## FKS SUBTRACTION IN MADGRAPH/MADEVENT

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in collaboration with Stefano Frixione, Fabio Maltoni & Tim Stelzer arXiv:0908.4272 [hep-ph]

CERN PhenClub, September 24, 2009

$\delta_O$	$a_{\mathcal{S}} = b_{\mathcal{S}}$	$\xi_{cut} = \xi_{\max}$	$\xi_{cut} = 0.3$	$\xi_{cut} = 0.1$	$\xi_{cut} = 0.01$		
		useenergy=.true.					
	1.0	$3.5988 \pm 0.0146$	$3.6173 \pm 0.0122$	$3.6190 \pm 0.0140$	$3.6126 \pm 0.0141$		
2	1.5	$3.6085 \pm 0.0126$	$3.5942 \pm 0.0143$	$3.5956 \pm 0.0115$	$3.5989 \pm 0.0133$		
	2.0	$3.6127 \pm 0.0121$	$3.6122 \pm 0.0158$	$3.6020 \pm 0.0147$	$3.5956 \pm 0.0144$		
	1.0	$3.6196 \pm 0.0142$	$3.6012 \pm 0.0139$	$3.5888 \pm 0.0142$	$3.5833 \pm 0.0130$		
0.6	1.5	$3.5941 \pm 0.0123$	$3.6012 \pm 0.0139$	$3.6009 \pm 0.0138$	$3.6047 \pm 0.0114$		
	2.0	$3.6066 \pm 0.0120$	$3.6111 \pm 0.0117$	$3.6053 \pm 0.0110$	$3.5950 \pm 0.0150$		
	1.0	$3.6350 \pm 0.0151$	$3.5927 \pm 0.0145$	$3.5813 \pm 0.0128$	$3.5811 \pm 0.0146$		
0.2	1.5	$3.6020 \pm 0.0119$	$3.6086 \pm 0.0133$	$3.6104 \pm 0.0127$	$3.5993 \pm 0.0119$		
	2.0	$3.5815 \pm 0.0140$	$3.5966 \pm 0.0136$	$3.5938 \pm 0.0121$	$3.6079 \pm 0.0125$		
	1.0	$3.6053 \pm 0.0202$	$3.5998 \pm 0.0181$	$3.5988 \pm 0.0122$	$3.6088 \pm 0.0165$		
0.06	1.5	$3.6144 \pm 0.0161$	$3.5986 \pm 0.0140$	$3.5847 \pm 0.0119$	$3.5884 \pm 0.0126$		
	2.0	$3.5990 \pm 0.0166$	$3.6016 \pm 0.0158$	$3.6014 \pm 0.0147$	$3.6191 \pm 0.0133$		
			useenergy	y=.false.			
	1.0	$3.6078 \pm 0.0164$	$3.6149 \pm 0.0162$	$3.6145 \pm 0.0158$	$3.6085 \pm 0.0140$		
2	1.5	$3.5695 \pm 0.0156$	$3.5841 \pm 0.0180$	$3.5975 \pm 0.0165$	$3.5986 \pm 0.0142$		
	2.0	$3.5921 \pm 0.0125$	$3.6260 \pm 0.0211$	$3.6034 \pm 0.0134$	$3.6007 \pm 0.0149$		
	1.0	$3.5891 \pm 0.0199$	$3.5786 \pm 0.0164$	$3.6084 \pm 0.0232$	$3.5956 \pm 0.0151$		
0.6	1.5	$3.6083 \pm 0.0152$	$3.5944 \pm 0.0136$	$3.6040 \pm 0.0123$	$3.6018 \pm 0.0147$		
	2.0	$3.5838 \pm 0.0141$	$3.5633 \pm 0.0154$	$3.5964 \pm 0.0129$	$3.5920 \pm 0.0158$		
	1.0	$3.5976 \pm 0.0171$	$3.5790 \pm 0.0166$	$3.5702 \pm 0.0155$	$3.6155 \pm 0.0132$		
0.2	1.5	$3.5804 \pm 0.0163$	$3.5925 \pm 0.0136$	$3.6012 \pm 0.0137$	$3.6091 \pm 0.0138$		
	2.0	$3.5978 \pm 0.0148$	$3.5749 \pm 0.0144$	$3.5825 \pm 0.0128$	$3.5902 \pm 0.0145$		
	1.0	$3.6122 \pm 0.0170$	$3.5942 \pm 0.0158$	$3.5743 \pm 0.0146$	$3.5962 \pm 0.0167$		
0.06	1.5	$3.6064 \pm 0.0198$	$3.5977 \pm 0.0136$	$3.6047 \pm 0.0115$	$3.5886 \pm 0.0123$		
	2.0	$3.5971 \pm 0.0169$	$3.6018 \pm 0.0136$	$3.5991 \pm 0.0148$	$3.6040 \pm 0.0148$		

- Our 'benchmark process': e+e- -> Z -> uubar ggg
- Result is independent of internal (non-physical) parameters

\*\* Also the integration uncertainty is independent of the choice for the internal parameters

# run-time: 1-4 minutes for each integration channel

**Table 1:** Cross section (in pb) and Monte Carlo integration errors for the (n + 1)-body process  $e^+e^- \rightarrow Z \rightarrow u\bar{u}ggg$ . See the text for details. CERN PhenClub, September 2009

$\delta_O$	$a_{\mathcal{S}} = b_{\mathcal{S}}$	$\xi_{cut} = \xi_{\max}$	$\xi_{cut} = 0.3$	$\xi_{cut} = 0.1$	$\xi_{cut} = 0.01$	
	useenergy=.true.					
	1.0	$3.5988 \pm 0.0146$	$3.6173 \pm 0.0122$	$3.6190 \pm 0.0140$	$3.6126 \pm 0.0141$	
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	<b>S1X-</b>	fold inc	rease o	t the sta		AIN.
	1.0	$3.6196 \pm 0.0142$	$3.6012 \pm 0.0139$	$3.5888 \pm 0.0142$	$3.5833 \pm 0.0130$	
0.6	1.5	$3.5941 \pm 0.0123$	$3.6012 \pm 0.0139$	$3.6009 \pm 0.0138$	$3.6047 \pm 0.0114$	
	2.0	$2.0000 \pm 0.0120$	$3.6111 \pm 0.0117$	$3.6053 \pm 0.0110$	$3.5950 \pm 0.0150$	
	1.0	$3.6350 \pm 0.0151$	$0.5927 \pm 0.0145$	$3.5813 \pm 0.0128$	$3.5811 \pm 0.0146$	
0.2	1.5	$2.6020 \pm 0.0113$	$2.6086 \pm 0.6$	27	$3.5993 \pm 0.0119$	
	2.0	$3.5815 \pm 0.0140$	3.5900 2 600	$07 \pm 0.0052$	$3.6079 \pm 0.0125$	•
	1.0	$3.6053 \pm 0.0202$	3.5998 5.000	$07 \pm 0.0053$	$3.6088 \pm 0.0165$	
0.06	1.5	$3.6144 \pm 0.0161$	$3.5986 \pm 0.5$	19	$3.5884 \pm 0.0126$	
	2.0	$3.5990 \pm 0.0166$	$3.6016 \pm 0.0158$	$3.6014 \pm 0.0147$	$3.6191 \pm 0.0133$	
	useenergy=.false.					
	1.0	$3.6078 \pm 0.0164$	$3.6149 \pm 0.0162$	$3.6145 \pm 0.0158$	$3.6085 \pm 0.0140$	
2	1.5	$3.5695 \pm 0.0156$	$3.5841 \pm 0.0180$	$3.5975 \pm 0.0165$	$3.5986 \pm 0.0142$	
	2.0	$3.5921 \pm 0.0125$	$3.6260 \pm 0.0211$	$3.6034 \pm 0.0134$	$3.6007 \pm 0.0149$	
	1.0	$3.5891 \pm 0.0199$	$3.5786 \pm 0.0164$	$3.6084 \pm 0.0232$	$3.5956 \pm 0.0151$	
0.6	1.5	$3.6083 \pm 0.0152$	$2.5544 \pm 0.015$	$3.6040 \pm 0.0123$	$3.6018 \pm 0.0147$	
	2.0	$3.5838 \pm 0.014$	$3.5633 \pm 0.0154$	$35964 \pm 0.0129$	$3.5920 \pm 0.0158$	
	1.0	$3.5976 \pm 0.0171$	3.5700 1 0 0100	$3.5702 \pm 0$		
0.2	1.5	$3.5804 \pm 0.0163$	$3.5925 \pm 0.0136$	3.6012 2 60	$96 \pm 0.0051$	Ν.
	2.0	$3.5978 \pm 0.0148$	$3.5749 \pm 0.0144$	3.5825	$86 \pm 0.0051$	
	1.0	$3.6122 \pm 0.0170$	$3.5942 \pm 0.0158$	$3.5743 \pm 0.01$	107	
0.06	1.5	$3.6064 \pm 0.0198$	$3.5977 \pm 0.0136$	$3.6047 \pm 0.0115$	$3.5886 \pm 0.0123$	
	2.0	$3.5971 \pm 0.0169$	$3.6018 \pm 0.0136$	$3.5991 \pm 0.0148$	$3.6040 \pm 0.0148$	

Our 'benchmark process': e+e- -> Z -> uubar ggg

Result is independent of internal (non-physical) parameters

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\* run-time: 1-4 minutes for each integration channel

**Table 1:** Cross section (in pb) and Monte Carlo integration errors for the (n + 1)-body process  $e^+e^- \rightarrow Z \rightarrow u\bar{u}ggg$ . See the text for details. CERN PhenClub, September 2009

(n+1)-body process	cross section	$\overline{N}_{\mathrm{FKS}}$	iterations $\times$ points	$N_{ m ch}$	$\epsilon$
$e^+e^- \to Z \to u\bar{u}gg$	$(0.4144 \pm 0.0006 \ (0.15\%)) \times 10^2$	3	$10 \times 50 \mathrm{k}$	6	0.536
$e^+e^- \to Z \to u\bar{u}ggg$	$(0.3601 \pm 0.0014 \ (0.38\%)) \times 10^{1}$	3	$10 \times 50 \mathrm{k}$	18	0.167
$e^+e^- \to Z \to u\bar{u}gggg$	$(0.8869 \pm 0.0054 \ (0.61\%)) \times 10^{-1}$	3	$10\times350 \rm k$	52	0.031
$e^+e^- \to \gamma^*/Z \to jjjj$	$(0.1801 \pm 0.0002 \ (0.12\%)) \times 10^3$	14	$10 \times 50 \mathrm{k}$	56	0.520
$e^+e^- \to \gamma^*/Z \to jjjjj$	$(0.1529 \pm 0.0004 \ (0.26\%)) \times 10^2$	30	$10 \times 50 \mathrm{k}$	328	0.171
$e^+e^- \rightarrow \gamma^*/Z \rightarrow jjjjjj$	$(0.3954 \pm 0.0015 \ (0.38\%)) \times 10^{0}$	55	$10\times350 \rm k$	2450	0.033
$e^+e^- \to Z \to t\bar{t}gg$	$(0.1219 \pm 0.0003 \ (0.24\%)) \times 10^{-1}$	3	$10 \times 10 \mathrm{k}$	6	0.899
$e^+e^- \to Z \to t\bar{t}ggg$	$(0.1521 \pm 0.0013 \ (0.83\%)) \times 10^{-2}$	3	$10 \times 10 \mathrm{k}$	18	0.708
$e^+e^- \to Z \to t\bar{t}gggg$	$(0.1108 \pm 0.0031 \ (2.76\%)) \times 10^{-3}$	3	$10 \times 20 k$	52	0.427
$e^+e^- \to Z \to t\bar{t}b\bar{b}g$	$(0.1972 \pm 0.0024 \ (1.23\%)) \times 10^{-4}$	4	$10 \times 10 \mathrm{k}$	16	1.000
$e^+e^- \to Z \to t\bar{t}b\bar{b}gg$	$(0.2157 \pm 0.0029 \ (1.34\%)) \times 10^{-4}$	5	$10 \times 10 \mathrm{k}$	120	0.824
$e^+e^- \to Z \to \tilde{t}_1 \tilde{\bar{t}}_1 ggg$	$(0.3712 \pm 0.0037 \ (1.00\%)) \times 10^{-8}$	3	$10 \times 10 \mathrm{k}$	18	0.764
$e^+e^- \to Z \to \tilde{g}\tilde{g}ggg$	$(0.1584 \pm 0.0020 \ (1.23 \ \%)) \times 10^{-1}$	2	$10 \times 10 \mathrm{k}$	9	0.753
$\mu^+\mu^- \to H \to gggg$	$(0.1404 \pm 0.0005 \ (0.34 \ \%)) \times 10^{-7}$	1	$10 \times 50 \mathrm{k}$	2	0.559
$\mu^+\mu^- \to H \to ggggg$	$(0.2575 \pm 0.0018 \ (0.69 \ \%)) \times 10^{-8}$	1	$10 \times 50 \mathrm{k}$	4	0.165
$\mu^+\mu^- \to H \to gggggg$	$(0.1186 \pm 0.0008 \ (0.70 \ \%)) \times 10^{-9}$	1	$10 \times 350 \mathrm{k}$	9	0.031

Compared to Born (without optimization relevant to separate treatment of different integration channels), error is 1.9-4.5 times larger with the same statistics\*

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\* 2 exceptions; ttbbg: 7 & ttgggg: 9 4





