What are the MC for?

1. High-$Q^2$ Scattering
2. Parton Shower
3. Hadronization
4. Underlying Event

Sherpa artist
What are the MC for?

1. High-$Q^2$ Scattering
   - where new physics lies
   - process dependent
   - first principles description
   - it can be systematically improved

2. Parton Shower

3. Hadronization

4. Underlying Event
What are the MC for?

1. High-$Q^2$ Scattering

2. Parton Shower

3. Hadronization

4. Underlying Event

- QCD - "known physics"
- universal/ process independent
- first principles description
Parton shower

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• Remember that parton-level cross sections for a hard process are inclusive in anything else.
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• And finally we want to turn partons into hadrons (hadronization)....
First Example

\[ e^+ e^- \rightarrow q\bar{q}g \]
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\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}
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x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}
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- Soft Divergencies
- Collinear Divergencies

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- Collinear limit

- Split our integral in two

\[ \frac{2 \, d\cos \theta_{13}}{\sin^2 \theta_{13}} = \frac{d\cos \theta_{13}}{1 - \cos \theta_{13}} + \frac{d\cos \theta_{13}}{1 + \cos \theta_{13}} \]

\[ \approx \frac{d \cos \theta_{13}}{(1 - \cos \theta_{13})} + \frac{d \cos \theta_{23}}{(1 - \cos \theta_{23})} \]

\[ \approx \frac{d \theta_{13}^2}{\theta_{13}^2} + \frac{d \theta_{23}^2}{\theta_{23}^2} \]
First Example

- Change the variable to $x_3$ and $\cos \theta_{13}$

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\approx \frac{d\theta_{13}^2}{\theta_{13}^2} + \frac{d\theta_{23}^2}{\theta_{23}^2}
$$

$$
d\sigma = \sigma_0 \sum \text{jets} \left( C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} \right) dz \frac{1 + (1 - z)^2}{z}
$$

 grote fraction of energy

**Generic Formula**
• Consider a process for which two particles are separated by a small angle $\theta$.

• In the limit of $\theta \to 0$ the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.
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The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability.
The process factorizes in the collinear limit. This procedure is universal!

\[
\frac{1}{(p_b + p_c)^2} \approx \frac{1}{2E_b E_c(1 - \cos \theta)} = \frac{1}{t}
\]

\[z = \frac{E_b}{E_a}\]
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soft

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Collinear factorization

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**soft** and **collinear** divergencies
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Soft and collinear divergencies

Collinear factorization:

\[
|M_{n+1}|^2 d\Phi_{n+1} \approx |M_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z)
\]

when \(\theta\) is small.
Collinear factorization

\[ |M_{n+1}|^2 d\Phi_{n+1} \approx |M_n|^2 d\Phi_n \frac{dt}{t} d\frac{d\phi}{2\pi} \frac{d}{2\pi} \alpha_s P_{a\rightarrow bc}(z) \]

\( t \) can be called the ‘evolution variable’ (will become clearer later): it can be the virtuality \( m^2 \) of particle \( a \) or its \( p_T^2 \) or \( E^2 \theta^2 \) …

\[
d\theta^2/\theta^2 = dm^2/m^2 = dp_T^2/p_T^2
\]

\[
m^2 \approx z(1-z)\theta^2 E_a^2
\]

\[
p_T^2 \approx zm^2
\]

It represents the hardness of the branching and tends to 0 in the collinear limit.

Different choice of ‘evolution parameter’ in different Parton-shower code
Collinear factorization

\[ |\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \approx |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\rightarrow bc}(z) \]

- \( z \) is the “energy variable”: it is defined to be the energy fraction taken by parton \( b \) from parton \( a \). It represents the energy sharing between \( b \) and \( c \) and tends to 1 in the soft limit (parton \( c \) going soft).

- \( \Phi \) is the azimuthal angle. It can be chosen to be the angle between the polarization of \( a \) and the plane of the branching.
The spin averaged (unregulated) splitting functions for the various types of branching are (Altarelli-Parisi):

\[
\hat{P}_{qq}(z) = C_F \left[ \frac{1 + z^2}{1 - z} \right], \\
\hat{P}_{gq}(z) = C_F \left[ \frac{1 + (1 - z)^2}{z} \right], \\
\hat{P}_{qq}(z) = T_R \left[ z^2 + (1 - z)^2 \right], \\
\hat{P}_{gg}(z) = C_A \left[ \frac{z}{1 - z} + \frac{1 - z}{z} + z(1 - z) \right].
\]

\[ C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}. \]
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**Comments:**

* Gluons radiate the most
* There are soft divergences in $z=1$ and $z=0$.
* $P_{qg}$ has no soft divergences.
• Each choice of argument for $\alpha_S$ is equally acceptable at the leading-logarithmic accuracy. However, there is a choice that allows one to resum certain classes of subleading logarithms.

• The higher order corrections to the partons splittings imply that the splitting kernels should be modified: $P_a \rightarrow_{bc}(z) \rightarrow P_a \rightarrow_{bc}(z) + \alpha_S P'_a \rightarrow_{bc}(z)$

For $g \rightarrow gg$ branchings $P'_a \rightarrow_{bc}(z)$ diverges as $-b_0 \log[z(1-z)]$ $P_a \rightarrow_{bc}(z)$ (just $z$ or $1-z$ if quark is present)

• Recall the one-loop running of the strong coupling:

$$\alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1 + \alpha_S(\mu^2)b_0 \log \frac{Q^2}{\mu^2}} \sim \alpha_S(\mu^2) \left(1 - \alpha_S(\mu^2)b_0 \log \frac{Q^2}{\mu^2}\right)$$

• We can therefore include the $P'(z)$ terms by choosing $p_T^2 \sim z(1-z)Q^2$ as argument of $\alpha_S$:

$$\alpha_S(Q^2) \left(P_a \rightarrow_{bc}(z) + \alpha_S(Q^2)P'_a \rightarrow_{bc}\right) = \alpha_S(Q^2) \left(1 - \alpha_S(Q^2)b \log z(1-z)\right) P_a \rightarrow_{bc}(z)$$

$$\sim \alpha_S(z(1-z)Q^2) P_a \rightarrow_{bc}(z)$$
Argument of $\alpha_S$

\[ |\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \approx |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} d\phi \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z) \]

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\[ \alpha_S(Q^2)\left( P_{a \rightarrow bc}(z) + \alpha_S(Q^2)P'_{a \rightarrow bc} \right) = \alpha_S(Q^2) \left( 1 - \alpha_S(Q^2)b \log z(1-z) \right) P_{a \rightarrow bc}(z) \sim \alpha_S(\mu^2) \left( 1 - \alpha_S(\mu^2)b \log z(1-z) \right) P_{a \rightarrow bc}(z) \]
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Collinear Limit

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To Remember

Collinear Limit

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- \( t \) is the evolution parameter (control the collinear behaviour)
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- **t** is the evolution parameter (control the collinear behaviour)
- **z** is the energy sharing variable
Collinear Limit

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- $t$ is the evolution parameter (control the collinear behaviour)
- $z$ is the energy sharing variable
- $\alpha_s$ need to be evaluated at the scale $t$
Collinear Limit

\[ |M_{n+1}|^2 d\Phi_{n+1} \approx |M_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\rightarrow bc}(z) \]

- **t** is the evolution parameter (control the collinear behaviour)
- **z** is the energy sharing variable
- **alpha_s** need to be evaluated at the scale \( t \)
- **P** is the splitting Kernel (control the soft behaviour)
Now consider $M_{n+1}$ as the new core process and use the recipe we used for the first emission in order to get the dominant contribution to the $(n+2)$-body cross section: add a new branching at angle much smaller than the previous one:

$$|M_{n+2}|^2 d\Phi_{n+2} \sim |M_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{d\phi'}{2\pi} \alpha_s P_{a \rightarrow bc}(z) \times \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{d\phi'}{2\pi} \alpha_s P_{b \rightarrow de}(z')$$

This can be done for an arbitrary number of emissions. The recipe to get the leading collinear singularity is thus cast in the form of an iterative sequence of emissions whose probability does not depend on the past history of the system: a ‘Markov chain’. No interference!!!
The dominant contribution comes from the region where the subsequently emitted partons satisfy the strong ordering requirement:

\[ \theta \gg \theta' \gg \theta'' \ldots \]

For the rate for multiple emission we get

\[
\sigma_{n+k} \propto \alpha_s^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \cdots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left( \frac{\alpha_s}{2\pi} \right)^k \log^k \left( \frac{Q^2}{Q_0^2} \right)
\]

where \( Q \) is a typical hard scale and \( Q_0 \) is a small infrared cutoff that separates perturbative from non-perturbative regimes.

Each power of \( \alpha_s \) comes with a logarithm. The logarithm can be easily large, and therefore it can lead to a breakdown of perturbation theory.
• What is the probability of no emission?
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\[ P_{\text{non-branching}}(t_i) = 1 - P_{\text{branching}}(t_i) = 1 - \frac{\delta t \alpha_s}{t_i} \frac{1}{2\pi} \int dz \hat{P}(z) \]
• What is the probability of no emission?

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Sudakov Form Factor

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\[ \simeq \lim_{N \to \infty} e^{\sum_{i=0}^{N} \left( -\frac{\delta t \alpha_s}{t_i 2\pi} \int dz \hat{P}(z) \right)} \]
Sudakov Form Factor

• What is the probability of no emission?

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• So the probability of no emission between two scales:

\[
P_{\text{no-branching}}(Q^2, t) = \lim_{N \to \infty} \prod_{i=0}^{N} \left( 1 - \frac{\delta t \alpha_s}{t_i} \int dz \hat{P}(z) \right) \]
\[
\approx \lim_{N \to \infty} e^{\sum_{i=0}^{N} \left( -\frac{\delta t \alpha_s}{t_i} \int dz \hat{P}(z) \right)} \]
\[
\approx e^{-\int_{t}^{Q^2} \frac{dt'}{t'} \int dz \frac{\alpha_s}{2\pi} \hat{P}(z)} \equiv e^{-\int_{t}^{Q^2} dp(t')} \]
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Sudakov form factor \( \Delta(Q^2, t) \)
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Sudakov form factor

\[ \Delta(Q^2, t) \]

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**Sudakov form factor**

\( \Delta(Q^2, t) \)
Sudakov Form Factor

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\[ \simeq e^{-\int_t^{Q^2} \frac{dt'}{t'} \int dz \frac{\alpha_s}{2\pi} \hat{P}(z)} \equiv e^{-\int_t^{Q^2} dp(t')} \]

→ Property: \( \Delta(A,B) = \Delta(A,C) \Delta(C,B) \)
The Sudakov form factor is the heart of the parton shower. It gives the probability that a parton does not branch between two scales.

Using this no-emission probability the branching tree of a parton is generated.

Define $dP_k$ as the probability for $k$ ordered splittings from leg a at given scales.

\[
\begin{align*}
    dP_1(t_1) &= \Delta(Q^2, t_1) \, dp(t_1) \Delta(t_1, Q^2_0), \\
    dP_2(t_1, t_2) &= \Delta(Q^2, t_1) \, dp(t_1) \, \Delta(t_1, t_2) \, dp(t_2) \, \Delta(t_2, Q^2_0) \Theta(t_1 - t_2), \\
    \cdots &= \cdots \\
    dP_k(t_1, \ldots, t_k) &= \Delta(Q^2, Q^2_0) \prod_{l=1}^{k} \, dp(t_l) \Theta(t_{l-1} - t_l)
\end{align*}
\]

$Q_0^2$ is the hadronization scale ($\sim 1$ GeV). Below this scale we do not trust the perturbative description for parton splitting anymore.
The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for $k$ splittings:

$$dP_k(t_1, \ldots, t_k) = \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l) \Theta(t_{l-1} - t_l)$$
The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for $k$ splittings:

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[ \int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, ...$$
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- Summing over all number of emissions
Unitarity

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- Summing over all number of emissions

\[ \sum_{k=0}^{\infty} P_k = \Delta(Q^2, Q_0^2) \sum_{k=0}^{\infty} \frac{1}{k!} \left[ \int_{Q_0^2}^{Q^2} dp(t) \right]^k = \Delta(Q^2, Q_0^2) \exp \left[ \int_{Q_0^2}^{Q^2} dp(t) \right] = 1 \]
The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for $k$ splittings:

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\]

- Hence, the total probability is conserved
• We have shown that the showers is unitary. However, how are the IR divergences cancelled explicitly? Let’s show this for the first emission:
Consider the contributions from (exactly) 0 and 1 emissions from leg a:

\[
\frac{d\sigma}{\sigma_n} = \Delta(Q^2, Q_0^2) + \Delta(Q^2, Q_0^2) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\rightarrow bc}(z)
\]

• Expanding to first order in \(\alpha_s\) gives

\[
\frac{d\sigma}{\sigma_n} \approx 1 - \sum_{bc} \int_{Q_0^2}^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\rightarrow bc}(z) + \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\rightarrow bc}(z)
\]

• Same structure of the two latter terms, with opposite signs: cancellation of divergences between the approximate virtual and approximate real emission cross sections.

• The probabilistic interpretation of the shower ensures that infrared divergences will cancel for each emission.
Final-state parton showers
With the Sudakov form factor, we can now implement a final-state parton shower in a Monte Carlo event generator!
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2. Given a virtual mass scale \( t_i \) and momentum fraction \( x_i \) at some stage in the evolution, generate the scale of the next emission \( t_{i+1} \) according to the Sudakov probability \( \Delta(t_i, t_{i+1}) \) by solving
   \[
   \Delta(t_{i+1}, t_i) = R
   \]
   where \( R \) is a random number (uniform on \([0, 1]\)).
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4. Otherwise, generate $z = z_i / z_{i+1}$ with a distribution proportional to $(\alpha_s / 2\pi)P(z)$, where $P(z)$ is the appropriate splitting function.
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4. Otherwise, generate $z = z_i/z_{i+1}$ with a distribution proportional to $(\alpha_s/2\pi)P(z)$, where $P(z)$ is the appropriate splitting function.

5. For each emitted particle, iterate steps 2-4 until branching stops.
There is a lot of freedom in the choice of evolution parameter $t$. It can be the virtuality $m^2$ of particle $a$ or its $p_T^2$ or $E^2\theta^2$ ... For the collinear limit they are all equivalent.

However, in the soft limit $(z \rightarrow 0,1)$ they behave differently.

Can we chose it such that we get the correct soft limit?

Soft gluon comes from the full event!

\[
\Delta(Q^2, t) = \exp \left[ - \sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z) \right]
\]
Angular ordering

Radiation inside cones around the original partons is allowed (and described by the eikonal approximation), outside the cones it is zero (after averaging over the azimuthal angle)
• Sudakov Form-Factor: Probability of No-emission between two scale.

\[ \Delta(Q^2, t) \sim e^{-\int_t^{Q^2} \frac{dt'}{t'} \, dz \, \frac{\alpha_s}{2\pi} \, \hat{P}(z)} \equiv e^{-\int_t^{Q^2} dp(t')} \]

• Probability of K-emission

\[ P_k \equiv \int dP_k(t_1, \ldots, t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[ \int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, \ldots \]

• Ensure that the parton shower is unitary
• Ensure cancelation of IR divergency
• Interference effect via Angular ordering
• So far, we have looked at final-state (time-like) splittings. For initial state, the splitting functions are the same.

• However, there is another ingredient: the parton density (or distribution) functions (PDFs). Naively: Probability to find a given parton in a hadron at a given momentum fraction \( x = p_z/P_z \) and scale \( t \).
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How do the PDFs evolve with increasing \( t \)?

\[
t \frac{\partial}{\partial t} f_i(x, t) = \int_x^1 \frac{d\zeta}{\zeta} \frac{\alpha_s}{2\pi} P_{ij}(\zeta) f_j\left(\frac{x}{\zeta}, t\right)
\]

DGLAP
Initial-state parton splittings

- Start with a quark PDF $f_0(x)$ at scale $t_0$. After a single parton emission, the probability to find the quark at virtuality $t > t_0$ is

$$f(x, t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) f_0 \left( \frac{x}{z} \right)$$

- After a second emission, we have

$$f(x, t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) \left[ f_0 \left( \frac{x}{z} \right) + \int_{t_0}^{t'} \frac{dt''}{t''} \frac{\alpha_s}{2\pi} \int_{x/z}^{1} \frac{dz'}{z'} P(z') f_0 \left( \frac{x}{zz'} \right) \right]$$
The DGLAP equation

- So for multiple parton splittings, we arrive at an integral-differential equation:

\[ t \frac{\partial}{\partial t} f_i(x, t) = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ij}(z) f_j \left( \frac{x}{z}, t \right) \]

- This is the famous DGLAP equation (where we have taken into account the multiple parton species i, j). The boundary condition for the equation is the initial PDFs \( f_{i0}(x) \) at a starting scale \( t_0 \) (around 2 GeV).

- These starting PDFs are fitted to experimental data.
To simulate parton radiation from the initial state, we start with the hard scattering, and then “deconstruct” the DGLAP evolution to get back to the original hadron: backwards evolution!

i.e. we undo the analytic resummation and replace it with explicit partons (e.g. in Drell-Yan this gives non-zero $p_T$ to the vector boson)

In backwards evolution, the Sudakovs include also the PDFs -- this follows from the DGLAP equation and ensures conservation of probability:

$$\Delta_{i}(x, t_1, t_2) = \exp \left\{ - \int_{t_1}^{t_2} dt' \sum_{j} \int_{x}^{1} dx' \frac{\alpha_s(t')}{2\pi} P_{ij} \left( \frac{x}{x'} \right) \frac{f_i(x', t')}{f_j(x, t')} \right\}$$

This represents the probability that parton $i$ will stay at the same $x$ (no splittings) when evolving from $t_1$ to $t_2$.

The shower simulation is now done as in a final state shower!
• The shower stops if all partons are characterized by a scale at the IR cut-off: $Q_0 \sim 1$ GeV.

• Physically, we observe hadrons, not (colored) partons.

• We need a non-perturbative model in passing from partons to colorless hadrons.

• There are two models (string and cluster), based on physical and phenomenological considerations.
A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.
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- General-purpose tools
- Complete exclusive description of the events: hard scattering, showering & hadronization (and underlying event)
- Reliable and well-tuned tools
- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD
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**Shower MC Generators: PYTHIA, HERWIG, SHERPA**
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**Shower MC Generators: PYTHIA, HERWIG, SHERPA**

"Note that a bunching tree is not a Feynman diagram: it represents the coherent sum of many real and virtual diagrams which are summed by the branching algorithm" (HERWIG manual)
The parton shower dresses partons with radiation. This makes the inclusive parton-level predictions (i.e. inclusive over extra radiation) completely exclusive.

- In the soft and collinear limits the partons showers are exact, but in practice they are used outside this limit as well.
- Partons showers are universal (i.e. independent from the process)
- Building block of the parton shower is the Sudakov
- There is a cut-off in the shower (below which we don’t trust perturbative QCD) at which a hadronization model takes over
Matching/Merging

Olivier Mattelaer
IPPP/Durham
In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)
1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description
Matrix Elements vs. Parton Showers

**ME**
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**Shower MC**
1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
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Approaches are complementary: merge them!
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**Approaches are complementary: merge them!**

**Difficulty:** avoid double counting, ensure smooth distributions
Goal for ME-PS merging/matching

2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia
Goal for ME-PS merging/matching

- Regularization of matrix element divergence

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Merging ME with PS

[Mangano]  
[Catani, Krauss, Kuhn, Webber]  
[Lönnblad]
Merging ME with PS

[References: Mangano, Catani, Krauss, Kuhn, Webber, Lönnblad]
Merging ME with PS

\[ \frac{k_T}{Q} < \frac{k_T}{Q} \]

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]
Merging ME with PS

Double counting between ME and PS easily avoided using phase space cut between the two: PS below cutoff, ME above cutoff.

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]
• So double counting problem easily solved, but what about getting smooth distributions that are independent of the precise value of $Q_c$?

• Below cutoff, distribution is given by PS - need to make ME look like PS near cutoff

• Let’s take another look at the PS!
Merging ME with PS

Why Matching?

Present matching approaches
CKKW matching in $e^+e^-$ collisions

Overview of the CKKW procedure
Clustering the $n$-jet event
Sudakov reweighting
Vetoed parton showers
Highest multiplicity treatment

Results of CKKW matching
mSherpa

Di$c$culties with practical implementations

The MLM procedure

Clustering the $n$-jet event

Find the two partons with smallest jet separation $y_{ij}$

If partons allowed to cluster by QCD splitting rules: combine partons to new particle (e.g. $q\bar{q}$, $qg$, $qg$)

Iterate 1-2 until 2-2 process reached ($e^+e^-$ $q\bar{q}$)

With the choice of the Durham jet measure, the jet separations $d_i = \frac{\Delta y_{ij}}{Q_0}$ at each branching corresponds closely to the $k_T$ of that branching, and is therefore suitable to use as argument for $s$ in the branching.
• How does the PS generate the configuration above?
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• Probability for the splitting at $t_1$ is given by

$$\left(\Delta_q(t_1, t_0)\right)^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$
Merging ME with PS

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\[
(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)
\]

and for the whole tree

\[
(\Delta_q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qq}(z')
\]
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• Probability for the splitting at \( t_1 \) is given by

\[
(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)
\]

and for the whole tree

\[
(\Delta_q(t_{cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')
\]
Merging ME with PS

- How does the PS generate the configuration above?
- Probability for the splitting at $t_1$ is given by

$$\left(\Delta_q(t_1, t_0)\right)^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$\left(\Delta_q(t_{\text{cut}}, t_0)\right)^2 \Delta_g(t_2, t_1) \left(\Delta_q(t_{\text{cut}}, t_2)\right)^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$
• How does the PS generate the configuration above?

• Probability for the splitting at $t_1$ is given by

\[
(\Delta_q(t_1, t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(\tilde{z})
\]

and for the whole tree

\[
(\Delta_q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(\tilde{z}) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(\tilde{z}')
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• How does the PS generate the configuration above?

• Probability for the splitting at \( t_1 \) is given by

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\]

and for the whole tree

\[
(\Delta_q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qq}(z')
\]
• How does the PS generate the configuration above?

• Probability for the splitting at $t_1$ is given by

$$\left( \Delta q(t_1, t_0) \right)^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$\left( \Delta q(t_{\text{cut}}, t_0) \right)^2 \Delta g(t_2, t_1) \left( \Delta q(t_{\text{cut}}, t_2) \right)^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$
Matching of Matrix Elements and Parton Showers

Lecture x:

Matching in $e^+e^-$ collisions

Why Matching?

Present matching approaches

CKKW matching in $e^+e^-$ collisions

Overview of the CKKW procedure

Clustering the $n$-jet event

Sudakov reweighting

Vetoed parton showers

Highest multiplicity treatment

Results of CKKW matching

Di $\text{Merm}^{\text{L}}$

procedure

Clustering the $n$-jet event

Find the two partons with smallest jet separation

If partons allowed to cluster by QCD splitting rules: combine partons to new particle (e.g. $q\bar{q}$, $qg$, $qg$)

Iterate 1-2 until 2 process reached ($e^+e^-$, $q\bar{q}$)

With the choice of the Durham jet measure, the jet separations $d_i = \frac{\Delta_y}{Q_0}$ at each branching corresponds closely to the $k_T$ of that branching, and is therefore suitable to use as argument for $s$ in the branching.

\[
(\Delta_q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')
\]
Matching of Matrix Elements and Parton Showers

Lecture x:

Matching in $e^-e^+$ collisions

Johan Als

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Diiculties with practical implementations

The MLM procedure

Clustering the $n$-jet event

Find the two partons with smallest jet separation $y_{ij}$

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Iterate 1-2 until 2 process reached ($e^-e^+$ $q\bar{q}$)

With the choice of the Durham jet measure, the jet separations $d_i = \frac{\alpha_s(t_0)}{\pi}$ at each branching corresponds closely to the $k_T$ of that branching, and is therefore suitable to use as argument for $s$ in the branching.

Corresponds to the matrix element

BUT with $\alpha_s$ evaluated at the scale of each splitting

$$(\Delta_q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$
Merging ME with PS

Corresponds to the matrix element
BUT with $\alpha_s$ evaluated at the scale of each splitting

Sudakov suppression due to not allowing additional radiation above the scale $t_{cut}$
Matching of Matrix Elements and Parton Showers

Lecture x:

Matching in $e^+e^-$ collisions

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Clustering the $n_s$-jet event

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The MLM procedure

Clustering the $n$-jet event

Find the two partons with smallest jet separation $y_{ij}$

If partons allowed to cluster by QCD splitting rules: combine partons to new particle (e.g. $q\bar{q}$, $qg$, $q$)

Iterate 1-2 until 2 process reached ($e^+e^-, q\bar{q}$, $qg$, $q$)

With the choice of the Durham jet measure, the jet separations $d_i = \| y_i Q_0 \| \| Q_0 \|$ at each branching corresponds closely to the $k_T$ of that branching, and is therefore suitable to use as argument for $s$ in the branching.

$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$
• To get an equivalent treatment of the corresponding matrix element, do as follows:

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1. Cluster the event using some clustering algorithm
   - this gives us a corresponding “parton shower history”
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To get an equivalent treatment of the corresponding matrix element, do as follows:

1. Cluster the event using some clustering algorithm - this gives us a corresponding “parton shower history”

2. Reweight $\alpha_s$ in each clustering vertex with the clustering scale

$$|\mathcal{M}|^2 \rightarrow |\mathcal{M}|^2 \frac{\alpha_s(t_1) \alpha_s(t_2)}{\alpha_s(t_0) \alpha_s(t_0)}$$
Merging ME with PS

- To get an equivalent treatment of the corresponding matrix element, do as follows:
  1. Cluster the event using some clustering algorithm - this gives us a corresponding “parton shower history”
  2. Reweight $\alpha_s$ in each clustering vertex with the clustering scale
     \[ |\mathcal{M}|^2 \rightarrow |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)} \]
  3. Use some algorithm to apply the equivalent Sudakov suppression
     \[ (\Delta_q(t_{cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(\text{cut}, t_2))^2 \]
Matching for initial state radiation

We want to simulate p\(p\) → Z + jets. We pick (according to the relative cross-section of the processes) a u\(^{-}\)d → Wd\(^{-}\)d event. We pick momenta according to the pdf-weighted matrix element:

\[ |M_{u^{-}d \rightarrow Wd^{-}d}(x_1, x_2, s_{(d_{ini})})|^2 \]

\[ f_{u}(x_1, d_{ini}) f_{\bar{d}}(x_2, d_{ini}) \]

We cluster the event using the boost-invariant \(k_T\) clustering scheme, to get nodes \(d_1\), \(d_2\), \(d_3\) as shown.

We apply the \(s\) and Sudakov weight:

\[ g(d_2, d_{ini}) \]
\[ g(d_1, d_{ini}) \]
\[ s(d_3) \]
\[ s(d_{ini}) \]
\[ s(d_1) \]
\[ s(d_2) \]

We apply initial-state radiation for the incoming u and \(\bar{d}\) starting at \(d_3 = M_{W}\), and final-state radiation for the outgoing d and \(\bar{d}\) starting at \(d_2\), but veto all emissions above \(d_{ini}\) (in both initial- and final-state showers).
• We are of course not interested in $e^+e^-$ but $p\bar{p}$ - what happens for initial state radiation?
• We are of course not interested in $e^+e^-$ but $p-p(\bar{p})$
  - what happens for initial state radiation?

• Let’s do the same exercise as before:

\[
\mathcal{P} = (\Delta I_q(t_{\text{cut}}, t_0))^2 \Delta g(t_2, t_1) (\Delta q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{qg}(z) \frac{\alpha_s(t_2)}{2\pi} P_{gq}(z')
\times \hat{\sigma}_{q\bar{q} \rightarrow e\nu}(\hat{s}, \ldots) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0)
\]
Matching for initial state radiation

- We are of course not interested in $e^+e^-$ but $p\bar{p}$ - what happens for initial state radiation?

- Let’s do the same exercise as before:

\[ P = \left( \Delta I_q(t_{\text{cut}}, t_0) \right)^2 \Delta g(t_2, t_1) \Delta q(t_{\text{cut}}, t_2) \Delta q(t_{\text{cut}}, t_0) \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1', t_1)}{f_q(x_1, t_1)} \frac{\alpha_s(t_2)}{2\pi} \frac{P_{gq}(z')}{z'} \times \hat{\sigma}_{q\bar{q}\rightarrow ev} \left( \hat{s}, \ldots \right) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0) \]
Matching for initial state radiation

- We are of course not interested in $e^+e^-$ but p-p(bar) - what happens for initial state radiation?

- Let’s do the same exercise as before:

\[
\mathcal{P} = (\Delta I_q(t_{\text{cut}}, t_0))^2 \Delta g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{q\bar{q}}(z) \frac{f_q(x_1, t_1)}{f_{\bar{q}}(x'_1, t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{q\bar{q}}(z') \\
\times \hat{\sigma}_{q\bar{q} \rightarrow e\nu}(\hat{s}, \ldots) f_q(x'_1, t_0) f_{\bar{q}}(x_2, t_0)
\]
• We are of course not interested in $e^+e^-$ but $p-p(\bar{p})$ - what happens for initial state radiation?

• Let’s do the same exercise as before:

$$\mathcal{P} = (\Delta I_q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(\hat{z})}{\hat{z}} \frac{f_q(x_1, t_1)}{f_q(x_1', t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{qg}(\hat{z}') \times \hat{\sigma}_{q\bar{q} \to e\nu} (\hat{s}, \ldots) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0)$$
• We are of course not interested in $e^+e^-$ but $p-p(\bar{p})$ - what happens for initial state radiation?

• Let’s do the same exercise as before:

$$\mathcal{P} = \left( \Delta I_q(t_{\text{cut}}, t_0) \right)^2 \Delta g(t_2, t_1) \left( \Delta q(t_{\text{cut}}, t_0) \right)^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{f_q(x_1, t_1)}{z} f_{\bar{q}}(x', t_1) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z') \times \hat{\sigma}_{q\bar{q} \rightarrow e\nu} (\hat{s}, ...) f_q(x'_1, t_0) f_{\bar{q}}(x_2, t_0)$$
• We are of course not interested in $e^+e^-$ but $p-p(\bar{p})$ - what happens for initial state radiation?

• Let’s do the same exercise as before:

$$P = (\Delta_I(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1) P_{qg}(z)}{2\pi} \frac{f_q(x_1, t_1)}{z} \frac{f_{q'}(x_1', t_1)}{2\pi} \frac{\alpha_s(t_2)}{P_{qg}(z')}$$

$$\times \hat{\sigma}_{q\bar{q} \rightarrow e\nu}(\hat{s}, ...) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0)$$
Matching for initial state radiation

\[
(\Delta_{Iq}(t_{cut}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{cut}, t_2))^2 \alpha_s(t_1) \frac{P_{gq}(z)}{2\pi} \frac{f_q(x_1, t_1)}{2\pi} \frac{\alpha_s(t_2)}{2\pi} P_{gq}(z')
\]

\[
\times \hat{\sigma}_{q\bar{q} \rightarrow e\nu}(\hat{s}, \ldots) f_q(x'_1, t_0) f_{\bar{q}}(x_2, t_0)
\]
(Δ_{lq}(t_{cut}, t_0))^2 Δ_g(t_2, t_1)(Δ_q(t_{cut}, t_2))^2 \frac{α_s(t_1)}{2π} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{2π} \frac{α_s(t_2)}{2π} \frac{P_{gq}(z')}{z'} \times \hat{σ}_{g\bar{q}\rightarrow e\nu}(\hat{s}, ...) f_q(x'_1, t_0) f_{\bar{q}}(x_2, t_0)

ME with $α_s$ evaluated at the scale of each splitting
Matching for initial state radiation

\[
(\Delta_{Iq}(t_{cut}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(t_{cut}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{gq}(z)}{z} \frac{f_q(x_1, t_1)}{f_q(x'_1, t_1)} \frac{\alpha_s(t_2)}{2\pi} P_{gq}(z')
\]

\[
\times \hat{\sigma}_{q\bar{q} \rightarrow e\nu} (\hat{s}, ...) f_q(x'_1, t_0) f_{\bar{q}}(x_2, t_0)
\]

ME with $\alpha_s$ evaluated at the scale of each splitting

PDF reweighting
Matching for initial state radiation

\[
(\Delta_{Iq}(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} \frac{P_{qq}(z)}{z} \frac{f_q(x_1, t_1)}{2\pi} \frac{\alpha_s(t_2)}{2\pi} \frac{P_{qq}(z')}{z'} \times \hat{\sigma}_{q\bar{q}\rightarrow ev}(\hat{s}, ...) f_q(x_1', t_0) f_{\bar{q}}(x_2, t_0)
\]

\[\text{ME with } \alpha_s \text{ evaluated at the scale of each splitting} \]

\[\text{PDF reweighting} \]

\[\text{Sudakov suppression due to non-branching above scale } t_{\text{cut}} \]
Matching for initial state radiation

We want to simulate $pp\rightarrow W + jets$. We pick (according to the relative cross-section of the processes) a $u\bar{d}$ event. We pick momenta according to the pdf-weighted matrix element $|M_{u\bar{d}}W_{d\bar{d}}(x_1,x_2,s_{ini})|^2 f_{u}(x_1,d_{ini}) f_{\bar{d}}(x_2,d_{ini})$. We cluster the event using the boost-invariant $k_T$ clustering scheme, to get nodes $d_1, d_2, d_3$ as shown. We apply the $s$ and Sudakov weight $(q(d_3,d_{ini}))^2 g(d_2,d_{ini}) g(d_1,d_{ini})^2 (q(d_2,d_{ini})) (s(d_2)) (s(d_{ini})) (s(d_1))$. We apply initial-state radiation for the incoming $u$ and $\bar{d}$ starting at $d_3 = M_W$, and final-state radiation for the outgoing $d$ and $\bar{d}$ starting at $d_2$, but veto all emissions above $d_{ini}$ (in both initial- and final-state showers).
- Again, use a clustering scheme to get a parton shower history
Matching for initial state radiation

- Again, use a clustering scheme to get a parton shower history

![Diagram showing parton showering and clustering](image)
Matching for initial state radiation

- Again, use a clustering scheme to get a parton shower history
- Now, reweight both due to $\alpha_s$ and PDF

\[ |\mathcal{M}|^2 \rightarrow |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)} f_q(x'_1, t_0) \frac{f_q(x'_1, t_1)}{f_q(x_1, t)} \]
• Again, use a clustering scheme to get a parton shower history
• Now, reweight both due to $\alpha_s$ and PDF

$$|\mathcal{M}|^2 \rightarrow |\mathcal{M}|^2 \frac{\alpha_s(t_1) \alpha_s(t_2) f_q(x'_1, t_0)}{\alpha_s(t_0) \alpha_s(t_0) f_q(x'_1, t_1)}$$

• Remember to use first clustering scale on each side for PDF scale:

$$\mathcal{P}_{\text{event}} = \hat{\sigma}(x_1, x_2, p_3, p_4, \ldots) f_q(x_1, t_1) f_{\bar{q}}(x_2, t_0)$$
• We still haven’t specified how to apply the Sudakov reweighting to the matrix element

• Three general schemes available in the literature:
  ➡ CKKW scheme [Catani,Krauss,Kuhn,Webber 2001; Krauss 2002]
  ➡ Lönnblad scheme (or CKKW-L) [Lönnblad 2002]
  ➡ MLM scheme [Mangano unpublished 2002; Mangano et al. 2007]
CKKW matching

[Catani, Krauss, Kuhn, Webber 2001]

[Krauss 2002]
Apply the required Sudakov suppression

\[(\Delta I_q(t_{\text{cut}}, t_0))^2 \Delta g(t_2, t_1)(\Delta q(t_{\text{cut}}, t_2))^2\]

analytically, using the best available (NLL) Sudakovs.

[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]
CKKW matching

- Apply the required Sudakov suppression

\[ \left( \Delta I_q(t_{\text{cut}}, t_0) \right)^2 \Delta g(t_2, t_1) \left( \Delta g(t_{\text{cut}}, t_2) \right)^2 \]

analytically, using the best available (NLL) Sudakovs.

- Perform “truncated showering”: Run the parton shower starting at \( t_0 \), but forbid any showers above the cutoff scale \( t_{\text{cut}} \).
**CKKW matching**

- Apply the required Sudakov suppression

\[
(\Delta_I(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_g(t_{\text{cut}}, t_2))^2
\]

analytically, using the best available (NLL) Sudakovs.

- Perform “truncated showering”: Run the parton shower starting at \(t_0\), but forbid any showers above the cutoff scale \(t_{\text{cut}}\).

\[\Delta_I(q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_g(t_{\text{cut}}, t_2))^2\]

[Catani, Krauss, Kuhn, Webber 2001]
[Krauss 2002]
CKKW matching

- Apply the required Sudakov suppression

\[(\Delta_{Iq}(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1) \Delta_q(t_{\text{cut}}, t_2))^2\]

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- Perform “truncated showering”: Run the parton shower starting at \(t_0\), but forbid any showers above the cutoff scale \(t_{\text{cut}}\).
CKKW matching

- Apply the required Sudakov suppression
  \[(\Delta_Iq(t_{cut}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(t_{cut}, t_2))^2\]
  analytically, using the best available (NLL) Sudakovs.

- Perform “truncated showering”: Run the parton shower starting at \(t_0\), but forbid any showers above the cutoff scale \(t_{cut}\).

  ✓ Best theoretical treatment of matrix element
  - Requires dedicated PS implementation
  - Mismatch between analytical Sudakov and (non-NLL) shower

- Implemented in Sherpa (v. 1.1)
An example of the procedure

We want to simulate \(pp \rightarrow W + \text{jets}\). We pick (according to the relative cross-section of the processes) a \(u \bar{d}\) event.

We pick momenta according to the pdf-weighted matrix element:

\[
|M_{u \bar{d}}| \left( x_1, x_2, s_{\text{ini}} \right) \]

\[
f_{u}(x_1, d_{\text{ini}}) f_{\bar{d}}(x_2, d_{\text{ini}})
\]

We cluster the event using the boost-invariant \(k_T\) clustering scheme, to get nodes \(d_1, d_2, d_3\) as shown.

We apply the \(s\) and Sudakov weight:

\[
s(d_3, d_{\text{ini}})^2 g(d_2, d_{\text{ini}}) g(d_1, d_{\text{ini}}) s(d_2, d_{\text{ini}})^2 s(d_1, d_{\text{ini}})
\]

We apply initial-state radiation for the incoming \(u\) and \(\bar{d}\) starting at \(d_3 = M_W\), and final-state radiation for the outgoing \(d\) and \(\bar{d}\) starting at \(d_2\), but veto all emissions above \(d_{\text{ini}}\) (in both initial- and final-state showers).

[CKKW-L matching]

[Lönnblad 2002]

[Hoeche et al. 2009]
• Cluster back to “parton shower history”
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• Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
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• Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
• Veto the event if any shower is harder than the clustering scale for the next step (or $t_{\text{cut}}$, if last step)
• Cluster back to “parton shower history”
• Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
• Veto the event if any shower is harder than the clustering scale for the next step (or $t_{\text{cut}}$, if last step)
• Keep any shower emissions that are softer than the clustering scale for the next step
• Cluster back to “parton shower history”
• Perform showering step-by-step for each step in the parton shower history, starting from the clustering scale for that step
• Veto the event if any shower is harder than the clustering scale for the next step (or $t_{\text{cut}}$, if last step)
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• Veto the event if any shower is harder than the clustering scale for the next step (or $t_{\text{cut}}$, if last step)
• Keep any shower emissions that are softer than the clustering scale for the next step
CKKW-L matching

✓ Automatic agreement between Sudakov and shower

- Requires dedicated PS implementation
  ➡ Need multiple implementations to compare between showers

• Implemented in Ariadne, Sherpa (v. 1.2), and Pythia 8
We want to simulate $pp \rightarrow Z + \text{jets}$.

We pick (according to the relative cross-section of the processes) a $u\bar{d}W\bar{d}$ event.

We pick momenta according to the pdf-weighted matrix element

$$|M_{u\bar{d}W\bar{d}}(x_1, x_2, s_{\text{ini}})|^2 f_{u}(x_1, d_{\text{ini}}) f_{\bar{d}}(x_2, d_{\text{ini}})$$

We cluster the event using the boost-invariant $k_T$ clustering scheme, to get nodes $d_1, d_2, d_3$ as shown.

We apply the $q$ and Sudakov weight

$$2 g(d_2, d_{\text{ini}}) g(d_1, d_{\text{ini}}) (q(d_1, d_{\text{ini}}))^2 s(d_2) s(d_{\text{ini}}) s(d_1) s(d_{\text{ini}})$$

We apply initial-state radiation for the incoming $u$ and $\bar{d}$ starting at $d_3 = M_W$, and final-state radiation for the outgoing $d$ and $\bar{d}$ starting at $d_2$, but veto all emissions above $d_{\text{ini}}$ (in both initial- and final-state showers).

\[M.L.\ Mangano, \sim2002, \ 2007\]
\[J.A. \ et \ al \ 2007, \ 2008\]
The simplest way to do the Sudakov suppression is to run the shower on the event, starting from $t_0$!
The simplest way to do the Sudakov suppression is to run the shower on the event, starting from $t_0$!

\[
M \equiv \left| M_{\bar{u} d W d \bar{d}} (x_1, x_2, s_{\text{ini}}) \right|^2 f_{\bar{u}}(x_1, s_{\text{ini}}) f_{d}(x_2, s_{\text{ini}})
\]
The simplest way to do the Sudakov suppression is to run the shower on the event, starting from $t_0$!

Perform jet clustering after PS - if hardest jet $k_T > t_{\text{cut}}$ or there are jets not matched to partons, reject the event.
The simplest way to do the Sudakov suppression is to run the shower on the event, starting from $t_0$!

Perform jet clustering after PS - if hardest jet $k_{T1} > t_{cut}$ or there are jets not matched to partons, reject the event.

The resulting Sudakov suppression from the procedure is

$$ (\Delta_I q(t_{cut}, t_0))^2 (\Delta q(t_{cut}, t_0))^2 $$

which turns out to be a good enough approximation of the correct expression

$$ (\Delta_I q(t_{cut}, t_0))^2 \Delta g(t_2, t_1) (\Delta q(t_{cut}, t_2))^2 $$
MLM matching

- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from \( t_0 \! \)!

- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from \( t_0 \! \)!

- Simplest available scheme
- Allows matching with any shower, without modification
  - Sudakov suppression not exact, minor mismatch with shower
- Implemented in AlpGen, HELAC, MadGraph+Pythia 6

[J.A. et al 2007, 2008]
• In the previous, assumed we can simulate all parton multiplicities by the ME

• In practice, we can only do limited number of final-state partons with matrix element (up to 4-5 or so)

• For the highest jet multiplicity that we generate with the matrix element, we need to allow additional jets above the matching scale $t_{\text{cut}}$, since we will otherwise not get a jet-inclusive description – but still can’t allow PS radiation harder than the ME partons

⇒ Need to replace $t_{\text{cut}}$ by the clustering scale for the softest ME parton for the highest multiplicity
We have a number of choices to make in the above procedure. The most important are:

1. The clustering scheme used to determine the parton shower history of the ME event
2. What to use for the scale $Q^2$ (factorization scale)
3. How to divide the phase space between parton showers and matrix elements
Back to the “matching goal”

- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions

2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia
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Emissions from PS

Parton shower

Matching scale

\[ \log(DJR) \]
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Summary of Matching Procedure

1. Generate ME events (with different parton multiplicities) using parton-level cuts (\(p_T^{\text{ME}}/\Delta R\) or \(k_T^{\text{ME}}\))

2. Cluster each event and reweight \(\alpha_s\) and PDFs based on the scales in the clustering vertices

3. Apply Sudakov factors to account for the required non-radiation above clustering cutoff scale and generate parton shower emissions below clustering cutoff:
   a. (CKKW) Analytical Sudakovs + truncated showers
   b. (CKKW-L) Sudakovs from truncated showers
   c. (MLM) Sudakovs from reclustered shower emissions

4. Apply separation cut
Comparing to experiment: $W+$jets

- Very good agreement at Tevatron (left) and LHC (right)

- Matched samples obtained via different matching schemes (MLM and CKKW) consistent within the expected uncertainties.

- Pure parton shower (Pythia) doesn’t describe the data beyond 1st jet.
Example: Simulation of pp→W with 0, 1, 2 jets (comfortable on a laptop)

```
mg5> generate p p > w+, w+ > l+ vl @0
mg5> add process p p > w+ j, w+ > l+ vl @1
mg5> add process p p > w+ j j, w+ > l+ vl @2
mg5> output
```

In run_card.dat:
```
... 1 = ickkw
... 0 = ptj
... 15 = xqcut
```

Matching automatically done when run through MadEvent and Pythia!
• By default, $k_T$-MLM matching is run if $xQcut > 0$, with the matching scale $Qcut = \max(xQcut*1.4, xQcut+10)$

• For shower-$k_T$, by default $Qcut = xQcut$

• If you want to change the Pythia setting for matching scale or switch to shower-$k_T$ matching:

```
In pythia_card.dat:
...
! This sets the matching scale, needs to be > xQcut
Qcut = 30
! This switches from $k_T$-MLM to shower-$k_T$ matching
! Note that MSTP(81)>=20 needed (pT-ordered shower)
SHOWERKT = T
```
How to do validate the matching

• The matching scale (Q CUT) should typically be chosen around $1/6-1/2 \times$ hard scale (so $x_{\text{qcut}}$ correspondingly lower)

• The matched cross section (for $X+0,1,...$ jets) should be close to the unmatched cross section for the 0-jet sample (found on the process HTML page)

• The differential jet rate plots should be smooth

• When Q CUT is varied (within the region of validity), the matched cross section or differential jet rates should not vary significantly
• This are the clustering scales in the kt-jet clustering scheme

• DJR1: pT of the last remaining jet

• DJR2: The minimum between the pT of the second to last remaining jet and the kt between the last two jet.

• Only radiative jet (not those from decay) should enter those plot.
Matching Validation

$W+$jets production at the Tevatron for MadGraph+Pythia
($k_T$-jet MLM scheme, $q^2$-ordered Pythia showers)

$Q^{\text{match}} = 10$ GeV

$Q^{\text{match}} = 30$ GeV

$\log(\text{Differential jet rate for } 1 \rightarrow 2 \text{ radiated jets } \sim p_T(2\text{nd jet}))$
Matching Validation

W+jets production at the Tevatron for MadGraph+Pythia
($k_T$-jet MLM scheme, $q^2$-ordered Pythia showers)
Matching Validation

$W+jets$ production at the Tevatron for MadGraph+Pythia
($k_T$-jet MLM scheme, $q^2$-ordered Pythia showers)
Matching Validation

$W+\text{jets}$ production at the Tevatron for MadGraph+Pythia
($k_T$-jet MLM scheme, $q^2$-ordered Pythia showers)

Jet distributions smooth, and stable when we vary the matching scale!
In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)
In a matched sample these differences are irrelevant since the behavior at high pt is dominated by the matrix element.
Lecture Summary

• Despite the apparent enormous complexity of simulation of complete collider events, nature has kindly allowed us to factorize the simulation into separate steps

• The Monte Carlo method allows us to step-by-step simulate hard scattering, parton shower, particle decays, hadronization, and underlying event

• Jet matching between matrix elements and parton showers gives crucial improvement of simulation of background as well as signal processes

• Running matching with MadGraph + Pythia is very easy, but the results should always be checked for consistency

• Matching is mandatory at NLO (actually without merging)