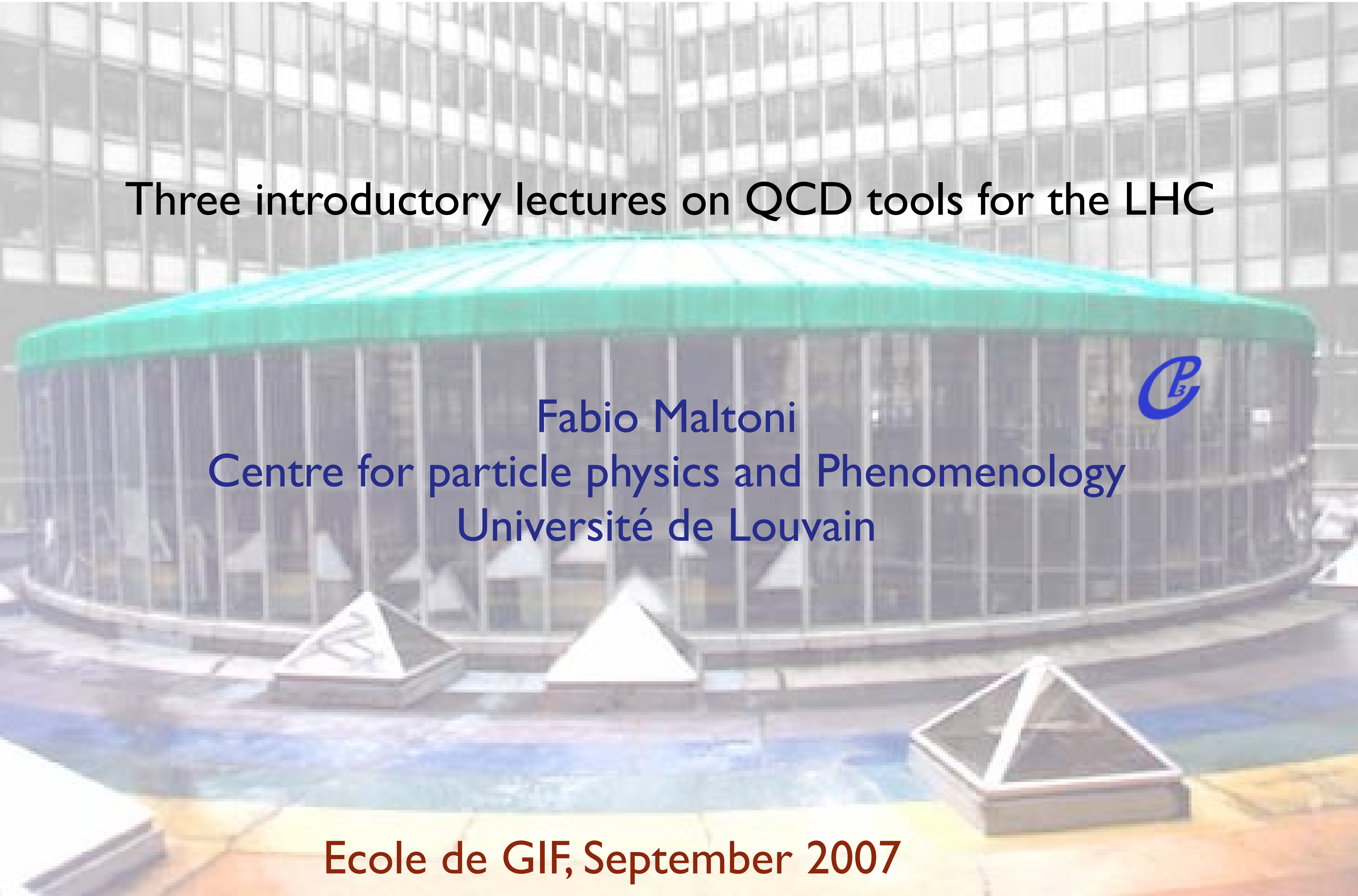
Outils de prédiction en pQCD



Three introductory lectures on QCD tools for the LHC

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Preliminary statements

- QCD is the richest part of our current description of the fundamental interactions.
- Two very different regimes: perturbative vs non-perturbative.
- Many tools and techniques have been developed...
- Tevatron is running now and LHC will start next year. Discoveries are around the corner...better stay focused!!

Why do we believe in QCD?

- QCD is a non-abelian gauge theory, is renormalizable, is asymptotically free, is a one-parameter theory [Once you measure α_S you know everything fundamental about (perturbative) QCD].
- It explains the low energy properties of the hadrons, justifies the observed spectrum and catch the most important dynamical properties.
- It explains scaling (and BTW anything else we have seen up to now!!) at high energies.
- It leaves EW interaction in place since the SU(3) commutes with SU(2) x U(1). There is no mixing and there are no enhacements of parity violating effect or flavor changing currents.
- It gives a hope for unification of fundamental interactions.

Excellent!

So are we done?

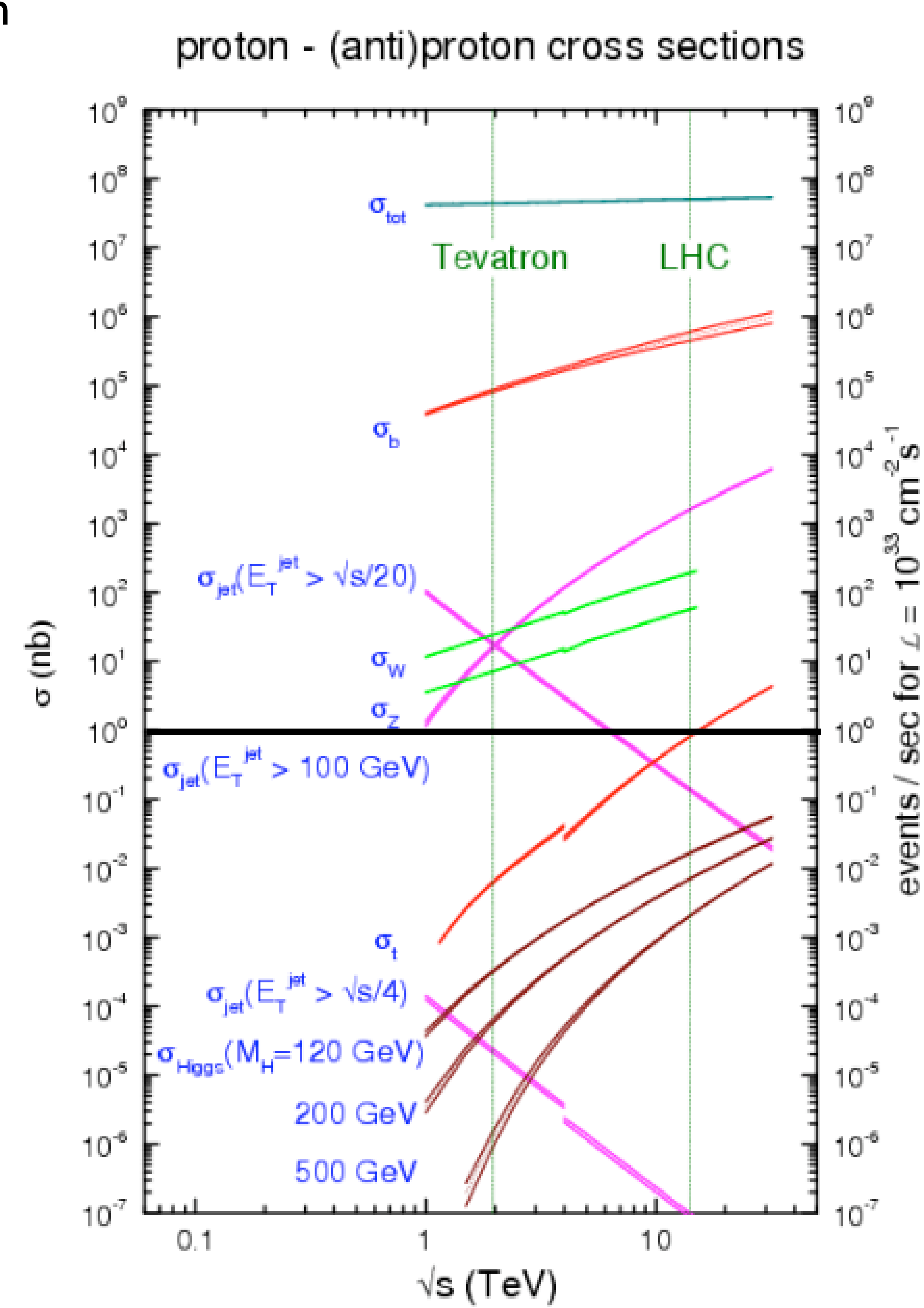
The big picture

Let's look at the cross section for producing bottom quarks/W,Z,jets,top, and Higgs.

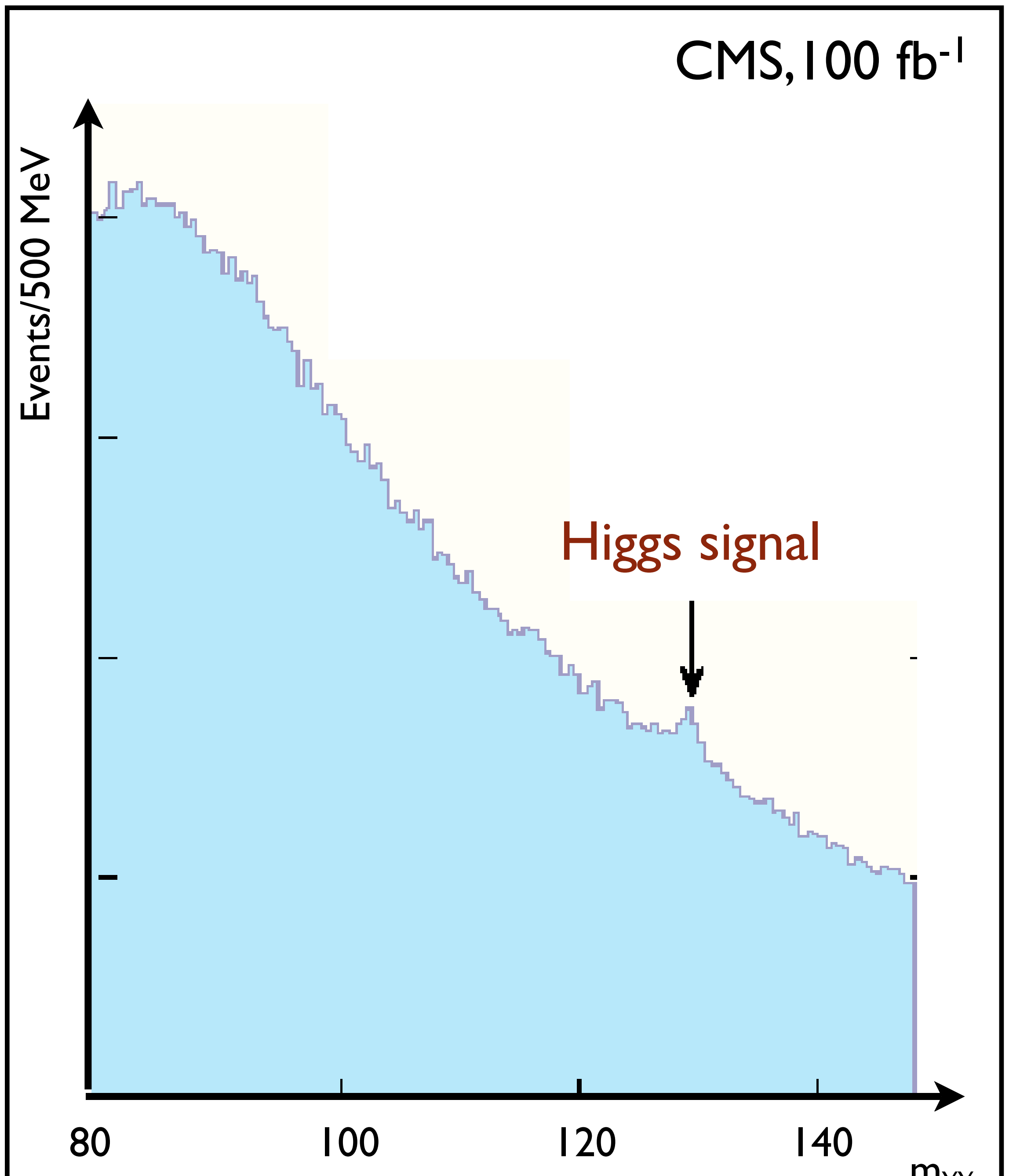
LHC physics = QCD + ϵ

Need to understand QCD backgrounds well!

We need not only an accurate normalization but also the kinematical distributions! Example...



Example #1: $gg \to H \to \gamma\gamma$



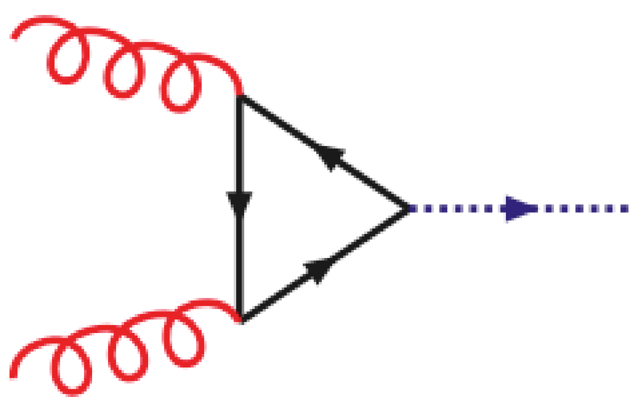
Huge background from QCD.

$qq \to \gamma\gamma$ known at NLO (DIPHOX) including fragmentation contributions
[Binoth, Guillet, Pilon, Werlen. 2000]

$gg \to \gamma\gamma$ direct known at NLO (two-loop)
[Bern, Dixon, Schmidt. 2002]

On the other hand this is an example where for discovery it doesn't need an accurate theoretical prediction for the background. Data modeling will suffice.

Example #1: $gg \to H \to \gamma\gamma$

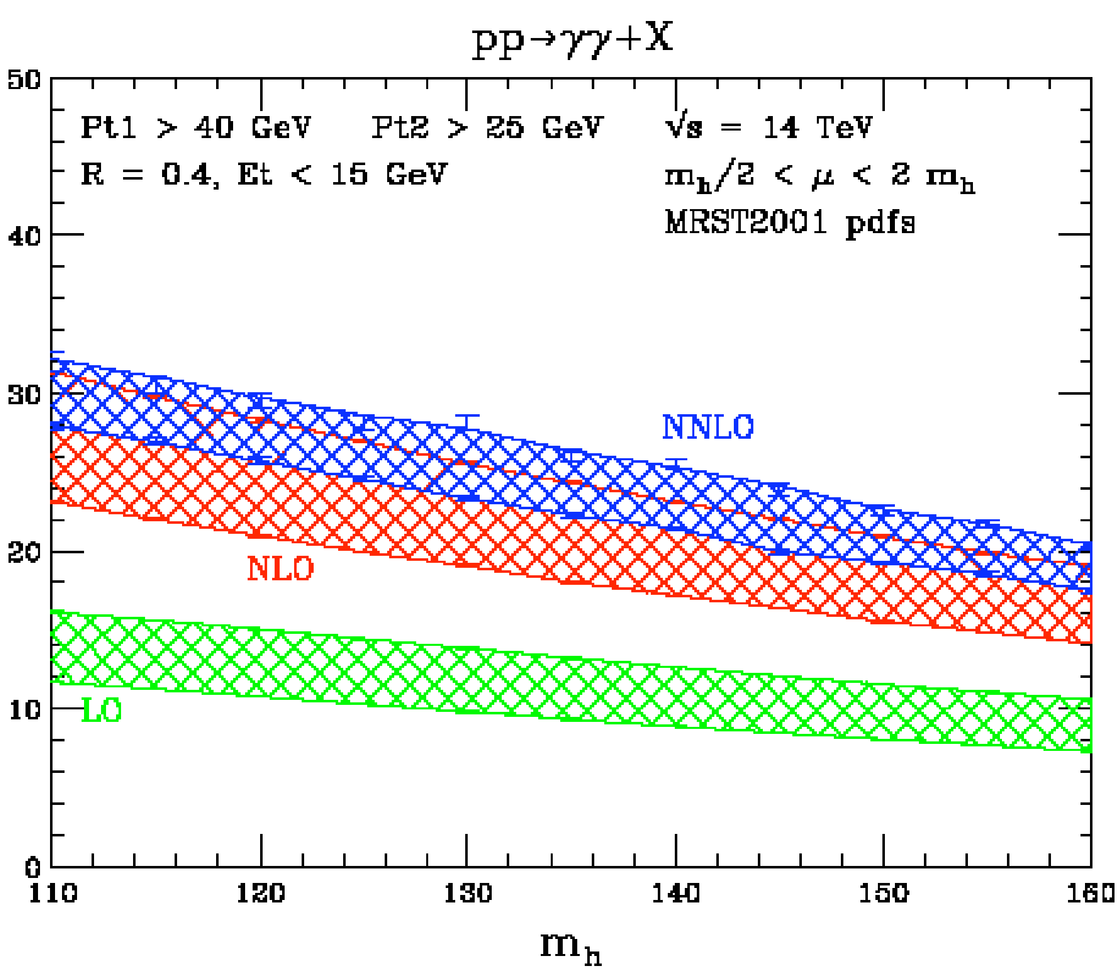


Dominant production mechanism at hadron colliders. "Heavy particle counter!". We need to predict well if we want to extract information from it.

QCD corrections:
[Daswon.1991] [Djouadi, Graudenz, Spira, Zerwas. 1991]
[Kramer, Laenen, Spira.1998] [Catani, De Florian, Grazzini.2001]
[Harlander, Kilgore.2001,2002] [Anastasiou, Melnikov.2002]
[Ravindran,Smith,Van Neerven. 2003]
[Catani, De Florian, Grazzini, Nason.2003]

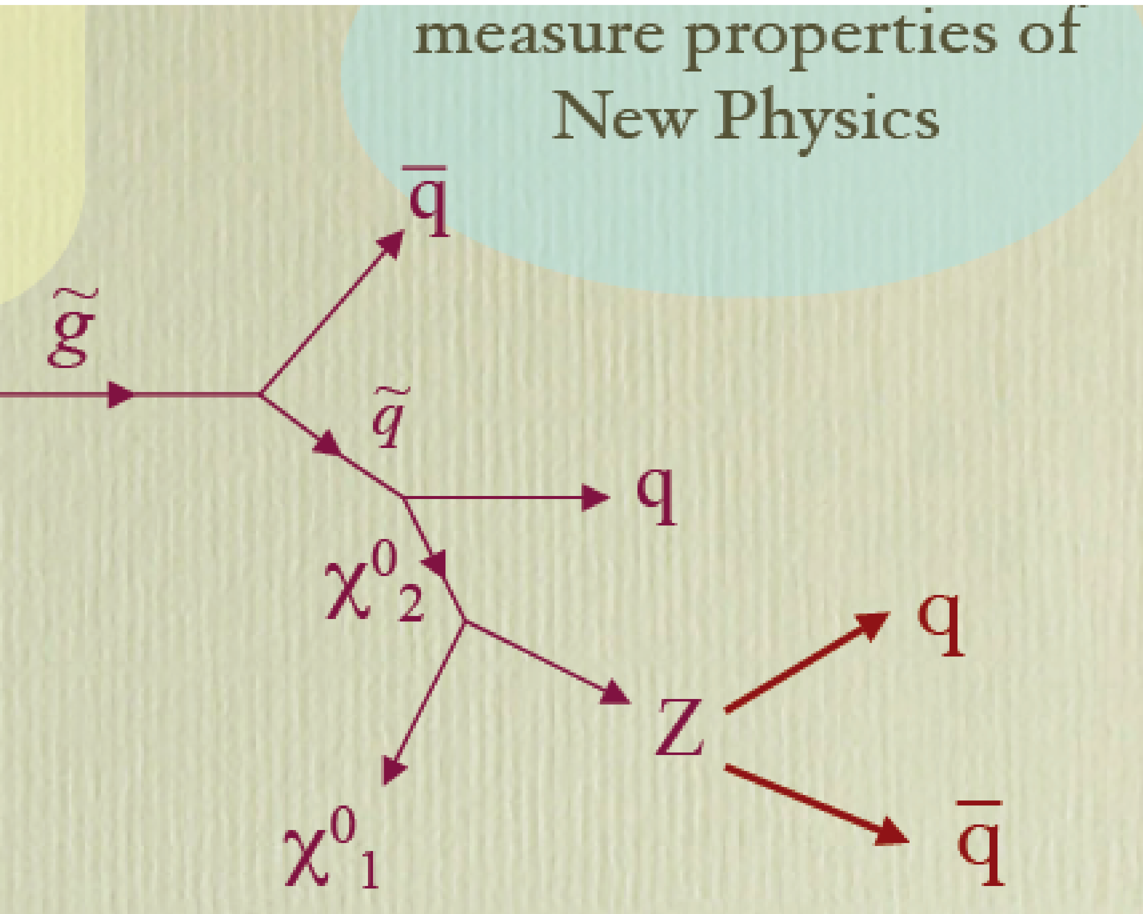
Two-loop EW corrections:
[Djouadi, Gambino, Kniehl. 1998]
[Aglietti, Bonciani, Degrassi,Vicini. 2004]
[Degrassi, FM. 2004]

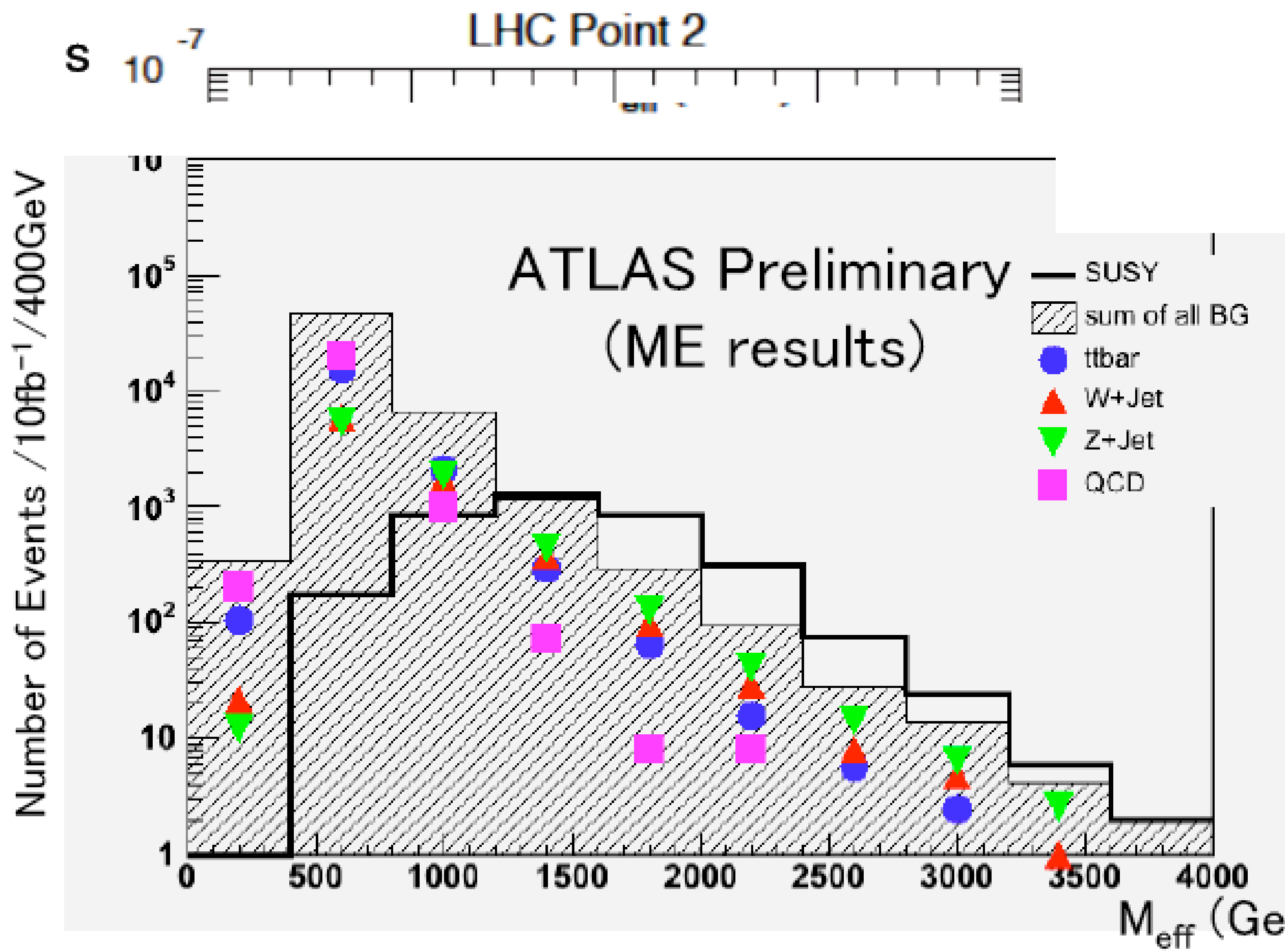
PDF evolution at NNLO ("Guinness of QCD"):
[Moch,Vogt,Vermaseren, 2004]



Best QCD predictions at present:
> Fully exclusive (PS interfaced) prediction at NLO+NLL [Frixione, Webber. 2003]
> Fully exclusive prediction at NNLO (first ever) [Anastasiou, Melnikov, Petriello. 2004]
> Resummed pt distribution at NLO+NNLL

Example #2: early discovery SuperSymmetry at the LHC





$= \not{E}_T + \sum P_T$

Signal: gluinos decay into jets +mE_T.

Background: t tbar+jets,(Z,W)+jets, jets. Very difficult to estimate theoretically: many parton calculation (2 → 8 gluons = 10 millions Feynman diagrams !!). Now MC's for this are available...

no QCD ⇒ no PARTY !

The big picture

Let's look at the cross section for producing bottom quarks/W,Z,jets,top, and Higgs.

LHC physics = QCD + ϵ

Need to understand QCD backgrounds well!

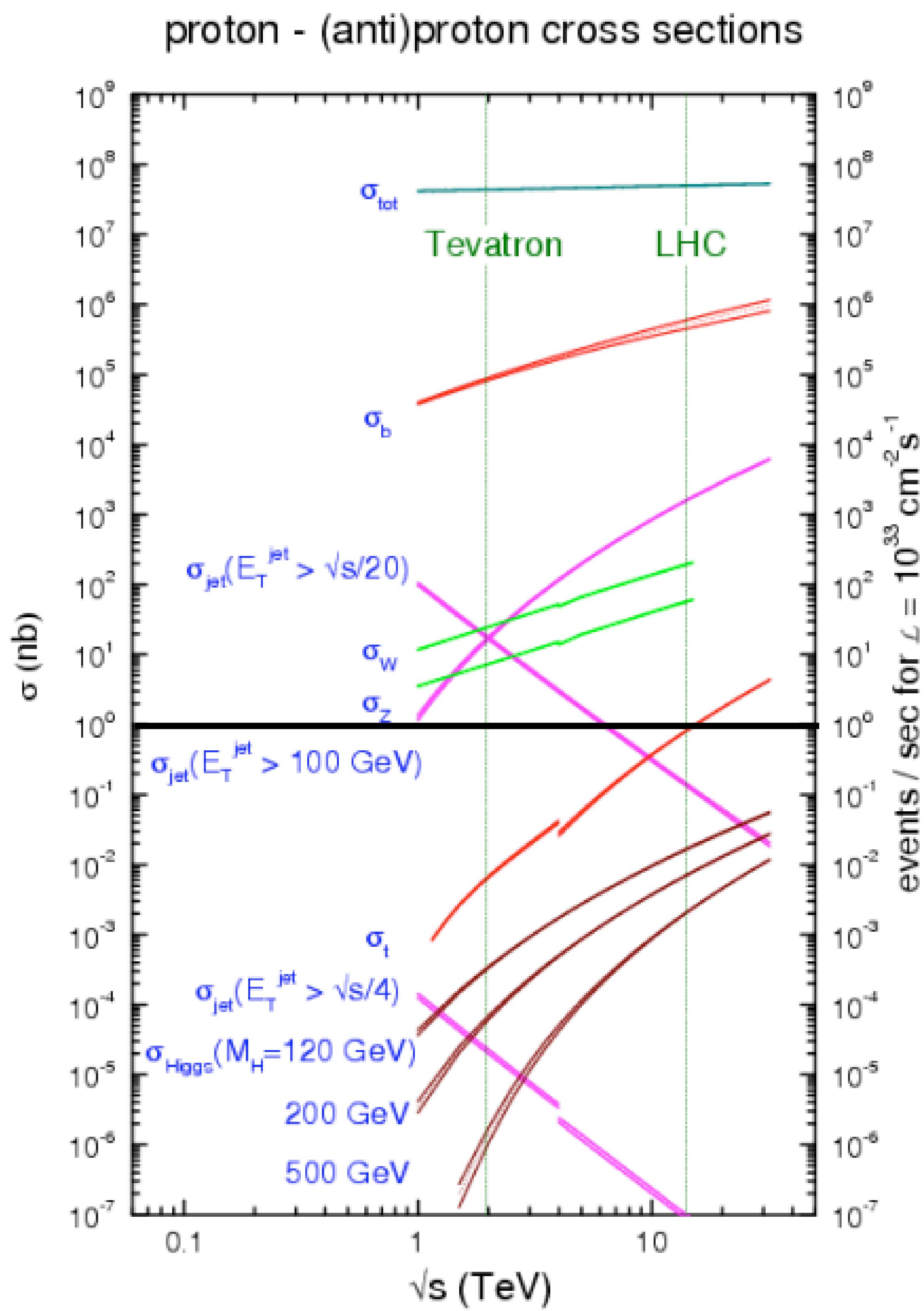
We need not only an accurate normalization but also the kinematical distributions!

QCD factorization theorem for short-distance inclusive processes:

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

Two ingredients necessary:

1. Parton Distribution functions (from exp, but evolution from th).
2. Short distance coefficients as an expansion in α_S (from th).



Progress in the PDF

PDF measured at HERA and fixed-target experiments. x dependence from data.
Q^2 dependence from DGLAP evolution.

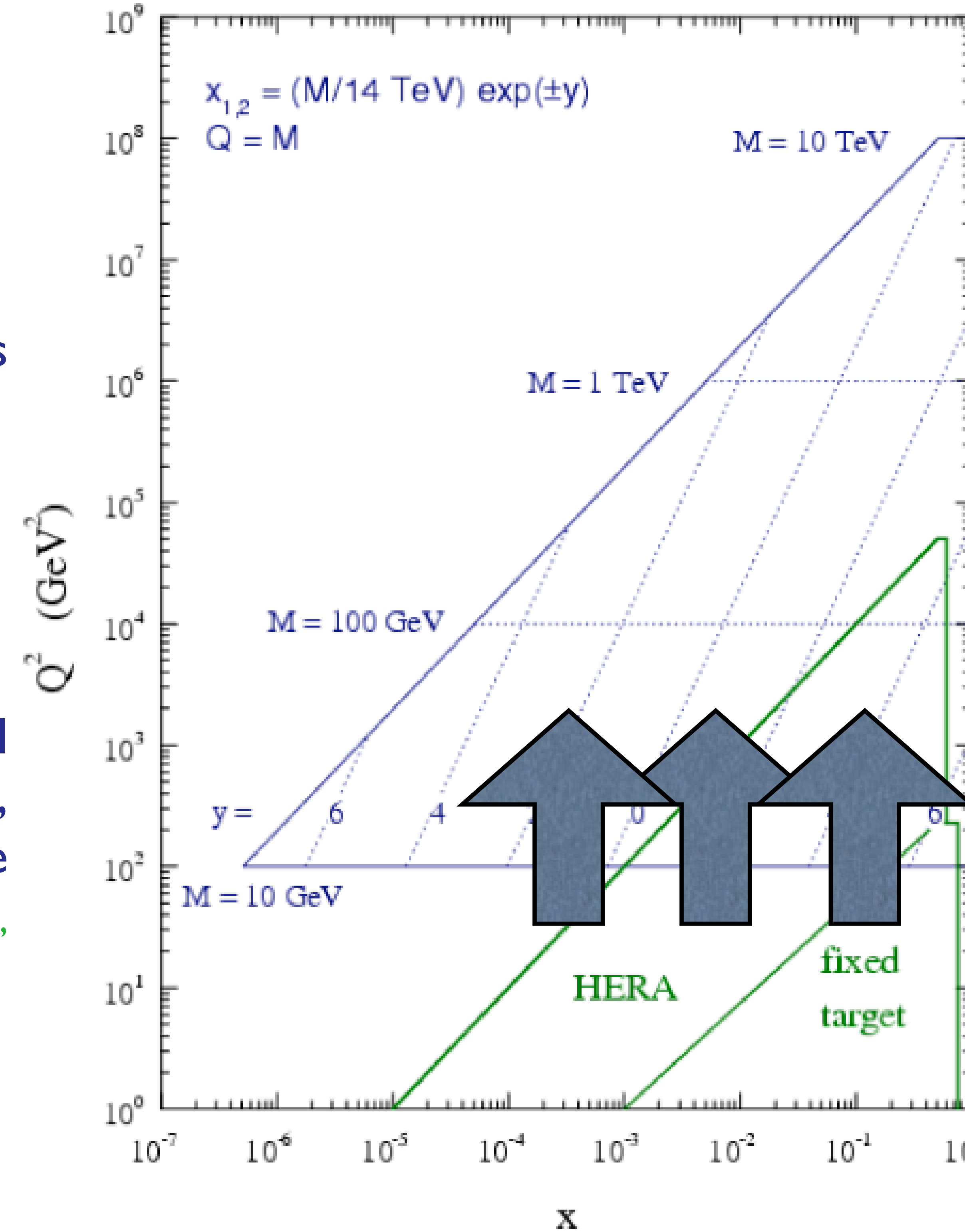
Recently:

NNLO calculation of the 3-loop splitting kernels ("the hardest calculation in QCD")
[Moch, Vermaseren, Vogt. 2004]

Together with short distance NNLO calculation first sets of NNLO PDF sets. [MRST and Alekhin, 2004]

PDF's with errors: Various "traditional methods",[CTEQ and MRST, 2003]. Also new approaches, the functional space [Giele, Keller, Kosower.2001] and the Neural Network approach [Del Debbio, Forte, La Torre, Piccione, Rojo. 2002,2005].

Issues:
1. small-x effects
2. Heavy flavors pdf



Progress in the short distance coeff's

First way:

- Include higher order terms in our fixed-order calculations (LO→NLO→NNLO...)
- ⇒ $$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \ldots$$
- Obtain the tree-level results for many partons final states

TH

Comments:

1. The theoretical errors systematically decrease.
2. Pure theoretical point of view.
3. A lot of new techniques and universal algorithms have been developed.
4. Final description only in terms of partons (IR safe observables)⇒not suitable to experimentalists...

Progress in the short distance coeff's

Second way:

- Describe final states with high multiplicities starting from $2 \to 1$ or 2 procs, using parton showers, and then an hadronization model.

EXP

Comments:

1. Fully exclusive final state description suitable for detector simulations.
2. Normalization is very uncertain
3. Very crude kinematic distributions for multi-parton final states.
4. Improvements are only at the model level.

Progress in the short distance coeff's

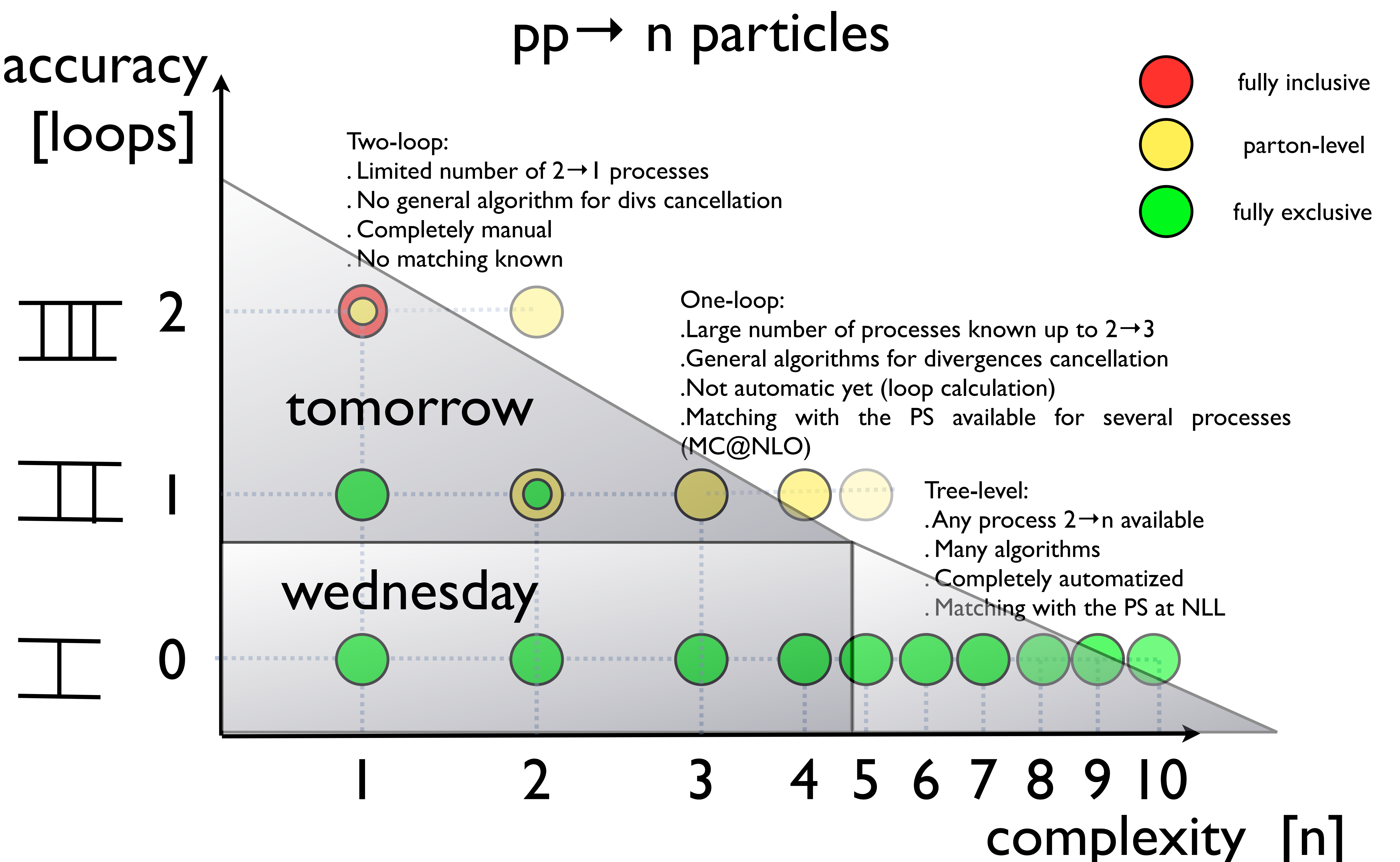
New trend: TH & EXP

Match fixed-order calculations and parton showers to obtain the most accurate predictions in a detector simulation friendly way!

Two directions:

1. Get fully exclusive description of events correct at NLO in the normalization and distributions.
2. Get fully exclusive description of many parton events correct at LO (LL) in all the phase space.

Theory status



Outline

- Basics
- Improving the accuracy: NLO and NNLO
- Improving the flexibility: Matrix elements MC's

Lecture material + exercises can be found at:

http://cp3wks05.fynu.ucl.ac.be/twiki/bin/view/Library/GIFSchool

Basics

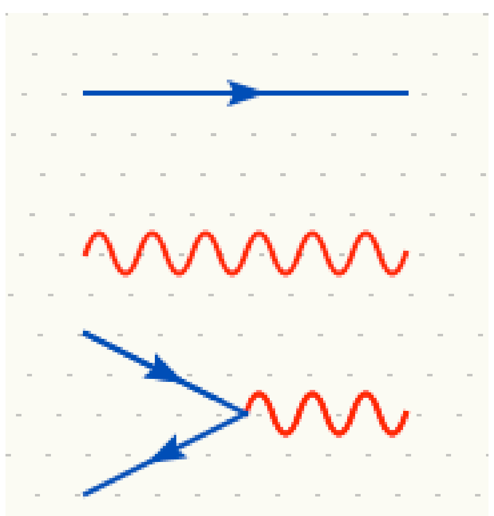
i.e., how to make computations in pQCD

- From QED to QCD
- Color Algebra
- Helicity techniques and recursion

From QED to QCD: abelian vs. non-abelian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi - eQ\bar{\psi}\not{A}\psi$$

where $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$



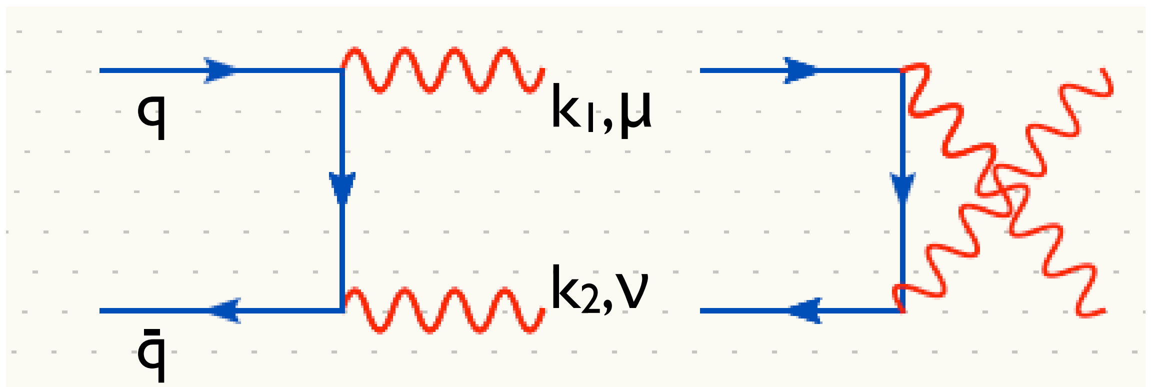
$$= \frac{i}{\not{p} - m + i\epsilon} = i\frac{\not{p} + m}{p^2 - m^2 + i\epsilon}$$

$$= -i\frac{g_{\mu\nu}}{p^2 + i\epsilon} \text{ (Feynman gauge)}$$

$$= -ie\gamma_\mu Q \quad (Q = -1 \text{ for the electron, } Q = 2/3 \text{ for the u-quark, etc}$$

From QED to QCD

We want to focus on how gauge invariance is realized in practice.
Let's start with the computation of a simple proces $e^+e^- \to \gamma\gamma$.
There are two diagrams:



$= D_1 + D_2$

$$\frac{i}{e^2} M_\gamma \equiv D_1 + D_2 = \bar{v}(\bar{q})\, \not{\epsilon}_2 \frac{1}{\not{q} - \not{k}_1} \not{\epsilon}_1\, u(q) \; + \; \bar{v}(\bar{q}) \not{\epsilon}_1 \frac{1}{\not{q} - \not{k}_2} \not{\epsilon}_2\, n(q) \equiv M_{\mu\nu} \epsilon_1^\mu \epsilon_2^\nu$$

Gauge invariance demands that

$$\epsilon_2^\nu \partial^\mu M_{\mu\nu} = \epsilon_1^\mu \partial^\nu M_{\mu\nu} = 0$$

$M_\mu \equiv M_{\mu\nu}\epsilon_2^\nu$ is in fact the current that couples to the photon k_1. Charge conservation requires $\partial_\mu M^\mu = 0$:

$$\partial_\mu M^\mu = 0 \quad \Rightarrow \quad \frac{d}{dt} \int M^0 d^3x = \int \partial_0 M^0 \, d^3x$$
$$= \int \vec{\nabla} \cdot \vec{M} \; d^3x = \int_{S \to \infty} \vec{M} \cdot d\vec{\Sigma} = 0$$

In momentum space, this means

$$k_1^\mu M_\mu = 0 \,.$$

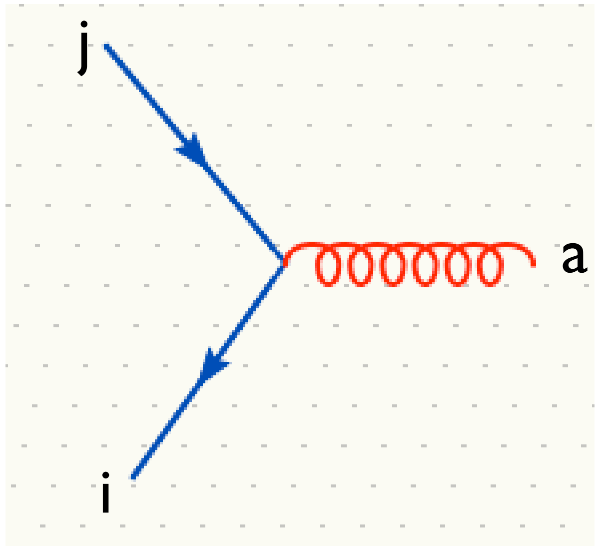
From QED to QCD

$$\begin{aligned} k_1^\mu \epsilon_2^\nu M_{\mu\nu} &= \bar{v}(\bar{q})\not{\epsilon}_2 \frac{1}{\not{q} - \not{k}_1}(\not{k}_1 - \not{q})u(q) + \bar{v}(\bar{q})(\not{k}_1 - \not{\bar{q}}) \frac{1}{\not{k}_1 - \not{\bar{q}}}\not{\epsilon}_2 u(q) \\ &= -\bar{v}(\bar{q})\not{\epsilon}_2 u(q) + \bar{v}(\bar{q})\not{\epsilon}_2 u(q) = 0 \end{aligned}$$

Only the sum of the two diagrams is gauge invariant.
For the amplitude to be gauge invariant it is enough that one of the polarizations is longitudinal. The state of the other gauge boson is irrelevant.

Let's try now to generalize what we have done for SU(3). In this case we take the (anti-)quarks
to be in the (anti-)fundamental representation of SU(3), 3 and 3^*. Then the current is in a $3 \otimes 3^* = 1 \oplus 8$. The singlet is like a photon, so we identify the gluon with the octet and generalize the QED vertex to :

$$-ig_s t^a_{ij}\gamma^\mu$$



with $[t^a, t^b] = if^{abc}t^c$

So now let's calculate qq → gg and we obtain

$$\frac{i}{g_s^2} M_g \equiv (t^b t^a)_{ij} D_1 + (t^a t^b)_{ij} D_2$$

$$M_g = (t^a t^b)_{ij} M_\gamma - g^2 f^{abc} t^c_{ij} D_1$$

From QED to QCD

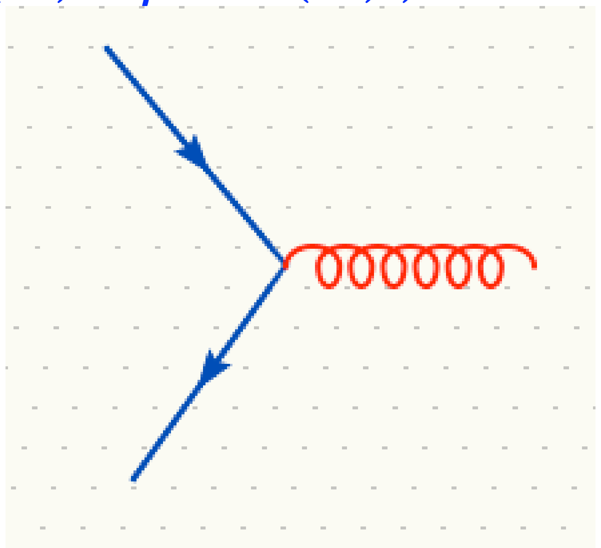
To satisfy gauge invariance we still need:

$$k_1^\mu {\epsilon_2}^\nu M_g^{\mu,\nu} = k_2^\nu \epsilon_1^\mu M_g^{\mu,\nu} = 0.$$

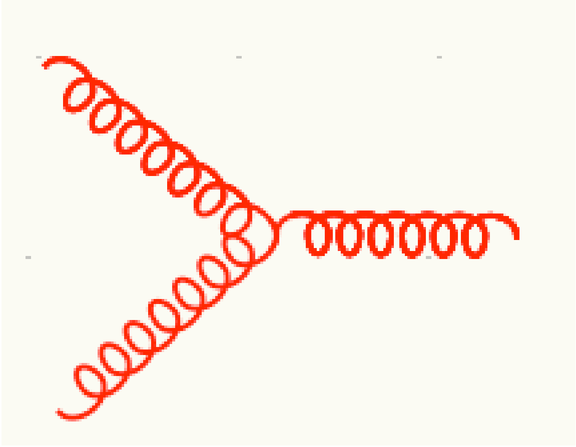
But in this case one piece is left out

$$k_{1\mu} M_g^\mu = -g_s^2 f^{abc} t^c_{ij} \bar{v}_i(\bar{q}) \not{\epsilon}_2 u_i(q)$$

$$k_{1\mu} M_g^\mu = i(-g_s f^{abc} \epsilon_2^\mu)(-i g_s t^c_{ij} \bar{v}_i(\bar{q}) \gamma_\mu u_i(q))$$

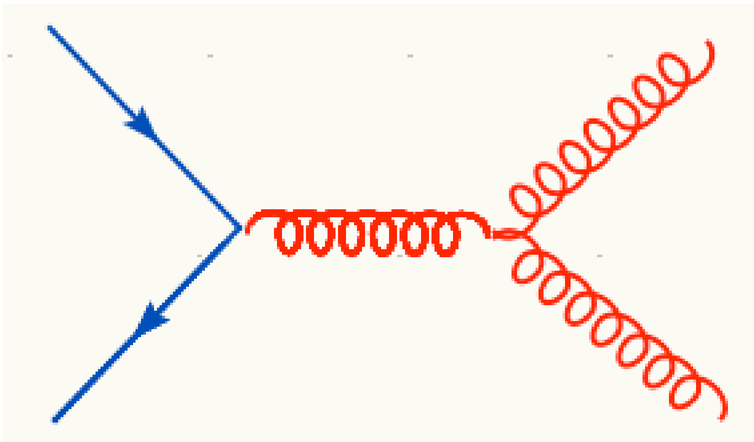


We indeed see that we interpret as the normal vertex times a new 3 gluon vertex:



$$-g_s f^{abc} V_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$

From QED to QCD



$$-ig_s^2 D_3 = \left(-ig_s t^a_{ij} \bar{v}_i(\bar{q})\gamma^\mu u_j(q)\right) \times \left(\frac{-i}{p^2}\right) \times$$

$$\left(-gf^{abc} V_{\mu\nu\rho}(-p, k_1, k_2)\epsilon_1^\nu(k_1)\epsilon_2^\rho(k_2)\right)$$

How do we write down the Lorentz part for this new interaction? We can impose

1. Lorentz invariance : only structure of the type $g_{\mu\nu}\, p_\rho$ are allowed
2. fully anti-symmetry : only structure of the type remain $g_{\mu 1 \mu 2}\ (k_1)_{\mu 3}$ are allowed...
3. dimensional analysis : only one power of the momentum.

that uniquely constrain the form of the vertex:

$$V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = V_0\left[(p_1 - p_2)_{\mu_3} g_{\mu_1\mu_2} + (p_2 - p_3)_{\mu_1} g_{\mu_2\mu_3} + (p_3 - p_1)_{\mu_2} g_{\mu_3\mu_1}\right]$$

With the above expression we obtain a contribution to the gauge variation:

$$k_1 \cdot D_3 = g^2 f^{abc} t^c V_0 \left[\bar{v}(\bar{q})\not{\epsilon}_2 u(q) - \frac{k_2 \cdot \epsilon_2}{2k_1 \cdot k_2}\bar{v}(\bar{q})\not{k}_1 u(q)\right]$$

The first term cancels the gauge variation of D_1+ D_2 if V_0=1, the second term is zero IFF the other gluon is physical!!

[EXERCISE]: Derive the form of the four-gluon vertex using the same euristic method

The QCD Lagrangian

By direct inspection and by using the form non-abelian covariant derivation, we can check that indeed non-abelian gauge symmetry implies self-interactions. This is not surprising since the gluon itself is charged (In QED the photon is not!)

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_f \bar{\psi}_i^{(f)} (i\partial\!\!\!/ - m_f) \psi_i^{(f)} - \bar{\psi}_i^{(f)} (g_s t^a_{ij} A\!\!\!/_a) \psi_j^{(f)}$$

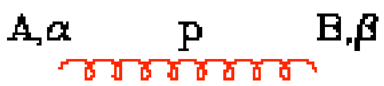
Gauge Fields and their interact.

Matter

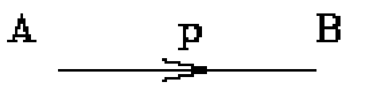
Interaction

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

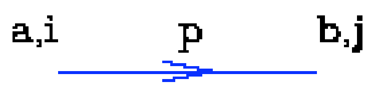
QCD Feynman rules



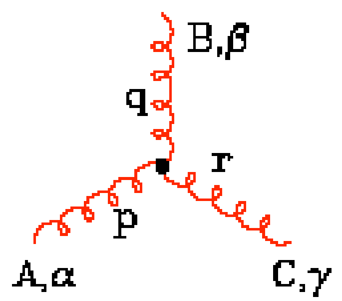
$$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^{\alpha}p^{\beta}}{p^{2}+i\epsilon}\right]\frac{i}{p^{2}+i\epsilon}$$



$$\delta^{AB}\,\frac{i}{(p^{2}+i\epsilon)}$$

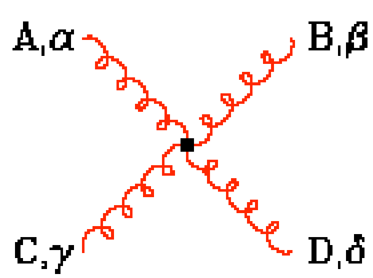


$$\delta^{ab}\,\frac{i}{(\not{p}-m+i\epsilon)_{ji}}$$

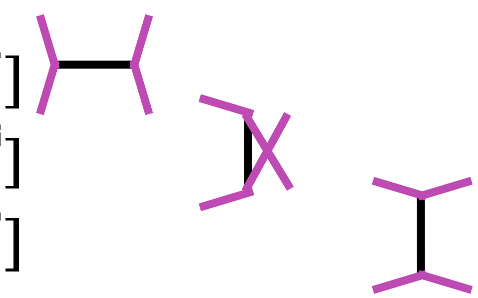


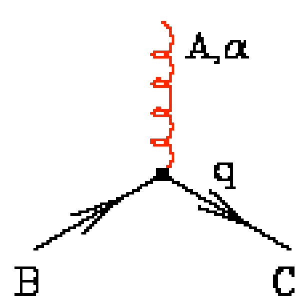
$$-g\,f^{ABC}\left[(p-q)^{\gamma}g^{\alpha\beta}+(q-r)^{\alpha}g^{\beta\gamma}+(r-p)^{\beta}g^{\gamma\alpha}\right]$$

(all momenta incoming)



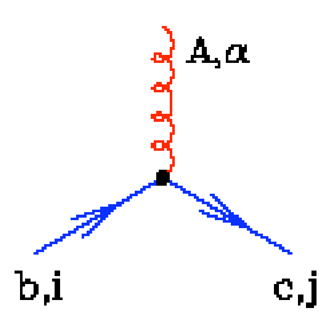
$$-ig^{2}\,f^{XAC}f^{XBD}\,[g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$
$$-ig^{2}\,f^{XAD}f^{XBC}\,[g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$$
$$-ig^{2}\,f^{XAB}f^{XCD}\,[g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$





$$g\,f^{ABC}q^{\alpha}$$

What is this?



$$-ig\,(t^{A})_{cb}\,(\gamma^{\alpha})_{ji}$$

From QED to QCD: physical states

Consider again QED:

$$\sum_{\epsilon_1} |M|^2 = \left(\sum_{\epsilon_1} \epsilon_1^{\mu} \epsilon_1^{\nu *} \right) M_\mu M_\nu^*$$

The two independent physical polarizations of a photon with momentum $k = (k_0; 0, 0, k_0)$ are given by $\epsilon^{\mu}_{L,R} = (0; 1, \pm i, 0)/\sqrt{2}$. They satisfy the standard normalization properties:

$$\epsilon_L \cdot \epsilon_L^* = -1 = \epsilon_R \cdot \epsilon_R^* \qquad \epsilon_L \cdot \epsilon_R^* = 0$$

We can write the sum over physical polarizations in a convenient form by introducing the vector $\bar{k} = (k_0; 0, 0, -k_0)$:

$$\sum_{i=L,R} \epsilon_i^{\mu} \epsilon_i^{\nu *} \equiv \begin{pmatrix} 0 & & \vec{0} & \\ & 1 & 0 & 0 \\ \vec{0} & 0 & 1 & 0 \\ & 0 & 0 & 0 \end{pmatrix} = -g_{\mu\nu} + \frac{k_\mu \bar{k}_\nu + k_\nu \bar{k}_\mu}{k \cdot \bar{k}}$$

In QED the second term can be safely dropped, since $k^\mu \cdot M_\mu = 0$. In fact the longitudinal and time-like component cancel each other, no matter what the choice for ϵ_2 is. The production of any number of unphysical photons vanishes.

For gluons the situation is different, since $k_1 \cdot M \sim \epsilon_2 \cdot k_2$. So the production of two unphysical gluons is not zero!!

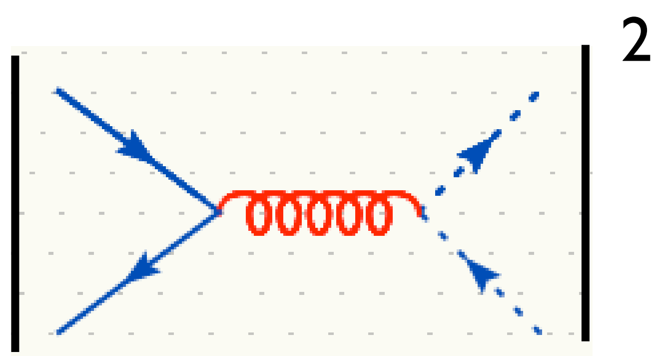
From QED to QCD: physical states

In the case of non-Abelian theories it is therefore important to restrict the sum over polarizations (and the off-shell propagators) to the physical degrees of freedom.

Alternatevely, one has to undertake a formal study of the implications of gauge-fixing in non-physical gauges. The outcome of this approach is the appearance of two color-octet scalar degrees of freedom that have the peculiar property that behave like fermions.

Ghost couple only to gluons and appear in internal loops and as external states (in place of two gluons). Since they break the spin-statistic theorem their contribution can be negative, which is what is require to cancel the the non-physical dof in the general case.

Adding the ghost contribution gives

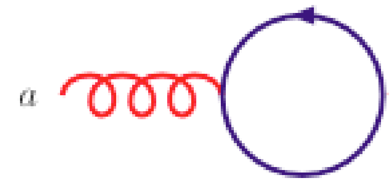


$$= -\left| i g_s^2 f^{abc} t^a \frac{1}{2k_1 \cdot k_2} \bar{v}(\bar{q}) \not{k}_1 u(q) \right|^2$$

which exactly cancels the non-physical polarization in a covariant gauge.

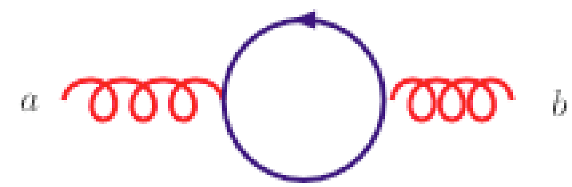
Color algebra

$$\mathrm{Tr}(t^a) = 0$$



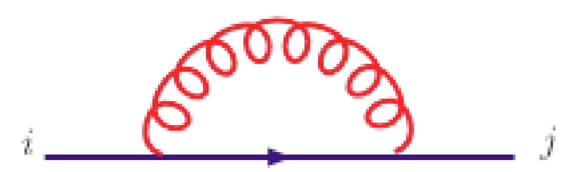
= 0

$$\mathrm{Tr}(t^a t^b) = T_R \delta^{ab}$$



= T_R *

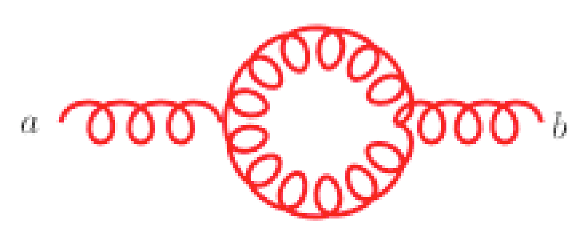
$$(t^a t^a)_{ij} = C_F \delta_{ij}$$



= C_F *

$$\sum_{cd} f^{acd} f^{bcd}$$

$$= (F^c F^c)_{ab} = C_A \delta_{ab}$$

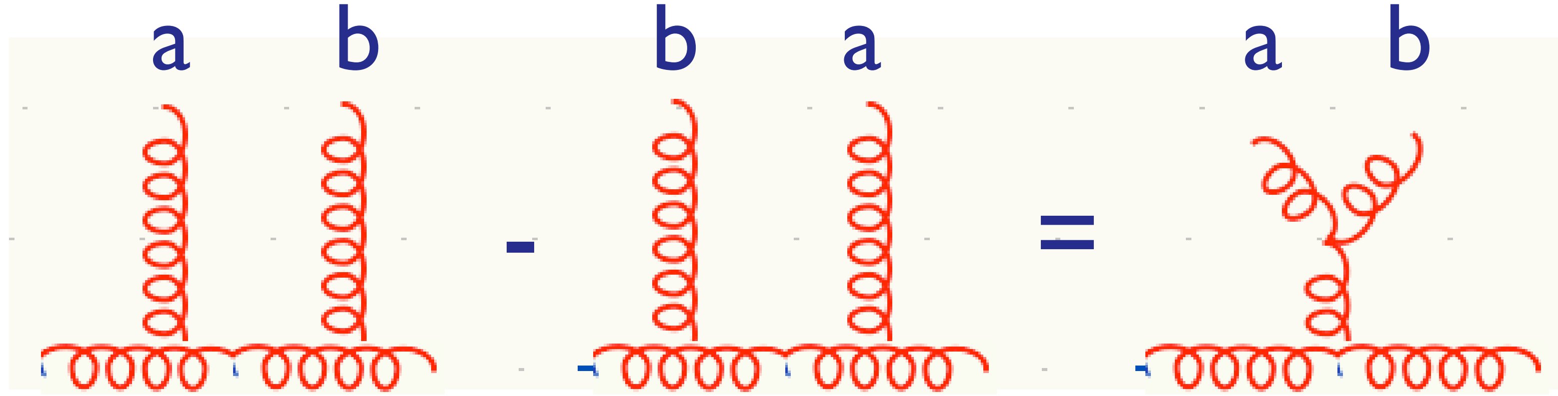


= C_A*

Color algebra

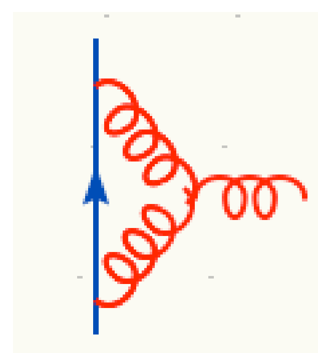
$$[t^a, t^b] = if^{abc}t^c$$

$$[F^a, F^b] = if^{abc}F^c$$

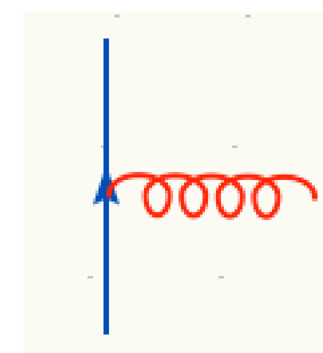


1-loop verteces

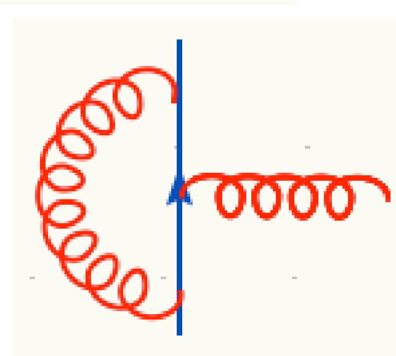
$$if^{abc}(t^b t^c)_{ij} = \frac{C_A}{2} t^a_{ij}$$



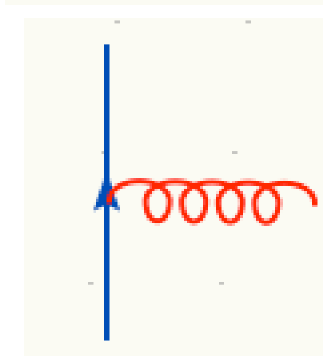
= $C_A/2$ *



$$(t^b t^a t^b)_{ij} = (C_F - \frac{C_A}{2}) t^a_{ij}$$

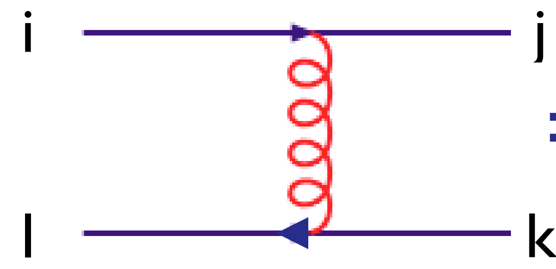


= -1/2/Nc *

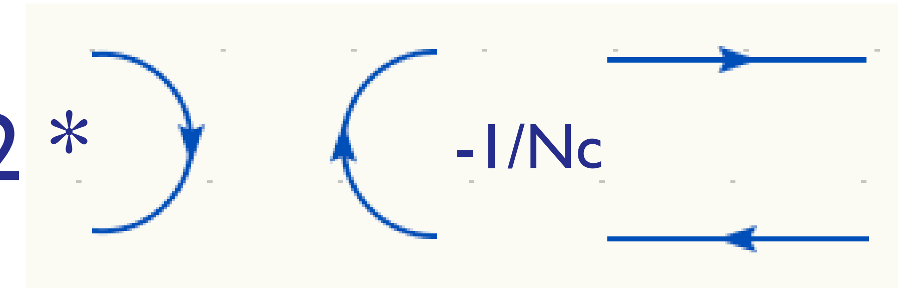


Color algebra: The Fierz identity

$$t^a_{ij} t^a_{kl} = \frac{1}{2}(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl})$$

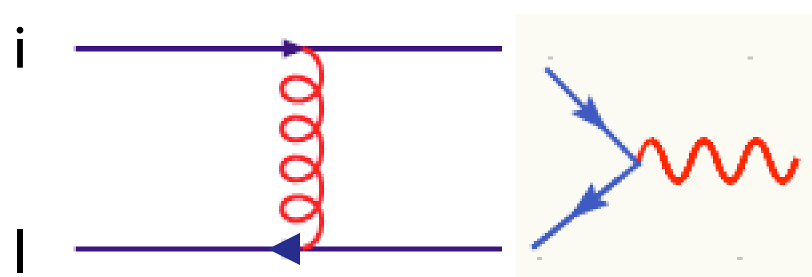


= 1/2 *



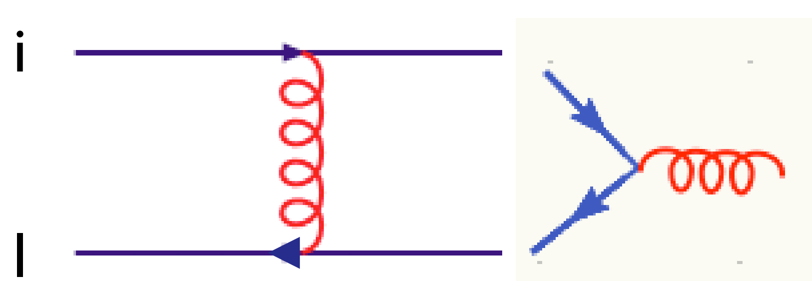
Problem: Show that the one-gluon exchange between quark-antiquark pair can be attractive or repulsive. Calculate the relative strength.

Solution: a q qb pair can be in a singlet state (photon) or in octet (gluon) : $3 \otimes \bar{3} = 1 \oplus 8$



$$\frac{1}{2}(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{lk})\delta_{ki} = \frac{1}{2}\delta_{lj}(N_c - \frac{1}{N_c}) = C_F\delta_{lj}$$

>0, attractive



$$\frac{1}{2}(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{lk})t^a_{ki} = -\frac{1}{2N_c}t^a_{lj}$$

<0, repulsive

Color algebra: 't Hooft double line

A different way to write the QCD lagrangian is not to introduce any matrix in the fundamental representation, but keep the fields in the NxN representation:

$$\mathcal{L} = -\frac{1}{4}(\mathcal{F}^{\mu\nu})^i_j(\mathcal{F}_{\mu\nu})^j_i + i\bar{\psi}_i\gamma^\mu(\delta^i_j\partial_\mu + i\frac{g}{\sqrt{2}}(\mathcal{A}_\mu)^i_j)\psi^j - m\bar{\psi}_i\psi^i$$

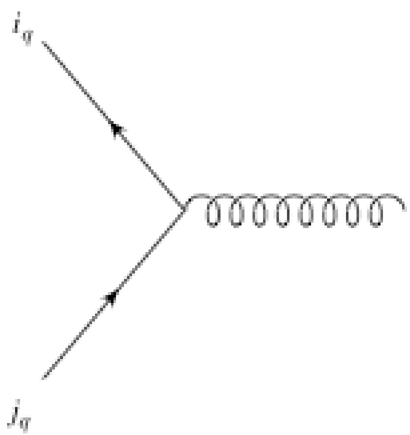
The main differences are:

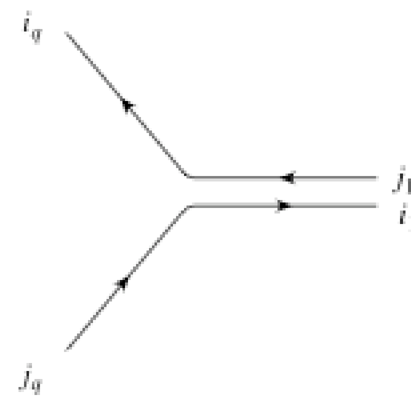
1. The color structure of the gluon propagator is not a delta:

$$\langle(\mathcal{A}_\mu)^{i_1}_{j_1}(\mathcal{A}_\nu)^{i_2}_{j_2}\rangle \propto \delta^{i_1}_{j_2}\delta^{i_2}_{j_1} - \frac{1}{N}\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}$$

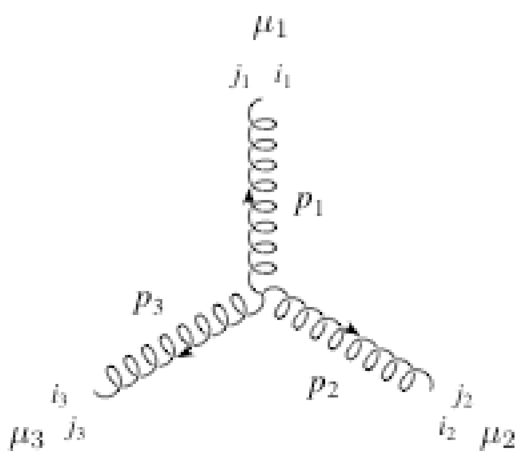
2. The Feynman rules:

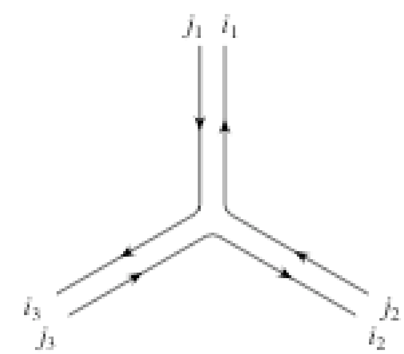
Color algebra: ‘t Hooft double line





$$i\frac{g}{\sqrt{2}}\gamma_1^{\mu}\delta_{j_1}^{i_q}\delta_{j_q}^{i_1}$$

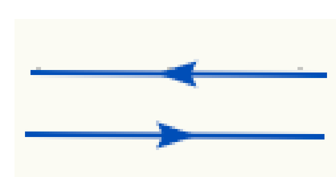




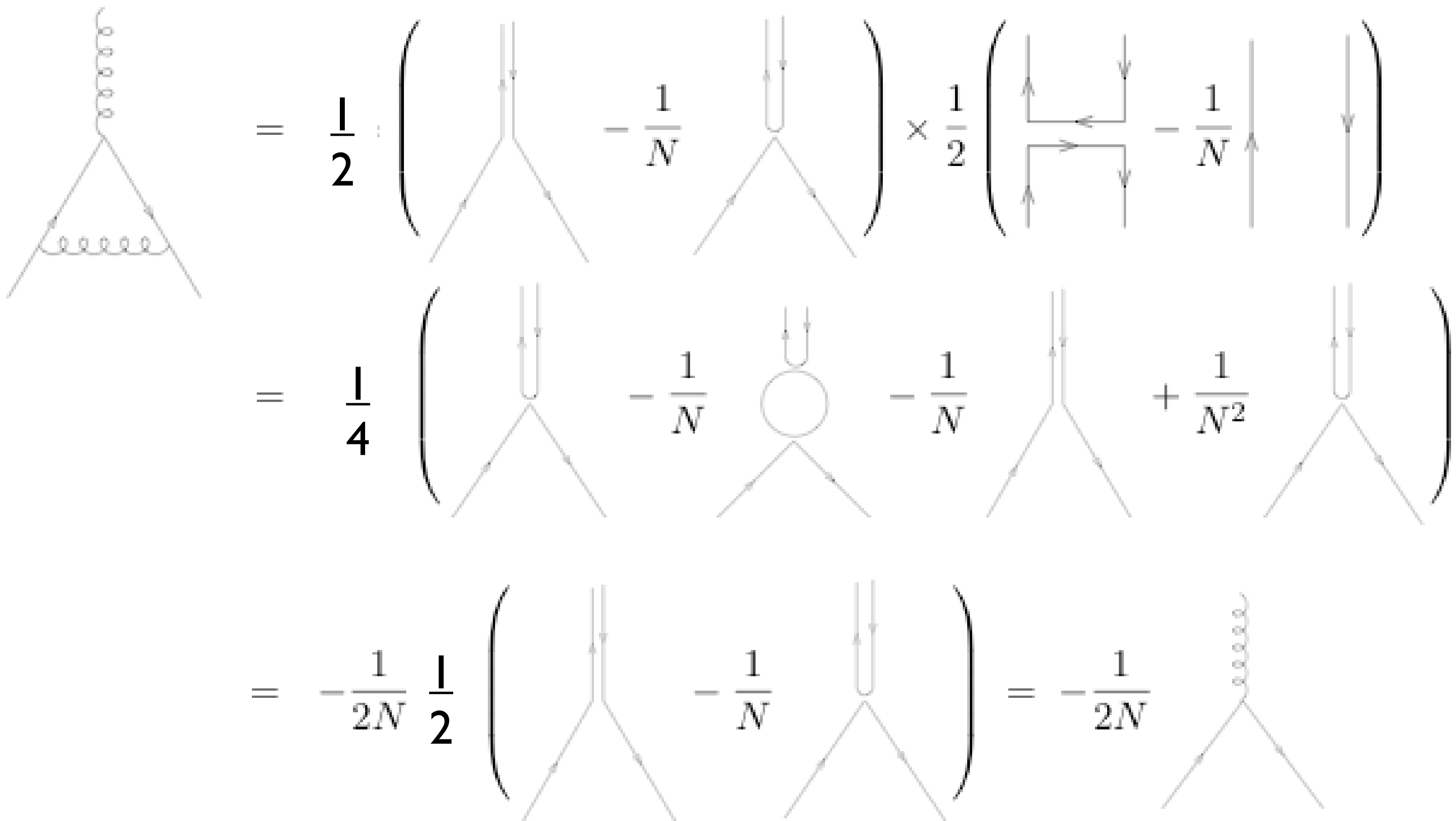
$$i\frac{g}{\sqrt{2}}\sum K^{\mu_1\mu_2\mu_3}\delta_{j_1}^{i_3}\delta_{j_2}^{i_1}\delta_{j_3}^{i_2}$$

This formulation leads to a graphical representation of the simplifications occuring in the large Nc limit, even though it is exactly equivalent to the usual one.

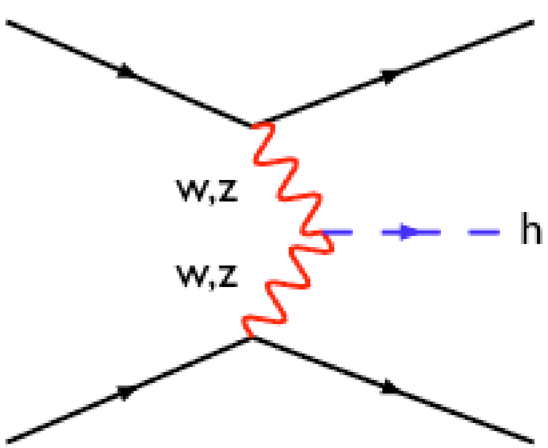
In the large Nc limit, a gluon behaves as a quark-antiquark pair. In addition it behaves classically, in the sense that quantum interference, which are effects of order $1/Nc^2$ are neglected. Many QCD algorithms and codes (such a the parton showers) are based on this picture.

 ≈ 1/2 

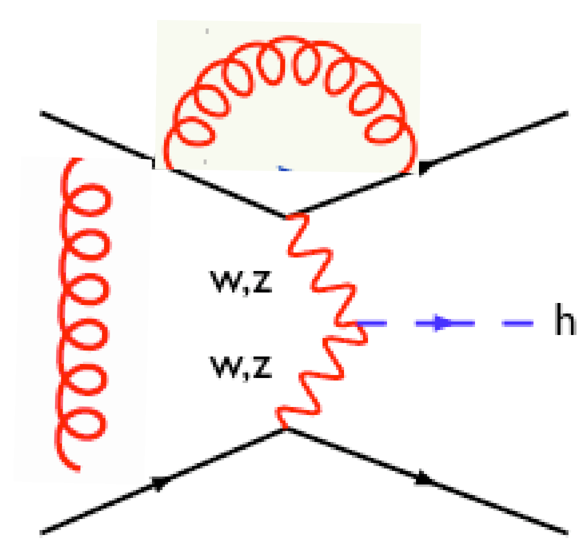
Color algebra: two exercises



Consider WBF: at LO there is no exchange of color between the quark lines:



$\delta_{ij}\delta_{kl}$



$$C_F\delta_{ij}\delta_{kl} \Rightarrow$$

$$M_{\mathrm{tree}}M^*_{1-\mathrm{loop}} = C_F N_c^2 \simeq N_c^3$$

$$\frac{1}{2}(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{kl}) \Rightarrow$$

Also at NLO there is no color exchange! With one little exception.... $M_{\mathrm{tree}}M^*_{1-\mathrm{loop}} = 0$

Basics

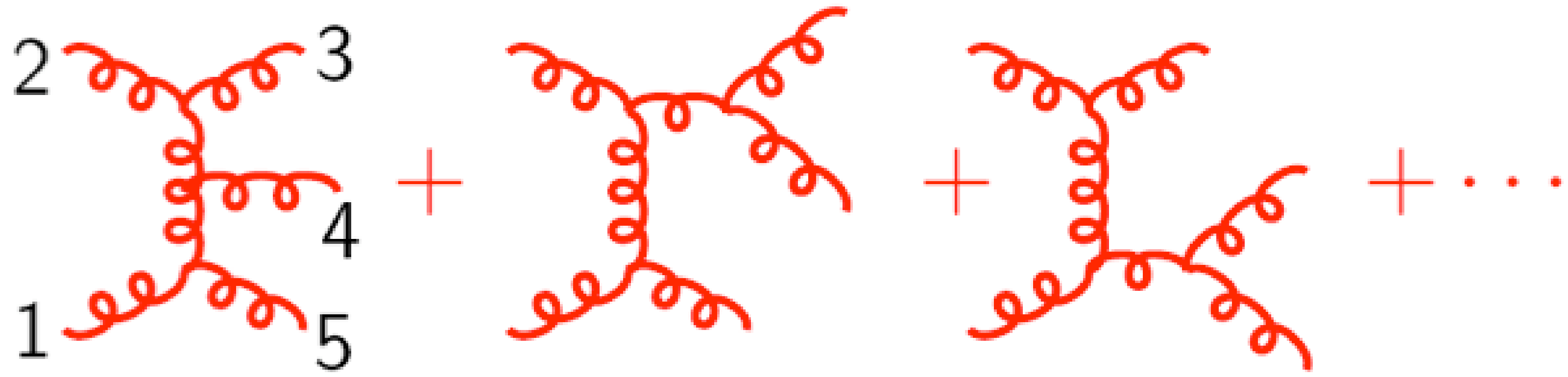
i.e., how to make computations in pQCD

- From QED to QCD
- Color Algebra
- Helicity techniques and recursion

i.e., how difficult is to make computations in QCD!!

Example: a simple calculation?

Consider a simple 5 gluon amplitude:



There are 25 diagrams with a complicated tensor structure, so you get....

Example: a simple calculation?

A(k1,e1,k2,e2,k3,e3,k4,e4,k5,e5) = + Tr(Ta1,Ta2,Ta3,Ta4,Ta5) * (1/2*den(2*k1.k2)*k1.e2*e1.e3*e4.e5 - den(2*k1.k2)*k1.e2*e1.e4*e3.e5
+ 1/2*den(2*k1.k2)*k1.e2*e1.e5*e3.e4 - 1/4*den(2*k1.k2)*k1.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*k1.e4*e1.e2*e3.e5 - 1/4*den(2*k1.k2)*k1.e5*e1.e2*e3.e4
- 1/2*den(2*k1.k2)*k2.e1*e2.e3* e4.e5 + den(2*k1.k2)*k2.e1*e2.e4*e3.e5 - 1/2*den(2*k1.k2)*k2.e1*e2.e5 *e3.e4 + 1/4*den(2*k1.k2)*k2.e3*e1.e2*e4.e5
- 1/2*den(2*k1.k2)*k2.e4* e1.e2*e3.e5 + 1/4*den(2*k1.k2)*k2.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)* den(2*k3.k4)*k1.k3*k1.e2*e1.e5*e3.e4
- 1/2*den(2*k1.k2)*den(2*k3.k4)* k1.k3*k2.e1*e2.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k2.e5* e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k3.e4*e1.e2*e3.e5
+ 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k4.e3*e1.e2*e4.e5 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.k3*k4.e5*e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*
k3.k4)*k1.k4*k1.e2*e1.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4* k2.e1*e2.e5*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4*k2.e5*e1.e2*
e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4*k3.e4*e1.e2*e3.e5 + 1/2* den(2*k1.k2)*den(2*k3.k4)*k1.k4*k3.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*

+ Tr(Ta1,Ta2,Ta3,Ta4,Ta5) * (1/2*den(2*k1.k2)*k1.e2*e1.e3*e4.e5

k1.e5*k3.e1*e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k3.e4*e1.e3 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k4.e1*e3.e4
- den(2* k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k4.e3*e1.e4 + 1/2*den(2*k1.k2)*den(2* k3.k4)*k1.e2*k2.k3*e1.e5*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k2.k4*e1.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e3*k3.e4*e1.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e4*k4.e3*e1.e5 - 1/2*den(2*
k1.k2)*den(2*k3.k4)*k1.e2*k2.e5*k3.e1*e3.e4 + den(2*k1.k2)*den(2* k3.k4)*k1.e2*k2.e5*k3.e4*e1.e3 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k2.e5*k4.e1*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e5*k4.e3*e1.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.k5*e1.e5*e3.e4 +
den(2* k1.k2)*den(2*k3.k4)*k1.e2*k3.e1*k3.e4*e3.e5 - den(2*k1.k2)*den(2* k3.k4)*k1.e2*k3.e1*k4.e3*e4.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k3.e1*k4.e5*e3.e4 - den(2*k … k1.k2)*
den(2*k3.k4)*k1.e2*k3.e4* … *k5.e3
*e1.e5 - den(2*k1.k2)*den(2 …)*den(2*
k3.k4)*k1.e2*k3.e5*k5.e1*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2* k4.k5*e1.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e1*k4.e3*e4.e5
+ den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e3*k4.e5*e1.e4 + den(2*k1.k2)* den(2*k3.k4)*k1.e2*k4.e3*k5.e1*e4.e5 - den(2*k1.k2)*den(2*k3.k4)*
k1.e2*k4.e3*k5.e4*e1.e5 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e5* k5.e1*e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e3*k2.e1*k3.e4*e2.e5
- den(2*k1.k2)*den(2*k3.k4)*k1.e3*k2.e5*k3.e4*e1.e2 + 1/2*den(2*k1.k2)* den(2*k3.k4)*k1.e3*k3.e4*k3.e5*e1.e2 + 1/2*den(2*k1.k2)*den(2*k3.k4)*
k1.e3*k3.e4*k4.e5*e1.e2 - den(2*k1.k2)*den(2*k3.k4)*k1.e4*k2.e1*k4.e3 *e2.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e4*k2.e5*k4.e3*e1.e2
- 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e4*k3.e5*k4.e3*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e4*k4.e3*k4.e5*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*
k1.e5*k2.k3*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.k4* e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k3.e2*e3.e4
- den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k3.e4*e2.e3 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k4.e2*e3.e4 + den(2*k1.k2)*den(2*
k3.k4)*k1.e5*k2.e1*k4.e3*e2.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e5* k2.e3*k3.e4*e1.e2 - den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e4*k4.e3*e1.e2
+ 1/4*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k3.k5*e1.e2*e3.e4 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.e5*k3.e4*k5.e3*e1.e2 - 1/4*den(2*k1.k2)*den(2*
k3.k4)*k1.e5*k4.k5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5* k4.e3*k5.e4*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k3*k2.e1*e2.e5*
e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k3*k3.e4*e1.e2*e3.e5 - 1/2* den(2*k1.k2)*den(2*k3.k4)*k2.k3*k4.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*
den(2*k3.k4)*k2.k3*k4.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)* k2.k4*k2.e1*e2.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k3.e4*
e1.e2*e3.e5 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k3.e5*e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k4.e3*e1.e2*e4.e5 - 1/2*den(2*
k1.k2)*den(2*k3.k4)*k2.k5*k3.e4*e1.e2*e3.e5 + 1/4*den(2*k1.k2)*den(2* k3.k4)*k2.k5*k3.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k5*
k4.e3*e1.e2*e4.e5 - 1/4*den(2*k1.k2)*den(2*k3.k4)*k2.k5*k4.e5*e1.e2* e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e3*k3.e4*e2.e5 -
den(2* k1.k2)*den(2*k3.k4)*k2.e1*k2.e4*k4.e3*e2.e5 + 1/2*den(2*k1.k2)*den(2* k3.k4)*k2.e1*k2.e5*k3.e2*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k2.e1*
k2.e5*k3.e4*e2.e3 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e5*k4.e2* e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e5*k4.e3*e2.e4

Brute force is not an option!

Solution

Keep track of all the quantum numbers,
(momenta, spin and color)
and organize them in
efficient way, by choosing appropriate basis.

The helicity method

Pioneering work of Berends, Gastmans, Troost, Wu in the '80, where they introduce the techniques of helicity amplitudes

$$u_\pm(k) = \frac{1}{2}(1 \pm \gamma_5)u(k)$$

$$\overline{u_-(k_i)}u_+(k_j) = \langle k_i - | k_j + \rangle \equiv \langle ij \rangle = \sqrt{s_{ij}}e^{-i\phi}$$

$$\overline{u_+(k_i)}u_-(k_j) = \langle k_i + | k_j - \rangle \equiv [ij] = -\sqrt{s_{ij}}e^{i\phi}$$

Using these objects, Xu, Zhang and Chang (1987) introduced simple vector polarizations

$$\varepsilon_\mu^+(k;q) = \frac{\langle q^- | \gamma_\mu | k^- \rangle}{\sqrt{2}\,\langle q\, k \rangle}, \qquad \varepsilon_\mu^-(k,q) = \frac{\langle q^+ | \gamma_\mu | k^+ \rangle}{\sqrt{2}\,[k\, q]}$$

gauge vector

It's just a more sophisticated version of the circular polarization. Choosing appropriately the gauge vector, expressions simplify dramatically.

Stripping color out

Inspired by the way gauge theories appear as the zero-slope limits of (open) string theories, it has been suggested to decompose the full amplitude as a sum of gauge invariant Subamplitudes times color coefficients:

$$\mathcal{A}_n(g_1,\ldots,g_n) = g^{n-2} \sum_{\sigma\in S_{n-1}} \mathrm{Tr}(\mathsf{t}^{a_1}\,\mathsf{t}^{a_{\sigma_2}}\cdots\mathsf{t}^{a_{\sigma_n}}) A_n(1,\sigma_2,\ldots,\sigma_n)$$

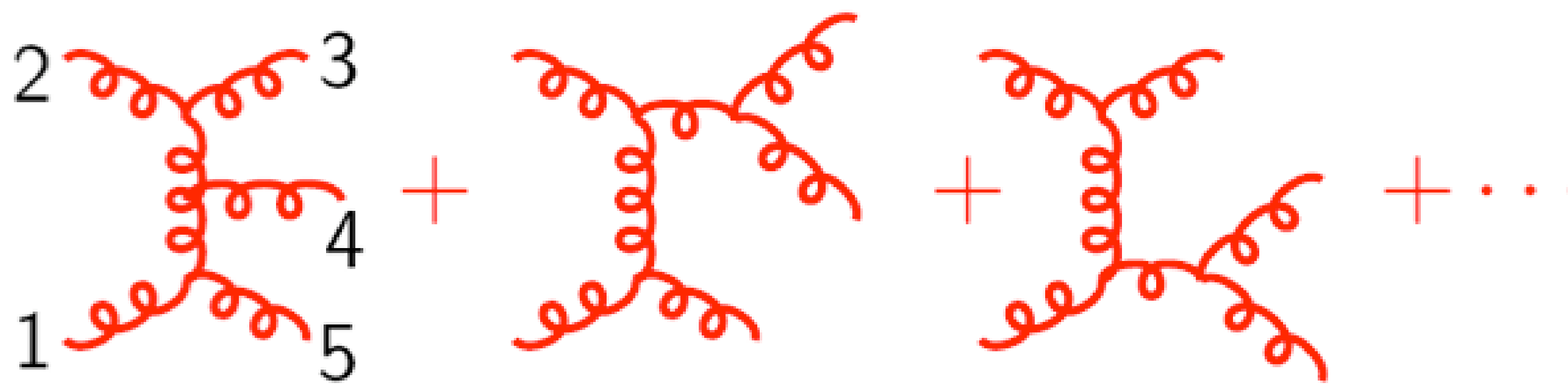
where the formula $if^{abc} = \mathrm{Tr}(\mathsf{t}^a,[\mathsf{t}^b,\mathsf{t}^c])$ has been repeatedly used to reduce the f's into traces of lambdas and the Fierz identities to cancel traces of length l<n.

Analogously for quarks:

$$\mathcal{A}_n(q_1,g_2,\ldots,g_{n-1},\bar{q}_n) = g^{n-2} \sum_{\sigma\in S_{n-2}} (\mathsf{t}^{a_{\sigma_2}}\cdots\mathsf{t}^{a_{\sigma_{n-1}}})^i_j A_n(1_q,\sigma_2,\ldots,\sigma_{n-2},n_{\bar{q}})$$

Example

Consider a simple 5 gluon amplitude:



There are 25 diagrams with a complicated tensor structure, but only 10 for a color flow and even less w/ helicities

$$A_5(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0$$

MHV amplitude

$$A_5(1^-, 2^-, 3^+, 4^+, 5^+) = i \frac{\langle 1\,2 \rangle^4}{\langle 1\,2 \rangle \langle 2\,3 \rangle \langle 3\,4 \rangle \langle 4\,5 \rangle \langle 5\,1 \rangle}$$

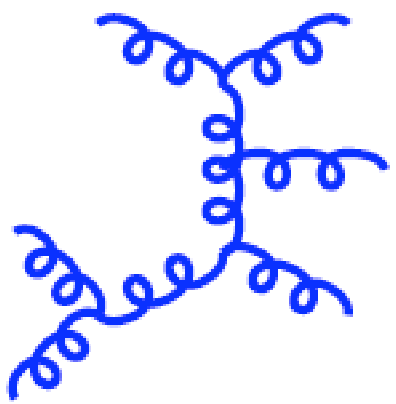
Number of diagrams for a n-gluon amplitude

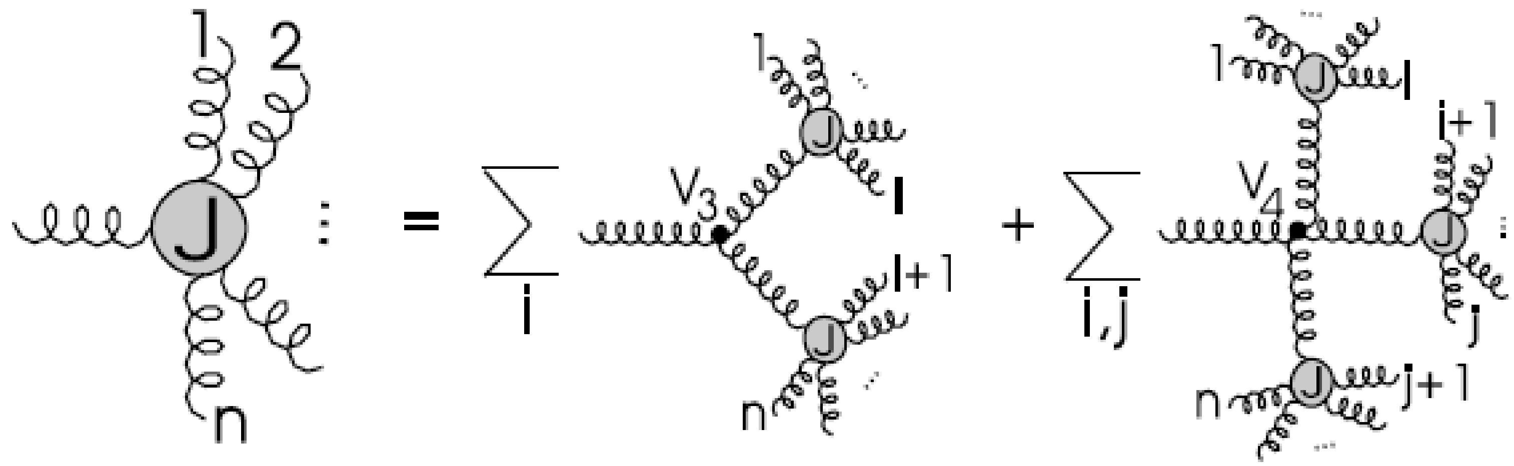
n	full Amp	partial Amp
4	4	3
5	25	10
6	220	36
7	2485	133
8	34300	501
9	559405	1991
10	10525900	7335
11	224449225	28199
12	5348843500	108280

$(2n)!$ $\qquad$ 3.8^n

Recursive relations

Feynman diagram beg to be evaluated recursively





J^μ is the Berends-Giele current. For MHV can solve analytically!

$$J^\mu(1^-, 2^+, \ldots, n^+) \;=\; \frac{\langle 1^- | \gamma^\mu \not{P}_{2,n} | 1^+ \rangle}{\sqrt{2}\,\langle 1\,2\rangle \cdots \langle n\,1\rangle} \sum_{m=3}^{n} \frac{\langle 1^- | \not{k}_m \not{P}_{1,m} | 1^+ \rangle}{P_{1,m-1}^2 P_{1,m}^2} \,,$$

Dotting with ε^- on the free leg and cleaning up gives:

$$A_n^{\text{tree}}(1^-, 2^-, 3^+, 4^+, \ldots, n^+) \;=\; i\, \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle \cdots \langle n\,1\rangle}$$

Parke-Taylor amplitude is proven!

Infinite number of Feynman diagrams solved at once!

Number of diagrams for n-gluon amplitudes

n	full Amp	partial Amp	BG
4	4	3	3
5	25	10	10
6	220	36	35
7	2485	133	70
8	34300	501	126
9	559405	1991	210
10	10525900	7335	330
11	224449225	28199	495
12	5348843500	108280	715
	$(2n)!$	3.8^n	n^4

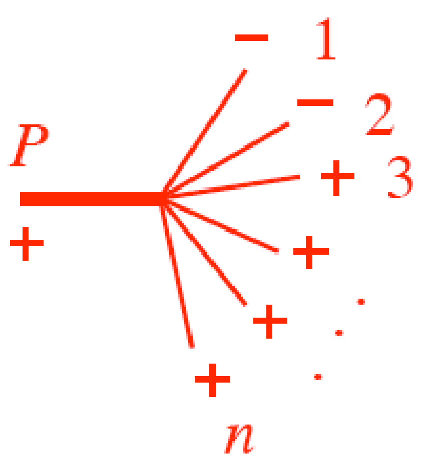
The factorial growth is tamed to a polynomial one!

Note, however, one still needs to sum over color, an operation which sets the complexity back to exponential.

What about analytic results?

- Until two years ago, only the analytic formulas for MHV amplitudes for any n was known.
- Recently unexpected stunning progress has been achieved triggered by Witten on the analytic calculation of tree-level and loop amplitudes from topological string theories.
- The recipe Cachazo, Bo Feng, Svreck, Witten et al. found is that the other helicity configuration can be obtained by sewing together "modified off-shell" MHV amplitudes, which can be thought a "building blocks"

$$V(1^-,2^-,3^+,\dots,n^+,P^+) = \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle \cdots \langle n-1,\,n\rangle\,\langle n\,P\rangle\,\langle P\,1\rangle}$$



Continue spinor off-shell ($P^2 \neq 0$): $\langle i\,P\rangle = \eta \sum_{j=1}^{n} \langle i^- | \not{k}_j | q^- \rangle$
where $P = k_1 + k_2 + \cdots k_n$ and q auxiliary, satisfying $q^2 = 0$.

Tree-level status

• Complexity of plain vanilla Feynman calculations grows factorially

• Standard techniques based on calculating simpler guauge invariant objects by a recursive techniques are very powerful and reduce the complexity from factorial to polinomial.

•In any case the calculation through partial amplitudes is not as efficient as the direct calculation of the full amplitude at fixed color through numerical recursive relations [ALPGEN, Moretti, Caravaglios, Mangano, Pittau, 1998; HELAC, Draggiotis, Kleiss, Papadopoulos, 1998], which has only an exponential growth.

• New twistor tree-level BCF or CSW, without or with color, relations don't improve on the "old" Berends-Giele recursive relations.
[Dinsdale, Wernick, Weinzierl, 2006; Duhr, Hoeche, FM, 2006].

Basics

summary

- Performing calculations in pQCD is difficult and still an art.
- Accurate and flexible tools are needed to improve our chances to make discoveries at the LHC.
- We have reviewed:
  - How gauge invariance works in QCD
  - The role of color
  - Basic techniques to calculate efficiently amplitudes in QCD

Lecture material + exercises can be found at:

http://cp3wks05.fynu.ucl.ac.be/twiki/bin/view/Library/GIFSchool

Outline

- Basics: how to make calculations in pQCD
- Improving the accuracy: NLO and NNLO
- Improving the flexibility:
  Matrix elements MC's

NLO and NNLO

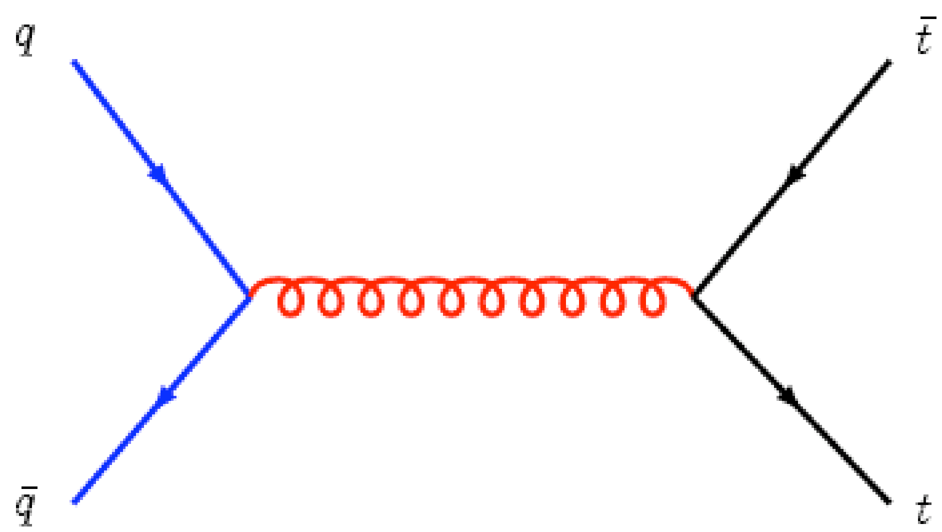
Improving the accuracy

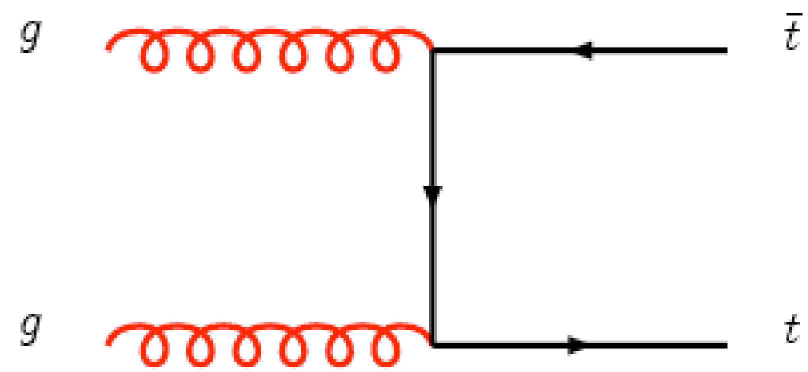
- Motivation
- NLO in proton proton collisions: $pp \to H+X$
- General approach and available tools
- Towards NNLO

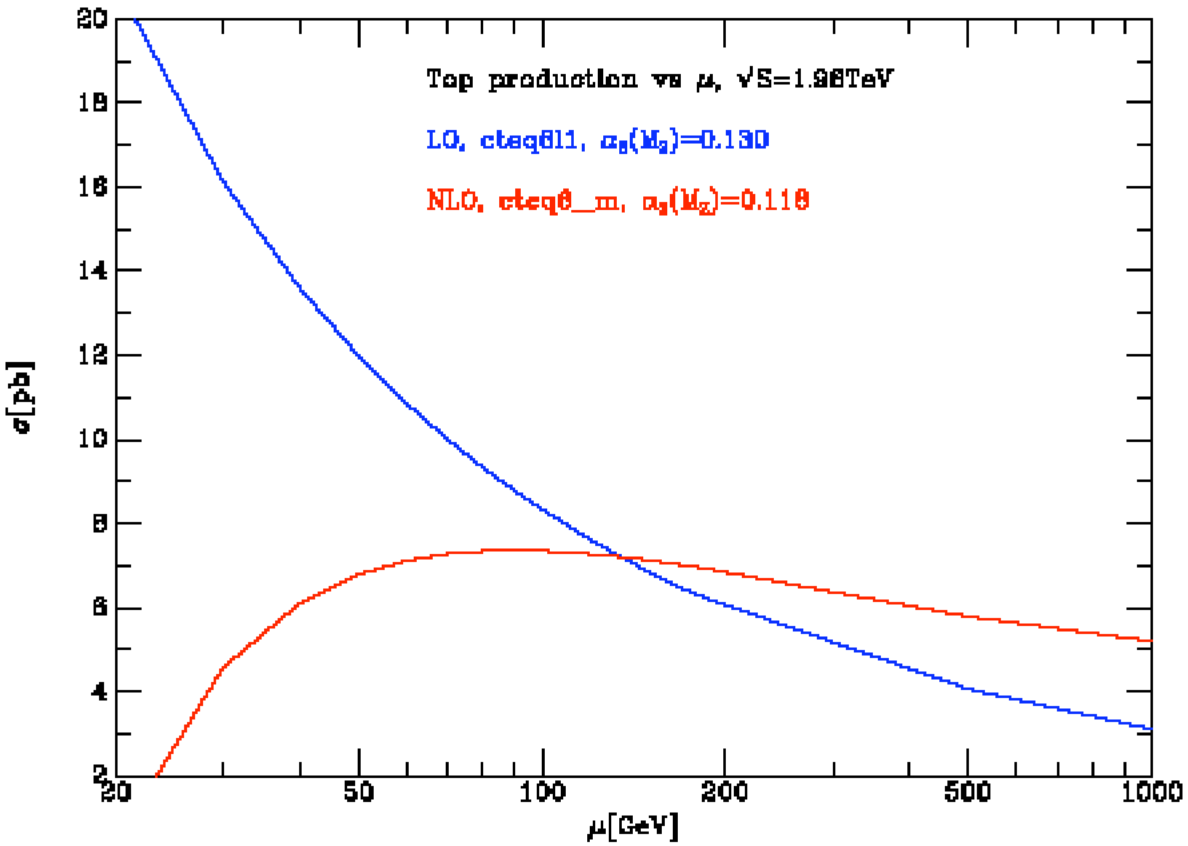
NLO calculations

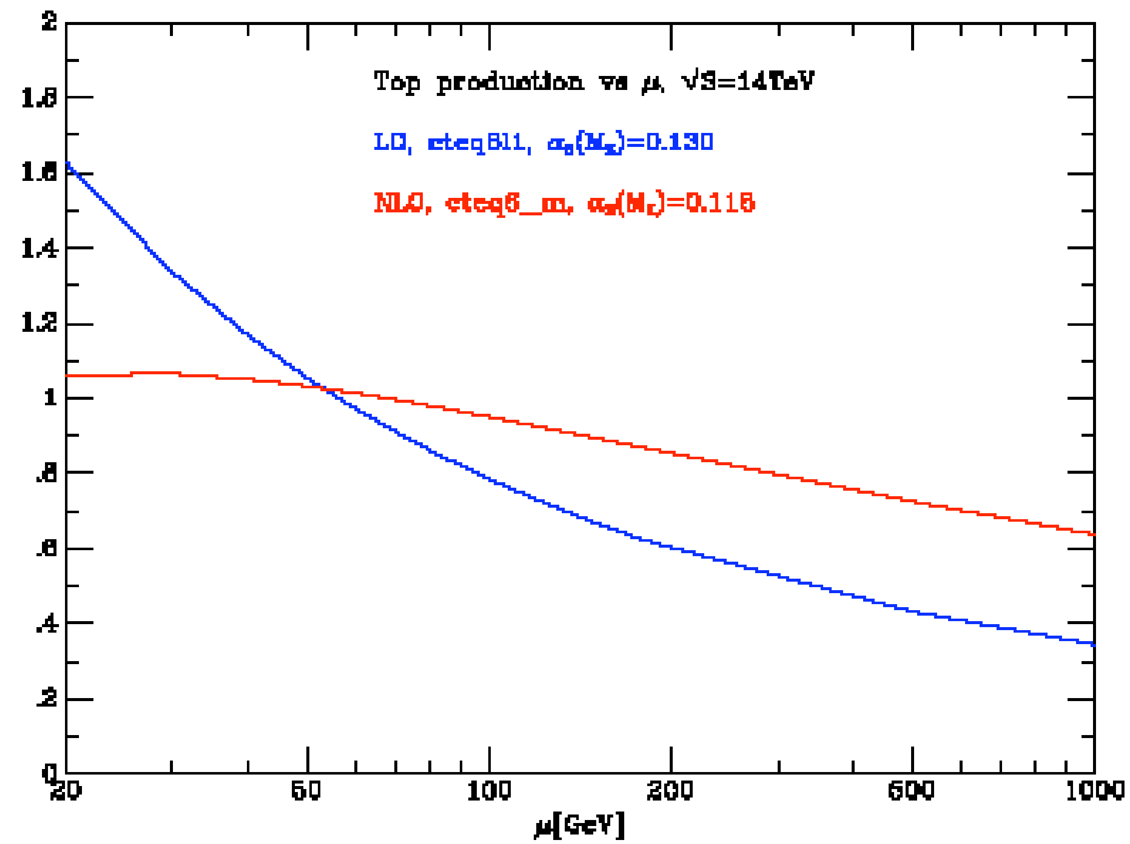
- NLO calculations are needed to perform measurements where the knowledge of total and differential rates is essential. This is true not only for the signal but also for the backgrounds.
- Standard NLO programs do not produce unweighted events and therefore are not suitable for direct experimental analysis.
- In fact, it can be highly non-trivial to establish an accurate connection between what is computed at the partonic level and what is measured (hadronic quantities).
- Comparison with data can be done once detector and hadronization effects have been deconvoluted.
- Be aware that there are many possibly dangerous (mal)practices in the exp community (K-factor, reiweithing of distributions,...)
- Suggestion: always consult with the authors of the code in case of doubts...

Tevatron vs LHC



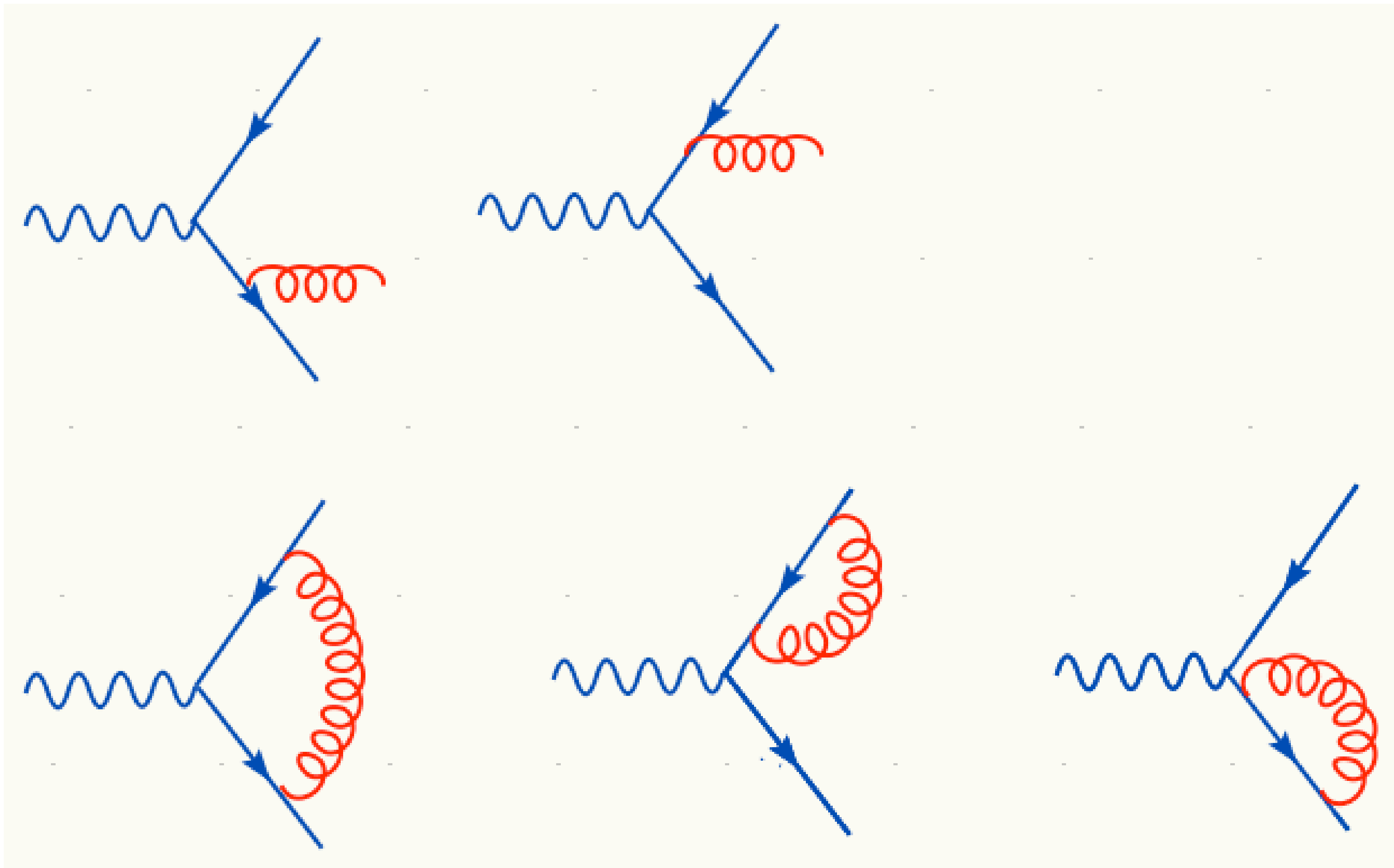






Inclusion of higher order corrections leads to a stabilization of the prediction.
At the LHC scale dependence is more difficult to estimate.

The elements of NLO calculation



Real

Virtual

The KLN theorem states that divergences appear because some of the final state are physically degenerate but we treated them as different. A final state with a soft gluon is nearly degenerate with a final state with no gluon at all (virtual).

$$\int \frac{d^d k}{(2\pi)^d} \cdots$$

$$\sigma^{\rm NLO} = \int_R |M_{real}|^2 d\Phi_3 + \int_V 2Re\left(M_0 M^*_{virt}\right) d\Phi_2 = \text{finite!}$$

Infrared divergences

Infrared divergences arise from interactions that happen a long time after the creation of the quark/antiquark pair.

When distances become comparable to the hadron size of ~1 Fermi, quasi-free partons of the perturbative calculation are confined/hadronized non-perturbatively.

We have seen that in total cross sections such divergences cancel. But what about for other quantities?

Well obviously the only possibility is to try to use the pQCD calculations for quantities that are not sensitive to the to the long-distance physics.

Can we formulate a criterium that is valid in general?

YES! It is called INFRARED SAFETY

Infrared-safe quantities

DEFINITION: quantities are that are insensitive to soft and collinear branching.

For these quantities, an extension of the general theorem (KLN) exists which proves that infrared divergences cancel betwen real and virtual or are simply removed by kinematic factors.

Such quantities are determined primarily by hard, short-distance physics. Long-distance effects give power corrections, suppressed by the inverse power of a large momentum scale (which must be present in the first place to justify the use of PT).

EXAMPLES: total rates & cross sections, jet distrubutions, shape variables...

NLO codes calculate IR safe quantities

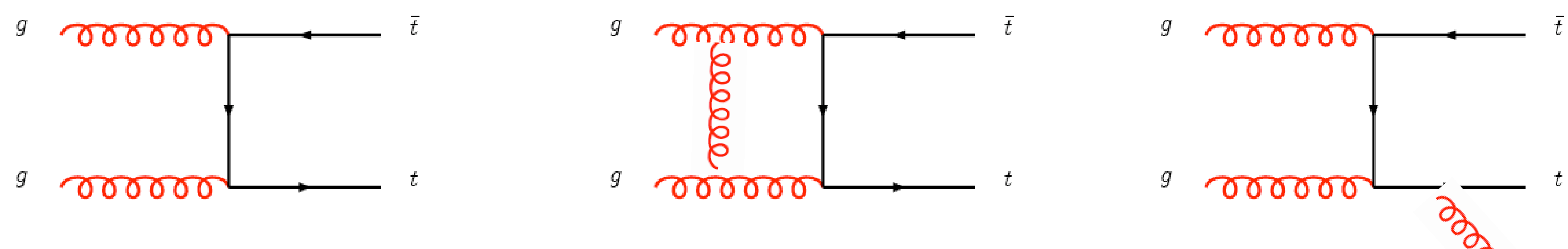
Something to remember well

Calling a code "a NLO code" is an abuse of language and can be confusing.

A NLO calculation always refers to an IR-safe observable.

An NLO code will, in general, be able to produce results for several quantities and distributions, only some of which will be at NLO accuracy.

Example: Suppose we use the NLO code for $pp \to t\bar{t}$



LO Virt Real

- ☞ Total cross section, $\sigma(tt)$ .... ✓
- ☞ P_T of one top quark .... ✓
- ☞ P_T of the tt pair .... ✗
- ☞ P_T of the jet .... ✗
- ☞ tt invariant mass, m(tt) .... ✓
- ☞ $\Delta\Phi(tt)$ .... ✗

Anatomy of pp→Higgs at NLO

- LO : 1-loop calculation and HEFT
- NLO in the HEFT
  - Virtual corrections and renormalization
  - Real corrections and IS singularities
- Cross sections at the LHC

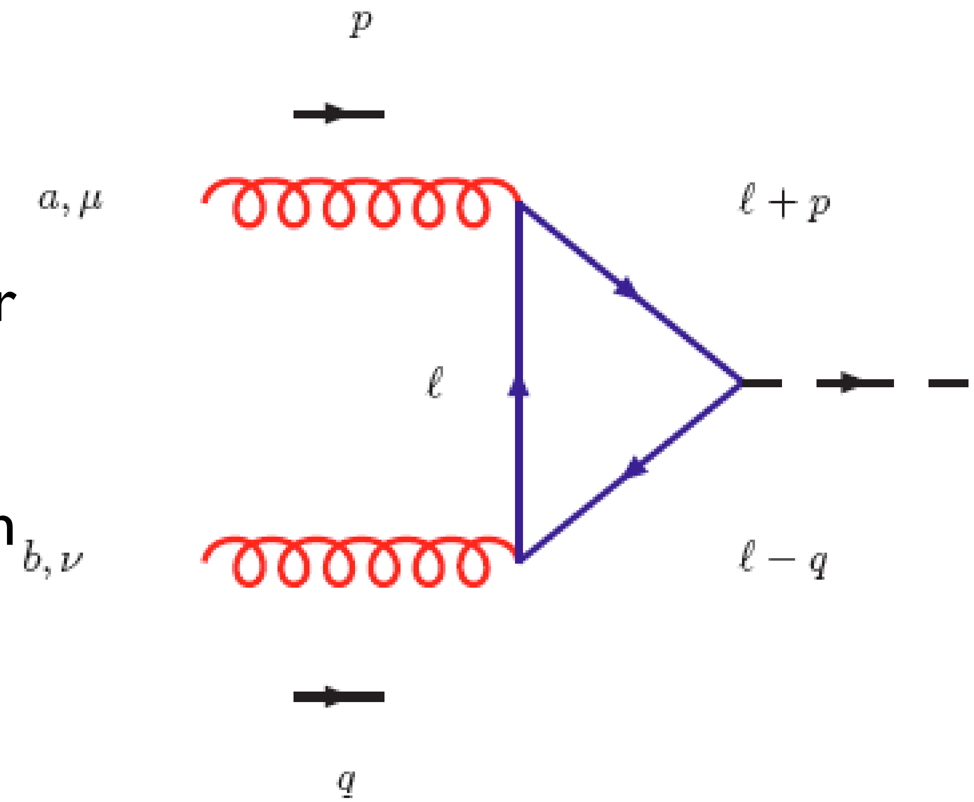
pp→H at LO

This is a "simple" 2→1 process.

However, at variance with pp→W, the LO order process already proceeds through a loop.

In this case, this means that the loop calculation has to give a finite result!

Let's do the calculation!



$$i\mathcal{A} = -(-ig_s)^2 \mathrm{Tr}(t^a t^b) \left(\frac{-im_t}{v}\right) \int \frac{d^d\ell}{(2\pi)^n} \frac{T^{\mu\nu}}{\mathrm{Den}} (i)^3 \epsilon_\mu(p) \epsilon_\nu(q)$$

where

$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell + p)^2 - m_t^2][(\ell - q)^2 - m_t^2]$$

We combine the denominators into one by using $\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1-x-y)]^3}$

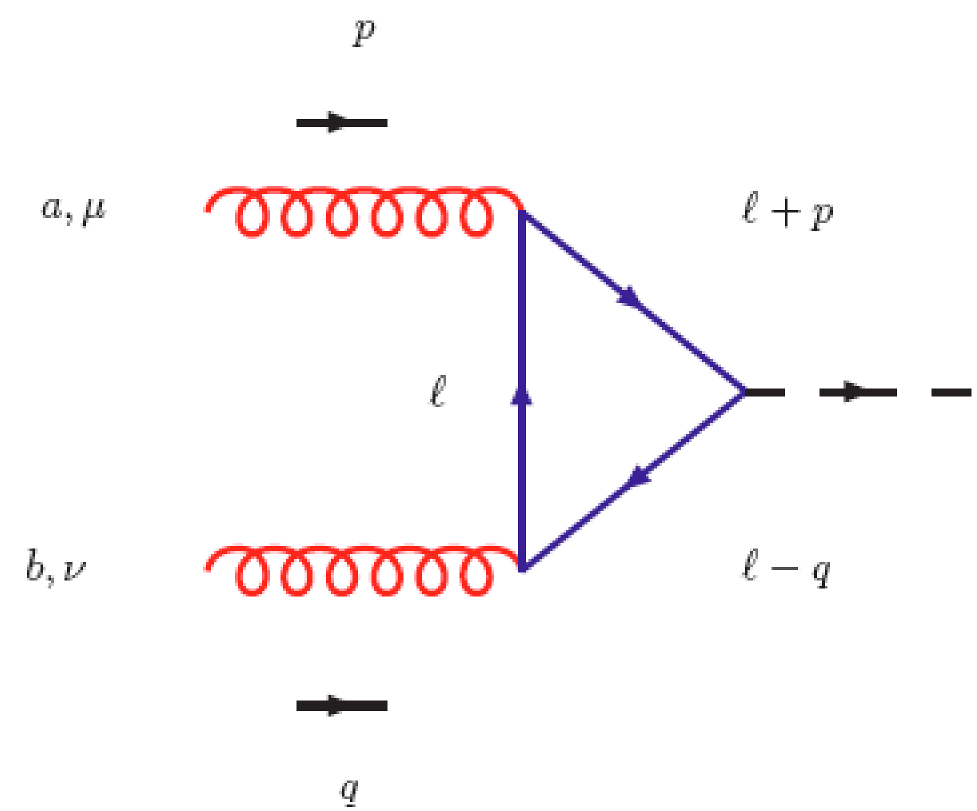
$$\frac{1}{\mathrm{Den}} = 2\int dx\ dy \frac{1}{[\ell^2 - m_t^2 + 2\ell \cdot (px - qy)]^3}.$$

pp→H at LO

We shift the momentum:

$$\ell' = \ell + px - qy$$

$$\frac{1}{\text{Den}} \to 2\int dx\ dy \frac{1}{[\ell'^{\,2} - m_t^2 + M_H^2 xy]^3}.$$



And now the tensor in the numerator:

$$T^{\mu\nu} = \text{Tr}\left[(\ell + m_t)\gamma^\mu(\ell + p + m_t)(\ell - q + m_t)\gamma^\nu)\right]$$

$$= 4m_t\left[g^{\mu\nu}(m_t^2 - \ell^2 - \frac{M_H^2}{2}) + 4\ell^\mu\ell^\nu + p^\nu q^\mu\right]$$

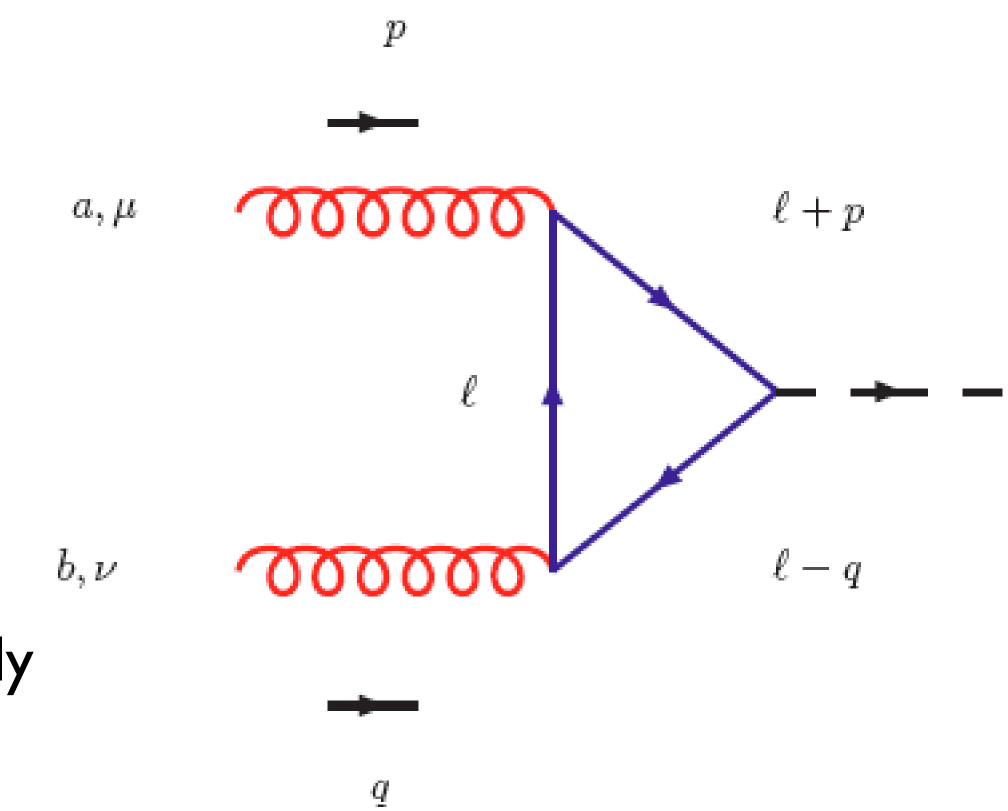
where I used the fact that the external gluons are on-shell. This trace is proportional to m_t ! This is due to the spin flip caused by the scalar coupling.

Now we shift the loop momentum also here, we drop terms linear in the loop momentum (they are odd and vanish) and

pp→H at LO

we perform the tensor decomposition using:

$$\int d^dk \frac{k^\mu k^\nu}{(k^2-C)^m} = \frac{1}{d} g^{\mu\nu} \int d^dk \frac{k^2}{(k^2-C)^m}$$



So I can write an expression which depends only on scalar loop integrals:

$$i\mathcal{A} = -\frac{2g_s^2 m_t^2}{v}\delta^{ab}\int \frac{d^d\ell'}{(2\pi)^d}\int dxdy\Big\{g^{\mu\nu}\Big[m^2+\ell'^2\Big(\frac{4-d}{d}\Big)+M_H^2(xy-\frac{1}{2})\Big]$$

$$+p^\nu q^\mu(1-4xy)\Big\}\frac{2dxdy}{(\ell'^2-m_t^2+M_H^2xy)^3}\epsilon_\mu(p)\epsilon_\nu(q).$$

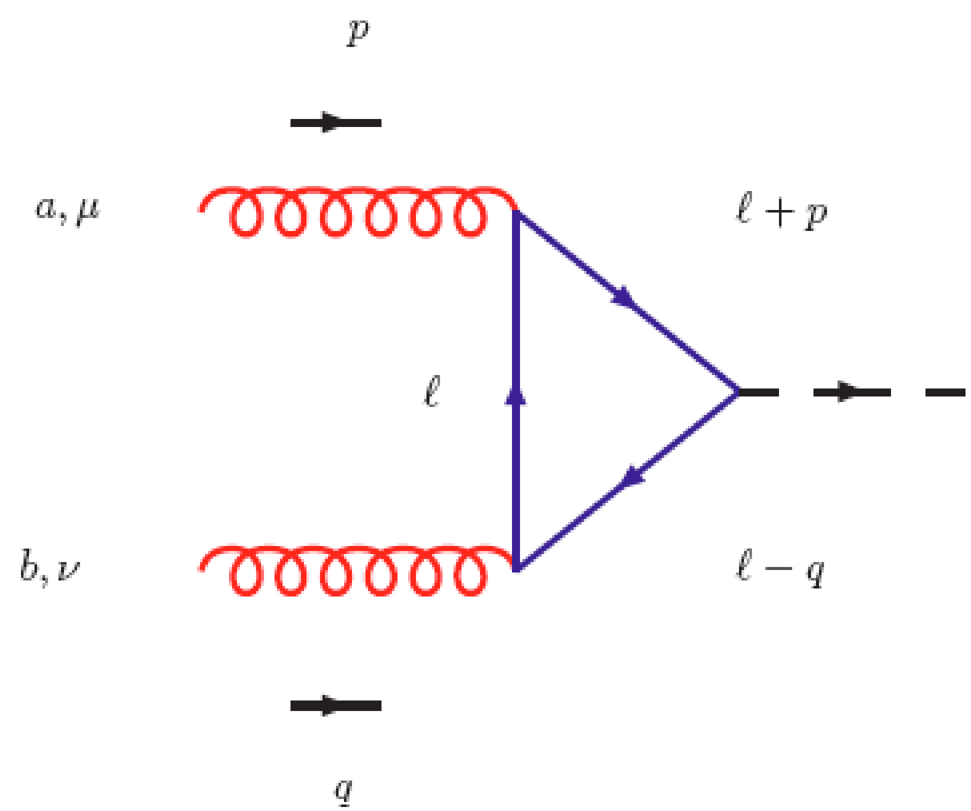
There's a term which apparently diverges....??

Ok, Let's look the scalar integrals up in a table (or calculate them!)

pp→H at LO

$$\int \frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^2-C)^3} = \frac{i}{32\pi^2}(4\pi)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon}(2-\epsilon)C^{-\epsilon}$$

$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-C)^3} = -\frac{i}{32\pi^2}(4\pi)^\epsilon \Gamma(1+\epsilon)C^{-1-\epsilon}.$$



where d=4-2eps. By substituting we arrive at a very simple final result!!

$$\mathcal{A}(gg\to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

Comments:

* The final dependence of the result is mt^2 : one from the Yukawa coupling, one from the spin flip.
* The tensor structure could have been guessed by gauge invariance.
* The integral depends on mt and mh.

LO cross section

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, g(x_1, \mu_f) g(x_2, \mu_f)\, \hat{\sigma}(gg \to H)$$

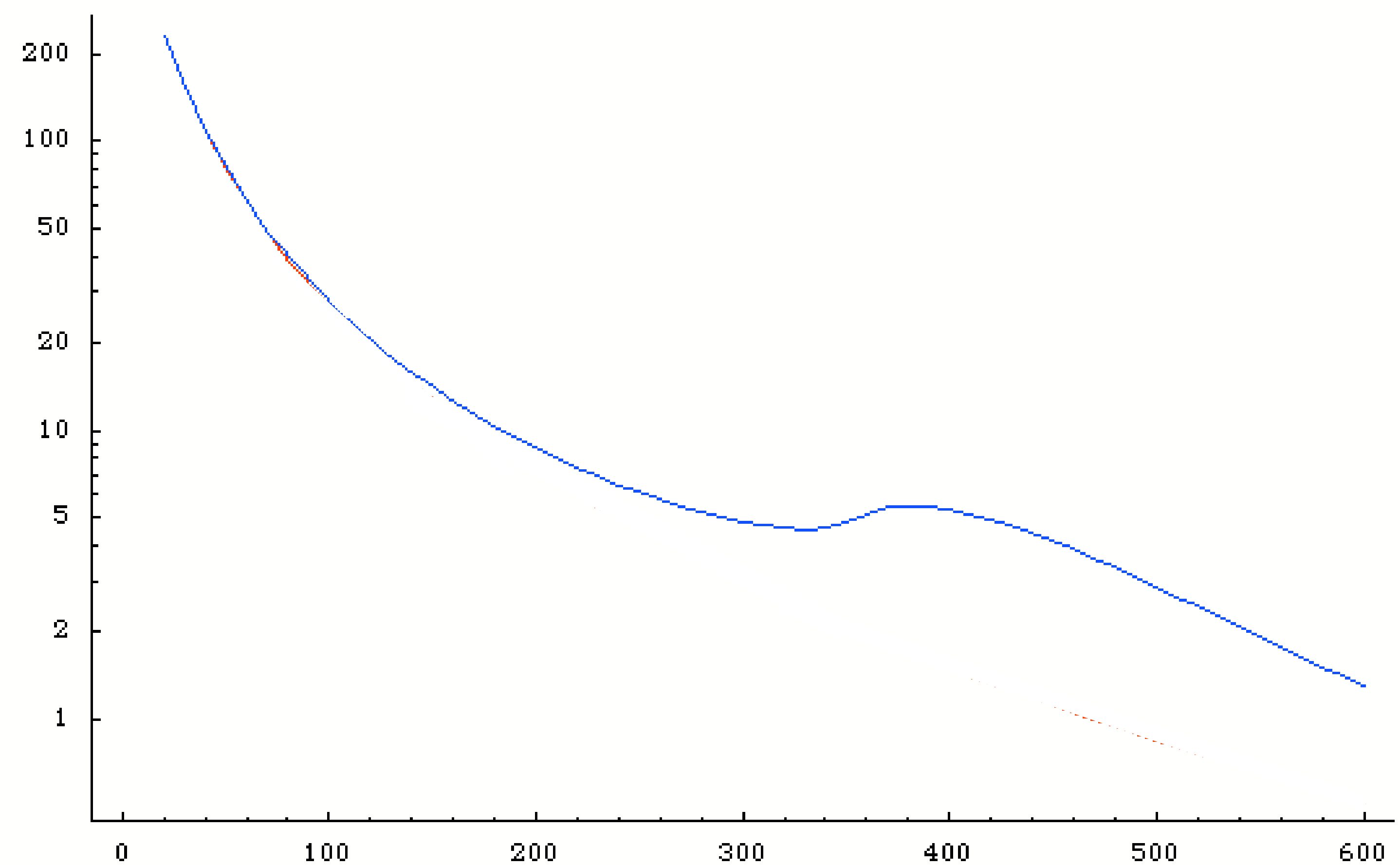
$$x_1 \equiv \sqrt{\tau} e^{y} \quad x_2 \equiv \sqrt{\tau} e^{-y} \quad \tau = x_1 x_2 \qquad \tau_0 = M_H^2/S \quad z = \tau_0/\tau$$

$$= \frac{\alpha_S^2}{64\pi v^2} \mid I\left(\frac{M_H^2}{m^2}\right) \mid^2 \tau_0 \int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy g(\sqrt{\tau_0} e^{y}) g(\sqrt{\tau_0} e^{-y})$$

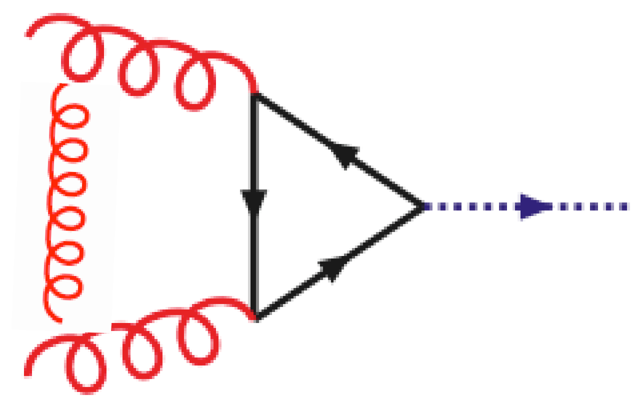
The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

I(x) has both a real and imaginary part, which develops at mh=2mt.

This causes a bump in the cross section.



pp →H @ NLO



At NLO we have to include an extra parton (virtual or real).

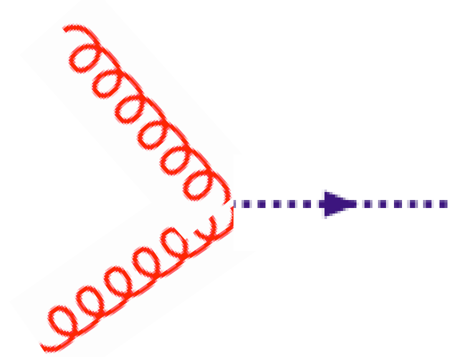
The virtuals will become a two-loop calculation!!

Can we avoid that?

Let's consider the case where the Higgs is light:

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

$$\xrightarrow{m \gg M_H} \quad -\frac{\alpha_S}{3\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\epsilon_\mu(p)\epsilon_\nu(q).$$



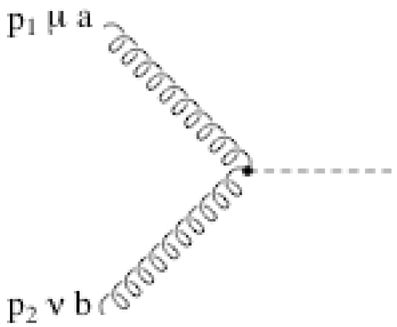
This looks like a local vertex, ggH.

The top quark has disappeared from the low energy theory but it has left something behind (non-decoupling).

Higgs effective field theory

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}\left(1-\frac{\alpha_S}{3\pi}\frac{H}{v}\right)G^{\mu\nu}G_{\mu\nu}$$

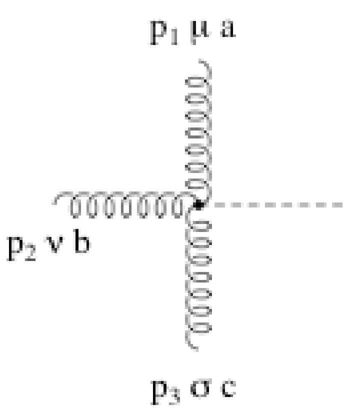
This is an effective non-renormalizable theory (no top) which describes the Higgs couplings to QCD.



(a)

$iA\delta^{ab}H^{\mu\nu}(p_1,p_2)$

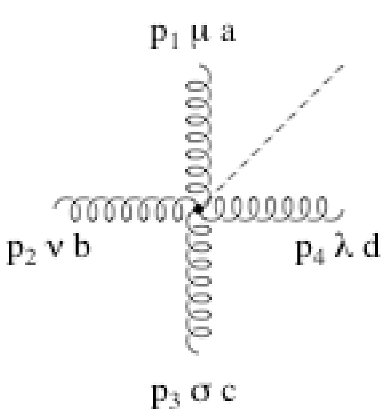
$$H^{\mu\nu}(p_1,p_2) = g^{\mu\nu}p_1\cdot p_2 - p_1^\nu p_2^\mu .$$



(b)

$-Agf^{abc}V^{\mu\nu\sigma}(p_1,p_2,p_3)$

$$V^{\mu\nu\rho}(p_1,p_2,p_3) = (p_1-p_2)^\rho g^{\mu\nu} + (p_2-p_3)^\mu g^{\nu\rho} + (p_3-p_1)^\nu g^{\rho\mu},$$



(c)

$-iAg^2X^{abcd}_{\mu\nu\sigma\lambda}$

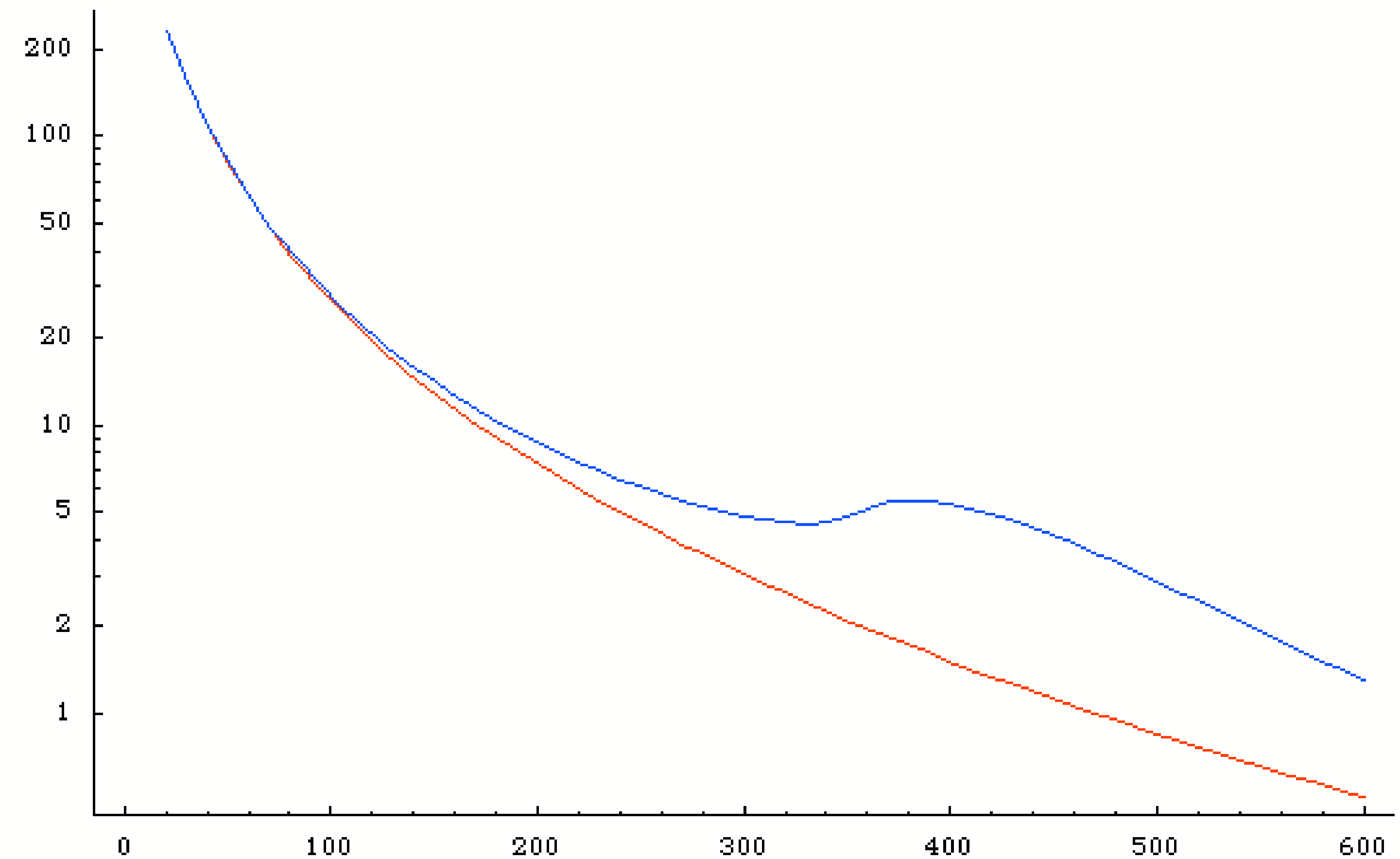
$$\begin{aligned} X^{\mu\nu\rho\sigma}_{abcd} &= f_{abe}f_{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f_{ace}f_{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f_{ade}f_{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma}). \end{aligned}$$

LO cross section: full vs HEFT

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, g(x_1, \mu_f) g(x_2, \mu_f)\, \hat{\sigma}(gg \to H)$$

The accuracy of the calculation in the HEFT calculation can be directly assessed by taking the limit m→∞.

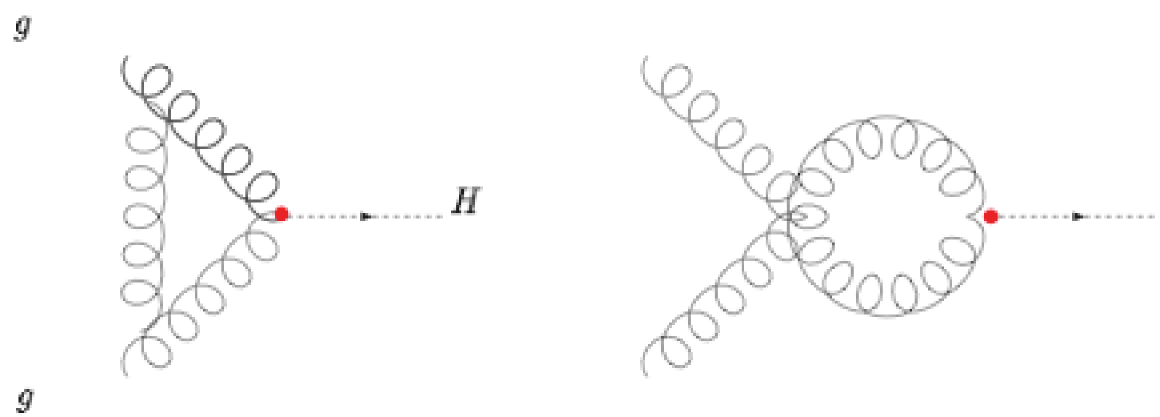
For light Higgs is better than 10%.



So, if we are interested in a light Higgs we use the HEFT and simplify our life.
If we do so, the NLO calculation becomes a standard 1-loop calculation, similar to Drell-Yan at NLO.

We can do it!!

Virtual contributions



Out of 8 diagrams, only two are non-zero (in dimensional regularization), a bubble and a triangle.

They can be easily written down by hand.

Then the integration over the tensor decomposition into scalar integrals and loop integration has to be performed.

$$\mathcal{L}_{\rm eff}^{\rm NLO} = \left(1 + \frac{11}{4}\frac{\alpha_S}{\pi}\right)\frac{\alpha_S}{3\pi}\frac{H}{v}G^{\mu\nu}G_{\mu\nu}$$

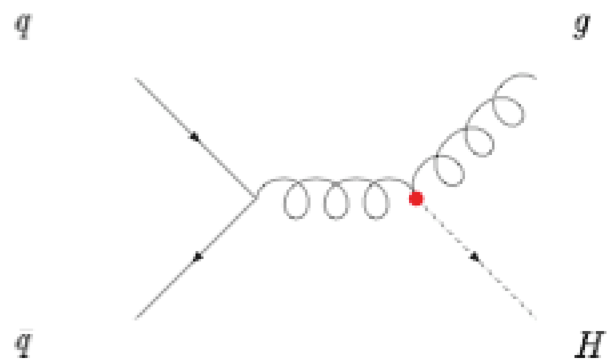
One also have to consider that the coefficient of the HEFT receive corrections which have to be included in the result.

The result is:

$$\sigma_{\rm virt} = \sigma_0\;\delta(1-z)\;\left[1 + \frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma\left(-\frac{2}{\epsilon^2} + \frac{11}{3} + \pi^2\right)\right]\,,$$

$$\sigma_{\rm Born} = \frac{\alpha_S^2}{\pi}\frac{m_H^2}{576 v^2 s}(1+\epsilon+\epsilon^2)\mu^{2\epsilon}\;\delta(1-z) \equiv \sigma_0\;\delta(1-z) \qquad z = m_H^2/s$$

Real contributions I



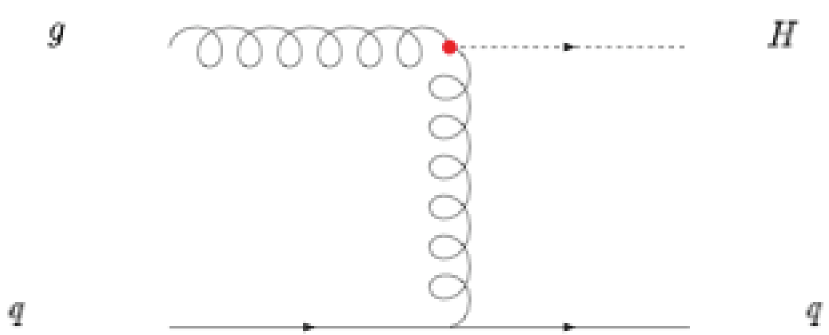
$$\overline{|\mathcal{M}|^2} = \frac{4}{81}\frac{\alpha_S^3}{\pi v^2}\,\frac{(\hat{u}^2+\hat{t}^2)-\epsilon(\hat{u}+\hat{t})^2}{\hat{s}}$$

Integrating over phase space (cms angle theta)

$$\hat{t} = -\hat{s}(1-z)(1-\cos\theta)/2$$
$$\hat{u} = -\hat{s}(1-z)(1+\cos\theta)/2$$

$$\sigma_{\mathrm{real}}(q\bar{q}) = \sigma_0\frac{\alpha_S}{2\pi}\frac{64}{27}\frac{(1-z)^3}{z}$$

finite!



$$\overline{|\mathcal{M}|^2} = -\frac{1}{54(1-\epsilon)}\frac{\alpha_S^3}{\pi v^2}\,\frac{(\hat{u}^2+\hat{s}^2)-\epsilon(\hat{u}+\hat{s})^2}{\hat{t}}$$

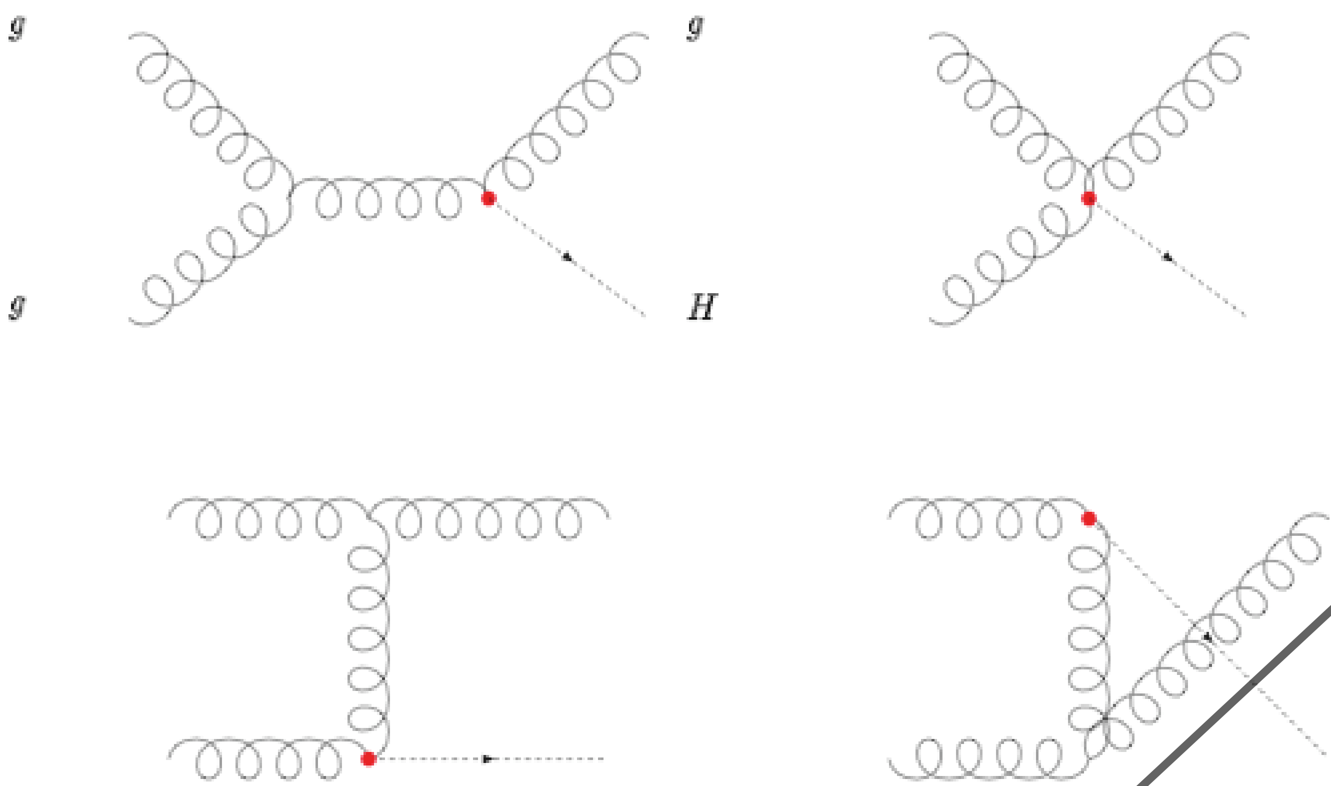
Integrating over the D-dimensional phase space the collinear singularity manifests a pole in 1/eps

$$\sigma_{\mathrm{real}} = \sigma_0\frac{\alpha_S}{2\pi}C_F\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma\left[-\frac{1}{\epsilon}p_{gq}(z)+\frac{(1-z)(7z-3)}{2z}+p_{gq}(z)\log\frac{(1-z)^2}{z}\right]$$

$$\left(\ \sigma_{\mathrm{c.t.}}^{\mathrm{coll.}} = \sigma_0\frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}\frac{c_\Gamma}{\epsilon}P_{gq}(z)\right]\ \right)$$

$$\begin{aligned}\sigma^{\overline{\mathrm{MS}}}(qg) &= \sigma_{\mathrm{real}}+\sigma_{\mathrm{c.t.}}^{\mathrm{coll.}}\\ &= \sigma_0\frac{\alpha_S}{2\pi}C_F\left[p_{gq}(z)\log\frac{m_H^2}{\mu_F^2}+p_{gq}(z)\log\frac{(1-z)^2}{z}+\frac{(1-z)(7z-3)}{2z}\right]\end{aligned}$$

Real contributions II



$$\sigma_{\text{real}} = \sigma_0 \frac{\alpha_S}{2\pi} C_A \left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma \left[\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\frac{b_0}{C_A} - \frac{\pi^2}{3} \right) \delta(1-z) - \frac{2}{\epsilon} p_{gg}(z) - \frac{11}{3}\frac{(1-z)^3}{z} - 4\frac{(1-z)^2(1+z^2)+z^2}{z(1-z)} \log z + 4\frac{1+z^4+(1-z)^4}{z}\left(\frac{\log(1-z)}{1-z}\right)_+ \right] .$$

This is the last piece: the result at the end must be finite!

2/eps cancels with the virtual contribution ✓

This is the renormalization of the coulping!!

$$\sigma^{\text{UV}}_{\text{c.t.}} = 2\ \sigma_{\text{Born}} \frac{\alpha_S}{2\pi} \left[- \left(\frac{\mu^2}{\mu^2_{\text{UV}}}\right)^{\epsilon} c_\Gamma \frac{b_0}{\epsilon} \right] ✓$$

This is an initial-state divergence to be reabsorbed in the pdf

$$\sigma^{\text{coll.}}_{\text{c.t.}} = 2\ \sigma_0 \frac{\alpha_S}{2\pi} \left[\left(\frac{\mu^2}{\mu^2_F}\right)^{\epsilon} \frac{c_\Gamma}{\epsilon} P_{gg}(z) \right] ✓$$

Final results = we made it!!

$$\sigma(pp \to H) = \sum_{ij} \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 f_i(x_1, \mu_f) f_j(x_2, \mu_f) \hat{\sigma}(ij)[\mu_f/m_h, \mu_r/m_h, \alpha_S(\mu_r)]$$

The final cross section is the sum of three channels: q qbar, q g, and g g.

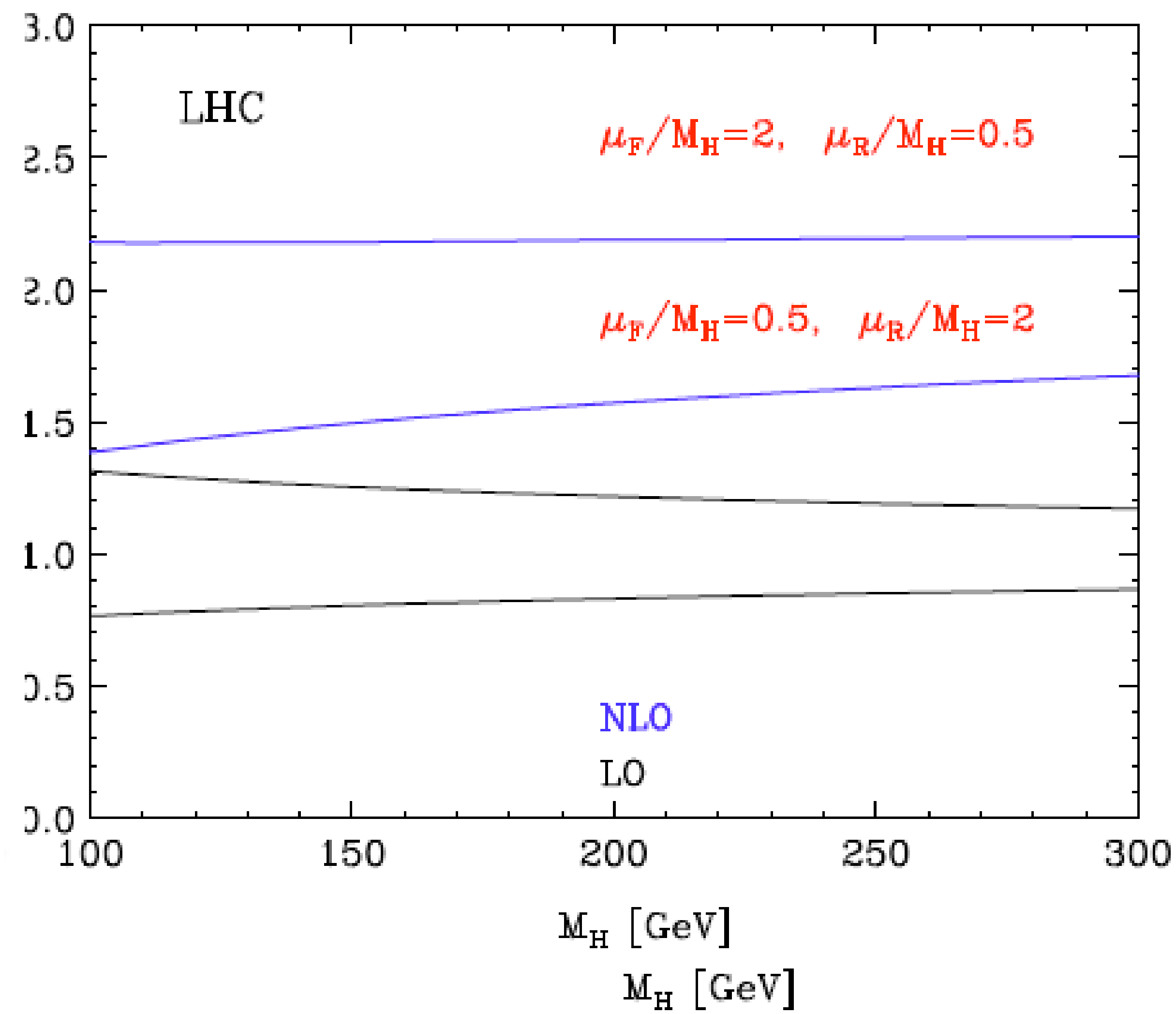
The short distance cross section at NLO depends explicitly on the subtraction scales (renormalization and factorization).

The explicit integration over the pdf's is trivial (just mind the plus distributions).

The result is that the corrections are huge!

K factor is ~2 and scale dependence not really very much improved.

Is perturbation theory valid?
NNLO is mandatory...



NLO and NNLO

Improving the accuracy

- Motivation
- NLO in proton proton collisions: $pp \to H+X$
- General approach and available tools
- Towards NNLO

General algorithm for calculations of observables at NLO

As we discussed, the form of the soft and collinear terms are UNIVERSAL, i.e., they don't depend on the short distance coefficients, but only on the color and spin of the partons partecipating soft or collinear limit.

Therefore it is conceivable to have an algorithm that can handle any process, once the real and virtual contributions are computed.

There are several such algorithms avaiable, but the conceptually simplest is the Subtraction Method [Catani & Seymour ; Catani, Dittmaier, Seymour, Trocsanyi]

$$\begin{aligned} \sigma_{ab}^{LO} &= \int_m d\sigma_{ab}^{B} \\ \sigma_{ab}^{NLO} &= \int_{m+1} d\sigma_{ab}^{R} + \int_m d\sigma_{ab}^{V} \end{aligned}$$

General algorithm for calculations of observables at NLO

One can use the universality to construct a set of counterterms

$$d\sigma^{ct} = \sum_{ct} \int_m d\sigma^B \otimes \int_1 dV_{ct}$$

which only depend on the partons involved in the divergent regions, dσ^B denotes the approriate colour and spin projection of the Born-level cross section and the counter terms are independent on the process under considerations.

These counter terms cancell all non-integrable singularities in dσ^R, so that one can write

$$\sigma_{ab}^{NLO} = \int_{m+1} [d\sigma_{ab}^R - d\sigma_{ab}^{ct}] + \int_{m+1} d\sigma_{ab}^{ct} + \int_m d\sigma_{ab}^V$$

where the space integration in the first term can be performed numerically in four dimensions and the integral of the counter terms can be done once for all.

An (incomplete) list of NLO codes

- NLOJET++ [Nagy] $pp \to$ (2,3) jets
- AYLEN/EMILIA [de Florian, Dixon, Kunszt, Signer] $pp \to (W,Z) + (W,Z,\gamma)$
- DIPHOX/EPHOX [Aurenche, Binoth, Fontannaz, Guillet, Heinrich, Pilon, Werlen] $pp \to \gamma + 1$ jet, $pp \to \gamma\gamma$, $\gamma^* p \to \gamma + 1$ jet
- MCFM [Campbell, Ellis] $pp \to (W,Z) + (0,1,2)$ jets, $pp \to (W,Z) + b\bar{b}, \ldots$
- heavy-quark production [Mangano, Nason, Ridolfi] $pp \to Q\bar{Q}$
- single-top production [Harris, Laenen, Phaf, Sullivan, Weinzierl] $pp \to Q\bar{q}$
- associated Higgs production with $t\bar{t}$ [Dawson, Jackson, Orr, Reina, Wackeroth, Beenakker, Dittmaier, Kramer, Plumper, Spira, Zerwas] $pp \to HQ\bar{Q}$
- VBFNLO [Figy, Zeppenfeld, C.O.] $pp \to (W,Z,H,WW,ZZ,WZ) + 2$ jets, QCD corrections to electroweak production, when typical vector-boson fusion cuts are applied
- di-photon production [del Duca, Maltoni, Nagy, Trocsanyi] $pp \to \gamma\gamma + 1$ jet

For a more complete list, and the corresponding web pages, see:

`http://www.cedar.ac.uk/hepcode`

Example:MCFM

Downloadable general purpose NLO code (Campbell & Ellis)

$p\bar{p} \to W^\pm/Z$	$p\bar{p} \to W^+ + W^-$
$p\bar{p} \to W^\pm + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^\pm + \gamma$	$p\bar{p} \to W^\pm/Z + H$
$p\bar{p} \to W^\pm + g^\star\,(\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^\pm/Z + 1\ \text{jet}$	$p\bar{p} \to W^\pm/Z + 2\ \text{jets}$
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H + 1\ \text{jet}$
$p\bar{p}(VV) \to H + 2\ \text{jets}$	$p\bar{p} \to t + q$
$p\bar{p} \to H + b$	$p\bar{p} \to Z + b$

- ☞ Plus all single-top channels, Wc, WQJ, ZQJ,...
- ☞ Extendable/sizeable library of processes,

  relevant for signal and background studies, including spin correlations.
- ☞ Cross sections and distributions at NLO are provided
- ☞ Easy and flexible choice of parameters/cuts (input card).

Bottlenecks and the future of NLO

- The construction and the numerical integration over phase space of the ($d\sigma^R$-counterterms) can be done in an automated way.
- The integration over the singular phase-space regions of $d\sigma^{ct}$ are done once for all. They are universal and process independent functions.
- The analytic calculation of scalar loop integrals is complicated and process-specific.
- A working, fast, completly general algorithm for the tensor reduction of the virtual integrals is challenging.
- This is a VERY active field of research, with a lot of progress achieved in the last one or two years. New approaches for numerical evaluation of the scalar integrals and also of the tensor decomposition proposed.
- Several new general algorithms to interface NLO calculations with parton showers have been also proposed
- Full automatization of NLO calculations interfaced with showers (~ Pythia@NLO) at the horizon.

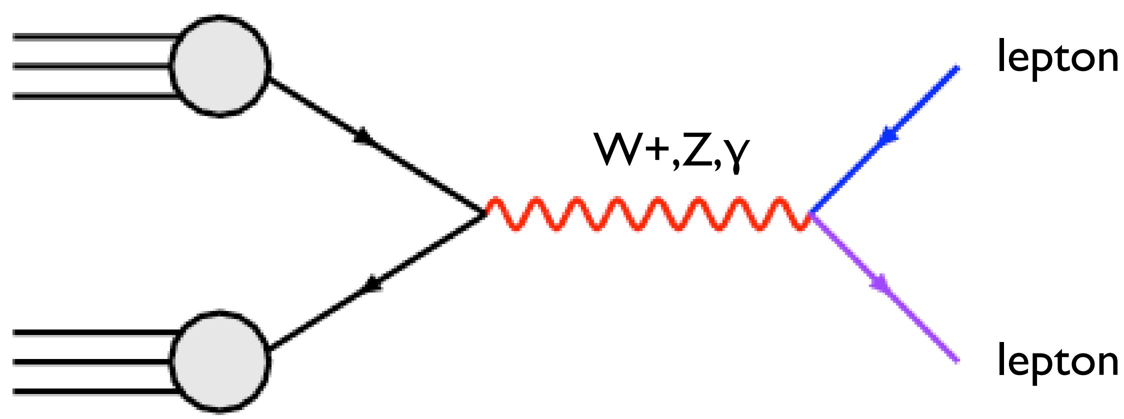
What about NNLO?

- At present only 2→1 calculations available, all of them (parton) exclusive final state.
- From loop integrals to phase space integrals...all of them are an art!
- General algorithms and checked only in e+e- →3j at NNLO.

Let's consider two physics cases:

a. Drell-Yan
b. Higgs

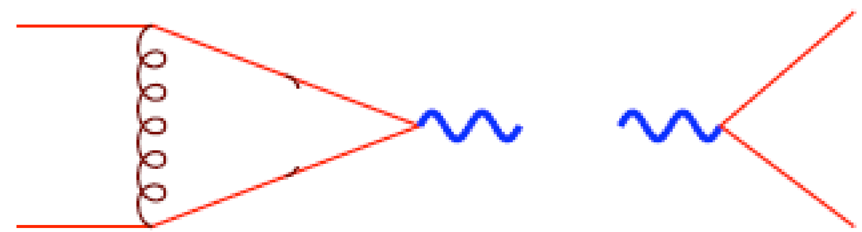
Drell-Yan



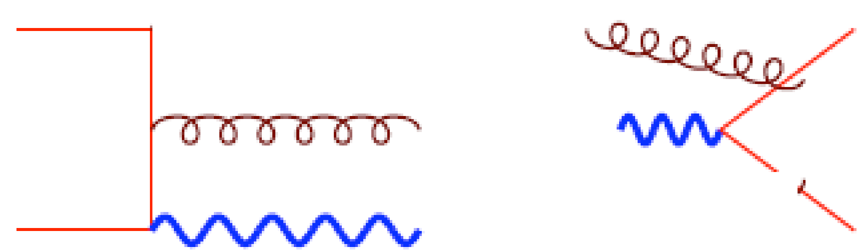
- Clean final state (no hadrons from the hard process).
- Nice test of QCD and EW interactions. The cross sections are known up to NNLO (QCD) and at NLO (EW).
- Measure m_W to be used in the EW fits together with the top mass to guess the Higgs mass.
- Constraint the PDF
- Channel to search for new heavy gauge bosons or new kind of interactions

Elements of pp→W NLO calculation

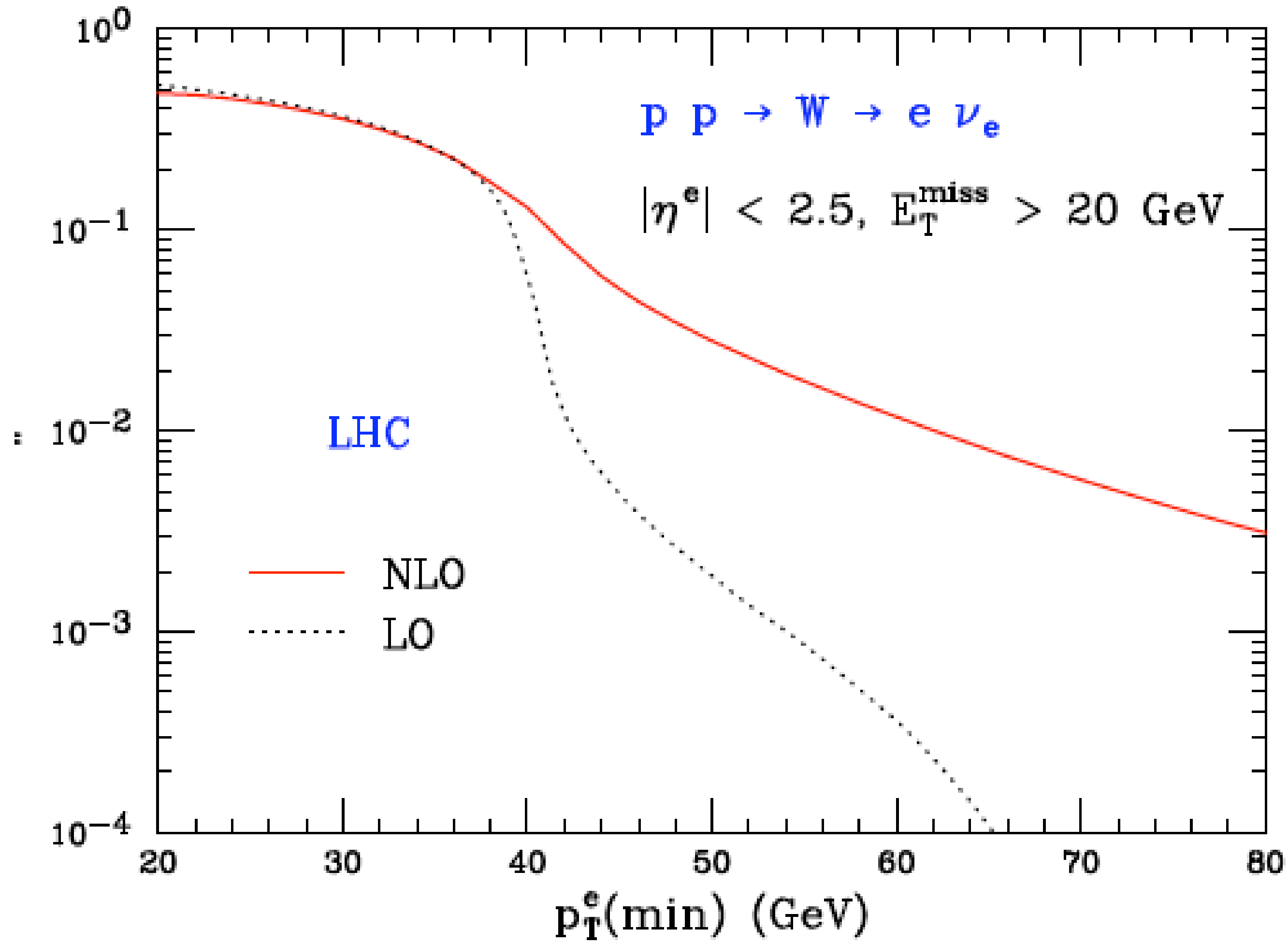
- Virtual



- Real



Drell-Yan @ NLO



✓ $$A_W = \frac{1}{\sigma^{(tot)}} \int_{p^e_T(\min)}^{\sqrt{S}/2} dp^e_T \, \frac{d\sigma}{dp^e_T}(\text{cuts})$$

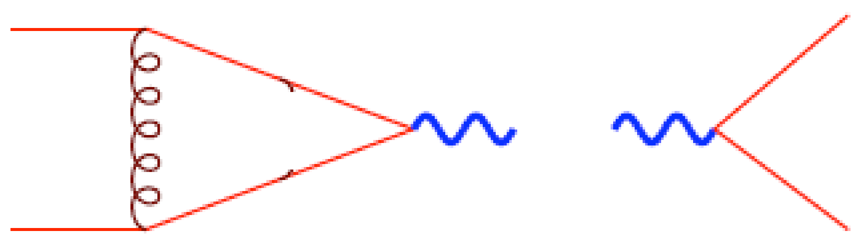
✓ $$K(x) = \frac{d\sigma_{NLO}/dx}{d\sigma_{LO}/dx}$$

K factors STRONGLY phase-space dependent.

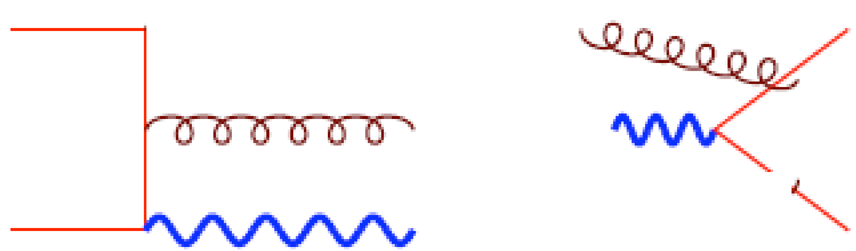
Lepton spin correlations have to be taken account correctly!

Elements of pp→W NLO calculation

- Virtual

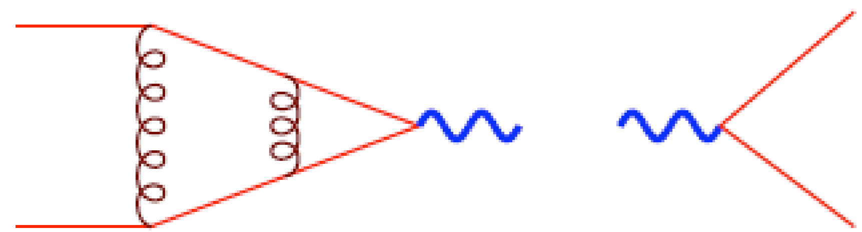


- Real



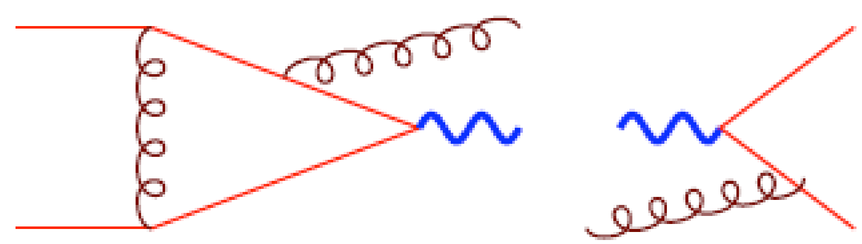
Elements of pp→W NNLO calculation

- Virtual-Virtual



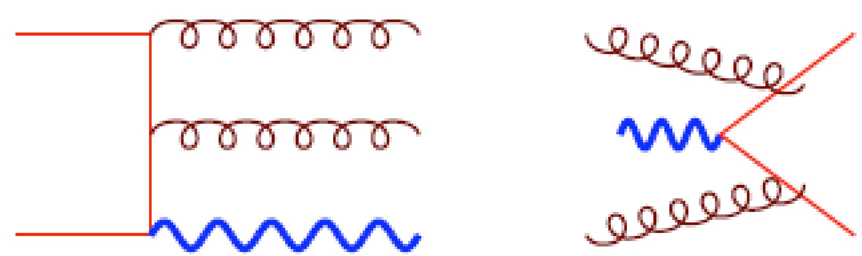
+ 100 terms

- Real-Virtual



+ 300 terms

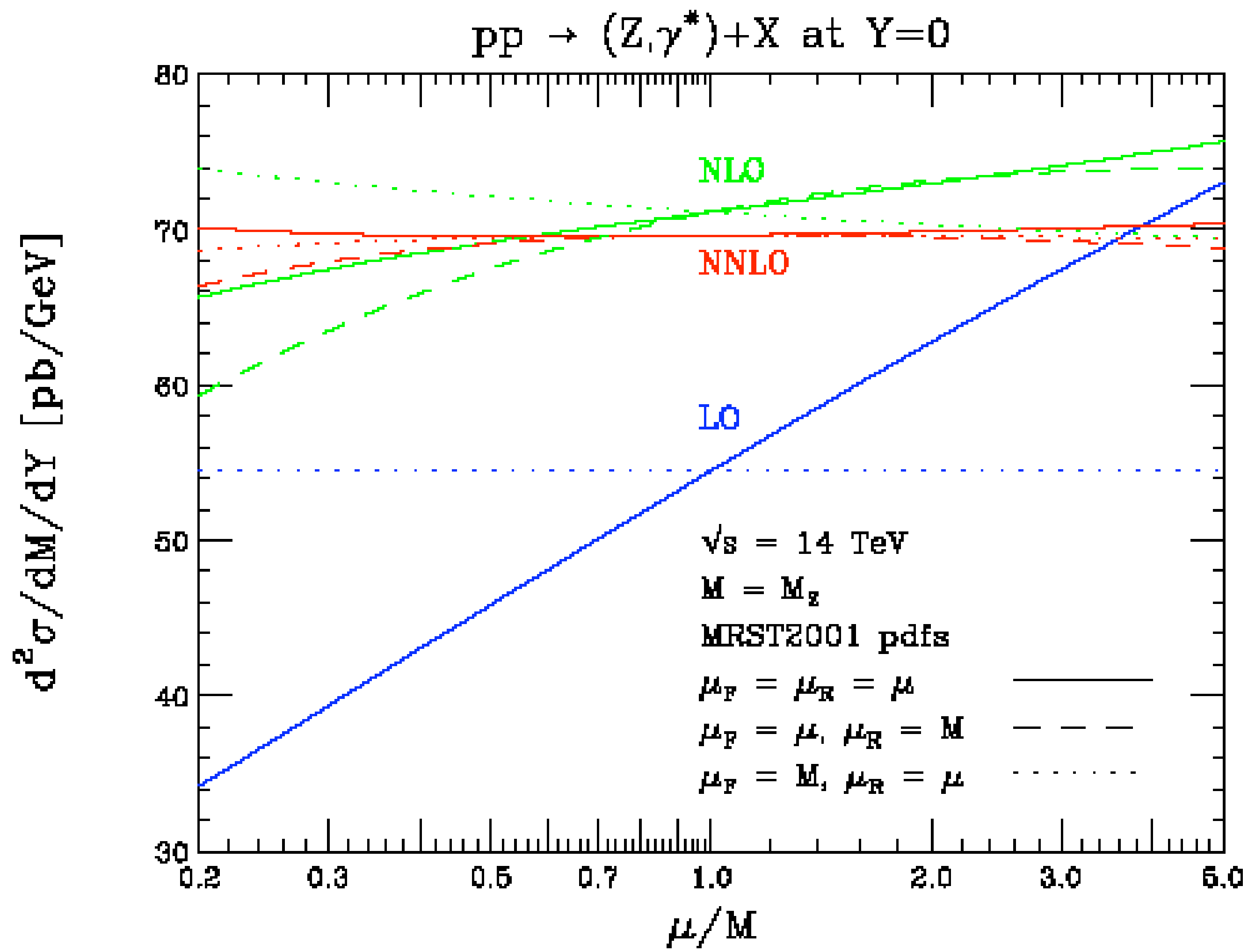
- Real-Real



+ 500 terms

⇒ Need clever algorithms to handle!

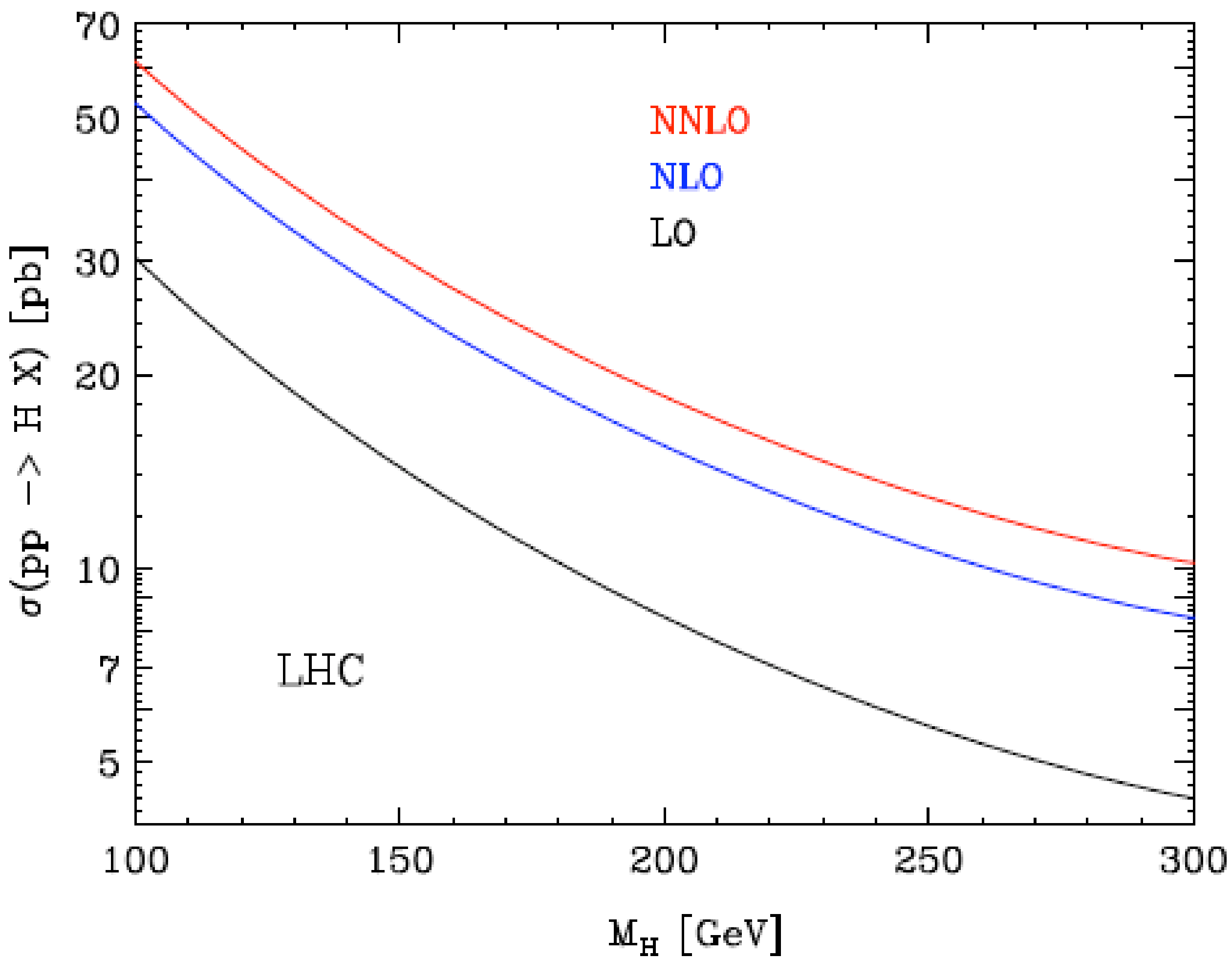
The NNLO result

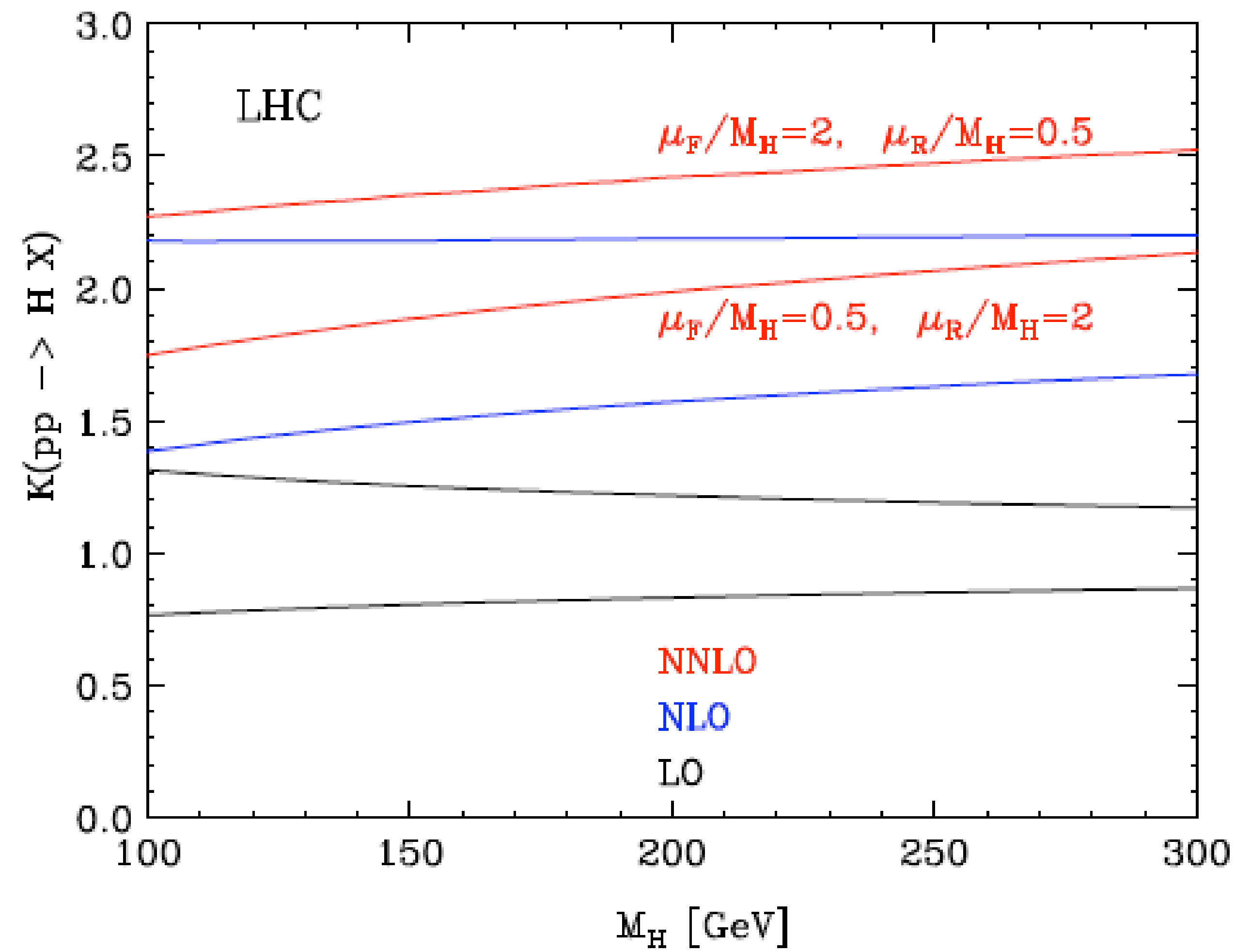


- Precision predictions at NNLO
- Also miss qualitative effects at lower orders
  - Few initial channels open; sensitivity to pdfs underestimated
  - Few jets in final state
  - Jets modeled by too few partons
  - Incorrect kinematics, e.g., no p_T

[Anastasiou, Dixon, Melnikov, Petriello. 2004]

pp→H at NNLO



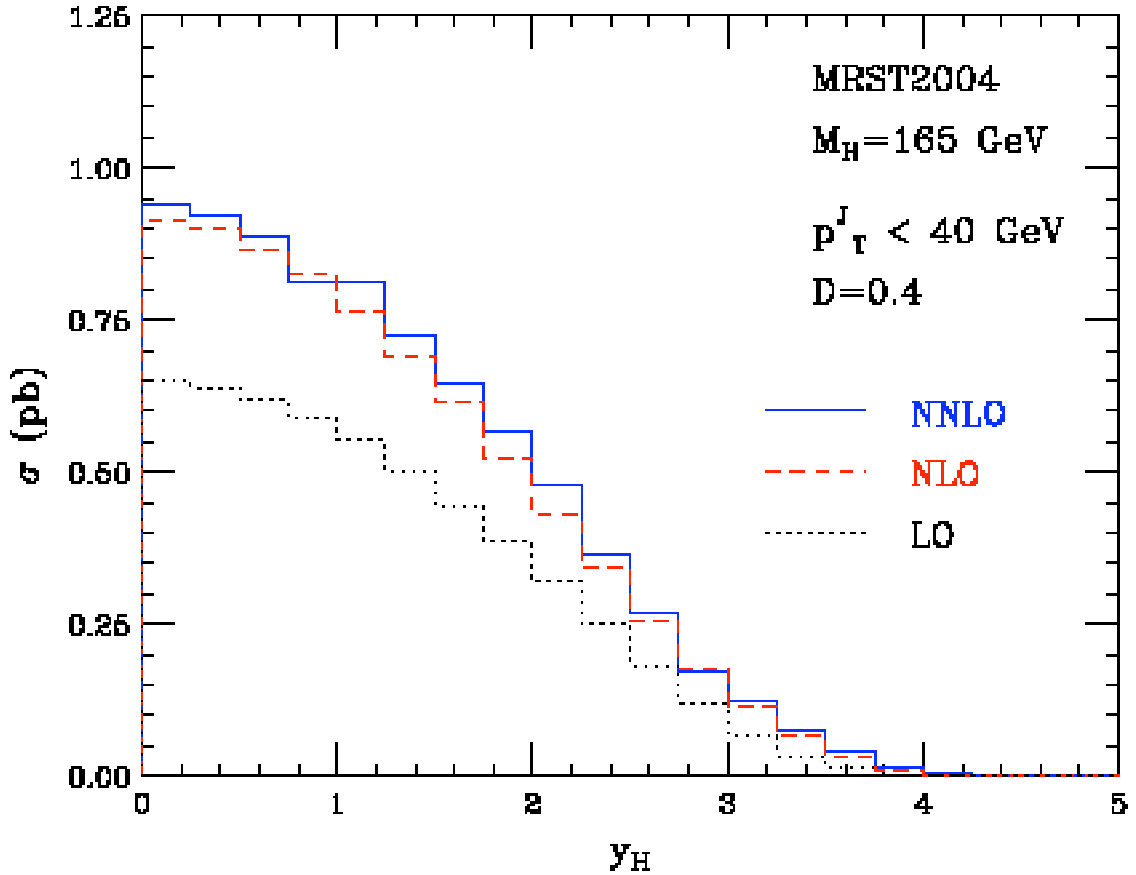


Is the series well behaved? $\Longrightarrow$ YES NNLO 15%

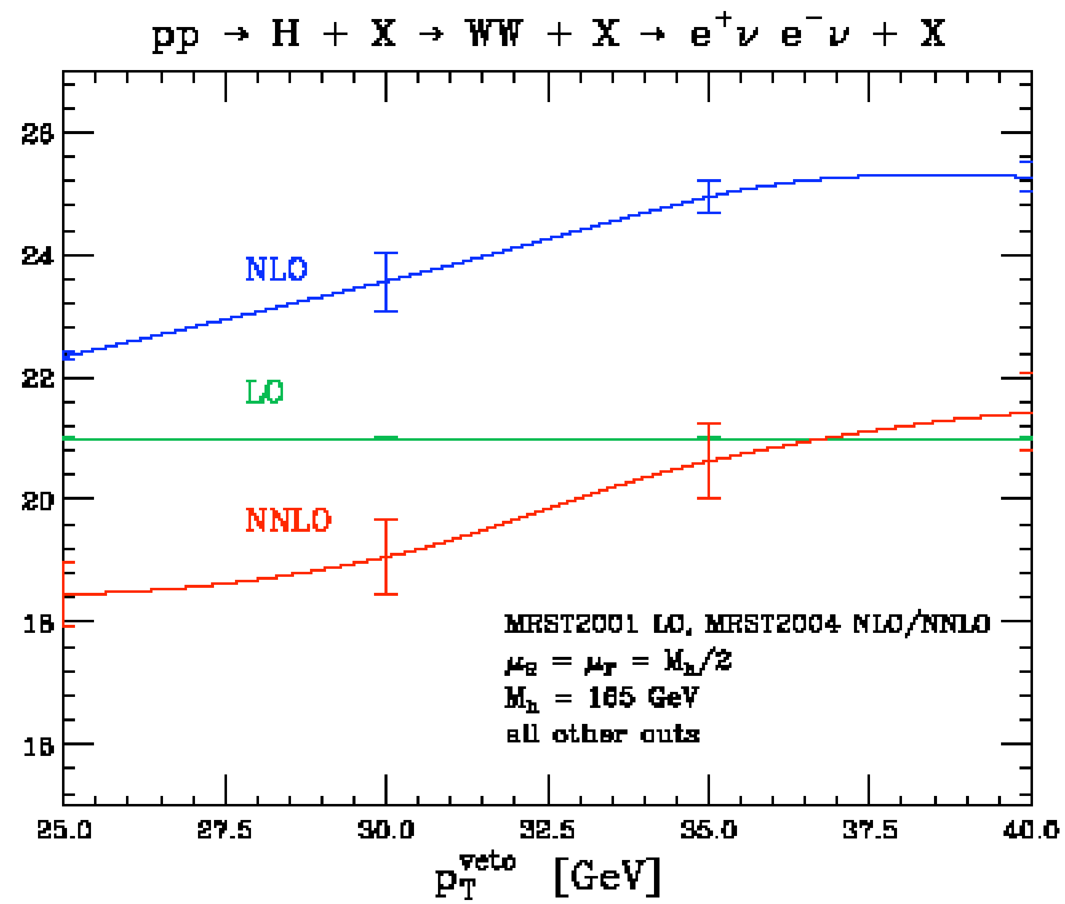
The current TH QCD uncertainty on the total cross section is about 10%.

What about our predictions for limited areas of the phase space?

pp→H at NNLO



[Catani, grazzini, 2007]



[Anastasiou, Melnikov, Petriello. 2005]
[Anastasiou, Dissertori, Stockli. 2007]

From LO to NLO and NNLO

summary

- Predictions at NLO are the first providing reliable predictions for IR observable and their uncertaintes.
- NLO calculations are a tough job and not complete automatization is available yet.
- We have detailed the calculation of $pp \to H+X$ at NLO
- NNLO are our current frontier of precision QCD and are available only for a very small set of process at hadron colliders.

Lecture material + exercises can be found at:

http://cp3wks05.fynu.ucl.ac.be/twiki/bin/view/Library/GIFSchool

Outline

- Basics
- Improving the accuracy: NLO and NNLO
- Improving the flexibility:
  Matrix elements MC's

What's a matrix-element based generator?

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

- Matrix element calculators provide our first estimation of rates for inclusive final states.

- Extra radiation is included: it is described by the PDF's in the initial state and by the definition of a final state parton, which at LO represents all possible final state evolutions.

- Due to the above approximations a cross section at LO can strongly depend on the factorization and renormalization scales.

- Any tree-level calculation for a final state F can be promoted to the exclusive F + X through a shower. However, a naive sum of final states with different jet multiplicities would lead to double counting. Matching needed...

The technical challenges

How do we calculate a LO cross section for 3 jets at the LHC?

I. Identify all subprocesses (gg→ggg, qg→qgg....) in

$$\sigma(pp \to 3j) = \sum_{ijk} \int f_i(x_1) f_j(x_2) \hat{\sigma}(ij \to k_1 k_2 k_3)$$

easy

II. For each one, calculate the amplitude:

$$\mathcal{A}(\{p\}, \{h\}, \{c\}) = \sum_i D_i$$

difficult

III. Square the amplitude, sum over spins & color, integrate over the phase space (D ~ 3n)

$$\hat{\sigma} = \frac{1}{2\hat{s}} \int d\Phi_p \sum_{h,c} |\mathcal{A}|^2$$

very hard

SM

General structure

subprocs handler

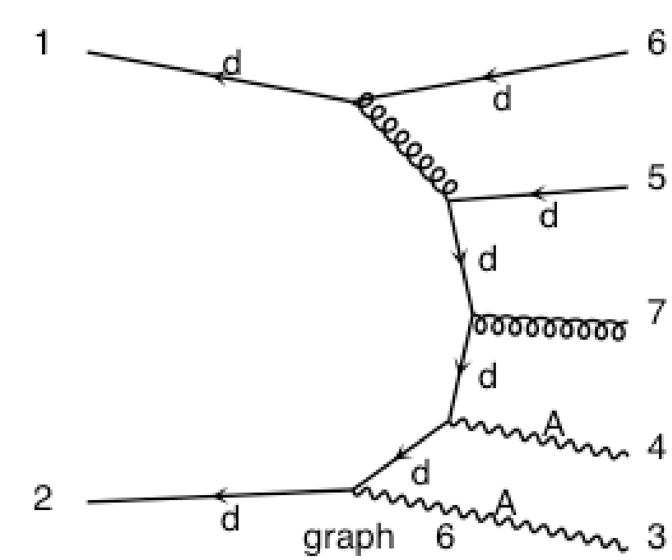
Includes all possible subprocess leading to a given multi-jet final state automatically or manually (done once for all)

d~ d -> a a u u~ g
d~ d -> a a c c~ g
s~ s -> a a u u~ g
s~ s -> a a c c~ g

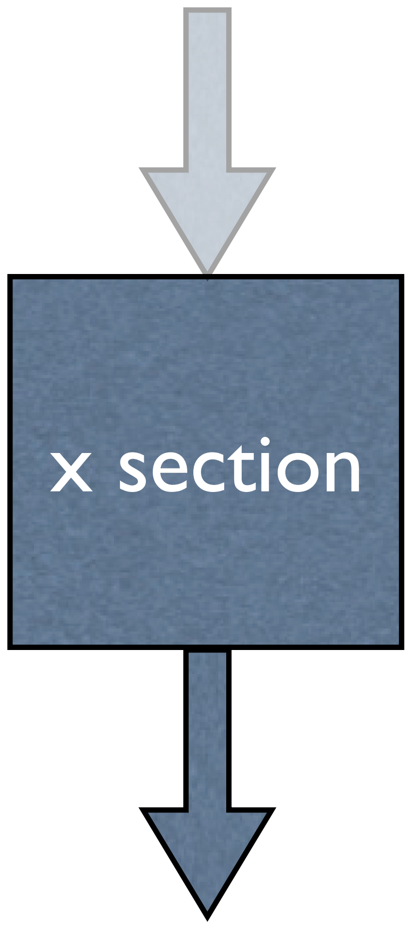
ME calculator

"Automatically" generates a code to calculate |M|^2 for arbitrary processes with many partons in the final state.

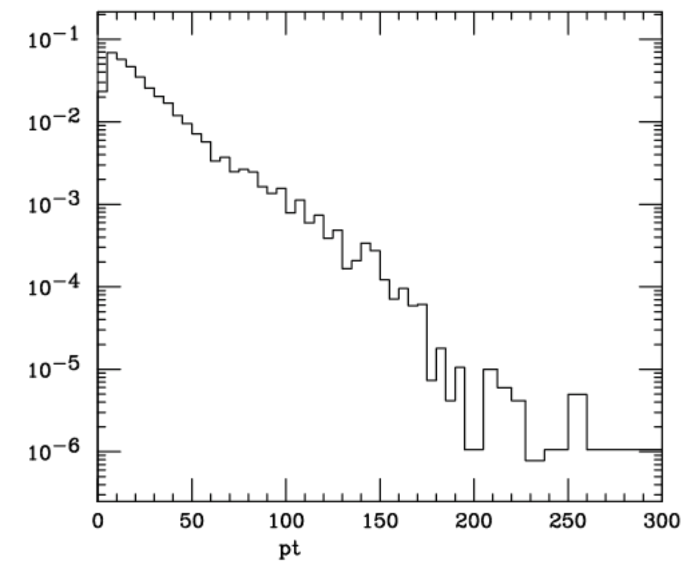
Most use Feynman diagrams w/ tricks to reduce the factorial growth, others have recursive relations to reduce the complexity to exponential. ☺



General structure



Integrate the matrix element over the phase space using a multi-channel technique and using parton-level cuts.



How does this work?

MC basics:

from integration to event generation

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$Dim[\Phi(n)] \sim 3n$$

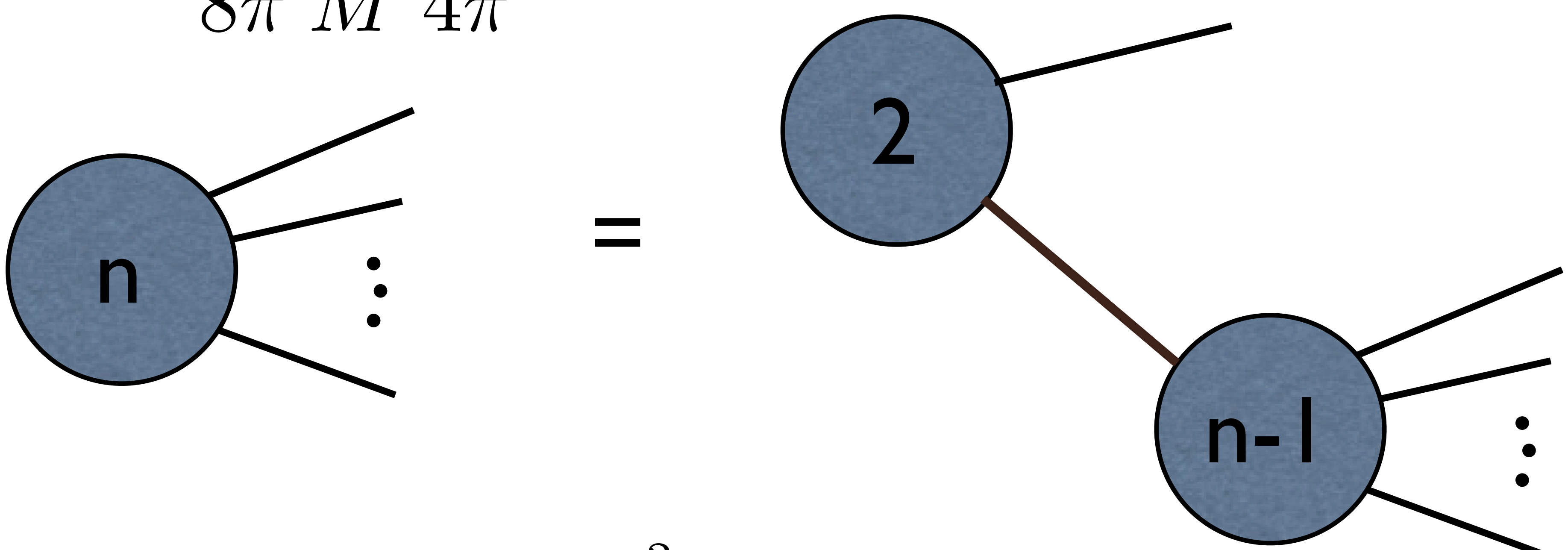
$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

General and flexible method is needed

Phase Space

$$d\Phi_n = \left[\Pi_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 (2E_i)}\right] (2\pi)^4 \delta^{(4)}(p_0 - \sum_{i=1}^{n} p_i)$$

$$d\Phi_2(M) = \frac{1}{8\pi} \frac{2p}{M} \frac{d\Omega}{4\pi}$$



$$d\Phi_n(M) = \frac{1}{2\pi} \int_0^{(M-\mu)^2} d\mu^2 d\Phi_2(M) d\Phi_{n-1}(\mu)$$

Exercises:

MC 101

This is a set of solved exercises (borrowed from M. Seymour) for practising the basic notions of MonteCarlos.

- Write the simplest integration function based on the definition of average and error
- Apply an analytic transformation : importance sampling
- Von Neumann's rejection method : plain vanilla
- Von Neumann's rejection method : improved
- Dimensionality of the phase space of 1-> n particles
- Useful functions
- Install Vegas
- Top decay
- q1 q2~ → tt~ production
- Representation of the grid in VEGAS. Each red square has the same area

Integrals as averages

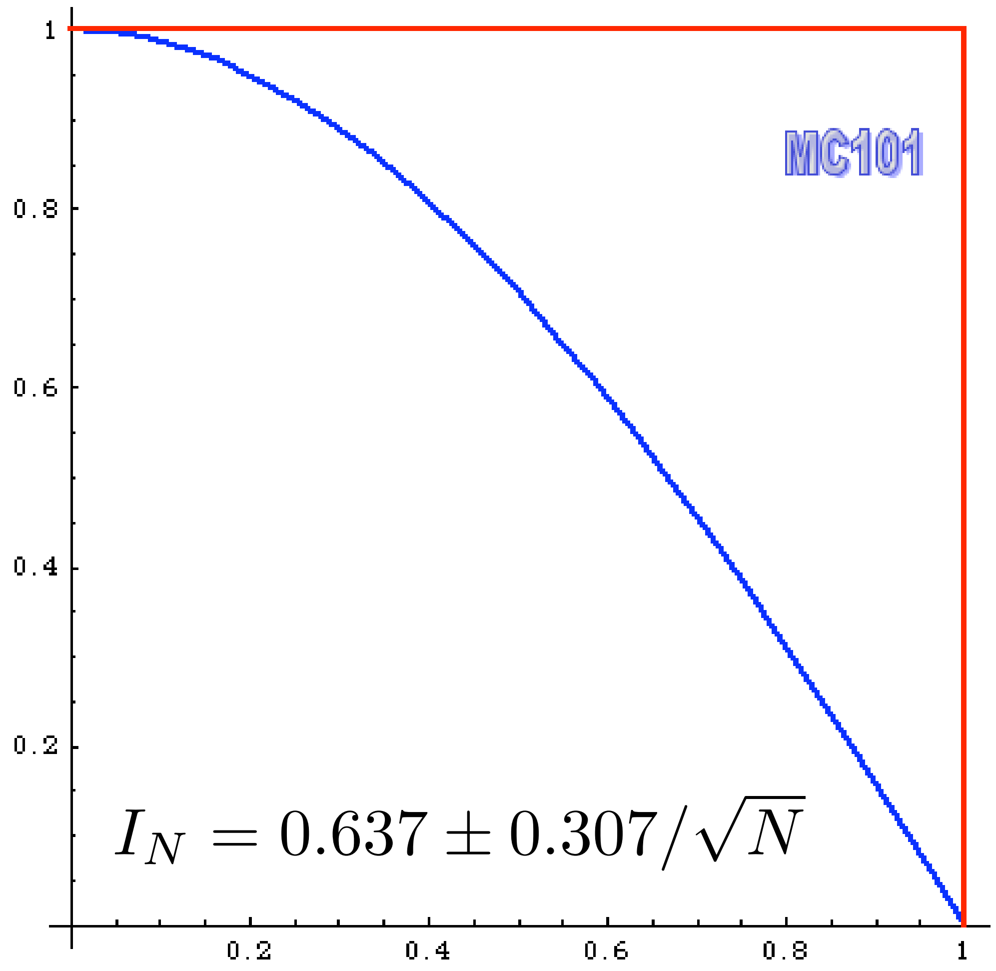
$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

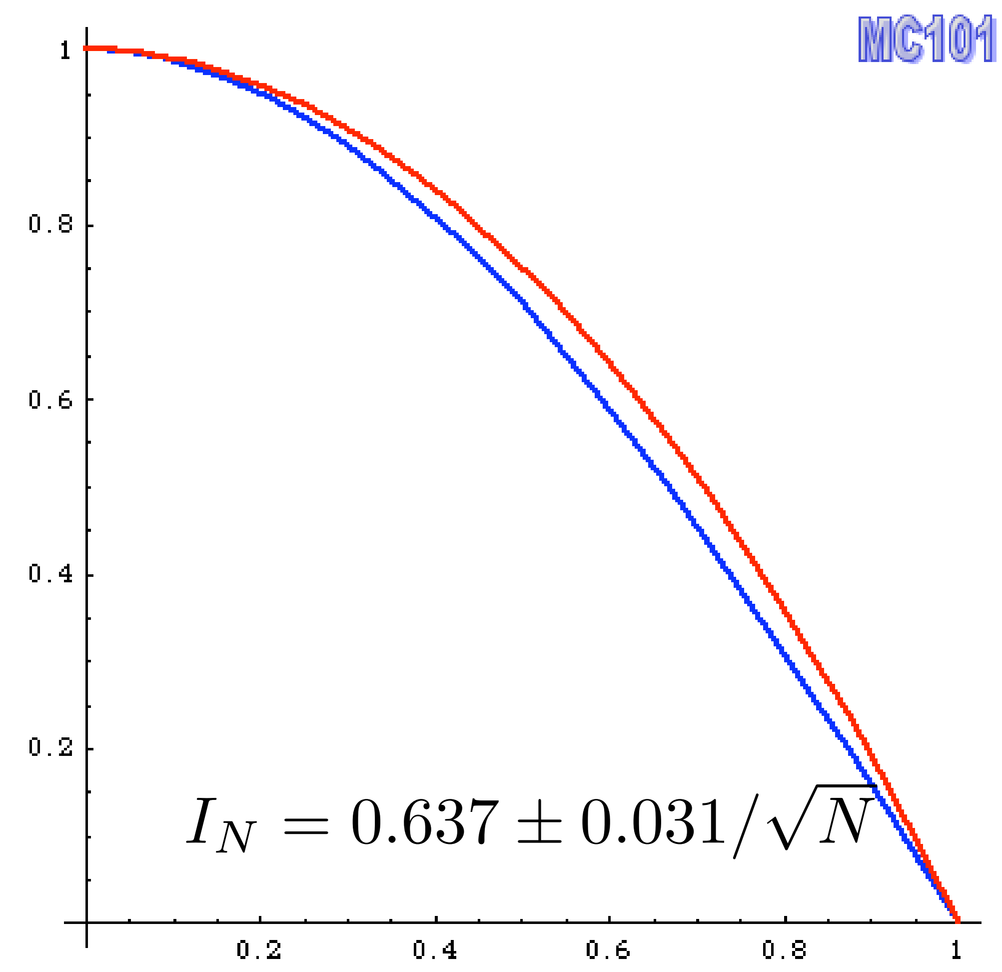
$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

$$I = I_N \pm \sqrt{V_N/N}$$

- ☞ Convergence is slow but it can be estimated easily
- ☞ Error does not depend on # of dimensions!
- ☞ Improvement by minimizing V_N.
- ☞ Optimal/Ideal case: $f(x)=C \Rightarrow V_N=0$

Importance Sampling





$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$I = \int_0^1 dx(1-x^2)\frac{\cos\frac{\pi}{2}x}{1-x^2}$$

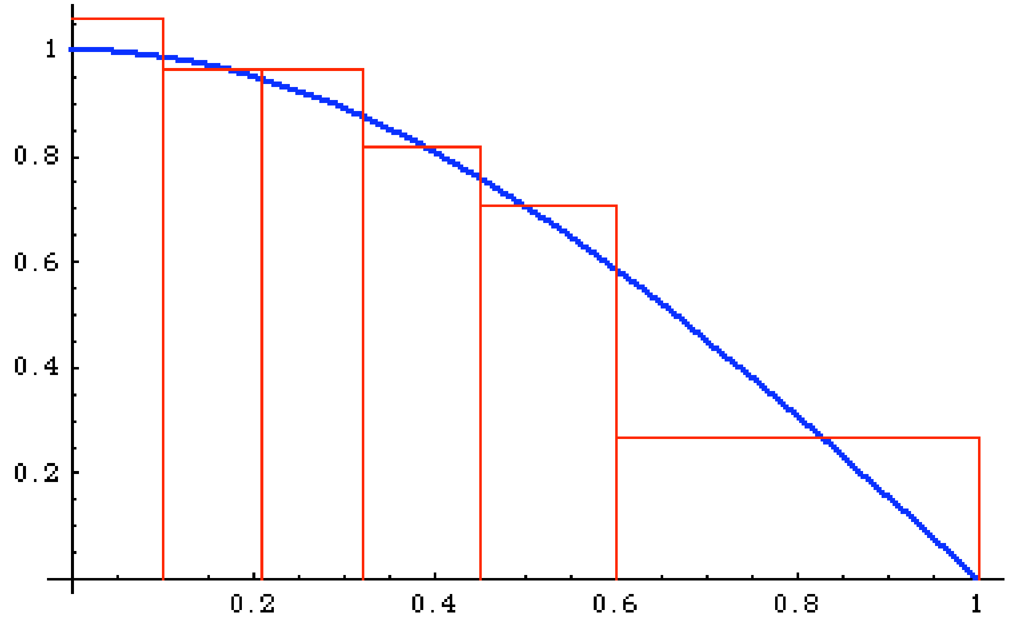
$$= \int_{\xi_1}^{\xi_2} d\xi \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2} \Rightarrow \simeq 1$$

Importance Sampling

but... you need to know too much about f(x)!

idea: learn during the run and build a step-function approximation p(x) of f(x) ⟹ VEGAS

MC101



many bins where f(x) is large

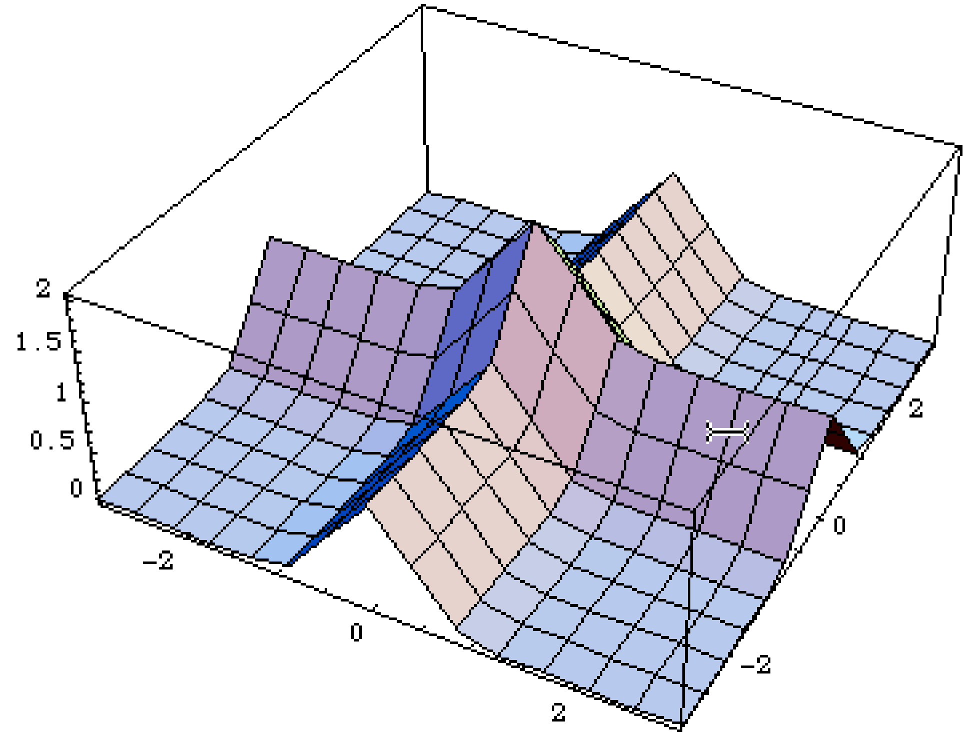
$$p(x) = \frac{1}{N_b \Delta x_i}, \quad x_i - \Delta x_i < x < x_i$$

Importance Sampling

can be generalized to n dimensions:

$$\vec{p(x)} = p(x) \cdot p(y) \cdot p(z) \ldots$$

but the peaks of $f(\vec{x})$ need to be "aligned" to the axis!



This is ok...

Importance Sampling

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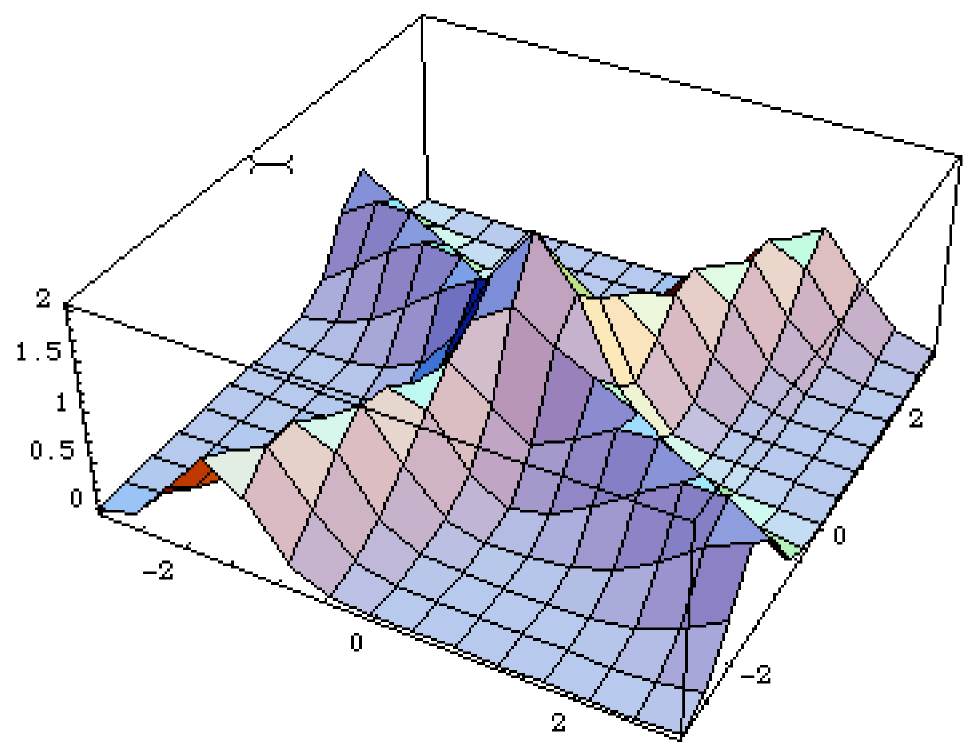

This is not ok...

Importance Sampling

can be generalized to n dimensions:

$$\vec{p(x)} = p(x) \cdot p(y) \cdot p(z) \ldots$$

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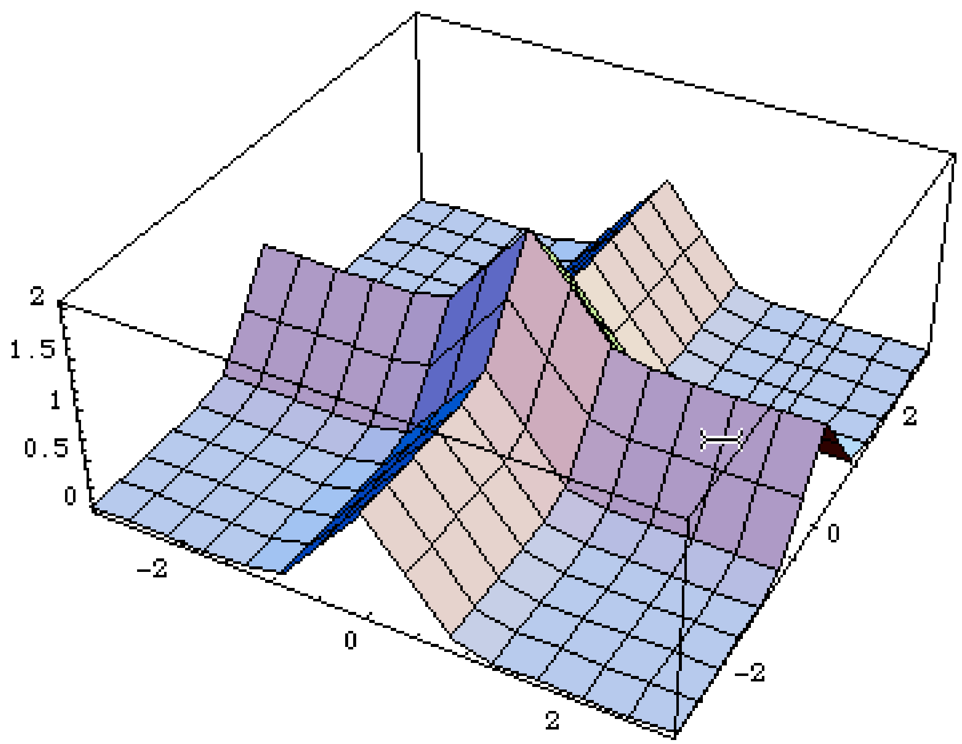
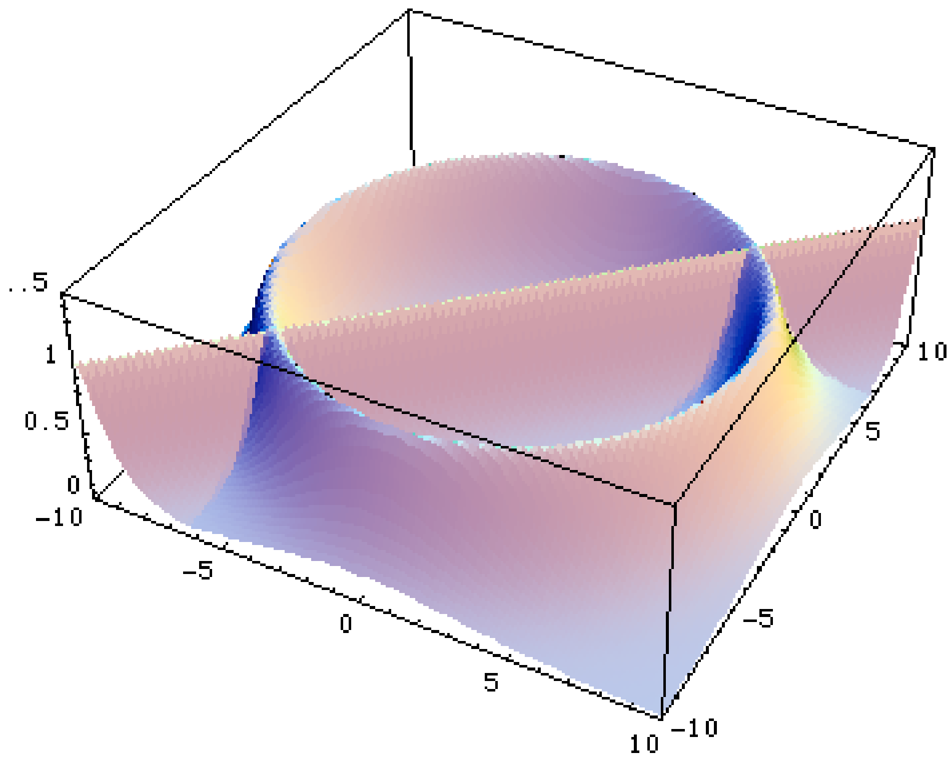


but it is sufficient to make a change of variables!

Multi-channel



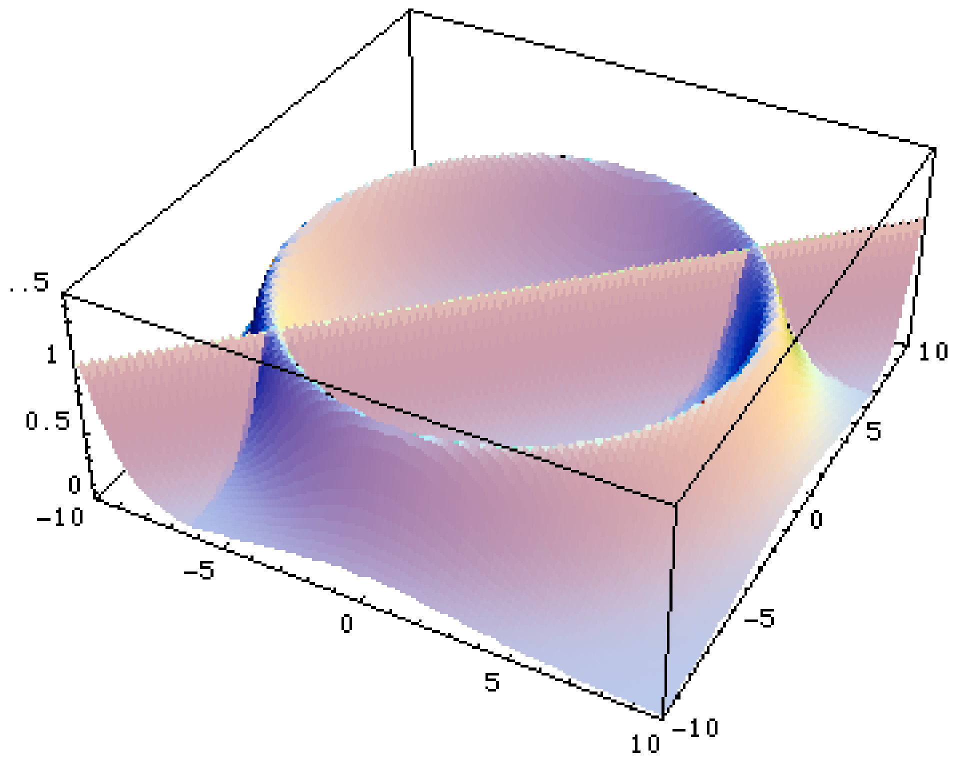
In this case there is no unique tranformation: Vegas is bound to fail!

Solution: use different transformations= channels

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

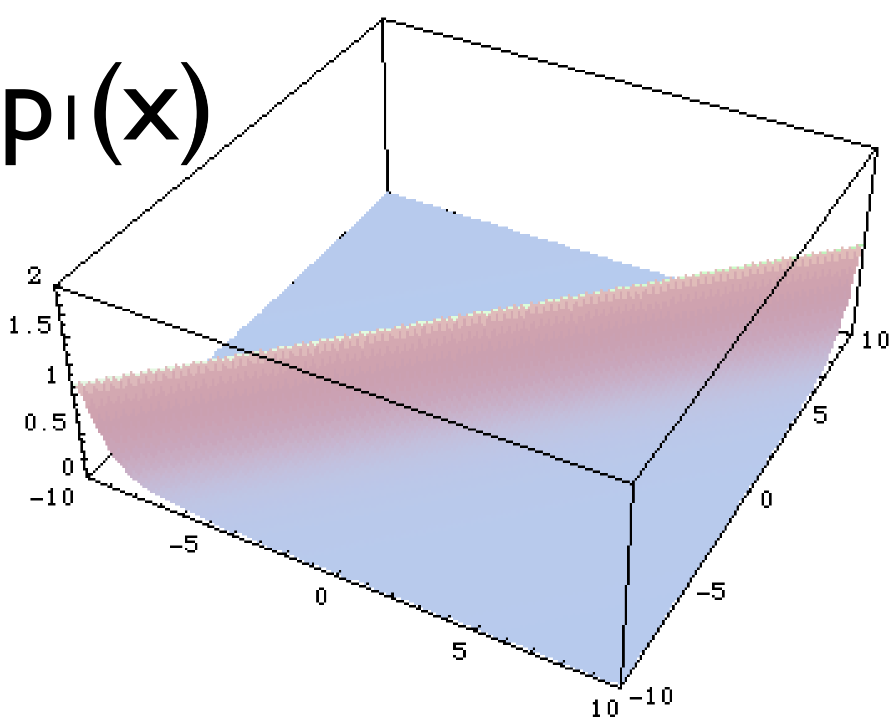
with each $p_i(x)$ taking care of one "peak" at the time

Multi-channel

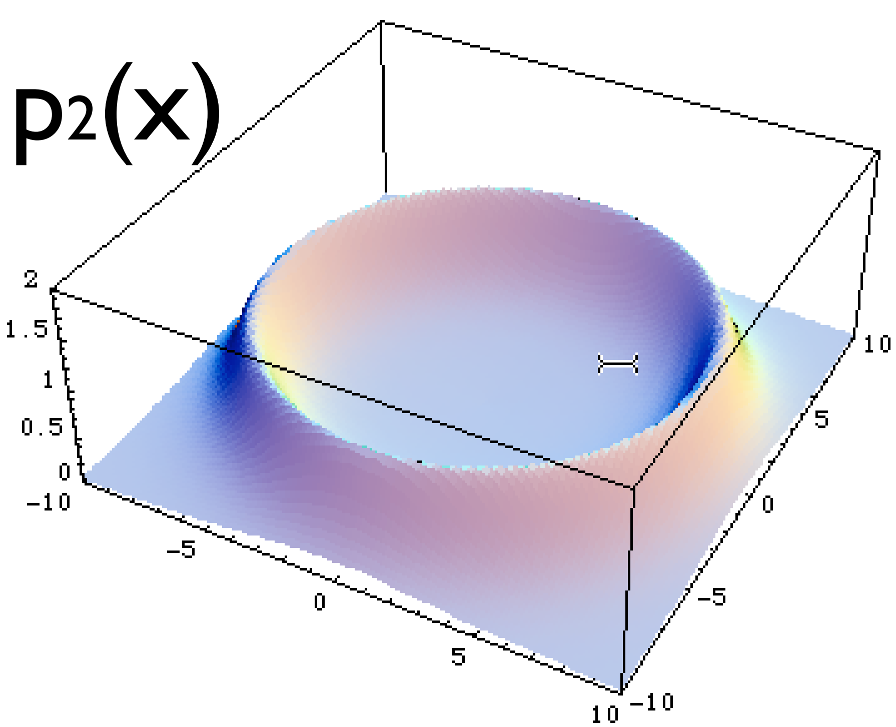


In this case there is no unique tranformation: Vegas is bound to fail!

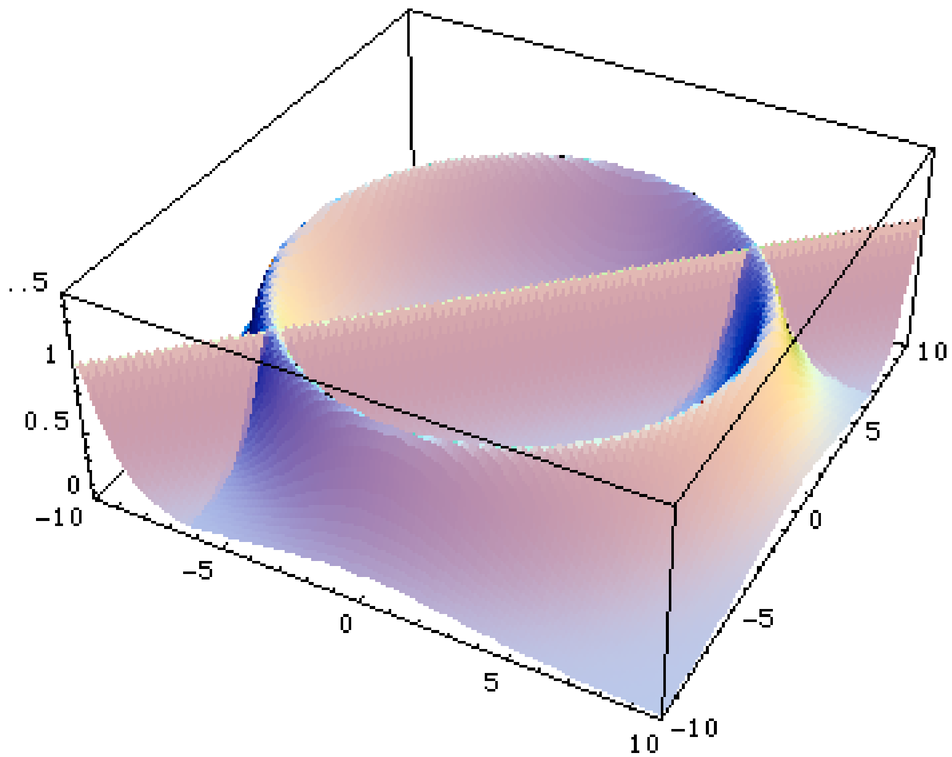
$p_1(x)$



$p_2(x)$



Multi-channel



In this case there is no unique tranformation: Vegas is bound to fail!

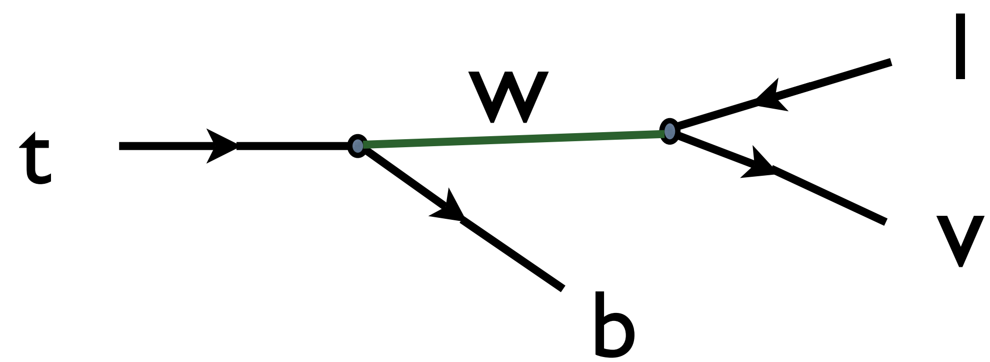
But if you know where the peaks are (=in which variables) we can use different transformations= channels:

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

$$I = \int f(x) dx = \sum_{i=1}^{n} \alpha_i \int \frac{f(x)}{p(x)} p_i(x) dx$$

Exercise: top decay

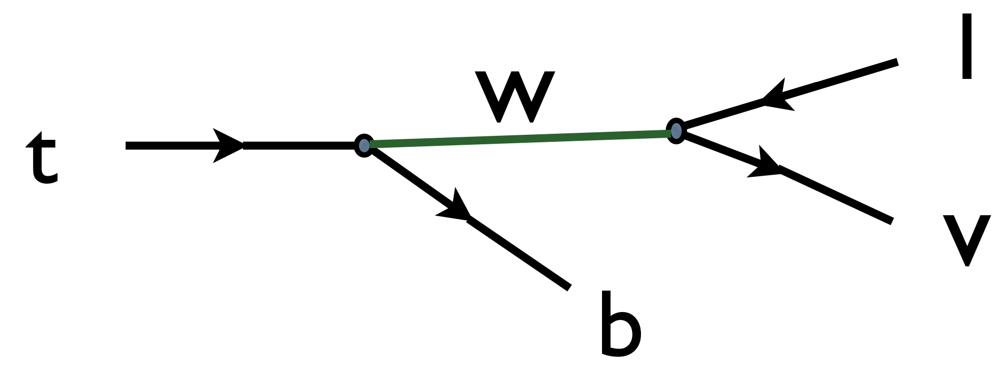
MC101

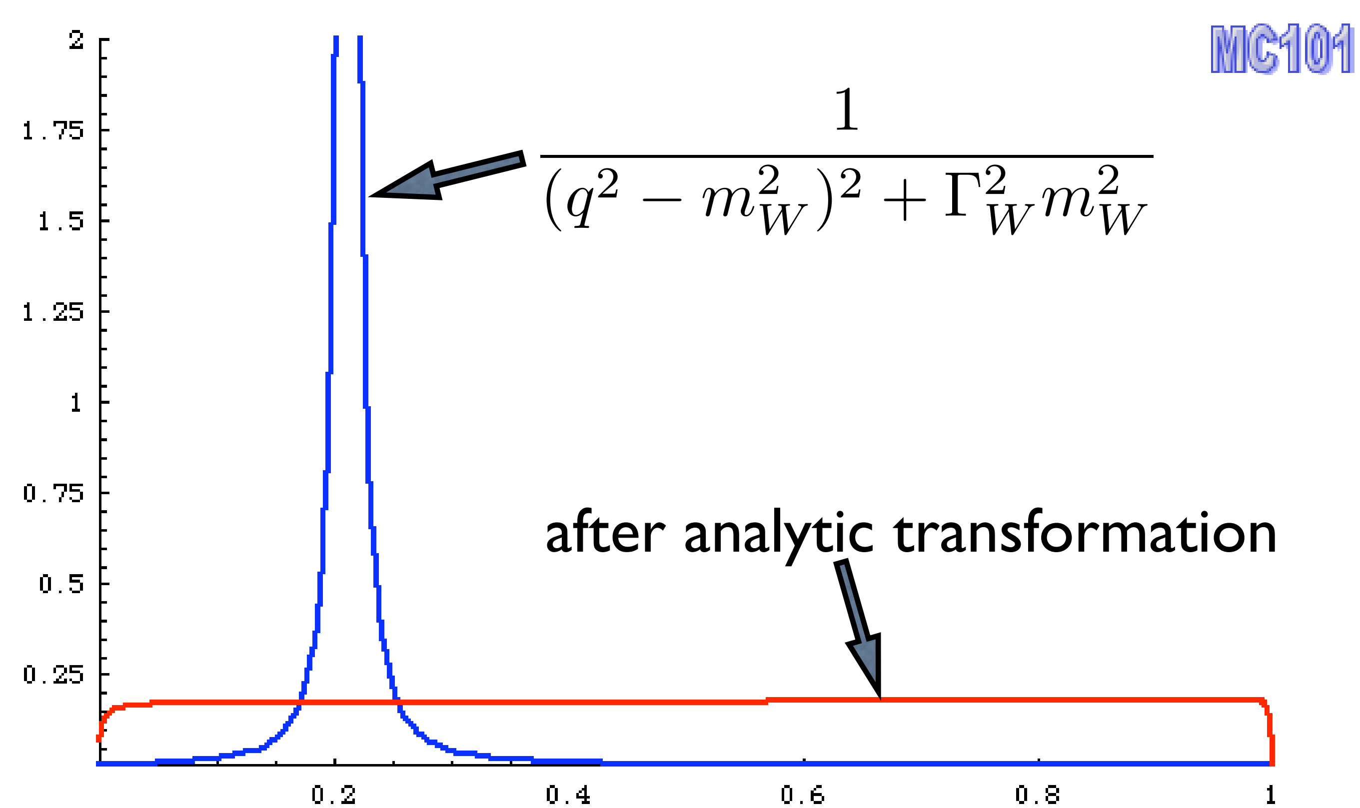


- Easy but non-trivial
- Breit-Wigner peak to be "flattened"

$$\frac{1}{(q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2}$$

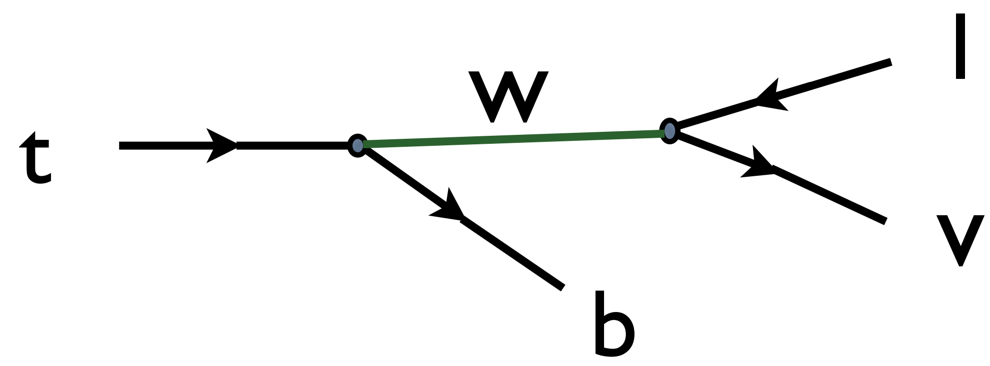
Exercise: top decay





Exercise: top decay

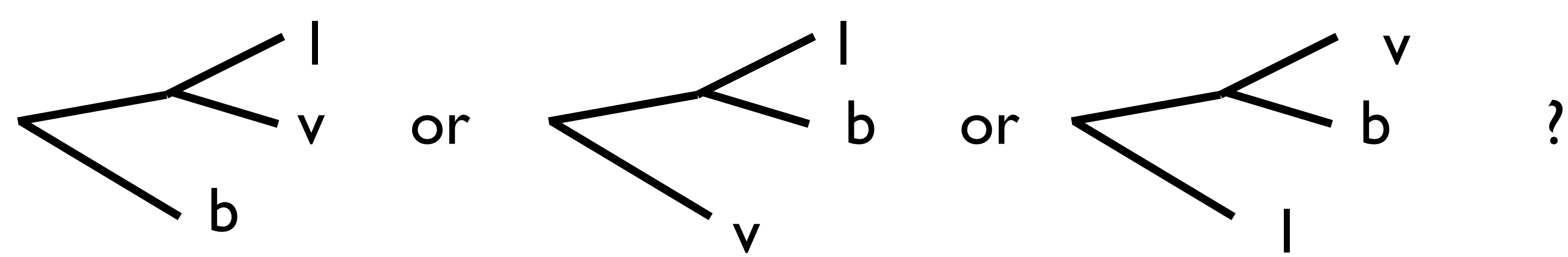
MC101



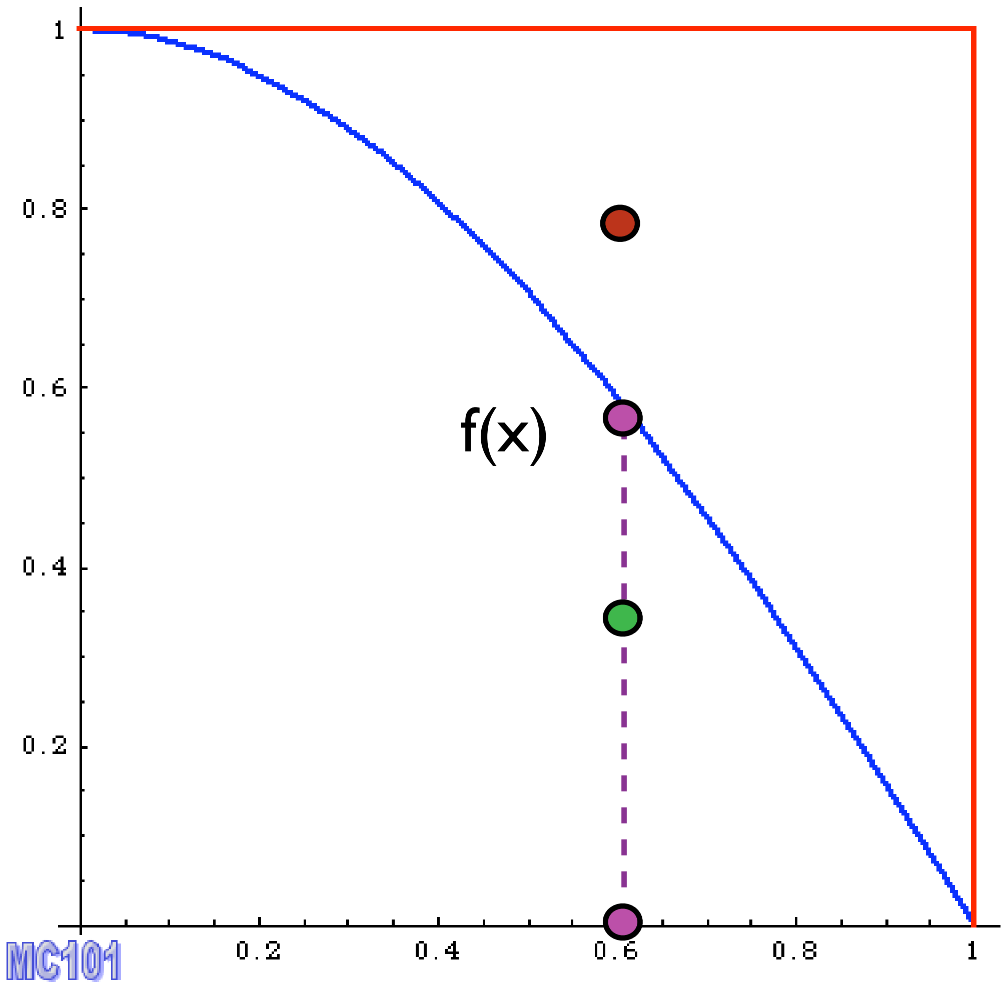
- Easy but non-trivial
- Breit-Wigner peak to be "flattened :

$$\frac{1}{(q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2}$$

- Choose the right "channel" for the phase space



Event generation



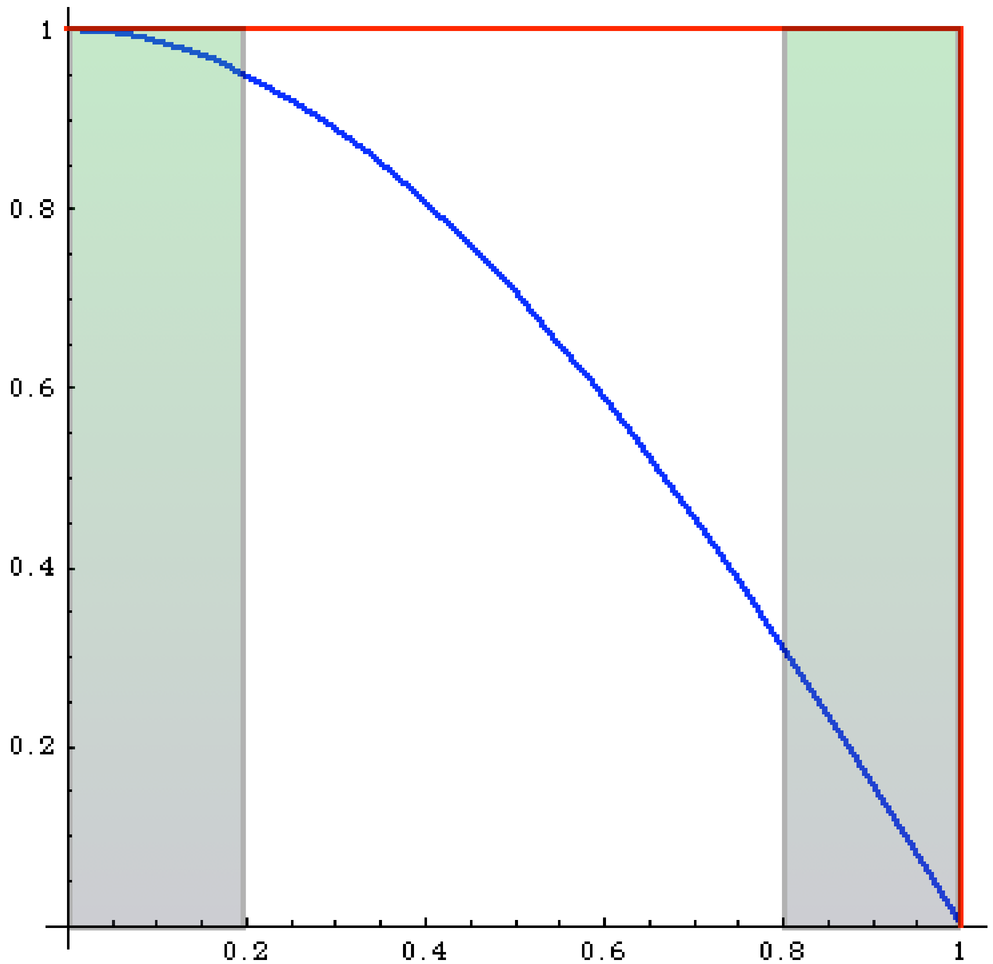
Alternative way

1. pick x
2. calculate f(x)
3. pick 0<y<fmax
4. Compare:
   if f(x)>y accept event,

   else reject it.

$$I = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

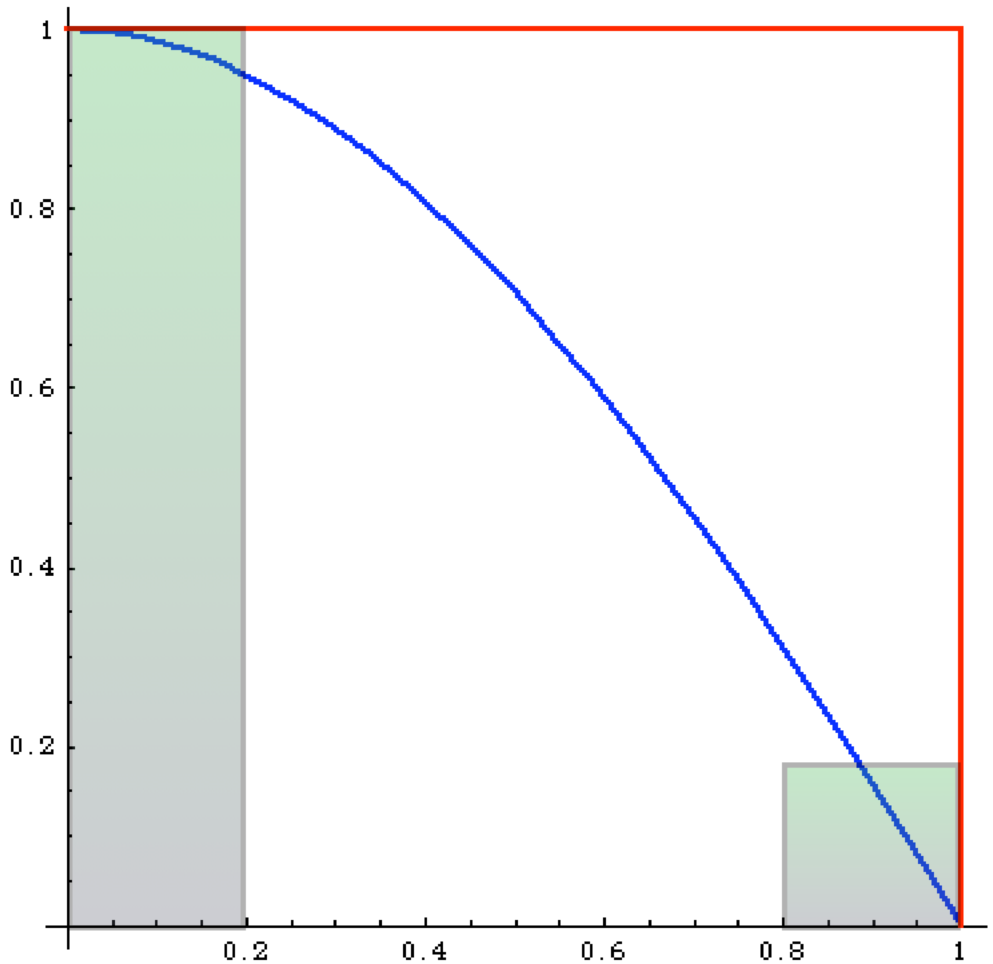


What's the difference?

before:

same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



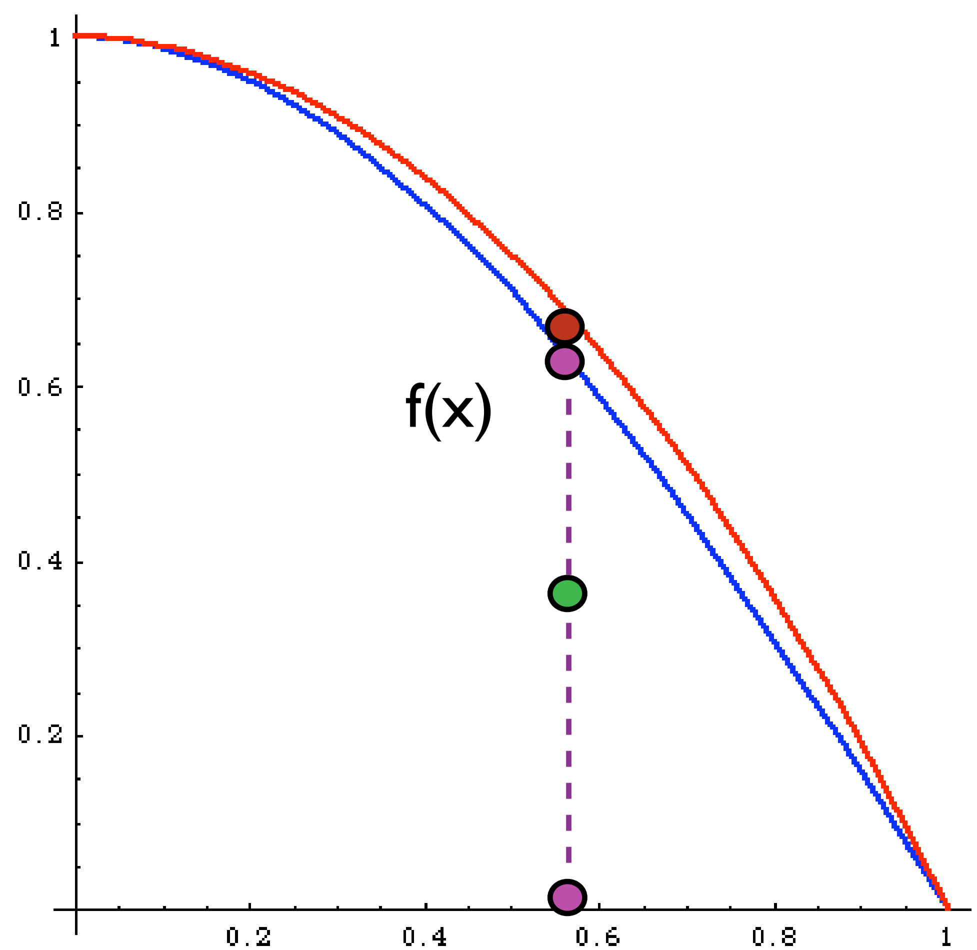
What's the difference?

after:

events is proportional to the probability of areas of phase space:
events have all the same weight ("unweighted")

Events distributed as in Nature

Event generation



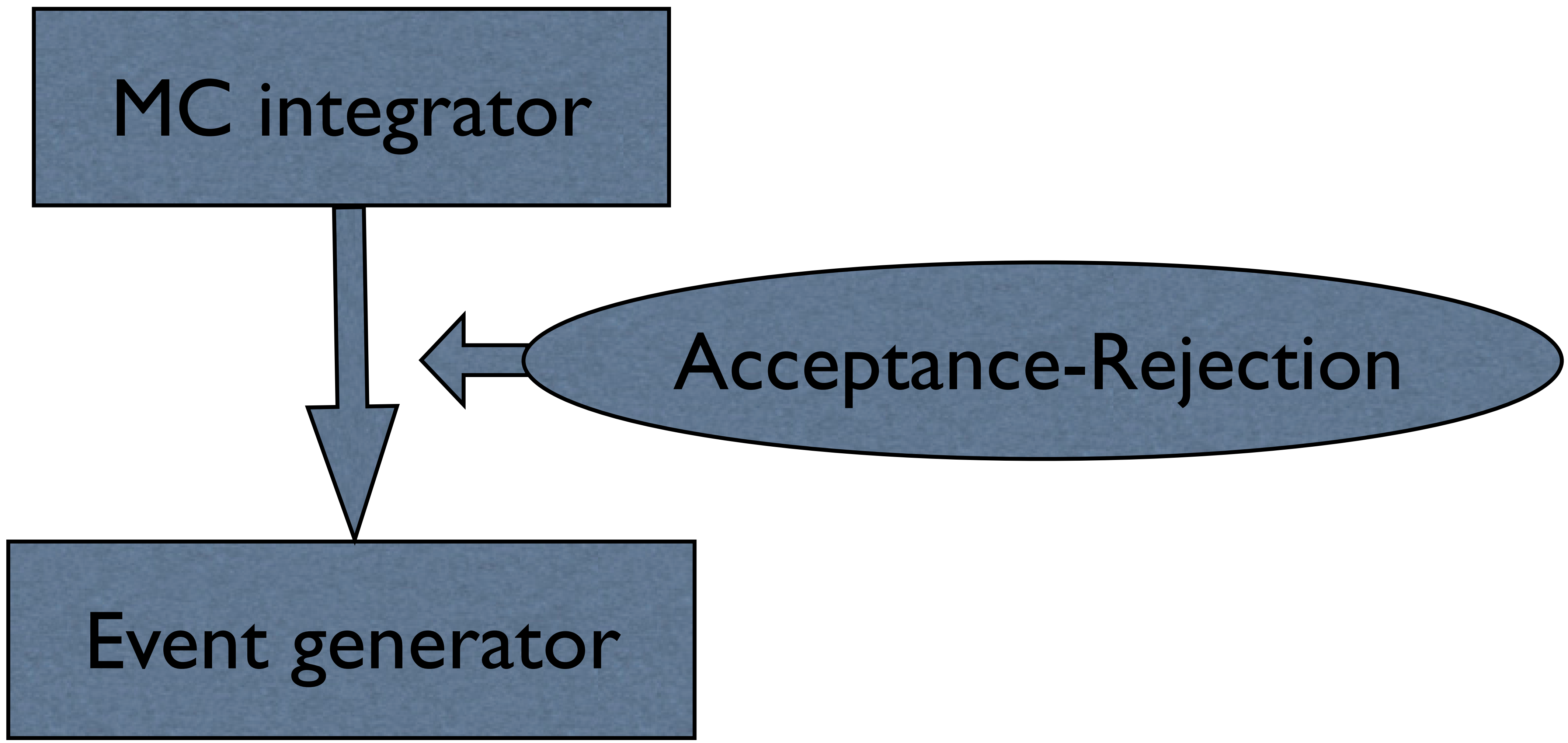
Improved

1. pick x distributed as p(x)
2. calculate f(x) and p(x)
3. pick 0<y<1
4. Compare:
   if f(x)>y p(x) accept event,

else reject it.

much better efficiency!!!

Event generation



☞ This is possible only if $f(x)<\infty$ AND has definite sign!

Monte Carlo Event Generator: definiton

At the most basic level a Monte Carlo event generator is a program which produces particle physics events with the same probability as they occur in nature (virtual collider).

In practice it performs a large number of (sometimes very difficult) integrals and then unweights to give the four momenta of the particles that interact with the detector (simulation).

Note that, at least among theorists, the definition of a "Monte Carlo program" also includes codes which don't provide a fully exclusive information on the final state but only cross sections or distributions at the parton level, even when no unweighting can be performed. I will refer to these kind of codes as "MC integrators".

SM

General structure

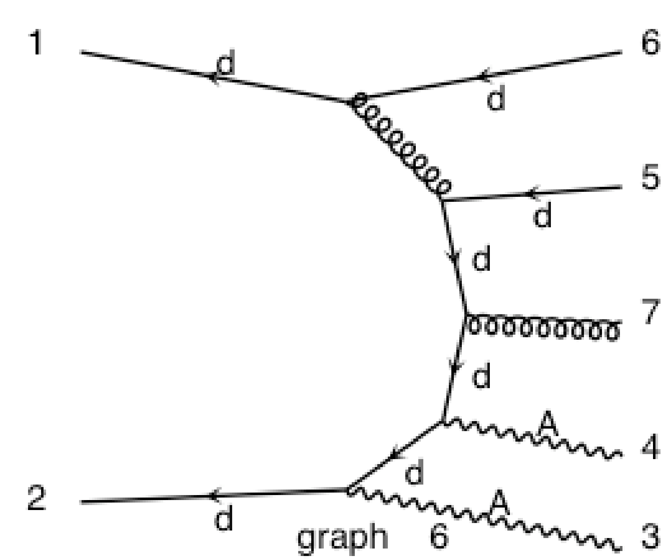
subprocs handler

Includes all possible subprocess leading to a given multi-jet final state automatically or manually (done once for all)

d~ d -> a a u u~ g
d~ d -> a a c c~ g
s~ s -> a a u u~ g
s~ s -> a a c c~ g

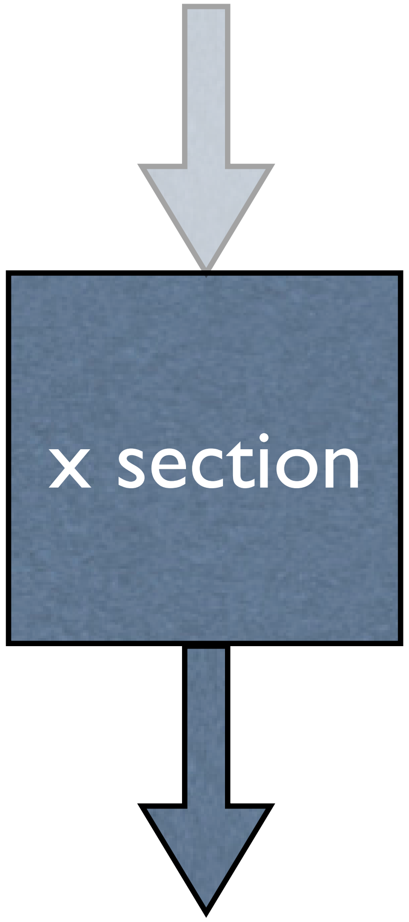
ME calculator

"Automatically" generates a code to calculate |M|^2 for arbitrary processes with many partons in the final state.

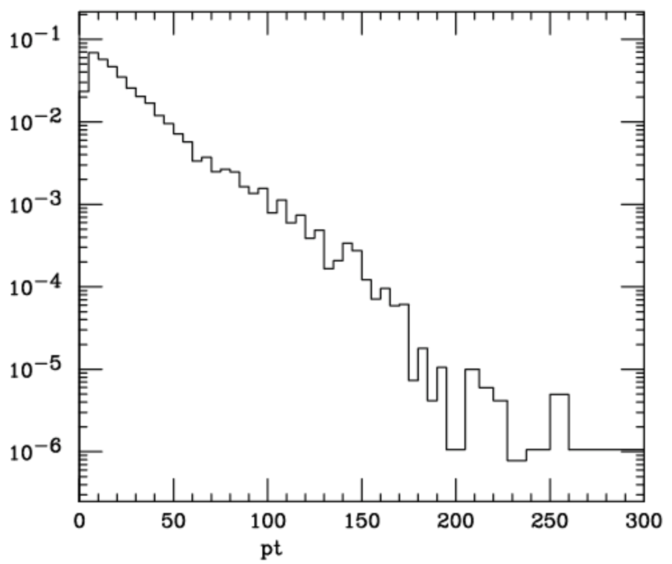


Most use Feynman diagrams w/ tricks to reduce the factorial growth, others have recursive relations to reduce the complexity to exponential. ☺

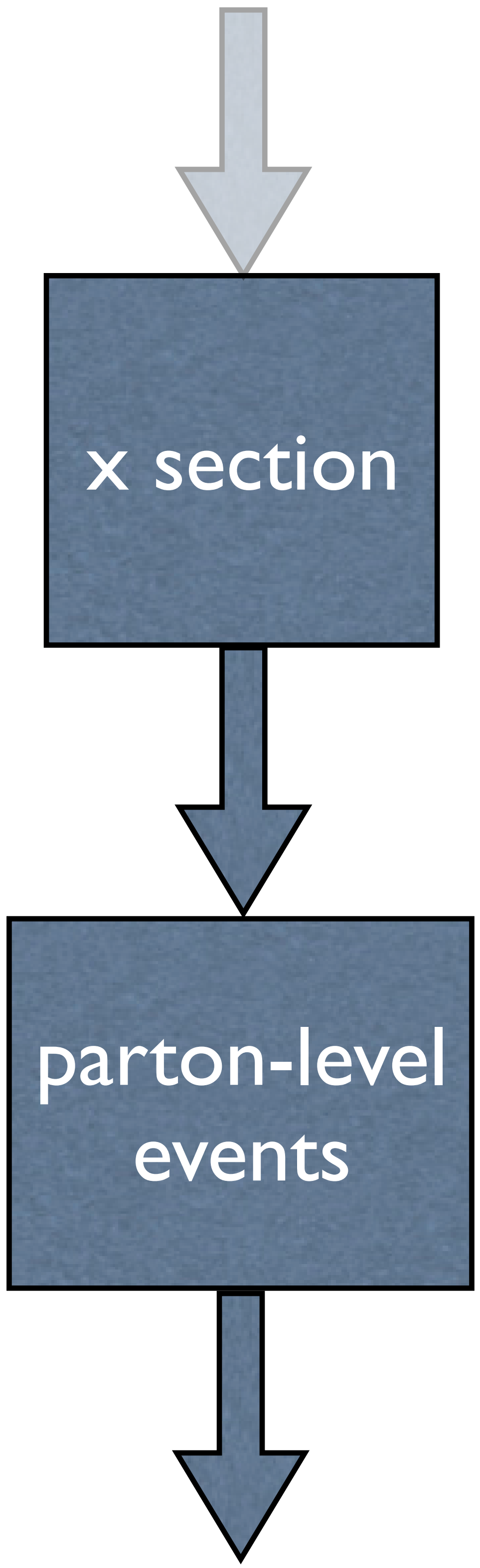
General structure



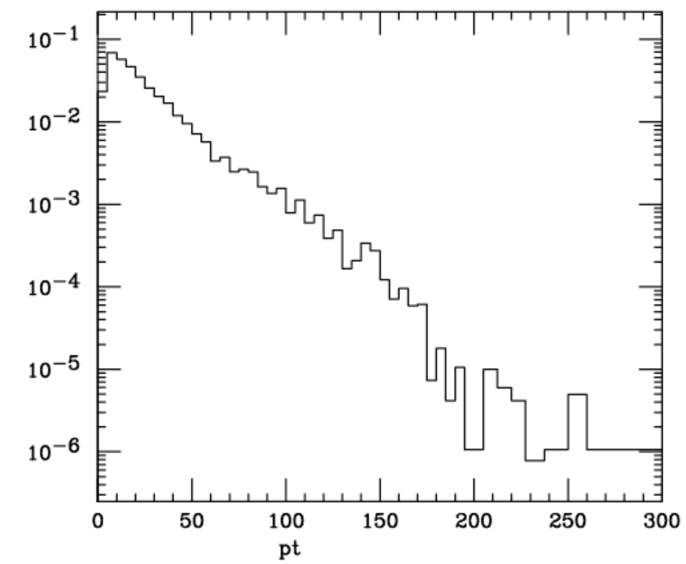
Integrate the matrix element over the phase space using a multi-channel technique and using parton-level cuts.



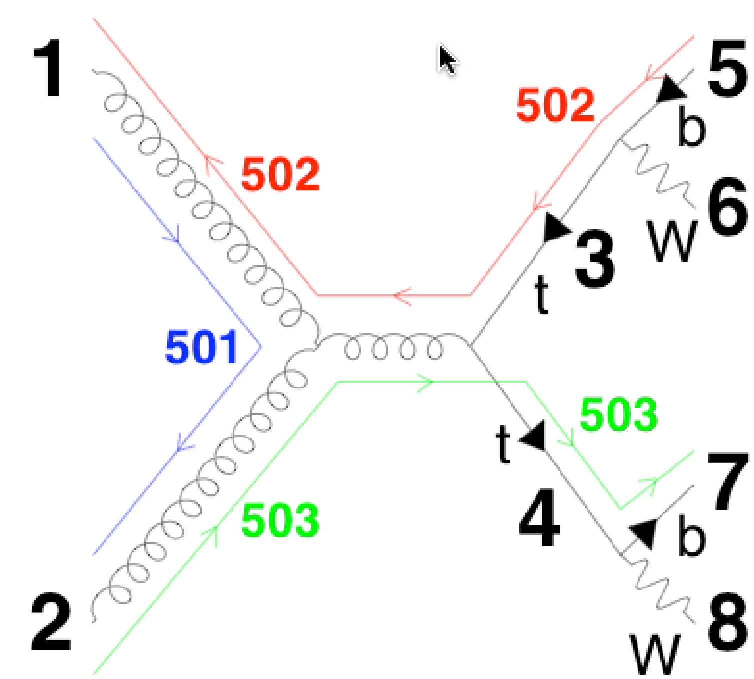
General structure



Integrate the matrix element over the phase space using a multi-channel technique and using parton-level cuts.



Events are obtained by unweighting. These are at the parton-level. Information on particle id, momenta, spin, color is given in the Les Houches format.



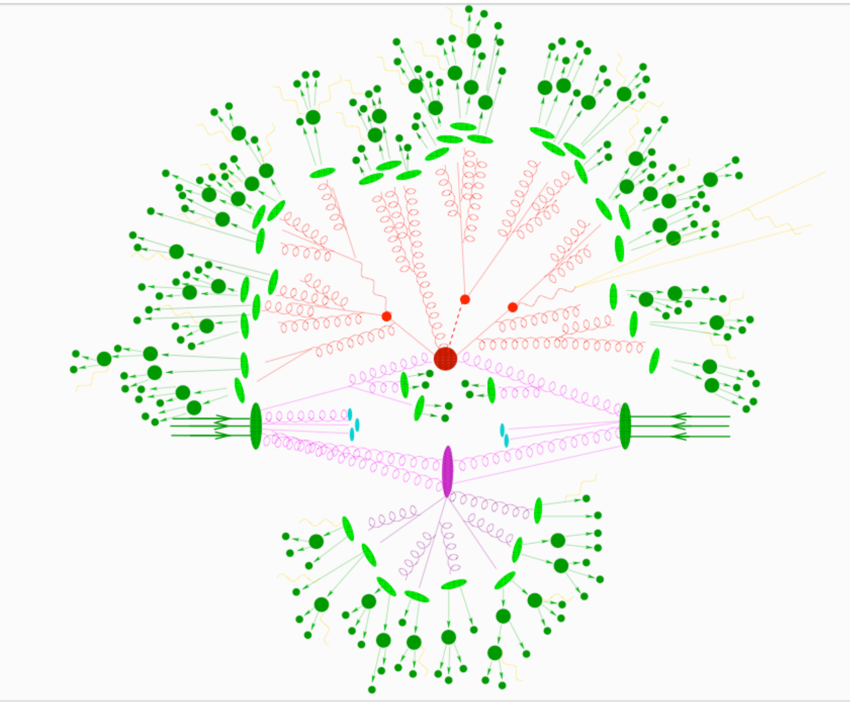
General structure

Shower & Hadro

Events in the LH format are passed to the showering and hadronization⇒

high multiplicity hadron-level events

This has to be done through a consistent procedure (matching)!

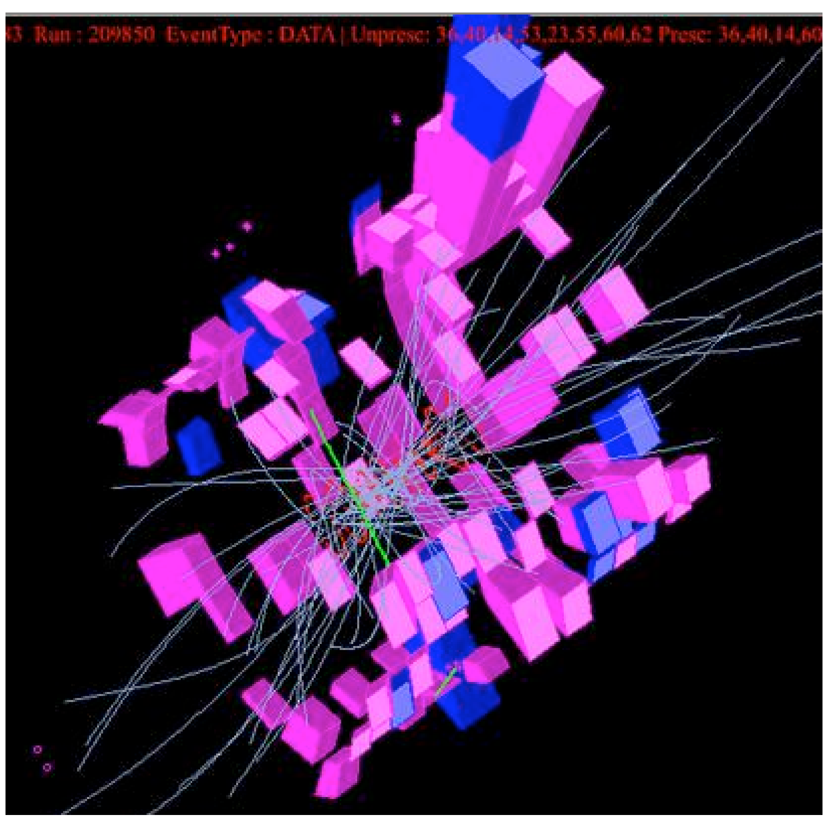


th

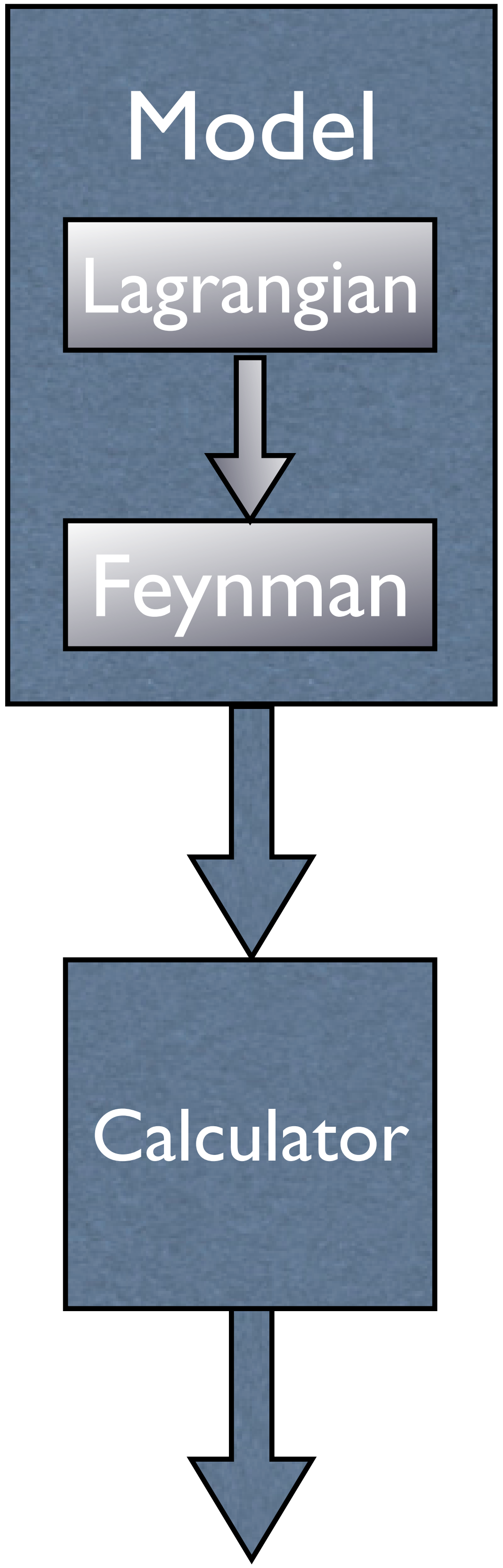
exp

Detector simulation & reco

Events in stdhep format are passed through fast or full simulation, and physical objects (leptons, photons, jet, b-jets, taus) are reconstructed.



Add-on for BSM



Les Houches interface

Invent a model, renormalizable or not, with new physics. Write the Lagrangian and the Feyman Rules.

The particles content, the type of interactions and the analytic form of the couplings in the Feynman rules define the model at tree level.

SUSY, Little Higgs, Higgsless, GUT, Extra dimensions (flat, warped, universal,...)

Parameters Calculator.
Given the "primary" couplings, all relevant quantities are calculated: masses, widths and the values of the couplings in the Feynman rules.

Caution: tree-level relations have to be satisfied to avoid gauge violations and/or wrong branching ratios.

FeynHiggs, ISAJET, NMHDecay, SOFTSUSY, SPHENO, SUSPECT, SDECAY...

Types of SM codes available

Several codes exist for the SM, built using different philosophies

TYPE	Characteristics	Examples
"One" Process	Highly dedicated, manual work, optimized, specific problems addressed	VecBos TopRex Phantom
Library	Semi automatic, modular structure, author-driven efficient	Gr@PPA AlpGen
Multi-purpose	High automatization, user-driven, huge versatility	Sherpa CompHep MadGraph Whizard

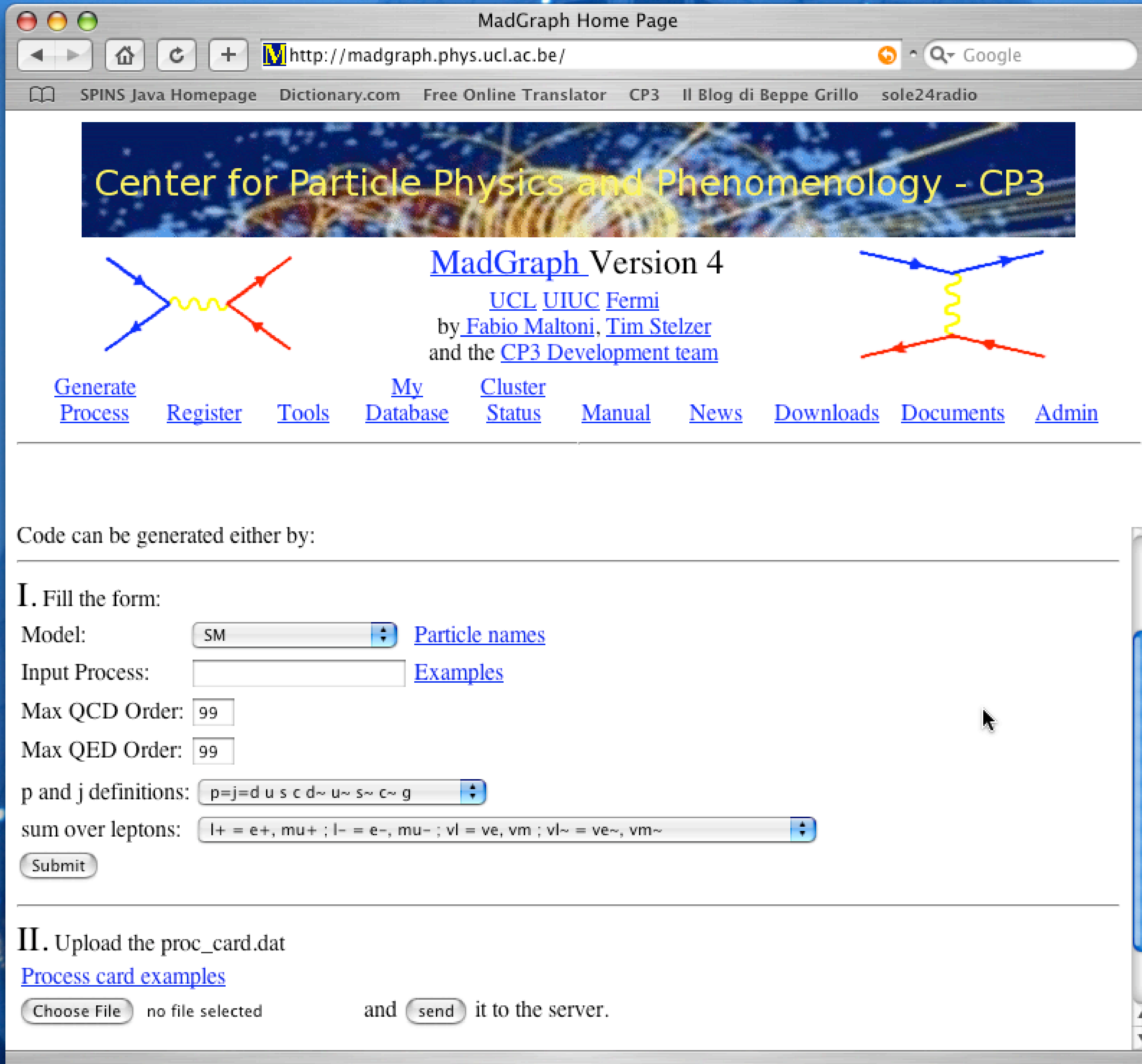
Madgraph/MadEvent

- The new web generation:
  - User inputs model/parameters/cuts.
  - Code runs in parallel on modest farms.
  - Returns cross section, plots, parton-level events.
  - BSM physics (MSSM, 2HDM,...) + returns Pythia and PGS events!
- Advantages:
  - Reduces overhead to getting results
  - Events can easily be shared/stored
  - Quick response to user requests and to new ideas!

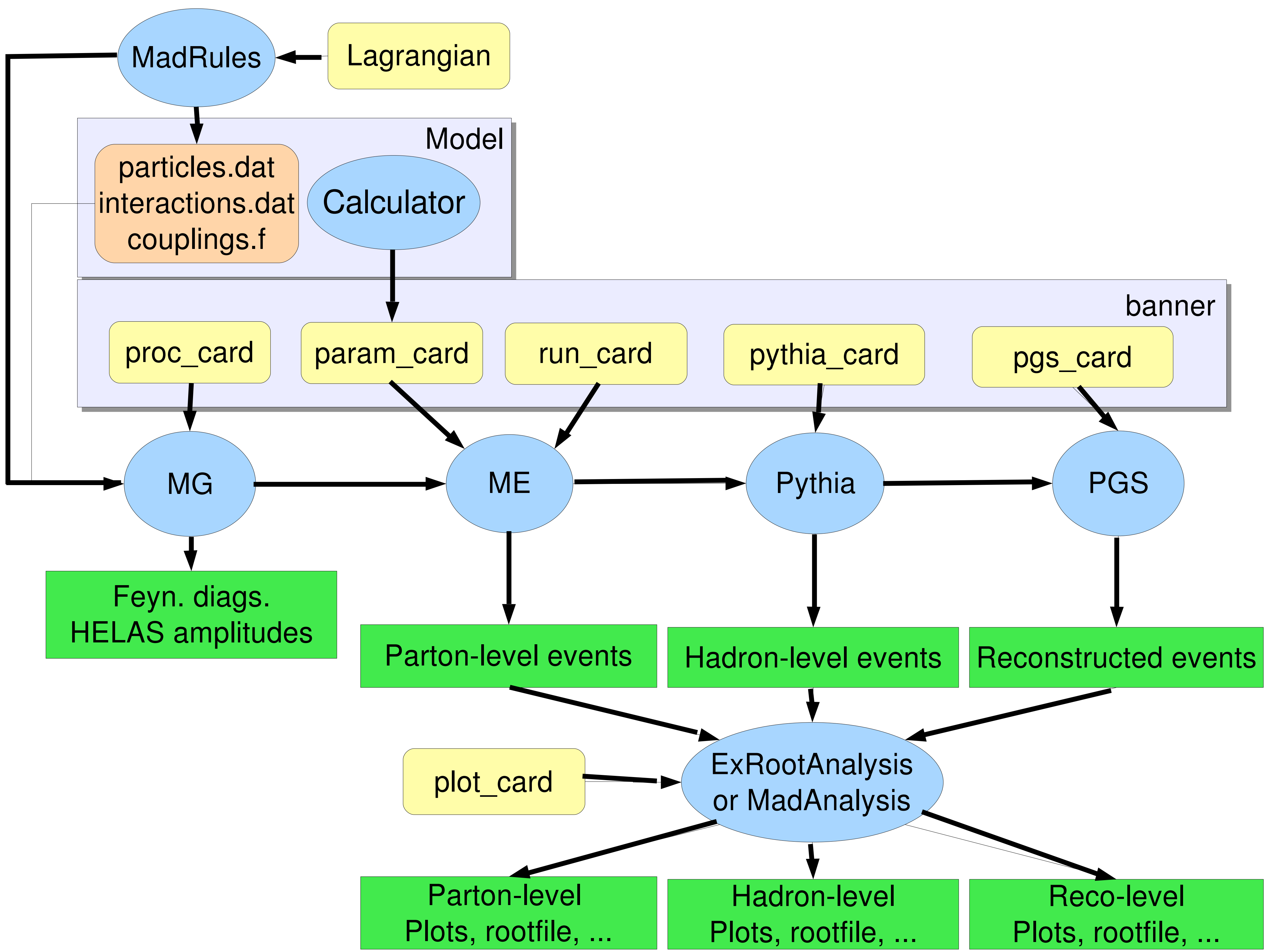
http://madgraph.phys.ucl.ac.be Belgian

http://madgraph.hep.uiuc.edu U.S.

http://madgraph.roma2.infn.it Italian


MadGraph Home Page
http://madgraph.phys.ucl.ac.be/
Google
SPINS Java Homepage
Dictionary.com
Free Online Translator
CP3
Il Blog di Beppe Grillo
sole24radio
Center for Particle Physics and Phenomenology - CP3
MadGraph Version 4
UCL UIUC Fermi
by Fabio Maltoni, Tim Stelzer
and the CP3 Development team
Generate Process
Register
Tools
My Database
Cluster Status
Manual
News
Downloads
Documents
Admin
Code can be generated either by:
I. Fill the form:
Model:
SM
Particle names
Input Process:
Examples
Max QCD Order: 99
Max QED Order: 99
p and j definitions: p=j=d u s c d~ u~ s~ c~ g
sum over leptons: l+ = e+, mu+ ; l- = e-, mu- ; vl = ve, vm ; vl~ = ve~, vm~
Submit
II. Upload the proc_card.dat
Process card examples
Choose File no file selected and send it to the server.

MadGraph/MadEvent Flow



Conclusions

- Performing calculations in pQCD is difficult and still an art.
- Accurate and flexible tools are needed to improve our chances to make discoveries at the LHC.
- Matrix element based MC's are tools that try to make the best out of tree-level calculations and parton showers.
- Recent progress in the field has been impressive, with many new sophisticated tools and techniques made available to the exp community.
- In this respect "life is harder" mostly for the exps: many issues have to be dealt with (which is the best tool to use in a given analysis, how are the systematics assessed, how the comparison with the data performed) and answers are not always easy.
- Minimal approach: understand the basics well (asymptotic freedom, infrared safety, factorization) and stay connected to the TH community.

Lecture material + exercises can be found at:

http://cp3wks05.fynu.ucl.ac.be/twiki/bin/view/Library/GIFSchool