Electroweak Interactions : Neutral currents in neutrino-lepton elastic scattering experiments

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1 The Glashow-Weinberg-Salam electroweak theory

1.1 Introduction

At the end of the fifties weak interactions were well described by the V-A theory, introduced in 1957 by Feynman and Gell-Mann. Expanding the Fermi theory for beta decay of 4-point interactions to the case of weak interactions violating parity (as observed by Wu in the early fifties), they concluded that only $\gamma^\mu(1-\gamma^5)$ currents were allowed in that kind of processes. The beta decays are described by the lagrangians :

$$L^{(\beta)} = -\frac{G_F^{(\beta)}}{\sqrt{2}} j^\mu_{hadr} j^{lept}_\mu = -\frac{G_F^{(\beta)}}{\sqrt{2}} \bar{p}\gamma^\mu(1-a\gamma^5)n\bar{e}\gamma_\mu(1-\gamma^5)\nu \quad (1)$$

$$L^{(\mu)} = -\frac{G_F^{(\mu)}}{\sqrt{2}} j^\mu_{lept} j^{lept}_\mu = -\frac{G_F^{(\beta)}}{\sqrt{2}} \bar{\nu}_\mu\gamma^\mu(1-\gamma^5)\mu\bar{e}\gamma_\mu(1-\gamma^5)\nu_e \quad (2)$$

where $G_F^{(\beta)} \simeq G_F^{(\mu)} \simeq G_F = 1,16 \times 10^{-5}\ GeV^{-2}$ is the Fermi constant. Still, neutrino scattering cross sections were shown to display a problematic energy behaviour, violating unitarity. This initiated various ideas to cure the problem of infinities : guided by QED as a gauge theory, attempts were thus made during the sixties to construct a gauge theory of weak interactions. The intermediate charged vector boson W, although its existence was not yet observed, was complemented with a neutral intermediate vector boson, the Z, to achieve the required cancellations, even if its implication in the known processes was at that time considered unlikely, since no neutral currents were ever observed until then. These assumptions paved the way for unified renormalizable theories of electroweak interactions, which became one of the outstanding successes of elementary particle physics : one of these theories was the standard electroweak theory of Glashow, Weinberg and Salam (GWS). In particular, one of the true success of the GWS theory was the prediction of the existence of neutral currents.

The structure of such currents follows from the fact that both the weak and the electromagnetic interactions are unified into a single electroweak interaction in the framework of a gauge theory, based upon the SU(2)×U(1) group. The GWS theory is based on the assumption of the existence of charged and neutral intermediate vector bosons and was constructed so that a gauge invariant $SU(2)_L \times U(1)_Y$ interaction takes place between the various fundamental fermions (leptons and quarks, assumed massless). The so-called electroweak interactions are then mediated by three massive vector bosons (W^+; W^-;Z^0) plus the massless photon γ. The gauge group, $SU(2)_L \times U(1)_Y$, must therefore have four generators : however, gauge invariance relates interactions to massless bosons only. The fact that the (W^+; W^-;Z^0) bosons mediating the weak force are massive (and the fact that we want a renormalizable theory) required to include an additional theoretical principle.

This principle was introduced through the spontaneous symmetry breaking of the electroweak gauge group $SU(2)_L \times U(1)_Y$ to the remaining unbroken abelian group $U(1)_Q$ of electromagnetism. (This mechanism, developped by Higgs, Brout & Englert also explained, as a consequence of the spontaneous breakdown of the underlying symmetry, how fermions and intermediate bosons get their masses.) In 1967, Weinberg and Salam constructed the $SU(2)_L \times U(1)_Y$ model of electroweak interactions of leptons, introducing in the same time a spontaneous breakdown of the gauge symmetry (it was proved by t'Hooft in 1972 that models of this type were renormalizable). The model was then generalized to quarks using the mechanism proposed by Glashow, Iliopoulos and Maiani.

Still, the charged and neutral currents described by the GWS theory required to introduce a new physical observable. This free parameter (its value can be predicted from Grand Unified Theories, but remains a free parameter when considering only the Standard Model), the Weinberg angle θ_W enters in the definitions of the neutral currents properties through $\mathrm{Si}\,n^2\theta_W$. Those neutral currents, related to the neutral massive boson Z^0, were discovered at CERN in 1973 in an experiment using the large bubble chamber "Gargamelle". Such experiments were also those in which the deep-inelastic muonless neutrino processes $\nu_\mu + N \rightarrow \nu_\mu + X$ and $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$ scatterings were observed.

As soon as 1962, accelerator neutrino experiments already gave the first evidence for lepton 'flavour' and for the separate identity of electron and muon neutrinos, ν_e and ν_μ. The early examples of these two types of event were observed in spark chambers. After these experiments, it became possible to perform a complete phenomenological analysis of all the neutral current data. The result of such an analysis was that one could uniquely determine all the coefficients appearing in the most general phenomenological V-A expressions written for hadron and lepton neutral currents. It was shown that this unique solution is in agreement with the GWS theory.

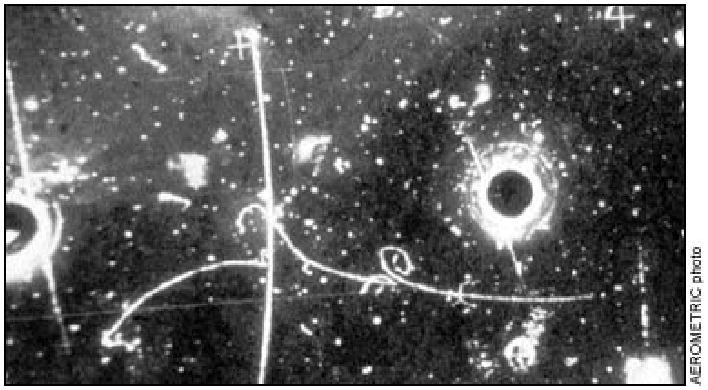
Nearly ten years later, in 1973, the existence of neutrino interactions without any charged lepton in the final state were seen for the first time in a bubble chamber experiment at CERN. These were termed neutral-current events and ascribed to reactions of the form :

$$\begin{aligned}
\nu_e + N &\rightarrow \nu_e + X \\
\bar{\nu}_e + N &\rightarrow \bar{\nu}_e + X \\
\nu_\mu + N &\rightarrow \nu_\mu + X \\
\bar{\nu}_\mu + N &\rightarrow \bar{\nu}_\mu + X
\end{aligned}$$

In addition to these semi-leptonic interactions, purely leptonic neutral-current interactions of the type :

$$\begin{aligned}
\nu_\mu + e^- &\rightarrow \nu_\mu + e^- \\
\bar{\nu}_\mu + e^- &\rightarrow \bar{\nu}_\mu + e^-
\end{aligned}$$

were also observed : they will be our point of interest in the following. The first event of this type was found in the Gargamelle bubble chamber at CERN, even before the evidence for the semi-leptonic neutral currents had been established ! Below one finds one example of a neutral-current event involving hadrons in the Gargamelle bubble chamber. The discovery of neutral-current events was important in providing the first evidence in favour of the unified model of weak and electromagnetic interactions, established some years before by Glashow, Weinberg and Salam.



(Exercise : find the neutral current in the above picture)

This observation, among others, confirmed most of the theoretical predictions of the GSW theory. We will now give a detailed discussion of the above processes, most especially the neutral current induced processes involving the scattering of muon neutrinos (antineutnnos) on electrons. Notice that neutral currents contribute also to other purely leptonic processes, such as $\nu_e + e^- \rightarrow \nu_e + e^-$ or $e^+ + e^- \rightarrow l^+ + l^-$ with $l = e, \mu, \tau$.

1.2 A reminder : the $SU(2)_L \times U(1)_Y$ gauge theory

One of the main cornerstones of the GWS theory is that it is able to explain the qualitative experimental facts that the $W^{\pm}$ bosons couple to L (left-handed) fermions (and R (right-handed) antifermions), that the Z^0 boson couples differently to L-fermions and R-fermions, but γ couples with the same strength to L-fermions and R-fermions, provided they are charged particles. The $SU(2)_L$ subgroup includes the left-handed fermions as irreducible representations, known to be doublets under $SU(2)_L$:

$$L = P_L \Psi_{leptons} = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L ; Q = P_L \Psi_{quarks} = \begin{bmatrix} u \\ d \end{bmatrix}_L \tag{3}$$

The Lie group SU(2) is of dimension $2^2 - 1 = 3$, it is related to the coupling constant g, and the $SU(2)_L$ representation matrix generators acting on these fields are $T_i = \tau_i/2$, proportional to the Pauli matrices :

$$\tau_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} ; \tau_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} ; \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{with} \quad [\tau_i, \tau_j] = 2i\varepsilon^{ijk}\tau_k \tag{4}$$

They are associated to the corrsponding vector gauge boson fields W^i_μ with $i = 1, 2, 3$.The right-handed fermions are all singlets under $SU(2)_L$:

$$e_R; u_R; d_R \tag{5}$$

Meanwhile, the $U(1)_Y$ subgroup is known as weak hypercharge and has a coupling constant $g\prime$ and a vector boson B_μ, sometimes known as the hyperphoton. The hypercharge Y is a conserved charge just like the electric charge Q, and takes the form of a diagonal matrix, with values to be defined later as verifying $Q = T_3 + Y/2$. Following the general results of Yang-Mills gauge theories, the pure-gauge part of the electroweak Lagrangian density is :

$$L_{gauge} = -\frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{6}$$

with the field strengths :

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\varepsilon^{ijk} W^j_\mu W^k_\nu \tag{7}$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{8}$$

The totally antisymmetric ε_{ijk} are the structure constants for $SU(2)_L$. This L_{gauge} provides the kinetic terms for the vector fields, and will give rise to the Feynman rules associated to their self-interactions.

The interactions between the various electroweak gauge bosons with fermions are determined by the covariant derivative. For example, the covariant derivatives acting on the lepton fields are :

$$D_\mu \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L = (\partial_\mu + ig' B_\mu Y_{l_L} + ig W^i_\mu T^i) \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L \tag{9}$$

$$D_\mu e_R = (\partial_\mu + ig' B_\mu Y_{l_R}) e_R \tag{10}$$

Y_{l_L} and Y_{l_R} are the weak hypercharges of left-handed leptons and right-handed leptons, and 2×2 unit matrices are understood to go with the ∂_μ and B_μ terms. The weak hypercharges of all other fermions are then fixed. Using the explicit form of the SU(2)$_L$ generators in terms of Pauli matrices, the covariant derivative of left-handed leptons is :

$$D_\mu \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L = \partial_\mu \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L + i[\frac{g'}{2} Y_{l_L} \begin{bmatrix} B_\mu & 0 \\ 0 & B_\mu \end{bmatrix} + \frac{g}{2} \begin{bmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{bmatrix}] \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L \tag{11}$$

Therefore, the covariant derivatives of the lepton fields can be summarized as :

$$D_\mu \nu_{e_L} = \partial_\mu \nu_e + i[\frac{g'}{2} Y_{l_L} B_\mu + \frac{g}{2} W^3_\mu] \nu_{e_L} + i\frac{g}{2}(W^1_\mu - iW^2_\mu) e_L \tag{12}$$

$$D_\mu e_L = \partial_\mu e_L + i[\frac{g'}{2} Y_{l_L} B_\mu - \frac{g}{2} W^3_\mu] e_L + i\frac{g}{2}(W^1_\mu + iW^2_\mu) \nu_{e_L} \tag{13}$$

$$D_\mu e_R = \partial_\mu e_R - i\frac{g'}{2} Y_{l_L} B_\mu \tag{14}$$

The covariant derivative of a field must carry the same electric charge as the field itself, in order for charge to be conserved. Evidently, then, $(W^1_\mu - iW^2_\mu)$ must carry electric charge +1 and $(W^1_\mu + iW^2_\mu)$ must carry electric charge -1, so these must be identified with the W$^\pm$ bosons of the weak interactions. The interaction Lagrangian is therefore :

$$\begin{aligned} L_I &= i \begin{bmatrix} \bar{\nu}_{e_L} & \bar{e}_L \end{bmatrix} \gamma^\mu D_\mu \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L \\ &= -\frac{g}{2} \bar{\nu}_{e_L} \gamma^\mu e_L (W^1_\mu - iW^2_\mu) - \frac{g}{2} \bar{e}_L \gamma^\mu \nu_e (W^1_\mu + iW^2_\mu) + ... \end{aligned} \tag{15}$$

The fields for the $W^\pm$ gauge bosons are defined by :

$$W^+_\mu = \frac{1}{\sqrt{2}}(W^1_\mu - iW^2_\mu) \tag{16}$$

$$W^-_\mu = \frac{1}{\sqrt{2}}(W^1_\mu + iW^2_\mu) \tag{17}$$

The vector bosons B_μ and W^3_μ are both electrically neutral. As a result of spontaneous symmetry breaking, we will find that they mix. In other words, the fields with well-defined masses are not B_μ and W^3_μ, but are are orthogonal linear combinations of these two gauge eigenstate fields. One is the photon field A_μ, and the other one is the massive Z boson vector field, Z_μ. Expressing our previous expressions in terms of those two fields is achieved thanks to a simple θ_W rotation in field space, where θ_W is known as the weak mixing Weinberg angle :

$$\begin{bmatrix} W^3_\mu \\ B_\mu \end{bmatrix} = \begin{bmatrix} Cos\theta_W & \mathrm{Si}\,n\theta_W \\ -\mathrm{Si}\,n\theta_W & Cos\theta_W \end{bmatrix} \begin{bmatrix} Z_\mu \\ A_\mu \end{bmatrix} \tag{18}$$

$$\Rightarrow$$

$$\begin{bmatrix} Z_\mu \\ A_\mu \end{bmatrix} = \begin{bmatrix} Cos\theta_W & -\mathrm{Si}\,n\theta_W \\ \mathrm{Si}\,n\theta_W & Cos\theta_W \end{bmatrix} \begin{bmatrix} W^3_\mu \\ B_\mu \end{bmatrix} \tag{19}$$

We now require that the resulting theory has the correct photon coupling to fermions, by requiring that the field A_μ appears in the covariant derivatives in the way dictated by QED. The covariant derivative of the right-handed electron field can be written :

$$D_\mu e_R = \partial_\mu e_R - ig' Cos\theta_W A_\mu e_R + ig' \mathrm{Si}\,n\theta_W Z_\mu e_R \tag{20}$$

Comparing to $D_\mu e_R = \partial_\mu e_R - ieA_\mu e_R$ from QED, we conclude that :

$$g' Cos\theta_W = e \tag{21}$$

Similarly, one finds, using our previous results, that :

$$D_\mu e_L = \partial_\mu e_R + i(\frac{g'}{2} Y_{l_L} Cos\theta_W - \frac{g}{2} \mathrm{Si}\, n\theta_W) A_\mu e_L + ... \tag{22}$$

Again, comparing to the prediction of QED that $D_\mu e_L = \partial_\mu e_L - ieA_\mu e_L$, we obtain the electromagnetic coupling :

$$\frac{g}{2} \mathrm{Si}\, n\theta_W - \frac{g'}{2} Y_{l_L} Cos\theta_W = e \tag{23}$$

In the same way :

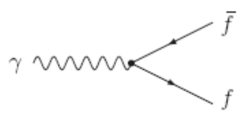
$$D_\mu \nu_e = \partial_\mu \nu_e + i(\frac{g'}{2} Y_{l_L} Cos\theta_W + \frac{g}{2} \mathrm{Si}\, n\theta_W) A_\mu \nu_e \tag{24}$$

where the "..." represent W and Z terms. However, we know that the neutrino has no electric charge, and therefore its covariant derivative cannot involve the photon. So, the coefficient of A_μ must vanish, and we find :

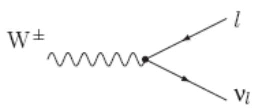
$$g \,\mathrm{Si}\, n\theta_W + g' Y_{l_L} Cos\theta_W = 0 \tag{25}$$

where $Y_{l_L} = -1$.

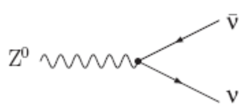
1.3 Memento : Feynman Rules



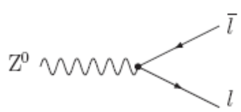
$-\mathrm{i}eQ_f\gamma^\mu$



$-\frac{\mathrm{i}g}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$

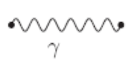


$-\frac{\mathrm{i}g}{4\cos\theta_W}\gamma^\mu(1-\gamma_5)$

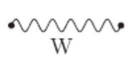


$\frac{\mathrm{i}g}{4\cos\theta_W}\gamma^\mu(1-4\sin^2\theta_W-\gamma_5)$

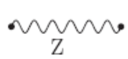
$\frac{\mathrm{i}}{\not{p}-m+\mathrm{i}\varepsilon}$



$-\frac{\mathrm{i}g_{\mu\nu}}{p^2+\mathrm{i}\varepsilon}$



$-\frac{\mathrm{i}g_{\mu\nu}}{p^2-M_W^2+\mathrm{i}\varepsilon}$



$-\frac{\mathrm{i}g_{\mu\nu}}{p^2-M_Z^2+\mathrm{i}\varepsilon}$

Some useful electroweak relations :

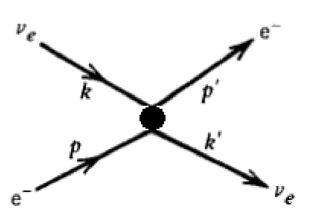
$$\begin{aligned}
m_W &= \frac{gv}{2} && (26)\\
m_Z &= \frac{m_w}{Cos\theta_W} && (27)\\
\frac{G_F}{\sqrt{2}} &= \frac{1}{8}\frac{g^2}{m_W^2} && (28)\\
v &= 246\ GeV && (29)
\end{aligned}$$

2 Neutrino scattering experiments

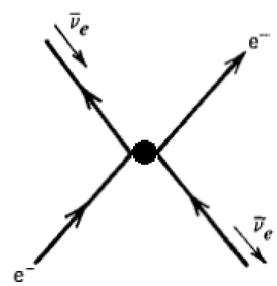
2.1 Charged Currents

Before moving to the neutral current experiments, it could be relevant to consider first neutrino scattering involving charged currents. Those are especially useful to understand the next section : although the experiments exposing the violation of parity in weak interactions provide some important highlights in the development of particle physics, parity violation and its V-A structure can now be demonstrated experimentally much more directly. Indeed, neutrinos (more particularly muon neutrinos) can be experimentally "prepared" in intense beams, scattered off targets (hadronic, or even leptonic targets) to probe the structure of the weak interaction. Recent technological achievements opens up the possibility of exhibiting the V-A structure ($\gamma^\mu(1-\gamma^5)$) of the weak coupling by measuring the angular distribution of neutrino scattering. An analogy could be made with the experimental confirmation of the V (vector) structure of the electromagnetic vertex from the measurements of the angular distributions in Bhabba/Möller ($ee \to ee$) or Mott ($e\mu \to e\mu$) scattering.

The diagrams we will investigate are shown below, with the corresponding particle four-momenta within the effective field theory context :



(a) Neutrino-electron scattering



(b) Antineutrino-electron scattering

assuming a charged current interaction. The amplitude for the first one, $\nu_e + e^- \to \nu_e + e^-$ is, in the effective theory (4-point vertex without propagator) :

$$M = \frac{G_F}{\sqrt{2}}[\bar{\nu}_e(k')\gamma_\mu(1-\gamma^5)e(p)][\bar{e}(p')\gamma^\mu(1-\gamma^5)\nu_e(k)] \tag{30}$$

The computation of the related cross section follows the same lines as the Mott scattering (described in details, for example, in the "Peskin & Schroeder", section 5.1.) except that the vectorial current "γ_μ" is replaced by the V-A current "$\gamma_\mu(1-\gamma^5)$". Squaring, summing over the final state spins and averaging over the initial state spins, one gets :

$$|\bar{M}|^2 \equiv \frac{1}{2}\sum_{spins}|M|^2 = \frac{G_F^2}{4}Tr[\gamma_\mu(1-\gamma^5)\not{p}\gamma^\nu(1-\gamma^5)\not{k}'] \times Tr[\gamma_\mu(1-\gamma^5)\not{k}\gamma^\nu(1-\gamma^5)\not{p}'] \tag{31}$$

Computing the traces in the CM frame such that $s=(k+p)^2=(k'+p')^2=2k\cdot p=2k'\cdot p'$, one finds :

$$\begin{aligned}
|\bar{M}|^2 &= 64G_F^2(k\cdot p)(k'\cdot p')\\
&= 16G_F^2 s^2 && (32)
\end{aligned}$$

The cross section for a $2 \rightarrow 2$ scattering is :

$$\frac{d\sigma}{d\Omega} = \frac{|\bar{M}|^2}{64\pi^2 s} \tag{33}$$

Integrating :

$$\sigma(\nu_e + e^- \rightarrow \nu_e + e^-) = \frac{G_F^2 s}{\pi} \tag{34}$$

In analogy, we can compute the results for the second diagram, $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ (s-channel) :

$$|\bar{M}|^2 = 16 G_F^2 t^2 \tag{35}$$

$$t^2 = s^2(1 - Cos\theta)^2 \tag{36}$$

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{16\pi^2}(1 - Cos\theta)^2 \tag{37}$$

$$\sigma(\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-) = \frac{G_F^2 s}{3\pi} \tag{38}$$

were introduced the usual Mandelstam parameters

$$\begin{aligned} s &= (p_1 + p_2)^2 = (p_3 + p_4)^2, \\ t &= (p_1 - p_3)^2 = (p_2 - p_4)^2, \\ u &= (p_1 - p_4)^2 = (p_2 - p_3)^2, \end{aligned} \tag{39}$$

where

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2 \tag{40}$$

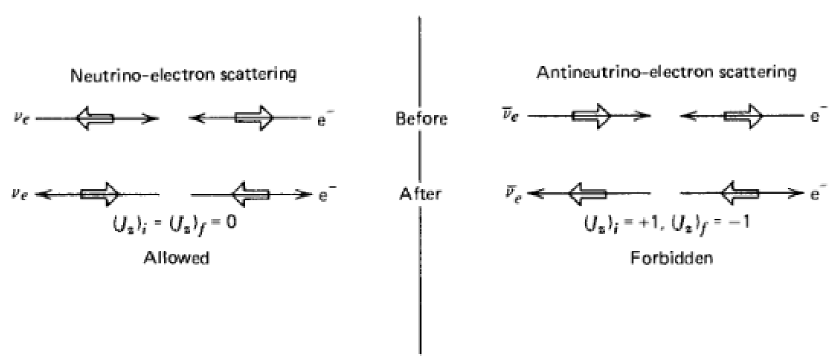
So, what we find above is that the process involving electronic neutrinos is three times weaker than the one involving electronic anti-neutrinos, which is one of the many "strange" asymetries of the weak interaction :

$$\frac{\sigma(\nu_e + e^- \rightarrow \nu_e + e^-)}{\sigma(\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-)} = \frac{1}{3} \tag{41}$$

Interestingly, one can also show that if one had a "V+A" structure rather than a "V-A" one, both differential cross sections would be equal to

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{4\pi^2}(1 + cos\ \theta)^2\ ! \tag{42}$$

This at-first-sight astonishing result comes from the fact $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ (obtained by crossing the neutrinos in $\nu_e + e^- \rightarrow \nu_e + e^-$ up to the exchange $s \longleftrightarrow t$) maintains an angular distribution proportional to $cos\ \theta$ in its corresponding differential cross-section, while $\nu_e + e^- \rightarrow \nu_e + e^-$ does not. Therefore, $\bar{\nu}_e$ scattering is unlikely to happen in the backward region ($d\sigma/d\Omega$ vanishes for $cos\ \theta = 1$), as could have been anticipated from angular momentum conservation. As far as helicity arguments are concerned, this is equivalent to require the process $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ to proceed only in the $J = 1$ state/scenario, i.e. only one of the three allowed helicity states. On the other hand, ν_e belongs to the $J = 0$ case for which no θ dependence follows, and therefore no such suppression.



The remaining question is, obviously, why do weak interaction violate parity ? That is another problem to solve...

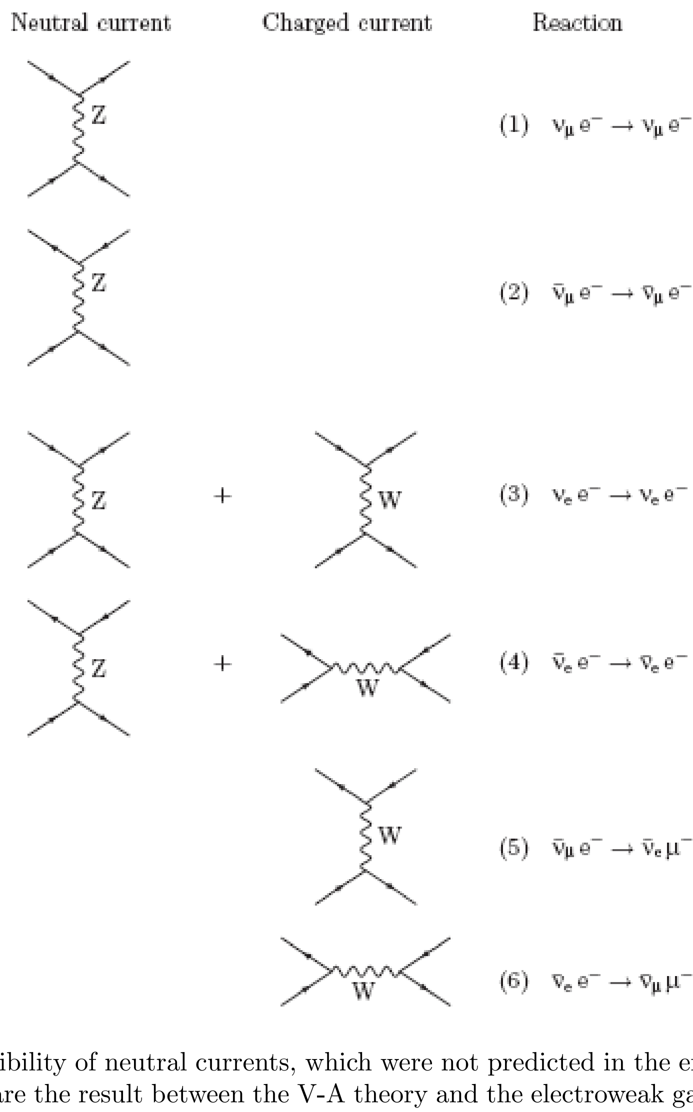
2.2 Neutral Currents

One of the cleanest evidence for the electroweak theory was the discovery of neutral currents. From the above lagrangian, we have seen how one can describe processes involving charged current interactions between neutrinos and electrons. Obviously, we can also investigate other leptonic scattering processes such as

$$\nu_\mu + e^- \to \nu_e + \mu^-$$

for which there is no neutral current contribution (cross section), but which involves a change in lepton flavor : however, the cross section is the same as for the electronic scattering process, $G_F^2 s/4\pi^2$, and are even easier to study since high-energy neutrino beams are predominantly produced as muonic (ν_μ) by the experimentalists. Still, this does not involve neutral current contributions, which we can now introduce by describing the above processes within the context of $SU(2)_L \times U(1)_Y$.

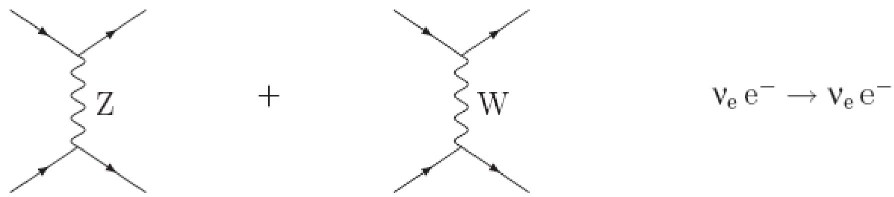
As described previously, the overall strength of the neutral-current interaction can be determined only by the scalar coupling G_F, but at higher energies such experiments imply neutral-current interactions depending on g^2 and $\mathrm{Si}\,n^2\theta_W$. Indeed, such processes have been thoroughly investigated, and the agreement after two and a half decades of research is indeed impressive for the GSW theory. This section will focus on detailing leptonic neutral-current reactions. They are part of six other processes of the same kind, as listed below :



This allows the possibility of neutral currents, which were not predicted in the effective V-A field theory. We can therefore compare the result between the V-A theory and the electroweak gauge theory, as described in the introduction. The present exercise will be to obtain the cross sections for these various channels, which expressions must be completely compatible with the few observed scatterings involving (anti)neutrinos.

As an illustration, let's consider the elastic scattering :

$$\nu_e(k) + e^-(p) \to \nu_e(k') + e^-(p')$$



The amplitude for the corresponding 4-point vertex in the V-A theory would then be :

$$\begin{aligned} iM &= \frac{G_F}{\sqrt{2}}[\bar{\nu}_l\gamma_\mu(1-\gamma^5)\nu_l\ \bar{l}\gamma^\mu(1-\gamma^5)l] \\ &= \frac{G_F}{\sqrt{2}}[\bar{\nu}_l\gamma_\mu(1-\gamma^5)l\ \bar{l}\gamma^\mu(1-\gamma^5)\nu_l], \end{aligned} \tag{43}$$

easy to compute with the usual algebra. (To get the above result, one used the following trick : charged-current reactions can be transformed into the charge-retaining form vertex structure by Fierz's reordering theorem. A special form of the theorem states that, when at least one of the couplings is $(1 \pm \gamma^5)$, then we can interchange the first and the third (or second and fourth) spinors without prejudice.) On the other hand, we can perform the same calculation in the electroweak theory as a comparison, and study a process involving simultaneously the charged current and neutral current propagators. Most especially, one can compute how to recover the effective result in the limit $q^2 \ll m^2_{W,Z}$ for such a case. Let's first remind the general structure of Charged Current (CC) and Neutral Current (NC) amplitudes : copying the QED procedure for the related currents leads to

$$j_\mu = j^\dagger_\mu = \frac{j^1_\mu + ij^2_\mu}{2}, \tag{44}$$

and

$$iM^{CC} \sim \frac{G_F}{\sqrt{2}} j^\mu j^\dagger_\mu = [\frac{g}{\sqrt{2}}j^\mu][\frac{1}{m^2_W}][\frac{g}{\sqrt{2}}j^\dagger_\mu] \tag{45}$$

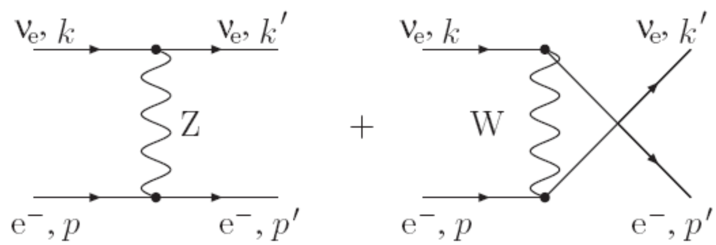
with $1/m^2_W$ the approximation for the W propagator at low q^2. The same can be performed for the neutral currents :

$$iM^{CC} \sim \frac{\rho}{2}\frac{G_F}{\sqrt{2}} j^\mu_0 j^\dagger_{\mu 0} = [\frac{g}{\cos\theta_W}j^\mu_0][\frac{1}{m^2_Z}][\frac{g}{\cos\theta_W}j^\dagger_{\mu 0}];$$

for which we define the ρ parameter as usual,

$$\rho = \frac{m^2_W}{m^2_Z\cos^2\theta_W}. \tag{46}$$

Let's now compute the following process :



Z and W exchange in neutrino–electron scattering

For this, we need to keep in mind that the structure of the $Z \to f\bar{f}$ vertex factor involves both vector and axial couplings, defined with respect to the weak isospin and the electrical charge of the considered fermion. Following these conventions, neutral currents-only processes involving leptons can be described by the effective amplitude :

$$M^{NC}(\nu\ l \to \nu\ l) = \frac{G_F}{\sqrt{2}}[\bar{\nu}\gamma_\alpha(1-\varepsilon\gamma^5)\nu][\bar{l}\gamma^\alpha(g_V^l - g_A^l\gamma^5)l] \tag{47}$$

where $\varepsilon = 1$ for neutrinos and $\varepsilon = -1$ for anti-neutrinos (all examples above involve a neutrino or antineutrino whose vertex is of the form $\gamma^\mu(1-\varepsilon\gamma^5)$). Evidently, not all reactions are of the same form as the above amplitude, especially those occuring at higher energies since several of them include both charged- and neutral- current reactions. Still, the total amplitude for $\nu\ e \to \nu\ e$ is :

$$\begin{aligned}
M &= M_Z + M_W, \\
M_Z &\equiv (\frac{ig}{4Cos\theta_W})^2\frac{-i}{q^2-m_z^2}[\bar{u}(k')\gamma_\mu(1-\gamma^5)u(k)] \otimes [\bar{u}(p')\gamma^\mu\gamma^5 - (1-4\,\mathrm{Si}\,n^2\theta_W)\gamma^\mu u(p)], \\
M_W &= (\frac{ig}{2\sqrt{2}})^2\frac{-i}{q^2-m_W^2}[\bar{u}(p')\gamma_\mu(1-\gamma^5)u(k)] \otimes [\bar{u}(k')\gamma^\mu(1-\gamma^5)u(p)].
\end{aligned} \tag{48}$$

Notice the correspondence between the electroweak theory and the V-A theory :

Effective couplings for several reactions

		Electroweak theory		V–A theory	
Reaction	ε	g_V	g_A	g_V	g_A
$\nu_\mu + e^- \to \nu_\mu + e^-$	+1	$-\frac{1}{2}+2s^2$	$-\frac{1}{2}$	0	0
$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$	−1	$-\frac{1}{2}+2s^2$	$-\frac{1}{2}$	0	0
$\nu_e + e^- \to \nu_e + e^-$	+1	$+\frac{1}{2}+2s^2$	$+\frac{1}{2}$	1	1
$\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$	−1	$+\frac{1}{2}+2s^2$	$+\frac{1}{2}$	1	1
$\nu_\mu + e^- \to \mu^- + \nu_e$	+1	1	1	1	1

With this one easily understands why the neutral currents mediated processes $\nu_\mu e \to \nu_\mu e$ $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ were not to be considered in the effective theory. Now coming back to our above calculation (assuming that the electron mass is negligible and with $\varepsilon = 1$), we compute the differential cross section as

$$\begin{aligned}
|\bar{M}|^2 &\equiv \frac{1}{2}\sum_{spins}|M|^2 = \frac{1}{2}\sum_{spins} MM^* \\
&= \frac{G_F^2}{4}Tr[\gamma_\mu(1-\gamma^5)\not{k}\gamma_\nu(1-\gamma^5)\not{k}'] \times Tr[\gamma^\mu(g_V - g_A\gamma^5)(\not{p}+m)\gamma^\nu(g_V-g_A\gamma^5)(\not{p}'+m)]
\end{aligned} \tag{49}$$

The expression can be factorized into two tensors :

$$|\bar{M}|^2 = \frac{G_F^2}{2}A_{\mu\nu}B^{\mu\nu} \tag{50}$$

$$A_{\mu\nu} = Tr[\gamma_\mu(1-\gamma^5)\not{k}\gamma_\nu\not{k}'] \tag{51}$$

$$B_{\mu\nu} = Tr[(g_V+g_A)^2\gamma_\mu\not{p}\gamma^\nu\not{p}' + 2g_Vg_A\gamma^2\gamma_\mu\not{p}\gamma_\nu\not{p}'] \tag{52}$$

Developing the traces and contracting the two tensors, we observe that products with different symmetries in μ and ν vanish, such as :

$$|\bar{M}|^2 = 16G_F^2[(g_V+g_A)^2(k\cdot p)(k'\cdot p') + (g_V-g_A)^2(k\cdot p')(k'\cdot p)] \tag{53}$$

Performing the phase space integration and the computation of the cross section in the laboratory frame (this case is easier since the electron is at rest) :

$$d\sigma = \frac{|\bar{M}|^2}{2m_e 2E_\nu} d\phi^{(2)} \tag{54}$$

$$d\phi^{(2)} = (2\pi)^4 \delta^4(k + p - p' - k') \frac{d^3k'}{(2\pi)^3 2E'_\nu} \frac{d^3p'}{(2\pi)^3 2E'_e} \tag{55}$$

Let's note that the delta function gives a relation between the scattering angle and the energy transfer, since :

$$\begin{aligned} \int d^3k' \delta(m_e^2 - (k - k' + p)^2) &= \pi \frac{E'_\nu}{E_\nu} dE'_\nu \\ 1 - Cos\theta &= \frac{E - E'}{EE'} m_e \end{aligned} \tag{56}$$

Finally :

$$\frac{d\sigma}{dE'} = \frac{G_F^2 m_e}{2\pi} [(g_V + g_A)^2 + (g_V - g_A)^2 (\frac{E'_\nu}{E_\nu})^2] \tag{57}$$

In the more general case where $a \neq 1$ and $m_e \neq 0$, the final result would have been :

$$\begin{aligned} \frac{d\sigma}{dE'} &= \frac{G_F^2 m_e}{2\pi} [(g_V + a g_A)^2 + (g_V - a g_A)^2 (\frac{E'_\nu}{E_\nu})^2 - \frac{m_e \beta}{E_\nu^2} (g_V^2 - {g_A}^2)] \\ \text{with } \beta &= E_\nu - E'_\nu \end{aligned} \tag{58}$$

Let's note that the above result can also be used to describe reactions involving charged currents. For instance, one has immediately :

$$\frac{d\sigma}{dE'} (\nu_\mu + e^- \to \nu_e + \mu^-) = \frac{2G_F^2 m_e}{\pi} \tag{59}$$

to integrate over the minimal and maximal possible energies.

3 Conclusion

Departing from our above results, one can check as an exercise that :

$$\begin{aligned} \sigma(\bar{\nu}_e e &\to \bar{\nu}_e e) = \alpha[4 \operatorname{Si}n^4\theta_W + \frac{1}{3}(1 + 2 \operatorname{Si}n^2\theta_W)^2] \\ \sigma(\nu_e e &\to \nu_e e) = \alpha[\frac{4}{3} \operatorname{Si}n^4\theta_W + (1 + 2 \operatorname{Si}n^2\theta_W)^2] \\ \sigma(\bar{\nu}_\mu e &\to \bar{\nu}_\mu e) = \alpha[4 \operatorname{Si}n^4\theta_W + \frac{1}{3}(1 - 2 \operatorname{Si}n^2\theta_W)^2] \\ \sigma(\bar{\nu}_\mu e &\to \bar{\nu}_\mu e) = \alpha[\frac{4}{3} \operatorname{Si}n^4\theta_W + (1 - 2 \operatorname{Si}n^2\theta_W)^2] \\ \text{with } \alpha &= \frac{G_F^2 m_e E_\nu}{2\pi} \end{aligned} \tag{60}$$

Much can be said about these results : first, one notices that the cross sections involving either neutrinos or anti-neutrinos are, again, not equal (charged- and neutral-current reactions include different propagators and spinors).

In conclusion, even if very weak (i.e. with a very low cross section), such processes are very interesting since they only involve leptons, and provide a rich amount of tests for every theoretical electroweak unification scheme. All these results also confirm that the neutral currents involve a vectorial V part and an axial A part, allowing for $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ to happen in the context of $SU(2)_L \otimes U(1)_Y$, but not in the effective field V-A theory. This is therefore a very important improvement, settling down the possibility of observable neutral currents in scattering experiments.

To conclude, let's quote Dieter Haidt "[...] the fact that so many neutrino experiments were running can historically be linked with the fact that physicists were to be confronted with the "neutral current observation" challenge without any preparation ! It must be noted that the searches for neutral currents in the previous neutrino experiments resulted in discouraging upper limits and were interpreted in a way, that the community believed in their nonexistence and the experimentalists rather turned to the investigation of other existing questions, such as the exciting observation of the proton's substructure at SLAC, or what structure would be revealed by the W in a neutrino experiment [...] At that time, the word neutral current was not even pronounced and ironically, as seen from today, the search for neutral currents was solely an also-ran low in the priority list".

And still, these neutral currents were lying beneath : this is how research succeeded

4 Some references

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