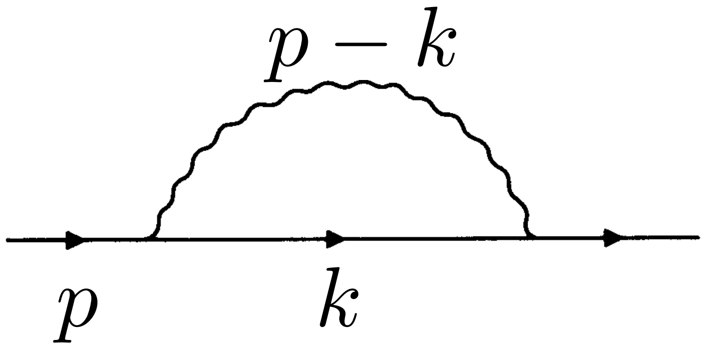
Next-to-Leading Order with FeynRules

Celine Degrande (IPPP, Durham University)
Lund 2014

Plan

- Renormalization
- KLN Theorem
- Rational Terms
- FeynRules at NLO

Electron self-energy



$$= (-ie)^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \frac{i\left(\not{k} + m\right)}{k^2 - m^2 + i\epsilon} \gamma^\nu \frac{-i\eta_{\mu\nu}}{(p-k)^2 + i\epsilon}$$

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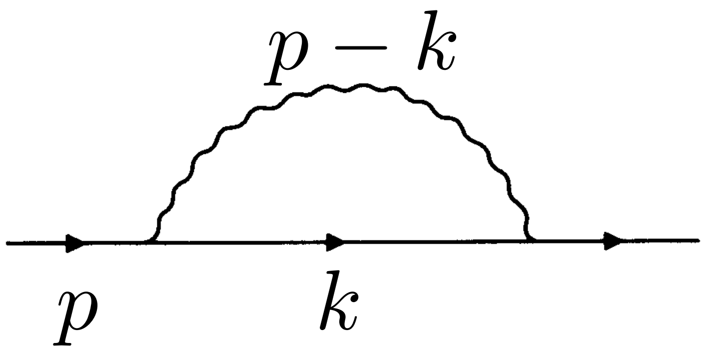

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Feynman parameter : $$\frac{1}{AB} = \int_0^1 dx \frac{1}{(Ax+B(1-x))}$$

$$= -e^2 \int_0^1 dx \int \frac{d^4l}{(2\pi)^4} \frac{\gamma^\mu \left(\not{l}+x\not{p}+m\right)\gamma_\mu}{\left(l^2-\Delta+i\epsilon\right)^2} \longleftarrow \begin{cases} l \equiv k - xp \\ \Delta = (1-x)(m^2 - xp^2) \end{cases}$$

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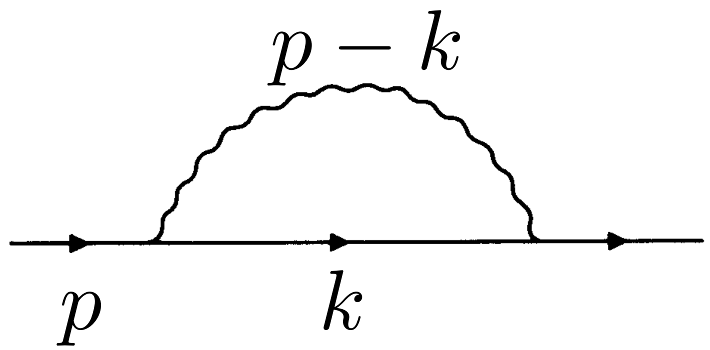

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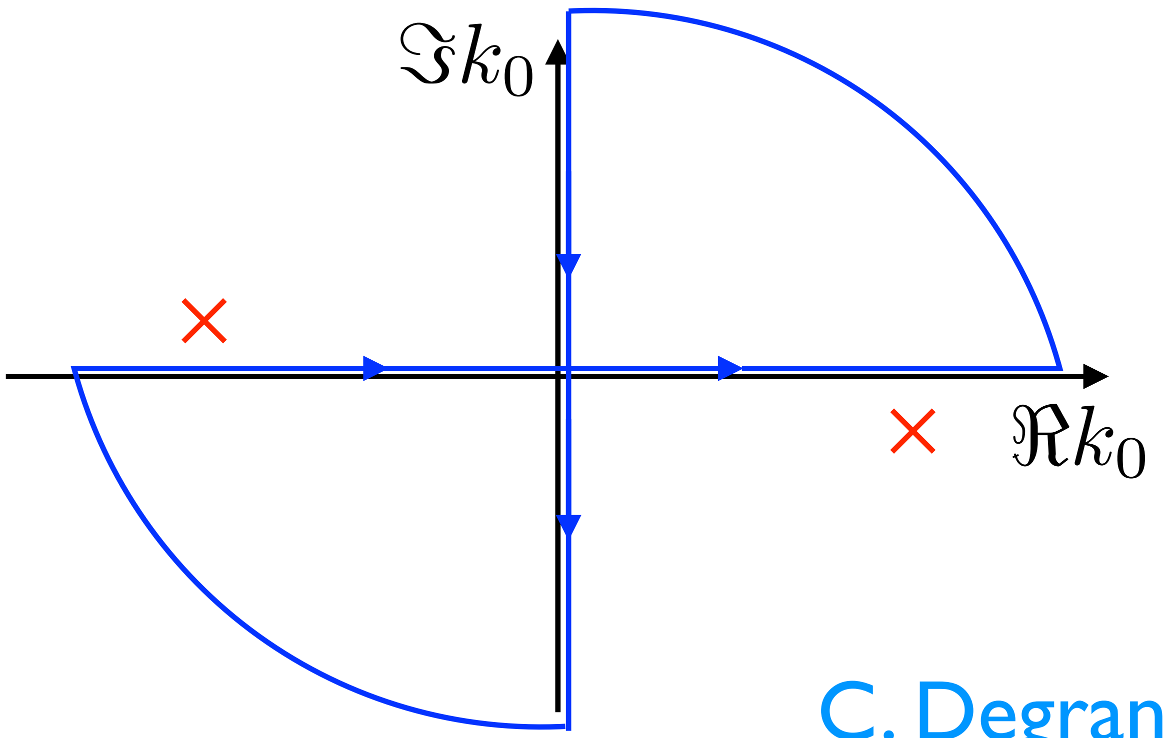
Feynman parameter : $\displaystyle \frac{1}{AB} = \int_0^1 dx \frac{1}{(Ax + B(1-x))}$

$$= -e^2 \int_0^1 dx \int \frac{d^4l}{(2\pi)^4}\frac{\gamma^\mu\left(\not l + x\not p + m\right)\gamma_\mu}{\left(l^2 - \Delta + i\epsilon\right)^2} \longleftarrow \left\{ \begin{array}{l} l \equiv k - xp \\ \Delta = (1-x)(m^2 - xp^2) \end{array} \right.$$

Minkowski to Euclidian space

$$\int_{-\infty}^{+\infty} dl_0 = -\int_{i\infty}^{-i\infty} dl_0 \underset{\uparrow}{=} i\int_{-\infty}^{+\infty} dl_0^E$$

$$\boxed{l_0^E \equiv -il_0, \quad \mathbf{1}^E \equiv \mathbf{1}}$$



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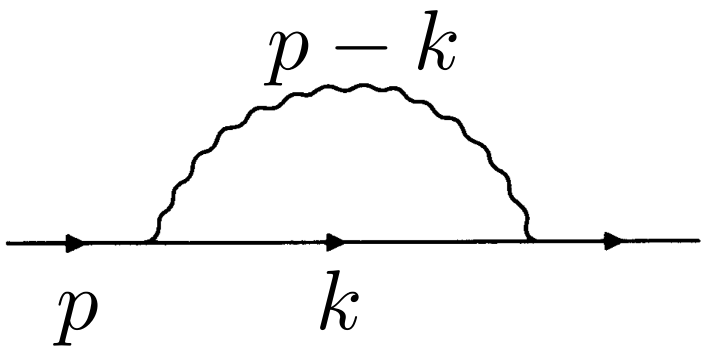
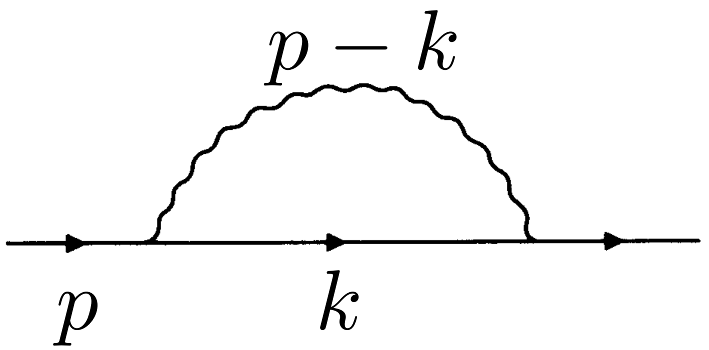
Electron self-energy



$$= -ie^2 \int_0^1 dx \int \frac{d^4 l^E}{(2\pi)^4} \frac{\gamma^\mu \left(\cancelto{0}{\not{l}^{E}} + x\not{p} + m\right) \gamma_\mu}{\left((l^E)^2 + \Delta - i\epsilon\right)^2}$$

$$\boxed{l^2 = -l_0^{E\,2} - \mathbf{l}^{E\,2} = -l^{E\,2}}$$

Electron self-energy



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$$\uparrow$$

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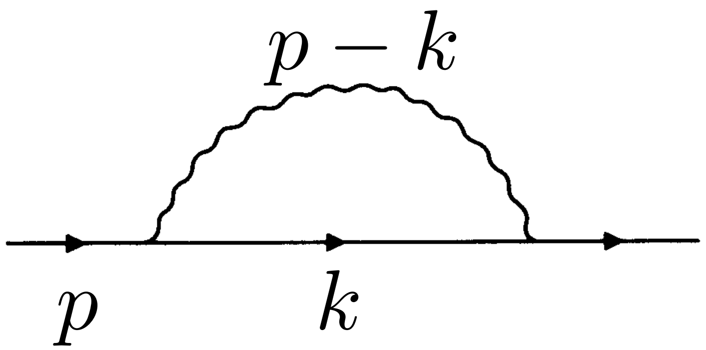
Dimensional regularization

$$= -ie^2 \int_0^1 dx \gamma^\mu \left(x\not{p} + m\right) \gamma_\mu \int \frac{d^d l^E}{(2\pi)^d} \frac{1}{\left((l^E)^2 + \Delta\right)^2}$$

Electron self-energy



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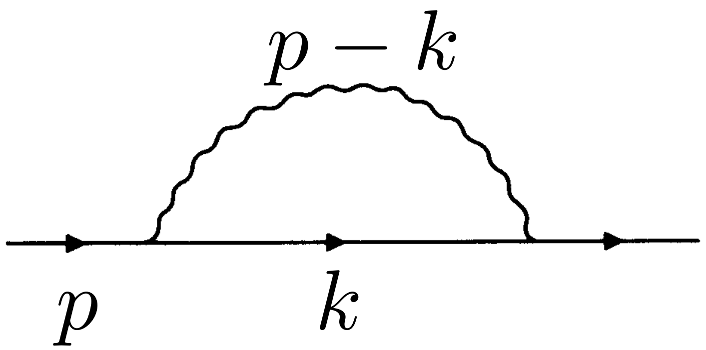

$$= -ie^2 \int_0^1 dx \int \frac{d^4 l^E}{(2\pi)^4} \frac{\gamma^\mu \left(\overbrace{\not{l}^E}^{0} + x\not{p} + m\right)\gamma_\mu}{\left((l^E)^2 + \Delta - i\epsilon\right)^2}$$

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$$\boxed{\int \frac{d\Omega_d}{(2\pi)^d}\frac{1}{2}} \int_0^\infty \frac{dl^{E\,2} \left(l^{E\,2}\right)^{d/2-1}}{\left(l^{E\,2} + \Delta\right)^2} = \boxed{\frac{1}{(4\pi)^{d/2}\Gamma\left(d/2\right)}} \left(\frac{1}{\Delta}\right)^{2-d/2} \int_0^1 dz z^{1-d/2}(1-z)^{d/2-1}$$

$$\boxed{z \equiv \frac{\Delta}{l^{E\,2} + \Delta}}$$

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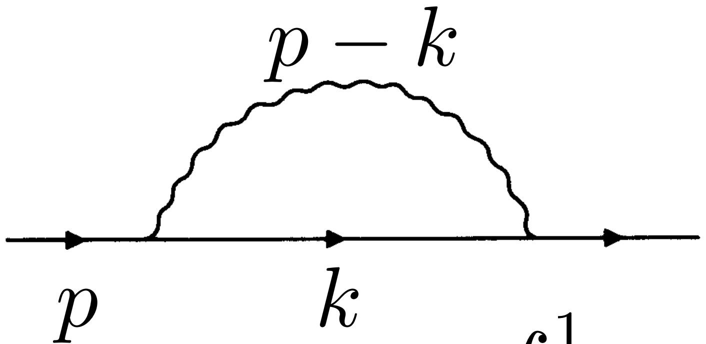
Electron self-energy

$$B(a,b) \equiv \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \equiv \int dz z^{a-1}(1-z)^{b-1}$$



$$= -ie^2 \int_0^1 dx \gamma^\mu \left(x\not{p} + m\right) \gamma_\mu \frac{1}{(4\pi)^{d/2}} \frac{\Gamma\left(2-d/2\right)}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{2-d/2}$$

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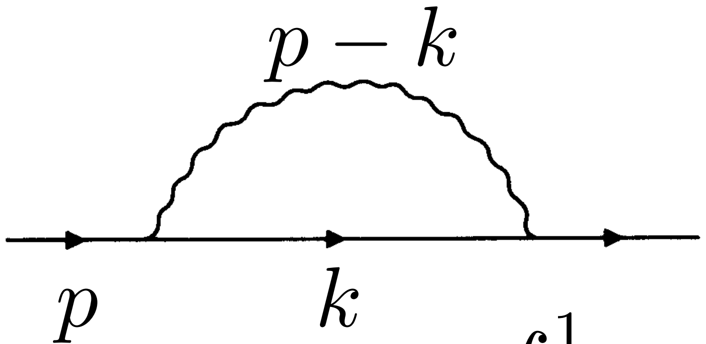

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$$\Big\downarrow \; d \equiv 4 - 2\epsilon$$

$$\frac{1}{(4\pi)^2} \left(\frac{1}{\epsilon} - \gamma + \log(4\pi) - \log\Delta\right)$$

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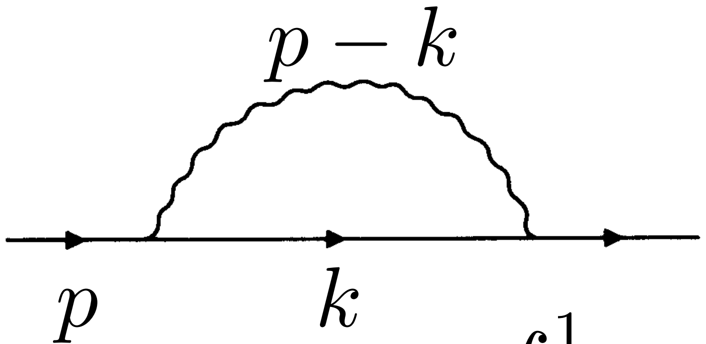

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$$\frac{1}{\bar{\epsilon}}$$

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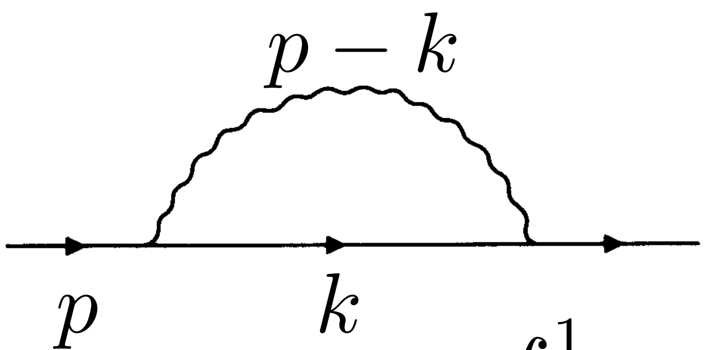

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$$= -i \frac{e^2}{(4\pi)^2} \frac{1}{\bar{\epsilon}} \left(-\not{p} + 4m\right) + \text{finite}$$

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Renormalization

$$\begin{aligned} x_0 &\rightarrow x + \delta x, \\ \phi_0 &\rightarrow (1 + \frac{1}{2}\delta Z_{\phi\phi})\phi + \sum_{\chi} \frac{1}{2}\delta Z_{\phi\chi}\chi \end{aligned}$$

$$\mathcal{L}_{Dirac} \rightarrow \bar{\psi}\left(\not{\partial} - m\right)\left(1 + \delta Z_{\psi\psi}\right)\psi - \delta m \bar{\psi}\psi$$

Renormalization

$$x_0 \quad \to \quad x + \boxed{\delta x},$$

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real constants

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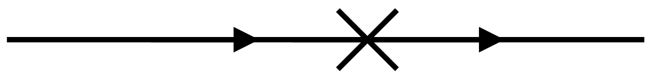

$$i\left(\not p - m\right)\delta Z_{\psi\psi} - i\delta m$$

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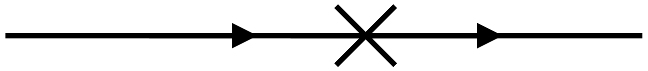

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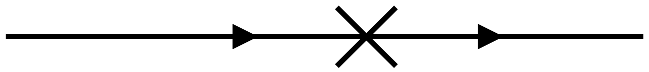

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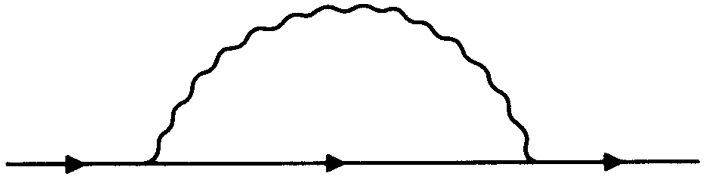

=0 $\frac{1}{\bar{\epsilon}}$ +finite

$$\delta Z_{\psi\psi} = -\frac{e^2}{(4\pi)^2}\frac{1}{\bar{\epsilon}} + \ldots$$

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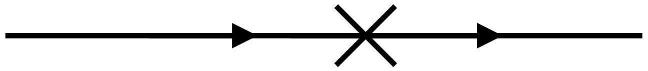

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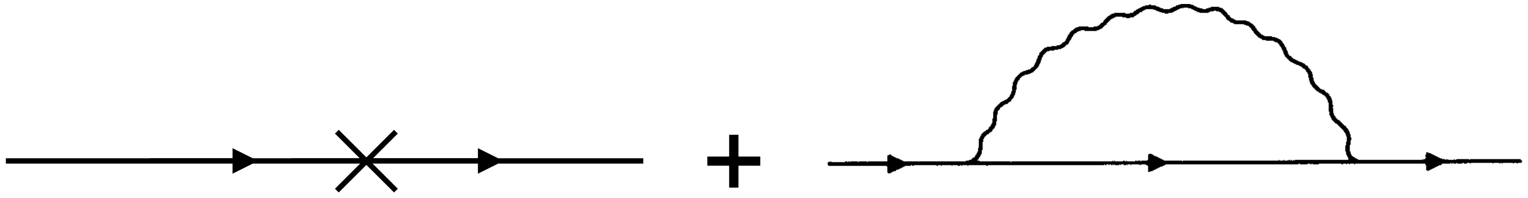

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Arbitrary:
Renormalization
scheme

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Operator dimension

- Fermion fields : 3/2
- Boson fields : 1
- derivatives : 1

Dimension 4

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2\right) \qquad \mathcal{L} = \bar{\psi}(x)\left[i\not{\partial} - m\right]\psi(x)$$

$$D_\mu = \partial_\mu - ieQA_\mu$$

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Definition : dimension of the operator is the sum of the dimensions of its fields and derivatives

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Dimension 4

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d=2

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Definition : dimension of the operator is the sum of the dimensions of its fields and derivatives

Exclude the dimension of the coefficient!

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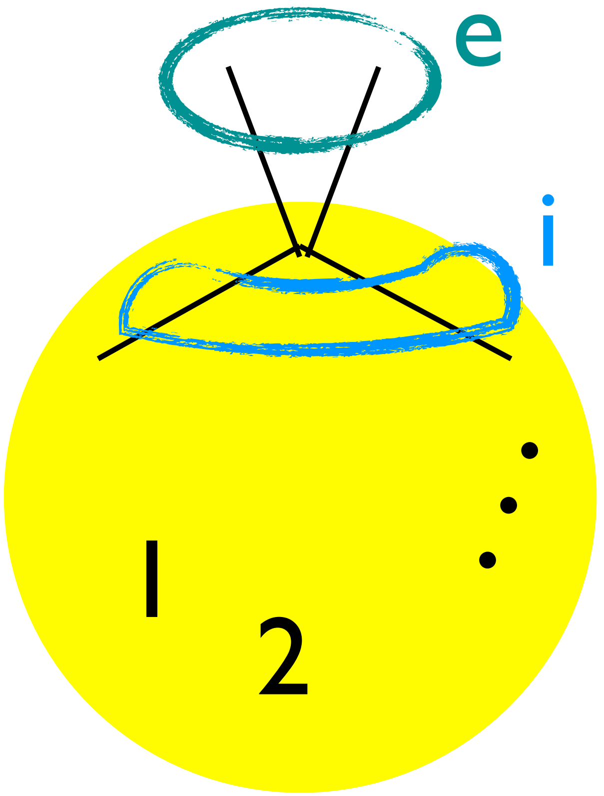
Divergent amplitudes

Each vertex: p^{d-e-i}



Divergent amplitudes

Each vertex: p^{d-e-i}

One loop: V propagators

Each internal fermion propagator: p^{-1}

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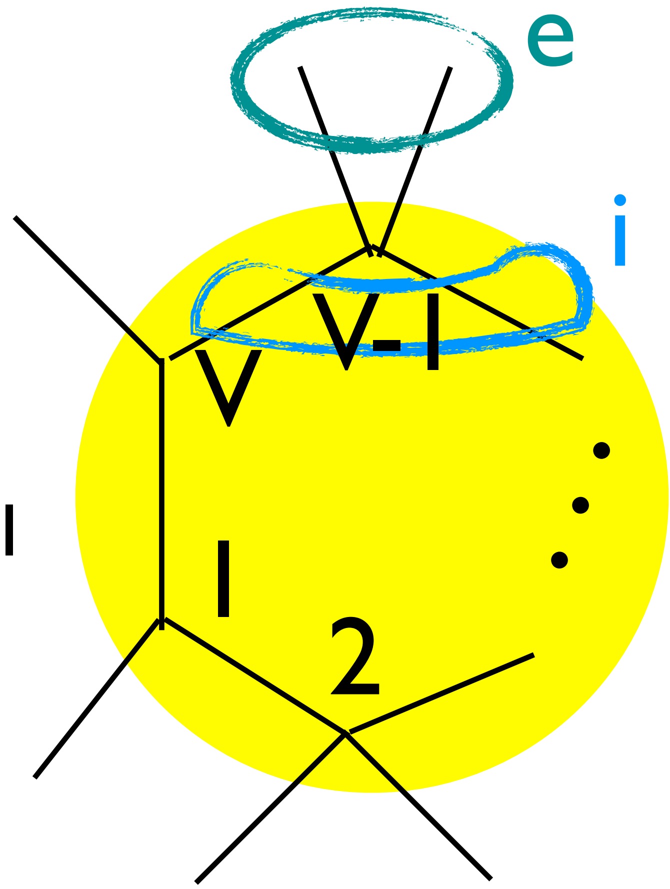

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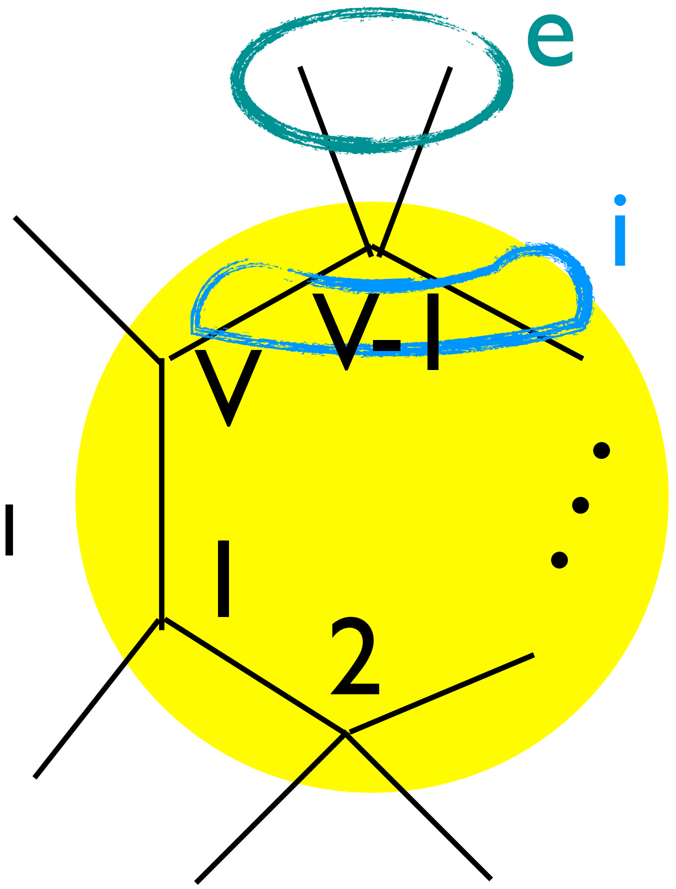

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$I = \sum i = 3F+2B$ (F/B # of internal fermions/bosons)

$F+B=V$

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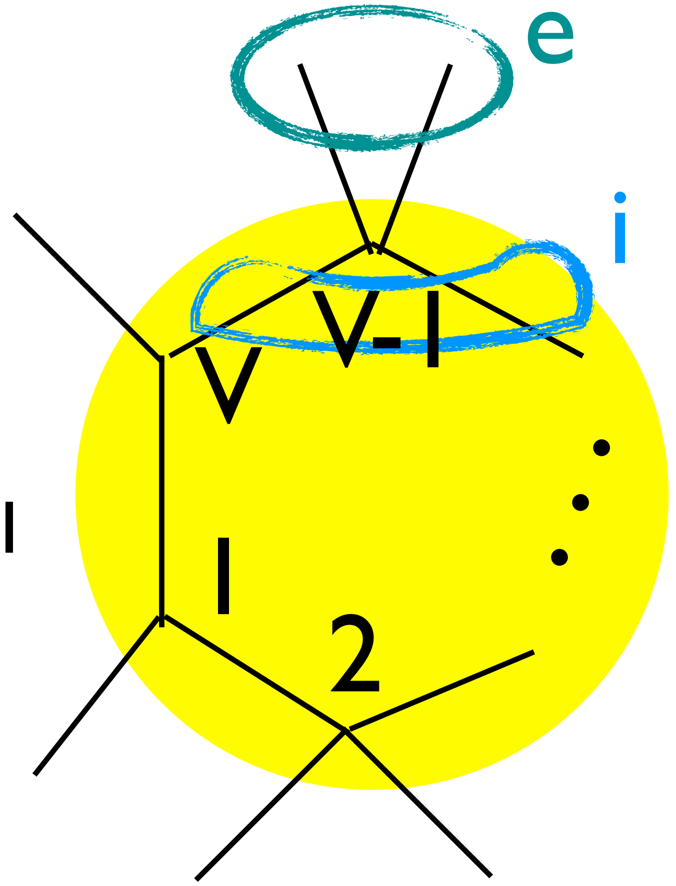

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Each extra loop: one extra propagator

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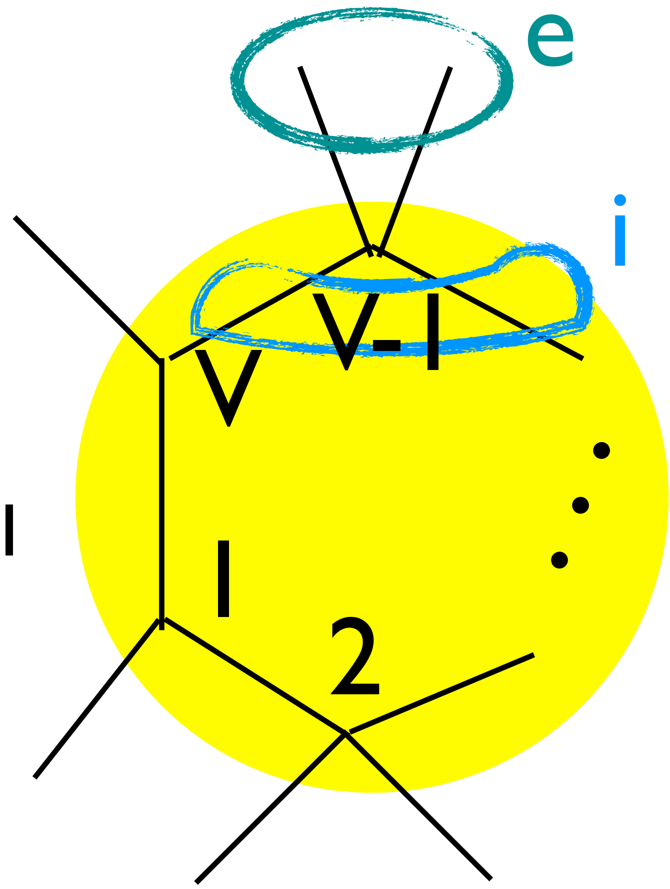

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$$A(D,E,L,V) \sim \left(\int d^4p\right)^L p^{D-E-I-2B-F} \sim \left(\int d^4p\right)^L p^{D-E-4V-4(L-1)}$$

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Renormalization

$$A(D,E,L,V) \sim \left(\int d^4p\right)^L p^{D-E-4V-4(L-1)}$$

Renormalizable model if d ≤ 4 for all operators

D ≤ 4V : A~$\Lambda^{4\text{-}E}$ diverges only if E ≤4

- All the divergence are absorbed by the fields and external parameters redefinitions
- Yang-mills theories
- With spontaneous symmetry breaking

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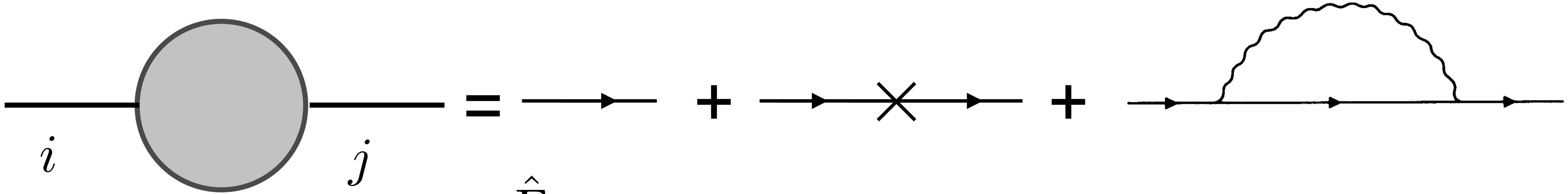
On-shell scheme

Renormalized mass = Physical mass

Two-point function vanishes on-shell (No loop corrections on the external legs)

Cancel the mixings



$$= \hat{\Gamma}_{ij}$$

$$\widetilde{\Re}\ \hat{\Gamma}^{f}_{ij}(p)u_j(p)\Big|_{p^2=m^2_{f,j}} = 0,$$

$$\lim_{p^2\to m^2_{f,i}} \frac{\not{p} + m_{f,i}}{p^2 - m^2_{f,i}}\, \widetilde{\Re}\ \hat{\Gamma}^{f}_{ii}(p)u_i(p) = iu_i(p),$$

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Renormalization conditions

$$\widetilde{\Re}\ \bar{u}_i(p')\hat{\Gamma}^f_{ij}(p')\Big|_{p'^2=m^2_{f,i}} = 0,$$

$$\lim_{p'^2\to m^2_{f,i}} \bar{u}_i(p')\ \widetilde{\Re}\,\hat{\Gamma}^f_{ii}(p')\frac{\not{p}'+m_{f,i}}{p'^2-m^2_{f,i}} = i\bar{u}_i(p').$$

$$\hat{\Gamma}_{ij} = \ i\delta_{ij}\left(\not{p}-m_i\right) + i\left[f^L_{ij}\left(p^2\right)\not{p}\gamma_- + f^R_{ij}\left(p^2\right)\not{p}\gamma_+ + f^{SL}_{ij}\left(p^2\right)\gamma_- + f^{SR}_{ij}\left(p^2\right)\gamma_+\right]$$

$$\begin{aligned}
\tilde{\Re}\left[f^L_{ij}\left(p^2\right)m_i + f^{SR}_{ij}\left(p^2\right)\right]\Big|_{p^2=m_i^2} &= 0\\
\tilde{\Re}\left[f^R_{ij}\left(p^2\right)m_i + f^{SL}_{ij}\left(p^2\right)\right]\Big|_{p^2=m_i^2} &= 0\\
\tilde{\Re}\left[2m_i\frac{\partial}{\partial p^2}\left[\left(f^L_{ii}\left(p^2\right)+f^R_{ii}\left(p^2\right)\right)m_i + f^{SL}_{ii}\left(p^2\right) + f^{SR}_{ii}\left(p^2\right)\right] + f^L_{ii}\left(p^2\right) + f^R_{ii}\left(p^2\right)\right]\Big|_{p^2=m_i^2} &= 0
\end{aligned}$$

Similar for the vectors and scalars

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Electron renormalization

$$f^L(p^2) = f^R(p^2) = \delta Z_{\psi\psi} - e^2 \int_0^1 dx(2-d)x \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(\epsilon)}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{\epsilon}$$

$$f^{SL}(p^2) = f^{SR}(p^2) = -m\delta Z_{\psi\psi} - \delta m - e^2 \int_0^1 dx \frac{m\, d}{(4\pi)^{d/2}} \frac{\Gamma(\epsilon)}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{\epsilon}$$

Electron renormalization

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Gamma algebra in d dimension!

Electron renormalization

$$f^L(p^2) = f^R(p^2) = \delta Z_{\psi\psi} - e^2 \int_0^1 dx(2-d)x \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(\epsilon)}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{\epsilon}$$

$$f^{SL}(p^2) = f^{SR}(p^2) = -m\delta Z_{\psi\psi} - \delta m - e^2 \int_0^1 dx \frac{m\, d}{(4\pi)^{d/2}} \frac{\Gamma(\epsilon)}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{\epsilon}$$

$$\delta m = -\frac{e^2}{(4\pi)^2} \left(\frac{3}{\bar{\epsilon}} + 4 - 6\log\left(\frac{m}{\mu}\right)\right)$$

Eqs 1 and 2

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Electron renormalization

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Eqs 1 and 2

$$\Delta = (1-x)(m^2 - xp^2)$$

$$\left.\frac{\partial \Delta^{-\epsilon}}{\partial p^2}\right|_{p^2=m^2} = \epsilon x(1-x)\Delta^{-\epsilon-1}\big|_{p^2=m^2} = \epsilon x(1-x)^{-1-2\epsilon} m^{-2-2\epsilon}$$

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Electron renormalization

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After integration over x

$$\frac{\epsilon\Gamma(2)\Gamma(-2\epsilon)}{\Gamma(2-2\epsilon)}$$

$$\delta Z_{\psi\psi} = -\frac{e^2}{(4\pi)^2}\left(\frac{1}{\bar{\epsilon}_{UV}} + \boxed{\frac{2}{\bar{\epsilon}_{IR}}} + 4 - 6\log\left(\frac{m}{\mu}\right)\right)$$

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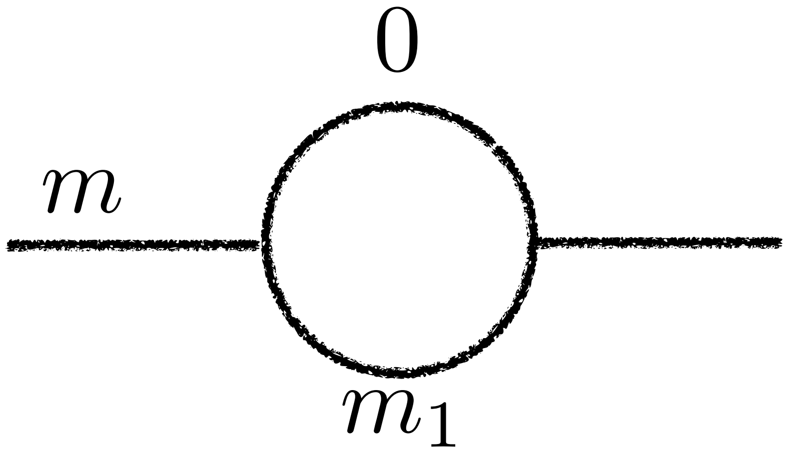
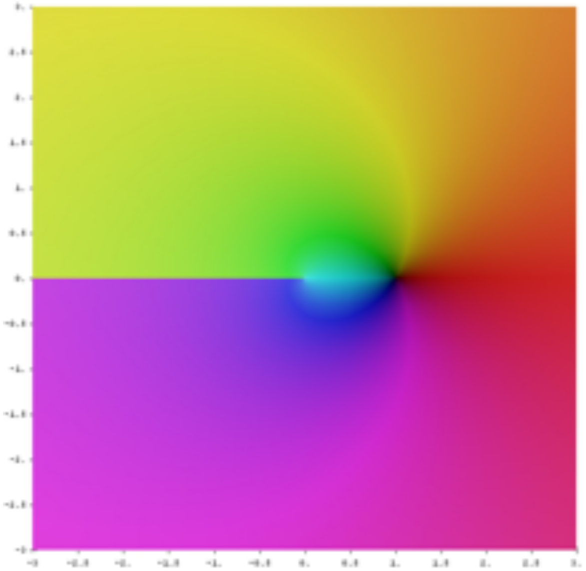
Real/Complex masses



Real masses

$$m_1^2 > m^2 \qquad \Re\left(\log\left[m_1^2 - p^2\right]\right)\Big|_{p^2=m^2}$$

$$m_1^2 < m^2 \qquad \Re\left(\log\left[p^2 - m_1^2\right] + i\pi\right)\Big|_{p^2=m^2}$$



Epsilon prescription of the propagator

Mass corrections are complex if the particle can decay

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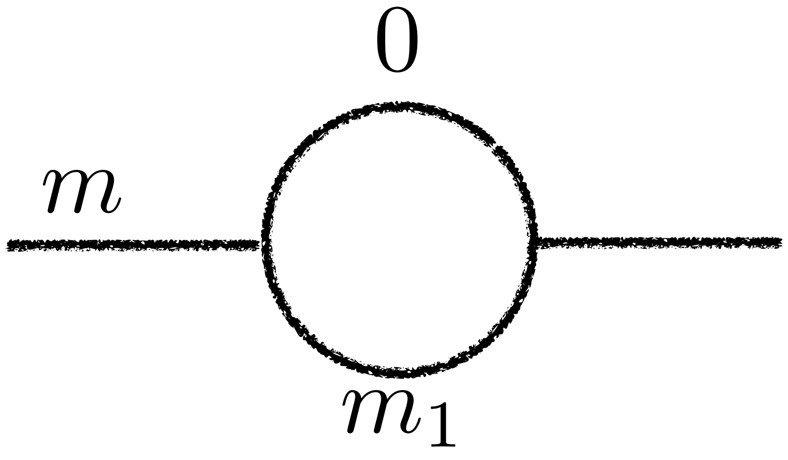
Real/Complex masses



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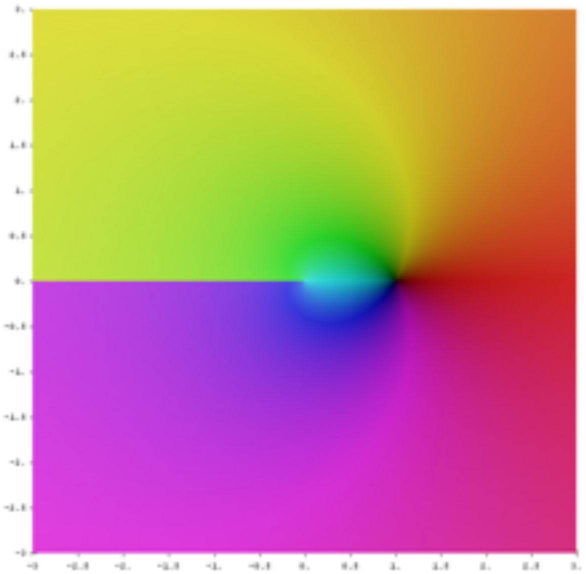
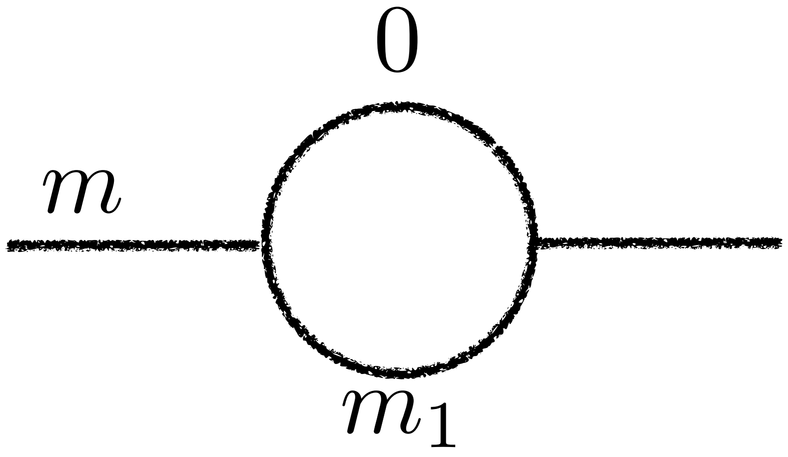


$m \in \mathbb{R}$

Mass corrections are complex if the particle can decay

Epsilon prescription of the propagator

Real/Complex masses



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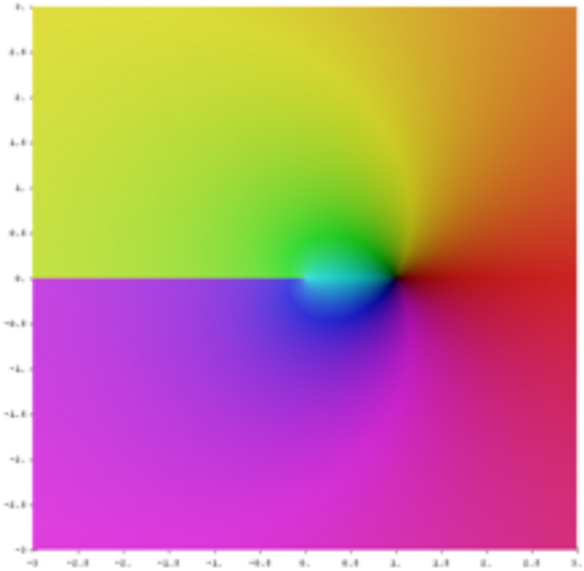
Mass corrections are complex if the particle can decay



$m \in \mathbb{R}$

Epsilon prescription of the propagator

Complex masses

$$\log\left[m_1^2 - p^2\right]\Big|_{p^2=m^2} \qquad\qquad m^2 \to m^2 - im\Gamma$$

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Complex mass scheme

$$\left.\begin{array}{r}\phi_0 \to (1+\frac{1}{2}\delta Z_{\phi\phi})\phi \\ \phi_0^\dagger \to (1+\frac{1}{2}\delta Z_{\phi\phi}^*)\phi^\dagger\end{array}\right\} \Rightarrow \partial^\mu\phi_0\partial_\mu\phi_0^\dagger \to (1+\Re\delta Z_{\phi\phi})\partial^\mu\phi\partial_\mu\phi^\dagger$$

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$$m \to m + \delta m$$

Now with complex masses

Complex mass scheme

$$\cancel{\left.\begin{array}{r}\phi_0 \to (1+\frac{1}{2}\delta Z_{\phi\phi})\phi \\ \phi_0^\dagger \to (1+\frac{1}{2}\delta Z^*_{\phi\phi})\phi^\dagger\end{array}\right\} \Rightarrow \partial^\mu\phi_0\partial_\mu\phi_0^\dagger \to (1+\Re\delta Z_{\phi\phi})\partial^\mu\phi\partial_\mu\phi^\dagger}$$

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$m \to m + \delta m$

Now with complex masses

Hermitian → $\mathcal{L}_0$

$$\mathcal{L}_0 = \mathcal{L} + \delta\mathcal{L}$$

Not hermitian → $\mathcal{L}$

C. Degrande

Complex mass scheme

$$\cancel{\left.\begin{array}{r}\phi_0 \to (1+\frac{1}{2}\delta Z_{\phi\phi})\phi \\ \phi_0^\dagger \to (1+\frac{1}{2}\delta Z^*_{\phi\phi})\phi^\dagger\end{array}\right\} \Rightarrow \partial^\mu\phi_0\partial_\mu\phi_0^\dagger \to (1+\Re\delta Z_{\phi\phi})\partial^\mu\phi\partial_\mu\phi^\dagger}$$

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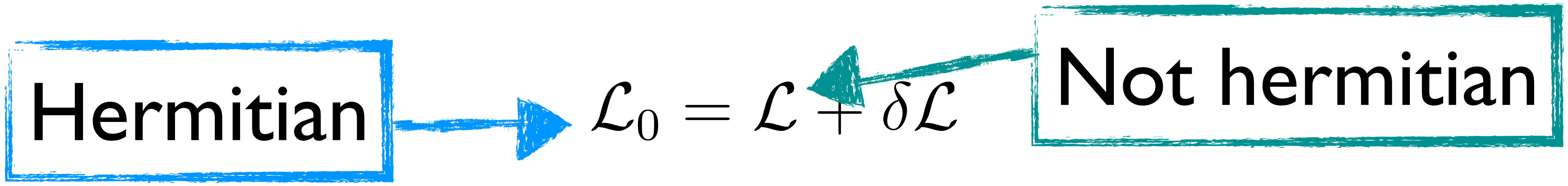
Now with complex masses



Hermitian

$$\mathcal{L}_0 = \mathcal{L} + \delta\mathcal{L}$$

Not hermitian

$$\cancel{\widetilde{\Re}}\left[f_{ij}^{L}\left(p^2\right)m_i + f_{ij}^{SR}\left(p^2\right)\right]\Big|_{p^2=m_i^2} = 0$$

$$\cancel{\widetilde{\Re}}\left[f_{ij}^{R}\left(p^2\right)m_i + f_{ij}^{SL}\left(p^2\right)\right]\Big|_{p^2=m_i^2} = 0$$

$$\cancel{\widetilde{\Re}}\left[2m_i\frac{\partial}{\partial p^2}\left[\left(f_{ii}^{L}\left(p^2\right)+f_{ii}^{R}\left(p^2\right)\right)m_i + f_{ii}^{SL}\left(p^2\right)+f_{ii}^{SR}\left(p^2\right)\right] + f_{ii}^{L}\left(p^2\right)+f_{ii}^{R}\left(p^2\right)\right]\Big|_{p^2=m_i^2} = 0$$

Similar for the vectors and scalars

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Renormalization conditions

Zero momentum scheme available for the gauge couplings

$$\begin{aligned}\Gamma^{\mu}_{FFV}\left(p_1,p_2\right) \quad = \quad & igT^a\delta_{f_1,f_2}\left[\gamma^{\mu}\left(\frac{\delta g}{g}+\frac{1}{2}\delta Z_{VV}+\frac{1}{2}\delta Z^R_{FF}+\frac{1}{2}\delta Z^L_{FF}+\frac{g'_V}{2g}\delta Z_{V'V}\right)\right.\\ & +\gamma^{\mu}\gamma_5\left(\frac{1}{2}\delta Z^R_{FF}-\frac{1}{2}\delta Z^L_{FF}+\frac{g'_A}{2g}\delta Z_{V'V}\right)\\ & \left.+\left(\gamma^{\mu}h^V\left(k^2\right)+\gamma^{\mu}\gamma_5 h^A\left(k^2\right)+\frac{\left(p_1-p_2\right)^{\mu}}{2m}h^S\left(k^2\right)+\frac{k_{\mu}}{2m}h^P\left(k^2\right)\right)\right]\end{aligned}$$

$$\begin{aligned}\frac{\delta g}{g}+\frac{1}{2}\delta Z_{VV}+\frac{1}{2}\delta Z^R_{FF}+\frac{1}{2}\delta Z^L_{FF}+\frac{g'_V}{2g}\delta Z_{V'V}+h^V\left(0\right)+h^S\left(0\right) &= 0\\ \frac{1}{2}\delta Z^R_{FF}-\frac{1}{2}\delta Z^L_{FF}+\frac{g'_A}{2g}\delta Z_{V'V}+h^A\left(0\right) &= 0.\end{aligned}$$

By gauge invariance

$$\frac{\delta g}{g}+\frac{1}{2}\delta Z_{VV}+\frac{g'_V}{2g}\delta Z_{V'V}+\frac{g'_A}{2g}\delta Z_{V'V}=0$$

C. Degrande

Renormalization conditions

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& +\gamma^\mu\gamma_5\left(\frac{1}{2}\delta Z^R_{FF}-\frac{1}{2}\delta Z^L_{FF}+\frac{g'_A}{2g}\delta Z_{V'V}\right)\\
& \left.+\left(\gamma^\mu h^V\left(k^2\right)+\gamma^\mu\gamma_5 h^A\left(k^2\right)+\frac{\left(p_1-p_2\right)^\mu}{2m}h^S\left(k^2\right)+\frac{k_\mu}{2m}h^P\left(k^2\right)\right)\right]
\end{aligned}
$$

$$\Big\downarrow$$

$$
\begin{aligned}
\frac{\delta g}{g}+\frac{1}{2}\delta Z_{VV}+\frac{1}{2}\delta Z^R_{FF}+\frac{1}{2}\delta Z^L_{FF}+\frac{g'_V}{2g}\delta Z_{V'V}+h^V\left(0\right)+h^S\left(0\right) \quad &= \quad 0\\
\frac{1}{2}\delta Z^R_{FF}-\frac{1}{2}\delta Z^L_{FF}+\frac{g'_A}{2g}\delta Z_{V'V}+h^A\left(0\right) \quad &= \quad 0.
\end{aligned}
$$

By gauge invariance

$$\Big\downarrow$$

$$\boxed{\frac{\delta g}{g}+\frac{1}{2}\delta Z_{VV}+\frac{g'_V}{2g}\delta Z_{V'V}+\frac{g'_A}{2g}\delta Z_{V'V}=0}$$

Only from two-point functions

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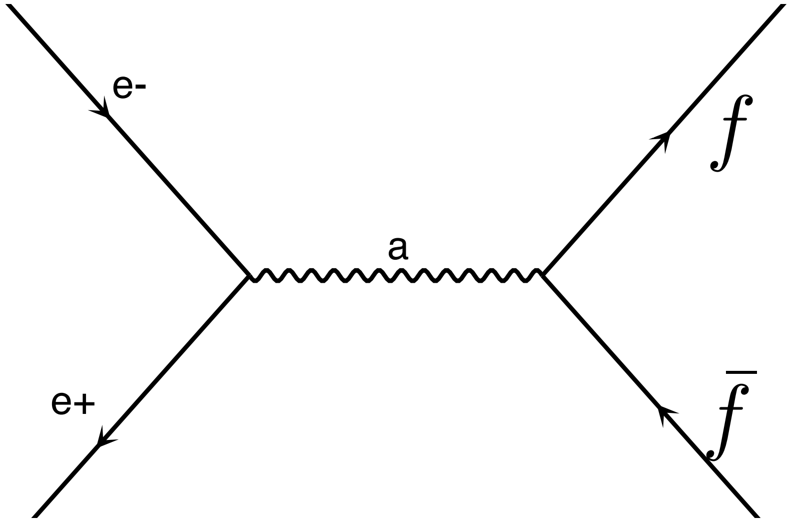
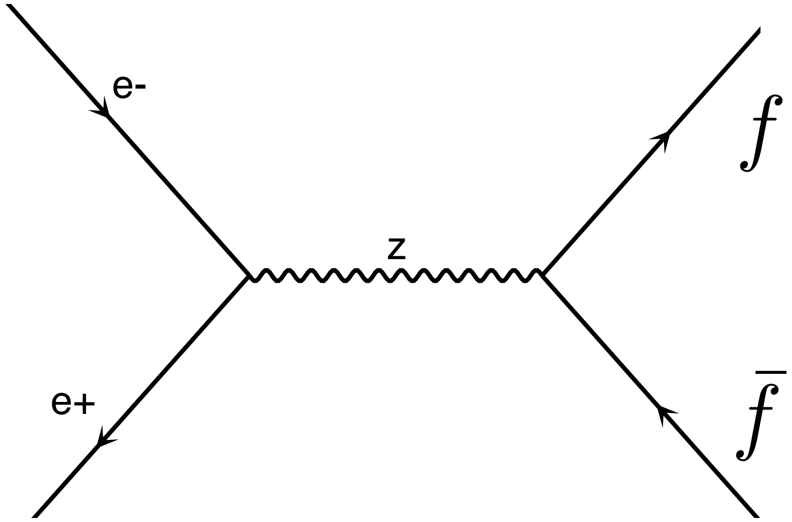
Plan

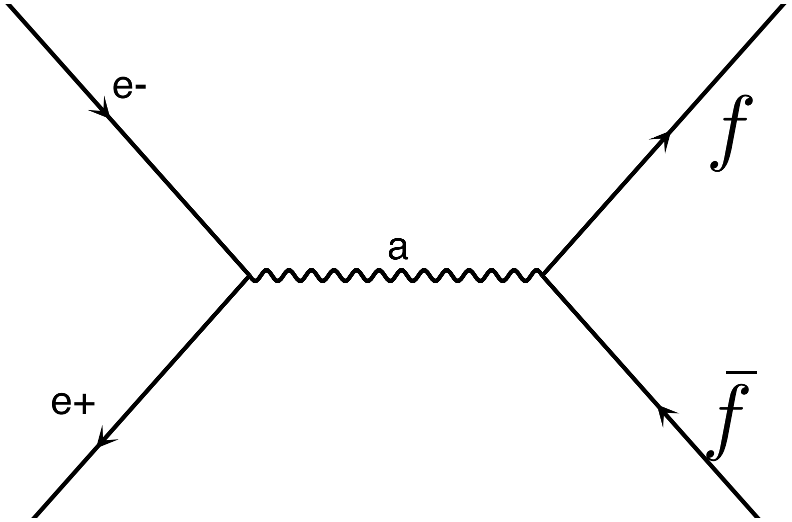
- Renormalization
- KLN Theorem
- Rational Terms
- FeynRules at NLO

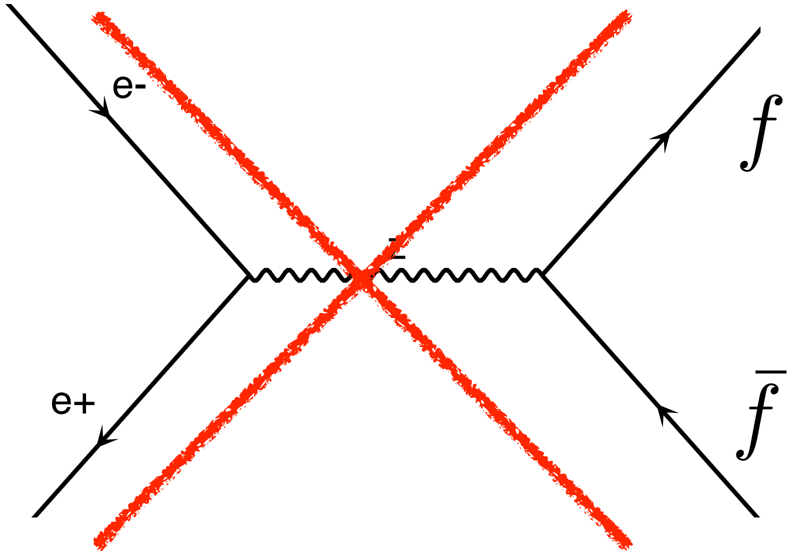
Infrared divergences





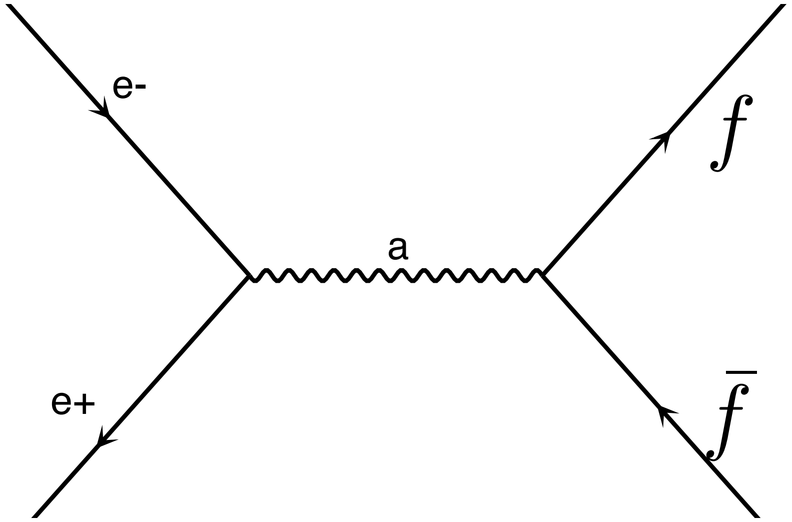
Infrared divergences

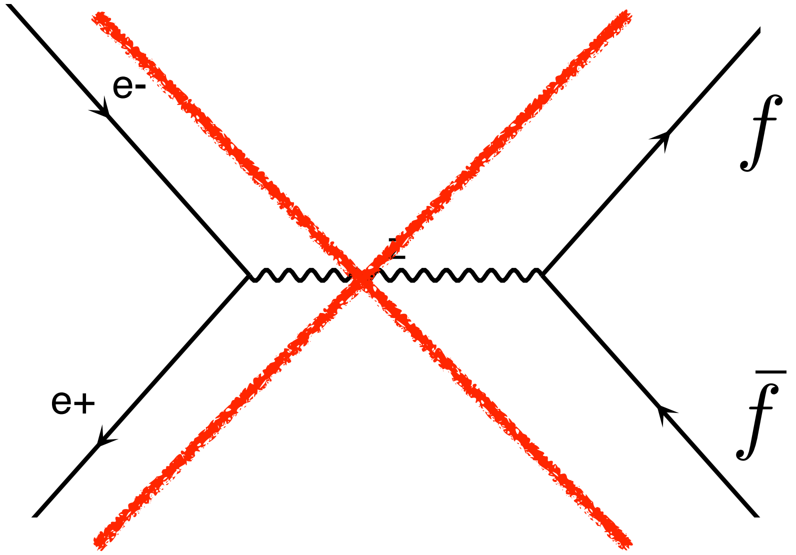




Well below the Z mass

Infrared divergences

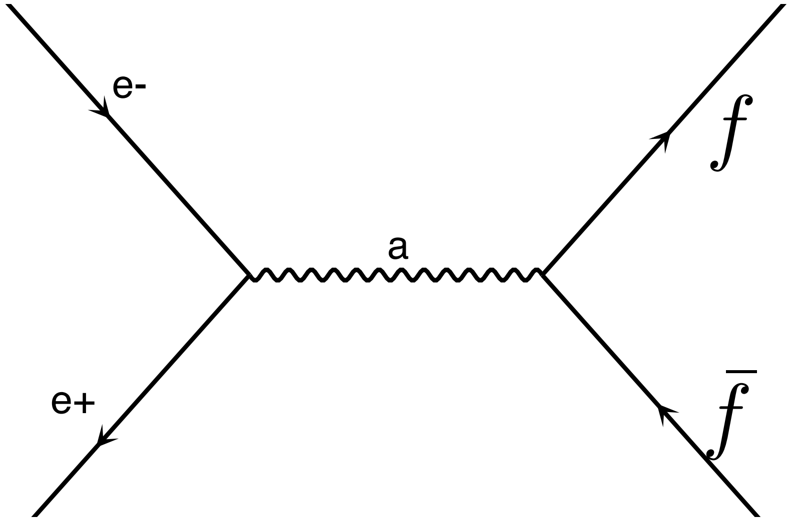


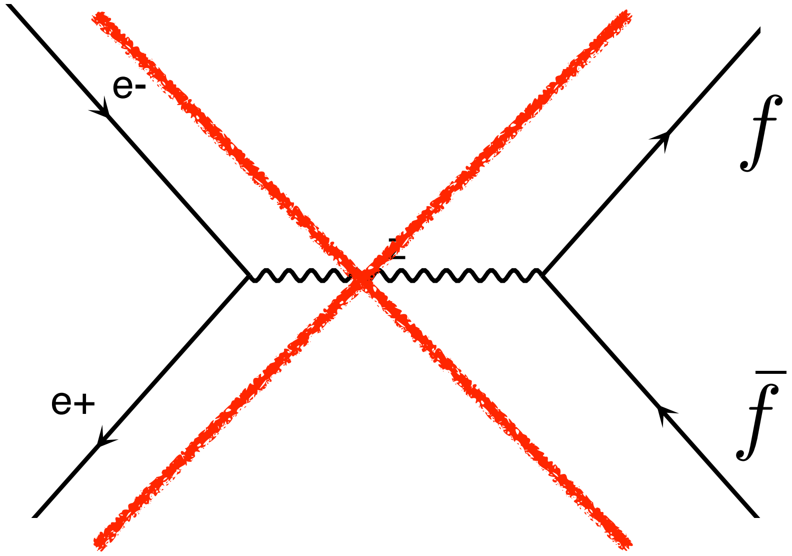


Well below the Z mass

$$\sigma_0 = \frac{4\pi\alpha^2}{3s}\, Q_f^2$$

Infrared divergences





Well below the Z mass

$$\sigma_0 = \frac{4\pi\alpha^2}{3s}\, Q_f^2$$

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sum_q \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q Q_q^2$$

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KLN illustration

Virtual corrections

$$\sigma^V = \sigma_0 \frac{2\alpha_S}{\pi} H(\epsilon) \left[-\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right]$$

where $$H(\epsilon) = \frac{3(1+\epsilon)}{(3+2\epsilon)\Gamma(2+2\epsilon)(4\pi)^{2\epsilon}} = 1 + \mathcal{O}(\epsilon)$$

Real corrections

$$\sigma^{q\bar{q}g} = \sigma_0 \frac{2\alpha_S}{\pi} H(\epsilon) \left[\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon) \right]$$

$$R \ = \ 3 \sum_q Q_q^2 \ \left\{ 1 + \frac{\alpha_S}{\pi} + \mathcal{O}(\alpha_S^2) \right\}$$

Pole cancellation provides a check

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Plan

- Renormalization
- KLN Theorem
- Rational Terms
- FeynRules at NLO

Loop computation

$$\begin{aligned}\mathcal{A}^{1-loop} &= \sum_i d_i \text{ Box}_i + \sum_i c_i \text{ Triangle}_i + \sum_i b_i \text{ Bubble}_i \\ &+ \sum_i a_i \text{ Tadpole}_i + R\end{aligned}$$

- Box, Triangle, Bubble and Tadpole are known scalar integrals
- Loop computation = find the coefficients
  - Unitarity
  - Multiple cuts
  - Tensor reduction (OPP)

R_2

$$\bar{A}(\bar{q}) = \frac{1}{(2\pi)^4}\int d^d\bar{q}\,\frac{\bar{N}(\bar{q})}{\bar{D}_0\bar{D}_1\ldots\bar{D}_{m-1}}, \qquad \bar{D}_i = (\bar{q}+p_i)^2 - m_i^2$$

$$\underbrace{\bar{N}(\bar{q})}_{d} = \underbrace{N(q)}_{4} + \underbrace{\tilde{N}(\tilde{q}, q, \epsilon)}_{\varepsilon}$$

$$R_2 \equiv \lim_{\epsilon\to 0}\frac{1}{(2\pi)^4}\int d^d\bar{q}\,\frac{\tilde{N}(\tilde{q}, q, \epsilon)}{\bar{D}_0\bar{D}_1\ldots\bar{D}_{m-1}}$$

Finite set of vertices that can be computed once for all

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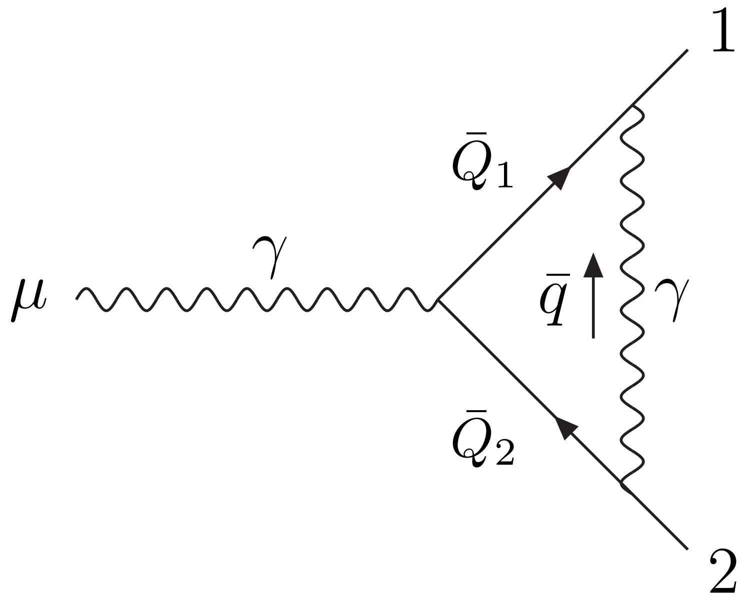
R_2 example



$$\bar{Q}_1 = \bar{q} + p_1 = Q_1 + \tilde{q}$$
$$\bar{Q}_2 = \bar{q} + p_2 = Q_2 + \tilde{q}$$

$$\bar{D}_0 = \bar{q}^2$$
$$\bar{D}_1 = (\bar{q} + p_1)^2$$
$$\bar{D}_2 = (\bar{q} + p_2)^2$$

'**t Hooft Veltman scheme**

$$\bar{\eta}^{\bar{\mu}\,\bar{\nu}}\bar{\eta}_{\bar{\mu}\,\bar{\nu}} = d,$$
$$\bar{\gamma}^{\bar{\mu}}\bar{\gamma}_{\bar{\mu}} = d\,\mathbb{1},$$

$$\begin{aligned} \bar{N}(\bar{q}) &\equiv e^3 \left\{ \bar{\gamma}_{\bar{\beta}} \, (\bar{\not{Q}}_1 + m_e) \, \gamma_\mu \, (\bar{\not{Q}}_2 + m_e) \, \bar{\gamma}^{\bar{\beta}} \right\} \\ &= e^3 \left\{ \gamma_\beta (\not{Q}_1 + m_e) \gamma_\mu (\not{Q}_2 + m_e) \gamma^\beta \right. \\ &- \left. \epsilon \, (\not{Q}_1 - m_e) \gamma_\mu (\not{Q}_2 - m_e) + \epsilon \tilde{q}^2 \, \gamma_\mu - \tilde{q}^2 \, \gamma_\beta \gamma_\mu \gamma^\beta \right\} \end{aligned}$$

$$\int d^n\bar{q} \frac{\tilde{q}^2}{\bar{D}_0\bar{D}_1\bar{D}_2} = -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon)\,,$$
$$\int d^n\bar{q} \frac{q_\mu q_\nu}{\bar{D}_0\bar{D}_1\bar{D}_2} = -\frac{i\pi^2}{2\epsilon} g_{\mu\nu} + \mathcal{O}(1)$$

$$\mathrm{R}_2 = -\frac{ie^3}{8\pi^2}\gamma_\mu$$

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R_1

Due to the ϵ dimensional parts of the denominators

Like for the 4 dimensional part but with a different set of integrals

$$\begin{aligned}
\int d^n\bar{q}\frac{\tilde{q}^2}{\bar{D}_i\bar{D}_j} &= -\frac{i\pi^2}{2}\left[m_i^2+m_j^2-\frac{(p_i-p_j)^2}{3}\right]+\mathcal{O}(\epsilon)\,,\\
\int d^n\bar{q}\frac{\tilde{q}^2}{\bar{D}_i\bar{D}_j\bar{D}_k} &= -\frac{i\pi^2}{2}+\mathcal{O}(\epsilon)\,,\\
\int d^n\bar{q}\frac{\tilde{q}^4}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_l} &= -\frac{i\pi^2}{6}+\mathcal{O}(\epsilon)\,.
\end{aligned}$$

Only R = R_1+R_2 is gauge invariant ⇨ Check

Plan

- Renormalization
- KLN Theorem
- Rational Terms
- FeynRules at NLO

FR@NLO

- Goal : Automate the one-loop computation for BSM models
- Required ingredients :
  - Tree-level vertices
  - R2 vertices (OPP)
  - UV counterterm vertices
- Solution : UFO at NLO

C. Degrande

FR@NLO

- Goal : Automate the one-loop computation for BSM models
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  - Tree-level vertices Done(FeynRules)
  - R2 vertices (OPP)
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C. Degrande

FR@NLO

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- Required ingredients :
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Done(FeynRules)

  - R2 vertices (OPP)
  - UV counterterm vertices

Missing

- Solution : UFO at NLO

C. Degrande

FR@NLO

- Goal : Automate the one-loop computation for BSM models
- Required ingredients :
    - Tree-level vertices

      Done(FeynRules)
    - R2 vertices (OPP)
    - UV counterterm vertices

      Missing
- Solution : UFO at NLO

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UV counterterm vertices

External parameters

$$\begin{aligned} x_0 &\to x + \delta x, \\ \phi_0 &\to (1 + \frac{1}{2}\delta Z_{\phi\phi})\phi + \sum_{\chi} \frac{1}{2}\delta Z_{\phi\chi}\chi \end{aligned}$$

Same for the conjugate field

One renormalization constant for each fermion chirality

Internal parameters are renormalised by replacing the external parameters in their expressions

$\mathcal{L}_0 = \mathcal{L} + \delta\mathcal{L}$ ← vertices after solving the reno. cond.

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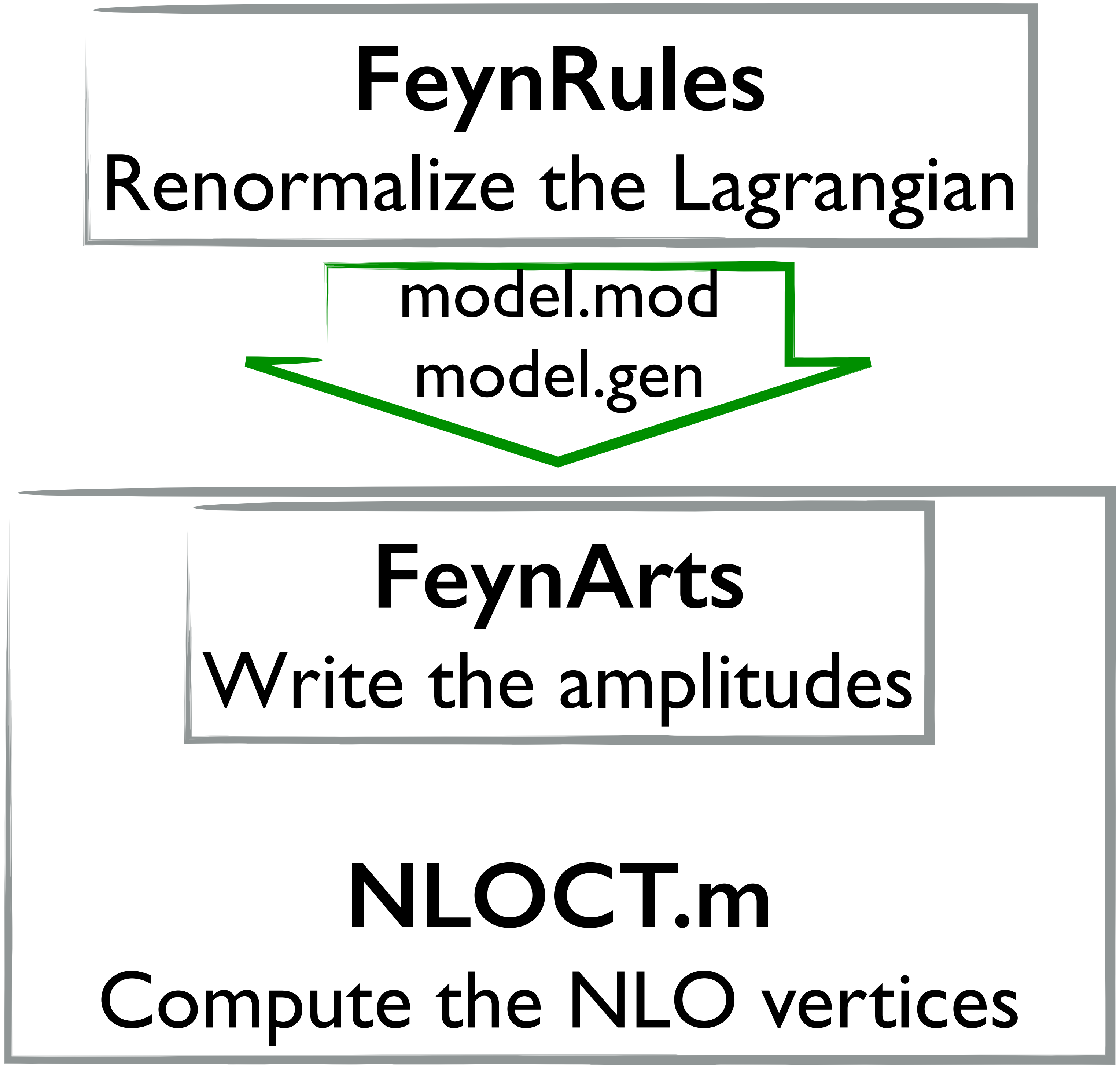
How does it work?



FeynRules

Renormalize the Lagrangian

model.mod
model.gen

FeynArts

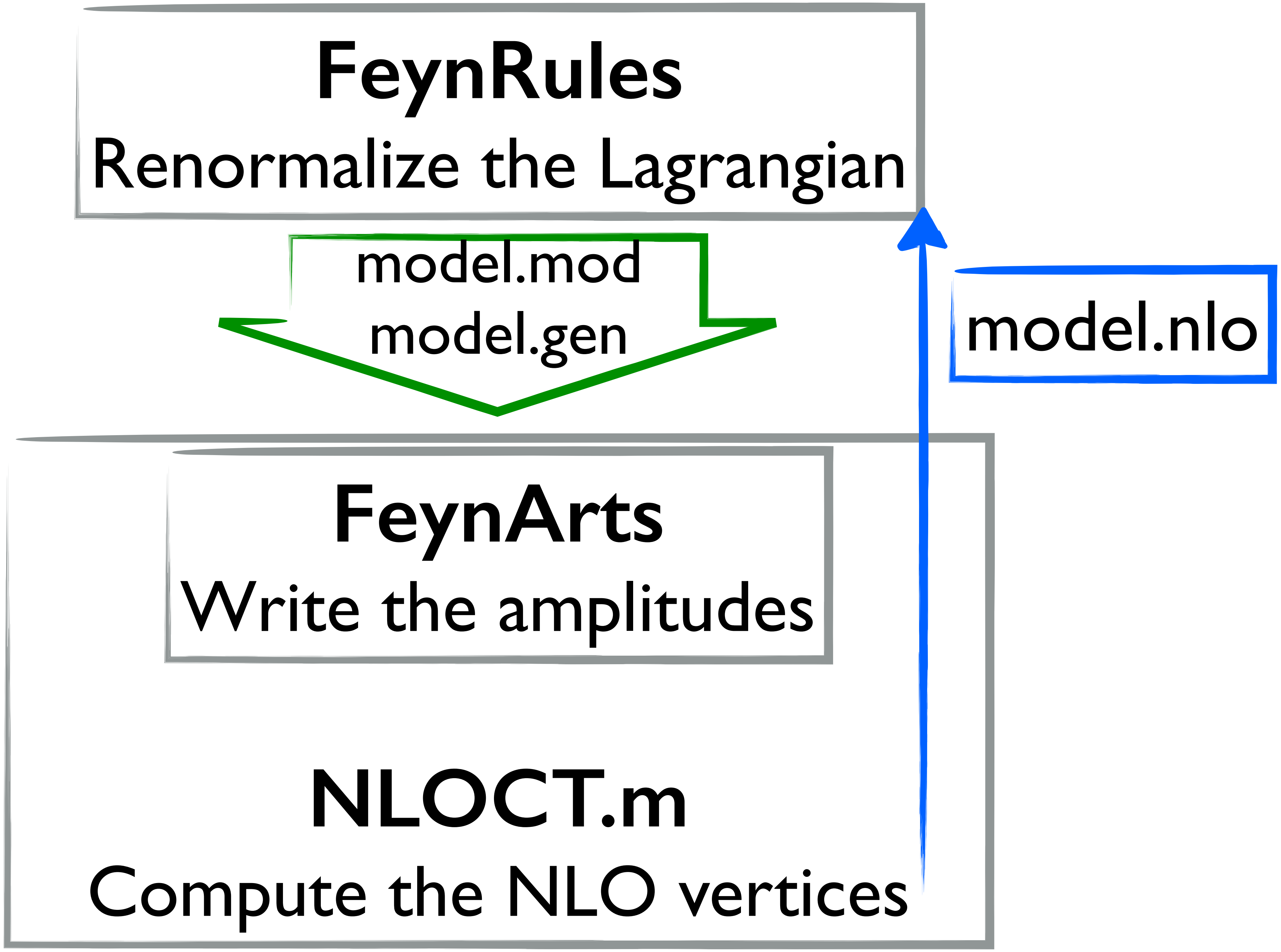
Write the amplitudes

NLOCT.m

Compute the NLO vertices

C. Degrande

How does it work?



FeynRules

Renormalize the Lagrangian

model.mod

model.gen

FeynArts

Write the amplitudes

NLOCT.m

Compute the NLO vertices

model.nlo

C. Degrande

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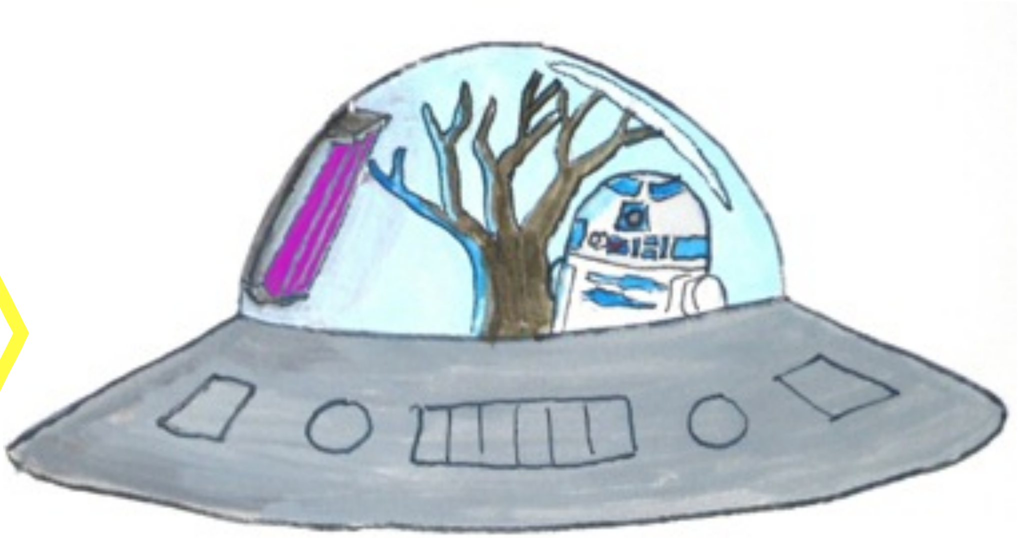

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How does it work?

FeynRules :

```
...
Lren  =  OnShellRenormalization[  LSM  ,   QCDOnly ->True];
WriteFeynArtsOutput[  Lren  , Output -> "SMrenoL",
GenericFile -> False]
```

How does it work?

FeynRules :

```
...
Lren  =  OnShellRenormalization[  LSM  ,   QCDOnly ->True];
WriteFeynArtsOutput[  Lren  ,  Output -> "SMrenoL",
GenericFile -> False]
```

FeynArts / NLOCT :

```
WriteCT[  "SMrenoL/SMrenoL" ,      "Lorentz",  Output->
“SMQCDreno",   QCDonly -> True]
```

How does it work?

FeynRules :

```
...
Lren  =  OnShellRenormalization[  LSM   ,   QCDOnly ->True];
WriteFeynArtsOutput[  Lren   ,  Output -> "SMrenoL",
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```

FeynArts / NLOCT :

```
WriteCT[  "SMrenoL/SMrenoL" ,      "Lorentz",  Output->
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How does it work?

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...
Lren  =  OnShellRenormalization[  LSM  ,   QCDOnly ->True];
WriteFeynArtsOutput[  Lren  ,  Output -> "SMrenoL",
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```

FeynArts / NLOCT :

```
WriteCT[  "SMrenoL/SMrenoL" ,    "Lorentz", Output->
“SMQCDreno",   QCDonly -> True]
```

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FeynRules :

```
...
Lren  =  OnShellRenormalization[  LSM  ,  QCDOnly ->True];
WriteFeynArtsOutput[  Lren  ,  Output -> "SMrenoL",
GenericFile -> False]
```

FeynArts / NLOCT :

```
WriteCT[  "SMrenoL/SMrenoL" ,     "Lorentz",  Output->
“SMQCDreno",   QCDonly -> True]
```

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How does it work?

FeynRules :

```
...
Lren  =  OnShellRenormalization[  LSM  ,   QCDOnly ->True];
WriteFeynArtsOutput[  Lren   ,  Output -> "SMrenoL",
GenericFile -> False]
```

FeynArts / NLOCT :

```
WriteCT[  "SMrenoL/SMrenoL" ,      "Lorentz",  Output->
“SMQCDreno",   QCDonly -> True]
```

FeynRules :

```
...
Get["SMQCDreno.nlo"];
WriteUFO[ LSM ,  UVCounterterms -> UV$vertlist ,
 R2Vertices -> R2$vertlist]
```

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How does it work?

FeynRules :

```
...
Lren  =  OnShellRenormalization[  LSM  ,   QCDOnly ->True];
WriteFeynArtsOutput[  Lren   ,  Output -> "SMrenoL",
GenericFile -> False]
```

FeynArts / NLOCT :

```
WriteCT[  "SMrenoL/SMrenoL" ,       "Lorentz",  Output->
“SMQCDreno",   QCDonly -> True]
```

FeynRules :

```
...
Get["SMQCDreno.nlo"];
WriteUFO[ LSM ,  UVCounterterms -> UV$vertlist ,
 R2Vertices -> R2$vertlist]
```

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model.nlo

Model information (FR+FeynArts model/generic files)

```
R2$vertlist = {
{{{anti[u], 1}, {u, 2}}, ((-I/12)*gs^2*
IndexDelta[Index[Colour, Ext[1]], Index[Colour, Ext[2]]]*IPL[{u, G}]*
(TensDot[SlashedP[2], ProjM][Index[Spin, Ext[1]], Index[Spin, Ext[2]]] +
TensDot[SlashedP[2], ProjP][Index[Spin, Ext[1]], Index[Spin, Ext[2]]])/Pi^2},
...
}
```

~FeynRules syntaxe

UV$vertlist (ε is FR$Eps)

```
FR$InteractionOrderPerturbativeExpansion = {{QCD, 1}, {QED, 0}};
```

QCDOnly

NLOCT$assumptions

WriteCT[…,Assumptions->{…}]

UFO@NLO

- CT_vertices.py

UV

```
V_1 = CTVertex(name = 'V_1',
               type = 'R2',
               particles = [ P.g, P.g, P.g ],
               color = [ 'f(1,2,3)' ],
               lorentz = [ L.VVV2 ],
               loop_particles = [ [ [P.b], [P.c], [P.d], [P.s], [P.t], [P.u] ], [ [P.g] ] ],
               couplings = {(0,0,0):C.R2GC_273_53,(0,0,1):C.R2GC_273_54})
```

- CT_couplings.py

Pole

```
UVGC_271_34 = Coupling(name = 'UVGC_271_34',
                       value = {-1:'( 0 if MB else -(complex(0,1)*G**2)/(24.*cmath.pi**2) ) +
(complex(0,1)*G**2)/(24.*cmath.pi**2)',0:'( -(complex(0,1)*G**2*reglog(MB/MU_R))/
(12.*cmath.pi**2) if MB else 0 )'},
                       order = {'QCD':2})
```

Finite

- In coupling_order.py

```
QCD = CouplingOrder(name = 'QCD',
                    expansion_order = 99,
                    hierarchy = 1,
                    perturbative_expansion = 1)

QED = CouplingOrder(name = 'QED',
                    expansion_order = 99,
                    hierarchy = 2)
```

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Restrictions/Assumptions

- Renormalizable Lagrangian, maximum dimension of the operators is 4
- Feynman Gauge
- $\{\gamma_\mu, \gamma_5\} = 0$
- ‘t Hooft-Veltman scheme
- On-shell/complex mass scheme for the masses and wave functions
- $\overline{\text{MS}}$ by default for everything else (zero-momentum possible for fermion gauge boson interaction)

NLOCT

- Amplitudes from FeynArts (discard irrelevant diagrams like ghost boxes)
- Compute terms at the generic level

$$\vec{c}\cdot\vec{L}=\sum_i c_i L_i$$

- Feynman parameters
- Remove terms with an odd or too low rank
- Gather loop momentum

$$\begin{aligned} q^\mu q^\nu q^\rho q^\sigma &\to q^4\frac{1}{d(d+2)}\left(\eta^{\mu\nu}\eta^{\rho\sigma}+\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\rho\nu}\right)\\ q^\mu q^\nu &\to q^2\frac{1}{d}\eta^{\mu\nu}. \end{aligned}$$

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NLOCT

- Replace momentum integrals

$$
\begin{aligned}
\int d^dq \frac{\epsilon}{q^2-m^2}\bigg|_{R_2} &= i\pi^2 m^2, \\
\int d^dq \frac{\epsilon}{\left(q^2-\Delta\right)^2}\bigg|_{R_2} &= i\pi^2, \\
\int d^dq \frac{q^2\left(a\,\epsilon+b\right)}{\left(q^2-\Delta\right)^2}\bigg|_{R_2} &= i\pi^2(2a-b)\Delta, \\
\int d^dq \frac{q^2\left(a\,\epsilon+b\right)}{\left(q^2-\Delta\right)^3}\bigg|_{R_2} &= i\pi^2\left(a-\frac{1}{2}b\right), \\
\int d^dq \frac{q^4\left(a\,\epsilon+b\right)}{\left(q^2-\Delta\right)^4}\bigg|_{R_2} &= i\pi^2\left(a-\frac{5}{6}b\right),
\end{aligned}
$$

$$
\begin{aligned}
\mu^{2\epsilon}\int d^dq \frac{a\epsilon+b}{q^2-m^2}\bigg|_{UV} &= i\pi^2 m^2\left(\frac{b}{\bar{\epsilon}}+a+b-b\log\left(\frac{m^2}{\mu^2}\right)\right), \\
\mu^{2\epsilon}\int d^dq \frac{a\epsilon+b}{\left(q^2-\Delta\right)^2}\bigg|_{UV} &= i\pi^2\boxed{(a\epsilon+b)}\left(\frac{1}{\bar{\epsilon}}-\boxed{\log\left(\frac{\Delta}{\mu^2}\right)}\right), \\
\mu^{2\epsilon}\int d^dq \frac{q^2\left(a\,\epsilon+b\right)}{\left(q^2-\Delta\right)^2}\bigg|_{UV} &= i\pi^2\boxed{(2a\epsilon+b\epsilon+2b)}\left(\frac{1}{\bar{\epsilon}}-\boxed{\log\left(\frac{\Delta}{\mu^2}\right)}\right)\Delta, \\
\mu^{2\epsilon}\int d^dq \frac{q^2\left(a\,\epsilon+b\right)}{\left(q^2-\Delta\right)^3}\bigg|_{UV} &= i\pi^2\frac{b}{\bar{\epsilon}}, \\
\mu^{2\epsilon}\int d^dq \frac{q^4\left(a\,\epsilon+b\right)}{\left(q^2-\Delta\right)^4}\bigg|_{UV} &= i\pi^2\frac{b}{\bar{\epsilon}},
\end{aligned}
$$

- Integrate over the Feynman parameters (but for the two-point UV finite terms)
- Replace masses and couplings by their values for each field insertion

C. Degrande

NLOCT

- Replace momentum integrals

$$
\begin{aligned}
\int d^dq \frac{\epsilon}{q^2-m^2}\bigg|_{R_2} &= i\pi^2 m^2, \\
\int d^dq \frac{\epsilon}{\left(q^2-\Delta\right)^2}\bigg|_{R_2} &= i\pi^2, \\
\int d^dq \frac{q^2\,(a\,\epsilon+b)}{\left(q^2-\Delta\right)^2}\bigg|_{R_2} &= i\pi^2(2a-b)\Delta, \\
\int d^dq \frac{\boxed{q^2}\,(a\,\epsilon+b)}{\left(q^2-\Delta\right)^3}\bigg|_{R_2} &= i\pi^2\left(a-\frac{1}{2}b\right), \\
\int d^dq \frac{\boxed{q^4}\,(a\,\epsilon+b)}{\left(q^2-\Delta\right)^4}\bigg|_{R_2} &= i\pi^2\left(a-\frac{5}{6}b\right),
\end{aligned}
$$

$$
\begin{aligned}
\mu^{2\epsilon}\int d^dq \frac{a\epsilon+b}{q^2-m^2}\bigg|_{UV} &= i\pi^2 m^2\left(\frac{b}{\bar{\epsilon}}+a+b-b\log\left(\frac{m^2}{\mu^2}\right)\right), \\
\mu^{2\epsilon}\int d^dq \frac{a\epsilon+b}{\left(q^2-\Delta\right)^2}\bigg|_{UV} &= i\pi^2\boxed{(a\epsilon+b)}\left(\frac{1}{\bar{\epsilon}}-\boxed{\log\left(\frac{\Delta}{\mu^2}\right)}\right), \\
\mu^{2\epsilon}\int d^dq \frac{q^2\,(a\,\epsilon+b)}{\left(q^2-\Delta\right)^2}\bigg|_{UV} &= i\pi^2\boxed{(2a\epsilon+b\epsilon+2b)}\left(\frac{1}{\bar{\epsilon}}-\boxed{\log\left(\frac{\Delta}{\mu^2}\right)}\right)\Delta, \\
\mu^{2\epsilon}\int d^dq \frac{\boxed{q^2}\,(a\,\epsilon+b)}{\left(q^2-\Delta\right)^3}\bigg|_{UV} &= i\pi^2\frac{b}{\bar{\epsilon}}, \\
\mu^{2\epsilon}\int d^dq \frac{\boxed{q^4}\,(a\,\epsilon+b)}{\left(q^2-\Delta\right)^4}\bigg|_{UV} &= i\pi^2\frac{b}{\bar{\epsilon}},
\end{aligned}
$$

- Integrate over the Feynman parameters (but for the two-point UV finite terms)
- Replace masses and couplings by their values for each field insertion

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NLOCT

- Perform the color algebra for triplets and octets

End R_2

- Write the renormalization conditions (fix p^2)
- Do the integration over the feynman parameters for the UV-finite parts

$$b_0\left(p^2, m_1, m_2\right) \equiv \int_0^1 dx \log\left(\frac{p^2(x-1)x + x\left(m_1^2 - m_2^2\right) + m_2^2 - i\epsilon_p}{\mu^2}\right)$$

$$b_0\left(0,0,0\right) = \frac{1}{\bar{\epsilon}}$$

- Solve the renormalization conditions
- Replace the counterterms by their values in the CT vertices

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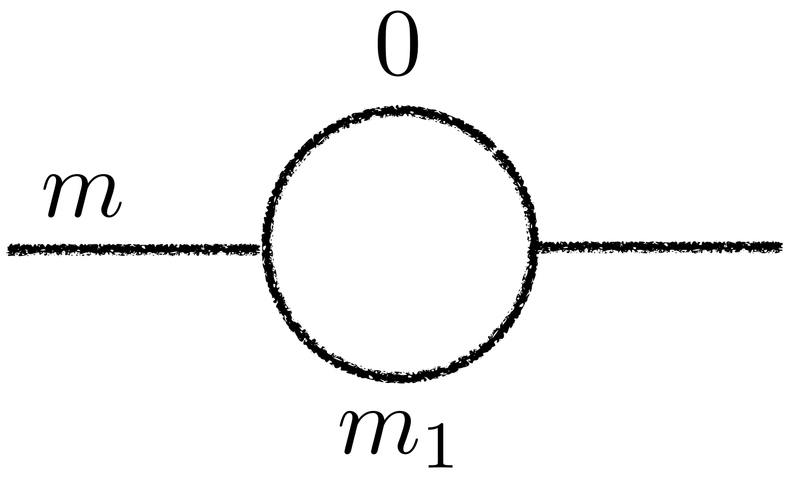
Real/Complex masses



Real masses

$$m \in \mathbb{R} \quad m_1^2 < m^2 \quad \xcancel{\Re}\left(\log\left[p^2 - m_1^2\right] \xcancel{+ i\pi}\right)\Big|_{p^2=m^2}$$

$$m_1^2 > m^2 \quad \xcancel{\Re}\left(\log\left[m_1^2 - p^2\right]\right)\Big|_{p^2=m^2}$$

Complex masses

$$\log\left[m_1^2 - p^2\right]\Big|_{p^2=m^2}$$

All cases are kept unless the users put some assumptions

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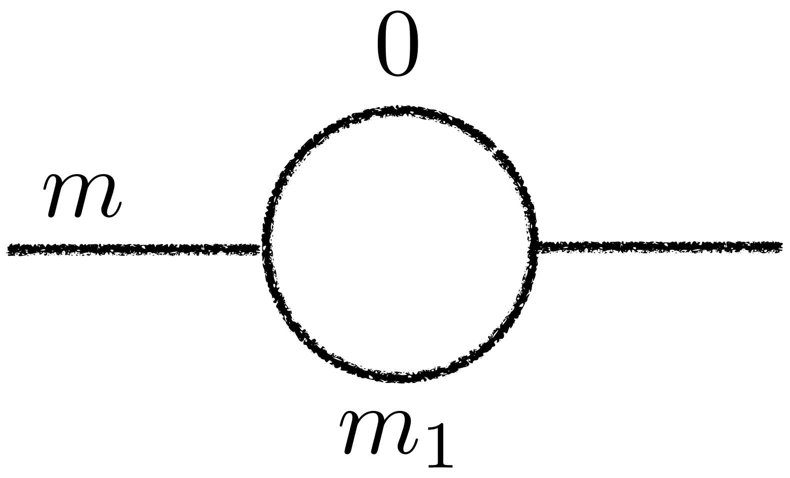
Real/Complex masses



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$$m_1^2 > m^2 \quad \cancel{\Re}\left(\log\left[m_1^2 - p^2\right]\right)\Big|_{p^2=m^2}$$

Complex masses

$$\log\left[m_1^2 - p^2\right]\Big|_{p^2=m^2}$$

Faster!

All cases are kept unless the users put some assumptions

Renormalization options

```
FR$LoopSwitches = {{Gf, MW}};
```

Switch internal masses with an external parameter appearing in its expression

Renormalization options

```
FR$LoopSwitches = {{Gf, MW}};
```

Switch int external parameter

appearing in its exp

Before calling OnShellRenormalization

Renormalization options

```
FR$LoopSwitches = {{Gf, MW}};
```

Switch int[illegible] external parameter appearing in its exp[illegible]

Before calling OnShellRenormalization

<u>OnShellRenormalization options</u>

QCDOnly : Only the coloured fields and their masses and the couplings with QCD if True

FlavorMixing : Forbid all the mixing or allow only some of them

Exclude4ScalarCT : No CT for the 4 scalars vertices (but keep the 4 scalars TL)

Simplify2Point : Put the quadratic part of the Lagrangian in canonical form (Avoided if False)

WriteCT options

`WriteCT[<model>,<genericfile>,options]`

OnShellRenormalization options

QCDOnly : Only QCD corrections

Assumptions : Mass spectrum for the UV counterterms

Exclude4ScalarCT : No computation of the CT for the 4 scalars vertices (but keep the 4 scalars TL)

ZeroMom : {coupling, vertex} use zero momentum for coupling on vertex

ComplexMass : complex mass scheme if True

R2 :Validation

- tested* on the SM (QCD:P. Draggiotis et al. +QED:M.V. Garzelli et al)
- tested* on MSSM (QCD:H.-S. Shao, Y.-J. Zhang) : test the Majorana

*Analytic comparison of the expressions

UV Validation

- SM QCD : tested* (W. Beenakker, S. Dittmaier, M. Kramer, B. Plumper)
- SM EW : tested* (expressions given by H.-S. Shao from A. Denner)

*Analytic comparison of the expressions

Tests in event generators

- aMC@NLO
- The SM QCD has been tested by V. Hirschi (Comparison with the built-in version)
- The MSSM QCD and SM EW are tested by H.-S. Shao and V. Hirschi
- 2HDM QCD is currently tested (p p > S, H^+ t)
  - gauge invariance
  - pole cancelation

SM tests

```
                                   === Finite ===
          Process            Stored ML5 opt      ML5 opt            ML5 default        Relative diff.     Result
d d~ > w+ w-  g       -1.2565695610e+01   -1.2565705416e+01   -1.2565696276e+01   3.9018817097e-07     Pass

                                   === Born ===
          Process            Stored ML5 opt      ML5 opt            ML5 default        Relative diff.     Result
 d d~ > w+ w-  g        1.8518318521e-06    1.8518318521e-06    1.8518318521e-06   8.0617231411e-15    Pass

                                 === Single pole ===
          Process            Stored ML5 opt      ML5 opt            ML5 default        Relative diff.     Result
d d~ > w+ w-  g       -1.9397426502e+01   -1.9397426502e+01   -1.9397426504e+01   5.5894073017e-11     Pass

                                 === Double pole ===
          Process            Stored ML5 opt      ML5 opt            ML5 default        Relative diff.     Result
d d~ > w+ w-  g       -5.6666666667e+00   -5.6666666667e+00   -5.6666666667e+00   3.0015206007e-14     Pass

                                   === Summary ===
                           1/1 passed, 0/1 failed=== Finite ===
         Process           Stored MadLoop v4   ML5 opt            ML5 default        Relative diff.     Result
 d~ d > a g g         -5.3971186943e+01   -5.3971193753e+01   -5.3971189940e+01   6.3091071914e-08    Pass

                                   === Born ===
         Process           Stored MadLoop v4   ML5 opt            ML5 default        Relative diff.     Result
  d~ d > a g g         6.4168774056e-05    6.4168764370e-05    6.4168764370e-05   7.5467680882e-08    Pass

                                 === Single pole ===
         Process           Stored MadLoop v4   ML5 opt            ML5 default        Relative diff.     Result
 d~ d > a g g         -3.7439549398e+01   -3.7439549398e+01   -3.7439549397e+01   6.8122965983e-12    Pass

                                 === Double pole ===
         Process           Stored MadLoop v4   ML5 opt            ML5 default        Relative diff.     Result
 d~ d > a g g         -8.6666666667e+00   -8.6666666667e+00   -8.6666666667e+00   2.2443585452e-14    Pass

                                   === Summary ===
                           1/1 passed, 0/1 failed=== Finite ===
         Process           Stored MadLoop v4   ML5 opt            ML5 default        Relative diff.     Result
 d~ d > z g g         -5.3769573669e+01   -5.3769573347e+01   -5.3769566412e+01   6.7475496780e-08    Pass
```

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SM tests

```
                              === Born ===
     Process          Stored MadLoop v4   ML5 opt           ML5 default        Relative diff.     Result
 d~ d > z g g      3.1531233900e-04    3.1531235770e-04    3.1531235770e-04    2.9654886777e-08    Pass

                           === Single pole ===
     Process          Stored MadLoop v4   ML5 opt           ML5 default        Relative diff.     Result
d~ d > z g g      -3.7464897007e+01   -3.7464897007e+01   -3.7464897007e+01   4.2333025503e-12    Pass

                           === Double pole ===
     Process          Stored MadLoop v4   ML5 opt           ML5 default        Relative diff.     Result
d~ d > z g g      -8.6666666667e+00   -8.6666666667e+00   -8.6666666667e+00   2.1316282073e-14    Pass

                            === Summary ===
                    1/1 passed, 0/1 failed=== Finite ===
     Process          Stored MadLoop v4   ML5 opt           ML5 default        Relative diff.     Result
d~ d > z z g      -5.9990384275e+00   -5.9990511729e+00   -5.9990379587e+00   1.1013604745e-06    Pass

                              === Born ===
     Process          Stored MadLoop v4   ML5 opt           ML5 default        Relative diff.     Result
 d~ d > z z g      2.2616997126e-06    2.2617000449e-06    2.2617000449e-06    7.3450366526e-08    Pass

                           === Single pole ===
     Process          Stored MadLoop v4   ML5 opt           ML5 default        Relative diff.     Result
d~ d > z z g      -1.5469587040e+01   -1.5469587040e+01   -1.5469587040e+01   1.5226666708e-11    Pass

                           === Double pole ===
     Process          Stored MadLoop v4   ML5 opt           ML5 default        Relative diff.     Result
d~ d > z z g      -5.6666666667e+00   -5.6666666667e+00   -5.6666666667e+00   2.6645352591e-15    Pass

                            === Summary ===
                    1/1 passed, 0/1 failed=== Finite ===
     Process          Stored MadLoop v4   ML5 opt           ML5 default        Relative diff.     Result
g g > h t t~      2.9740187004e+01    2.9740187005e+01    2.9740187036e+01    5.3265970697e-10    Pass
```

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SM tests

```
                        === Born ===
    Process       Stored MadLoop v4   ML5 opt          ML5 default       Relative diff.     Result
g g > h t t~      1.1079653971e-07    1.1079653974e-07    1.1079653974e-07    1.3190849004e-10    Pass

                        === Single pole ===
    Process       Stored MadLoop v4   ML5 opt          ML5 default       Relative diff.     Result
g g > h t t~      -7.0825709000e+00   -7.0825709000e+00   -7.0825709000e+00   5.0901237085e-13    Pass

                        === Double pole ===
    Process       Stored MadLoop v4   ML5 opt          ML5 default       Relative diff.     Result
g g > h t t~      -6.0000000000e+00   -6.0000000000e+00   -6.0000000000e+00   1.7023419711e-15    Pass

                        === Summary ===
                  1/1 passed, 0/1 failed=== Finite ===
    Process       Stored MadLoop v4   ML5 opt          ML5 default       Relative diff.     Result
g g > z t t~      3.6409017466e+01    3.6409021125e+01    3.6409021117e+01    5.0242920154e-08    Pass

                        === Born ===
    Process       Stored MadLoop v4   ML5 opt          ML5 default       Relative diff.     Result
g g > z t t~      7.0723041711e-07    7.0723046101e-07    7.0723046101e-07    3.1039274206e-08    Pass

                        === Single pole ===
    Process       Stored MadLoop v4   ML5 opt          ML5 default       Relative diff.     Result
g g > z t t~      -7.1948086812e+00   -7.1948086773e+00   -7.1948086773e+00   2.7349789963e-10    Pass

                        === Double pole ===
    Process       Stored MadLoop v4   ML5 opt          ML5 default       Relative diff.     Result
g g > z t t~      -6.0000000000e+00   -6.0000000000e+00   -6.0000000000e+00   2.5165055225e-15    Pass

                        === Summary ===
                  1/1 passed, 0/1 failed=== Finite ===
    Process       Stored ML5 opt      ML5 opt          ML5 default       Relative diff.     Result
d d~ > w+ w- g    -1.2565695610e+01   -1.2565705416e+01   -1.2565696276e+01   3.9018817097e-07    Pass
```

C. Degrande

SM tests

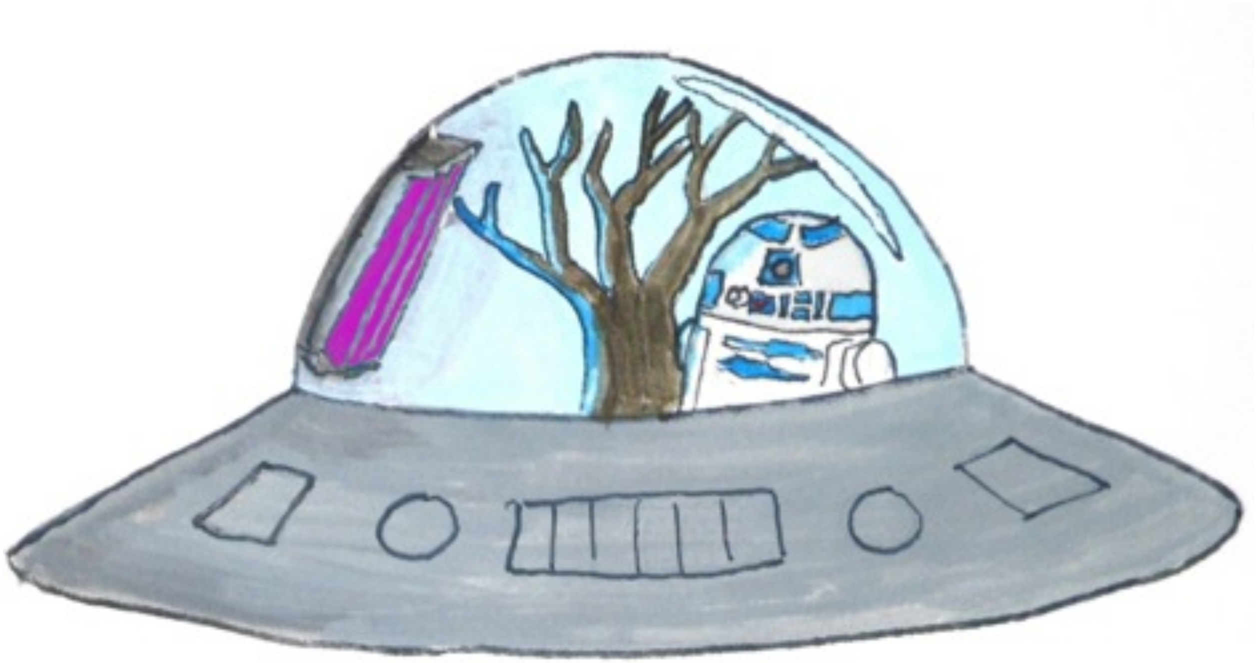
```
                         === Born ===
     Process       Stored ML5 opt    ML5 opt        ML5 default      Relative diff.    Result
d d~ > w+ w- g   1.8518318521e-06  1.8518318521e-06  1.8518318521e-06  8.0617231411e-15   Pass

                       === Single pole ===
     Process       Stored ML5 opt    ML5 opt        ML5 default      Relative diff.    Result
d d~ > w+ w- g   -1.9397426502e+01  -1.9397426502e+01  -1.9397426504e+01  5.5894073017e-11   Pass

                       === Double pole ===
     Process       Stored ML5 opt    ML5 opt        ML5 default      Relative diff.    Result
d d~ > w+ w- g   -5.6666666667e+00  -5.6666666667e+00  -5.6666666667e+00  3.0015206007e-14   Pass

                        === Summary ===
                 1/1 passed, 0/1 failed=== Finite ===
     Process       Stored ML5 opt    ML5 opt        ML5 default      Relative diff.    Result
d~ d > a g g     -1.1504816412e+01  -1.1504816557e+01  -1.1504815497e+01  4.6089385415e-08   Pass

                         === Born ===
     Process       Stored ML5 opt    ML5 opt        ML5 default      Relative diff.    Result
 d~ d > a g g     2.3138920858e-06  2.3138920858e-06  2.3138920858e-06  4.3012538015e-15   Pass

                       === Single pole ===
     Process       Stored ML5 opt    ML5 opt        ML5 default      Relative diff.    Result
d~ d > a g g     -2.8637049838e+01  -2.8637049838e+01  -2.8637049838e+01  1.5718407645e-13   Pass

                       === Double pole ===
     Process       Stored ML5 opt    ML5 opt        ML5 default      Relative diff.    Result
d~ d > a g g     -8.6666666667e+00  -8.6666666667e+00  -8.6666666667e+00  1.7421961310e-15   Pass

                        === Summary ===
                 1/1 passed, 0/1 failed=== Finite ===
     Process       Stored ML5 opt    ML5 opt        ML5 default      Relative diff.    Result
d~ d > z g g     -1.0306105482e+01  -1.0306105654e+01  -1.0306102645e+01  1.4600800434e-07   Pass
```

+2/3

C. Degrande

Summary/Prospect

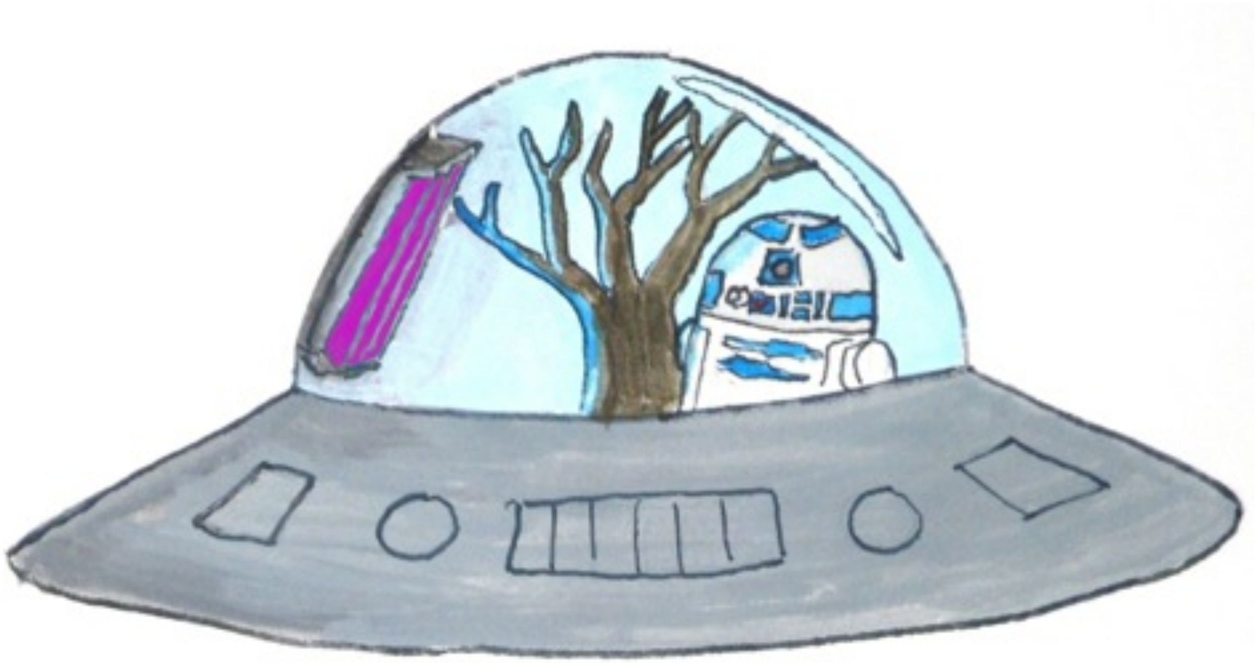
- Automatic BSM@NLO
  - renormalizable
  - Feynman gauge



C. Degrande

Summary/Prospect

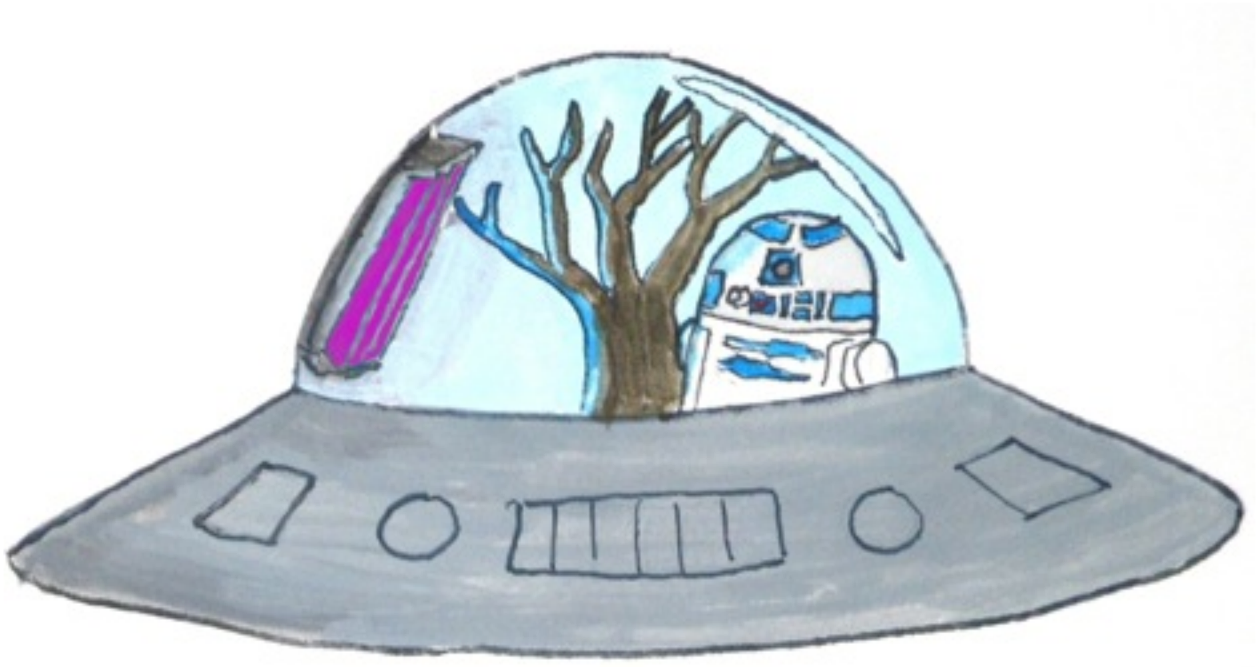
- Automatic BSM@NLO
  - renormalizable
  - Feynman gauge
- Next version
  - EFT
  - Any gauge
  - other renormalization scheme (EW)



C. Degrande

Summary/Prospect

- Automatic BSM@NLO
  - renormalizable
  - Feynman gauge
- Next version
  - EFT
  - Any gauge
  - other renormalization scheme (EW)
- With the help of the FeynRules and Madgraph_aMC@NLO teams



C. Degrande