Monte Carlo's: event simulation for the LHC

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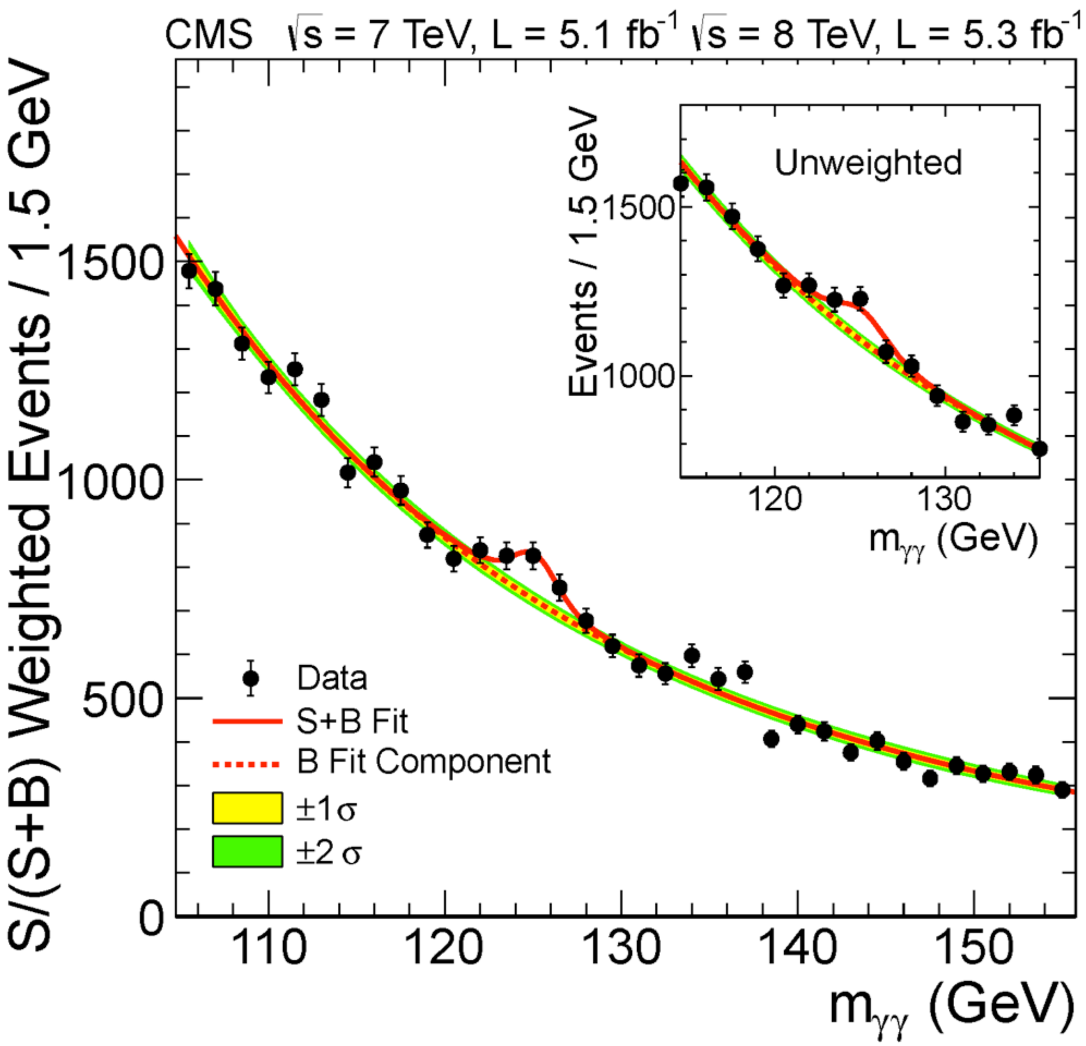
Lecture I

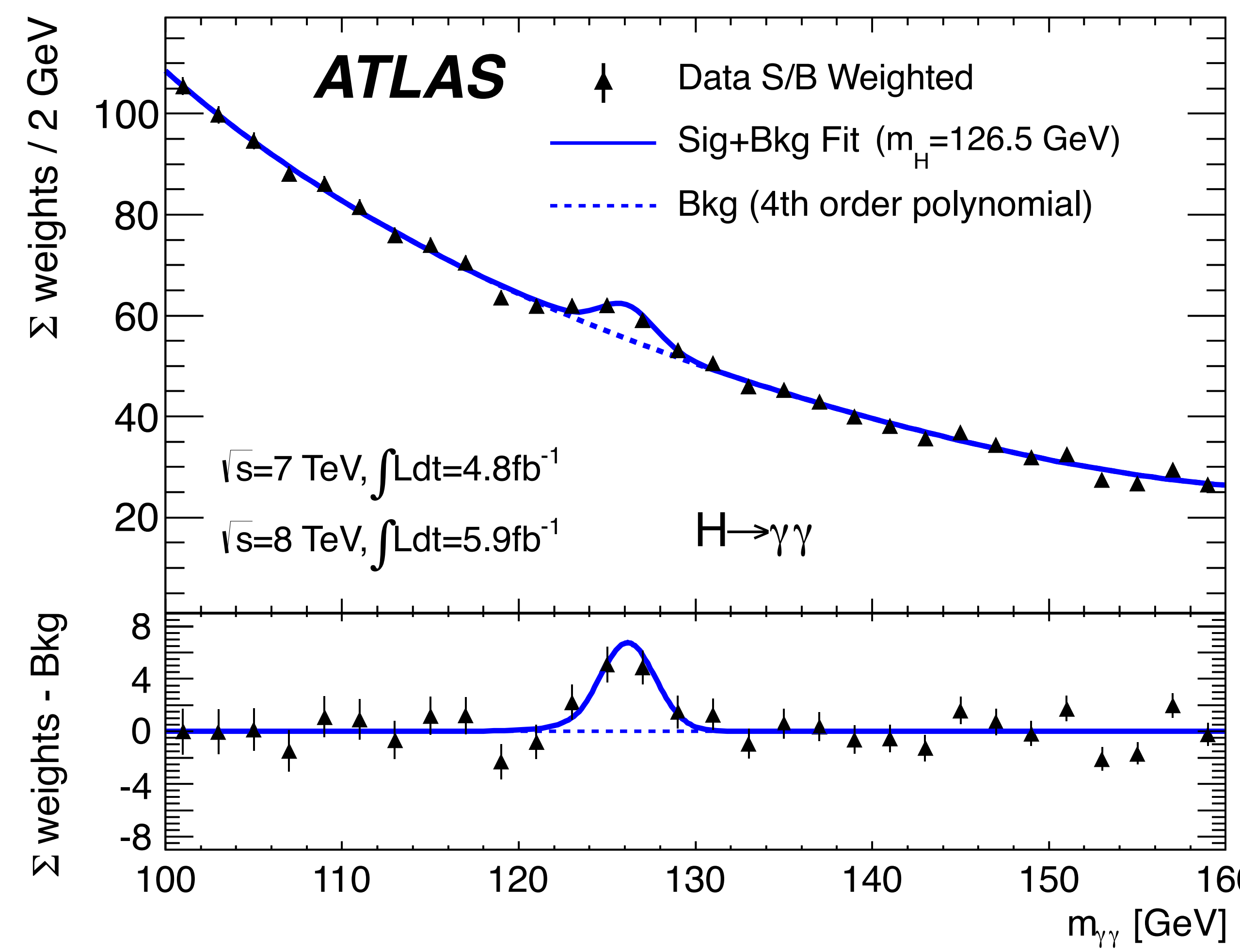
The hottest news in HEP

- A new **force** has been discovered, the first ever seen* not related to a gauge symmetry.
- Its **mediator** looks a lot like the scalar predicted in the SM

*fundamental, ie with elementary mediators.

Independence day 2012





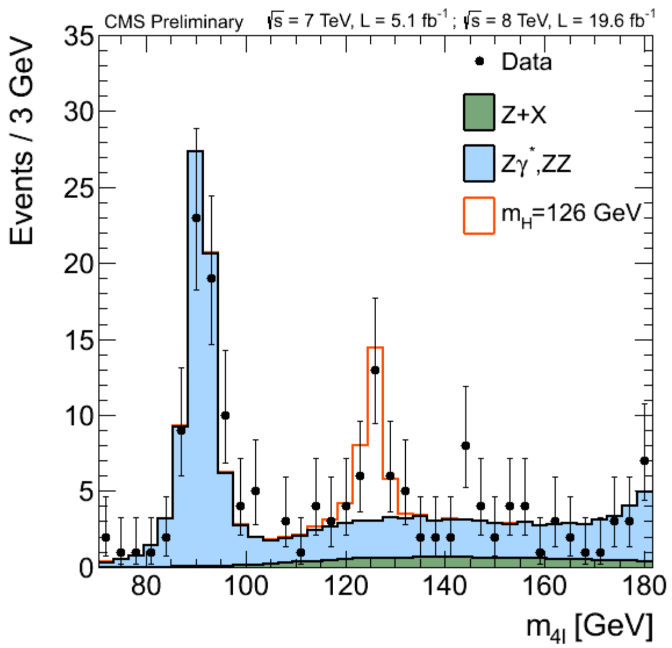
Clear evidence for a new resonance!

Now reaching >10 σ

Discoveries at hadron colliders

peak

$pp \to H \to 4l$

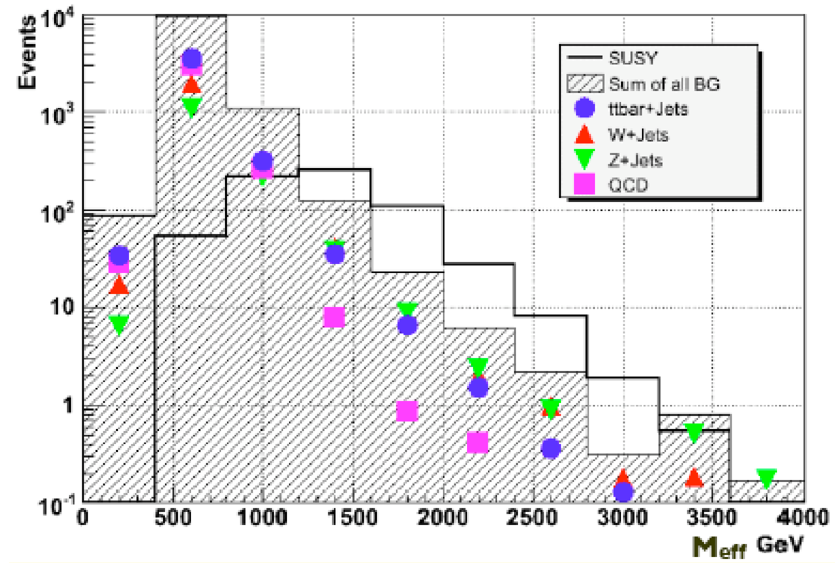


"easy"

Background directly measured from data. TH needed only for parameter extraction (Normalization, acceptance,...)

shape

$pp \to \tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q} \to jets + \not{E}_T$

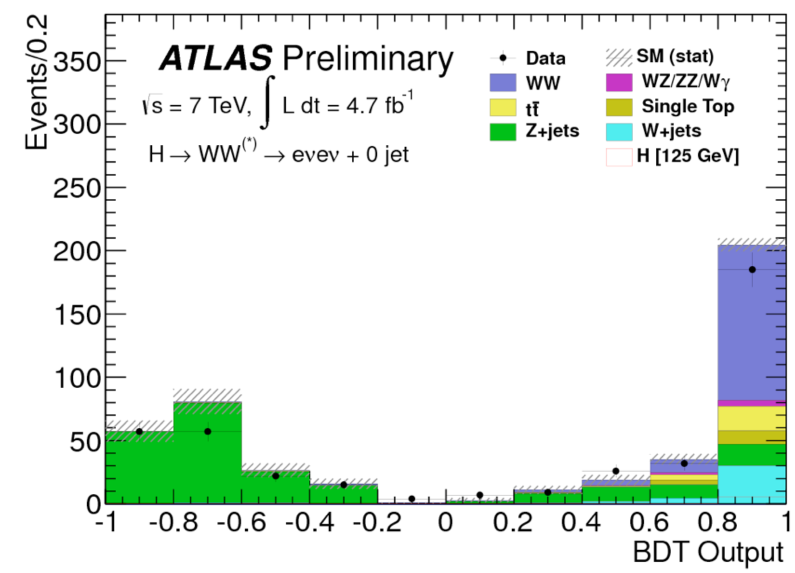


hard

Background shapes needed. Flexible MC for both signal and background tuned and validated with data.

discriminant

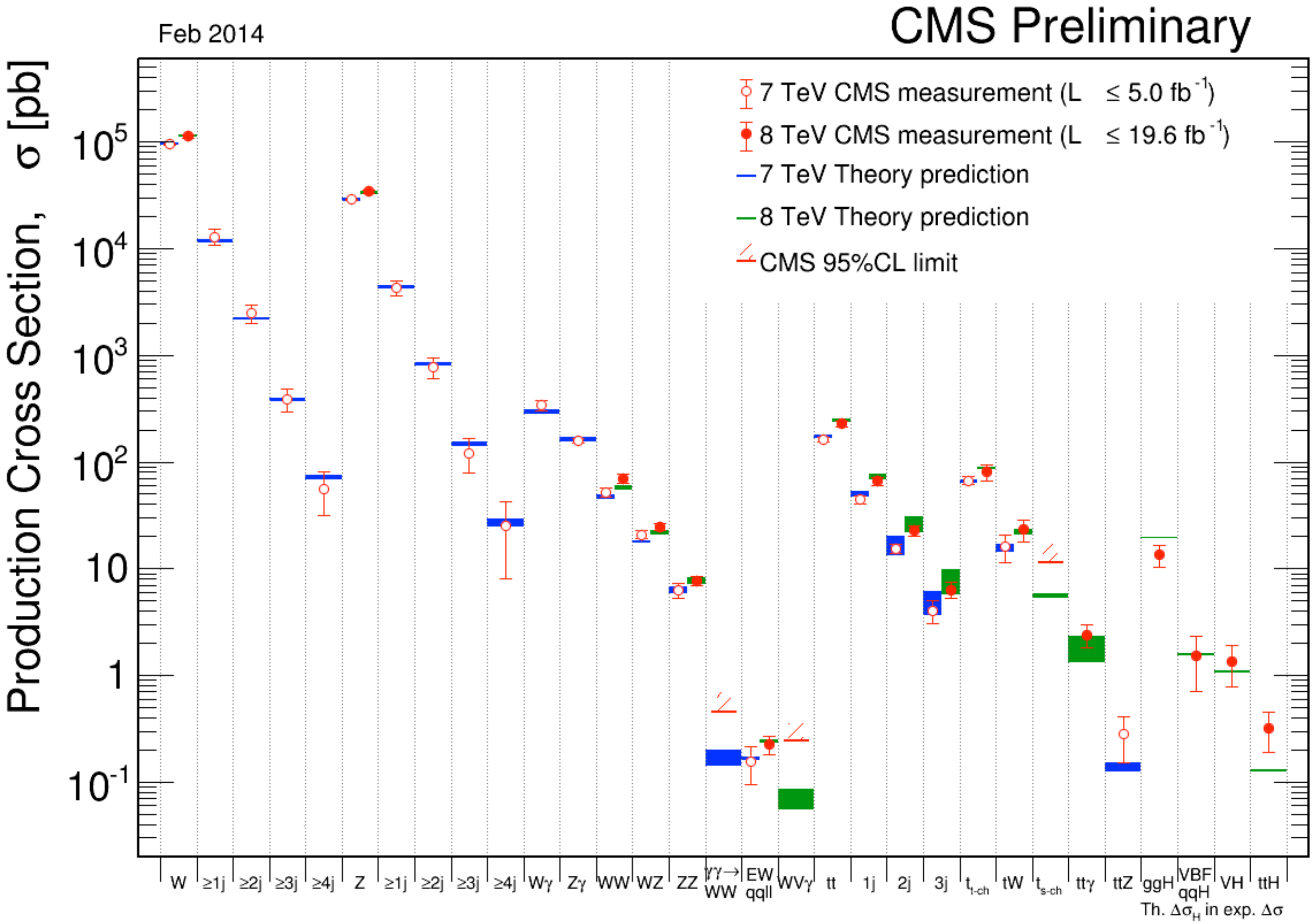
$pp \to H \to W^+W^-$



very hard

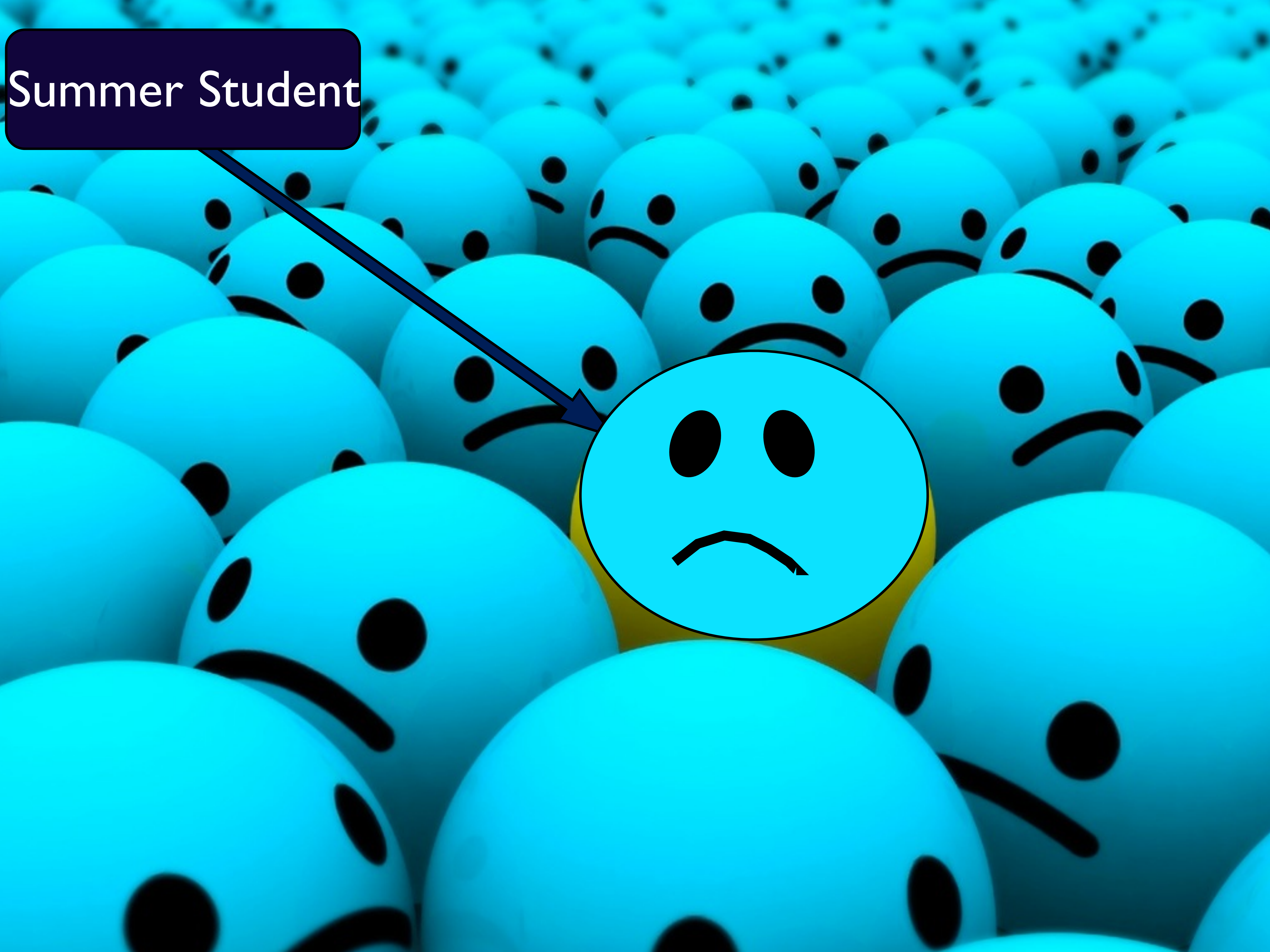
Background normalization and shapes known very well. Interplay with the best theoretical predictions (via MC) and data.

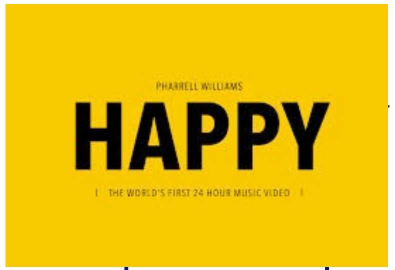
Challenges for LHC physicists



Even this plot actually needs theory input (and the total quoted uncertainty in the measurements does have a contribution from theory)!!!

No sign of new physics (so far)!


Summer Student

WHY  ?

- **Optimism**: New Physics could be hiding there already, **I might be the one** to dig it out.

- **Democratization**: No evidence of most beaten BSM proposals, means more and more room for diversification. Possibility for small teams to make a big discovery.

- **Ingenuity/Creativity**: From new signatures to smart and new analysis techniques (MVA), and combination with non-collider searches (DM, Flavor...).

- We need MC that are able to predict the pheno of the Unexpected:

    - **Massification** (the practice of making luxury products available to the mass market) : MC's in the hands of every th/exp might turn out to be the best overall strategy for discovering the Unexpected.

    - **Accuracy**: accurate simulations for both SM and BSM are a must.

Challenges for LHC physicists

- Accurate and experimental friendly predictions for collider physics range from being very useful to strictly necessary.
- Confidence on possible excesses, evidences and eventually discoveries builds upon an intense (and often non-linear) process of description/prediction of data via MC's.
- Both **measurements** and **exclusions** rely on accurate predictions.

New generation (LHC) of MC tools

Theory

Lagrangian
Gauge invariance
QCD
Partons
NLO
Resummation
...

Detector simulation
Pions, Kaons, ...
Reconstruction
B-tagging efficiency
Boosted decision tree
Neural network
...

Experiment

The LHC simulation chain

- Idea
- Lagrangian
- FeynRules
- ME Generator
- Signal & Bkg
- Events
- PS+Had
- Detect. Sim.
- Data

PLAN

- Basics : LO predictions
- Event generation
- Exclusive predictions : Parton Showers
- The simulation frontier

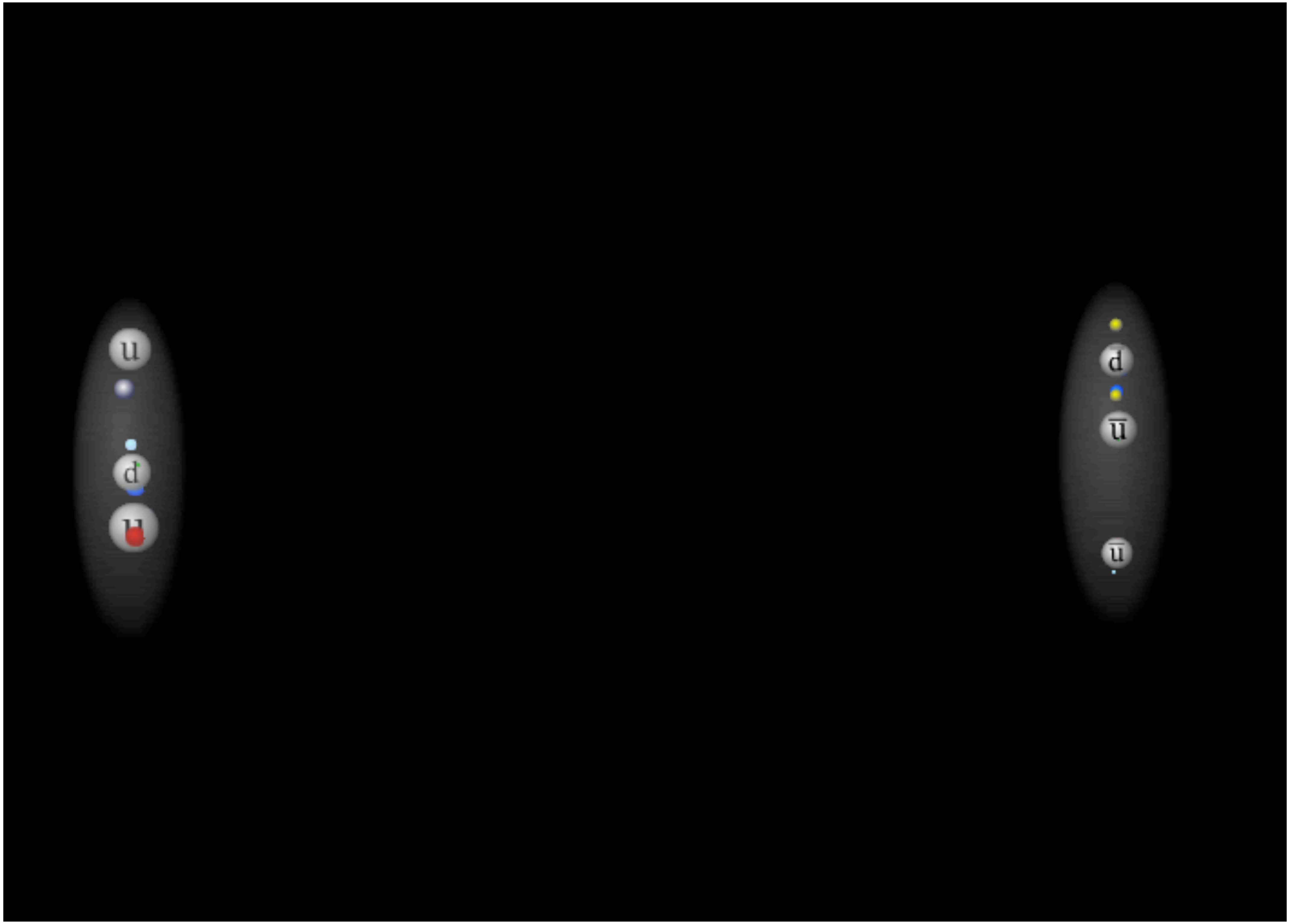
REAL PLAN

- How to calculate the probability of producing a given final state (top, Higgs, W's, Z's jets, squarks, gluinos,...) in a given configuration (phase space point) or total cross sections.
- How to turn these calculations into parton-level even generators (not yet realistic)
- How to make realistic simulations in terms of physical initial states (pp) and final states (hadrons, leptons, neutrinos, photons...)
- Current tools

TRY IT OUT YOURSELF

Wiki with exercises on MC integration event generation:

https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/MCSummerCERN14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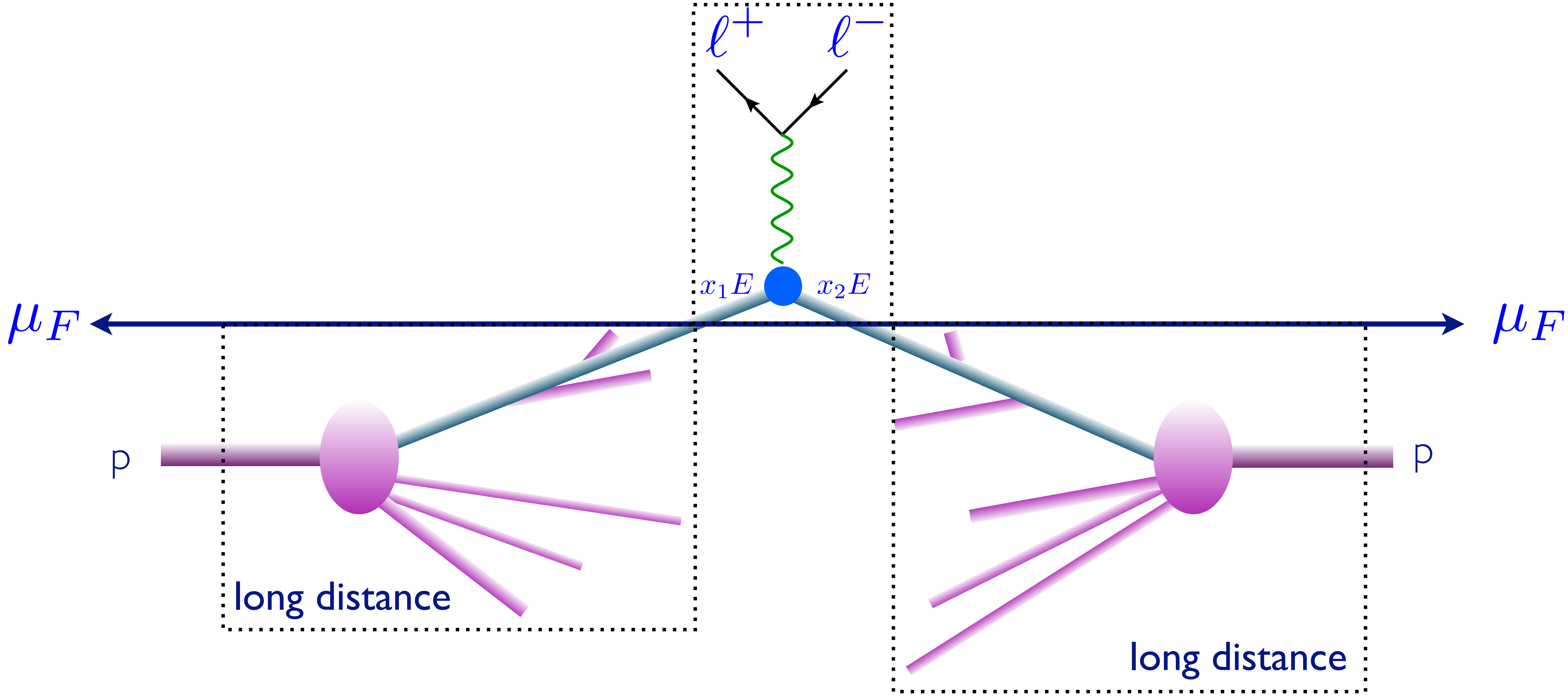
Master formula for the LHC



$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\text{FS}}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

Master formula for the LHC

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

Two ingredients necessary:

1. Parton distribution functions : non perturbative
(fit from experiments, but evolution from theory)

2. Parton-level cross section: short distance coefficients as an expansion in α_S (from theory)

Perturbative expansion

$\hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

LO predictions | **NLO corrections** | **NNLO corrections** | **NNNLO corrections**

- Including higher corrections improves predictions and reduces theoretical uncertainties

Predictions at LO

How do we calculate a LO cross section for 3 jets at the LHC?

I. Identify all subprocesses (gg→ggg, qg→qgg....) in:

$$\sigma(pp \to 3j) = \sum_{ijk} \int f_i(x_1) f_j(x_2) \hat{\sigma}(ij \to k_1 k_2 k_3)$$

easy

II. For each one, calculate the amplitude:

$$\mathcal{A}(\{p\},\{h\},\{c\}) = \sum_i D_i$$

difficult

III. Square the amplitude, sum over spins & color, integrate over the phase space (D ~ 3n)

$$\hat{\sigma} = \frac{1}{2\hat{s}} \int d\Phi_p \sum_{h,c} |\mathcal{A}|^2$$

quite hard

Phase-space integral

- Calculations of cross section or decay widths involve integrations over phase space of very complex functions

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

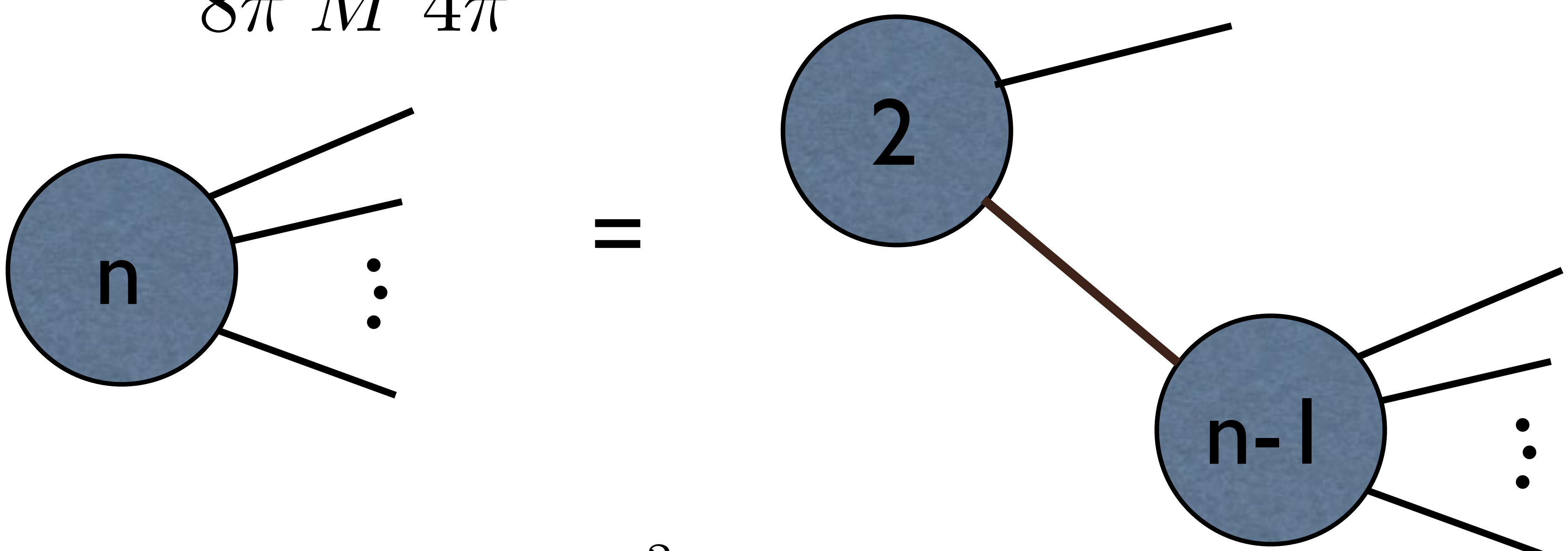
$Dim[\Phi(n)] \sim 3n$

General and flexible method is needed:
Numerical (Monte Carlo) integration

Phase-space

$$d\Phi_n = \left[\Pi_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 (2E_i)}\right] (2\pi)^4 \delta^{(4)}(p_0 - \sum_{i=1}^{n} p_i)$$

$$d\Phi_2(M) = \frac{1}{8\pi} \frac{2p}{M} \frac{d\Omega}{4\pi}$$



$$d\Phi_n(M) = \frac{1}{2\pi} \int_0^{(M-\mu)^2} d\mu^2 d\Phi_2(M) d\Phi_{n-1}(\mu)$$

Integrals as averages

$$I = \int_{x_1}^{x_2} f(x)dx \Rightarrow I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \Rightarrow V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

$$I = I_N \pm \sqrt{V_N/N}$$

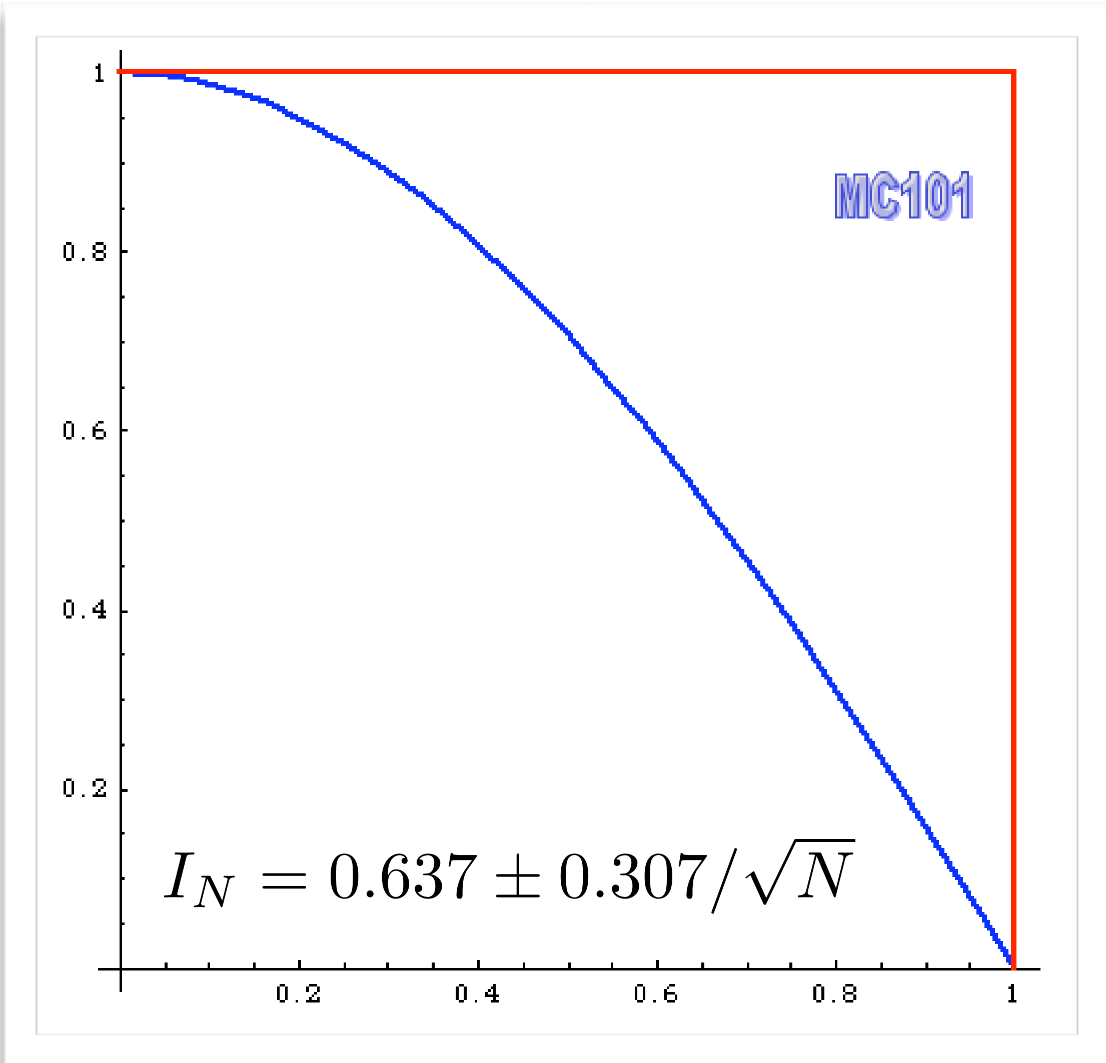
☞ Convergence is slow but it can be estimated easily

☞ Error does not depend on # of dimensions!

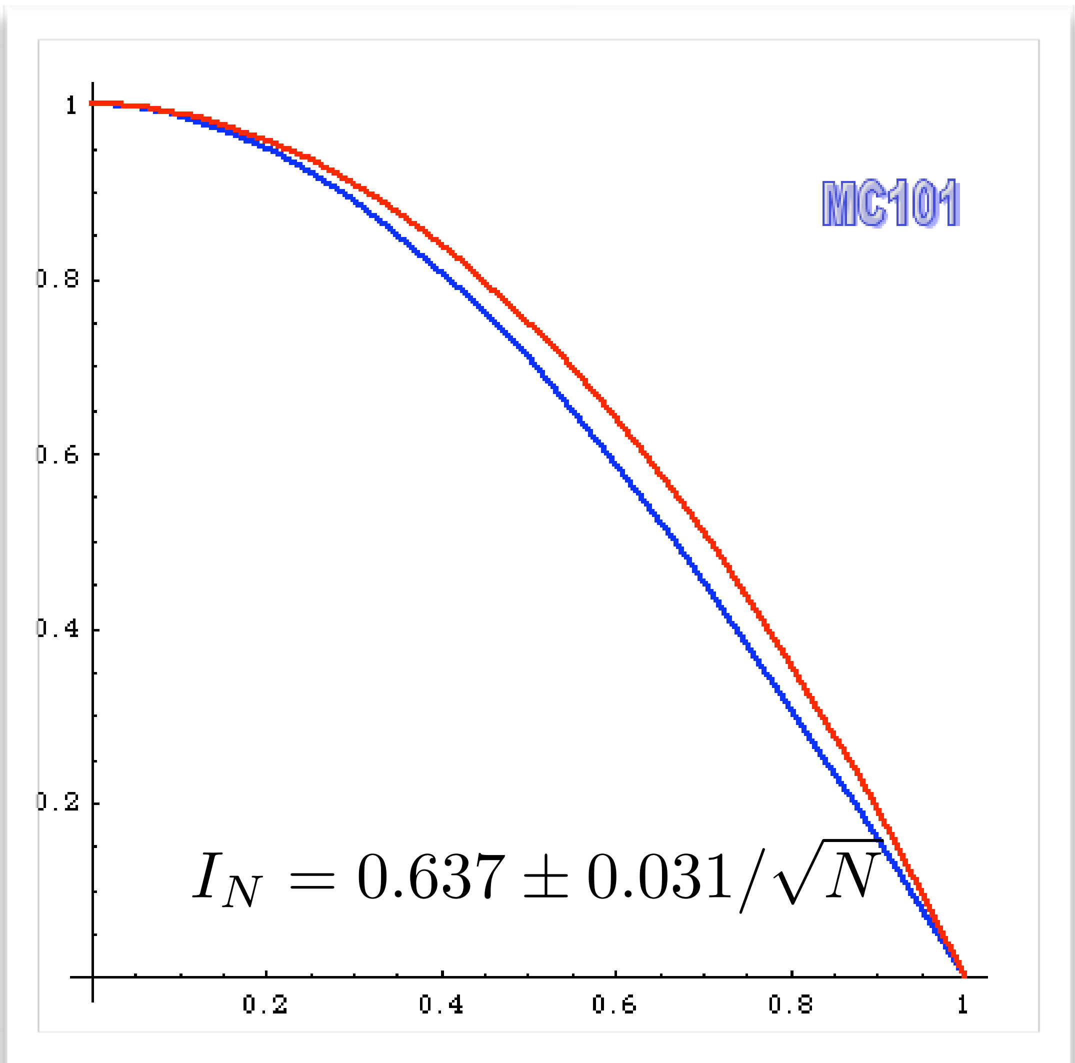
☞ Improvement by minimizing V_N

☞ Optimal/Ideal case: f(x) = Constant $\Rightarrow V_N = 0$

Importance sampling



$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



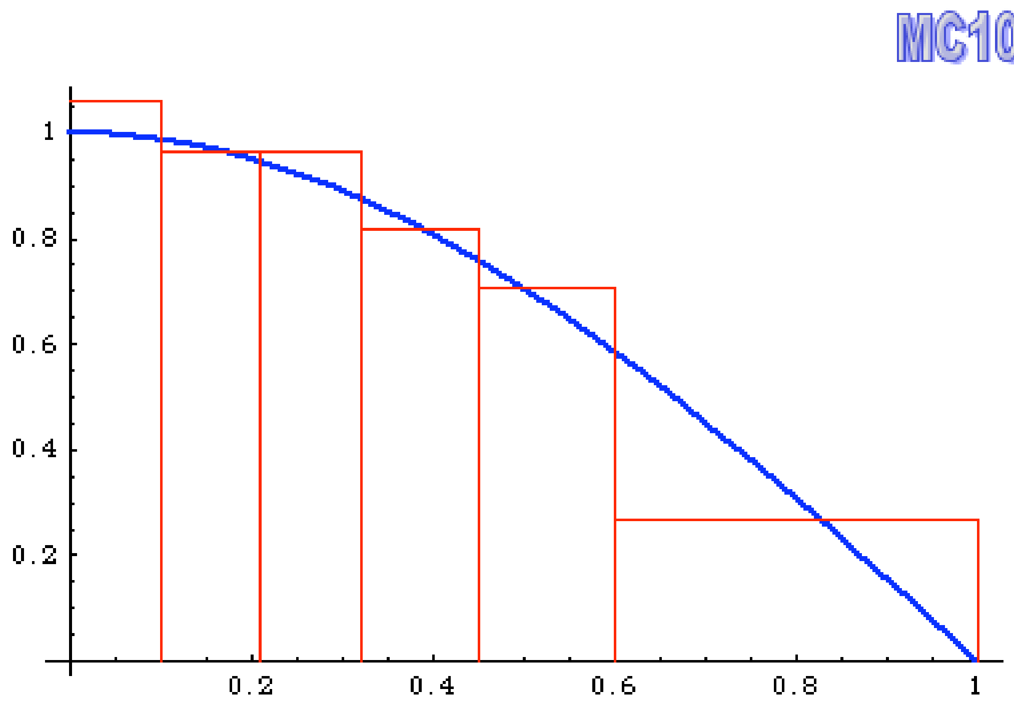
$$I = \int_0^1 dx(1-x^2)\frac{\cos\frac{\pi}{2}x}{1-x^2}$$

$$= \int_{\xi_1}^{\xi_2} d\xi \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2} \Rightarrow \simeq 1$$

Importance sampling

But... you need to know too much about f(x)!

Idea: learn during the run and build a step-function approximation p(x) of f(x) ⟹ VEGAS



more bins where f(x) is large

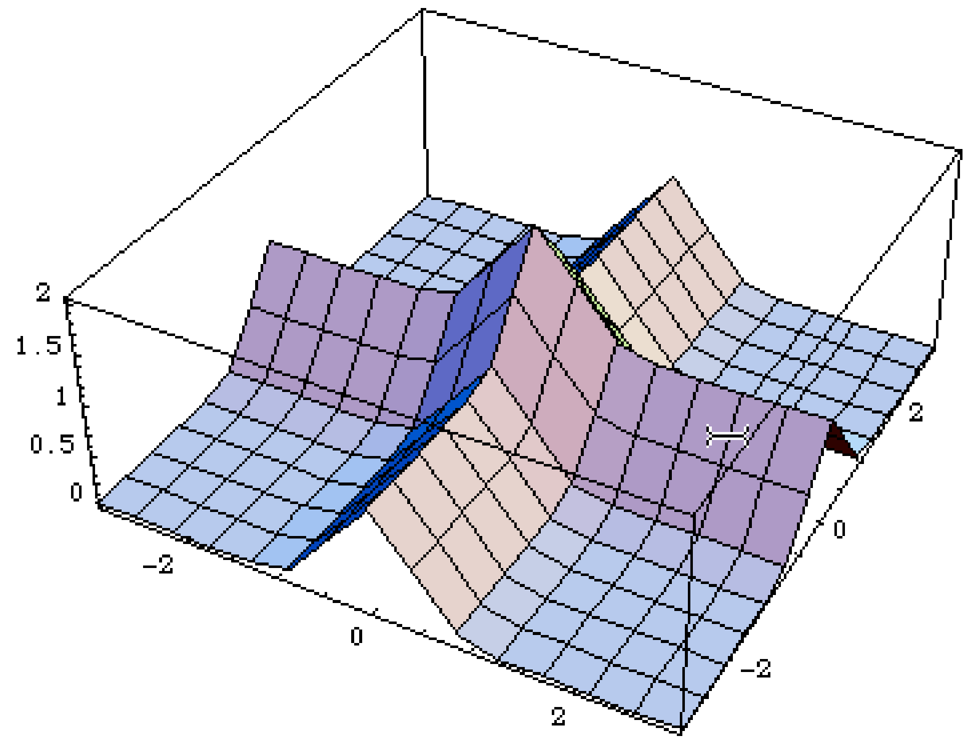
$$p(x) = \frac{1}{N_b \Delta x_i}, \quad x_i - \Delta x_i < x < x_i$$

Importance Sampling

can be generalized to n dimensions:

$$p(\vec{x}) = p(x) \bullet p(y) \bullet p(z) \ldots$$

but the peaks of $f(\vec{x})$ need to be "aligned" to the axis!



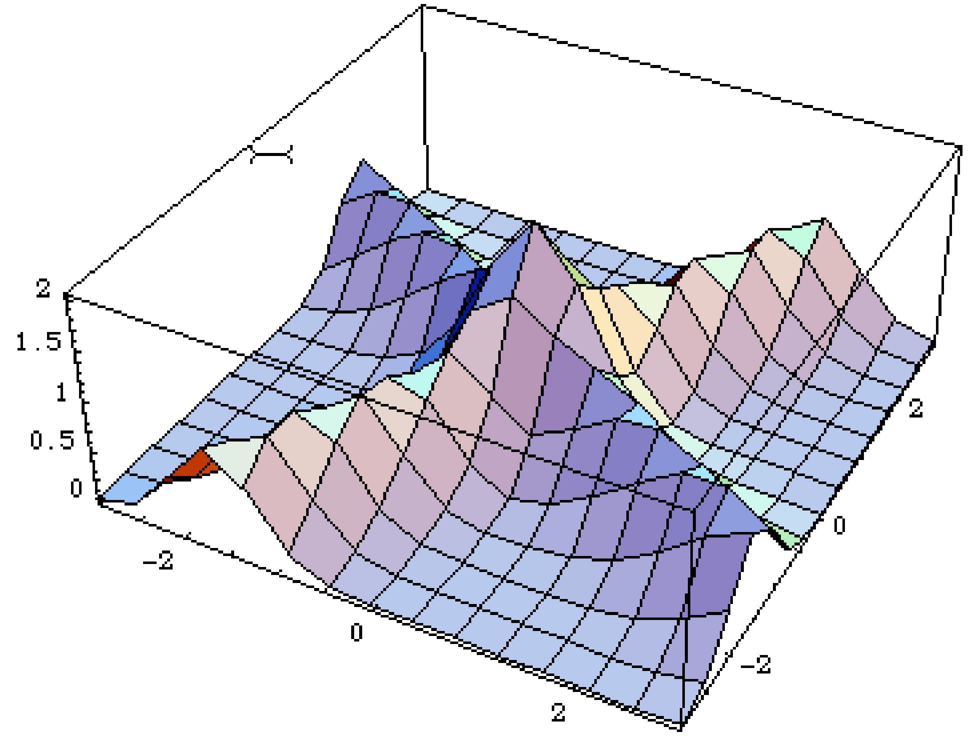
This is ok...

Importance Sampling

can be generalized to n dimensions:

$$p(\vec{x}) = p(x) \cdot p(y) \cdot p(z) \ldots$$

but the peaks of $f(\vec{x})$ need to be "aligned" to the axis!



This is not ok...

Importance Sampling

can be generalized to n dimensions:

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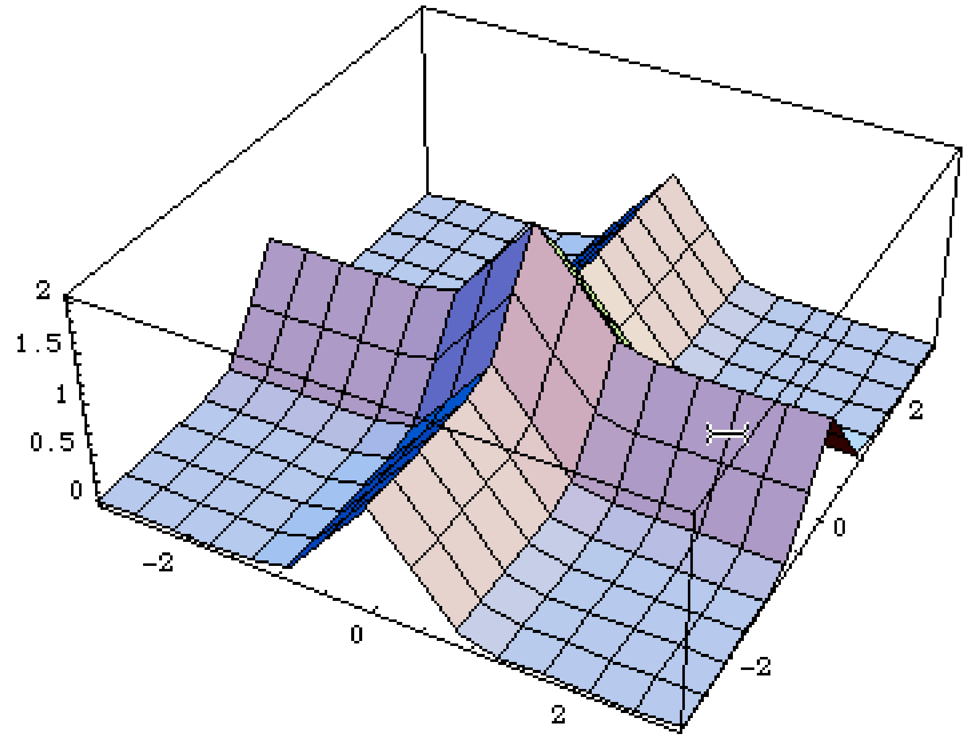
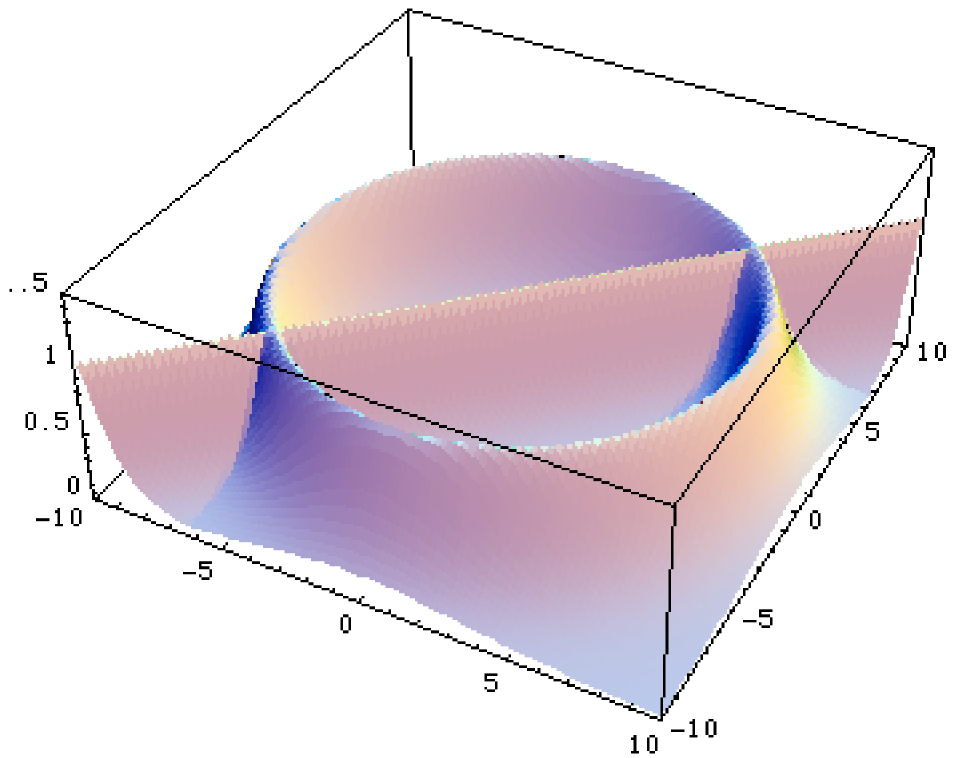


but it is sufficient to make a change of variables!

Multi-channel



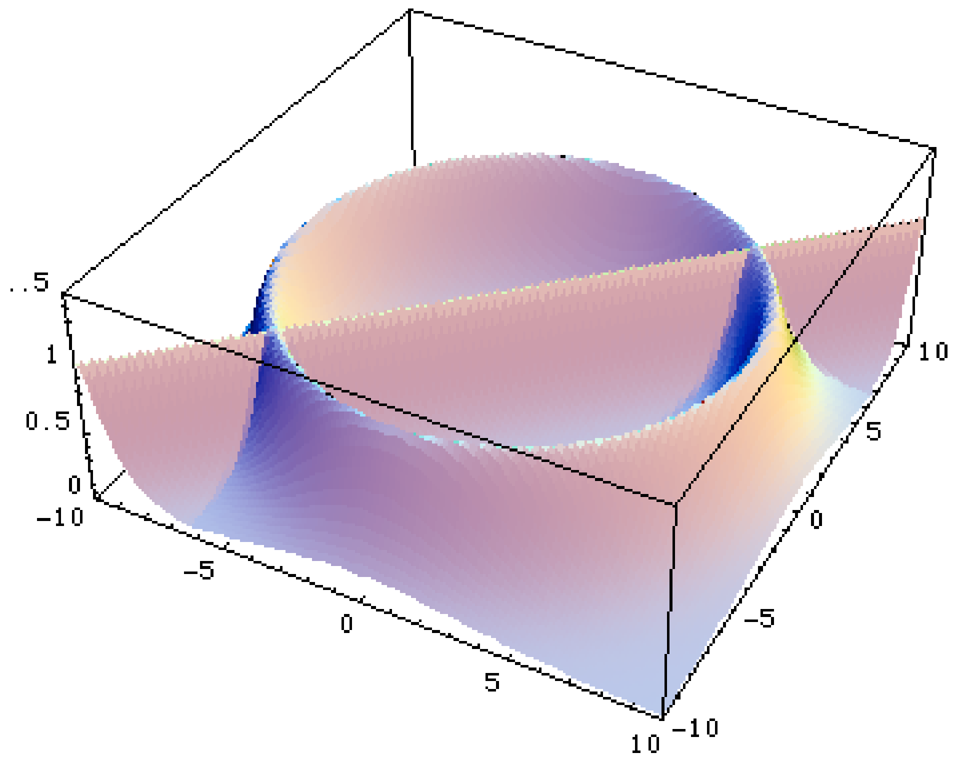
In this case there is no unique tranformation: Vegas is bound to fail!

Solution: use different transformations= channels

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

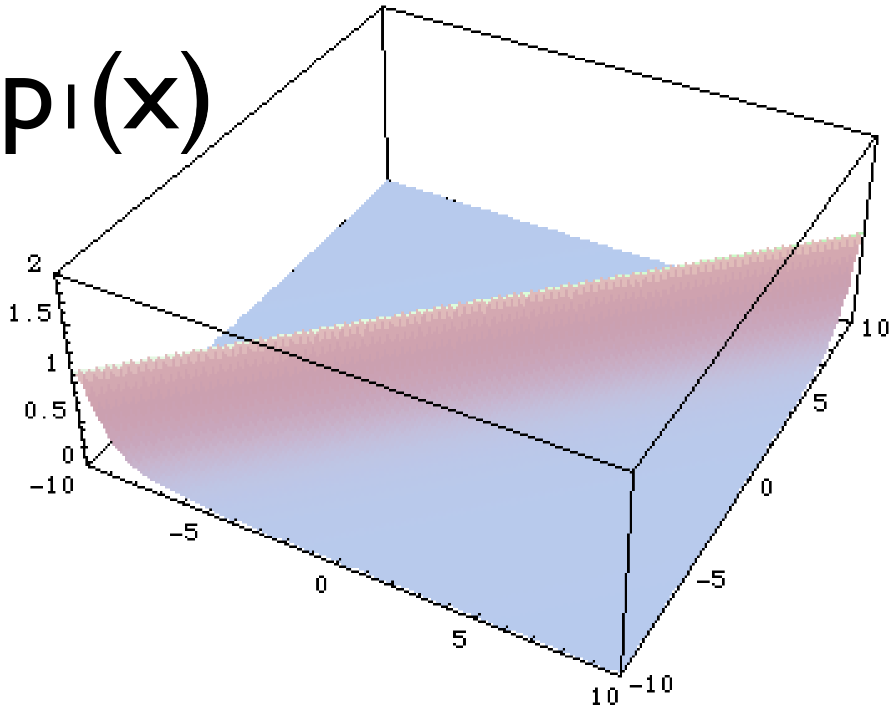
with each $p_i(x)$ taking care of one "peak" at the time

Multi-channel

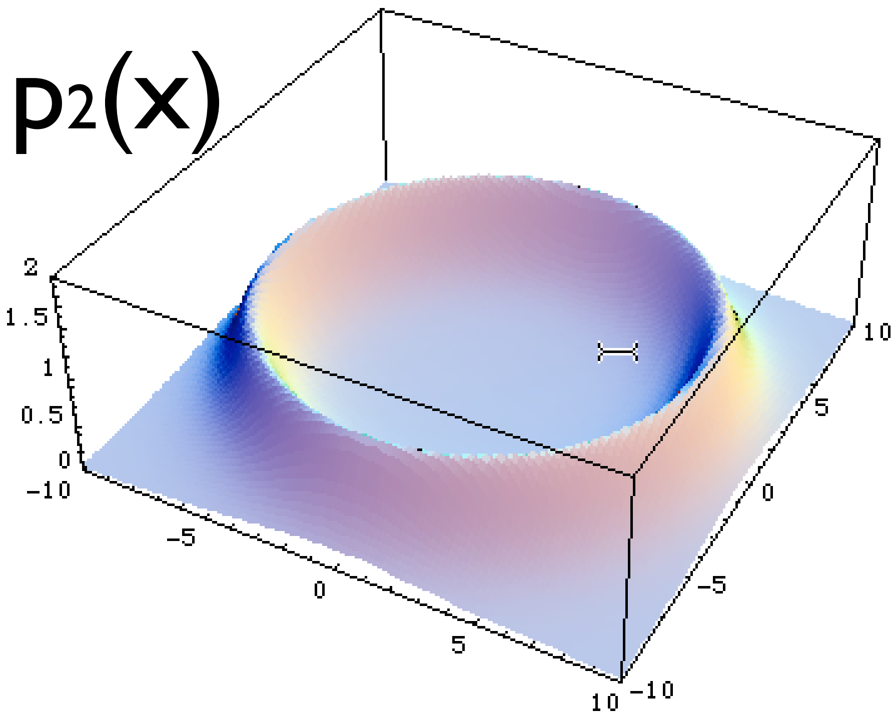


In this case there is no unique tranformation: Vegas is bound to fail!

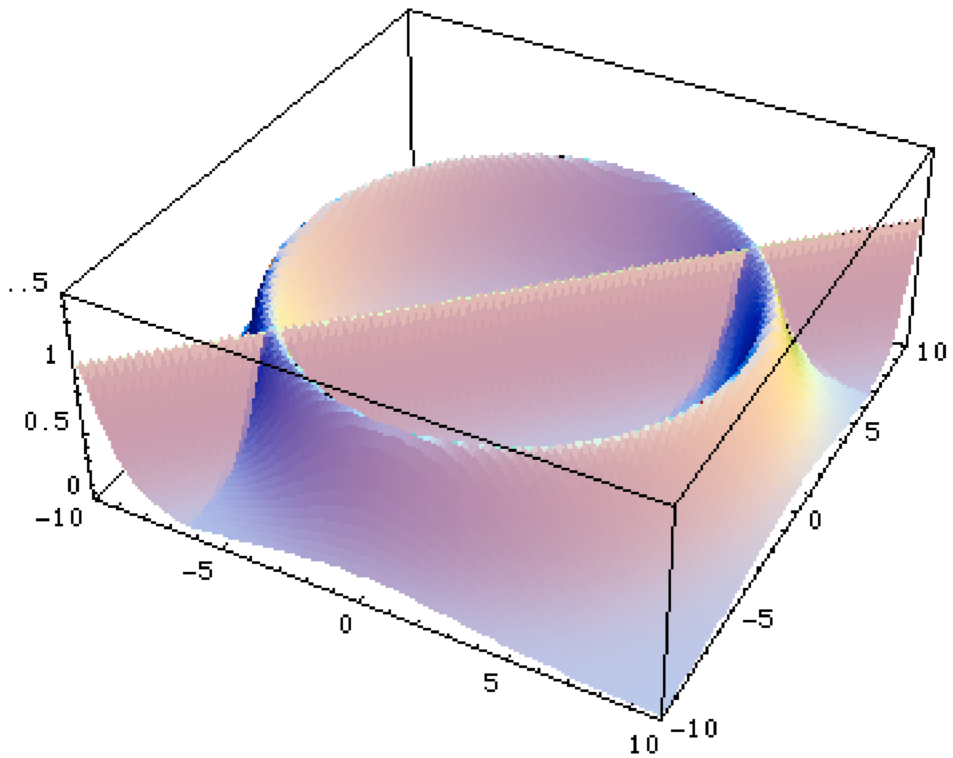
$p_1(x)$



$p_2(x)$



Multi-channel



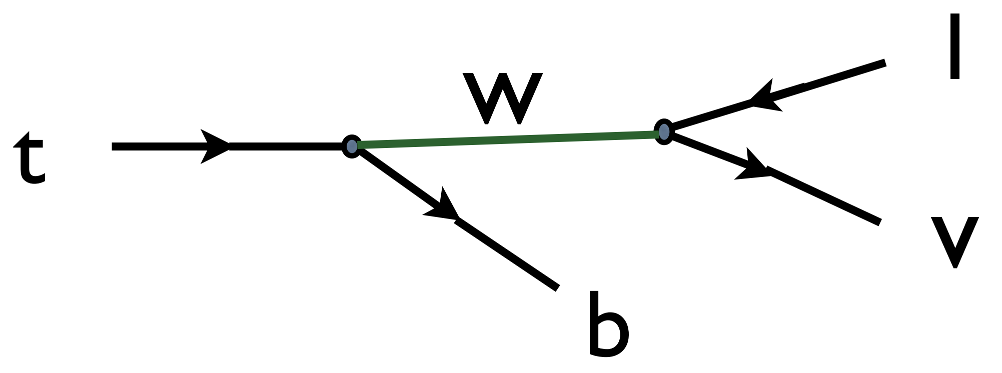
In this case there is no unique tranformation: Vegas is bound to fail!

But if you know where the peaks are (=in which variables) we can use different transformations= channels:

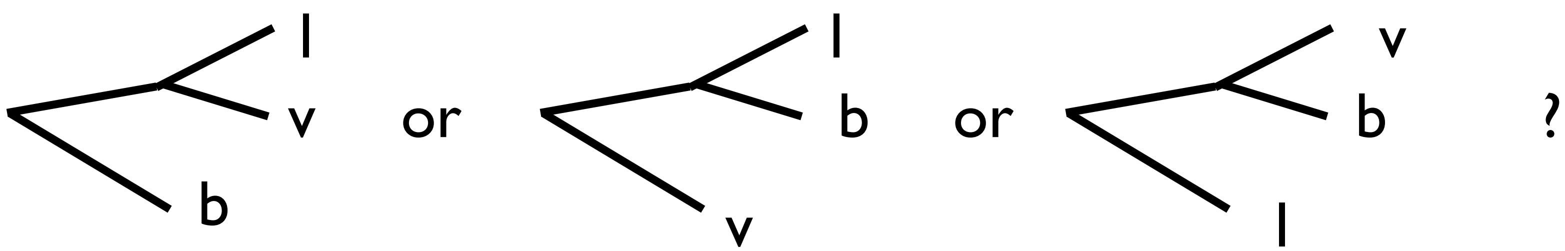
$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \textbf{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

$$I = \int f(x)dx = \sum_{i=1}^{n} \alpha_i \int \frac{f(x)}{p(x)} p_i(x) dx$$

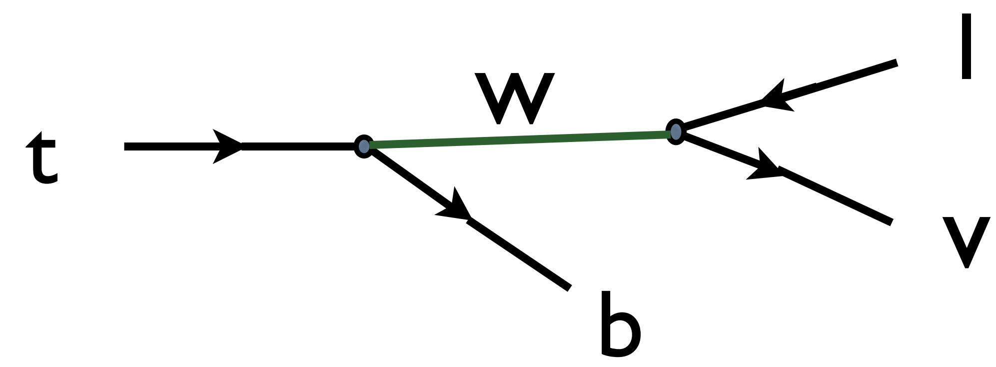
Exercise: Top decay

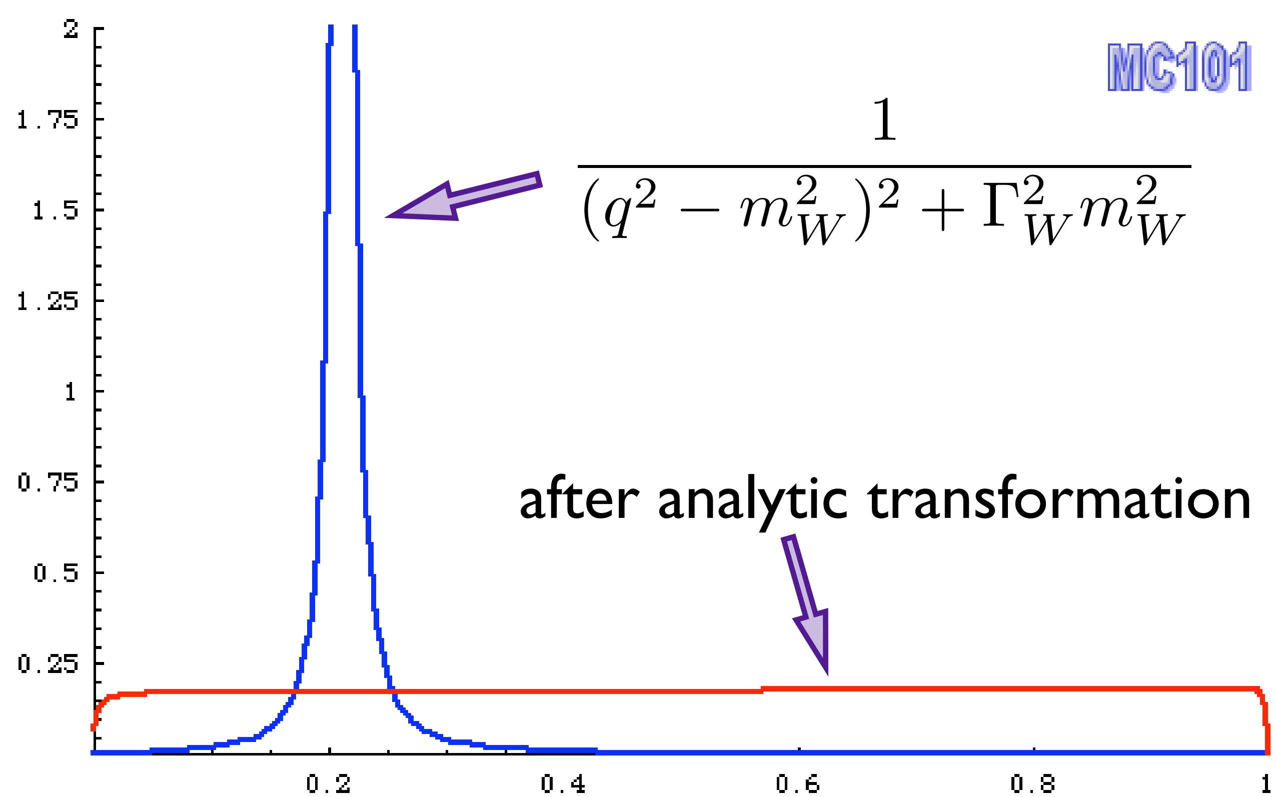


- Easy but non-trivial
- Breit-Wigner peak $$\frac{1}{(q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2}$$ to be "flattened":
- Choose the right "channel" for the phase space:



Exercise: Top decay

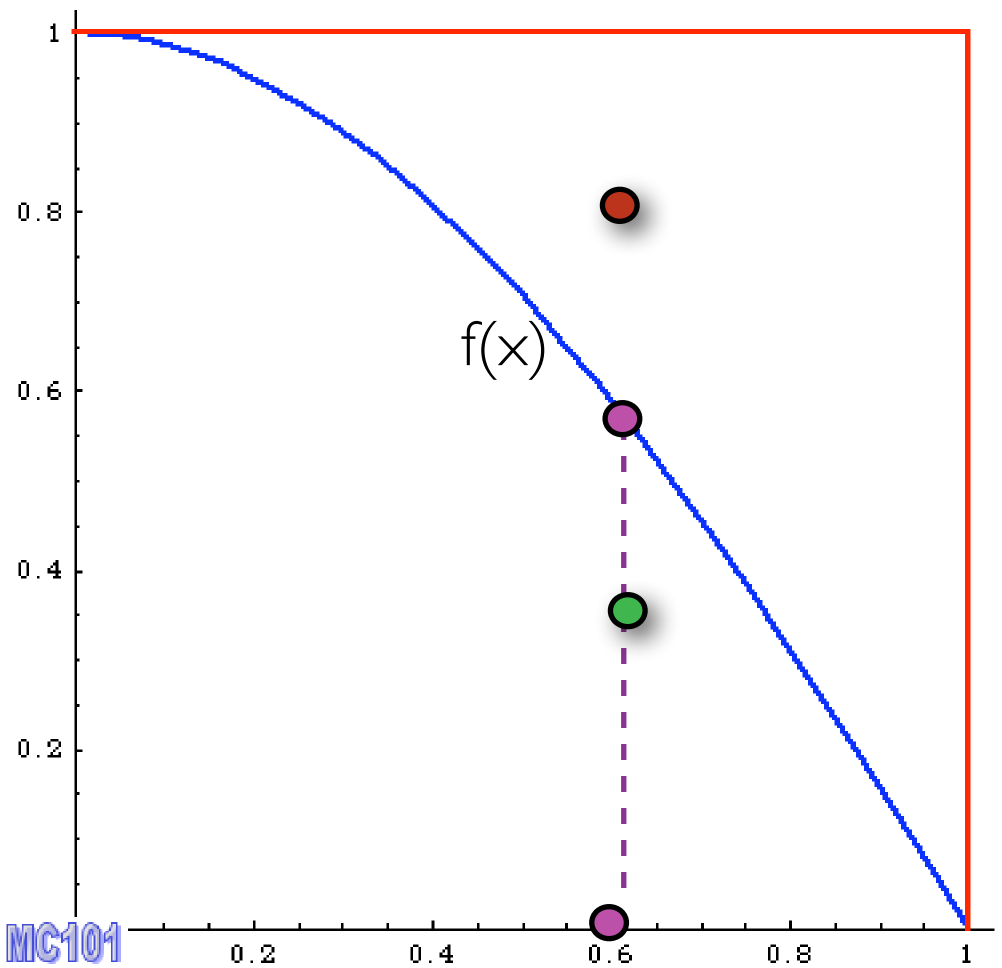




Event generation

- Every phase-space point computed in this way, can be seen as an event (=collision) in a detector
- However, they still carry the "weight" of the matrix elements:
  - events with large weights where the cross section is large
  - events with small weights where the cross section is small
- In nature, the events don't carry a weight:
  - more events where the cross section is large
  - less events where the cross section is small
- How to go from weighted events to unweighted events?

Event generation



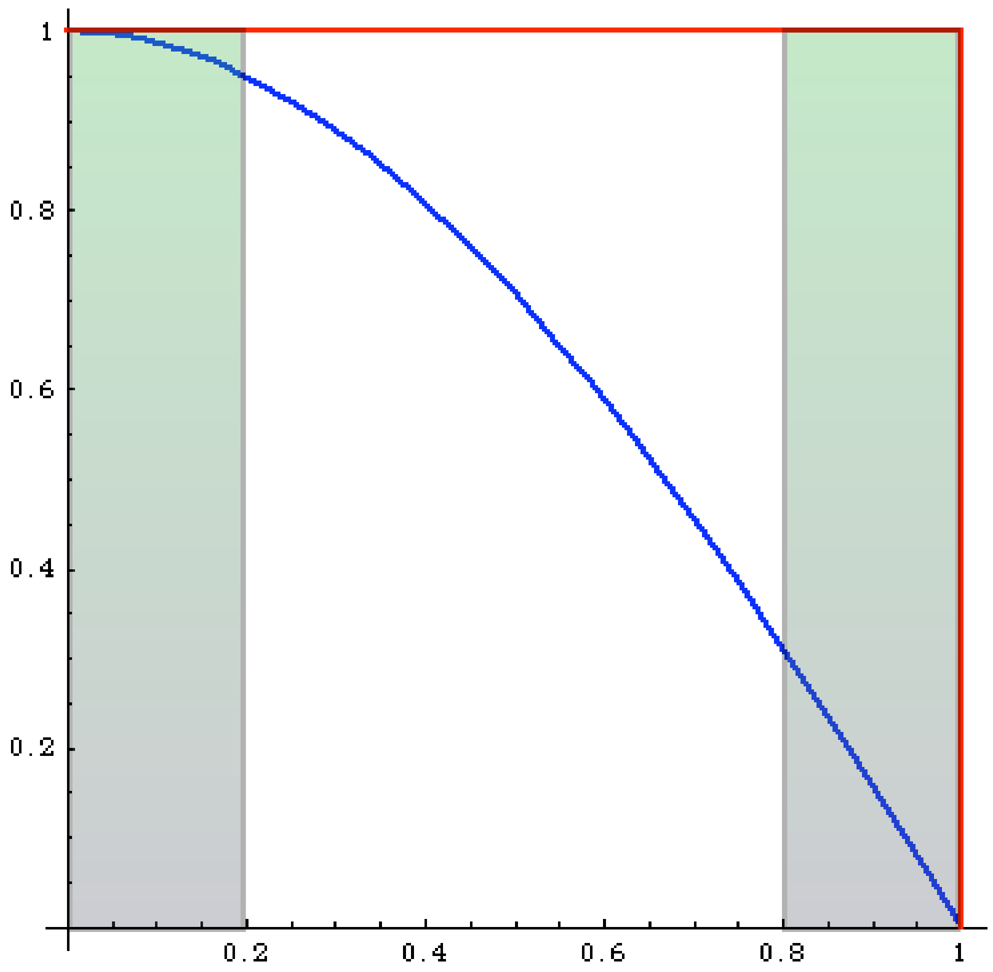
Alternative way

1. (randomly) pick x
2. calculate f(x)
3. (randomly) pick 0<y<fmax
4. Compare:
   if f(x)>y accept event,

   else reject it.

$$\text{Integral} = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation



What's the difference?

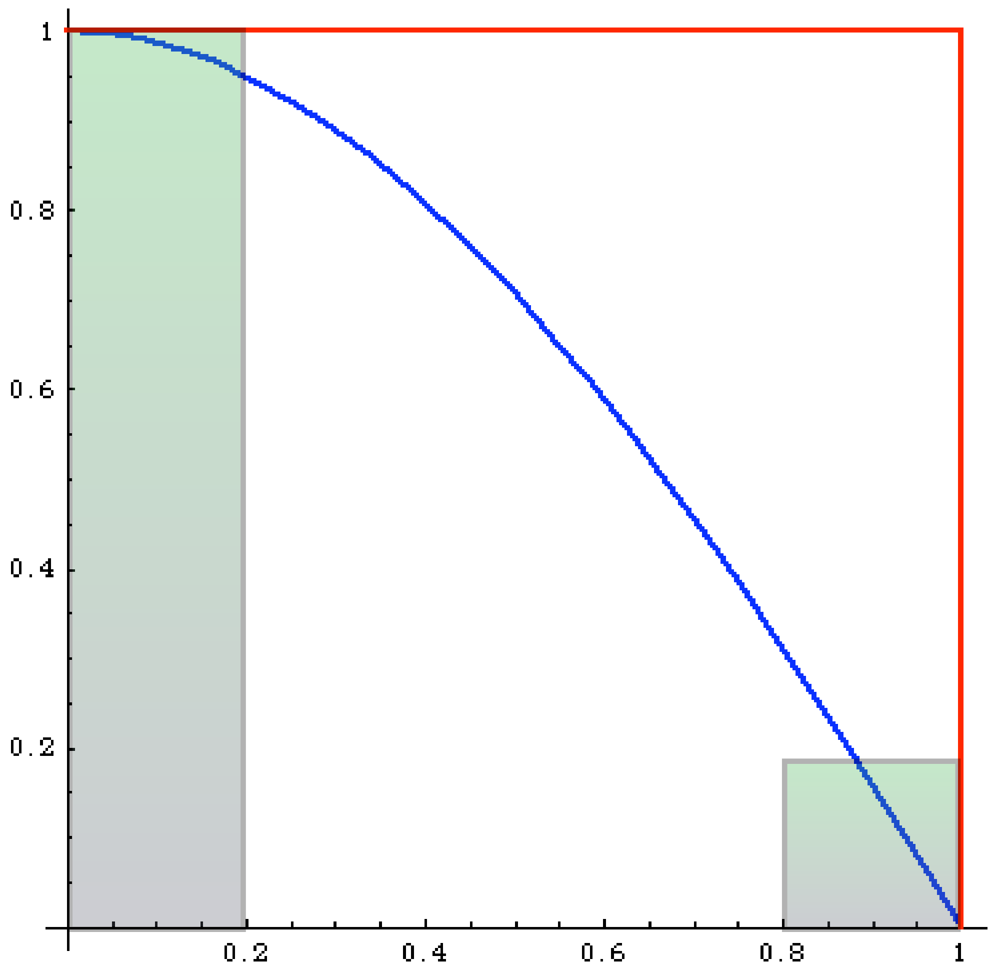
before:

Same # of events in areas of phase space with very different probabilities:

Events must have different weights:

$$w_i = p(x_i)$$

Event generation



What's the difference?

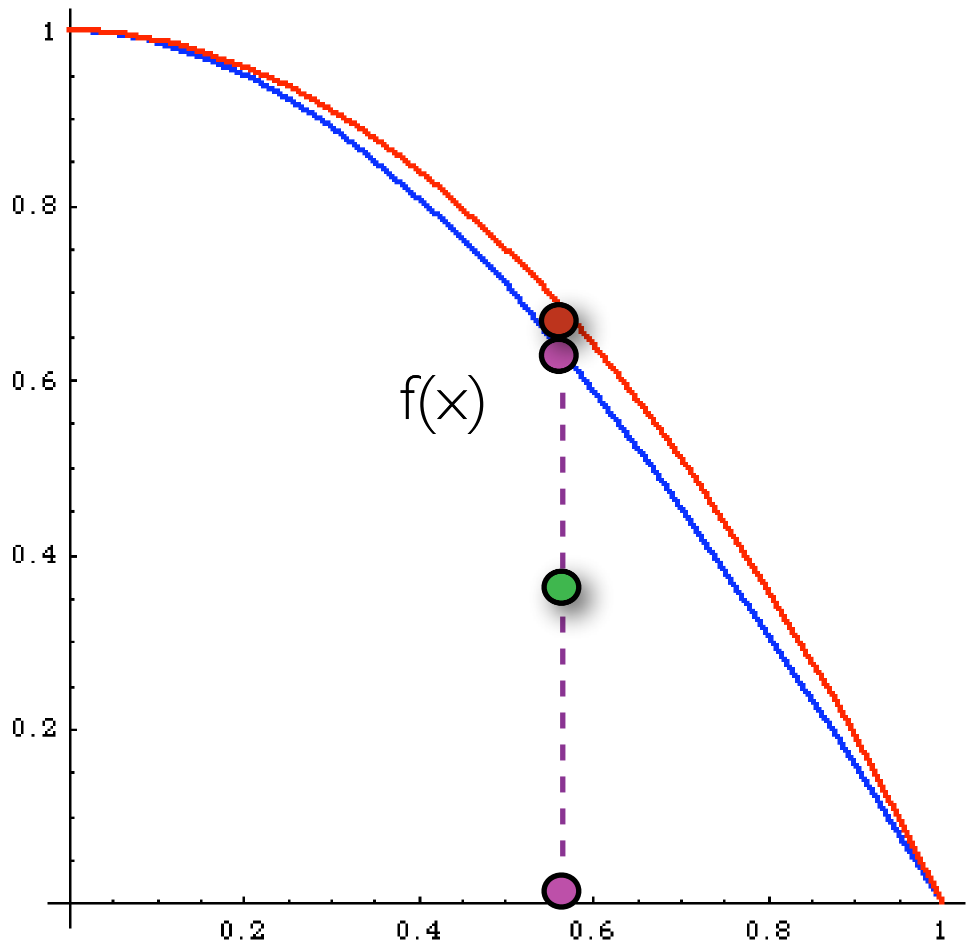
after:

events is proportional to the probability of areas of phase space:

Events have all the same weight ("unweighted")

Events distributed as in Nature

Event Generation



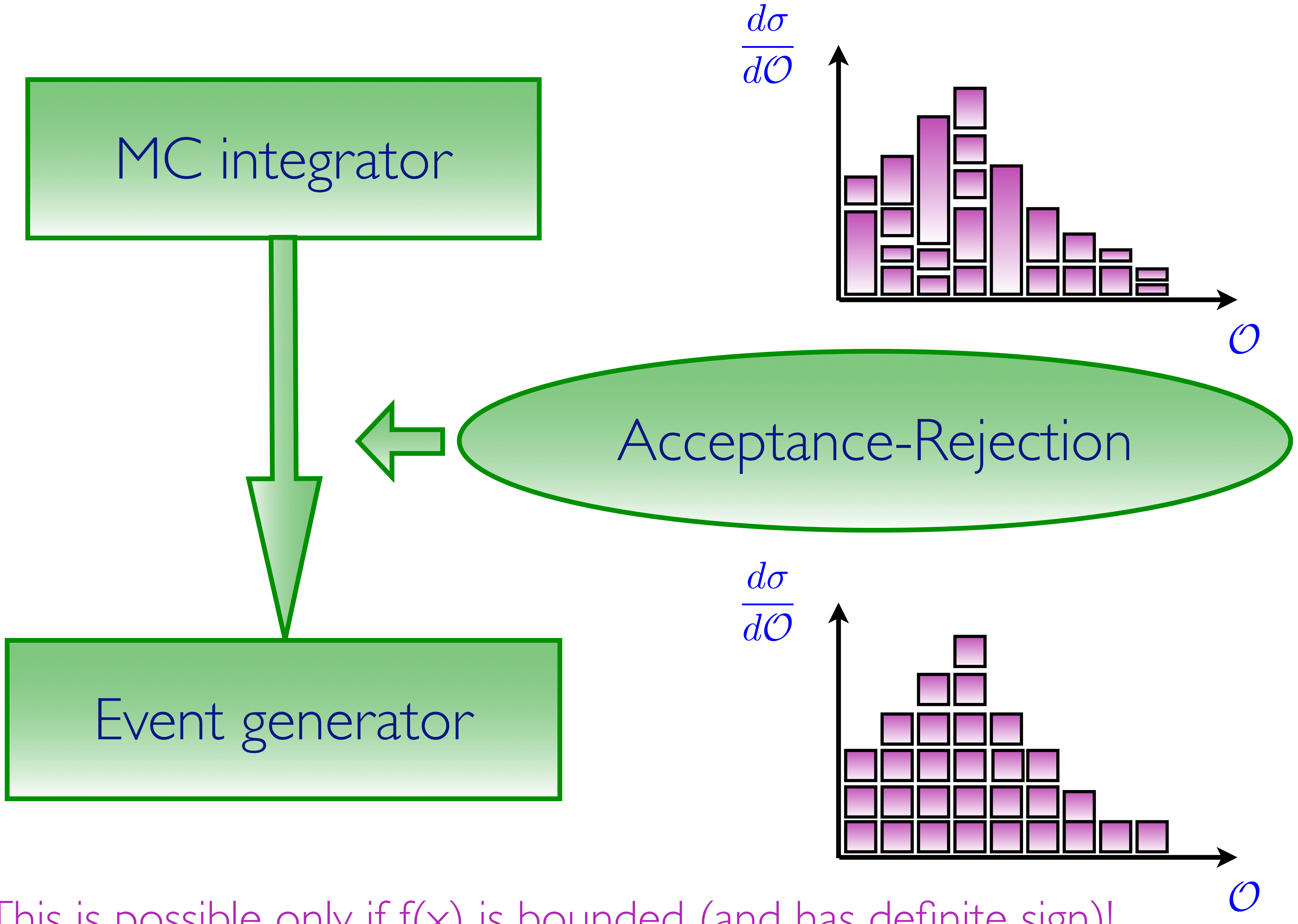
Improved

1. pick x distributed as p(x)
2. calculate f(x) and p(x)
3. pick 0<y<1
4. Compare:
   if f(x)>y p(x) accept event,

else reject it.

much better efficiency!!!

Event generation



☞ This is possible only if f(x) is bounded (and has definite sign)!

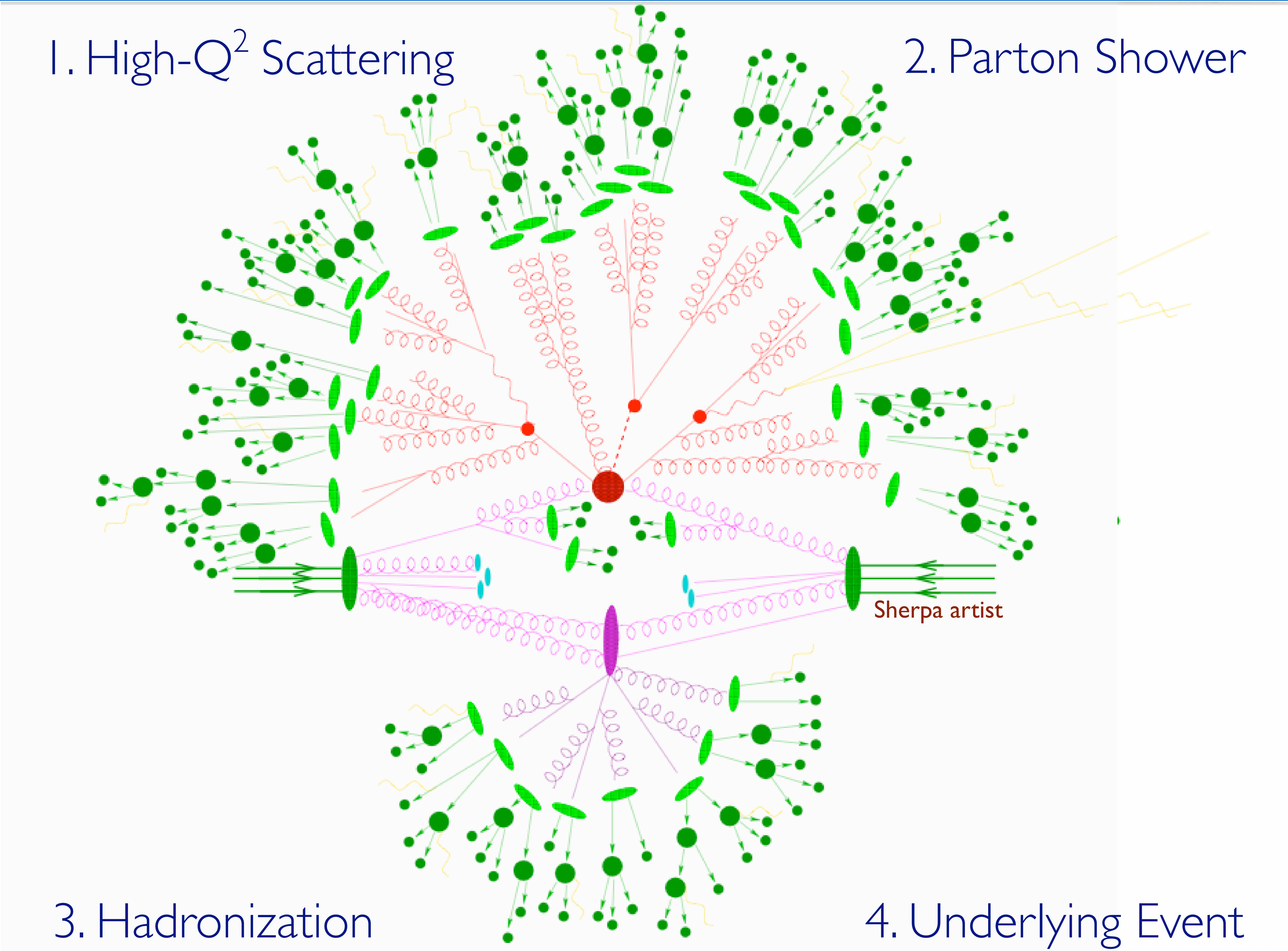
MC Event generator: definition

At the most basic level a Monte Carlo event generator is a program which produces particle physics events with the same probability as they occur in nature (virtual collider).

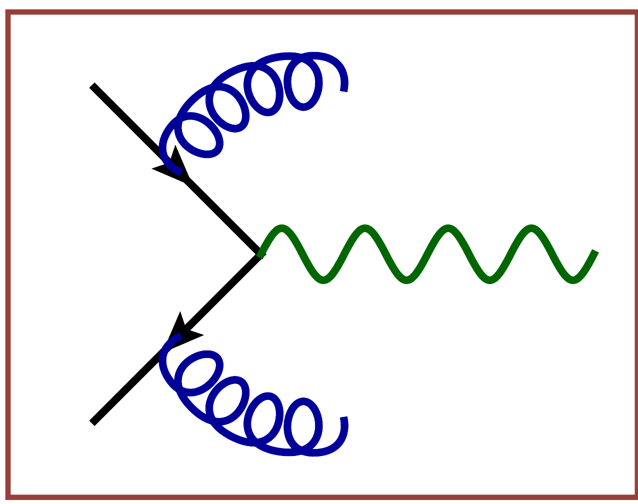
In practice it performs (a possibly large) number of (sometimes very difficult) integrals and then unweights to give the four momenta of the particles that interact with the detector (simulation).

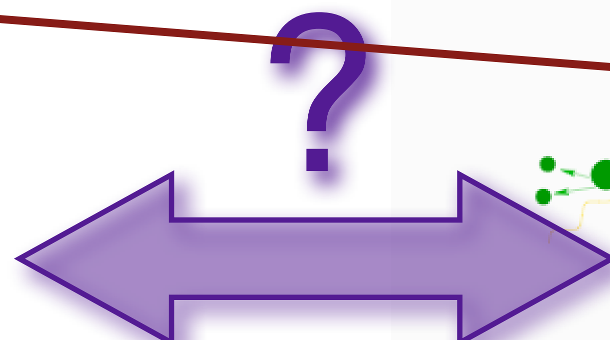
Note that, at least among theorists, the definition of a "Monte Carlo program" also includes codes which don't provide a fully exclusive information on the final state but only cross sections or distributions at the parton level, even when no unweighting can be performed (typically at NLO).

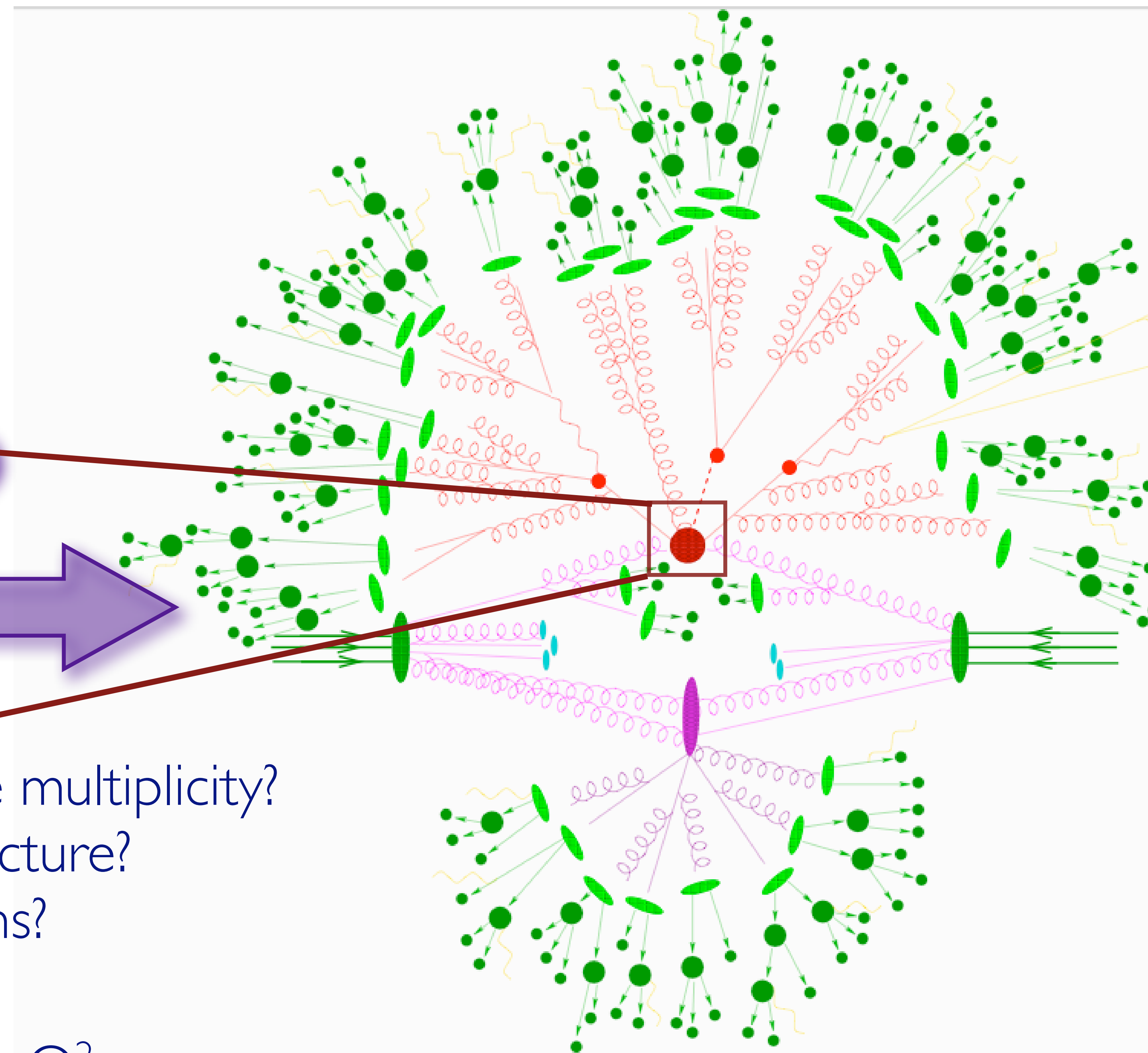
I will refer to these kind of codes as "MC integrators".


1. High-Q^2 Scattering
2. Parton Shower
Sherpa artist
3. Hadronization
4. Underlying Event

Limits of fixed-order predictions







- Particle multiplicity?
- Jet structure?
- Hadrons?

High Q^2

Summary

- Having accurate and flexible simulations tools available for the LHC is a necessity (even more now!!)
- At LO event generation is technically challenging, yet conceptually straightforward.

Credits

To organize this presentation I have benefited from lectures (and actual slides), talks and discussions with many people.
In particular:

- Mike Seymour (MC basics)
- Claude Duhr (FeynRules)
- Johan Alwall (ME+PS merging)
- Rikkert Frederix, Paolo Torrielli (NLO+PS)
- Stefano Frixione, Michelangelo Mangano, Paolo Nason (for QCD, PS, LO, NLO, and more...)
- Many more….

Whom I all warmly thank!!