



LHC Phenomenology with Madgraph

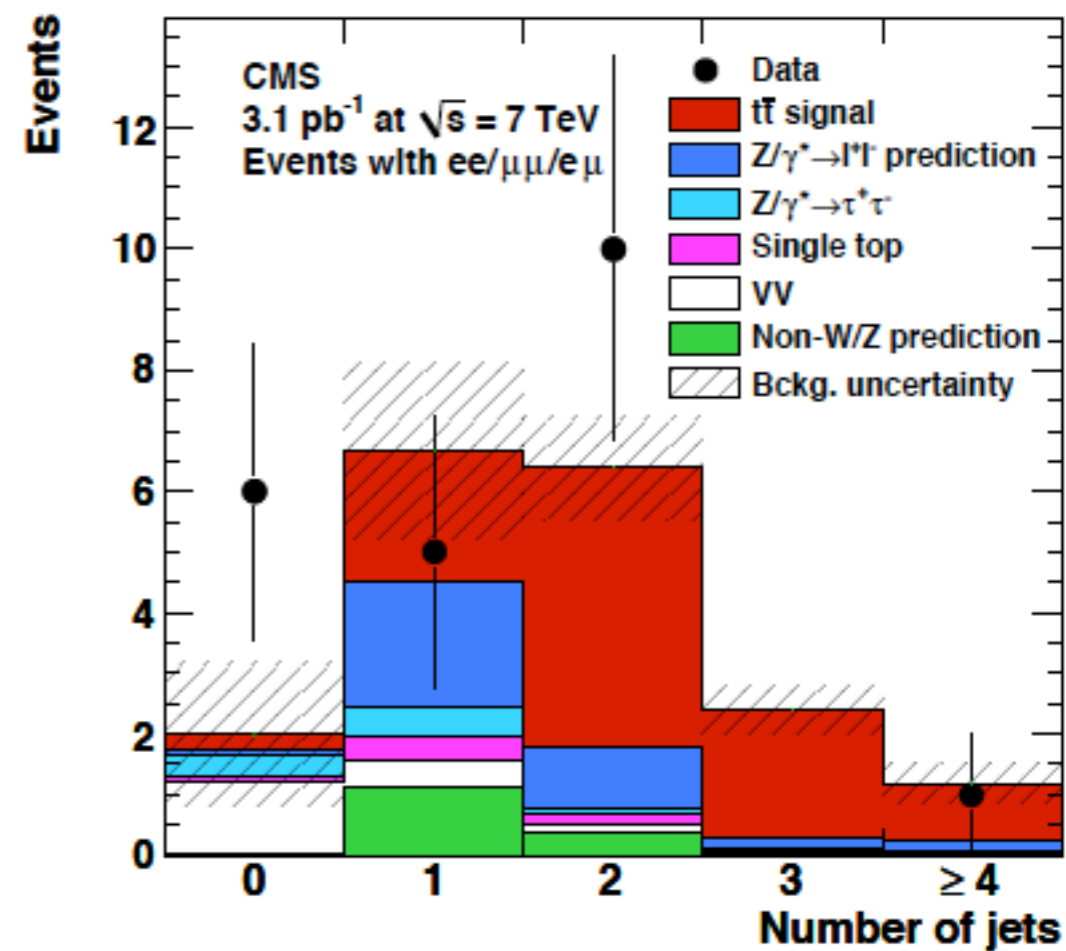
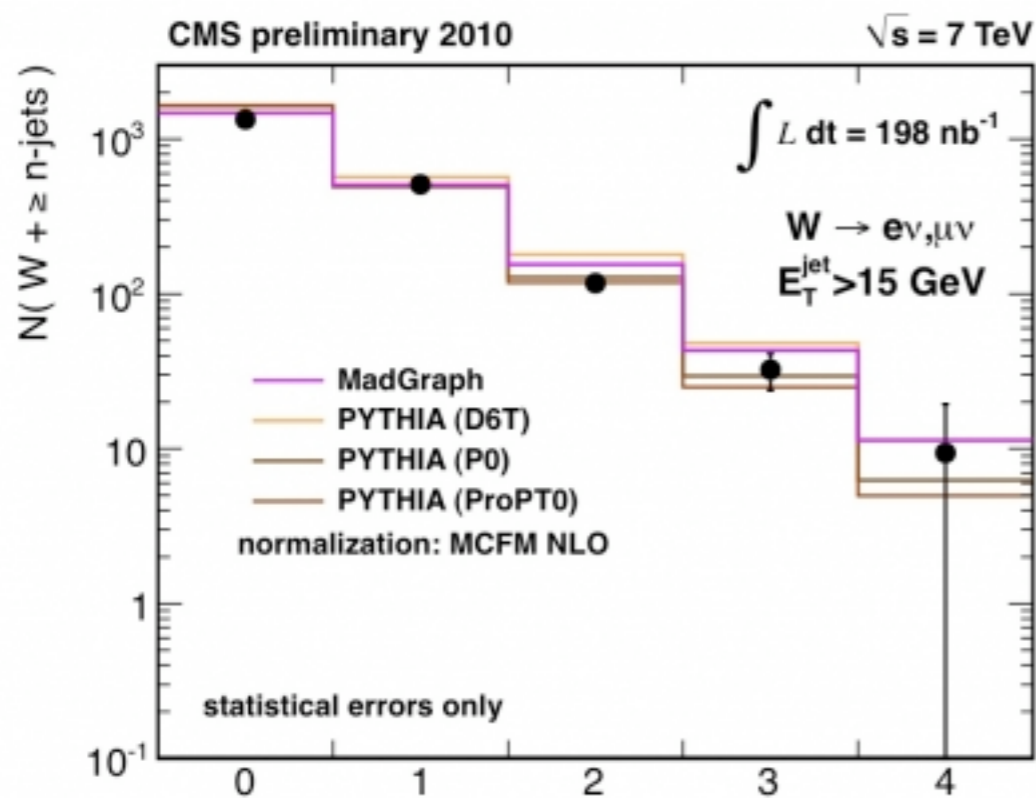
Introduction to MC's for the LHC

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Université Catholique de Louvain



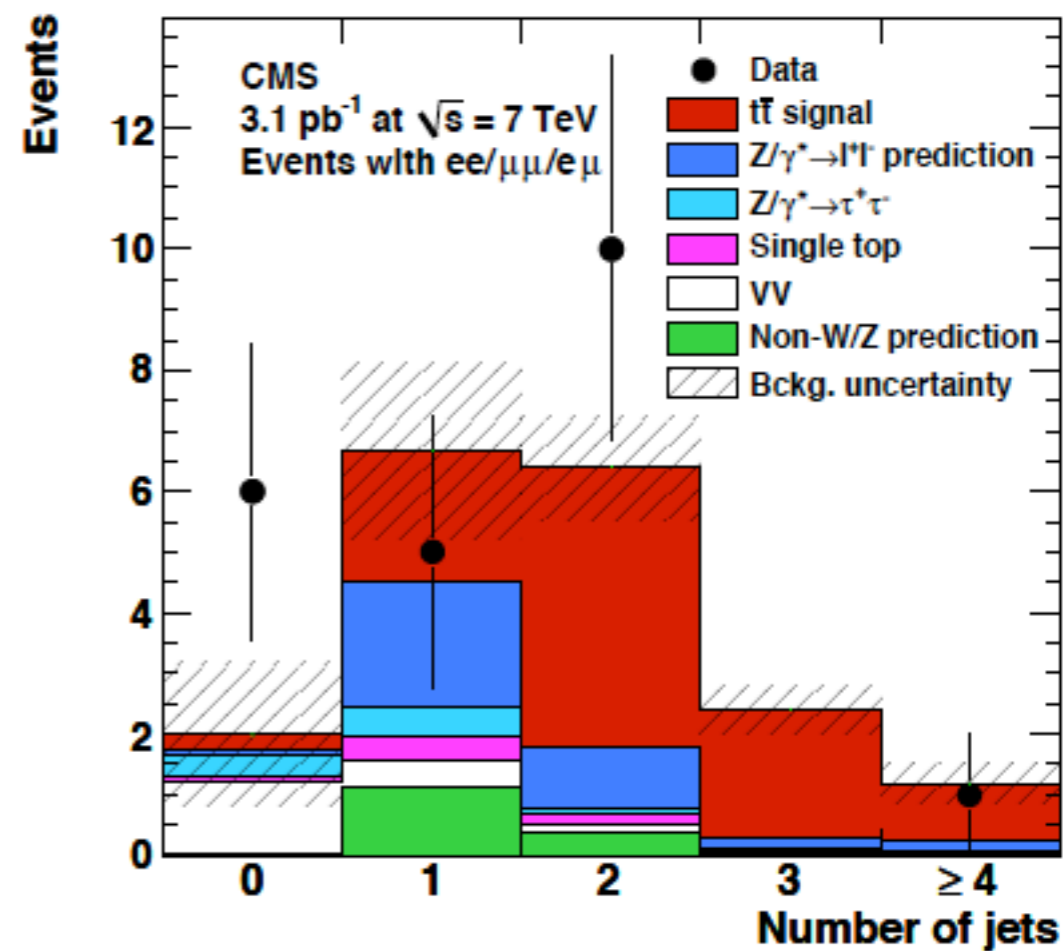
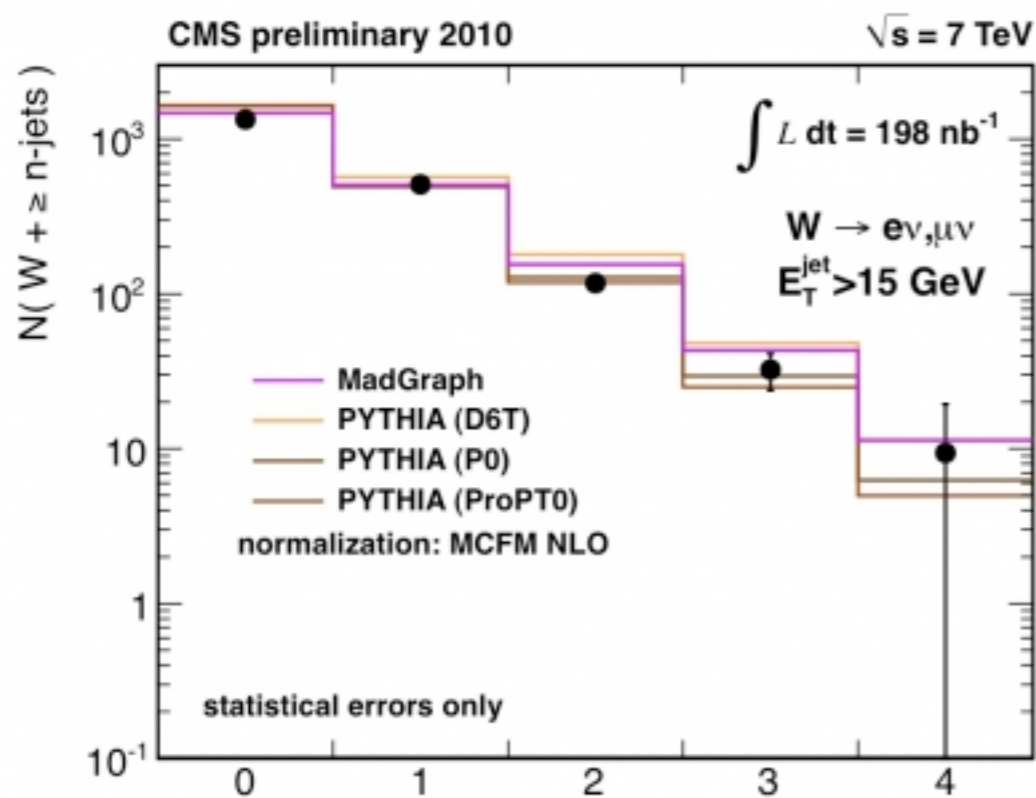
Claims and Aims





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LHC data is there!!!!





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Claims and Aims

1. There has been a number of key theoretical results recently in the quest of achieving the best possible description of events at the LHC.
2. Monte Carlo development is a VERY active line of theoretical research in particle phenomenology with a community growing from $O(10)$ to $O(100)$ in the last years.
3. MC's embody our level of understanding of high energy interactions. They naturally provide a framework where theory and experiment naturally meet.



Claims and Aims



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1. Provide an overview on the recent achievements in our capabilities of making USEFUL predictions for the LHC and give an idea on the imminent future.
2. Give you the minimal set of TH concepts necessary to understand what MC generators are and do.
3. Be ready to work on the LHC data...



Claims and (your) Aims

Many exercises on LHC phenomenology available on the MadGraph Wiki.

<http://cp3wks05.fynu.ucl.ac.be/twiki/bin/view/Physics/Copenhagen2010>

Claims and (your) Aims



Think

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Test : How much do I know about MC's?



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Test : How much do I know about MC's?



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Score	Result	Comment
5	Addict	Always keep in mind that there are also other interesting pheno activities in the field.
4	Excellent	No problem in following these lectures.
3	Fair	Check out carefully the missed topics.
≤ 2	Room for improvement (not passing the Turing test)	Enroll in a MC crash course at your home institution.
5 No clue	No clue	 <p>It's Time to Call 911 1 2 3 4 5 6 7 8 9 * 0 # What to do in an EMERGENCY</p>

A simple plan

- Physics challenges at the LHC
- Basics : QCD and MC's methods
- The new generation of MC tools
- New simulations for New Physics

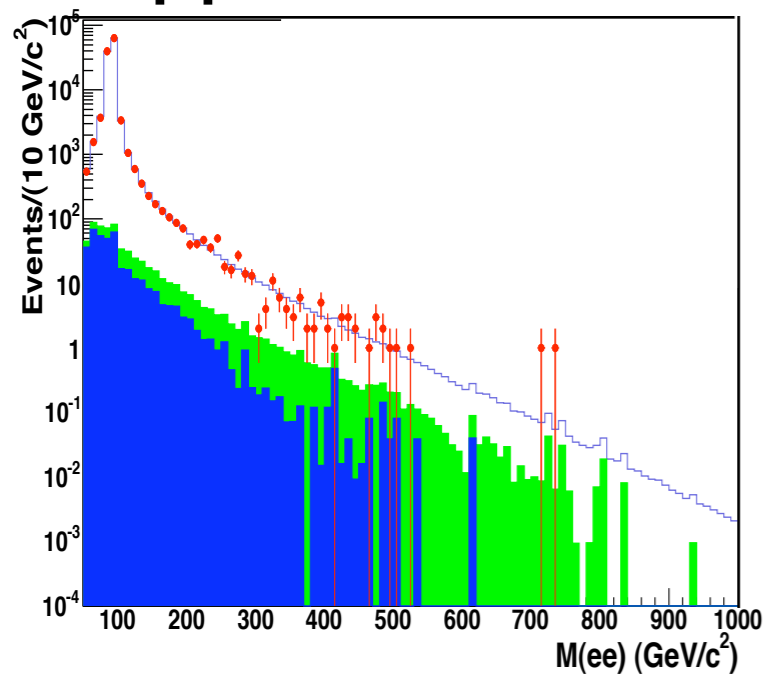
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Discoveries at hadron colliders

peak

$$pp \rightarrow Z' \rightarrow e^+e^-$$

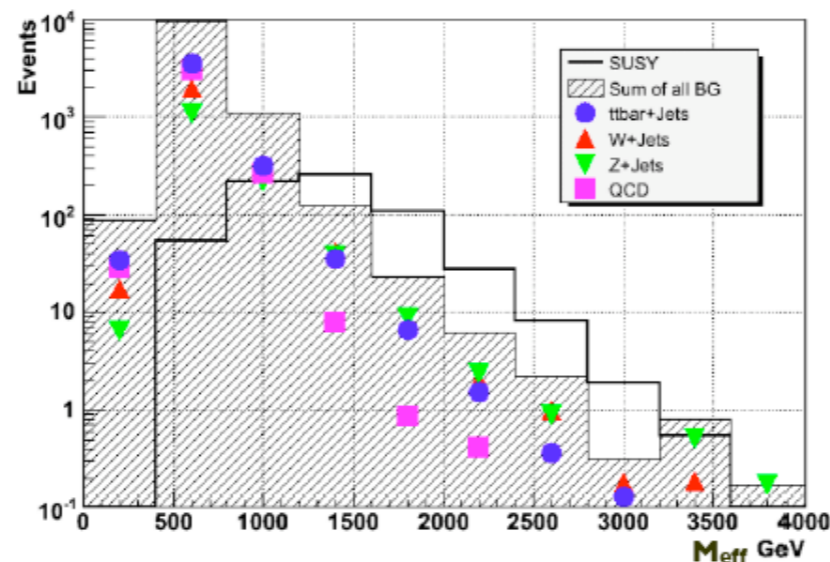


“easy”

Background directly measured from data. TH needed only for parameter extraction (Normalization, acceptance,...)

shape

$$pp \rightarrow \tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q} \rightarrow \text{jets} + \cancel{E}_T$$

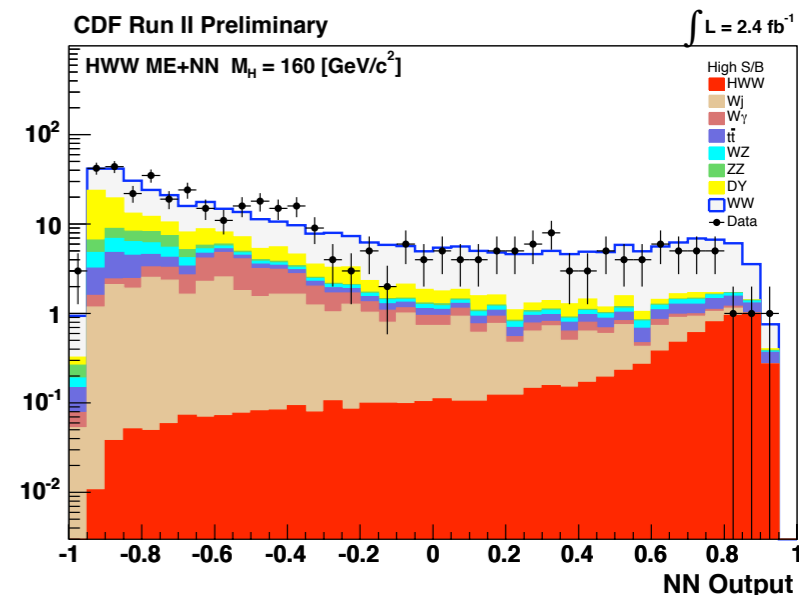


hard

Background shapes needed. Flexible MC for both signal and background tuned and validated with data.

rate

$$pp \rightarrow H \rightarrow W^+W^-$$

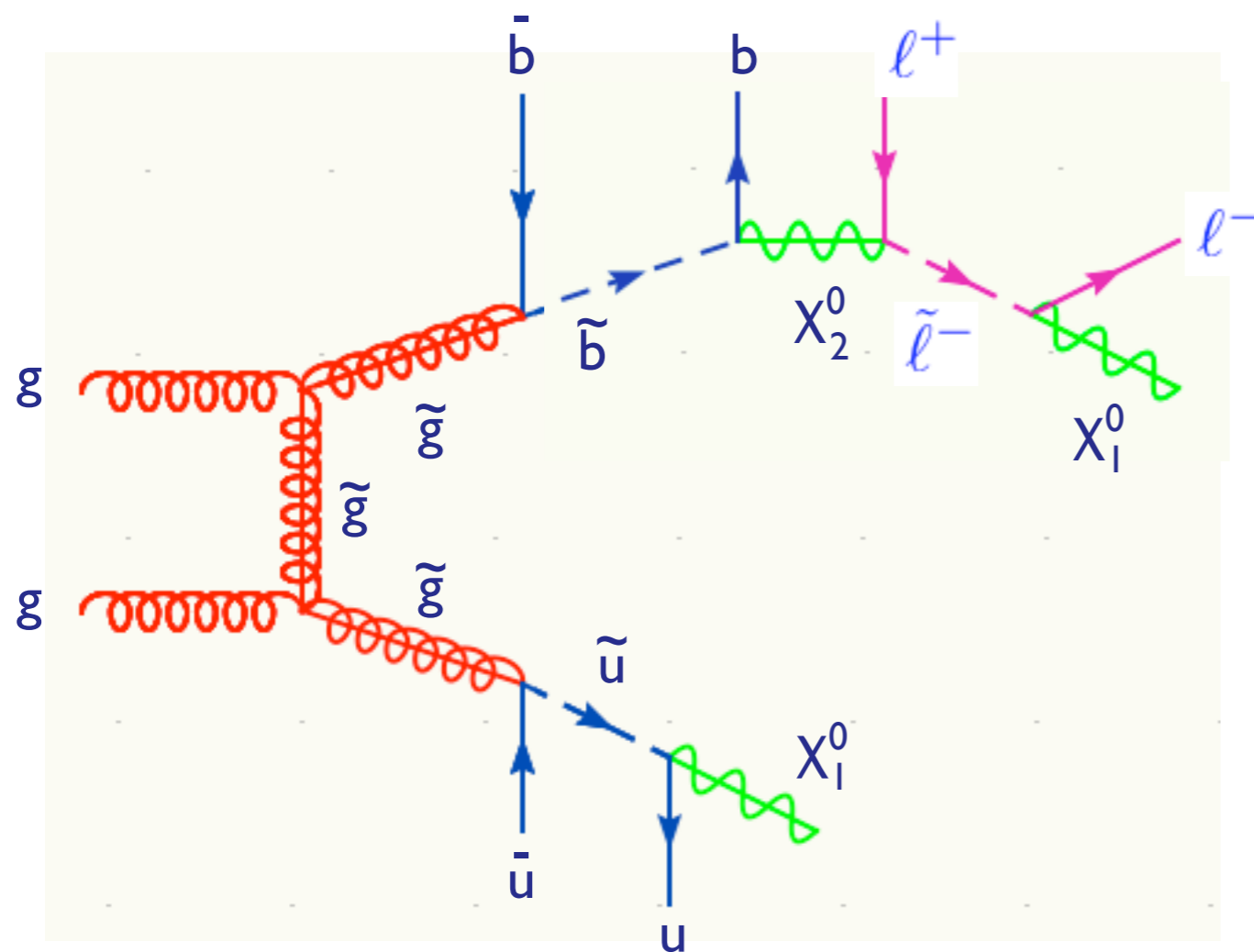


very hard

Background normalization and shapes known very well. Interplay with the best theoretical predictions (via MC) and data.

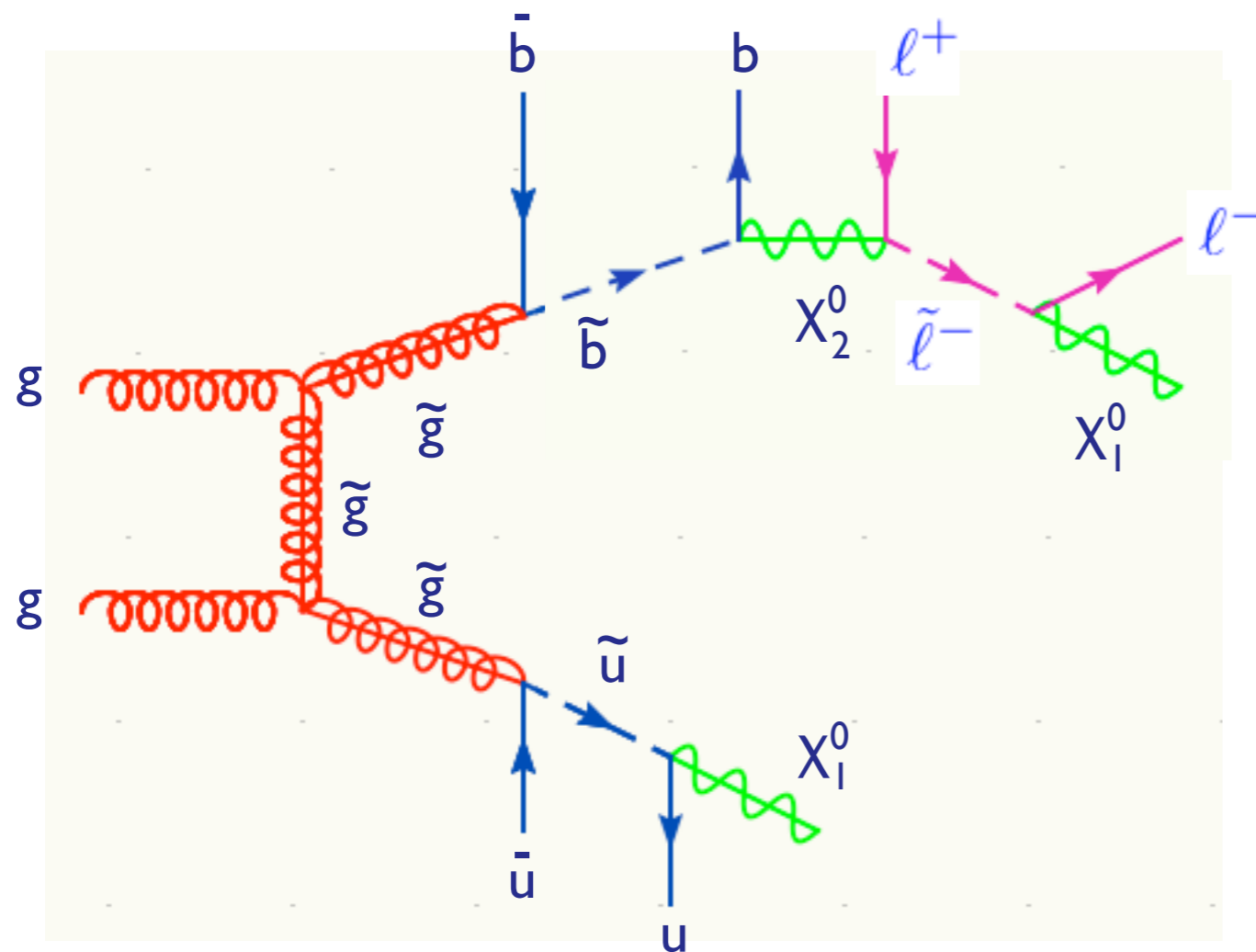
A new challenge

Consider SUSY-like inclusive searches: heavy colored states decaying through a chain into jets, leptons and missing E_T ...

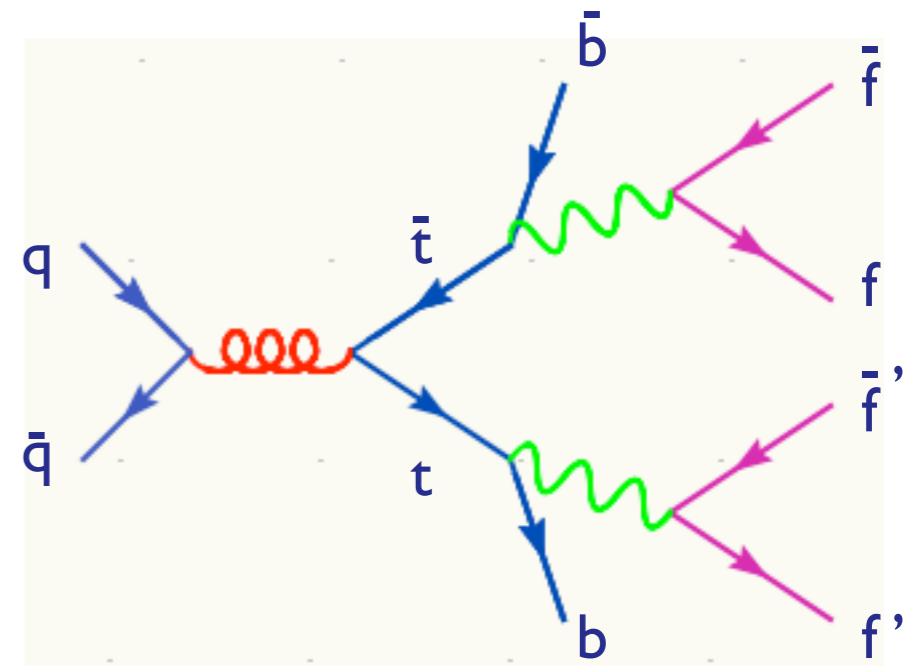


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Consider SUSY-like inclusive searches: heavy colored states decaying through a chain into jets, leptons and missing E_T ... We have already a very good example of a similar discovery!

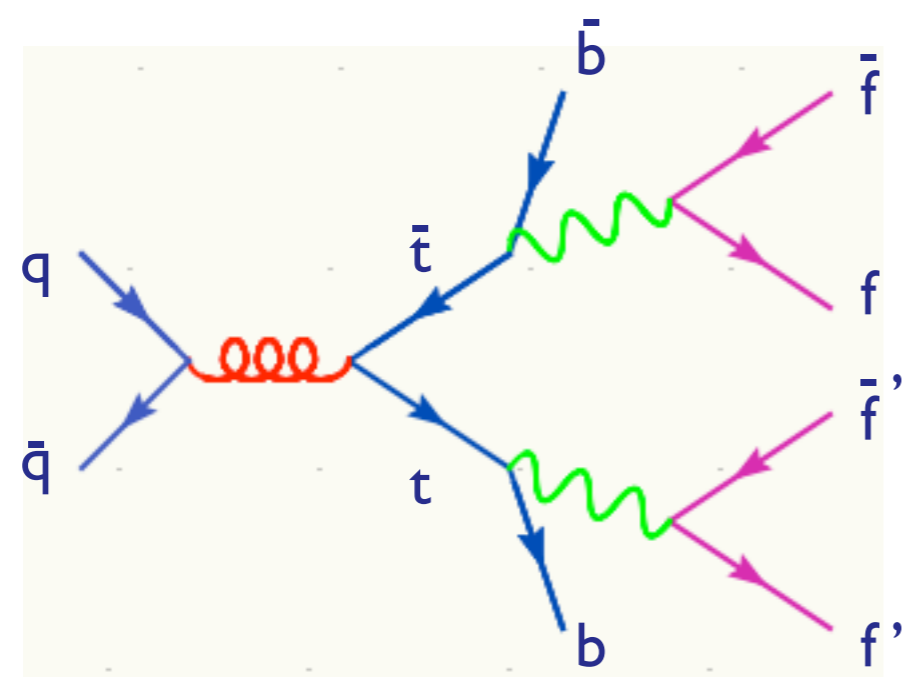


VS



Follow the same approach of CDF in 1995 to establish first evidence of an excess wrt to SM-top and then consistency with SM top production [$mt=174$, $t \rightarrow b\bar{\nu}$, $\sigma(tt)$], works for the SM Higgs, but in general beware that...

A lesson from the top



How did it go?

0. The only unknown was the top mass!

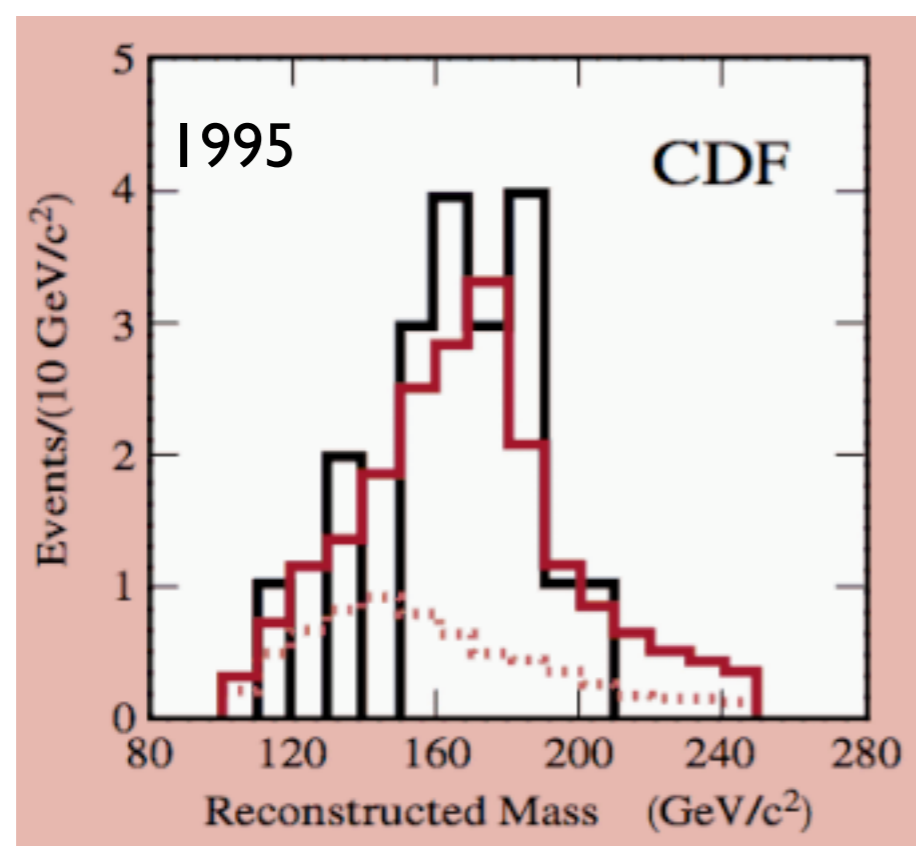
1. The experimentally easiest channel for triggering/reconstruction/background-control was chosen.

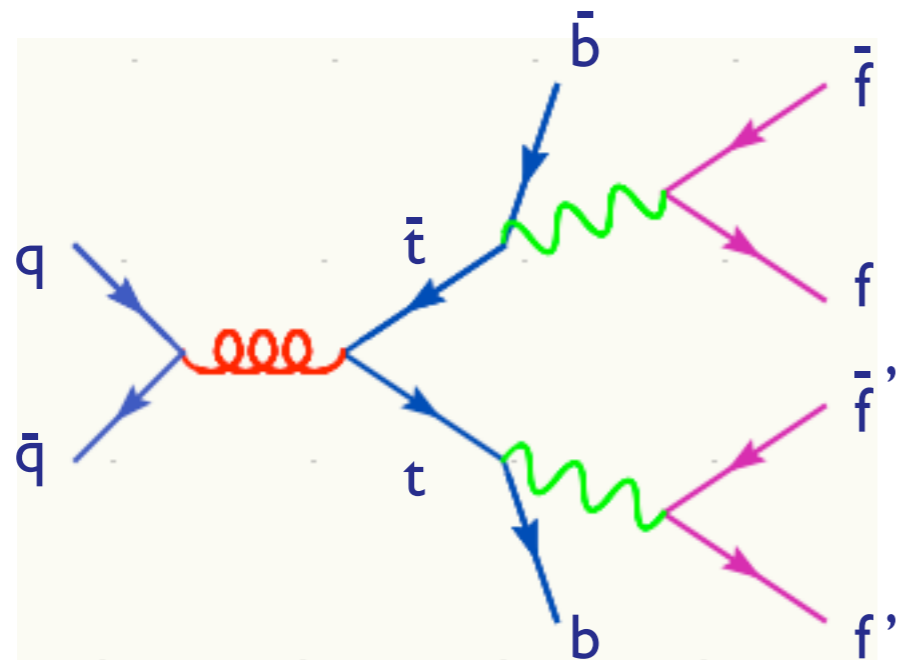
2. Mass reconstruction employed

3. Backgrounds estimated via control samples with heavy flavors and also via MC ratio's.

4. Number of events consistent with the cross section expectation from QCD

Handful of events was enough!



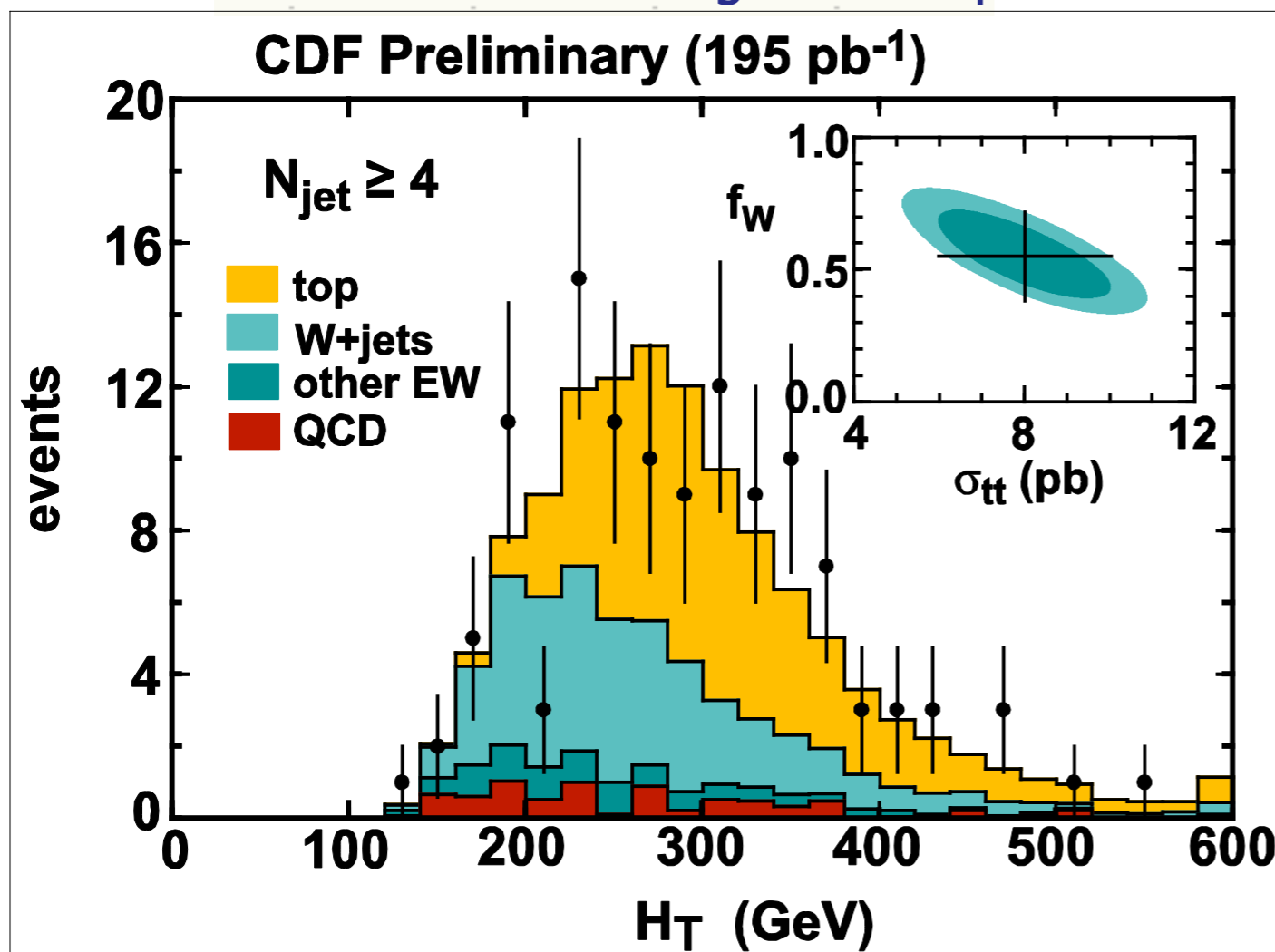


Immediately confirmed in Run II, also by the most inclusive measurements, H_T .

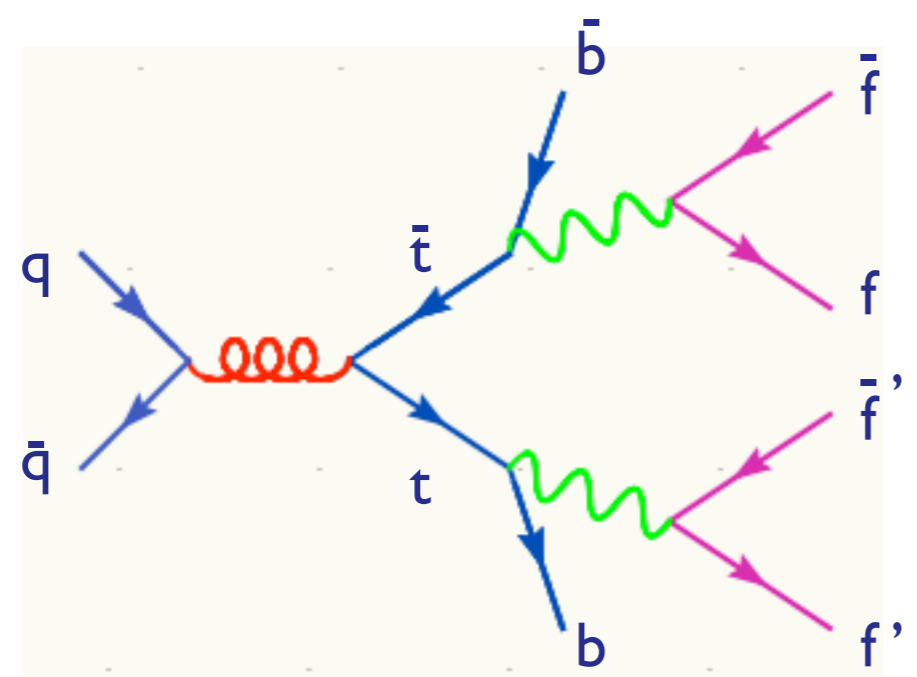
Other channels start to be considered as the statistics increases to have a consistent picture.

Cleaner and cleaner samples more exclusive studies:

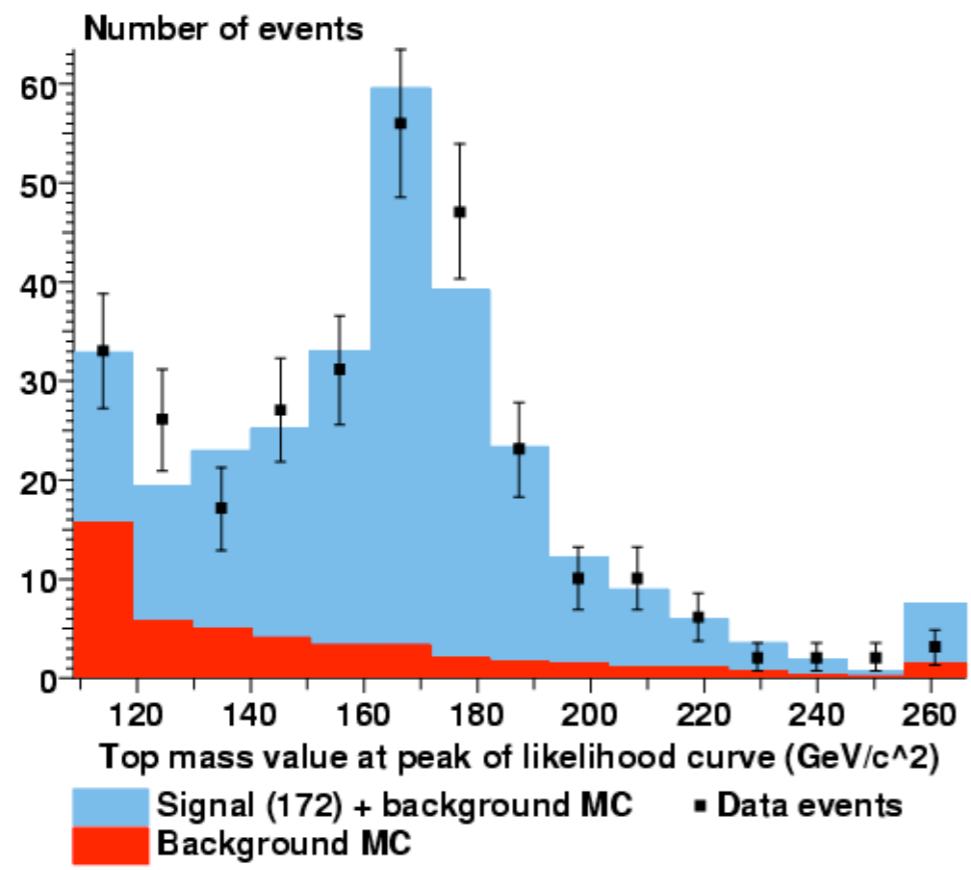
1. W Polarization
2. BR's ratio's
3. Top Quark charge
4. Differential m_{tt} distribution
5. Search for new physics!!



A lesson from the top



CDF Run 2 Preliminary 1.7/fb



Summary:

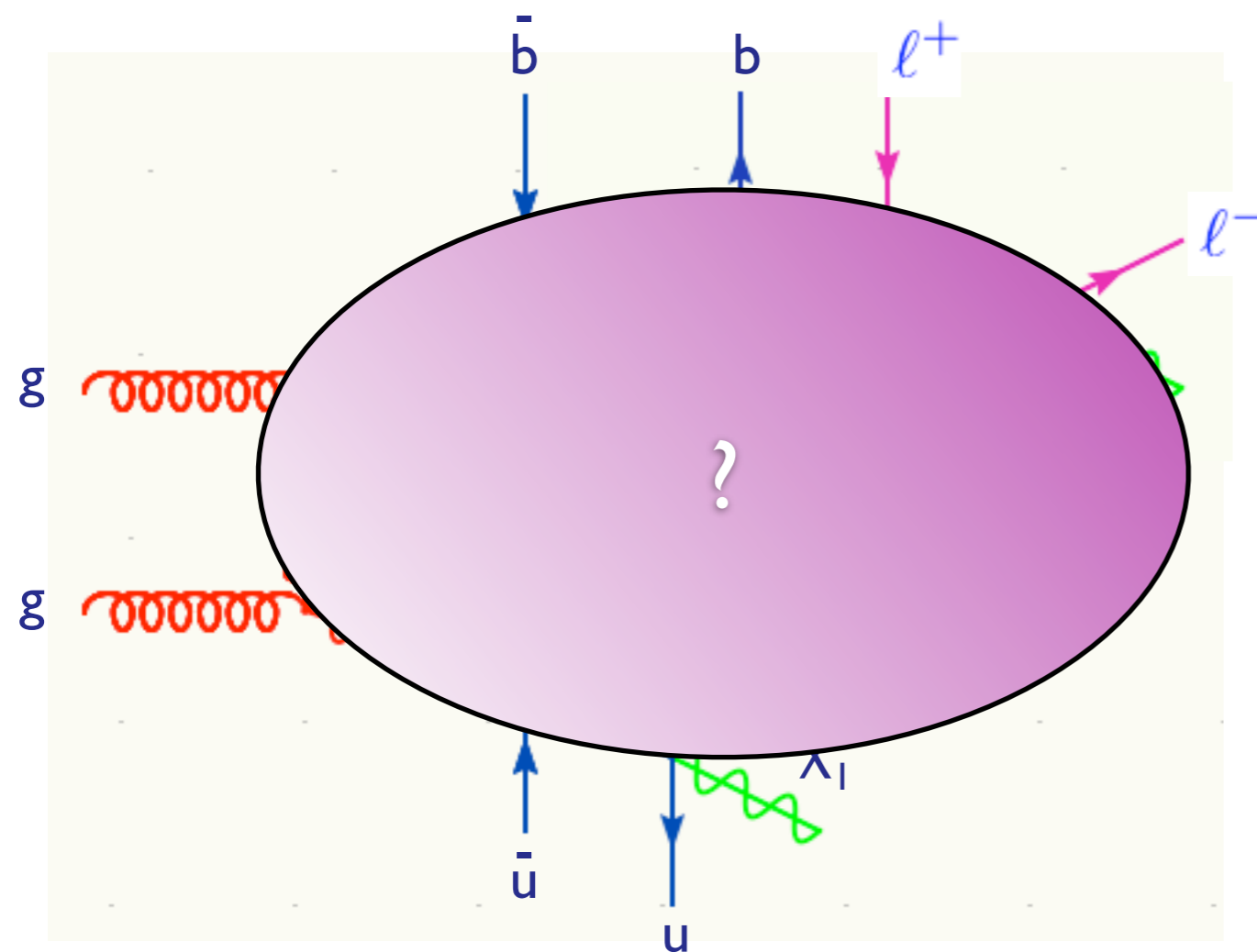
1. More than 15-year long story
2. At all stages MC's played a role.
3. Now all studies, including the mass measurements, are strongly based on our simulation tools, i.e., matrix element methods.

More sophisticated analysis need more sophisticated MC's...

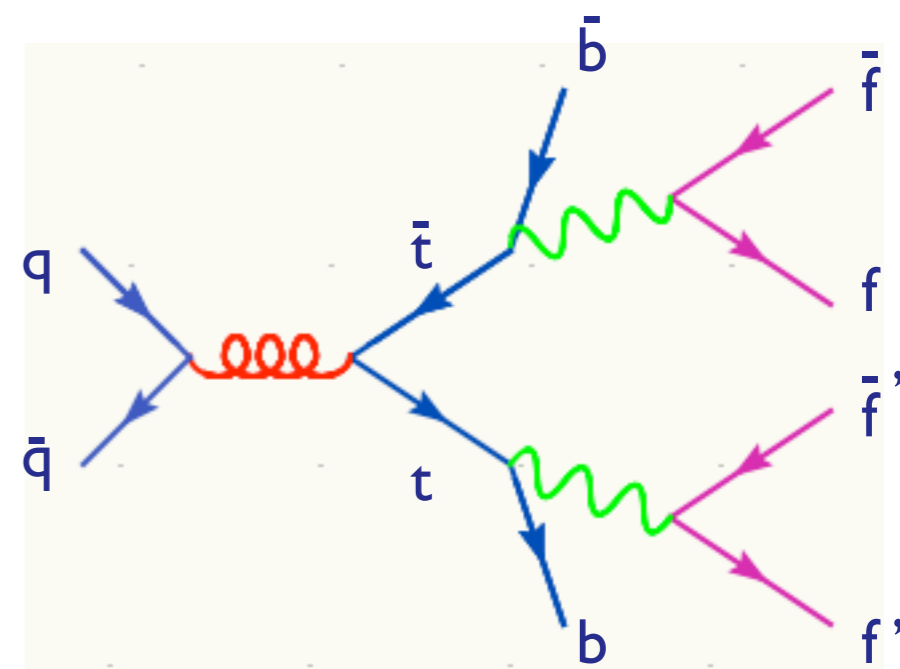
Is this strategy directly applicable to new heavy state searches?

A new challenge

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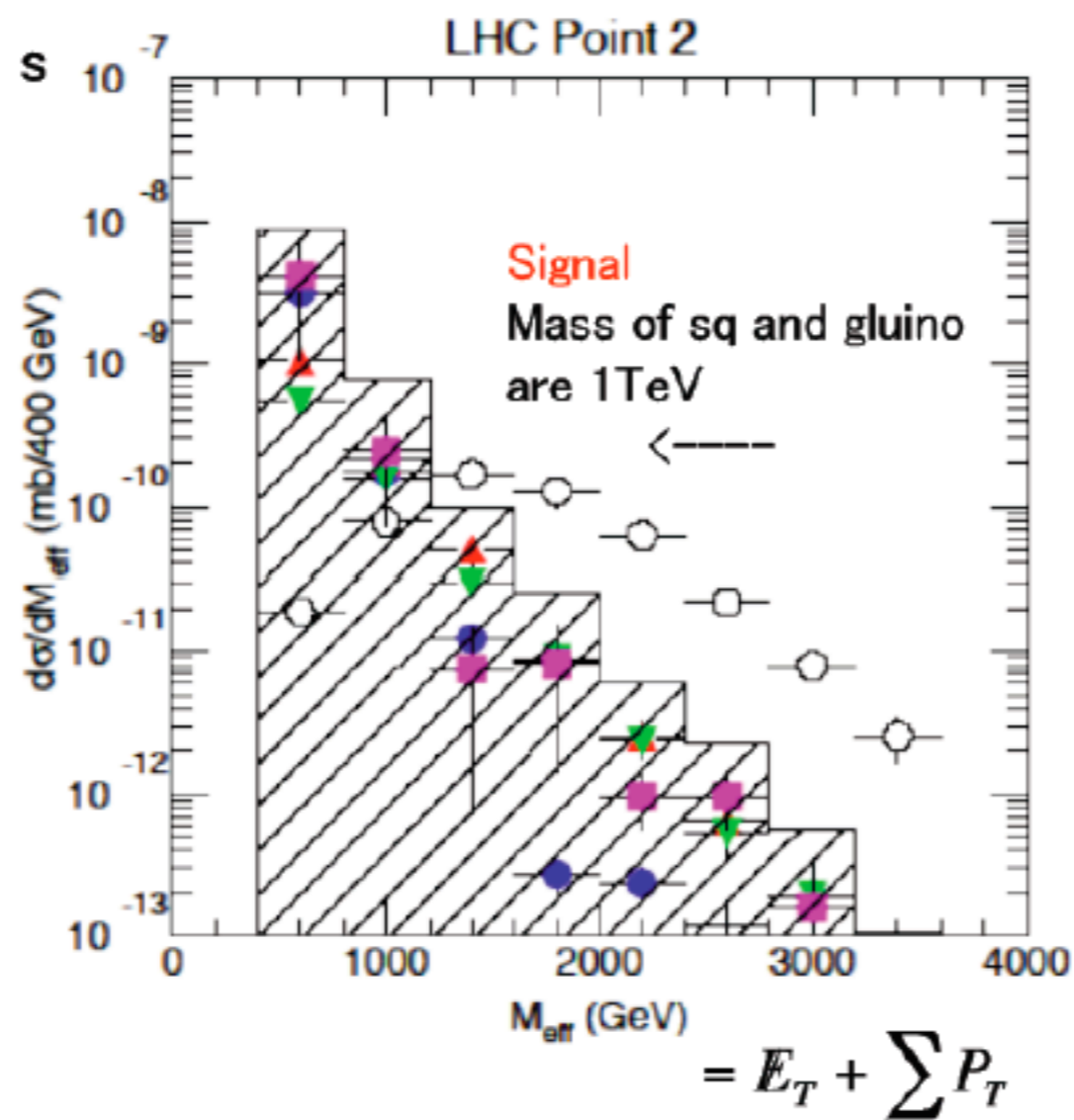


VS



Follow the same approach of CDF in 1995 to establish first evidence of an excess wrt to SM-top and then consistency with SM top production [$mt=174$, $t \rightarrow bl\nu$, $\sigma(tt)$], works for the SM Higgs, but in general beware that... **we don't know what to expect!**

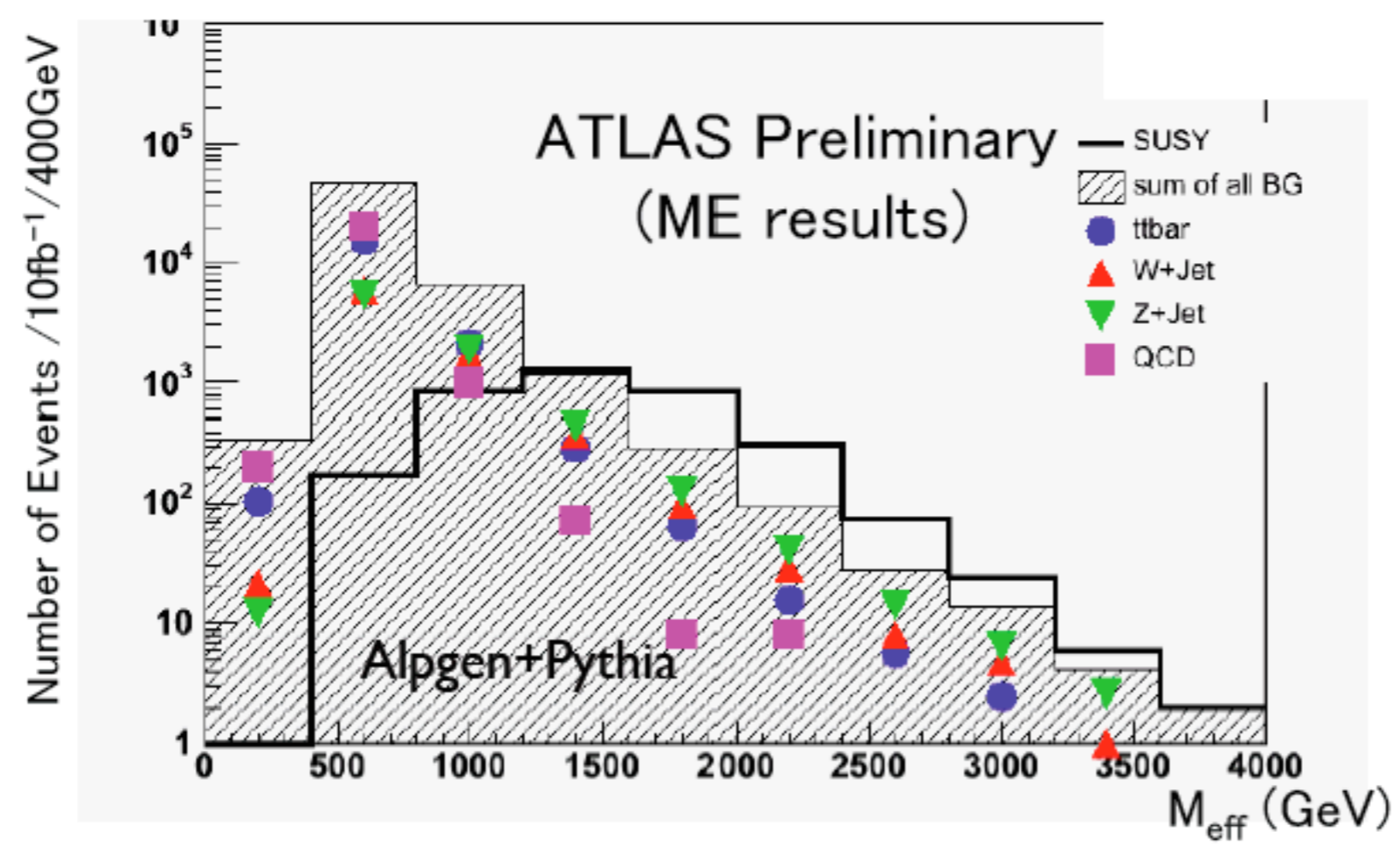
Example: early discovery SuperSymmetry at the LHC



“Old MC”

Background: $t \bar{t} + \text{jets}, (Z, W) + \text{jets}, \text{jets}$. Very difficult to estimate theoretically: many parton calculation ($2 \rightarrow 8$ gluons = 10 millions Feynman diagrams diagrams!!). Now MC's for this are available...

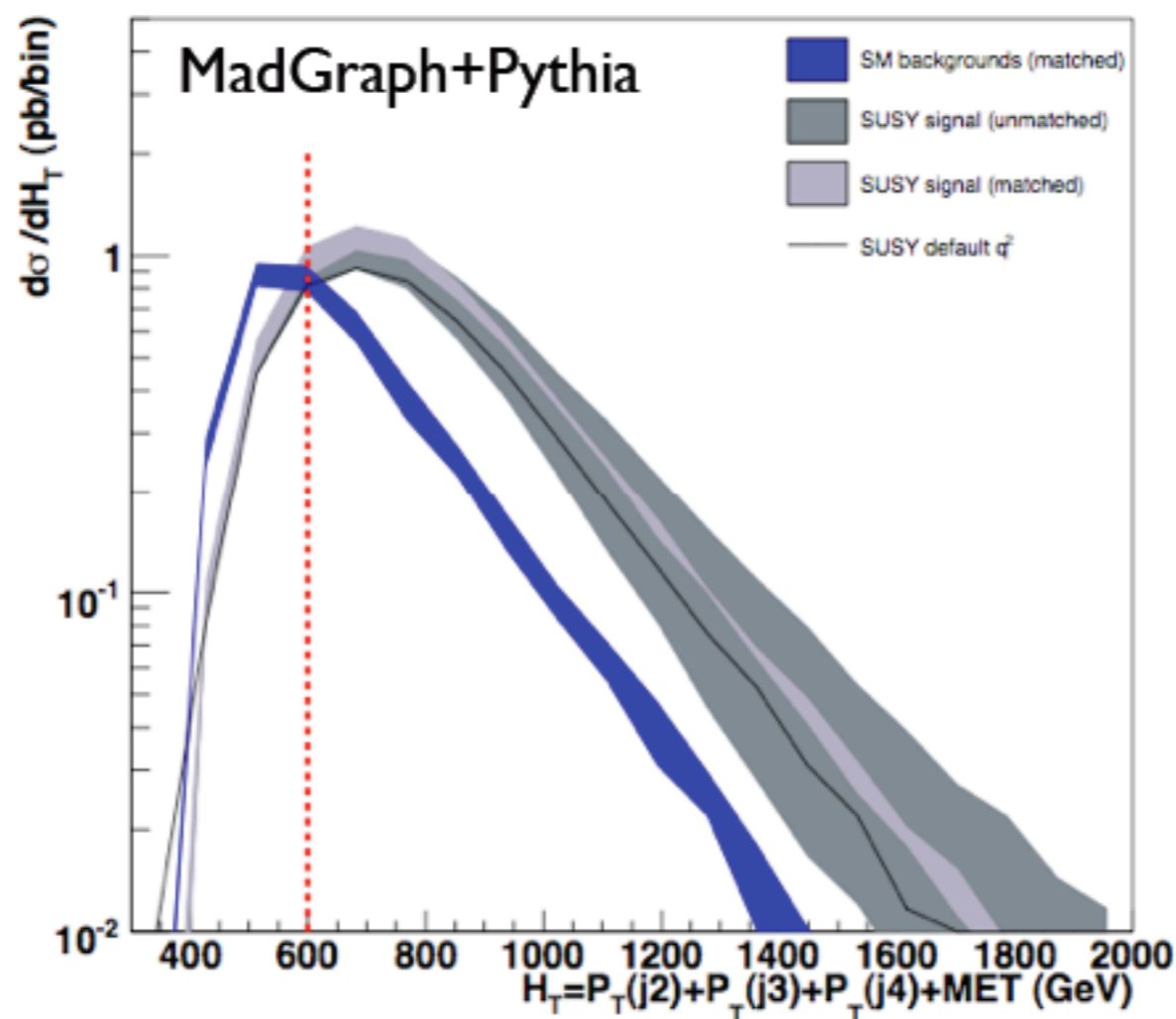
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“New MC
for the BKG”

Background: $t\bar{t}$ +jets, (Z,W)+jets, jets. Very difficult to estimate theoretically: many parton calculation ($2 \rightarrow 8$ gluons = 10 millions Feynman diagrams diagrams!!). Now MC's for this are available...

Example: early discovery SuperSymmetry at the LHC



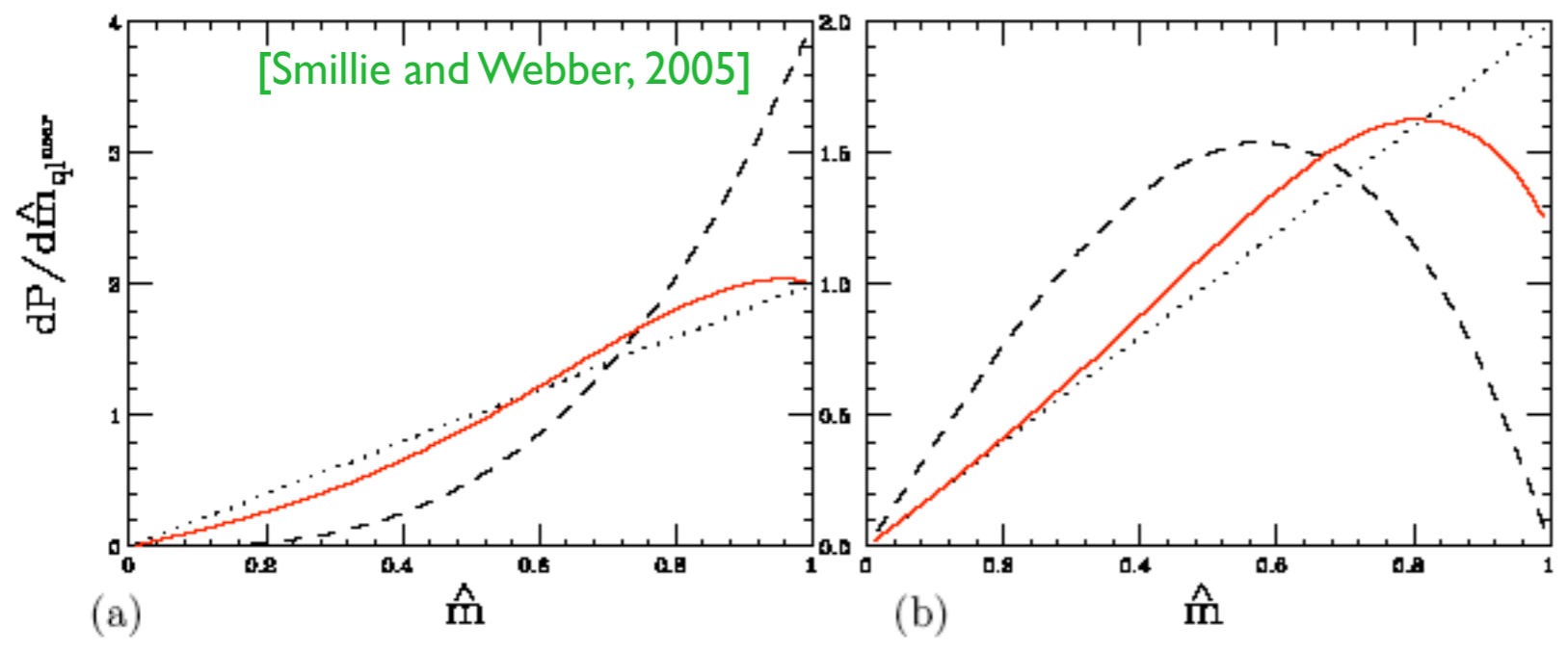
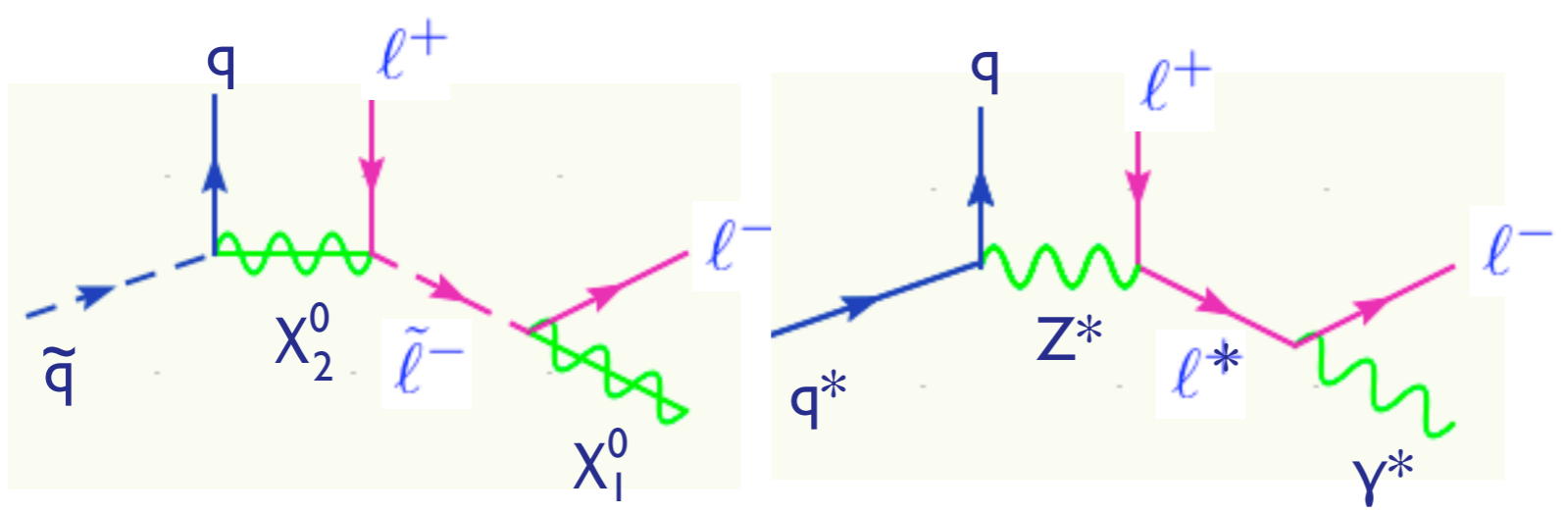
“New MC
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Texte: signal matched ME+PS. Predictability improved. Same theoretical status as the background.

Example: SUSY vs UED

Information on the mass of the intermediate states can be obtained through the study of kinematical edges. The shape of the edges can give information on the spin of the intermediate states. Compare for instance SUSY and UED:



Beware that most of the MC's make some of or all the following simplifications:

1. production and decay are factorized.
2. Spin is ignored.
3. Chains proceed only through $1 \rightarrow 2$ decays.
4. The narrow width approximation is employed.
5. Non-resonant diagrams are ignored.

Flexible and powerful ME tools are needed to check and in case go beyond the above approximations!

The path towards discoveries

$$\text{LHC physics} = \text{QCD} + \epsilon$$

1. Rediscover the known SM at the LHC (top's, W's, Z's) + jets.

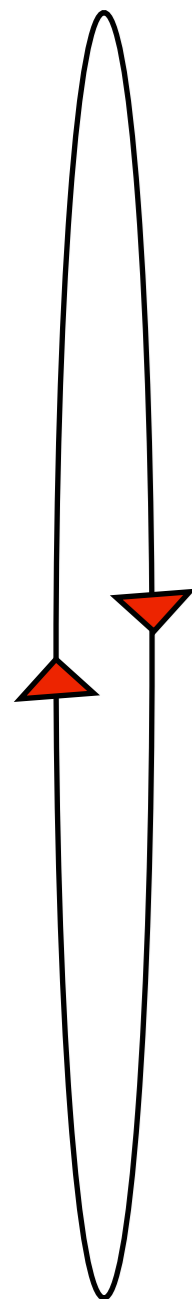
New regime for QCD. Exclusive **description** for rich and energetic final states with flexible MC to be validated and tuned to control samples. Shapes for multi-jet final states and normalization for key process important. Accurate **predictions** (NLO, NNLO) needed only for standard candle cross sections.

2. Identify excess(es) over SM

Importance of a good theoretical description depends on the nature of the physics discovered: from none (resonances) to fundamental (inclusive SUSY).

3. Identify the nature of BSM: from coarse information to measurements of mass spectrum, quantum numbers, couplings.

Not fully worked out strategy. Several approaches proposed (MARMOSSET, VISTA,...). Only in the final phase accurate QCD predictions and MC tools for SM as well as for the BSM signals will be needed.





Bottom-line

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No QCD \Rightarrow No Party

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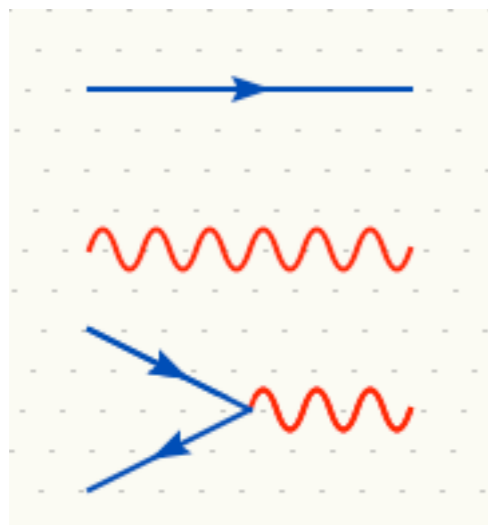
Minimal QCD: Basics

- From QED to QCD
- Color Algebra
- Helicity techniques and recursion
- Tools for tree-level calculations

From QED to QCD: abelian vs. non-abelian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi - eQ\bar{\psi}A\psi$$

where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$



$$= \frac{i}{\not{p} - m + i\epsilon} = i\frac{\not{p} + m}{p^2 - m^2 + i\epsilon}$$

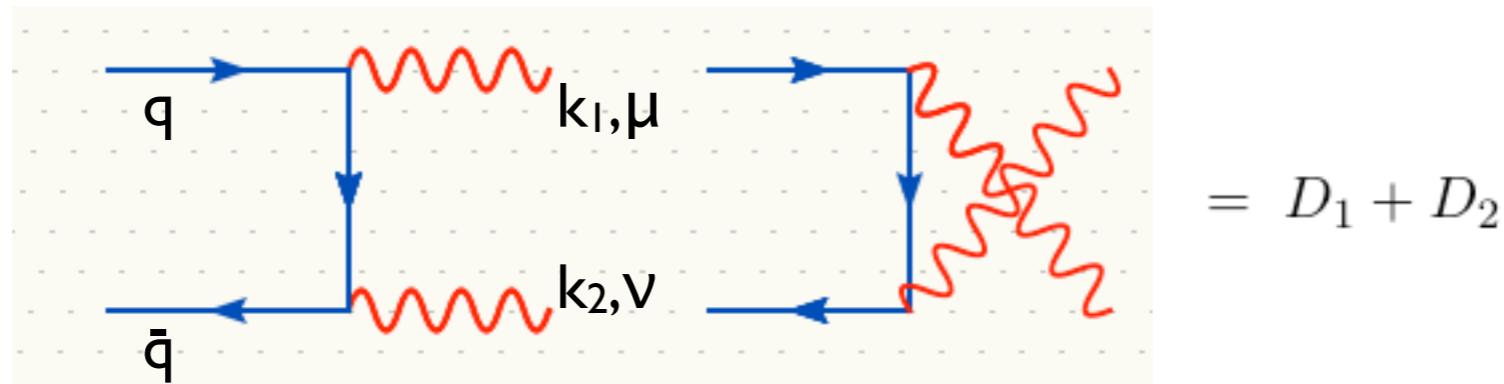
$$= -i\frac{g_{\mu\nu}}{p^2 + i\epsilon} \text{ (Feynman gauge)}$$

$$= -ie\gamma_{\mu}Q \quad (Q = -1 \text{ for the electron, } Q = 2/3 \text{ for the u-quark, etc})$$

From QED to QCD

We want to focus on how gauge invariance is realized in practice.

Let's start with the computation of a simple process $e^+e^- \rightarrow \gamma\gamma$. There are two diagrams:



$$\frac{i}{e^2} M_\gamma \equiv D_1 + D_2 = \bar{v}(\bar{q}) \not{\epsilon}_2 \frac{1}{\not{q} - \not{k}_1} \not{\epsilon}_1 u(q) + \bar{v}(\bar{q}) \not{\epsilon}_1 \frac{1}{\not{q} - \not{k}_2} \not{\epsilon}_2 u(q) \equiv M_{\mu\nu} \epsilon_1^\mu \epsilon_2^\nu$$

Gauge invariance demands that

$$\epsilon_2^\nu \partial^\mu M_{\mu\nu} = \epsilon_1^\mu \partial^\nu M_{\mu\nu} = 0$$

$M_\mu \equiv M_{\mu\nu} \epsilon_2^\nu$ is in fact the current that couples to the photon k_1 . Charge conservation requires $\partial_\mu M^\mu = 0$:

$$\begin{aligned} \partial_\mu M^\mu = 0 &\Rightarrow \frac{d}{dt} \int M^0 d^3x = \int \partial_0 M^0 d^3x \\ &= \int \vec{\nabla} \cdot \vec{M} d^3x = \int_{S \rightarrow \infty} \vec{M} \cdot d\vec{\Sigma} = 0 \end{aligned}$$

From QED to QCD

$$\begin{aligned}
 k_1^\mu \epsilon_2^\nu M_{\mu\nu} &= \bar{v}(\bar{q}) \not{\epsilon}_2 \frac{1}{\not{q} - \not{k}_1} (\not{k}_1 - \not{q}) u(q) + \bar{v}(\bar{q}) (\not{k}_1 - \not{q}) \frac{1}{\not{k}_1 - \not{q}} \not{\epsilon}_2 u(q) \\
 &= -\bar{v}(\bar{q}) \not{\epsilon}_2 u(q) + \bar{v}(\bar{q}) \not{\epsilon}_2 u(q) = 0
 \end{aligned}$$

Only the sum of the two diagrams is gauge invariant.

For the amplitude to be gauge invariant it is enough that one of the polarizations is longitudinal. The state of the other gauge boson is irrelevant.

Let's try now to generalize what we have done for SU(3). In this case we take the (anti-)quarks

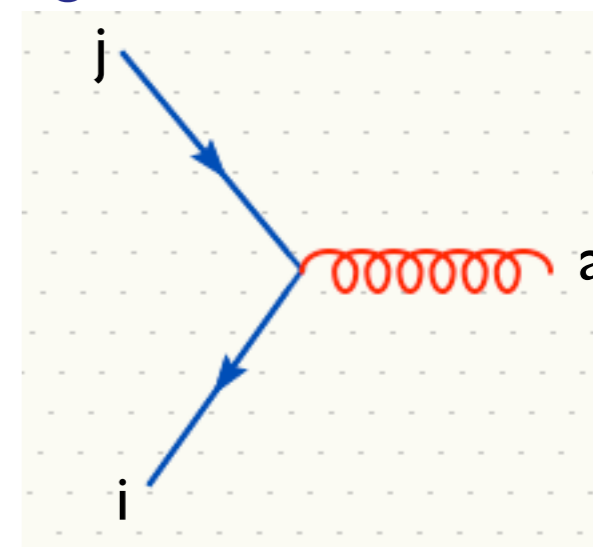
to be in the (anti-)fundamental representation of SU(3), 3 and 3*. Then the current is in a $3 \otimes 3^* = 1 \oplus 8$. The singlet is like a photon, so we identify the gluon with the octet and generalize the QED vertex to :

$$\text{with } [t^a, t^b] = i f^{abc} t^c \qquad -ig_s t_{ij}^a \gamma^\mu$$

So now let's calculate $qq \rightarrow gg$ and we obtain

$$\frac{i}{g_s^2} M_g \equiv (t^b t^a)_{ij} D_1 + (t^a t^b)_{ij} D_2$$

$$M_g = (t^a t^b)_{ij} M_\gamma - g^2 f^{abc} t_{ij}^c D_1$$



From QED to QCD

To satisfy gauge invariance we still need:

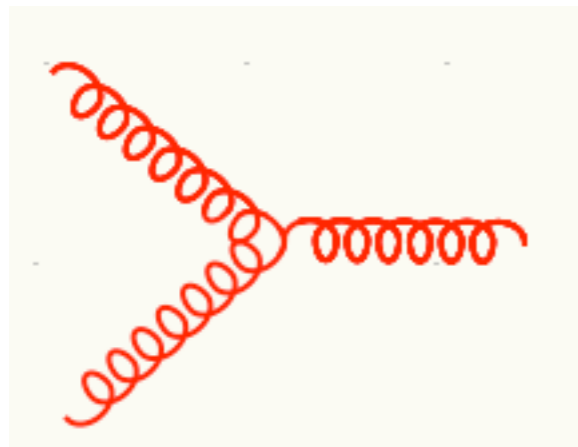
$$k_1^\mu \epsilon_2^\nu M_g^{\mu,\nu} = k_2^\nu \epsilon_1^\mu M_g^{\mu,\nu} = 0.$$

But in this case one piece is left out

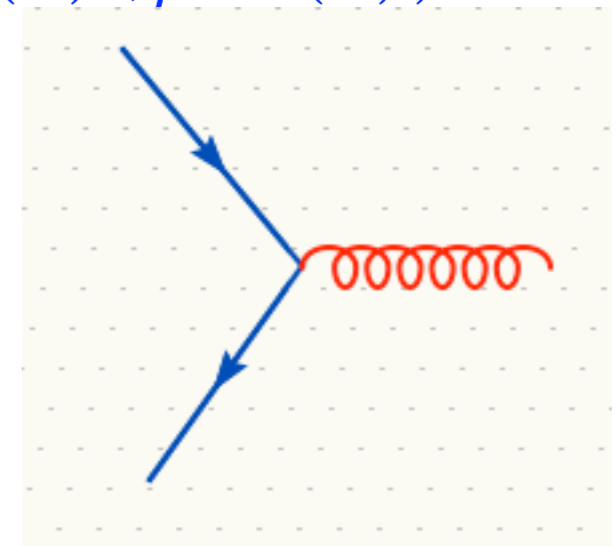
$$k_{1\mu} M_g^\mu = -g_s^2 f^{abc} t_{ij}^c \bar{v}_i(\bar{q}) \not{\epsilon}_2 u_i(q)$$

$$k_{1\mu} M_g^\mu = i(-g_s f^{abc} \epsilon_2^\mu) (-ig_s t_{ij}^c \bar{v}_i(\bar{q}) \gamma_\mu u_i(q))$$

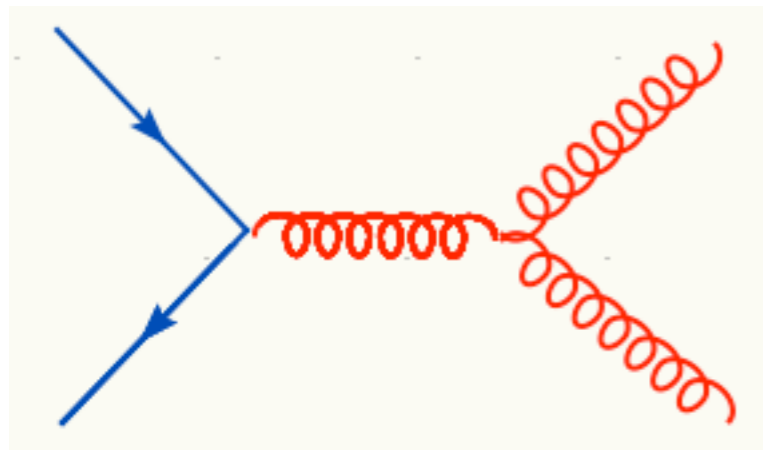
We indeed see that we interpret as the normal vertex times a new 3 gluon vertex:



$$-g_s f^{abc} V_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$



From QED to QCD



$$-ig_s^2 D_3 = \left(-ig_s t_{ij}^a \bar{v}_i(\bar{q}) \gamma^\mu u_j(q) \right) \times \left(\frac{-i}{p^2} \right) \times \left(-gf^{abc} V_{\mu\nu\rho}(-p, k_1, k_2) \epsilon_1^\nu(k_1) \epsilon_2^\rho(k_2) \right)$$

How do we write down the Lorentz part for this new interaction? We can impose

1. Lorentz invariance : only structure of the type $g_{\mu\nu} p_\rho$ are allowed
2. fully anti-symmetry : only structure of the type remain $g_{\mu_1\mu_2} (k_1)_{\mu_3}$ are allowed...
3. dimensional analysis : only one power of the momentum.

that uniquely constrain the form of the vertex:

$$V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = V_0 \left[(p_1 - p_2)_{\mu_3} g_{\mu_1\mu_2} + (p_2 - p_3)_{\mu_1} g_{\mu_2\mu_3} + (p_3 - p_1)_{\mu_2} g_{\mu_3\mu_1} \right]$$

With the above expression we obtain a contribution to the gauge variation:

$$k_1 \cdot D_3 = g^2 f^{abc} t^c V_0 \left[\bar{v}(\bar{q}) \not{\epsilon}_2 u(q) - \frac{k_2 \cdot \epsilon_2}{2k_1 \cdot k_2} \bar{v}(\bar{q}) \not{k}_1 u(q) \right]$$

The first term cancels the gauge variation of $D_1 + D_2$ if $V_0=1$, the second term is zero IFF the other gluon is physical!!

[EXERCISE]: Derive the form of the four-gluon vertex using the same heuristic method



The QCD Lagrangian

By direct inspection and by using the form non-abelian covariant derivation, we can check that indeed non-abelian gauge symmetry implies self-interactions. This is not surprising since the gluon itself is charged (In QED the photon is not!)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \sum_f \bar{\psi}_i^{(f)} (i\partial - m_f) \psi_i^{(f)} - \bar{\psi}_i^{(f)} (g_s t_{ij}^a A_a) \psi_j^{(f)}$$

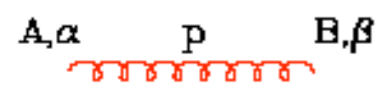
Gauge
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interact.

➔

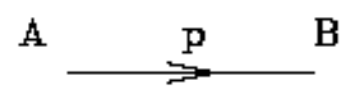
$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$

Matter

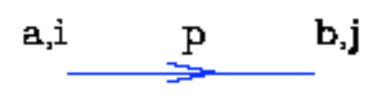
Interaction



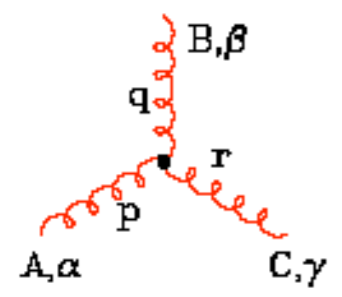
$$\delta^{AB} \left[-g^{\alpha\beta} + (1-\lambda) \frac{p^\alpha p^\beta}{p^2 + i\epsilon} \right] \frac{i}{p^2 + i\epsilon}$$



$$\delta^{AB} \frac{i}{(p^2 + i\epsilon)}$$

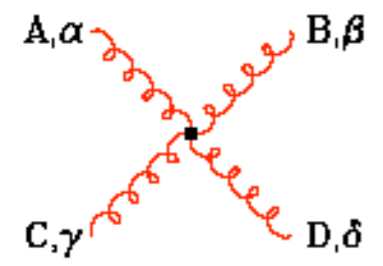


$$\delta^{ab} \frac{i}{(\not{p} - m + i\epsilon)_\mu}$$



$$-g f^{ABC} [(p-q)^\gamma g^{\alpha\beta} + (q-r)^\alpha g^{\beta\gamma} + (r-p)^\beta g^{\gamma\alpha}]$$

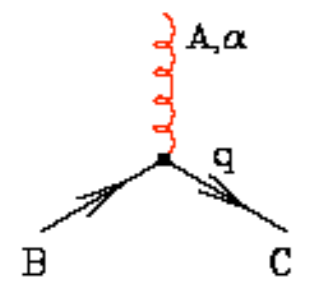
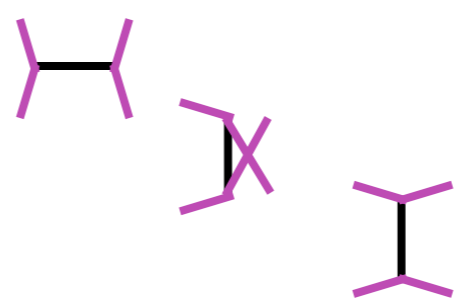
(all momenta incoming)



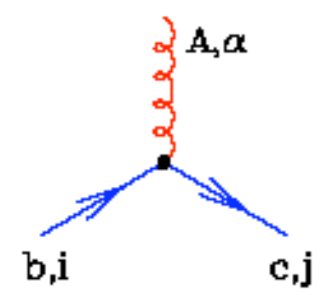
$$-ig^2 f^{XAC} f^{XBD} [g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\delta} g^{\beta\gamma}]$$

$$-ig^2 f^{XAD} f^{XBC} [g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\gamma} g^{\beta\delta}]$$

$$-ig^2 f^{XAB} f^{XCD} [g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\gamma}]$$



$$g f^{ABC} q^\alpha$$



$$-ig (t^A)_{cb} (\gamma^\alpha)_\mu$$

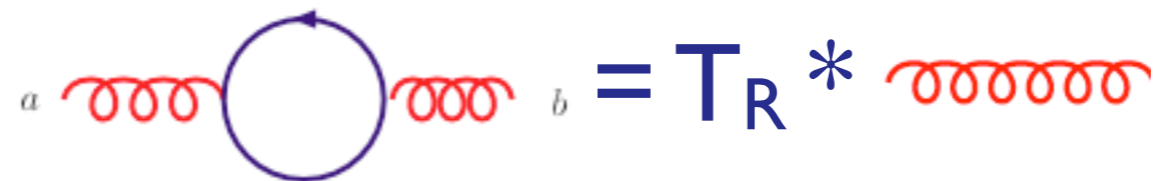


Color algebra

$$\text{Tr}(t^a) = 0$$



$$\text{Tr}(t^a t^b) = T_R \delta^{ab}$$

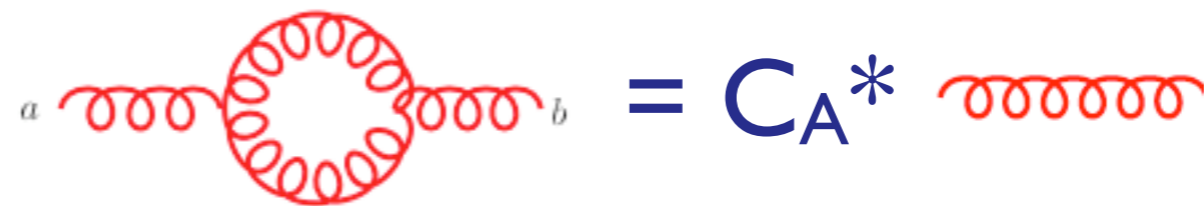


$$(t^a t^a)_{ij} = C_F \delta_{ij}$$



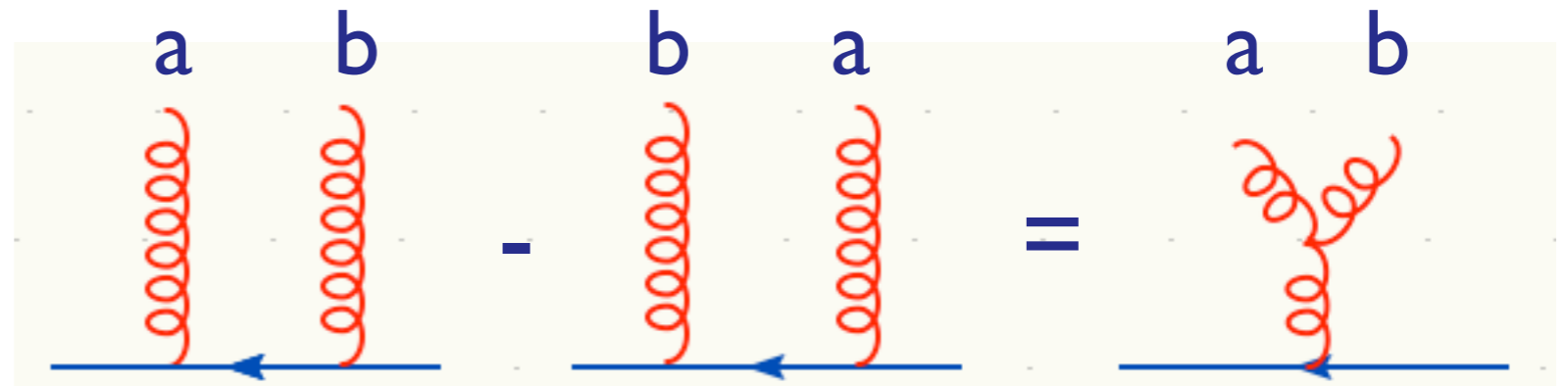
$$\sum_{cd} f^{acd} f^{bcd}$$

$$= (F^c F^c)_{ab} = C_A \delta_{ab}$$



Color algebra

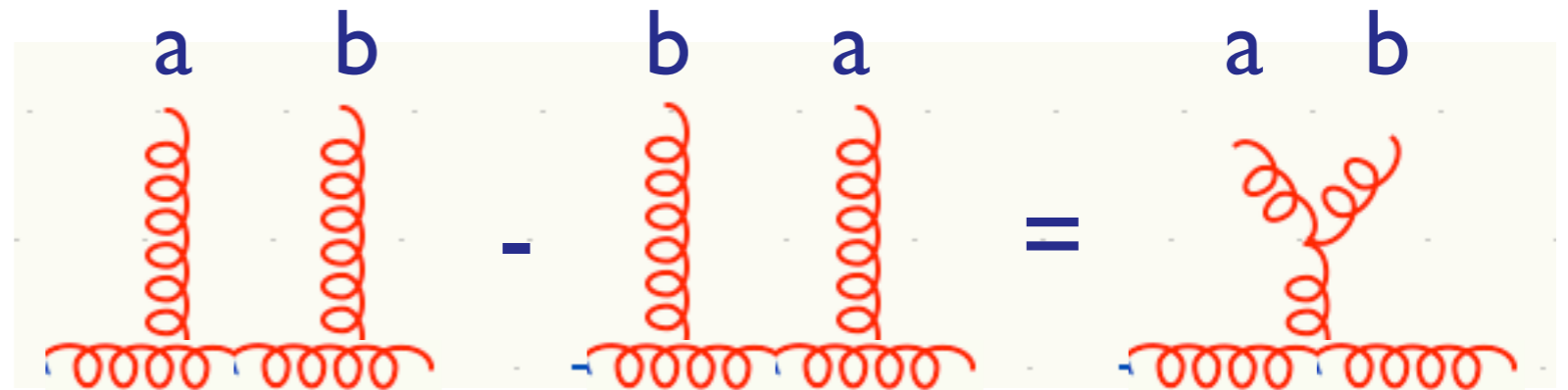
$$[t^a, t^b] = i f^{abc} t^c$$



Color algebra

$$[t^a, t^b] = i f^{abc} t^c$$

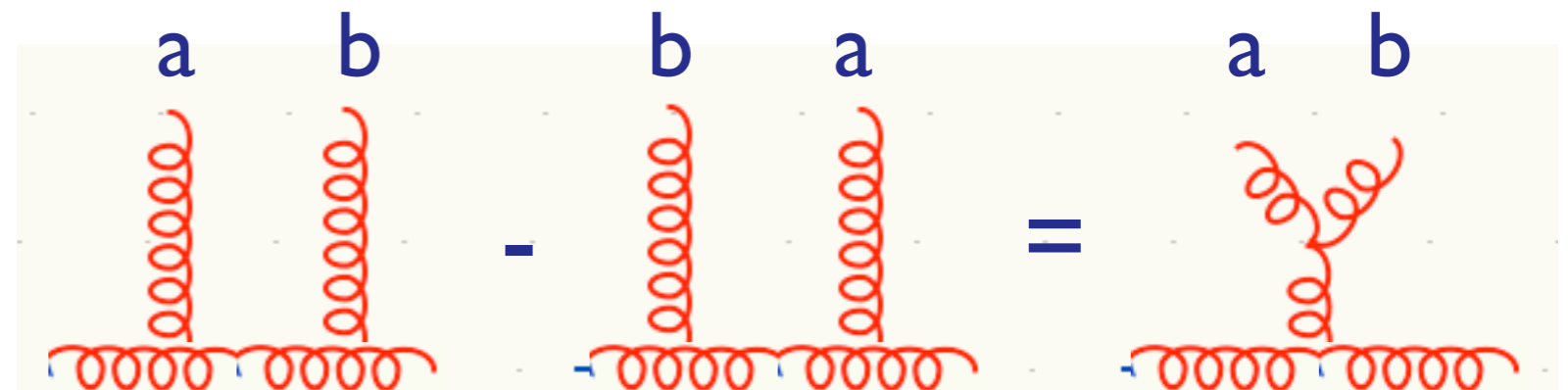
$$[F^a, F^b] = i f^{abc} F^c$$



Color algebra

$$[t^a, t^b] = i f^{abc} t^c$$

$$[F^a, F^b] = i f^{abc} F^c$$

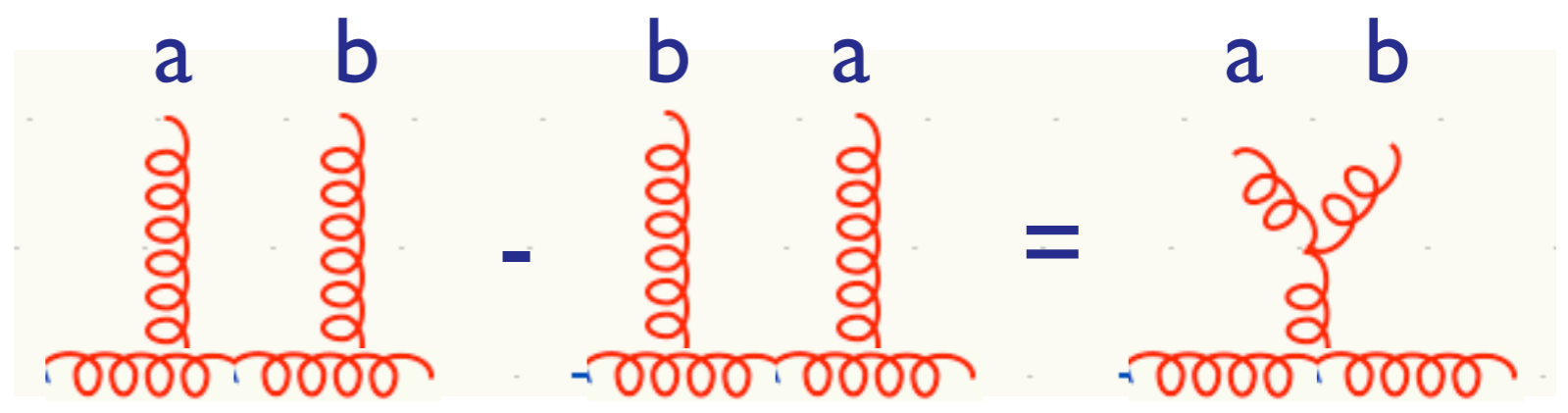


1-loop vertices

Color algebra

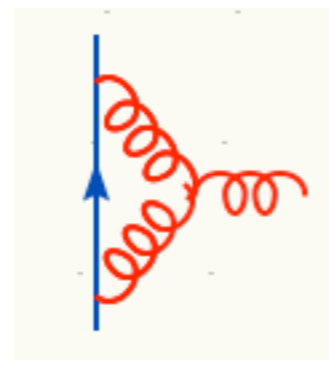
$$[t^a, t^b] = i f^{abc} t^c$$

$$[F^a, F^b] = i f^{abc} F^c$$

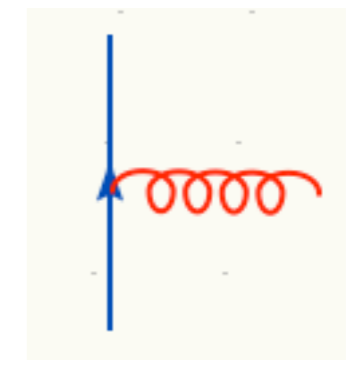


1-loop vertexes

$$i f^{abc} (t^b t^c)_{ij} = \frac{C_A}{2} t_{ij}^a$$



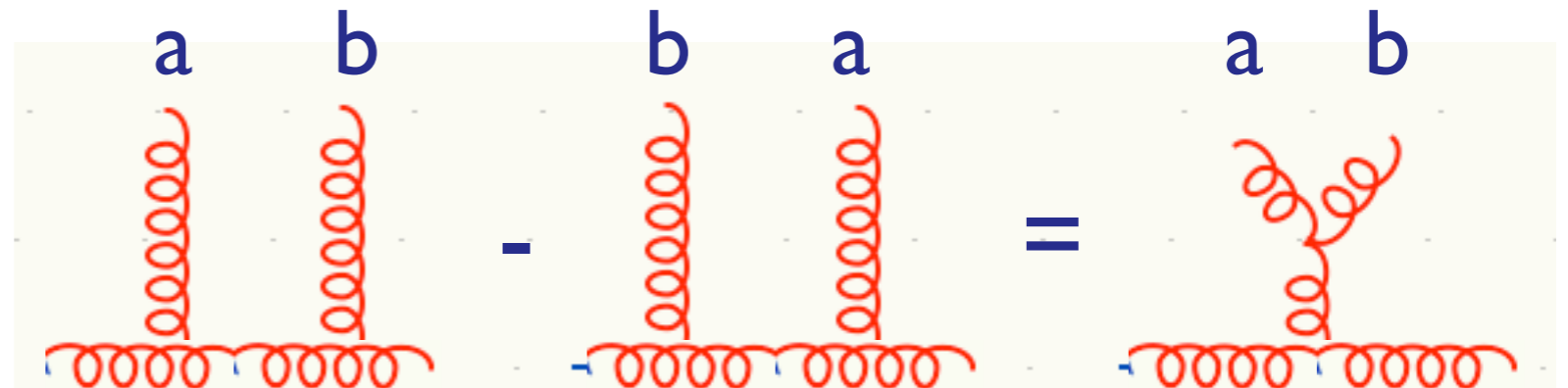
$$= C_A/2 *$$



Color algebra

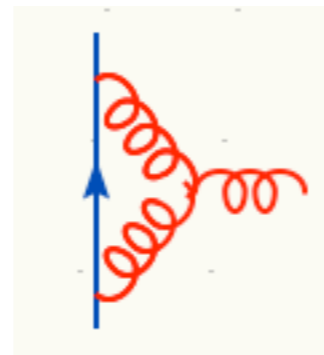
$$[t^a, t^b] = i f^{abc} t^c$$

$$[F^a, F^b] = i f^{abc} F^c$$

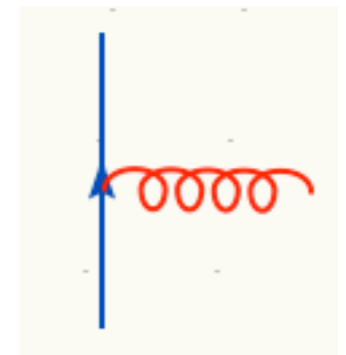


I-loop vertexes

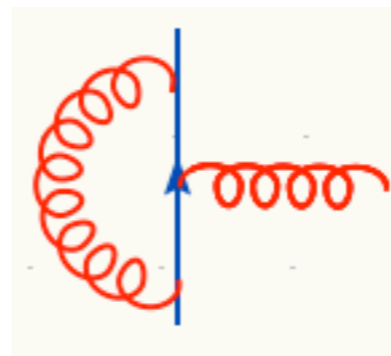
$$i f^{abc} (t^b t^c)_{ij} = \frac{C_A}{2} t_{ij}^a$$



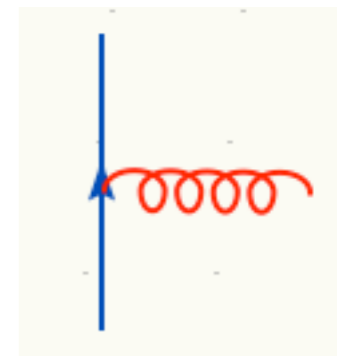
$$= C_A/2 *$$



$$(t^b t^a t^b)_{ij} = (C_F - \frac{C_A}{2}) t_{ij}^a$$

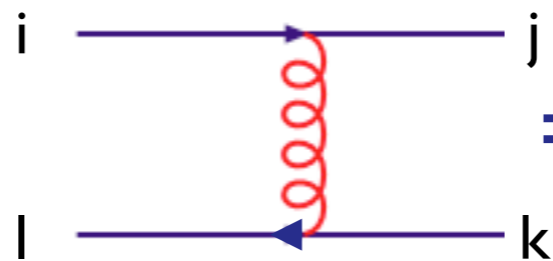


$$= -1/2/N_c *$$

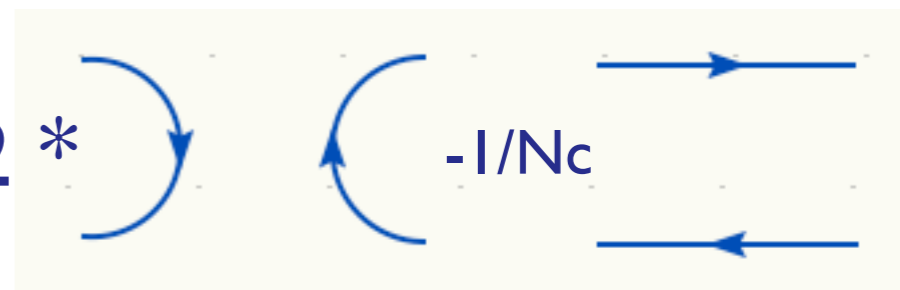


Color algebra: The Fierz identity

$$t_{ij}^a t_{kl}^a = \frac{1}{2} (\delta_{il} \delta_{kj} - \frac{1}{N_c} \delta_{ij} \delta_{kl})$$

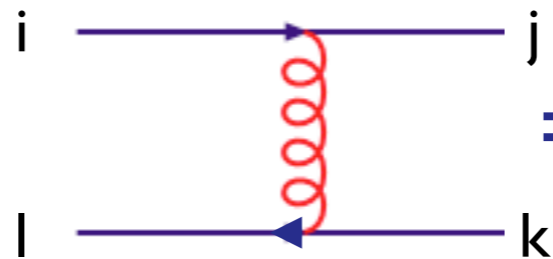


= 1/2 *

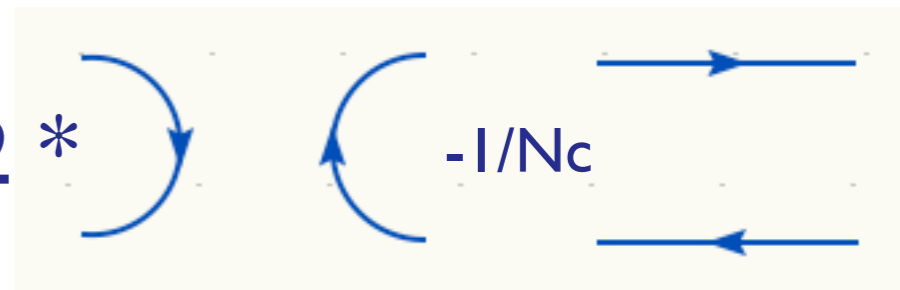


Color algebra: The Fierz identity

$$t_{ij}^a t_{kl}^a = \frac{1}{2} (\delta_{il} \delta_{kj} - \frac{1}{N_c} \delta_{ij} \delta_{kl})$$



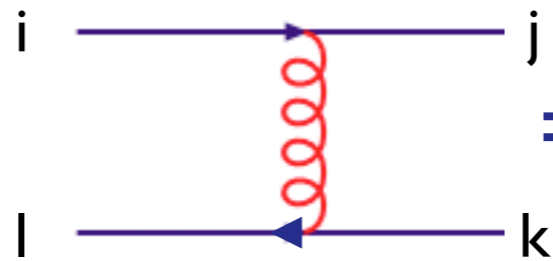
$$= 1/2 *$$



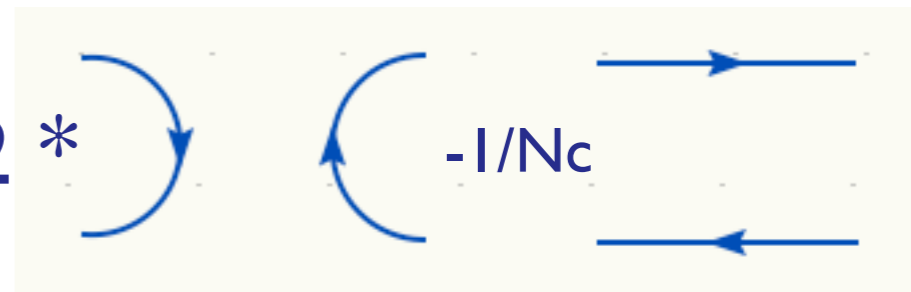
Problem: Show that the one-gluon exchange between quark-antiquark pair can be attractive or repulsive. Calculate the relative strength.

Color algebra: The Fierz identity

$$t_{ij}^a t_{kl}^a = \frac{1}{2} (\delta_{il} \delta_{kj} - \frac{1}{N_c} \delta_{ij} \delta_{kl})$$

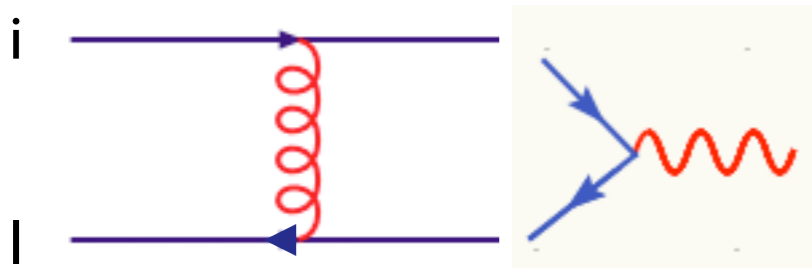


$$= 1/2 *$$

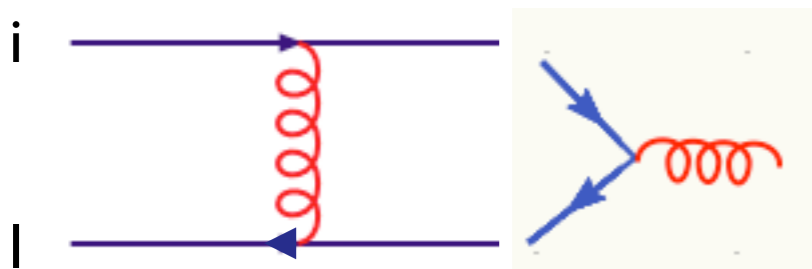


Problem: Show that the one-gluon exchange between quark-antiquark pair can be attractive or repulsive. Calculate the relative strength.

Solution: a q qb pair can be in a singlet state (photon) or in octet (gluon) : $3 \otimes 3 = 1 \oplus 8$



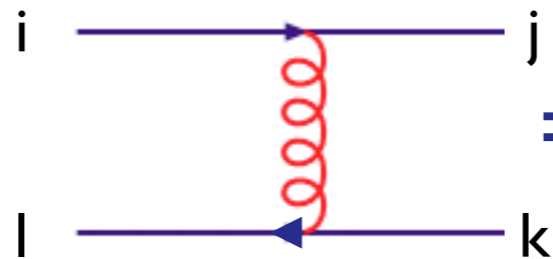
$$\frac{1}{2} (\delta_{ik} \delta_{lj} - \frac{1}{N_c} \delta_{ij} \delta_{lk}) \delta_{ki} = \frac{1}{2} \delta_{lj} (N_c - \frac{1}{N_c}) = C_F \delta_{lj}$$



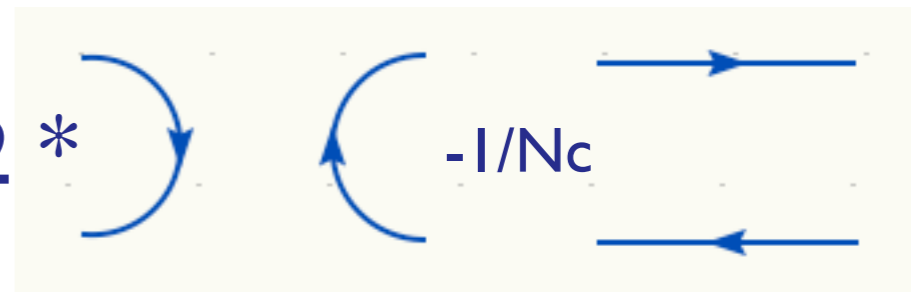
$$\frac{1}{2} (\delta_{ik} \delta_{lj} - \frac{1}{N_c} \delta_{ij} \delta_{lk}) t_{ki}^a = -\frac{1}{2N_c} t_{lj}^a$$

Color algebra: The Fierz identity

$$t_{ij}^a t_{kl}^a = \frac{1}{2} (\delta_{il} \delta_{kj} - \frac{1}{N_c} \delta_{ij} \delta_{kl})$$

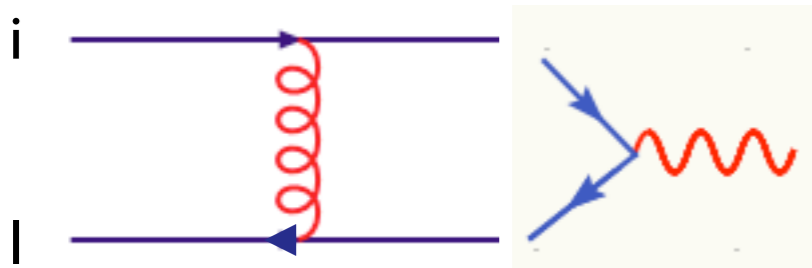


$$= 1/2 *$$



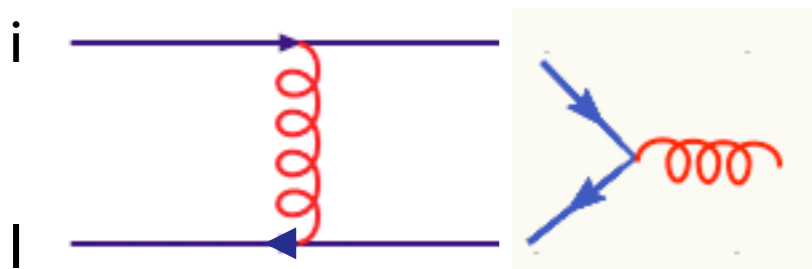
Problem: Show that the one-gluon exchange between quark-antiquark pair can be attractive or repulsive. Calculate the relative strength.

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$$\frac{1}{2} (\delta_{ik} \delta_{lj} - \frac{1}{N_c} \delta_{ij} \delta_{lk}) \delta_{ki} = \frac{1}{2} \delta_{lj} (N_c - \frac{1}{N_c}) = C_F \delta_{lj}$$

>0, attractive



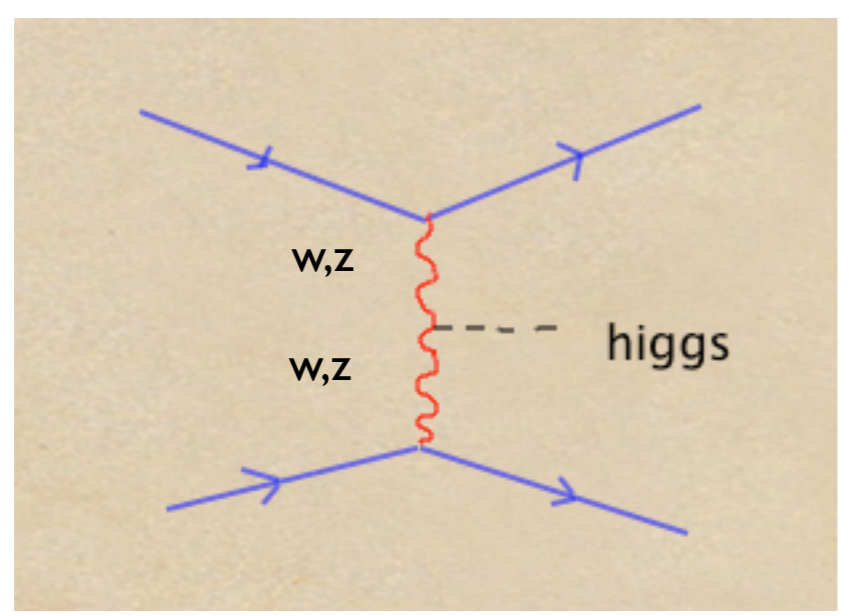
$$\frac{1}{2} (\delta_{ik} \delta_{lj} - \frac{1}{N_c} \delta_{ij} \delta_{lk}) t_{ki}^a = -\frac{1}{2N_c} t_{lj}^a$$

<0, repulsive

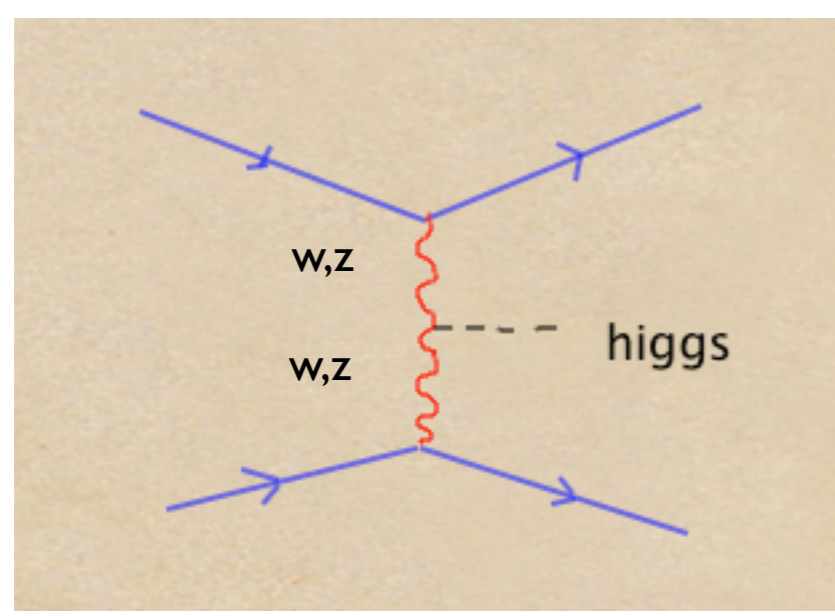


Example: WBF fusion

Example: WWBF fusion

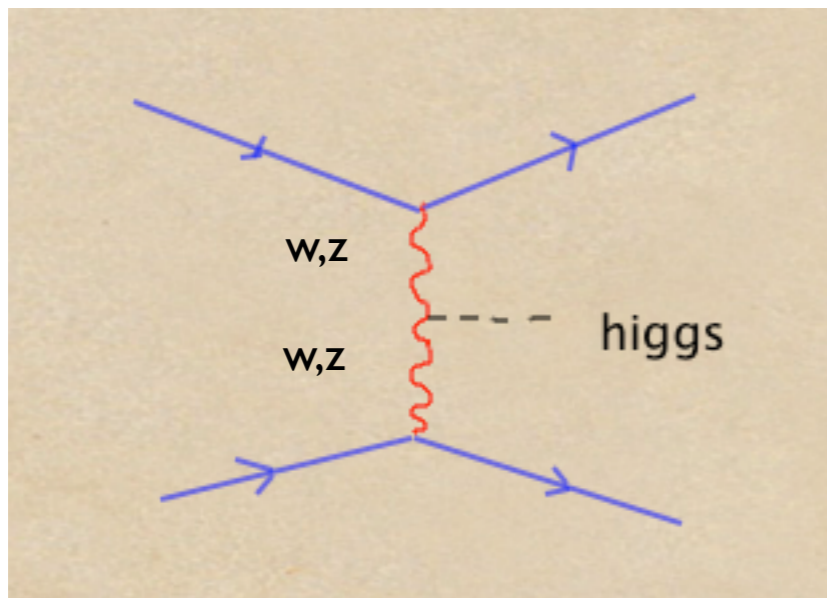


Example: WBF fusion



Facts:

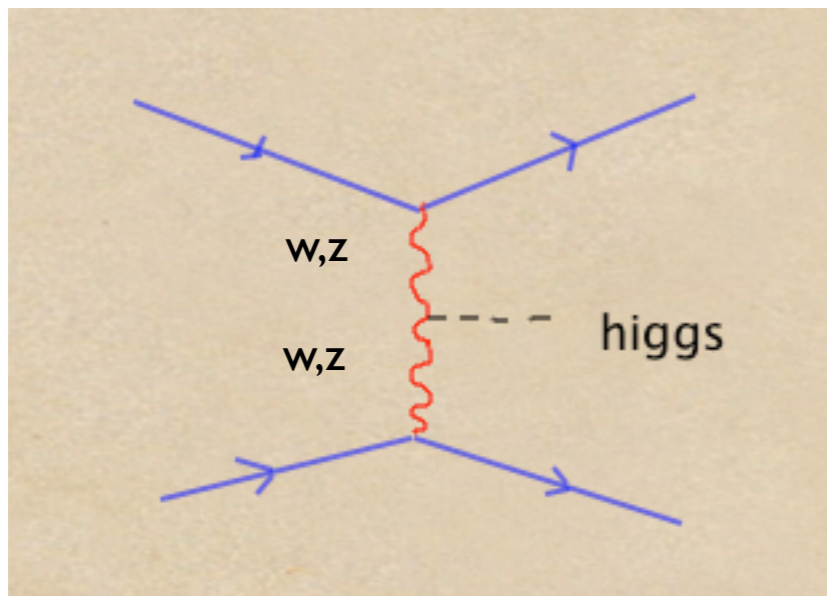
Example: WBF fusion



Facts:

1. Important channel for light Higgs both for discovery and measurement

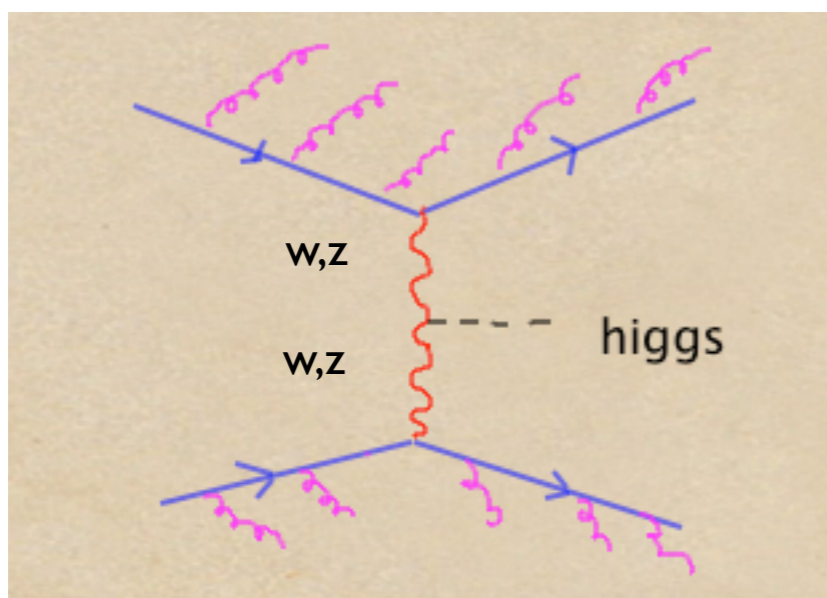
Example: WBF fusion



Facts:

1. Important channel for light Higgs both for discovery and measurement
2. Color singlet exchange in the t-channel

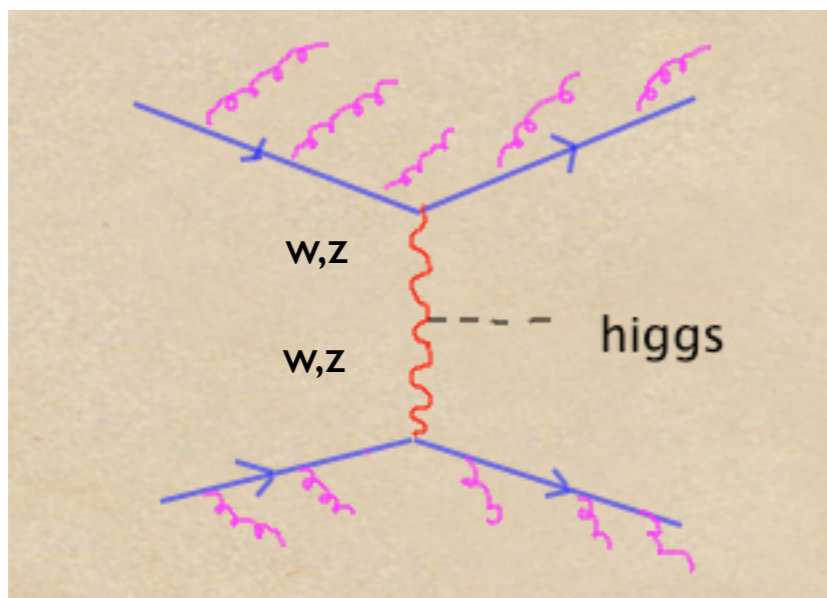
Example: WBF fusion



Facts:

1. Important channel for light Higgs both for discovery and measurement
2. Color singlet exchange in the t-channel

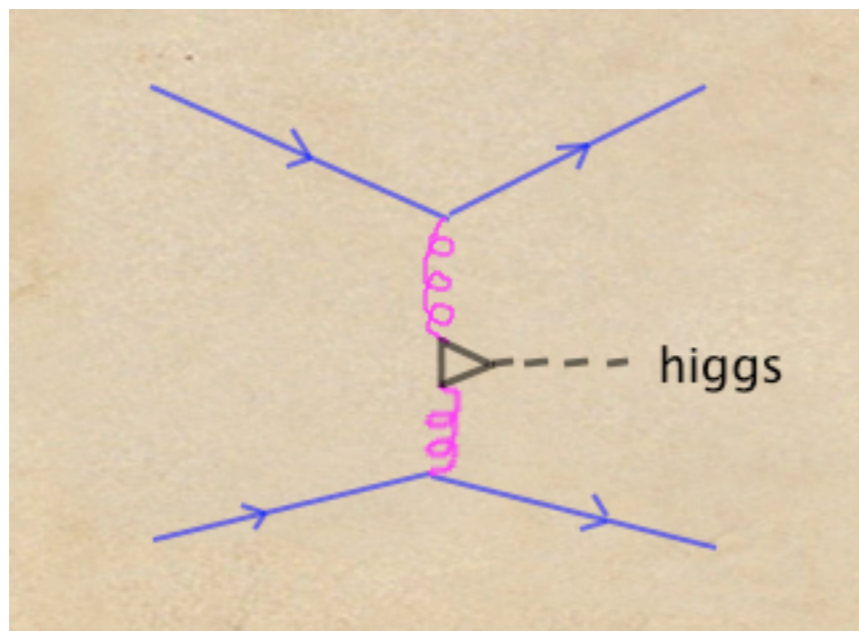
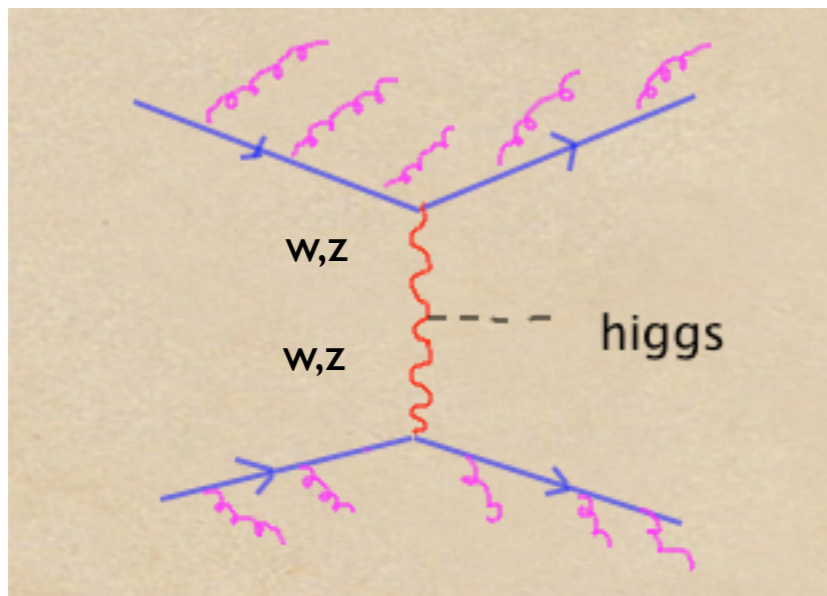
Example: WBF fusion



Facts:

1. Important channel for light Higgs both for discovery and measurement
2. Color singlet exchange in the t-channel
3. Characteristic signature:
forward-backward jets + RAPIDITY GAP

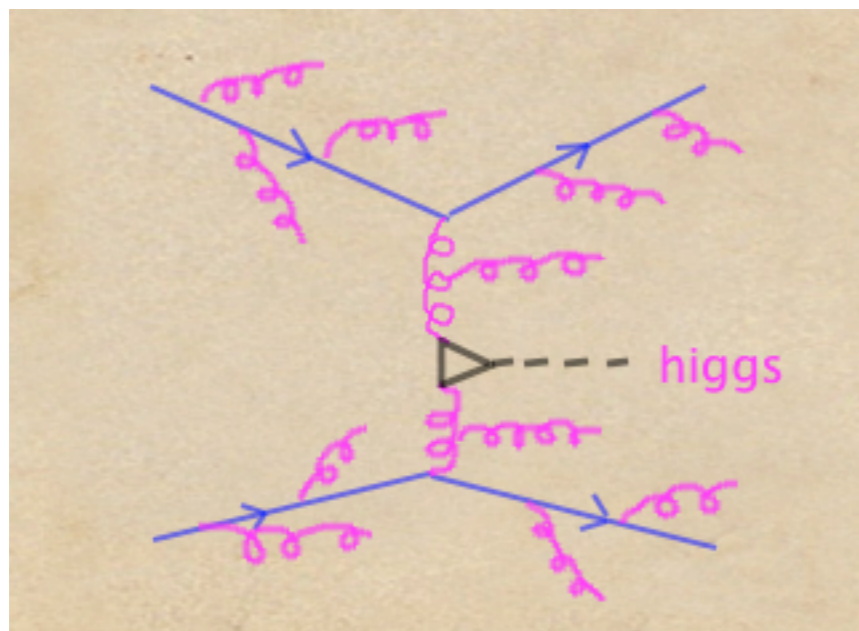
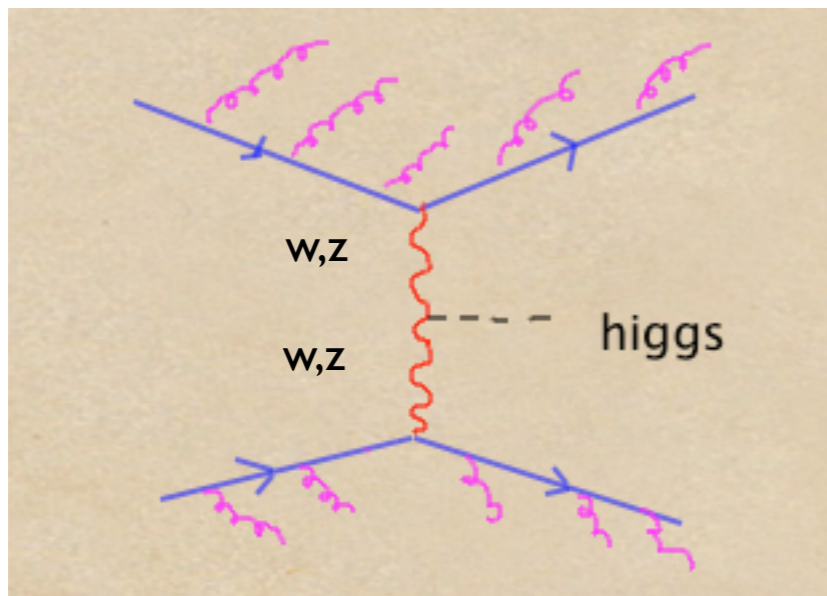
Example: WBF fusion



Facts:

1. Important channel for light Higgs both for discovery and measurement
2. Color singlet exchange in the t-channel
3. Characteristic signature: forward-backward jets + RAPIDITY GAP
4. QCD production is a background to precise measurements of couplings

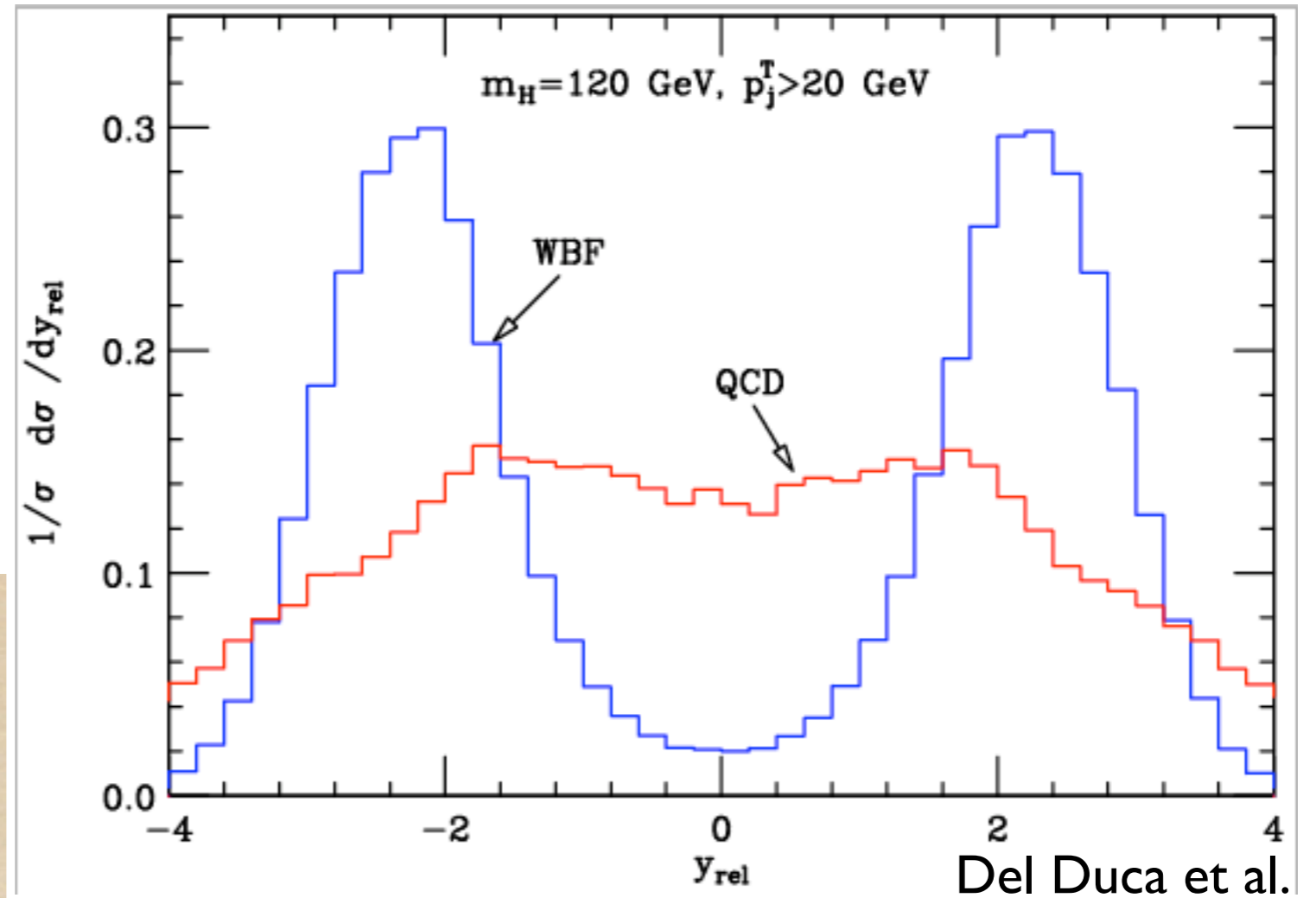
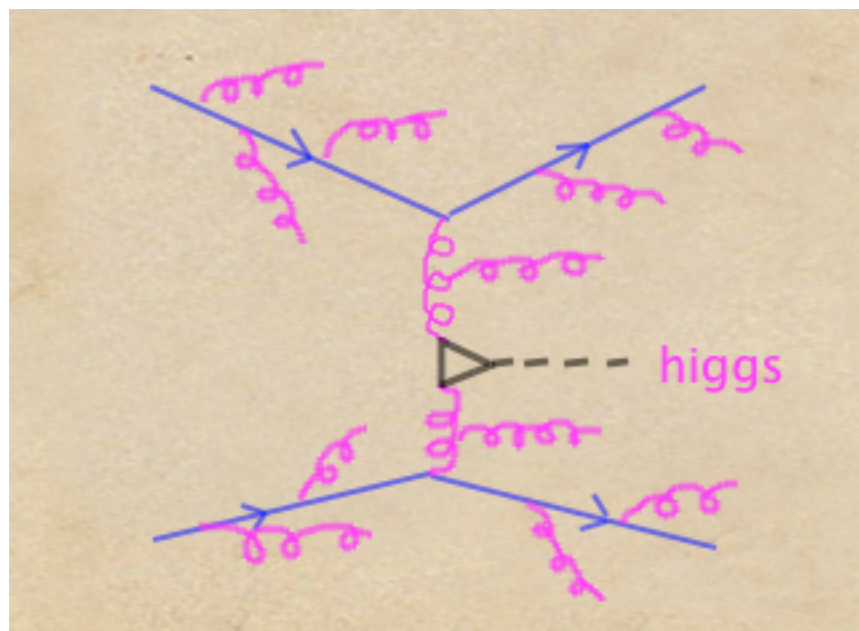
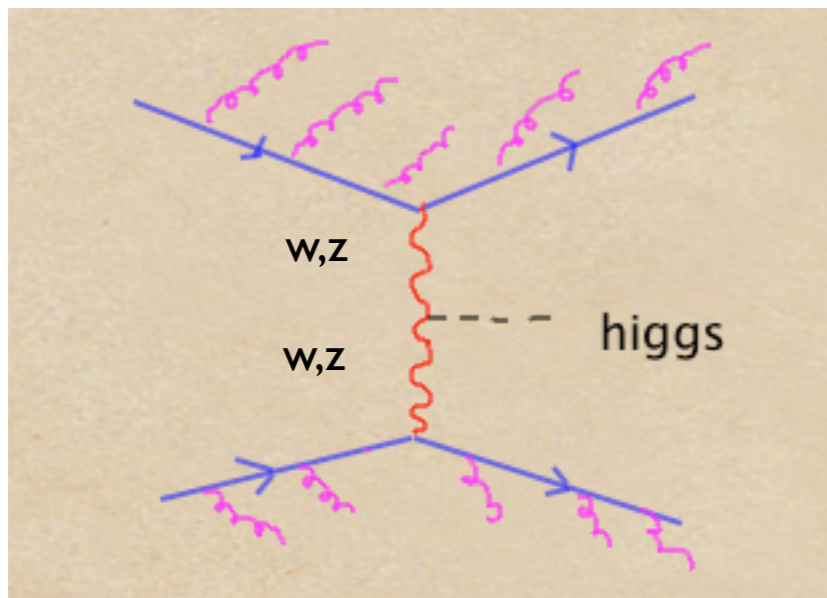
Example: WBF fusion



Facts:

1. Important channel for light Higgs both for discovery and measurement
2. Color singlet exchange in the t-channel
3. Characteristic signature: forward-backward jets + RAPIDITY GAP
4. QCD production is a background to precise measurements of couplings

Example: WBF fusion



Third jet distribution



Example: WBF fusion

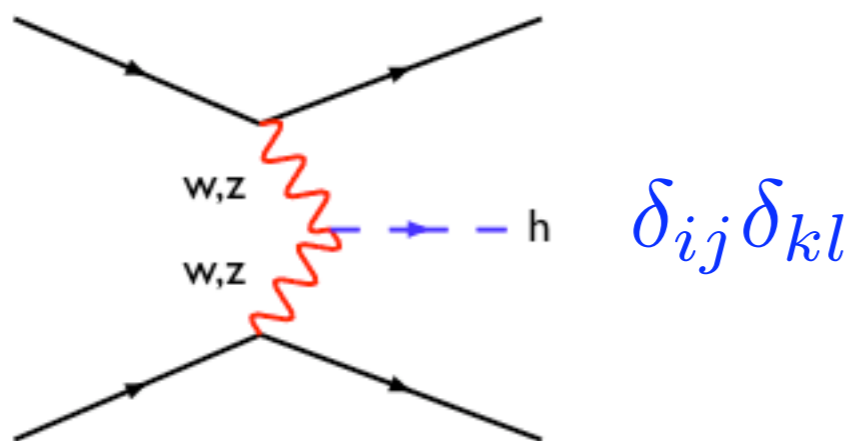


Example: WBF fusion

Consider WBF: at LO there is no exchange of color between the quark lines:

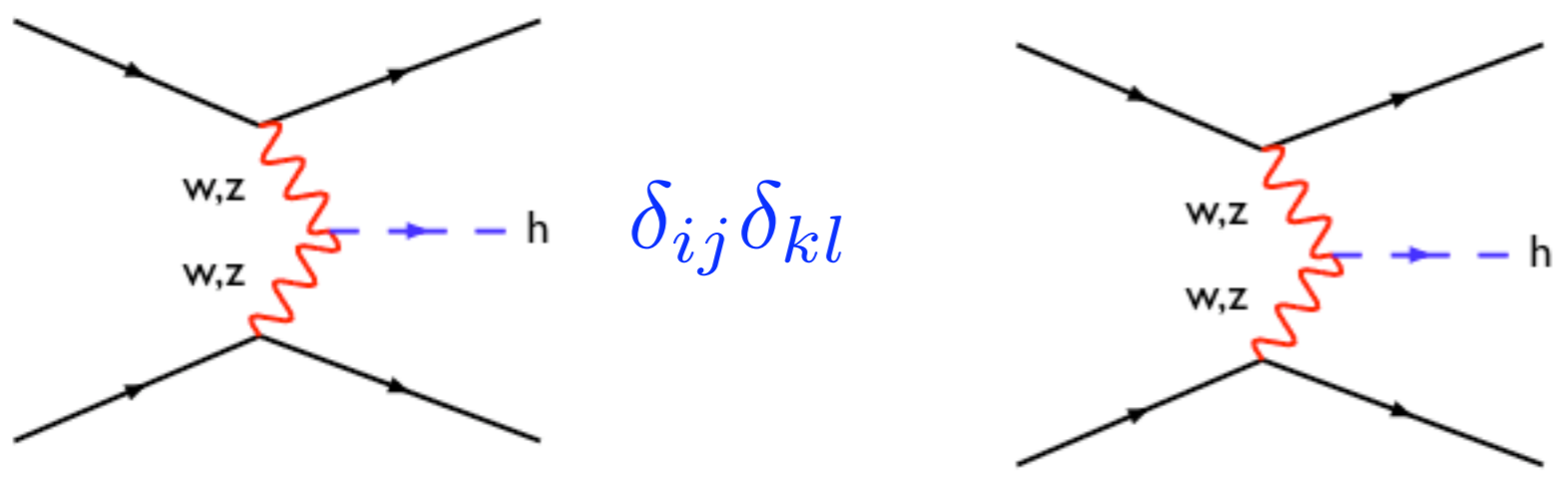
Example: WBF fusion

Consider WBF: at LO there is no exchange of color between the quark lines:



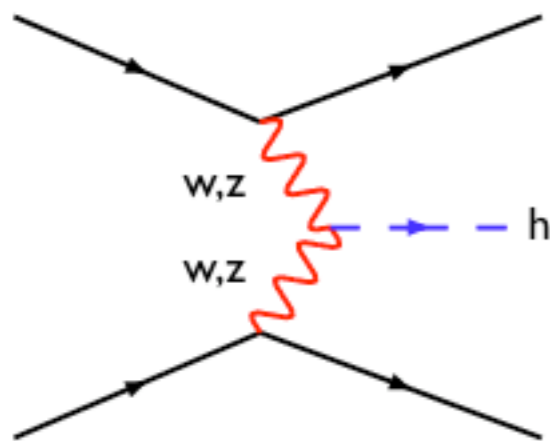
Example: WBF fusion

Consider WBF: at LO there is no exchange of color between the quark lines:

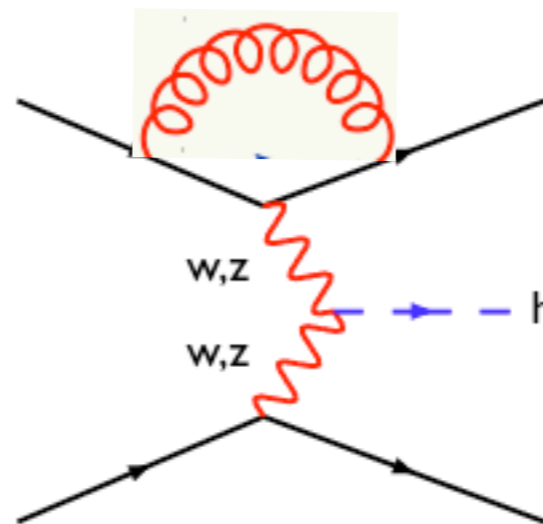


Example: WBF fusion

Consider WBF: at LO there is no exchange of color between the quark lines:

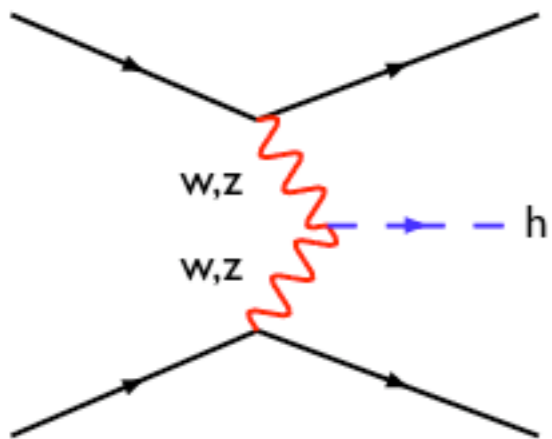


$$\delta_{ij} \delta_{kl}$$

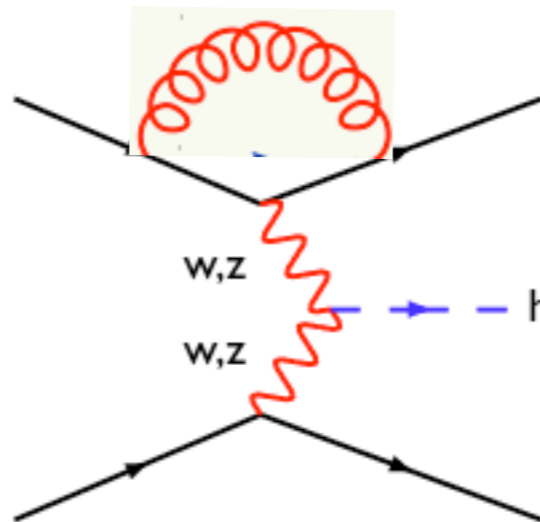


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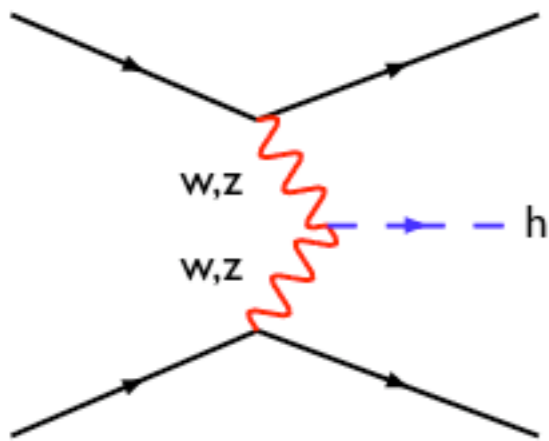
$$\delta_{ij} \delta_{kl}$$



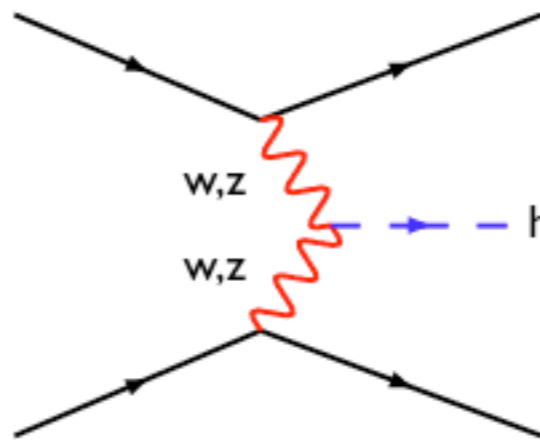
$$C_F \delta_{ij} \delta_{kl} \Rightarrow$$
$$M_{\text{tree}} M_{1\text{-loop}}^* = C_F N_c^2 \simeq N_c^3$$

Example: WBF fusion

Consider WBF: at LO there is no exchange of color between the quark lines:



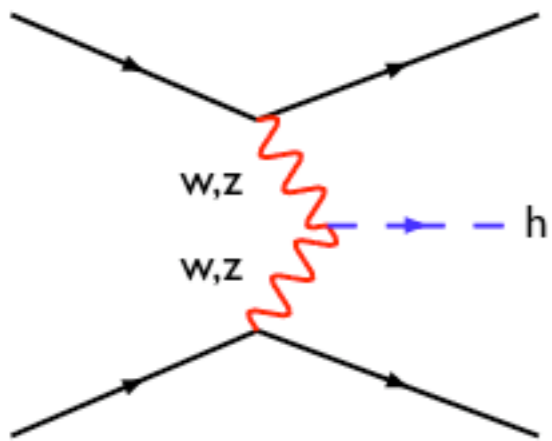
$$\delta_{ij} \delta_{kl}$$



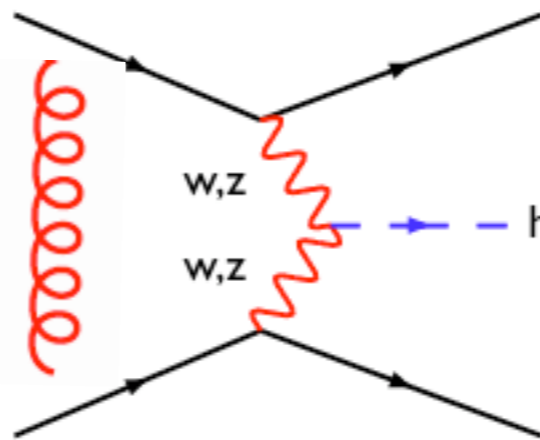
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Example: WBF fusion

Consider WBF: at LO there is no exchange of color between the quark lines:



$$\delta_{ij} \delta_{kl}$$

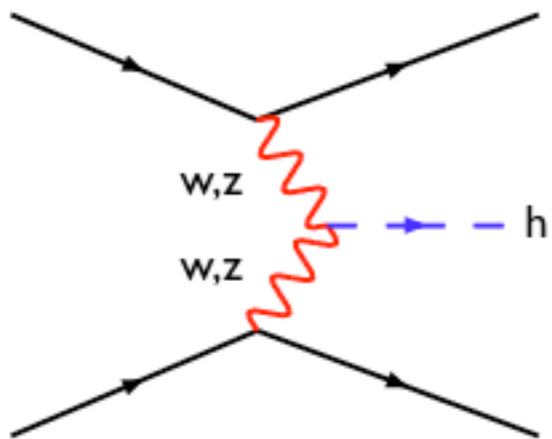


$$C_F \delta_{ij} \delta_{kl} \Rightarrow$$

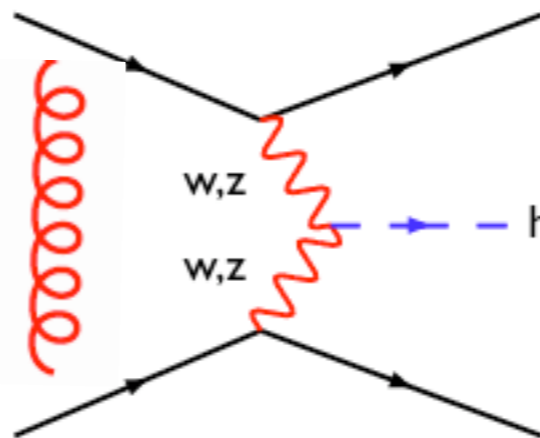
$$M_{\text{tree}} M_{1\text{-loop}}^* = C_F N_c^2 \simeq N_c^3$$

Example: WBF fusion

Consider WBF: at LO there is no exchange of color between the quark lines:



$$\delta_{ij} \delta_{kl}$$



$$C_F \delta_{ij} \delta_{kl} \Rightarrow$$

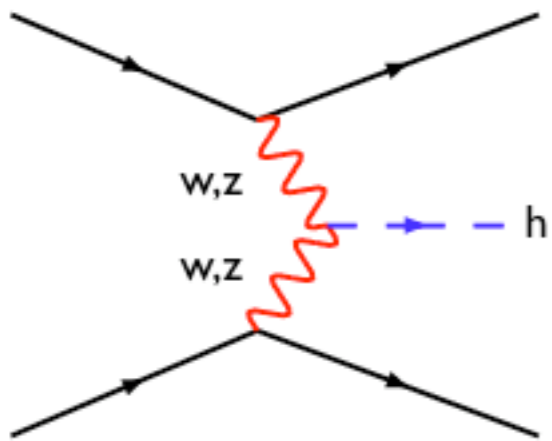
$$M_{\text{tree}} M_{1\text{-loop}}^* = C_F N_c^2 \simeq N_c^3$$

$$\frac{1}{2} (\delta_{ik} \delta_{lj} - \frac{1}{N_c} \delta_{ij} \delta_{kl}) \Rightarrow$$

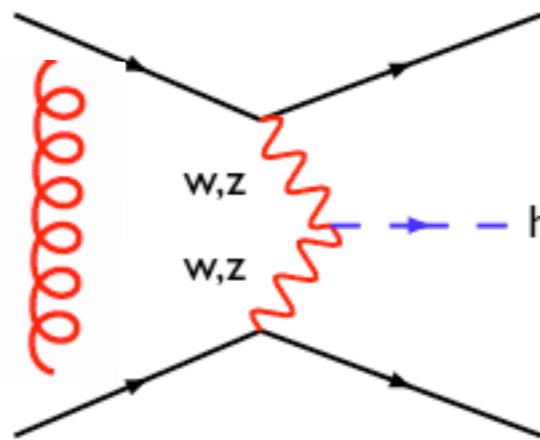
$$M_{\text{tree}} M_{1\text{-loop}}^* = 0$$

Example: WBF fusion

Consider WBF: at LO there is no exchange of color between the quark lines:



$$\delta_{ij} \delta_{kl}$$



$$C_F \delta_{ij} \delta_{kl} \Rightarrow$$

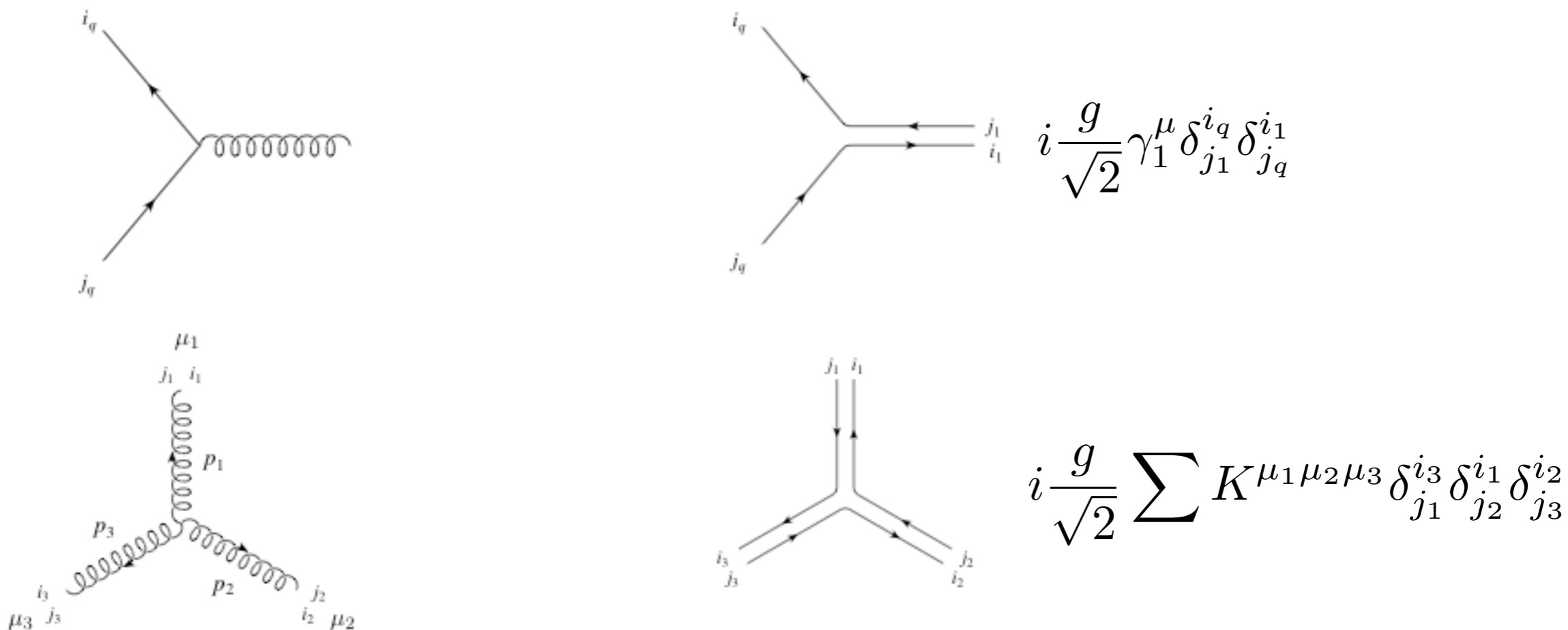
$$M_{\text{tree}} M_{1\text{-loop}}^* = C_F N_c^2 \simeq N_c^3$$

$$\frac{1}{2} (\delta_{ik} \delta_{lj} - \frac{1}{N_c} \delta_{ij} \delta_{kl}) \Rightarrow$$

$$M_{\text{tree}} M_{1\text{-loop}}^* = 0$$

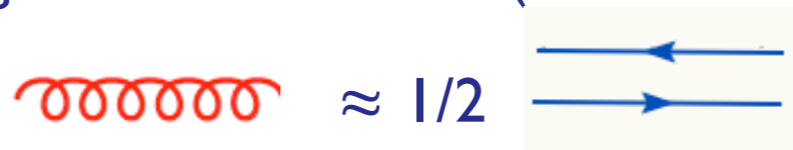
Also at NLO there is no color exchange! With one little exception....

Color algebra: 't Hooft double line



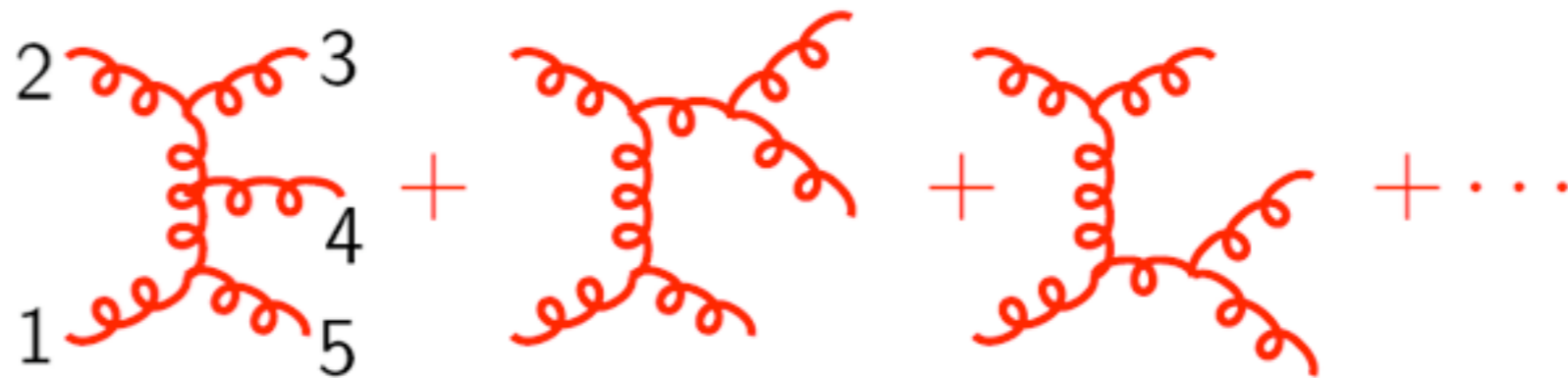
This formulation leads to a graphical representation of the simplifications occurring in the large N_c limit, even though it is exactly equivalent to the usual one.

In the large N_c limit, a gluon behaves as a quark-antiquark pair. In addition it behaves classically, in the sense that quantum interference, which are effects of order $1/N_c^2$ are neglected. Many QCD algorithms and codes (such as the parton showers) are based on this picture.



Example: a simple calculation?

Consider a simple 5 gluon amplitude:



There are 25 diagrams with a complicated tensor structure, so you get....



Solution

Keep track of all the quantum numbers,
(momenta, spin and color)
and organize them in
efficient way, by choosing appropriate basis.

The helicity method

Pioneering work of Berends, Gastmans, Troost, Wu in the '80, where they introduce the techniques of helicity amplitudes

$$u_{\pm}(k) = \frac{1}{2}(1 \pm \gamma_5)u(k)$$

$$\overline{u_{-}(k_i)}u_{+}(k_j) = \langle k_i - | k_j + \rangle \equiv \langle ij \rangle = \sqrt{s_{ij}}e^{-i\phi}$$

$$\overline{u_{+}(k_i)}u_{-}(k_j) = \langle k_i + | k_j - \rangle \equiv [ij] = -\sqrt{s_{ij}}e^{i\phi}$$

Using these objects, Xu, Zhang and Chang (1987) introduced simple vector polarizations

$$\varepsilon_{\mu}^{+}(k; q) = \frac{\langle q^{-} | \gamma_{\mu} | k^{-} \rangle}{\sqrt{2} \langle q k \rangle}, \quad \varepsilon_{\mu}^{-}(k, q) = \frac{\langle q^{+} | \gamma_{\mu} | k^{+} \rangle}{\sqrt{2} [k q]}$$

gauge vector

It's just a more sophisticated version of the circular polarization. Choosing appropriately the gauge vector, expressions simplify dramatically.

Stripping color out

Inspired by the way gauge theories appear as the zero-slope limits of (open) string theories, it has been suggested to decompose the full amplitude as a sum of **gauge invariant Subamplitudes** times color coefficients:

$$\mathcal{A}_n(g_1, \dots, g_n) = g^{n-2} \sum_{\sigma \in S_{n-1}} \text{Tr}(t^{a_1} t^{a_{\sigma_2}} \dots t^{a_{\sigma_n}}) A_n(1, \sigma_2, \dots, \sigma_n)$$

where the formula $if^{abc} = \text{Tr}(t^a, [t^b, t^c])$ has been repeatedly used to reduce the f's into traces of lambdas and the Fierz identities to cancel traces of length $l < n$.

Analogously for quarks:

$$\mathcal{A}_n(q_1, g_2, \dots, g_{n-1}, \bar{q}_n) = g^{n-2} \sum_{\sigma \in S_{n-2}} (t^{a_{\sigma_2}} \dots t^{a_{\sigma_{n-1}}})_{ij}^i A_n(1_q, \sigma_2, \dots, \sigma_{n-2}, n_{\bar{q}})$$

Example

Consider a simple 5 gluon amplitude:



There are 25 diagrams with a complicated tensor structure, but only 10 for a color flow and even less w/ helicities

$$A_5(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0$$

$$A_5(1^-, 2^-, 3^+, 4^+, 5^+) = i \frac{\langle 1 2 \rangle^4}{\langle 1 2 \rangle \langle 2 3 \rangle \langle 3 4 \rangle \langle 4 5 \rangle \langle 5 1 \rangle}$$

MHV amplitude





Number of diagrams for a n-gluon amplitude

n	full Amp	partial Amp
4	4	3
5	25	10
6	220	36
7	2485	133
8	34300	501
9	559405	1991
10	10525900	7335
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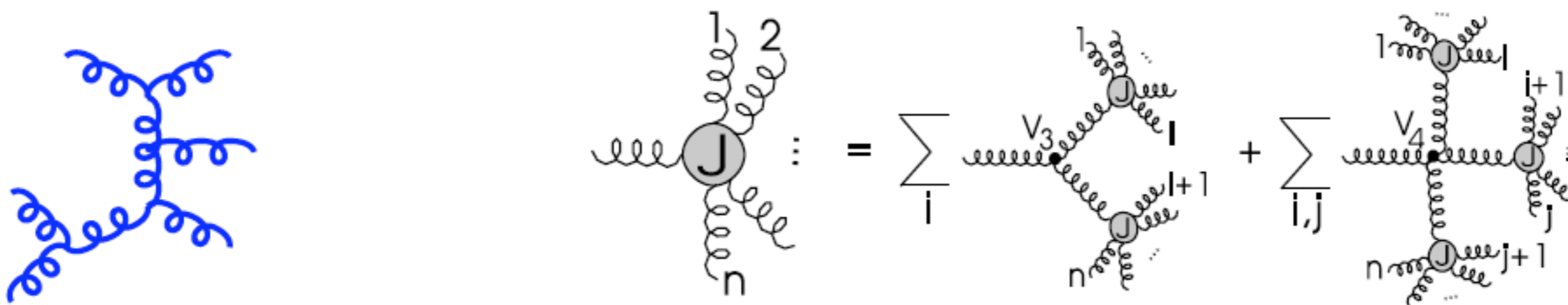
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$$3.8^n$$

Recursive relations

Feynman diagrams beg to be evaluated recursively



J^μ is the Berends-Giele current. For MHV can solve analytically!

$$J^\mu(1^-, 2^+, \dots, n^+) = \frac{\langle 1^- | \gamma^\mu \not{P}_{2,n} | 1^+ \rangle}{\sqrt{2} \langle 1 2 \rangle \cdots \langle n 1 \rangle} \sum_{m=3}^n \frac{\langle 1^- | \not{k}_m \not{P}_{1,m} | 1^+ \rangle}{P_{1,m-1}^2 P_{1,m}^2},$$

Dotting with ε^- on the free leg and cleaning up gives:

$$A_n^{\text{tree}}(1^-, 2^-, 3^+, 4^+, \dots, n^+) = i \frac{\langle 1 2 \rangle^4}{\langle 1 2 \rangle \cdots \langle n 1 \rangle}$$

Parke-Taylor
amplitude is proven!

Infinite number of Feynman diagrams solved at once!



Number of diagrams for n-gluon amplitudes

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4	4	3	3
5	25	10	10
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Note, however, one still needs to sum over color, an operation which sets the complexity back to exponential.



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Master QCD formula

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \rightarrow X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

Two ingredients necessary:

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$$\hat{\sigma}_{ab \rightarrow X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Leading order

Next-to-leading order

Next-to-next-to-leading order

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from integration to event generation



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General and flexible method is needed



Phase Space



Phase Space

$$d\Phi_n = \left[\prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 (2E_i)} \right] (2\pi)^4 \delta^{(4)} \left(p_0 - \sum_{i=1}^n p_i \right)$$



Phase Space

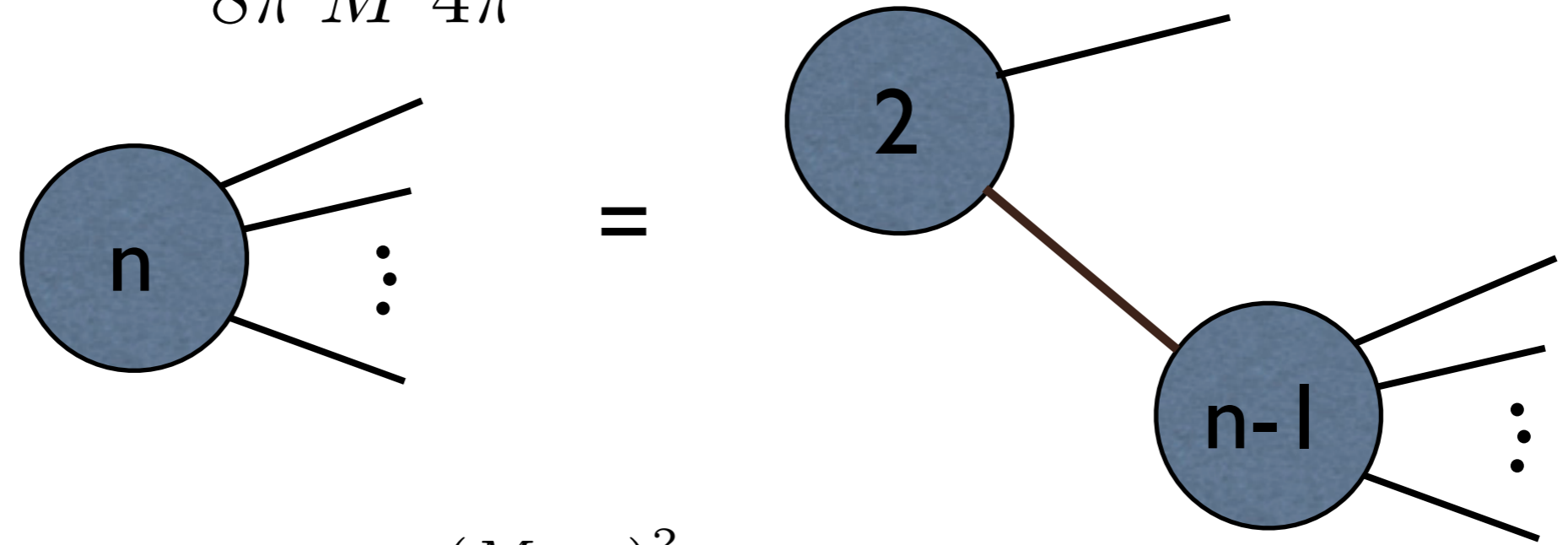
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$$d\Phi_n(M) = \frac{1}{2\pi} \int_0^{(M-\mu)^2} d\mu^2 d\Phi_2(M) d\Phi_{n-1}(\mu)$$



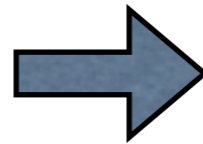
Integrals as averages



Integrals as averages

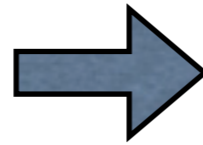


$$I = \int_{x_1}^{x_2} f(x) dx$$



$$I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x_i)$$

$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2$$



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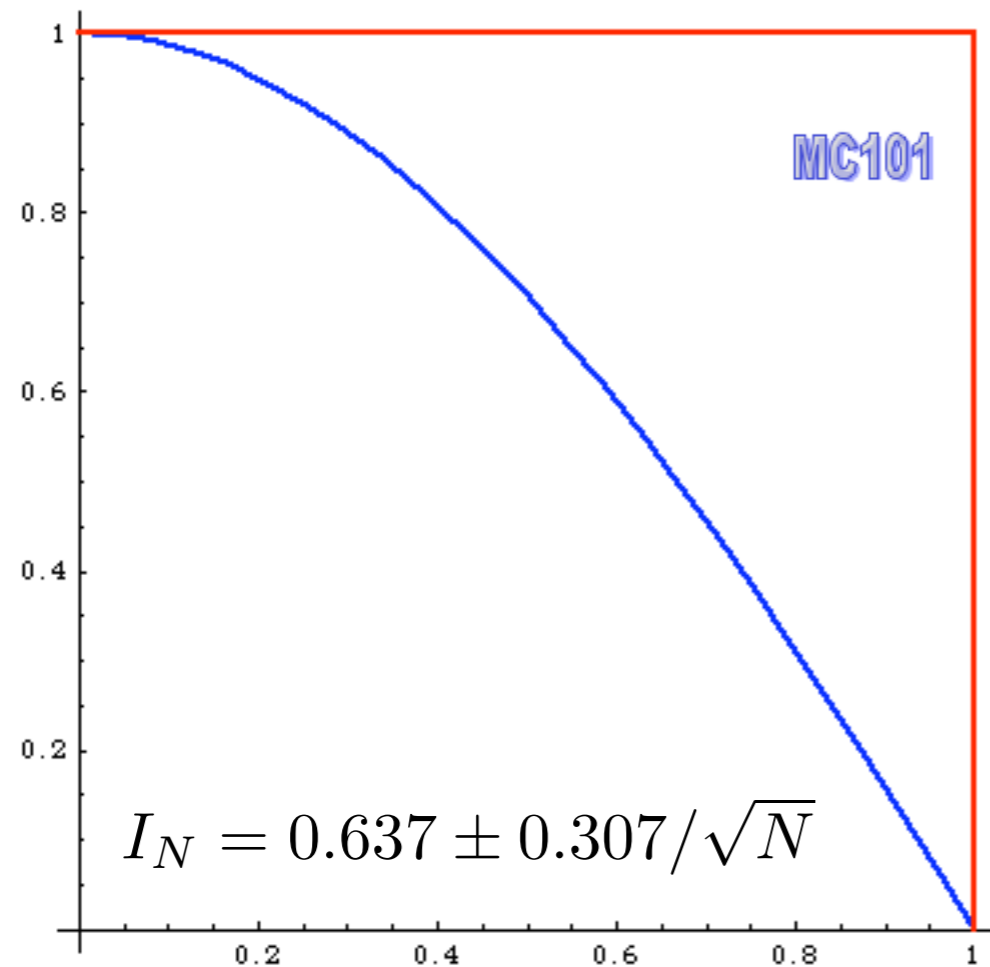
- 👉 Convergence is slow but it can be estimated easily
- 👉 Error does not depend on # of dimensions!
- 👉 Improvement by minimizing V_N .
- 👉 Optimal/Ideal case: $f(x)=C \Rightarrow V_N=0$



Importance Sampling



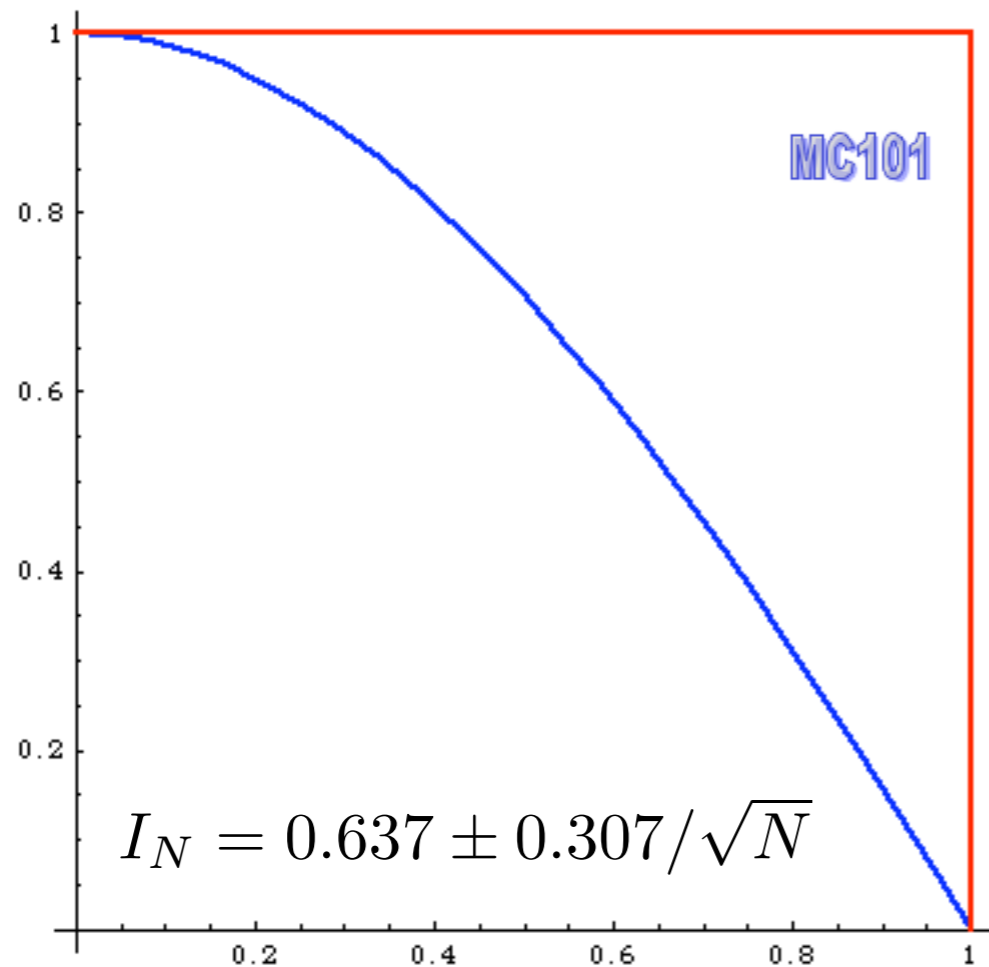
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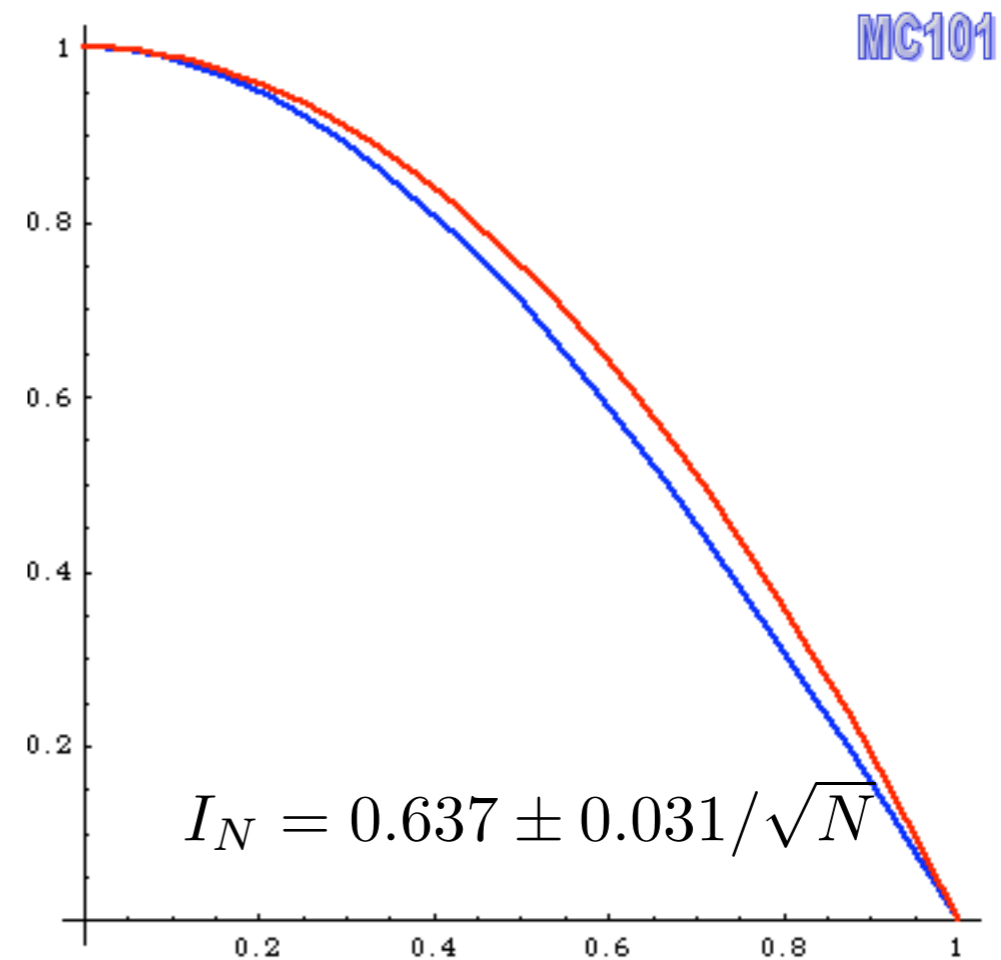
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$



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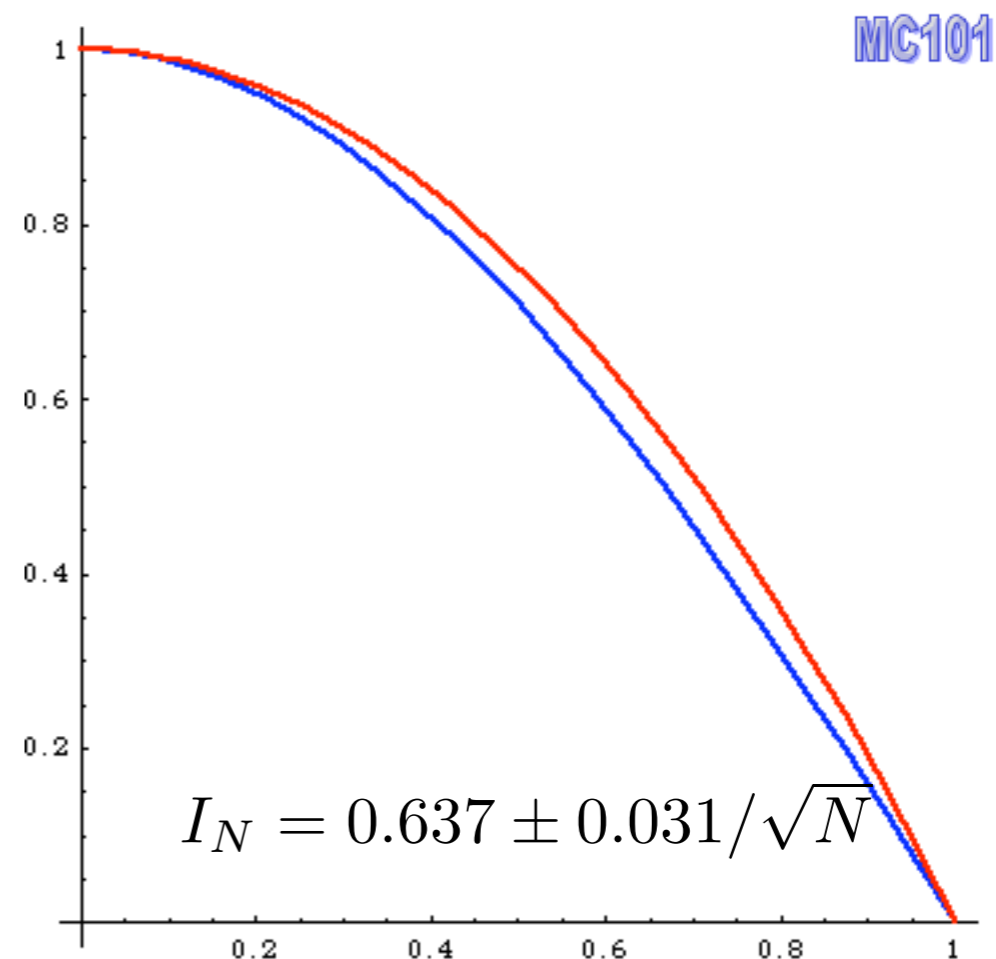
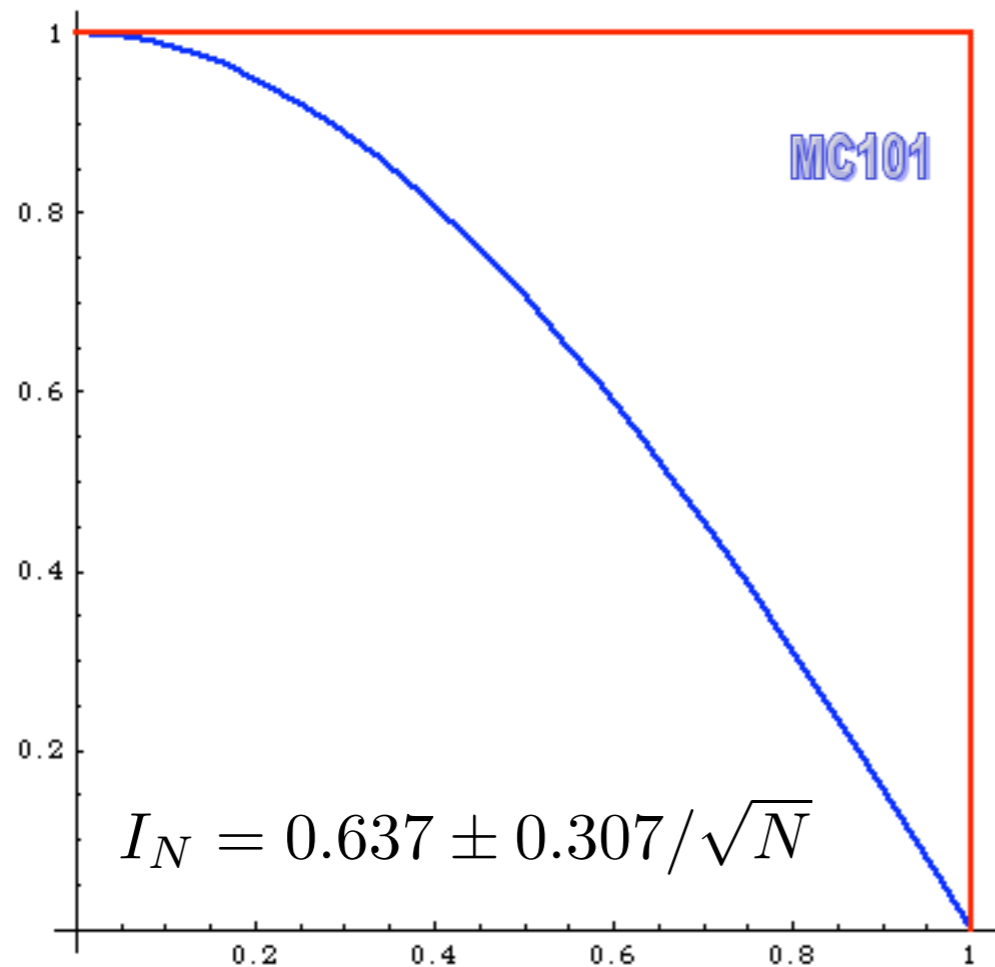
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$$I = \int_0^1 dx (1 - x^2) \frac{\cos \frac{\pi}{2} x}{1 - x^2}$$



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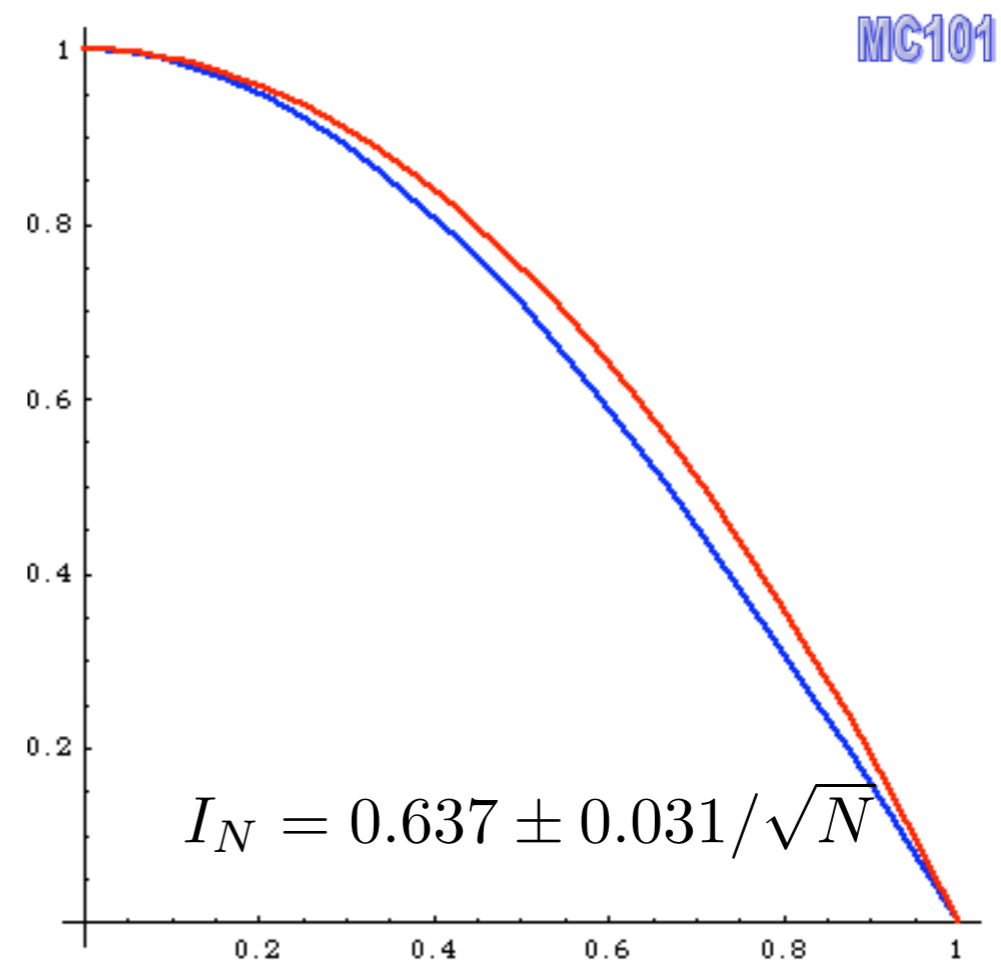
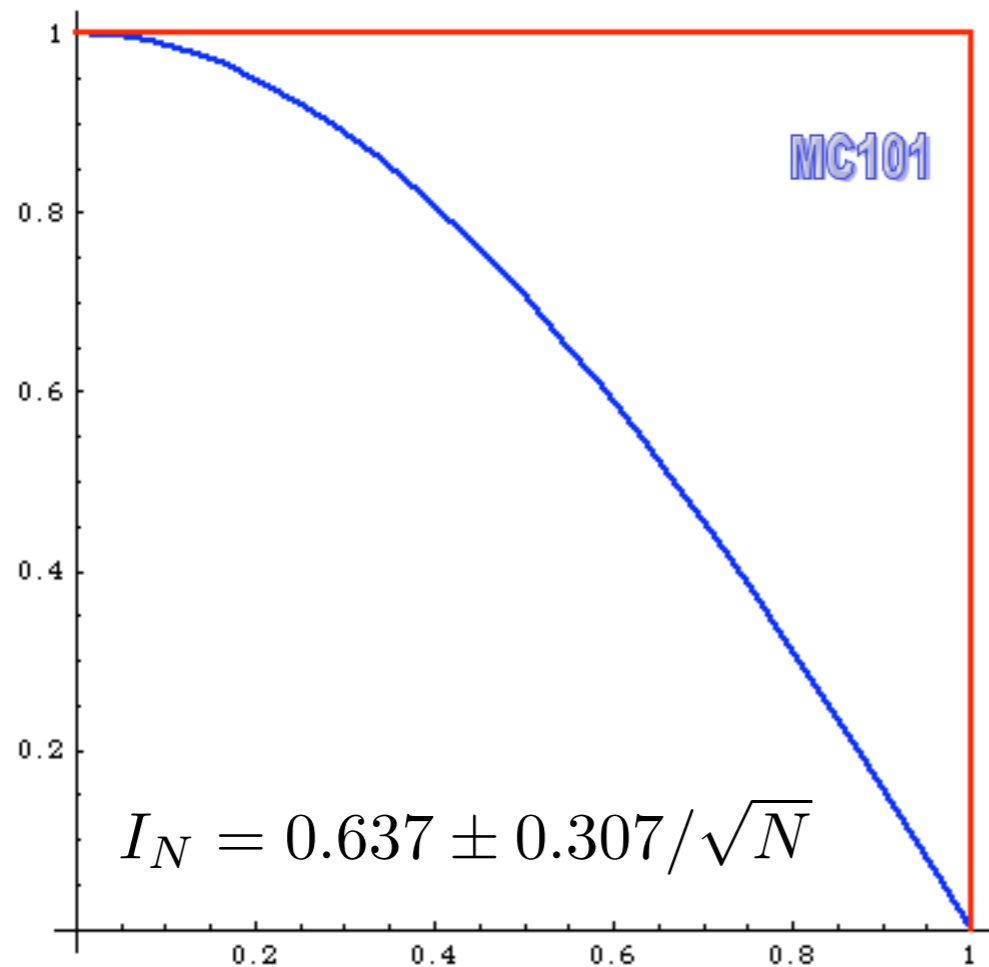


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$$\begin{aligned} I &= \int_0^1 dx (1 - x^2) \frac{\cos \frac{\pi}{2} x}{1 - x^2} \\ &= \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2} \end{aligned}$$



Importance Sampling



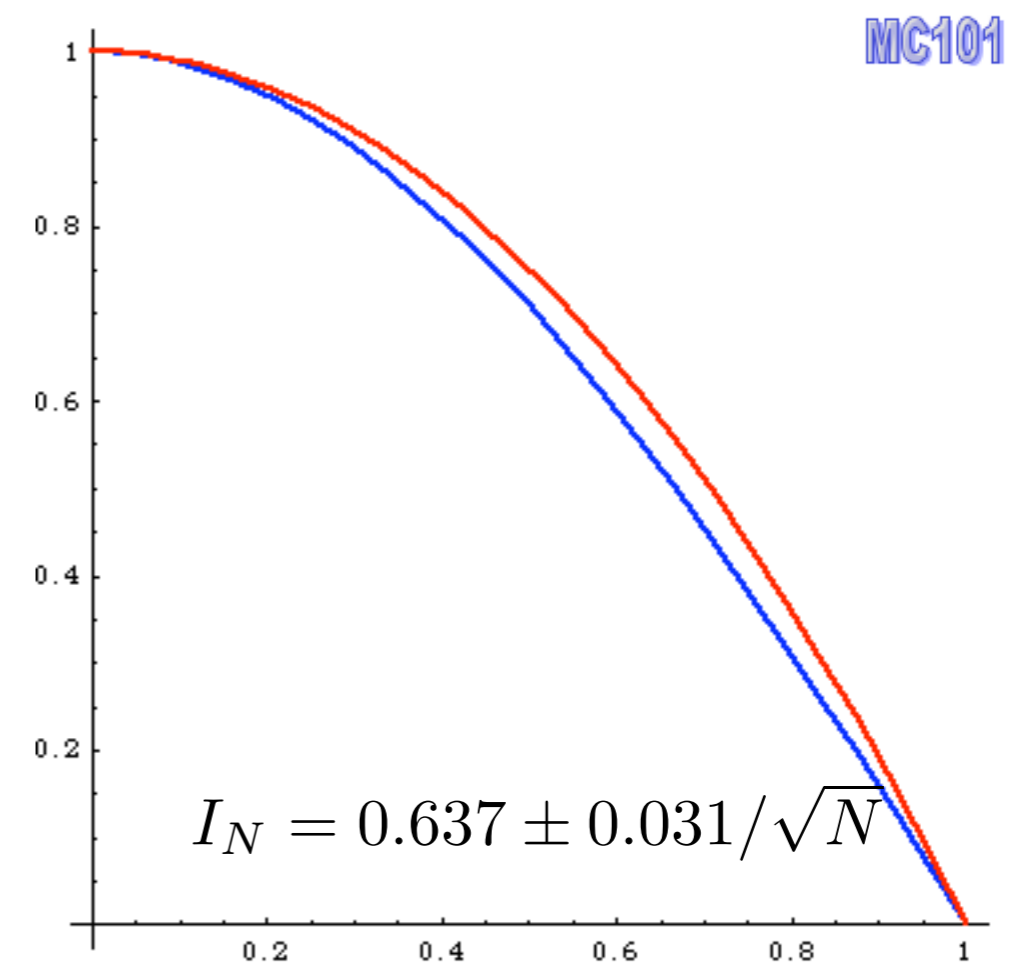
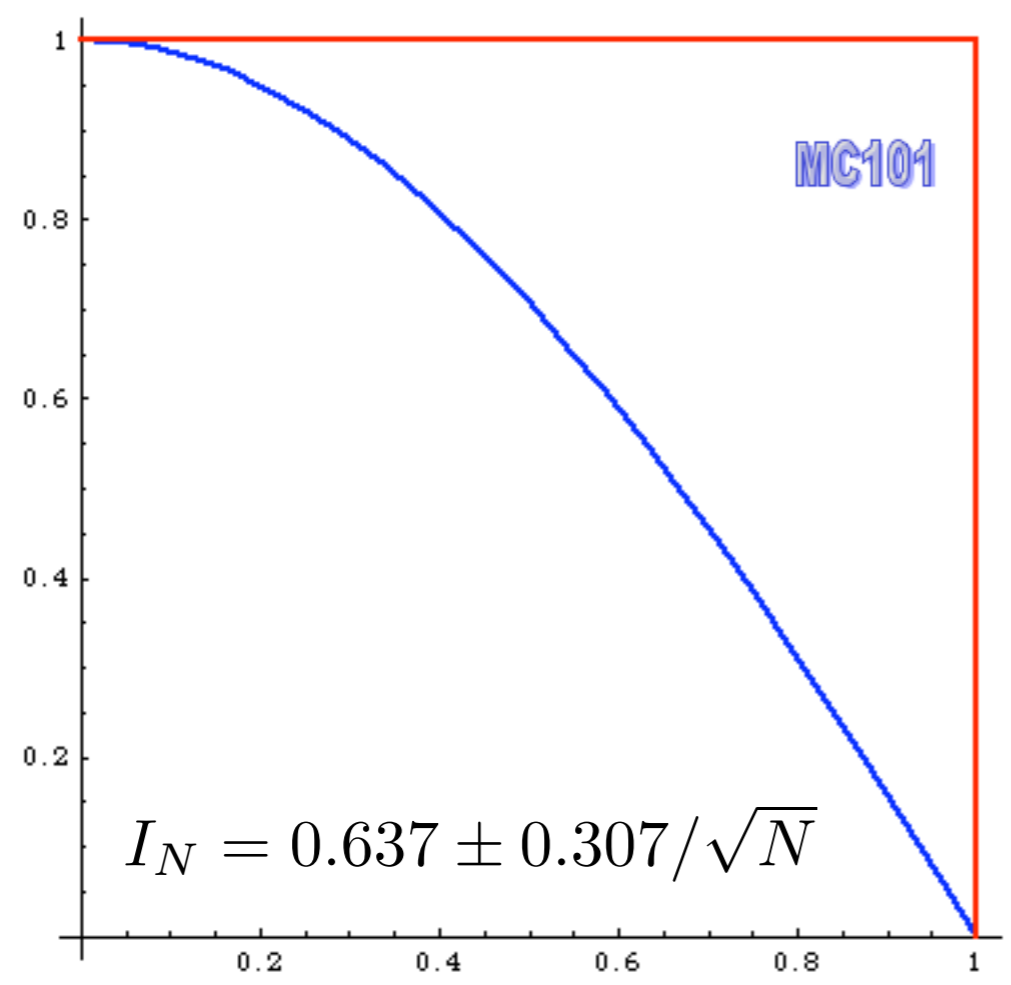
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Importance Sampling

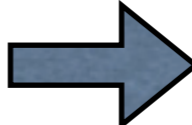


Importance Sampling

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idea: learn during the run and build a step-function
approximation $p(x)$ of $f(x)$  VEGAS

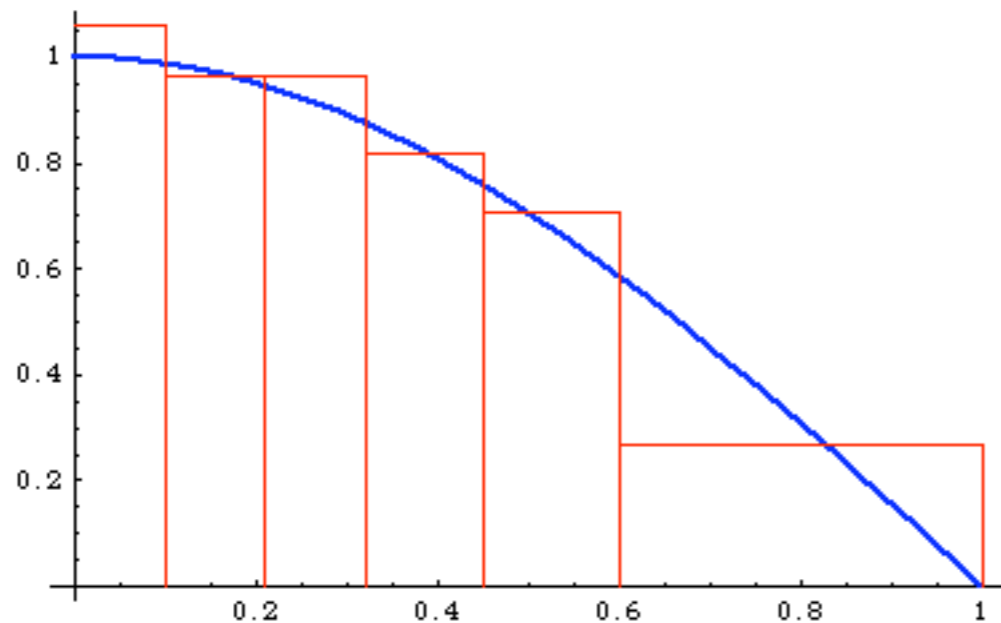
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MC101



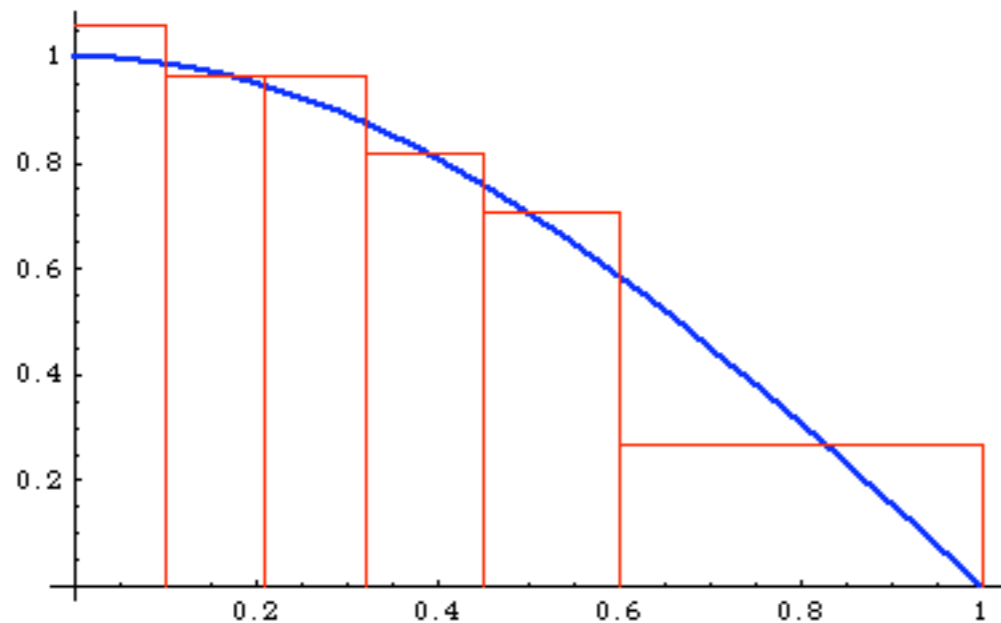
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MC101



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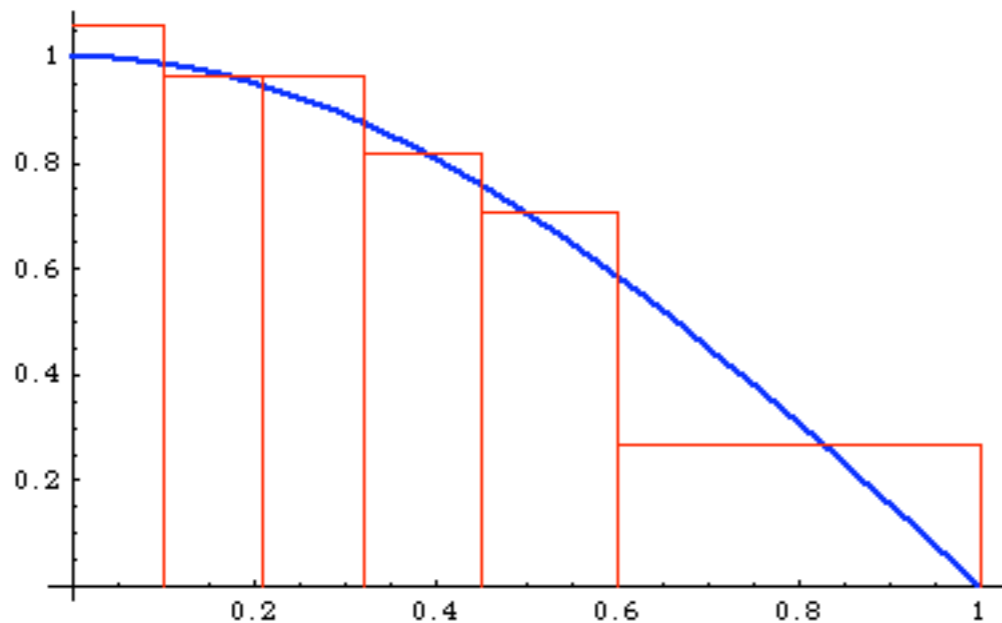
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MC101



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$$p(x) = \frac{1}{N_b \Delta x_i}, \quad x_i - \Delta x_i < x < x_i$$



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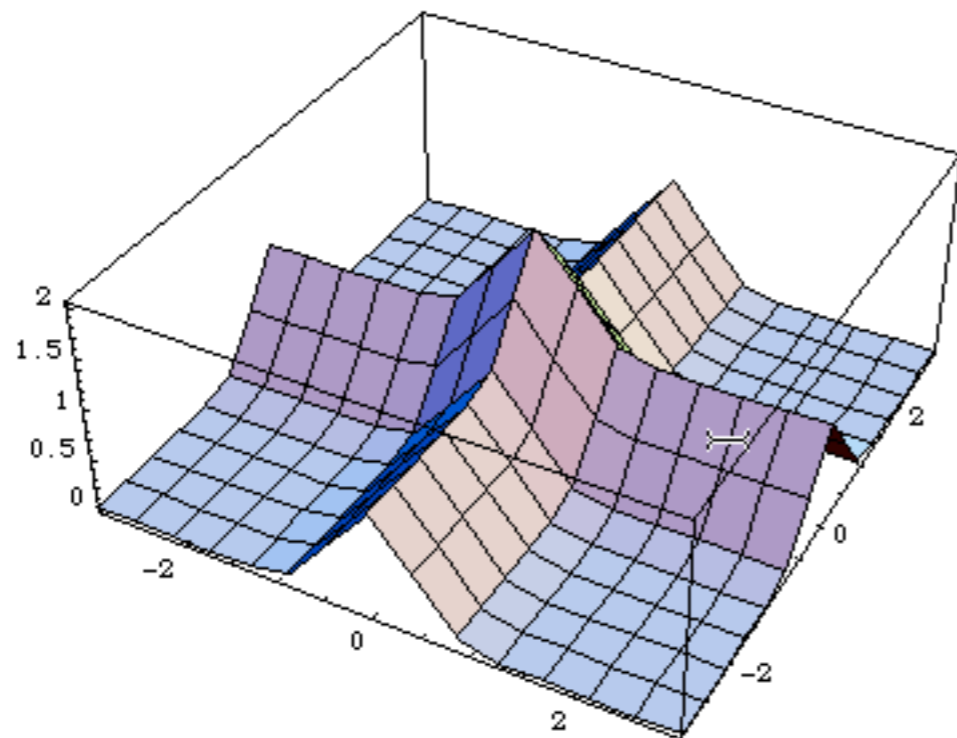
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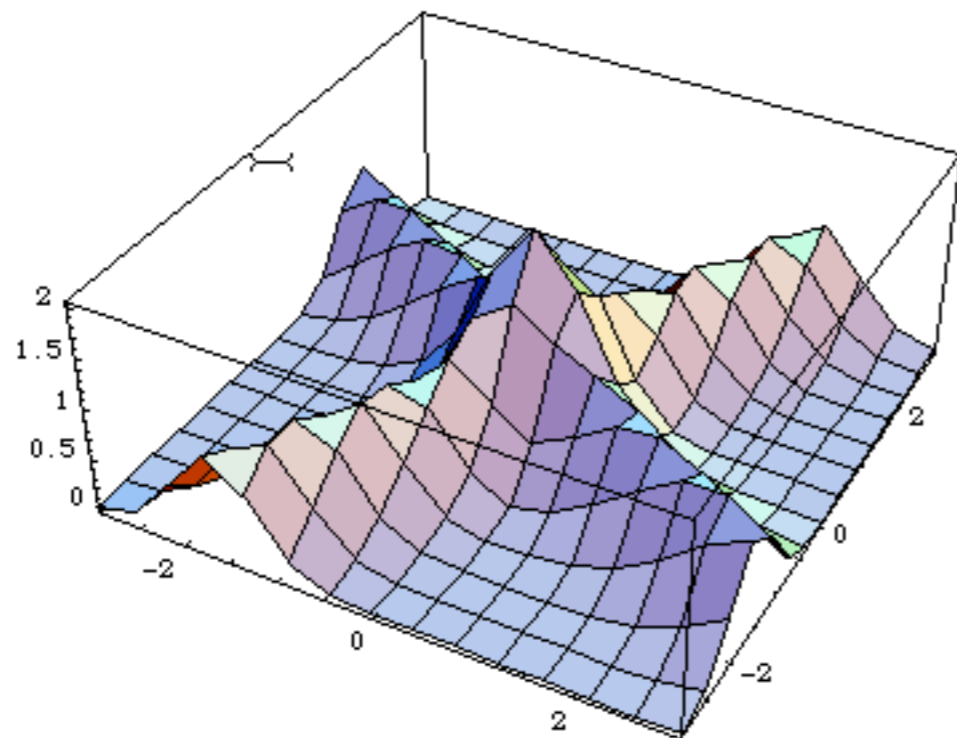
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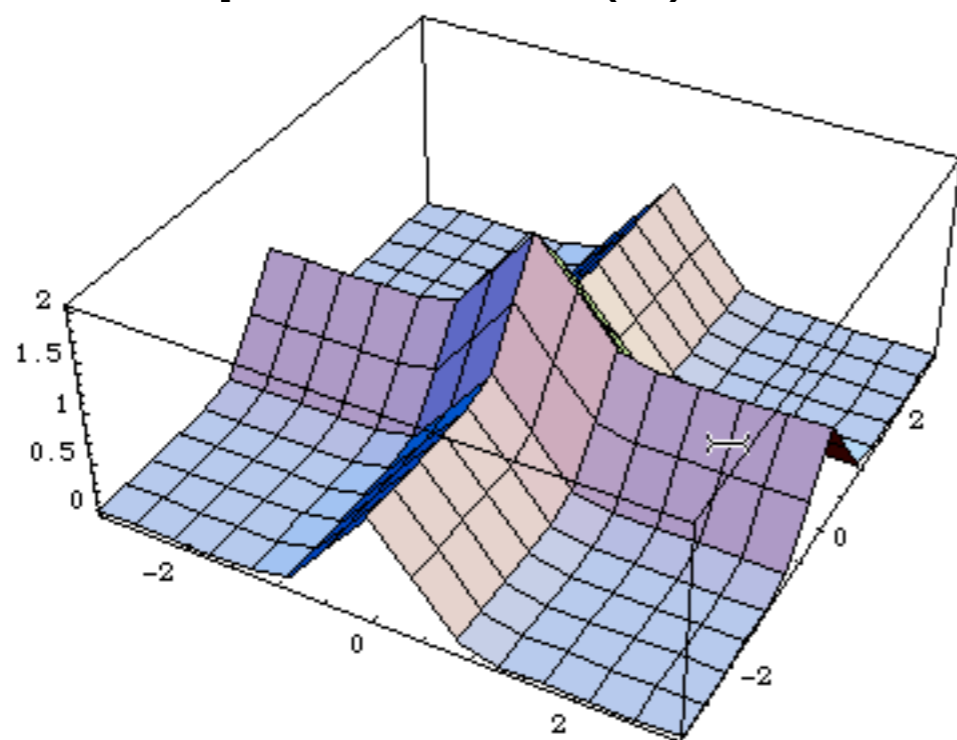
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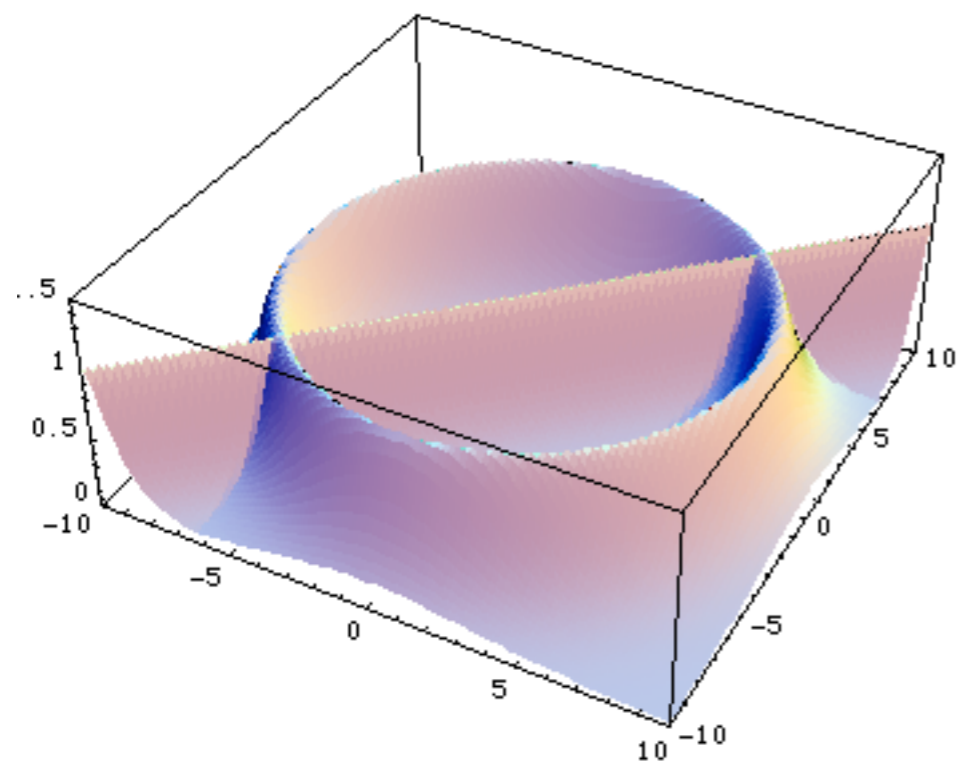


but it is sufficient to make
a change of variables!

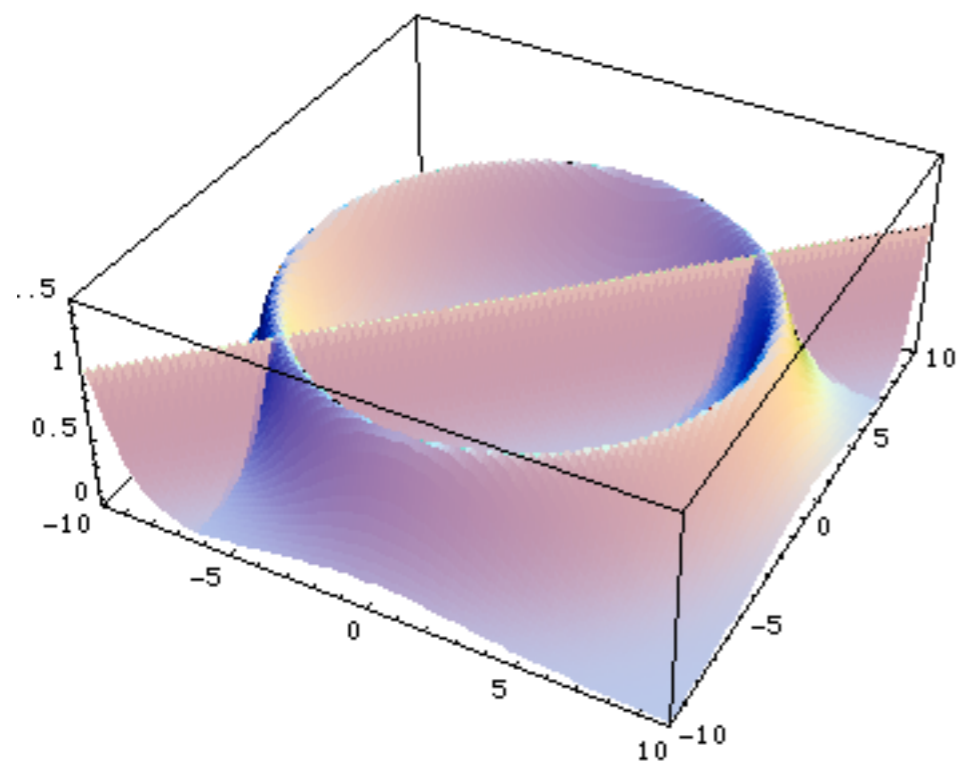


Multi-channel

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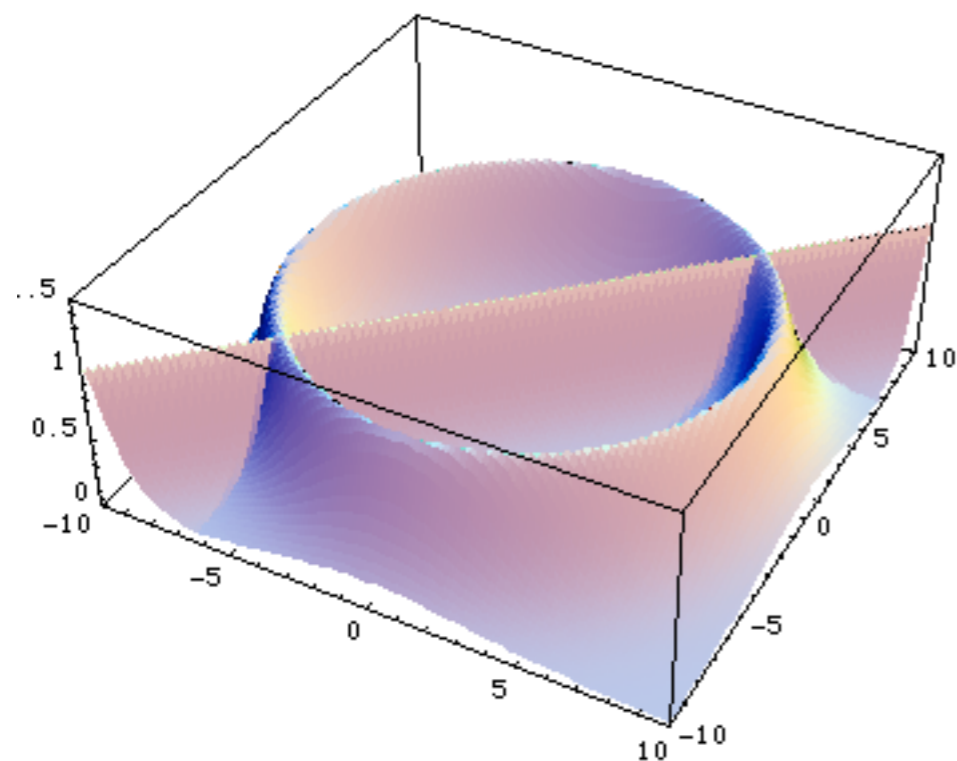


Multi-channel



In this case there is no unique transformation:
Vegas is bound to fail!

Multi-channel



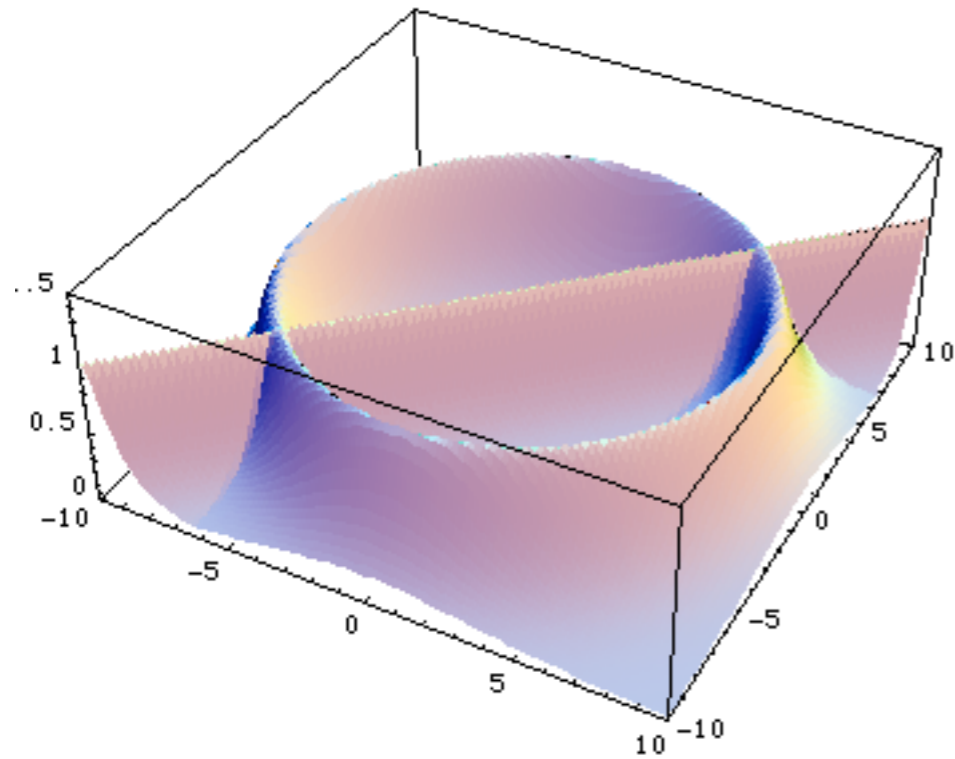
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Solution: use different transformations= channels

$$p(x) = \sum_{i=1}^n \alpha_i p_i(x) \quad \text{with} \quad \sum_{i=1}^n \alpha_i = 1$$

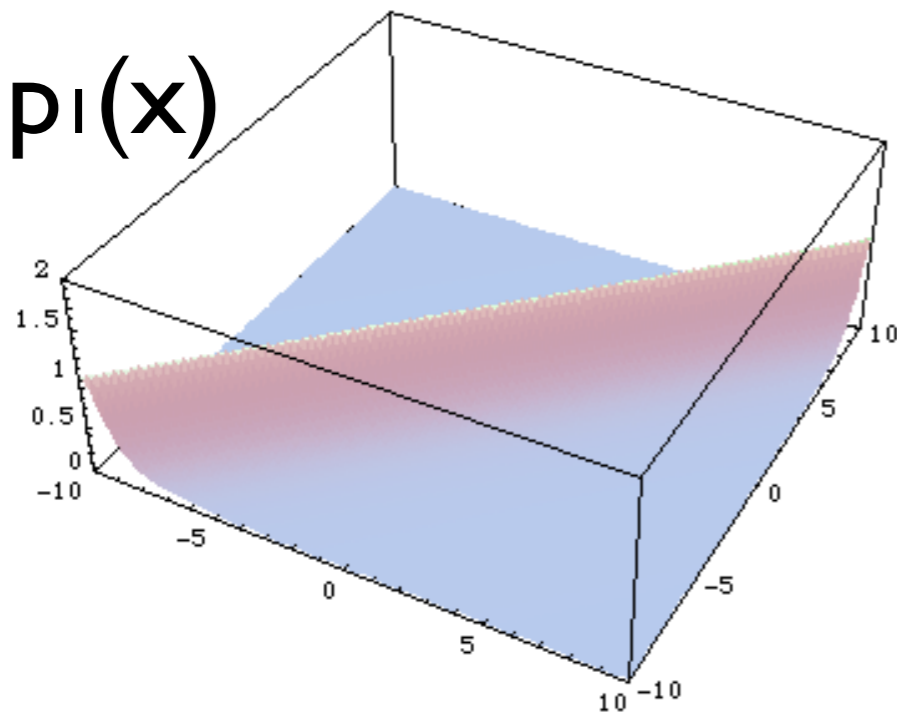
with each $p_i(x)$ taking care of one “peak” at the time

Multi-channel

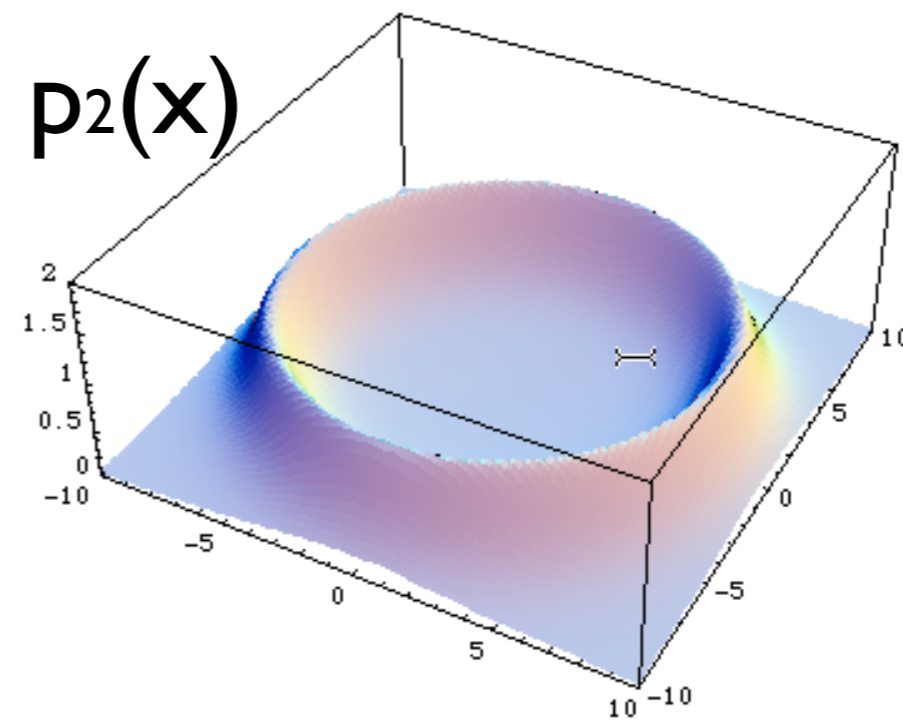


In this case there is no unique transformation: Vegas is bound to fail!

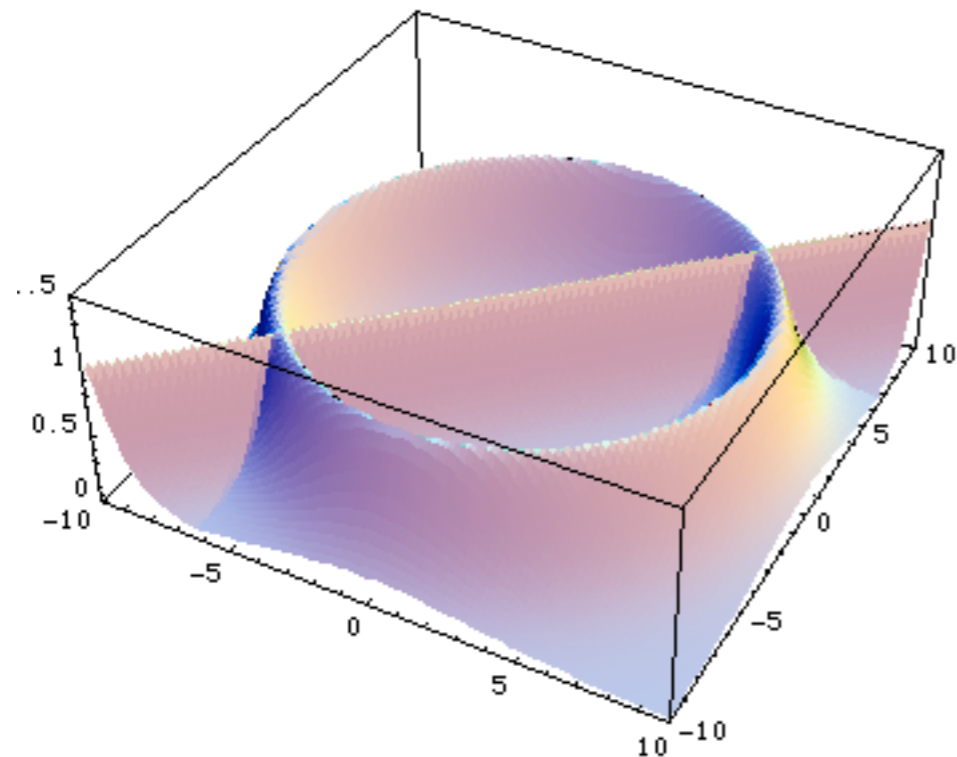
$p_1(x)$



$p_2(x)$



Multi-channel



In this case there is no unique transformation:
Vegas is bound to fail!

But if you know where the peaks are (=in which variables) we can use different transformations= channels:

$$p(x) = \sum_{i=1}^n \alpha_i p_i(x) \quad \text{with} \quad \sum_{i=1}^n \alpha_i = 1$$

$$I = \int f(x) dx = \sum_{i=1}^n \alpha_i \int \frac{f(x)}{p(x)} p_i(x) dx$$

Multi-channel

- **Advantages**
 - The integral does not depend on the α_i but the variance does and can be minimised by a careful choice
- **Drawbacks**
 - Need to calculate all g_j values for each point
 - Each phase space channel must be invertible
 - N coupled equations for α_i so it might only work for small number of channels

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Very popular method!



Multi-channel based on single diagrams

Consider the integration of an amplitude $|M|^2$ at tree level which lots of diagrams contribute to. If there were a basis of functions,

$$f = \sum_{i=1}^n f_i \quad \text{with} \quad f_i \geq 0, \quad \forall i,$$

such that:

1. we know how to integrate each one of them,
2. they describe all possible peaks,

then the problem would be solved:

$$I = \int d\vec{\Phi} f(\vec{\Phi}) = \sum_{i=1}^n \int d\vec{\Phi} g_i(\vec{\Phi}) \frac{f_i(\vec{\Phi})}{g_i(\vec{\Phi})} = \sum_{i=1}^n I_i,$$



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Does such a basis exist? **YES!** $f_i = \frac{|A_i|^2}{\sum_i |A_i|^2} |A_{\text{tot}}|^2$



Multi-channel based on single diagrams*

- Key Idea
 - Any single diagram is “easy” to integrate
 - Divide integration into pieces, based on diagrams
- Get N independent integrals
 - Errors add in quadrature so no extra cost
 - No need to calculate “weight” function from other channels.
 - Can optimize # of points for each one independently
 - Parallel in nature
- What about interference?
 - Never creates “new” peaks, so we’re OK!



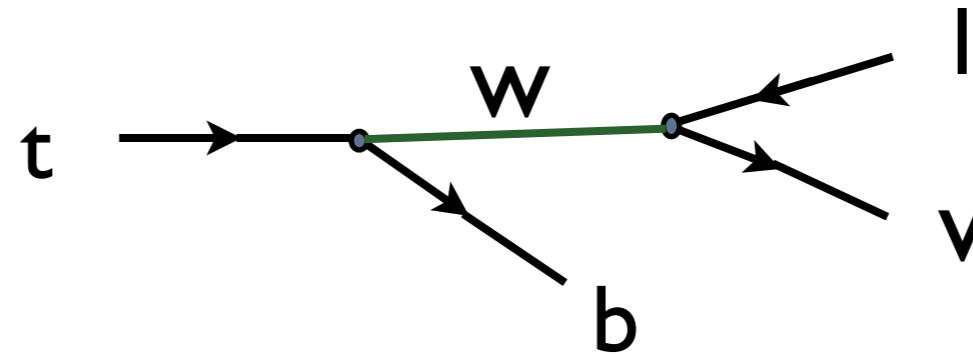
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*Method used in MadGraph

Exercise: top decay

MC101

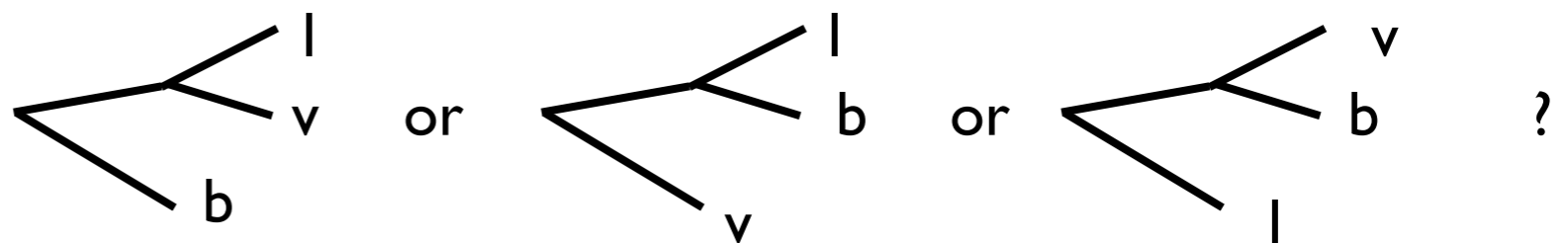


- Easy but non-trivial

- Breit-Wigner peak
“flattened :

$$\frac{1}{(q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2} \text{to be}$$

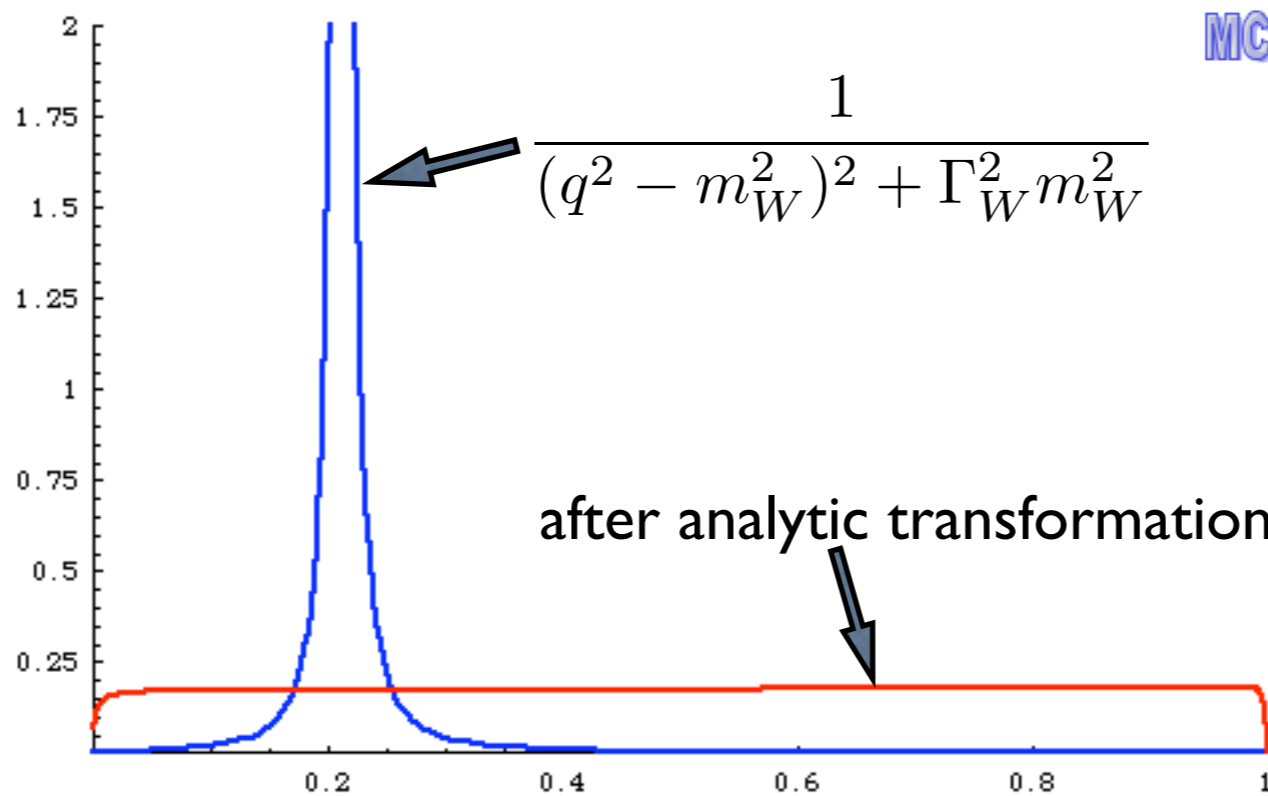
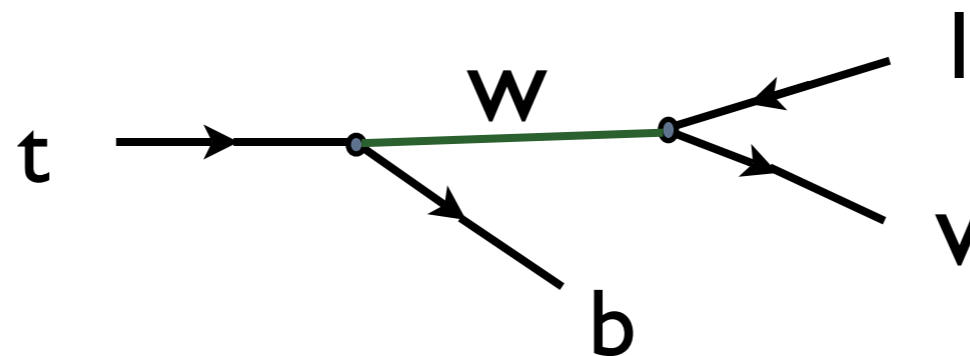
- Choose the right “channel” for the phase space



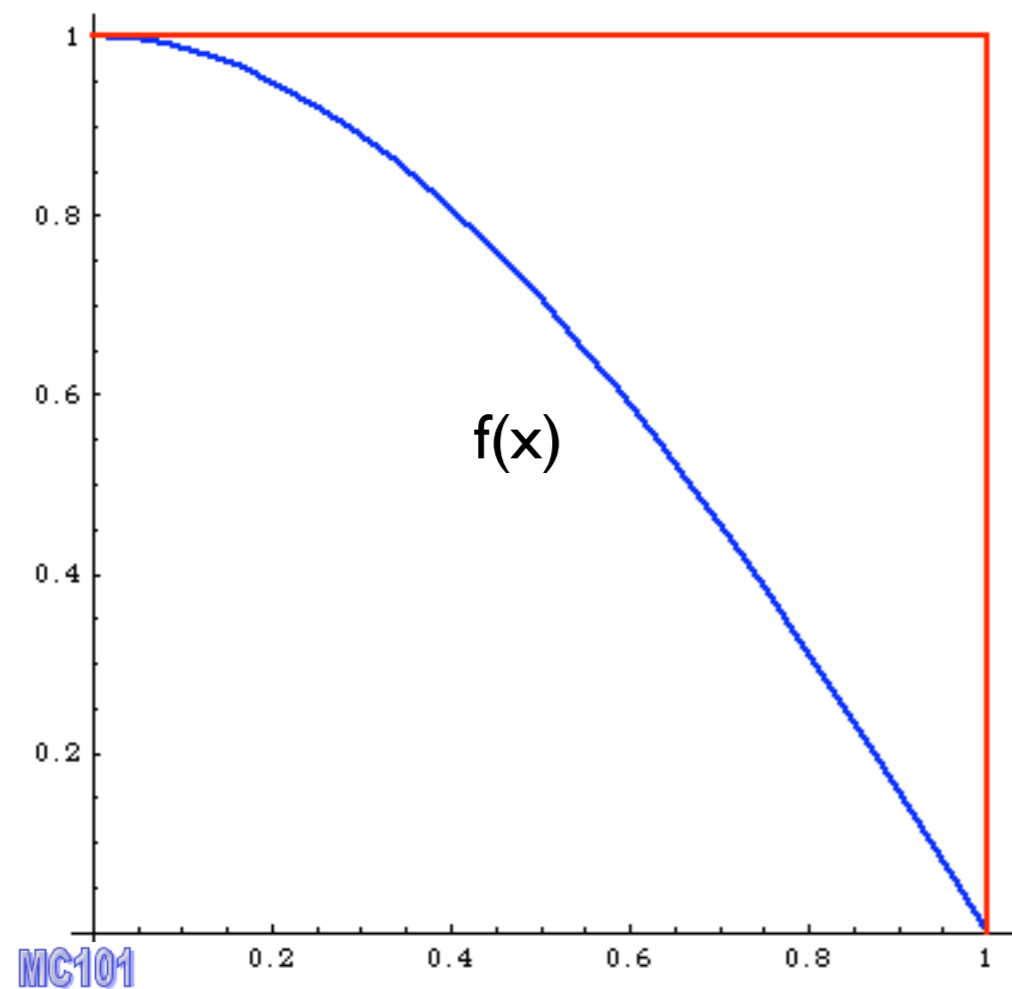


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MC101



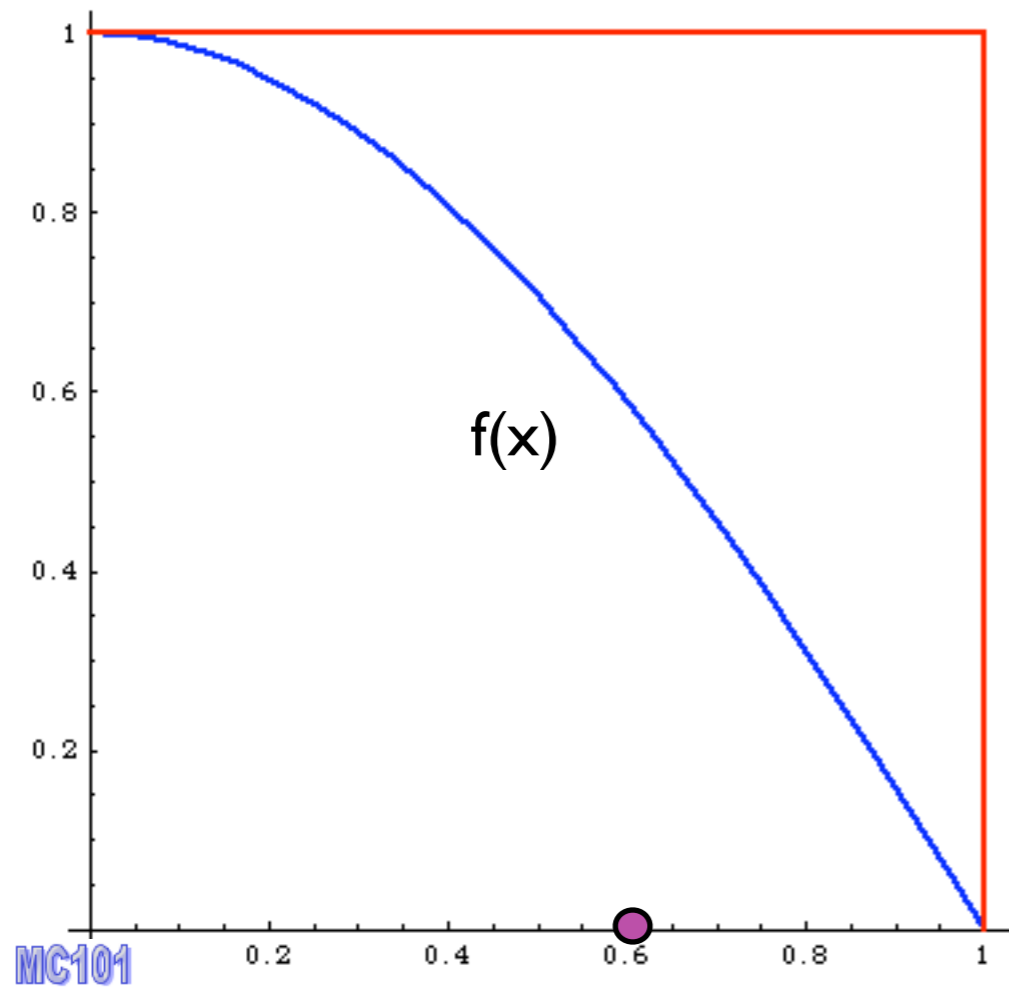
Event generation



Alternative way



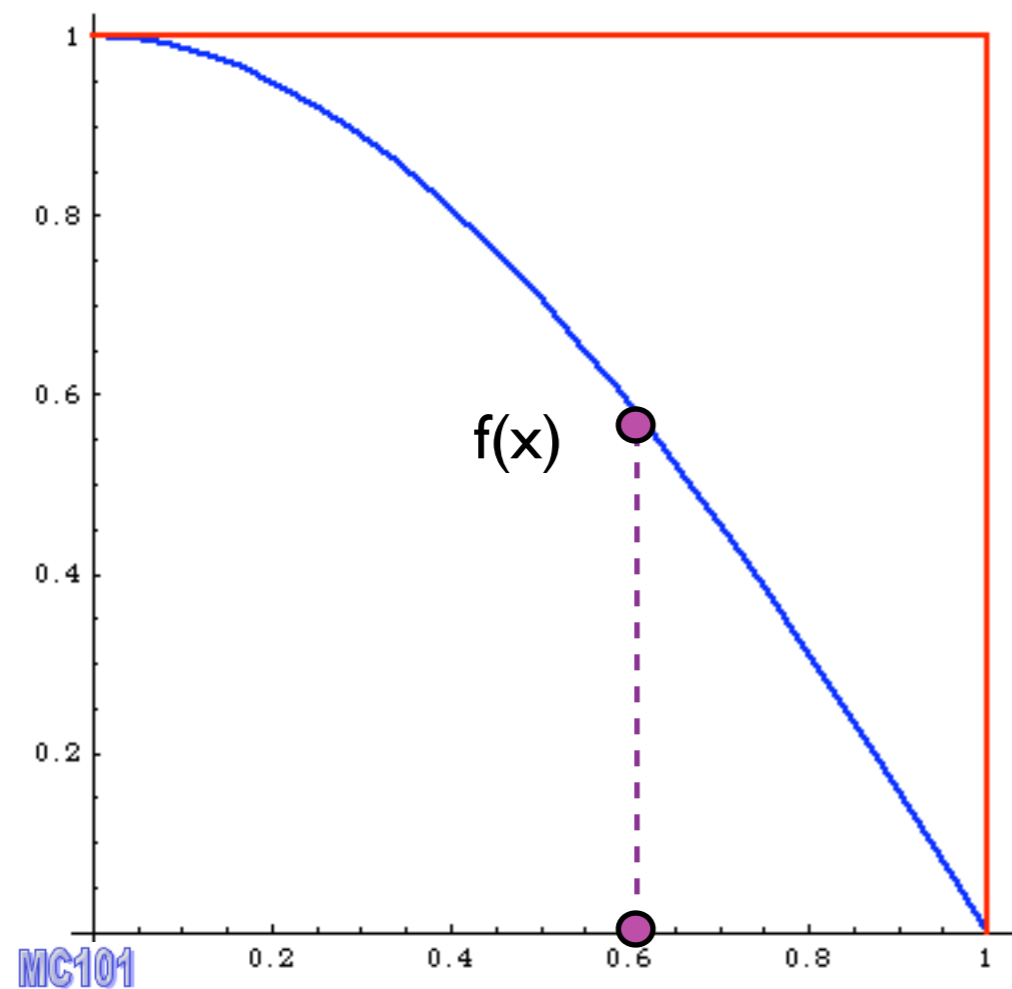
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Alternative way

1. pick x

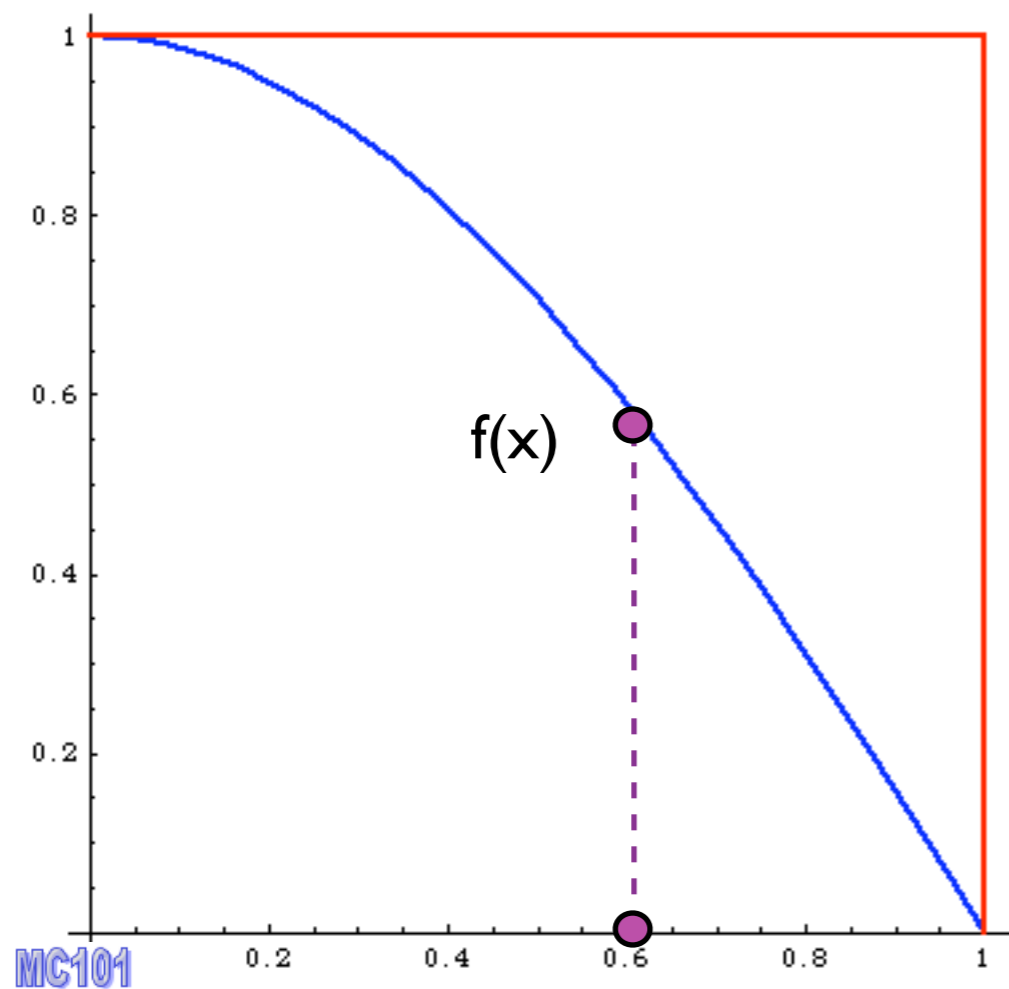
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Alternative way

1. pick x
2. calculate $f(x)$

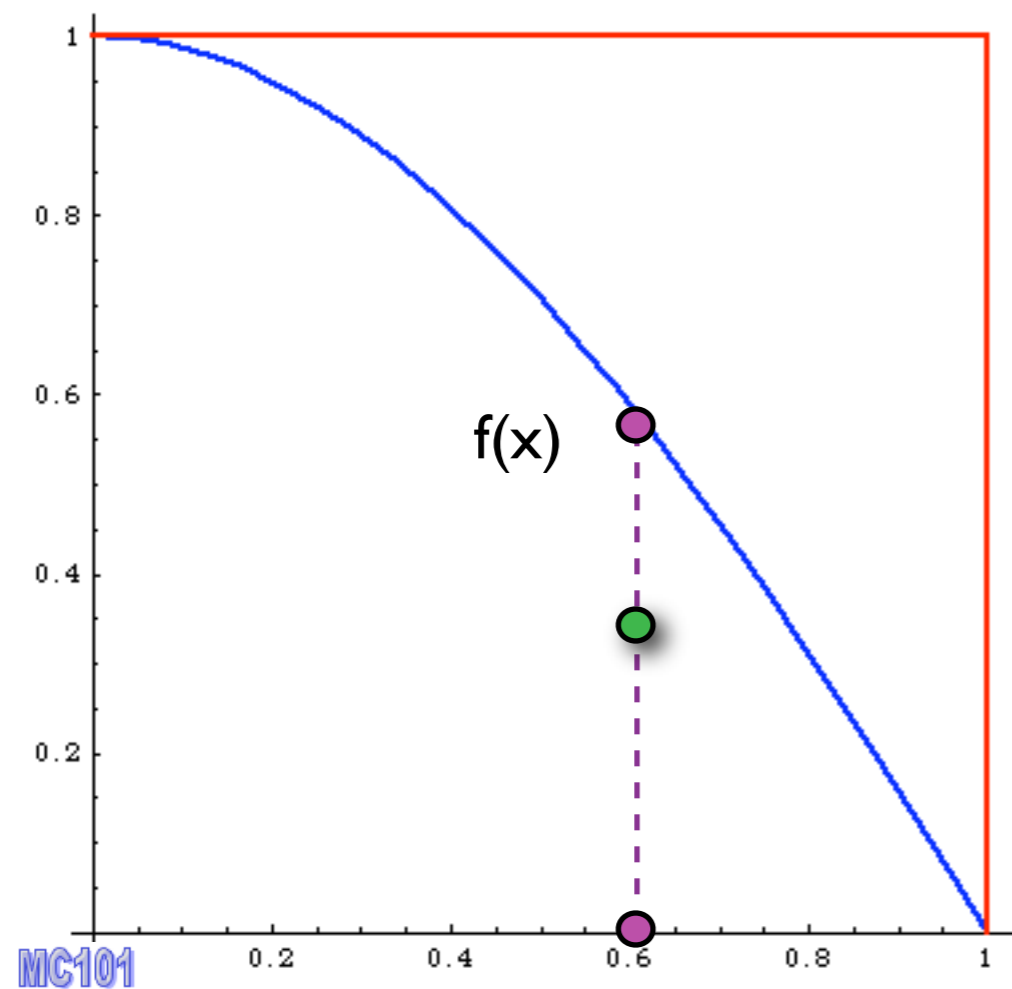
Event generation



Alternative way

1. pick x
2. calculate $f(x)$
3. pick $0 < y < f_{\max}$

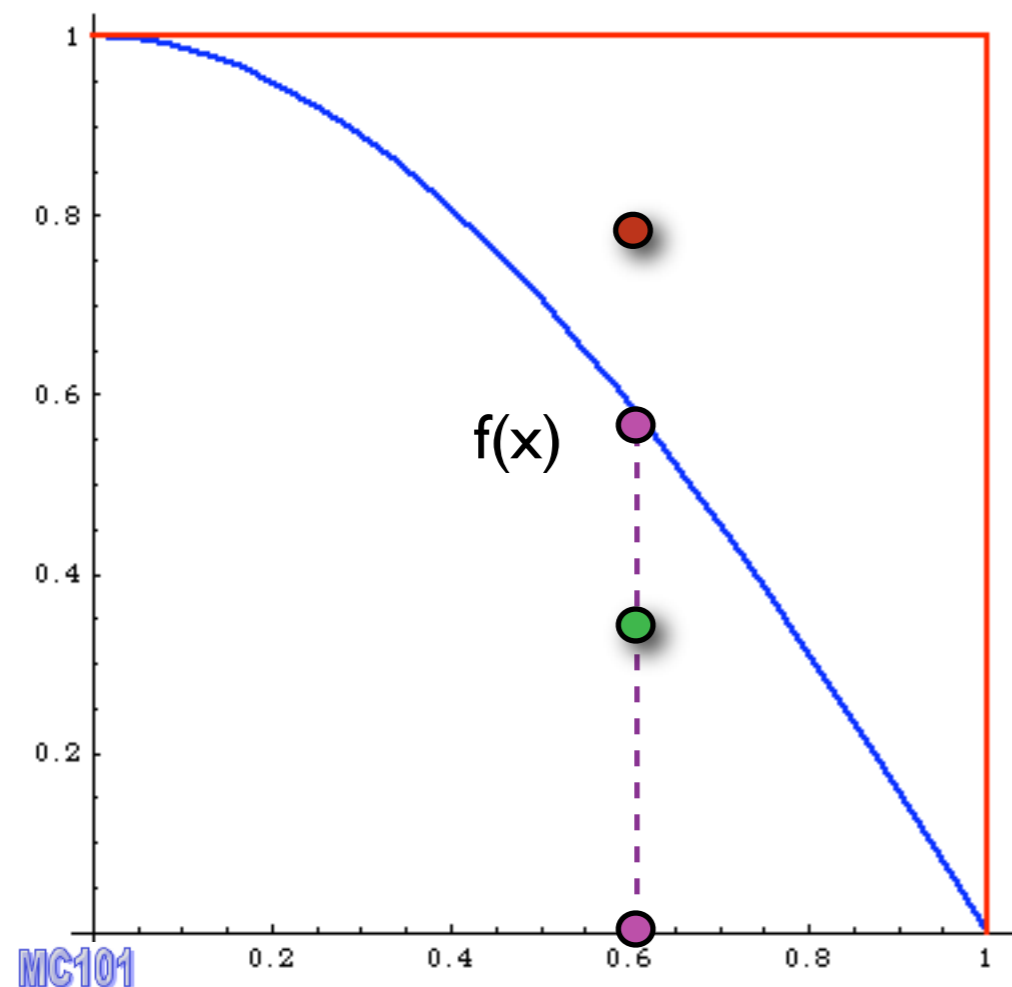
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4. Compare:
if $f(x) > y$ accept event,

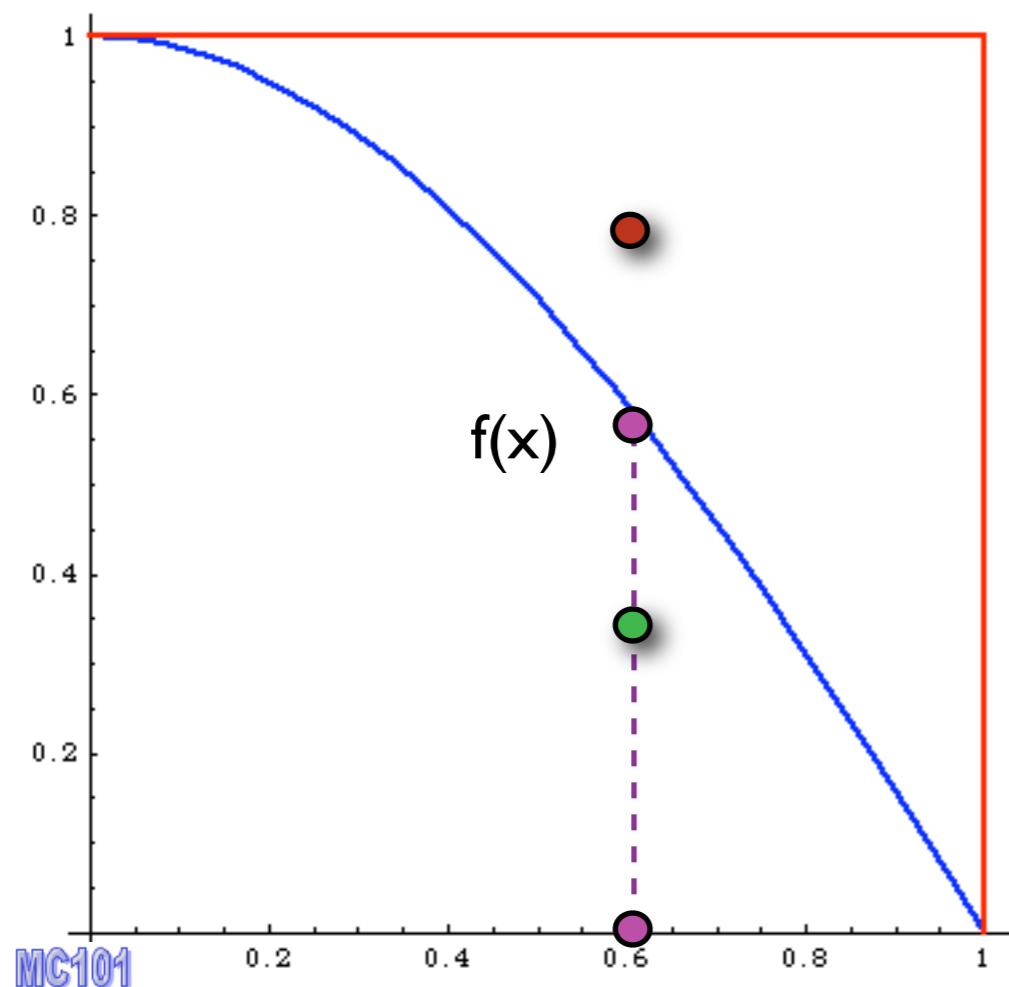
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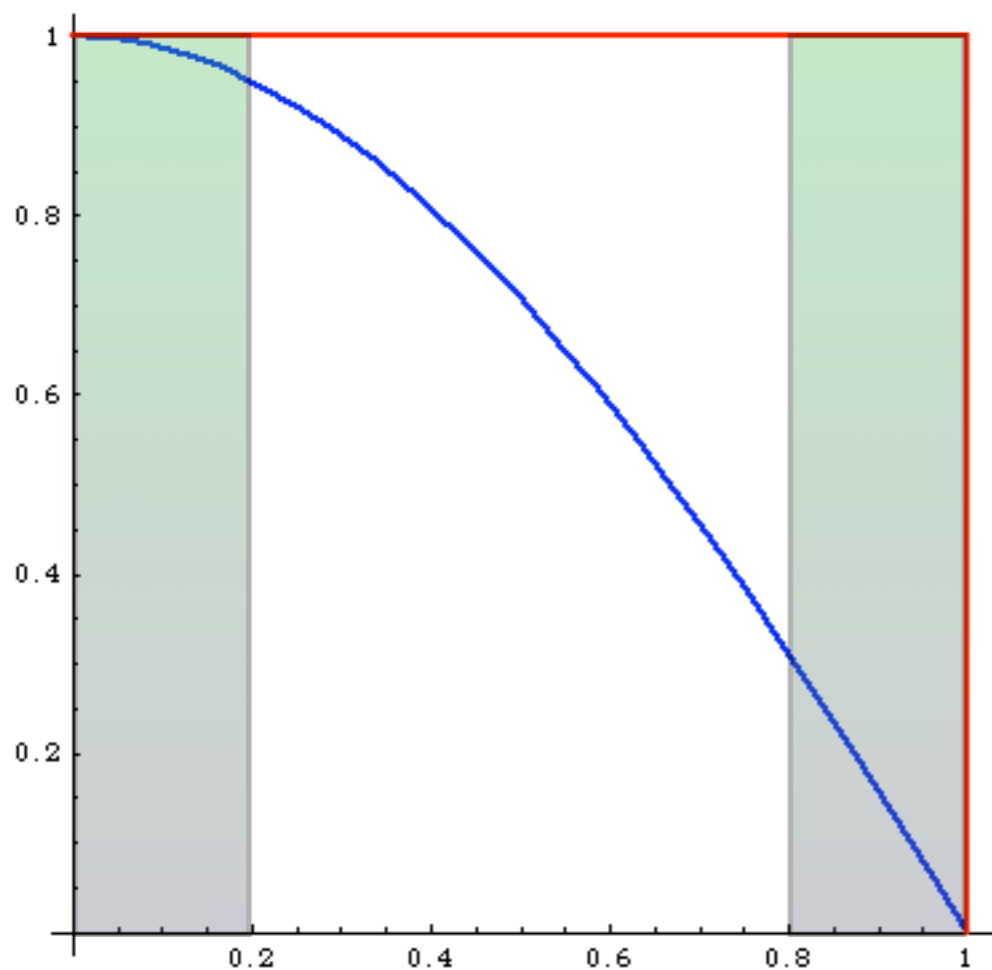


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if $f(x) > y$ accept event,
else reject it.

$$| = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

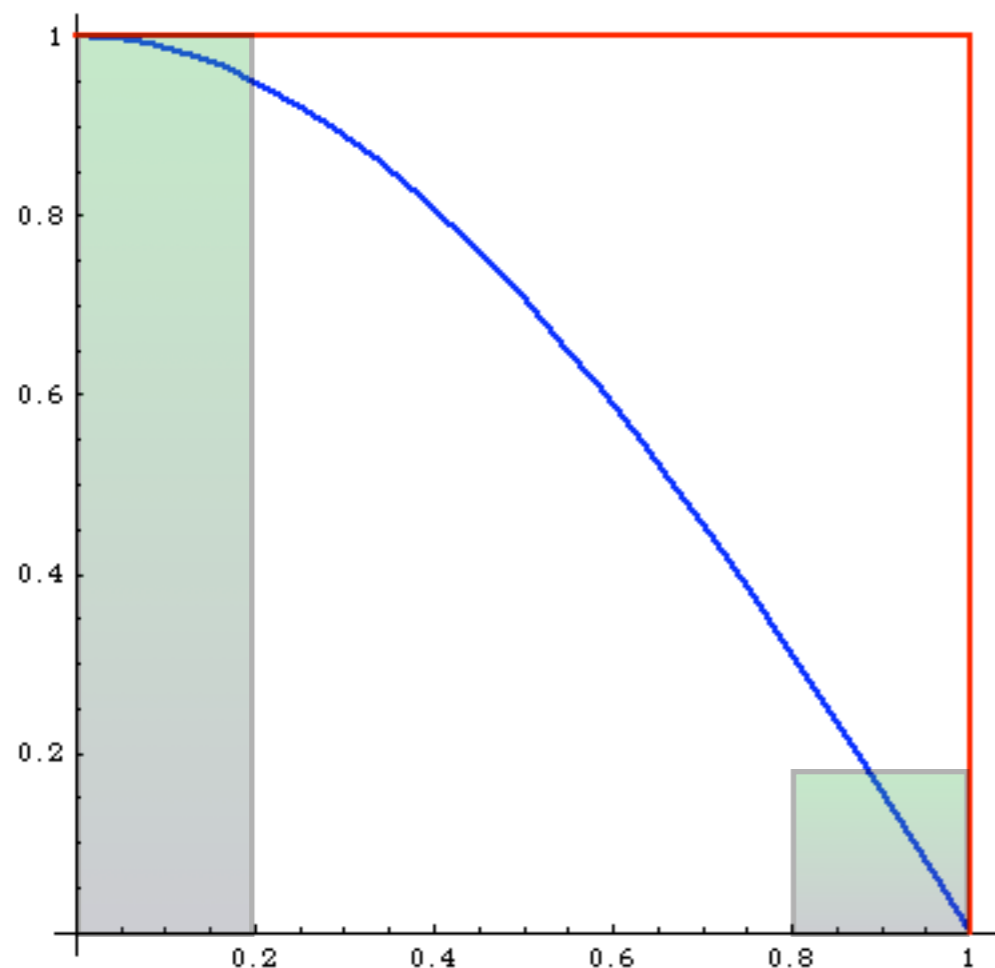


What's the difference?

before:

same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



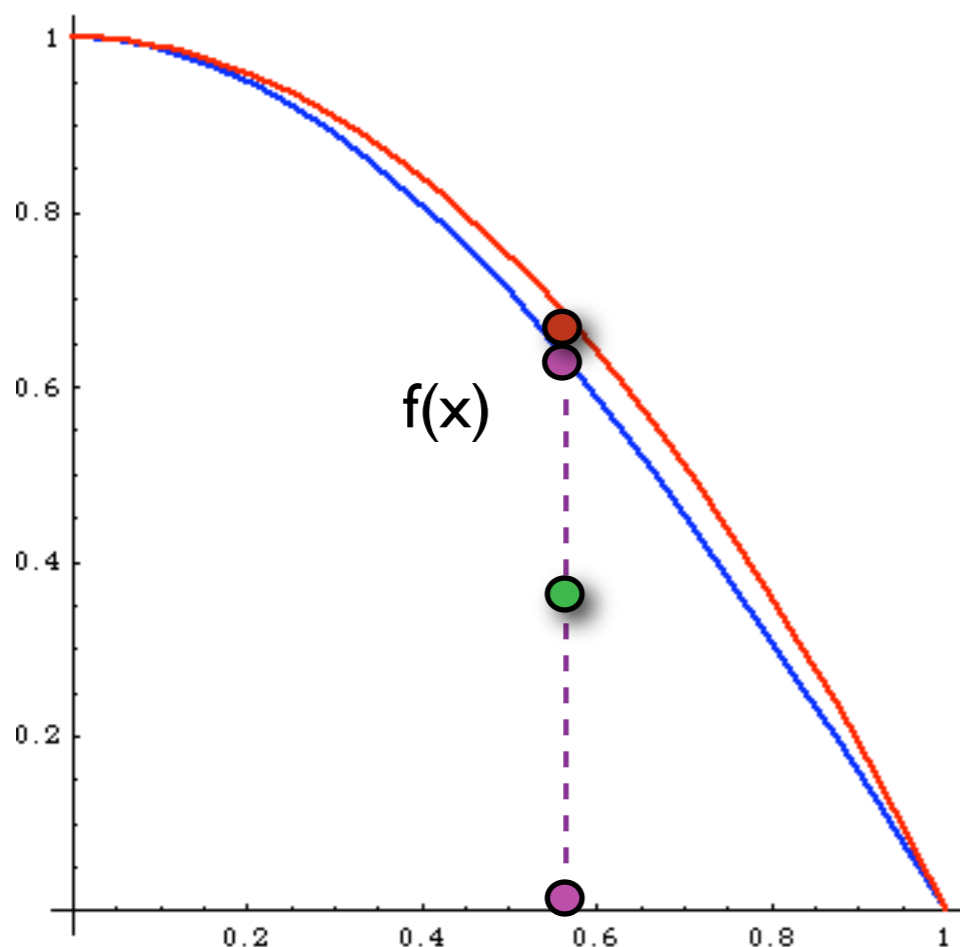
What's the difference?

after:

events is proportional to the probability of areas of phase space:
events have all the same weight ("unweighted")

Events distributed as in Nature

Event generation



Improved

1. pick x distributed as $p(x)$
2. calculate $f(x)$ and $p(x)$
3. pick $0 < y < 1$
4. Compare:
if $f(x) > y p(x)$ accept event,
else reject it.

much better efficiency!!!



Event generation

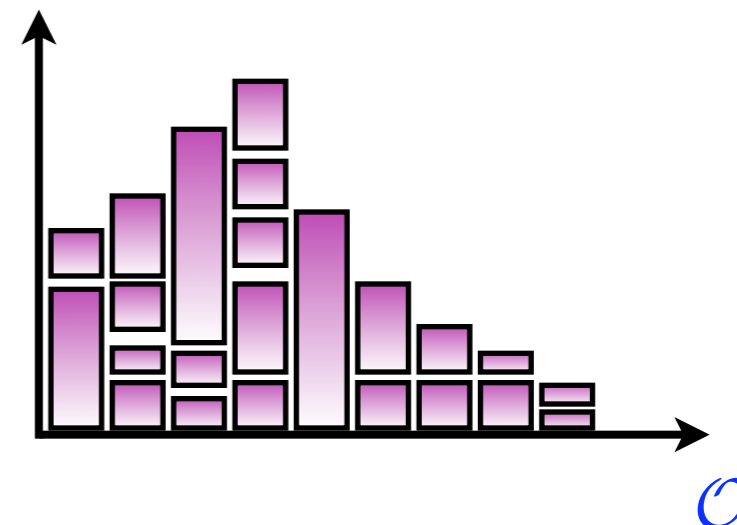
Event generation

MC integrator

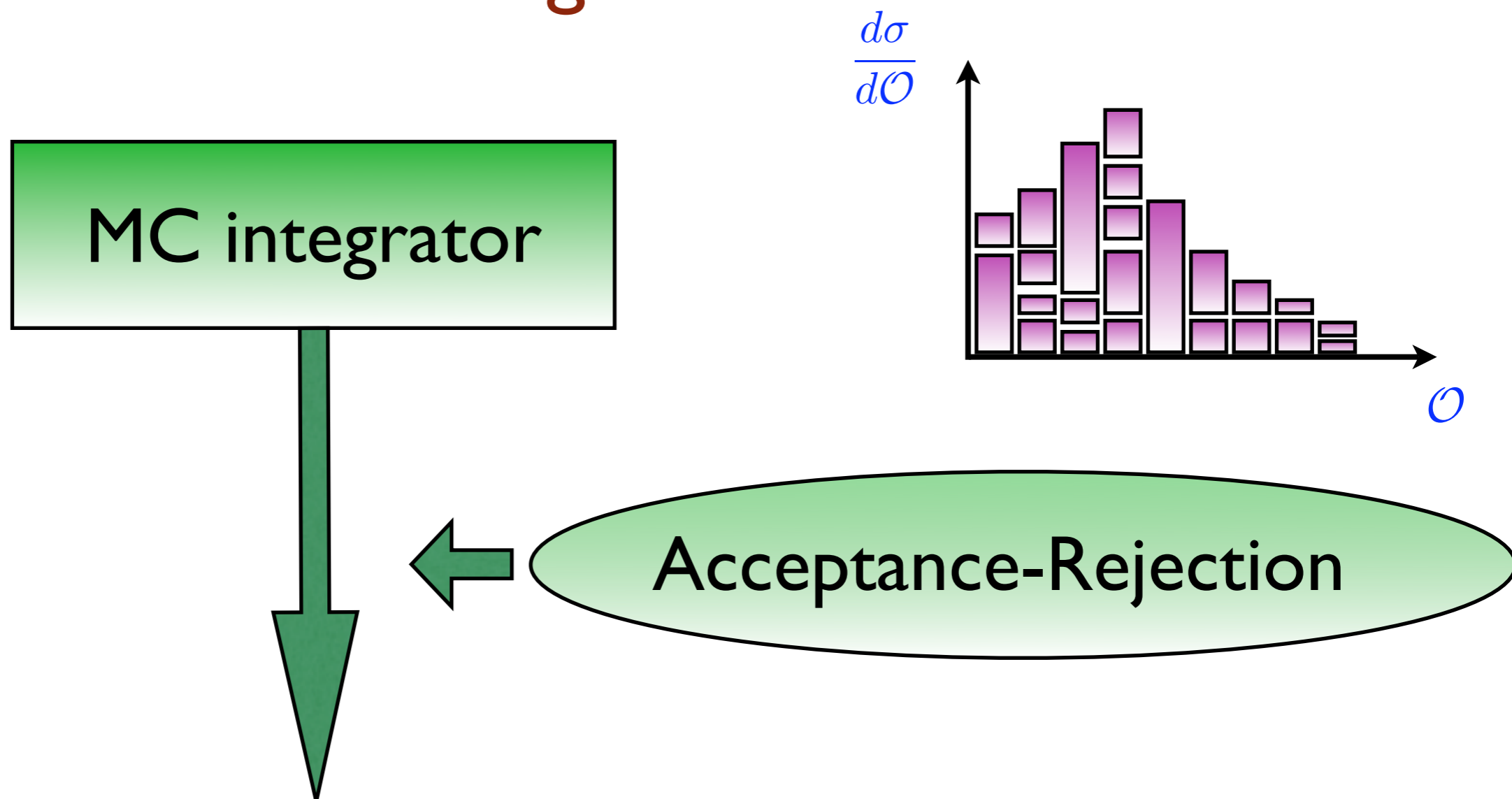
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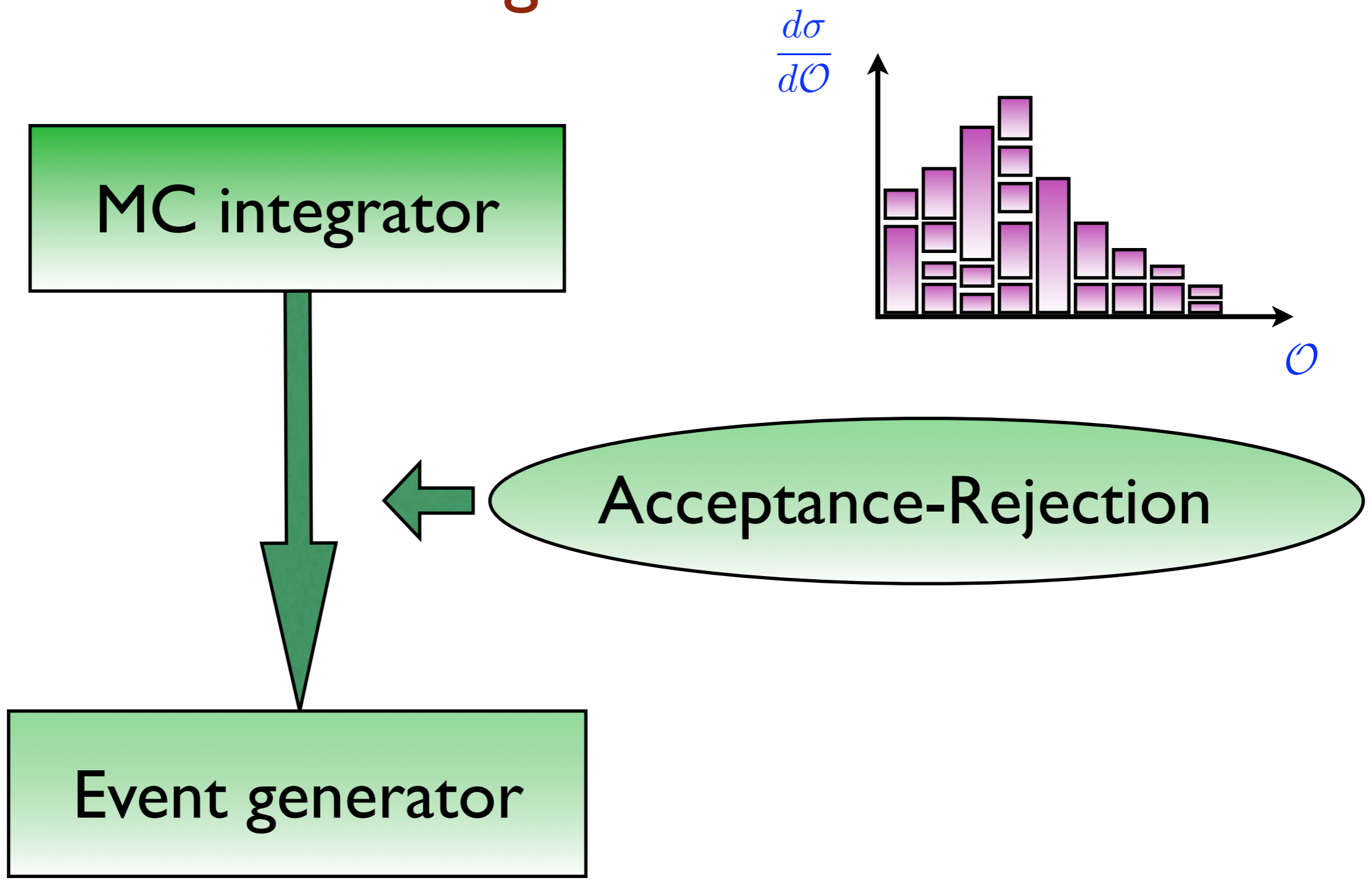
$$\frac{d\sigma}{d\mathcal{O}}$$



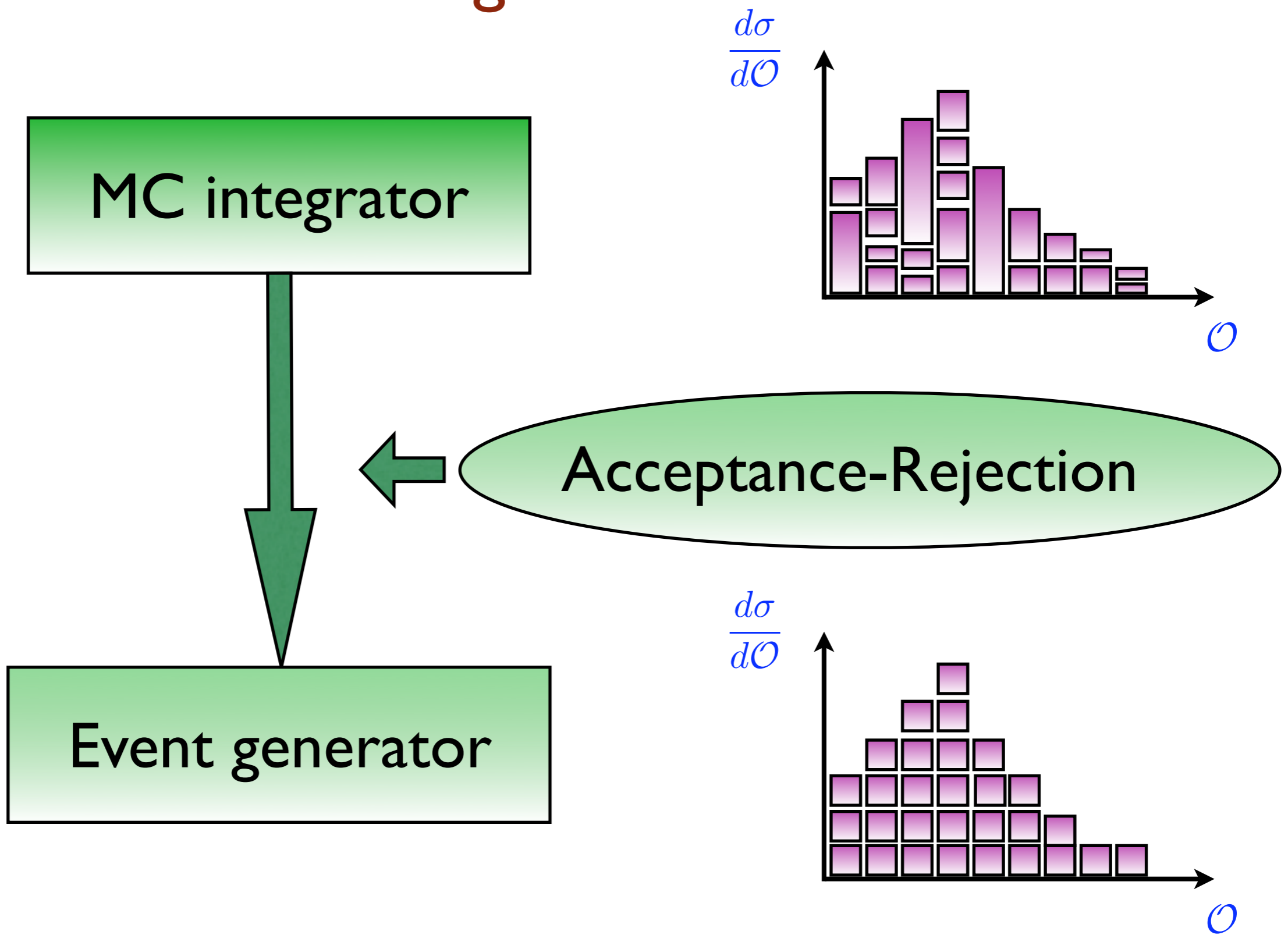
Event generation



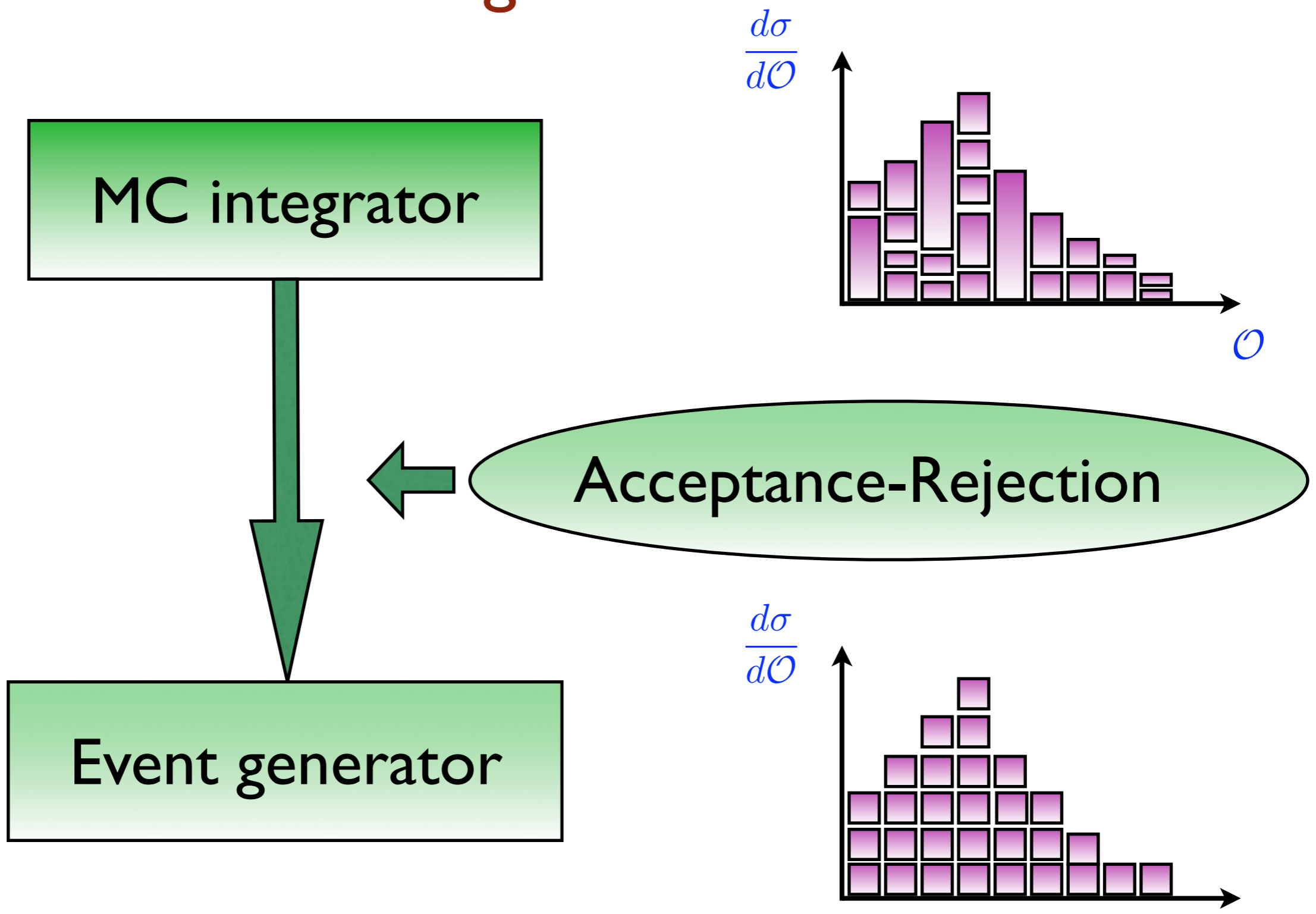
Event generation



Event generation



Event generation



☞ This is possible only if $f(x) < \infty$ AND has definite sign!



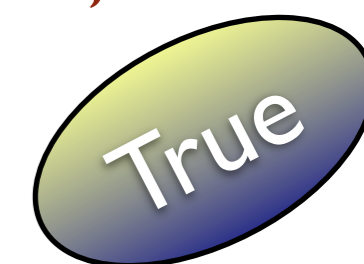
#1

A MC generator produces “unweighted” events ,
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Monte Carlo Event Generator: definiton

At the most basic level a Monte Carlo event generator is a program which produces particle physics events with the same probability as they occur in nature (virtual collider).

In practice it performs a large number of (sometimes very difficult) integrals and then unweights to give the four momenta of the particles that interact with the detector (simulation).

Note that, at least among theorists, the definition of a “Monte Carlo program” also includes codes which don’t provide a fully exclusive information on the final state but only cross sections or distributions at the parton level, even when no unweighting can be performed. I will refer to these kind of codes as “MC integrators”.

Summary of tree-level computations

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \rightarrow X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

- Matrix element calculators provide our first estimation of rates for **inclusive** final states.
- Extra radiation **is** included: it is described by the PDF's in the initial state and by the definition of a final state parton, which at LO represents all possible final state evolutions.
- Due to the above approximations a cross section at LO can strongly depend on the factorization and renormalization scales.
- Any tree-level calculation for a final state F can be promoted to the exclusive F + X through a shower. More on this soon...

LO : the technical challenges

How do we calculate a LO cross section for 3 jets at the LHC?

I. Identify all subprocesses ($gg \rightarrow ggg$, $qg \rightarrow qgg$) in

$$\sigma(pp \rightarrow 3j) = \sum_{ijk} \int f_i(x_1) f_j(x_2) \hat{\sigma}(ij \rightarrow k_1 k_2 k_3)$$

easy

II. For each one, calculate the amplitude:

$$\mathcal{A}(\{p\}, \{h\}, \{c\}) = \sum_i D_i$$

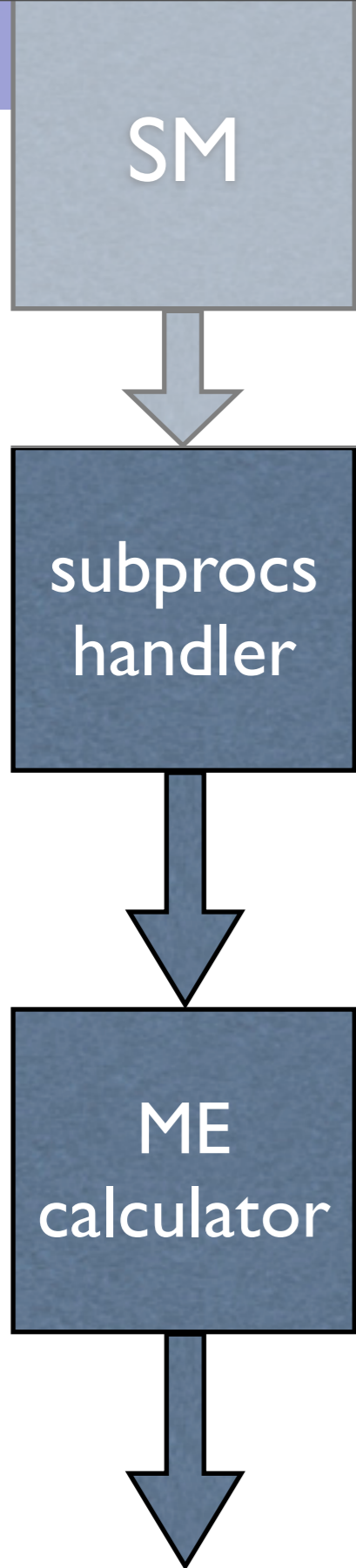
difficult

III. Square the amplitude, sum over spins & color, integrate over the phase space ($D \sim 3n$)

$$\hat{\sigma} = \frac{1}{2\hat{s}} \int d\Phi_p \sum_{h,c} |\mathcal{A}|^2$$

very hard

General structure

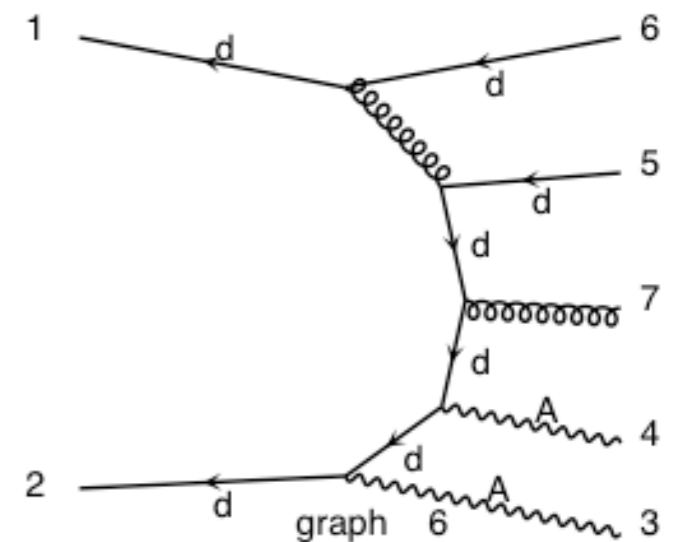


Includes all possible subprocess leading to a given multi-jet final state automatically or manually (done once for all)

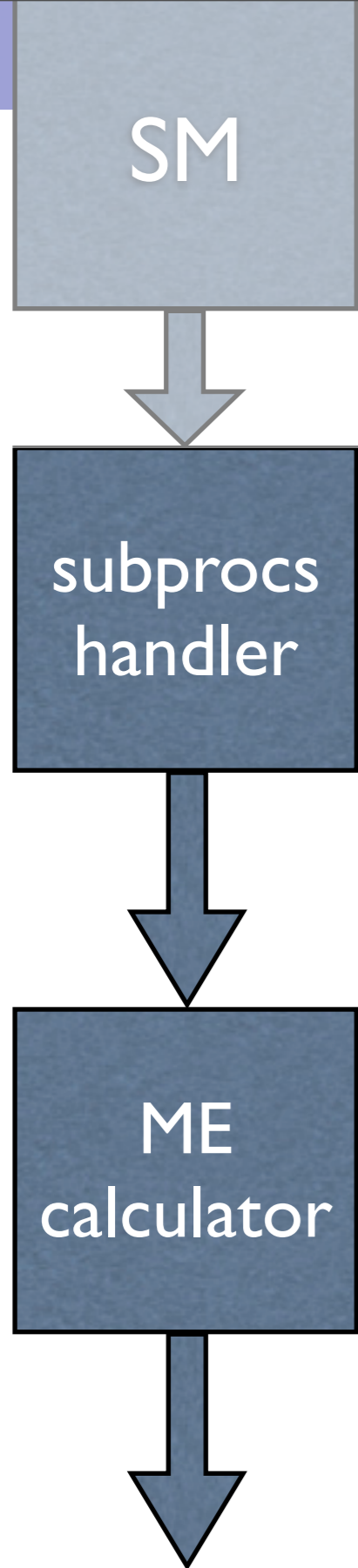
“Automatically” generates a code to calculate $|M|^2$ for arbitrary processes with many partons in the final state.

Most use Feynman diagrams w/ tricks to reduce the factorial growth, others have recursive relations to reduce the complexity to exponential. ☺

$d \sim d \rightarrow a a u u \sim g$
 $d \sim d \rightarrow a a c c \sim g$
 $s \sim s \rightarrow a a u u \sim g$
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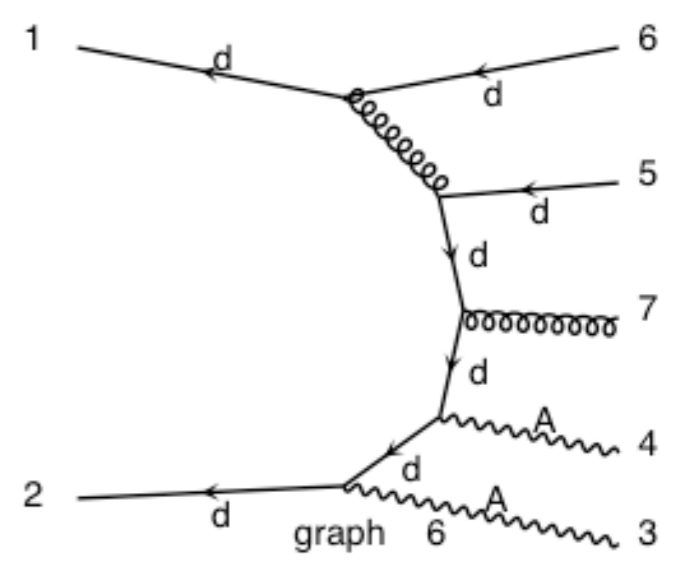


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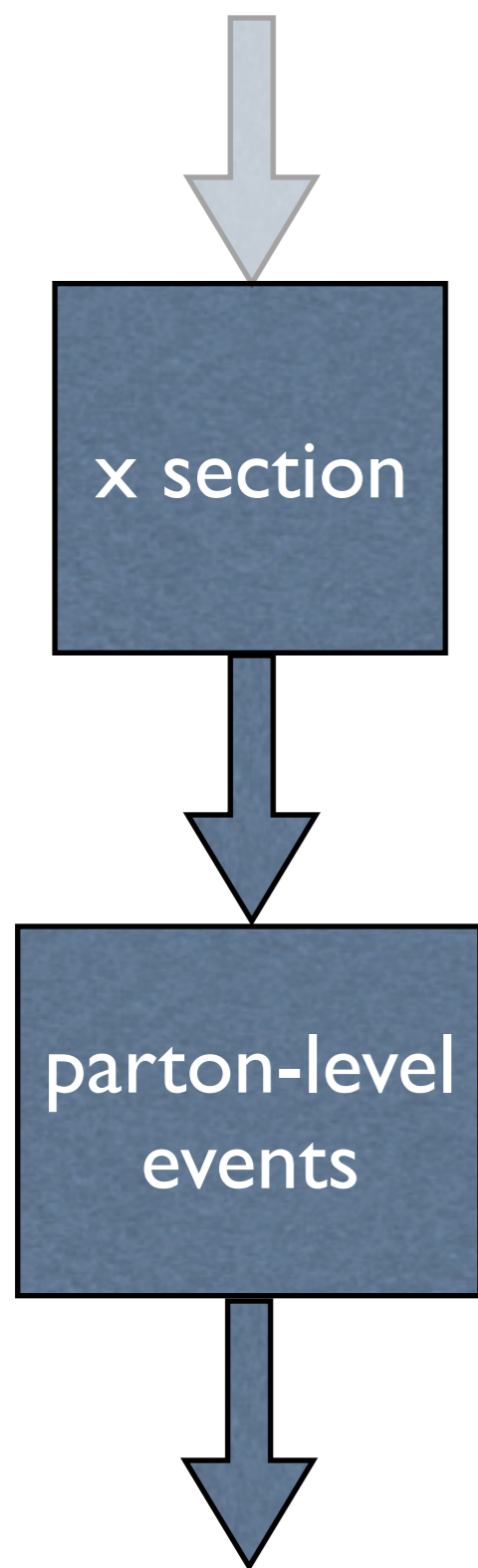
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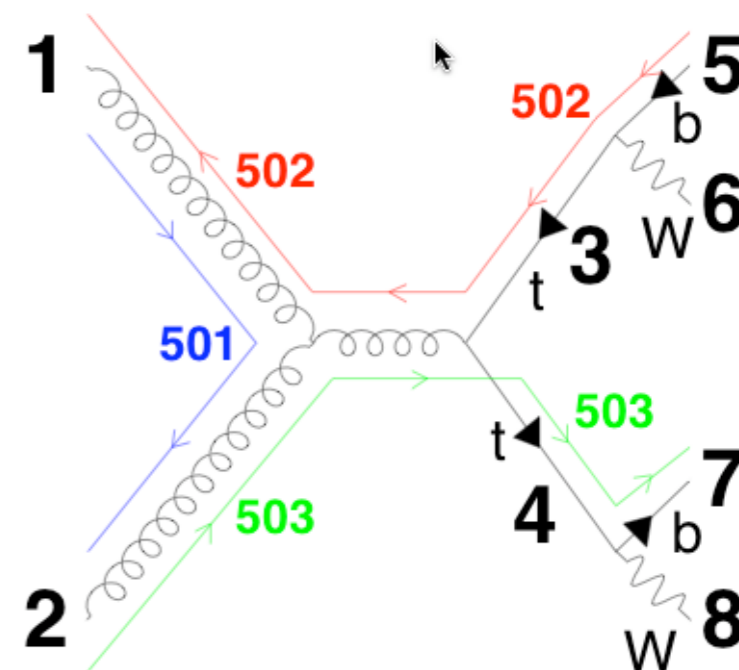
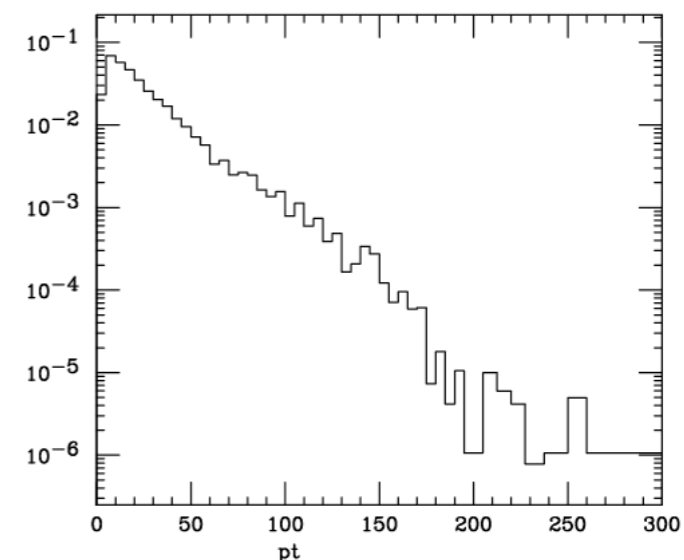


General structure



Integrate the matrix element over the phase space using a multi-channel technique and using parton-level cuts.

Events are obtained by unweighting. These are at the parton-level. Information on particle id, momenta, spin, color is given in the Les Houches format.



Parton level computation at LO

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \rightarrow X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

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A simple plan

- Physics challenges at the LHC
- Basics : QCD and MC's methods
- The new generation of MC tools
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Master QCD formula

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Two ingredients necessary:

1. Parton Distribution functions (from exp, but evolution from th).
2. Short distance coefficients as an expansion in α_S (from th).

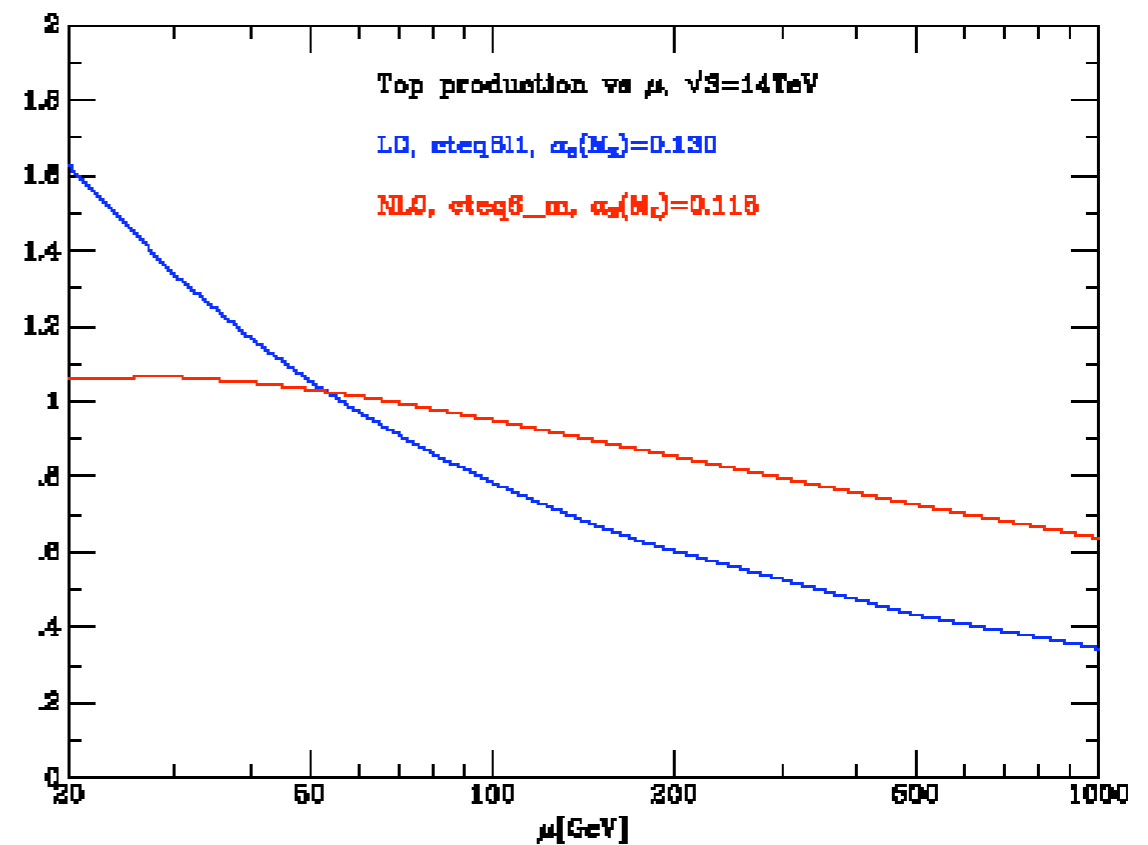
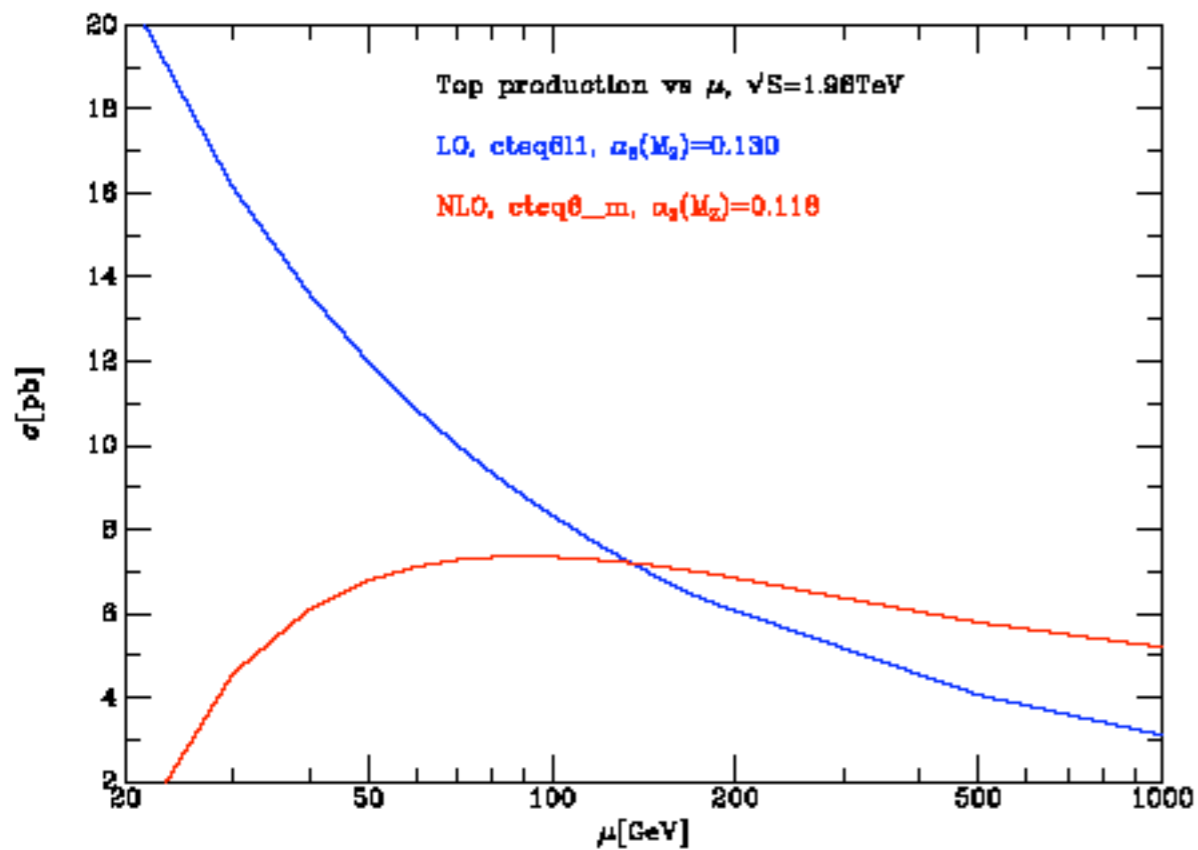
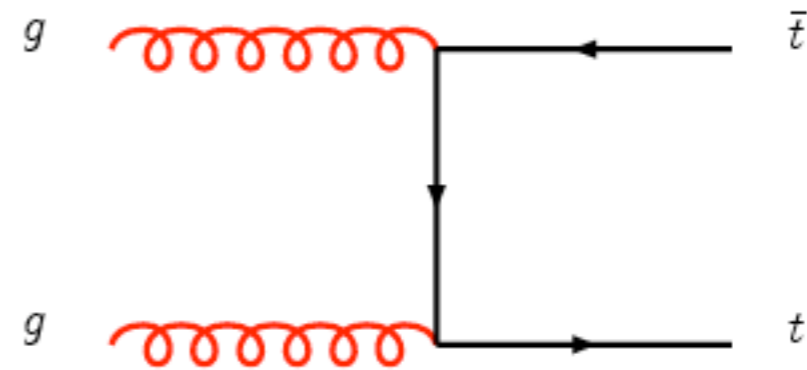
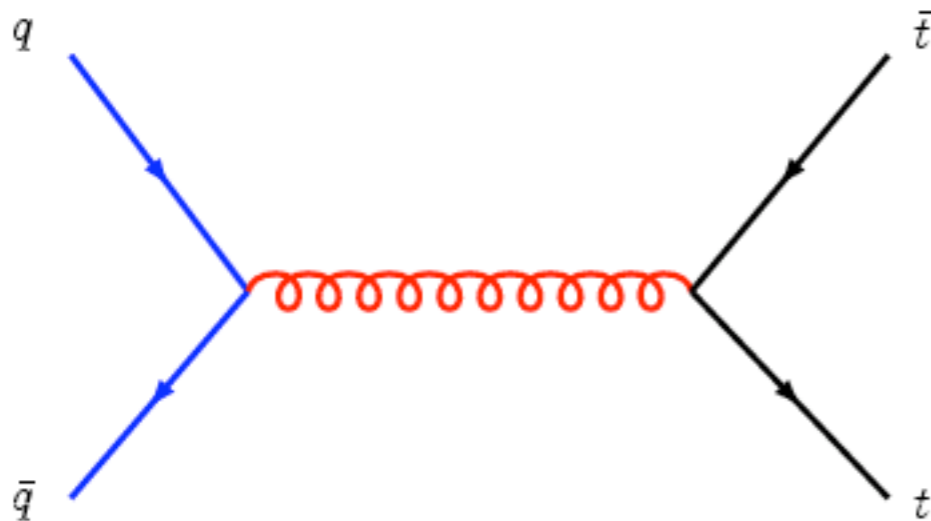
$$\hat{\sigma}_{ab \rightarrow X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Leading order

Next-to-leading order

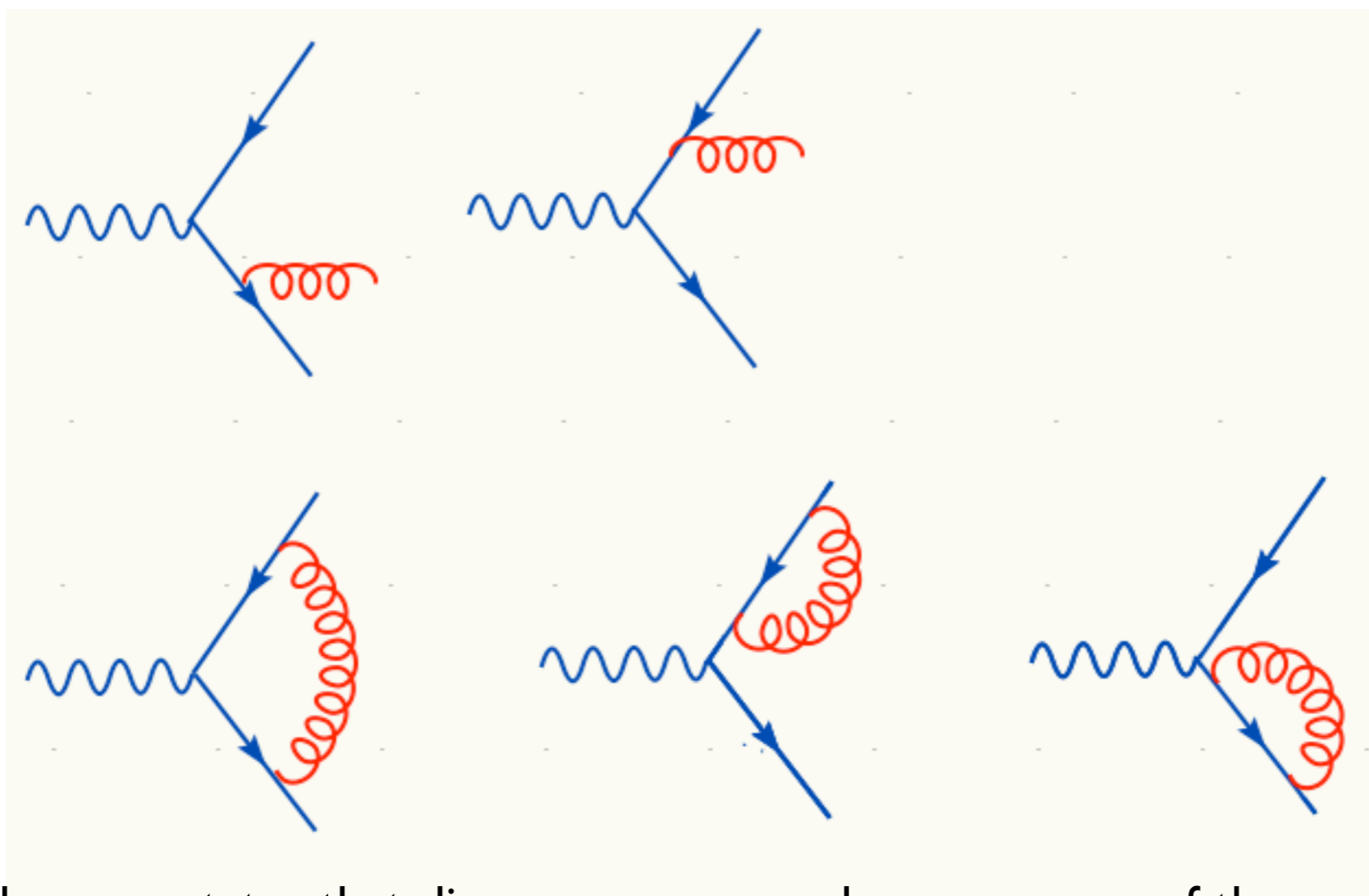
Next-to-next-to-leading order

Tevatron vs LHC



Inclusion of higher order corrections leads to a stabilization of the prediction.
At the LHC scale dependence is more difficult to estimate.

The elements of NLO calculation



Real

Virtual

The KLN theorem states that divergences appear because some of the final state are physically degenerate but we treated them as different. A final state with a soft gluon is nearly degenerate with a final state with no gluon at all (virtual).

$$\sigma^{\text{NLO}} = \int_R |M_{\text{real}}|^2 d\Phi_3 + \int_V 2\text{Re} (M_0 M_{\text{virt}}^*) d\Phi_2 = \text{finite!}$$

$$\int \frac{d^d k}{(2\pi)^d} \dots$$



Infrared-safe quantities

DEFINITION: quantities are that are insensitive to soft and collinear branching.

For these quantities, an extension of the general theorem (KLN) exists which proves that infrared divergences cancel between real and virtual or are simply removed by kinematic factors.

Such quantities are determined primarily by hard, short-distance physics. Long-distance effects give power corrections, suppressed by the inverse power of a large momentum scale (which must be present in the first place to justify the use of PT).

EXAMPLES: total rates & cross sections, jet distributions, shape variables...

**NLO codes calculate IR safe quantities
and return histograms (calculators)**



Something to remember well

Calling a code “a NLO code” is an abuse of language and can be confusing.

A NLO calculation always refers to **an IR-safe observable**.

An NLO code will, in general, be able to produce results for several quantities and distributions, only some of which will be at NLO accuracy.

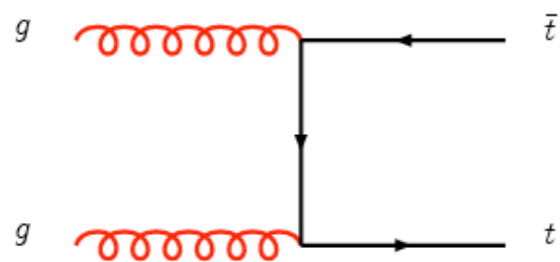
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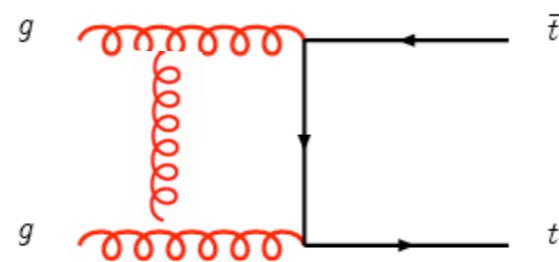
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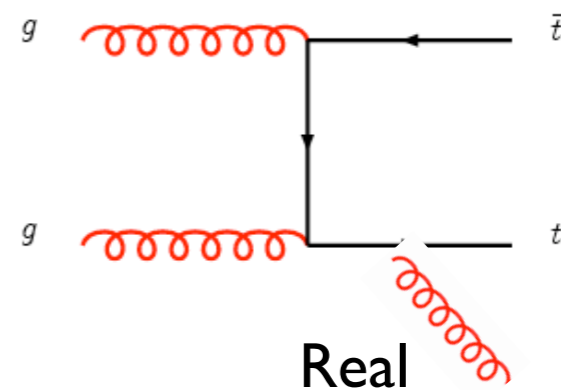
Example: Suppose we use the NLO code for $pp \rightarrow t\bar{t}$



LO



Virt



Real

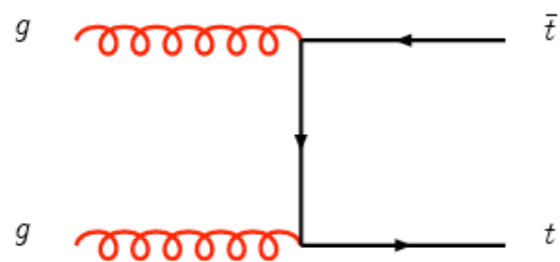
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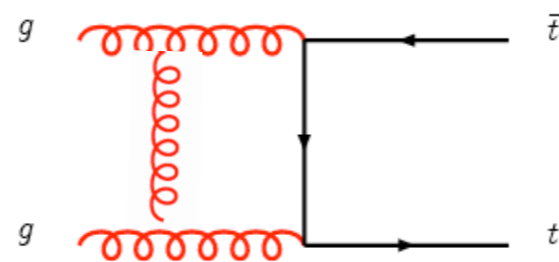
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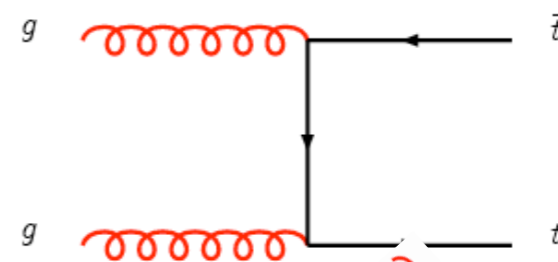
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- ☞ $t\bar{t}$ invariant mass, $m(t\bar{t})$
- ☞ $\Delta\Phi(t\bar{t})$

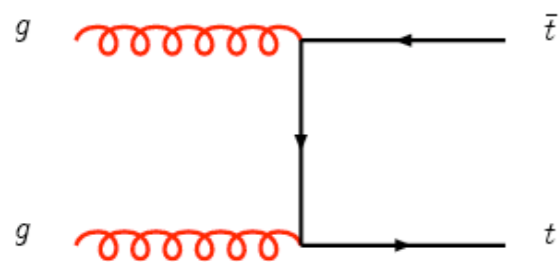
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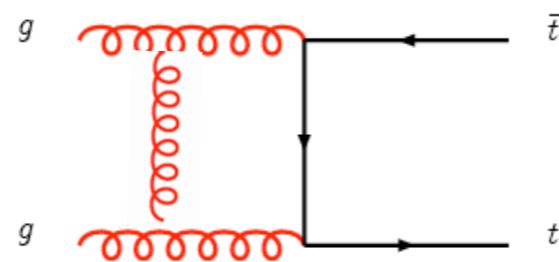
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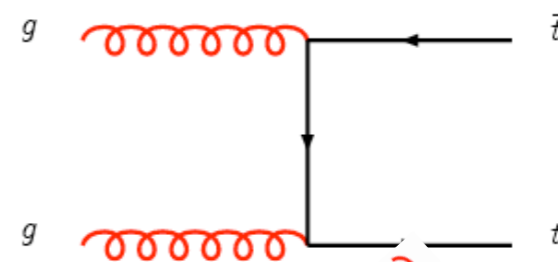
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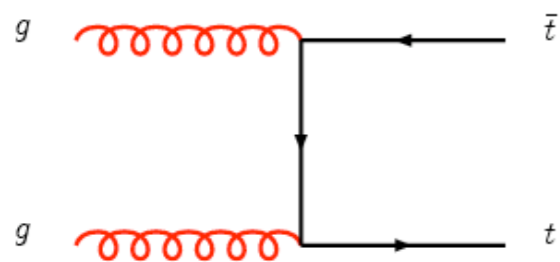
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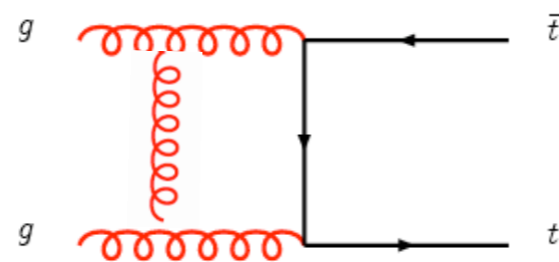
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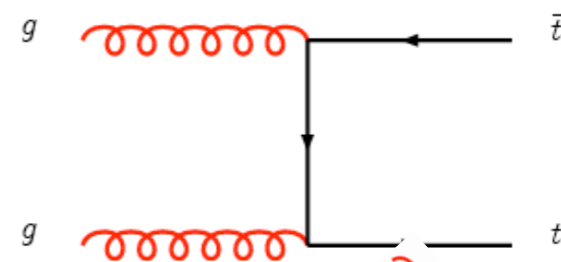
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LO



Virt



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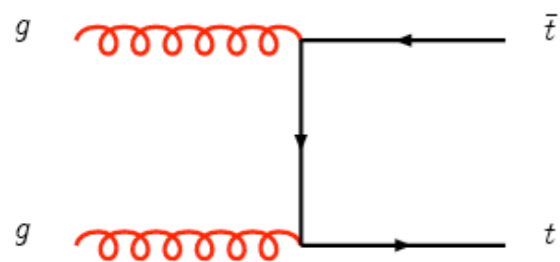
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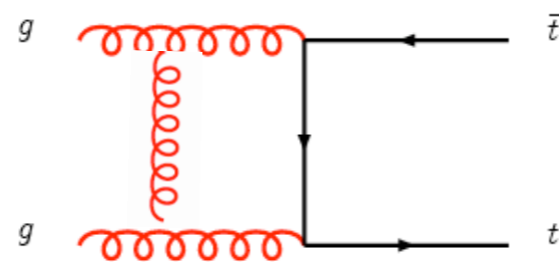
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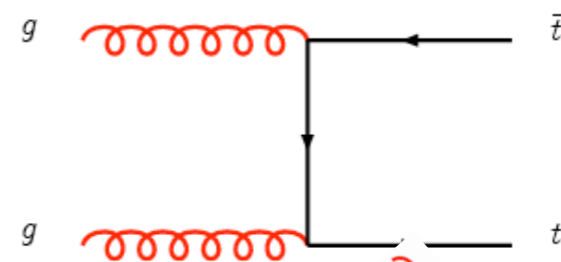
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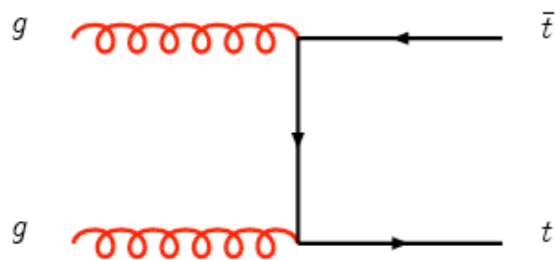
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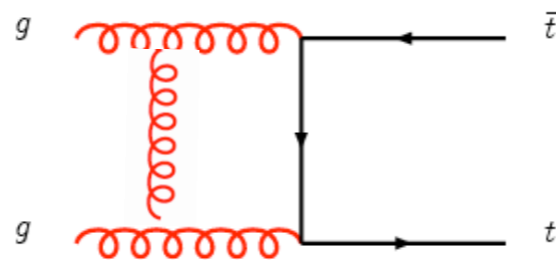
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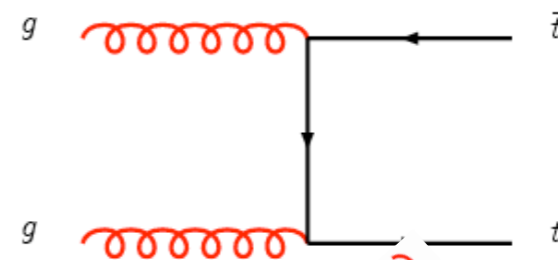
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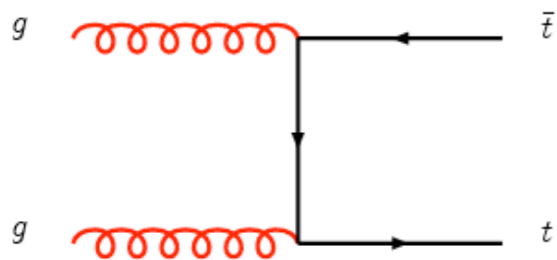
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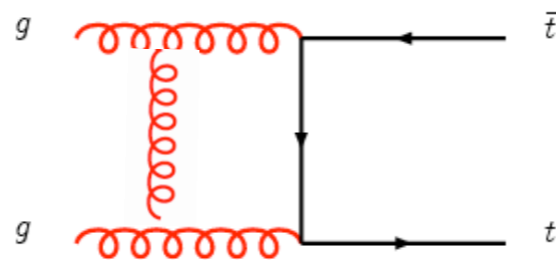
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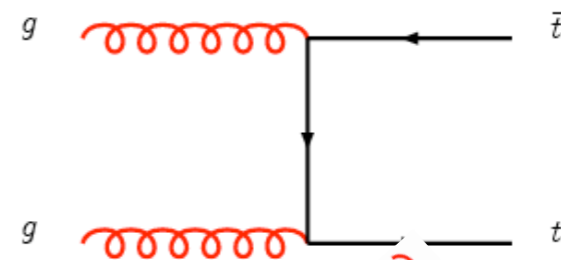
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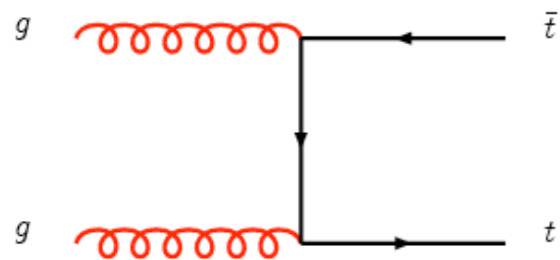
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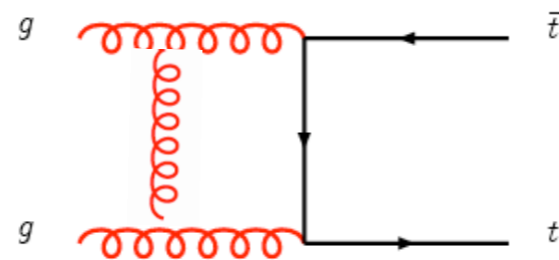
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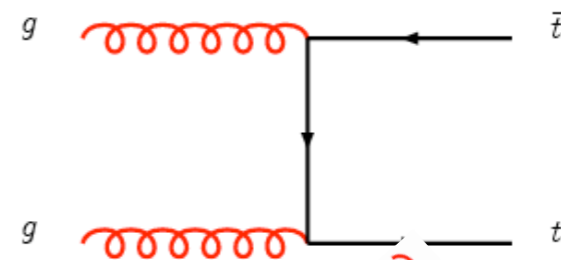
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A calculations/code at NLO for a process provides NLO predictions for any IR safe observable.



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- Dedicated codes or families of codes available.

Example:MC FM

Downloadable general purpose NLO code (Campbell & Ellis)

$$p\bar{p} \rightarrow W^\pm / Z$$

$$p\bar{p} \rightarrow W^\pm + Z$$

$$p\bar{p} \rightarrow W^\pm + \gamma$$

$$p\bar{p} \rightarrow W^\pm + g^* (\rightarrow b\bar{b})$$

$$p\bar{p} \rightarrow W^\pm / Z + 1 \text{ jet}$$

$$p\bar{p}(gg) \rightarrow H$$

$$p\bar{p}(VV) \rightarrow H + 2 \text{ jets}$$

$$p\bar{p} \rightarrow H + b$$

$$p\bar{p} \rightarrow W^+ + W^-$$

$$p\bar{p} \rightarrow Z + Z$$

$$p\bar{p} \rightarrow W^\pm / Z + H$$

$$p\bar{p} \rightarrow Z b\bar{b}$$

$$p\bar{p} \rightarrow W^\pm / Z + 2 \text{ jets}$$

$$p\bar{p}(gg) \rightarrow H + 1 \text{ jet}$$

$$p\bar{p} \rightarrow t + q$$

$$p\bar{p} \rightarrow Z + b$$

- ➡ Plus all single-top channels, Wc, WQJ, ZQJ,...
- ➡ Extendable/sizeable library of processes,
relevant for signal and background studies, including spin correlations.
- ➡ Cross sections and distributions at NLO are provided
- ➡ Easy and flexible choice of parameters/cuts (input card).

NLO MC's integrators: summary

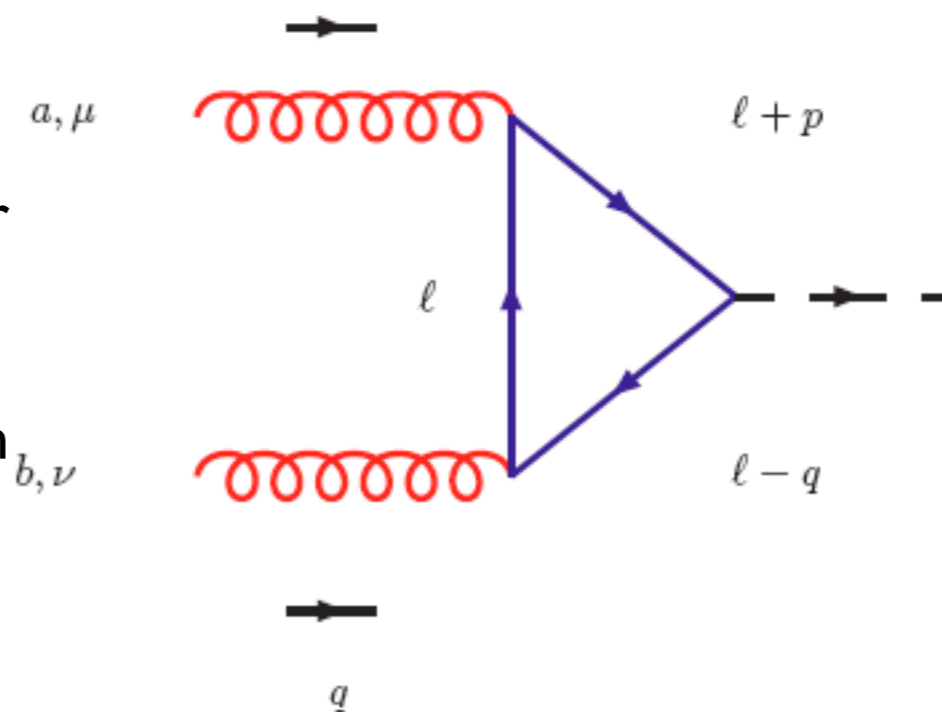
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- These are NOT event generators!!
- Dedicated codes or families of codes available.
- Automatization in sight.

$pp \rightarrow H$ at LO p

This is a “simple” $2 \rightarrow 1$ process.

However, at variance with $pp \rightarrow W$, the LO order process already proceeds through a loop.

In this case, this means that the loop calculation has to give a finite result!



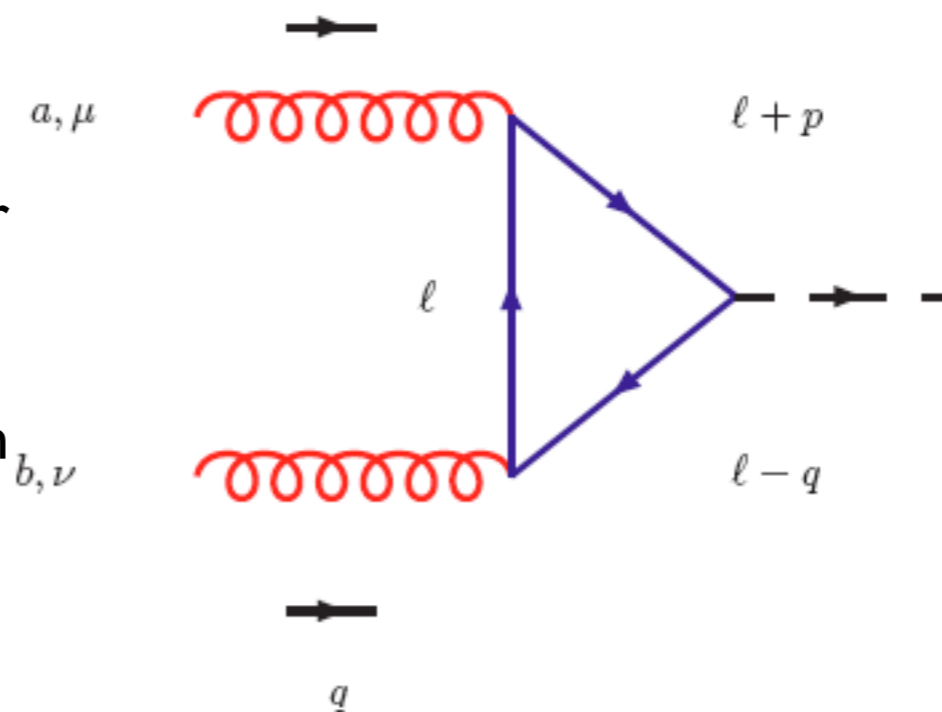
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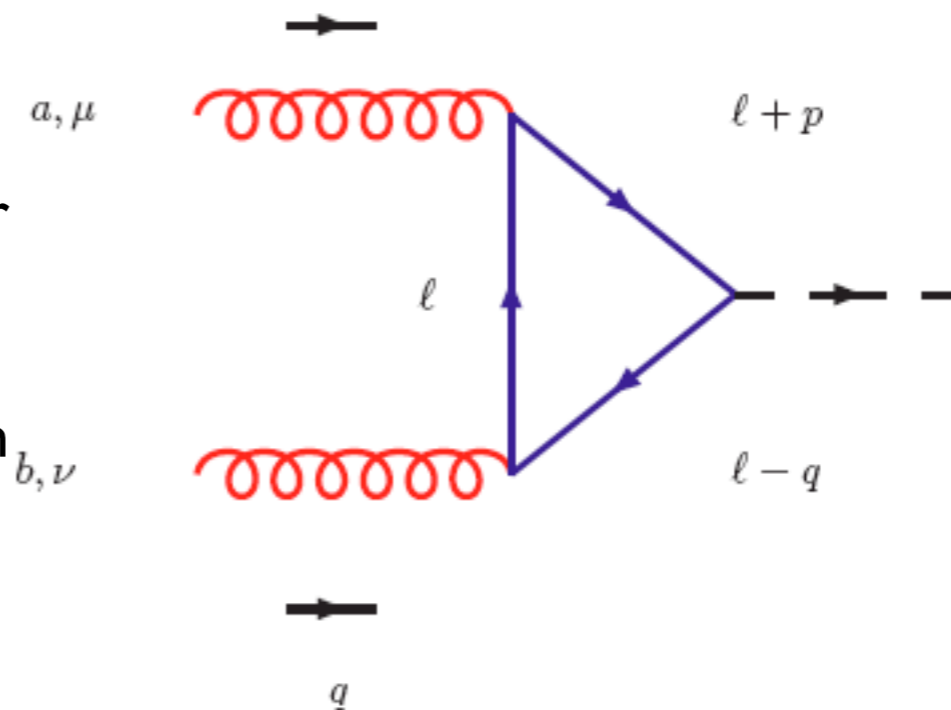


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$$i\mathcal{A} = -(-ig_s)^2 \text{Tr}(t^a t^b) \left(\frac{-im_t}{v} \right) \int \frac{d^d \ell}{(2\pi)^n} \frac{T^{\mu\nu}}{\text{Den}} (i)^3 \epsilon_\mu(p) \epsilon_\nu(q)$$

where

$$\text{Den} = (\ell^2 - m_t^2)[(\ell + p)^2 - m_t^2][(\ell - q)^2 - m_t^2]$$



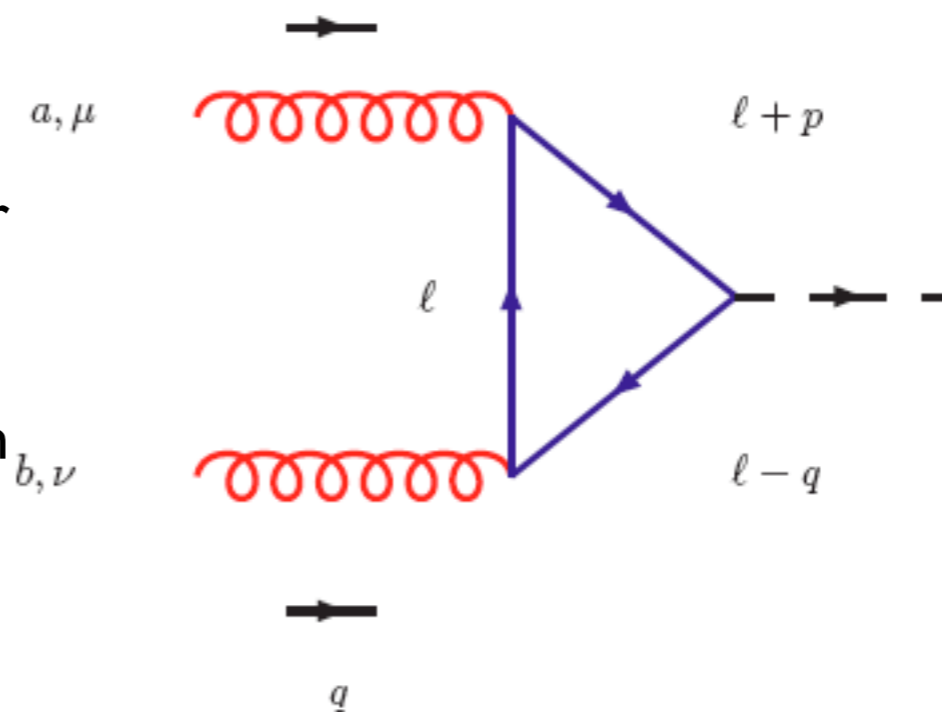
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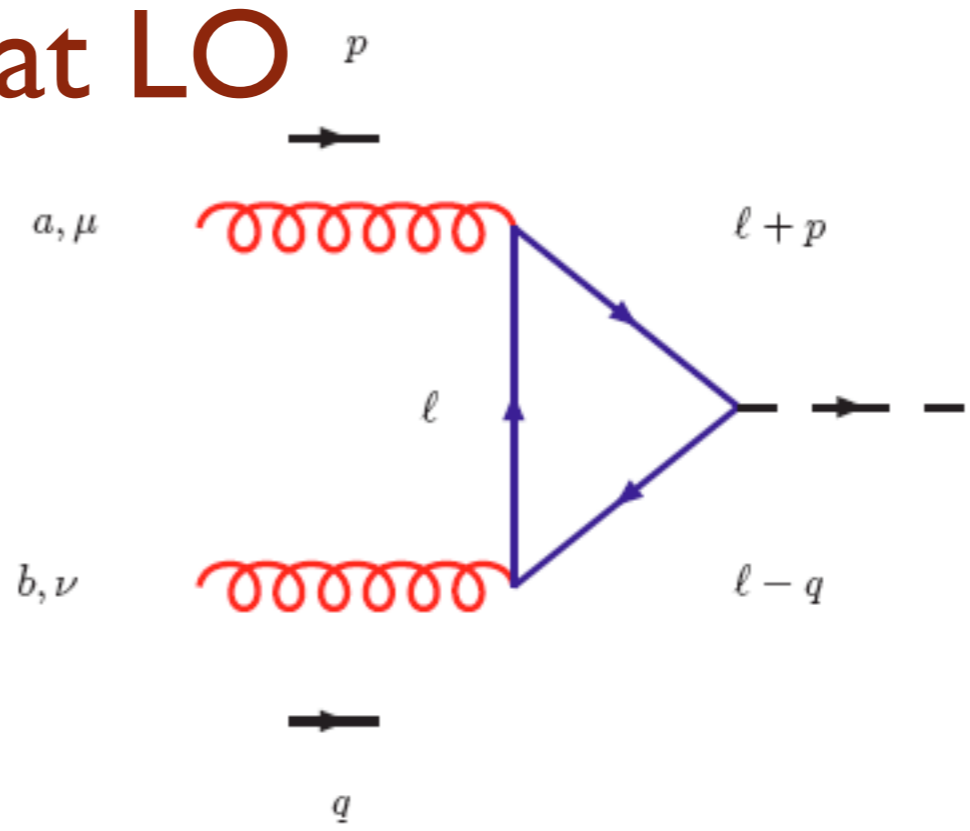
We combine the denominators into one by using $\frac{1}{ABC} = 2 \int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1-x-y)]^3}$

$$\frac{1}{\text{Den}} = 2 \int dx dy \frac{1}{[\ell^2 - m_t^2 + 2\ell \cdot (px - qy)]^3}$$

pp → H at LO

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - C)^3} = \frac{i}{32\pi^2} (4\pi)^\epsilon \frac{\Gamma(1 + \epsilon)}{\epsilon} (2 - \epsilon) C^{-\epsilon}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - C)^3} = -\frac{i}{32\pi^2} (4\pi)^\epsilon \Gamma(1 + \epsilon) C^{-1-\epsilon}.$$



where $d=4-2\epsilon$. By substituting we arrive at a very simple final result!!

$$\mathcal{A}(gg \rightarrow H) = -\frac{\alpha_S m_t^2}{\pi v} \delta^{ab} \left(g^{\mu\nu} \frac{M_H^2}{2} - p^\nu q^\mu \right) \int dx dy \left(\frac{1 - 4xy}{m_t^2 - m_H^2 xy} \right) \epsilon_\mu(p) \epsilon_\nu(q).$$

Comments:

- * The final dependence of the result is mt^2 : one from the Yukawa coupling, one from the spin flip.
- * The tensor structure could have been guessed by gauge invariance.
- * The integral depends on mt and mh .

LO cross section

$$\sigma(pp \rightarrow H) = \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 g(x_1, \mu_f) g(x_2, \mu_f) \hat{\sigma}(gg \rightarrow H)$$

$$x_1 \equiv \sqrt{\tau} e^y \quad x_2 \equiv \sqrt{\tau} e^{-y} \quad \tau = x_1 x_2 \quad \tau_0 = M_H^2/S \quad z = \tau_0/\tau$$

$$= \frac{\alpha_S^2}{64\pi v^2} \left| I\left(\frac{M_H^2}{m^2}\right) \right|^2 \tau_0 \int_{\log \sqrt{\tau_0}}^{-\log \sqrt{\tau_0}} dy g(\sqrt{\tau_0} e^y) g(\sqrt{\tau_0} e^{-y})$$

The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

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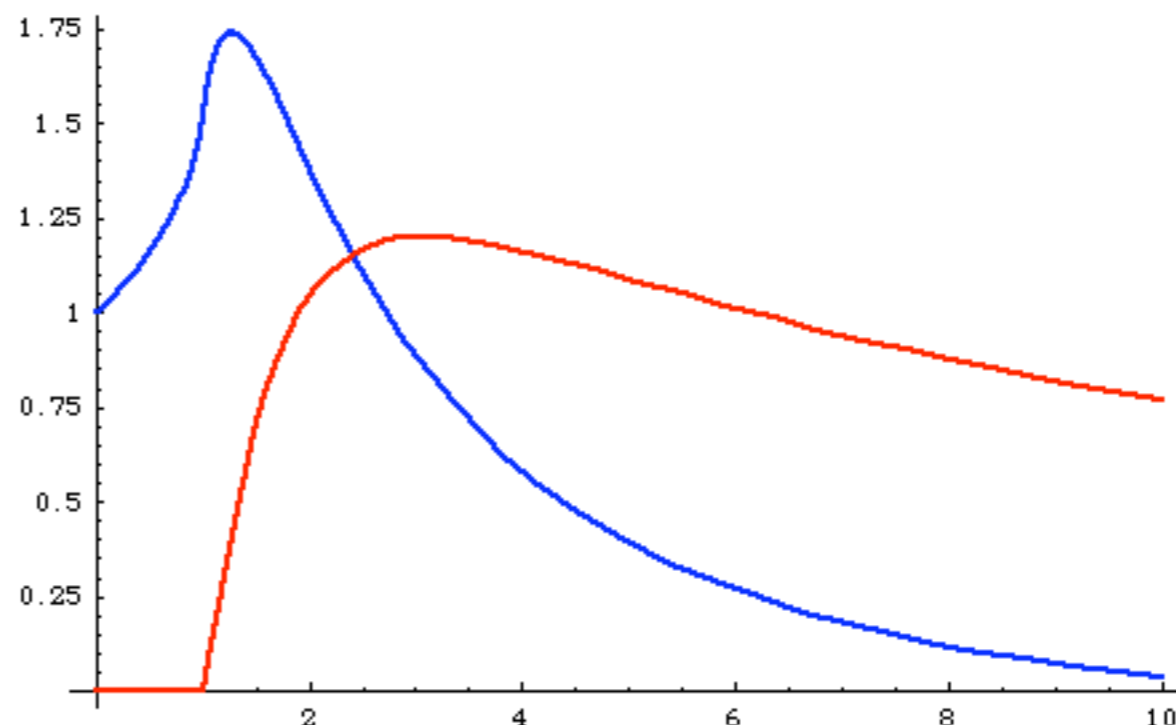
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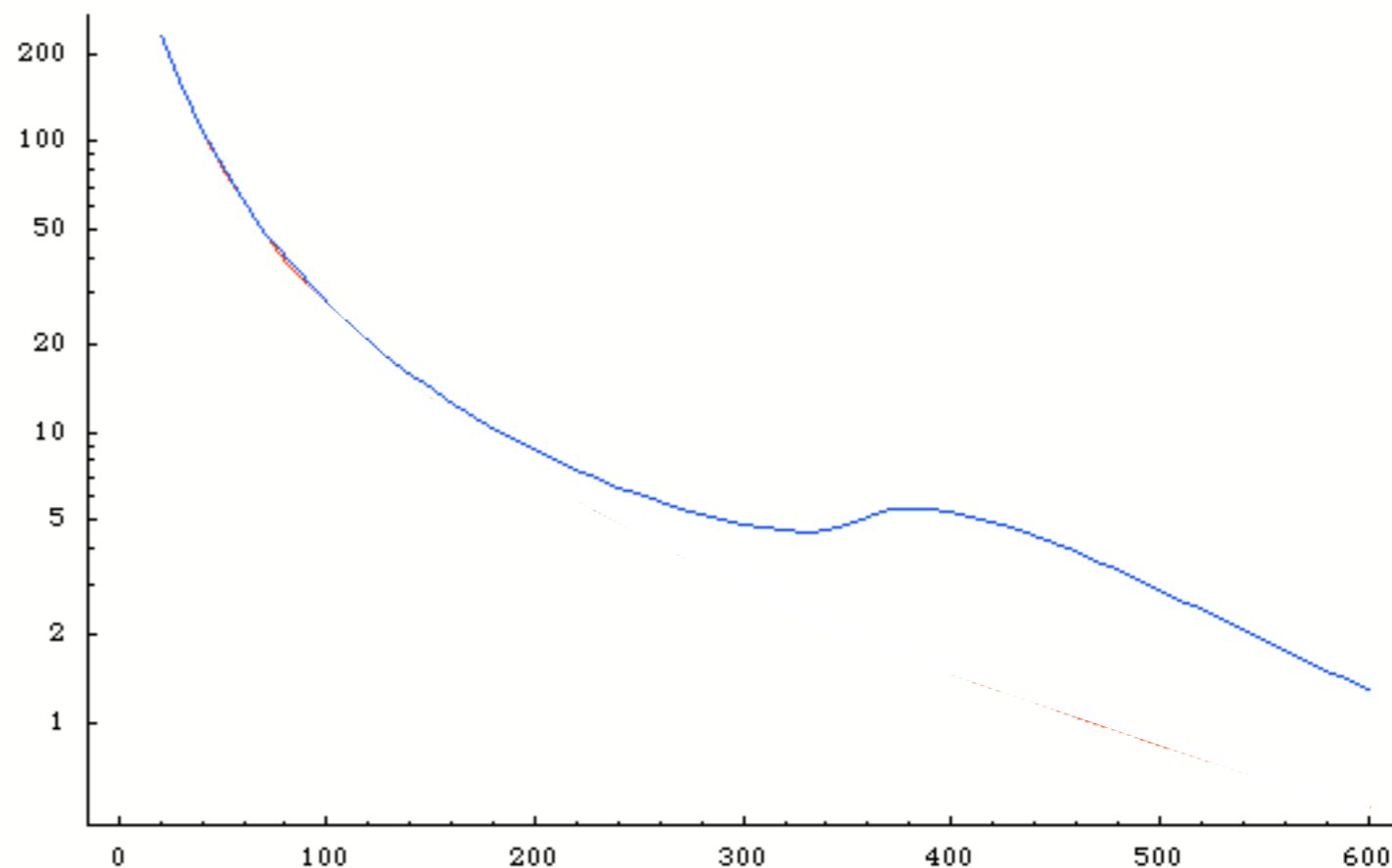
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This causes a bump in the cross section.

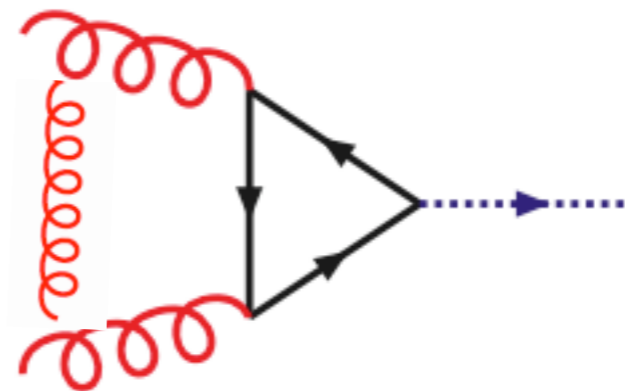


$pp \rightarrow H @ \text{NLO}$

At NLO we have to include an extra parton (virtual or real).

The virtuals will become a two-loop calculation!!

Can we avoid that?



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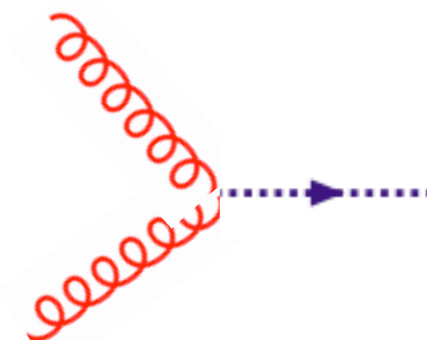
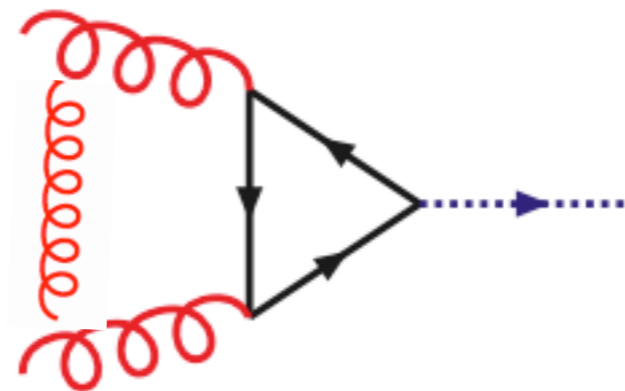
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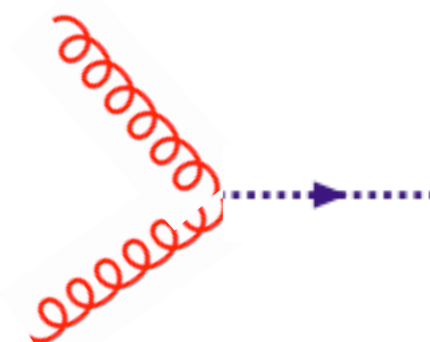
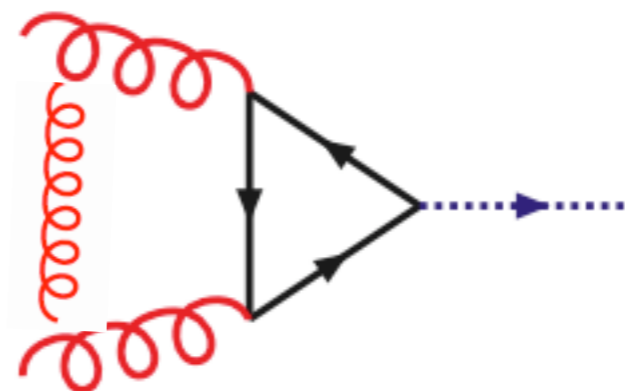
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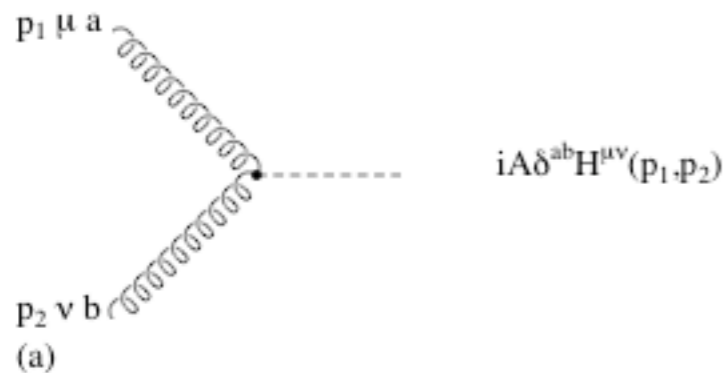
This looks like a local vertex, ggH.

The top quark has disappeared from the low energy theory but it has left something behind (non-decoupling).

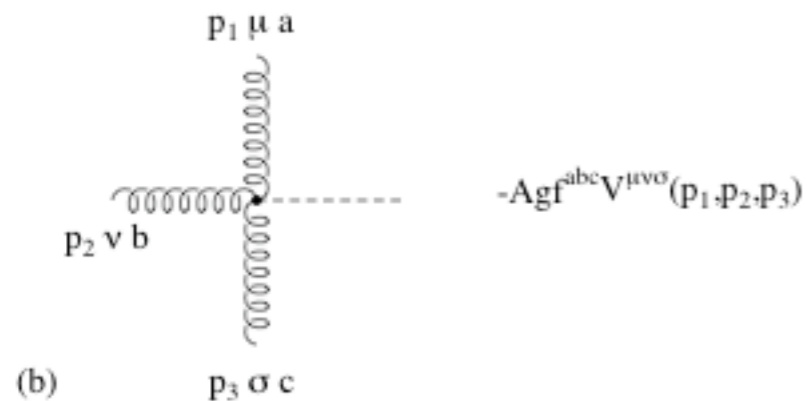
Higgs effective field theory

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} \left(1 - \frac{\alpha_S}{3\pi} \frac{H}{v} \right) G^{\mu\nu} G_{\mu\nu}$$

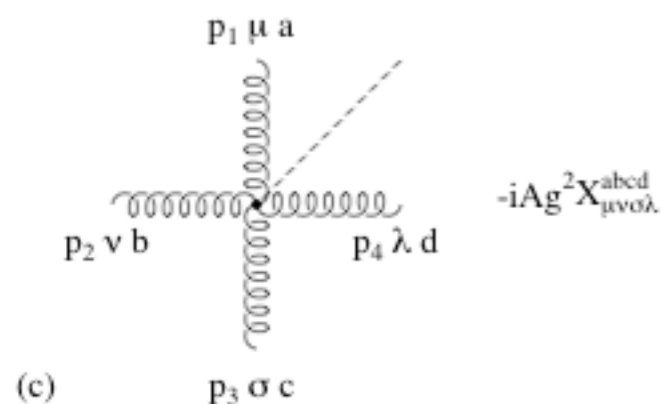
This is an effective non-renormalizable theory (no top) which describes the Higgs couplings to QCD.



$$H^{\mu\nu}(p_1, p_2) = g^{\mu\nu} p_1 \cdot p_2 - p_1^\nu p_2^\mu.$$



$$V^{\mu\nu\rho}(p_1, p_2, p_3) = (p_1 - p_2)^\rho g^{\mu\nu} + (p_2 - p_3)^\mu g^{\nu\rho} + (p_3 - p_1)^\nu g^{\rho\mu},$$



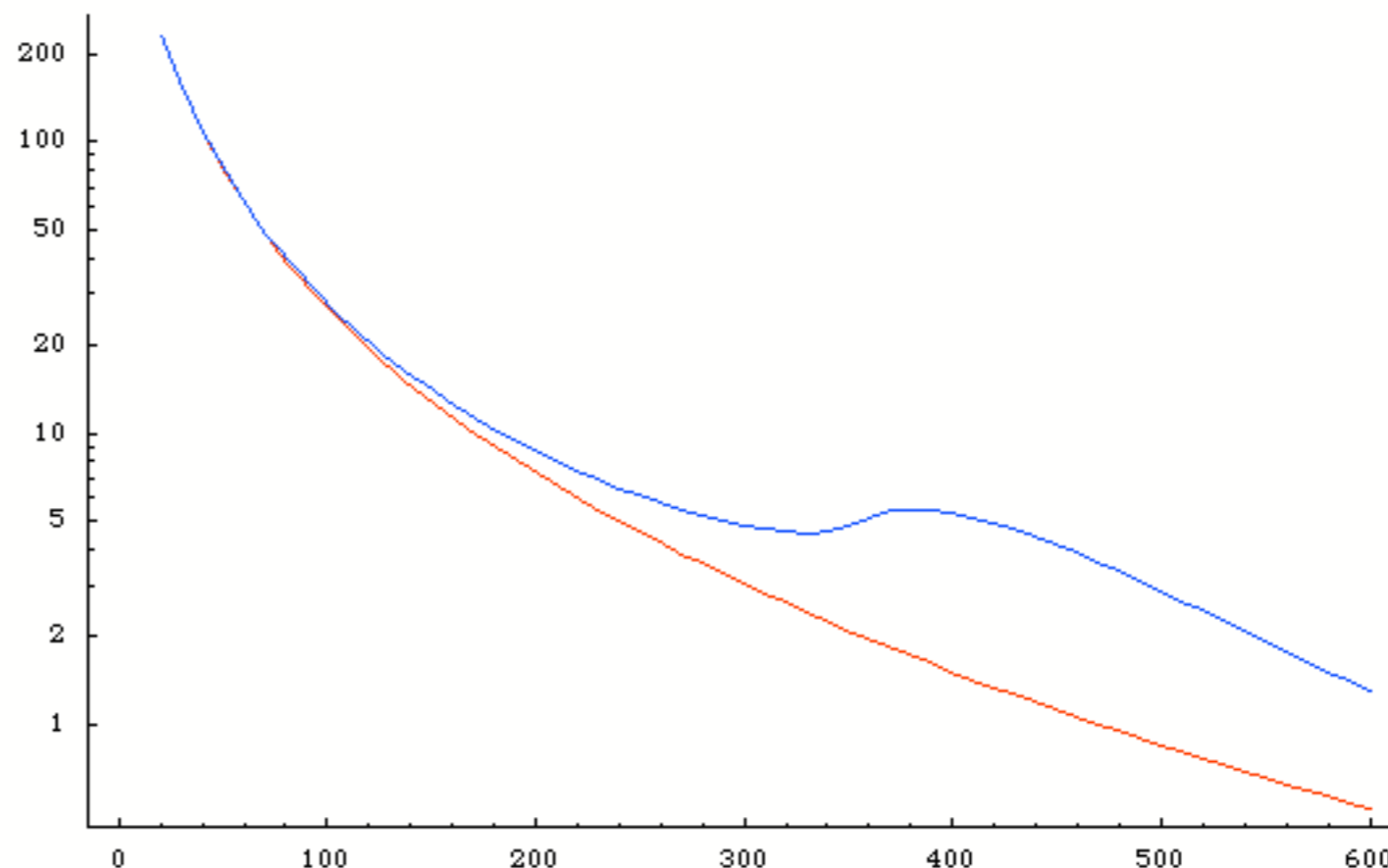
$$\begin{aligned} X_{abcd}^{\mu\nu\rho\sigma} = & f_{abe} f_{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ & + f_{ace} f_{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ & + f_{ade} f_{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}). \end{aligned}$$

LO cross section: full vs HEFT

$$\sigma(pp \rightarrow H) = \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 g(x_1, \mu_f) g(x_2, \mu_f) \hat{\sigma}(gg \rightarrow H)$$

The accuracy of the calculation in the HEFT calculation can be directly assessed by taking the limit $m \rightarrow \infty$.

For light Higgs is better than 10%.



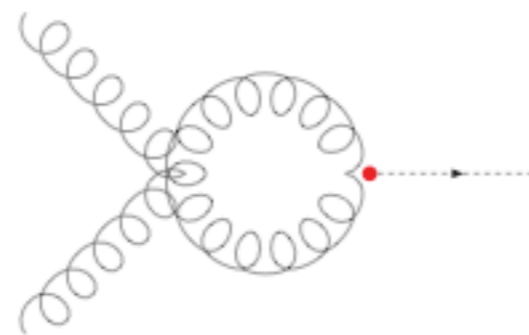
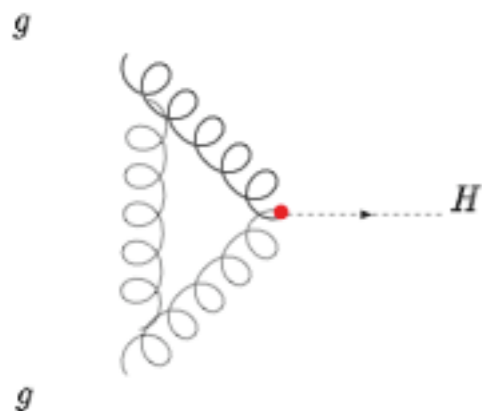
So, if we are interested in a light Higgs we use the HEFT and simplify our life. If we do so, the NLO calculation becomes a standard 1-loop calculation, similar to Drell-Yan at NLO.

We can do it!!



Virtual contributions

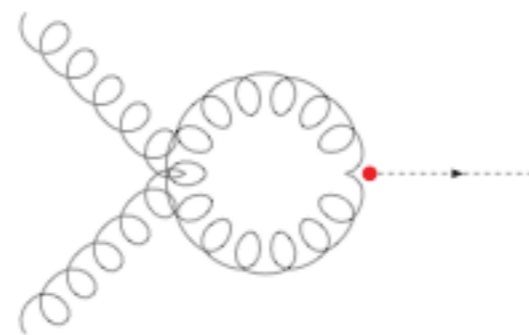
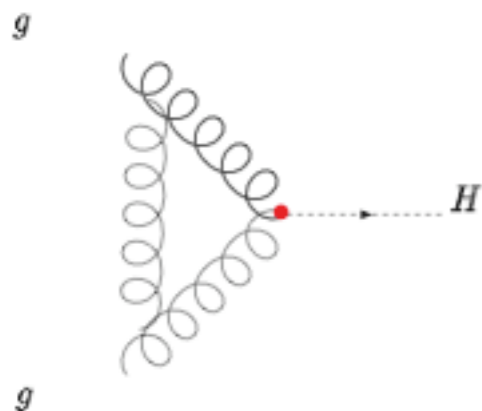
Virtual contributions



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They can be easily written down by hand.

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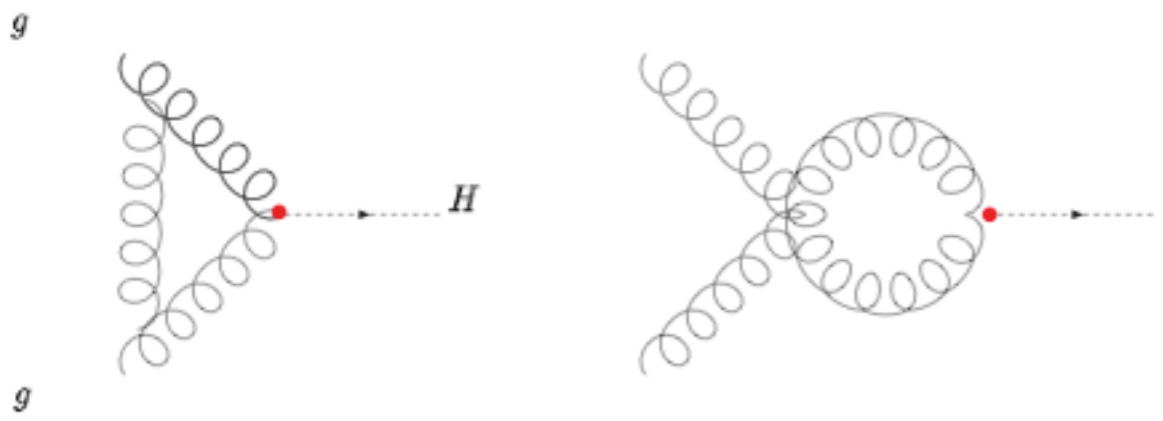


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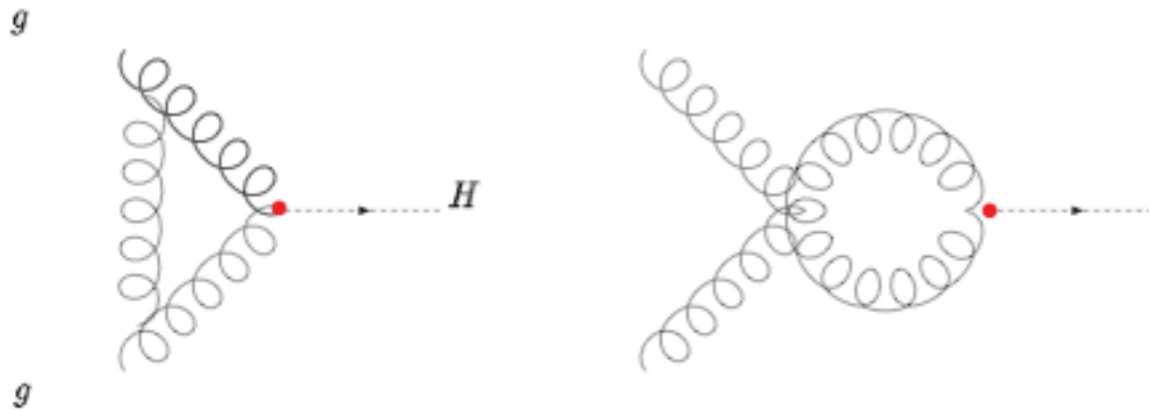
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One also have to consider that the coefficient of the HEFT receive corrections which have to be included in the result.

$$\mathcal{L}_{\text{eff}}^{\text{NLO}} = \left(1 + \frac{11}{4} \frac{\alpha_S}{\pi} \right) \frac{\alpha_S}{3\pi} \frac{H}{v} G^{\mu\nu} G_{\mu\nu}$$

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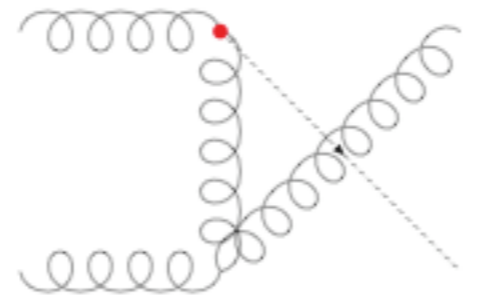
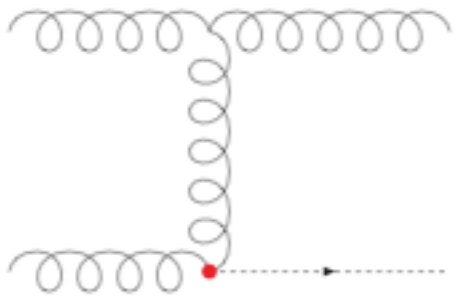
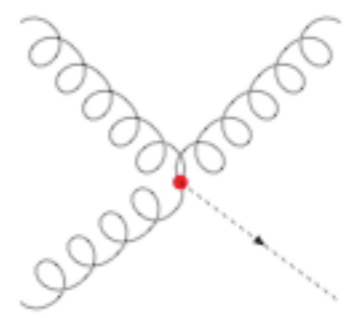
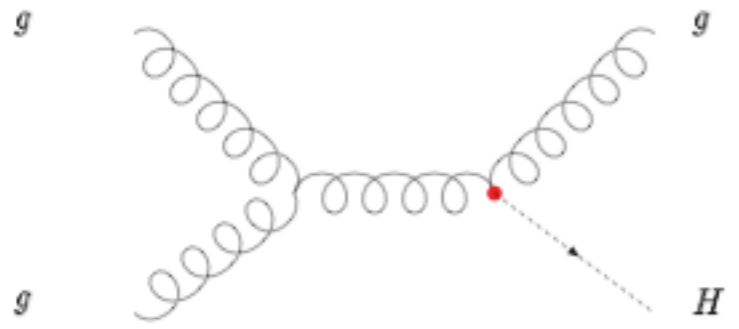
$$\mathcal{L}_{\text{eff}}^{\text{NLO}} = \left(1 + \frac{11}{4} \frac{\alpha_S}{\pi} \right) \frac{\alpha_S}{3\pi} \frac{H}{v} G^{\mu\nu} G_{\mu\nu}$$

The result is:

$$\sigma_{\text{virt}} = \sigma_0 \delta(1-z) \left[1 + \frac{\alpha_S}{2\pi} C_A \left(\frac{\mu^2}{m_H^2} \right)^\epsilon c_\Gamma \left(-\frac{2}{\epsilon^2} + \frac{11}{3} + \pi^2 \right) \right],$$

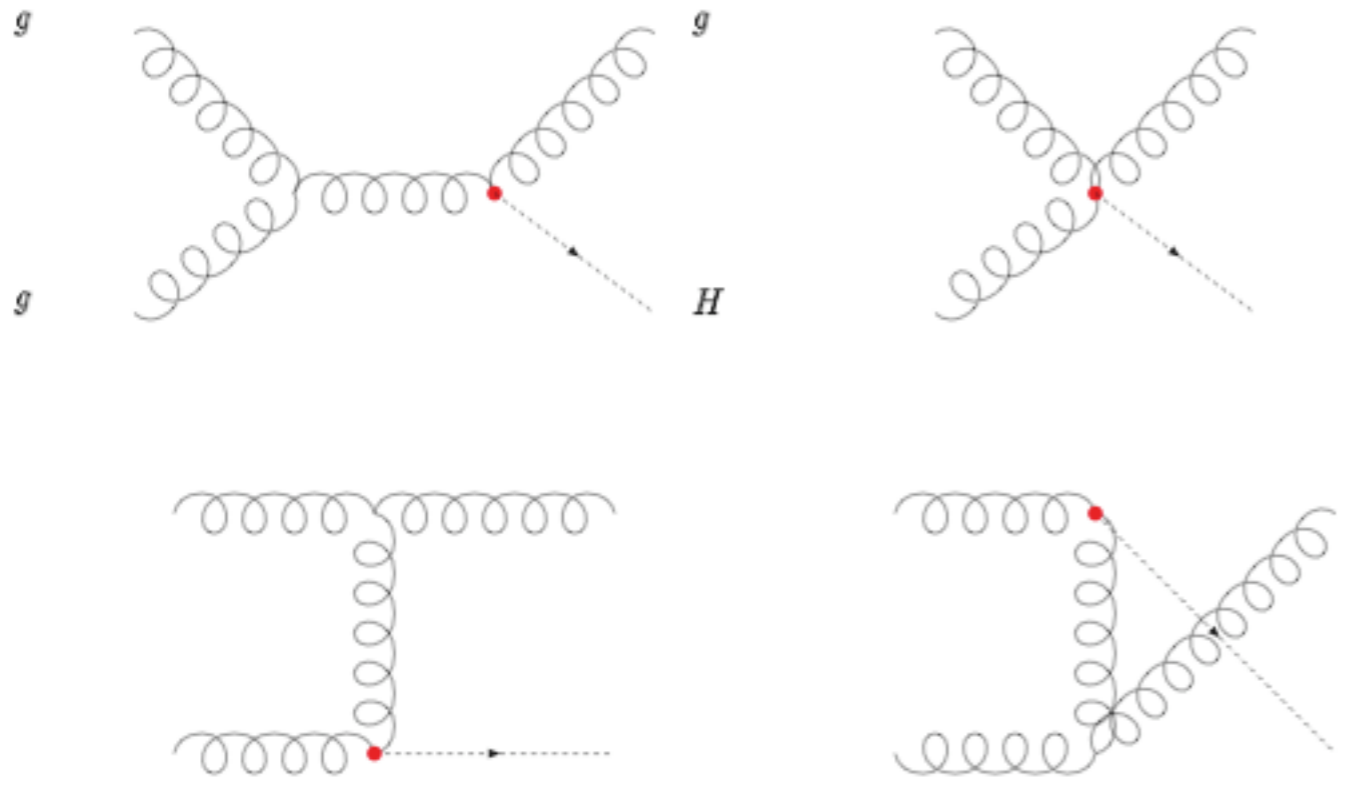
$$\sigma_{\text{Born}} = \frac{\alpha_S^2}{\pi} \frac{m_H^2}{576 v^2 s} (1 + \epsilon + \epsilon^2) \mu^{2\epsilon} \delta(1-z) \equiv \sigma_0 \delta(1-z) \quad z = m_H^2/s$$

Real contributions



This is the last piece: the result at the end must be finite!

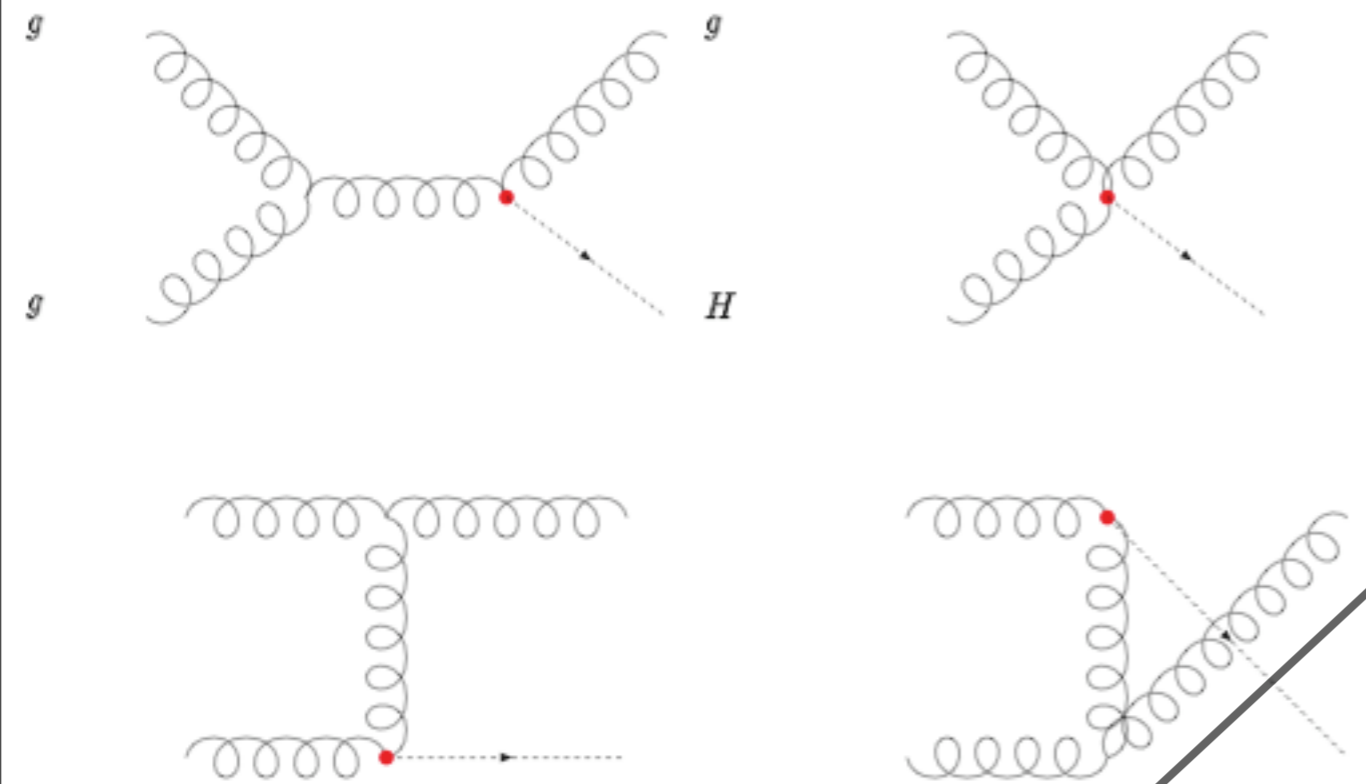
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$$\begin{aligned}
 \sigma_{\text{real}} = & \sigma_0 \frac{\alpha_S}{2\pi} C_A \left(\frac{\mu^2}{m_H^2} \right)^\epsilon e_\Gamma \left[\left(\frac{2}{\epsilon^2} + \frac{2 b_0}{\epsilon C_A} - \frac{\pi^2}{3} \right) \delta(1-z) \right. \\
 & - \frac{2}{\epsilon} p_{gg}(z) - \frac{11}{3} \frac{(1-z)^3}{z} - 4 \frac{(1-z)^2(1+z^2) + z^2}{z(1-z)} \log z \\
 & \left. + 4 \frac{1+z^4 + (1-z)^4}{z} \left(\frac{\log(1-z)}{1-z} \right)_+ \right].
 \end{aligned}$$

Real contributions

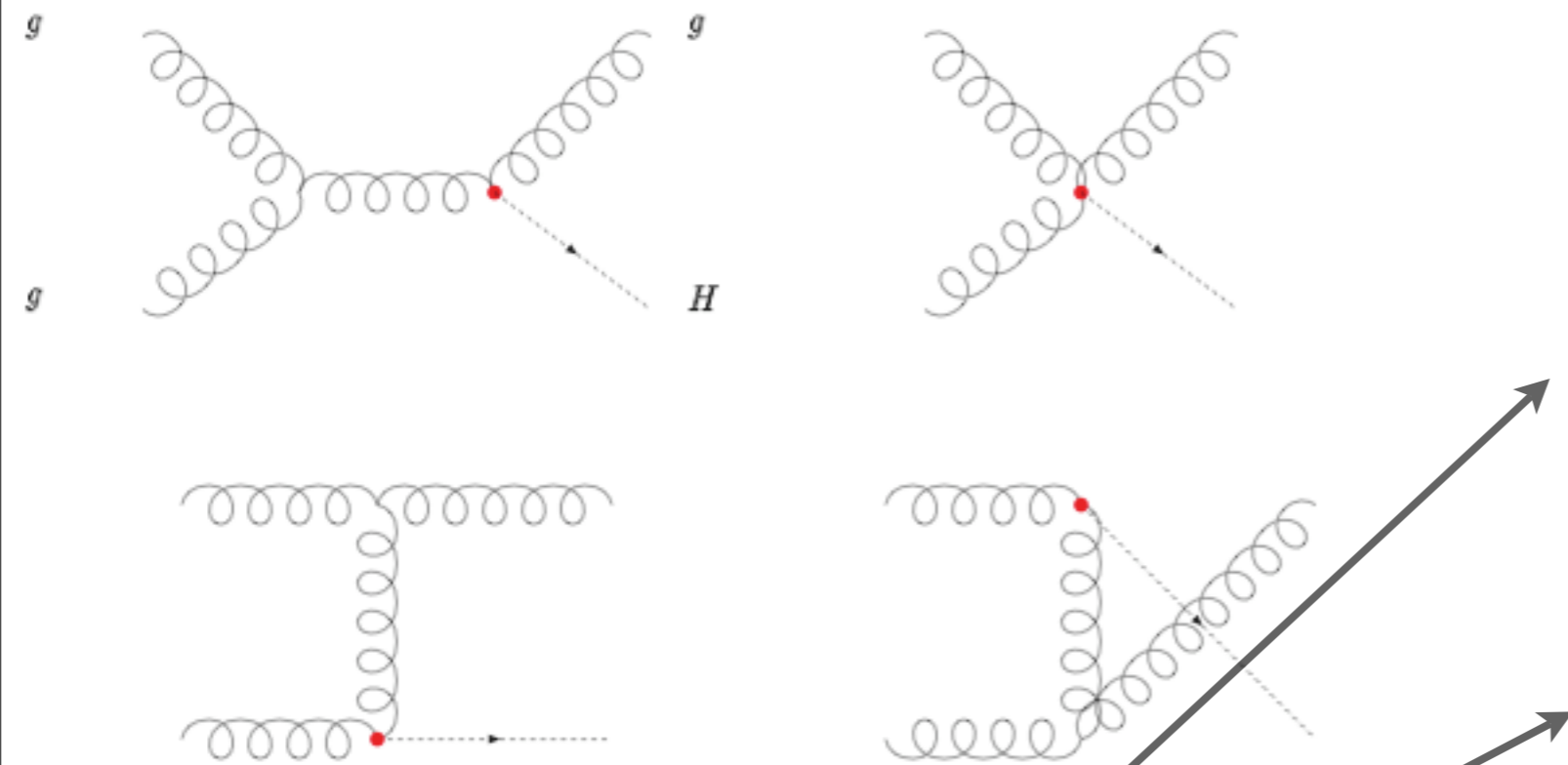


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2/eps cancels with the virtual contribution ✓

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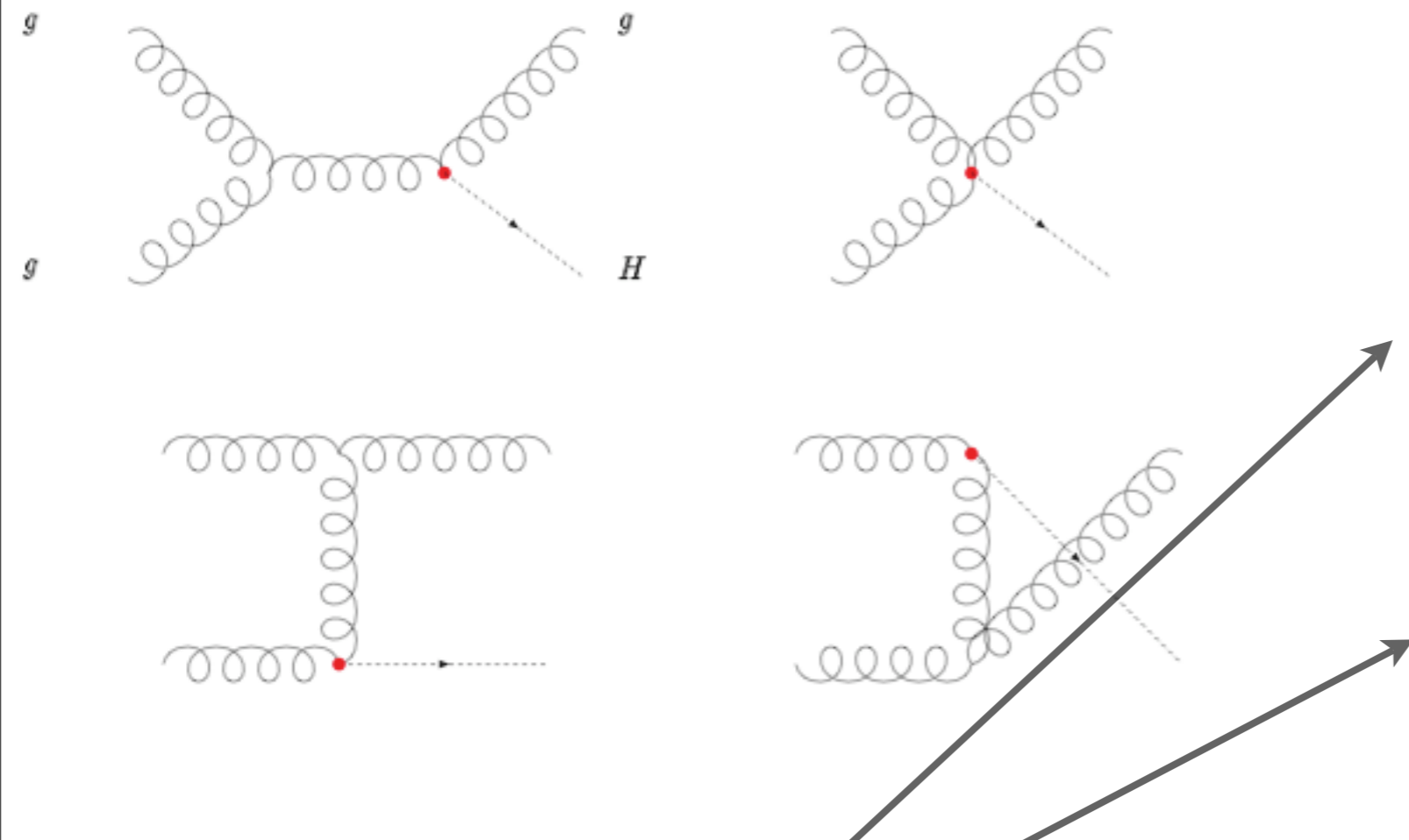
$2/\epsilon$ cancels with the virtual contribution ✓

This is the renormalization of the coupling!!

$$\sigma_{\text{c.t.}}^{\text{UV}} = 2 \sigma_{\text{Born}} \frac{\alpha_S}{2\pi} \left[- \left(\frac{\mu^2}{\mu_{\text{UV}}^2} \right)^\epsilon c_\Gamma \frac{b_0}{\epsilon} \right] \checkmark$$

$$\sigma_{\text{real}} = \sigma_0 \frac{\alpha_S}{2\pi} C_A \left(\frac{\mu^2}{m_H^2} \right)^\epsilon c_\Gamma \left[\left(\frac{2}{\epsilon^2} - \frac{2 b_0}{\epsilon C_A} - \frac{\pi^2}{3} \right) \delta(1-z) - \frac{2}{\epsilon} p_{gg}(z) - \frac{11}{3} \frac{(1-z)^3}{z} - 4 \frac{(1-z)^2(1+z^2) + z^2}{z(1-z)} \log z + 4 \frac{1+z^4 + (1-z)^4}{z} \left(\frac{\log(1-z)}{1-z} \right)_+ \right]$$

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This is an initial-state divergence to be reabsorbed in the pdf

$$\sigma_{\text{c.t.}}^{\text{coll.}} = 2 \sigma_0 \frac{\alpha_S}{2\pi} \left[\left(\frac{\mu^2}{\mu_F^2} \right)^\epsilon \frac{c_\Gamma}{\epsilon} P_{gg}(z) \right] \checkmark$$

Final results = we made it!!

$$\sigma(pp \rightarrow H) = \sum_{ij} \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 f_i(x_1, \mu_f) f_j(x_2, \mu_f) \hat{\sigma}(ij) [\mu_f/m_h, \mu_r/m_h, \alpha_S(\mu_r)]$$

The final cross section is the sum of three channels: q qbar, q g, and g g.

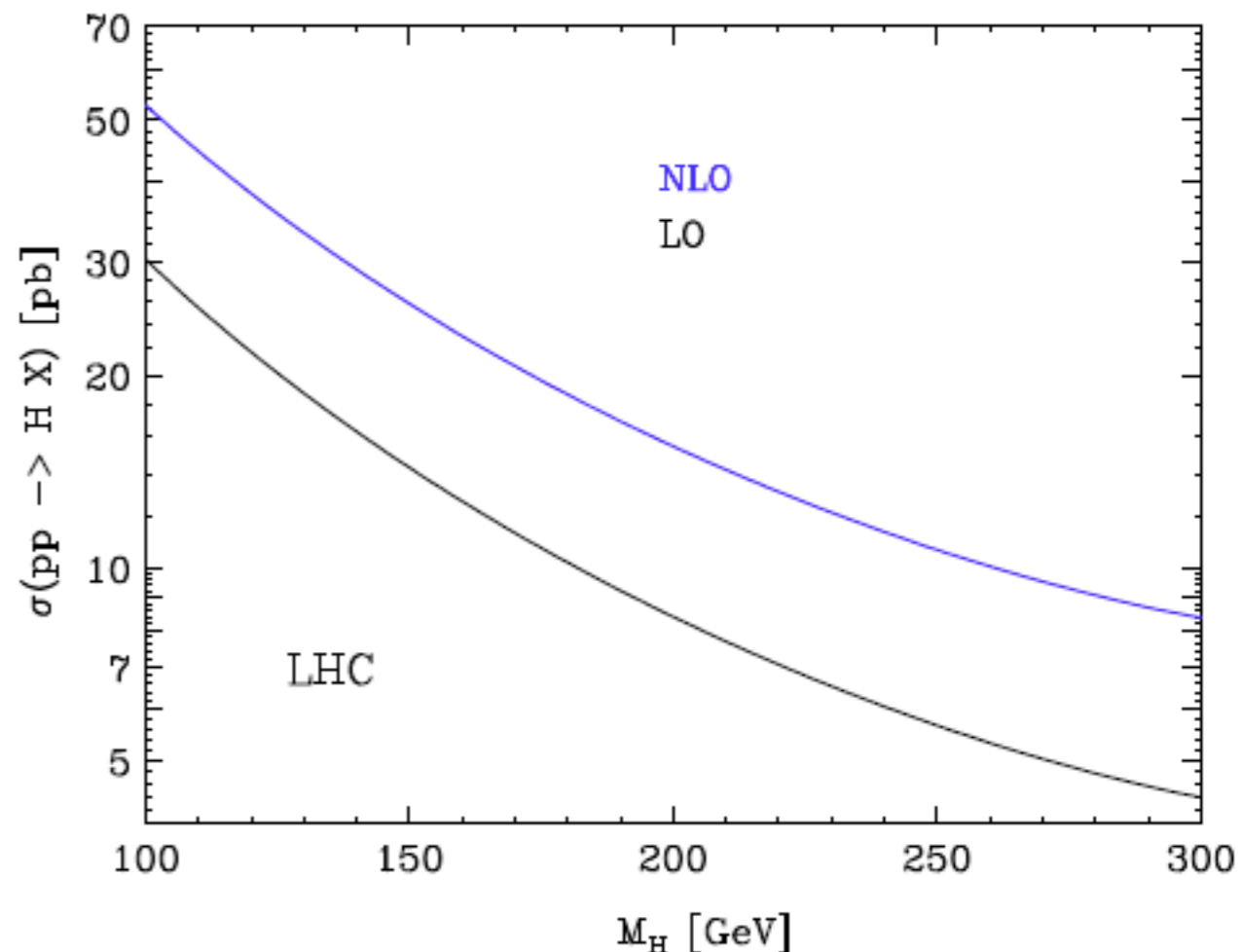
The short distance cross section at NLO depends explicitly on the subtraction scales (renormalization and factorization).

The explicit integration over the pdf's is trivial (just mind the plus distributions).

The result is that the corrections are huge!

K factor is ~ 2 and scale dependence not really very much improved.

Is perturbation theory valid?
NNLO is mandatory...



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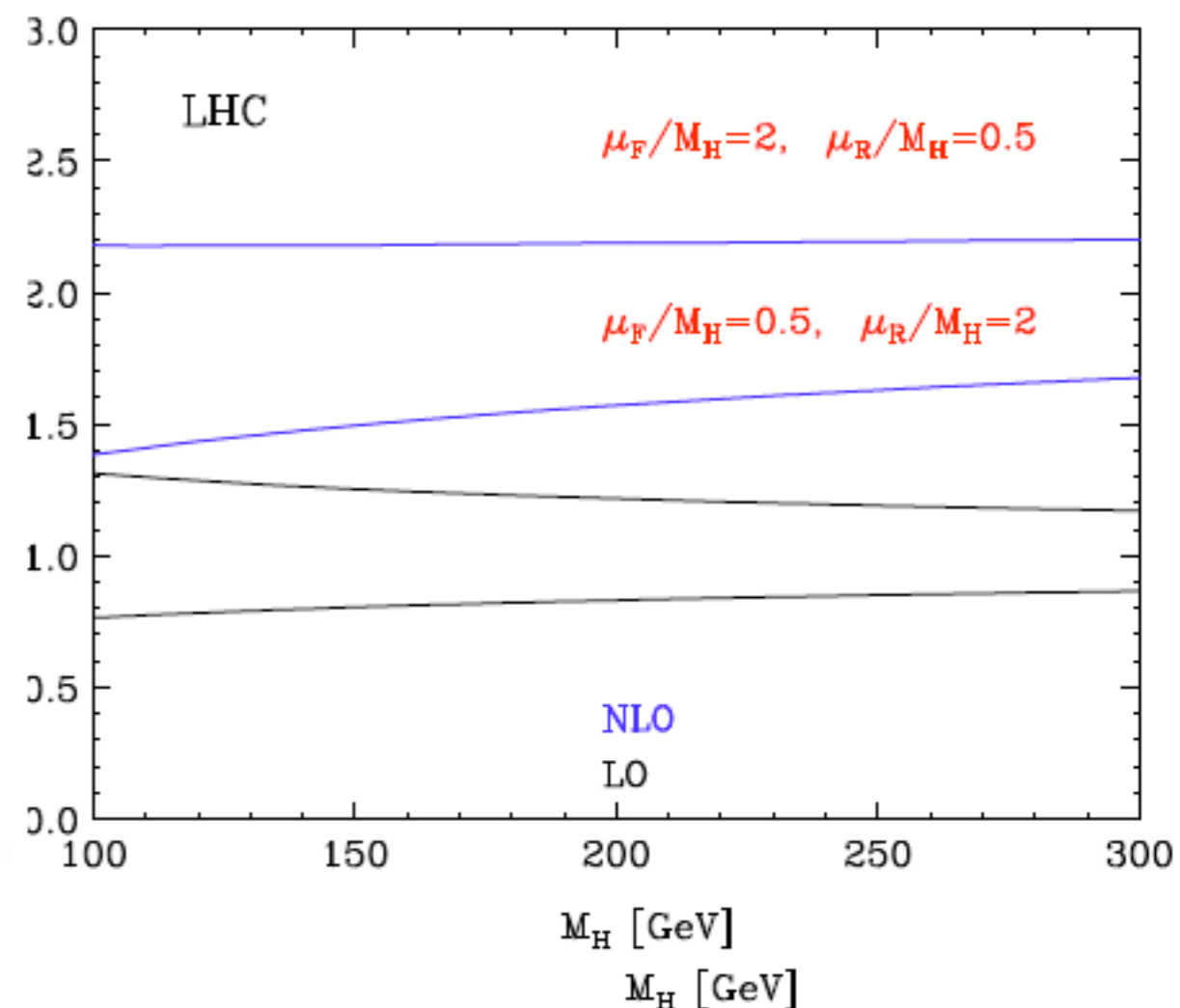
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A simple plan

- **Intro:** the LHC challenge
- **Minimal QCD:** basics
- **Precision QCD:** from NLO to NNLO
- **Useful QCD:** Parton Shower approach
- **Best QCD:** Merging Fixed Order with PS

A simple plan

- Intro: the LHC challenge
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Summary of last lecture

The adjective “NLO” refers to IR-safe observables which are calculable in pQCD.



General algorithm for calculations of observables at NLO

As we discussed, the form of the soft and collinear terms are UNIVERSAL, i.e., they don't depend on the short distance coefficients, but only on the color and spin of the partons participating soft or collinear limit.

Therefore it is conceivable to have an algorithm that can handle any process, once the real and virtual contributions are computed.

There are several such algorithms available, but the conceptually simplest is the Subtraction Method [Catani & Seymour ; Catani, Dittmaier, Seymour, Trocsanyi]

$$\begin{aligned}\sigma_{ab}^{LO} &= \int_m d\sigma_{ab}^B \\ \sigma_{ab}^{NLO} &= \int_{m+1} d\sigma_{ab}^R + \int_m d\sigma_{ab}^V\end{aligned}$$



General algorithm for calculations of observables at NLO

One can use the universality to construct a set of counterterms

$$d\sigma^{ct} = \sum_{ct} \int_m d\sigma^B \otimes \int_1 dV_{ct}$$

which only depend on the partons involved in the divergent regions, $d\sigma^B$ denotes the appropriate colour and spin projection of the Born-level cross section and the counter terms are independent on the process under considerations.

These counter terms cancel all non-integrable singularities in $d\sigma^R$, so that one can write

$$\sigma_{ab}^{NLO} = \int_{m+1} [d\sigma_{ab}^R - d\sigma_{ab}^{ct}] + \int_{m+1} d\sigma_{ab}^{ct} + \int_m d\sigma_{ab}^V$$

where the space integration in the first term can be performed numerically in four dimensions and the integral of the counter terms can be done once for all.

An (incomplete) list of NLO codes

- NLOJET++ [Nagy] $pp \rightarrow (2,3) \text{ jets}$
- AYLEN/EMILIA [de Florian, Dixon, Kunszt, Signer] $pp \rightarrow (W, Z) + (W, Z, \gamma)$
- DIPHOX/EPHOX [Aurenche, Binoth, Fontannaz, Guillet, Heinrich, Pilon, Werlen] $pp \rightarrow \gamma + 1 \text{ jet}, pp \rightarrow \gamma\gamma, \gamma^* p \rightarrow \gamma + 1 \text{ jet}$
- MCFM [Campbell, Ellis] $pp \rightarrow (W, Z) + (0,1,2) \text{ jets}, pp \rightarrow (W, Z) + b\bar{b}, \dots$
- heavy-quark production [Mangano, Nason, Ridolfi] $pp \rightarrow Q\bar{Q}$
- single-top production [Harris, Laenen, Phaf, Sullivan, Weinzierl] $pp \rightarrow Q\bar{q}$
- associated Higgs production with $t\bar{t}$ [Dawson, Jackson, Orr, Reina, Wackerroth, Beenakker, Dittmaier, Kramer, Plumper, Spira, Zerwas] $pp \rightarrow H Q\bar{Q}$
- VBFNLO [Figy, Zeppenfeld, C.O.] $pp \rightarrow (W, Z, H, WW, ZZ, WZ) + 2 \text{ jets}$, QCD corrections to electroweak production, when typical vector-boson fusion cuts are applied
- di-photon production [del Duca, Maltoni, Nagy, Trocsanyi] $pp \rightarrow \gamma\gamma + 1 \text{ jet}$

For a **more complete list**, and the corresponding web pages, see:

<http://www.cedar.ac.uk/hepcode>

Example: MCFM

Downloadable general purpose NLO code (Campbell & Ellis)

$$p\bar{p} \rightarrow W^\pm / Z$$

$$p\bar{p} \rightarrow W^\pm + Z$$

$$p\bar{p} \rightarrow W^\pm + \gamma$$

$$p\bar{p} \rightarrow W^\pm + g^* (\rightarrow b\bar{b})$$

$$p\bar{p} \rightarrow W^\pm / Z + 1 \text{ jet}$$

$$p\bar{p}(gg) \rightarrow H$$

$$p\bar{p}(VV) \rightarrow H + 2 \text{ jets}$$

$$p\bar{p} \rightarrow H + b$$

$$p\bar{p} \rightarrow W^+ + W^-$$

$$p\bar{p} \rightarrow Z + Z$$

$$p\bar{p} \rightarrow W^\pm / Z + H$$

$$p\bar{p} \rightarrow Z b\bar{b}$$

$$p\bar{p} \rightarrow W^\pm / Z + 2 \text{ jets}$$

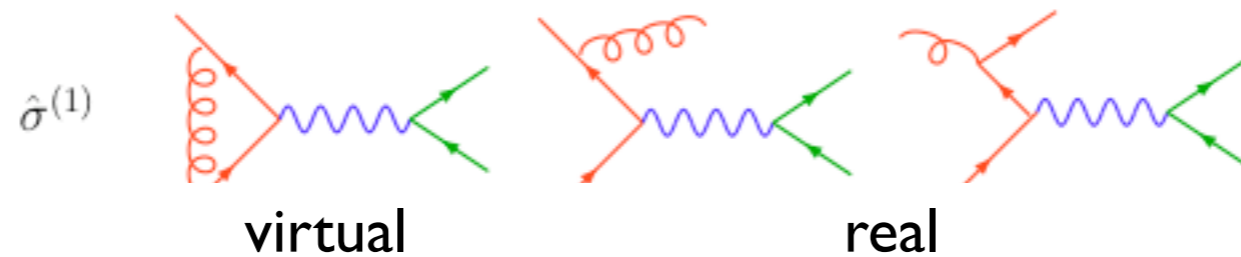
$$p\bar{p}(gg) \rightarrow H + 1 \text{ jet}$$

$$p\bar{p} \rightarrow t + q$$

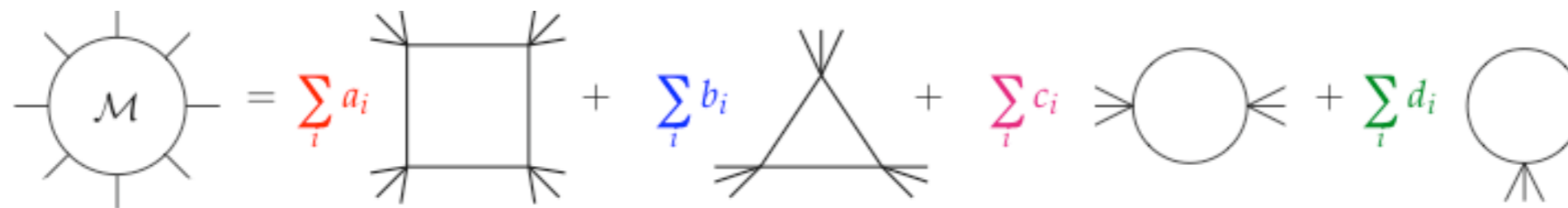
$$p\bar{p} \rightarrow Z + b$$

- ➡ Plus all single-top channels, Wc , WQJ , ZQJ ,...
- ➡ Extendable/sizeable library of processes, relevant for signal and background studies, including spin correlations.
- ➡ Cross sections and distributions at NLO are provided
- ➡ Easy and flexible choice of parameters/cuts (input card).

Next-to-leading order : Loops



Any one-loop amplitude can be written as (PV decomposition):



$$\mathcal{M} = \sum_i a_i(D) \text{Boxes}_i + \sum_i b_i(D) \text{Triangles}_i + \sum_i c_i(D) \text{Bubbles}_i + \sum_i d_i(D) \text{Tadpoles}_i$$

* All the scalar loop integrals are known and now easily available [Ellis, Zanderighi]

* Open issue is to compute the D-dimensional coefficient in the expansion:
 large number of terms forbid a direct evaluation with symbolic algebra. In addition normally large gauge cancellation, inverse Gram determinants, spurious phase-space singularities lead to **numerical instabilities**.

Sometimes it is better to calculate

$$\mathcal{M} = \sum_i a_i(4) \text{Boxes}_i + \sum_i b_i(4) \text{Triangles}_i + \sum_i c_i(4) \text{Bubbles}_i + \sum_i d_i(4) \text{Tadpoles}_i + R$$

Where **R** is a rational function

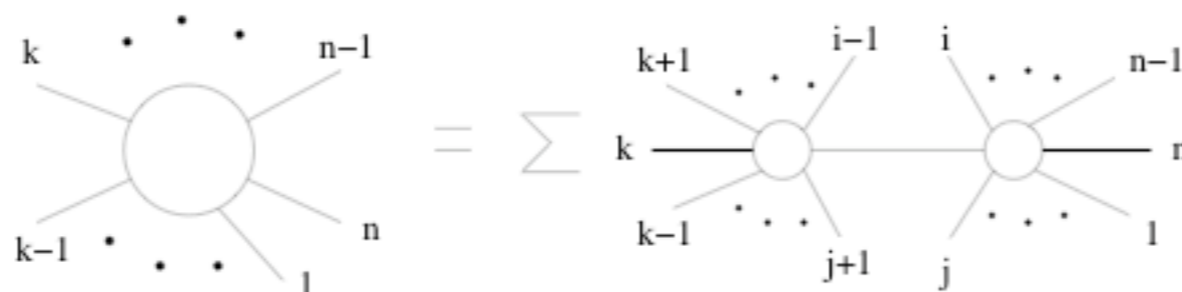
Progress in loops

Several new developments coming from the idea

A scattering amplitude is an analytic function of the external momenta and (most) its structure can be reconstructed from the poles and the branch cuts.

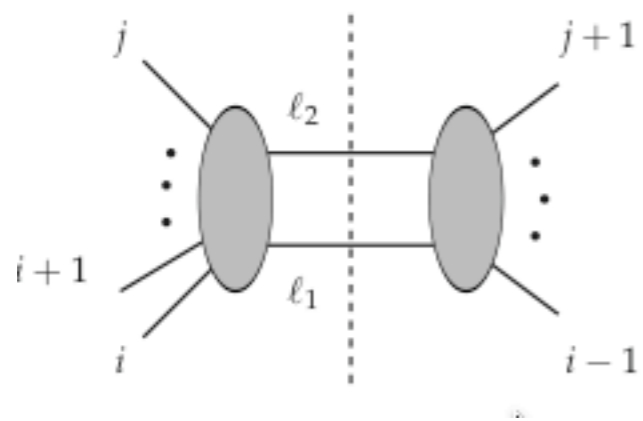
— **LOOPS** can be calculated from tree-level amplitudes

✓ **POLES** : lower number of external lines. Cauchy residue theorem



[Cachazo, Svrcek, Witten]
[Witten]
[Britto, Cachazo, Feng]

✓ **BRANCH CUTS** : lower number of loops



$$\text{Disc} = \int d^4\Phi A^{\text{tree}}(\ell_1, i, \dots, j, \ell_2) A^{\text{tree}}(-\ell_2, j+1, \dots, i-1, -\ell_1)$$

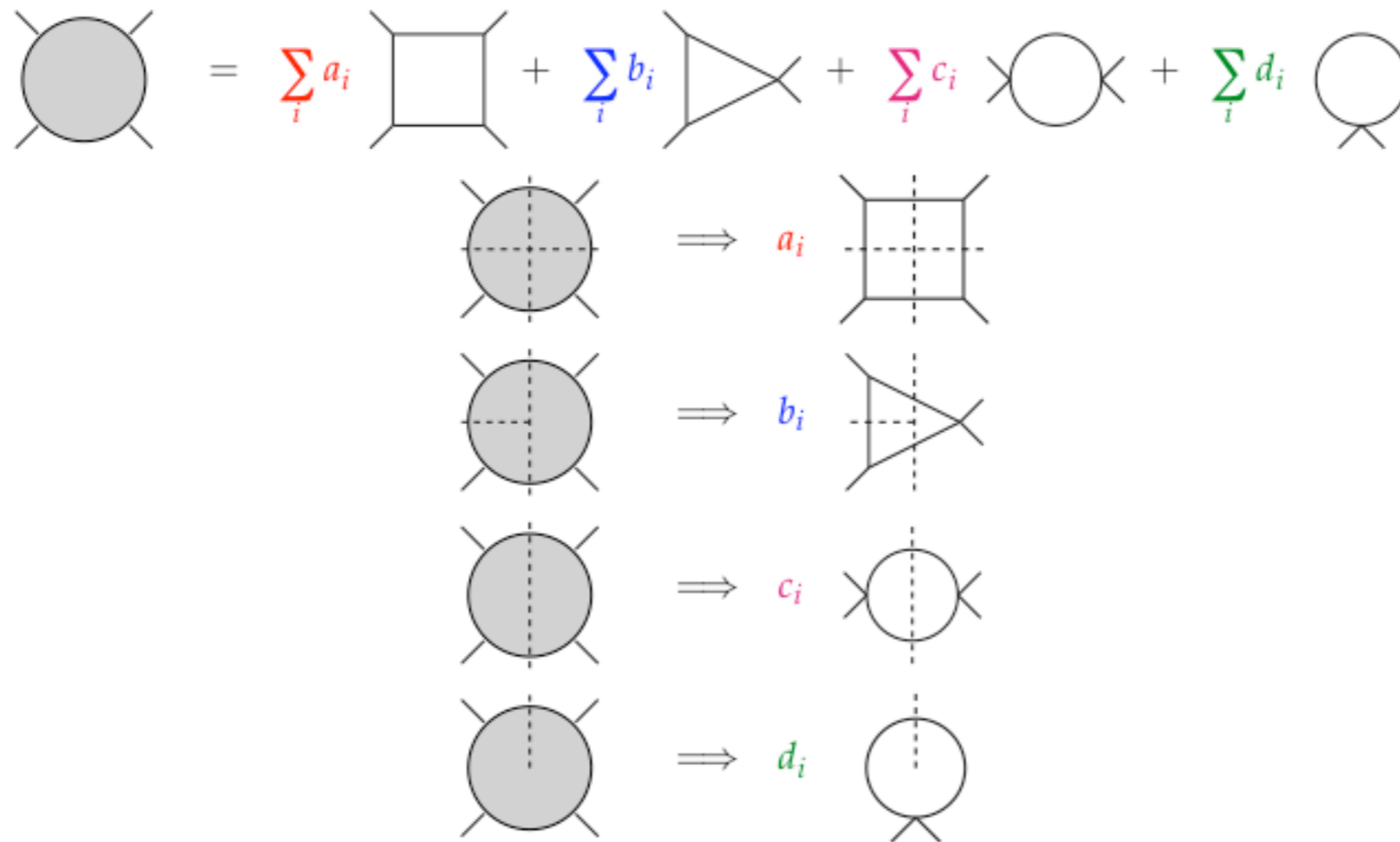
$$d^4\Phi = d^4\ell_1 d^4\ell_2 \delta^{(4)}(\ell_1 + \ell_2 - P_{ij}) \delta^{(+)}(\ell_1^2) \delta^{(+)}(\ell_2^2)$$

$$\delta^{(+)}(p^2) = \delta(p^2) \theta(p_0) \quad \text{on-shell condition}$$

[Vermaseren, van Neerven]
[Bern, Dixon, Dunbar, Kosower]
[Britto, Cachazo, Feng]

Generalized unitarity

[Bern, Dixon, Kosower]
[Britto, Cachazo, Feng]
[Anastasiou, Kunszt, Mastrolia]



Three and four particle cuts are non zero due to the continuation of momenta into complex values!



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- Be aware that there are many possibly dangerous (mal)practices in the exp community (K-factor, reweighting of distributions,...)
- Suggestion: always consult with the authors of the code in case of doubts...

What about NNLO?

- At present only $2 \rightarrow 1$ calculations available, all of them (parton) exclusive final state.
- From loop integrals to phase space integrals...all of them are an art!
- General algorithms and checked only in $e^+e^- \rightarrow 3j$ at NNLO.

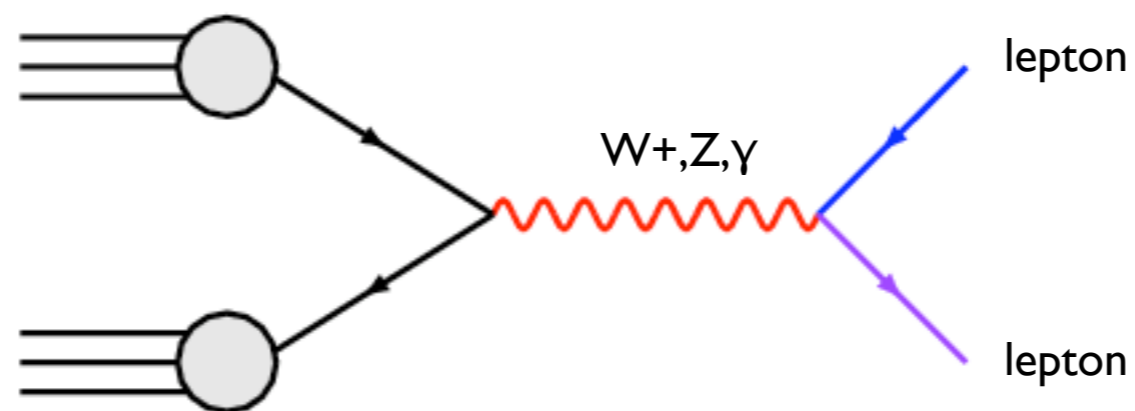
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Let's consider two physics cases:

- a. Drell-Yan
- b. Higgs

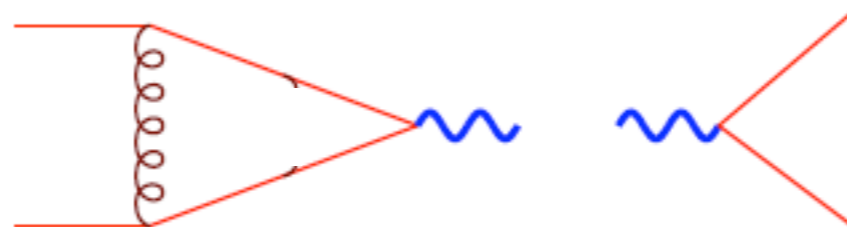
Drell-Yan



- Clean final state (no hadrons from the hard process).
- Nice test of QCD and EW interactions. The cross sections are known up to NNLO (QCD) and at NLO (EW).
- Measure m_W to be used in the EW fits together with the top mass to guess the Higgs mass.
- Constraint the PDF
- Channel to search for new heavy gauge bosons or new kind of interactions

Elements of $pp \rightarrow W$ NLO calculation

- Virtual

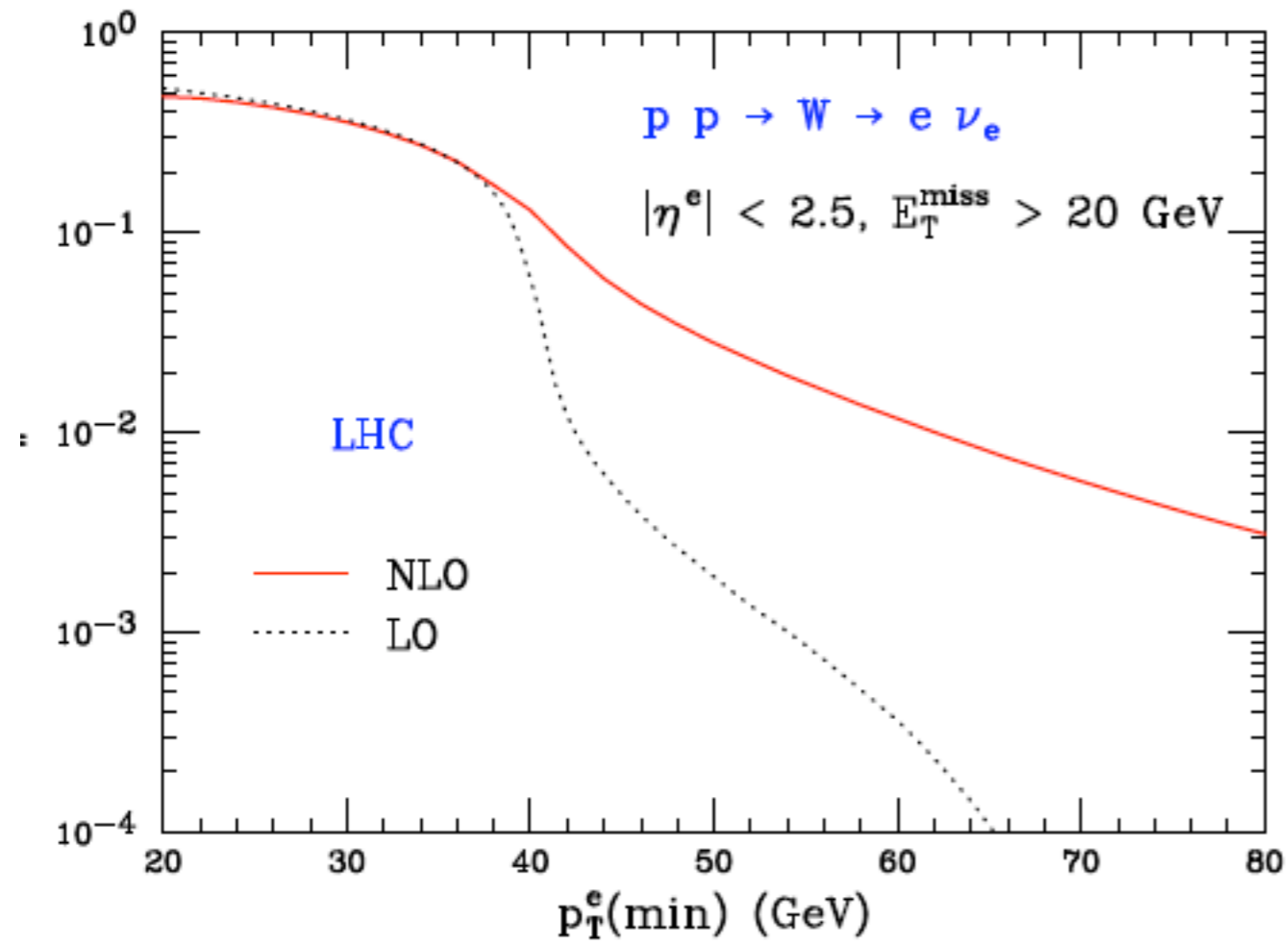


- Real





Drell-Yan @ NLO



✓ $A_W = \frac{1}{\sigma^{(tot)}} \int_{p_T^e(\text{min})}^{\sqrt{s}/2} dp_T^e \frac{d\sigma}{dp_T^e} (\text{cuts})$

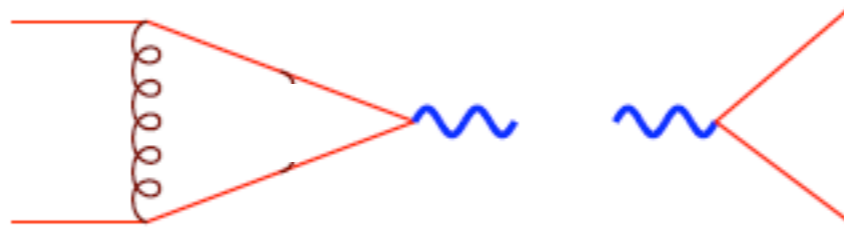
✓ $K(x) = \frac{d\sigma_{\text{NLO}}/dx}{d\sigma_{\text{LO}}/dx}$

K factors **STRONGLY** phase-space dependent.

Lepton **spin correlations** have to be taken account correctly!

Elements of $pp \rightarrow W$ NLO calculation

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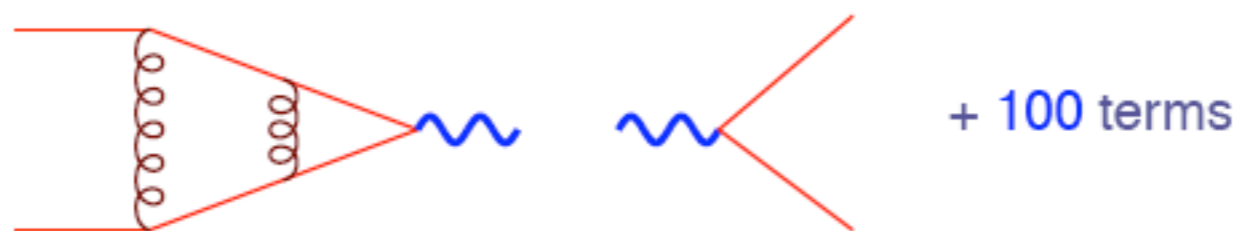


- Real

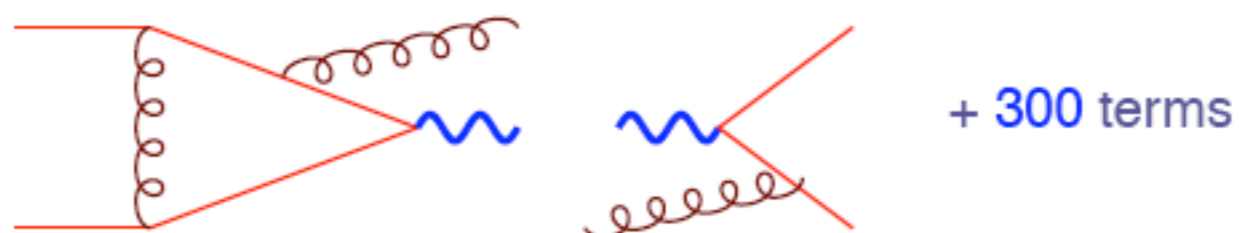


Elements of $pp \rightarrow W$ NNLO calculation

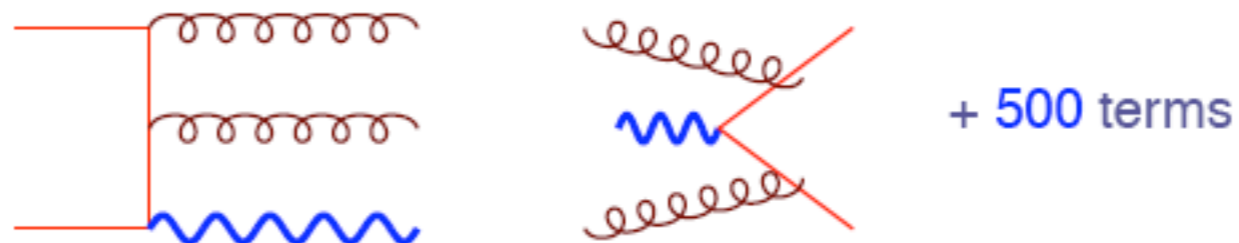
Virtual-Virtual



Real-Virtual

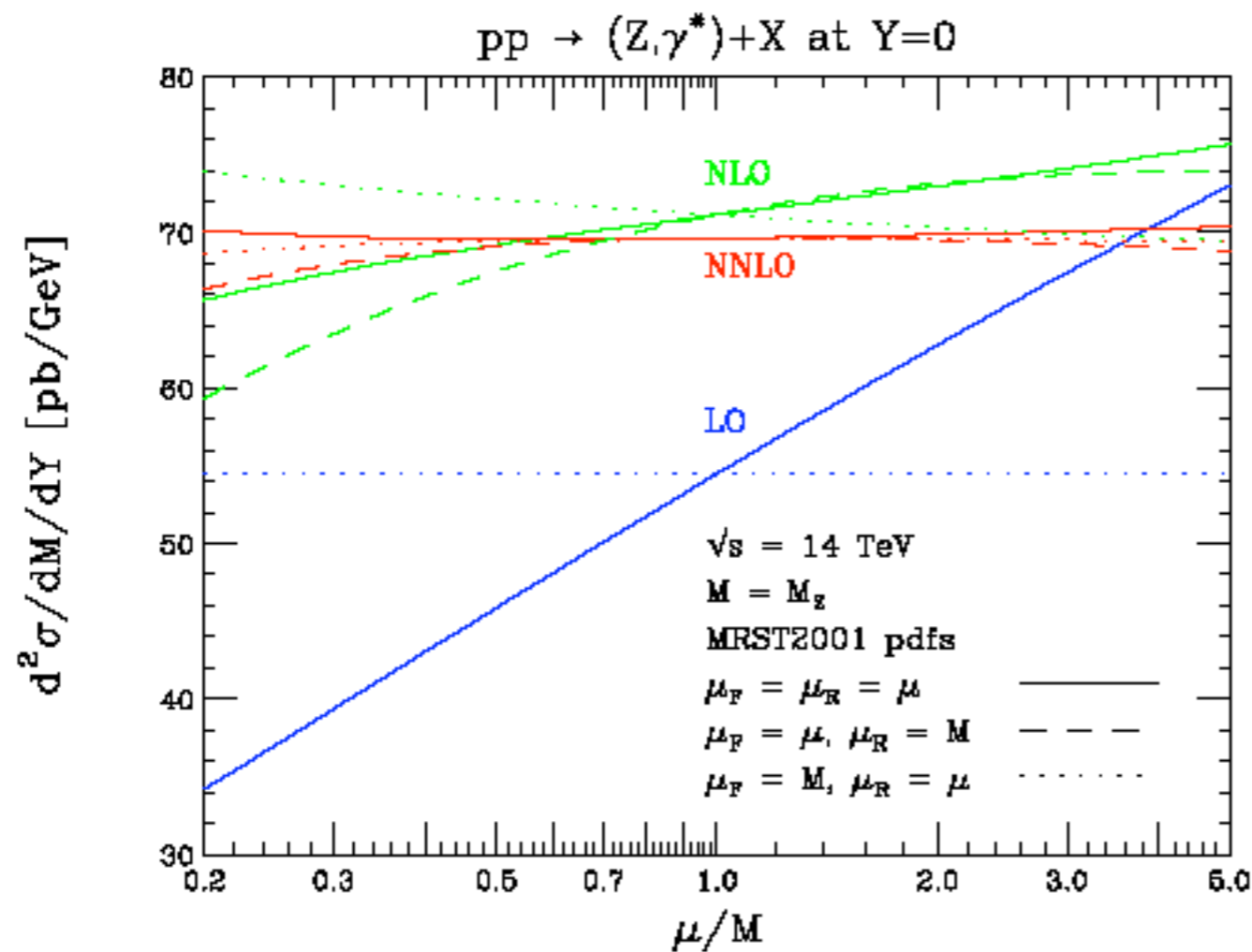


Real-Real



⇒ Need clever algorithms to handle!

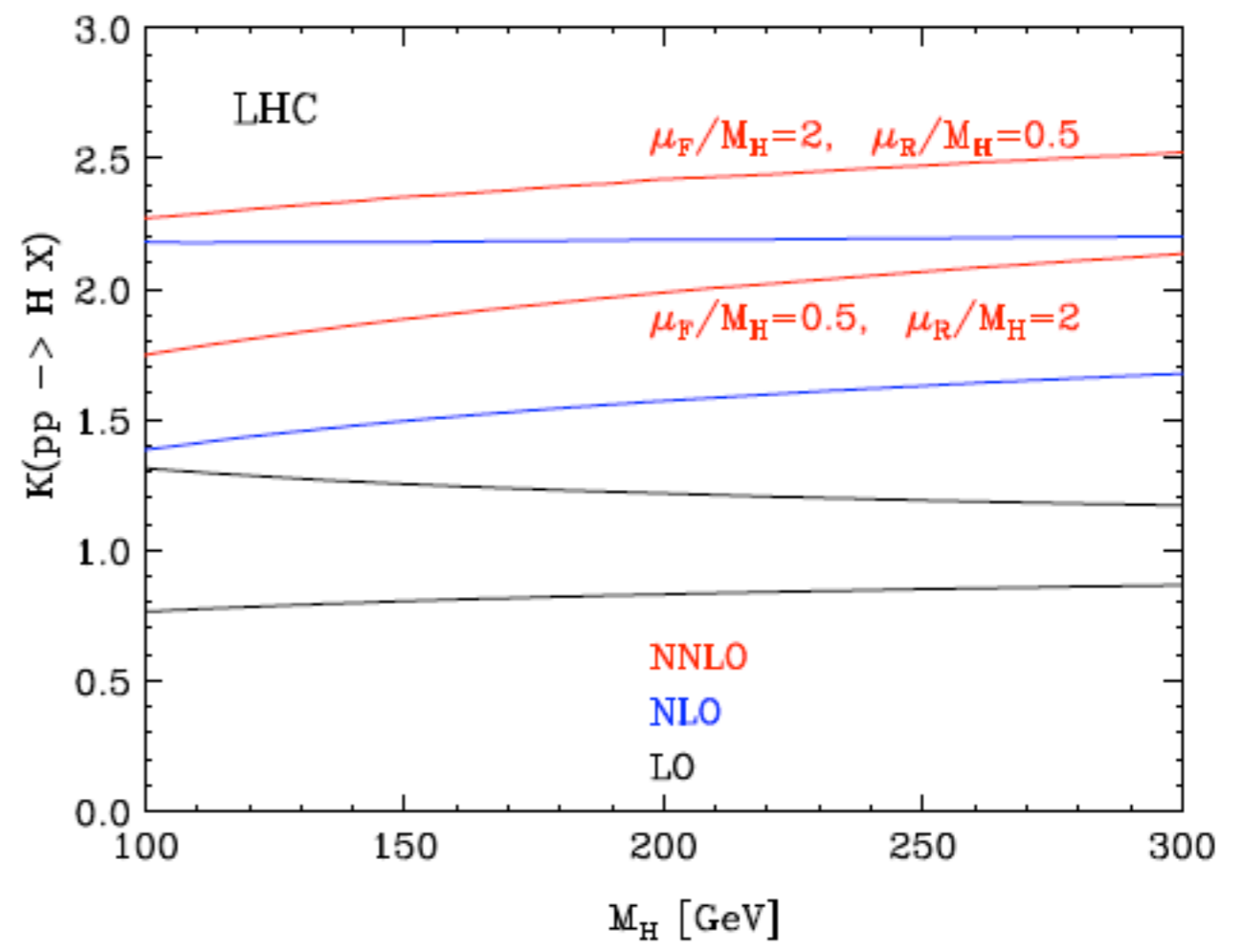
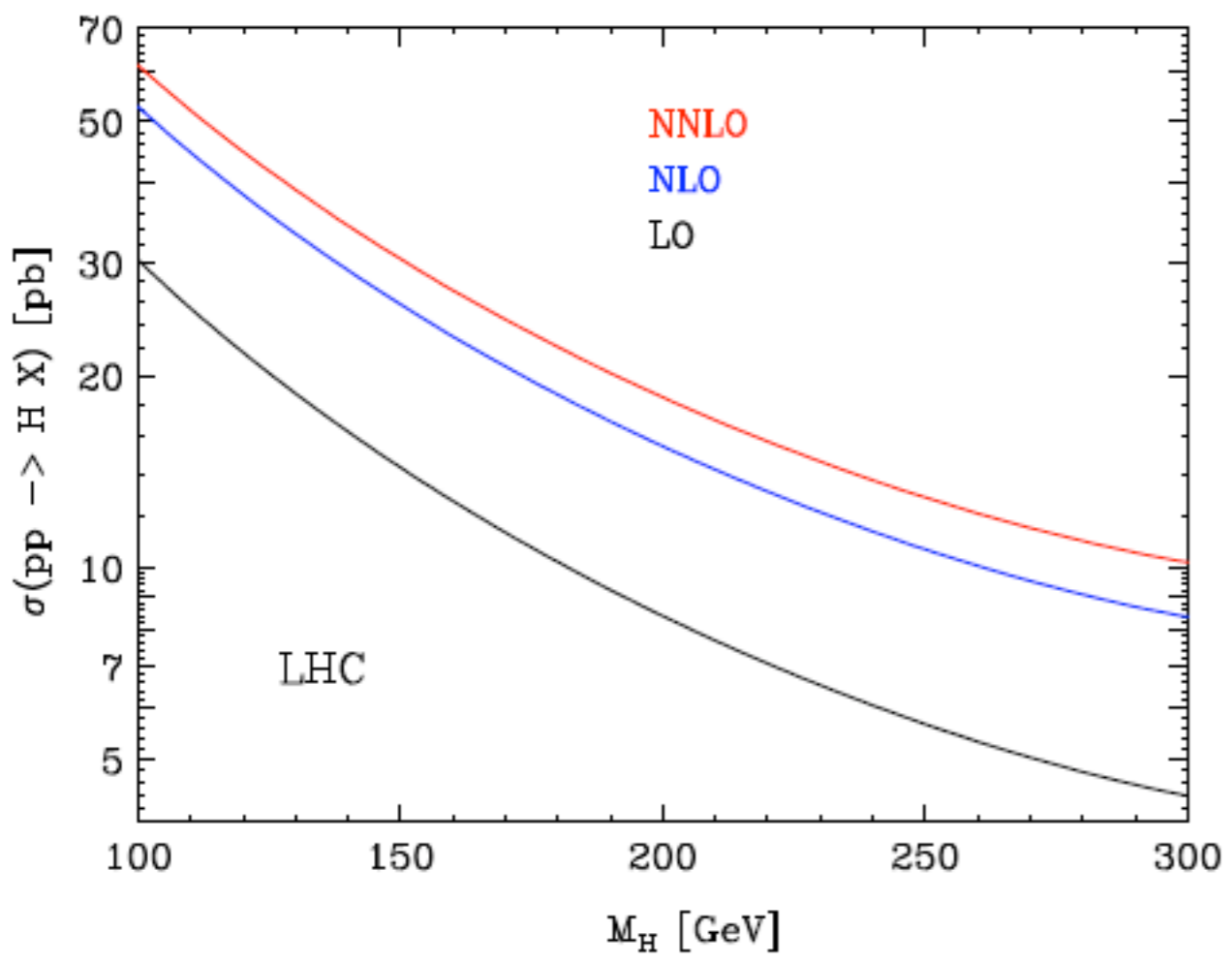
The NNLO result



- Precision predictions at NNLO
- Also miss qualitative effects at lower orders
 - Few initial channels open; sensitivity to pdfs underestimated
 - Few jets in final state
 - Jets modeled by too few partons
 - Incorrect kinematics, e.g., no p_T

[Anastasiou, Dixon, Melnikov, Petriello. 2004]

pp → H at NNLO



Is the series well behaved? \Rightarrow YES NNLO 15%

The current TH QCD uncertainty on the total cross section is about 10%.

What about our predictions for limited areas of the phase space?



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- Frontier of precision QCD calculations.
- NNLO calculations are needed for very special cases, such as standard candles and/or precision physics.
- Still an art. General algorithm not yet in place.
- Handful of results available, mostly in private codes (few exceptions!).

A simple plan

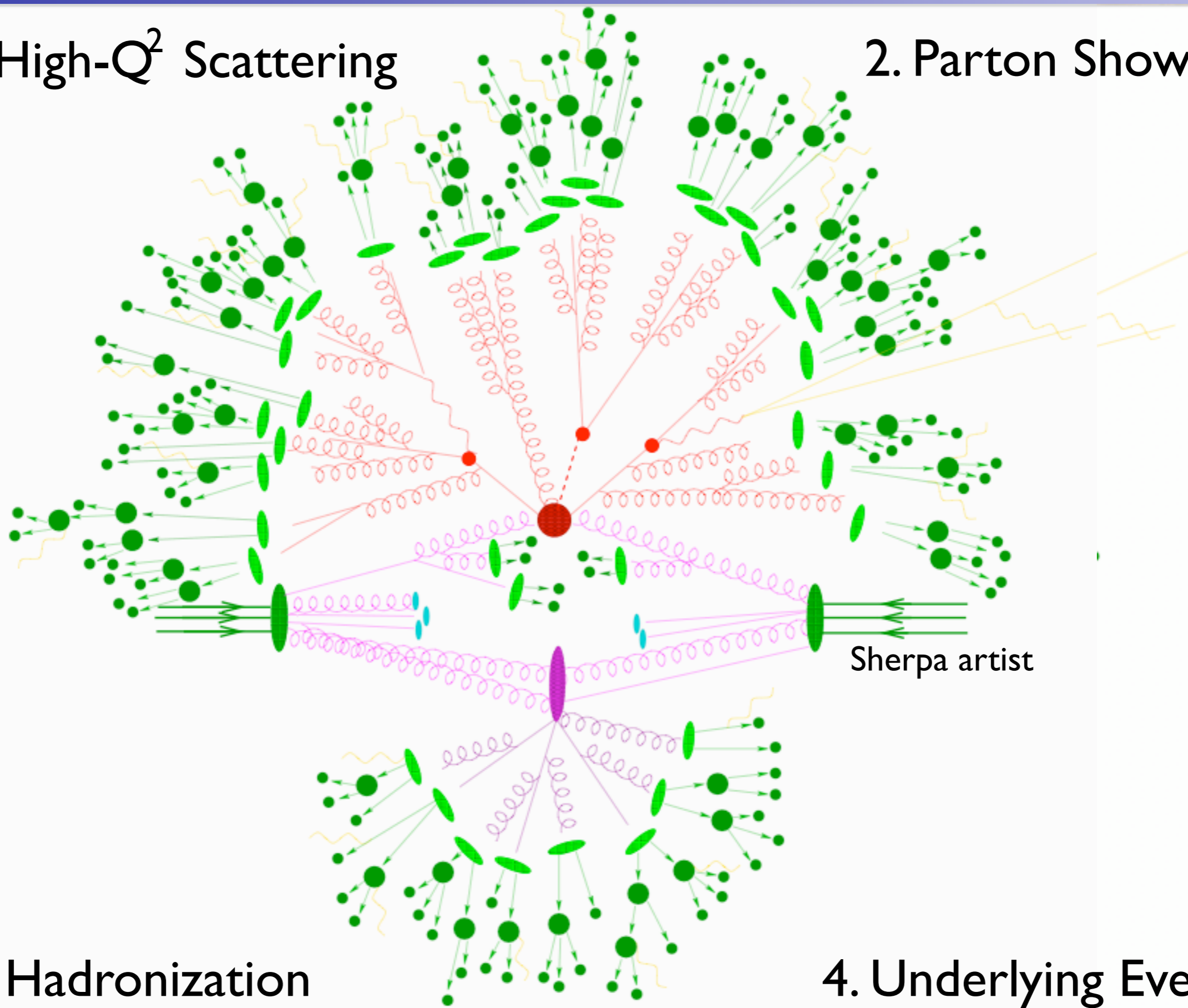
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A simple plan

- Intro: the LHC challenge
- Minimal QCD: basics
- Precision QCD: from NLO to NNLO
- Useful QCD: Parton Shower approach **now**
- Best QCD: Merging Fixed Order with PS

1. High- Q^2 Scattering

2. Parton Shower



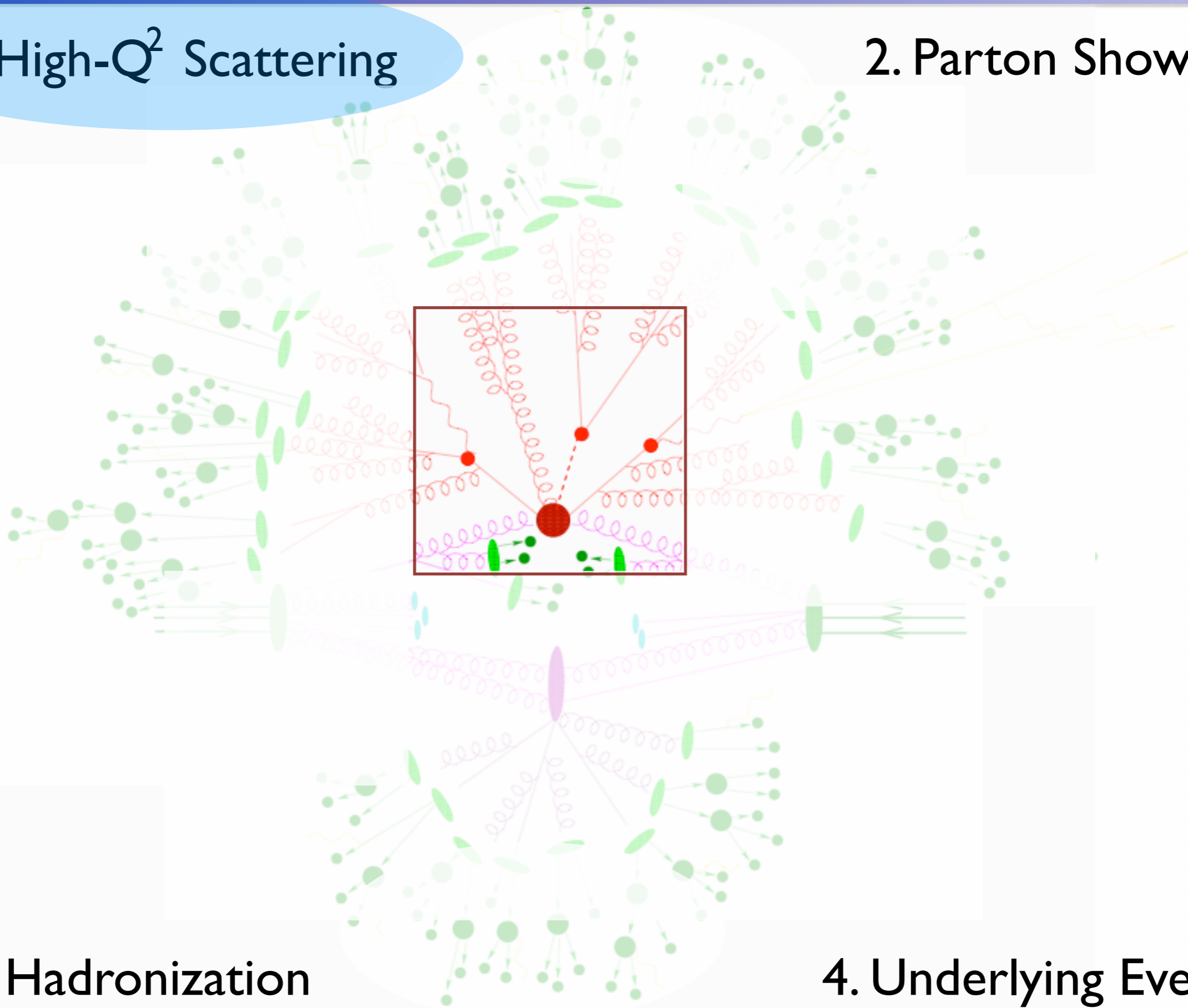
3. Hadronization

4. Underlying Event



I. High- Q^2 Scattering

2. Parton Shower



3. Hadronization

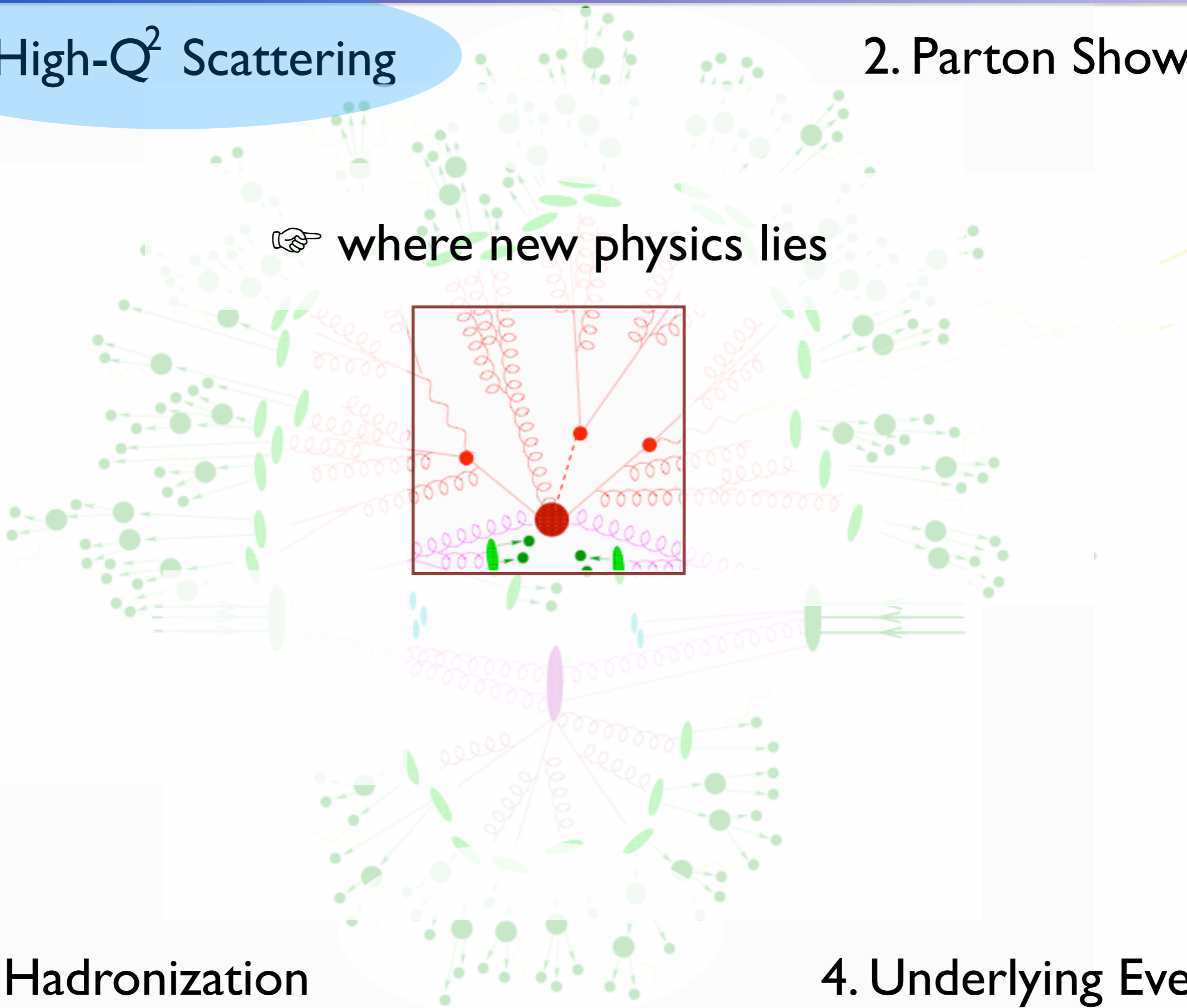
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I. High- Q^2 Scattering

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👉 where new physics lies



3. Hadronization

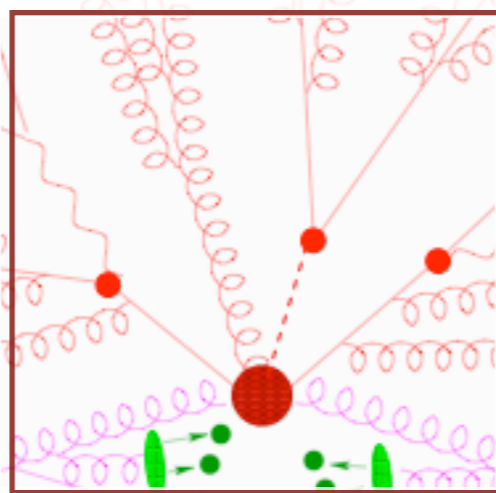
4. Underlying Event



I. High- Q^2 Scattering

2. Parton Shower

☞ where new physics lies



☞ process dependent

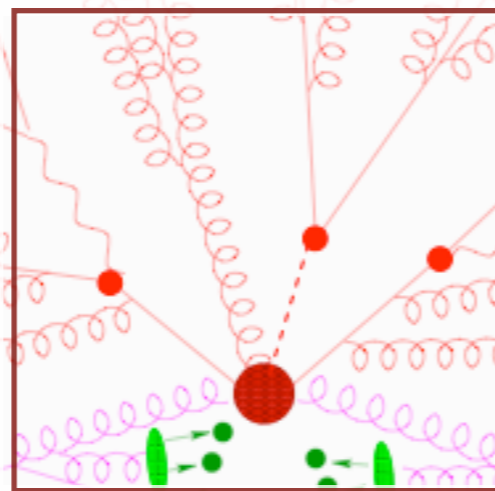
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☞ where new physics lies



☞ process dependent

☞ first principles description

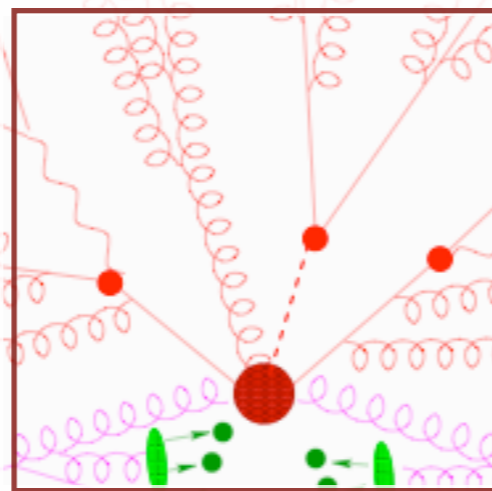
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☞ where new physics lies



☞ process dependent

☞ first principles description

☞ it can be systematically improved

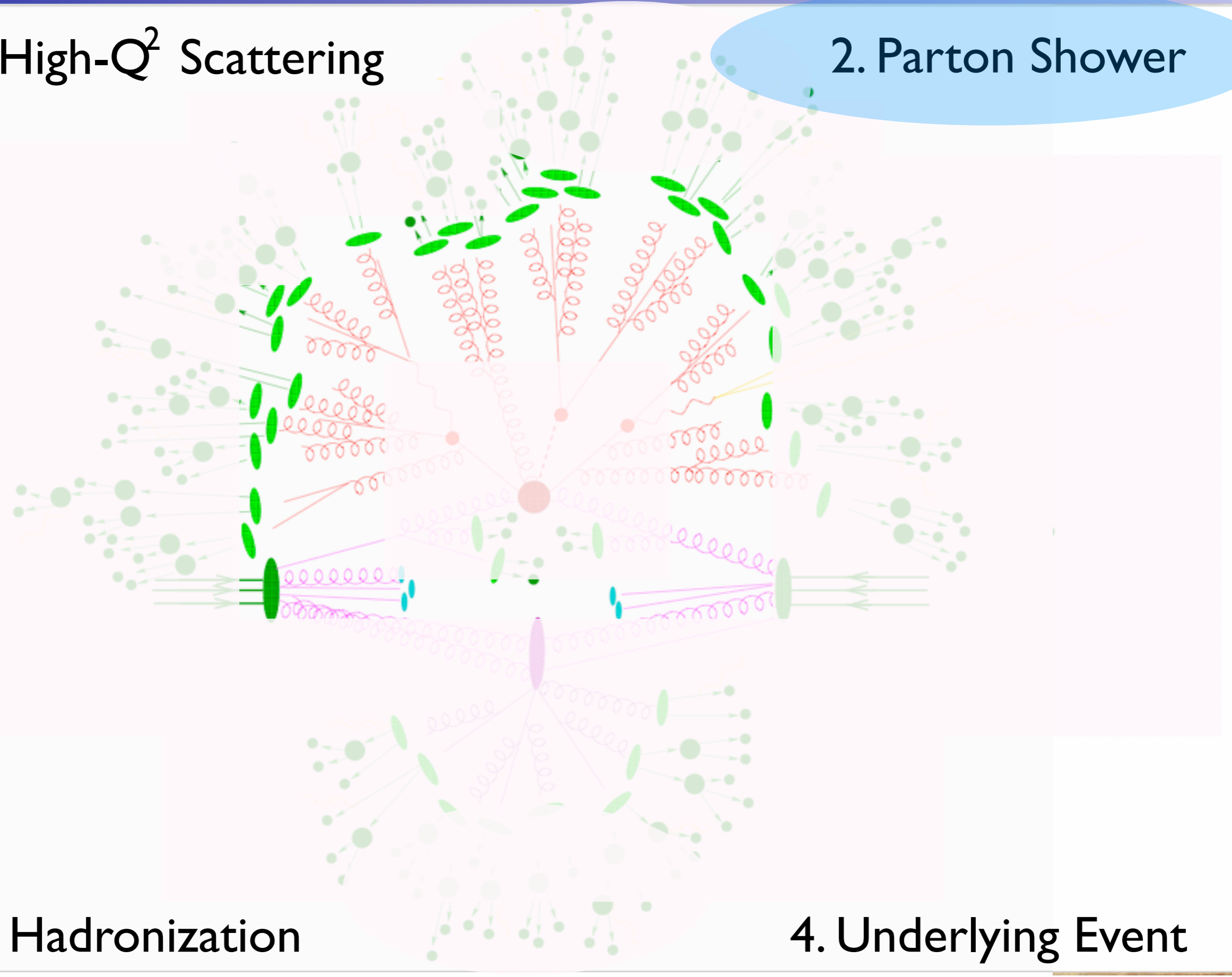
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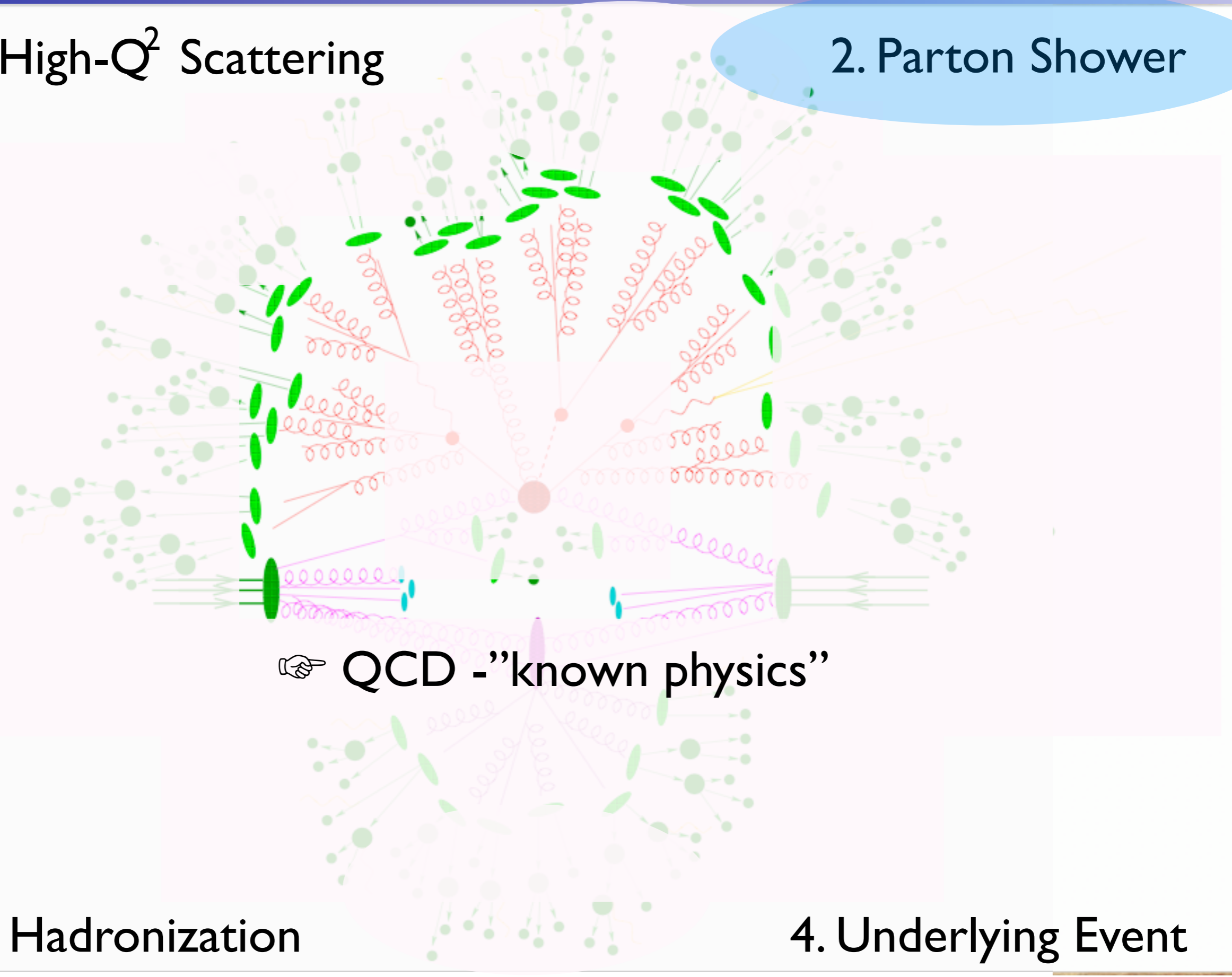
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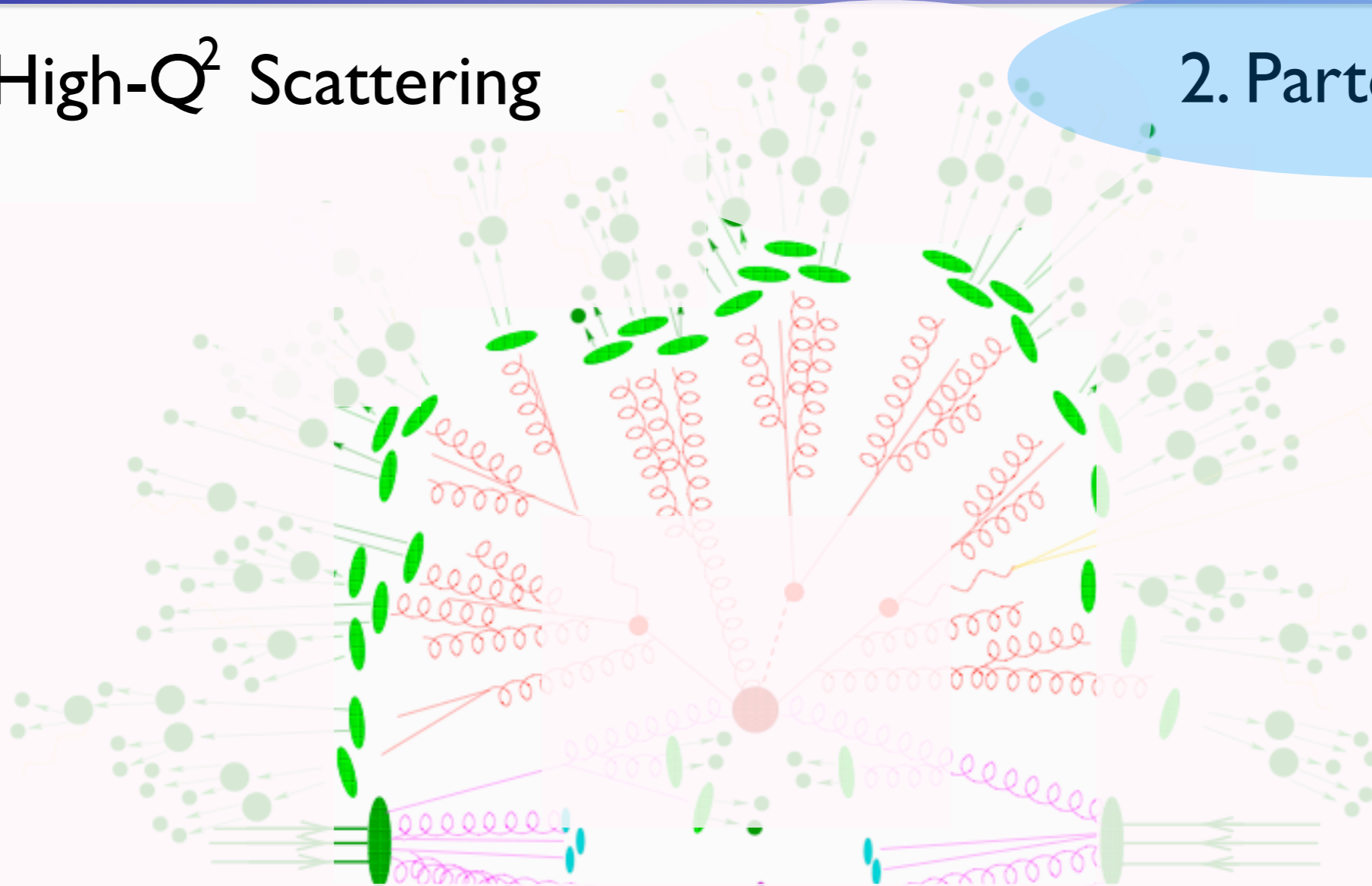
☞ QCD - "known physics"

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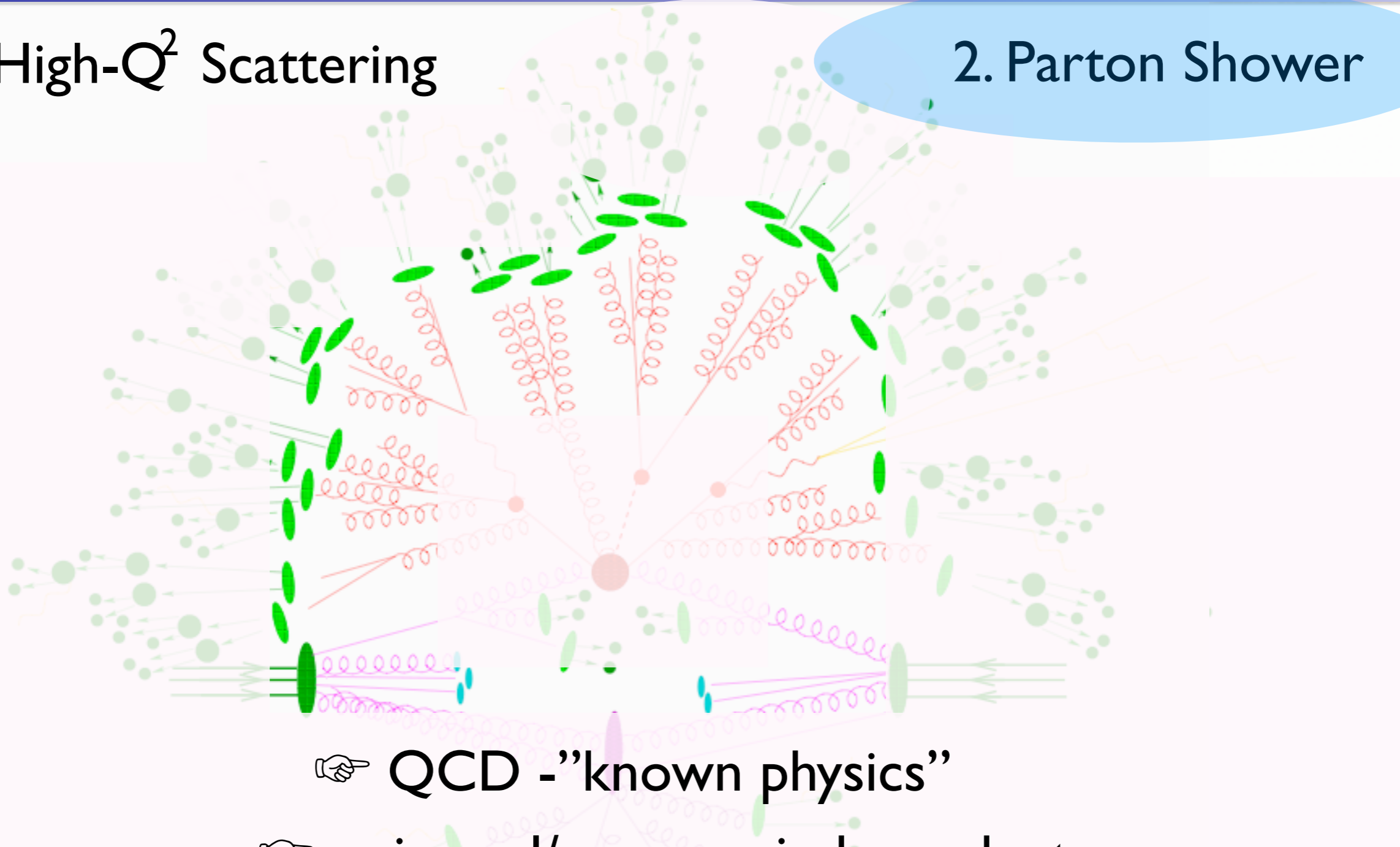
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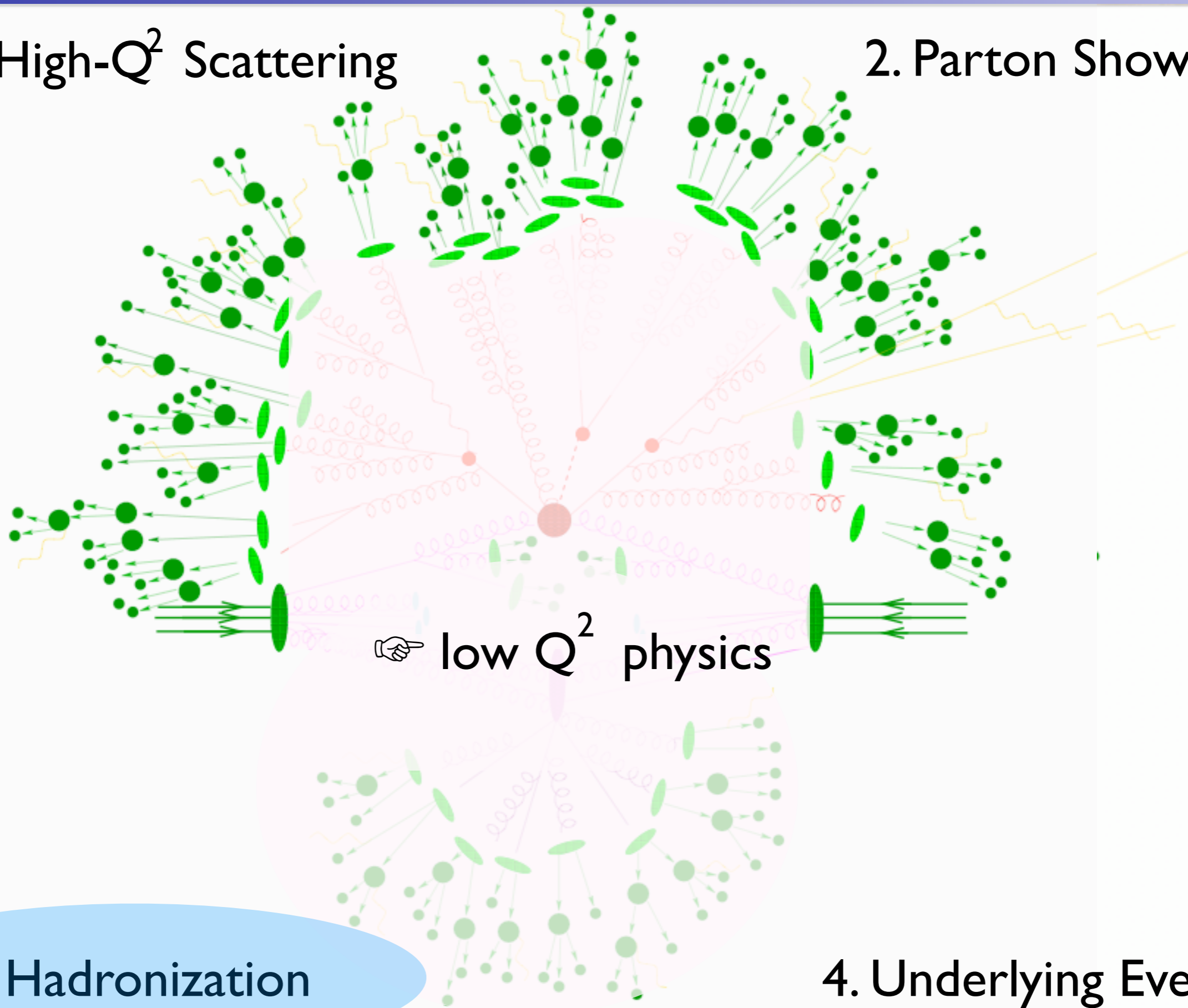
☞ first principles description

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low Q^2 physics

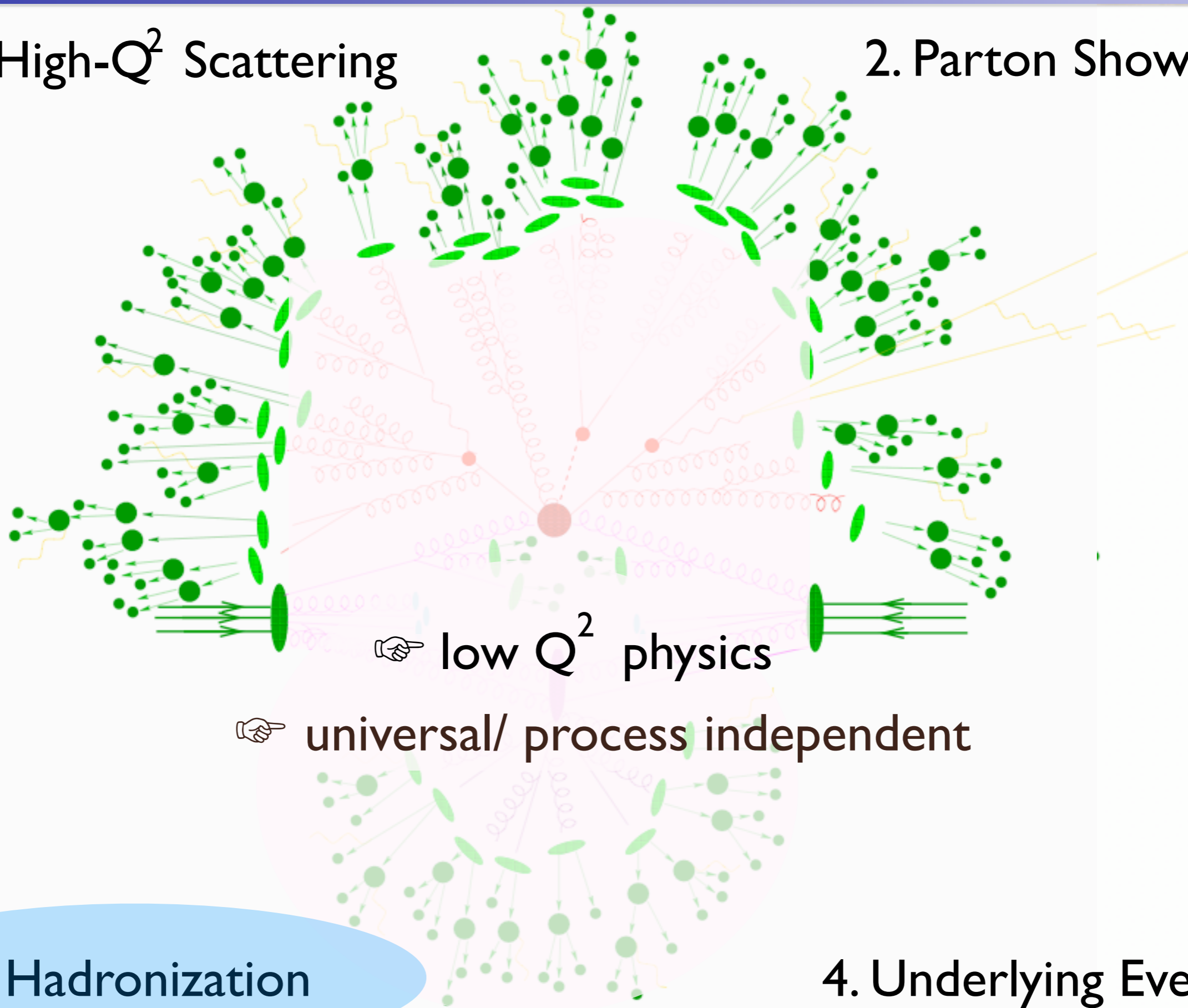
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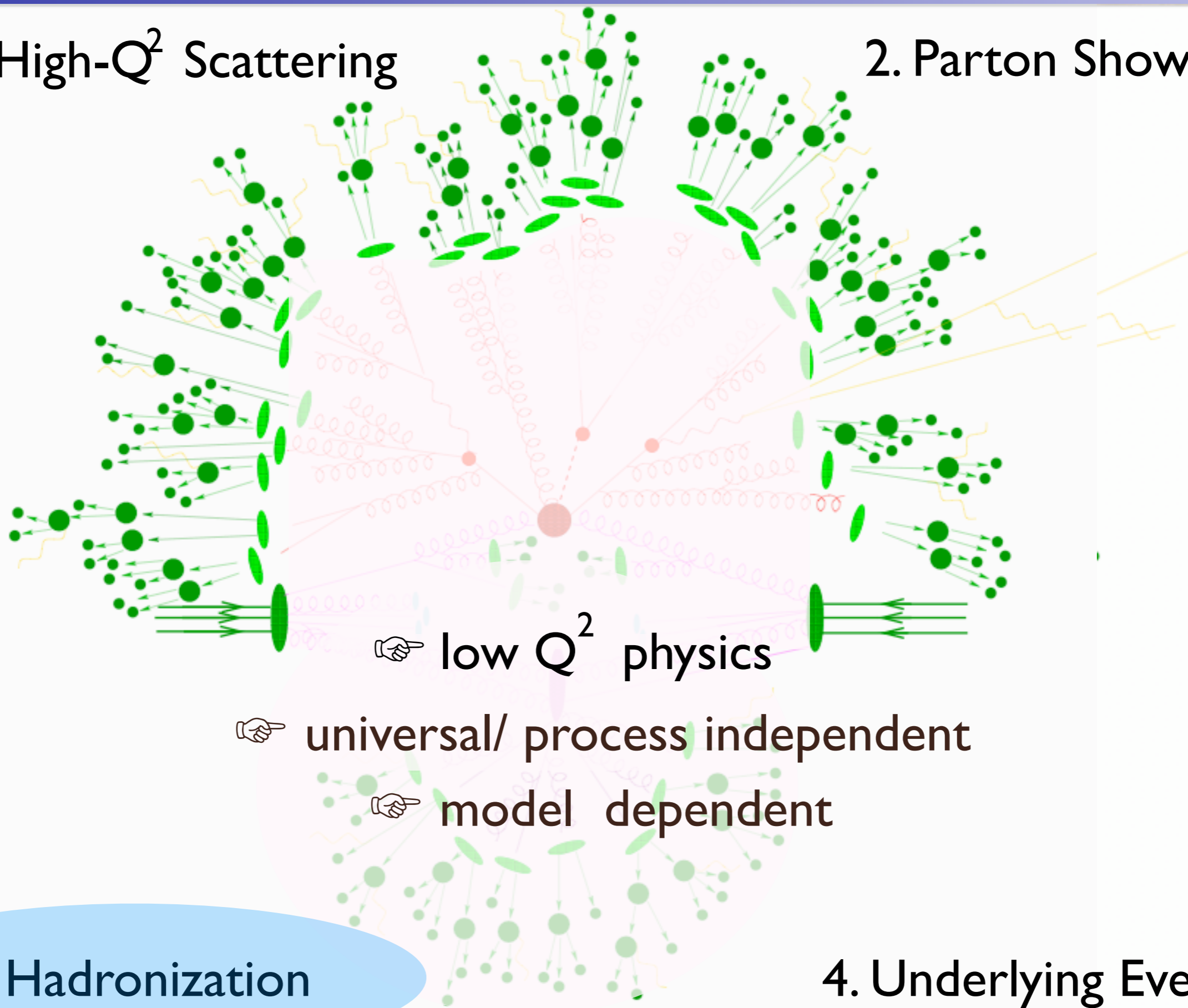
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low Q^2 physics

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model dependent

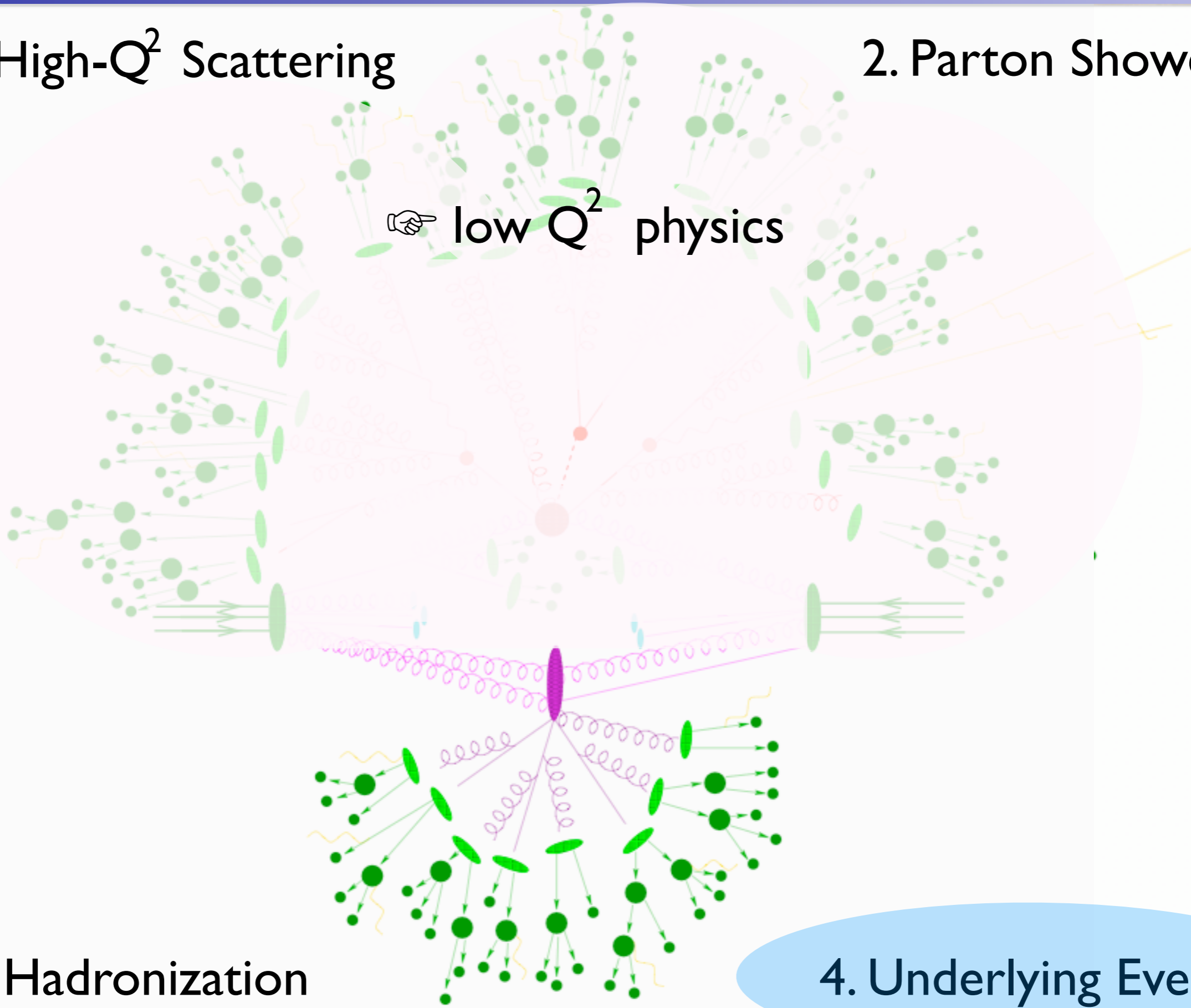
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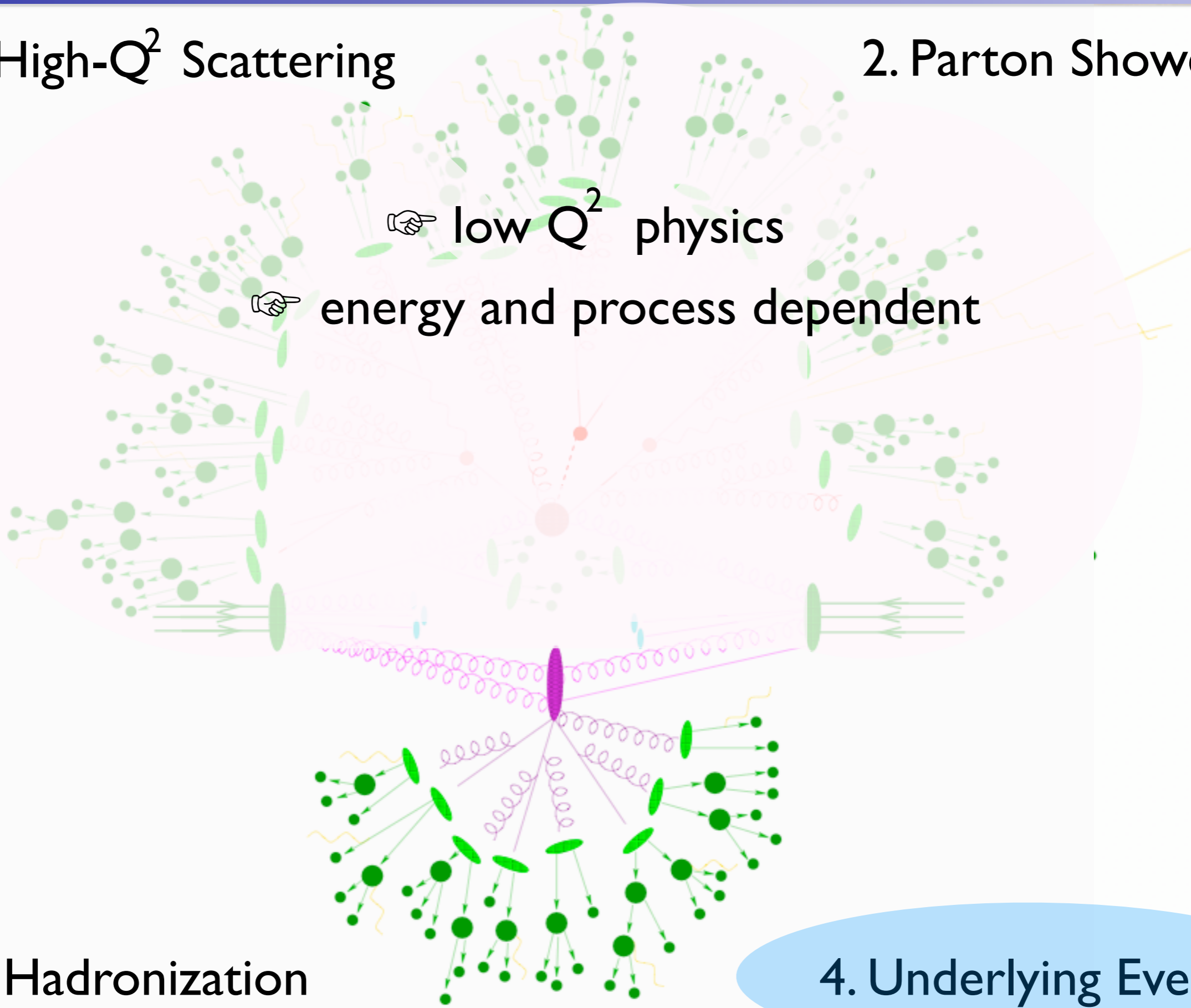
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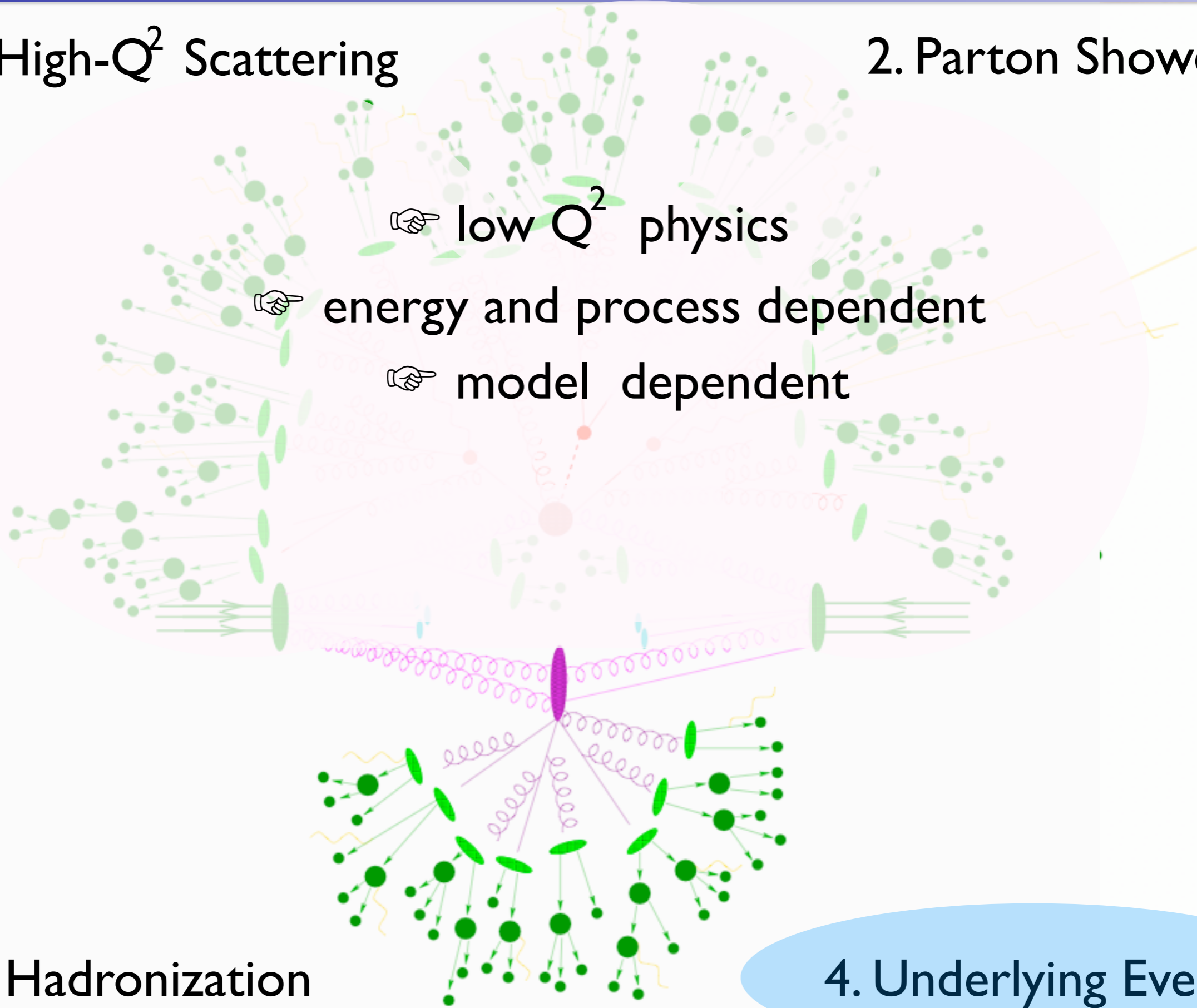
energy and process dependent

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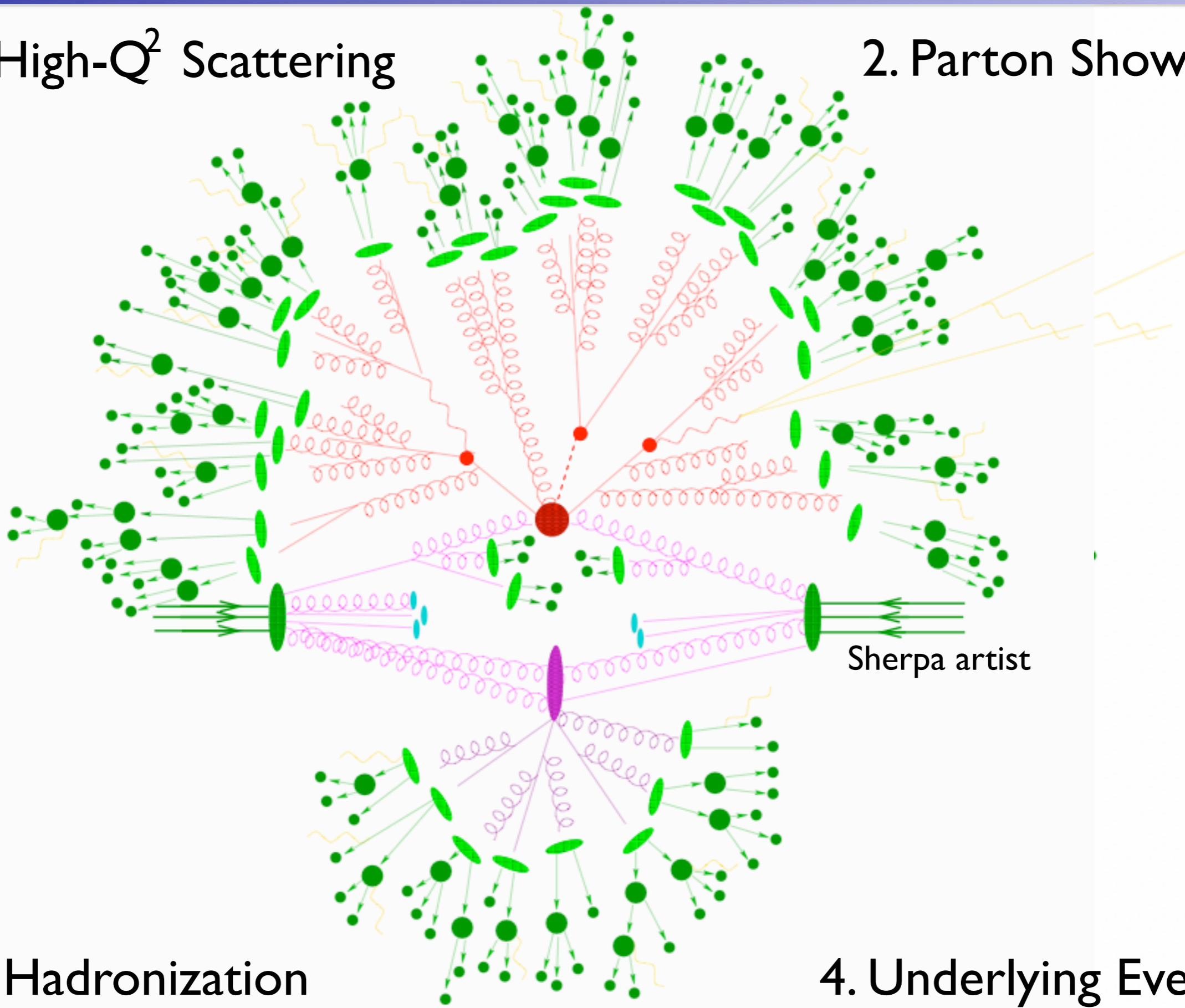
model dependent

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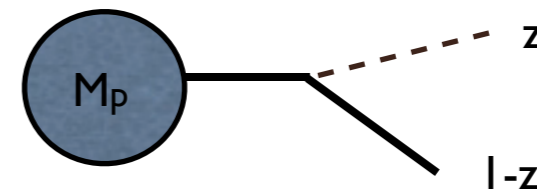
3. Hadronization

4. Underlying Event

Parton Shower MC event generators

ME involving $q \rightarrow q g$ (or $g \rightarrow gg$) are strongly enhanced when they are close in the phase space:

$$\frac{1}{(p_q + p_g)^2} \simeq \frac{1}{2E_q E_g (1 - \cos \theta)}$$



Collinear factorization:

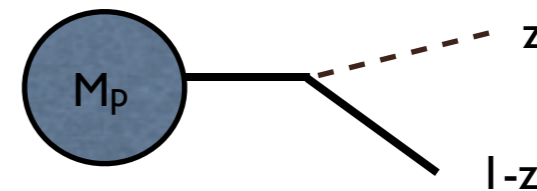
$$|M_{p+1}|^2 d\Phi_{p+1} \simeq |M_p|^2 d\Phi_p \frac{dt}{t} \frac{\alpha_S}{2\pi} P(z) dz d\phi$$

1. Allows for a parton shower (Markov process) evolution
2. The evolution resums the dominant leading-log contributions
3. By adding angular ordering the main quantum (interference) effects are also included

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Both **soft**

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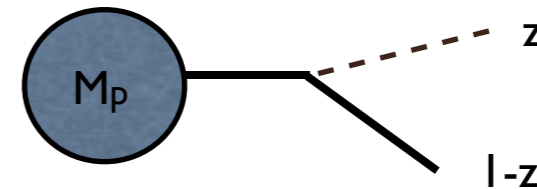
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Both **soft** and collinear **divergences**: very different nature!

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Parton branching

The spin averaged (unregulated) splitting functions for the various types of branching are:

$$\hat{P}_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)} \right],$$

$$\hat{P}_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right],$$

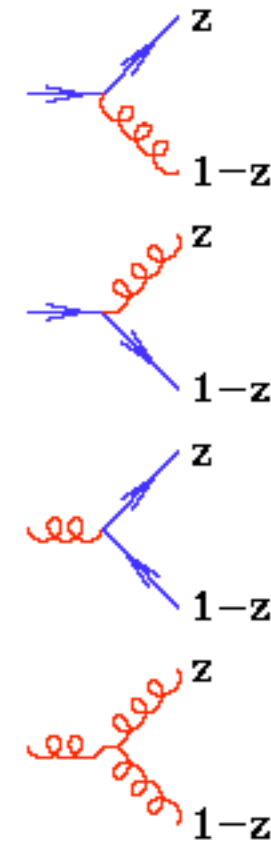
$$\hat{P}_{qg}(z) = T_R \left[z^2 + (1-z)^2 \right],$$

$$\hat{P}_{gg}(z) = C_A \left[\frac{z}{(1-z)} + \frac{1-z}{z} + z(1-z) \right].$$

$$C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}.$$

Comments:

- * Gluons radiate the most
- * There soft divergences in $z=1$ and $z=0$.
- * P_{qg} has no soft divergences.





Sudakov Form factor

Conservation of total probability:

$$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$$

“multiplicativeness” in “time” evolution:

$$\mathcal{P}_{\text{nothing}}(0 < t \leq T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1) \mathcal{P}_{\text{nothing}}(T_1 < t \leq T)$$

Subdivide further, with $T_i = (i/n)T$, $0 \leq i \leq n$:

$$\begin{aligned} \mathcal{P}_{\text{nothing}}(0 < t \leq T) &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \leq T_{i+1}) \\ &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} \left(1 - \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1}) \right) \\ &= \exp \left(- \lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1}) \right) \\ &= \exp \left(- \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right) = \Delta(T) \\ \implies d\mathcal{P}_{\text{first}}(T) &= d\mathcal{P}_{\text{something}}(T) \exp \left(- \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right) \end{aligned}$$



#3

The “Sudakov form factor” directly quantifies how likely is for a parton to undergo branching.

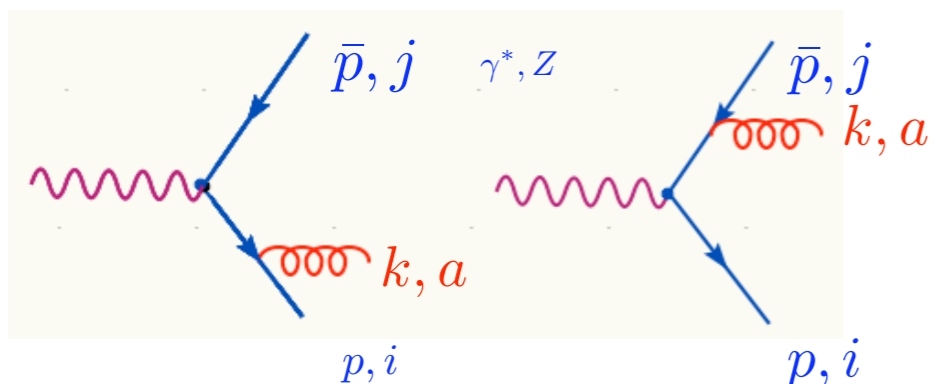


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False

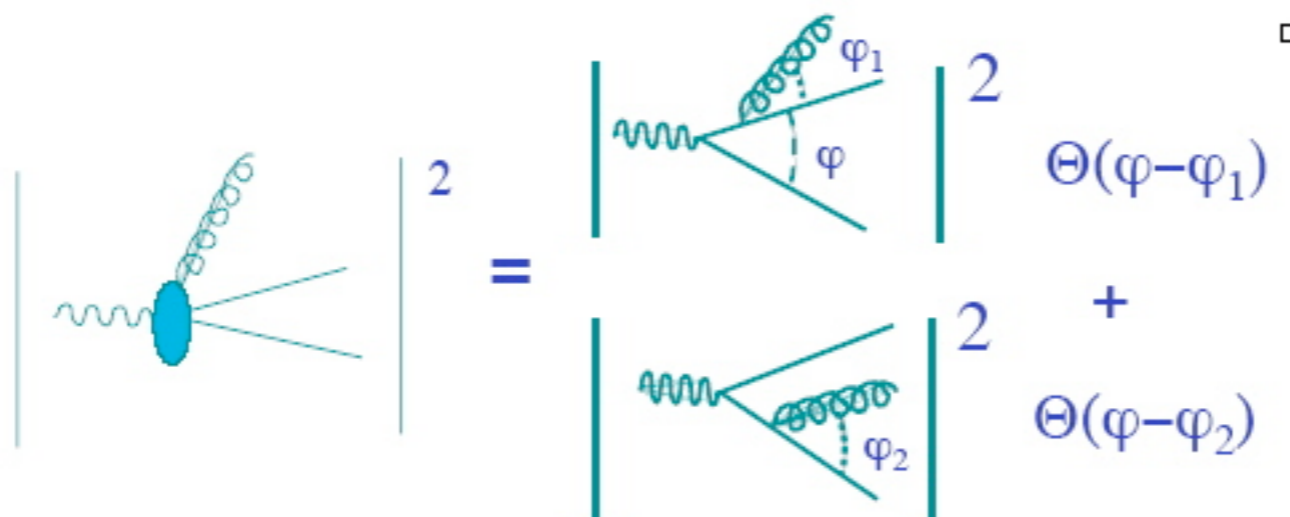
Angular ordering



$$d\sigma_g = \sum |A_{soft}|^2 \frac{d^3k}{(2\pi)^3 2k^0} \sum |A_0|^2 \frac{-2p^\mu \bar{p}^\nu}{(pk)(\bar{p}k)} g^2 \sum \epsilon_\mu \epsilon_\nu^* \frac{d^3k}{(2\pi)^3 2k^0}$$

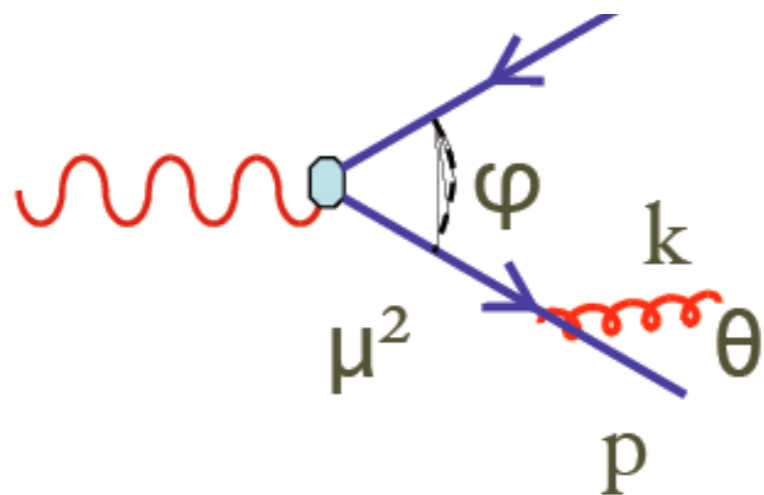
$$= d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{ik})(1 - \cos \theta_{jk})} d\cos \theta$$

You can easily prove that:



Radiation happens only for angles smaller than the color connected (antenna) opening angle!

The physics of angular ordering



Lifetime of the virtual intermediate state:

$$\tau < \gamma/\mu = E/\mu^2 = 1/\theta k_{\perp}$$

Distance between q and \bar{q} after τ :

$$d = \varphi\tau = (\varphi/\theta) 1/k_{\perp}$$

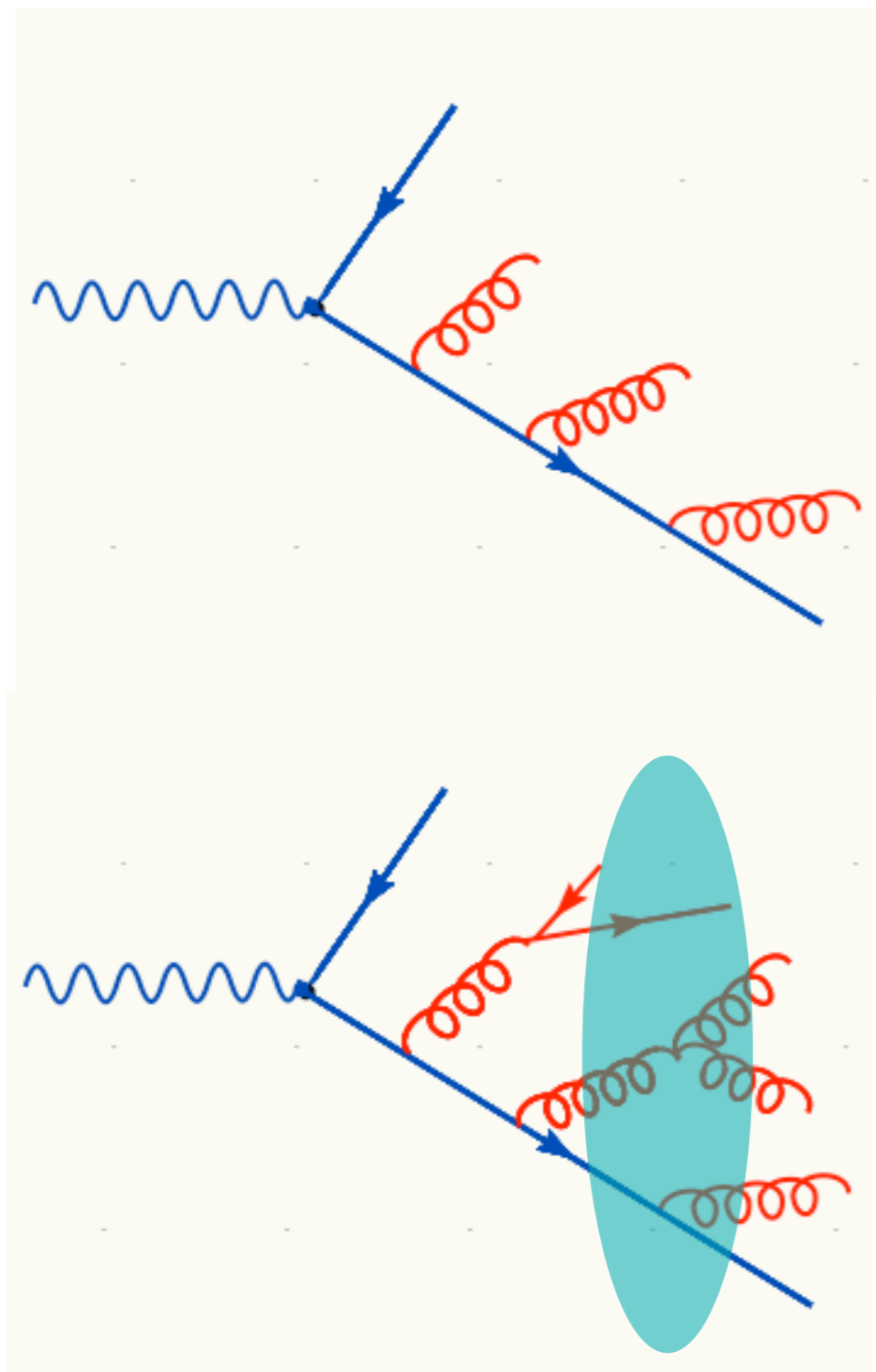
$$\begin{aligned} \mu^2 &= (p+k)^2 = 2 E k_0 (1 - \cos\theta) \\ &\sim E k_0 \theta^2 \sim E k_{\perp} \theta \end{aligned}$$

If the transverse wavelength of the emitted gluon is longer than the separation between q and \bar{q} , the gluon emission is suppressed, due to the fact that the $q \bar{q}$ system will appear as color neutral (a dipole-like emission is suppressed).

Therefore $d > 1/k_{\perp}$, which implies

$$\theta < \varphi$$

Angular ordering



The construction can be iterated to the next emission, with the result that the emission angles keep getting smaller and smaller.

In fact one can generalize the treatment before to a generic parton of color charge Q_k splitting into two partons i and j , $Q_k=Q_i+Q_j$. The result is that inside the cones i and j emit as independent charges, and outside their angular-order cones the emission is coherent and can be treated as if it was directly from color charge Q_k .

This has an effect on the multiplicity of hadrons in jets (INTRAjet radiation), since the radiation is more suppressed with respect to the total phase space available, which one would get from an incoherent radiation. Color ordering enforces coherence and leads to the proper evolution with energy of particle multiplicities.

#2

MC's are based on a classical approximation (Markov Chain), QM effects are not properly described.

Quantum effects are included:

1. **Exactly** in the hard scattering matrix element.
2. **Approximately** by the angular-ordering of the shower

#2

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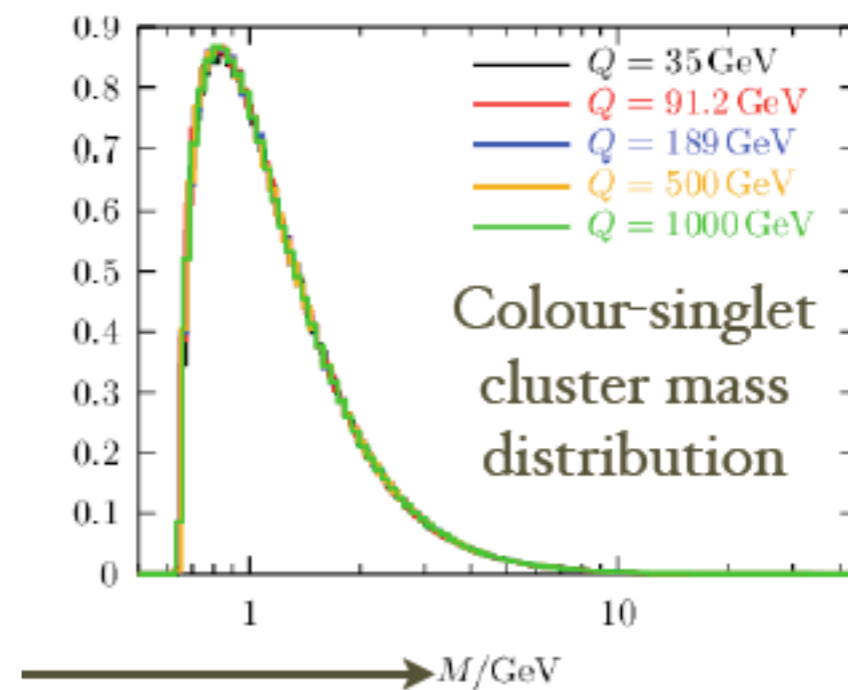
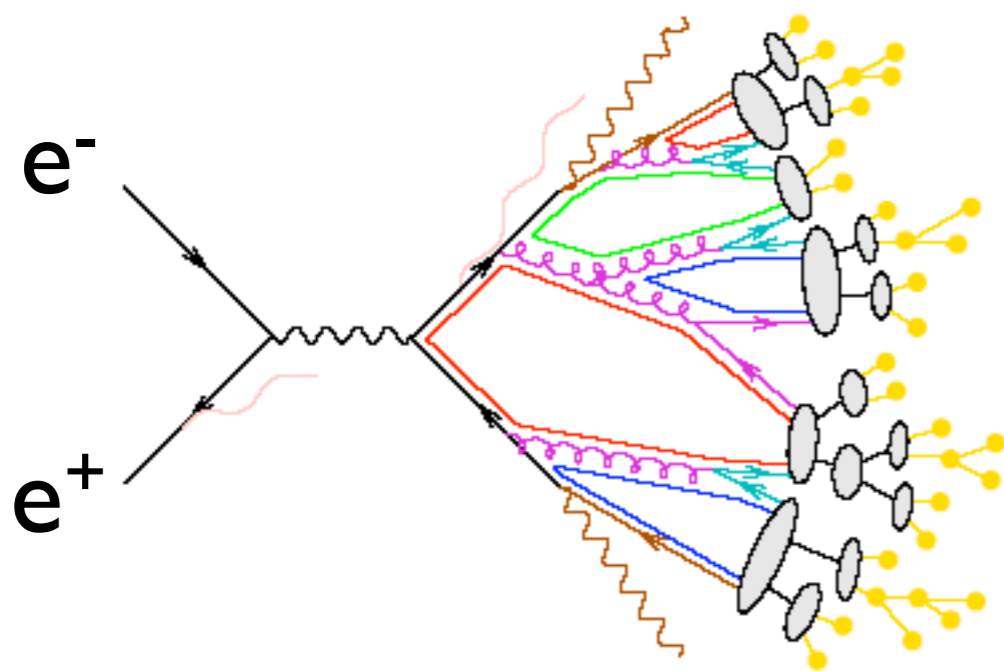
False

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Monte Carlo approach to PS

The structure of the perturbative evolution, including angular ordering, leads naturally to the clustering in phase-space of color-singlet parton pairs (preconfinement). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass color singlet clusters.



A simple plan

- Physics challenges at the LHC
- Basics : QCD and MC's methods
- The new generation of MC tools
- New simulations for New Physics

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Parton Shower MC event generators

- General-purpose tools
- Always the first exp choice
- Complete exclusive description of the events: hard scattering, showering & hadronization, underlying event
- Reliable and well tuned tools.

most famous: PYTHIA, HERWIG, SHERPA

- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD
[Nagy, Soper, 2005; Giele, Kosower, Skands, 2007; Krauss, Schumman, 2007]

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How we (used to) make predictions?

First way:

- For low multiplicity include higher order terms in our fixed-order calculations (LO→NLO→NNLO...)

$$\Rightarrow \hat{\sigma}_{ab \rightarrow X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

- For high multiplicity use the tree-level results



Comments:

1. The theoretical errors systematically decrease.
2. Pure theoretical point of view.
3. A lot of new techniques and universal algorithms are developed.
4. Final description only in terms of partons and calculation of IR safe observables \Rightarrow not directly useful for simulations

How we (used to) make predictions?

Second way:

- Describe final states with high multiplicities starting from $2 \rightarrow 1$ or $2 \rightarrow 2$ procs, using parton showers, and then an hadronization model.



Comments:

1. Fully exclusive final state description for detector simulations
2. Normalization is very uncertain
3. Very crude kinematic distributions for multi-parton final states
4. Improvements are only at the model level.

ME vs PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Frixione, Nason, Webber]

ME



- 1. parton-level description
- 2. fixed order calculation
- 3. quantum interference exact
- 4. valid when partons are hard and well separated
- 5. needed for multi-jet description

Shower MC



- 1. hadron-level description
- 2. resums large logs
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- 4. valid when partons are collinear and/or soft
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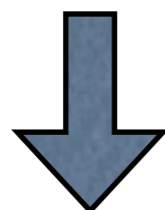


Difficulty: avoid double counting

ME vs PS

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Approaches are complementary: merge them!

Difficulty: avoid double counting

How to improve our predictions?

New trend:

TH & EXP

Match fixed-order calculations and parton showers to obtain the most accurate predictions in a detector simulation friendly way!

Two directions:

1. Get fully exclusive description of many parton events correct at LO (LL) in all the phase space.

ME+PS

2. Get fully exclusive description of events correct at NLO in the normalization and distributions.

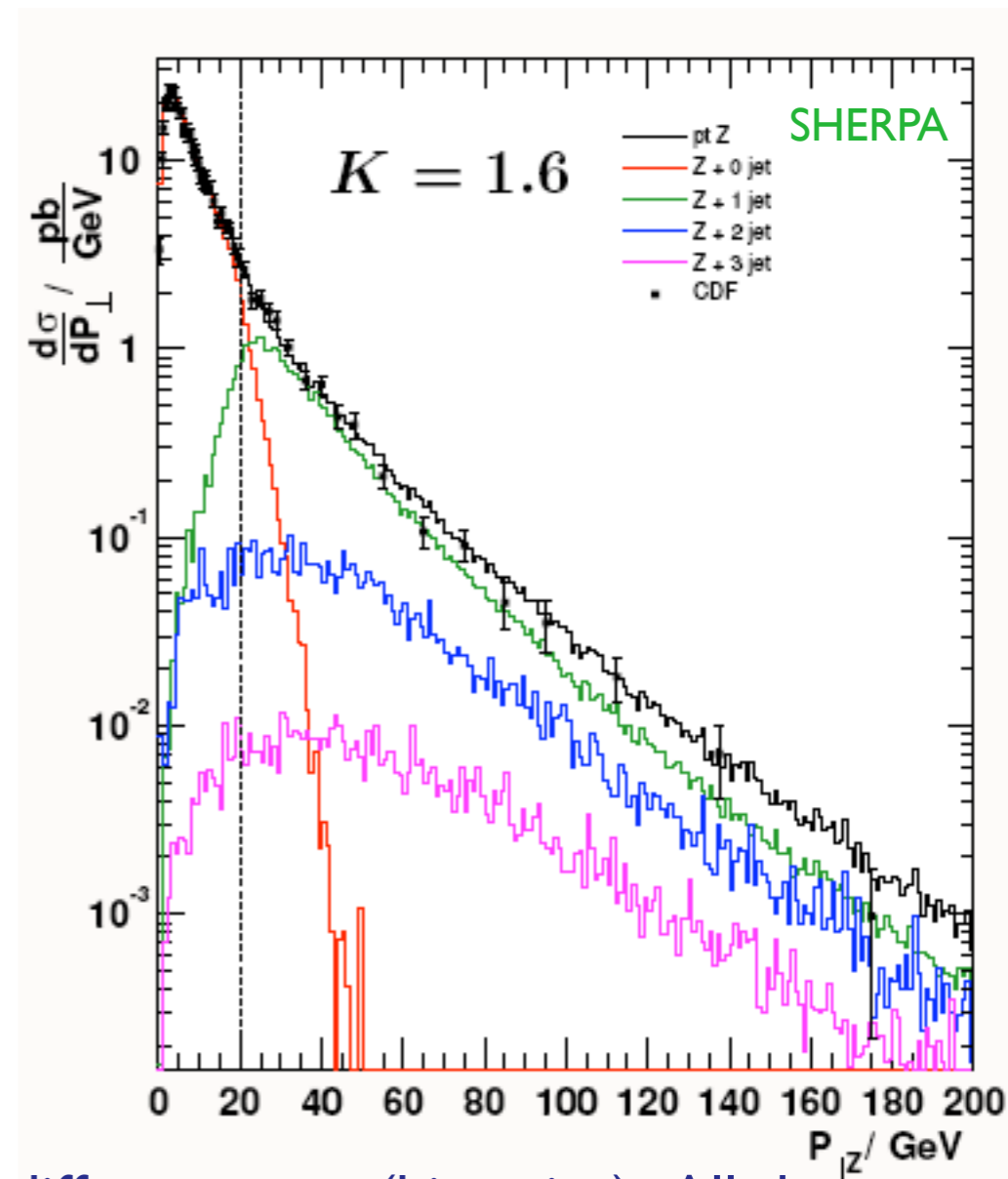
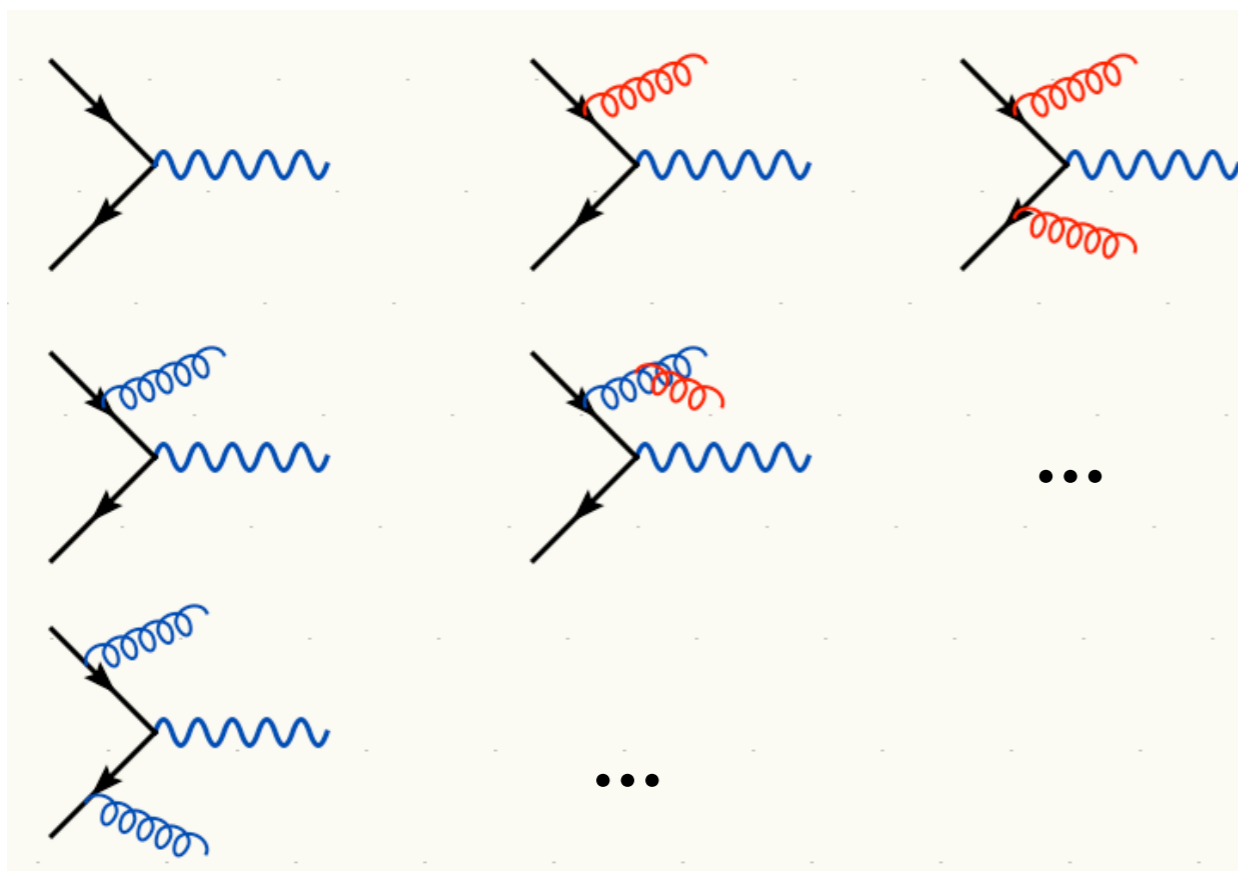
NLO_wPS

Merging fixed order with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]

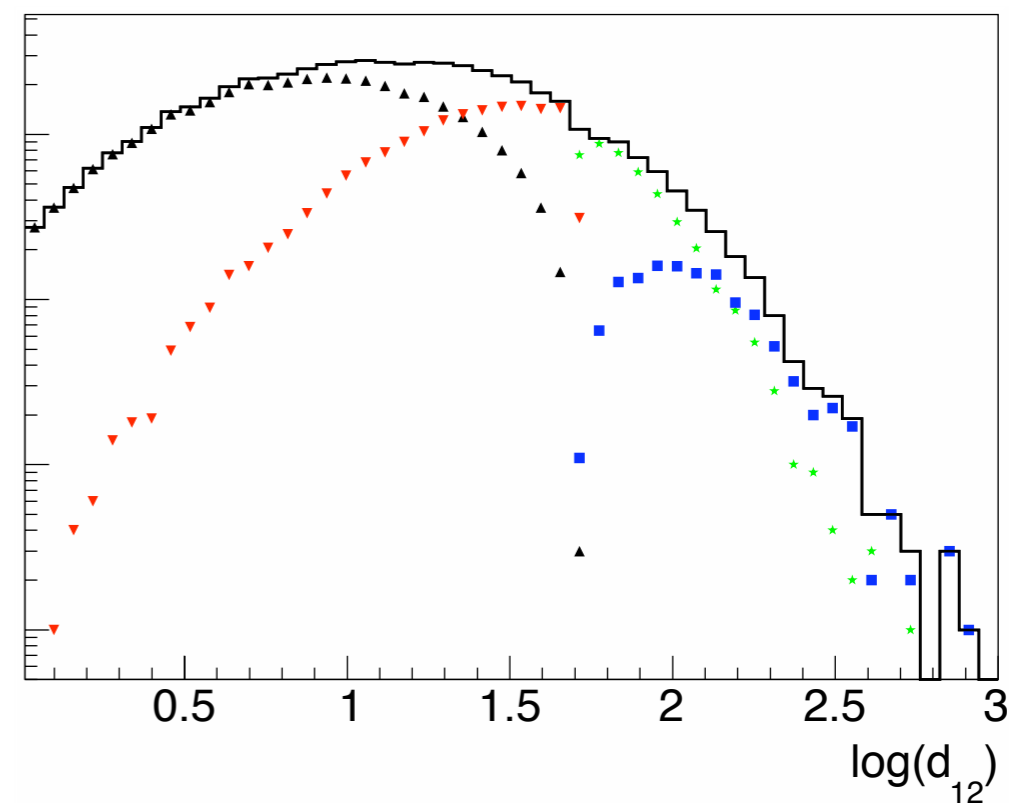
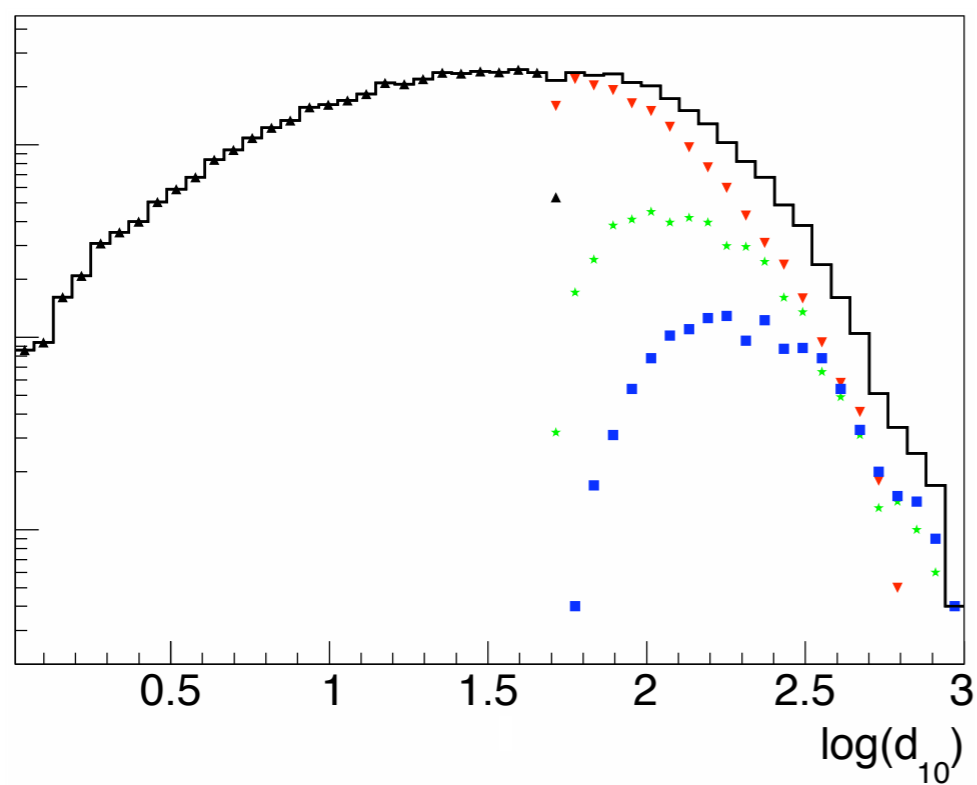
PS →

ME



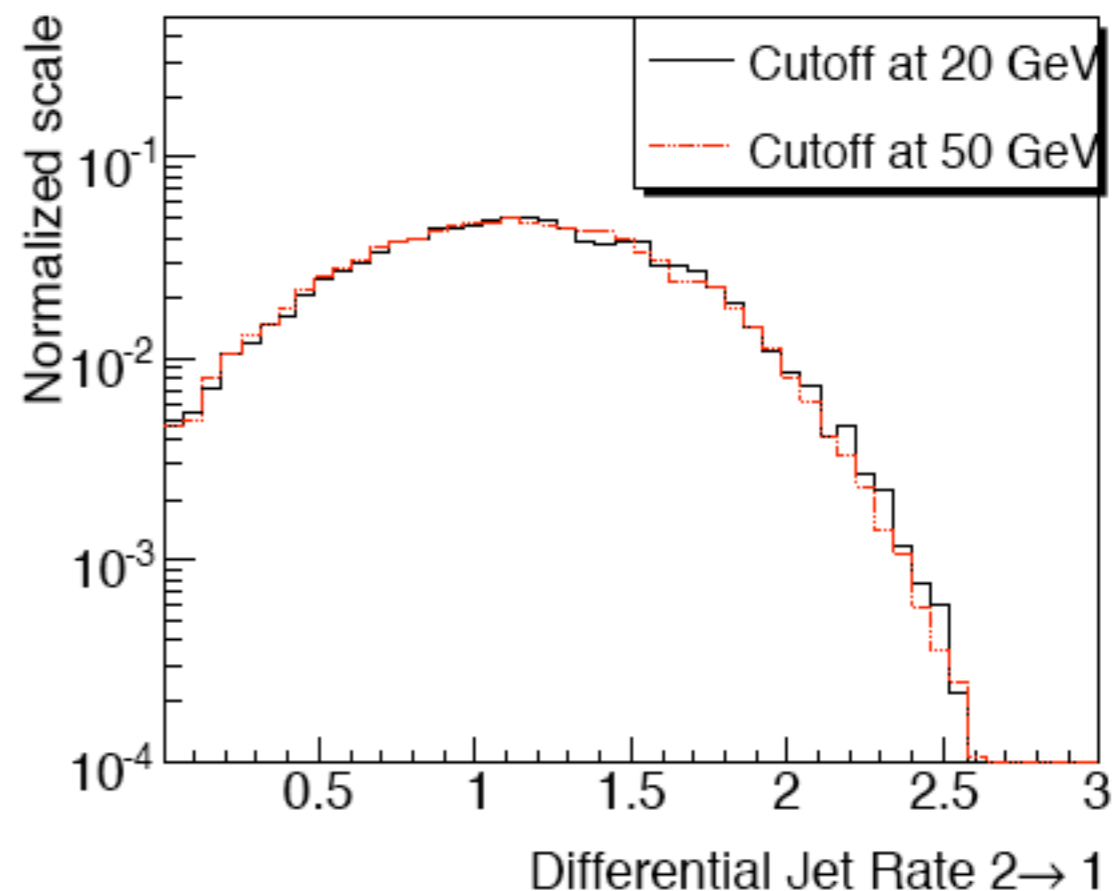
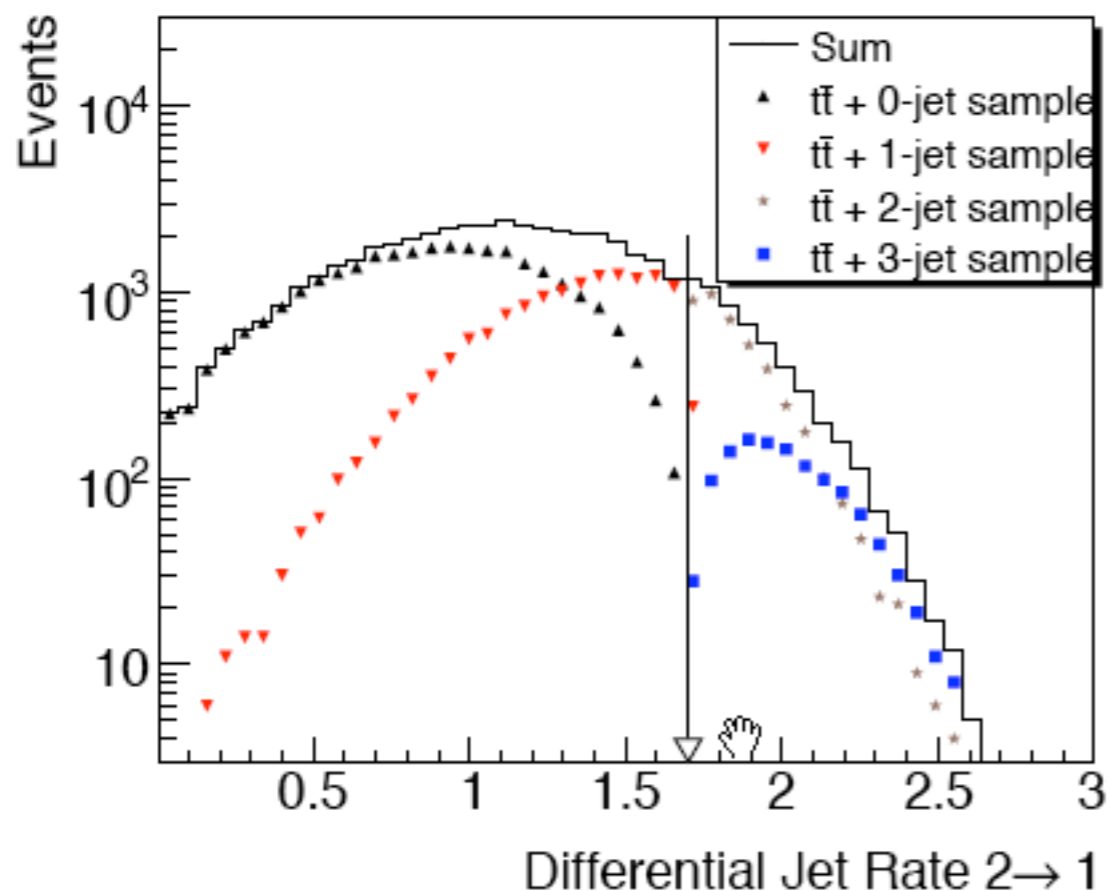
Double counting of configurations that can be obtained in different ways (histories). All the matching algorithms (CKKW, MLM,...) apply criteria to select only one possibility based on the hardness of the partons. As the result events are exclusive and can be added together into an inclusive sample. Distributions are accurate but overall normalization still “arbitrary”.

Sanity checks: differential jet rates



Jet rates are smooth at the cutoff scale

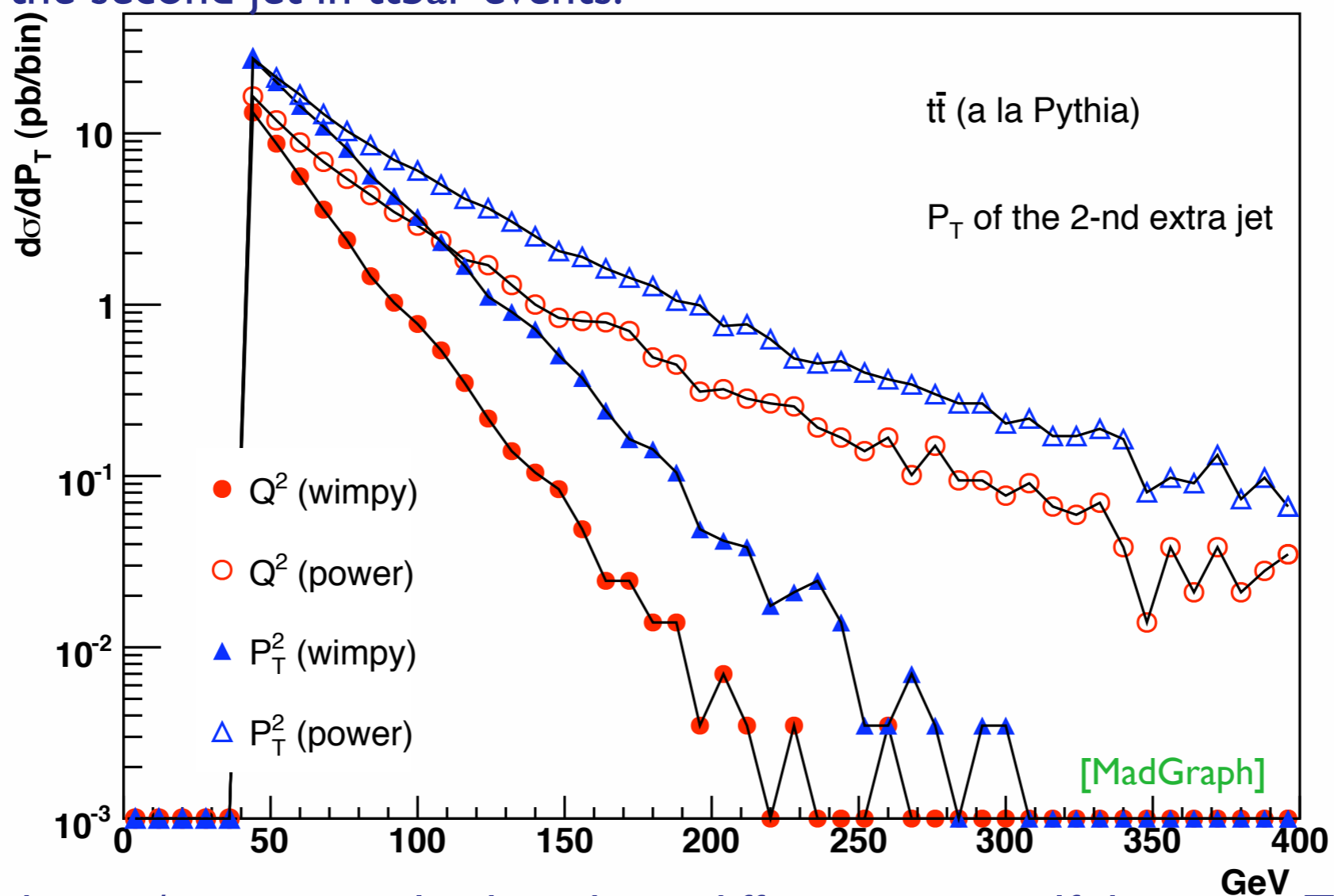
Sanity checks: differential jet rates



Jet rates are independent of the cutoff scale

PS alone vs matched samples

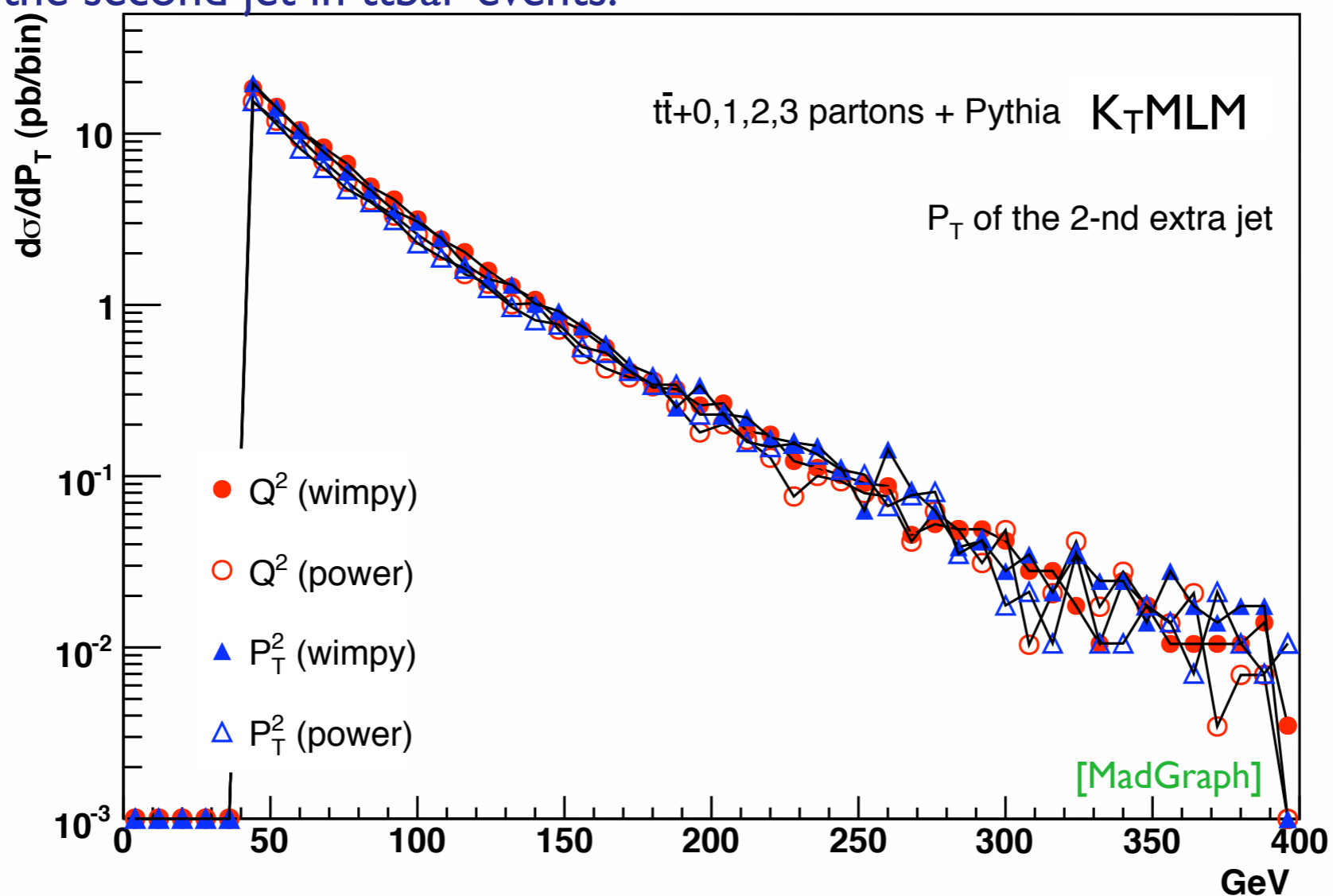
A MC Shower like Pythia produces inclusive samples covering all phase space. However, there are regions of the phase space (ex. high pt tails) which cannot be described well by the log enhanced (shower) terms in the QCD expansion and lead to ambiguities. Consider for instance the high-pt distribution of the second jet in $t\bar{t}$ events:



Changing some choices/parameters leads to huge differences \Rightarrow self diagnosis. Trying to tune the log terms to make up for it is not a good idea \Rightarrow mess up other regions/shapes, process dependence.

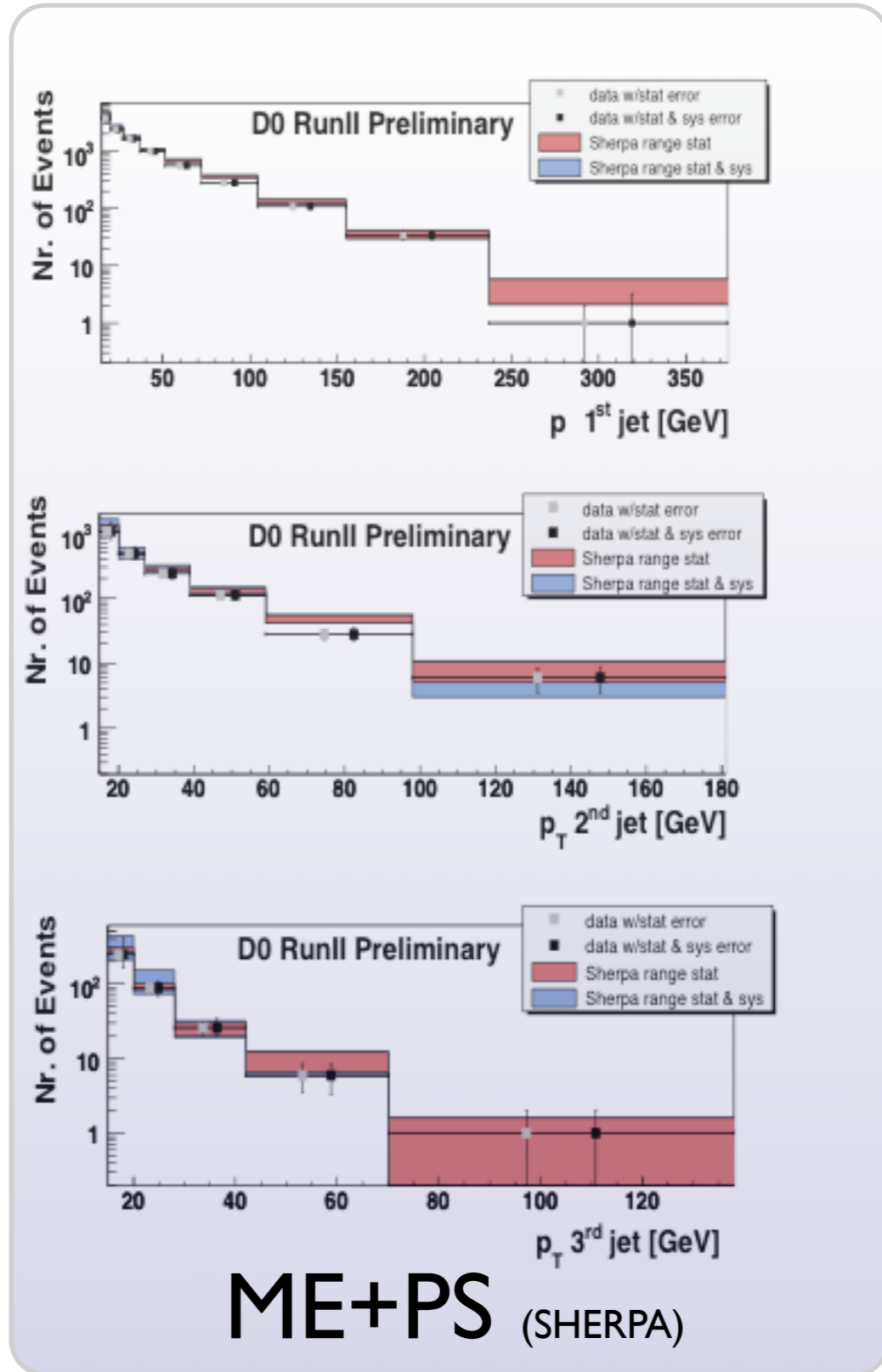
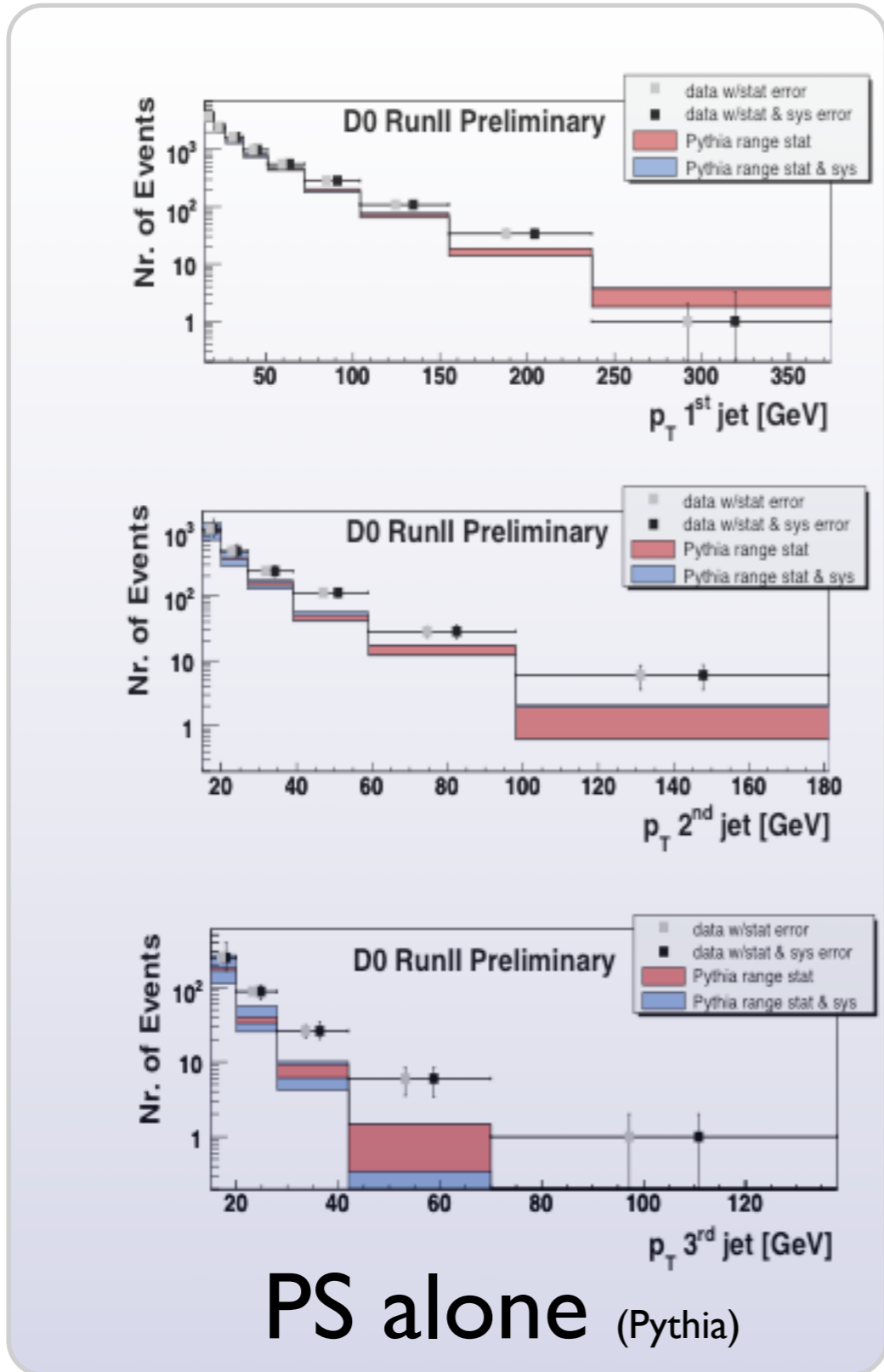
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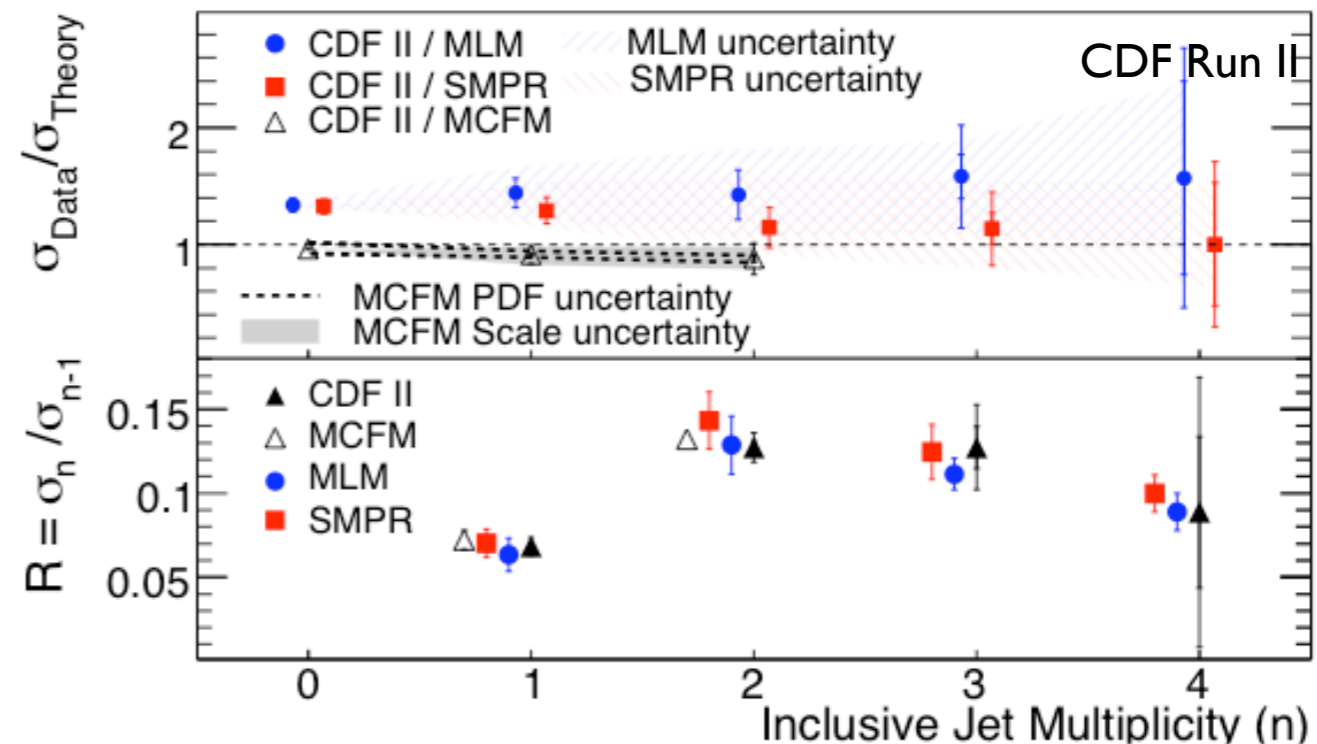
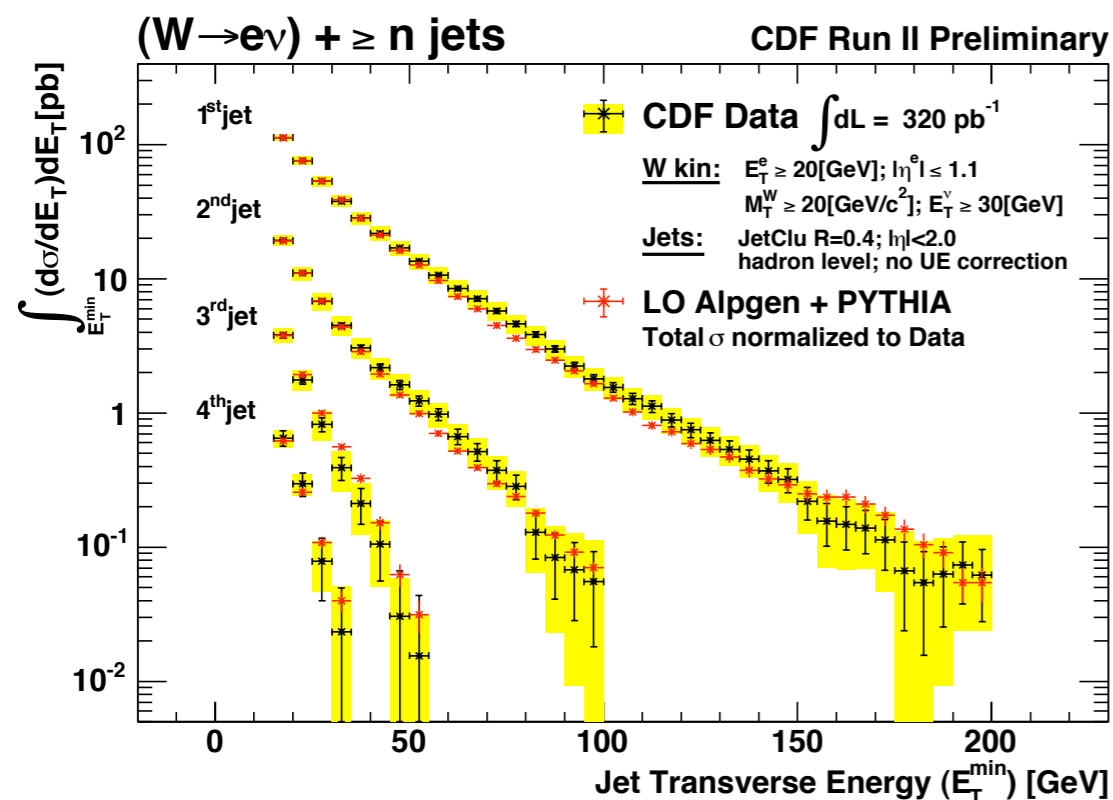


In a matched sample these differences are irrelevant since the behaviour at high pt is dominated by the matrix element. LO+LL is more reliable. (Matching uncertainties not shown.)

PS alone vs matched samples : Z+jets at D0



W+jets at CDF



* Very good agreement in shapes (left) and in relative normalization (right).

* NLO rates in outstanding agreement with data.

* Matched samples obtained via different matching schemes (MLM and CKKW) consistent within the expected uncertainties. Differences might arise in more exclusive quantities.

NLO_wPS

Problem of double counting becomes even more severe at NLO

- * Real emission from NLO and PS has to be counted once
- * Virtual contributions in the NLO and Sudakov should not overlap

Current available (and working) solutions:

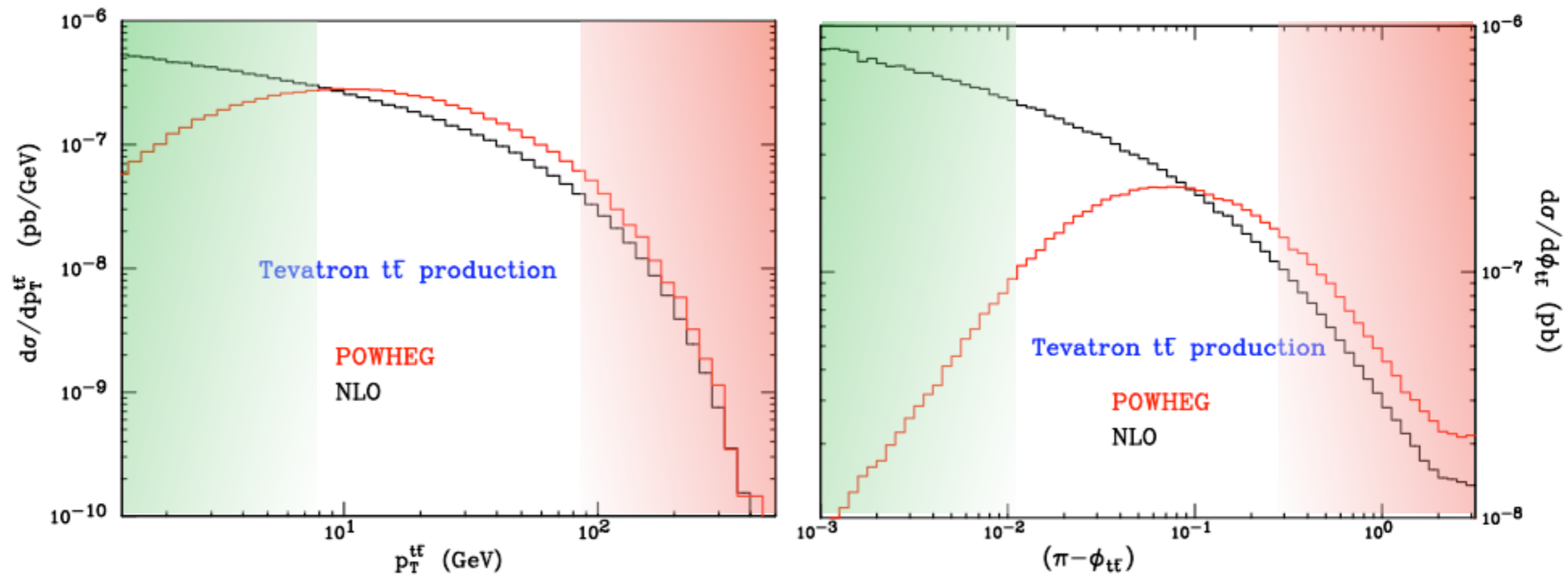
MC@NLO [Frixione, Webber, 2003; Frixione, Nason, Webber, 2003]

- Matches NLO to HERWIG angular-ordered PS.
- “Some” work to interface an NLO calculation to HERWIG.
Uses only FKS subtraction scheme.
- Some events have negative weights.
- Sizable library of procs now.

POWHEG [Nason 2004; Frixione, Nason, Oleari, 2007]

- Is independent from the PS. It can be interfaced to PYTHIA or HERWIG.
- Can use existing NLO results.
- Generates only positive unit weights.
- For top only ttbar (with spin correlations) is available so far.

ttbar : NLOwPS vs NLO



- * Soft/Collinear resummation of the $p_T(tt) \rightarrow 0$ region.
- * At high $p_T(tt)$ it approaches the tt +parton (tree-level) result.
- * When $\Phi(tt) \rightarrow 0$ ($\Phi(tt) \rightarrow \pi$) the emitted radiation is hard (soft).
- * Normalization is FIXED and non trivial!!

NLOwPS

“Best” tools when NLO calculation is available (i.e. low jet multiplicity).

* Main points:

- * NLOwPS provide a consistent way to include K-factors into MC's
- * Scale dependence is meaningful
- * Allows a correct estimate of the PDF errors.
- * Non-trivial dynamics beyond LO included for the first time.

N.B.: The above is true for observables which are at NLO to start with!!!

* Current limitations:

- * Considerable manual work for the implementation of a new process.
- * Only SM.
- * Only available for low multiplicity.



A look into the future



A look into the future

Leading Order :

- * fully automatized + matching algorithms
- * Continuously improved : new ideas for showers, better hadronization/ underlying events), better matrix element generators, new matching schemes.
- * Several fully working frameworks available.



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Basics covered, room for improvement



A look into the future



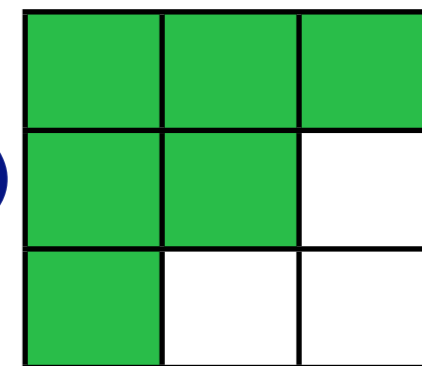
Leading Order :

- * fully automatized + matching algorithms
- * Continuously improved : new ideas for showers, better hadronization/underlying events), better matrix element generators, new matching schemes.
- * Several fully working frameworks available.

Basics covered, room for improvement

Next-to-leading Order :

- * Automatic generation of reals + counterterms available
- * General framework for interfacing to the shower (POWHEG)
- * Impressive results in automatic 1-loop computations
[BlackHat, Rocket, Cuttools]



A look into the future



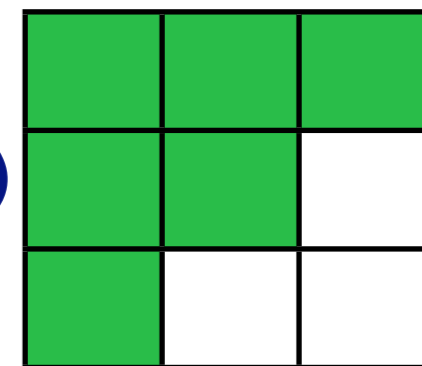
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Fully automatized NLOwPS in sight

Tools

New generation of integrated/interfaced tools:

- Example I:

SHERPA, Matrix Element generator + PS with CKKW

- Example II:

AlpGen + Pythia or Herwig, ME + PS with MLM

MadGraph + Pythia or Herwig, ME + PS with MLM (or CKKW)

...

- Example III :

MC@NLO, NLO + HERWIG with Frixione-Webber method.

POWHEG tools, NLO + PS with Nason et al. method (POWHEG).

HERWIG++, NLO + new HERWIG with POWHEG method.

More tools/techniques for merging under continuous development:

VINCIA, **GENEVA**, Dipole Showers, CKKW-NLO,....



#5

Tree-level based MC's are less accurate
than those at NLO.



#5

Tree-level based MC's are less accurate
than those at NLO.

It depends



Again, it depends on the observable. A NLO code can only provide some observable at NLO, and in particular not those with extra-jets.

Example : $t\bar{t}$ in NLOwPS vs $t\bar{t}$ + jets matched.

Overall normalization : better $t\bar{t}$ at NLO

pt of the first jet : the same accuracy

pt of the 2nd jet : better the $t\bar{t}$ matched sample

So, if I am considering $t\bar{t}$ as a signal, most probably NLOwPS is the best tool.

While if I am considering $t\bar{t}$ as a background to SUSY or to $t\bar{t}h$ with $h \rightarrow b\bar{b}$, the matched sample is a better approximation.



Status : SM

$pp \rightarrow n$ particles



Status : SM

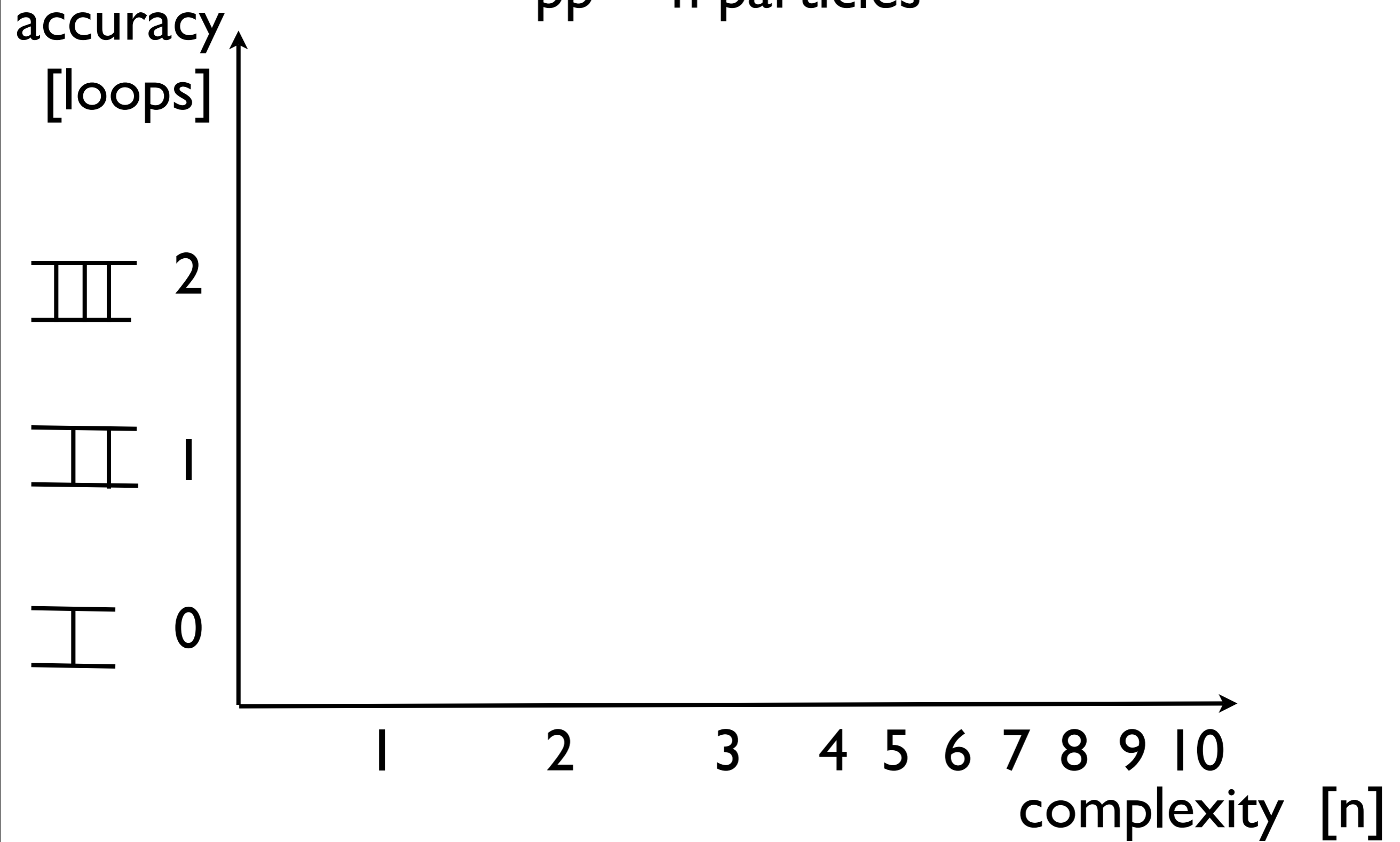
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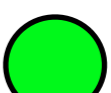
$pp \rightarrow n$ particles

accuracy
[loops]

III 2
II 1
I 0

1 2 3 4 5 6 7 8 9 10

complexity [n]



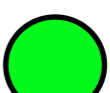
-  fully inclusive
-  parton-level
-  fully exclusive



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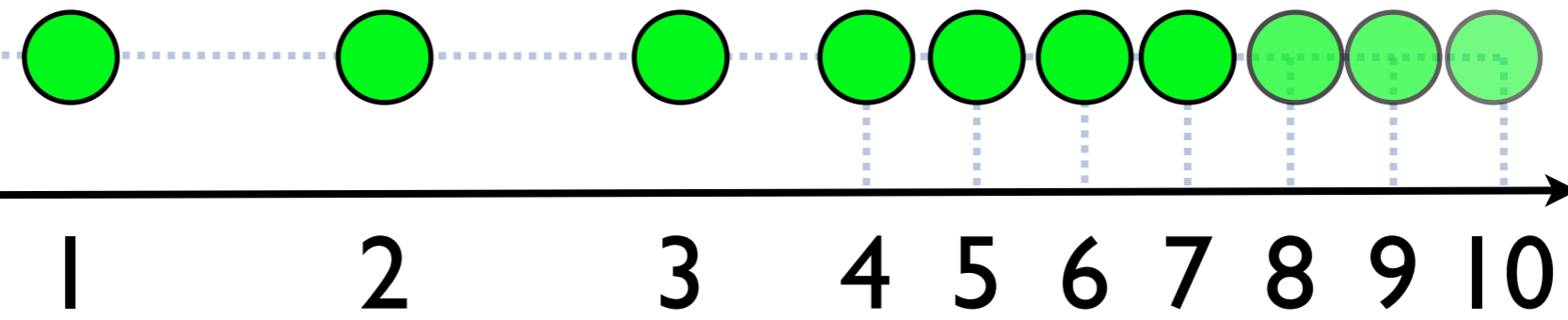
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accuracy
[loops]

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III 2
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Tree-level:
.Any process $2 \rightarrow n$ available
.Many algorithms
.Completely automatized
.Matching with the PS at NLL



complexity [n]



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accuracy
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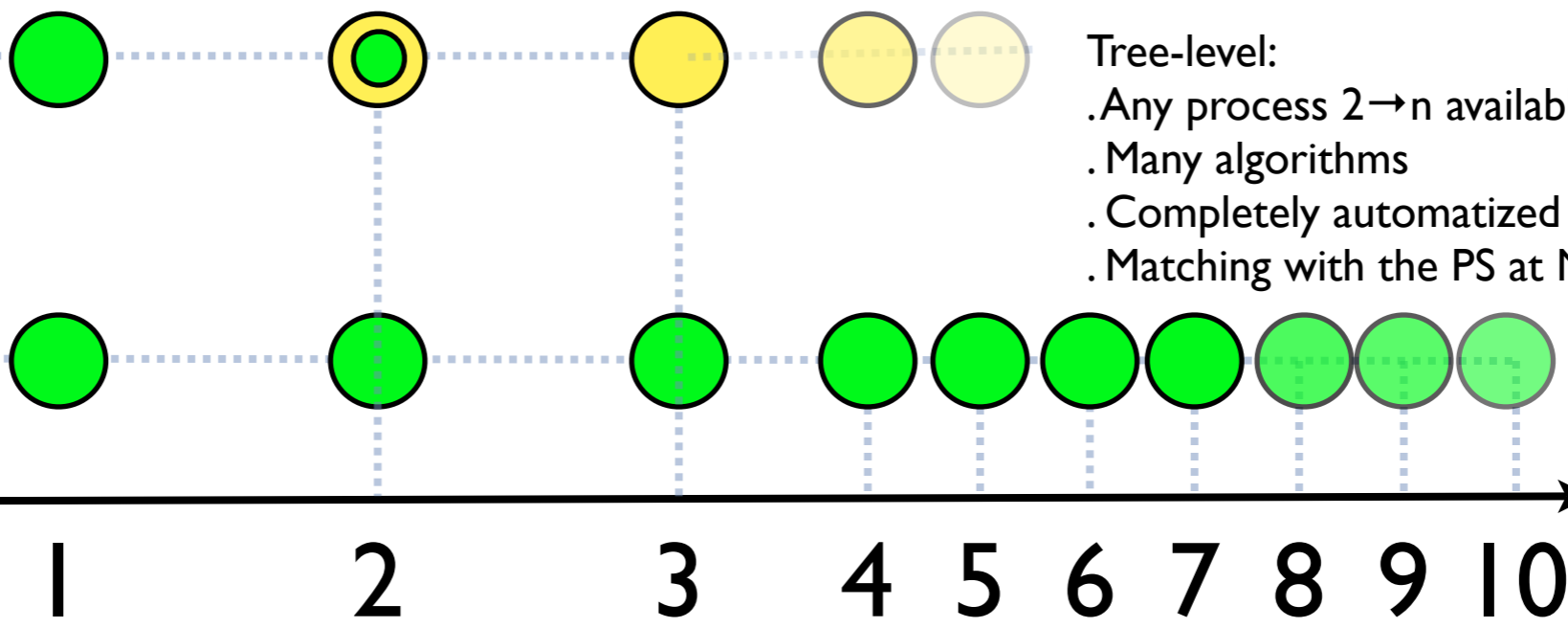
III 2

II 1

I 0

One-loop:
 .Large number of processes known up to $2 \rightarrow 3$
 .General algorithms for divergences cancellation
 .Automatization in sight
 .Matching with the PS in MC@NLO e POWHEG

Tree-level:
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complexity [n]



Status : SM

$pp \rightarrow n$ particles

accuracy
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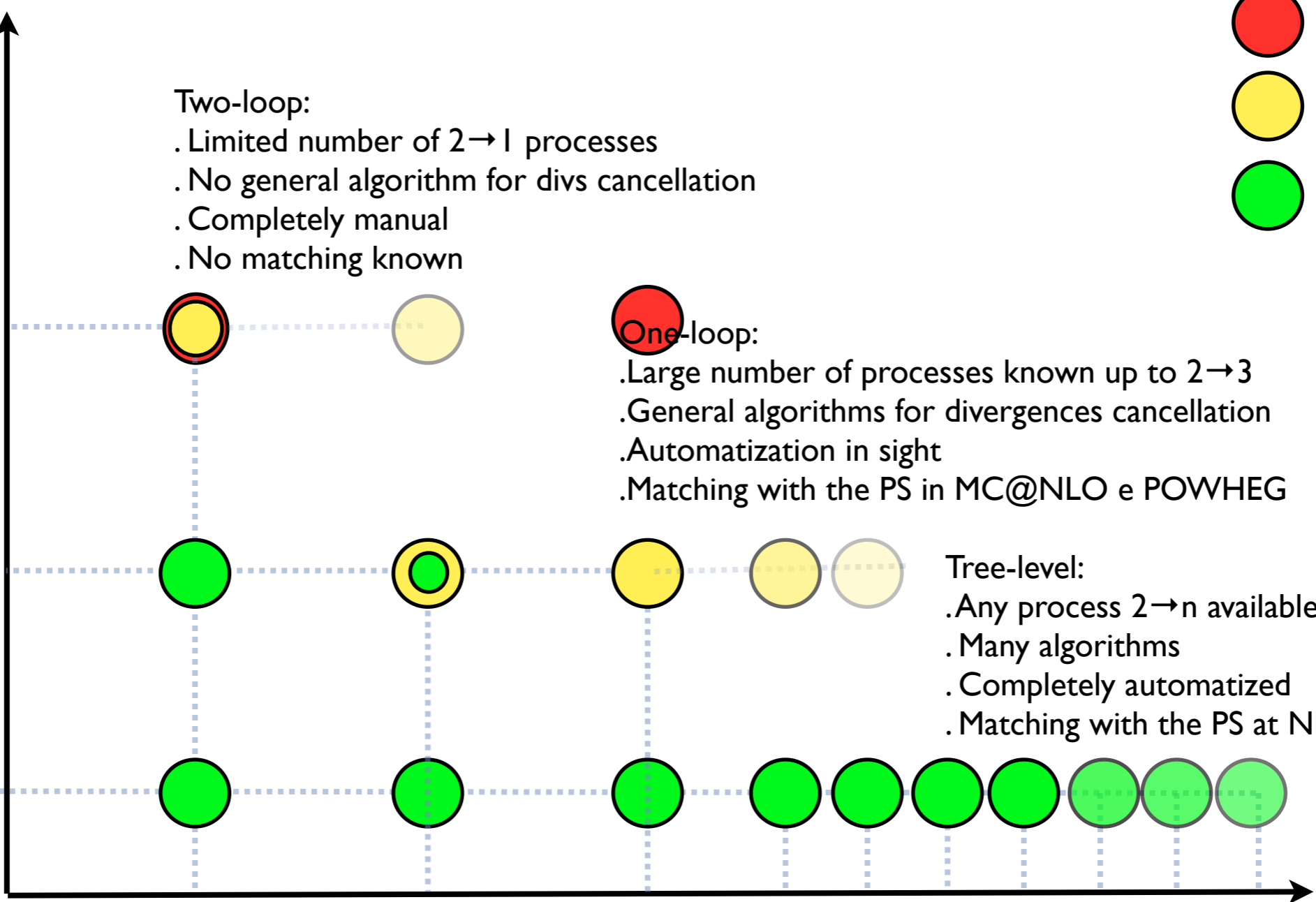
III
II
I
0

Two-loop:
 . Limited number of $2 \rightarrow 1$ processes
 . No general algorithm for divs cancellation
 . Completely manual
 . No matching known

One-loop:
 . Large number of processes known up to $2 \rightarrow 3$
 . General algorithms for divergences cancellation
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complexity [n]

A simple plan

- Physics challenges at the LHC
- Basics : QCD and MC's methods
- The new generation of MC tools
- New simulations for New Physics

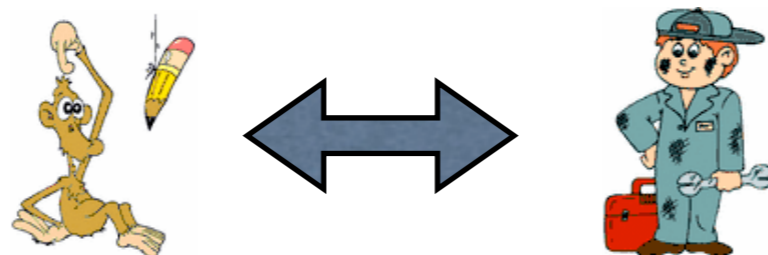
A simple plan

- Physics challenges at the LHC
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What about BSM?

Two main (related) issues:

1. A plethora of BSM proposals exist to be compared with data. It will be essential to have an efficient, validated MC framework for theorists to communicate with experimentalists their idea (and viceversa).

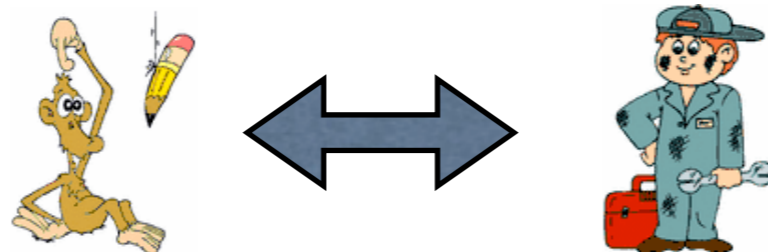


2. Once models are available in multipurpose MC's, new detailed studies are possible that allow to bring to the BSM signatures the same level of sophistication achieved for the SM.

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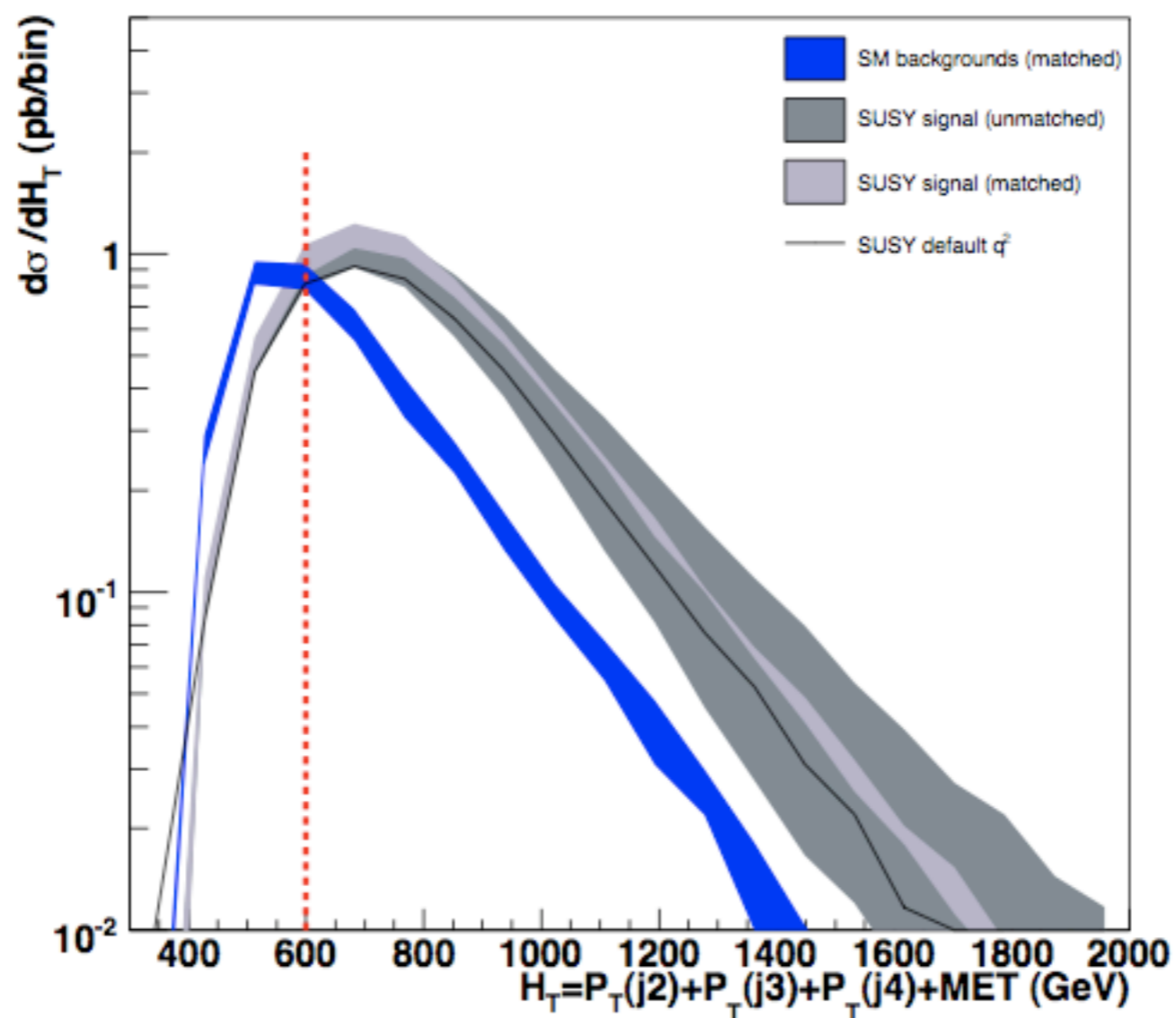
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BSM @ LHC : present

[Alwall, de Visscher, FM, 2009]



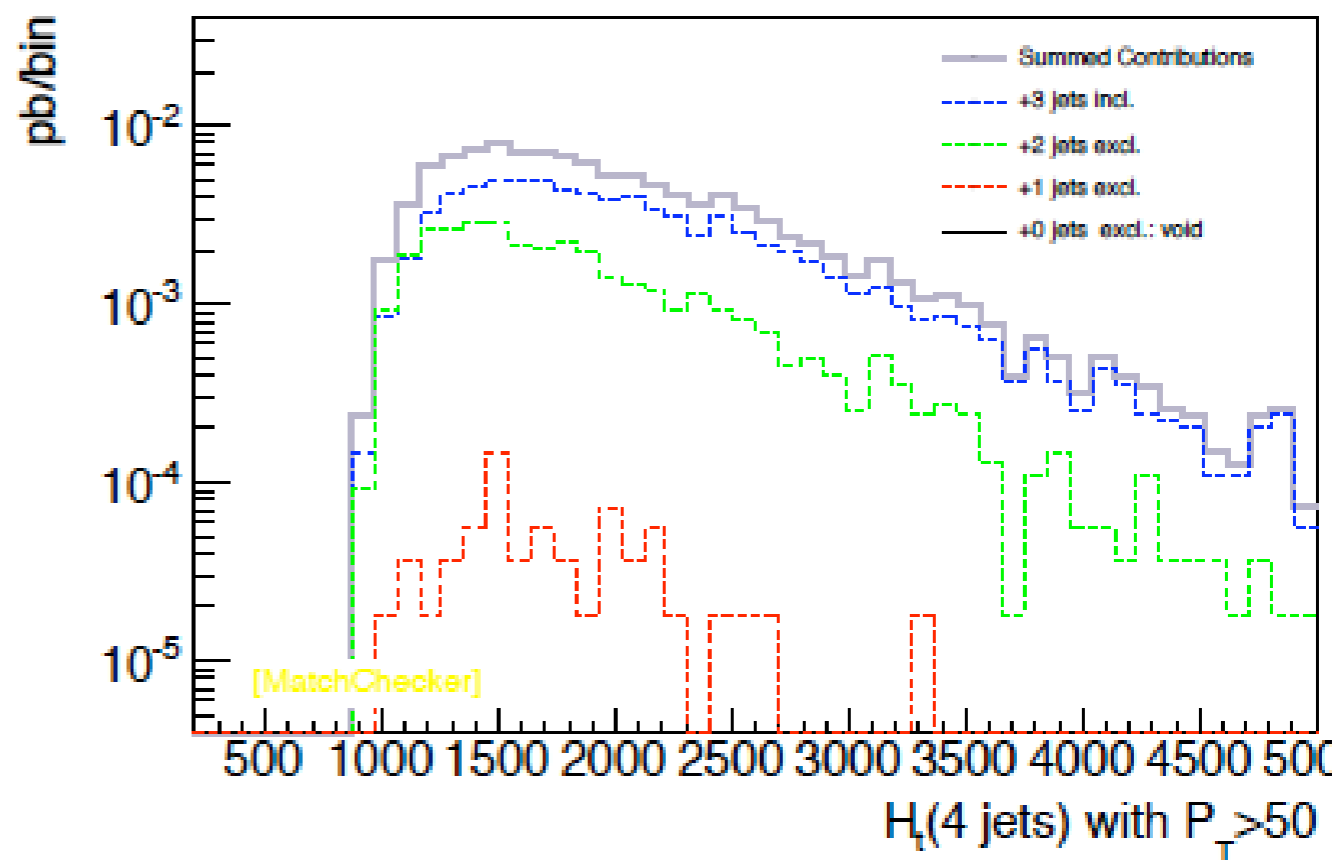
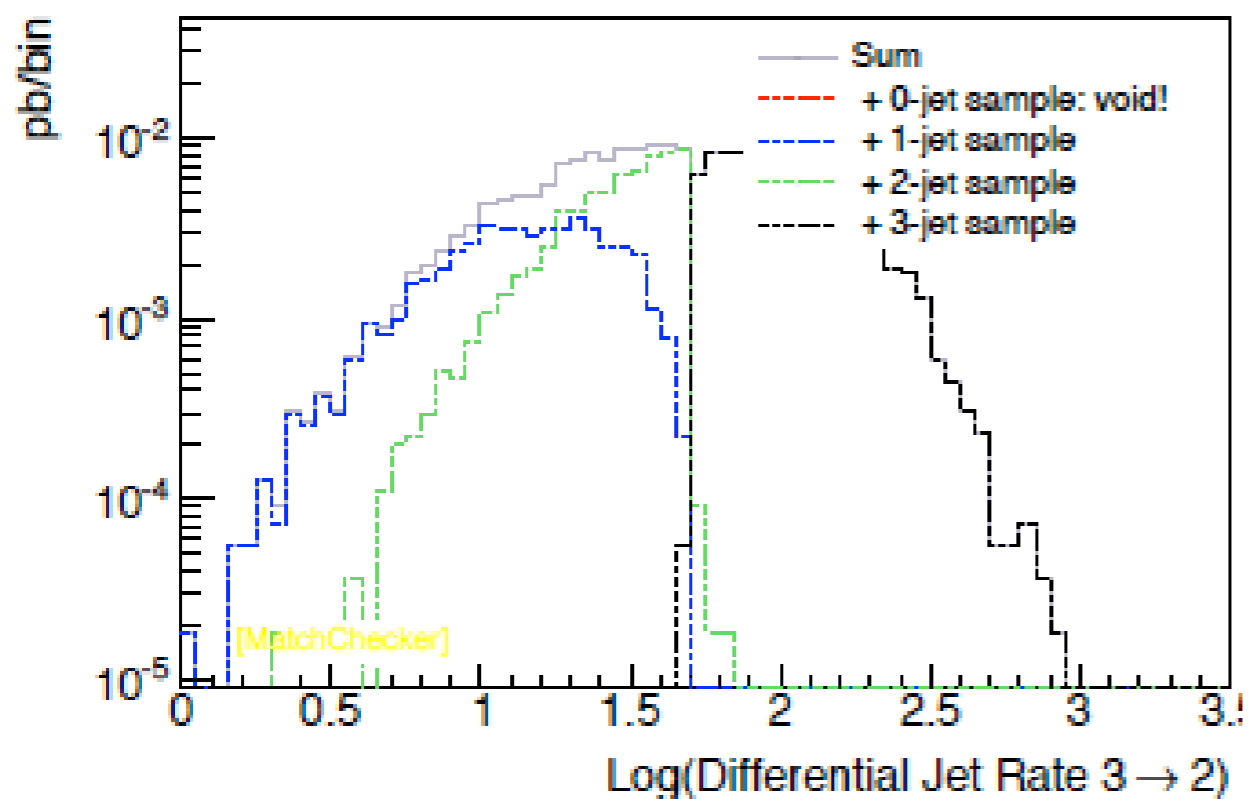
Both signal and background matched!

Sizable reduction of the uncertainties. Overall picture unchanged for SPS I a.

Gravitons

[K. Hagiwara, J. Kanzaki, Q. Li and K. Mawatari, 2009]
 [P. de Aquino, K. Hagiwara, Q. Li, F. M.]

- Fixed mass gravitons (RS and also $m_G=0$)
- ADD gravitons also available : challenging due peculiar “propagator” : this is automatically handled in MG now.



Works out of the box..



BSM : status and outlook

$pp \rightarrow n$ particles

accuracy
[loops]

III
II
I
0

2
1
0

- fully inclusive
- parton-level
- fully exclusive
- ◇ + SM

NLO:
 . $2 \rightarrow 1$ (SM) and $2 \rightarrow 2$
 . Fully inclusive ("K factors only")
 . Completely manual

NLO
 . Automatization in sight

Tree-level:
 . Any process $2 \rightarrow n + i$ sm
 . Feynman-diagram based
 . Completely automatized
 . Double counting
 . Merging ME&PS
 . NEW!

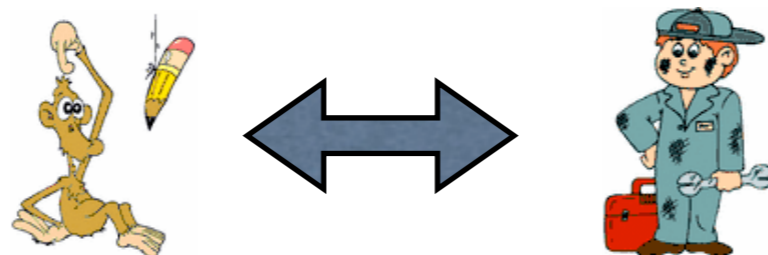
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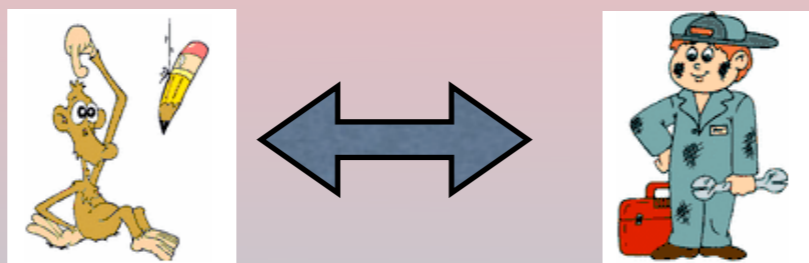


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A Roadmap (with roadblocks) for BSM @ the LHC

TH

EXP

Idea

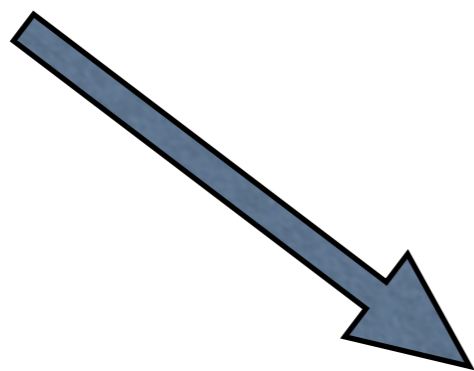
Data

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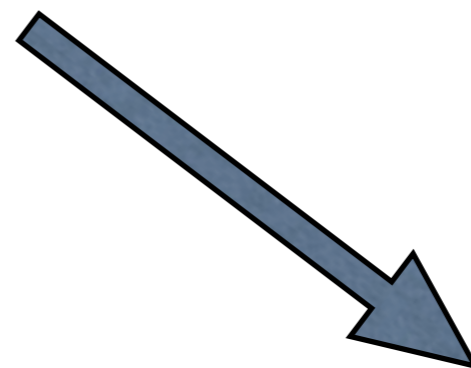
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Data

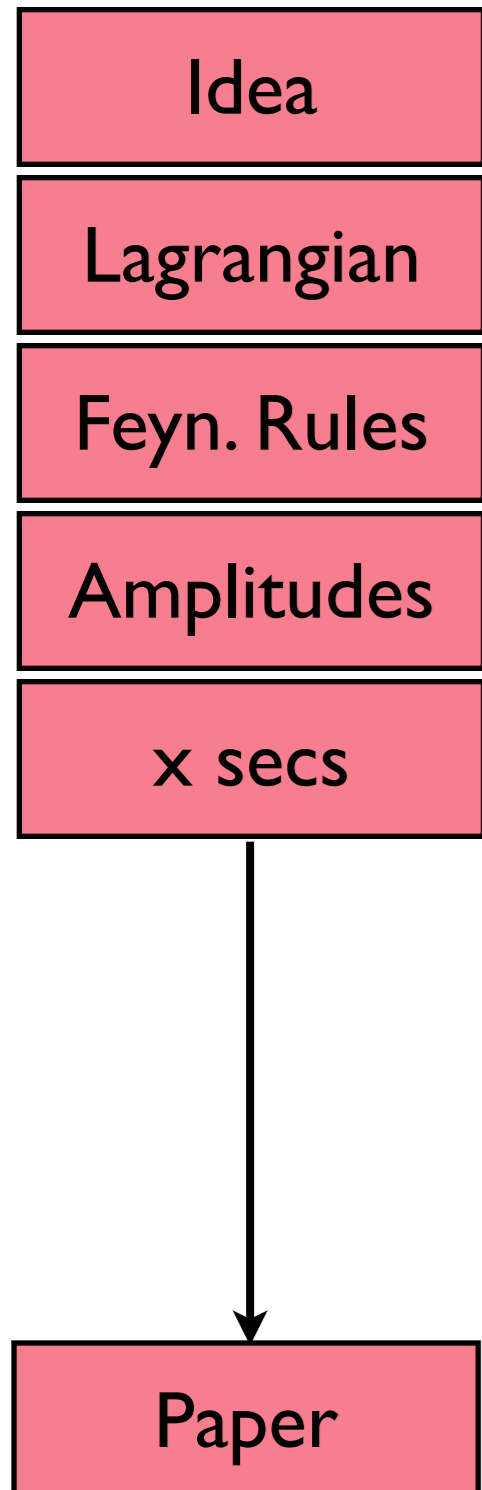
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PHENO

Idea

Lagrangian

Feyn. Rules

Amplitudes

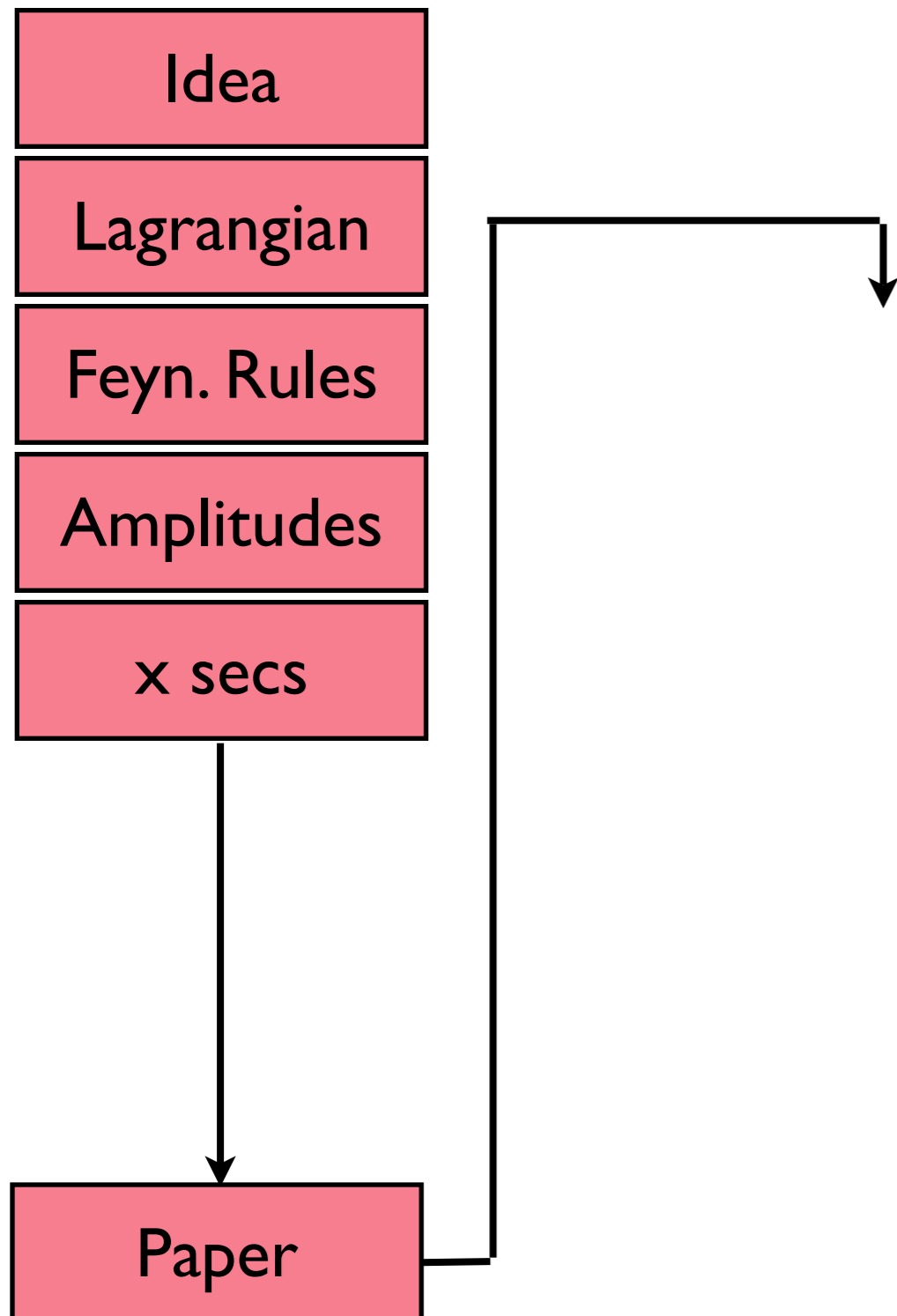
x secs

Paper

A Roadmap (with roadblocks) for BSM @ the LHC

TH

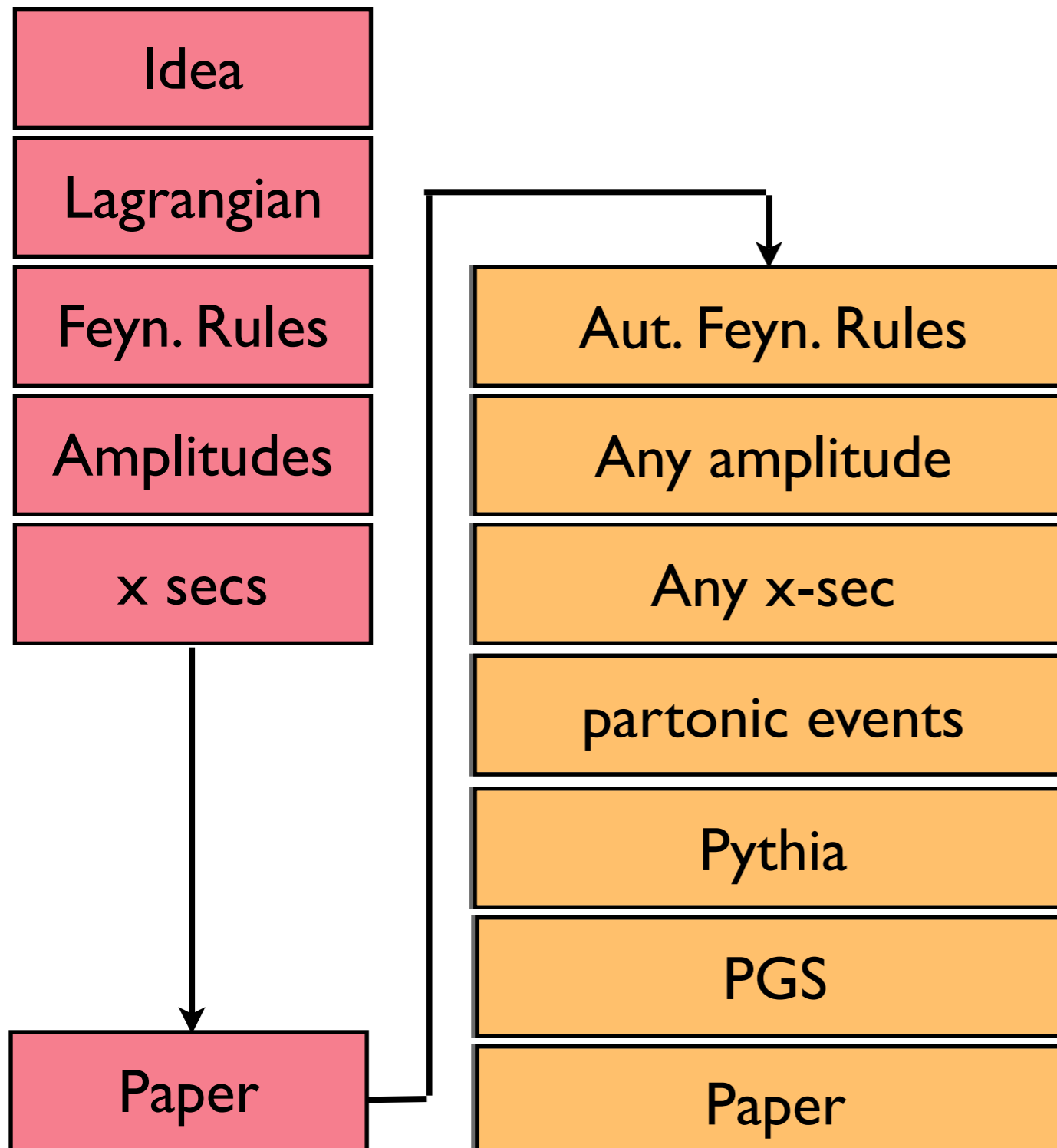
PHENO



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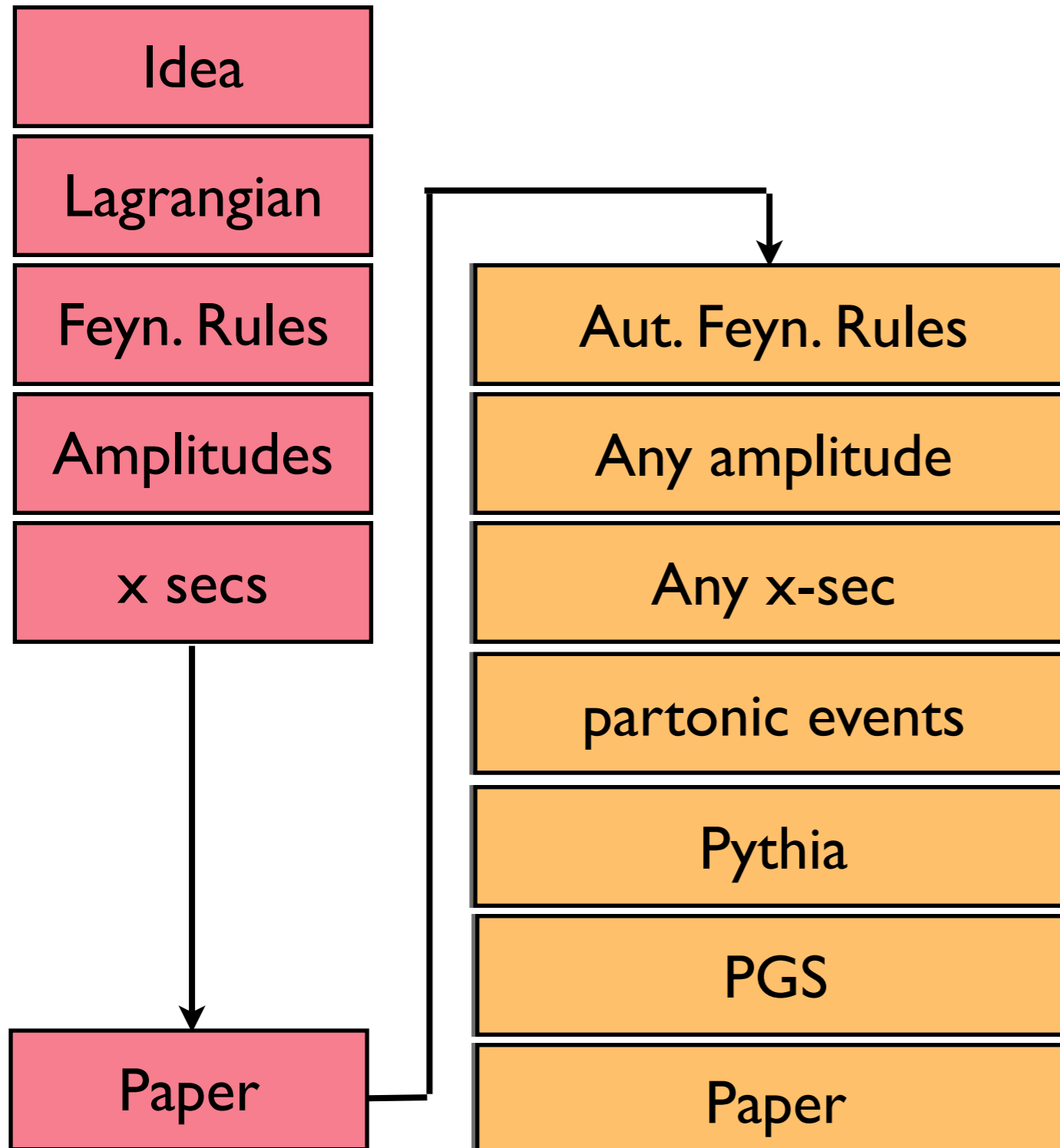


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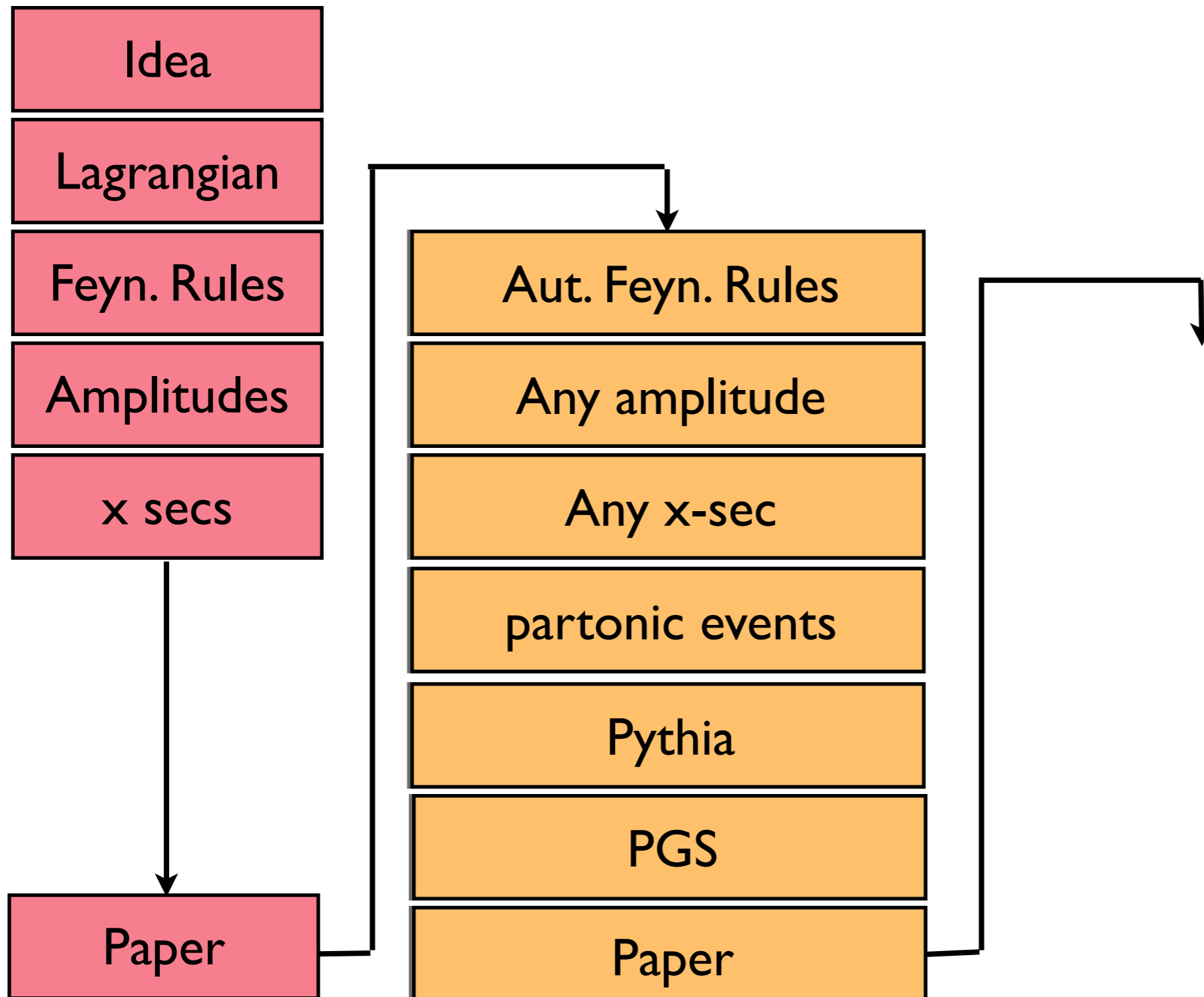


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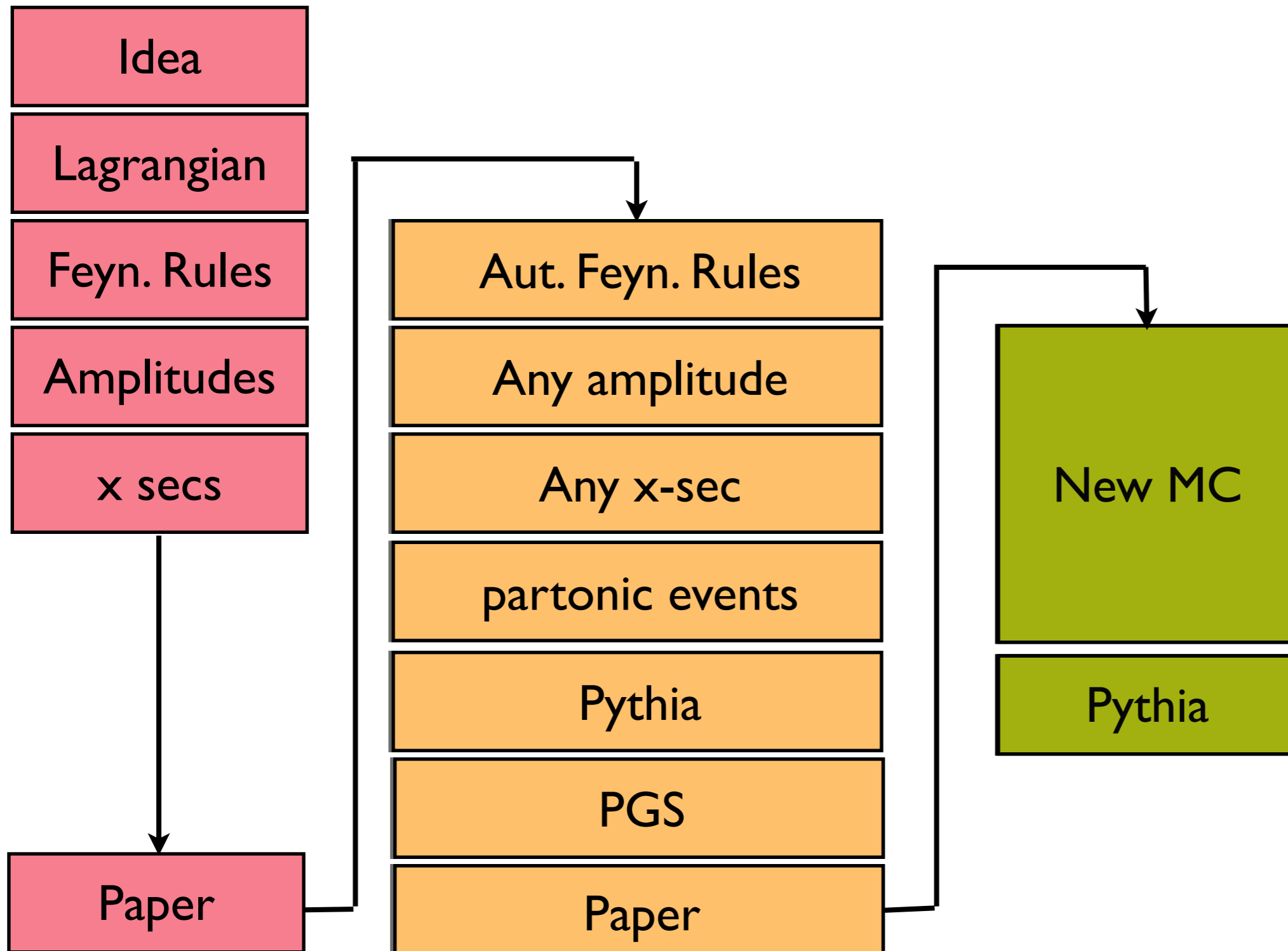


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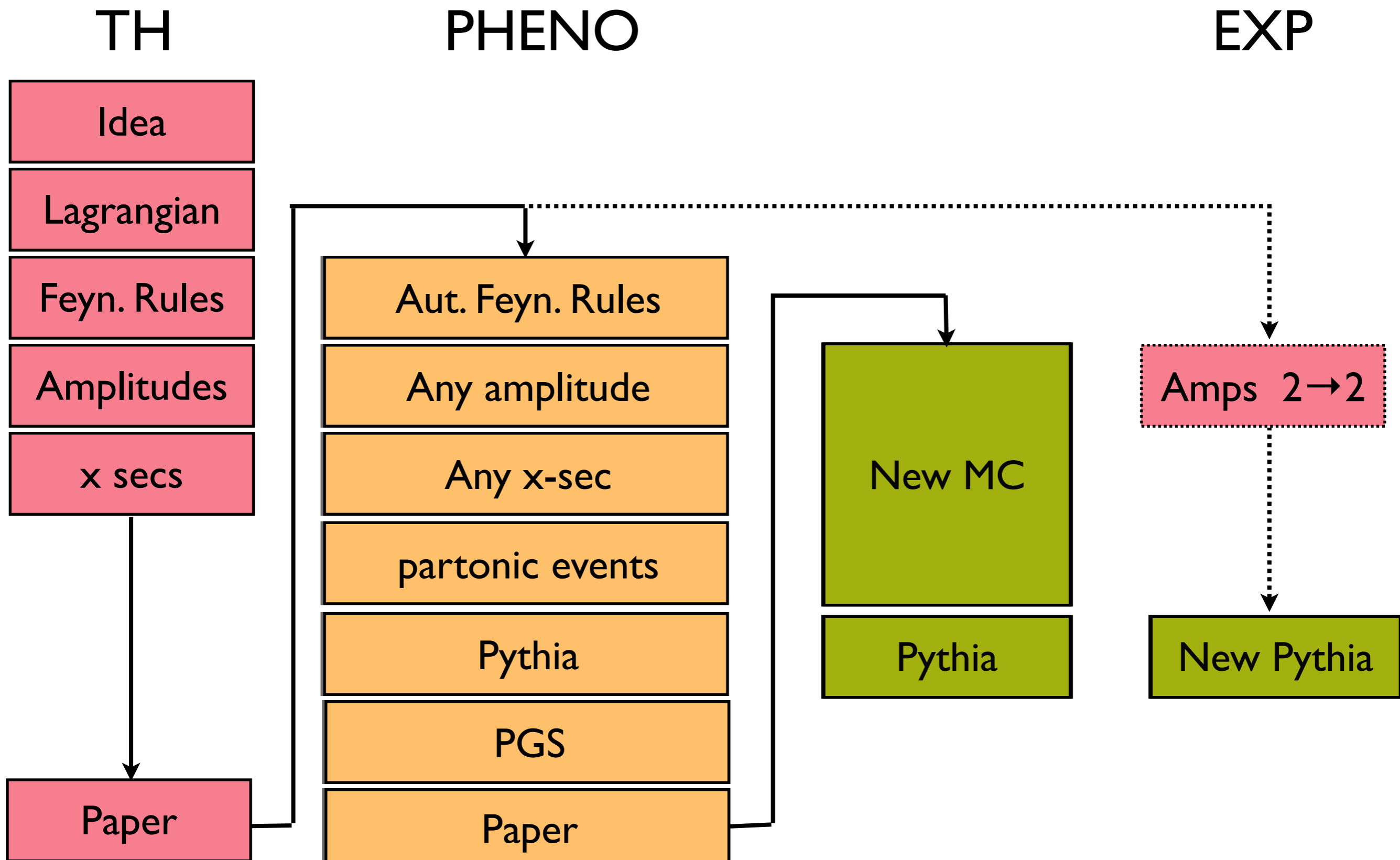
TH

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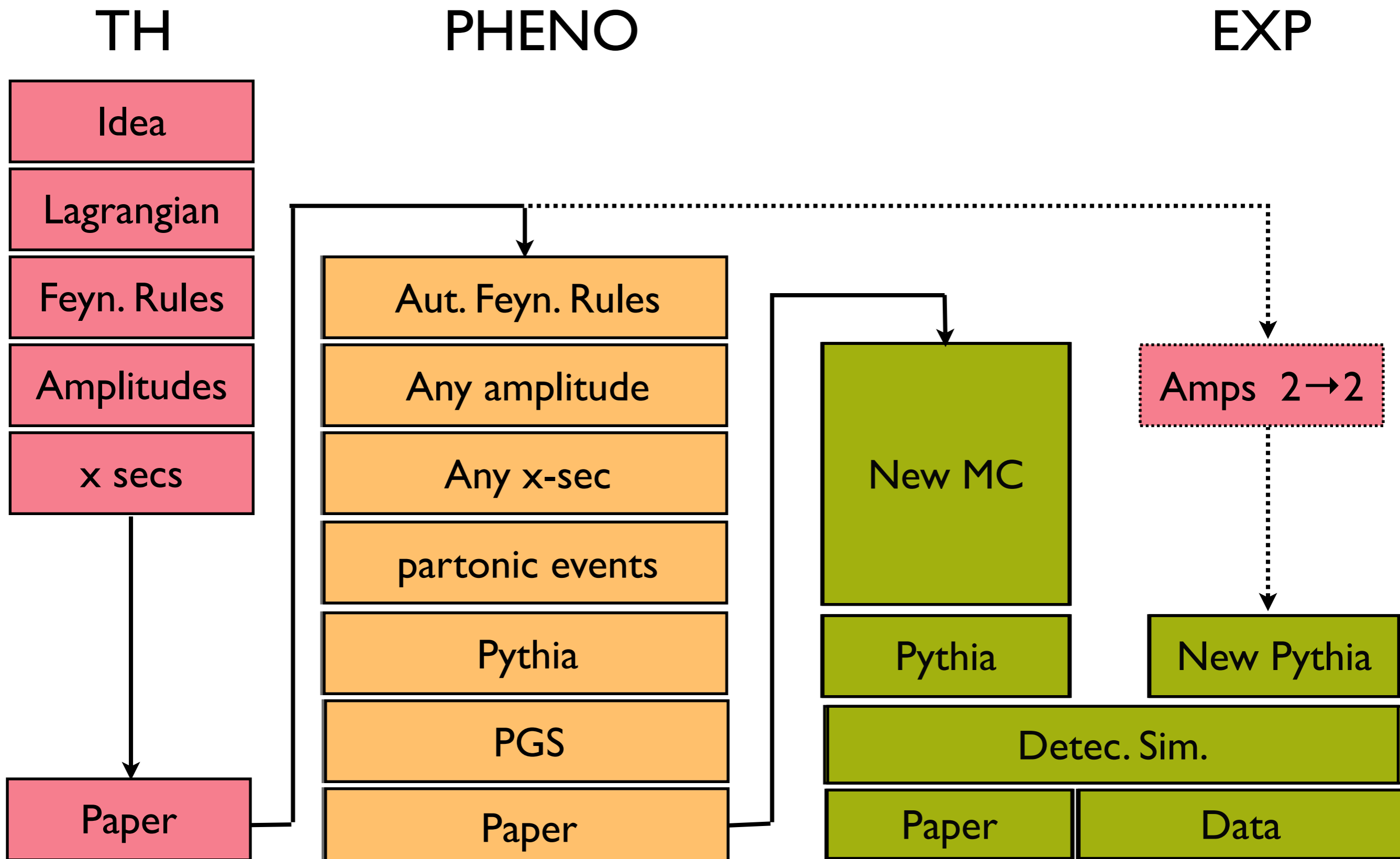
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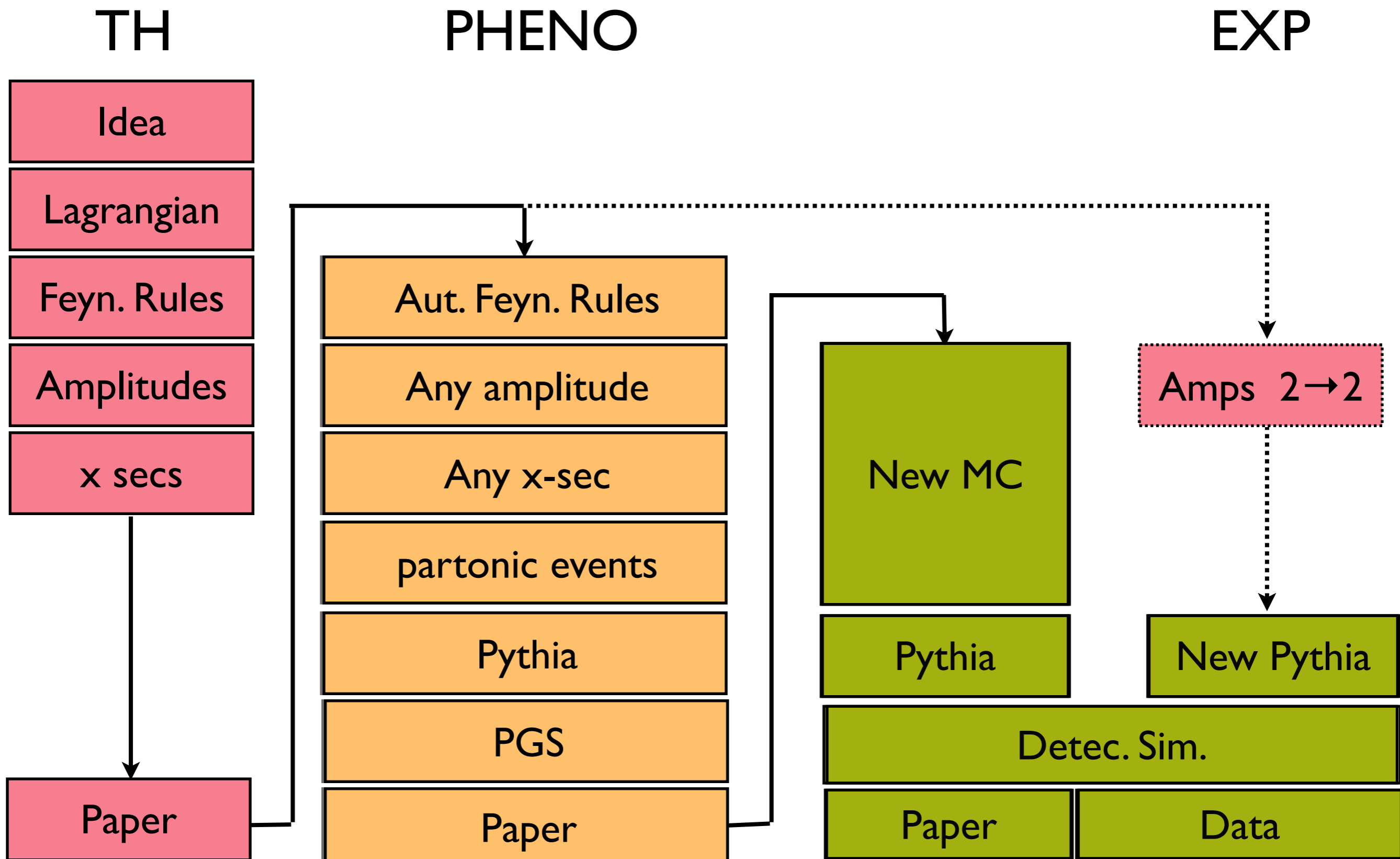




A Roadmap (with roadblocks) for BSM @ the LHC

- Workload is tripled!
- Long delays due to localized expertises and error prone. Painful validations are necessary at each step.
- It leads to a proliferation of private MC tools/ sample productions impossible to maintain, document and reproduce on the mid- and long-term.
- Just publications is a very inefficient way of communicating between TH/PHENO/EXP.

A Roadmap (with roadblocks) for BSM @ the LHC



A Roadmap (with roadblocks) for BSM @ the LHC

TH

Idea

Lagrangian

PHENO

Aut. Feyn. Rules

Any amplitude

Any x-sec

partonic events

EXP

Pythia

Detec. Sim.

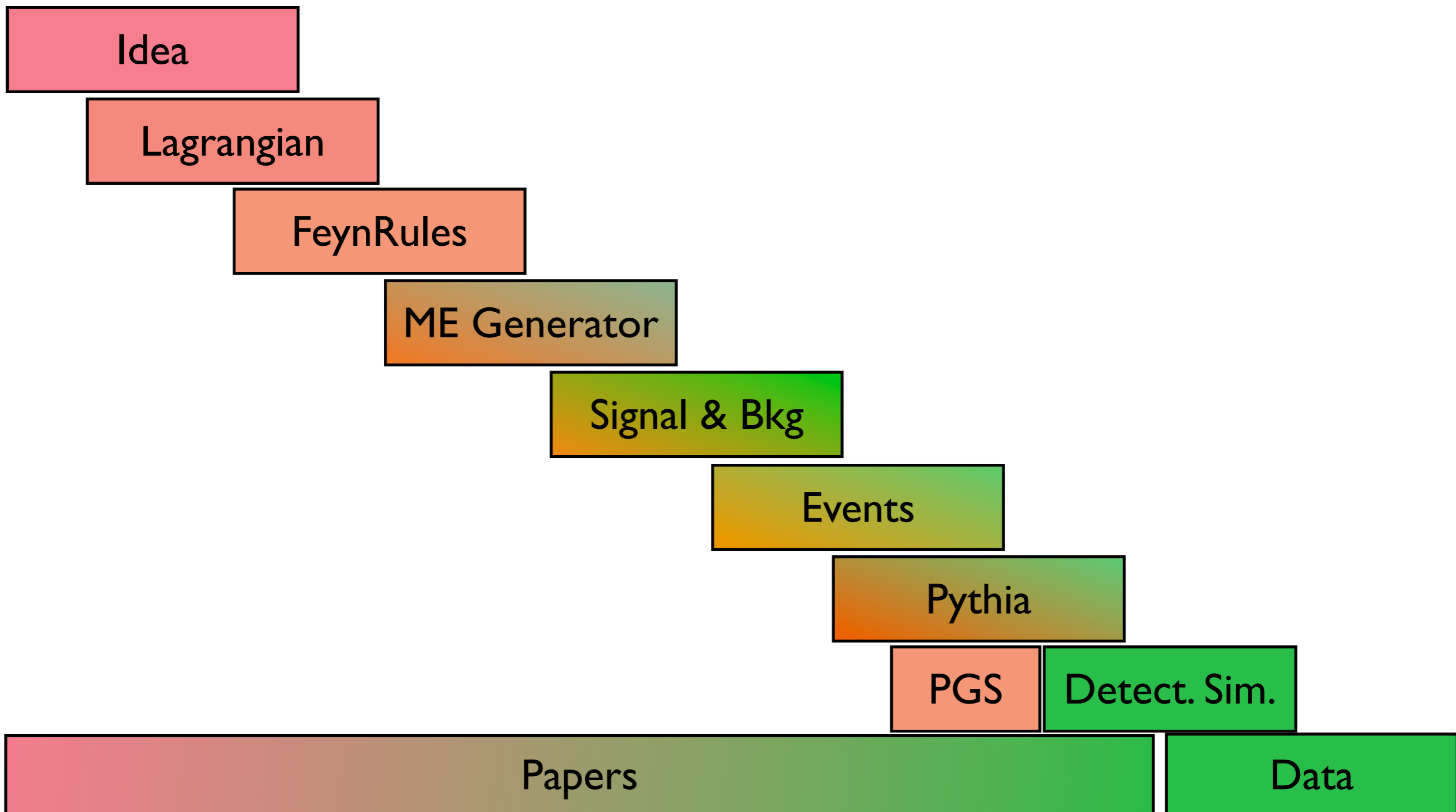
Data



A Roadmap for BSM @ the LHC

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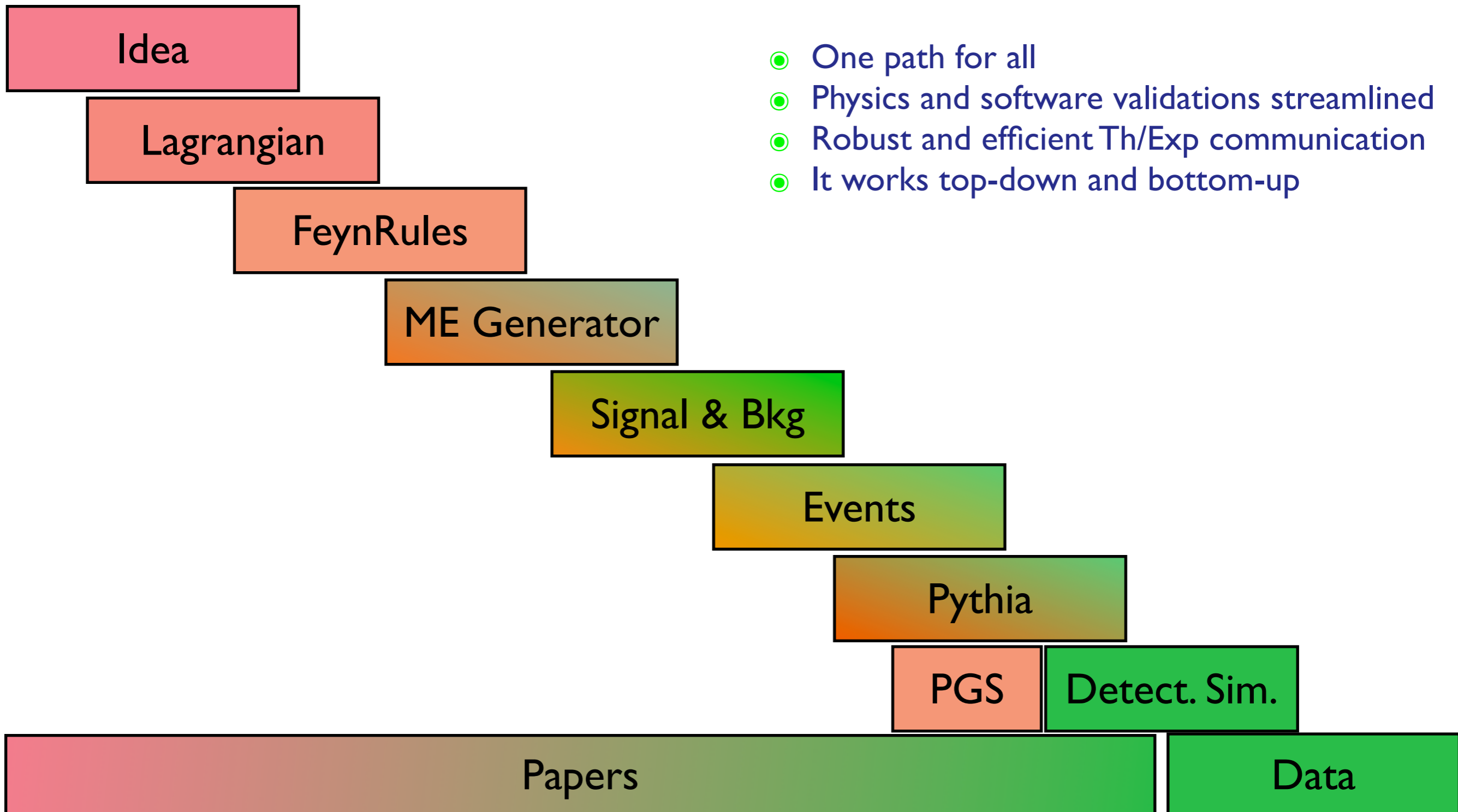




A Roadmap for BSM @ the LHC

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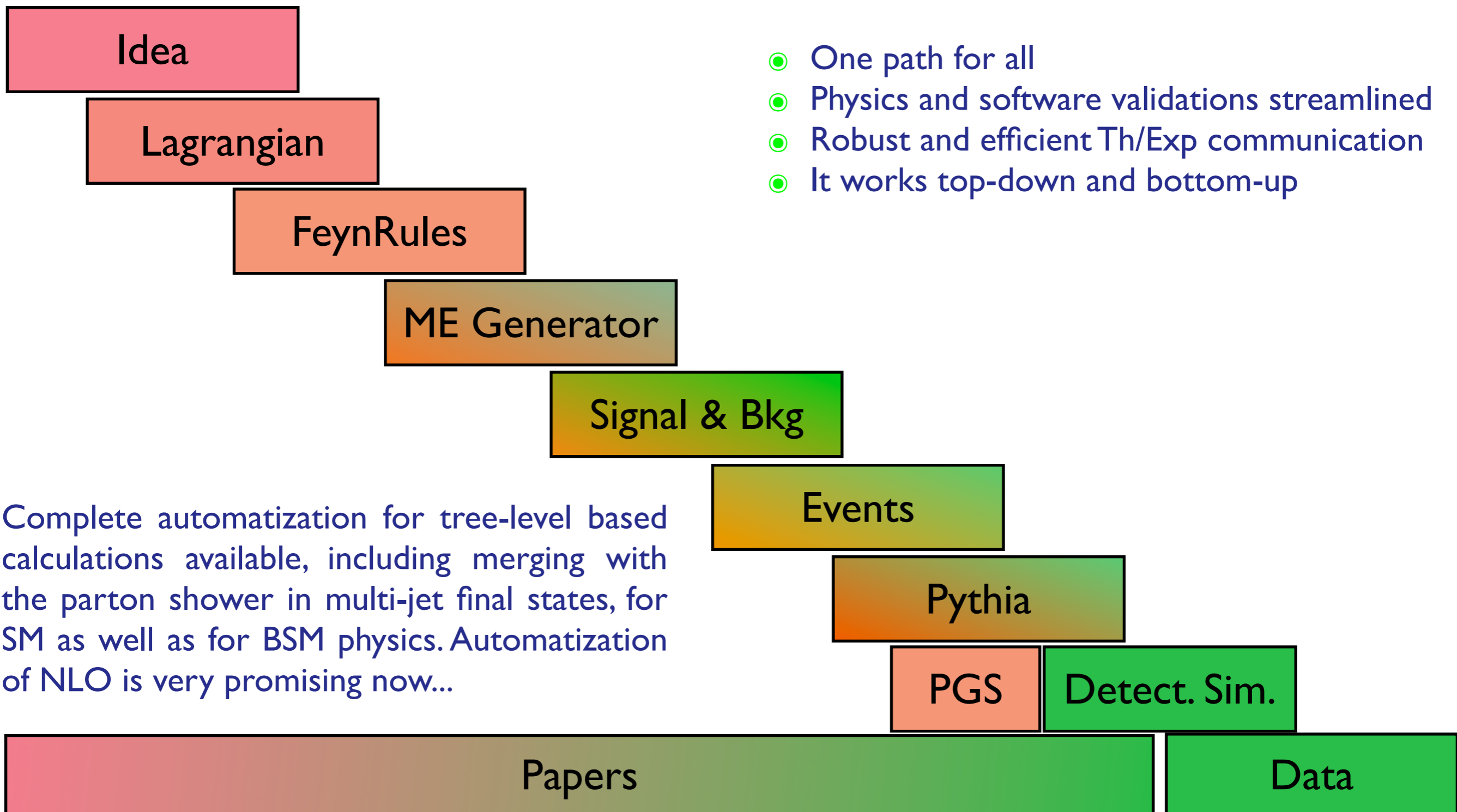
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- Physics and software validations streamlined
- Robust and efficient Th/Exp communication
- It works top-down and bottom-up



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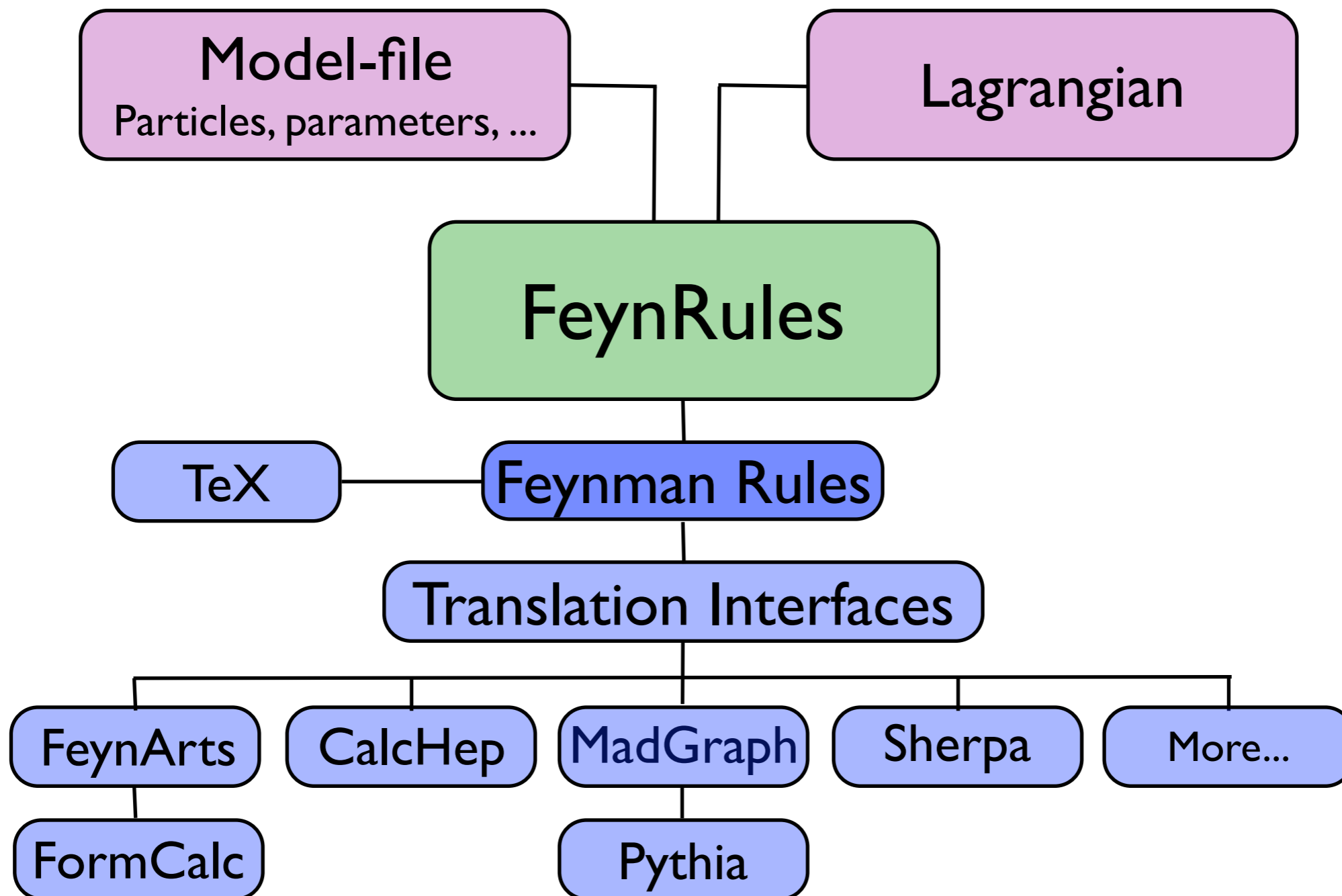
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Complete automatization for tree-level based calculations available, including merging with the parton shower in multi-jet final states, for SM as well as for BSM physics. Automatization of NLO is very promising now...



The FeynRules Project

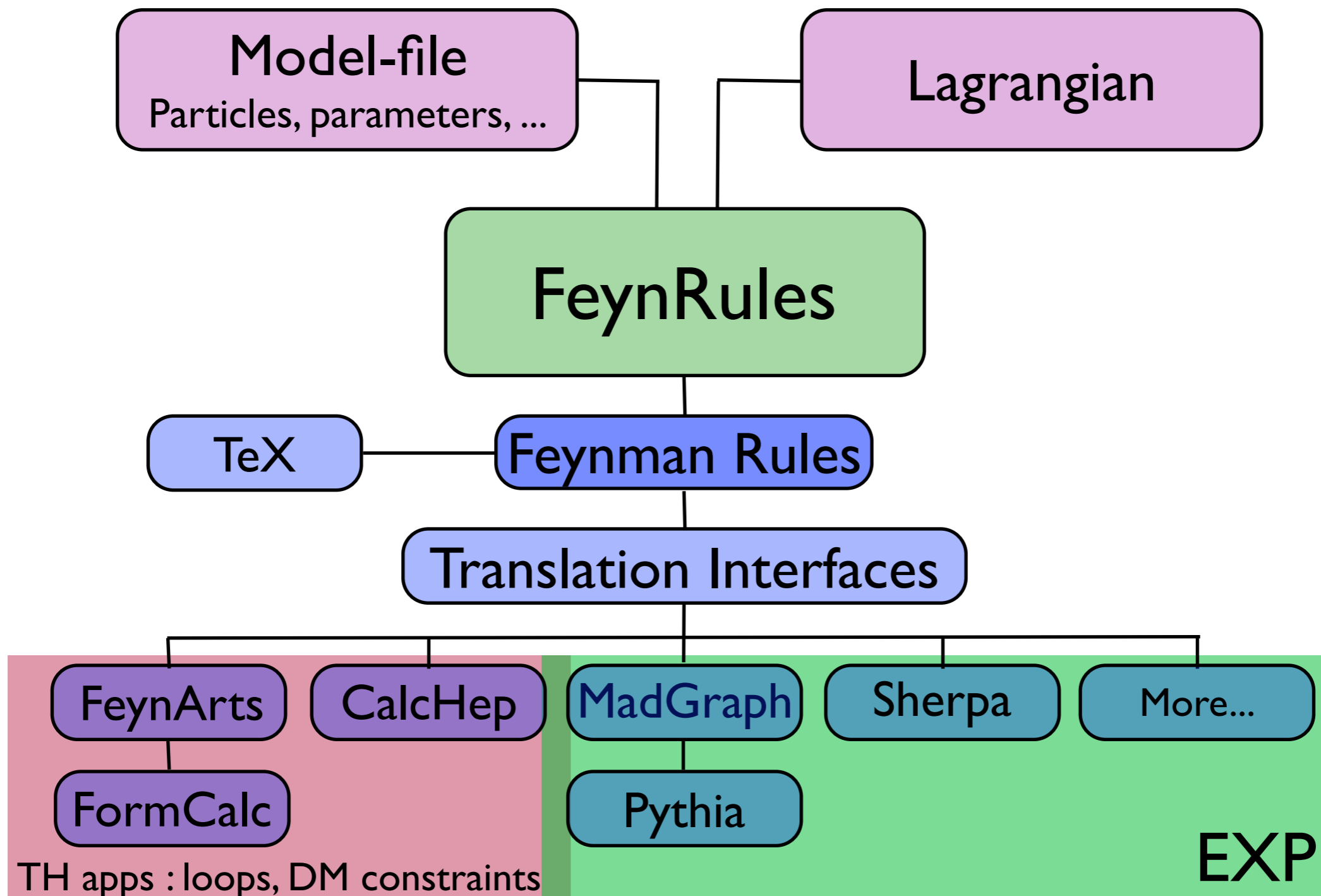
[Christensen, Duhr, 2008; Christensen, et al.2009]





The FeynRules Project

[Christensen, Duhr, 2008; Christensen, et al.2009]



Conclusions

- The need for better description and more reliable predictions for SM processes for the LHC has motivated a significant increase of theoretical and phenomenological activity in the last years, leading to several important achievements.
- A new generation of tools and techniques has been is available. Among the most useful is the matching between fixed-order and parton-shower both at tree-level and at NLO.
- Fully efficient and flexible BSM simulation chain being completed. Same level of sophistication as SM processes attained.
- Shift in paradigm: useful TH predictions in the form of tools that can be used by EXP's. Communication and collaboration between THs & EXPs easier \Rightarrow emergence of an integrated LHC community.