GIF School, September 2007

pp $\rightarrow$ Higgs: a case study

Fabio Maltoni

Abstract

The NLO calculation for pp $\rightarrow$ Higgs and the corresponding results for the LHC are illustrated.

1 Leading order with a finite top mass

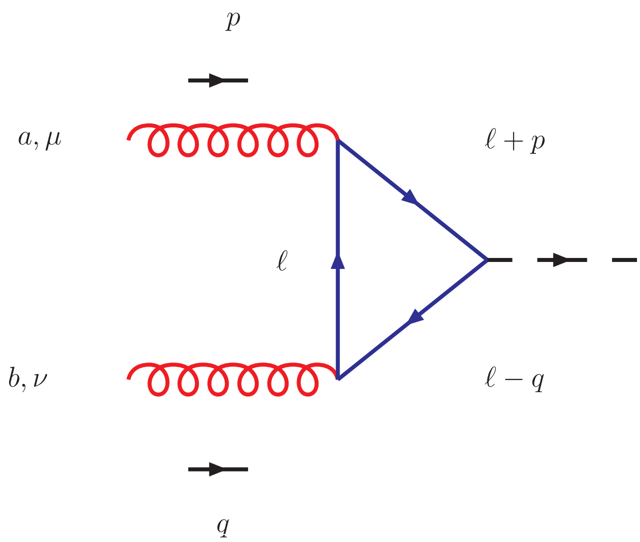

Figure 1: Representative Feynman diagram for the process $gg \to H$. Another diagram, the one with the gluons exchanged, contributes to the total amplitude.

The primary production mechanism for a Higgs boson in hadronic collisions is through gluon fusion, $gg \to H$, which is shown in Fig. 1. The loop contains all massive colored particles in the model. Consider only the top quark. To evaluate the diagram of Fig. 1 (there are actually two diagrams, the one shown and another one with the gluons exchanged. They give the same contribution so we'll just multiply our final result by two), use use dimensional regularization in $D = 4 - 2\epsilon$ dimensions.

(a) Using the QCD Feynman rules write the expression for the amplitude corresponding to the diagram of Fig. 1:

$$i\mathcal{A} = -(-ig_s)^2 \mathrm{Tr}(t^a t^b) \left(\frac{-im_t}{v}\right) \int \frac{d^d\ell}{(2\pi)^n} \frac{T^{\mu\nu}}{\mathrm{Den}} (i)^3 \epsilon_\mu(p) \epsilon_\nu(q) \tag{1}$$

where the overall minus sign is due to the closed fermion loop.[1] The denominator is $\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$.

(b) Use the usual Feynman parametrization method to combine the denominators of the loop integral into one, using the following:

$$\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1-x-y)]^3} \tag{2}$$

and so the denominator becomes,

$$\frac{1}{\mathrm{Den}} = 2\int dx\, dy \frac{1}{[\ell^2 - m_t^2 + 2\ell\cdot(px - qy)]^3}. \tag{3}$$

[1] $\epsilon_\mu(p)$ are the transverse gluon polarizations.

(c) Shift the integration momenta to $\ell' = \ell + px - qy$ so the denominator takes the form

$$\frac{1}{\text{Den}} \to 2\int dx\, dy \frac{1}{[\ell'^{\,2} - m_t^2 + M_H^2 xy]^3}. \tag{4}$$

(d) Evaluate the numerator of the loop integral in the shifted loop momentum:

$$\begin{aligned} T^{\mu\nu} &= \text{Tr}\Big[(\ell + m_t)\gamma^\mu(\ell + p + m_t)(\ell - q + m_t)\gamma^\nu)\Big] \\ &= 4m_t\Big[g^{\mu\nu}(m_t^2 - \ell^2 - \frac{M_H^2}{2}) + 4\ell^\mu\ell^\nu + p^\nu q^\mu\Big] \end{aligned} \tag{5}$$

Use the fact that that for transverse gluons, $\epsilon(p) \cdot p = 0$ and so terms proportional to the external momenta, p_μ or q_ν, can be dropped. You should find that the trace is proportional to the quark mass. This can be easily understood as an effect of the spin-flip coupling of the Higgs. Gluons or photons do not change the spin of the fermion, while the Higgs does. If the quark circulating in the loop is massless then the trace vanishes due to helicity conservation. This is the reason why even when the Yukawa coupling of the light quark and the Higgs is enhanced (such as in SUSY or 2HDM with large $\tan(\beta)$), the contribution is anyway suppressed by the kimatical mass.

(e) Shift momenta in the numerator, drop terms linear in ℓ' and use the relation

$$\int d^dk \frac{k^\mu k^\nu}{(\ell^2 - C)^m} = \frac{1}{d} g^{\mu\nu} \int d^dk \frac{k^2}{(k^2 - C)^m} \tag{6}$$

to write the amplitude in the form

$$\begin{aligned} i\mathcal{A} &= -\frac{2g_s^2 m_t^2}{v}\delta^{ab} \int \frac{d^d\ell'}{(2\pi)^d} \int dxdy \Big\{ g^{\mu\nu} \Big[m^2 + \ell'^2 \Big(\frac{4-d}{d}\Big) + M_H^2(xy - \frac{1}{2}) \Big] \\ &+ p^\nu q^\mu (1 - 4xy) \Big\} \frac{2dxdy}{(k'^2 - m_t^2 + M_H^2 xy)^3} \epsilon_\mu(p)\epsilon_\nu(q). \end{aligned} \tag{7}$$

(f) Compute the integral of Eq. 7 by using the well known formulas of dimensional regularization

$$\begin{aligned} \int \frac{d^dk}{(2\pi)^d} \frac{k^2}{(k^2 - C)^3} &= \frac{i}{32\pi^2}(4\pi)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon}(2 - \epsilon)C^{-\epsilon} \\ \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - C)^3} &= -\frac{i}{32\pi^2}(4\pi)^\epsilon \Gamma(1+\epsilon) C^{-1-\epsilon}. \end{aligned} \tag{8}$$

You should find that your result is finite.

(g) Compare your result with the known result:

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\Big(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\Big) \int dxdy \Big(\frac{1 - 4xy}{m_t^2 - m_H^2 xy}\Big)\epsilon_\mu(p)\epsilon_\nu(q). \tag{9}$$

(Note that we have multiplied by 2 in Eq. (9) to include the diagram where the gluon legs are crossed.) The Feynman integral of Eq. (9) can easily be performed to find an analytic result if desired. Note that the tensor structure could have been predicted from the start by using the fact that $p^\mu \mathcal{A}^{\mu\nu} = q^\nu \mathcal{A}^{\mu\nu} = 0$.

(h) Define $I(a)$ as

$$I(a) \equiv \int_0^1 dx \int_0^{1-x} dy \frac{1-4xy}{1-axy}. \tag{10}$$

and express the amplitude in terms of such an expression. Plot the function $I(a)$ and verify that it goes quickly to its limiting values when $a \to 0$ and $a \to \infty$. Numerically, the heavy fermion mass limit is an extremely good approximation even for $m \sim M_H$. From this plot we can also see that the contribution of light quarks to gluon fusion of the Higgs boson is irrelevant. In fact we have,

$$I(a) \longrightarrow_{a\to\infty} \sim -\frac{1}{2a}\log^2(a). \tag{11}$$

Therefore, for the Standard Model, only the top quark is numerically important when computing Higgs boson production from gluon fusion.

(i) It is particularly interesting to consider the case when the fermion in the loop is much more massive than the Higgs boson, $M_H \ll m_t$. In this case we find,

$$\mathcal{A}(gg \to H) \longrightarrow_{m>>M_H} -\frac{\alpha_S}{3\pi v}\delta^{ab}\Big(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\Big)\epsilon_\mu(p)\epsilon_\nu(q). \tag{12}$$

We see that the production process $gg \to H$ is independent of the mass of the heavy fermion in the loop in the limit $M_H \ll m_t$. Hence it counts the number of heavy generations and is a window into new physics at scales much above the energy being probed. This is a contradiction of our intuition that heavy particles should decouple and not affect the physics at lower energy. The reason the heavy fermions do not decouple is, of course, because the Higgs boson couples to the fermion mass.

(l) Cross section at the LHC. Resonant production of a heavy Higgs can be found from the standard formula:

$$\begin{aligned} \hat{\sigma} &= \frac{1}{2s}\overline{|\mathcal{A}|^2}\frac{d^3P}{(2\pi)^3 2E_H}(2\pi)^4\delta^4(p+q-P) \\ &= \frac{1}{2s}\overline{|\mathcal{A}|^2}2\pi\delta(s-m_H^2), \end{aligned} \tag{13}$$

using

$$\begin{aligned} \delta^{ab}\delta^{ab} &= N_c^2 - 1 \\ \Big(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\Big)^2 &= \frac{m_H^4}{2} \end{aligned} \tag{14}$$

$$\overline{|\mathcal{A}|^2} = \frac{1}{4}\frac{1}{(N_c^2-1)^2}|\mathcal{A}|^2 . \tag{15}$$

Verify that the result is

$$\hat{\sigma}(gg \to H) = \frac{\alpha_S^2}{64\pi v^2} \mid I\Big(\frac{M_H^2}{m^2}\Big) \mid^2 \tau_0\delta(\tau - \tau_0)$$

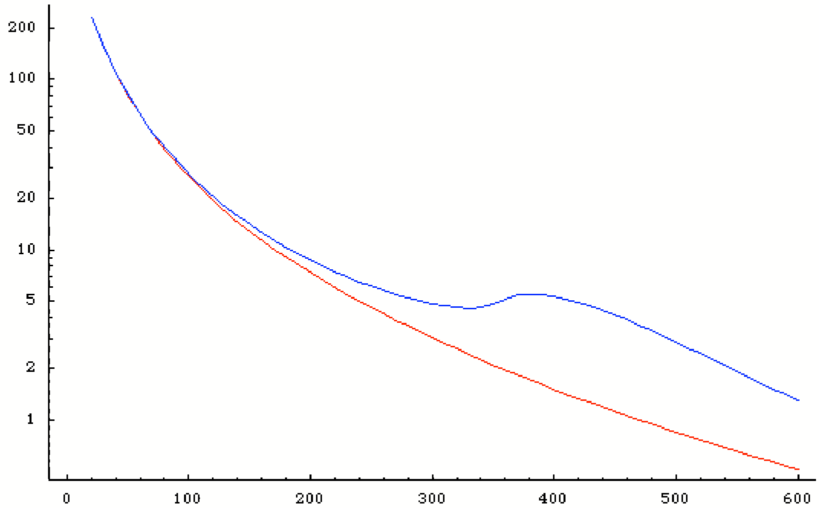

Figure 2: LO cross section for $pp \to H$ at LO at the LHC (pb) as a function of the Higgs mass (GeV).The red (lower) curve is the large top-mass limit, while the blue (upper) curve is the exact result.

where $s = x_1 x_2 S \equiv \tau S$ is the parton-parton energy squared, we have defined

$$z \equiv \frac{M_H^2}{s} = \frac{M_H^2}{\tau S} = \frac{\tau_0}{\tau} \tag{16}$$

with $\tau_0 = M_H^2/S$ and the integral I is defined by Eq. (10).

(m) To find the physical cross section we must integrate with the distribution of gluons in a proton,

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 g(x_1) g(x_2) \hat{\sigma}(gg \to H), \tag{17}$$

where $g(x)$ is the distribution of gluons in the proton. Perform the change of variables $x_1 \equiv \sqrt{\tau} e^y$, $x_2 \equiv \sqrt{\tau} e^{-y}$, and $\tau = x_1 x_2$. Find the Jacobian and the change of the integration limits and show that the result can be written as:

$$\sigma(pp \to H) \;=\; \frac{\alpha_S^2}{64\pi v^2} \mid I\left(\frac{M_H^2}{m^2}\right) \mid^2 \tau_0 \int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy g(\sqrt{\tau_0} e^y) g(\sqrt{\tau_0} e^{-y}) \tag{18}$$

Often the above integral over the parton distribution is given the name of gluon-gluon parton luminosity.

(n) Using the pdf's from the CTEQ collaboration, CTEQ5L (Fortran,C or Mathematica) compute the gluon-gluon luminosity and the LO Higgs cross section at the LHC. Compare with the results shown in Fig. 2.

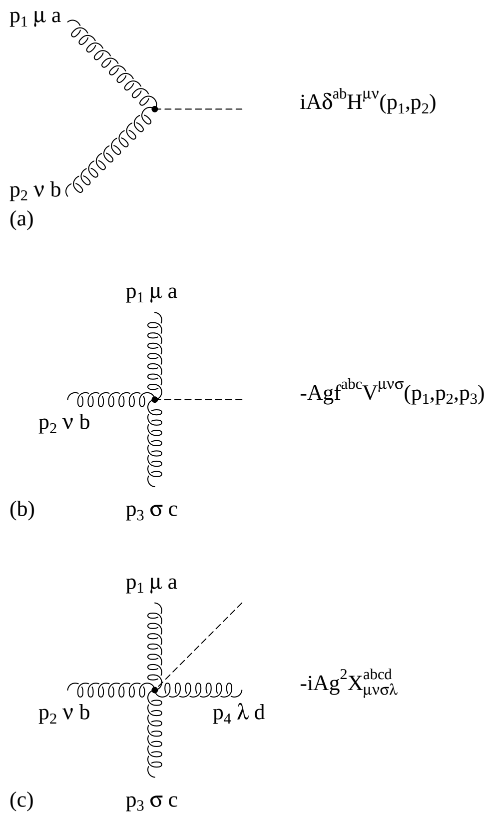

Figure 3: Feynman rules in the EFT where the top is integrated out. Gluon momenta are outgoing.

(n) Higgs Effective field theory.

A striking feature of our result for Higgs boson production from gluon fusion is that it is independent of the heavy quark mass for a light Higgs boson. In fact Eq. (12) can be derived from the effective vertex,

$$\begin{aligned} \mathcal{L}_{\rm eff} &= \frac{\alpha_S}{12\pi} G^A_{\mu\nu} G^{A\ \mu\nu} \left(\frac{H}{v}\right) \\ &= \frac{\beta_F}{g_s} G^A_{\mu\nu} G^{A\ \mu\nu} \left(\frac{H}{2v}\right)(1-\delta), \end{aligned}$$

where

$$\beta_F = \frac{g_s^3 N_H}{24\pi^2} \tag{19}$$

is the contribution of heavy fermion loops to the $SU(3)$ beta function and $\delta = 2\alpha_S/\pi$.[2]

[2]The $(1-\delta)$ term arises from a subtlety in the use of the low energy theorem. Since the Higgs coupling to the heavy fermions is $M_f(1+\frac{H}{v})\overline{f}f$, the counterterm for the Higgs Yukawa coupling is fixed in terms of the renormalization of the fermion mass and wavefunction. The beta function, on the other hand, is evaluated at $q^2=0$. The $1-\delta$ term corrects for this mismatch.

(N_H is the number of heavy fermions with $m >> M_H$.) The effective Lagrangian of Eq. (19) gives ggH, $gggH$ and $ggggH$ vertices and can be used to compute the radiative corrections of $\mathcal{O}(\alpha_S^3)$ to gluon production. The correction in principle involve 2-loop diagrams. However, using the effective vertices from Eq. (19), the $\mathcal{O}(\alpha_S^3)$ corrections can be found from a 1-loop calculation. To fix the notation we shall use

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} A H G^A_{\mu\nu} G^{A\ \mu\nu}, \tag{20}$$

where $G^A_{\mu\nu}$ is the field strength of the SU(3) color gluon field and H is the Higgs-boson field. The effective coupling A is given by

$$A = \frac{\alpha_S}{3\pi v}\left(1 + \frac{11}{4}\frac{\alpha_S}{\pi}\right), \tag{21}$$

where v is the vacuum expectation value parameter, $v^2 = (G_F\sqrt{2})^{-1} = (246)^2\ \mathrm{GeV}^2$ and the α_S correction is included, as discussed above. The effective Lagrangian generates vertices involving the Higgs boson and two, three or four gluons. The associated Feynman rules are displayed in Fig. 3 The two-gluon–Higgs-boson vertex is proportional to the tensor

$$H^{\mu\nu}(p_1, p_2) = g^{\mu\nu} p_1 \cdot p_2 - p_1^\nu p_2^\mu. \tag{22}$$

The vertices involving three and four gluons and the Higgs boson are proportional to their counterparts from pure QCD:

$$V^{\mu\nu\rho}(p_1, p_2, p_3) = (p_1 - p_2)^\rho g^{\mu\nu} + (p_2 - p_3)^\mu g^{\nu\rho} + (p_3 - p_1)^\nu g^{\rho\mu}, \tag{23}$$

and

$$\begin{aligned} X^{\mu\nu\rho\sigma}_{abcd} &= f_{abe} f_{cde}(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f_{ace} f_{bde}(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ &+ f_{ade} f_{bce}(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}). \end{aligned} \tag{24}$$

2 $gg \to$ Higgs @ NLO

In this section we study the process $gg \to H$ at NLO, in the large top-quark mass limit. All results given below are in Conventional Dimensional Regularization (CDR). Using the effective Lagrangian for the gluon-gluon and gluon-Higgs interactions:

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}\left(1 - \frac{\alpha_S}{3\pi}\frac{H}{v}\right) G^{\mu\nu} G_{\mu\nu} \tag{25}$$

one finds

$$\begin{aligned} \sigma_{\rm Born} &= \frac{\alpha_S^2}{\pi}\frac{m_H^2}{576 v^2 s}(1 + \epsilon + \epsilon^2)\mu^{2\epsilon}\ \delta(1-z) \\ &\equiv \sigma_0\ \delta(1-z)\,, \end{aligned} \tag{26}$$

where $z = m_H^2/s$ as defined previously. Note that we defined σ_0 as containing an explicit factor z. At NLO, for $gg \to H$, there are both virtual and real contributions. In the virtuals one should also take into account that the $\mathcal{L}_{\text{eff}}$ gets corrected by the exchange of virtual gluons inside the top-quark loop, so that the interaction becomes:

$$\mathcal{L}_{\text{eff}}^{\text{NLO}} = \left(1 + \frac{11}{4}\frac{\alpha_S}{\pi}\right)\frac{\alpha_S}{3\pi}\frac{H}{v}G^{\mu\nu}G_{\mu\nu} \tag{27}$$

2.1 $gg \to H$: virtual corrections

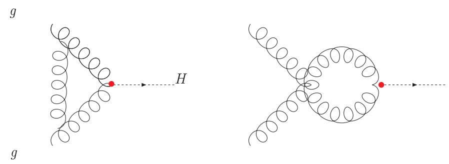

Figure 4: Feynman diagrams giving virtual contributions in the infinite top-quark mass limit

The non-zero virtual diagrams are two, the vertex correction and the bubble with the four gluon vertex. Their sum (plus the α_S corrections from Eq. (27) gives:

$$\sigma_{\text{virt}} = \sigma_0\ \delta(1-z)\ \left[1 + \frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma\left(-\frac{2}{\epsilon^2} + \frac{11}{3} + \pi^2\right)\right], \tag{28}$$

where c_Γ where

$$c_\Gamma = (4\pi)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma(1-\epsilon)^2}{\Gamma(1-2\epsilon)}. \tag{29}$$

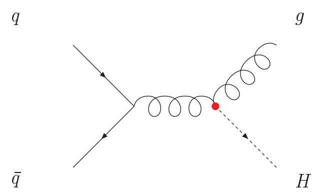

Figure 5: Feynman diagrams giving $q\bar{q}$ real contributions in the infinite top-quark mass limit. These contributions are finite.

2.2 Real Contribution: quark anti-quark initial state

This contribution, shown in Fig. 5 is finite and can be calculated directly in four dimensions. The amplitude is

$$\overline{|\mathcal{M}|^2} = \frac{4}{81}\frac{\alpha_S^3}{\pi v^2}\,\frac{(u^2+t^2)-\epsilon(u+t)^2}{s}\,, \tag{30}$$

to be integrated over the D-dimensional phase space

$$d\Phi_2 = \frac{1}{8\pi}\left(\frac{4\pi}{s}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\; z^{\epsilon}(1-z)^{1-2\epsilon}\; v^{-\epsilon}(1-v)^{-\epsilon}dv \tag{31}$$

where $v = 1/2(1+\cos\theta)$ and $z = M_H^2/s$ as usual. Using

$$t \;=\; -s(1-z)(1-v) \tag{32}$$

$$u \;=\; -s(1-z)v \tag{33}$$

and taking the limit $\epsilon \longrightarrow 0$ gives:

$$\sigma_{\rm real}(q\bar{q}) = \sigma_0\frac{\alpha_S}{2\pi}\frac{64}{27}\frac{(1-z)^3}{z} \tag{34}$$

2.3 Real Contribution: quark gluon initial state

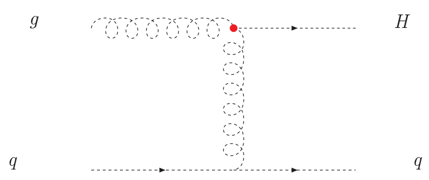

Figure 6: Feynman diagrams giving qg real contributions in the infinite top-quark mass limit.

Let us consider now the contribution from the diagrams with an initial quark, i.e., the process $gq \rightarrow Hq$. The amplitude is

$$\overline{|\mathcal{M}|^2} = -\frac{1}{54(1-\epsilon)}\frac{\alpha_S^3}{\pi v^2}\,\frac{(u^2+s^2)-\epsilon(u+s)^2}{t} \tag{35}$$

and integrating it over the D-dimensional phase space Eq. (31) we get

$$\sigma_{\rm real} = \sigma_0 \,\frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{m_H^2}\right)^\epsilon c_\Gamma \,\left[-\frac{1}{\epsilon} p_{gq}(z) + z - \frac{3}{2}\frac{(1-z)^2}{z} + p_{gq}(z)\log\frac{(1-z)^2}{z}\right], \quad (36)$$

We perform the factorization of the collinear divergences adding the counterterm:

$$\sigma^{\rm coll.}_{\rm c.t.} = \sigma_0 \frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu_F^2}\right)^\epsilon \frac{c_\Gamma}{\epsilon} P_{gq}(z)\right] \quad (37)$$

such that we get the result in the $\overline{\rm MS}$ scheme (note that our definition of σ_0, Eq. (26), contains a factor z):

$$\begin{aligned}\sigma^{\overline{\rm MS}}(qg) &= \sigma_{\rm real} + \sigma^{\rm coll.}_{\rm c.t.} \\ &= \sigma_0 \,\frac{\alpha_S}{2\pi} C_F \,\left[p_{gq}(z)\log\frac{m_H^2}{\mu_F^2} + p_{gq}(z)\log\frac{(1-z)^2}{z} + z - \frac{3}{2}\frac{(1-z)^2}{z}\right]. \quad (38)\end{aligned}$$

2.4 Real Contribution: gluon-gluon initial state

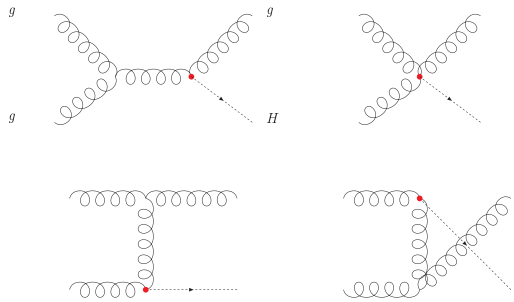

Figure 7: Feynman diagrams giving gg real contributions in the infinite top-quark mass limit.

For the real correction we have to integrate the $gg \to Hg$ amplitude

$$\overline{|\mathcal{M}|^2} = \frac{1}{24(1-\epsilon)^2}\frac{\alpha_S^3}{\pi v^2}\frac{(m_H^8+s^4+t^4+u^4)(1-2\epsilon)+\frac{1}{2}\epsilon(m_H^4+s^2+t^2+u^2)^2}{stu} \quad (39)$$

over the D-dimensional phase space Eq.(31). This gives

$$\begin{aligned}\sigma_{\rm real} &= \sigma_0\,\frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\,c_\Gamma\,\left[\left(\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\frac{b_0}{C_A}-\frac{\pi^2}{3}\right)\delta(1-z)\right.\\ &\quad -\frac{2}{\epsilon}p_{gg}(z)-\frac{11}{3}\frac{(1-z)^3}{z}-4\frac{(1-z)^2(1+z^2)+z^2}{z(1-z)}\log z\\ &\quad \left.+\,4\frac{1+z^4+(1-z)^4}{z}\left(\frac{\log(1-z)}{1-z}\right)_+\right]. \qquad (40)\end{aligned}$$

Once again the above result is looks the same in both regularization schemes. Adding it up, we get:

$$\begin{aligned}\sigma_{\rm real}+\sigma_{\rm virt} &= \sigma_{\rm Born}+\sigma_0\,\frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\,c_\Gamma\,\left[\left(\frac{2}{\epsilon}\frac{b_0}{C_A}+\frac{2\pi^2}{3}\right)\delta(1-z)\right.\\ &\quad -\frac{2}{\epsilon}p_{gg}(z)-\frac{11}{3}\frac{(1-z)^3}{z}-4\frac{(1-z+z^2)^2}{z(1-z)}\log z\\ &\quad \left.+\,8\frac{(1-z+z^2)^2}{z}\left(\frac{\log(1-z)}{1-z}\right)_+\right]. \qquad (41)\end{aligned}$$

At variance with the Drell-Yan process, there is a left-over divergence proportional to $\delta(1-z)$. This is associated to the renormalization of the strong coupling. Using Eq. (57) and Eq. (**??**) we can write the following counterterm:

$$\sigma^{\rm UV}_{\rm c.t.} = 2\;\sigma_{\rm Born}\frac{\alpha_S}{2\pi}\left[-\left(\frac{\mu^2}{\mu^2_{\rm UV}}\right)^{\epsilon}c_\Gamma\frac{b_0}{\epsilon}\right] \qquad (42)$$

The factorization of the collinear divergence is handled is the usual way adding the counterterm:

$$\sigma^{\rm coll.}_{\rm c.t.} = 2\;\sigma_0\frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}\frac{c_\Gamma}{\epsilon}P_{gg}(z)\right] \qquad (43)$$

such that we get the usual result in the $\overline{\rm MS}$ scheme (note that our definition of σ_0, Eq. (26), contains a factor z):

$$\begin{aligned}\sigma^{\overline{\rm MS}}(gg) &= \sigma_0\,\frac{\alpha_S}{2\pi}C_A\left[\left(\frac{11}{3}+\frac{2}{3}\pi^2-2\frac{b_0}{C_A}\log\frac{m_H^2}{\mu^2_{\rm UV}}\right)\delta(1-z)\right.\\ &\quad -\frac{11}{3}\frac{(1-z)^3}{z}+2p_{gg}\log\frac{m_H^2}{\mu_F^2}-4\frac{(1-z+z^2)^2}{z(1-z)}\log z\\ &\quad \left.+\,8\frac{(1-z+z^2)^2}{z}\left(\frac{\log(1-z)}{1-z}\right)_+\right]. \qquad (44)\end{aligned}$$

3 Results

4 Conclusions

We have shown in detail how to calculate the QCD inclusive cross section for Higgs production at hadron colliders. We have used dimensional regularization and the $\overline{MS}$ subtraction

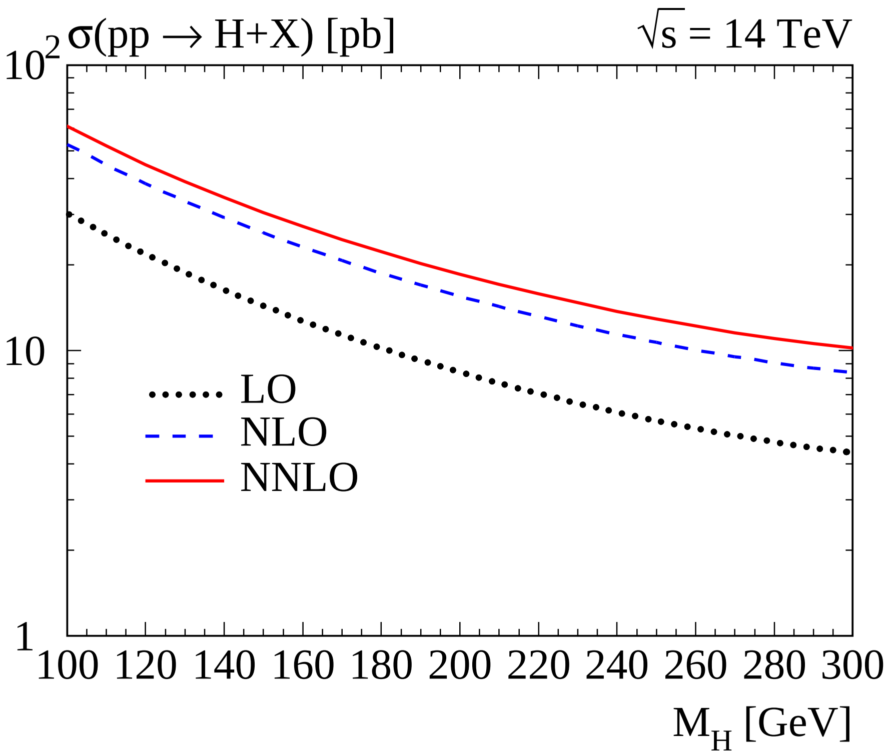

Figure 8: Cross section for Higgs procduction from gluon fusion at the LHC.

scheme for UV and collinear divergences.

5 Appendix

We define the 4-dimensional splitting functions as in (4.94) of the ESW book:

$$P_{qq}(z) = C_F\, p_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right] \quad (45)$$

$$P_{qg}(z) = T_R\, p_{qg}(z) = T_R \left[z^2 + (1-z)^2\right] \quad (46)$$

$$P_{gq}(z) = C_F\, p_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z}\right] \quad (47)$$

$$P_{gg}(z) = C_A\, p_{gg}(z) = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + b_0\, \delta(1-z)\,, \quad (48)$$

where $b_0 = 11/6\, C_A - 2n_f T_F/3$. We also define the following quantities as the extension of the splitting functions in Conventional Dimensional Regularization :

$$P_{ij}^{\mathrm{CDR}}(z) = P_{ij}(z) + \epsilon P_{ij}^{\epsilon}(z) \quad (49)$$

where

$$P_{qq}^{\epsilon}(z) = C_F\, p_{qq}^{\epsilon}(z) = -C_F(1-z) \quad (50)$$

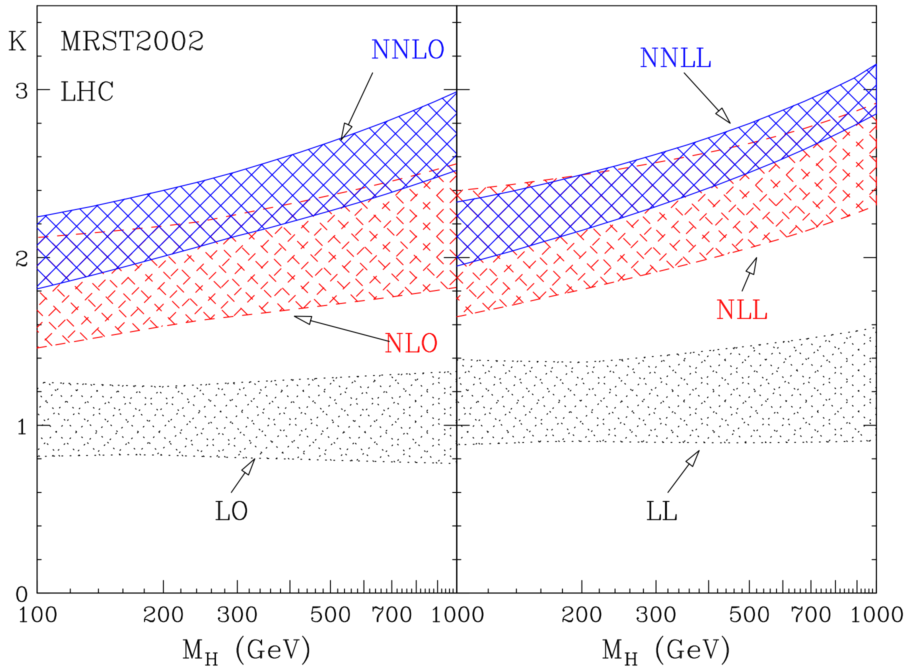

Figure 9: K-factors for Higgs production from gluon fusion at the LHC.

$$P^{\epsilon}_{qg}(z) = T_R\, p^{\epsilon}_{qg}(z) = -T_R 2z(1-z) \tag{51}$$
$$P^{\epsilon}_{gq}(z) = C_F\, p^{\epsilon}_{gq}(z) = -C_F z \tag{52}$$
$$P^{\epsilon}_{gg}(z) = 0 \tag{53}$$

Factorization of the collinear divergences is performed through the addition of the following counterterm for each parton in the initial state:

$$\mathrm{CT}^{\mathrm{CDR}} = \sigma_0^{\mathrm{CDR}} \frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}\frac{c_\Gamma}{\epsilon}P_{ij}(z)\right] \tag{54}$$

where $\sigma_0^{\mathrm{SCHEME}}$ is the LO cross section and its value depends on the scheme (see the example for Drell-Yan)]. In CDR, when there is a collinear divergence, the cross section behaves as

$$\sigma^{\mathrm{coll}}_{\mathrm{real}} \sim -\frac{1}{\epsilon}P^{\mathrm{CDR}}_{ij}(z)\sigma_0^{\mathrm{CDR}} + \text{other terms}\,. \tag{55}$$

Adding the counterterm (**??**), leaves a finite part

$$\sigma^{\overline{\mathrm{MS}}}_{\mathrm{real}} \sim -P^{\epsilon}_{ij}(z)\;(\sigma_0^{\mathrm{CDR}}|_{\epsilon\to 0}) + \text{other terms}\,. \tag{56}$$

6 Strong coupling renormalization

In this section we just state the rule. The $\overline{\mathrm{MS}}$ ultraviolet counterterm for the scattering amplitude at 1-loop is:

$$\frac{n}{\epsilon}\left[-b_0\,\frac{\alpha_S}{4\pi}\,c_\Gamma \mathcal{A}^{\mathrm{tree}}\right], \tag{57}$$

where $b_0 = 11/6\, C_A - 2 n_f T_F/3$ and n is the order of the tree-level amplitude in g_s. The above counterterm is defined in CDR.