Automatic Generation of Quarkonium Amplitudes in NRQCD

Lunch seminar

14 November 2007

Pierre Artoisenet

Center of Particle Physics and Phenomenology

Université Catholique de Louvain

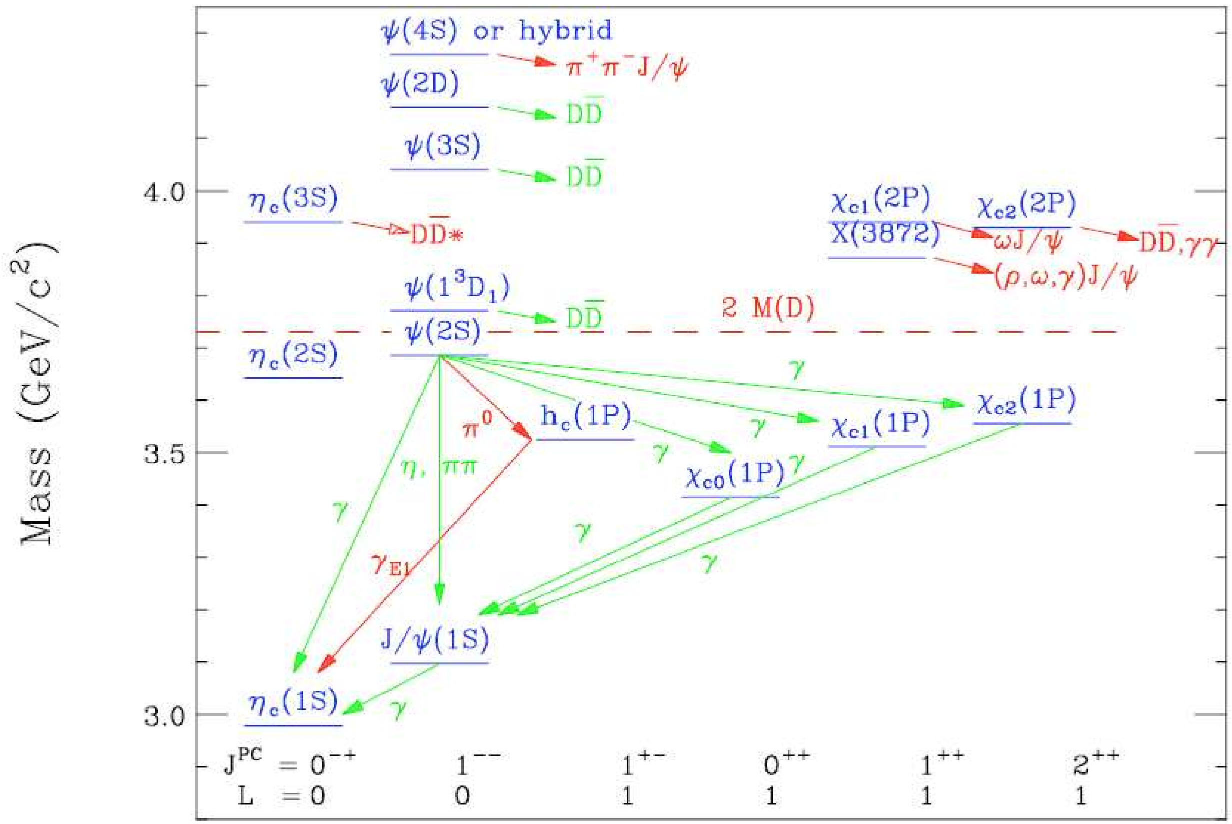
PLAN

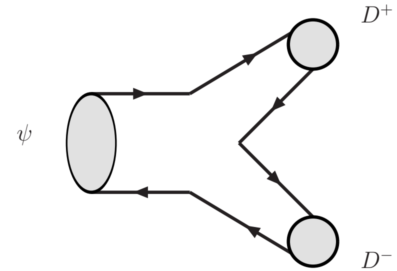
- Introduction: the heavy quarkonium system
- MadOnia: a new code for quarkonium production
- Illustration
- Ongoing studies: J/ψ and Υ hadroproduction
- Conclusion and Perspectives

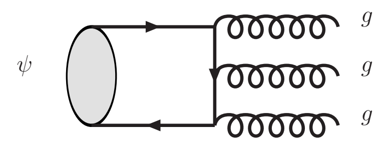
Introduction: the heavy quarkonium system

Heavy quarkonium system

- charmonium: $c\bar{c}$ bound state ($\frac{v^2}{c^2} \approx 0.3$)

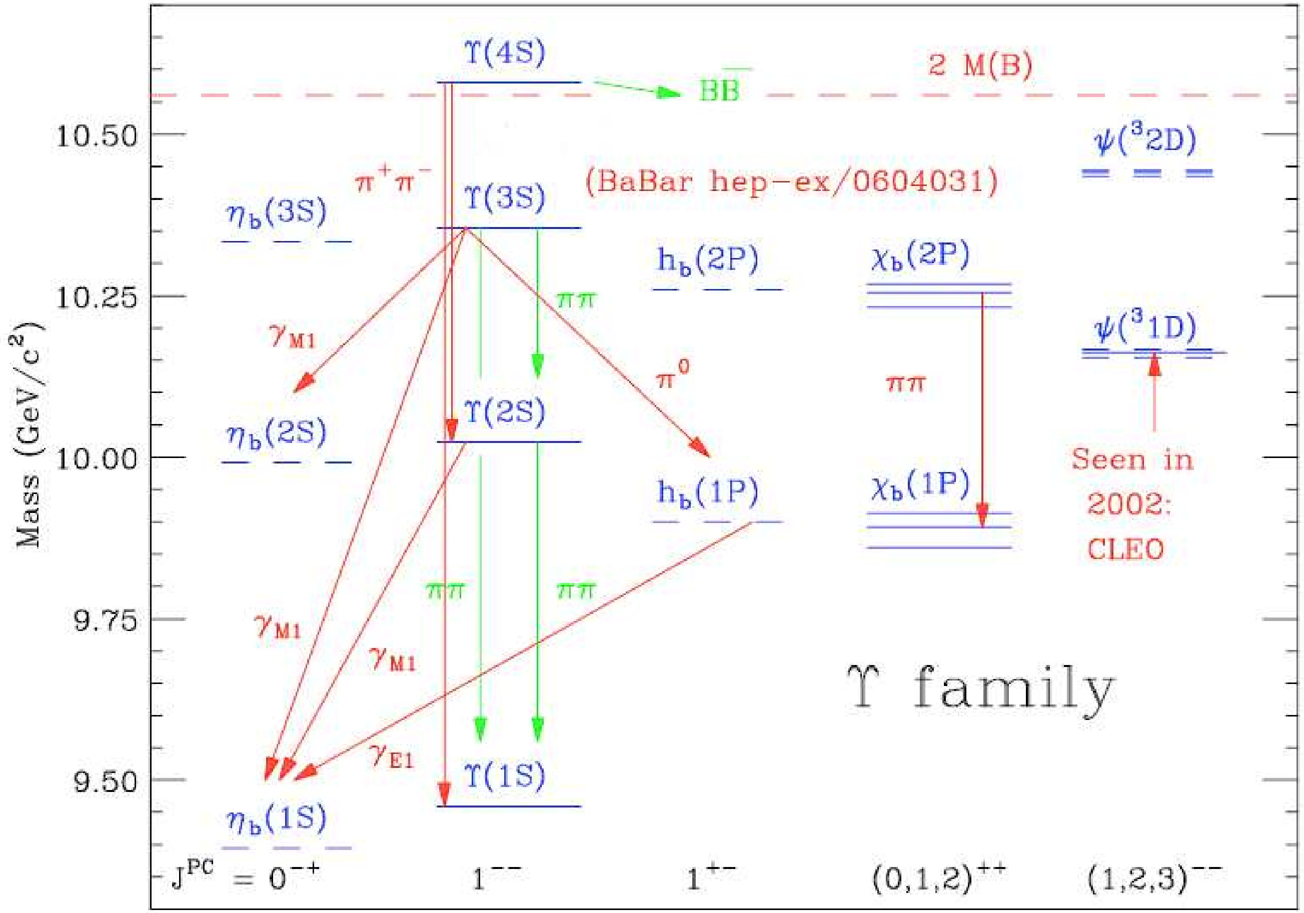






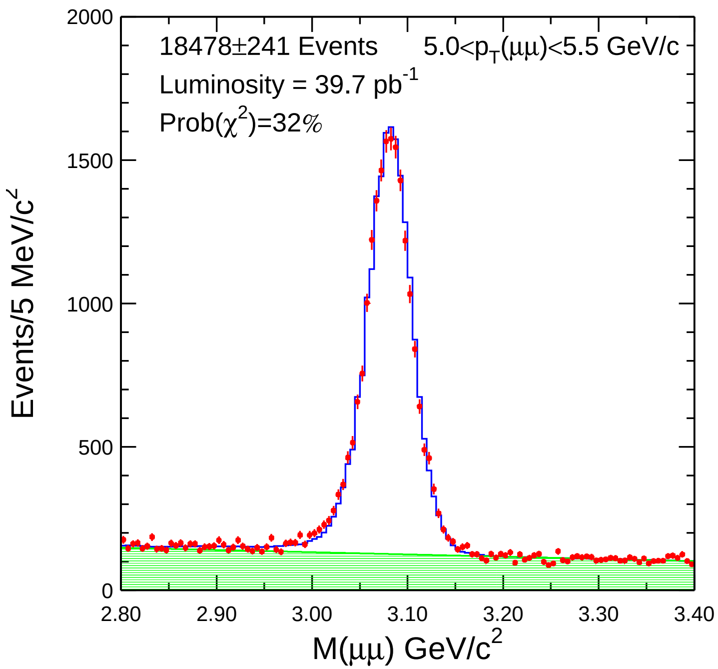
Heavy quarkonium system

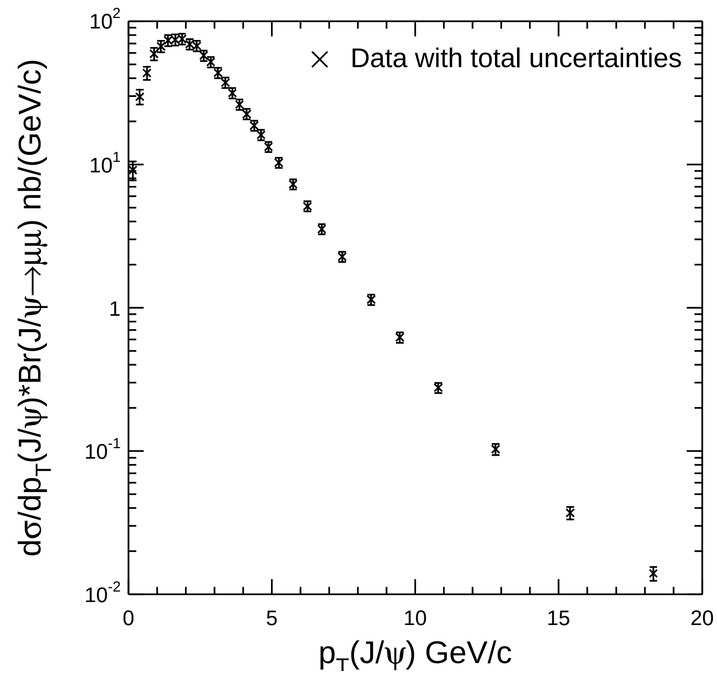
- bottomonium: $b\bar{b}$ bound state ($\frac{v^2}{c^2} \approx 0.1$)



Heavy quarkonium system

- experimental observation
  - $\mathcal{Q}(1^{--}) \to l^+ l^-$
    - very clean signature in hadron colliders
      example: J/ψ production at the Tevatron II ($\sqrt{s} = 1.96$ TeV)

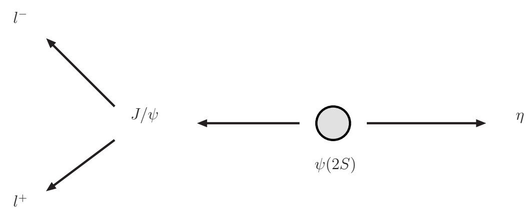




Phys. Rev. D 71 032001 (2005)

Heavy quarkonium system

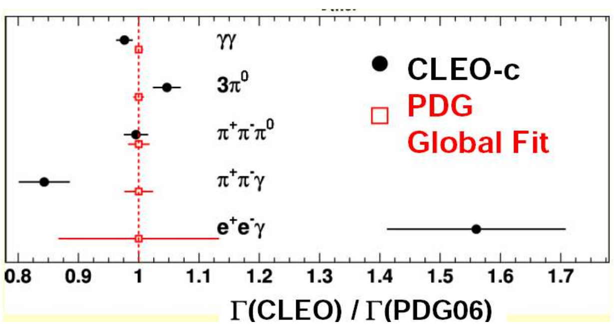
- experimental observation
  - $\mathcal{Q}(1^{--}) \to l^+ l^-$
    - can be used as a probe for exclusive measurement
      example: $e^+e^- \to \psi(2S) \to J/\psi\eta$ (A. Lopez eta al, PRL 99 122001 (2007))

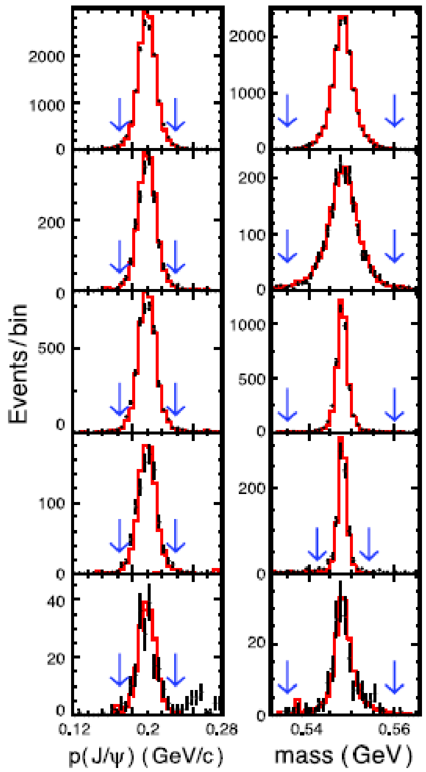


27M $\psi(2S)$

$$Br(\psi(2S) \to J/\psi\eta) = 3.1\%$$

$$Br(J/\psi \to l^+l^-) = 12\%$$





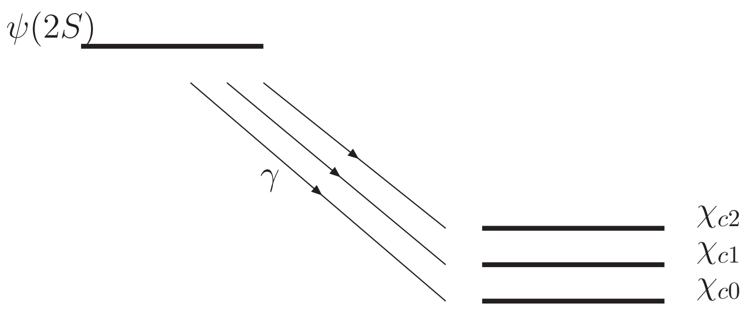
Heavy quarkonium system

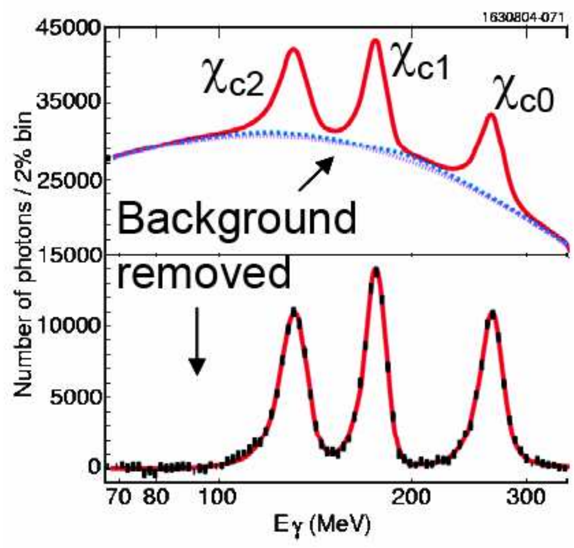
- experimental observation
  - $\mathcal{Q}(1^{--}) \to l^+ l^-$
  - $\chi_{c,b}$ states can be observed through radiative decays (E transitions)

Heavy quarkonium system

- experimental observation
  - $\mathcal{Q}(1^{--}) \to l^+ l^-$
  - $\chi_{c,b}$ states can be observed through radiative decays (E transitions)

    Example 1: measurement of χ_c decays at Cleo





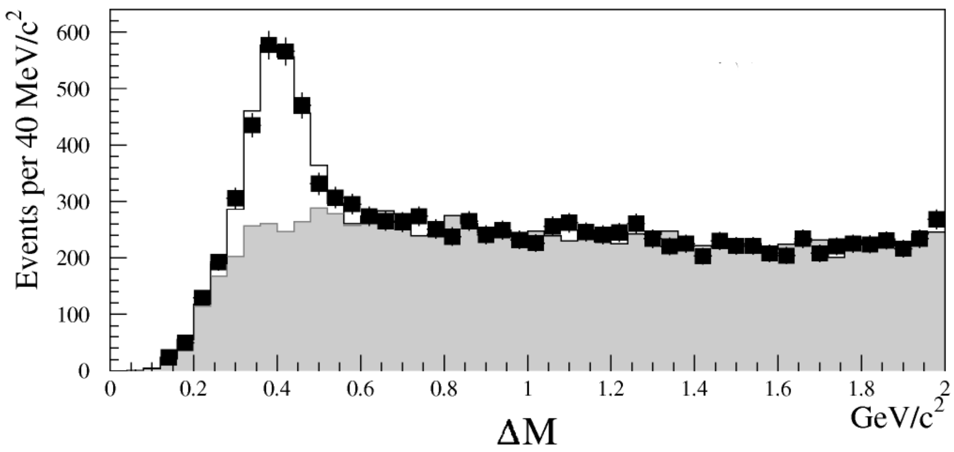
χ_{cj} decays to $\gamma\gamma$, $\pi\pi$, KK, $\eta^{(\prime)}\eta^{(\prime)}$, baryon/anti-baryon, multibody final states (H. Mahlke, Charmonium results from Cleo)

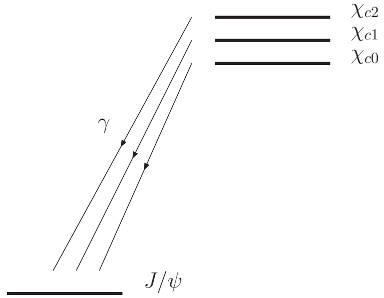
Heavy quarkonium system

- experimental observation
  - $\mathcal{Q}(1^{--}) \rightarrow l^+ l^-$
  - $\chi_{c,b}$ states can be observed through radiative decays (E transitions)

    Example 2: $p\bar{p} \rightarrow \chi_c + X$, $\chi_c \rightarrow J/\psi\gamma$

$$\Delta M = m(\mu^+\mu^-\gamma) - m(\mu^+\mu^-)$$

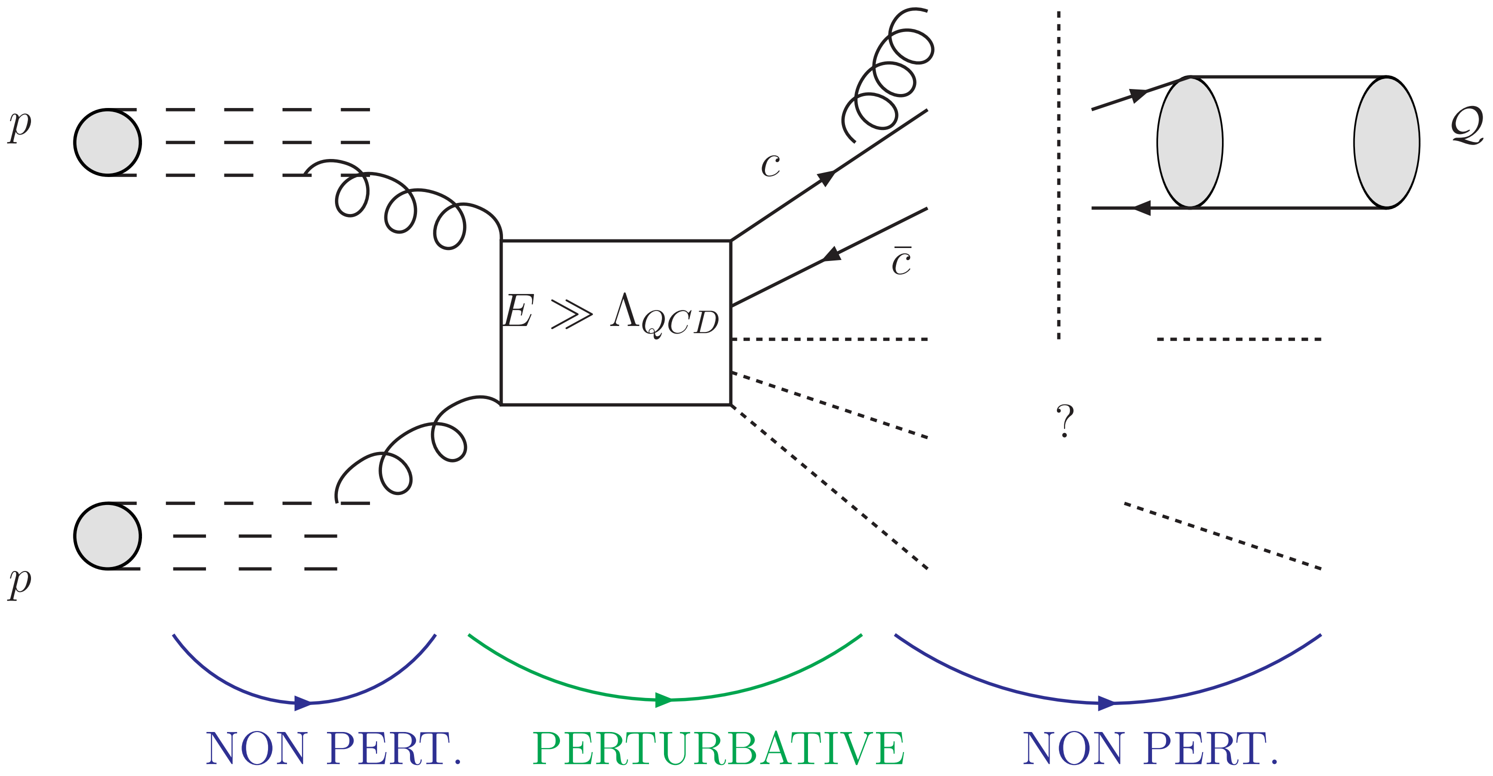




F. Abe et al., Phys. Rev. Lett. 79, 578

Theoretical aspects

aim: factorize the non perturbative effects in a process-independent way



Theoretical aspects

- Color Singlet Model

$$\sigma_{\mathcal{Q}} = \sigma(c\bar{c}(^{2S+1}L_J^{[1]}))|\psi(0)|^2$$

the perturbative $c\bar{c}$ pair has the same quantum numbers "as in the bound state".

- Color Evaporation Model

$$\sigma_{onium} = \frac{1}{9}\int_{2m_c}^{2m_D} \frac{\sigma_{c\bar{c}}}{dm} dm, \quad \sigma_{\mathcal{Q}} = \rho_{\mathcal{Q}}\sigma_{onium}$$

the perturbative $c\bar{c}$ pair is created without any constrain.

- Non relativistic QCD

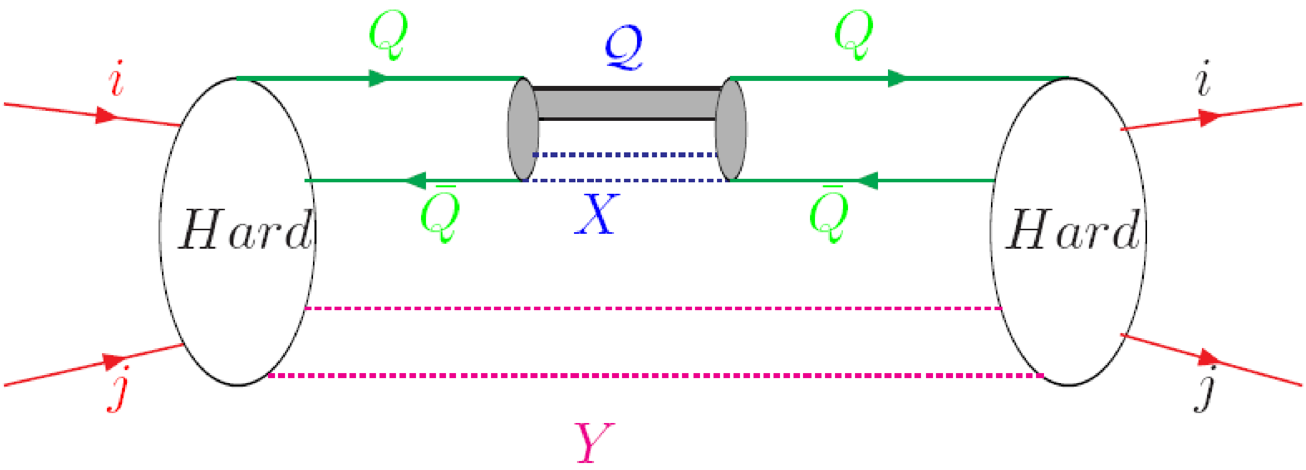
$$\sigma(\mathcal{Q}) = \sum_n \hat{\sigma}(c\bar{c}(n))\langle\mathcal{O}^{\mathcal{Q}}(n)\rangle_\Lambda$$

the perturbative $c\bar{c}$ pair is *a priori* in an arbitrary state n, but $c\bar{c}(n) \to \mathcal{Q} \sim v^{f(n,\mathcal{Q})}$

NRQCD factorization

- Factorization at the level of the squared amplitude:

$$\sum_n \underbrace{\langle i,j|T|YQ\bar{Q}_n\rangle}_{\mathcal{A}^*(ij\to YQ\bar{Q}_n)}\underbrace{\langle Q\bar{Q}_n|\mathcal{Q}X\rangle\langle \mathcal{Q}X|Q\bar{Q}_n\rangle}_{\langle 0|\mathcal{O}^{\mathcal{Q}}(n)|0\rangle}\underbrace{\langle Q\bar{Q}_nY|T|i,j\rangle}_{\mathcal{A}(ij\to YQ\bar{Q}_n)}$$



soft partons (X) are included in the long distance part

$\longrightarrow$ the intermediate $Q\bar{Q}$ pair can be in a color-octet state

MadOnia: a new code for quarkonium production

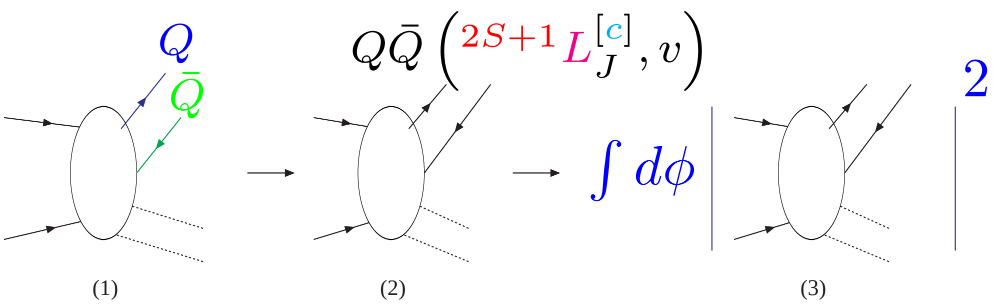
The purpose of MadOnia

- expression of cross sections within NRQCD:

$$\sigma(ij \to \mathcal{Q} + X) \quad = \quad \sum_n \hat{\sigma}(ij \to Q\bar{Q}(n) + X)\langle \mathcal{O}^{\mathcal{Q}}(n)\rangle_\Lambda$$

  - $\langle \mathcal{O}^{\mathcal{Q}}(n)\rangle$ are the long distance matrix elements
  - $\hat{\sigma}(i + j \to Q\bar{Q}(n) + X)$ are the short distance cross sections

- MadOnia: automatic tree-level computation of $\hat{\sigma}(ij \to Q\bar{Q}(n) + X)$



(1) open quark amplitude (MadGraph)

(2) projected amplitude (MadOnia)

(3) phase-space integration (unweighting $\longrightarrow$ MC event generator)

Capabilities and Validation

- capabilities:
  - universality: MadOnia generates any helicity amplitude

$$\mathcal{M}\left(ij \to Q\bar{Q}\left({}^{2S+1}L_J^{[c]}\right) + X\right)$$

    at tree-level, for any model that can be implemented in MadGraph
  - it keeps track of quantum numbers on event-by-event basis $\longrightarrow$ events ready for showering and hadronization (in particular, calculation in terms of color-ordered amplitudes).
  - $Q\bar{Q}'$ production: the quark and the anti-quark can be of different flavour (such as B_c)
  - double quarkonium production (ex: $e^+e^- \to J/\psi\eta_c$)
  - relativistic corrections for S-wave state production can be computed

Capabilities and Validation

- validation:
  - gauge invariance has been checked
  - charge conjugation conservation:

$$
\begin{aligned}
A({}^1S_0^{[1]} + (2k+1)\gamma) &= 0 \\
A({}^3S_1^{[1]} + (2k)\gamma) &= 0 \\
A({}^1P_1^{[1]} + (2k)\gamma) &= 0 \\
A({}^3P_1^{[1]} + (2k)\gamma) &= 0 \\
A({}^3P_{0,2}^{[1]} + (2k+1)\gamma) &= 0
\end{aligned}
$$

  - comparison with analytical amplitudes point by point in the phase space

$$ij \to \mathcal{Q}k$$

with i, j, k = quarks or gluons, for all S- and P-wave states, color-singlet and color-octet transitions

Illustration

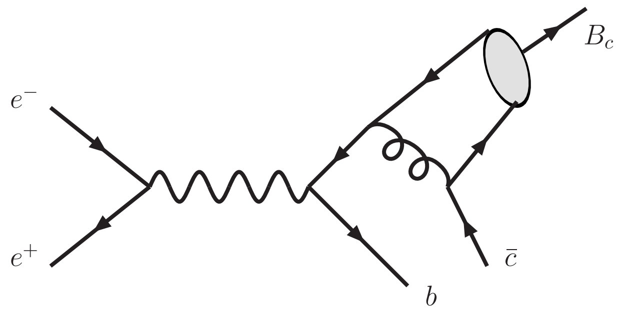
Illustration

- example: B_c production from e^+e^-

Illustration

- example: B_c production from e^+e^-

$$e^+e^- \longrightarrow b\bar{c}B_c({}^3S_1^{[1]})$$
$$e^+e^- \longrightarrow b\bar{c}B_c({}^1S_0^{[1]})$$



Illustration

- example: B_c production from e^+e^-
  - enter the process: fill the input file proc_card.dat

```
# Begin PROCESS # This is TAG. Do not modify this line

e+e->bc~cb~[3S11]   @0       # First Process
QCD=99              # Max QCD couplings
QED=2               # Max QED couplings
end_coup            # End the couplings input

e+e->bc~cb~[1S01]   @1       # Second Process
QCD=99              # Max QCD couplings
QED=2               # Max QED couplings
end_coup            # End the couplings input

done                # this tells MG there are no more procs

# End PROCESS  # This is TAG. Do not modify this line
#*********************************************************************
# Model information                                                  *
#*********************************************************************
# Begin MODEL  # This is TAG. Do not modify this line
sm
# End   MODEL  # This is TAG. Do not modify this line
```

Illustration

- example: B_c production from e^+e^-
  - enter the process: fill the input file proc_card.dat

```
# Begin PROCESS # This is TAG. Do not modify this line

e+e->bc~cb~[3S11]    @0         # First Process
QCD=99                # Max QCD couplings
QED=2                 # Max QED couplings
end_coup              # End the couplings input

e+e->bc~cb~[1S01]    @1         # Second Process
QCD=99                # Max QCD couplings
QED=2                 # Max QED couplings
end_coup              # End the couplings input

done                  # this tells MG there are no more procs

# End PROCESS  # This is TAG. Do not modify this line
#*************************************************************
# Model information
#*************************************************************
# Begin MODEL  # This is TAG. Do not modify this line
sm
# End   MODEL  # This is TAG. Do not modify this line
```

Illustration

- example: B_c production from e^+e^-
  - enter the process: fill the input file proc_card.dat

```
# Begin PROCESS # This is TAG. Do not modify this line

e+e->bc~cb~[3S11]   @0        # First Process
QCD=99               # Max QCD couplings
QED=2                # Max QED couplings
end_coup             # End the couplings input

e+e->bc~cb~[1S01]   @1        # Second Process
QCD=99               # Max QCD couplings
QED=2                # Max QED couplings
end_coup             # End the couplings input

done                 # this tells MG there are no more procs

# End PROCESS  # This is TAG. Do not modify this line
#***************************************************************
# Model information
#***************************************************************
# Begin MODEL  # This is TAG. Do not modify this line
sm
# End   MODEL  # This is TAG. Do not modify this line
```

Illustration

- example: B_c production from e^+e^-
  - enter the process: fill the input file proc_card.dat

```
# Begin PROCESS # This is TAG. Do not modify this line

e+e->bc~cb~[3S11]   @0       # First Process
QCD=99              # Max QCD couplings
QED=2               # Max QED couplings
end_coup            # End the couplings input

e+e->bc~cb~[1S01]   @1       # Second Process
QCD=99              # Max QCD couplings
QED=2               # Max QED couplings
end_coup            # End the couplings input

done                # this tells MG there are no more procs

# End PROCESS  # This is TAG. Do not modify this line
#**************************************************************
# Model information
#**************************************************************
# Begin MODEL  # This is TAG. Do not modify this line
sm
# End   MODEL  # This is TAG. Do not modify this line
```

Illustration

- example: B_c production from e^+e^-
  - enter the process: fill the input file proc_card.dat

```
# Begin PROCESS # This is TAG. Do not modify this line

e+e->bc~cb~[3S11]   @0       # First Process
QCD=99              # Max QCD couplings
QED=2               # Max QED couplings
end_coup            # End the couplings input

e+e->bc~cb~[1S01]   @1       # Second Process
QCD=99              # Max QCD couplings
QED=2               # Max QED couplings
end_coup            # End the couplings input

done                # this tells MG there are no more procs

# End PROCESS  # This is TAG. Do not modify this line
#************************************************************
# Model information
#************************************************************
# Begin MODEL  # This is TAG. Do not modify this line
sm
# End   MODEL  # This is TAG. Do not modify this line
```

Illustration

- example: B_c production from e^+e^-
  - Output:
    MadOnia generates a fortran code that gives the squared matrix element summed/averaged over polarization degrees of freedom at an arbitrary phase-space point:

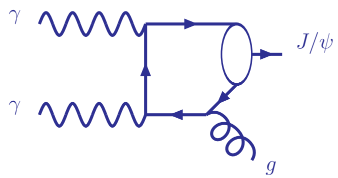
$$\frac{1}{4}\sum_{\lambda_1,\ldots,\lambda_5}|M(e^+(p_1)e^-(p_2)\to b(p_3)\bar{c}(p_4)B_c(p_5))|^2$$

  - interface with a phase-space generator $\longrightarrow$ cross sections

Illustration

- J/ψ production from $\gamma\gamma$ collisions (Lep II, $\sqrt{s} = 196$ GeV)

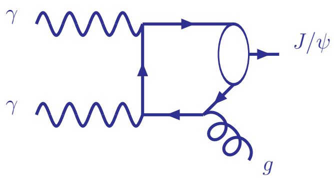
$$J/\psi^{[8]}g$$



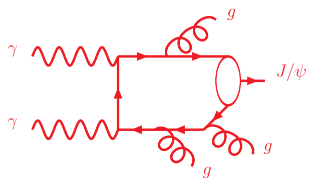
Illustration

- J/ψ production from $\gamma\gamma$ collisions (Lep II, $\sqrt{s} = 196$ GeV)

$J/\psi^{[8]}g$



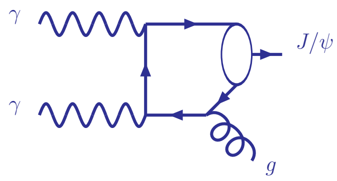
$J/\psi^{[1]}ggg$



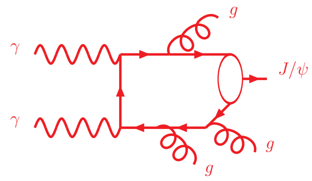
Illustration

- J/ψ production from $\gamma\gamma$ collisions (Lep II, $\sqrt{s} = 196$ GeV)

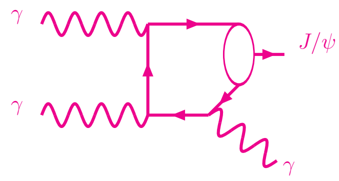
$J/\psi^{[8]} g$



$J/\psi^{[1]} ggg$



$J/\psi^{[1]} \gamma$



Illustration

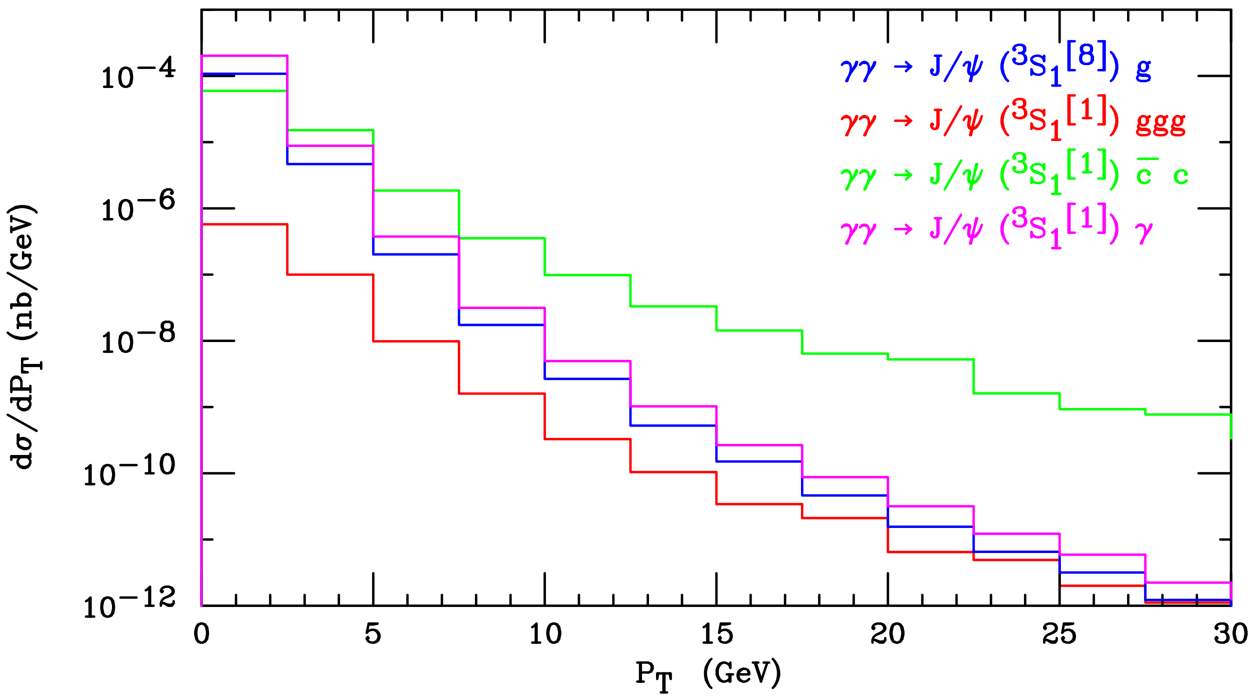
- J/ψ production from $\gamma\gamma$ collisions (Lep II, $\sqrt{s} = 196$ GeV)

$J/\psi^{[8]}g$ $J/\psi^{[1]}ggg$ $J/\psi^{[1]}\gamma$ $J/\psi^{[1]}c\bar{c}$

Illustration

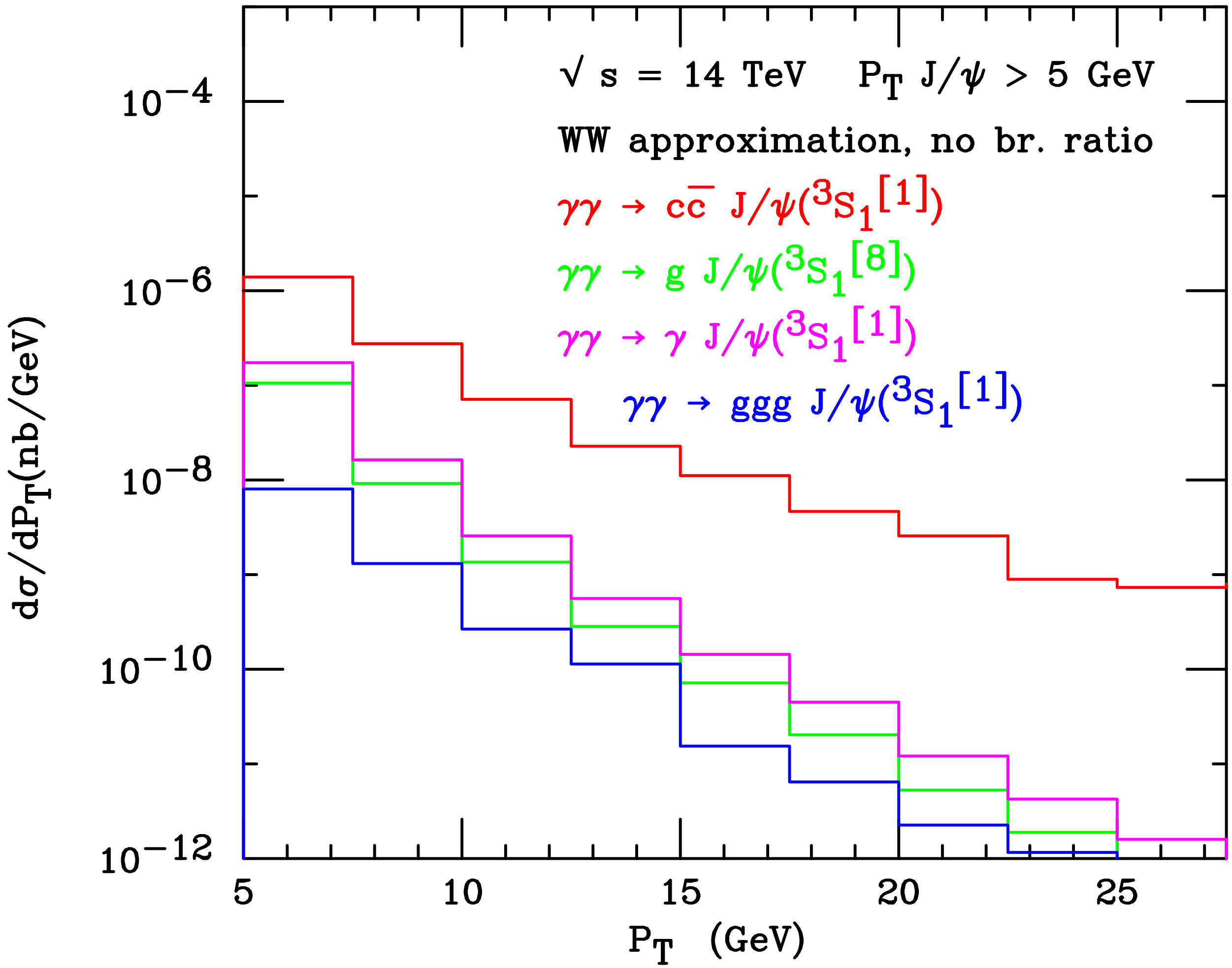
- J/ψ production from $\gamma\gamma$ collisions (Lep II, $\sqrt{s} = 196$ GeV)

$J/\psi^{[8]}g$ $\quad$ $J/\psi^{[1]}ggg$ $\quad$ $J/\psi^{[1]}\gamma$ $\quad$ $J/\psi^{[1]}c\bar{c}$



Illustration

- J/ψ production from $\gamma\gamma$ collisions (LHC, $\sqrt{s} = 14$ TeV)

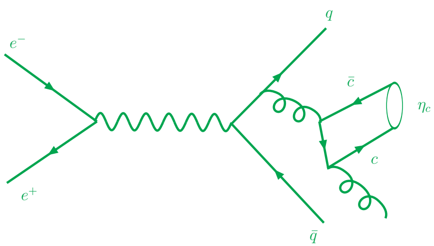
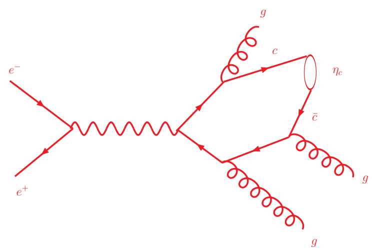
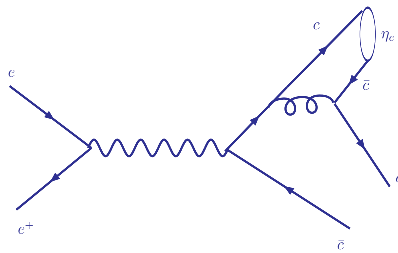


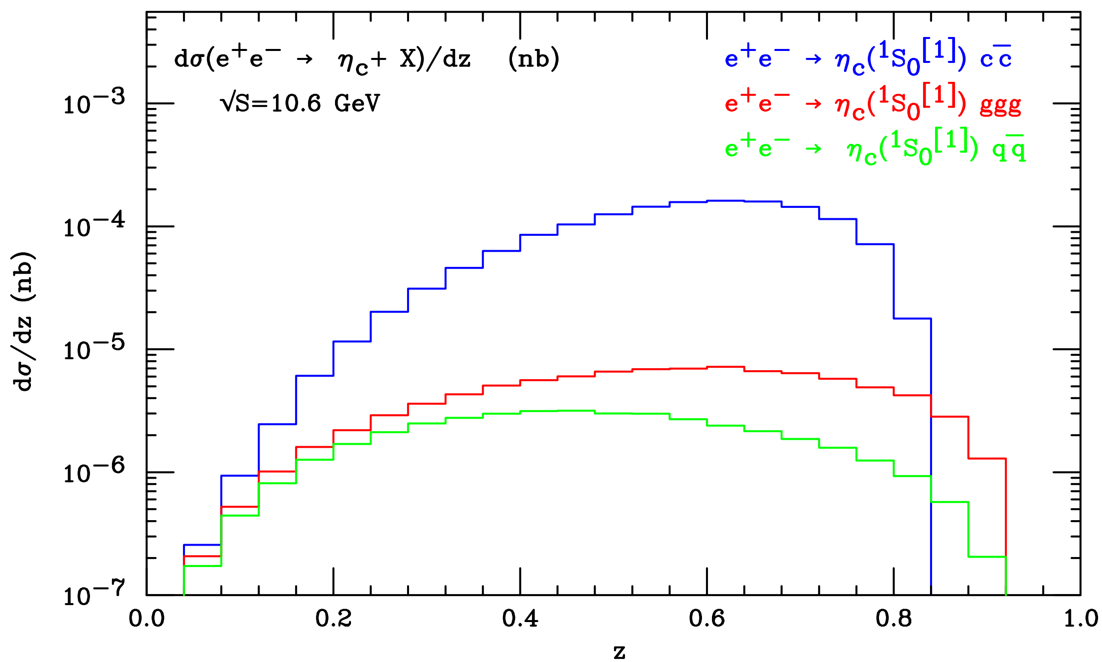
Illustration

- $e^+e^- \to \eta_c + X$ @ 10.6 GeV

  subprocesses:

$e^+e^- \to \eta_c q\bar{q}g$ $\qquad$ $e^+e^- \to \eta_c ggg$ $\qquad$ $e^+e^- \to \eta_c c\bar{c}$



$\sigma(\eta_c c\bar{c}) = 58.7$ fb

$\sigma(\eta_c ggg) = 3.72$ fb

$\sigma(\eta_c gq\bar{q}) = 1.63$ fb

Remarks:

$\sigma(J/\psi c\bar{c}) = 148$ fb

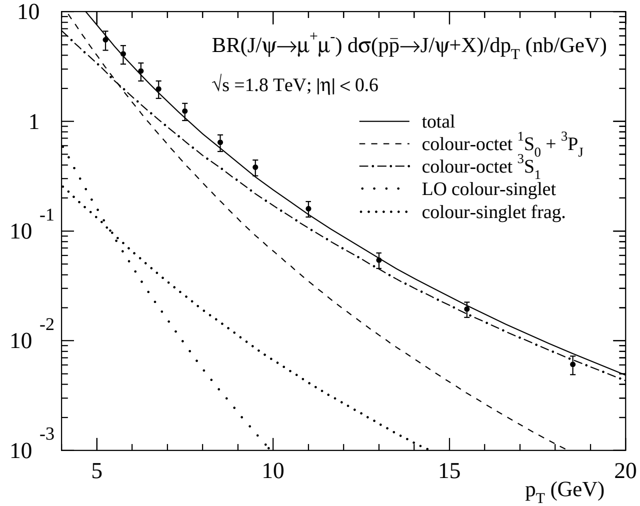
$\sigma(J/\psi gg) = 266$ fb

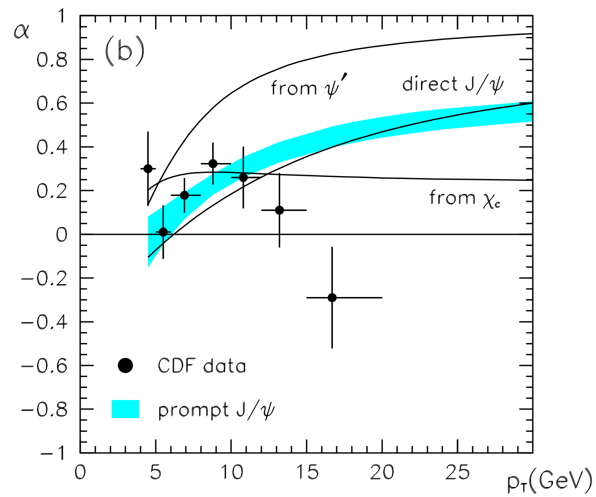
Ongoing studies: J/ψ and Υ hadroproduction

J/ψ production at the Tevatron

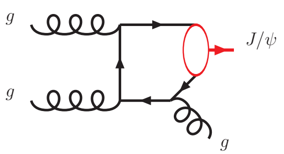
- inclusive production: current status

[from M. Kramer, Prog. Part. Nucl. Phys. 47: 141-201,2001.]

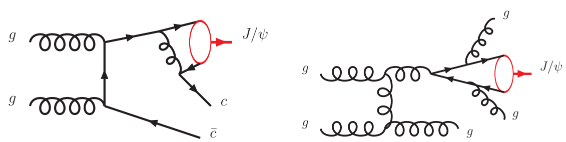




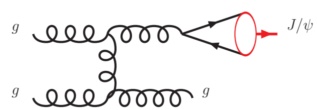
LO color-singlet



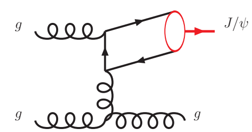
color-singlet frag.



color-octet $^3S_1$



color-octet $^1S_0$



cross section in the fragmentation approximation

$$d\sigma_{J/\psi}(P) \simeq \int_0^1 dz\, d\sigma_c\left(\frac{P}{z}, \mu_{frag}\right) D_{c\to J/\psi}(z, \mu_{frag})$$

and similarly for the fragmentation from a gluon.

J/ψ production at the Tevatron

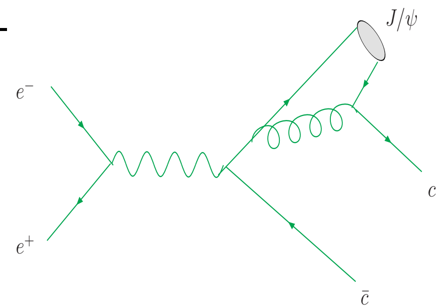
- associated J/ψ production

$$p\bar{p} \longrightarrow J/\psi c\bar{c}$$

motivation:

- situation in e^+e^- annihilation

	$\sigma(e^+e^- \to J/\psi + c\bar{c})$
Belle	$0.87^{+0.21}_{-0.19} \pm 0.17$ pb
NRQCD prediction	0.0897 pb
(LO in v and α_s)	



Color transfer between the active charm-quark pair and one of the spectator charm quark might lead to an enhancement

J/ψ production at the Tevatron

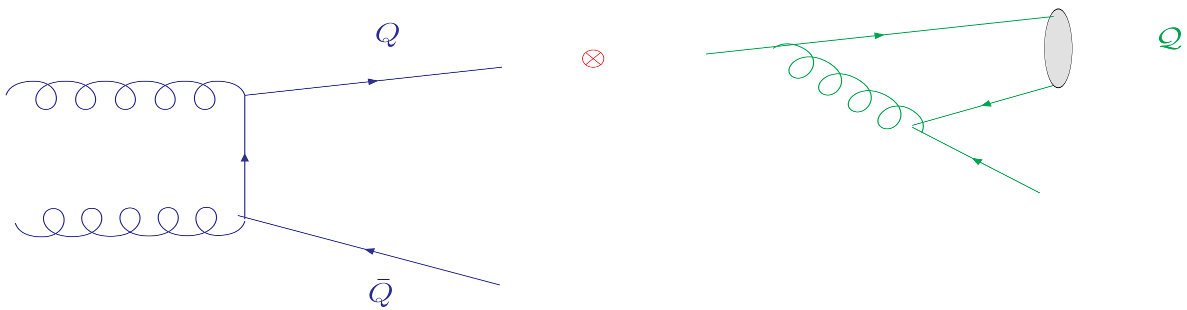
- associated J/ψ production

$$p\bar{p} \longrightarrow J/\psi c\bar{c}$$

motivation:

- includes topologies that are not taken into account in the fragmentation approximation

$$d\sigma_{\mathcal{Q}}(P) \simeq \int_0^1 dz d\sigma_{Q_i}(\frac{P}{z}, \mu_{frag}) D_{Q_i \to \mathcal{Q}}(z, \mu_{frag})$$



J/ψ production at the Tevatron

- associated J/ψ production

$$p\bar{p} \longrightarrow J/\psi c\bar{c}$$

motivation:

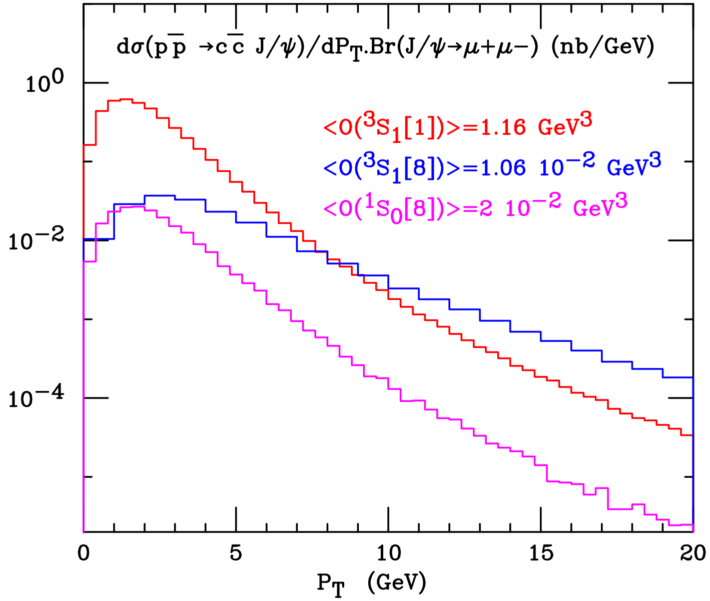
  - part of the α_s^4 color-singlet inclusive J/ψ production
  - it could be tested experimentally
  - this new channel offers the opportunity to check the universality of the Long Distance Matrix Elements

J/ψ production at the Tevatron

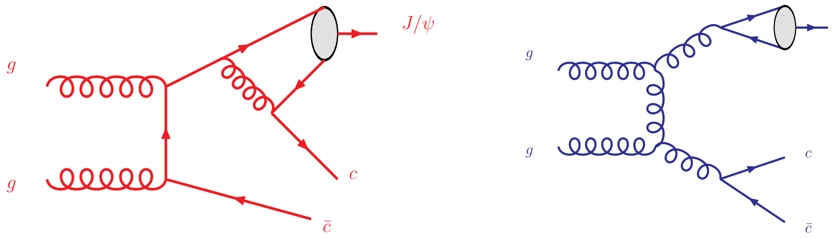
- associated J/ψ production

$$p\bar{p} \longrightarrow J/\psi c\bar{c}$$

P_T distributions:



dominant topologies in the region $P_T \gg m_c$:



J/ψ production at the Tevatron

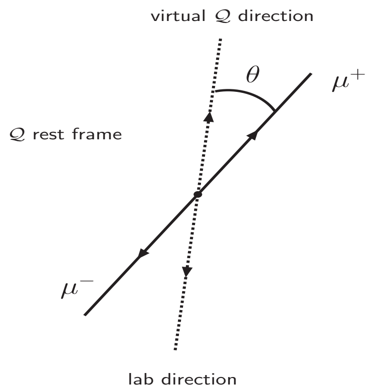
- associated J/ψ production

$$p\bar{p} \longrightarrow J/\psi c\bar{c}$$

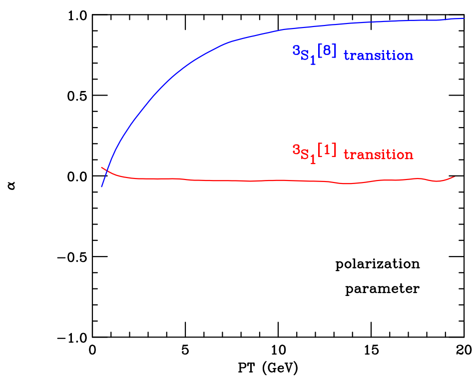
J/ψ polarization:

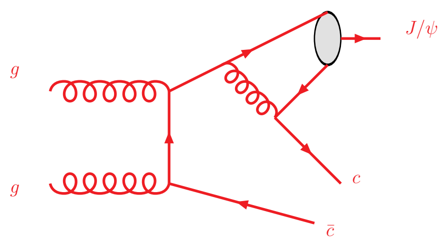
extracted from the angular distribution of the produced leptons

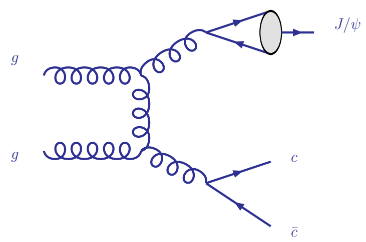
$$I(\cos\theta) = \frac{3}{2(\alpha+3)}(1+\alpha\,\cos^2\theta)$$



$$\alpha = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$$





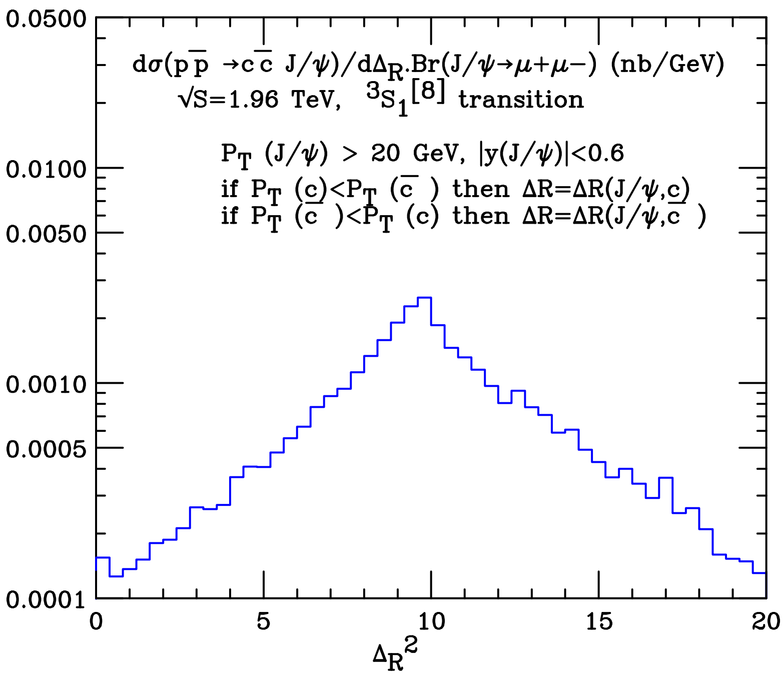


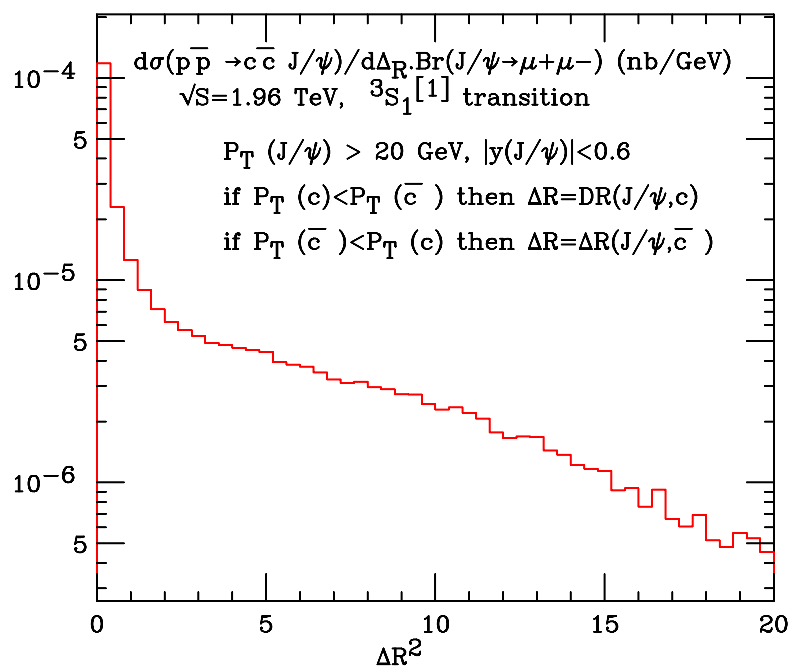
J/ψ production at the Tevatron

- associated J/ψ production

$$p\bar{p} \longrightarrow J/\psi c\bar{c}$$

angular separation $\Delta R(J/\psi, c)$:



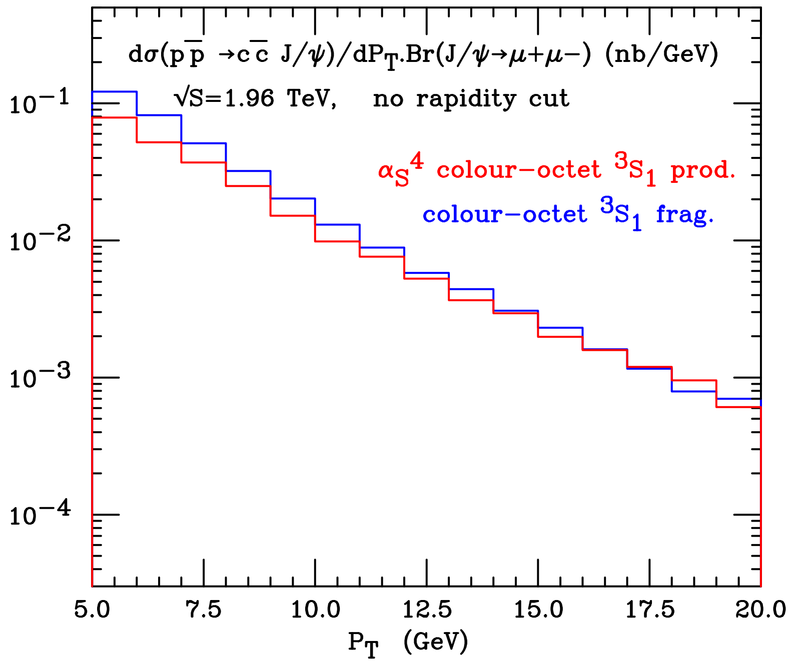


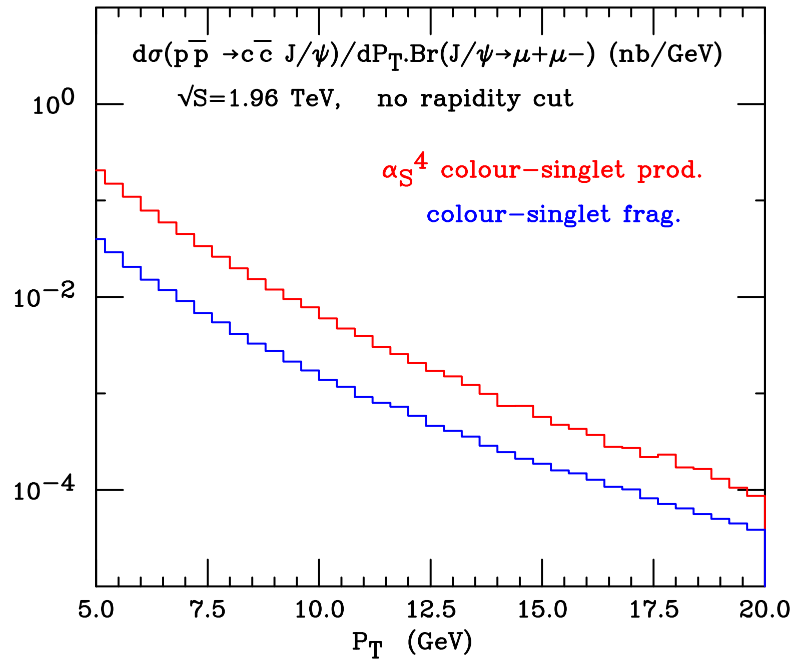
J/ψ production at the Tevatron

- associated J/ψ production

$$p\bar{p} \longrightarrow J/\psi c\bar{c}$$

comparison with the fragmentation approximation:





$\Upsilon + 3$ jets at the Tevatron

- subprocesses:

dg_uuxdbbx3S11	gd_uuxdbbx3S11	gu_uuuxbbx3S11	ug_uddxbbx3S11	uux_uuxgbbx3S11
uxu_ddxgbbx3S11	du_udgbbx3S11	gdx_uuxdxbbx3S11	gux_uuxuxbbx3S11	ug_uggbbx3S11
uxd_uxdgbbx3S11	uxu_gggbbx3S11	dux_uxdgbbx3S11	gg_gggbbx3S11	gux_uxddxbbx3S11
ug_uuuxbbx3S11	uxdx_uxdxgbbx3S11	uxu_uuxgbbx3S11	dxg_uuxdxbbx3S11	gg_uuxgbbx3S11
gux_uxggbbx3S11	uu_uugbbx3S11	uxg_uuxuxbbx3S11	uxux_uxuxgbbx3S11	dxu_udxgbbx3S11
gu_uddxbbx3S11	ud_udgbbx3S11	uux_ddxgbbx3S11	uxg_uxddxbbx3S11	dxux_uxdxgbbx3S11
gu_uggbbx3S11	udx_udxgbbx3S11	uux_gggbbx3S11	uxg_uxggbbx3S11	

$\Upsilon + 3$ jets at the Tevatron

- subprocesses:

dg_uuxdbbx3S11	gd_uuxdbbx3S11	gu_uuuxbbx3S11	ug_uddxbbx3S11	uux_uuxgbbx3S11
uxu_ddxgbbx3S11	du_udgbbx3S11	gdx_uuxdxbbx3S11	gux_uuxuxbbx3S11	ug_uggbbx3S11
uxd_uxdgbbx3S11	uxu_gggbbx3S11	dux_uxdgbbx3S11	gg_gggbbx3S11	gux_uxddxbbx3S11
ug_uuuxbbx3S11	uxdx_uxdxgbbx3S11	uxu_uuxgbbx3S11	dxg_uuxdxbbx3S11	gg_uuxgbbx3S11
gux_uxggbbx3S11	uu_uugbbx3S11	uxg_uuxuxbbx3S11	uxux_uxuxgbbx3S11	dxu_udxgbbx3S11
gu_uddxbbx3S11	ud_udgbbx3S11	uux_ddxgbbx3S11	uxg_uxddxbbx3S11	dxux_uxdxgbbx3S11
gu_uggbbx3S11	udx_udxgbbx3S11	uux_gggbbx3S11	uxg_uxggbbx3S11	

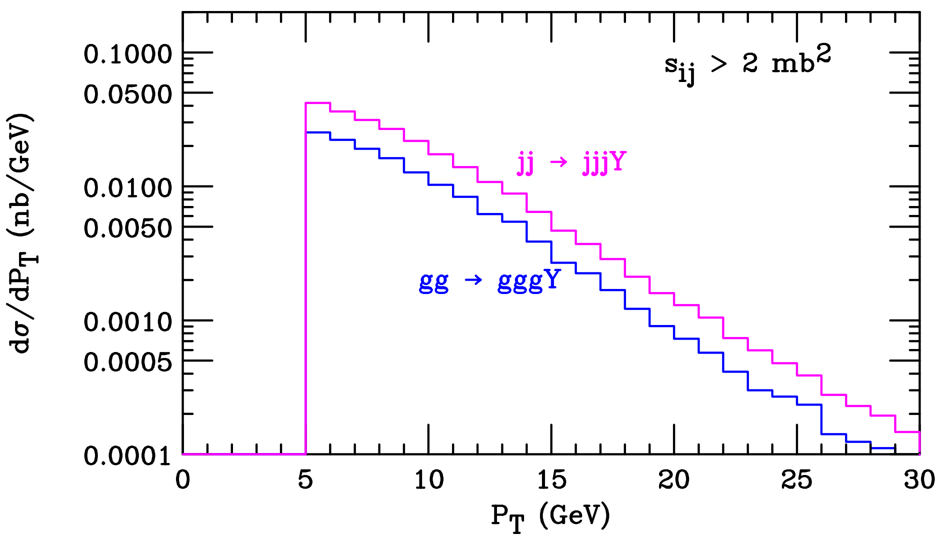
$\approx$ 2000 Feynman diagrams before projection

Υ + 3 jets at the Tevatron

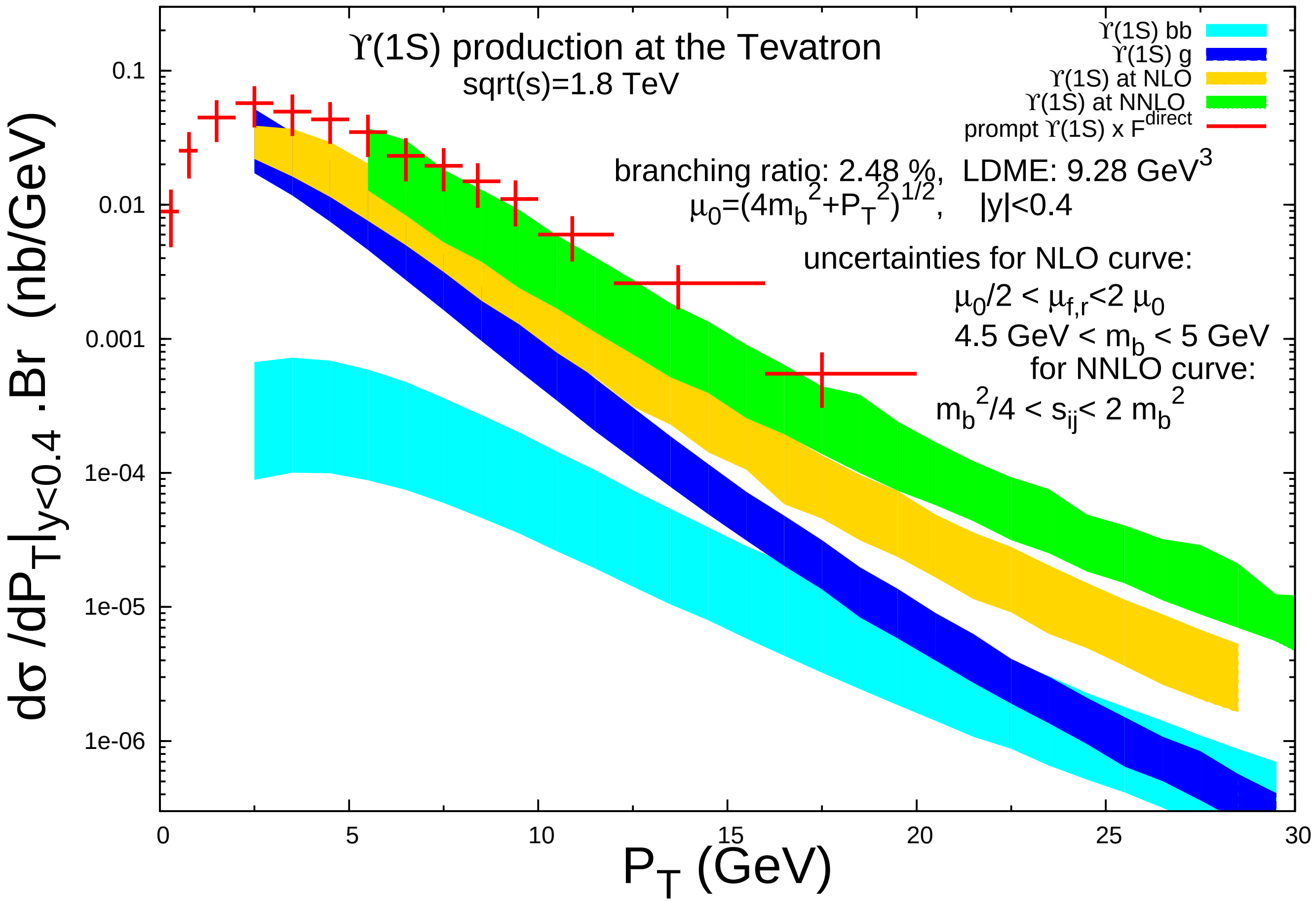
- subprocesses:

dg_uuxdbbx3S11	gd_uuxdbbx3S11	gu_uuuxbbx3S11	ug_uddxbbx3S11	uux_uuxgbbx3S11
uxu_ddxgbbx3S11	du_udgbbx3S11	gdx_uuxdxbbx3S11	gux_uuxuxbbx3S11	ug_uggbbx3S11
uxd_uxdgbbx3S11	uxu_gggbbx3S11	dux_uxdgbbx3S11	gg_gggbbx3S11	gux_uxddxbbx3S11
ug_uuuxbbx3S11	uxdx_uxdxgbbx3S11	uxu_uuxgbbx3S11	dxg_uuxdxbbx3S11	gg_uuxgbbx3S11
gux_uxggbbx3S11	uu_uugbbx3S11	uxg_uuxuxbbx3S11	uxux_uxuxgbbx3S11	dxu_udxgbbx3S11
gu_uddxbbx3S11	ud_udgbbx3S11	uux_ddxgbbx3S11	uxg_uxddxbbx3S11	dxux_uxdxgbbx3S11
gu_uggbbx3S11	udx_udxgbbx3S11	uux_gggbbx3S11	uxg_uxggbbx3S11	

$\approx$ 2000 Feynman diagrams before projection



Υ + 3 jets at the Tevatron



Conclusion & Perspectives

- MadOnia is an amplitude generator for quarkonium production within NRQCD which is:
  - universal (new models can be defined)
  - user-friendly
  - flexible
- Examples of application:
  - $\gamma\gamma \to J/\psi + X$ at Lep II
  - $e^+e^- \to \eta_c + X$ at B factories
  - $p\bar{p} \to J/\psi + c\bar{c}$ at the Tevatron
  - $p\bar{p} \to \Upsilon + 3$ jets at the Tevatron
- work in progress: event generator with interfaces to Pythia and Herwig