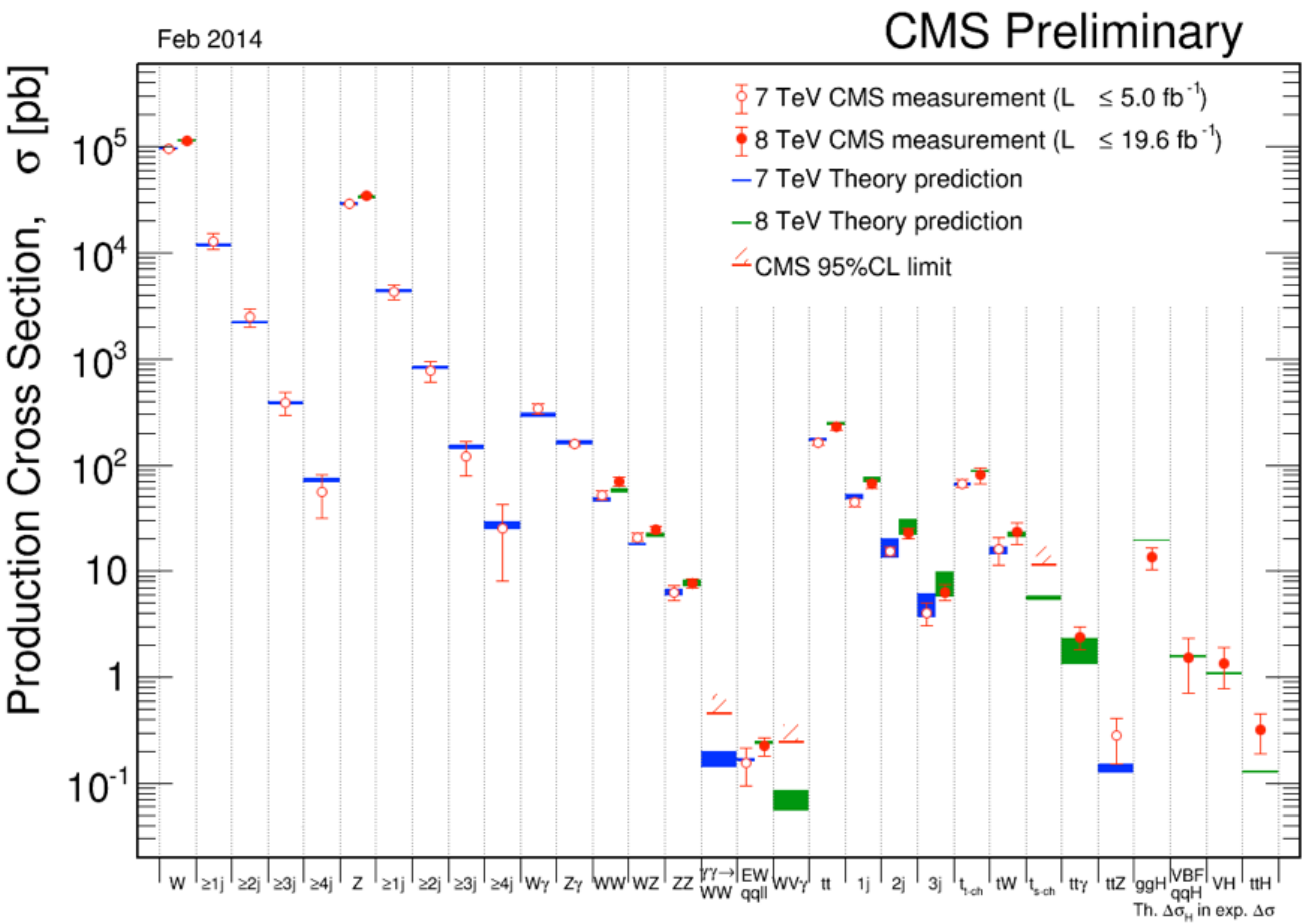
Monte-Carlo Generation

Olivier Mattelaer
IPPP/Durham

Week Plan

- Monday: FeynRules
- Tuesday: MadGraph5@LO
- Wednesday: Matching/Merging
- Thursday: NLO
- Friday: Unleashed the tools

Standard Model



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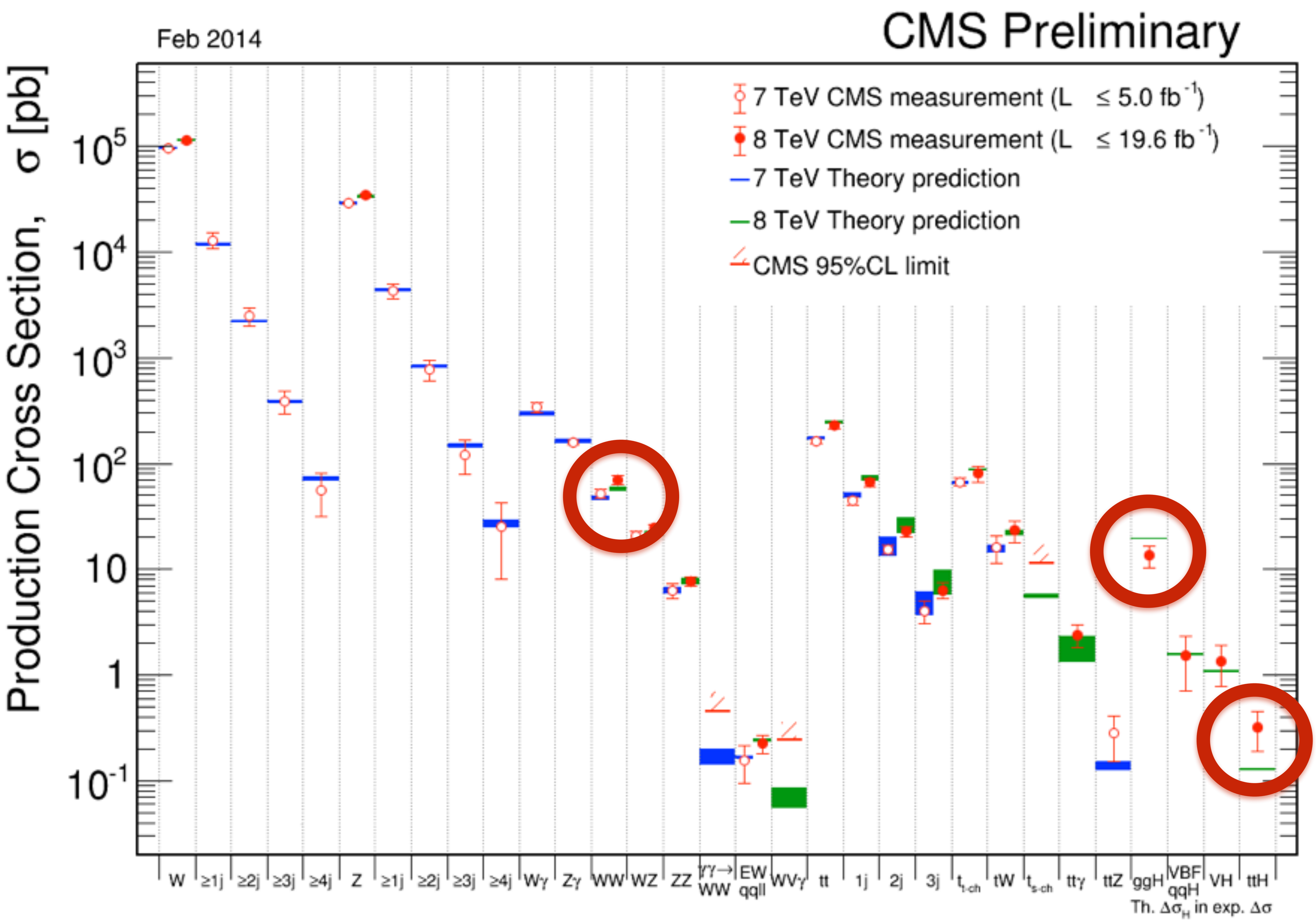
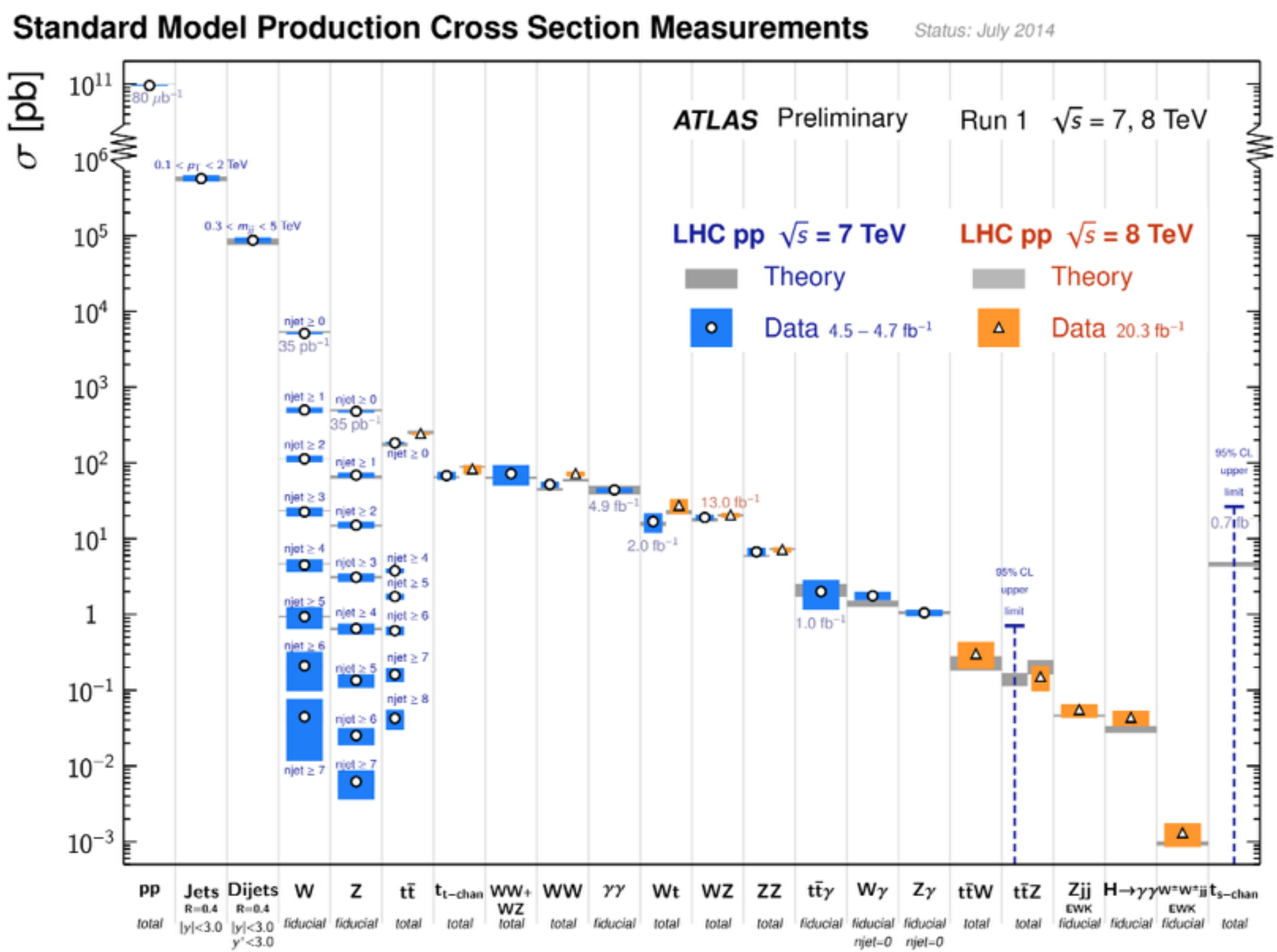


ATLAS



Standard Model Production Cross Section Measurements
Status: July 2014
σ [pb]
ATLAS Preliminary
Run 1 √s = 7, 8 TeV
LHC pp √s = 7 TeV
Theory
Data 4.5 – 4.7 fb⁻¹
LHC pp √s = 8 TeV
Theory
Data 20.3 fb⁻¹
80 μb⁻¹
35 pb⁻¹
4.9 fb⁻¹
2.0 fb⁻¹
13.0 fb⁻¹
1.0 fb⁻¹
0.7 fb
95% CL upper limit
pp
Jets
Dijets
W
Z
tt̄
t_t-chan
WW+WZ
WW
γγ
Wt
WZ
ZZ
tt̄γ
Wγ
Zγ
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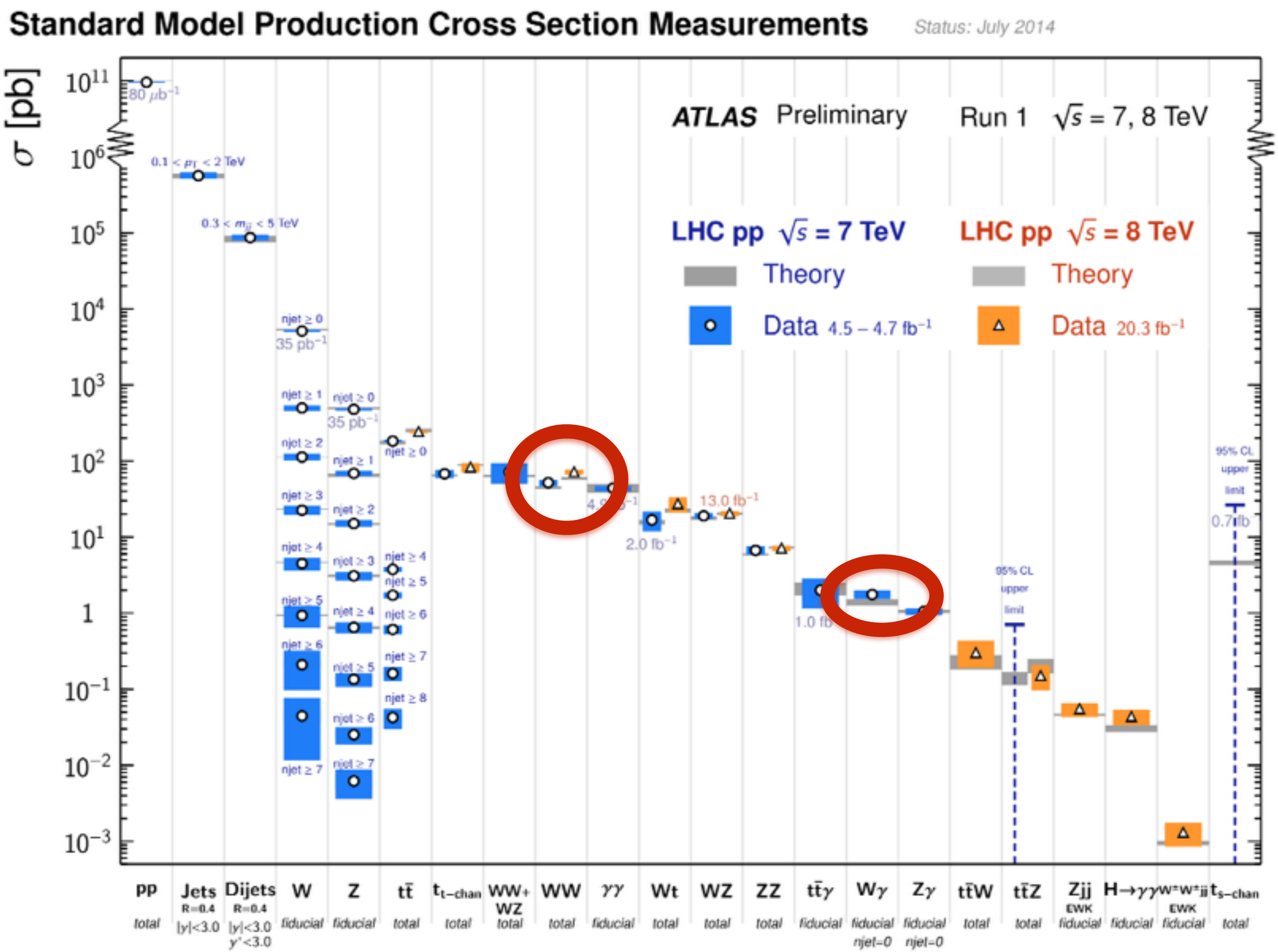
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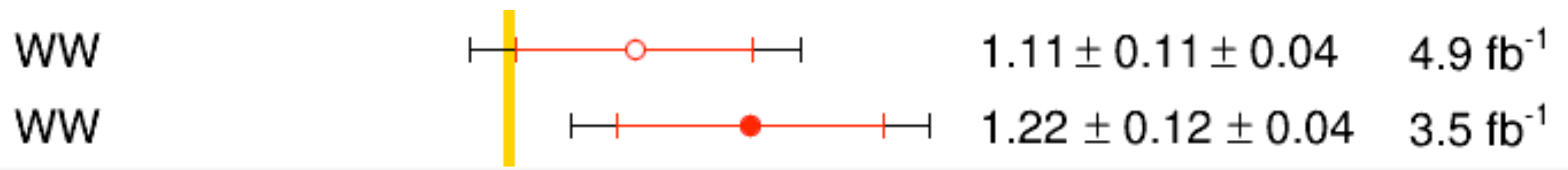
CMS



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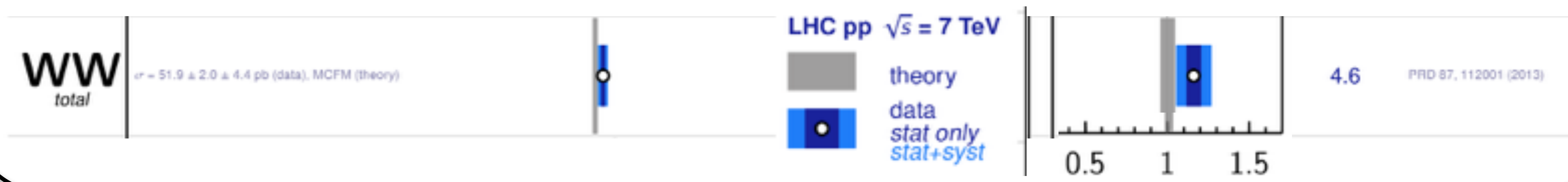
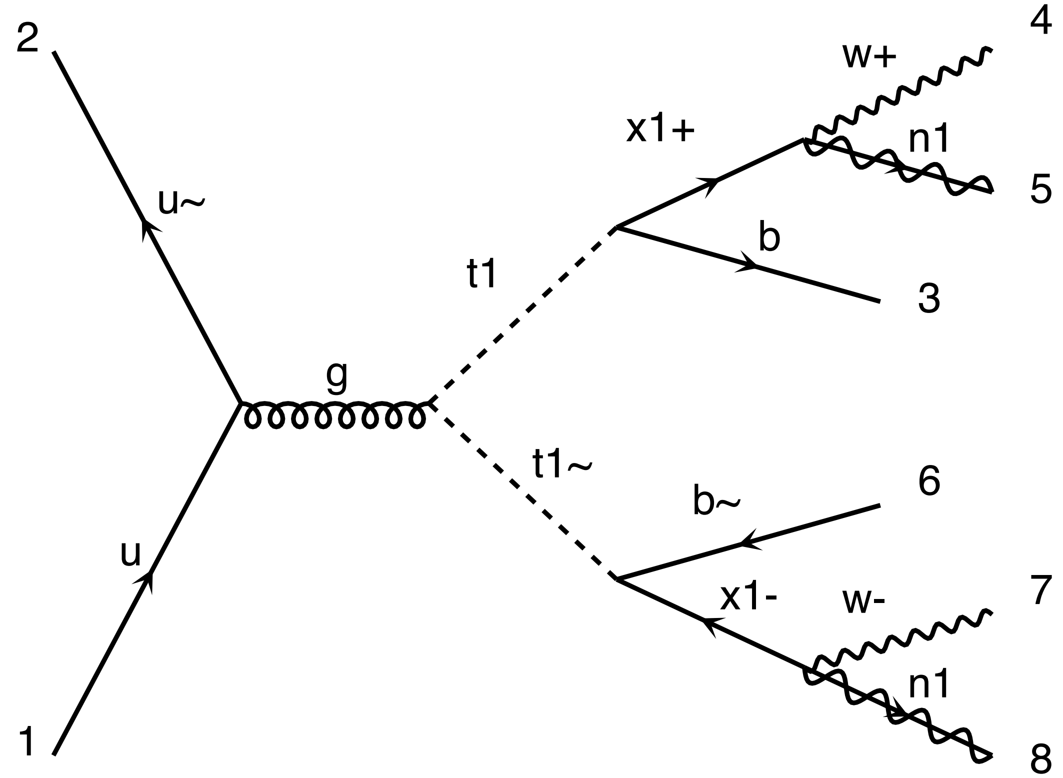


COMBINE

- Both seems indicates a 15-20% excess
- Not significant at all
- Need more data / theoretical precision

SUSY Like Explanation



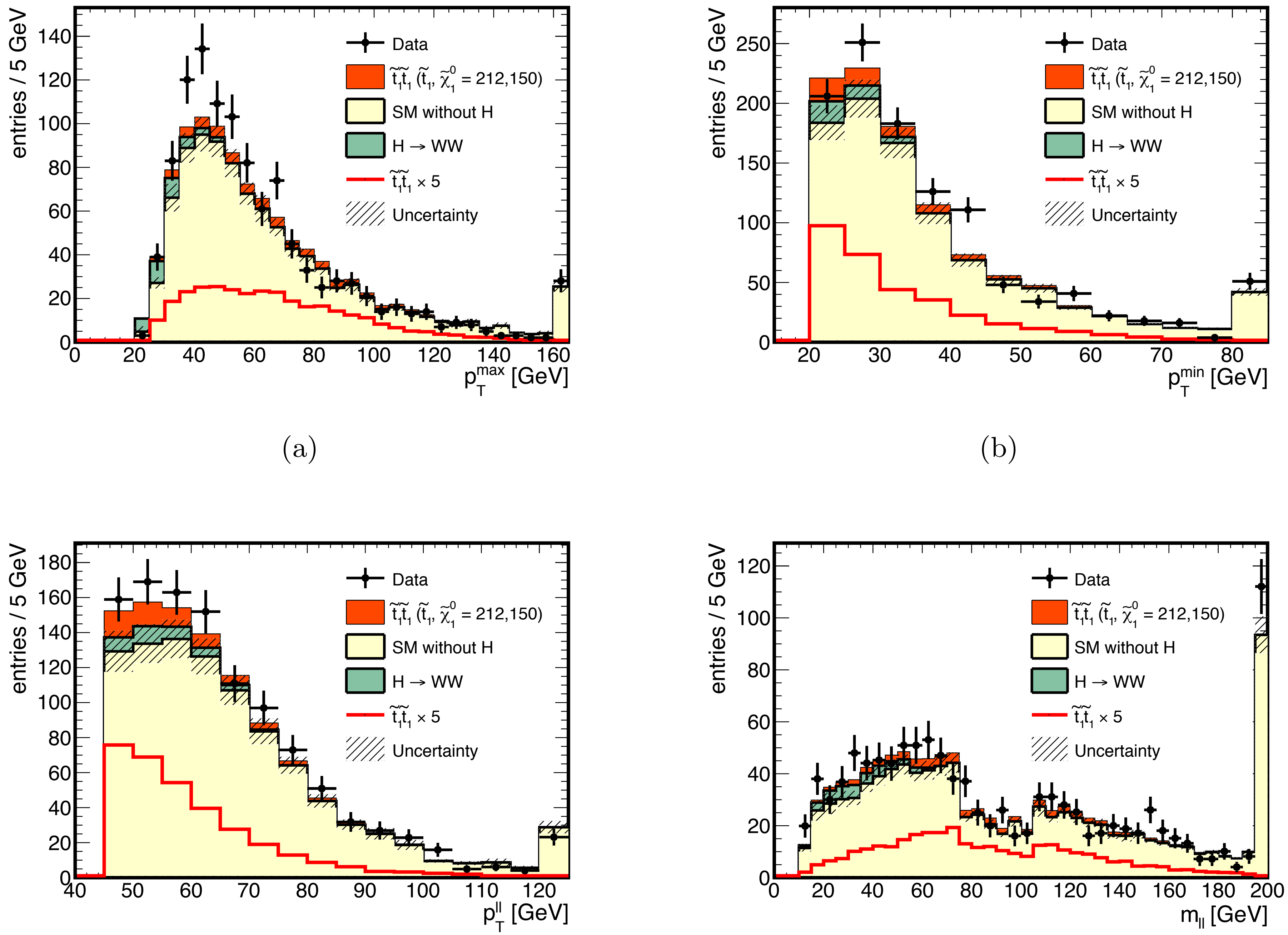
Kim, Rolbiecki, sakurai, Tattersal [1406.0858]

Compressed Spectrum $M_{\tilde{t}} \approx M_{\chi^+}$

- Soft b/b~
  - ➡ not observed
- evade direct searches constraints

Check the model!

- Monte-Carlo!



(a) (b)

Kim, Rolbiecki, sakurai, Tattersal [1406.0858]

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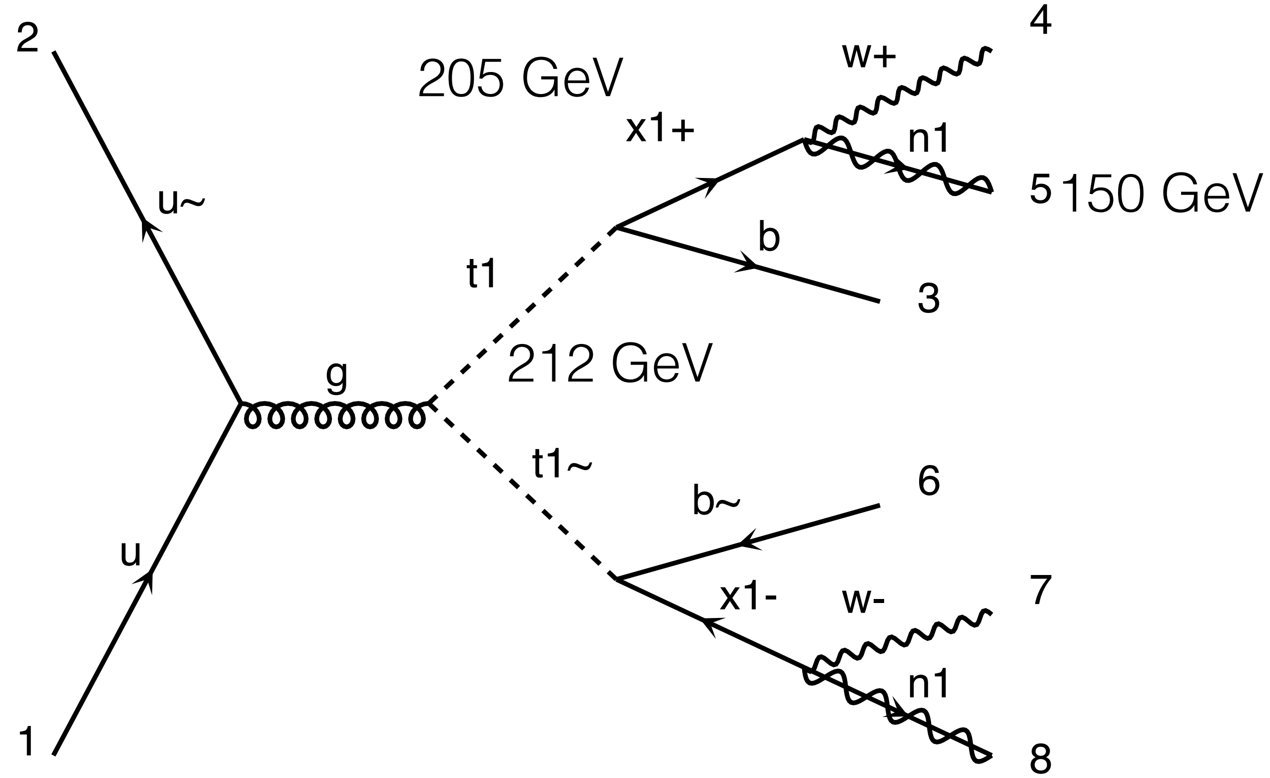

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Modelling Excesses

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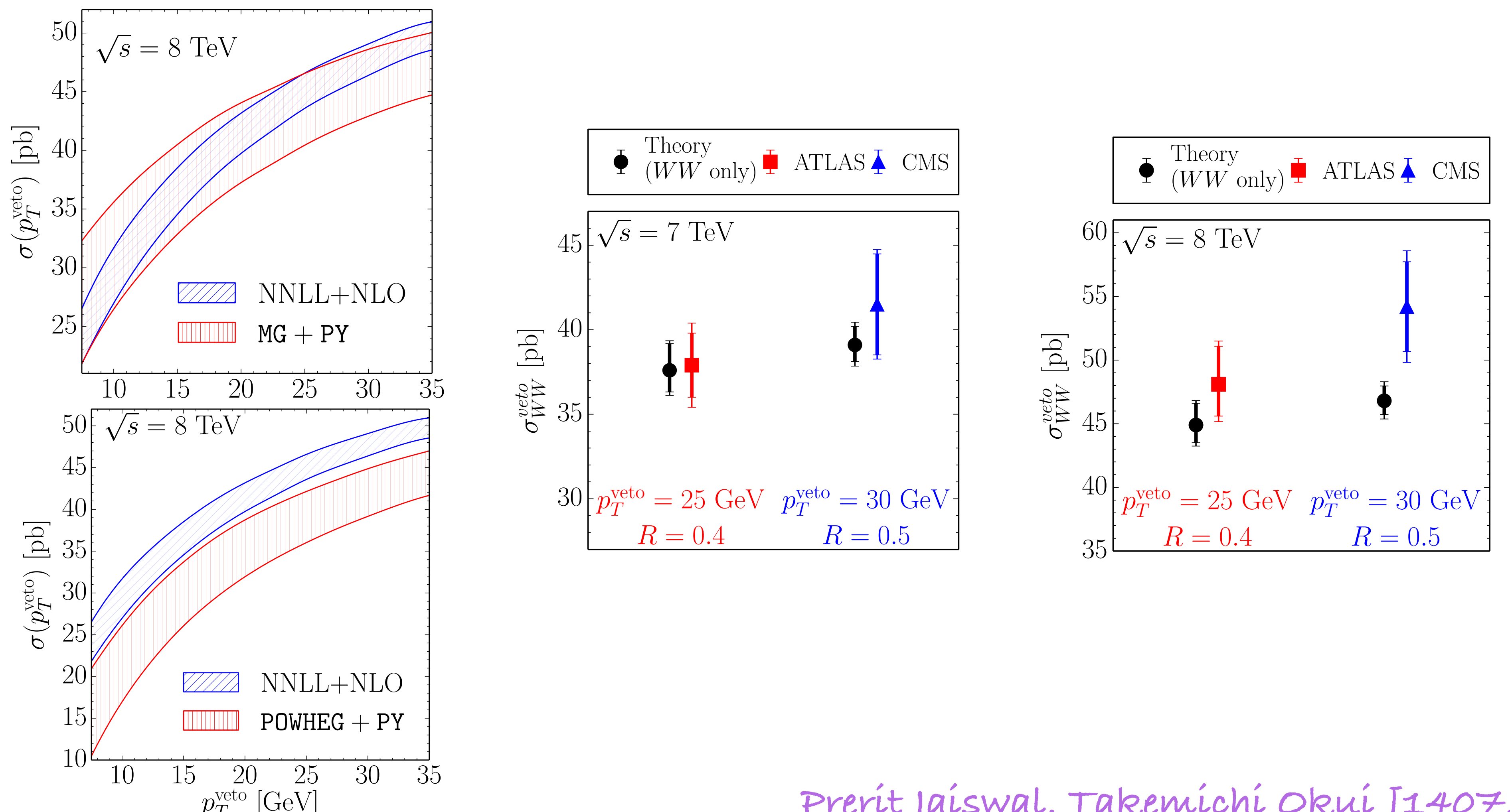

Prerit Jaiswal, Takemichi Okui [1407.4537]

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2. Exhaust SM explanations for the excess
3. Think of possible new physics explanations
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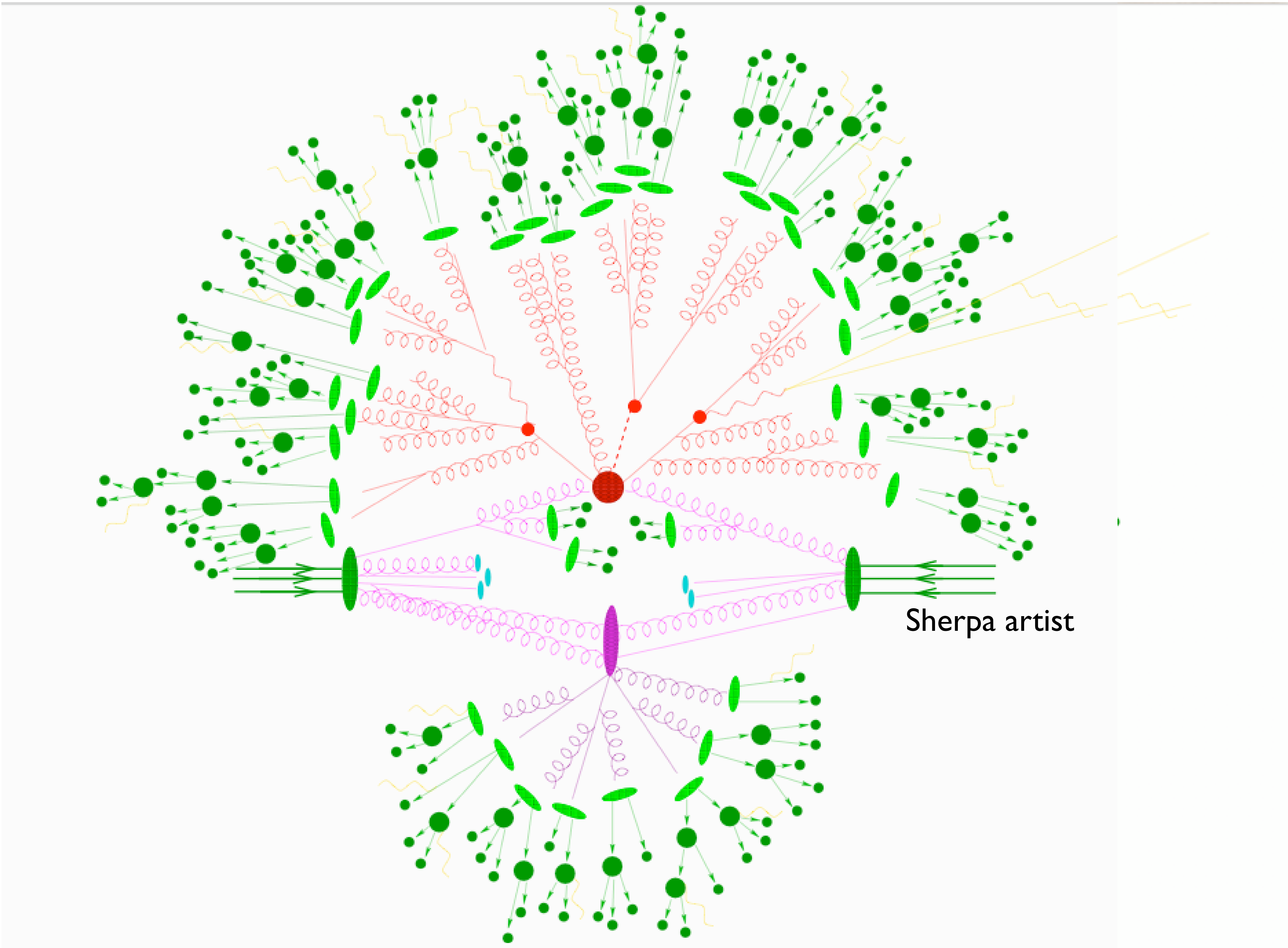
4. Find range of model parameters that can explain excess
   ➡ Typically, using Monte Carlo simulations
5. Find other observables (collider as well as flavor/EWP/cosmology) where the explanation can be verified/falsified
   ➡ Note that indirect constraints (flavor/EWP/cosmology) typically modified by additional particles in the spectrum

Simulation of collider events

What are the MC for?

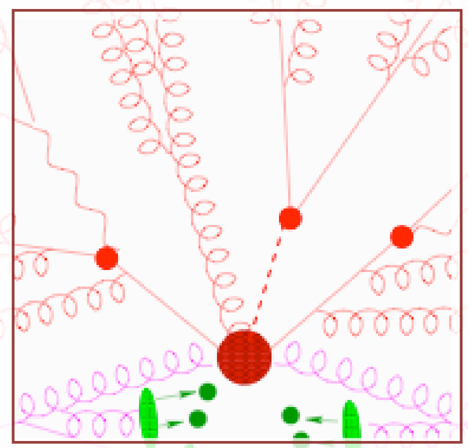


What are the MC for?

1. High-Q^2 Scattering

2. Parton Shower

☞ where new physics lies



☞ process dependent

☞ first principles description

☞ it can be systematically improved

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

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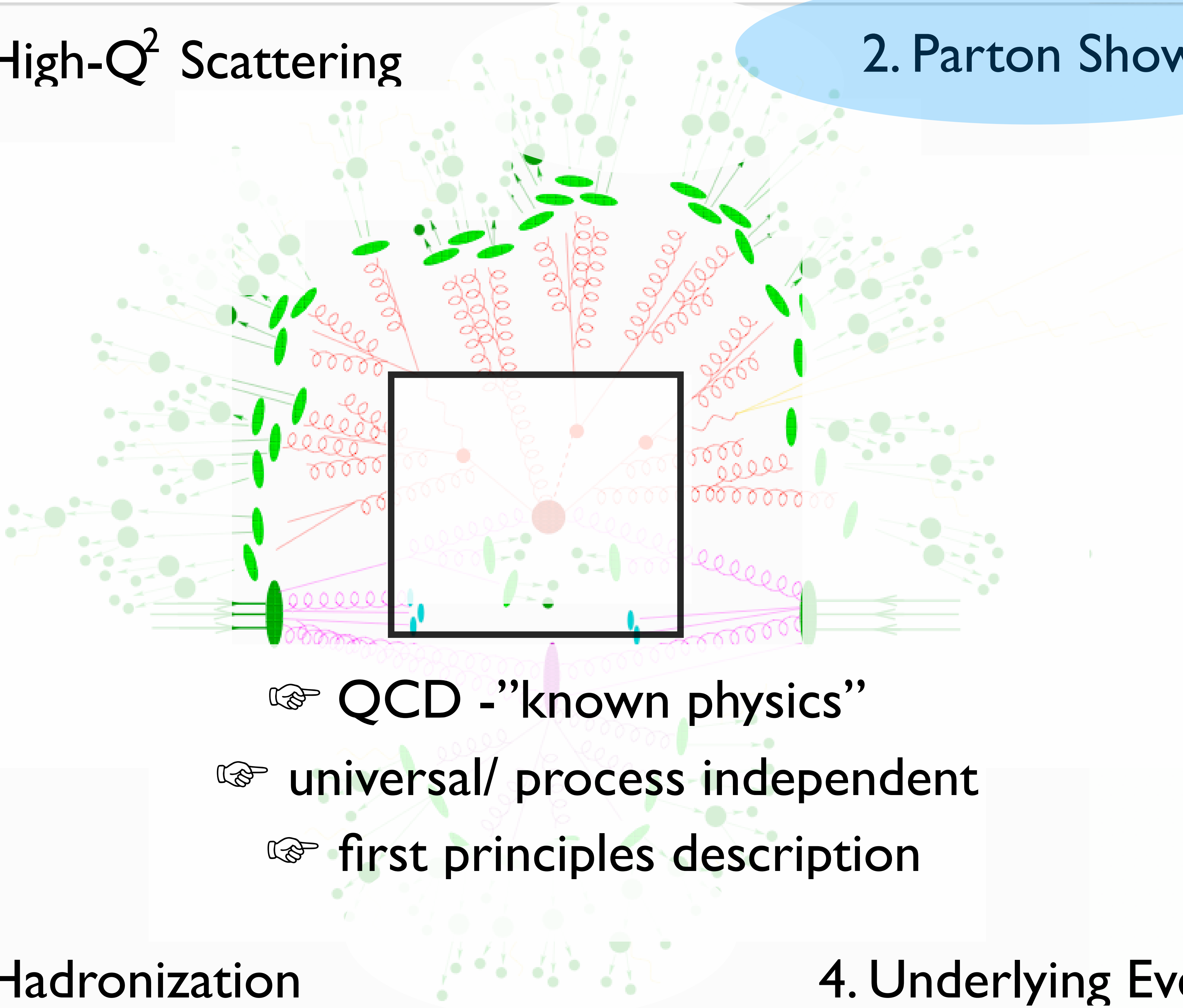

☞ QCD -"known physics"

☞ universal/ process independent

☞ first principles description

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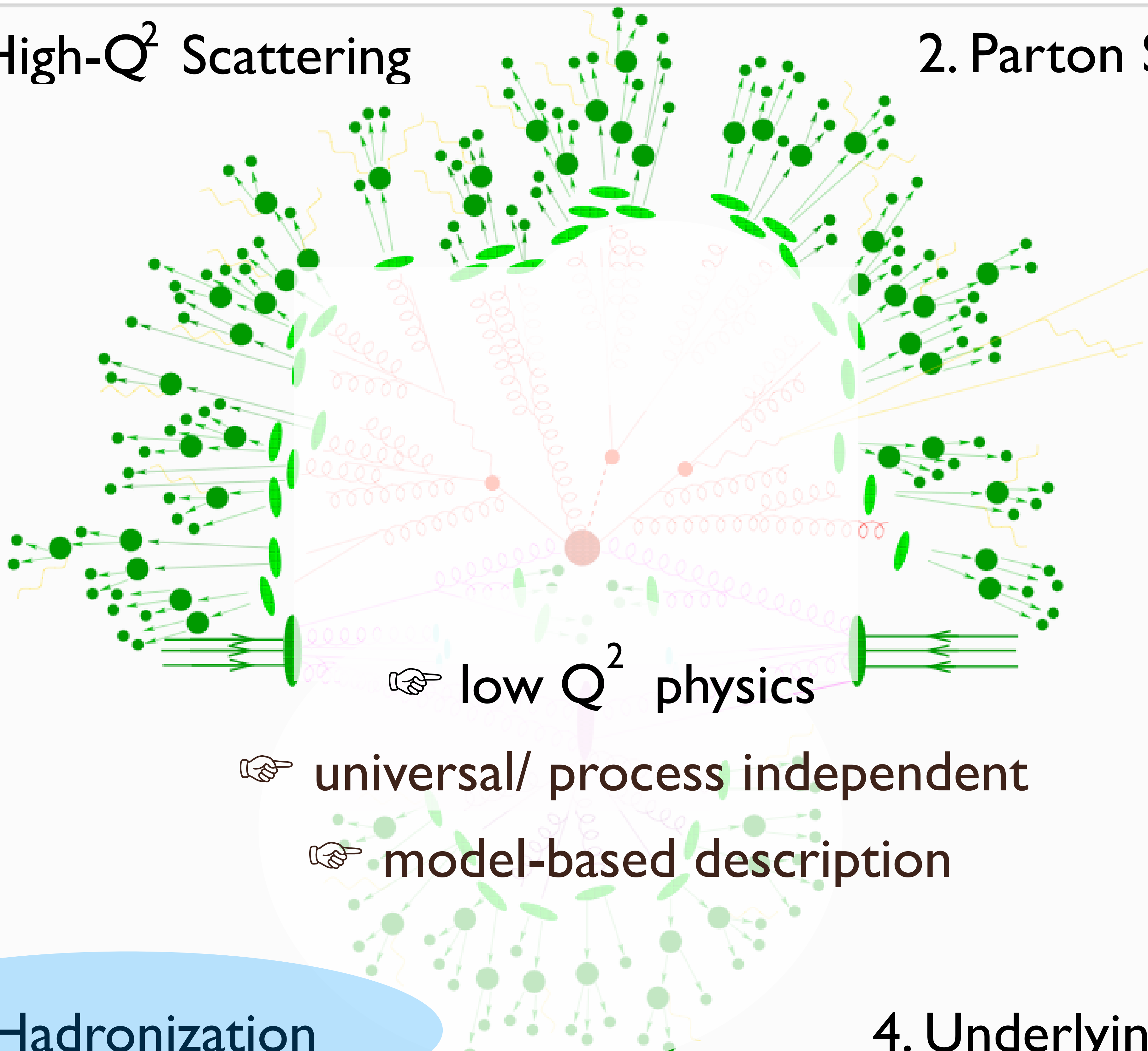

- ☞ low Q^2 physics
- ☞ universal/ process independent
- ☞ model-based description

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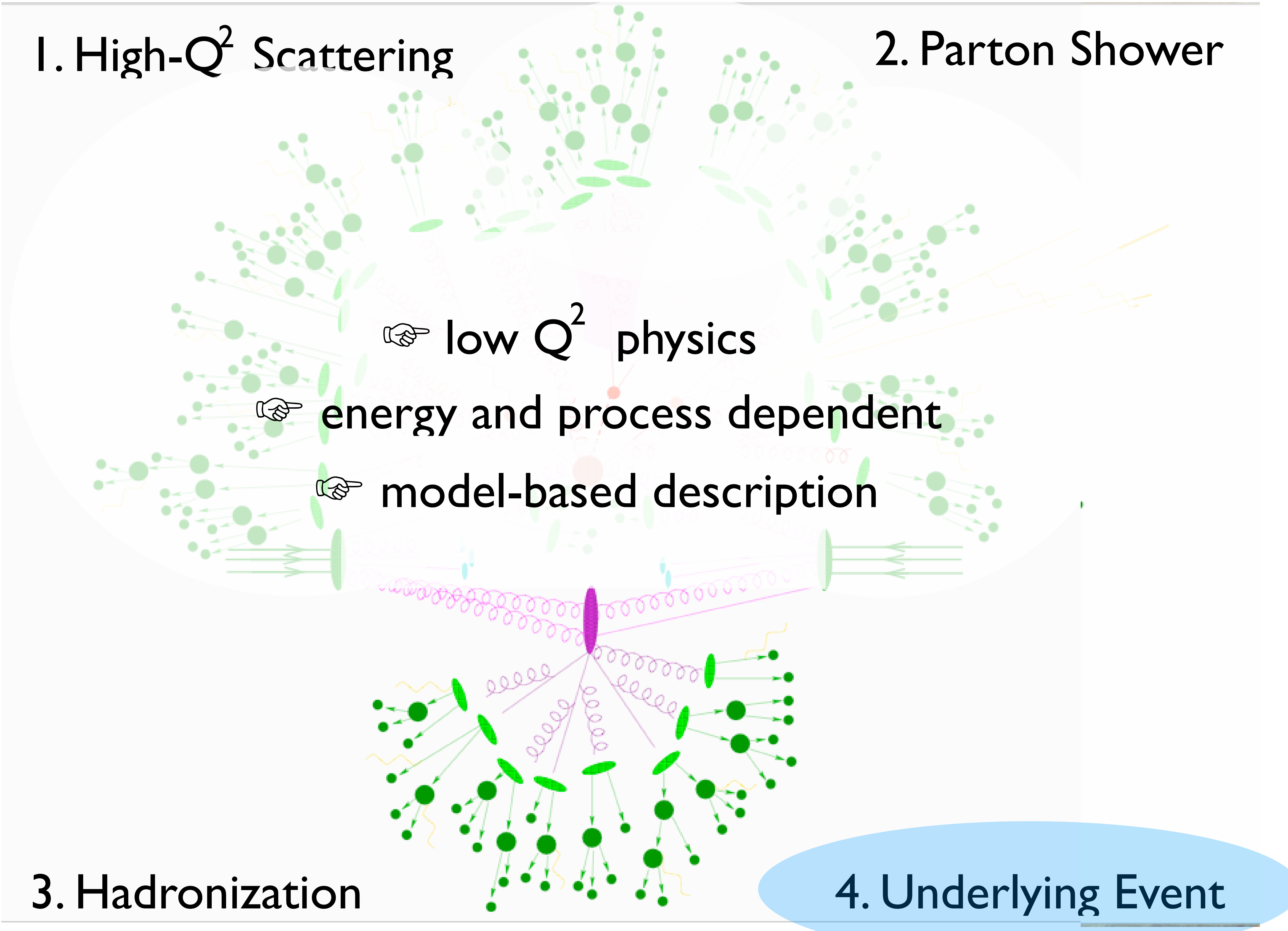
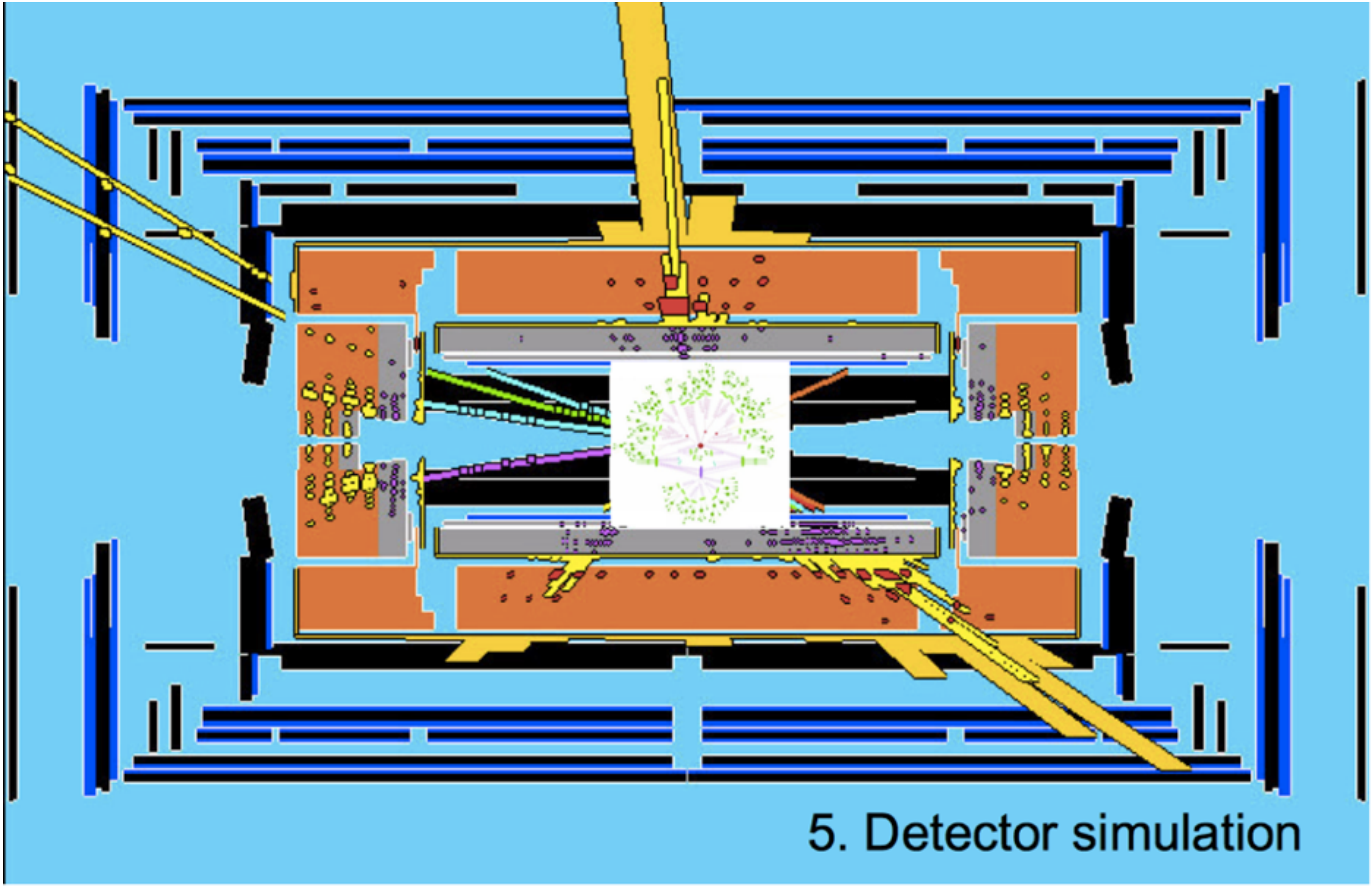


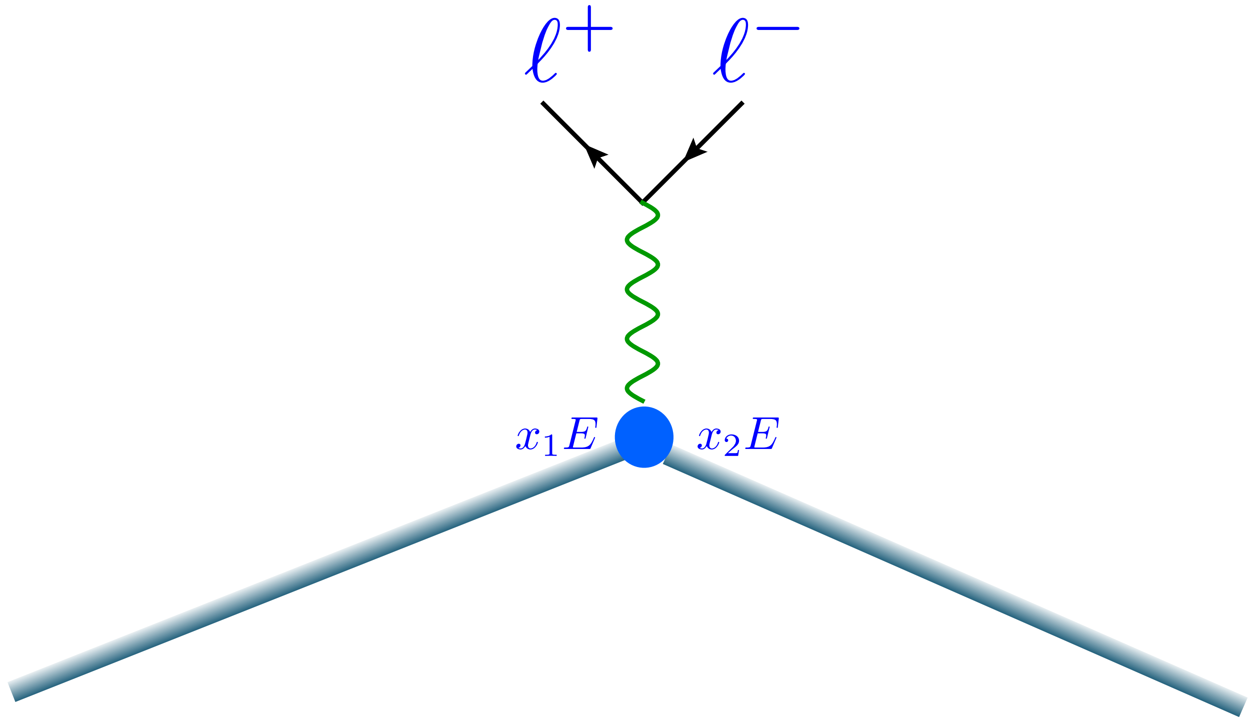
☞ low Q^2 physics

☞ energy and process dependent

☞ model-based description

What are the MC for?

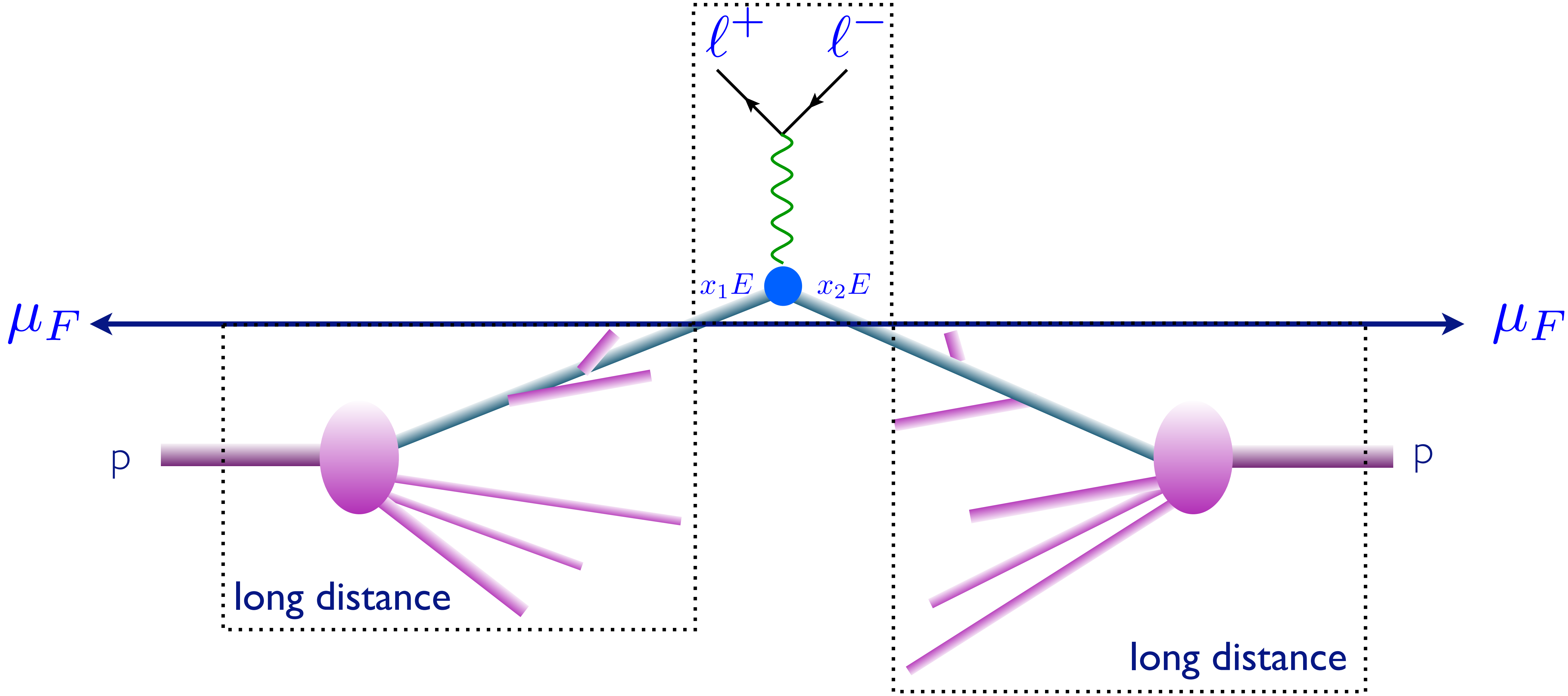




$$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Parton-level cross section

Master formula for the LHC

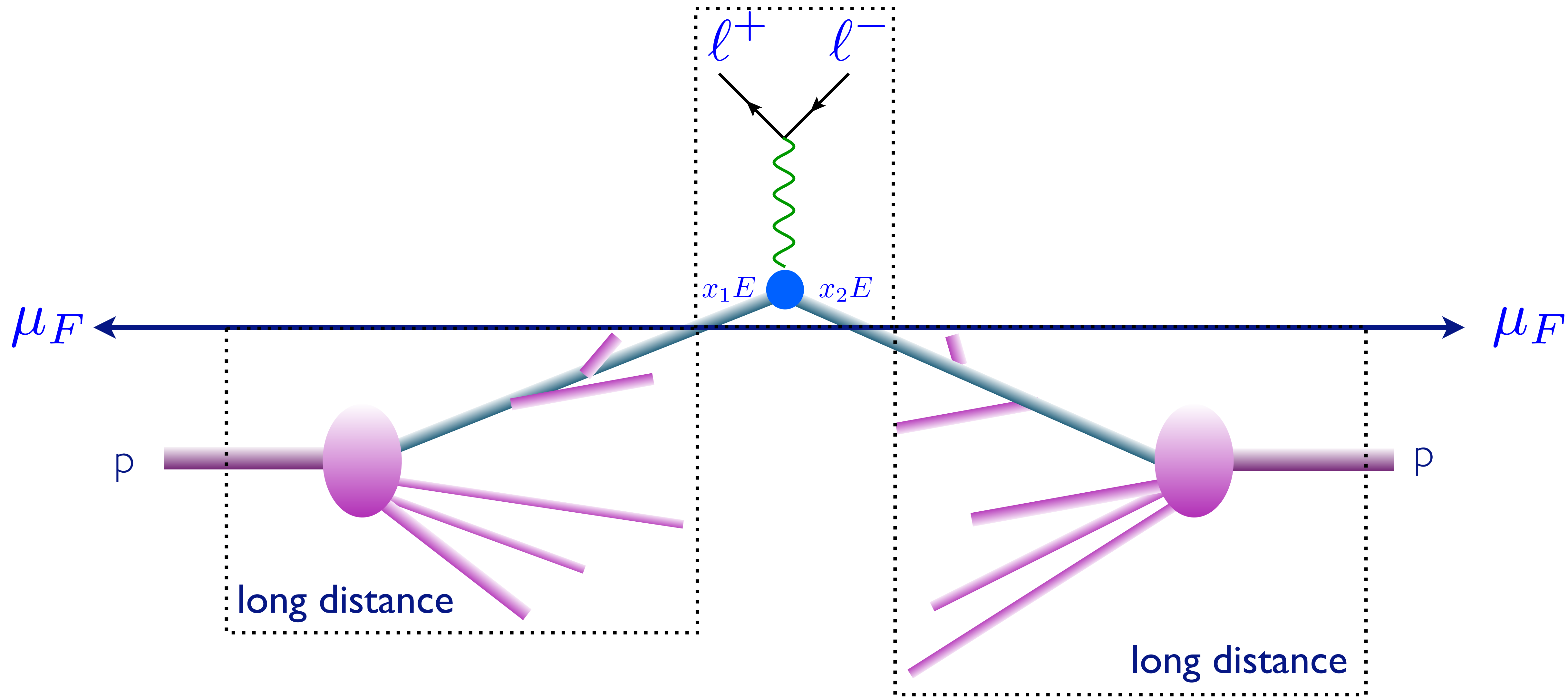


$$f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

Parton density functions

Parton-level cross section

Master formula for the LHC



$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

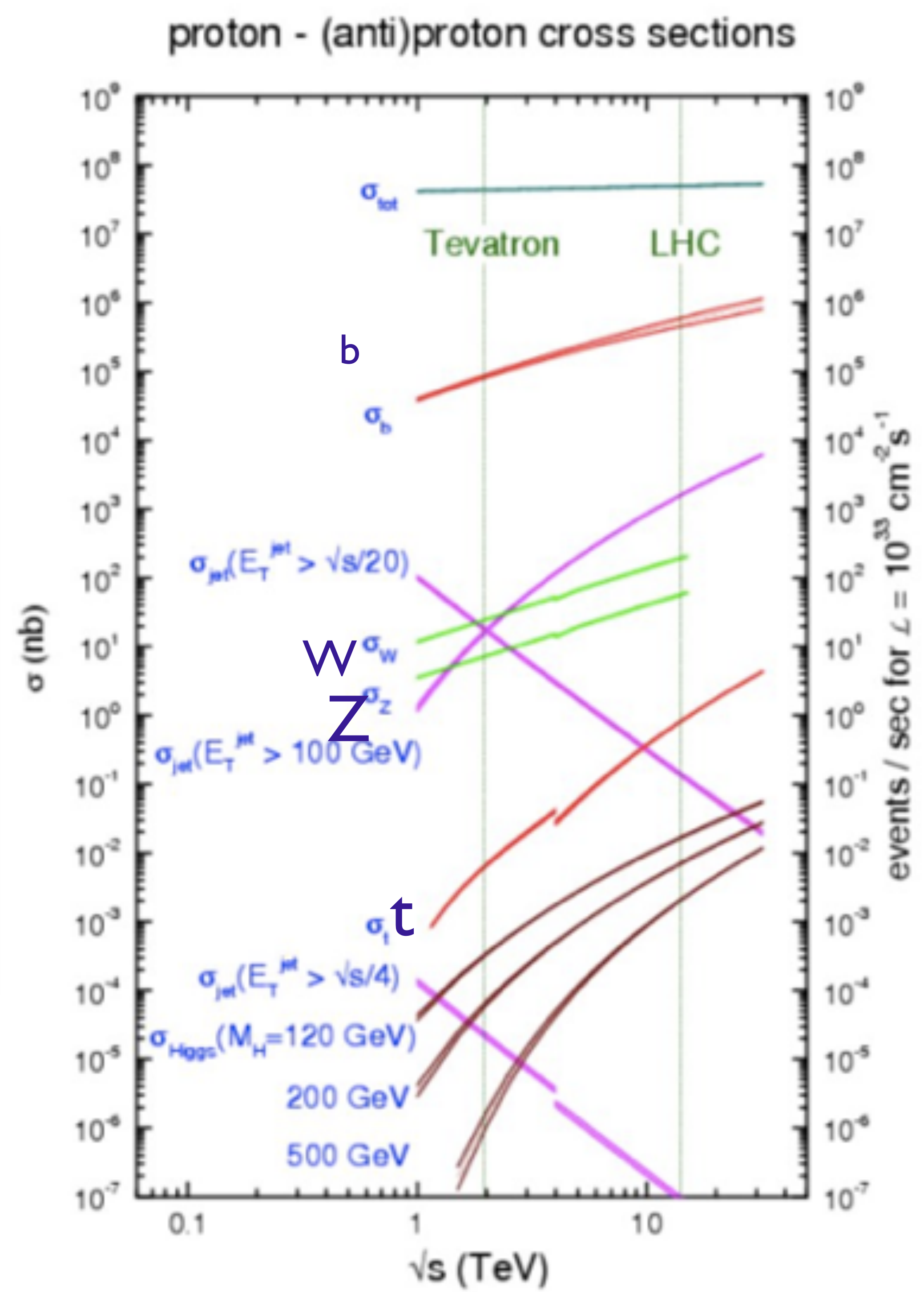
Hadron Colliders

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat\sigma_{ab\to X}(\hat s, \mu_F, \mu_R)$$

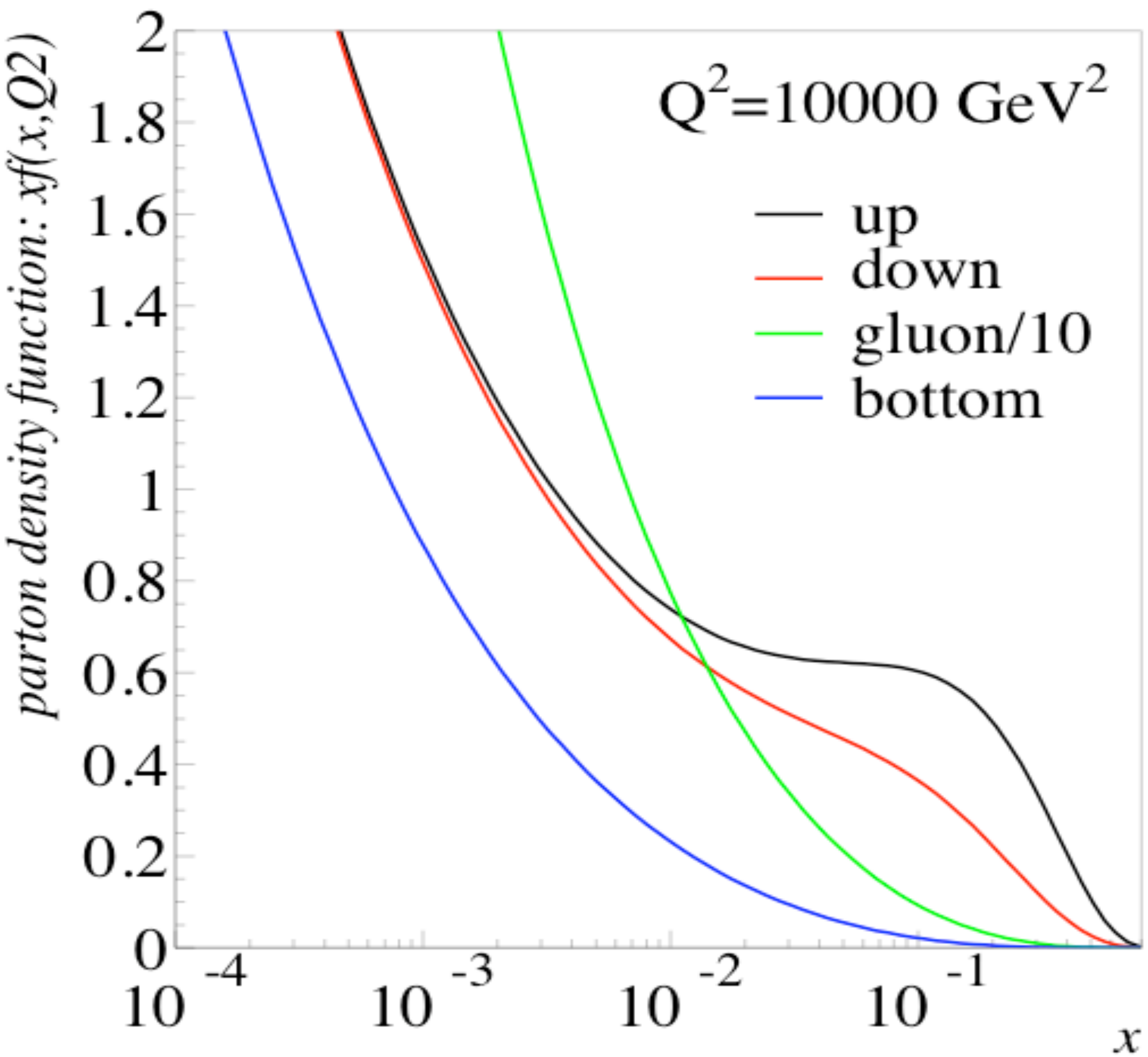
First: Understand our processes!

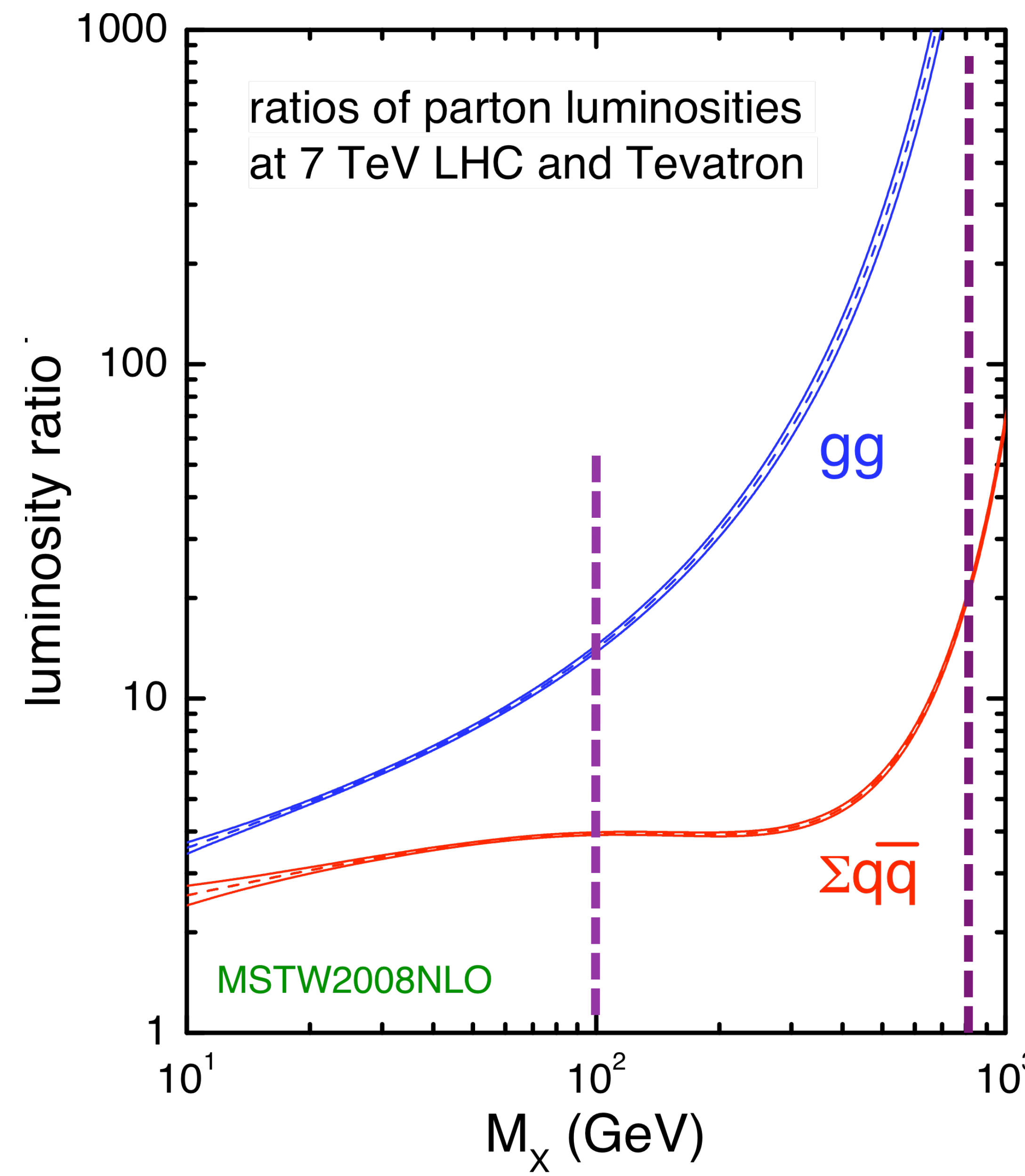
Cross sections at a collider depend on:

- Coupling strength
- Coupling to what?
  (light quarks, gluons, heavy quarks, EW gauge bosons?)
- Mass
- Single production/pair production



Parton densities

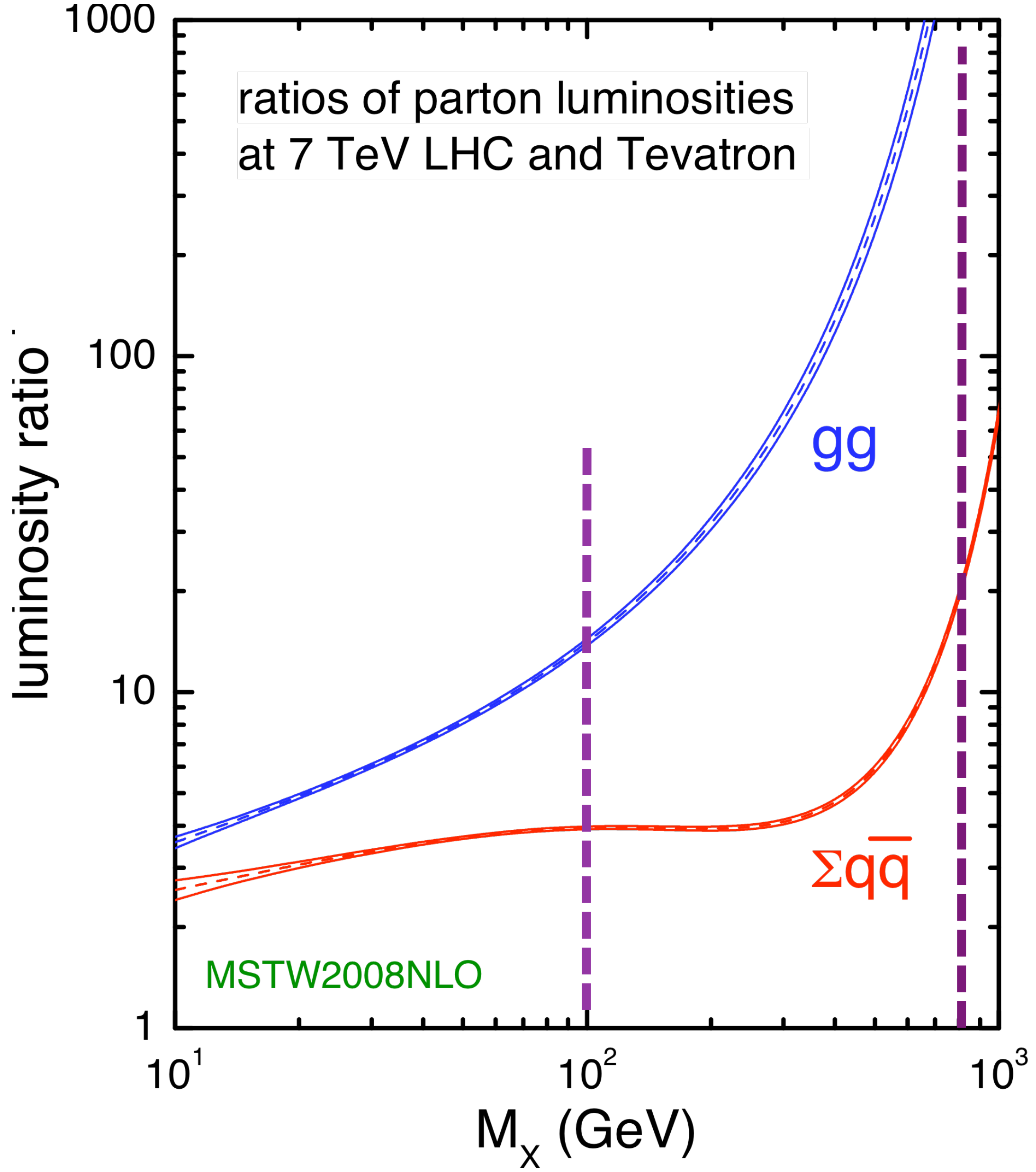


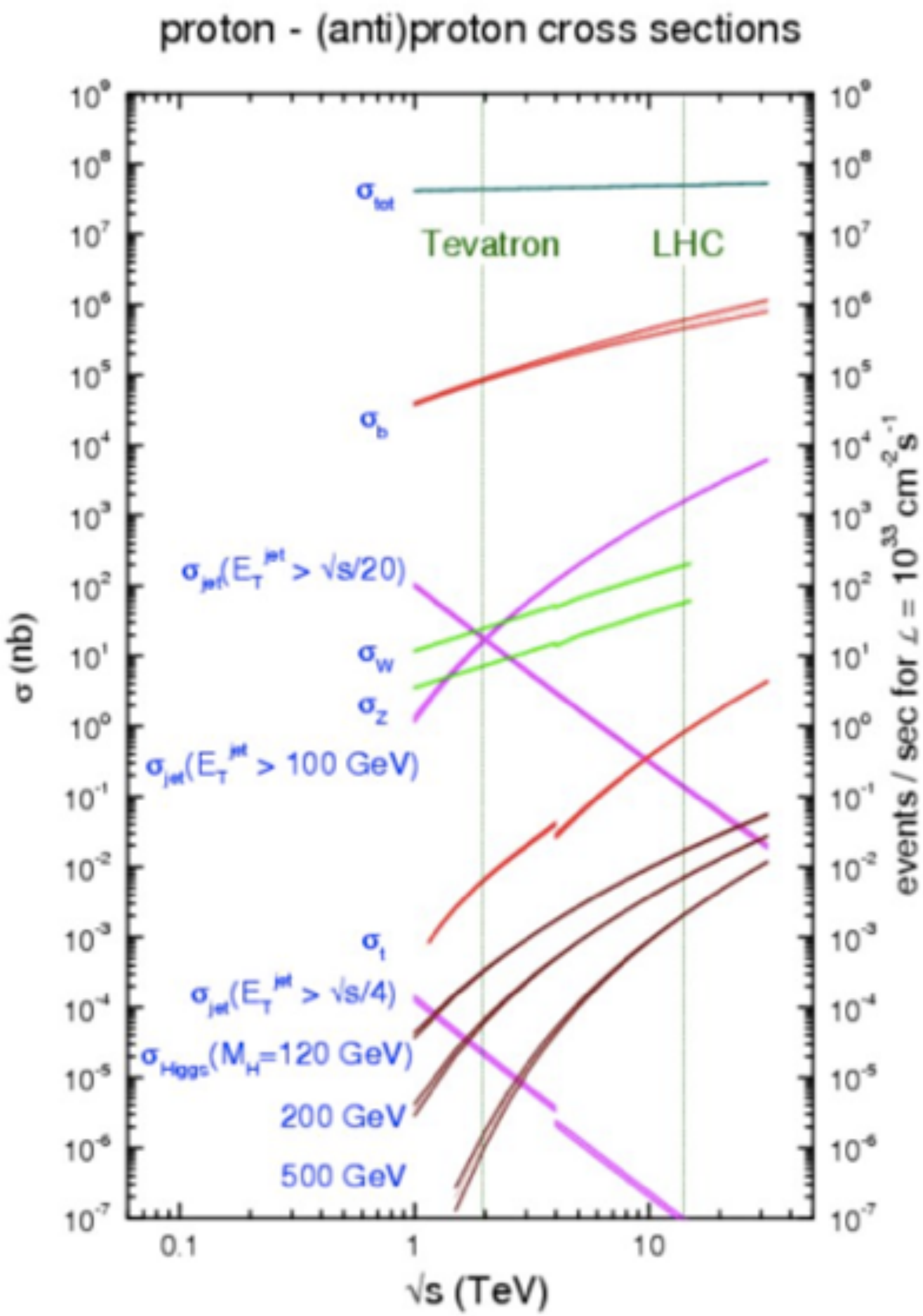


At small x (small ŝ), gluon domination.
At large x valence quarks

LHC formidable at large mass –
For low mass, Tevatron backgrounds smaller

Back to the processes





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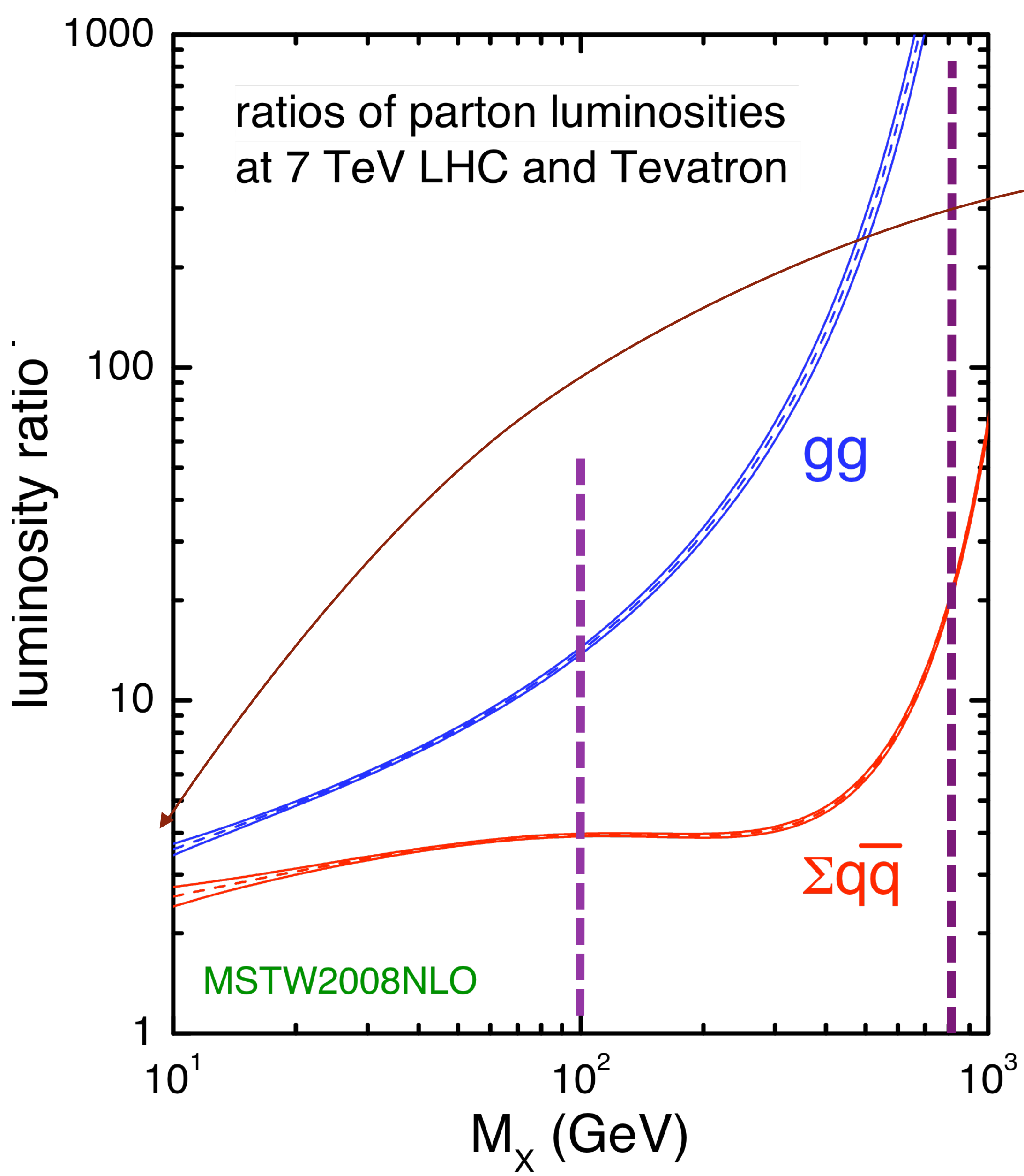

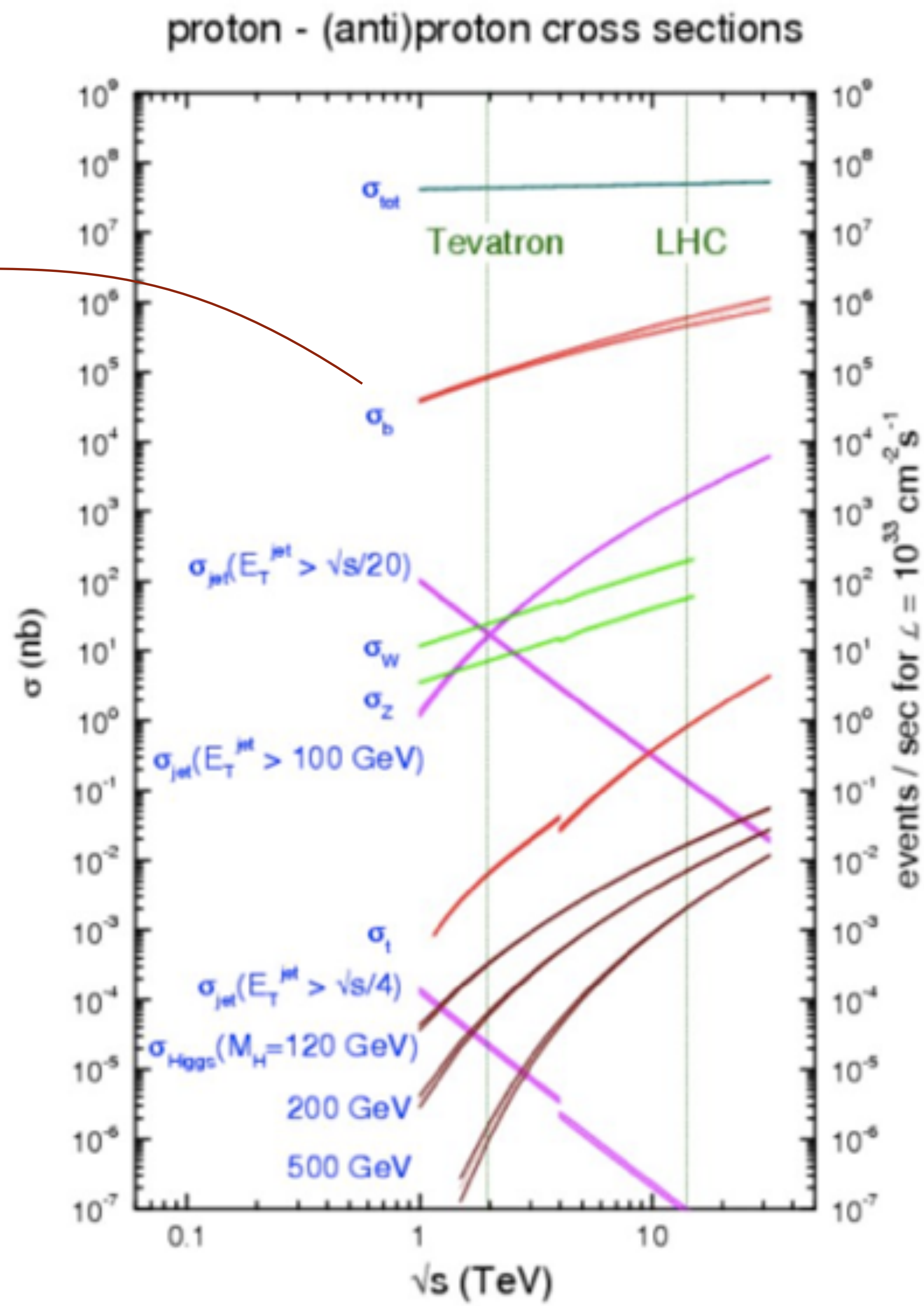

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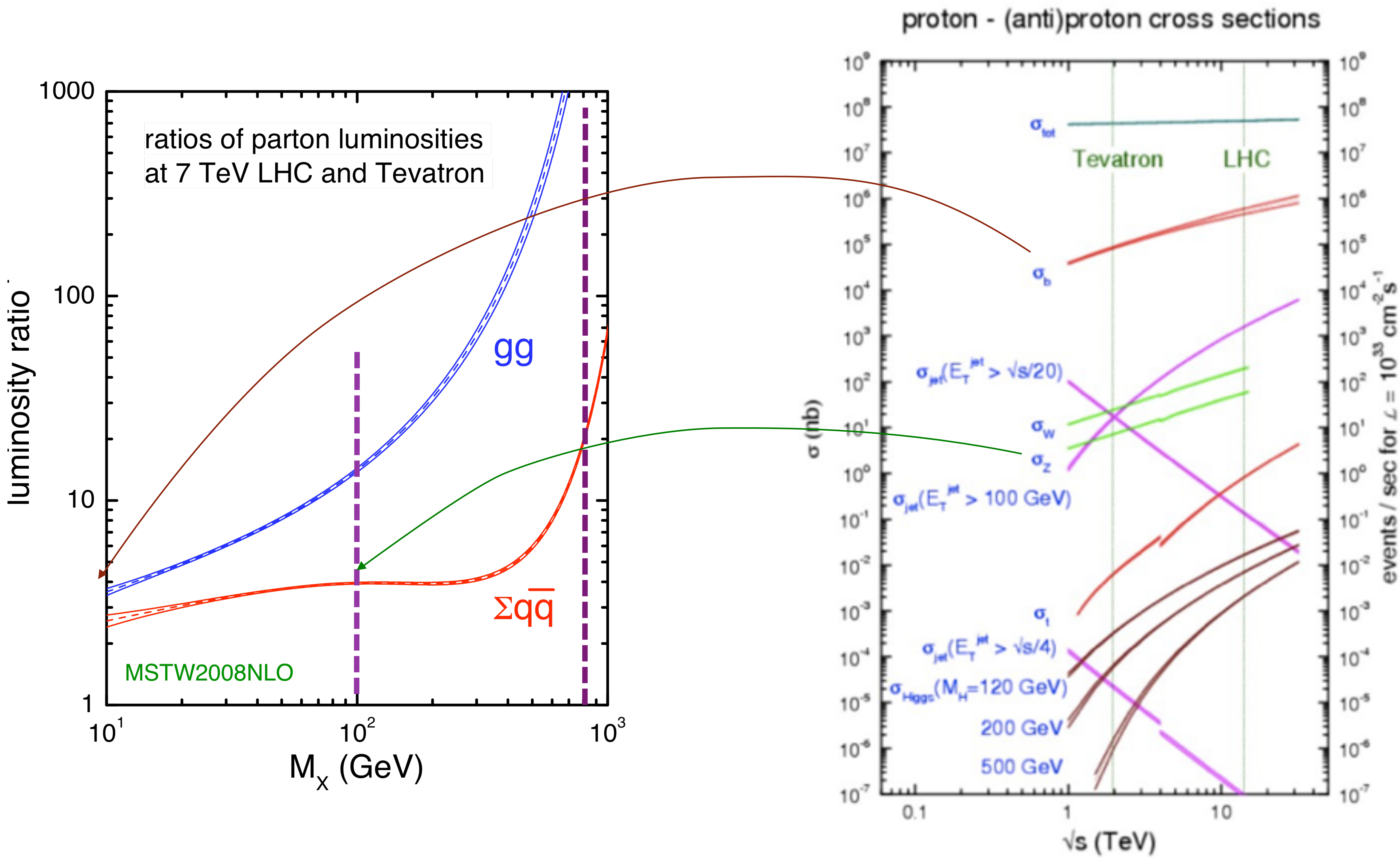

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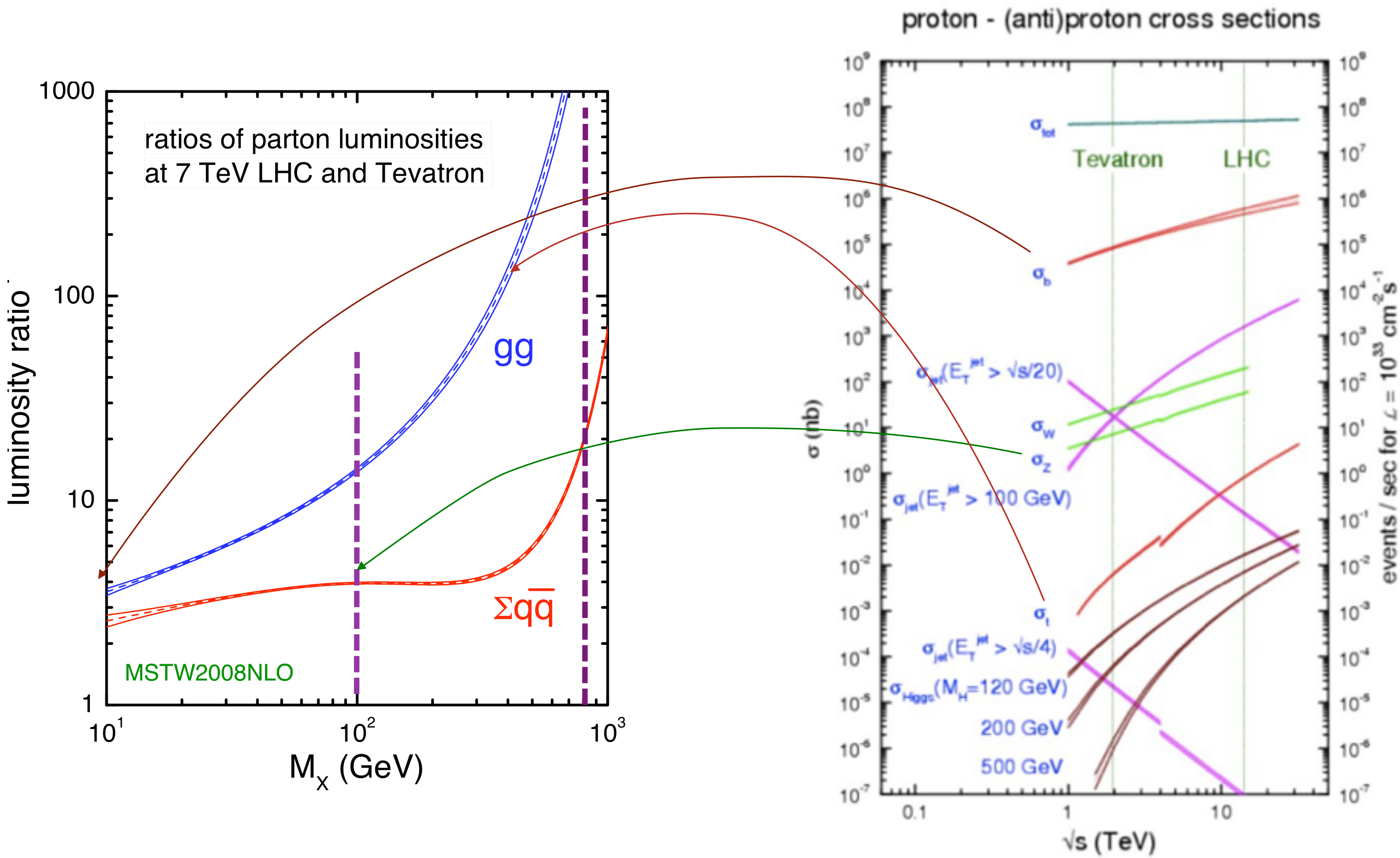
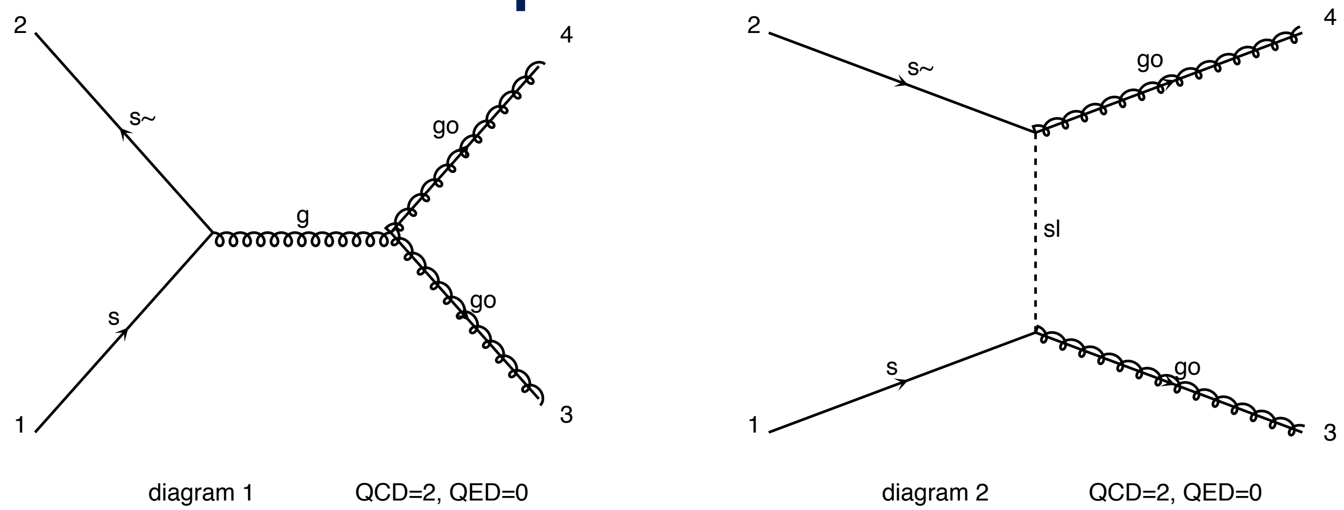


Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

- cross-section

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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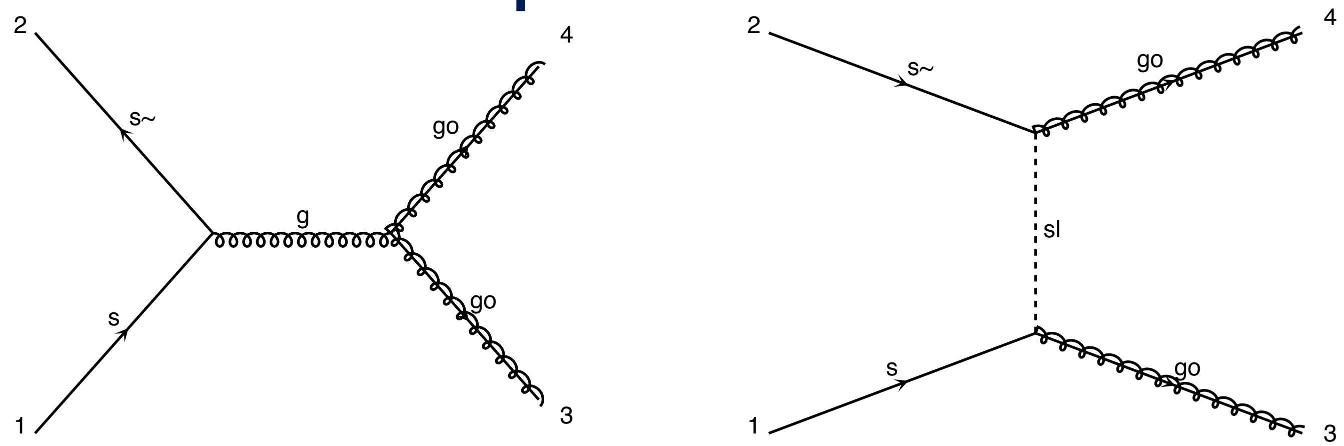

diagram 1 QCD=2, QED=0

diagram 2 QCD=2, QED=0

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Easy enough

Hard

Very Hard (in general)

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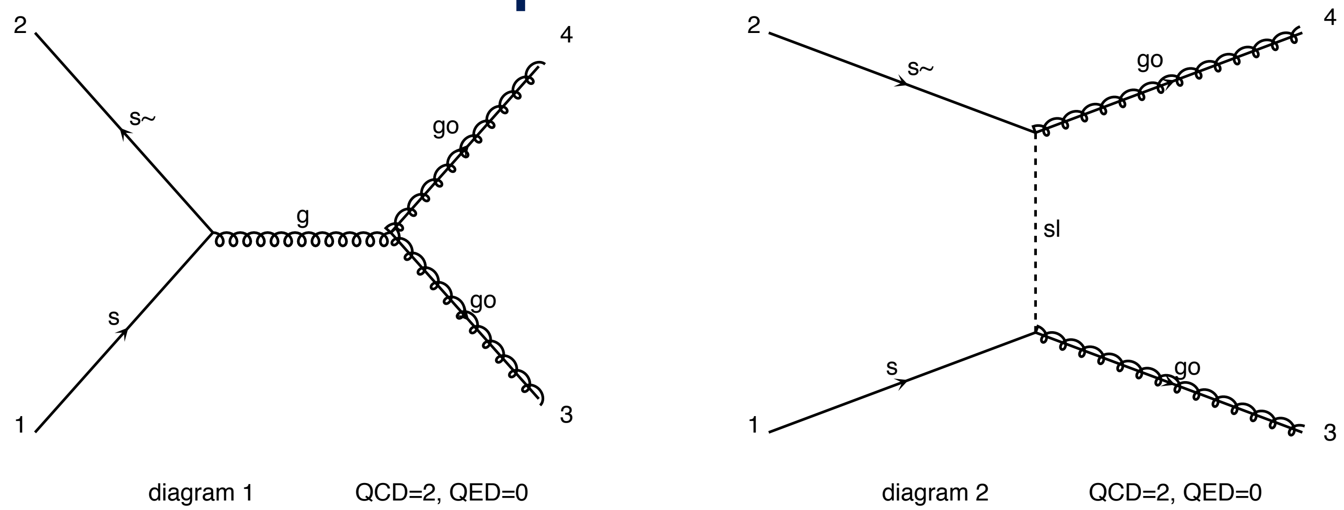

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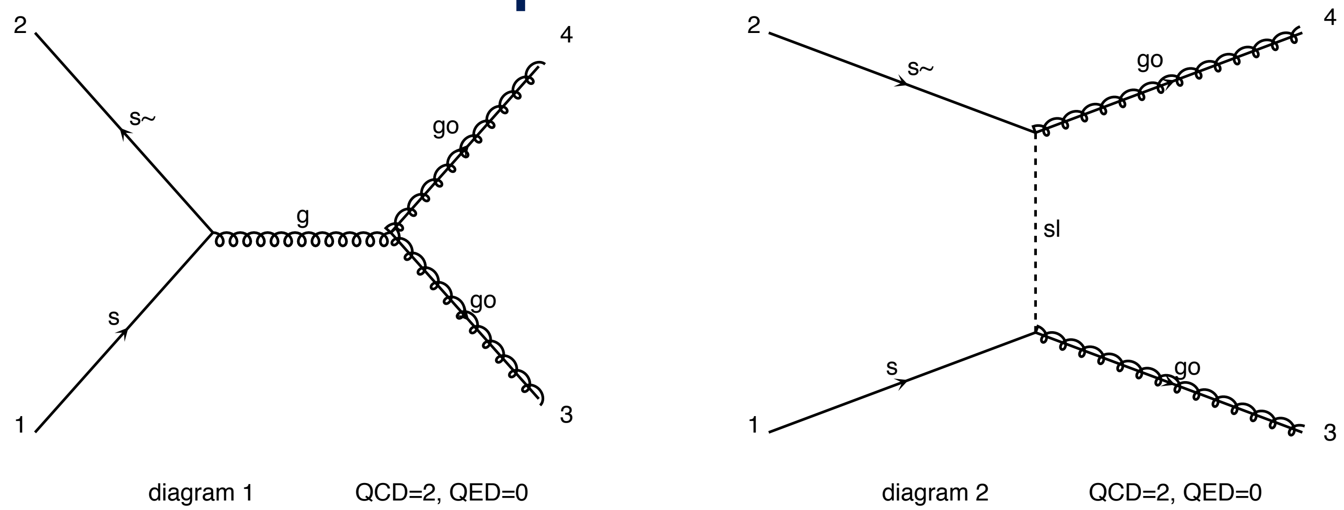

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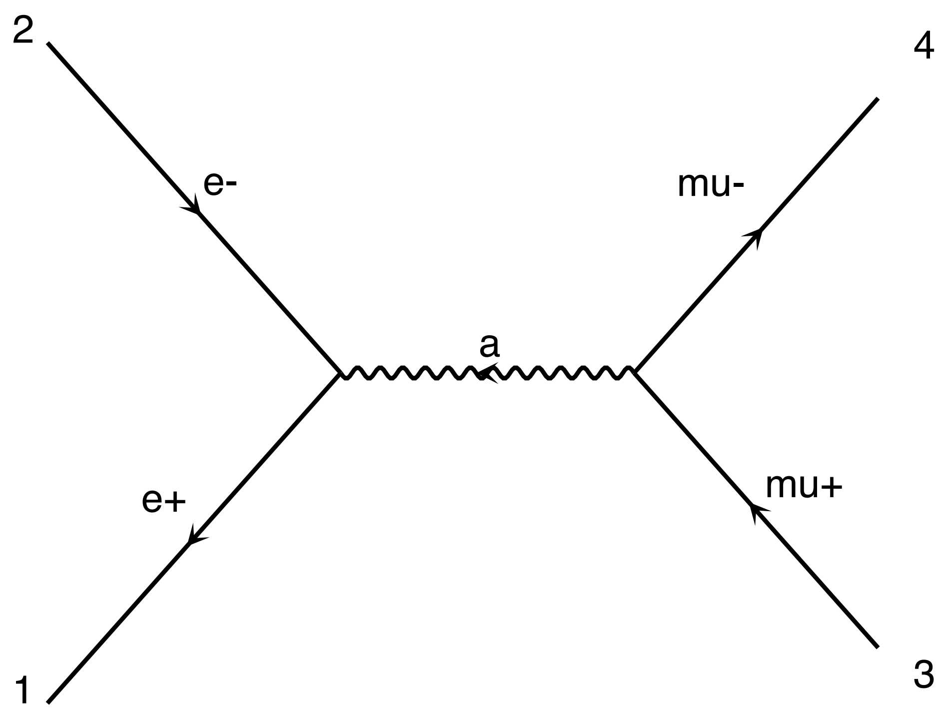
Very Hard (in general)

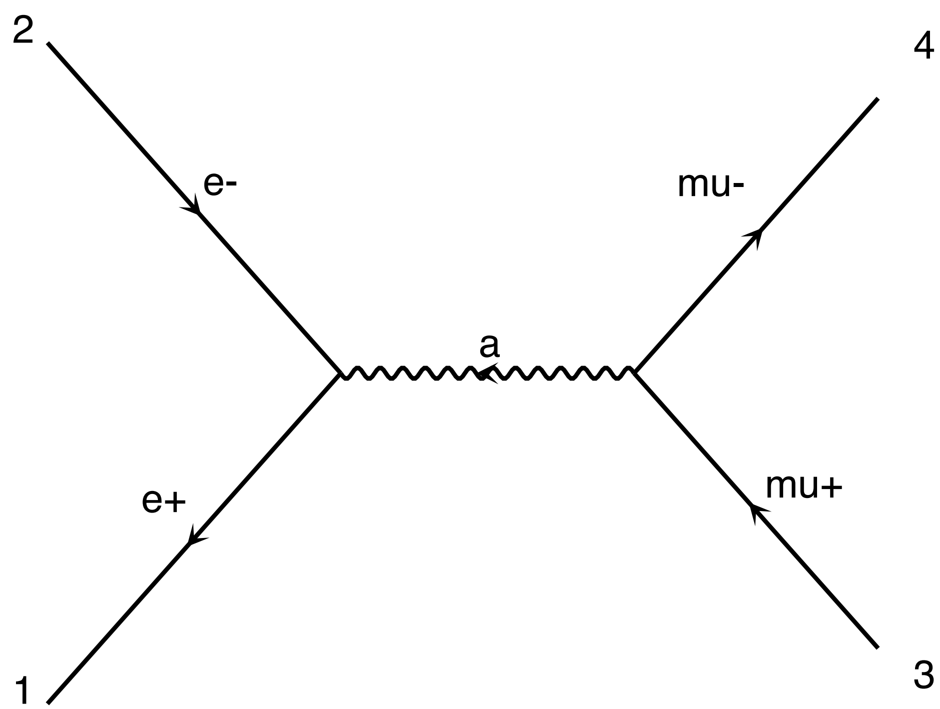
After

Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

Matrix Element



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$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

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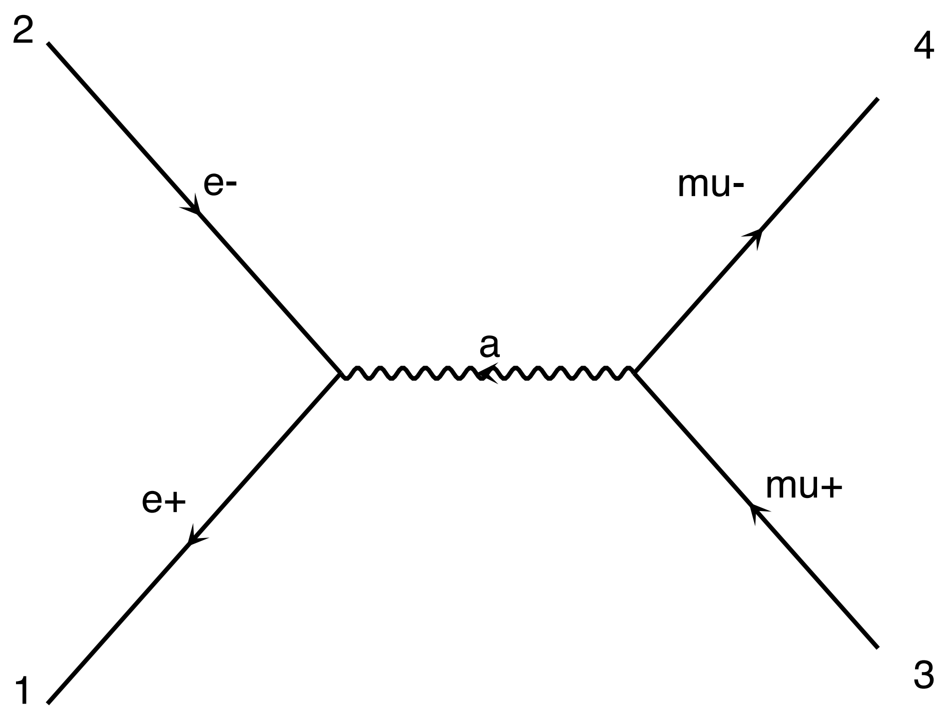

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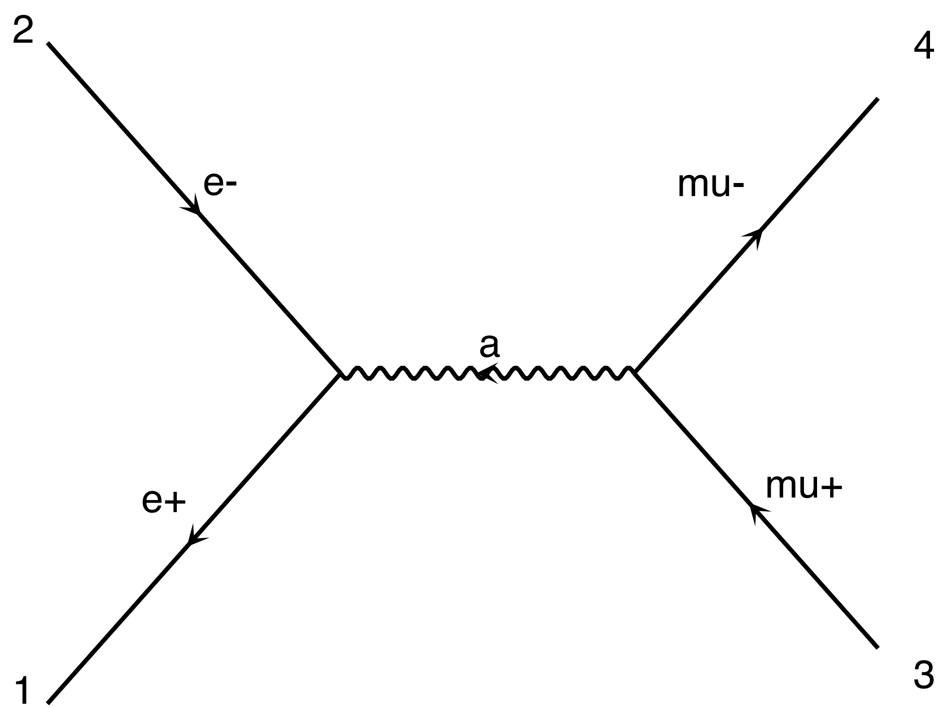

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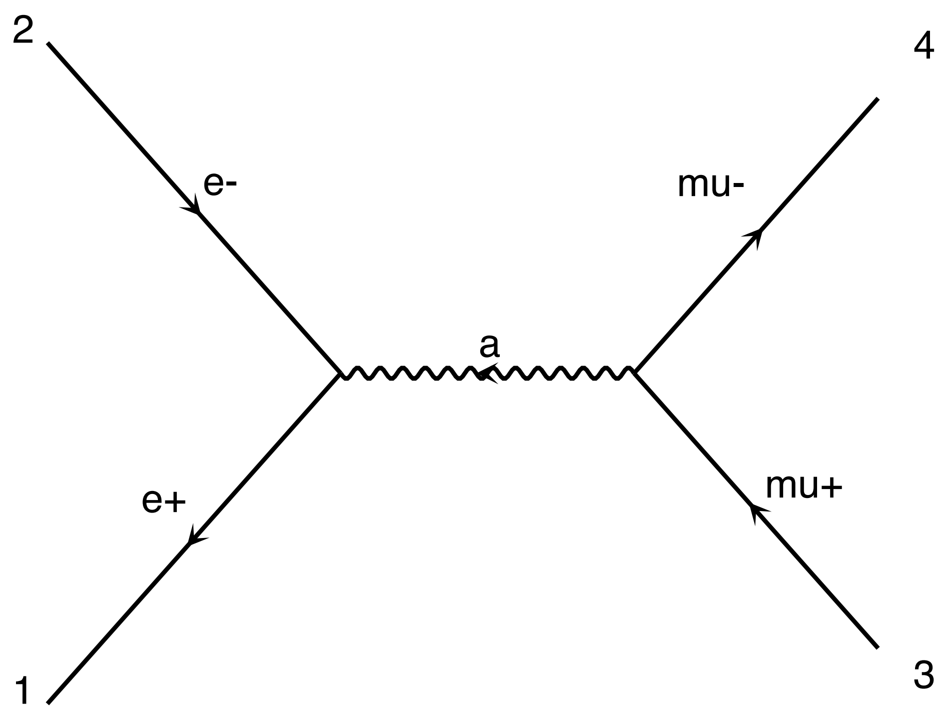
Matrix Element



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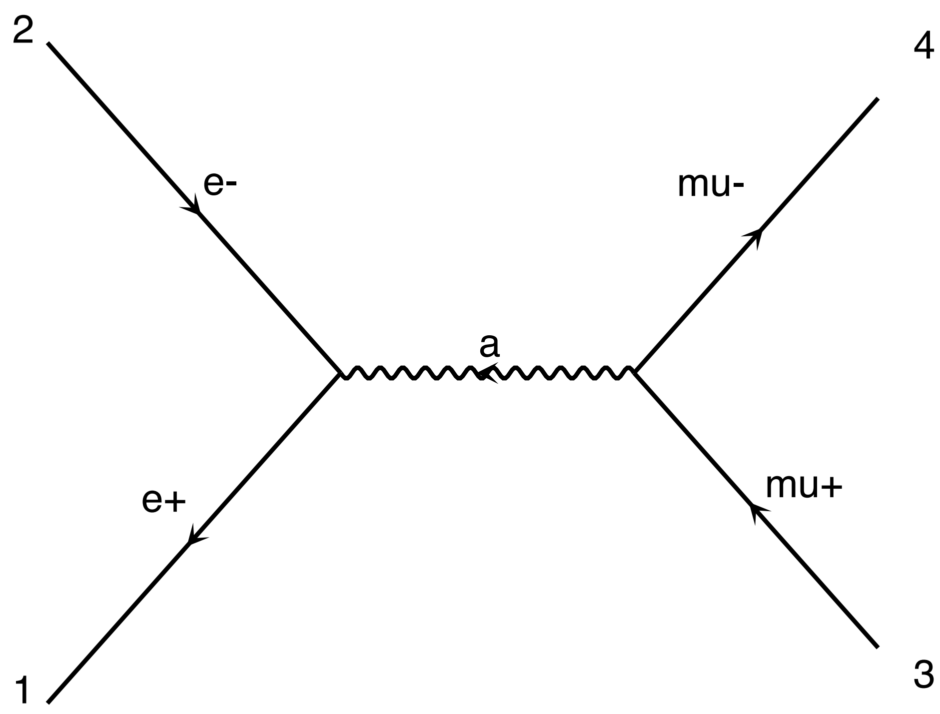
$$\rightarrow \frac{8e^4}{q^4} [(p_1 . p_3)(p_2 . p_4) + (p_1 . p_4)(p_2 . p_3)]$$

Matrix Element



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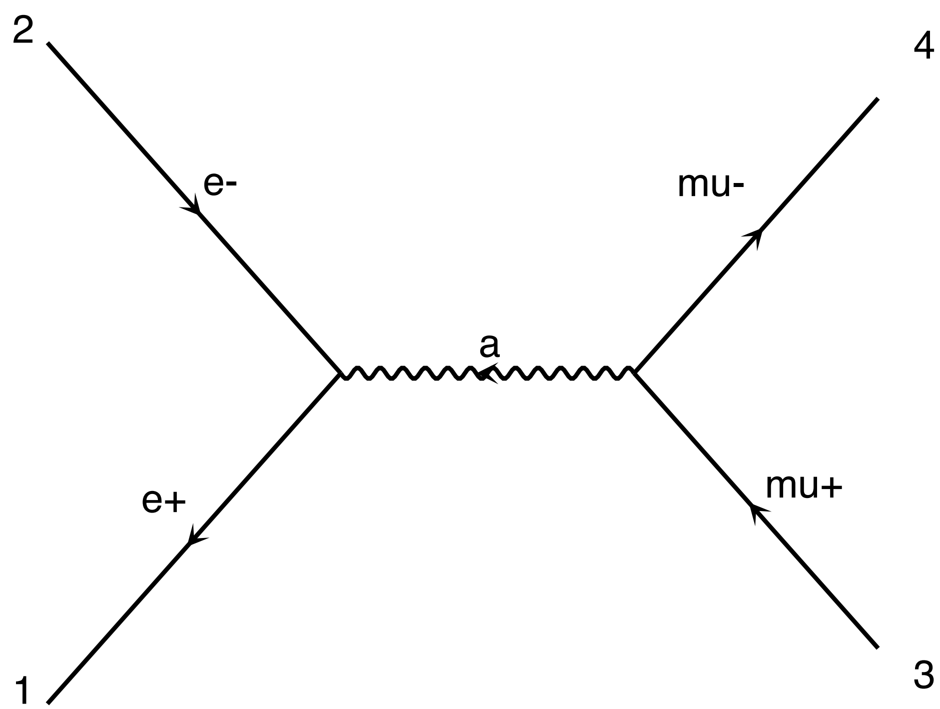
Very Efficient !!!

Matrix Element



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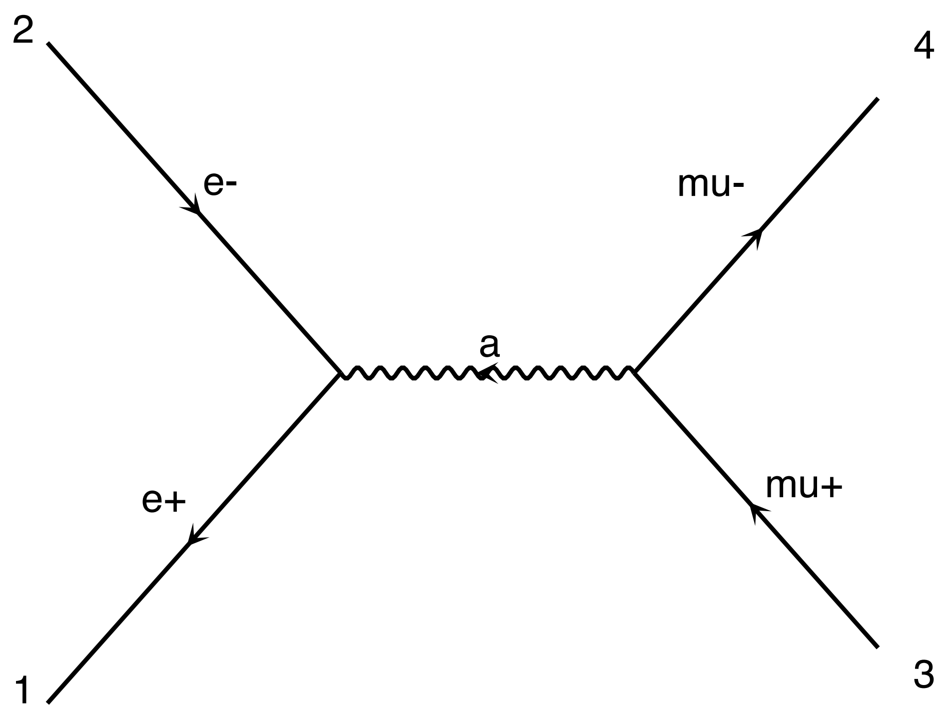
Only for 2→2 and 2→3

Matrix Element



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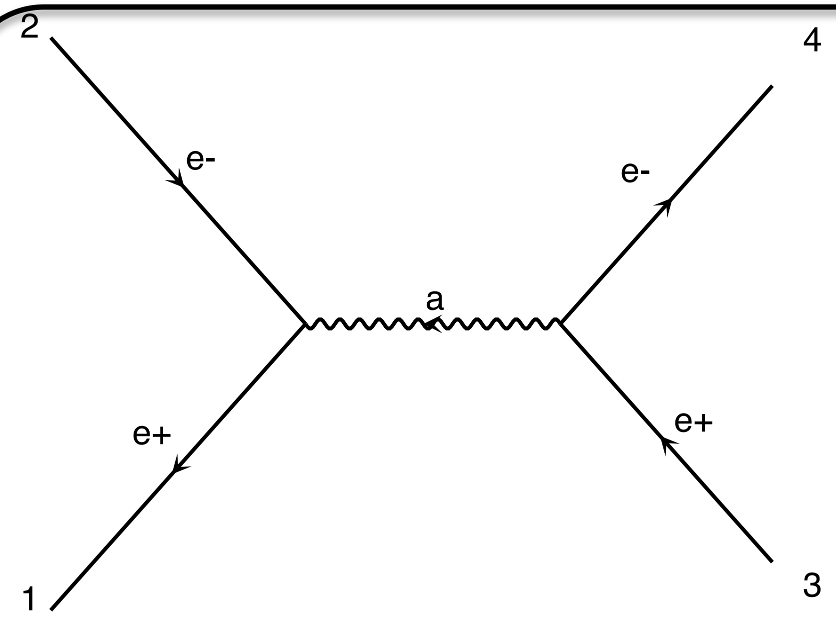
Very Efficient !!!

Only for 2→2 and 2→3

Because the number of terms rises as N^2

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



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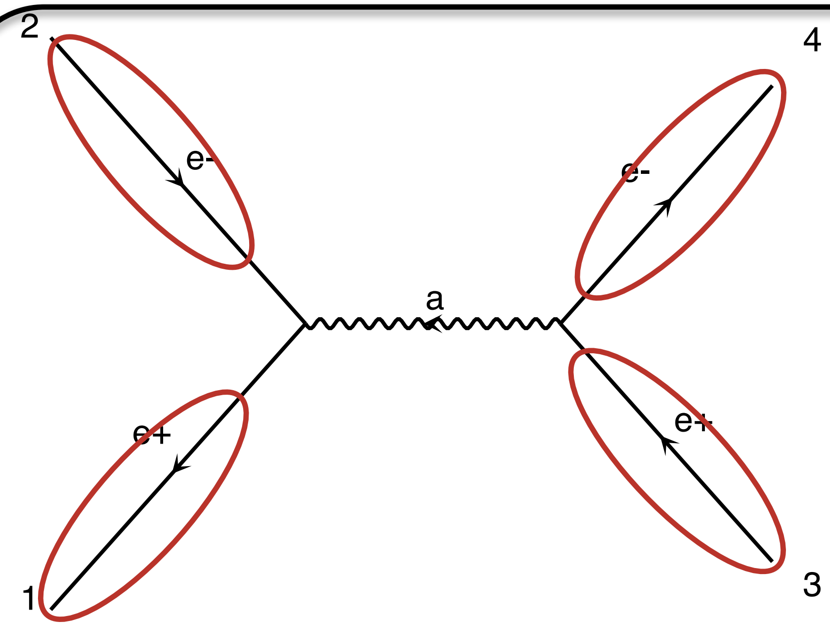

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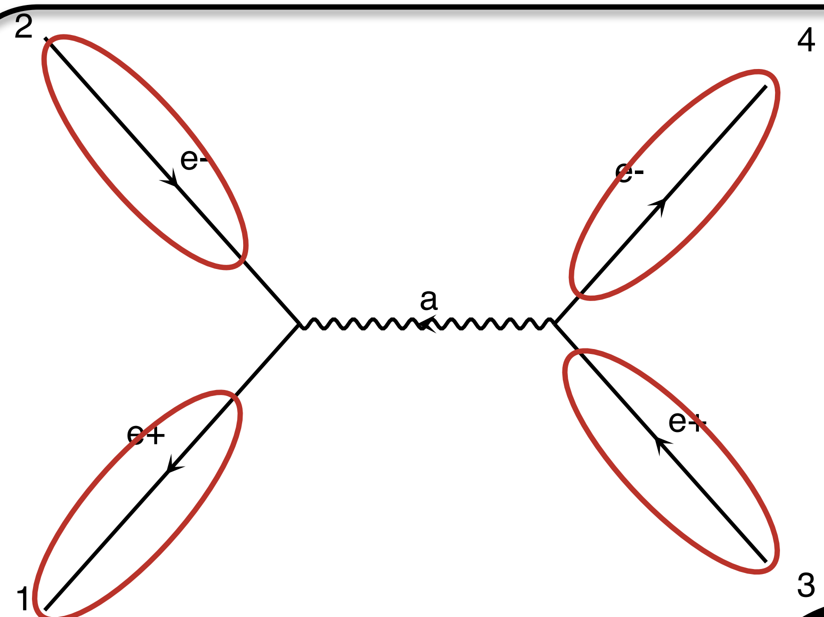
Numbers for given helicity and momenta

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

Lines present in the code.

$$\bar{v}_1 = fct(\vec{p}_1, m_1)$$
$$u_2 = fct(\vec{p}_2, m_2)$$
$$v_3 = fct(\vec{p}_3, m_3)$$
$$\bar{u}_4 = fct(\vec{p}_4, m_4)$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
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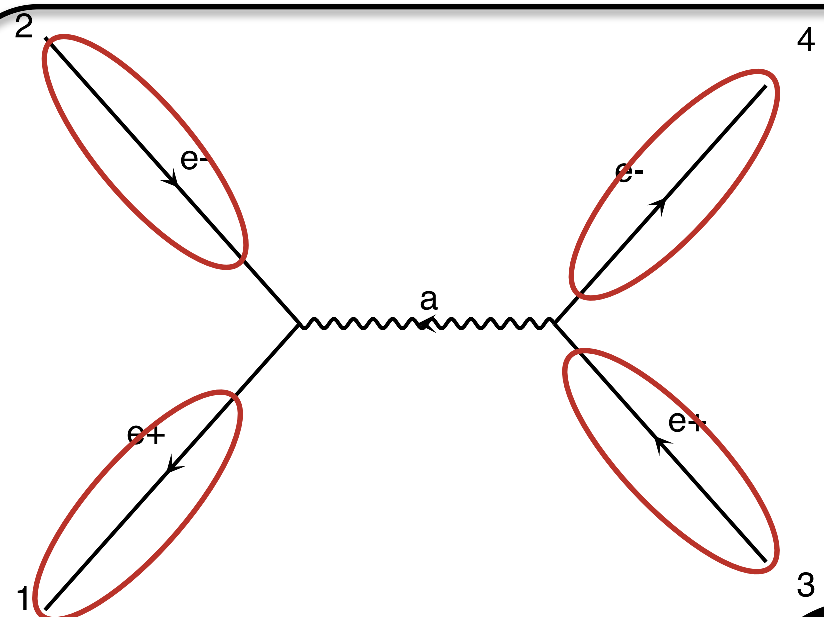


$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

Lines present in the code.

$$\left\{ \begin{array}{l} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{array} \right.$$

$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_\lambda(\vec{p}) \\ \omega_\lambda(p)\chi_\lambda(\vec{p}) \end{pmatrix}$$

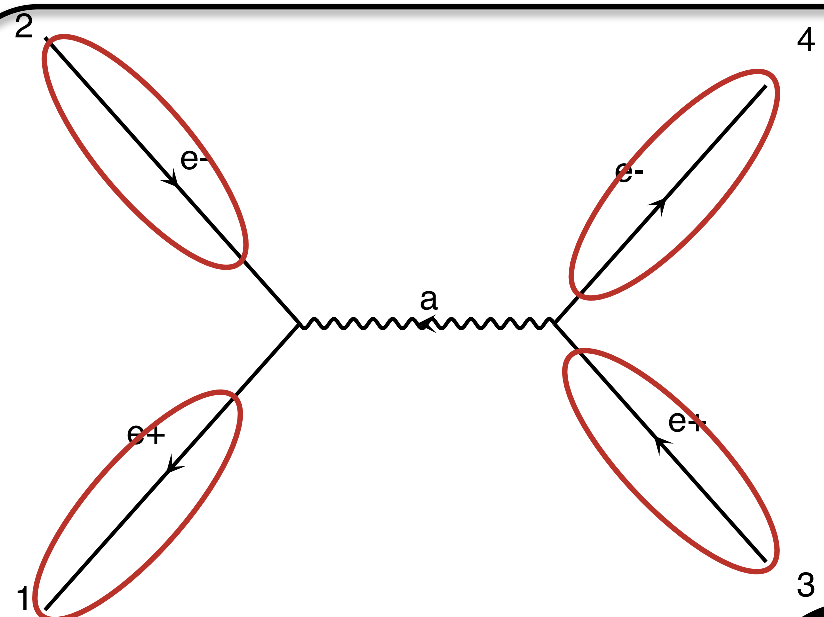
$$\omega_\pm(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} |\vec{p}| + p_z \\ p_x + ip_y \end{pmatrix},$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} -p_x + ip_y \\ |\vec{p}| + p_z \end{pmatrix}.$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^{\nu} u)$$

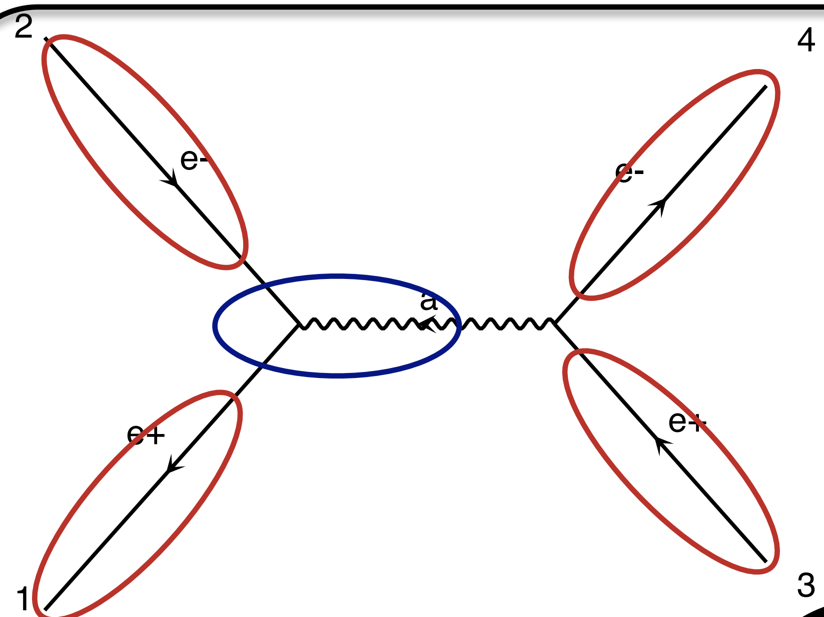
Numbers for given helicity and momenta

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{cases}$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

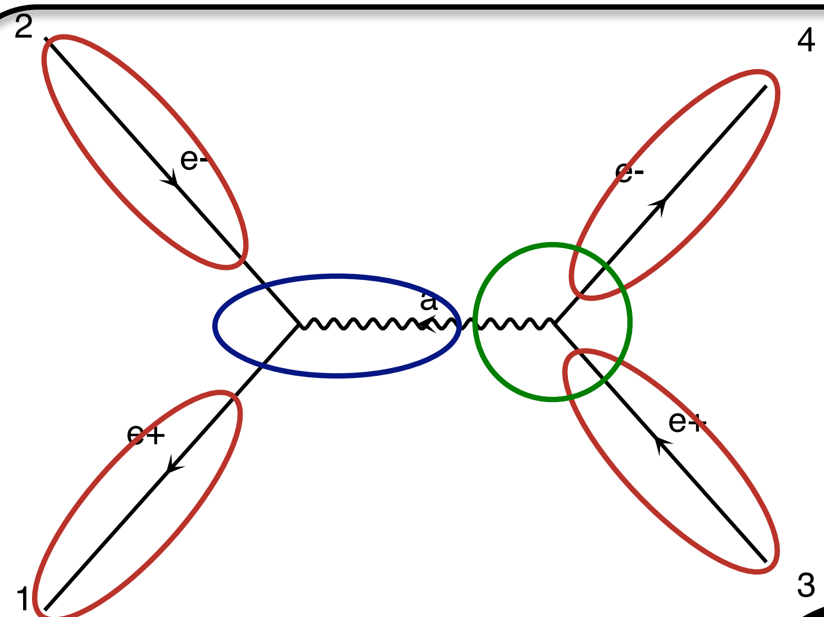
Calculate propagator wavefunctions

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a) \end{cases}$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v)\, \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

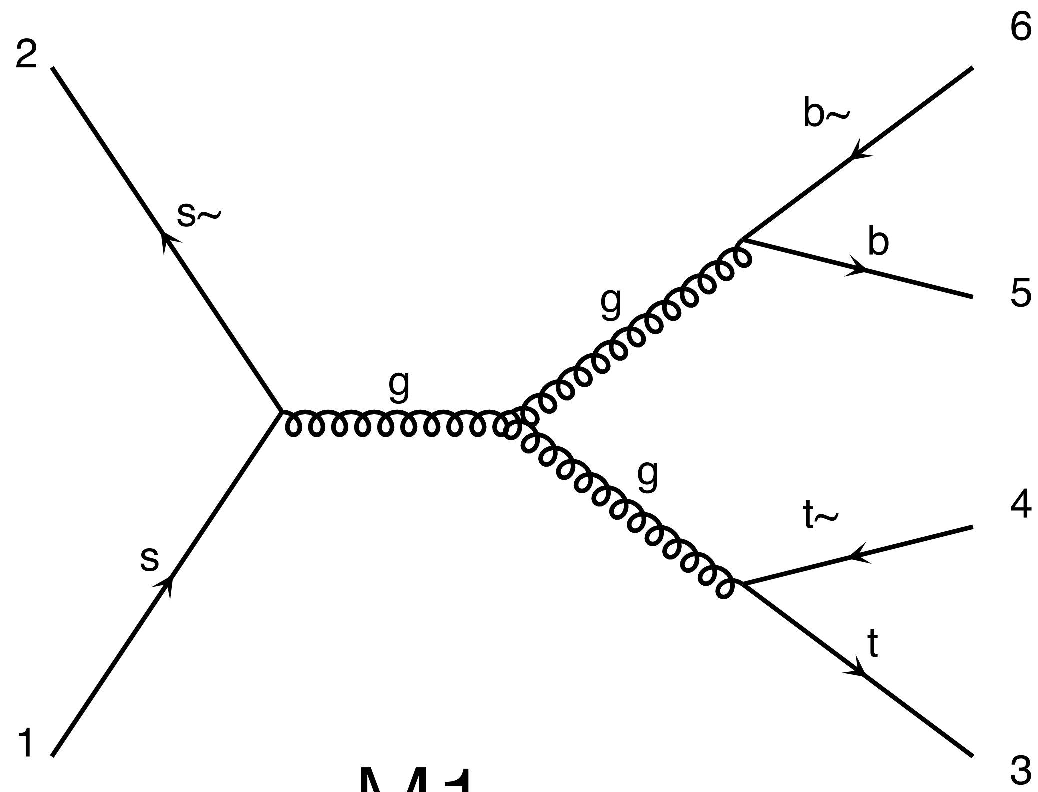
Finally evaluate amplitude (c-number)

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a) \\ \mathcal{M} = fct(v_3, \bar{u}_4, W_a) \end{cases}$$

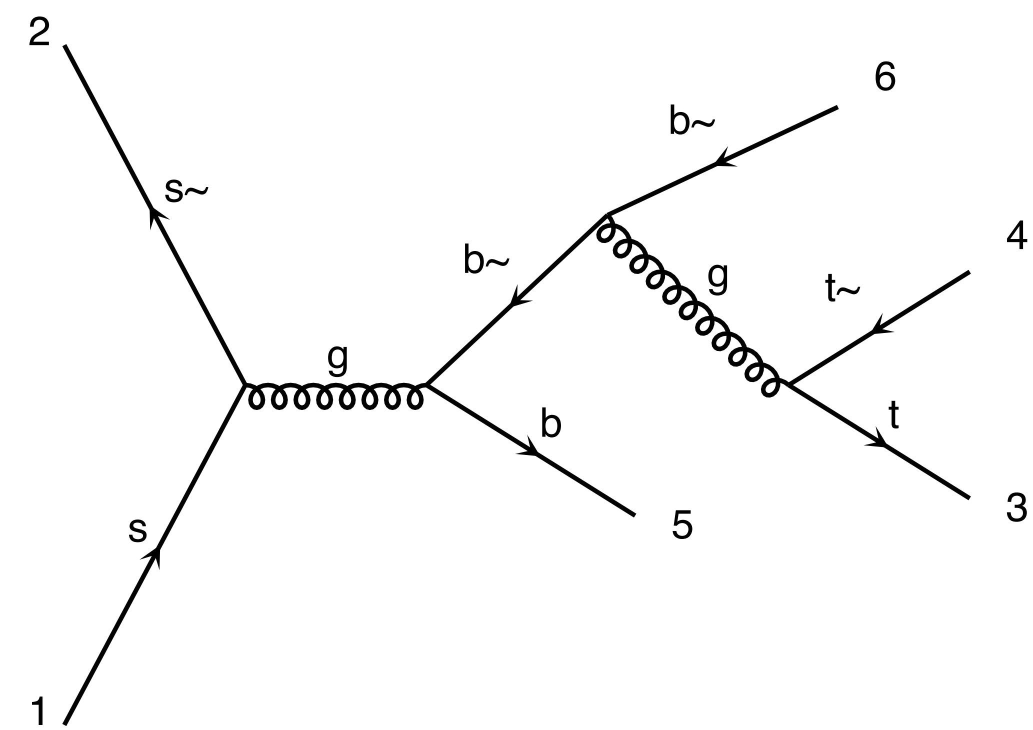
Real case

Known



M1

Number of routines: 0



M2

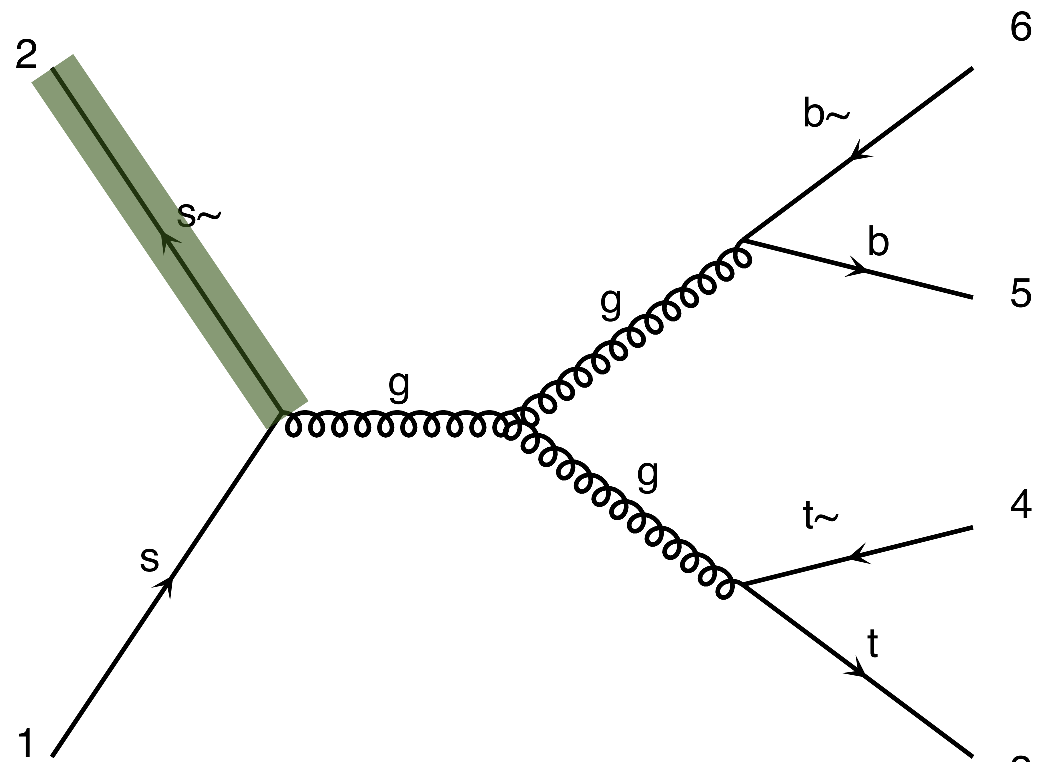
Number of routines: 0

Number of routines for both:0

$$|M|^2 = |M_1 + M_2|^2$$

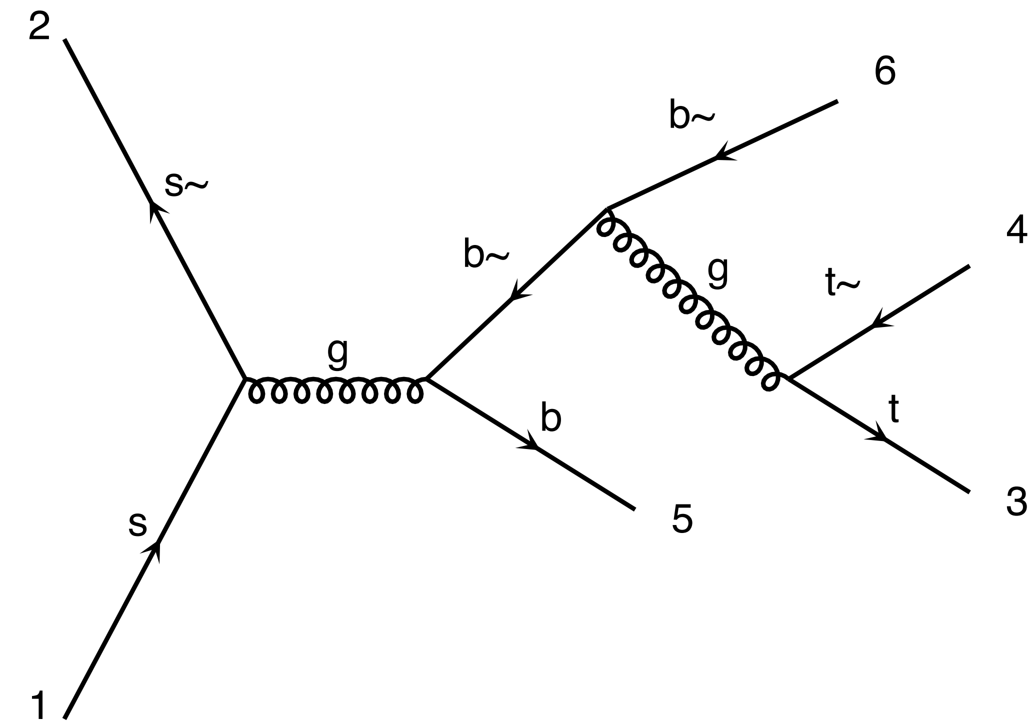
Real case

Known



M1

Number of routines: 1



M2

Number of routines: 0

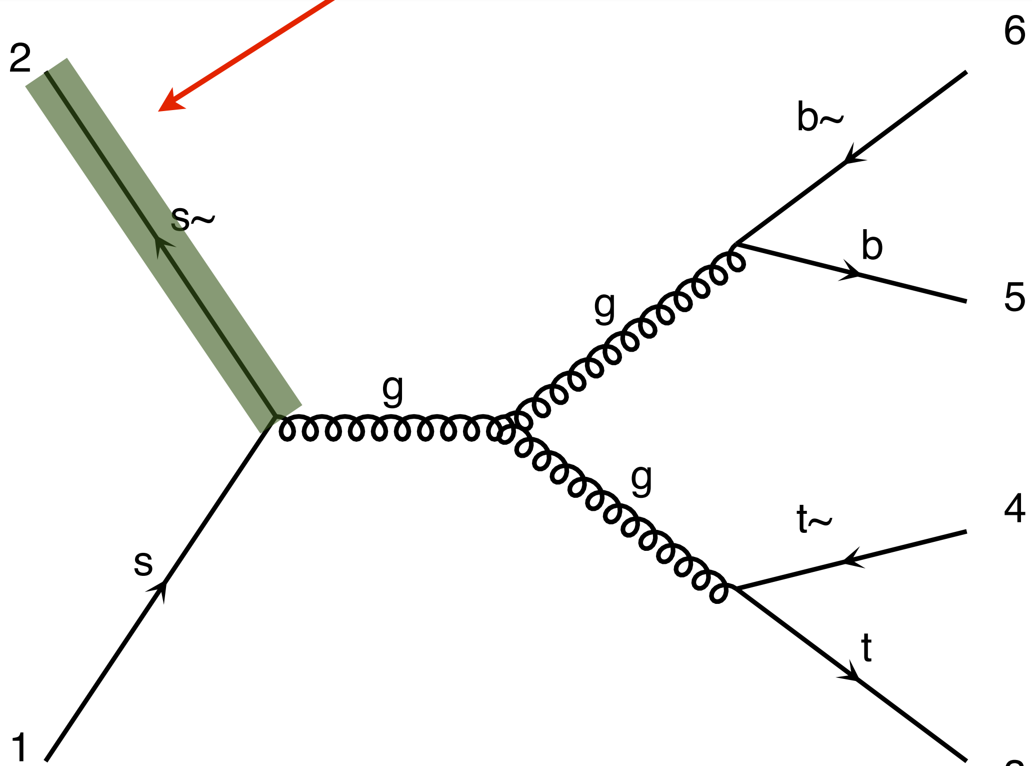
Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

Real case

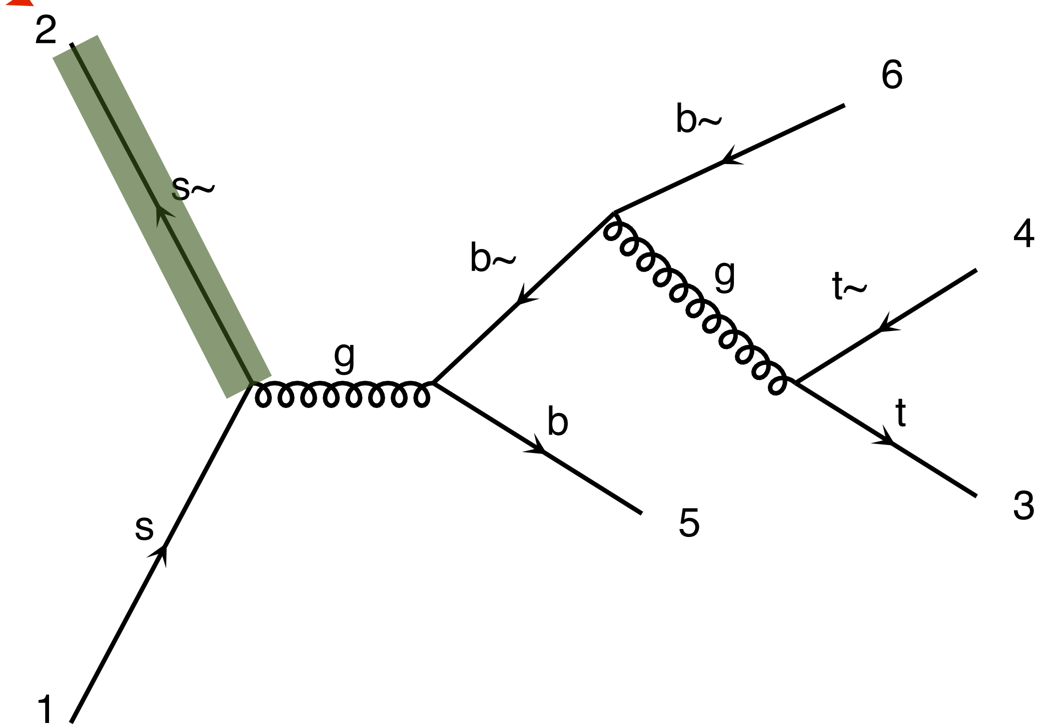
Identical

Known



M1

Number of routines: 1



M2

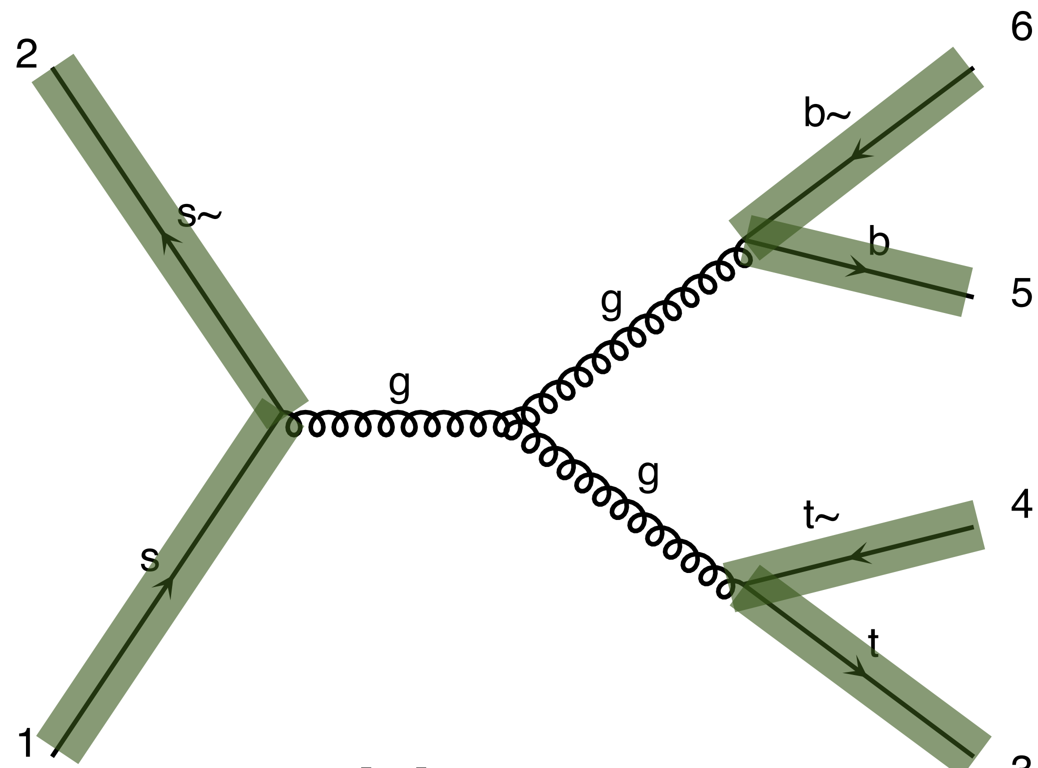
Number of routines: 1

Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

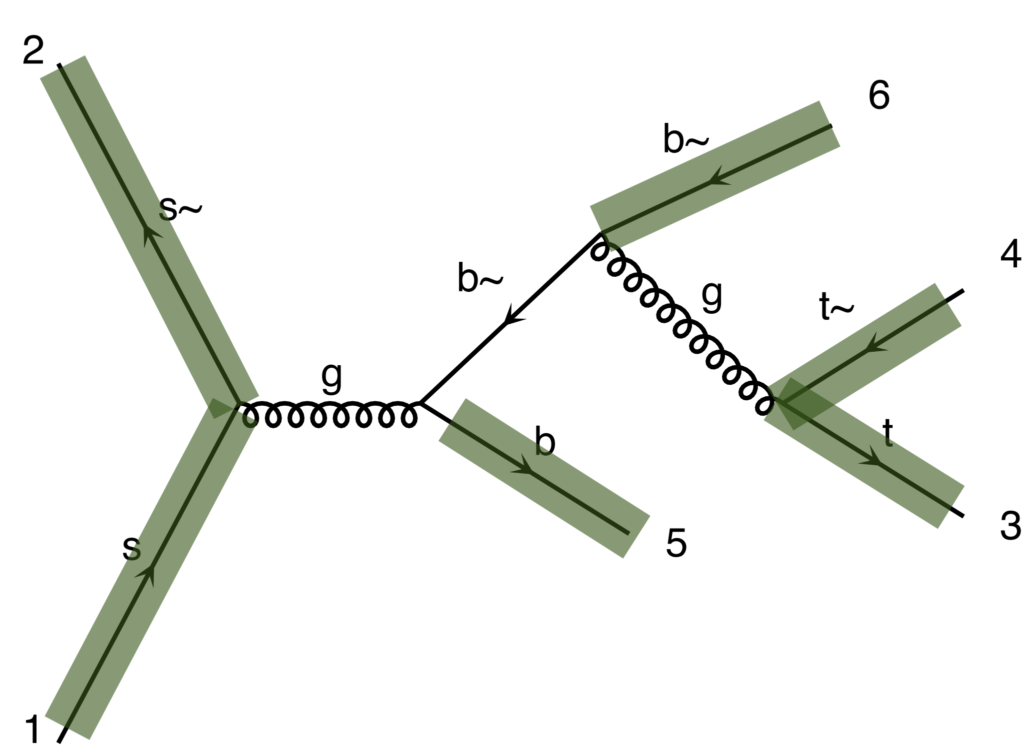
Real case

Known



M1

Number of routines: 6



M2

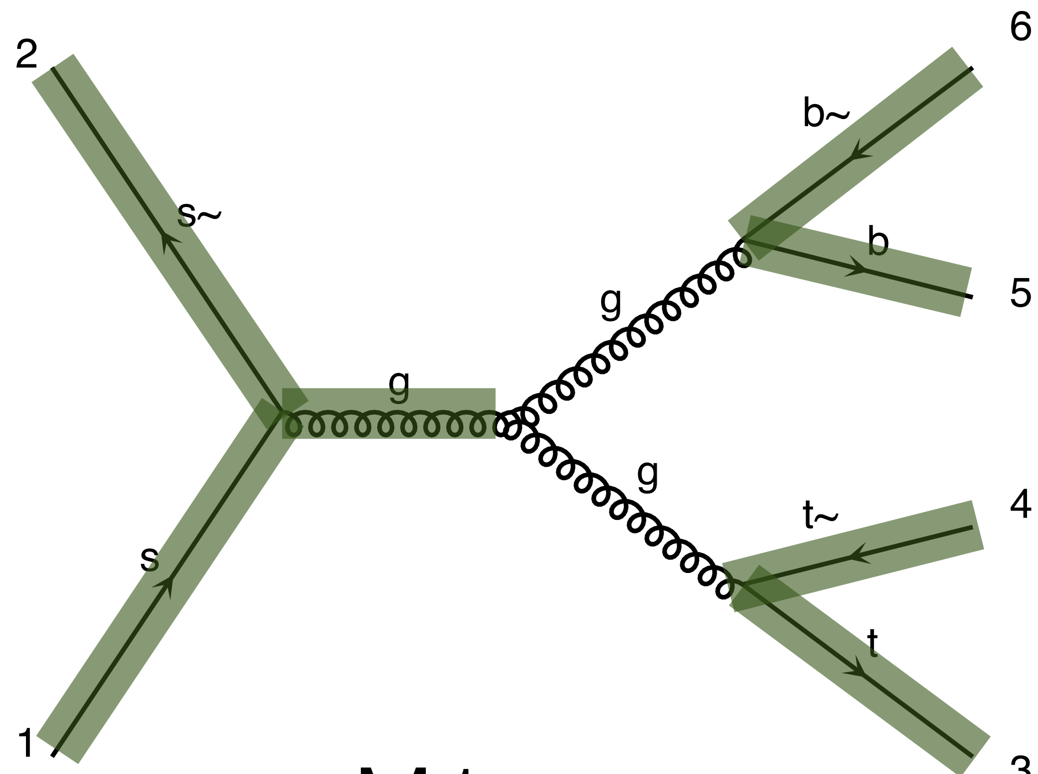
Number of routines: 6

Number of routines for both: 6

$$|M|^2 = |M_1 + M_2|^2$$

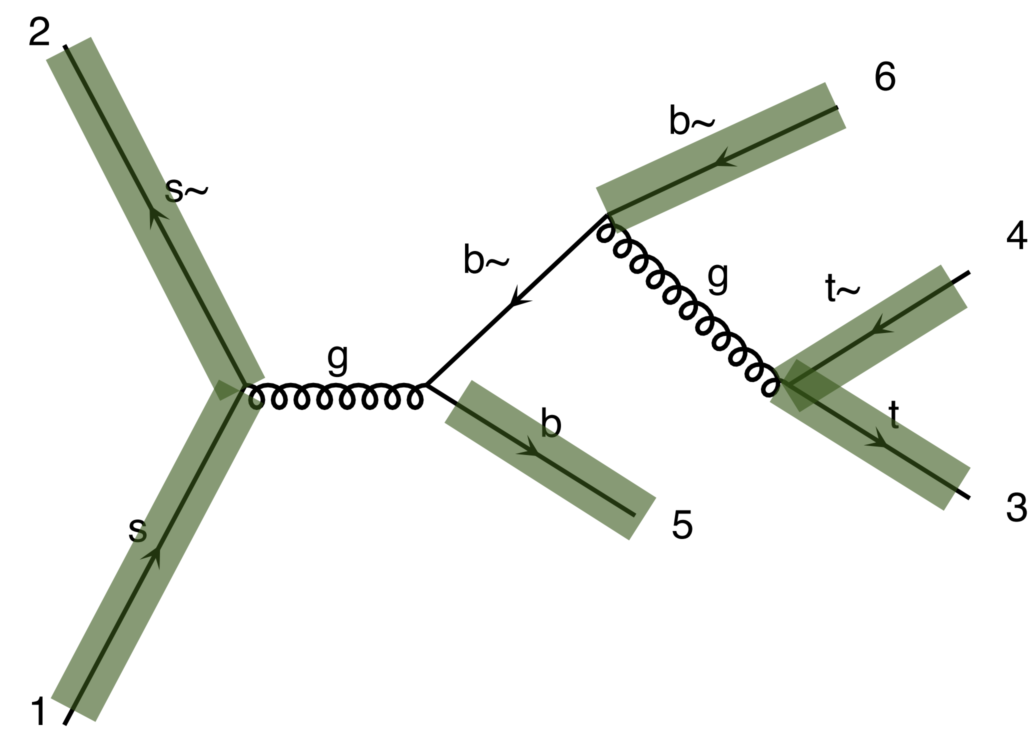
Real case

Known



M1

Number of routines: 7



M2

Number of routines: 6

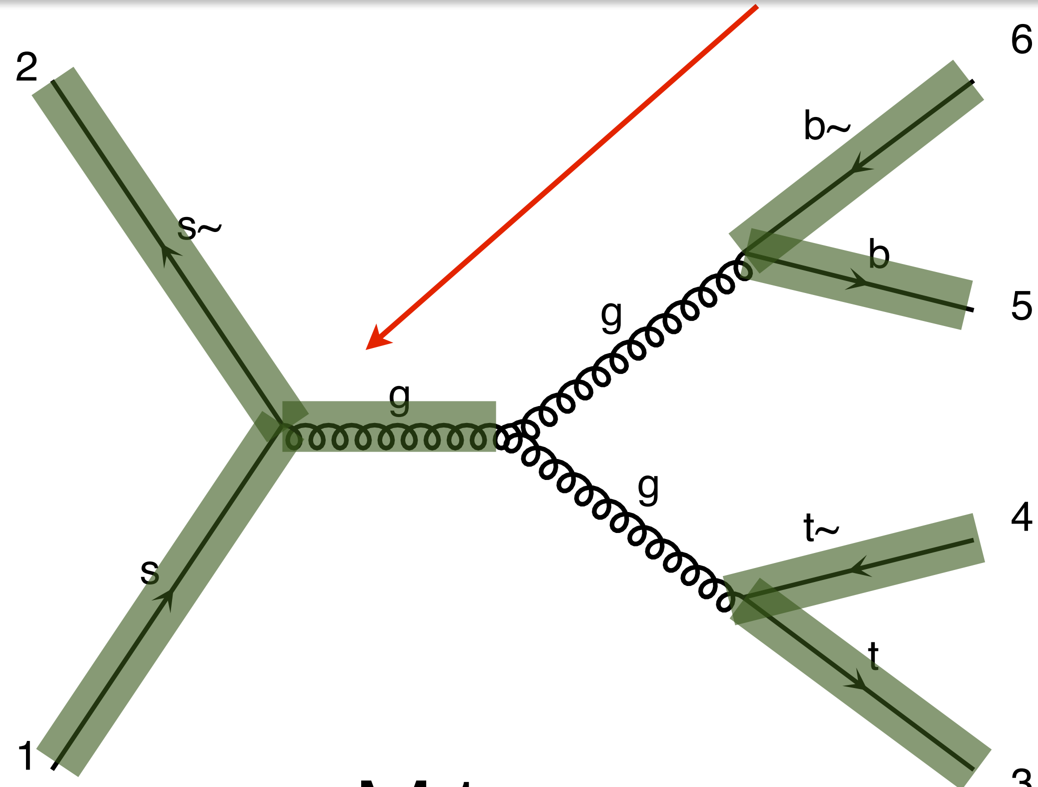
Number of routines for both:7

$$|M|^2 = |M_1 + M_2|^2$$

Real case

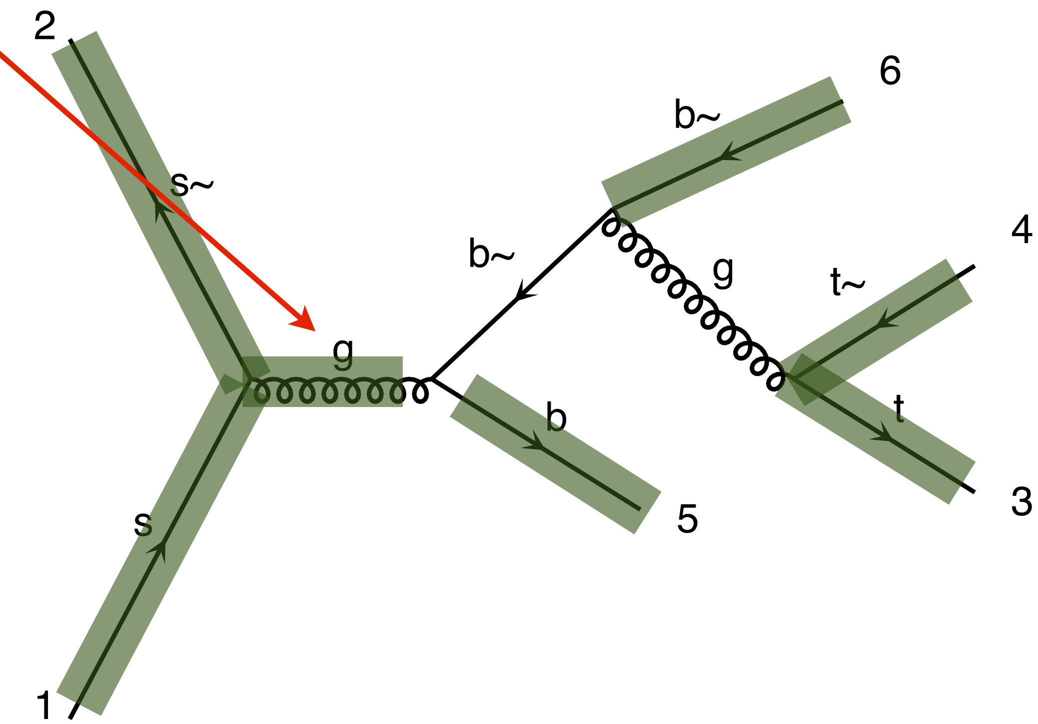
Known

Identical



M1

Number of routines: 7



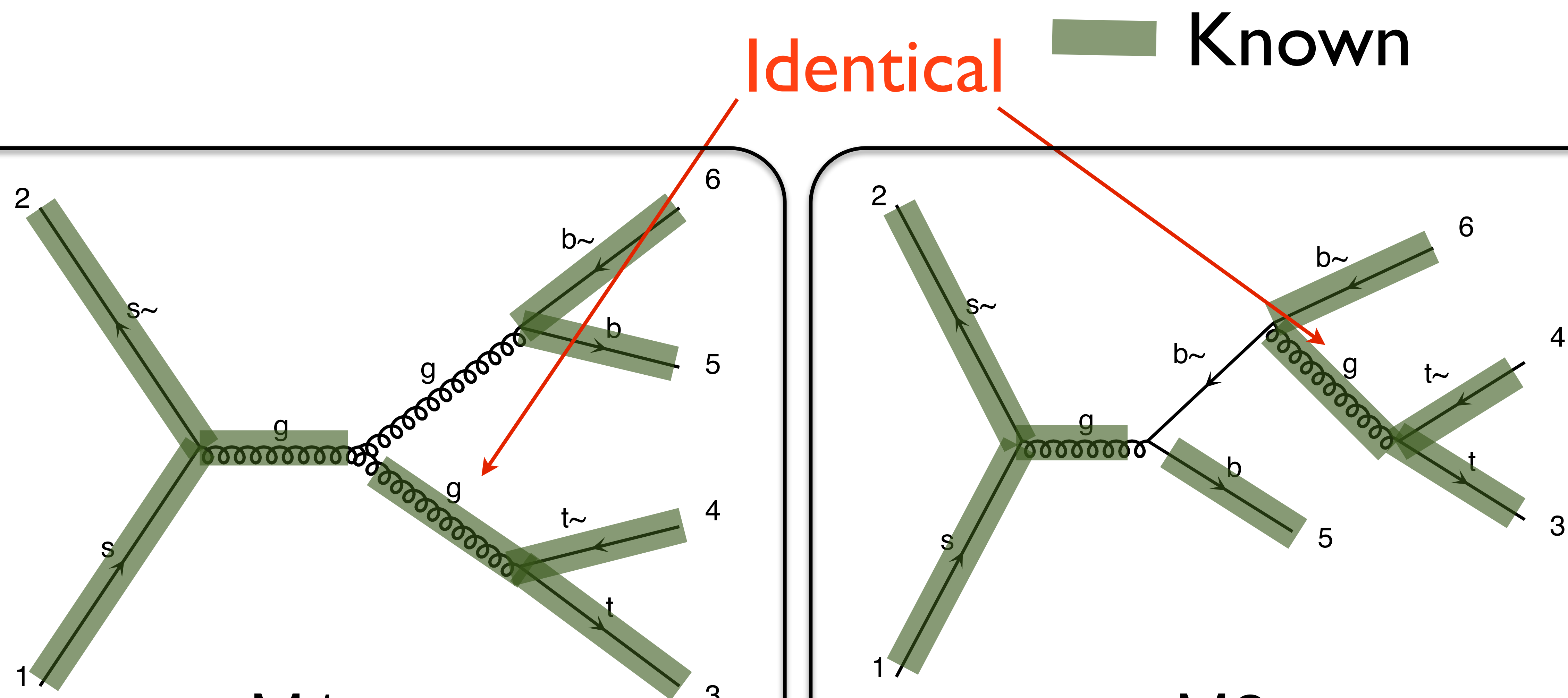
M2

Number of routines: 7

Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

Real case



M1

Number of routines: 8

M2

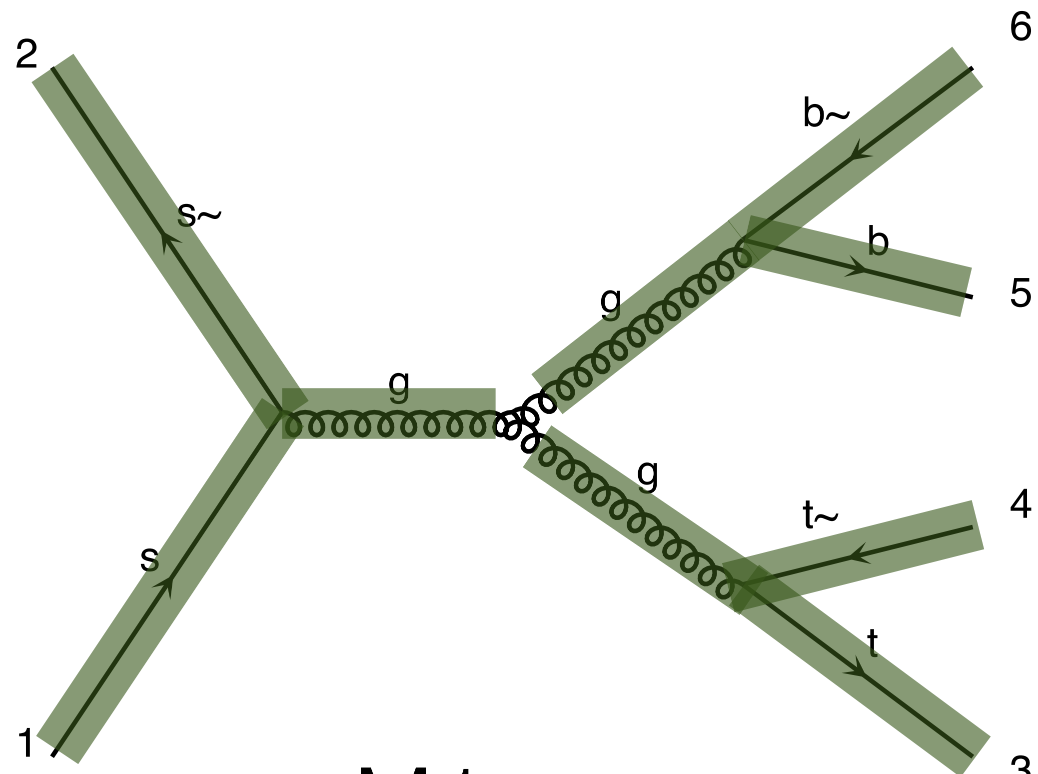
Number of routines: 8

Number of routines for both: 8

$$|M|^2 = |M_1 + M_2|^2$$

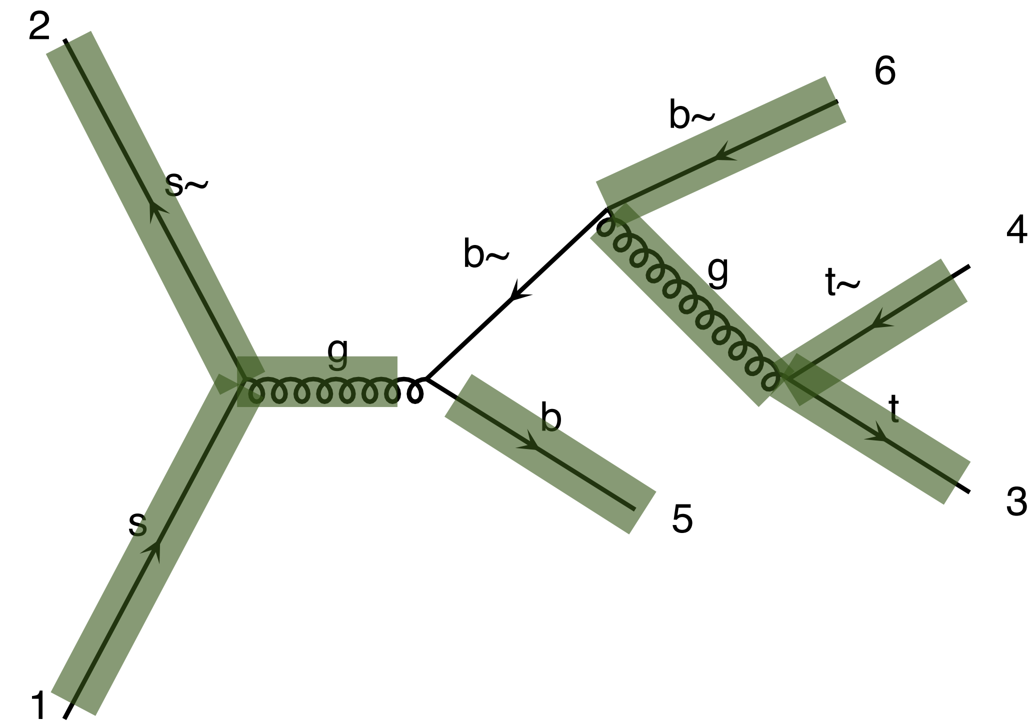
Real case

Known



M1

Number of routines: 9



M2

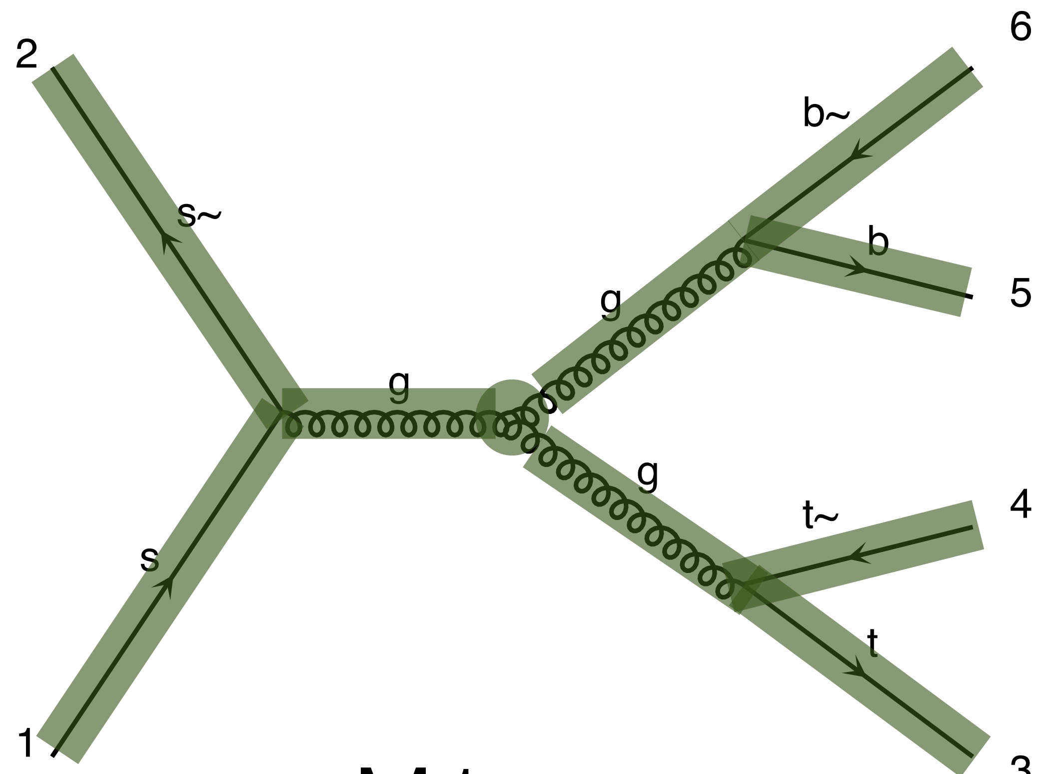
Number of routines: **8**

Number of routines for both: 9

$$|M|^2 = |M_1 + M_2|^2$$

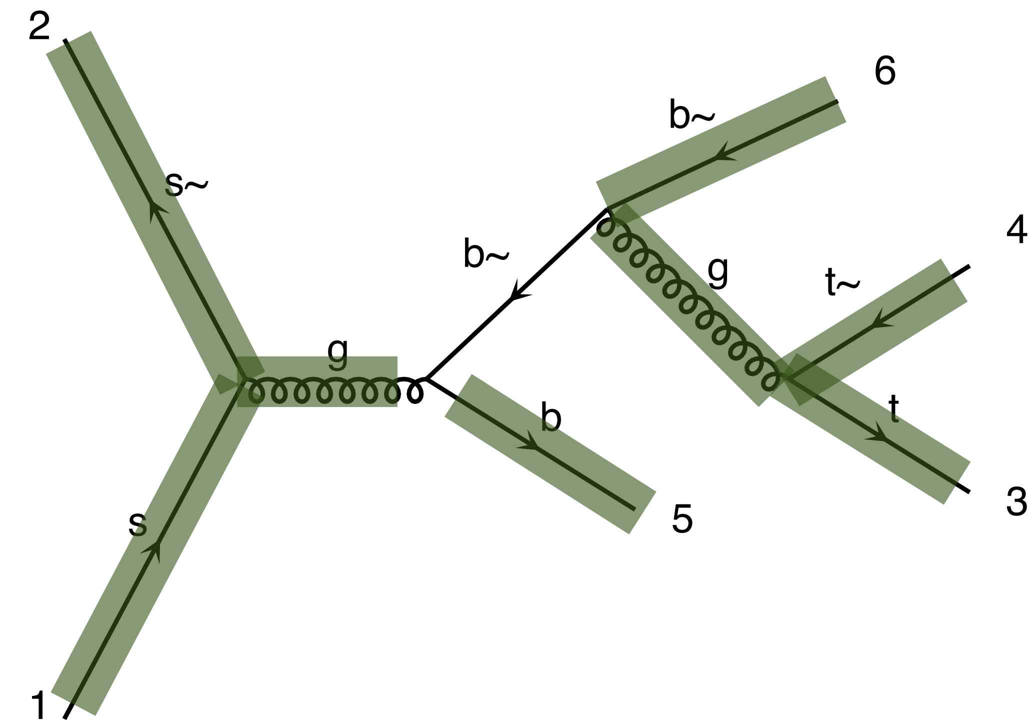
Real case

Known



M1

Number of routines: 10



M2

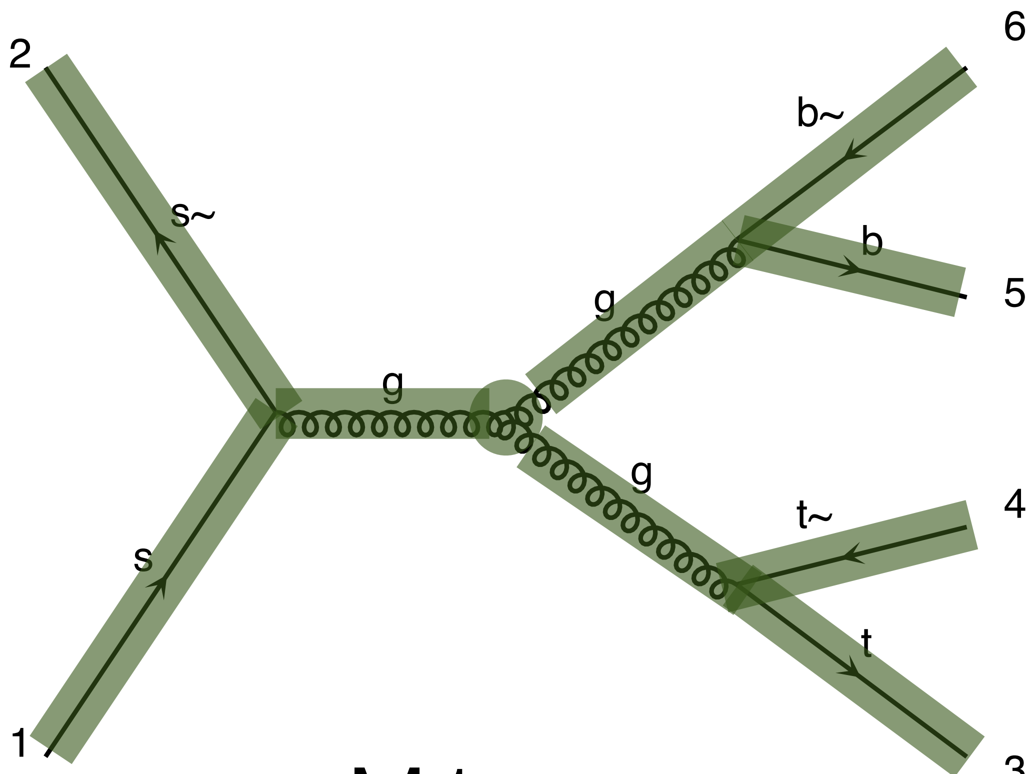
Number of routines: 8

Number of routines for both: 10

$$|M|^2 = |M_1 + M_2|^2$$

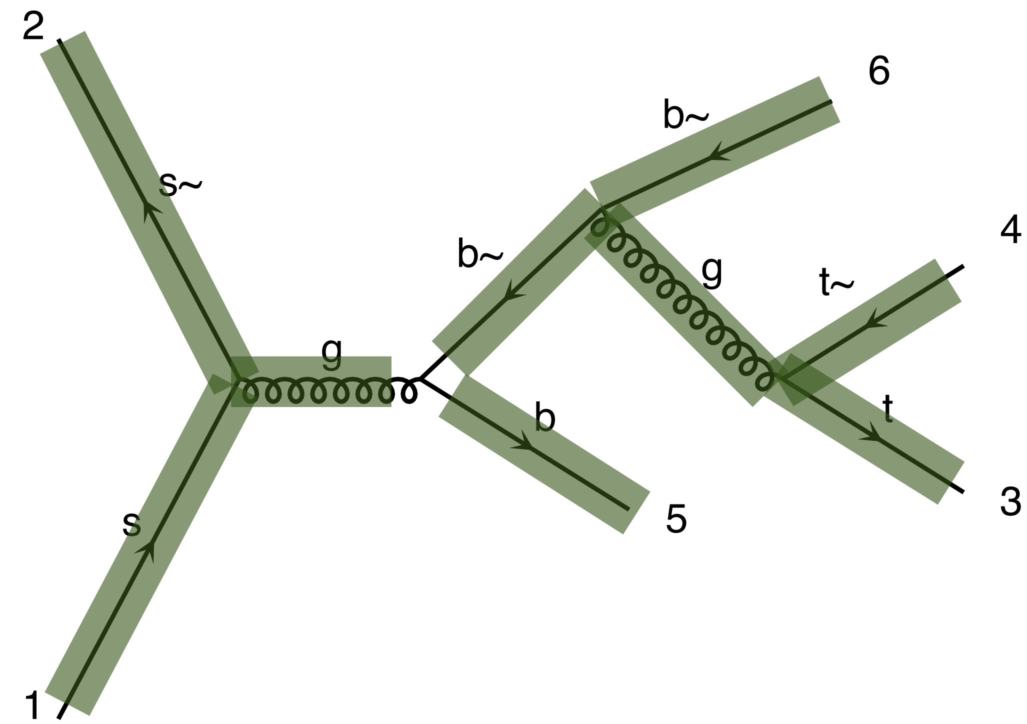
Real case

Known



M1

Number of routines: 10



M2

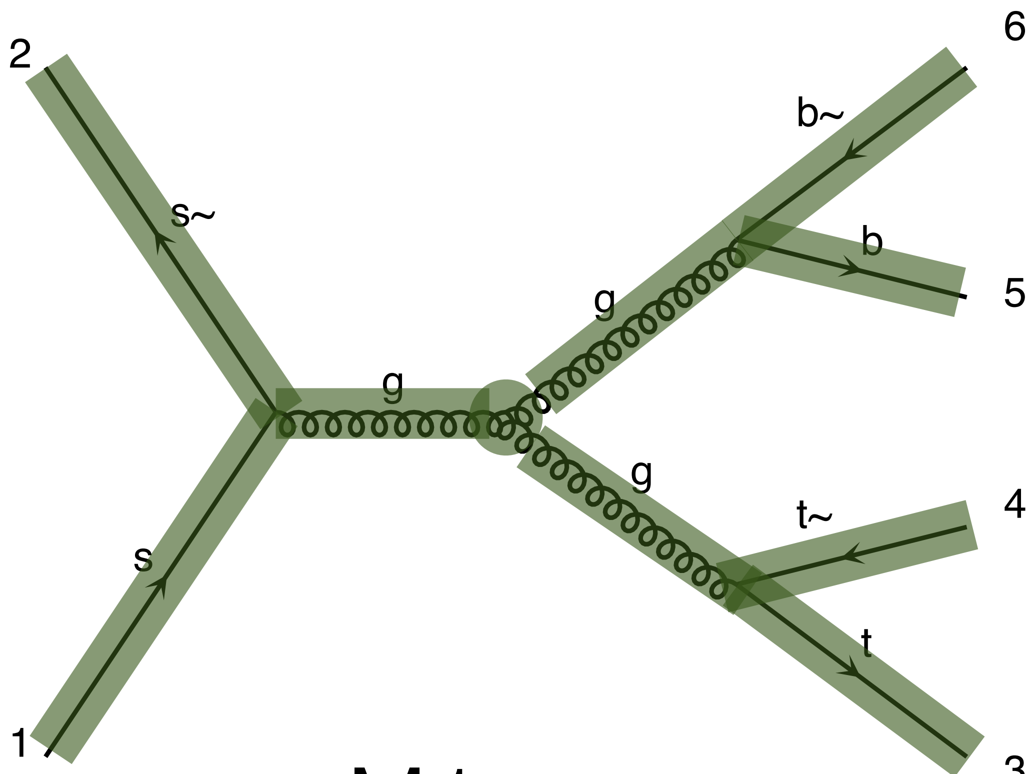
Number of routines: 9

Number of routines for both: 11

$$|M|^2 = |M_1 + M_2|^2$$

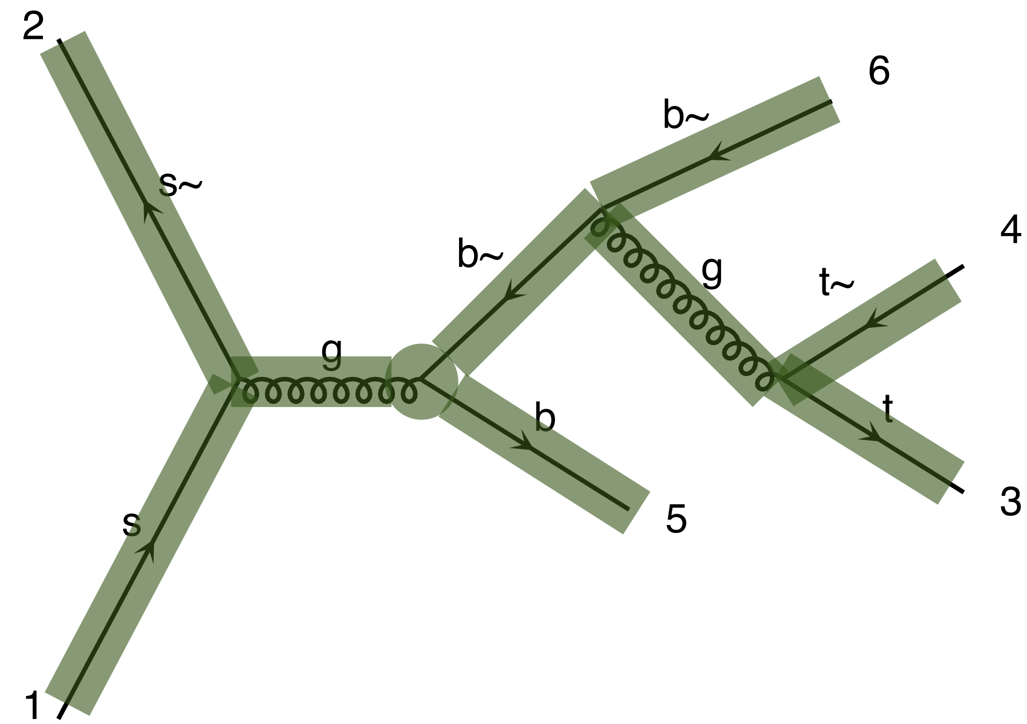
Real case

Known



M1

Number of routines: 10



M2

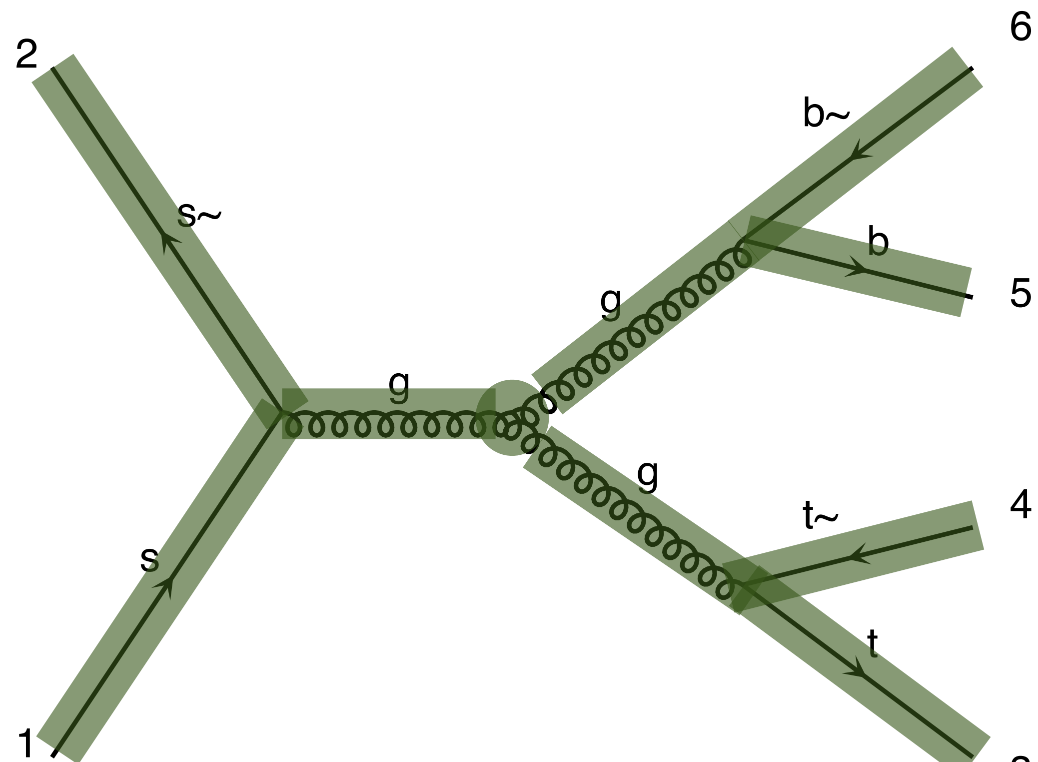
Number of routines: 10

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

Real case

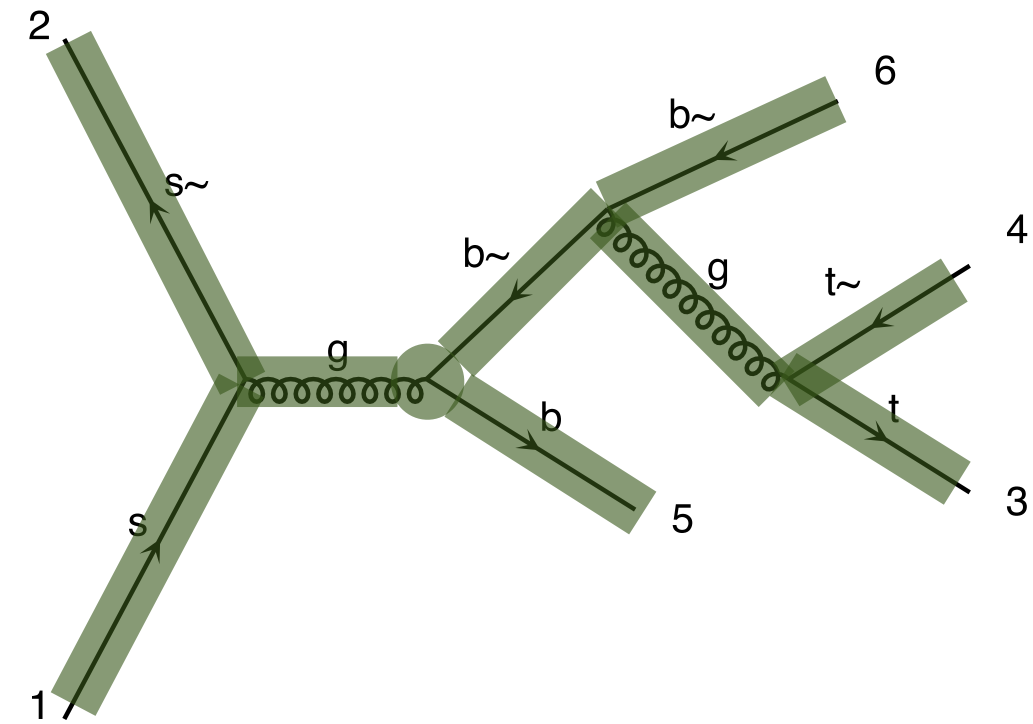
Known



M1

Number of routines: 10

2(N+1)



M2

Number of routines: 10

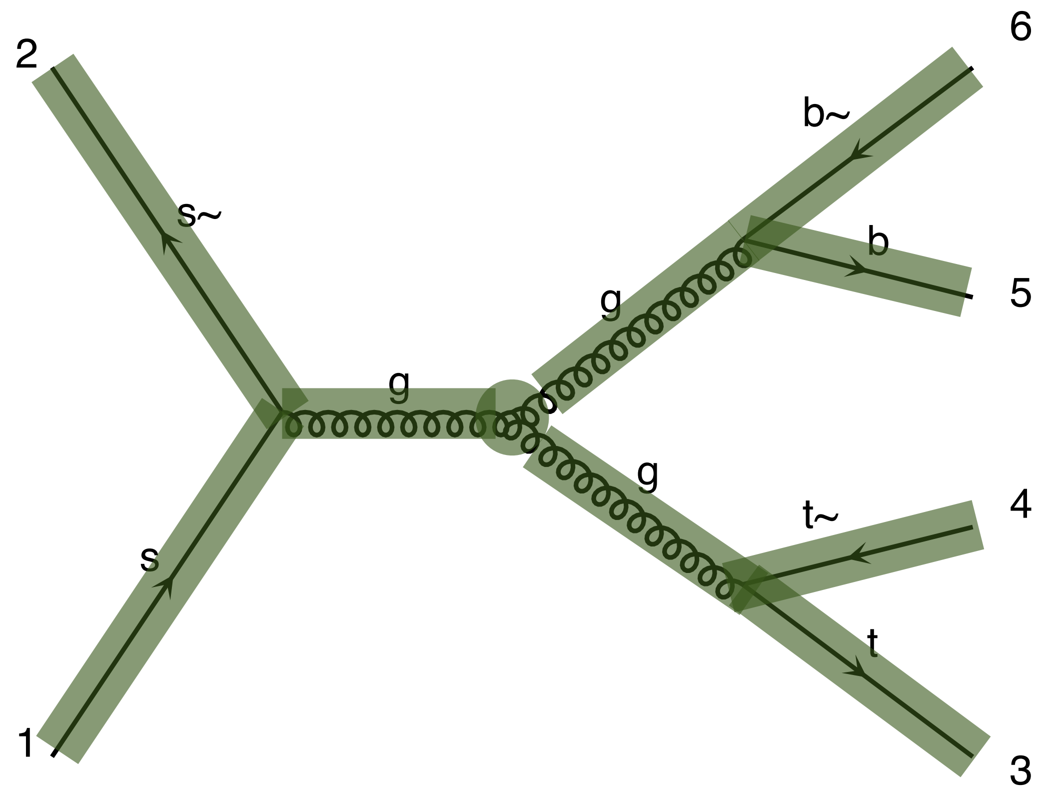
2(N+1)

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

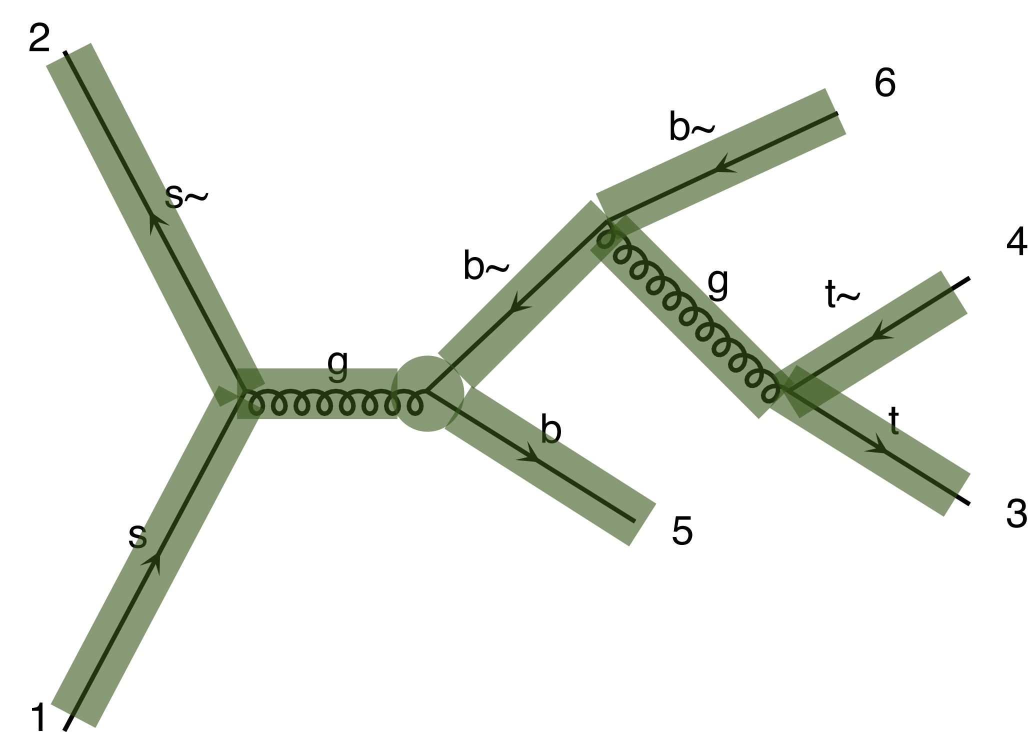
Real case

Known



Number of routines: 10

2(N+1)



Number of routines: 10

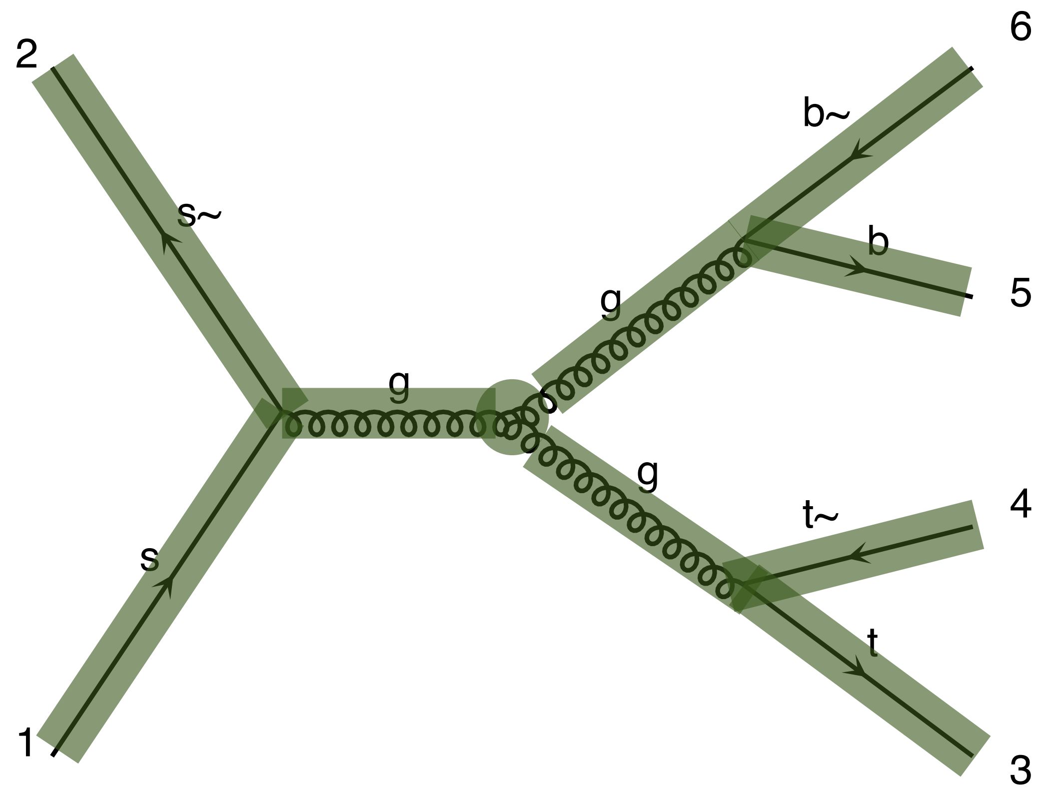
2(N+1)

Number of routines for both: 12

N!*2(N+1) ⟶ N!

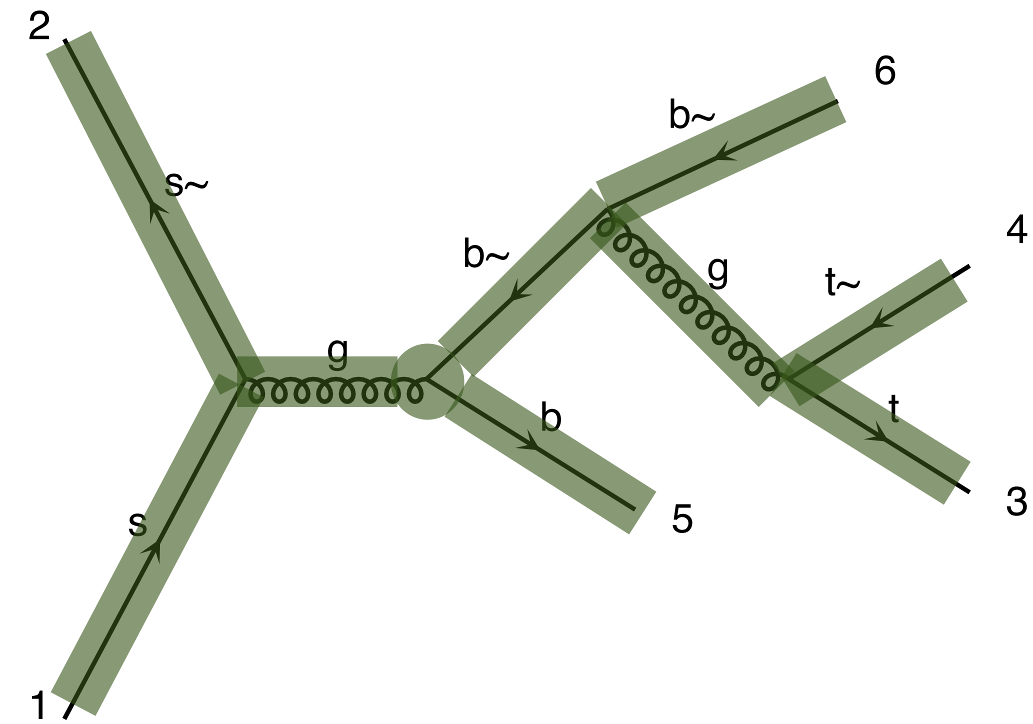
Real case

Known



Number of routines: 10

2(N+1)



Number of routines: 10

2(N+1)

Number of routines for both: 12

N!*2(N+1) ⟶ N! —recursion⟶ 2^N

Helicity amplitudes

- Thanks to new diagram generation algorithm, wf recycling much more efficient in MG5 than MG4

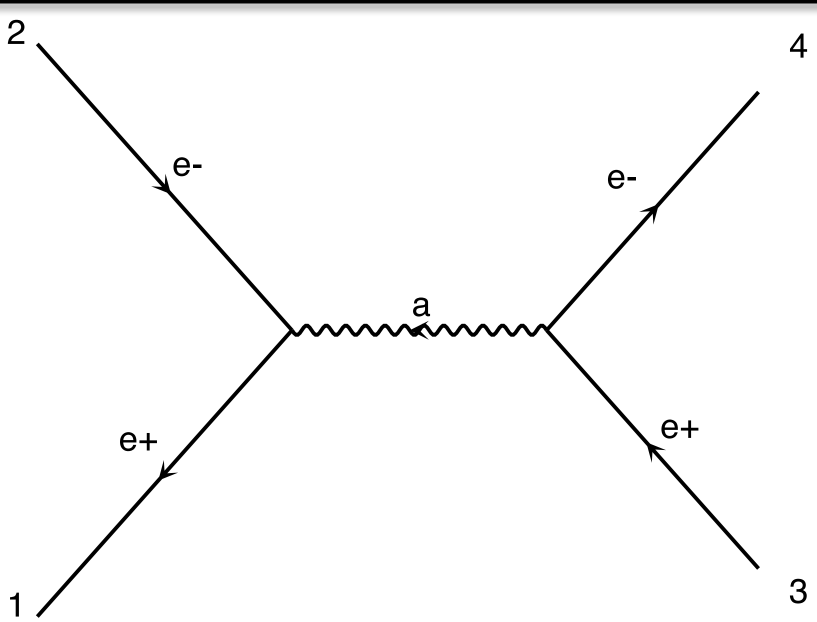
Process	Amplitudes	Wavefunctions		Run time	
		MG 4	MG 5	MG 4	MG 5
$u\bar{u} \to e^+e^-$	2	6	6	$< 6\mu s$	$< 6\mu s$
$u\bar{u} \to e^+e^-e^+e^-$	48	62	32	0.22 ms	0.14 ms
$u\bar{u} \to e^+e^-e^+e^-e^+e^-$	3474	3194	301	46.5 ms	19.0 ms
$u\bar{u} \to d\bar{d}$	1	5	5	$< 4\mu s$	$< 4\mu s$
$u\bar{u} \to d\bar{d}g$	5	11	11	27 μs	27 μs
$u\bar{u} \to d\bar{d}gg$	38	47	29	0.42 ms	0.31 ms
$u\bar{u} \to d\bar{d}ggg$	393	355	122	10.8 ms	6.75 ms
$u\bar{u} \to u\bar{u}gg$	76	84	40	1.24 ms	0.80 ms
$u\bar{u} \to u\bar{u}ggg$	786	682	174	35.7 ms	17.2 ms
$u\bar{u} \to d\bar{d}d\bar{d}$	14	28	19	84 μs	83 μs
$u\bar{u} \to d\bar{d}d\bar{d}g$	132	178	65	1.88 ms	1.15 ms
$u\bar{u} \to d\bar{d}d\bar{d}gg$	1590	1782	286	141 ms	34.4 ms
$u\bar{u} \to d\bar{d}d\bar{d}d\bar{d}$	612	758	141	42.5 ms	6.6 ms

no recycling

300,000 ($u\bar{u} \to e^+e^-e^+e^-e^+e^-$)

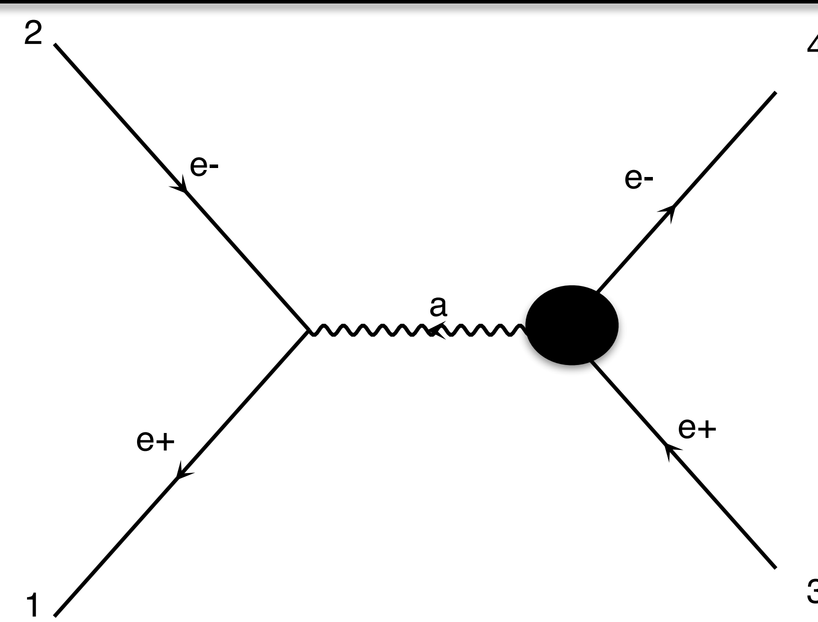
5500 ($u\bar{u} \to d\bar{d}d\bar{d}d\bar{d}$)

Time for matrix element evaluation on a Sony Vaio TZ laptop



$$\mathcal{M} = \bar{u}\gamma^\mu v P_{\mu\nu} \bar{u}\gamma^\nu v$$

$$\bar{u}_1 = fct(\vec{p}_1, m)$$
$$v_2 = fct(\vec{p}_2, m)$$
$$\bar{u}_3 = fct(\vec{p}_3, m)$$
$$v_4 = fct(\vec{p}_4, m)$$
$$W_a = fct(\bar{u}_1, v_2, M_a, \Gamma_a)$$
$$\mathcal{M} = fct(\bar{u}_3, v_4, W_a)$$



$$\mathcal{M} = \bar{u}\gamma^\mu v P_{\mu\nu} \bar{u}\Gamma^\mu v$$

$$\bar{u}_1 = fct(\vec{p}_1, m)$$
$$v_2 = fct(\vec{p}_2, m)$$
$$\bar{u}_3 = fct(\vec{p}_3, m)$$
$$v_4 = fct(\vec{p}_4, m)$$
$$W_a = fct(\bar{u}_1, v_2, M_a, \Gamma_a)$$
$$\mathcal{M} = fct(\bar{u}_3, v_4, W_a)$$

HELAS

- **Original HELicity Amplitude Subroutine library**

[Murayama, Watanabe, Hagiwara]

HELAS

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- **One routine by Lorentz structure**
  - ➡ **MSSM** [cho, al] hep-ph/0601063 (2006)
  - ➡ **HEFT** [Frederix] (2007)
  - ➡ **Spin 2** [Hagiwara, al] 0805.2554 (2008)
  - ➡ **Spin 3/2** [Mawatari, al] 1101.1289 (2011)

HELAS

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Chiral Perturbation

BNV Model

SLIH

Effective Field Theory

NMSSM

Chromo-magnetic operator

Full HEFT

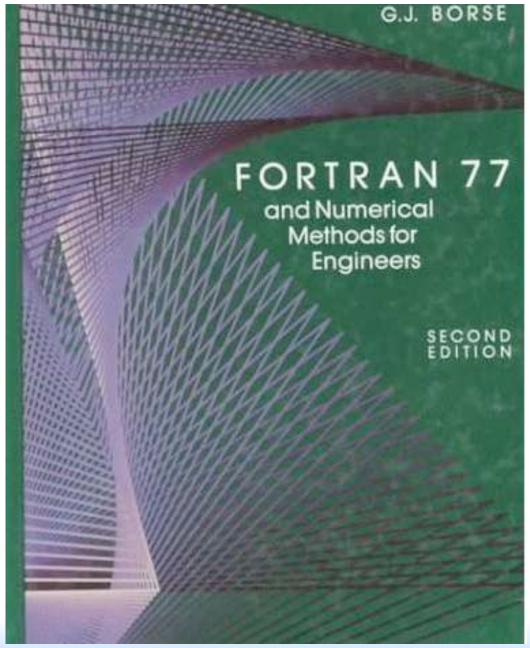
Black Holes

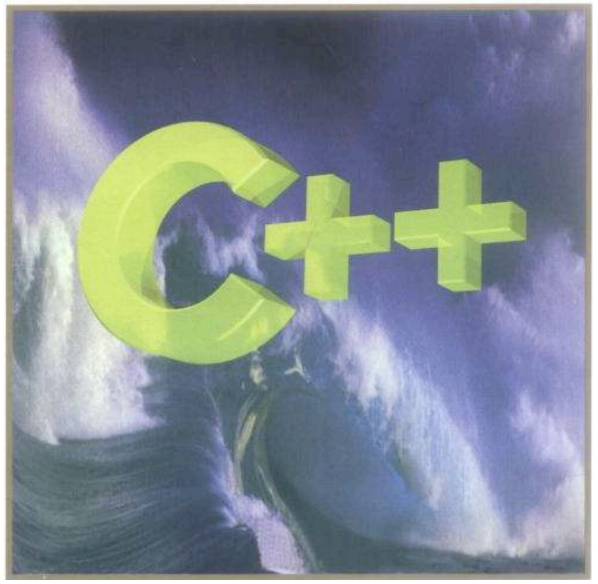
ALOHA

ALOHA ~~Google~~ translate

From: UFO To: Helicity Translate

Type text or a website address or translate a document.





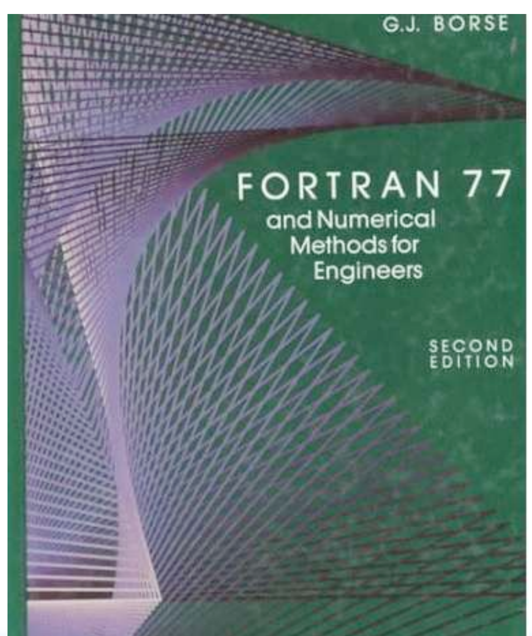
ALOHA

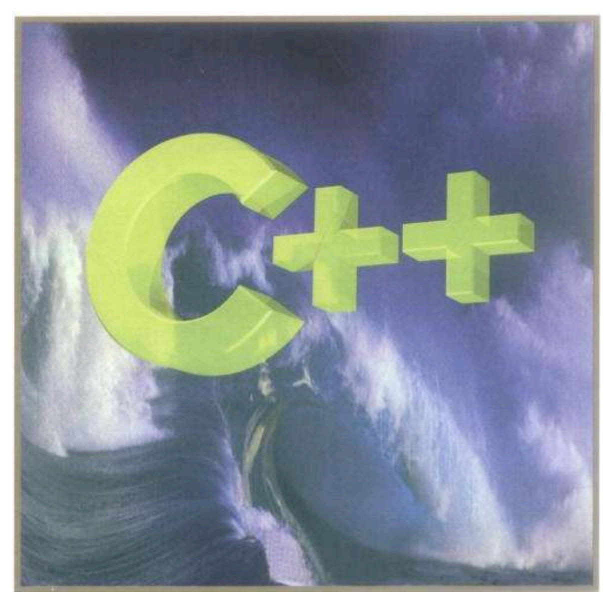
ALOHA ~~Google~~ translate

From: UFO To: Helicity Translate

Basically, any new operator can be handle by MG5/Pythia8 out of the box!

Type text or a website address or translate a document.





ALOHA DETAIL

Input

```
FFV1 = Lorentz(name = 'FFV1',
               spins = [ 2, 2, 3 ],
               structure = 'Gamma(3,2,1)')
```

Output

```
C     This File is Automatically generated by ALOHA
C     The process calculated in this file is:
C     Gamma(3,2,1)
C
      SUBROUTINE FFV1_0(F1,F2,V3,C,VERTEX)
      IMPLICIT NONE
      DOUBLE COMPLEX F1(6)
      DOUBLE COMPLEX F2(6)
      DOUBLE COMPLEX V3(6)
      DOUBLE COMPLEX C
      DOUBLE COMPLEX VERTEX


      VERTEX = C*( (F2(1)*( (F1(3)*( (0, -1)*V3(1)+(0, 1)*V3(4)))
     $ +(F1(4)*( (0, 1)*V3(2)+V3(3)))))+( (F2(2)*( (F1(3)*( (0, 1)
     $ *V3(2)-V3(3)))+(F1(4)*( (0, -1)*V3(1)+(0, -1)*V3(4)))))
     $ +( (F2(3)*( (F1(1)*( (0, -1)*V3(1)+(0, -1)*V3(4)))+(F1(2)
     $ *( (0, -1)*V3(2)-V3(3)))))+(F2(4)*( (F1(1)*( (0, -1)*V3(2)
     $ +V3(3)))+(F1(2)*( (0, -1)*V3(1)+(0, 1)*V3(4)))))))

      END
```

ALOHA

- Compute those Function Analytically
- Code in Python
- Can handle
    - ➡ all spin up to 2
    - ➡ custom propagator
    - ➡ majorana (but in 4 fermion operator)
    - ➡ Any dimensional operator
- Only use in MadGraph5_aMC@NLO
- Plan to have similar tools for the other generator

Monte Carlo Integration and Generation

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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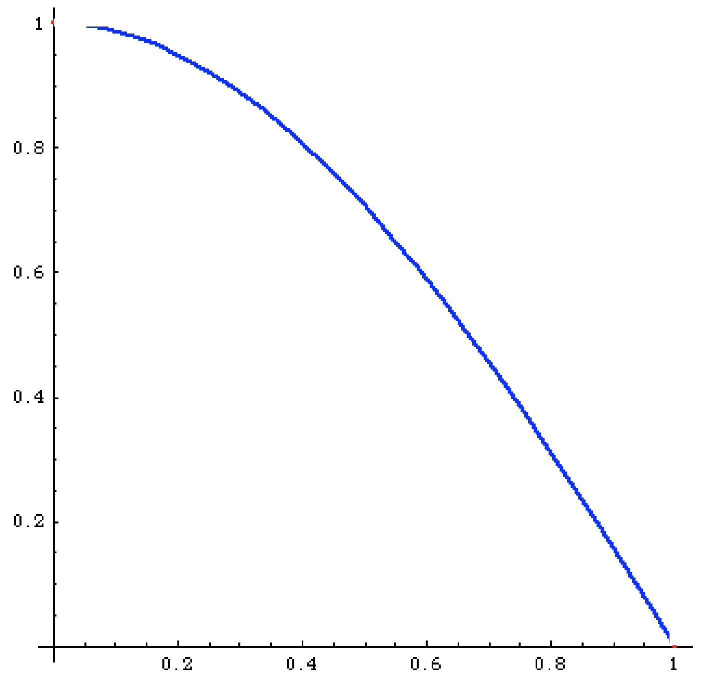
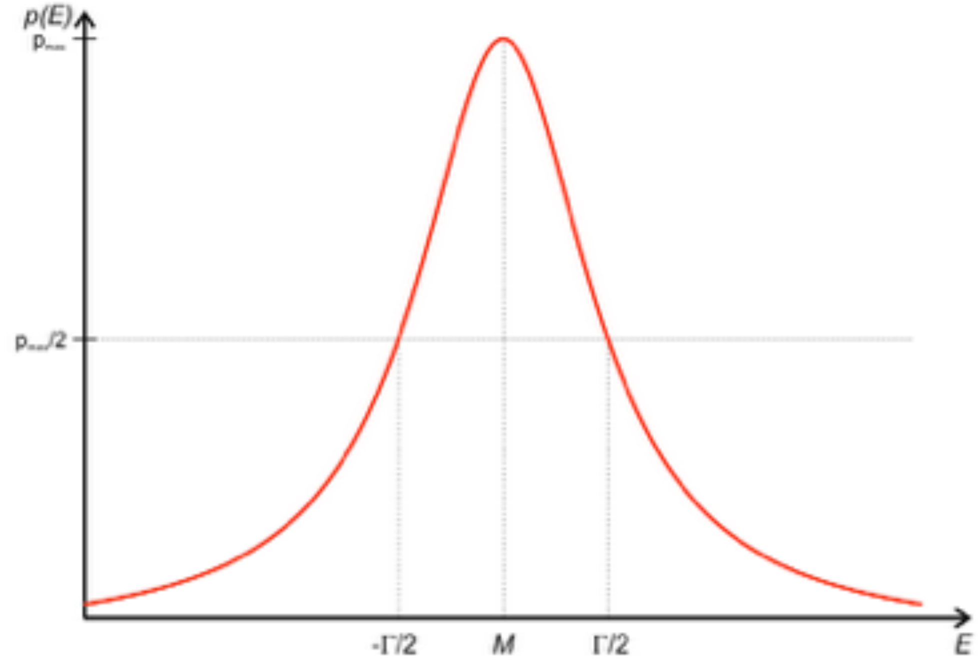
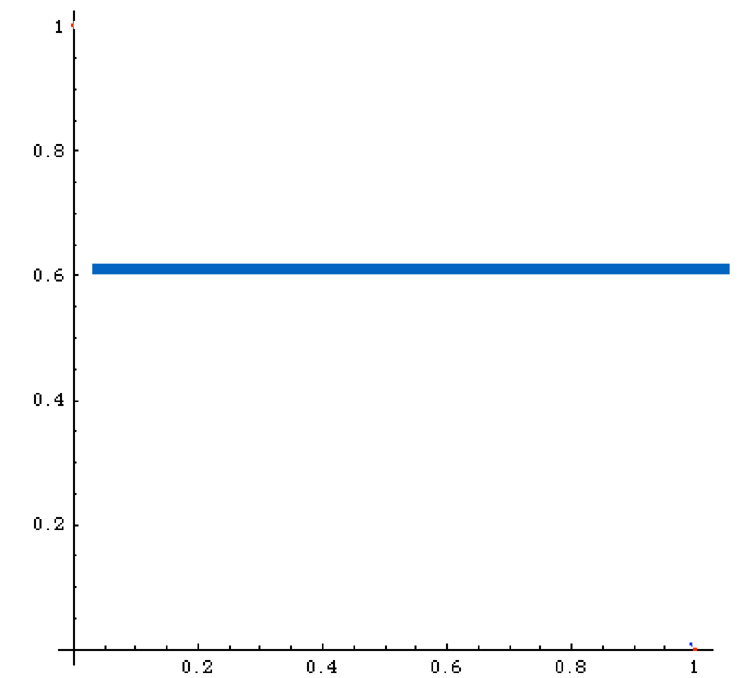
General and flexible method is needed

Integration

$$\int dx\, C$$







Integration

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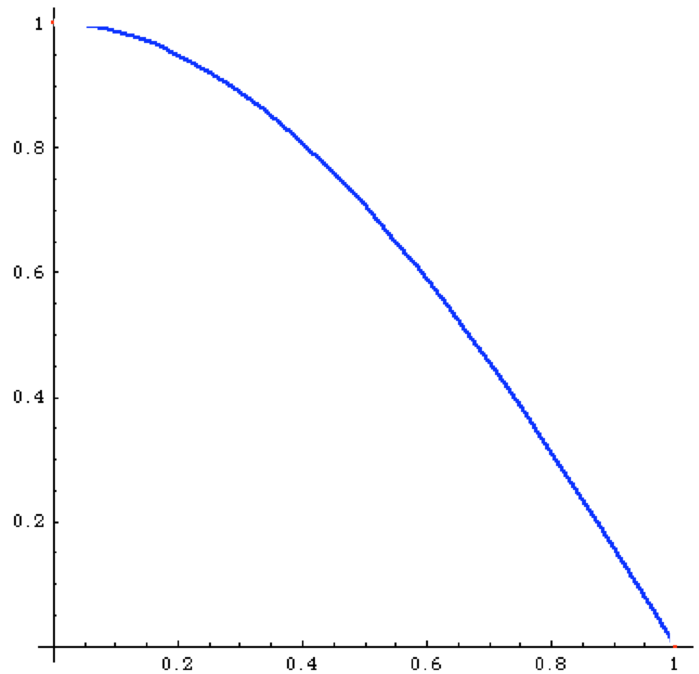
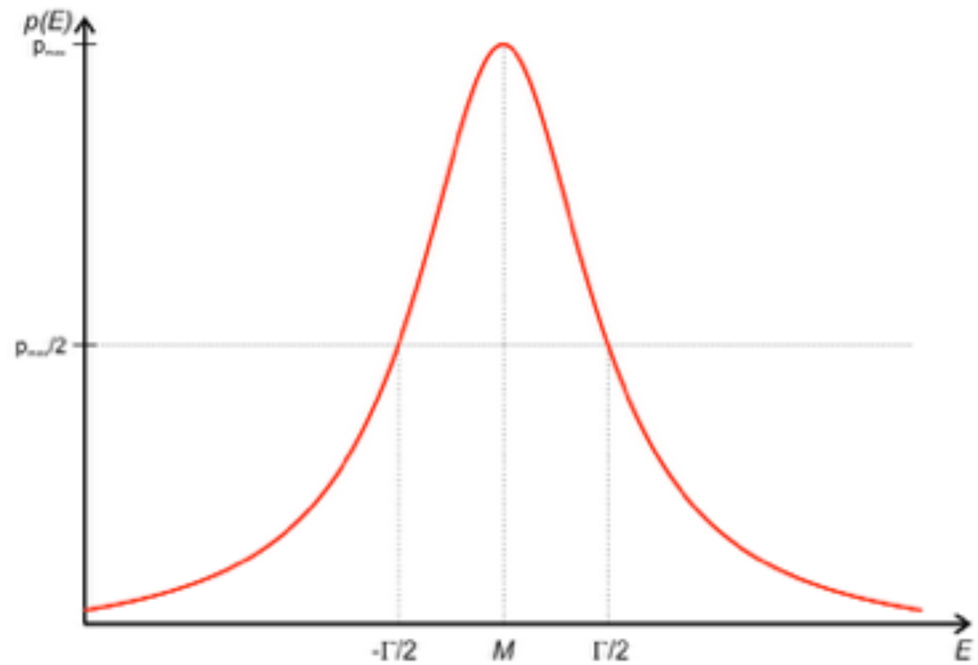
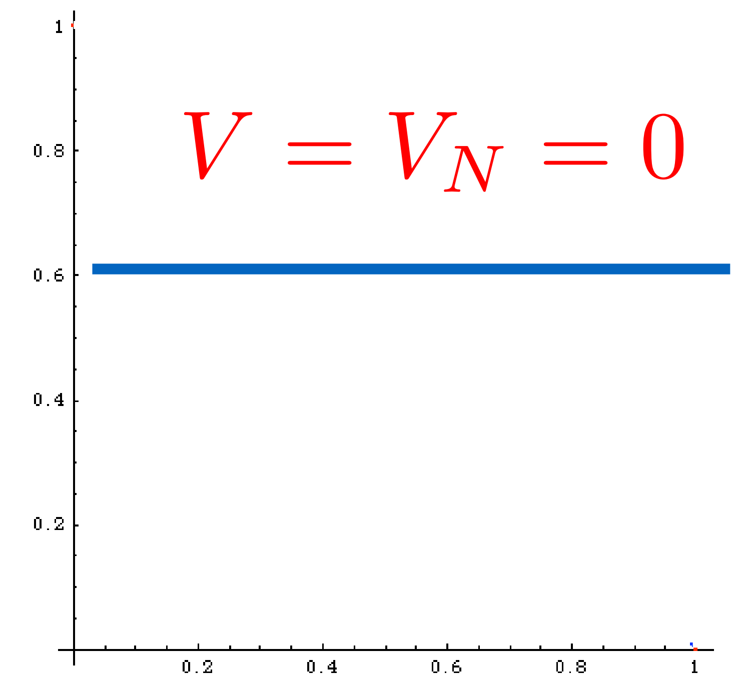




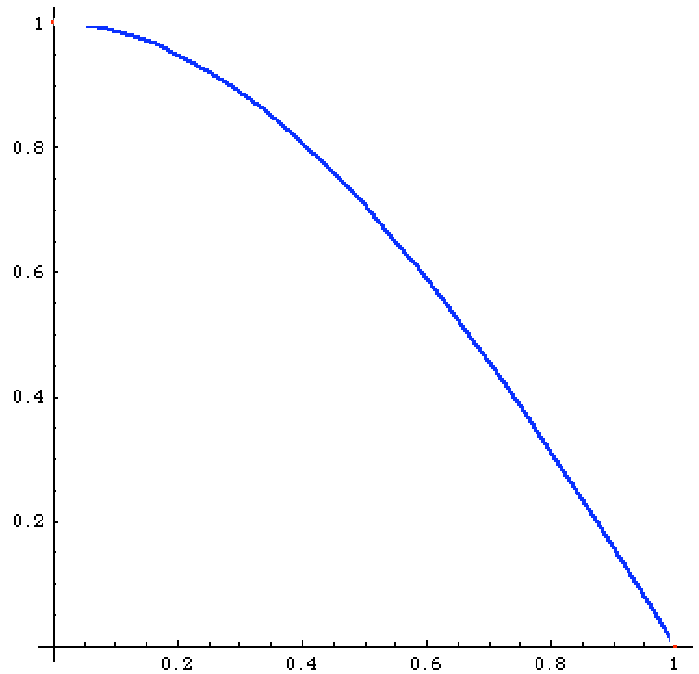


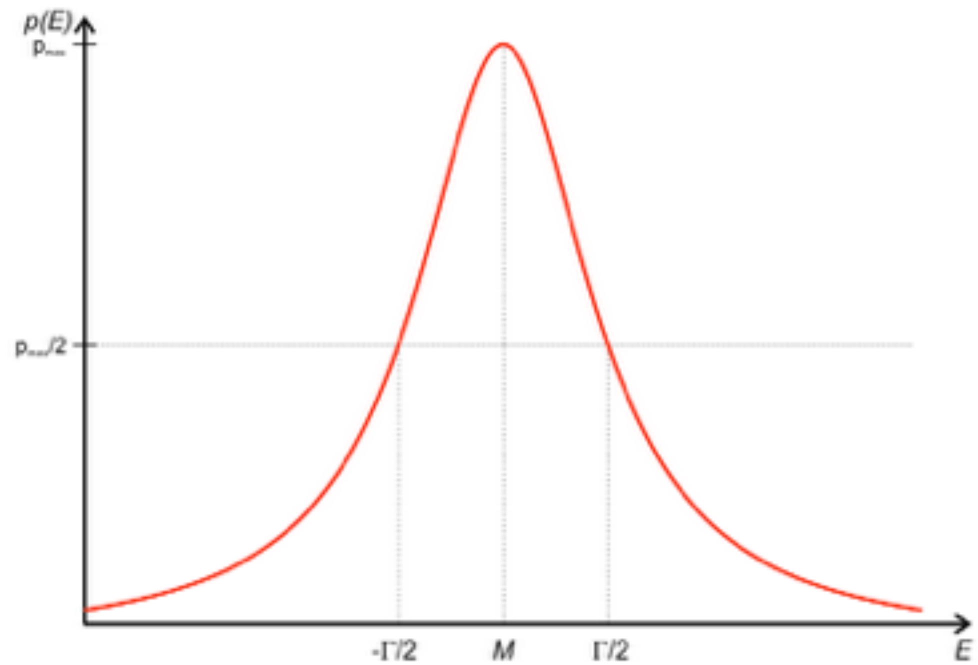
Method of evaluation

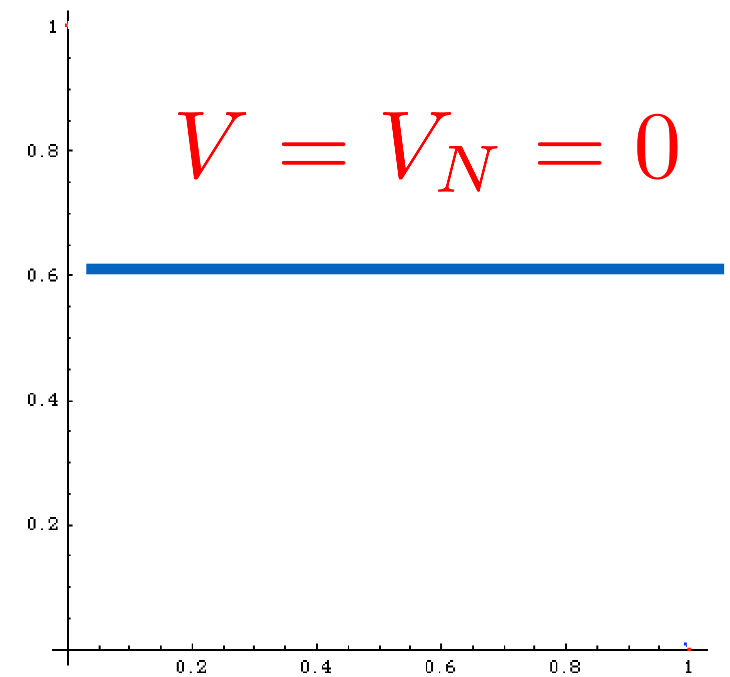
- MonteCarlo
- Trapezium $1/N^2$
- Simpson

Integration

$$\int dx\, C$$







	simpson	MC
3	0.638	0.3
5	0.6367	0.8
20	0.63662	0.6
100	0.636619	0.65
1000	0.636619	0.636

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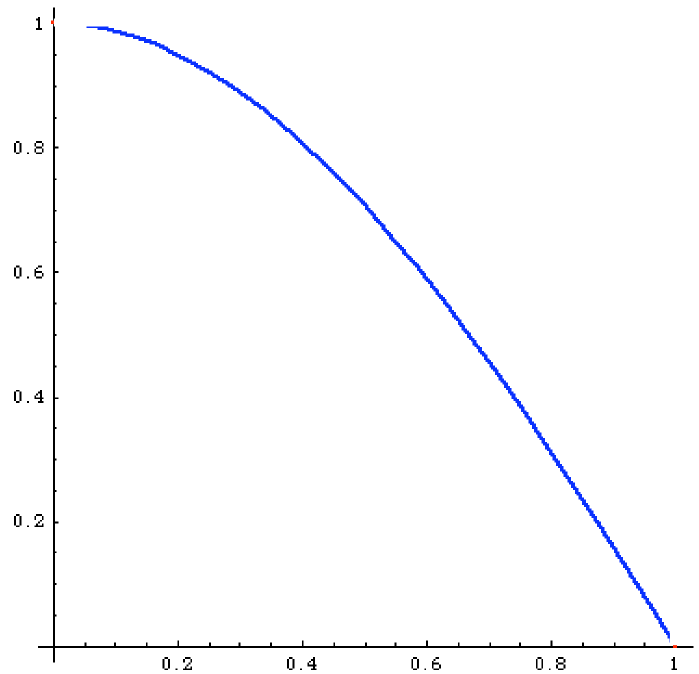

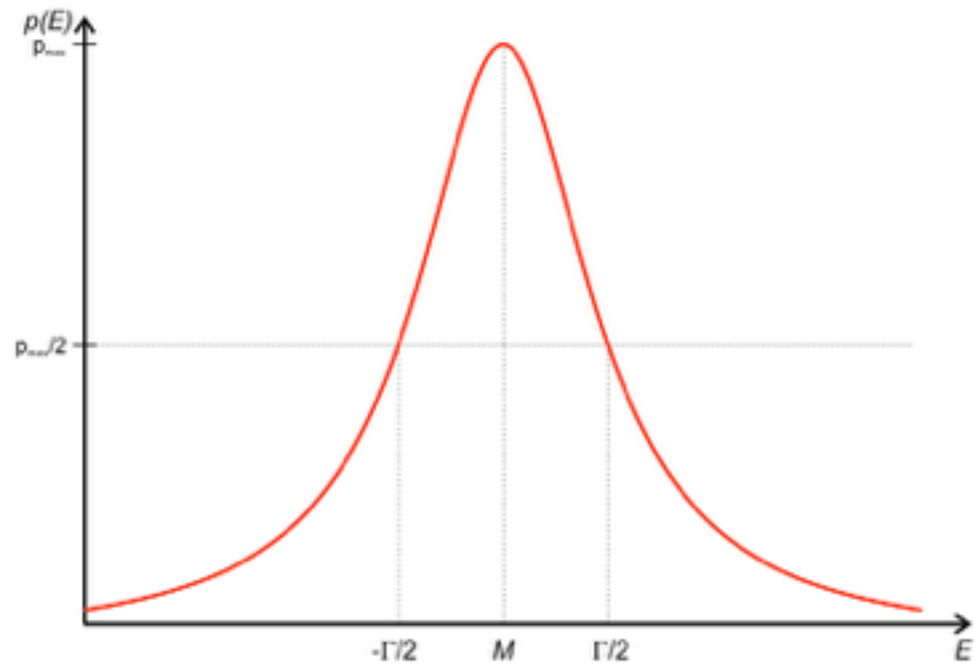

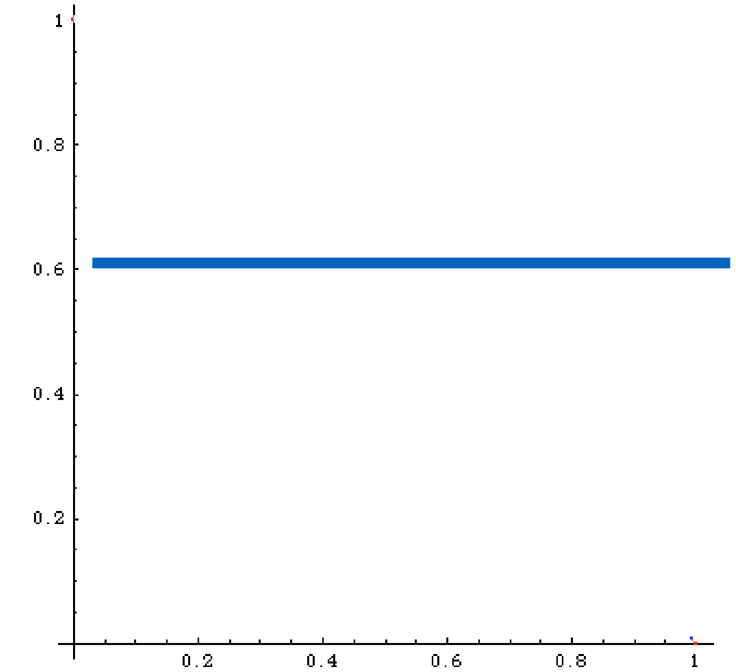

Method of evaluation

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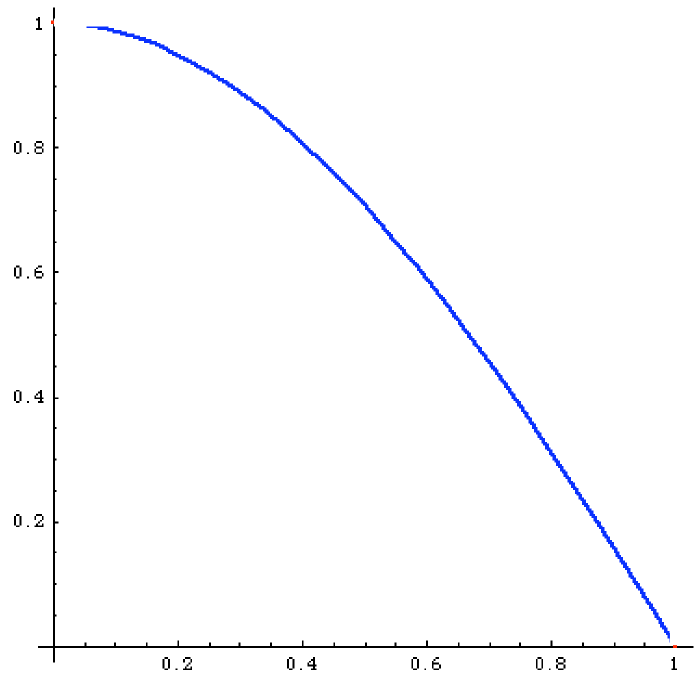
$1/N^2$

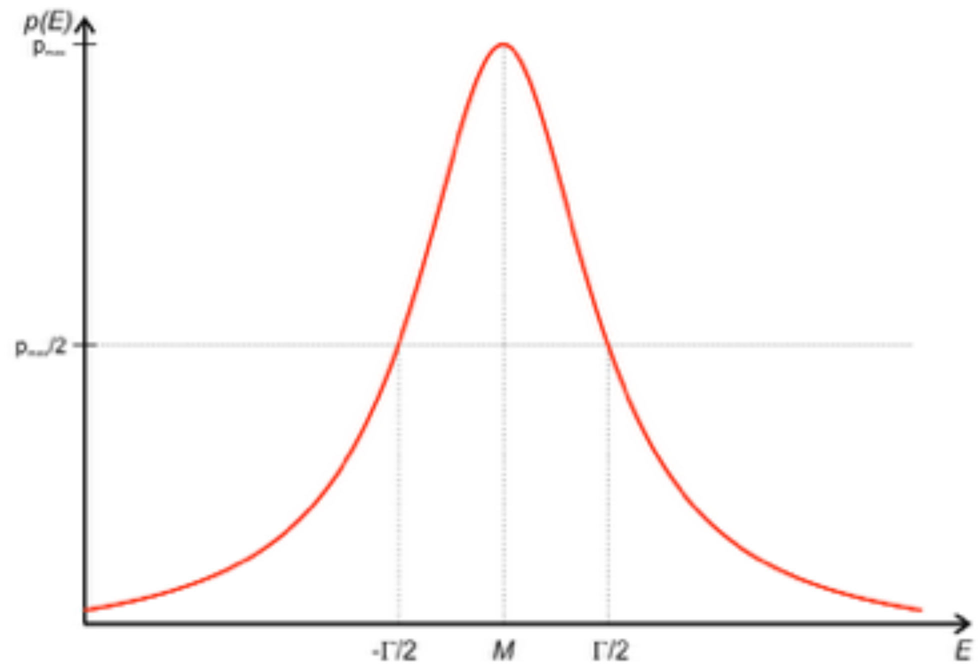
More Dimension →

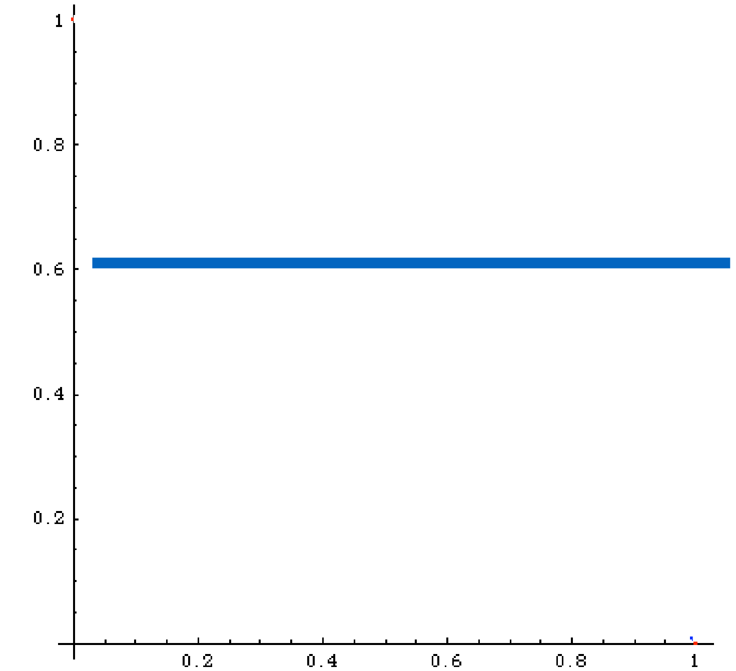
$1/N^{4/d}$

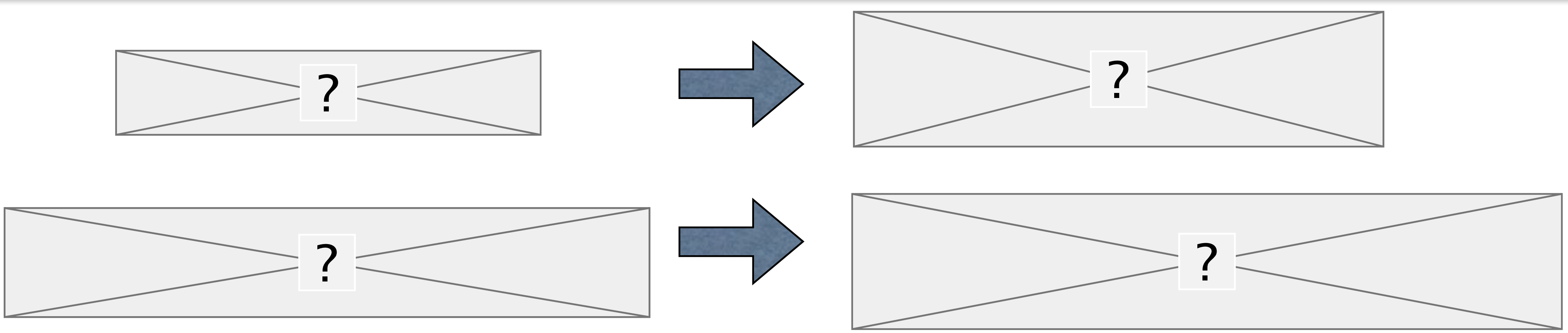
Integration

$$\int dx\, C$$









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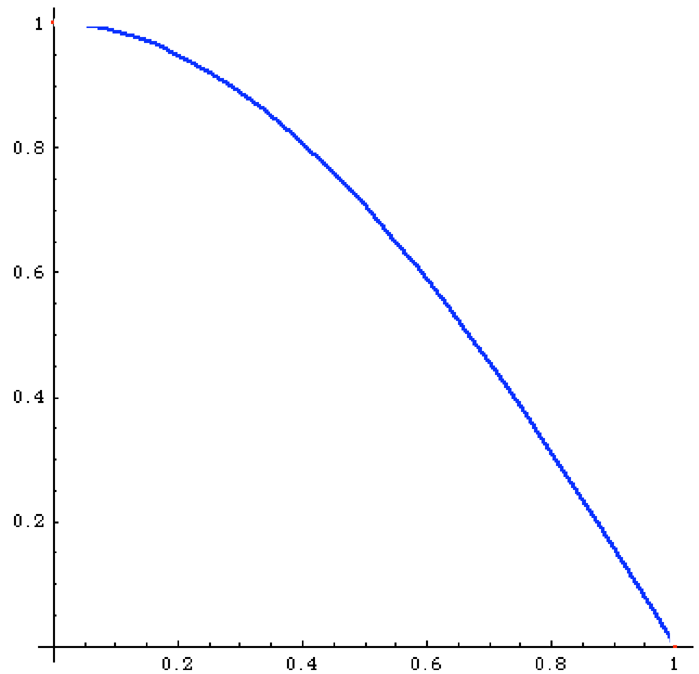

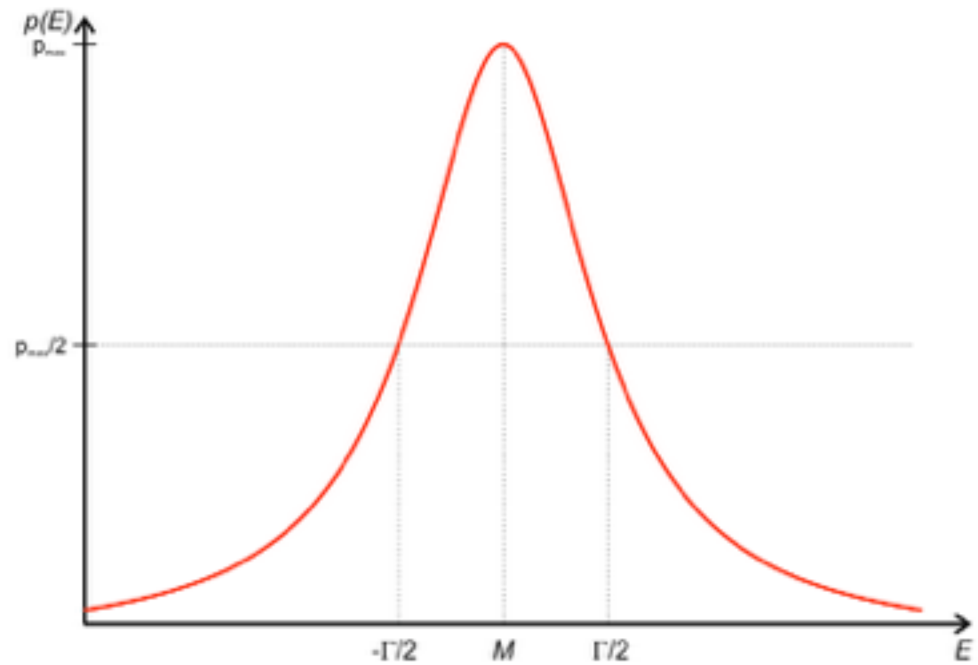

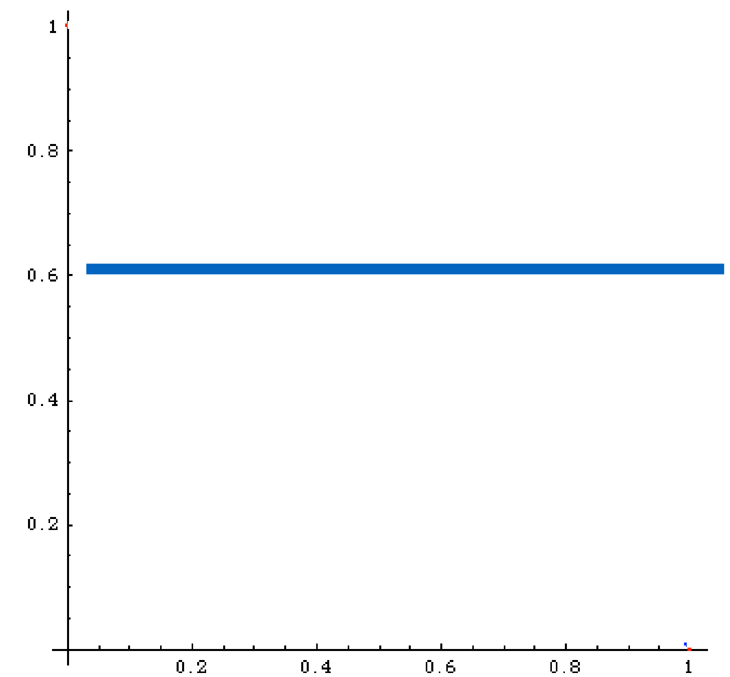

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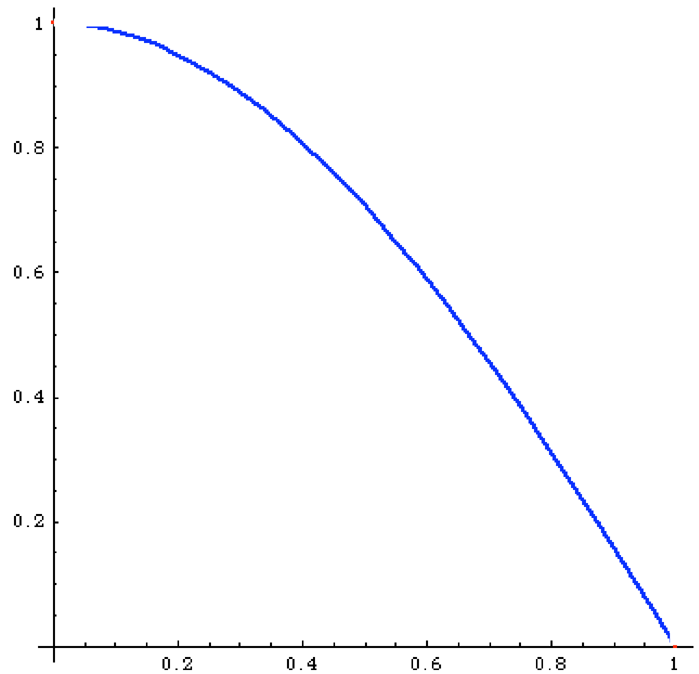
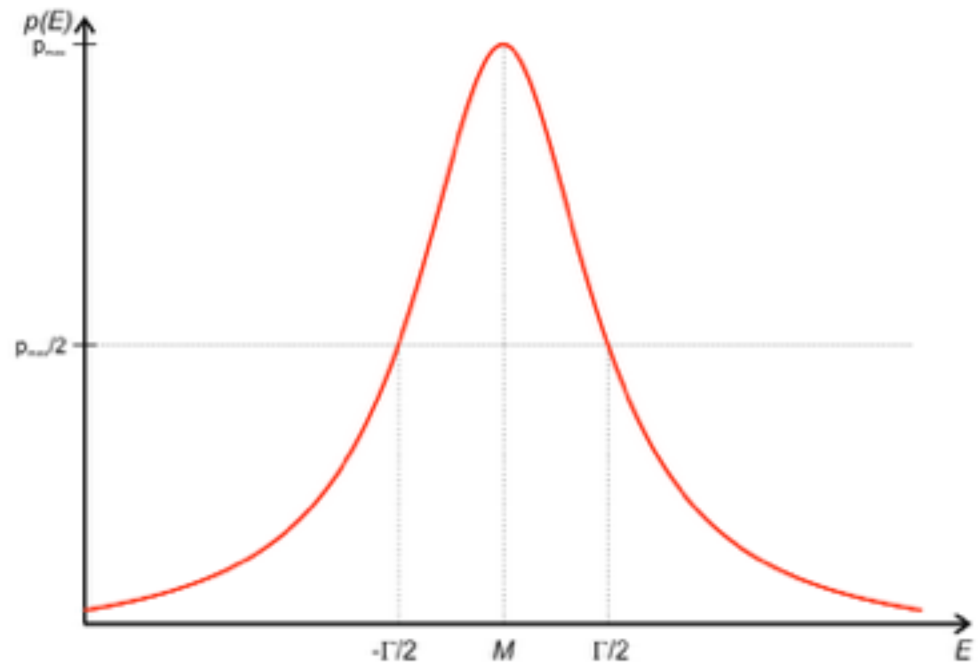
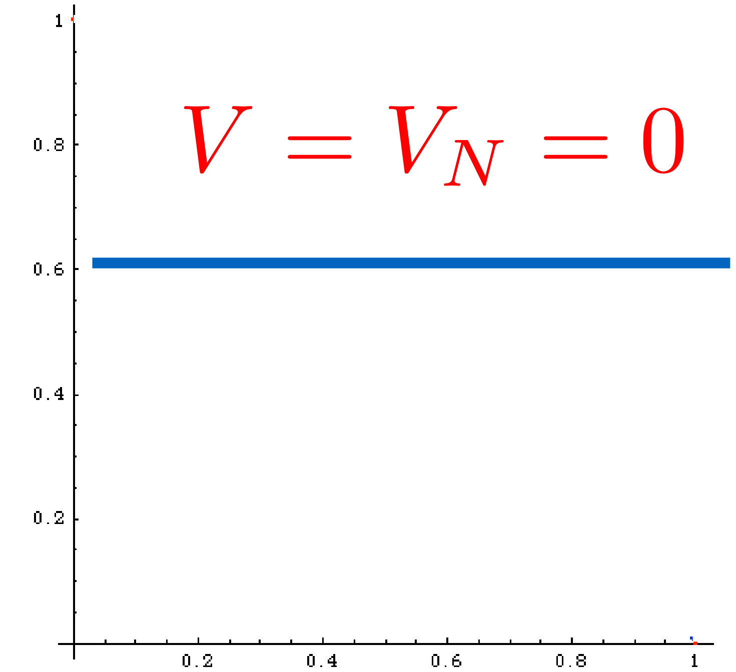
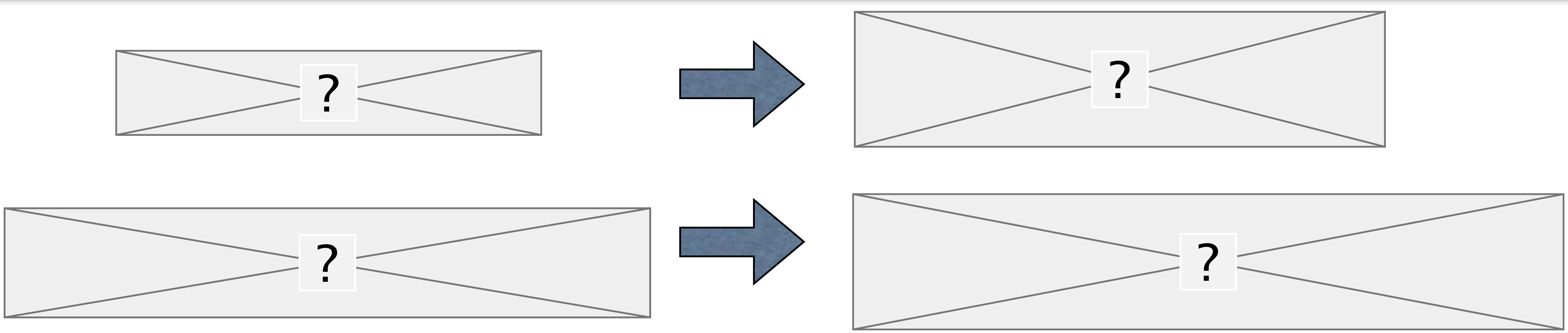






$$V = V_N = 0$$



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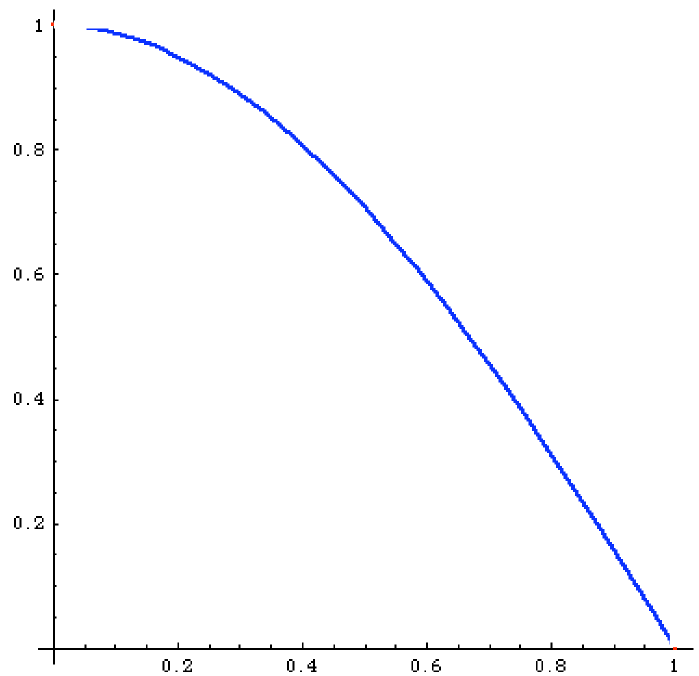
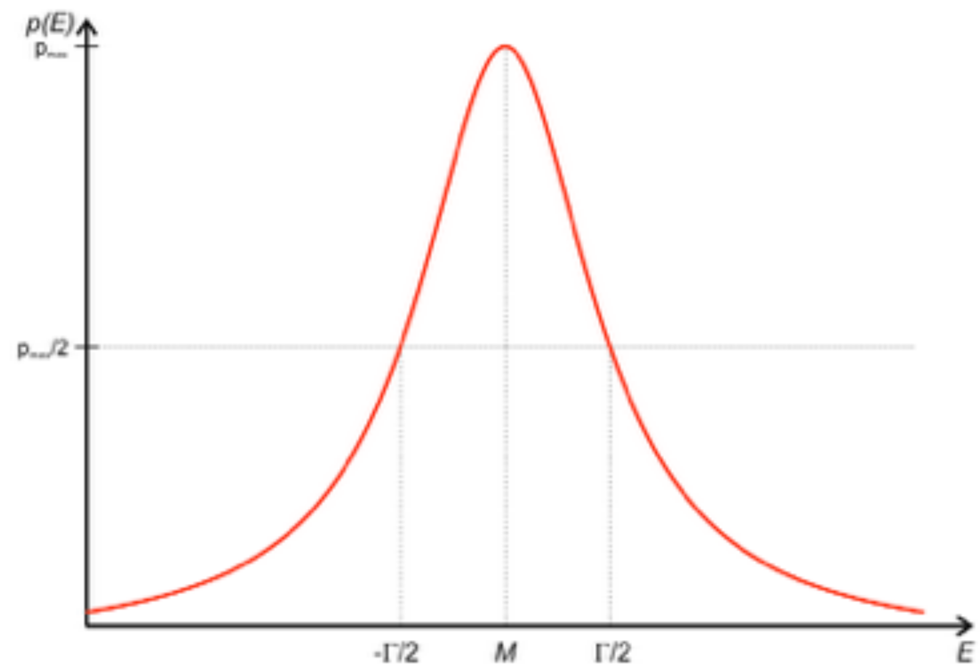
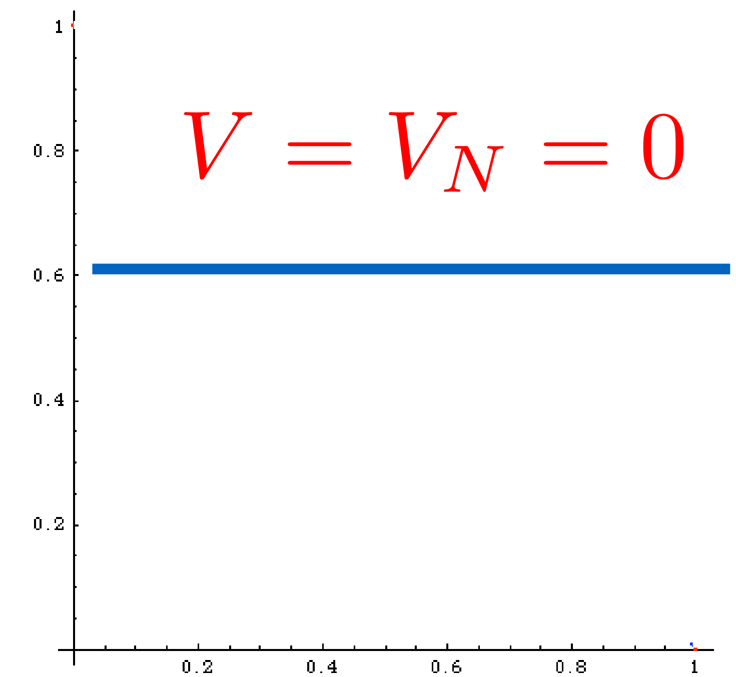


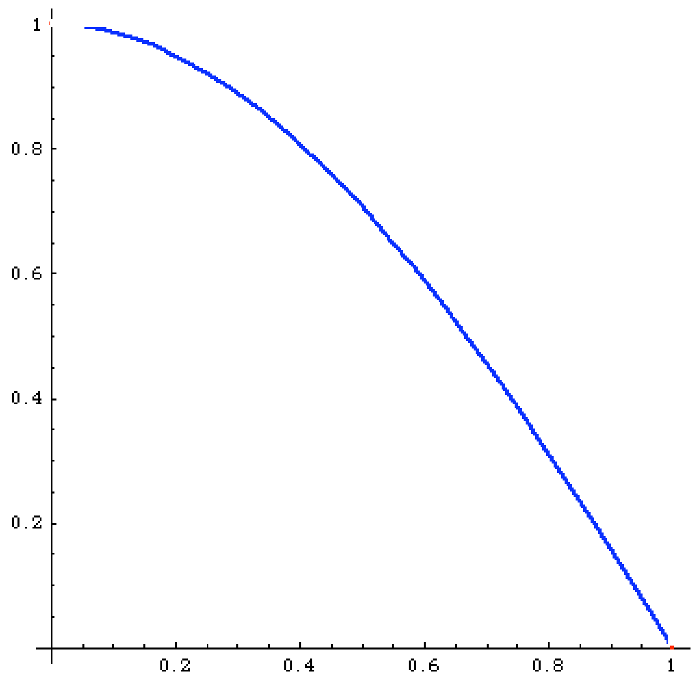




$$V = V_N = 0$$

Can be minimized!

Importance Sampling



$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

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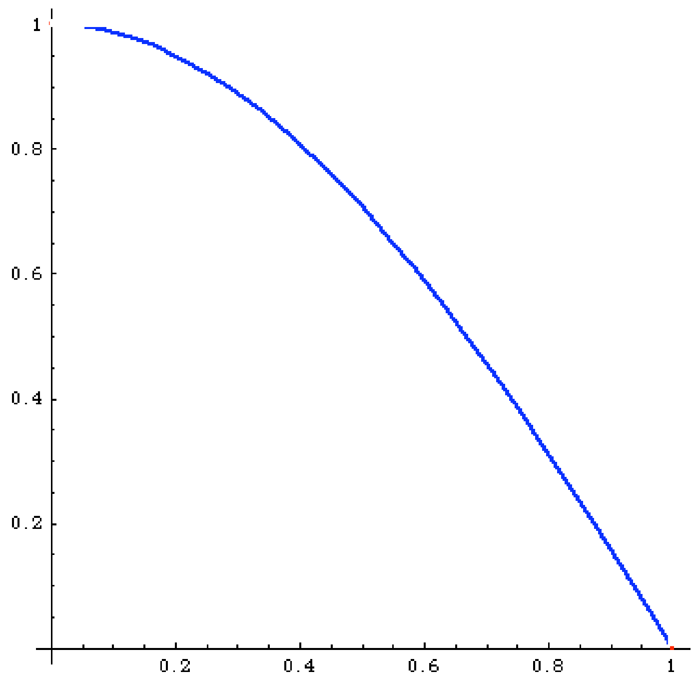

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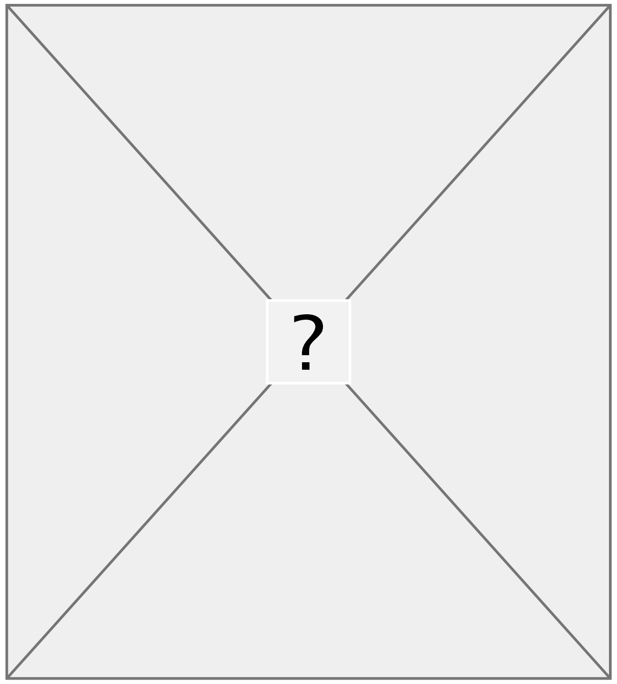

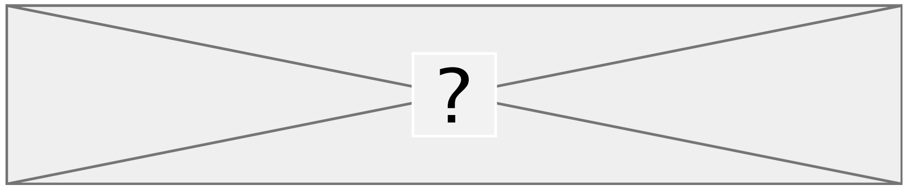

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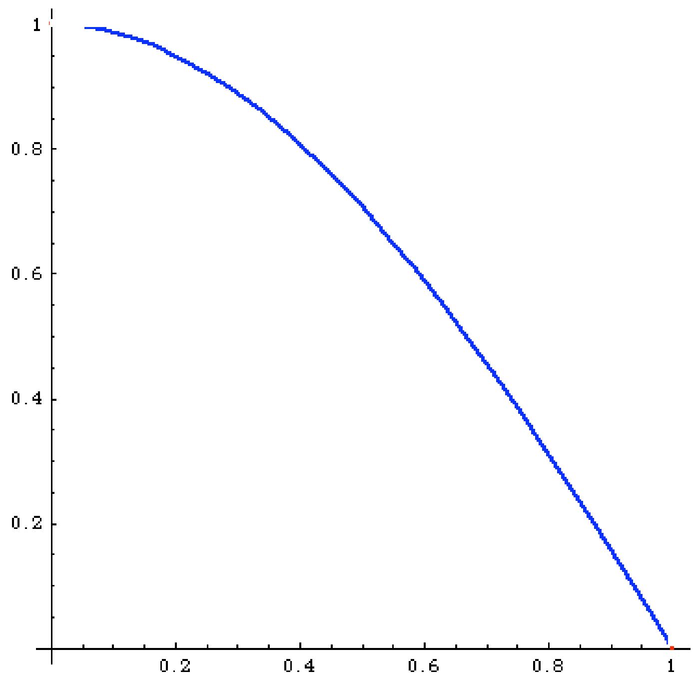

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

$$= \int_{\xi_1}^{\xi_2} d\xi \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2}$$

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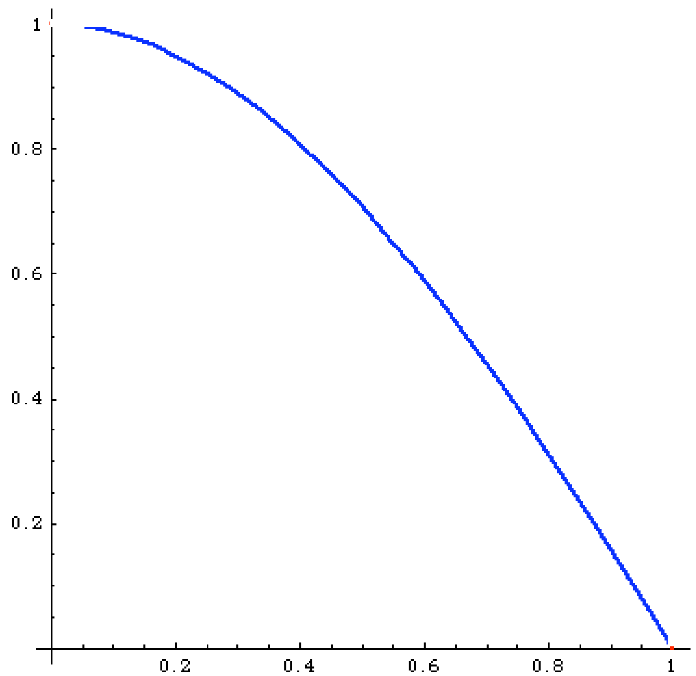

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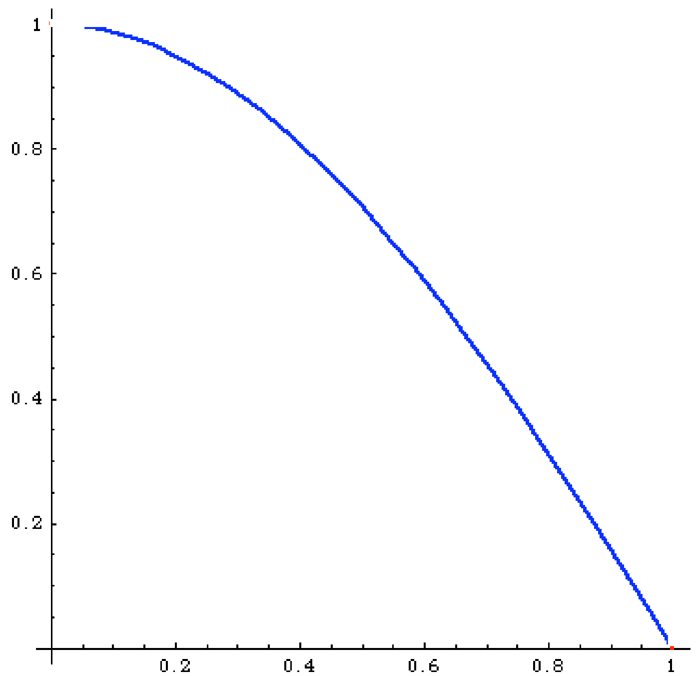

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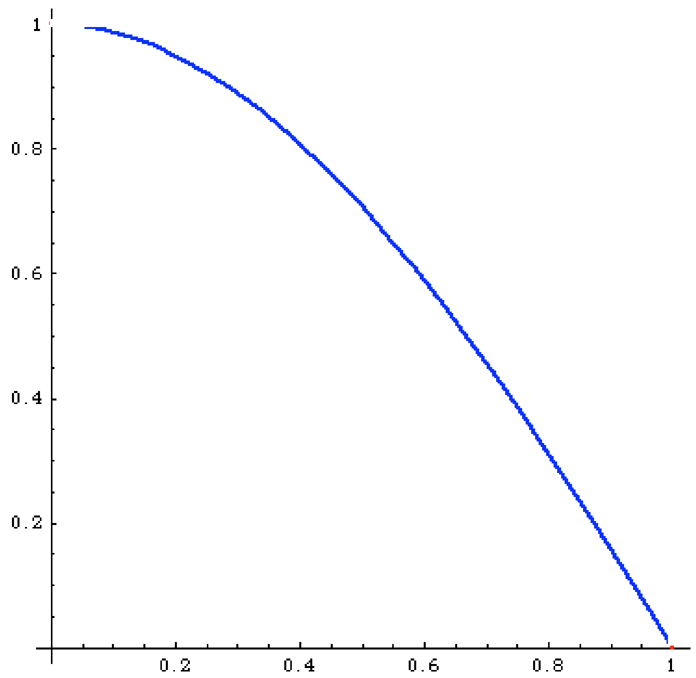

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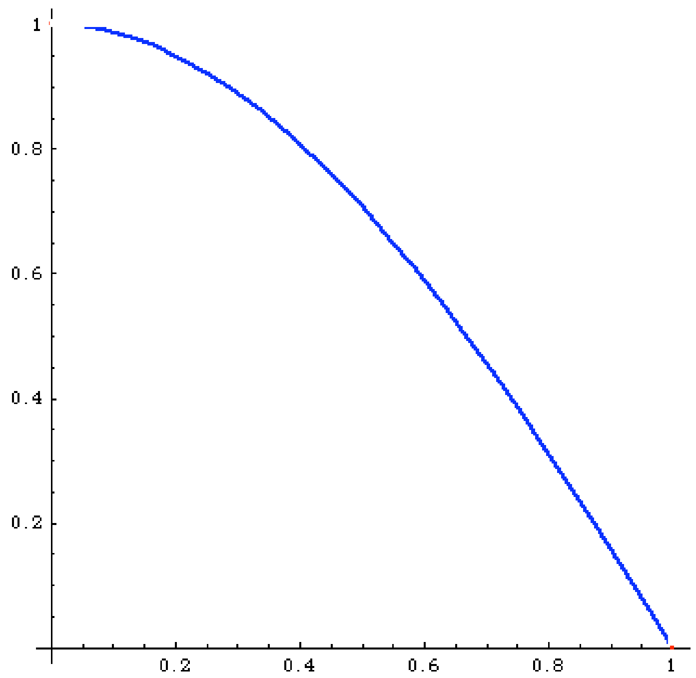

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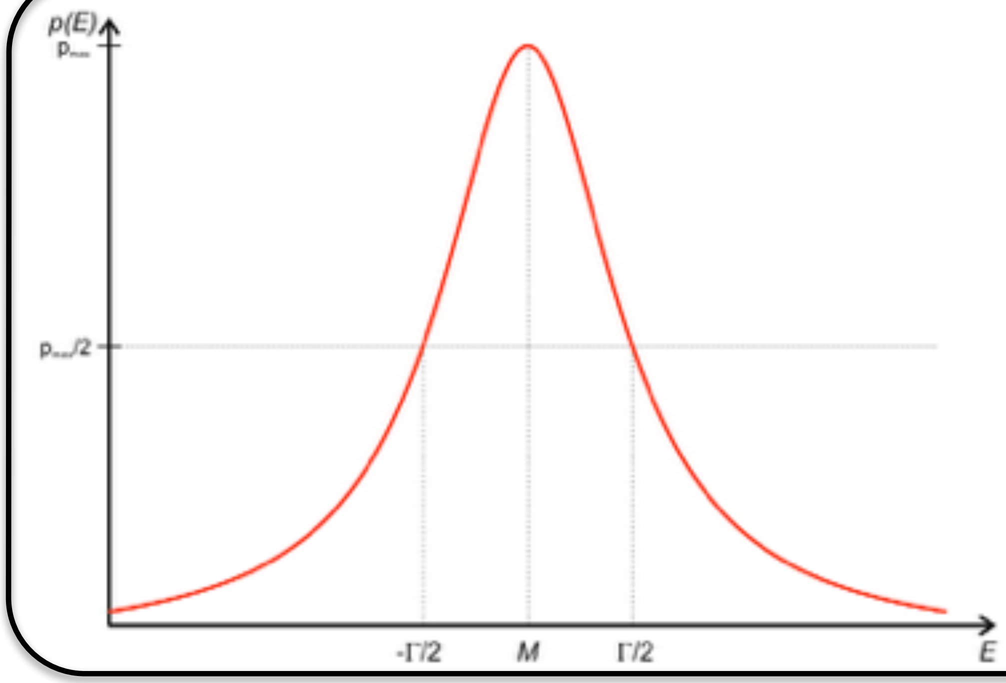
The Phase-Space parametrization is important to have an efficient computation!

Importance Sampling



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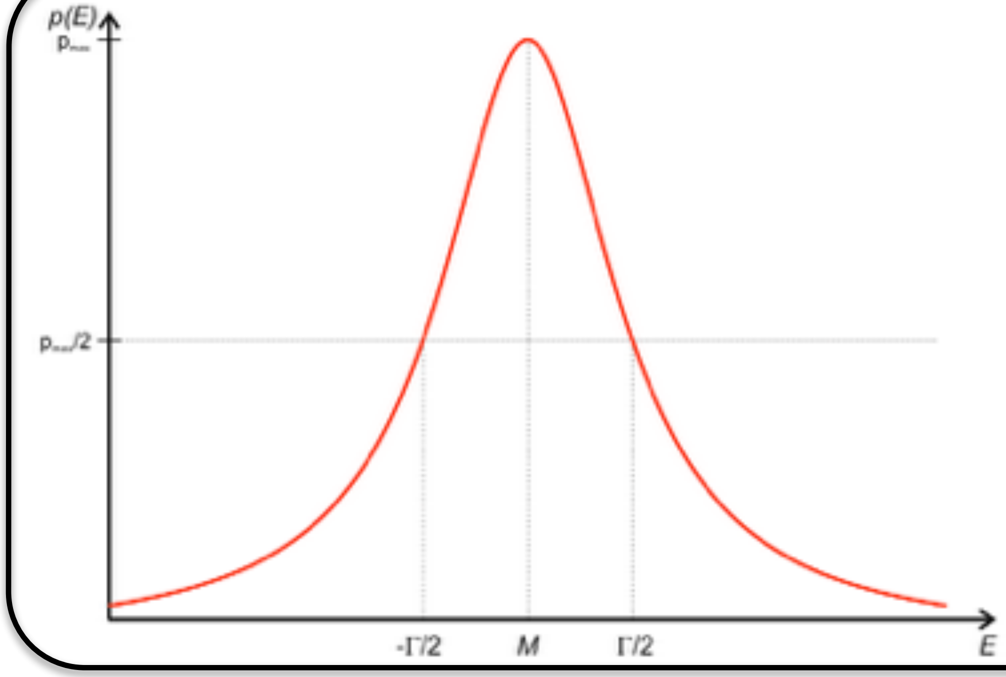

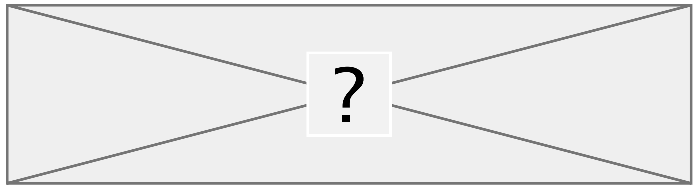

$$\xi = arctan\left(\frac{q^2 - M^2}{\Gamma M}\right)$$

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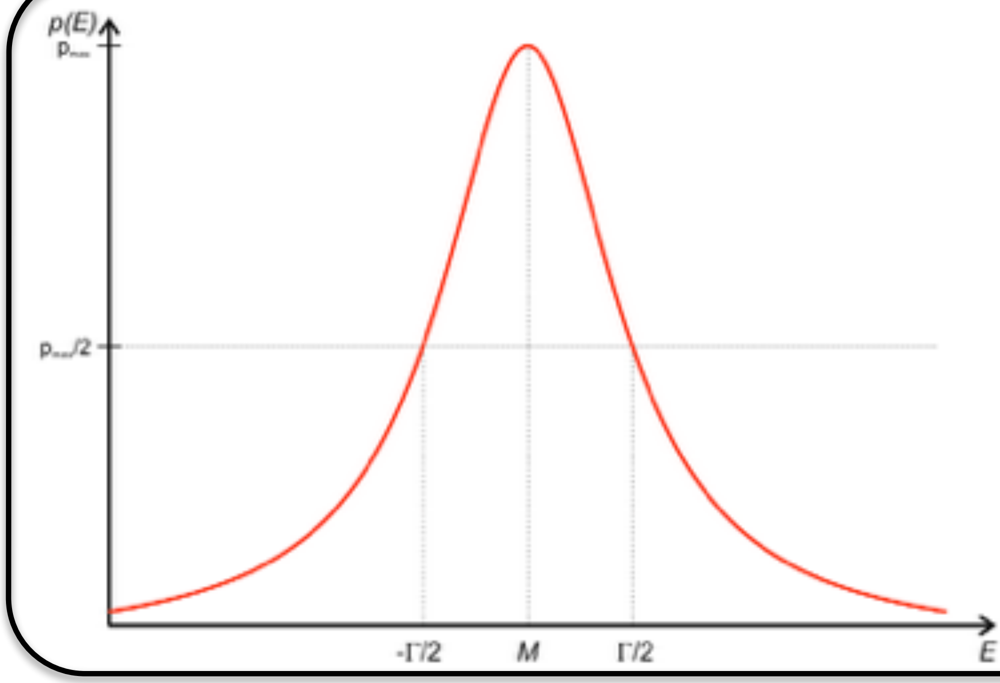

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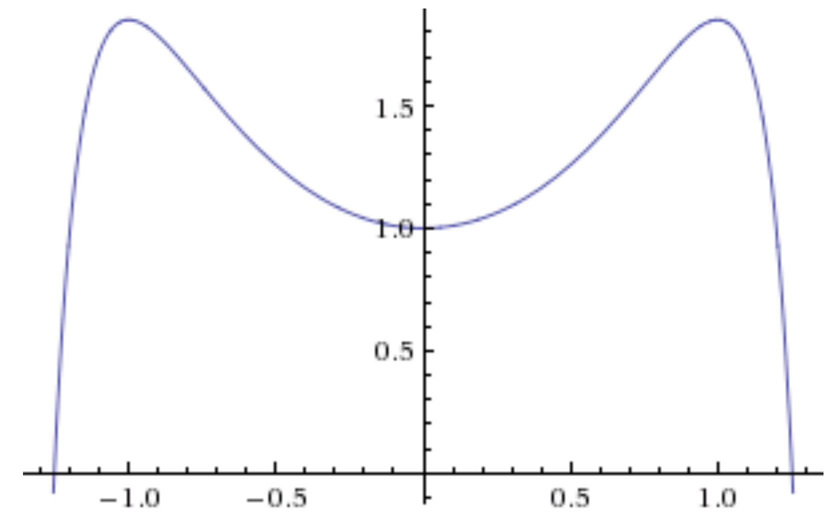

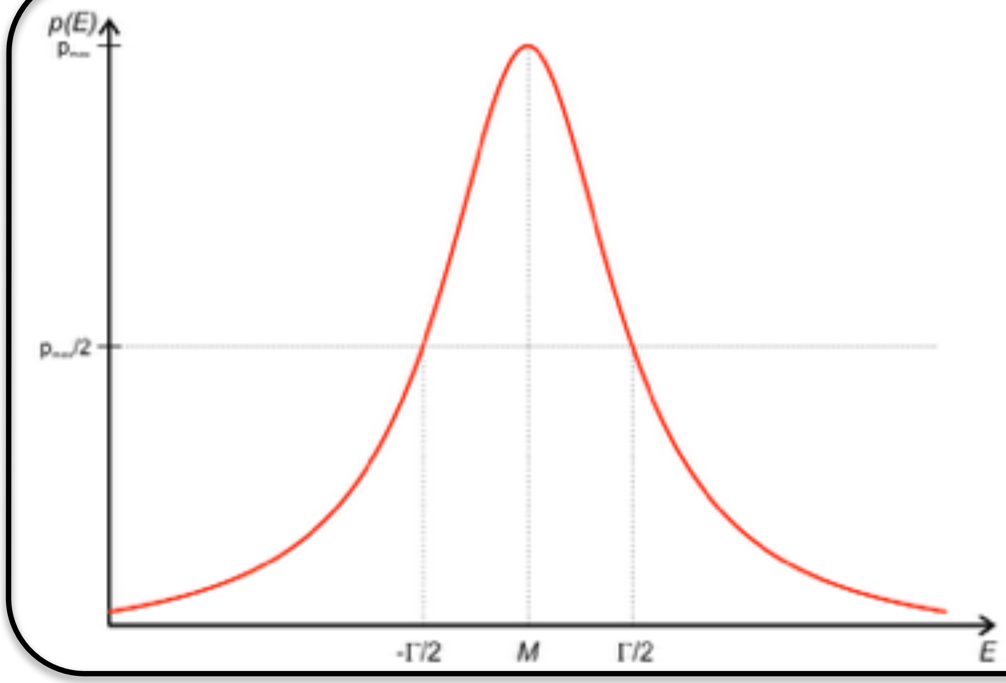

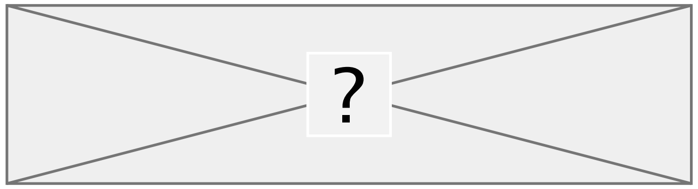

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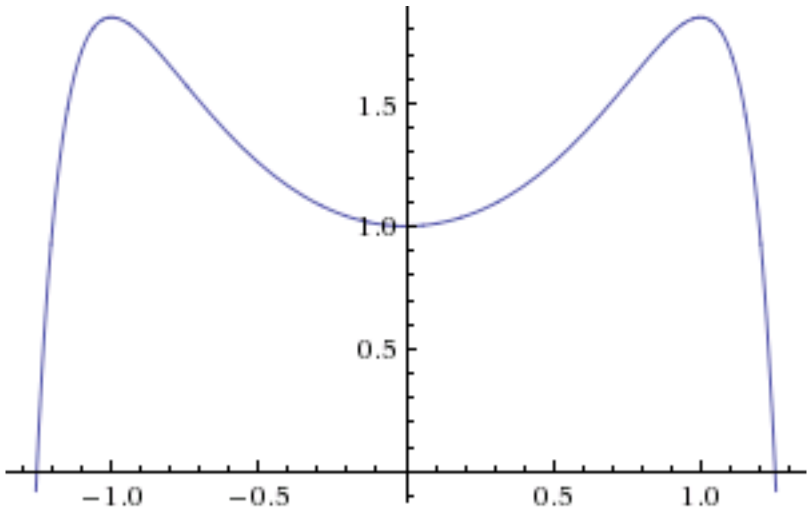
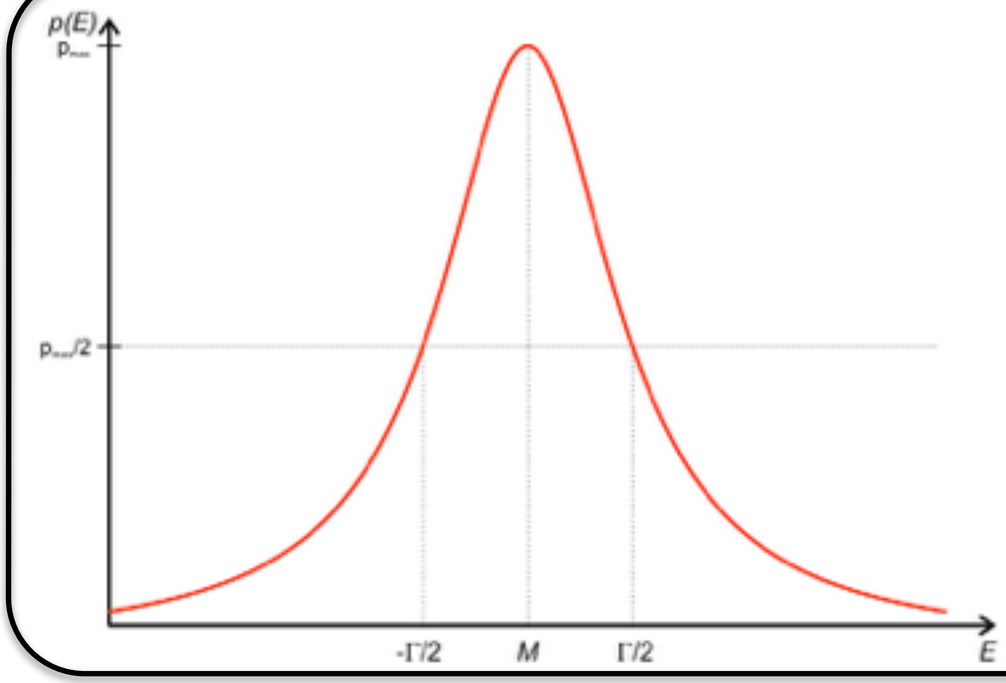
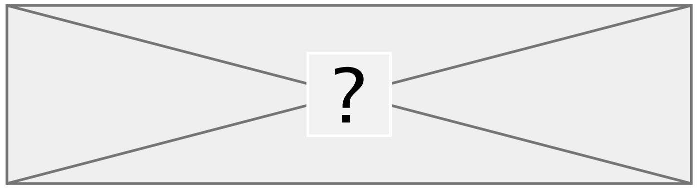


Why Importance Sampling?

Probability of using that point p(x)





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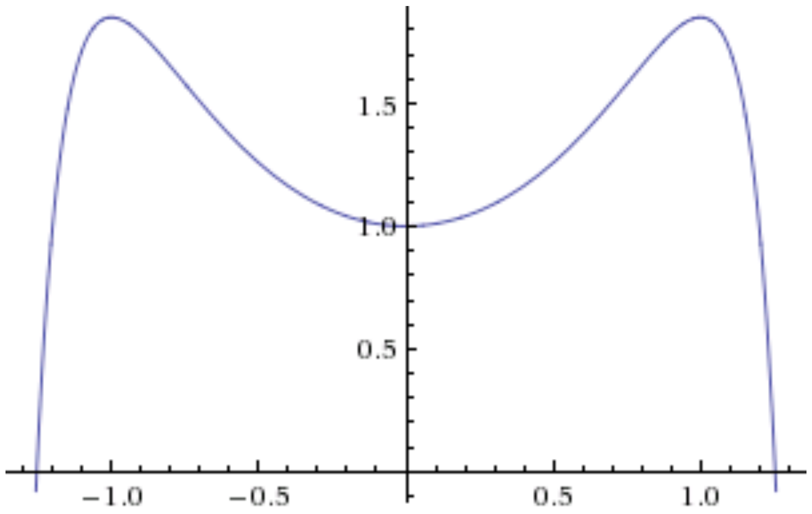

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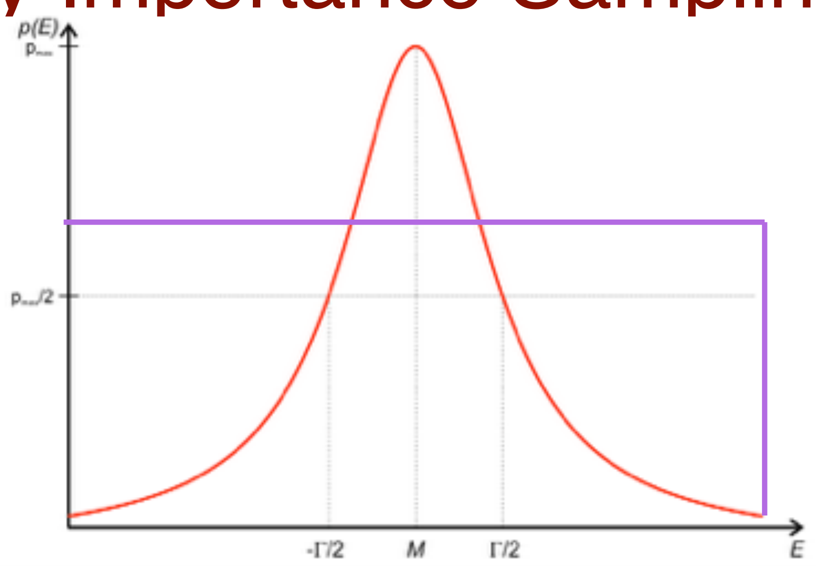

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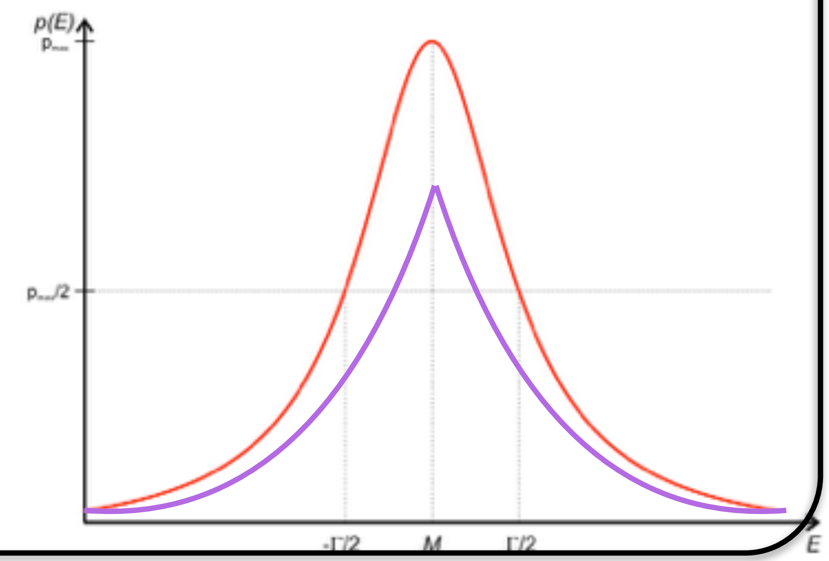

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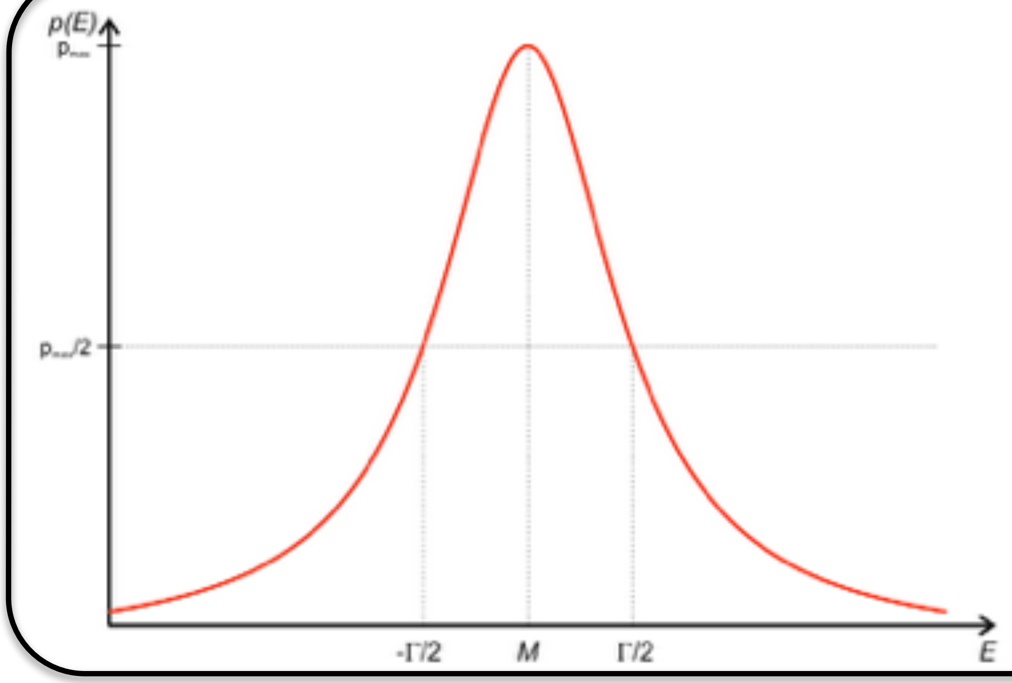

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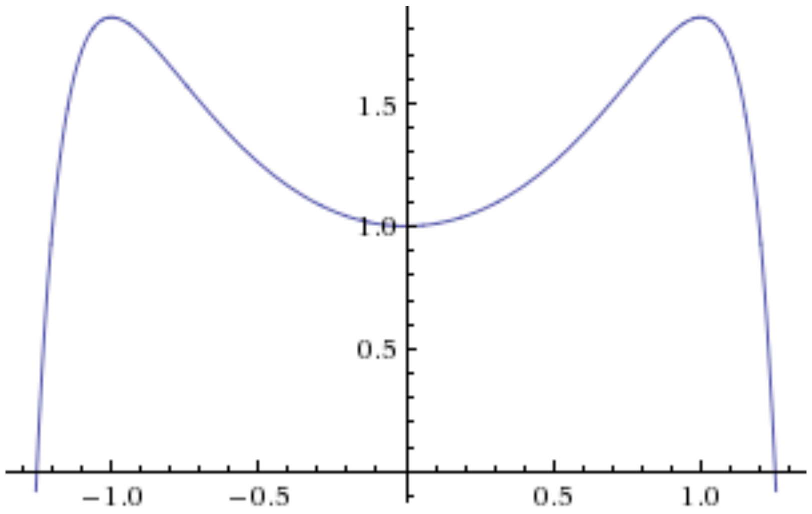

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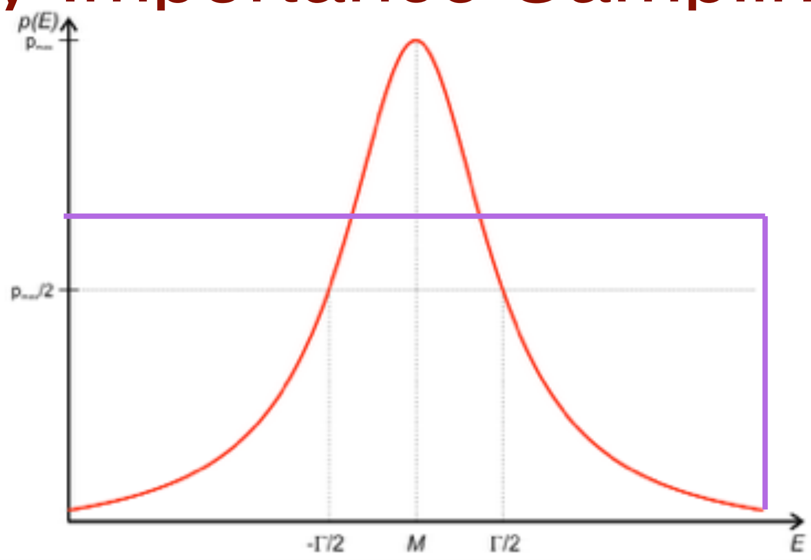

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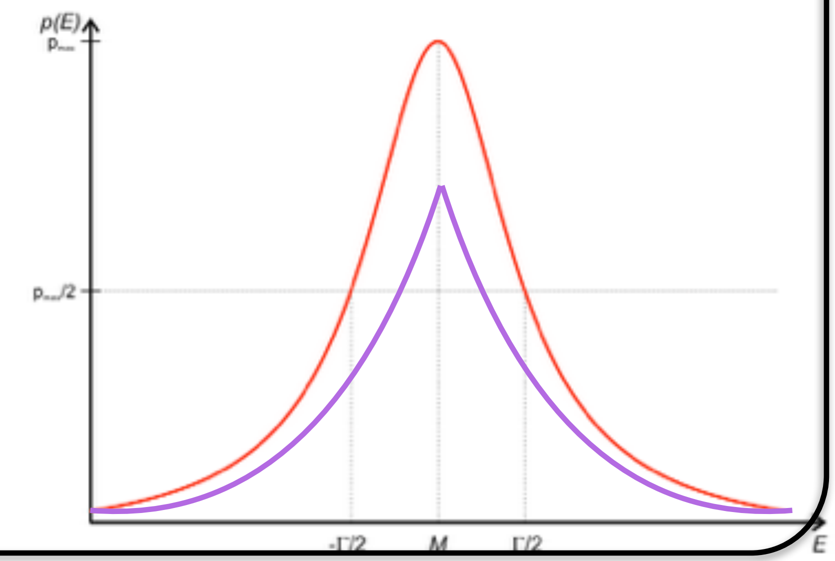

The change of variable ensure that the evaluation of the function is done where the function is the largest!

Importance Sampling

Key Point

- Generate the random point in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is flatter in this new variable.
- Needs to know an approximate function.

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

Algorithm

1. Creates bin such that each of them have the same contribution.

   ➡Many bins where the function is large

2. Use the approximate for the importance sampling method.

VEGAS

More than one Dimension

- VEGAS works only with 1(few) dimension
  - ➡memory problem

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Solution

- Use projection on the axis

$$\vec{p}(x) = p(x) \cdot p(y) \cdot p(z) \ldots$$

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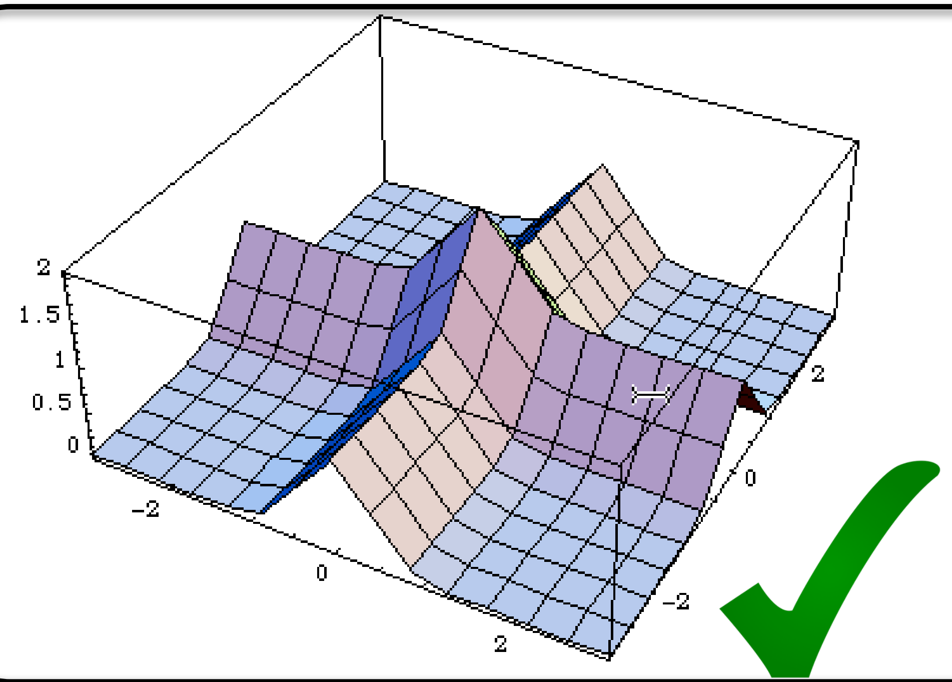

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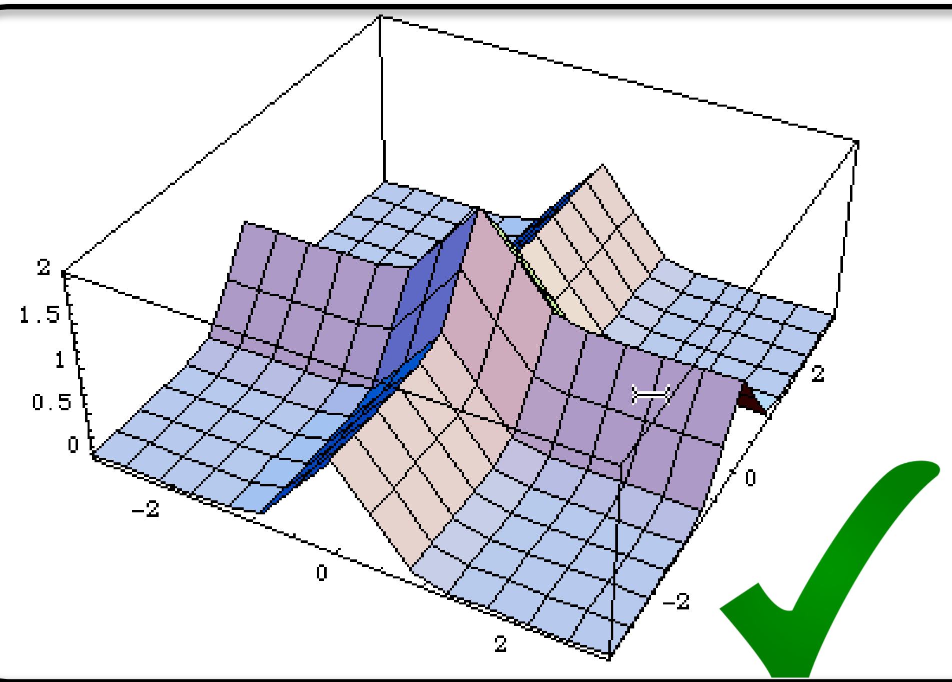
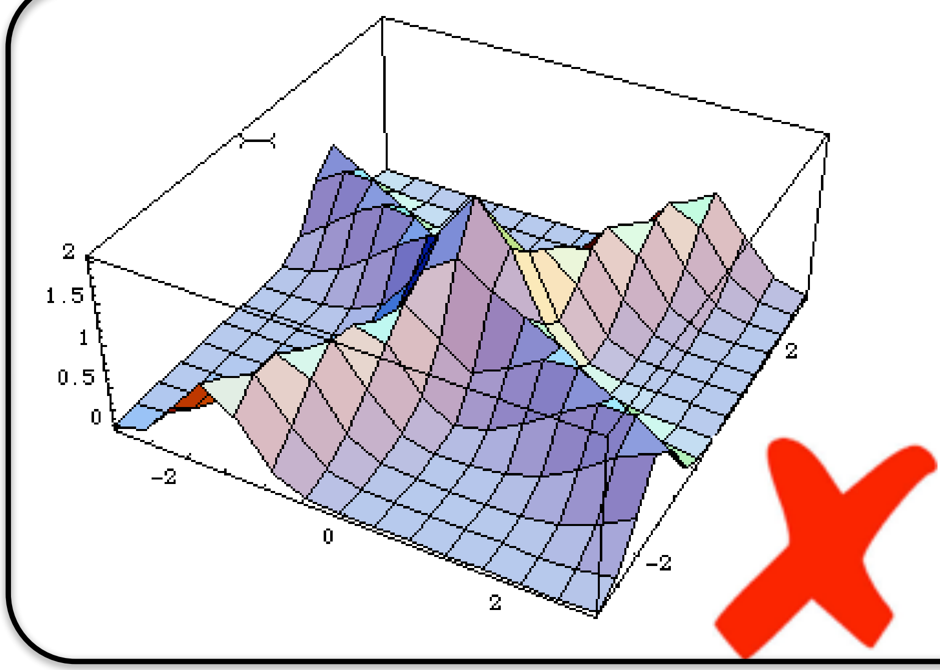
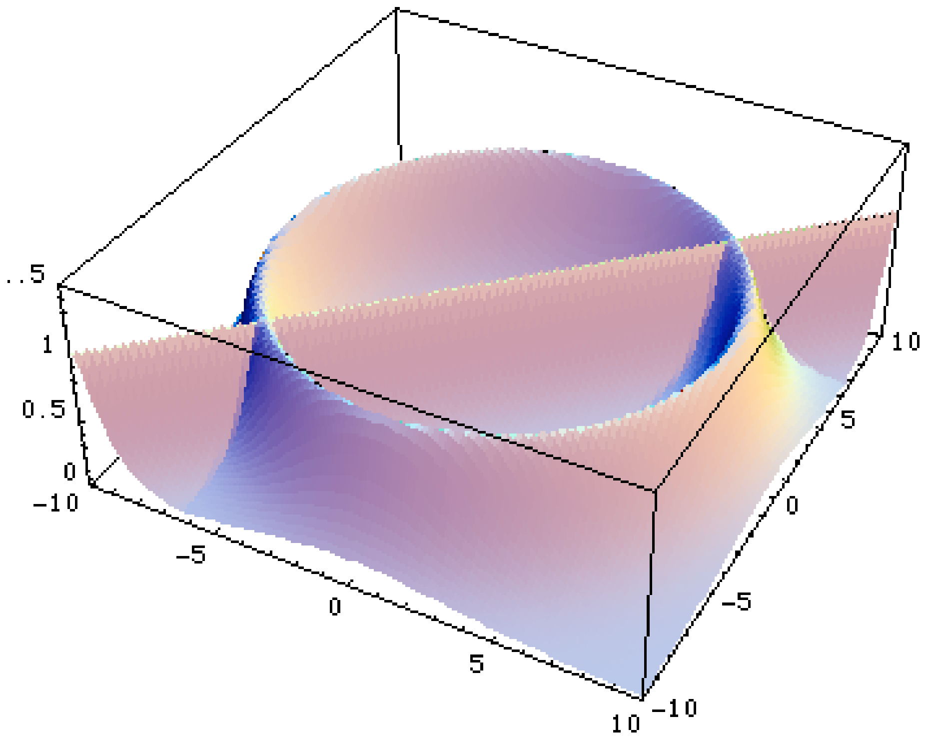




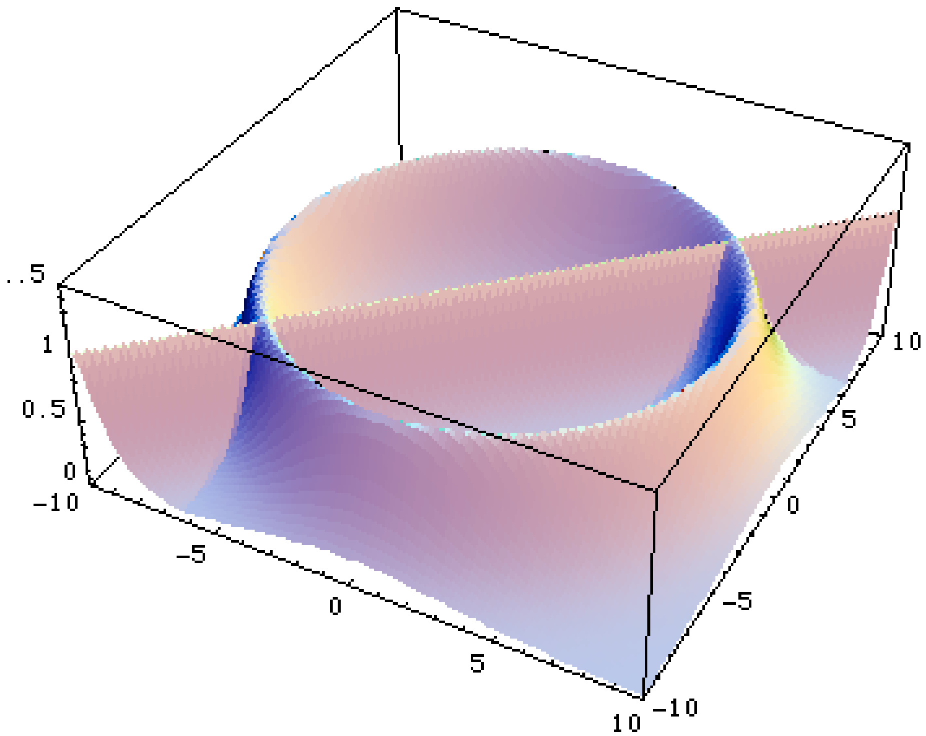
- We need to ensure the factorization !
  - ➡Additional change of variable

Multi-channel



What do we do if there is no transformation that aligns all integrand peaks to the chosen axes?
Vegas is bound to fail!

Multi-channel



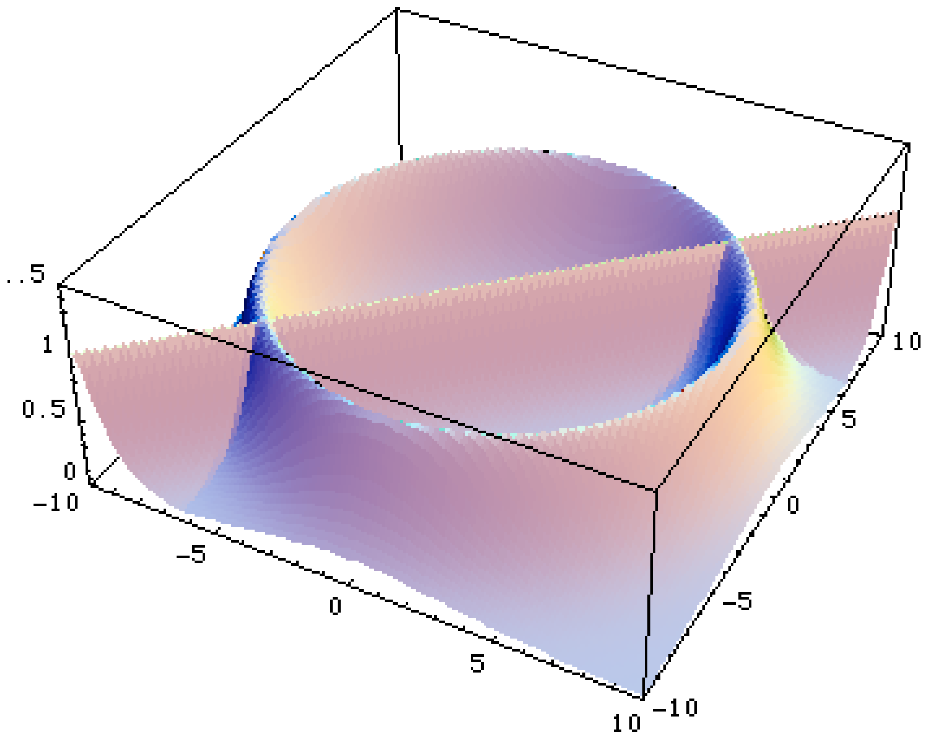
What do we do if there is no transformation that aligns all integrand peaks to the chosen axes? Vegas is bound to fail!

Solution: use different transformations = channels

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

with each $p_i(x)$ taking care of one "peak" at the time

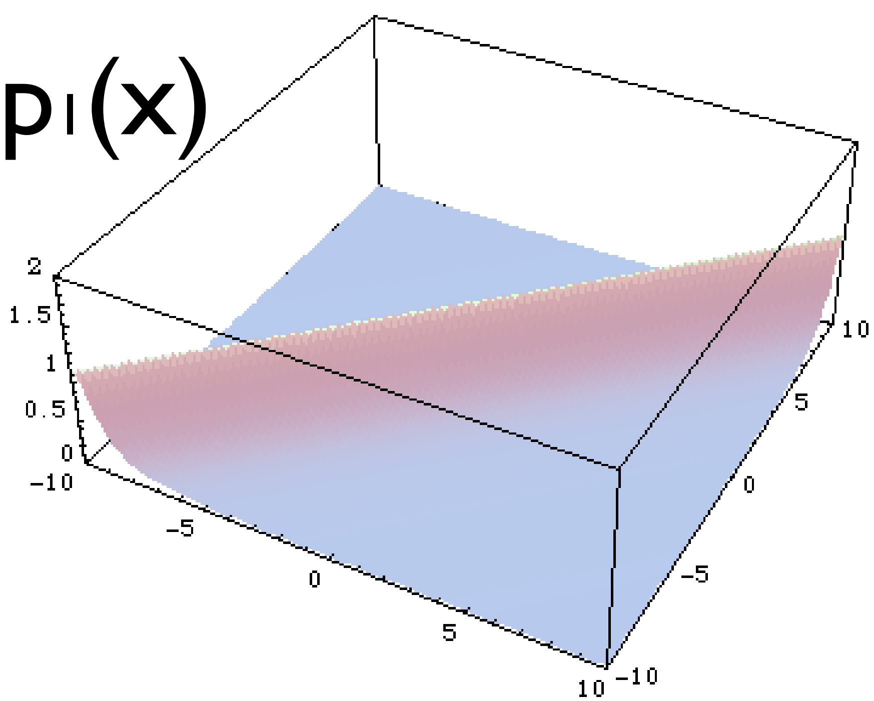
Multi-channel



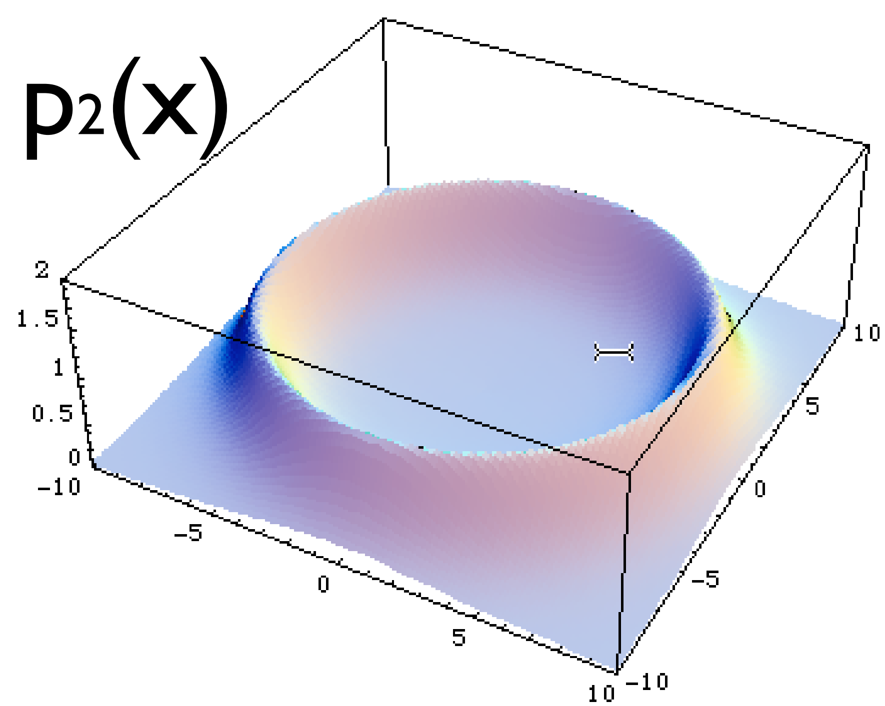
$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x)$$

with $$\sum_{i=1}^{n} \alpha_i = 1$$

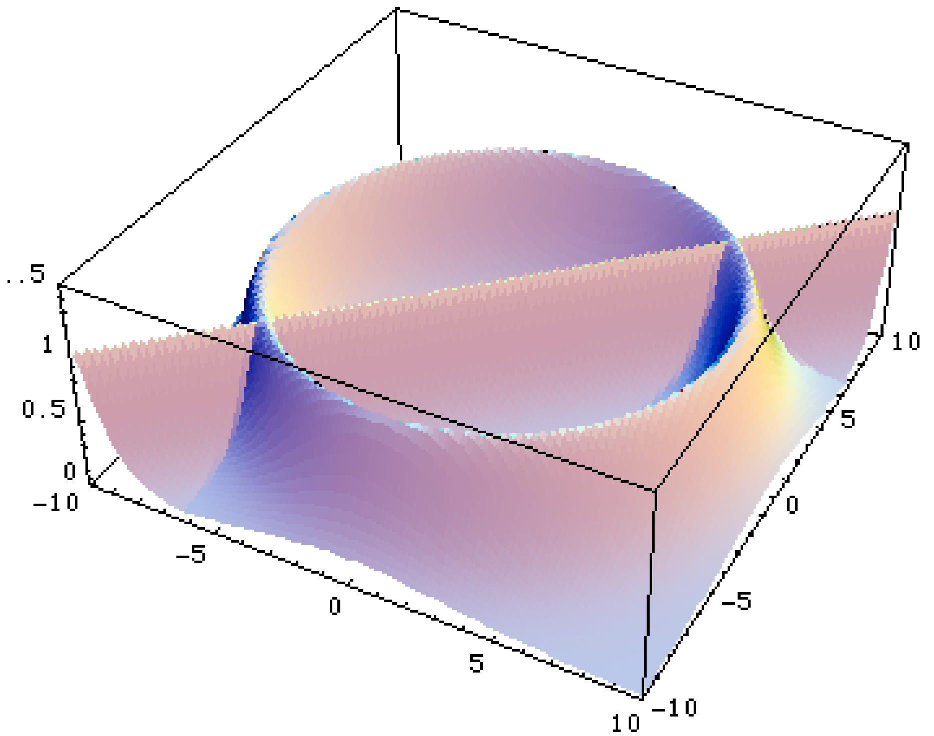
$p_1(x)$



$p_2(x)$



Multi-channel

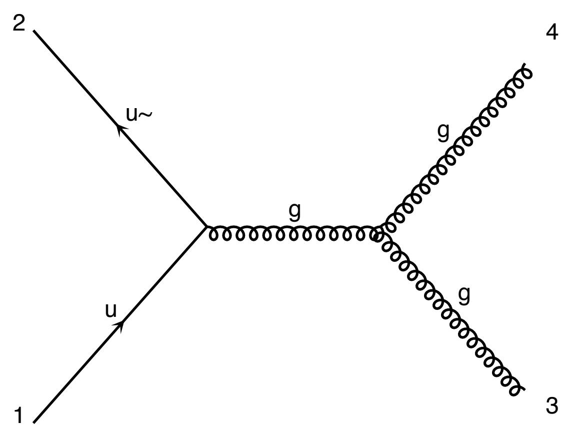


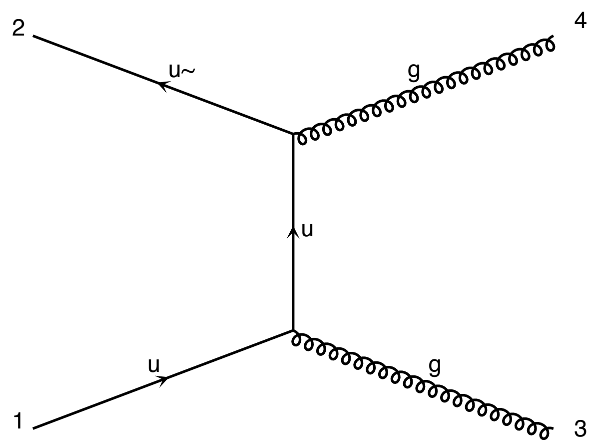
$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x)$$

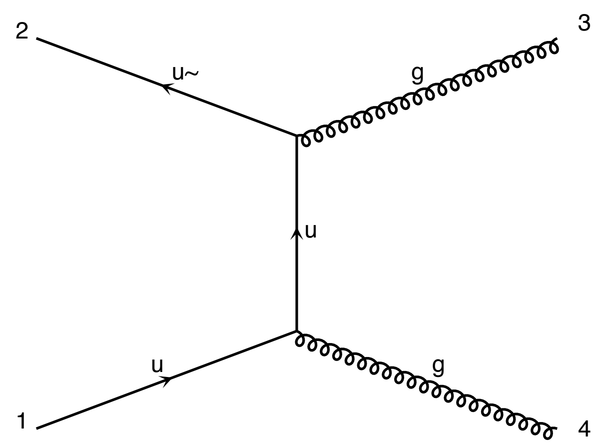
with
$$\sum_{i=1}^{n} \alpha_i = 1$$

Then,

$$I = \int f(x)dx = \sum_{i=1}^{n} \alpha_i \int \frac{f(x)}{p(x)} p_i(x)dx$$







$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2} \qquad \propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2} \qquad \propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

Multi-channel based on single diagrams*

*Method used in MadGraph

Does a basis exist?

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

*Method used in MadGraph

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$$\approx 1$$

*Method used in MadGraph

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≈ 1

Key Idea

- Any single diagram is "easy" to integrate (pole structures/ suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

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≈ 1

Key Idea

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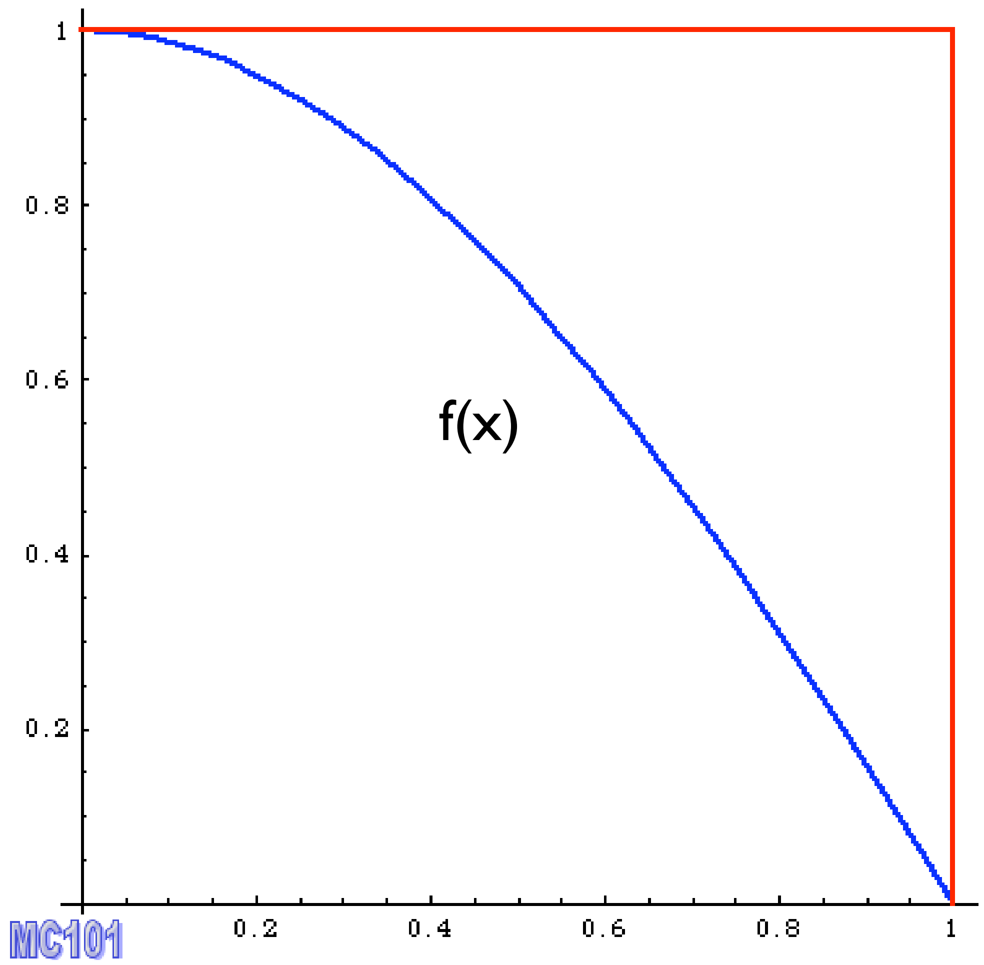
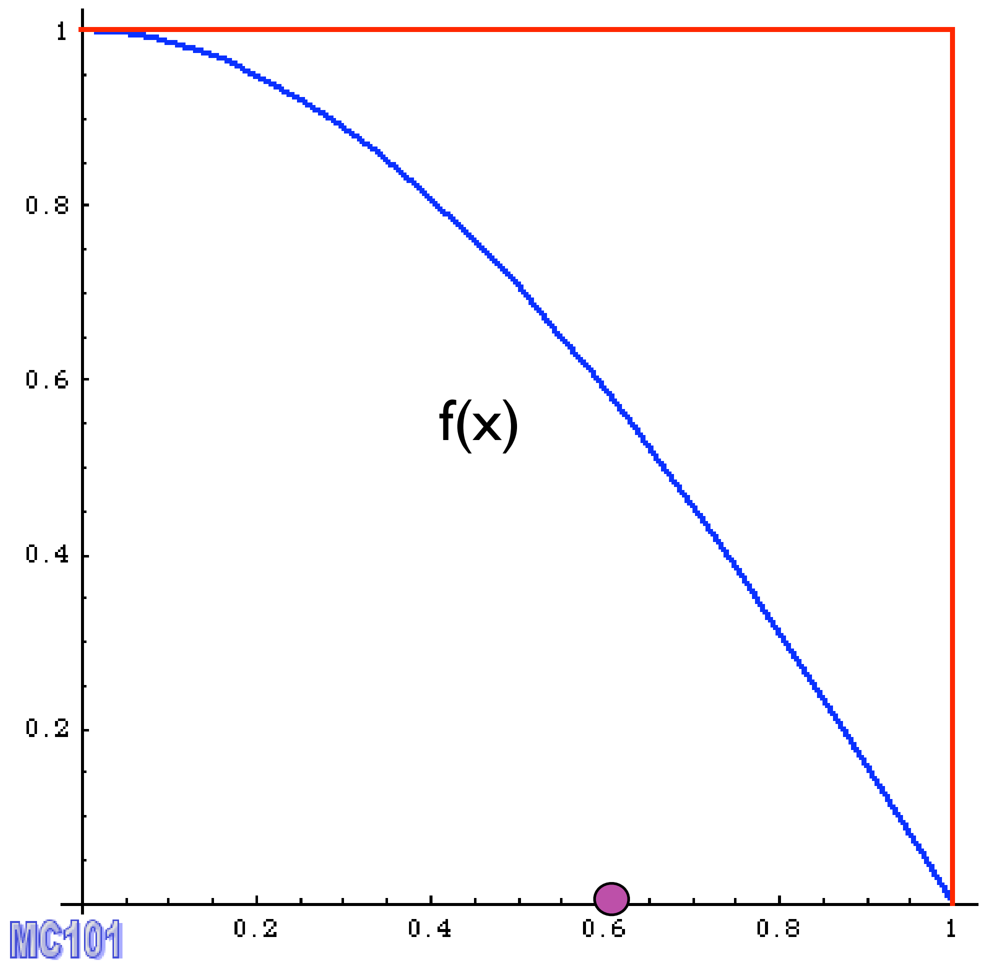
N Integral

- Errors add in quadrature so no extra cost
- "Weight" functions already calculated during $|M|^2$ calculation
- Parallel in nature

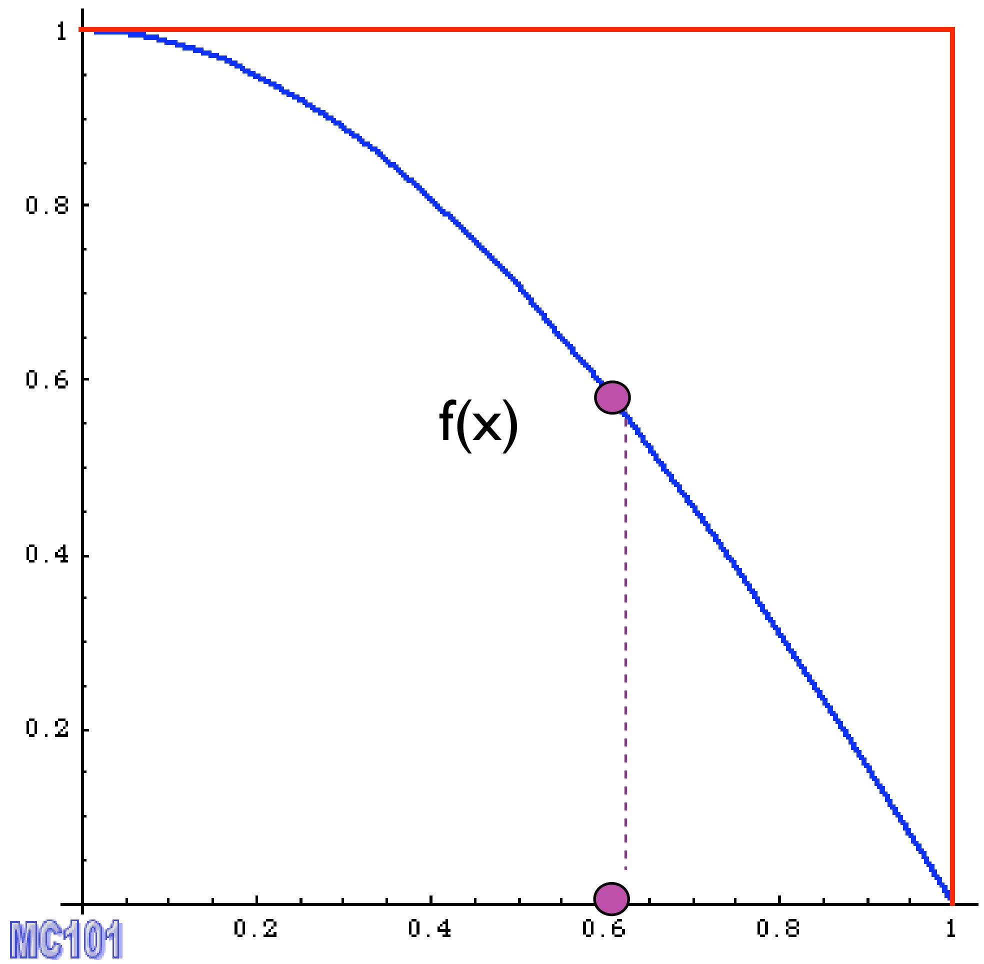
Event generation



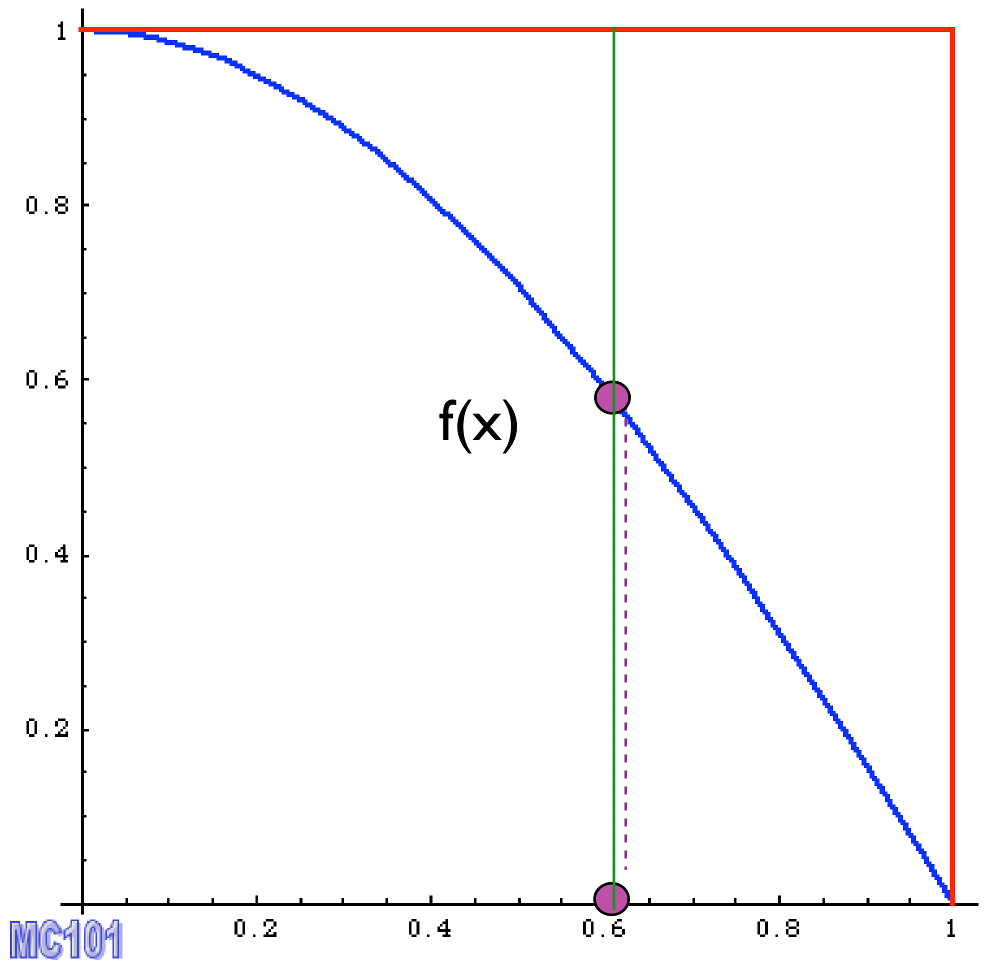


1. pick x

Event generation

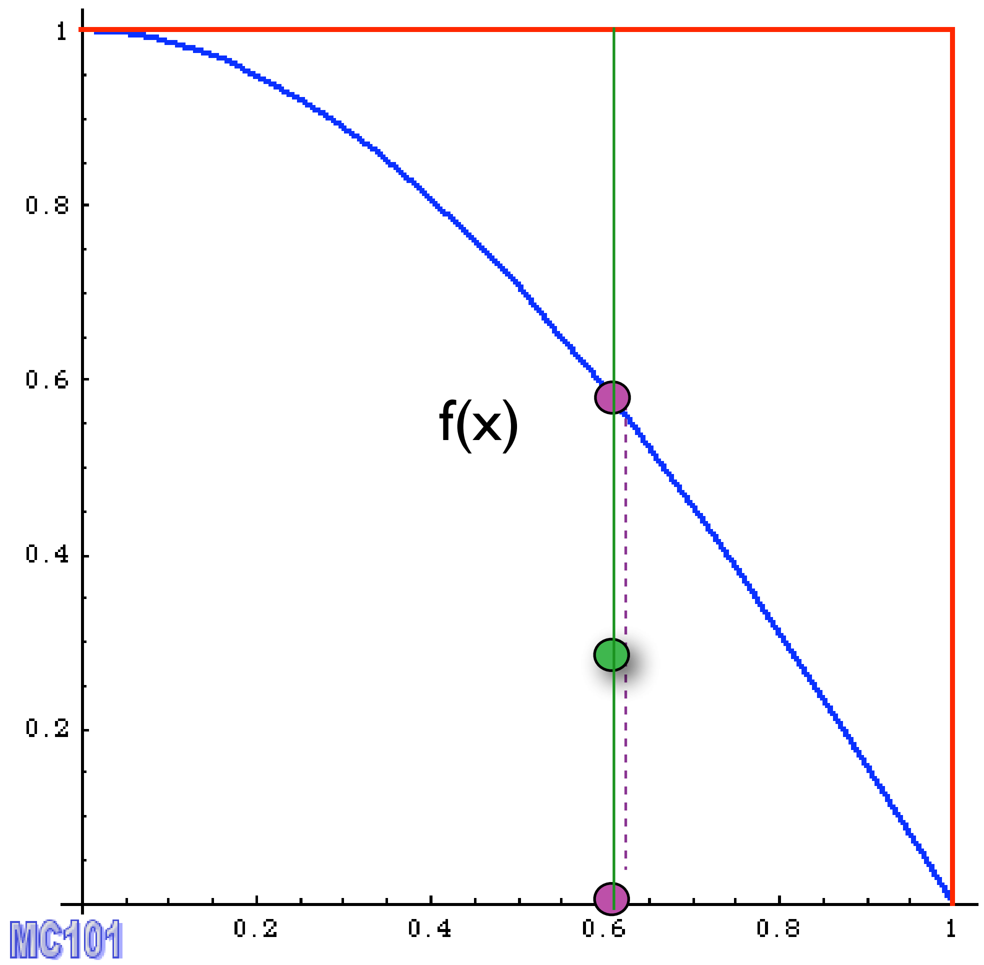


1. pick x
2. calculate f(x)



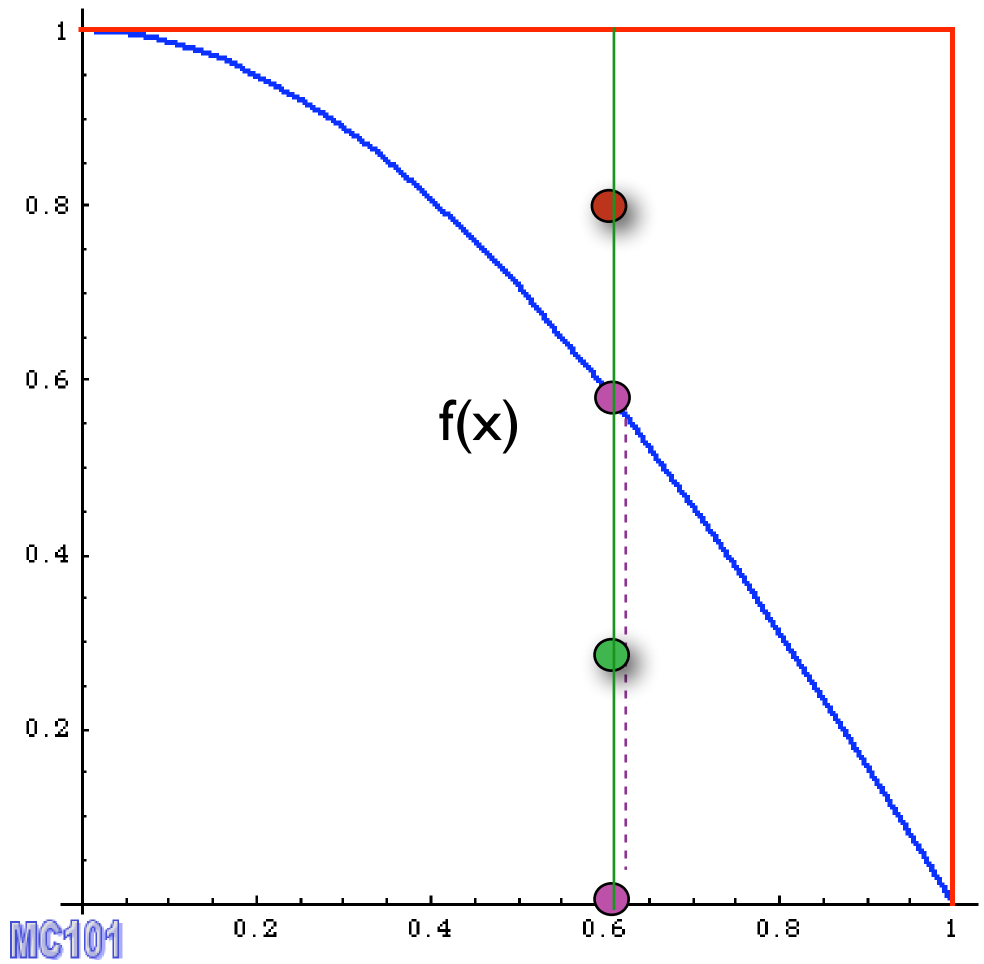
1. pick x
2. calculate f(x)
3. pick 0<y<fmax

Event generation



1. pick x
2. calculate f(x)
3. pick 0<y<fmax
4. Compare:
   if f(x)>y accept event,

Event generation



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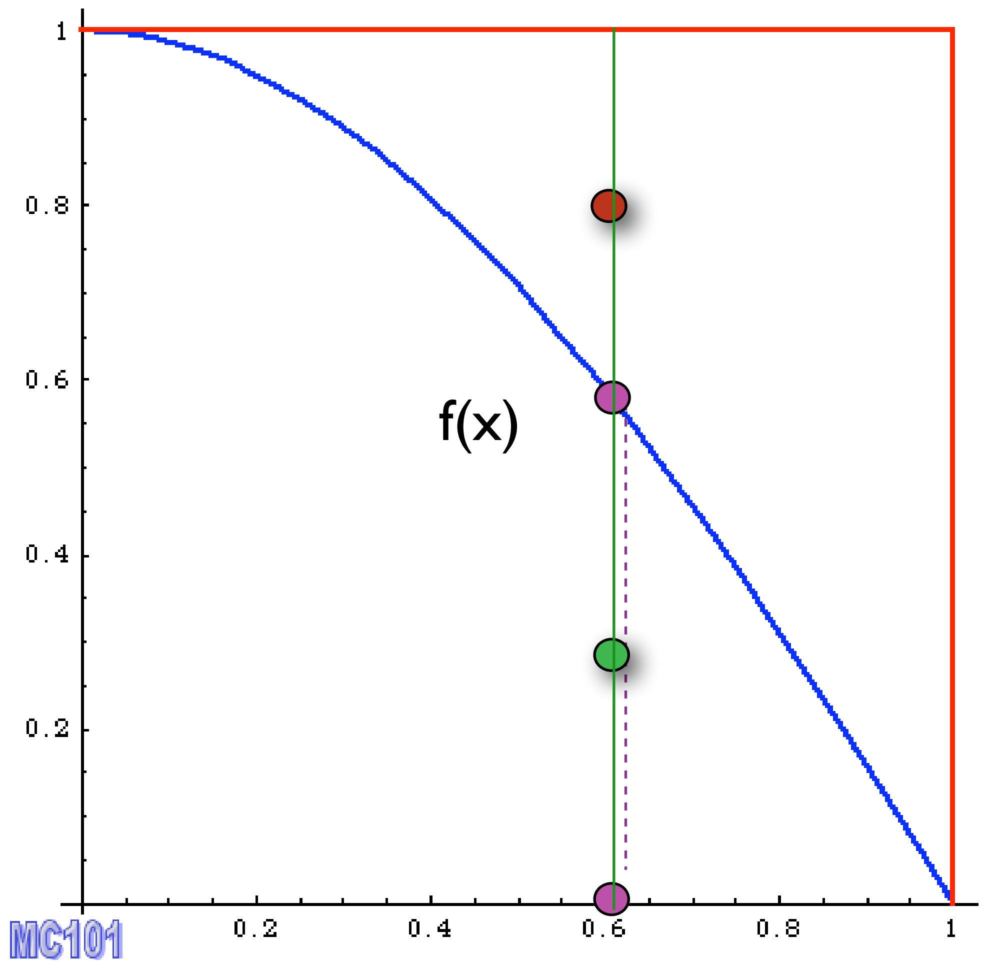
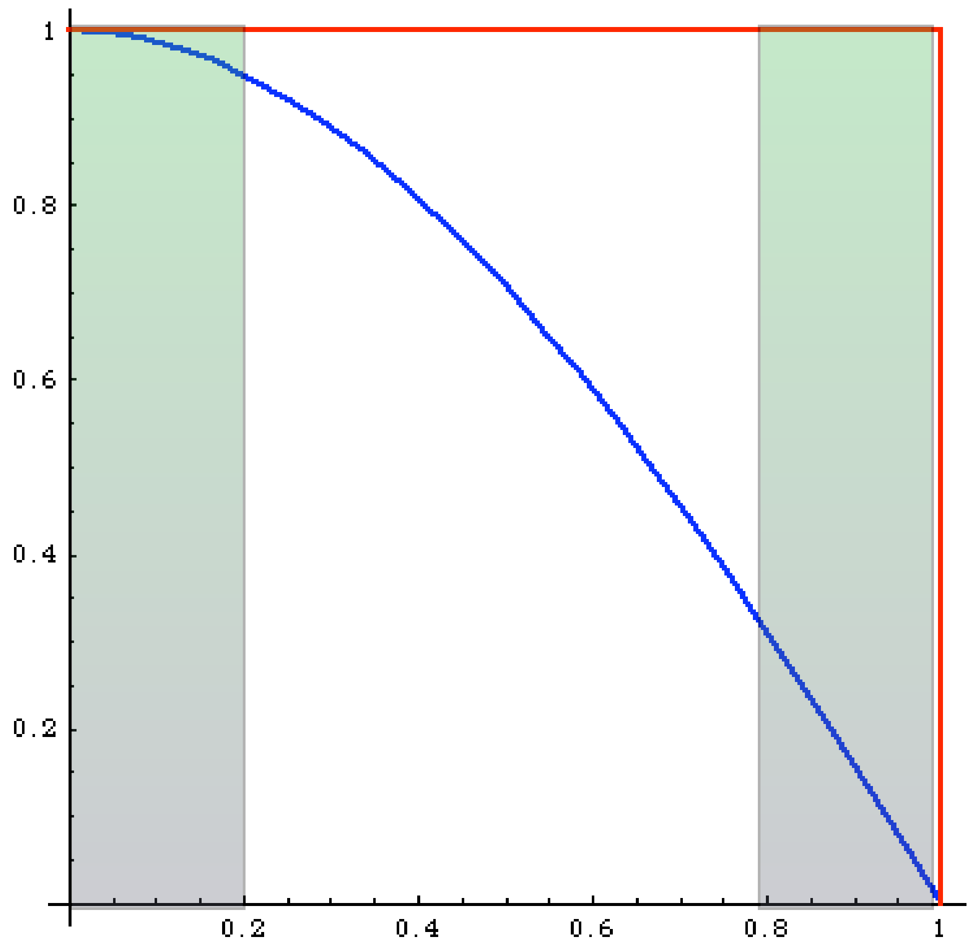


1. pick x
2. calculate f(x)
3. pick 0<y<fmax
4. Compare:
   if f(x)>y accept event,

   else reject it.

$$I = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

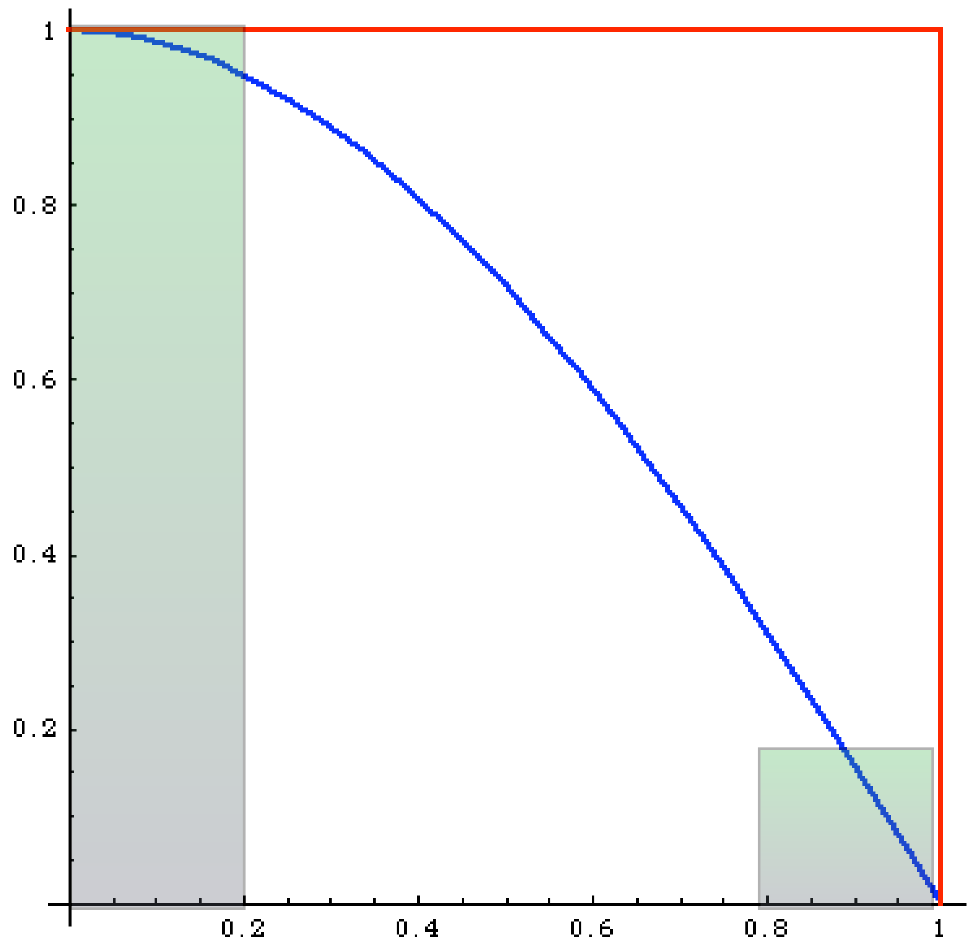


What's the difference between weighted and unweighted?

Weighted:

Same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



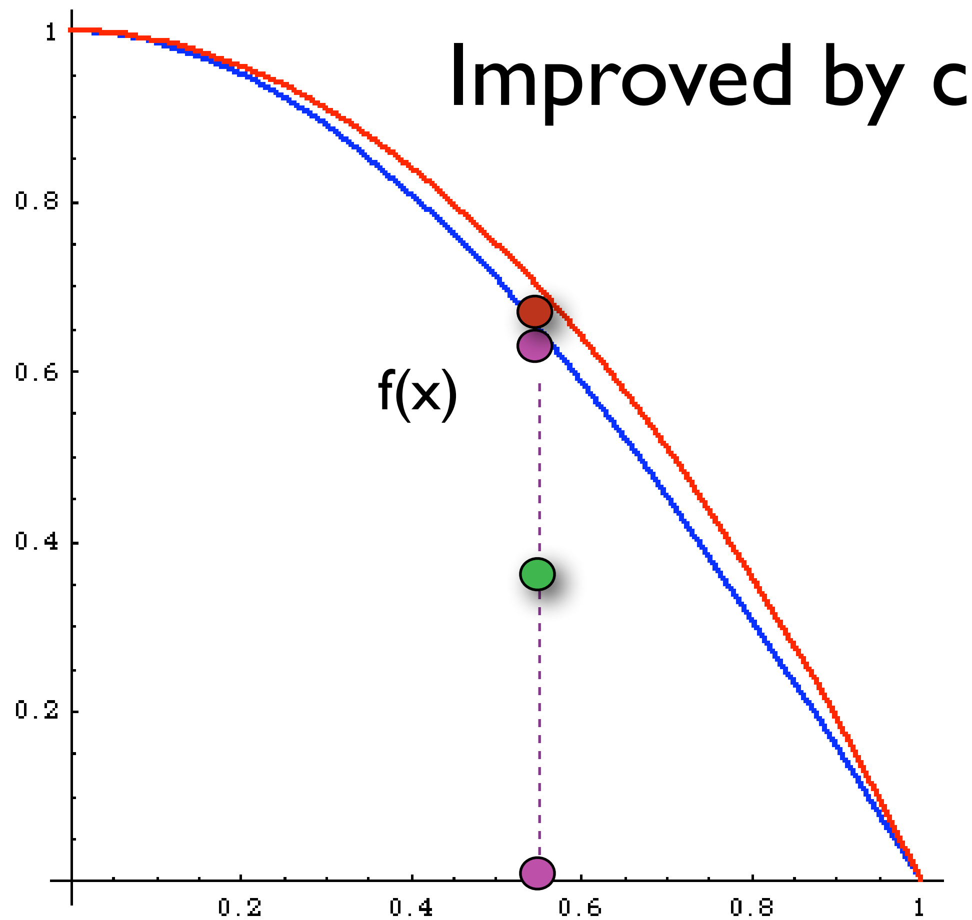
What's the difference between weighted and unweighted?

Unweighted:

events is proportional to the probability of areas of phase space:
events have all the same weight (”unweighted”)

Events distributed as in nature

Event generation



Improved by combining with importance sampling:

1. pick x distributed as p(x)
2. calculate f(x) and p(x)
3. pick 0<y<1
4. Compare:
   if f(x)>y p(x) accept event,

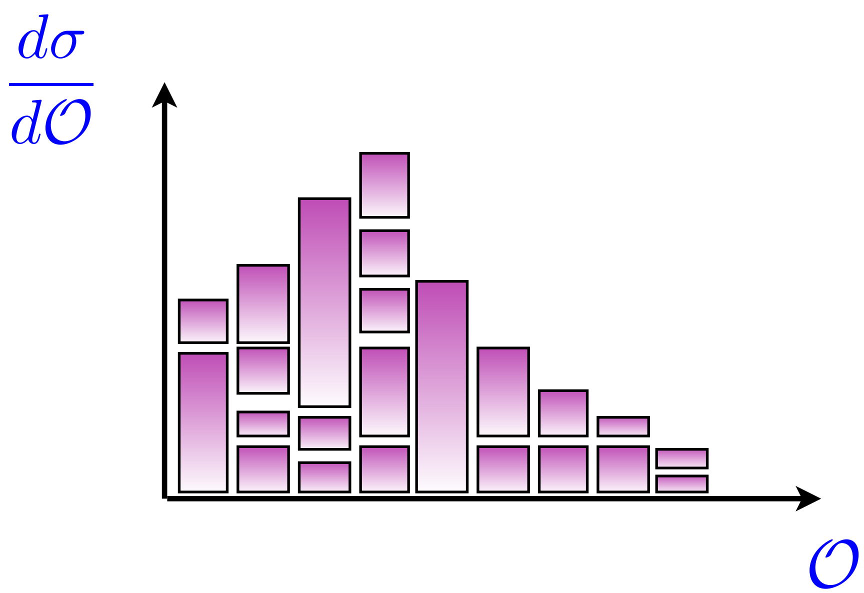
else reject it.

much better efficiency!!!

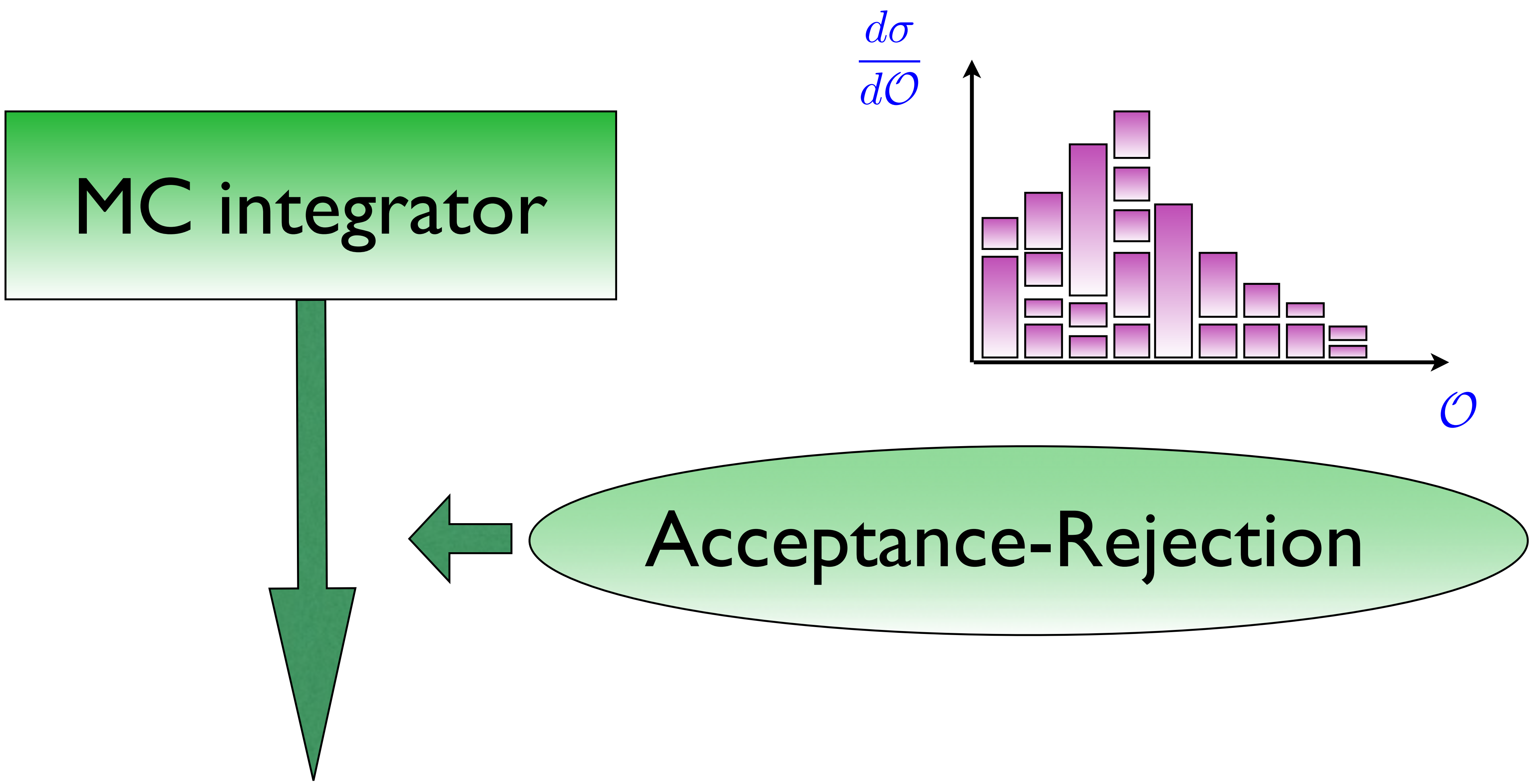
Event generation

MC integrator

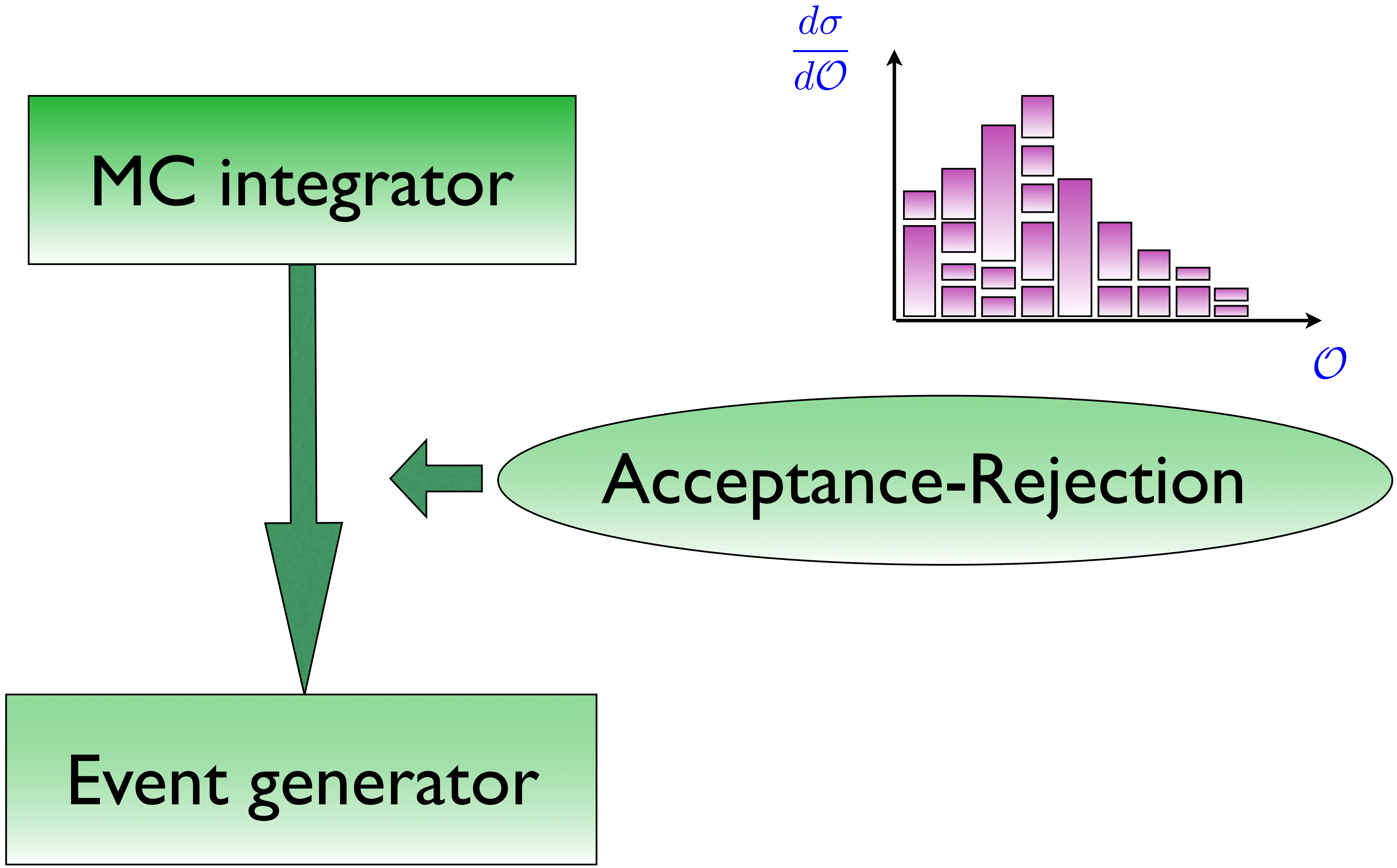
MC integrator



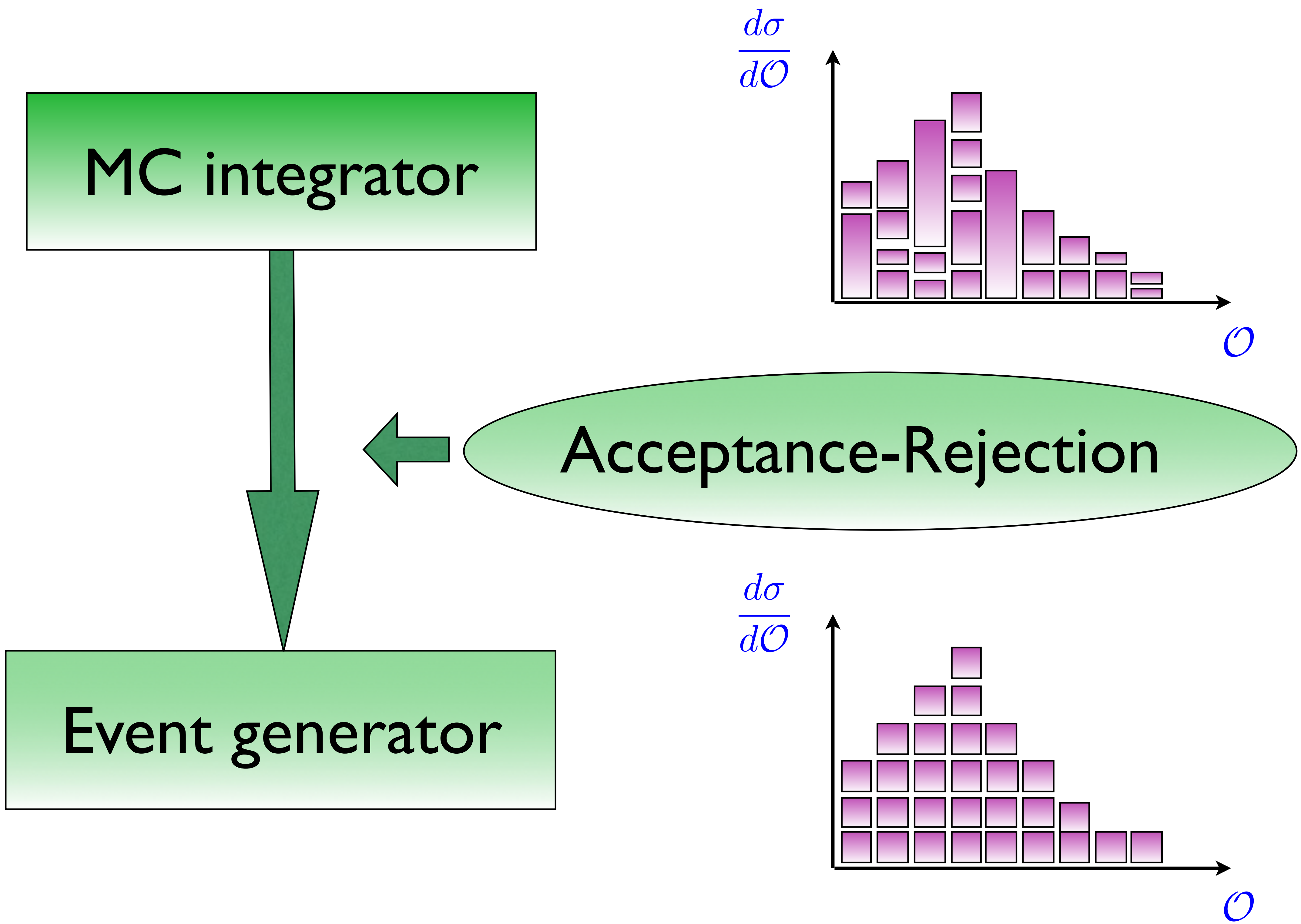
Event generation

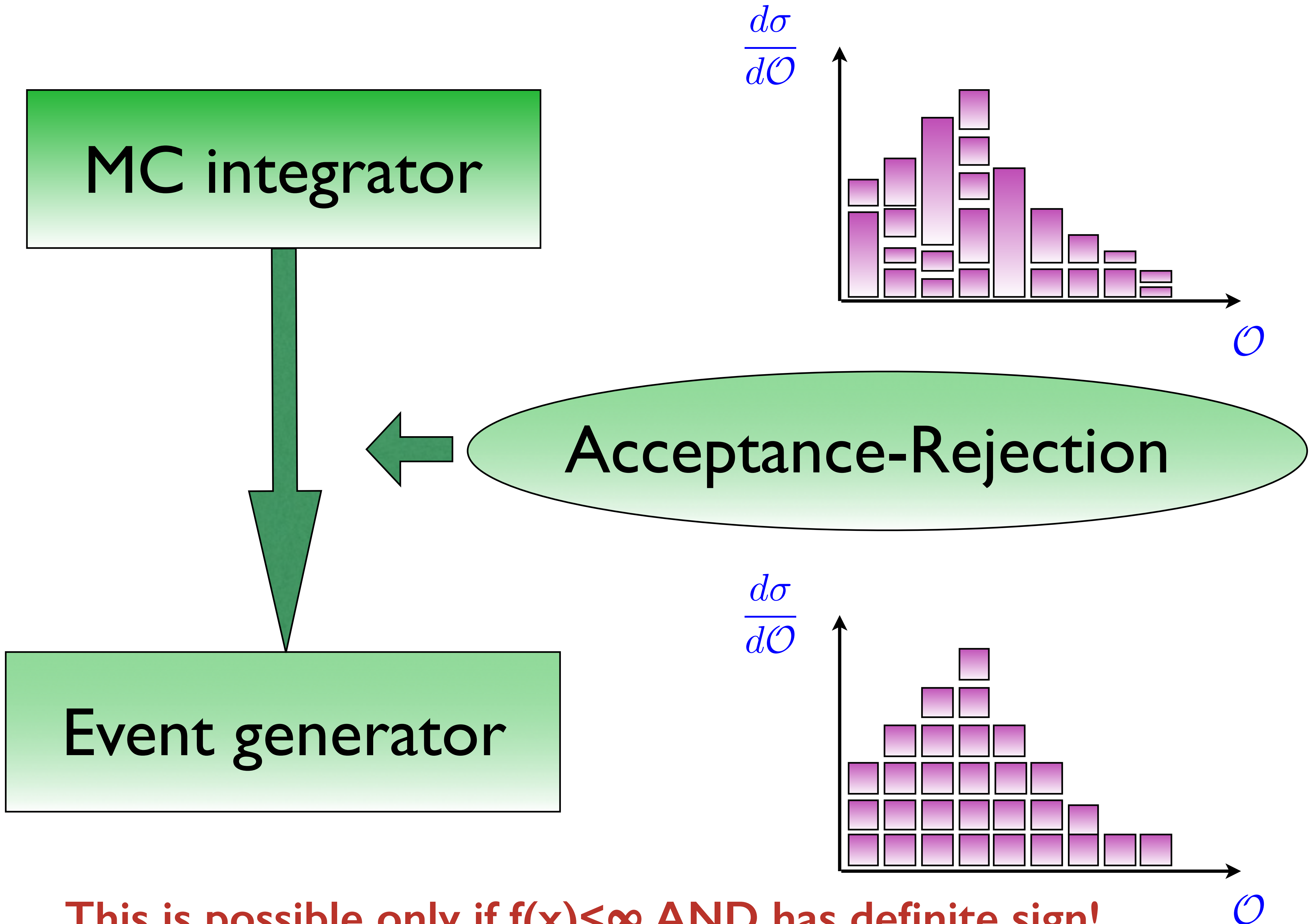


Event generation



Event generation





☞ **This is possible only if f(x)<∞ AND has definite sign!**

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension
- Need to know the function
  - Impact on cut

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension
- Need to know the function
  - Impact on cut

Good Point

- Complex area of Integration
- Easy Error estimate
- quick estimation of the integral
- Possibility to have unweighted events

What have we learned!

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral | Parton density functions | Parton-level cross section

- The Importance of PDF
  - ➡ Defines the physics
- Evaluation of Matrix Element
  - ➡ Numerical method faster than analytical formula
  - ➡ Use of Helicity Amplitude
- Phase Space Integration
  - ➡ Need to know in advance what we integrate. Be careful with strong cuts!