DHELAS subroutines for Higgs effective couplings

Rikkert Frederix

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Abstract

Here are all the new DHELAS subroutines needed for the computation of the Higgs effective couplings to gluons. This includes all vertices of one or two (pseudo-)scalar Higgs's connected to two, three and four gluons. We used non-propagating internal particles to write the higher-then-four-point effective couplings in terms of three- and four-point couplings.

Contents

1 Effective Lagrangian for the (pseudo-)scalar Higgs's to gluons

The (pseudo-)scalar effective couplings to massless vector bosons can be included in the Standard Model by adding the following terms to the Lagrangian

$$\mathcal{L}^1_{(P)SC} = -\frac{1}{4} g_H \Phi G^a_{\mu\nu} G^a_{\mu\nu} + g_A \Phi_A G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}, \tag{1}$$

where g_h and g_A are the coupling constants, Φ is the scalar boson, Φ_A is the pseudo-scalar boson and $G^a_{\mu\nu}$ is the field strength for a massless vector boson

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{2}$$

and $\tilde{G}^a_{\mu\nu}$ is the dual of $G^a_{\mu\nu}$

$$\tilde{G}^a_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}. \tag{3}$$

For the two (pseudo-)scalar effective couplings we add the following terms to the Lagrangian

$$\mathcal{L}^2_{(P)SC} = -\frac{1}{8v} g_H (\Phi\Phi + \Phi_A \Phi_A) G^a_{\mu\nu} G^a_{\mu\nu} + \frac{1}{2v} g_A \Phi \Phi_A G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}, \tag{4}$$

where v is the vacuum expectation value of the scalar.

2 DHELAS subroutines for one (pseudo-)scalar effective couplings

These are the new DHELAS subroutines needed for the (pseudo-)scalar Higgs effective couplings. Of course, they can be used to compute other diagrams or (effective) couplings with the same kinematical structure.

2.1 Vertices I: VVSH

The vector-vector-scalar (VVSH) vertex subroutines are defined by:

$$\mathcal{L}_{VVSH} = -\frac{1}{4} g_h \Phi \big(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu\big)^2 + g_A \Phi_A \epsilon^{\mu\nu\rho\sigma} (\partial_\mu G^a_\nu)(\partial_\rho G^a_\sigma), \tag{5}$$

where g_h and g_A are the coupling constants, Φ is the scalar boson, Φ_A is the pseudo-scalar boson and G^a_μ is a *massless* vector boson.

2.1.1 VVSHXX

This subroutine computes the amplitude of the vector-vector-scalar bosons vertex. This subroutine will be called as

CALL VVSHXX(GA,GB,SC,GH, VERTEX).

We have four inputs GA, GB, SC and GH and one output VERTEX. Note that the ordering of the massless vector bosons is important. Different ordering can lead to a difference in sign for the pseudo-scalar part.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first vector boson and its four-momentum.

2. complex GB(6). This is a complex 6 dimensional array which contains the wavefunction of the second vector boson and its four-momentum.

3. complex SC(3). This is a complex 3 dimensional array which contains the scalar boson wavefunction and its four-momentum.

4. complex GH(2). This is a complex 2 dimensional array which contains the coupling of the (pseudo-) scalar boson with the two massless vector bosons. The first element, GH(1), is the coupling constant of the scalar and the second element, GH(2), the coupling constant of the pseudo-scalar with the massless vector bosons.

The outputs:

1. complex VERTEX. This is a complex number which is the amplitude of the vector-vector-scalar vertex.

What this subroutine computes here is the following T-matrix:

$$\mathrm{VERTEX} = g_h\, S\,(-G_1 \cdot G_2 p \cdot q - G_1 \cdot q\, G_2 \cdot p) + g_A S\, G_1^\mu G_2^\nu \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma, \tag{6}$$

where we used the notation

$$g_h = \mathrm{GH}(1) \tag{7}$$
$$g_A = \mathrm{GH}(2) \tag{8}$$
$$G_1^\mu = \mathrm{GA}(\mu+1) \tag{9}$$
$$G_2^\mu = \mathrm{GB}(\mu+1) \tag{10}$$
$$S = \mathrm{SC}(1) \tag{11}$$
$$p^\mu = (\Re e\mathrm{GA}(5), \Re e\mathrm{GA}(6), \Im m\mathrm{GA}(6), \Im m\mathrm{GA}(5)) \tag{12}$$
$$q^\mu = (\Re e\mathrm{GB}(5), \Re e\mathrm{GB}(6), \Im m\mathrm{GB}(6), \Im m\mathrm{GB}(5)). \tag{13}$$

2.1.2 JVSHXX

This subroutine computes an off-shell massless vector boson current from the vector-vector-scalar vertex attached with the vector boson propagator, from the scalar and the other vector boson. This subroutine will be called as

CALL JVSHXX(GA,SC,GH,XM,XW, JVSH)

We have five inputs GA, SC, GH, XM and XW and one output JVSH.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the vector boson and its four-momentum.

2. complex SC(3). This is a complex 3 dimensional array which contains the scalar wavefunction and its four-momentum.

3. complex GH(2). This is a complex 2 dimensional array which contains the coupling of the (pseudo-) scalar boson with the two massless vector bosons. The first element, GH(1), is the coupling constant of the scalar and the second element, GH(2), the coupling constant of the pseudo-scalar with the massless vector bosons.

4. real XM. This real number is not used.

5. real XW. This real number is not used.

The outputs:

1. complex JVSH(6). This is a complex six-dimensional array which contains the off-shell massless vector boson current from the vector-vector-scalar vertex attached with the massless vector boson propagator, combined with its four-momentum.

What this subroutine computes is the following portion of the Feynman amplitude:

$$j^\nu = \frac{1}{q^2}\, S\Big(- g_h(q\cdot p\, G_1^\nu + q\cdot G_1 p^\nu) + g_A \eta^{\nu\tau} G_1^\mu \epsilon_{\mu\tau\rho\sigma} p^\rho q^\sigma \Big), \tag{14}$$

where

$$\mathrm{JVSH}(5) = \mathrm{GA}(5) + \mathrm{SC}(2), \tag{15}$$
$$\mathrm{JVSH}(6) = \mathrm{GA}(6) + \mathrm{SC}(3). \tag{16}$$

Here we used the notation

$$j^\mu = \mathrm{JVSH}(\mu+1) \tag{17}$$
$$g_h = \mathrm{GH}(1) \tag{18}$$
$$g_A = \mathrm{GH}(2) \tag{19}$$
$$\eta^{\nu\tau} = \mathrm{diag}[1,-1,-1,-1] \tag{20}$$
$$G_1^\mu = \mathrm{GA}(\mu+1) \tag{21}$$
$$S = \mathrm{SC}(1) \tag{22}$$
$$p^\mu = (\Re e\mathrm{GA}(5), \Re e\mathrm{GA}(6), \Im m\mathrm{GA}(6), \Im m\mathrm{GA}(5)) \tag{23}$$
$$q^\mu = -(\Re e\mathrm{JVSH}(5), \Re e\mathrm{JVSH}(6), \Im m\mathrm{JVSH}(6), \Im m\mathrm{JVSH}(5)). \tag{24}$$

2.1.3 HVVHXX

This subroutine computes the off-shell (pseudo-)scalar current from the vector-vector-scalar vertex attached with the scalar propagator, from the two vector-bosons. This subroutine will be called as

CALL HVVHXX(GA,GB,GH,MASS,WIDTH, HVVH)

We have five inputs GA, GB, GH, MASS and WIDTH and one output HVVH. Note that the ordering of the massless scalar bosons is important. A different ordering leads to a different sign for the pseudo-scalar part.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first vector boson and its four-momentum.

2. complex GB(6). This is a complex 6 dimensional array which contains the wavefunction of the second vector boson and its four-momentum.

3. complex GH(2). This is a complex 2 dimensional array which contains the coupling of the (pseudo-) scalar boson with the two massless vector bosons. The first element, GH(1), is the coupling constant of the scalar and the second element, GH(2), the coupling constant of the pseudo-scalar with the massless vector bosons.

4. real MASS. This real number is the mass of the outgoing scalar current.

5. real WIDTH. This real number is the width of the outgoing scalar current.

The outputs:

1. complex HVVH(3). This is a complex three-dimensional array which contains the off-shell scalar current from the vector-vector-scalar vertex attached with the scalar propagator, combined with its four-momentum.

What this subroutine computes is the following portion of the Feynman amplitude:

$$\mathrm{HVVH}(1) = \frac{1}{q^2 - m^2 + im\Gamma} \Big(g_h (G_1 \cdot G_2 \, p_1 \cdot p_2 - G_1 \cdot p_2 \, G_2 \cdot p_1) - g_A G_1^\mu G_2^\nu \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma \Big), \tag{25}$$

where

$$\mathrm{HVVH}(2) = \mathrm{GA}(5) + \mathrm{GB}(5), \tag{26}$$

$$\mathrm{HVVH}(2) = \mathrm{GA}(6) + \mathrm{GB}(6). \tag{27}$$

Here we used the notation

$$g_h = \mathrm{GH}(1) \tag{28}$$

$$g_A = \mathrm{GH}(2) \tag{29}$$

$$G_1^\mu = \mathrm{GA}(\mu + 1) \tag{30}$$

$$G_2^\mu = \mathrm{GB}(\mu + 1) \tag{31}$$

$$p_1^\mu = (\Re e\mathrm{GA}(5), \Re e\mathrm{GA}(6), \Im m\mathrm{GA}(6), \Im m\mathrm{GA}(5)) \tag{32}$$

$$p_2^\mu = (\Re e\mathrm{GB}(5), \Re e\mathrm{GB}(6), \Im m\mathrm{GB}(6), \Im m\mathrm{GB}(5)) \tag{33}$$

$$q^\mu = -(\Re e\mathrm{HVVH}(2), \Re e\mathrm{HVVH}(3), \Im m\mathrm{HVVH}(3), \Im m\mathrm{HVVH}(2)) \tag{34}$$

$$m = \mathrm{MASS} \tag{35}$$

$$\Gamma = \mathrm{WIDTH}. \tag{36}$$

2.2 Vertices II: VVVS

The three-vector-scalar vertex (VVVS) is defined by

$$\mathcal{L}_{VVVS} = -g g_h f_{abc} \Phi (\partial_\mu G_\nu^a) G^{\mu\, b} G^{\nu\, c} + g g_A f_{abc} \Phi_A \epsilon^{\mu\nu\rho\sigma} (\partial_\mu G_\nu^a) G_\rho^b G_\sigma^c, \tag{37}$$

where g is the first coupling constant, g_h is the coupling constant for the scalar boson Φ to the vector bosons and g_A is the coupling constant for the pseudo-scalar boson Φ_A to the vector bosons. G_μ^a is a *massless* vector boson.

2.2.1 VVVSXX

This subroutine computes an amplitude from the three-vector-scalar vertex. This subroutine will be called as

CALL VVVSXX(GA,GB,GC,SC,G1,G2, VERTEX)

We have six inputs GA, GB, GC, SC, G1 and G2 and one output VERTEX. Note that the ordering of the massless vectors (i.e; GA, GB and GC) is important. Changing the order can lead to a change in sign.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first massless vector boson and its four-momentum.

2. complex GB(6). This is a complex 6 dimensional array which contains the wavfunction of the second massless vector boson and its four-momentum.

3. complex GC(6). This is a complex 6 dimensional array which contains the wavefunction of the third massless vector boson and its four-momentum.

4. complex SC(3). This is a complex 3 dimensional array which contains the scalar boson wavefunction and its four-momentum.

5. real G1. This real number is the first coupling constant.

6. complex G2(2). This is a complex two dimensional array which contains part of the coupling constant for the coupling of the (pseudo-)scalar boson with the three massless vector bosons. The first element, G2(1), is the coupling constant of the scalar and the second element, G2(2), the coupling constant of the pseudo-scalar with the massless vector bosons.

The outputs:

1. complex VERTEX. This complex number is the amplitude of the three-vector-scalar vertex.

What this subroutine computes is the following T-matrix:

$$\begin{aligned}\text{VERTEX} = gS\Big[g_h\Big(&G_1\cdot G_2(p_1\cdot G_3 - p_2\cdot G_3) + G_2\cdot G_3(p_2\cdot G_1 - p_3\cdot G_1) + \\ &+ G_3\cdot G_1(p_3\cdot G_2 - p_1\cdot G_2)\Big) + g_A G_1^\mu G_2^\nu G_3^\rho \epsilon_{\mu\nu\rho\sigma}(p_1+p_2+p_3)^\sigma\Big]\end{aligned} \tag{38}$$

where we used the notation:

$$g = \text{G1} \tag{39}$$
$$g_h = \text{G2(1)} \tag{40}$$
$$g_A = \text{G2(2)} \tag{41}$$
$$G_1^\mu = \text{GA}(\mu+1) \tag{42}$$
$$G_2^\mu = \text{GB}(\mu+1) \tag{43}$$
$$G_3^\mu = \text{GC}(\mu+1) \tag{44}$$
$$S = \text{SC(1)} \tag{45}$$
$$p_1^\mu = (\Re e\text{GA(5)}, \Re e\text{GA(6)}, \Im m\text{GA(6)}, \Im m\text{GA(5)}) \tag{46}$$
$$p_2^\mu = (\Re e\text{GB(5)}, \Re e\text{GB(6)}, \Im m\text{GB(6)}, \Im m\text{GB(5)}) \tag{47}$$
$$p_3^\mu = (\Re e\text{GC(5)}, \Re e\text{GC(6)}, \Im m\text{GC(6)}, \Im m\text{GC(5)}). \tag{48}$$

2.2.2 JVVSXX

This subroutine computes an off-shell massless vector boson current from the three-vector-scalar vertex attached with the vector boson propagator, from the scalar boson and the other two vector bosons. This subroutine will be called as

CALL JVVSXX(GA,GB,SC,G1,G2,XM,XW, JVVS)

We have seven inputs GA, GB, SC, G1, G2, XM and XW and one output JVVS. Note that the ordering of the vector boson inputs (i.e. GA and GB) is important. Changing the order can lead to a change in sign.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first massless vector boson and its four-momentum.

2. complex GB(6). This is a complex 6 dimensional array which contains the wavefunction of the second massless vector boson and its four-momentum.

3. complex SC(3). This is a complex 3 dimensional array which contains the scalar boson wavefunction and its four-momentum.

4. real G1. This is a real number which is the first coupling constant.
5. complex G2(2). This is a complex 2 dimensional array which contains part of the coupling constant for the coupling of the (pseudo-) scalar boson with the three massless vector bosons. The first element, G2(1), is the coupling constant of the scalar and the second element, G2(2), the coupling constant of the pseudo-scalar with the massless vector bosons.
6. real XM. This real number is not used.
7. real XW. This real number is not used.

The outputs:

1. complex JVVS(6). This is a complex six-dimensional array which contains the off-shell massless vector boson current from the three-vector-scalar vertex attached with the vector boson propagator, combined with its four-momentum.

What this subroutine computes is the following portion of the Feynman amplitude:

$$j^\mu = g\frac{1}{q^2}\,S\Big[g_h\Big(G_1\cdot G_2(p_1^\mu - p_2^\mu) + (p_2-q)\cdot G_1 G_2^\mu + (q-p_1)\cdot G_2 G_1^\mu\Big) + g_A \eta^{\mu\tau} G_1^\nu G_2^\rho \epsilon_{\mu\tau\rho\sigma}(p_1+p_2+q)^\sigma\Big] \tag{49}$$

where

$$\text{JVVS}(5) = \text{GA}(5) + \text{GB}(5) + \text{SC}(2), \tag{50}$$
$$\text{JVVS}(6) = \text{GA}(6) + \text{GB}(6) + \text{SC}(3). \tag{51}$$

Here we used the notation

$$j^\mu = \text{JVVS}(\mu+1) \tag{52}$$
$$g = \text{G1} \tag{53}$$
$$g_h = \text{G2}(1) \tag{54}$$
$$g_A = \text{G2}(2) \tag{55}$$
$$\eta^{\nu\tau} = \text{diag}[1,-1,-1,-1] \tag{56}$$
$$G_1^\mu = \text{GA}(\mu+1) \tag{57}$$
$$G_2^\mu = \text{GB}(\mu+1) \tag{58}$$
$$S = \text{SC}(1) \tag{59}$$
$$p_1^\mu = (\Re e\text{GA}(5), \Re e\text{GA}(6), \Im m\text{GA}(6), \Im m\text{GA}(5)) \tag{60}$$
$$p_2^\mu = (\Re e\text{GB}(5), \Re e\text{GB}(6), \Im m\text{GB}(6), \Im m\text{GB}(5)) \tag{61}$$
$$q^\mu = -(\Re e\text{JVVS}(5), \Re e\text{JVVS}(6), \Im m\text{JVVS}(6), \Im m\text{JVVS}(5)). \tag{62}$$

2.2.3 HVVVXX

This subroutine computes an off-shell (pseudo-)scalar boson current from the three-vector-scalar vertex attached with the scalar propagator, from the three massless vector bosons. This subroutine will be called as

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CALL HVVVXX(GA,GB,GC,G1,G2,MASS,WIDTH, HVVV)
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We have seven inputs GA, GB, GC, G1, G2, MASS and WIDTH and one output HVVV. Note that the ordering of the vector boson inputs (i.e. GA, GB and GC) is important. Changing the order can lead to a change in sign.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first massless vector boson and its four-momentum.

2. complex GB(6). This is a complex 6 dimensional array which contains the wavefunction of the second massless vector boson and its four-momentum.

3. complex GC(6). This is a complex 6 dimensional array which contains the wavefunction of the third massless vector boson and its four-momentum.

4. real G1. This is a real number which is the first coupling constant.

5. complex G2(2). This is a complex 2 dimensional array which contains part of the coupling constant for the coupling of the (pseudo-) scalar boson with the three massless vector bosons. The first element, G2(1), is the coupling constant of the scalar and the second element, G2(2), the coupling constant of the pseudo-scalar with the massless vector bosons.

6. real MASS. This real number is the mass of the scalar boson.

7. real WIDTH. This real number is the width of the scalar boson.

The outputs:

1. complex HVVV(3). This is a complex three-dimensional array which contains the off-shell scalar boson current from the three-vector-scalar vertex attached with the scalar boson propagator, combined with its four-momentum.

What this subroutine computes is the following portion of the Feynman amplitude:

$$\mathrm{HVVV}(1) = -g\frac{1}{q^2 - m^2 + im\Gamma}\Big[g_h\Big(G_1 \cdot G_2(p_1 - p_2) \cdot G_3 + G_2 \cdot G_3(p_2 - p_3) \cdot G_1 + \\ + G_1 \cdot G_3(p_3 - p_1) \cdot G_2\Big) + g_A G_1^\mu G_2^\nu G_3^\rho \epsilon_{\mu\nu\rho\sigma}(p_1 + p_2 + p_3)^\sigma\Big], \quad (63)$$

where

$$\mathrm{HVVV}(2) = \mathrm{GA}(5) + \mathrm{GB}(5) + \mathrm{GC}(5), \quad (64)$$
$$\mathrm{HVVV}(3) = \mathrm{GA}(6) + \mathrm{GB}(6) + \mathrm{GC}(6). \quad (65)$$

Here we used the notation

$$g = \mathrm{G1} \quad (66)$$
$$g_h = \mathrm{G2(1)} \quad (67)$$
$$g_A = \mathrm{G2(2)} \quad (68)$$
$$G_1^\mu = \mathrm{GA}(\mu + 1) \quad (69)$$
$$G_2^\mu = \mathrm{GB}(\mu + 1) \quad (70)$$
$$G_3^\mu = \mathrm{GC}(\mu + 1) \quad (71)$$
$$p_1^\mu = (\Re e\mathrm{GA}(5), \Re e\mathrm{GA}(6), \Im m\mathrm{GA}(6), \Im m\mathrm{GA}(5)) \quad (72)$$
$$p_2^\mu = (\Re e\mathrm{GB}(5), \Re e\mathrm{GB}(6), \Im m\mathrm{GB}(6), \Im m\mathrm{GB}(5)) \quad (73)$$
$$p_3^\mu = (\Re e\mathrm{GC}(5), \Re e\mathrm{GC}(6), \Im m\mathrm{GC}(6), \Im m\mathrm{GC}(5)) \quad (74)$$
$$q^\mu = -(\Re e\mathrm{HVVV}(2), \Re e\mathrm{HVVV}(3), \Im m\mathrm{HVVV}(3), \Im m\mathrm{HVVV}(2)) \quad (75)$$
$$m = \mathrm{MASS} \quad (76)$$
$$\Gamma = \mathrm{WIDTH}. \quad (77)$$

2.3 Vertices III: VVT

The vector-vector-tensor vertex (VVT) is defined as 'half' the four-vector vertex in such a way that the sum of two vector-vector-tensor vertices connected by the tensor particle exchanged through s-, t- and u-channels is precisely equal to the four-vector vertex. Thus, the vector-vector-tensor interaction term of the Lagrangian is

$$\mathcal{L}_{VVT} = igf_{abc}G^a_\mu G^b_\nu T^{c\mu\nu}, \tag{78}$$

where g is the coupling constant, G^a_μ is a *massless* vector boson, and $T^{a\mu\nu}$ is the massless, non-propagating tensor particle.

2.3.1 VVTXX

This subroutine computes an amplitude from the vector-vector-tensor vertex. This subroutine will be called as

CALL VVTXXX(GA,GB,TC,G, VERTEX)

We have four inputs GA, GB, TC, and G and one output VERTEX.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first massless vector boson and its four-momentum.
2. complex GB(6). This is a complex 6 dimensional array which contains the wavefunction of the second massless vector boson and its four-momentum.
3. complex TC(18). This is a complex 18 dimensional array which contains the wavefunction of the tensor particle and its four-momentum.
4. complex G(2). This is a 2 dimensional array of which only the first component is used. G(1) is the coupling constant of the vector-vector-tensor vertex. G(2) is not used.

The outputs:

1. complex VERTEX. This complex number is the amplitude of the vector-vector-tensor vertex.

What this subroutine computes is the following T-matrix:

$$\text{VERTEX} = \frac{g}{\sqrt{2}} T_{\mu\nu}(G^\mu_1 G^\nu_2 - G^\nu_1 G^\mu_2) \tag{79}$$

where we used the notation:

$$g = \text{G}(1) \tag{80}$$
$$G_{1\mu} = \text{GA}(\mu + 1) \tag{81}$$
$$G_{2\mu} = \text{GB}(\mu + 1) \tag{82}$$
$$T_{\mu\nu} = \text{TC}(4\mu + \nu + 1). \tag{83}$$

2.3.2 JVTXX

This subroutine computes an off-shell massless vector boson current from the vector-vector-tensor vertex attached with the vertex propagator, from the tensor particle and the massless vector boson. This subroutine will be called as

CALL JVTXXX(GA,TC,G,XM,XW, JVT)

We have five inputs GA, TC, G, XM and XW and one output JVT.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the massless vector boson and its four-momentum.

2. complex TC(18). This is a complex 18 dimensional array which contains the wavefunction of the tensor particle and its four-momentum.

3. complex G(2). This is a 2 dimensional array of which only the first component is used. G(1) is the coupling constant of the vector-vector-tensor vertex. G(2) is not used.

4. real XM. This real number is not used.

5. real XW. This real number is not used.

The outputs:

1. complex JVT(6). This is a complex six-dimensional array which contains the off-shell massless vector boson current from the vector-vector-tensor vertex attached with the vector boson propagator, combined with its four-momentum.

What this subroutine computes is the following portion of the Feynman amplitude:

$$j^\mu = -\frac{g}{\sqrt{2}}\frac{1}{q^2}(G_\nu T^{\nu\mu} - T^{\mu\nu}G_\nu), \tag{84}$$

where

$$\text{JVT}(5) = \text{GA}(5) + \text{TC}(17), \tag{85}$$
$$\text{JVT}(6) = \text{GA}(6) + \text{TC}(18). \tag{86}$$

Here we used the notation

$$j^\mu = \text{JVT}(\mu + 1) \tag{87}$$
$$g = \text{G}(1) \tag{88}$$
$$G^\mu = \text{GA}(\mu + 1) \tag{89}$$
$$T^{\mu\nu} = \text{TC}(4\mu + \nu + 1) \tag{90}$$
$$q^\mu = -(\Re e\text{JVT}(17), \Re e\text{JVT}(18), \Im m\text{JVT}(18), \Im m\text{JVT}(17)). \tag{91}$$

2.3.3 UVVXXX

This subroutine computes an off-shell tensor current from the vector-vector-tensor vertex from the two massless vector bosons. This subroutine will be called as

CALL UVVXXX(GA,GB,G,XM,XW, UVV)

We have five inputs GA, GB, G, XM and XW and one output UVV.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first massless vector boson and its four-momentum.

2. complex GB(6). This is a complex 6 dimensional array which contains the wavefunction of the second massless vector boson and its four-momentum.

3. complex G(2). This is a 2 dimensional array of which only the first component is used. G(1) is the coupling constant of the vector-vector-tensor vertex. G(2) is not used.

4. real XM. This real number is not used.

5. real XW. This real number is not used.

The outputs:

1. complex UVV(18). This is a complex 18 dimensional array which contains the tensor particle current from the vector-vector-tensor vertex combined with its four-momentum. Note that the tensor particle does *not* propagate.

What this subroutine computes is the following portion of the Feynman amplitude:

$$j^{\mu\nu} = \frac{g}{\sqrt{2}}(G_1^\mu G_2^\nu - G_2^\mu G_1^\nu), \tag{92}$$

where we used the notation

$$j^{\mu\nu} = \mathrm{UVV}(4\mu + \nu + 1) \tag{93}$$
$$g = \mathrm{G}(1) \tag{94}$$
$$G_1^\mu = \mathrm{GA}(\mu + 1) \tag{95}$$
$$G_2^\mu = \mathrm{GB}(\mu + 1). \tag{96}$$

2.4 Vertices IV: TTS

The tensor-tensor-scalar vertex (TTS) is designed to be the scalar boson contribution to the four-vector-scalar vertex. Because the four-vector vertex can be written is two vector-vector-tensor vertices connected by the tensor particle, the scalar contribution to the four-vector-scalar vertex can be described by a tensor-tensor-scalar interaction

$$\mathcal{L}_{TTS} = g_h \Phi\, T_a^{\mu\nu} T^a_{\mu\nu}, \tag{97}$$

where Φ is the scalar boson and $T^a_{\mu\nu}$ the tensor particle.

2.4.1 TTSXXX

This subroutine computes an amplitude from the tensor-tensor-scalar vertex. This subroutine will be called as

CALL TTSXXX(TC1,TC2,SC,GH, VERTEX)

We have four inputs TC1, TC2, SC, and GH and one output VERTEX.

The inputs:

1. complex TC1(18). This is a complex 18 dimensional array which contains the wavefunction of the first tensor particle and its four-momentum.
2. complex TC2(18). This is a complex 18 dimensional array which contains the wavefunction of the second tensor particle and its four-momentum.
3. complex SC(3). This is a complex 3 dimensional array which contains the wavefunction of the scalar particle and its four-momentum.
4. complex GH(2). This is a two dimensional array, which contains the coupling constant. GH(1) is the coupling constant of the scalar boson with the two tensor particles and GH(2) is not used.

The outputs:

1. complex VERTEX. This complex number is the amplitude of the tensor-tensor-scalar vertex.

What this subroutine computes is the following T-matrix:

$$\mathrm{VERTEX} = -g_h\, S\, T_{1\mu\nu} T_2^{\mu\nu} \tag{98}$$

where we used the notation:

$$g_h = \mathrm{GH}(1) \tag{99}$$
$$S = \mathrm{SC}(1) \tag{100}$$
$$T_1^{\mu\nu} = \mathrm{TC1}(4\mu + \nu + 1) \tag{101}$$
$$T_2^{\mu\nu} = \mathrm{TC2}(4\mu + \nu + 1). \tag{102}$$

2.4.2 UTSXXX

This subroutine computes an off-shell tensor current from the tensor-tensor-scalar vertex from the other tensor particle and the scalar boson. This subroutine will be called as

CALL UTSXXX(TC,SC,GH,XM,XW, UTS)

We have five inputs TC, SC, GH, XM and XW and one output UTS.
The inputs:

1. complex TC(18). This is a complex 18 dimensional array which contains the wavefunction of the tensor particle and its four-momentum.
2. complex SC(3). This is a complex 3 dimensional array which contains the wavefunction of the scalar particle and its four-momentum.
3. complex GH(2). This is a two dimensional array, which contains the coupling constant. GH(1) is the coupling constant of the scalar boson with the two tensor particles and GH(2) is not used.
4. real XM. This real number is not used.
5. real XW. This real number is not used.

The outputs:

1. complex UTS(18). This is a complex 18 dimensional array which contains the tensor particle current from the tensor-tensor-scalar vertex combined with its four-momentum. Note that the tensor particle does *not* propagate.

What this subroutine computes is the following portion of the Feynman amplitude:

$$j^{\mu\nu} = -g_h \, S \, T^{\mu\nu}, \tag{103}$$

where we used the notation

$$g_h = \mathrm{GH}(1) \tag{104}$$
$$j^{\mu\nu} = \mathrm{UTS}(4\mu + \nu + 1) \tag{105}$$
$$T^{\mu\nu} = \mathrm{TC}(4\mu + \nu + 1) \tag{106}$$
$$S = \mathrm{SC}(1). \tag{107}$$

2.4.3 HTTXXX

This subroutine computes an off-shell scalar boson current from the tensor-tensor-scalar vertex from the two tensor particles. This subroutine will be called as

CALL HTTXXX(T1,T2,GH,MASS,WIDTH, HTT)

We have five inputs T1, T2, GH, MASS and WIDTH and one output HTT.
The inputs:

1. complex T1(18). This is a complex 18 dimensional array which contains the wavefunction of the first tensor particle and its four-momentum.
2. complex T2(18). This is a complex 18 dimensional array which contains the wavefunction of the second tensor particle and its four-momentum.
3. complex GH(2). This is a two dimensional array, which contains the coupling constant. GH(1) is the coupling constant of the scalar boson with the two tensor particles and GH(2) is not used.
4. real MASS. This real number is the mass of the outgoing scalar boson current.
5. real WIDTH. This real number is the width of the outgoing scalar boson current.

The outputs:

1. complex HTT(3). This is a complex three-dimensional array which contains the scalar boson current from the tensor-tensor-scalar vertex attached with the scalar boson propagator and combined with its four-momentum.

What this subroutine computes is the following portion of the Feynman amplitude:

$$\mathrm{HTT}(1) = g_h \frac{1}{q^2 - m^2 + im\Gamma} T_1^{\mu\nu} T_{2\mu\nu}, \tag{108}$$

where

$$\mathrm{HTT}(2) = \mathrm{T1}(17) + \mathrm{T1}(17), \tag{109}$$
$$\mathrm{HTT}(3) = \mathrm{T2}(18) + \mathrm{T2}(18). \tag{110}$$

Here we used the notation

$$g_h = \mathrm{GH}(1) \tag{111}$$
$$T_1^{\mu\nu} = \mathrm{T1}(4\mu + \nu + 1) \tag{112}$$
$$T_2^{\mu\nu} = \mathrm{T2}(4\mu + \nu + 1) \tag{113}$$
$$q^\mu = -(\Re e\mathrm{HTT}(2), \Re e\mathrm{HTT}(3), \Im m\mathrm{HTT}(3), \Im m\mathrm{HTT}(2)) \tag{114}$$
$$m = \mathrm{MASS} \tag{115}$$
$$\Gamma = \mathrm{WIDTH}. \tag{116}$$

3 DHELAS subroutines for two (pseudo-)scalar effective couplings

These are the new DHELAS subroutines needed for the two (pseudo-)scalar Higgs effective couplings. Of course, they can be used to compute other diagrams or (effective) couplings with the same kinematical structure.

3.1 Vertices V: VVSSH

The vector-vector-scalar-scalar (VVSSH) vertex subroutines are defined by:

$$\mathcal{L}_{VVSSH} = -\frac{1}{4}\big(g_{hh}\Phi\Phi + g_{AA}\Phi_A\Phi_A\big)\big(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu\big)^2 + g_{hA}\Phi\Phi_A \epsilon^{\mu\nu\rho\sigma}(\partial_\mu G^a_\nu)(\partial_\rho G^a_\sigma), \tag{117}$$

where g_{hh}, g_{hA} and g_{AA} are the coupling constants, Φ is the scalar boson, Φ_A is the pseudo-scalar boson and G^a_μ is a *massless* vector boson.

3.1.1 VVSSHX

This subroutine computes the amplitude of the vector-vector-scalar bosons vertex. This subroutine will be called as

CALL VVSSHXX(GA,GB,S1,S2,GHH ,VERTEX).

We have five inputs GA, GB, S1, S2 and GHH and one output VERTEX.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first vector boson and its four-momentum.
2. complex GB(6). This is a complex 6 dimensional array which contains the wavefunction of the second vector boson and its four-momentum.
3. complex S1(3). This is a complex 3 dimensional array which contains the first scalar boson wavefunction and its four-momentum.
4. complex S2(3). This is a complex 3 dimensional array which contains the second scalar boson wavefunction and its four-momentum.
5. complex GHH(2). This is a complex 2 dimensional array which contains the coupling constant for the coupling of the (pseudo-) scalar bosons with the massless vector bosons. The first element, GHH(1), is the coupling constant of the two scalars or two pseudo-scalars and the second element, GHH(2), the coupling constant of one scalar and one pseudo-scalar with the massless vector bosons.

The outputs:

1. complex VERTEX. This is a complex number which is the amplitude of the vector-vector-scalar-scalar vertex.

What this subroutine computes here is the following T-matrix:

$$\text{VERTEX} = S_1 S_2 \Bigg[g_h (G_1 \cdot G_2 p \cdot q - G_1 \cdot q \, G_2 \cdot p) - g_A G_1^\mu G_2^\nu \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma \Bigg], \tag{118}$$

where we used the notation

$$g_h = \text{GHH(1)} \tag{119}$$
$$g_A = \text{GHH(2)} \tag{120}$$
$$G_1^\mu = \text{GA}(\mu+1) \tag{121}$$
$$G_2^\mu = \text{GB}(\mu+1) \tag{122}$$
$$S_1 = \text{S1(1)} \tag{123}$$
$$S_2 = \text{S2(1)} \tag{124}$$
$$p^\mu = (\Re e\text{GA}(5), \Re e\text{GA}(6), \Im m\text{GA}(6), \Im m\text{GA}(5)) \tag{125}$$
$$q^\mu = (\Re e\text{GB}(5), \Re e\text{GB}(6), \Im m\text{GB}(6), \Im m\text{GB}(5)). \tag{126}$$

3.1.2 JVSSHX

This subroutine computes an off-shell massless vector boson current from the vector-vector-scalar-scalar vertex attached with the vector boson propagator, from the two scalars and the other vector boson. This subroutine will be called as

CALL JVSSHX(GA,S1,S2,GHH,XM,XW, JVSSH)

We have six inputs GA, S1, S2, GHH, XM and XW and one output JVSSH.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the vector boson and its four-momentum.
2. complex S1(3). This is a complex 3 dimensional array which contains the first scalar wavefunction and its four-momentum.
3. complex S2(3). This is a complex 3 dimensional array which contains the second scalar wavefunction and its four-momentum.
4. complex GHH(2). This is a complex 2 dimensional array which contains the coupling constant for the coupling of the (pseudo-) scalar bosons with the massless vector bosons. The first element, GHH(1), is the coupling constant of the two scalars or two pseudo-scalars and the second element, GHH(2), the coupling constant of one scalar and one pseudo-scalar with the massless vector bosons.
5. real XM. This real number is not used.
6. real XW. This real number is not used.

The outputs:

1. complex JVSSH(6). This is a complex six-dimensional array which contains the off-shell massless vector boson current from the vector-vector-scalar-scalar vertex attached with the massless vector boson propagator, combined with its four-momentum.

What this subroutine computes is the following portion of the Feynman amplitude:

$$j^\nu = \frac{1}{q^2}\, S_1 S_2 \left(g_h (q\cdot p\, G_1^\nu + q\cdot G_1 p^\nu) - \eta^{\nu\tau} g_A G_1^\mu \epsilon_{\mu\tau\rho\sigma} p^\rho q^\sigma \right), \tag{127}$$

where

$$\mathrm{JVSSH}(5) = \mathrm{GA}(5) + \mathrm{SC}(2), \tag{128}$$
$$\mathrm{JVSSH}(6) = \mathrm{GA}(6) + \mathrm{SC}(3). \tag{129}$$

Here we used the notation

$$j^\mu = \mathrm{JVSSH}(\mu+1) \tag{130}$$
$$g_h = \mathrm{GHH}(1) \tag{131}$$
$$g_A = \mathrm{GHH}(2) \tag{132}$$
$$\eta^{\nu\tau} = \mathrm{diag}[1,-1,-1,-1] \tag{133}$$
$$G_1^\mu = \mathrm{GA}(\mu+1) \tag{134}$$
$$S_1 = \mathrm{S1}(1) \tag{135}$$
$$S_2 = \mathrm{S2}(1) \tag{136}$$
$$p^\mu = (\Re e\mathrm{GA}(5), \Re e\mathrm{GA}(6), \Im m\mathrm{GA}(6), \Im m\mathrm{GA}(5)) \tag{137}$$
$$q^\mu = -(\Re e\mathrm{JVSSH}(5), \Re e\mathrm{JVSSH}(6), \Im m\mathrm{JVSSH}(6), \Im m\mathrm{JVSSH}(5)). \tag{138}$$

3.1.3 HVVSHX

This subroutine computes an off-shell (pseudo-)scalar boson current from the vector-vector-scalar-scalar vertex attached with the scalar propagator, from the two massless vector bosons and the other scalar. This subroutine will be called as

```
CALL HVVSHX(GA,GB,SC,GHH,MASS,WIDTH, HVVSH)
```

We have six inputs GA, GB, SC, GHH, MASS and WIDTH and one output HVVSH.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first massless vector boson and its four-momentum.
2. complex GB(6). This is a complex 6 dimensional array which contains the wavefunction of the second massless vector boson and its four-momentum.
3. complex SC(3). This is a complex 3 dimensional array which contains the wavefunction of the incoming scalar boson and its four-momentum.
4. complex GHH(2). This is a complex 2 dimensional array which contains the coupling constant for the coupling of the (pseudo-) scalar bosons with the massless vector bosons. The first element, GHH(1), is the coupling constant of the two scalars or two pseudo-scalars and the second element, GHH(2), the coupling constant of one scalar and one pseudo-scalar with the massless vector bosons.
5. real MASS. This real number is the mass of the scalar boson.
6. real WIDTH. This real number is the width of the scalar boson.

The outputs:

1. complex HVVSH(3). This is a complex three-dimensional array which contains the off-shell scalar boson current from the vector-vector-scalar-scalar vertex attached with the scalar boson propagator, combined with its four-momentum.

What this subroutine computes is the following portion of the Feynman amplitude:

$$\mathrm{HVVSH}(1) = S\frac{1}{q^2-m^2+im\Gamma}\Big[-g_h(G_1\cdot G_2 p_1\cdot p_2 - G_1\cdot p_2\, G_2\cdot p_1) + g_A G_1^\mu G_2^\nu \epsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\Big], \tag{139}$$

where

$$\mathrm{HVVSH}(2) = \mathrm{GA}(5)+\mathrm{GB}(5)+\mathrm{SC}(2), \tag{140}$$
$$\mathrm{HVVSH}(3) = \mathrm{GA}(6)+\mathrm{GB}(6)+\mathrm{SC}(3). \tag{141}$$

Here we used the notation

$$g_h = \mathrm{GHH}(1) \tag{142}$$
$$g_A = \mathrm{GHH}(2) \tag{143}$$
$$G_1^\mu = \mathrm{GA}(\mu+1) \tag{144}$$
$$G_2^\mu = \mathrm{GB}(\mu+1) \tag{145}$$
$$S = \mathrm{SC}(1) \tag{146}$$
$$p_1^\mu = (\Re e\mathrm{GA}(5), \Re e\mathrm{GA}(6), \Im m\mathrm{GA}(6), \Im m\mathrm{GA}(5)) \tag{147}$$
$$p_2^\mu = (\Re e\mathrm{GB}(5), \Re e\mathrm{GB}(6), \Im m\mathrm{GB}(6), \Im m\mathrm{GB}(5)) \tag{148}$$
$$q^\mu = -(\Re e\mathrm{HVVSH}(2), \Re e\mathrm{HVVSH}(3), \Im m\mathrm{HVVSH}(3), \Im m\mathrm{HVVSH}(2)) \tag{149}$$
$$m = \mathrm{MASS} \tag{150}$$
$$\Gamma = \mathrm{WIDTH}. \tag{151}$$

3.2 Vertices VI: VVVTL

The three-vector-internal vertex (VVVTL) is defined to be the vector part of the three-vector-scalar-scalar vertex. A non-propagating internal particle L connects the vector part with the scalar part. Therefore it is defined by

$$\mathcal{L}_{VVVTL} = -gg_h f_{abc}\, L\, (\partial_\mu G^a_\nu) G^{\mu\, b} G^{\nu\, c} + gg_A f_{abc} L_A \epsilon^{\mu\nu\rho\sigma} (\partial_\mu G^a_\nu) G^b_\rho G^c_\sigma, \tag{152}$$

where g is the overal coupling constant, g_h is the coupling constant for the scalar internal particle L to the massless vector bosons and g_A is the coupling constant for the pseudo-scalar internal particle L_A to the massless vector bosons. G^a_μ is a *massless* vector boson. Note that for the three-gluon-two-scalar or three-gluon-two-pseudo-scalar vertices we need the coupling with the scalar internal particle, and for the three-gluon-scalar-pseudo-scalar vertex we need the pseudo-scalar internal particle.

3.2.1 VVVTLX

This subroutine computes an amplitude from the three-vector-internal vertex. This subroutine will be called as

CALL VVVTLX(GA,GB,GC,SC,G1,G2, VERTEX)

We have six inputs GA, GB, GC, SC, G1 and G2 and one output VERTEX. Note that the ordering of the massless vectors (i.e; GA, GB and GC) is important. Changing the order can lead to a change in sign.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first massless vector boson and its four-momentum.
2. complex GB(6). This is a complex 6 dimensional array which contains the wavfunction of the second massless vector boson and its four-momentum.
3. complex GC(6). This is a complex 6 dimensional array which contains the wavefunction of the third massless vector boson and its four-momentum.
4. complex SC(3). This is a complex 3 dimensional array which contains the internal particle wavefunction and its four-momentum.
5. real G1. This real number is the overal coupling constant.
6. complex G2(2). This is a complex two dimensional array which contains part of the coupling constant for the coupling of the internal particle with the three massless vector bosons. The first element, G2(1), is the coupling constant of the scalar and the second element, G2(2), the coupling constant of the pseudo-scalar with the massless vector bosons.

The outputs:

1. complex VERTEX. This complex number is the amplitude of the three-vector-internal vertex.

What this subroutine computes is the following T-matrix:

$$\begin{aligned} \text{VERTEX} = gS\Big[g_h \Big(& G_1 \cdot G_2 (p_1 \cdot G_3 - p_2 \cdot G_3) + G_2 \cdot G_3 (p_2 \cdot G_1 - p_3 \cdot G_1) + \\ & + G_3 \cdot G_1 (p_3 \cdot G_2 - p_1 \cdot G_2) \Big) + g_A G_1^\mu G_2^\nu G_3^\rho \epsilon_{\mu\nu\rho\sigma} (p_1 + p_2 + p_3)^\sigma \Big] \end{aligned} \tag{153}$$

where we used the notation:

$$g = \text{G1} \quad (154)$$

$$g_h = \text{G2}(1) \quad (155)$$

$$g_A = \text{G2}(2) \quad (156)$$

$$G_1^\mu = \text{GA}(\mu+1) \quad (157)$$

$$G_2^\mu = \text{GB}(\mu+1) \quad (158)$$

$$G_3^\mu = \text{GC}(\mu+1) \quad (159)$$

$$S = \text{SC}(1) \quad (160)$$

$$p_1^\mu = (\Re e\text{GA}(5), \Re e\text{GA}(6), \Im m\text{GA}(6), \Im m\text{GA}(5)) \quad (161)$$

$$p_2^\mu = (\Re e\text{GB}(5), \Re e\text{GB}(6), \Im m\text{GB}(6), \Im m\text{GB}(5)) \quad (162)$$

$$p_3^\mu = (\Re e\text{GC}(5), \Re e\text{GC}(6), \Im m\text{GC}(6), \Im m\text{GC}(5)). \quad (163)$$

3.2.2 JVVTLX

This subroutine computes an off-shell massless vector boson current from the three-vector-internal vertex attached with the vector boson propagator, from the internal particle and the other two vector bosons. This subroutine will be called as

CALL JVVTLX(GA,GB,SC,G1,G2,XM,XW, JVVT)

We have seven inputs GA, GB, SC, G1, G2, XM and XW and one output JVVT. Note that the ordering of the vector boson inputs (i.e. GA and GB) is important. Changing the order can lead to a change in sign.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first massless vector boson and its four-momentum.
2. complex GB(6). This is a complex 6 dimensional array which contains the wavefunction of the second massless vector boson and its four-momentum.
3. complex SC(3). This is a complex 3 dimensional array which contains the internal particle wavefunction and its four-momentum.
4. real G1. This is a real number which is the overal coupling constant.
5. complex G2(2). This is a complex 2 dimensional array which contains part of the coupling constant for the coupling of the internal particle with the three massless vector bosons. The first element, G2(1), is the coupling constant of the scalar and the second element, G2(2), the coupling constant of the pseudo-scalar with the massless vector bosons.
6. real XM. This real number is not used.
7. real XW. This real number is not used.

The outputs:

1. complex JVVT(6). This is a complex six-dimensional array which contains the off-shell massless vector boson current from the three-vector-internal vertex attached with the vector boson propagator, combined with its four-momentum.

What this subroutine computes is the following portion of the Feynman amplitude:

$$j^\mu = g\frac{1}{q^2}\,S\left[g_h\Big(G_1\cdot G_2(p_1^\mu - p_2^\mu) + (p_2-q)\cdot G_1 G_2^\mu + (q-p_1)\cdot G_2 G_1^\mu\Big) + g_A\eta^{\mu\tau}G_1^\nu G_2^\rho \epsilon_{\mu\tau\rho\sigma}(p_1+p_2+q)^\sigma\right] \quad (164)$$

where

$$\text{JVVT}(5) = \text{GA}(5) + \text{GB}(5) + \text{SC}(2), \tag{165}$$

$$\text{JVVT}(6) = \text{GA}(6) + \text{GB}(6) + \text{SC}(3). \tag{166}$$

Here we used the notation

$$j^\mu = \text{JVVT}(\mu + 1) \tag{167}$$

$$g = \text{G1} \tag{168}$$

$$g_h = \text{G2}(1) \tag{169}$$

$$g_A = \text{G2}(2) \tag{170}$$

$$\eta^{\nu\tau} = \text{diag}[1, -1, -1, -1] \tag{171}$$

$$G_1^\mu = \text{GA}(\mu + 1) \tag{172}$$

$$G_2^\mu = \text{GB}(\mu + 1) \tag{173}$$

$$S = \text{SC}(1) \tag{174}$$

$$p_1^\mu = (\Re e\text{GA}(5), \Re e\text{GA}(6), \Im m\text{GA}(6), \Im m\text{GA}(5)) \tag{175}$$

$$p_2^\mu = (\Re e\text{GB}(5), \Re e\text{GB}(6), \Im m\text{GB}(6), \Im m\text{GB}(5)) \tag{176}$$

$$q^\mu = -(\Re e\text{JVVT}(5), \Re e\text{JVVT}(6), \Im m\text{JVVT}(6), \Im m\text{JVVT}(5)). \tag{177}$$

3.2.3 UVVVLX

This subroutine computes an internal particle current from the three-vector-internal vertex from the three massless vector bosons. Note that the internal particle does not propagate, so there is no multiplication with a propagator. This subroutine will be called as

CALL UVVVLX(GA,GB,GC,G1,G2,XM,XW, UVVV)

We have seven inputs GA, GB, GC, G1, G2, XM and XW and one output UVVV. Note that the ordering of the vector boson inputs (i.e. GA, GB and GC) is important. Changing the order can lead to a change in sign.

The inputs:

1. complex GA(6). This is a complex 6 dimensional array which contains the wavefunction of the first massless vector boson and its four-momentum.
2. complex GB(6). This is a complex 6 dimensional array which contains the wavefunction of the second massless vector boson and its four-momentum.
3. complex GC(6). This is a complex 6 dimensional array which contains the wavefunction of the third massless vector boson and its four-momentum.
4. real G1. This is a real number which is the overal coupling constant.
5. complex G2(2). This is a complex 2 dimensional array which contains part of the coupling constant for the coupling of the internal particle with the three massless vector bosons. The first element, G2(1), is the coupling constant of the scalar and the second element, G2(2), the coupling constant of the pseudo-scalar with the massless vector bosons.
6. real XM. This real number is not used.
7. real XW. This real number is not used.

The outputs:

1. complex UVVV(3). This is a complex three-dimensional array which contains the internal particle current from the three-vector-internal vertex combined with its four-momentum. Note that the internal particle does *not* propagate.

What this subroutine computes is the following portion of the Feynman amplitude:

$$\text{UVVV}(1) = g\Big[g_h\Big(G_1 \cdot G_2(p_1 - p_2) \cdot G_3 + G_2 \cdot G_3(p_2 - p_3) \cdot G_1 + \\ + G_1 \cdot G_3(p_3 - p_1) \cdot G_2\Big) + g_A G_1^\mu G_2^\nu G_3^\rho \epsilon_{\mu\nu\rho\sigma}(p_1 + p_2 + p_3)^\sigma\Big], \quad (178)$$

where

$$\text{UVVV}(2) = \text{GA}(5) + \text{GB}(5) + \text{GC}(5), \quad (179)$$

$$\text{UVVV}(3) = \text{GA}(6) + \text{GB}(6) + \text{GC}(6). \quad (180)$$

Here we used the notation

$$g = \text{G1} \quad (181)$$

$$g_h = \text{G2}(1) \quad (182)$$

$$g_A = \text{G2}(2) \quad (183)$$

$$G_1^\mu = \text{GA}(\mu + 1) \quad (184)$$

$$G_2^\mu = \text{GB}(\mu + 1) \quad (185)$$

$$G_3^\mu = \text{GC}(\mu + 1) \quad (186)$$

$$p_1^\mu = (\Re e\text{GA}(5), \Re e\text{GA}(6), \Im m\text{GA}(6), \Im m\text{GA}(5)) \quad (187)$$

$$p_2^\mu = (\Re e\text{GB}(5), \Re e\text{GB}(6), \Im m\text{GB}(6), \Im m\text{GB}(5)) \quad (188)$$

$$p_3^\mu = (\Re e\text{GC}(5), \Re e\text{GC}(6), \Im m\text{GC}(6), \Im m\text{GC}(5)). \quad (189)$$

3.3 Vertices VII: SSTL

This vertex is defined to be the scalar part of the three-vector-scalar-scalar coupling. The scalar part is connected with the vector part by a non-propagating internal particle L. Thus, the vertex is defined by

$$\mathcal{L}_{SSTL} = g_v(\Phi\Phi + \Phi_A\Phi_A)L + g_v\Phi\Phi_A L_A, \quad (190)$$

where g_v is the coupling constant, Φ and Φ_A are the scalar boson and pseuddo-scalar boson respectively. L is the scalar internal particle and L_A the pseudo-scalar internal particle. The internal particles do *not* propagate.

3.3.1 SSTLXX

This subroutine computes an amplitude from the scalar-scalar-internal vertex. This subroutine will be called as

CALL SSTLXX(S1,S2,L1,GV, VERTEX)

We have four inputs S1, S2, L1 and GV and one output VERTEX.

The inputs:

1. complex S1(3). This is a complex 3 dimensional array which contains the wavefunction of the first scalar boson and its four-momentum.

2. complex S2(3). This is a complex 3 dimensional array which contains the wavfunction of the second scalar boson and its four-momentum.

3. complex L1(3). This is a complex 3 dimensional array which contains the wavefunction of the internal particle and its four-momentum.

4. real GV. This real number is the coupling constant.

The outputs:

1. complex VERTEX. This complex number is the amplitude of the scalar-scalar-internal vertex.

What this subroutine computes is the following T-matrix:

$$\text{VERTEX} = -g_v S_1 S_2 L \tag{191}$$

where we used the notation:

$$g_v = \text{GV} \tag{192}$$
$$S_1 = \text{S1(1)} \tag{193}$$
$$S_2 = \text{S2(1)} \tag{194}$$
$$L = \text{L1(1)}. \tag{195}$$

3.3.2 HSTLXX

This subroutine computes an internal particle current from the scalar-scalar-internal vertex, from the two scalar bosons. This subroutine will be called as

CALL HSTLXX(S1,S2,GV,XM,XW, HSSL)

We have five inputs S1, S2, GV, XM and XW and one output HSSL.
The inputs:

1. complex S1(3). This is a complex 3 dimensional array which contains the wavefunction of the first scalar boson and its four-momentum.
2. complex S2(3). This is a complex 3 dimensional array which contains the wavfunction of the second scalar boson and its four-momentum.
3. real GV. This real number is the coupling constant.
4. real XM. This real number is not used.
5. real XW. This real number is not used.

The outputs:

1. complex HSTL(3). This complex three-dimensional array consists of the internal particle current combined with its four-momentum. Note that the internal particle does *not* propagate.

What this subroutine computes is the following portion of the Fenyman amplitude:

$$\text{HSTL(1)} = -g_v S_1 S_2 \tag{196}$$

and

$$\text{HSTL(2)} = \text{S1(2)} + \text{S1(2)}, \tag{197}$$
$$\text{HSTL(3)} = \text{S2(3)} + \text{S2(3)}. \tag{198}$$

Here we used the notation:

$$g_v = \text{GV} \tag{199}$$
$$S_1 = \text{S1(1)} \tag{200}$$
$$S_2 = \text{S2(1)}. \tag{201}$$

3.3.3 USSLXX

This subroutine computes a scalar boson current from the scalar-scalar-internal vertex attached with the scalar boson propagator, from the other scalar boson and the internal particle. This subroutine will be called as

CALL USSLXX(SC,SI,GV,MASS,WIDTH, USSL)

We have five inputs SC, SI, GV, MASS and WIDTH and one output USSL.

The inputs:

1. complex SC(3). This is a complex 3 dimensional array which contains the wavefunction of the scalar boson and its four-momentum.
2. complex SI(3). This is a complex 3 dimensional array which contains the wavefunction of the internal particle and its four-momentum.
3. real GV. This real number is the coupling constant.
4. real MASS. This real number is the mass of the outgoing scalar current.
5. real WIDTH. This real number is the width of the outgoing scalar current.

The outputs:

1. complex USSL(3). This complex three-dimensional array consists of the scalar boson current attached with the scalar boson propagator combined with its four-momentum.

What this subroutine computes is the following portion of the Fenyman amplitude:

$$\mathrm{USSL}(1) = g_v \frac{1}{q^2 - m^2 + im\Gamma} S\, L \tag{202}$$

and

$$\mathrm{USSL}(2) = \mathrm{SC}(2) + \mathrm{SI}(2), \tag{203}$$

$$\mathrm{USSL}(3) = \mathrm{SC}(3) + \mathrm{SI}(3). \tag{204}$$

Here we used the notation:

$$g_v = \mathrm{GV} \tag{205}$$

$$S = \mathrm{SC}(1) \tag{206}$$

$$L = \mathrm{SI}(1) \tag{207}$$

$$q^\mu = -(\Re e\mathrm{USSL}(2), \Re e\mathrm{USSL}(3), \Im m\mathrm{USSL}(3), \Im m\mathrm{USSL}(2)) \tag{208}$$

$$m = \mathrm{MASS} \tag{209}$$

$$\Gamma = \mathrm{WIDTH}. \tag{210}$$

3.4 Vertices VIII: TTSS

The tensor-tensor-scalar-scalar vertex (TTSS) is designed to be the scalar boson contribution to the four-vector-scalar-scalar vertex. Because the four-vector vertex can be written is two vector-vector-tensor vertices connected by the tensor particle, the scalar contribution to the four-vector-scalar-scalar vertex can be described by a tensor-tensor-scalar-scalar interaction

$$\mathcal{L}_{TTSS} = g_h g_v \Phi\Phi\, T_a^{\mu\nu} T^a_{\mu\nu}, \tag{211}$$

where Φ is the scalar boson and $T^a_{\mu\nu}$ the tensor particle and g_v and g_h are the coupling constants.

3.4.1 TTSSXX

This subroutine computes an amplitude from the tensor-tensor-scalar-scalar vertex. This subroutine will be called as

CALL TTSSXX(TC1,TC2,S1,S2,GHH,GV, VERTEX)

We have six inputs TC1, TC2, S1, S2, GHH and GV and one output VERTEX.

The inputs:

1. complex TC1(18). This is a complex 18 dimensional array which contains the wavefunction of the first tensor particle and its four-momentum.
2. complex TC2(18). This is a complex 18 dimensional array which contains the wavefunction of the second tensor particle and its four-momentum.
3. complex S1(3). This is a complex 3 dimensional array which contains the wavefunction of the first scalar particle and its four-momentum.
4. complex S2(3). This is a complex 3 dimensional array which contains the wavefunction of the second scalar particle and its four-momentum.
5. complex GHH(2). This is a two dimensional array, which contains part of the coupling constant. GHH(1) is the coupling constant of the two scalar or two pseudo-scalar bosons with the two tensor particles and GHH(2) is not used.
6. real GV. This real number is the overal coupling constant.

The outputs:

1. complex VERTEX. This complex number is the amplitude of the tensor-tensor-scalar-scalar vertex.

What this subroutine computes is the following T-matrix:

$$\text{VERTEX} = g_h g_v \, S_1 S_2 \, T_{1\mu\nu} T_2^{\mu\nu} \tag{212}$$

where we used the notation:

$$g_h = \text{GHH}(1) \tag{213}$$
$$g_v = \text{GV} \tag{214}$$
$$S_1 = \text{S1}(1) \tag{215}$$
$$S_2 = \text{S2}(1) \tag{216}$$
$$T_1^{\mu\nu} = \text{TC1}(4\mu + \nu + 1) \tag{217}$$
$$T_2^{\mu\nu} = \text{TC2}(4\mu + \nu + 1). \tag{218}$$

3.4.2 UTSSXX

This subroutine computes an off-shell tensor current from the tensor-tensor-scalar-scalar vertex from the other tensor particle and the two scalar bosons. This subroutine will be called as

CALL UTSSXX(TC,S1,S2,GHH,GV,XM,XW, UTSS)

We have seven inputs TC, S1, S2, GHH, GV, XM and XW and one output UTSS.

The inputs:

1. complex TC(18). This is a complex 18 dimensional array which contains the wavefunction of the tensor particle and its four-momentum.

2. complex S1(3). This is a complex 3 dimensional array which contains the wavefunction of the first scalar particle and its four-momentum.

3. complex S2(3). This is a complex 3 dimensional array which contains the wavefunction of the second scalar particle and its four-momentum.

4. complex GHH(2). This is a two dimensional array, which contains part of the coupling constant. GHH(1) is the coupling constant of the two scalar or two pseudo-scalar bosons with the two tensor particles and GHH(2) is not used.

5. real GV. This real number is the overal coupling constant.

6. real XM. This real number is not used.

7. real XW. This real number is not used.

The outputs:

1. complex UTSS(18). This is a complex 18 dimensional array which contains the tensor particle current from the tensor-tensor-scalar-scalar vertex combined with its four-momentum. Note that the tensor particle does *not* propagate.

What this subroutine computes is the following portion of the Feynman amplitude:

$$j^{\mu\nu} = g_h g_v \, S_1 S_2 \, T^{\mu\nu}, \tag{219}$$

and

$$\mathrm{UTSS}(17) = \mathrm{TC}(17) + \mathrm{S1}(2) + \mathrm{S2}(2), \tag{220}$$
$$\mathrm{UTSS}(18) = \mathrm{TC}(18) + \mathrm{S1}(3) + \mathrm{S2}(3). \tag{221}$$

Here we used the notation

$$j^{\mu\nu} = \mathrm{UTSS}(4\mu + \nu + 1) \tag{222}$$
$$g_h = \mathrm{GHH}(1) \tag{223}$$
$$g_v = \mathrm{GV} \tag{224}$$
$$T^{\mu\nu} = \mathrm{TC}(4\mu + \nu + 1) \tag{225}$$
$$S_1 = \mathrm{S1}(1) \tag{226}$$
$$S_2 = \mathrm{S2}(1). \tag{227}$$

3.4.3 HTTSXX

This subroutine computes an off-shell scalar boson current from the tensor-tensor-scalar-scalar vertex from the tensor particles and the other scalar. This subroutine will be called as

CALL HTTSXX(T1,T2,SC,GHH,GV,MASS,WIDTH, HTTS)

We have seven inputs T1, T2, SC, GHH, GV, MASS and WIDTH and one output HTTS.

The inputs:

1. complex T1(18). This is a complex 18 dimensional array which contains the wavefunction of the first tensor particle and its four-momentum.

2. complex T2(18). This is a complex 18 dimensional array which contains the wavefunction of the second tensor particle and its four-momentum.

3. complex SC(3). This is a complex 3 dimensional array which contains the wavefunction of the scalar particle and its four-momentum.

4. complex GHH(2). This is a two dimensional array, which contains part of the coupling constant. GHH(1) is the coupling constant of the two scalar or two pseudo-scalar bosons with the two tensor particles and GHH(2) is not used.
5. real GV. This real number is the overal coupling constant.
6. real XM. This real number is the mass of the outgoing scalar current.
7. real XW. This real number is the width of the outgoing scalar current.

The outputs:

1. complex HTTS(3). This is a complex 3 dimensional array which contains the scalar boson current from the tensor-tensor-scalar-scalar vertex attached with the scalar propagator, combined with its four-momentum.

What this subroutine computes is the following portion of the Feynman amplitude:

$$\mathrm{HTTS}(1) = -g_h g_v \frac{1}{q^2 - m^2 + im\Gamma} \, S \, T_1^{\mu\nu} T_{2\mu\nu}, \tag{228}$$

and

$$\mathrm{HTTS}(2) = \mathrm{T1}(17) + \mathrm{T2}(17) + \mathrm{SC}(2), \tag{229}$$
$$\mathrm{HTTS}(3) = \mathrm{T2}(18) + \mathrm{T2}(18) + \mathrm{SC}(3). \tag{230}$$

Here we used the notation

$$g_h = \mathrm{GHH}(1) \tag{231}$$
$$g_v = \mathrm{GV} \tag{232}$$
$$T_1^{\mu\nu} = \mathrm{T1}(4\mu + \nu + 1) \tag{233}$$
$$T_2^{\mu\nu} = \mathrm{T2}(4\mu + \nu + 1) \tag{234}$$
$$S = \mathrm{SC}(1) \tag{235}$$
$$m = \mathrm{MASS} \tag{236}$$
$$\Gamma = \mathrm{WIDTH}. \tag{237}$$