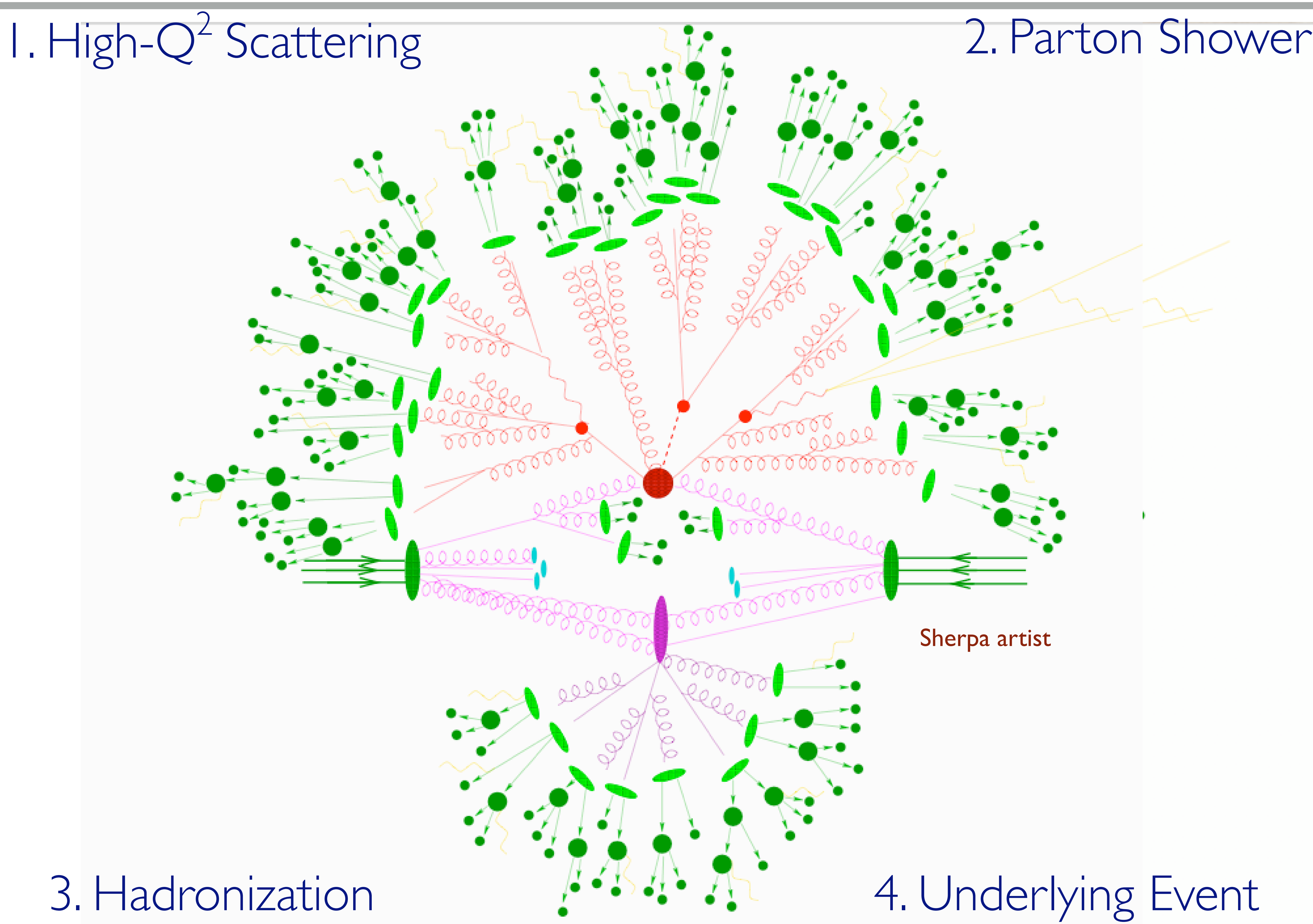
Parton Shower

Olivier Mattelaer
IPPP/Durham

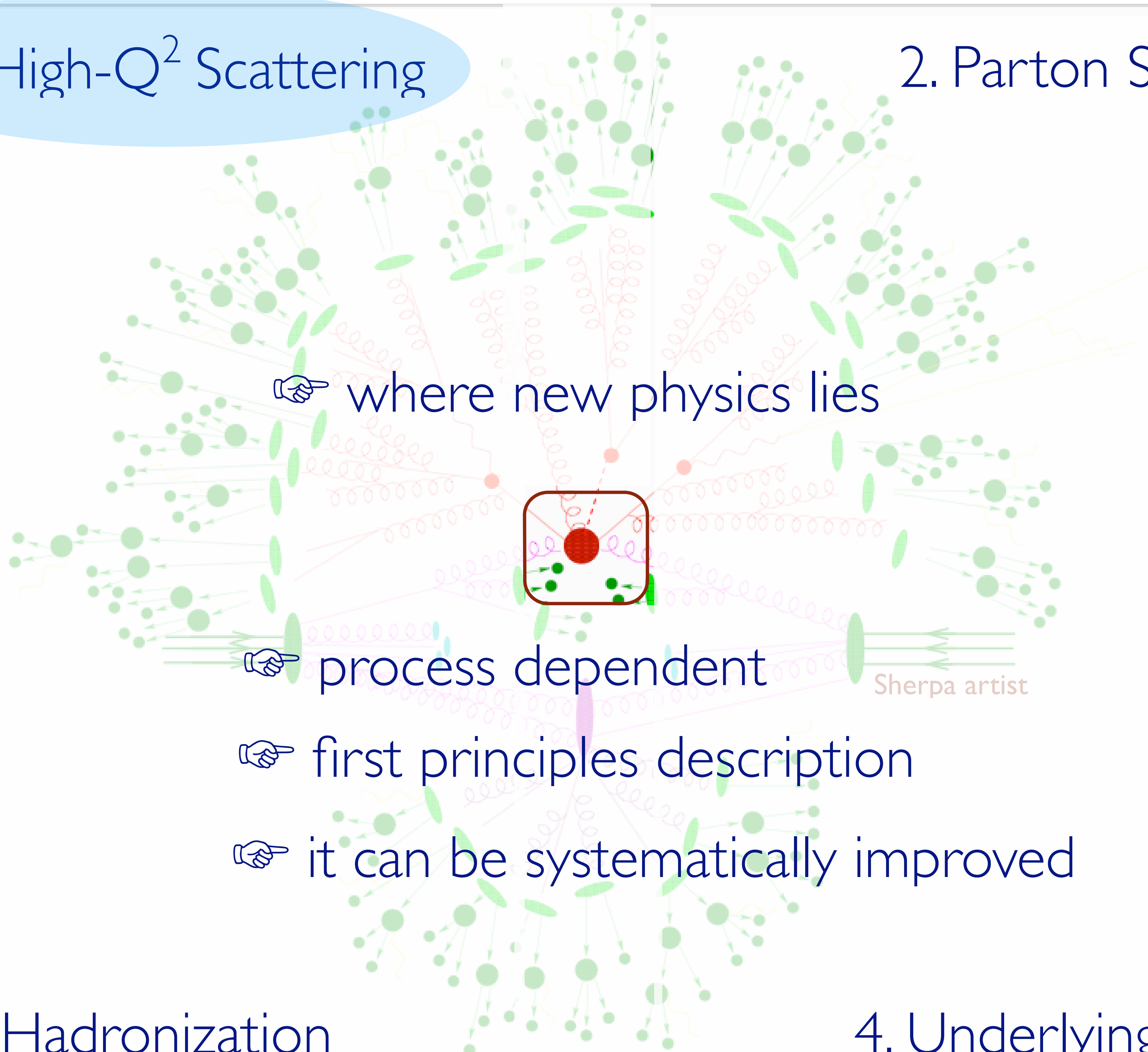
What are the MC for?



What are the MC for?

1. High-Q^2 Scattering

2. Parton Shower



- ☞ where new physics lies
- ☞ process dependent
- ☞ first principles description
- ☞ it can be systematically improved

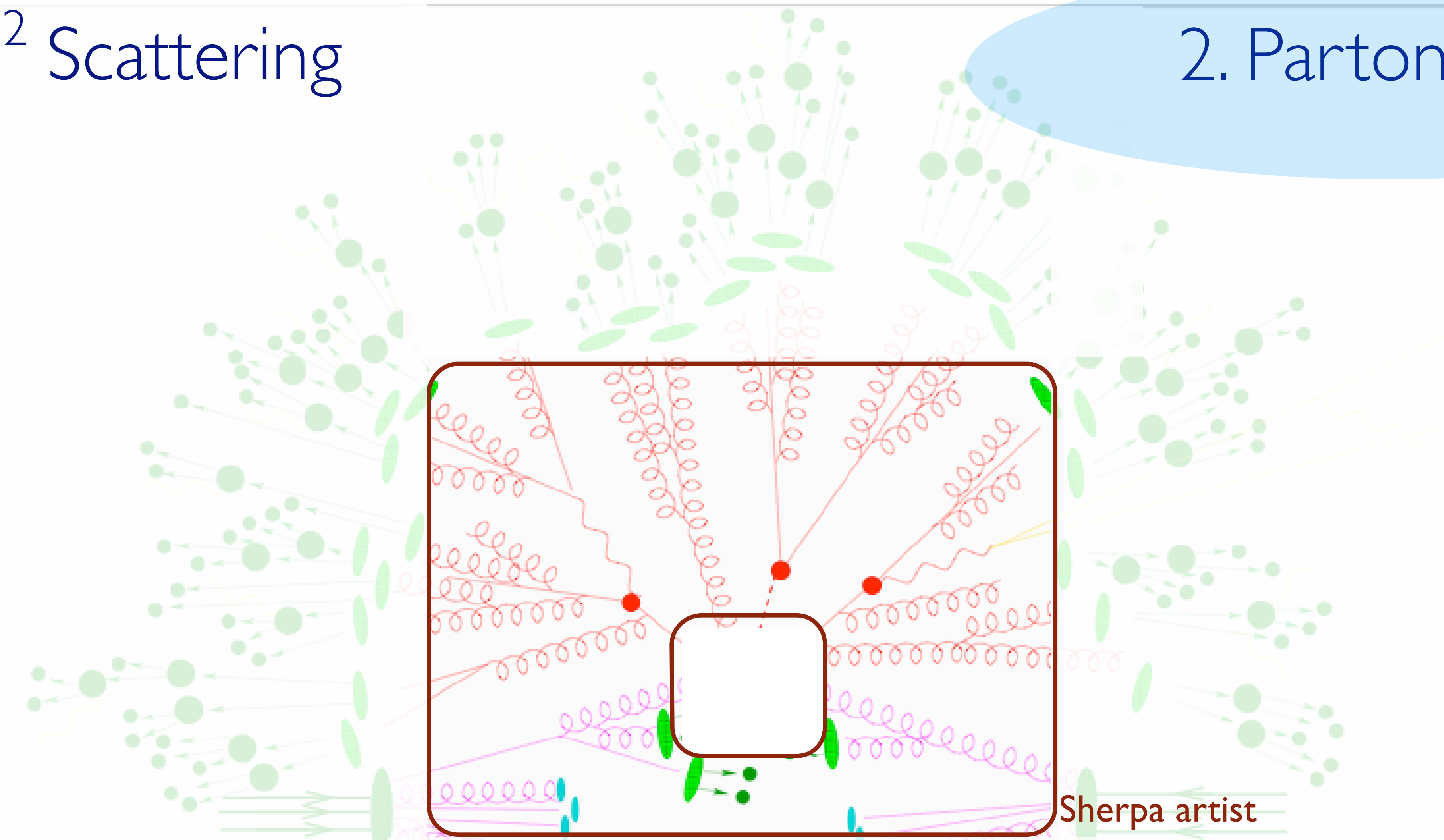
3. Hadronization

4. Underlying Event

What are the MC for?

1. High-Q^2 Scattering

2. Parton Shower



- ☞ QCD -"known physics"
- ☞ universal/ process independent
- ☞ first principles description

3. Hadronization

4. Underlying Event

Parton shower

- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation

Parton shower

- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation
- This effect should be **unitary:** the inclusive cross section shouldn't change when extra radiation is added

Parton shower

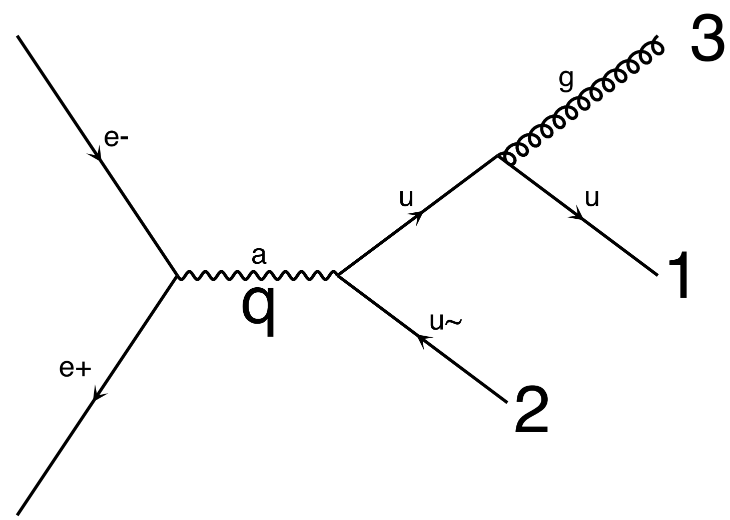
- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation
- This effect should be **unitary:** the inclusive cross section shouldn't change when extra radiation is added
- Remember that parton-level cross sections for a hard process are inclusive in anything else.
  E.g. for LO Drell-Yan production **all** radiation is included via PDFs (apart from non-perturbative power corrections)

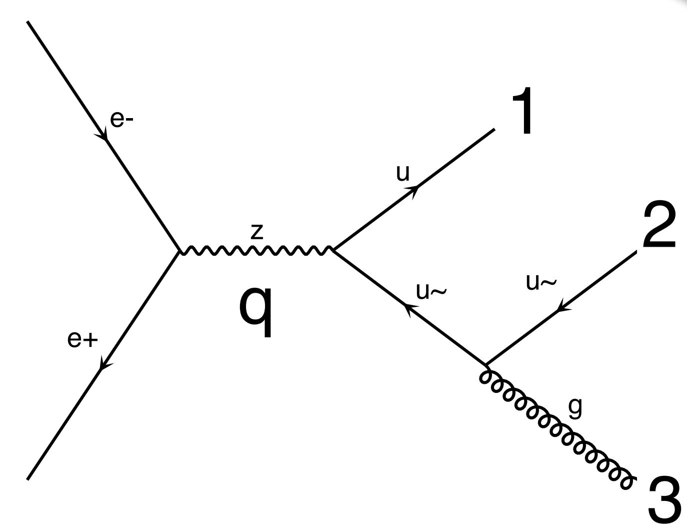
Parton shower

- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation
- This effect should be **unitary:** the inclusive cross section shouldn't change when extra radiation is added
- Remember that parton-level cross sections for a hard process are inclusive in anything else.
  E.g. for LO Drell-Yan production **all** radiation is included via PDFs (apart from non-perturbative power corrections)
- And finally we want to turn partons into hadrons (hadronization)....

First Example

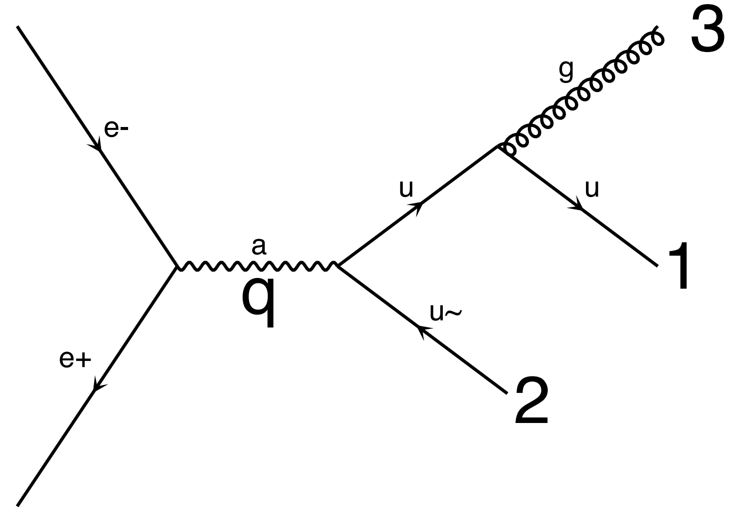
$$e^+e^- \to q\bar{q}g$$

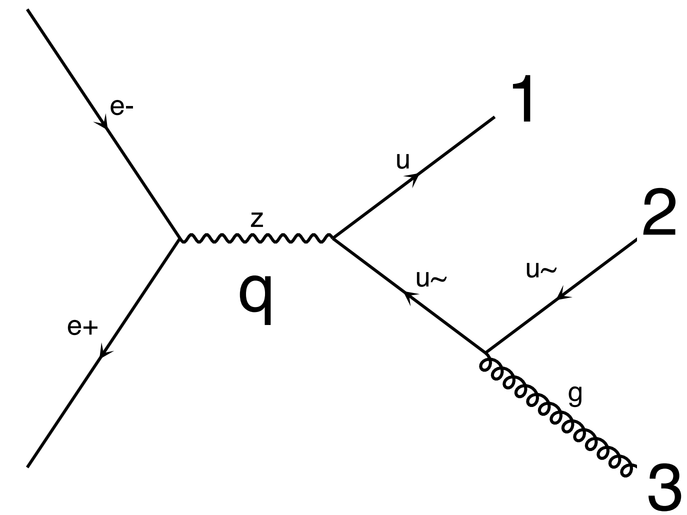




First Example

$$e^+e^- \to q\bar{q}g$$





$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$

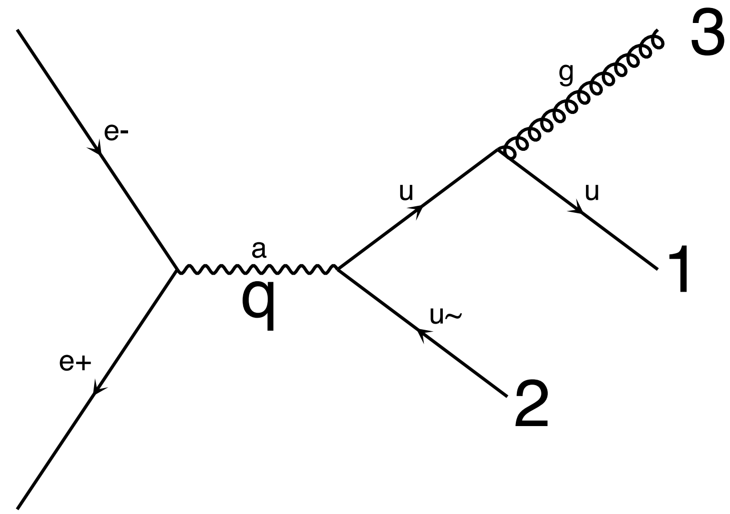
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$

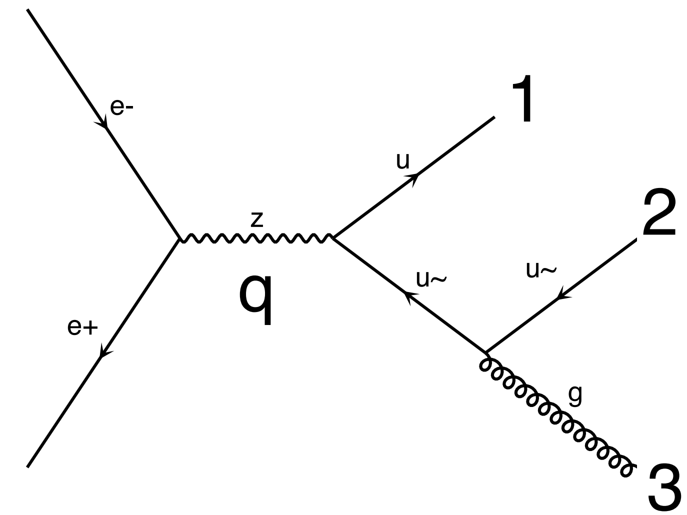
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$

$$x_1 + x_2 + x_3 = 2$$

First Example

$$e^+e^- \to q\bar{q}g$$





$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

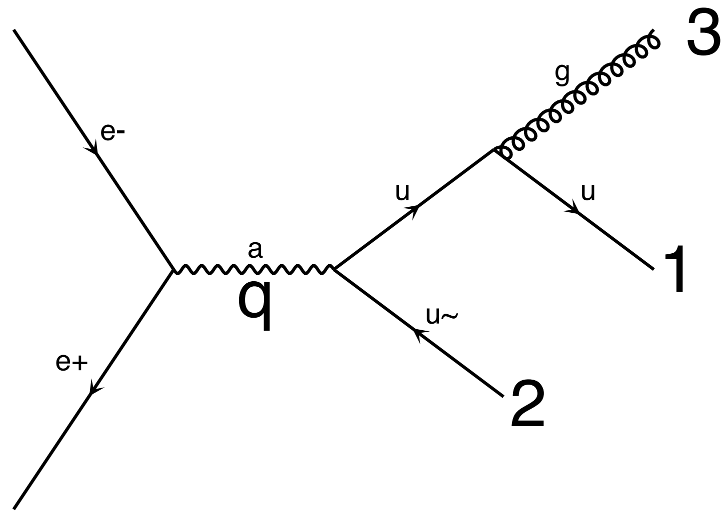
- Divergent at $x_1 = 1$ and $x_2 = 1$
  - Soft Divergencies
  - Collinear Divergencies

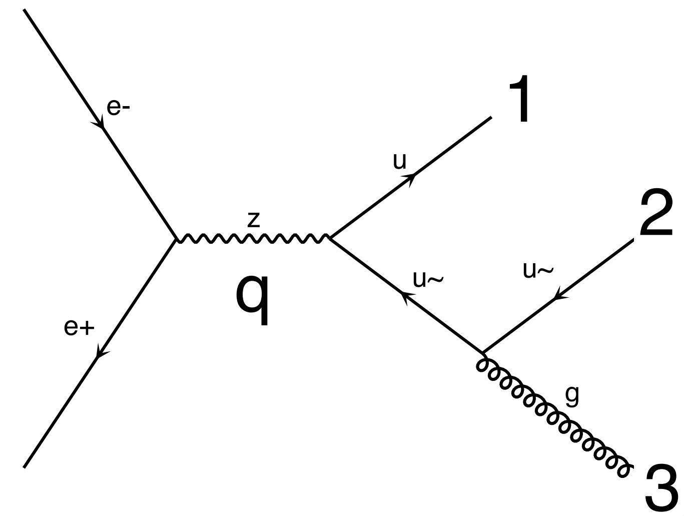
$$(1-x_1) = \frac{x_2 x_3}{2}(1 - cos\theta_{23})$$

$$(1-x_2) = \frac{x_1 x_3}{2}(1 - cos\theta_{13})$$

First Example

$$e^+e^- \to q\bar{q}g$$





$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$

$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$

$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$

$$x_1 + x_2 + x_3 = 2$$

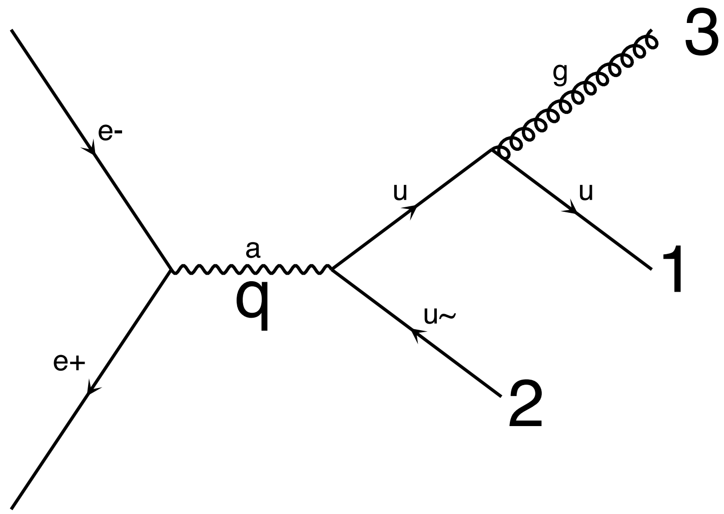
- Divergent at $x_1 = 1$ and $x_2 = 1$
  - Soft Divergencies
  - Collinear Divergencies

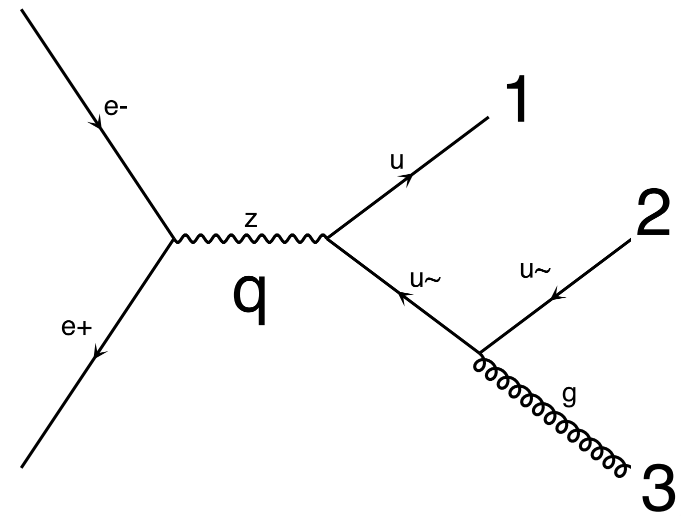
$$(1 - x_1) = \frac{x_2 x_3}{2}(1 - cos\theta_{23})$$

$$(1 - x_2) = \frac{x_1 x_3}{2}(1 - cos\theta_{13})$$

First Example

$$e^+e^- \to q\bar{q}g$$





$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

- Divergent at $x_1 = 1$ and $x_2 = 1$
  - Soft Divergencies
  - Collinear Divergencies

$$(1-x_1) = \frac{x_2 x_3}{2}(1 - cos\theta_{23})$$

$$(1-x_2) = \frac{x_1 x_3}{2}(1 - cos\theta_{13})$$

First Example

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

First Example

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

- Change the variable to x_3 and $\cos\theta_{13}$

$$\frac{d\sigma}{dx_3 d\cos\theta_{13}} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \left(\frac{2}{\sin^2\theta_{13}} \frac{1-(1-x_3)^2}{x_3} - x_3 \right)$$

First Example

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

- Change the variable to x_3 and $\cos\theta_{13}$

$$\frac{d\sigma}{dx_3 d\cos\theta_{13}} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \left(\frac{2}{\sin^2\theta_{13}} \frac{1-(1-x_3)^2}{x_3} - x_3 \right)$$

First Example

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

- Change the variable to x_3 and $\cos\theta_{13}$

$$\frac{d\sigma}{dx_3 d\cos\theta_{13}} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \left(\frac{2}{\sin^2\theta_{13}} \frac{1-(1-x_3)^2}{x_3} - x_3 \right)$$

First Example

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

- Change the variable to x_3 and $\cos\theta_{13}$

$$\frac{d\sigma}{dx_3 d\cos\theta_{13}} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \left(\frac{2}{\sin^2\theta_{13}} \frac{1-(1-x_3)^2}{x_3} - x_3 \right)$$

- Collinear limit
  - Split our integral in two

$$\frac{2\, dcos\theta_{13}}{sin^2\theta_{13}} = \frac{dcos\theta_{13}}{1 - cos\theta_{13}} + \frac{dcos\theta_{13}}{1 + cos\theta_{13}}$$
$$\approx \frac{d\cos\theta_{13}}{(1-\cos\theta_{13})} + \frac{d\cos\theta_{23}}{(1-\cos\theta_{23})}$$
$$\approx \frac{d\theta_{13}^2}{\theta_{13}^2} + \frac{d\theta_{23}^2}{\theta_{23}^2}$$

First Example

- Change the variable to x_3 and $\cos\theta_{13}$

$$\frac{d\sigma}{dx_3 d\cos\theta_{13}} = \sigma_0 C_F \frac{\alpha_s}{2\pi}\left(\frac{2}{\sin^2\theta_{13}}\frac{1-(1-x_3)^2}{x_3} - x_3\right)$$

- Collinear limit
  - Split our integral in two

$$\frac{2\,dcos\theta_{13}}{sin^2\theta_{13}} = \frac{dcos\theta_{13}}{1-cos\theta_{13}} + \frac{dcos\theta_{13}}{1+cos\theta_{13}}$$

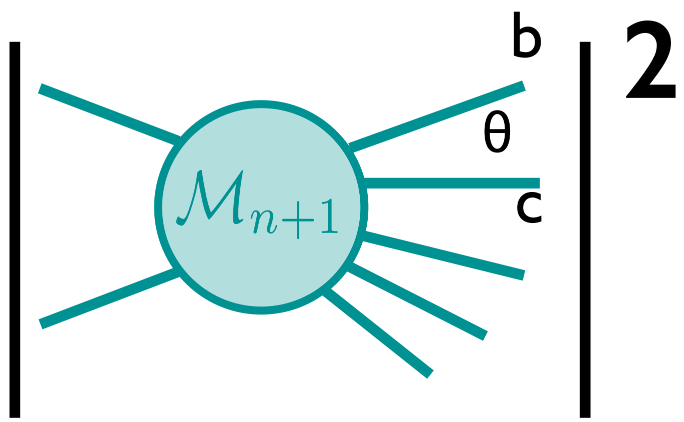
$$\approx \frac{d\cos\theta_{13}}{(1-\cos\theta_{13})} + \frac{d\cos\theta_{23}}{(1-\cos\theta_{23})}$$

$$\approx \frac{d\theta_{13}^2}{\theta_{13}^2} + \frac{d\theta_{23}^2}{\theta_{23}^2}$$

$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

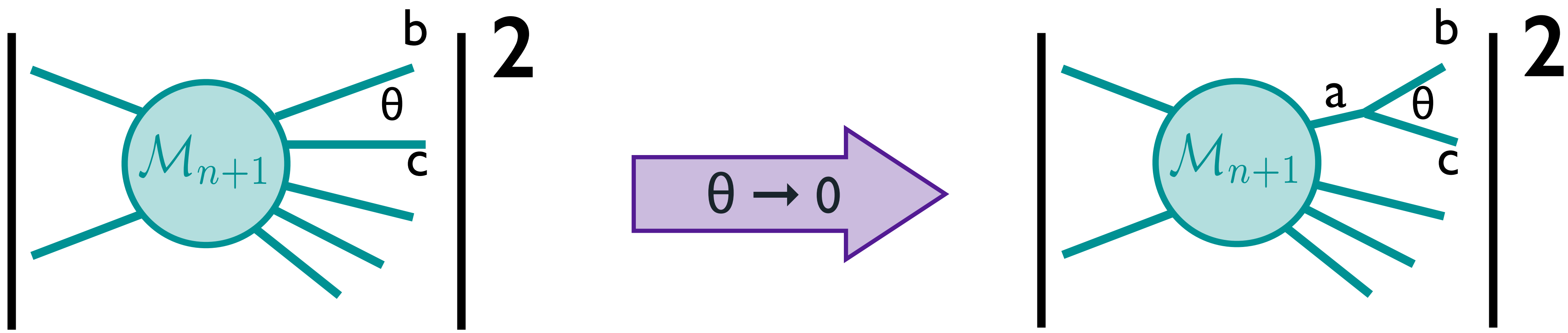
☞ z fraction of energy

☞ **Generic Formula**



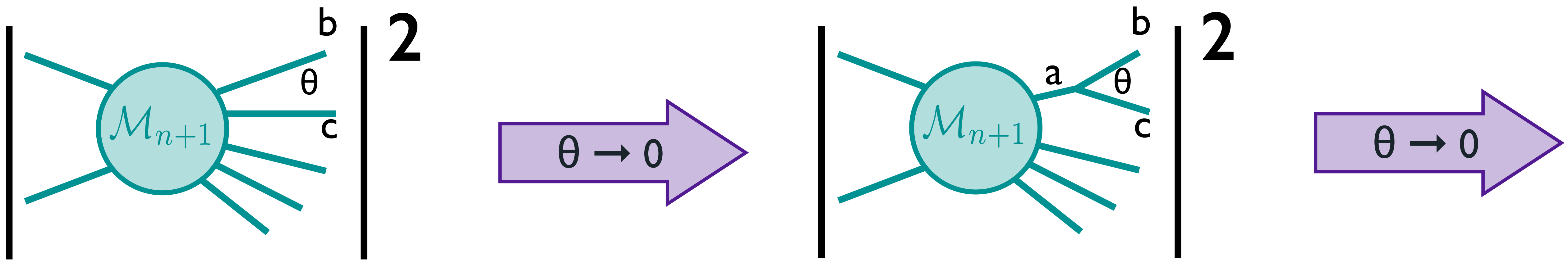
- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.

Collinear factorization



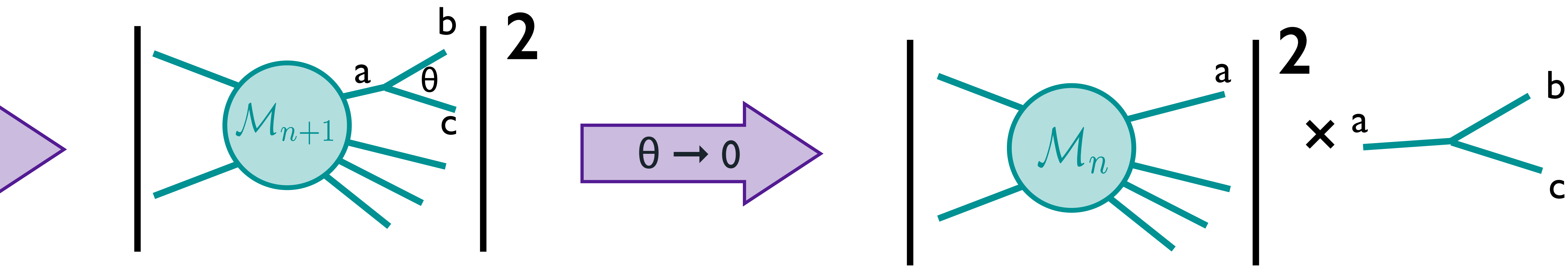
- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.

Collinear factorization



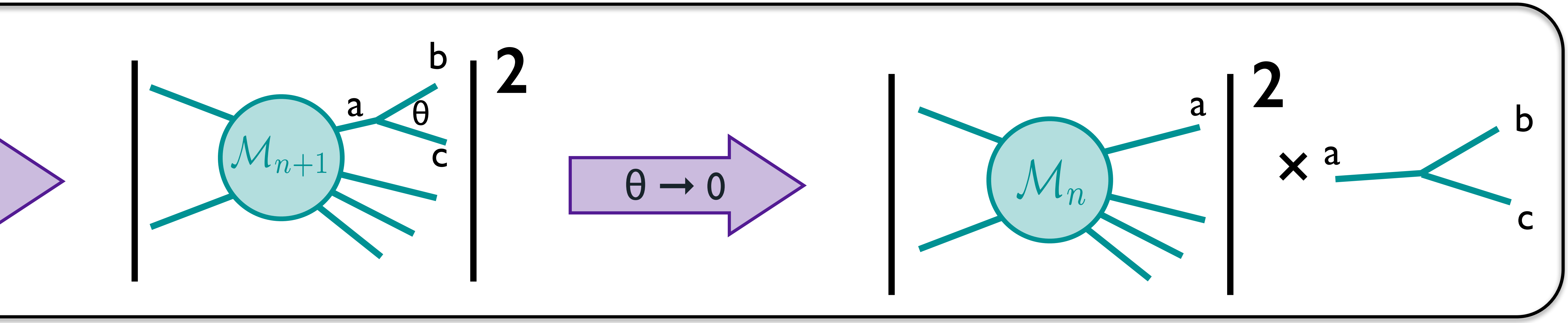
- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.

Collinear factorization



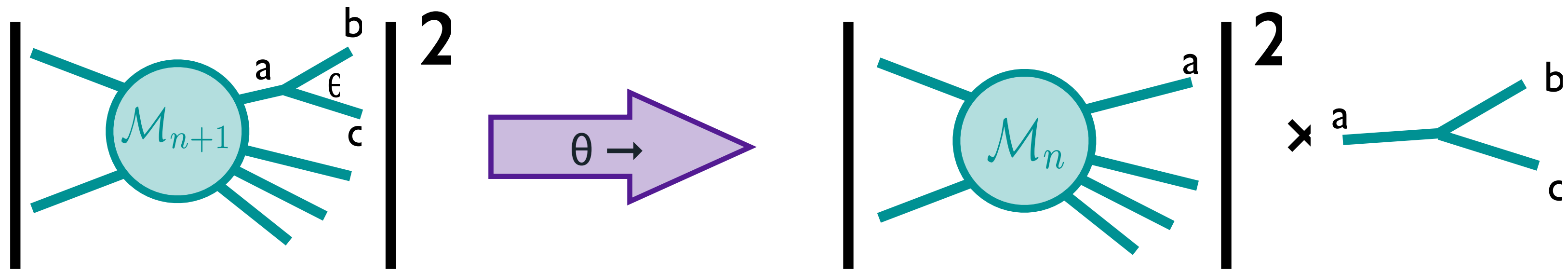
- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.

Collinear factorization



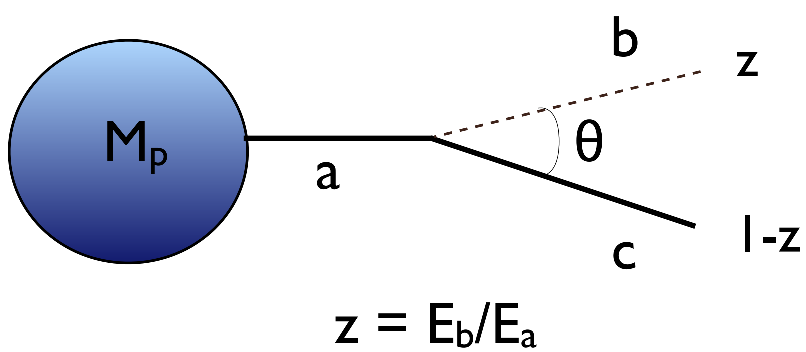
- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.
- The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability.

Collinear factorization

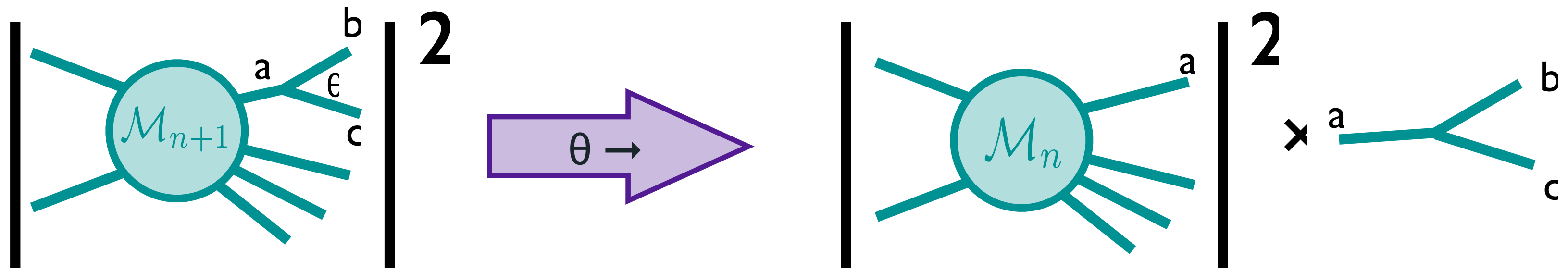


- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b+p_c)^2} \simeq \frac{1}{2E_bE_c(1-\cos\theta)} = \frac{1}{t}$$



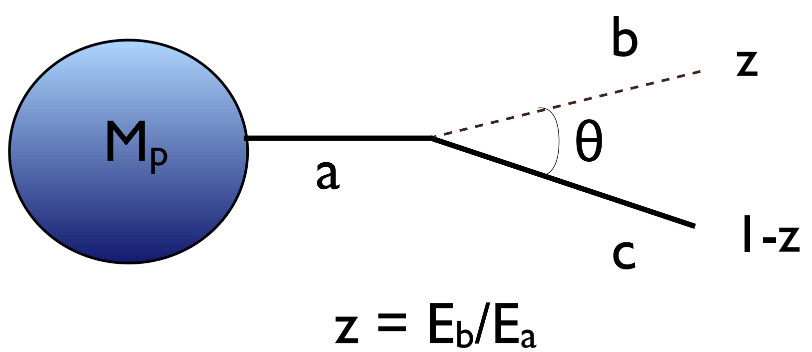
Collinear factorization



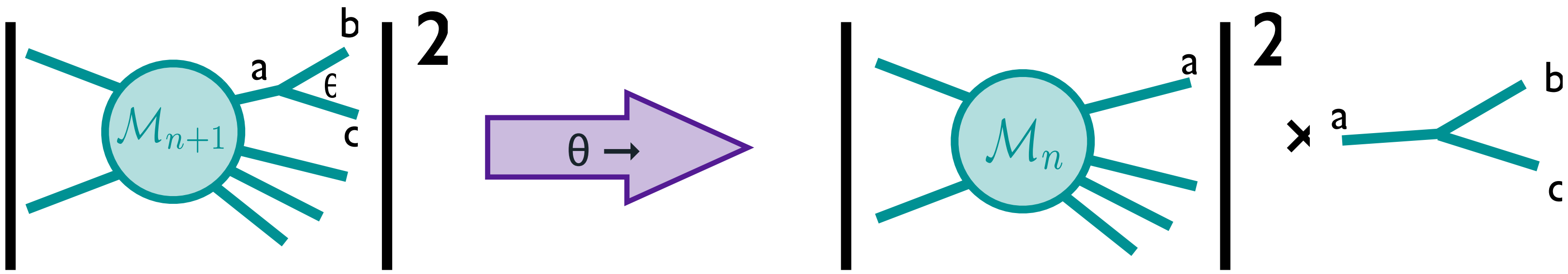
- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b+p_c)^2} \simeq \frac{1}{2E_bE_c(1-\cos\theta)} = \frac{1}{t}$$

soft

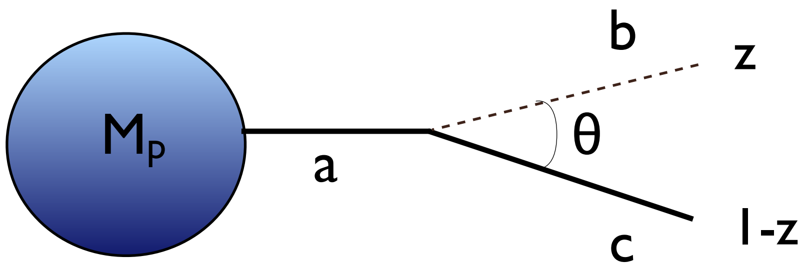


Collinear factorization



- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b+p_c)^2} \simeq \frac{1}{2E_bE_c(1-\cos\theta)} = \frac{1}{t}$$

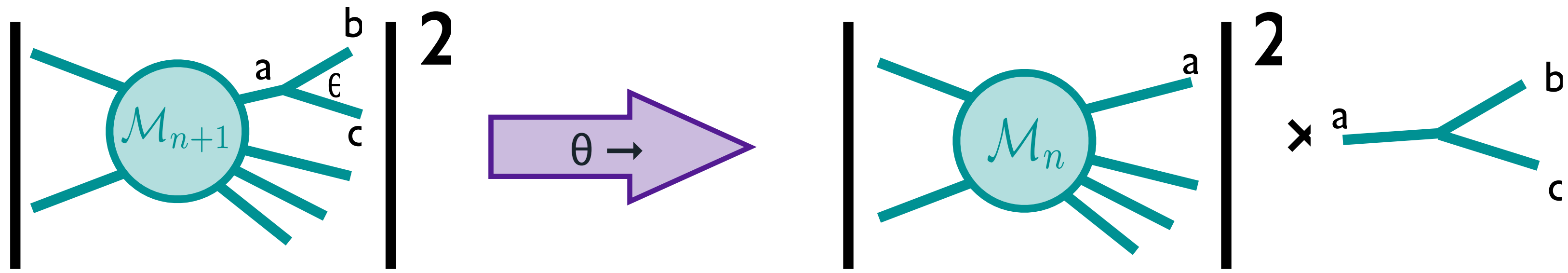


$z = E_b/E_a$

soft and collinear

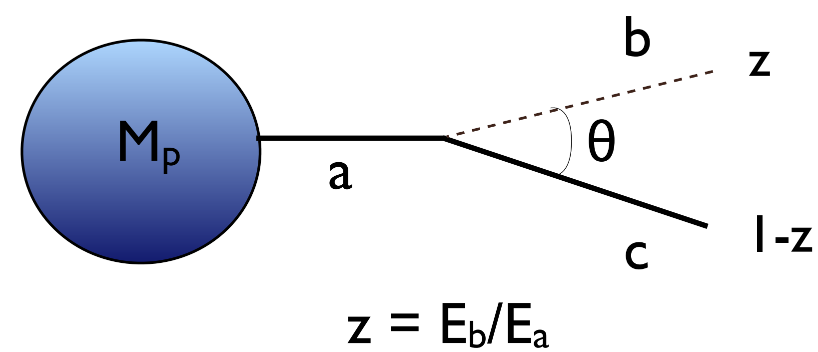
divergencies

Collinear factorization



- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b+p_c)^2} \simeq \frac{1}{2E_bE_c(1-\cos\theta)} = \frac{1}{t}$$



soft and **collinear**

divergencies

Collinear factorization:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \;\simeq\; |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

when θ is small.

Collinear factorization

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- **t** can be called the 'evolution variable' (will become clearer later): it can be the virtuality **m^2** of particle a or its **p_T^2** or **$E^2\theta^2$** …

$$d\theta^2/\theta^2 = dm^2/m^2 = dp_T^2/p_T^2$$

$$m^2 \simeq z(1-z)\theta^2 E_a^2$$
$$p_T^2 \simeq zm^2$$

  - It represents the hardness of the branching and tends to 0 in the collinear limit.
  - Different choice of 'evolution parameter' in different Parton-shower code

Collinear factorization

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- **z** is the "energy variable": it is defined to be the energy fraction taken by parton **b** from parton **a.** It represents the energy sharing between **b** and **c** and tends to 1 in the soft limit (parton c going soft)
- **Φ** is the azimuthal angle. It can be chosen to be the angle between the polarization of a and the plane of the branching.

Parton Shower basics

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

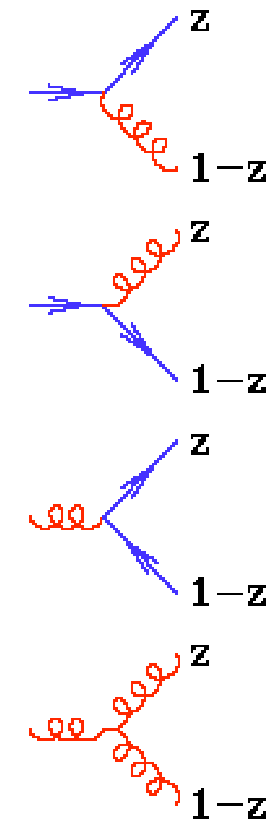
The spin averaged (unregulated) splitting functions for the various types of branching are (Altarelli-Parisi):

$$\hat{P}_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)}\right],$$

$$\hat{P}_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z}\right],$$

$$\hat{P}_{qg}(z) = T_R \left[z^2+(1-z)^2\right],$$

$$\hat{P}_{gg}(z) = C_A \left[\frac{z}{(1-z)} + \frac{1-z}{z} + z(1-z)\right].$$



$C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}.$

Parton Shower basics

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

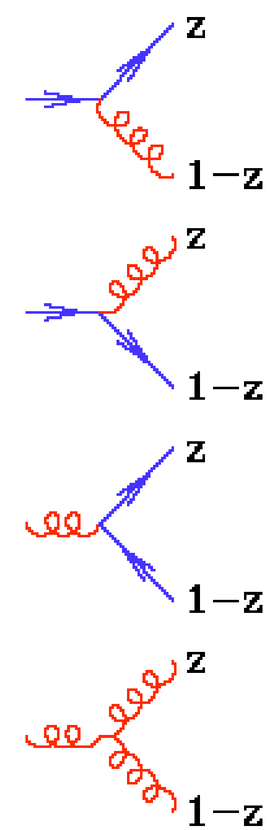
The spin averaged (unregulated) splitting functions for the various types of branching are (Altarelli-Parisi):

$$\hat{P}_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)}\right],$$

$$\hat{P}_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z}\right],$$

$$\hat{P}_{qg}(z) = T_R \left[z^2+(1-z)^2\right],$$

$$\hat{P}_{gg}(z) = C_A \left[\frac{z}{(1-z)} + \frac{1-z}{z} + z(1-z)\right].$$



$C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}.$

Comments:

* Gluons radiate the most
* There are soft divergences in z=1 and z=0.
* P_{qg} has no soft divergences.

Argument of α_S

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Each choice of argument for α_S is equally acceptable at the leading-logarithmic accuracy. However, there is a choice that allows one to resum certain classes of subleading logarithms.

- The higher order corrections to the partons splittings imply that the splitting kernels should be modified: $P_{a \longrightarrow bc}(z) \longrightarrow P_{a \longrightarrow bc}(z) + \alpha_s P'_{a \longrightarrow bc}(z)$

For $g \longrightarrow gg$ branchings $P'_{a \longrightarrow bc}(z)$ diverges as $-b_0 \log[z(1-z)]\ P_{a \longrightarrow bc}(z)$
(just z or 1-z if quark is present)

- Recall the one-loop running of the strong coupling:

$$\alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1 + \alpha_S(\mu^2) b_0 \log \frac{Q^2}{\mu^2}} \sim \alpha_S(\mu^2)\left(1 - \alpha_S(\mu^2) b_0 \log \frac{Q^2}{\mu^2}\right)$$

- We can therefore include the $P'(z)$ terms by choosing $p_T^2 \sim z(1-z)Q^2$ as argument of α_S:

$$\begin{aligned}\alpha_S(Q^2)\left(P_{a\to bc}(z) + \alpha_S(Q^2) P'_{a\to bc}\right) &= \alpha_S(Q^2)\left(1 - \alpha_S(Q^2) b \log z(1-z)\right) P_{a\to bc}(z) \\ &\sim \alpha_S(z(1-z)Q^2) P_{a\to bc}(z)\end{aligned}$$

Argument of α_S

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Each choice of argument for α_S is equally acceptable at the leading-logarithmic accuracy. However, there is a choice that allows one to resum certain classes of subleading logarithms.

- The higher order corrections to the partons splittings imply that the splitting kernels should be modified: $P_{a\longrightarrow bc}(z) \longrightarrow P_{a\longrightarrow bc}(z) + \alpha_s P'_{a\longrightarrow bc}(z)$

For $g \longrightarrow gg$ branchings $P'_{a\longrightarrow bc}(z)$ diverges as $-b_0 \log[z(1-z)]\ P_{a\longrightarrow bc}(z)$ (just z or 1-z if quark is present)

- Recall the one-loop running of the strong coupling:

$$\alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1+\alpha_S(\mu^2) b_0 \log\frac{Q^2}{\mu^2}} \sim \alpha_S(\mu^2)\left(1-\alpha_S(\mu^2) b_0 \log\frac{Q^2}{\mu^2}\right)$$

- We can therefore include the $P'(z)$ terms by choosing $p_T^2 \sim z(1-z)Q^2$ as argument of α_S:

$$\begin{aligned}\alpha_S(Q^2)\left(P_{a\to bc}(z) + \alpha_S(Q^2) P'_{a\to bc}\right) &= \alpha_S(Q^2)\left(1-\alpha_S(Q^2) b \log z(1-z)\right) P_{a\to bc}(z)\\ &\sim \alpha_S(\quad t \quad) P_{a\to bc}(z)\end{aligned}$$

To Remember

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \;\simeq\; |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_\mathrm{S}}{2\pi} P_{a\to bc}(z)$$

To Remember

Collinear Limit

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

To Remember

Collinear Limit

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- **t** is the evolution parameter (control the collinear behaviour)

To Remember

Collinear Limit

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- **t** is the evolution parameter (control the collinear behaviour)
- **z** is the energy sharing variable

To Remember

Collinear Limit

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

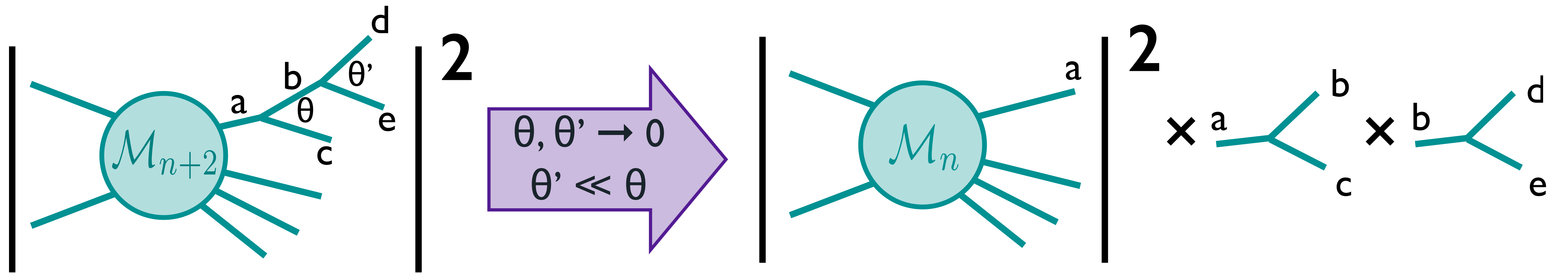
- **t** is the evolution parameter (control the collinear behaviour)
- **z** is the energy sharing variable
- **alpha_s** need to be evaluated at the scale t

To Remember

Collinear Limit

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- **t** is the evolution parameter (control the collinear behaviour)
- **z** is the energy sharing variable
- **alpha_s** need to be evaluated at the scale t
- **P** is the splitting Kernel (control the soft behaviour)

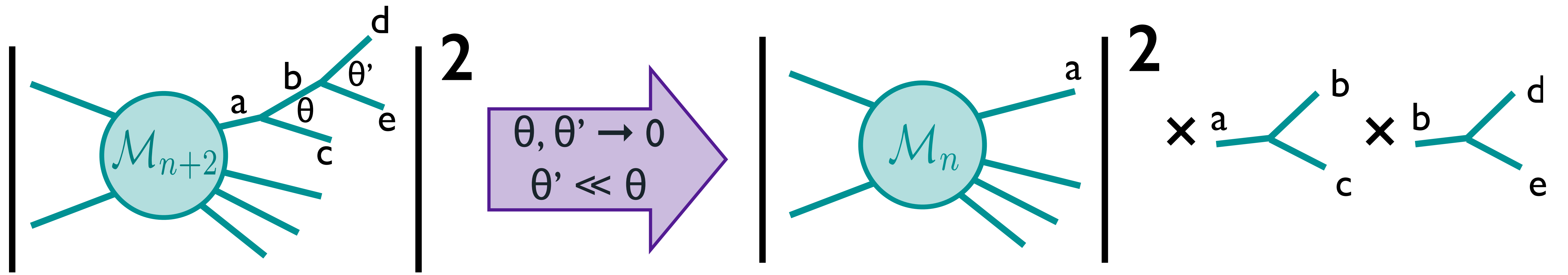


- Now consider $\mathbf{M_{n+1}}$ as the new core process and use the recipe we used for the first emission in order to get the dominant contribution to the (n+2)-body cross section: add a new branching at angle much smaller than the previous one:

$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) \times \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_S}{2\pi} P_{b\to de}(z')$$

- This can be done for an arbitrary number of emissions. The recipe to get the leading collinear singularity is thus cast in the form of an iterative sequence of emissions whose probability does not depend on the past history of the system: a 'Markov chain'. No interference!!!

Multiple emission



- The dominant contribution comes from the region where the subsequently emitted partons satisfy the strong ordering requirement: $\theta \gg \theta' \gg \theta''$...

  For the rate for multiple emission we get

$$\sigma_{n+k} \propto \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \cdots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

  where Q is a typical hard scale and Q_0 is a small infrared cutoff that separates perturbative from non perturbative regimes.

- Each power of α_s comes with a logarithm. The logarithm can be easily large, and therefore it can lead to a breakdown of perturbation theory.

Sudakov Form Factor

- What is the probability of no emission?

Sudakov Form Factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz \hat{P}(z)$$

Sudakov Form Factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i} \frac{\alpha_s}{2\pi} \int dz \hat{P}(z)$$

- So the probability of no emission between two scales:

Sudakov Form Factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz\hat{P}(z)$$

- So the probability of no emission between two scales:

$$P_{no-branching}(Q^2, t) = \lim_{N\to\infty}\prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)$$

Sudakov Form Factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz\hat{P}(z)$$

- So the probability of no emission between two scales:

$$P_{no-branching}(Q^2, t) = \lim_{N\to\infty}\prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)$$

$$\simeq \lim_{N\to\infty} e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)}$$

Sudakov Form Factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz\hat{P}(z)$$

- So the probability of no emission between two scales:

$$P_{no-branching}(Q^2, t) = \lim_{N\to\infty}\prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)$$

$$\simeq \lim_{N\to\infty} e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)}$$

$$\simeq e^{-\int_t^{Q^2}\frac{dt'}{t'}\int dz\frac{\alpha_S}{2\pi}\hat{P}(z)} \equiv e^{-\int_t^{Q^2}dp(t')}$$

Sudakov Form Factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz\hat{P}(z)$$

- So the probability of no emission between two scales:

$$P_{no-branching}(Q^2,t) = \lim_{N\to\infty}\prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)$$

$$\simeq \lim_{N\to\infty} e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)}$$

Sudakov form factor

$\Delta(Q^2,t)$

$$\simeq e^{-\int_t^{Q^2}\frac{dt'}{t'}\int dz\frac{\alpha_S}{2\pi}\hat{P}(z)} \equiv e^{-\int_t^{Q^2}dp(t')}$$

Sudakov Form Factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz \hat{P}(z)$$

- So the probability of no emission between two scales:

$$P_{no-branching}(Q^2, t) = \lim_{N\to\infty} \prod_{i=0}^{N} \left(1 - \frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz \hat{P}(z)\right)$$

$$\simeq \lim_{N\to\infty} e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz \hat{P}(z)\right)}$$

Sudakov form factor

$$\Delta(Q^2, t) \qquad \simeq e^{-\int_t^{Q^2} \frac{dt'}{t'}\int dz \frac{\alpha_S}{2\pi}\hat{P}(z)} \equiv e^{-\int_t^{Q^2} dp(t')}$$

Sudakov Form Factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz\hat{P}(z)$$

- So the probability of no emission between two scales:

$$P_{no-branching}(Q^2,t) = \lim_{N\to\infty}\prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)$$

$$\simeq \lim_{N\to\infty} e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)}$$

Sudakov form factor

$\Delta(Q^2,t)$

$$\simeq e^{-\int_t^{Q^2}\frac{dt'}{t'}\int dz\frac{\alpha_S}{2\pi}\hat{P}(z)} \equiv e^{-\int_t^{Q^2}dp(t')}$$

Sudakov Form Factor

- What is the probability of no emission?

$$\mathcal{P}_{\mathrm{non-branching}}(t_i) = 1 - \mathcal{P}_{\mathrm{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz \hat{P}(z)$$

- So the probability of no emission between two scales:

$$P_{no-branching}(Q^2,t) = \lim_{N\to\infty}\prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz \hat{P}(z)\right)$$

$$\simeq \lim_{N\to\infty} e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz \hat{P}(z)\right)}$$

Sudakov form factor

$$\Delta(Q^2,t) \simeq e^{-\int_t^{Q^2}\frac{dt'}{t'}\int dz \frac{\alpha_S}{2\pi}\hat{P}(z)} \equiv e^{-\int_t^{Q^2} dp(t')}$$

➡Property: $\Delta(A,B) = \Delta(A,C)\,\Delta(C,B)$

Parton shower

- The Sudakov form factor is the heart of the parton shower. It gives the probability that a parton does not branch between two scales
- Using this no-emission probability the branching tree of a parton is generated.
- Define **dP_k** as the probability for k ordered splittings from leg a at given scales

$$
\begin{aligned}
dP_1(t_1) &= \Delta(Q^2, t_1)\ dp(t_1)\Delta(t_1, Q_0^2), \\
dP_2(t_1, t_2) &= \Delta(Q^2, t_1)\ dp(t_1)\ \Delta(t_1, t_2)\ dp(t_2)\ \Delta(t_2, Q_0^2)\Theta(t_1 - t_2), \\
\ldots &= \ldots \\
dP_k(t_1, \ldots, t_k) &= \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)
\end{aligned}
$$

- **Q_0^2** is the hadronization scale (~1 GeV). Below this scale we do not trust the perturbative description for parton splitting anymore.

Unitarity

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

Unitarity

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, ...$$

Unitarity

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2)\frac{1}{k!}\left[\int_{Q_0^2}^{Q^2} dp(t)\right]^k, \quad \forall k = 0, 1, ...$$

- Summing over all number of emissions

Unitarity

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, ...$$

- Summing over all number of emissions

$$\sum_{k=0}^{\infty} P_k = \Delta(Q^2, Q_0^2) \sum_{k=0}^{\infty} \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k = \Delta(Q^2, Q_0^2) \exp \left[\int_{Q_0^2}^{Q^2} dp(t) \right] = 1$$

Unitarity

$$dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, ...$$

- Summing over all number of emissions

$$\sum_{k=0}^{\infty} P_k = \Delta(Q^2, Q_0^2) \sum_{k=0}^{\infty} \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k = \Delta(Q^2, Q_0^2) \exp \left[\int_{Q_0^2}^{Q^2} dp(t) \right] = 1$$

- Hence, the total probability is conserved

singularities

- We have shown that the showers is unitary. However, how are the IR divergences cancelled explicitly? Let's show this for the first emission:
  Consider the contributions from (exactly) 0 and 1 emissions from leg a:

$$\frac{d\sigma}{\sigma_n} = \Delta(Q^2, Q_0^2) + \Delta(Q^2, Q_0^2) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Expanding to first order in α_s gives

$$\frac{d\sigma}{\sigma_n} \simeq 1 - \sum_{bc} \int_{Q_0^2}^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) + \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Same structure of the two latter terms, with opposite signs: cancellation of divergences between the approximate virtual and approximate real emission cross sections.

- The probabilistic interpretation of the shower ensures that infrared divergences will cancel for each emission.

Final-state parton showers

Final-state parton showers

With the Sudakov form factor, we can now implement a final-state parton shower in a Monte Carlo event generator!

Final-state parton showers

With the Sudakov form factor, we can now implement a final-state parton shower in a Monte Carlo event generator!

1. Start the evolution at the virtual mass scale t_0 (e.g. the mass of the decaying particle) and momentum fraction $z_0 = 1$

Final-state parton showers

With the Sudakov form factor, we can now implement a final-state parton shower in a Monte Carlo event generator!

1. Start the evolution at the virtual mass scale t_0 (e.g. the mass of the decaying particle) and momentum fraction $z_0 = 1$

2. Given a virtual mass scale t_i and momentum fraction x_i at some stage in the evolution, generate the scale of the next emission t_{i+1} according to the Sudakov probability $\Delta(t_i,t_{i+1})$ by solving
$\Delta(t_{i+1},t_i) = R$
where R is a random number (uniform on [0, 1]).

Final-state parton showers

With the Sudakov form factor, we can now implement a final-state parton shower in a Monte Carlo event generator!

1. Start the evolution at the virtual mass scale t_0 (e.g. the mass of the decaying particle) and momentum fraction $z_0 = 1$
2. Given a virtual mass scale t_i and momentum fraction x_i at some stage in the evolution, generate the scale of the next emission t_{i+1} according to the Sudakov probability $\Delta(t_i, t_{i+1})$ by solving
   $\Delta(t_{i+1}, t_i) = R$
   where R is a random number (uniform on [0, 1]).
3. If $t_{i+1} < t_{cut}$ it means that the shower has finished.

Final-state parton showers

With the Sudakov form factor, we can now implement a final-state parton shower in a Monte Carlo event generator!

1. Start the evolution at the virtual mass scale t_0 (e.g. the mass of the decaying particle) and momentum fraction $z_0 = 1$

2. Given a virtual mass scale t_i and momentum fraction x_i at some stage in the evolution, generate the scale of the next emission t_{i+1} according to the Sudakov probability $\Delta(t_i,t_{i+1})$ by solving
$\Delta(t_{i+1},t_i) = R$
where R is a random number (uniform on [0, 1]).

3. If $t_{i+1} < t_{\mathrm{cut}}$ it means that the shower has finished.

4. Otherwise, generate $z = z_i/z_{i+1}$ with a distribution proportional to $(\alpha_s/2\pi)P(z)$, where $P(z)$ is the appropriate splitting function.

Final-state parton showers

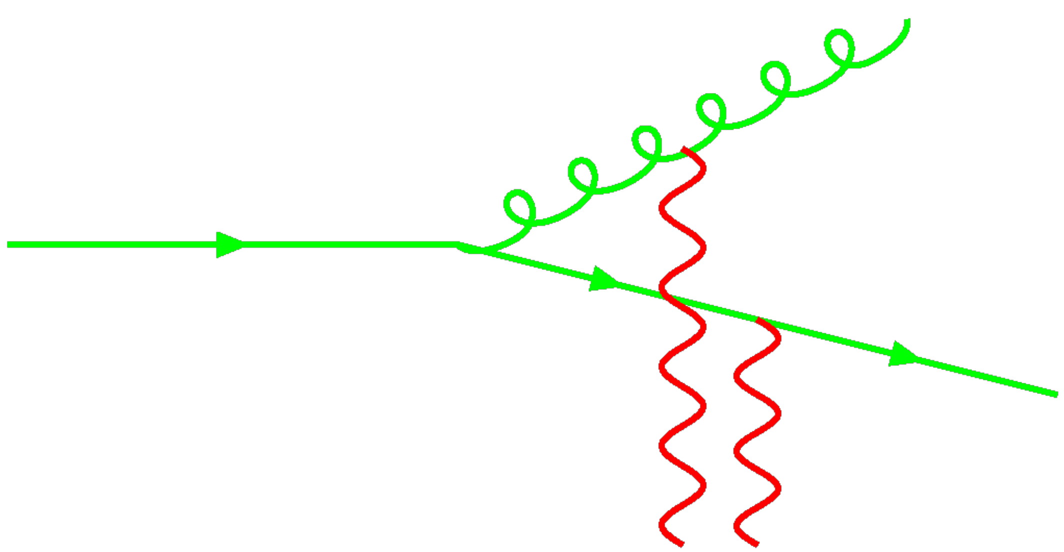
With the Sudakov form factor, we can now implement a final-state parton shower in a Monte Carlo event generator!

1. Start the evolution at the virtual mass scale t_0 (e.g. the mass of the decaying particle) and momentum fraction $z_0 = 1$
2. Given a virtual mass scale t_i and momentum fraction x_i at some stage in the evolution, generate the scale of the next emission t_{i+1} according to the Sudakov probability $\Delta(t_i,t_{i+1})$ by solving
   $\Delta(t_{i+1},t_i) = R$
   where R is a random number (uniform on [0, 1]).
3. If $t_{i+1} < t_{cut}$ it means that the shower has finished.
4. Otherwise, generate $z = z_i/z_{i+1}$ with a distribution proportional to $(\alpha_s/2\pi)P(z)$, where $P(z)$ is the appropriate splitting function.
5. For each emitted particle, iterate steps 2-4 until branching stops.

Soft Limit

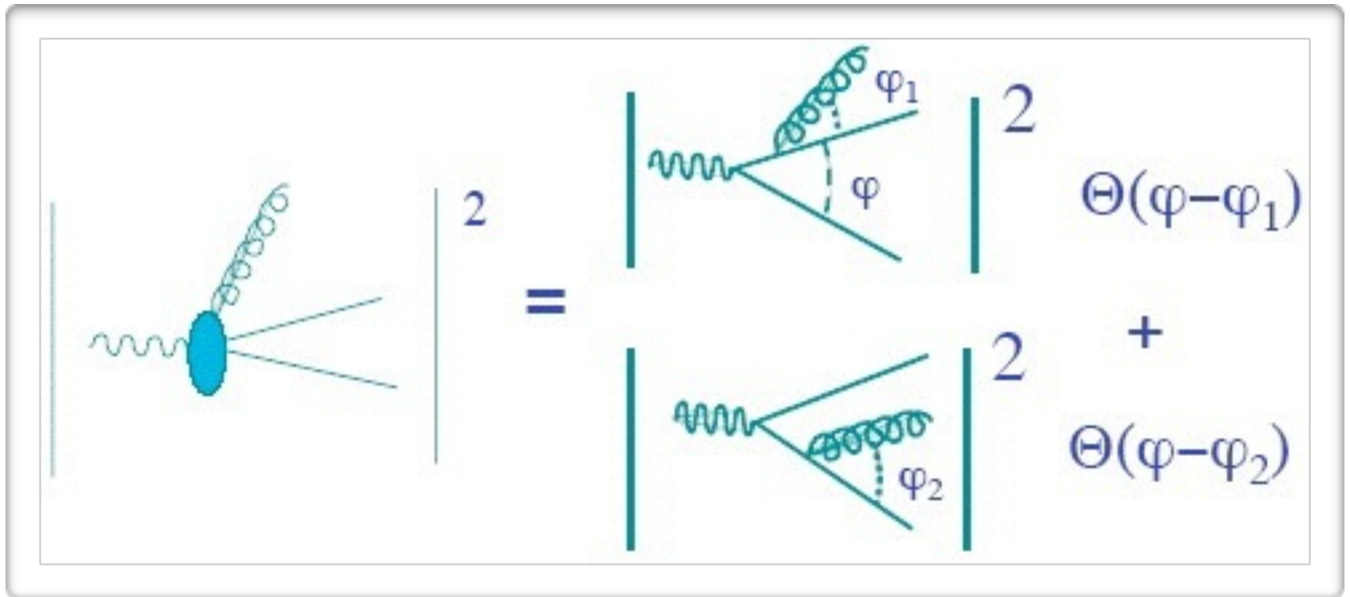
$$\Delta(Q^2,t) = \exp\left[-\sum_{bc}\int_t^{Q^2}\frac{dt'}{t'}dz\frac{d\phi}{2\pi}\frac{\alpha_S}{2\pi}P_{a\to bc}(z)\right]$$

- There is a lot of freedom in the choice of evolution parameter t. It can be the virtuality m^2 of particle a or its p_T^2 or $E^2\theta^2$ ... For the collinear limit they are all equivalent
- However, in the soft limit ($z \longrightarrow 0,1$) they behave differently
- Can we chose it such that we get the correct soft limit?
- Soft gluon comes from the full event!

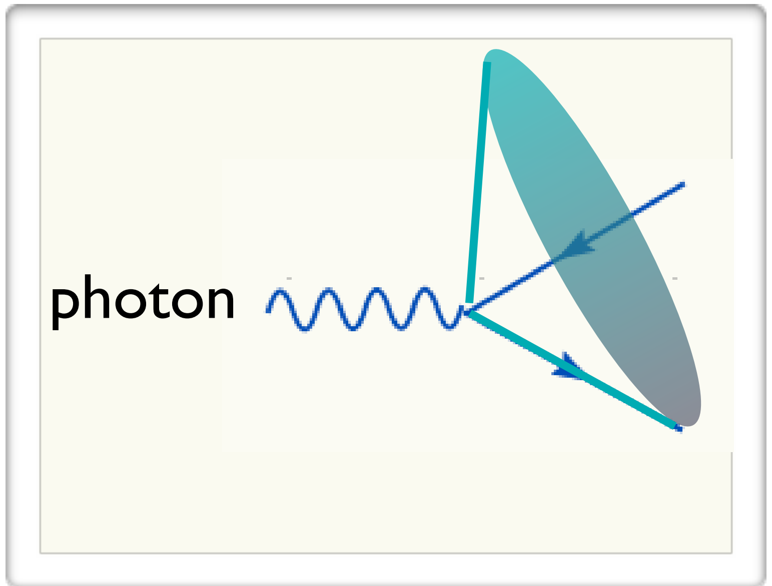


- Quantum Interference

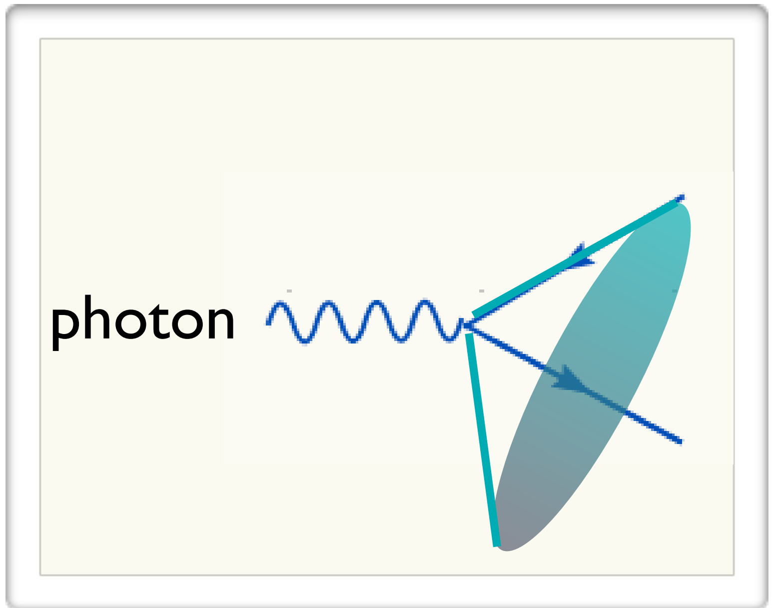
Angular ordering



Radiation inside cones around the original partons is allowed (and described by the eikonal approximation), outside the cones it is zero (after averaging over the azimuthal angle)



+



To Remember

- Sudakov Form-Factor: Probablility of No-emission between two scale.

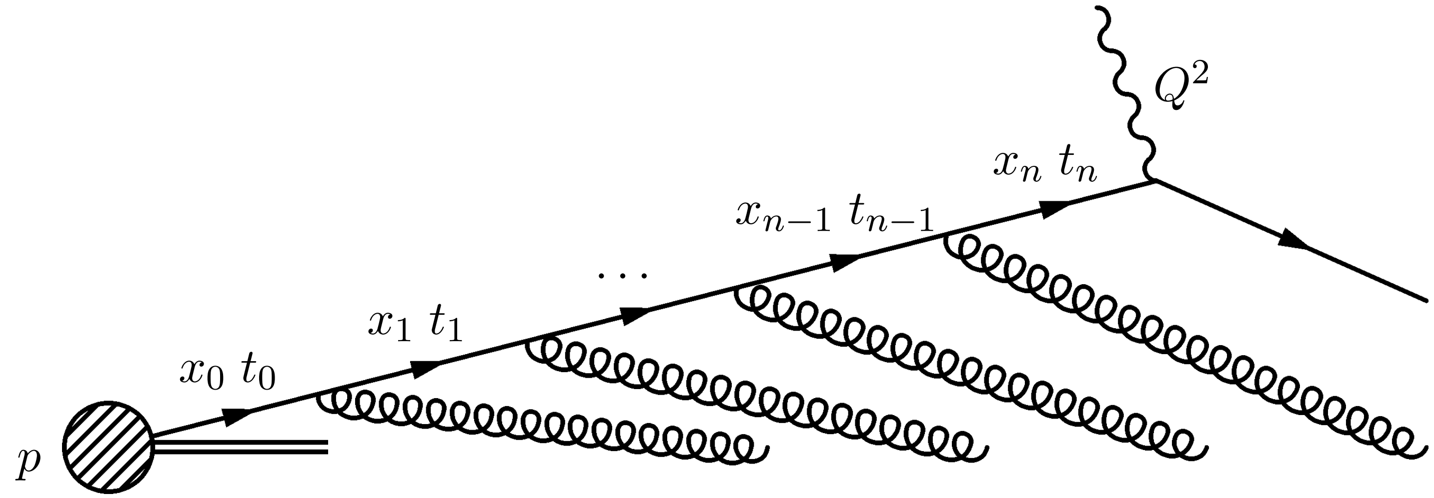
$$\Delta(Q^2, t) \simeq e^{-\int_t^{Q^2} \frac{dt'}{t'} dz \frac{\alpha_S}{2\pi} \hat{P}(z)} \equiv e^{-\int_t^{Q^2} dp(t')}$$

- Probalitity of K-emission

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, ...$$

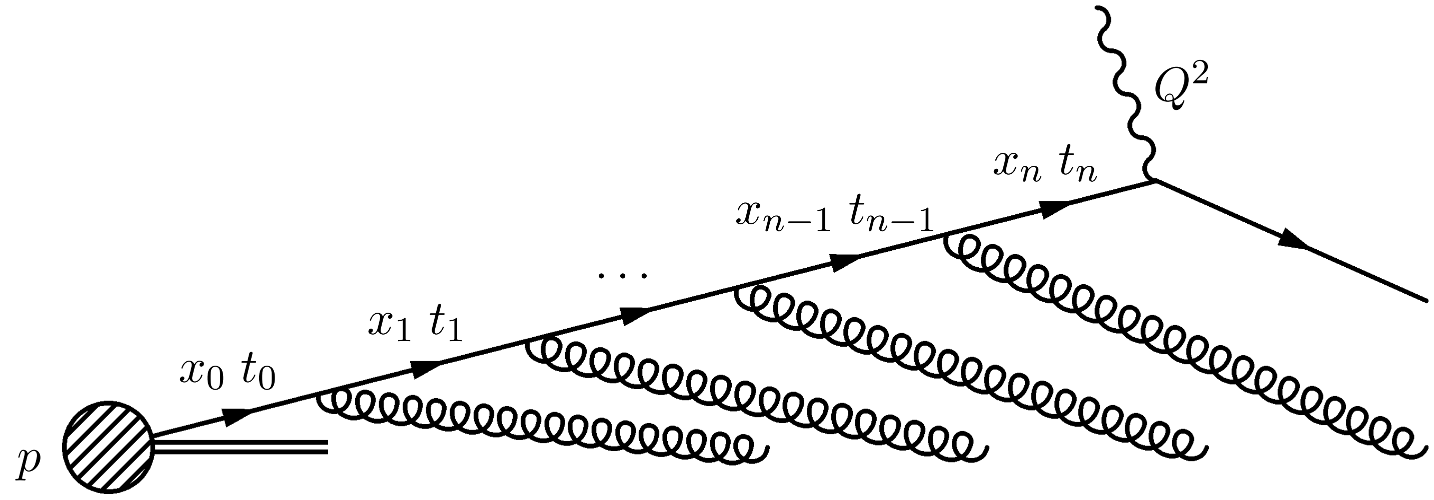
- Ensure that the parton shower is unitary
- Ensure cancelation of IR divergency
- Interference effect via Angular ordering

Initial-state



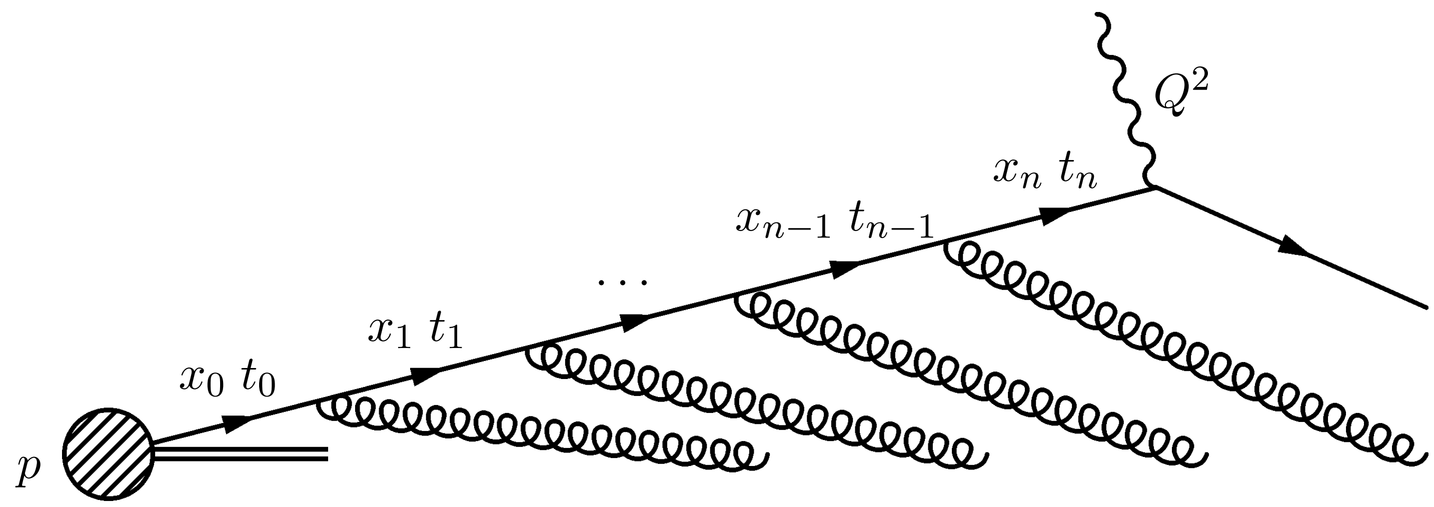
- So far, we have looked at final-state (time-like) splittings. For initial state, the splitting functions are the same
- However, there is another ingredient: the parton density (or distribution) functions (PDFs). Naively: Probability to find a given parton in a hadron at a given momentum fraction $\mathbf{x = p_z/P_z}$ and scale $\mathbf{t}$.

Initial-state



- So far, we have looked at final-state (time-like) splittings. For initial state, the splitting functions are the same
- However, there is another ingredient: the parton density (or distribution) functions (PDFs). Naively: Probability to find a given parton in a hadron at a given momentum fraction **x** = p_z/ P_z and scale **t**.
- How do the PDFs evolve with increasing **t**?

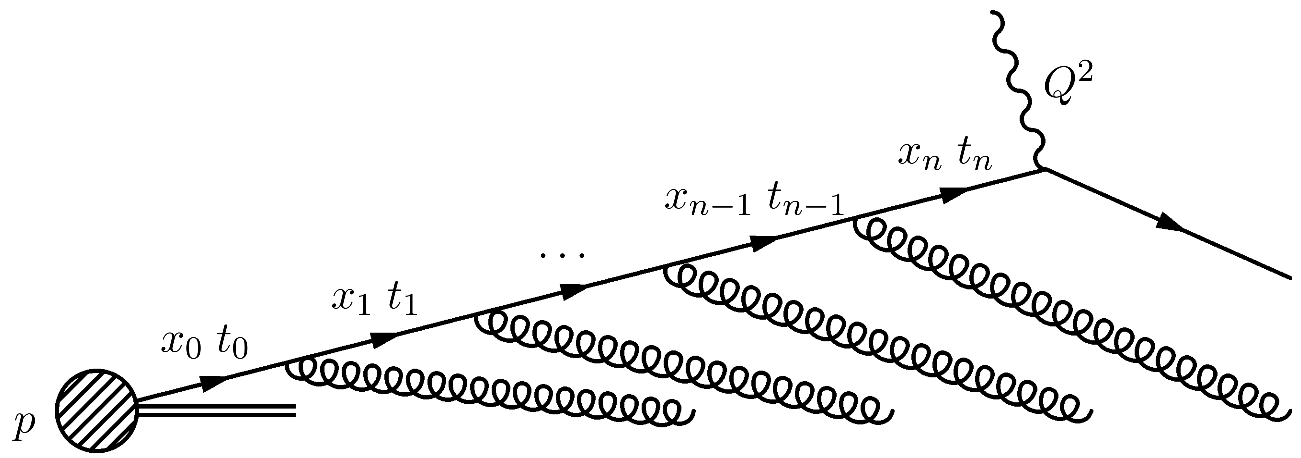
Initial-state



- So far, we have looked at final-state (time-like) splittings. For initial state, the splitting functions are the same
- However, there is another ingredient: the parton density (or distribution) functions (PDFs). Naively: Probability to find a given parton in a hadron at a given momentum fraction **x = p_z/ P_z** and scale **t**.
- How do the PDFs evolve with increasing **t**?

$$t\frac{\partial}{\partial t} f_i(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P_{ij}(z) f_j\left(\frac{x}{z}, t\right) \quad \text{DGLAP}$$

Initial-state parton splittings



- Start with a quark PDF **$f_0(x)$** at scale **t_0**. After a single parton emission, the probability to find the quark at virtuality **$t > t_0$** is

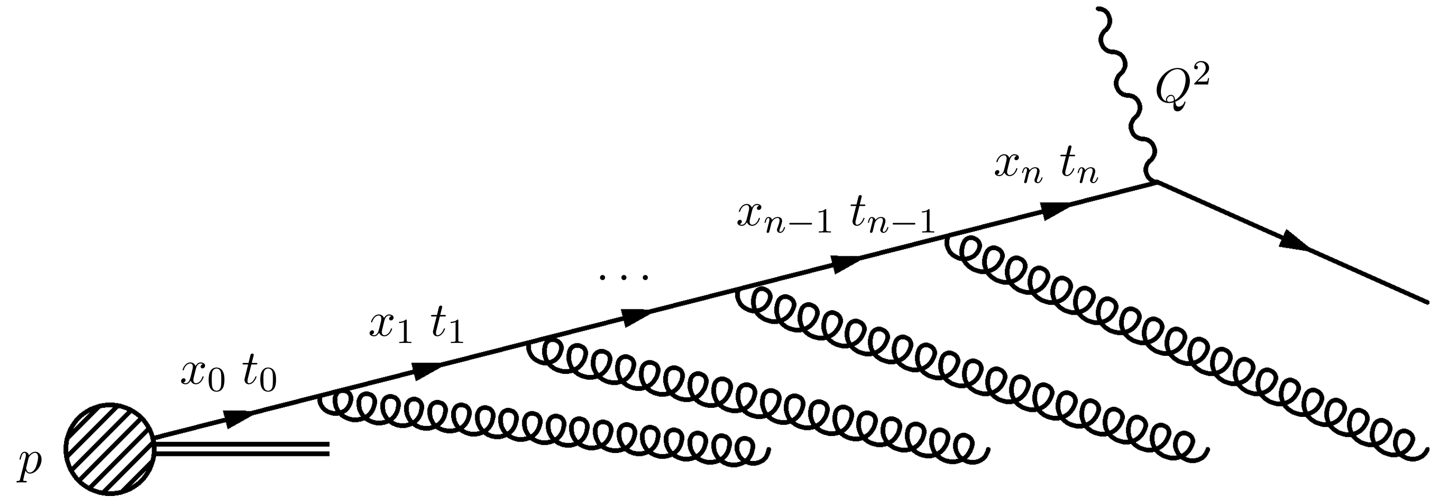
$$f(x,t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) f_0\left(\frac{x}{z}\right)$$

- After a second emission, we have

$$f(x,t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) \left\{ f_0\left(\frac{x}{z}\right) + \int_{t_0}^{t'} \frac{dt''}{t''} \frac{\alpha_s}{2\pi} \int_{x/z}^1 \frac{dz'}{z'} P(z') f_0\left(\frac{x}{zz'}\right) \right\}$$

$f(x/z, t')$

The DGLAP equation



- So for multiple parton splittings, we arrive at an integral-differential equation:

$$t\frac{\partial}{\partial t}f_i(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi}P_{ij}(z)f_j\left(\frac{x}{z},t\right)$$

- This is the famous DGLAP equation (where we have taken into account the multiple parton species i, j). The boundary condition for the equation is the initial PDFs $f_{i0}(x)$ at a starting scale t_0 (around 2 GeV).

- These starting PDFs are fitted to experimental data.

parton showers

- To simulate parton radiation from the initial state, we start with the hard scattering, and then "deconstruct" the DGLAP evolution to get back to the original hadron: backwards evolution!
  - i.e. we undo the analytic resummation and replace it with explicit partons (e.g. in Drell-Yan this gives non-zero p_T to the vector boson)
- In backwards evolution, the Sudakovs include also the PDFs -- this follows from the DGLAP equation and ensures conservation of probability:

$$\Delta_{Ii}(x,t_1,t_2) = \exp\left\{-\int_{t_1}^{t_2} dt' \sum_j \int_x^1 \frac{dx'}{x'} \frac{\alpha_s(t')}{2\pi} P_{ij}\left(\frac{x}{x'}\right) \frac{f_i(x',t')}{f_j(x,t')}\right\}$$

This represents the probability that parton **i** will stay at the same **x** (no splittings) when evolving from $\mathbf{t_1}$ to $\mathbf{t_2}$.

- The shower simulation is now done as in a final state shower!

Hadronization

- The shower stops if all partons are characterized by a scale at the IR cut-off: $Q_0 \sim 1$ GeV.
- Physically, we observe hadrons, not (colored) partons.
- We need a non-perturbative model in passing from partons to colorless hadrons.
- There are two models (string and cluster), based on physical and phenomenological considerations.

A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

Parton Shower MC event generators

A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

- General-purpose tools
- Complete exclusive description of the events: hard scattering, showering & hadronization (and underlying event)
- Reliable and well-tuned tools
- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD

Parton Shower MC event generators

A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

- General-purpose tools
- Complete exclusive description of the events: hard scattering, showering & hadronization (and underlying event)
- Reliable and well-tuned tools
- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD

Shower MC Generators: PYTHIA, HERWIG, SHERPA

Parton Shower MC event generators

A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

- General-purpose tools
- Complete exclusive description of the events: hard scattering, showering & hadronization (and underlying event)
- Reliable and well-tuned tools
- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD

Shower MC Generators: PYTHIA, HERWIG, SHERPA

"Note that a banching tree is not a Feynman diagram: it represents the coherent sum of many real and virtual diagrams which are summed by the branching algorithm" (HERWIG manual)

To Remember

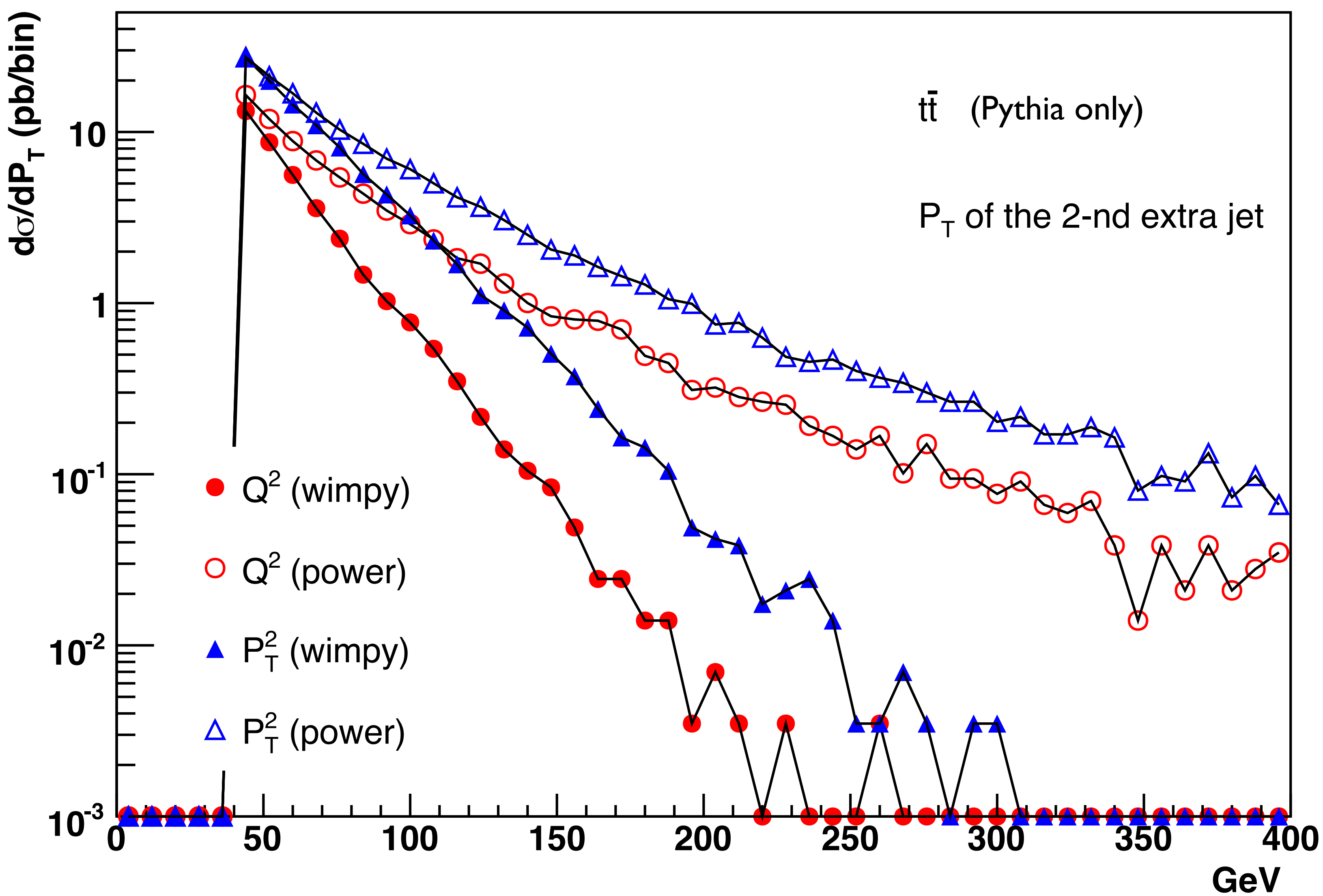
- The parton shower dresses partons with radiation. This makes the inclusive parton-level predictions (i.e. inclusive over extra radiation) completely exclusive
  - In the soft and collinear limits the partons showers are exact, but in practice they are used outside this limit as well.
  - Partons showers are universal (i.e. independent from the process)
  - Building block of the parton shower is the Sudakov
- There is a cut-off in the shower (below which we don't trust perturbative QCD) at which a hadronization model takes over

Matching/Merging

Olivier Mattelaer
IPPP/Durham

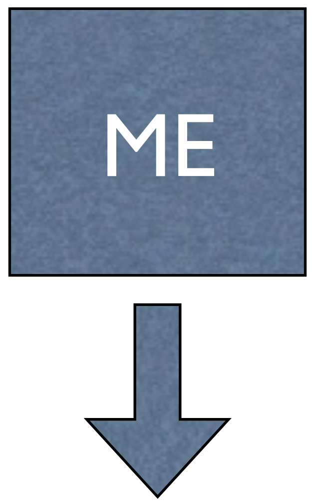
PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)



Matrix Elements vs. Parton Showers

Matrix Elements vs. Parton Showers



1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Matrix Elements vs. Parton Showers

ME	Shower MC
1. Fixed order calculation 2. Computationally expensive 3. Limited number of particles 4. Valid when partons are hard and well separated 5. Quantum interference correct 6. Needed for multi-jet description	1. Resums logs to all orders 2. Computationally cheap 3. No limit on particle multiplicity 4. Valid when partons are collinear and/or soft 5. Partial interference through angular ordering 6. Needed for hadronization

Matrix Elements vs. Parton Showers

ME

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Shower MC

1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are collinear and/or soft
5. Partial interference through angular ordering
6. Needed for hadronization

Approaches are complementary: merge them!

Matrix Elements vs. Parton Showers

ME

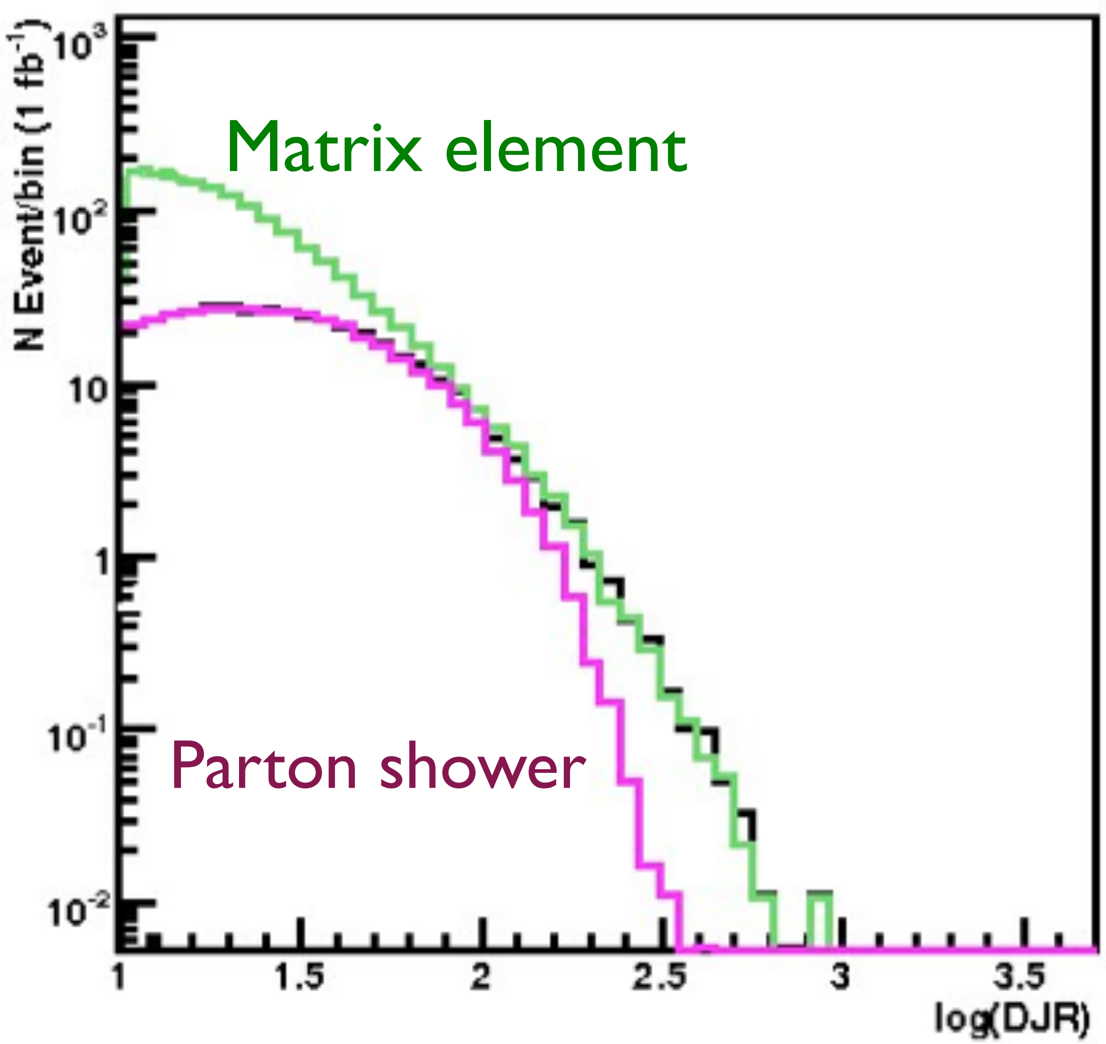
1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Shower MC

1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are collinear and/or soft
5. Partial interference through angular ordering
6. Needed for hadronization

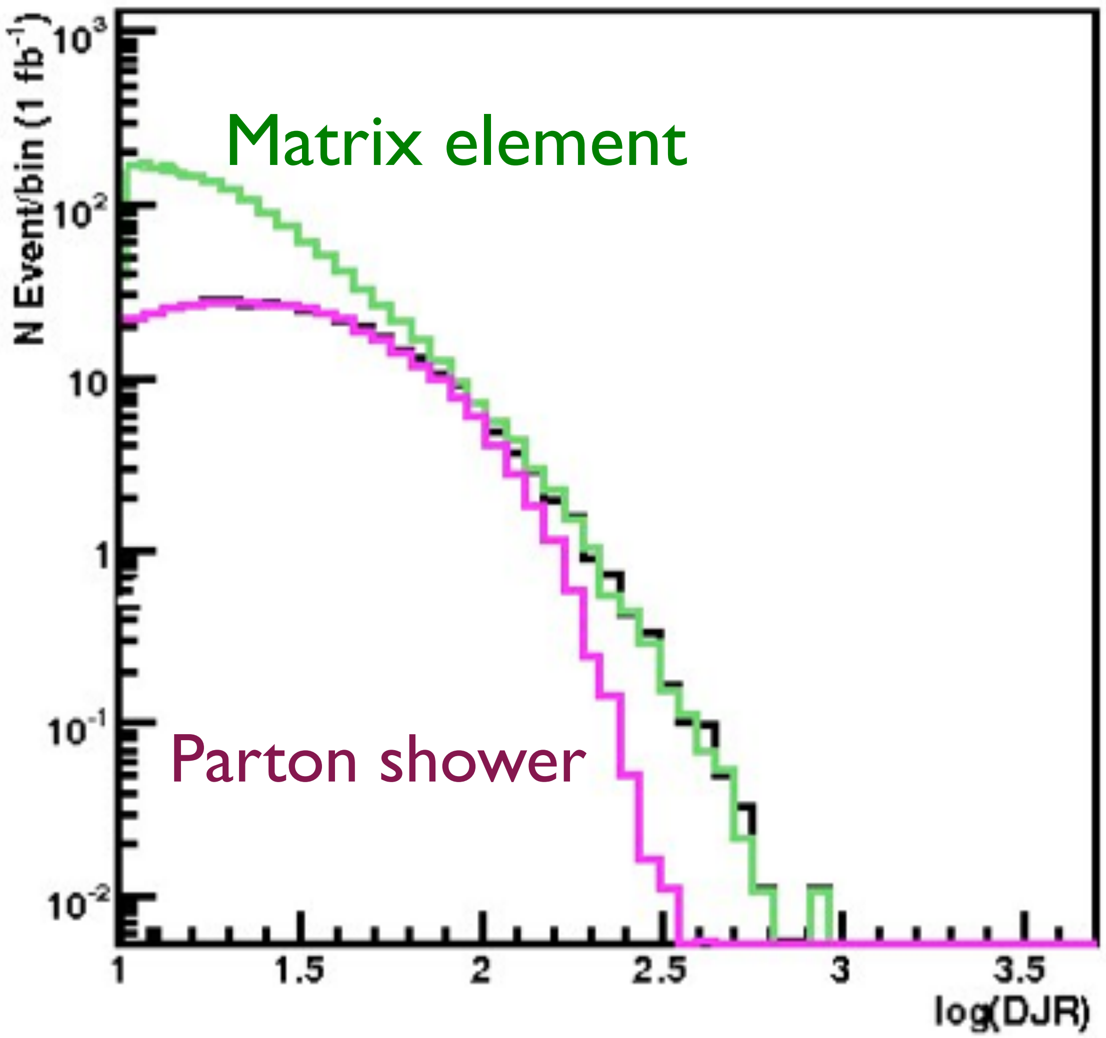
Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

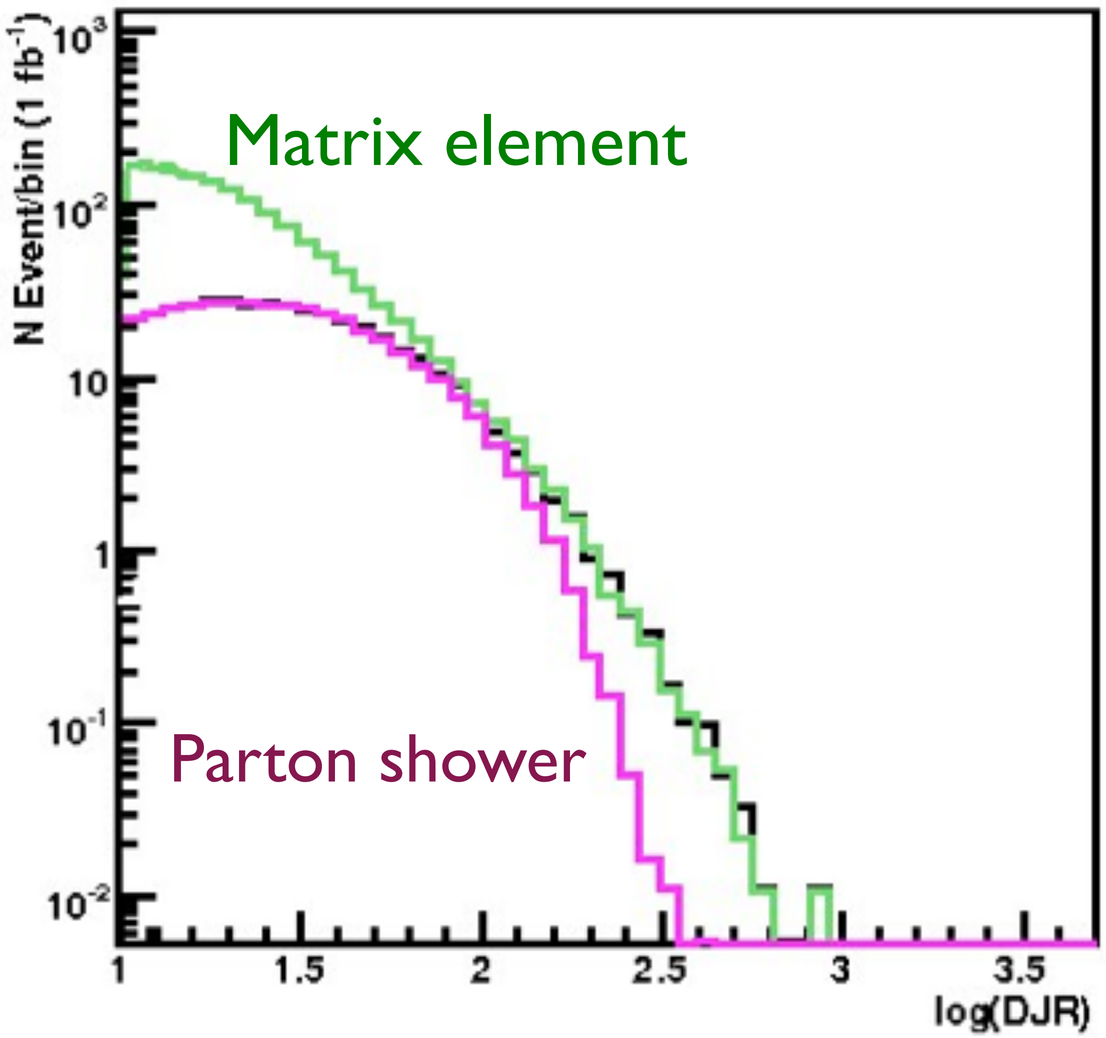
- Regularization of matrix element divergence



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Goal for ME-PS merging/matching

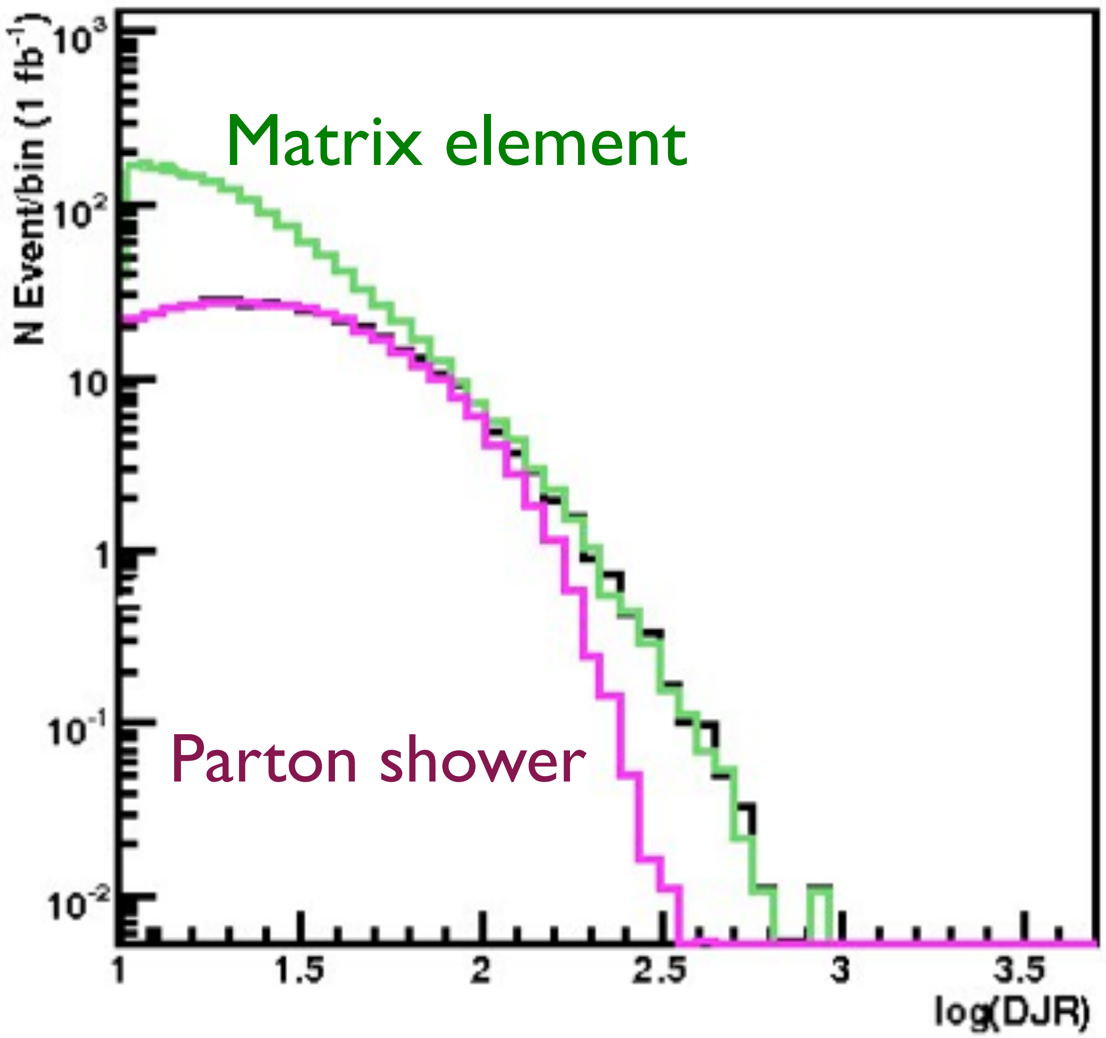
- Regularization of matrix element divergence
- Correction of the parton shower for large momenta



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Goal for ME-PS merging/matching

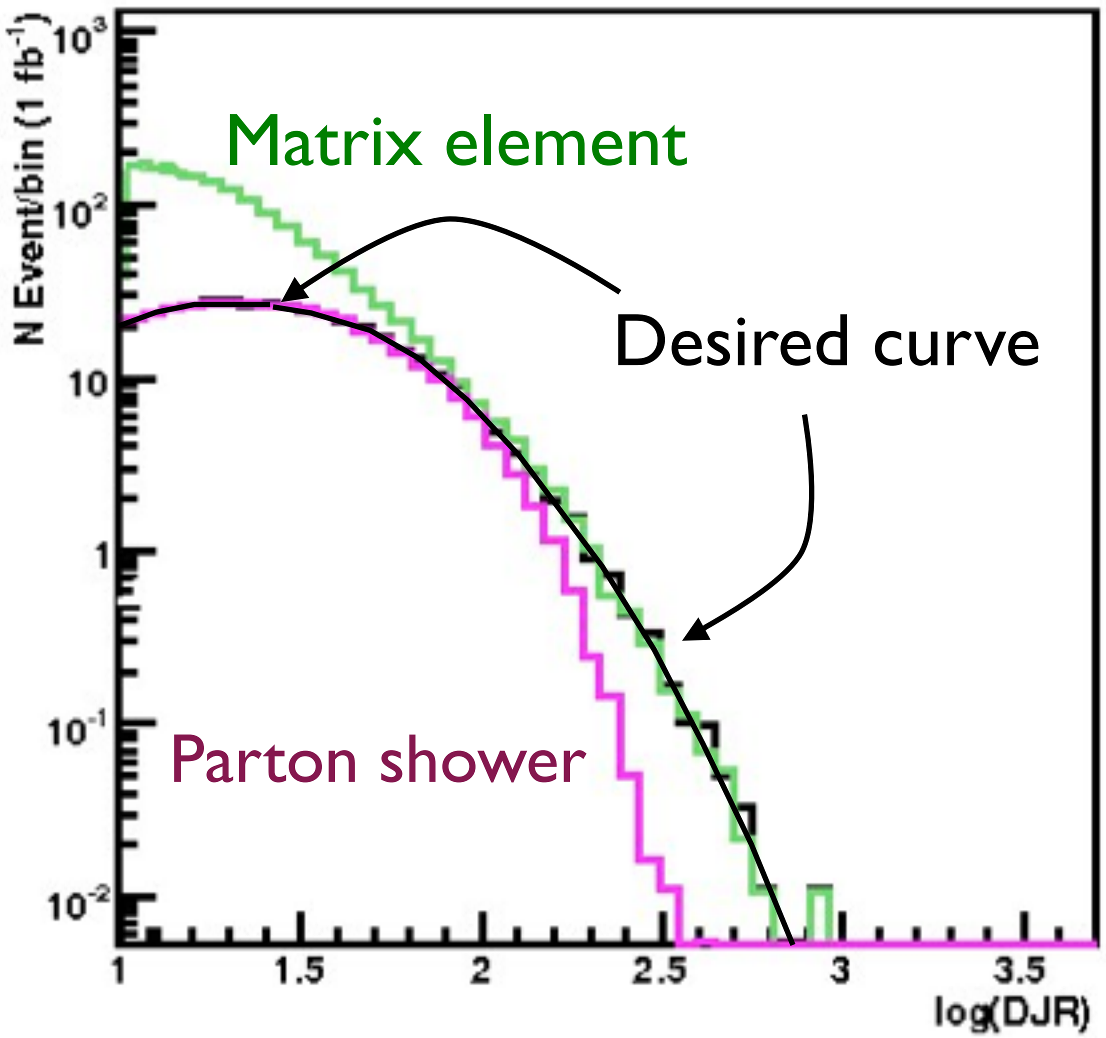
- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Goal for ME-PS merging/matching

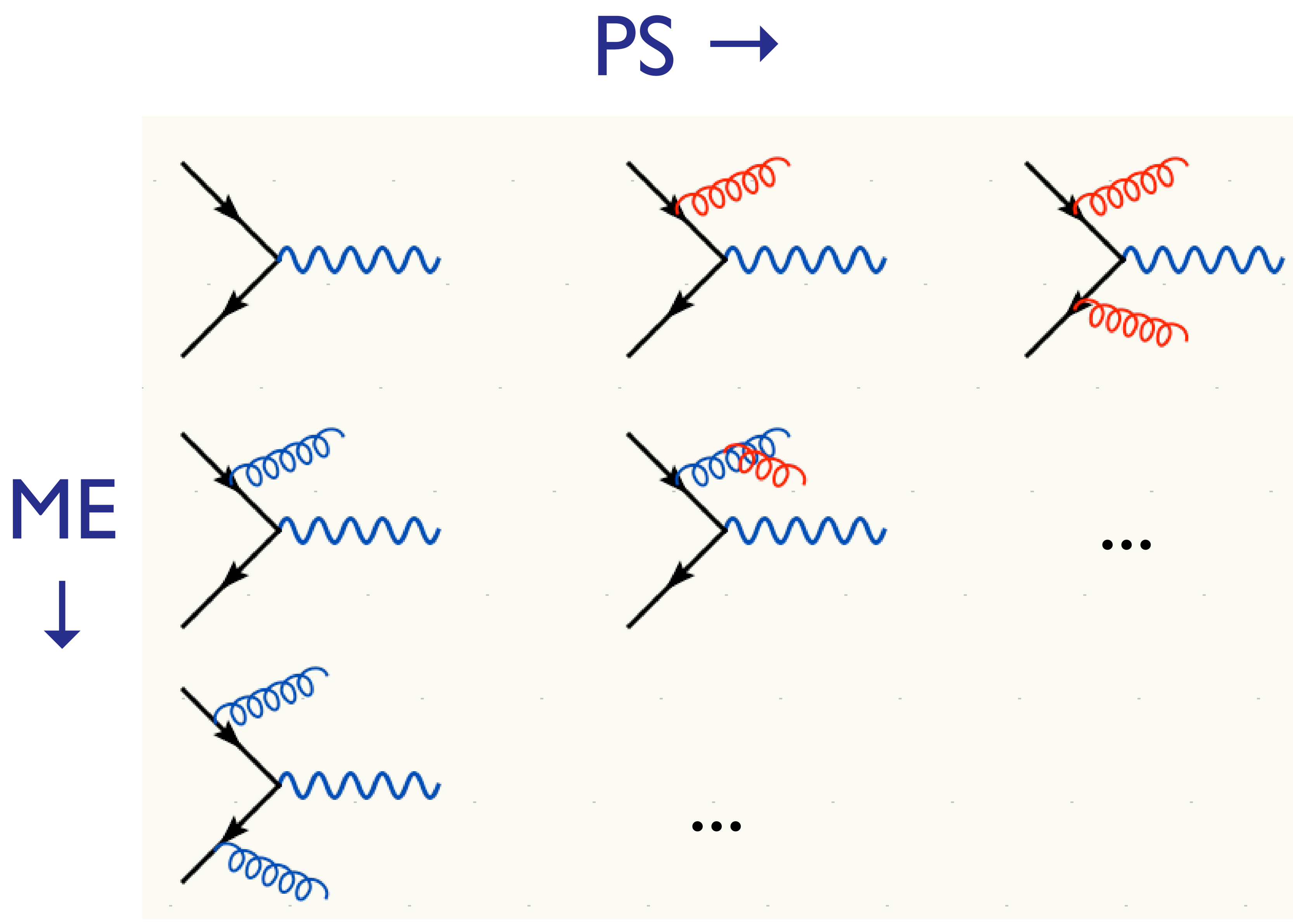
- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Merging ME with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]

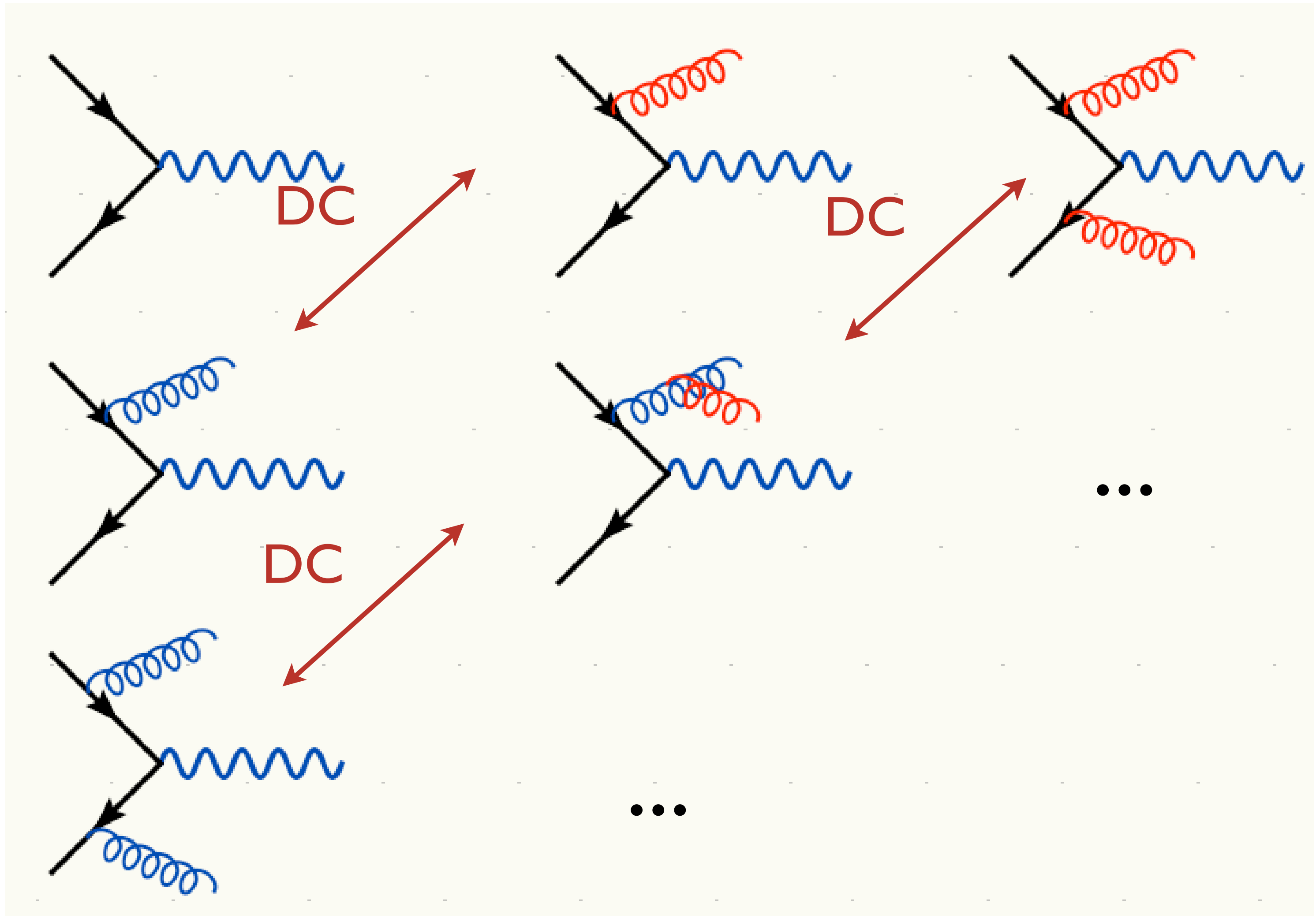


Merging ME with PS

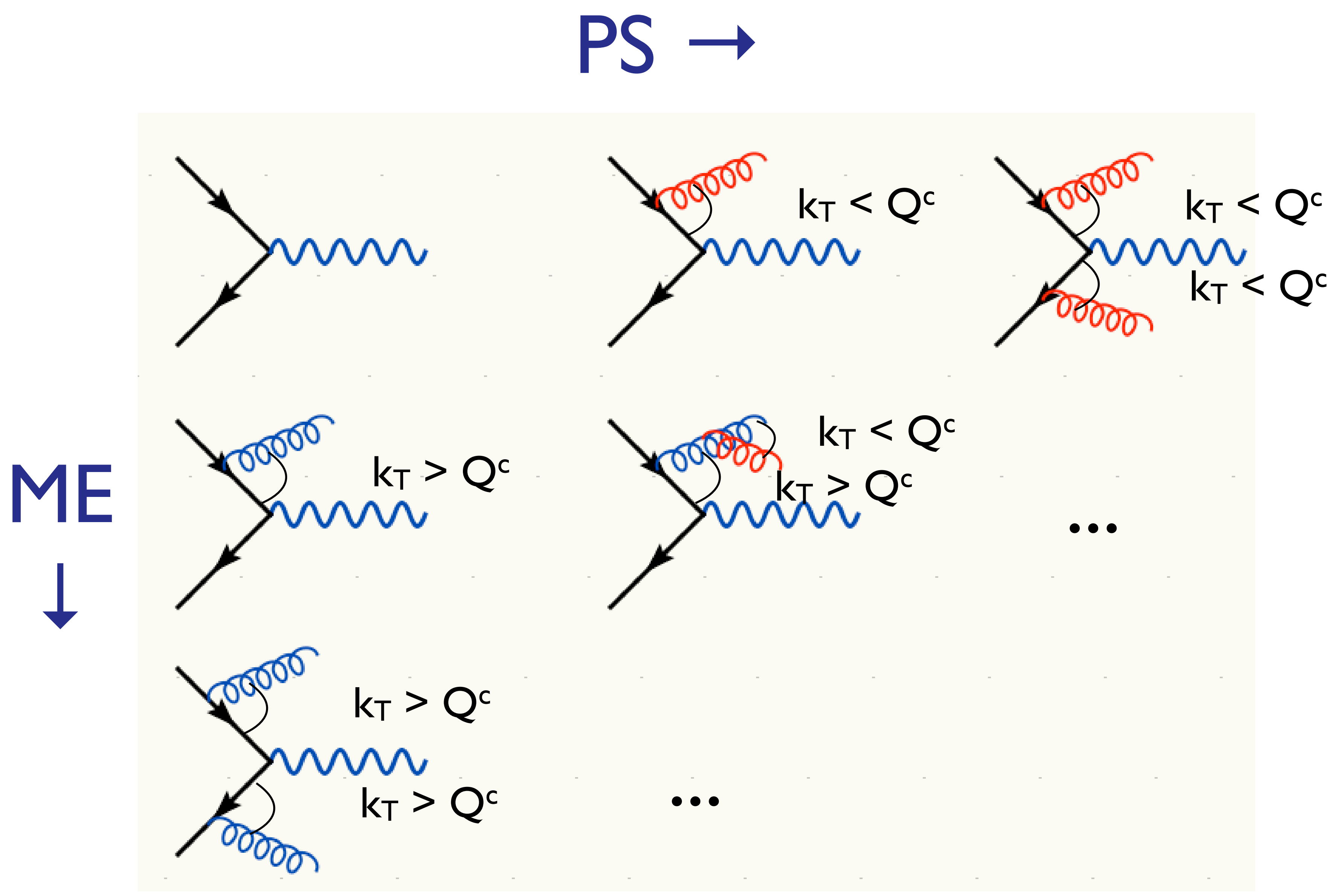
[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]

PS →

ME ↓



[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]

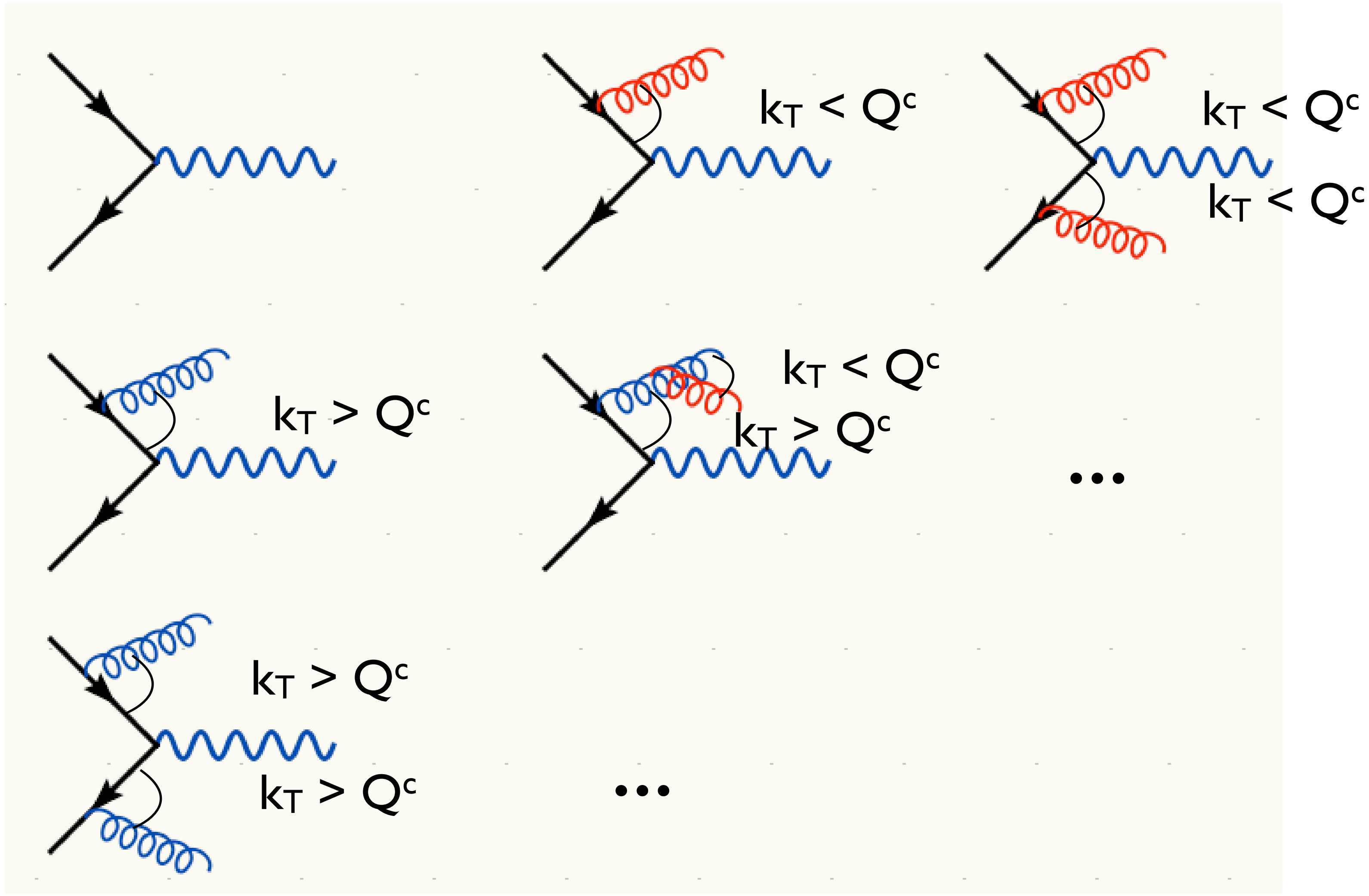


Merging ME with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]

PS →

ME ↓

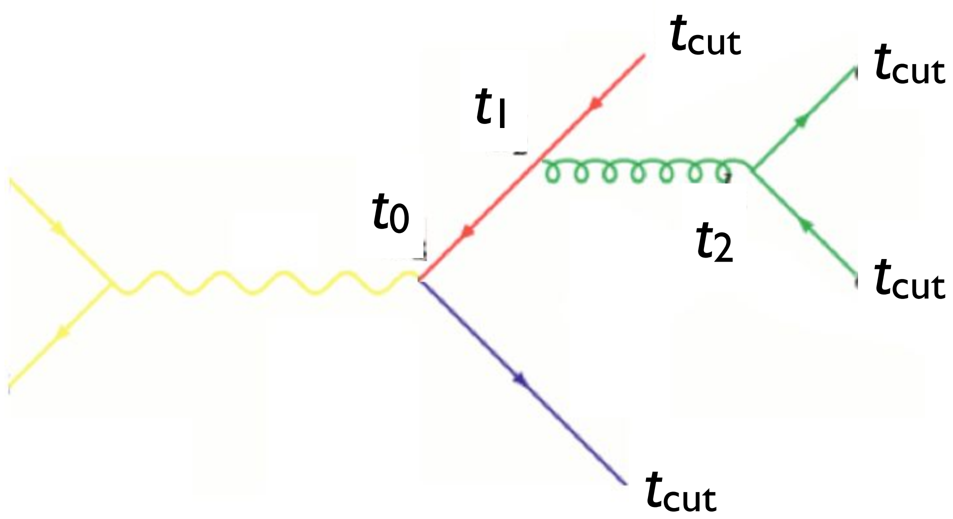


Double counting between ME and PS easily avoided using phase space cut between the two: PS below cutoff, ME above cutoff.

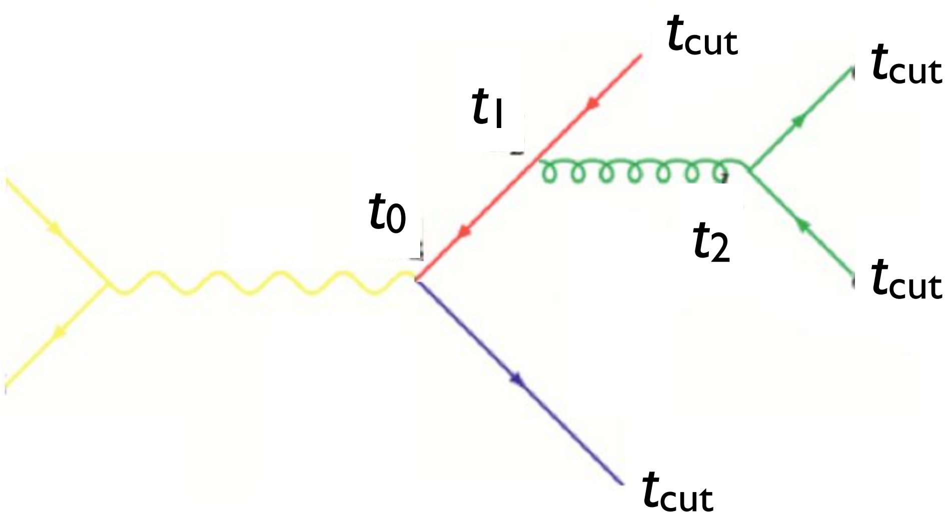
Merging ME with PS

- So double counting problem easily solved, but what about getting smooth distributions that are independent of the precise value of Q^c?
- Below cutoff, distribution is given by PS
  - need to make ME look like PS near cutoff
- Let's take another look at the PS!

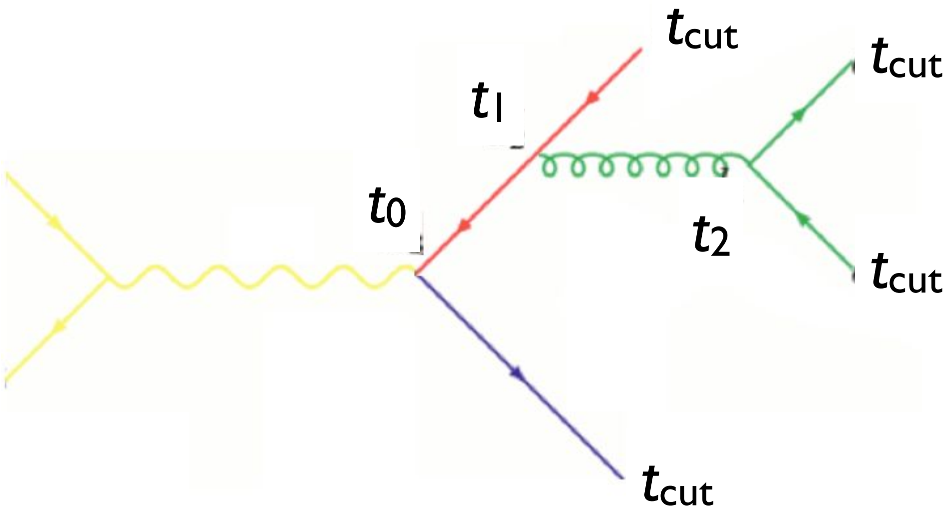
Merging ME with PS



Merging ME with PS

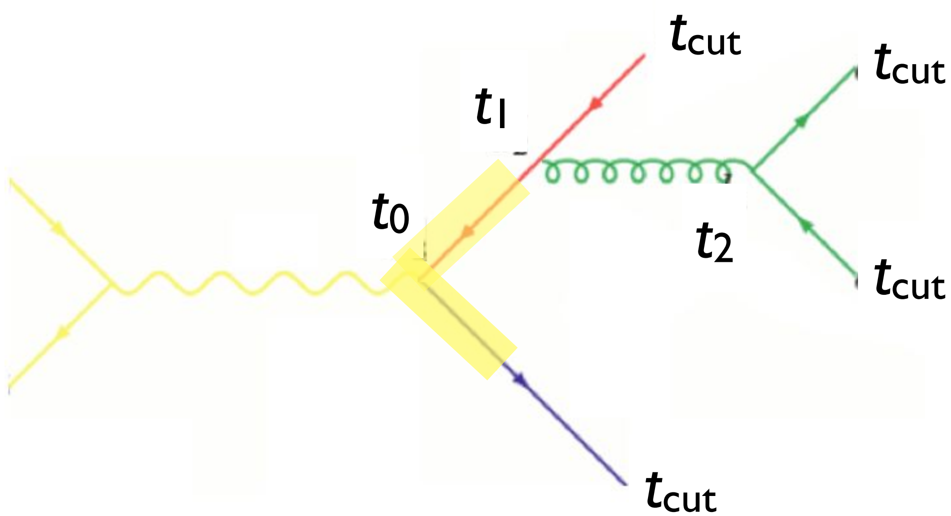


- How does the PS generate the configuration above?



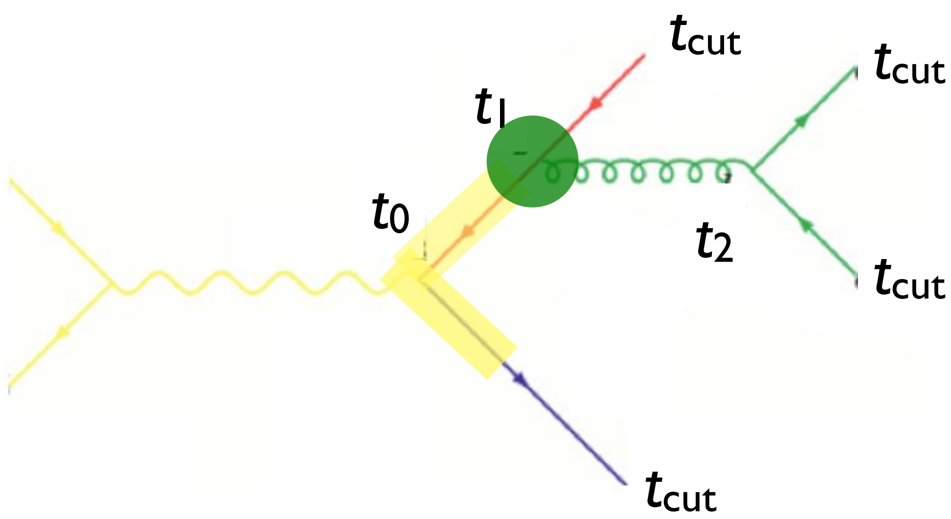
- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$



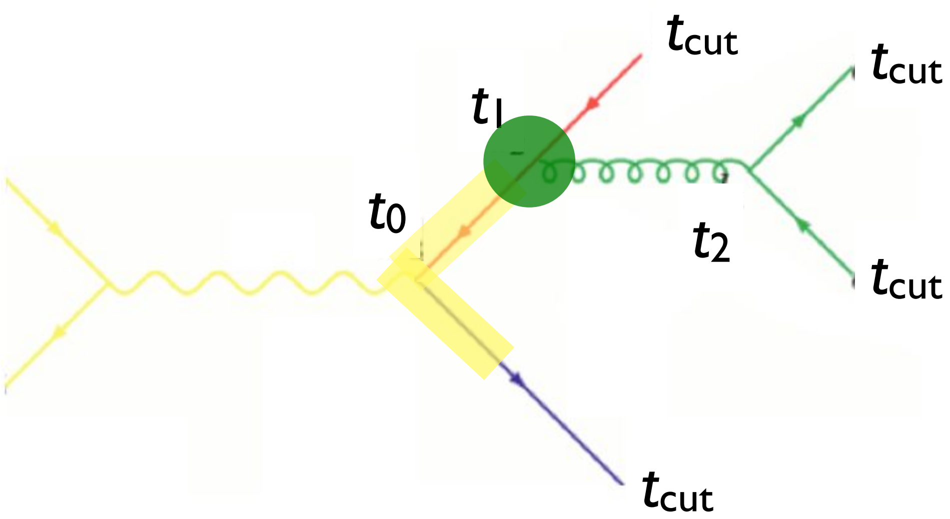
- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$



- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

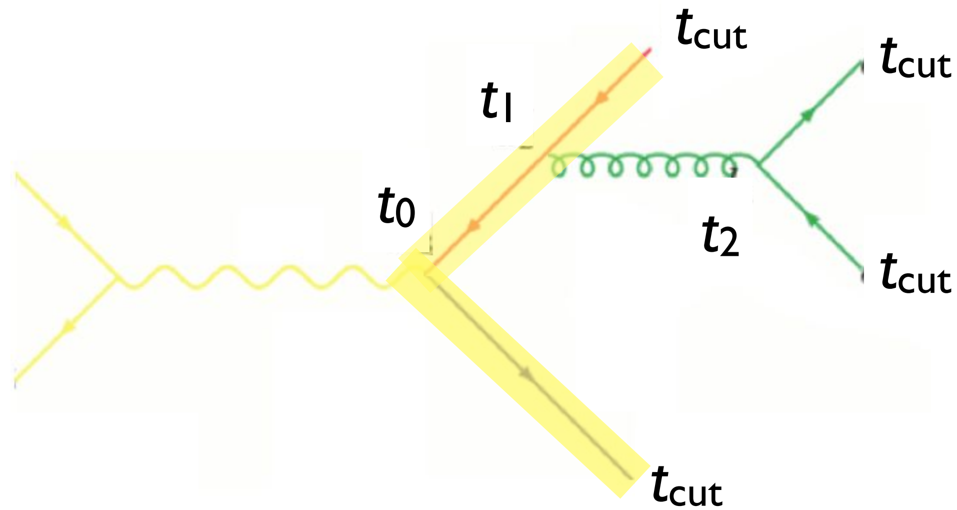


- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$(\Delta_q(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$



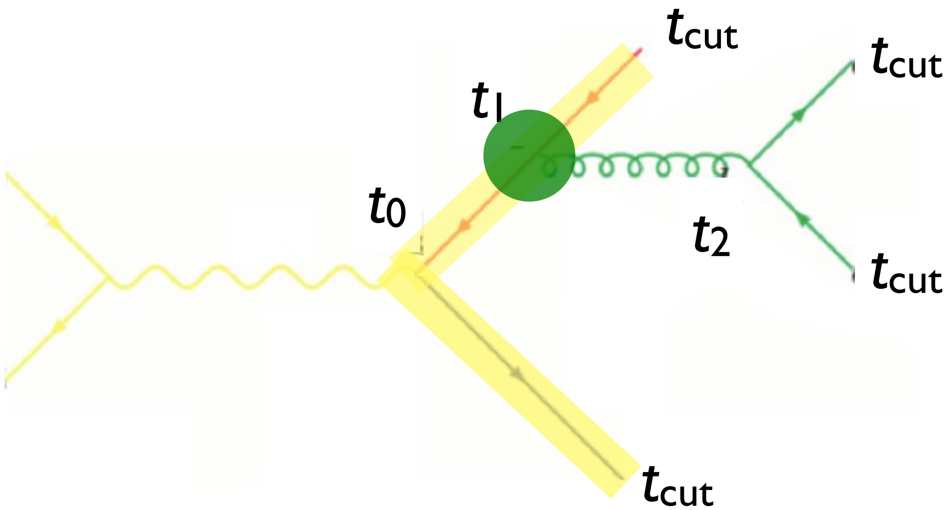
- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$(\Delta_q(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

Merging ME with PS

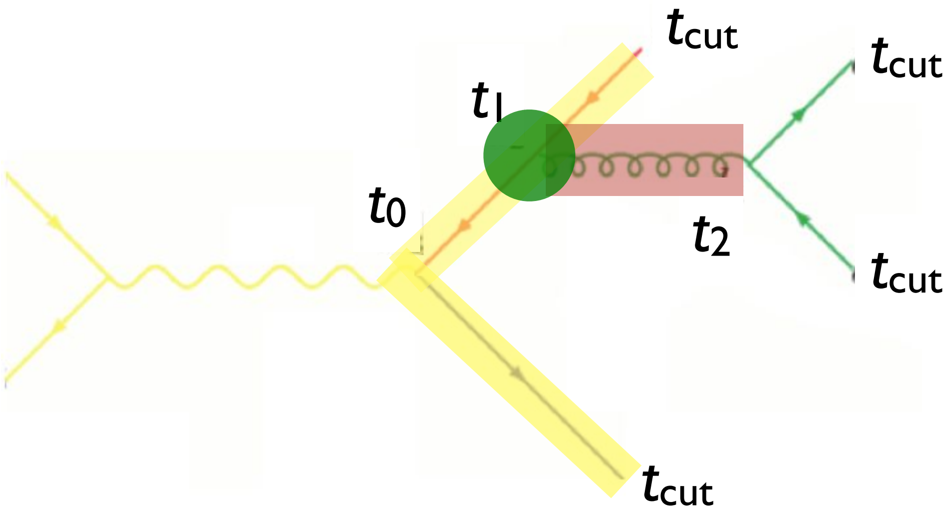


- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$(\Delta_q(t_{cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$



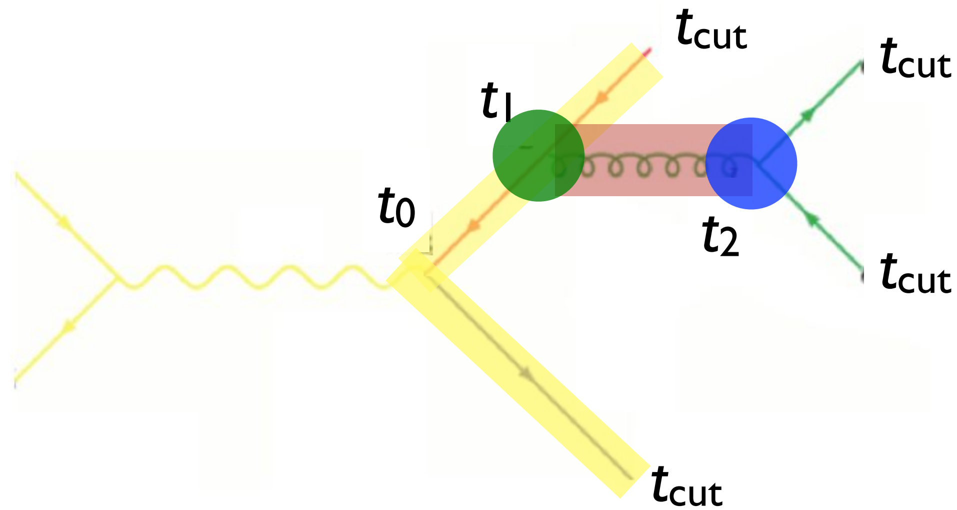
- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$(\Delta_q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

Merging ME with PS

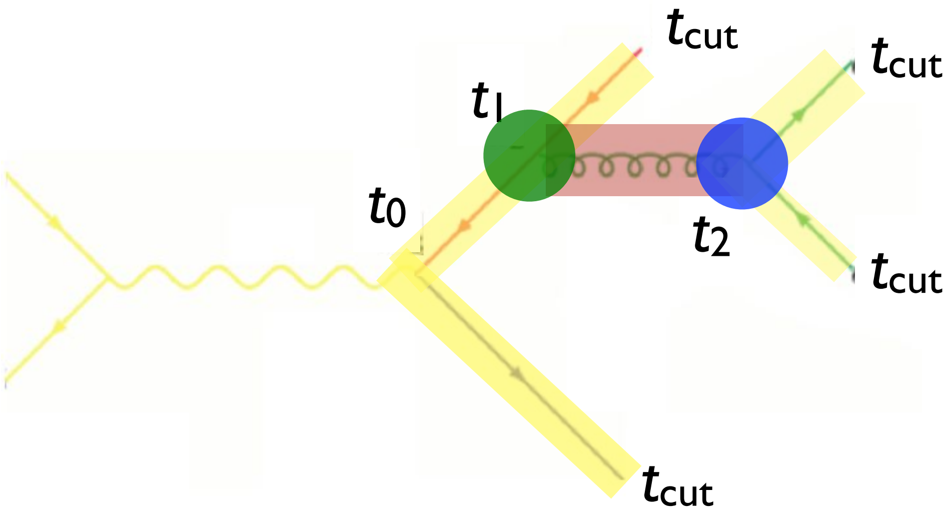


- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$(\Delta_q(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

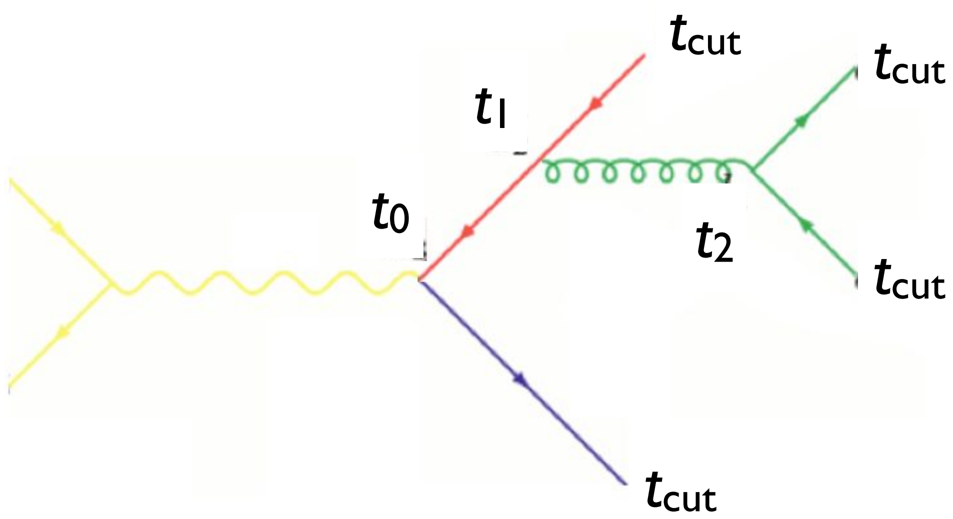


- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

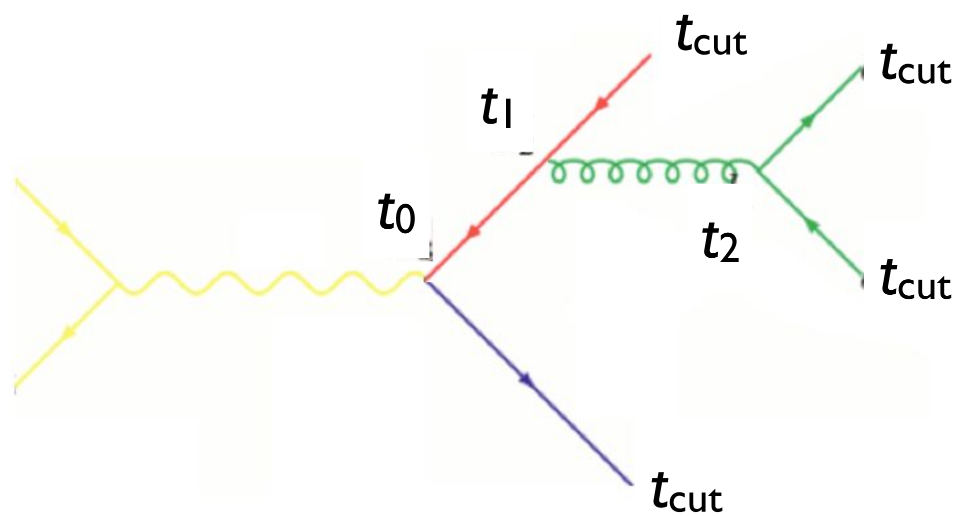
and for the whole tree

$$(\Delta_q(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$



$$(\Delta_q(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

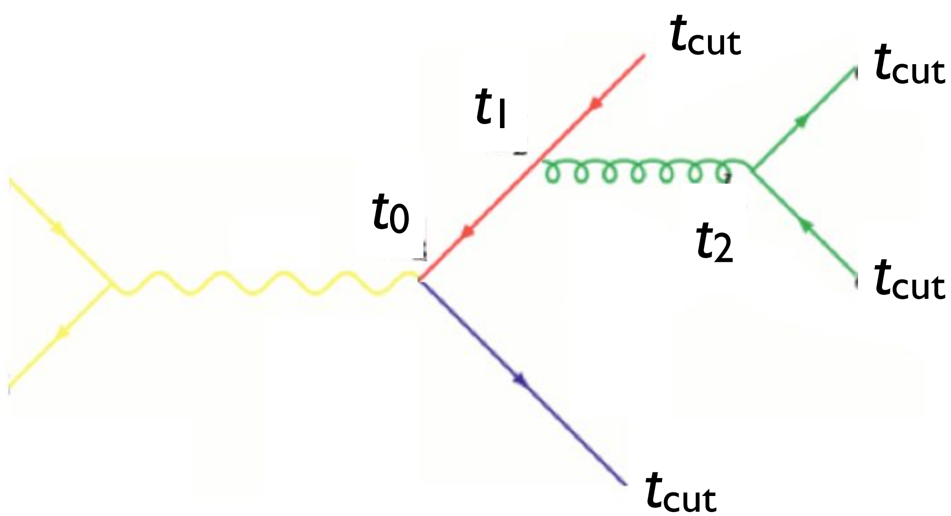
Merging ME with PS



$$(\Delta_q(t_{\rm cut},t_0))^2\Delta_g(t_2,t_1)(\Delta_q(_{\rm cut},t_2))^2\boxed{\frac{\alpha_s(t_1)}{2\pi}P_{gq}(z)\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')}$$

Corresponds to the matrix element
BUT with α_s evaluated at the scale of each splitting

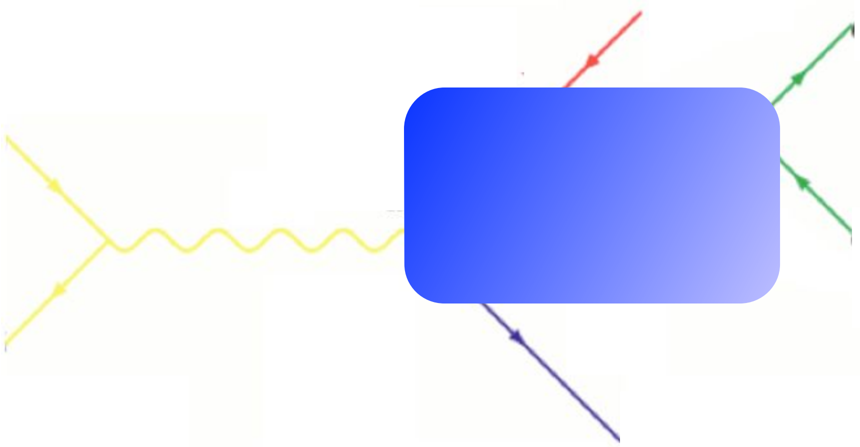
Merging ME with PS



$$(\Delta_q(t_{\rm cut},t_0))^2\Delta_g(t_2,t_1)(\Delta_q(_{\rm cut},t_2))^2\frac{\alpha_s(t_1)}{2\pi}P_{gq}(z)\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$

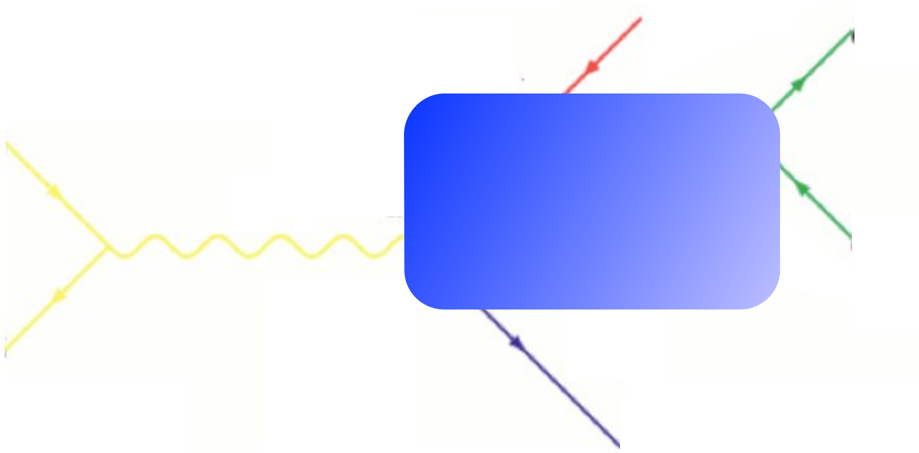
Corresponds to the matrix element
BUT with α_s evaluated at the scale of each splitting

Sudakov suppression due to not allowing additional radiation above the scale t_{cut}



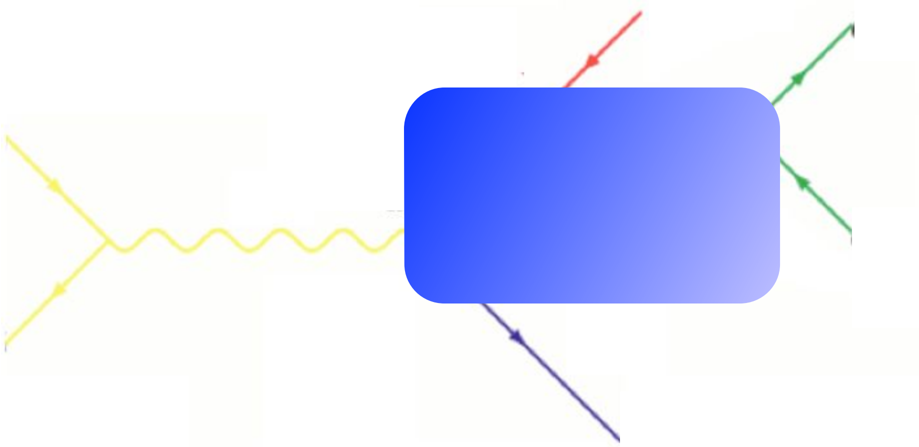
$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, ...)$$

Merging ME with PS



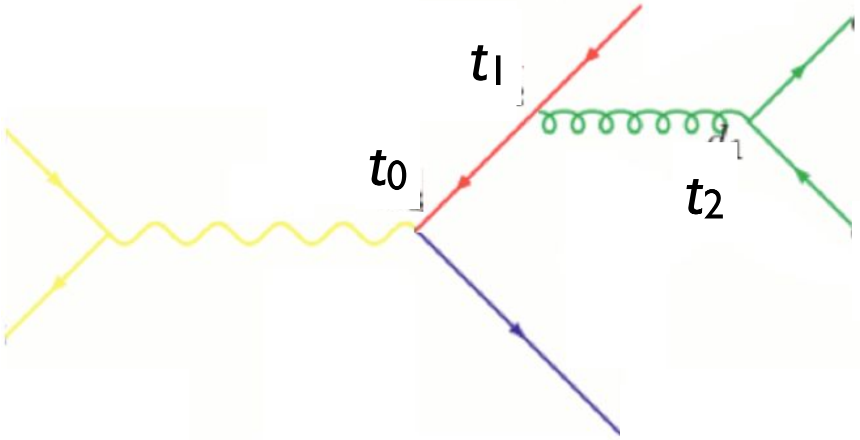
$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$$

- To get an equivalent treatment of the corresponding matrix element, do as follows:



$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$$

- To get an equivalent treatment of the corresponding matrix element, do as follows:
  1. Cluster the event using some clustering algorithm
     - this gives us a corresponding "parton shower history"

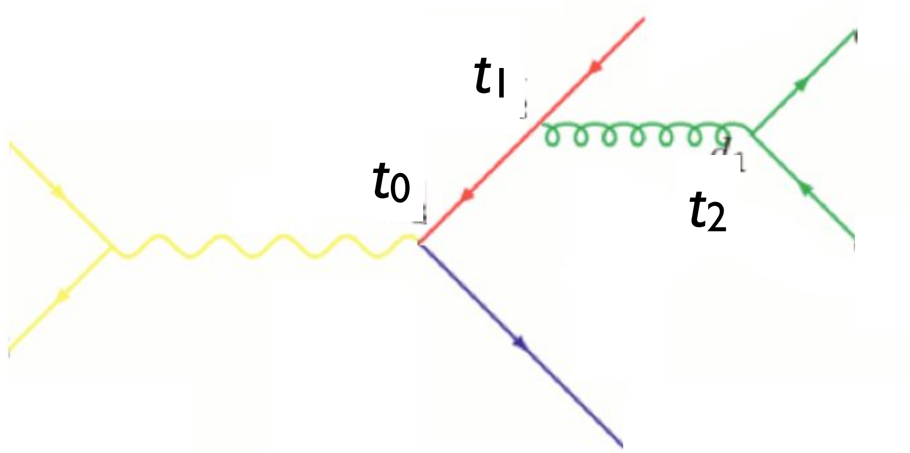


$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$$

- To get an equivalent treatment of the corresponding matrix element, do as follows:

1. Cluster the event using some clustering algorithm
   - this gives us a corresponding "parton shower history"

Merging ME with PS

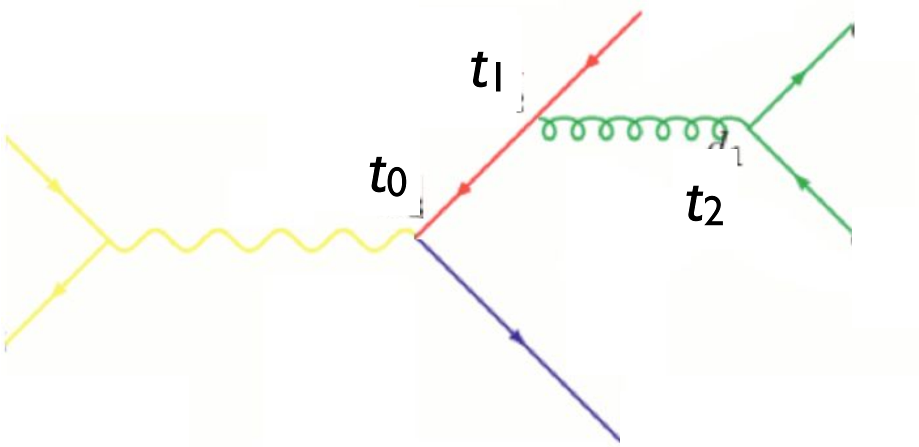


$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$$

- To get an equivalent treatment of the corresponding matrix element, do as follows:
  1. Cluster the event using some clustering algorithm
     - this gives us a corresponding "parton shower history"
  2. Reweight α_s in each clustering vertex with the clustering scale

$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)}$$

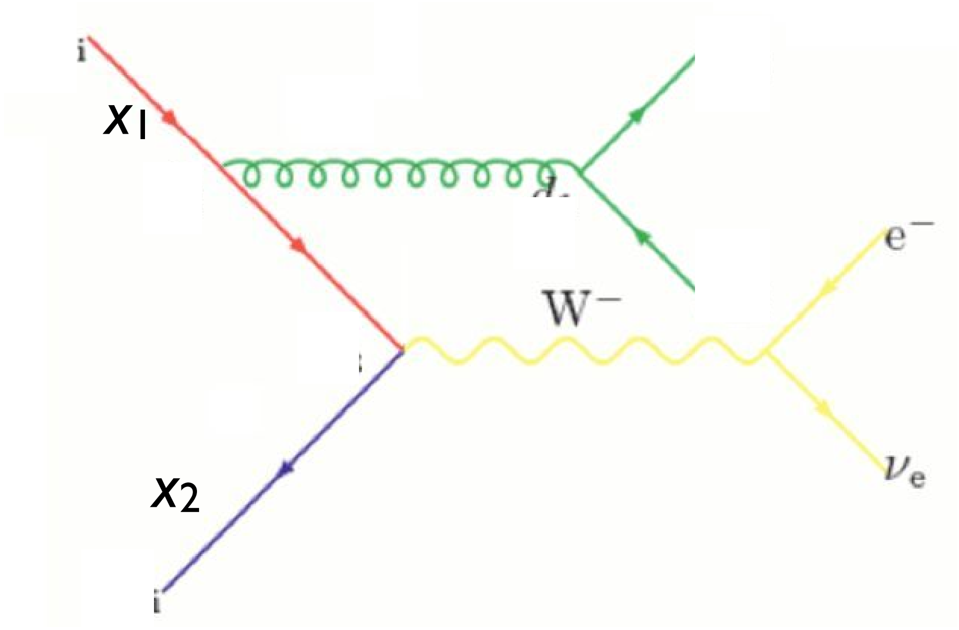
Merging ME with PS



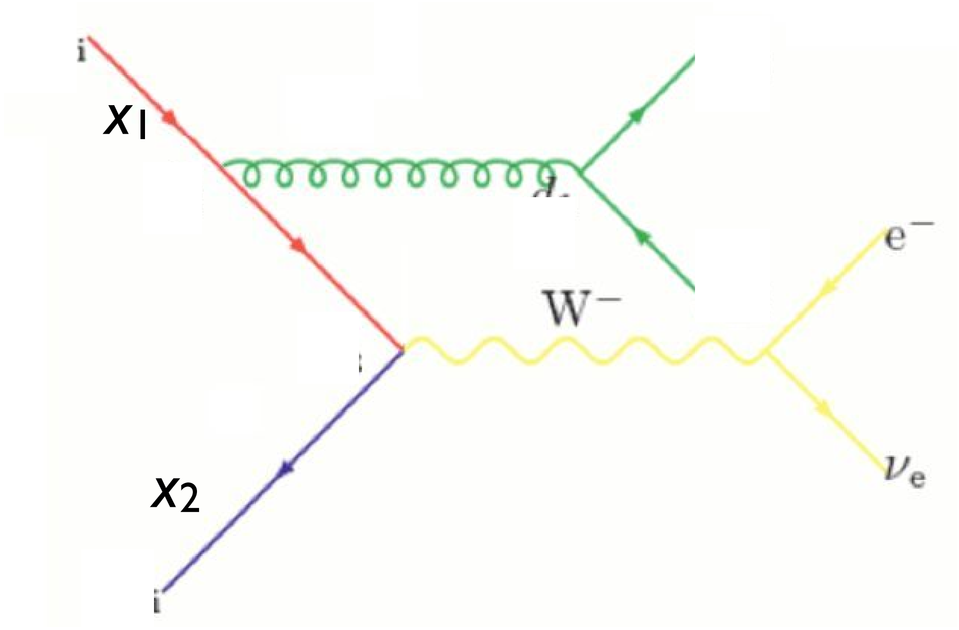
$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$$

- To get an equivalent treatment of the corresponding matrix element, do as follows:
  1. Cluster the event using some clustering algorithm
     - this gives us a corresponding "parton shower history"
  2. Reweight α_s in each clustering vertex with the clustering scale
     $$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)}$$
  3. Use some algorithm to apply the equivalent Sudakov suppression $(\Delta_q(t_{\mathrm{cut}}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(_{\mathrm{cut}}, t_2))^2$

Matching for initial state radiation



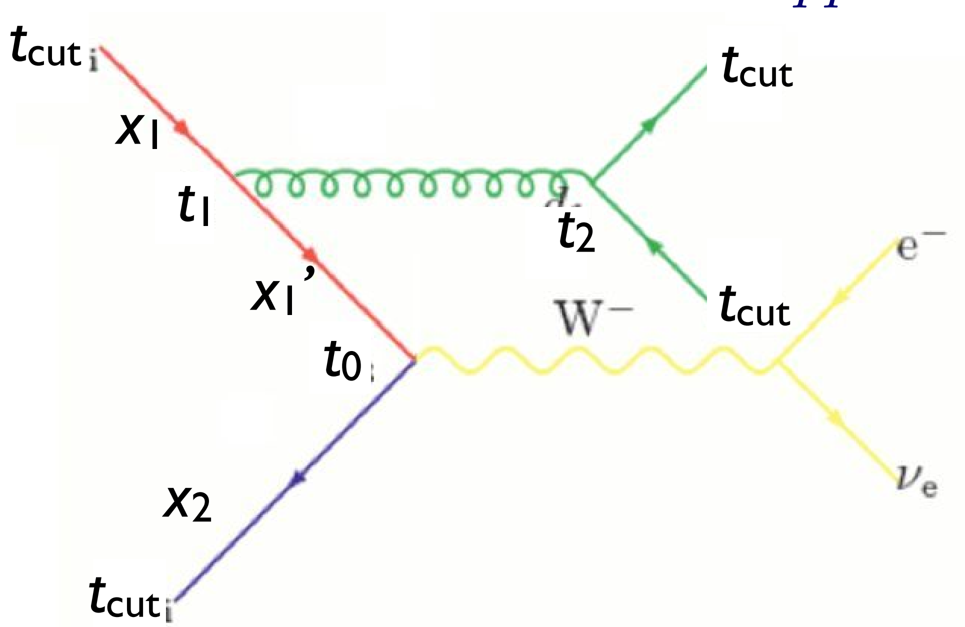
- We are of course not interested in e^+e^- but p-p(bar) - what happens for initial state radiation?



Matching for initial state radiation

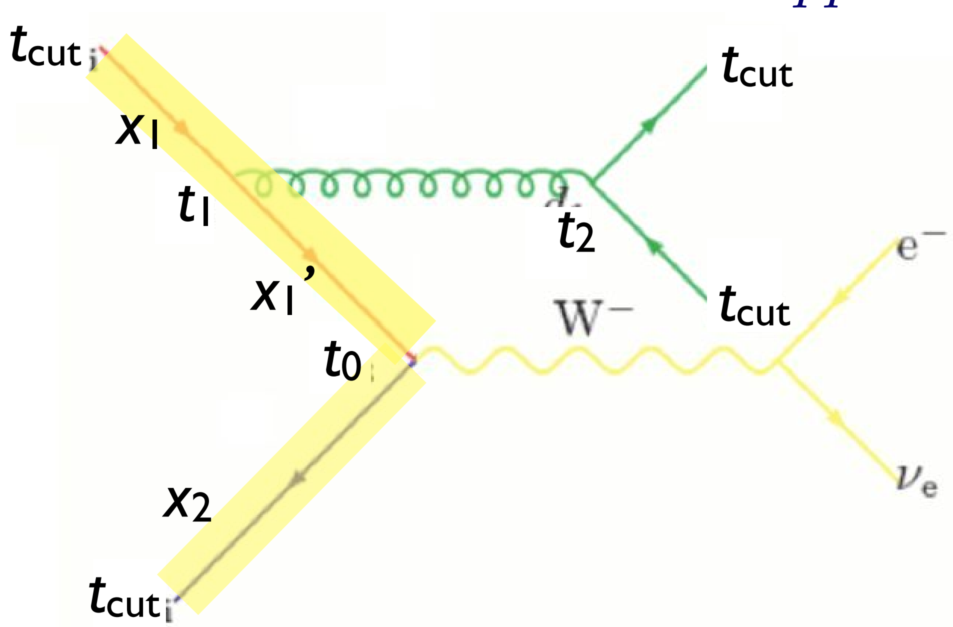
- We are of course not interested in e^+e^- but p-p(bar) - what happens for initial state radiation?
- Let's do the same exercise as before:

$$\mathcal{P} = (\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi}\frac{P_{gq}(z)}{z}\frac{f_q(x_1, t_1)}{f_q(x_1', t_1)}\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$
$$\times \hat\sigma_{q\bar q\to e\nu}(\hat s, \ldots) f_q(x_1', t_0) f_{\bar q}(x_2, t_0)$$



- We are of course not interested in e^+e^- but p-p(bar) - what happens for initial state radiation?
- Let's do the same exercise as before:

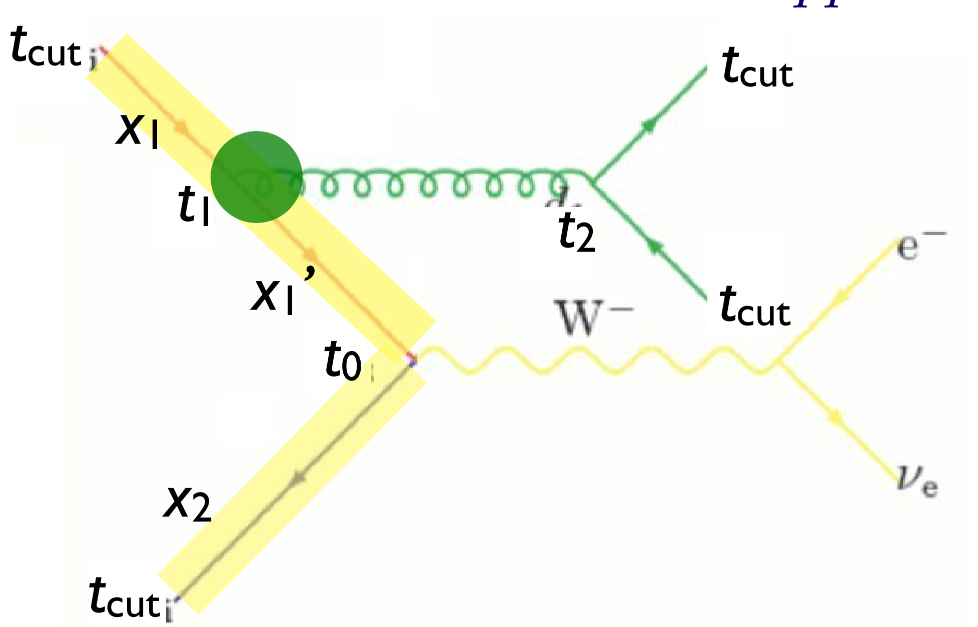
$$\mathcal{P} = (\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x'_1, t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z') \times \hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s}, \ldots) f_q(x'_1, t_0) f_{\bar{q}}(x_2, t_0)$$



- We are of course not interested in e^+e^- but p-p(bar) - what happens for initial state radiation?

- Let's do the same exercise as before:

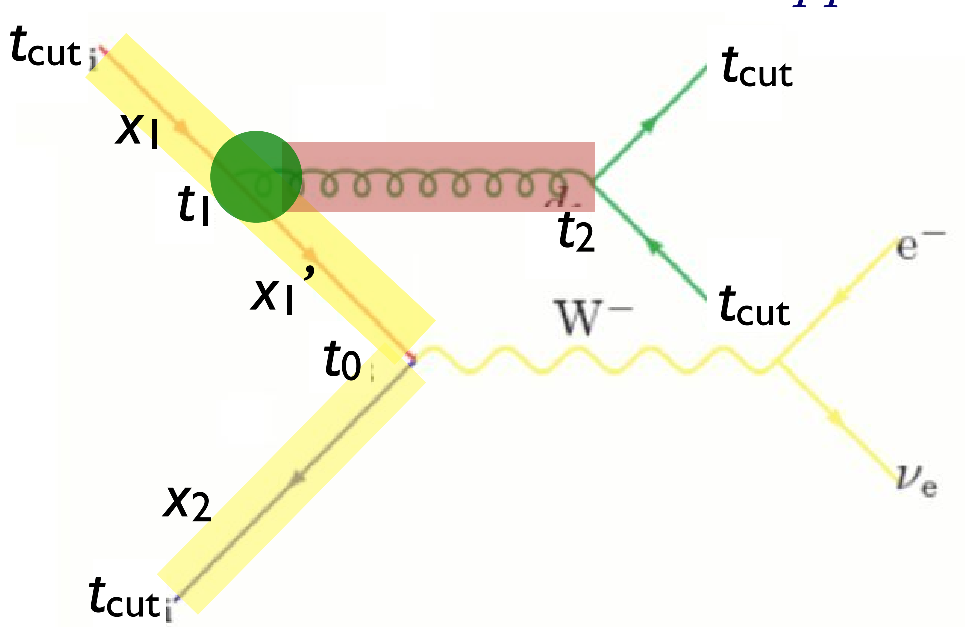
$$\mathcal{P} = (\Delta_{Iq}(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x_1', t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$
$$\times \hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s}, \ldots) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0)$$



- We are of course not interested in e$^+$e$^-$ but p-p(bar) - what happens for initial state radiation?

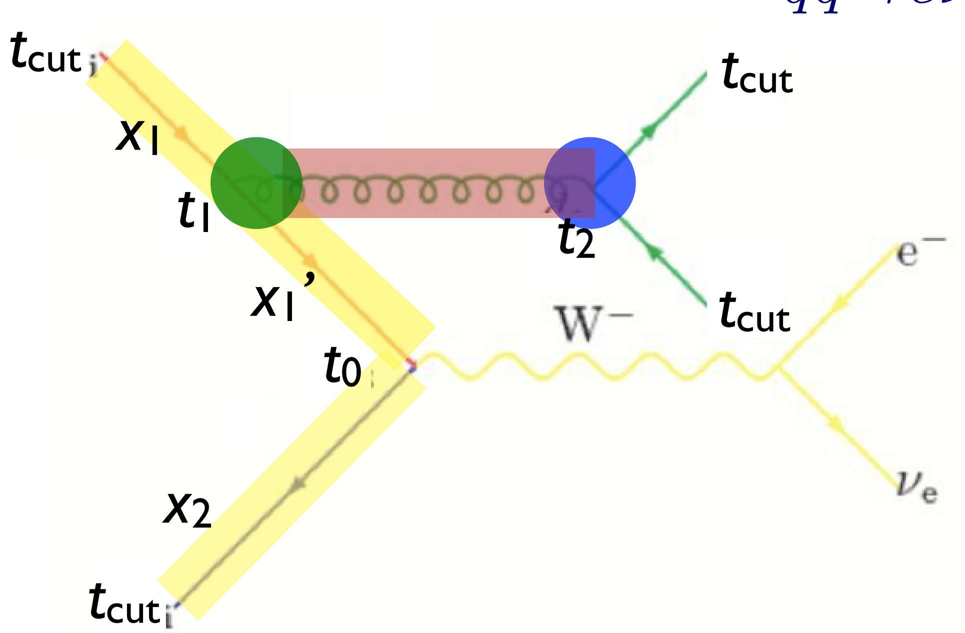
- Let's do the same exercise as before:

$$\mathcal{P} = (\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi}\frac{P_{gq}(z)}{z}\frac{f_q(x_1, t_1)}{f_q(x_1', t_1)}\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$
$$\times \hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s}, \ldots) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0)$$



- We are of course not interested in e^+e^- but p-p(bar) - what happens for initial state radiation?
- Let's do the same exercise as before:

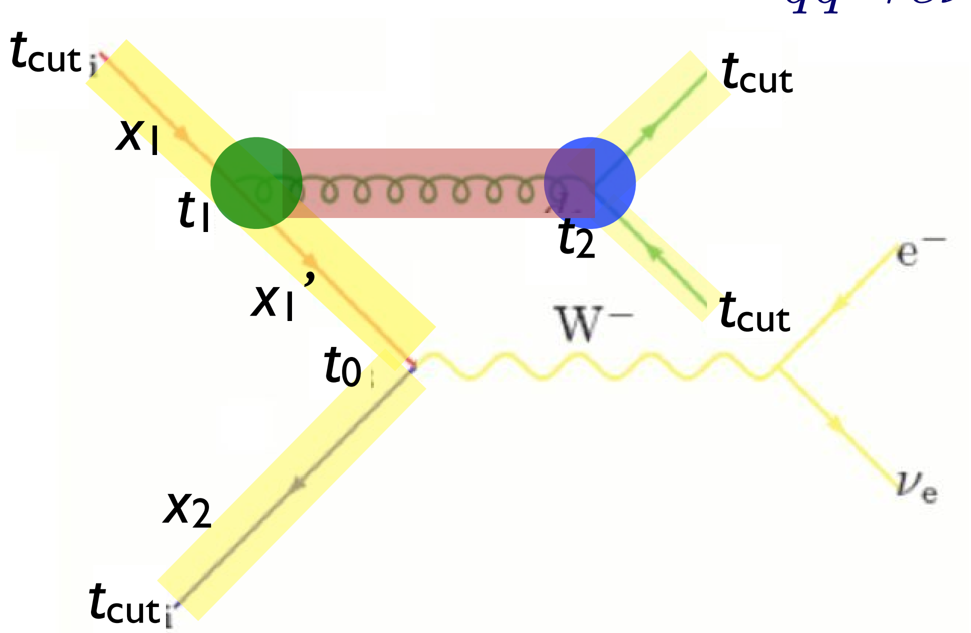
$$\mathcal{P} = (\Delta_{Iq}(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x_1', t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$
$$\times \hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s}, \ldots) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0)$$



Matching for initial state radiation

- We are of course not interested in e^+e^- but p-p(bar) - what happens for initial state radiation?
- Let's do the same exercise as before:

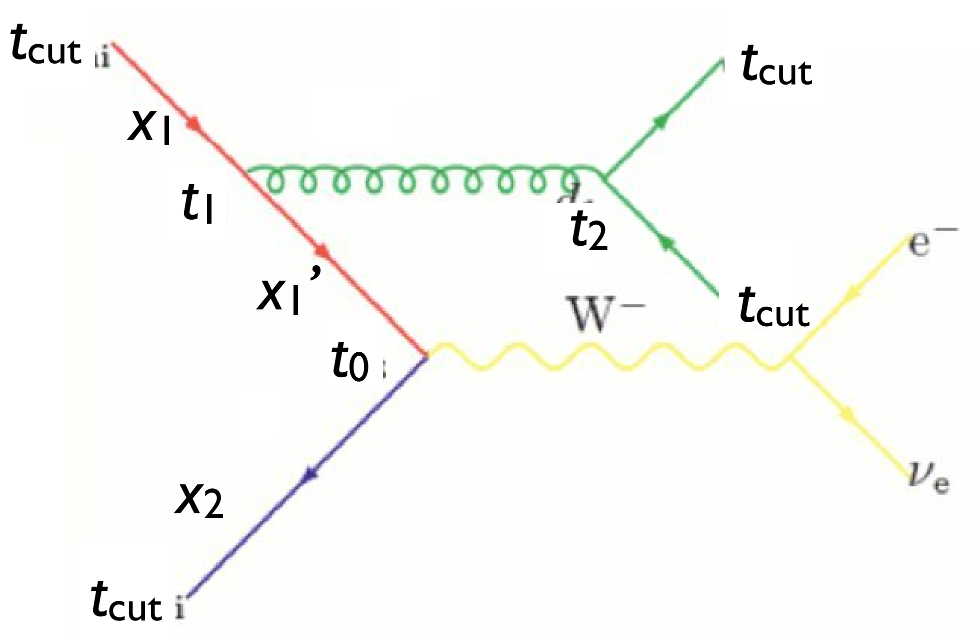
$$\mathcal{P} = (\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\rm cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x'_1, t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$
$$\times \hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s}, \ldots) f_q(x'_1, t_0) f_{\bar{q}}(x_2, t_0)$$



Matching for initial state radiation

$$(\Delta_{Iq}(t_{\rm cut},t_0))^2\Delta_g(t_2,t_1)(\Delta_q(t_{\rm cut},t_2))^2\frac{\alpha_s(t_1)}{2\pi}\frac{P_{gq}(z)}{z}\frac{f_q(x_1,t_1)}{f_q(x_1',t_1)}\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$

$$\times\hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s},\ldots)f_q(x_1',t_0)f_{\bar{q}}(x_2,t_0)$$

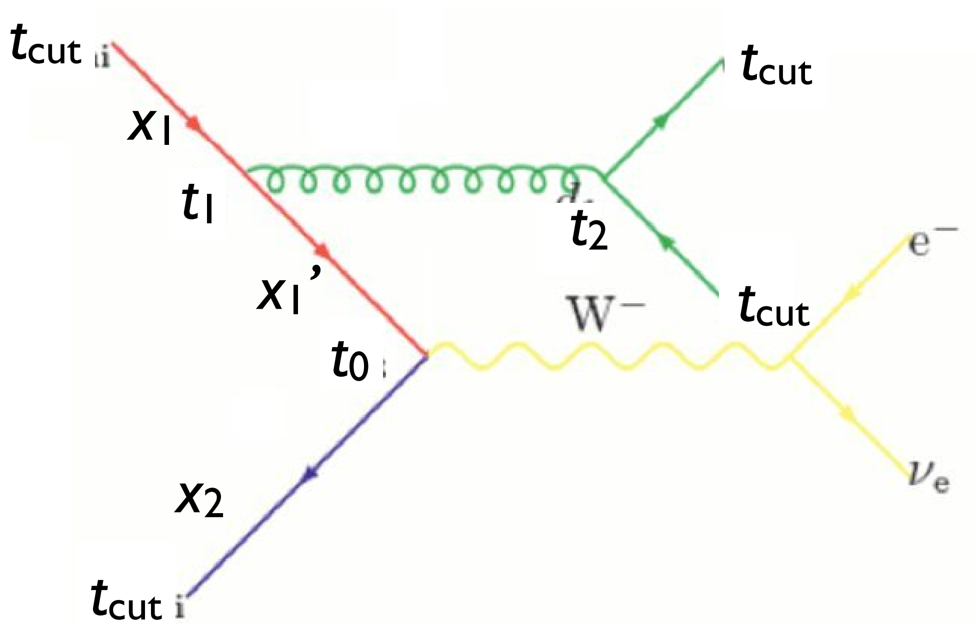


Matching for initial state radiation

$$(\Delta_{Iq}(t_{\rm cut},t_0))^2\Delta_g(t_2,t_1)(\Delta_q(t_{\rm cut},t_2))^2\frac{\alpha_s(t_1)}{2\pi}\frac{P_{gq}(z)}{z}\frac{f_q(x_1,t_1)}{f_q(x_1',t_1)}\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$

$$\times\hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s},\ldots)f_q(x_1',t_0)f_{\bar{q}}(x_2,t_0)$$

ME with α_s evaluated at the scale of each splitting



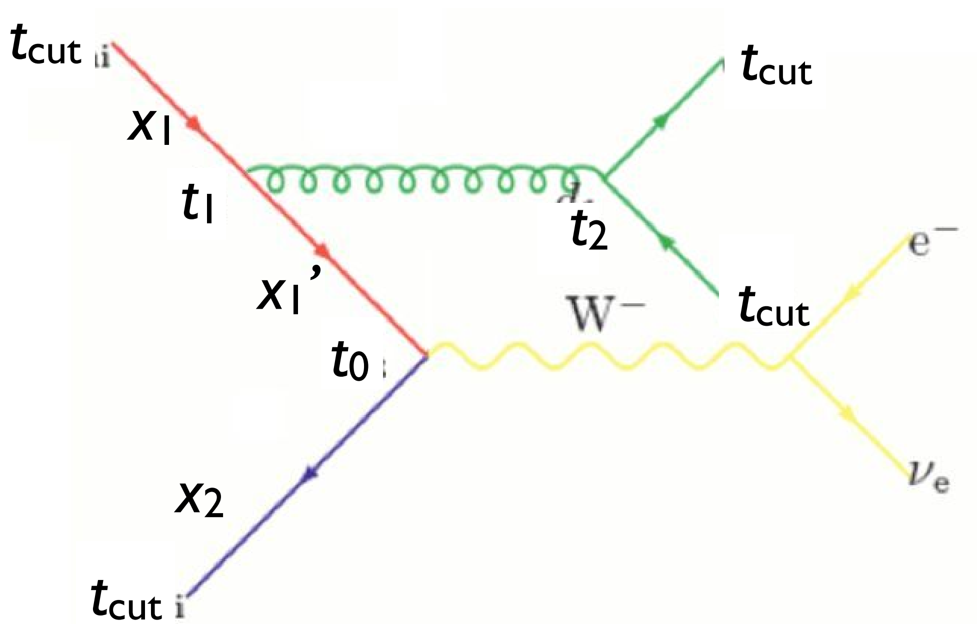
Matching for initial state radiation

$$(\Delta_{Iq}(t_{\rm cut},t_0))^2\Delta_g(t_2,t_1)(\Delta_q(t_{\rm cut},t_2))^2\frac{\alpha_s(t_1)}{2\pi}\frac{P_{gq}(z)}{z}\frac{f_q(x_1,t_1)}{f_q(x_1',t_1)}\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$

$$\times\hat{\sigma}_{q\bar{q}\to e\nu}(\hat{s},\ldots)f_q(x_1',t_0)f_{\bar{q}}(x_2,t_0)$$

ME with α_s evaluated at the scale of each splitting

PDF reweighting



Matching for initial state radiation

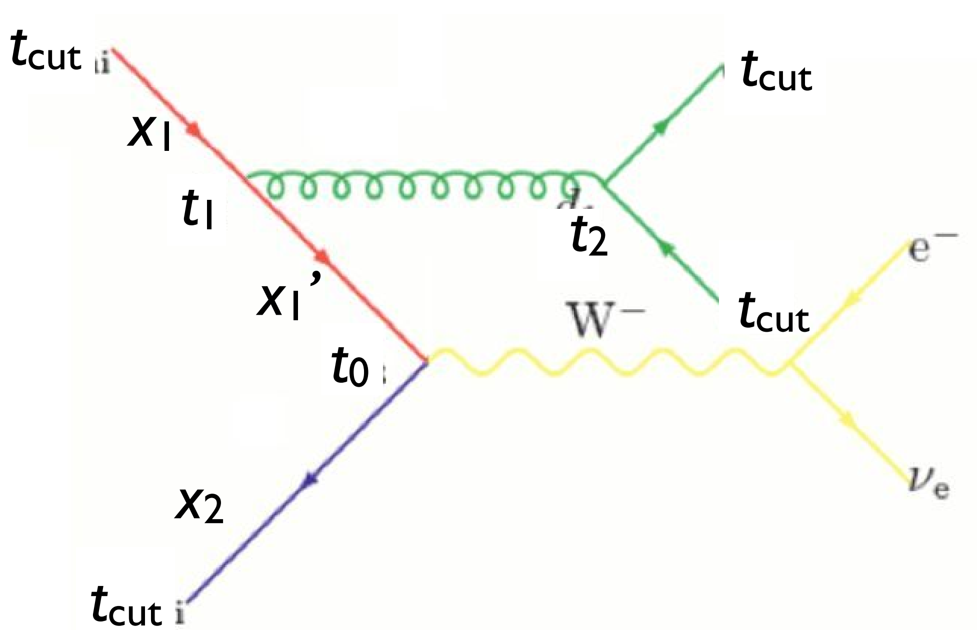
$$(\Delta_{Iq}(t_{\rm cut},t_0))^2\Delta_g(t_2,t_1)(\Delta_q(t_{\rm cut},t_2))^2 \frac{\alpha_s(t_1)}{2\pi}\frac{P_{gq}(z)}{z}\frac{f_q(x_1,t_1)}{f_q(x_1',t_1)}\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$

$$\times\hat\sigma_{q\bar q\to e\nu}(\hat s,\ldots)f_q(x_1',t_0)f_{\bar q}(x_2,t_0)$$

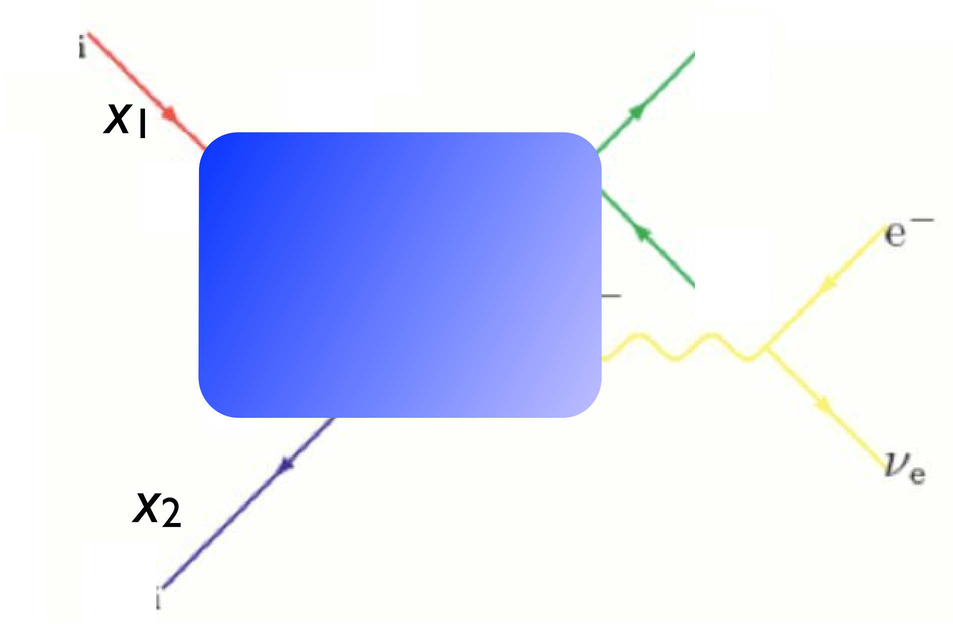
ME with α_s evaluated at the scale of each splitting

PDF reweighting

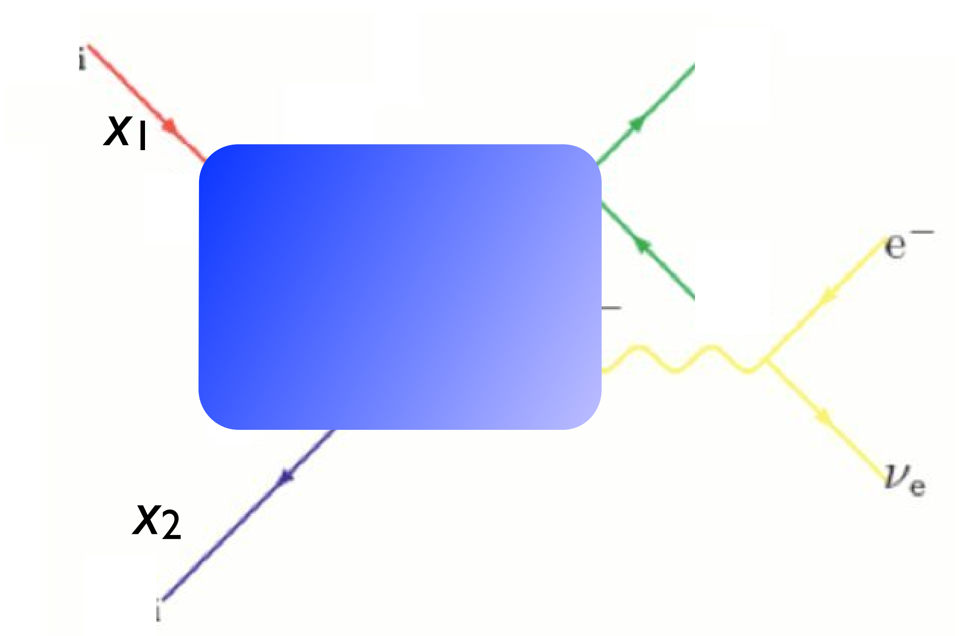
Sudakov suppression due to non-branching above scale $t_{\rm cut}$



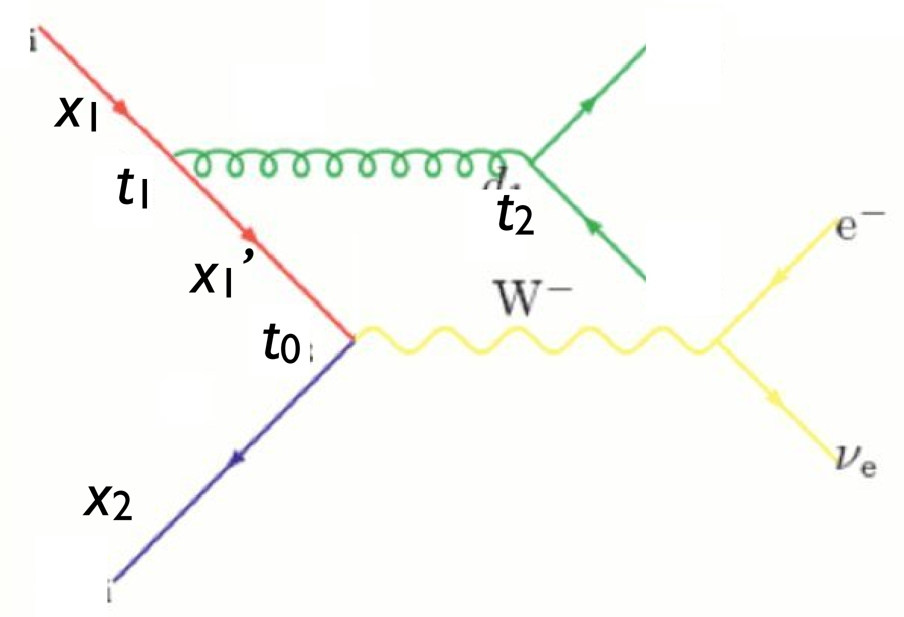
Matching for initial state radiation



- Again, use a clustering scheme to get a parton shower history

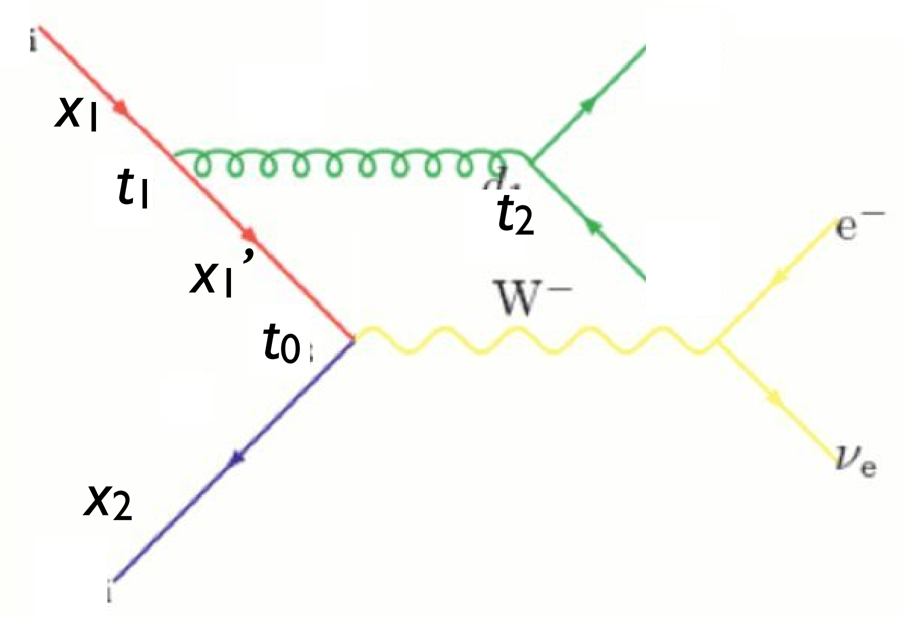


- Again, use a clustering scheme to get a parton shower history



- Again, use a clustering scheme to get a parton shower history
- Now, reweight both due to α_s and PDF

$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)} \frac{f_q(x_1', t_0)}{f_q(x_1', t_1)}$$

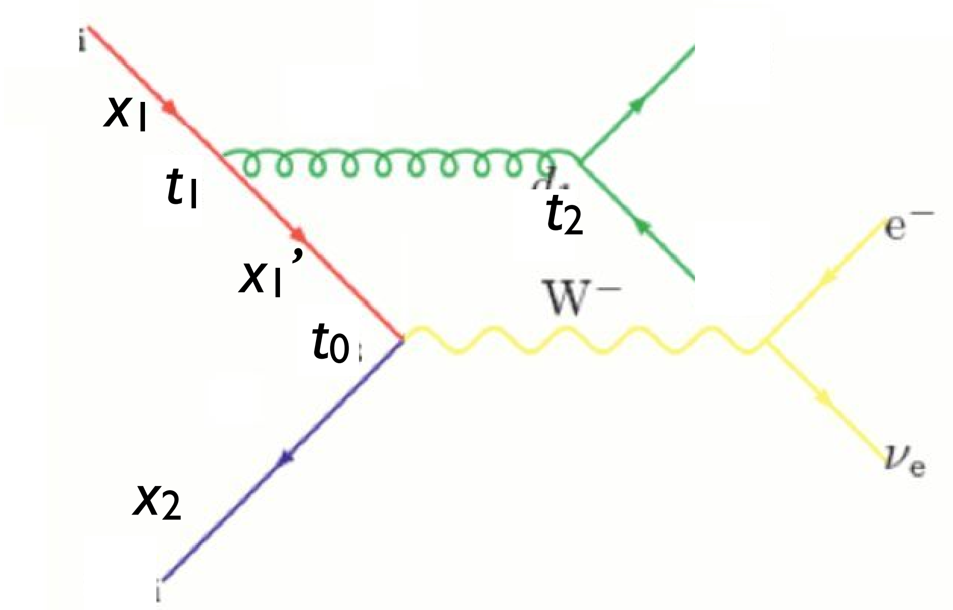


- Again, use a clustering scheme to get a parton shower history
- Now, reweight both due to α_s and PDF

$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)} \frac{f_q(x_1', t_0)}{f_q(x_1', t_1)}$$

- Remember to use first clustering scale on each side for PDF scale:

$$\mathcal{P}_{\text{event}} = \hat{\sigma}(x_1, x_2, p_3, p_4, \dots) f_q(x_1, t_1) f_{\bar{q}}(x_2, t_0)$$



Matching schemes

- We still haven't specified how to apply the Sudakov reweighting to the matrix element
- Three general schemes available in the literature:
  - ➡ CKKW scheme [Catani,Krauss,Kuhn,Webber 2001; Krauss 2002]
  - ➡ Lönnblad scheme (or CKKW-L) [Lönnblad 2002]
  - ➡ MLM scheme [Mangano *unpublished* 2002; Mangano et al. 2007]

CKKW matching

[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]

CKKW matching

[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]

- Apply the required Sudakov suppression

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2$$

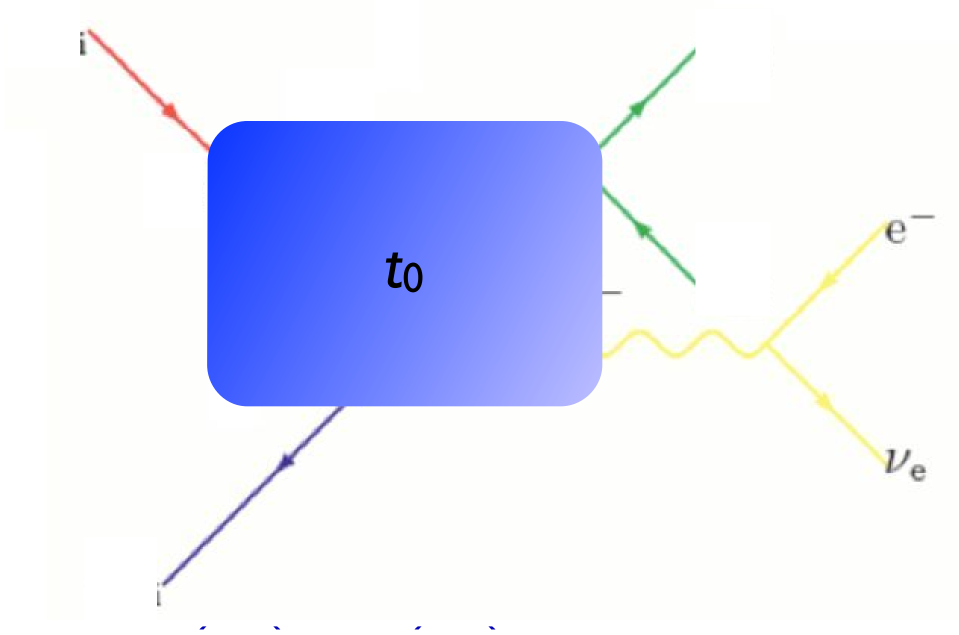
analytically, using the best available (NLL) Sudakovs.

[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]

- Apply the required Sudakov suppression

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2$$

  analytically, using the best available (NLL) Sudakovs.
- Perform "truncated showering": Run the parton shower starting at t_0, but forbid any showers above the cutoff scale t_{cut}.



CKKW matching

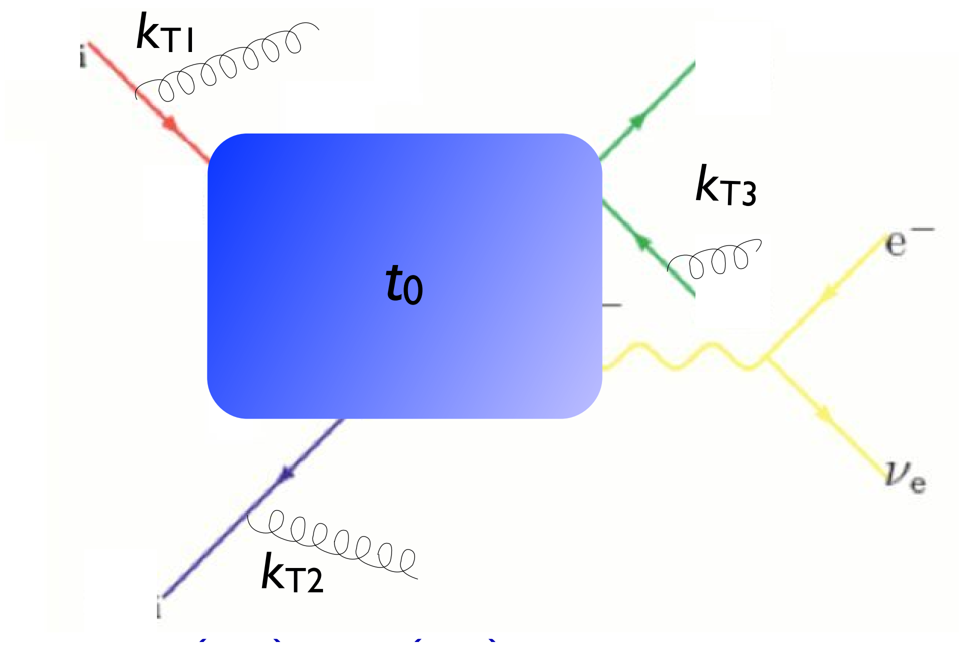
[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]

- Apply the required Sudakov suppression

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2$$

analytically, using the best available (NLL) Sudakovs.

- Perform "truncated showering": Run the parton shower starting at t_0, but forbid any showers above the cutoff scale $t_{\rm cut}$.



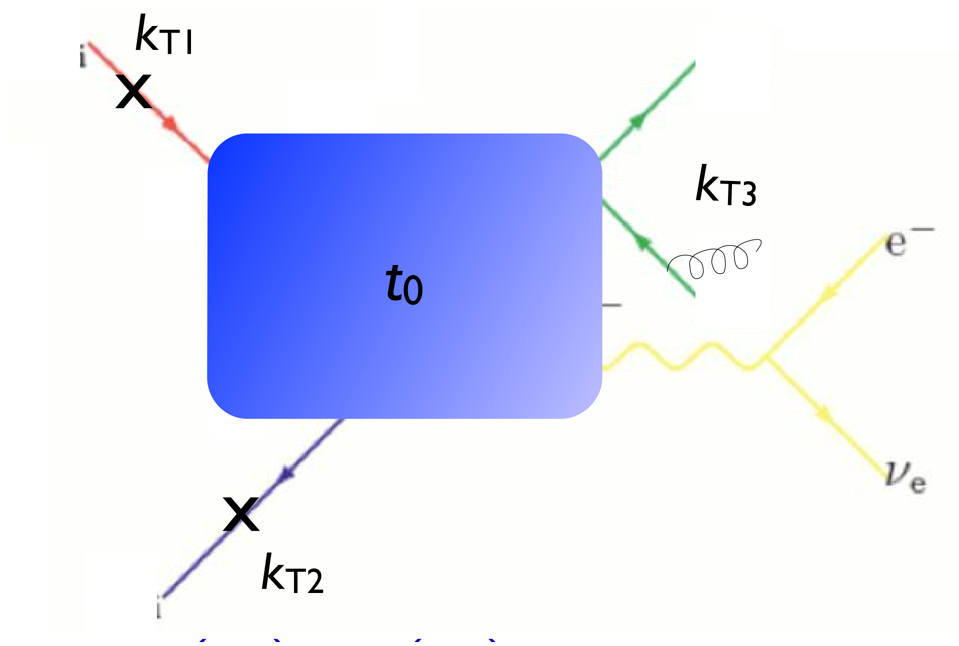
[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]

- Apply the required Sudakov suppression

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2$$

analytically, using the best available (NLL) Sudakovs.

- Perform "truncated showering": Run the parton shower starting at t_0, but forbid any showers above the cutoff scale $t_{\rm cut}$.



CKKW matching

[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]

- Apply the required Sudakov suppression

$$(\Delta_{Iq}(t_{\rm cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\rm cut}, t_2))^2$$

analytically, using the best available (NLL) Sudakovs.

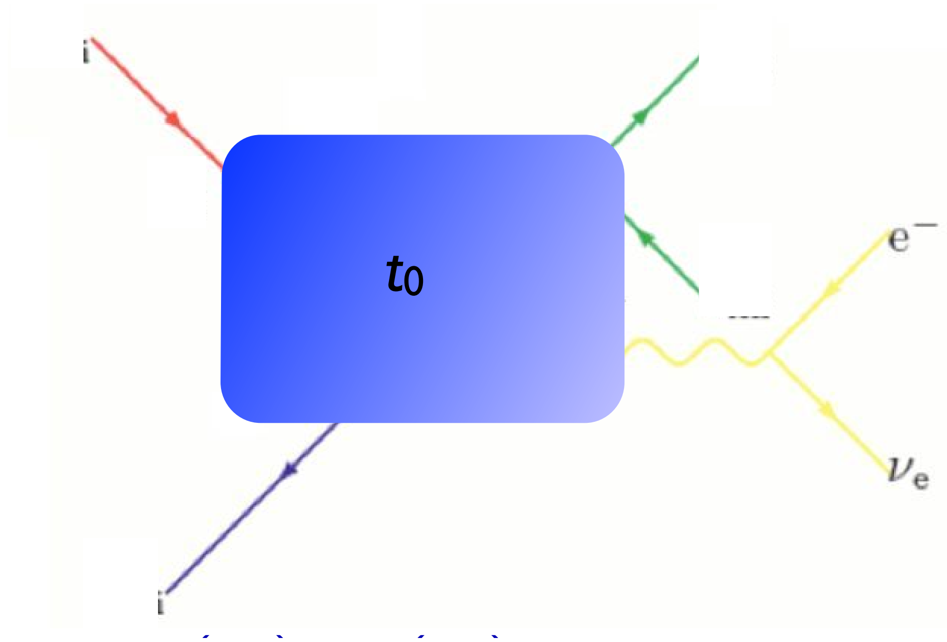
- Perform "truncated showering": Run the parton shower starting at t_0, but forbid any showers above the cutoff scale $t_{\rm cut}$.

✓ Best theoretical treatment of matrix element

- Requires dedicated PS implementation
- Mismatch between analytical Sudakov and (non-NLL) shower
- Implemented in Sherpa (v. 1.1)

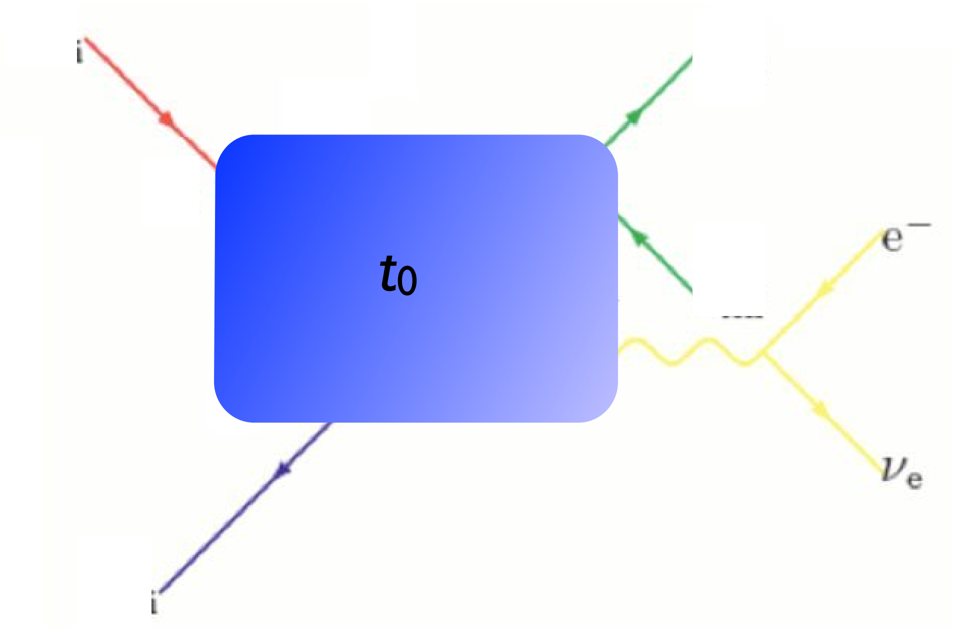
CKKW-L matching

[Lönnblad 2002]
[Hoeche et al. 2009]



CKKW-L matching

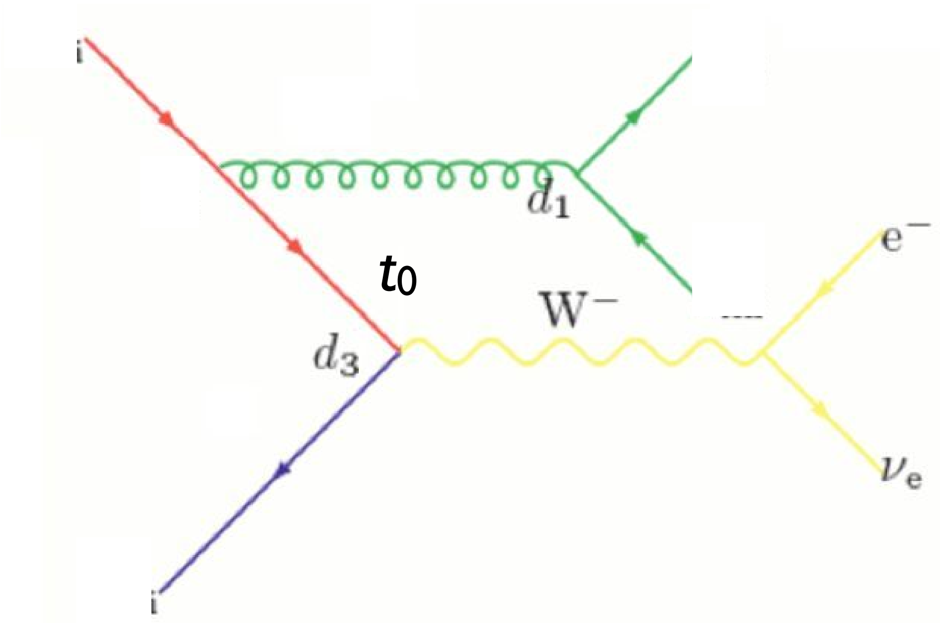
[Lönnblad 2002]
[Hoeche et al. 2009]



- Cluster back to "parton shower history"

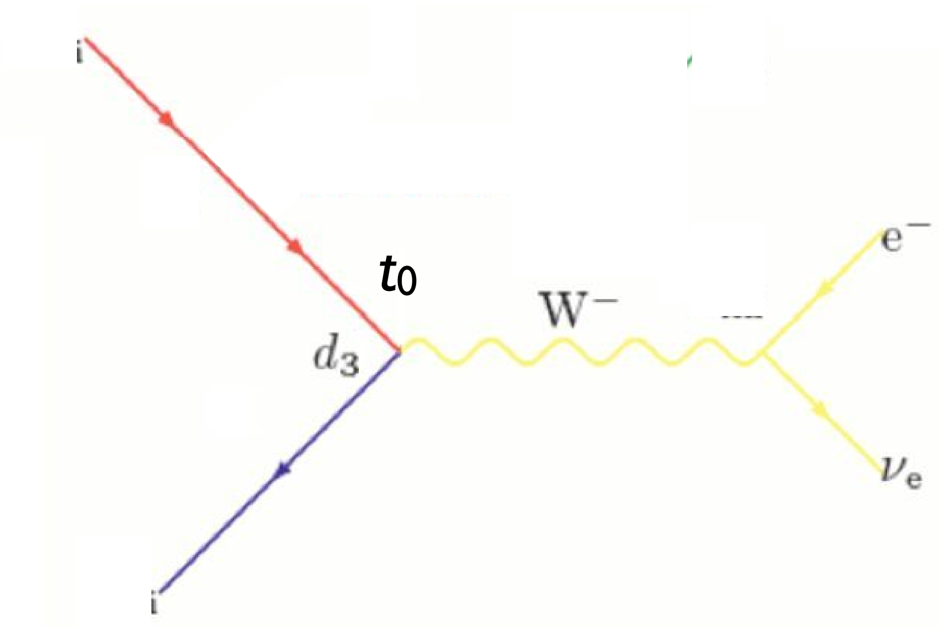
CKKW-L matching

[Lönnblad 2002]
[Hoeche et al. 2009]



- Cluster back to "parton shower history"

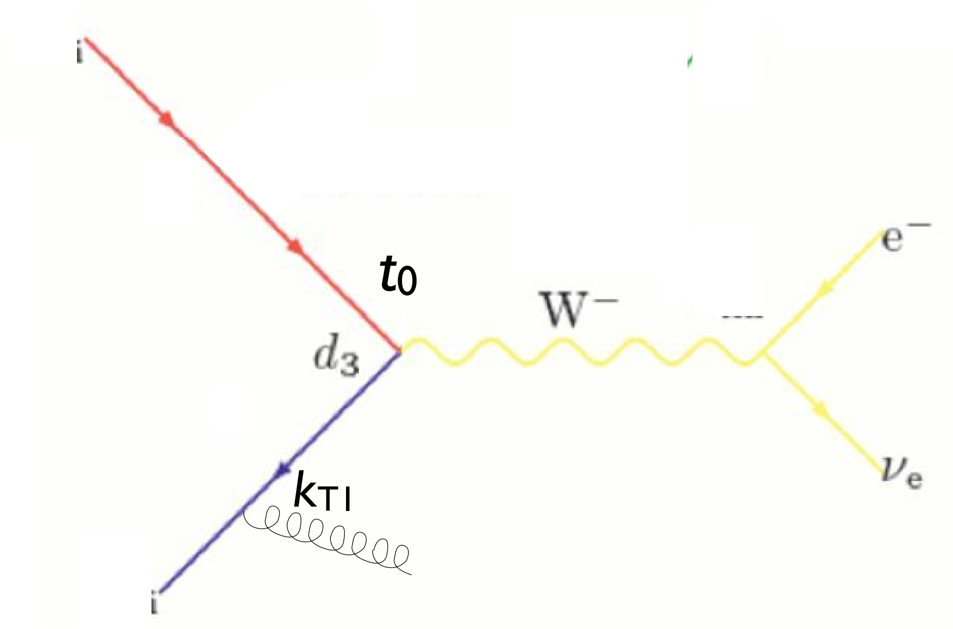
[Lönnblad 2002]
[Hoeche et al. 2009]



- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step

CKKW-L matching

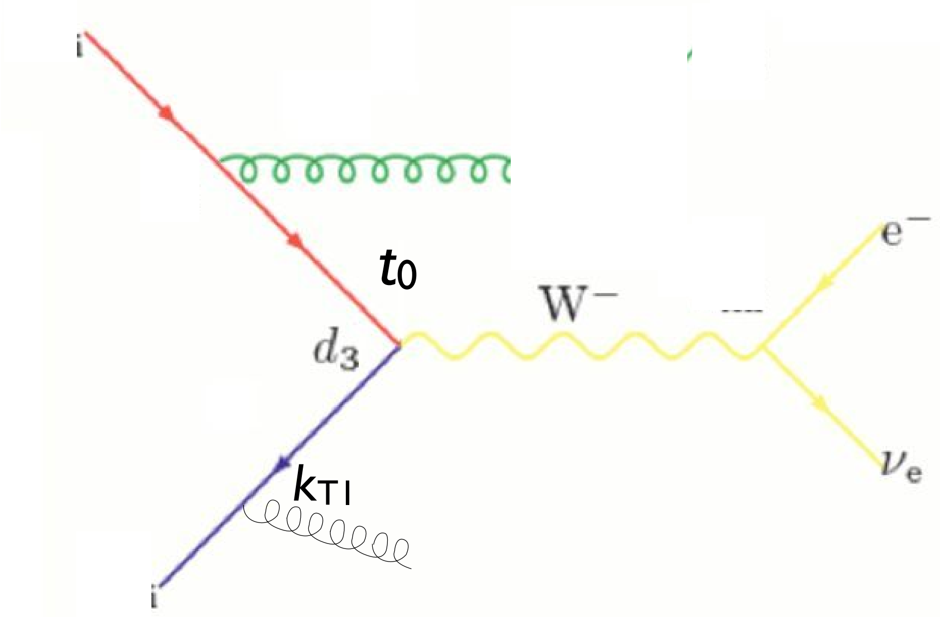
[Lönnblad 2002]
[Hoeche et al. 2009]



- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step

CKKW-L matching

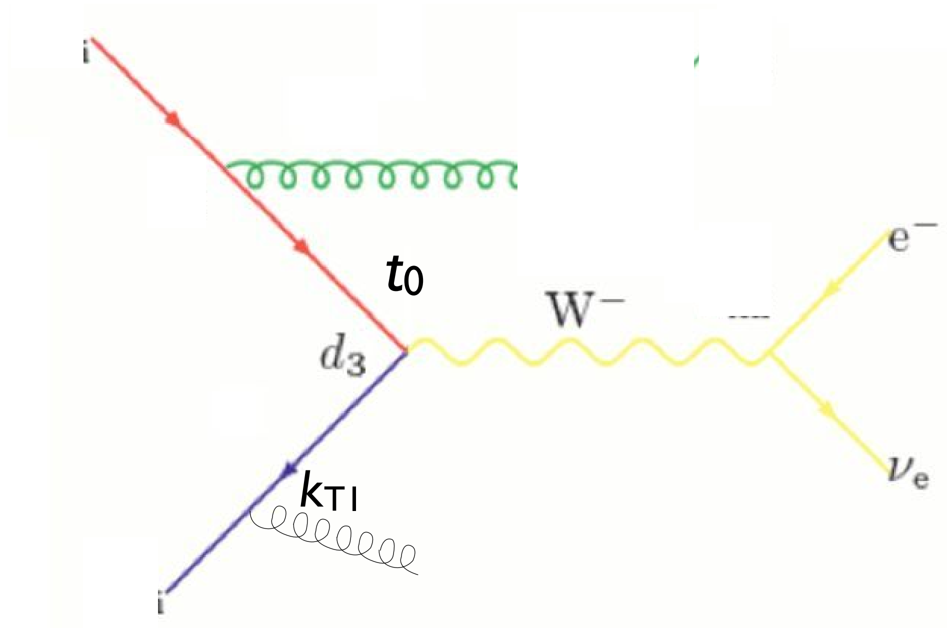
[Lönnblad 2002]
[Hoeche et al. 2009]



- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step

CKKW-L matching

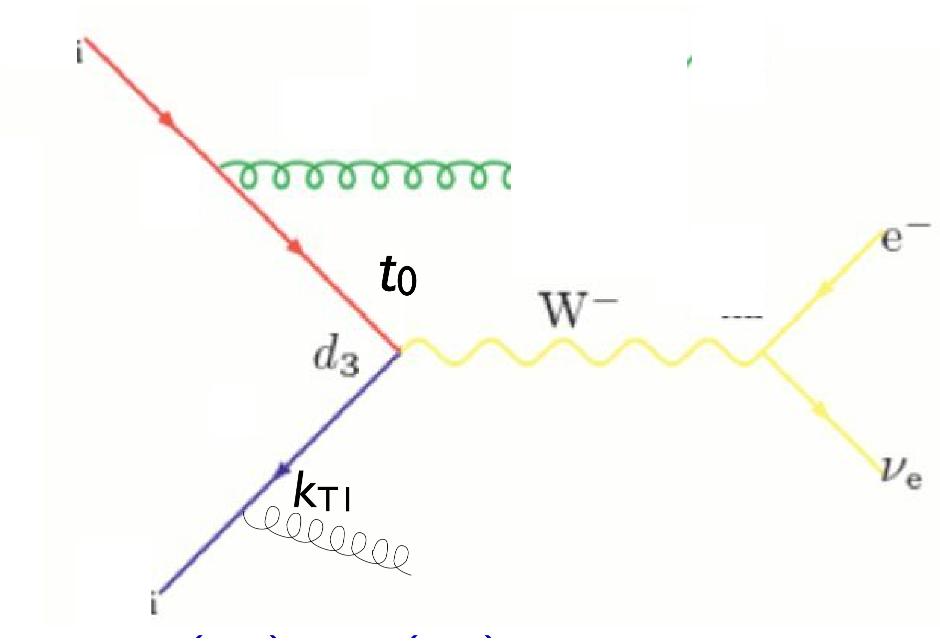
[Lönnblad 2002]
[Hoeche et al. 2009]



- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
- Veto the event if any shower is harder than the clustering scale for the next step (or t_{cut}, if last step)

CKKW-L matching

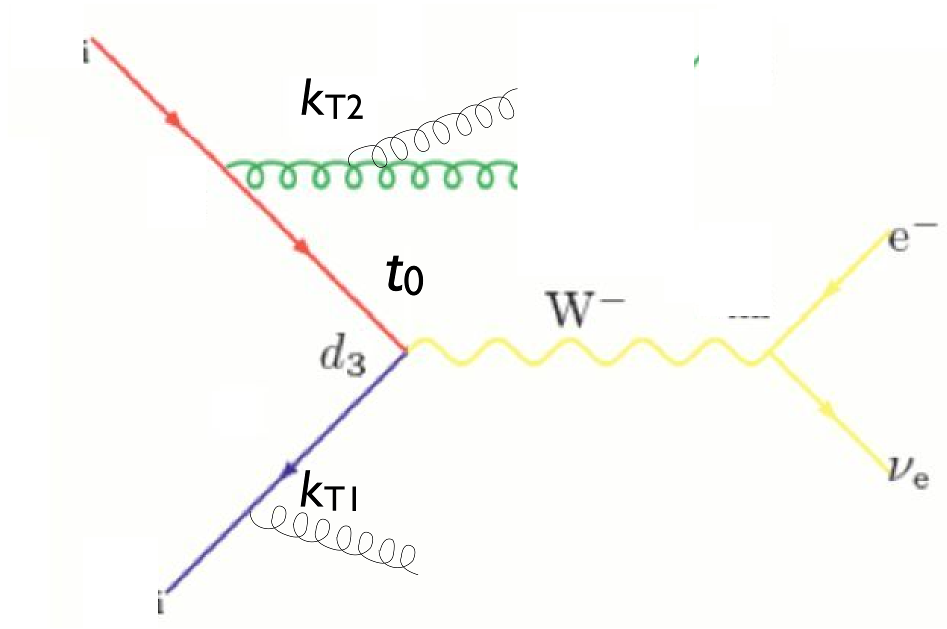
[Lönnblad 2002]
[Hoeche et al. 2009]



- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
- Veto the event if any shower is harder than the clustering scale for the next step (or t_{cut}, if last step)
- Keep any shower emissions that are softer than the clustering scale for the next step

CKKW-L matching

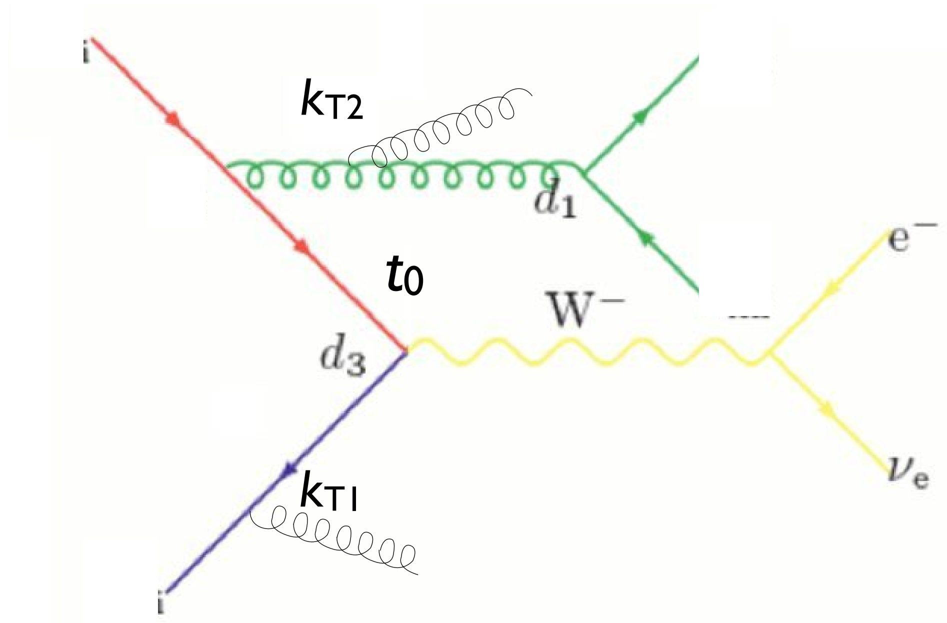
[Lönnblad 2002]
[Hoeche et al. 2009]



- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
- Veto the event if any shower is harder than the clustering scale for the next step (or t_{cut}, if last step)
- Keep any shower emissions that are softer than the clustering scale for the next step

CKKW-L matching

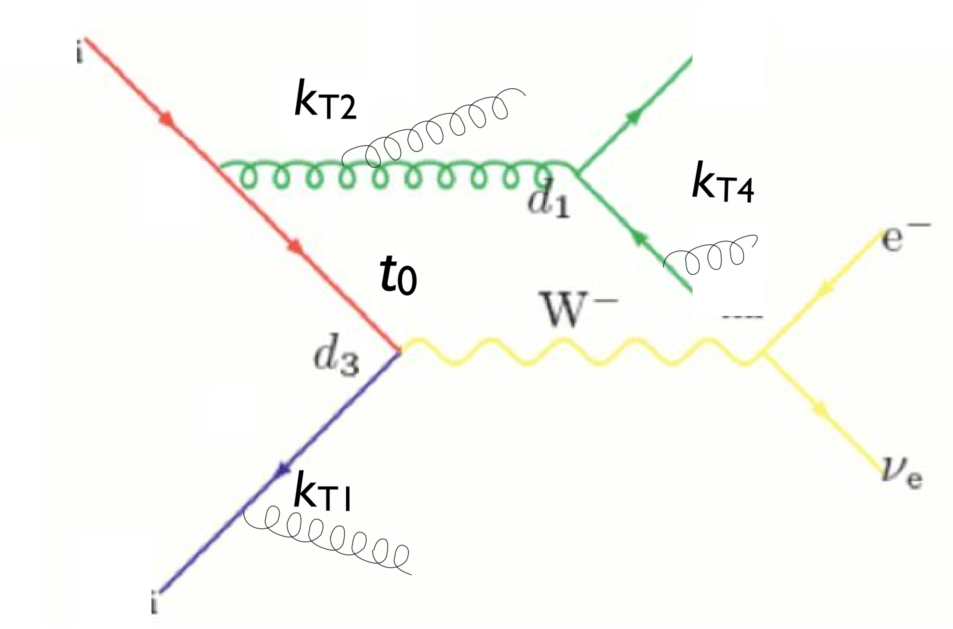
[Lönnblad 2002]
[Hoeche et al. 2009]



- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
- Veto the event if any shower is harder than the clustering scale for the next step (or t_{cut}, if last step)
- Keep any shower emissions that are softer than the clustering scale for the next step

CKKW-L matching

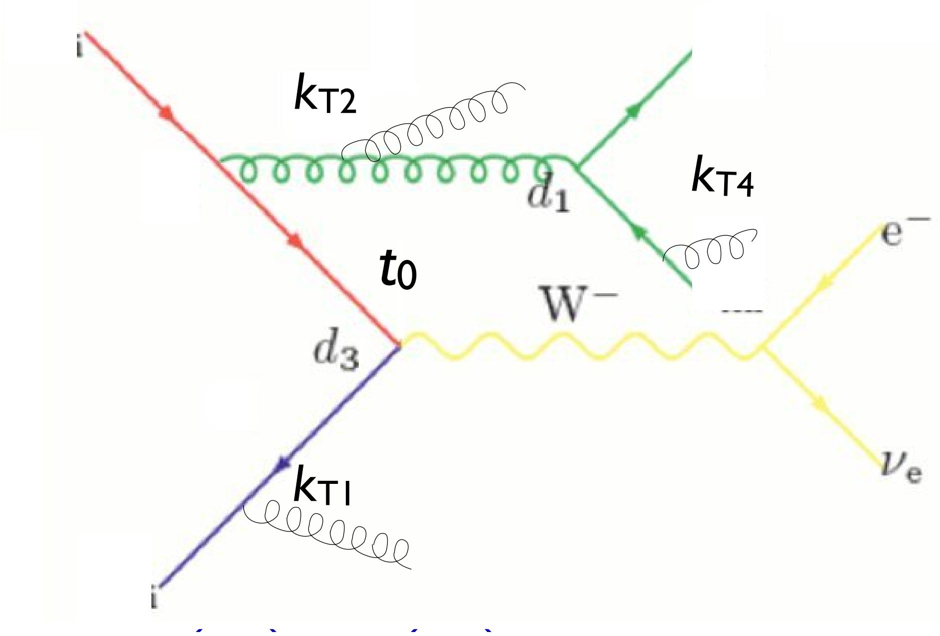
[Lönnblad 2002]
[Hoeche et al. 2009]



- Cluster back to "parton shower history"
- Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
- Veto the event if any shower is harder than the clustering scale for the next step (or t_{cut}, if last step)
- Keep any shower emissions that are softer than the clustering scale for the next step

CKKW-L matching

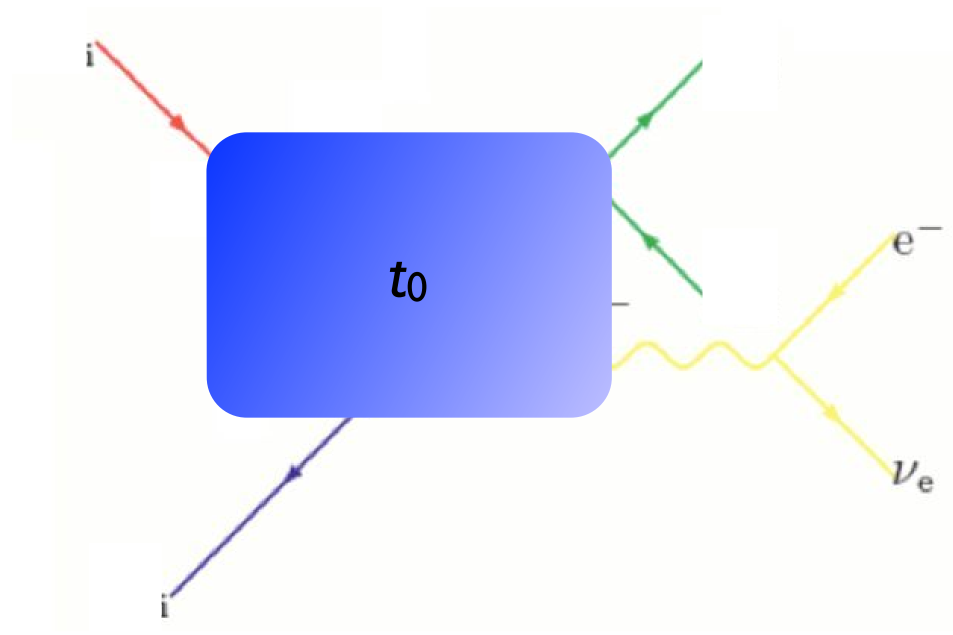
[Lönnblad 2002]
[Hoeche et al. 2009]



- ✓ Automatic agreement between Sudakov and shower
- Requires dedicated PS implementation
  - ➡ Need multiple implementations to compare between showers
- Implemented in Ariadne, Sherpa (v. 1.2), and Pythia 8

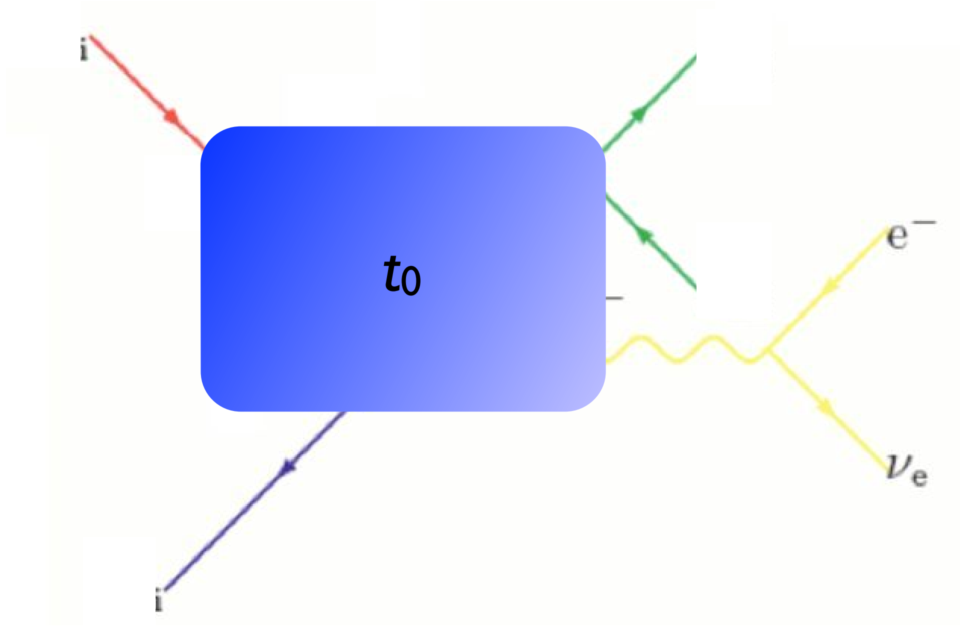
MLM matching

[M.L. Mangano, ~2002, 2007]
[J.A. et al 2007, 2008]



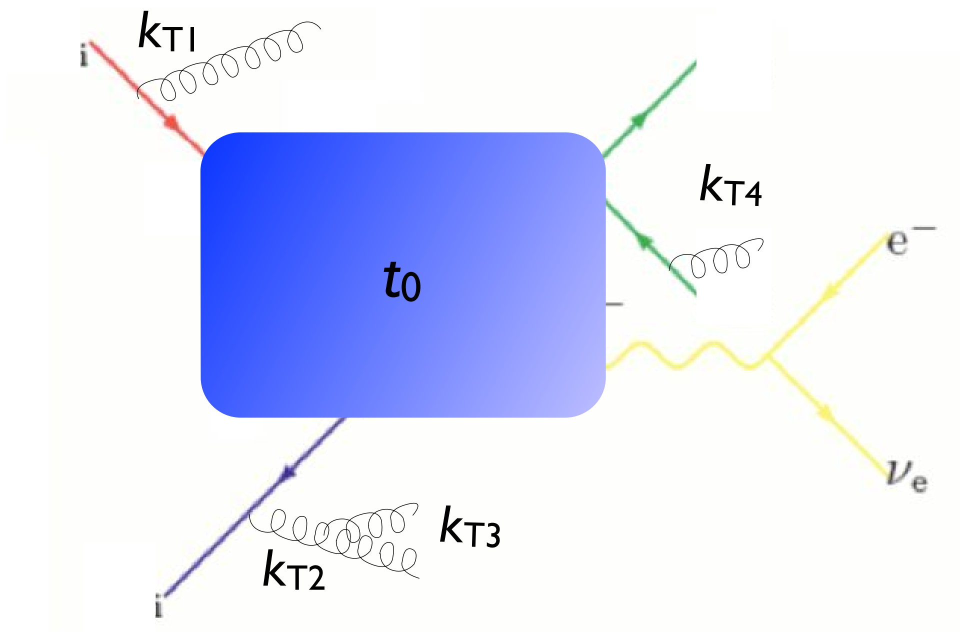
[M.L. Mangano, ~2002, 2007]
[J.A. et al 2007, 2008]

- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t_0!



[M.L. Mangano, ~2002, 2007]
[J.A. et al 2007, 2008]

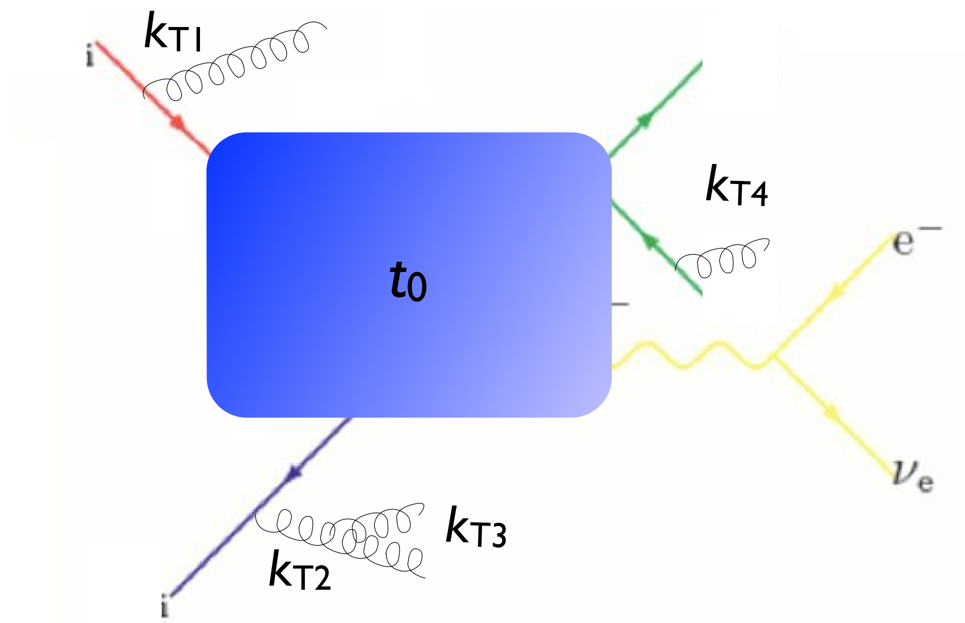
- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t_0!



MLM matching

[M.L. Mangano, ~2002, 2007]
[J.A. et al 2007, 2008]

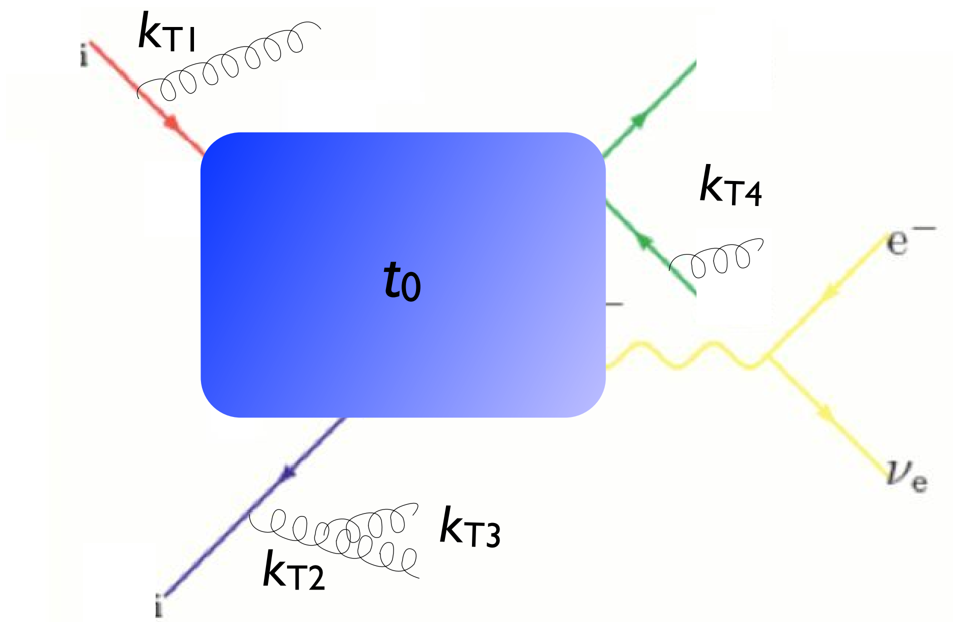
- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t_0!



- Perform jet clustering after PS - if hardest jet $k_{T1} > t_{cut}$ or there are jets not matched to partons, reject the event

[M.L. Mangano, ~2002, 2007]
[J.A. et al 2007, 2008]

- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t_0!



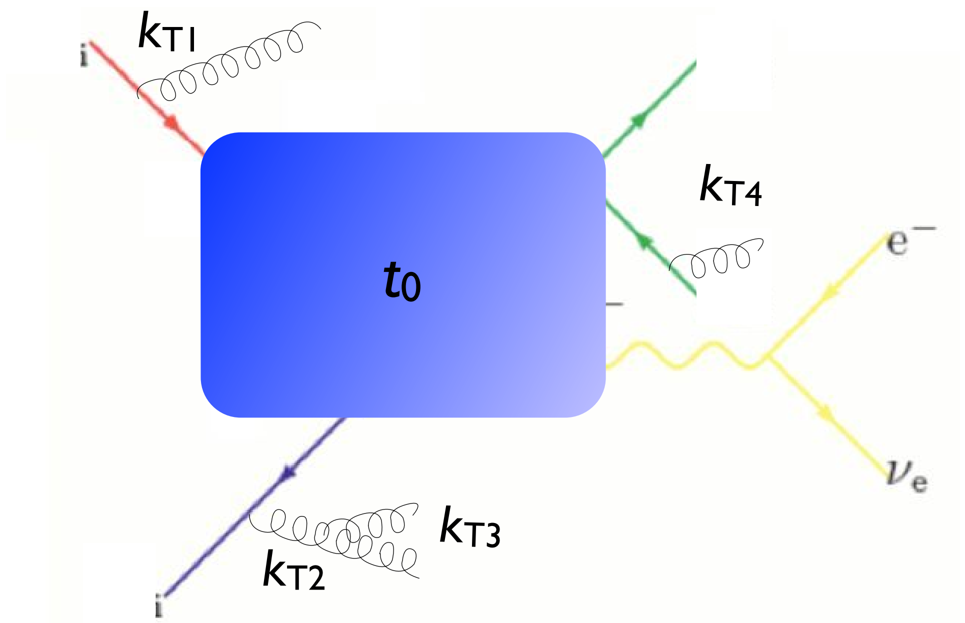
- Perform jet clustering after PS - if hardest jet $k_{T1} > t_{cut}$ or there are jets not matched to partons, reject the event
- The resulting Sudakov suppression from the procedure is

$$(\Delta_{Iq}(t_{cut}, t_0))^2 (\Delta_q(t_{cut}, t_0))^2$$

which turns out to be a good enough approximation of the correct expression $$(\Delta_{Iq}(t_{cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{cut}, t_2))^2$$

[M.L. Mangano, ~2002, 2007]
[J.A. et al 2007, 2008]

- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t_0!



✓ Simplest available scheme

✓ Allows matching with any shower, without modification

➡ Sudakov suppression not exact, minor mismatch with shower

- Implemented in AlpGen, HELAC, MadGraph+Pythia 6

Highest multiplicity sample

- In the previous, assumed we can simulate all parton multiplicities by the ME
- In practice, we can only do limited number of final-state partons with matrix element (up to 4-5 or so)
- For the highest jet multiplicity that we generate with the matrix element, we need to allow additional jets above the matching scale t_{cut}, since we will otherwise not get a jet-inclusive description – but still can't allow PS radiation harder than the ME partons

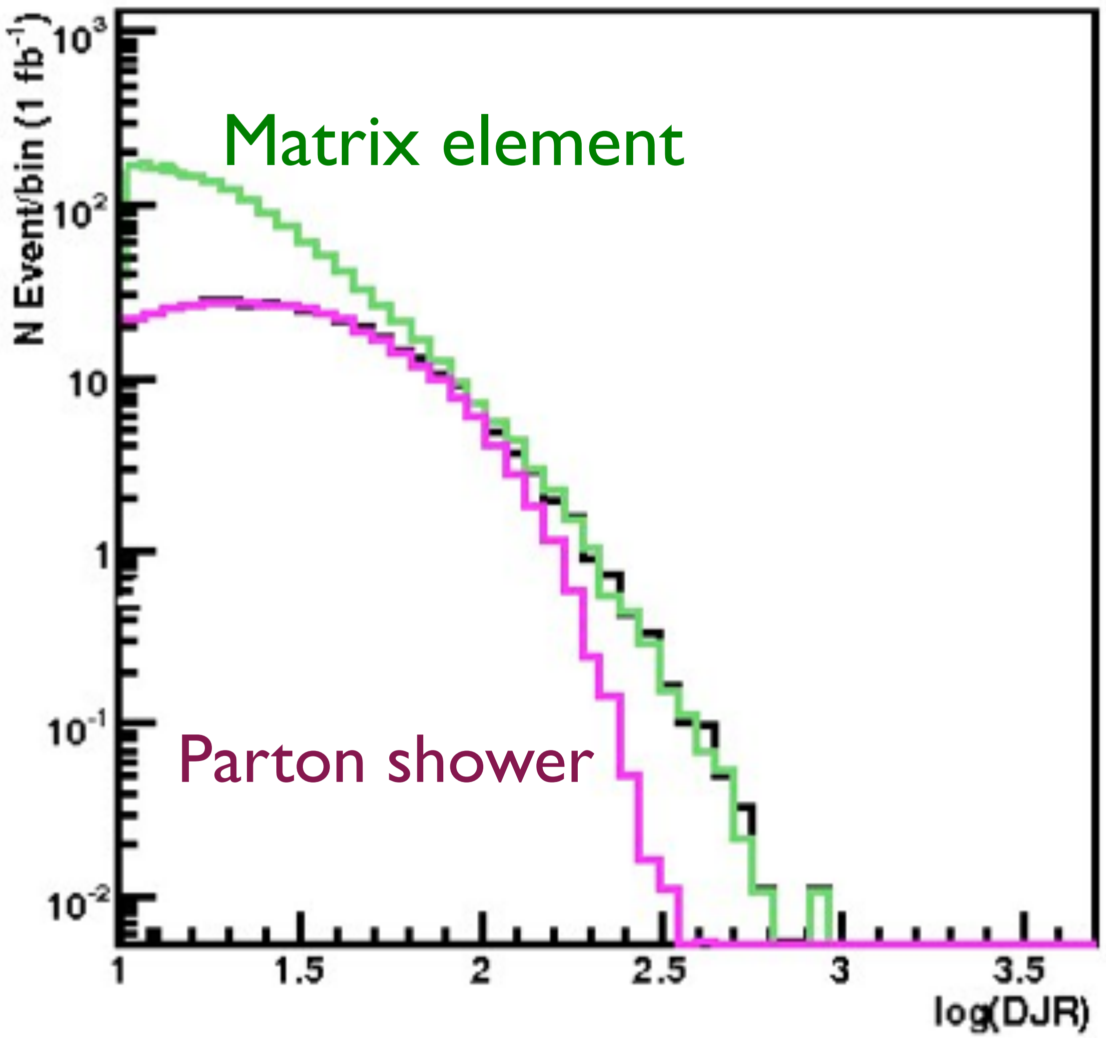
➡ Need to replace t_{cut} by the clustering scale for the softest ME parton for the highest multiplicity

matching schemes

- We have a number of choices to make in the above procedure. The most important are:
  1. The clustering scheme used to determine the parton shower history of the ME event
  2. What to use for the scale Q^2 (factorization scale)
  3. How to divide the phase space between parton showers and matrix elements

Back to the "matching goal"

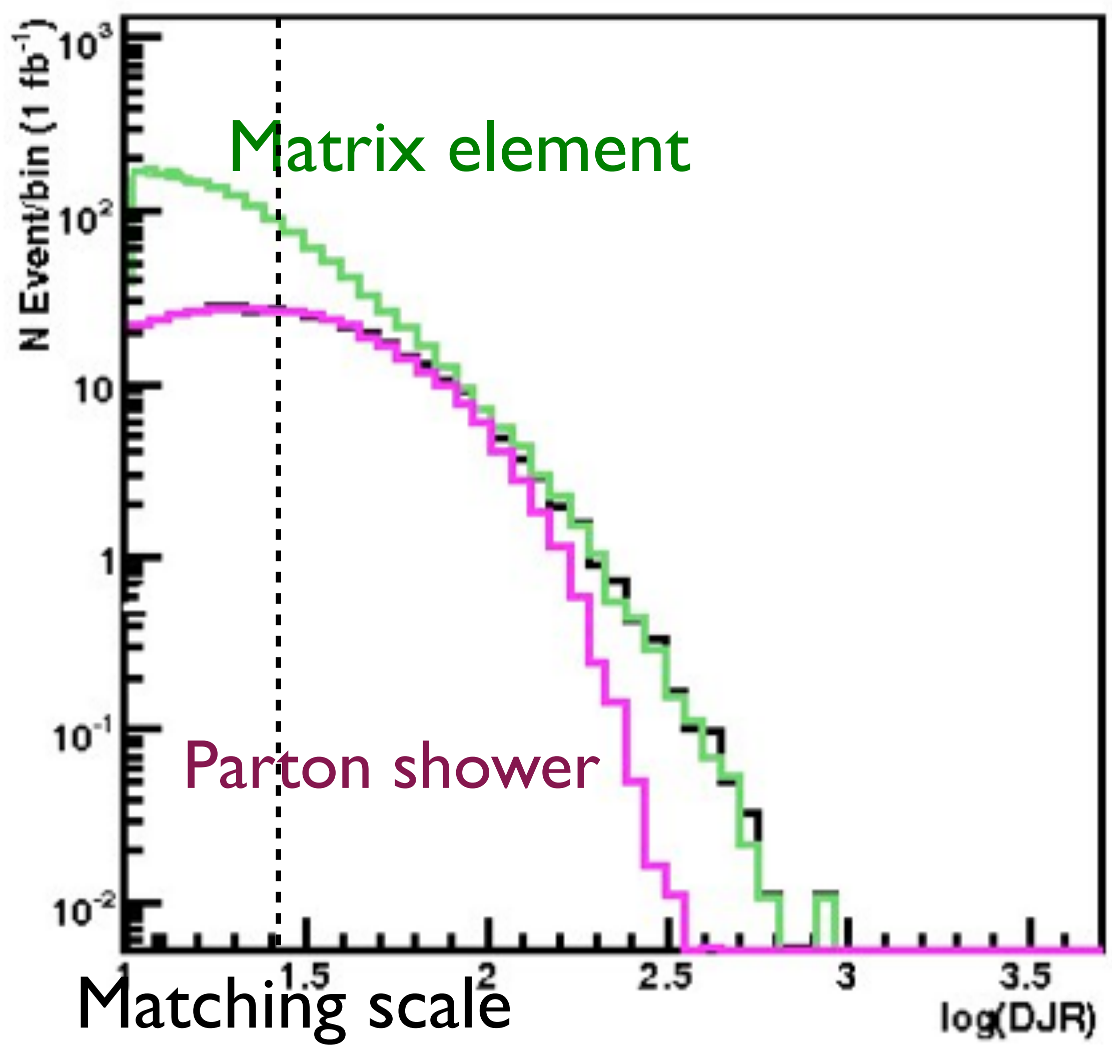
- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Back to the "matching goal"

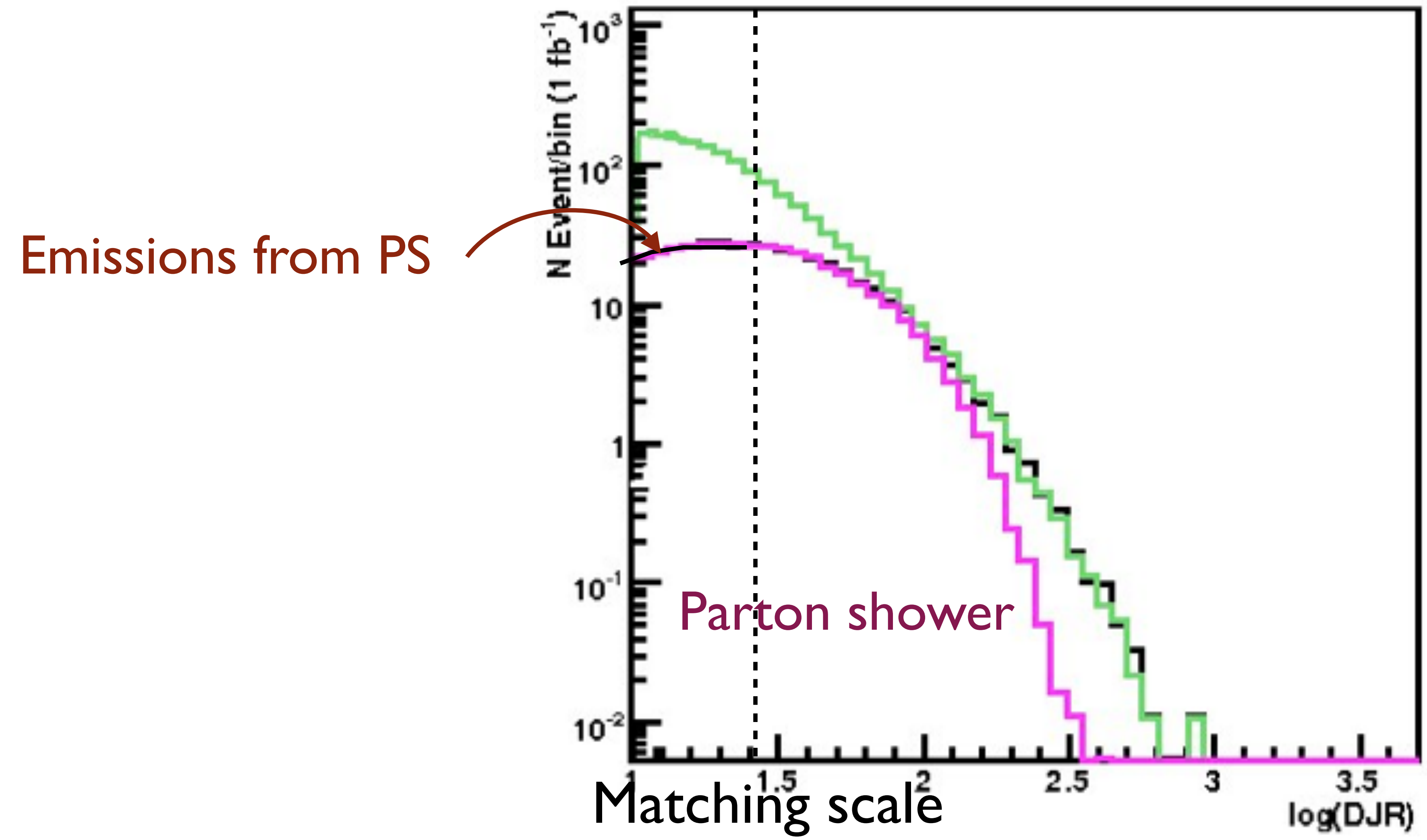
- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Back to the “matching goal”

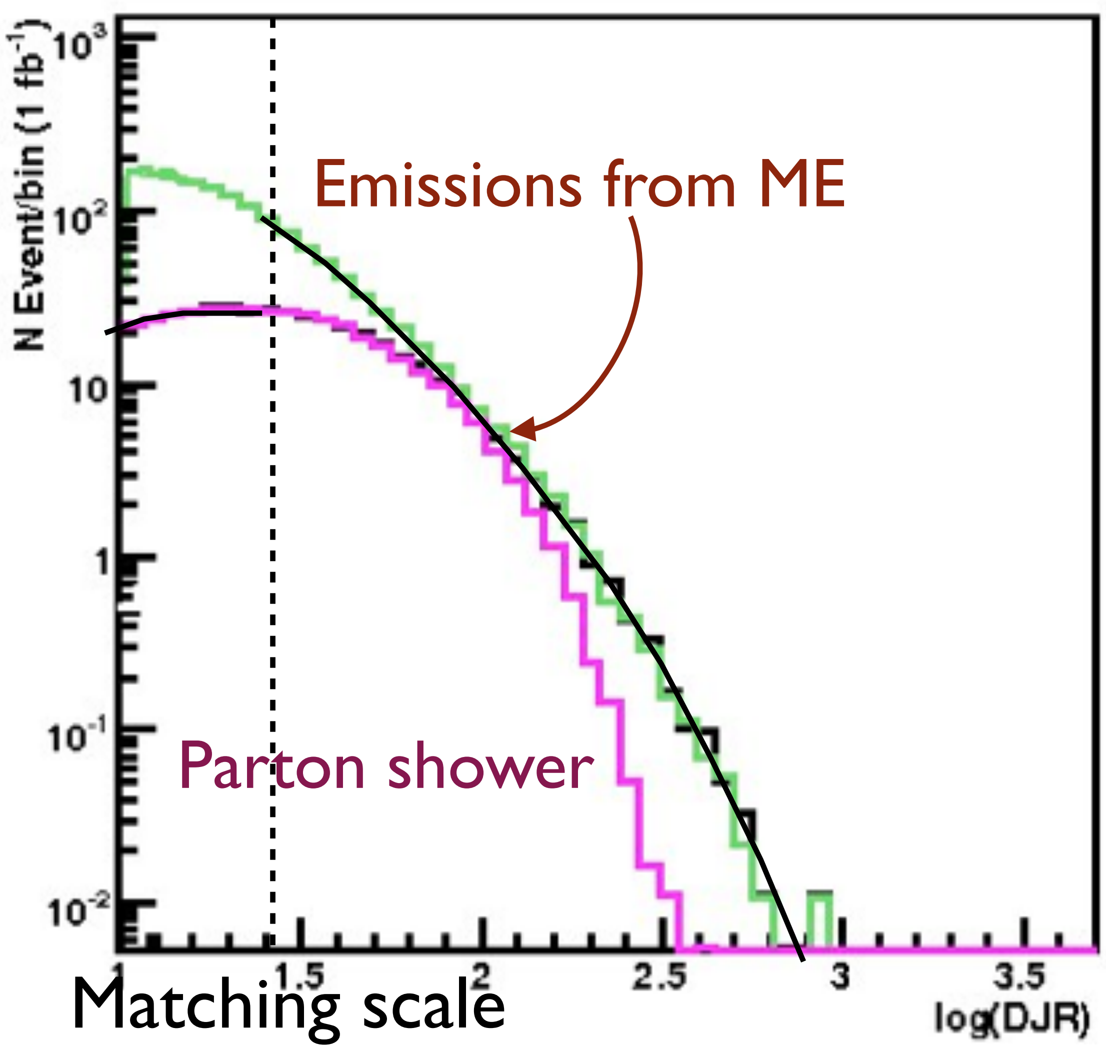
- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Back to the "matching goal"

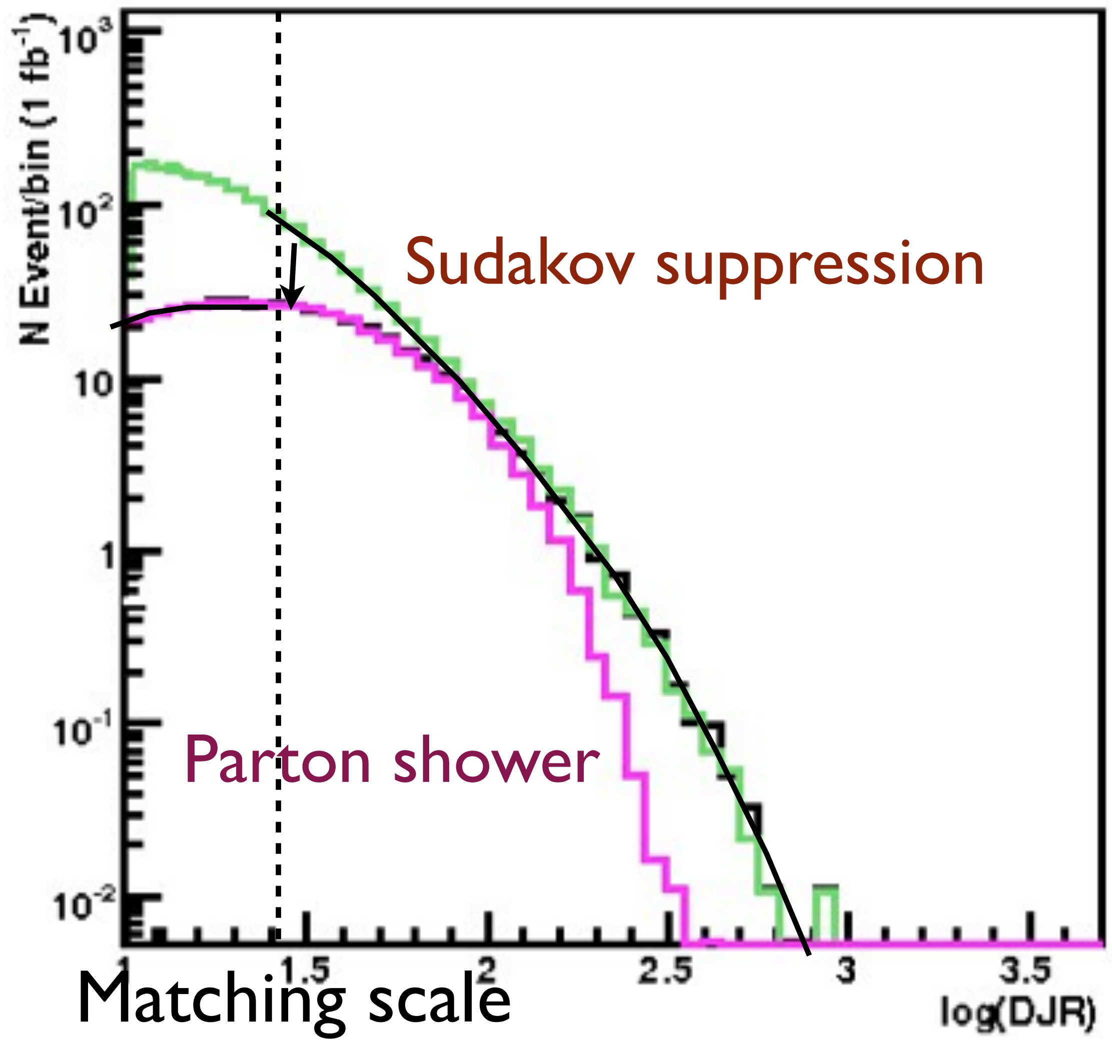
- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Back to the "matching goal"

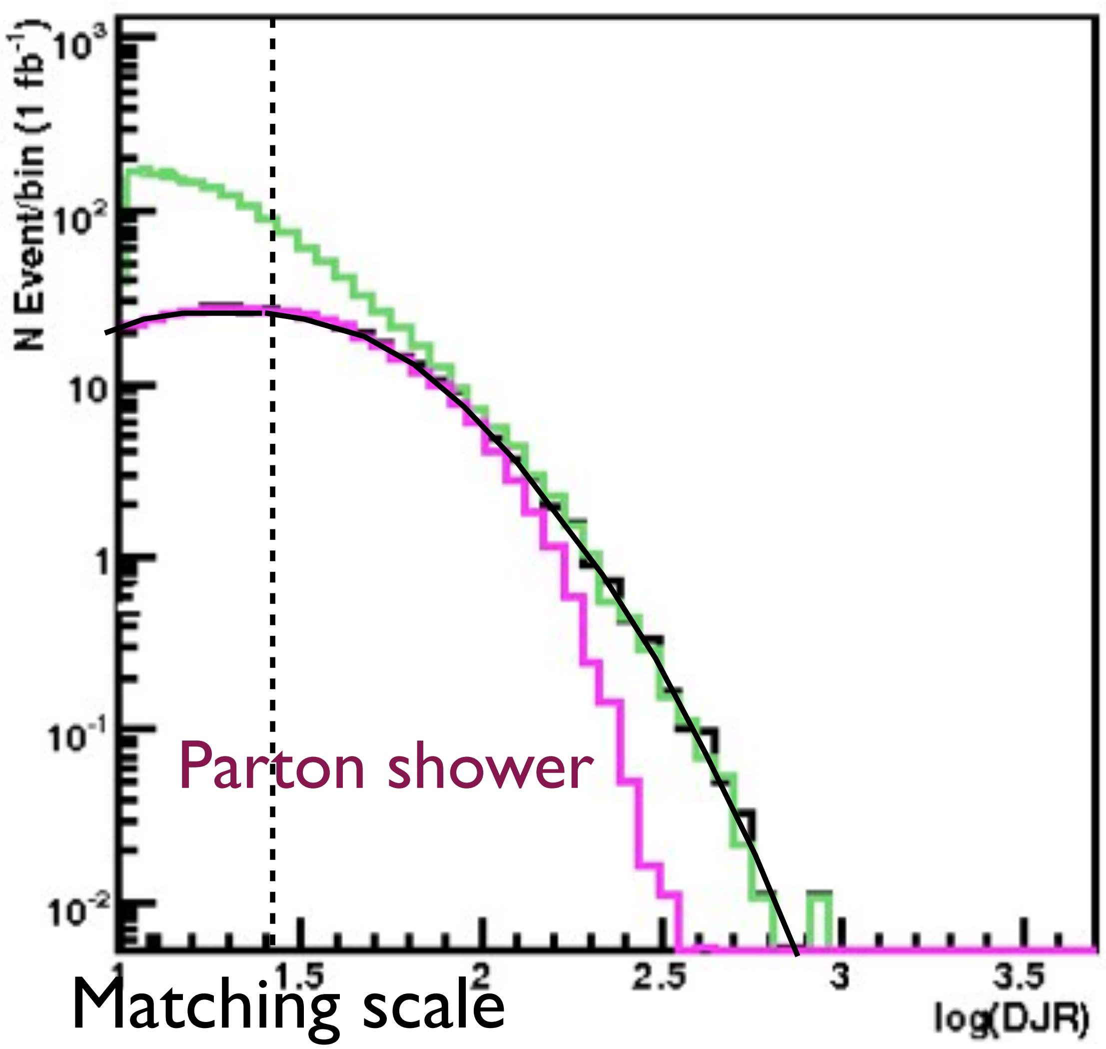
- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Back to the "matching goal"

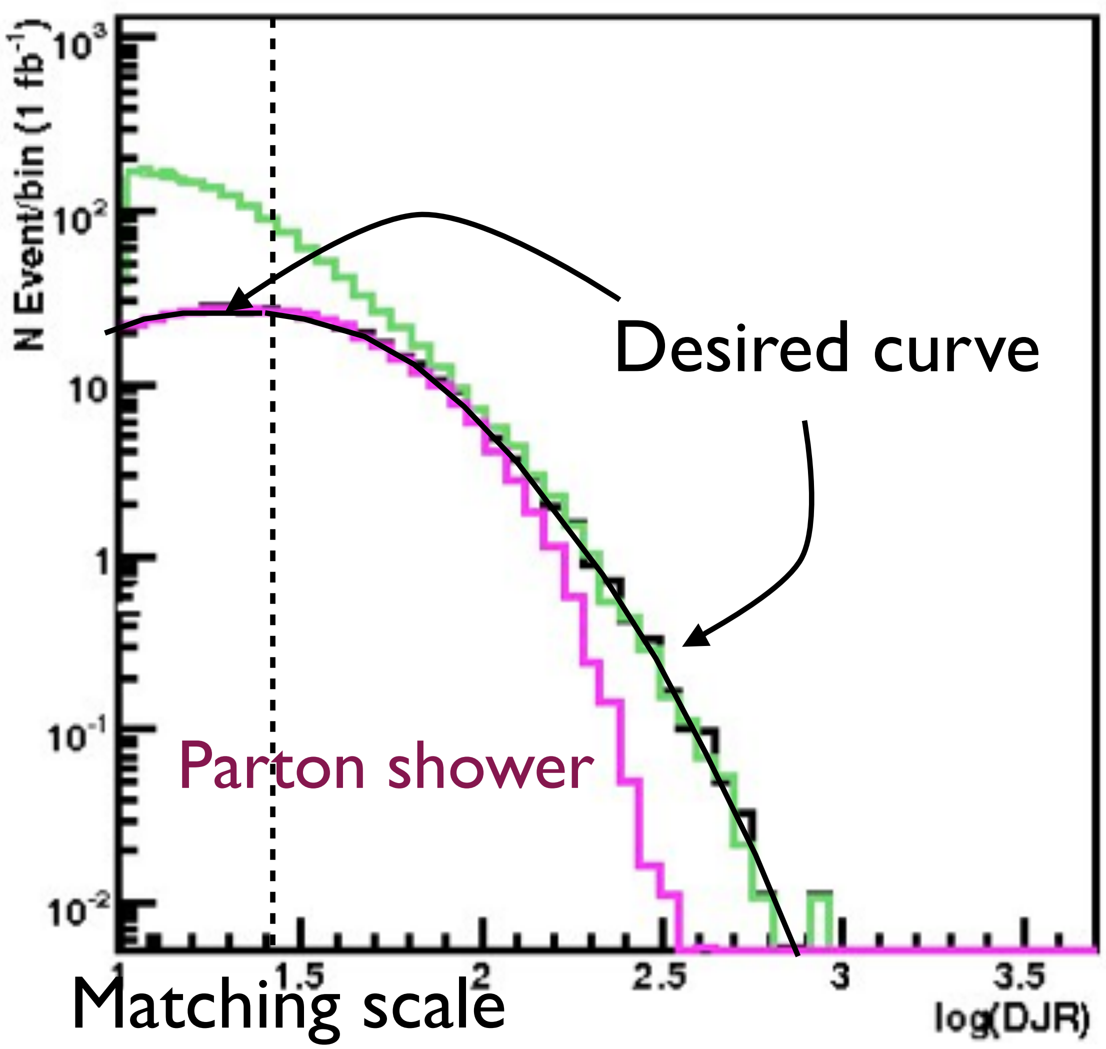
- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Back to the “matching goal”

- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions

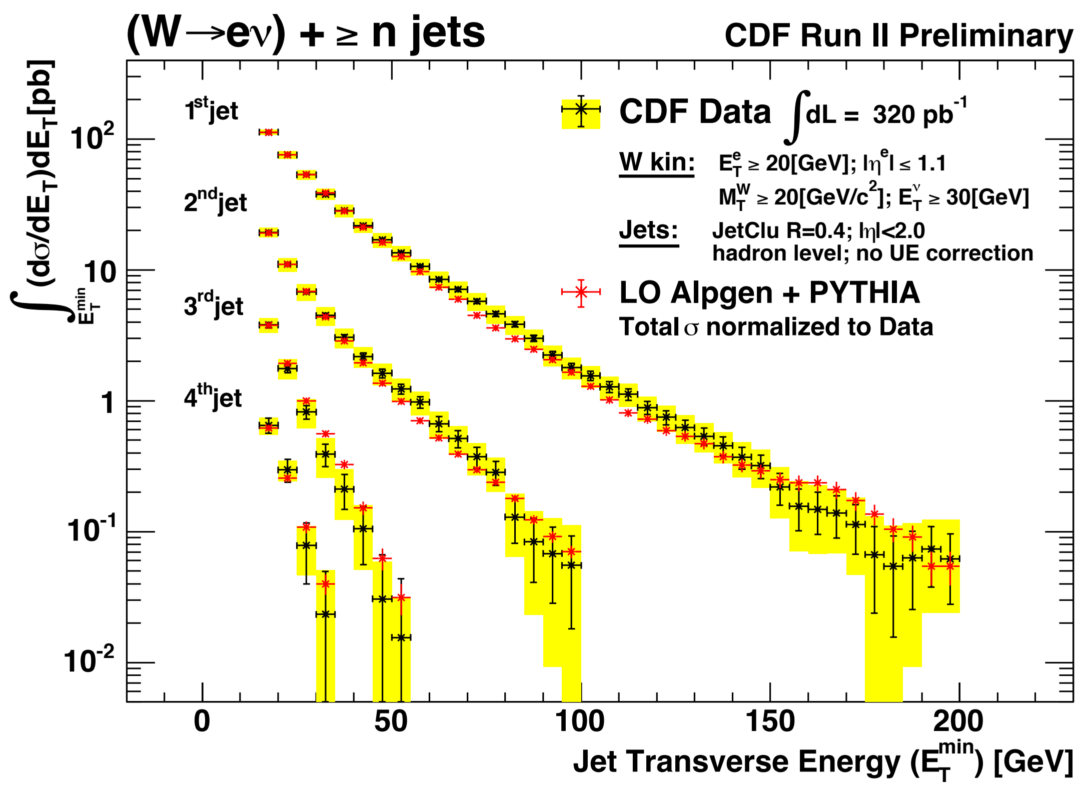


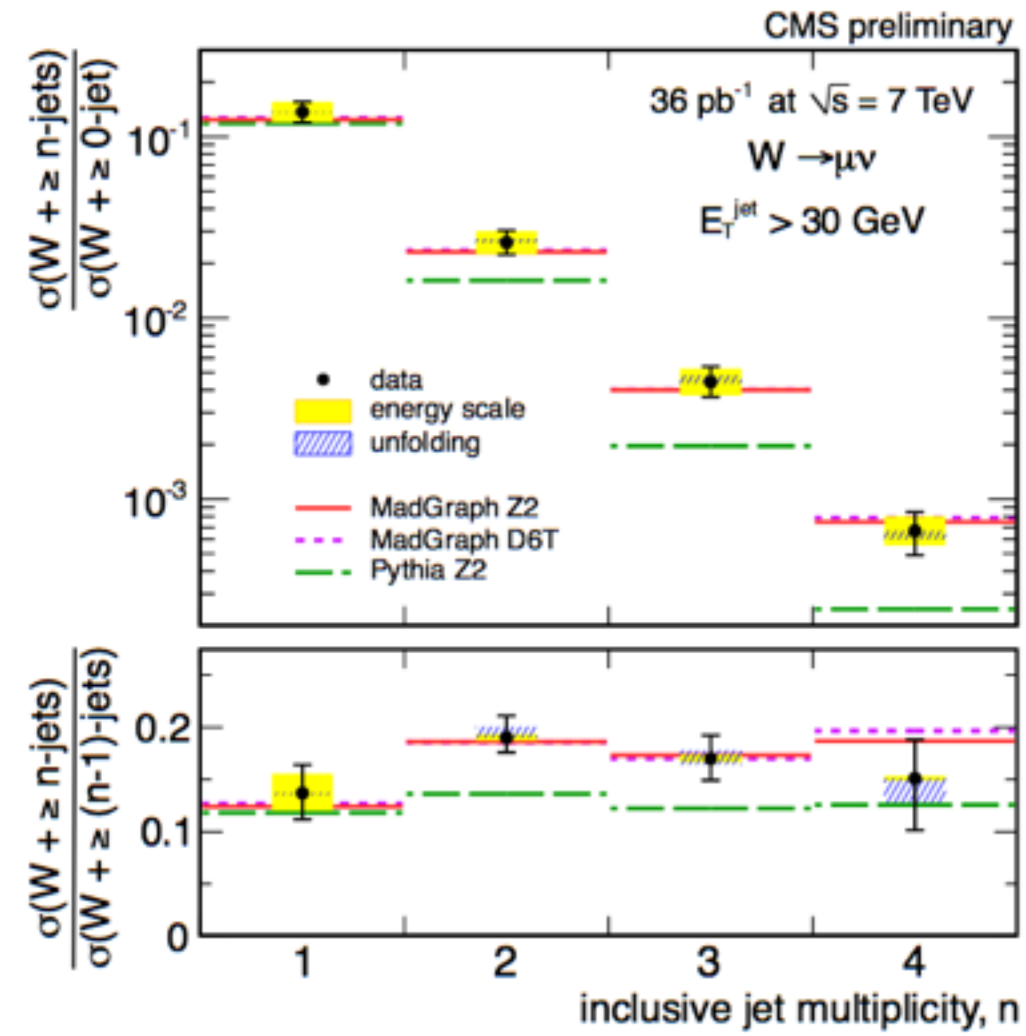
2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Summary of Matching Procedure

1. Generate ME events (with different parton multiplicities) using parton-level cuts ($p_T^{ME}/\Delta R$ or k_T^{ME})
2. Cluster each event and reweight α_s and PDFs based on the scales in the clustering vertices
3. Apply Sudakov factors to account for the required non-radiation above clustering cutoff scale and generate parton shower emissions below clustering cutoff:
   a. (CKKW) Analytical Sudakovs + truncated showers
   b. (CKKW-L) Sudakovs from truncated showers
   c. (MLM) Sudakovs from reclustered shower emissions
4. Apply separation cut

Comparing to experiment: W+jets





- Very good agreement at Tevatron (left) and LHC (right)
- Matched samples obtained via different matching schemes (MLM and CKKW) consistent within the expected uncertaintes.
- Pure parton shower (Pythia) doesn't describe the data beyond 1st jet.

Example: Simulation of pp→W with 0, 1, 2 jets (comfortable on a laptop)

```
mg5> generate p p > w+, w+ > l+ vl @0
mg5> add process p p > w+ j, w+ > l+ vl @1
mg5> add process p p > w+ j j, w+ > l+ vl @2
mg5> output
```

```
In run_card.dat:
…
  1 = ickkw
…
  0 = ptj
…
 15 = xqcut
```

- ickkw: Matching on
- ptj: No cone matching
- xqcut: k_T matching scale

Matching automatically done when run through MadEvent and Pythia!

matching in MadGraph+Pythia

- By default, k_T-MLM matching is run if xqcut > 0, with the matching scale QCUT = max(xqcut*1.4, xqcut+10)
- For shower-kT, by default QCUT = xqcut
- If you want to change the Pythia setting for matching scale or switch to shower-k_T matching:

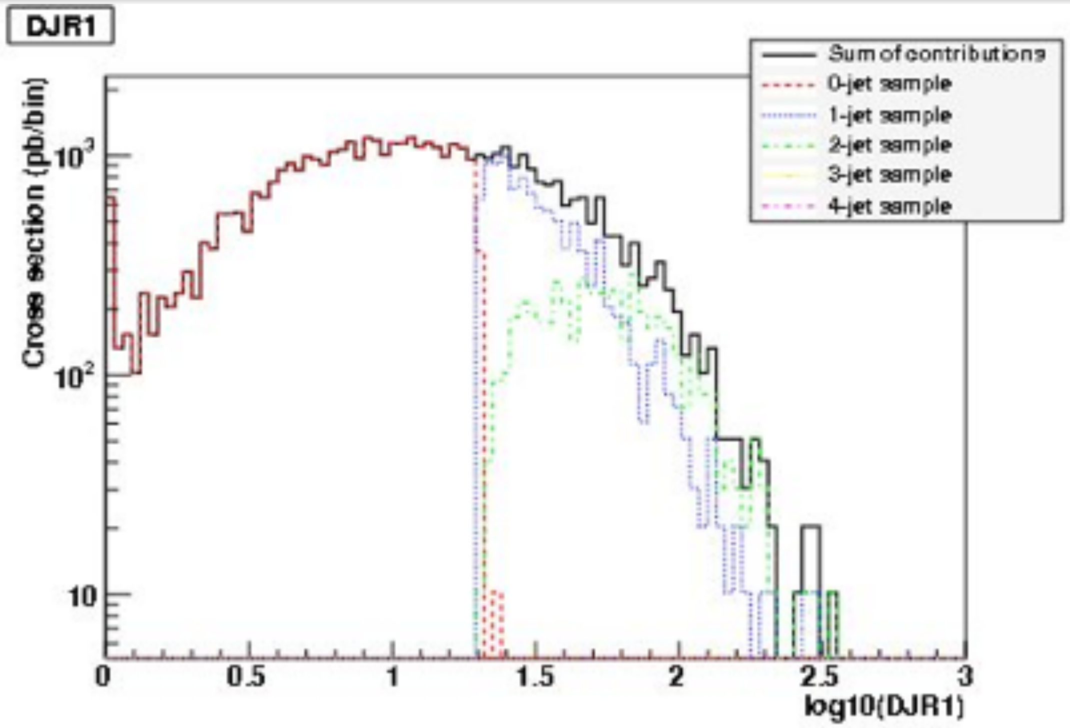
```
In pythia_card.dat:
…
! This sets the matching scale, needs to be > xqcut
QCUT = 30
! This switches from kT-MLM to shower-kT matching
! Note that MSTP(81)>=20 needed (pT-ordered shower)
SHOWERKT = T
```

How to do validate the matching

- The matching scale (QCUT) should typically be chosen around 1/6-1/2 x hard scale (so xqcut correspondingly lower)
- The matched cross section (for X+0,1,... jets) should be close to the unmatched cross section for the 0-jet sample (found on the process HTML page)
- The differential jet rate plots should be smooth
- When QCUT is varied (within the region of validity), the matched cross section or differential jet rates should not vary significantly

Differential Jet Rate Plot

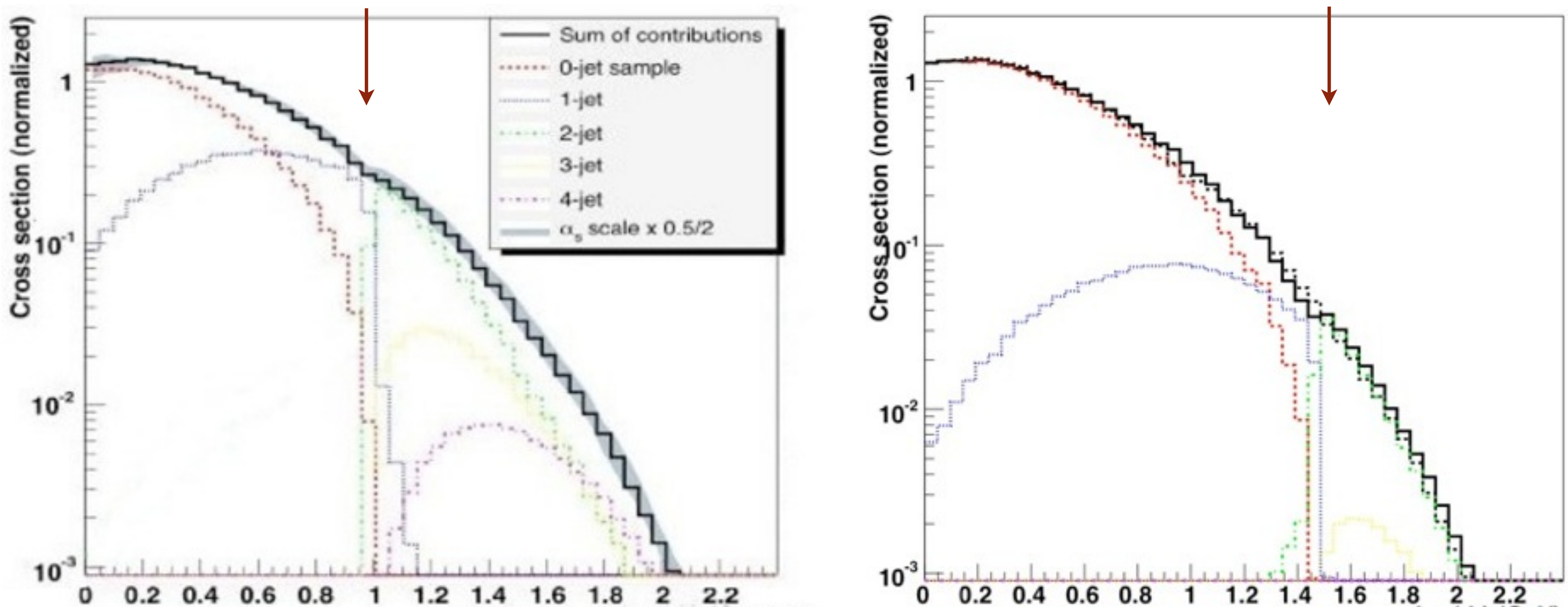
- This are the clustering scales in the kt-jet clustering scheme
- DJR1: pT of the last remaining jet
- DJR2: The minimum between the pT of the second to last remaining jet and the kt between the last two jet.
- Only radiative jet (not those from decay) should enter those plot.



Matching Validation

W+jets production at the Tevatron for MadGraph+Pythia
(k_T-jet MLM scheme, q^2-ordered Pythia showers)

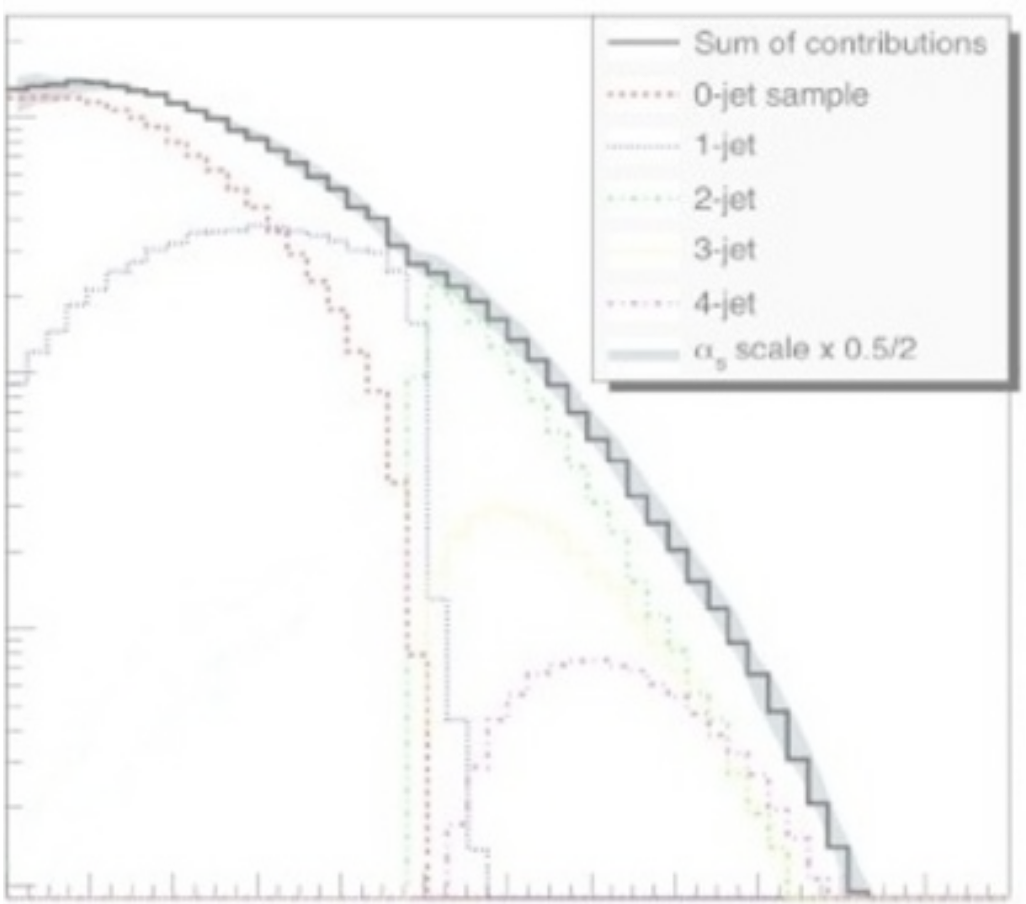
Q^{match} = 10 GeV Q^{match} = 30 GeV

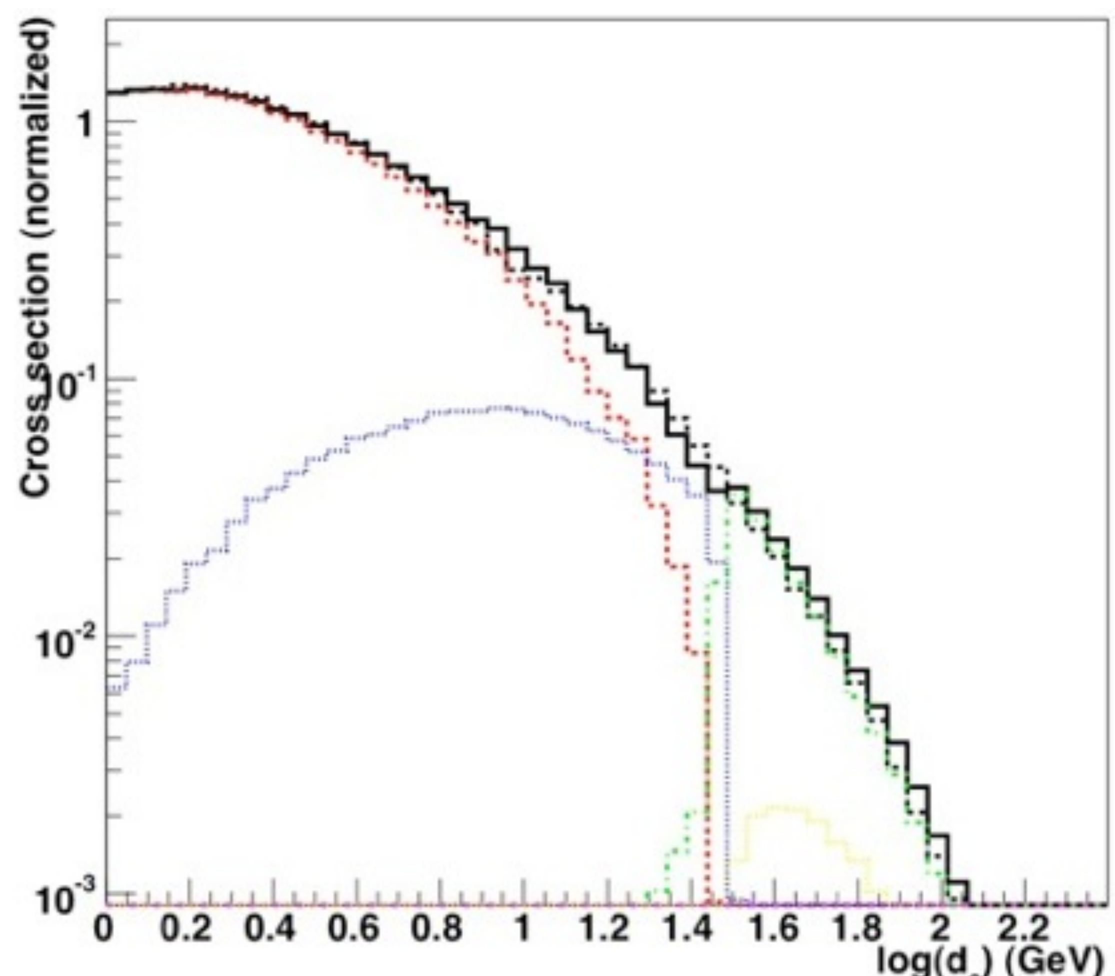


log(Differential jet rate for 1 → 2 radiated jets ~ pT(2nd jet))

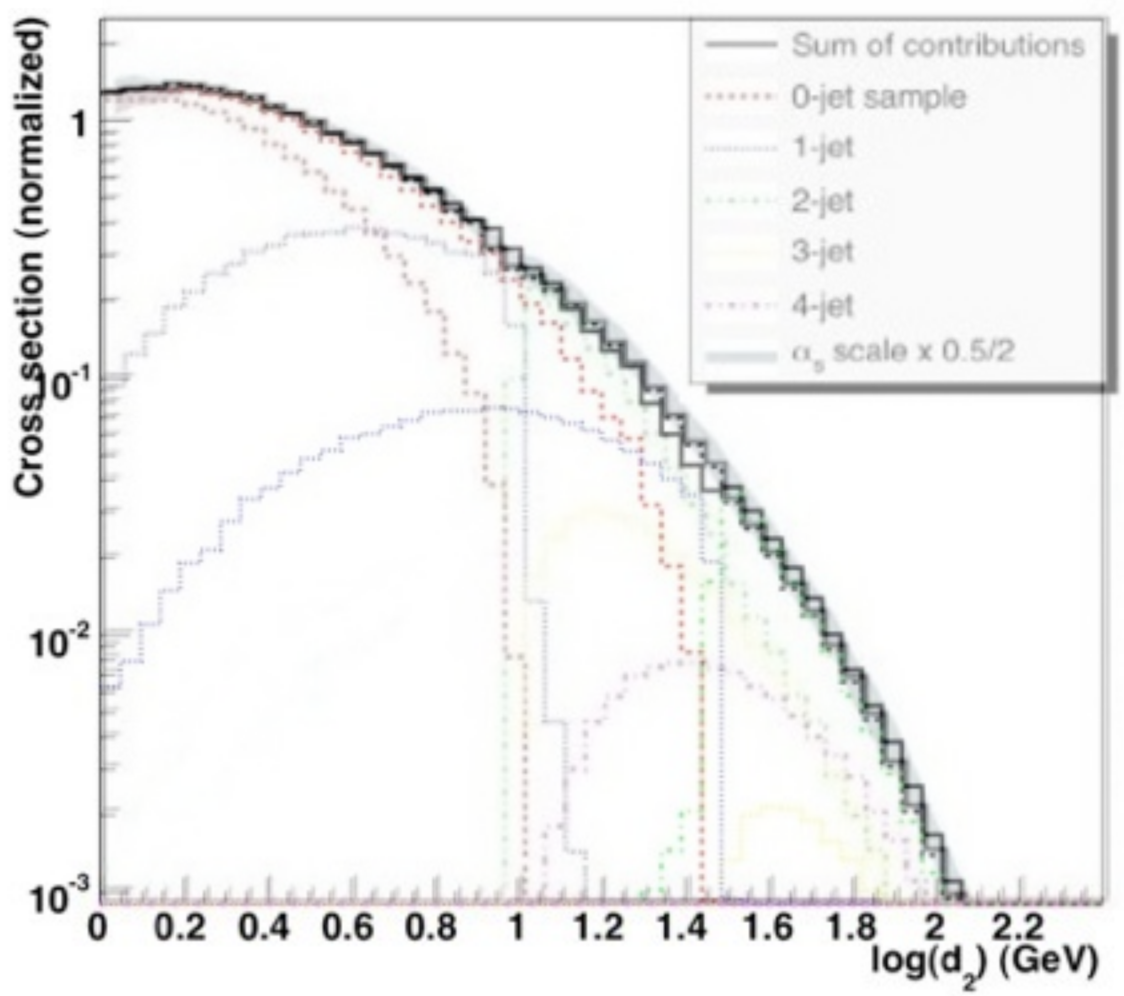
Matching Validation

W+jets production at the Tevatron for MadGraph+Pythia
(k_T-jet MLM scheme, q^2-ordered Pythia showers)



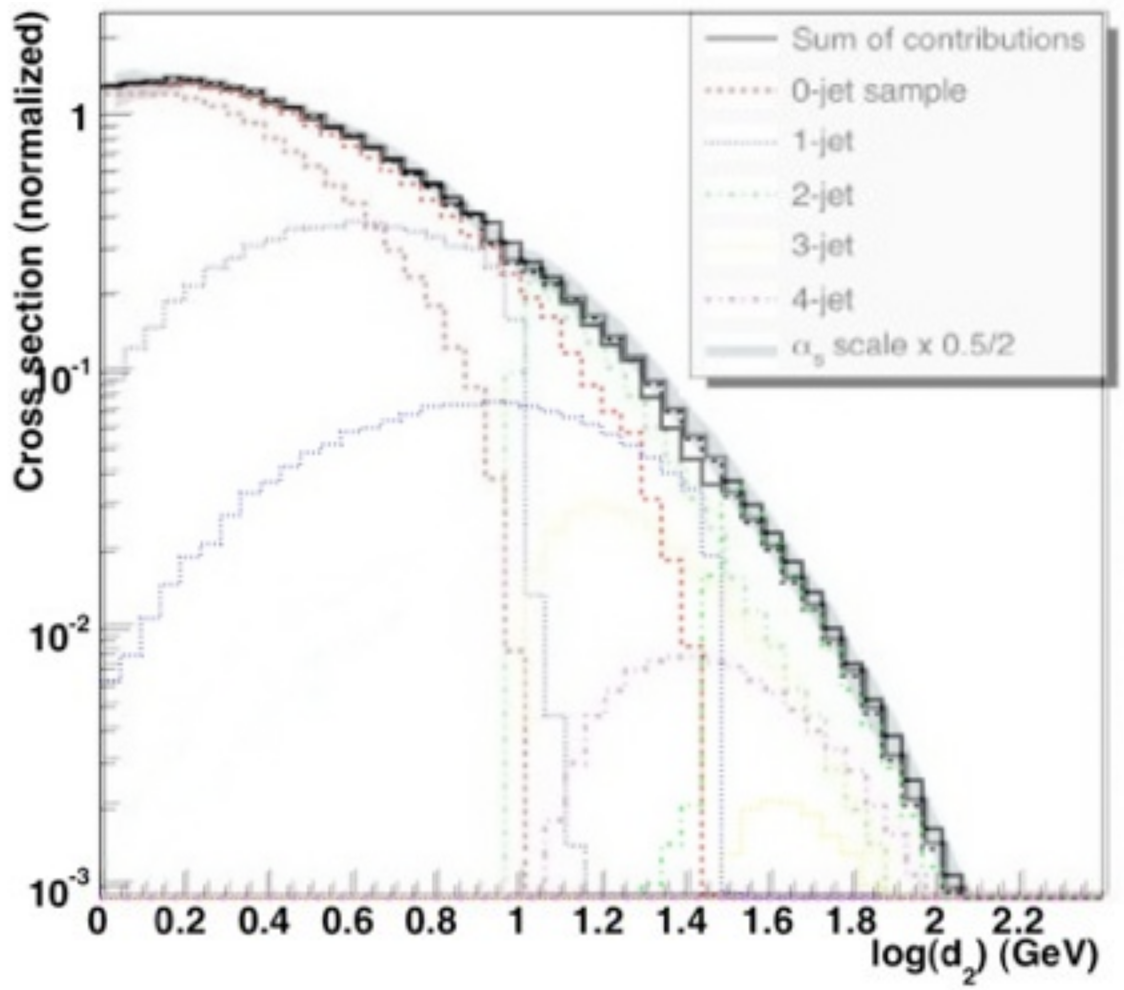


W+jets production at the Tevatron for MadGraph+Pythia
(k_T-jet MLM scheme, q^2-ordered Pythia showers)



Matching Validation

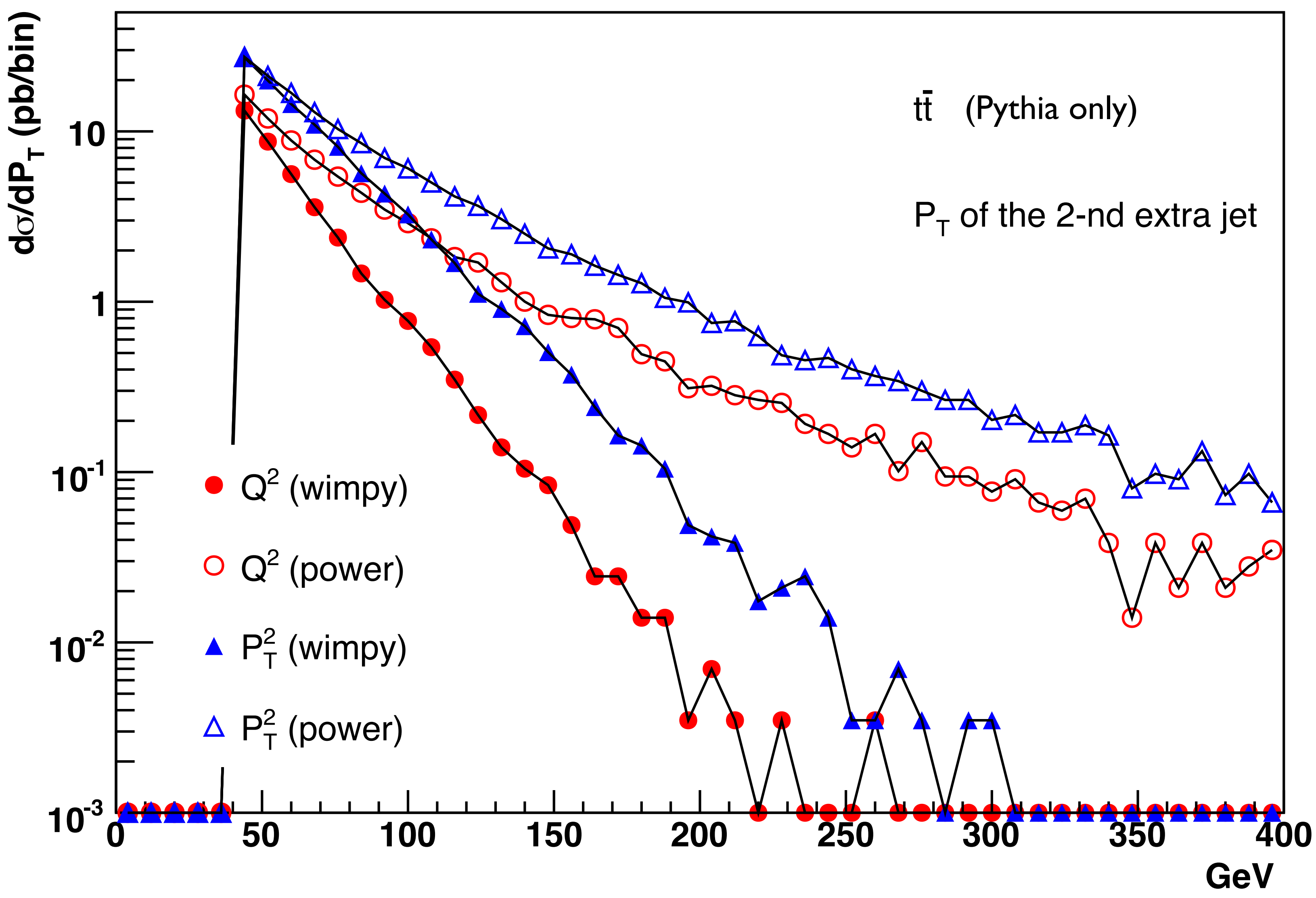
W+jets production at the Tevatron for MadGraph+Pythia
(k_T-jet MLM scheme, q^2-ordered Pythia showers)



Jet distributions smooth, and stable when we vary the matching scale!

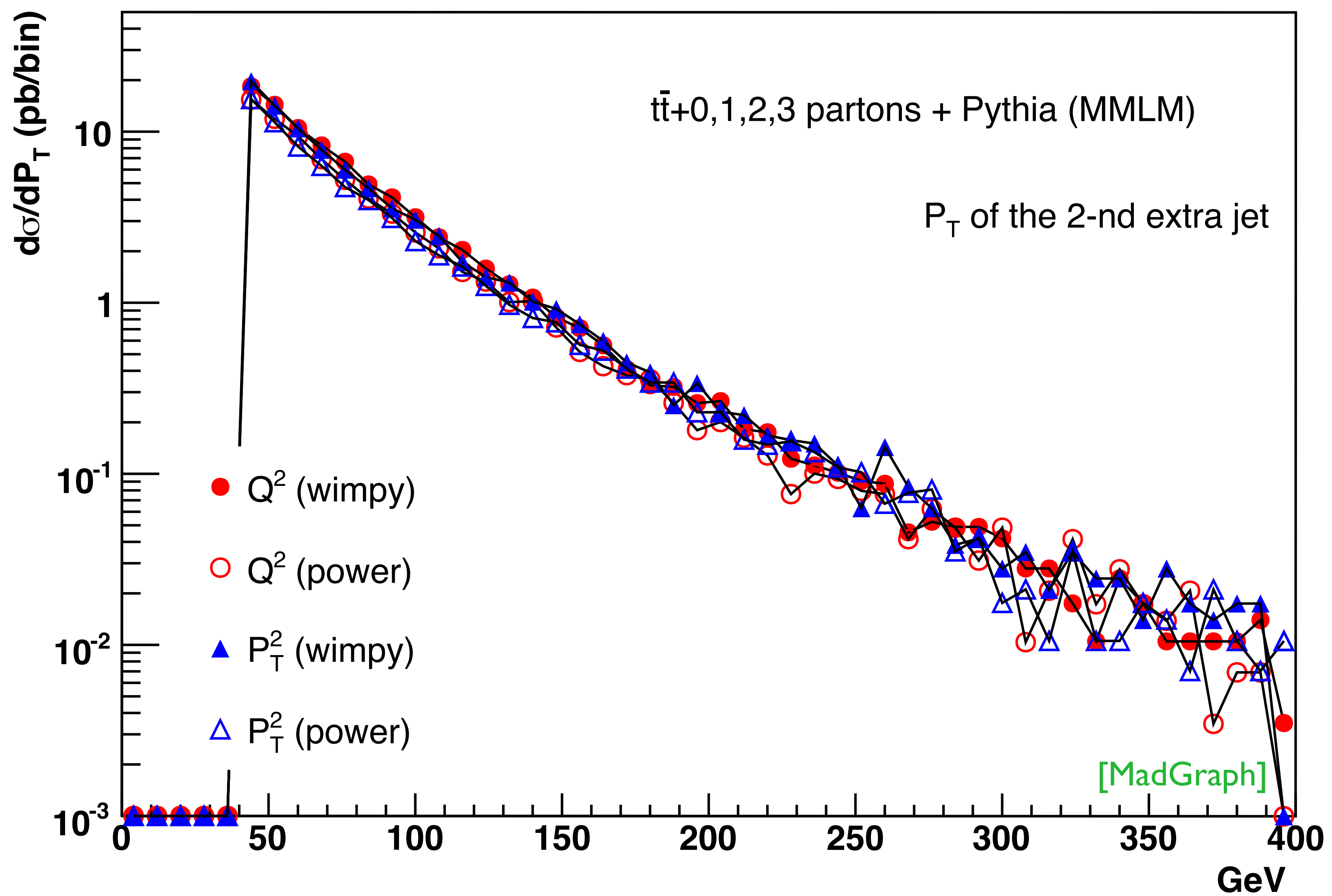
PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)



PS alone vs ME matching

In a matched sample these differences are irrelevant since the behavior at high pt is dominated by the matrix element.



Lecture Summary

- Despite the apparent enormous complexity of simulation of complete collider events, nature has kindly allowed us to factorize the simulation into separate steps
- The Monte Carlo method allows us to step-by-step simulate hard scattering, parton shower, particle decays, hadronization, and underlying event
- Jet matching between matrix elements and parton showers gives crucial improvement of simulation of background as well as signal processes
- Running matching with MadGraph + Pythia is very easy, but the results should always be checked for consistency
- Matching is mandatory at NLO (actually without merging)