MadWeight

a new tool for event-reweighting and mass determination

Olivier Mattelaer

University of Louvain

Pierre Artoisenet: UCL-CP3

Fabio Maltoni: UCL-CP3

Vincent Lemaître: UCL-CP3

Motivation and plan

- motivation : how to determine new physics at hadron colliders
  - excess of events
  - topology of the process
  - precise measurements (matrix element method)
- plan
  - weighting experimental events
  - how to evaluate the weights
  - MadWeight : automatic computation of the weights
  - check and capabilities

Weighting experimental events

- matrix element method : weighting events

$$P(\boldsymbol{x}, \alpha) = \qquad |M_\alpha|^2(\boldsymbol{x})$$

where

- $|M_\alpha|^2$ is the squared matrix element

Weighting experimental events

- matrix element method : weighting events

$$P(\boldsymbol{x}, \alpha) = \qquad |M_\alpha|^2(\boldsymbol{y}) \qquad W(\boldsymbol{x}, \boldsymbol{y})$$

where

- $|M_\alpha|^2$ is the squared matrix element
- $W(\boldsymbol{x}, \boldsymbol{y})$ is the resolution function

Weighting experimental events

- matrix element method : weighting events

$$P(\boldsymbol{x}, \alpha) = \frac{1}{\sigma} \int d\phi(\boldsymbol{y}) |M_\alpha|^2(\boldsymbol{y}) \qquad W(\boldsymbol{x}, \boldsymbol{y})$$

where

- $|M_\alpha|^2$ is the squared matrix element
- $W(\boldsymbol{x}, \boldsymbol{y})$ is the resolution function
- $d\phi(\boldsymbol{y})$ is the partonic phase-space measure

Weighting experimental events

- matrix element method : weighting events

$$P(\boldsymbol{x},\alpha) = \frac{1}{\sigma}\int d\phi(\boldsymbol{y})|M_\alpha|^2(\boldsymbol{y})dq_1dq_2f_1(q_1)f_2(q_2)W(\boldsymbol{x},\boldsymbol{y})$$

where

- $|M_\alpha|^2$ is the squared matrix element
- $W(\boldsymbol{x},\boldsymbol{y})$ is the resolution function
- $d\phi(\boldsymbol{y})$ is the partonic phase-space measure
- $f_1(q_1), f_2(q_2)$ are the Parton Distribution Functions

Weighting experimental events

- combine the weights into a likelihood

$$L(\alpha) = \prod_{i=1}^{N} P(\boldsymbol{x}_i;\alpha)$$

Weighting experimental events

- combine the weights into a likelihood

$$L(\alpha) = e^{-N \int P(\boldsymbol{x},\alpha) d\boldsymbol{x}} \prod_{i=1}^{N} P(\boldsymbol{x}_i; \alpha)$$

Weighting experimental events

- combine the weights into a likelihood

$$L(\alpha) = e^{-N \int P(\boldsymbol{x},\alpha) d\boldsymbol{x}} \prod_{i=1}^{N} P(\boldsymbol{x}_i;\alpha)$$

the best estimation of α is the one that maximizes L

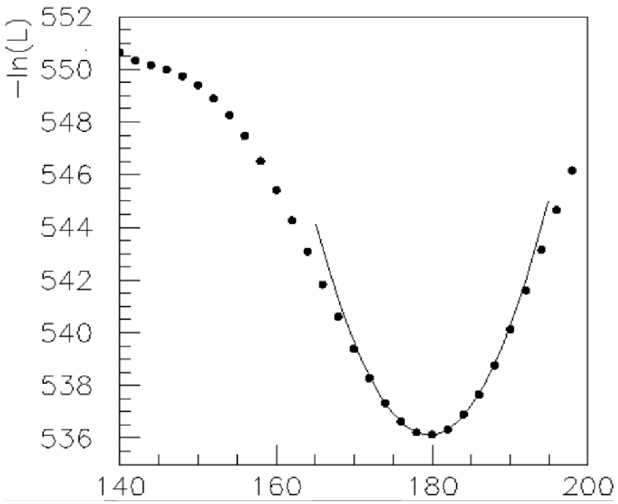
Weighting experimental events

- combine the weights into a likelihood

$$L(\alpha) = e^{-N\int P(\boldsymbol{x},\alpha)d\boldsymbol{x}} \prod_{i=1}^{N} P(\boldsymbol{x}_i;\alpha)$$

the best estimation of α is the one that maximizes L

example : top-quark mass measurement from $t\bar{t} \rightarrow l^+X$ sample at DØ



- 72 events
- $M_{top} = 180.1 \pm 3.6_{stat} \pm 4.0_{sys}$ Gev
- J. Estrada : Phd dissertation, University of Rochester (2001)

Weighting experimental events

- advantages :
  - it takes into account the full matrix element (in particular spin-correlation effects)
  - resolution of the detector is included
  - it is particularly usefull for processes with missing particles

- drawbacks :
  - the evaluation of the weight is time-consuming compare to other methods
  - what are the systematics errors ?

How to evaluate the weight ?

$$P(\boldsymbol{x},\alpha) = \frac{1}{\sigma}\int d\phi(\boldsymbol{y}) dq_1 dq_2 \underbrace{f_1(q_1) f_2(q_2)}_{experimental} |M_\alpha|^2(\boldsymbol{y}) \underbrace{W(\boldsymbol{x},\boldsymbol{y})}_{experimental}$$

- transfer functions
  - fitted by Monte-Carlo
  - no restriction in the code
- Parton Distribution Functions
  - extrapolated from data
  - different libraries

How to evaluate the weight ?

$$P(\boldsymbol{x},\alpha) = \frac{1}{\sigma}\int d\phi(\boldsymbol{y}) dq_1 dq_2 \underbrace{f_1(q_1) f_2(q_2)}_{experimental} \underbrace{|M_\alpha|^2(\boldsymbol{y})}_{MadGraph} \underbrace{W(\boldsymbol{x},\boldsymbol{y})}_{Monte\,Carlo}$$

How to evaluate the weight ?

$$P(\boldsymbol{x},\alpha) = \underbrace{\frac{1}{\sigma}\int d\phi(\boldsymbol{y}) dq_1 dq_2}_{MadWeight} \underbrace{f_1(q_1) f_2(q_2)}_{experimental} \underbrace{|M_\alpha|^2(\boldsymbol{y})}_{MadGraph} \underbrace{W(\boldsymbol{x},\boldsymbol{y})}_{Monte\,Carlo}$$

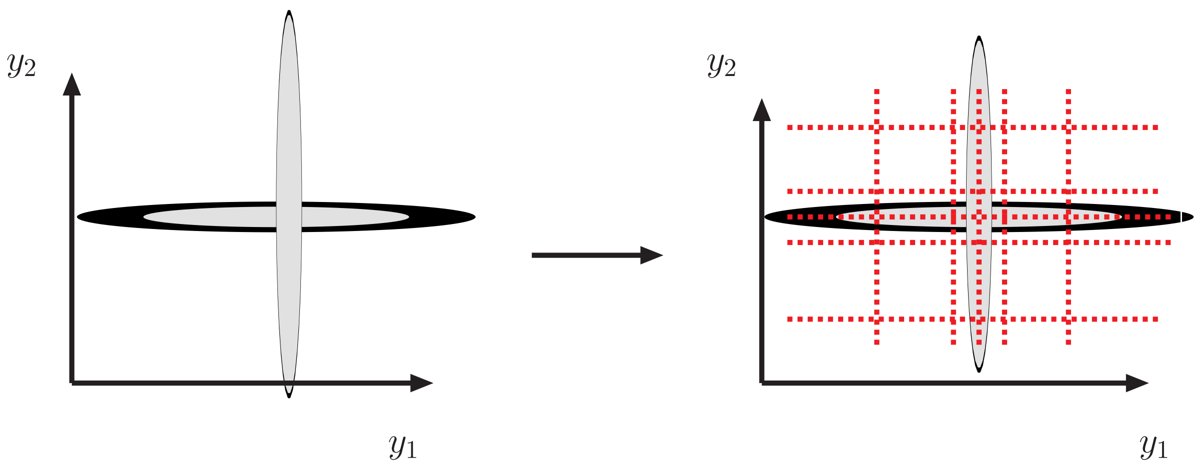
numerical integration : very difficult due to the *un-aligned peaks* in the integrand

$$|M_\alpha(\boldsymbol{y})|^2 = |\widetilde{M_\alpha(\boldsymbol{y})}|^2 \prod_j BW(m_j^*, m_{j,p}, \Gamma_j)$$

$$W(\boldsymbol{x},\boldsymbol{y}) \approx \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(x_i-y_i)^2}{2\sigma_i^2}}$$

Monte Carlo technics

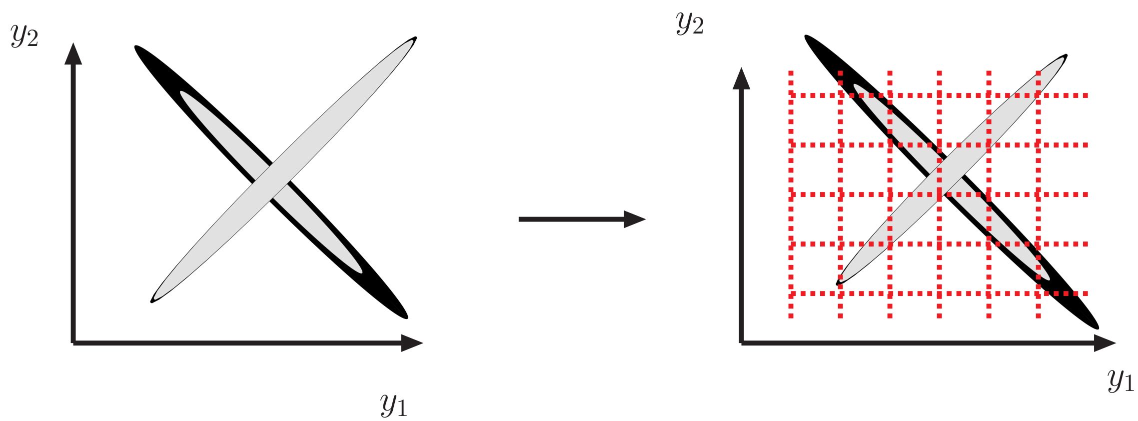
- efficiency of an adaptative MC integration :
  - case 1 : any peak is aligned along a single direction of the P-S parametrization



$\longrightarrow$ the adaptative Monte-Carlo P-S integration is very efficient

Monte Carlo technics

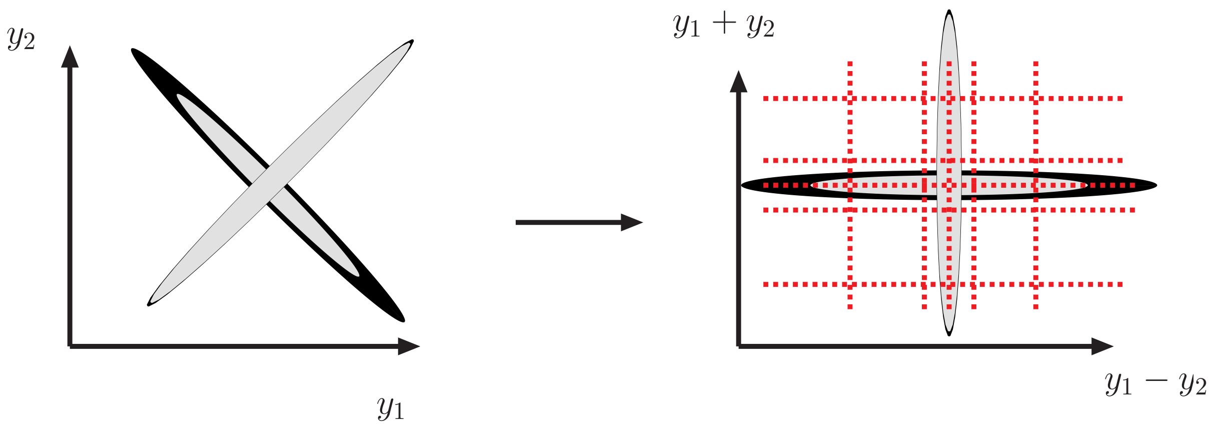
- efficiency of an adaptative MC integration :
  - case 2 : some peaks are not aligned along a single direction of the P-S parametrization



$\longrightarrow$ the adaptative Monte-Carlo P-S integration converges slowly

Monte Carlo technics

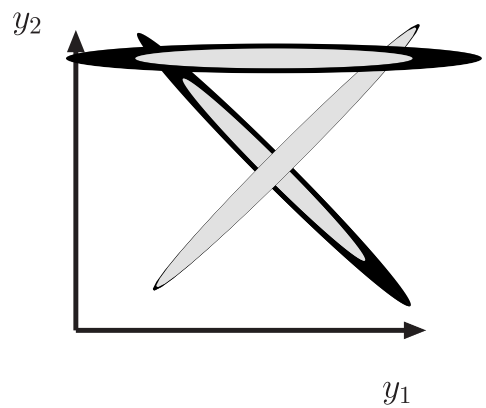
- efficiency of an adaptative MC integration :
  - possible solution : perform a change of variables



$\longrightarrow$ the adaptative Monte-Carlo P-S integration is very efficient

Monte Carlo technics

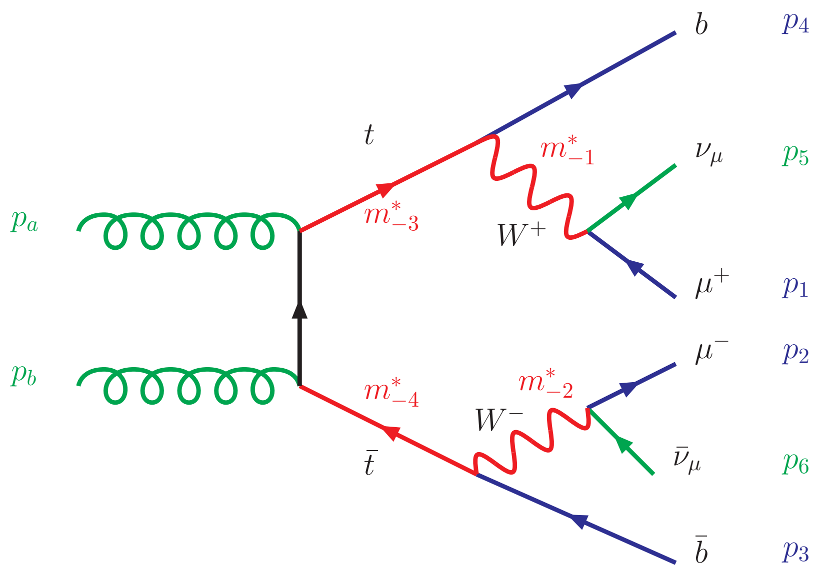
- efficiency of an adaptative MC integration :
  - case 3 : there are more peaks than phase-space variables



$\longrightarrow$ the efficiency depends of the shape, relative position, ... of the peaks

Phase-space generation

- ideal case : $t\bar{t}$



- peaks in $|M_\alpha(\boldsymbol{y})|^2$ controlled by $m^*_{-1}, \ldots, m^*_{-4}$ (4 variables)
- peaks in $W(\boldsymbol{x}, \boldsymbol{y})$ controlled by $\theta_i, \phi_i, |p_i|^2 \quad i \in \{1, 2, 3, 4\}$ (12 variables)
- $dim[d\phi] = 16, \rightarrow$ each peak can be aligned along a single variable of integration

Phase-space generation

which parametrization do we use ?

- natural parametrization

$$d\phi = \prod_{i=1}^{4} \frac{d^3p_i}{(2\pi)^3 2E_i} \prod_{i=5}^{6} \frac{d^3p_i}{(2\pi)^3 2E_i} dx_1 dx_2 \delta^4 \left(p_a + p_b - \sum_j p_j \right)$$

where all the peaks in $W(\boldsymbol{x}, \boldsymbol{y})$ are aligned

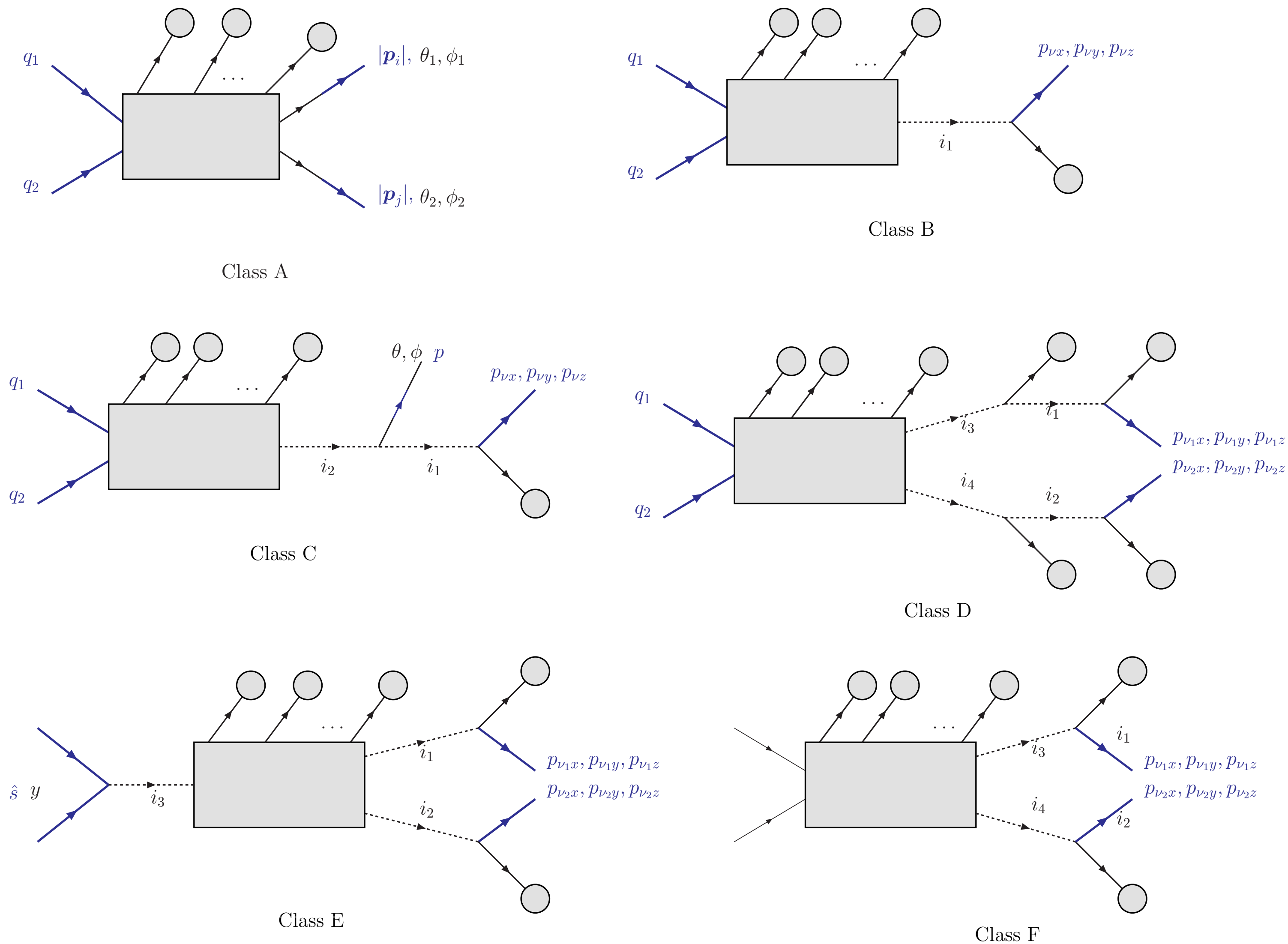
- we apply local changes of variables to reach the parametrization

$$d\phi = \prod_{i=1}^{4} d\theta_i d\phi_i d|\boldsymbol{p}_i| \prod_{j=1}^{4} dm^{*2}_{-j} \times J$$

where each Breit-Wigner distribution is also aligned

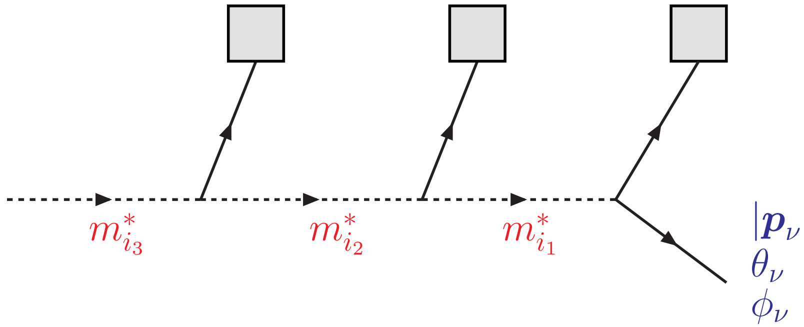
MadWeight : changes of variables

- changes of variables to restore energy momentum conservation

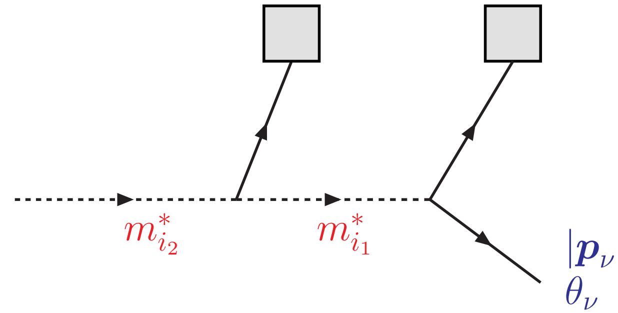


MadWeight : changes of variables

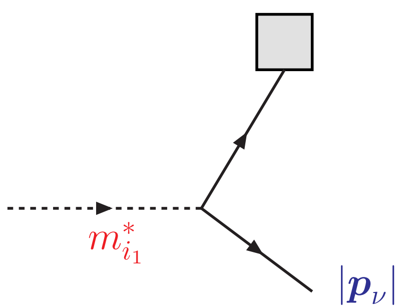
- auxiliary changes of variables :



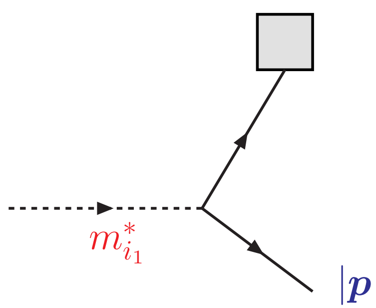
block A



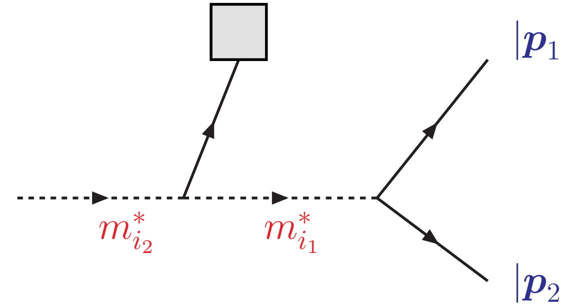
block B



block C



block D



block E

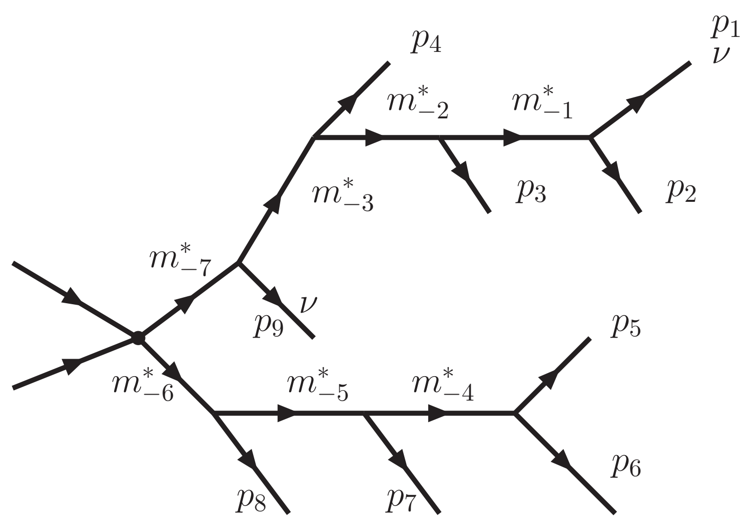
MadWeight code

in general in MadWeight algorithm,

- the phase-space is splitted into *blocks*, each of them is associated to a specific local change of variables
- we only consider analytic changes of variables
- we always keep the visible angles in the phase-space parametrization (they are assumed to be well reconstructed).
- the decomposition into blocks depends on the topology, on the widths of the Breit-Wigner distributions, and on the shape of the resolution function.

Decay chain example

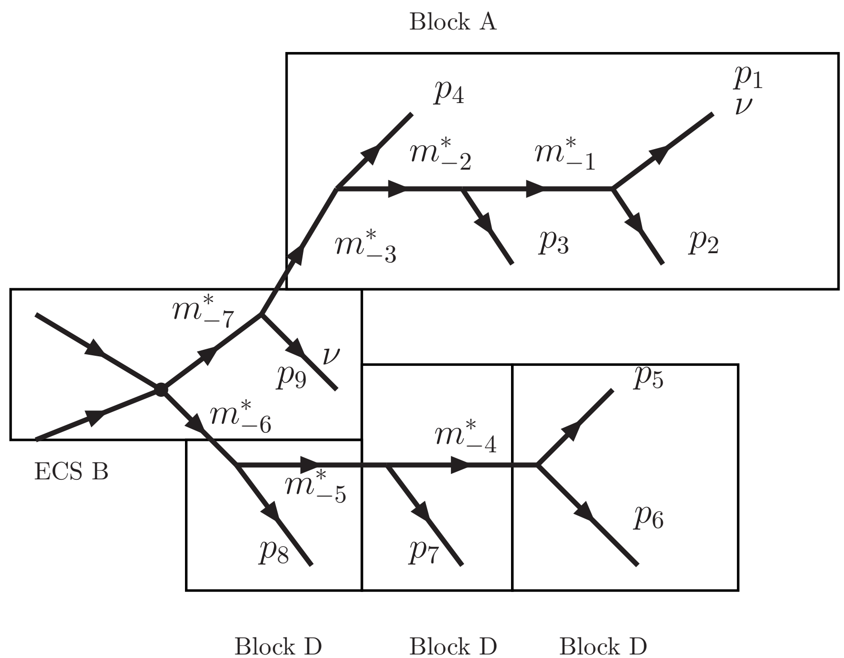
let us consider a specific example of decay chain :



- peaks in $|M_\alpha(\boldsymbol{y})|^2$ controlled by $m^*_{-1}, \ldots, m^*_{-7}$ (7 variables)
- peaks in $W(\boldsymbol{x}, \boldsymbol{y})$ controlled by
  $\theta_i, \phi_i, |p_i|^2 \quad i \in \{2, 3, 4, 5, 6, 7, 8\}$ (21 variables)
- $dim[d\phi] = 25$, $\rightarrow$ some peaks must be left misaligned

Decay chain example

- each local change of variables is performed successively



final parametrization :

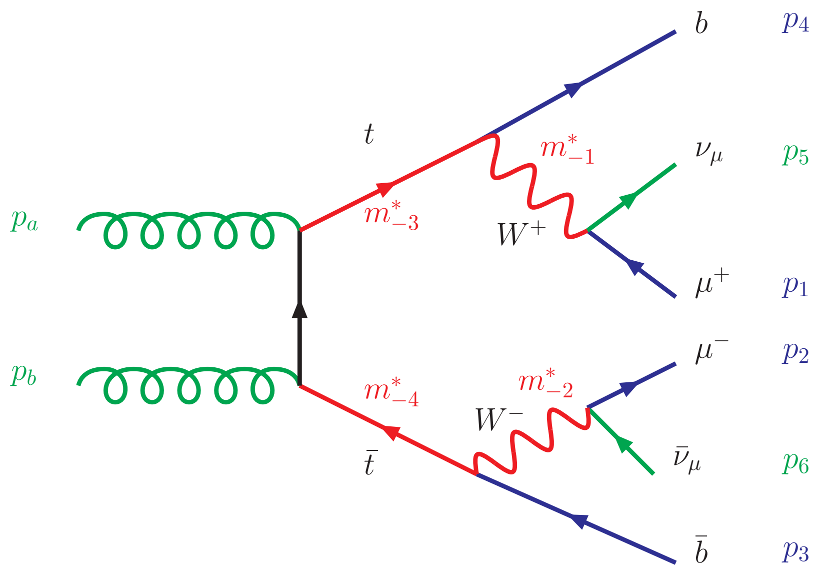
$$d\phi = d|\boldsymbol{p}_2|d|\boldsymbol{p}_3|d|\boldsymbol{p}_4|d|\boldsymbol{p}_6| \prod_{i=2}^{8} d\theta_i d\phi_i \prod_{j=1}^{7} dm^{*2}_{-j} \times J$$

Check and capabilities

- can we find back the pole mass from MC sample ?

Check and capabilities

- can we find back the pole mass from MC sample ?
- search for the top-quark mass

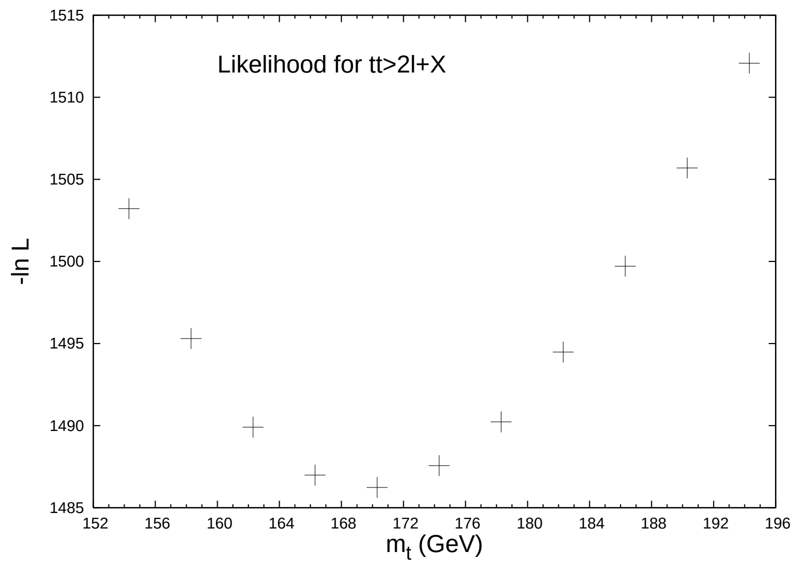


Check and capabilities

- can we find back the pole mass from MC sample ?
- search for the top-quark mass
- 30 Monte Carlo events (MadGraph/Pythia/PGS)

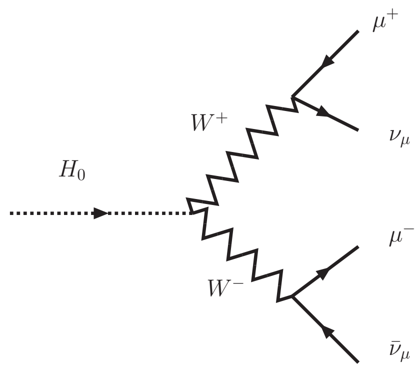
Check and capabilities

- can we find back the pole mass from MC sample ?
- search for the top-quark mass
- 30 Monte Carlo events (MadGraph/Pythia/PGS)
- input : $m_{top} = 174.3$ Gev, output : $m_{top} = 170.3 \pm 4.0$ Gev



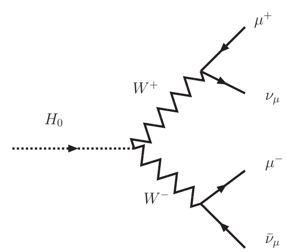
Check of capabilities

- search for the Higgs mass

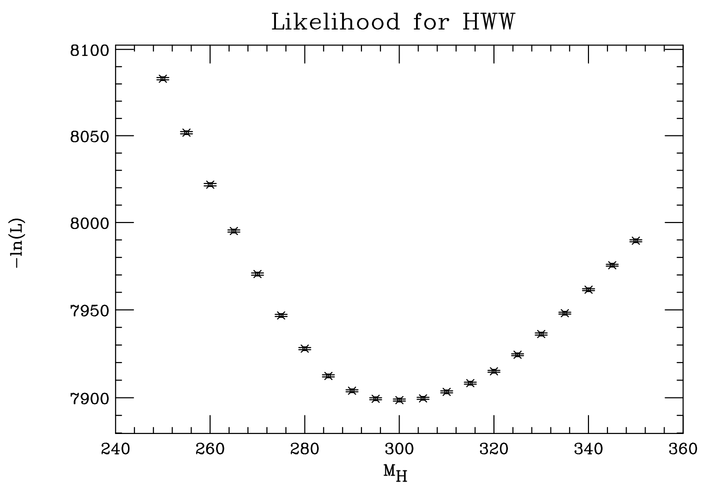


Check of capabilities

- search for the Higgs mass



- 500 Monte Carlo events (MadGraph/Pythia/PGS)
- input : $m_{Higgs} = 300$ Gev, output : $m_{Higgs} = 300 \pm 5$ Gev



Conclusion

- the Matrix Element method provides the best discriminator on an event-by-event basis
- both theoritical ($|M|^2$) and experimental ($\boldsymbol{x}$, $W(\boldsymbol{x}, \boldsymbol{y})$) information is used
- the computation of the weights requires a specific phase space generator : MadWeight
  - it finds the best phase-space parametrisation(s)
  - all changes of variables are local and analytical
  - our code works for all decay chains
- first release candidate available on request : olivier.mattelaer@uclouvain.be & pierre.artoisenet@uclouvain.be