Parton Shower

Olivier Mattelaer
IPPP/Durham

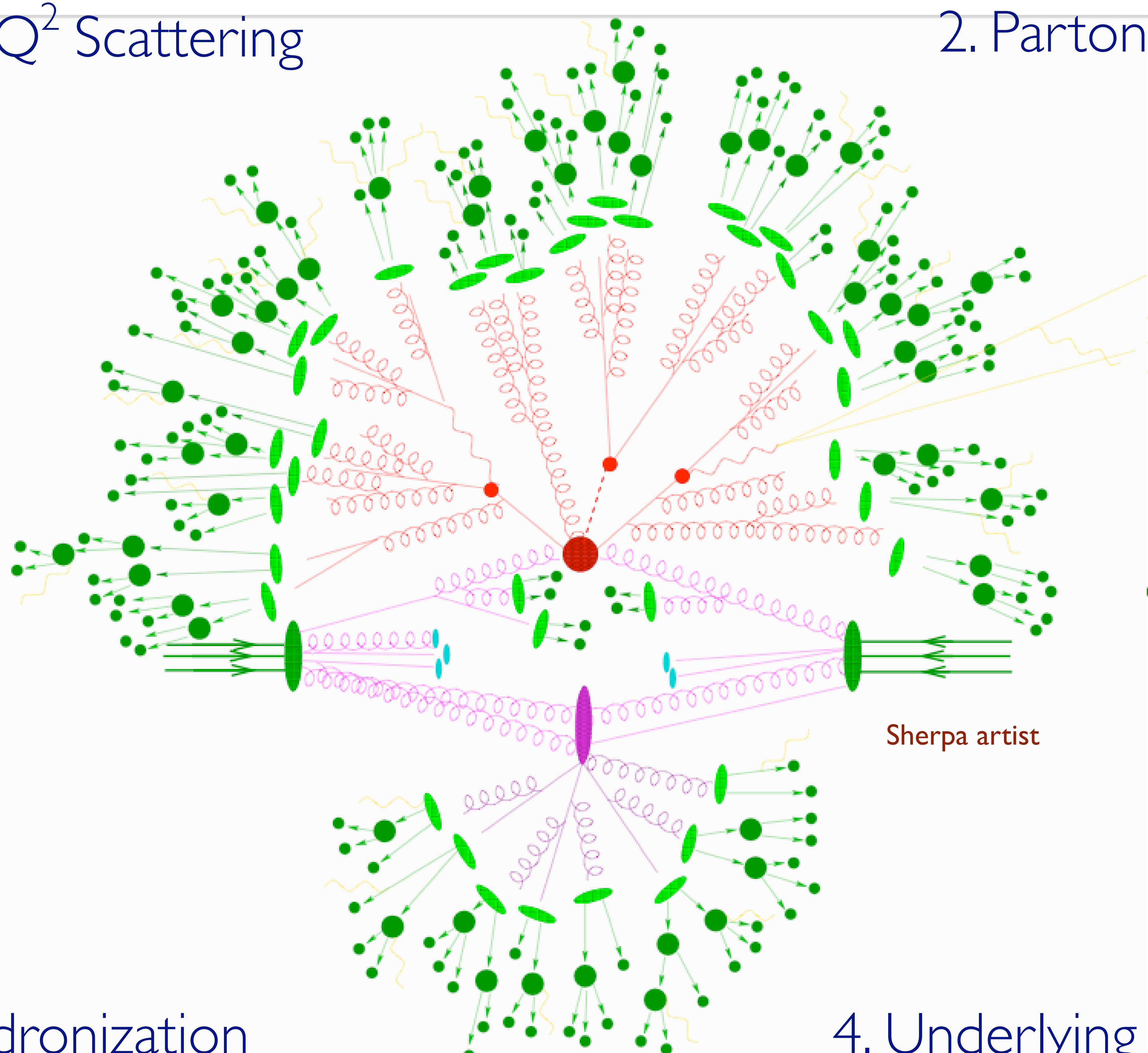
Week Plan

- Monday: FeynRules
- Tuesday: MadGraph5@LO
- Wednesday: Matching/Merging
- Thursday: NLO
- Friday: Unleashed the tools

What are the MC for?

1. High-Q^2 Scattering

2. Parton Shower



3. Hadronization

4. Underlying Event

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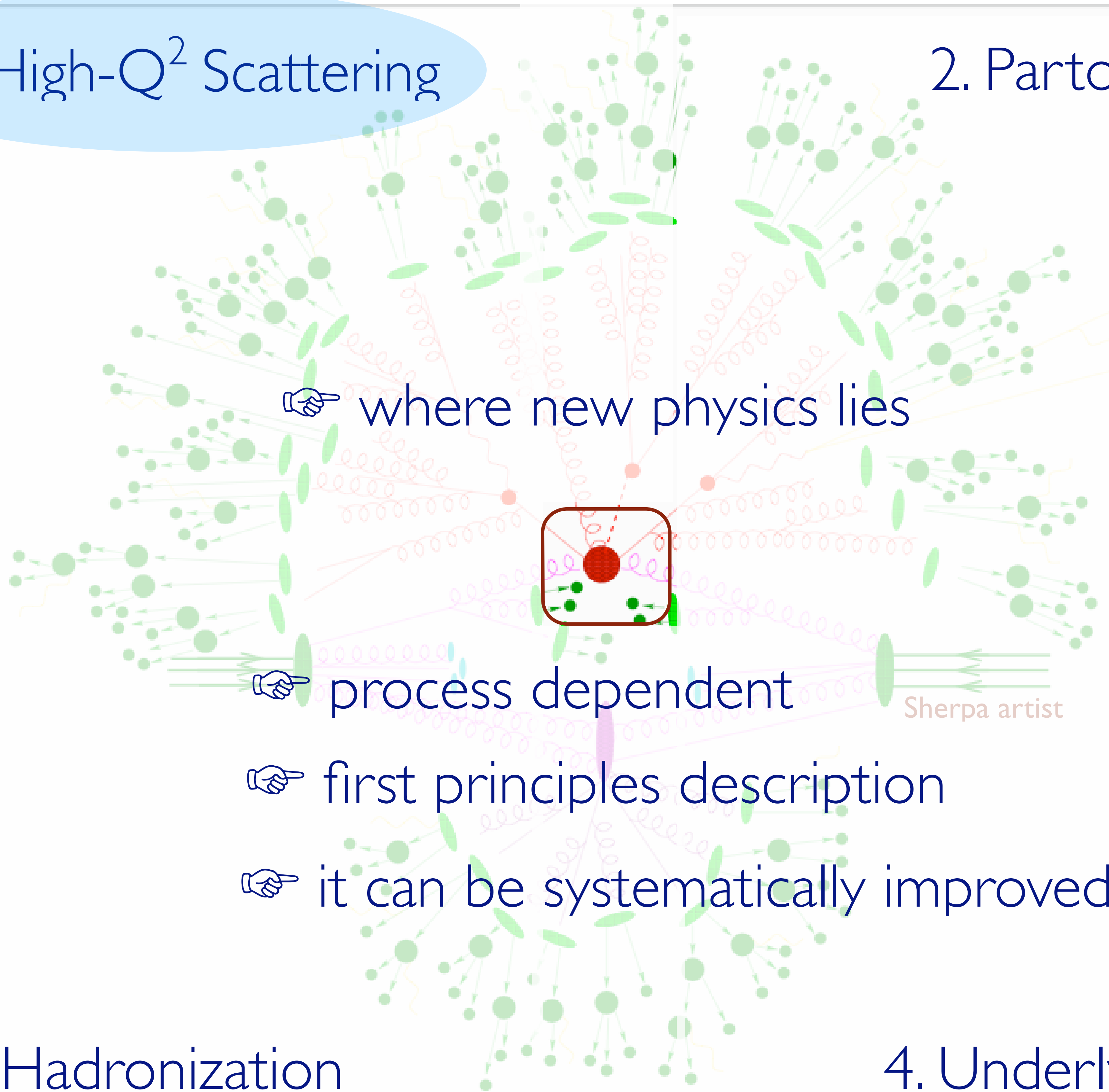

☞ where new physics lies

☞ process dependent

☞ first principles description

☞ it can be systematically improved

3. Hadronization

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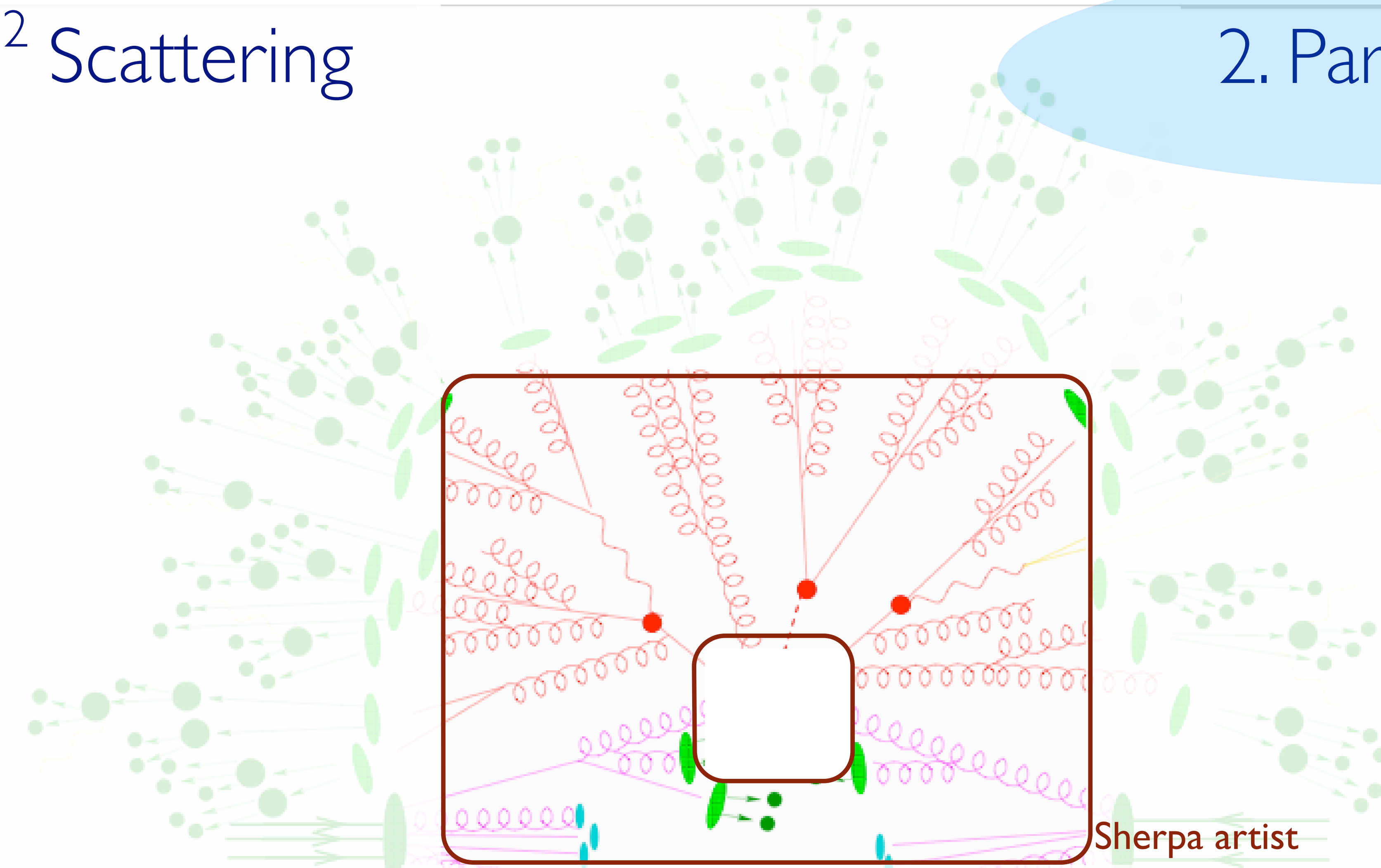

- ☞ QCD -"known physics"
- ☞ universal/ process independent
- ☞ first principles description

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4. Underlying Event

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- Remember that parton-level cross sections for a hard process are inclusive in anything else.
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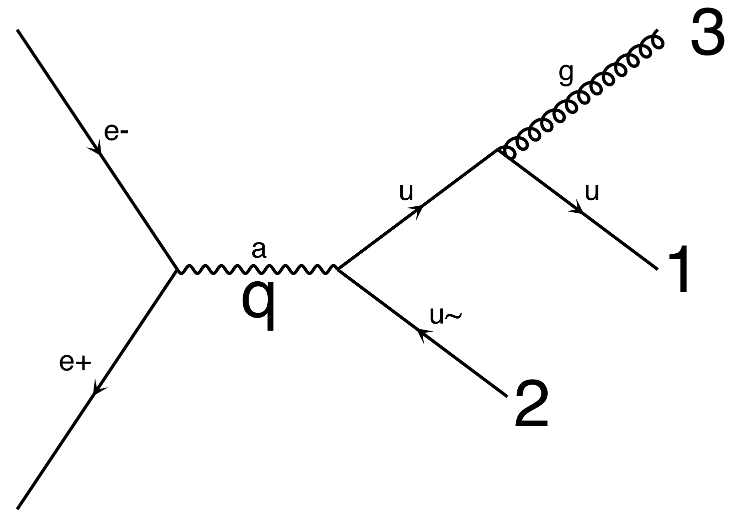
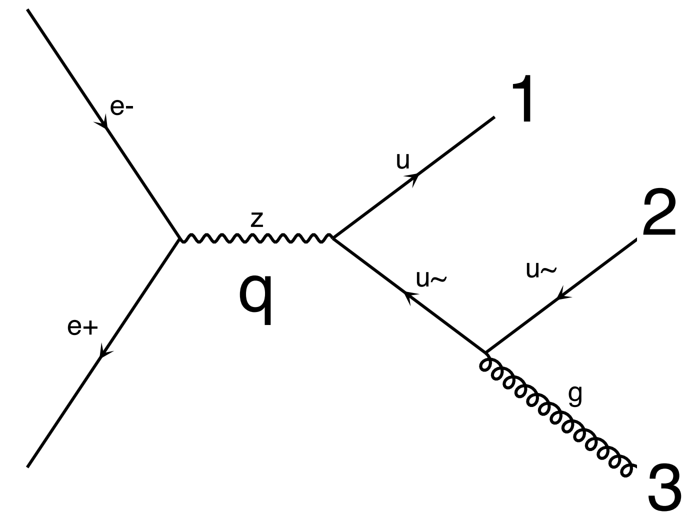
- Remember that parton-level cross sections for a hard process are inclusive in anything else.
  E.g. for LO Drell-Yan production **all** radiation is included via PDFs (apart from non-perturbative power corrections)
- And finally we want to turn partons into hadrons (hadronization)....

$$e^+e^- \to q\bar{q}g$$





First Example

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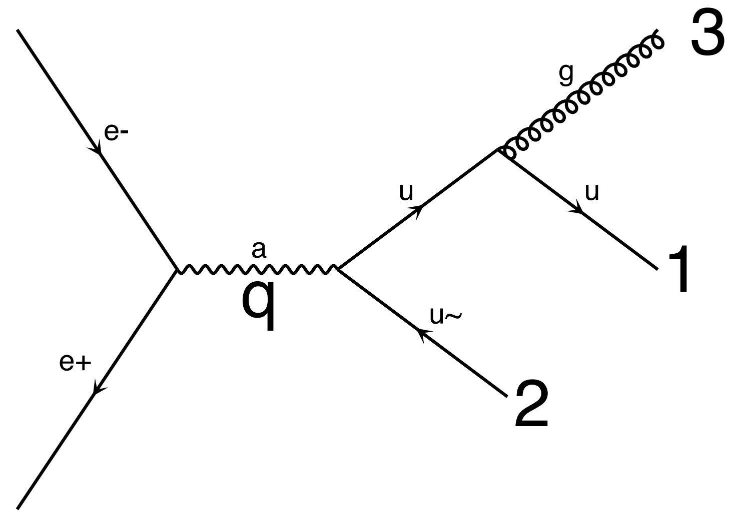

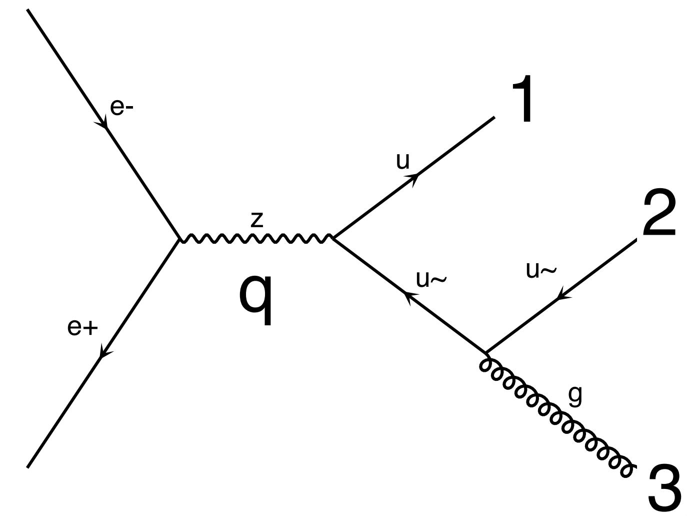

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

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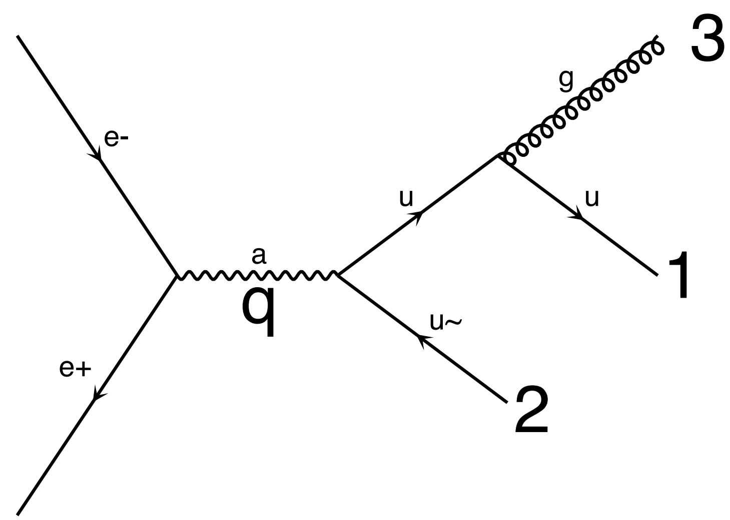

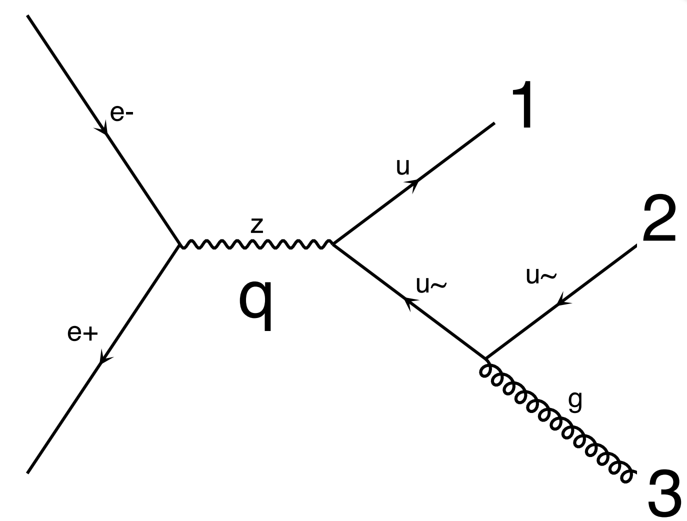

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- Divergent at $x_1 = 1$ and $x_2 = 1$
  - Soft Divergencies
  - Collinear Divergencies

$$(1-x_1) = \frac{x_2 x_3}{2}(1 - cos\theta_{23})$$

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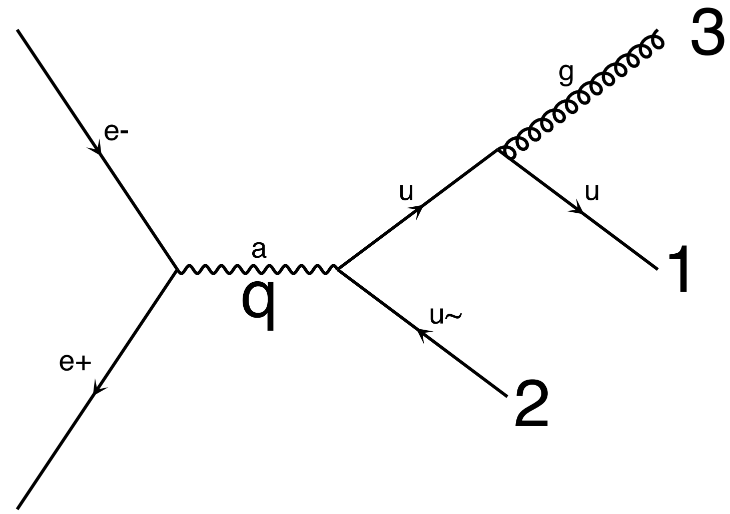

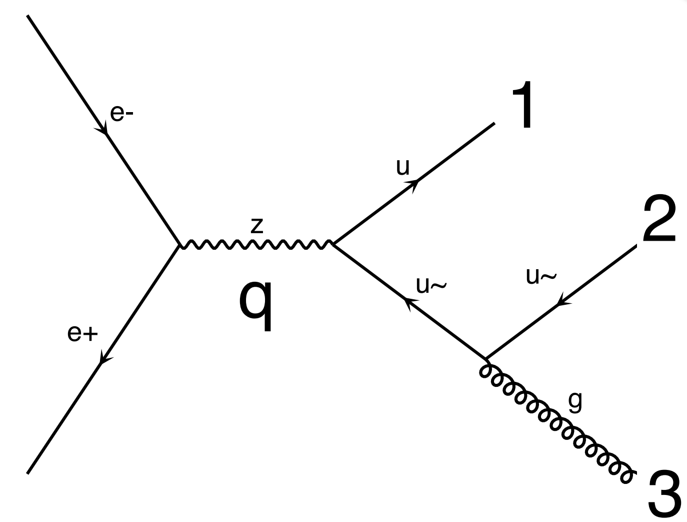

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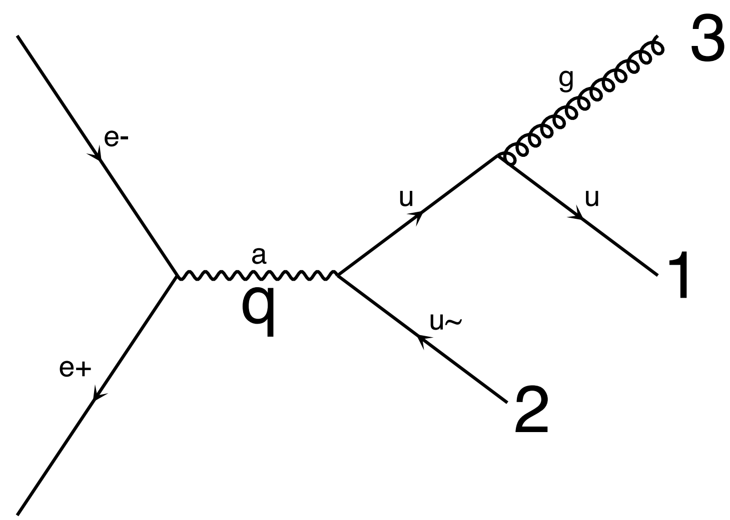

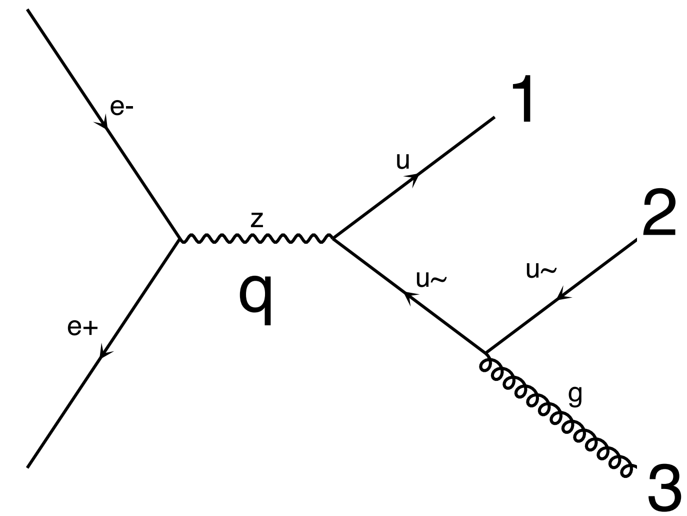

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  - Split our integral in two

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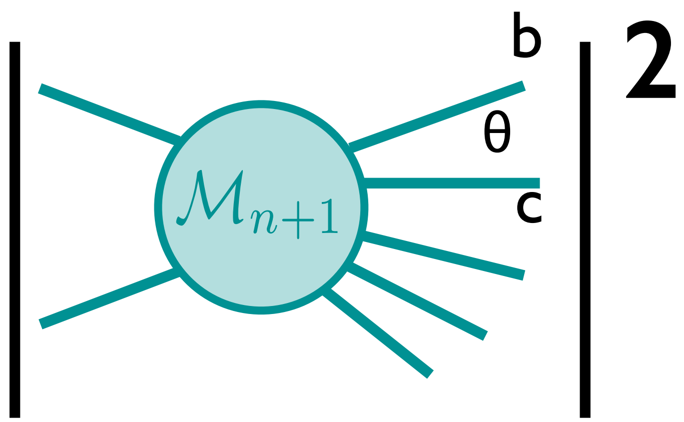
$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

☞ z fraction of energy

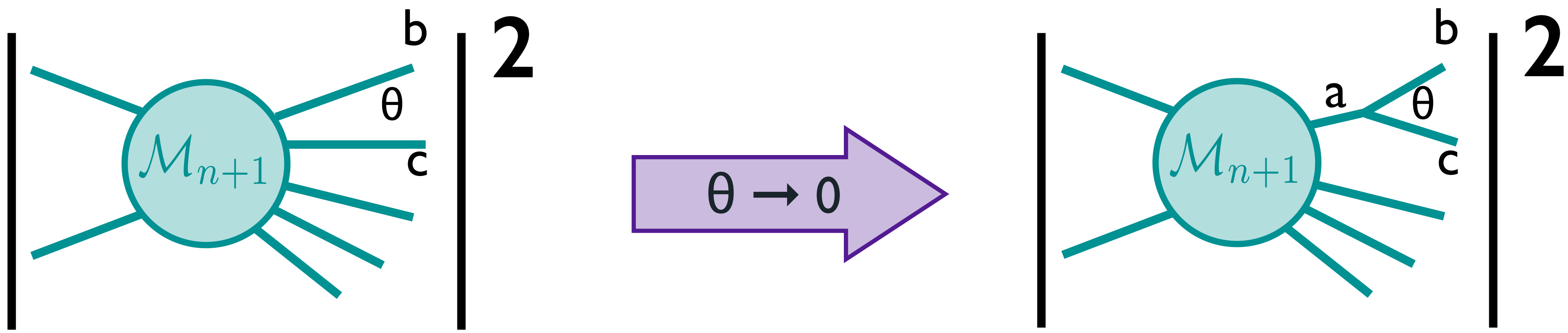
☞ **Generic Formula**

Collinear factorization



- Consider a process for which two particles are separated by a small angle θ.
- In the limit of $\theta \to 0$ the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.

Collinear factorization



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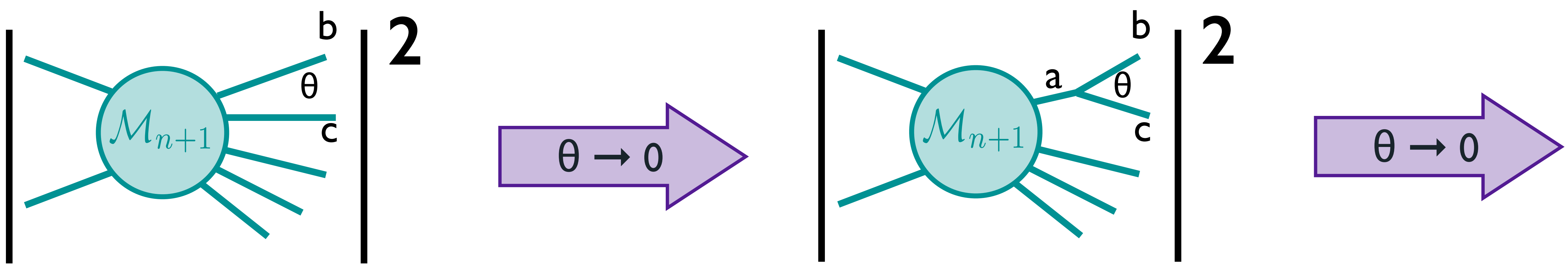

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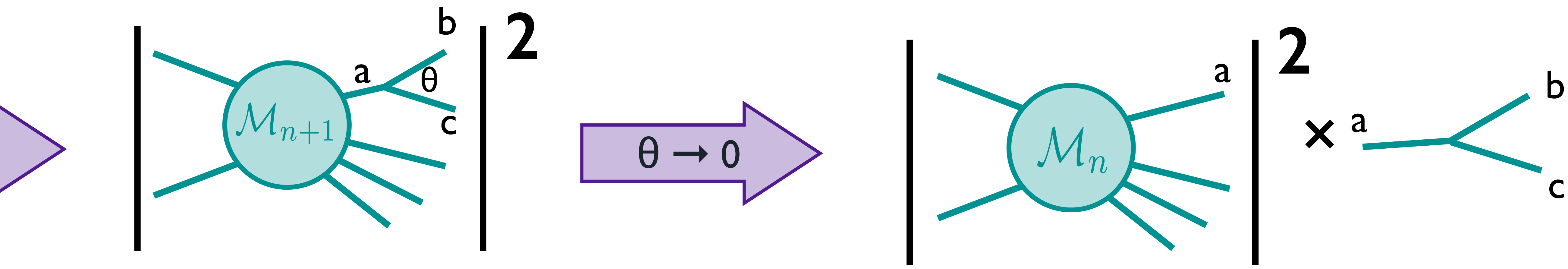

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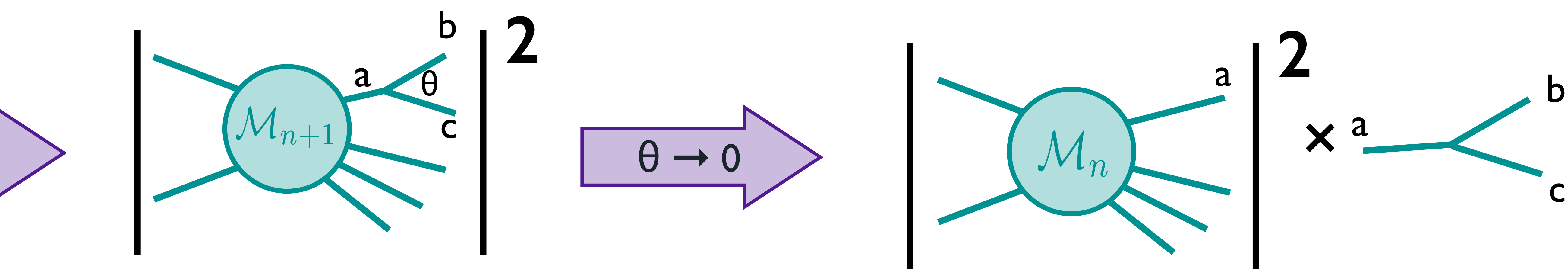

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- The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability.

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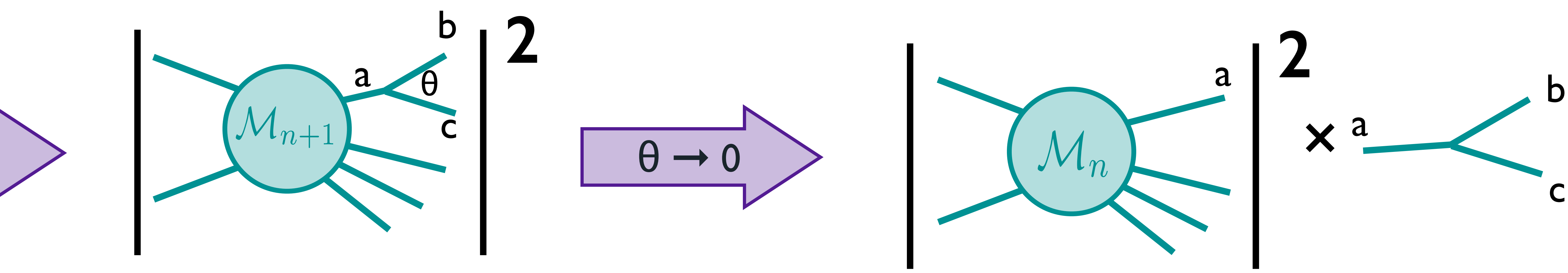

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- The first task of Monte Carlo physics is to make this statement quantitative.

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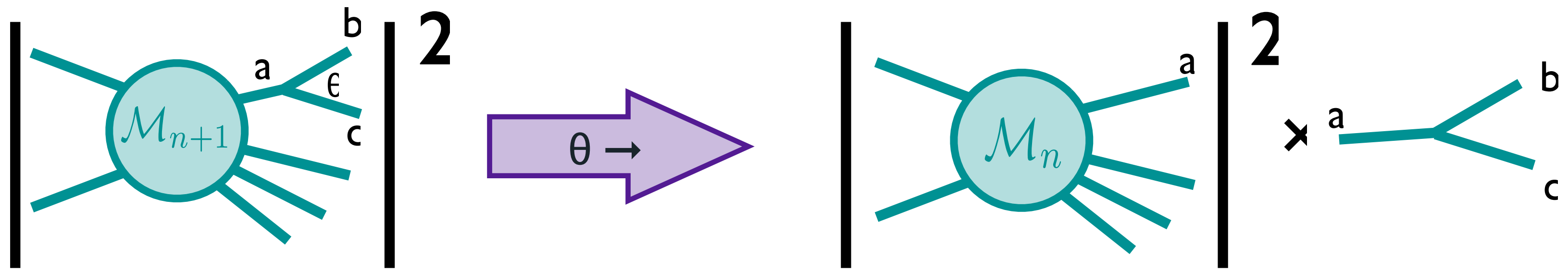
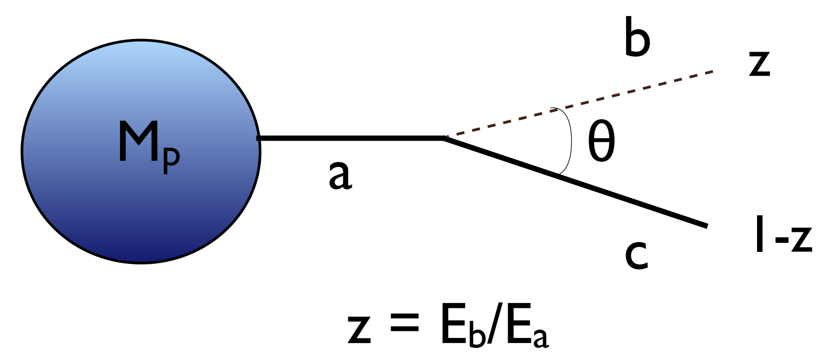


- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b + p_c)^2} \simeq \frac{1}{2E_bE_c(1-\cos\theta)} = \frac{1}{t}$$



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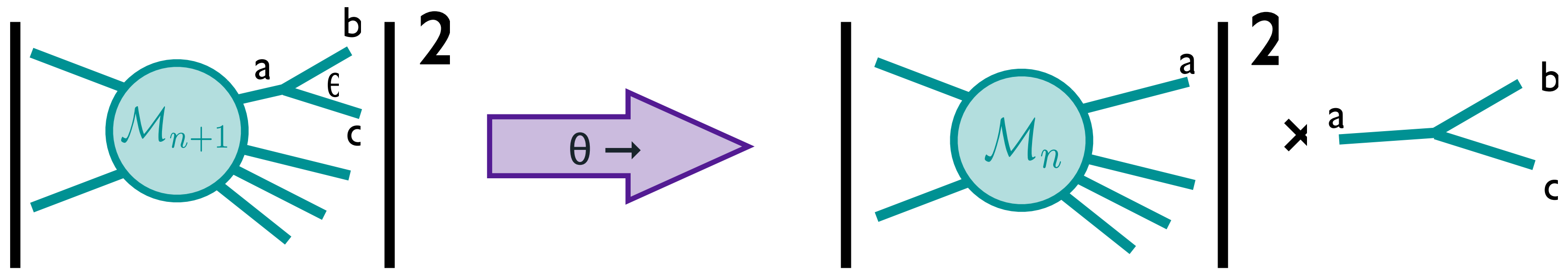

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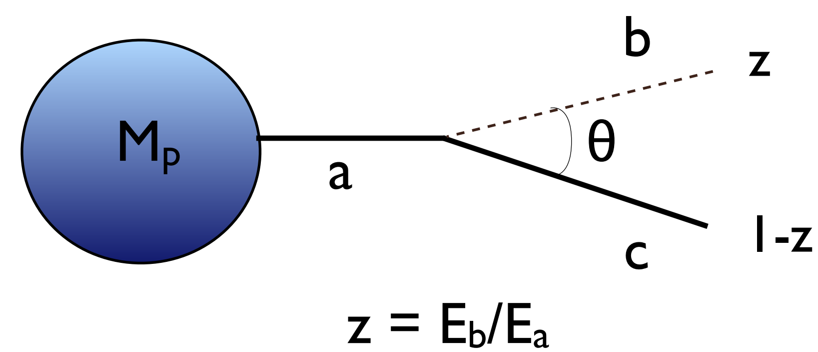

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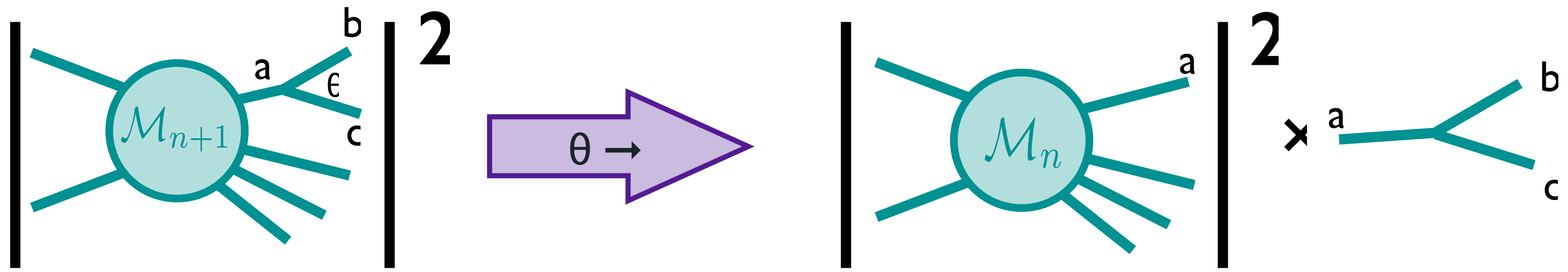

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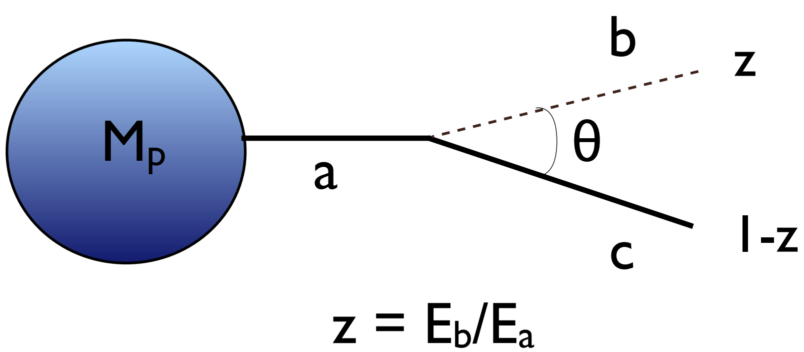

soft and collinear

divergencies

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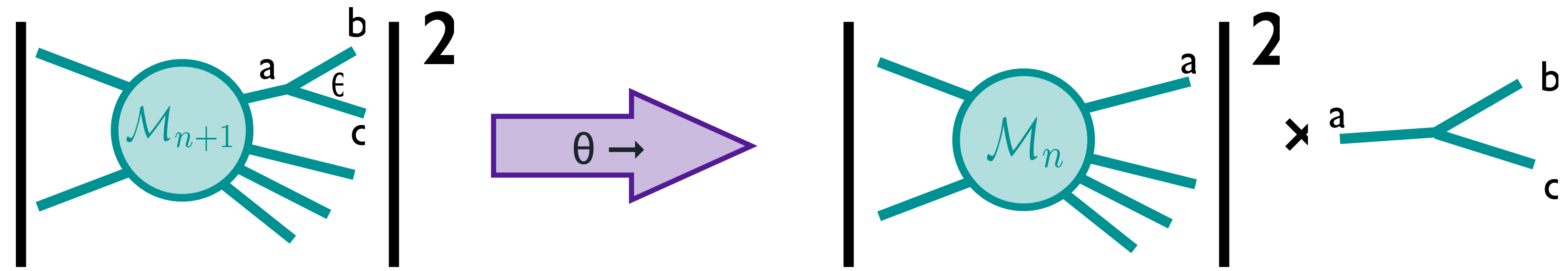

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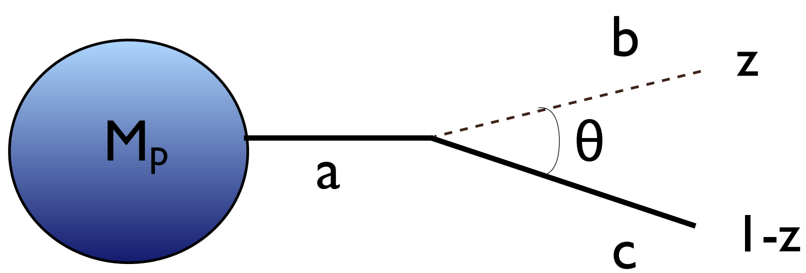

$z = E_b/E_a$

soft and collinear

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Collinear factorization:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

when θ is small.

Collinear factorization

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- **t** can be called the 'evolution variable' (will become clearer later): it can be the virtuality **m²** of particle a or its **p_T^2** or **$E^2\theta^2$** ...

$$d\theta^2/\theta^2 = dm^2/m^2 = dp_T^2/p_T^2$$

$$m^2 \simeq z(1-z)\theta^2 E_a^2$$
$$p_T^2 \simeq zm^2$$

  - It represents the hardness of the branching and tends to 0 in the collinear limit.
  - Different choice of 'evolution parameter' in different Parton-shower code

Collinear factorization

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- **z** is the "energy variable": it is defined to be the energy fraction taken by parton **b** from parton **a.** It represents the energy sharing between **b** and **c** and tends to 1 in the soft limit (parton c going soft)
- **Φ** is the azimuthal angle. It can be chosen to be the angle between the polarization of a and the plane of the branching.

Parton Shower basics

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

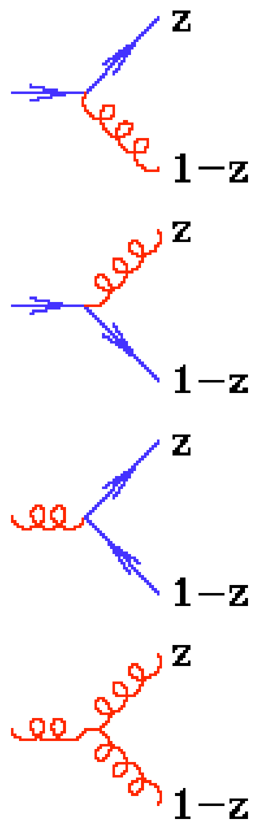
The spin averaged (unregulated) splitting functions for the various types of branching are (Altarelli-Parisi):

$$\hat{P}_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)}\right],$$

$$\hat{P}_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$\hat{P}_{qg}(z) = T_R\left[z^2+(1-z)^2\right],$$

$$\hat{P}_{gg}(z) = C_A\left[\frac{z}{(1-z)}+\frac{1-z}{z}+z(1-z)\right].$$



$C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}.$

Parton Shower basics

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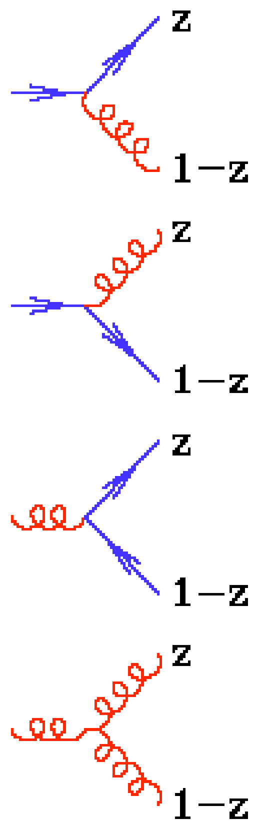
\hat{P}_{gg}(z) &= C_A\left[\frac{z}{(1-z)}+\frac{1-z}{z}+z(1-z)\right].
\end{aligned}$$



$C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}.$

Comments:

* Gluons radiate the most

* There are soft divergences in z=1 and z=0.

* P_{qg} has no soft divergences.

Argument of α_S

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Each choice of argument for α_S is equally acceptable at the leading-logarithmic accuracy. However, there is a choice that allows one to resum certain classes of subleading logarithms.

- The higher order corrections to the partons splittings imply that the splitting kernels should be modified: $P_{a \longrightarrow bc}(z) \longrightarrow P_{a \longrightarrow bc}(z) + \alpha_s P'_{a \longrightarrow bc}(z)$

For $g \longrightarrow gg$ branchings $P'_{a \longrightarrow bc}(z)$ diverges as $-b_0 \log[z(1-z)]\ P_{a \longrightarrow bc}(z)$ (just z or 1-z if quark is present)

- Recall the one-loop running of the strong coupling:

$$\alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1 + \alpha_S(\mu^2) b_0 \log \frac{Q^2}{\mu^2}} \sim \alpha_S(\mu^2)\left(1 - \alpha_S(\mu^2) b_0 \log \frac{Q^2}{\mu^2}\right)$$

- We can therefore include the $P'(z)$ terms by choosing $p_T^2 \sim z(1-z)Q^2$ as argument of α_S:

$$\begin{aligned}\alpha_S(Q^2)\left(P_{a\to bc}(z) + \alpha_S(Q^2) P'_{a\to bc}\right) &= \alpha_S(Q^2)\left(1 - \alpha_S(Q^2) b \log z(1-z)\right) P_{a\to bc}(z) \\ &\sim \alpha_S(z(1-z)Q^2) P_{a\to bc}(z)\end{aligned}$$

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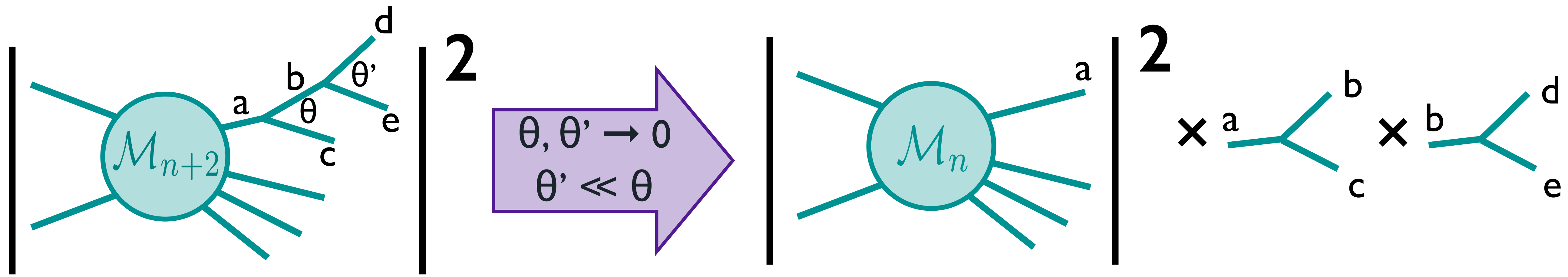
Multiple emission

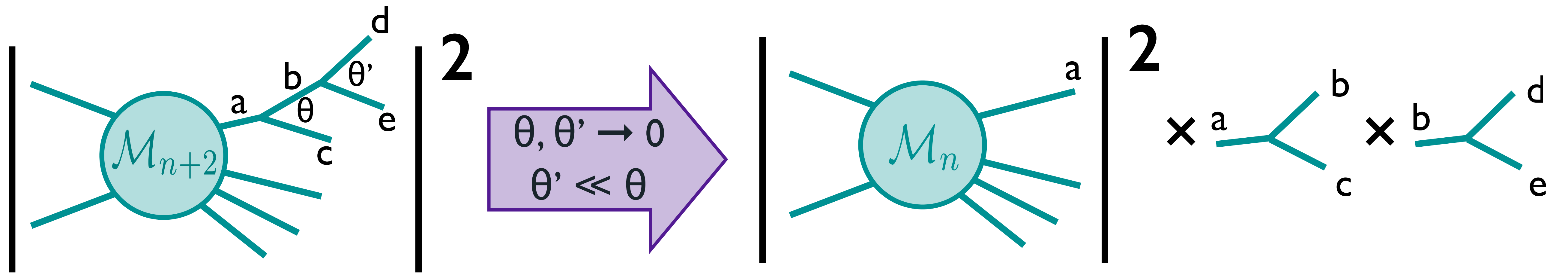


- Now consider **M_{n+1}** as the new core process and use the recipe we used for the first emission in order to get the dominant contribution to the (n+2)-body cross section: add a new branching at angle much smaller than the previous one:

$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) \times \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_S}{2\pi} P_{b\to de}(z')$$

- This can be done for an arbitrary number of emissions. The recipe to get the leading collinear singularity is thus cast in the form of an iterative sequence of emissions whose probability does not depend on the past history of the system: a 'Markov chain'. No interference!!!

Multiple emission



- The dominant contribution comes from the region where the subsequently emitted partons satisfy the strong ordering requirement: θ » θ' » θ''...
  For the rate for multiple emission we get

$$\sigma_{n+k} \propto \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \dots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

  where Q is a typical hard scale and Q_0 is a small infrared cutoff that separates perturbative from non perturbative regimes.

- Each power of α_s comes with a logarithm. The logarithm can be easily large, and therefore it can lead to a breakdown of perturbation theory.

Absence of interference

- The collinear factorization picture gives a branching sequence for a given leg starting from the hard subprocess all the way down to the non-perturbative region.
- Suppose you want to describe two such histories from two different legs: these two legs are treated in a completely uncorrelated way. And even within the same history, subsequent emissions are uncorrelated.
- The collinear picture completely misses the possible interference effects between the various legs. The extreme simplicity comes at the price of quantum inaccuracy.
- Nevertheless, the collinear picture captures the leading contributions: it gives an excellent description of an arbitrary number of (collinear) emissions:
  - It is a "resummed computation"
  - It bridges the gap between fixed-order perturbation theory and the non-perturbative hadronization.

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➡Property: $\Delta(A,B) = \Delta(A,C)\,\Delta(C,B)$

Parton shower

- The Sudakov form factor is the heart of the parton shower. It gives the probability that a parton does not branch between two scales
- Using this no-emission probability the branching tree of a parton is generated.
- Define **dP_k** as the probability for k ordered splittings from leg a at given scales

$$
\begin{aligned}
dP_1(t_1) &= \Delta(Q^2,t_1)\ dp(t_1)\Delta(t_1,Q_0^2), \\
dP_2(t_1,t_2) &= \Delta(Q^2,t_1)\ dp(t_1)\ \Delta(t_1,t_2)\ dp(t_2)\ \Delta(t_2,Q_0^2)\Theta(t_1-t_2), \\
\ldots &= \ldots \\
dP_k(t_1,...,t_k) &= \Delta(Q^2,Q_0^2)\prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1}-t_l)
\end{aligned}
$$

- **Q_0^2** is the hadronization scale (~1 GeV). Below this scale we do not trust the perturbative description for parton splitting anymore.

Unitarity

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

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- Hence, the total probability is conserved

singularities

- We have shown that the showers is unitary. However, how are the IR divergences cancelled explicitly? Let's show this for the first emission:
  Consider the contributions from (exactly) 0 and 1 emissions from leg a:

$$\frac{d\sigma}{\sigma_n} = \Delta(Q^2, Q_0^2) + \Delta(Q^2, Q_0^2) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Expanding to first order in α_s gives

$$\frac{d\sigma}{\sigma_n} \simeq 1 - \sum_{bc} \int_{Q_0^2}^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) + \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Same structure of the two latter terms, with opposite signs: cancellation of divergences between the approximate virtual and approximate real emission cross sections.

- The probabilistic interpretation of the shower ensures that infrared divergences will cancel for each emission.

Final-state parton showers

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Veto Algorithm

1. find overestimate of the branching probability

$$\bar{P}(z) \geq \hat{\boldsymbol{P}}(z), \quad \bar{z}_{min} \leq z_{min}(t), \; z_{max}(t) \leq \bar{z}_{max}, \quad \bar{\alpha}_S \geq \alpha_S(t)$$

$$g(t) = \frac{\bar{\alpha}}{2\pi t} \int_{\bar{z}_{min}}^{\bar{z}_{max}} \bar{P}(z) \geq \int \frac{\alpha_S}{2\pi} \frac{1}{t} \hat{P}(z) = p(t)$$

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$$R = \bar{\Delta}(Q^2, t) \equiv e^{-\int_t^{Q^2} g(t')dt'}$$

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Standard unweighting needs $\dfrac{p(t)\Delta(Q^2, t)}{g(t)\bar{\Delta}(Q^2, t)} \leq 1$

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We have $\mathcal{P}(t) = g(t)\bar{\Delta}(Q^2, t)$ We need $\mathcal{P}(t) = p(t)\Delta(Q^2, t)$

Standard unweighting needs $\dfrac{p(t)\Delta(Q^2,t)}{g(t)\bar{\Delta}(Q^2,t)} \cancel{\leq} 1$ Standard unweighting does not work!

3. Special selection: Veto Algorithm

Veto Algorithm

1. Idea

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   - We want to compensate the over-estimate of the choice of the scale by not re-generate above that scale if the scale is rejected

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Is is what we want?

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$$\mathcal{P}_2(t) = p(t)\bar{\Delta}(t_0, t) \int_t^{t_0} dt_1 \left((g(t_1) - p(t_1)) \int_t^{t_1} dt_2 (g(t_2) - p(t_2))\right)$$

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$$= p(t)\bar{\Delta}(t_0,t)\frac{1}{2}\left(\int_t^{t_0} dt' g(t')-p(t')\right)^2$$

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$$\begin{aligned}
\mathcal{P}(t) = \sum_{n=0}^{\infty} \mathcal{P}_n(t) &= p(t)\bar{\Delta}(t_0, t)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\int_t^{t_0} dt' g(t') - p(t')\right)^n \\
&= p(t)\bar{\Delta}(t_0, t)e^{\int_t^{t_0} dt' g(t') - p(t')} \\
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&= p(t)\Delta(t_0, t)
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$$= p(t)\Delta(t_0,t)$$

So this is what we want!

Final-state parton showers

With the Sudakov form factor, we can now implement a final-state parton shower in a Monte Carlo event generator!

1. Start the evolution at the virtual mass scale t_0 (e.g. the mass of the decaying particle) and momentum fraction $z_0 = 1$

2. Given a virtual mass scale t_i and momentum fraction x_i at some stage in the evolution, generate the scale of the next emission t_{i+1} according to the Sudakov probability $\Delta(t_i,t_{i+1})$ by solving
$\Delta(t_{i+1},t_i) = R$
where R is a random number (uniform on [0, 1]).

3. If $t_{i+1} < t_{cut}$ it means that the shower has finished.

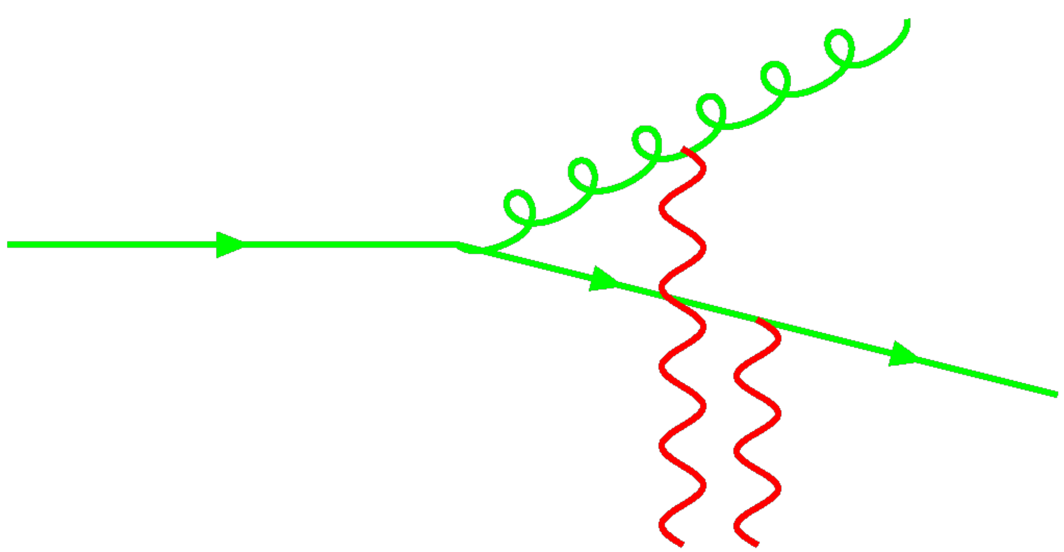
4. Otherwise, generate $z = z_i/z_{i+1}$ with a distribution proportional to $(\alpha_s/2\pi)P(z)$, where $P(z)$ is the appropriate splitting function.

5. For each emitted particle, iterate steps 2-4 until branching stops.

Soft Limit

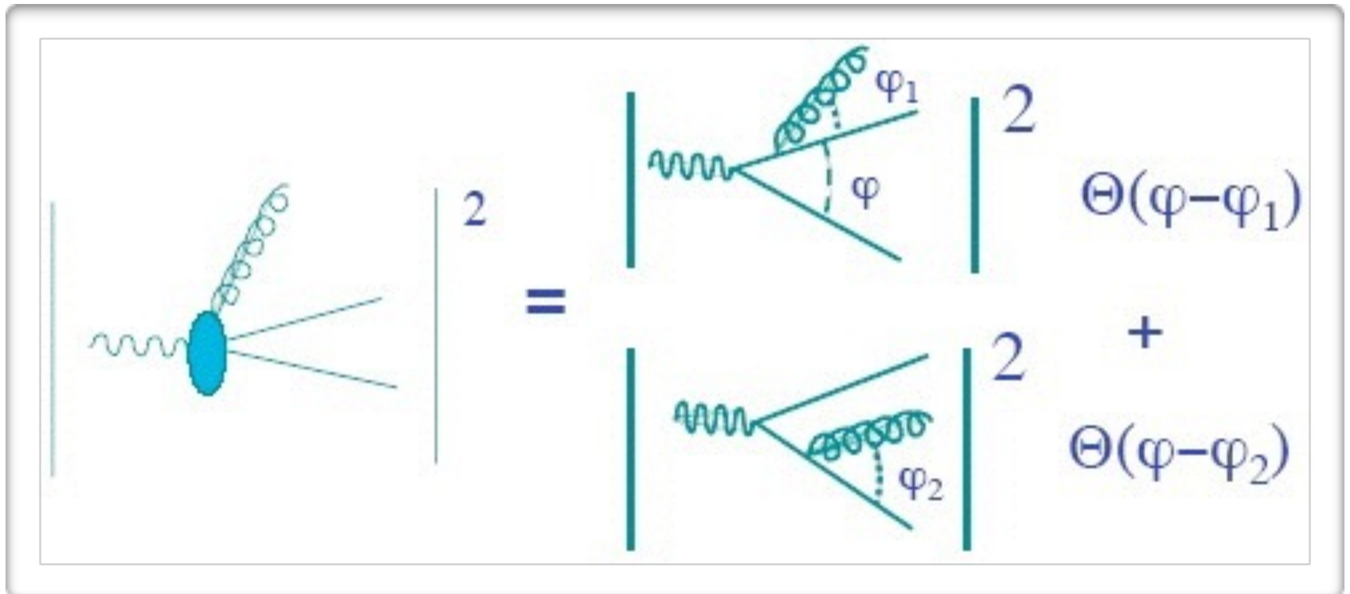
$$\Delta(Q^2,t) = \exp\left[-\sum_{bc}\int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi}\frac{\alpha_s}{2\pi} P_{a\to bc}(z)\right]$$

- There is a lot of freedom in the choice of evolution parameter t. It can be the virtuality m^2 of particle a or its p_T^2 or $E^2\theta^2$ ... For the collinear limit they are all equivalent
- However, in the soft limit ($z \longrightarrow 0,1$) they behave differently
- Can we chose it such that we get the correct soft limit?
- Soft gluon comes from the full event!

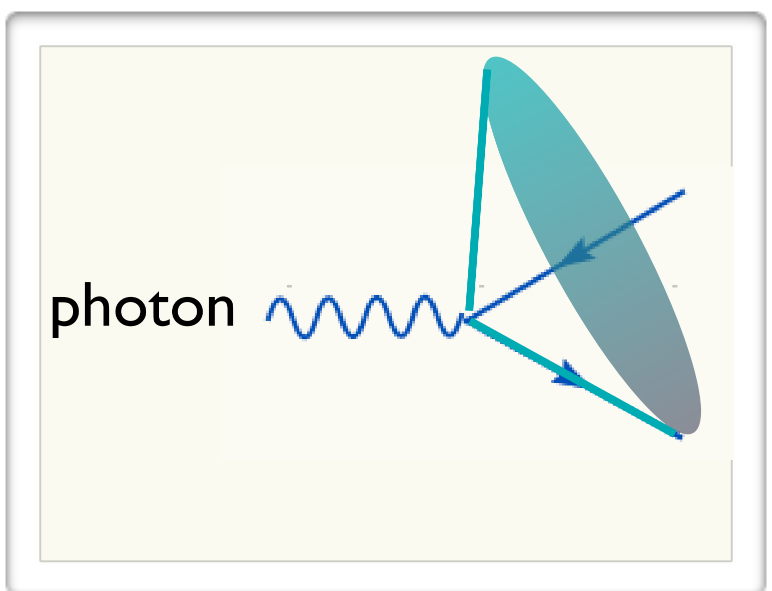


- Quantum Interference

Angular ordering



Radiation inside cones around the original partons is allowed (and described by the eikonal approximation), outside the cones it is zero (after averaging over the azimuthal angle)



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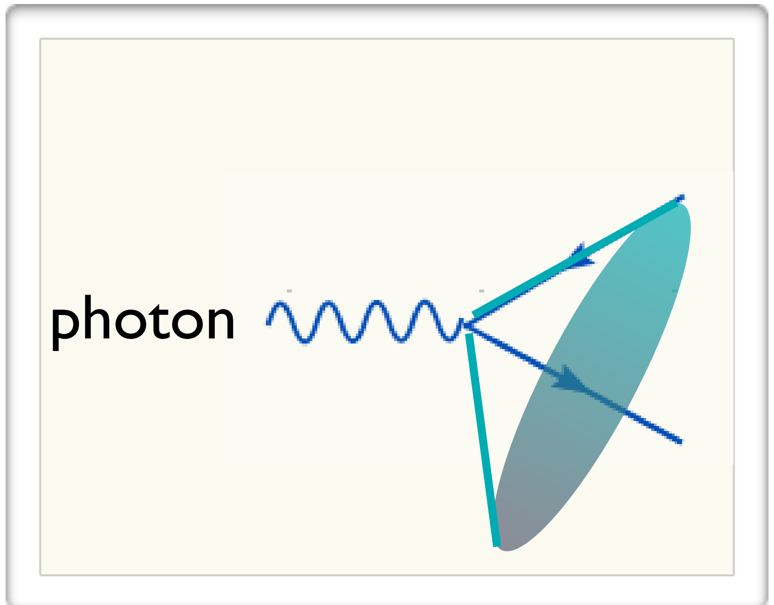
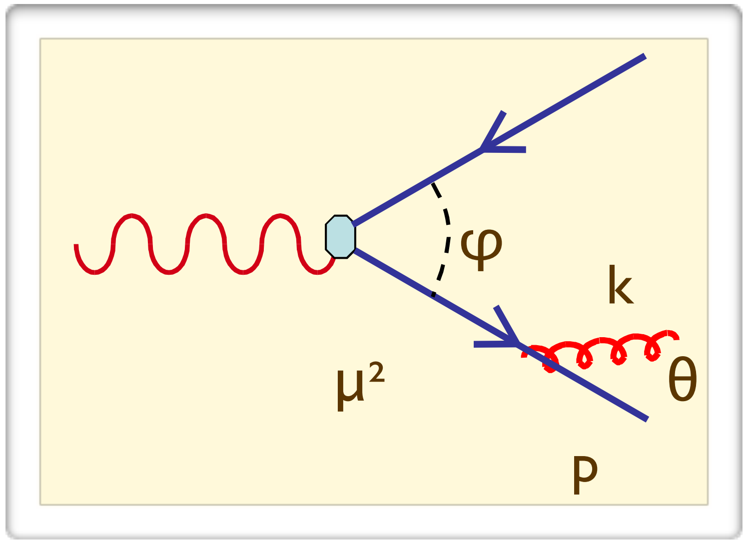


Intuitive explanation



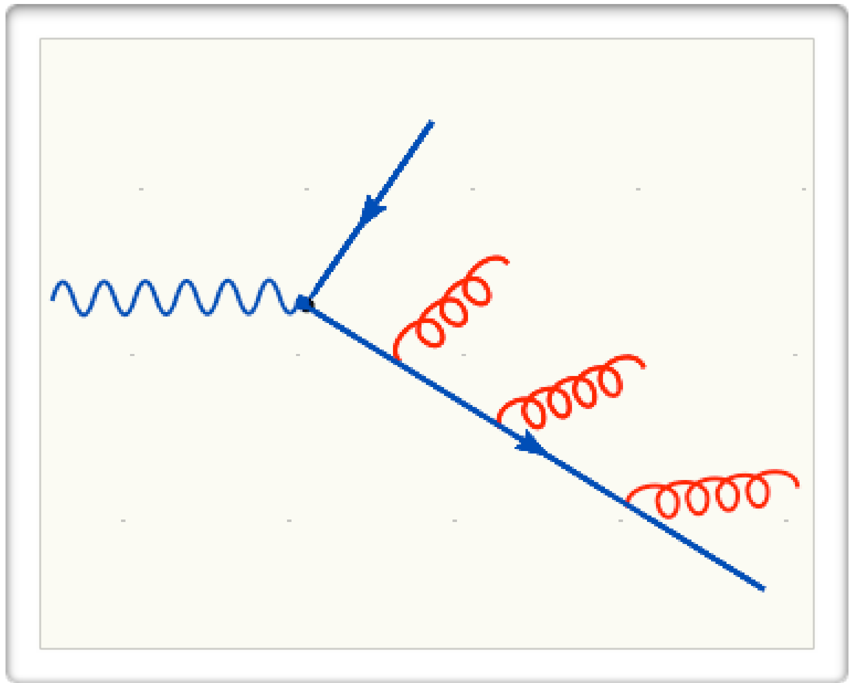
- Lifetime of the virtual intermediate state: $\tau < \gamma/\mu = E/\mu^2 = 1/(k_0\theta^2) = 1/(k_\perp\theta)$
- Distance between q and qbar after τ: $d = \varphi\tau = (\varphi/\theta)\ 1/k_\perp$

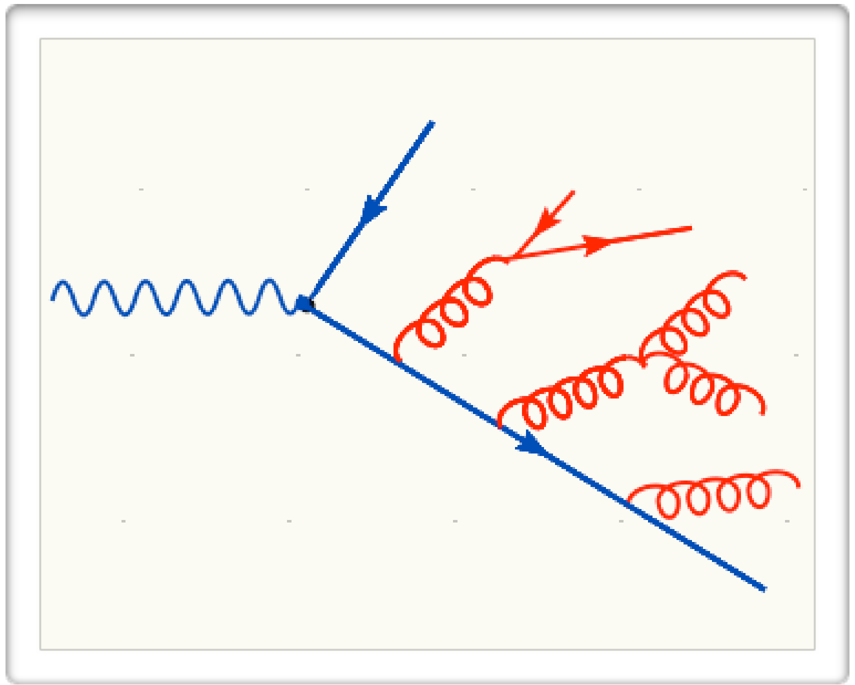
$$\mu^2 = (p+k)^2 = 2E\, k_0\, (1\text{-}\cos\theta)$$
$$\sim E\, k_0\, \theta^2 \sim E\, k_\perp\, \theta$$

If the transverse wavelength of the emitted gluon is longer than the separation between q and qbar, the gluon emission is suppressed, because the q qbar system will appear as colour neutral (i.e. dipole-like emission, suppressed)

Therefore $d > 1/k_\perp$, which implies $\theta < \varphi$.

Angular ordering





- The construction can be iterated to the next emission, with the result that the emission angles keep getting smaller and smaller.
- One can generalize it to a generic parton of color charge Q_k splitting into two partons i and j, $Q_k=Q_i+Q_j$. The result is that inside the cones i and j emit as independent charges, and outside their angular-ordered cones the emission is coherent and can be treated as if it was directly from color charge Q_k.
- KEY POINT FOR THE MC!
- Angular ordering is automatically satisfied in θ ordered showers! (and easy to account for in p_T ordered showers).

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1. A quantum effect coming from the interference of different Feynman diagrams.

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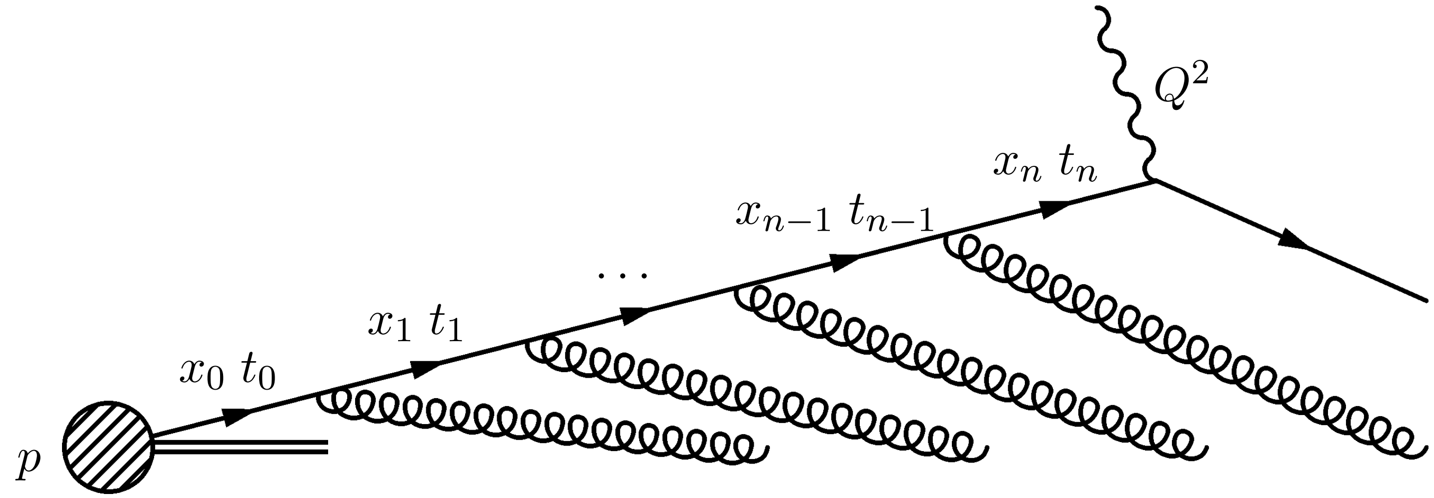
3. It is not an exclusive property of QCD (i.e., it is also present in QED) but in QCD produces very non-trivial effects, depending on how particles are color connected.

Initial-state



- So far, we have looked at final-state (time-like) splittings. For initial state, the splitting functions are the same
- However, there is another ingredient: the parton density (or distribution) functions (PDFs). Naively: Probability to find a given parton in a hadron at a given momentum fraction **$x = p_z / P_z$** and scale **t**.

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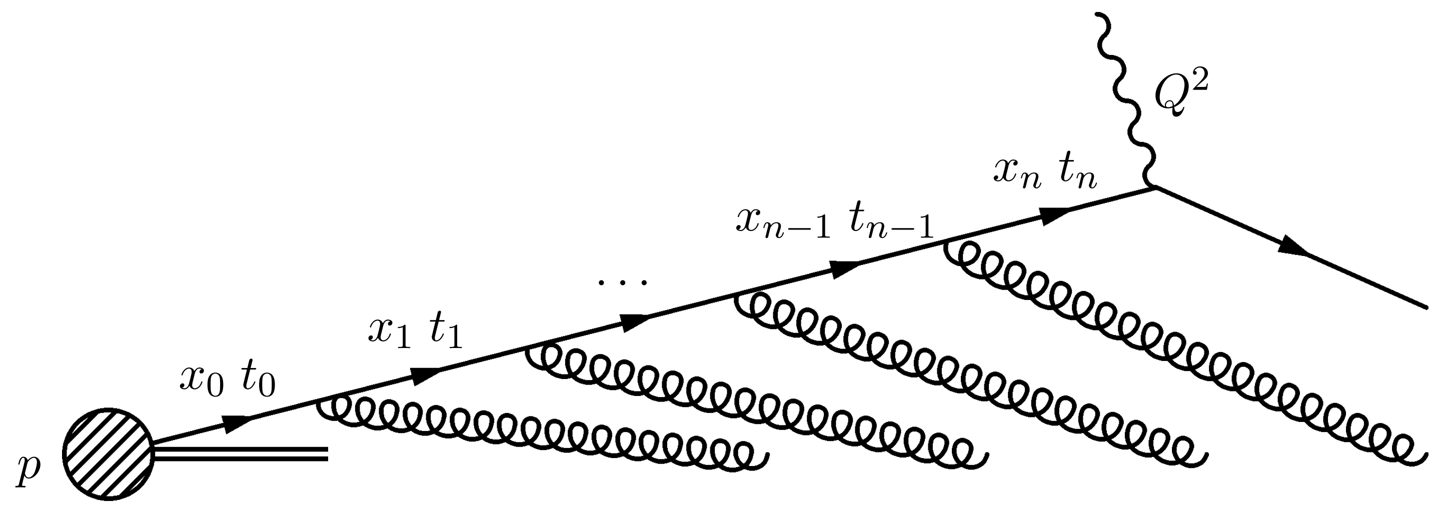

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- How do the PDFs evolve with increasing **t**?

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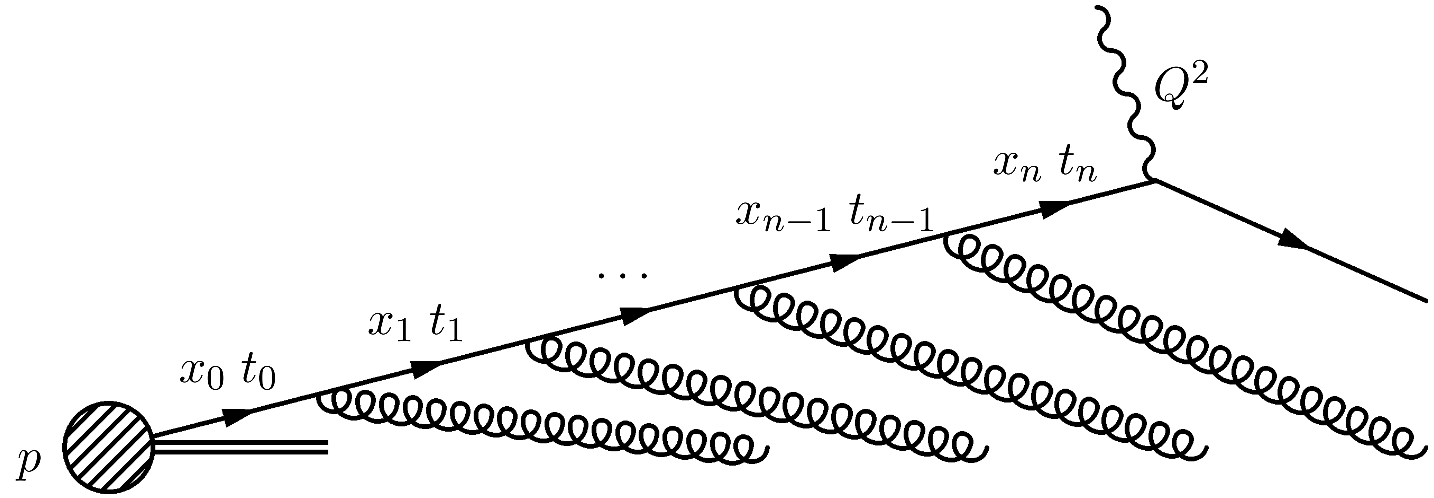

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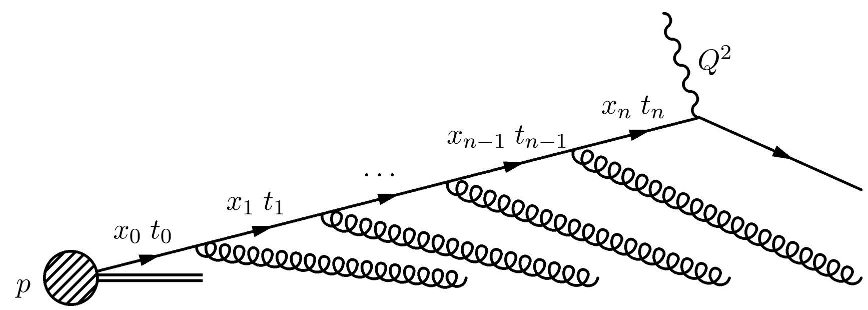
- How do the PDFs evolve with increasing **t**?

$$t\frac{\partial}{\partial t} f_i(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P_{ij}(z) f_j\left(\frac{x}{z},t\right) \qquad \text{DGLAP}$$

Initial-state parton splittings



- Start with a quark PDF **$f_0(x)$** at scale **t_0**. After a single parton emission, the probability to find the quark at virtuality **$t > t_0$** is

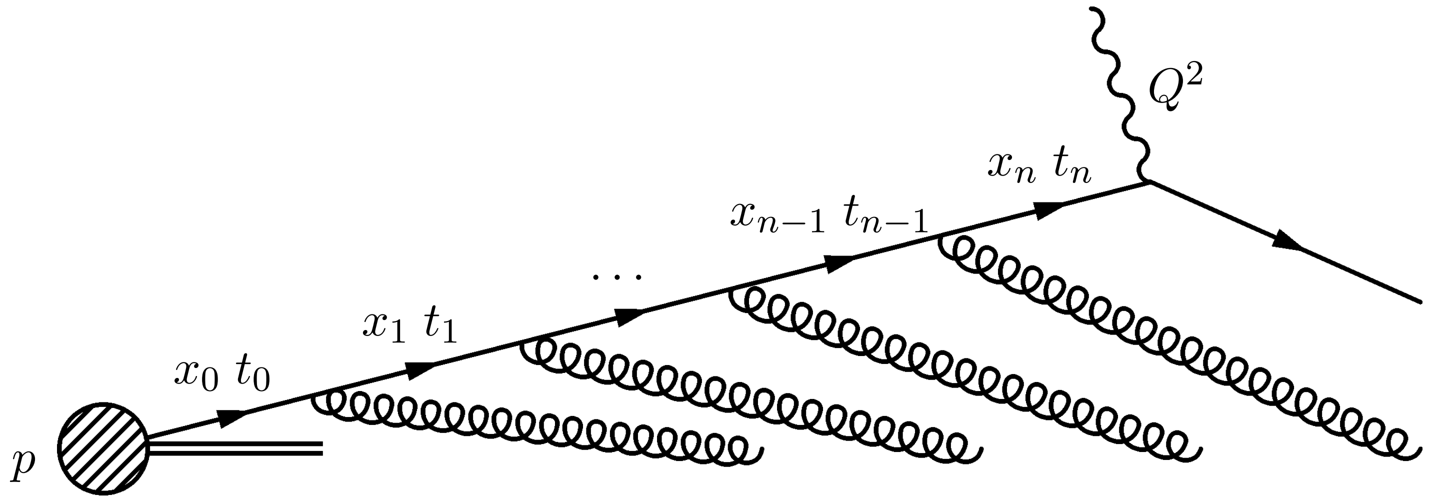
$$f(x,t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) f_0\left(\frac{x}{z}\right)$$

- After a second emission, we have

$$f(x,t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) \Big\{ f_0\left(\frac{x}{z}\right) + \int_{t_0}^{t'} \frac{dt''}{t''} \frac{\alpha_s}{2\pi} \int_{x/z}^1 \frac{dz'}{z'} P(z') f_0\left(\frac{x}{zz'}\right) \Big\}$$

f(x/z, t')

The DGLAP equation



- So for multiple parton splittings, we arrive at an integral-differential equation:

$$t\frac{\partial}{\partial t}f_i(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi}P_{ij}(z)f_j\left(\frac{x}{z},t\right)$$

- This is the famous DGLAP equation (where we have taken into account the multiple parton species i, j). The boundary condition for the equation is the initial PDFs $f_{i0}(x)$ at a starting scale t_0 (around 2 GeV).

- These starting PDFs are fitted to experimental data.

parton showers

- To simulate parton radiation from the initial state, we start with the hard scattering, and then "deconstruct" the DGLAP evolution to get back to the original hadron: backwards evolution!
  - i.e. we undo the analytic resummation and replace it with explicit partons (e.g. in Drell-Yan this gives non-zero p_T to the vector boson)
- In backwards evolution, the Sudakovs include also the PDFs -- this follows from the DGLAP equation and ensures conservation of probability:

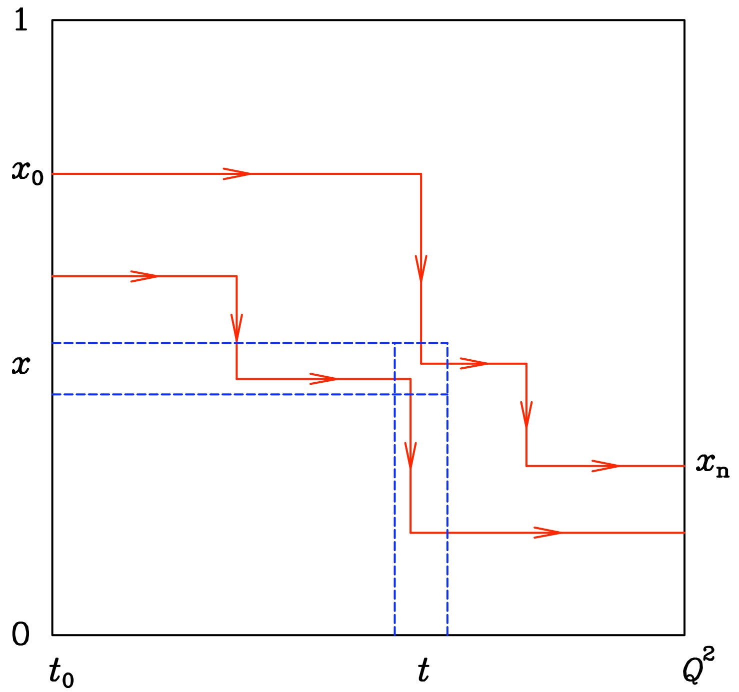
$$\Delta_{Ii}(x, t_1, t_2) = \exp\left\{ -\int_{t_1}^{t_2} dt' \sum_j \int_x^1 \frac{dx'}{x'} \frac{\alpha_s(t')}{2\pi} P_{ij}\left(\frac{x}{x'}\right) \frac{f_i(x', t')}{f_j(x, t')} \right\}$$

  This represents the probability that parton **i** will stay at the same **x** (no splittings) when evolving from **t₁** to **t₂**.

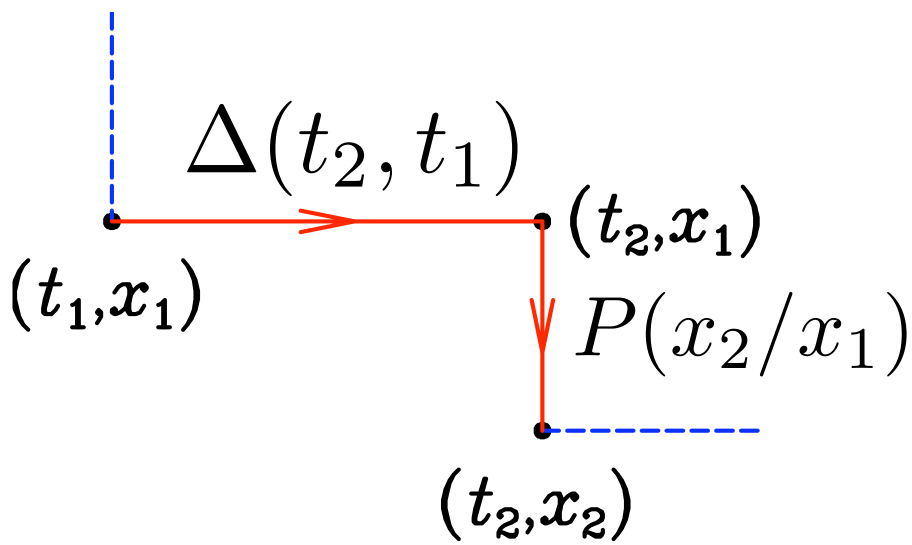
- The shower simulation is now done as in a final state shower!

Parton Shower

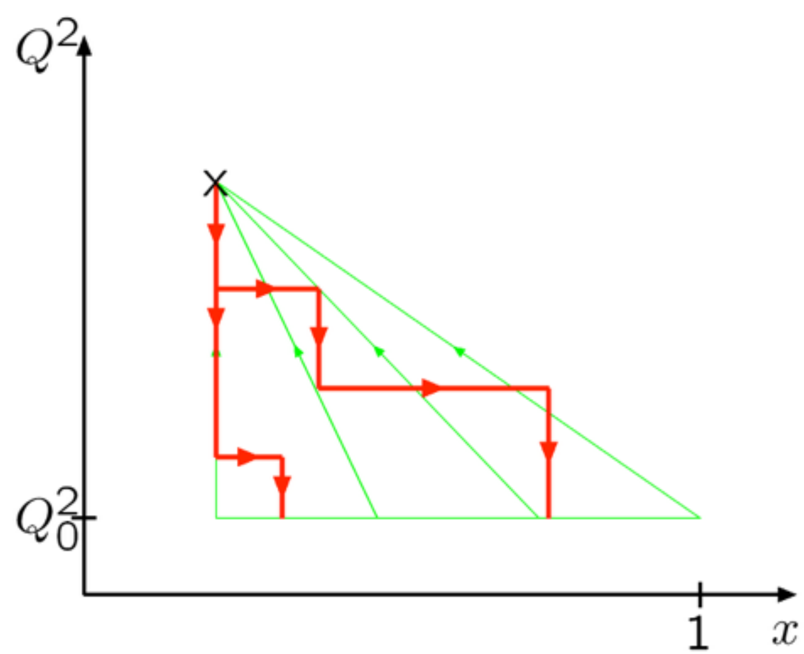
Final State Parton Shower



Basic 2-step:



Initial State Parton Shower



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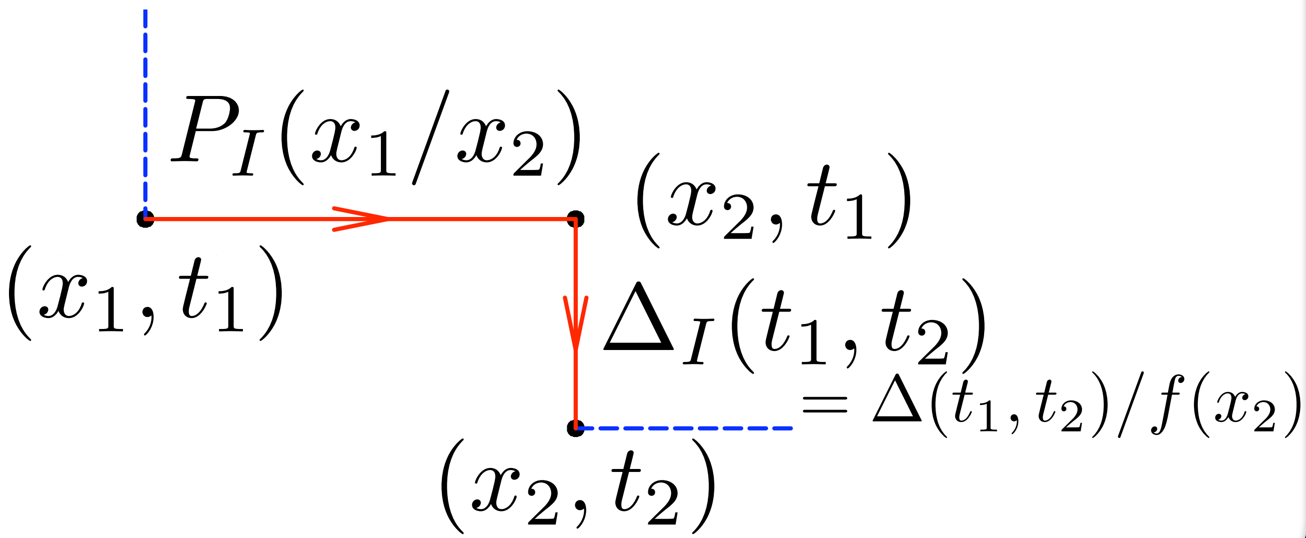
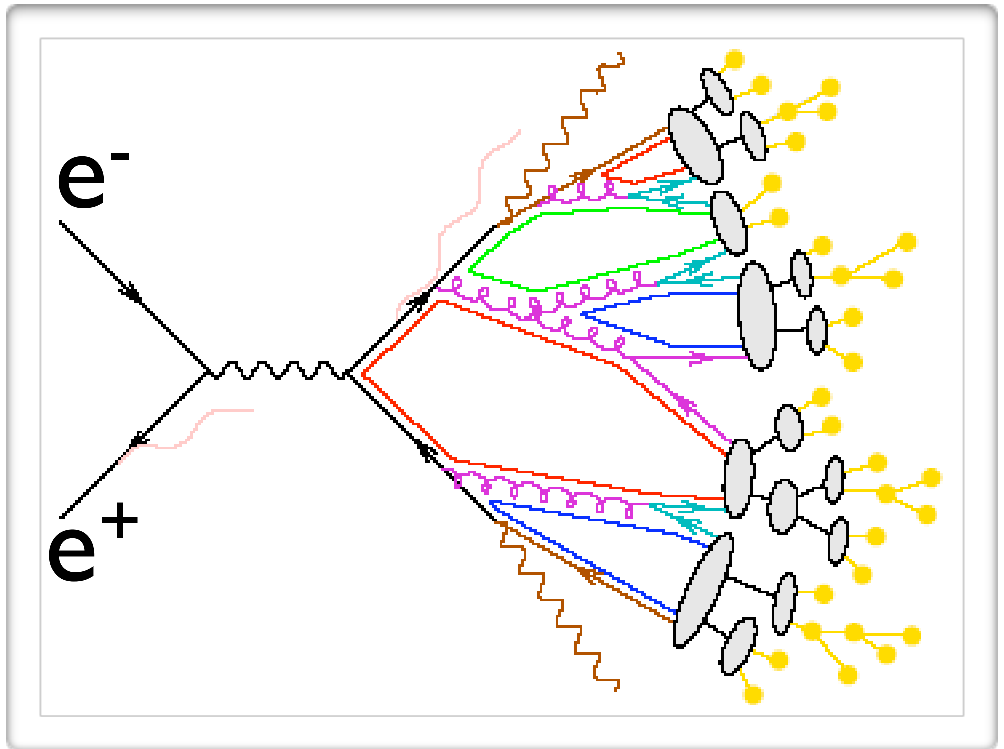


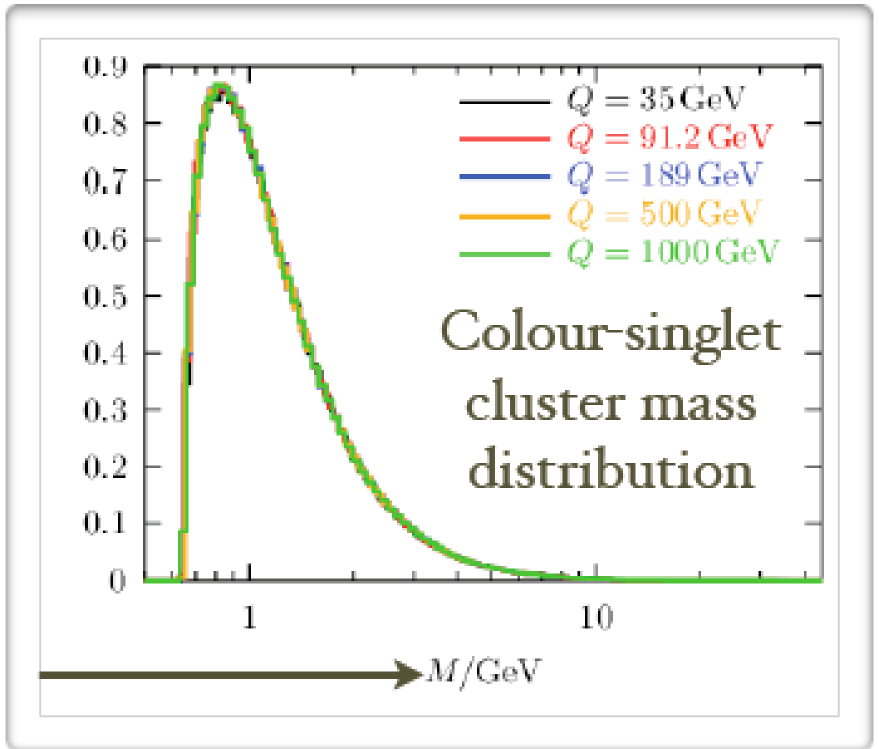
Hadronization

- The shower stops if all partons are characterized by a scale at the IR cut-off: $Q_0 \sim 1$ GeV.
- Physically, we observe hadrons, not (colored) partons.
- We need a non-perturbative model in passing from partons to colorless hadrons.
- There are two models (string and cluster), based on physical and phenomenological considerations.

Cluster model

The structure of the perturbative evolution including angular ordering, leads naturally to the clustering in phase-space of color-singlet parton pairs (preconfinement). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass color singlet clusters.





String model

From lattice QCD one sees that the color confinement potential of a quark-antiquark grows linearly with their distance: V(r) ~ kr, with k ~ 0.2 GeV. This is modeled with a string with uniform tension (energy per unit length) k that gets stretched between the qq pair.

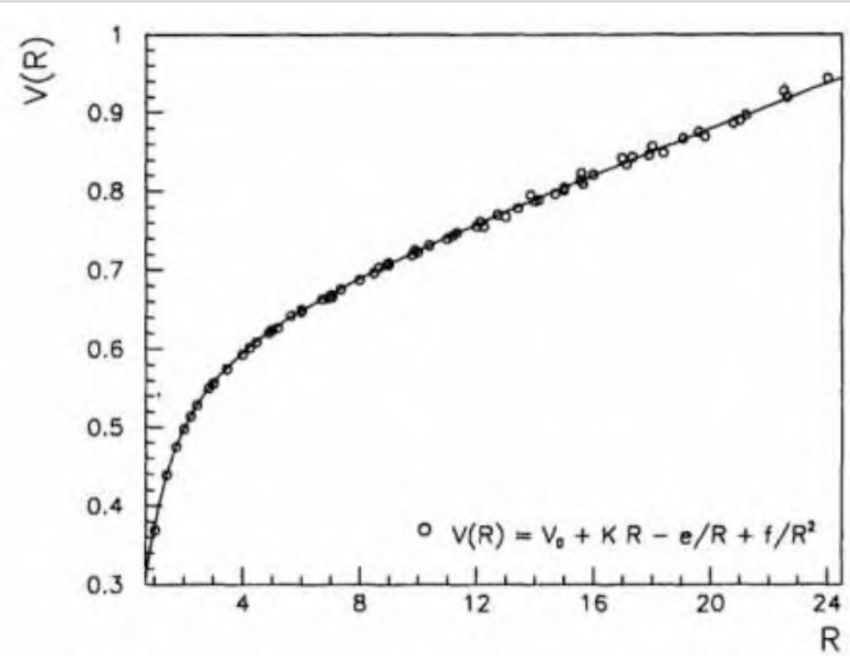
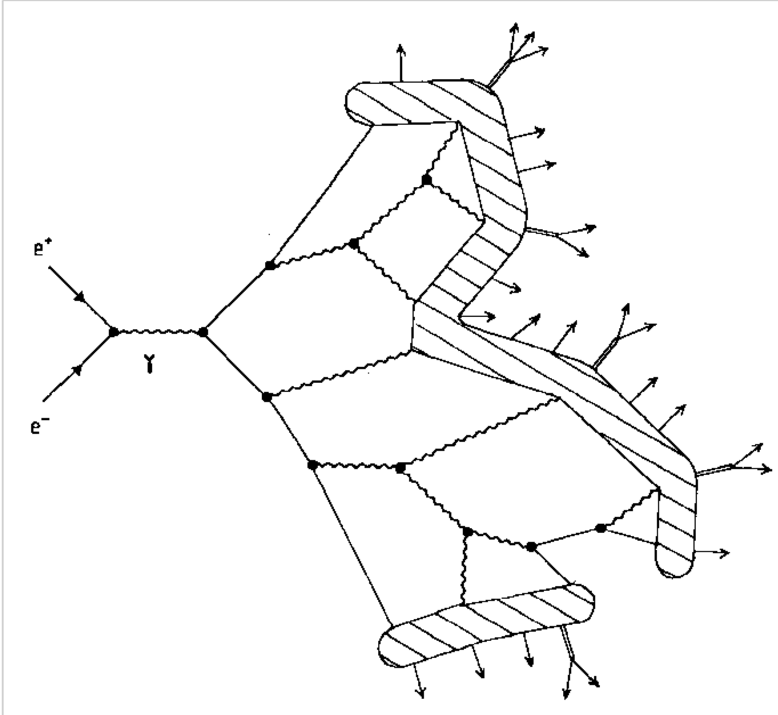


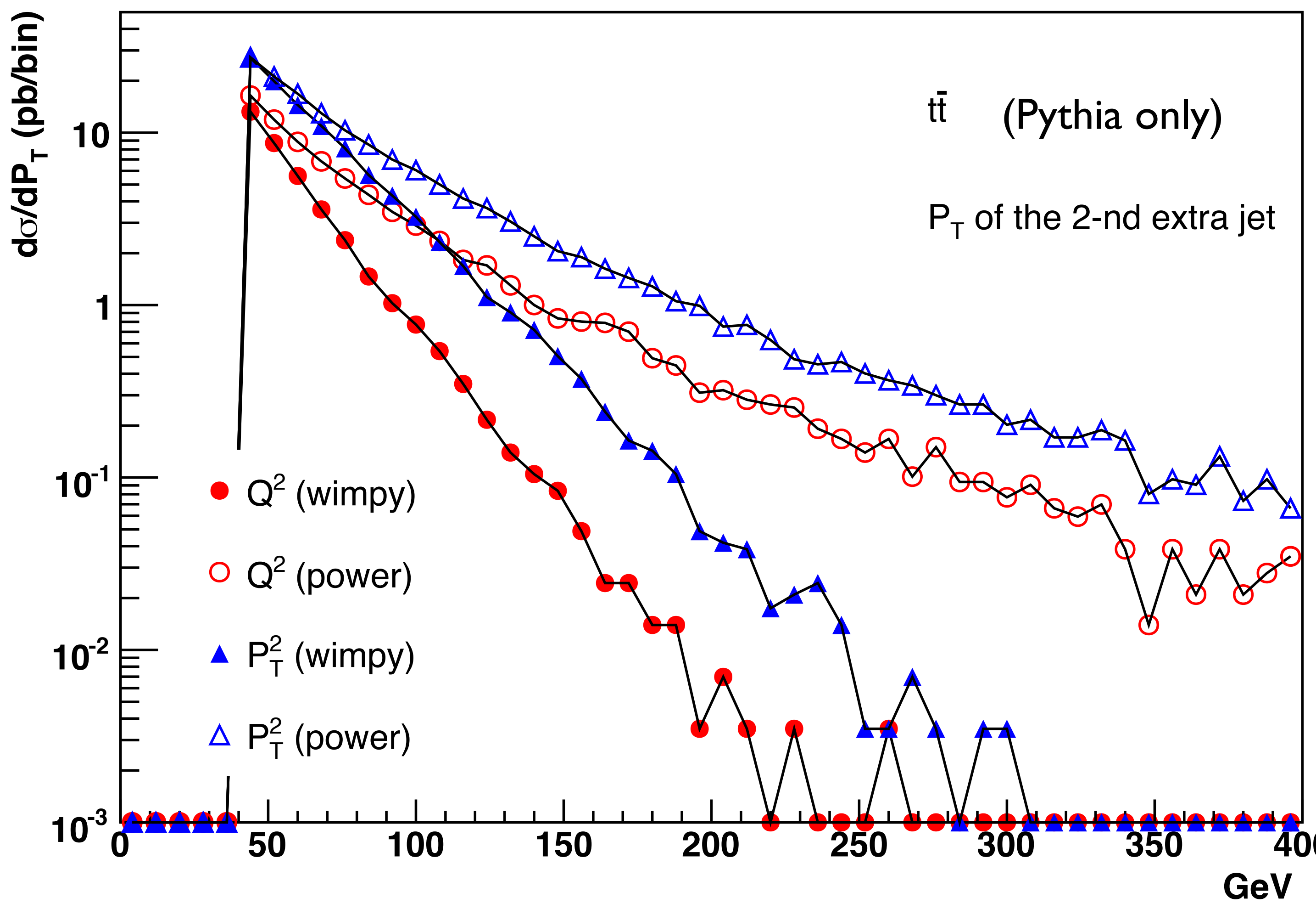
Fig. 2.9. QCD potential *vs.* R (in lattice units) from lattice QCD. Figure from ref. [23].



When quark-antiquarks are too far apart, it becomes energetically more favorable to break the string by creating a new qq pair in the middle.

Shower starting scale

Varying the shower starting scale ('wimpy' or 'power') and the evolution parameter ('Q^2' or 'p_T^2') a whole range of predictions can be made:



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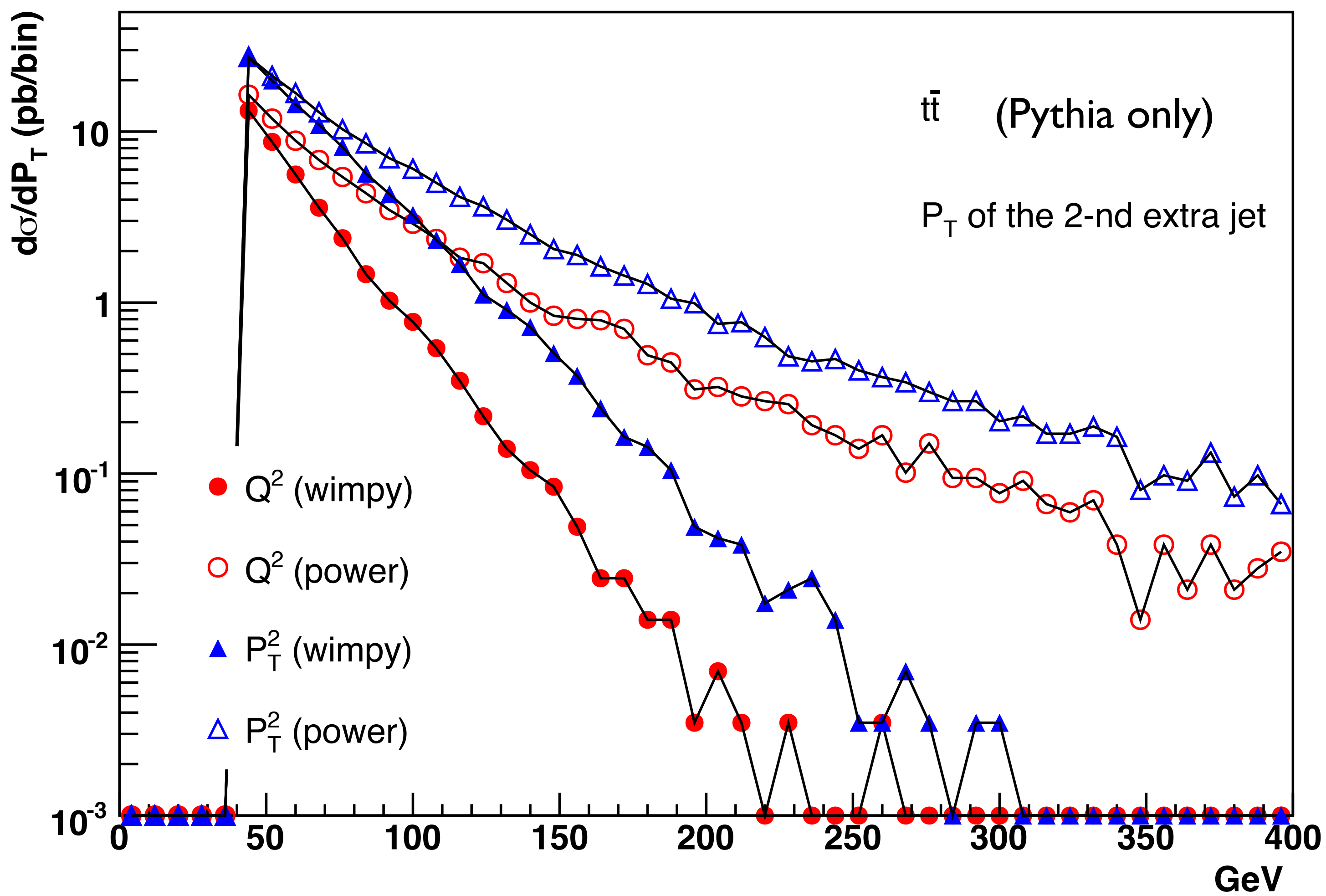

Ideal to describe the data: one can tune the parameters and fit it!
But is this really what we want...Does it work for other procs?

Parton Shower MC event generators

A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

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- General-purpose tools
- Complete exclusive description of the events: hard scattering, showering & hadronization (and underlying event)
- Reliable and well-tuned tools
- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD

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Shower MC Generators: PYTHIA, HERWIG, SHERPA

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Shower MC Generators: PYTHIA, HERWIG, SHERPA

"Note that a banching tree is not a Feynman diagram: it represents the coherent sum of many real and virtual diagrams which are summed by the branching algorithm" (HERWIG manual)

Herwig

- All HERWIG versions implement the angular-ordering: subsequent emissions are characterized by smaller and smaller angles.

HERWIG 6: $$t = \frac{p_b \cdot p_c}{E_b E_c} \simeq 1 - \cos\theta$$

HERWIG++: $$t = \frac{(p_{b\perp})^2}{z^2(1-z)^2} = t(\theta)$$

- With angular ordering the parton shower does not populate the full phase space: empty regions of the phase space, called "dead zones", will arise.

- It may seem that the presence of dead zones is a weakness, but it is not so: they implement correctly the collinear approximation, in the sense that they constrain the shower to live uniquely in the region where it is reliable.
  Matrix element corrections (MLM/CKKW matching) remove the dead-zones

- Hadronization: cluster model.

Pythia

- Choice of evolution variables for Fortran and C++ versions:

  PYTHIA 6: $t = (p_b + p_c)^2 \sim z(1-z)\theta^2 E_a^2$

  PYTHIA 8: $t = (p_b)_\perp^2$

- Simpler variables, but decreasing angles not guaranteed: PYTHIA rejects the events that do not respect the angular ordering. In practice equivalent to angular ordering (in particular for Pythia 8)

- Not implementing directly angular ordering, the phase space can be filled entirely (even without matrix element corrections), so one can have the so called "power shower" (use with a certain care: it uses the collinear/soft approximation for from the region where it is valid)

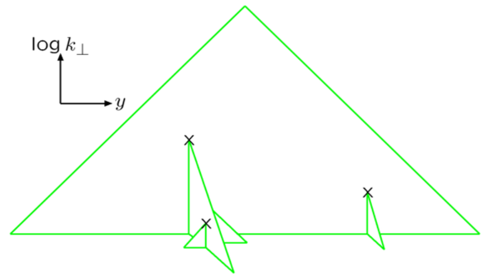
- Hadronization: string model.

Sherpa

- SHERPA uses a different kind of shower not based on the collinear $1 \longrightarrow 2$ branching, but on more complex $2 \longrightarrow 3$ elementary process: emission of the daughter off a color dipole

- The real emission matrix element squared is decomposed into a sum of terms $D_{ij,k}$ (dipoles) that capture the soft and collinear singularities in the limits i collinear to j, i soft (k is the spectator), and a factorization formula is deduced in the leading color approximation:

$$D_{ij,k} \rightarrow B \frac{\alpha_S}{p_i \cdot p_j} K_{ij,k}$$



- The shower is developed from a Sudakov form factor

$$\Delta = \exp\left(- \int \frac{dt}{t} \int dz \; \alpha_S \; K_{ij,k} \right)$$

- It treats correctly the soft gluon emission off a color dipole, so angular ordering is built in.

- Hadronization: cluster model (default) and string model

Summary

- The parton shower dresses partons with radiation. This makes the inclusive parton-level predictions (i.e. inclusive over extra radiation) completely exclusive
  - In the soft and collinear limits the partons showers are exact, but in practice they are used outside this limit as well.
  - Partons showers are universal (i.e. independent from the process)
- There is a cut-off in the shower (below which we don't trust perturbative QCD) at which a hadronization model takes over
  - Hadronization models are universal and independent from the energy of the collision