GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN
Ciemat
Centro de Investigaciones Energéticas, Medioambientales y Tecnológicas

RENAFAE
CERN

Basics of QCD for the LHC

Lecture IV

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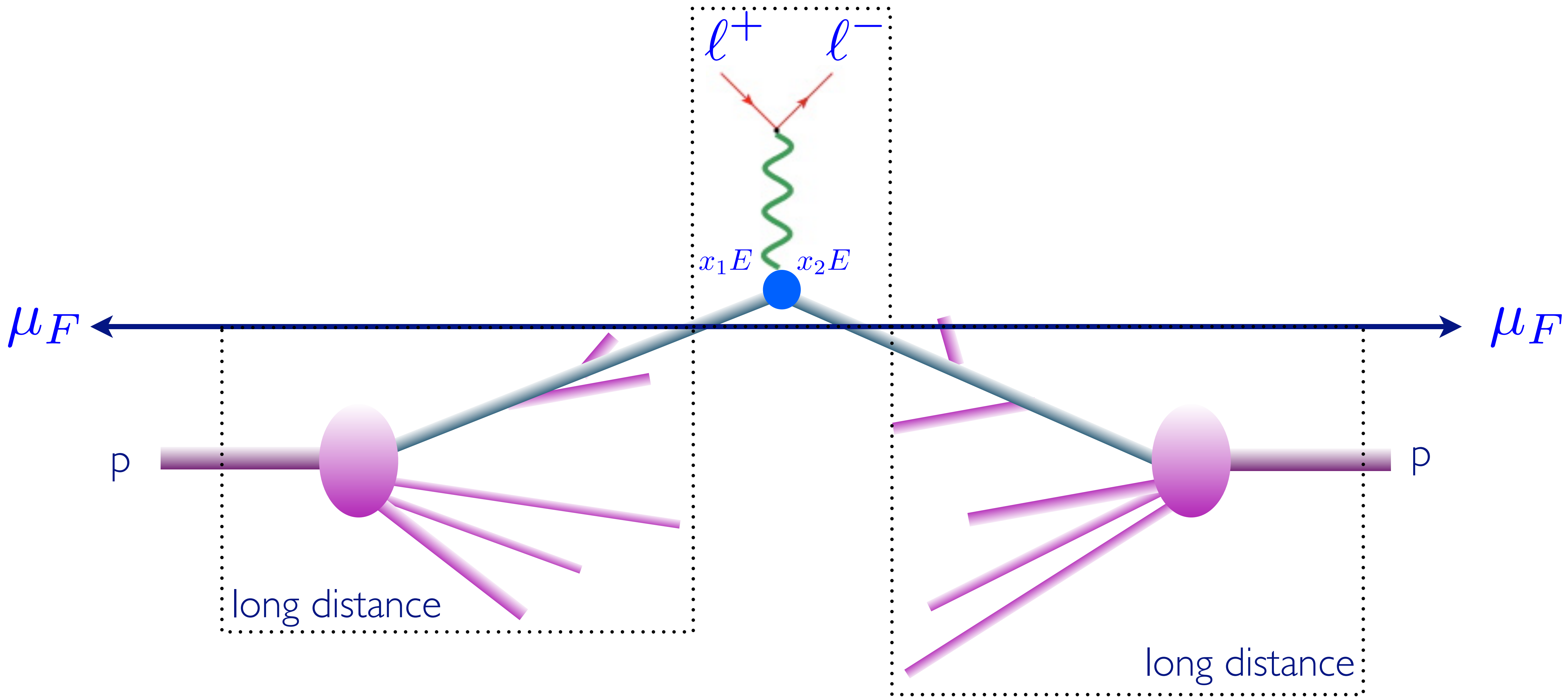
Lectures

1. Intro and QCD fundamentals
2. QCD in the final state
3. QCD in the initial state
4. From accurate QCD to useful QCD

LHC master formula



$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

How do we make predictions?

1. Fixed order computations: from LO to NNLO — TH-Accurate
2. Parton showers and fully exclusive simulations — EXP-Useful

In practice we use public codes, which are often very-loosely called Monte Carlo's, that implement various results/approaches. In general the predictions of NLO and NNLO calculations are given in terms of **distributions** of infrared safe observables (histograms), while proper Monte Carlo Generators give out **events**. Keep this difference in mind!

LHC master formula

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

Two ingredients necessary:

1. Parton Distribution functions (from exp, but evolution from th).

2. Short distance coefficients as an expansion in α_S (from th).

$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Leading order

Next-to-leading order

Next-to-next-to-leading order

Predictions at LO

How do we calculate a LO cross section for 3 jets at the LHC?

I. Identify all subprocesses (gg→ggg, qg→qgg....) in:

$$\sigma(pp \to 3j) = \sum_{ijk} \int f_i(x_1) f_j(x_2) \hat{\sigma}(ij \to k_1 k_2 k_3)$$

easy

II. For each one, calculate the amplitude:

$$\mathcal{A}(\{p\},\{h\},\{c\}) = \sum_i D_i$$

difficult

III. Square the amplitude, sum over spins & color, integrate over the phase space (D ~ 3n)

$$\hat{\sigma} = \frac{1}{2\hat{s}} \int d\Phi_p \sum_{h,c} |\mathcal{A}|^2$$

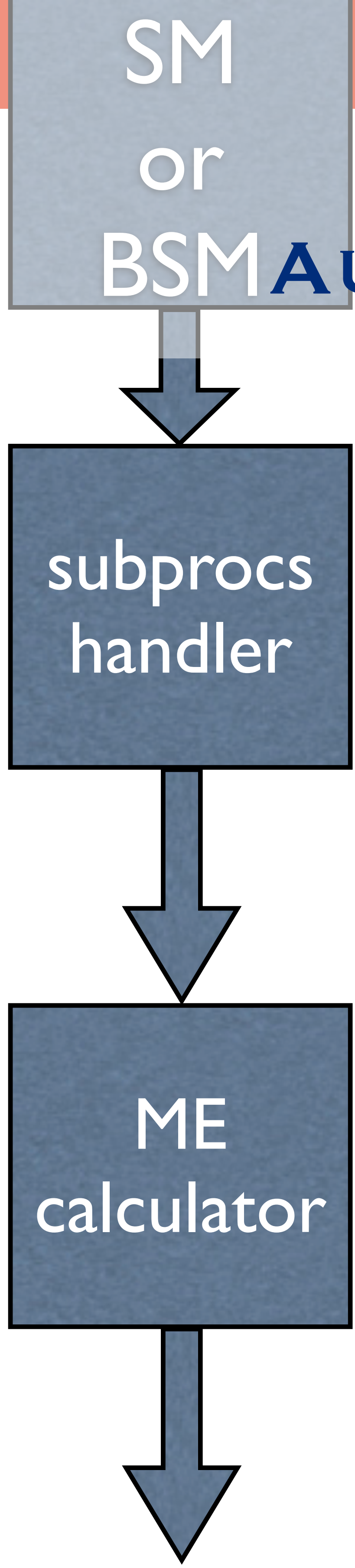
quite hard

How difficult is to calculate $|A|^2$ for arbitrary processes?

Problem of generating the matrix elements for any process of interest has been solved in full generality and it has been automatized!

More than that, also the integration over phase of such matrix elements can be achieved in an automatic way (non-trivial problem not discussed here)!

Several public tools exist:
CompHEP/CalcHEP/MadGraph/SHERPA/Whizard/....



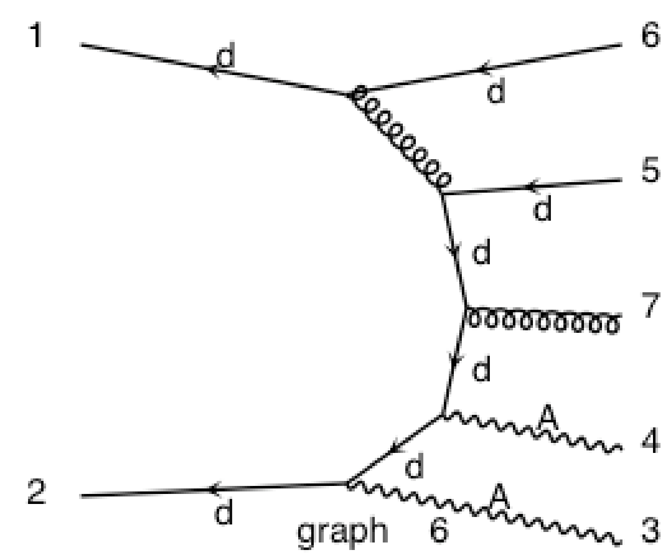
Automatic LO cross sections

Includes all possible subprocess leading to a given multi-jet final state automatically or manually (done once for all)

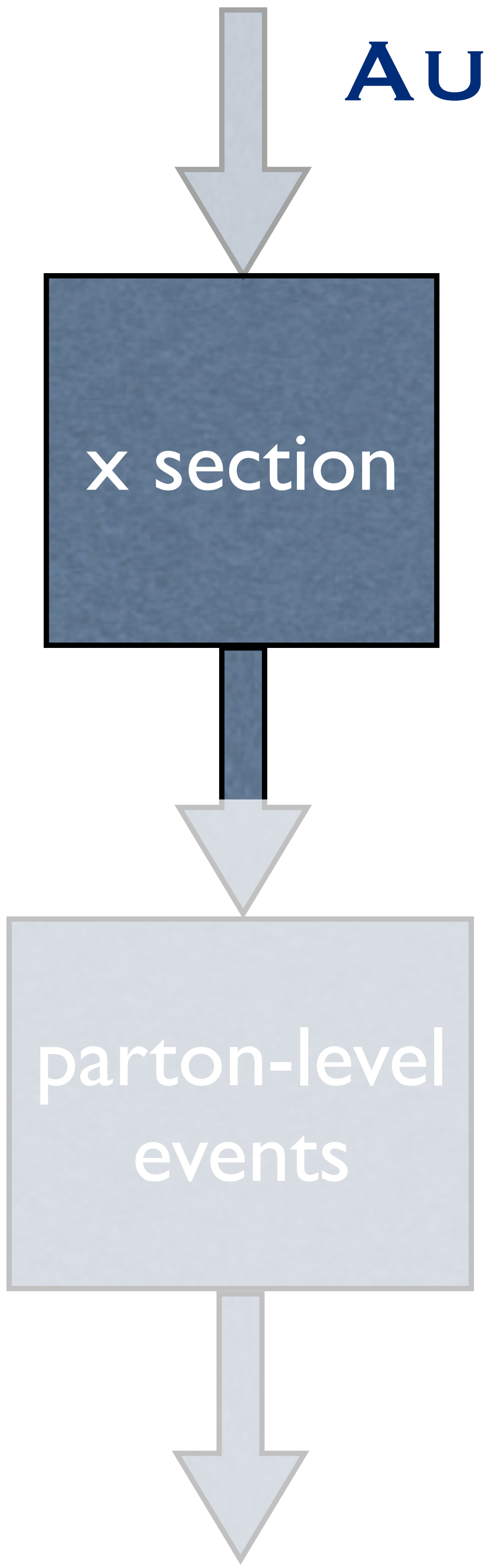
d~ d -> a a u u~ g
d~ d -> a a c c~ g
s~ s -> a a u u~ g
s~ s -> a a c c~ g

"Automatically" generates a code to calculate |M|^2 for arbitrary processes with many partons in the final state.

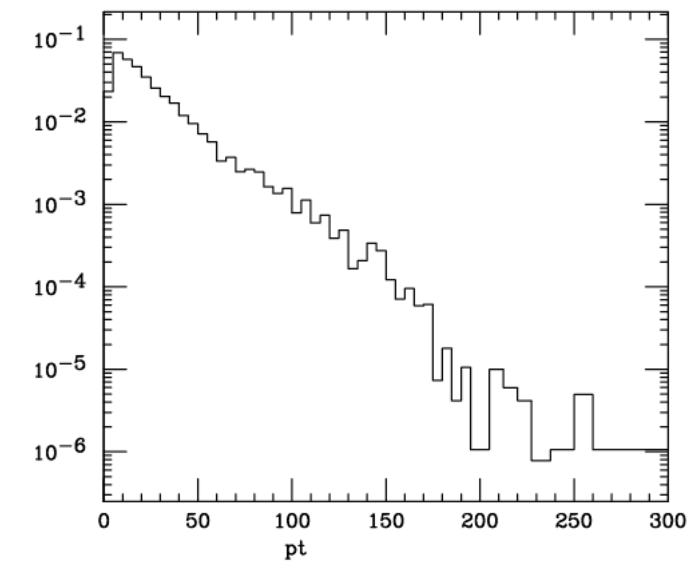
Most use Feynman diagrams w/ tricks to reduce the factorial growth, others have recursive relations to reduce the complexity to exponential. ☺



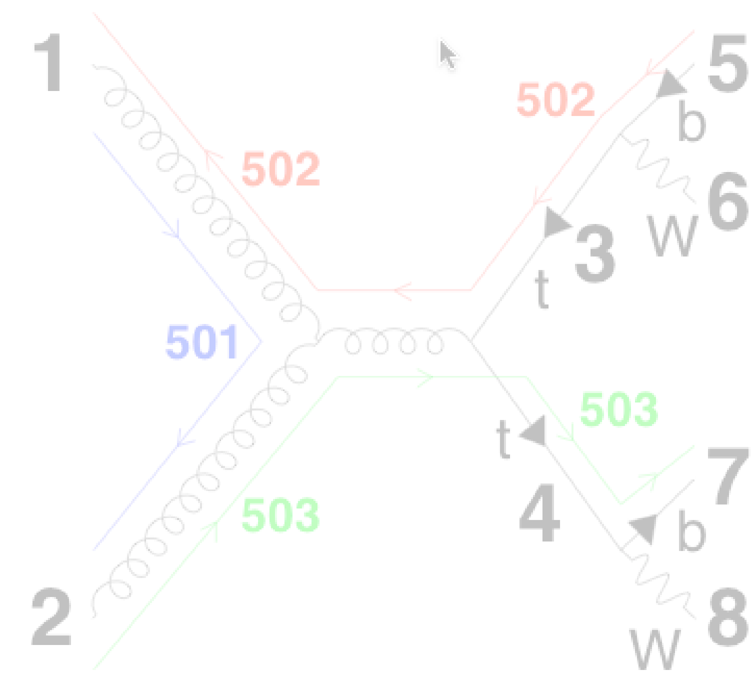
Automatic LO cross sections



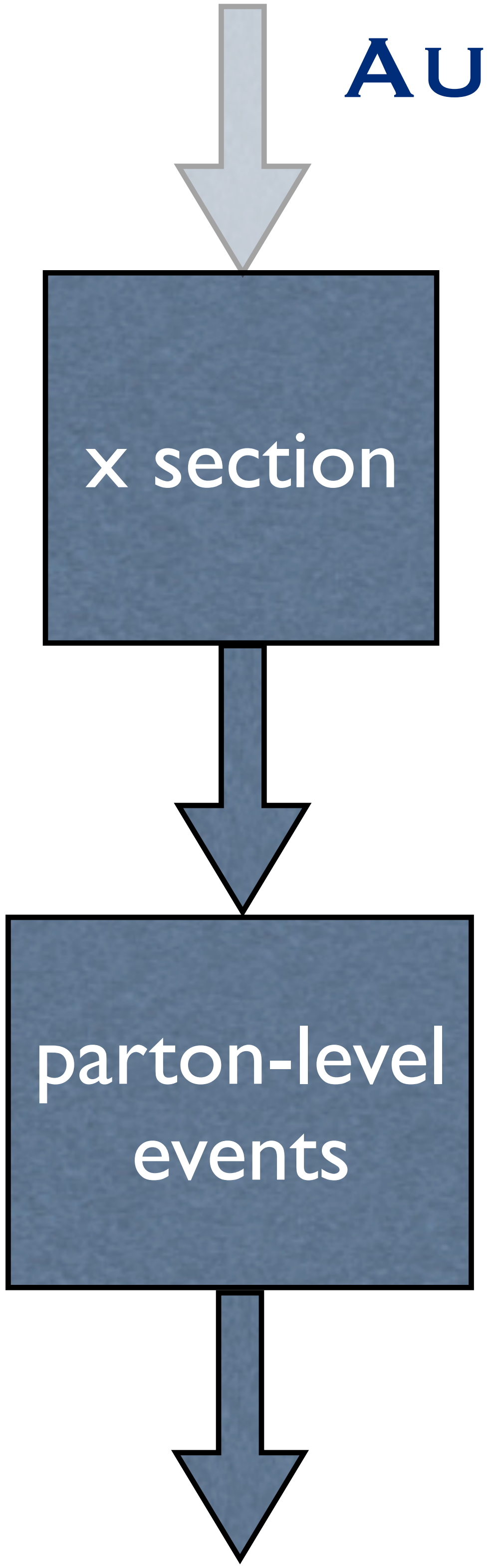
Integrate the matrix element over the phase space using a multi-channel technique and using parton-level cuts.



Events are obtained by unweighting. These are at the parton-level. Information on particle id, momenta, spin, color is given in the Les Houches format.



Automatic LO cross sections



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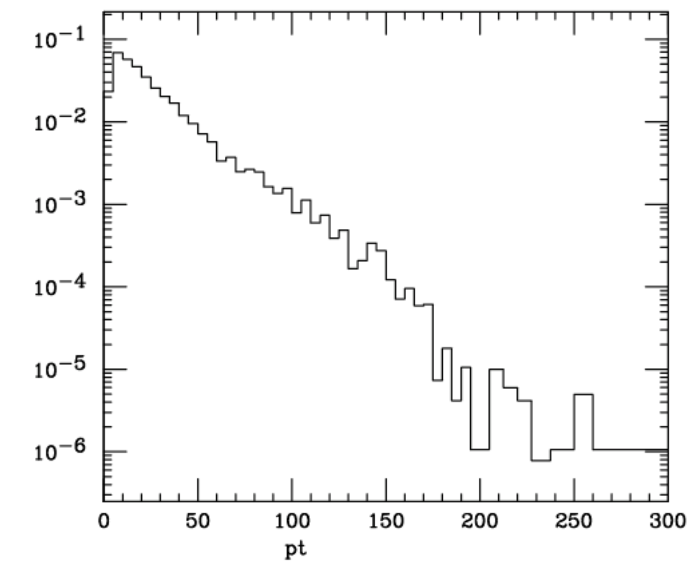

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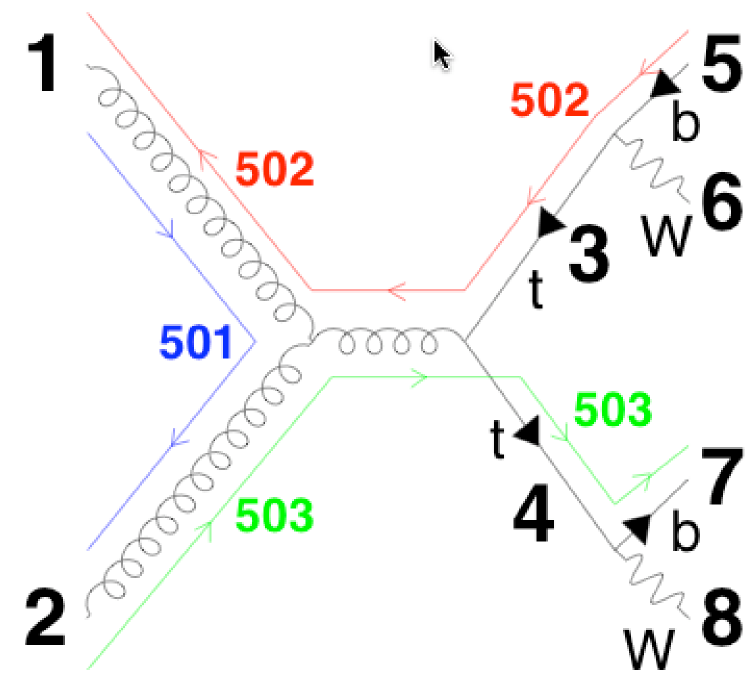


LO predictions : final remarks

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1,x_2,\alpha_S(\mu_R^2),\frac{Q^2}{\mu_F^2},\frac{Q^2}{\mu_R^2})$$

- By calculating the short distance coefficient at tree-level we obtain the first estimate of rates for inclusive final states.

- Even at LO **extra radiation is included:** it is described by the PDF's in the initial state and by the definition of a final state parton, which at LO represents all possible final state evolutions.

- Due to the above approximations a cross section at LO can strongly depend on the factorization and renormalization scales.

- Predictions can be systematically improved, at NLO and NNLO, by including higher order corrections in the short distance and in the evolution of the PDF's.

Predictions at NLO

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

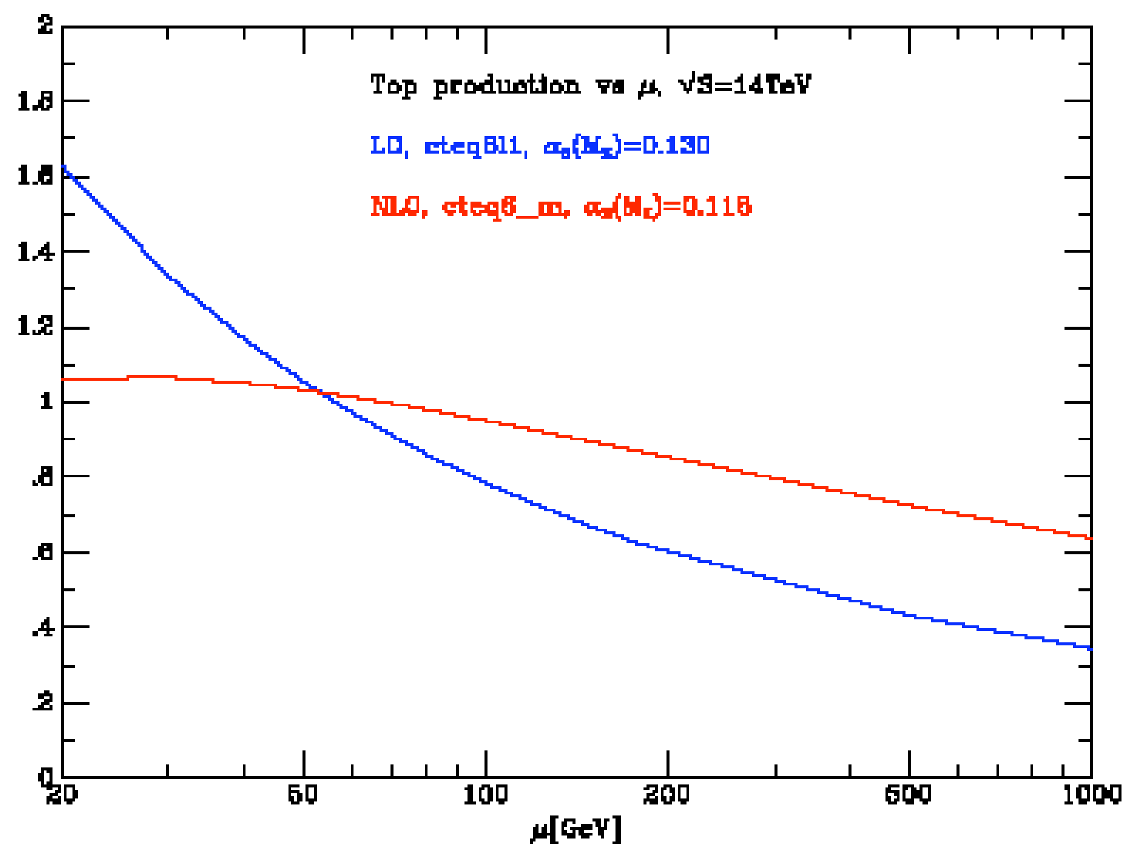
$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Why?

1. First order where scale dependences are compensated by the running of α_S and the evolution of the PDF's: FIRST RELIABLE ESTIMATE OF THE TOTAL CROSS SECTION.

2. The impact of extra radiation is included. For example, jets now have a structure.

3. New effects coming up from higher order terms (e.g., opening up of new production channels or phase space dimensions) can be evaluated.

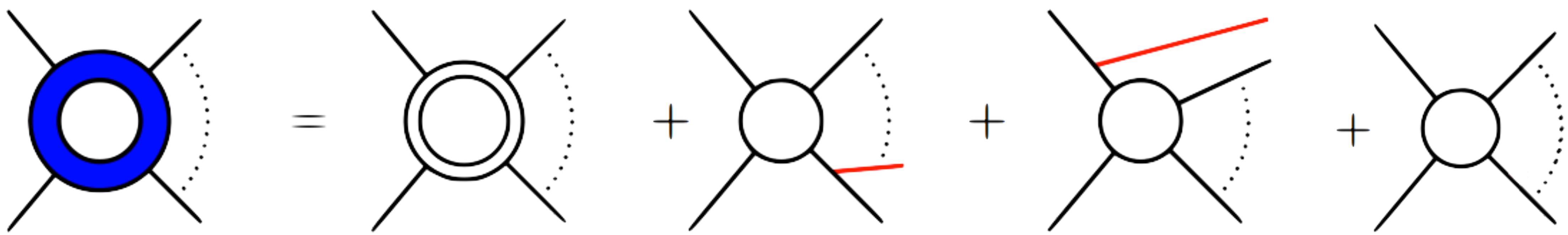


Predictions at NLO

How?

1. Get the "ingredients"
2. "Method" to combine them to calculate infrared observables

Ingredients:

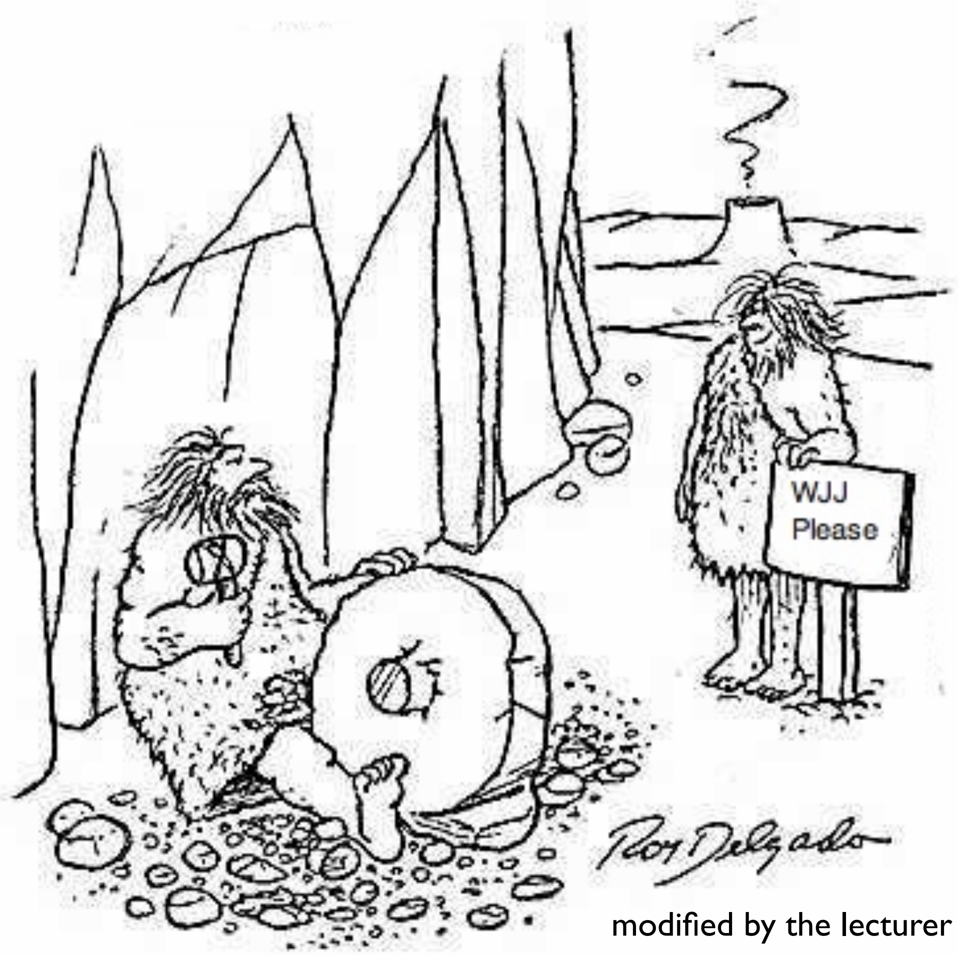


Virtual part   Real emission part   Born

$$\int \mathrm{d}\sigma^{(\mathrm{NLO})} O(\Phi) = \int \mathrm{d}\Phi_B V(\Phi_B) O(\Phi_B) + \int \mathrm{d}\Phi_R R(\Phi_R)\; O(\Phi_R) \int \mathrm{d}\Phi_B B(\Phi_B) O(\Phi_B)$$

Loops have been for long the bottleneck of NLO computations, with their calculations taking years of manual and symbolic work to get the correct results.

Predictions at NLO



modified by the lecturer

Generalized Unitarity
(ex. BlackHat, Rocket,...)

Integrand Reduction
(ex. CutTools, Samurai)

Tensor Reduction
(ex. Golem)

Thanks to new amazing results, some of them inspired by string theory developments, now the computation of loops has been extended to high-multiplicity processes or/and automated.

t

Predictions at NLO

How?

1. Get the "ingredients"
2. "Method" to combine them to calculate infrared observables

Method: Universal subtraction

$$\int \mathrm{d}\sigma^{(\mathrm{NLO})} O(\Phi) = \int \mathrm{d}\Phi_B V(\Phi_B) O(\Phi_B) + \int \mathrm{d}\Phi_R R(\Phi_R)\, O(\Phi_R) \int \mathrm{d}\Phi_B B(\Phi_B) O(\Phi_B)$$

$$= \int \mathrm{d}\Phi_B \left[B(\Phi_B) + V(\Phi_B) + \int \mathrm{d}\Phi_{R|B} S(\Phi_R) \right] O(\Phi_B)$$

$$+ \int \mathrm{d}\Phi_R \left[R(\Phi_R)\, O(\Phi_R) - S(\Phi_R) O(\Phi_B) \right]$$

Local universal counterterms have been identified whose integral on the extra radiation variable is analytically known and that can be used to make reals and virtuals separately finite.

Predictions at NLO

MCFM: downloadable general purpose NLO code [Campbell & Ellis+ collaborators]

Final state	Notes	Reference
W/Z		
diboson (W/Z/γ)	photon fragmentation, anomalous couplings	hep-ph/9905386, arXiv:1105.0020
Wbb	massless b-quark massive b quark	hep-ph/9810489 arXiv:1011.6647
Zbb	massless b-quark	hep-ph/0006304
W/Z+1 jet		
W/Z+2 jets		hep-ph/0202176, hep-ph/0308195
Wc	massive c-quark	hep-ph/0506289
Zb	5-flavour scheme	hep-ph/0312024
Zb+jet	5-flavour scheme	hep-ph/0510362

Final state	Notes	Reference
H (gluon fusion)		
H+1 jet (g.f.)	effective coupling	
H+2 jets (g.f.)	effective coupling	hep-ph/0608194, arXiv:1001.4495
WH/ZH		
H (WBF)		hep-ph/0403194
Hb	5-flavour scheme	hep-ph/0204093
t	s- and t-channel (5F), top decay included	hep-ph/0408158
t	t-channel (4F)	arXiv:0903.0005, arXiv:0907.3933
Wt	5-flavour scheme	hep-ph/0506289
top pairs	top decay included	

- ☞ ~30 processes
- ☞ First results implemented in 1998 ...this is 13 years worth of work of several people (~4M$)
- ☞ Cross sections and parton-level distributions at NLO are provided
- ☞ One general framework. However, each process implemented by hand

Predictions at NLO

Completely automatically generated NLO codes for a variety of processes via **MadLoop+MadFKS**

Total sample cross sections at the LHC for 26 sample procs

Code generation time:
a few hours

Running time:
two weeks on a cluster

	Process	μ	n_{lf}	Cross section (pb) LO	NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

Predictions at NLO

- NLO calculations have historically presented two types of challenges: the loop calculations and the construction of a numerical code resilient to the cancellation of the divergences.
- Both issues have now basically solved in general and **many NLO calculations can now be done in an automatic way.**
- Several public codes that compute IR-safe quantities (cross sections, jet rates, ...) at the parton level are available.
- Be careful : NLO codes are NOT all event generators!!

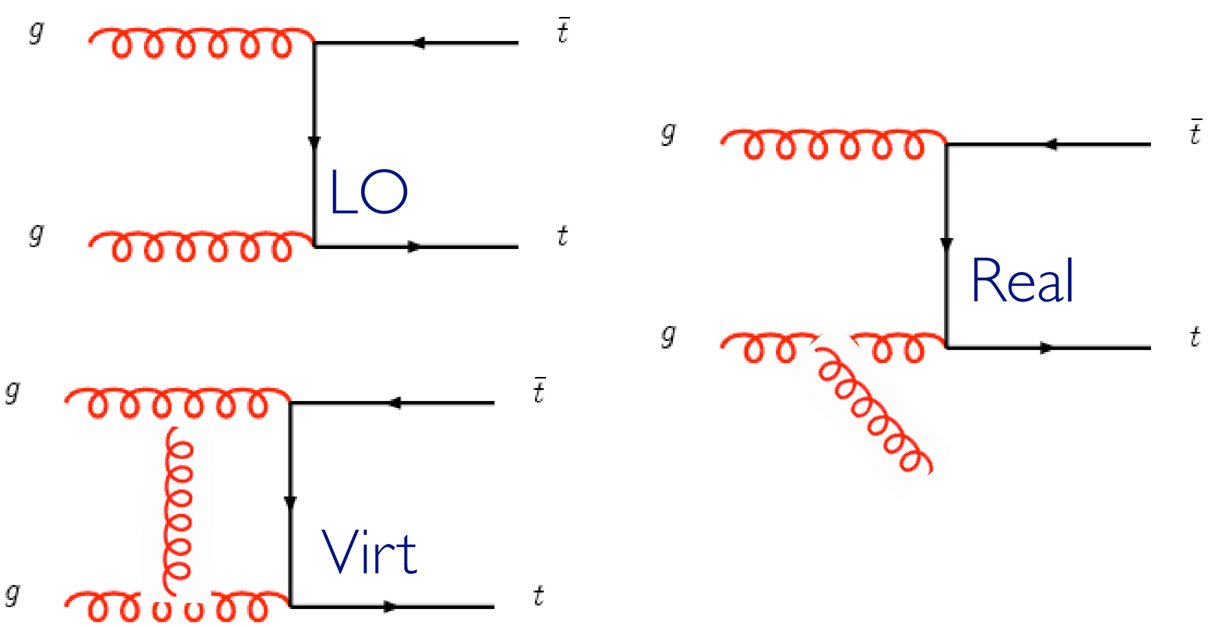
Predictions at NLO

Warning!

Calling a code "a NLO code" is an abuse of language and can be confusing.
A NLO calculation always refers to an IR-safe observable, when the genuine α_S corrections to this observable on top of the LO estimate are known.

An NLO code will, in general, be able to produce results for several quantities and distributions, only some of which will be at NLO accuracy.

Example: Suppose we use the NLO code for pp → tt



- ☞ Total cross section, σ(tt) .... ✓
- ☞ P_T >0 of one top quark.................. ✓
- ☞ P_T >0 of the tt pair .... ✗
- ☞ P_T >0 of the jet................................ ✗
- ☞ tt invariant mass, m(tt) .... ✓
- ☞ ΔΦ(tt)>0 .... ✗

Hands-on!

Experience a "simple" NLO calculation yourself

pp→Higgs+x at NLO

- LO : 1-loop calculation and HEFT
- NLO in the HEFT
  - Virtual corrections and renormalization
  - Real corrections and IS singularities
- Cross sections at the LHC

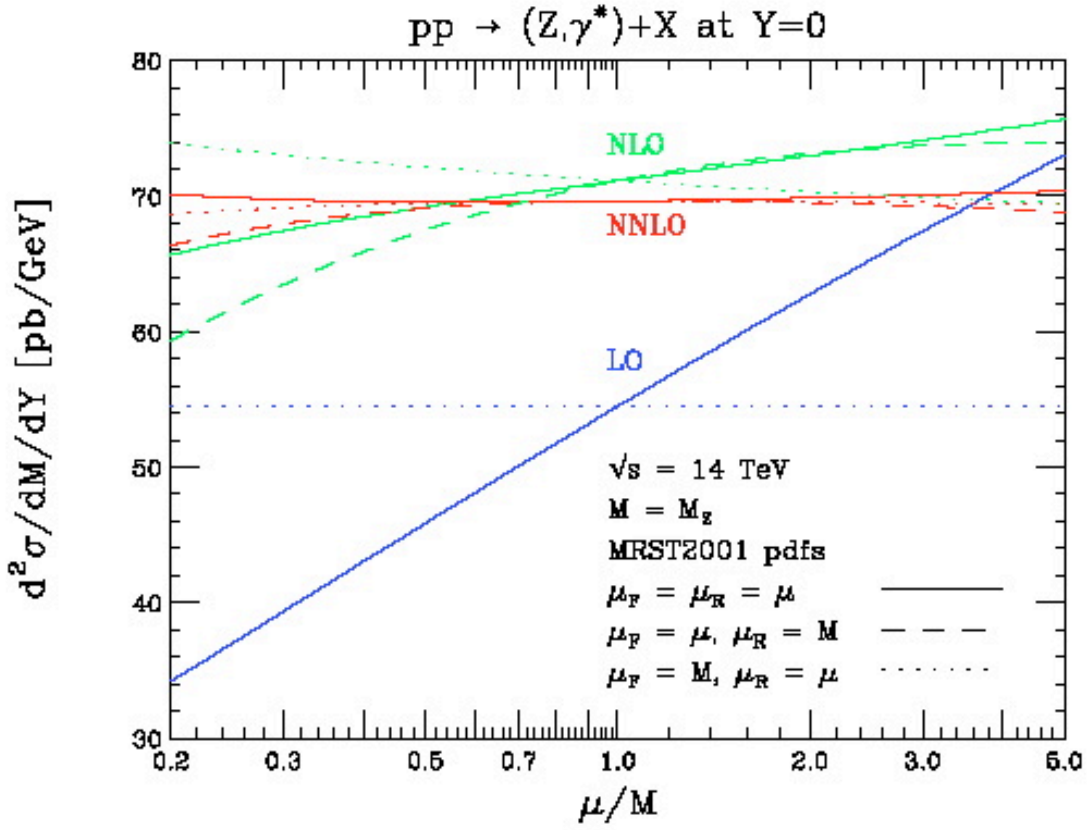
Predictions at NNLO

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1,x_2,\alpha_S(\mu_R^2),\frac{Q^2}{\mu_F^2},\frac{Q^2}{\mu_R^2})$$

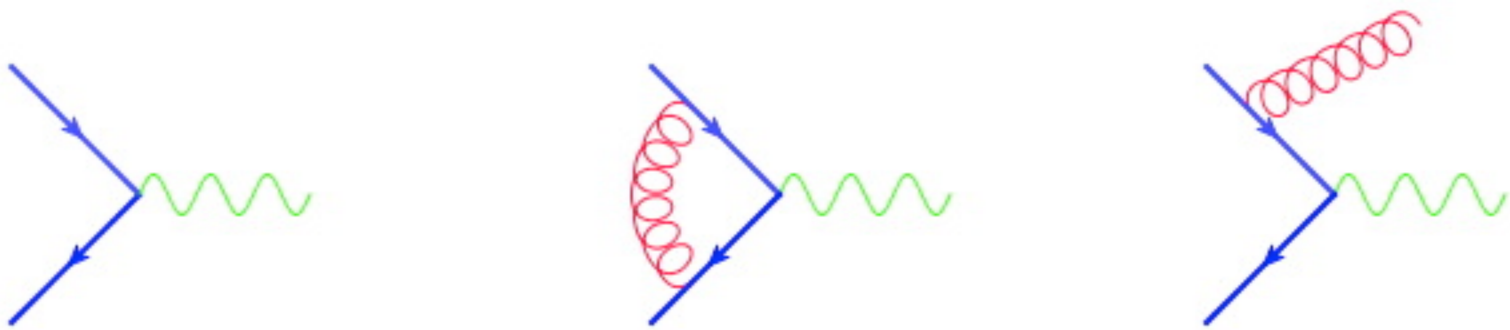
$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \ldots$$

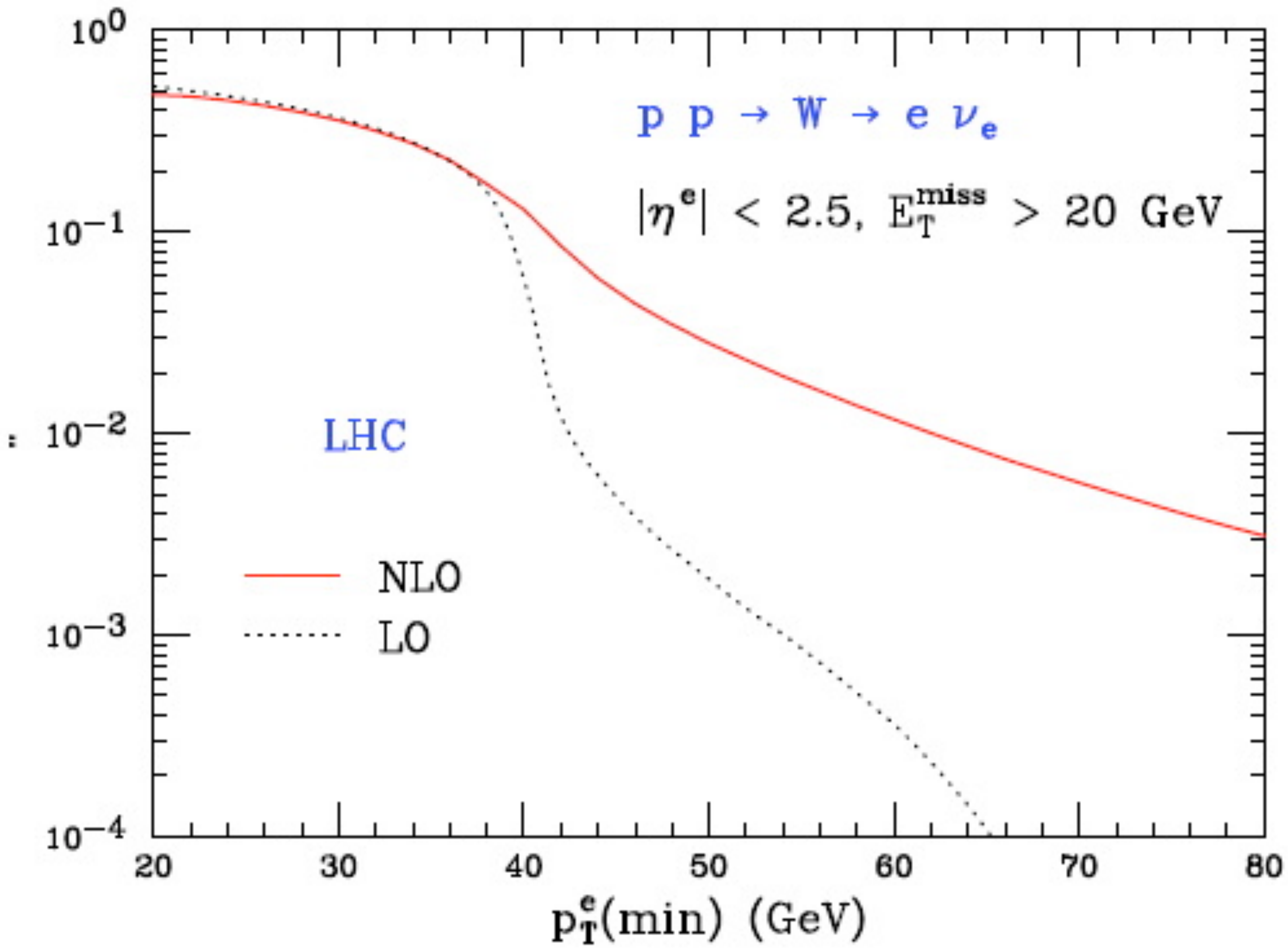
Why?

- A NNLO computation gives control on the uncertainties of a perturbative calculation.
- It's "mandatory" if NLO corrections are very large to check the behaviour of the perturbative series
- It's the best we have! It is needed for Standard Candles and for really exploiting all the available information, for example that of NNLO PDF's.



Drell-Yan predictions at NLO



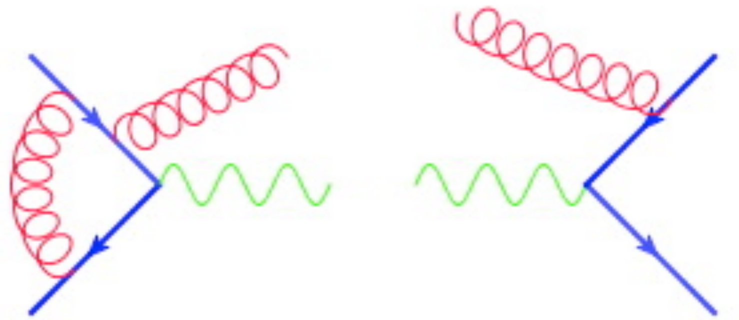


- At LO the W has no p_T, therefore the pt of the lepton has a sharp cutoff.
- The "K-factor" looks like enormous at high p_T. When this happens it means that the observable you are looking at it is actually at LO not at NLO!
- It is important to keep the spin correlations of the lepton in the calculation.

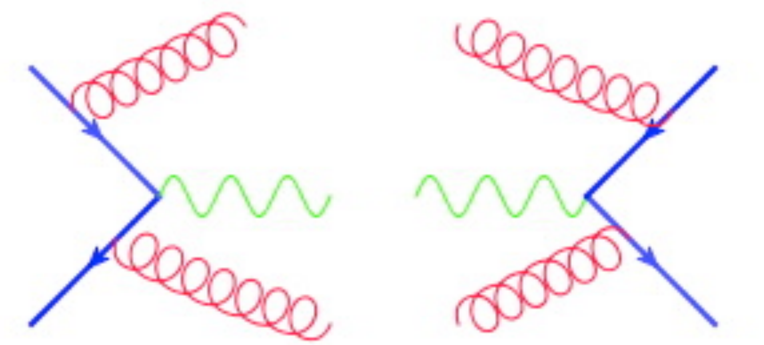
Drell-Yan predictions at NNLO

- Virtual-Virtual : O(100) terms

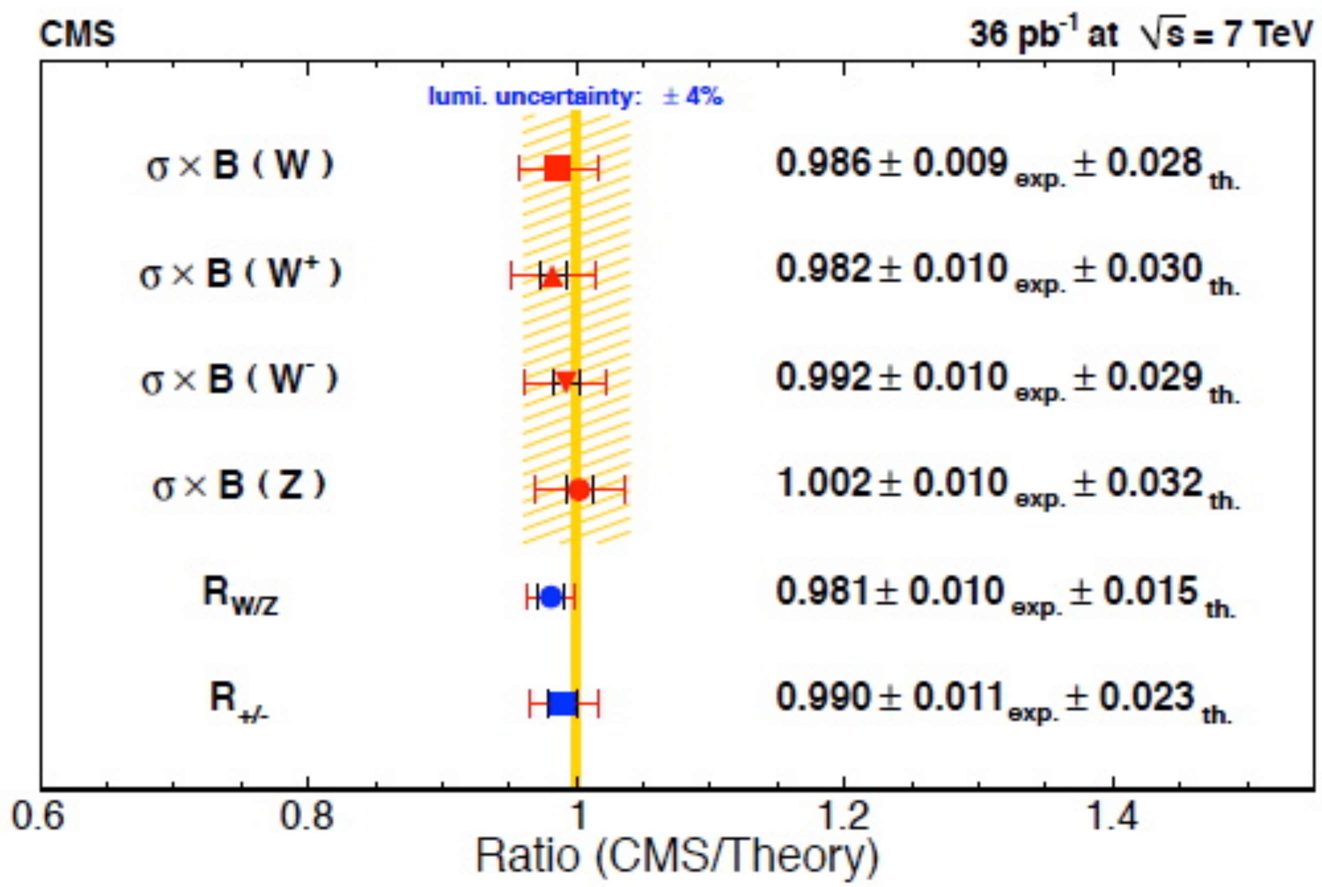
- Real-Virtual : O(300) terms



- Real-Real : O(500) terms



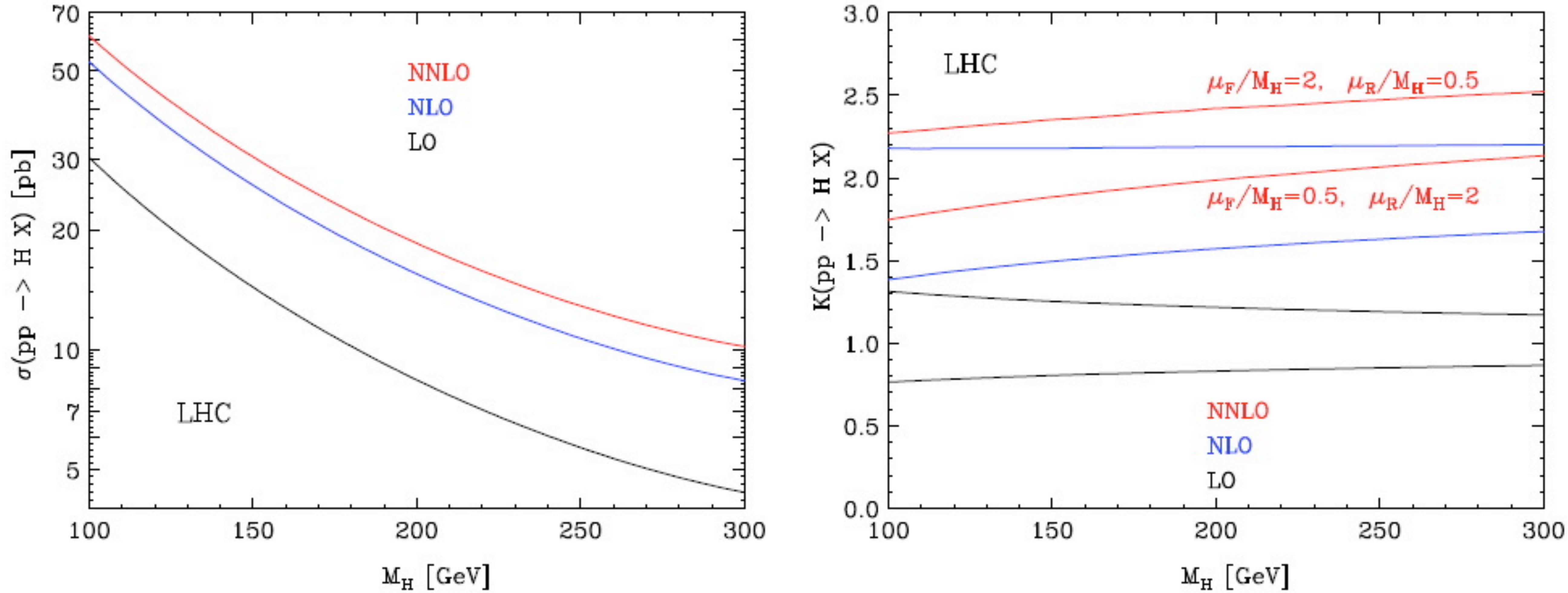
Drell-Yan predictions at NNLO



[TH = Anastasiou, Dixon, Melnikov, Petriello. 2004]

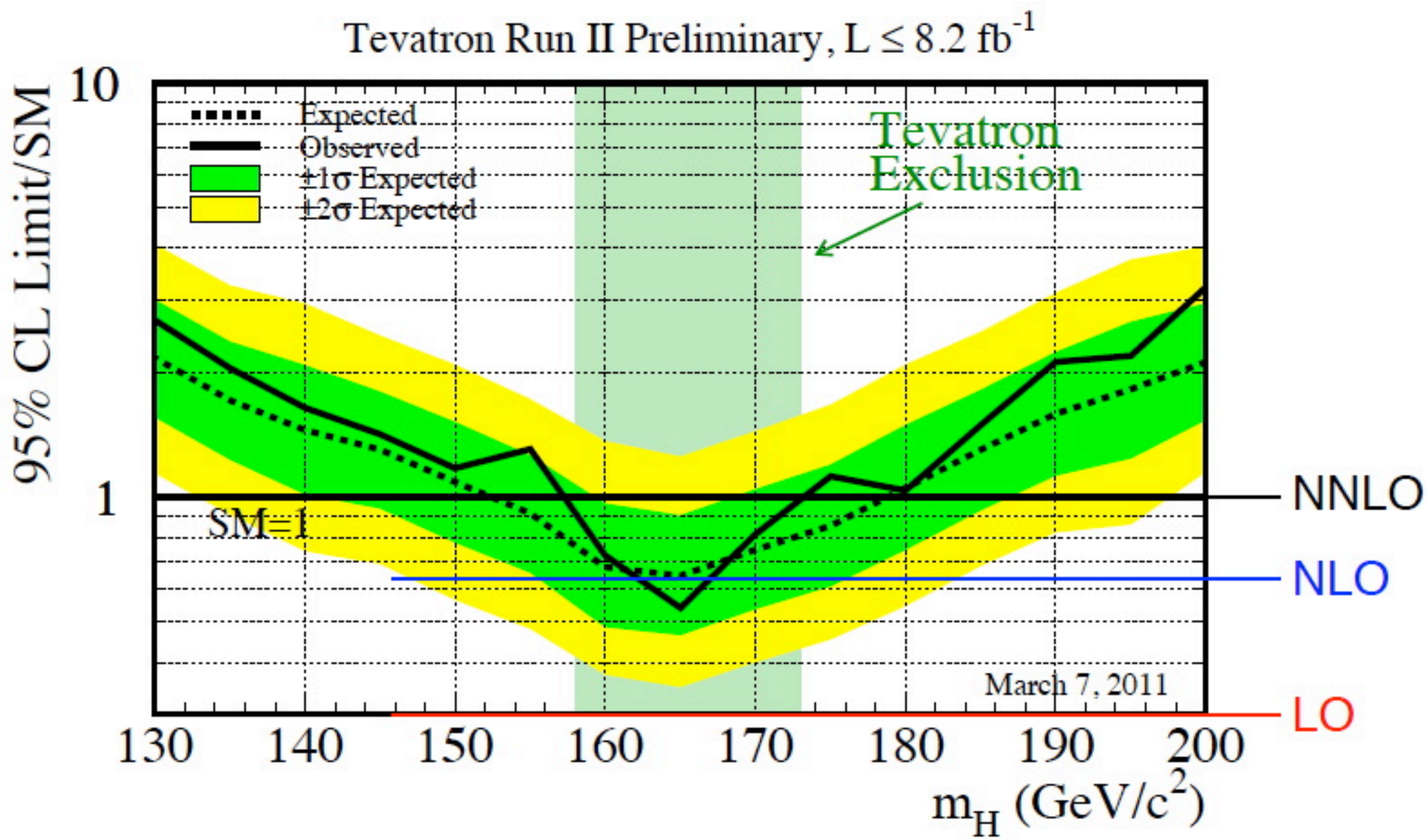
- Impressive improvement of the scale dependence.
- High-p_T end of the electron and extra jet known at NLO accuracy

Higgs predictions at NNLO



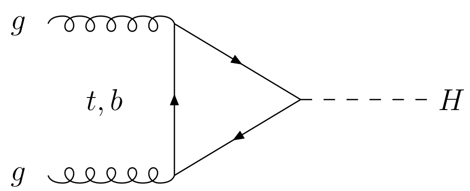
- The perturbative series stabilizes.
- NLO estimation of higher orders effects by scale uncertainty works reasonably well

Higgs predictions at NNLO

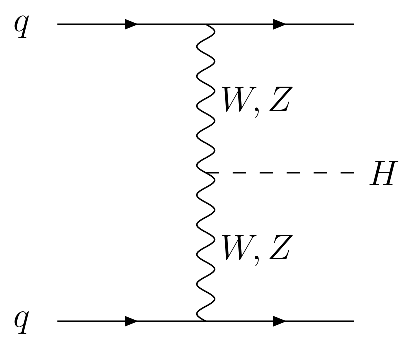


be careful : just illustrative example, not very precise

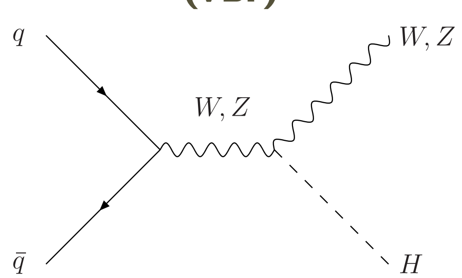
Higgs predictions at 7/8 TeV



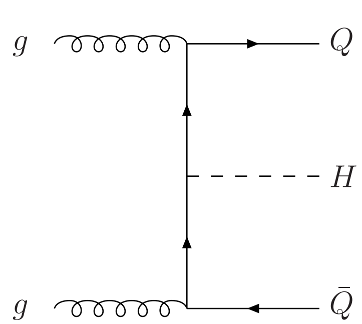
GluonFusion



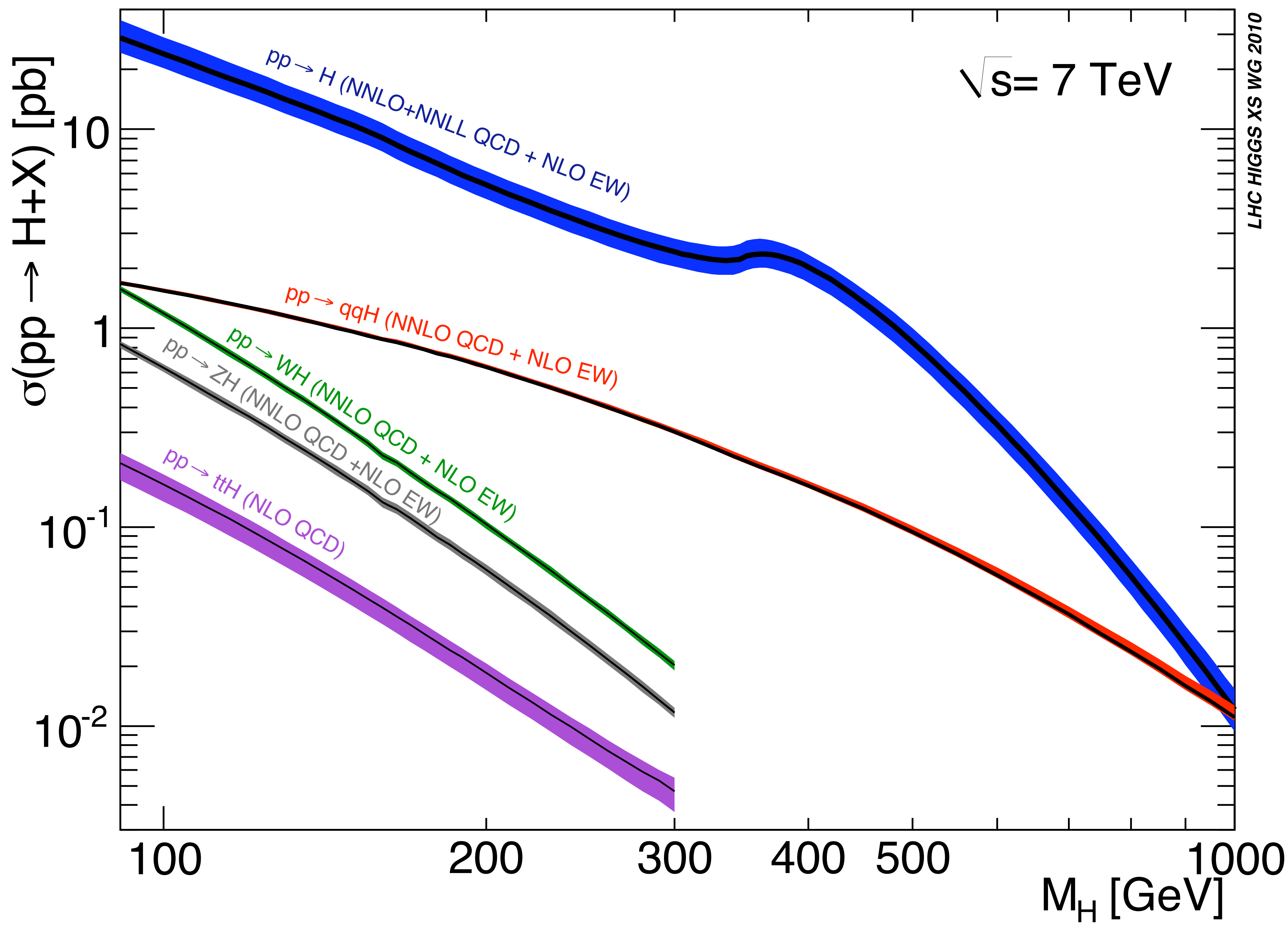
vector boson fusion (VBF)



associated production with vector bosons



associated production with heavy quarks



Predictions at NNLO : final remarks

- Handful of precious predictions at NNLO now available for Higgs and Drell-Yan processes at the parton level for distributions.

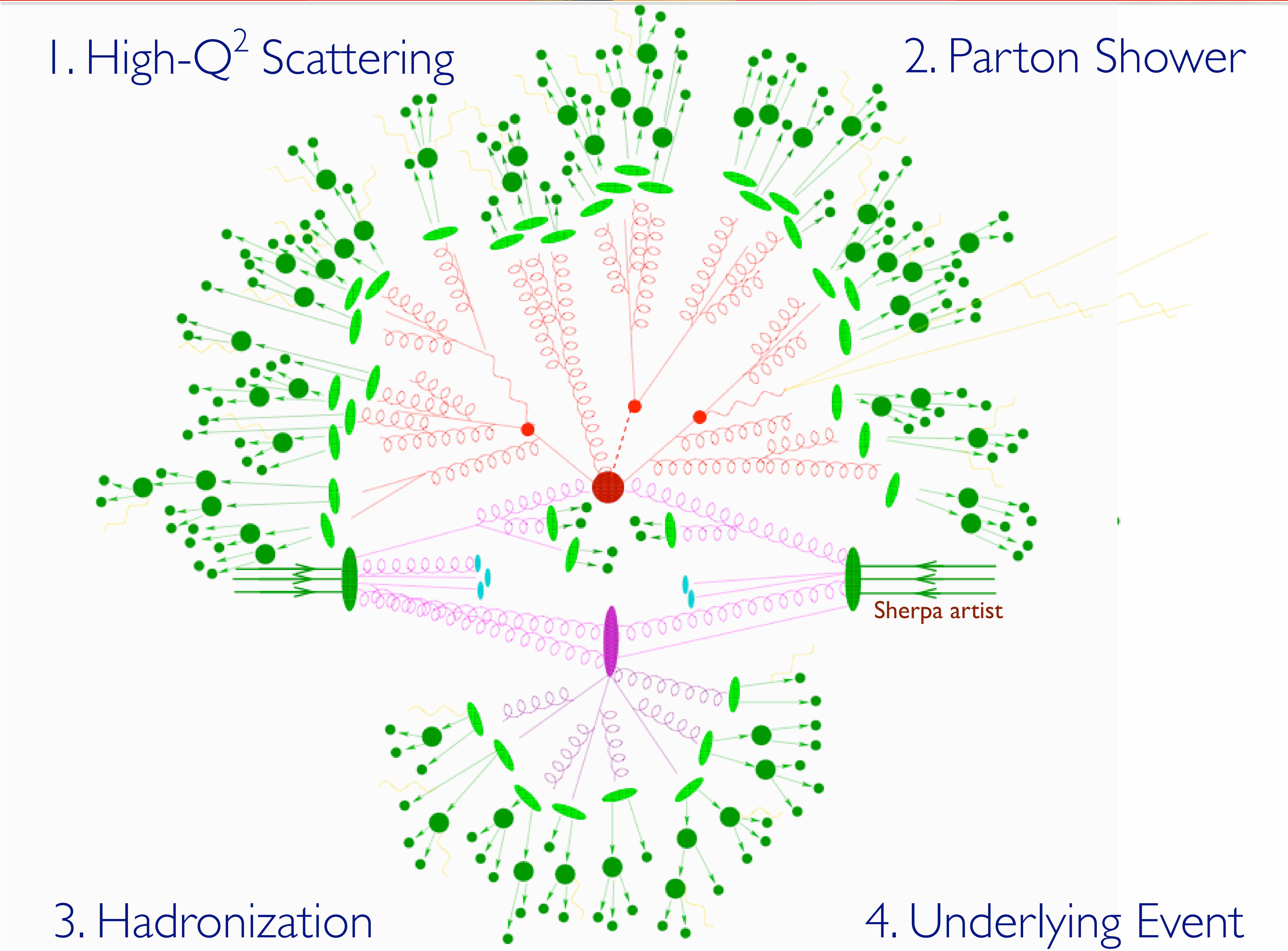
- Others (VV, ttbar) in progress and in sight.

NNLO stays to the LHC era
as
NLO stayed to the Tevatron era

How do we make predictions?

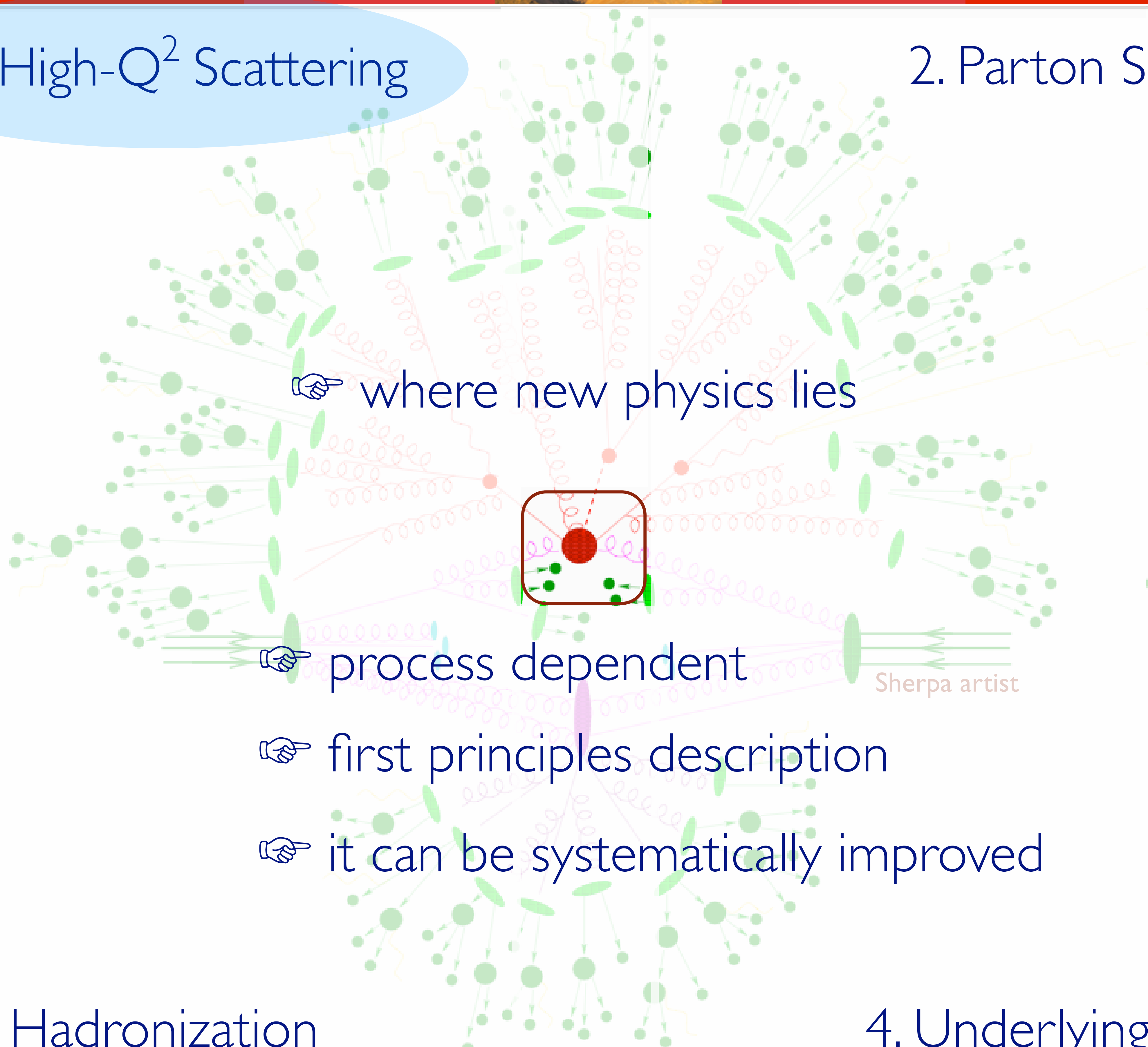
1. Fixed order computations: from LO to NNLO — TH-Accurate
2. Parton showers and fully exclusive simulations — EXP-Useful

In other words we enter here the realm of the proper Monte Carlo Event generators!


1. High-Q^2 Scattering
2. Parton Shower
3. Hadronization
4. Underlying Event
Sherpa artist

1. High-Q^2 Scattering

2. Parton Shower



☞ where new physics lies

☞ process dependent

☞ first principles description

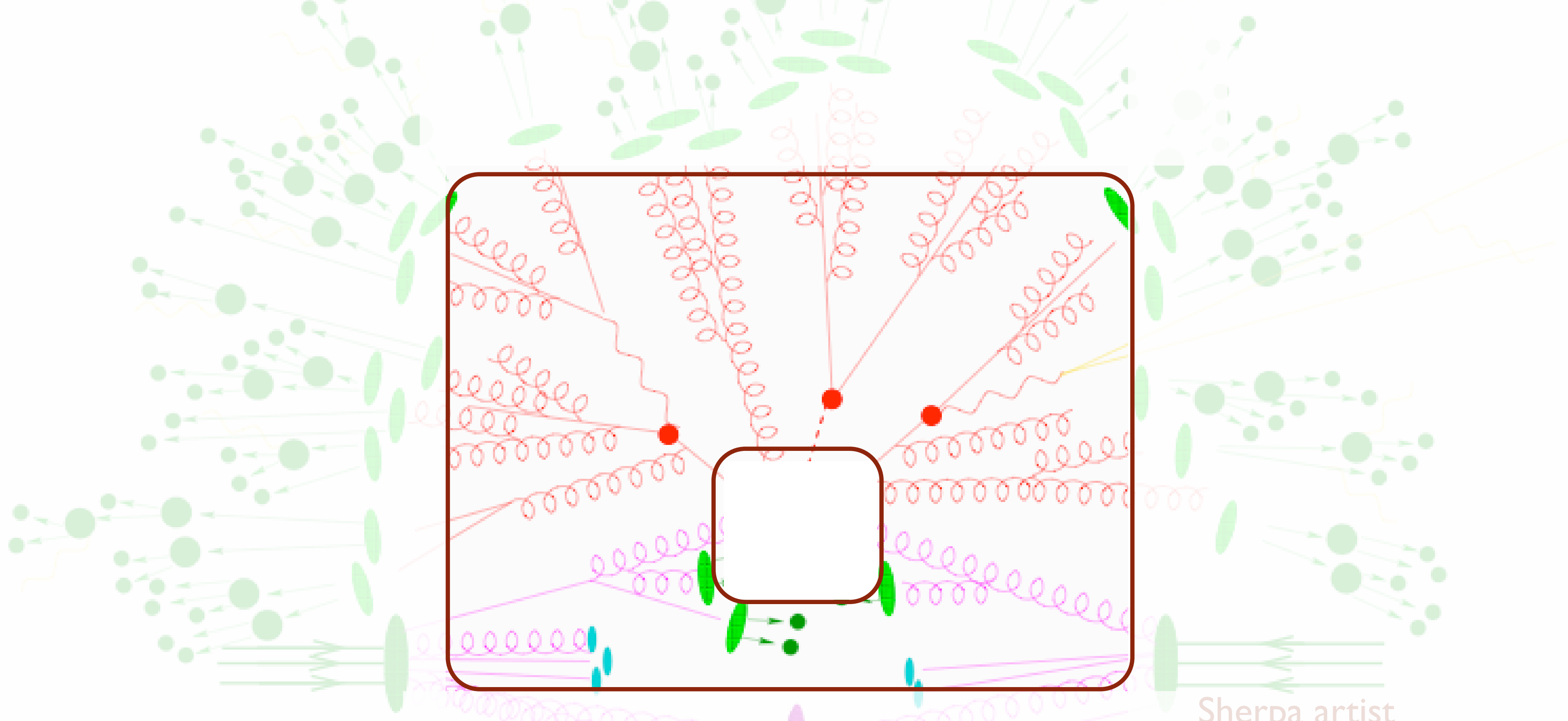
☞ it can be systematically improved

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



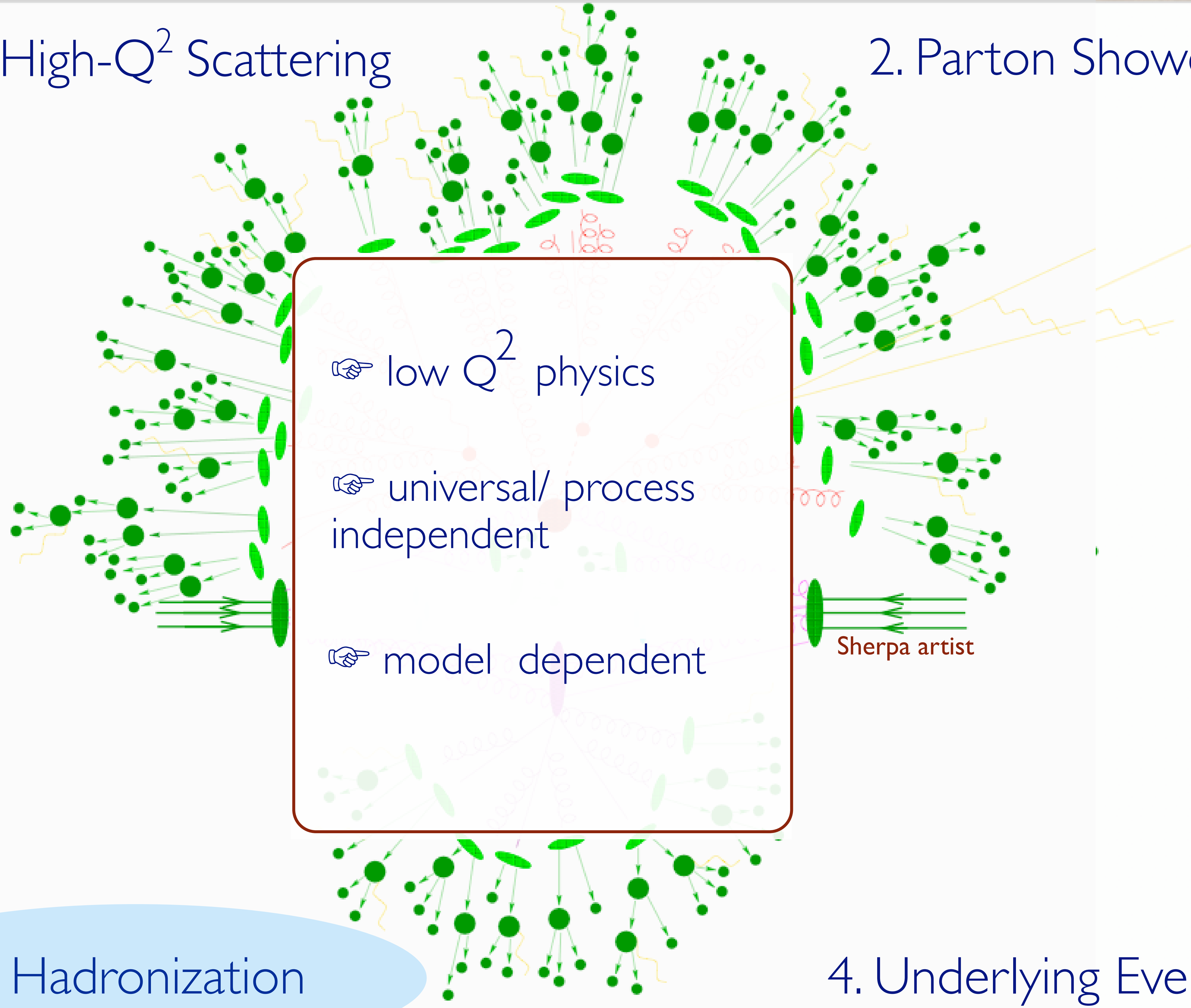
- ☞ QCD -"known physics"
- ☞ universal/ process independent
- ☞ first principles description

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



- ☞ low Q^2 physics
- ☞ universal/ process independent
- ☞ model dependent

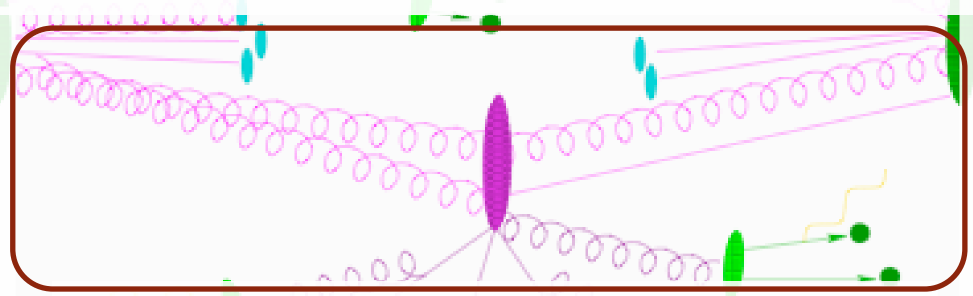
3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower

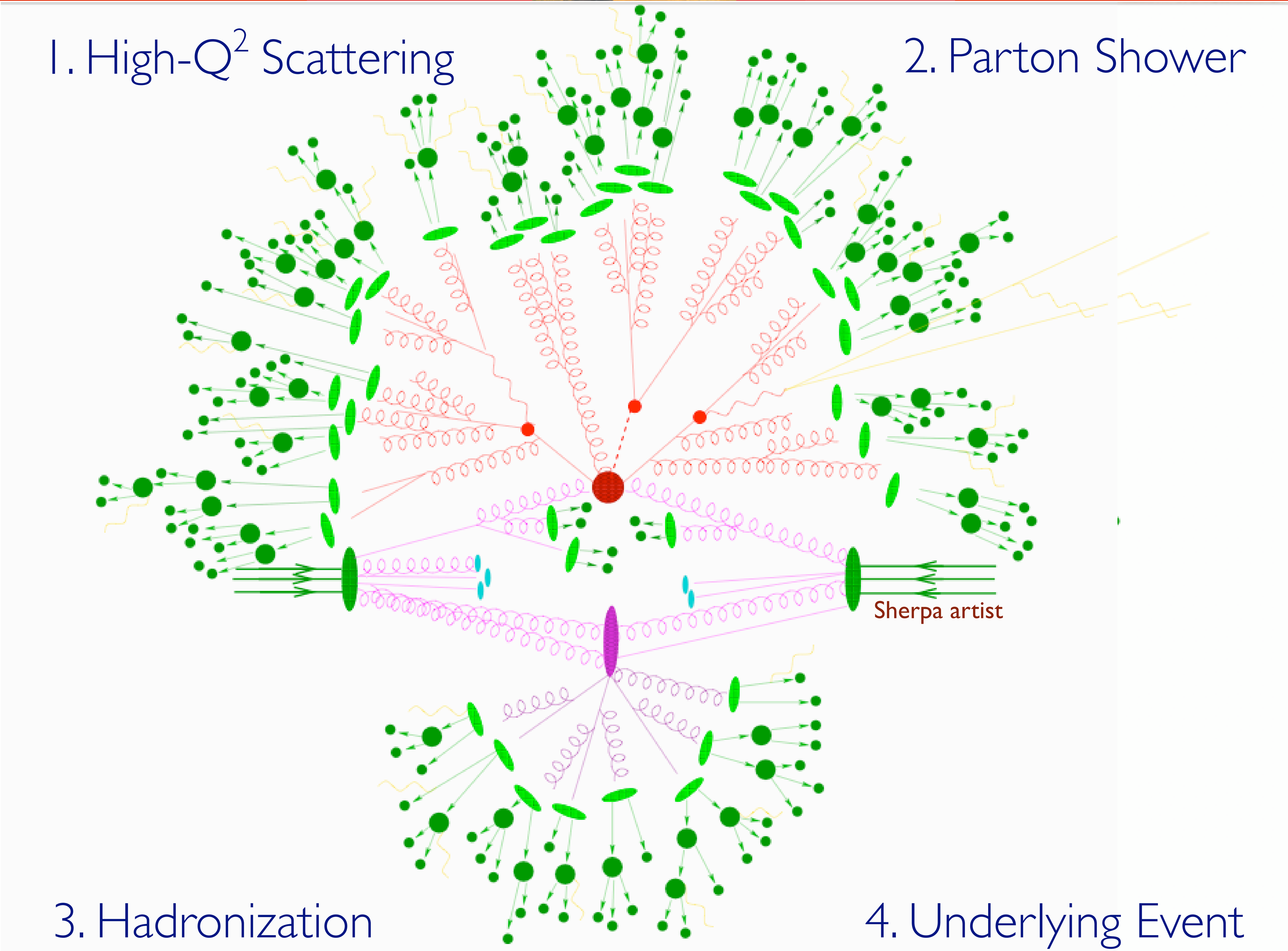
- ☞ low Q^2 physics
- ☞ energy and process dependent
- ☞ model dependent



Sherpa artist

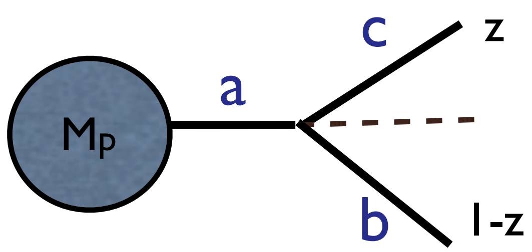
3. Hadronization

4. Underlying Event


1. High-Q^2 Scattering
2. Parton Shower
Sherpa artist
3. Hadronization
4. Underlying Event

Parton showers

ME involving q ➝q g (or g ➝ gg) are strongly enhanced when they are close in the phase space:



$$z = E_b/E_a \, , t = k_a^2$$

$$\begin{aligned} \theta &= \theta_b + \theta_c \\ &= \frac{\theta_b}{1-z} = \frac{\theta_c}{z} \\ &= \frac{1}{E_a}\sqrt{\frac{t}{z(1-z)}} \end{aligned}$$

$$\frac{1}{(p_q+p_g)^2} \simeq \frac{1}{2E_q E_g(1-\cos\theta)}$$

$$\begin{aligned} d\sigma_{N+1} &= d\sigma_N \frac{dt}{t}\frac{d\phi}{2\pi} dz \frac{\alpha_S}{2\pi} \left|K_{ba}(z)\right|^2 \\ d\bar{\sigma}_{N+1} &= d\bar{\sigma}_N \frac{dt}{t} dz \frac{\alpha_S}{2\pi} P_{ba}(z) \end{aligned}$$

In the collinear limit the cross section factorizes. The splitting can be iterated.

Parton showers

It is easy to iterate the branching process:

$$a(t) \longrightarrow b(z) + c\,, \quad b(t') \longrightarrow d(z') + e$$

$$d\bar{\sigma}_{N+2} = d\bar{\sigma}_N \frac{dt}{t} dz \frac{dt'}{t'} dz' \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ba}(z) P_{db}(z')$$

This is a generalized Markov process (in the continuum), where the probability of the system to change (discontinuosly) to another state, depends only on present state and not how it got there:

$$\tau_1 < \ldots < \tau_n \quad \Longrightarrow$$
$$P\Big(x(\tau_n) < x_n | x(\tau_{n-1}), \ldots, x(\tau_1)\Big) = P(x(\tau_n) < x_n | x(\tau_{n-1}))$$

No memory!

Parton showers

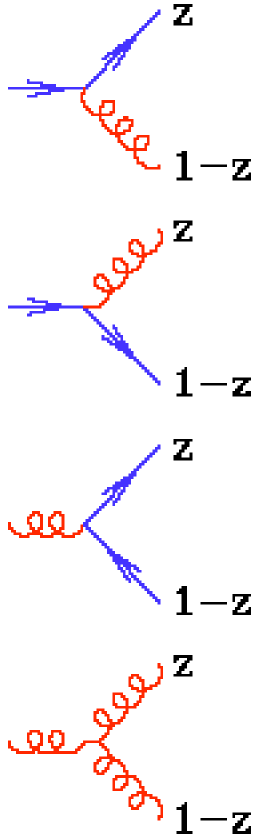
The spin averaged (unregulated) splitting functions for the various types of branching ar

$$\hat{P}_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)}\right],$$

$$\hat{P}_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$\hat{P}_{qg}(z) = T_R\left[z^2+(1-z)^2\right],$$

$$\hat{P}_{gg}(z) = C_A\left[\frac{z}{(1-z)}+\frac{1-z}{z}+z\,(1-z)\right].$$



$$C_F = \tfrac{4}{3}, C_A = 3, T_R = \tfrac{1}{2}.$$

Comments:
* Gluons radiate the most
* There are soft divergences in z=1 and z=0.
* Pqg has no soft divergences.

Parton showers

Following a given line in a branching tree, it is clear that contributions coming from the strongly-ordered region will be leading:

$$Q^2 \gg t_1 \gg t_2 \gg \dots t_N \gg Q_0^2$$

$$\sigma_N \propto \sigma_0 \alpha_s^N \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \dots \int_{Q_0^2}^{t_{N-1}} \frac{dt_N}{t_N} = \sigma_0 \frac{\alpha_s^N}{N!} \left(\log \frac{Q^2}{Q_0^2} \right)^N$$

Denote by $\Phi_a[E, Q^2]$

the ensemble of parton cascades initiated by a parton a of energy E and emerging from a hard process with scale Q^2 (Generating functional). Also, define

$$\Delta(Q_1^2, Q_2^2)$$

as the probability that a **does not branch** for virtualities $Q_1^2 > t > Q_2^2$

Parton showers

With this, it easy to write a formula that takes into account all the branches associated to a parton a:

$$\Phi_a[E, Q^2] = \Delta_a(Q^2, Q_0^2)\Phi_a[E, Q_0^2] + \int_{Q_0^2}^{Q^2} \frac{dt}{t}\Delta_a(Q^2, t)\sum_b \int dz \frac{\alpha_S}{2\pi} P_{ba}(z)\Phi_b[zE, t]\Phi_c[(1-z)E, t]$$

Simple interpretation. First term describes the evolution to Q_0, where no branching has occurred. The second term is the contribution coming from evolving with no branching up to a given t and then branching there. Now conservation of probability imposes that:

$$1 = \Delta_a(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dt}{t}\Delta_a(Q^2, t)\sum_b \int dz \frac{\alpha_S}{2\pi} P_{ba}(z)$$

Which can be solved to give an explicit expression for **Δ**.

Parton showers

$$\Delta_a(Q^2, Q_0^2) = \exp\left[- \int_{Q_0^2}^{Q^2} \frac{dt}{t} \sum_b \int dz \frac{\alpha_S}{2\pi} P_{ab}(z) \right]$$

Which gives an explicit expression for the Sudakov form factor, i.e. la probability that a parton will not branch in going from the virtuality Q^2 to Q^2_0 .

Proof:

derive the conservation of probability equation

$$0 = \frac{d\Delta_a}{dQ_0^2}(Q^2, Q_0^2) - \frac{\mathcal{P}_a}{Q_0^2} \Delta_a(Q^2, Q_0^2)\,, \qquad \mathcal{P}_a = \sum_b \int dz \frac{\alpha_S}{2\pi} P_{ba}(z)$$

and impose the initial condition

$$\Delta_a(Q^2, Q^2) = 1$$

Note that: $$\Delta_a(Q^2, t) = \frac{\Delta_a(Q^2, Q_0^2)}{\Delta_a(t, Q_0^2)}$$ and therefore sometimes the second argument is not used.

Parton showers

Formulation in terms of Sudakov form factor is well suited to computer implementation, and is the basis of parton shower Monte Carlo programs. Let's rewrite the formula using p_T and a parton-level event at the Born level:

$$d\sigma^{\mathrm{PS}} = d\Phi_B B(\Phi_B) \left[\Delta(p_\perp^{\min}) + d\Phi_{R|B} \Delta(p_T(\Phi_{R|B})) \frac{R^{\mathrm{PS}}(\Phi_R)}{B(\Phi_B)} \right]$$

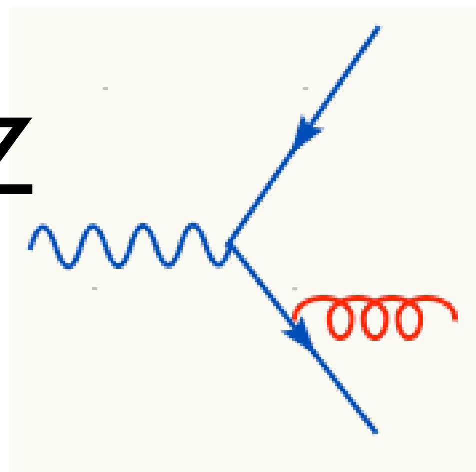
$$\Delta(p_T) = \exp\left[-\int d\Phi_{R|B} \frac{R^{\mathrm{PS}}(\Phi_R)}{B(\Phi_B)} \Theta(p_T(\Phi_R) - p_T) \right] \ . \quad R^{\mathrm{PS}}(\Phi) = P(\Phi_{R|B})\, B(\Phi_B).$$

Monte Carlo branching algorithm operates as follows. Given an initial configuration (parton-level event at the Born level), a parton is chosen, a rnd value of p_T is chosen accordingly to the probability of non-emission down to p_T. If it is larger than a p_T^{min}, than a branching occurs at p_T, and x is generated according to the splitting function $P(\Phi_{R|B})$ (as well as a flat azimuthal angle). An extra parton is now included and the process starts from there.

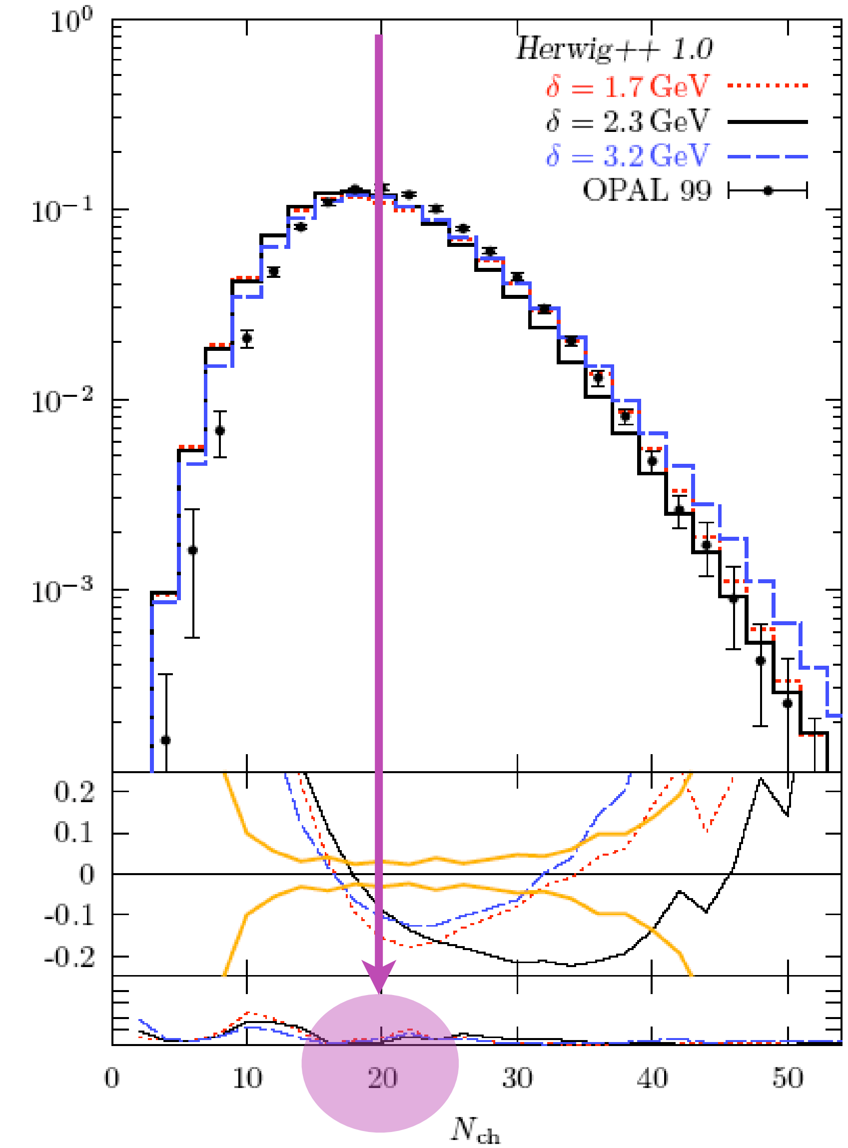
Due to successive branching, a parton cascade or shower develops. Each outgoing line is source of a new cascade, until all lines have stopped branching. At this stage, which depends on p_T^{min}, outgoing partons have to be converted into hadrons.

Parton showers

γ^*, Z



?



A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

Parton showers

Note that we can define the following quantities with mass squared dimensions

$$\begin{aligned} Q^2 &= z(1-z)\theta^2 E^2 \\ p_T^2 &= z^2(1-z)^2\theta^2 E^2 \\ \tilde{t} &= \theta^2 E^2 \end{aligned}$$

and obtain

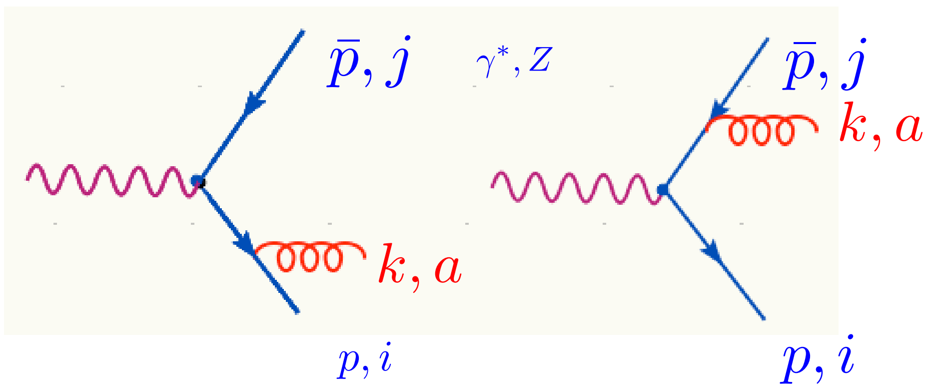
$$\frac{d\theta^2}{\theta^2} = \frac{dQ^2}{Q^2} = \frac{dp_T^2}{p_T^2} = \frac{d\tilde{t}}{\tilde{t}}$$

Different MC programs make different choices for the variable. HERWIG uses θ, while Pythia uses p_T.

This fact has an important consequence: the evolution parameter of the shower is not uniquely defined. This is because the scales chosen above have all the same angular behavior, provided that z is not too close to 0 or 1.

Differences stem from the SOFT region. It is therefore necessary to study what happens for soft emissions to find the optimal choice.

Angular ordering



$$d\sigma_{qqg} = C_F \frac{\alpha_S}{2\pi} \sigma^{\mathrm{Born}} d\cos\theta \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} d\cos\theta$$

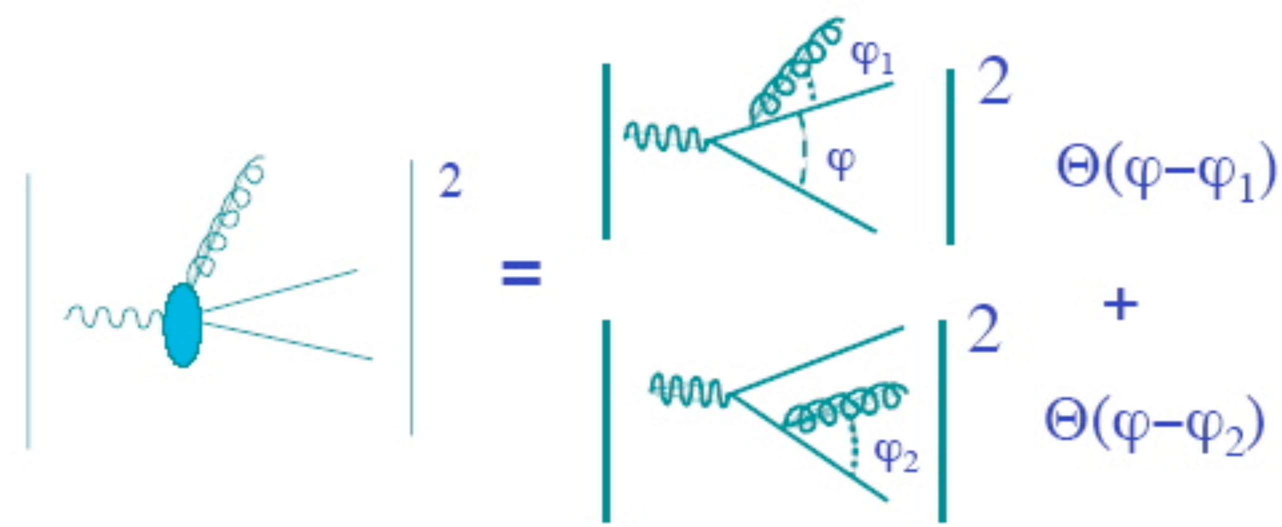
You can easily prove that:

$$\frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} = \frac{1}{2}\left[\frac{\cos\theta_{jk}-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} + \frac{1}{(1-\cos\theta_{jk})}\right] + \frac{1}{2}[i \to j]$$

The probabilistic interpretation of W_i and W_j is achieved simply by azimuthal averaging:

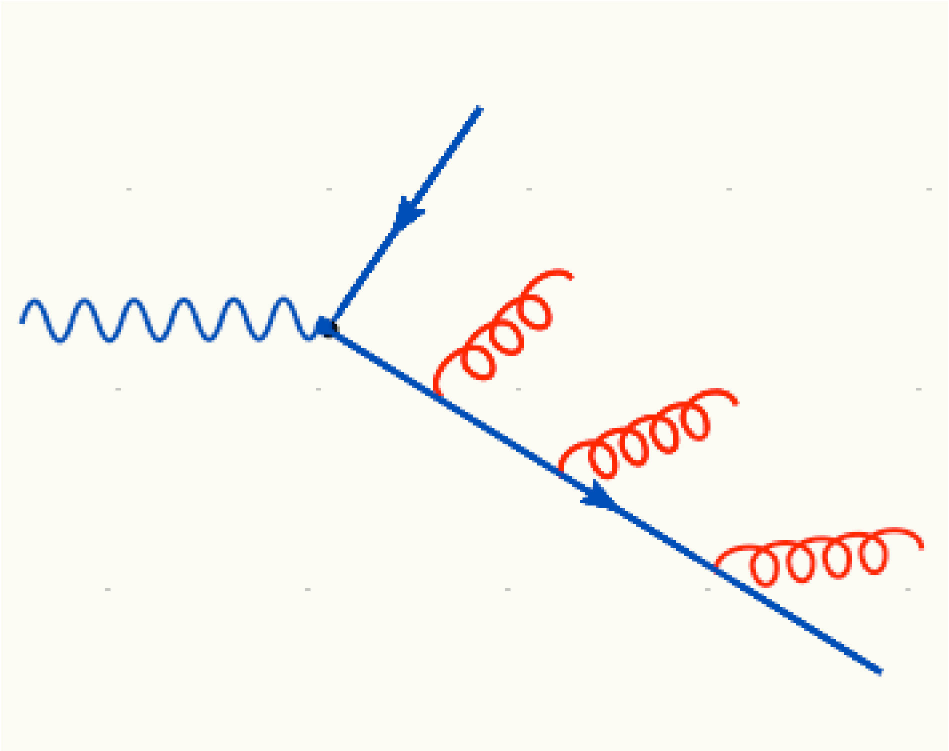
$$\int \frac{d\phi}{2\pi} W_i = \frac{1}{1-\cos\theta_{ik}} \quad \text{if} \quad \theta_{ik} < \theta_{ij}\,, 0\,\text{otherwise}$$

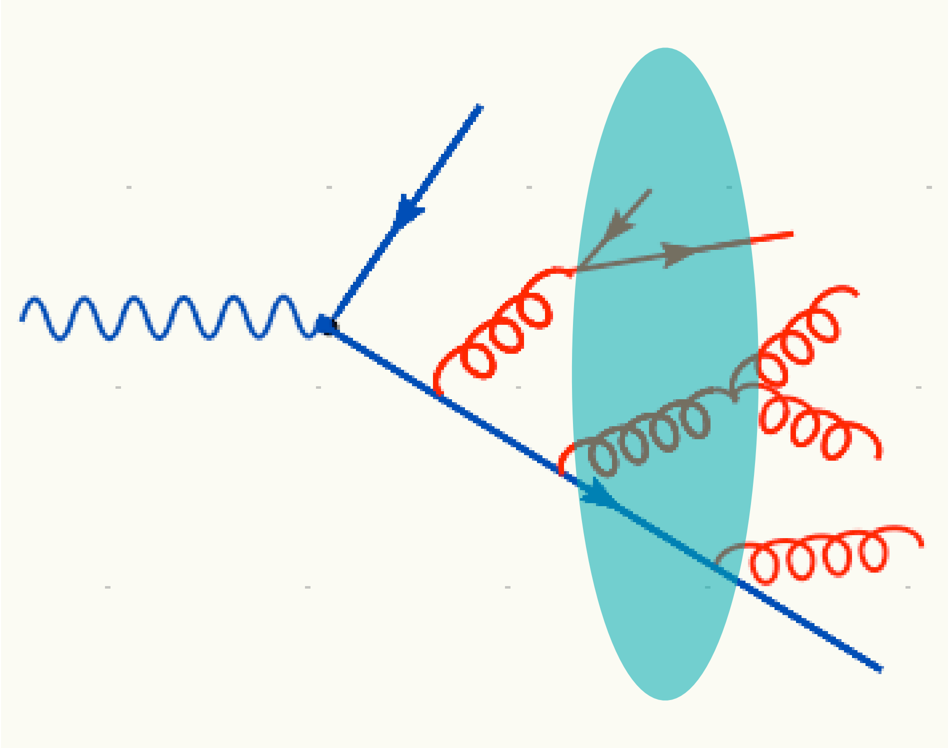
And the same for W_j



Radiation happens only for angles smaller than the color connected (antenna) opening angle!

Angular ordering





The construction can be iterated to the next emission, with the result that the emission angles keep getting smaller and smaller.

One can generalize it to a generic parton of color charge Qk splitting into two partons i and j , Qk=Qi +Qj. The result is that inside the cones i and j emit as independent charges, and outside their angular-order cones the emission is coherent and can be treated as if it was directly from color charge Qk.

KEY POINT FOR THE MC!

Angular ordering is automatically satisfied in p_T and θ ordered showers!

Angular ordering

Angular ordering is:

1. A quantum effect coming from the interference of different Feynman diagrams.

2. Nevertheless it can be expressed in "a classical fashion" (square of a amplitude is equal to the sum of the squares of two special "amplitudes"). The classical limit is the dipole-radiation.

3. It is not an exclusive property of QCD (i.e., it is also present in QED) but in QCD produces very non-trivial effects, depending on how particles are color connected.

Parton Shower MC event generators

A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

- General-purpose tools
- Always the first exp choice
- Complete exclusive description of the events: hard scattering, showering & hadronization, underlying event
- Reliable and well tuned tools.
- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD.

Complete MC Generators: PYTHIA, HERWIG, SHERPA

CAN WE MAKE ACCURATE AND REALISTIC PREDICTIONS?

Accurate:

- For low multiplicity include higher order terms in our fixed-order calculations (LO→NLO→NNLO...)

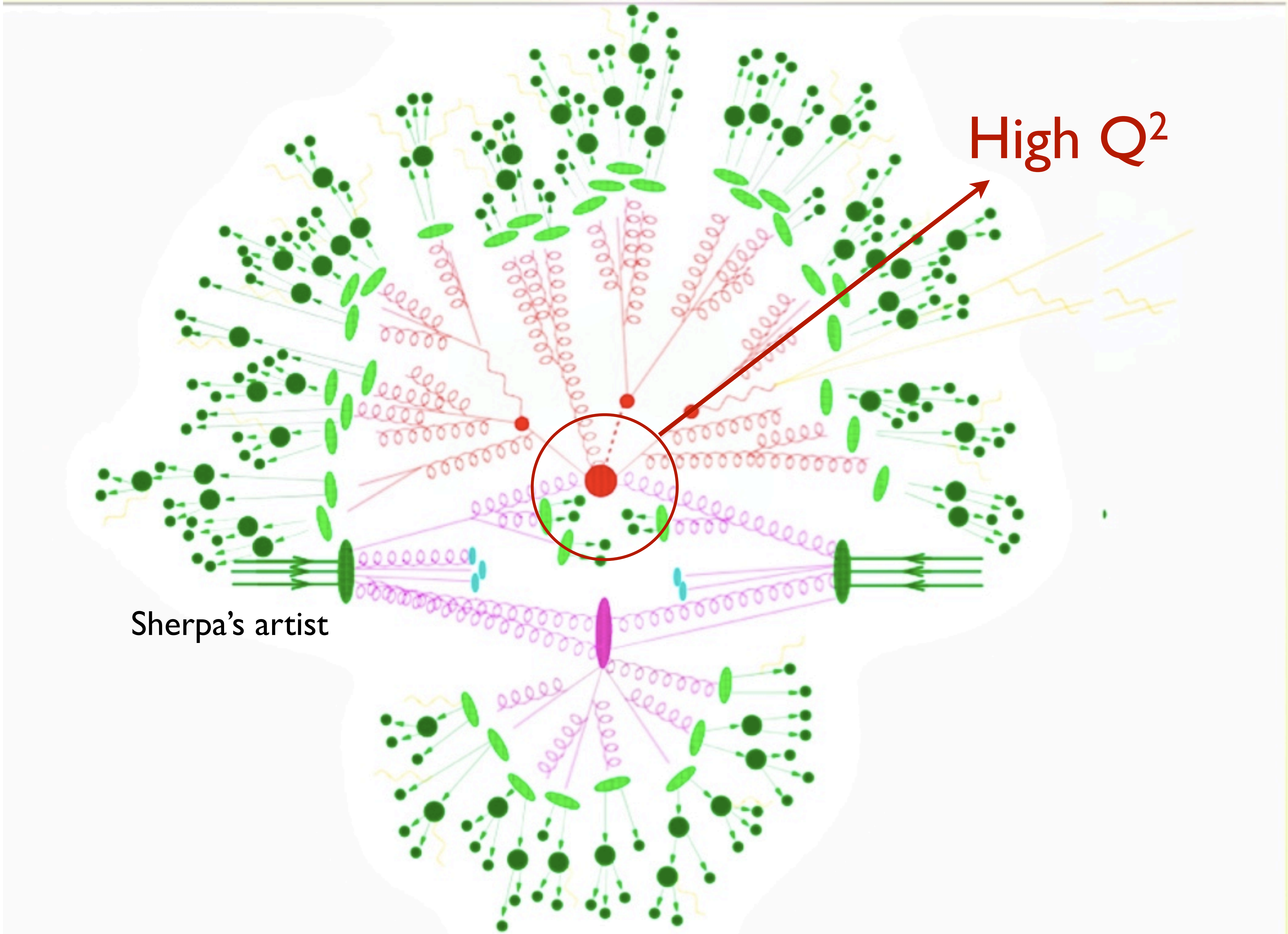
$$\Rightarrow \hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

- We use the corresponding evolution in the PDF's

Comments:

1. The theoretical errors systematically decrease.
2. A lot of new techniques and universal algorithms have been developed.
3. The frontier is now NNLO!
4. Final description only in terms of partons

and calculation of IR safe observables ⇒ not directly useful for exp simulations.


High Q^2
Sherpa's artist

CAN WE MAKE ACCURATE AND REALISTIC PREDICTIONS?

Realistic:

- Describe final states with high multiplicities starting from 2 → 1 or 2 → 2 procs, using parton showers

$$d\sigma^{\mathrm{PS}} = d\Phi_B B(\Phi_B) \left[\Delta(p_\perp^{\mathrm{min}}) + d\Phi_{R|B} \Delta(p_T(\Phi_{R|B})) \frac{R^{\mathrm{PS}}(\Phi_R)}{B(\Phi_B)} \right]$$

$$\Delta(p_T) = \exp\left[-\int d\Phi_{R|B} \frac{R^{\mathrm{PS}}(\Phi_R)}{B(\Phi_B)} \Theta(p_T(\Phi_R) - p_T) \right] \quad \cdot \quad R^{\mathrm{PS}}(\Phi) = P(\Phi_{R|B})\, B(\Phi_B).$$

(no or first emisssion) and then a hadronization model.

Comments:

1. Fully exclusive final state description for detector simulations
2. Normalization is very uncertain
3. Very crude kinematic distributions for multi-parton final states
4. Improvements are only at the model level.

CAN WE MAKE ACCURATE AND REALISTIC PREDICTIONS?

Fixed order prediction	Shower MC
1. parton-level description	1. hadron-level description
2. fixed order calculation	2. resums large logs
3. quantum interference exact	3. quantum interference through angular ordering
4. valid for few partons	4. valid when partons are collinear and/or soft
4. NLO results available	5. nedeed for realistic studies

Approaches are complementary: merge them!

Difficulty: avoid double counting

CAN WE MAKE ACCURATE AND REALISTIC PREDICTIONS?

New Trend:

Match fixed-order calculations and parton showers to obtain the most accurate predictions in a detector simulation friendly way!

Three directions:

1. Get fully exclusive description of many parton events correct at LO (LL) in all the phase space. **MEPS**

2. Get fully exclusive description of events correct at NLO in the normalization and distributions. **NLOwPS**

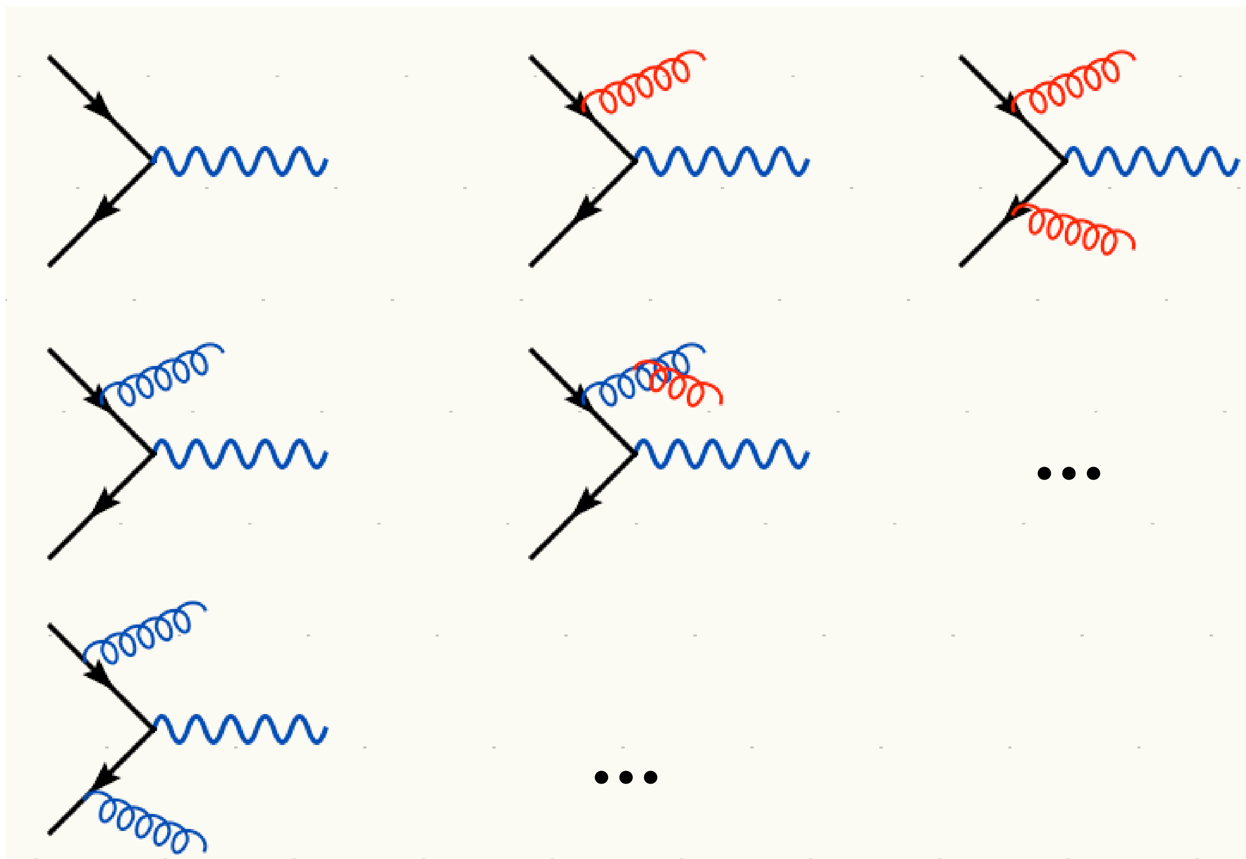
3. Get fully exclusive descriptions at NLO with multi-parton matching for higher multiplicities. **Merging@NLO**

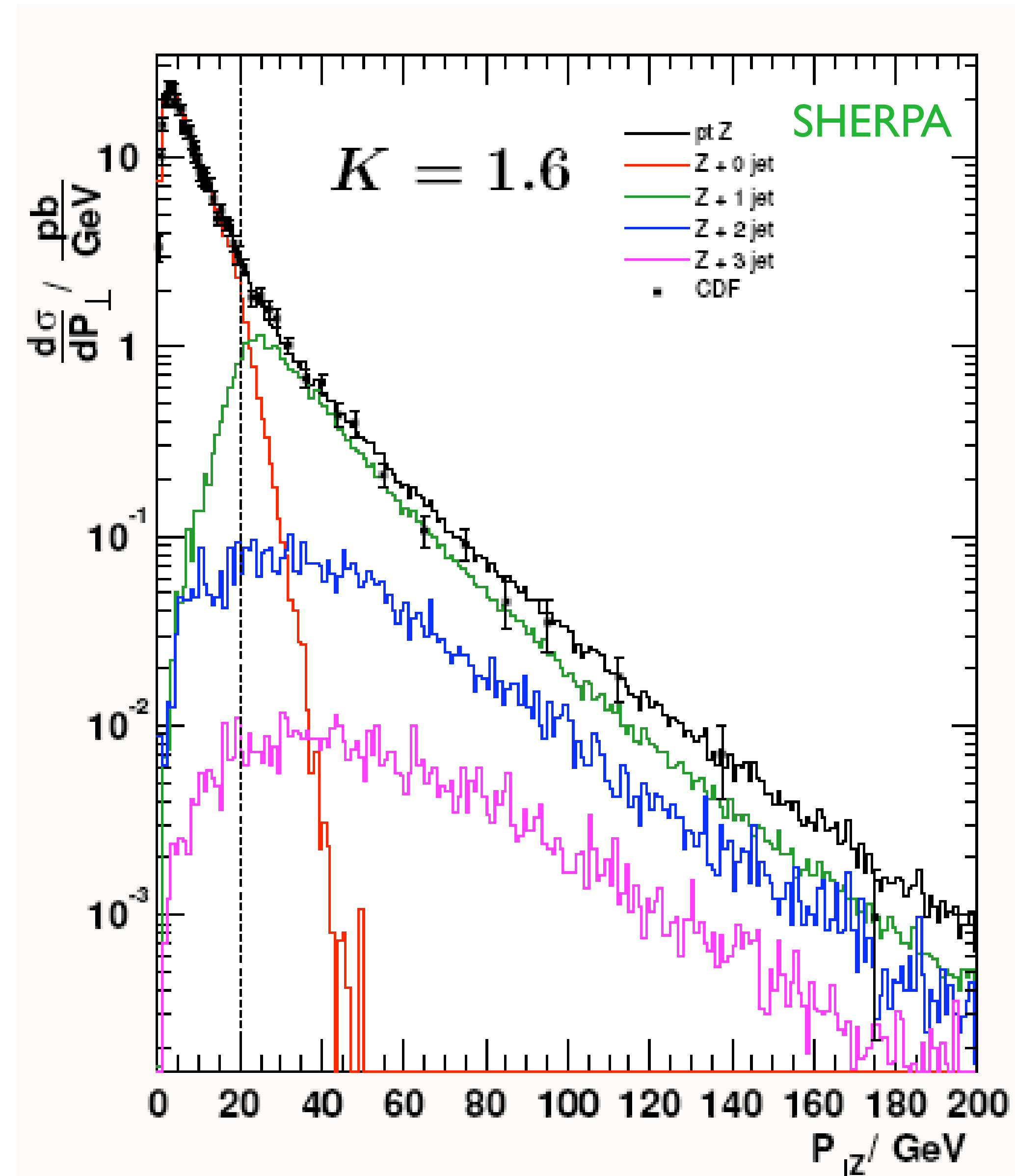
Merging fixed order with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]

PS →

ME ↓

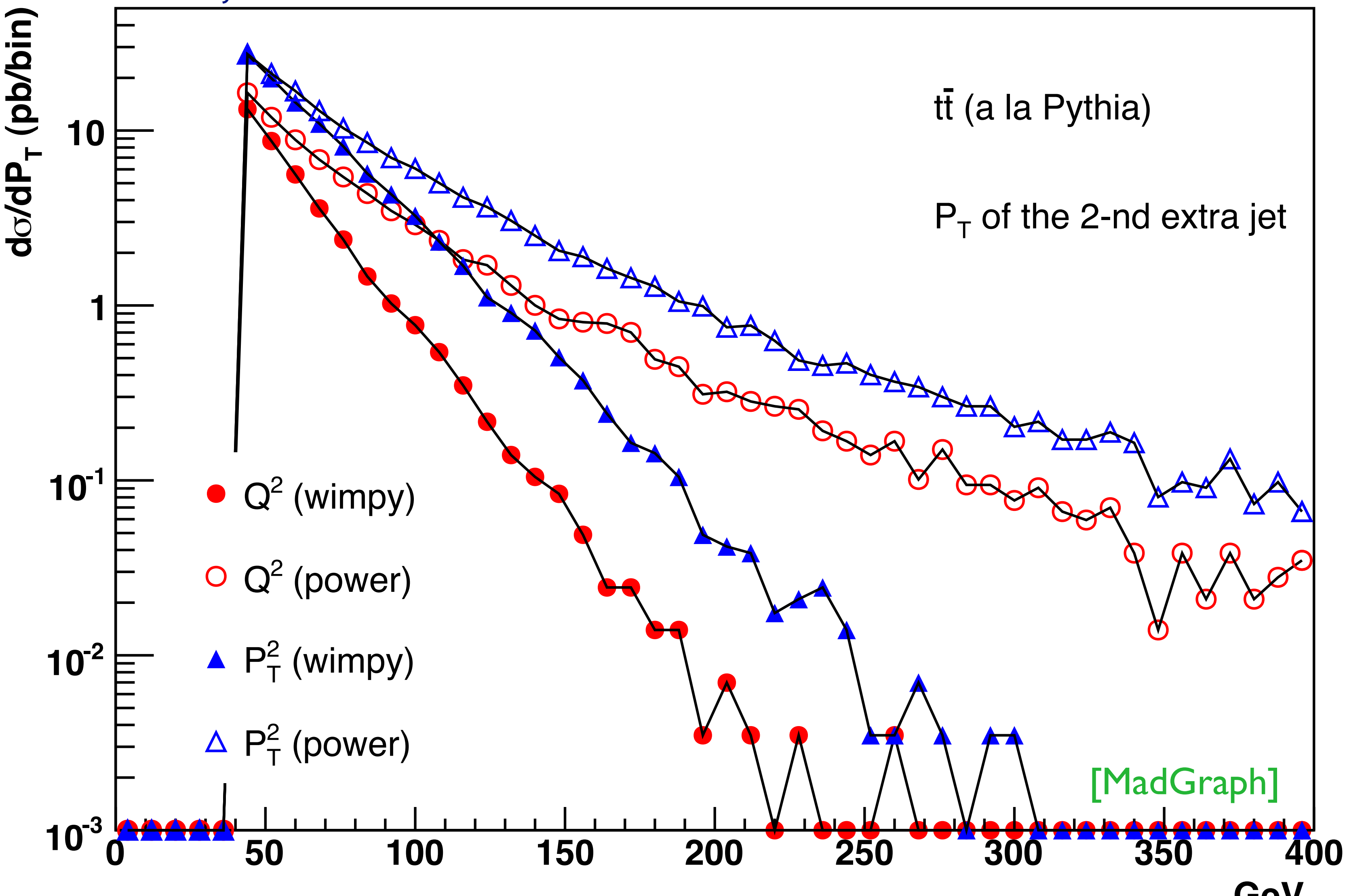




Double counting of configurations that can be obtained in different ways (histories). All the matching algorithms (CKKW, MLM,...) apply criteria to select only one possibility based on the hardness of the partons. As the result events are exclusive and can be added together into an inclusive sample. Distributions are accurate but overall normalization still "arbitrary".

PS alone vs matched samples

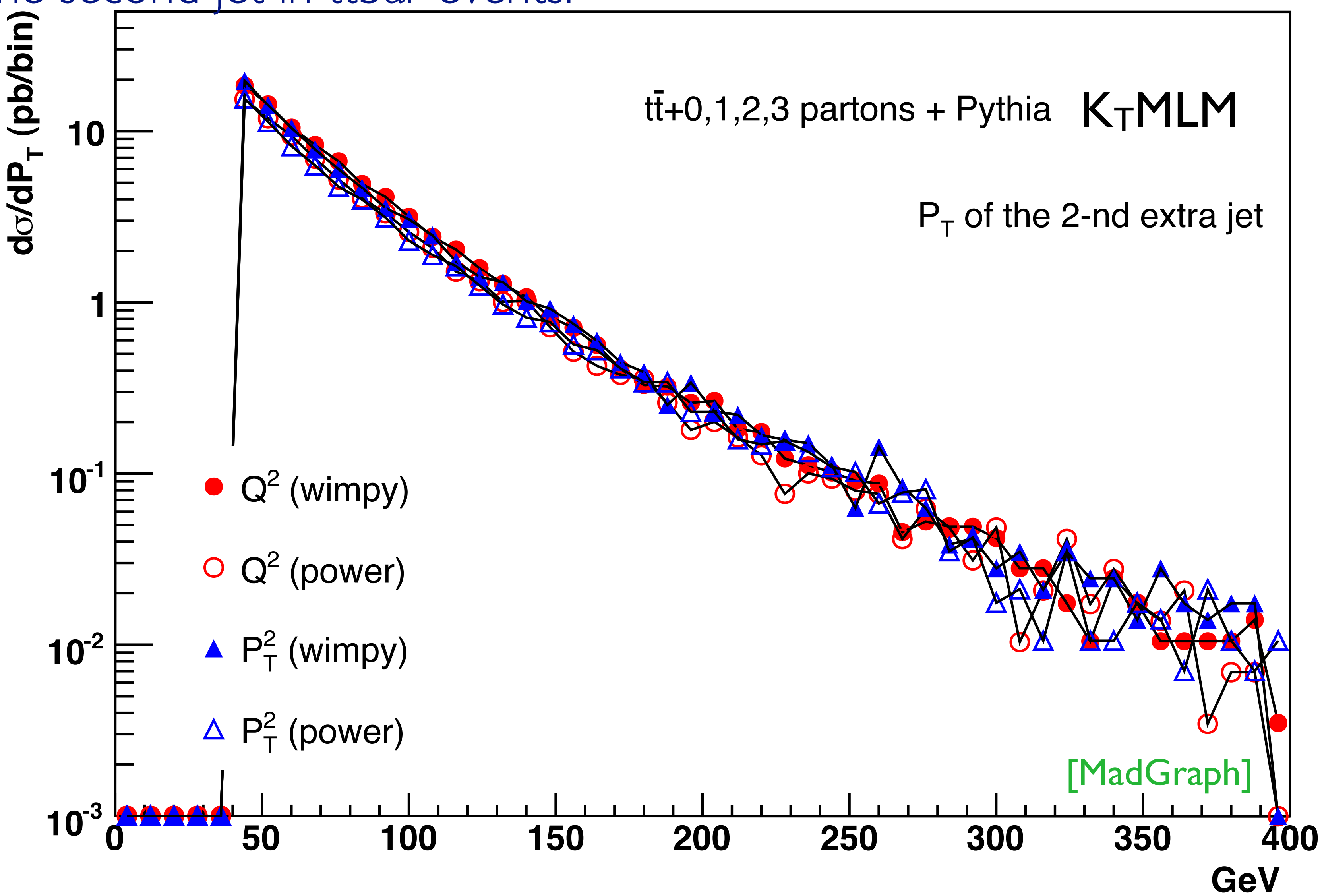
A MC Shower like Pythia produces inclusive samples covering all phase space. However, there are regions of the phase space (ex. high pt tails) which cannot be described well by the log enhanced (shower) terms in the QCD expansion and lead to ambiguities. Consider for instance the high-pt distribution of the second jet in ttbar events:



Changing some choices/parameters leads to huge differences ⇒ self diagnosis. Trying to tune the log terms to make up for it is not a good idea ⇒ mess up other regions/shapes, process dependence.

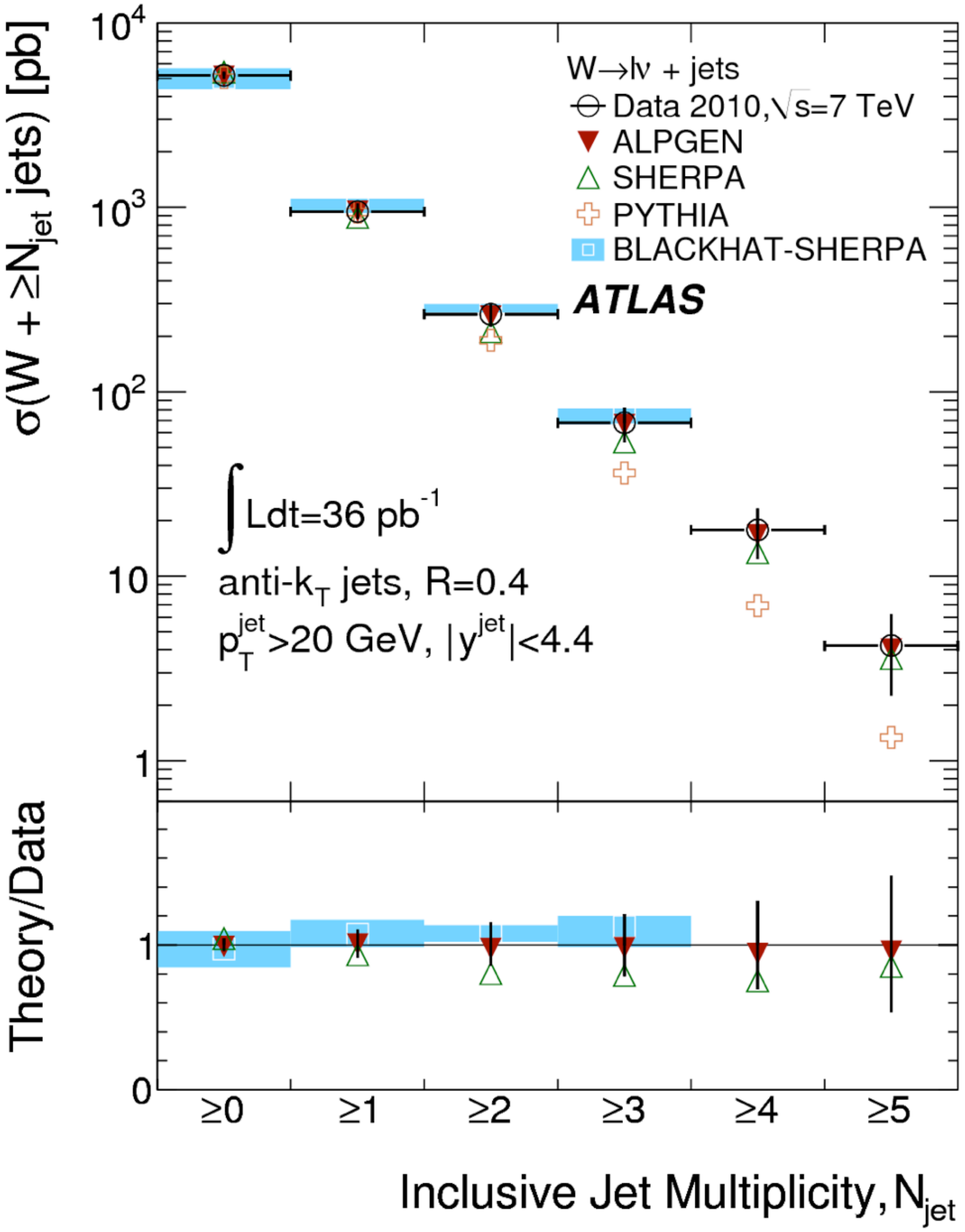
PS alone vs matched samples

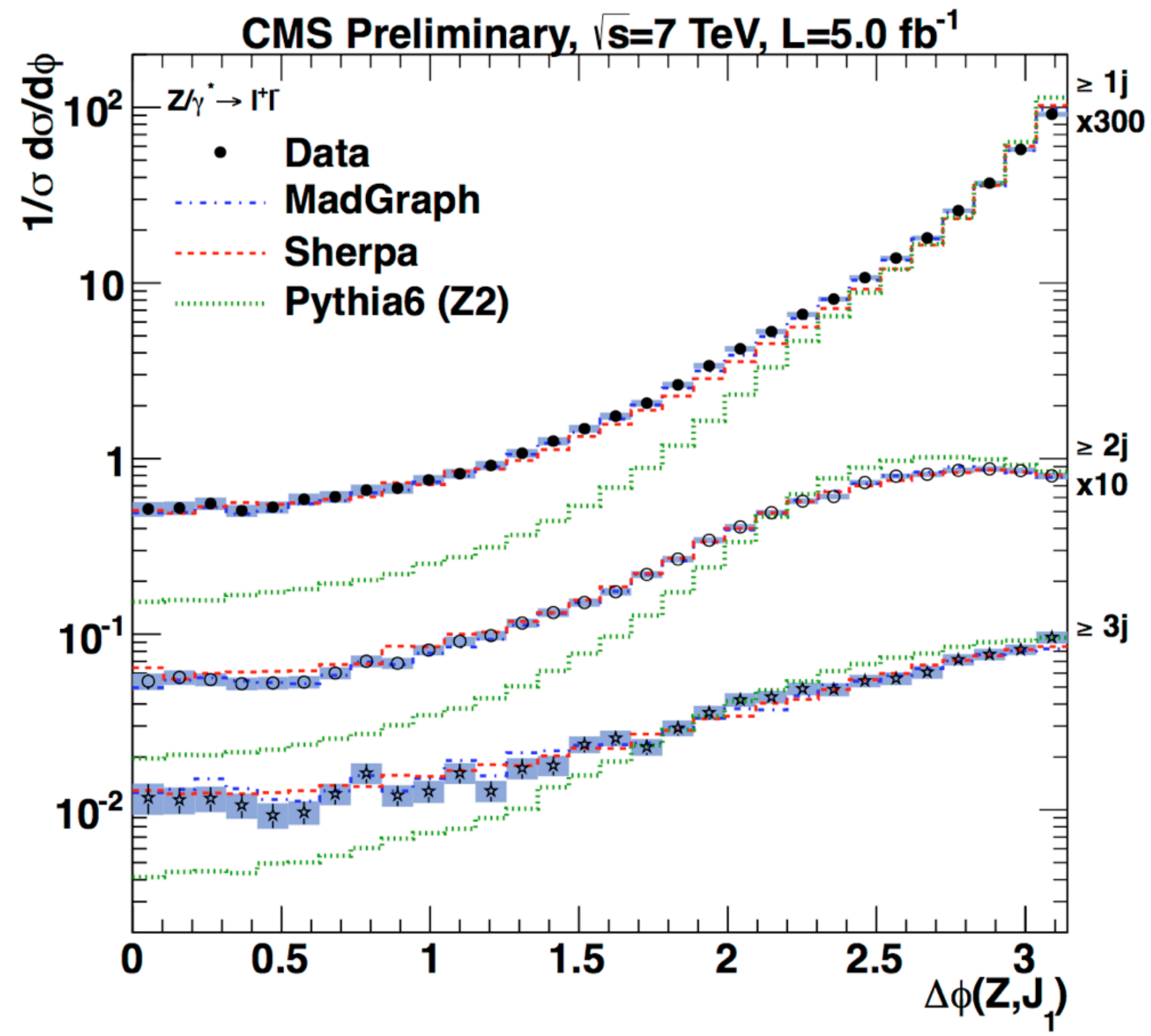
A MC Shower like Pythia produces inclusive samples covering all phase space. However, there are regions of the phase space (ex. high pt tails) which cannot be described well by the log enhanced (shower) terms in the QCD expansion and lead to ambiguities. Consider for instance the high-pt distribution of the second jet in ttbar events:



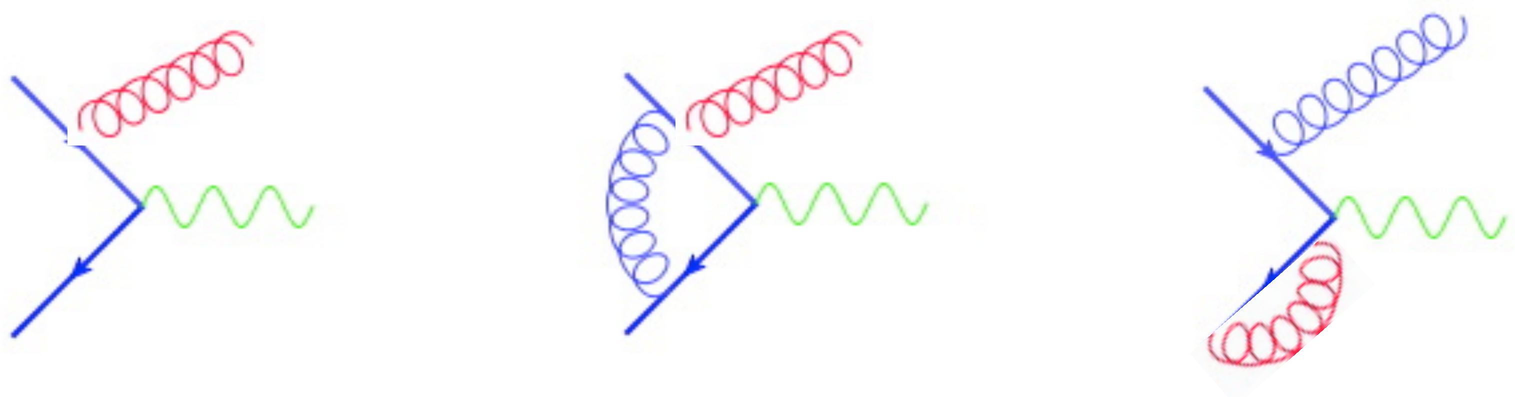
In a matched sample these differences are irrelevant since the behaviour at high pt is dominated by the matrix element. LO+LL is more reliable. (Matching uncertaintes not shown.)

TH/EXP comparison at the LHC





What about NLO?



$$d\sigma_{\rm NAIVE}^{\rm NLOwPS} = \left[d\Phi_B(B(\Phi_B) + V + S_{\rm ct}^{\rm int})\right] I_{\rm MC}^{n} + \left[d\Phi_B d\Phi_{R|B}(R - S_{ct})\right] I_{\rm MC}^{n+1}$$

This simple approach does not work:

- Instability: weights associated to I^n_{MC} and I^{n+1}_{MC} are divergent pointwise (infinite weights).

- Double counting: $d\sigma^{naive}_{NLOwPS}$ expanded at NLO does not coincide with NLO rate. Some configurations are dealt with by both the NLO and the PSMC.

Two solutions available

NLO WITH PS IN A NUTSHELL

$$d\sigma^{\mathrm{NLO+PS}} = d\Phi_B \bar{B}^s(\Phi_B)\left[\Delta^s(p_\perp^{\min}) + d\Phi_{R|B}\frac{R^s(\Phi_R)}{B(\Phi_B)}\Delta^s(p_T(\Phi))\right] + d\Phi_R R^f(\Phi_R)$$

integrates to 1 (unitarity)

with

$$\bar{B}^s = B(\Phi_B) + \left[V(\Phi_B) + \int d\Phi_{R|B} R^s(\Phi_{R|B})\right]$$

Full cross section at fixed Born kinematics (If F=1).

$$R(\Phi_R) = R^s(\Phi_R) + R^f(\Phi_R)$$

This formula is valid both for both MC@NLO and POWHEG

MC@NLO: $R^{\mathrm{s}}(\Phi) = P(\Phi_{R|B})\, B(\Phi_B)$

Needs exact mapping (ΦB,ΦR) → Φ

POWHEG: $R^{\mathrm{s}}(\Phi) = F R(\Phi)\,,\, R^{\mathrm{f}}(\Phi) = (1-F)R(\Phi)$

F=1 = Exponentiates the Real. It can be damped by hand.

MC@NLO AND POWHEG

MC@NLO

[Frixione, Webber, 2003;

- Matches NLO to HERWIG and HERWIG++ angular-ordered PS.
- Some events have negative weights.
- Large and well tested library of processes.

- Now available also for Pythia (Q^2) [Torrielli, Frixione, 1002.4293]
- Now automatized [Frederix, Frixione, Torrielli]
- Now available in aMC@NLO (see later)

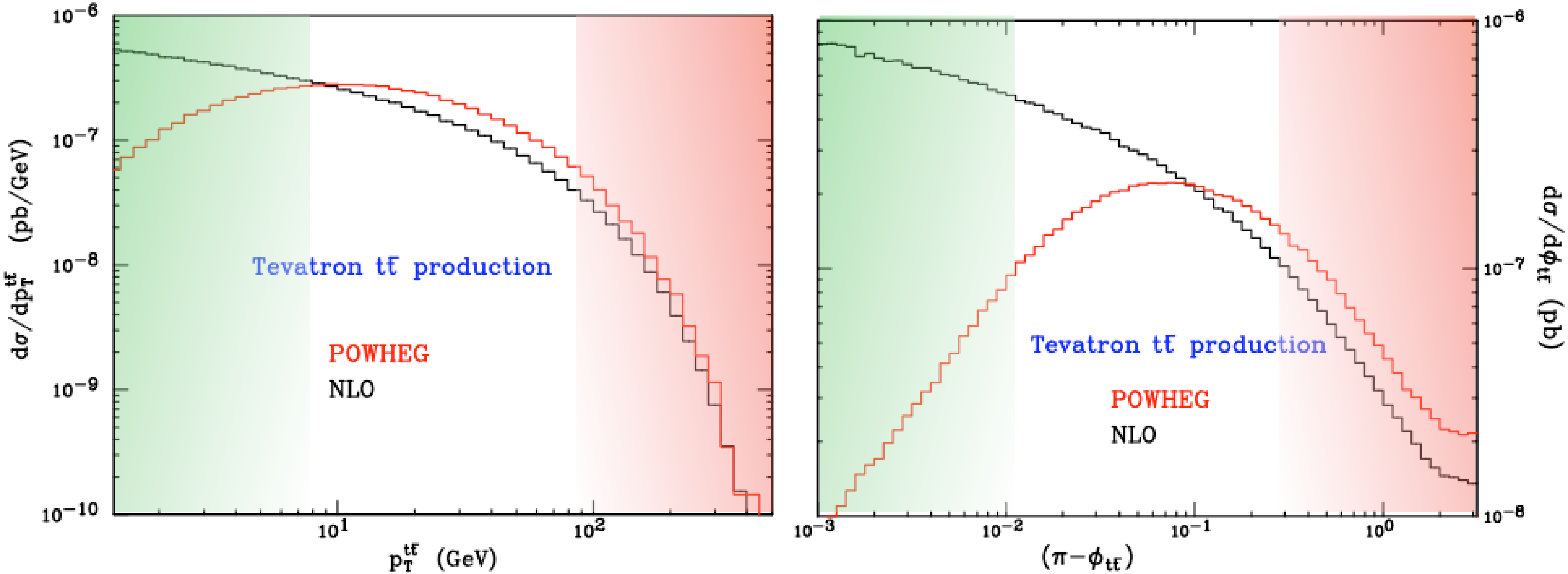
POWHEG

[Nason 2004;
Frixione, Nason, Oleari, 2007]

- Is independent* of the PS. It can be interfaced to PYTHIA, HERWIG or SHERPA.
- Generates only* positive unit weights.

- Can use existing NLO results via the POWHEG-Box [Aioli, Nason, Oleari, Re et al. 2009]
- Method used by HELAC, HERWIG++ and SHERPA [Kardos, Papadopoulos, Trocsanyi 1101.2672], [Hoeche,Krauss, Schooenner, Siegert, 1008.5399]

TTBAR : NLOwPS vs NLO



* Soft/Collinear resummation of the $p_T(tt) \to 0$ region.
* At high $p_T(tt)$ it approaches the tt+parton (tree-level) result.
* When $\Phi(tt)\to 0$ ($\Phi(tt)\to \pi$) the emitted radiation is hard (soft).
* Normalization is FIXED and non trivial!!

NLOwPS

"Best" tools when NLO calculation is available (i.e. low jet multiplicity).

* Main points:
  * NLOwPS provide a consistent to include K-factors into MC's
  * Scale dependence is meaningful
  * Allows a correct estimates of the PDF errors.
  * Non-trivial dynamics beyond LO included for the first time.

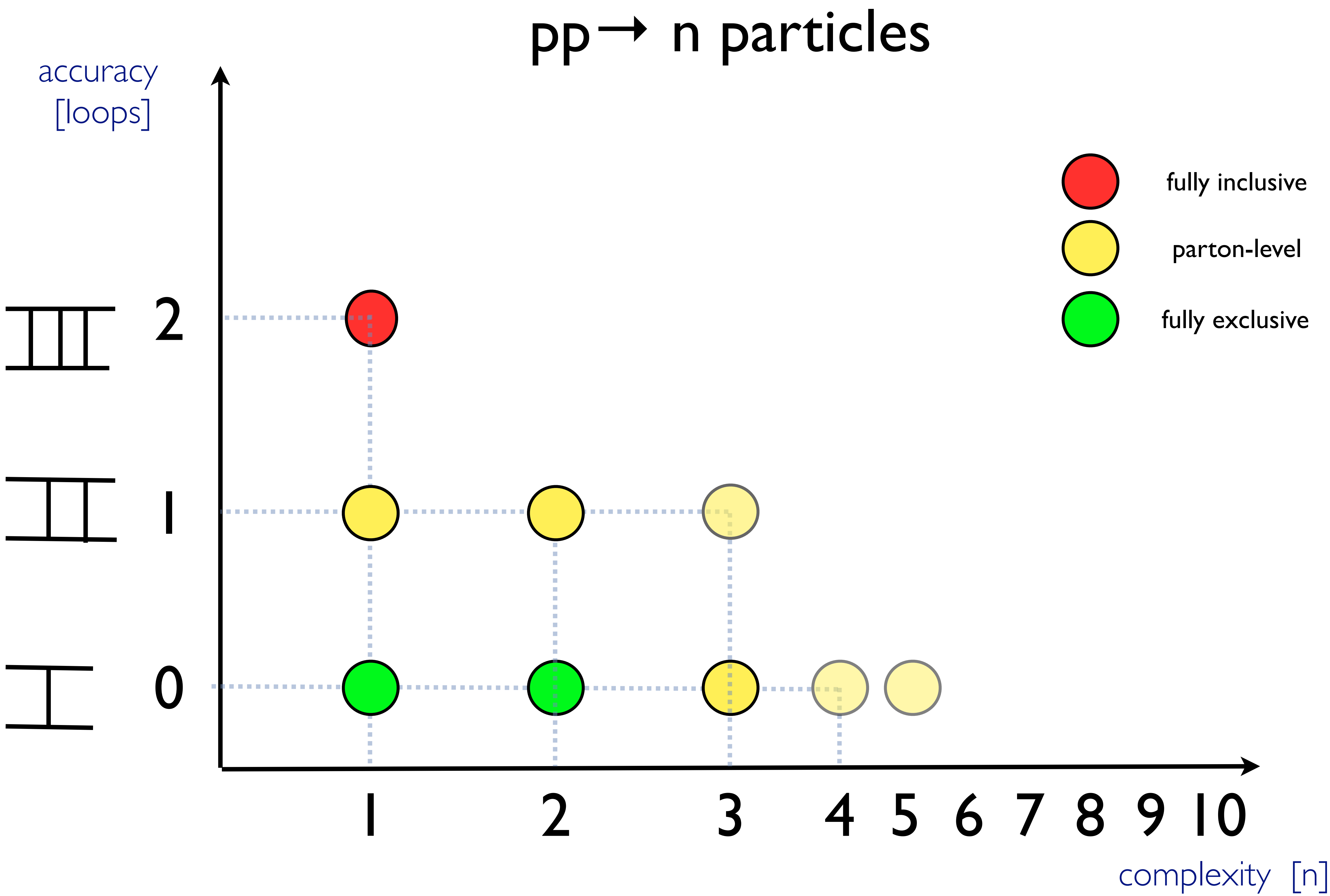
N.B. : The above is true for observables which are at NLO to start with!!!

* Current developments:
  * Upgrading of all available NLO computations to MC's in progres
  * Extendable to BSM without hurdles.
  * Only available for low multiplicity: improvements possible.

Conclusions

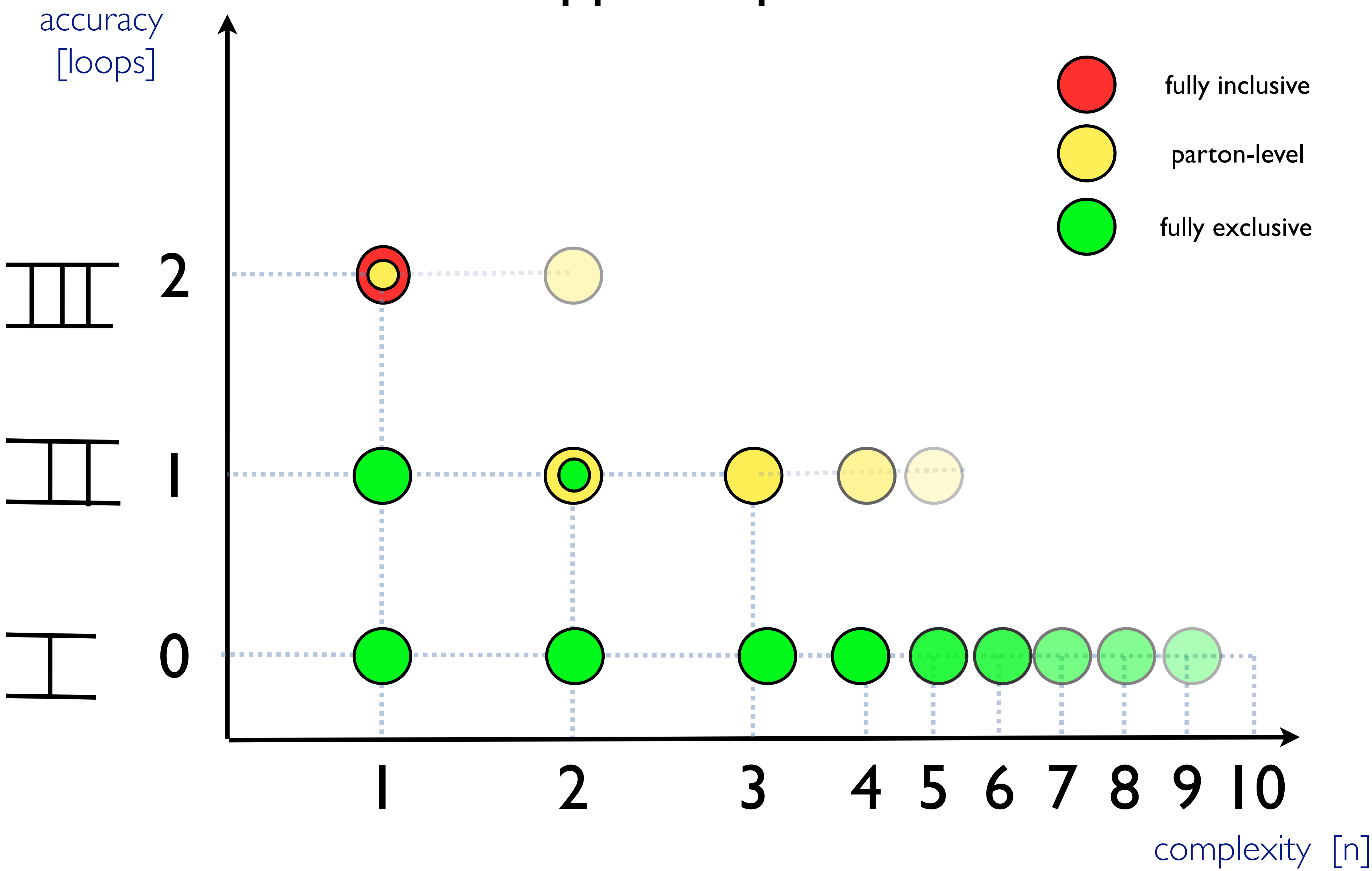
- The need for better description and more reliable predictions for SM processes for the LHC has motivated a significant increase of theoretical and phenomenological activity in the last years, leading to several important achievements in the field of QCD and MC's.
- A complete set of NLO computations is available, even in fully automatic form. Several NNLO results are being used already now and will be extended in the future.
- New techniques and codes available for interfacing at LO and NLO computations at fixed order to parton-shower has been proven for the SM.
- Unprecedented accuracy and flexibility achieved.

SM STATUS CIRCA 2002



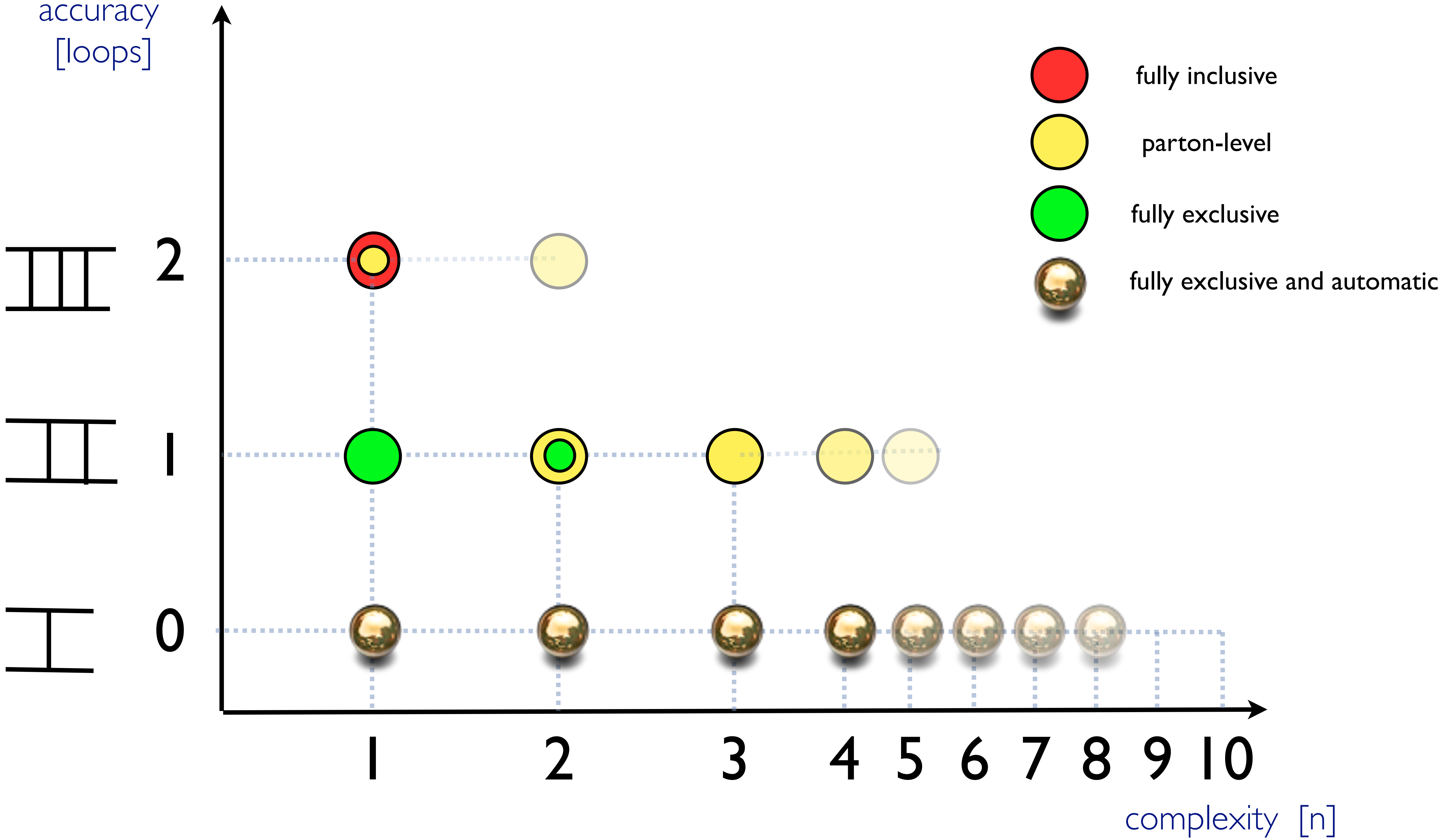
SM STATUS : SINCE 2007

pp → n particles



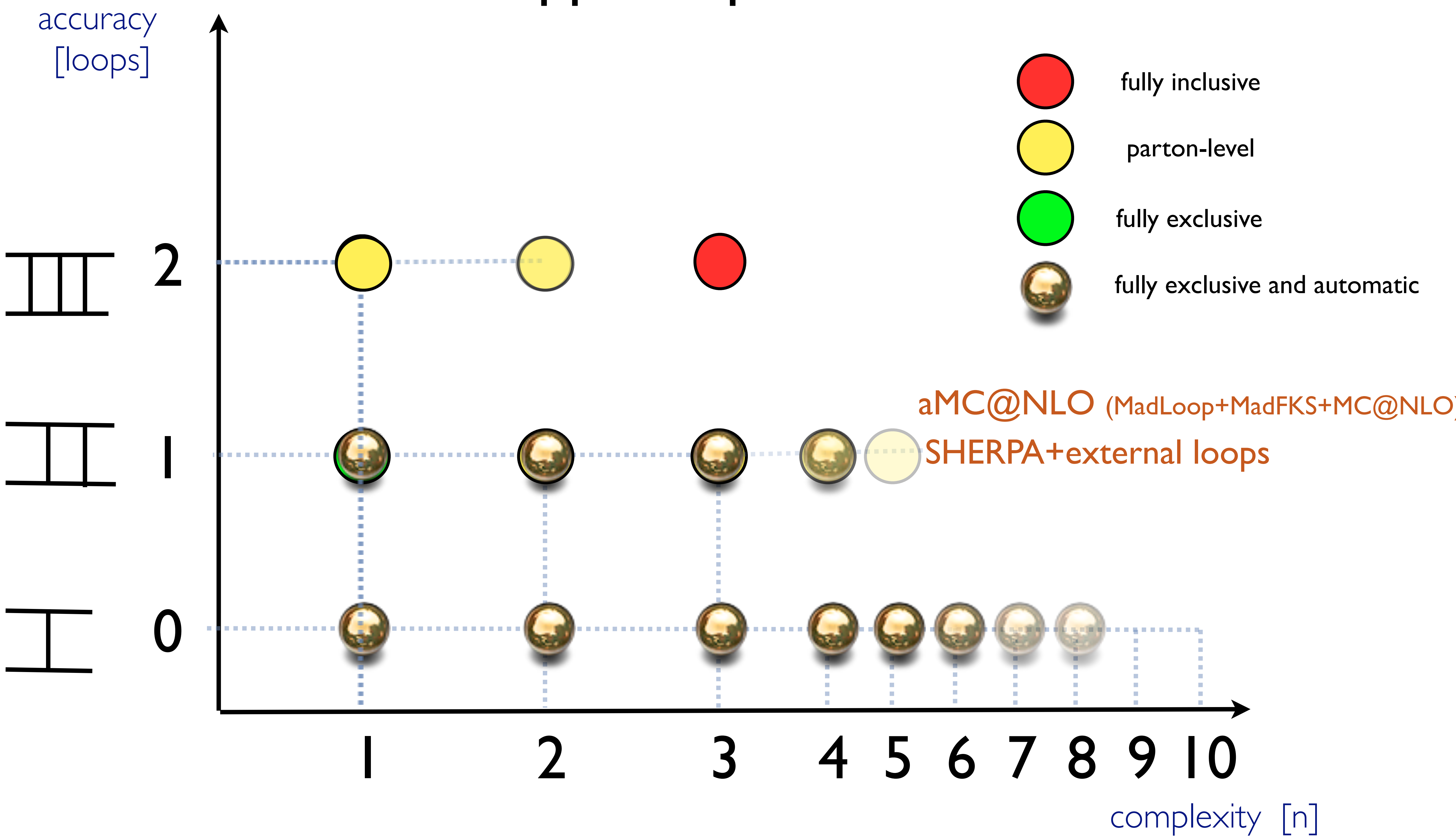
SM Status : since 2007

pp→ n particles



SM STATUS: NOW

pp→ n particles



Conclusions

- The need for better description and more reliable predictions for SM processes for the LHC has motivated a significant increase of theoretical and phenomenological activity in the last years, leading to several important achievements in the field of QCD and MC's.
- A new generation of tools and techniques is now available.
- A complete set of NLO computations is available, even in fully automatic form. Several NNLO results are being used already now and will be extended in the future.
- New techniques and codes available for interfacing at LO and NLO computations at fixed order to parton-shower has been proven for the SM.
- Unprecedented accuracy and flexibility achieved.
- EXP/TH interactions enhanced by a new framework where exps and theos speak the same language.