Simulation of BSM physics

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Plan

- Field theory : a short reminder
    - free fields (KG details, Fermion)
    - Scattering matrix in perturbation
    - Wick theorem to Feynman rules
- Why Monte-Carlo/automated tools?
- Lagrangian to the Feynman rules
    - Model file : Parameters, fields, gauge group and Lagrangian
    - Running FeynRules
- Demo

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Lagrangian density formalism

$$S = \int d^4x \mathcal{L}\left(\Phi_r, \partial_\mu \Phi_r, ...\right)$$

polynomial in the the fields and its derivatives

Equation of motion

$$\Phi_r(x) \to \Phi_r(x) + \delta\Phi_r(x)$$

$\delta\Phi_r(x)$ vanishes at infinity

$$\frac{\partial \mathcal{L}}{\partial \Phi_r(x)} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \Phi_r\right)}\right) = 0$$

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Lagrangian density formalism

field conjugate $$\pi_r(x) = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_r}$$

In field theory: it (anti-) commutes with the field
both are operators!

Hamiltonian density $$\mathcal{H}(x) \equiv \pi_r(x)\dot{\Phi}_r - \mathcal{L}(\Phi_r, \partial_\mu \Phi_r, \dots)$$

$$L = \int d^3x \mathcal{L}(x)$$

$$H = \int d^3x \mathcal{H}(x)$$

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Harmonic oscillator

$$H_{osc} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$$

with $[q, p] = i$ QM: Position and momentum are operators

Harmonic oscillator

$$H_{osc} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2 \xrightarrow[N = a^\dagger a]{\left.\begin{matrix} a \\ a^\dagger \end{matrix}\right\} = \frac{m\omega q \pm ip}{\sqrt{2m\omega}}} H_{osc} = \omega\left(N + \frac{1}{2}\right)$$

with $[q, p] = i$ **QM: Position and momentum are operators**

$$\left[a, a^\dagger\right] = 1 \qquad\qquad [a, a] = \left[a^\dagger, a^\dagger\right] = 0$$

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if $N|\alpha\rangle = \alpha|\alpha\rangle$ then $Na|\alpha\rangle = (\alpha - 1)a|\alpha\rangle\,, \quad Na^\dagger|\alpha\rangle = (\alpha + 1)a^\dagger|\alpha\rangle$

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N is positive definite, therefore it has a non-negative minimal eigenvalue α_0

$$a^\dagger a\left|\alpha_0\right\rangle = \alpha_0\left|\alpha_0\right\rangle$$

Therefore $a\left|\alpha_0\right\rangle = 0$ and $\alpha_0 = 0$

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Eigenvalues of N are integers $\left|n\right\rangle = \frac{\left(a^\dagger\right)^n}{\sqrt{n!}}\left|0\right\rangle \qquad E_n = \omega\left(n + \frac{1}{2}\right)$

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Klein-Gordon field

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right)$$

$$\left(\Box + m^2\right)\phi\left(x\right) = 0 \qquad \pi(x) = \dot{\phi}\left(x\right)$$

Real field becomes an hermitian operator

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Real field becomes an hermitian operator

Equal time commutation relations

$$\left[\phi\left(\mathbf{x},t\right),\, \dot{\phi}\left(\mathbf{x}',t\right)\right] = i\delta\left(\mathbf{x}-\mathbf{x}'\right)$$

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Periodic boundary conditions $\phi\left(0, x, y, t\right) = \phi\left(L, x, y, t\right), \quad \text{etc.}$

$$\phi(x) = \sum_{\mathbf{k}} \frac{a(\mathbf{k})e^{-ikx} + a^\dagger(\mathbf{k})e^{ikx}}{\sqrt{2V\omega_k}}$$

$$\mathbf{k} = \frac{2\pi}{L}\left(n_1, n_2, n_3\right), \quad n_i \in \mathbb{Z} \qquad\qquad \omega_k = k^0 = \sqrt{m^2 - \mathbf{k}^2}$$

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$\phi^+(x)$

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$$\phi(x) = \sum_{\mathbf{k}} \frac{a(\mathbf{k})e^{-ikx} + a^\dagger(\mathbf{k})e^{ikx}}{\sqrt{2V\omega_k}}$$

$\phi^+(x)$ → $a(\mathbf{k})e^{-ikx}$ term; $\phi^-(x)$ → $a^\dagger(\mathbf{k})e^{ikx}$ term

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Klein-Gordon field

Commutation relations on the field and its conjugate imply

$$\left[a\left(\mathbf{k}\right), a^{\dagger}\left(\mathbf{k}'\right)\right] = \delta_{kk'}$$

$$\left[a\left(\mathbf{k}\right), a\left(\mathbf{k}'\right)\right] = \left[a^{\dagger}\left(\mathbf{k}\right), a^{\dagger}\left(\mathbf{k}'\right)\right] = 0$$

One H.O. per mode!

Occupation number operators:

$$N\left(\mathbf{k}\right) = a^{\dagger}\left(\mathbf{k}\right) a\left(\mathbf{k}\right)$$

Creation and annihilation operators of particles with momentum k

$$H = \sum_{k} \omega_k \left(N(\mathbf{k}) + \frac{1}{2}\right)$$

Confirm the interpretation

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K-G field: vacuum

$$a(\mathbf{k})\,|0\rangle = 0, \ \text{all } \mathbf{k} \qquad\qquad E_0 = \frac{1}{2}\sum_k \omega_k$$

Infinite energy of the vacuum state !

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Infinite energy of the vacuum state !

Solution: Normal product

$$N\left[\phi(x)\phi(y)\right] \equiv \phi^+(x)\phi^+(y) + \phi^-(x)\phi^+(y) + \phi^-(y)\phi^+(x) + \phi^-(x)\phi^-(y)$$

Redefine the Lagrangian and all observables as their normal product, then the vacuum has zero energy, …

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One particle state	$a^\dagger(\mathbf{k})\,|0\rangle$
Two particles state	$a^\dagger(\mathbf{k})a^\dagger(\mathbf{k}')\,|0\rangle$
$\vdots$	$\vdots$

many particles in the same state = Bosons

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K-G field:covariant commutation relations

$$\left[\phi^+(x), \phi^+(y)\right] = \left[\phi^-(x), \phi^-(y)\right] = 0$$

$$[\phi(x), \phi(y)] = \left[\phi^+(x), \phi^-(y)\right] + \left[\phi^-(x), \phi^+(y)\right]$$

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$$\left[\phi^+(x), \phi^-(y)\right] = \frac{1}{2V} \sum_{kk'} \frac{1}{\sqrt{\omega_k \omega_{k'}}} \left[a\left(\mathbf{k}\right), a^\dagger\left(\mathbf{k}'\right)\right] e^{-ikx+ik'y} \overset{V\to\infty}{=} i\Delta^+\left(x-y\right)$$

$$\Delta^+\left(x\right) \equiv \frac{-i}{2(2\pi)^3} \int \frac{d^3\mathbf{k}}{\omega_k} e^{-ikx}$$

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$$\left[\phi^{-}(x), \phi^{+}(y)\right] = -\Delta^{+}\left(y-x\right) \equiv \Delta^{-}\left(x-y\right)$$

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$$\left[\phi^-(x), \phi^+(y)\right] = -\Delta^+\left(y-x\right) \equiv \Delta^-\left(x-y\right)$$

$$[\phi(x), \phi(y)] = i\Delta\left(x-y\right) = i\left(\Delta^+\left(x-y\right) + \Delta^-\left(x-y\right)\right) = \frac{-i}{(2\pi)^3} \int \frac{d^3\mathbf{k}}{\omega_k} \sin kx$$

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$$\Delta^+(x) \equiv \frac{-i}{2(2\pi)^3}\int\frac{d^3\mathbf{k}}{\omega_k}e^{-ikx}$$

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Solution to K-G eq. $\left(\Box + m^2\right)\Delta(x-y) = 0$

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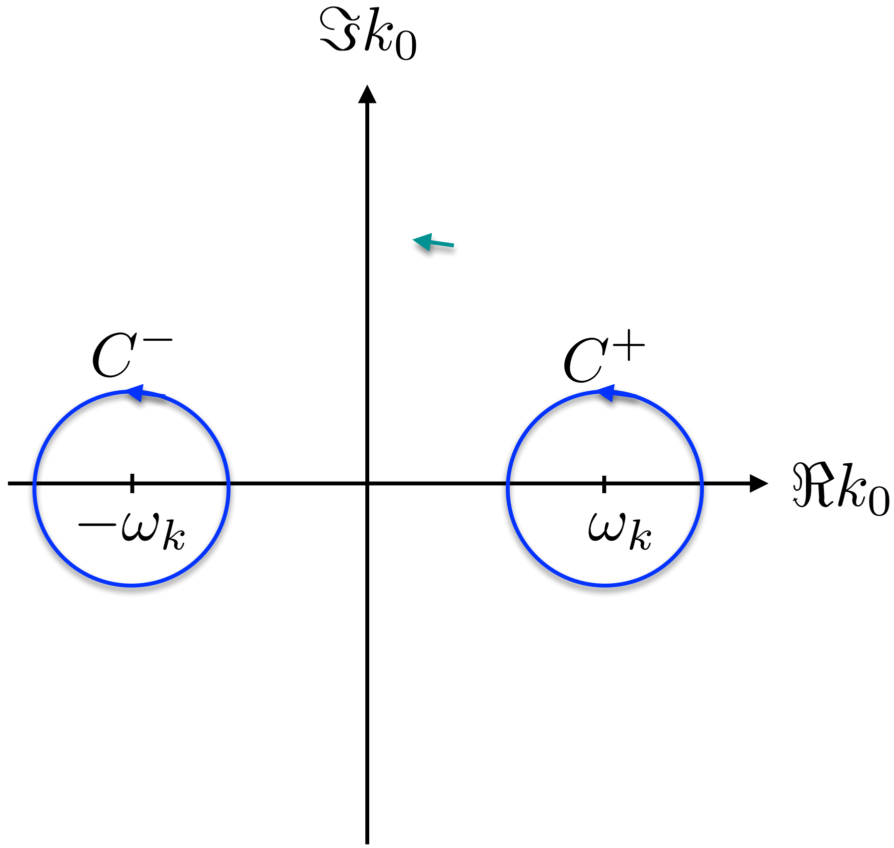
K-G field: commutator function

$$\Delta^{\pm}(x) = \frac{-1}{(2\pi)^4}\int_{C^{\pm}} d^4k \frac{e^{-ikx}}{k^2 - m^2}$$



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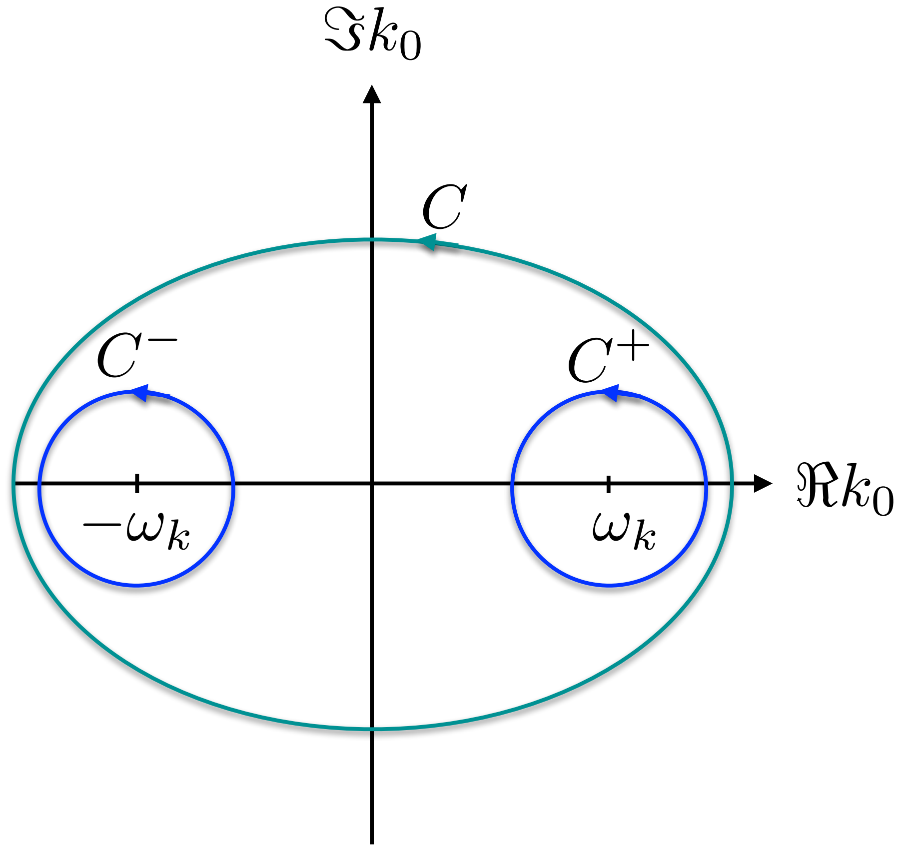

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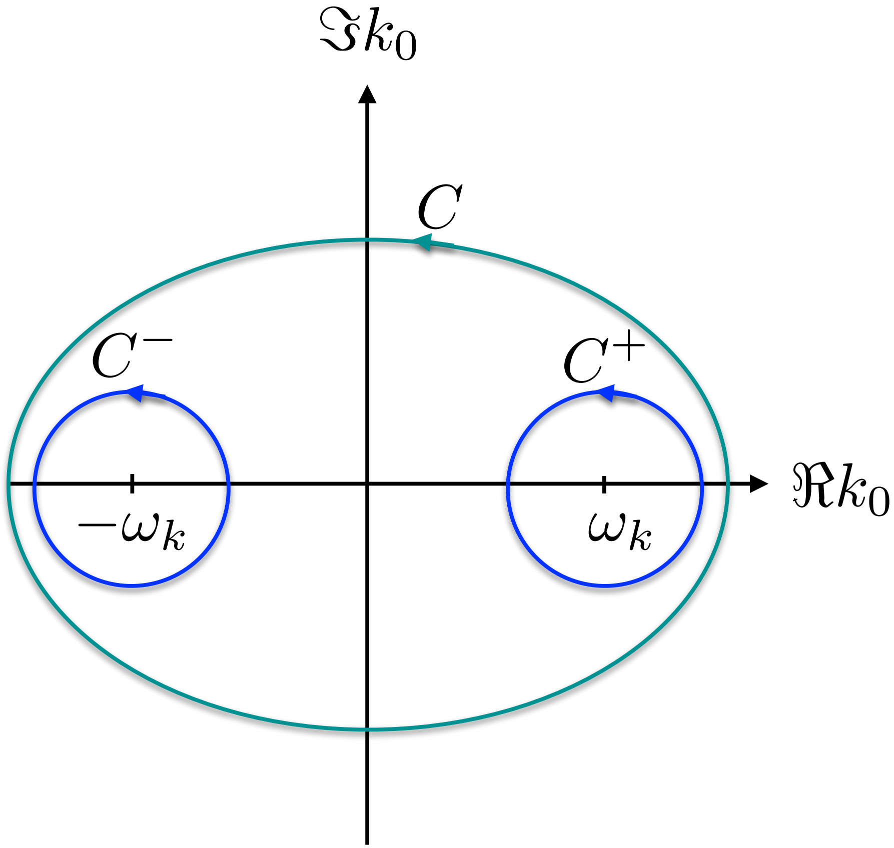

Microcausality

$$[\phi(\mathbf{x},t),\phi(\mathbf{y},t)] = i\Delta(\mathbf{x}-\mathbf{y},0) = 0$$

By Lorentz invariance

$$[\phi(x),\phi(y)] = i\Delta(x-y) = 0, \text{ for } (x-y)^2 < 0$$

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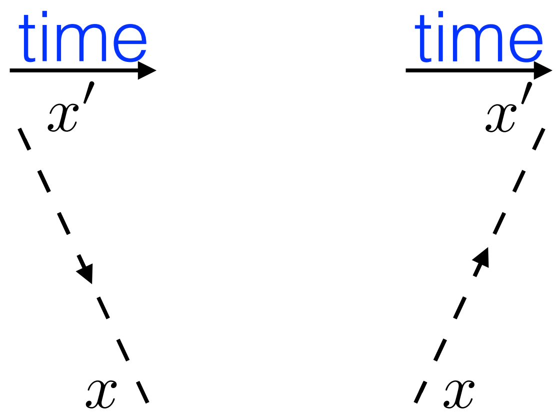
K-G field: Propagator

$$i\Delta_F(x) \equiv \langle 0| \, T\{\phi(x)\phi(x')\} \, |0\rangle$$

$$T\{\phi(x)\phi(x')\} = \begin{cases} \phi(x)\phi(x') & \text{if } t > t' \\ \phi(x')\phi(x) & \text{if } t' > t \end{cases}$$



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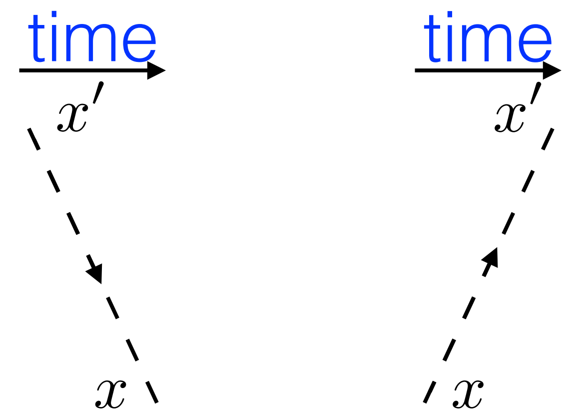
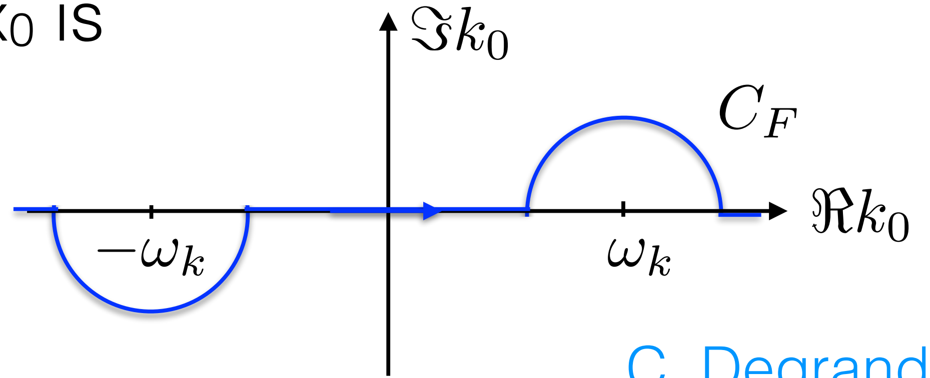


Since

$$i\Delta^+(x) = \langle 0| \left[\phi^+(x)\phi^-(x')\right] |0\rangle = \langle 0| \phi^+(x)\phi^-(x') |0\rangle = \langle 0| \phi(x)\phi(x') |0\rangle$$

$$i\Delta_F(x) = \theta(t)\Delta^+(x) - \theta(-t)\Delta^-(x) = \frac{1}{(2\pi)^4}\int_{C_F} d^4k \frac{e^{-ikx}}{k^2 - m^2}$$

Close by the top/bottom if x_0 is negative/positive



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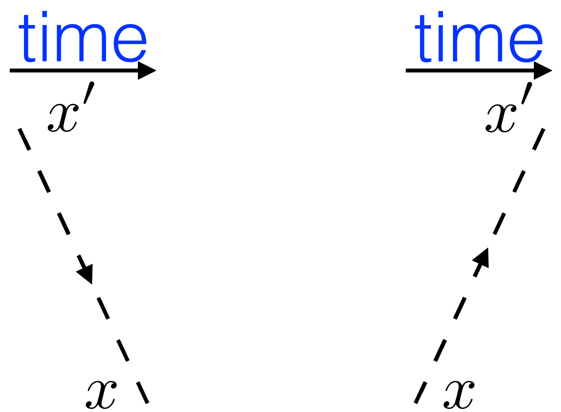

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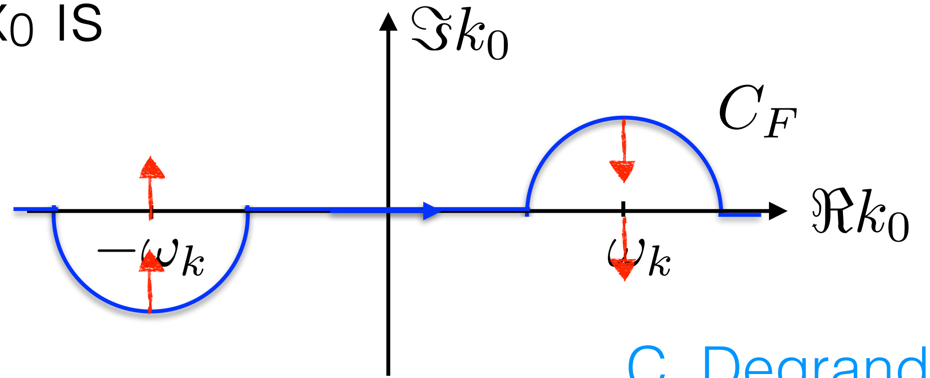

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Fermion operators

$$\{a_r, a_s^\dagger\} = a_r a_s^\dagger + a_s^\dagger a_r = \delta_{rs}, \quad \{a_r, a_s\} = \{a_r^\dagger, a_s^\dagger\} = 0$$

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$$[N_r, a_s] = -\delta_{rs} a_s, \quad [N_r, a_s^\dagger] = \delta_{rs} a_s^\dagger$$

remains creation and annihilation operators but

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remains creation and annihilation operators but

$$N_r^2 = a_r^\dagger a_r a_r^\dagger a_r = a_r^\dagger \left(1 - a_r^\dagger a_r\right) a_r = N_r$$

$$n_r = 0 \text{ or } n_r = 1$$

At most one particle in the same state = **Fermion**

Equivalently, the two particles states is antisymmetric

$$|1_r 1_s\rangle = a_r^\dagger a_s^\dagger |0\rangle = -a_s^\dagger a_r^\dagger |0\rangle = -|1_s 1_r\rangle$$

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Dirac field

$$\mathcal{L} = \bar{\psi}(x)\left[i\partial\!\!\!/ - m\right]\psi(x)$$

$$\left[i\partial\!\!\!/ - m\right]\psi(x) = 0$$

$$\pi_\alpha(x) = i\psi^\dagger_\alpha(x)$$

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$$\psi(x) = \sum_{rp}\sqrt{\frac{m}{VE_p}}\left[c_r(\mathbf{p})u_r(\mathbf{p})e^{-ipx} + d^\dagger_r(\mathbf{p})v_r(\mathbf{p})e^{ipx}\right]$$

Dirac algebra

$$(p\!\!\!/ - m)\,u_r(\mathbf{p}) = 0, \qquad (p\!\!\!/ + m)\,v_r(\mathbf{p}) = 0$$

$$u^\dagger_r(\mathbf{p})u_s(\mathbf{p}) = v^\dagger_r(\mathbf{p})v_s(\mathbf{p}) = \frac{E_p}{m}\delta_{rs}, \qquad u^\dagger_r(\mathbf{p})v_s(-\mathbf{p}) = 0$$

$$E_p = \sqrt{m^2 + \mathbf{p}^2}$$

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$$\pi_\alpha(x) = i\psi^\dagger_\alpha(x)$$

$$\psi(x) = \sum_{rp}\sqrt{\frac{m}{VE_p}}\left[c_r(\mathbf{p})u_r(\mathbf{p})e^{-ipx} + d^\dagger_r(\mathbf{p})v_r(\mathbf{p})e^{ipx}\right]$$

Dirac algebra

$$(p\!\!\!/ - m)\,u_r(\mathbf{p}) = 0, \qquad (p\!\!\!/ + m)\,v_r(\mathbf{p}) = 0$$

$$u^\dagger_r(\mathbf{p})u_s(\mathbf{p}) = v^\dagger_r(\mathbf{p})v_s(\mathbf{p}) = \frac{E_p}{m}\delta_{rs}, \qquad u^\dagger_r(\mathbf{p})v_s(-\mathbf{p}) = 0$$

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$$\{c_r(\mathbf{p}), c^\dagger_s(\mathbf{p})\} = \{d_r(\mathbf{p}), d^\dagger_s(\mathbf{p})\} = \delta_{rs}, \qquad \{\text{Other combi}\} = 0$$

$$N\left(\psi_\alpha\psi_\beta\right) = \psi^+_\alpha\psi^+_\beta + \psi^-_\alpha\psi^+_\beta - \psi^-_\beta\psi^+_\alpha + \psi^+_\alpha\psi^+_\beta$$

C. Degrande

Dirac field

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$$Q = q \sum_{rp} \left(N_r(\mathbf{p}) - \bar{N}_r(\mathbf{p}) \right)$$

Conserved charged from the U(1) symmetry of the Lagrangian

Field theory predicts antiparticles!

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Similar for a complex Klein-Gordon field

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Spin-statistics theorem is a consequence of QFT

Dirac field: fields commutation relations

$$\{\psi_\alpha(x), \psi_\beta(y)\} = \{\bar{\psi}_\alpha(x), \bar{\psi}_\beta(y)\} = 0$$

$$\left\{\psi^{\pm}_\alpha(x), \bar{\psi}^{\mp}_\beta(y)\right\} = i\left(i\not{\partial} + m\right)_{\alpha\beta} \Delta^{\pm}(x-y) \equiv iS^{\pm}(x-y)$$

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Same as for K-G

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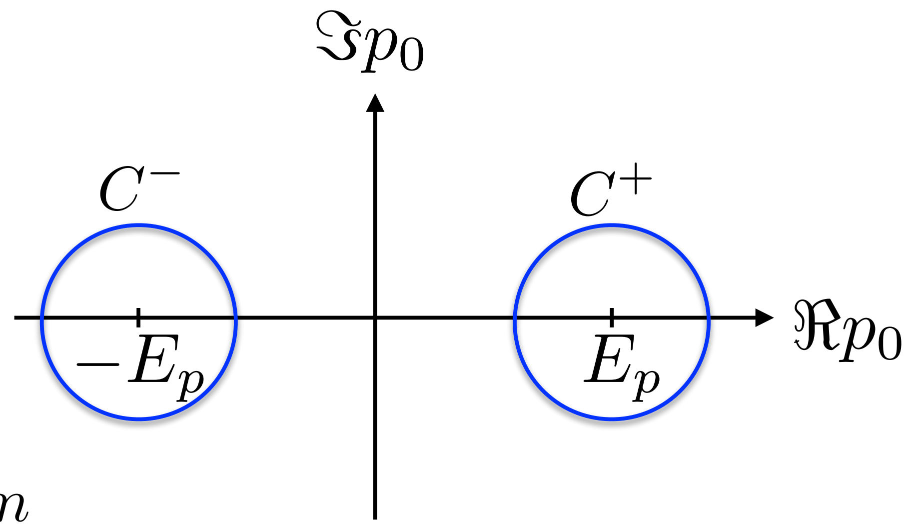

$$S^{\pm}(x) = \frac{-1}{(2\pi)^4} \int_{C^{\pm}} d^4p\, e^{-ipx} \frac{\not{p} + m}{p^2 - m^2}$$

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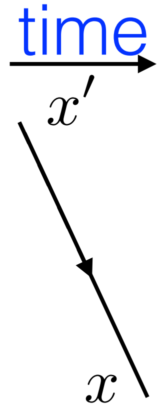
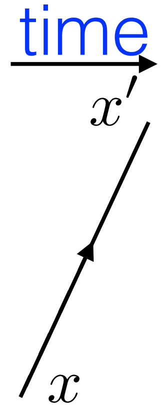
Dirac field: Propagator

$$iS_F(x) \equiv \langle 0|\, T\left\{\psi(x)\bar{\psi}(x')\right\}|0\rangle$$

$$T\left\{\psi(x)\bar{\psi}(x')\right\} = \begin{cases} \psi(x)\bar{\psi}(x') & \text{if } t > t' \\ -\bar{\psi}(x')\psi(x) & \text{if } t' > t \end{cases}$$





$$S_F(x) = \theta(t)S^+(x) - \theta(-t)S^-(x) = \frac{1}{(2\pi)^4}\int d^4p\, e^{-ipx}\frac{\not{p}+m}{p^2-m^2+i\epsilon}$$

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Interactions

Example: $\partial_\mu \to D_\mu = \partial_\mu - ieQA_\mu$ in the Dirac Lagrangian

$$A_\mu(x) \to A_\mu(x) + \partial_\mu f(x)$$

$$\psi(x) \to e^{ieQf(x)}\psi(x)$$

Gauge interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$$

$$\mathcal{L}_0 = N\left[\bar{\psi}(x)\left[i\partial\!\!\!/ - m\right]\psi(x) - \frac{1}{2}\partial_\nu A_\mu(x)\partial^\nu A^\mu(x)\right]$$

$$\mathcal{L}_I = N\left[eQ\bar{\psi}(x)A\!\!\!/\,\psi(x)\right]$$

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$$

$$\mathcal{H}_I = -\mathcal{L}_I$$

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Interaction picture

$$i\frac{d}{dt}|\Phi(t)\rangle = H_I(t)|\Phi(t)\rangle \qquad H_I(t) = e^{iH_0(t-t_0)}H_I^S e^{-iH_0(t-t_0)}$$

Schroedinger picture: only the states are time-dependent

Interaction picture: states are constant if there are no interactions

Scattering matrix: $|\Phi(\infty)\rangle = S|\Phi(-\infty)\rangle = S|i\rangle$

Transition probability: $|\langle f|\Phi(\infty)\rangle|^2$

Conservation of probability
= Unitarity of the S-matrix

$$\sum_f |S_{fi}|^2 = 1, \quad \langle f|S|i\rangle \equiv S_{fi}$$

Complete orthonormal set of states

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S-Matrix expansion

$$i\frac{d}{dt}\left|\Phi(t)\right\rangle = H_I(t)\left|\Phi(t)\right\rangle$$

$$\left|\Phi(t)\right\rangle = \left|i\right\rangle + (-i)\int_{-\infty}^{t} dt_1 H_I(t_1)\left|\Phi(t_1)\right\rangle$$

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If $H_I(t)$ is small (**Perturbation**)

$$\left|\Phi(t)\right\rangle = \left|i\right\rangle + (-i)\int_{-\infty}^{t} dt_1 H_I(t_1)\left|i\right\rangle + (-i)^2\int_{-\infty}^{t} dt_1\int_{-\infty}^{t_1} dt_2 H_I(t_1)H_I(t_2)\left|\Phi(t_2)\right\rangle$$

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$$S = \sum_{n=0}^{\infty}(-i)^n \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \ldots \int_{-\infty}^{t_{n-1}} dt_n H_I(t_1)H_I(t_2)\ldots H_I(t_n)$$

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Earlier time on the right

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Boson/Fermion op. as it they commute/anti-com

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Wick's theorem

to contribute to a transition $|i\rangle \to |f\rangle$, the element of S should annihilate the particles in $|i\rangle$, create the particles in $|f\rangle$ and may contain particles that are created and then absorbed

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$$T\{A_1 A_2 \ldots A_n\} = N[A_1 A_2 \ldots A_n]$$

$$+N[\underbracket{A_1 A_2} A_3 \ldots A_{n-1} A_n] + N[\underbracket{A_1 A_2 A_3} \ldots A_{n-1} A_n] + \cdots + N[A_1 A_2 A_3 \ldots \underbracket{A_{n-1} A_n}]$$

$$+ N[\underbracket{A_1 A_2}\underbracket{A_3 A_4} \ldots A_{n-1} A_n] + \cdots + N[A_1 A_2 \ldots \underbracket{A_{n-3} A_{n-2}}\underbracket{A_{n-1} A_n}]$$

$$+ \ldots$$

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Non vanishing contraction are propagators

$$\underbracket{\phi(x_1)\phi(x_2)} = i\Delta_F(x_1 - x_2)$$

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$$T\{N[AB\dots]_{x_1} \dots N[AB\dots]_{x_n}\} = T\{(AB\dots)_{x_1} \dots (AB\dots)_{x_n}\}_{\text{no eq. time. contr.}}$$

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The Feynman diagrams

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int dx_1 \int dx_2 \dots \int dx_n T\left\{\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\dots\mathcal{H}_I(x_n)\right\}$$

$$= \sum_{n=0}^{\infty} S^{(n)}$$

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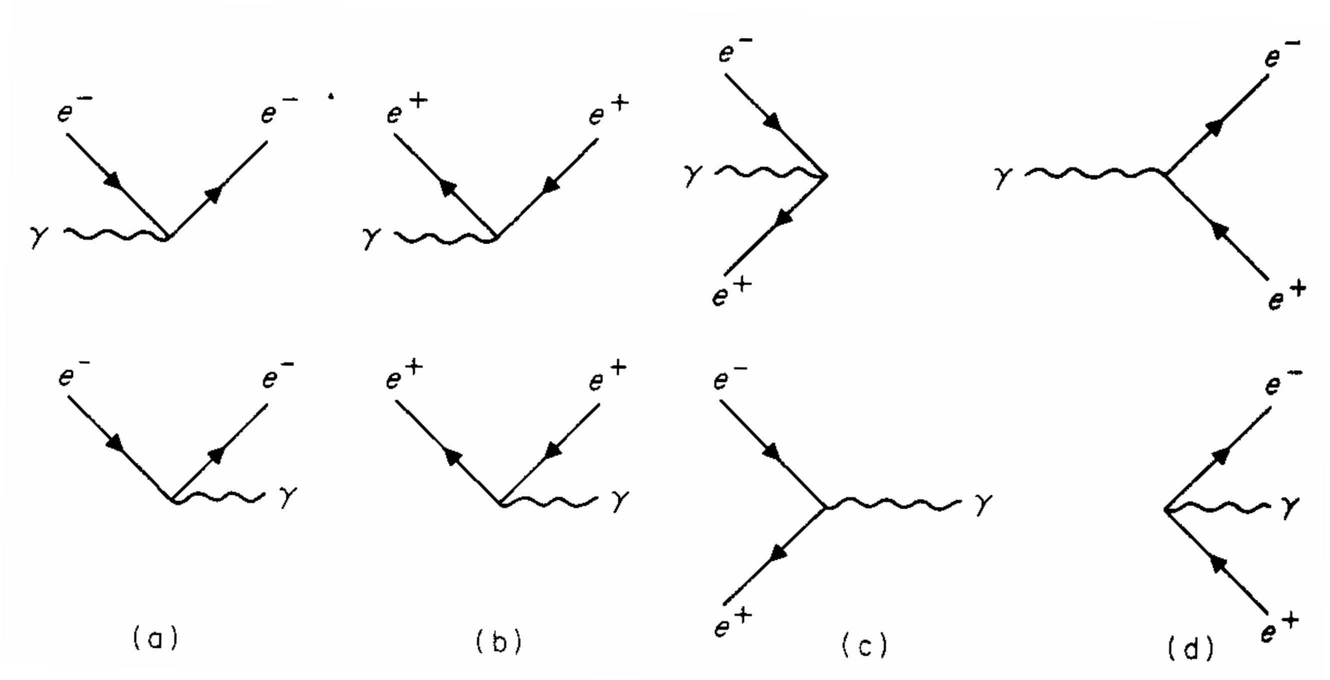
$$S^{(1)} = ieQ \int d^4x_1 N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\right] = ieQ \int d^4x_1 N\left[\left(\left(\bar{\psi}^+ + \bar{\psi}^-\right)\left(\not{A}^+ + \not{A}^-\right)\left(\psi^+ + \psi^-\right)\right)_{x_1}\right]$$



$$\langle f|\, S^{(1)}\, |i\rangle = 0$$

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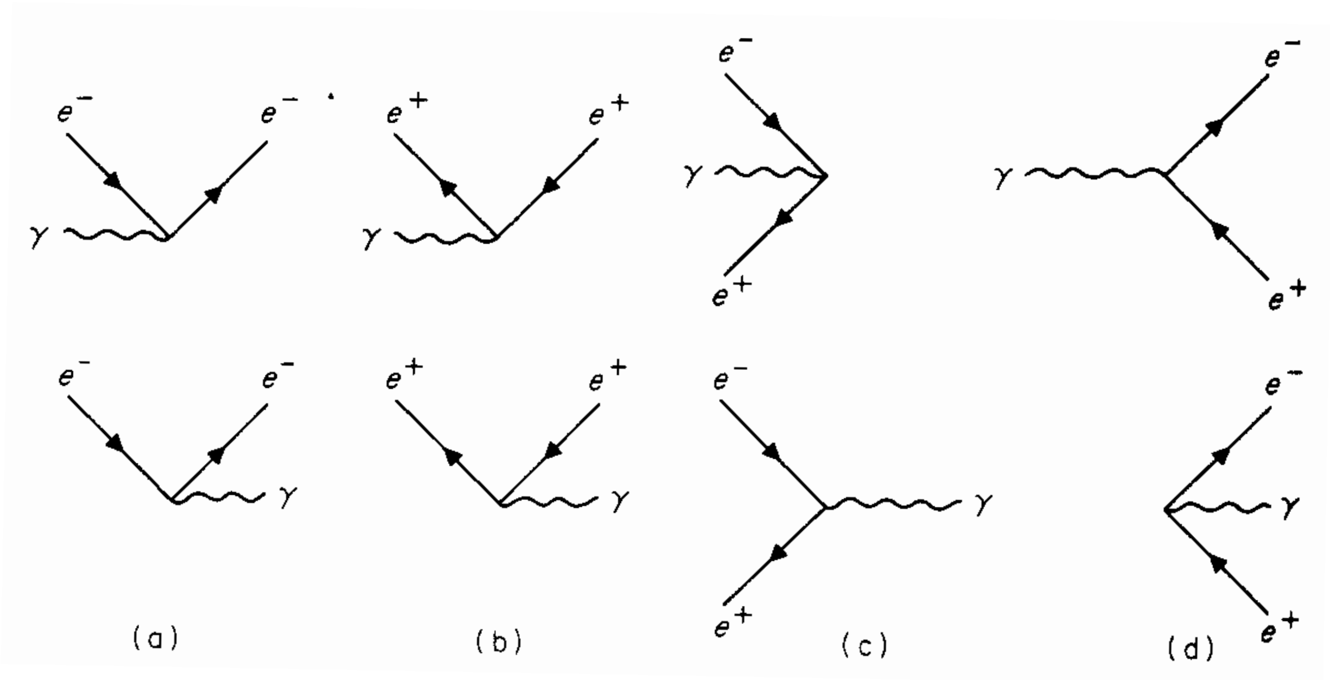
The Feynman diagrams

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int dx_1 \int dx_2 \dots \int dx_n T \left\{ \mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\dots\mathcal{H}_I(x_n) \right\}$$

$$= \sum_{n=0}^{\infty} S^{(n)}$$

$$S^{(1)} = ieQ \int d^4x_1 N \left[\left(\bar{\psi} \not{A} \psi \right)_{x_1} \right] = ieQ \int d^4x_1 N \left[\left(\left(\bar{\psi}^+ + \bar{\psi}^- \right) \left(\not{A}^+ + \not{A}^- \right) \left(\psi^+ + \psi^- \right) \right)_{x_1} \right]$$



No physical processes

$$\langle f | S^{(1)} | i \rangle = 0$$

C. Degrande

The Feynman diagrams

$$S^{(2)} = -\frac{(eQ)^2}{2!}\int\int d^4x_1 d^4x_2 N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right] + N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right] + N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

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$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

C. Degrande

The Feynman diagrams

$$S^{(2)} = -\frac{(eQ)^2}{2!}\int\int d^4x_1 d^4x_2 N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

Two diagrams like on the previous slide

$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right] + N\left[\left(\psi\not{A}\psi\right)_{x_1}\left(\psi\not{A}\psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right] + N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

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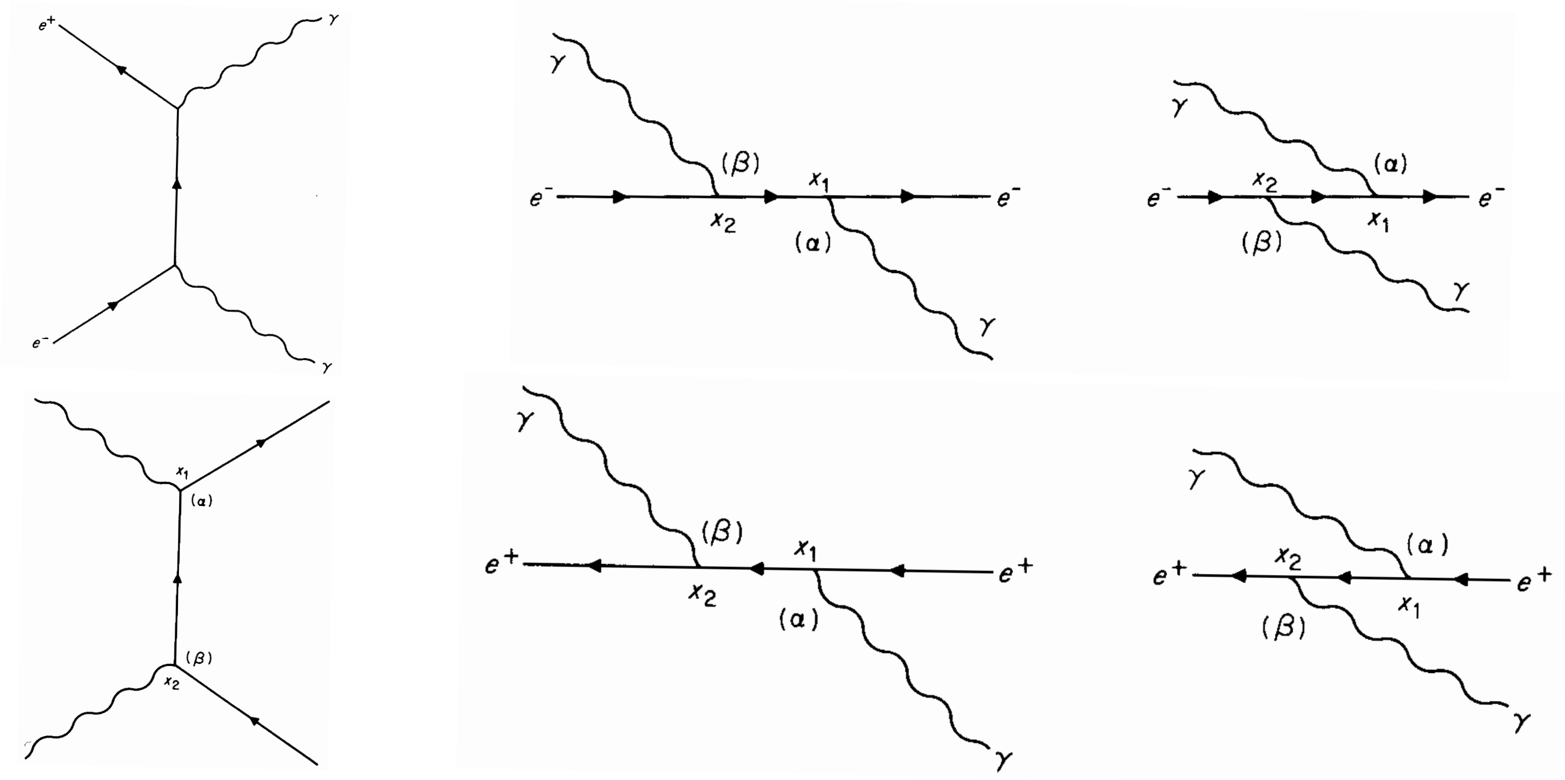
$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

C. Degrande

The Feynman diagrams

$$S^{(2)} = -\frac{(eQ)^2}{2!}\int\int d^4x_1 d^4x_2 N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]_{\psi_{x_1}\text{–}\bar{\psi}_{x_2}\ \text{contracted}} + N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]_{\bar{\psi}_{x_1}\text{–}\psi_{x_2}\ \text{contracted}}$$



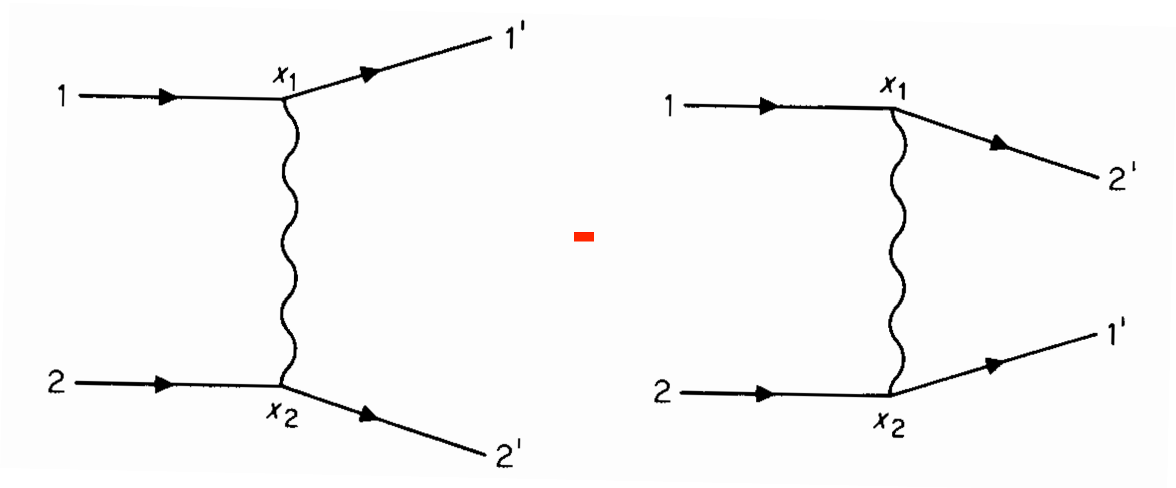
C. Degrande

The Feynman diagrams

$$S^{(2)} = -\frac{(eQ)^2}{2!}\int\int d^4x_1 d^4x_2 N\left[(\bar{\psi}\not{A}\psi)_{x_1}(\bar{\psi}\not{A}\psi)_{x_2}\right]$$

$$+ N\left[(\bar{\psi}\not{A}\underbrace{\psi)_{x_1}(\bar{\psi}}\not{A}\psi)_{x_2}\right] + N\left[(\underbrace{\bar{\psi}\not{A}\psi)_{x_1}(\bar{\psi}\not{A}\psi}_{})_{x_2}\right]$$

$$+ N\left[(\bar{\psi}\underbrace{\not{A}\psi)_{x_1}(\bar{\psi}\not{A}}\psi)_{x_2}\right]$$



Only topologically different diagrams

~~$+x_1 \leftrightarrow x_2$~~

$\overline{}$

~~2!~~

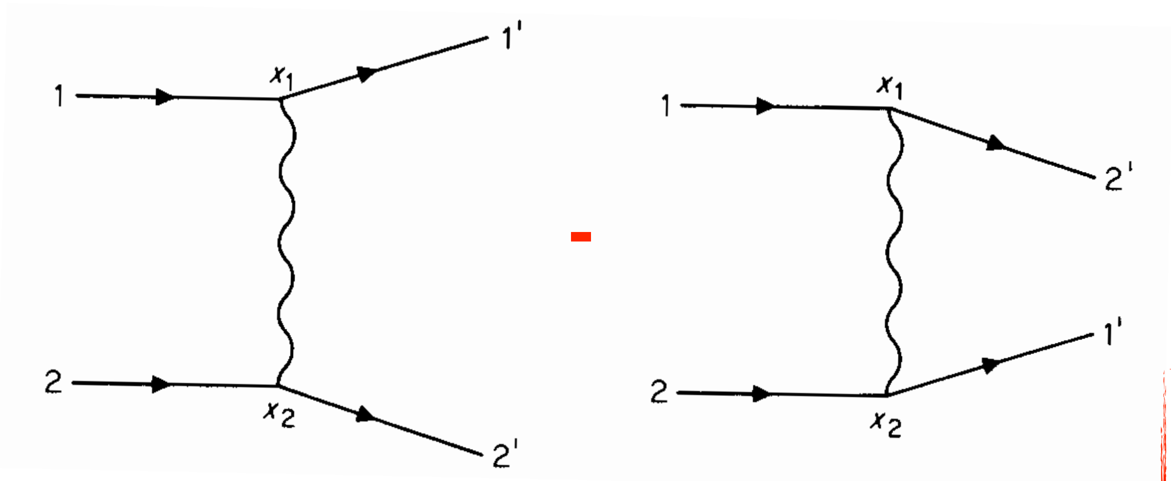
C. Degrande

The Feynman diagrams

$$S^{(2)} = -\frac{(eQ)^2}{2!}\int\int d^4x_1 d^4x_2 N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar{\psi}\not{A}\underbrace{\psi\right)_{x_1}\left(\bar{\psi}}\not{A}\psi\right)_{x_2}\right] + N\left[\left(\underbrace{\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi}\right)_{x_2}\right]$$

$$+ N\left[\left(\bar{\psi}\underbrace{\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}}\psi\right)_{x_2}\right]$$



Only topologically different diagrams

~~$+x_1 \leftrightarrow x_2$~~

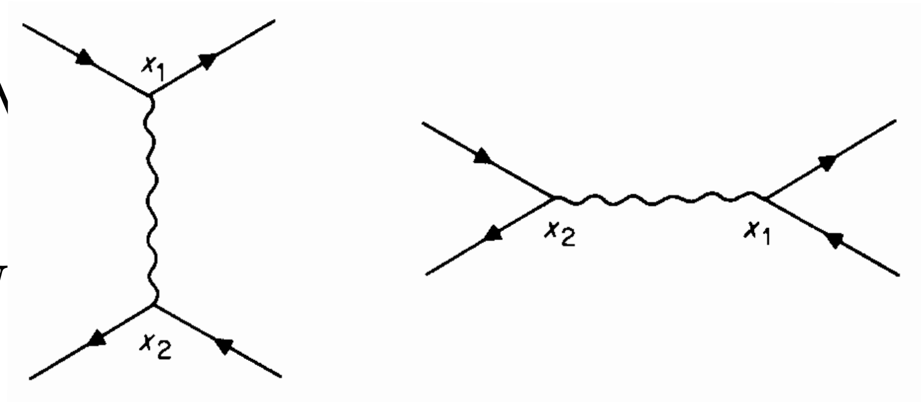
$$\langle 2'1'| = \langle 0|\, c(2')c(1')$$

~~2!~~

C. Degrande

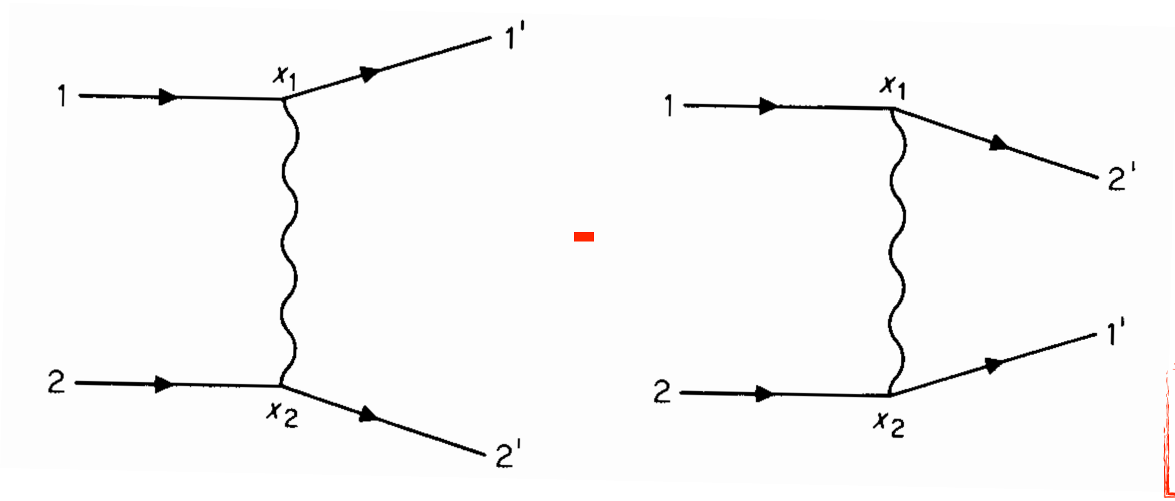
The Feynman diagrams

$$S^{(2)} = -\frac{(eQ)^2}{2!}\int\int d^4x_1 d^4x_2 N$$



$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right] + N$$

$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$



Only topologically different diagrams

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~~2!~~

C. Degrande

The Feynman diagrams

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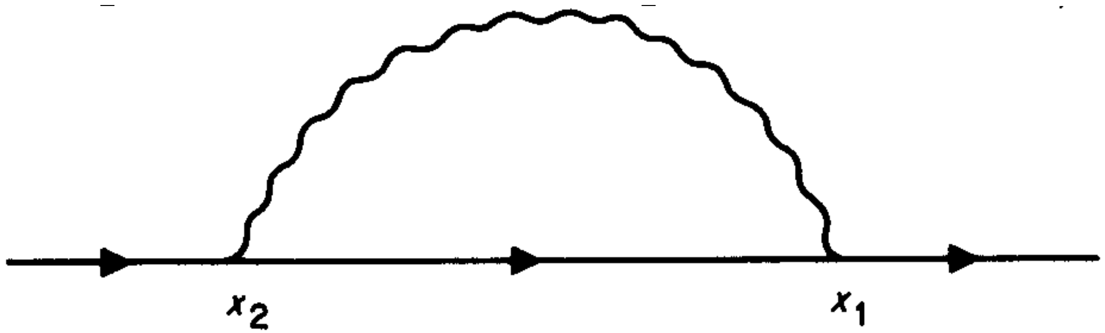
$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right] + N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right] + N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$



C. Degrande

The Feynman diagrams

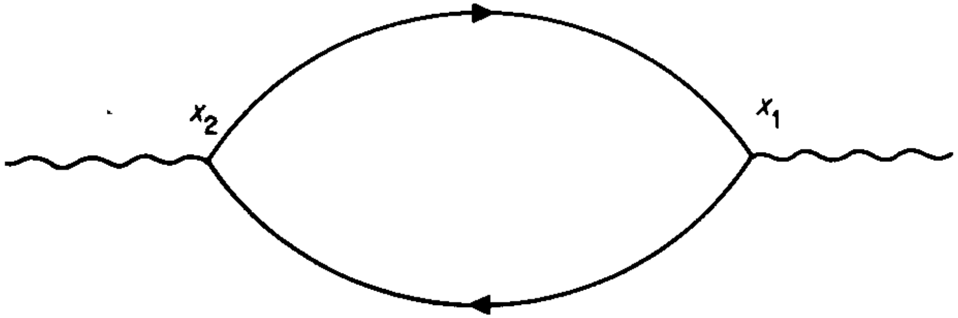
$$N\left[\left(\bar\psi_\alpha \not{A}^-_{\alpha\beta}\psi_\beta\right)_{x_1}\left(\bar\psi_\rho \not{A}^+_{\rho\sigma}\psi_\sigma\right)_{x_2}\right]$$

$$=(-1)\psi_\sigma(x_2)\bar\psi_\alpha(x_1)\not{A}^-_{\alpha\beta}(x_1)\psi_\beta(x_1)\bar\psi_\rho(x_2)\not{A}^+_{\rho\sigma}(x_2)$$

$$=(-1)Tr\left[iS_F(x_2-x_1)\not{A}^-(x_1)iS_F(x_1-x_2)\not{A}^+(x_2)\right]$$

$$+N\left[\left(\bar\psi\not{A}\psi\right)_{x_1}\left(\bar\psi\not{A}\psi\right)_{x_2}\right]$$

$$+N\left[\left(\bar\psi\not{A}\psi\right)_{x_1}\left(\bar\psi\not{A}\psi\right)_{x_2}\right]$$



C. Degrande

The Feynman diagrams

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$$+N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

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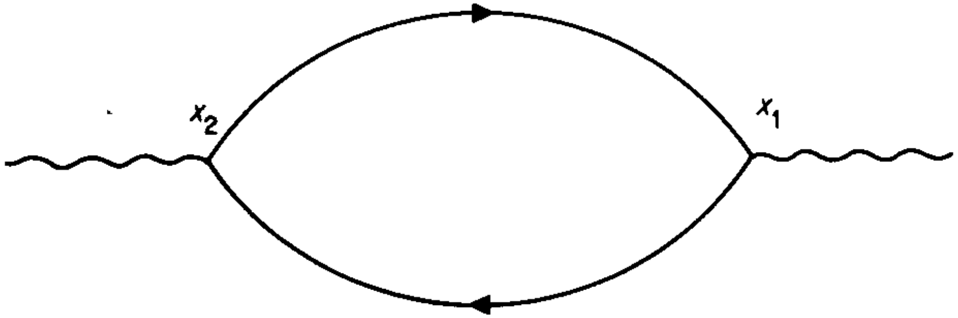

C. Degrande

The Feynman diagrams

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$$+ N\left[\left(\bar{\psi}\not{A}\psi\right)_{x_1}\left(\bar{\psi}\not{A}\psi\right)_{x_2}\right]$$

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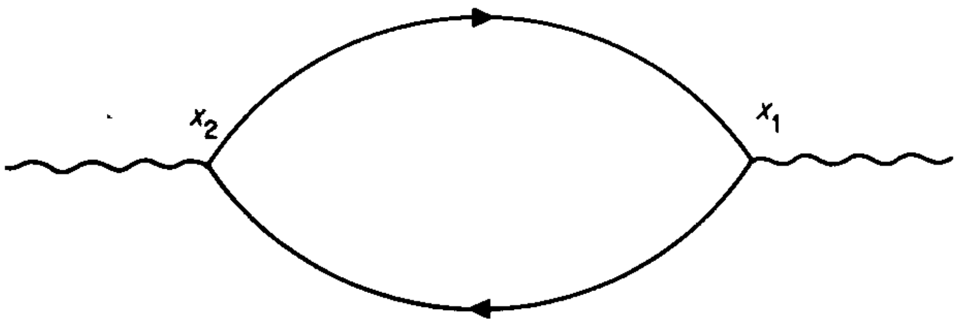

C. Degrande

The Feynman diagrams

$$S^{(2)} = -\frac{(eQ)^2}{2!}\int\int d^4x_1 d^4x_2 N\left[\left(\bar\psi \not{A} \psi\right)_{x_1}\left(\bar\psi \not{A} \psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar\psi \not{A} \psi\right)_{x_1}\left(\bar\psi \not{A} \psi\right)_{x_2}\right] + N\left[\left(\bar\psi \not{A} \psi\right)_{x_1}\left(\bar\psi \not{A} \psi\right)_{x_2}\right]$$

$$+ N\left[\left(\bar\psi \not{A} \psi\right)_{x_1}\left(\bar\psi \not{A} \psi\right)_{x_2}\right]$$

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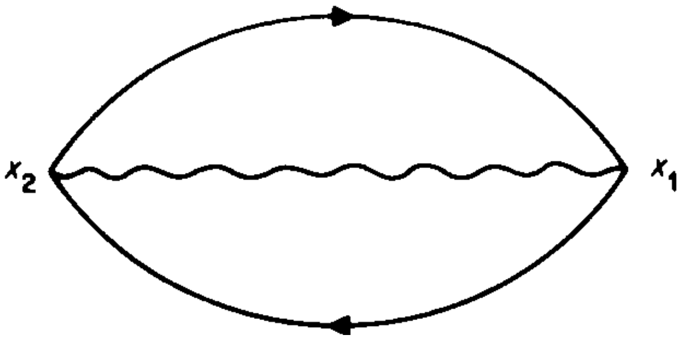


C. Degrande

Feynman Rules

- In momentum space

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

C. Degrande

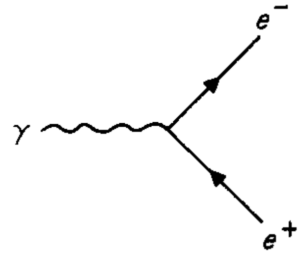
Feynman Rules

- In momentum space

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

  - For each vertex

$$-ieQ\gamma^\mu$$



C. Degrande

Feynman Rules

- In momentum space

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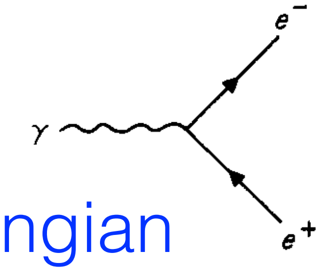

Extracted by FeynRules from the Lagrangian

Used in MadGraph5_aMC@NLO

Feynman Rules

- In momentum space

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

  - For each vertex

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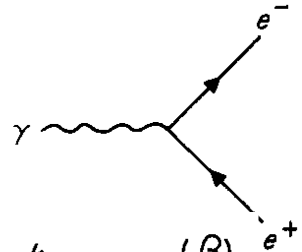
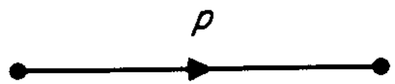


  - For each internal line

$$\frac{-i\eta_{\alpha\beta}}{k^2 + i\epsilon}$$

$$\frac{i}{\not{p} - m + i\epsilon}$$



C. Degrande

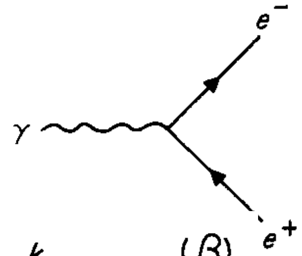
Feynman Rules

- In momentum space

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

  - For each vertex

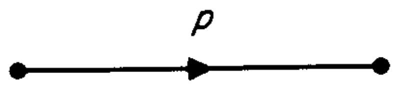
$$-ieQ\gamma^\mu$$



  - For each internal line

$$\frac{-i\eta_{\alpha\beta}}{k^2 + i\epsilon}$$

$$\frac{i}{\not{p} - m + i\epsilon}$$



Can be checked in FeynRules

Assumed in MadGraph5_aMC@NLO

C. Degrande

Feynman Rules

- In momentum space

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

  - For each vertex

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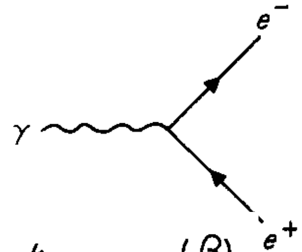

  - For each internal line

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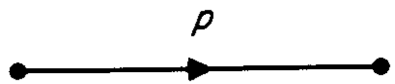
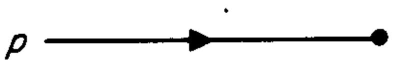
$$\frac{i}{\not p - m + i\epsilon}$$



  - For each external line

$u(\mathbf{p})$



$v(\mathbf{p})$

$\bar{u}(\mathbf{p})$

$\bar{v}(\mathbf{p})$

$\varepsilon(\mathbf{k})$

$\varepsilon^*(\mathbf{k})$

C. Degrande

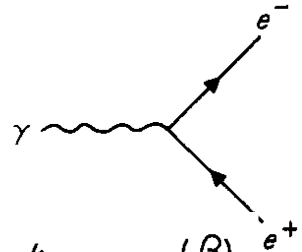
Feynman Rules

- In momentum space

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

- For each vertex

$$-ieQ\gamma^{\mu}$$



- For each internal line

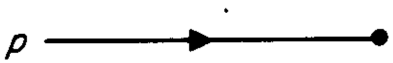
$$-i\eta_{\alpha\beta}$$

Can be checked in FeynRules

Assumed in MadGraph5_aMC@NLO

- For each external line

$u(\mathbf{p})$



$v(\mathbf{p})$

$\bar{u}(\mathbf{p})$

$\bar{v}(\mathbf{p})$

$\varepsilon(\mathbf{k})$

$\varepsilon^*(\mathbf{k})$

C. Degrande

Feynman Rules

- In momentum space

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

  - Spinor factor ordered against the fermion flow
  - Close fermion loop : -1 and Trace
  - For each loop, integration over the momentum not fixed by momentum conservation

$$\int \frac{d^4 p}{(2\pi)^4}$$

  - -1 for each exchange of fermions operators

C. Degrande

Feynman Rules

- In momentum space

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

In MG5_aMC@NLO

- Spinor factor ordered against the fermion flow
- Close fermion loop : -1 and Trace
- For each loop, integration over the momentum not fixed by momentum conservation

$$\int \frac{d^4p}{(2\pi)^4}$$

- -1 for each exchange of fermions operators

C. Degrande