FeynRules

New models phenomenology made easy

Claude Duhr

March 11, 2008
MC4BSM

- Why yet another tool..?
- FeynRules
- Example:
  - How to add a new sector to the SM
- Conclusion

Why yet another tool..?

- In general, a new model is given by a Lagrangian, containing all the particles and their mutual interactions.
- At some point, one would like to compare the model with experiment.

→ Needs in general some hard calculations:

- cross-sections
- decay rates
- radiative corrections

Why yet another tool..?

- Fortunately, several tools are available to do the calculations
  - MC generators (MadGraph, CalcHep, CompHEP, AMEGIC++)
  - FeynArts,...

New model → **Existing tools**

(Lagrangian, new particles,...)

(Programming language, files containing the new particles and interactions,...)

FeynRules

- Mathematica® based package that calculates Feynman rules from a Lagrangian.
- No special requirements on the form of the Lagrangian.
- Particle types supported so far: scalars, fermions (Dirac and Majorana), vectors, spin-2, ghosts.

FeynRules

- The FR model file contains all the information about the model:
  - Particles & fields
  - Parameters (masses, coupling constants,...)
  - mixing matrices
  - etc.

- The syntax of the FR model-files is an extension of syntax used in FeynArts.
- Feynman rules are calculated by Mathematica using the information from the model-file and the Lagrangian.
- The vertices can be exported into a TeX-file.

FeynRules

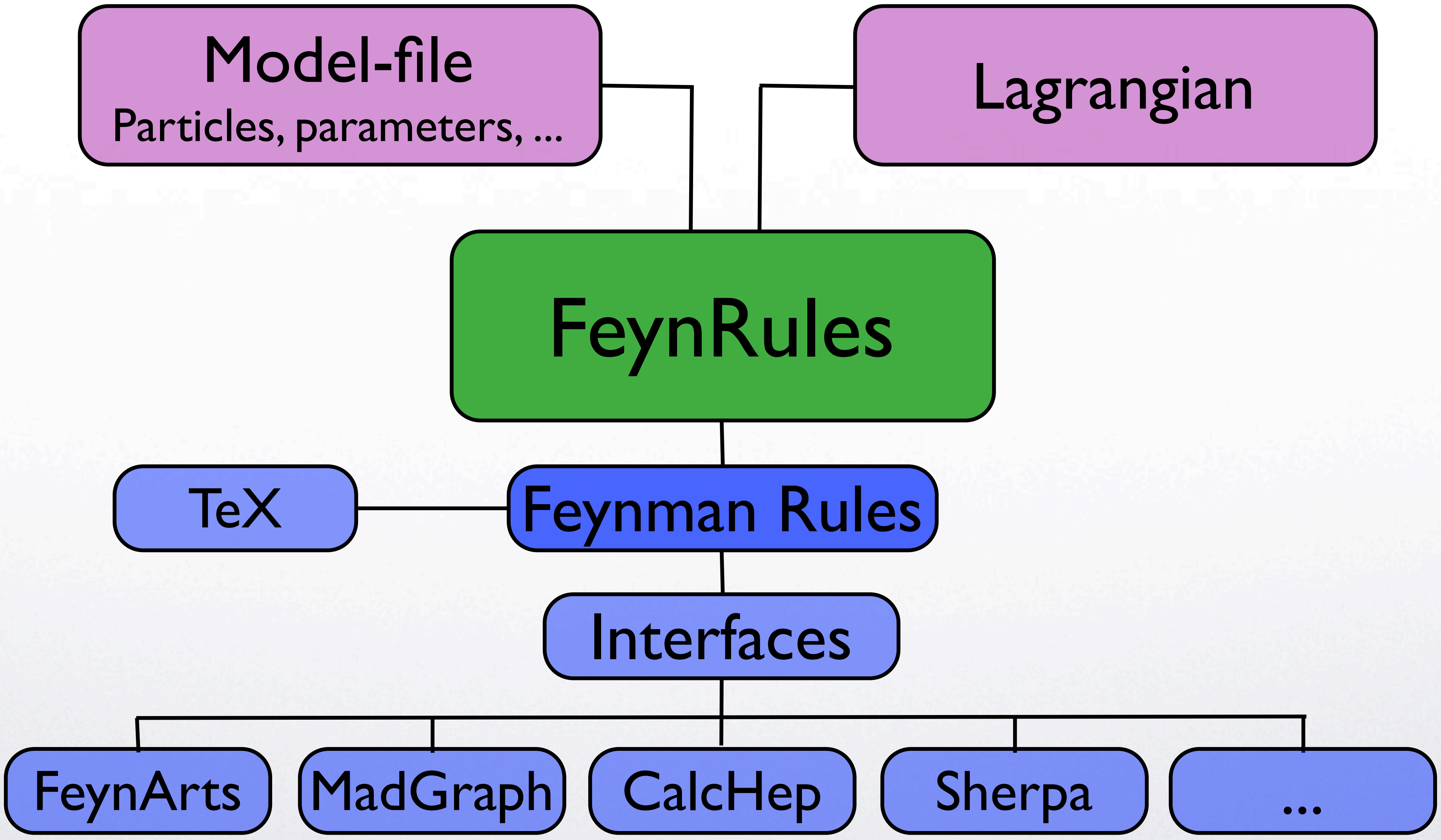
- The informations given in the model-file, together with the vertices obtained by FR, is generic enough to allow for an interface to other existing tools.

- FR creates all files needed to run the new model just by knowing the FR model-file and the Lagrangian.

- Interfaces available so far

  - FeynArts
  - MadGraph/MadEvent (CD, M. Herquet)
  - CalcHep/CompHep (CD, N. Christensen)
  - Sherpa (CD, S. Schumann)

FeynRules


Model-file
Particles, parameters, ...
Lagrangian
FeynRules
TeX
Feynman Rules
Interfaces
FeynArts
MadGraph
CalcHep
Sherpa
...

Validation

- Standard model: 29 key-processes tested against the stock version
    - FeynArts
    - MadGraph
    - CalcHep/CompHep: both in unitary and Feynman gauge
    - Sherpa: Validation procedure in progress
- 3-site model: 222 key-processes tested in CalcHep/CompHep

Validation

- Standard model: 29 key-processes tested against the stock version

Process	CalcHEP Stock	CalcHEP Feynman	CalcHEP Unitary	CompHEP Feynman
gg->gg	118910.	118910.	118910.	118910.
$u\bar{u}$->gg	237.87	237.87	237.87	237.83
$t\bar{t}$->gg	75.975	75.975	75.975	75.974
$e^+e^-\to\mu^+\mu^-$	0.45028	0.45028	0.45028	0.45021
$t\bar{t}\to u\bar{u}$	22.113	22.113	22.113	22.114
$u\bar{u}\to s\bar{s}$	11.6	11.6	11.6	11.599
$t\bar{t}\to W^+W^-$	19.91	19.91	19.91	19.91
$t\bar{t}$->ZZ	1.3017	1.3017	1.3017	1.3017
ZZ->ZZ	2.3797	2.3797	2.3797	2.3797
W^+W^-->ZZ	270.05	270.05	270.05	269.86
$W^+W^-\to W^+W^-$	1310.	1310.	1310.	1310.1
hh->hh	2.146	2.146	2.146	2.1456
hh->ZZ	65.714	65.714	65.714	65.714
$hh\to W^+W^-$	100.36	100.36	100.36	100.36

...

Validation

- 3-site model: 222 key-processes tested in CalcHep/CompHep

	Lanhep CalcHEP Feynman	Lanhep CalcHEP Unitary	FeynRules CalcHEP Feynman	FeynRules CalcHEP Unitary	FeynRules CompHEP Feynman
$u\bar{u}$->gg	170.5	170.5	170.5	170.5	170.49
u'$\overline{u'}$->gg	0.098763	0.098763	0.098763	0.098763	0.098761
$t\bar{t}$->γZ	1.1233	1.1233	1.1233	1.1233	1.1233
t'$\bar{t}$->γZ	0.033204	0.033204	0.033204	0.033204	0.033204
t'$\overline{t'}$->Z'Z'	1.887	1.887	1.887	1.887	1.887
$t\bar{b}$->ZW^{+}'	1.5603	1.5603	1.5603	1.5603	1.5604
$e\bar{e}$->e'$\bar{e}$	0.093127	0.093127	0.093127	0.093127	0.093127
e'$\bar{e}$->u'$\bar{u}$	2.3603	2.3603	2.3603	2.3603	2.3603
e$\overline{\nu_e}$->μ'$\overline{\nu_\mu{}'}$	0.0005618	0.0005618	0.0005618	0.0005618	0.00056181
e'$\overline{\nu_e{}'}$->d'$\overline{u'}$	2.5761	2.5761	2.5761	2.5761	2.5762
gg->gg	114310.	114310.	114310.	114310.	114310.
ZZ->Z'Z'	0	0	0	0	0
W^{+}W^{-}'->γZ	8.329	8.329	8.329	8.329	8.3288

...

Example: The Hill model

SM SCALAR AND EXTRA SINGLET(S)

J. J. VAN DER BIJ
Institut für Physik, Albert-Ludwigs Universität Freiburg, H. Herderstr. 3, 79104 Freiburg i.B., Deutschland

[arXiv:0707.0359]

$$L = -\frac{1}{2}(D_\mu \Phi)^\dagger (D_\mu \Phi) \; - \frac{\lambda_0}{8}(\Phi^\dagger \Phi - f_0^2)^2$$

$$- \frac{1}{2}(\partial_\mu H)^2 \; - \frac{\lambda_1}{8}(2 f_1 H - \Phi^\dagger \Phi)^2$$

Example: The Hill model

SM SCALAR AND EXTRA SINGLET(S)

J. J. VAN DER BIJ

Institut für Physik, Albert-Ludwigs Universität Freiburg, H. Herderstr. 3,

To include this new sector in FeynArts and Monte Carlo's, you just need to add a few lines to the SM model file.

0359]

$$L = -\frac{\;}{2}(D_\mu\Phi)^\dagger(D_\mu\Phi) - \frac{\;}{8}(\Phi^\dagger\Phi - f_0^2)^2$$

$$-\frac{1}{2}(\partial_\mu H)^2 - \frac{\lambda_1}{8}(2f_1 H - \Phi^\dagger\Phi)^2$$

The Hill model

$$L = -\frac{1}{2}(D_\mu \Phi)^\dagger (D_\mu \Phi) \;\; - \frac{\lambda_0}{8}(\Phi^\dagger \Phi - f_0^2)^2$$ SM Higgs

$$-\frac{1}{2}(\partial_\mu H)^2 \;\; - \frac{\lambda_1}{8}(2 f_1 H - \Phi^\dagger \Phi)^2$$ Hill field

---

$$\begin{aligned} f_0 &= 246 GeV \\ f_1 &= 600 GeV \\ \lambda_0 &= 0,1 \\ \lambda_1 &= 0,25 \end{aligned}$$

The Hill model

$$L = -\frac{1}{2}(D_\mu\Phi)^\dagger(D_\mu\Phi) \;\; - \frac{\lambda_0}{8}(\Phi^\dagger\Phi - f_0^2)^2$$ SM Higgs

$$- \frac{1}{2}(\partial_\mu H)^2 \;\; - \frac{\lambda_1}{8}(2f_1 H - \Phi^\dagger\Phi)^2$$ Hill field

---

- If the SM Higgs acquires a vev, then the Hill field acquires a vev too.

$$\langle\Phi\rangle \;\; = \;\; \begin{pmatrix} 0 \\ f_0 \end{pmatrix} \qquad\qquad \langle H\rangle \;\; = \;\; \frac{f_0^2}{2f_1}$$

The Hill model

$$L = -\frac{1}{2}(D_\mu\Phi)^\dagger(D_\mu\Phi) \;\; - \frac{\lambda_0}{8}(\Phi^\dagger\Phi - f_0^2)^2 \qquad \text{SM Higgs}$$

$$- \frac{1}{2}(\partial_\mu H)^2 \;\; - \frac{\lambda_1}{8}(2f_1 H - \Phi^\dagger\Phi)^2 \qquad \text{Hill field}$$

- The mass matrix of the two scalars is

$$M_H = \begin{pmatrix} (\lambda_0+\lambda_1)f_0^2 & -\lambda_1 f_0 f_1 \\ -\lambda_1 f_0 f_1 & \lambda_1 f_1^2 \end{pmatrix}$$

Just use Mathematica to diagonalize the matrix...

The Hill model

- Mass eigenvalues:

$$m_1 = 78,5 GeV$$
$$m_2 = 326 GeV$$

- Mass eigenstates:

$$h_1 = h\cos\alpha + H\sin\alpha$$
$$h_2 = -h\sin\alpha + H\cos\alpha$$

$$\Phi = \begin{pmatrix} 0 \\ f_0 + h \end{pmatrix} \qquad \alpha = 0,60321$$

The Hill model

- Consequences:

  - All SM Higgs (Yukawa, gauge) couplings get doubled.

$$t\bar{t}h \quad \rightarrow \quad t\bar{t}h_1, \qquad t\bar{t}h_2$$

  - All SM Higgs couplings get modified (mixing angle).

$$\frac{y_t}{\sqrt{2}} \quad \rightarrow \quad \frac{y_t \sin\alpha}{\sqrt{2}}, \quad \frac{y_t \cos\alpha}{\sqrt{2}}$$

  - New self-couplings among h1 and h2.

$$h_1 h_1 h_2 : \qquad -\cos\alpha\lambda_1(f_1\cos^2\alpha + 3f_0\sin\alpha\cos\alpha - 2f_1\sin^2\alpha)$$

The Hill model

- Consequences:
  - All SM Higgs (Yukawa, gauge) couplings get doubled.

$$t\bar{t}h \quad \to \quad t\bar{t}h_1, \qquad t\bar{t}h_2$$

  - All SM Higgs couplings get modified (mixing angle).

Lots of things need to be changed in the SM implementation to get the Hill model!

$\longrightarrow$ Let FeynRules do the job!

$$h_1 h_1 h_2 : \qquad -\cos\alpha\lambda_1(f_1\cos^2\alpha + 3f_0\sin\alpha\cos\alpha - 2f_1\sin^2\alpha)$$

Model building with FeynRules

- Step 1: Add all the parameters of the new sector to the model file:

```
f1 == {Value -> 600,
      InteractionOrder -> {QED, -1}},

l1 == {Value -> 0.25,
       InteractionOrder -> {QED, 2}},

ca == {Value -> 0.896242},
```

Cosine of the mixing angle

$$L = -\frac{1}{2}(D_\mu \Phi)^\dagger (D_\mu \Phi) - \frac{\lambda_0}{8}(\Phi^\dagger \Phi - f_0^2)^2 - \frac{1}{2}(\partial_\mu H)^2 - \frac{\lambda_1}{8}(2 f_1 H - \Phi^\dagger \Phi)^2$$

Model building with FeynRules

- Step 1: Add all the parameters of the new sector to the model file:

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ca == {Value -> 0.896242},
```

ca: Cosine of the mixing angle

$$L = -\frac{1}{2}(D_\mu\Phi)^\dagger(D_\mu\Phi)$$

$$-\frac{\lambda_1}{8}(2f_1 H - \Phi^\dagger\Phi)^2$$

Additional information needed e.g. for the MC integration.

Model building with FeynRules

- Step II: Add all the particles of the new sector to the model file:

Mass eigenstate

```
S[1] == {
  ClassName -> h1,
  SelfConjugate -> True,
  Mass -> {Mh1, 78.5}},

S[2] == {
  ClassName -> H,
  SelfConjugate -> True,
  Unphysical -> True,
  Definitions -> {H -> sa h1 +ca h2}}
```

Mixing

$$L = -\frac{1}{2}(D_\mu\Phi)^\dagger(D_\mu\Phi) - \frac{\lambda_0}{8}(\Phi^\dagger\Phi - f_0^2)^2 - \frac{1}{2}(\partial_\mu H)^2 - \frac{\lambda_1}{8}(2f_1 H - \Phi^\dagger\Phi)^2$$

Model building with FeynRules

- Step III: The lagrangian describing the new sector (Unitary gauge)

```
Φ = {0, h + f0}

LHill = -1 / 2 del[H, mu] ^2 -
  l1 / 8 (2 f1 H - HC[Φ].Φ) ^2
```

$$-\frac{1}{2}\,\partial_{\mathrm{mu}}(H)^2 - \frac{1}{8}\,\mathrm{l1}\left(2\,\mathrm{f1}\,H - \Phi^{\dagger}.\Phi\right)^2$$

$$L = -\frac{1}{2}(D_\mu\Phi)^\dagger(D_\mu\Phi)$$

$$-\frac{\lambda_0}{8}(\Phi^\dagger\Phi - f_0^2)^2$$

$$-\frac{1}{2}(\partial_\mu H)^2$$

$$-\frac{\lambda_1}{8}(2f_1 H - \Phi^\dagger\Phi)^2$$

Phenomenology with FeynRules

- Now we are ready to do some phenomenology...

- Let's consider the following process in the framework of Hill model

$$e^+e^- \to Z b\tilde{b} \quad \to \quad \mu^+\mu^- \, b\tilde{b}$$

  At a CoM energy of 500GeV.

- Let's first have a look at the one-loop corrections.

  → Use FeynArts

Phenomenology with FeynRules

- The results obtained by FeynRules can be easily exported to FeynArts:

```
WriteFeynArtsOutput["HillModel.mod",{LSM + LHill}, FlavorExpand → SU2W]
```

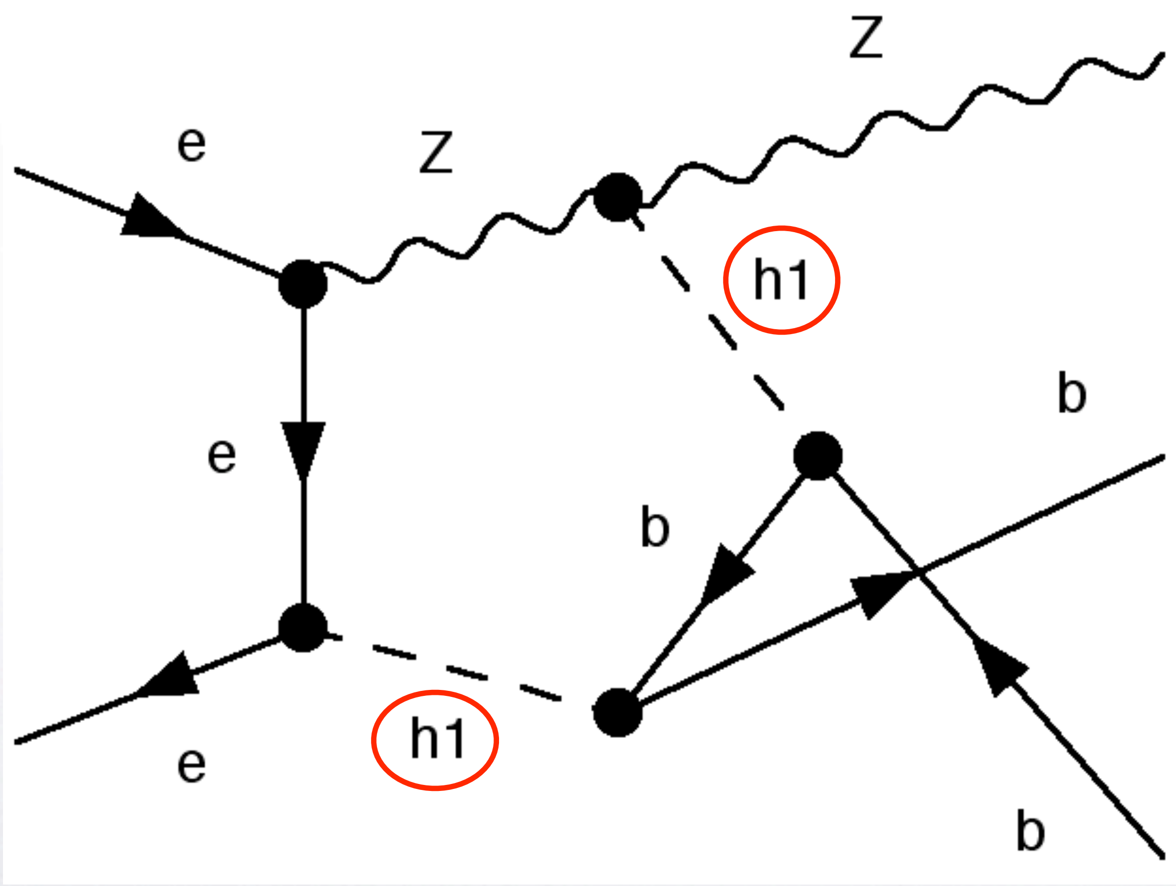
– – – FeynRules interface to FeynArts – – –

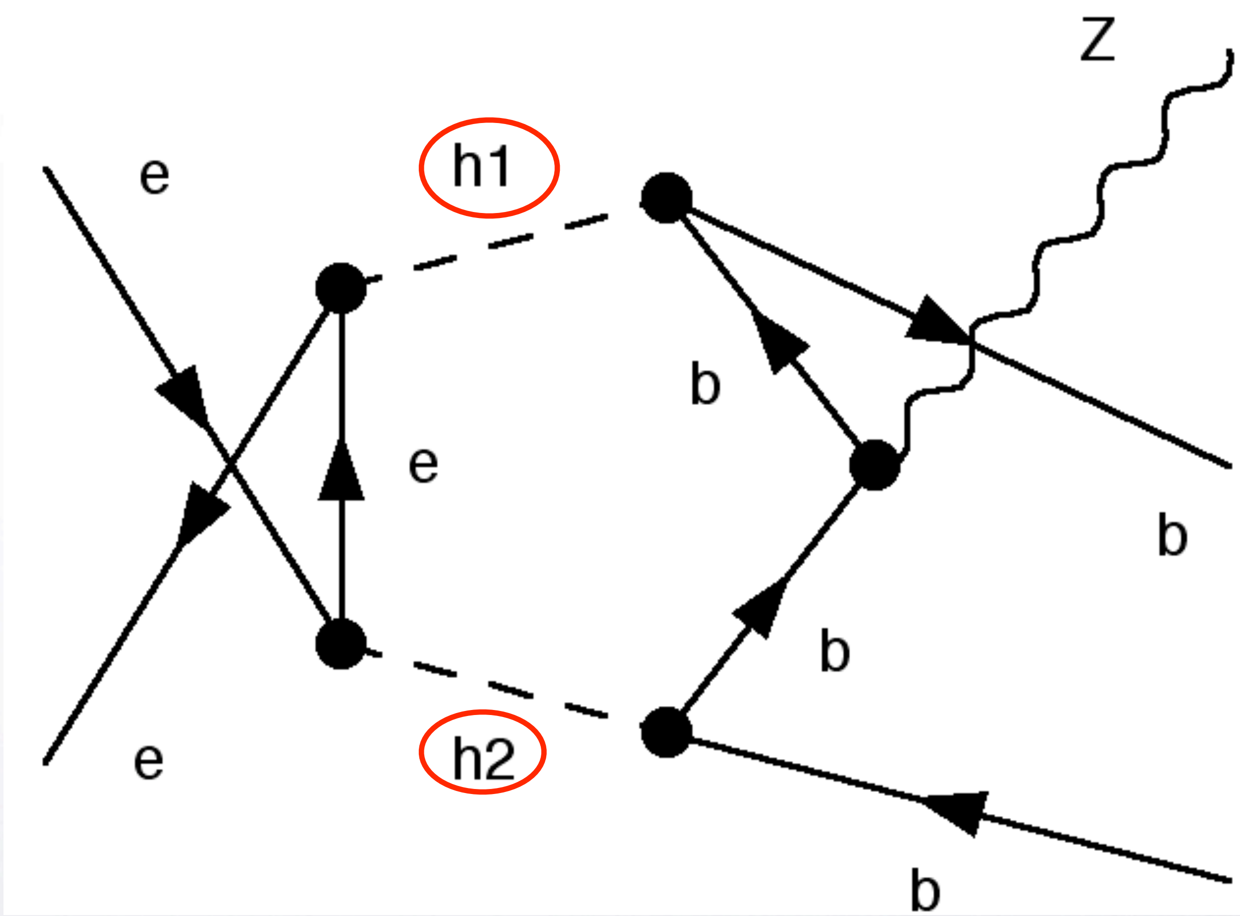
C. Duhr, 2007

- This produces a FeynArts model-file which can be read by FeynArts.

```
topo = CreateTopologies[1, 2 → 3,
   ExcludeTopologies → Internal];
Amp = InsertFields[topo,
         {F[2, {1}], -F[2, {1}]} → {V[2], F[4, {3}], -F[4, {3}]},
         Model → HillModel];
```

Phenomenology with FeynRules





Phenomenology with FeynRules

- The results obtained by FeynRules can be easily exported toMC generators:

```
WriteMGOutput[LSM + LHill]
```

– – – FeynRules interface to MadGraph – – –

C. Duhr, M. Herquet

- This produces all the files needed to implement the Hill model into an MC. Let’s have a look at our process!

Phenomenology with FeynRules

- The results obtained by FeynRules can be easily exported toMC generators:

```
WriteCHOutput[LSM + LHill]
```

– – – FeynRules interface to CalcHep – – –

Authors: N. Christensen, C. Duhr

- This produces all the files needed to implement the Hill model into an MC. Let's have a look at our process!

Phenomenology with FeynRules

- The results obtained by FeynRules can be easily exported toMC generators:

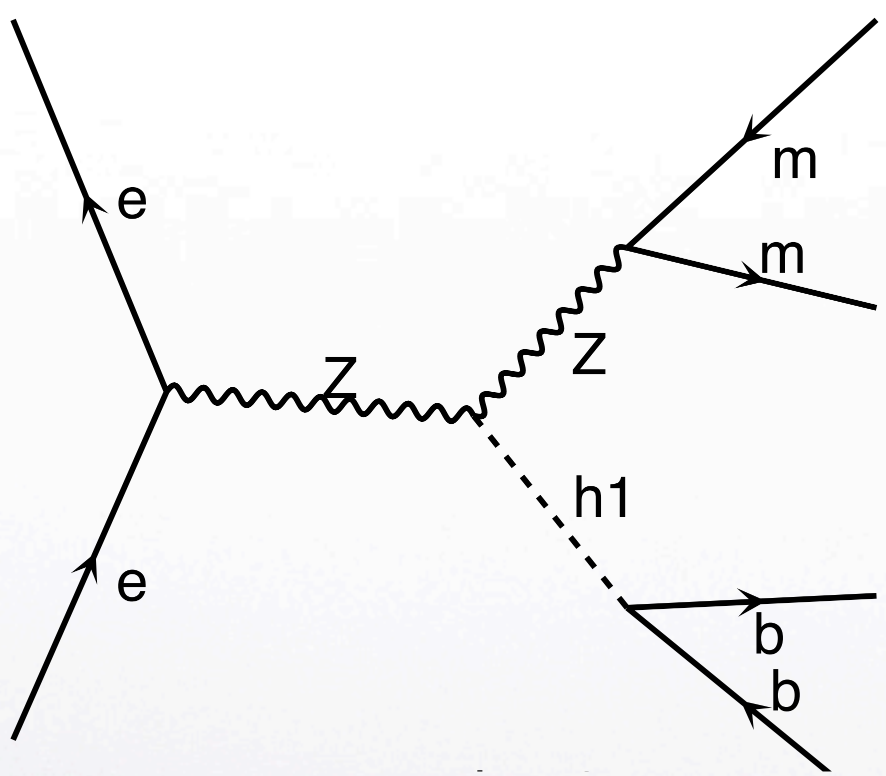
```
WriteSHOutput[LSM + LHill]
```

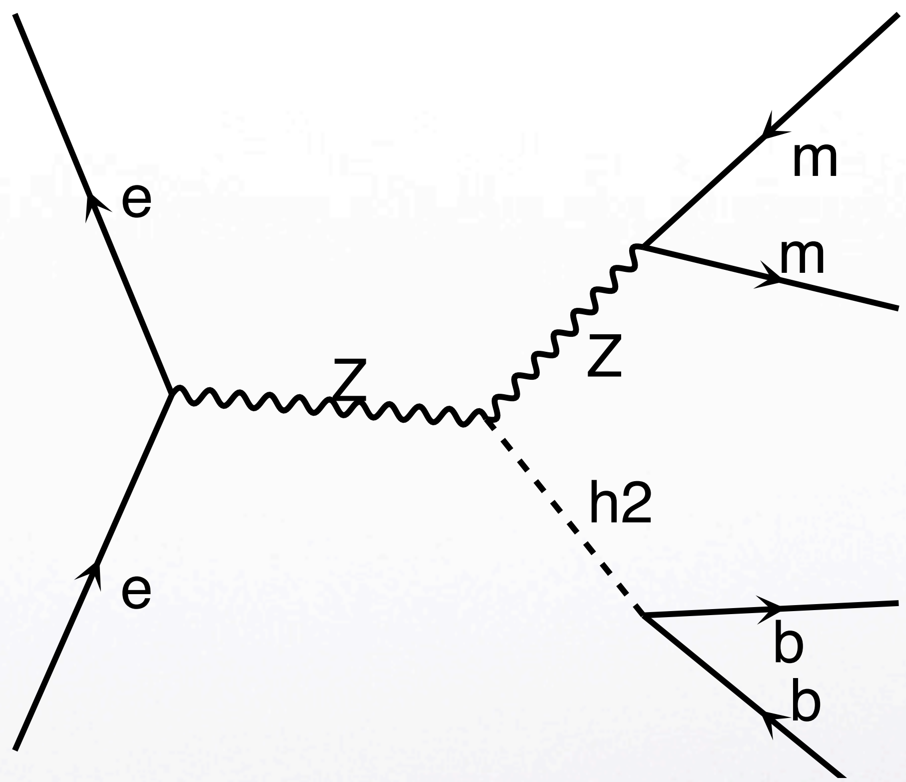
– – – FeynRules interface to Sherpa – – –

C. Duhr, S. Schumacher

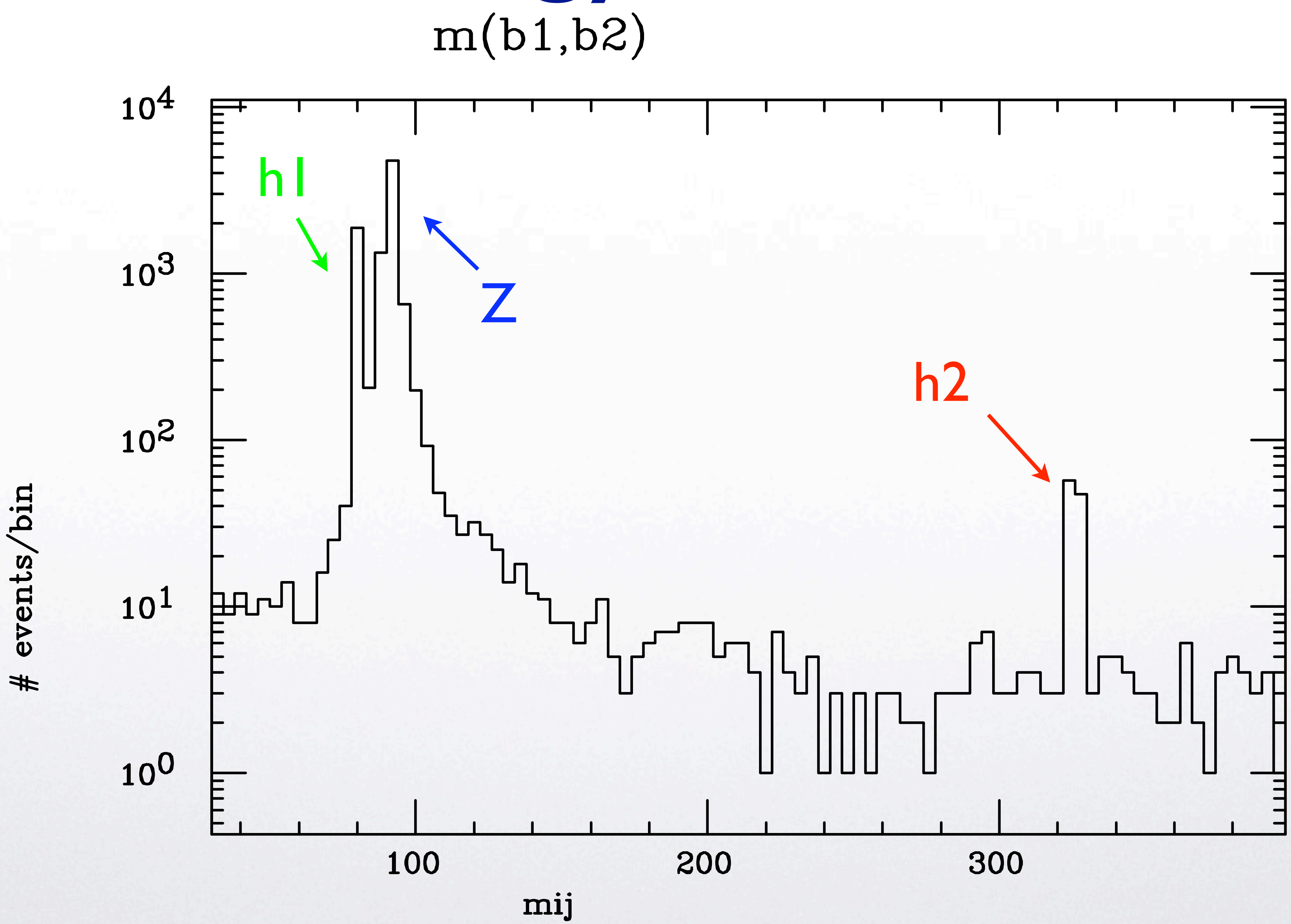
- This produces all the files needed to implement the Hill model into an MC. Let's have a look at our process!

Phenomenology with FeynRules

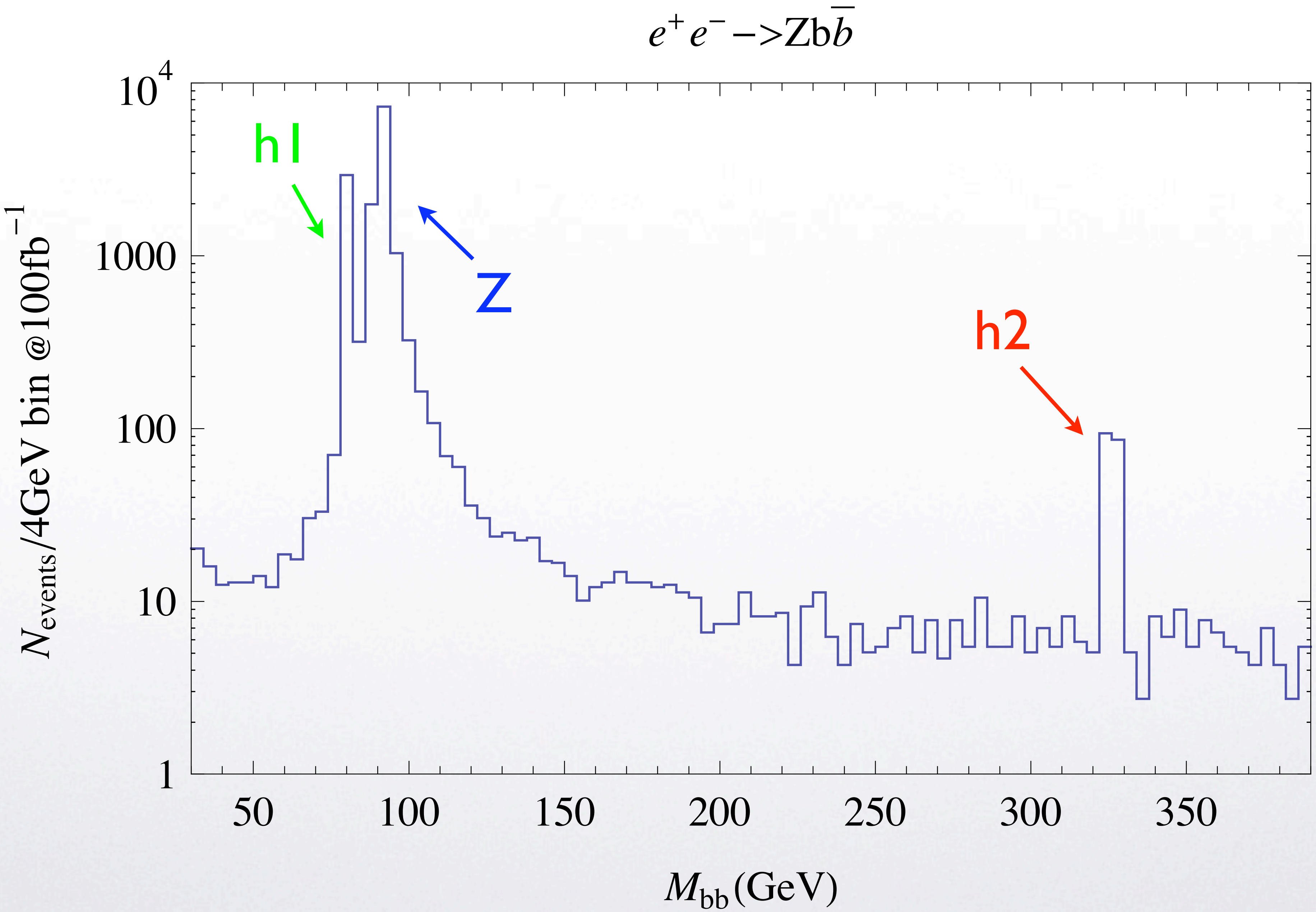




Phenomenology with MadGraph



Phenomenology with CalcHep



Conclusion

- FeynRules is a Mathematica®-based package to extract Feynman rules from a lagrangian.
- The output of FeynRules is completely generic and can be easily interfaced to other available codes.
- Available interfaces:
  - FeynArts
  - MadGraph/MadEvent
  - CalcHep/CompHep
  - Sherpa
  - ...
- The code can be downloaded from
  http://europa.fyma.ucl.ac.be/feynrules

Example of how to get vertices

The Strongly-Interacting Light Higgs

G. F. Giudice[a], C. Grojean[a,b], A. Pomarol[c], R. Rattazzi[a,d]

[hep-ph/0703164]

$$\frac{c_H}{2f^2}\partial^\mu\left(H^\dagger H\right)\partial_\mu\left(H^\dagger H\right)+\frac{c_T}{2f^2}\left(H^\dagger\overleftrightarrow{D^\mu}H\right)\left(H^\dagger\overleftrightarrow{D}_\mu H\right)$$

$$+\frac{ic_{HW}g}{16\pi^2 f^2}(D^\mu H)^\dagger\sigma^i(D^\nu H)W^i_{\mu\nu}+\frac{ic_{HB}g'}{16\pi^2 f^2}(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu}$$

Getting Feynman rules

$$\frac{c_H}{2f^2}\partial^\mu\left(H^\dagger H\right)\partial_\mu\left(H^\dagger H\right)$$

$$+\frac{c_T}{2f^2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right)\left(H^\dagger \overleftrightarrow{D}_\mu H\right)$$

$$+\frac{ic_{HW}\,g}{16\pi^2 f^2}(D^\mu H)^\dagger\sigma^i(D^\nu H)W^i_{\mu\nu}$$

$$+\frac{ic_{HB}\,g'}{16\pi^2 f^2}(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu}$$

```
Phi = {0, (v + H) / v}
```

$$\frac{c_H\,(2\,H\,\partial_{mu}[H] + 2\,v\,\partial_{mu}[H])^2}{8\,ff^2} +$$

$$\frac{c_T\,(\text{Phibar.DPhi[mu]} - \text{DPhibar[mu].Phi})^2}{2\,ff^2}$$

$$\frac{i\,c_{HB}\,g_1\,\text{DPhi[mu]}^\dagger.\text{DPhi[nu] FS[B, mu, nu]}}{16\,ff^2\,\pi^2} +$$

$$\frac{1}{16\,ff^2\,\pi^2}\,i\,c_{HW}\,g\,\text{DPhi[mu]}^\dagger.\sigma^i.\text{DPhi[nu]}\ \text{FS[Wi, mu, nu, i]}$$

Getting Feynman rules

Particle 1 : Scalar , H

Particle 2 : Vector , Z

Particle 3 : Vector , Z

Vertex:

$$\frac{1}{c_w^2\, ff^2\, \pi^2\, s_w^2\, v}\, i\, e\, \left(c_w^2 + s_w^2\right)$$

$$\left(-c_{HW}\, c_w^2\, g\, s_w\, p_1^{\mu_2}\, p_2^{\mu_3} - c_{HB}\, c_w\, g_1\, s_w^2\, p_1^{\mu_2}\, p_2^{\mu_3} - c_{HW}\, c_w^2\, g\, s_w\, p_1^{\mu_3}\, p_3^{\mu_2} - c_{HB}\, c_w\, g_1\, s_w^2\, p_1^{\mu_3}\, p_3^{\mu_2} + 32\, c_T\, c_w^2\, e\, \pi^2\, v^2\, \eta_{\mu_2,\mu_3} + 32\, c_T\, e\, \pi^2\, s_w^2\, v^2\, \eta_{\mu_2,\mu_3} + c_{HW}\, c_w^2\, g\, s_w\, \eta_{\mu_2,\mu_3}\, p_1 \cdot p_2 + c_{HB}\, c_w\, g_1\, s_w^2\, \eta_{\mu_2,\mu_3}\, p_1 \cdot p_2 + c_{HW}\, c_w^2\, g\, s_w\, \eta_{\mu_2,\mu_3}\, p_1 \cdot p_3 + c_{HB}\, c_w\, g_1\, s_w^2\, \eta_{\mu_2,\mu_3}\, p_1 \cdot p_3\right)$$

Getting Feynman rules

Particle 1 : Scalar , H

Particle 2 : Vector , A

Particle 3 : Scalar , H

Particle 4 : Vector , W

Particle 5 : Vector , $W^\dagger$

Vertex:

$$\frac{1}{16\, c_w\, ff^2\, \pi^2\, s_w^2\, v^2}$$

$$\begin{aligned} i\, e^2 \Big(& c_{HW}\, g\, s_w^2\, p_1^{\mu_5}\, \eta_{\mu_3,\mu_4} + c_{HW}\, g\, s_w^2\, p_2^{\mu_5}\, \eta_{\mu_3,\mu_4} + \\ & c_{HW}\, c_w^2\, g\, p_3^{\mu_5}\, \eta_{\mu_3,\mu_4} - c_{HB}\, c_w\, g_1\, s_w\, p_3^{\mu_5}\, \eta_{\mu_3,\mu_4} - \\ & c_{HW}\, c_w^2\, g\, p_4^{\mu_5}\, \eta_{\mu_3,\mu_4} - c_{HW}\, g\, s_w^2\, p_4^{\mu_5}\, \eta_{\mu_3,\mu_4} - \\ & c_{HW}\, g\, s_w^2\, p_1^{\mu_4}\, \eta_{\mu_3,\mu_5} - c_{HW}\, g\, s_w^2\, p_2^{\mu_4}\, \eta_{\mu_3,\mu_5} - \\ & c_{HW}\, c_w^2\, g\, p_3^{\mu_4}\, \eta_{\mu_3,\mu_5} + c_{HB}\, c_w\, g_1\, s_w\, p_3^{\mu_4}\, \eta_{\mu_3,\mu_5} + \\ & c_{HW}\, c_w^2\, g\, p_5^{\mu_4}\, \eta_{\mu_3,\mu_5} + c_{HW}\, g\, s_w^2\, p_5^{\mu_4}\, \eta_{\mu_3,\mu_5} + \\ & c_{HW}\, c_w^2\, g\, p_4^{\mu_3}\, \eta_{\mu_4,\mu_5} + c_{HW}\, g\, s_w^2\, p_4^{\mu_3}\, \eta_{\mu_4,\mu_5} - \\ & c_{HW}\, c_w^2\, g\, p_5^{\mu_3}\, \eta_{\mu_4,\mu_5} - c_{HW}\, g\, s_w^2\, p_5^{\mu_3}\, \eta_{\mu_4,\mu_5} \Big) \end{aligned}$$

Getting Feynman rules

Kaluza-Klein States from Large Extra Dimensions

Tao Han[a], Joseph D. Lykken[b] and Ren-Jie Zhang[a]

[a] *Department of Physics, University of Wisconsin, Madison, WI 53706*

[b] *Theory Group, Fermi National Accelerator Laboratory, Batavia, IL 60510*

[hep-ph/9811350]

Particle content:

- Spin 2 graviton, KK-scalars
- Fermions
- Scalars
- Gauge bosons

Getting Feynman rules

- Lagrangian coupling the fermions to the graviton and the KK-scalar:

$$\begin{aligned}\kappa^{-1}\mathcal{L}_{\mathrm{F}}^{\vec{n}}(\kappa) \;=\;& \frac{1}{2}\Big[(\widetilde{h}^{\vec{n}}\eta^{\mu\nu}-\widetilde{h}^{\mu\nu,\vec{n}})\overline{\psi}i\gamma_{\mu}D_{\nu}\psi - m_{\psi}\widetilde{h}^{\vec{n}}\overline{\psi}\psi + \frac{1}{2}\overline{\psi}i\gamma^{\mu}(\partial_{\mu}\widetilde{h}^{\vec{n}}-\partial^{\nu}\widetilde{h}^{\vec{n}}_{\mu\nu})\psi\Big] \\ &+\frac{3\omega}{2}\widetilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^{\mu}D_{\mu}\psi - 2\omega m_{\psi}\widetilde{\phi}^{\vec{n}}\overline{\psi}\psi + \frac{3\omega}{4}\partial_{\mu}\widetilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^{\mu}\psi\;. \qquad (44)\end{aligned}$$

- Very complicated structure as far as Feynman rules are concerned, but we are only a few steps away from the Feynman rules...

Getting Feynman rules

- Step 1: Add all the parameters in the lagrangian to the model file:

```
M$Parameters = {

   g, k, om, ...

}
```

$$
\begin{aligned}
\kappa^{-1}\mathcal{L}_{\rm F}^{\vec{n}}(\kappa) &= \frac{1}{2}\Big[(\widetilde{h}^{\vec{n}}\eta^{\mu\nu} - \widetilde{h}^{\mu\nu,\vec{n}})\overline{\psi}i\gamma_\mu D_\nu\psi \\
&\quad - m_\psi\widetilde{h}^{\vec{n}}\overline{\psi}\psi + \frac{1}{2}\overline{\psi}i\gamma^\mu(\partial_\mu\widetilde{h}^{\vec{n}} - \partial^\nu\widetilde{h}^{\vec{n}}_{\mu\nu})\psi\Big] \\
&\quad + \frac{3\omega}{2}\widetilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu D_\mu\psi - 2\omega m_\psi\widetilde{\phi}^{\vec{n}}\overline{\psi}\psi \\
&\quad + \frac{3\omega}{4}\partial_\mu\widetilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu\psi
\end{aligned}
$$

Getting Feynman rules

- Step II: Add all the particles in the lagrangian to the model file:

```
M$ClassesDescription = {

  Sp2[1] == {
     ClassName -> h,
     SelfConjugate -> True,
     Symmetric -> True}

  ...

}
```

$$
\begin{aligned}
\kappa^{-1}\mathcal{L}_{\mathrm{F}}^{\vec{n}}(\kappa) \;=\; & \frac{1}{2}\Big[(\widetilde{h}^{\vec{n}}\eta^{\mu\nu} - \widetilde{h}^{\mu\nu,\vec{n}})\overline{\psi}i\gamma_{\mu}D_{\nu}\psi \\
& - m_{\psi}\widetilde{h}^{\vec{n}}\overline{\psi}\psi + \frac{1}{2}\overline{\psi}i\gamma^{\mu}(\partial_{\mu}\widetilde{h}^{\vec{n}} - \partial^{\nu}\widetilde{h}^{\vec{n}}_{\mu\nu})\psi\Big] \\
& + \frac{3\omega}{2}\widetilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^{\mu}D_{\mu}\psi - 2\omega m_{\psi}\widetilde{\phi}^{\vec{n}}\overline{\psi}\psi \\
& + \frac{3\omega}{4}\partial_{\mu}\widetilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^{\mu}\psi
\end{aligned}
$$

Getting Feynman rules

- **Step III:** The lagrangian

```
LF = k (1 / 2
  ((h[ρ, ρ] ME[μ, ν] - h[μ, ν])
     (I HC[ψ]. Ga[μ].del[ψ, ν] - g G[ν, a] HC[ψ].Ga[μ].T[a].ψ) -
   mpsi h[μ, μ] HC[ψ].ψ + I / 2 HC[ψ]. Ga[μ].ψ
     (del[h[ν, ν], μ] - del[h[μ, ν], ν])) +
3 om / 2 φ (I HC[ψ]. Ga[μ].del[ψ, μ] - g G[μ, a] HC[ψ].Ga[μ].T[a].ψ) -
2 om mpsi φ HC[ψ].ψ + 3 om / 4 del[φ, μ] I HC[ψ]. Ga[μ].ψ)
```

$$k\Big(-2\,\mathrm{mpsi}\,\mathrm{om}\,\phi\,\psi^\dagger.\psi + \frac{3}{4}\,i\,\mathrm{om}\,\partial_\mu(\phi)\,\psi^\dagger.\gamma^\mu.\psi + \frac{3}{2}\,\mathrm{om}\,\phi\left(i\,\psi^\dagger.\gamma^\mu.\partial_\mu(\psi) - g\,\psi^\dagger.\gamma^\mu.T^a.\psi\,G_{\mu,a}\right) + \frac{1}{2}\Big(\frac{1}{2}\,i\left(\partial_\mu(h_{\nu,\nu}) - \partial_\nu(h_{\mu,\nu})\right)\psi^\dagger.\gamma^\mu.\psi - \mathrm{mpsi}\,\psi^\dagger.\psi\,h_{\mu,\mu} + \left(i\,\psi^\dagger.\gamma^\mu.\partial_\nu(\psi) - g\,\psi^\dagger.\gamma^\mu.T^a.\psi\,G_{\nu,a}\right)\left(h_{\rho,\rho}\,\eta_{\mu,\nu} - h_{\mu,\nu}\right)\Big)\Big)$$

$$\begin{aligned}\kappa^{-1}\mathcal{L}_{\mathrm{F}}^{\vec{n}}(\kappa) \;=\; & \frac{1}{2}\Big[(\tilde{h}^{\vec{n}}\eta^{\mu\nu} - \tilde{h}^{\mu\nu,\vec{n}})\overline{\psi} i\gamma_\mu D_\nu\psi \\ & - m_\psi \tilde{h}^{\vec{n}}\overline{\psi}\psi + \frac{1}{2}\overline{\psi} i\gamma^\mu(\partial_\mu\tilde{h}^{\vec{n}} - \partial^\nu\tilde{h}^{\vec{n}}_{\mu\nu})\psi\Big] \\ & + \frac{3\omega}{2}\tilde{\phi}^{\vec{n}}\overline{\psi} i\gamma^\mu D_\mu\psi - 2\omega m_\psi\tilde{\phi}^{\vec{n}}\overline{\psi}\psi \\ & + \frac{3\omega}{4}\partial_\mu\tilde{\phi}^{\vec{n}}\overline{\psi} i\gamma^\mu\psi\end{aligned}$$

Getting Feynman rules

- Step IV: The FeynmanRules

```
FeynmanRules[LF]
```

Calculating vertices...

4 vertices obtained.

(* *)

Vertex 1

Particle 1 : Scalar , ϕ

Particle 2 : Dirac , ψ

Particle 3 : Dirac , $\psi^\dagger$

Vertex:

$$\frac{1}{4} i\, k\, \text{om}\, \delta_{i_2,i_3} \left(3\, p_1^{\alpha 2}\, {\gamma^{\alpha 2}}_{s_3,s_2} + 6\, p_2^{\alpha 2}\, {\gamma^{\alpha 2}}_{s_3,s_2} - 8\, \text{mpsi}\, \delta_{s_2,s_3}\right)$$

Getting Feynman rules

- Step IV: The FeynmanRules

(* *)

Vertex 2

Particle 1 : Spin 2 , h

Particle 2 : Dirac , ψ

Particle 3 : Dirac , $\psi^\dagger$

Vertex:

$$-\frac{1}{8} i\, k\, \delta_{i_2,i_3}$$

$$\left(p_1^{\beta}\, {\gamma^{\alpha}}_{s_3,s_2} + 2\, p_2^{\beta}\, {\gamma^{\alpha}}_{s_3,s_2} + p_1^{\alpha}\, {\gamma^{\beta}}_{s_3,s_2} + 2\, p_2^{\alpha}\, {\gamma^{\beta}}_{s_3,s_2} - 2\, p_1^{\alpha 2}\, {\gamma^{\alpha 2}}_{s_3,s_2}\, \eta_{\alpha,\beta} - 4\, p_2^{\gamma 2}\, {\gamma^{\gamma 2}}_{s_3,s_2}\, \eta_{\alpha,\beta} + 4\, \mathrm{mpsi}\, \delta_{s_2,s_3}\, \eta_{\alpha,\beta}\right)$$

Getting Feynman rules

- Step IV: The FeynmanRules

(* *)

Vertex 3

Particle 1 : Scalar , ϕ

Particle 2 : Vector , G

Particle 3 : Dirac , ψ

Particle 4 : Dirac , $\psi^\dagger$

Vertex:

$$-\frac{3}{2}\, i\, g\, k\,\mathrm{om}\, \gamma^{\mu_2}{}_{s_4,s_3}\, T^{a_2}{}_{i_4,i_3}$$

Getting Feynman rules

- Step IV: The FeynmanRules

(*********************************)

Vertex 4

Particle 1 : Vector , G

Particle 2 : Spin 2 , h

Particle 3 : Dirac , ψ

Particle 4 : Dirac , $\psi^\dagger$

Vertex:

$$\frac{1}{4}\, i\, g\, k \left(\gamma^{\beta}{}_{s_4,s_3}\, \eta_{\mu_1,\alpha} + \gamma^{\alpha}{}_{s_4,s_3}\, \eta_{\mu_1,\beta} - 2\, \gamma^{\mu_1}{}_{s_4,s_3}\, \eta_{\alpha,\beta} \right) T^{a_1}{}_{i_4,i_3}$$

Getting Feynman rules

- Step III: The lagrangian

```
LF = k (1 / 2
 ((h[ρ, ρ] ME[μ, ν] - h[μ, ν])
    (I HC[ψ]. Ga[μ].del[ψ, ν] - g G[ν, a] HC[ψ].Ga[μ].T[a].ψ) -
   mpsi h[μ, μ] HC[ψ].ψ + I / 2 HC[ψ]. Ga[μ].ψ
    (del[h[ν, ν], μ] - del[h[μ, ν], ν])) +
3 om / 2 ϕ (I HC[ψ]. Ga[μ].del[ψ, μ] - g G[μ, a] HC[ψ].Ga[μ].T[a].ψ) -
2 om mpsi ϕ HC[ψ].ψ + 3 om / 4 del[ϕ, μ] I HC[ψ]. Ga[μ].ψ)
```

$$k\left(-2\,\mathrm{mpsi}\,\mathrm{om}\,\phi\,\psi^\dagger.\psi + \frac{3}{4}\,i\,\mathrm{om}\,\partial_\mu(\phi)\,\psi^\dagger.\gamma^\mu.\psi + \frac{3}{2}\,\mathrm{om}\,\phi\left(i\,\psi^\dagger.\gamma^\mu.\partial_\mu(\psi) - g\,\psi^\dagger.\gamma^\mu.T^a.\psi\,G_{\mu,a}\right) + \frac{1}{2}\left(\frac{1}{2}\,i\left(\partial_\mu(h_{\nu,\nu}) - \partial_\nu(h_{\mu,\nu})\right)\psi^\dagger.\gamma^\mu.\psi - \mathrm{mpsi}\,\psi^\dagger.\psi\,h_{\mu,\mu} + \left(i\,\psi^\dagger.\gamma^\mu.\partial_\nu(\psi) - g\,\psi^\dagger.\gamma^\mu.T^a.\psi\,G_{\nu,a}\right)\left(h_{\rho,\rho}\,\eta_{\mu,\nu} - h_{\mu,\nu}\right)\right)\right)$$

$$\kappa^{-1}\mathcal{L}_{\mathrm{F}}^{\vec{n}}(\kappa) = \frac{1}{2}\Big[(\tilde{h}^{\vec{n}}\eta^{\mu\nu} - \tilde{h}^{\mu\nu,\vec{n}})\overline{\psi}i\gamma_\mu D_\nu\psi - m_\psi\tilde{h}^{\vec{n}}\overline{\psi}\psi + \frac{1}{2}\overline{\psi}i\gamma^\mu(\partial_\mu\tilde{h}^{\vec{n}} - \partial^\nu\tilde{h}^{\vec{n}}_{\mu\nu})\psi\Big] + \frac{3\omega}{2}\tilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu D_\mu\psi - 2\omega m_\psi\tilde{\phi}^{\vec{n}}\overline{\psi}\psi + \frac{3\omega}{4}\partial_\mu\tilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu\psi$$

Getting Feynman rules

- Step IV: The FeynmanRules

```
FeynmanRules[LF]
```

Calculating vertices...

4 vertices obtained.

(* *)

Vertex 1

Particle 1 : Scalar , ϕ

Particle 2 : Dirac , ψ

Particle 3 : Dirac , $\psi^\dagger$

Vertex:

$$\frac{1}{4}\, i\, k\, \text{om}\, \delta_{i_2,i_3} \left(3\, p_1^{\alpha 2}\, {\gamma^{\alpha 2}}_{s_3,s_2} + 6\, p_2^{\alpha 2}\, {\gamma^{\alpha 2}}_{s_3,s_2} - 8\, \text{mpsi}\, \delta_{s_2,s_3}\right)$$

Getting Feynman rules

- Step IV: The FeynmanRules

(* *)

Vertex 2

Particle 1 : Spin 2 , h

Particle 2 : Dirac , ψ

Particle 3 : Dirac , $\psi^\dagger$

Vertex:

$$-\frac{1}{8}\, i\, k\, \delta_{i_2,i_3}$$

$$\left(p_1^\beta\, \gamma^\alpha{}_{s_3,s_2} + 2\, p_2^\beta\, \gamma^\alpha{}_{s_3,s_2} + p_1^\alpha\, \gamma^\beta{}_{s_3,s_2} + 2\, p_2^\alpha\, \gamma^\beta{}_{s_3,s_2} - 2\, p_1^{\alpha 2}\, \gamma^{\alpha 2}{}_{s_3,s_2}\, \eta_{\alpha,\beta}\right.$$

$$\left. - 4\, p_2^{\gamma 2}\, \gamma^{\gamma 2}{}_{s_3,s_2}\, \eta_{\alpha,\beta} + 4\, \mathrm{mpsi}\, \delta_{s_2,s_3}\, \eta_{\alpha,\beta}\right)$$

Getting Feynman rules

- Step IV: The FeynmanRules

(* *)

Vertex 3

Particle 1 : Scalar , ϕ

Particle 2 : Vector , G

Particle 3 : Dirac , ψ

Particle 4 : Dirac , $\psi^\dagger$

Vertex:

$$-\frac{3}{2}\, i\, g\, k\,\mathrm{om}\, \gamma^{\mu_2}{}_{s_4,s_3}\, T^{a_2}{}_{i_4,i_3}$$

Getting Feynman rules

- Step IV: The FeynmanRules

(***********************************)

Vertex 4

Particle 1 : Vector , G

Particle 2 : Spin 2 , h

Particle 3 : Dirac , ψ

Particle 4 : Dirac , $\psi^\dagger$

Vertex:

$$\frac{1}{4}\, i\, g\, k \left(\gamma^{\beta}{}_{s_4,s_3}\, \eta_{\mu_1,\alpha} + \gamma^{\alpha}{}_{s_4,s_3}\, \eta_{\mu_1,\beta} - 2\, \gamma^{\mu_1}{}_{s_4,s_3}\, \eta_{\alpha,\beta} \right) T^{a_1}{}_{i_4,i_3}$$