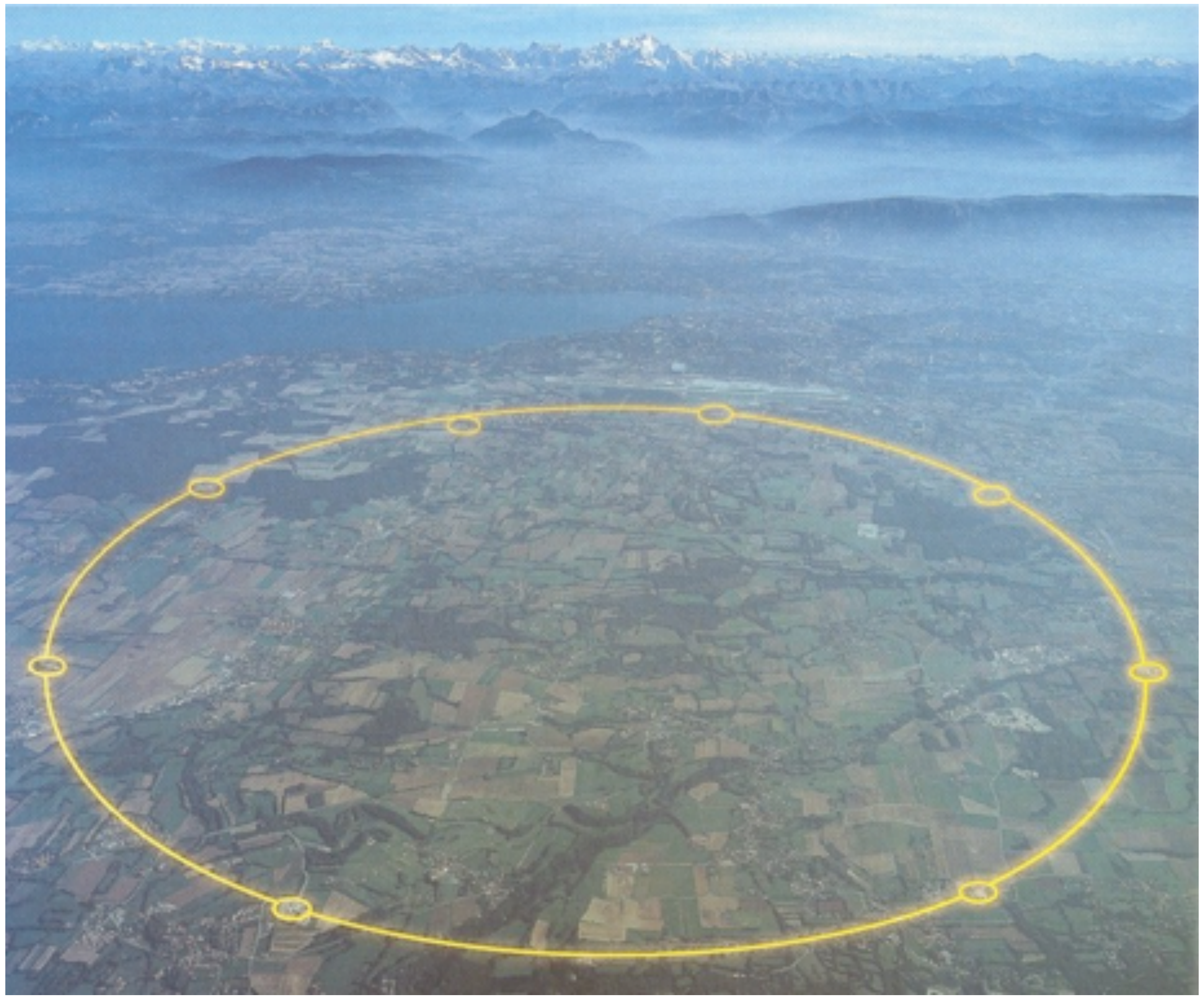
Monte-Carlo Generation

Olivier Mattelaer
IPPP/Durham

Introduction

Topic

- Collider Physics



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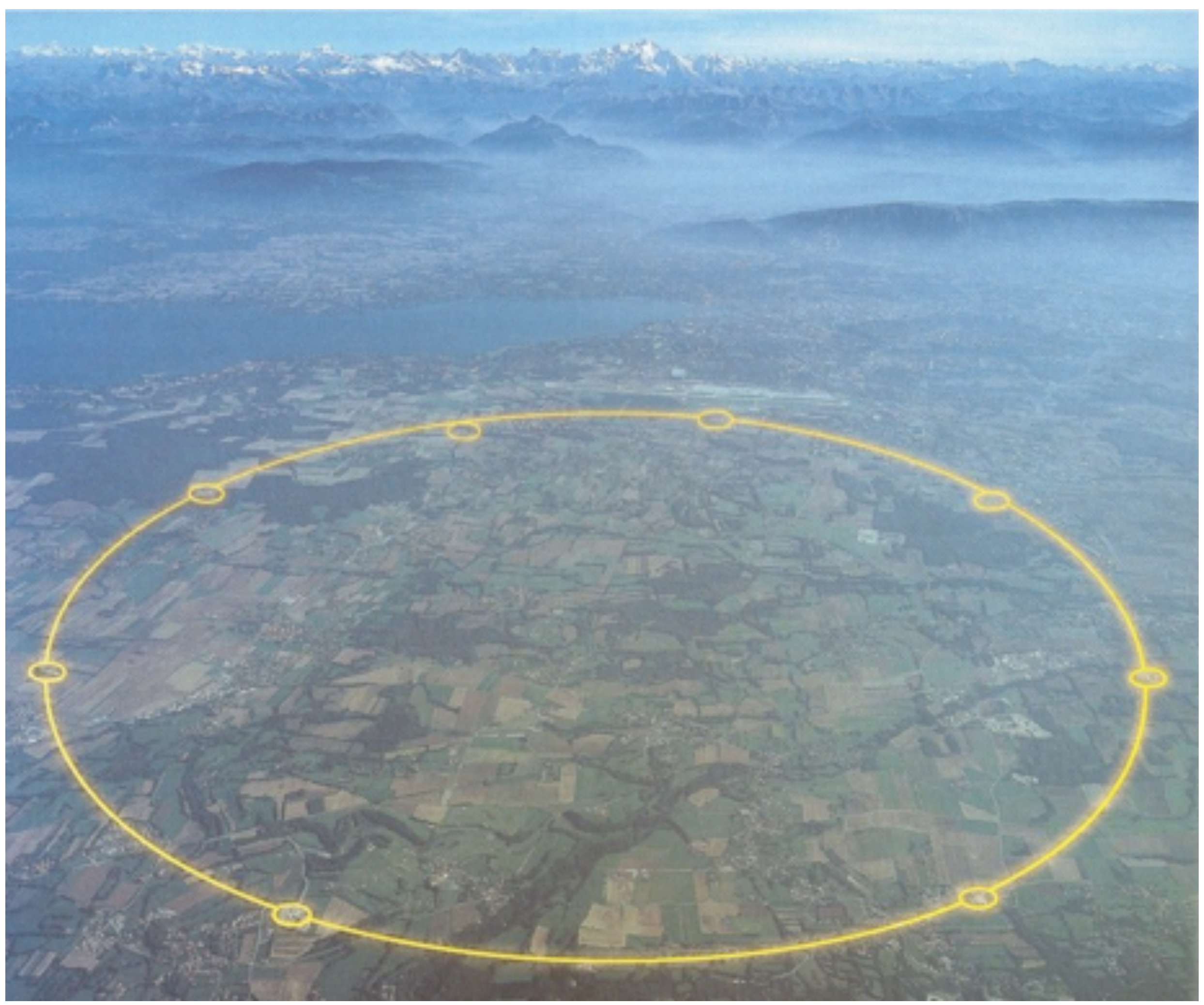
- Collider Physics
  - accelerating particle -> High Energy collision



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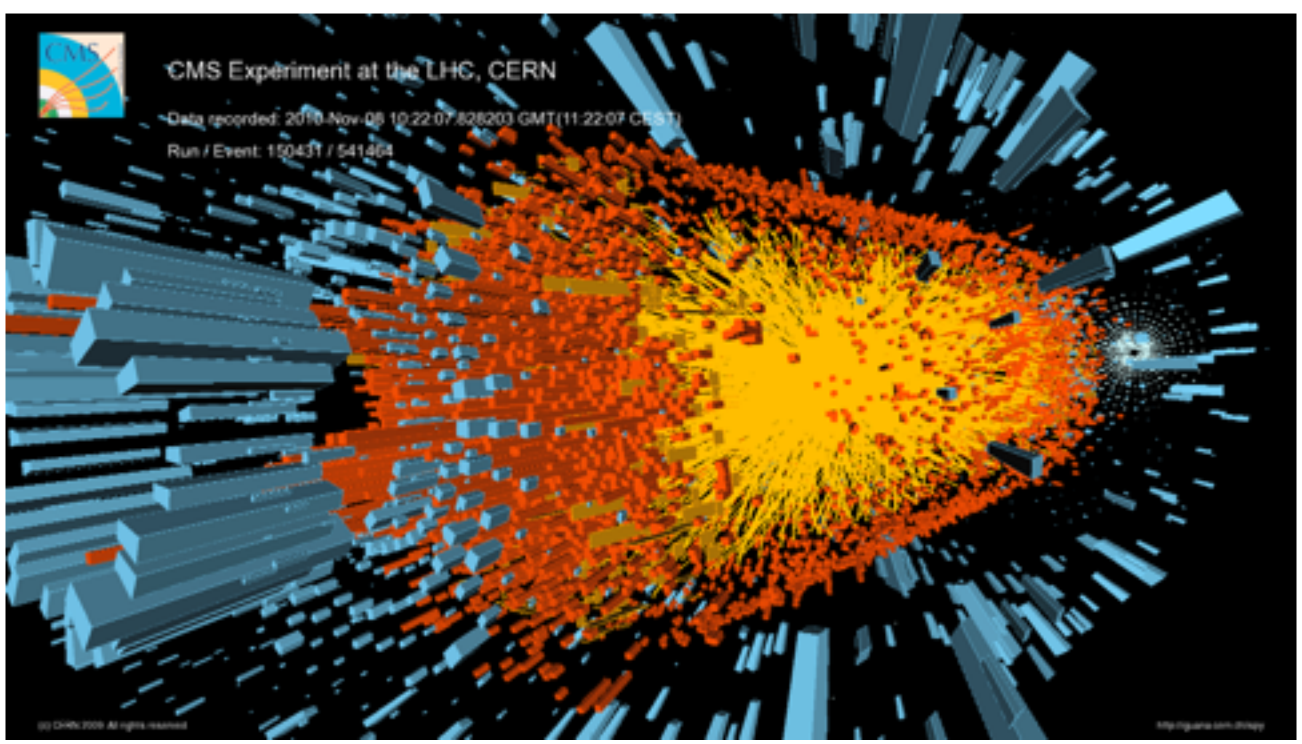
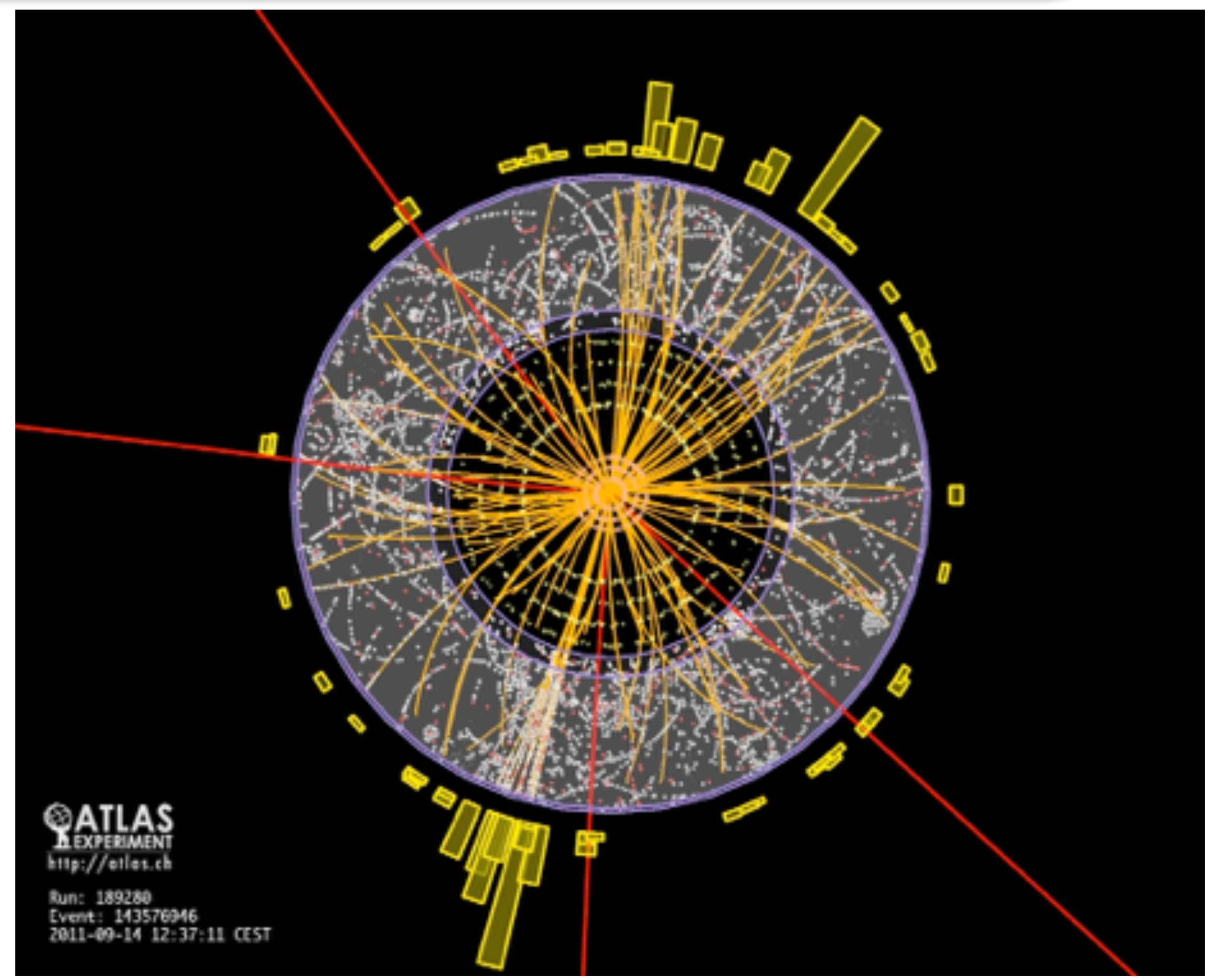
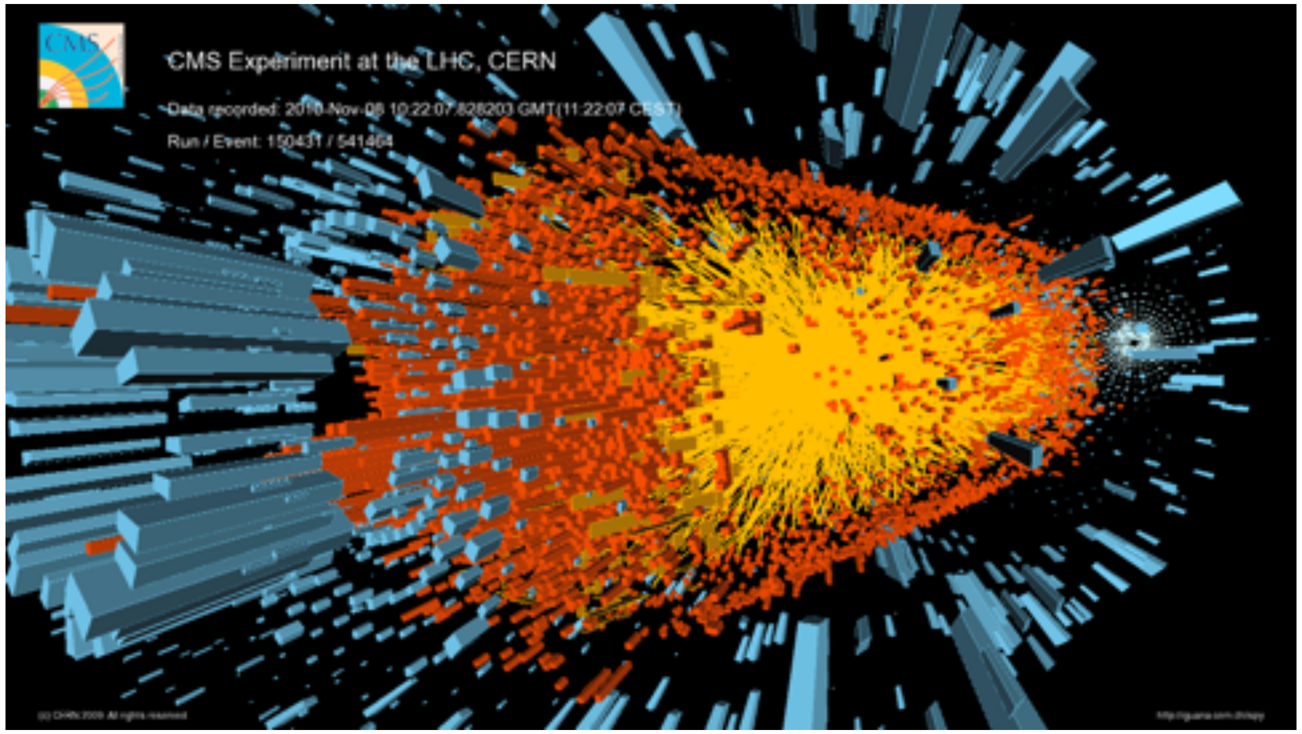


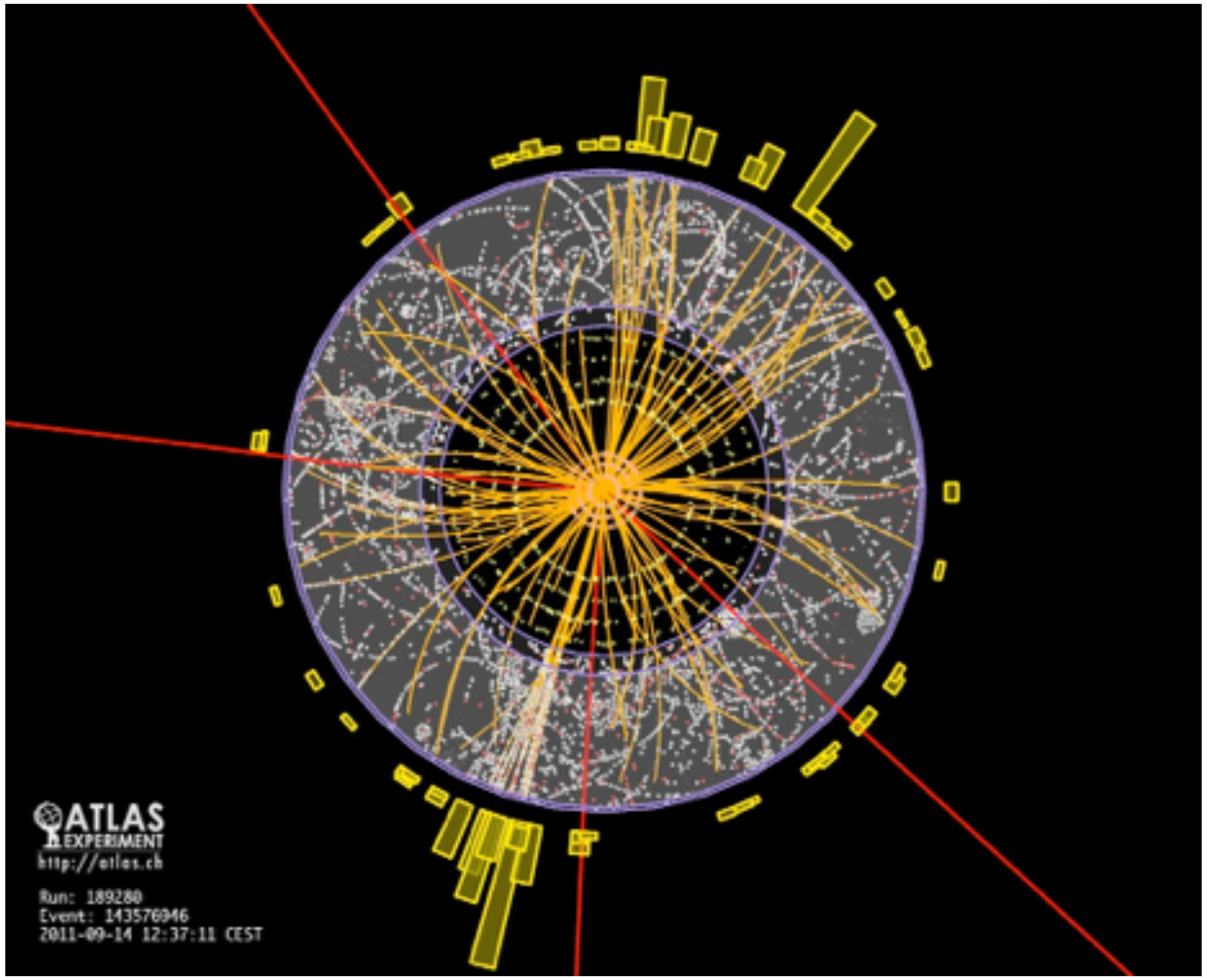


Introduction

Topic

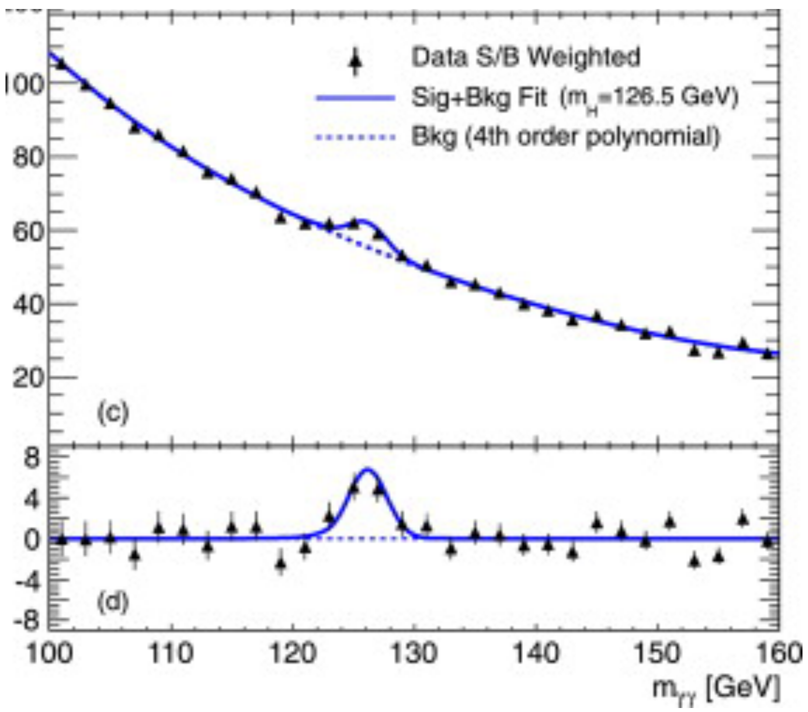
- Collider Physics
  - accelerating particle -> High Energy collision
- What do we need to predict/understand such collision?





Kind of measurement

Peak



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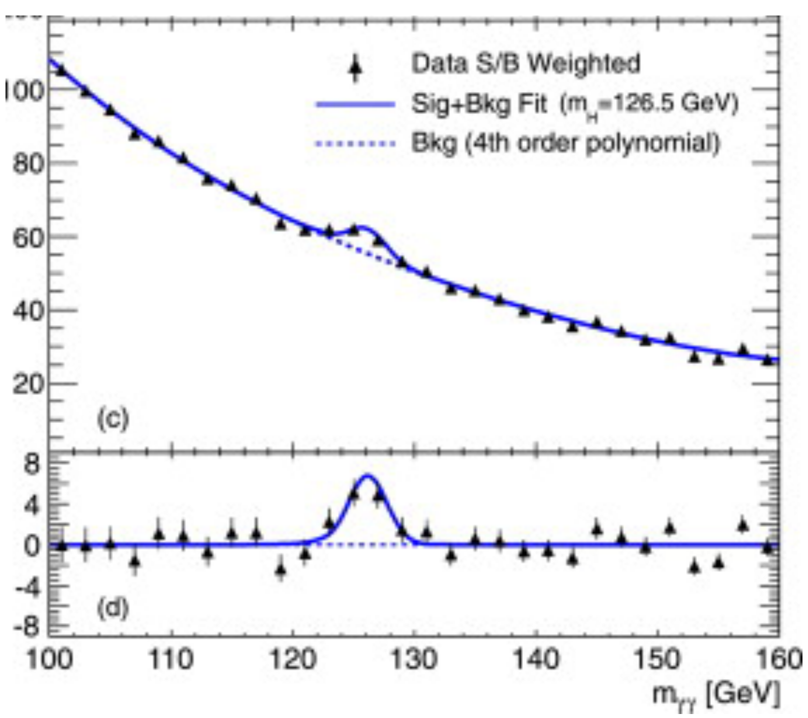
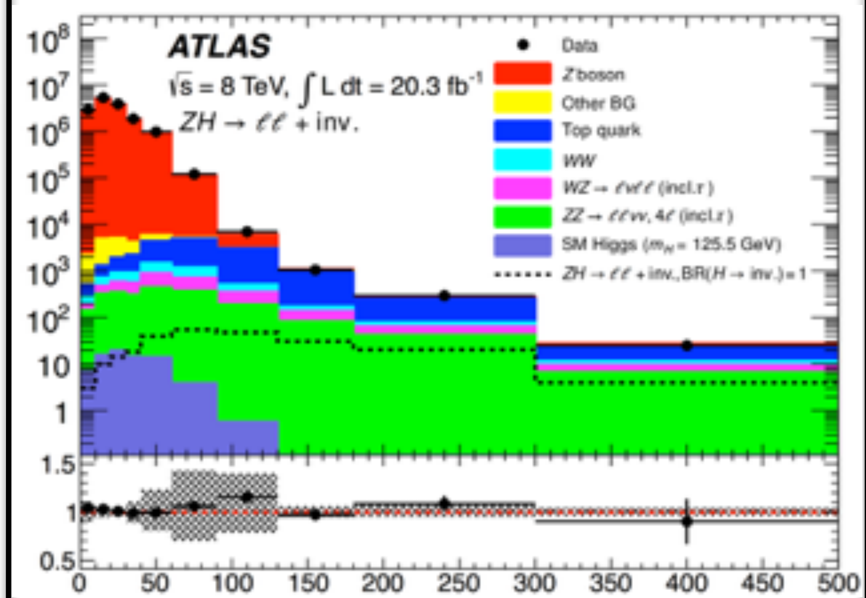


Shape



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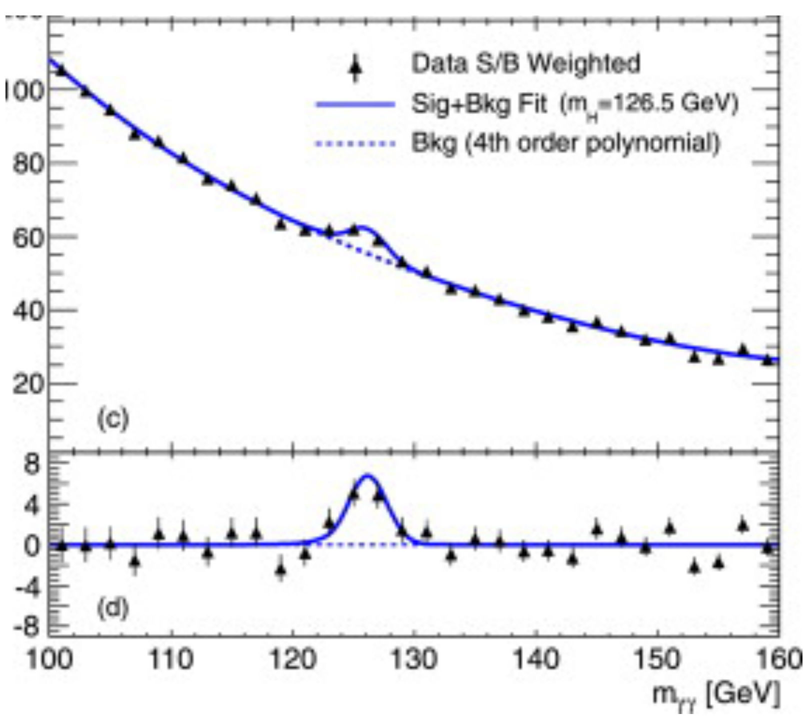

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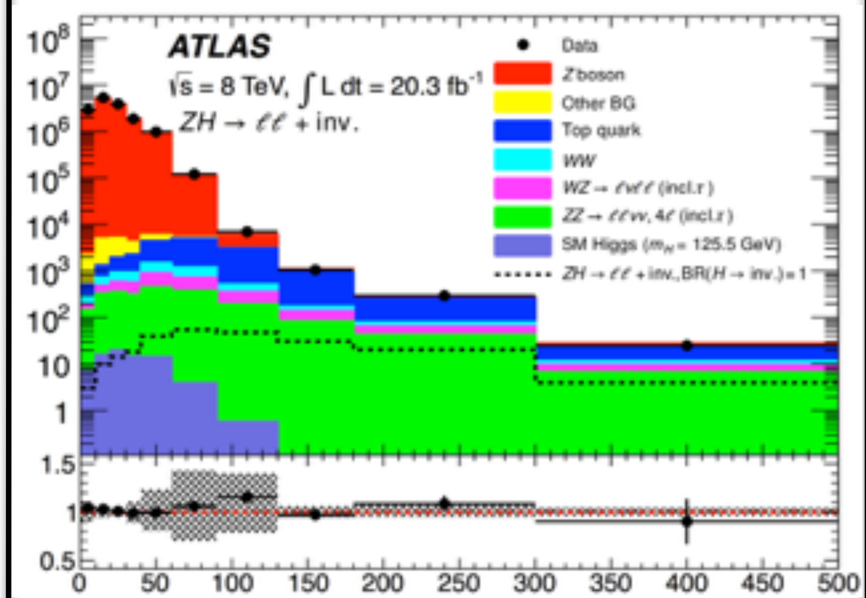
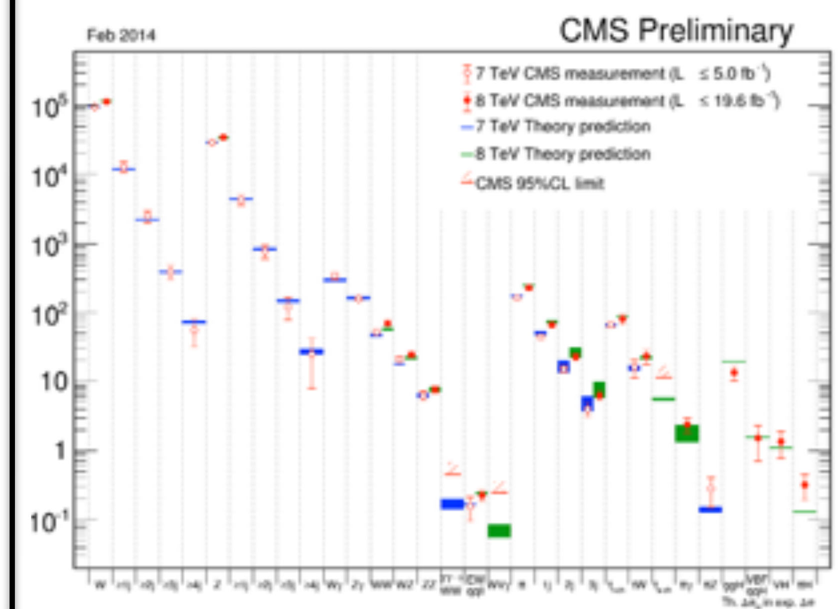


Rate



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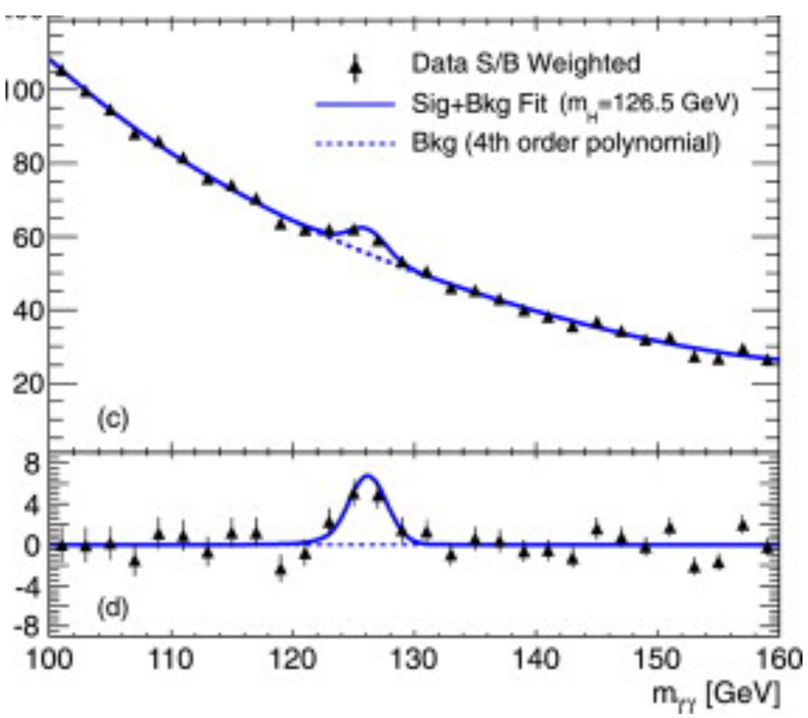
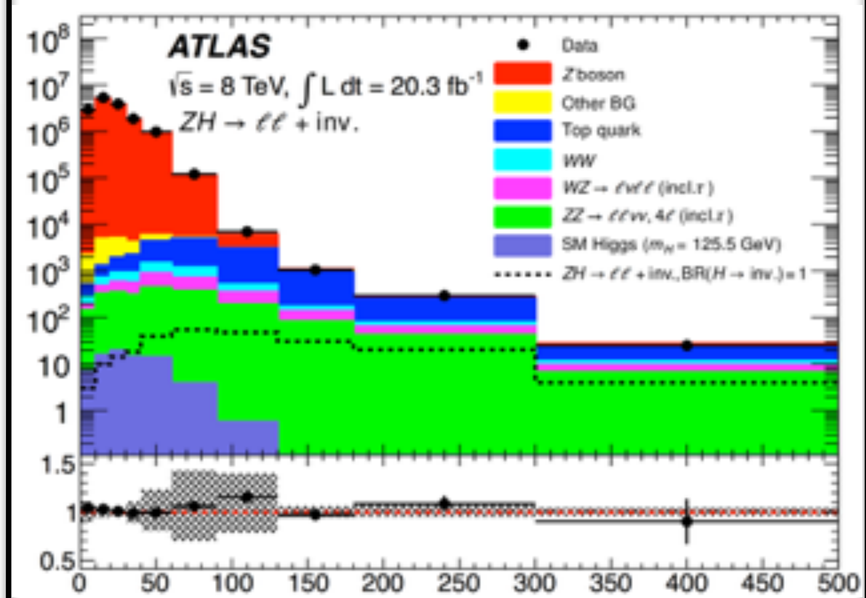


"EASY"

Shape



"HARD"

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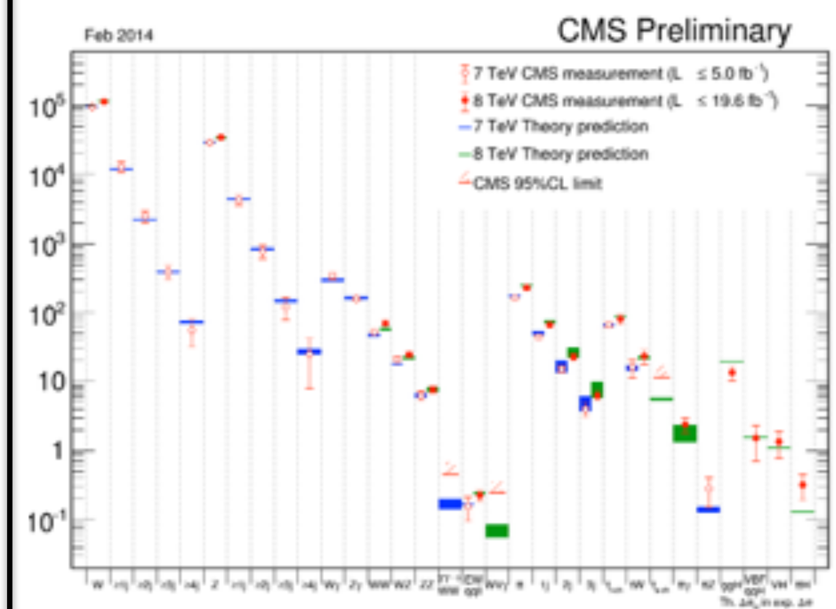

"VERY HARD"

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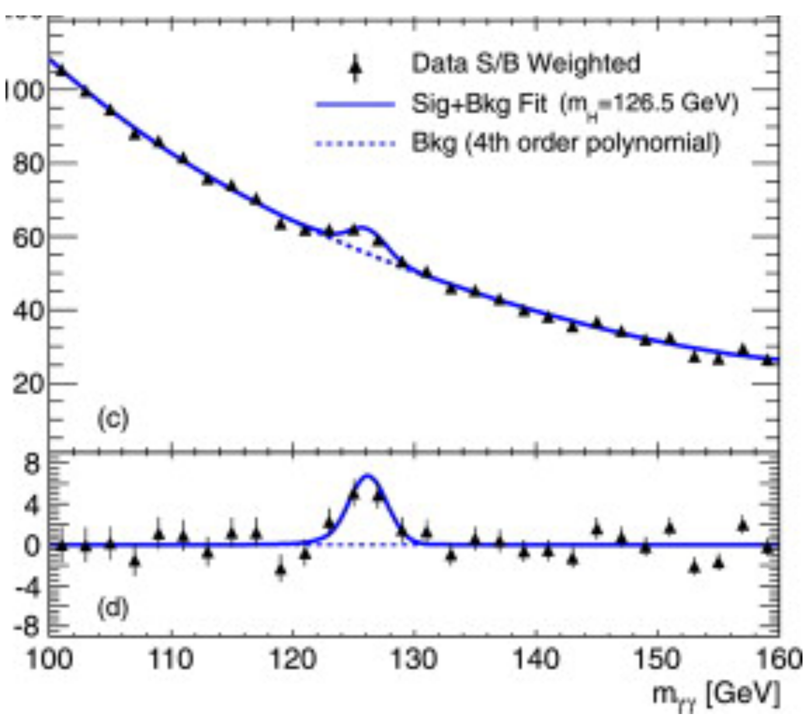

“EASY”

Background directly measured from **data**. Theory needed only for parameter extraction

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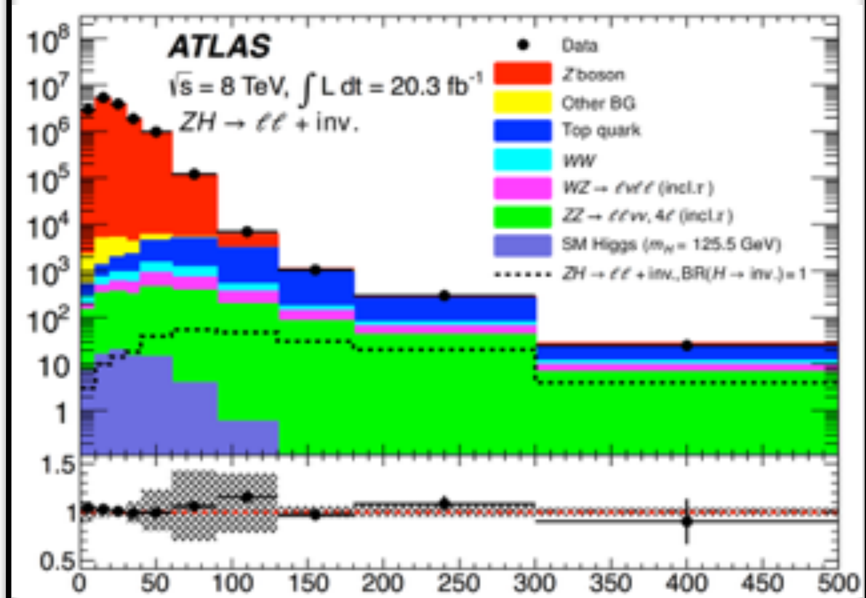

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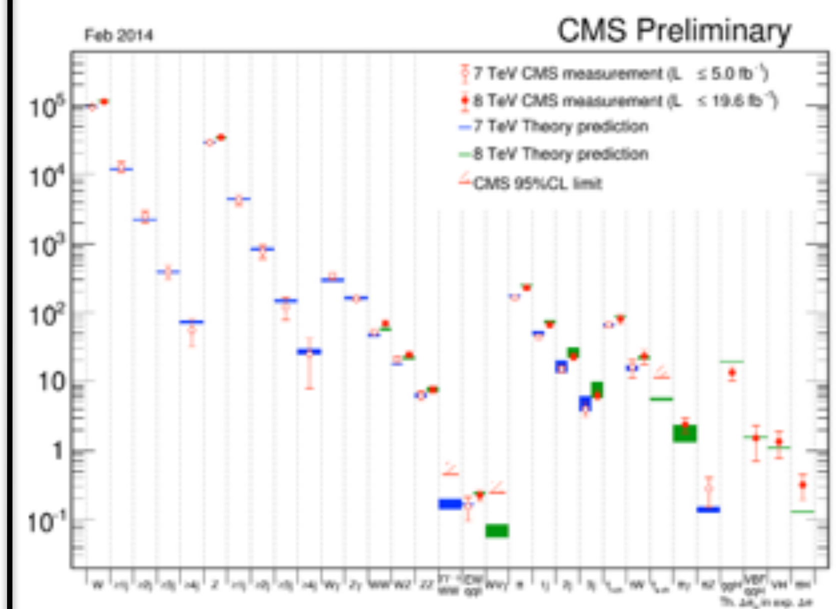

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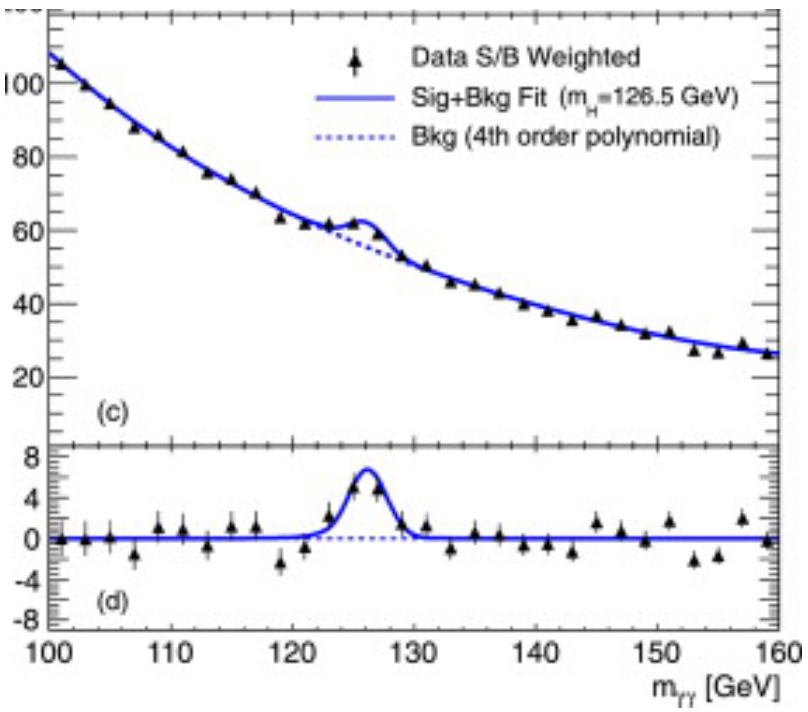

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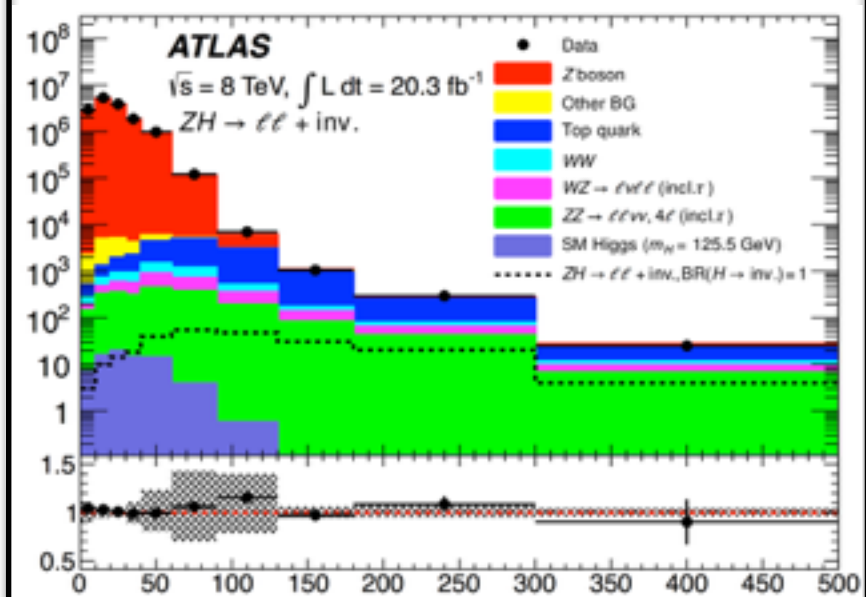
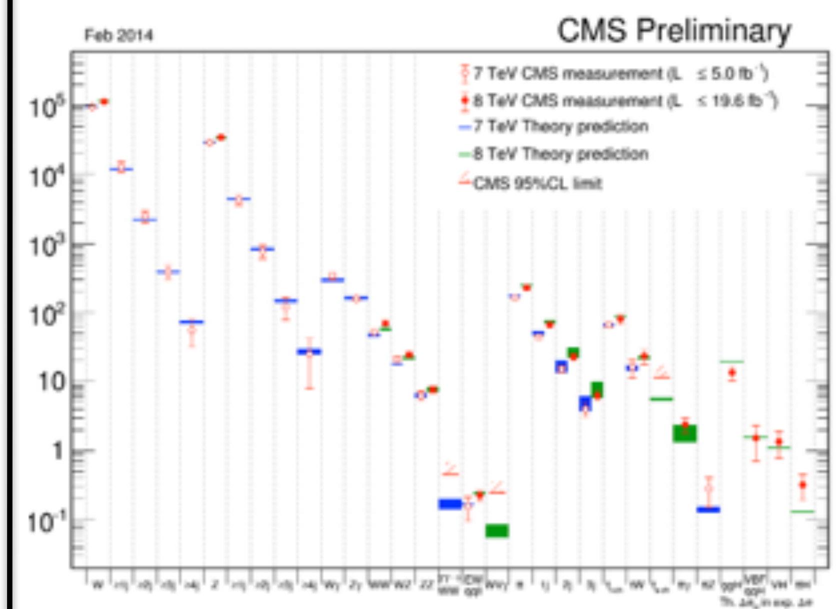


“HARD”

Background **SHAPE** needed. Flexible MC for both signal and background validated and tuned to data

Rate



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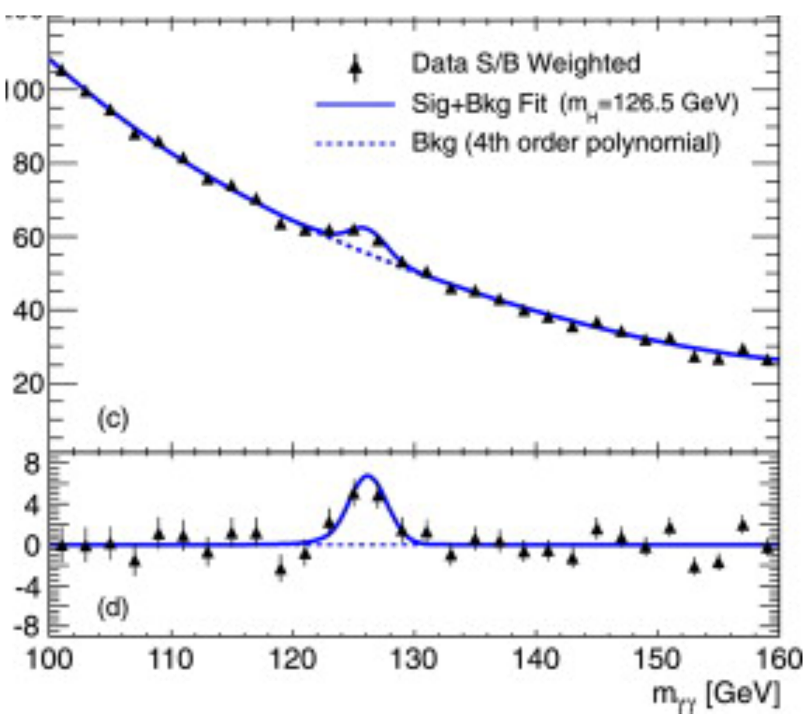

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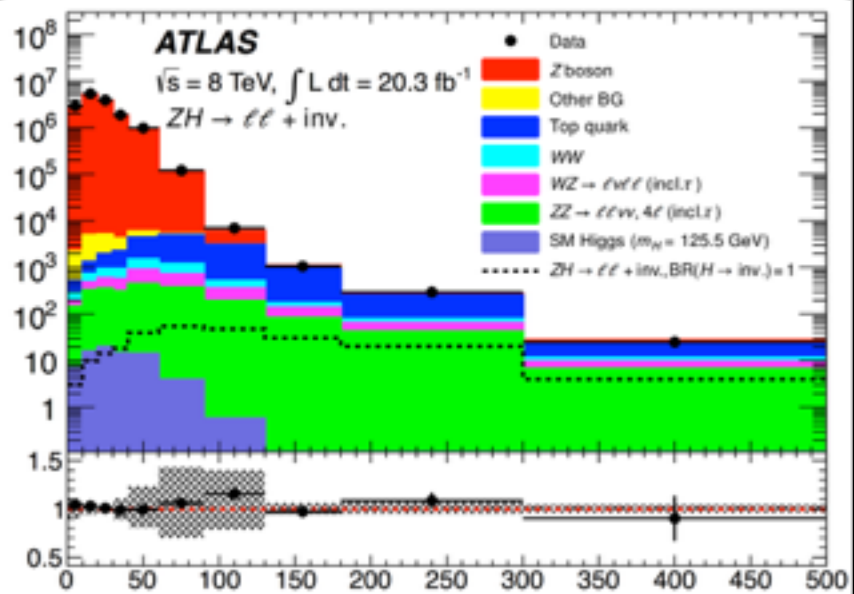

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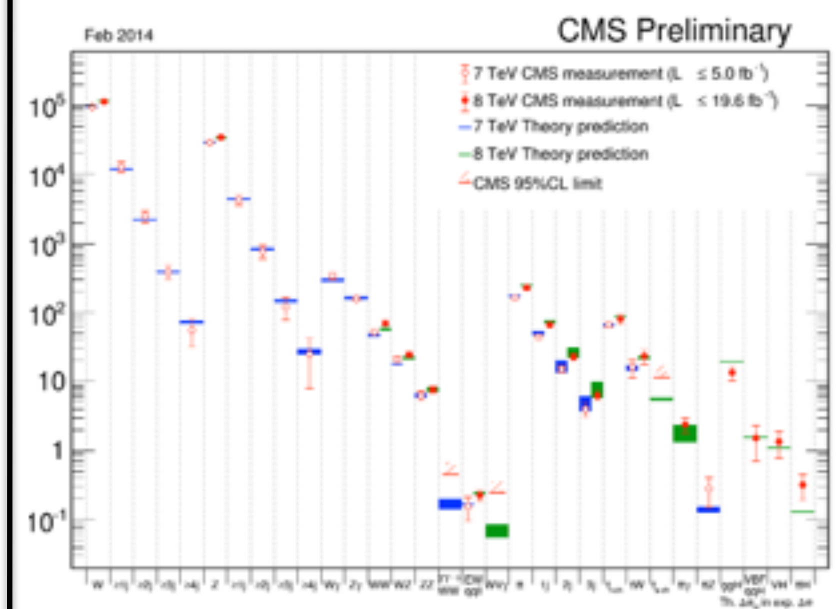
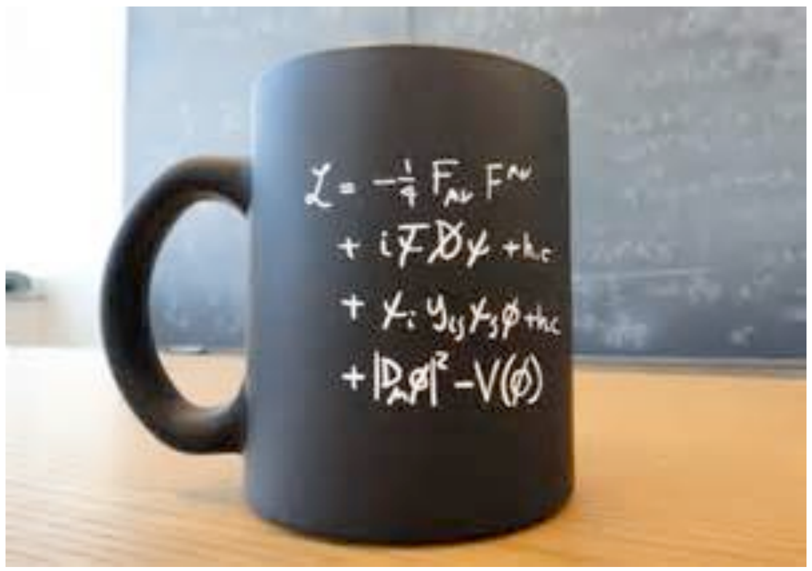


"VERY HARD"

Relies on prediction for both **shape** and **normalization**. Complicated interplay of best simulations and data

Theory side

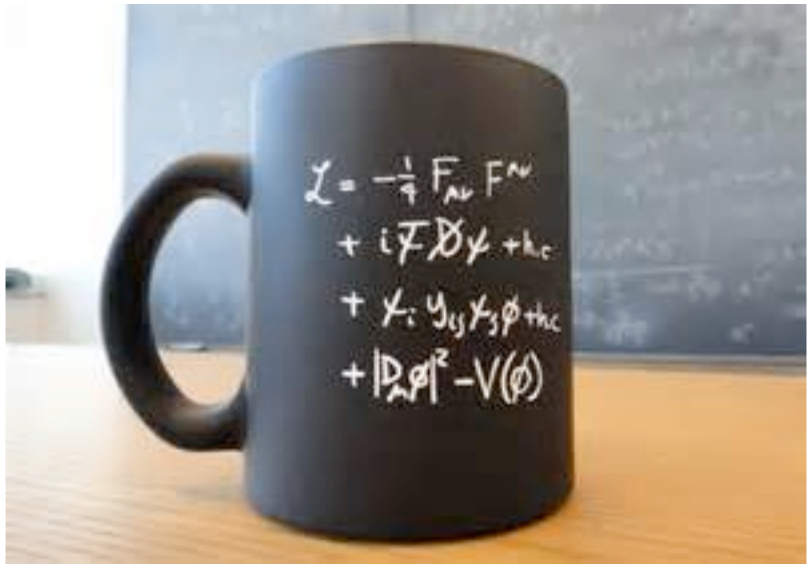
Lagrangian



- This is Where the new idea are expressed

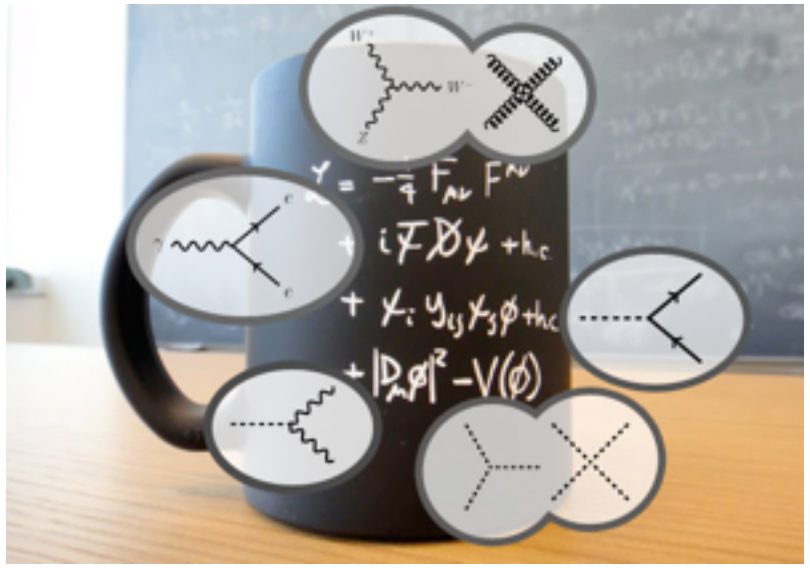
Theory side

Lagrangian

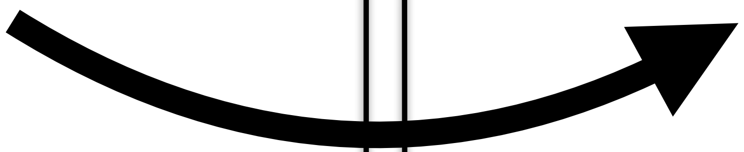


- This is Where the new idea are expressed

Feynman Rule



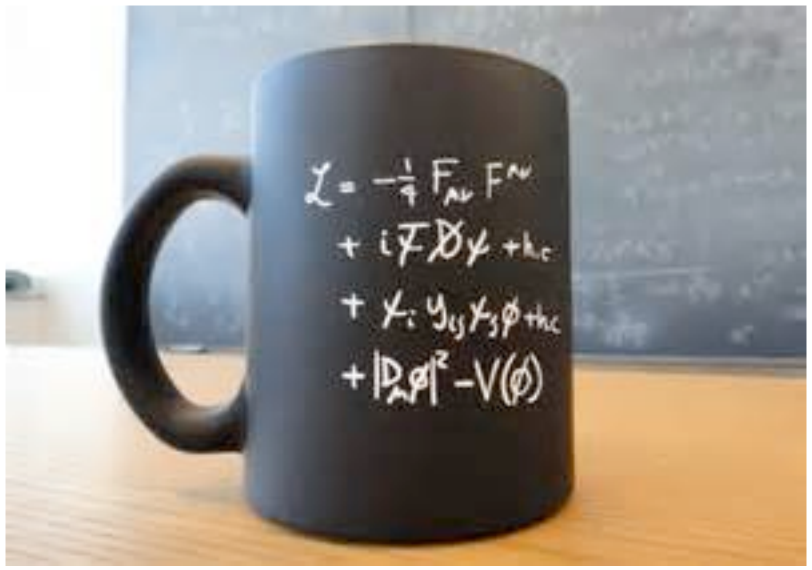
- Same information as the Lagrangian



FeynRules

Theory side

Lagrangian



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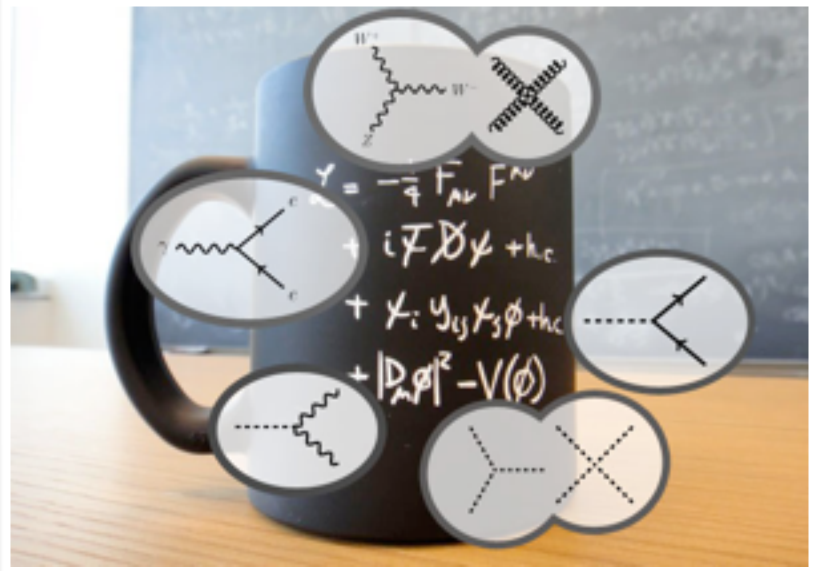
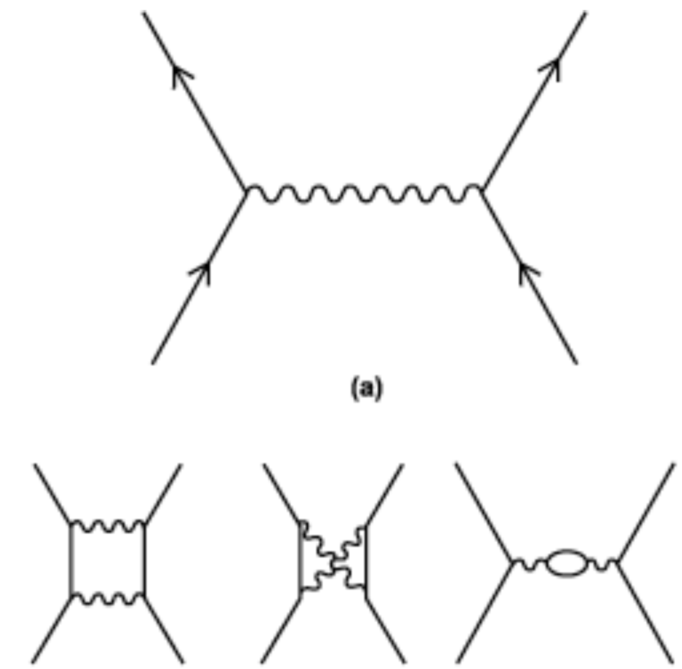


- Same information as the Lagrangian

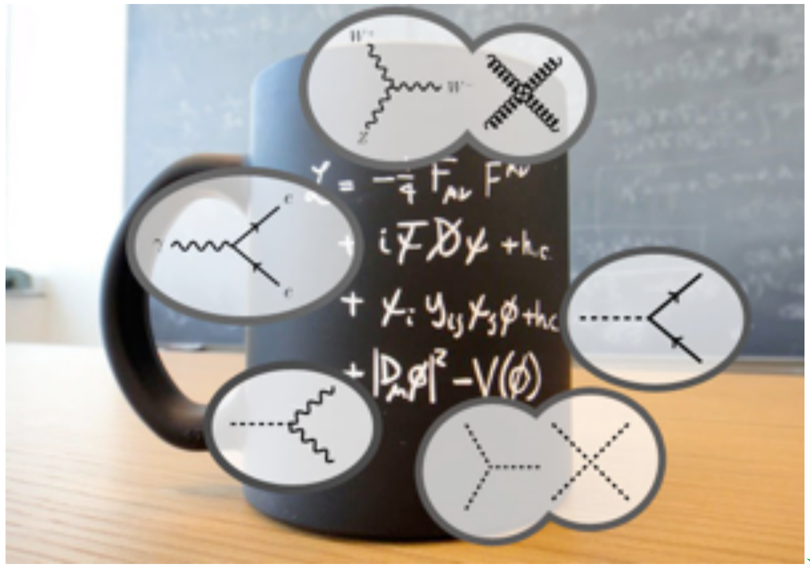
FeynRules

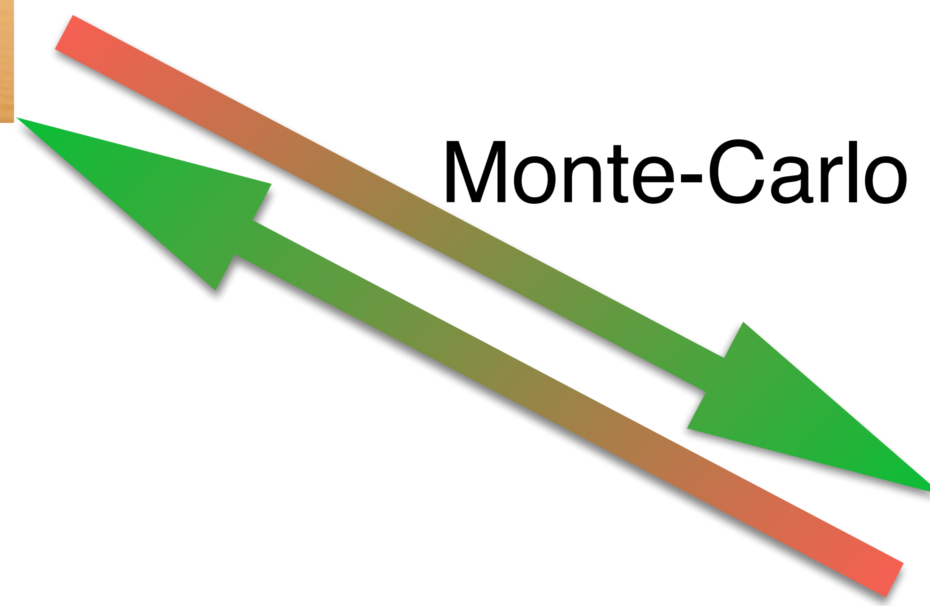
Cross-section

$$\frac{d\sigma}{d\ \cos\theta} = \left(\frac{d\sigma}{d\ \cos\theta}\right)_R \frac{(1+\cos\theta)/2}{\left[1+\frac{(1-\cos\theta)KE}{Mc^2}\right]}$$

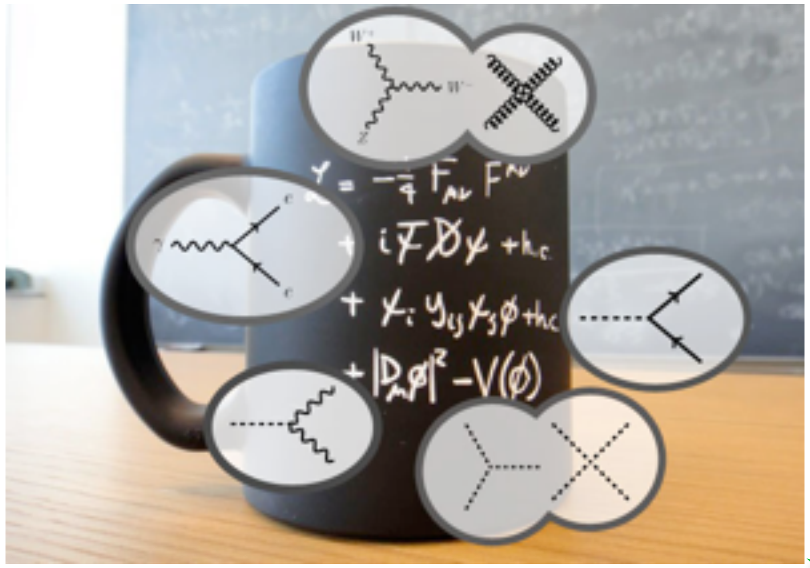


- What is the precision?





Monte-Carlo Physics



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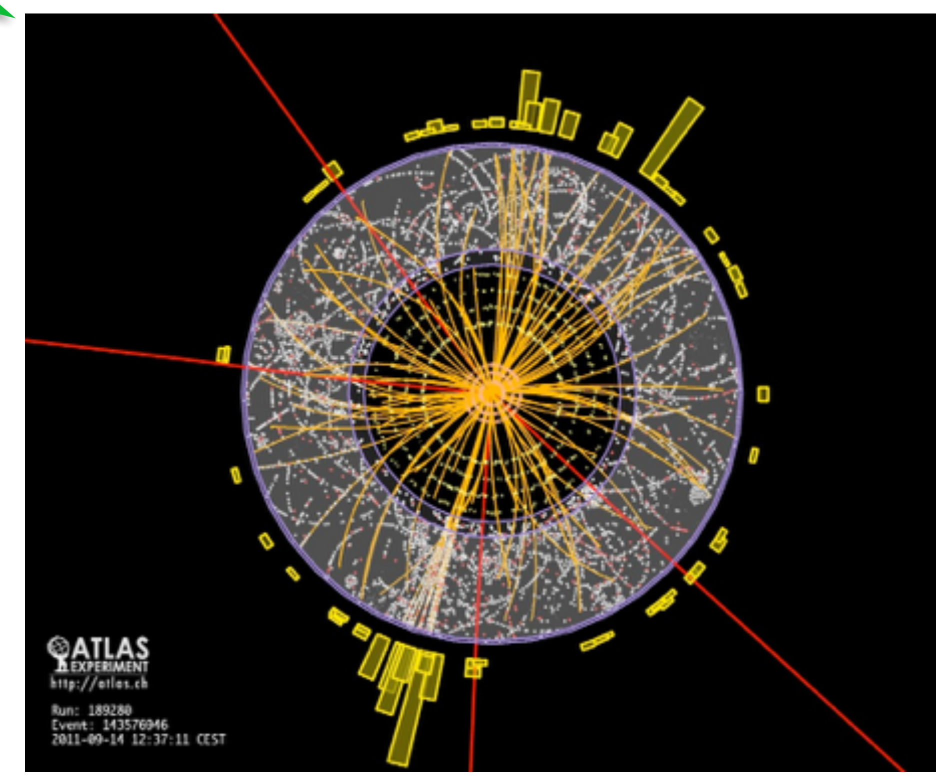
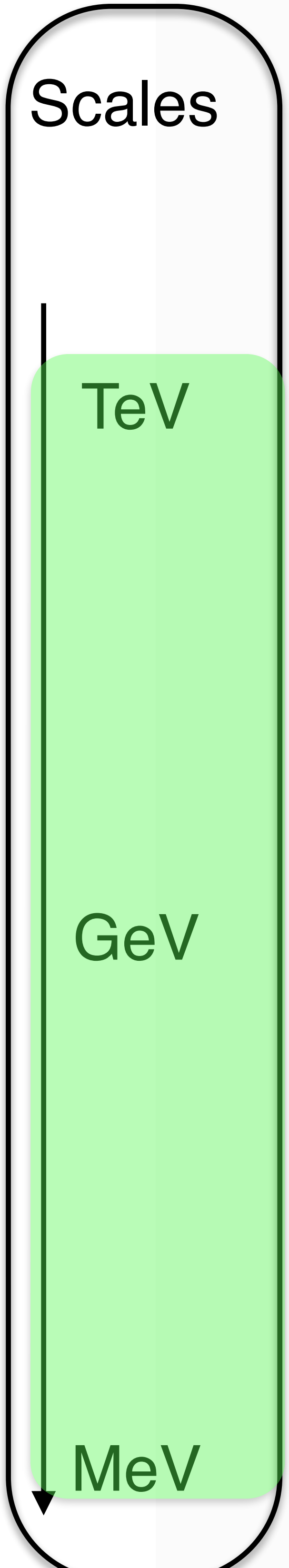
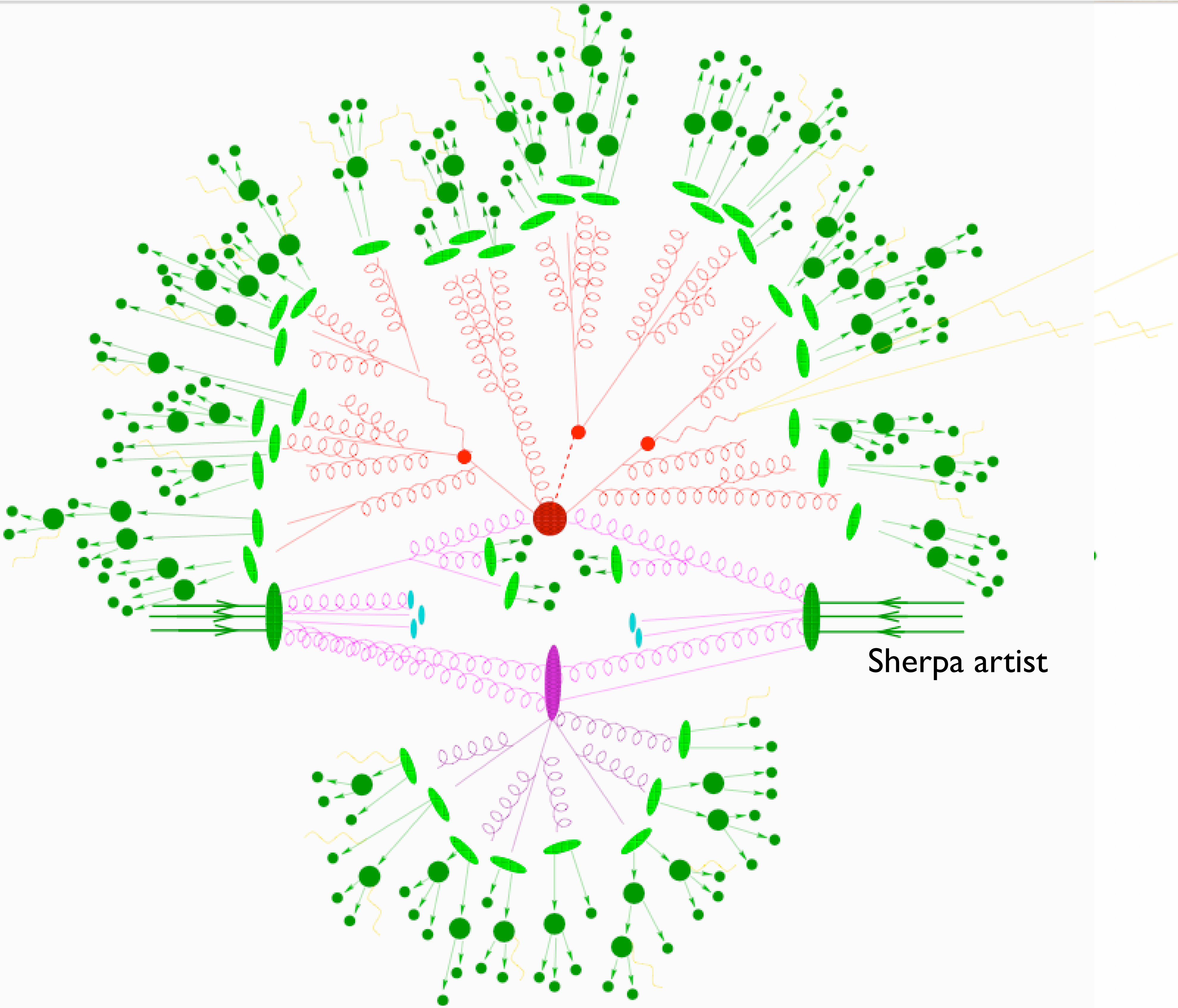


Simulation of collider events

What are the MC for?





What are the MC for?

Scales

TeV

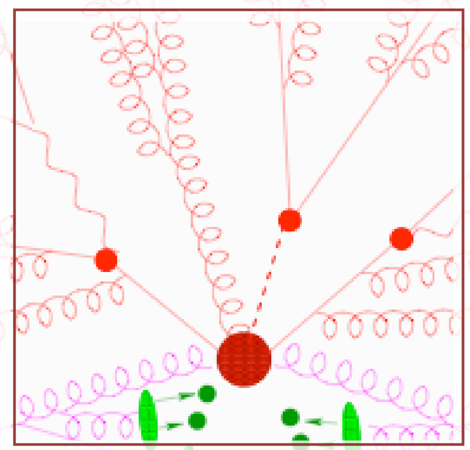
GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ where BSM physics lies



☞ process dependent

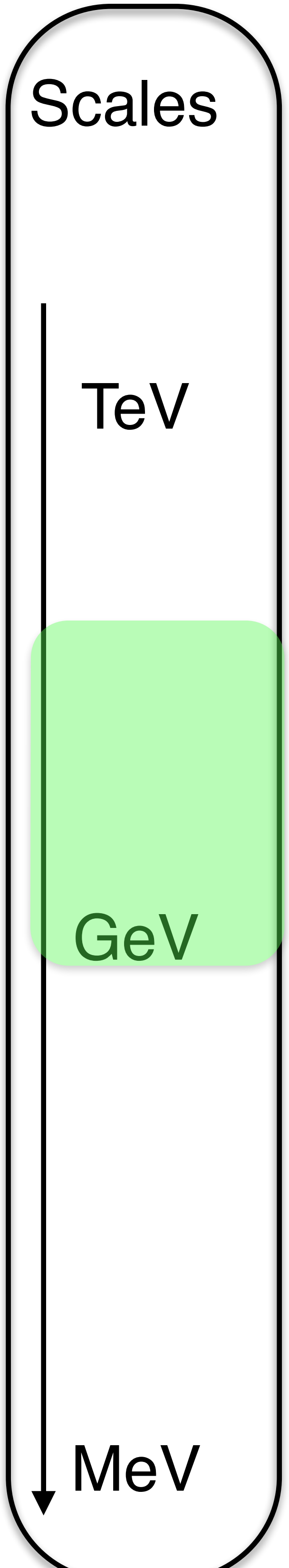
☞ first principles description

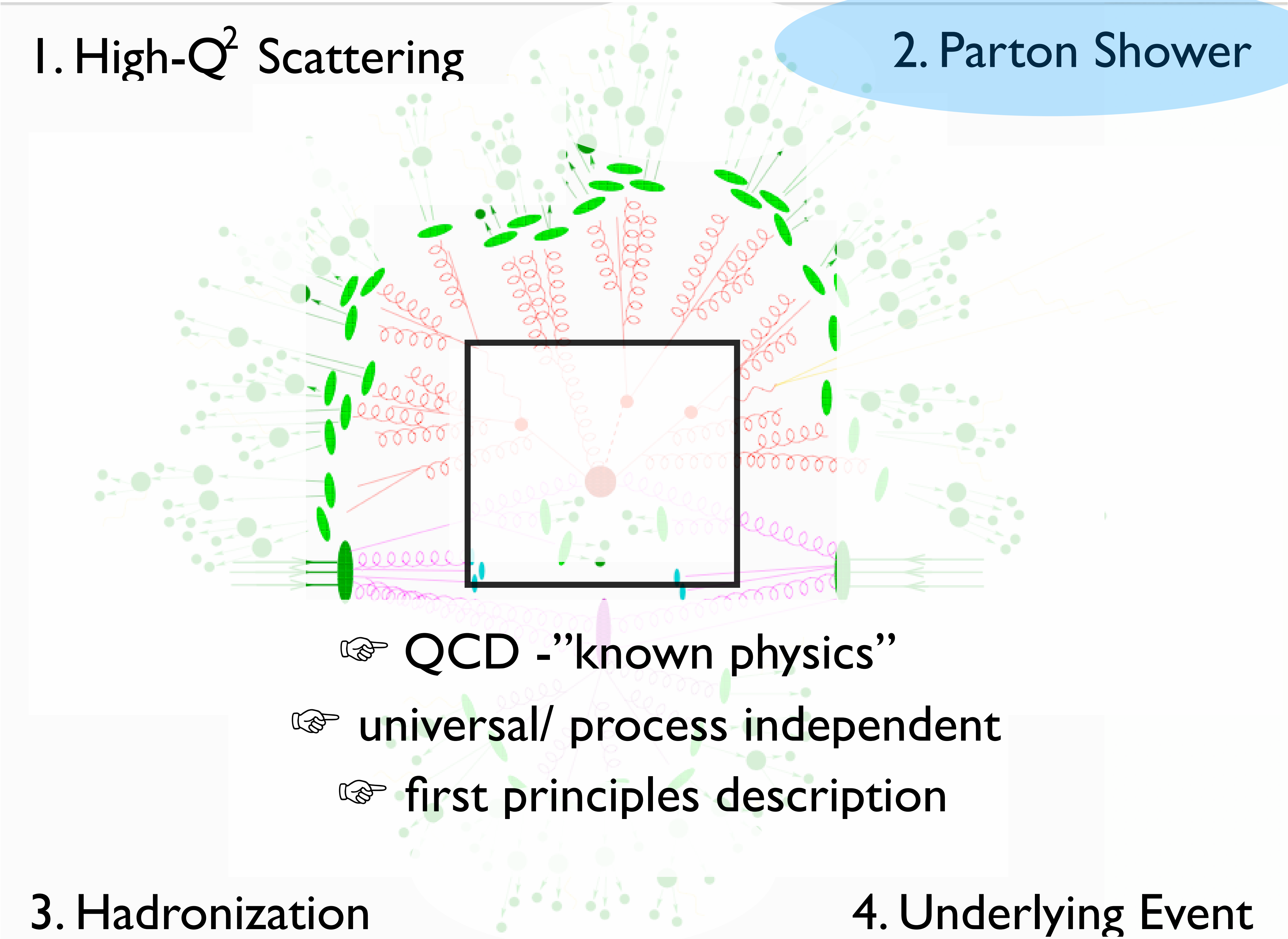
☞ it can be systematically improved

3. Hadronization

4. Underlying Event

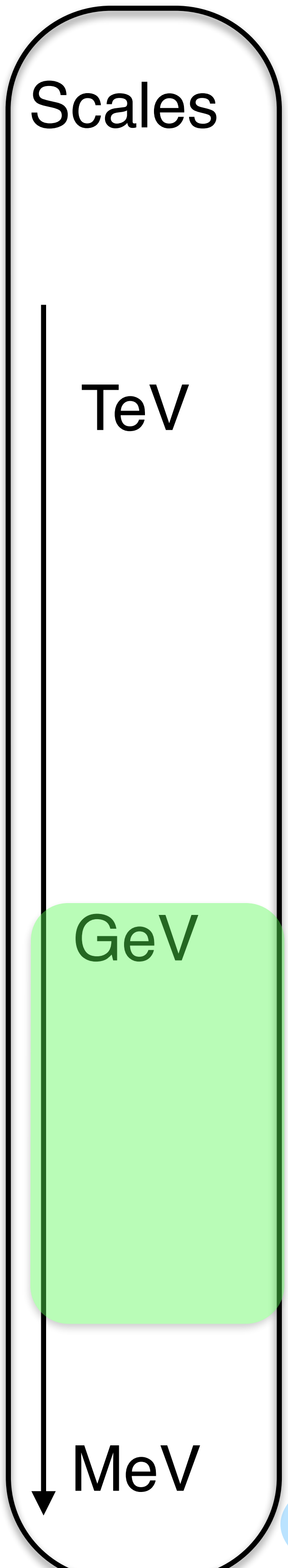
What are the MC for?





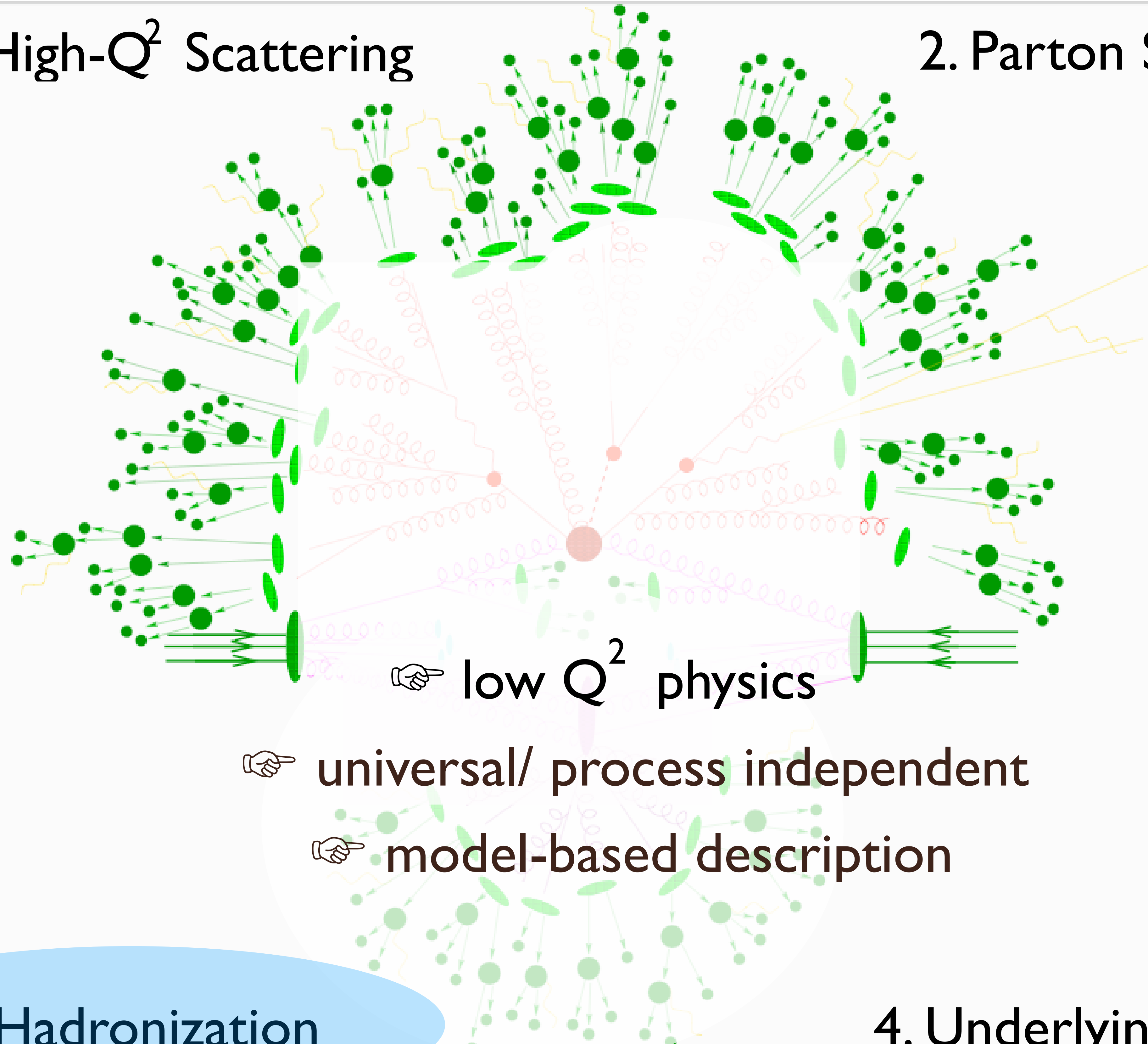
- ☞ QCD -"known physics"
- ☞ universal/ process independent
- ☞ first principles description

What are the MC for?



1. High-Q^2 Scattering

2. Parton Shower



- ☞ low Q^2 physics
- ☞ universal/ process independent
- ☞ model-based description

3. Hadronization

4. Underlying Event

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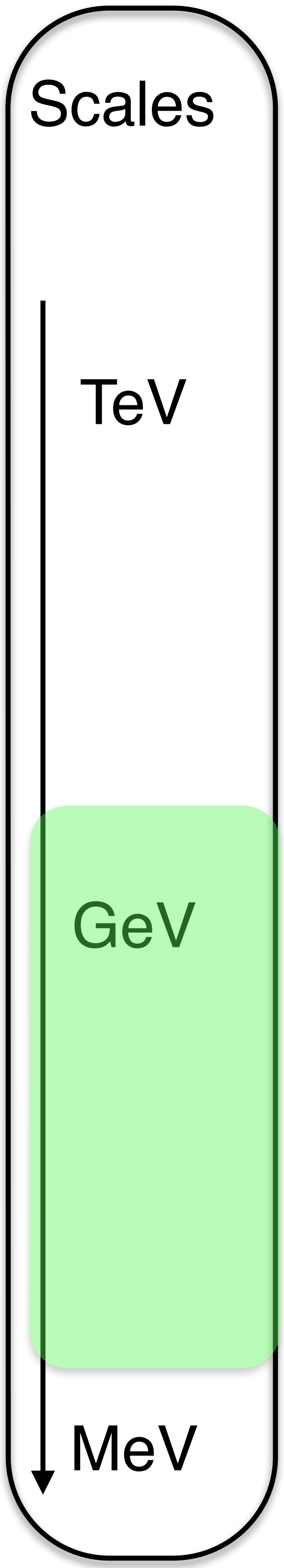

1. High-Q^2 Scattering

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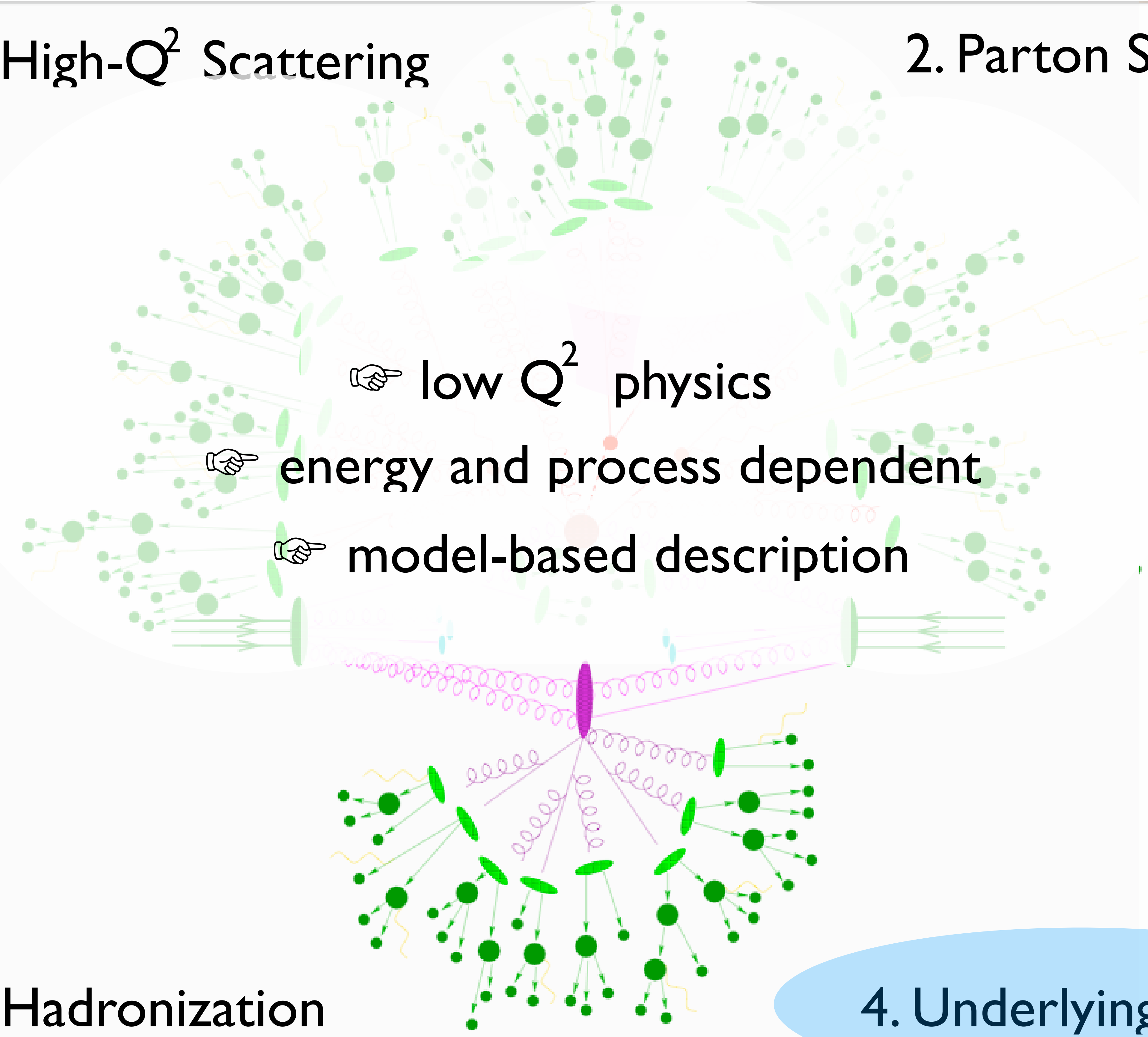
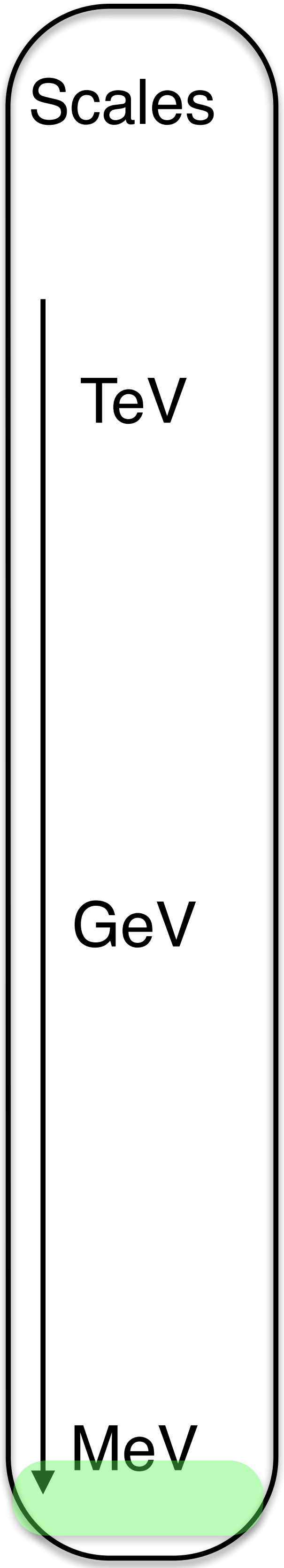


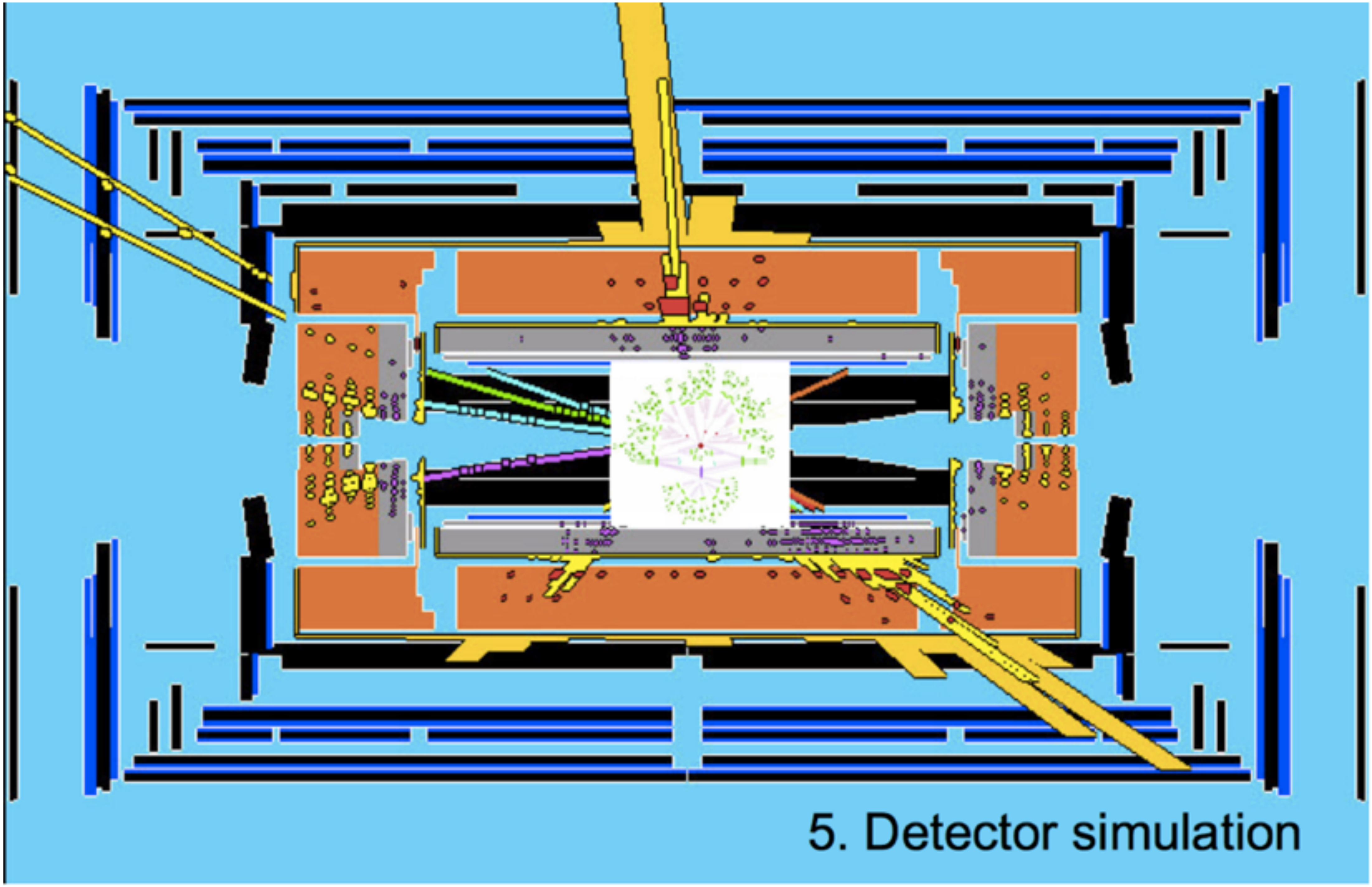
- ☞ low Q^2 physics
- ☞ energy and process dependent
- ☞ model-based description

3. Hadronization

4. Underlying Event

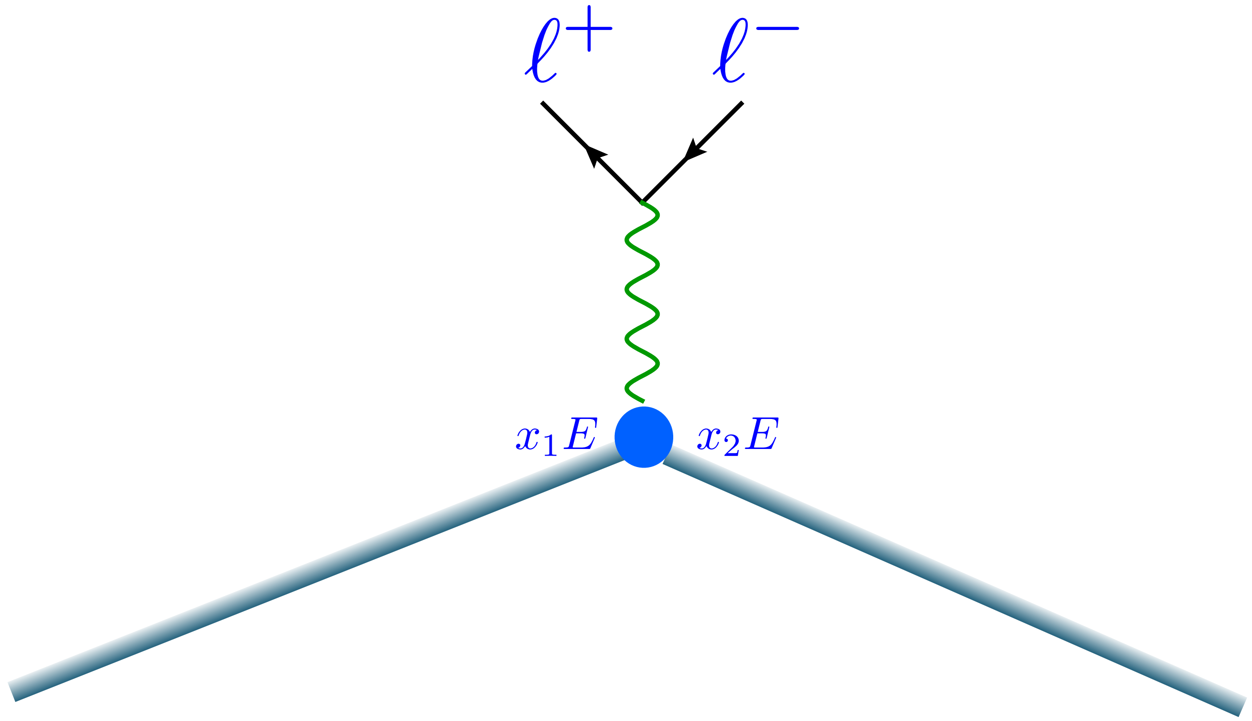
What are the MC for?





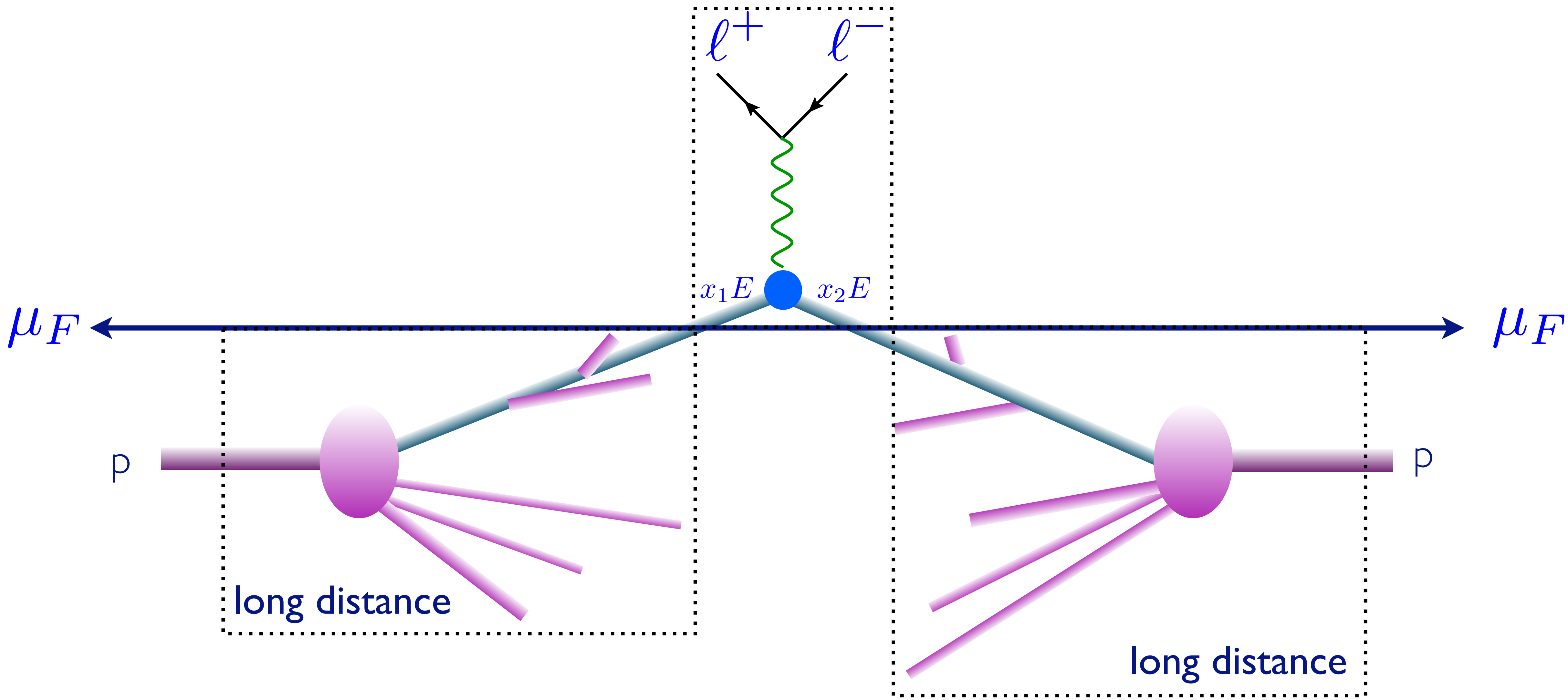
To Remember

- Multi-scale problem
  - ➡ New physics visible only at High scale
  - ➡ Problem split in different scale



$$\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

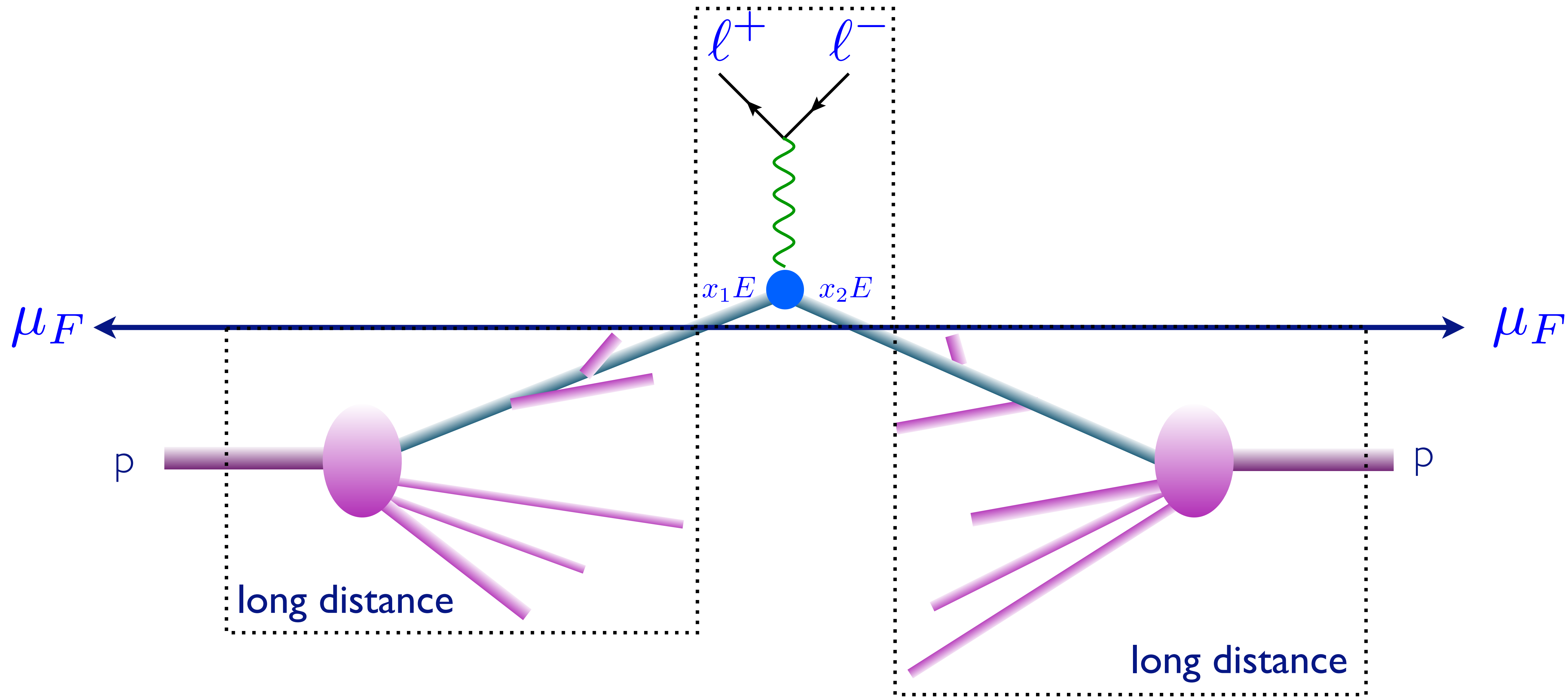
Parton-level cross section



$$f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Parton density functions

Parton-level cross section



$$\sum_{a,b}\int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\mathrm{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

Perturbative expansion

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LO predictions

Perturbative expansion

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LO predictions

NLO corrections

Perturbative expansion

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LO predictions

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LO predictions

NLO corrections

NNLO corrections

N3LO or NNNLO corrections

Perturbative expansion

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LO predictions | NLO corrections | NNLO corrections | N3LO or NNNLO corrections

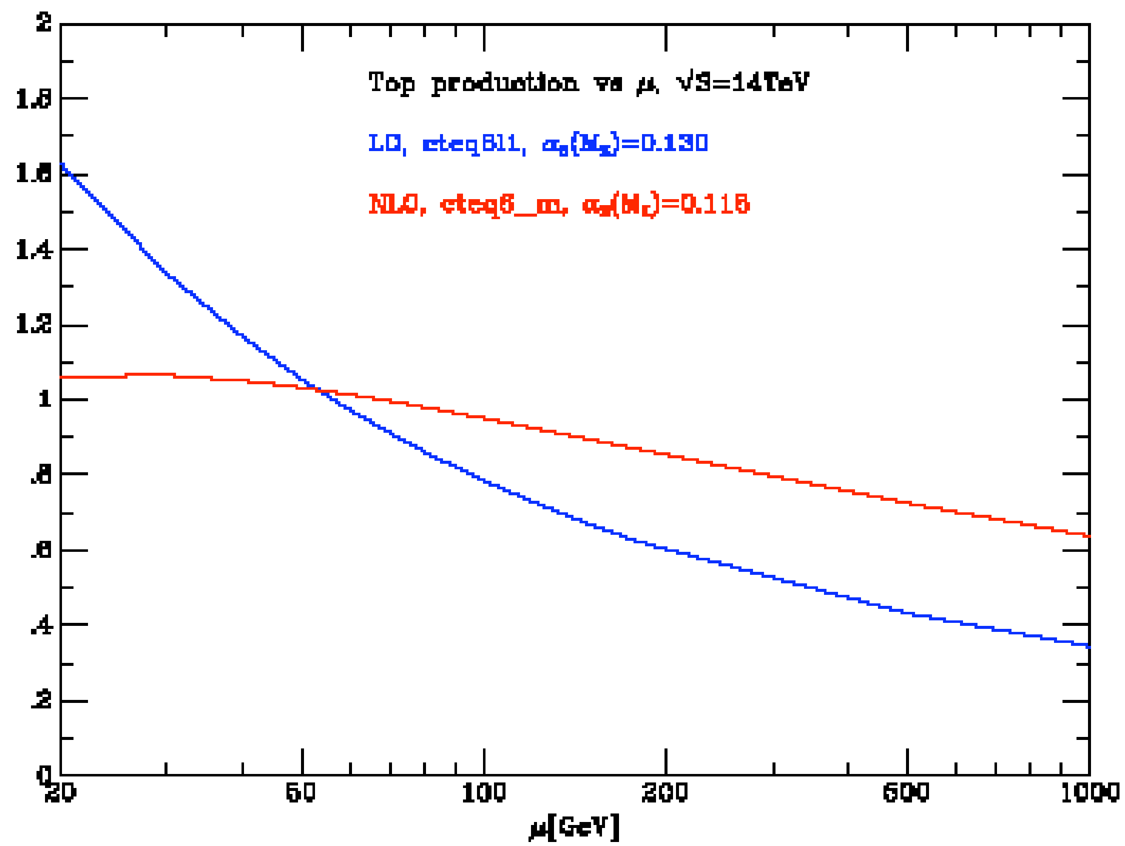
- Including higher corrections improves predictions and reduces theoretical uncertainties

Improved predictions

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \; f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, d\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

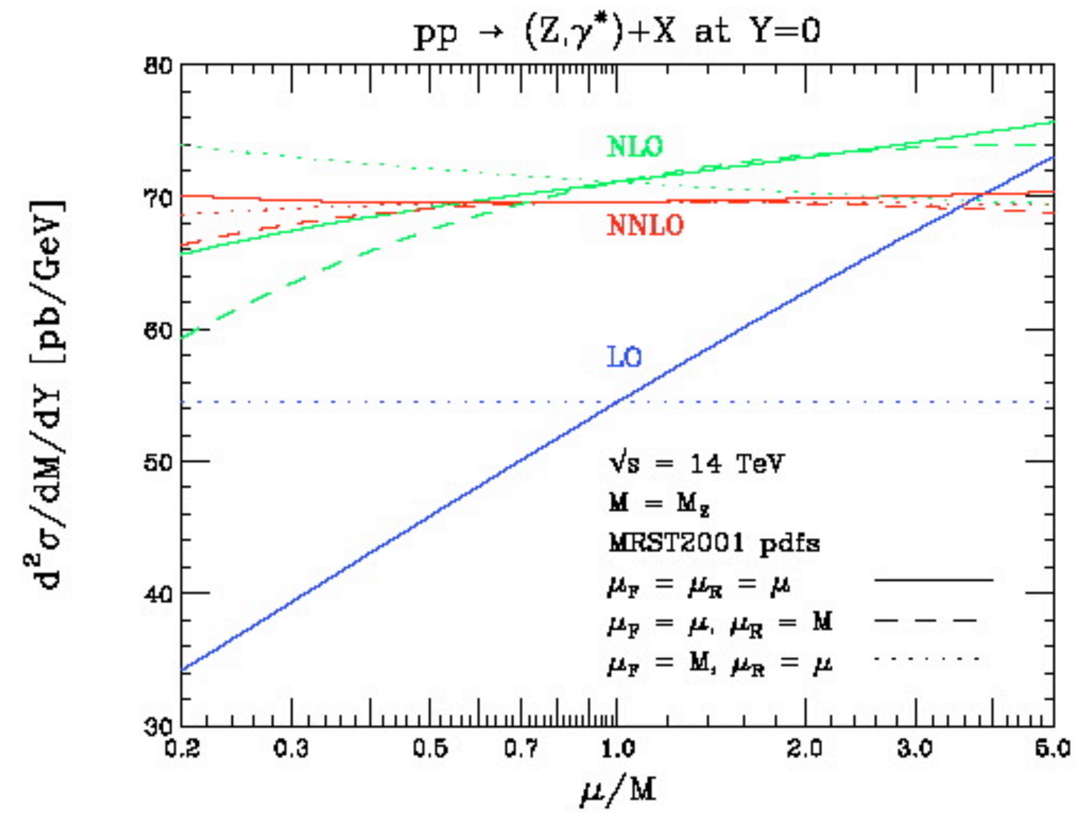
$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_s}{2\pi} \sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \ldots \right)$$

- Leading Order predictions can depend strongly on the renormalization and factorization scales
- Including higher order corrections reduces the dependence on these scales



Going NNLO...?

- NNLO is the current state-of-the-art. There are only a few results available: Higgs, Drell-Yan, ttbar
- Why do we need it?
  - control of the uncertainties in a calculation
  - It is "mandatory" if NLO corrections are very large to check the behavior of the perturbative series
  - It is needed for Standard Candles and very precise tests of perturbation theory, exploiting all the available information, e.g. for determining NNLO PDF sets



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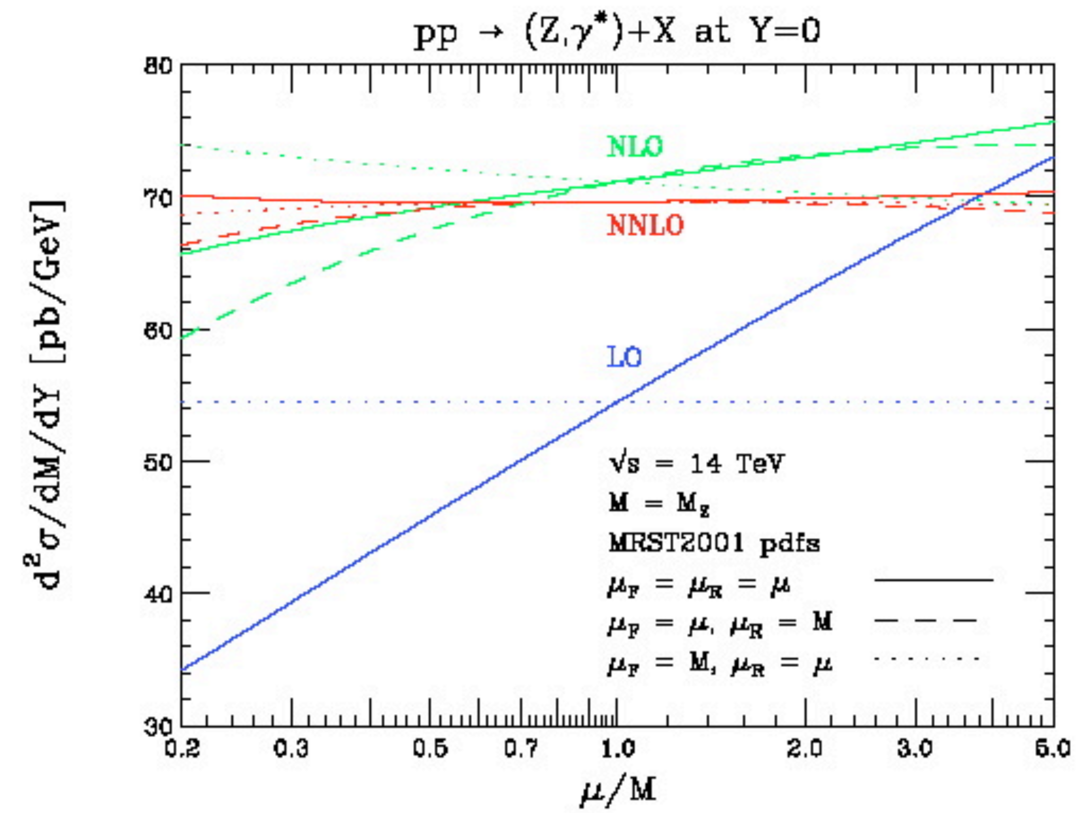
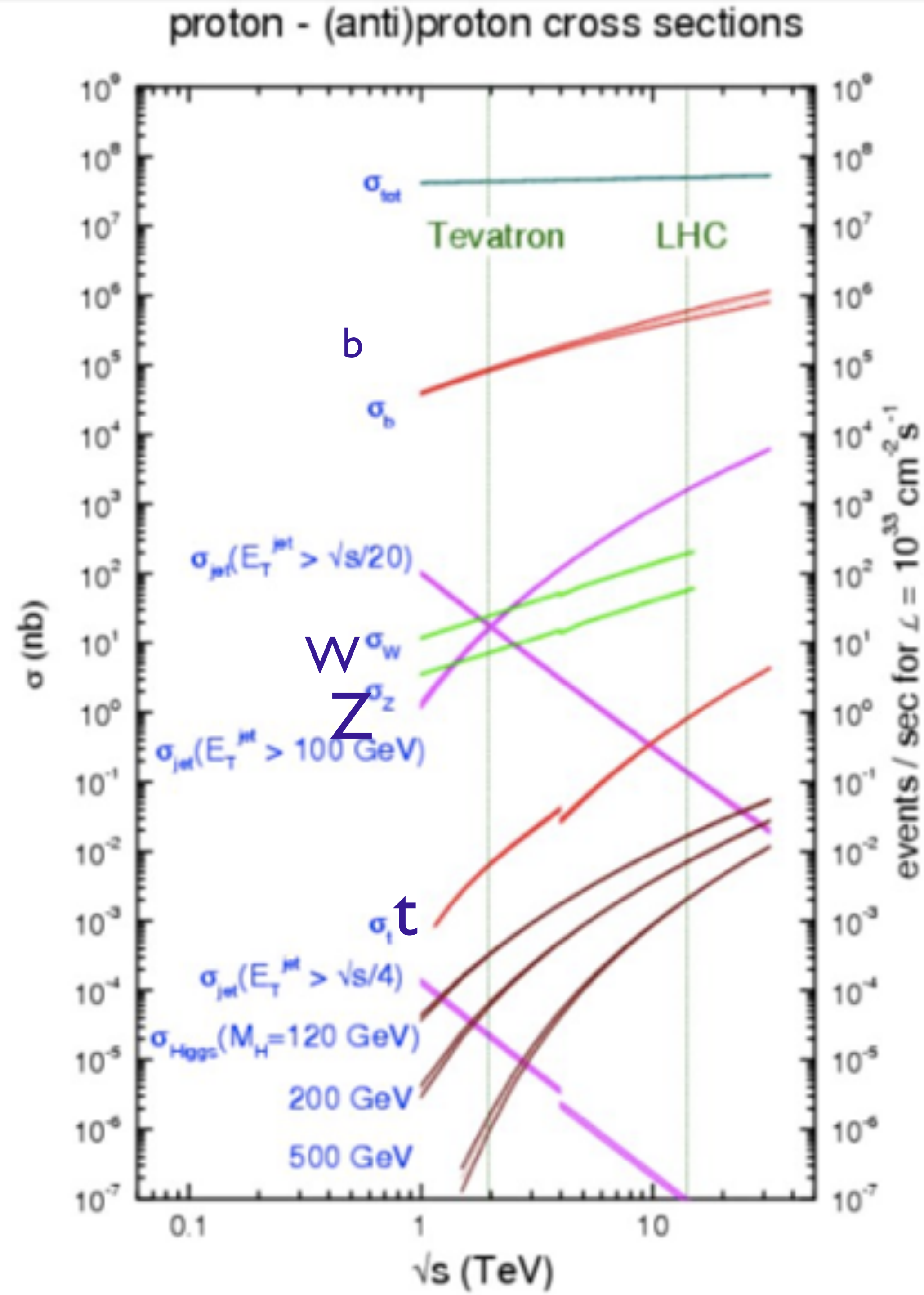


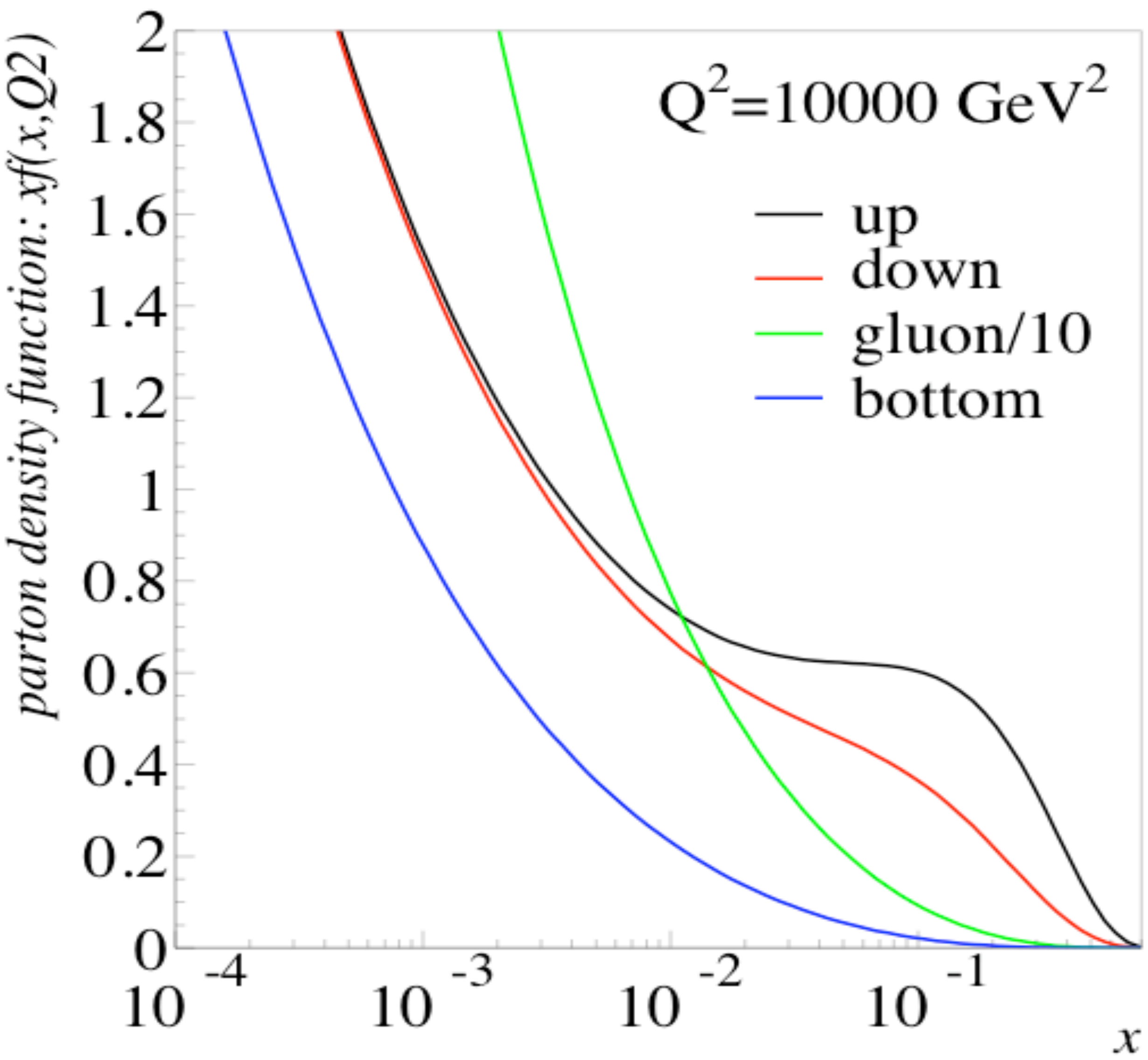
Let's focus on LO

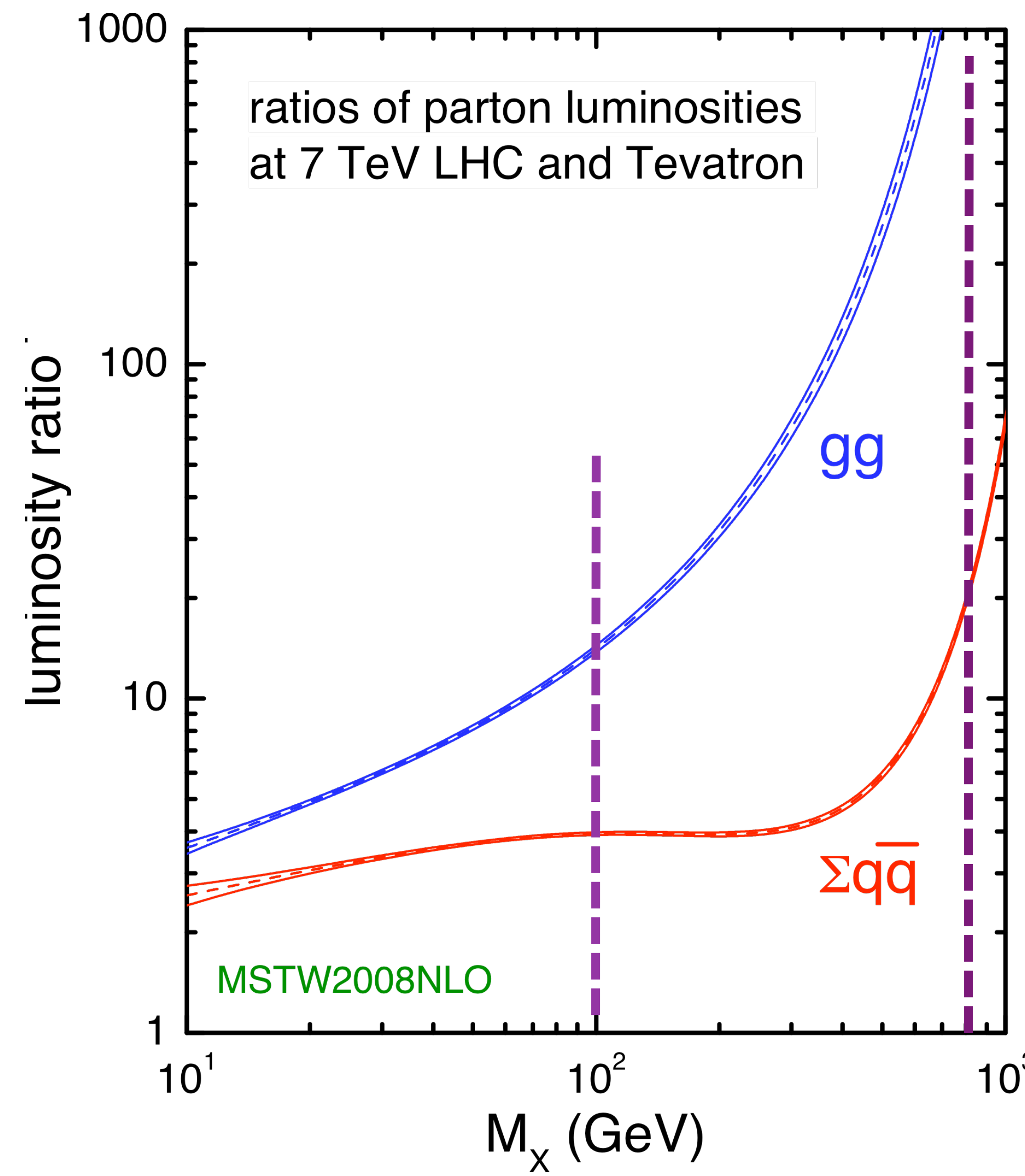
Hadron Colliders

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$



Parton densities

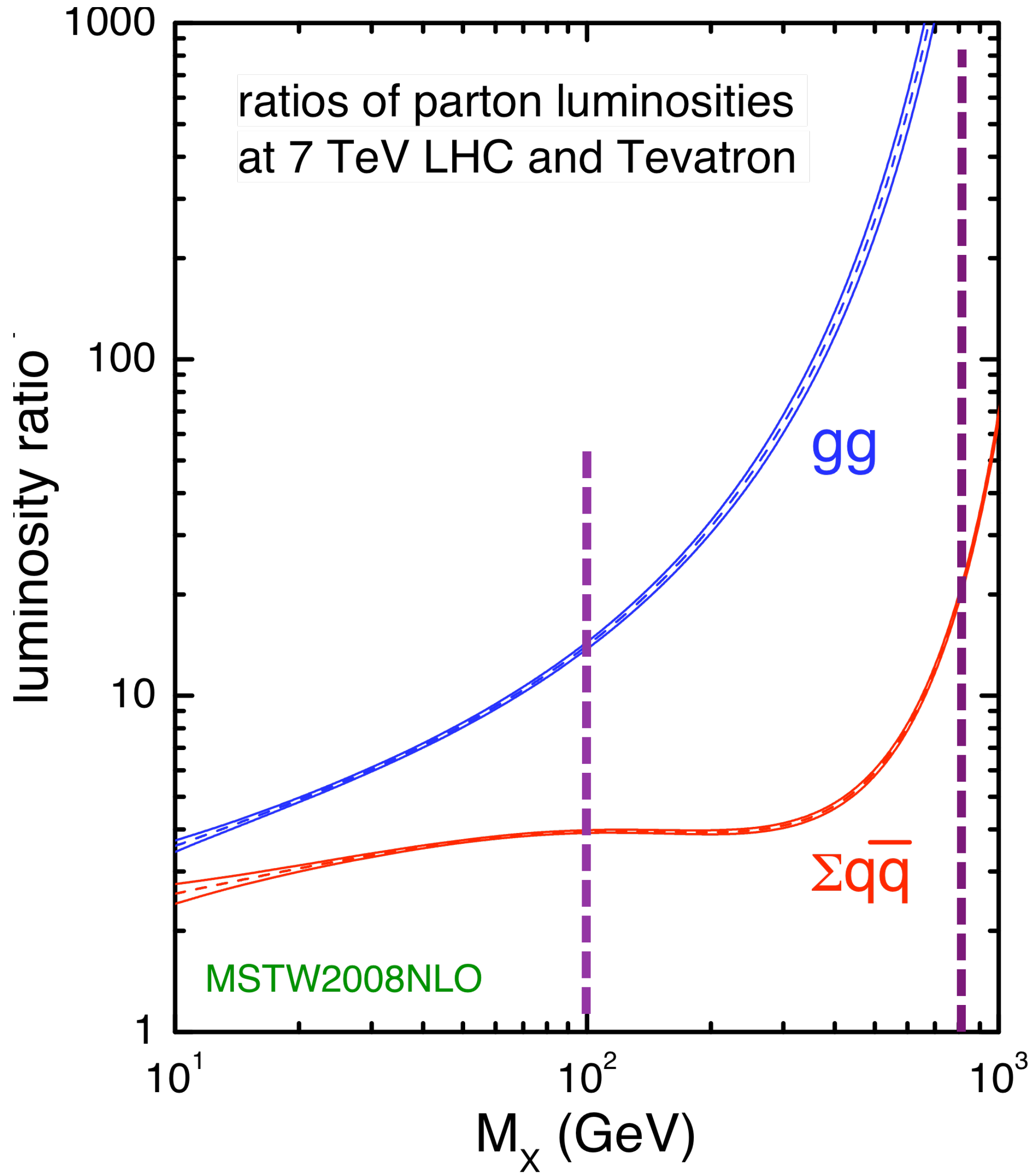


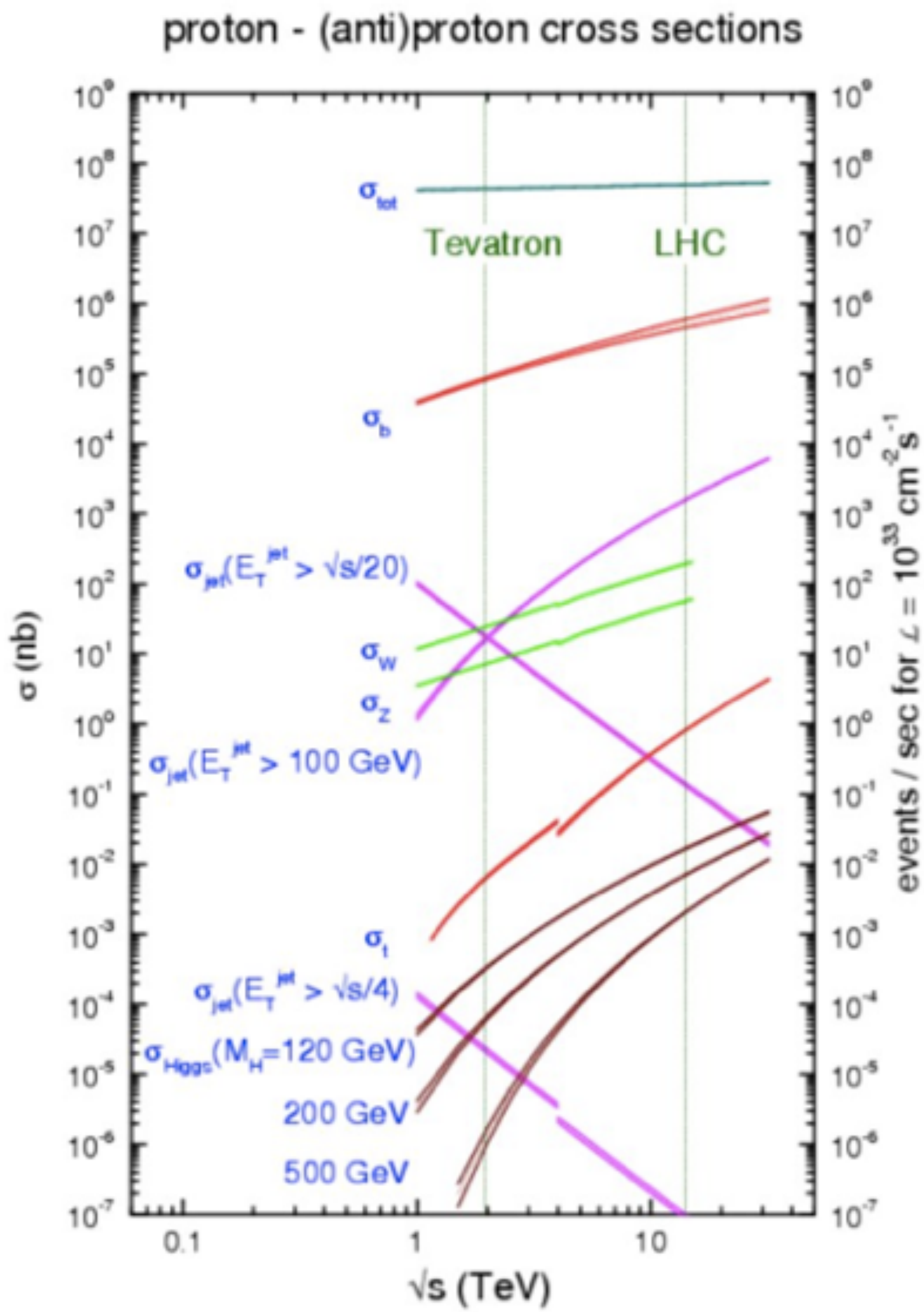


At small x (small ŝ), gluon domination.
At large x valence quarks

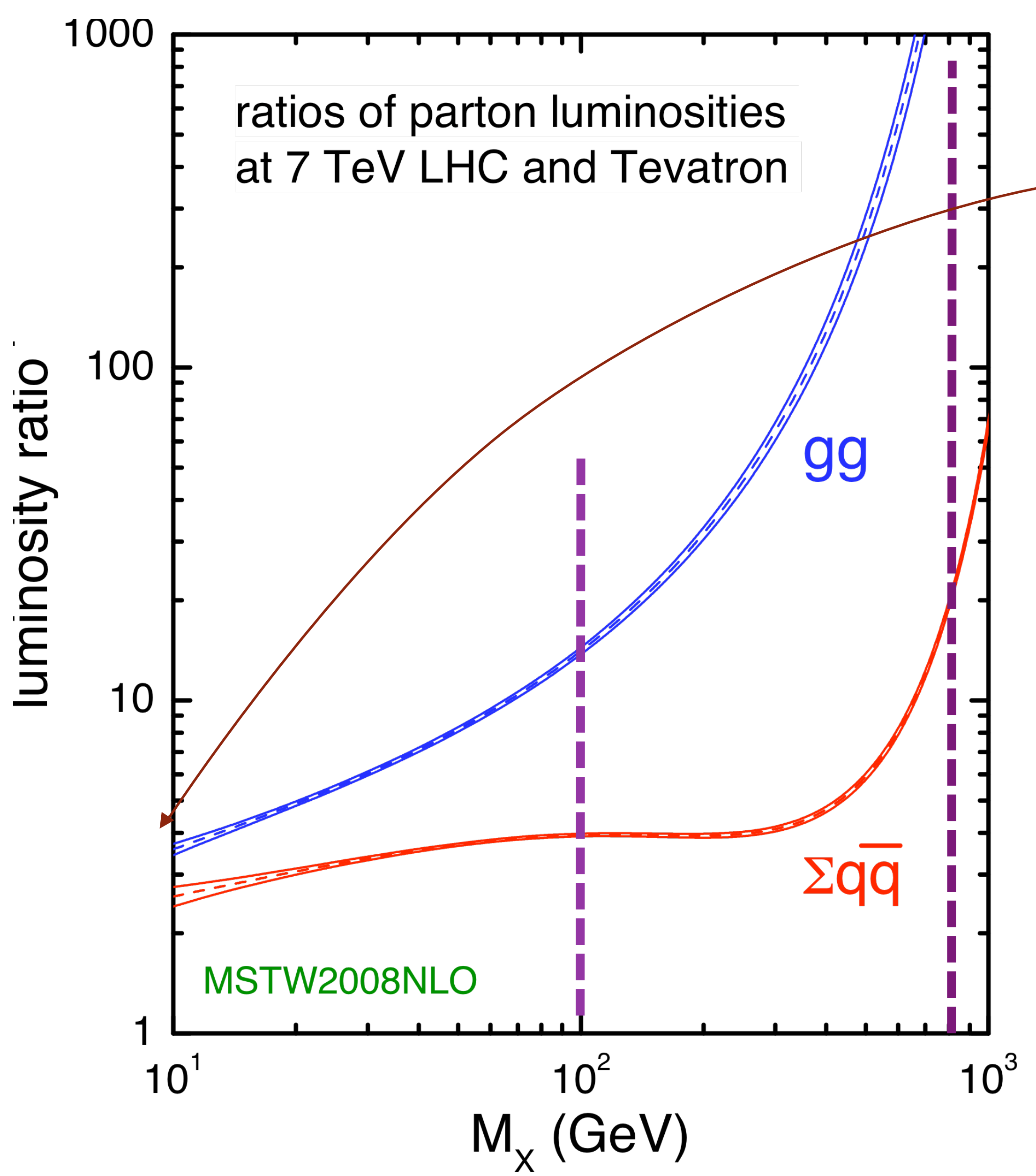
LHC formidable at large mass –
For low mass, Tevatron backgrounds smaller

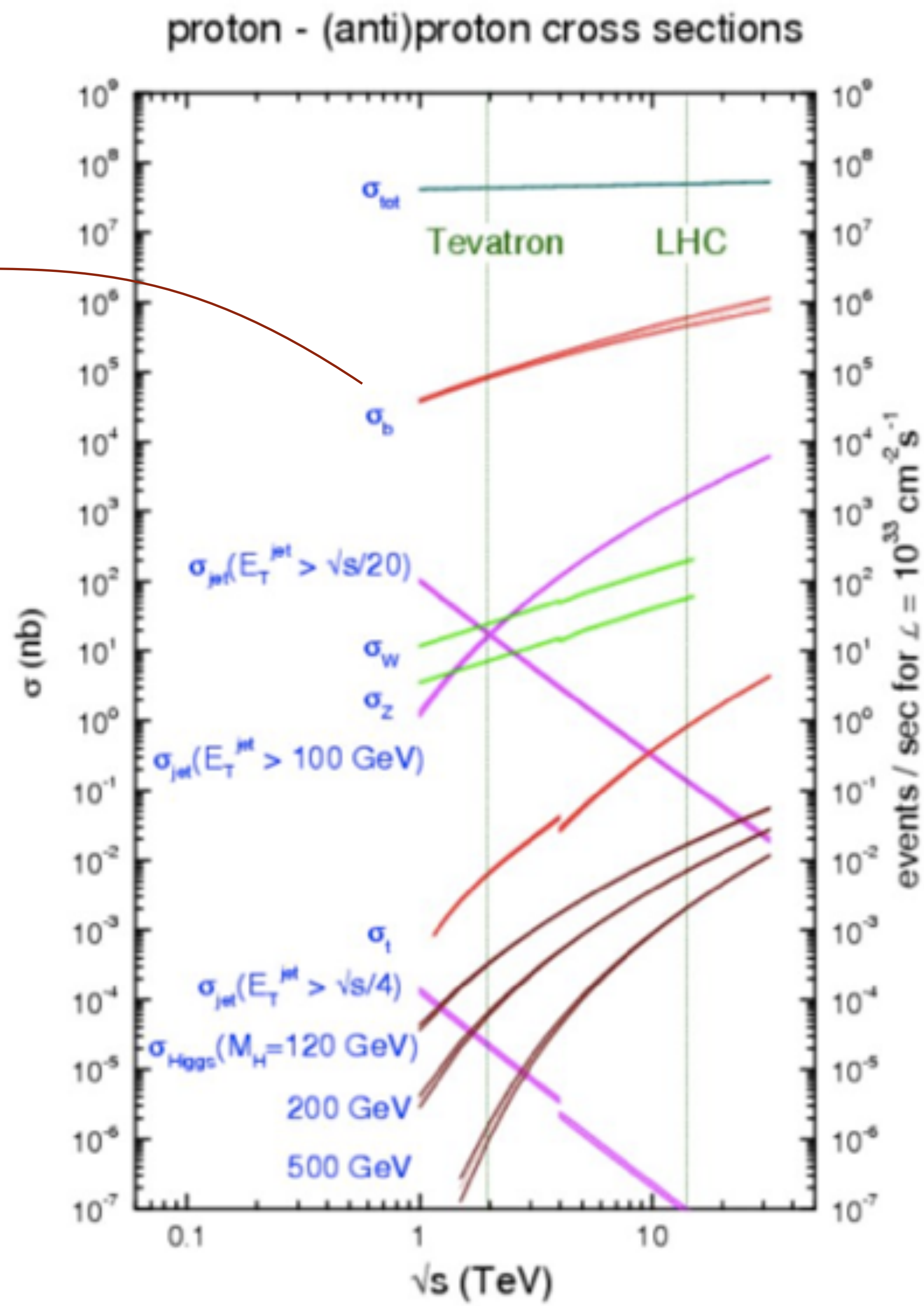
Back to the processes



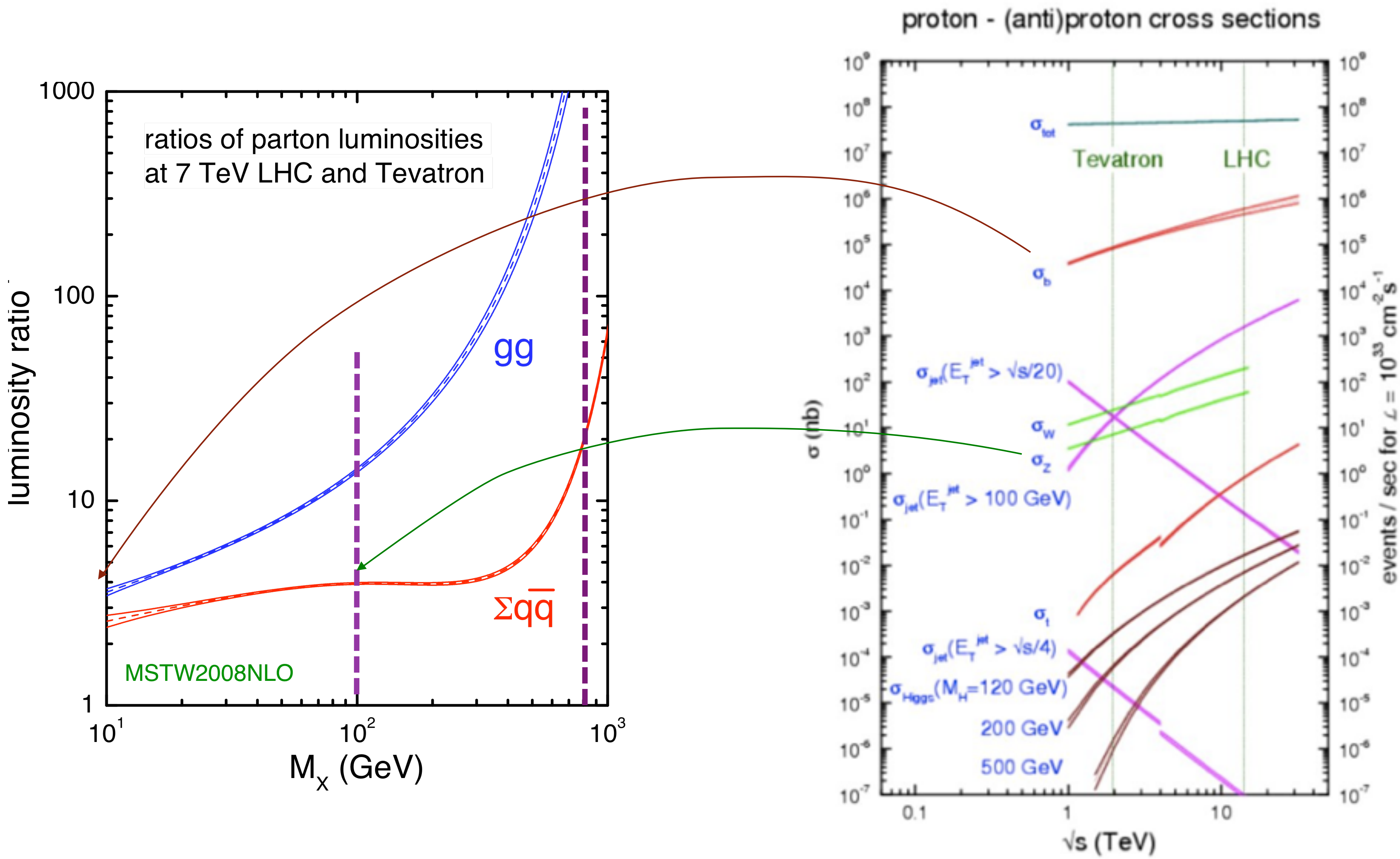


Back to the processes

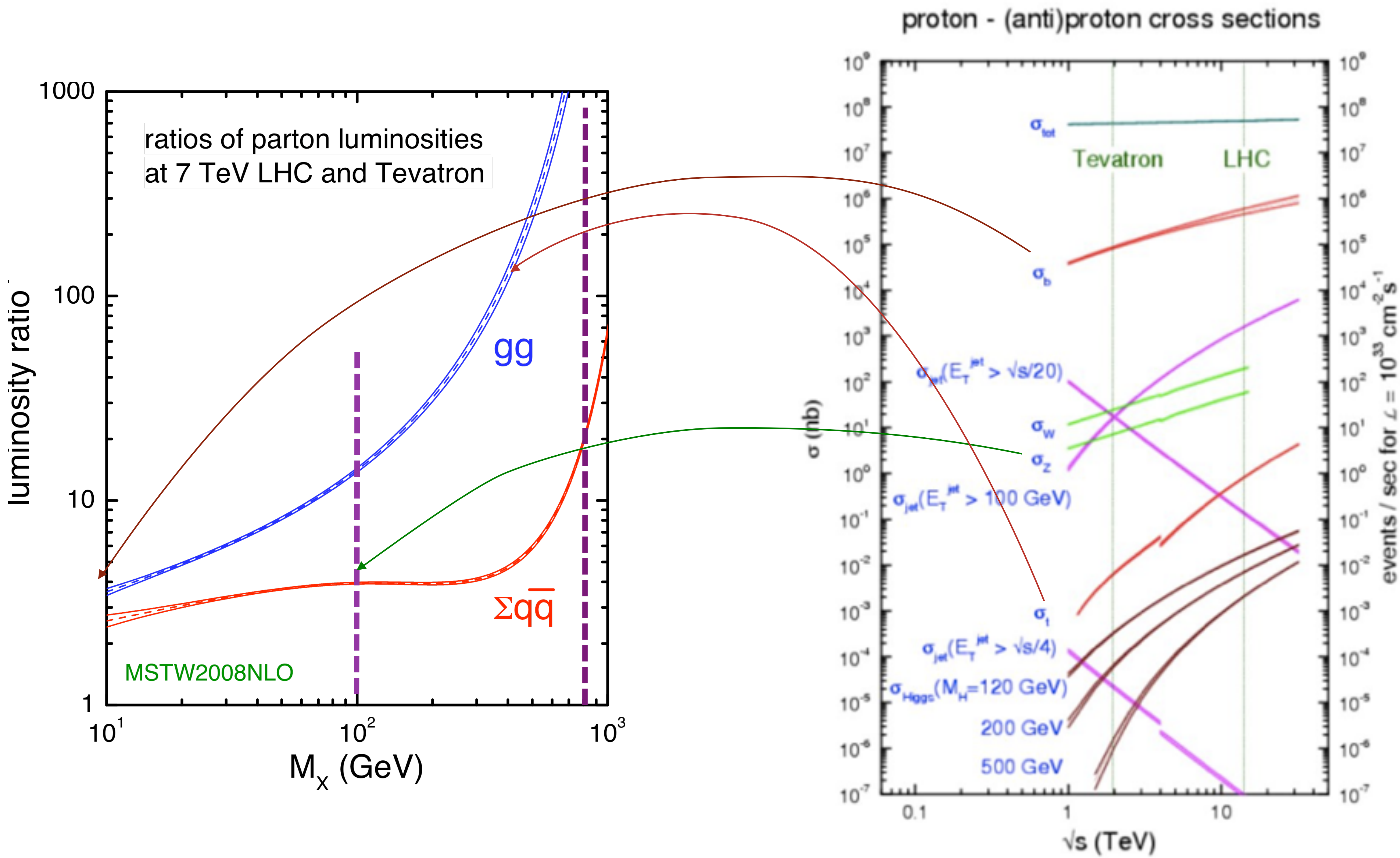




Back to the processes



Back to the processes



To Remember

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

Parton density functions

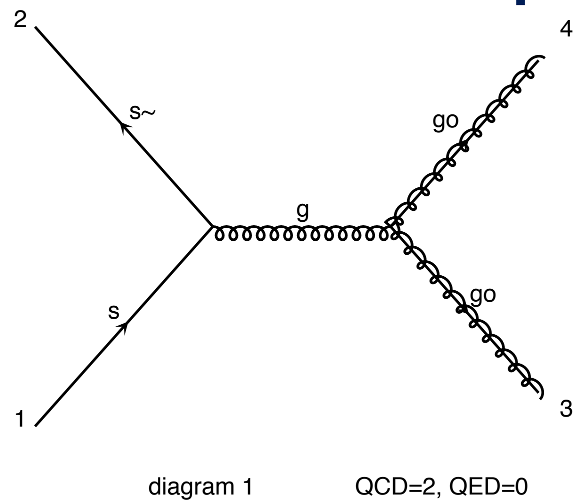
Parton-level cross section

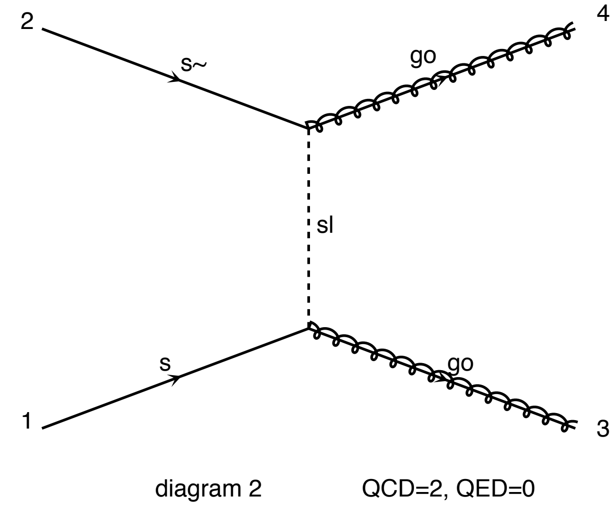
- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- NLO/NNLO: Reduce scale uncertainty linked to your division of your multi-scale problem

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism





- Evaluate the matrix-element

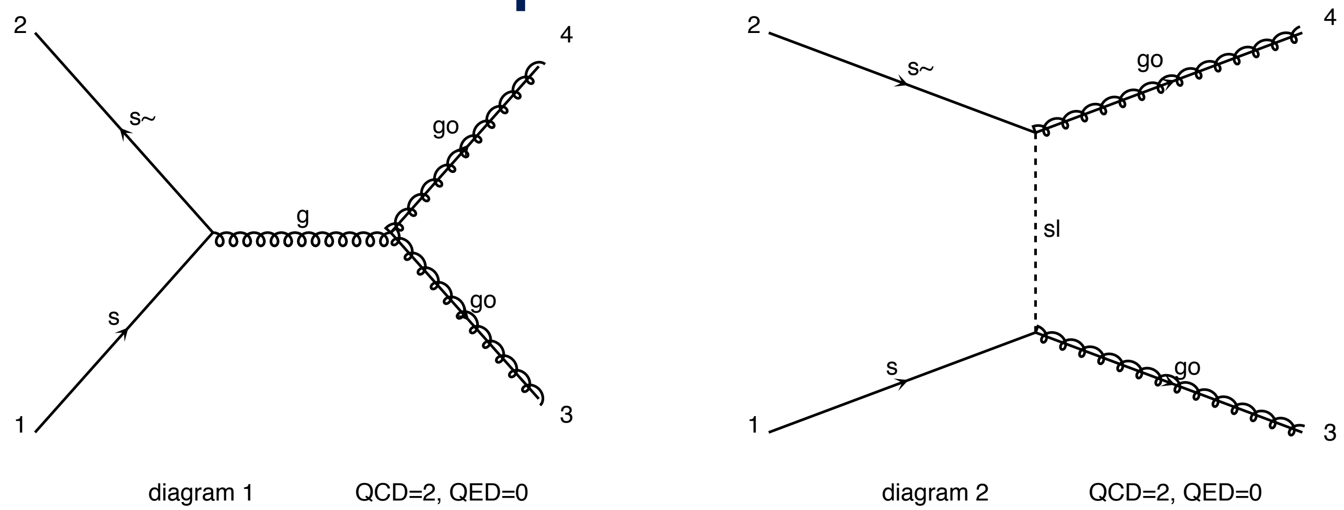
$$|\mathcal{M}|^2$$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

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$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

Very Hard (in general)

Matrix-Element

Calculate a given process (e.g. gluino pair)

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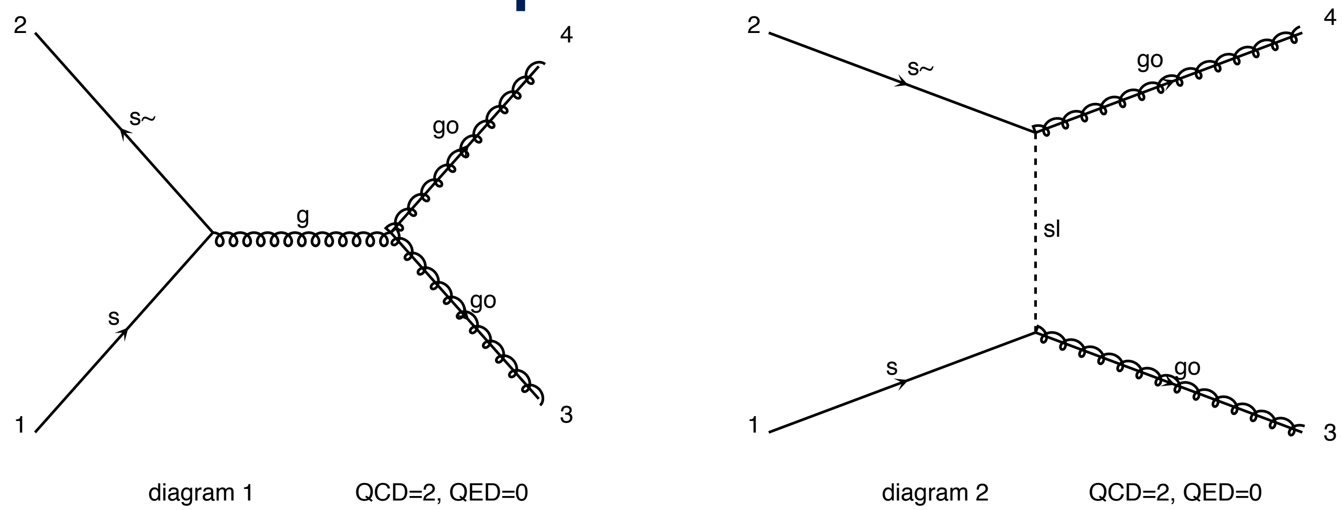


- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

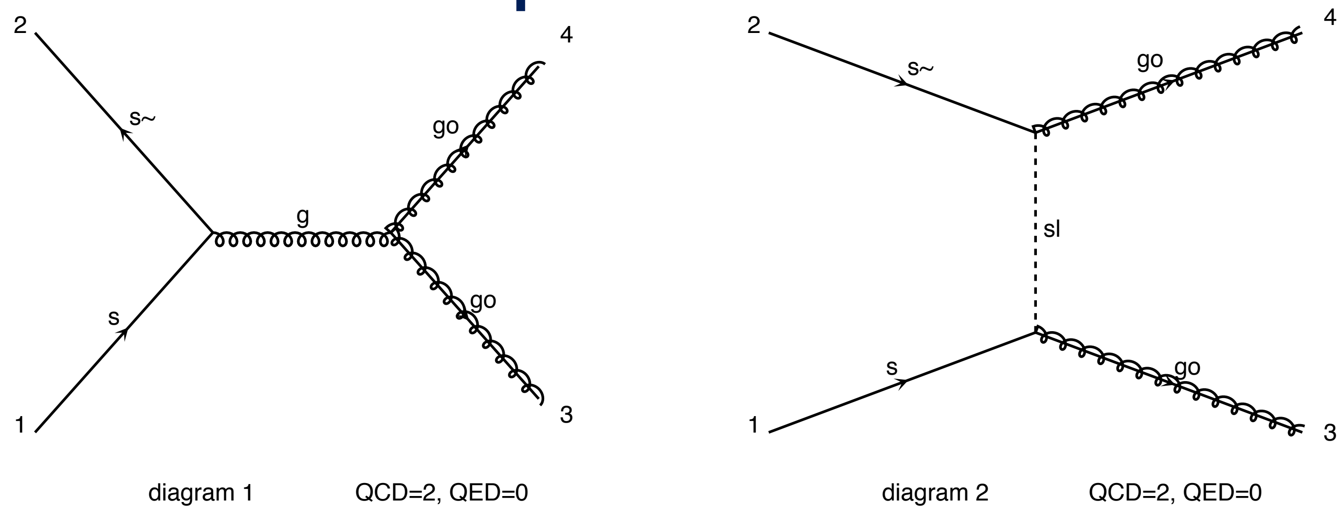
Next

Very Hard (in general)

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

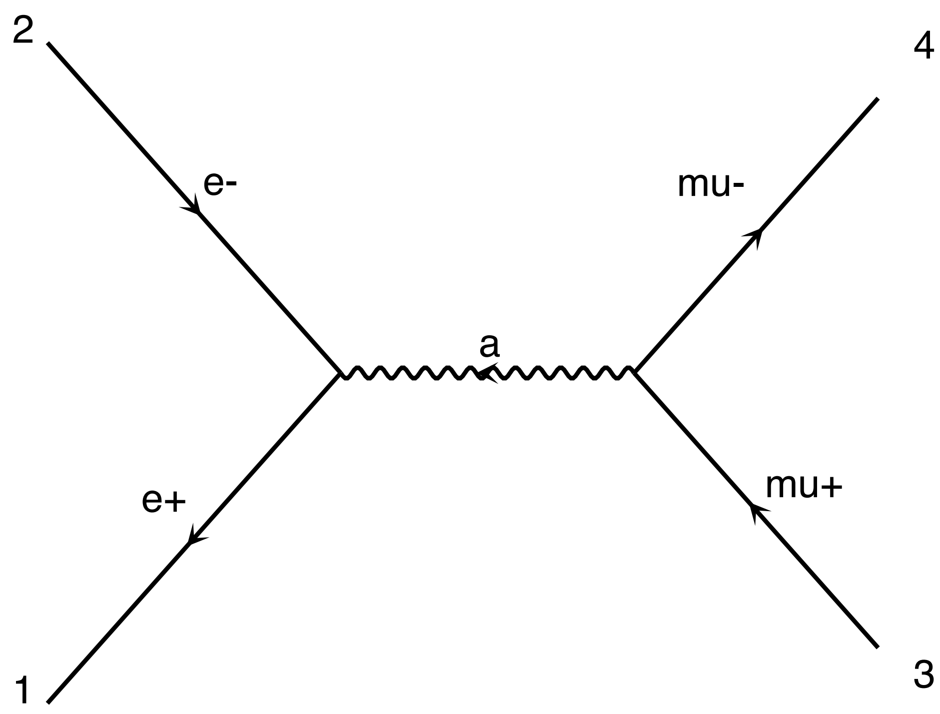
Hard

Next

Very Hard (in general)

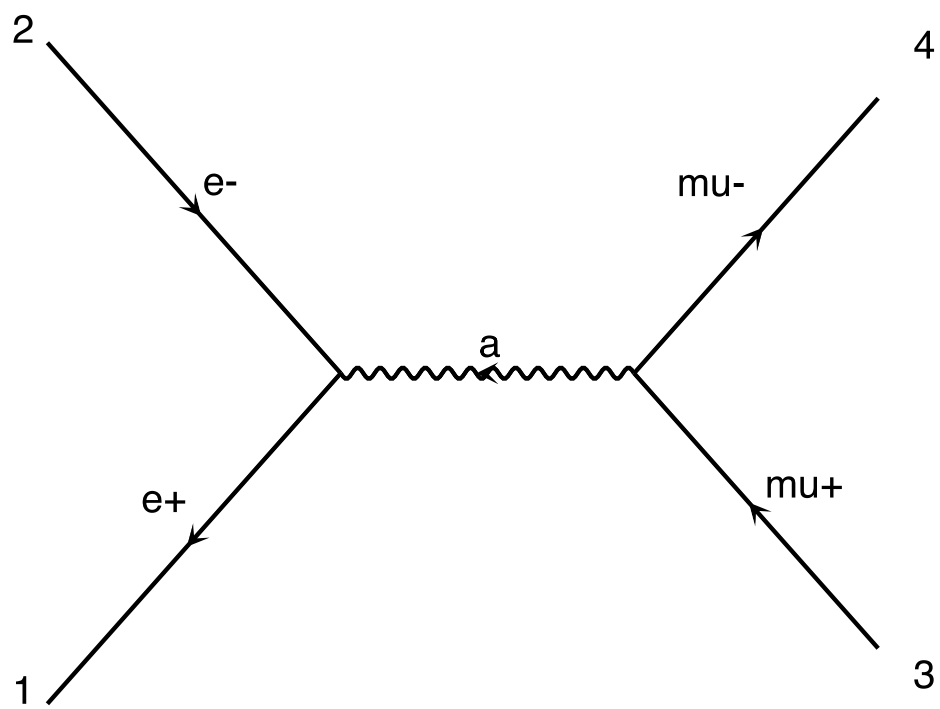
After

Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

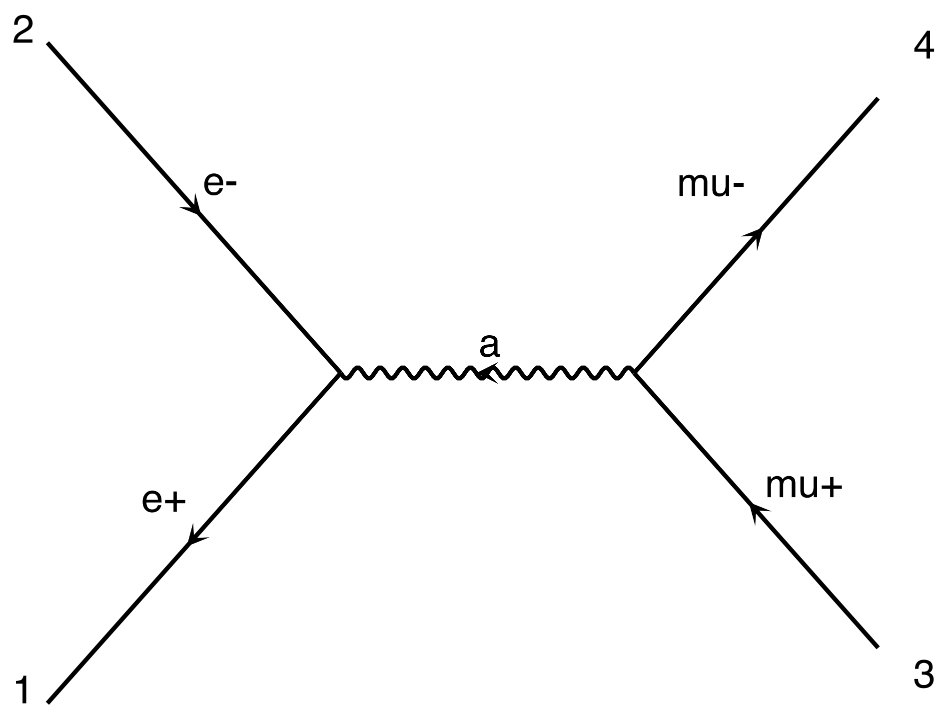
Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol} |\mathcal{M}|^2 = \frac{1}{4}\sum_{pol} \mathcal{M}^*\mathcal{M}$$

Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

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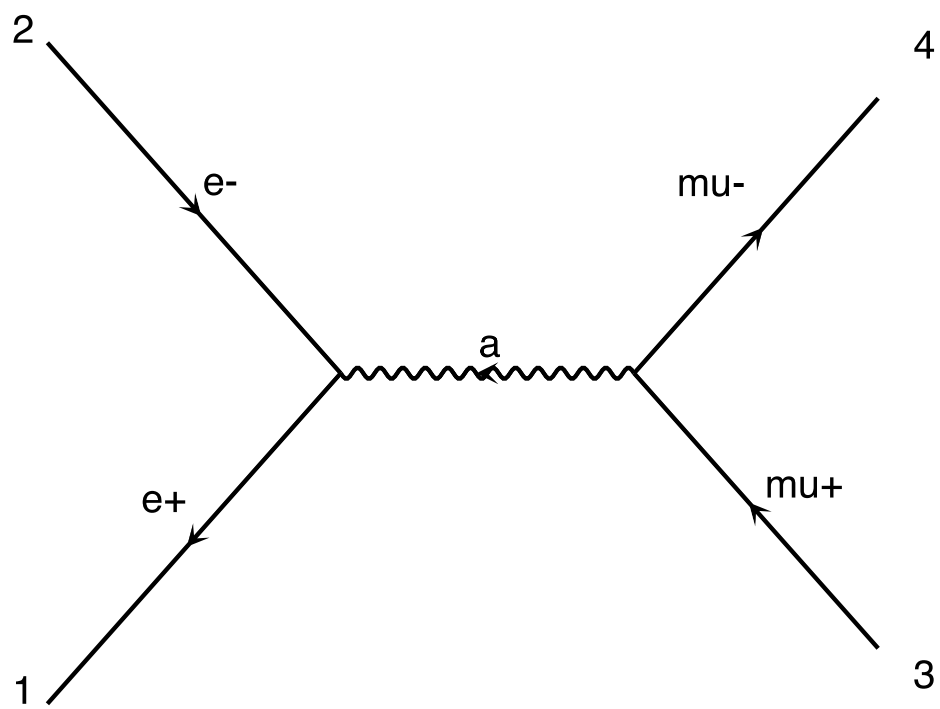
$$\sum_{pol}\bar{u}u = \not{p} + m$$

Matrix Element



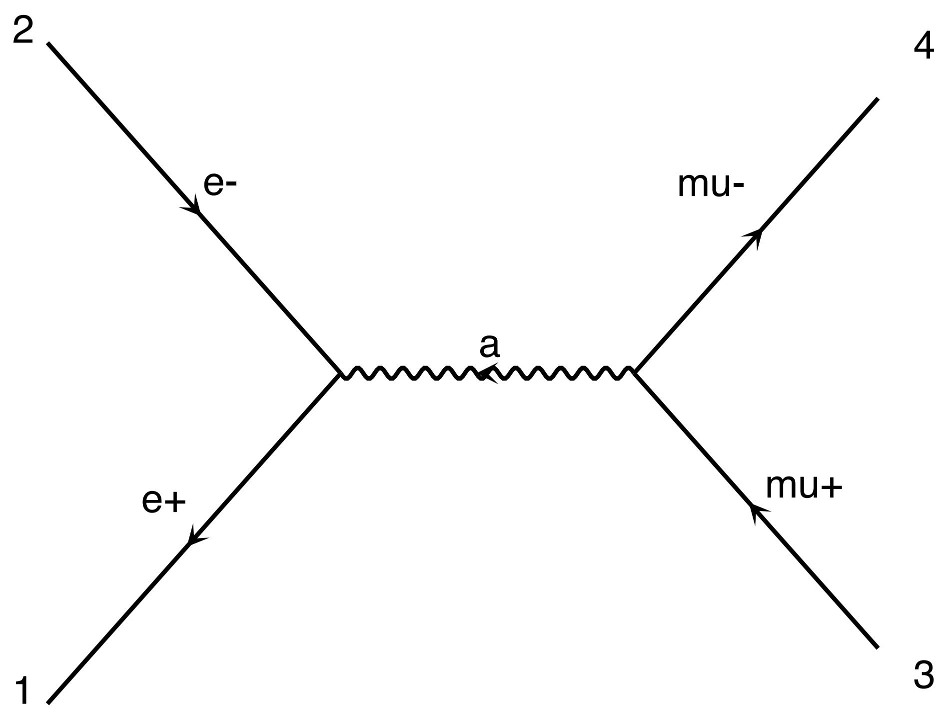
$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol} |\mathcal{M}|^2 = \frac{1}{4}\sum_{pol} \mathcal{M}^* \mathcal{M}$$

$$\sum_{pol} \bar{u}u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4} Tr[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu] Tr[\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu]$$

Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

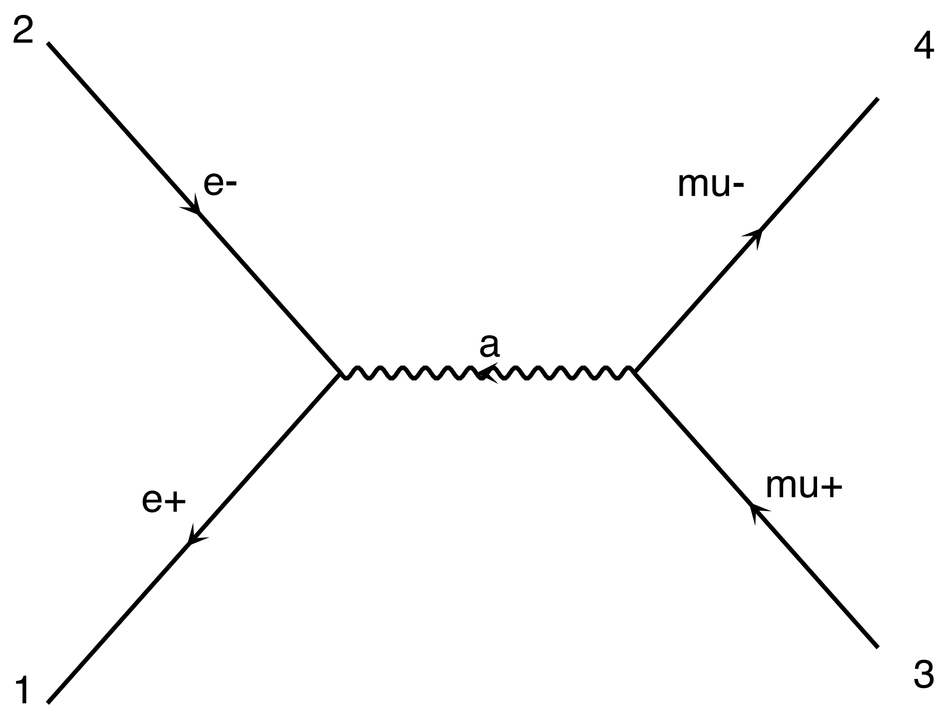
$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

$$\sum_{pol}\bar{u}u = \not{p} + m$$

→ $$\frac{e^4}{4q^4}Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

→ $$\frac{8e^4}{q^4}\left[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)\right]$$

Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

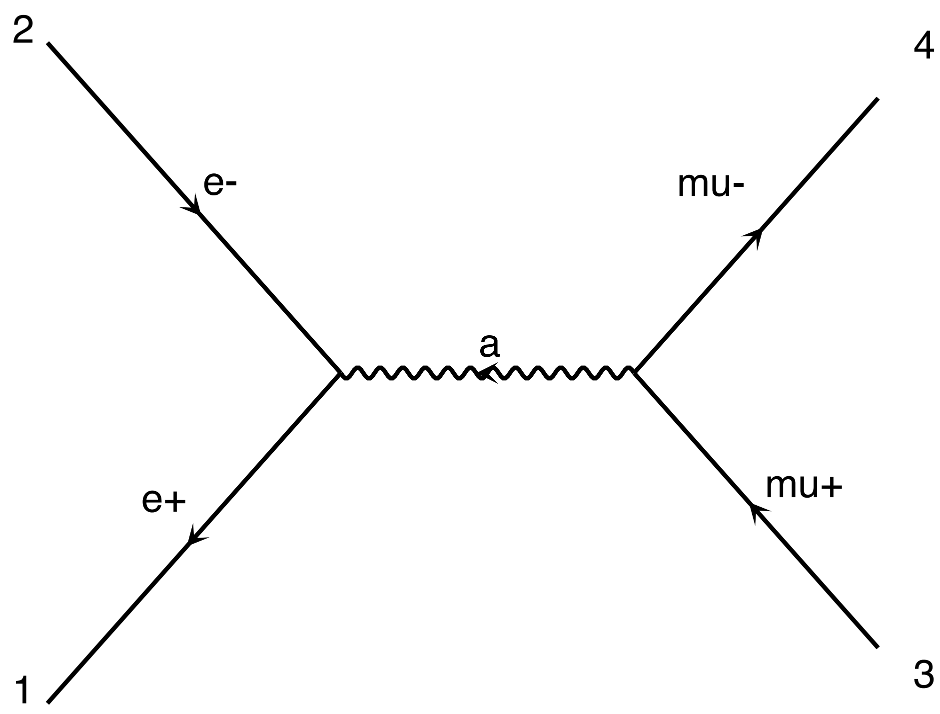
$$\sum_{pol}\bar{u}u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4}Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

$$\rightarrow \frac{8e^4}{q^4}\left[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)\right]$$

Very Efficient !!!

Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

$$\sum_{pol}\bar{u}u = \not{p} + m$$

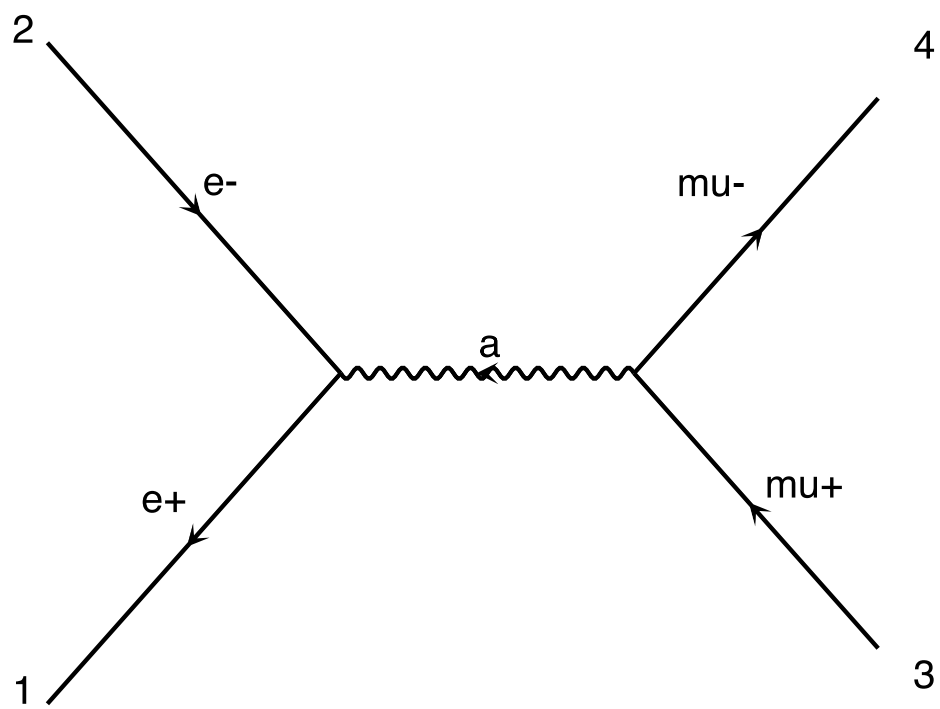
$$\rightarrow \frac{e^4}{4q^4}Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

$$\rightarrow \frac{8e^4}{q^4}[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Very Efficient !!!

Only for 2→2 and 2→3

Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

$$\sum_{pol}\bar{u}u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4} Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

$$\rightarrow \frac{8e^4}{q^4}[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

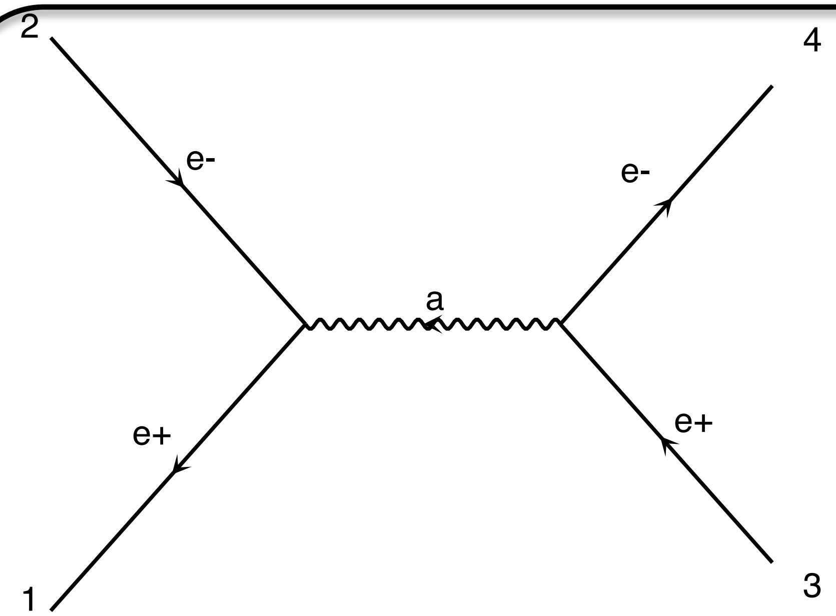
Very Efficient !!!

Only for 2→2 and 2→3

Because the number of terms rises as N^2

Idea

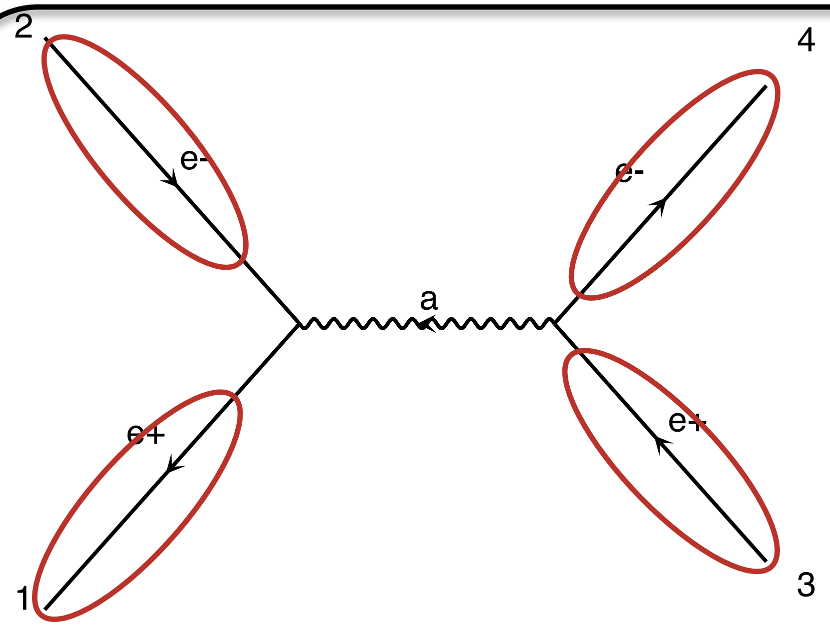
- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^{\nu} u)$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
  - ➡ Loop on Helicity and average the results

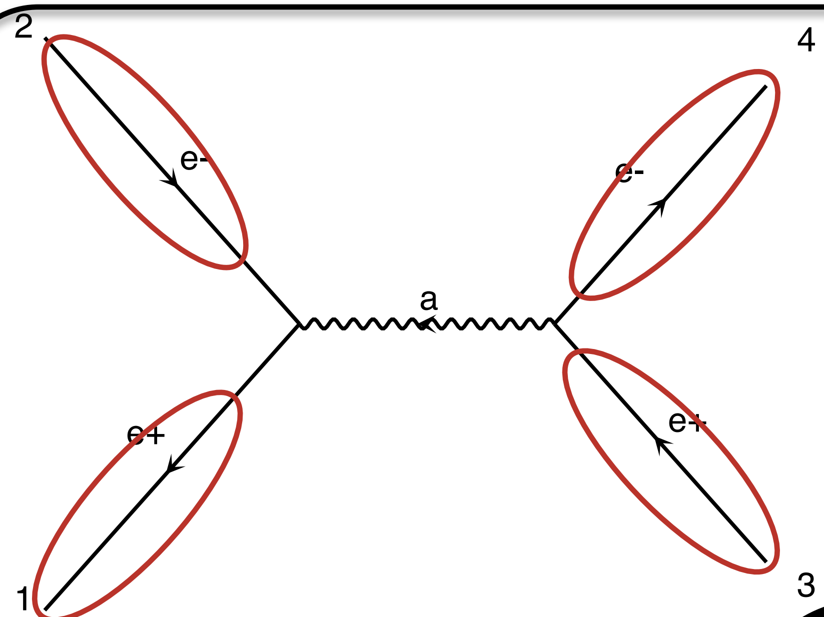


$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

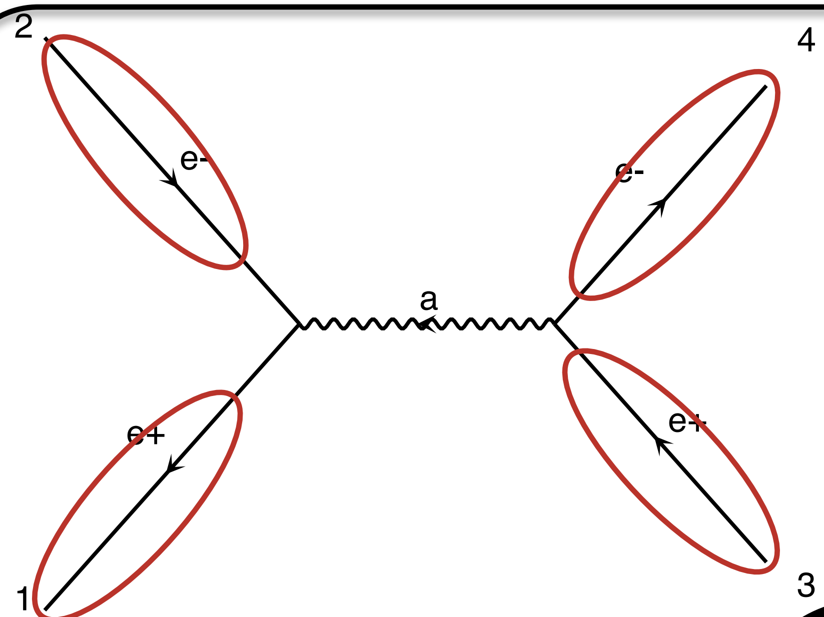
Numbers for given helicity and momenta

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{cases}$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

Lines present in the code.

$$\bar{v}_1 = fct(\vec{p}_1, m_1)$$
$$u_2 = fct(\vec{p}_2, m_2)$$
$$v_3 = fct(\vec{p}_3, m_3)$$
$$\bar{u}_4 = fct(\vec{p}_4, m_4)$$

$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_\lambda(\vec{p}) \\ \omega_\lambda(p)\chi_\lambda(\vec{p}) \end{pmatrix}$$

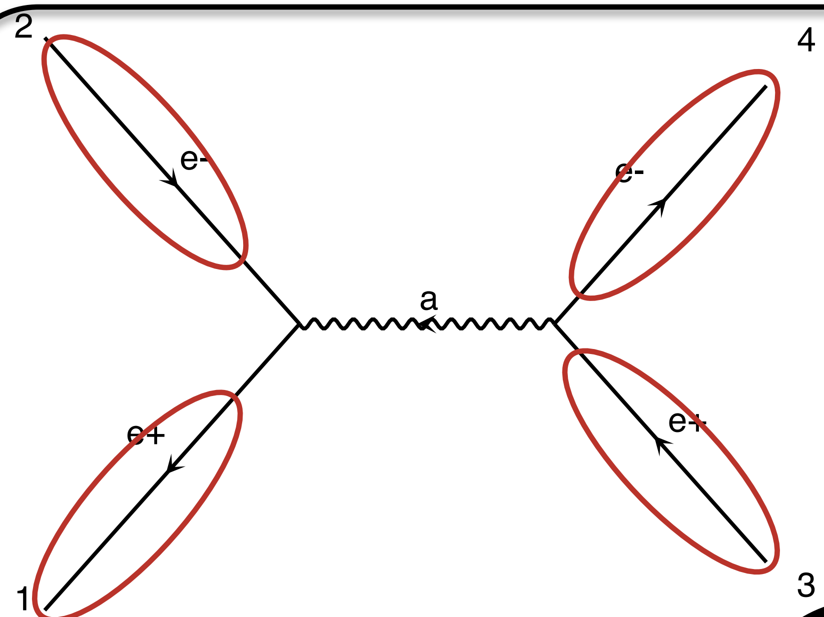
$$\omega_\pm(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} |\vec{p}| + p_z \\ p_x + ip_y \end{pmatrix},$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} -p_x + ip_y \\ |\vec{p}| + p_z \end{pmatrix}.$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

Lines present in the code.

$$\bar{v}_1 = fct(\vec{p}_1, m_1)$$
$$u_2 = fct(\vec{p}_2, m_2)$$
$$v_3 = fct(\vec{p}_3, m_3)$$
$$\bar{u}_4 = fct(\vec{p}_4, m_4)$$

Idea

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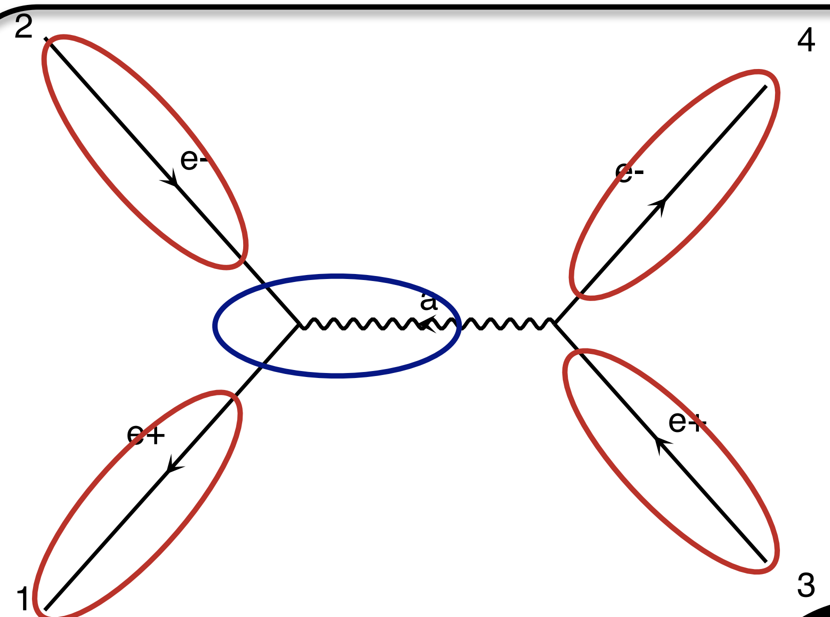


$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

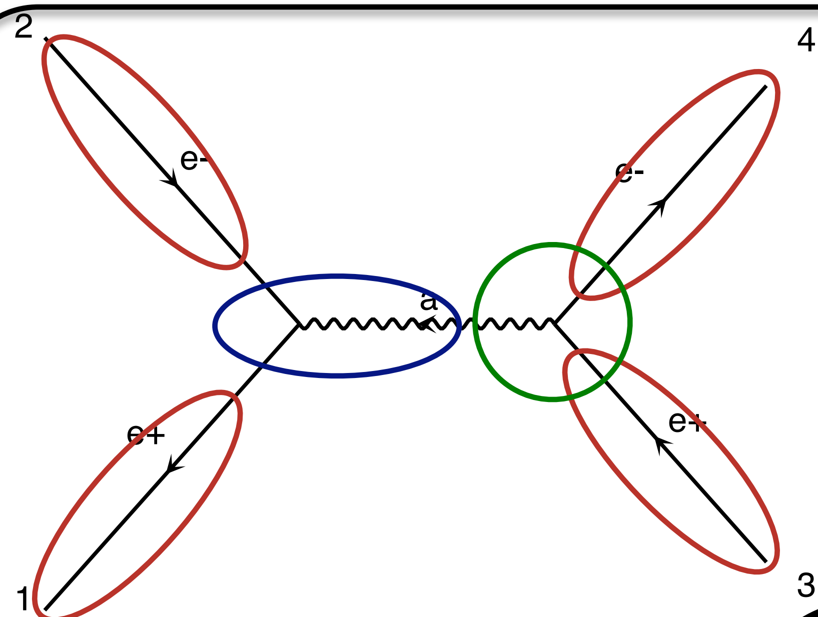
Calculate propagator wavefunctions

Lines present in the code.

$$\left\{\begin{array}{l} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a) \end{array}\right.$$

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

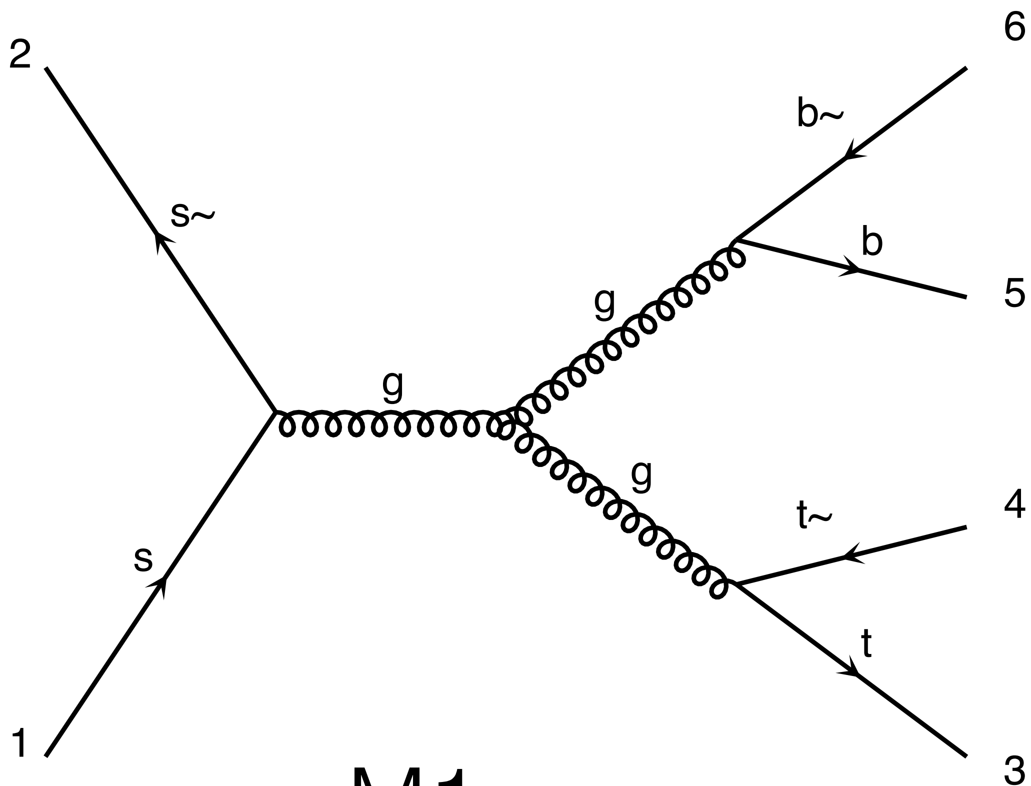
Finally evaluate amplitude (c-number)

Lines present in the code.

$$\left\{ \begin{array}{l} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a) \\ \mathcal{M} = fct(v_3, \bar{u}_4, W_a) \end{array} \right.$$

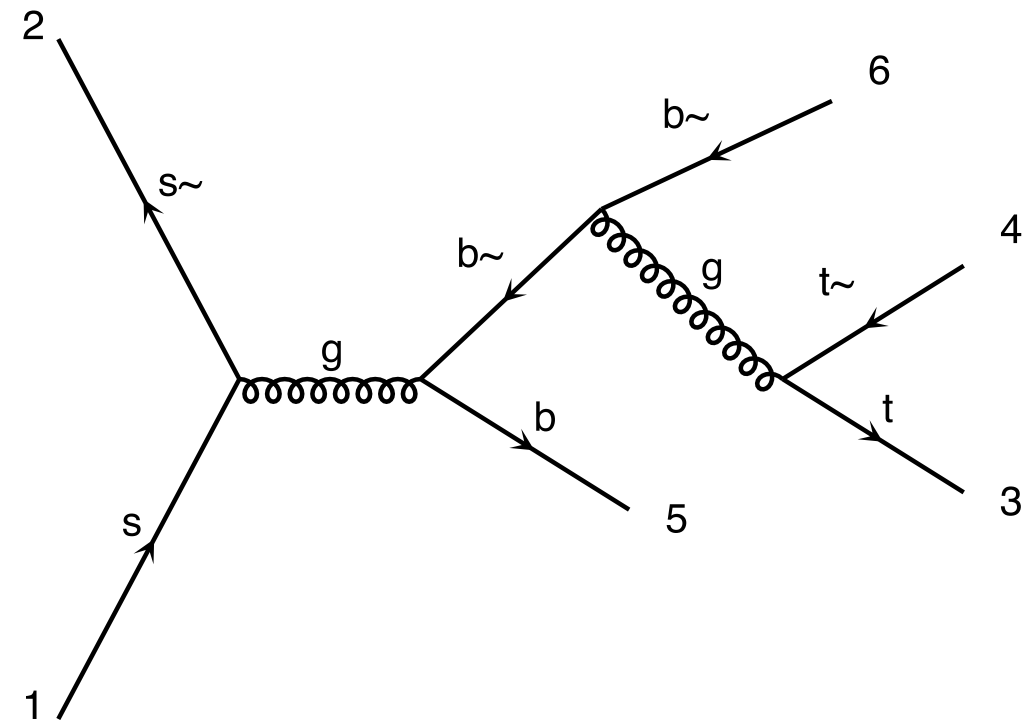
Real case

Known



M1

Number of routines: 0



M2

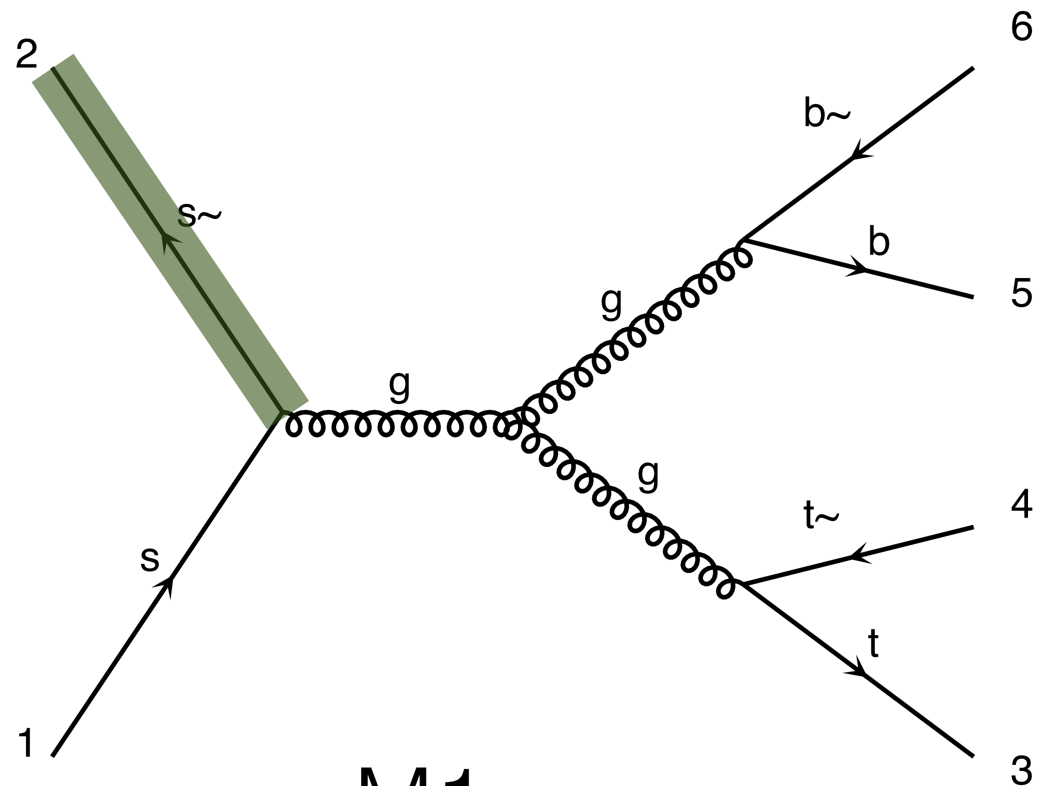
Number of routines: 0

Number of routines for both: 0

$$|M|^2 = |M_1 + M_2|^2$$

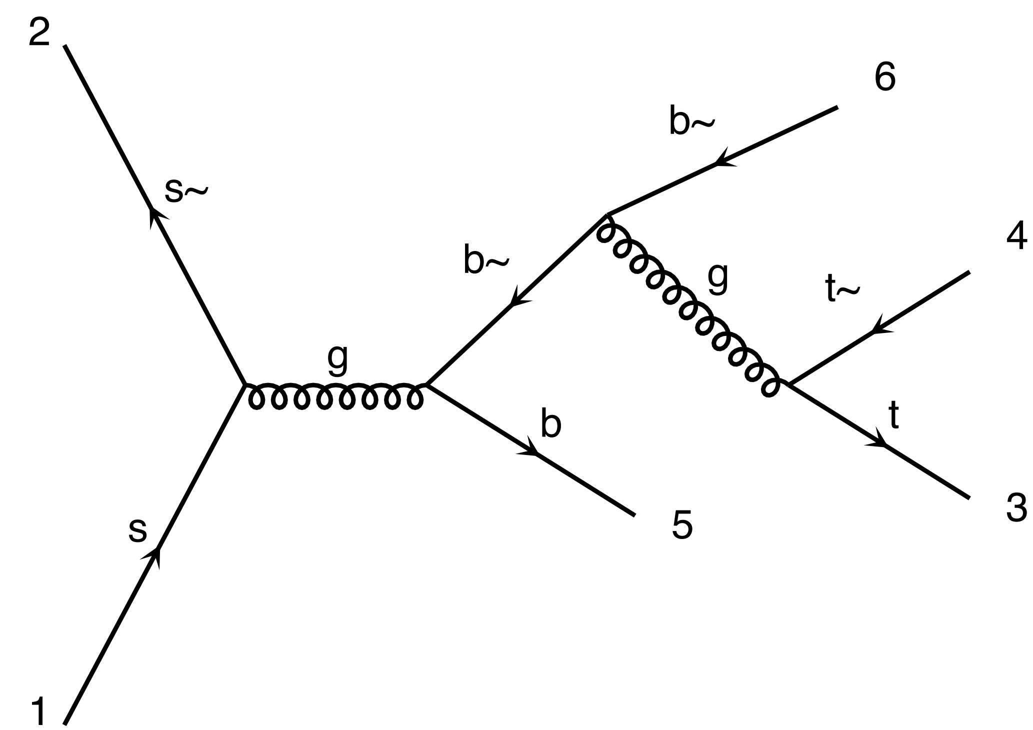
Real case

Known



M1

Number of routines: 1



M2

Number of routines: 0

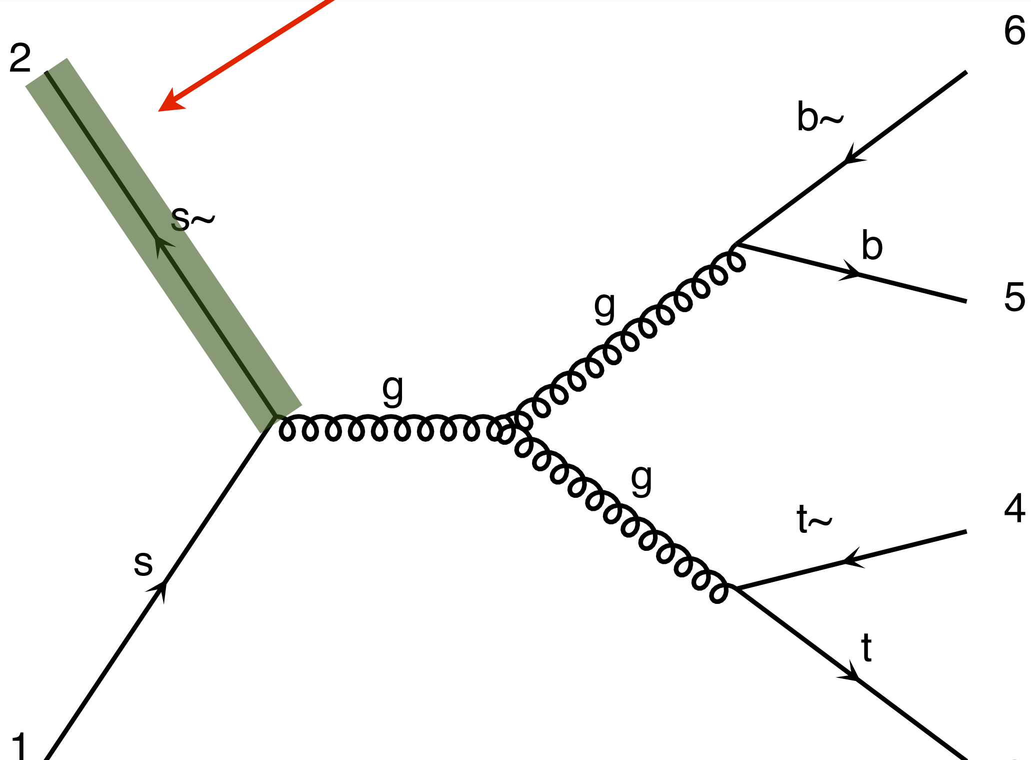
Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

Real case

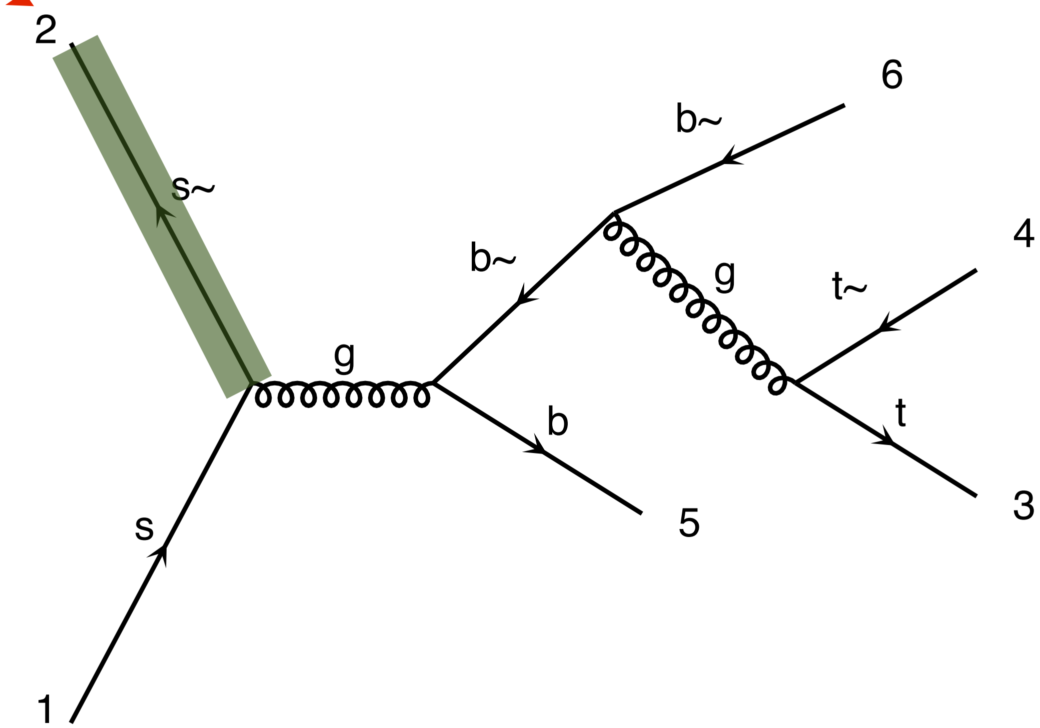
Identical

Known



M1

Number of routines: 1



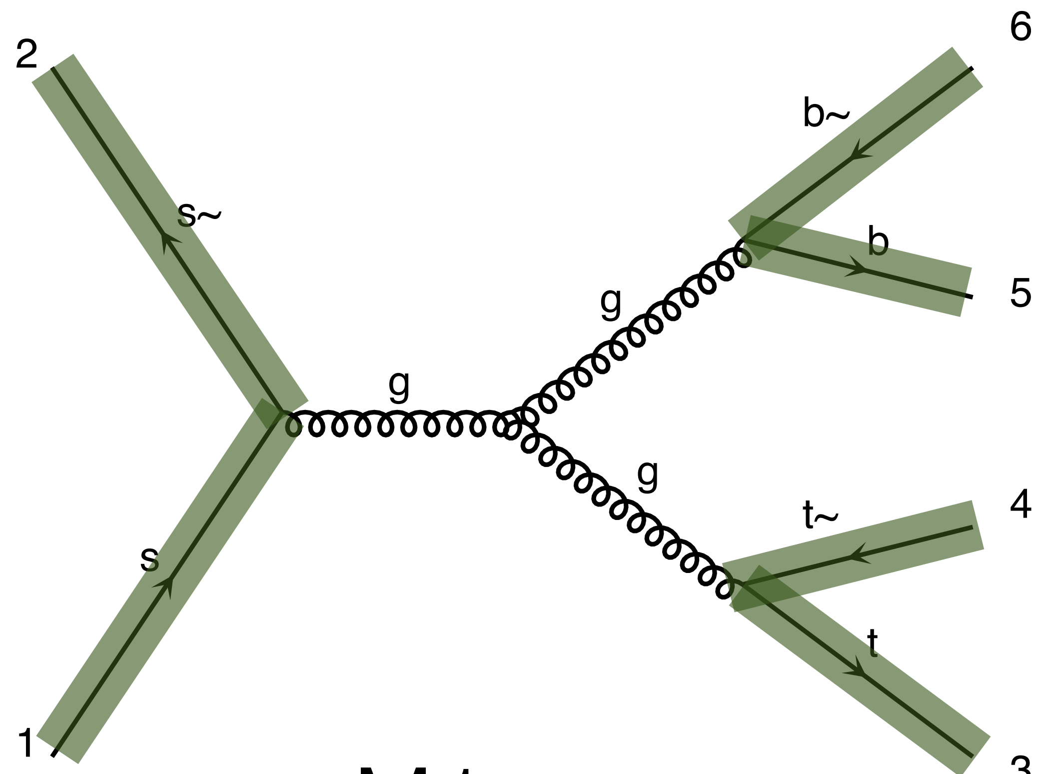
M2

Number of routines: 1

Number of routines for both: 1

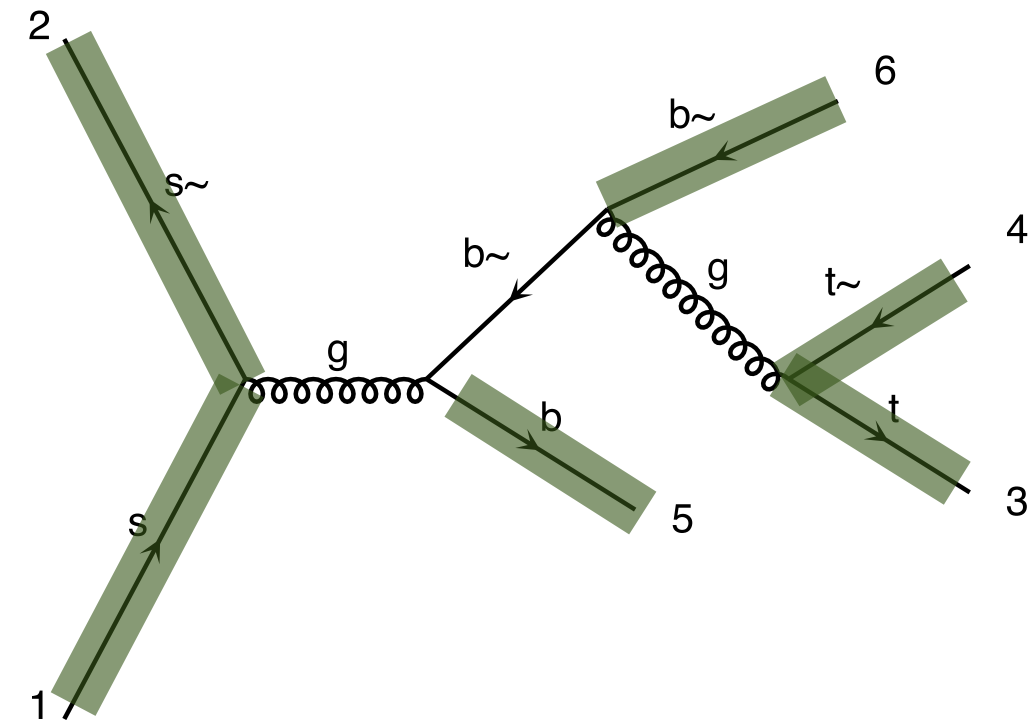
$$|M|^2 = |M_1 + M_2|^2$$

Known



M1

Number of routines: 6



M2

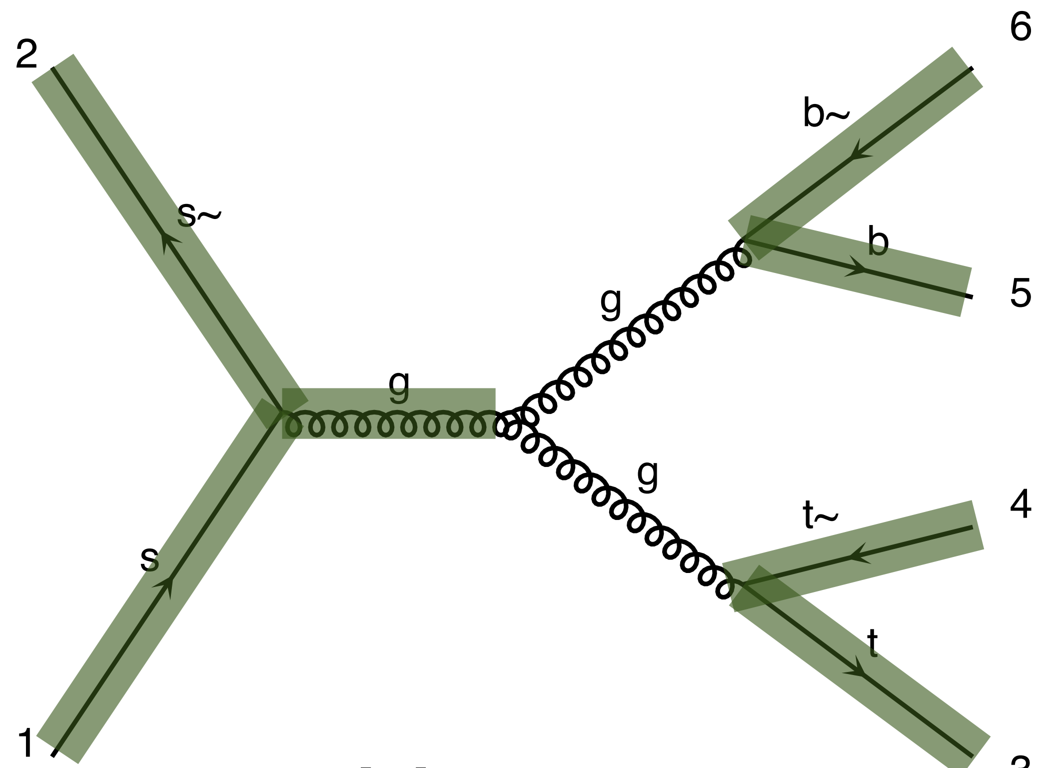
Number of routines: 6

Number of routines for both: 6

$$|M|^2 = |M_1 + M_2|^2$$

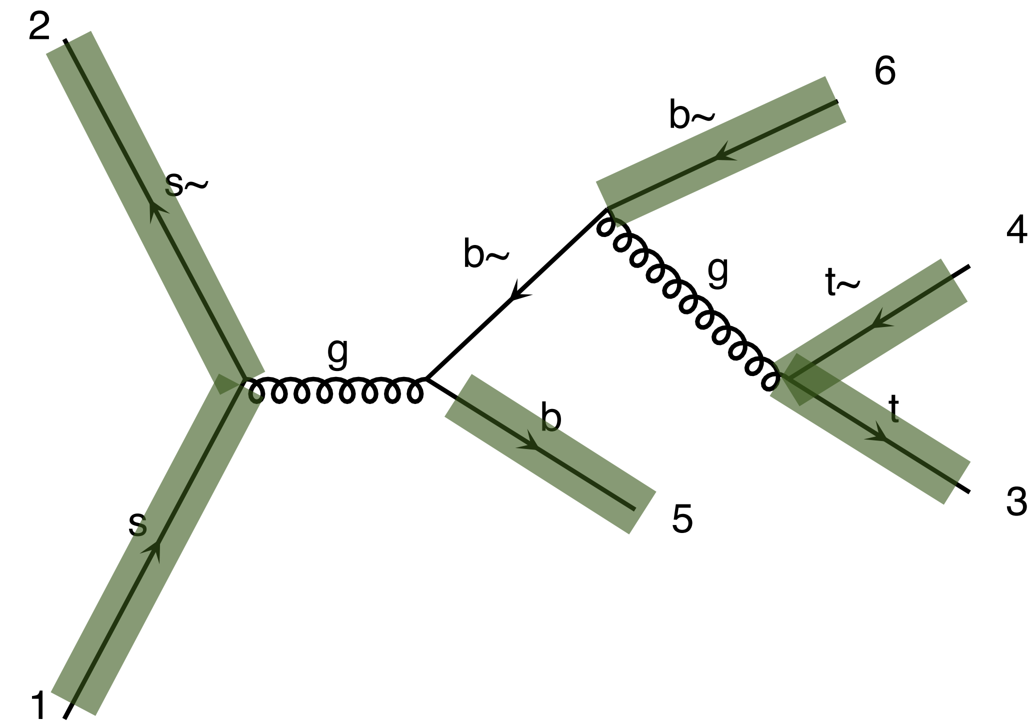
Real case

Known



M1

Number of routines: 7



M2

Number of routines: 6

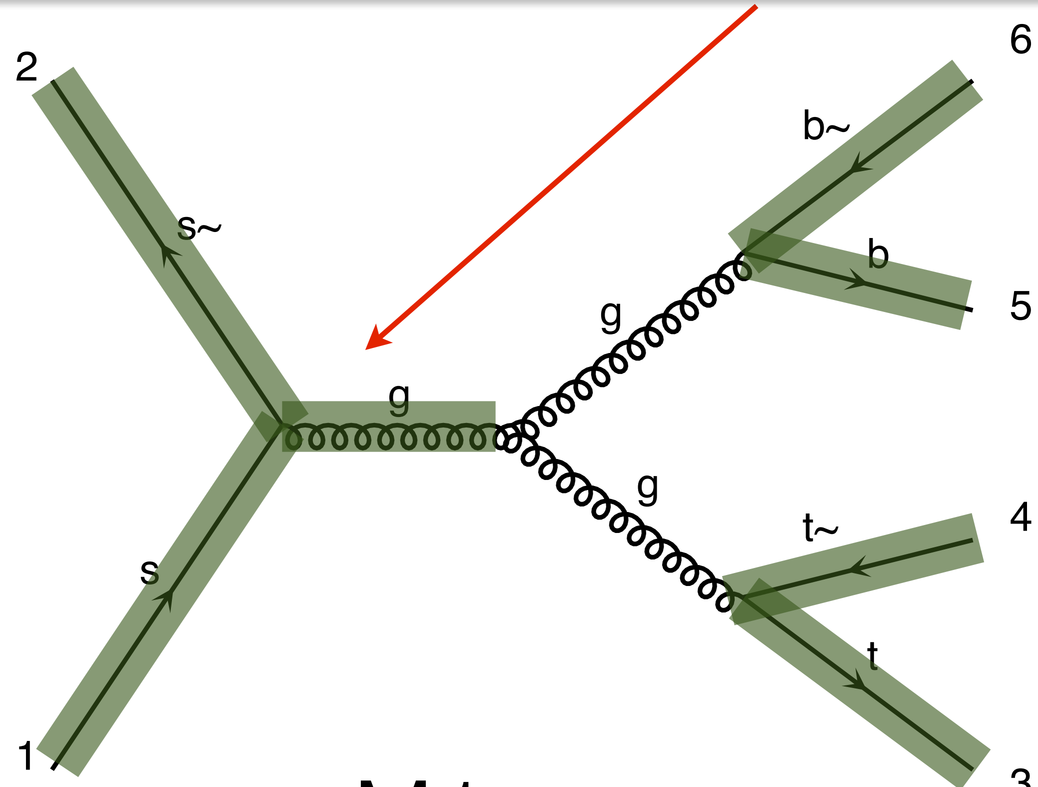
Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

Real case

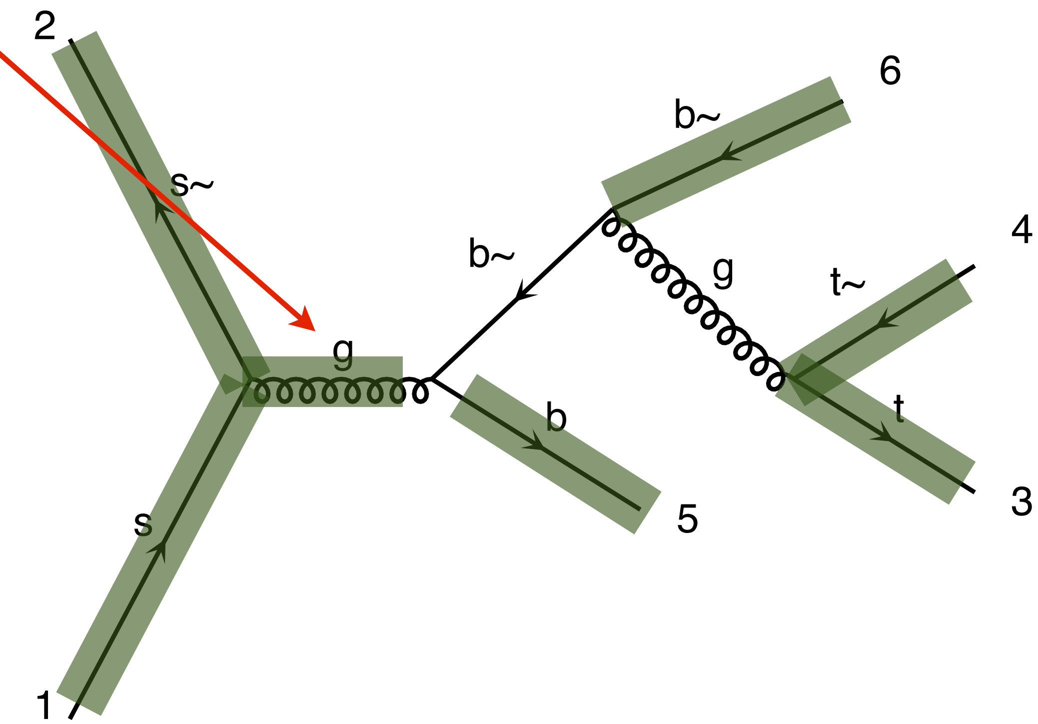
Known

Identical



M1

Number of routines: 7



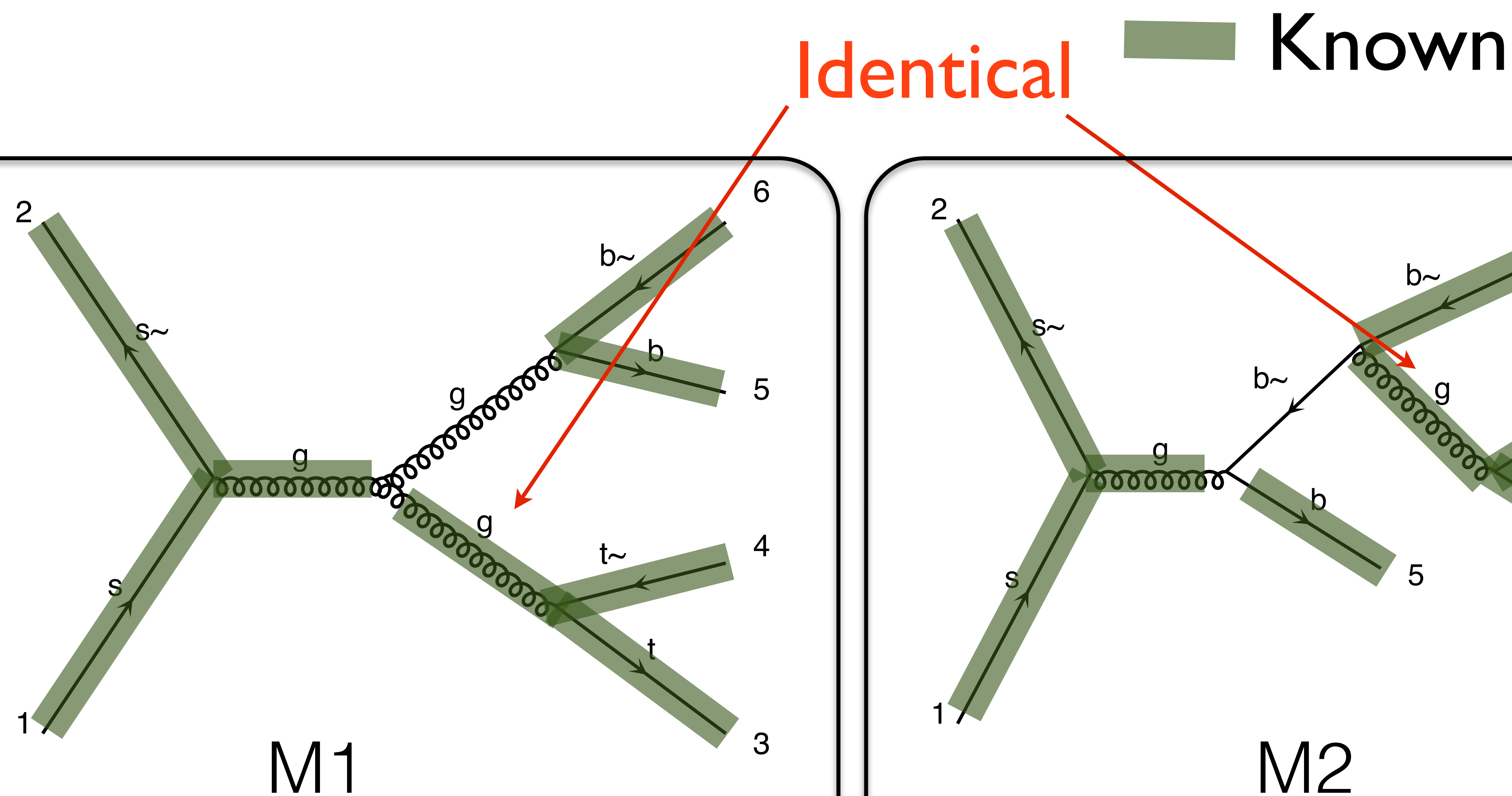
M2

Number of routines: 7

Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

Real case



Number of routines: 8

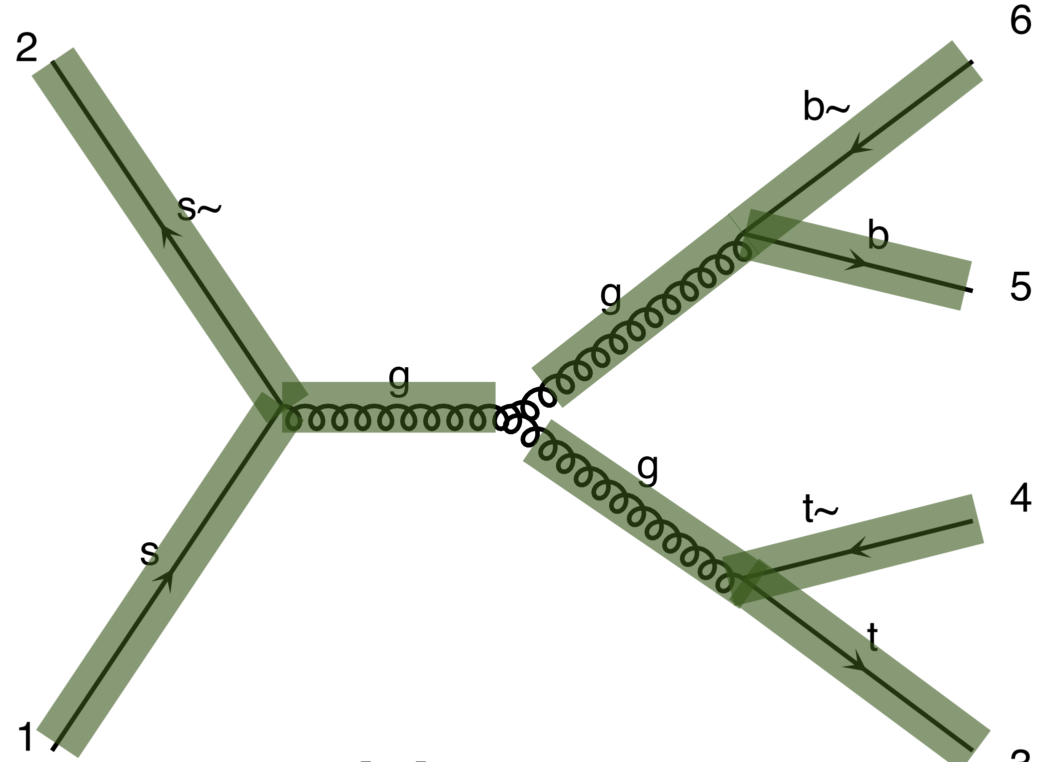
Number of routines: 8

Number of routines for both: 8

$$|M|^2 = |M_1 + M_2|^2$$

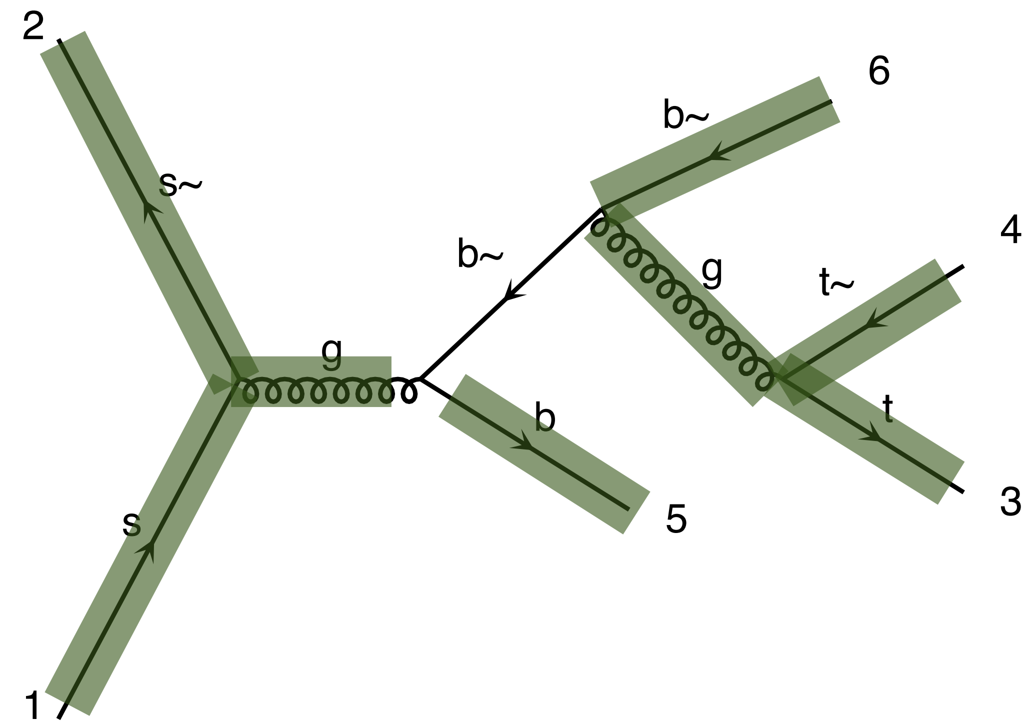
Real case

Known



M1

Number of routines: 9



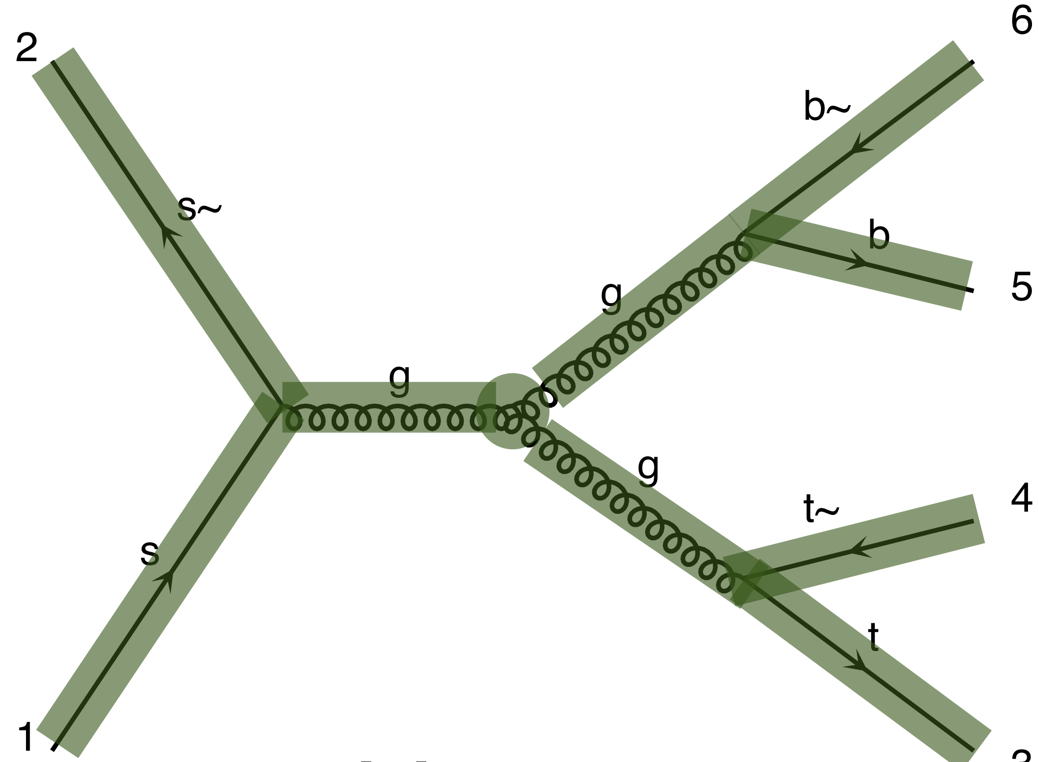
M2

Number of routines: 8

Number of routines for both: 9

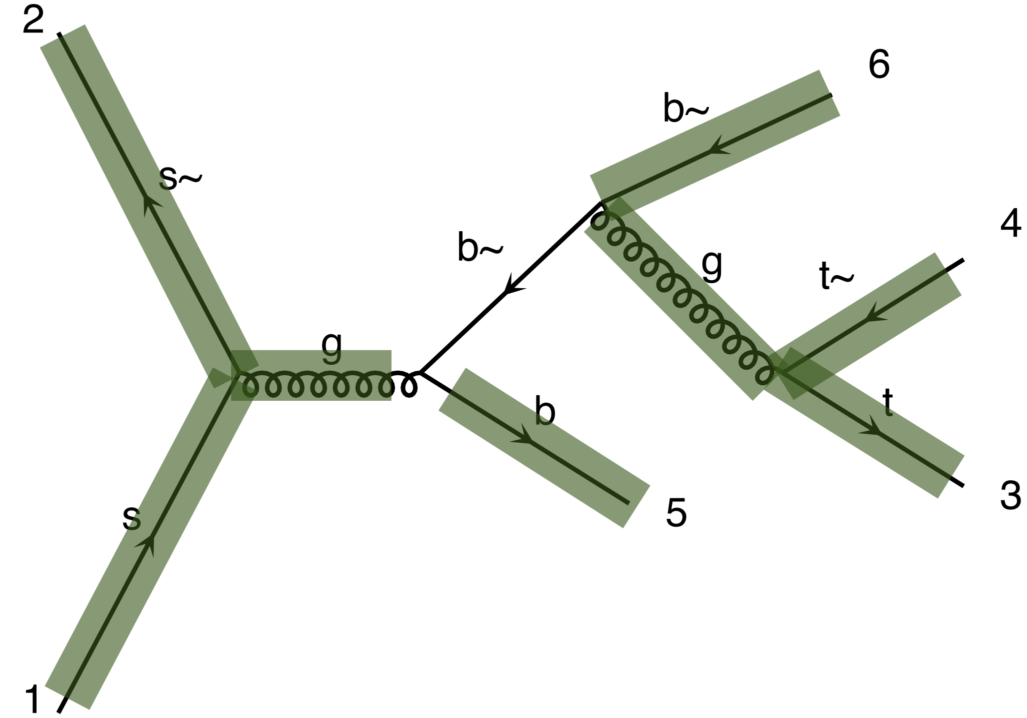
$$|M|^2 = |M_1 + M_2|^2$$

Known



M1

Number of routines: 10



M2

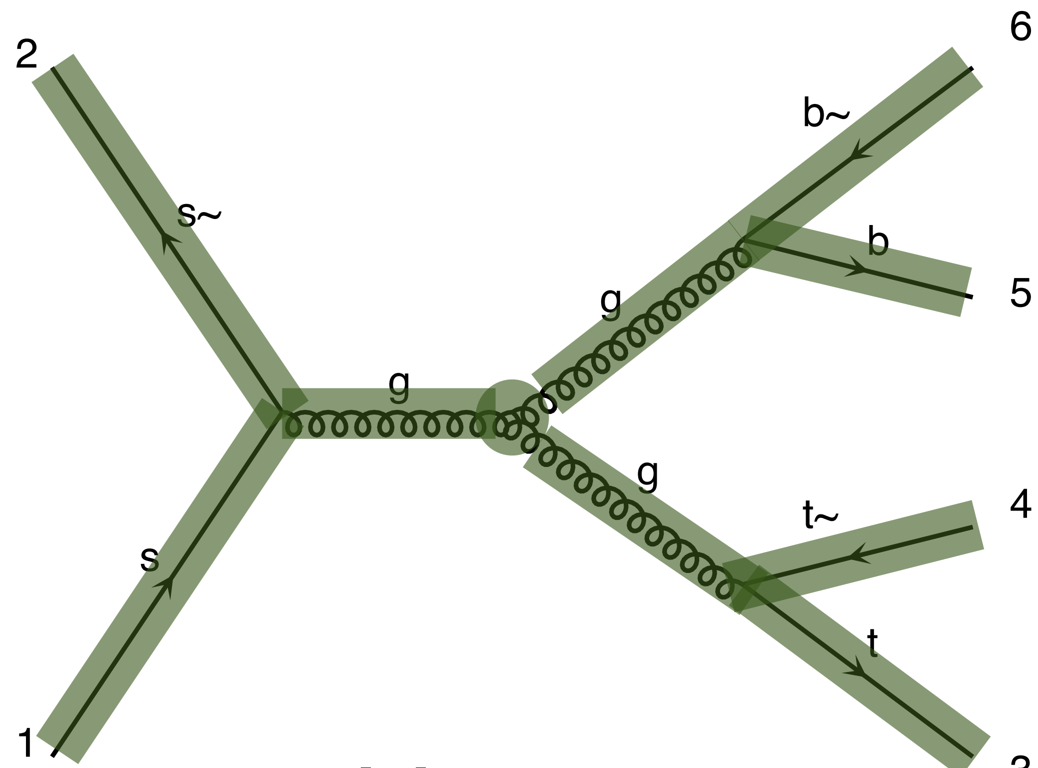
Number of routines: 8

Number of routines for both: 10

$$|M|^2 = |M_1 + M_2|^2$$

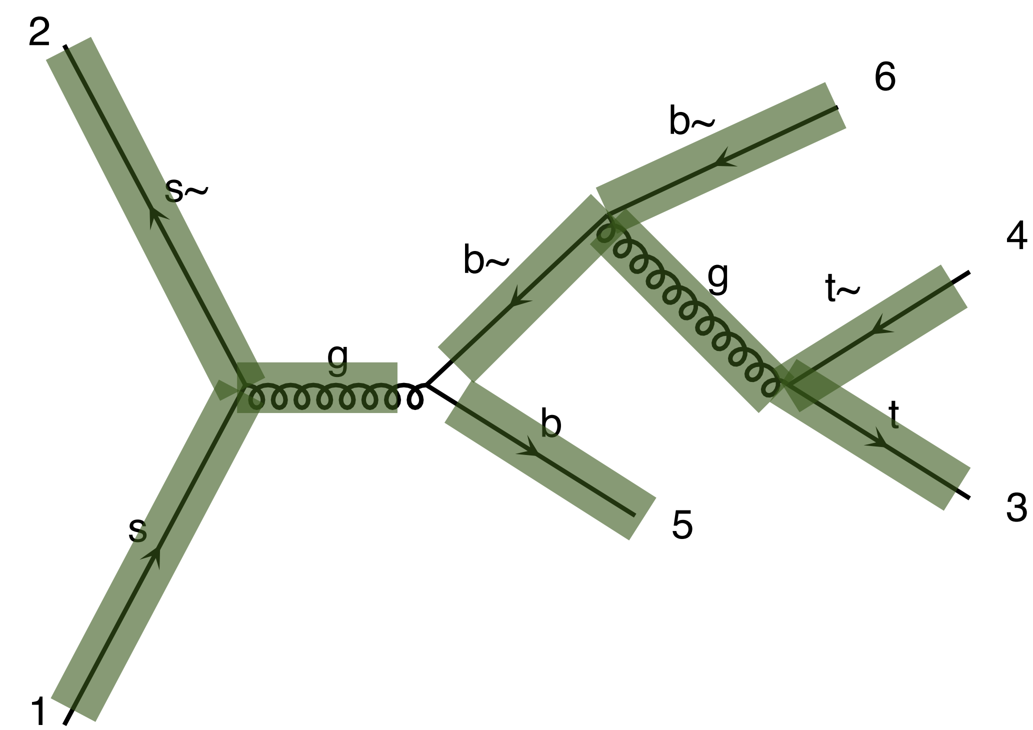
Real case

Known



M1

Number of routines: 10



M2

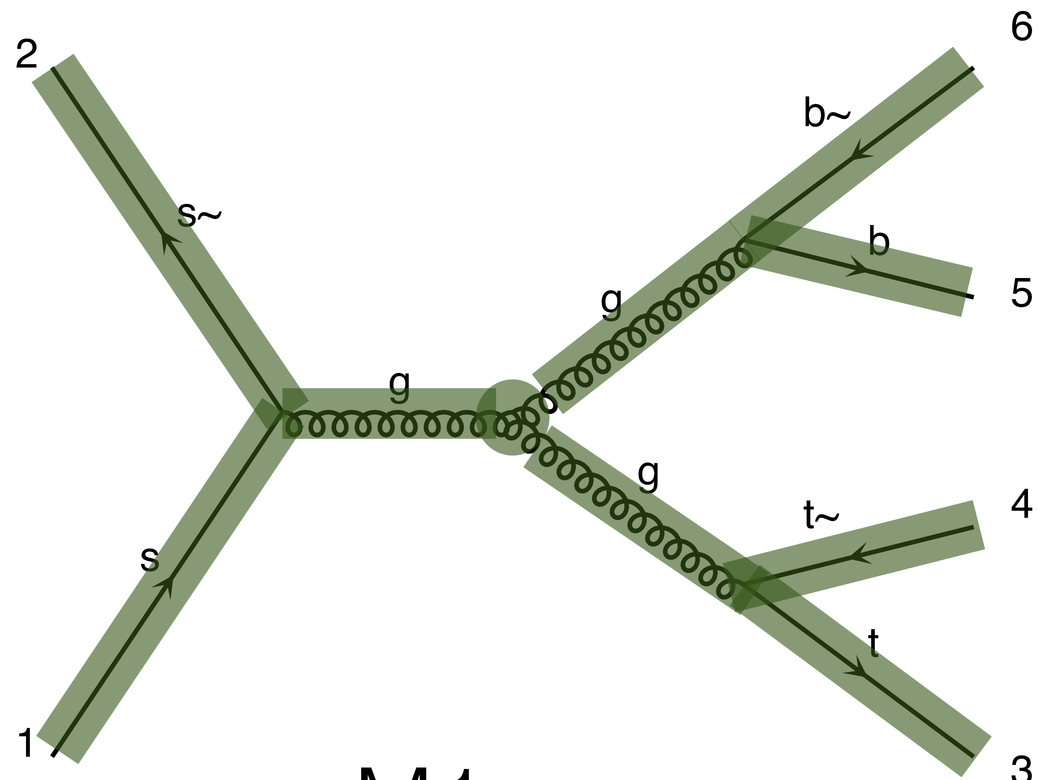
Number of routines: 9

Number of routines for both: 11

$$|M|^2 = |M_1 + M_2|^2$$

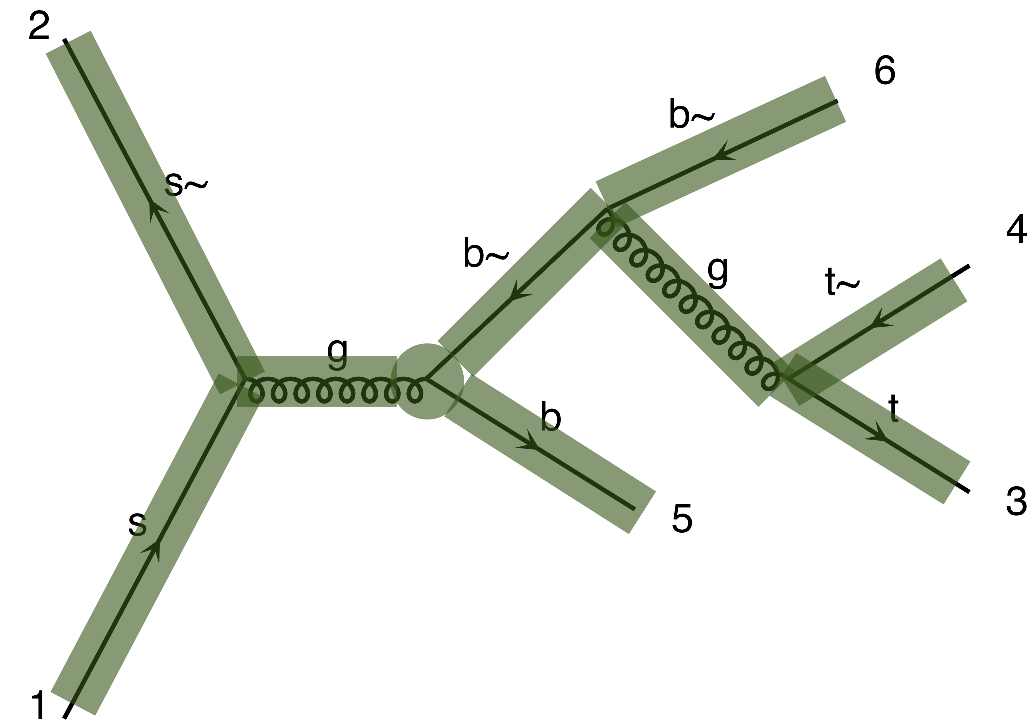
Real case

Known



M1

Number of routines: 10



M2

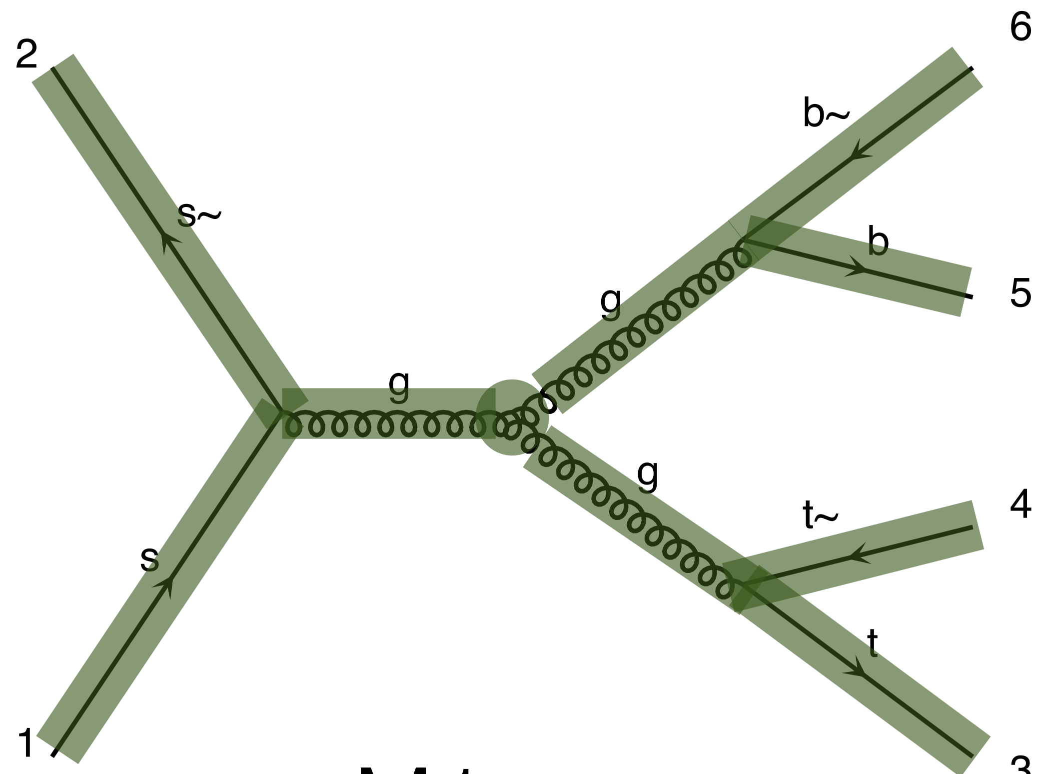
Number of routines: 10

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

Real case

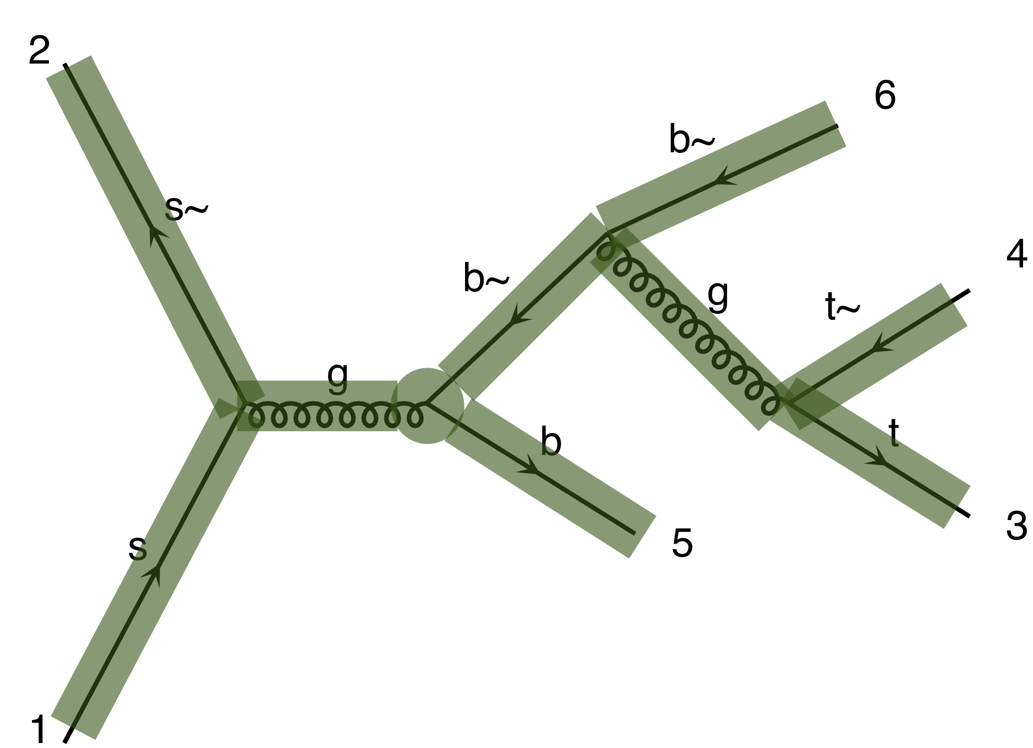
Known



M1

Number of routines: 10

2(N+1)



M2

Number of routines: 10

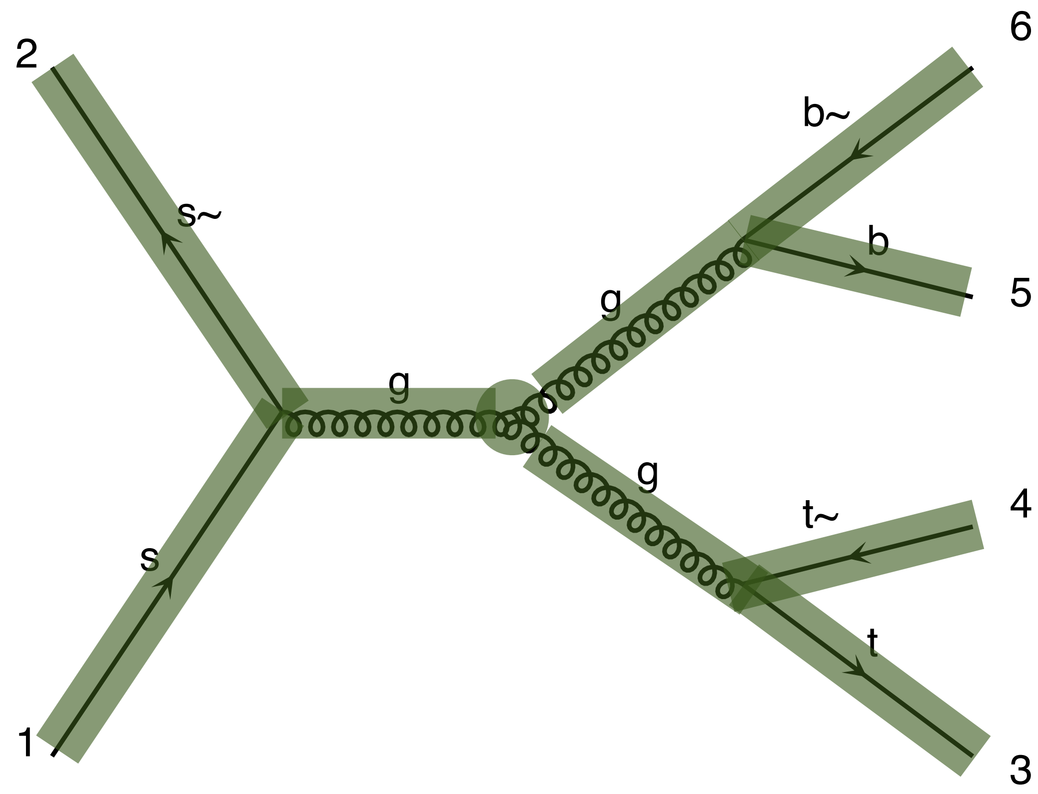
2(N+1)

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

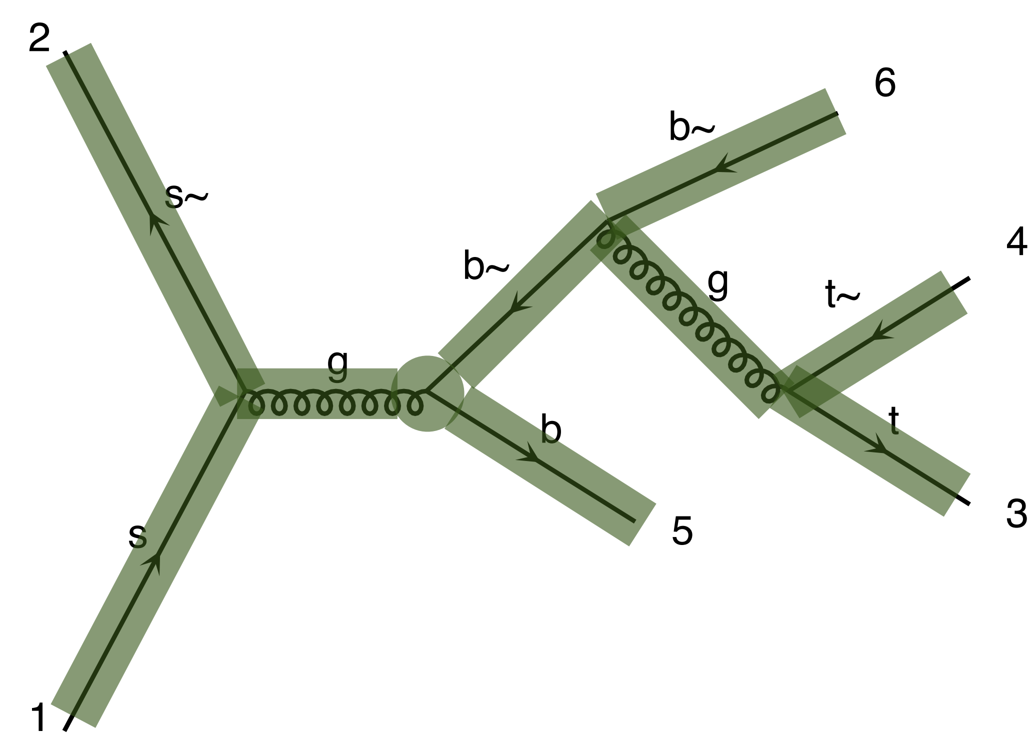
Real case

Known



Number of routines: 10

2(N+1)



Number of routines: 10

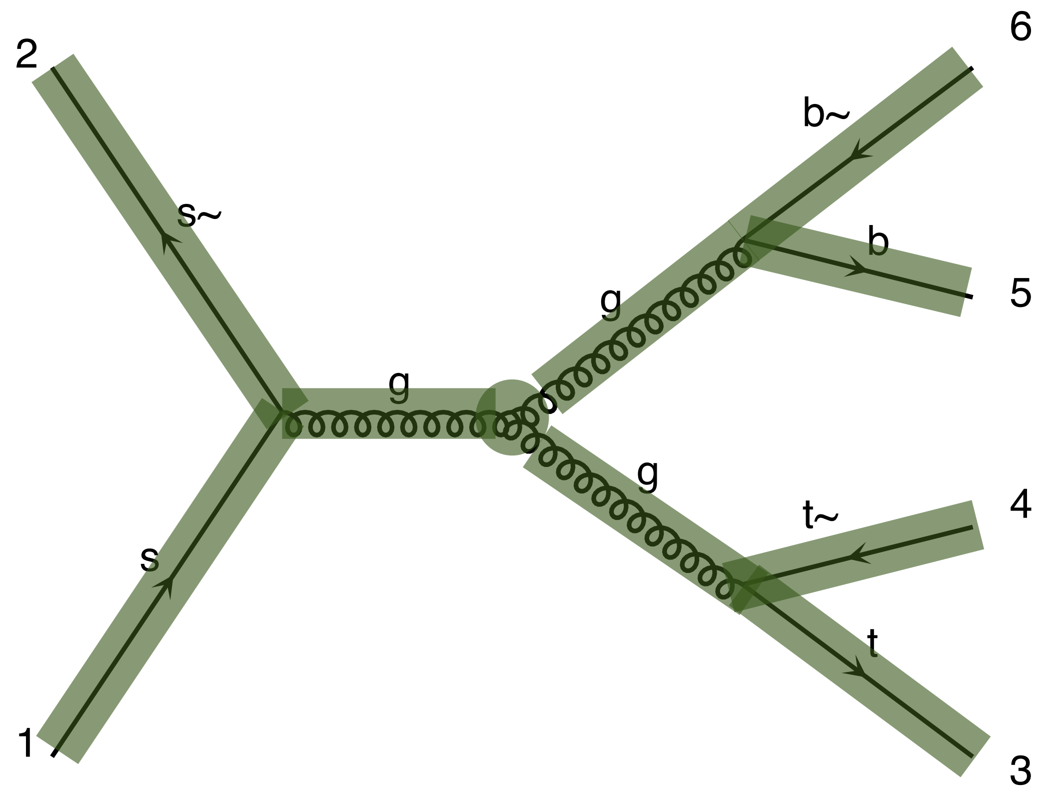
2(N+1)

Number of routines for both: 12

N!*2(N+1) ⟶ N!

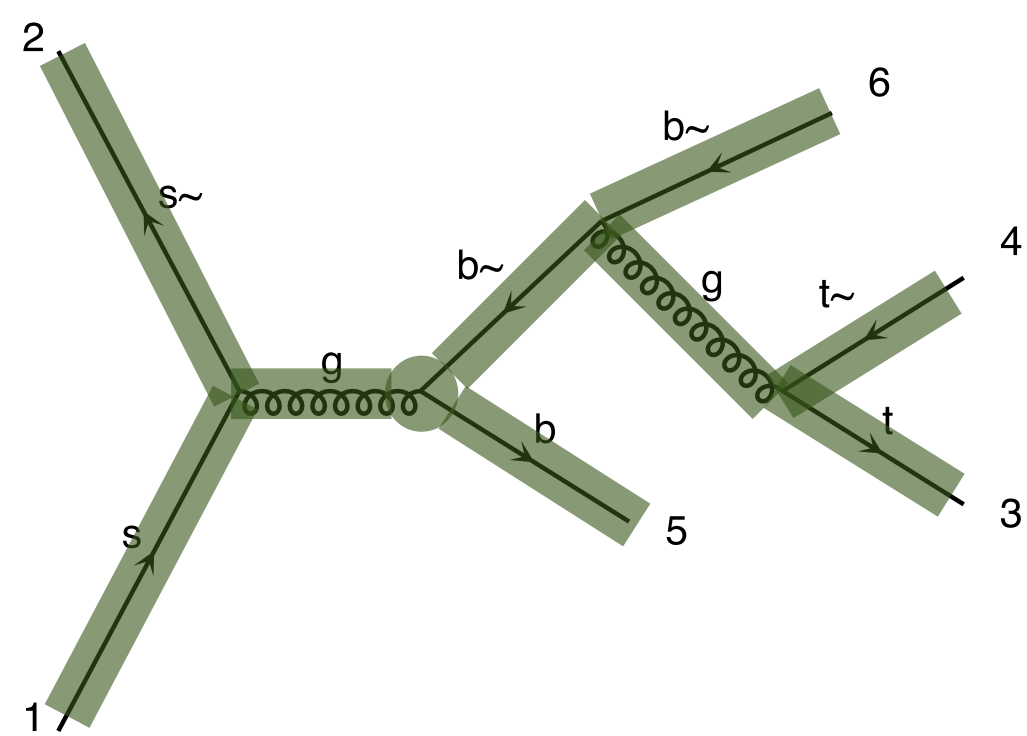
Real case

Known



Number of routines: 10

2(N+1)



Number of routines: 10

2(N+1)

Number of routines for both: 12

N!*2(N+1) ⟶ N! ⟶ (recursion) 2^N

Helicity amplitudes

- Thanks to new diagram generation algorithm, wf recycling much more efficient in MG5 than MG4

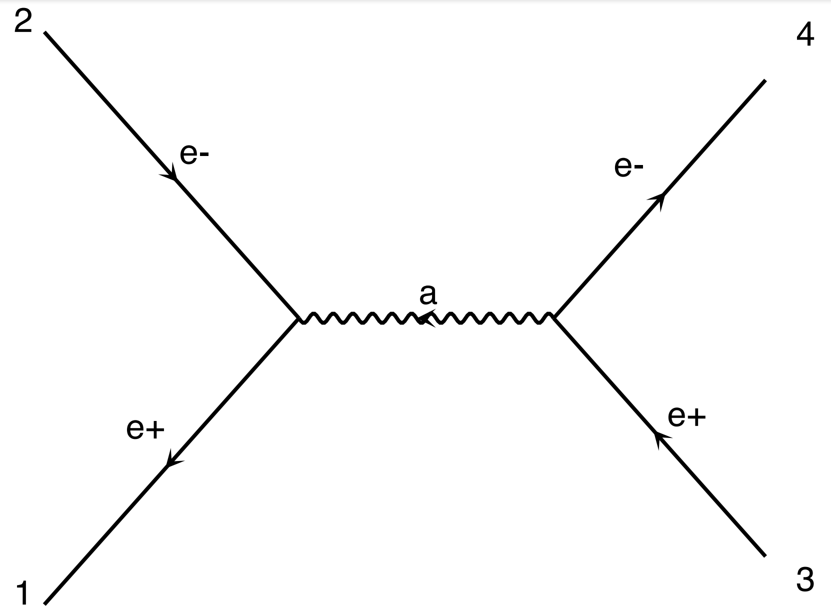
Process	Amplitudes	Wavefunctions		Run time	
		MG 4	MG 5	MG 4	MG 5
$u\bar{u} \to e^+e^-$	2	6	6	$< 6\mu s$	$< 6\mu s$
$u\bar{u} \to e^+e^-e^+e^-$	48	62	32	0.22 ms	0.14 ms
$u\bar{u} \to e^+e^-e^+e^-e^+e^-$	3474	3194	301	46.5 ms	19.0 ms
$u\bar{u} \to d\bar{d}$	1	5	5	$< 4\mu s$	$< 4\mu s$
$u\bar{u} \to d\bar{d}g$	5	11	11	27 μs	27 μs
$u\bar{u} \to d\bar{d}gg$	38	47	29	0.42 ms	0.31 ms
$u\bar{u} \to d\bar{d}ggg$	393	355	122	10.8 ms	6.75 ms
$u\bar{u} \to u\bar{u}gg$	76	84	40	1.24 ms	0.80 ms
$u\bar{u} \to u\bar{u}ggg$	786	682	174	35.7 ms	17.2 ms
$u\bar{u} \to d\bar{d}d\bar{d}$	14	28	19	84 μs	83 μs
$u\bar{u} \to d\bar{d}d\bar{d}g$	132	178	65	1.88 ms	1.15 ms
$u\bar{u} \to d\bar{d}d\bar{d}gg$	1590	1782	286	141 ms	34.4 ms
$u\bar{u} \to d\bar{d}d\bar{d}d\bar{d}$	612	758	141	42.5 ms	6.6 ms

no recycling

300,000

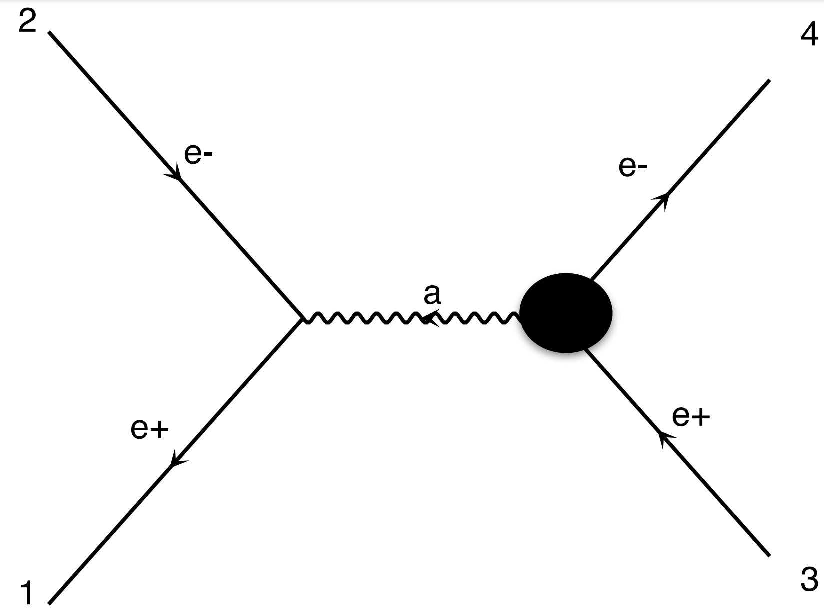
5500

Time for matrix element evaluation on a Sony Vaio TZ laptop



$$\mathcal{M} = \bar{u}\gamma^\mu v P_{\mu\nu} \bar{u}\gamma^\nu v$$

$$\bar{u}_1 = fct(\vec{p}_1, m)$$
$$v_2 = fct(\vec{p}_2, m)$$
$$\bar{u}_3 = fct(\vec{p}_3, m)$$
$$v_4 = fct(\vec{p}_4, m)$$
$$W_a = fct(\bar{u}_1, v_2, M_a, \Gamma_a)$$
$$\mathcal{M} = fct(\bar{u}_3, v_4, W_a)$$



$$\mathcal{M} = \bar{u}\gamma^\mu v P_{\mu\nu} \bar{u}\Gamma^\mu v$$

$$\bar{u}_1 = fct(\vec{p}_1, m)$$
$$v_2 = fct(\vec{p}_2, m)$$
$$\bar{u}_3 = fct(\vec{p}_3, m)$$
$$v_4 = fct(\vec{p}_4, m)$$
$$W_a = fct(\bar{u}_1, v_2, M_a, \Gamma_a)$$
$$\mathcal{M} = fct(\bar{u}_3, v_4, W_a)$$

HELAS

- **Original HELicity Amplitude Subroutine library**

[Murayama, Watanabe, Hagiwara]

HELAS

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- **One routine by Lorentz structure**
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  - ➡ **HEFT** [Frederix] (2007)
  - ➡ **Spin 2** [Hagiwara, al] 0805.2554 (2008)
  - ➡ **Spin 3/2** [Mawatari, al] 1101.1289 (2011)

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Chiral Perturbation

BNV Model

SLIH

Effective Field Theory

NMSSM

Chromo-magnetic operator

Full HEFT

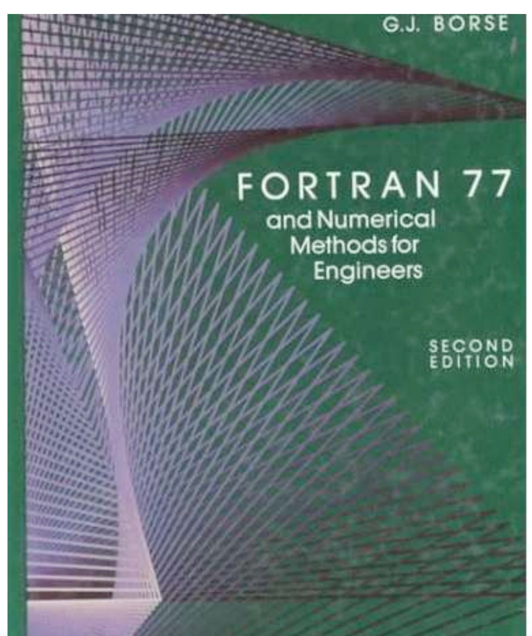
Black Holes

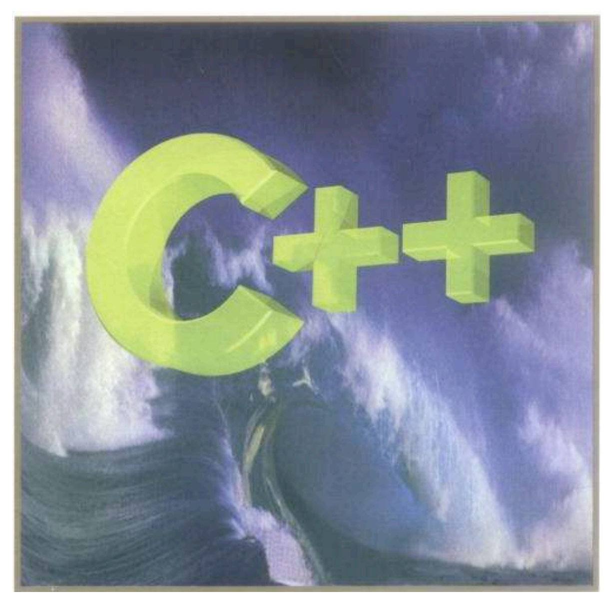
ALOHA

ALOHA
~~Google~~ translate

From: UFO ▾ ⇆ To: Helicity [Translate]

Type text or a website address or translate a document.





30

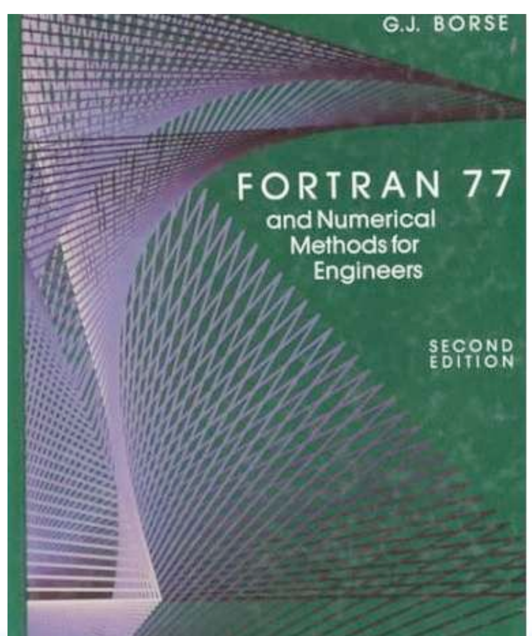
ALOHA

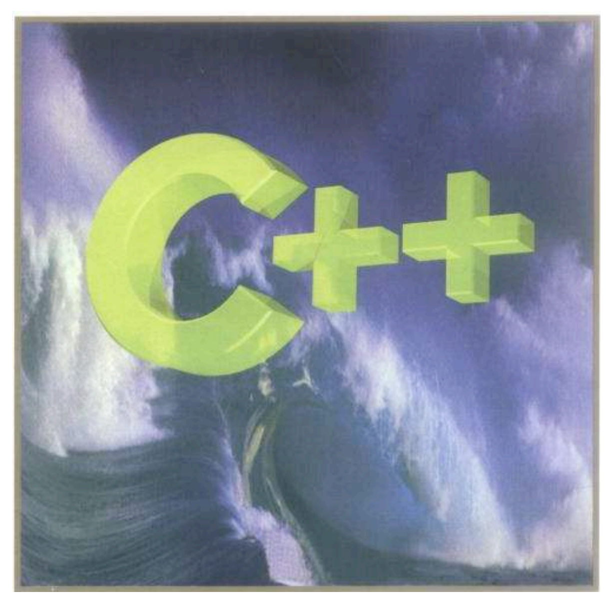
ALOHA
~~Google~~ translate

From: UFO ▾ To: Helicity Translate

Basically, any new operator can be handle by MG5/Pythia8 out of the box!

Type text or a website address or translate a document.





ALOHA DETAIL

Input

```
FFV1 = Lorentz(name = 'FFV1',
               spins = [ 2, 2, 3 ],
               structure = 'Gamma(3,2,1)')
```

Output

```
C       This File is Automatically generated by ALOHA
C       The process calculated in this file is:
C       Gamma(3,2,1)
C
        SUBROUTINE FFV1_0(F1,F2,V3,C,VERTEX)
        IMPLICIT NONE
        DOUBLE COMPLEX F1(6)
        DOUBLE COMPLEX F2(6)
        DOUBLE COMPLEX V3(6)
        DOUBLE COMPLEX C
        DOUBLE COMPLEX VERTEX



        VERTEX = C*( (F2(1)*( (F1(3)*( (0, -1)*V3(1)+(0, 1)*V3(4)))
       $ +(F1(4)*( (0, 1)*V3(2)+V3(3)))))+( (F2(2)*( (F1(3)*( (0, 1)
       $ *V3(2)-V3(3)))+(F1(4)*( (0, -1)*V3(1)+(0, -1)*V3(4)))))
       $ +( (F2(3)*( (F1(1)*( (0, -1)*V3(1)+(0, -1)*V3(4)))+(F1(2)
       $ *( (0, -1)*V3(2)-V3(3)))))+(F2(4)*( (F1(1)*( (0, -1)*V3(2)
       $ +V3(3)))+(F1(2)*( (0, -1)*V3(1)+(0, 1)*V3(4)))))))

        END
```

ALOHA

- Compute those Function Analytically
- Code in Python
- Can handle
  - ➡ all spin up to 2
  - ➡ custom propagator
  - ➡ majorana (but in 4 fermion operator)
  - ➡ Any dimensional operator
- Only use in MadGraph5_aMC@NLO
- Plan to have similar tools for the other generator

To Remember

- Numerical computation faster than analytical computation
- We are able to compute matrix-elment
  - ➡ for large number of final state
  - ➡ for any BSM theory
  - ➡ actually also for loop

Monte Carlo Integration and Generation

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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$$Dim[\Phi(n)] \sim 3n$$

Monte Carlo Integration

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$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

$Dim[\Phi(n)] \sim 3n$

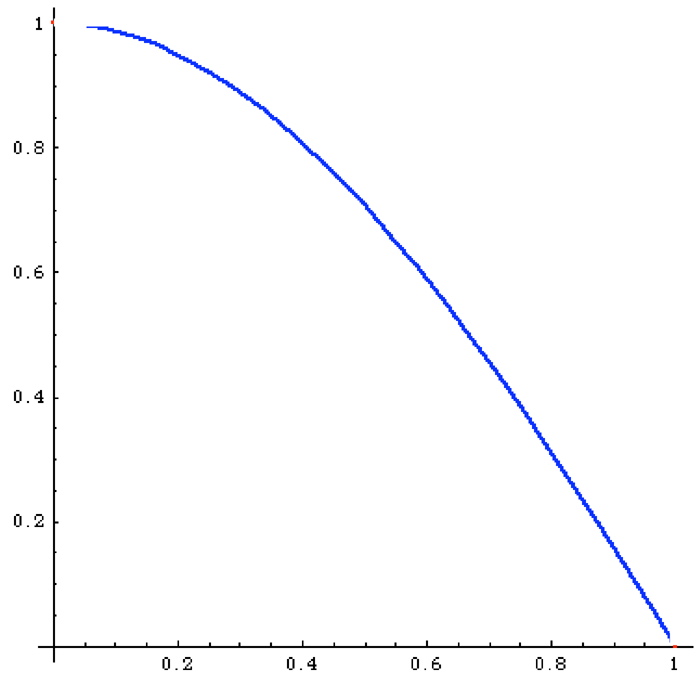
General and flexible method is needed

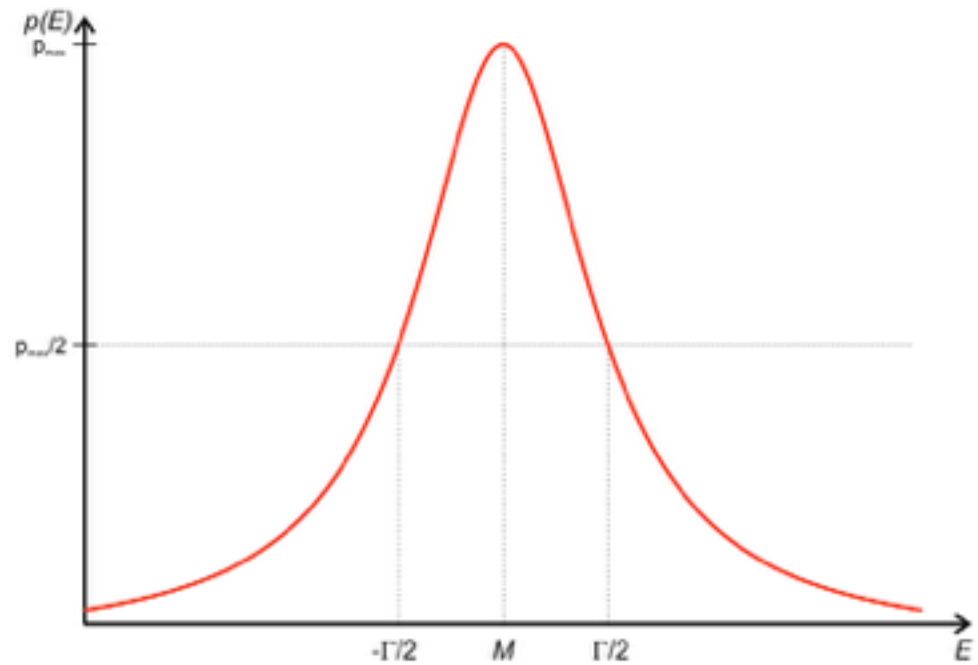
Integration

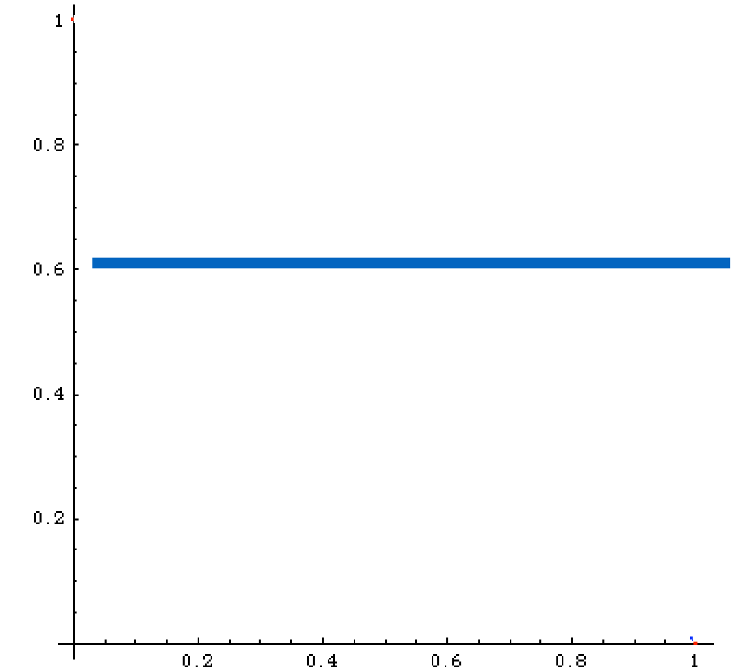
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

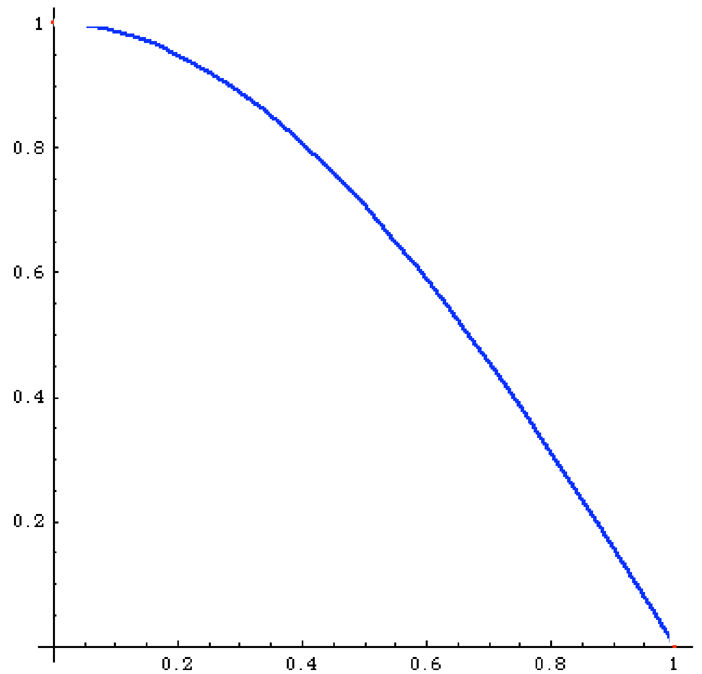
$$\int dx\, C$$



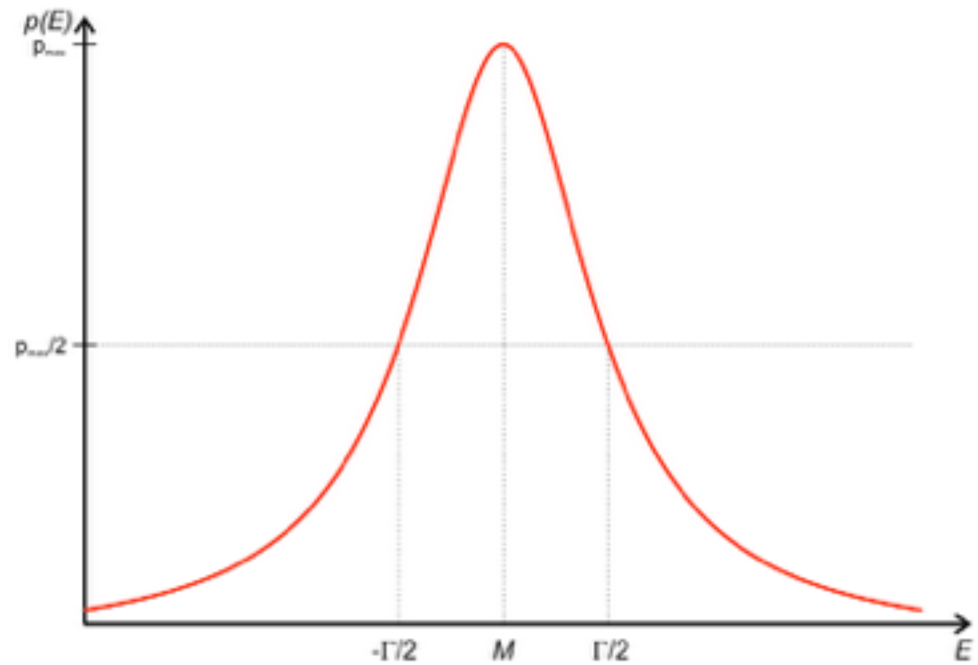




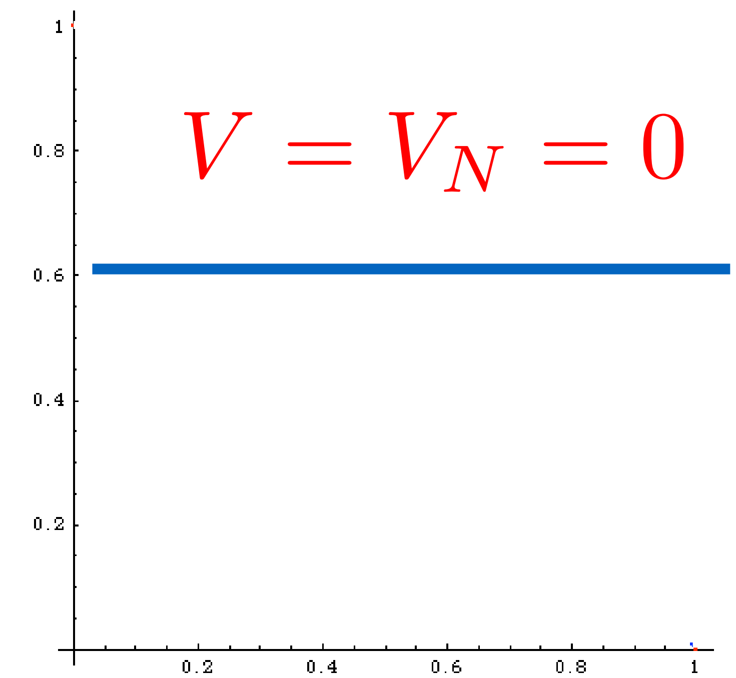
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



$$\int dx\, C$$

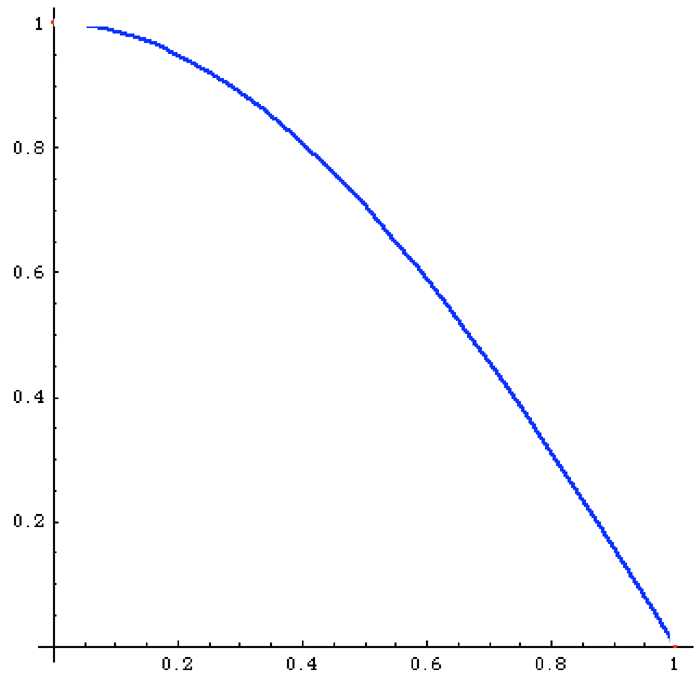


Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

Integration

$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



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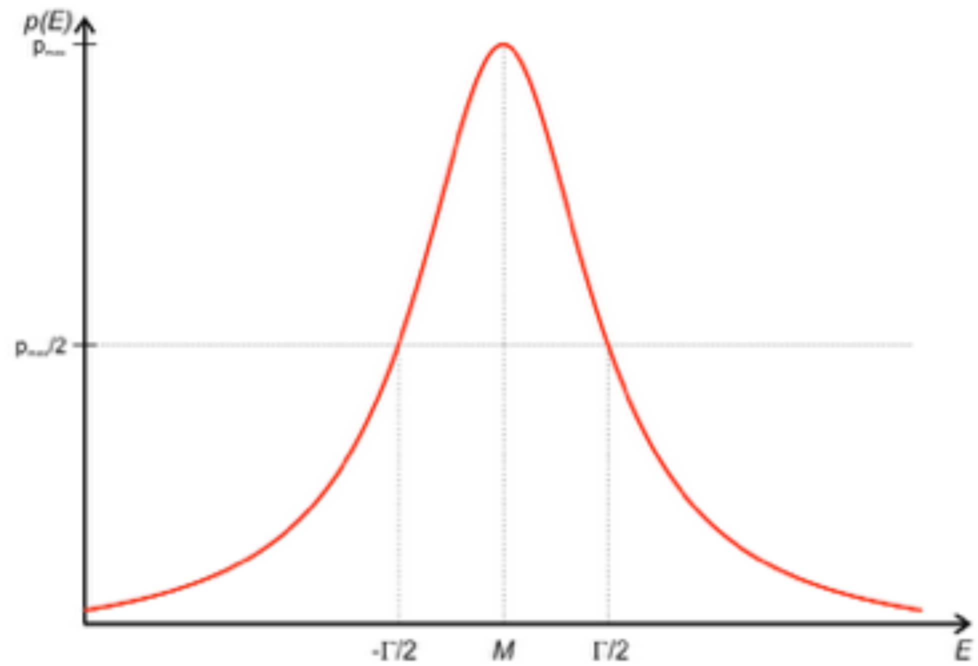

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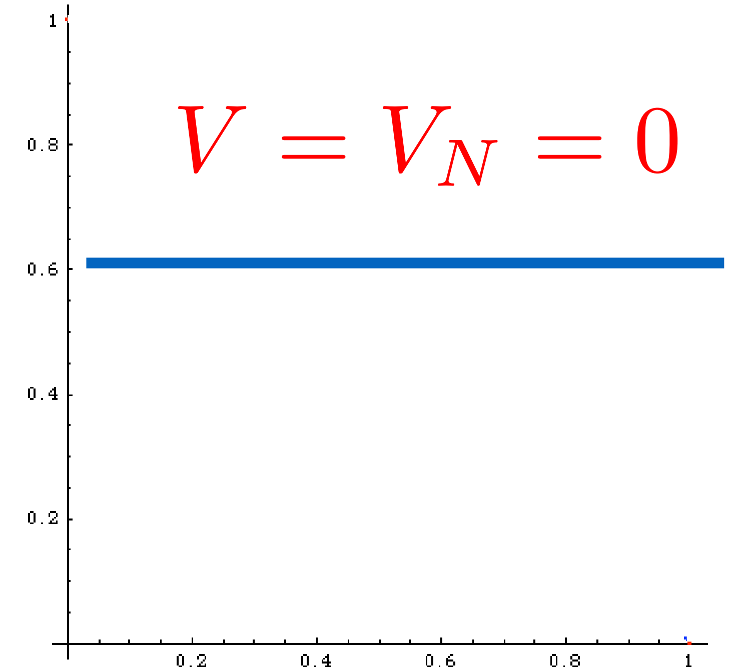

	simpson	MC
3	0,638	0,3
5	0,6367	0,8
20	0,63662	0,6
100	0,636619	0,65
1000	0,636619	0,636

Method of evaluation

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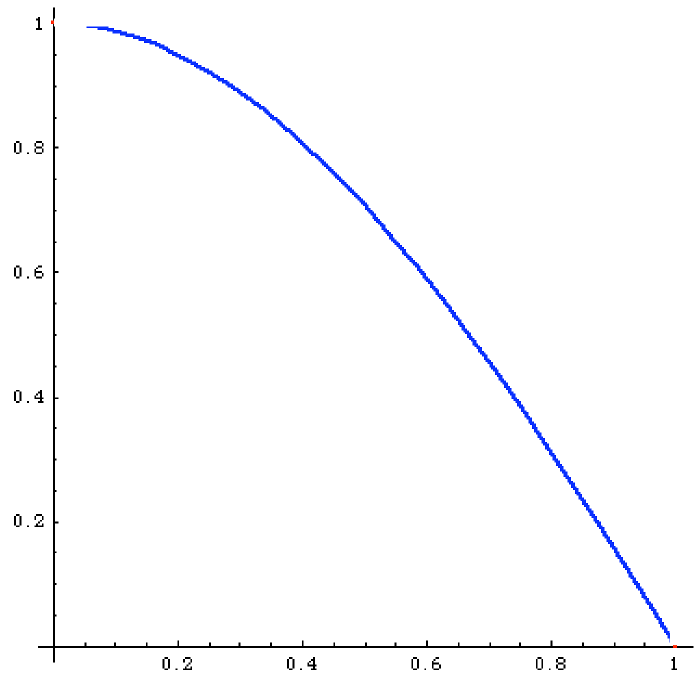
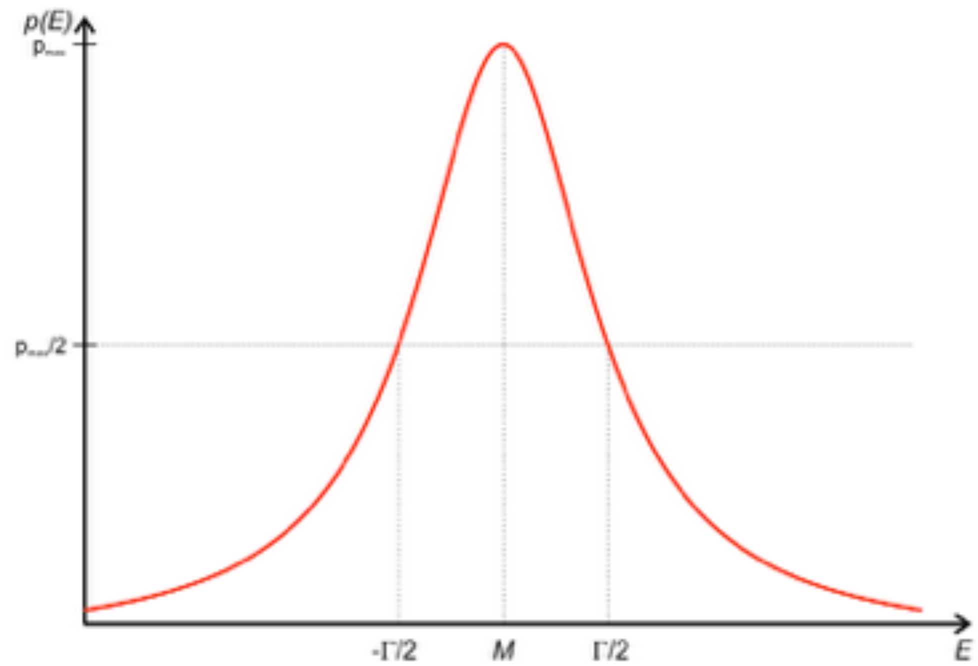
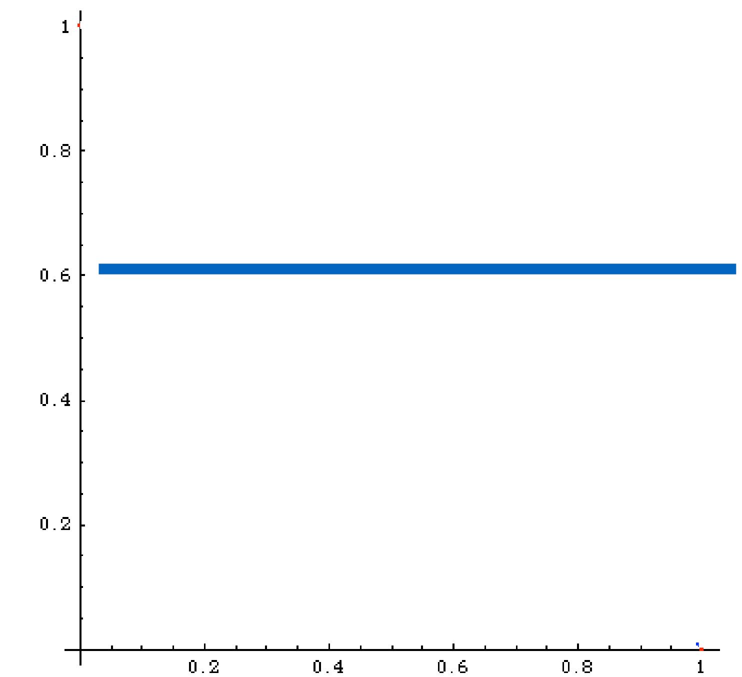






Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

More Dimension →

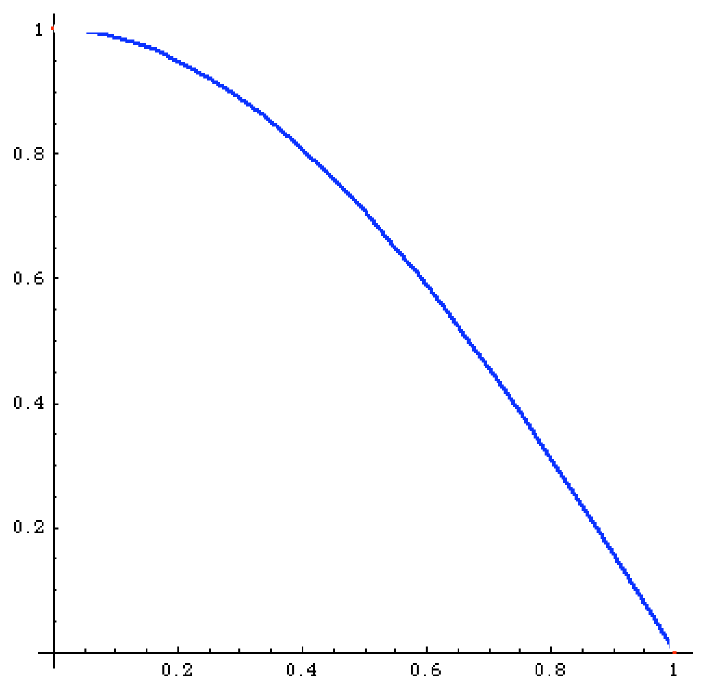
$1/\sqrt{N}$

$1/N^{2/d}$

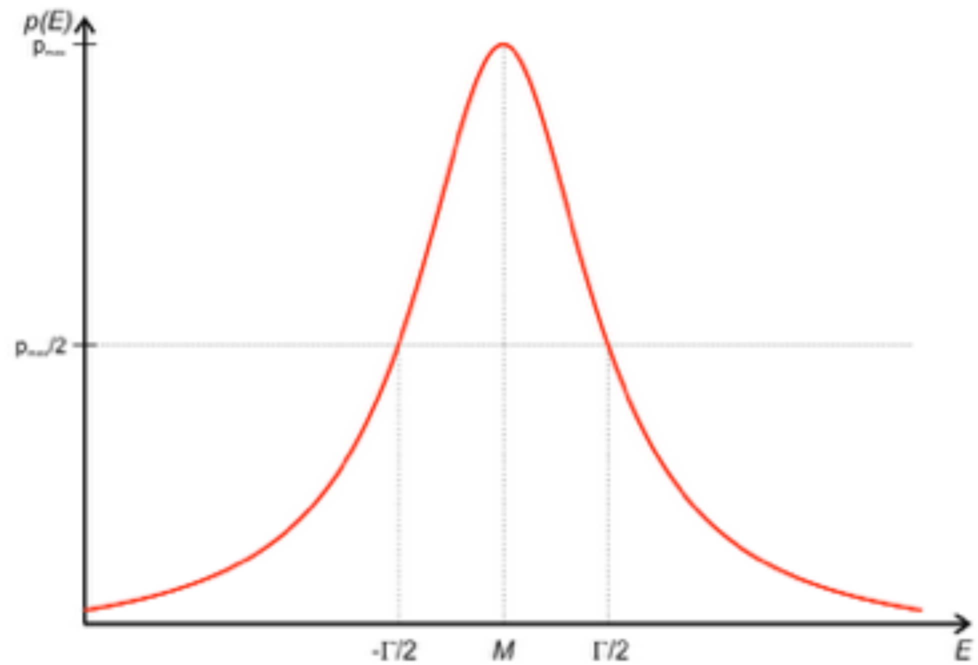
$1/N^{4/d}$

Integration

$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



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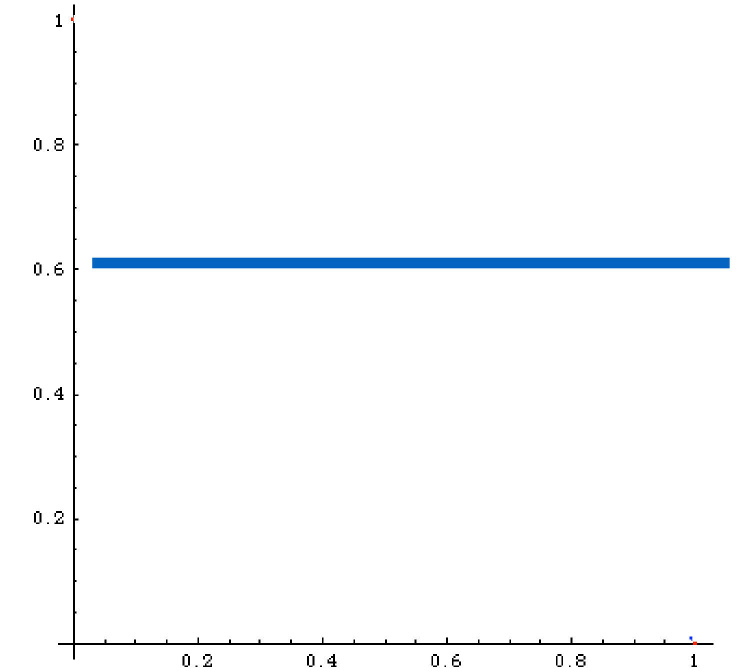

$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N} [f(x)]^2 - I_N^2$$

Integration

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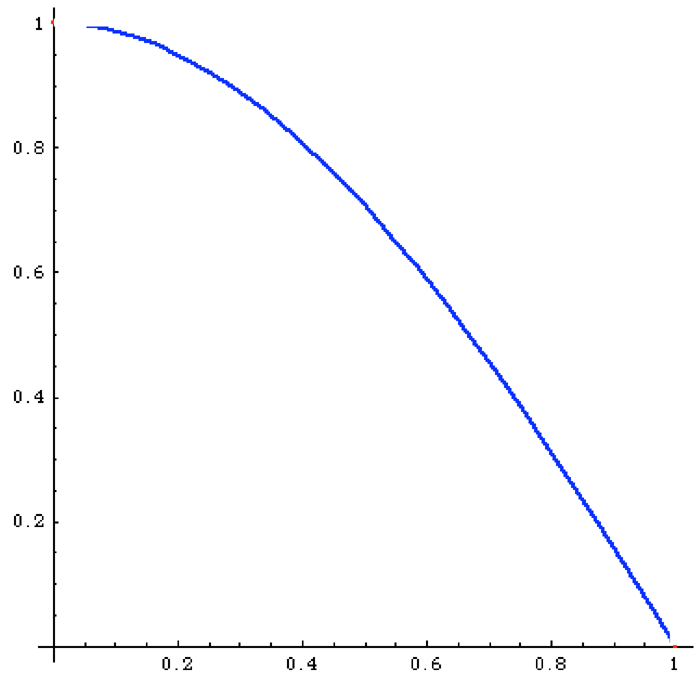

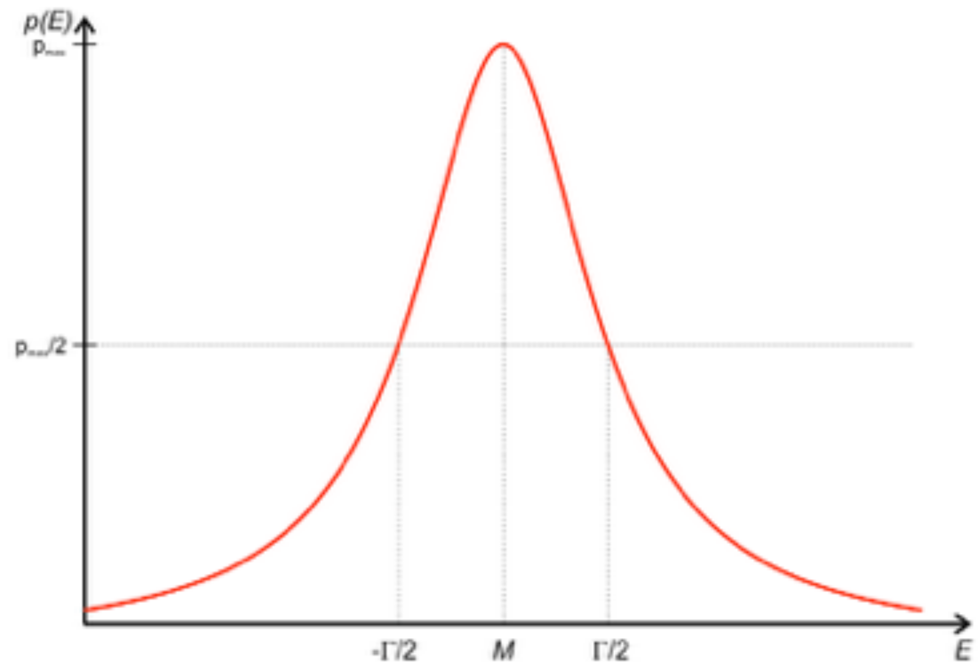

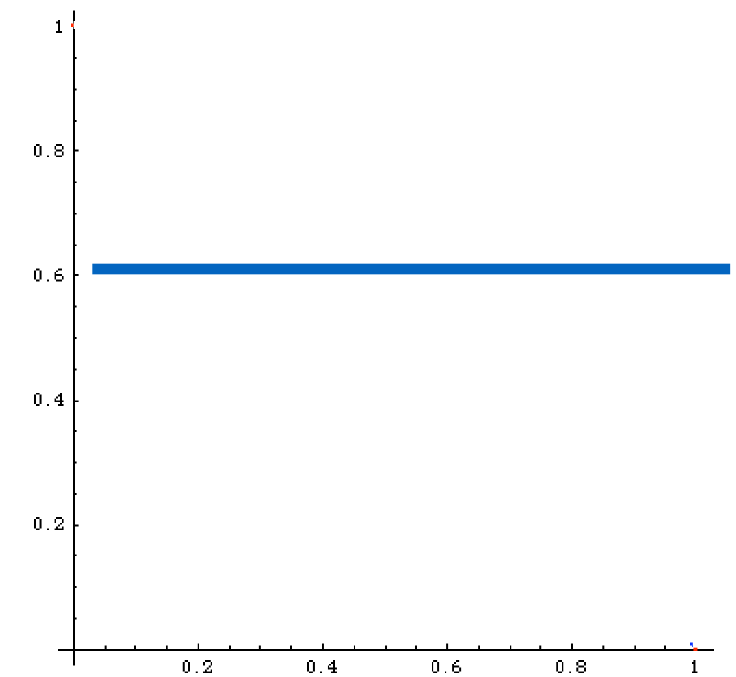

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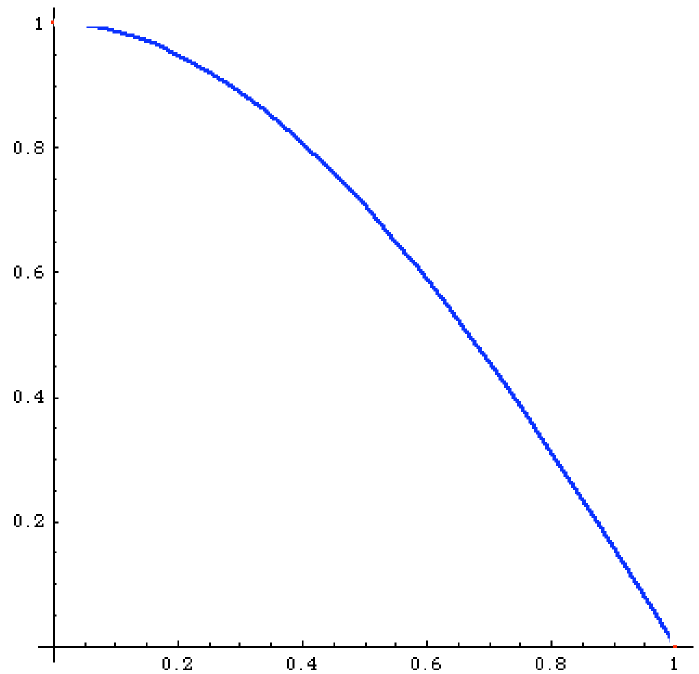
$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \Rightarrow V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

$$I = I_N \pm \sqrt{V_N/N}$$

Integration

$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



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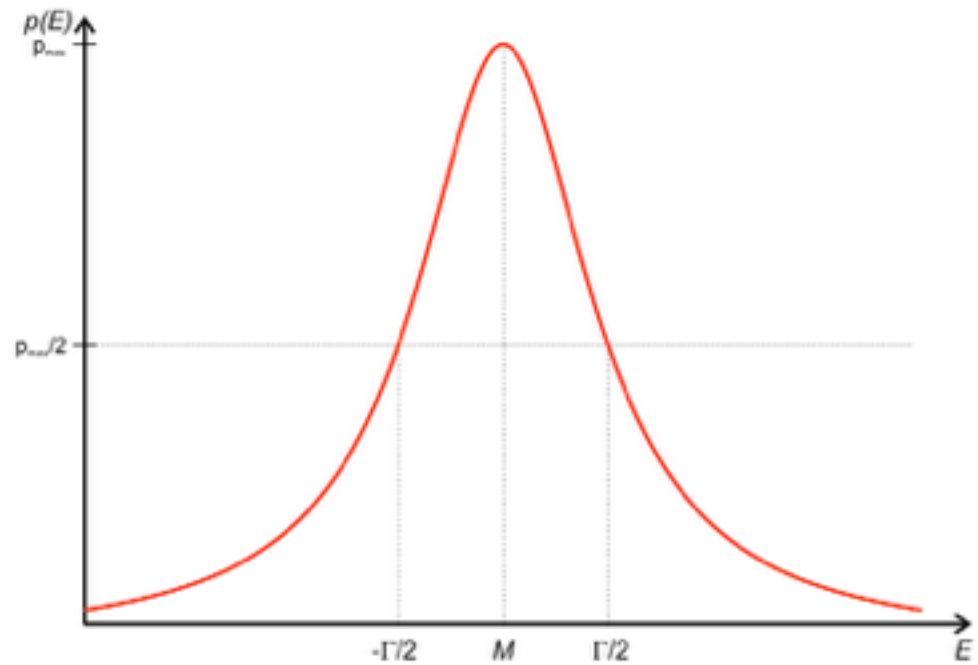
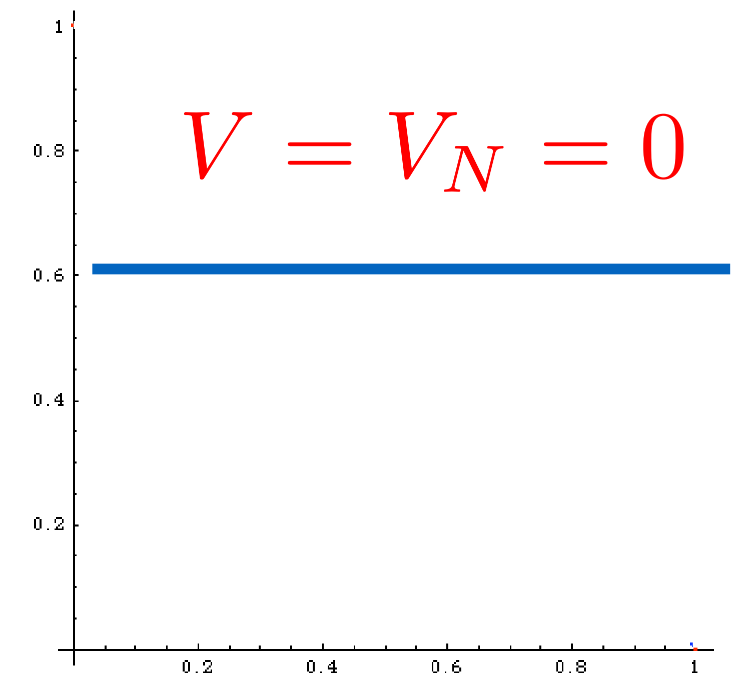


$$\int dx\, C$$



$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

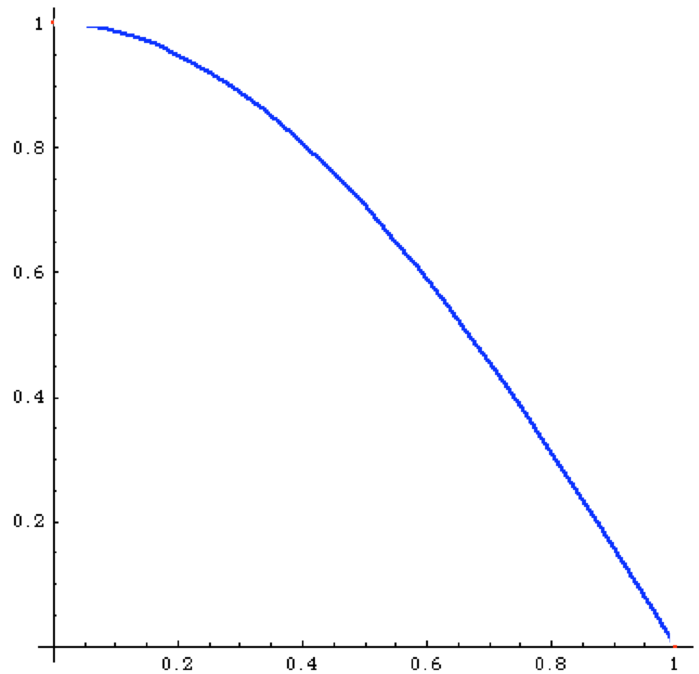
$$I = I_N \pm \sqrt{V_N/N}$$

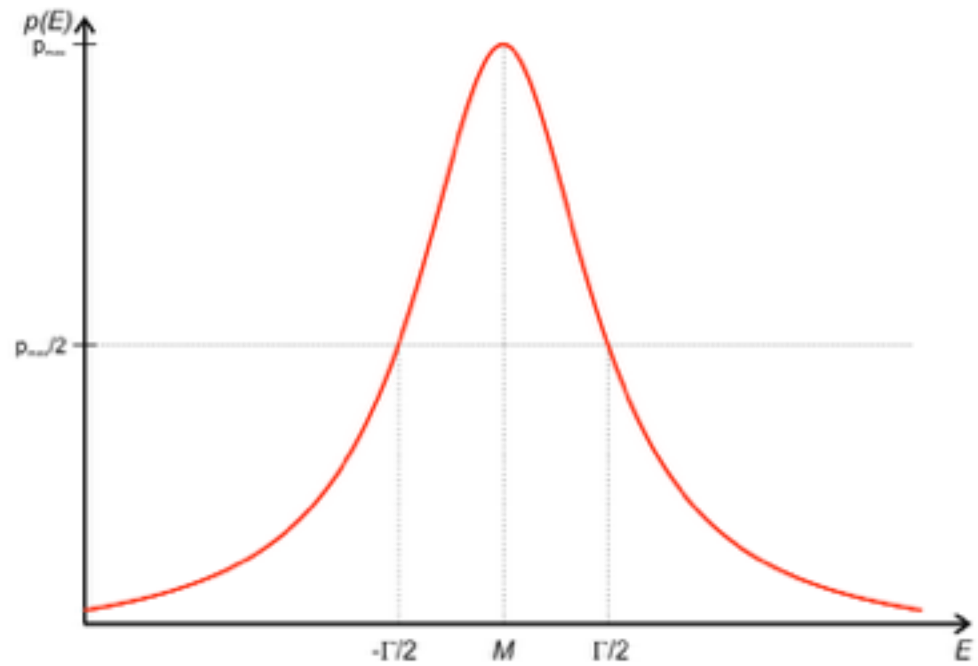
Integration

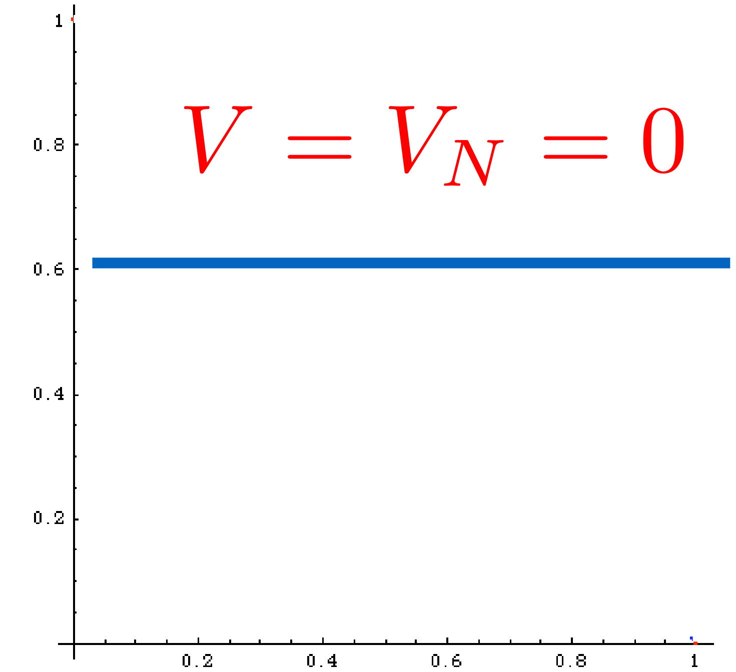
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$







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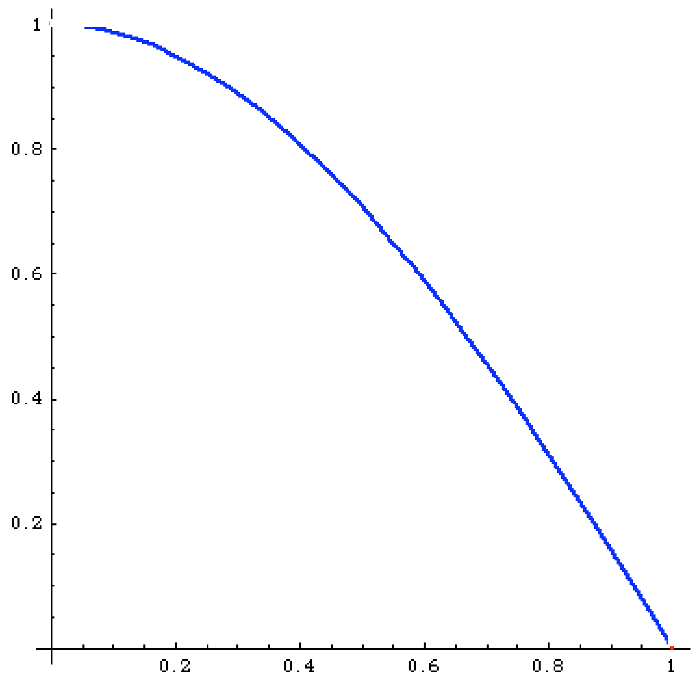
Can be minimized!

Importance Sampling



$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

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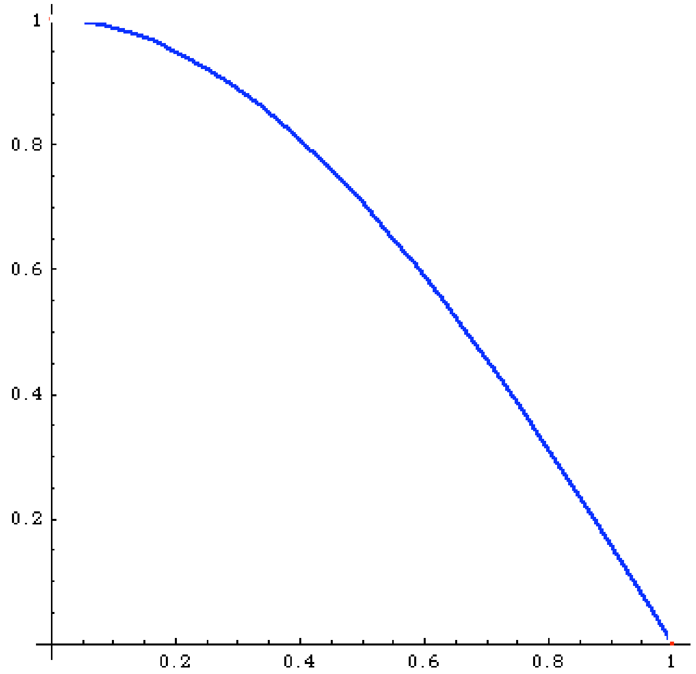

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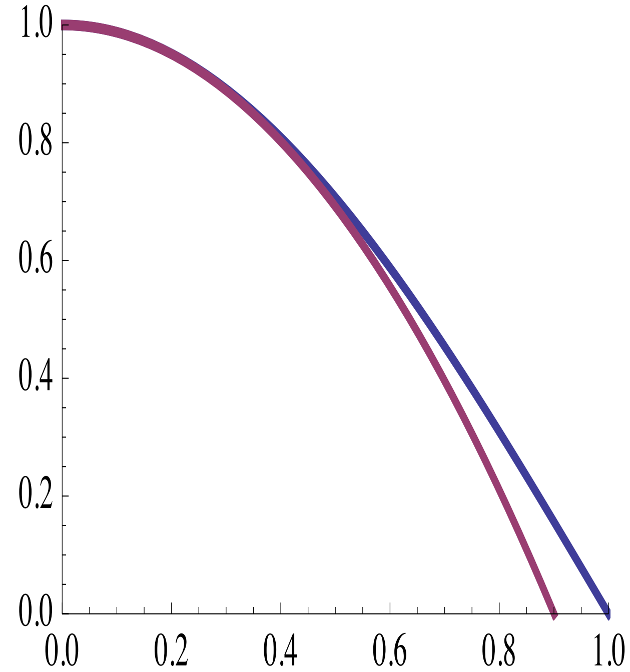

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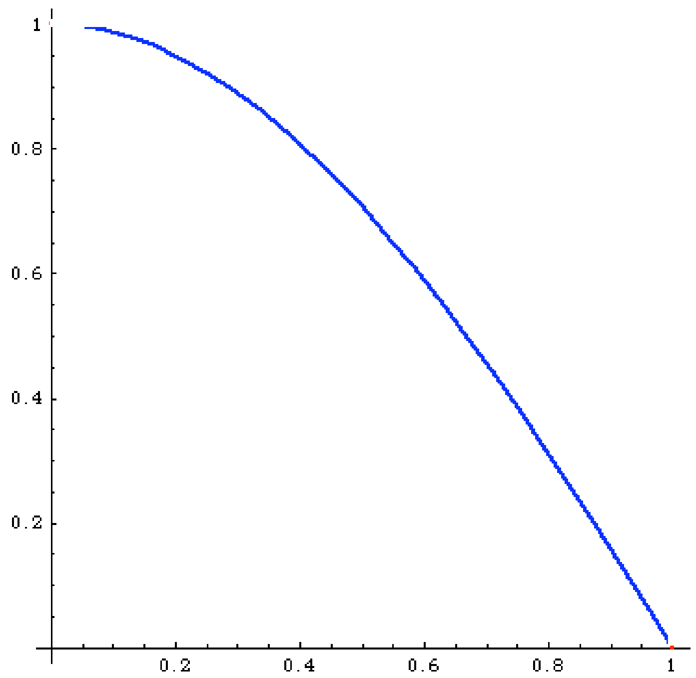

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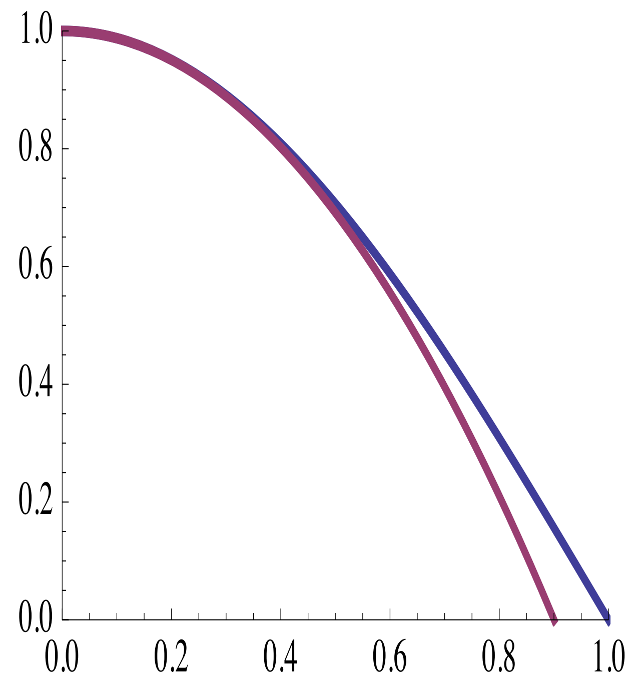

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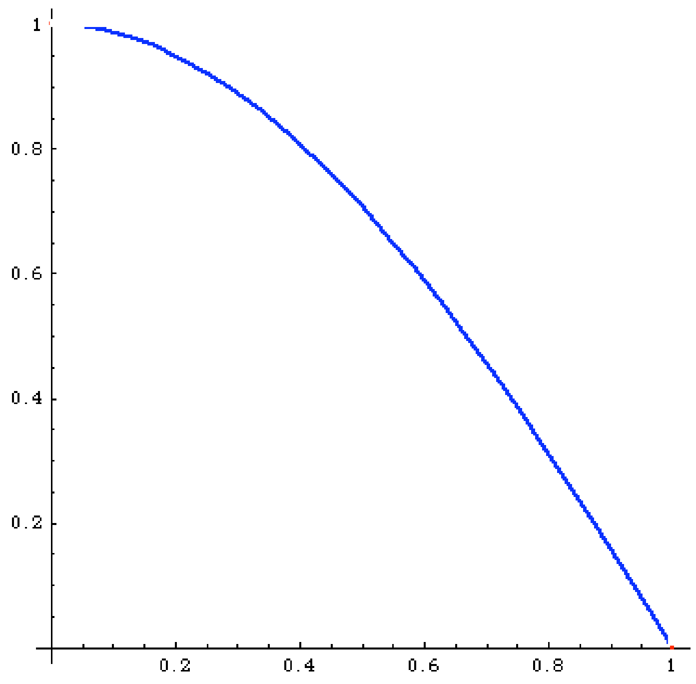

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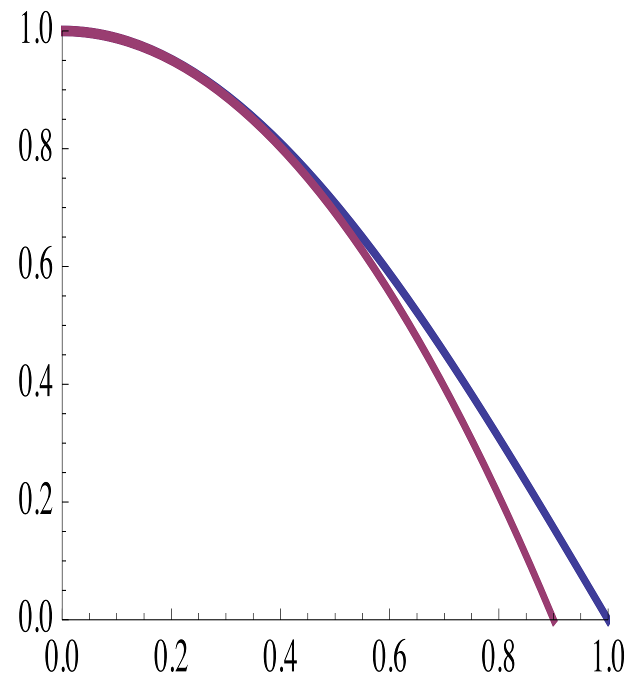

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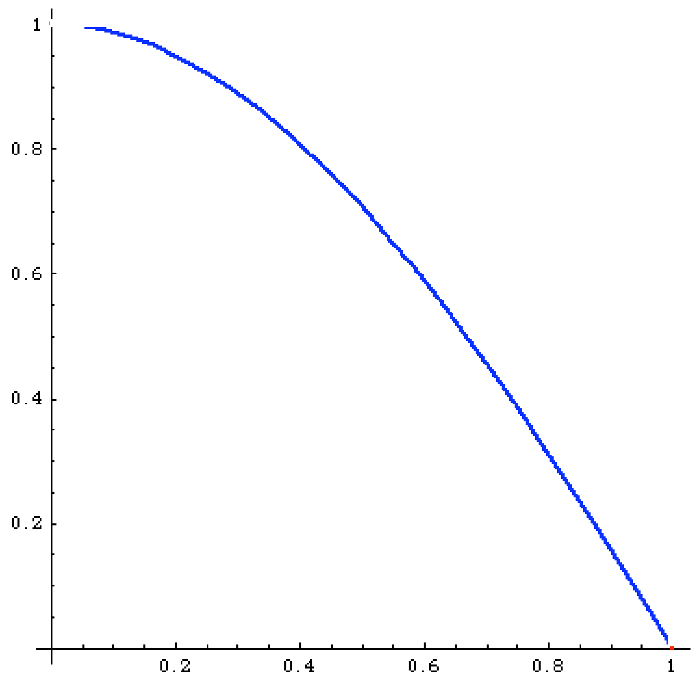
$\simeq 1$

Importance Sampling



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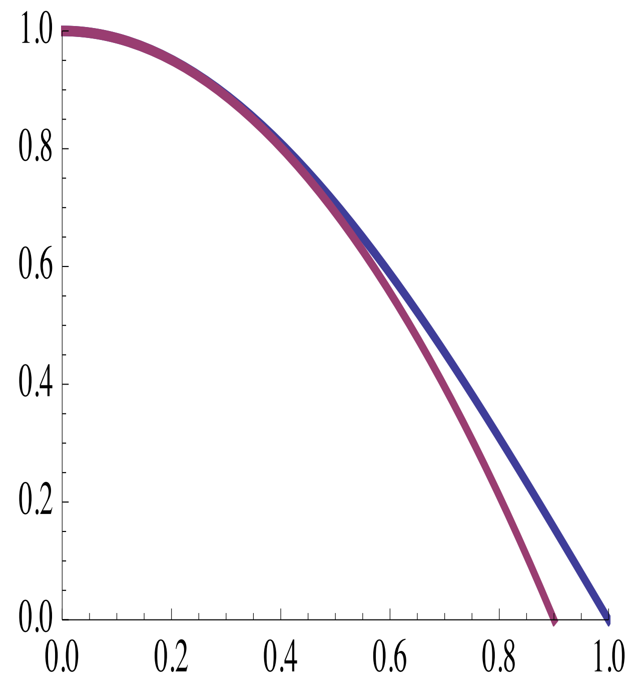

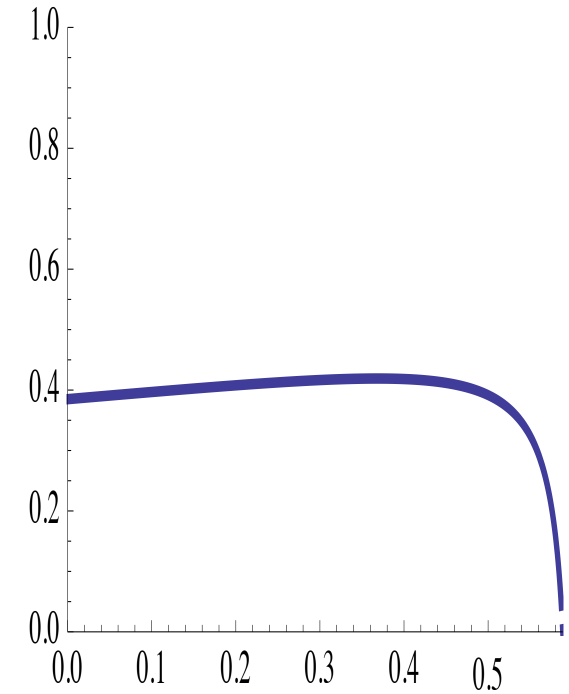

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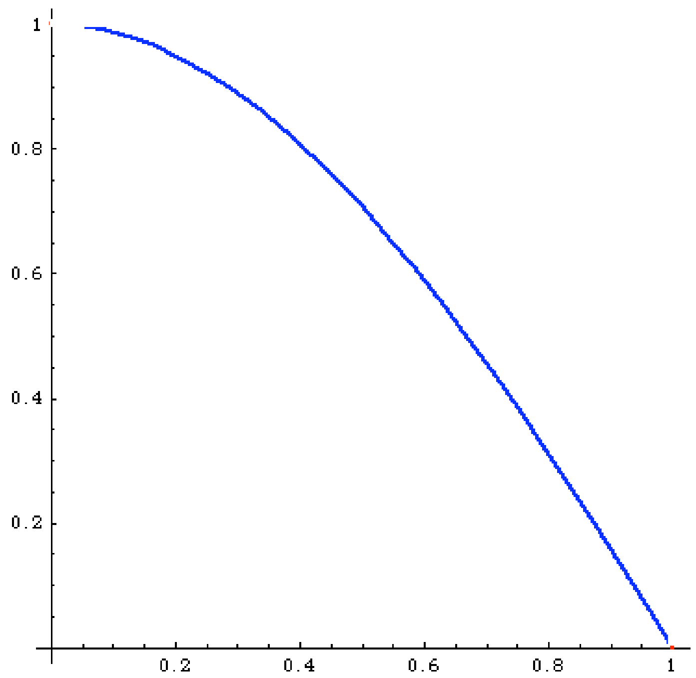

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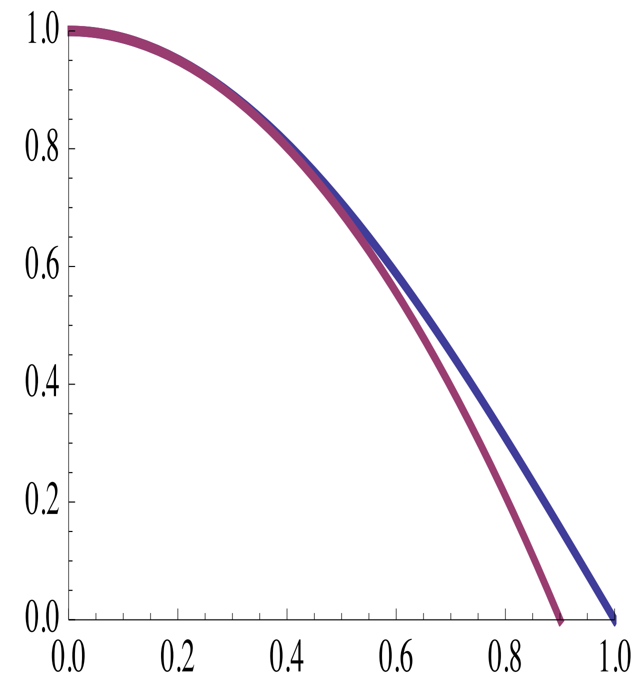

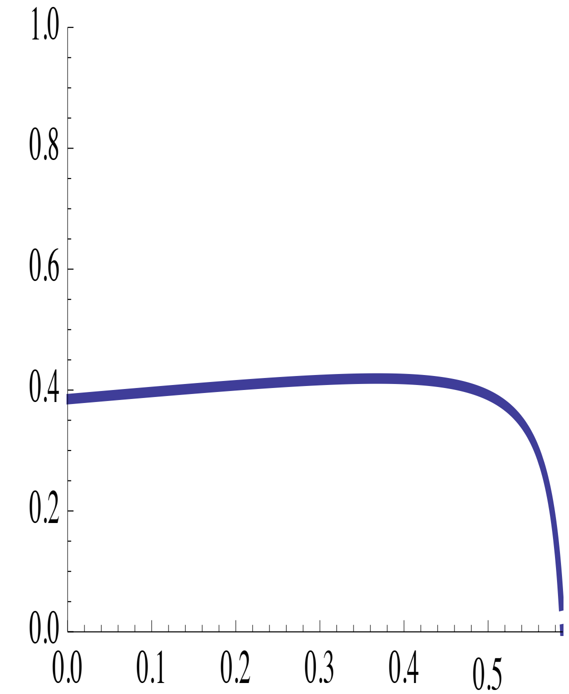

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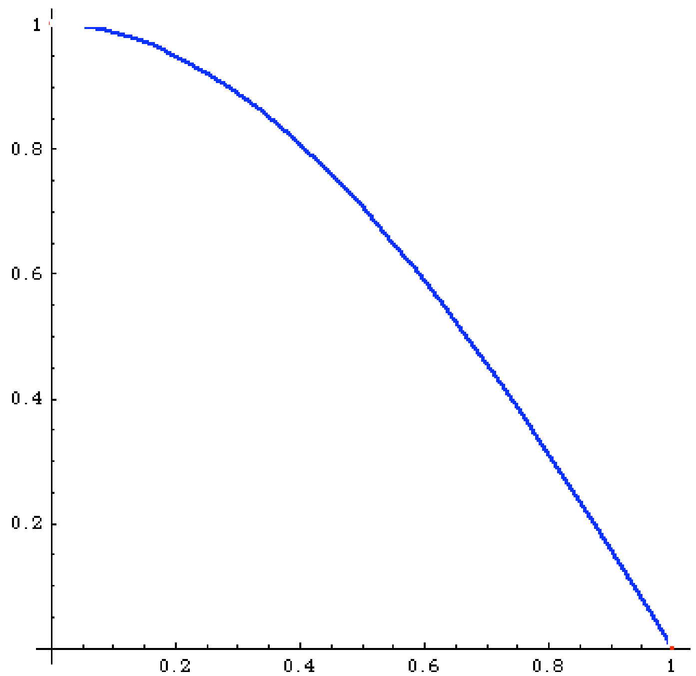

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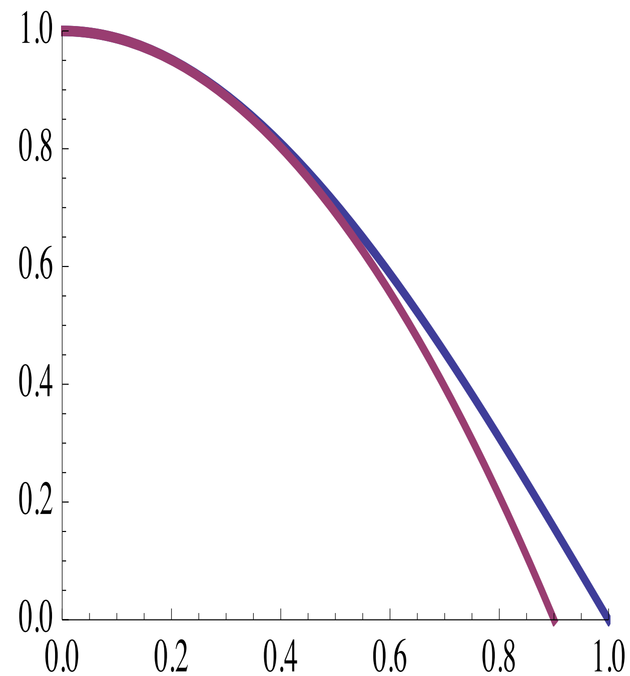

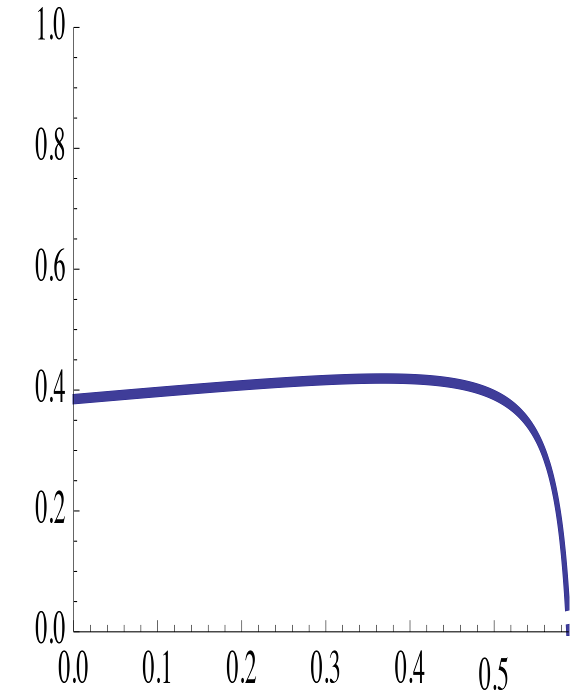

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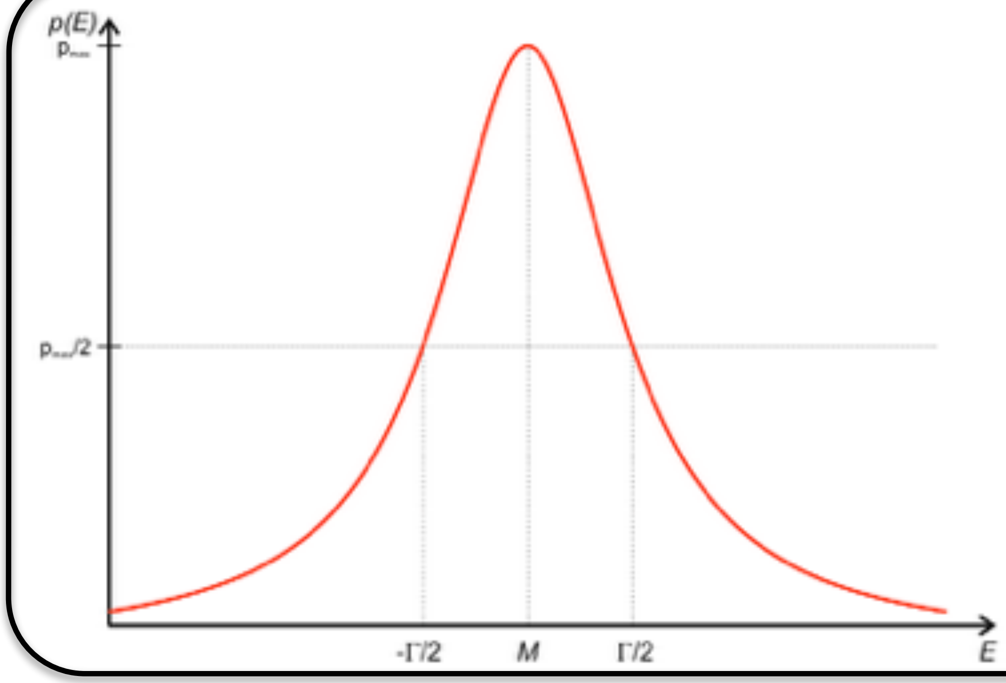
$\simeq 1$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

The Phase-Space parametrization is important to have an efficient computation!

Importance Sampling



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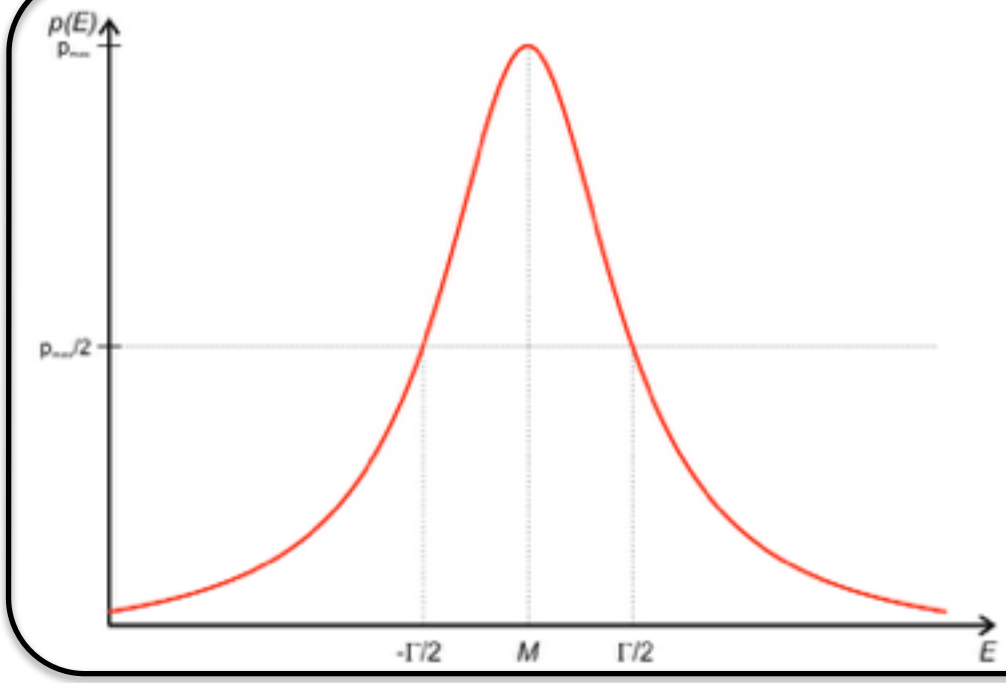

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\xi = arctan\left(\frac{q^2 - M^2}{\Gamma M}\right)$$

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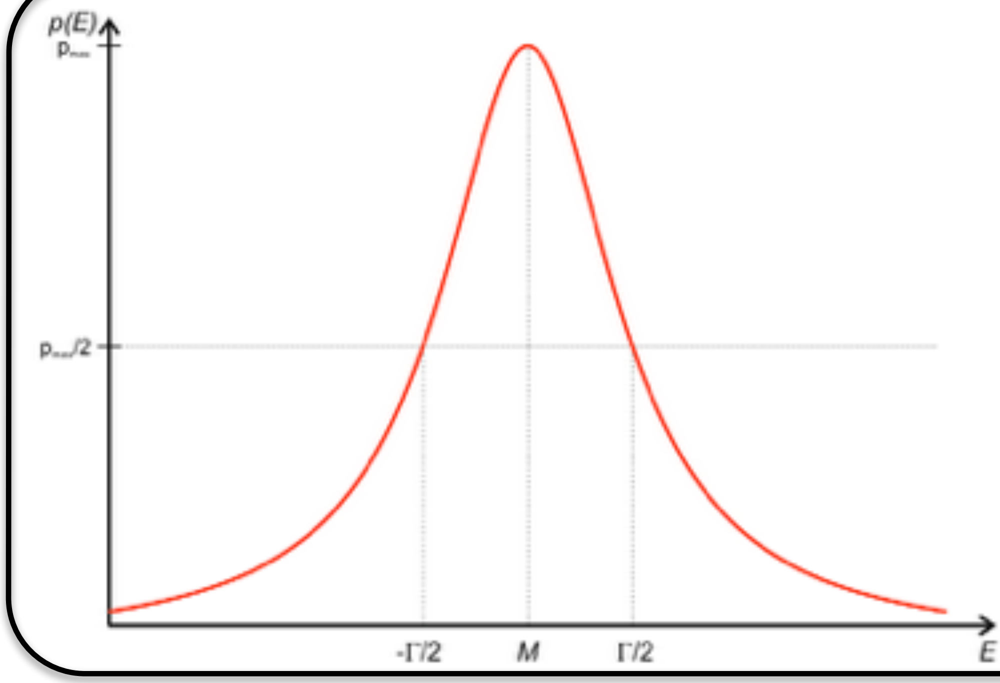

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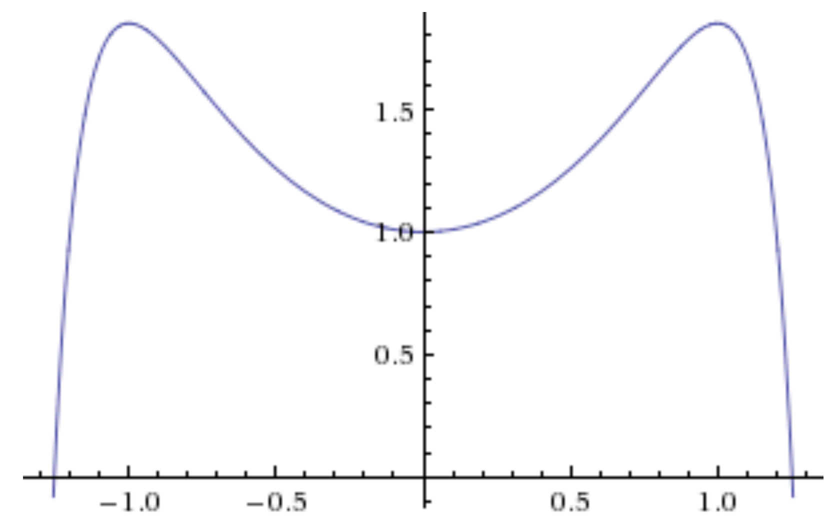

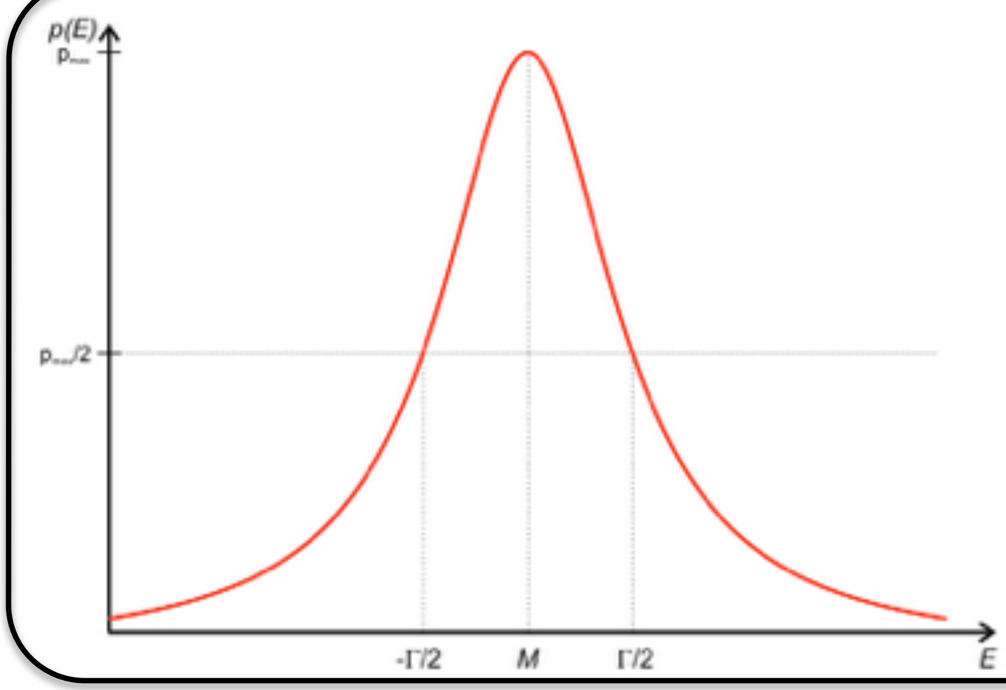

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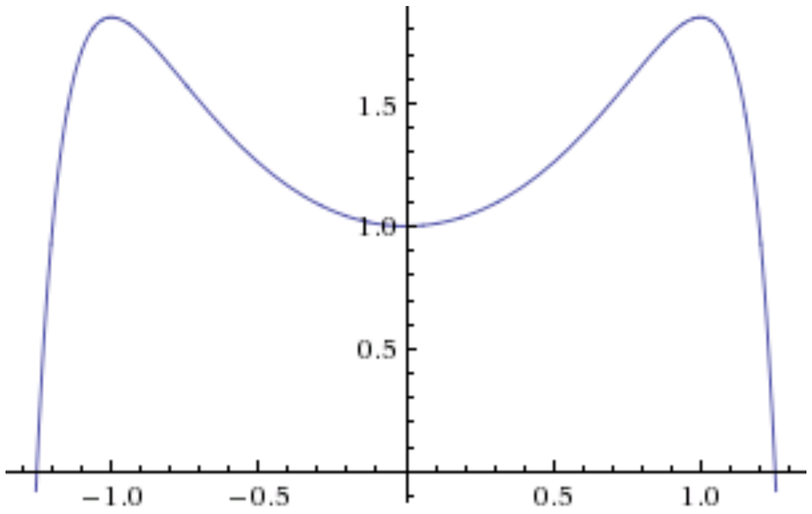
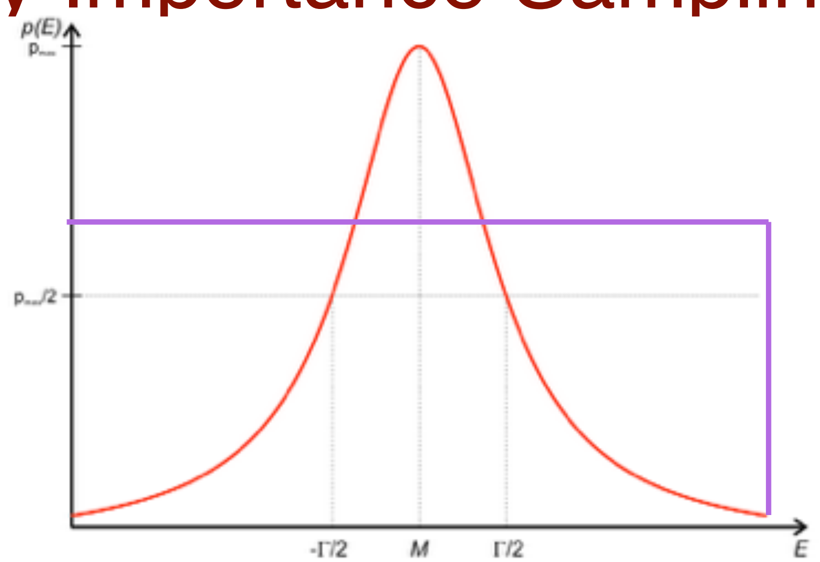
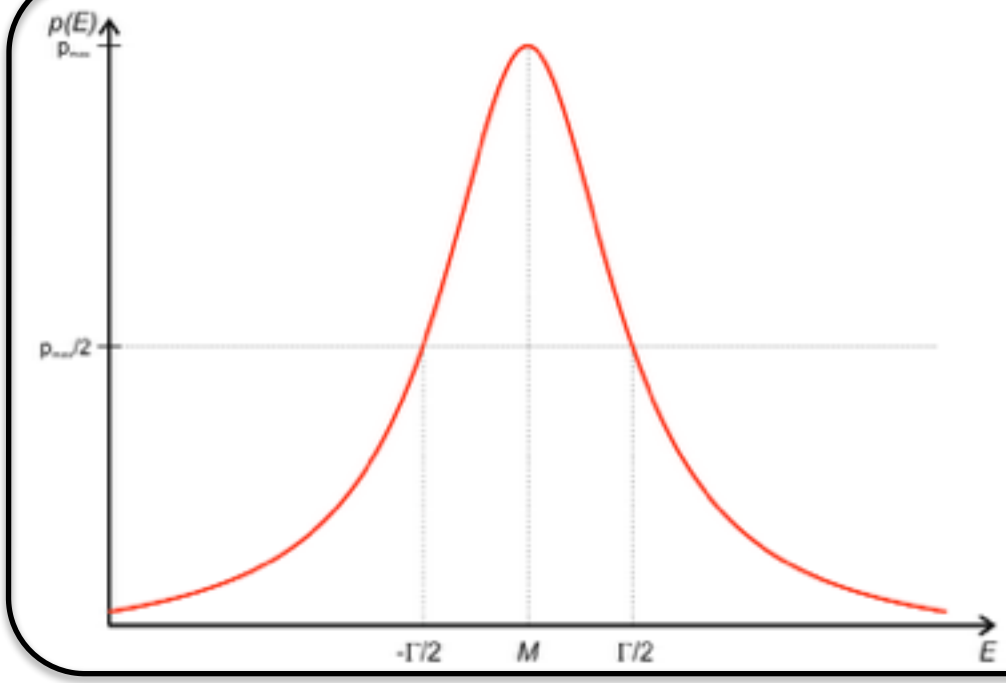


Why Importance Sampling?



Probability of using that point p(x)



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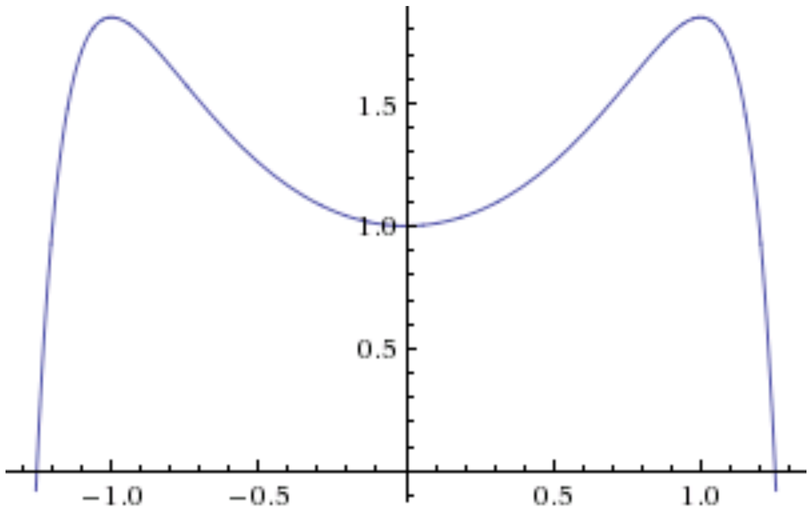

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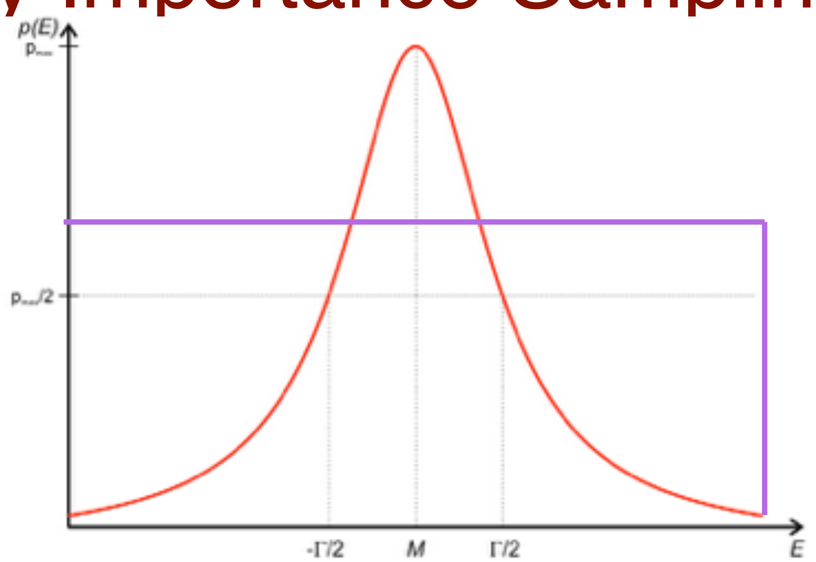

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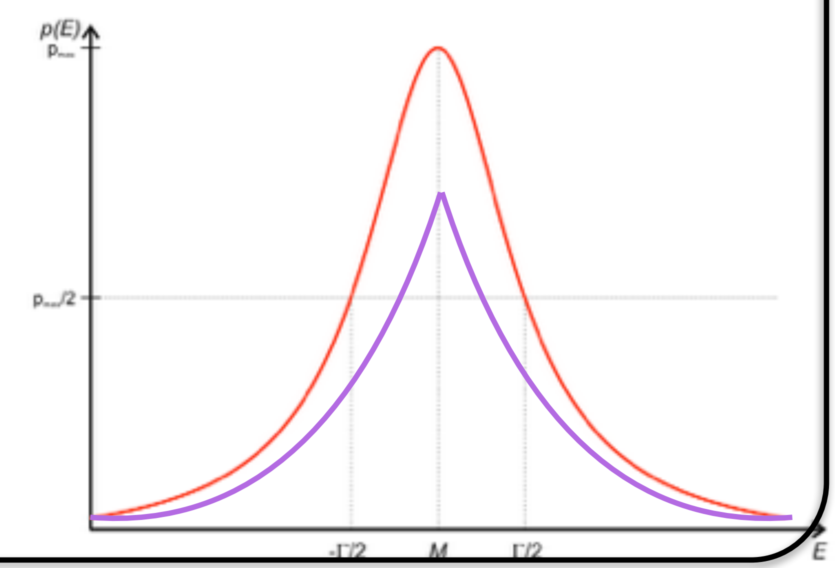

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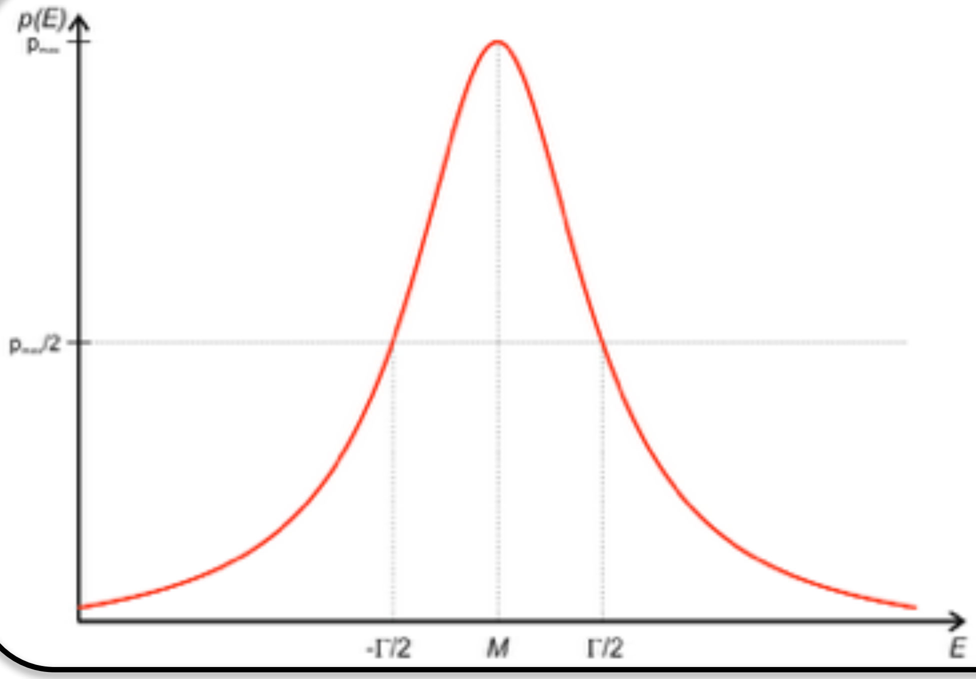

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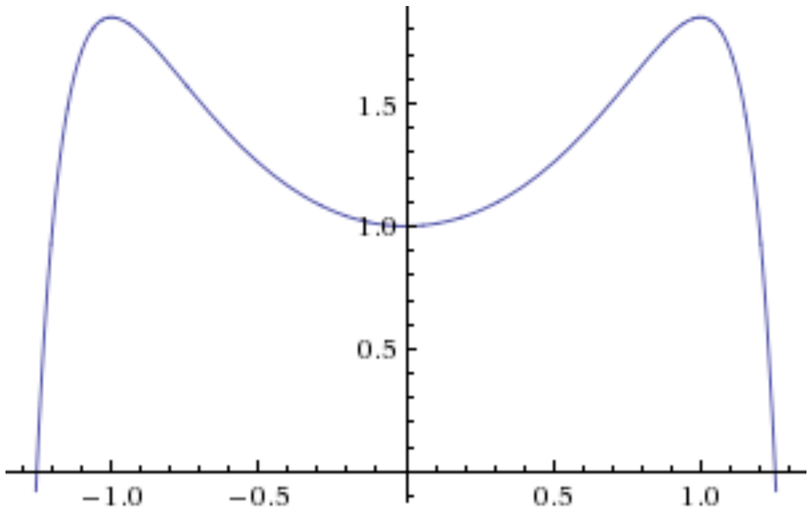

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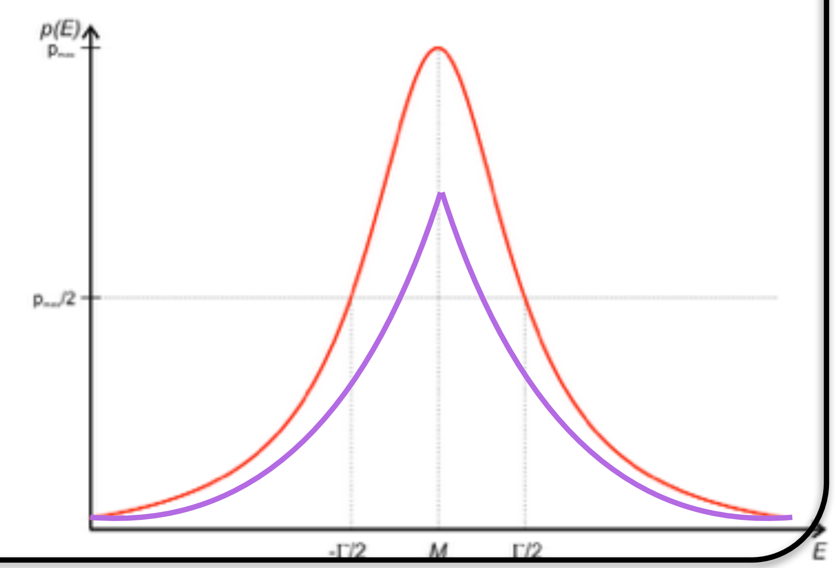

The change of variable ensure that the evaluation of the function is done where the function is the largest!

Importance Sampling

Key Point

- Generate the random point in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is flatter in this new variable.
- Needs to know an approximate function.

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

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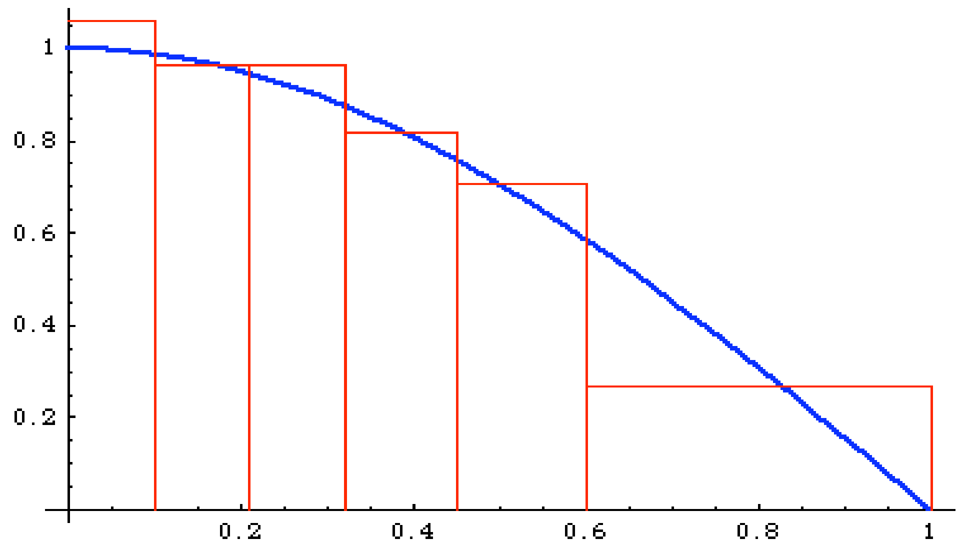

Algorithm

1. Creates bin such that each of them have the same contribution.

   ➡Many bins where the function is large

2. Use the approximate for the importance sampling method.

VEGAS

More than one Dimension

- VEGAS works only with 1(few) dimension
  - ➡memory problem

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Solution

- Use projection on the axis

$$\vec{p}(x) = p(x) \cdot p(y) \cdot p(z) \ldots$$

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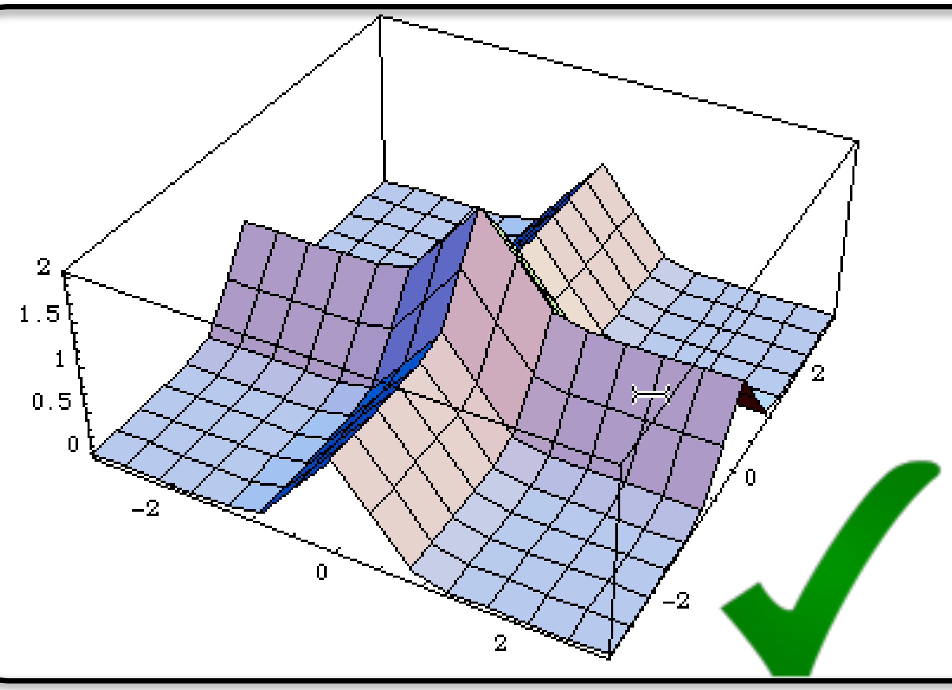

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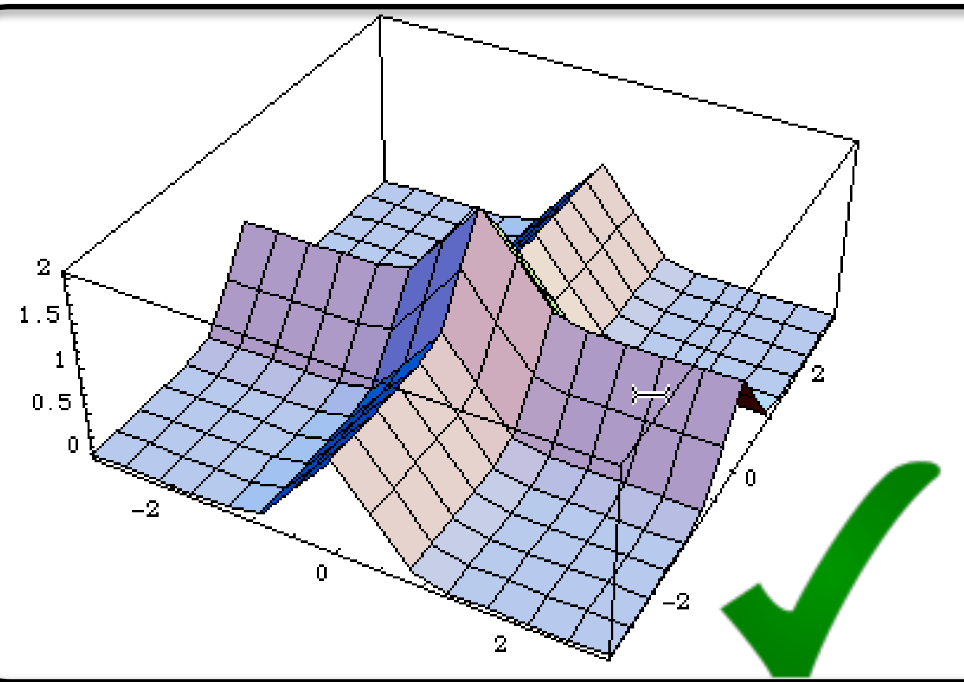
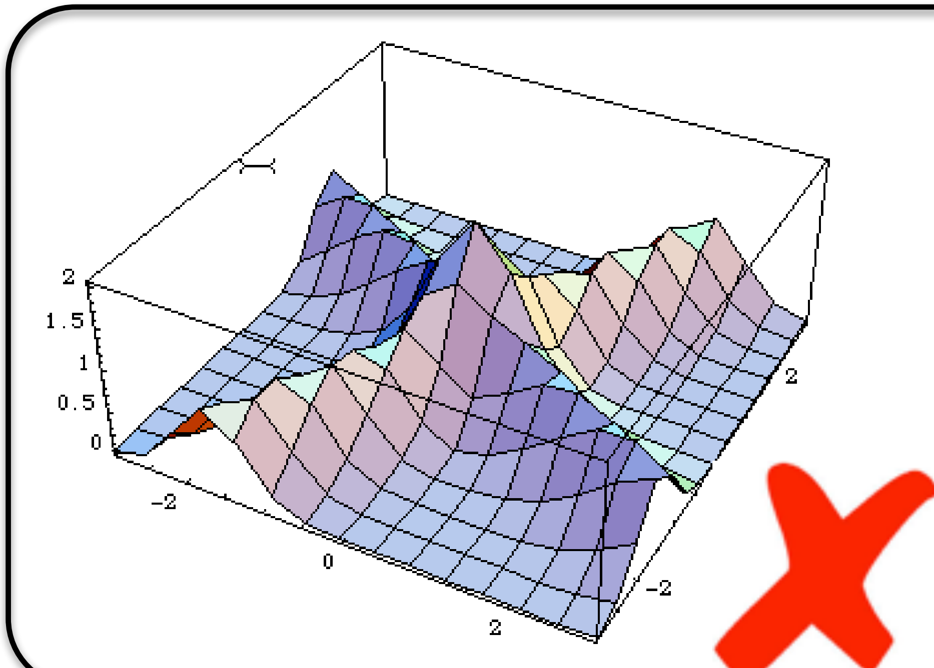


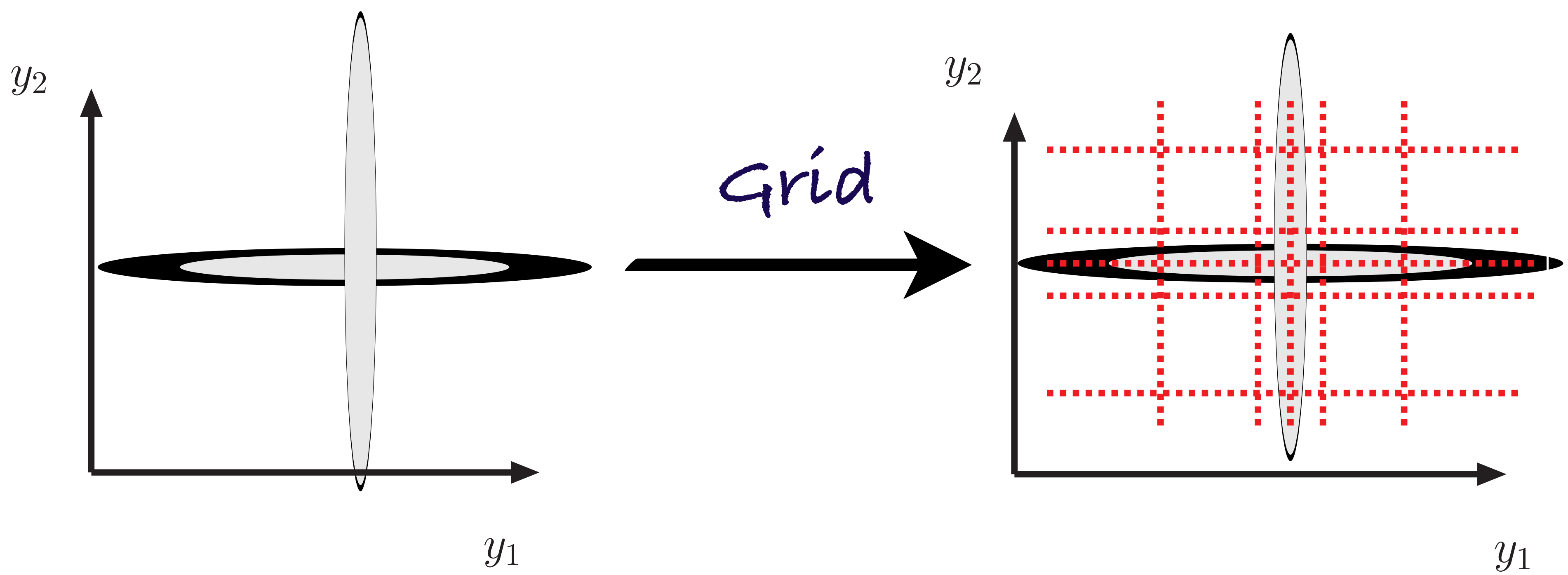


- We need to ensure the factorization !

➡Additional change of variable

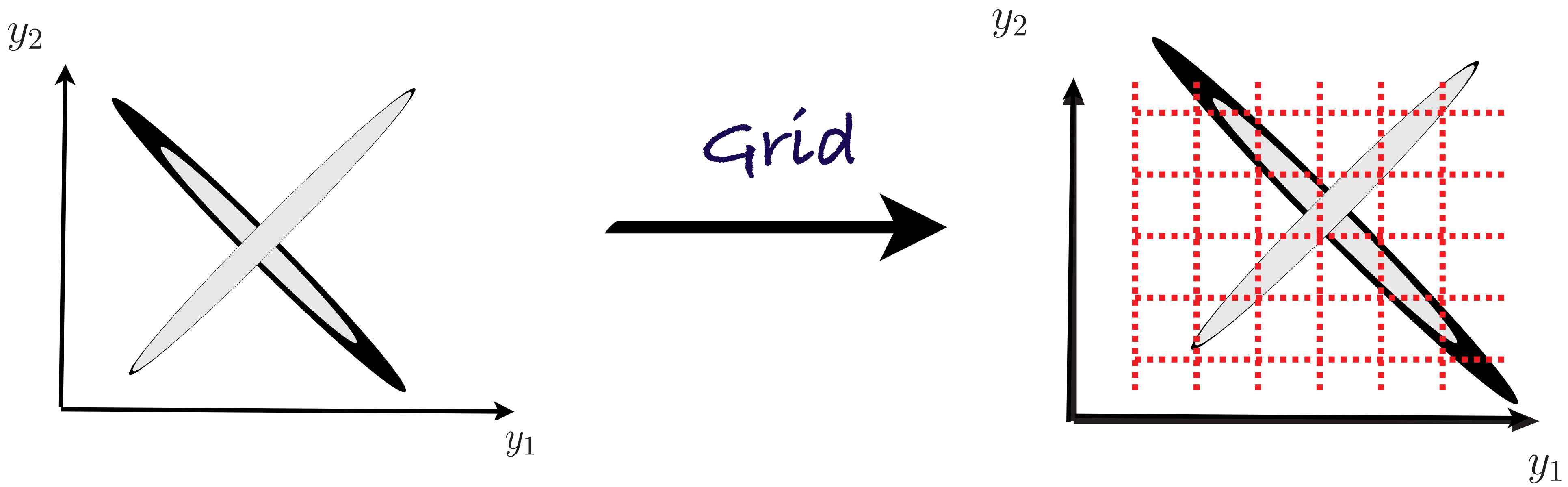
Monte-Carlo Integration

- The choice of the parameterisation has a strong impact on the efficiency



- The adaptive Monte-Carlo Technique picks point in interesting areas
  → The technique is efficient

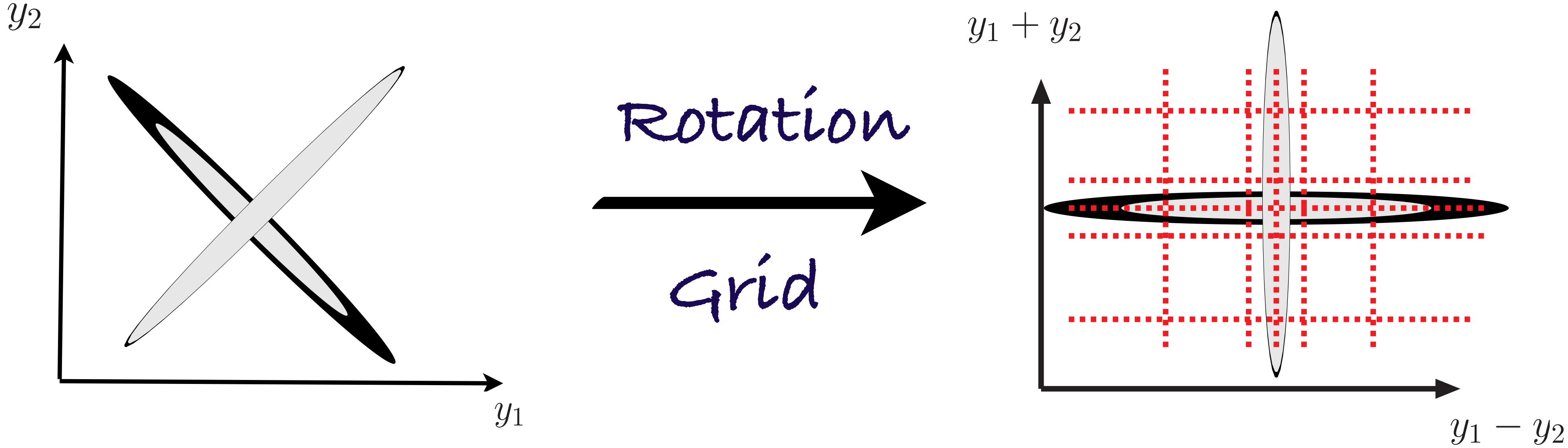
- The choice of the parametrization has a strong impact on the efficiency



- The adaptive Monte-Carlo Techniques picks points everywhere
  → The integral converges slowly

Monte-Carlo Integration

- The choice of the parametrization has a strong impact on the efficiency



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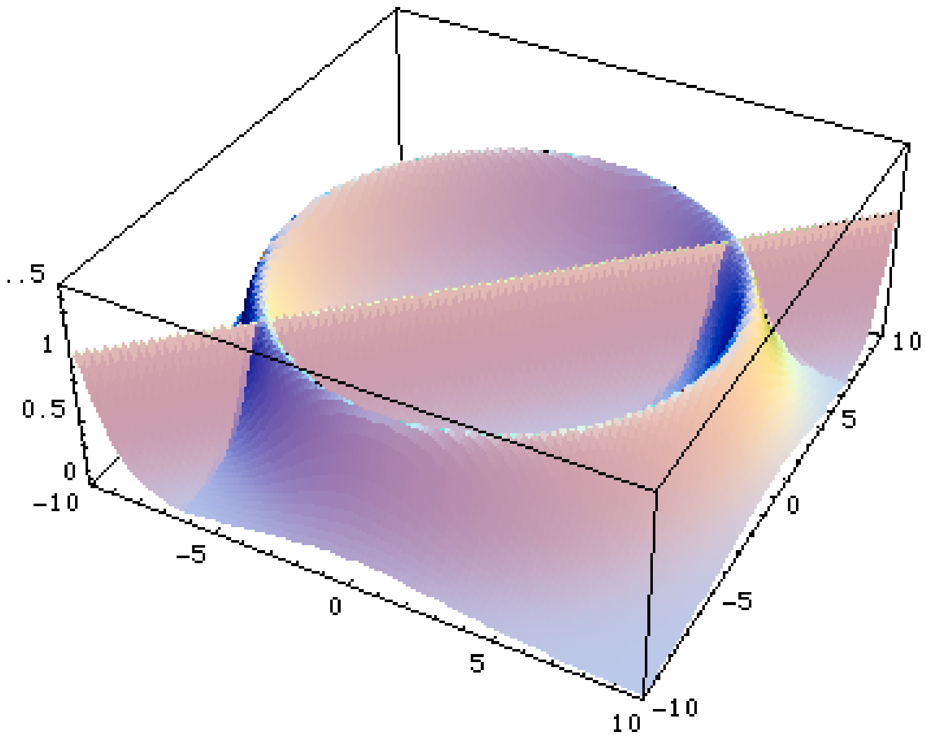
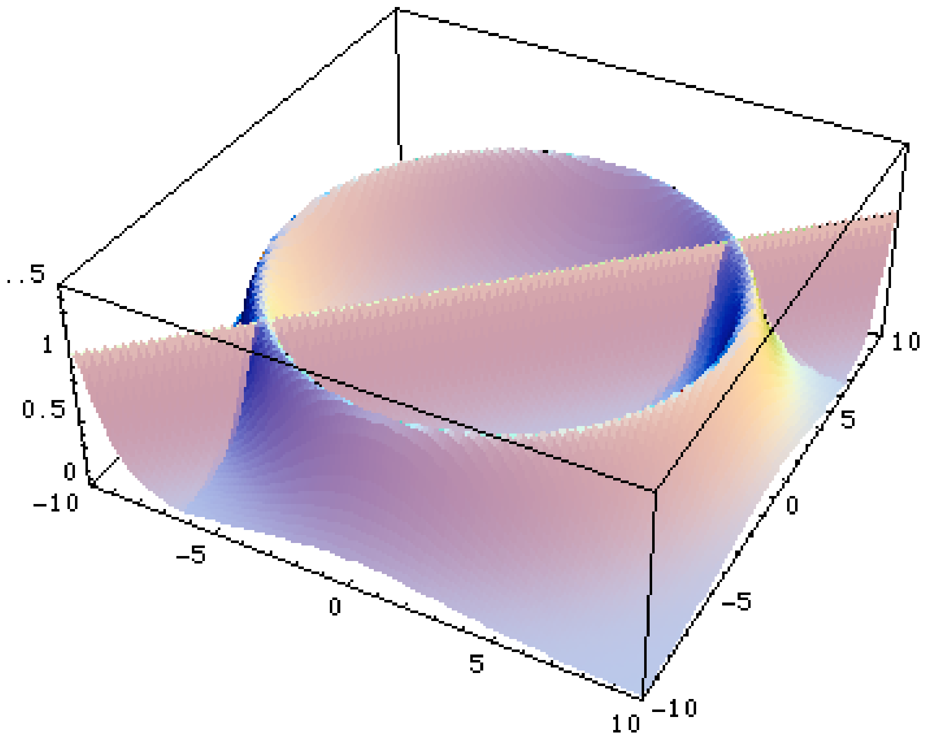
Multi-channel



What do we do if there is no transformation that aligns all integrand peaks to the chosen axes?
Vegas is bound to fail!

Multi-channel



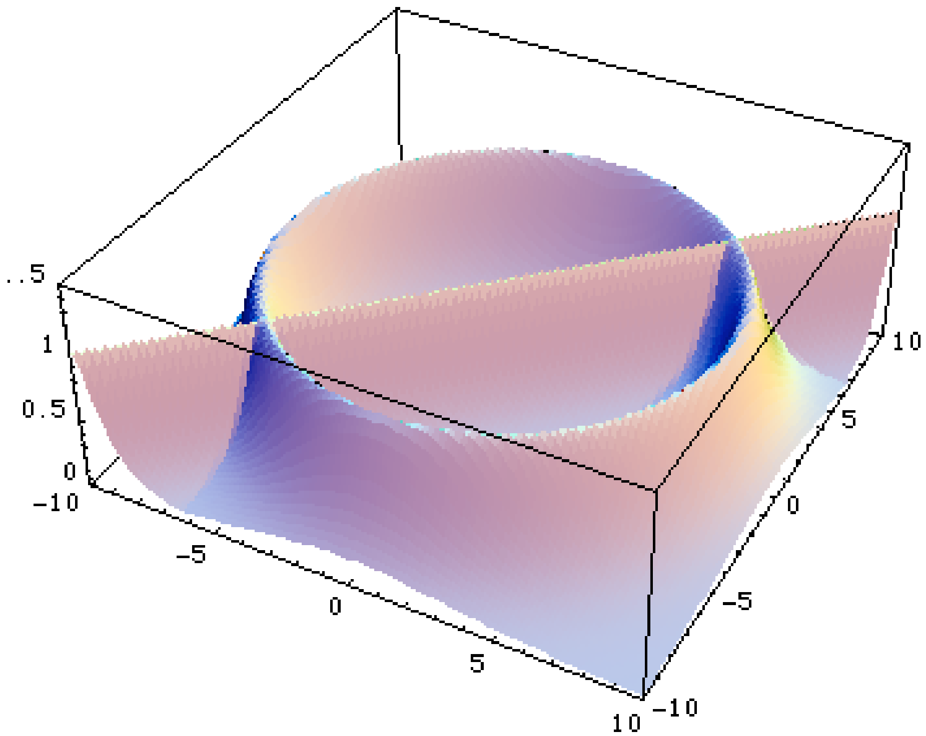
What do we do if there is no transformation that aligns all integrand peaks to the chosen axes? Vegas is bound to fail!

Solution: use different transformations = channels

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

with each $p_i(x)$ taking care of one "peak" at the time

Multi-channel

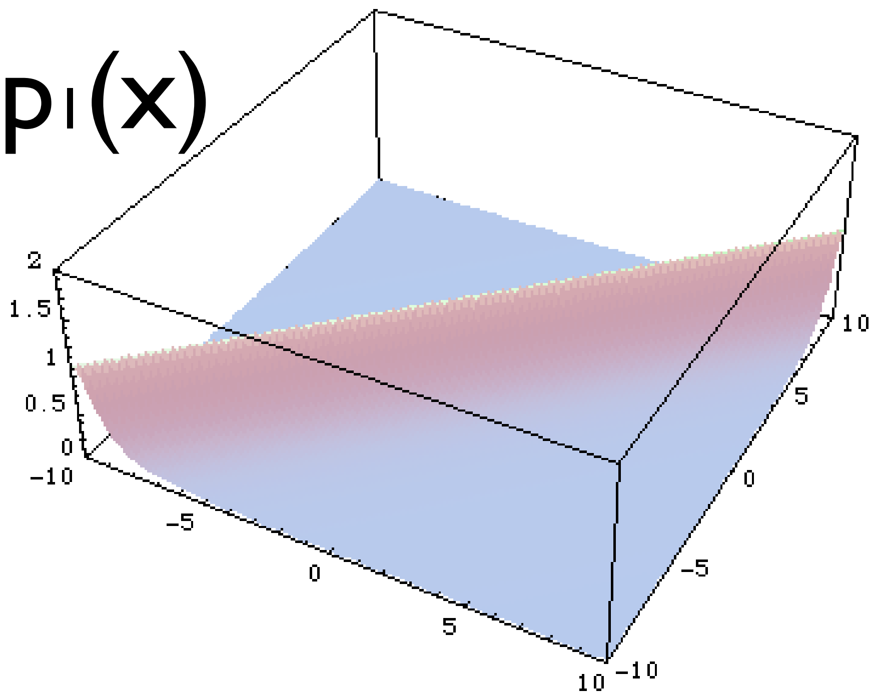


$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x)$$

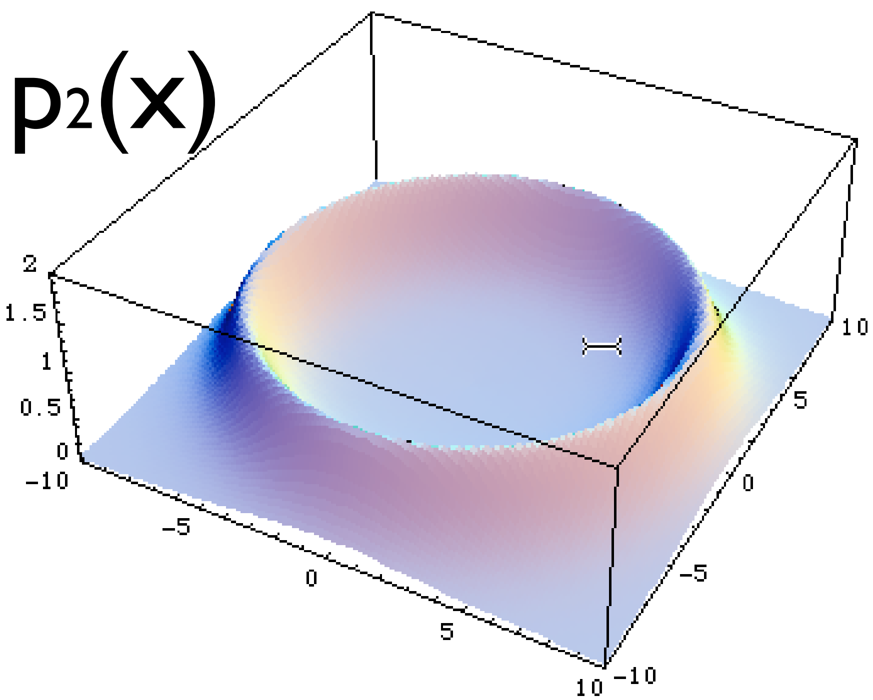
with

$$\sum_{i=1}^{n} \alpha_i = 1$$

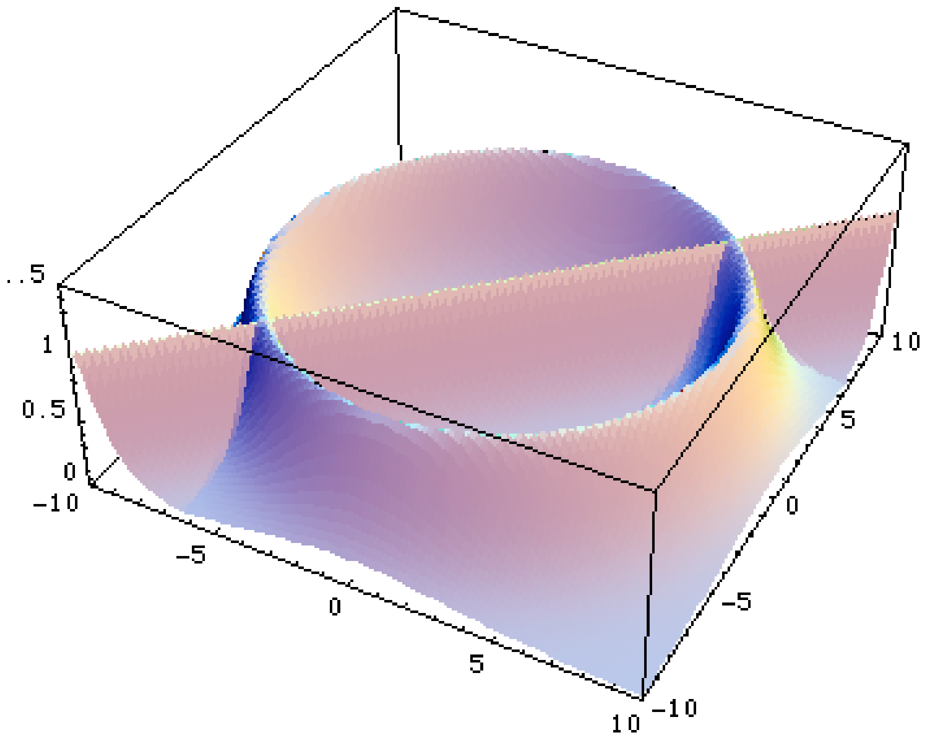
$p_1(x)$



$p_2(x)$



Multi-channel



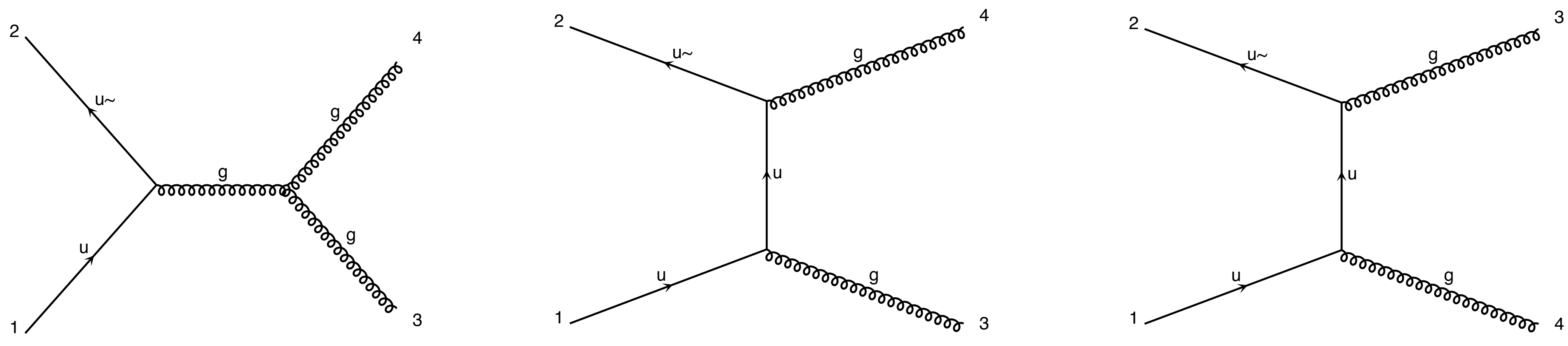
$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x)$$

with $$\sum_{i=1}^{n} \alpha_i = 1$$

Then,

$$I = \int f(x)dx = \sum_{i=1}^{n} \alpha_i \int \frac{f(x)}{p(x)} p_i(x) dx$$

$$\approx 1$$



$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1+p_2)^2} \qquad \propto \frac{1}{\hat{t}} = \frac{1}{(p_1-p_3)^2} \qquad \propto \frac{1}{\hat{u}} = \frac{1}{(p_1-p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

Does a basis exist?

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

Does a basis exist?

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$$\approx 1$$

Multi-channel based on single diagrams*

*Method used in MadGraph

Does a basis exist?

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

$$\frac{|M_{tot}|^2}{\sum_j |M_j|^2} \approx 1$$

Key Idea

- Any single diagram is "easy" to integrate (pole structures/ suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

*Method used in MadGraph

Does a basis exist?

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≈ 1

Key Idea

- Any single diagram is "easy" to integrate (pole structures/ suitable integration variables known from the propagators)
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N Integral

- Errors add in quadrature so no extra cost
- "Weight" functions already calculated during $|M|^2$ calculation
- Parallel in nature

To Remember

- Phase-Space integration are difficult
- We need to know the function
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P0_gg_hqq

s= 0.44288 ± 0.00268 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G7	0.1263	0.00102	8.002	256.0	2.03e+03
G6	0.1225	0.00132	16.002	760.0	6.21e+03
G2	0.08464	0.0011	32.002	1931.0	2.28e+04
G4	0.08122	0.00169	32.002	101.0	1.25e+03
G1	0.02821	0.000563	8.002	144.0	5.1e+03

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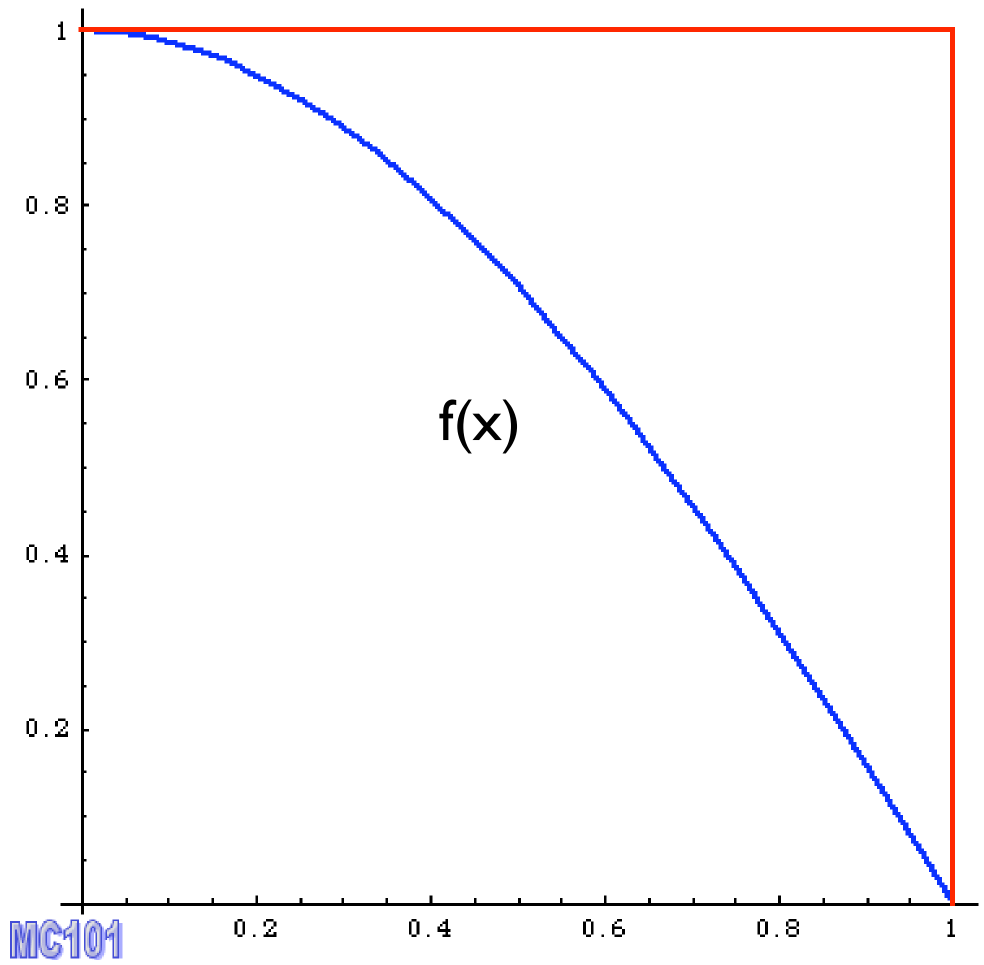
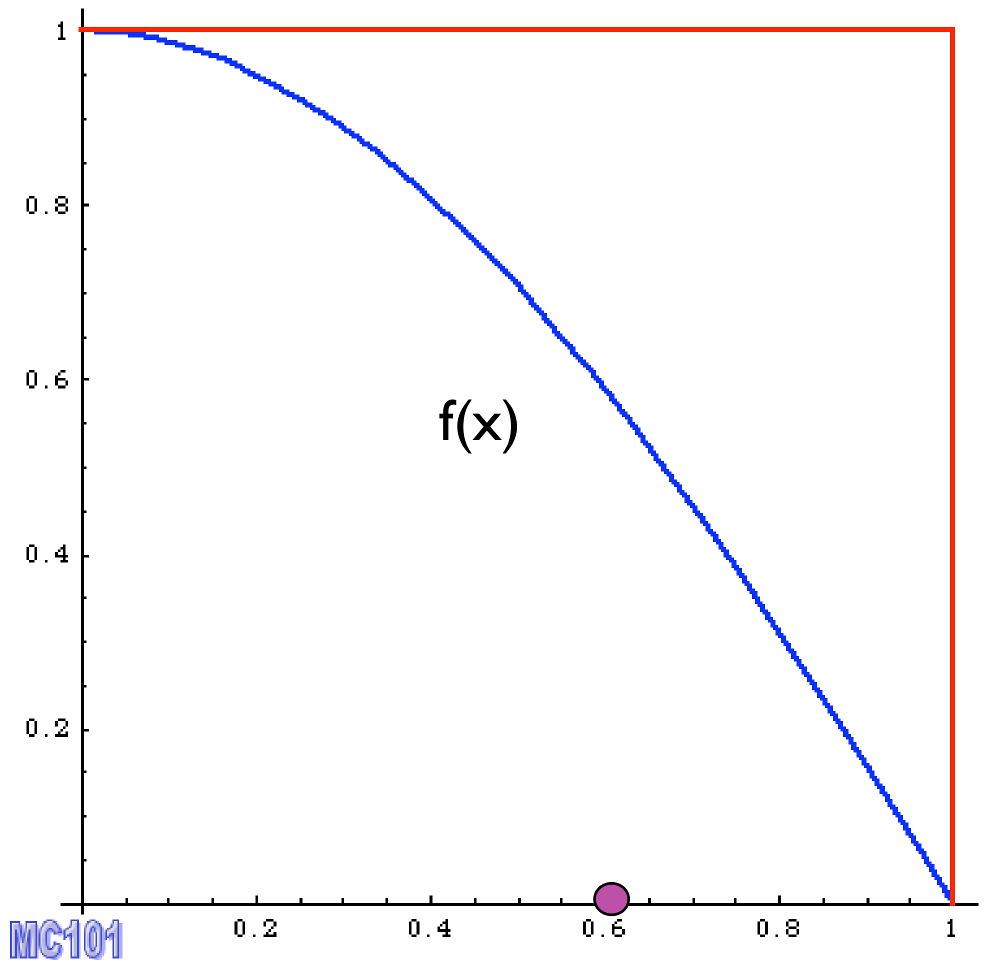
Event generation

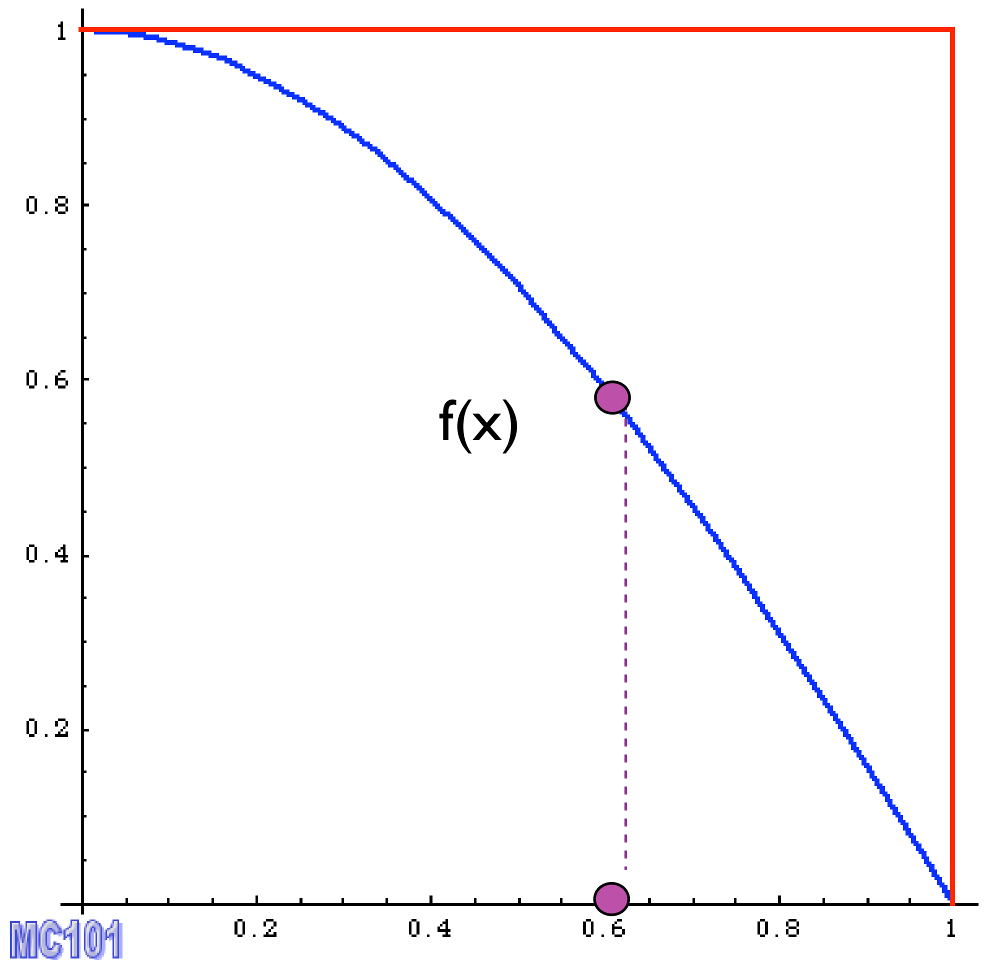


Event generation



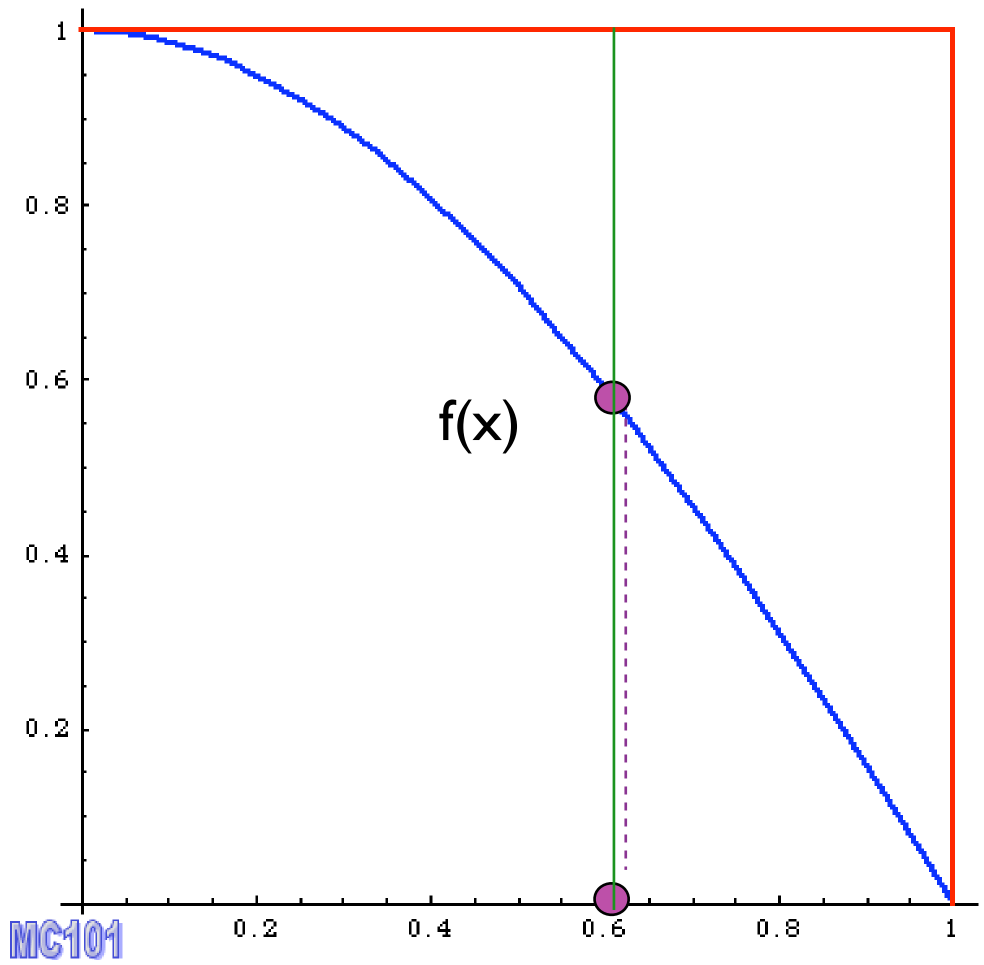
1. pick x

Event generation



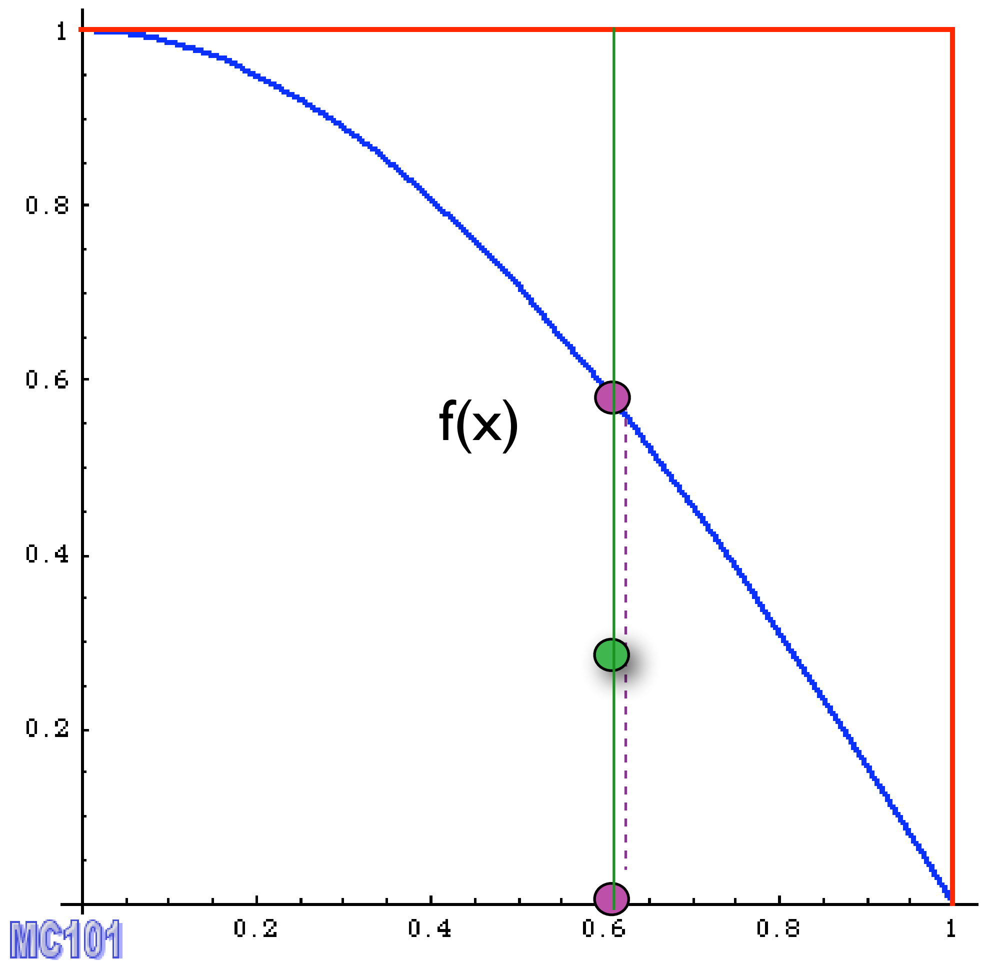
1. pick x
2. calculate f(x)

Event generation



1. pick x
2. calculate f(x)
3. pick 0<y<fmax

Event generation



1. pick x
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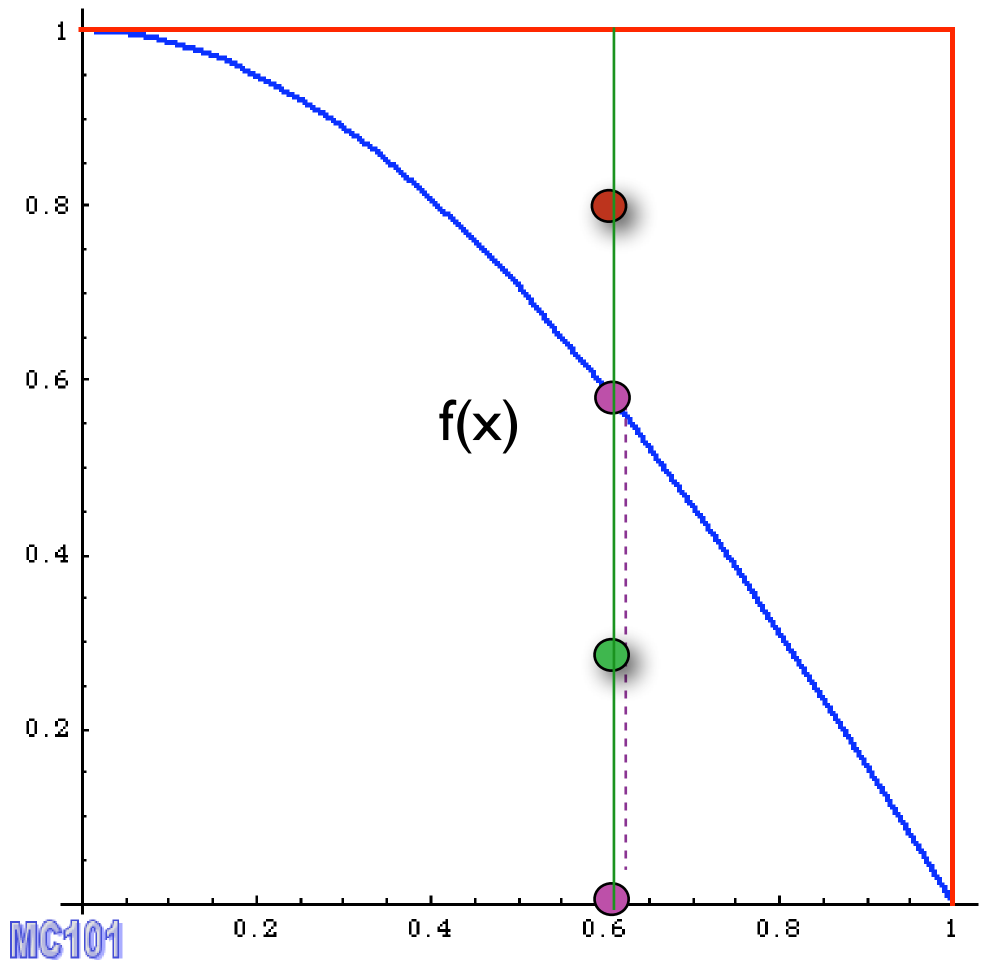
3. pick 0<y<fmax
4. Compare:
   if f(x)>y accept event,

Event generation



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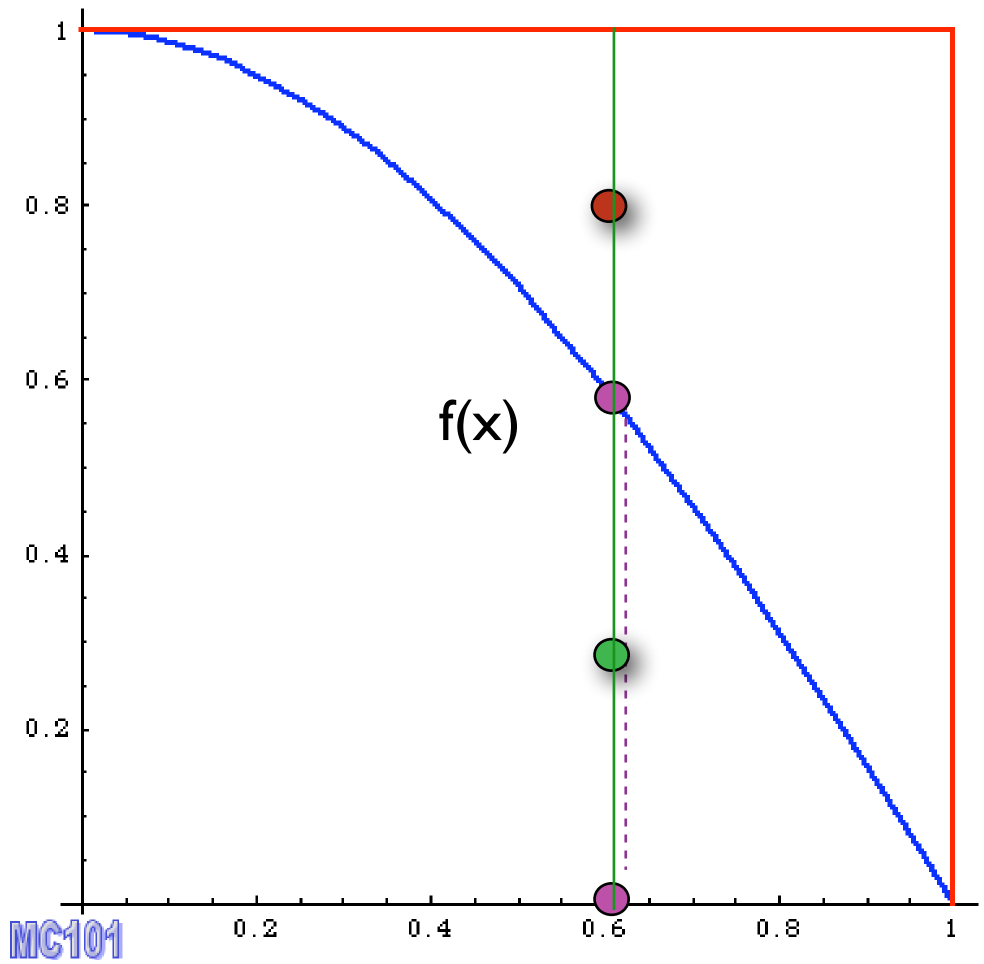

1. pick x
2. calculate f(x)
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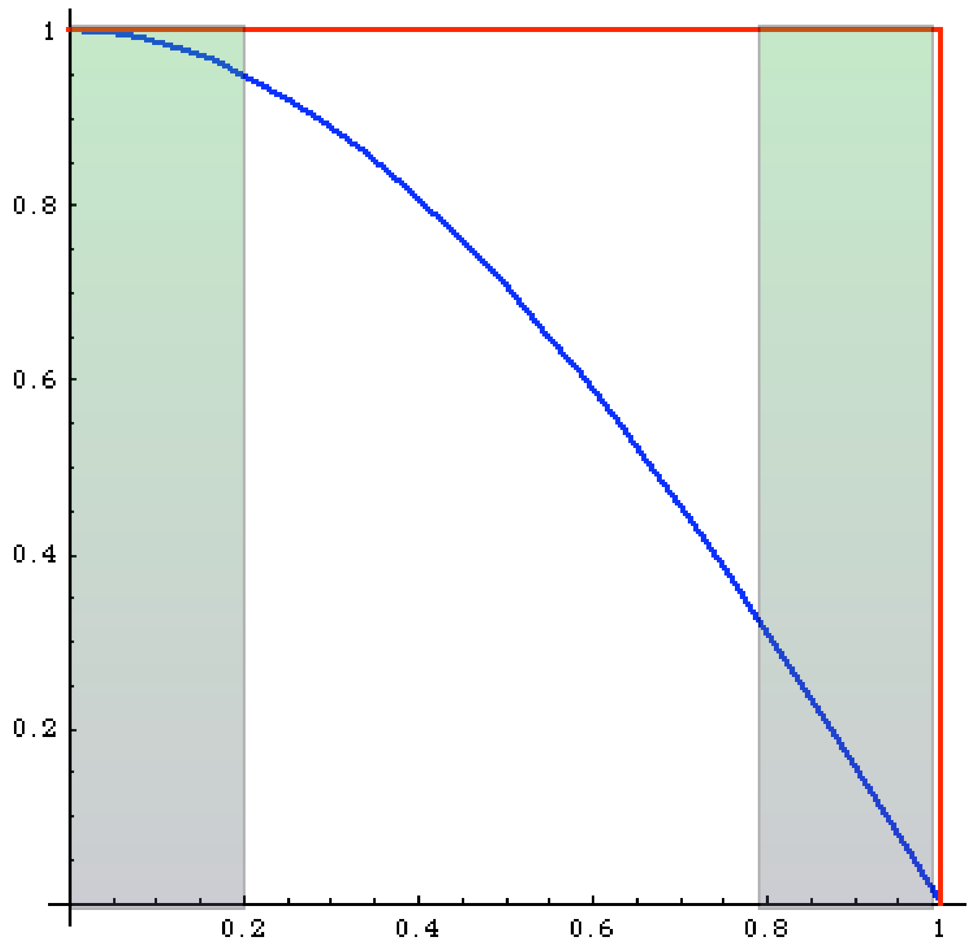
4. Compare:
   if f(x)>y accept event,

   else reject it.

$$I = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

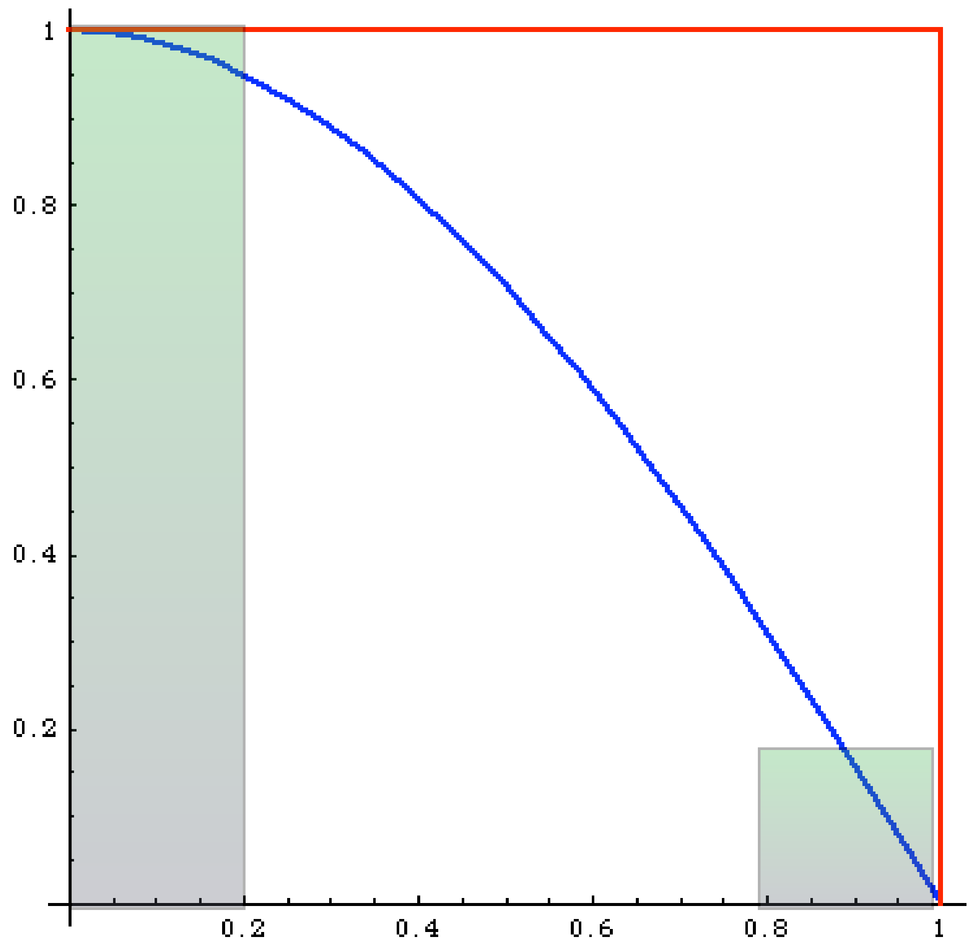


What's the difference between weighted and unweighted?

Weighted:

Same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



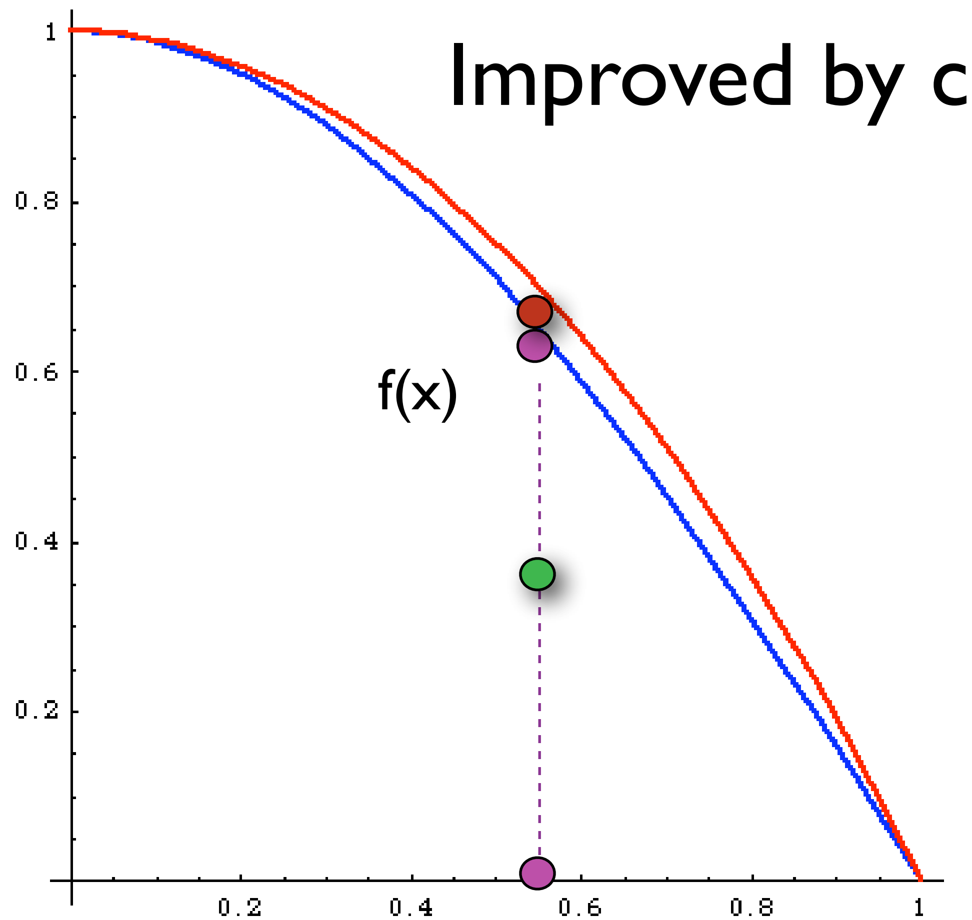
What's the difference between weighted and unweighted?

Unweighted:

events is proportional to the probability of areas of phase space:
events have all the same weight ("unweighted")

Events distributed as in nature

Event generation



Improved by combining with importance sampling:

1. pick x distributed as p(x)
2. calculate f(x) and p(x)
3. pick 0<y<1
4. Compare:
   if f(x)>y p(x) accept event,

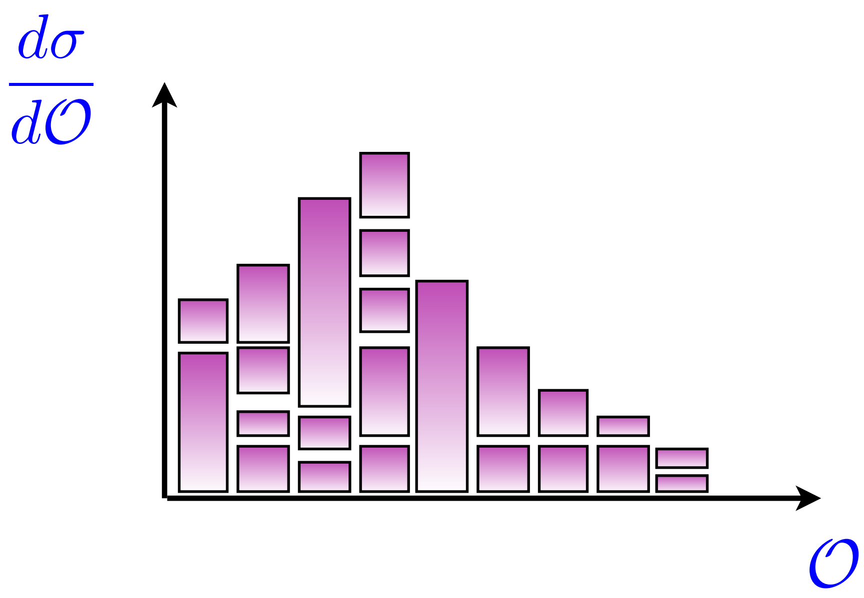
else reject it.

much better efficiency!!!

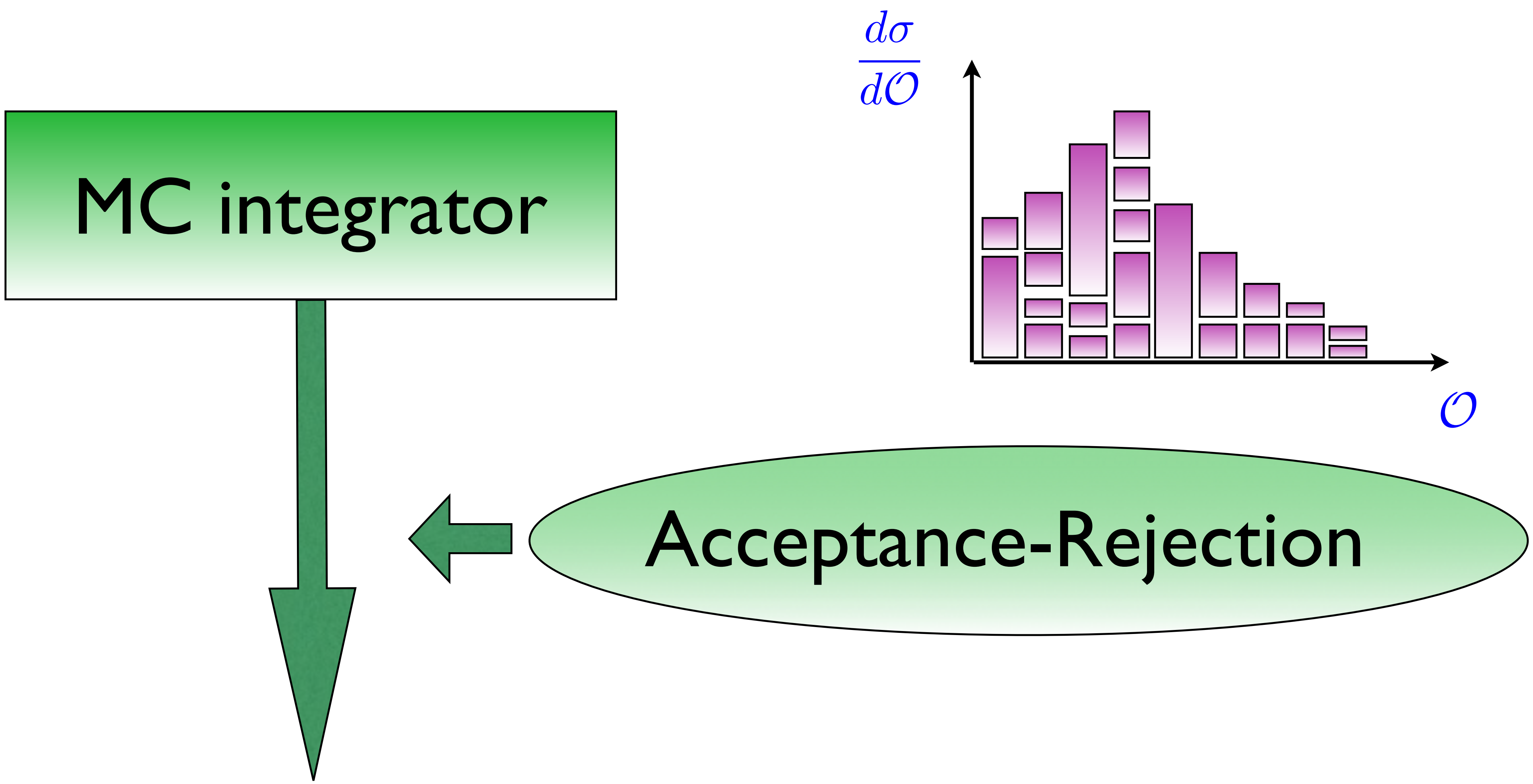
Event generation

MC integrator

MC integrator



Event generation



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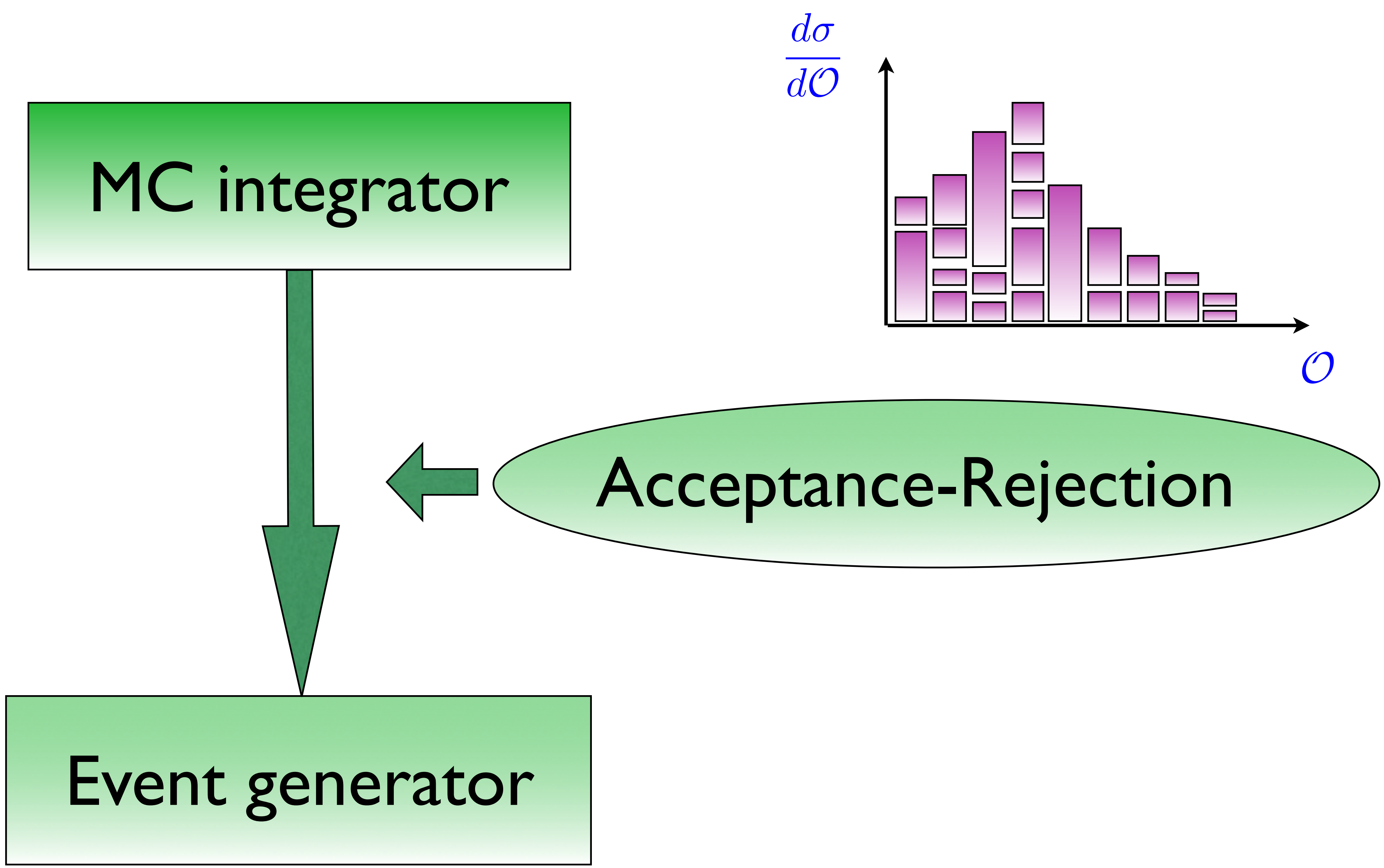
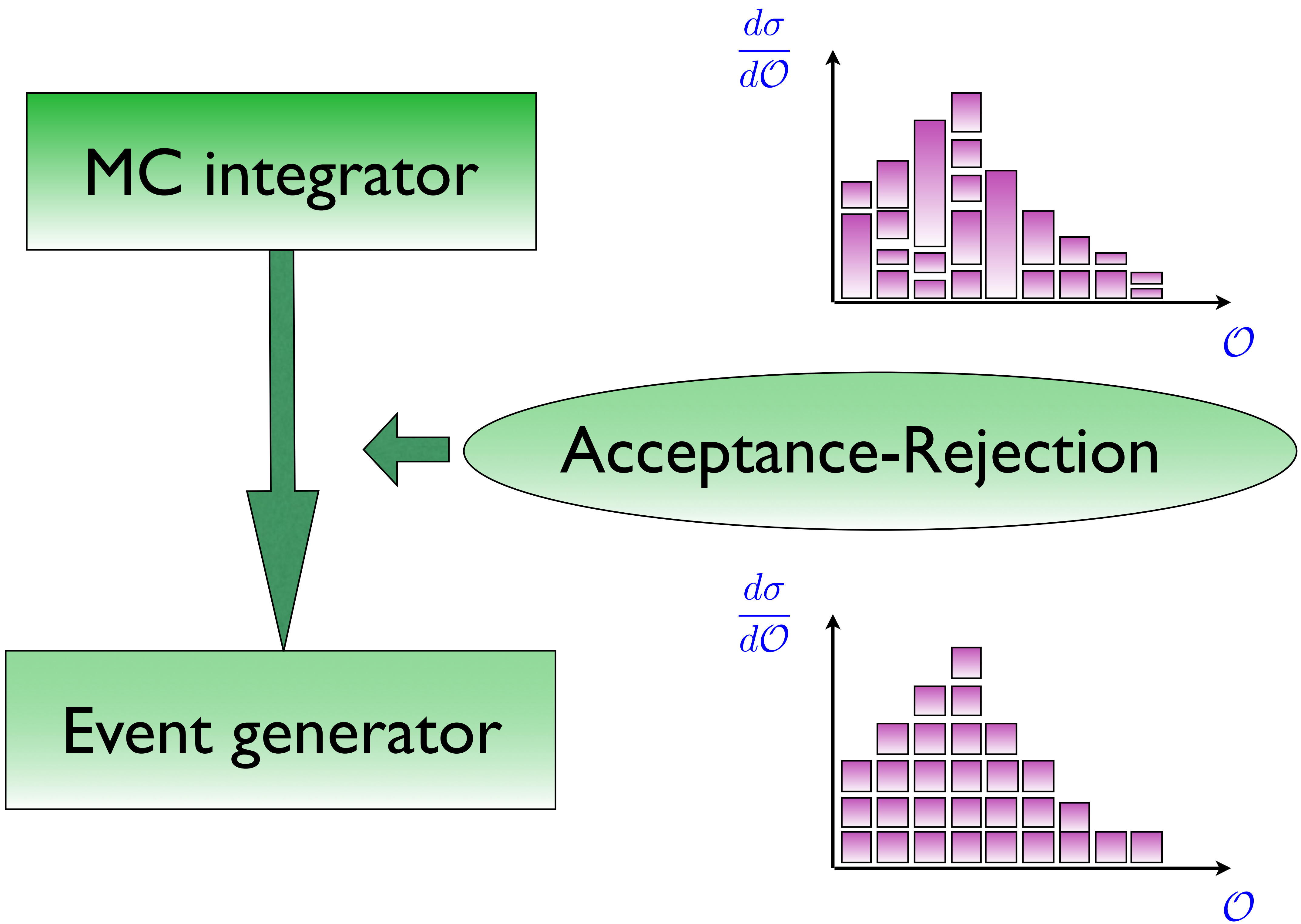


Event generation



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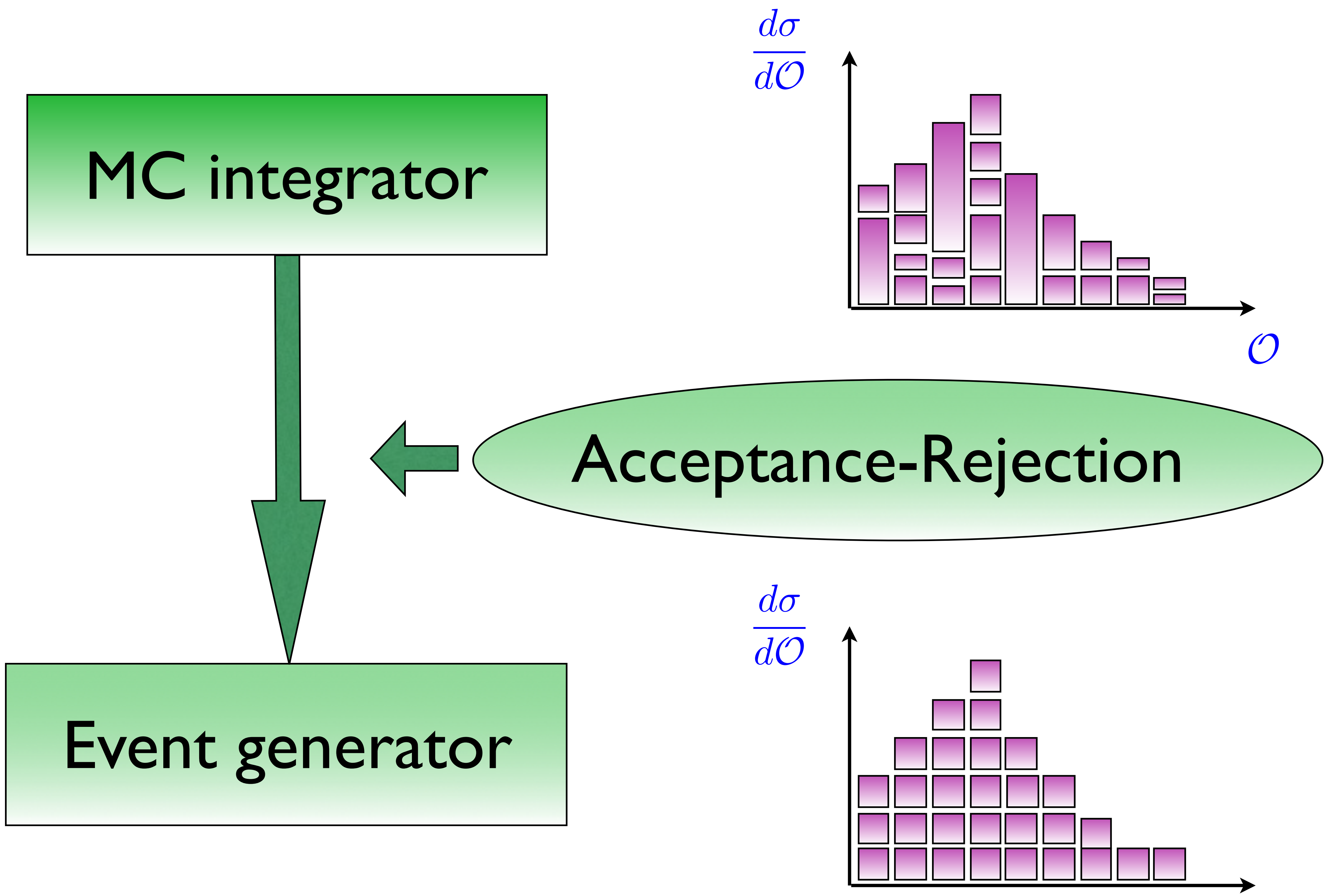

☞ **This is possible only if f(x)<∞ AND has definite sign!**

To Remember

- Sample of unweighted events
  - ➡ Events distributed like nature
  - ➡ Need the function to be
    - ❑ Borned
    - ❑ Always positive
  - ➡ More efficient if the integration is more efficient
    - ❑ Same dependencies in the cut

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension
- Need to know the function
  - Impact on cut

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension
- Need to know the function
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Good Point

- Complex area of Integration
- Easy Error estimate
- quick estimation of the integral
- Possibility to have unweighted events

MadGraph

1991	HELAS
1994	MadGraph
2002	MadEvent
2006	MG/MEv4
2011	MadGraph5
2014	MadGraph5_ aMC@NLO

- Computing Matrix Element for a fixed Helicity and sum over the felicities.

- Suite of Routine, which allow to write the matrix element for any (SM) process

MadGraph

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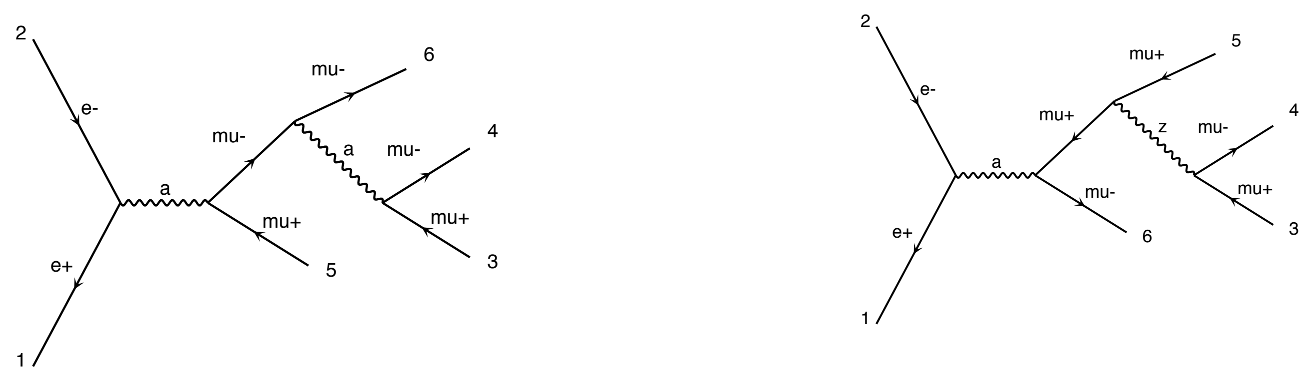
- Automate the creation of the diagram generation and the writing of the HELAS routine



MadGraph

1991 HELAS

MAD stands for Madison

199

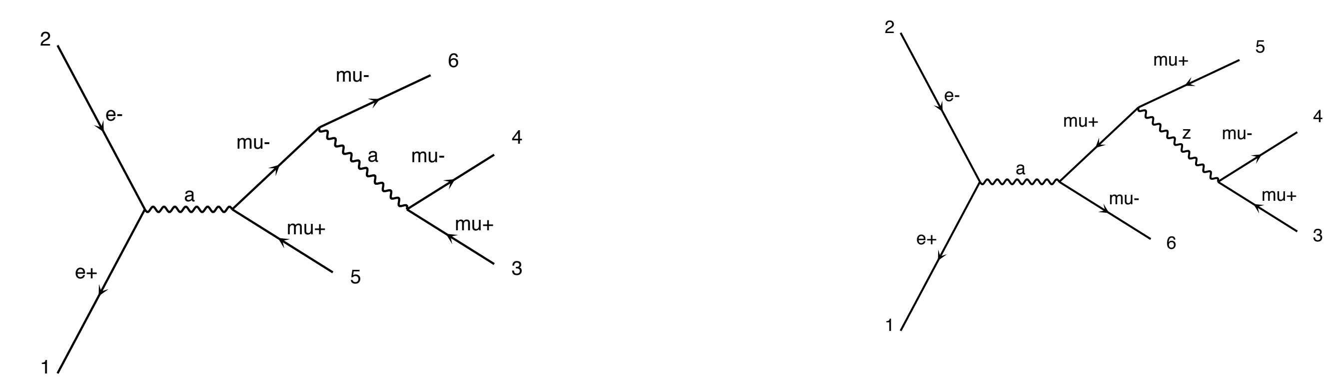
200

200

ram

2011 MadGraph5

2014 MadGraph5_ aMC@NLO



MadGraph

- 1991 HELAS
- 1994 MadGraph
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- 2006 MG/MEv4
- 2011 MadGraph5
- 2014 MadGraph5_aMC@NLO

- Multi-Channel Method!
- Automatic phase-space Integration
- Generation of Events

- Support for the MSSM (SMADGRAPH)

MadGraph

- 1991 HELAS
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- 2011 MadGraph5
- 2014 MadGraph5_ aMC@NLO

- Support for BSM
- Decay Chain
- Pass to a platform (MadOnia/MadWeight/…)
- Link to Pythia/PGS
- Matching/Merging

- Official/Main SM generator for CMS

MadGraph

1991	HELAS
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- Full restart of the MadGraph part in Python
- Fully Automatic BSM
- Various Output Format
- Huge Improvement

MadGraph

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- Fully Automatic computation at
  - NLO* (cross-section)
  - NLO* matched to PS

*NLO= NLO in QCD

MadGraph Functionality

- Leading Order Option
  - Support of BSM
    - Fermion Flow
  - Computation of the Width
  - Narrow width Approximation
    - Decay Chain
    - MadSpin
  - Systematics
- NLO
  - SM with merging

What have we learned!

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral — Parton density functions — Parton-level cross section

- The Importance of PDF
  - ➡ Defines the physics
- Evaluation of Matrix Element
  - ➡ Numerical method faster than analytical formula
  - ➡ cross-section prediction needs NLO
- Phase Space Integration
  - ➡ Need to know in advance what we integrate. Be careful with strong cuts!