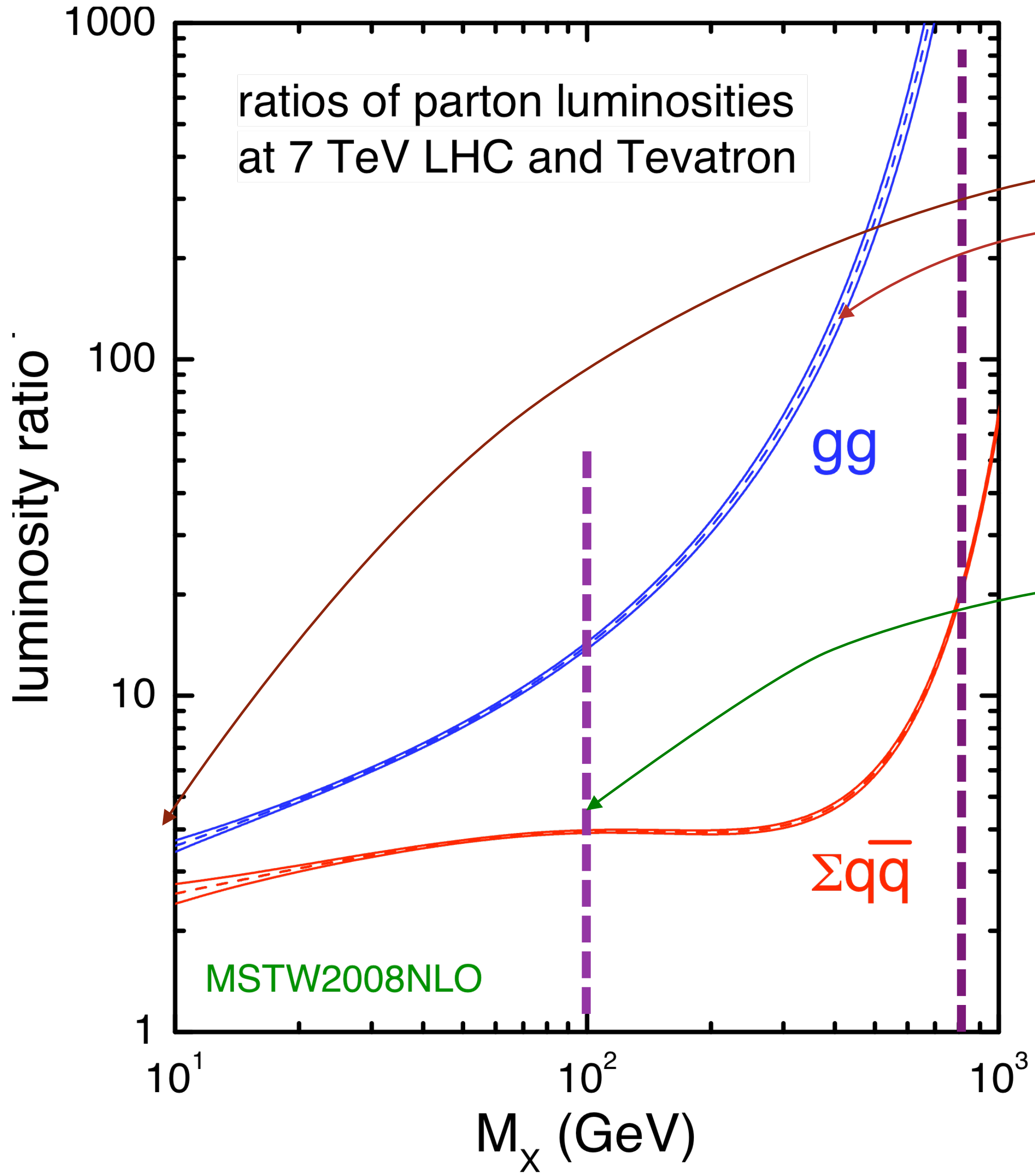
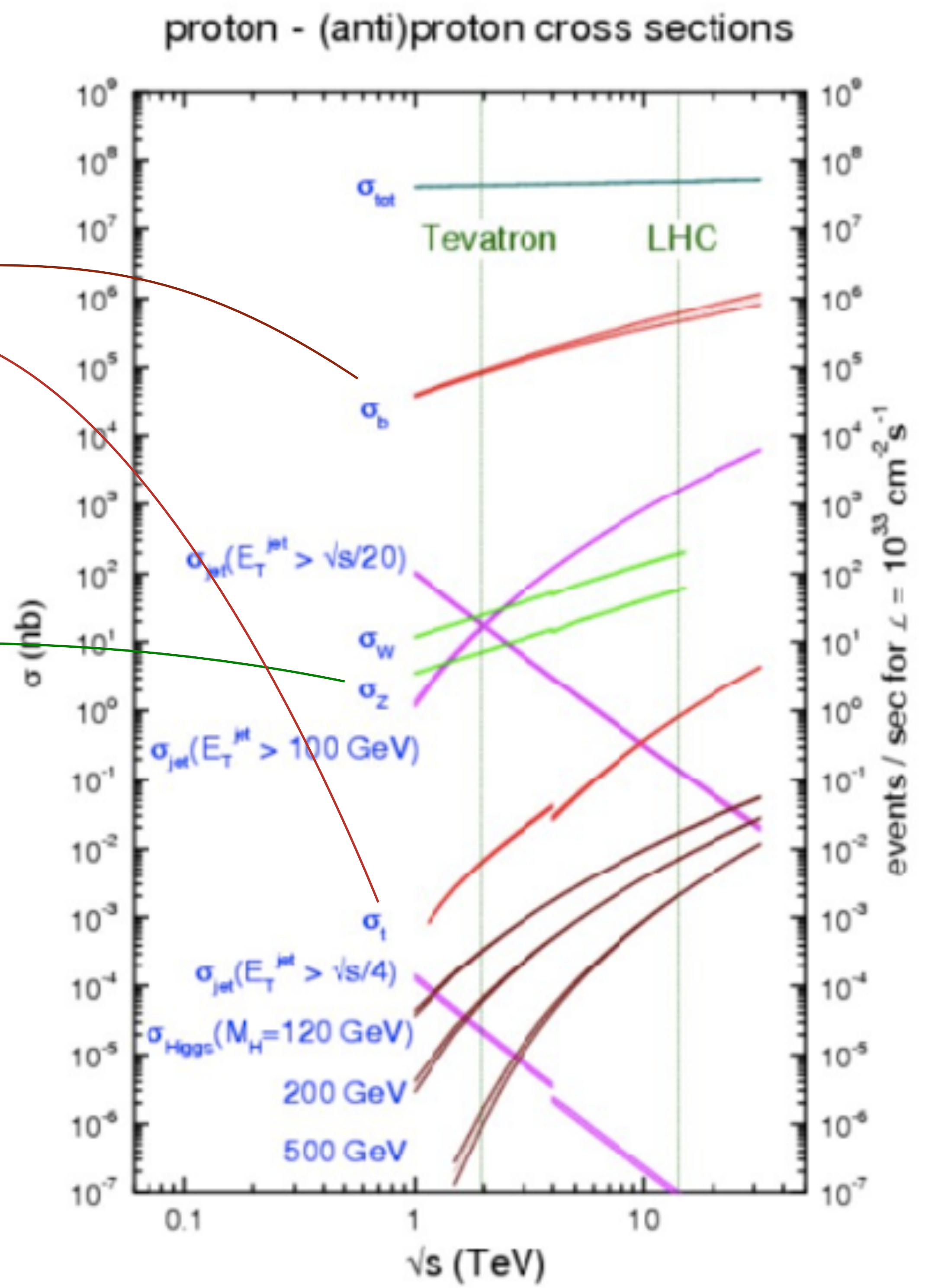




To Remember

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

Parton density functions

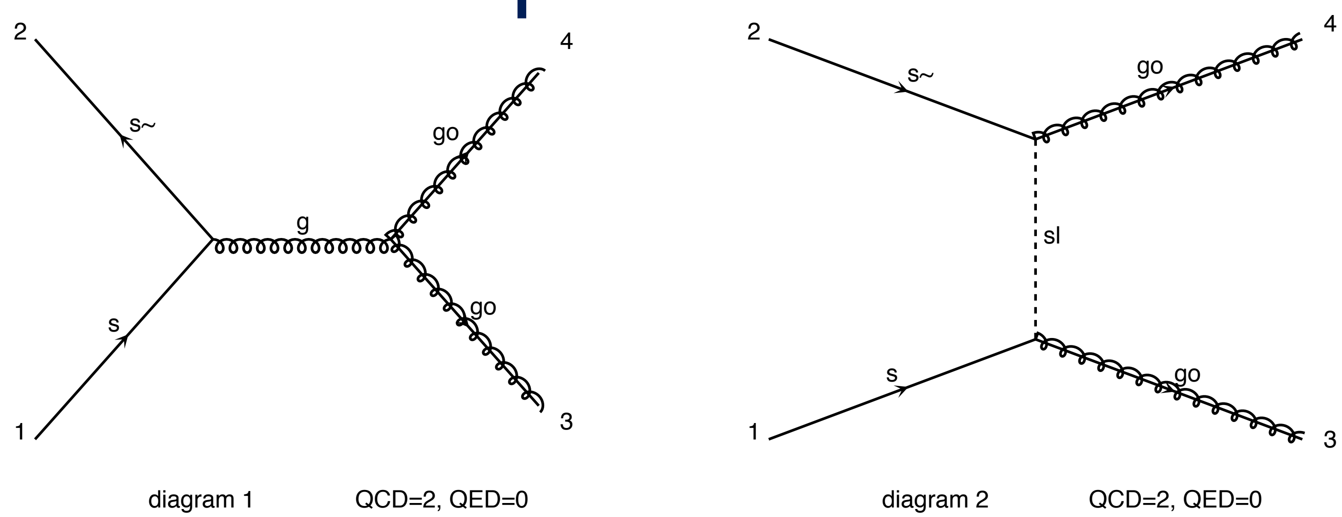
Parton-level cross section

- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- NLO/NNLO: Reduce scale uncertainty linked to your division of your multi-scale problem

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$$|\mathcal{M}|^2$$ ➡Need Feynman Rules!

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$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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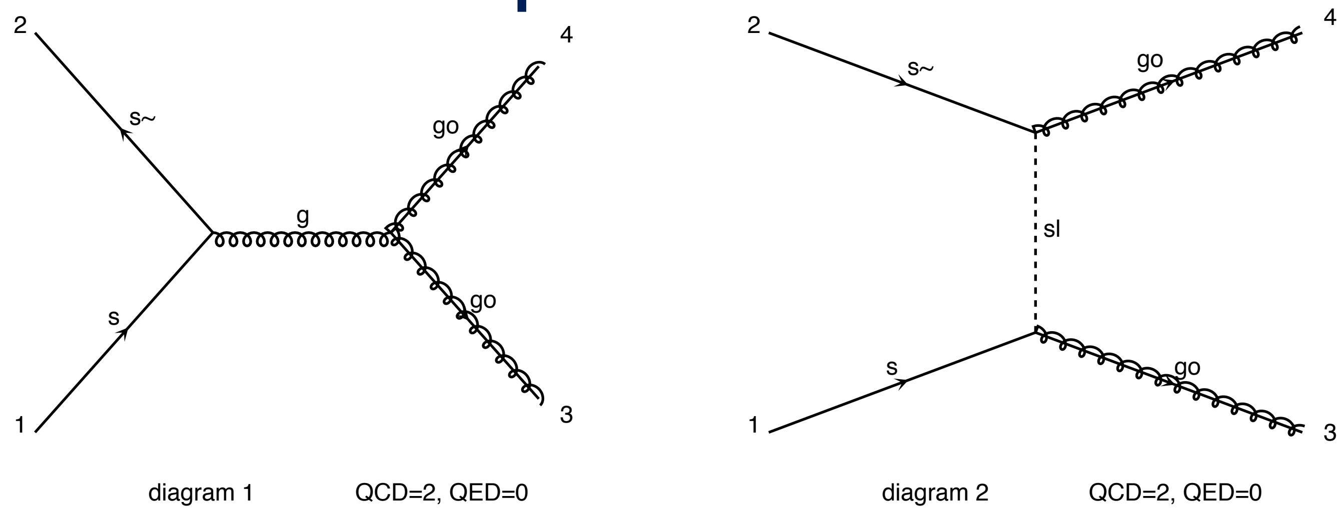

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Easy enough

Hard

Very Hard (in general)

Matrix-Element

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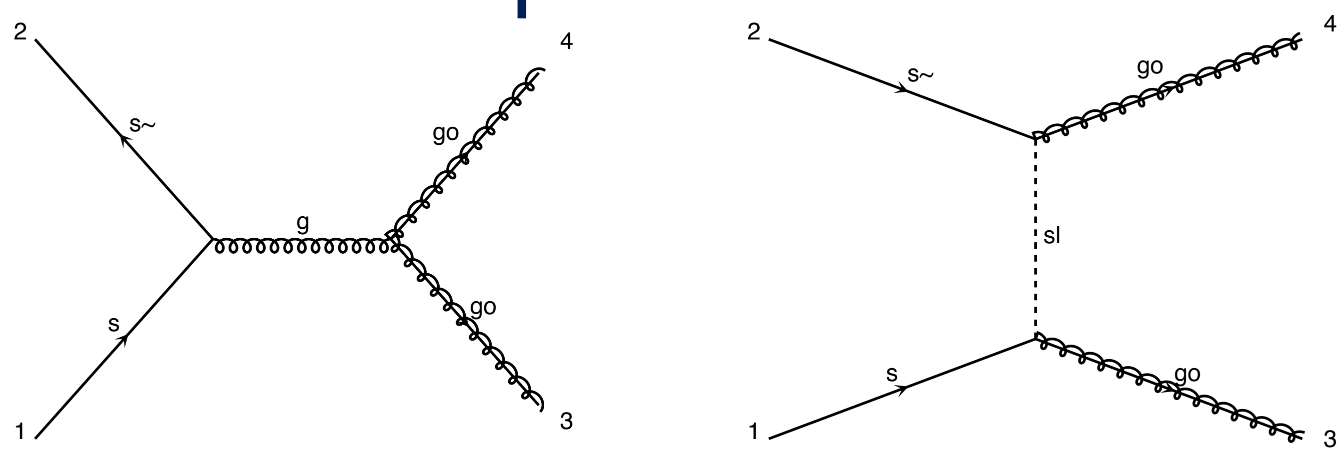

diagram 1 QCD=2, QED=0

diagram 2 QCD=2, QED=0

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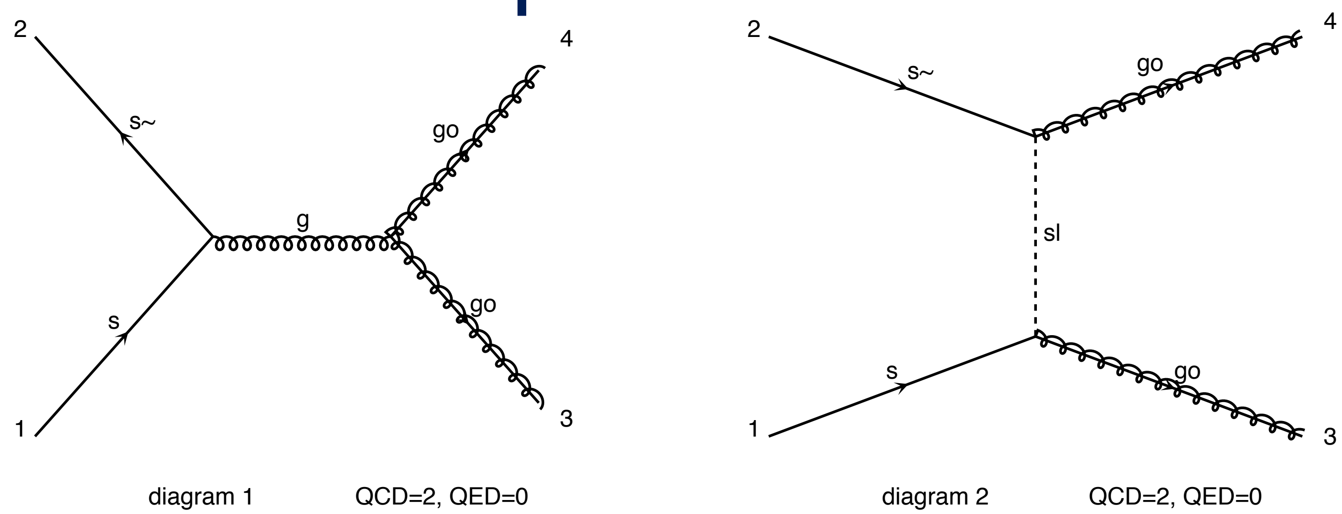

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Now

Monte Carlo Integration and Generation

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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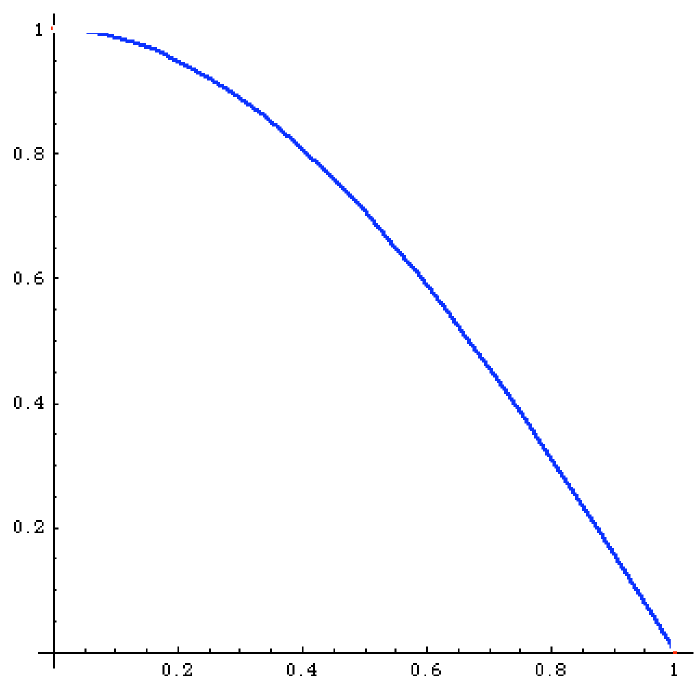
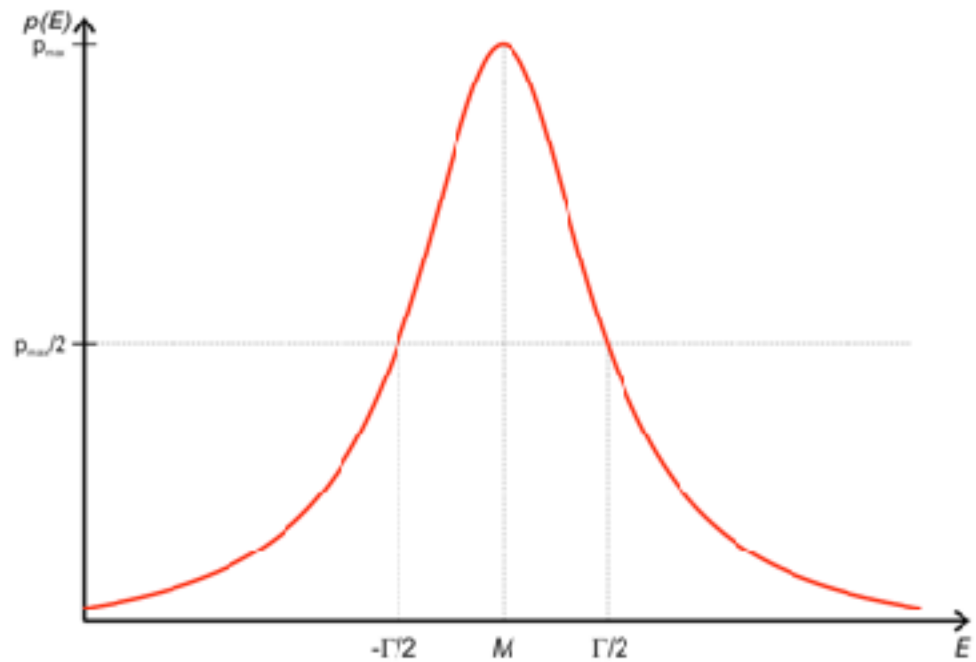
General and flexible method is needed

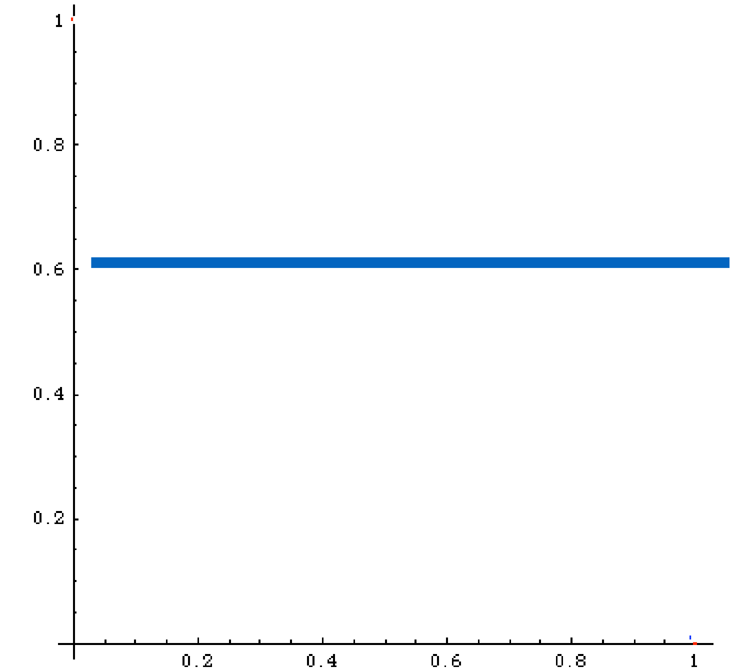
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$







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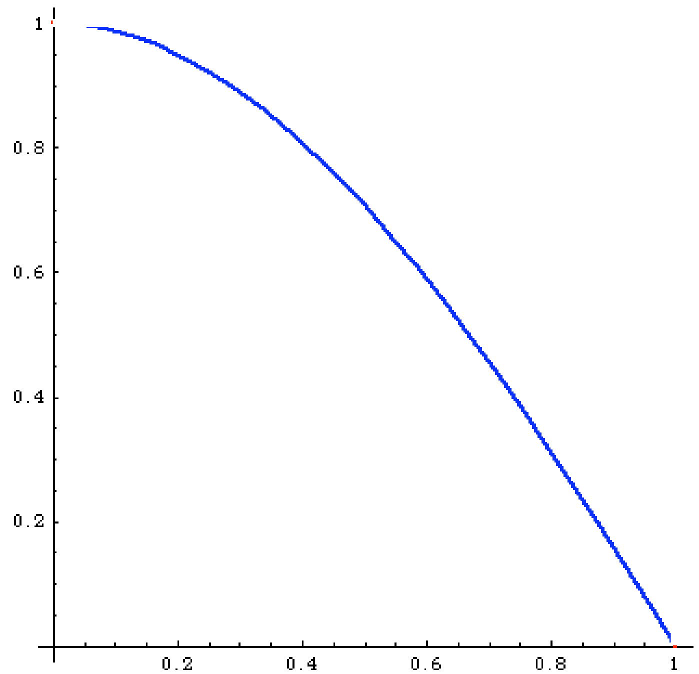

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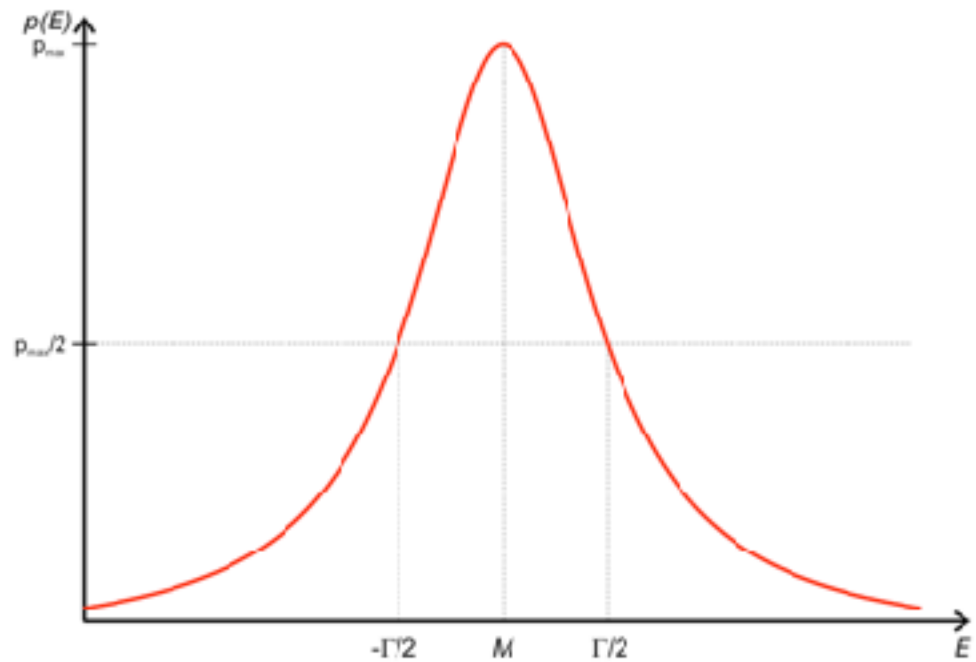

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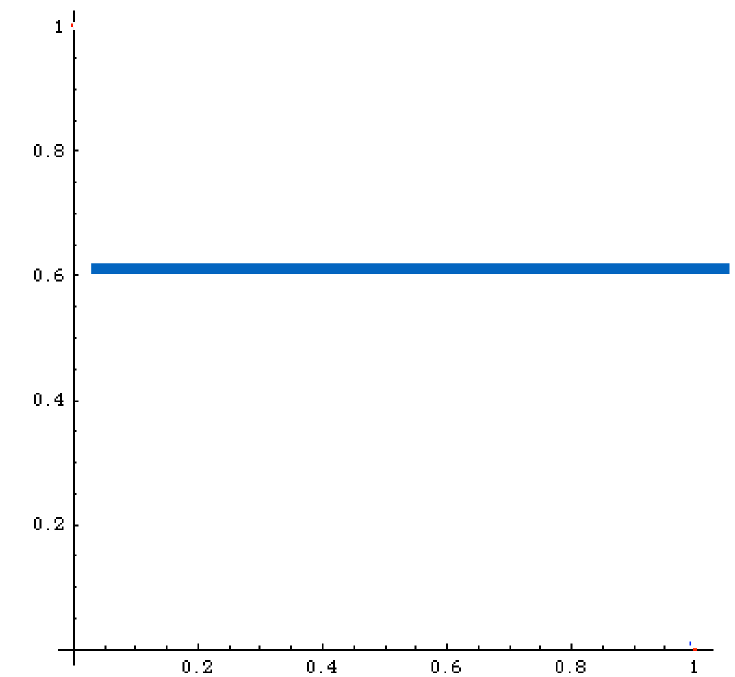

Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

Integration

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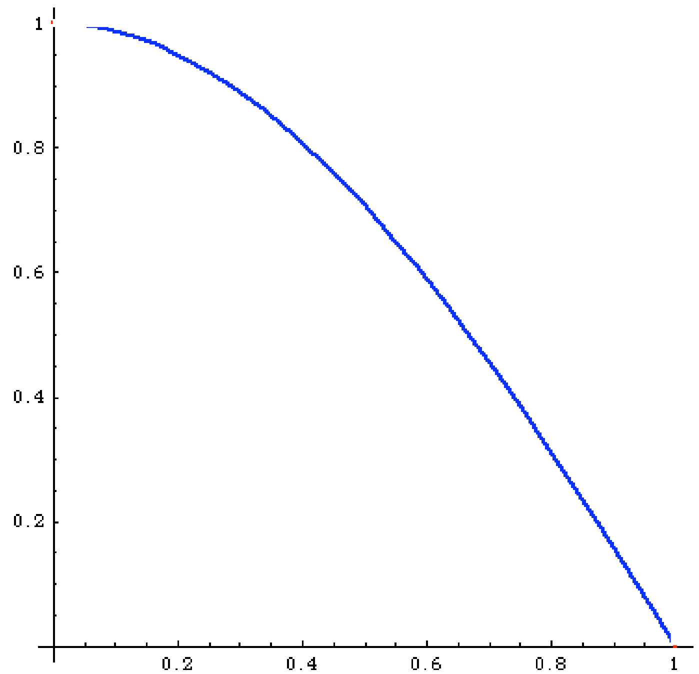

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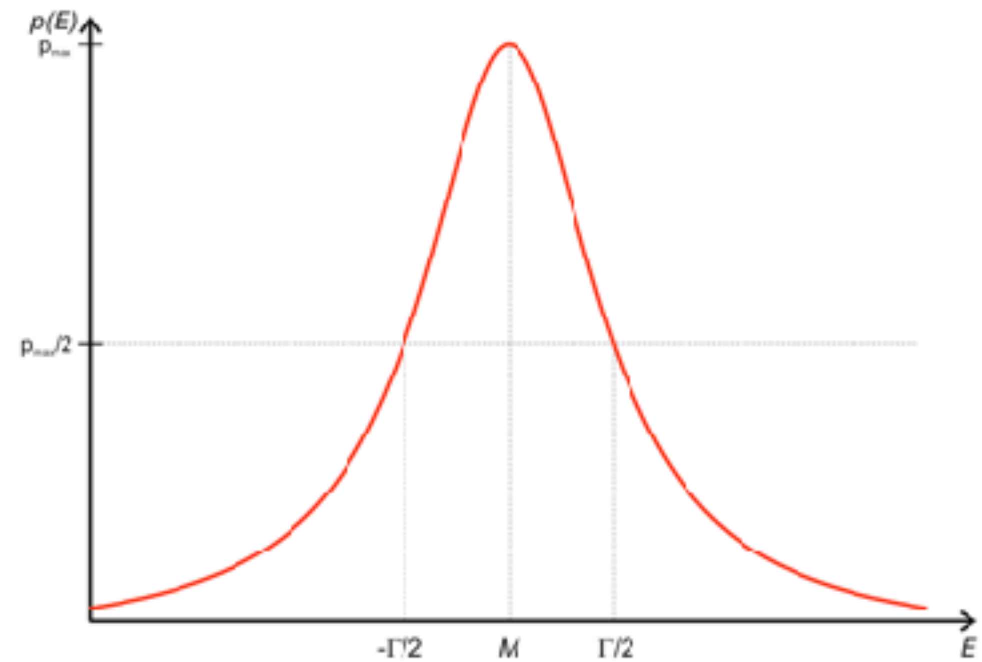

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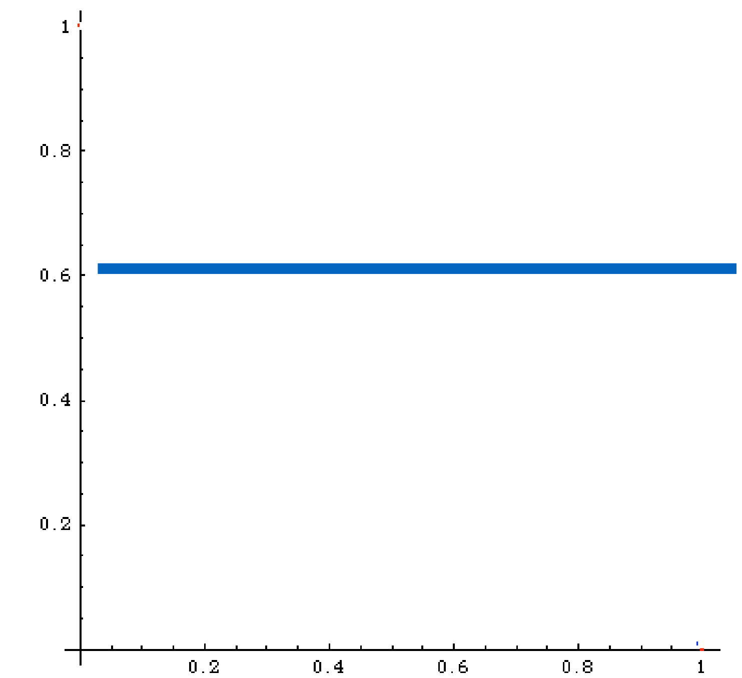

	simpson	MC
3	0.638	0.3
5	0.6367	0.8
20	0.63662	0.6
100	0.636619	0.65
1000	0.636619	0.636

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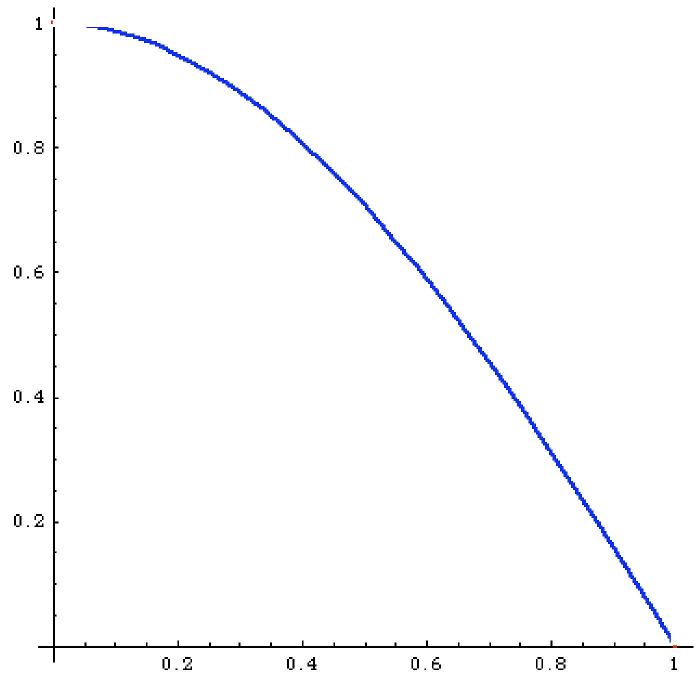

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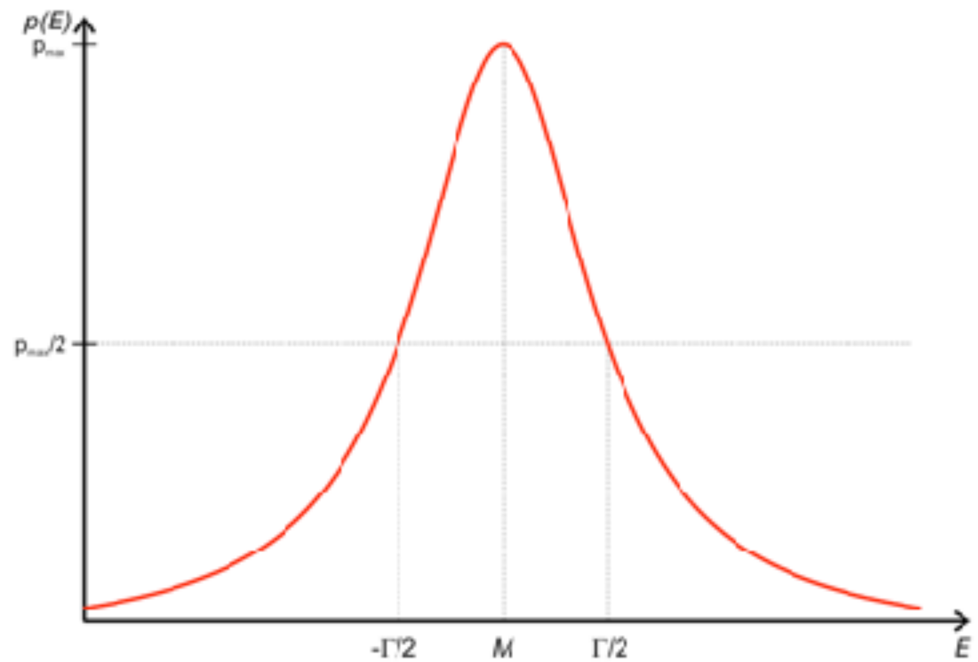

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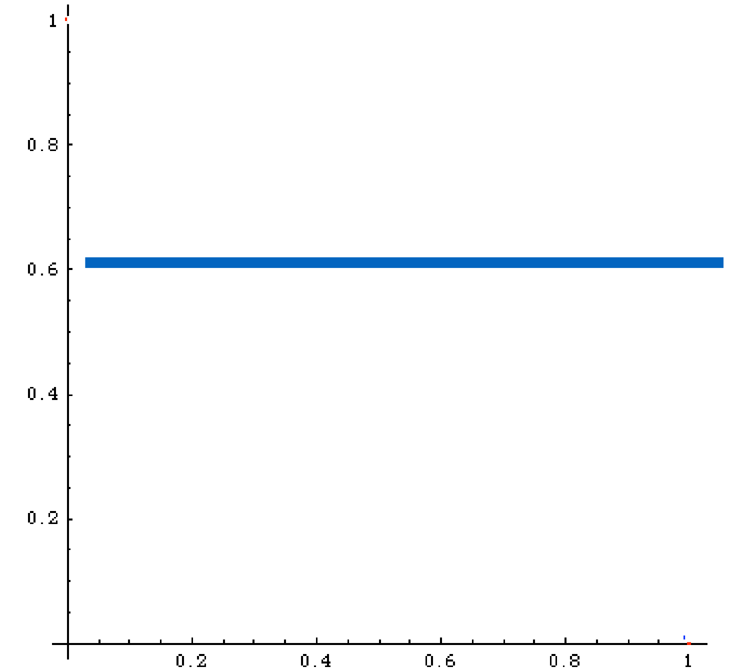

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- Simpson $1/N^4$

More Dimension →

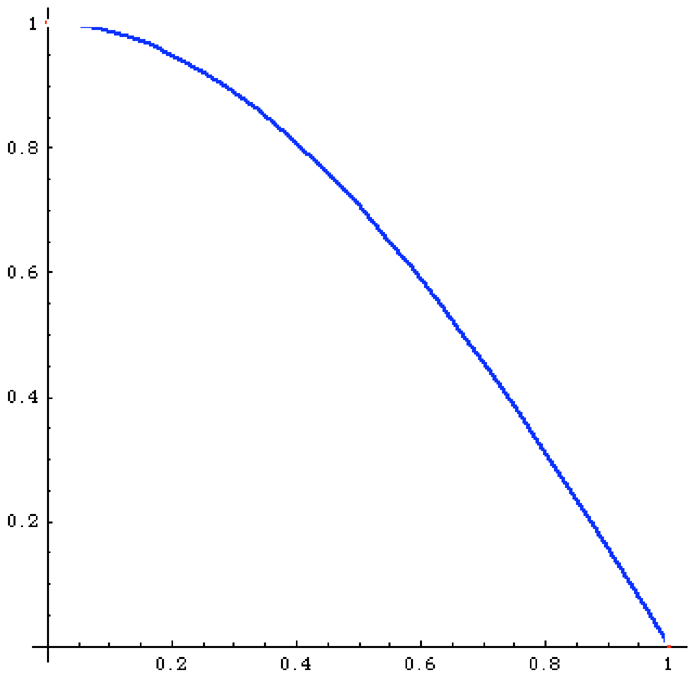
$1/\sqrt{N}$

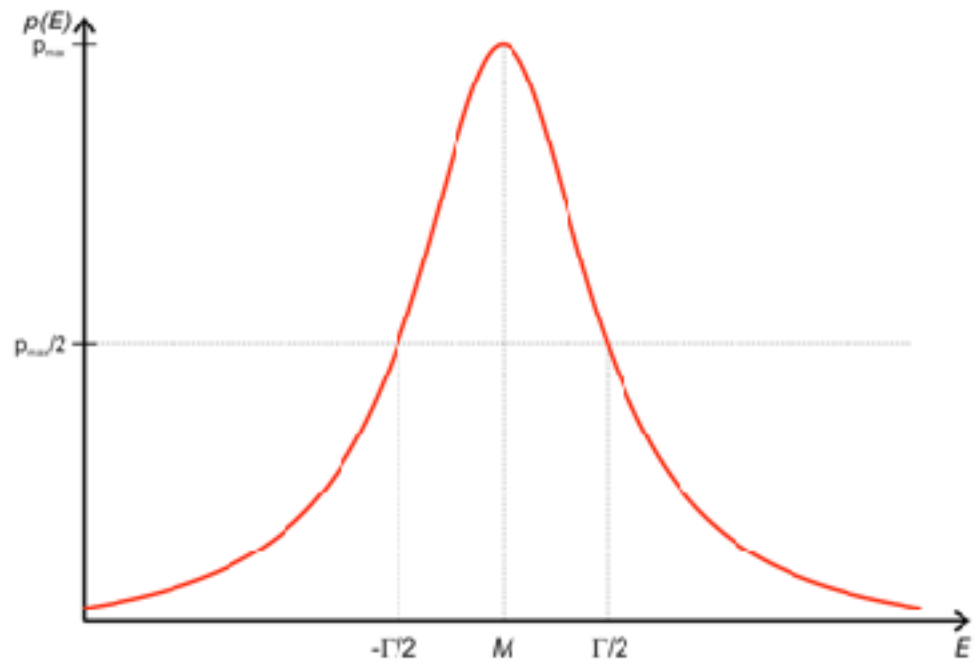
$1/N^{2/d}$

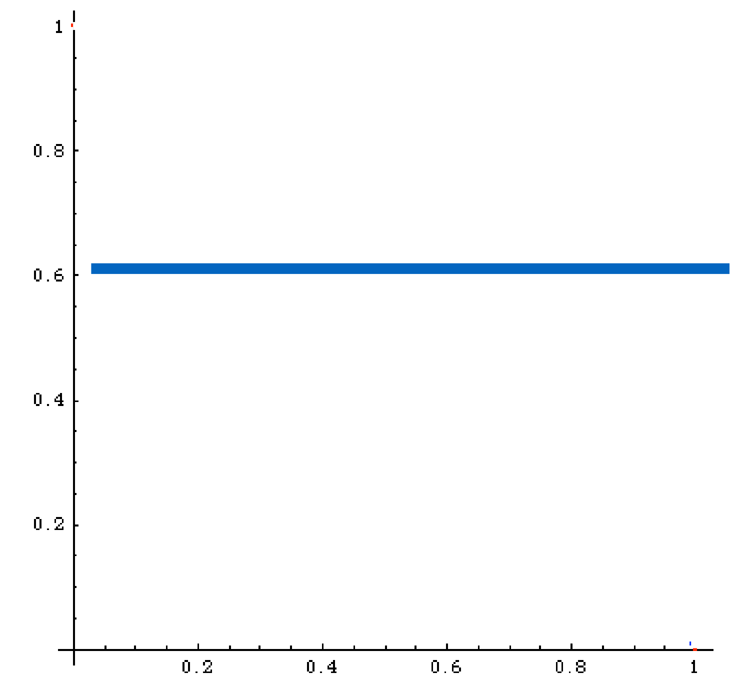
$1/N^{4/d}$

Integration

$$I = \int_0^1 dx \cos\frac{\pi}{2}x \qquad \int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2} \qquad \int dx\, C$$







$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

Integration

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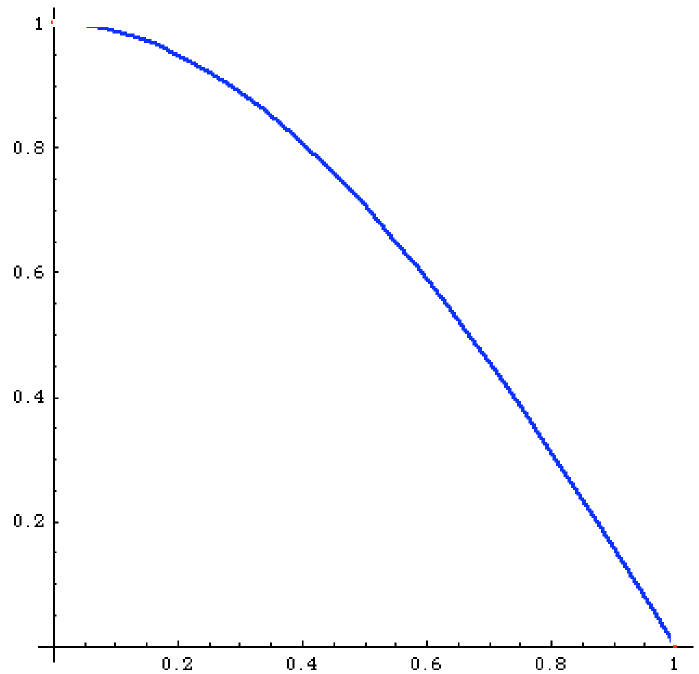

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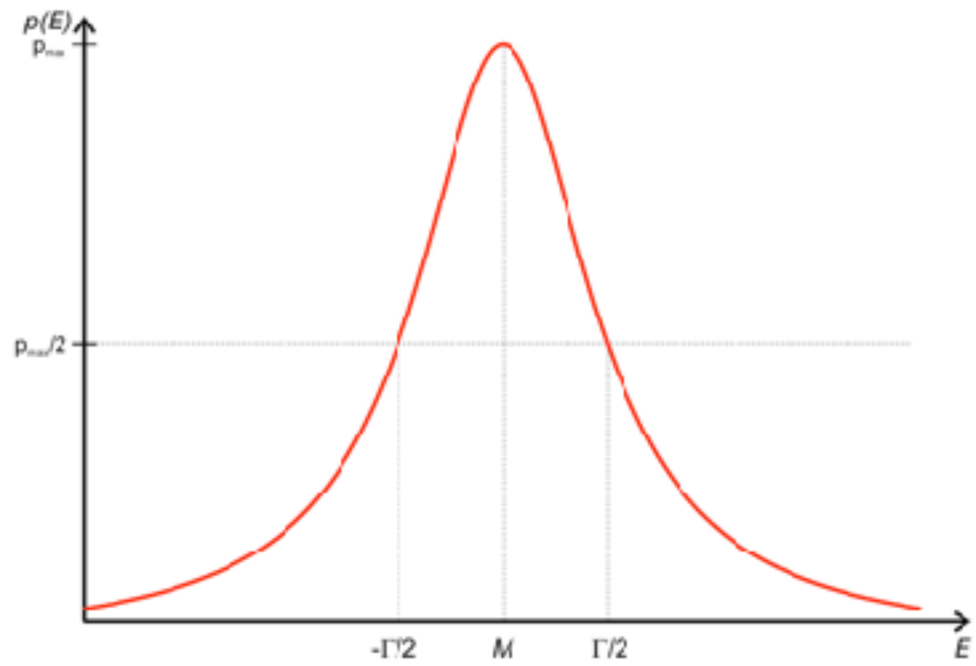
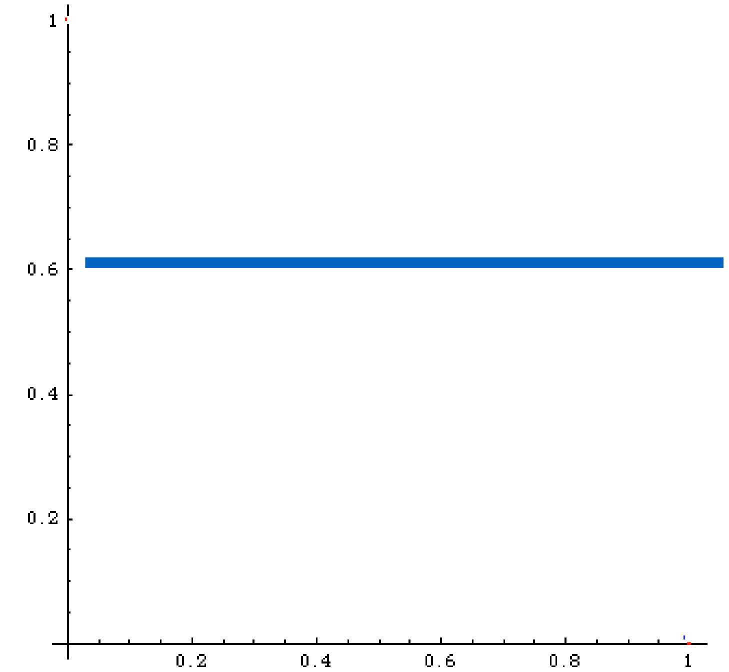


$$\int dx\, C$$



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$$I = I_N \pm \sqrt{V_N/N}$$

Integration

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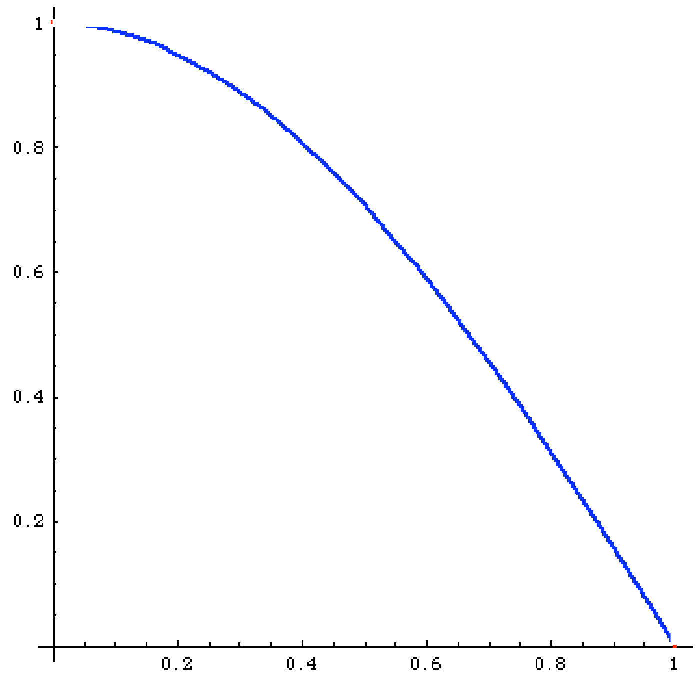

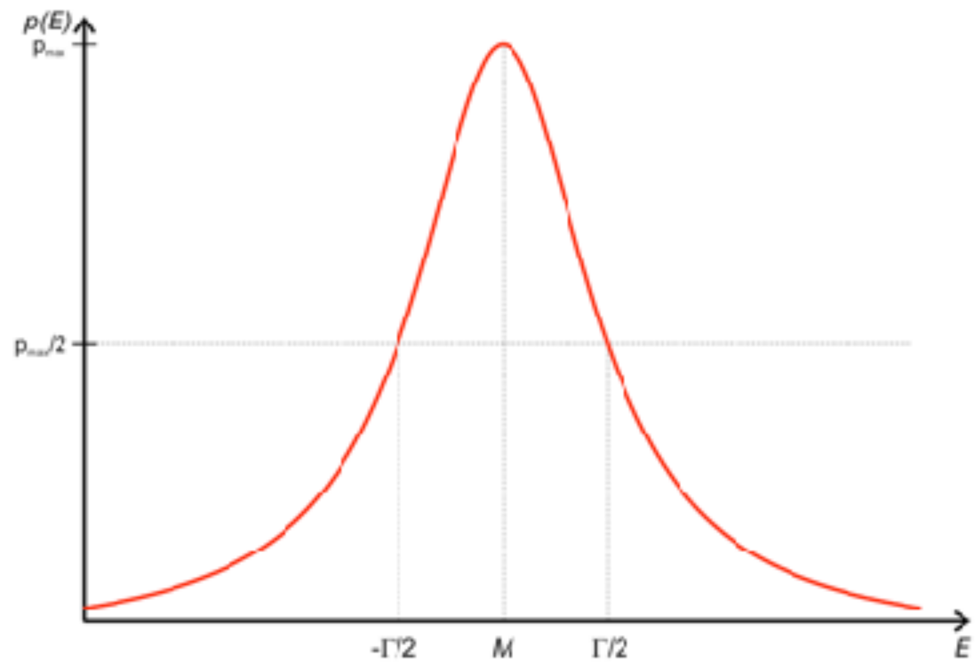

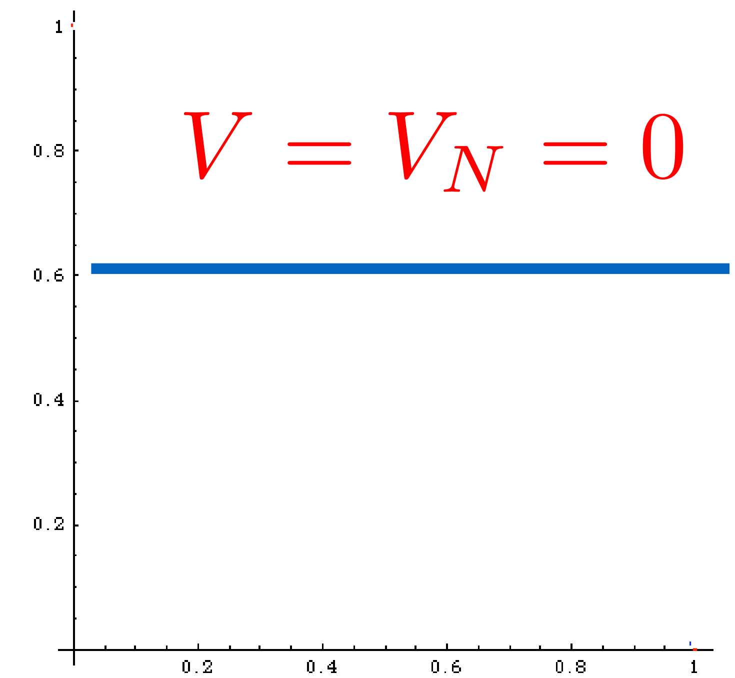

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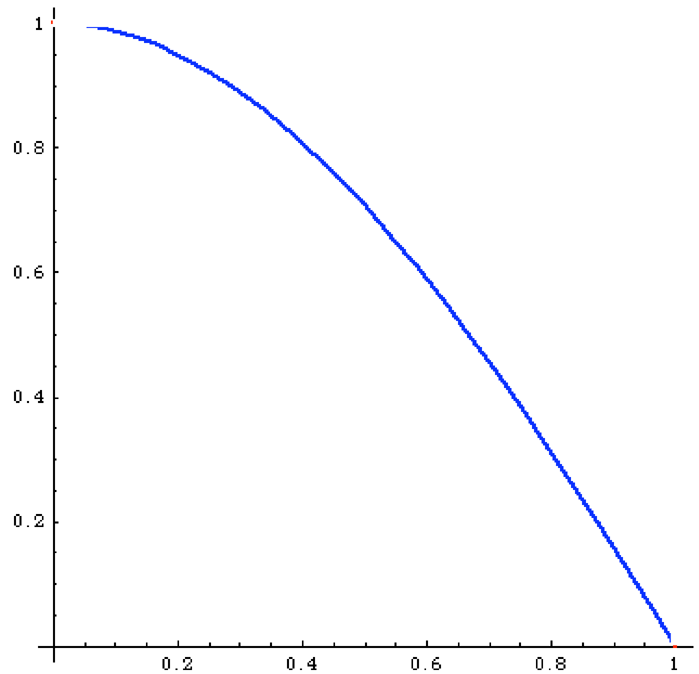

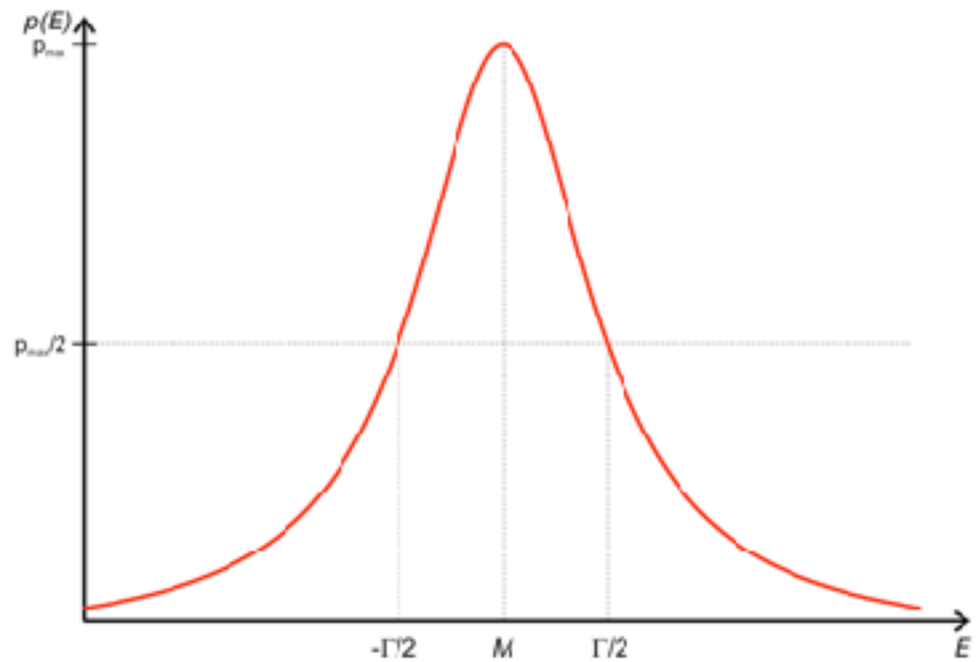

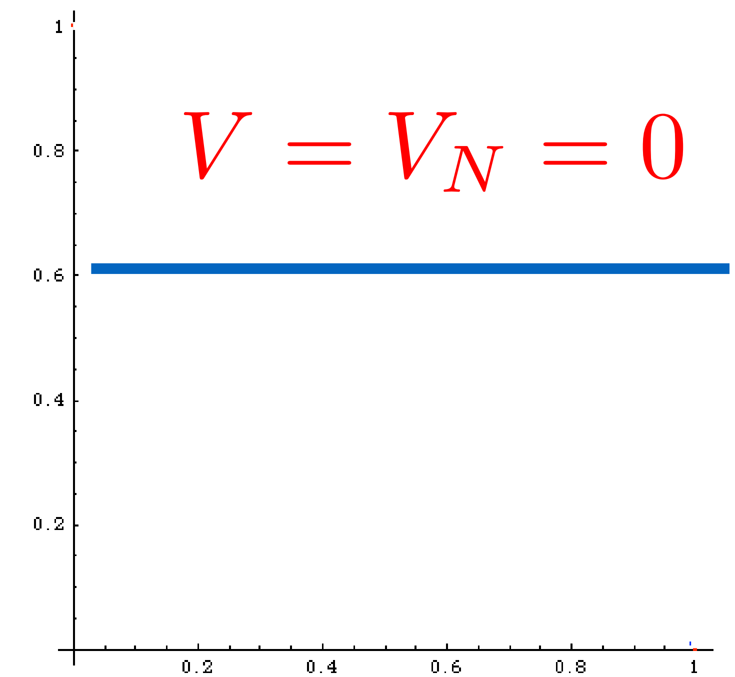

$$V = V_N = 0$$

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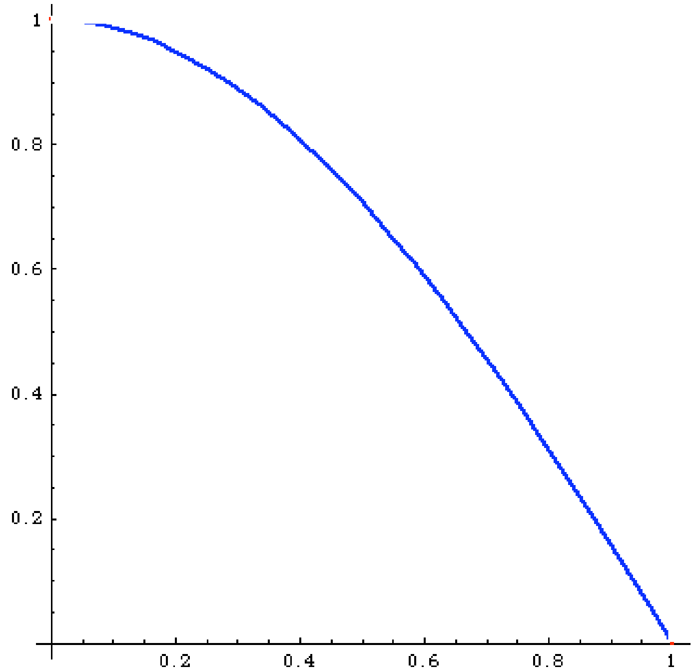
$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N} \sum_{i=1}^{N} [f(x)]^2 - I_N^2$$

$$I = I_N \pm \sqrt{V_N / N}$$

Can be minimized!

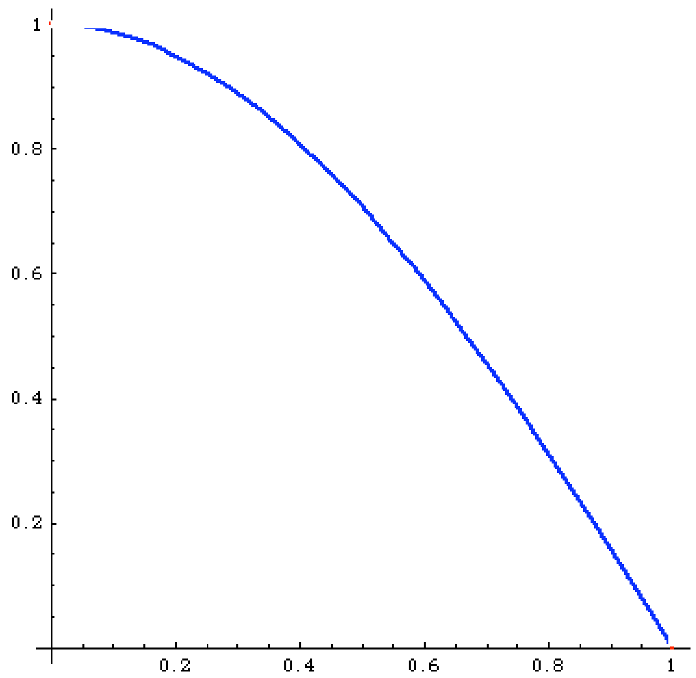
Importance Sampling



$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

Importance Sampling



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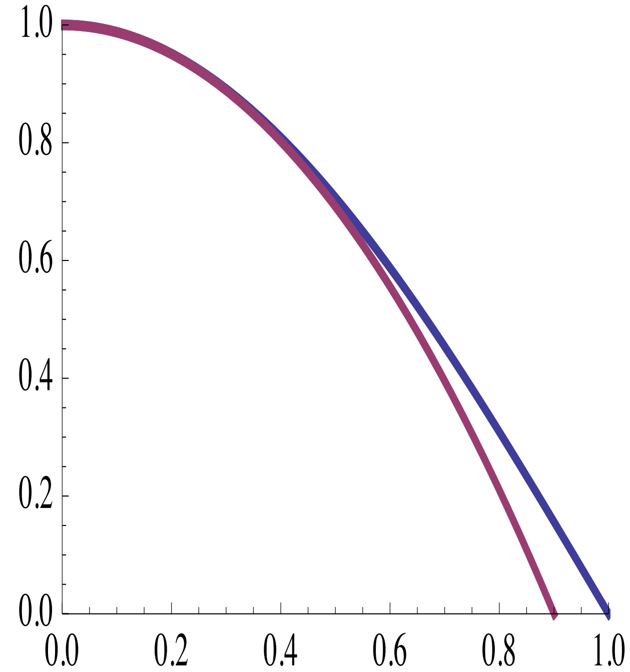

$$I = \int_0^1 dx(1 - cx^2)\frac{\cos\left(\frac{\pi}{2}x\right)}{(1 - cx^2)}$$

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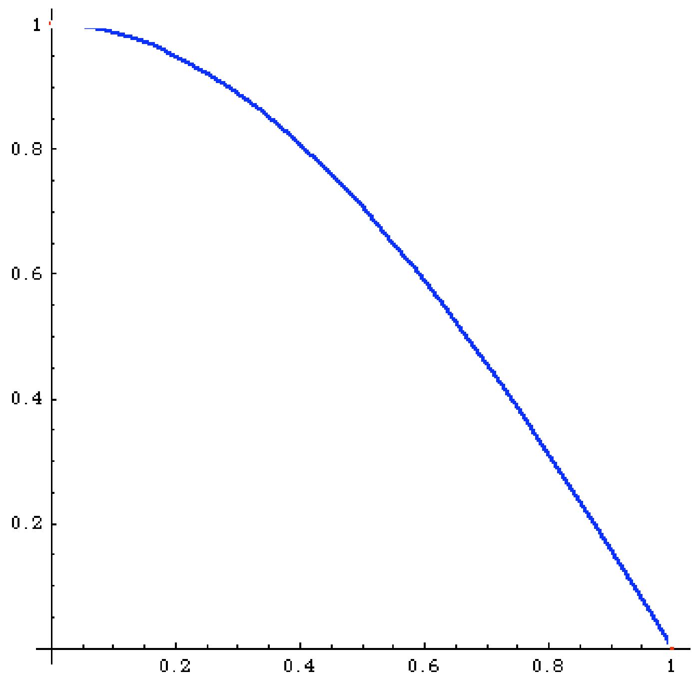

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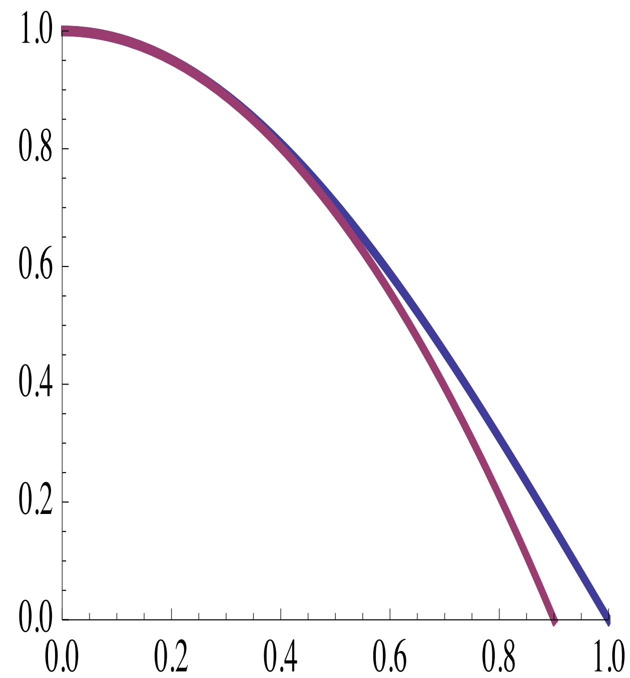

$$I = \int_0^1 dx(1-cx^2)\frac{\cos\left(\frac{\pi}{2}x\right)}{(1-cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2c}$$

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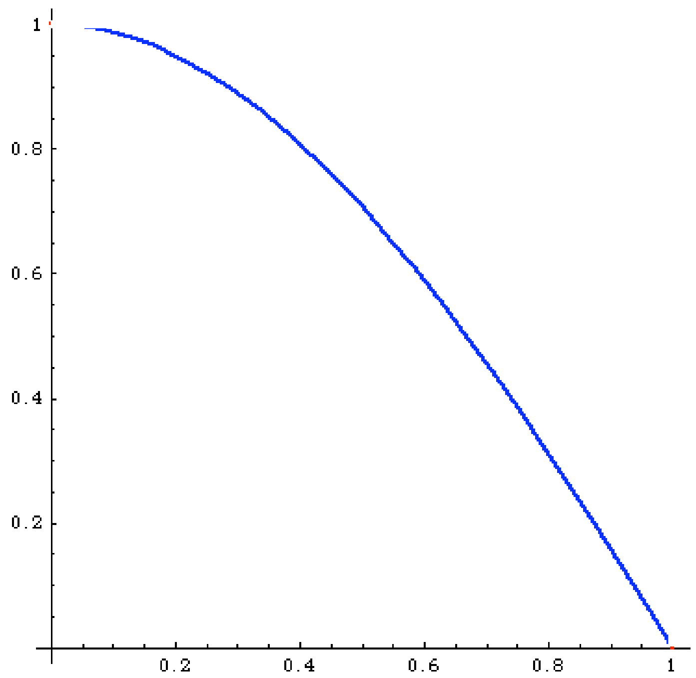

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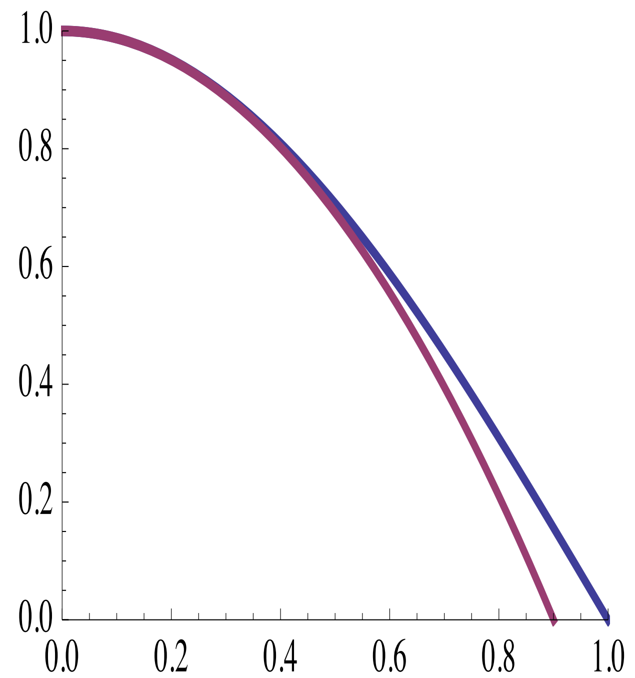

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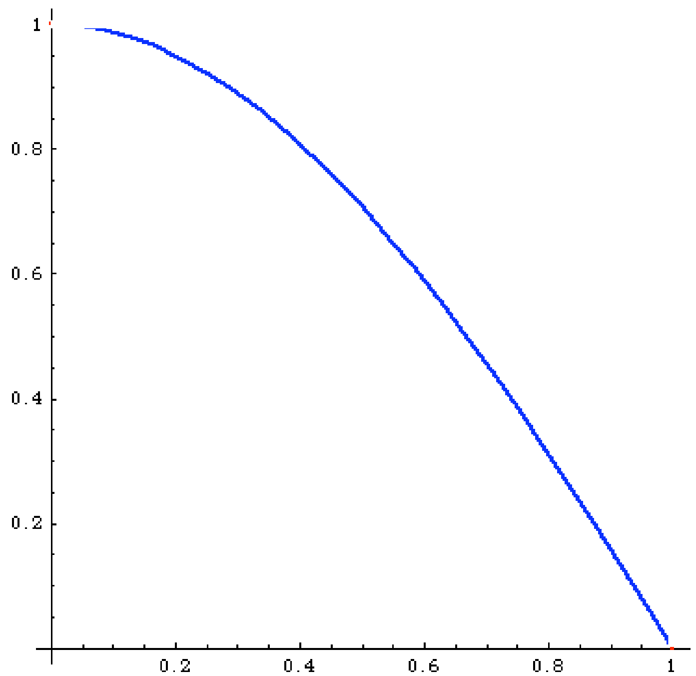
$\rightarrow \simeq 1$



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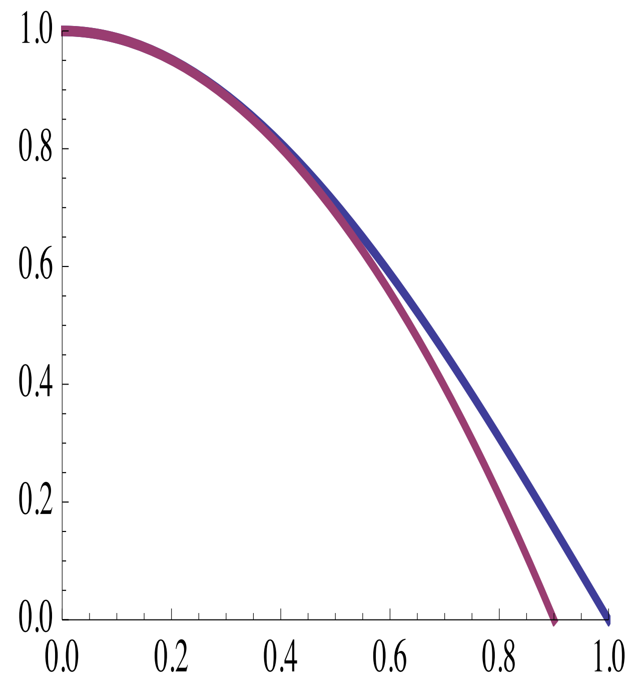

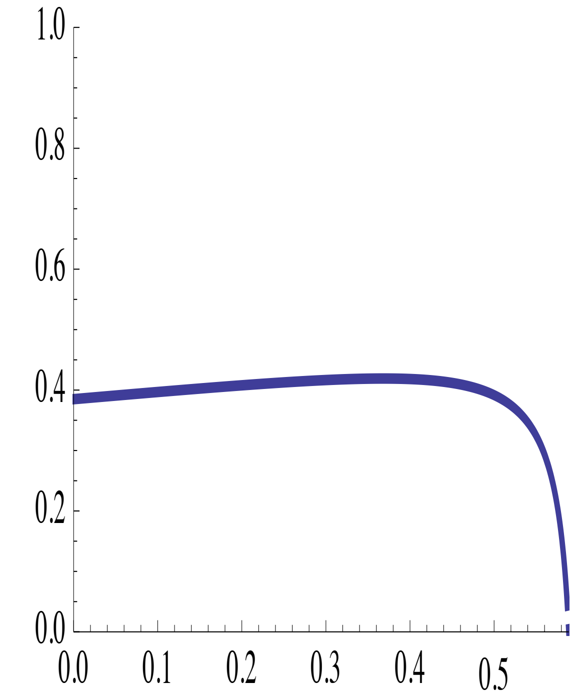

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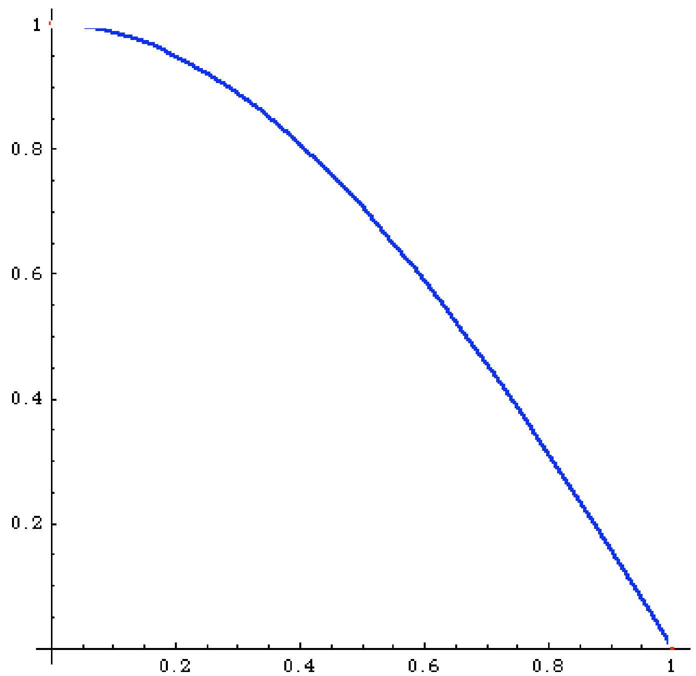

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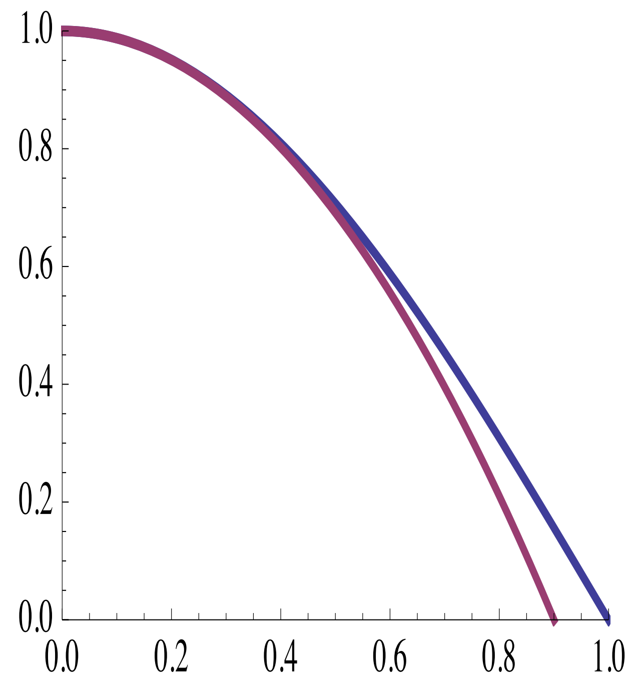

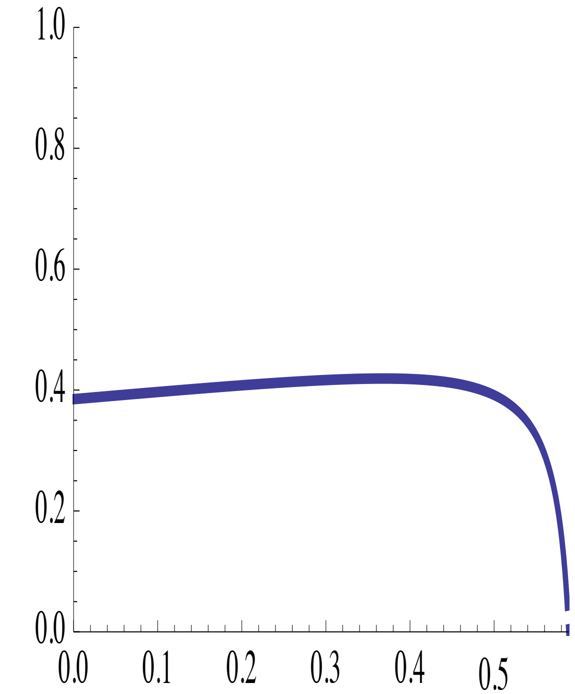

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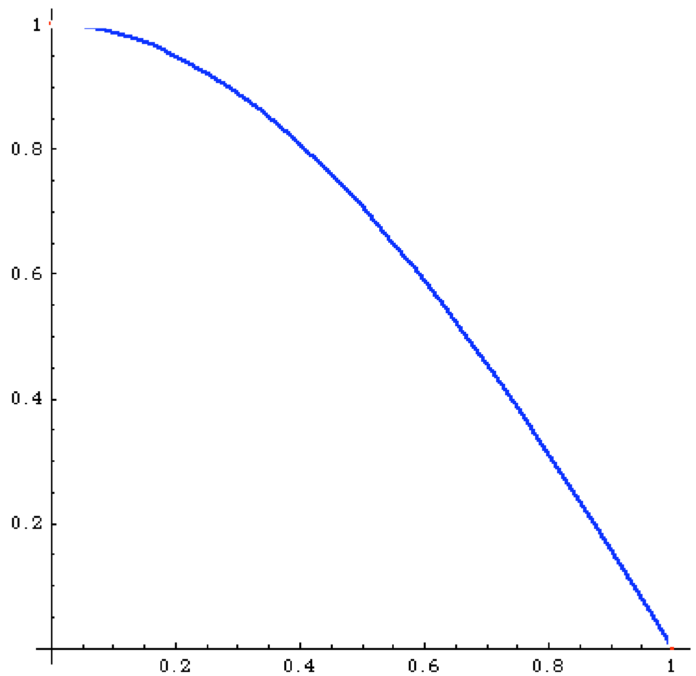
$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

Importance Sampling



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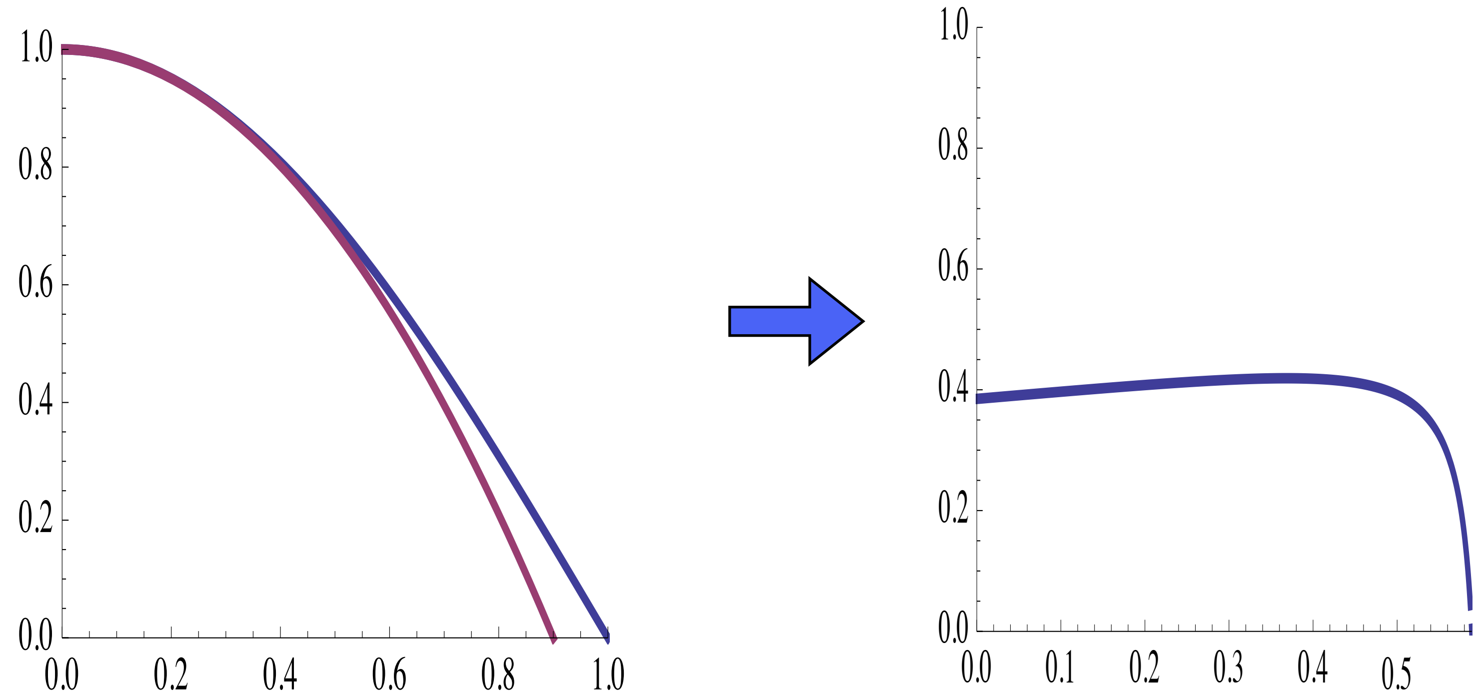

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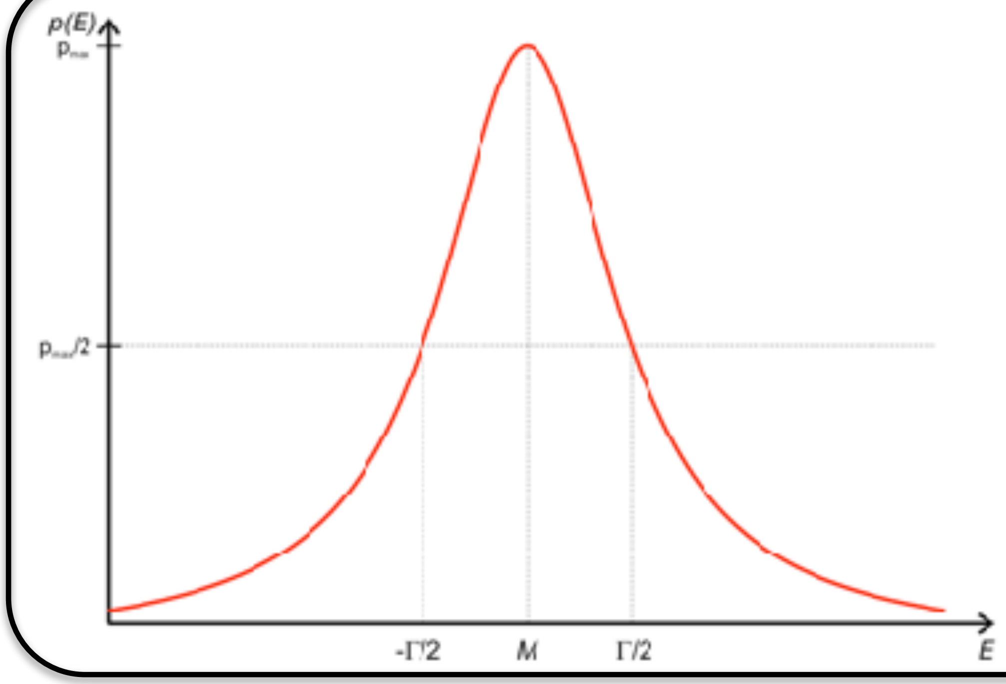
$$\simeq 1$$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

The Phase-Space parametrization is important to have an efficient computation!

Importance Sampling



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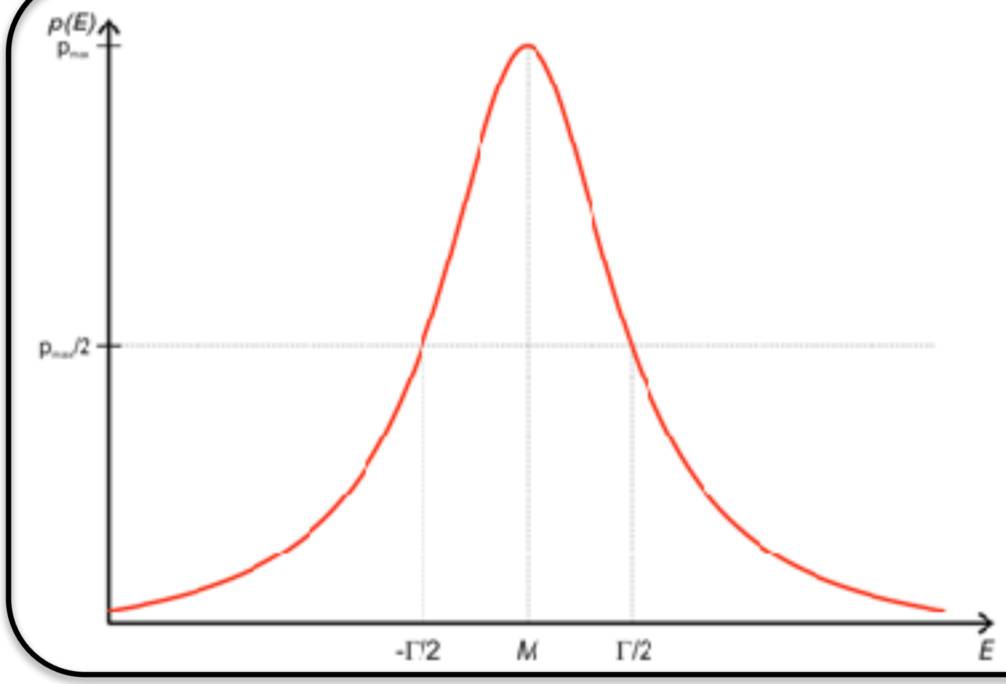
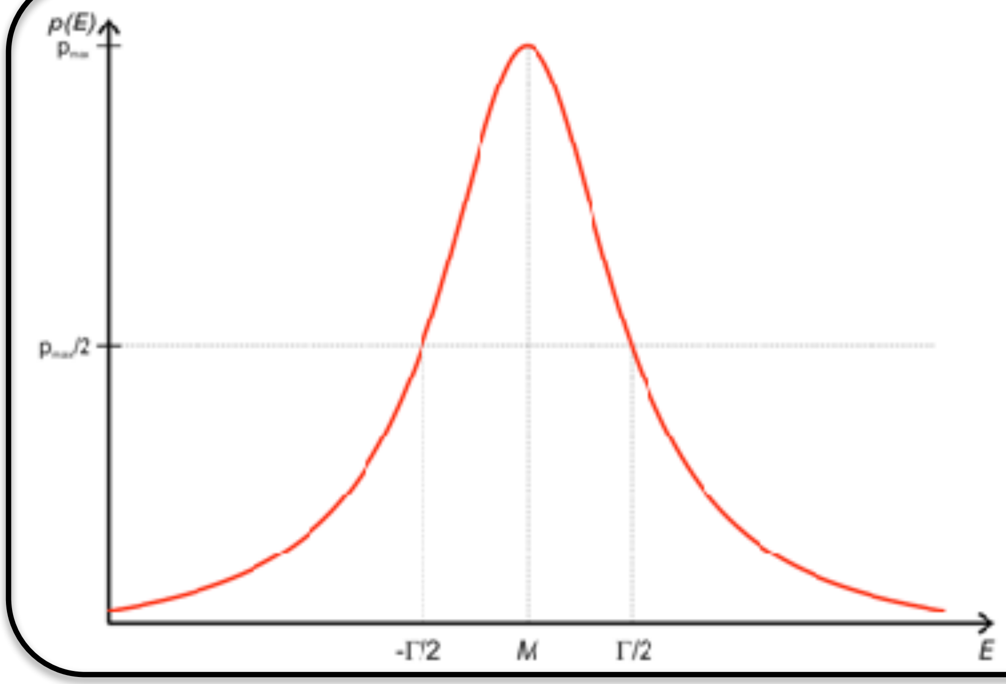


$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\xi = arctan\left(\frac{q^2 - M^2}{\Gamma M}\right)$$



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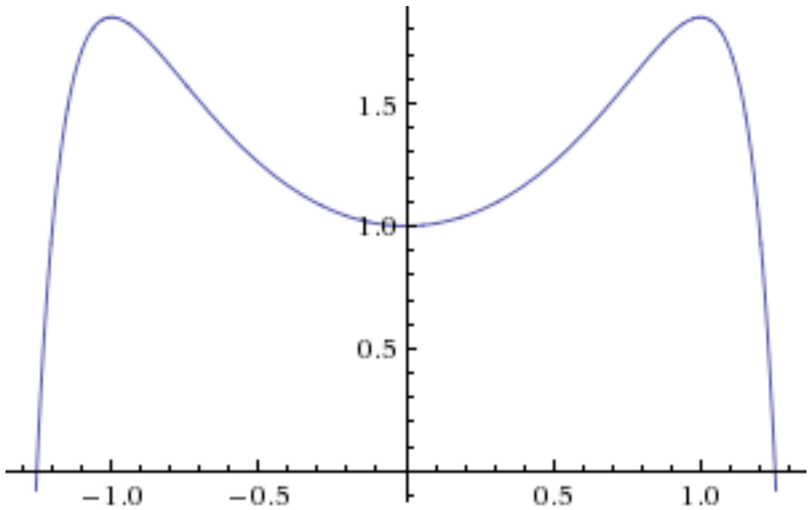

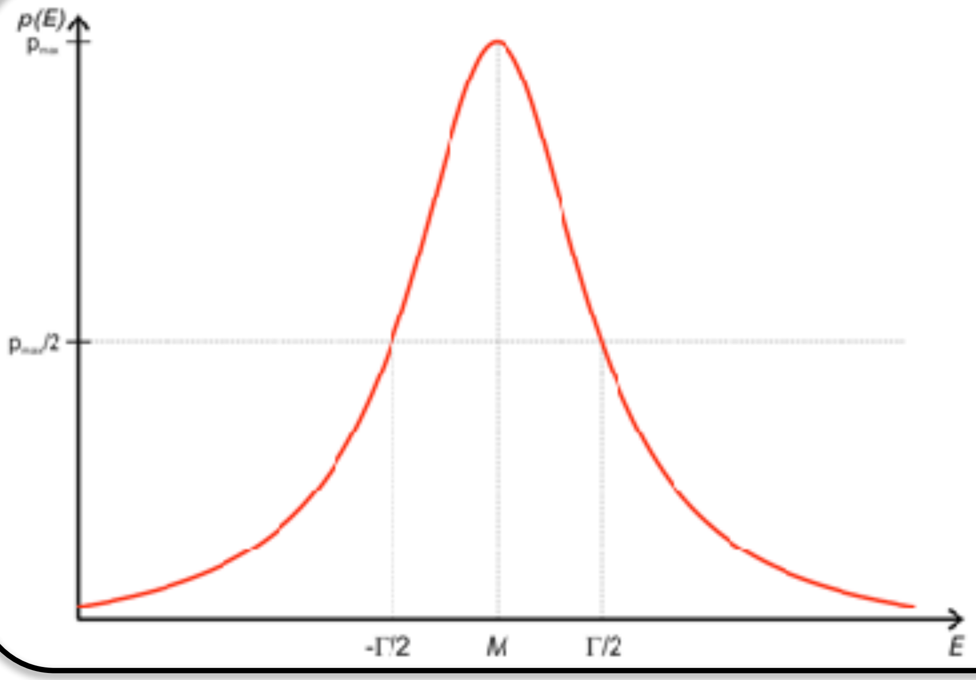

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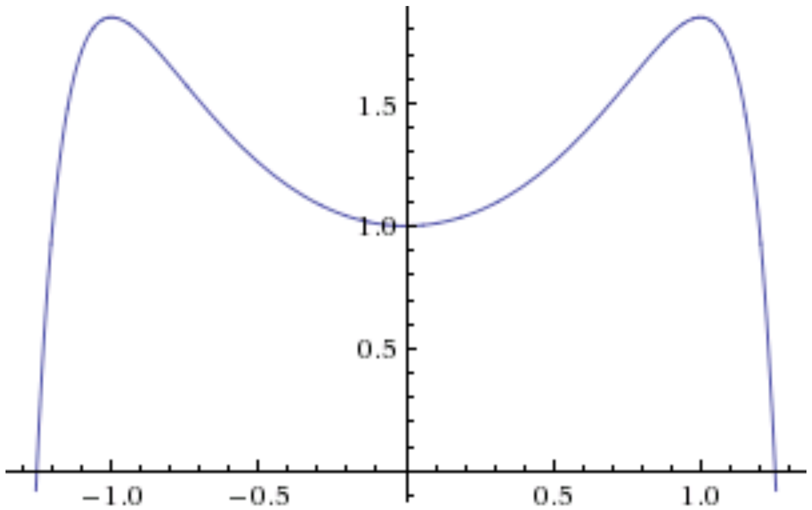
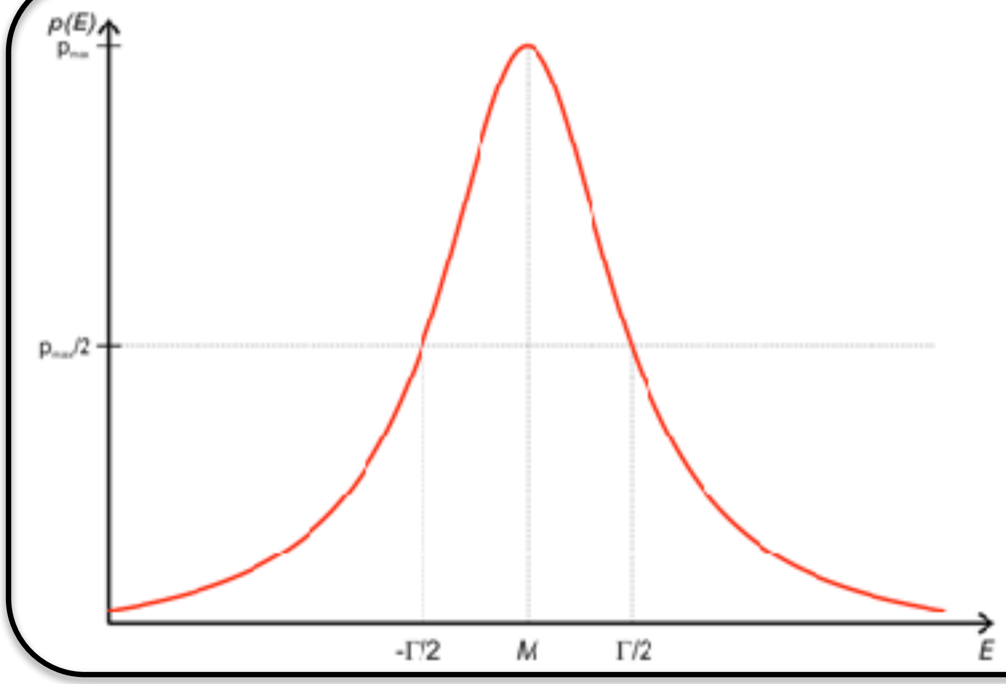


Why Importance Sampling?

Probability of using that point p(x)

Importance Sampling



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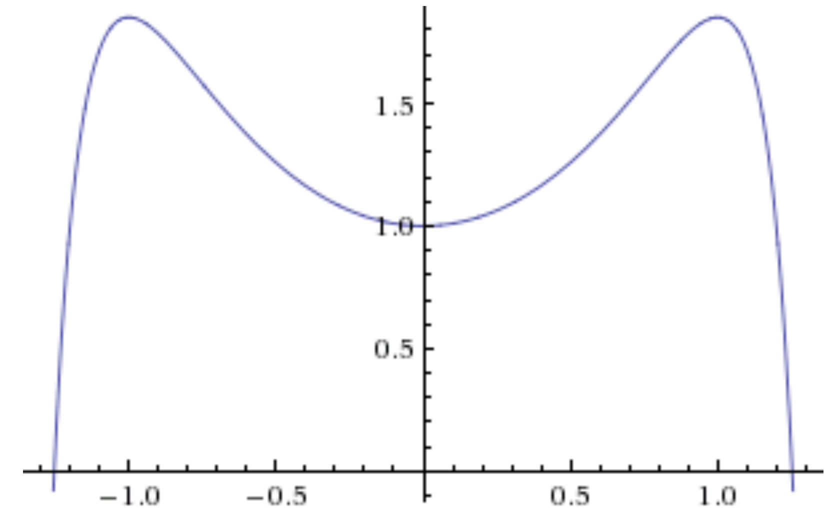

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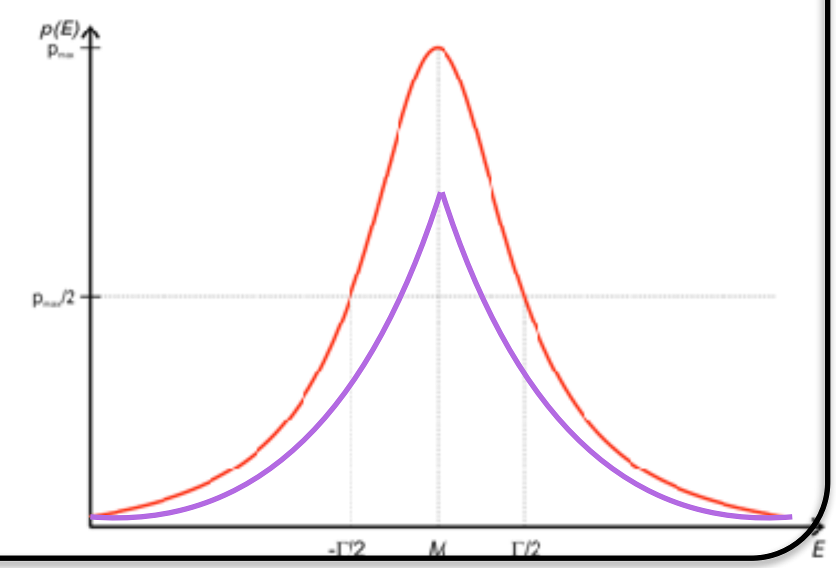

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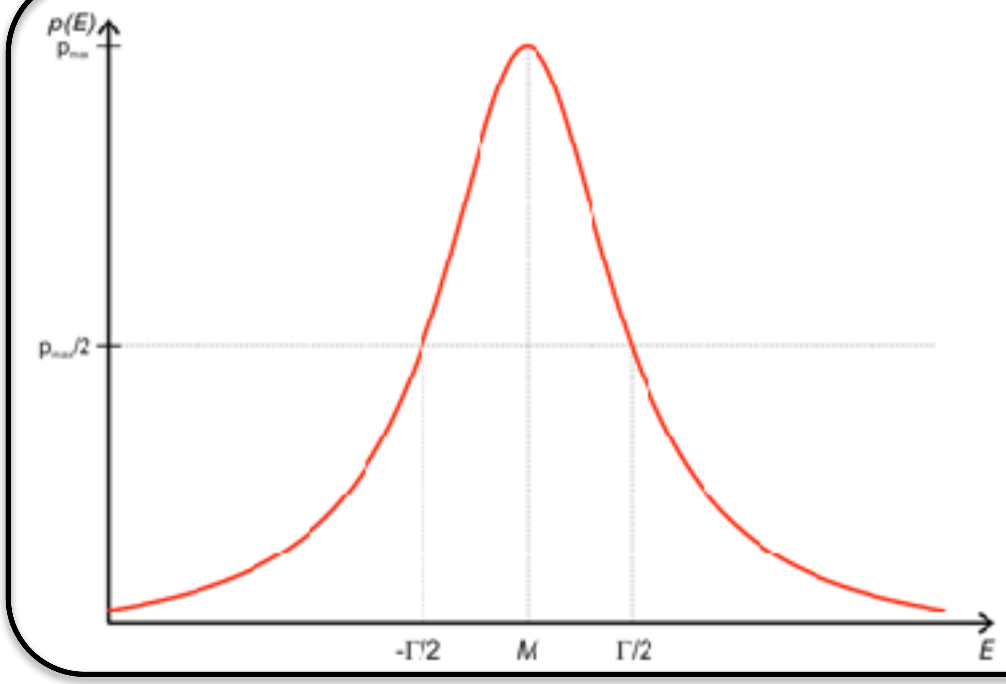

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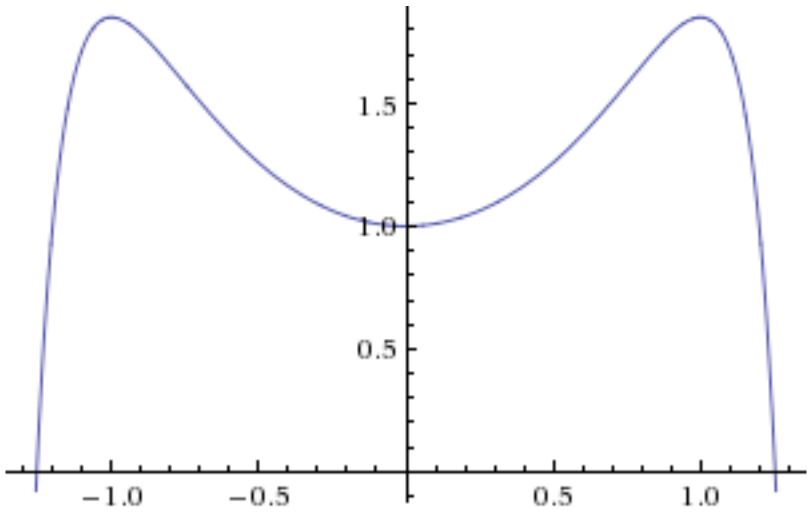

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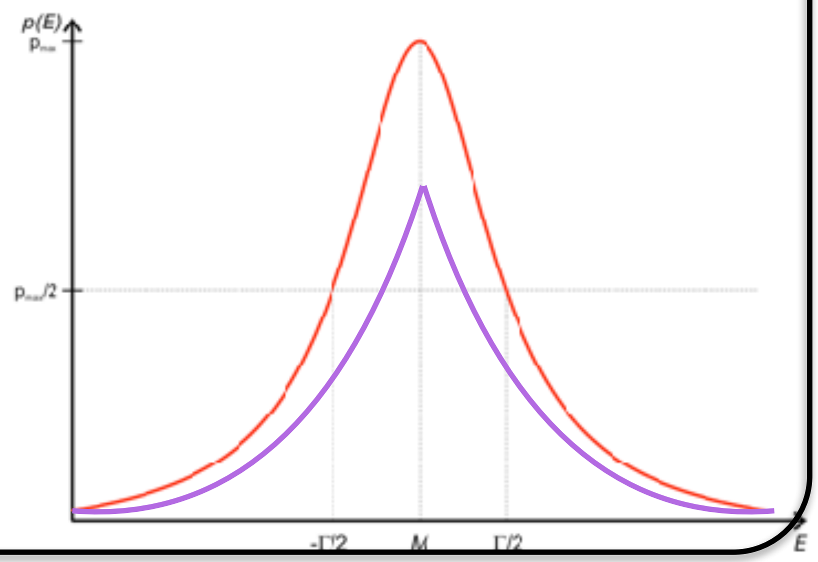

The change of variable ensure that the evaluation of the function is done where the function is the largest!

Importance Sampling

Key Point

- Generate the random point in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is flatter in this new variable.
- Needs to know an approximate function.

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

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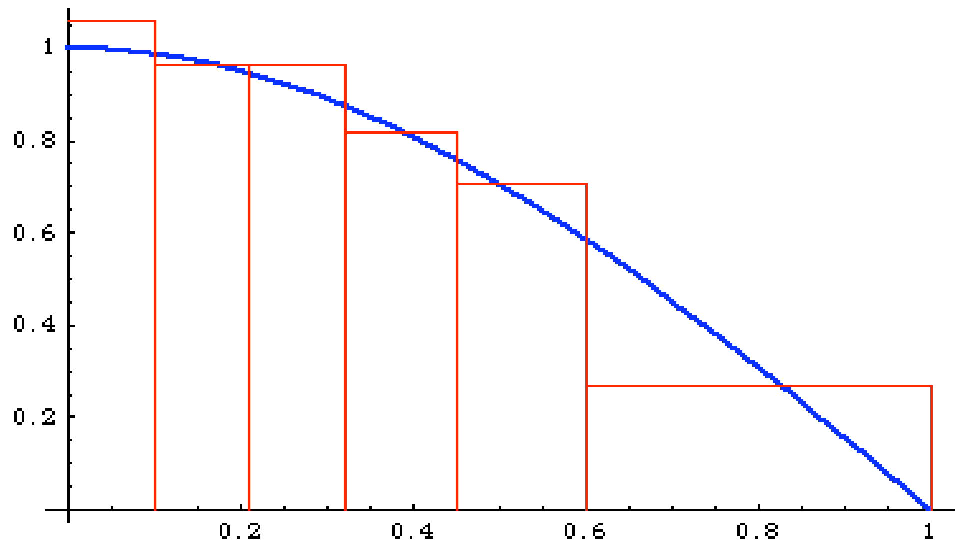

Algorithm

1. Creates bin such that each of them have the same contribution.

   ➡Many bins where the function is large

2. Use the approximate for the importance sampling method.

VEGAS

More than one Dimension

- adaptive methods works only with 1(few) dimension (memory problem)

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Solution

- Use projection on the axis

$$\vec{p}(x) = p(x) \cdot p(y) \cdot p(z) \ldots$$

VEGAS

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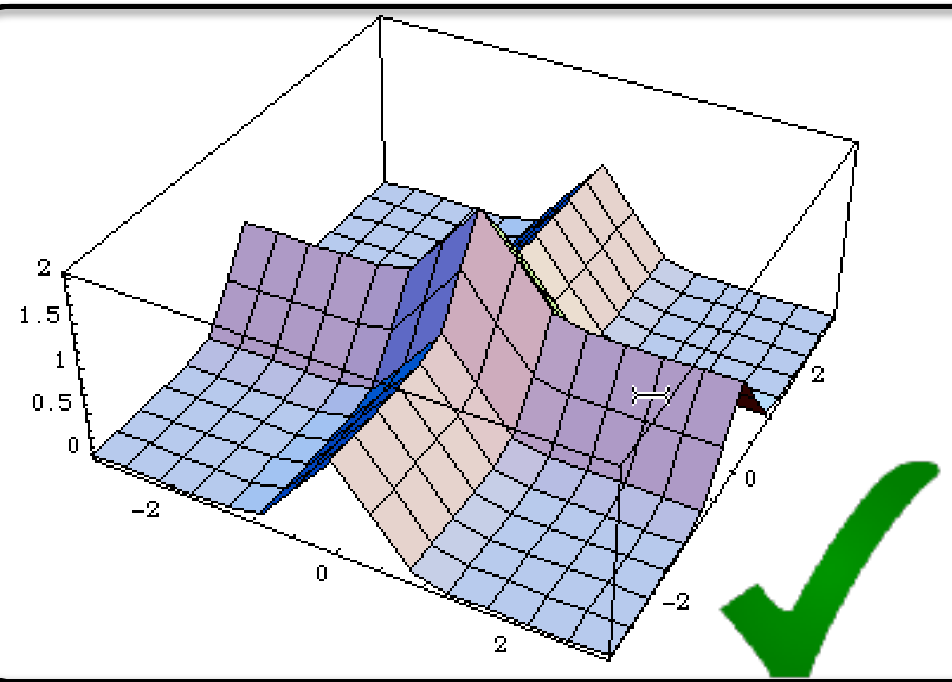

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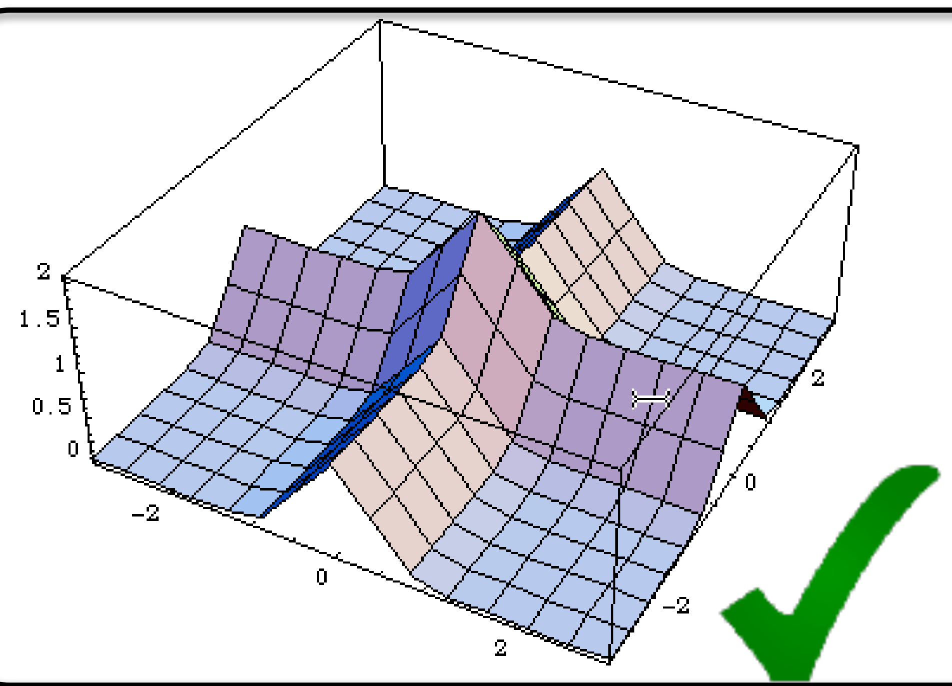
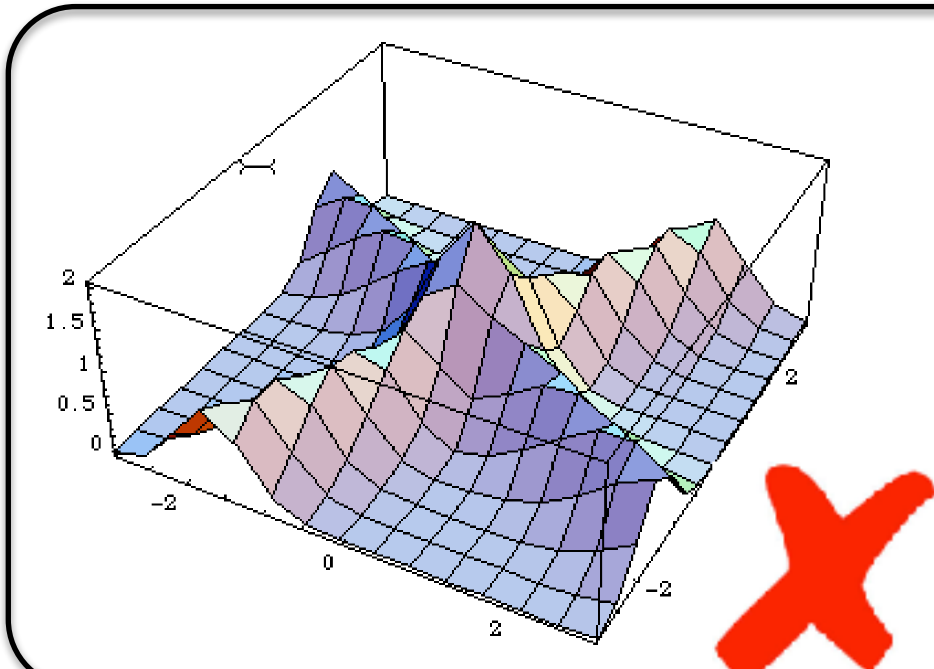


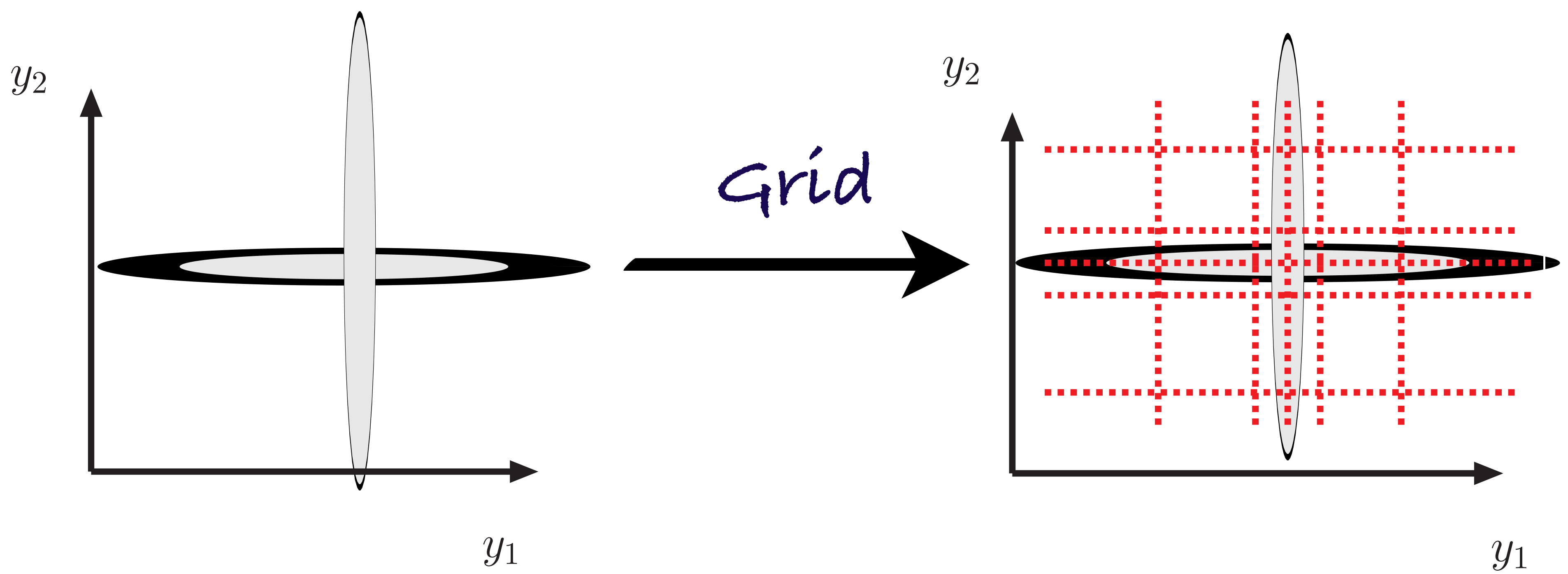


- We need to ensure the factorization !

➡ Additional change of variable

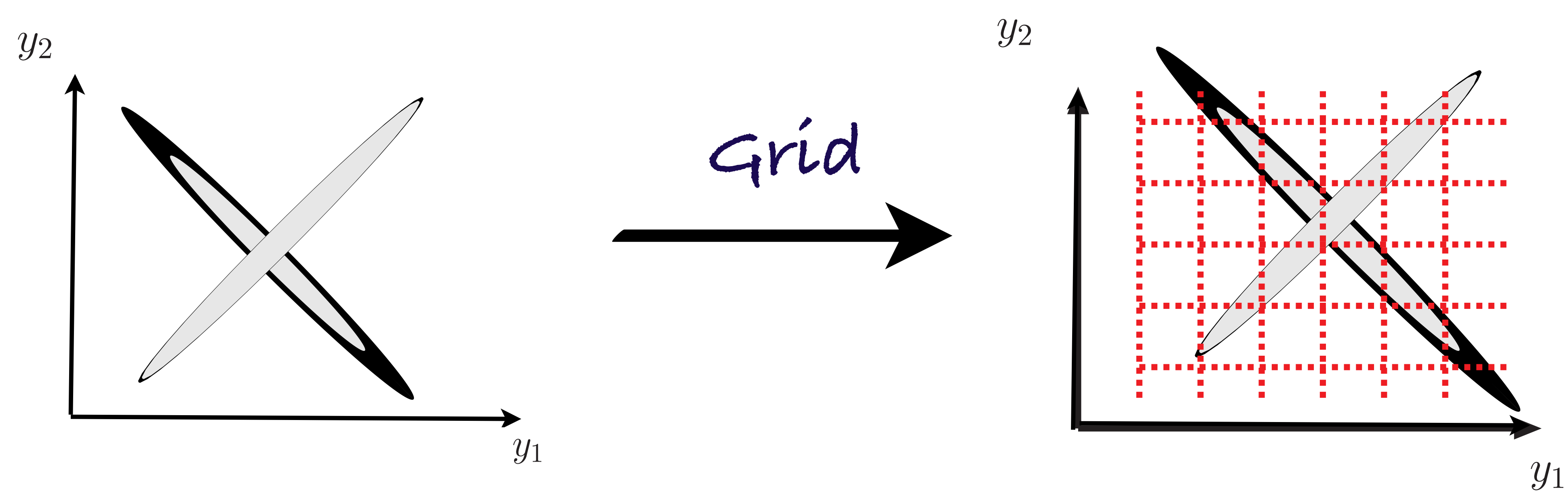
Monte-Carlo Integration

- The choice of the parameterisation has a strong impact on the efficiency



- The adaptive Monte-Carlo Technique picks point in interesting areas
  → The technique is efficient

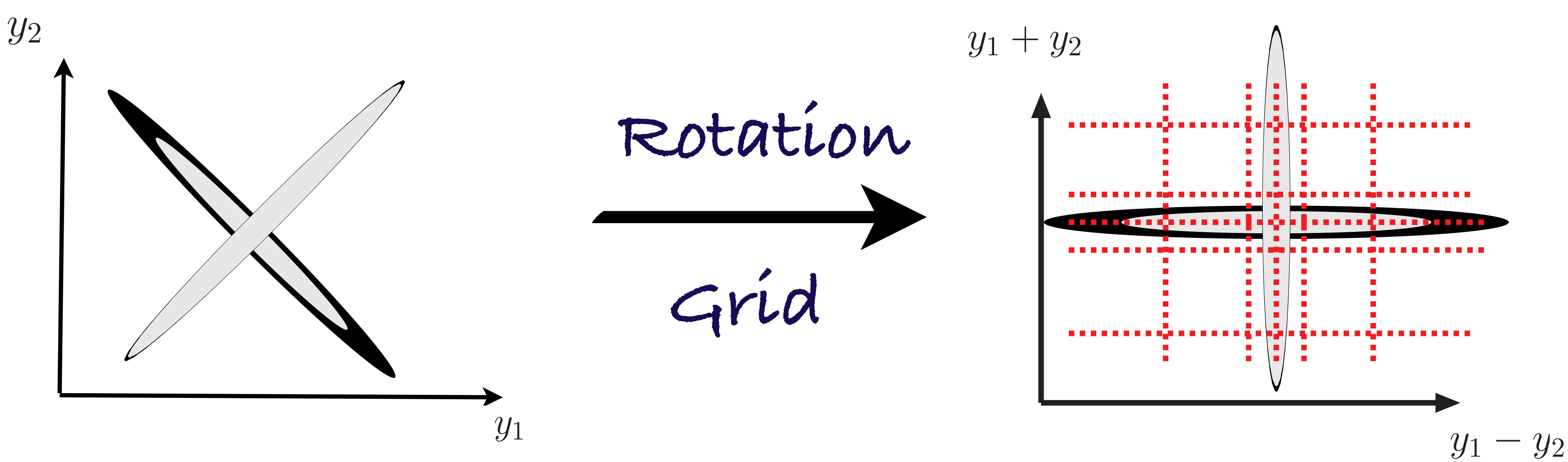
- The choice of the parametrization has a strong impact on the efficiency



- The adaptive Monte-Carlo Techniques picks points everywhere
  → The integral converges slowly

Monte-Carlo Integration

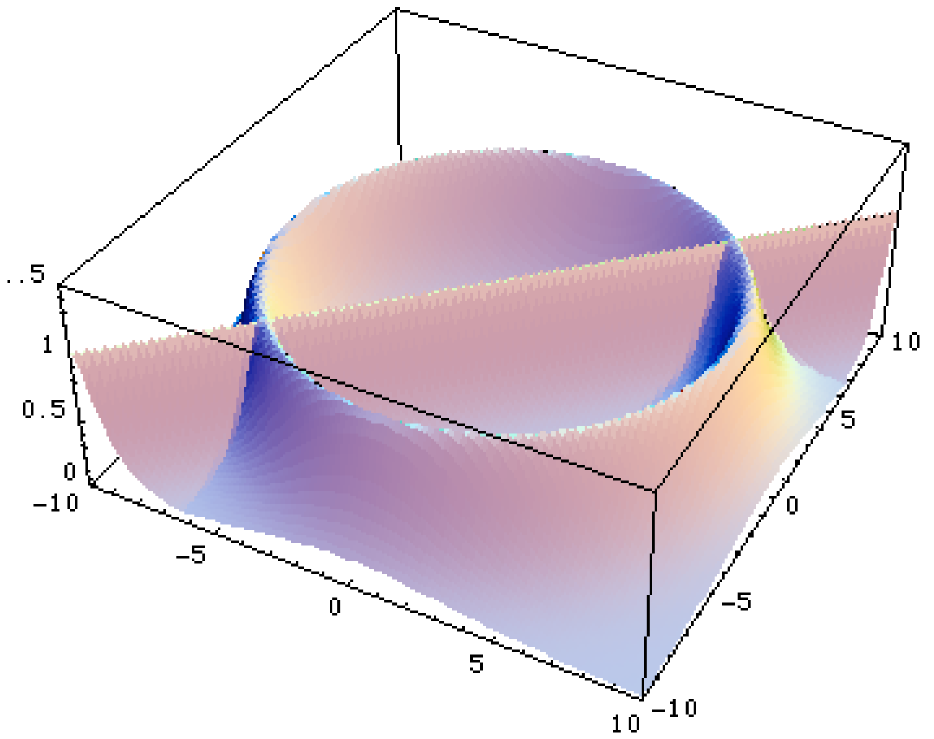
- The choice of the parametrization has a strong impact on the efficiency



- ☐ The adaptive Monte-Carlo Techniques picks point in interesting areas

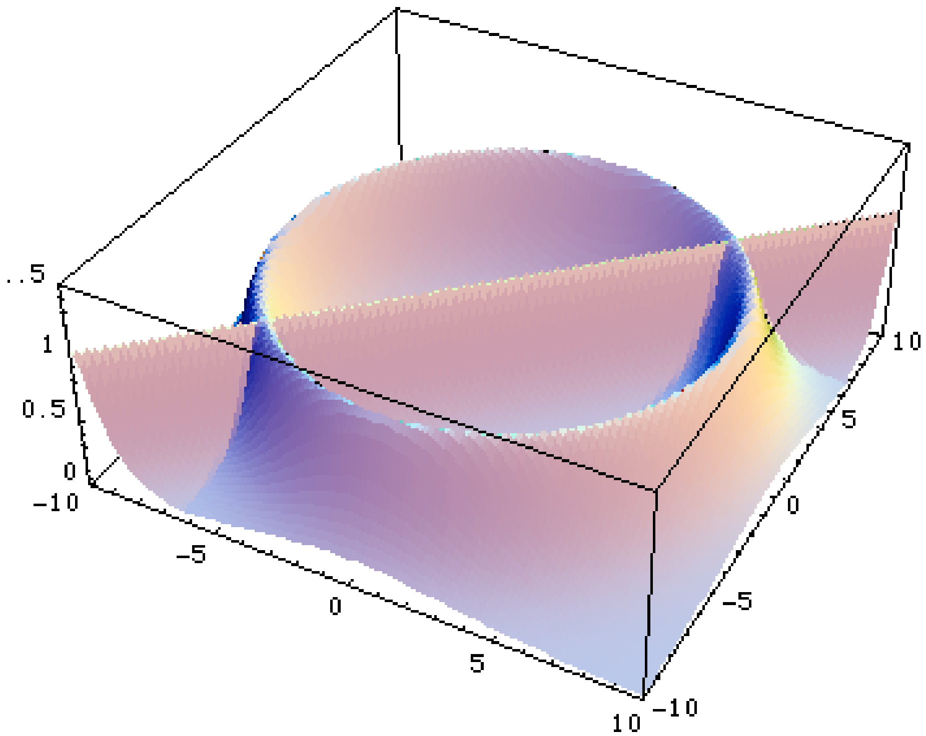
→ The technique is efficient

Multi-channel



What do we do if there is no transformation that aligns all integrand peaks to the chosen axes?
Vegas is bound to fail!

Multi-channel



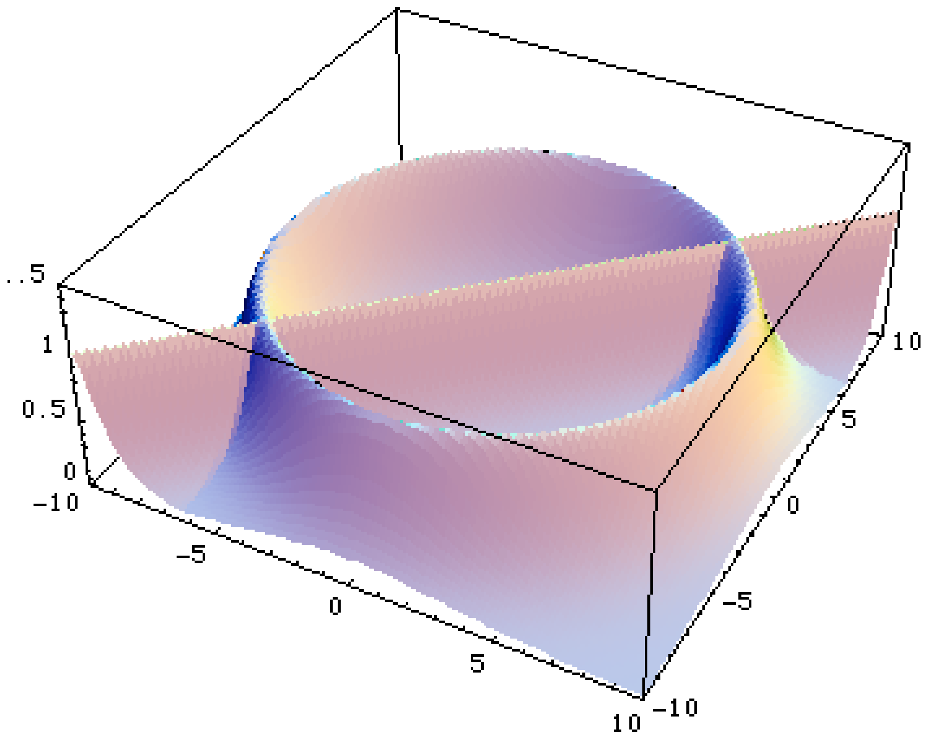
What do we do if there is no transformation that aligns all integrand peaks to the chosen axes? Vegas is bound to fail!

Solution: use different transformations = channels

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

with each $p_i(x)$ taking care of one "peak" at the time

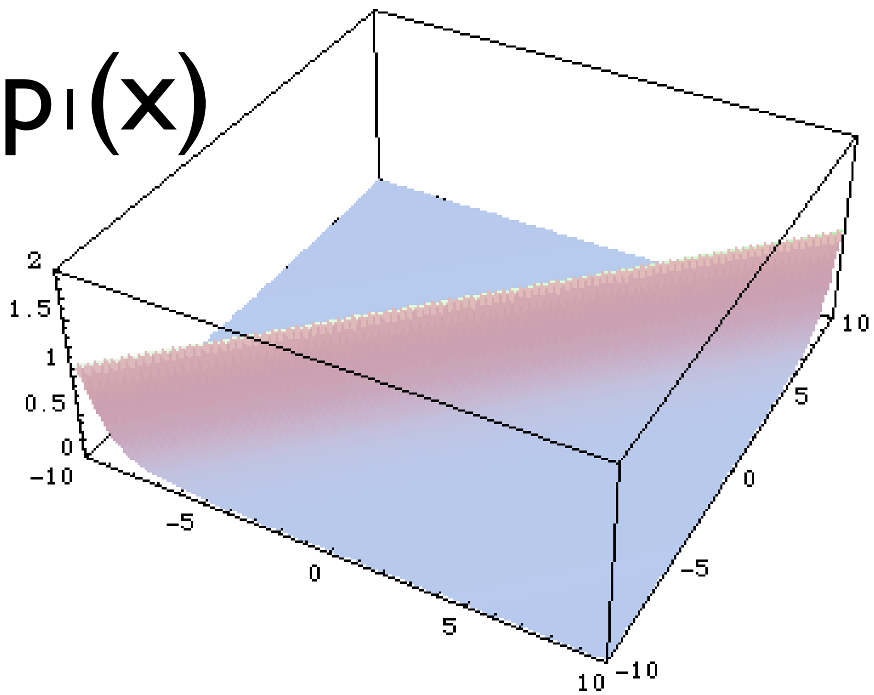
Multi-channel



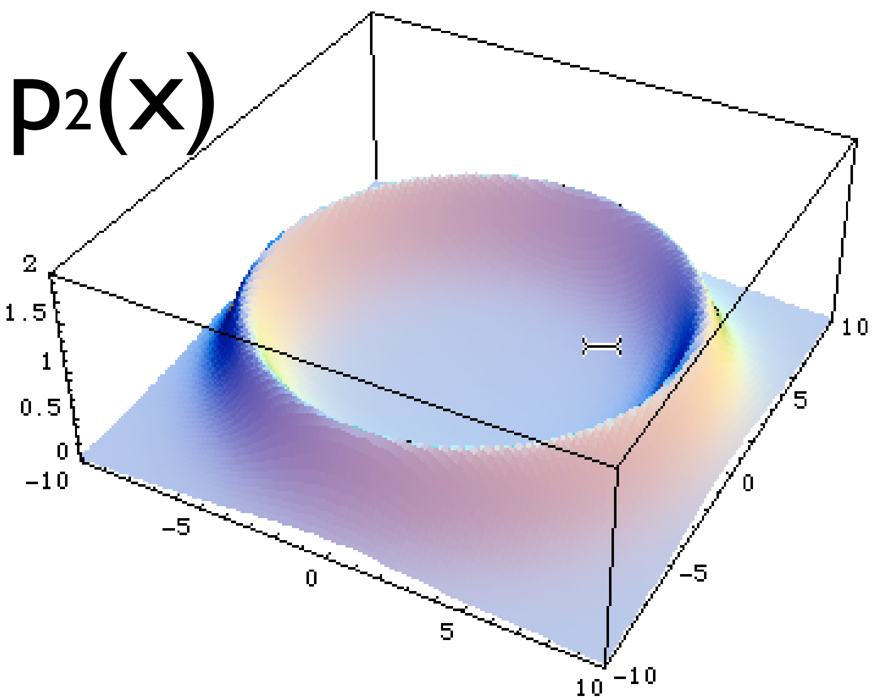
$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x)$$

with $$\sum_{i=1}^{n} \alpha_i = 1$$

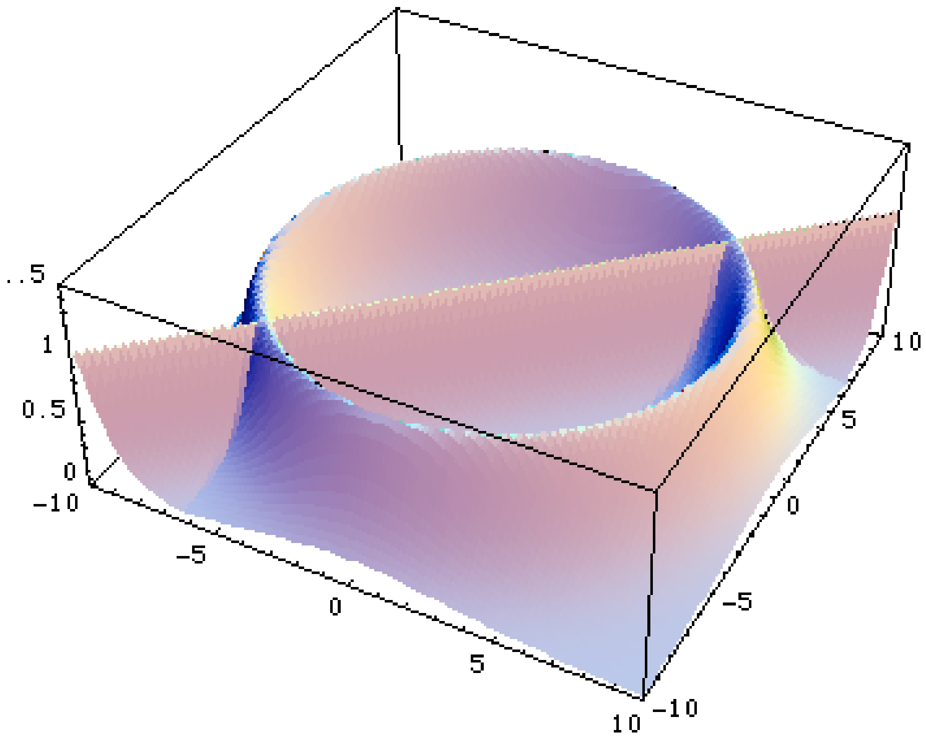
$p_1(x)$



$p_2(x)$



Multi-channel



$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x)$$

with
$$\sum_{i=1}^{n} \alpha_i = 1$$

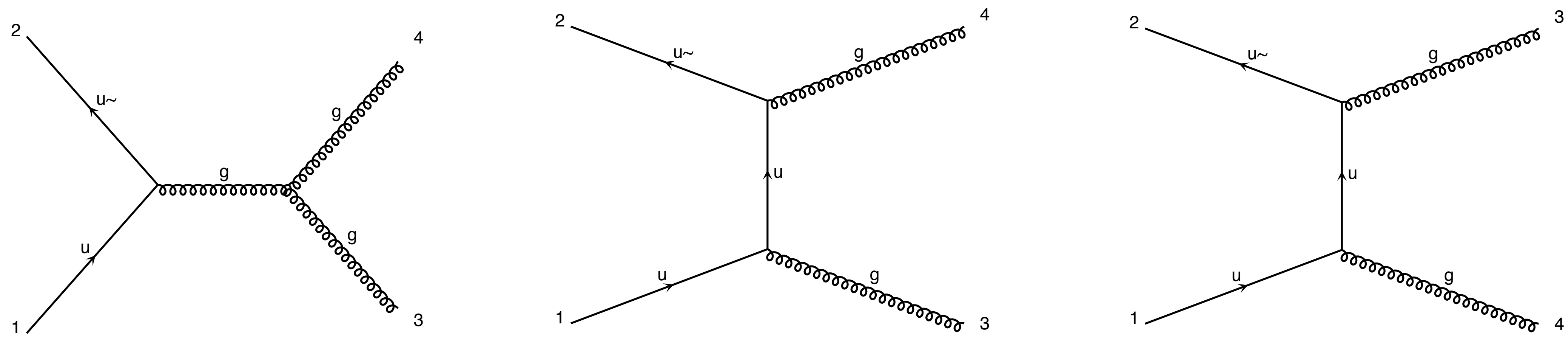
Then,

$$I = \int f(x)dx = \sum_{i=1}^{n} \alpha_i \int \frac{f(x)}{p(x)} p_i(x)dx$$

$$\frac{f(x)}{p(x)} \approx 1$$

Exemple of use

- Phase-Space integration
- Parton Shower
- Hadronization



$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2} \qquad \propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2} \qquad \propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

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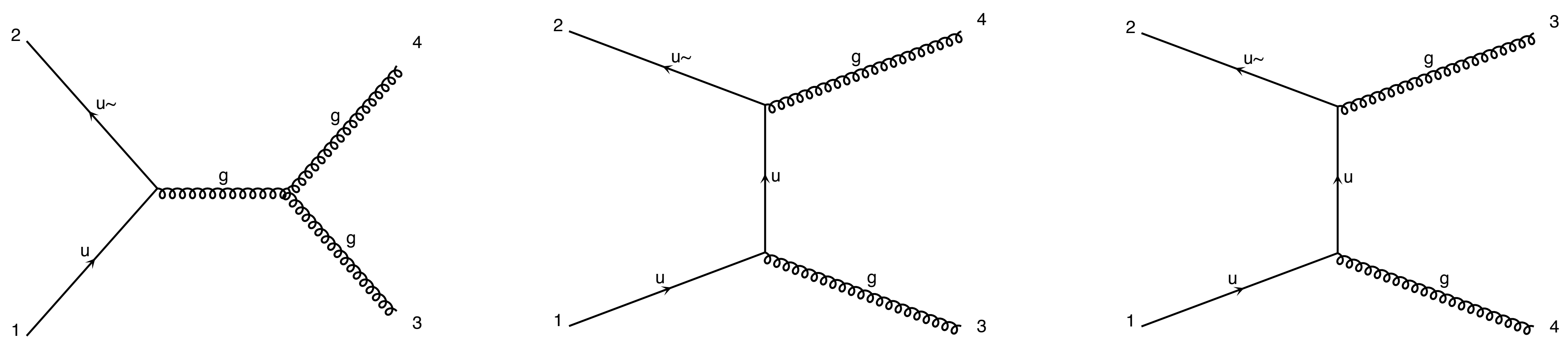

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Three very different pole structures contributing to the same matrix element.

Multi-channel based on single diagrams*

*Method used in MadGraph

Does a basis exist?

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

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$$\approx 1$$

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Key Idea

- Any single diagram is "easy" to integrate (pole structures/ suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
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N Integral

- Errors add in quadrature so no extra cost
- "Weight" functions already calculated during $|M|^2$ calculation
- Parallel in nature

To Remember

- Phase-Space integration are difficult
- We need to know the function
  - ➡ Be careful with cut (they change the function)
- Split the function in a sum (one for each structure) and integrate each of those separately
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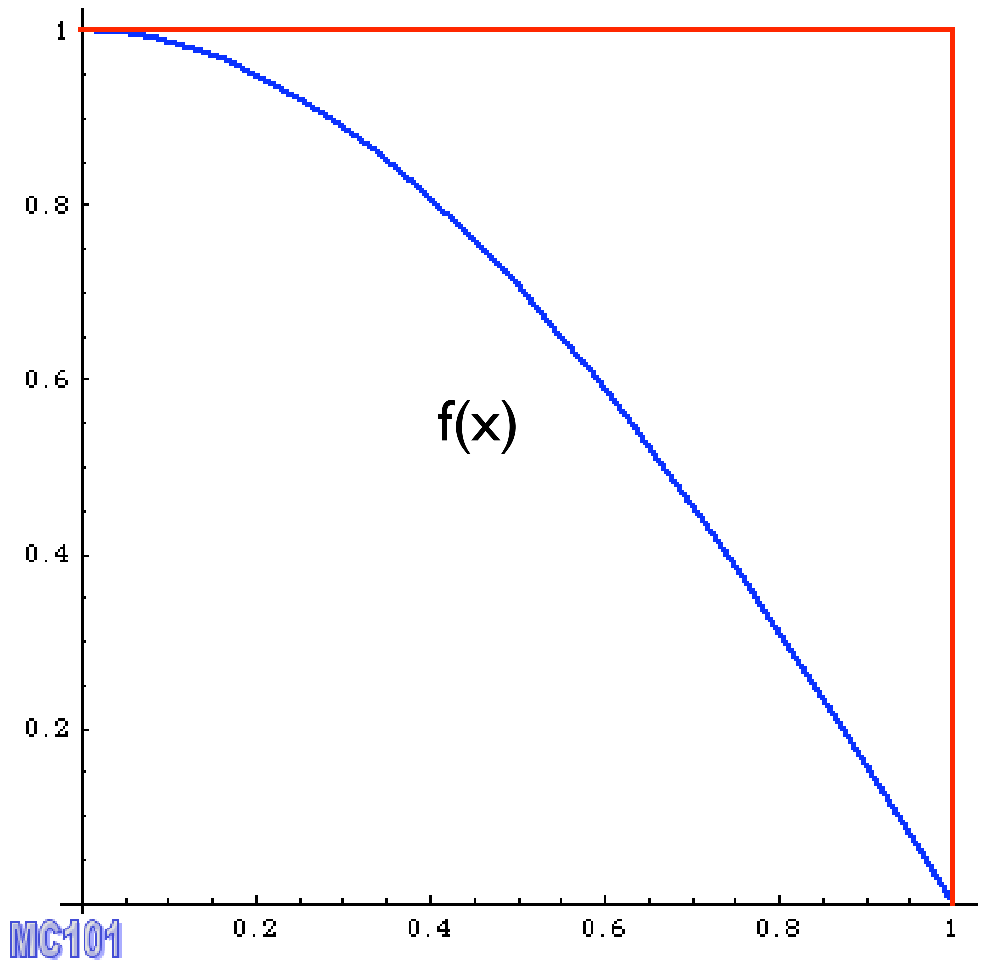
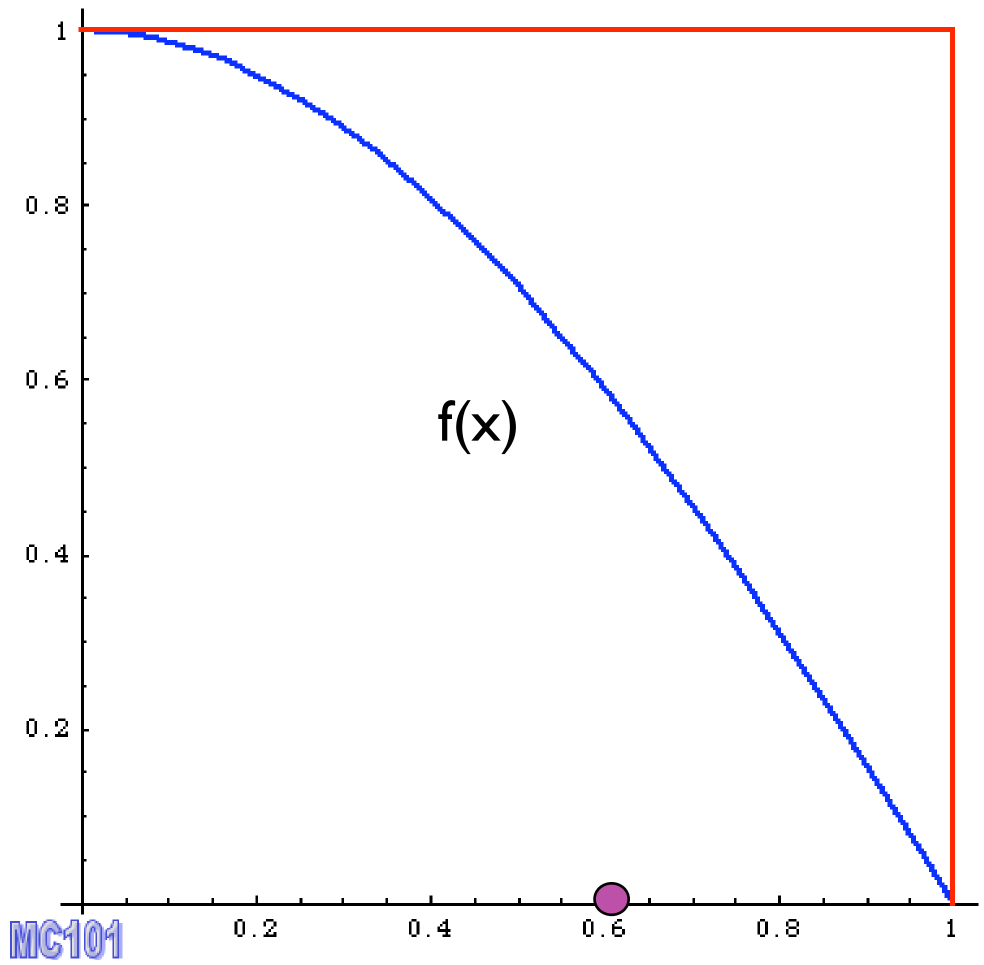
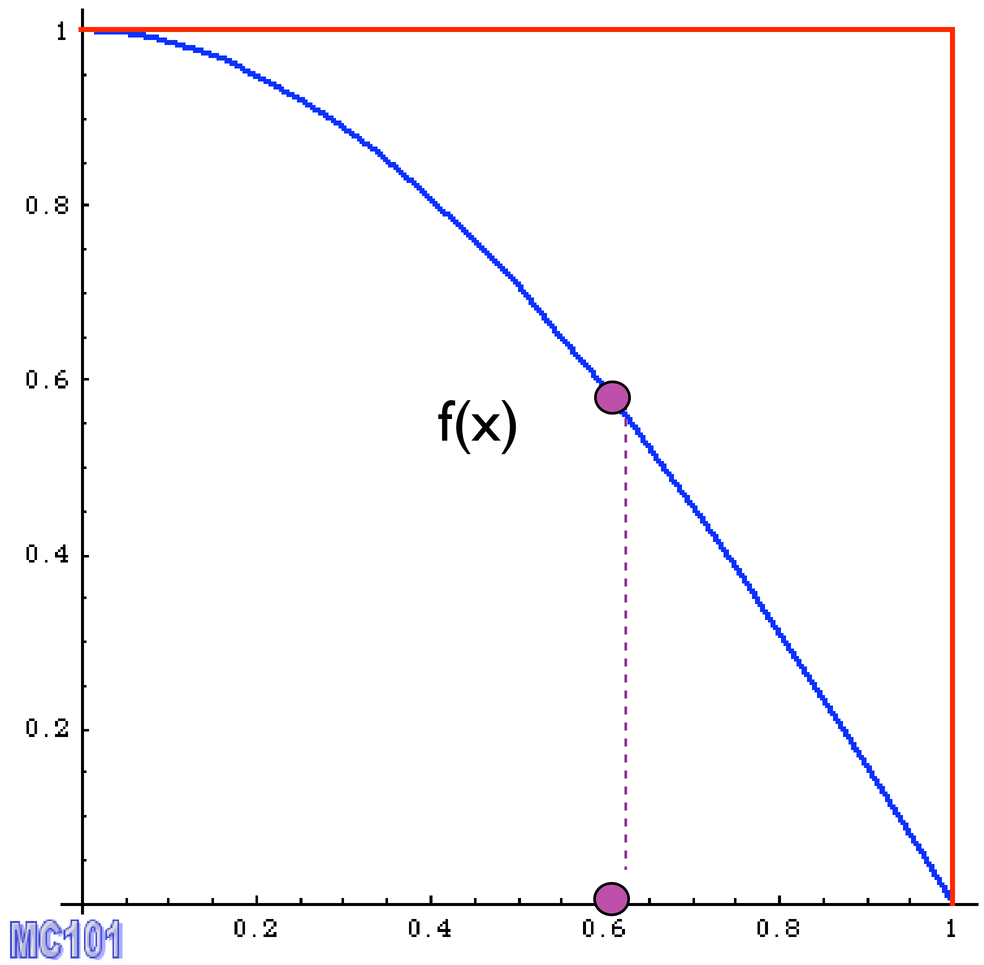

1
0.8
0.6
0.4
0.2
f(x)
MC101
0.2
0.4
0.6
0.8
1

Event generation



1. pick x

Event generation



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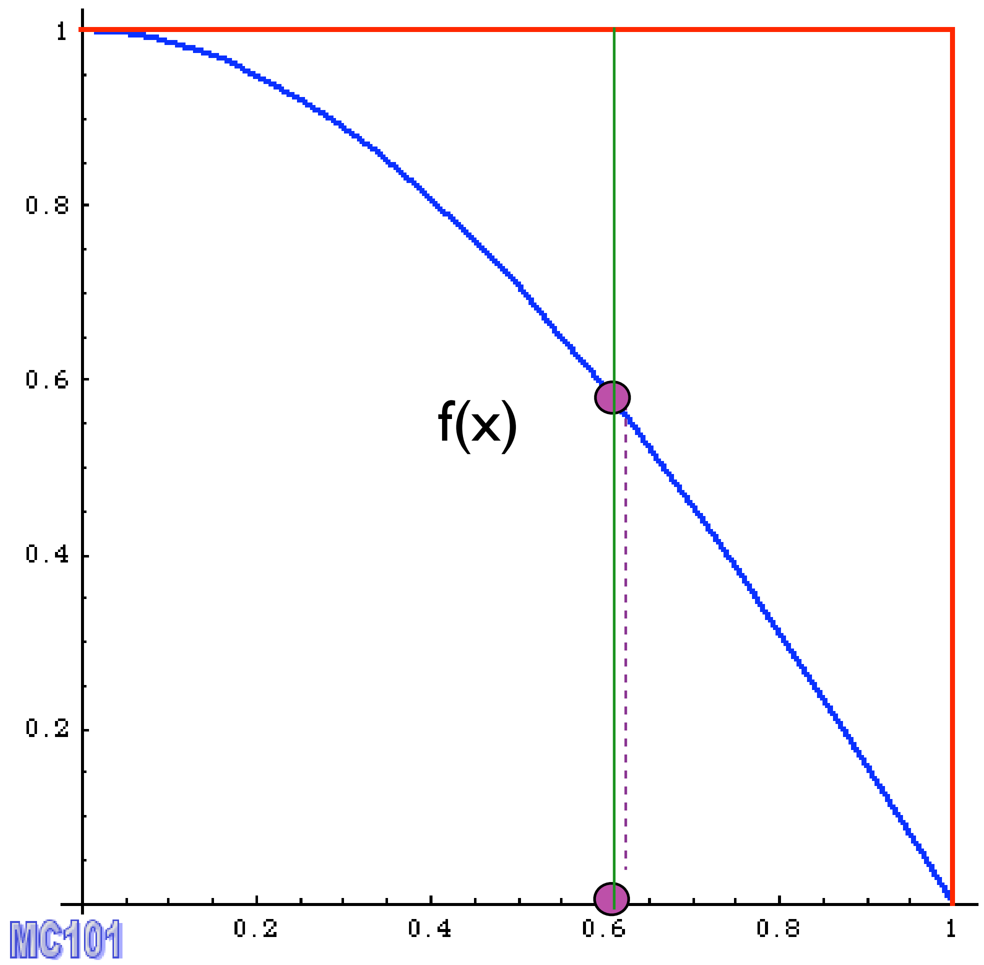
Event generation



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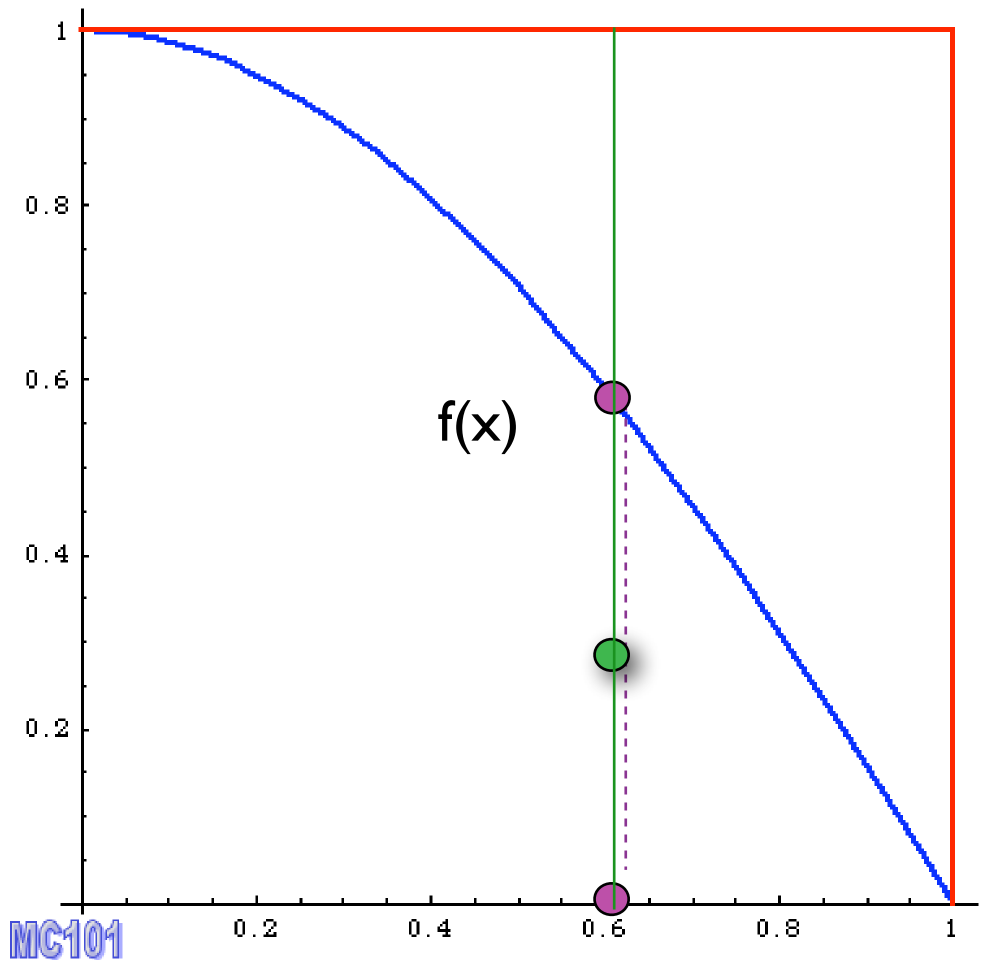

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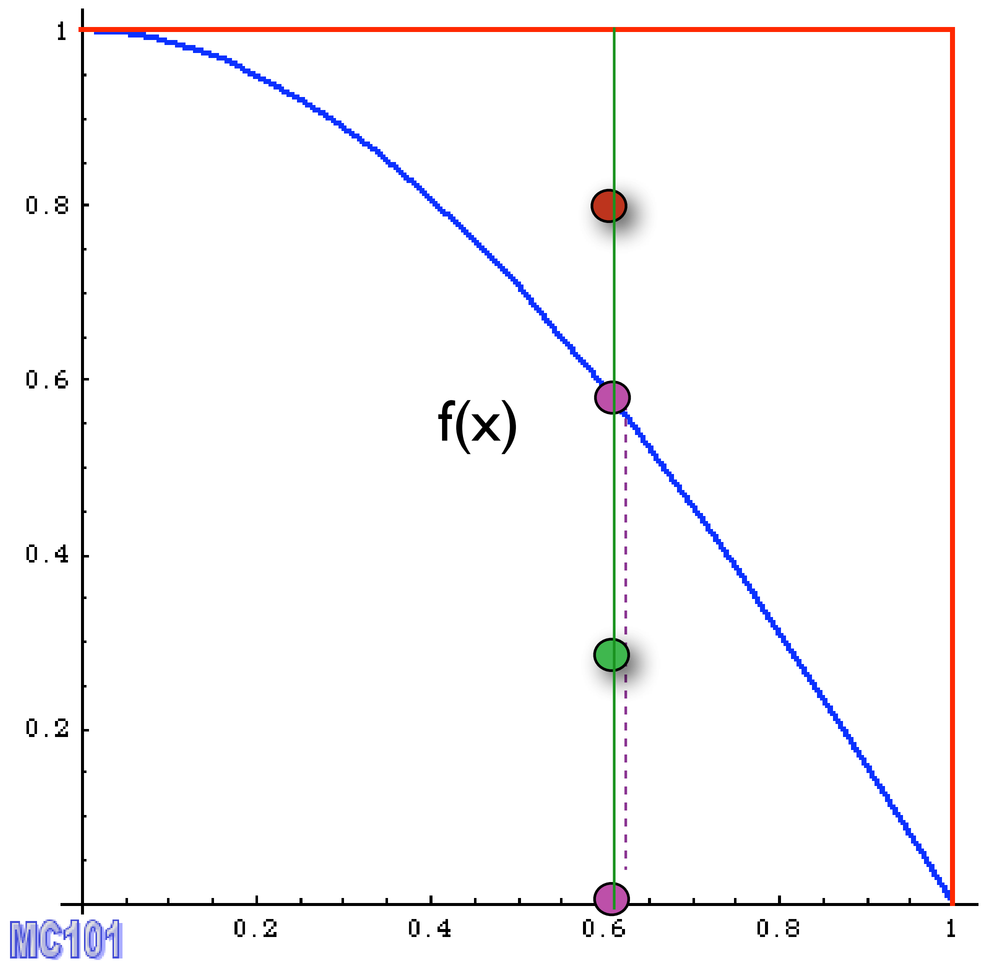
   if f(x)>y accept event,

Event generation



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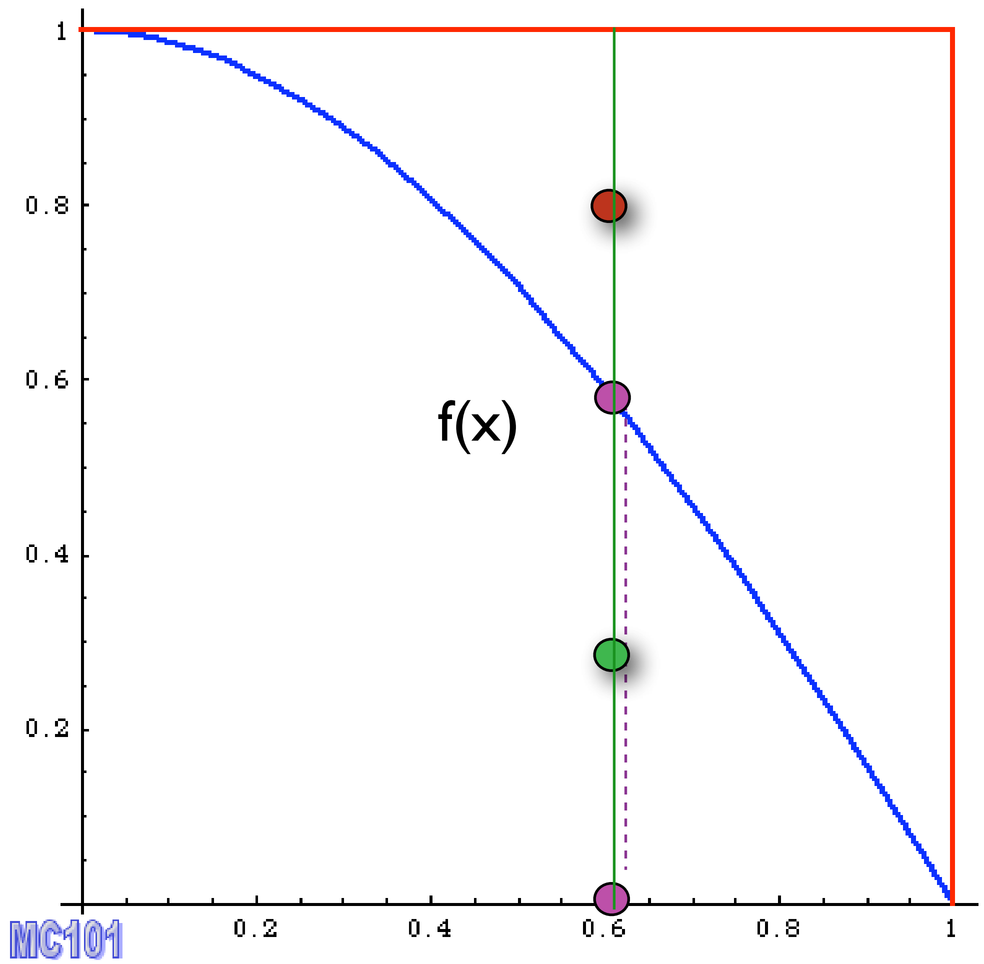

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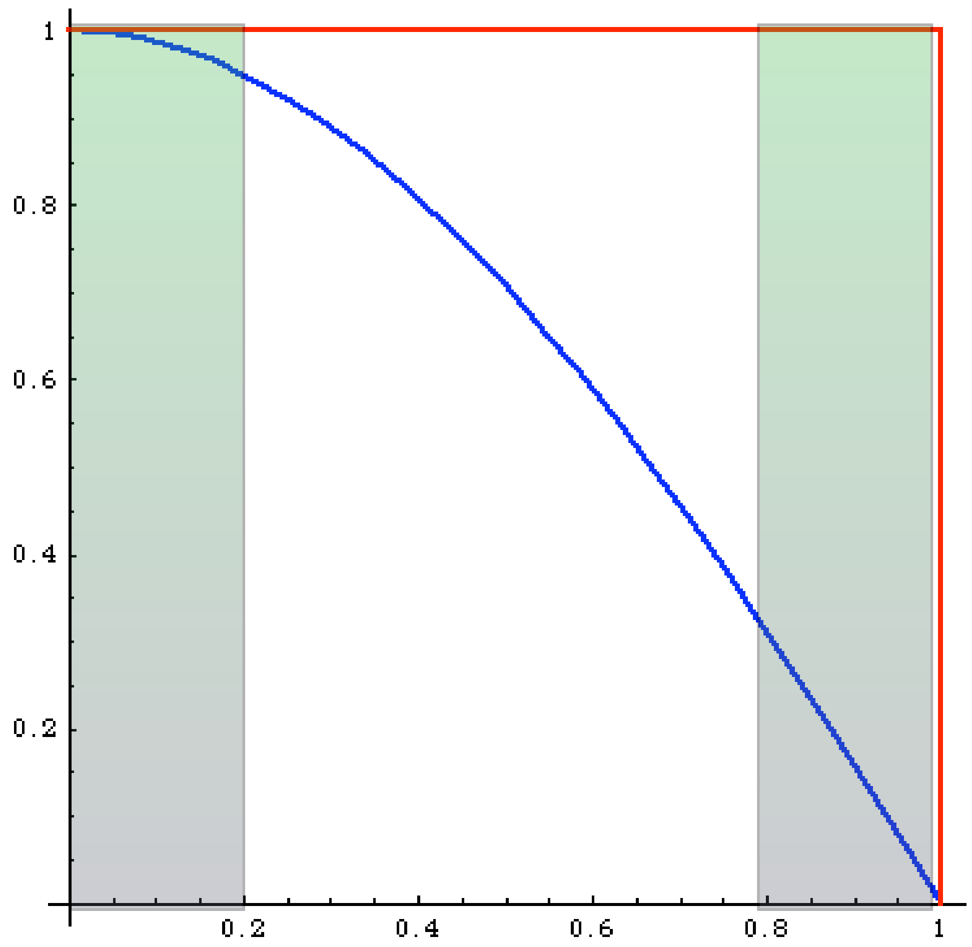
$$I = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

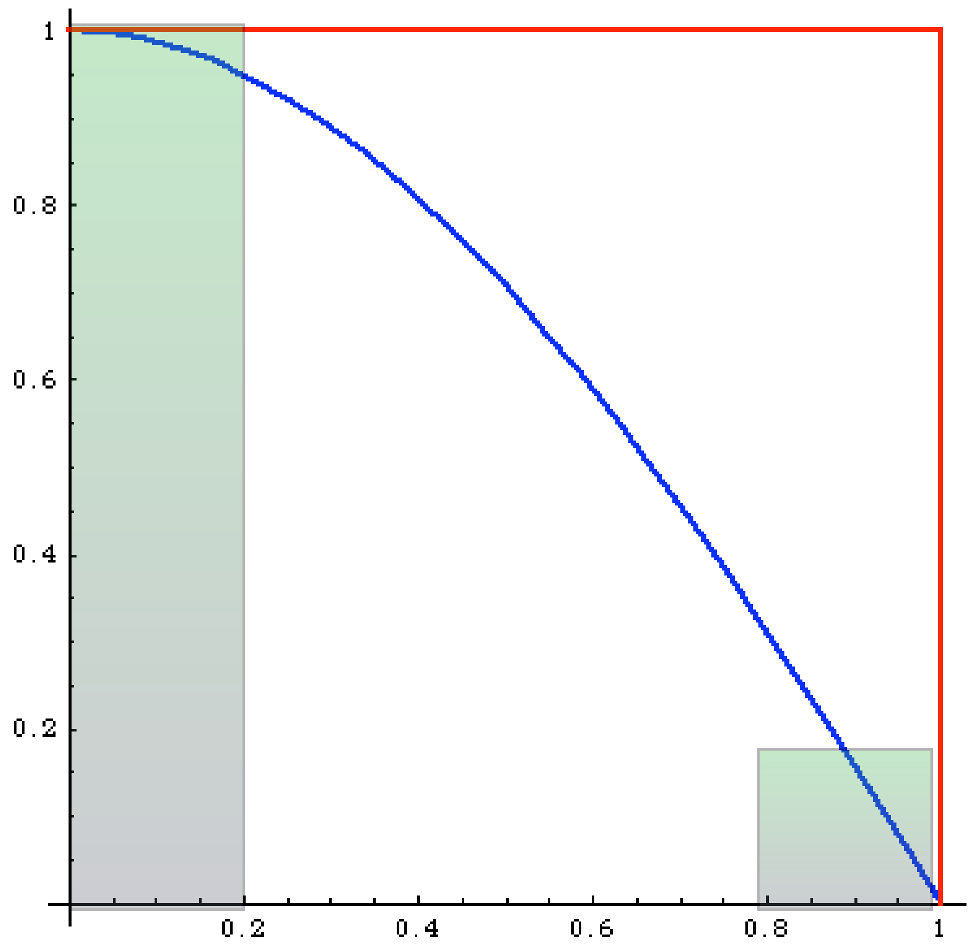


What's the difference between weighted and unweighted?

Weighted:

Same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



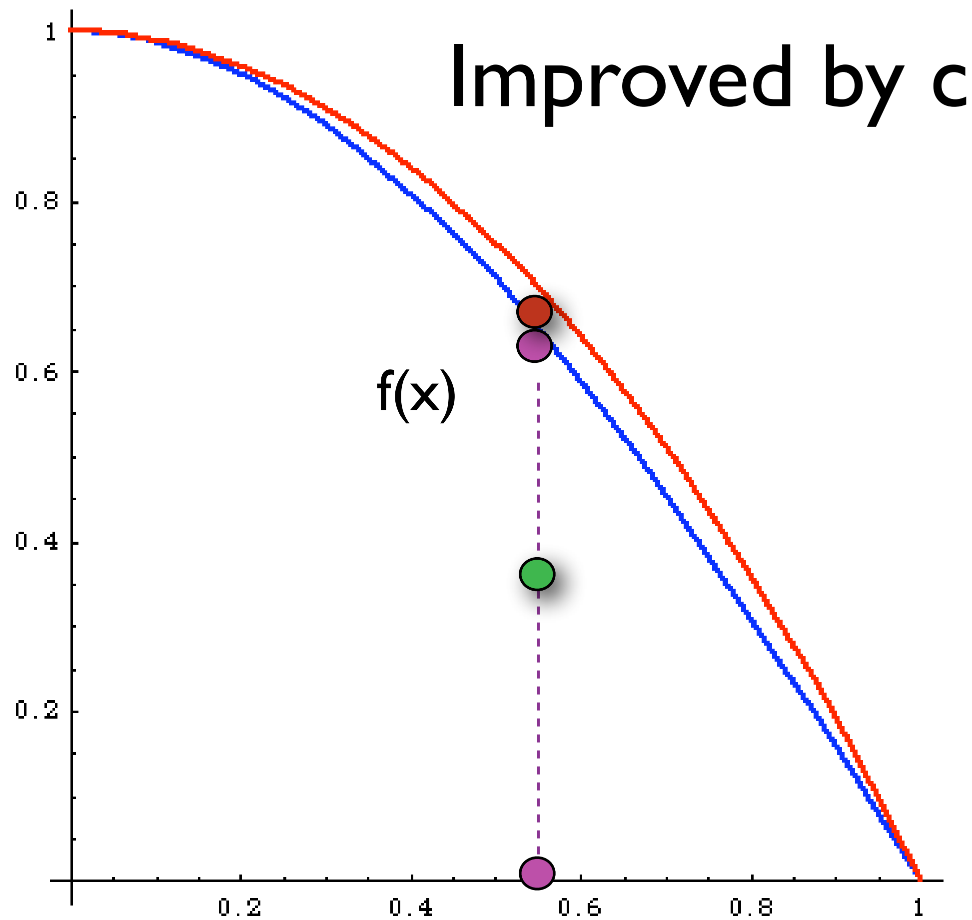
What's the difference between weighted and unweighted?

Unweighted:

events is proportional to the probability of areas of phase space:
events have all the same weight ("unweighted")

Events distributed as in nature

Event generation



Improved by combining with importance sampling:

1. pick x distributed as p(x)
2. calculate f(x) and p(x)
3. pick 0<y<f/p(max)
4. Compare:
   if f(x)>y p(x) accept event,
   else reject it.

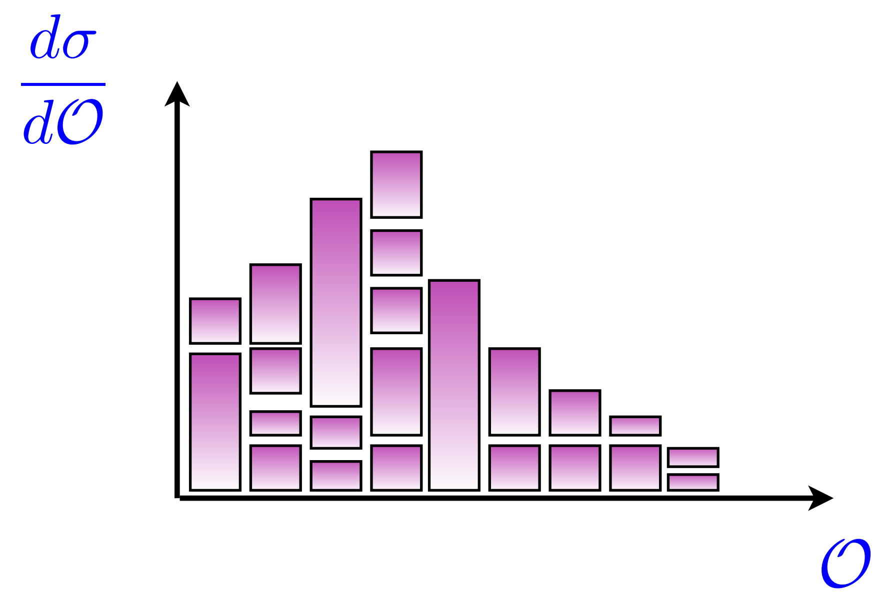
much better efficiency!!! (need to check this page likely wrong in the description)

Event generation

MC integrator

Event generation

MC integrator



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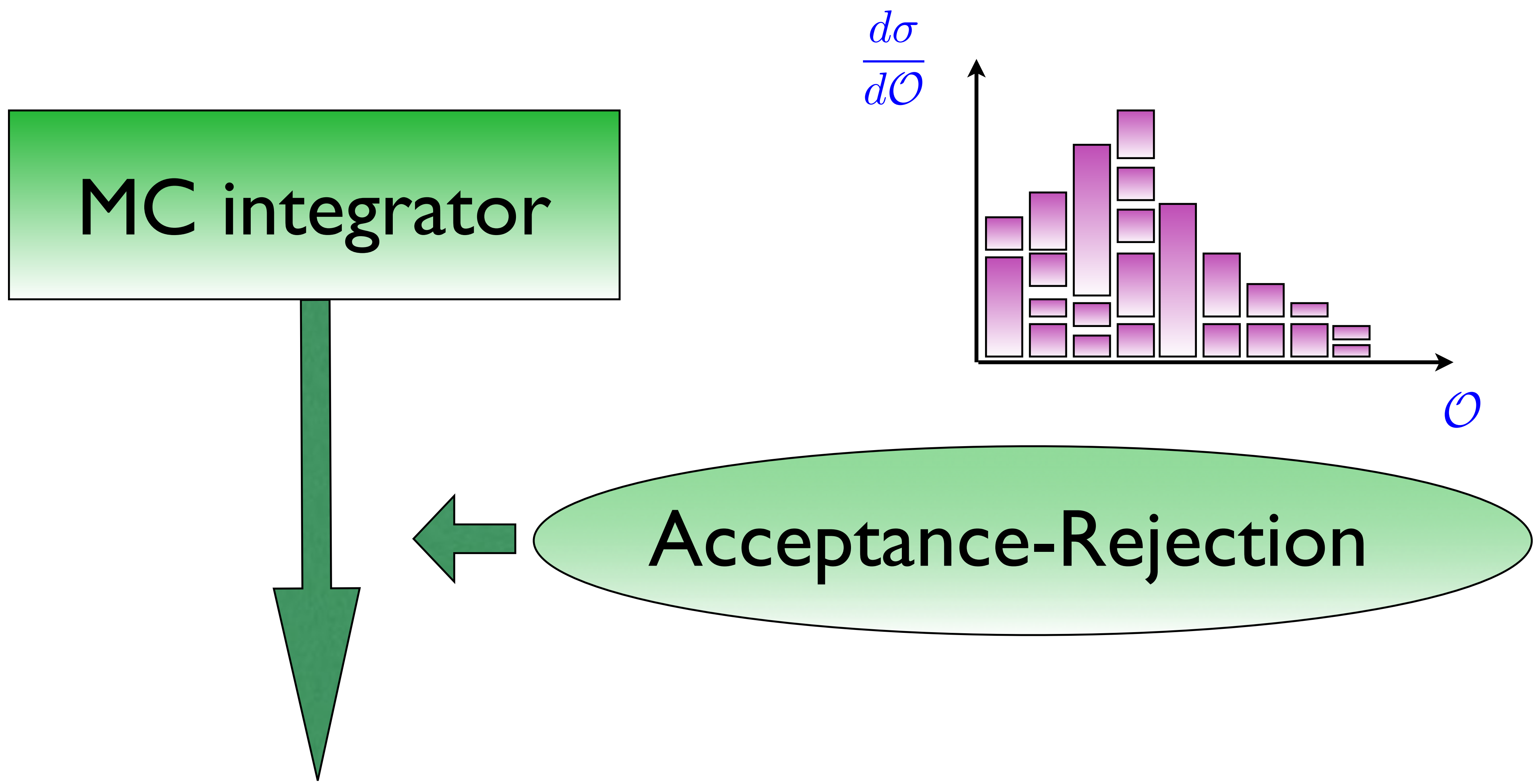

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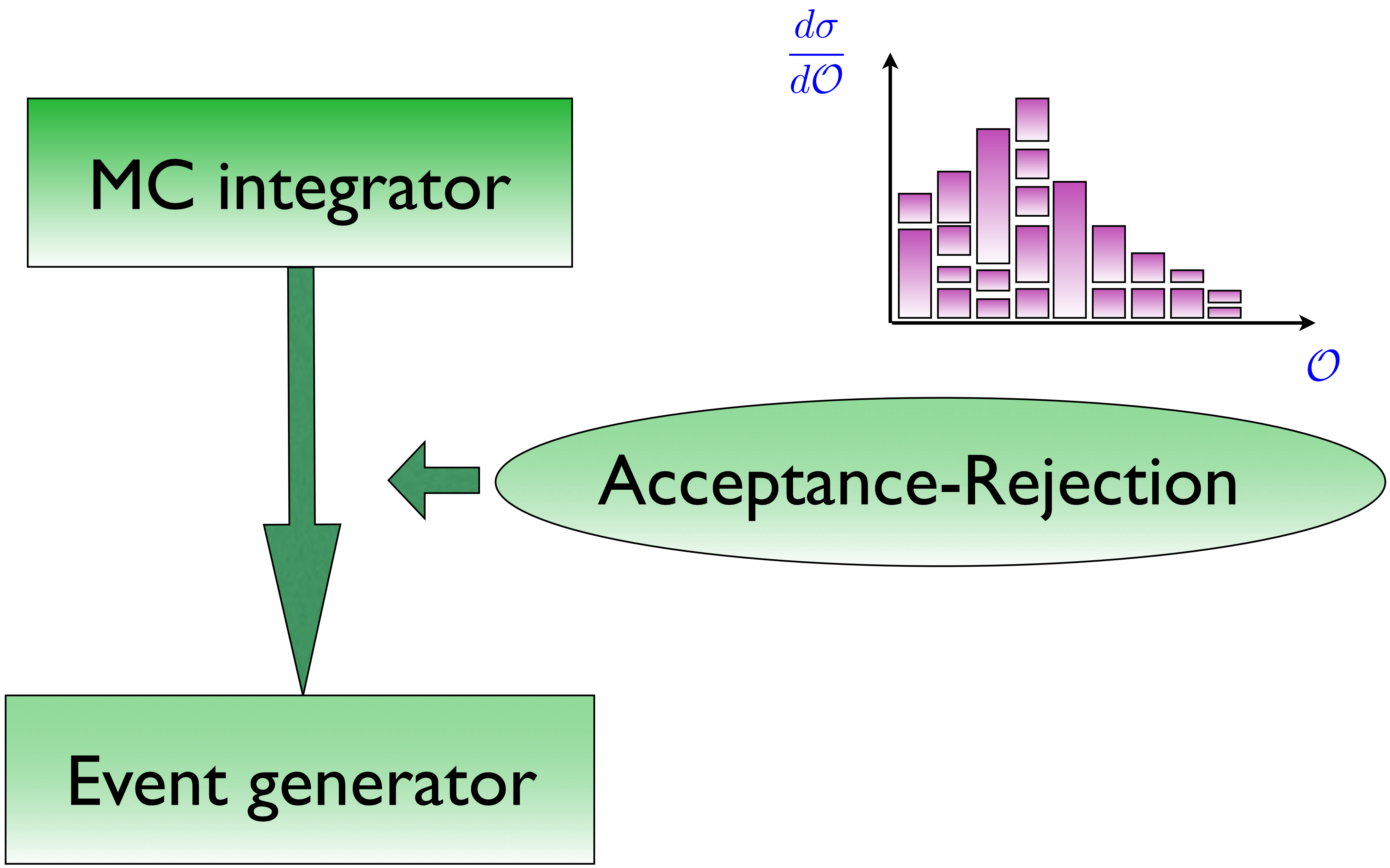
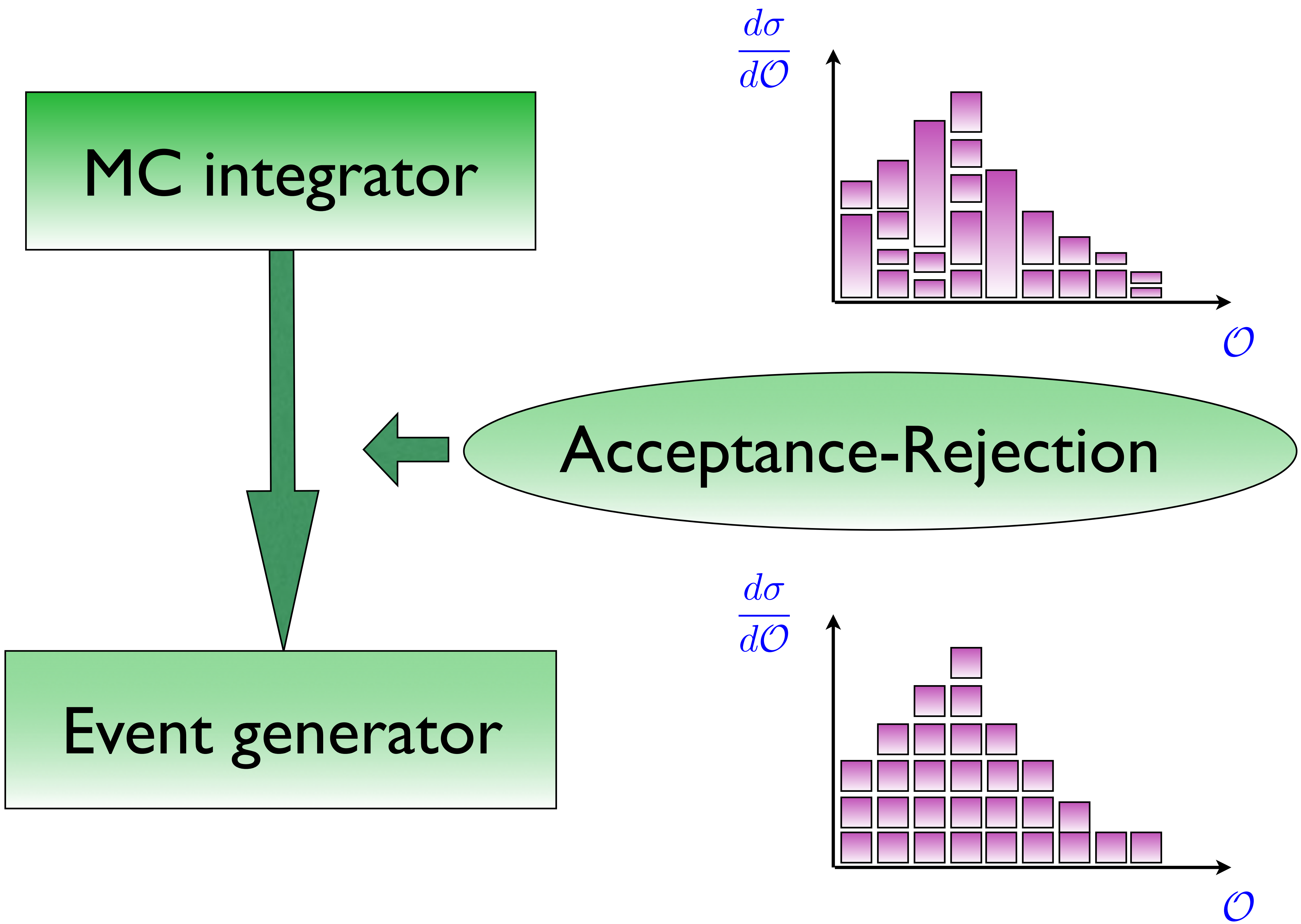
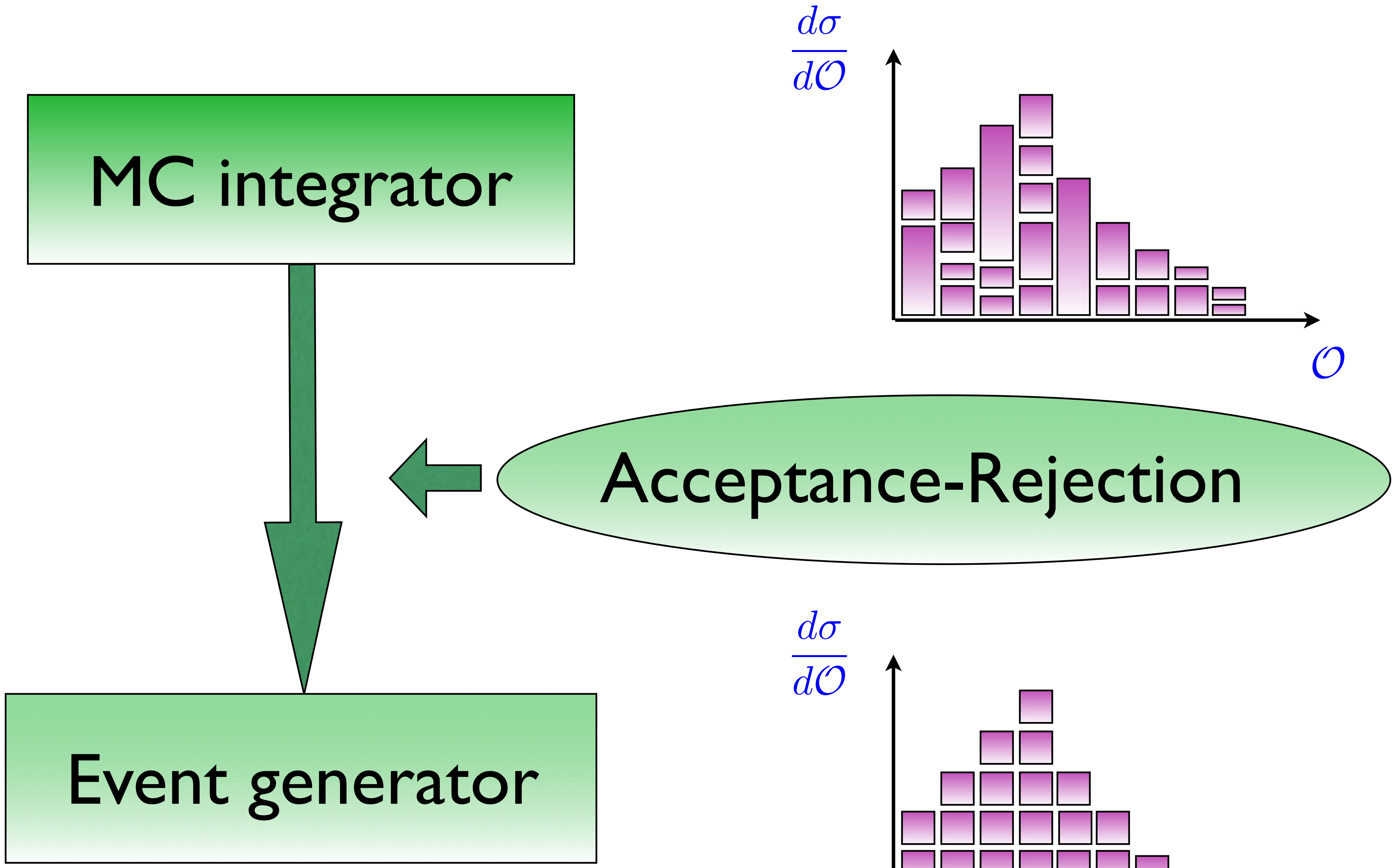



MC integrator
Acceptance-Rejection
Event generator
$\frac{d\sigma}{d\mathcal{O}}$
$\mathcal{O}$
$\frac{d\sigma}{d\mathcal{O}}$
$\mathcal{O}$

Event generation



☞ **This is possible only if f(x)<∞ AND has definite sign!**

To Remember

- Sample of unweighted events
  - ➡ Events distributed like nature
  - ➡ Need the function to be
    - ❑ Borned
    - ❑ Always positive
  - ➡ More efficient if the integration is more efficient
    - ❑ Same dependencies in the cut

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension
- Need to know the function
  - Impact on cut

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- Need to know the function
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Good Point

- Complex area of Integration
- Easy Error estimate
- quick estimation of the integral
- Possibility to have unweighted events

Type of MC Simulation

Parton shower

- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation

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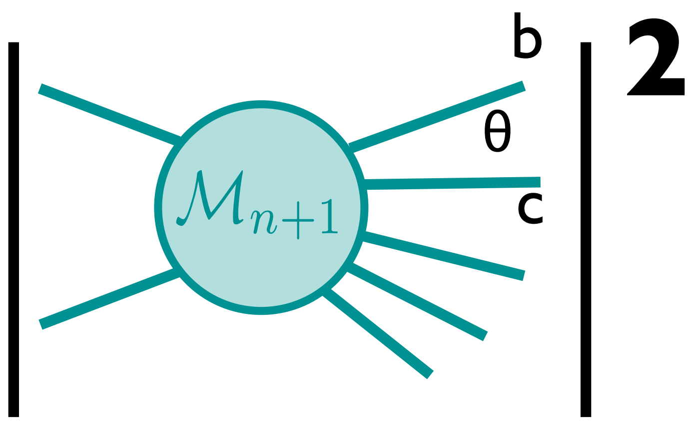
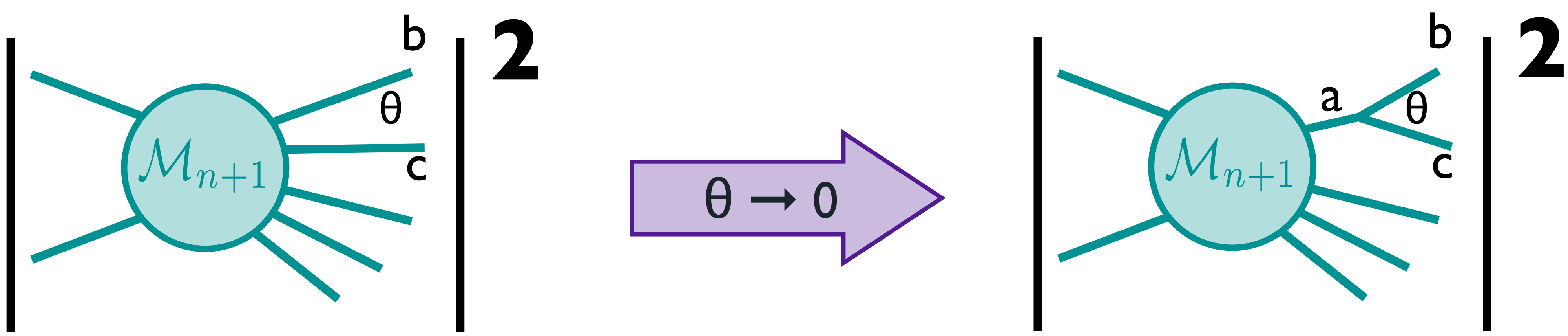
- Remember that parton-level cross sections for a hard process are inclusive in anything else.
  E.g. for LO Drell-Yan production **all** radiation is included via PDFs (apart from non-perturbative power corrections)
- And finally we want to turn partons into hadrons (hadronization)....

Collinear factorization



- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.

Collinear factorization



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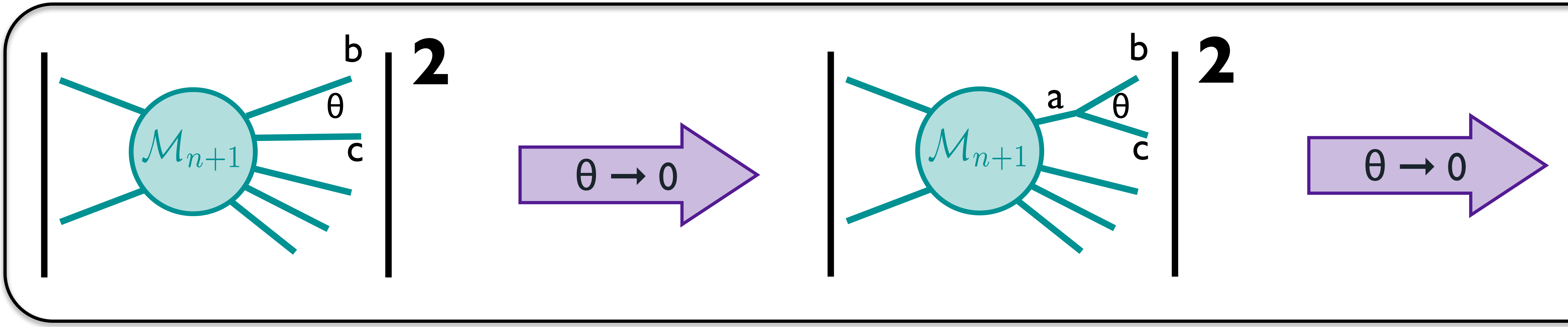

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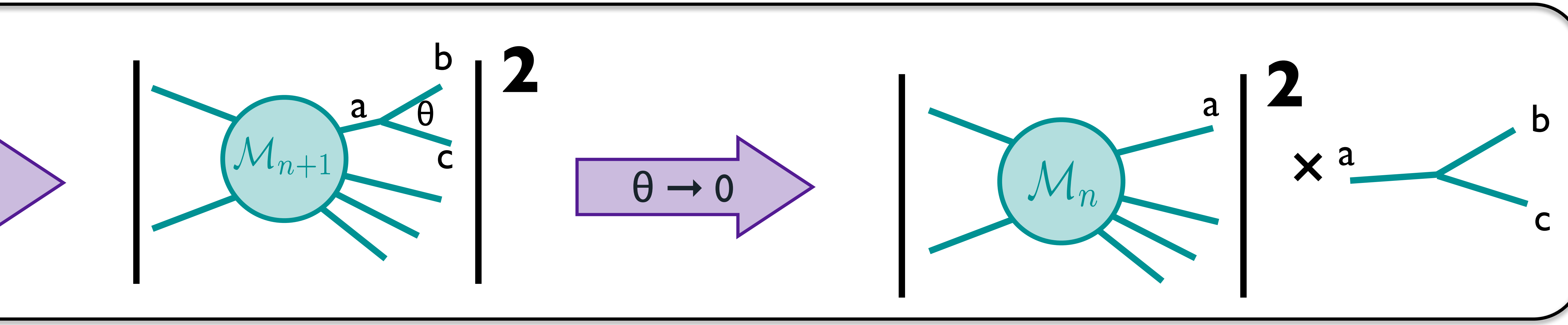

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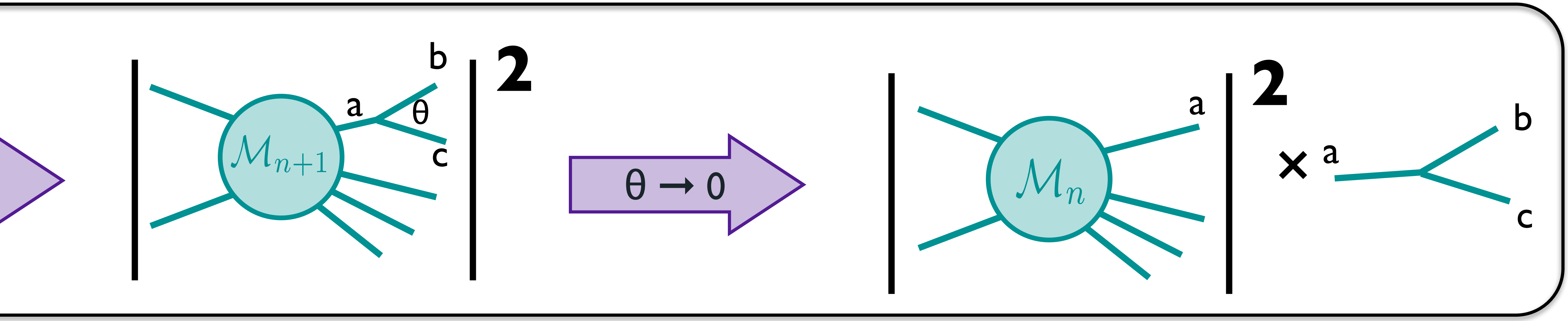
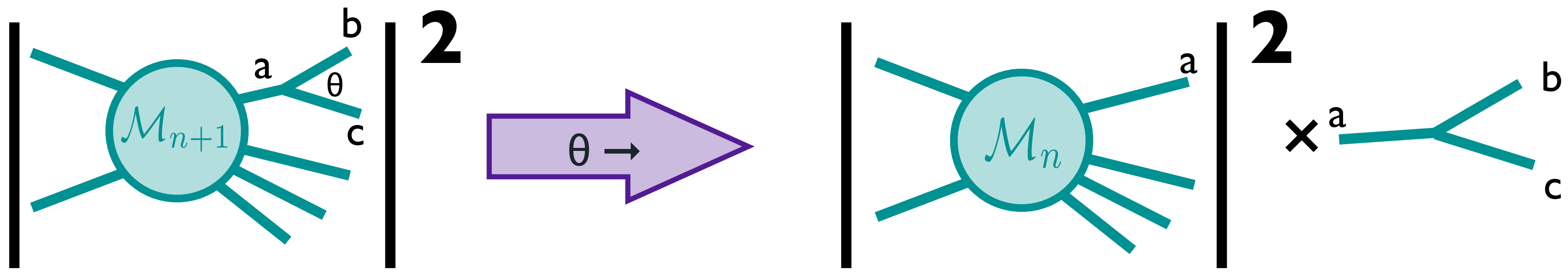


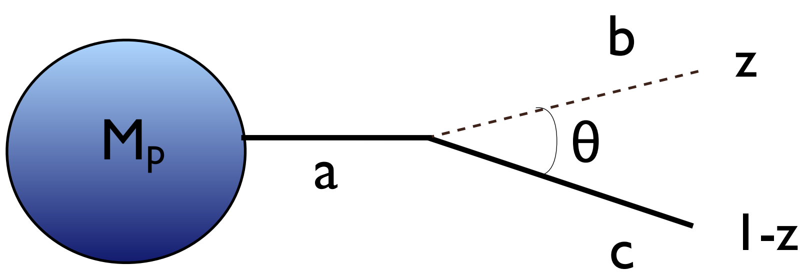
- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.
- The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability.

Collinear factorization



- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b+p_c)^2} \simeq \frac{1}{2E_bE_c(1-\cos\theta)} = \frac{1}{t}$$



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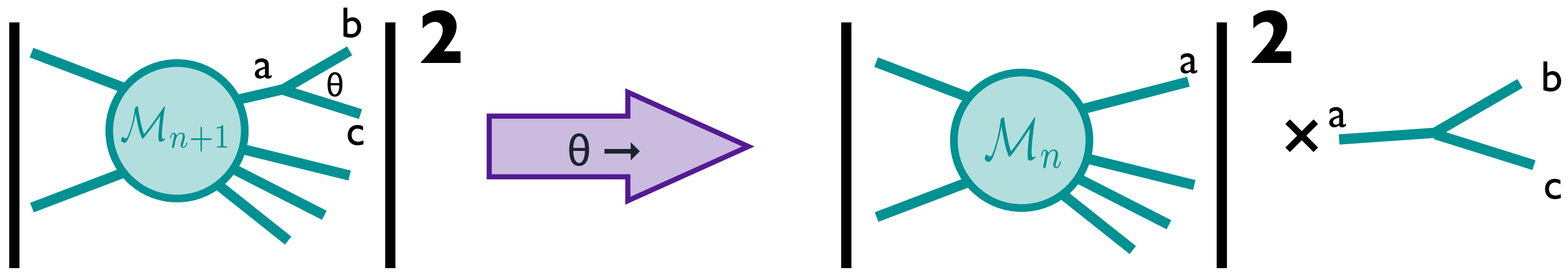

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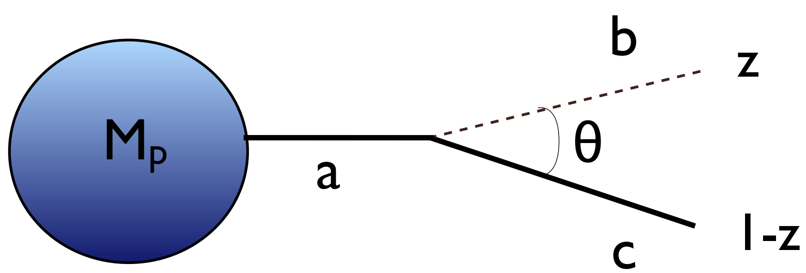
soft



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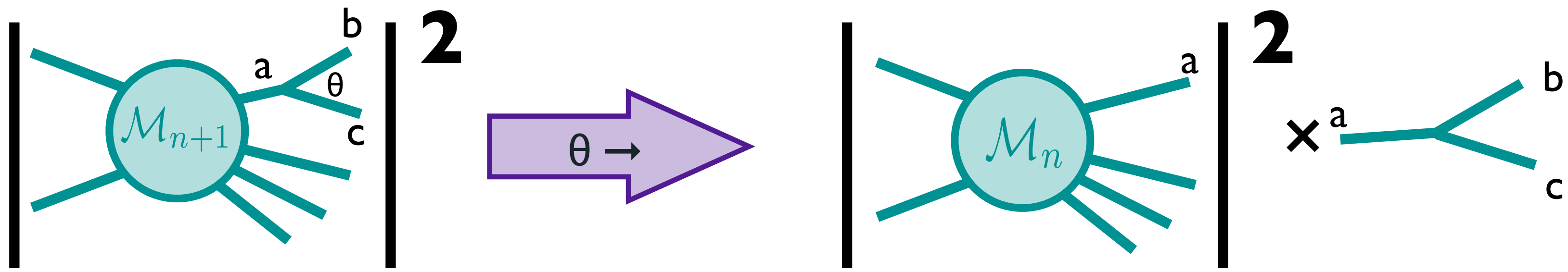

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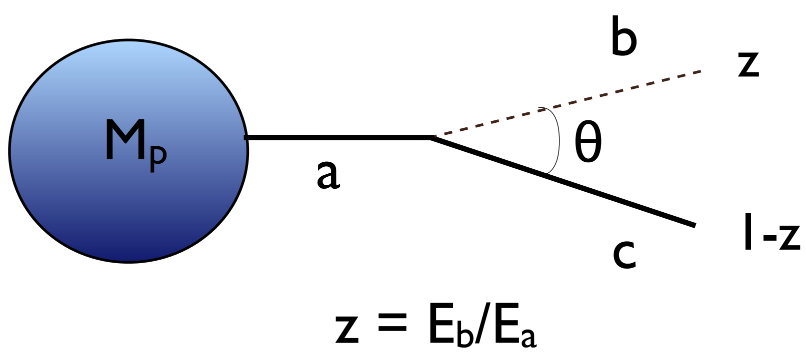
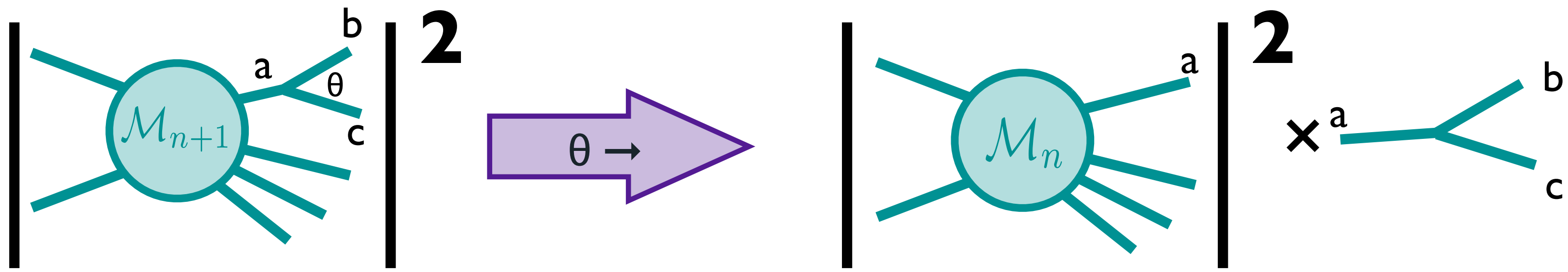


soft and collinear

divergencies

Collinear factorization



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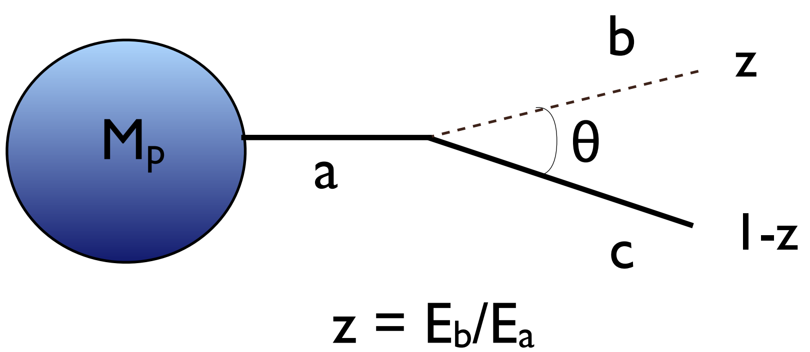


soft and **collinear**

divergencies

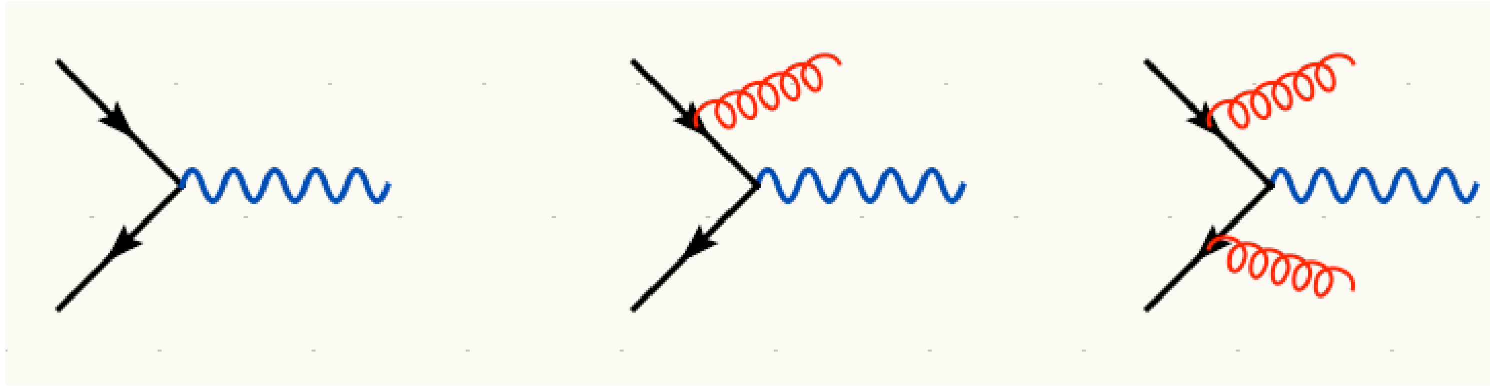
Collinear factorization:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

when θ is small.

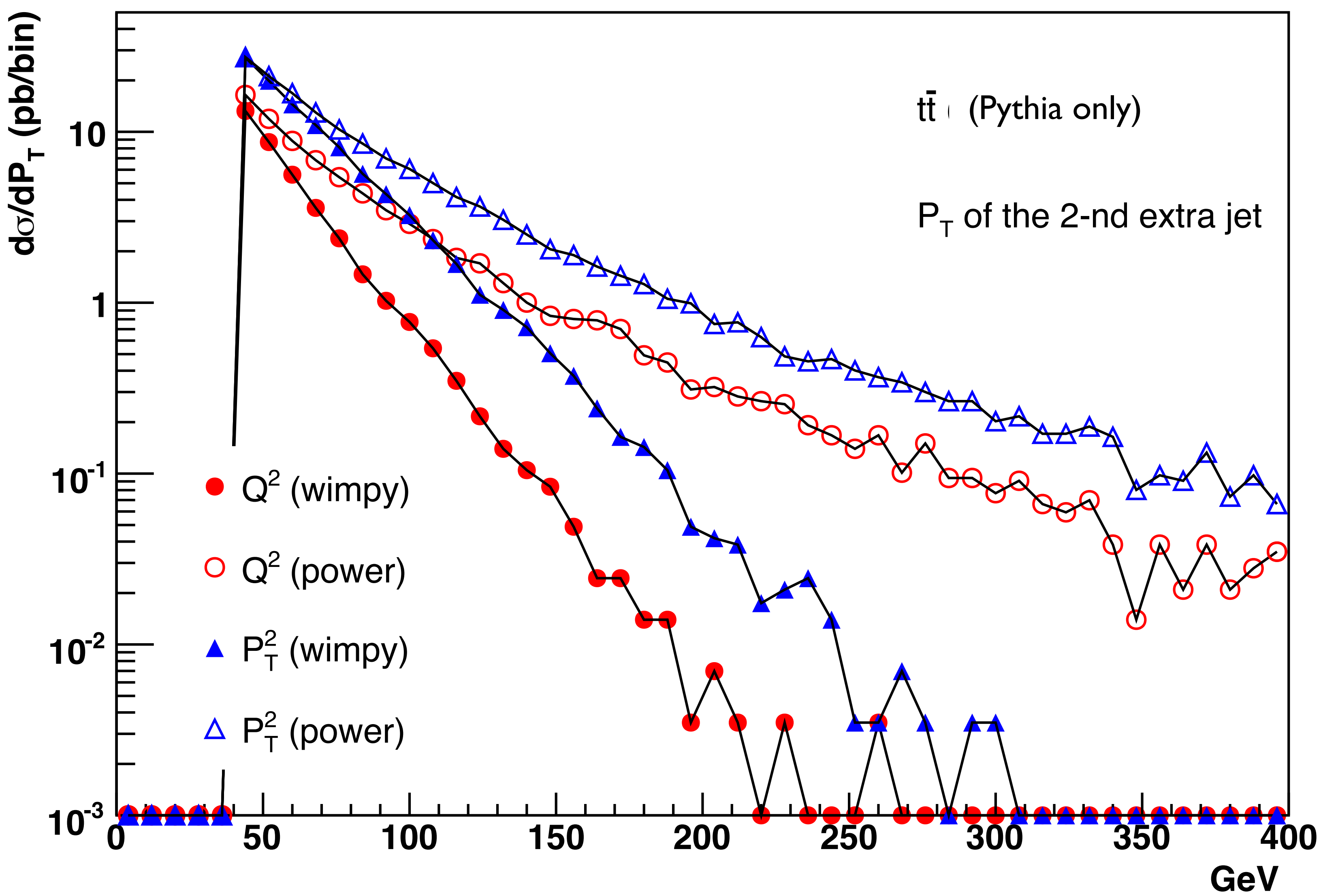
Merging ME with PS

PS →



PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)



Matrix Elements vs. Parton Showers

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ME

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

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Approaches are complementary: merge them!

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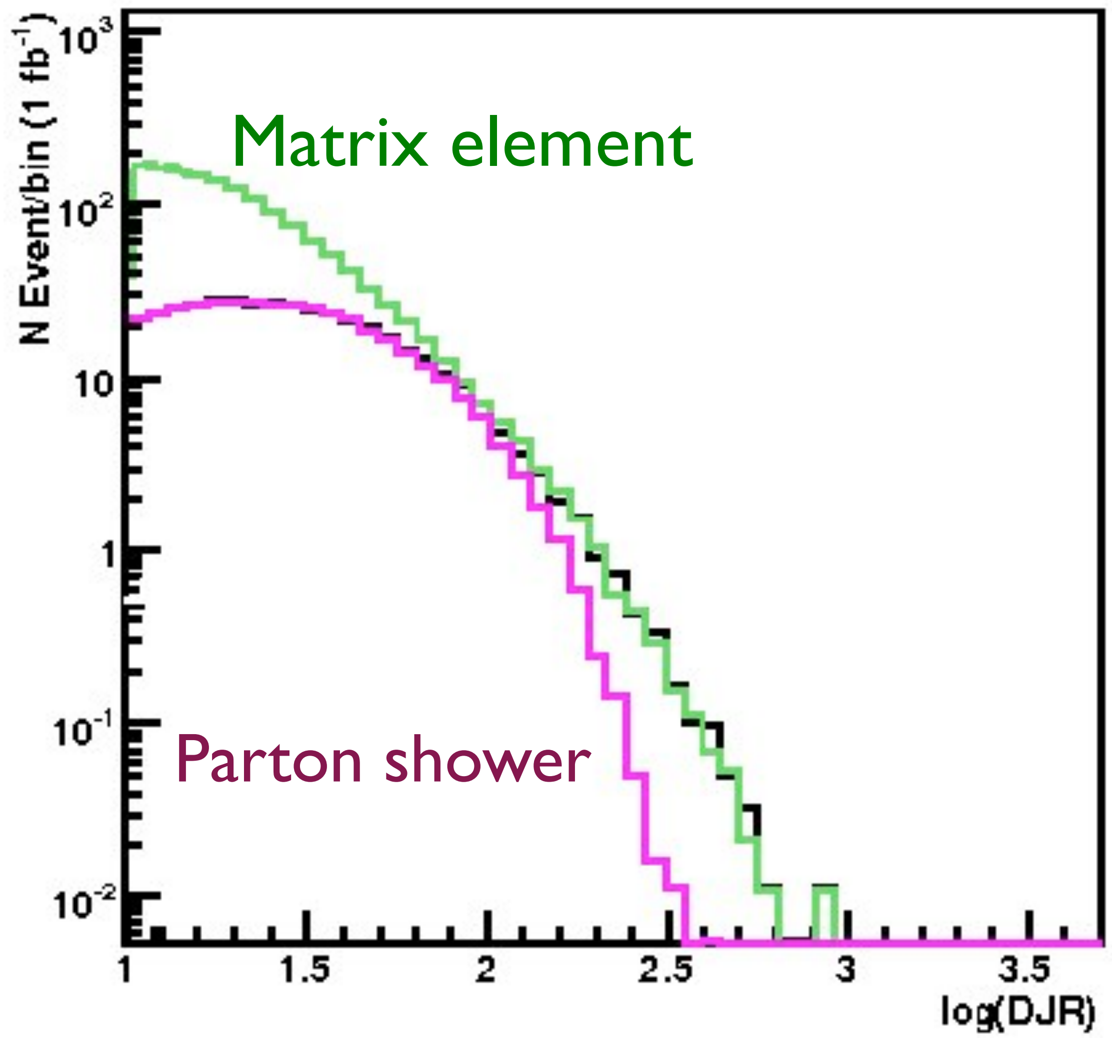
Approaches are complementary: merge them!

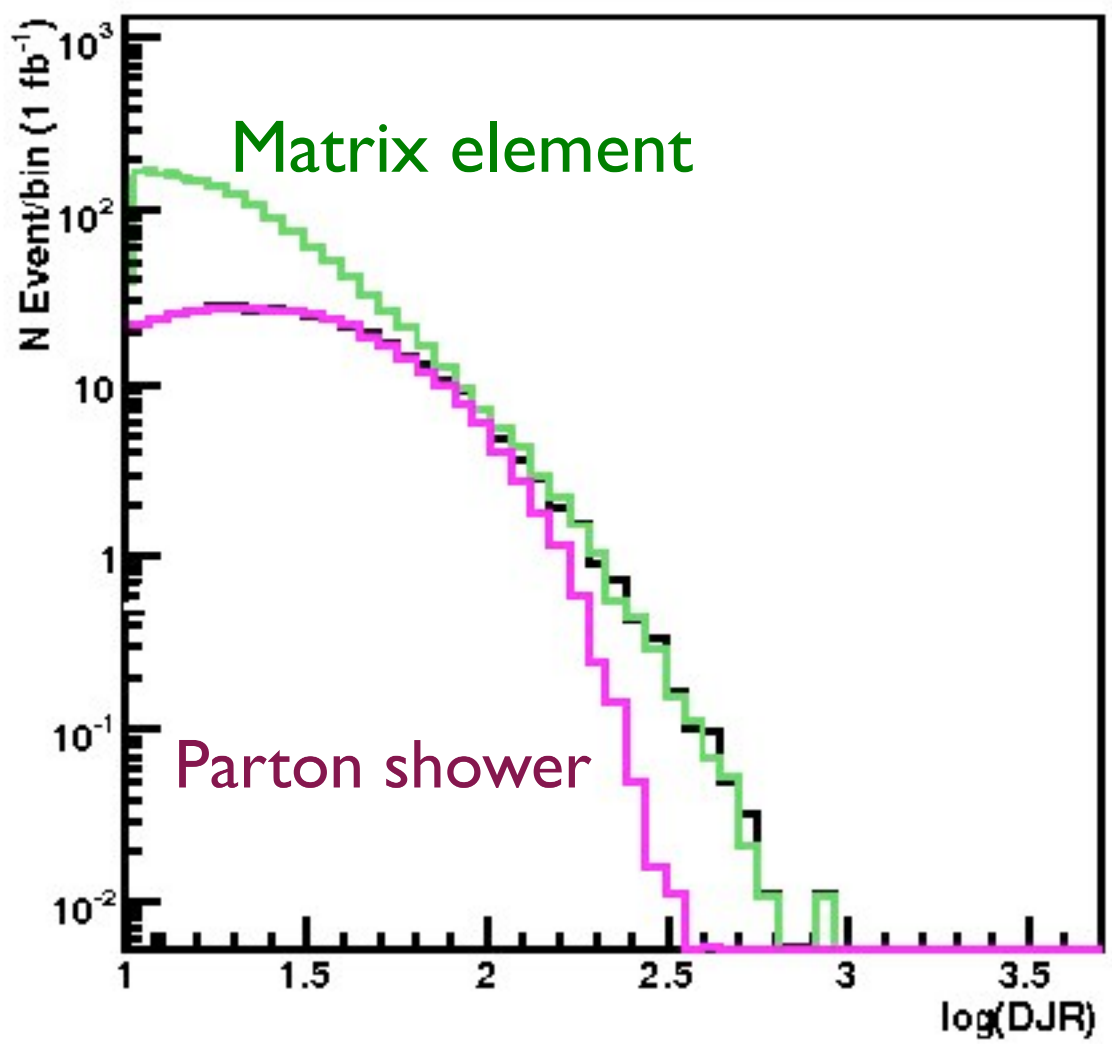
Difficulty: avoid double counting, ensure smooth distributions

Goal for ME-PS merging/matching



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

- Regularization of matrix element divergence



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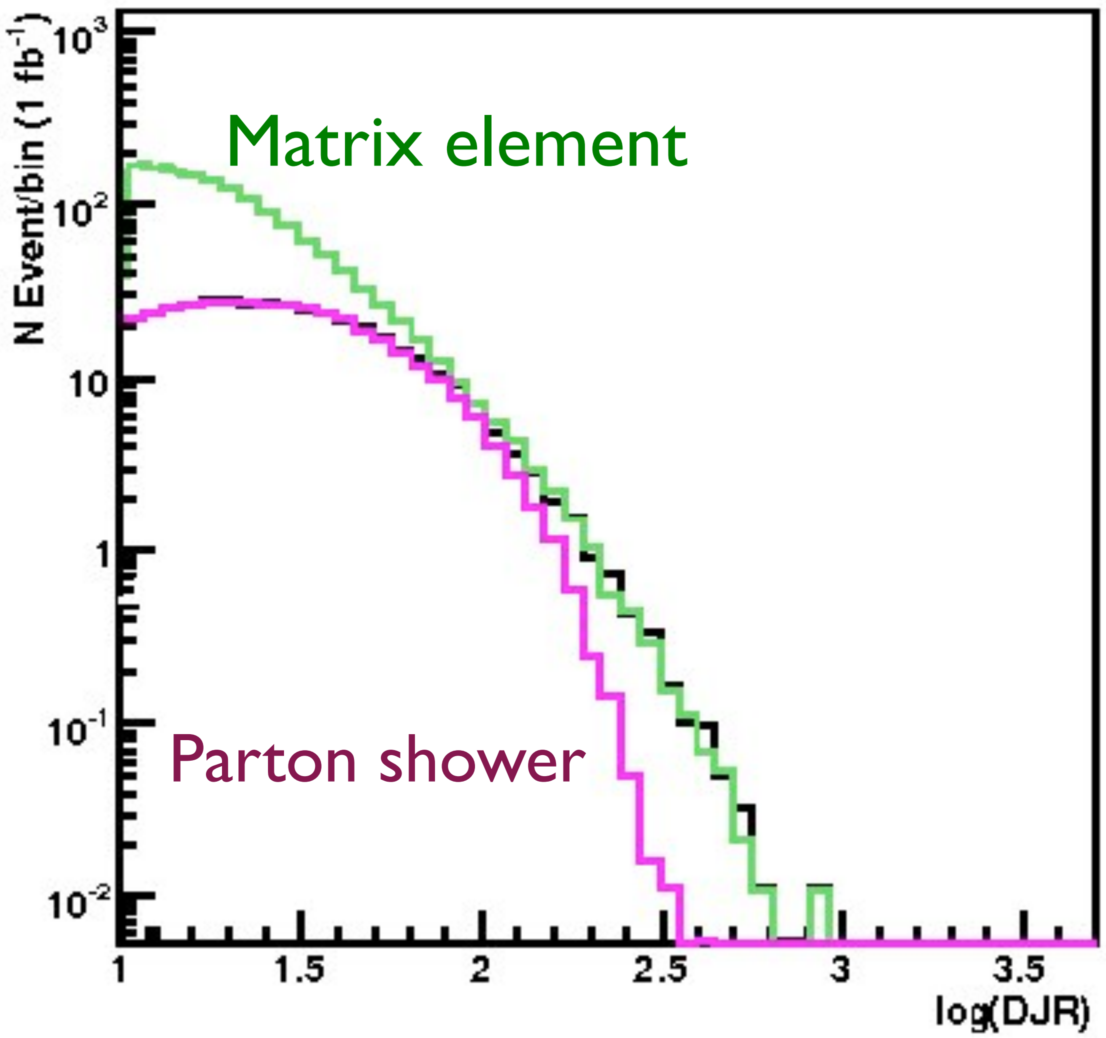

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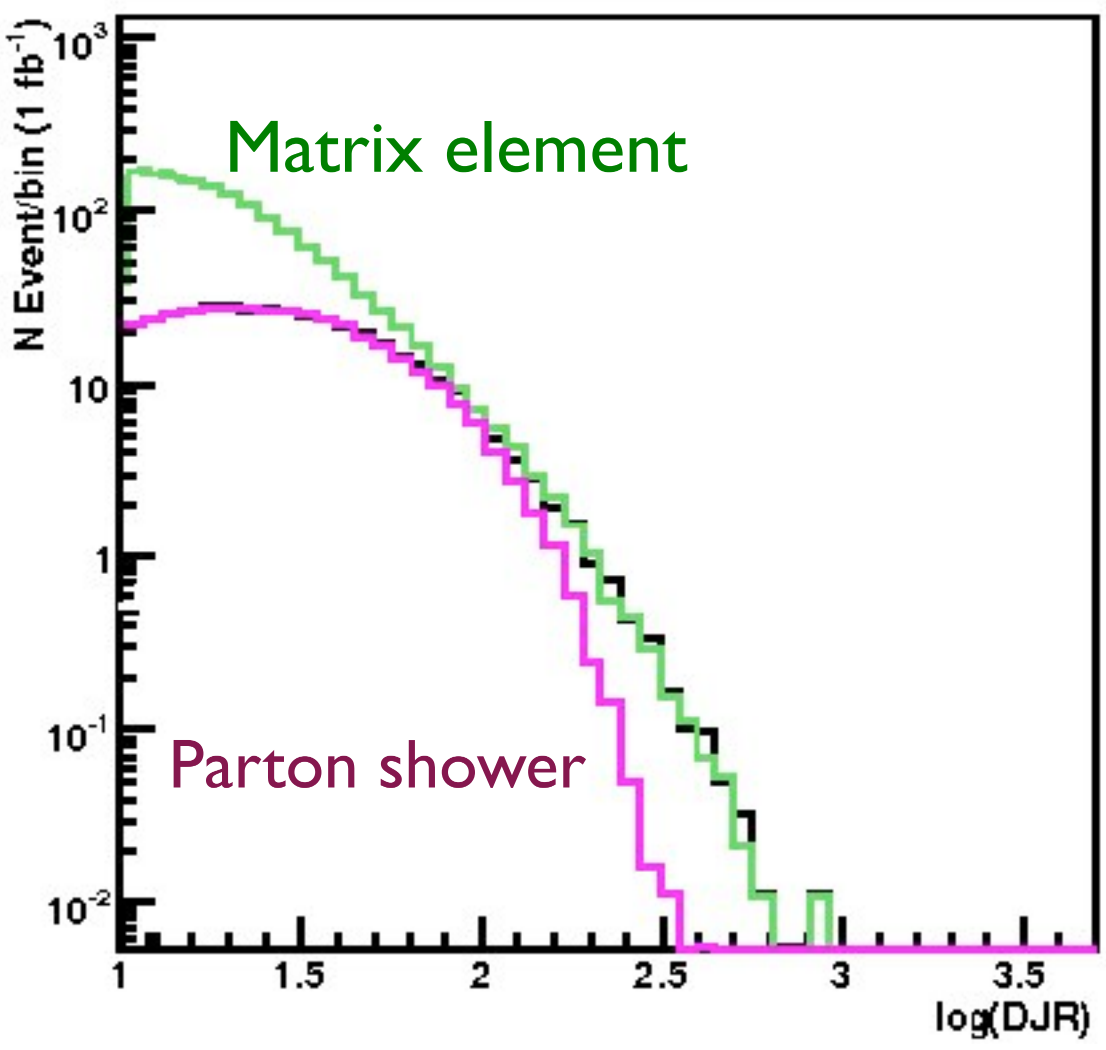

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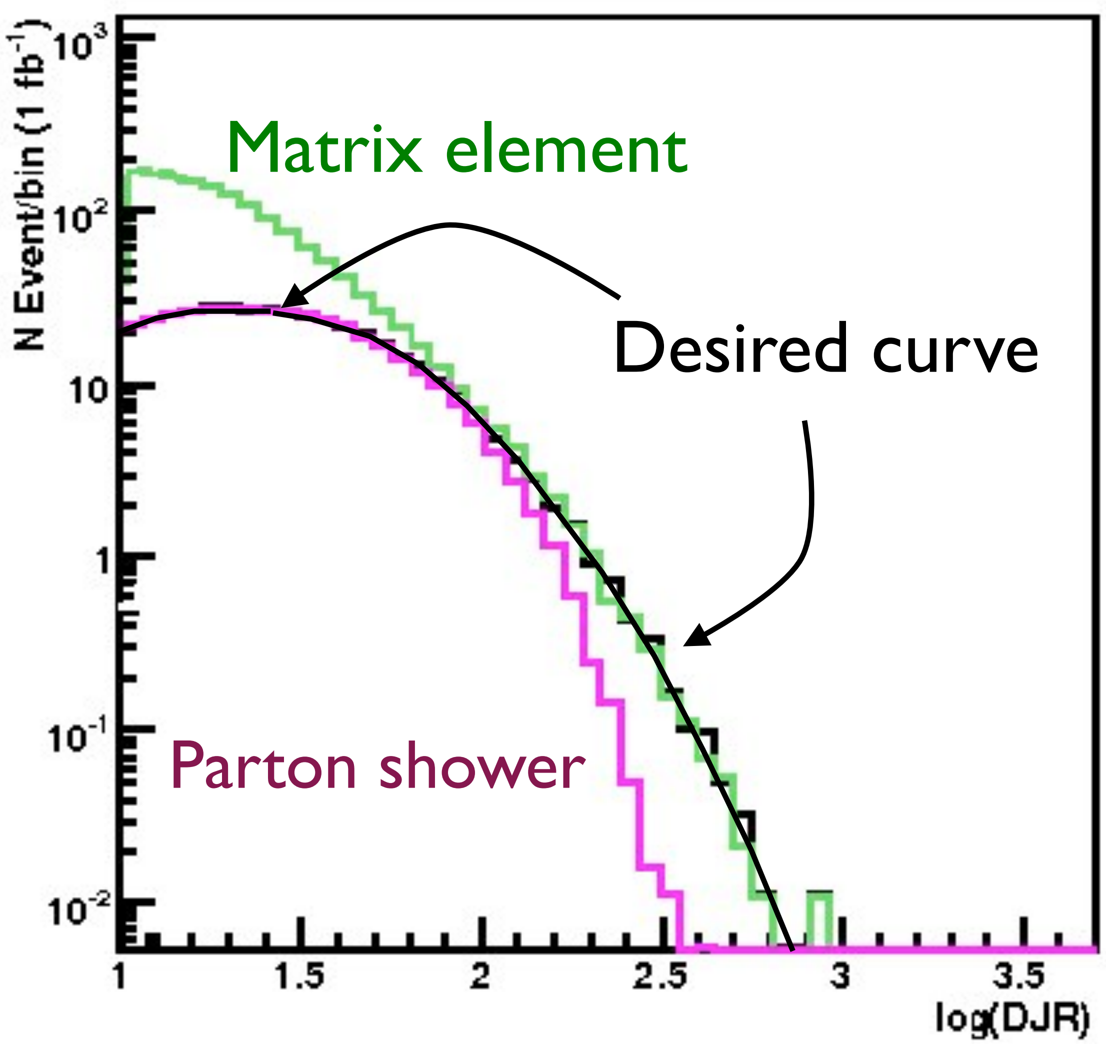

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Merging ME with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]

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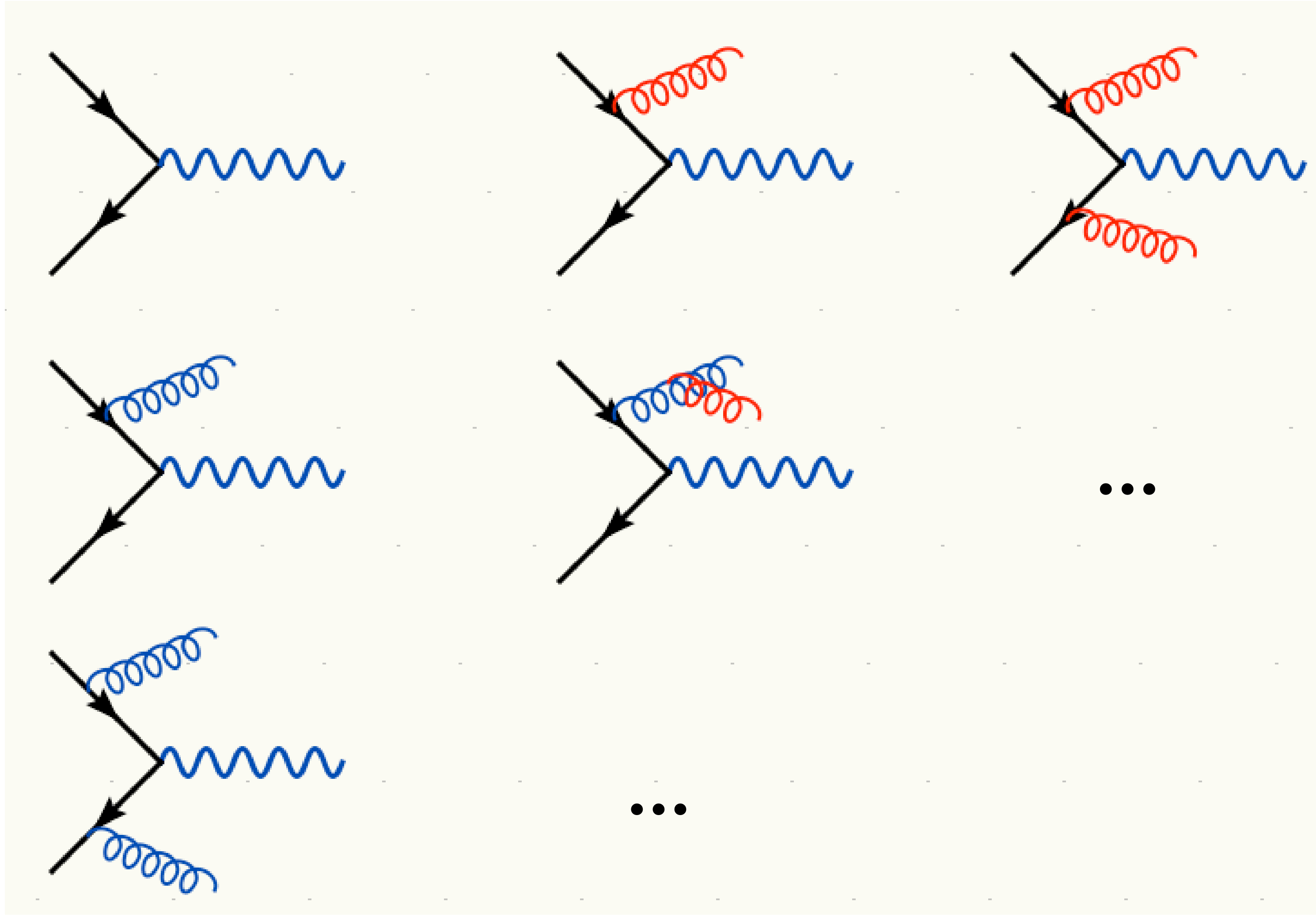

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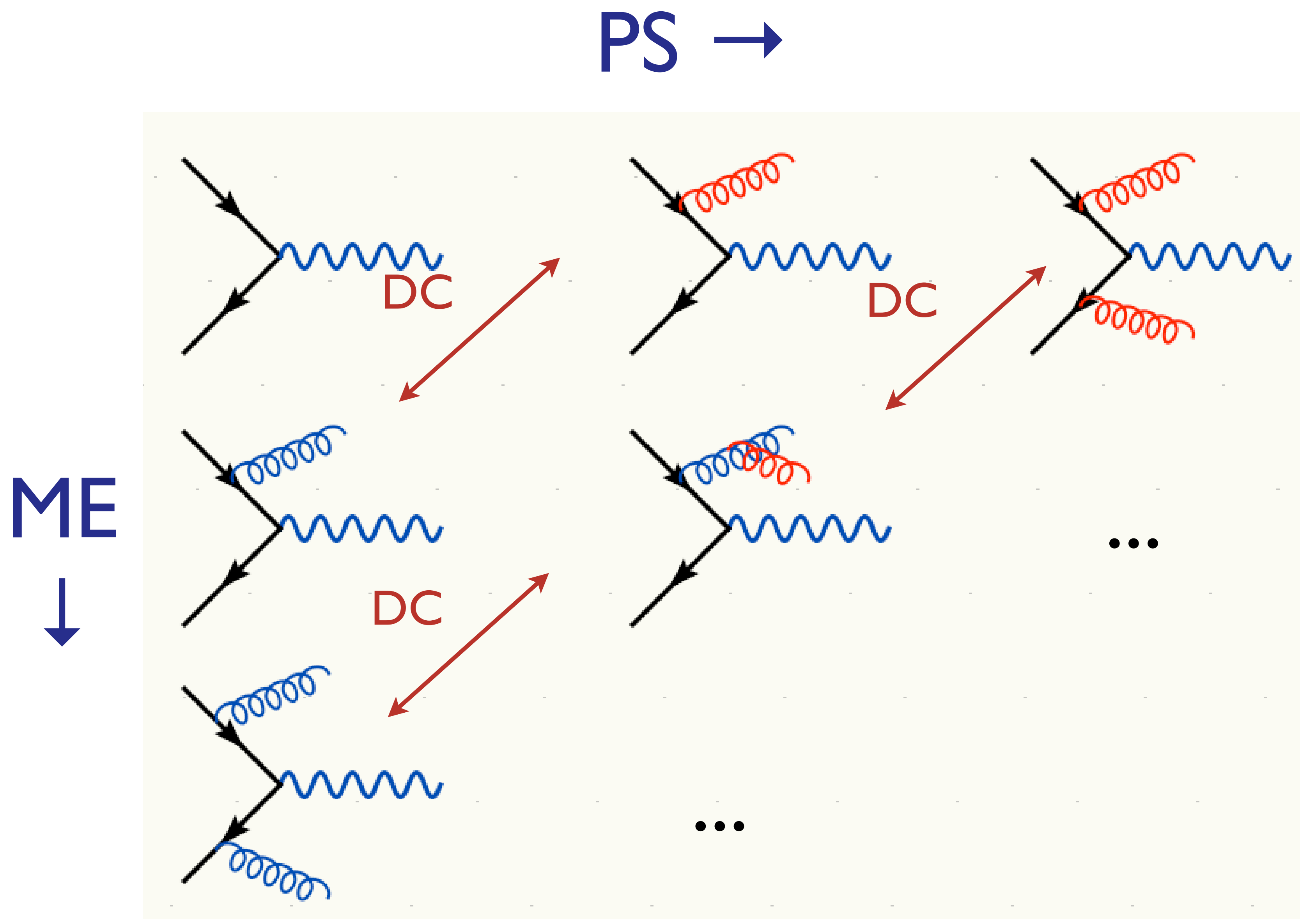
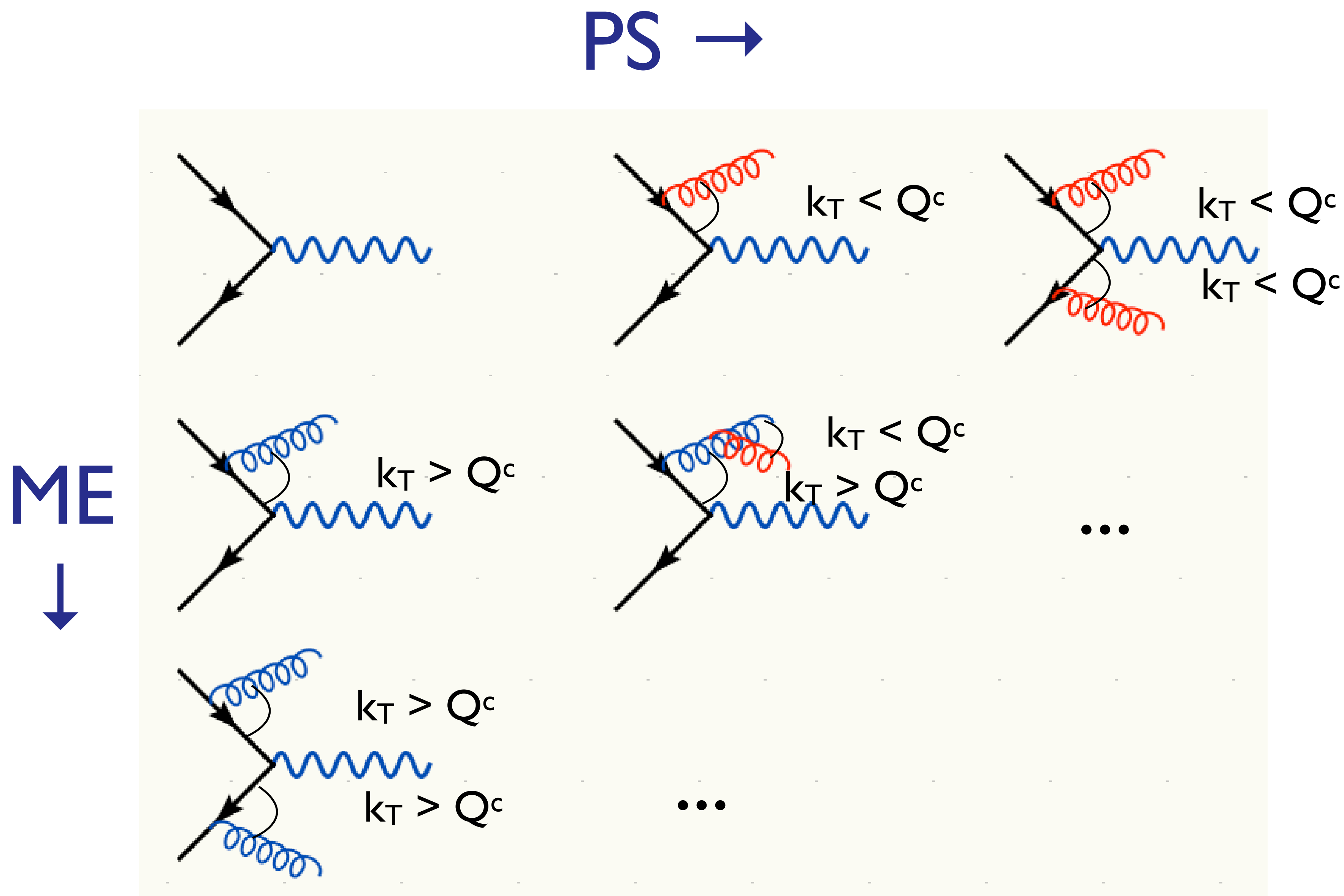


Merging ME with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]



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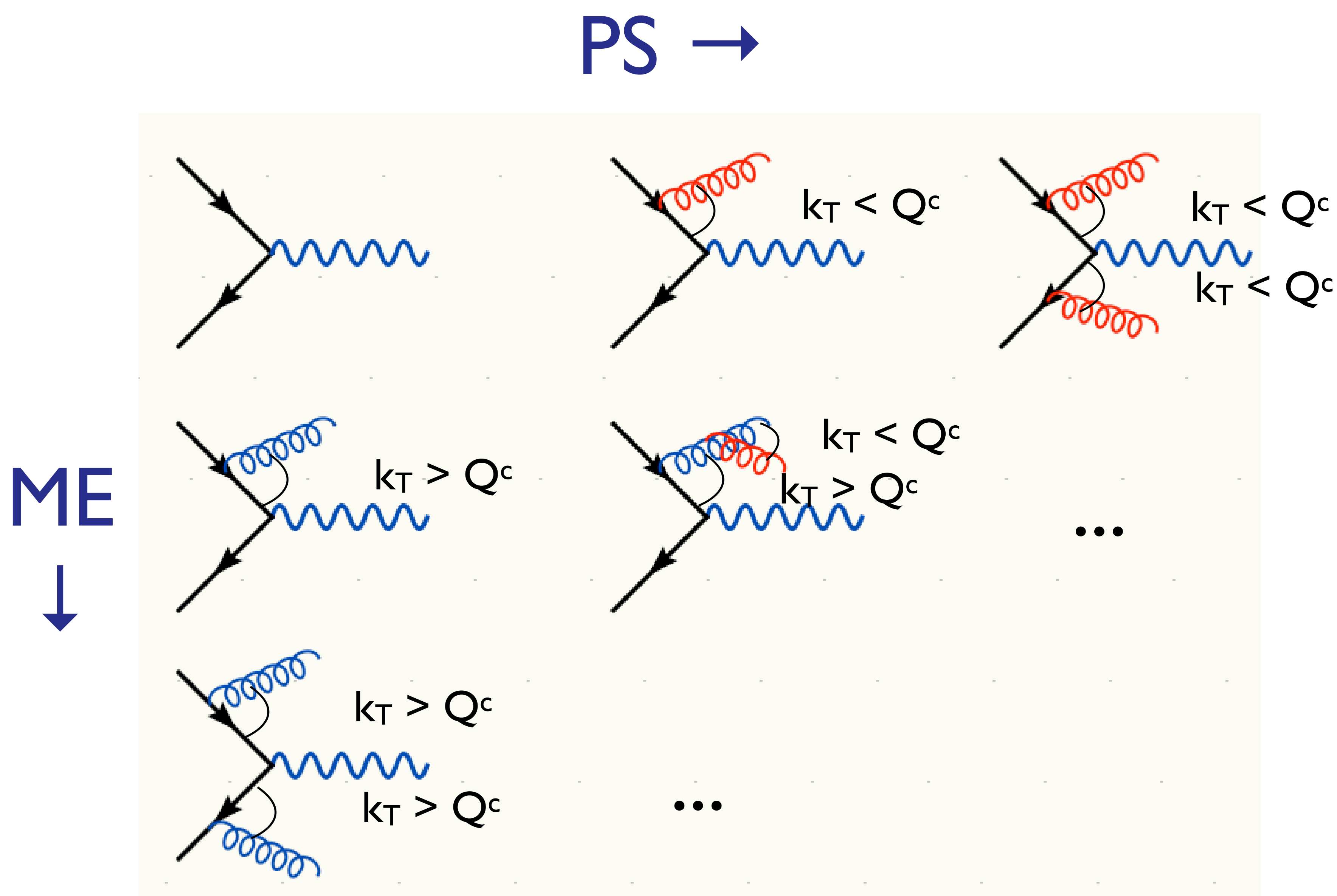
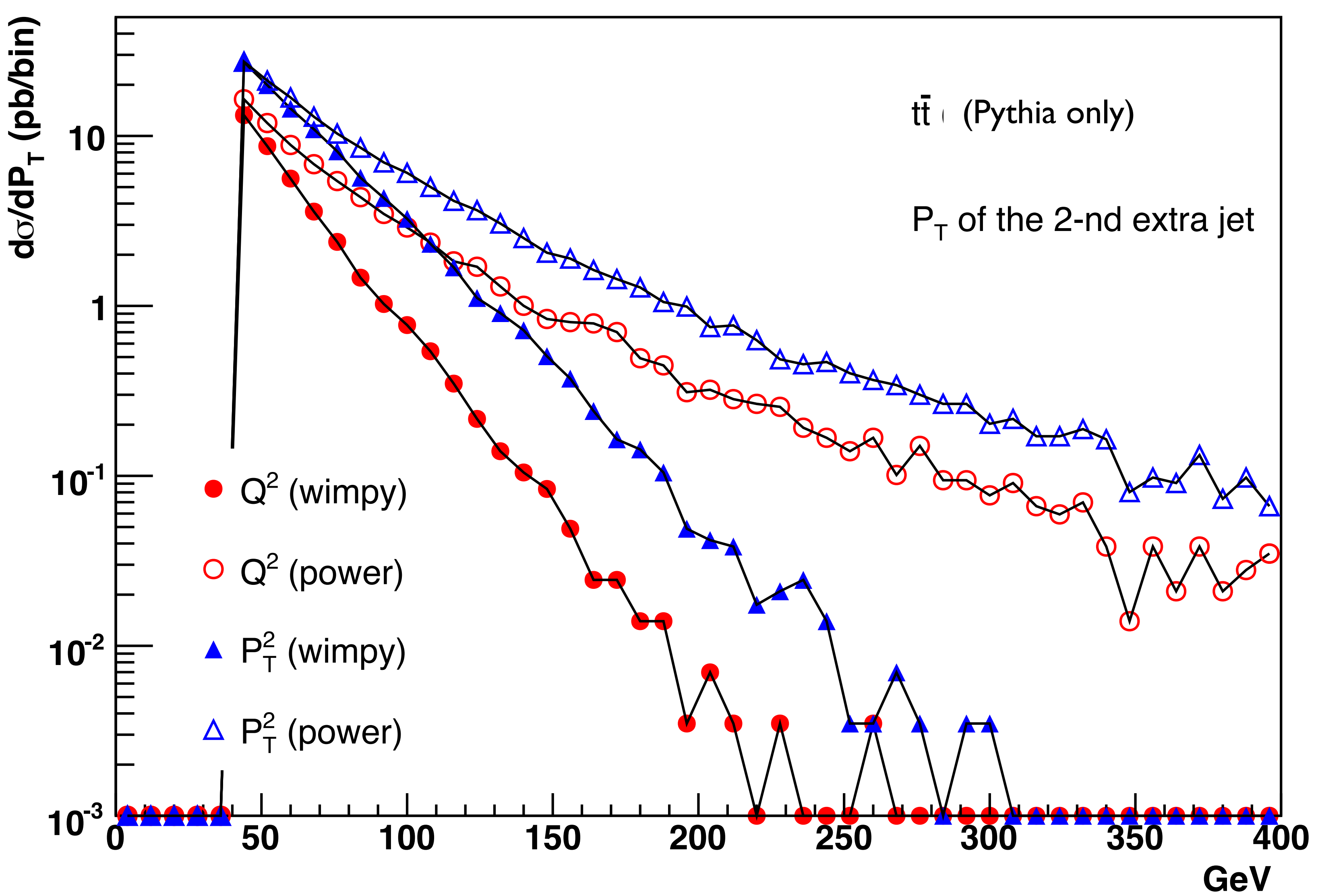


Double counting between ME and PS easily avoided using phase space cut between the two: PS below cutoff, ME above cutoff.

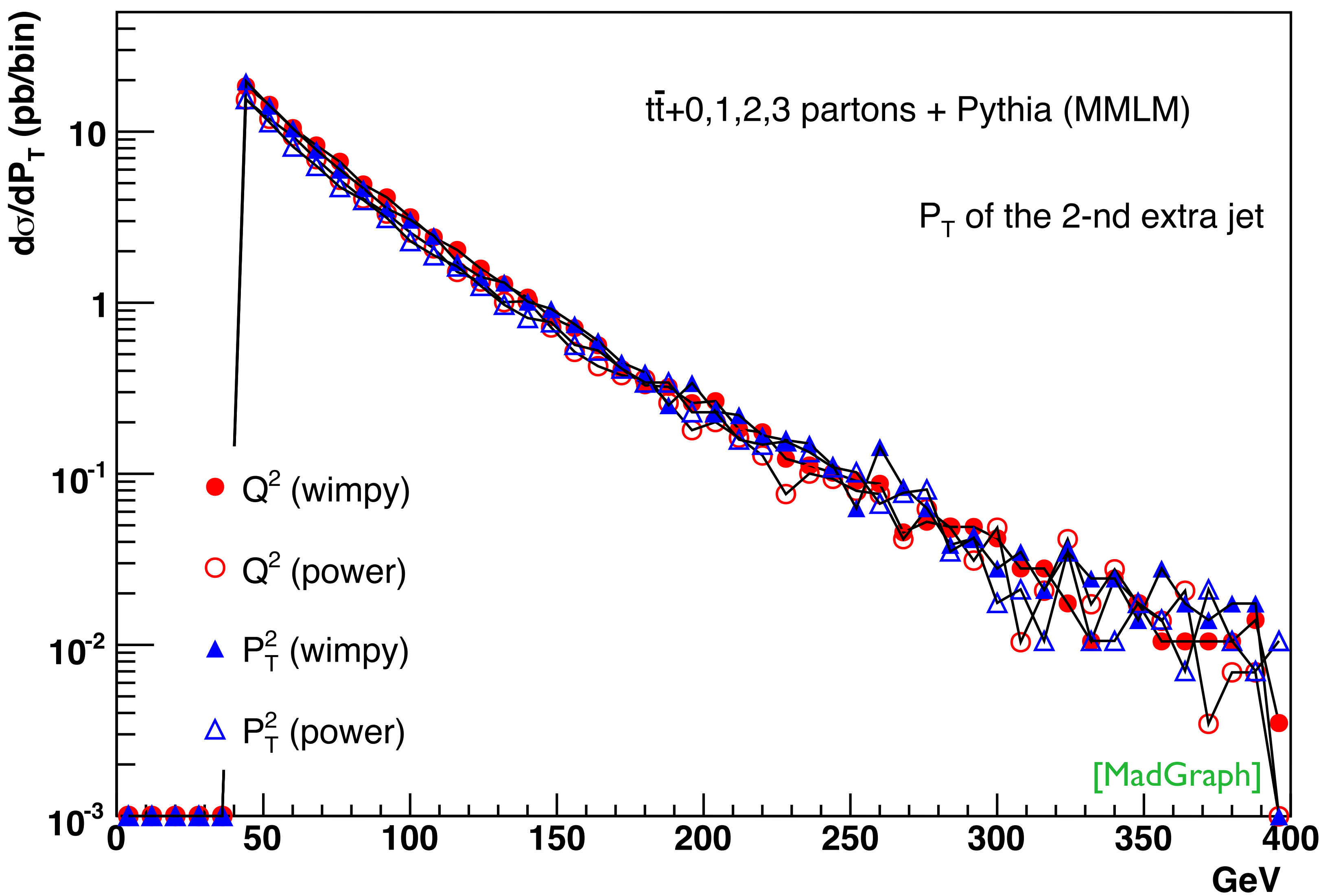
PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)



PS alone vs ME matching

In a matched sample these differences are irrelevant since the behavior at high pt is dominated by the matrix element.



Tools for MC Simulation

Which kind of MC?

- LO
  - ➡ fix order (plus parton-shower)
  - ➡ matched-merged
- NLO
  - ➡ POWHEG / MC@NLO
  - ➡ merged sample
- NNLO / re-summation / N3LO
- Default:
  - ➡ Do the most advanced possible generation.
  - ➡ Speed issue? check faster possibilities

NLO: merged sample

MadGraph5_aMC@NLO

Sherpa

… + Matchbox

- NLO in QCD
- Only SM support
- free list of process

- MC@NLO method

- MC@NLO and POWHEG

MadGraph5_aMC@NLO

- Need to link to a Shower program (Pythia8)
- FxFx / UnLOPS Merging

Sherpa

- Need to link the one loop tool
- UNLOPS merging

… + Matchbox

- Need to provide events
- Need to link to matrix element (both tree and loop)

NLO (one multiplicity)

POWHEG

- Fixed list of processes
- Some BSM

- POWHEG matching
  - Less negative events
  - Not pure NLO

VBF@NLO

- QED@NLO
- very dedicated

MadGraph5_aMC@NLO

- BSM possible

Sherpa

- SM

NLO (cross-section)

MCFM

- Fixed list of processes
- Some BSM
- No events generation

MadGraph5_aMC@NLO

Sherpa

Powheg

VBF@NLO

HPAIR

NJETS

LO (matched/merged)

Sherpa

- Fully built in
- Starts BSM supports
- CKKW-L
- ATLAS Default

… + Pythia

- MLM / UMEPS / CKKW CKKWL
- Full BSM supports
- CMS default (with MG5_aMC)

… + Herwig

- MLM /CKKW CKKWL
- Full BSM supports

LO

CODE	Main advantage	highest multiplicity
MG5_aMC	BSM	normal: 6 decay: 14
Sherpa	fast for QCD muli-leg	normal: 7 decay: 7
CalcHep	very fast for 2 > 2	normal: 3/4 decay: 6
Whizard	ILC physics	normal: 6 decay: 10
pythia	low multiplicity	normal: 3 decay: 100
herwig	low multiplicity	normal: 3 decay: 100

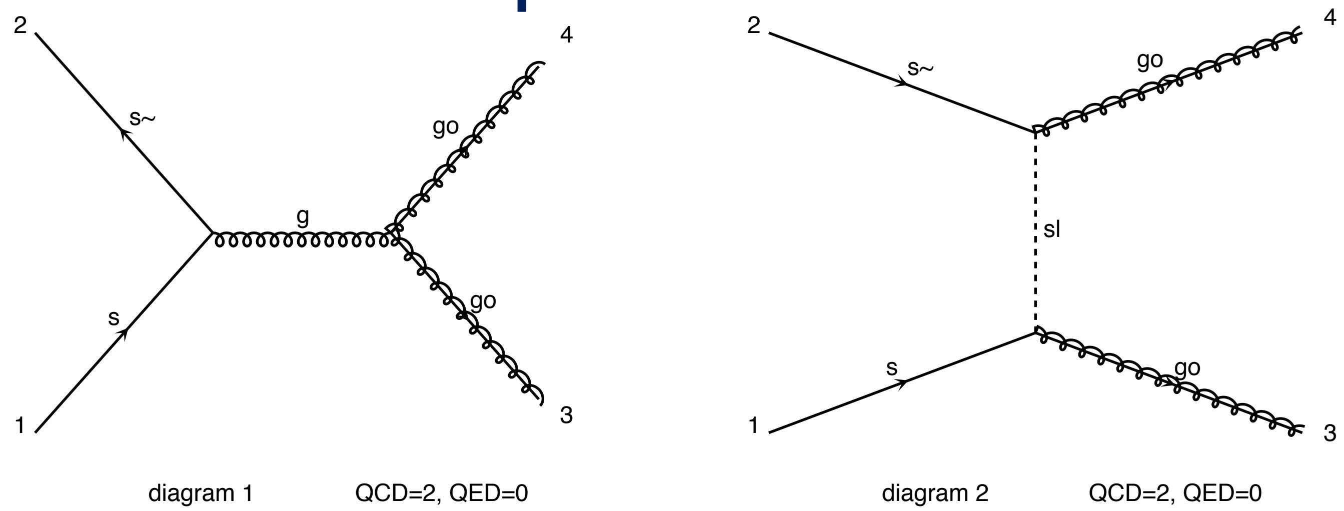
MadGraph5

Olivier Mattelaer
IPPP/Durham

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Calculate a given process (e.g. gluino pair)

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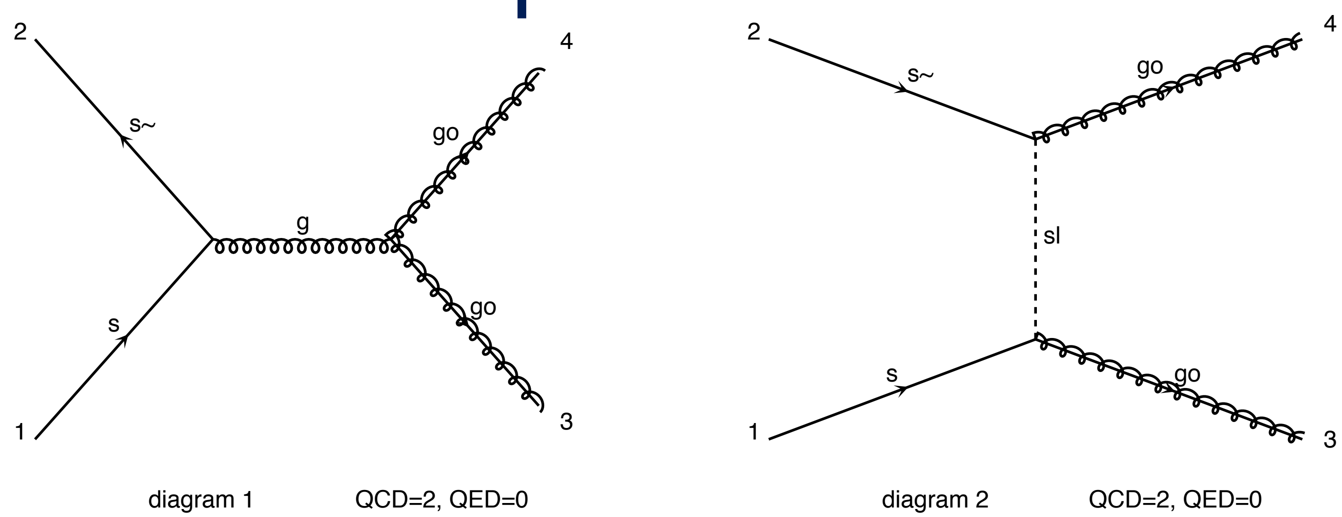

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Easy enough

Hard

Very Hard (in general)

Calculate a given process (e.g. gluino pair)

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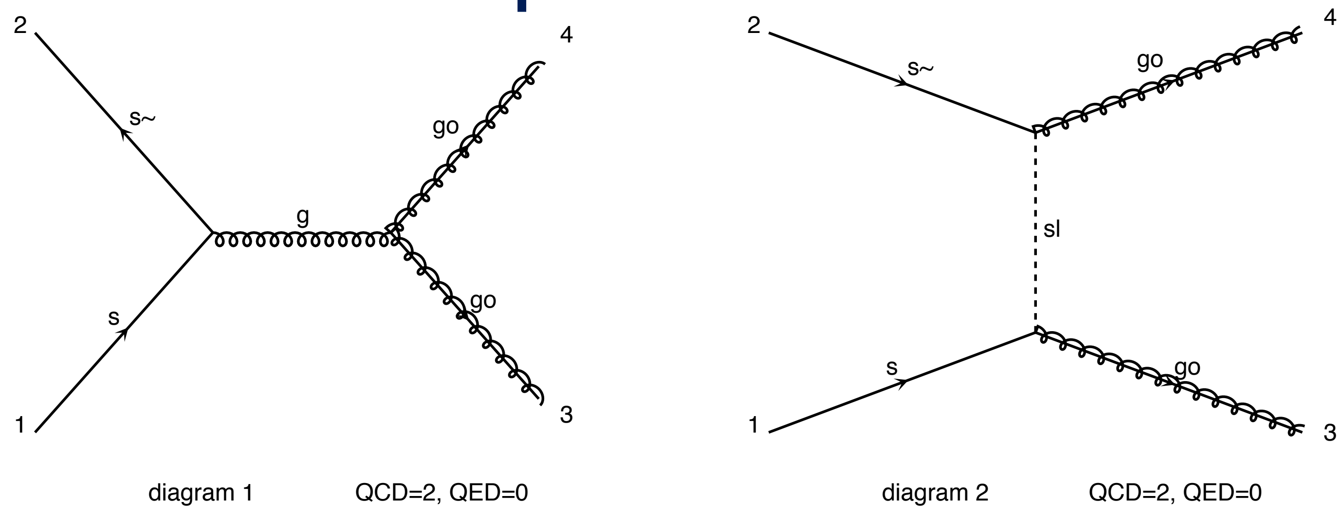

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Easy enough

Hard

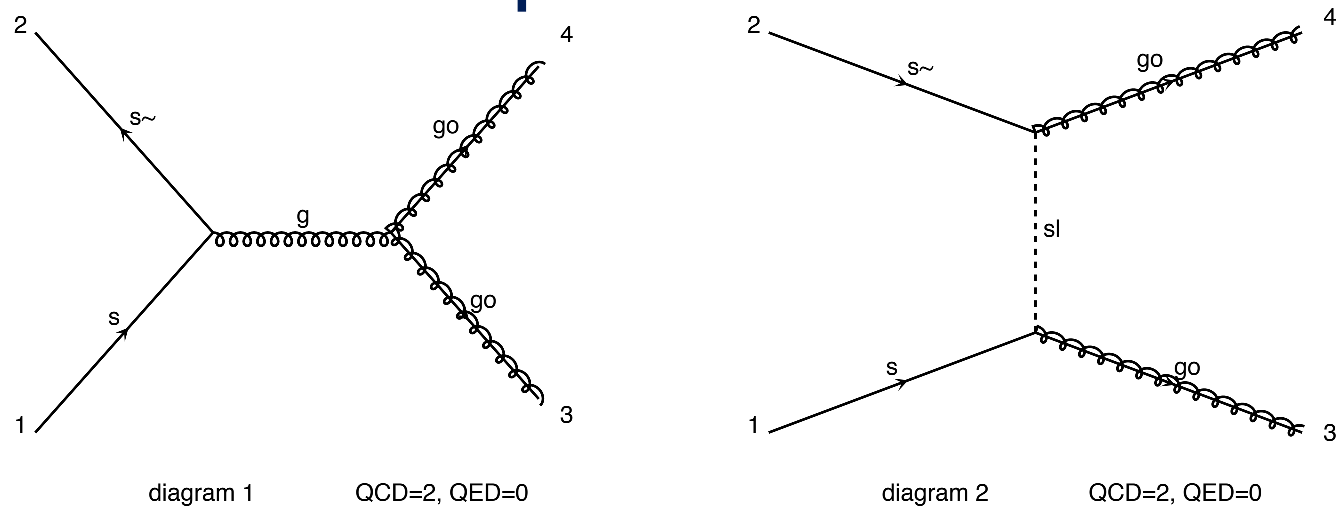
Now

Very Hard (in general)

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



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$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

Now

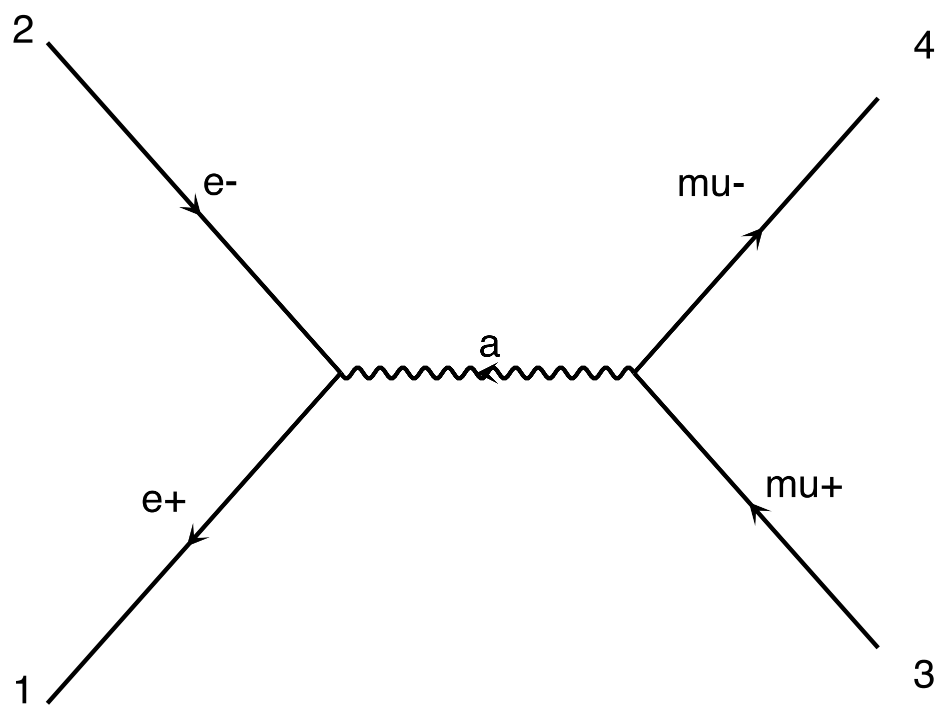
Very Hard (in general)

monday

Plan For Today

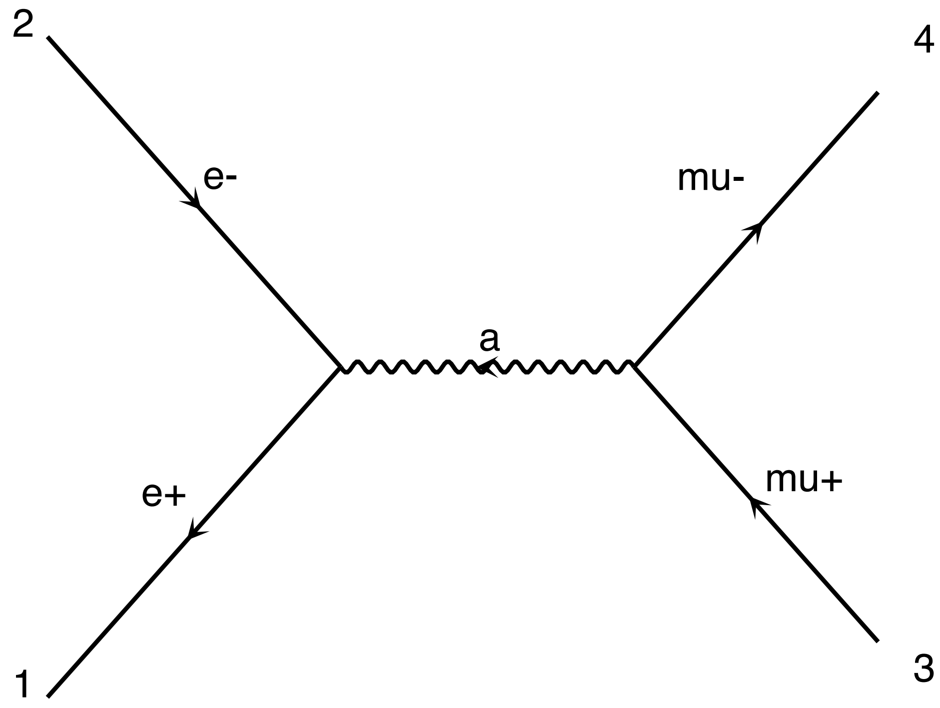
- Computation of the matrix-element
  - Tree Level
  - Loop
- Tools/functionality of MG5_aMC

Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

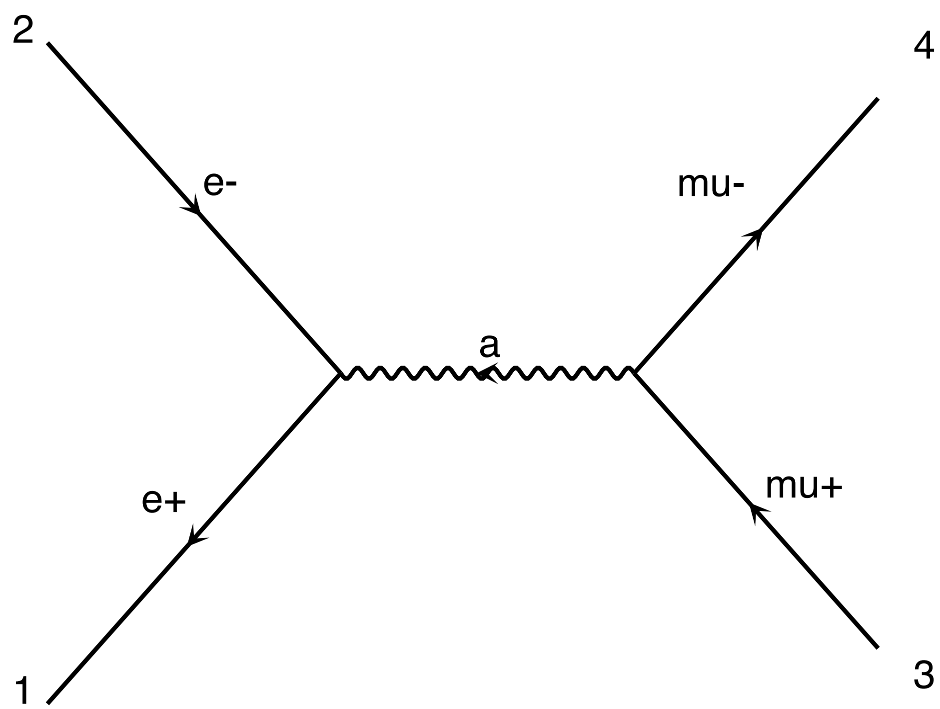
Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol} |\mathcal{M}|^2 = \frac{1}{4}\sum_{pol} \mathcal{M}^* \mathcal{M}$$

Matrix Element



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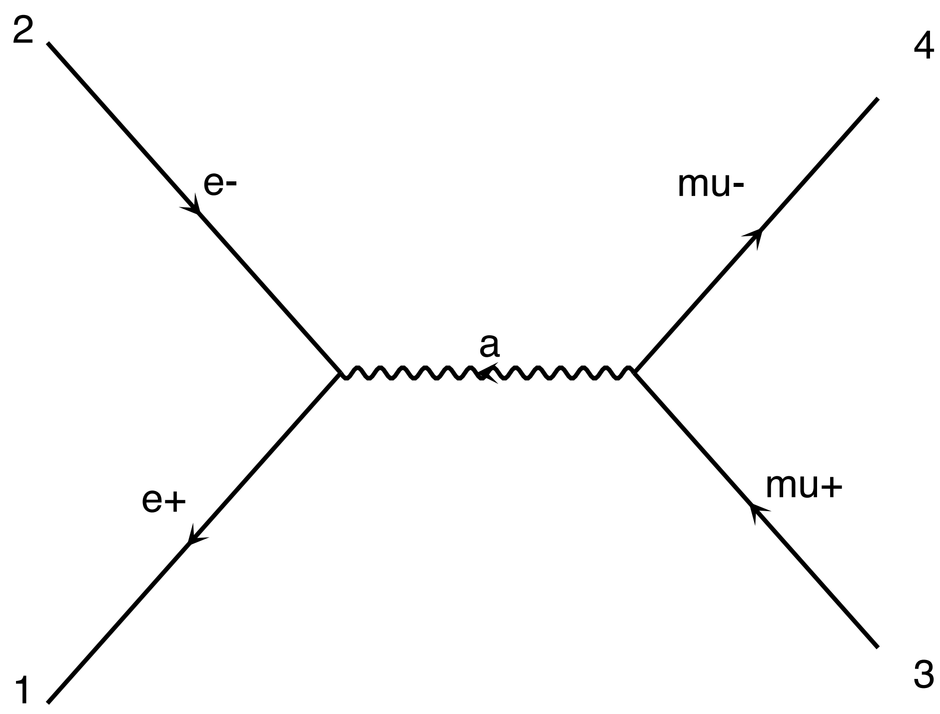
$$\sum_{pol}\bar{u}u = \not{p} + m$$

Matrix Element



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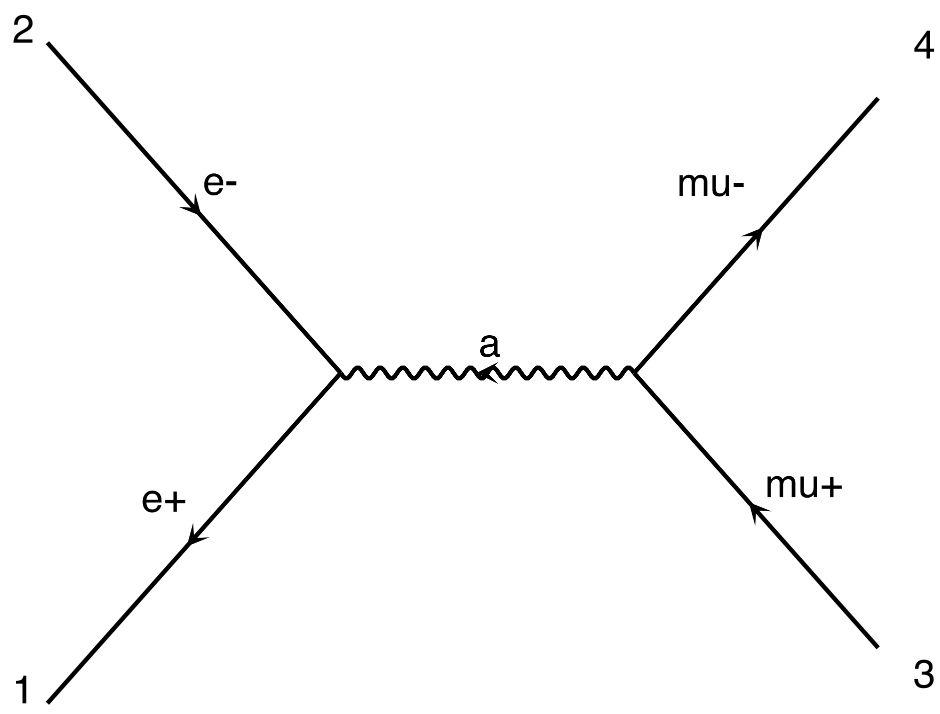
$$\sum_{pol}\bar{u}u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4} Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu] Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

Matrix Element



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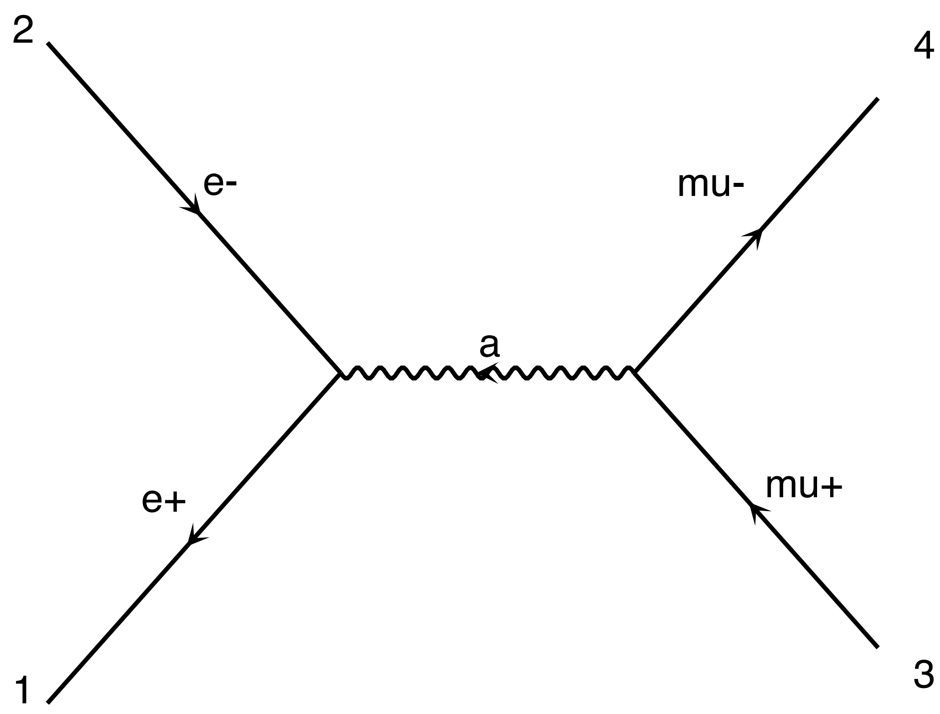
$$\rightarrow \frac{8e^4}{q^4}\left[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)\right]$$

Matrix Element



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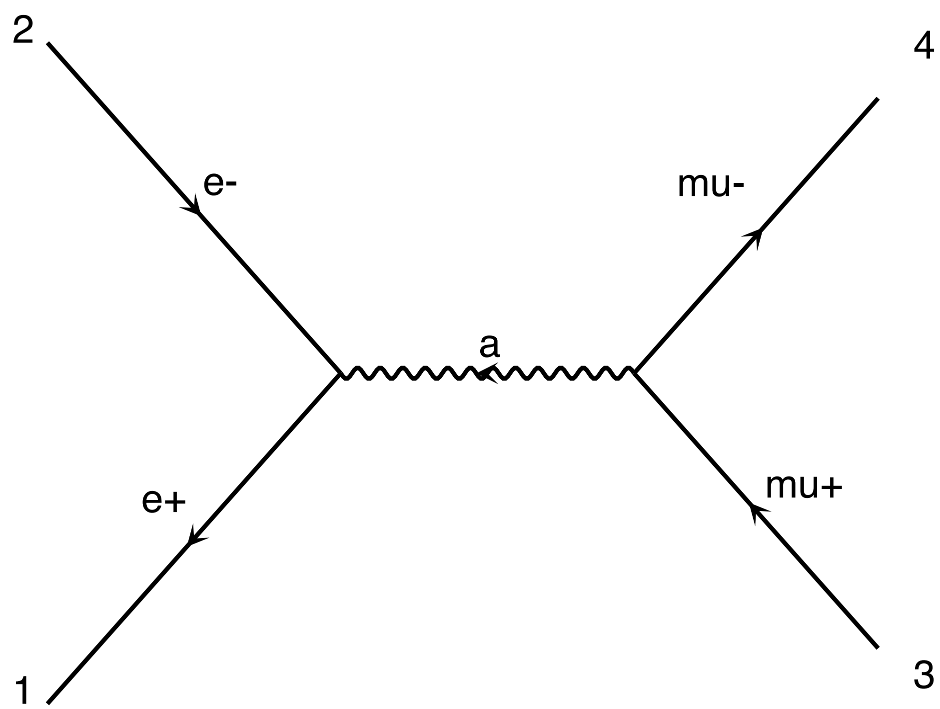
Very Efficient !!!

Matrix Element



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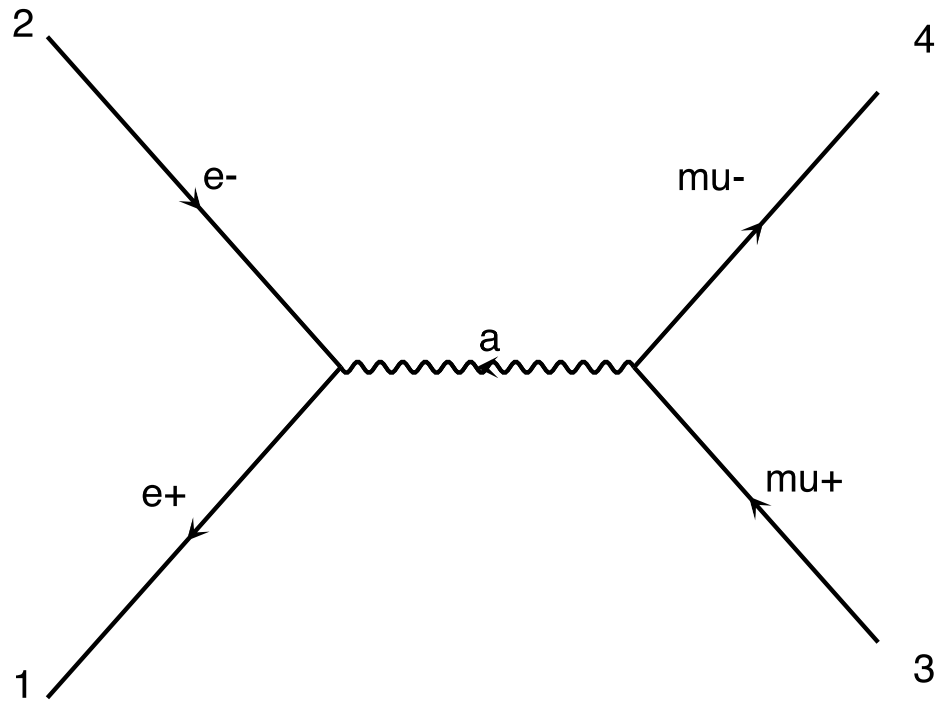
$$\rightarrow \frac{8e^4}{q^4}[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Very Efficient !!!

Only for 2→2 and 2→3

Matrix Element



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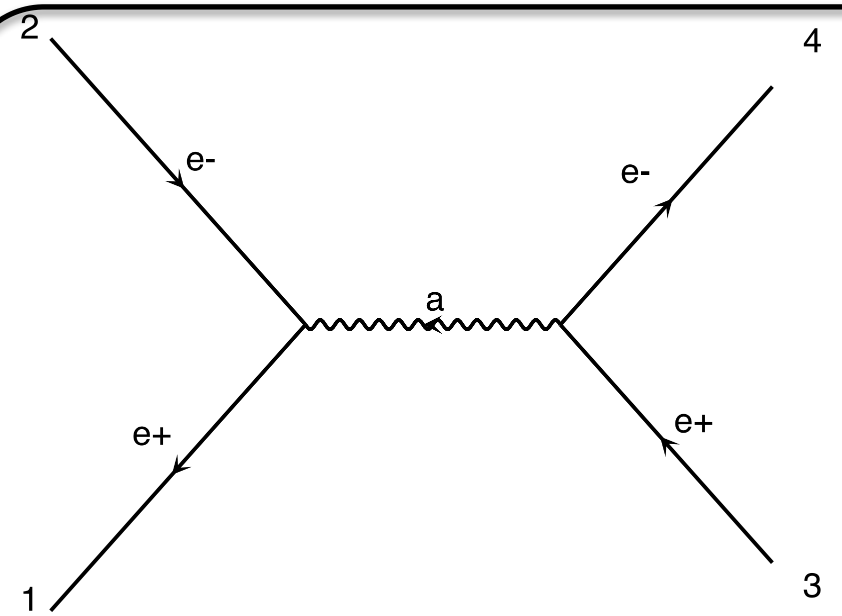
Very Efficient !!!

Only for 2→2 and 2→3

Because the number of terms rises as N^2

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\, e\, \gamma^\nu u)$$

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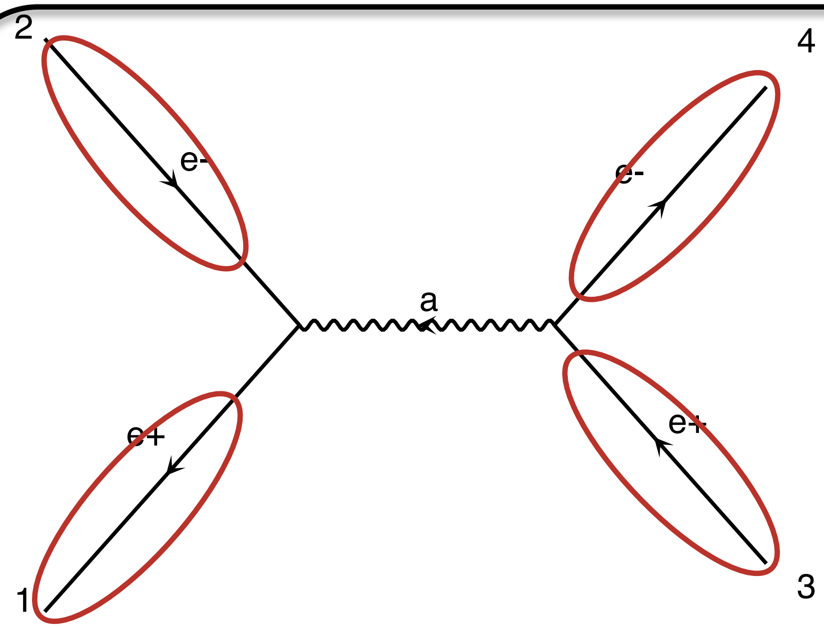

$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v)\, \frac{g_{\mu\nu}}{q^2}\, (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

Idea

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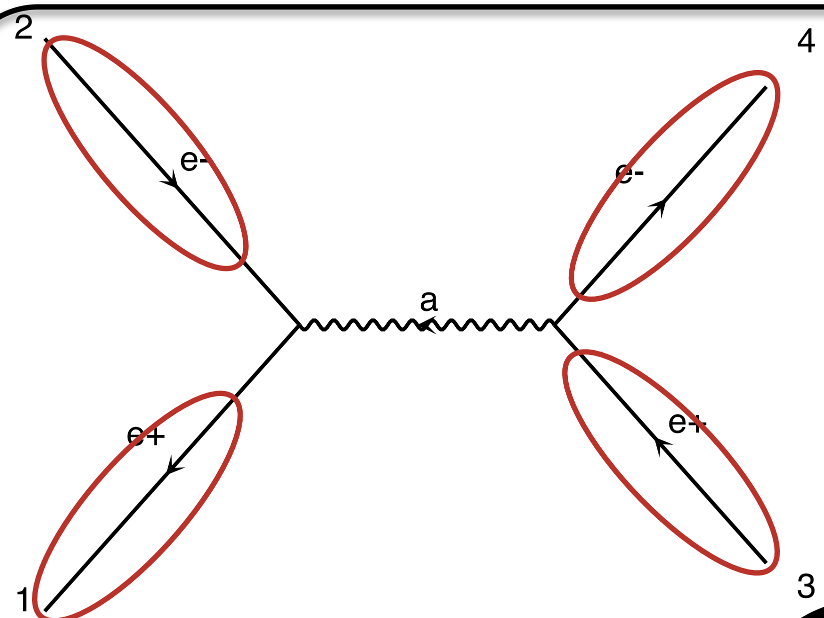

$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{cases}$$

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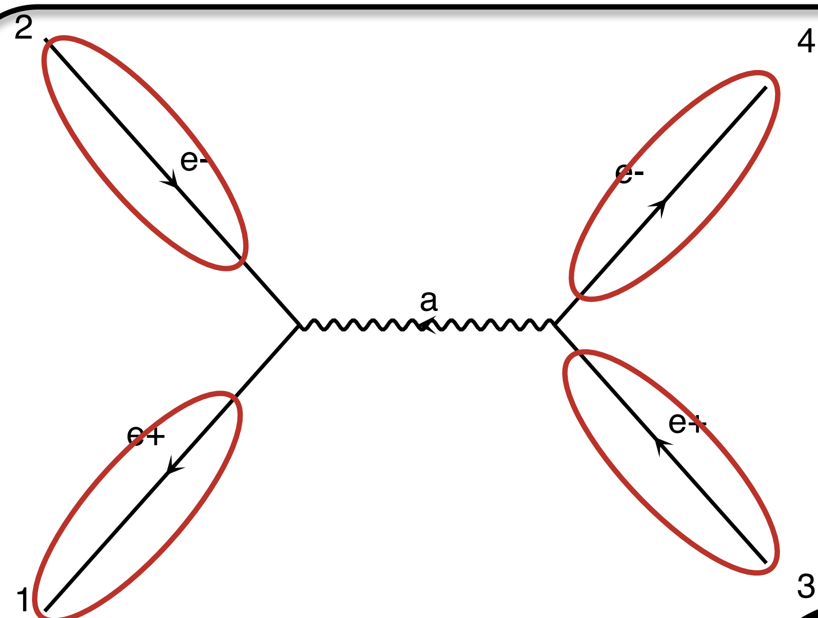

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$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_\lambda(\vec{p}) \\ \omega_\lambda(p)\chi_\lambda(\vec{p}) \end{pmatrix}$$

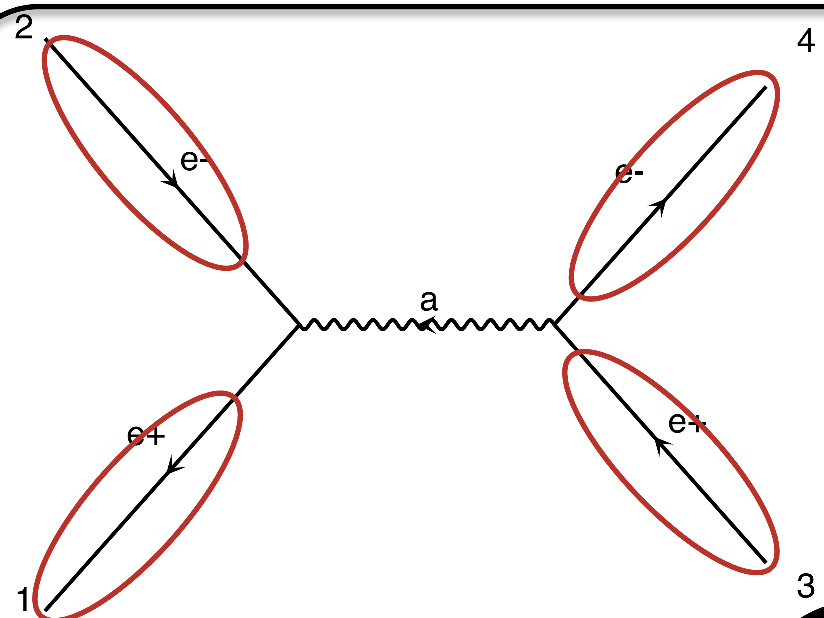
$$\omega_\pm(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} |\vec{p}| + p_z \\ p_x + ip_y \end{pmatrix},$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} -p_x + ip_y \\ |\vec{p}| + p_z \end{pmatrix}.$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
  - ➡ Loop on Helicity and average the results



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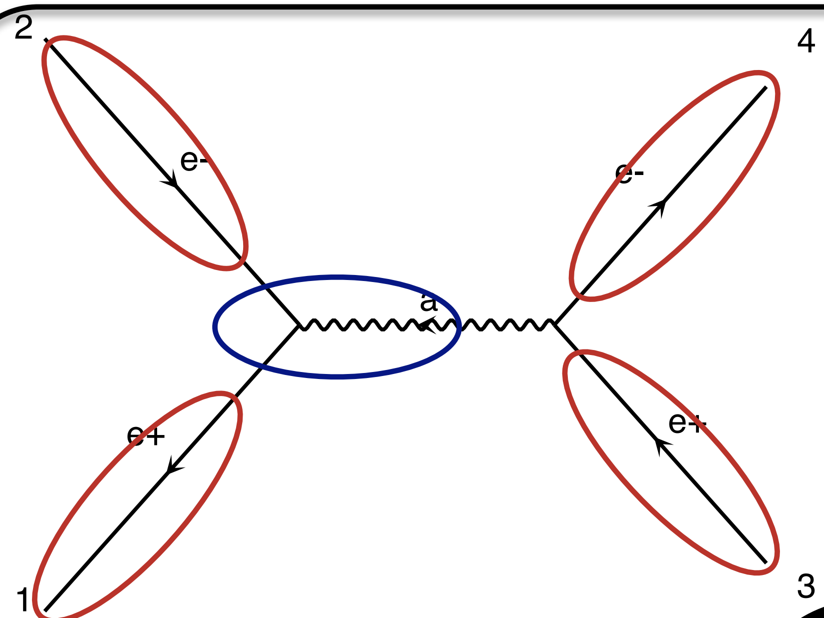

$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a) \end{cases}$$

Idea

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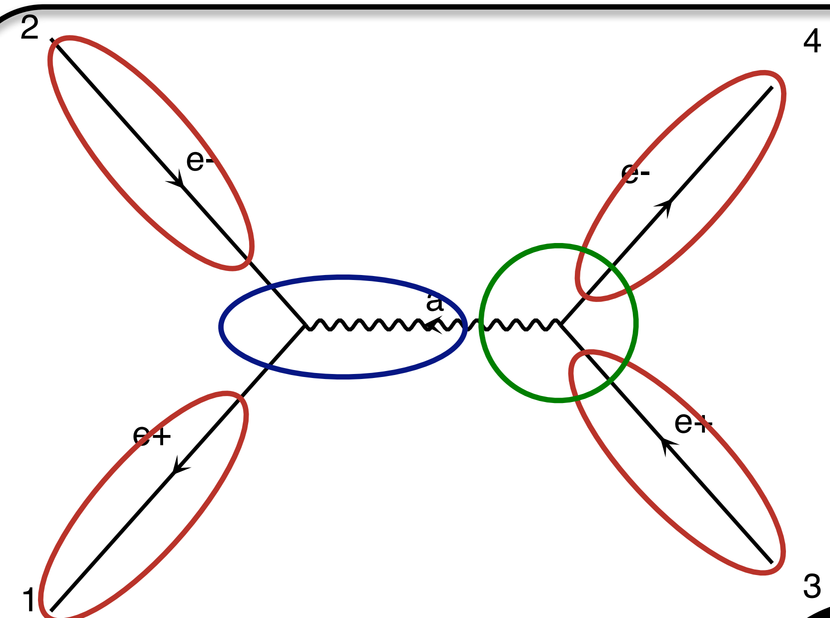


$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

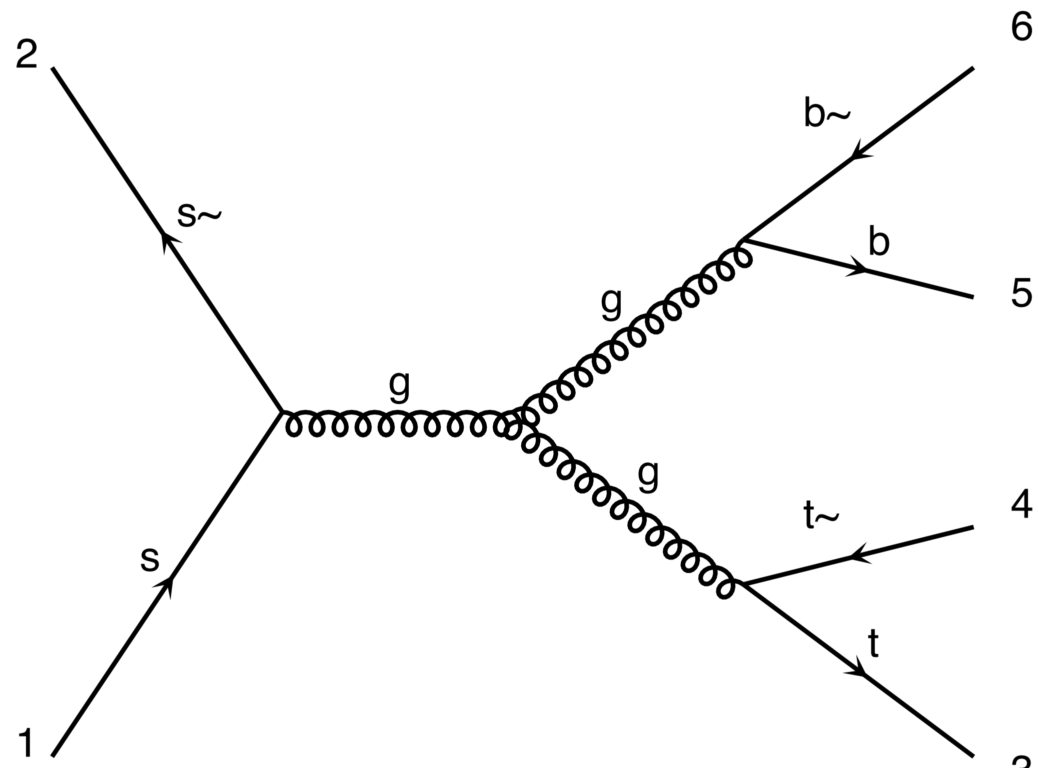
Finally evaluate amplitude (c-number)

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a) \\ \mathcal{M} = fct(v_3, \bar{u}_4, W_a) \end{cases}$$

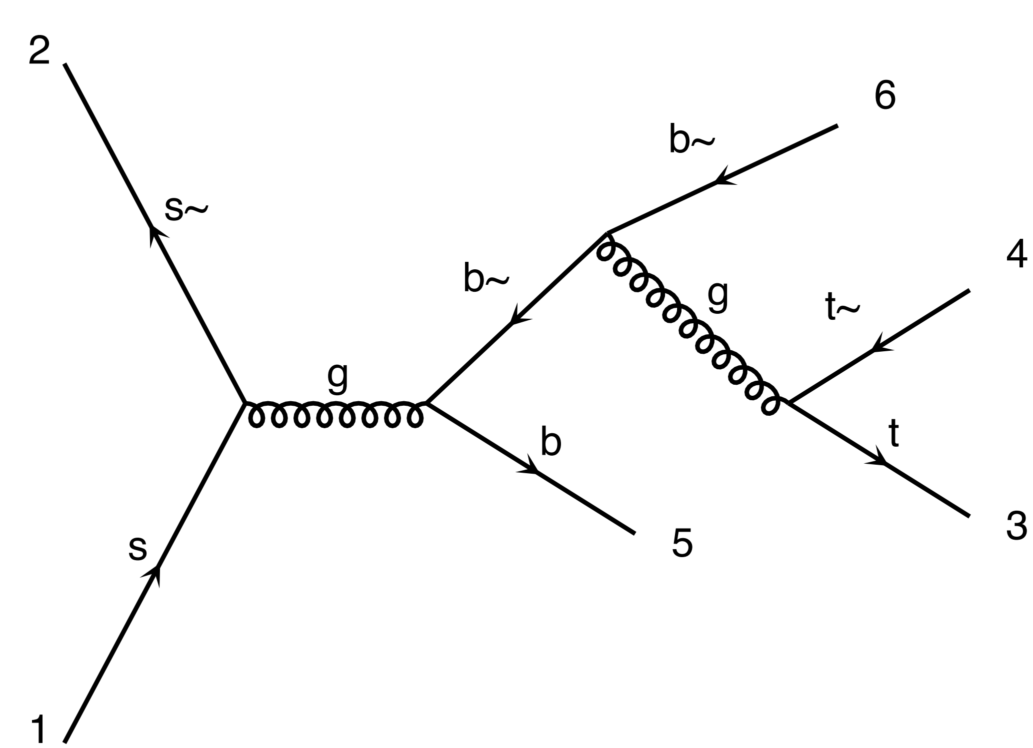
Real case

Known



M1

Number of routines: 0



M2

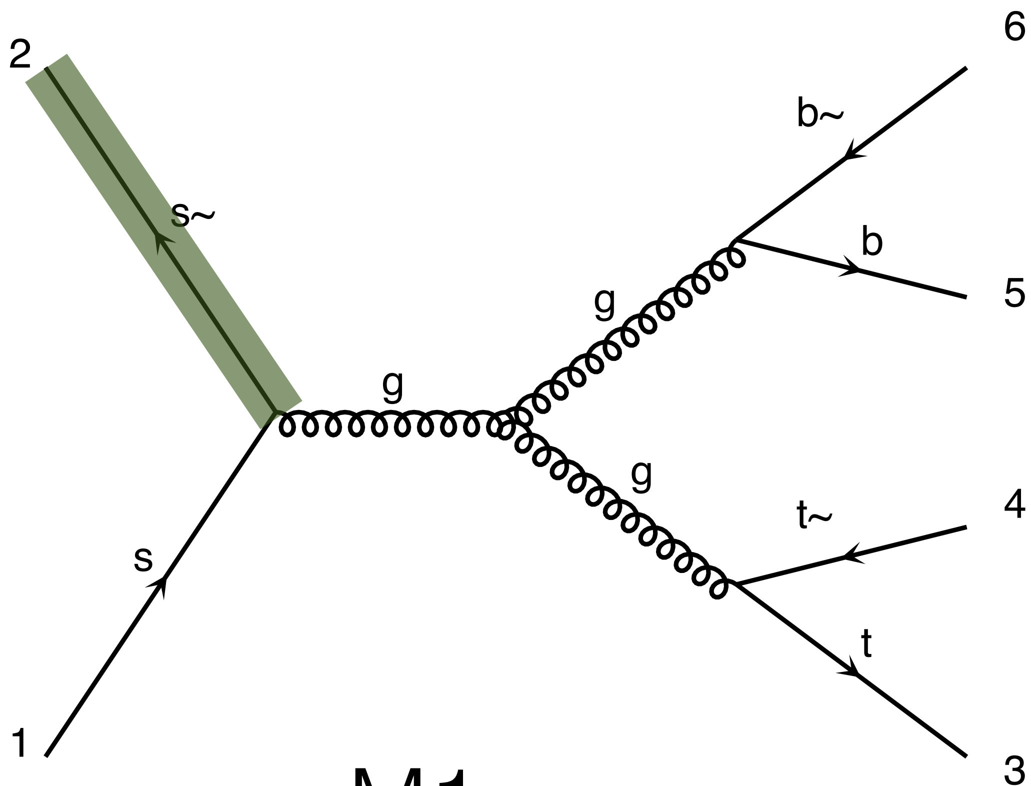
Number of routines: 0

Number of routines for both: 0

$$|M|^2 = |M_1 + M_2|^2$$

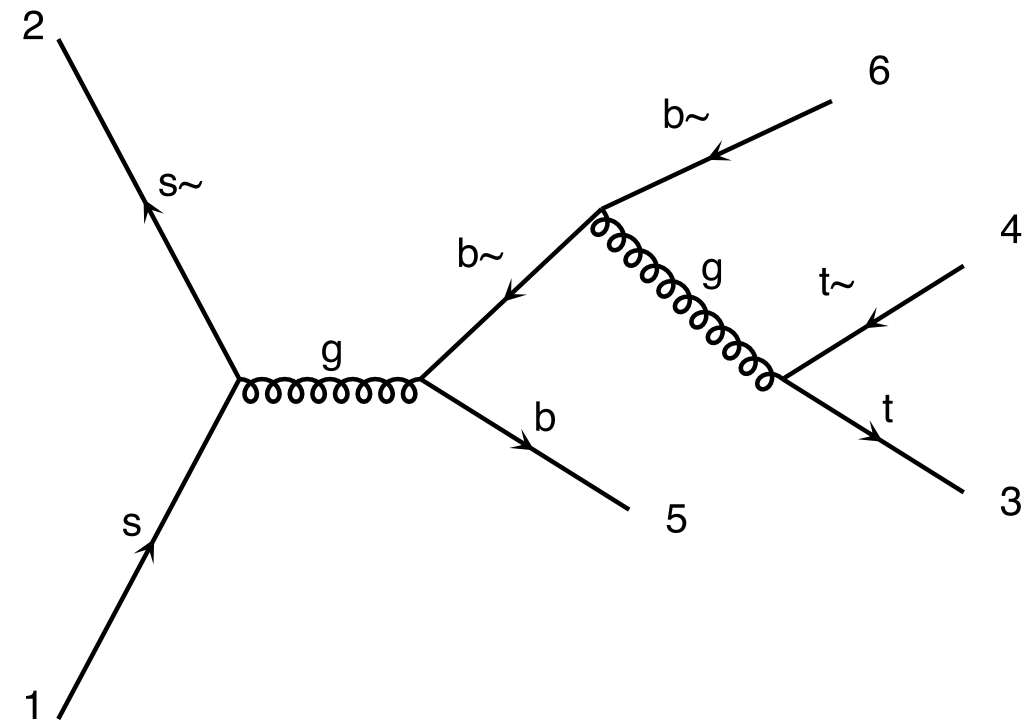
Real case

Known



M1

Number of routines: 1



M2

Number of routines: 0

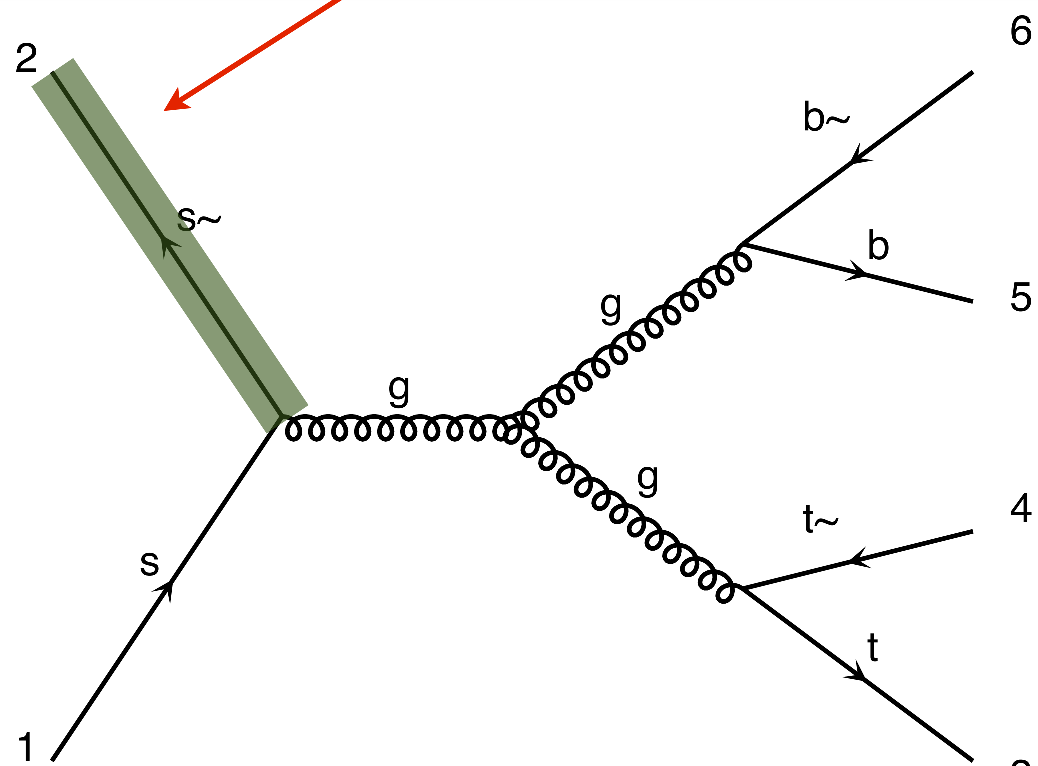
Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

Real case

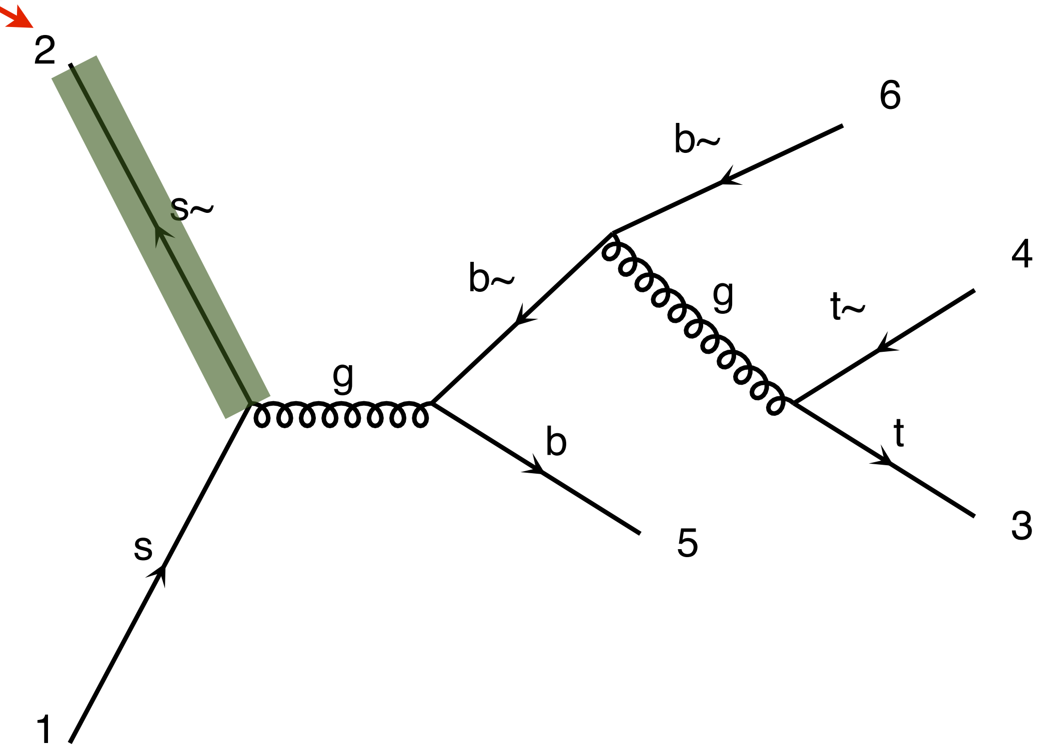
Identical

Known



M1

Number of routines: 1



M2

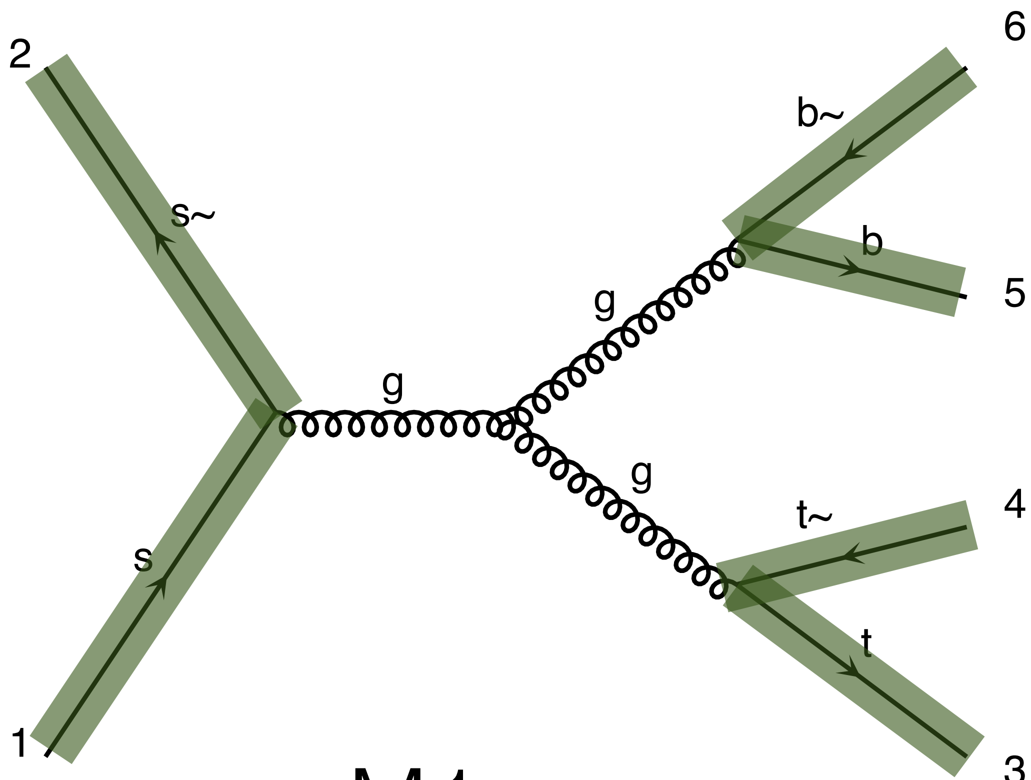
Number of routines: 1

Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

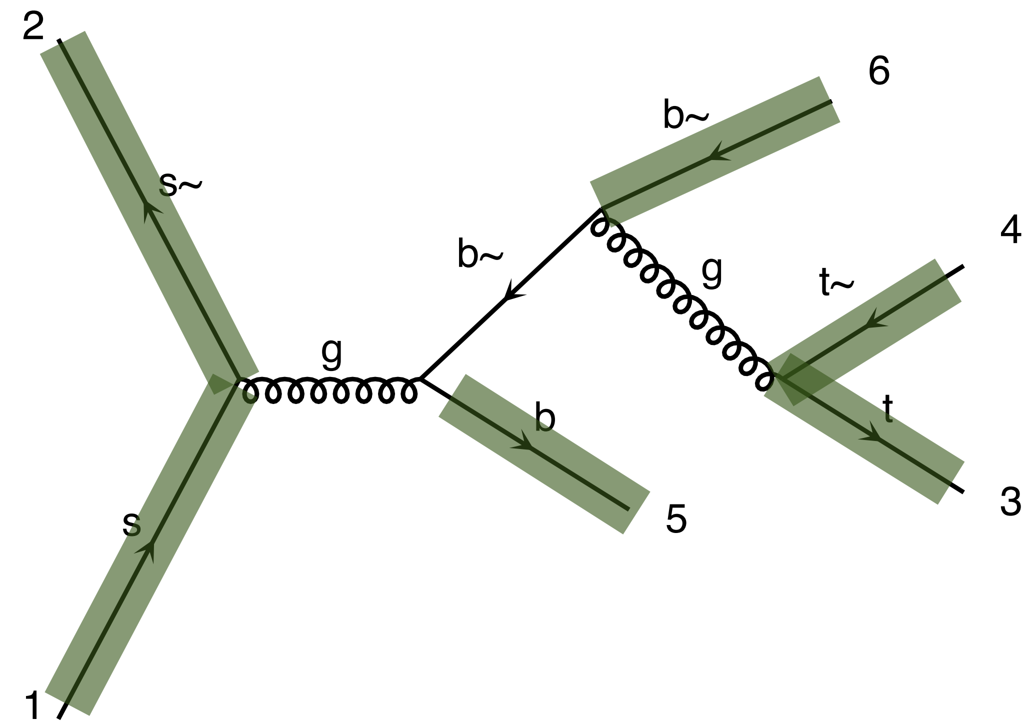
Real case

Known



M1

Number of routines: 6



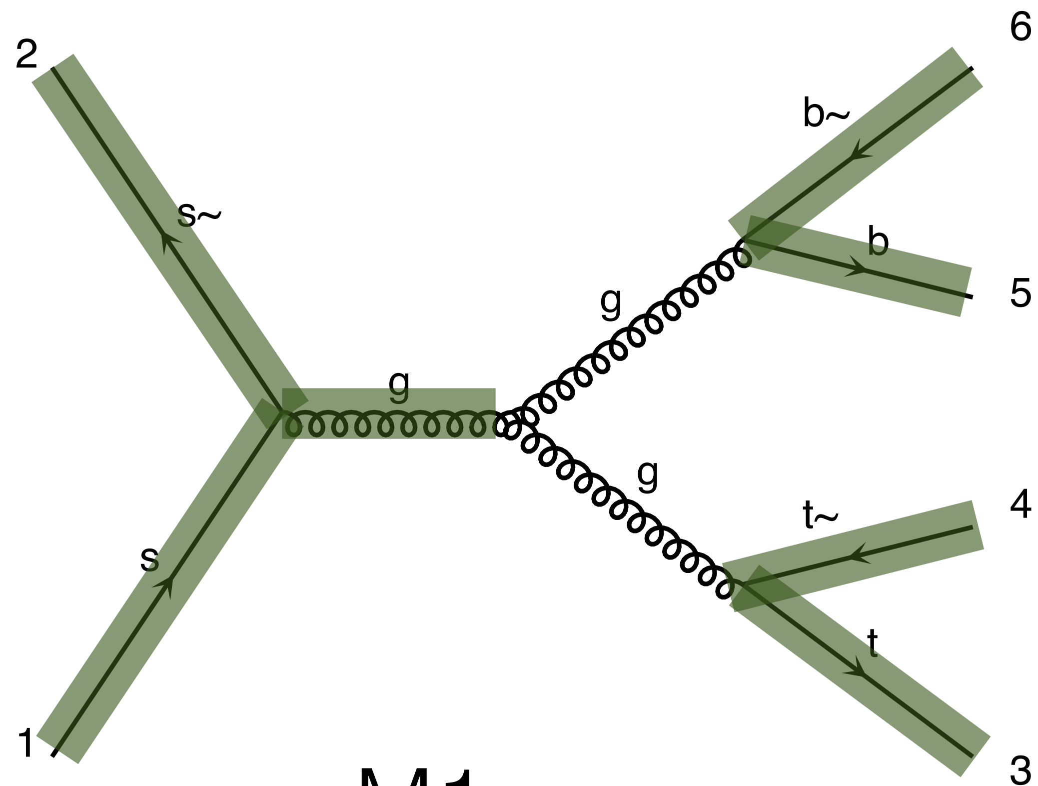
M2

Number of routines: 6

Number of routines for both: 6

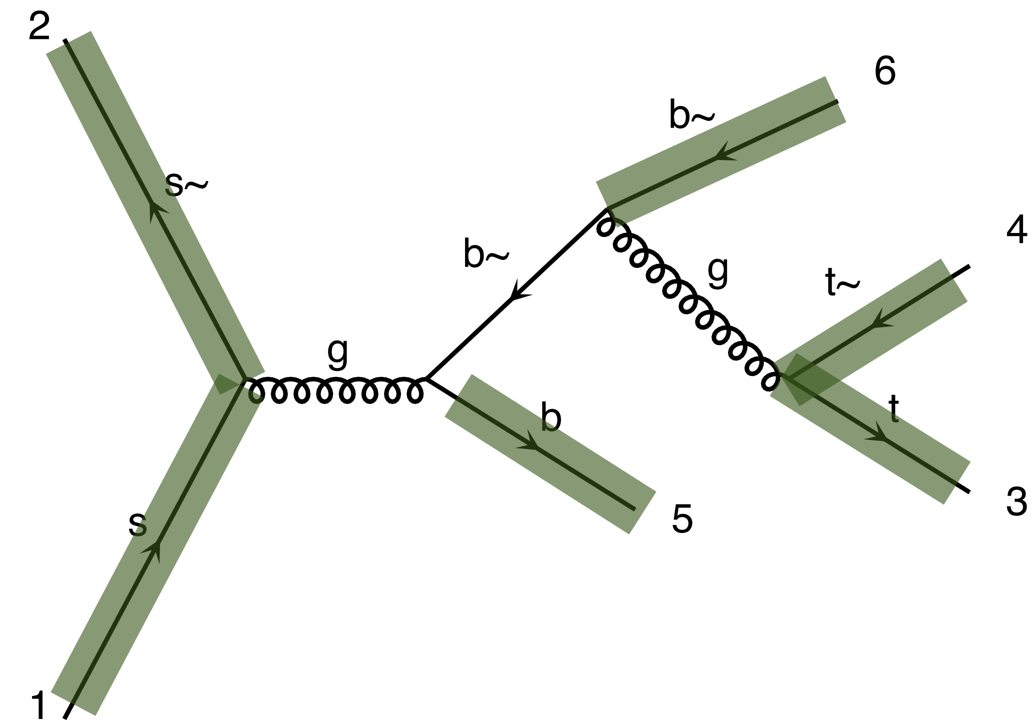
$$|M|^2 = |M_1 + M_2|^2$$

Known



M1

Number of routines: 7



M2

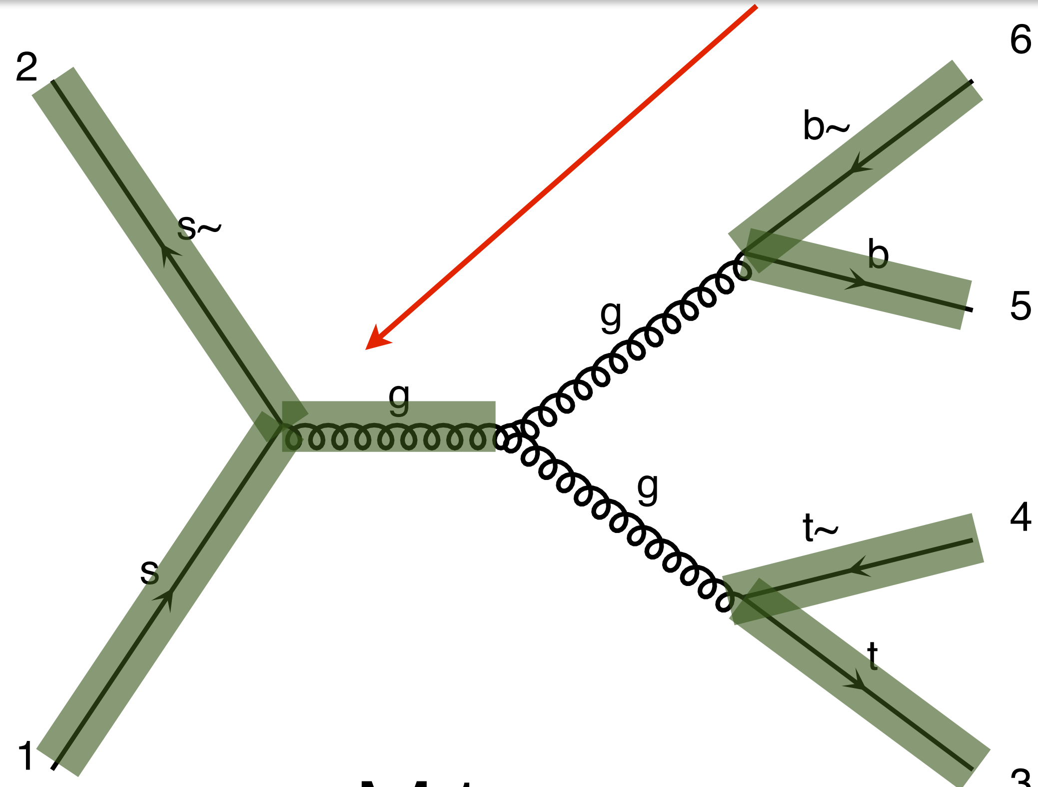
Number of routines: 6

Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

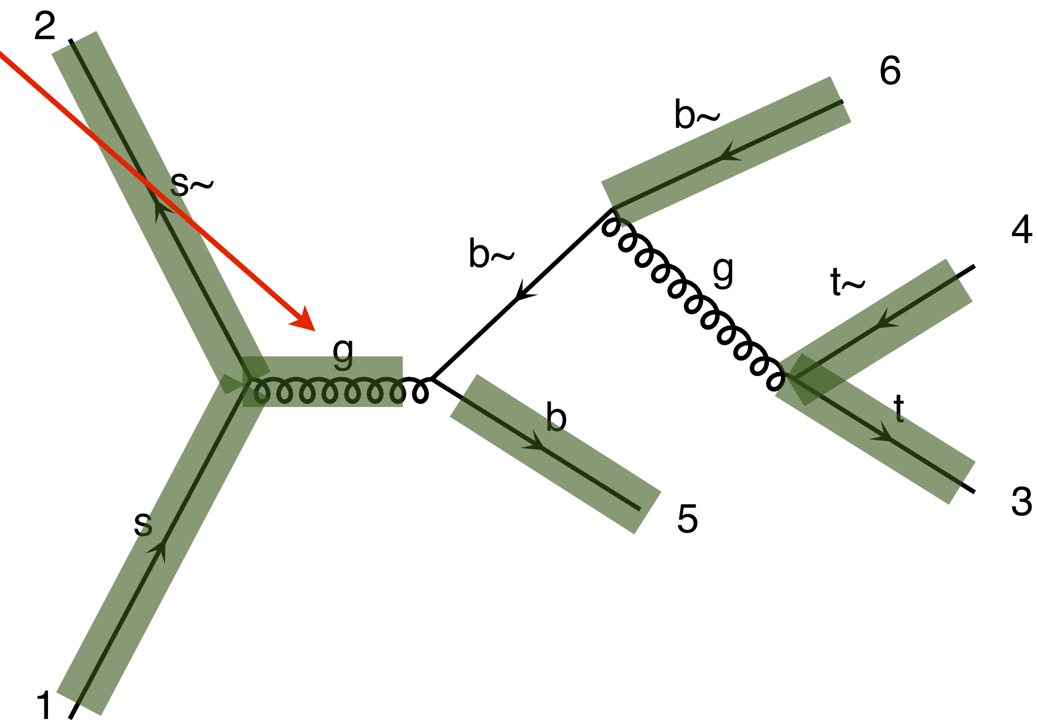
Known

Identical



M1

Number of routines: 7

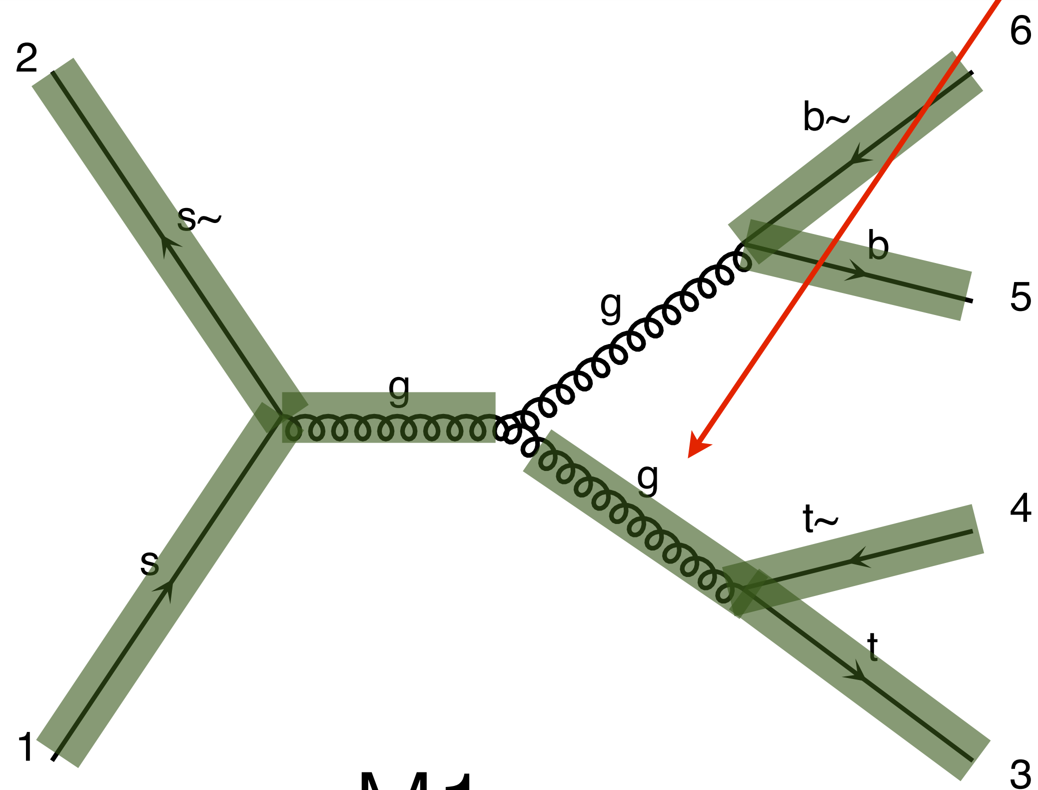


M2

Number of routines: 7

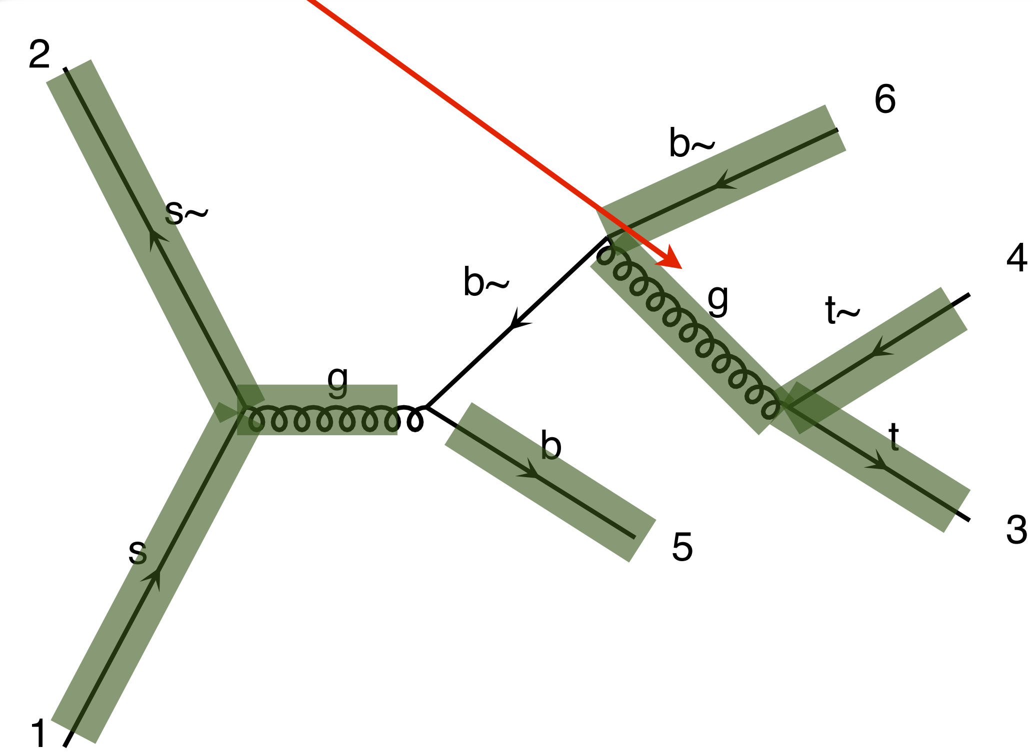
Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$



M1

Number of routines: 8



M2

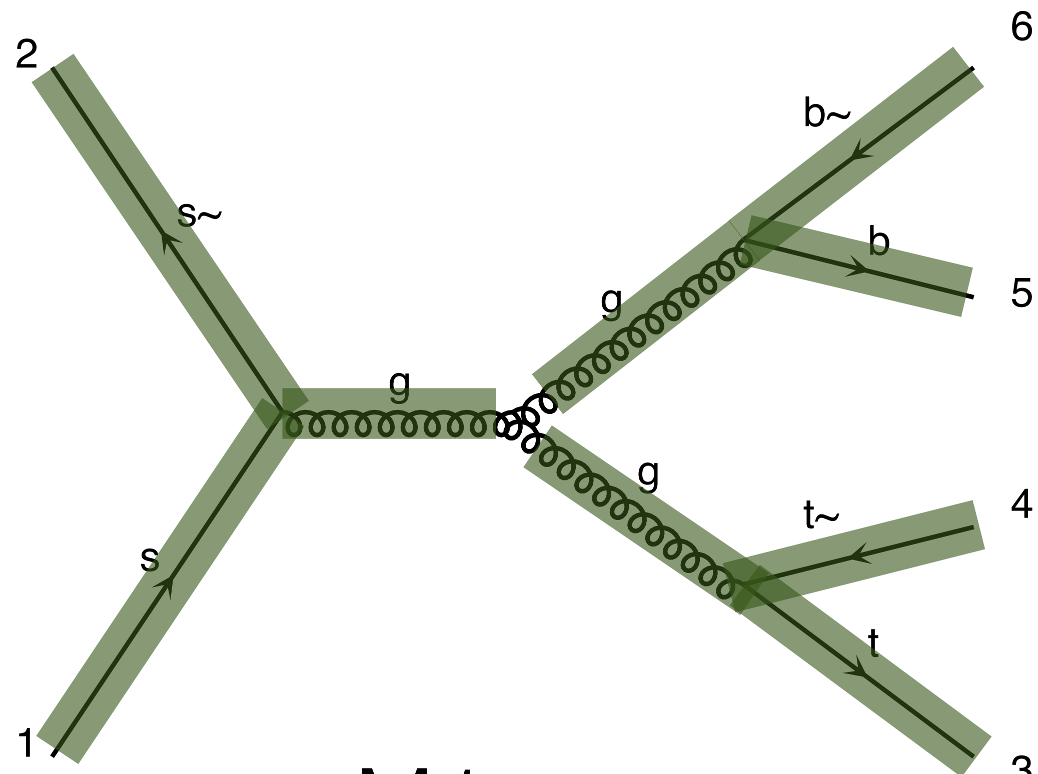
Number of routines: 8

Number of routines for both: 8

$$|M|^2 = |M_1 + M_2|^2$$

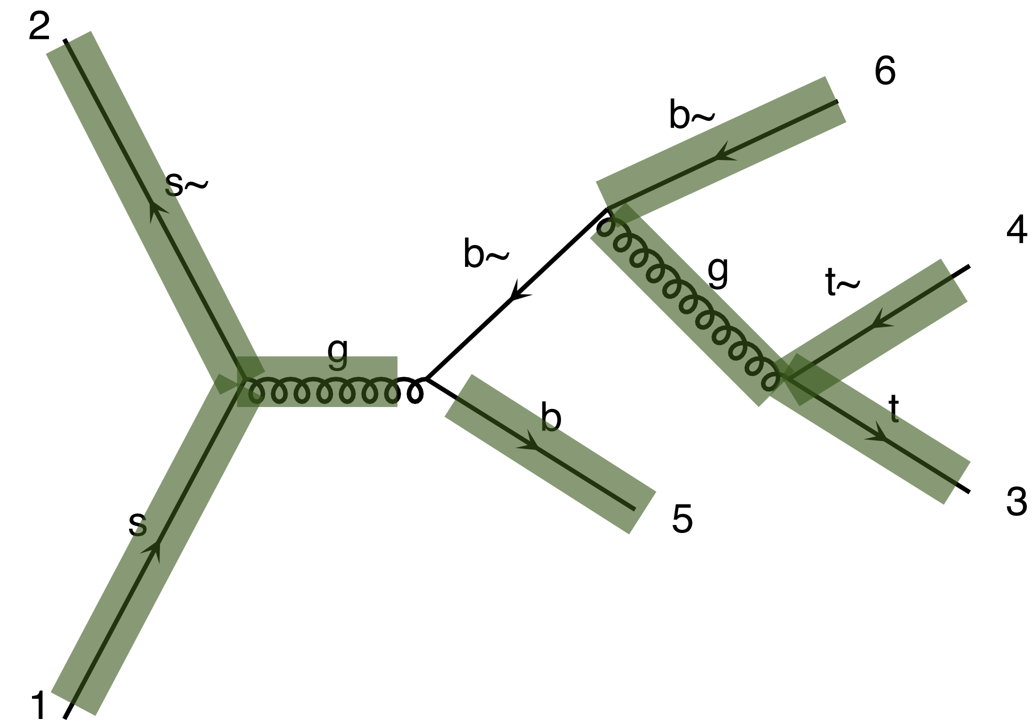
Real case

Known



M1

Number of routines: 9



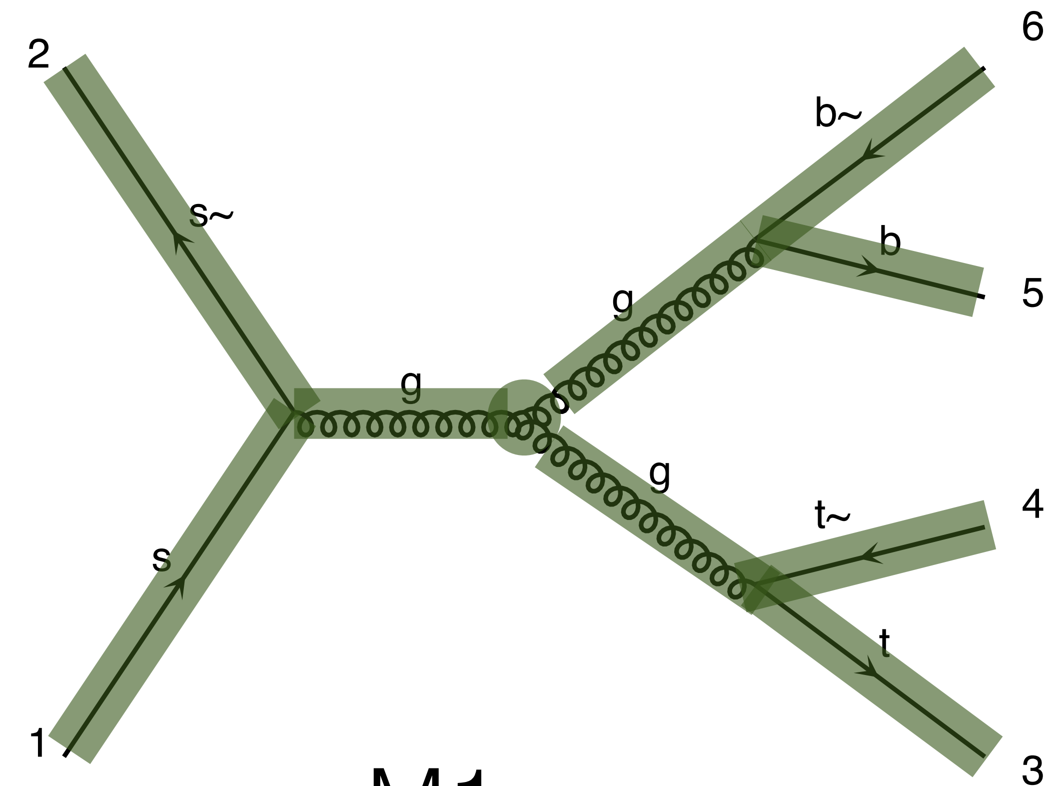
M2

Number of routines: 8

Number of routines for both: 9

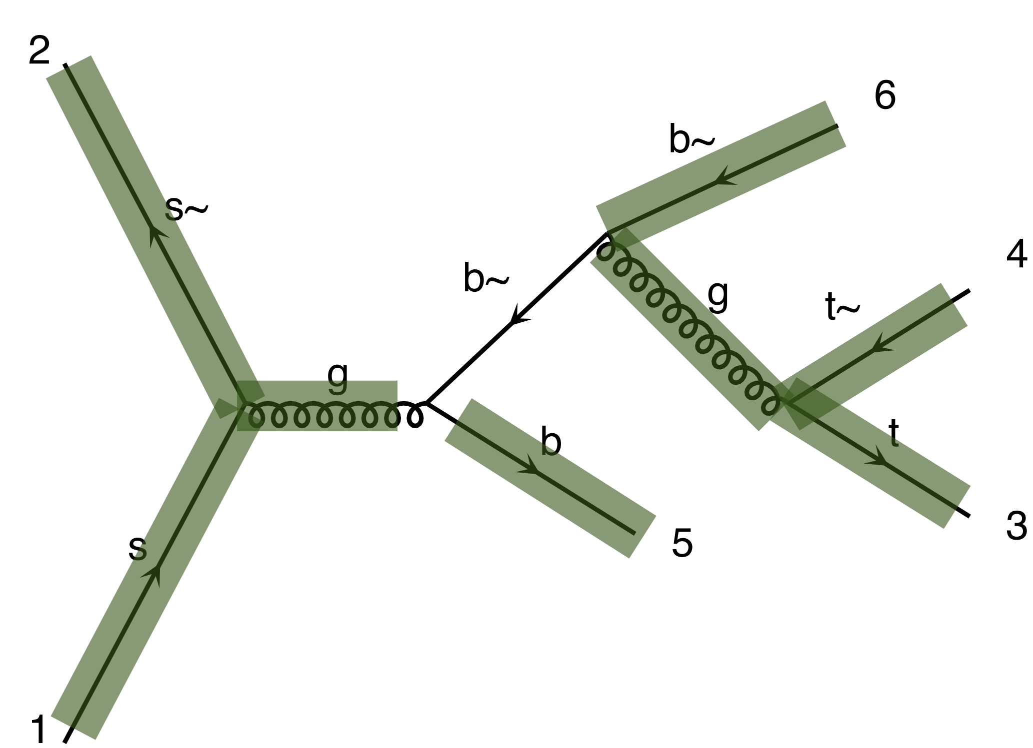
$$|M|^2 = |M_1 + M_2|^2$$

Known



M1

Number of routines: 10



M2

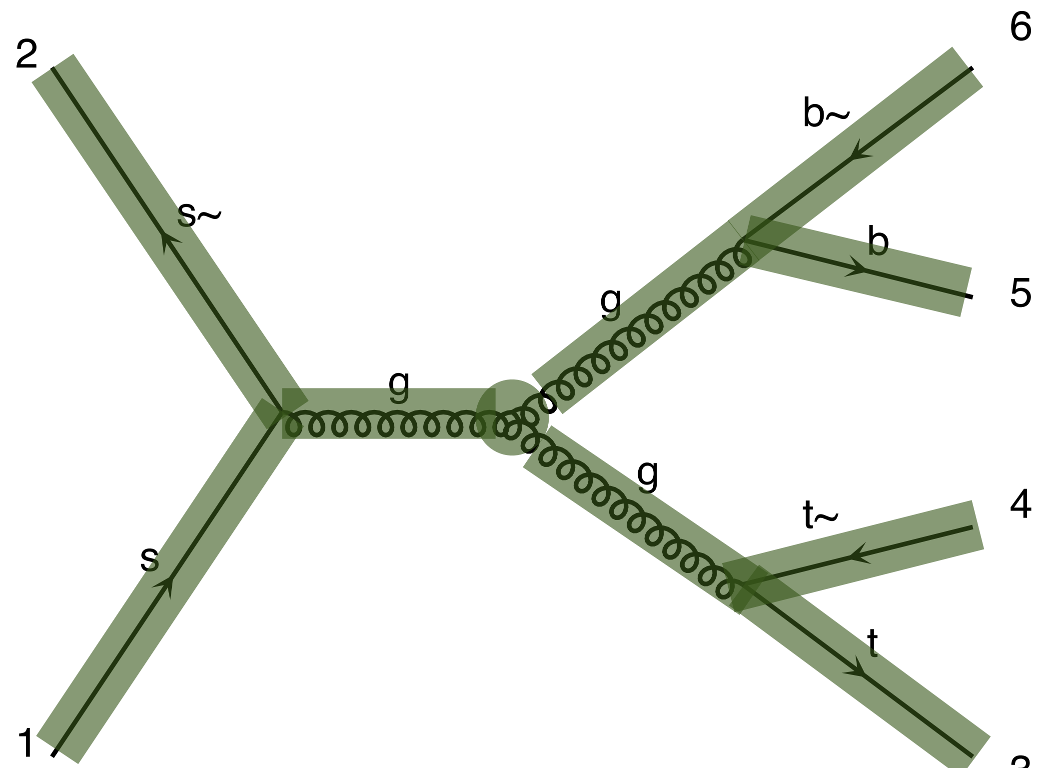
Number of routines: 8

Number of routines for both: 10

$$|M|^2 = |M_1 + M_2|^2$$

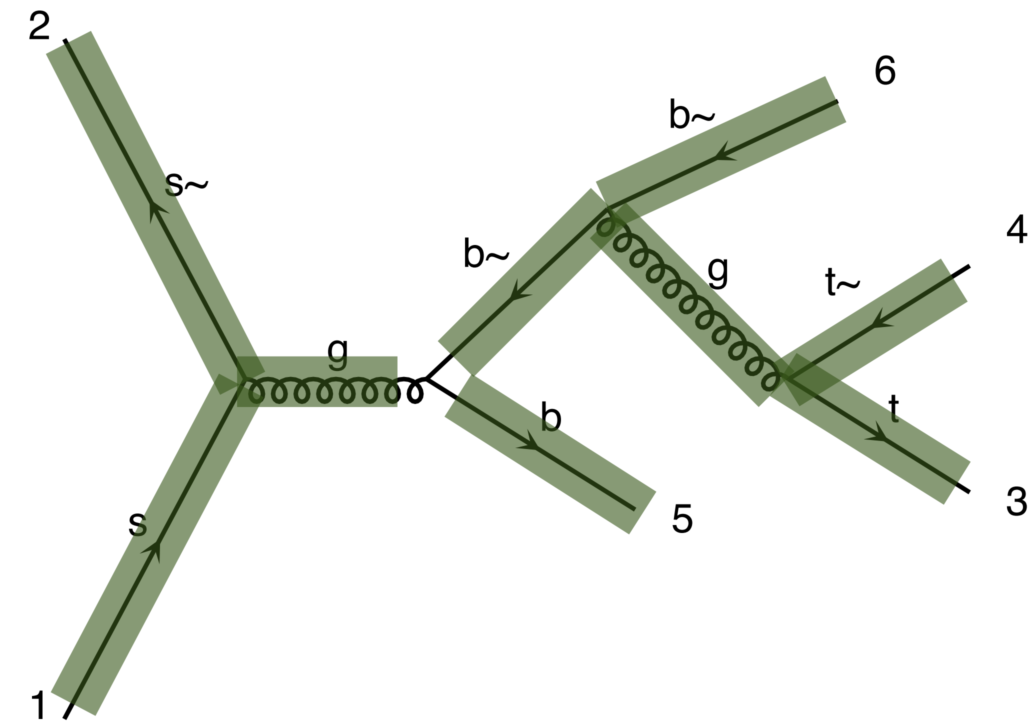
Real case

Known



M1

Number of routines: 10



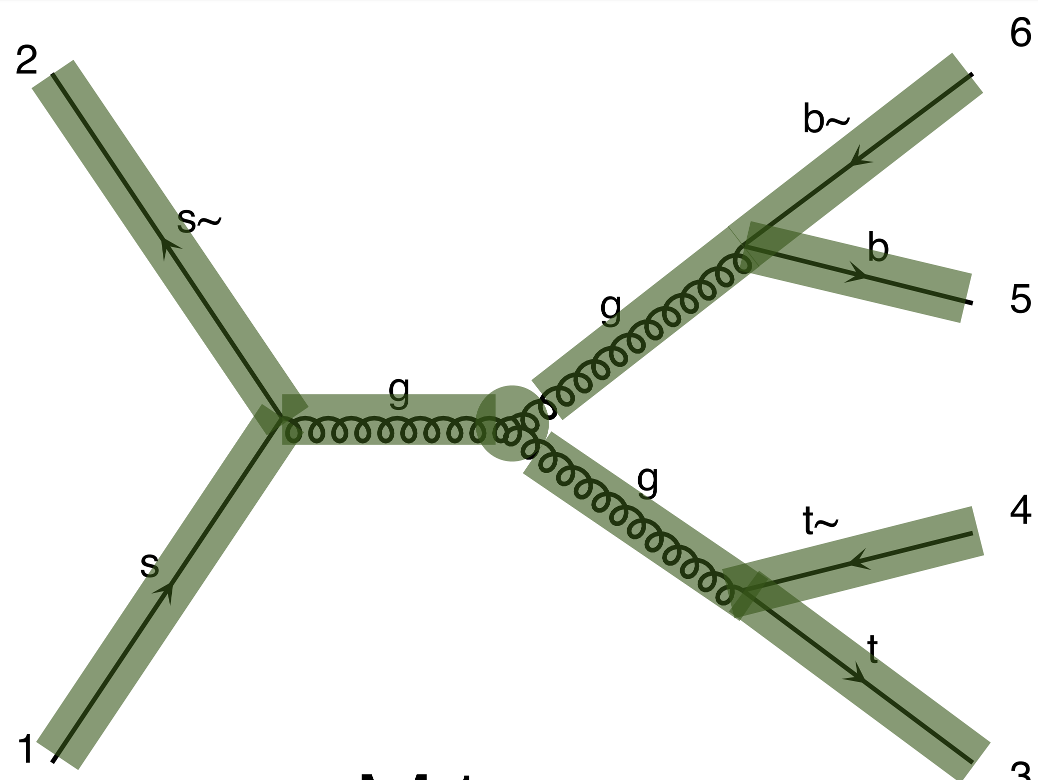
M2

Number of routines: 9

Number of routines for both: 11

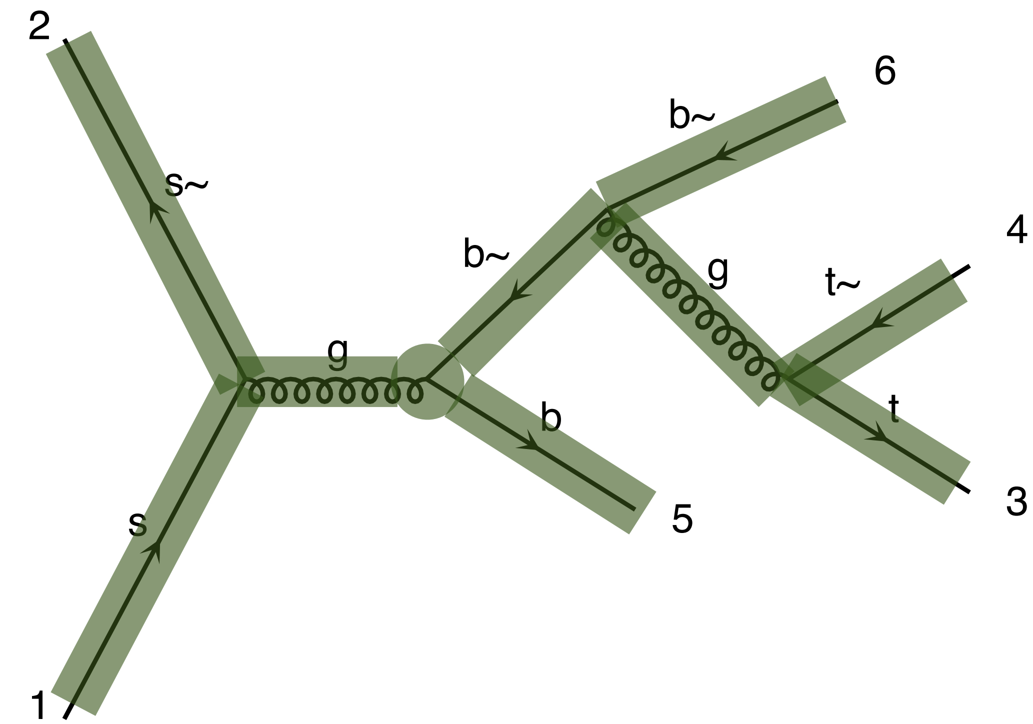
$$|M|^2 = |M_1 + M_2|^2$$

Known



M1

Number of routines: 10



M2

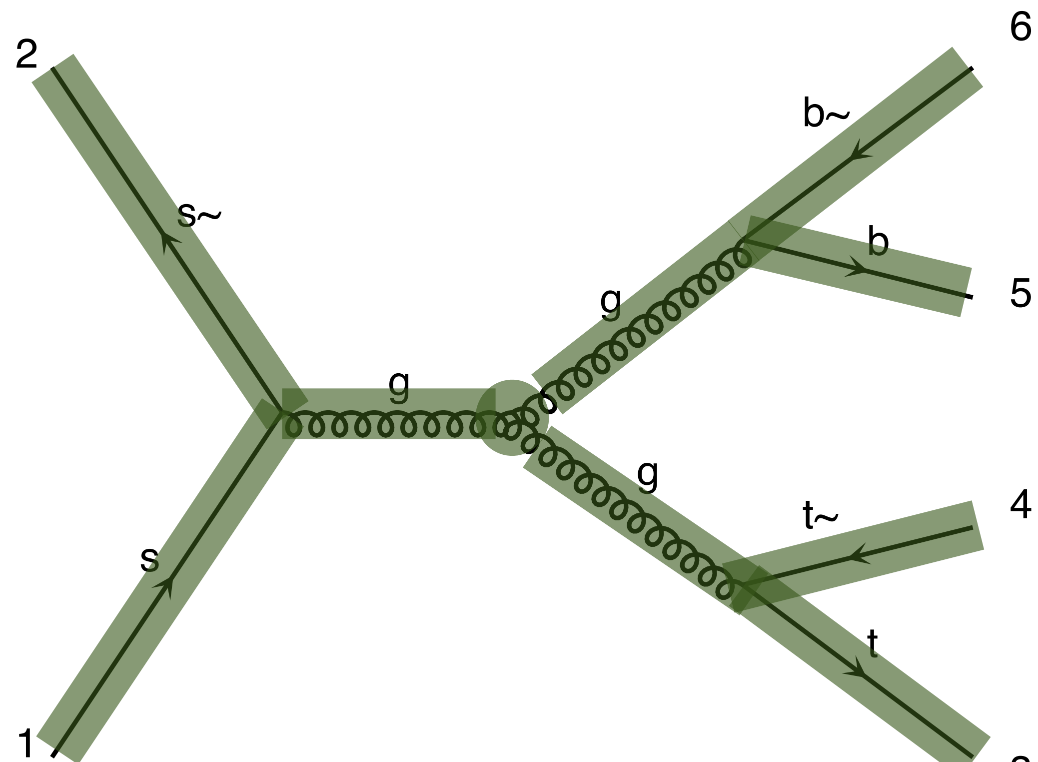
Number of routines: 10

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

Real case

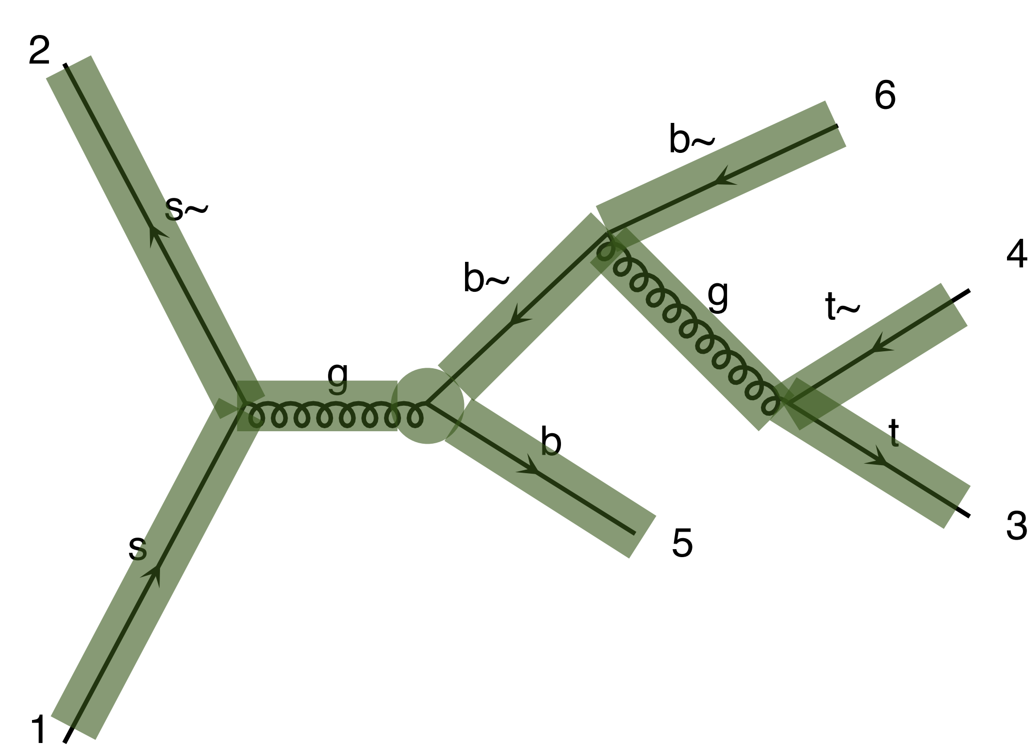
Known



M1

Number of routines: 10

2(N+1)



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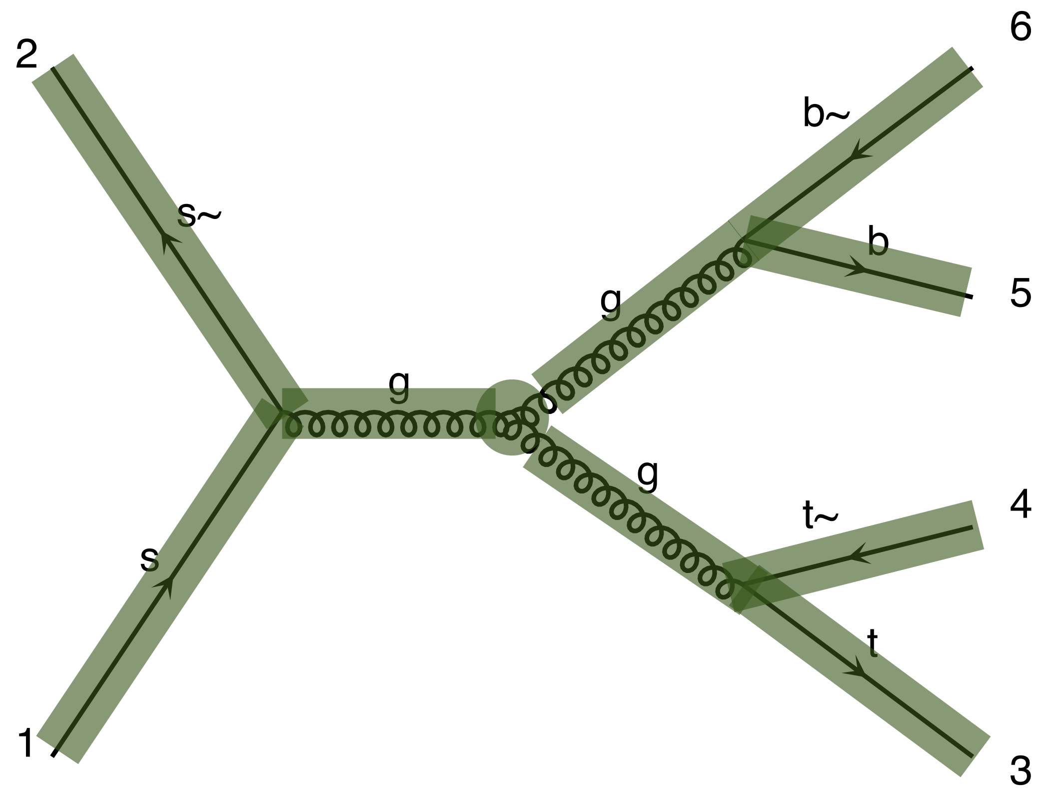

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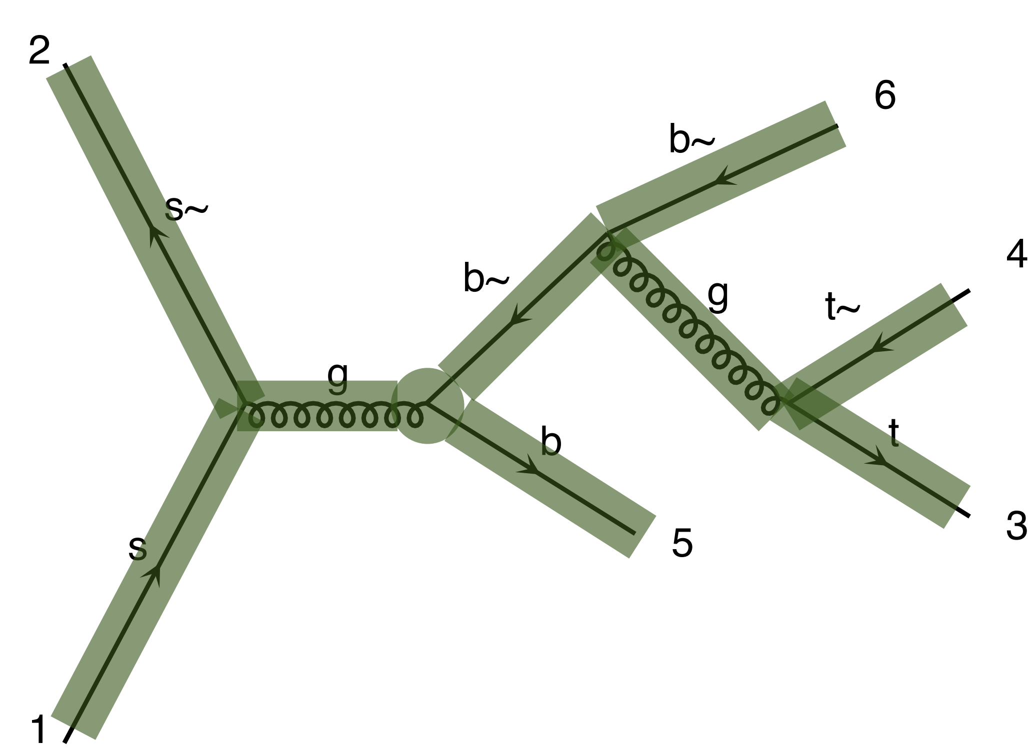
2(N+1)



Number of routines: 10

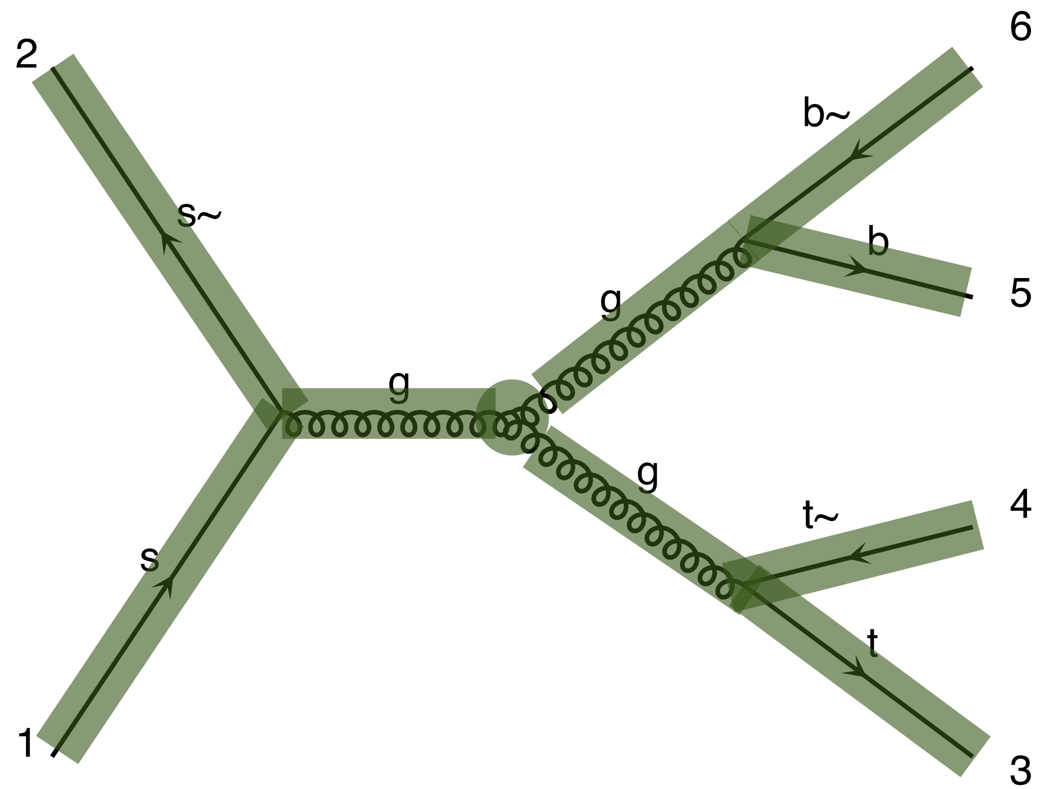
2(N+1)

Number of routines for both: 12

N!*2(N+1) ⟶ N!

Real case

Known



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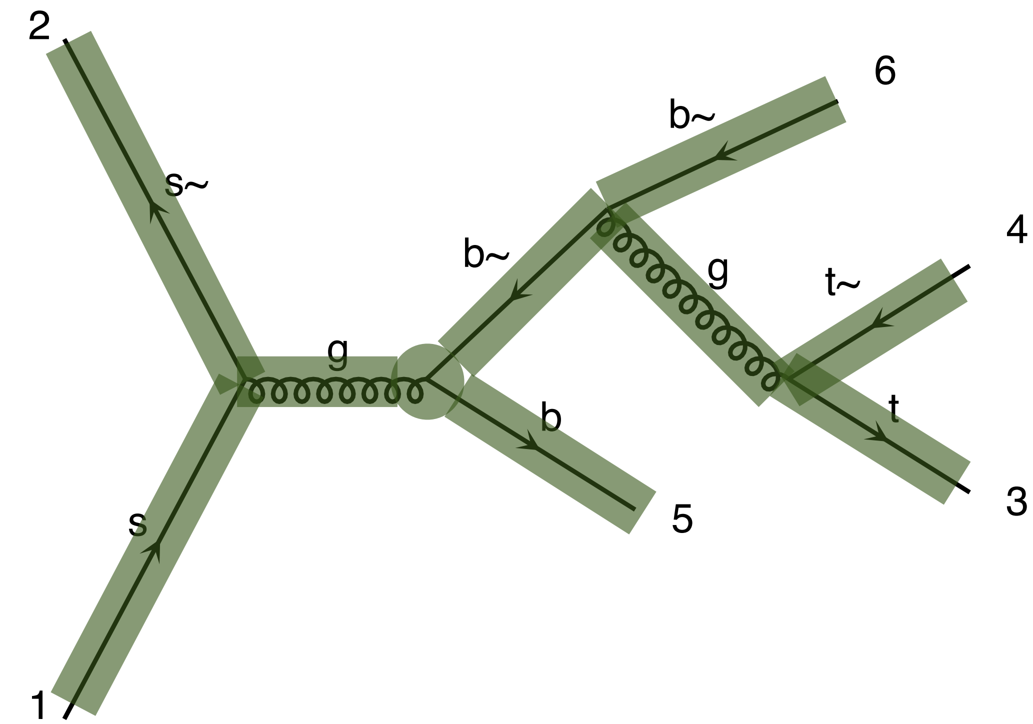

Number of routines: 10

2(N+1)

Number of routines for both: 12

N!*2(N+1) ⟶ N! ⟶ (recursion) 2^N

HELAS

- **Original HELicity Amplitude Subroutine library**

[Murayama, Watanabe, Hagiwara]

HELAS

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- **One routine by Lorentz structure**
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Chiral Perturbation

BNV Model

SLIH

Effective Field Theory

NMSSM

Chromo-magnetic operator

Full HEFT

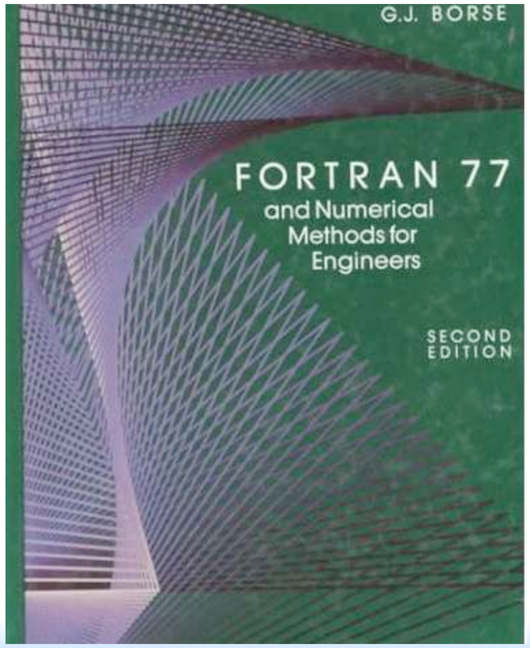
Black Holes

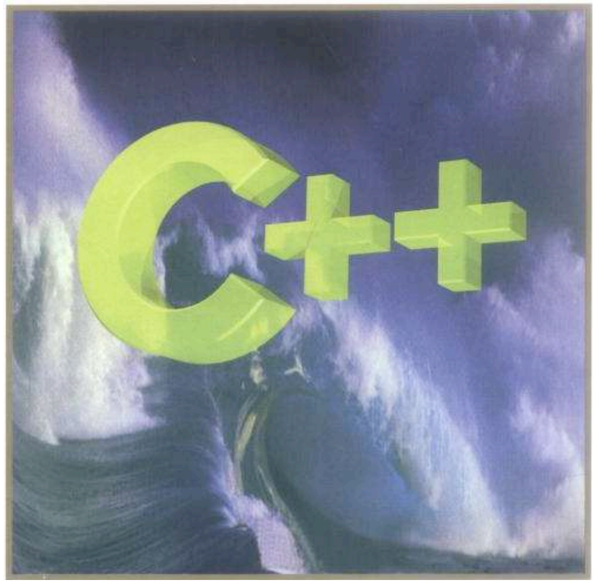
ALOHA

ALOHA ~~Google~~ translate

From: UFO To: Helicity Translate

Type text or a website address or translate a document.





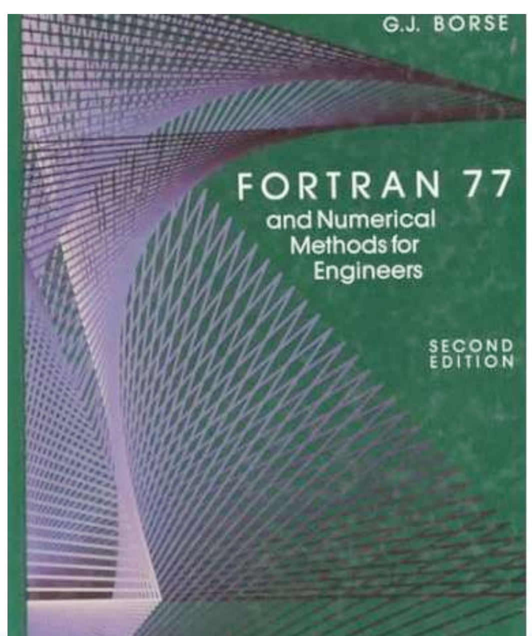
ALOHA

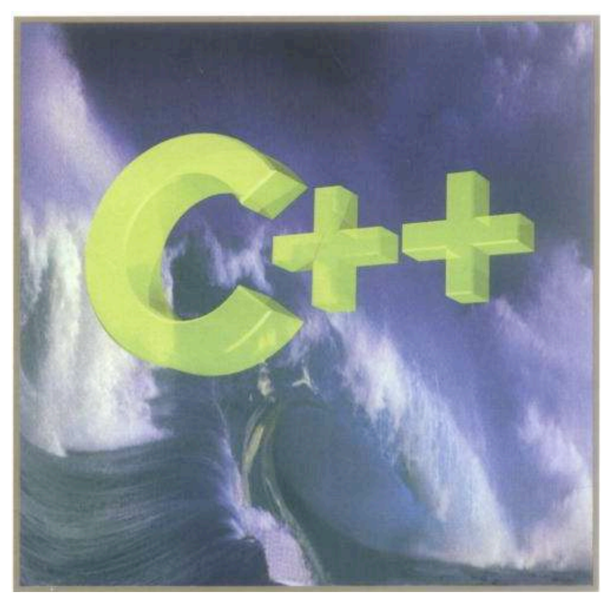
ALOHA ~~Google~~ translate

From: UFO ▾ To: Helicity Translate

Basically, any new operator can be handle by MG5/Pythia8 out of the box!

Type text or a website address or translate a document.





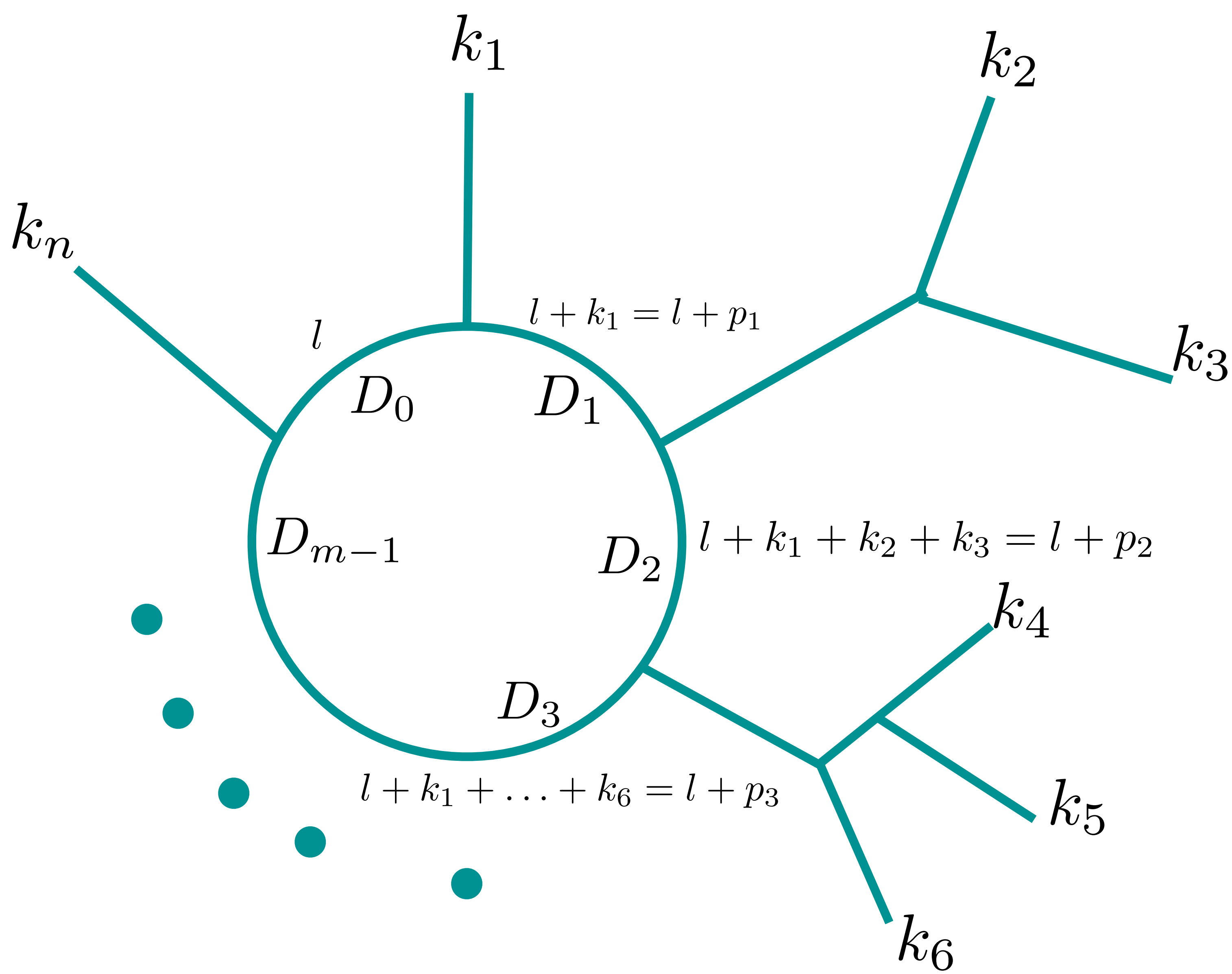
To Remember

- Numerical computation faster than analytical computation
- We are able to compute matrix-element
  - ➡ for large number of final state
  - ➡ for any BSM theory

Loop Computation

One Loop

- Consider this *m*-point loop diagram with *n* external momenta



One Loop

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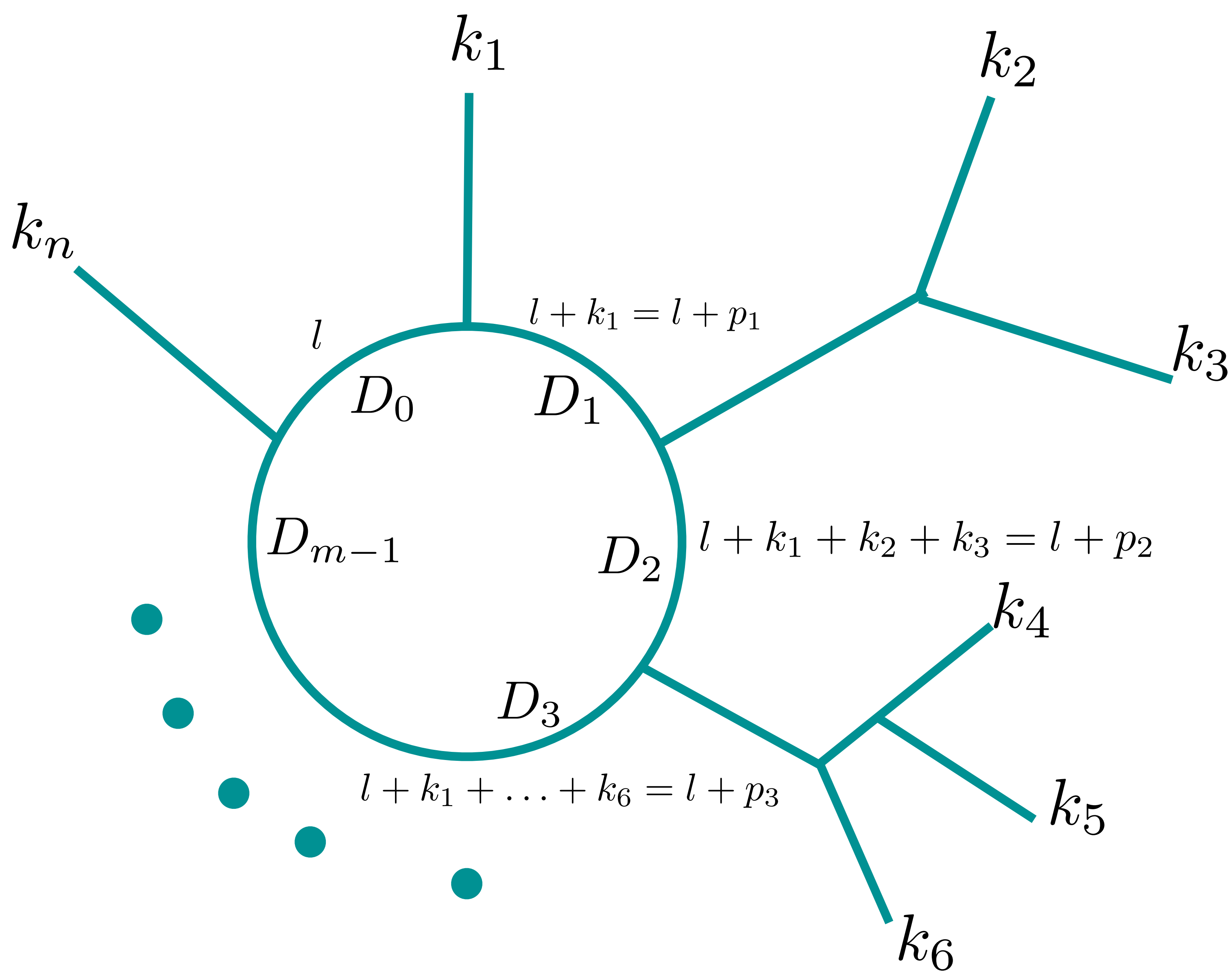

- The integral to compute is

$$\int d^d l \, \frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}}$$

$$D_i = (l + p_i)^2 - m_i^2$$

Integrand reduction

Key Point

- Any one-loop integral can be decomposed in scalar integrals
- The task is to find these coefficients efficiently (analytically or numerically)

Basis of scalar integrals

- The a, b, c, d and R coefficients depend only on external parameters and momenta

$$\mathcal{M}^{\text{1-loop}} = \sum_{i_0<i_1<i_2<i_3} d_{i_0 i_1 i_2 i_3} \text{Box}_{i_0 i_1 i_2 i_3} + \sum_{i_0<i_1<i_2} c_{i_0 i_1 i_2} \text{Triangle}_{i_0 i_1 i_2} + \sum_{i_0<i_1} b_{i_0 i_1} \text{Bubble}_{i_0 i_1} + \sum_{i_0} a_{i_0} \text{Tadpole}_{i_0} + R + \mathcal{O}(\epsilon)$$

$$D_i = (l+p_i)^2 - m_i^2$$

$$\text{Tadpole}_{i_0} = \int d^d l \frac{1}{D_{i_0}}$$

$$\text{Bubble}_{i_0 i_1} = \int d^d l \frac{1}{D_{i_0} D_{i_1}}$$

$$\text{Triangle}_{i_0 i_1 i_2} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2}}$$

$$\text{Box}_{i_0 i_1 i_2 i_3} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2} D_{i_3}}$$

- All these scalar integrals are known and available in computer libraries (FF [v. Oldenborgh], QCDLoop [Ellis, Zanderighi], OneLOop [v. Hameren])

Divergences

- The a, b, c, d and R coefficients depend only on external parameters and momenta

$$\mathcal{M}^{\text{1-loop}} = \sum_{i_0<i_1<i_2<i_3} d_{i_0 i_1 i_2 i_3} \text{Box}_{i_0 i_1 i_2 i_3} + \sum_{i_0<i_1<i_2} c_{i_0 i_1 i_2} \text{Triangle}_{i_0 i_1 i_2} + \sum_{i_0<i_1} b_{i_0 i_1} \text{Bubble}_{i_0 i_1} + \sum_{i_0} a_{i_0} \text{Tadpole}_{i_0} + R + \mathcal{O}(\epsilon)$$

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$$\text{Box}_{i_0 i_1 i_2 i_3} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2} D_{i_3}}$$

➡The coefficients d, c, b and a are finite and do not contain poles in $1/\epsilon$

➡The $1/\epsilon$ dependence is in the scalar integrals (and the UV renormalization)

➡When we have solved this system (and included the UV renormalization) we have the full dependence on the soft/collinear divergences in terms of coefficients in front of the poles. These divergences should cancel against divergences in the real emission corrections (according to KLN theorem)

$$\text{Virtual} \sim v_0 + \frac{v_1}{\epsilon} + \frac{v_2}{\epsilon^2}$$

Integrand reduction

Key Point

- Any one-loop integral can be decomposed in scalar integrals
- The task is to find these coefficients efficiently (analytically or numerically)

Two methods

- Passarino-Veltman
- OPP

Integrand reduction

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- Any one-loop integral can be decomposed in scalar integrals
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Two methods

- Passarino-Veltman
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Standard Approach

- Passarino-Veltman reduction:

$$\int d^d l \, \frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}} \to \sum_i \mathrm{coeff}_i \int d^d l \, \frac{1}{D_0 D_1 \cdots}$$

  - Reduce a general integral to "scalar integrals" by "completing the square"

- Passarino-Veltman reduction:

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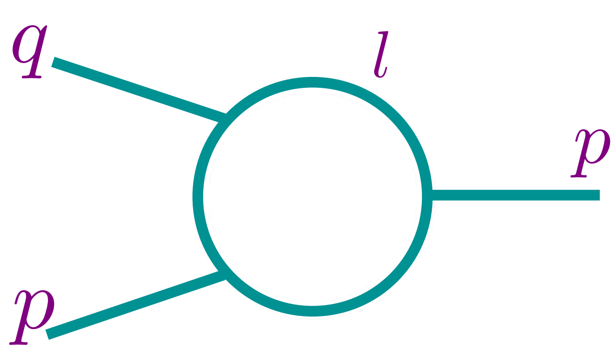
  - Reduce a general integral to "scalar integrals" by "completing the square"

- Let's do an example:
  Suppose we want to calculate this triangle integral



$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2 - m_1^2)((l+p)^2 - m_2^2)((l+q)^2 - m_3^2)}$$

Passarino-Veltman

Main Idea

$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2 - m_1^2)((l+p)^2 - m_2^2)((l+q)^2 - m_3^2)}$$

- The only independent four vectors are p^μ and q^μ . Therefore, the integral must be proportional to those. We can set-up a system of linear equations.

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$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2 - m_1^2)((l+p)^2 - m_2^2)((l+q)^2 - m_3^2)} = \begin{pmatrix} p^\mu & q^\mu \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

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Resolution (dropping the mass)

- contracting with 2*p and 2*q

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$$\int \frac{d^n l}{(2\pi)^n} \frac{2l\cdot p}{l^2(l+p)^2(l+q)^2}$$

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Gram Determinant: G

Resolution (dropping the mass)

- contracting with 2*p and 2*q

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$$\left(\begin{array}{c}2p_\mu\\2q_\mu\end{array}\right)(p^\mu\ q^\mu)\left(\begin{array}{c}C_1\\C_2\end{array}\right) = \left(\begin{array}{cc}2p\cdot p & 2p\cdot q\\2p\cdot q & 2q\cdot q\end{array}\right)\left(\begin{array}{c}C_1\\C_2\end{array}\right) = \left(\begin{array}{c}[2l\cdot p]\\ [2l\cdot q]\end{array}\right)$$

Gram Determinant: G

Passarino-Veltman

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- contracting with 2*p and 2*q

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$$\begin{pmatrix} 2p_\mu \\ 2q_\mu \end{pmatrix} (p^\mu \; q^\mu) \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 2p\cdot p & 2p\cdot q \\ 2p\cdot q & 2q\cdot q \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} [2l\cdot p] \\ [2l\cdot q] \end{pmatrix}$$

Gram Determinant: G

Resolution (dropping the mass)

- express the integral as simpler integral

$$\int \frac{d^n l}{(2\pi)^n}\frac{2l\cdot p}{l^2(l+p)^2(l+q)^2} = \int \frac{d^n l}{(2\pi)^n}\frac{(l+p)^2 - l^2 - p^2}{l^2(l+p)^2(l+q)^2}$$

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$$= \int \frac{d^nl}{(2\pi)^n}\frac{1}{l^2(l+q)^2} - \int \frac{d^nl}{(2\pi)^n}\frac{1}{(l+p)^2(l+q)^2} - p^2\int \frac{d^nl}{(2\pi)^n}\frac{1}{l^2(l+p)^2(l+q)^2}$$

Passarino-Veltman

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Scalar Integral: Know analytically

Resolution (dropping the mass)

- contracting with $2{*}p$ and $2{*}q$

$$[2l\cdot p] = \int \frac{d^n l}{(2\pi)^n}\frac{2l\cdot p}{l^2(l+p)^2(l+q)^2}$$

$$\begin{pmatrix} 2p_\mu \\ 2q_\mu \end{pmatrix}(p^\mu \; q^\mu)\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 2p\cdot p & 2p\cdot q \\ 2p\cdot q & 2q\cdot q \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} [2l\cdot p] \\ [2l\cdot q] \end{pmatrix}$$

Gram Determinant: G

Already computed

Resolution (dropping the mass)

- contracting with 2*p and 2*q

$$[2l\cdot p] = \int \frac{d^n l}{(2\pi)^n}\frac{2l\cdot p}{l^2(l+p)^2(l+q)^2}$$

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Gram Determinant: G

Final Step

- Inverting the Gram Determinant

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = G^{-1} \begin{pmatrix} [2l\cdot p] \\ [2l\cdot q] \end{pmatrix}$$

- We have an expression in term of scalar integral

$$\int \frac{d^n l}{(2\pi)^n}\frac{l^\mu}{(l^2-m_1^2)((l+p)^2-m_2^2)((l+q)^2-m_3^2)} = \begin{pmatrix} p^\mu & q^\mu \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

Already computed

Key Point

- Any one-loop integral can be decomposed in scalar integrals
- The task is to find these coefficients efficiently (analytically or numerically)

Two methods

- Passarino-Veltman
- OPP

Integrand reduction

Key Point

- Any one-loop integral can be decomposed in scalar integrals
- The task is to find these coefficients efficiently (analytically or numerically)

Two methods

- Passarino-Veltman
- OPP

OPP Reduction

- The decomposition to scalar integrals presented before works at the level of the integrals

$$\mathcal{M}^{\text{1-loop}} = \sum_{i_0<i_1<i_2<i_3} d_{i_0i_1i_2i_3}\text{Box}_{i_0i_1i_2i_3} + \sum_{i_0<i_1<i_2} c_{i_0i_1i_2}\text{Triangle}_{i_0i_1i_2} + \sum_{i_0<i_1} b_{i_0i_1}\text{Bubble}_{i_0i_1} + \sum_{i_0} a_{i_0}\text{Tadpole}_{i_0} + R + \mathcal{O}(\epsilon)$$

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- If we would know a similar relation at the **integrand** level, we would be able to manipulate the integrands and extract the coefficients without doing the integrals

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- If we would know a similar relation at the **integrand** level, we would be able to manipulate the integrands and extract the coefficients without doing the integrals

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}(l)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0 i_1 i_2} + \tilde{c}_{i_0 i_1 i_2}(l)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i + \sum_{i_0<i_1}^{m-1} \left[b_{i_0 i_1} + \tilde{b}_{i_0 i_1}(l)\right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i \neq i_0}^{m-1} D_i + \tilde{P}(l) \prod_{i}^{m-1} D_i$$

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Spurious term

spurious terms

- The functional form of the spurious terms is known (it depends on the rank of the integral and the number of propagators in the loop) [del Aguila, Pittau 2004]
  - for example, a box coefficient from a rank 1 numerator is

$$\tilde{d}_{i_0 i_1 i_2 i_3}(l) = \tilde{d}_{i_0 i_1 i_2 i_3}\, \epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma$$

    (remember that p_i is the sum of the momentum that has entered the loop so far, so we always have $p_0 = 0$)

  - The integral is zero

$$\int d^d l \frac{\tilde{d}_{i_0 i_1 i_2 i_3}(l)}{D_0 D_1 D_2 D_3} = \tilde{d}_{i_0 i_1 i_2 i_3} \int d^d l \frac{\epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma}{D_0 D_1 D_2 D_3} = 0$$

How it works...

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$

$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$

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To solve the OPP reduction, choosing special values for the loop momenta helps a lot

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For example, choosing *l* such that

$$D_0(l^\pm) = D_1(l^\pm) = \\ = D_2(l^\pm) = D_3(l^\pm) = 0$$

sets all the terms in this equation to zero except the first line

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$$+ \tilde{P}(l) \prod_{i}^{m-1} D_i$$

$$= 0$$

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There are two (complex) solutions to this equation due to the quadratic nature of the propagators

How it works...

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$

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Coefficient computed in a previous step

How it works...

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$

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Now we choose l such that

$$D_0(l^i) = D_1(l^i) = D_2(l^i) = 0$$

sets all the terms in this equation to zero except the first and second line

Coefficient computed in a previous step

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$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}(l) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i$$
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$$+ \tilde{P}(l) \prod_{i}^{m-1} D_i$$
$$= 0$$

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$$+\tilde{P}(l) \prod_{i}^{m-1} D_i$$

Now, choosing *l* such that

$$D_0(l^i) = D_1(l^i) = 0$$

sets all the terms in this equation to zero except the first, second and third line

Coefficient computed in a previous step

How it works...

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$

$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$

$$+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0i_1} + \tilde{b}_{i_0i_1}(l)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i$$

$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i\neq i_0}^{m-1} D_i$$

$$+\tilde{P}(l) \prod_{i}^{m-1} D_i$$

$$= 0$$

Now, choosing *l* such that

$$D_0(l^i) = D_1(l^i) = 0$$

sets all the terms in this equation to zero except the first, second and third line

Coefficient computed in a previous step

How it works...

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}(l)\right] \prod_{i \neq i_0,i_1,i_2,i_3}^{m-1} D_i$$
$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0 i_1 i_2} + \tilde{c}_{i_0 i_1 i_2}(l)\right] \prod_{i \neq i_0,i_1,i_2}^{m-1} D_i$$
$$+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0 i_1} + \tilde{b}_{i_0 i_1}(l)\right] \prod_{i \neq i_0,i_1}^{m-1} D_i$$
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$$+ \tilde{P}(l) \prod_{i}^{m-1} D_i$$

Now, choosing *l* such that

$$D_1(l^i) = 0$$

sets the last line to zero

Coefficient computed in a previous step

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Now, choosing arbitrary *l*

Coefficient computed in a previous step

How it works...

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}(l)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i$$

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$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i \neq i_0}^{m-1} D_i$$

$$+ \tilde{P}(l) \prod_{i}^{m-1} D_i$$

We have our Numerator!

Coefficient computed in a previous step

d dimensions

- In the previous consideration I was very sloppy in considering if we are working in 4 or d dimensions
- In general, external momenta and polarization vectors are in 4 dimensions; only the loop momentum is in d dimensions

- To be more correct, we compute the integral

$$\int d^d l \, \frac{N(l,\tilde{l})}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \cdots \bar{D}_{m-1}}$$

$$\bar{l} = l + \tilde{l}$$

($\bar{l}$: d dim; l: 4 dim; $\tilde{l}$: epsilon dim)

$$\bar{D}_i = (\bar{l} + p_i)^2 - m_i^2 = (l + p_i)^2 - m_i^2 + \tilde{l}^2 = D_i + \tilde{l}^2$$

$$l \cdot \tilde{l} = 0 \qquad \bar{l} \cdot p_i = l \cdot p_i \qquad \bar{l} \cdot \bar{l} = l \cdot l + \tilde{l} \cdot \tilde{l}$$

Implications

- The decomposition in terms of scalar integrals has to be done in d dimensions
- This is why the rational part R is needed

$$
\begin{aligned}
&\sum_{0\le i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) \int d^d\bar{\ell}\, \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}\bar{D}_{i_3}} \\
&+ \sum_{0\le i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) \int d^d\bar{\ell}\, \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}} \\
&+ \sum_{0\le i_0<i_1}^{m-1} b(i_0 i_1) \int d^d\bar{\ell}\, \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}} \\
&+ \sum_{i_0=0}^{m-1} a(i_0) \int d^d\bar{\ell}\, \frac{1}{\bar{D}_{i_0}} \\
&+ R\,.
\end{aligned}
$$

Rational terms

$$R = R_1 + R_2$$

Rational terms

- In the OPP method, they are split into two contributions, generally called

$$R = R_1 + R_2$$

- Both have their origin in the UV part of the model, but only R_1 can be directly computed in the OPP reduction and is given by the CutTools program

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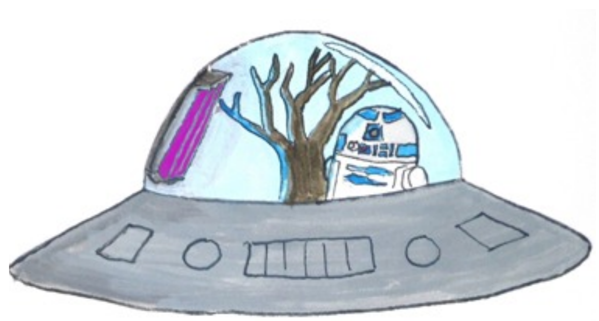


Integrand reduction

Key Point

- Any one-loop integral can be decomposed in scalar integrals
- The task is to find these coefficients efficiently (analytically or numerically)

Two methods

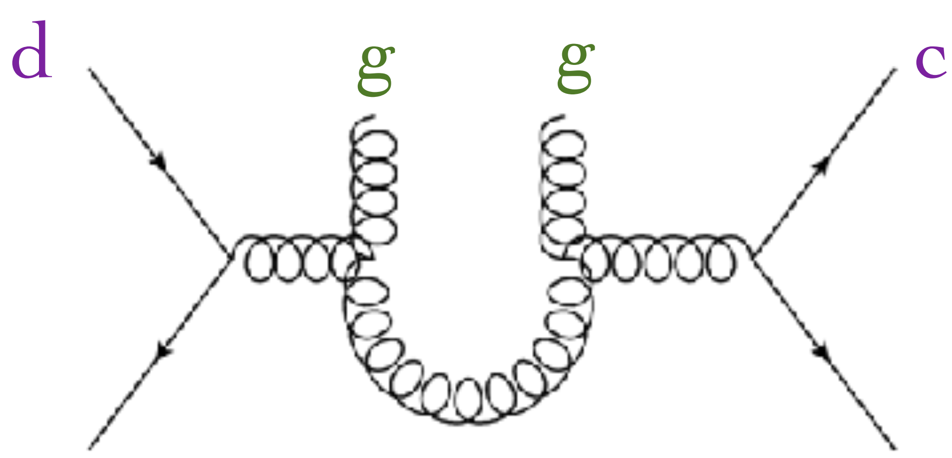
- Passarino-Veltman
- OPP

One Tool

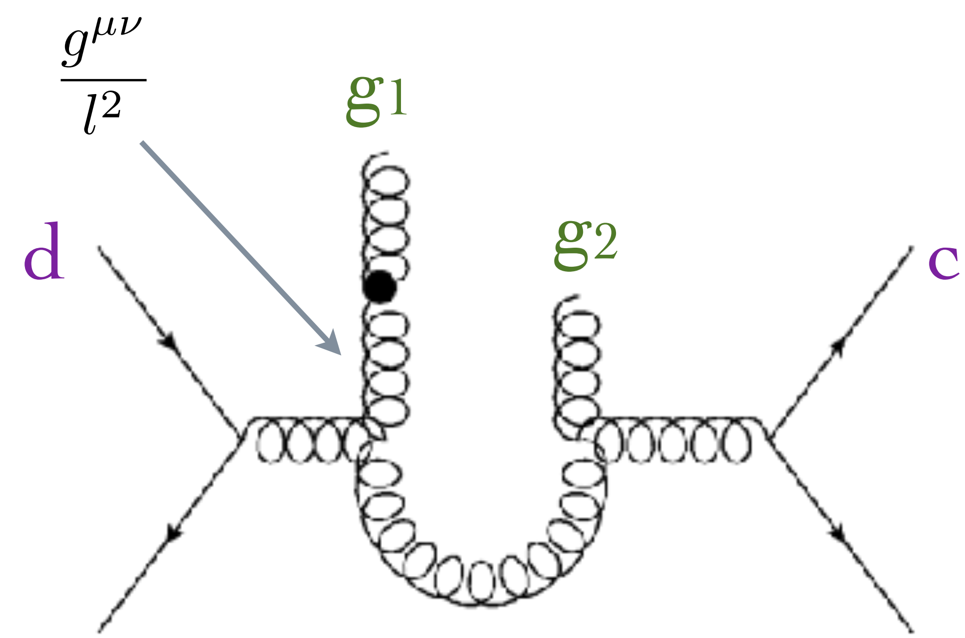
- MadLoop

Numerator

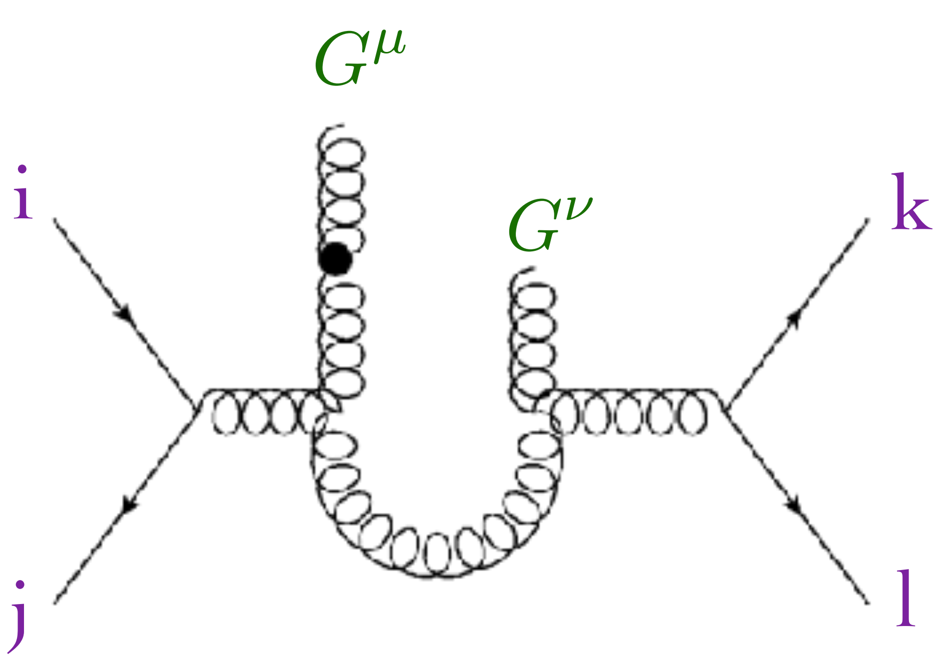
- We want to use (modified) HELAS method



⇨



- Closing the lorentz trace :



⇨

$$\delta^{\mu\nu} = \sum_{i=0}^{4} \underbrace{\delta^{\mu i}}_{G^\mu} \underbrace{\delta^{i\nu}}_{G^\nu}$$

External Wavefunction for HELAS

- Other modifications :
  - ➥ Allow for the loop momentum to be complex
  - ➥ Remove the denominator of the loop propagators
  - ➥ Close the color trace

A L O H A

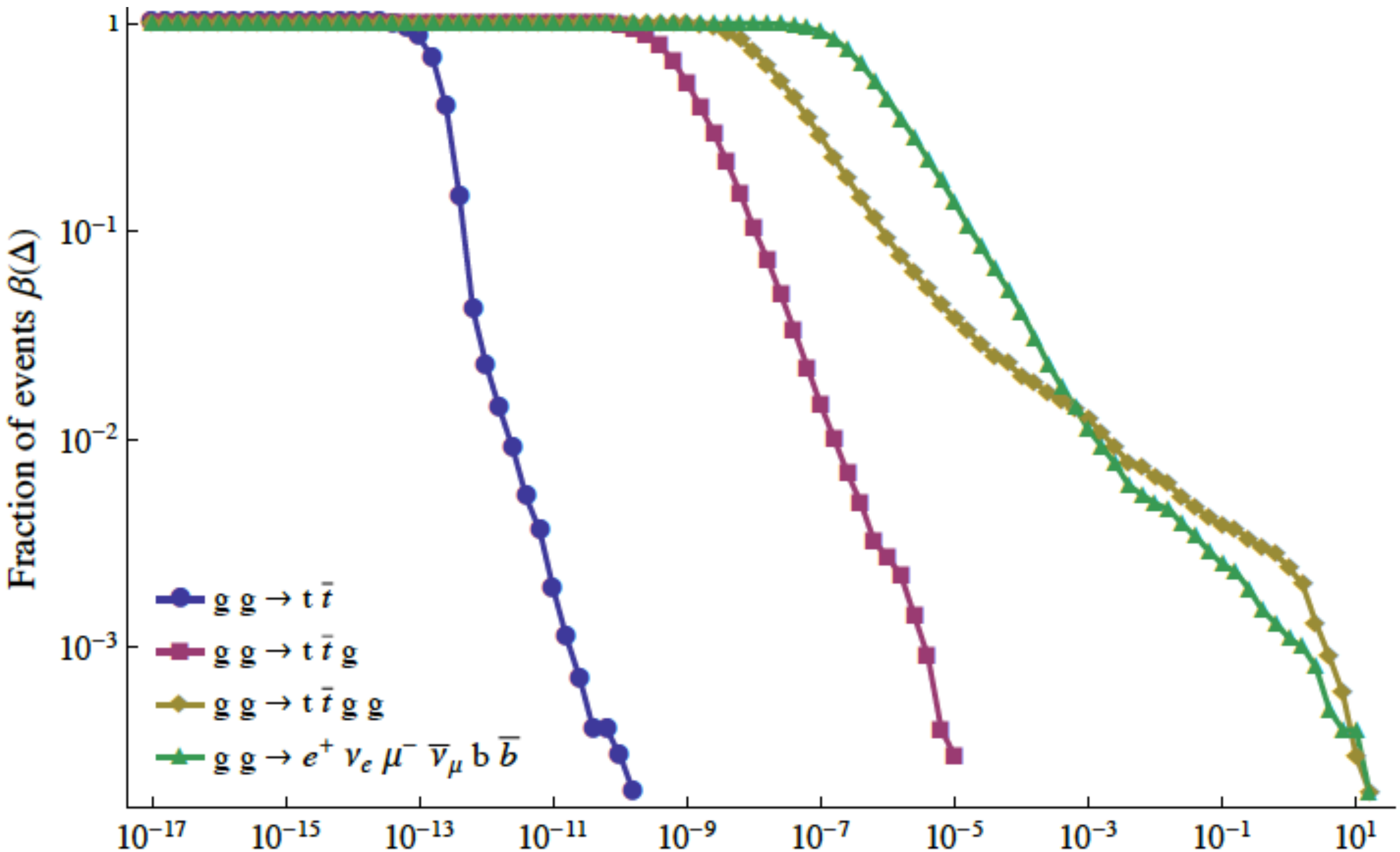
- Ok, now this gives you $\mathcal{N}(l^\mu)$, the integrand numerator to be fed to CT!

- But this is **SLOW**!!
- We have to compute this numerator ~ 50 times for each phase-space point!
- Idea instead of computing the numerator compute the polynomial form

$$\mathcal{N}(l^\mu) = \sum_{r=0}^{r_{max}} C^{(r)}_{\mu_0\mu_1\cdots\mu_r} l^{\mu_0} l^{\mu_1} \dots l^{\mu_r}$$

[S. Pozzorini & al. hep-ph/1111.5206]

Numerical Stability

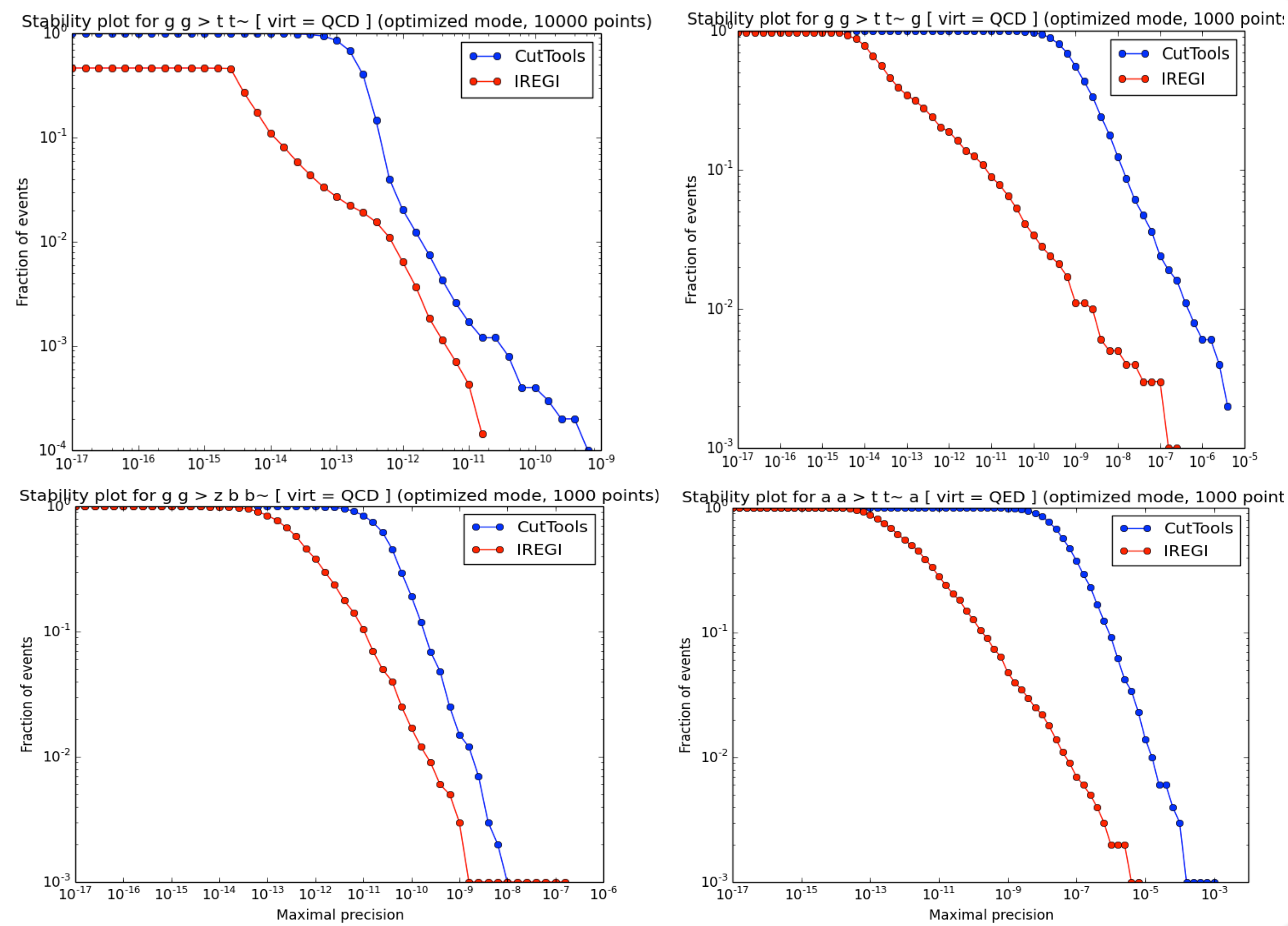


- For 2 to 4 processes, ~7% of the Phase-space point have a precision worse than 1e-3

➡ Previous solution pass to quadruple precision (extremelly slow)

IREGI

- New Solution use IREGI: a TIR program
  - ➡ Slower than previous method but faster than quadruple precision
  - ➡ Usually less uncertainty (and not for the same PS point)



[H.-shao]

To Remember

- The main trick is to decompose in scalar integral
- OPP: works at the integrand level
- TIR: works at the integral level
- Loop evaluation is very slow
- Loop evaluation can be “unstable”

Various package in MG5_aMC@NLO

Plan

exemple: HEFT

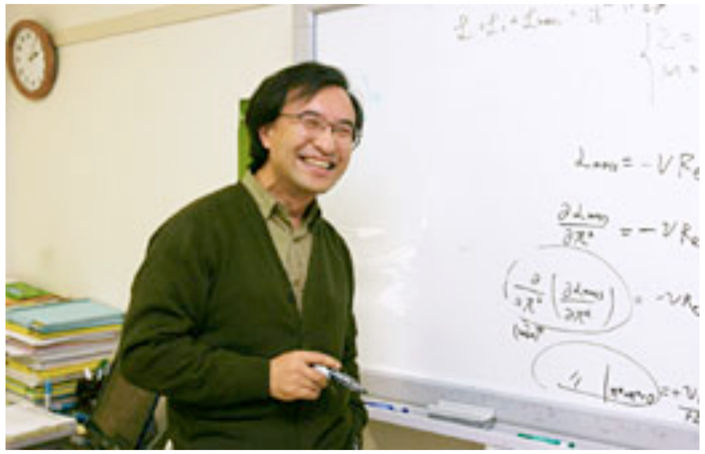
- Model Description
- Width Computation
- Decay Chain
- Interference

Will not be cover

- Re-Weighting method
- Scale Variation
- TauDecay
- MadDM
- MadWeight
- Standalone
- external matrix element provider (Pythia8 and Matchbox)

MadGraph

1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO



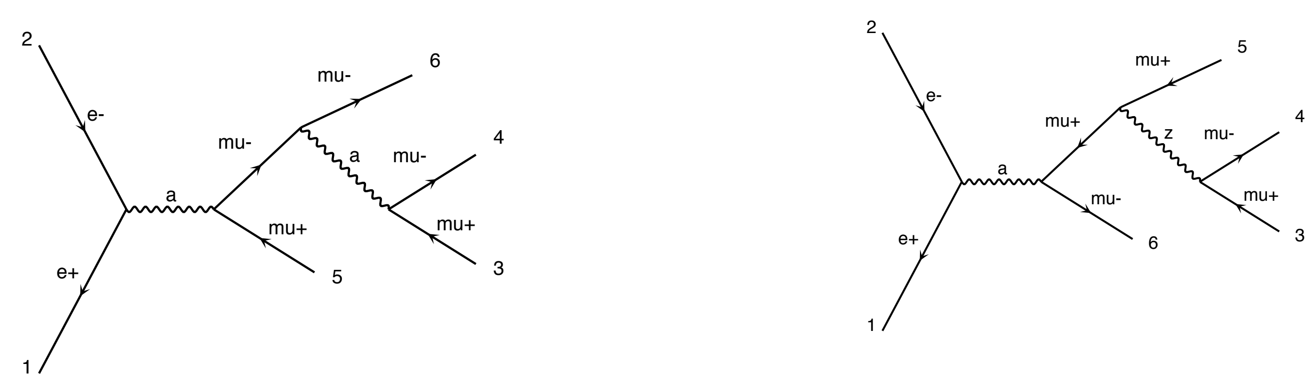
- Computing Matrix Element for a fixed Helicity and sum over the felicities.

- Suite of Routine, which allow to write the matrix element for any (SM) process

MadGraph

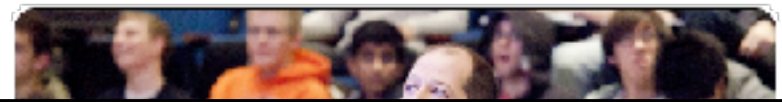
1991	HELAS
1994	**MadGraph**
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO

- Automate the creation of the diagram generation and the writing of the HELAS routine



MadGraph

1991 HELAS



199

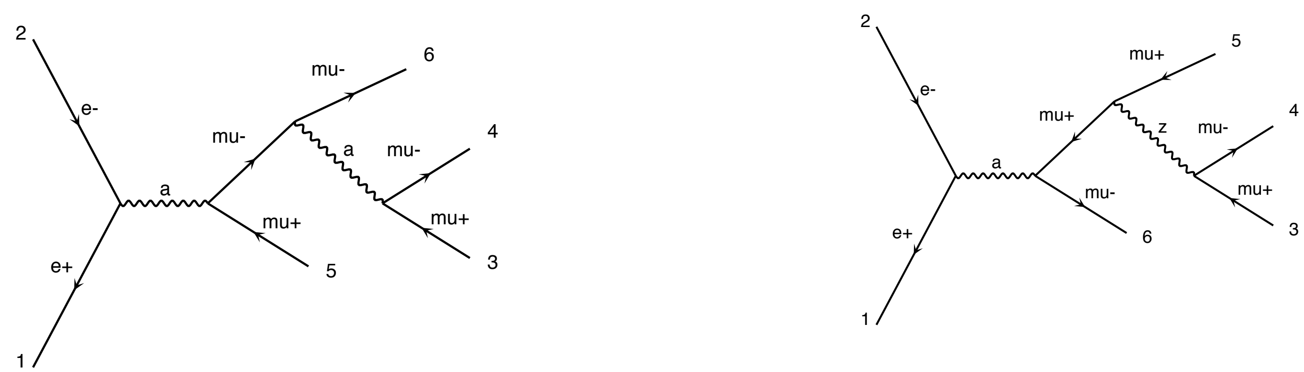
MAD stands for Madison

200

200

ram

2011 MadGraph5



2014 MadGraph5_ aMC@NLO

MadGraph

1991	HELAS
1994	MadGraph
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2011	MadGraph5
2014	MadGraph5_ aMC@NLO

- Multi-Channel Method!
- Automatic phase-space Integration
- Generation of Events

- Support for the MSSM (SMADGRAPH)

MadGraph

1991 HELAS

1994 MadGraph

2002 MadEvent

2006 MG/MEv4

2011 MadGraph5

2014 MadGraph5_aMC@NLO

- Support for BSM
- Decay Chain
- Pass to a platform (MadOnia/MadWeight/…)
- Link to Pythia/PGS
- Matching/Merging

- Official/Main SM generator for CMS

MadGraph

1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO

- Full restart of the MadGraph part in Python
- Fully Automatic BSM
- Various Output Format
- Huge Improvement

MadGraph

1991 HELAS

1994 MadGraph

2002 MadEvent

2006 MG/MEv4

2011 MadGraph5

2014 MadGraph5_aMC@NLO

- Fully Automatic computation at
  - NLO* (cross-section)
  - NLO* matched to PS

*NLO= NLO in QCD

Effective Field Theory

$$\mathcal{L} = \mathcal{L}_{SM} + \sum \frac{c_i}{\Lambda^2} \mathcal{O}_i$$

- Only few Operators for one process and different effects

Weak Boson production

Conserving CP

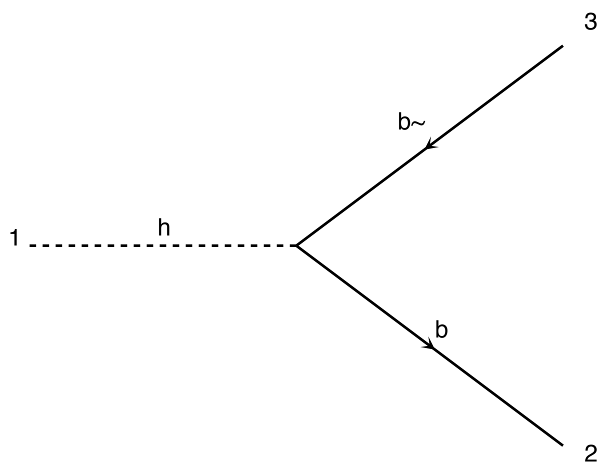
$$\mathcal{O}_{WWW} = \mathrm{Tr}[W_{\mu\nu} W^{\nu\rho} W_\rho^\mu]$$
$$\mathcal{O}_W = (D_\mu \Phi)^\dagger W^{\mu\nu} (D_\nu \Phi)$$
$$\mathcal{O}_B = (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi)$$

Not Conserving CP

$$\mathcal{O}_{\tilde{W}WW} = \mathrm{Tr}[\tilde{W}_{\mu\nu} W^{\nu\rho} W_\rho^\mu]$$
$$\mathcal{O}_{\tilde{W}} = (D_\mu \Phi)^\dagger \tilde{W}^{\mu\nu} (D_\nu \Phi)$$

2 body decay

$$\Gamma = \frac{1}{2MS} \int d\Phi_2 |\mathcal{M}|^2$$



2-body decay

2 body decay

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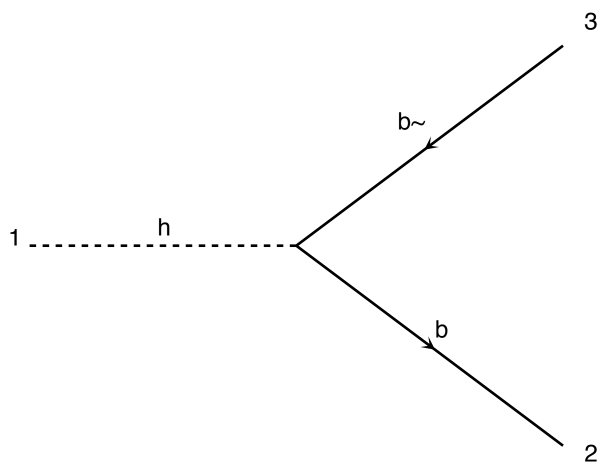

- By Lorentz Invariance the matrix element is constant over the phase-space.

$$\Gamma = \frac{\sqrt{\lambda(M^2, m_1^2, m_2^2)}|\mathcal{M}|^2}{16\pi S M^3}$$

$$\lambda(M^2, m_1^2, m_2^2) = \left(M^2 - m_1^2 - m_2^2\right)^2 - 4 m_1^2 m_2^2$$

2-body decay

2 body decay

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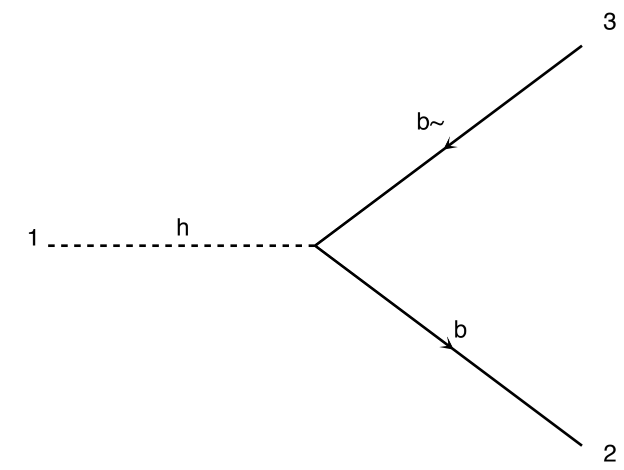

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$$\lambda(M^2, m_1^2, m_2^2) = \left(M^2 - m_1^2 - m_2^2\right)^2 - 4m_1^2 m_2^2$$

- Calculable analytically by FeynRules

MadWidth

hep-ph/1402.1178

2-body

- Use FeynRules formula (instateneous)

MadWidth

hep-ph/1402.1178

2-body

Fast-Estimation of 3-body

- Only use 2-body decay and PS factor

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hep-ph/1402.1178

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Relevant?

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MadWidth

hep-ph/1402.1178

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Relevant?

No

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DONE

MadWidth

hep-ph/1402.1178

2-body

Fast-Estimation of 3-body

- Only use 2-body decay and PS factor

Relevant?

No

Maybe

DONE

- Use FeynRules formula (instateneous)

Channel Generation

- Remove Sequence of 2-body/radiation diagram

MadWidth

hep-ph/1402.1178

2-body

Fast-Estimation of 3-body

- Only use 2-body decay and PS factor

Relevant?

No

Maybe

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Channel Generation

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DONE

Estimation of 3-body

- Based on the diagram. Approx. PS/Matrix-Element

Relevant?

MadWidth

hep-ph/1402.1178

2-body
- Use FeynRules formula (instateneous)

Fast-Estimation of 3-body
- Only use 2-body decay and PS factor

Relevant?

No

Maybe

Channel Generation
- Remove Sequence of 2-body/radiation diagram

Estimation of 3-body
- Based on the diagram. Approx. PS/Matrix-Element

Relevant?

No

DONE

MadWidth

hep-ph/1402.1178

Durham University

2-body

Fast-Estimation of 3-body

- Only use 2-body decay and PS factor

Relevant?

No

Maybe

Channel Generation

- Use FeynRules formula (instateneous)
- Remove Sequence of 2-body/radiation diagram

DONE

Estimation of 3-body

- Based on the diagram. Approx. PS/Matrix-Element

No

Relevant?

Yes?

Numerical Integration

MadWidth

hep-ph/1402.1178

2-body

Fast-Estimation of 4 body

- Only use 2-body decay and PS factor

Relevant?

No

Maybe

- Use FeynRules formula (instateneous)

Channel Generation

- Remove Sequence of 2-body/radiation diagram

DONE

No

Estimation of 4 body

- Based on the diagram. Approx. PS/Matrix-Element

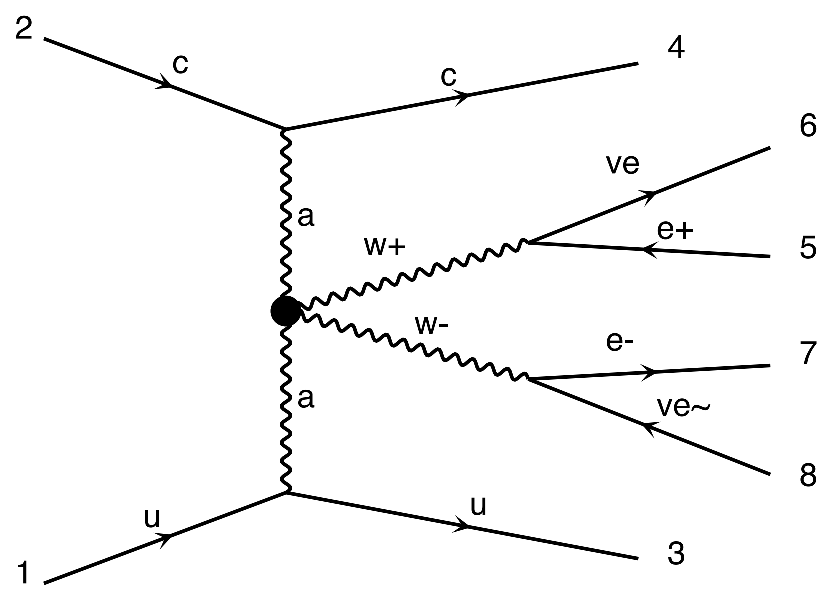
Relevant?

Yes?

Numerical Integration

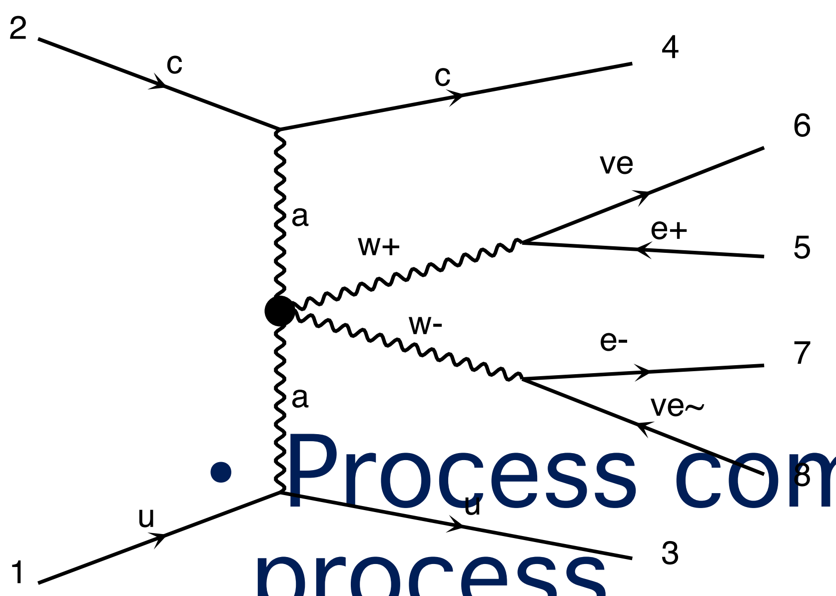
Decay

Resonant Diagram

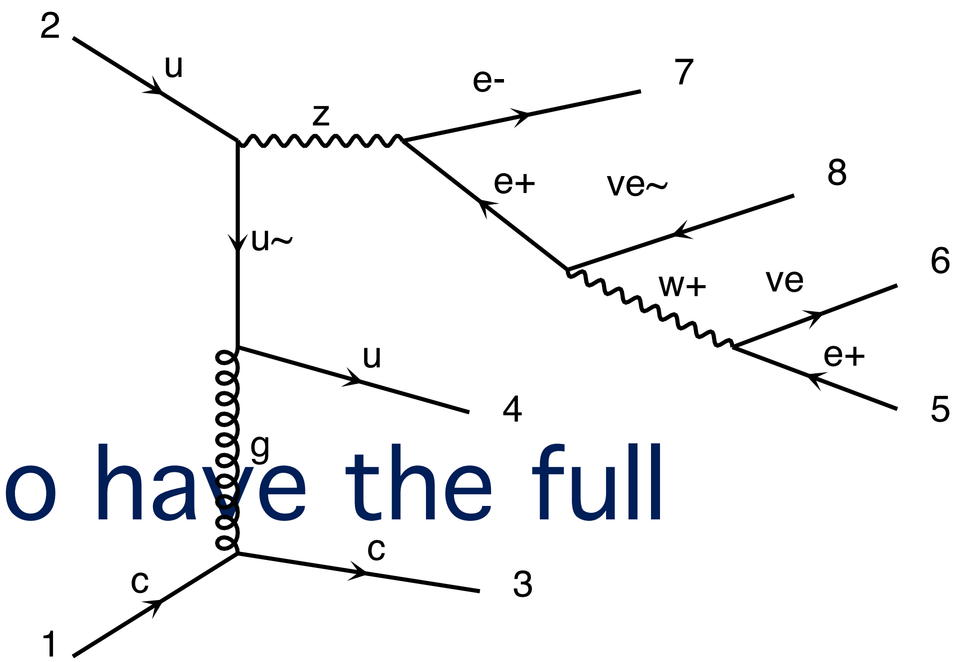


Decay

Resonant Diagram



Non Resonant Diagram



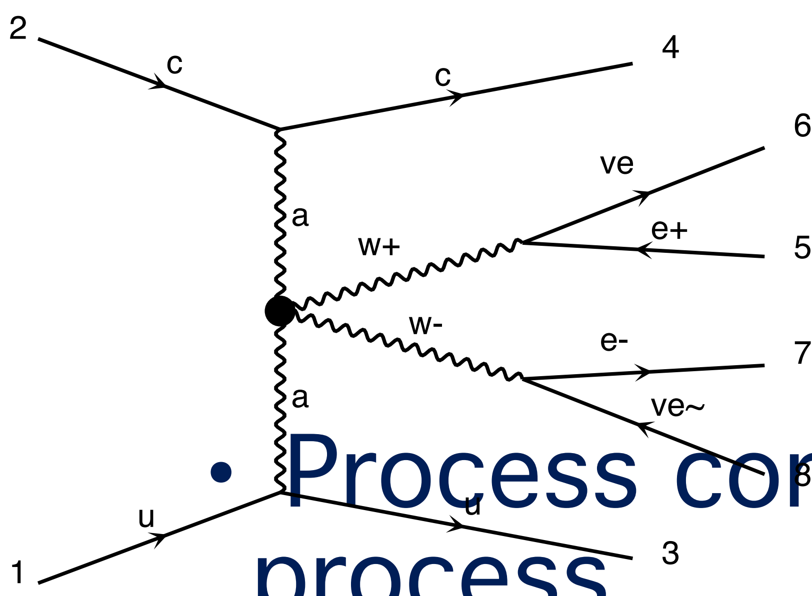
- Process complicated to have the full process

➡Including off-shell contribution

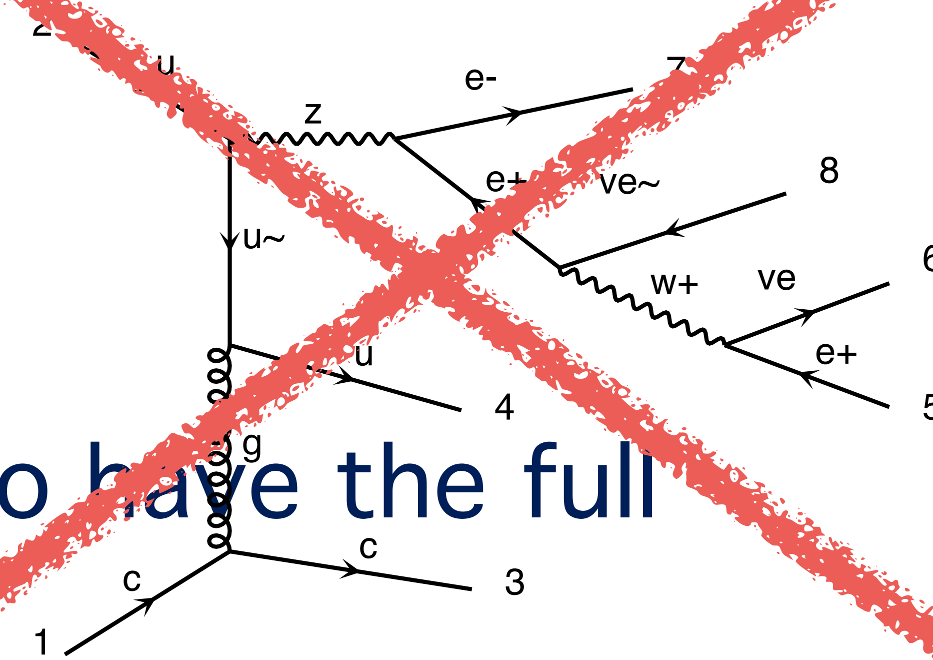
Problem

Decay

Resonant Diagram



Non Resonant Diagram



- Process complicated to have the full process

➡Including off-shell contribution

Problem

Solution

- Only keep on-shell contribution

Narrow-Width Approx.

Theory

$$\int dq^2 \left| \frac{1}{q^2 - M^2 - iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma}\, \delta(q^2 - M^2)$$

$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}(\frac{\Gamma}{M}))$$

Comment

Narrow-Width Approx.

Theory

$$\int dq^2 \left|\frac{1}{q^2 - M^2 - iM\Gamma}\right|^2 \approx \frac{\pi}{M\Gamma}\,\delta(q^2 - M^2)$$

- This is an Approximation!

$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}(\frac{\Gamma}{M}))$$

- This force the particle to be on-shell!

Comment

- Recover by re-introducing the Breit-wigner up-to a cut-off

Decay chains

- p p > t t~ w+, (t > w+ b, w+ > l+ vl), \
  (t~ > w- b~, w- > j j), \
  w+ > l+ vl

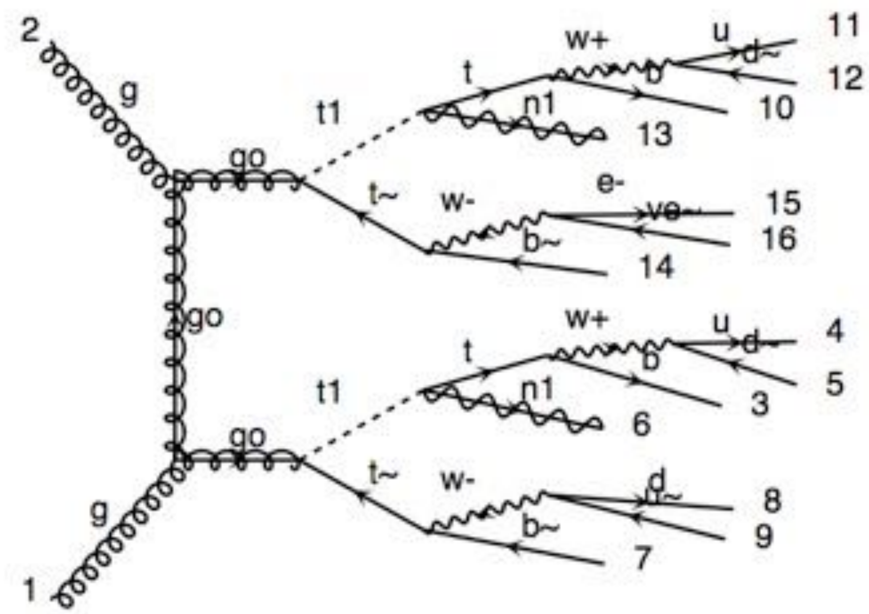
- Separately generate core process and each decay
  - Decays generated with the decaying particle as resulting wavefunction

- Iteratively combine decays and core processes

- Difficulty: Multiple diagrams in decays

Decay chains

- Decay chains retain full matrix element for the diagrams compatible with the decay
- Full spin correlations (within and between decays)
- Full width effects
- However, no interference with non-resonant diagrams
    - ➡ Description only valid close to pole mass
    - ➡ Cutoff at $|m \pm n\Gamma|$ where n is set in run_card.



Online Event Generation

file://localhost/Users/alwall/MadGraph/version_1_0_1/PROC_gogo_t1tx_tn1_ballall/HTML/crossx.html

Results for g g > go go , (go > t1 t~ , t~ > b~ all all / h+ , (t1 > t n1 , t > b all all / h+)) in the mssm

Available Results

Links	Events	Tag	Run	Collider	Cross section (pb)	Events
results banner	Parton-level LHE	fermi	test	p p 7000 x 7000 GeV	.33857E-03	10000

Main Page

Thanks to developments in MadEvent, also (very) long decay chains possible to simulate directly in MadGraph!

MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]

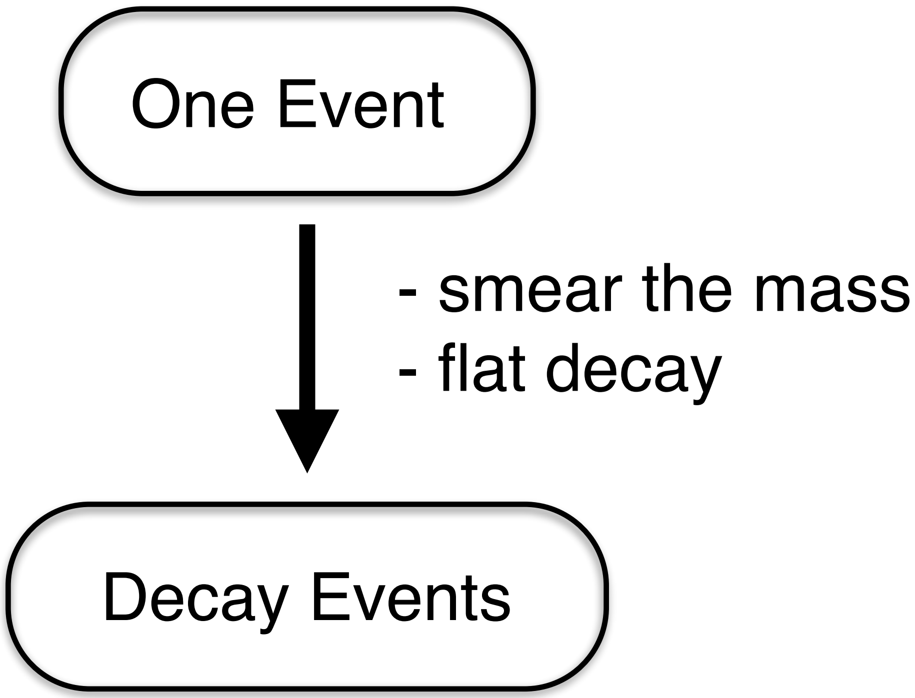
One Event

offshell	spin	unweighted
No	No	YES

MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]

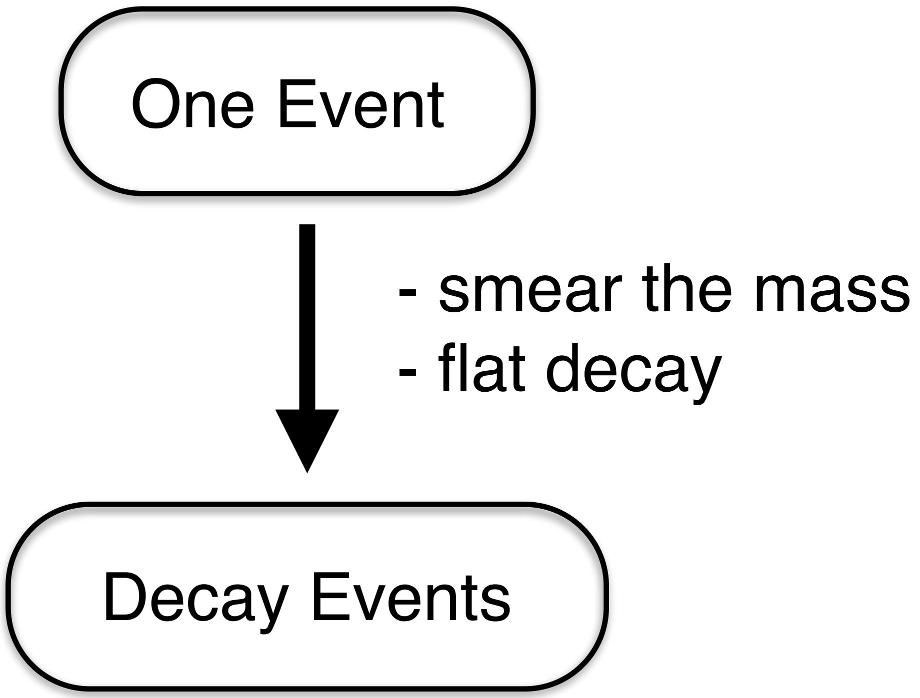


offshell	spin	unweighted
No	No	YES

MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]

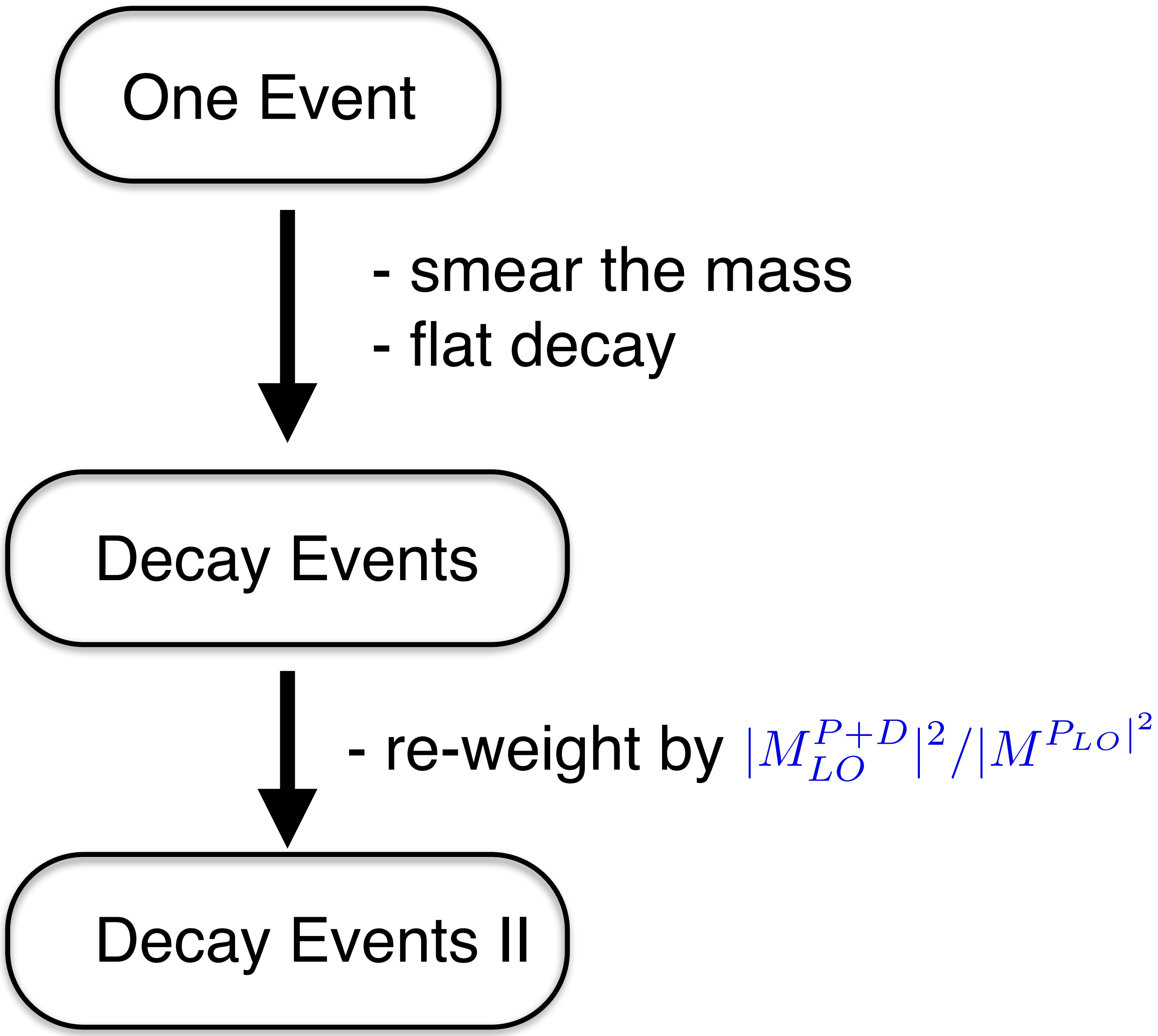


offshell	spin	unweighted
No	No	YES
YES	No	No

MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]



offshell	spin	unweighted
No	No	YES
YES	No	No

MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]

One Event

- smear the mass
- flat decay

Decay Events

- re-weight by $|M_{LO}^{P+D}|^2/|M^{P_{LO}}|^2$

Decay Events II

offshell	spin	unweighted
No	No	YES
YES	No	No
YES	YES	No

MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]

One Event

- smear the mass
- flat decay

Decay Events

- re-weight by $|M_{LO}^{P+D}|^2/|M^{P_{LO}}|^2$

Decay Events II

- accept/reject method
- reject the decay not the event

Final Sample

offshell	spin	unweighted
No	No	YES
YES	No	No
YES	YES	No

MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]

One Event

↓ - smear the mass
- flat decay

Decay Events

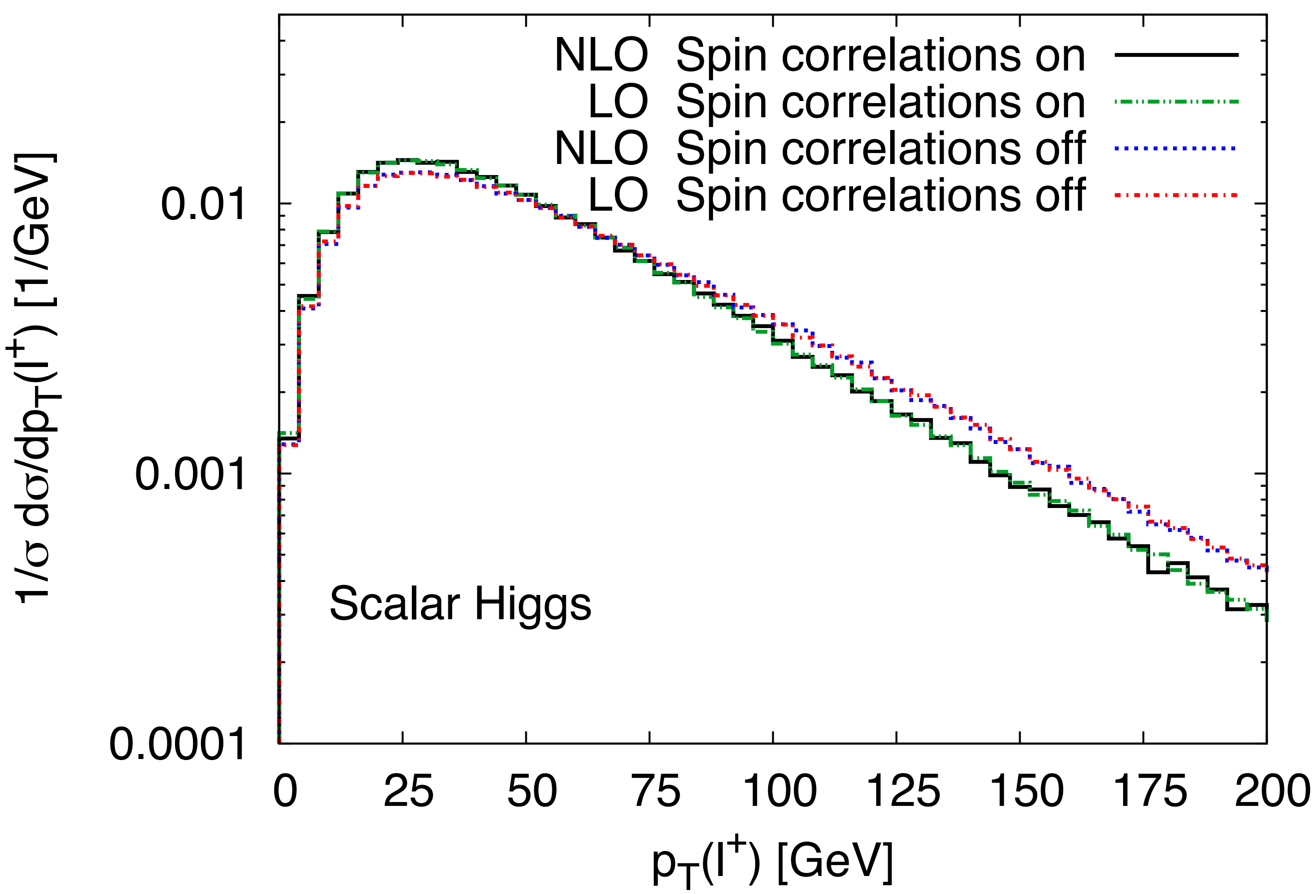
↓ - re-weight by $|M_{LO}^{P+D}|^2/|M^{P_{LO}}|^2$

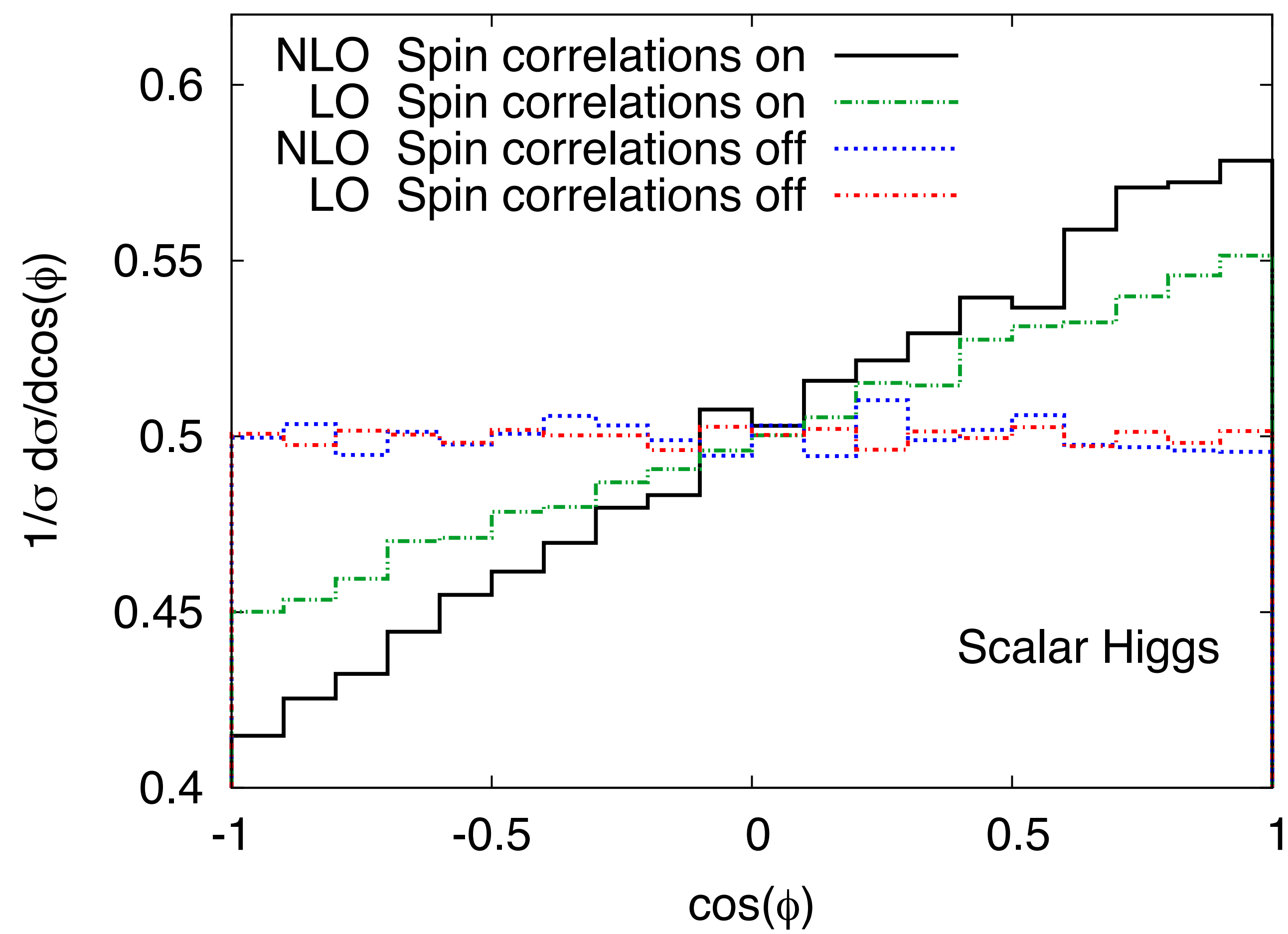
Decay Events II

↓ - accept/reject method
- reject the decay not the event

Final Sample

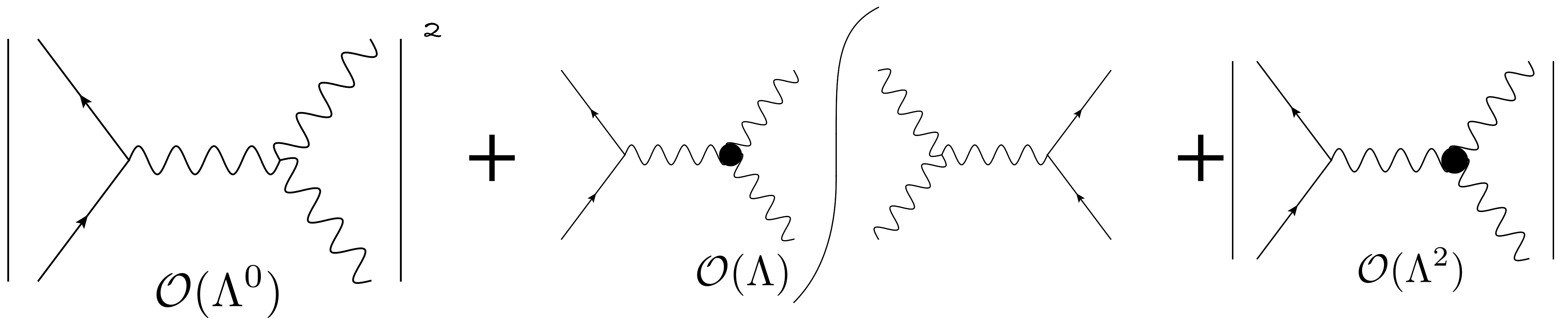
	offshell	spin	unweighted
One Event	No	No	YES
Decay Events	YES	No	No
Decay Events II	YES	YES	No
Final Sample	YES	YES	YES


NLO Spin correlations on
LO Spin correlations on
NLO Spin correlations off
LO Spin correlations off
Scalar Higgs
1/σ dσ/dp_T(l^+) [1/GeV]
p_T(l^+) [GeV]
0.01
0.001
0.0001
0
25
50
75
100
125
150
175
200


NLO Spin correlations on
LO Spin correlations on
NLO Spin correlations off
LO Spin correlations off
Scalar Higgs
1/σ dσ/dcos(ϕ)
cos(ϕ)
0.6
0.55
0.5
0.45
0.4
-1
-0.5
0
0.5
1

Non Definite positive

Motivation:

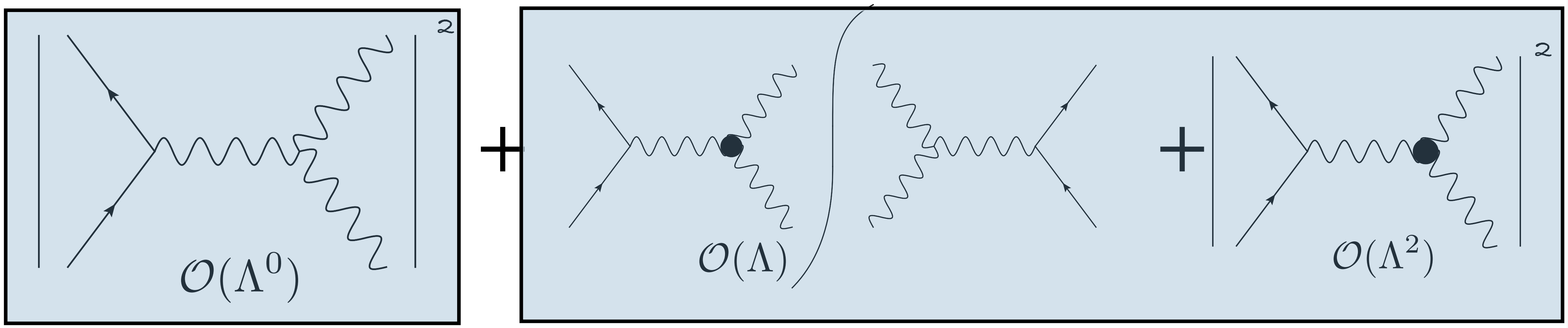


SM
Model independent
Dominant

BSM
Model dependent
Sub-Dominant

Non Definite positive

Motivation:

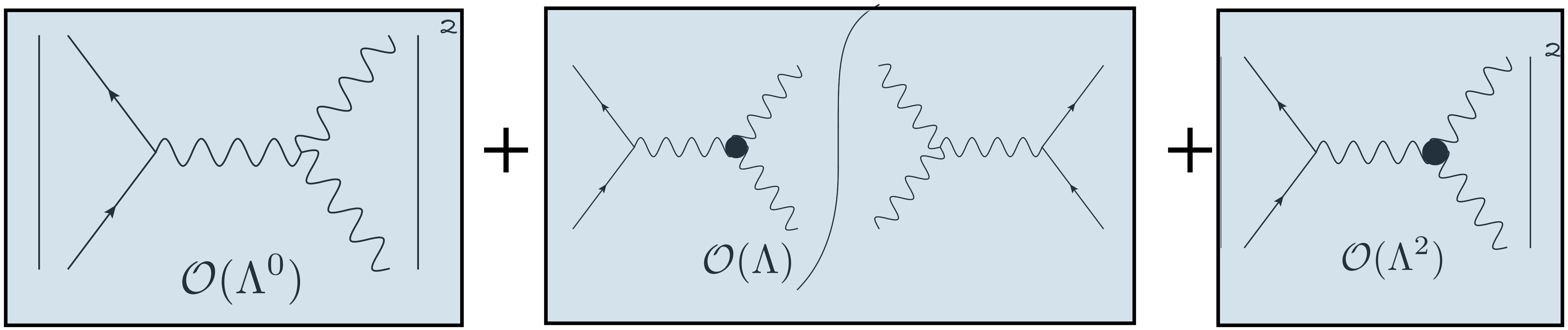


SM
Model independent
Dominant

BSM
Model dependent
Sub-Dominant

Non Definite positive

Motivation:

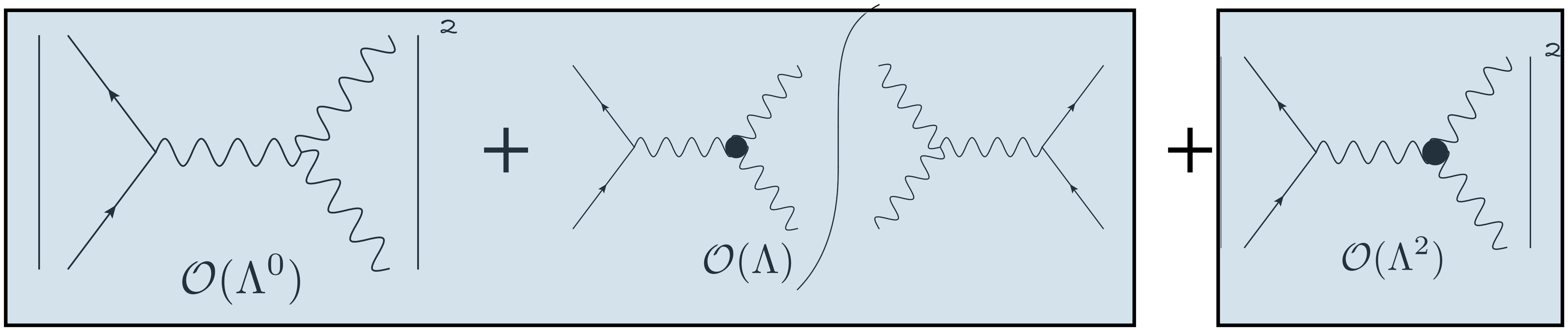


SM
Model independent
Dominant

BSM
Model dependent
Sub-Dominant

Non Definite positive

Motivation:

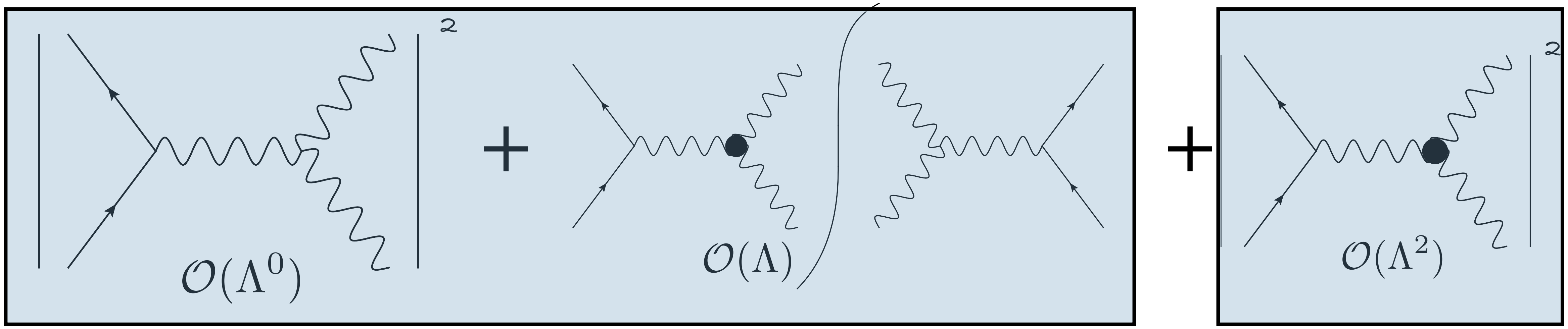


SM
Model independent
Dominant

BSM
Model dependent
Sub-Dominant

Non Definite positive

Motivation:



SM
Model independent
Dominant

BSM
Model dependent
Sub-Dominant

Idea:

- Compute them separately
- Have a new syntax for such selection (NP^2=)

Status:

- Not compatible with decay chains

Type of generation

	Tree (SM)	Tree (BSM)	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	NLO (EW) (BSM)	Loop Induced (SM)	Loop Induced (BSM)
Fix Order	✓	✓	✓	✓	✓	✗	✓	✓
+Parton Shower	✓	✓	✓	✓	✗	✗	✓	✓
Merged Sample	✓	✓	✓	?	✗	✗	✓	✓

Loop Induced:

Type of generation

	Tree (SM)	Tree (BSM)	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	NLO (EW) (BSM)	Loop Induced (SM)	Loop Induced (BSM)
Fix Order	✓	✓	✓	✓	✓	✗	✓	✓
+Parton Shower	✓	✓	✓	✓	✗	✗	✓	✓
Merged Sample	✓	✓	✓	?	✗	✗	✓	✓

Loop Induced:

- 2 to 2 processes: OK on a laptop
- 2 to 3 processes: OK on a small size cluster
- 2 to 4 processes: Specific case

MadGraph Functionality

- Leading Order Option
  - Support of BSM
  - Computation of the Width
  - Narrow width Approximation
    - Decay Chain
    - MadSpin
  - Systematics
- NLO
  - SM with merging

Tutorial

Olivier Mattelaer
IPPP/Durham

Tutorial map

Learning MG5

- follow the built-in tutorial
- cards meaning
- meaning of QCD/QED
- details of syntax ($/)
- script
- width computation
- decay chain

BSM CASE

- check the model
- width computation
- signal generation
  - decay chain
- merging sample generation
- background/NLO generation

Learning MG5_aMC

Where to find help?

- Ask me
- Use the command "help" / "help XXX"
  - ➡ "help" tell you the next command that you need to do.
- Launchpad:
  - ➡ https://answers.launchpad.net/madgraph5
  - ➡ FAQ: https://answers.launchpad.net/madgraph5/+faqs

What are those cards?

- Read the Cards and identify what they do
  - ➡ param_card: model parameters
  - ➡ run_card: beam/run parameters and cuts
    - ✦ https://answers.launchpad.net/madgraph5/+faq/2014

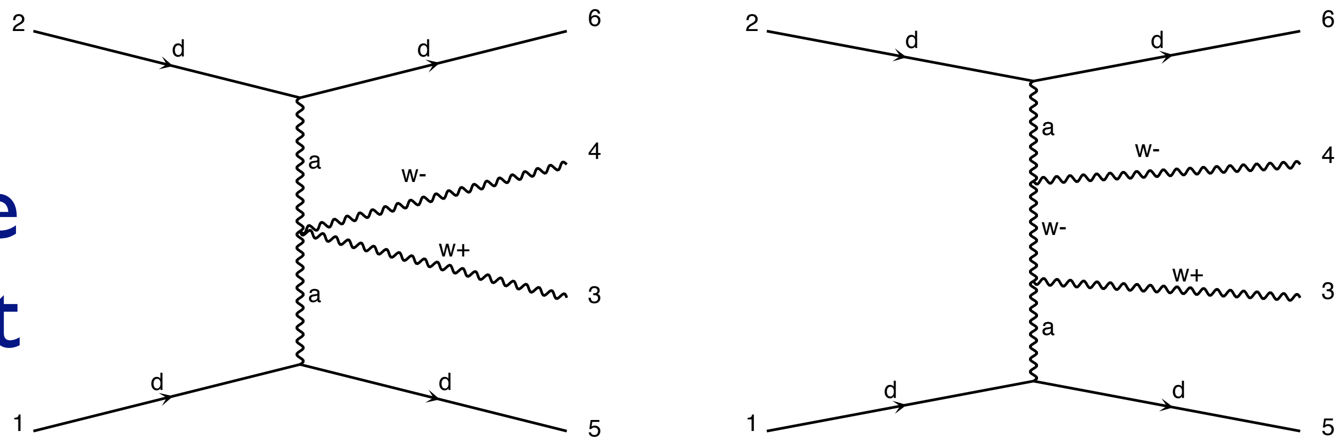
Exercise II: Cards Meaning

- How do you change
  - ➡ top mass
  - ➡ top width
  - ➡ W mass
  - ➡ beam energy
  - ➡ pt cut on the lepton

Exercise II : Syntax

- What's the meaning of the order QED/QCD
- What's the difference between
  - ➡ p p > t t~
  - ➡ p p > t t~ QED=2
  - ➡ p p > t t~ QED=0
  - ➡ p p > t t~ QCD=0
  - ➡ p p > t t~ QED<=2
  - ➡ p p > t t~ QCD^2==2
- Compute the cross-section for each of those and check the diagram

- Generate VBF process
- check that you have the diagram that you want



Exercise III: Syntax

- Generate the cross-section and the distribution (invariant mass) for
  - ➡ p p > e+ e-
  - ➡ p p > z, z > e+ e-
  - ➡ p p > e+ e- $ z
  - ➡ p p > e+ e- / z

Hint :To plot automatically distributions:

mg5> install MadAnalysis

- Use the invariant mass distribution to determine the meaning of each syntax.

Exercise IV: Automation/Width

- Compute the cross-section for the top pair production for 3 different mass points.
  - ➡ Do NOT use the interactive interface
    - ✦ hint: you can edit the param_card/run_card via the "set" command [After the launch]
    - ✦ hint: All command [including answer to question] can be put in a file. (run ./bin/mg5 PATH_TO_FILE)
  - ➡ Remember to change the value of the width
    - ✦ "set width 6 Auto" works
    - ✦ cross-check that it indeed returns the correct width

Examples

File:

```
import model EWDim6
generate p p > z z
ouput TUTO_DIM6
launch
  set nevents 5000
  set MZ 100
```

How to Run:

```
./bin/mg5_amc PATH
```

Exercise V: Decay Chain

- Generate p p > t t~ h, fully decayed (fully leptonic decay for the top)
  - ➡ Using the decay-chain formalism
  - ➡ Using MadSpin
- Compare cross-section
  - ➡ which one is the correct one?
  - ➡ Why are they different?
- Compare the shape.

BSM Tutorial

Exercise I: Check the model validity

- Check the model validity:
  - ➡ check p p > uv uv~
  - ➡ check p p > ev ev~
  - ➡ check p p > t t~ p1 p2

- This checks
  - ➡ gauge invariance
  - ➡ lorentz invariance
  - ➡ that various way to compute the matrix element provides the same answer

Exercise II: Width computation

- Check with MG the width computed with FR:
  - ➡ generate uv > all all; output; launch
  - ➡ generate ev > all all; output; launch
  - ➡ generate p1 > all all; output; launch
  - ➡ generate p2 > all all; output; launch
- Check with MadWidth
  - ➡ compute_widths uv ev p1 p2
  - ➡ (or Auto in the param_card)

FR Number
0.0706 GeV
0.00497 GeV
0 GeV
0.0224 GeV

- Muv = 400 GeV   Mev = 50 GeV λ=0.1
- m1 = 1GeV   m2 = 100GeV   m12 = 0.5 GeV

Exercise III:

- Compute cross-section and distribution
  - ➡ uv pair production with decay in top and Φ_1/Φ_2 (semi leptonic decay for the top
- Hint: The width of the new physics particles has to be set correctly in the param_card.
  - ➡ You can either use "Auto" arXiv:1402.1178
  - ➡ or use the value computed in exercise 1
- Hint: For sub-decay, you have to put parenthesis:
  - ➡ example:
    p p > t t~ w+, (t > w+ b, w+ >e+ ve), (t~ > b~ w-, w- > j j), w+ > l+ vl

Too Slow?

- Use MadSpin! arXiv:1212.3460
  - ➡ Use Narrow Width Approximation to factorize production and decay
- instead of
  - ➡ p p > t t~ w+, (t > w+ b, w+ >e+ ve), (t~ > b~ w-, w- > j j), w+ > l+ vl
- Do
  - ➡ p p > t t~ w+
- At the question:

```
The following switches determine which programs are run:
 1 Run the pythia shower/hadronization:                       pythia=OFF
 2 Run PGS as detector simulator:                                pgs=OFF
 3 Run Delphes as detector simulator:                    delphes=NOT INSTA
 4 Decay particles with the MadSpin module:                  madspin=OFF
 5 Add weight to events based on coupling parameters:       reweight=OFF
  Either type the switch number (1 to 5) to change its default setting,
  or set any switch explicitly (e.g. type 'madspin=ON' at the prompt)
  Type '0', 'auto', 'done' or just press enter when you are done.
 [0, 1, 2, 4, 5, auto, done, pythia=ON, pythia=OFF, ... ][60s to answer]
```

- At the next question edit the madspin_card and define the decay

Exercise IV: generate multiple multiplicity sample for pythia8

- We will do MLM matching
  - ➡ in the run_card.dat ickkw=1
  - ➡ the matching scale (Qcut) will be define in pythia
    - ✦ in madgraph we use xqcut which should be smaller than Qcut (but at least 10-20 GeV)

Exercise V: Have Fun

- Simulate Background
- Go to NLO (ask me the model)
- …

Solution Learning MG5_aMC

Exercise II: Cards Meaning

- How do you change
  - ➡ top mass
  - ➡ top width
  - ➡ W mass
  - ➡ beam energy
  - ➡ pt cut on the lepton

Param_card (top mass, top width, W mass)

Run_card (beam energy, pt cut on the lepton)

- top mass

```
####################################
## INFORMATION FOR MASS
####################################
Block mass
    6 1.730000e+02 # MT
   23 9.118800e+01 # MZ
   25 1.200000e+02 # MH
## Dependent parameters, given by model restrictions.
## Those values should be edited following the
## analytical expression. MG5 ignores those values
## but they are important for interfacing the output of MG5
## to external program such as Pythia.
  1 0.000000 # d : 0.0
  2 0.000000 # u : 0.0
  3 0.000000 # s : 0.0
  4 0.000000 # c : 0.0
  11 0.000000 # e- : 0.0
  12 0.000000 # ve : 0.0
  13 0.000000 # nu- : 0.0
  14 0.000000 # vm : 0.0
  16 0.000000 # vt : 0.0
  21 0.000000 # g : 0.0
  22 0.000000 # a : 0.0
  24 80.419002 # w+ : cmath.sqrt(MZ__exp__2/2. + cmath.sqrt(MZ__exp__4/4. - (aEW*cmath.pi*MZ__exp__2)/(Gf*sqrt__2)))
```

- W mass

```
#####################################
## INFORMATION FOR MASS
#####################################
Block mass
    5 4.700000e+00 # MB
    6 1.730000e+02 # MT
   15 1.777000e+00 # MTA
   23 9.118800e+01 # MZ
   25 1.200000e+02 # MH
## Dependent parameters, given by model restrictions.
## Those values should be edited following the
## analytical expression. MG5 ignores those values
## but they are important for interfacing the output of MG5
## to external program such as Pythia.
  1 0.000000 # d : 0.0
  2 0.000000 # u : 0.0
  3 0.000000 # s : 0.0
  4 0.000000 # c : 0.0
  11 0.000000 # e- : 0.0
  12 0.000000 # ve : 0.0
  13 0.000000 # mu- : 0.0
  14 0.000000 # vm : 0.0
  16 0.000000 # vt : 0.0
  21 0.000000 # g : 0.0
  22 0.000000 #
  24 80.419002 # w+ : cmath.sqrt(MZ__exp__2/2. + cmath.sqrt(MZ__exp__4/4. - (aEW*cmath.pi*MZ__exp__2)/(Gf*sqrt__2)))
```

W Mass is an internal parameter!

MG5 didn't use this value!

So you need to change MZ or Gf or alpha_EW

Exercise III: Syntax

- What's the meaning of the order QED/QCD
- What's the difference between
  - ➡ p p > t t~
  - ➡ p p > t t~ QED=2
  - ➡ p p > t t~ QED=0
  - ➡ p p > t t~ QCD^2==2

Solution I : Syntax

- What's the meaning of the order QED/QCD
  - ➡ By default MG5 takes the lowest order in QED!
  - ➡ p p > t t~ => p p > t t~ QED=0
  - ➡ p p > t t~ QED=2
    - ✦ additional diagrams (photon/z exchange)

p p > t t~

Cross section (pb)
555 ± 0.84

p p > t t~ QED=2

Cross section (pb)
555.8 ± 0.91

No significant QED contribution

- QED<=2 is the SAME as QED=2
  - ➡ quite often source of confusion since most of the people use the = syntax
- QCD^2==2
  - ➡ returns the interference between the QCD and the QED diagram

Cross section (pb)
5.455e-17 ± 4.7e-19 ± systematics

Solution I Syntax

- generate p p > w+ w- j j
  - 76 processes
  - 1432 diagrams
  - None of them are VBF

- generate p p > w+ w- j j QED = 2
  - 76 processes
  - 1432 diagrams
  - None of them are VBF

- generate p p > w+ w- j j QED = 4
  - 76 processes
  - 5332 diagrams
  - VBF present! + those not VBF

- generate p p > w+ w- j j QCD = 0
  - 60 processes
  - 3900 diagrams
  - VBF present!

- generate p p > w+ w- j j QCD = 2
  - 76 processes
  - 5332 diagrams

- generate p p > w+ w- j j QCD = 4
  - 76 processes
  - 5332 diagrams

Exercise IV: Syntax

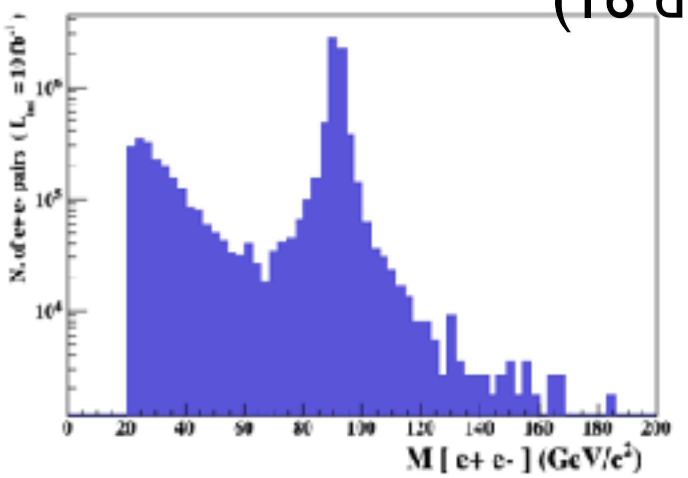
- Generate the cross-section and the distribution (invariant mass) for
  - ➡ p p > e+ e-
  - ➡ p p > z, z > e+ e-
  - ➡ p p > e+ e- $ z
  - ➡ p p > e+ e- / z

Hint :To have automatic distributions:

mg5> install MadAnalysis

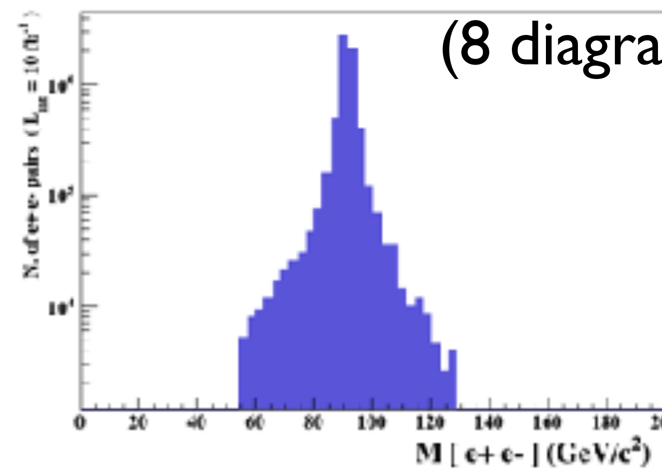
p p > e+ e-

(16 diagrams)



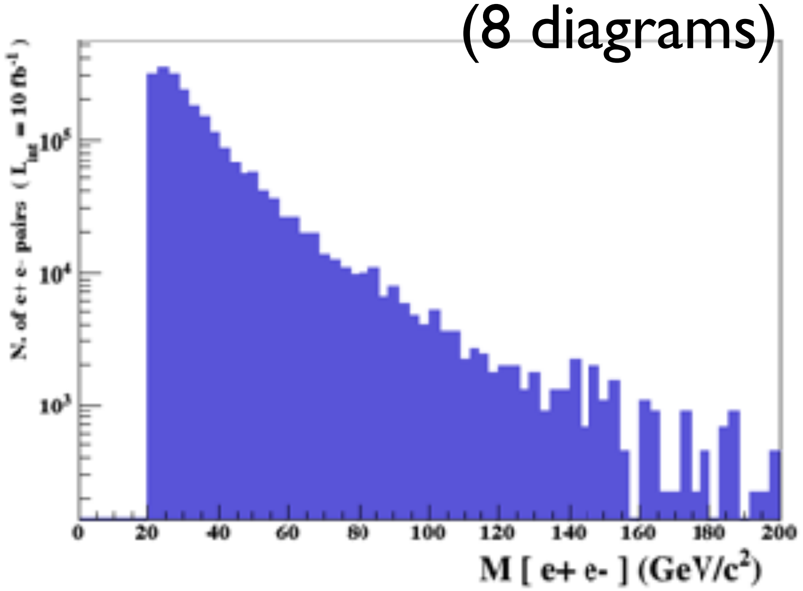
p p >z , z > e+ e-

(8 diagrams)



p p > e+ e- /z

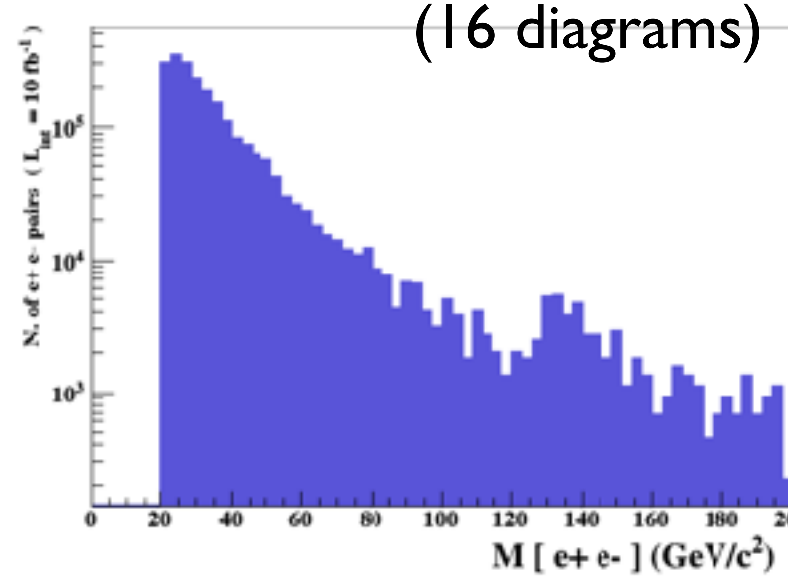
(8 diagrams)



No Z

p p > e+ e- $ z

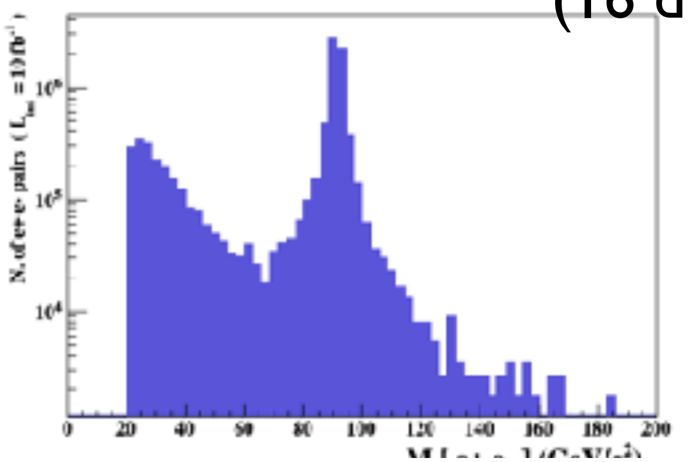
(16 diagrams)



Z- onshell veto

p p > e+ e-

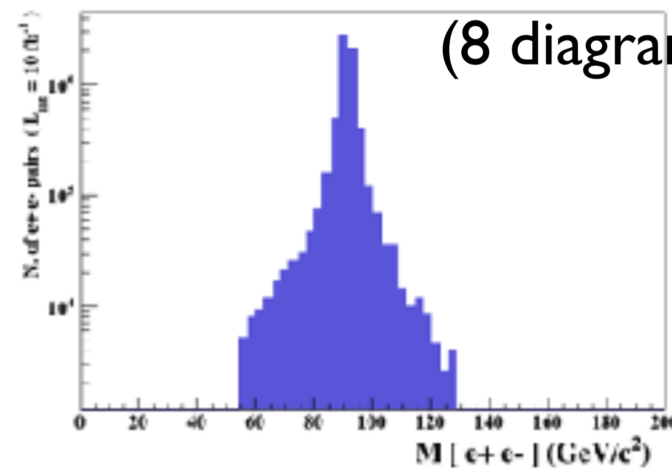
(16 diagrams)



Correct Distribution

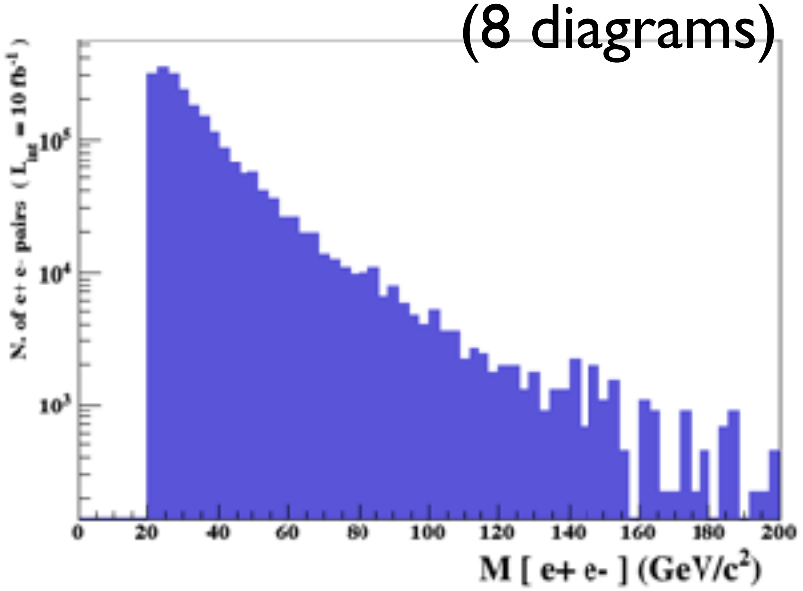
p p >z , z > e+ e-

(8 diagrams)



p p > e+ e- /z

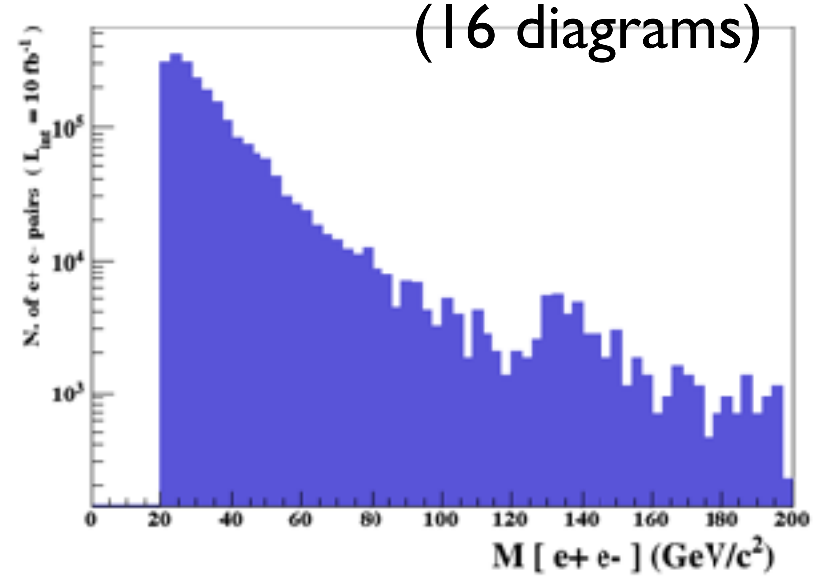
(8 diagrams)



No Z

p p > e+ e- $ z

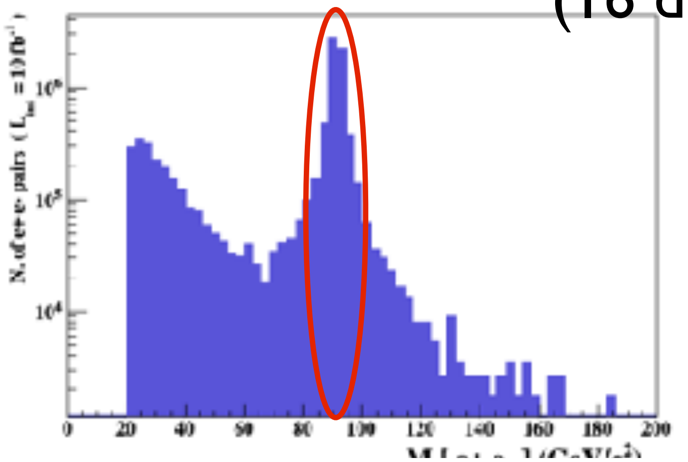
(16 diagrams)



Z- onshell veto

p p > e+ e-

(16 diagrams)

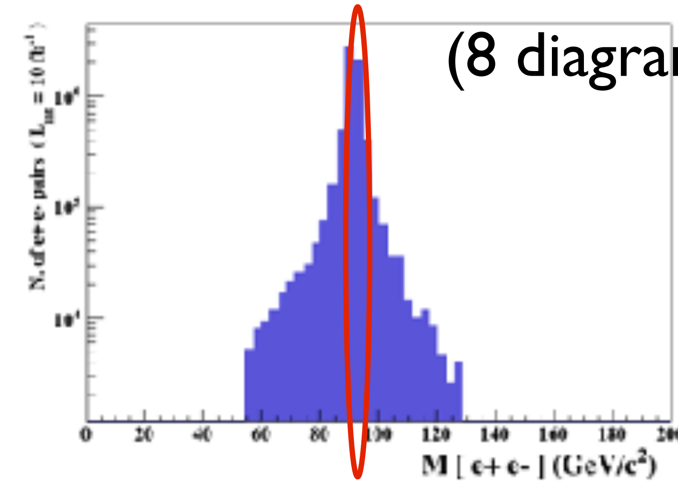


Correct Distribution

Z Peak

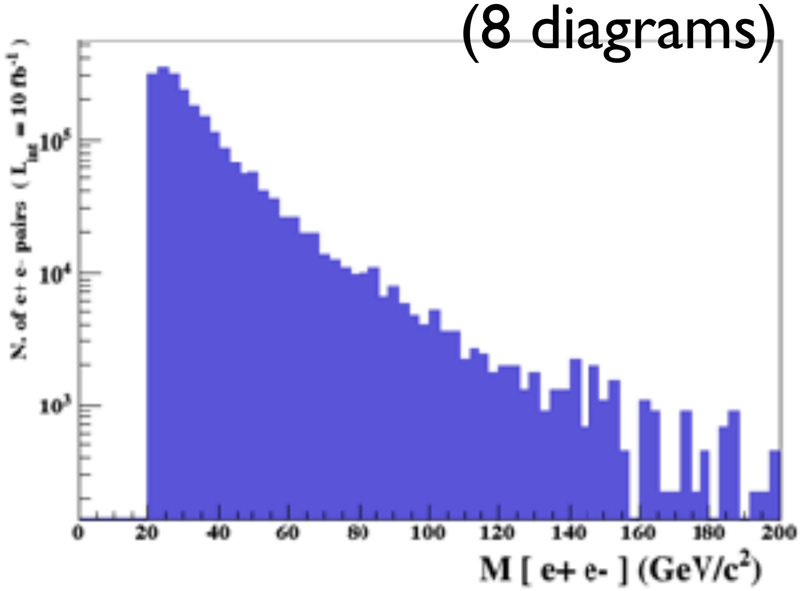
p p >z , z > e+ e-

(8 diagrams)



p p > e+ e- /z

(8 diagrams)

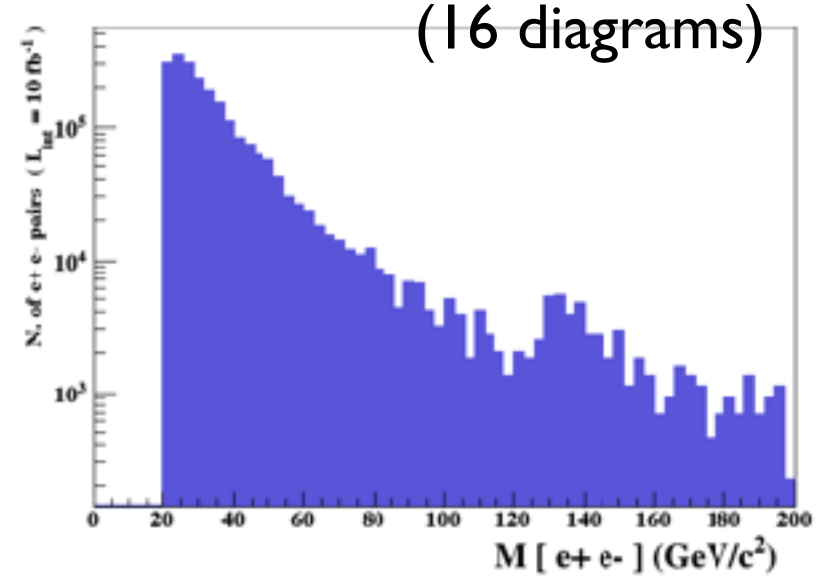


No Z

NO Z Peak

p p > e+ e- $ z

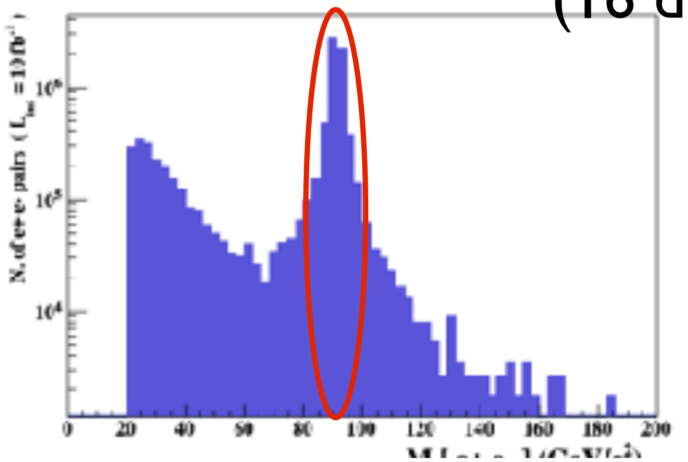
(16 diagrams)



Z- onshell veto

p p > e+ e-

(16 diagrams)

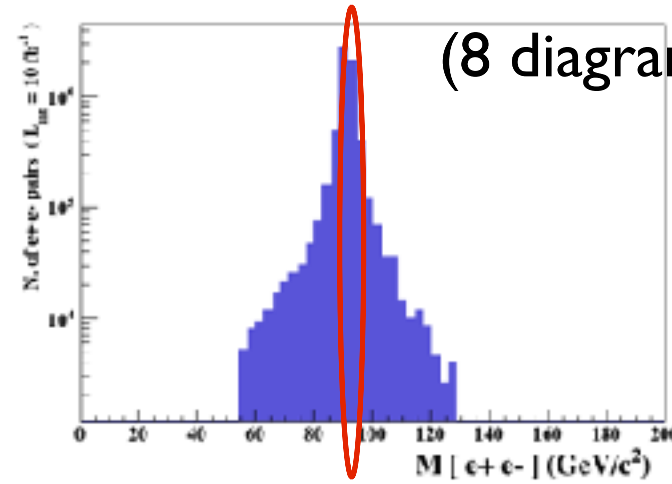


Correct Distribution

Z Peak

p p >z , z > e+ e-

(8 diagrams)

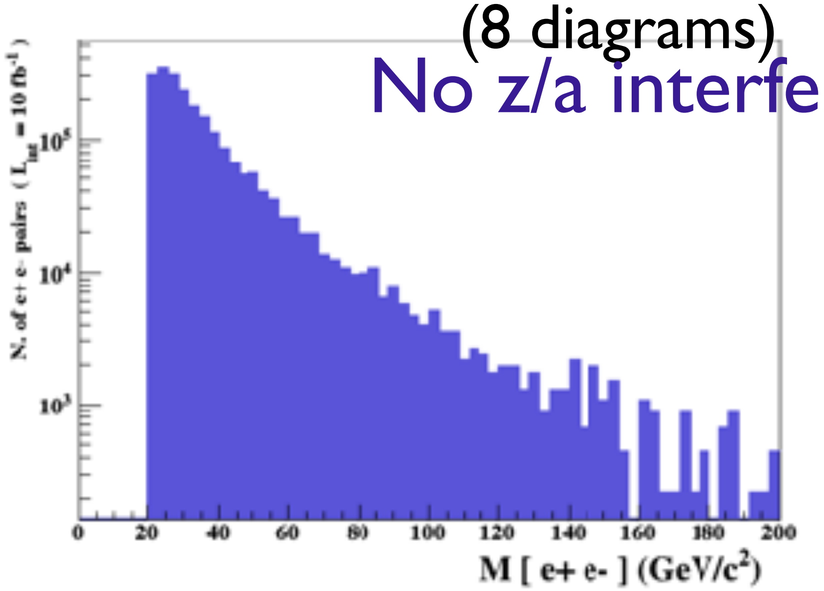


p p > e+ e- /z

(8 diagrams)

No z/a interference

NO Z Peak

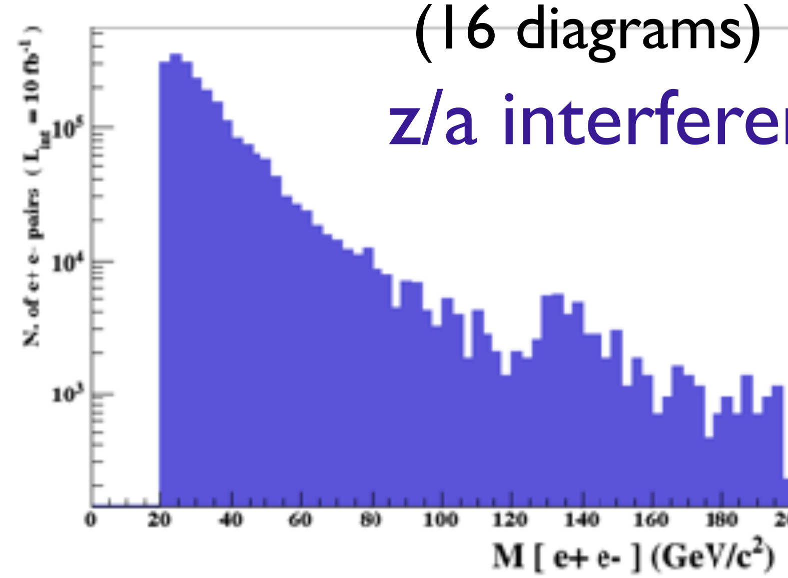


No Z

p p > e+ e- $ z

(16 diagrams)

z/a interference

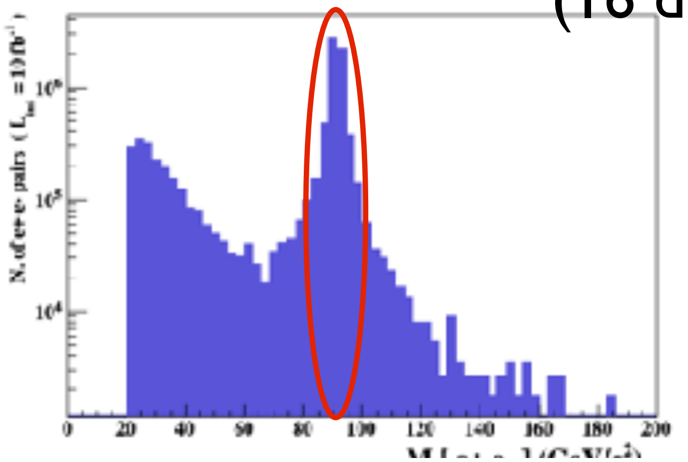


Z- onshell veto

Z Peak

p p > e+ e-

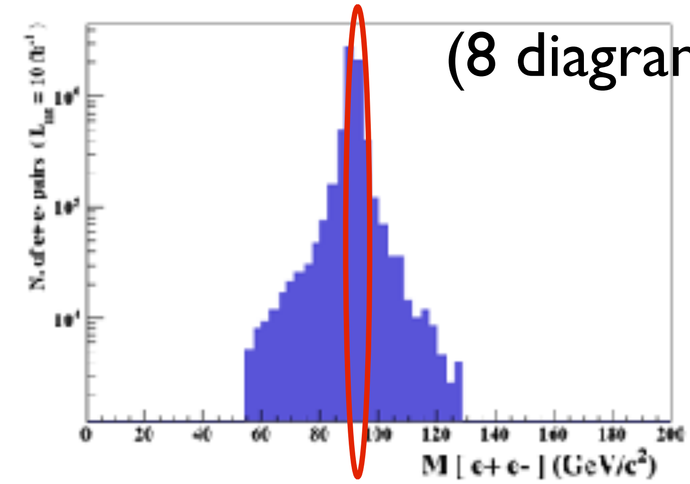
(16 diagrams)



Correct Distribution

p p >z , z > e+ e-

(8 diagrams)

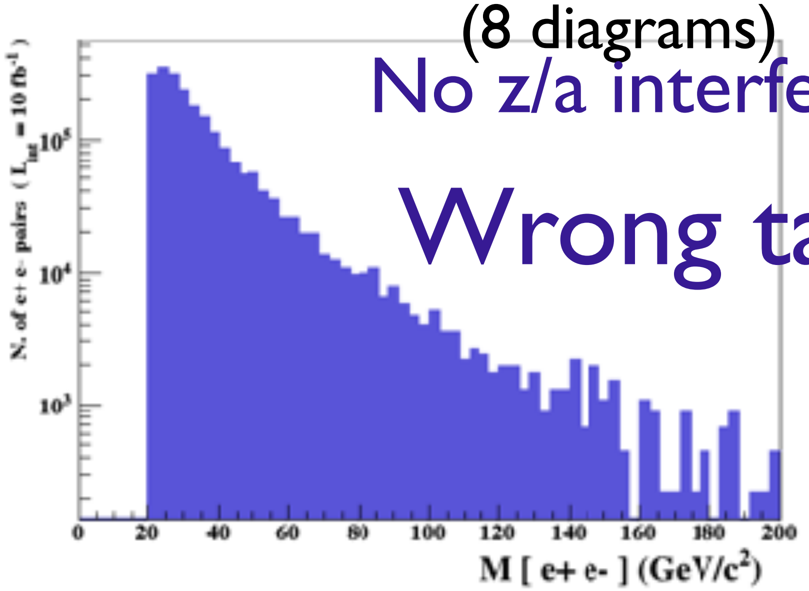


p p > e+ e- /z

(8 diagrams)

No z/a interference

Wrong tail



No Z

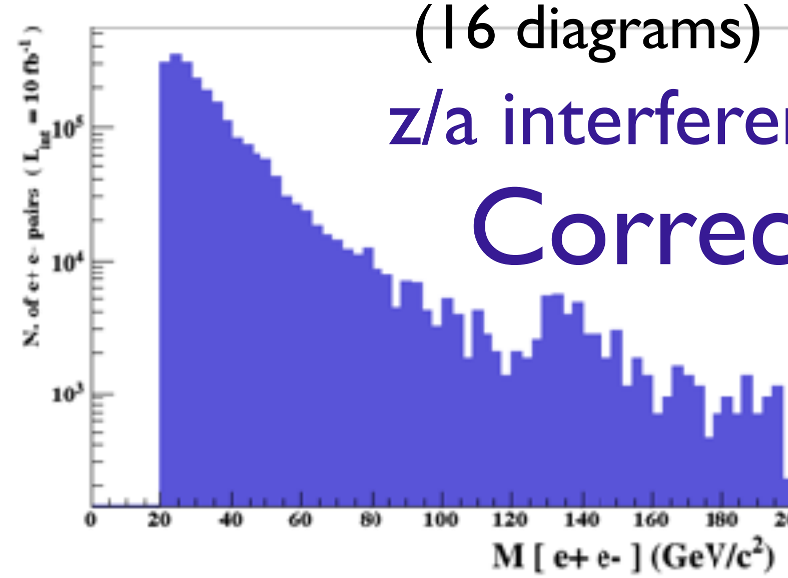
NO Z Peak

p p > e+ e- $ z

(16 diagrams)

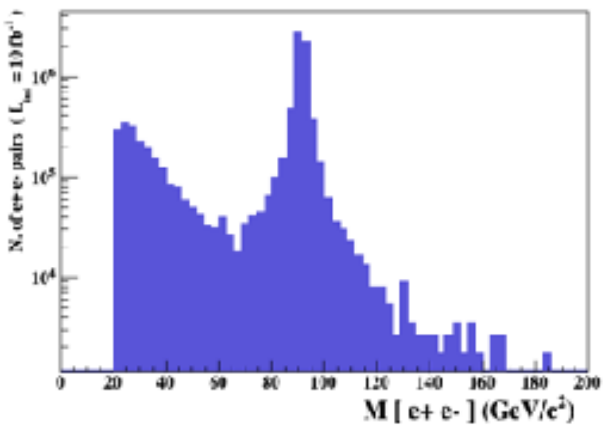
z/a interference

Correct tail



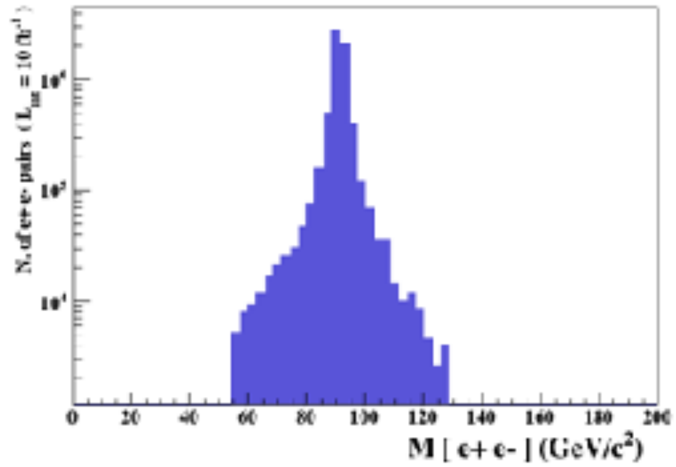
Z- onshell veto

p p > e+ e- | p p >z , z > e+ e- | p p > e+ e- $ z



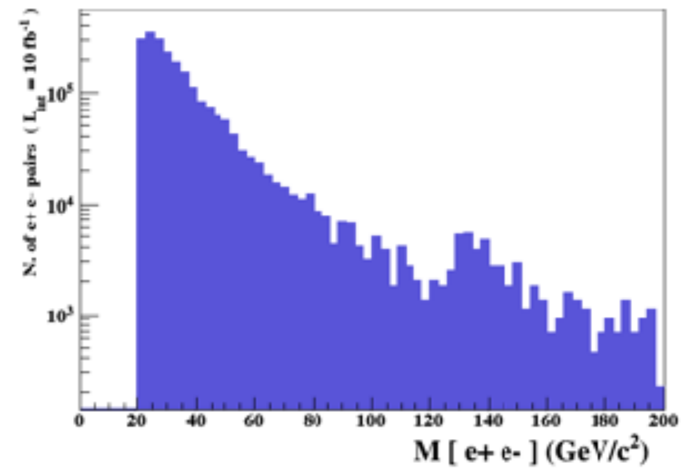
(16 diagrams)

=



(8 diagrams)

+



(16 diagrams)

Onshell cut: BW_cut

$$|M^* - M| < BW_{cut} * \Gamma$$

- The Physical distribution is (very close to) exact sum of the two other one.
- The "$" forbids the Z to be onshell but the photon invariant mass can be at MZ (i.e. on shell substraction).
- The "/" is to be avoid if possible since this leads to violation of gauge invariance.

WARNING

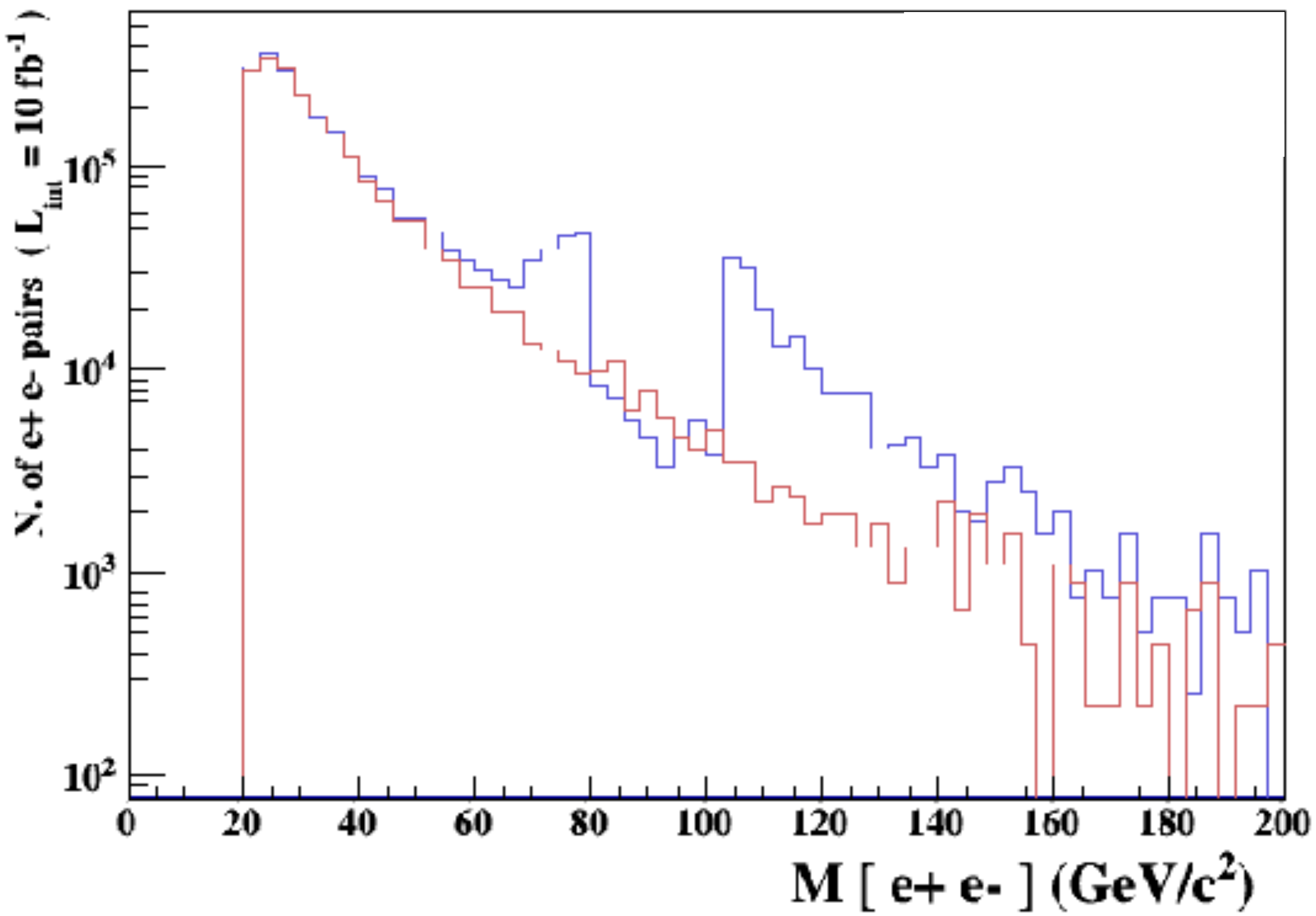
- NEXT SLIDE is generated with bw_cut =5
- This is TOO SMALL to have a physical meaning (15 the default value used in previous plot is better)
- This was done to illustrate more in detail how the "$" syntax works.

See previous slide warning

p p > e+ e- / Z

(red curve)

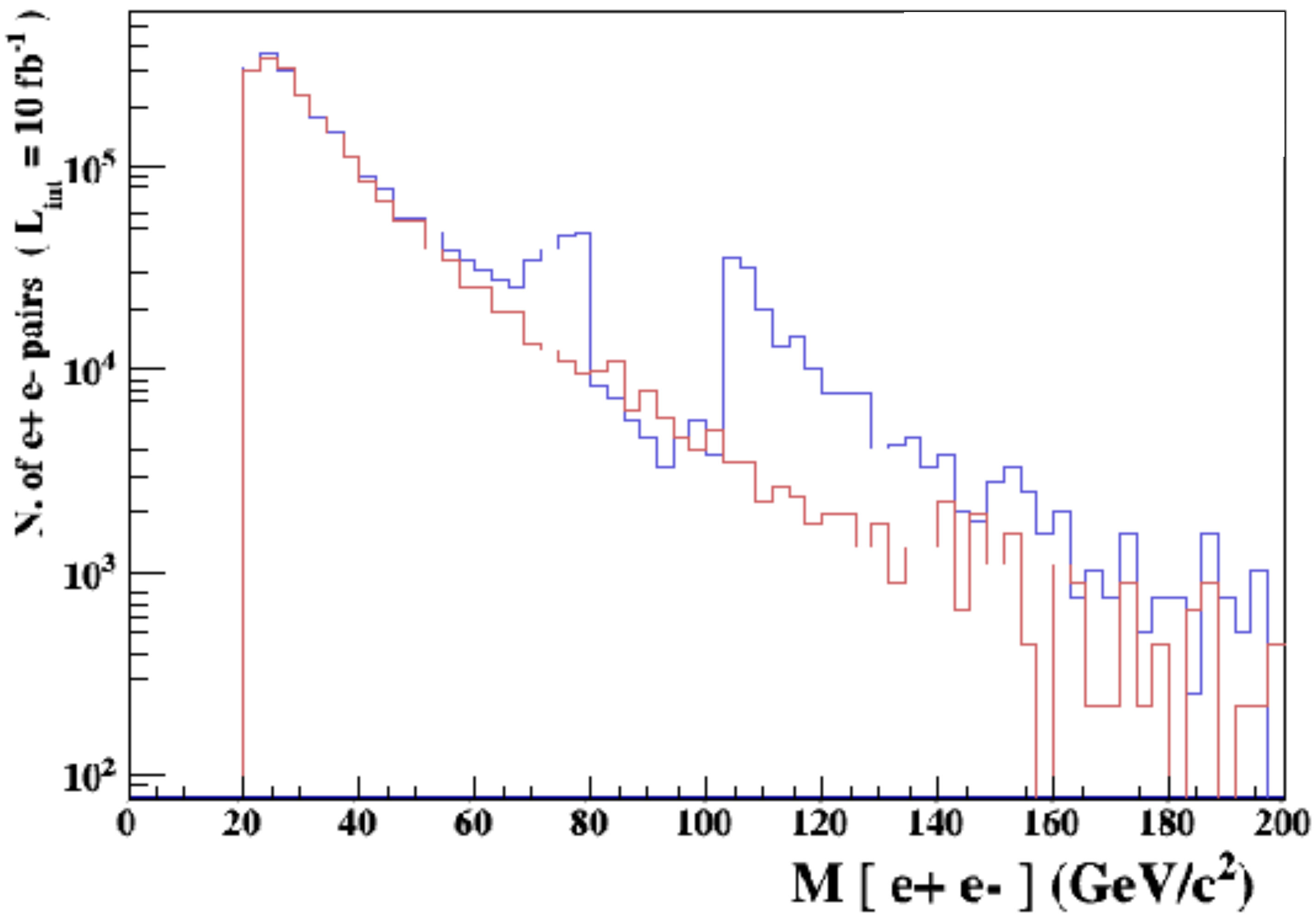
(blue curve)



See previous slide warning

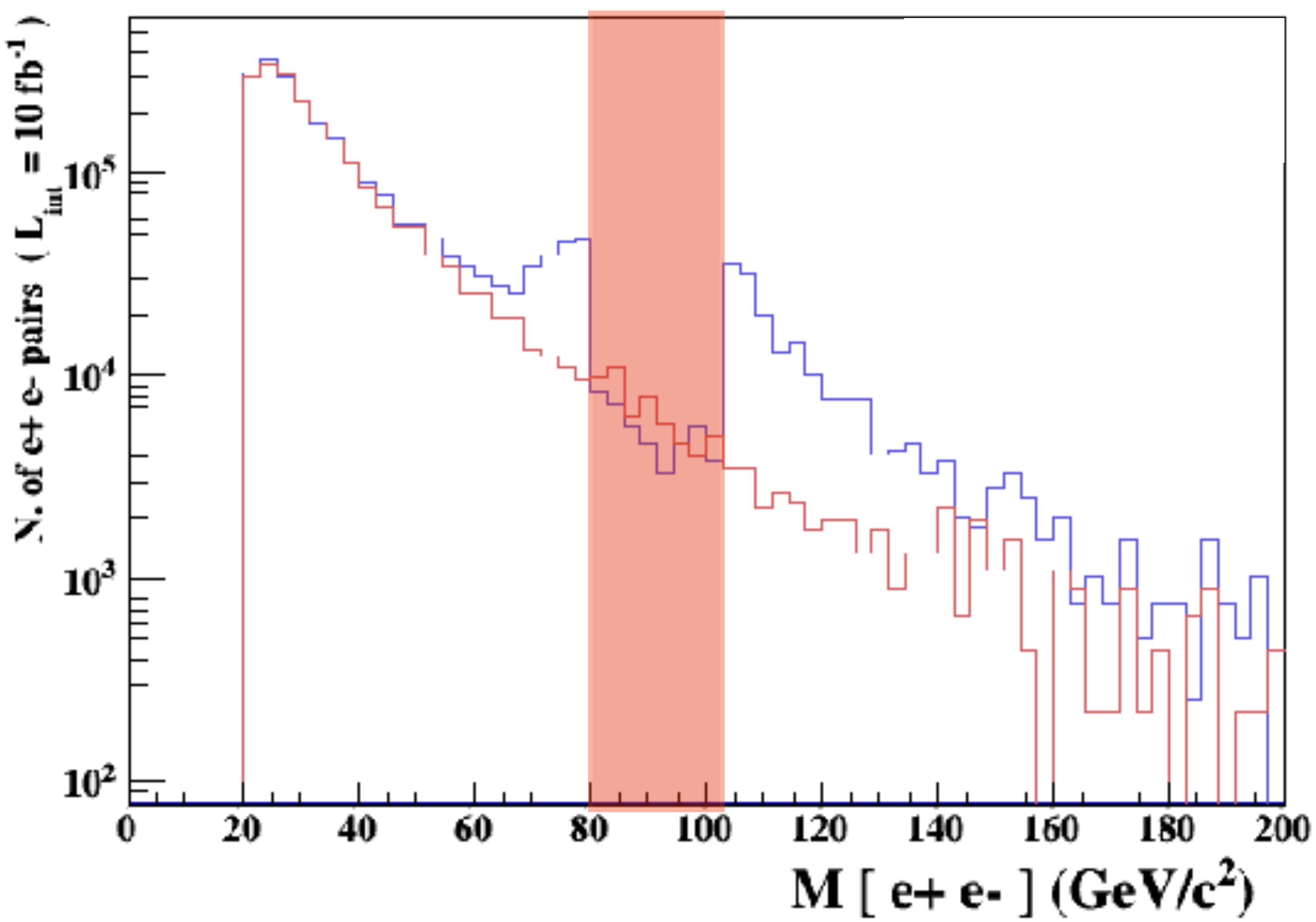
p p > e+ e- / Z (red curve)

adding p p > e+ e- $ Z (blue curve)



See previous slide warning

p p > e+ e- / Z (red curve) adding p p > e+ e- $ Z (blue curve)

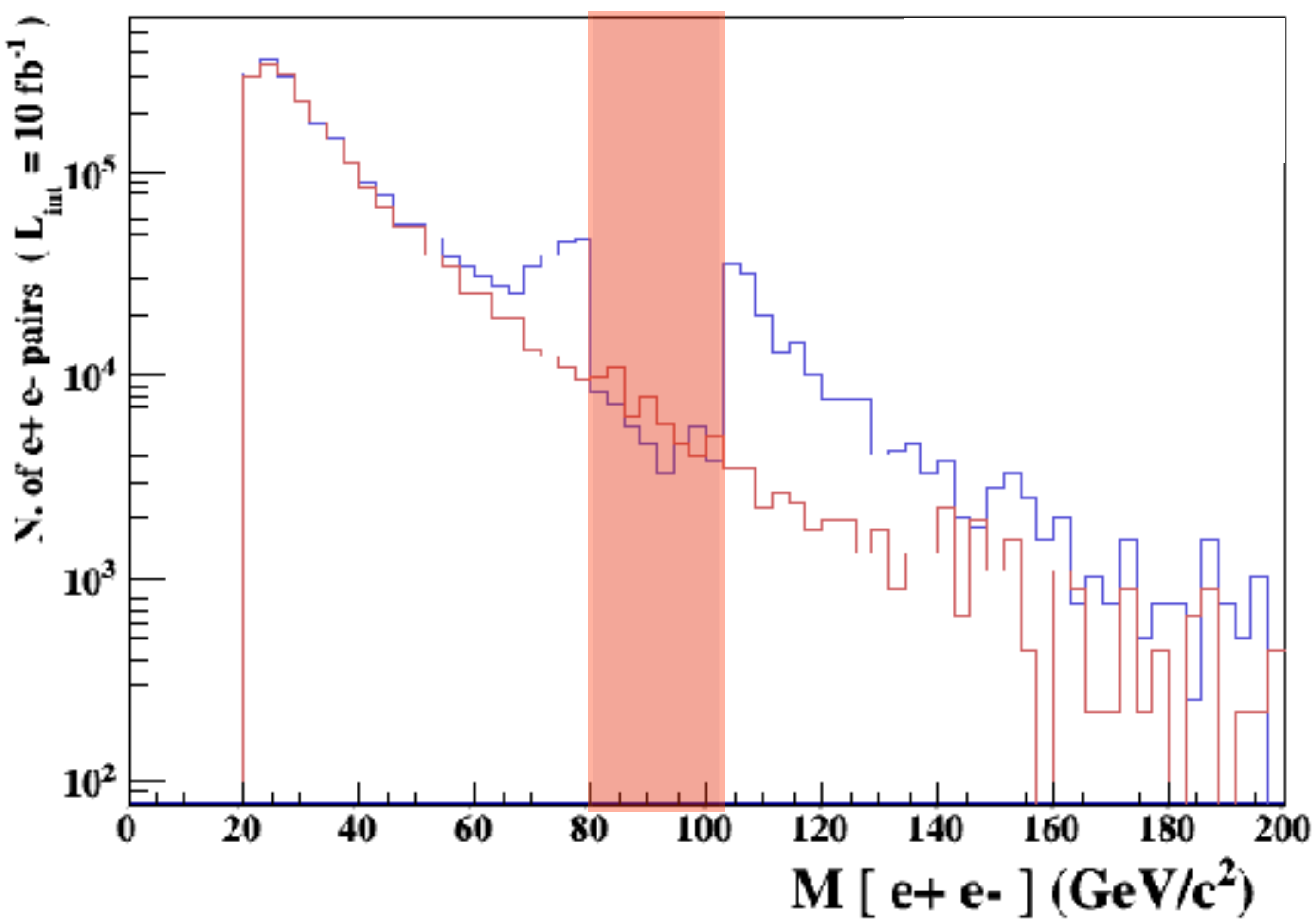


- Z onshell veto

5 times width area

See previous slide warning

p p > e+ e- / Z (red curve) adding p p > e+ e- $ Z (blue curve)



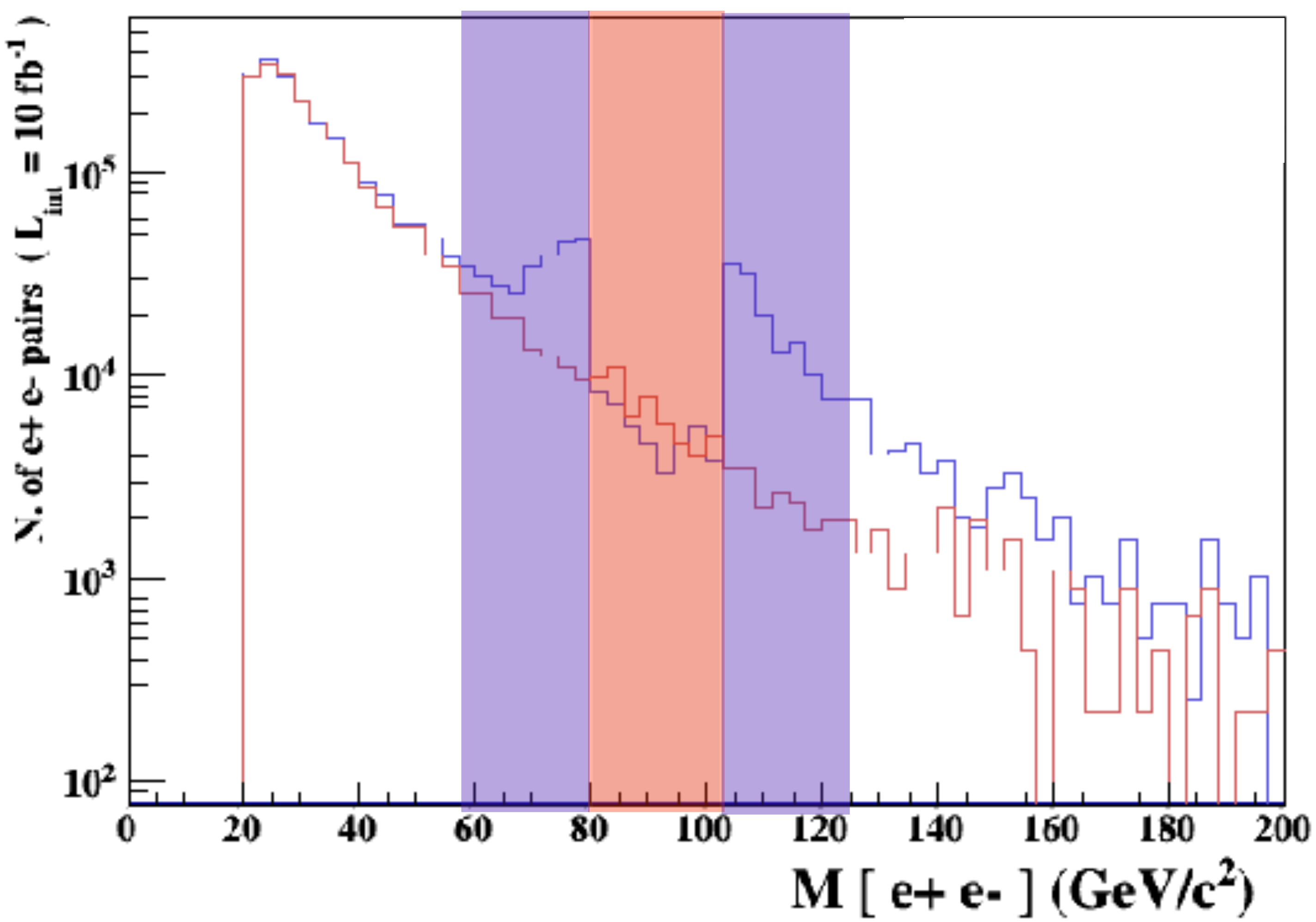
- Z onshell veto
- In veto area only photon contribution

5 times width area

See previous slide warning

p p > e+ e- / Z (red curve)

adding p p > e+ e- $ Z (blue curve)



- Z onshell veto
- In veto area only photon contribution
- area sensitive to z-peak

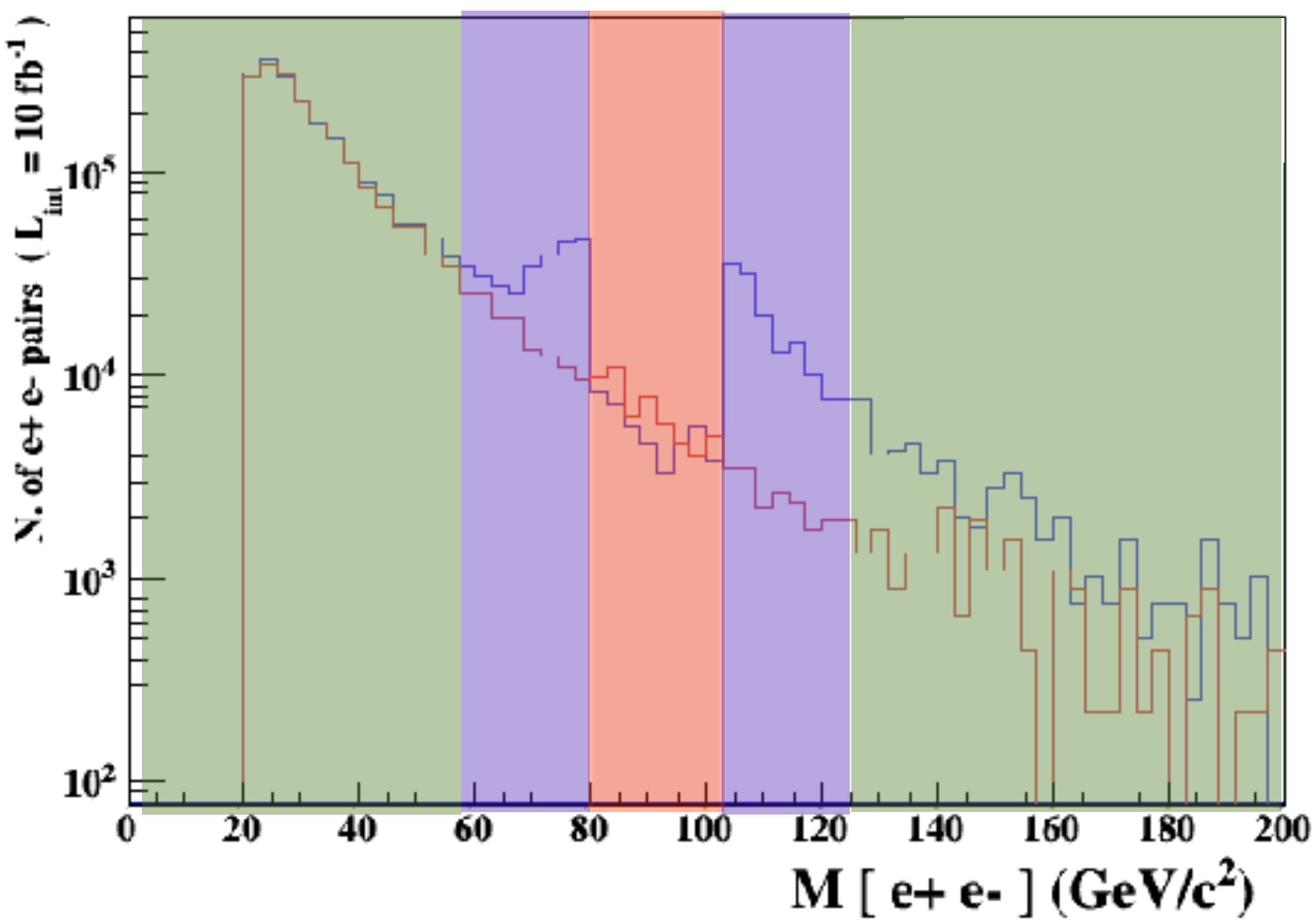
5 times width area

15 times width area

See previous slide warning

p p > e+ e- / Z
(red curve)

adding p p > e+ e- $ Z
(blue curve)



- Z onshell veto
- In veto area only photon contribution
- area sensitive to z-peak
- very off-shell Z, the difference between the curve is due to interference which are need to be KEPT in simulation.

5 times width area

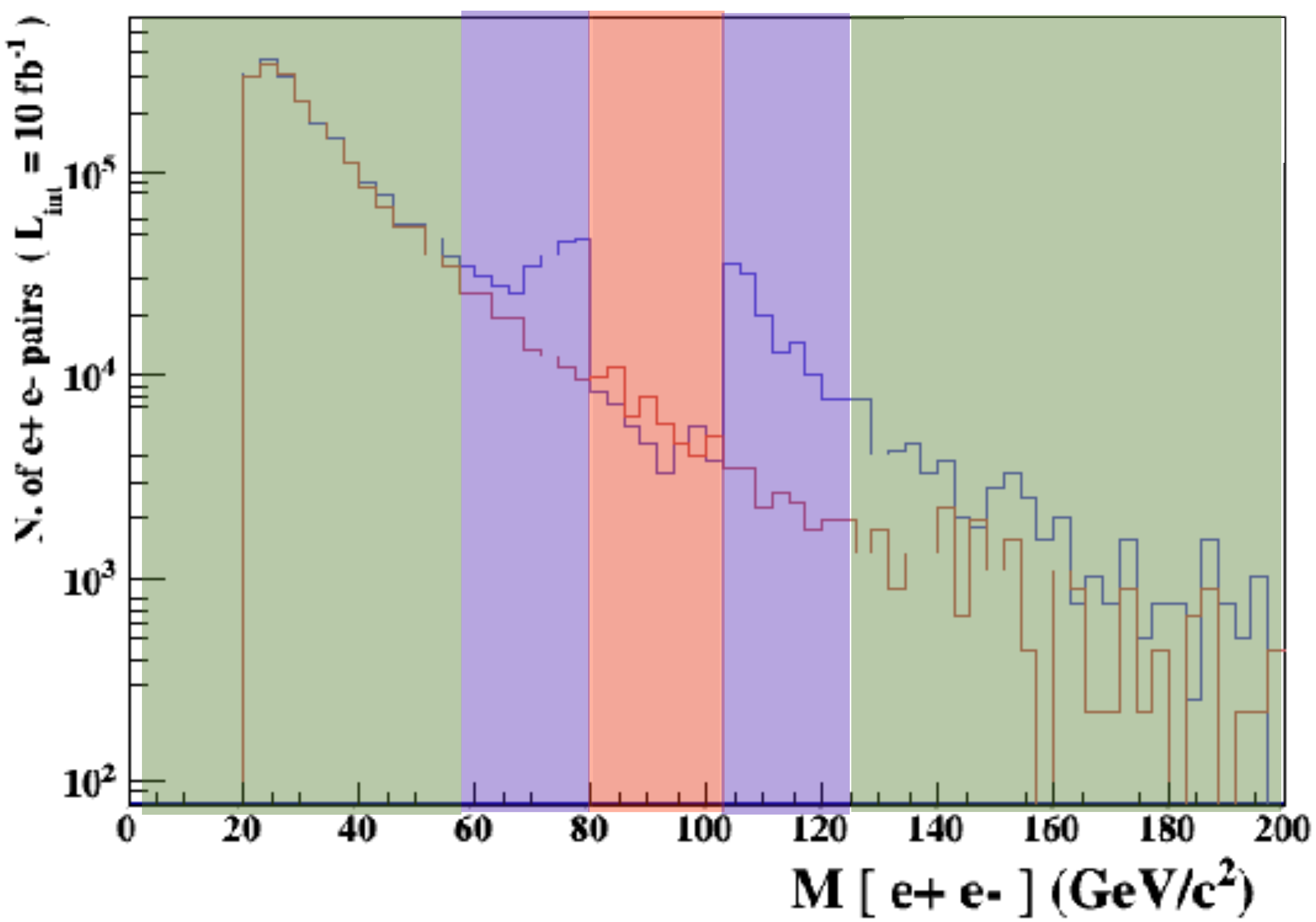
15 times width area

>15 times width area

See previous slide warning

p p > e+ e- / Z (red curve)

adding p p > e+ e- $ Z (blue curve)



- Z onshell veto
- In veto area only photon contribution
- area sensitive to z-peak
- very off-shell Z, the difference between the curve is due to interference which are need to be KEPT in simulation.

5 times width area

15 times width area

>15 times width area

The "$" can be use to split the sample in BG/SG area

- Syntax Like
  - ➡ p p > z > e+ e- (ask one S-channel z)
  - ➡ p p > e+ e- / z (forbids any z)
  - ➡ p p > e+ e- $$ z (forbids any z in s-channel)
- ARE NOT GAUGE INVARIANT !
- forgets diagram interference.
- can provides un-physical distributions.

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Avoid Those as much as possible!

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  - ➡ p p > z > e+ e- (ask one S-channel z)
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  - ➡ p p > e+ e- $$ z (forbids any z in s-channel)
- ARE NOT GAUGE INVARIANT !
- forgets diagram interference.
- can provides un-physical distributions.

Avoid Those as much as possible!

check physical meaning and gauge/Lorentz invariance if you do.

- Syntax like
  - p p > z, z > e+ e-   (on-shell z decaying)
  - p p > e+ e- $ z   (forbids s-channel z to be on-shell)
- Are linked to cut $|M^* - M| < BW_{cut} * \Gamma$
- Are more safer to use
- Prefer those syntax to the previous slides one

Exercise V: Automation

- Look at the cross-section for the previous process for 3 different mass points.
  - ➡ hint: you can edit the param_card/run_card via the "set" command [After the launch]
  - ➡ hint: All command [including answer to question] can be put in a file.

Exercise V: Automation

- File content:

```
import model sm
generate p p > t t~
output
launch
set mt 160
set wt Auto
done
launch
set mt 165
set wt Auto
launch
set mt 170
set wt Auto
launch
set mt 175
set wt Auto
launch
set mt 180
set wt Auto
launch
set mt 185
set wt Auto
```

- Run it by:
  - ./bin/mg5 PATH
    - (smarter than ./bin/mg5 < PATH)
- If an answer to a question is not present: Default is taken automatically

Exercise VI: Decay

MadSpin

- generate p p > t t~ h

MadSpin Card

- ➡ decay t > w+ b, w+ > e+ ve
- ➡ decay t~ >w- b~, w- > e- ve~
- ➡ decay h > b b~

2m18.214s

0.004707

MadGraph

- generate p p > t t~ h, (t > w+ b, w+ > e+ ve), (t~ >w- b~, w- > e- ve~), h > b b~

9m30.806s

0.003014

Different here because of cut (not cut should be applied since 2.3.0)