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Basics of QCD for the LHC

Lecture III

Fabio Maltoni
Center for Particle Physics and Phenomenology (CP3)
Université Catholique de Louvain

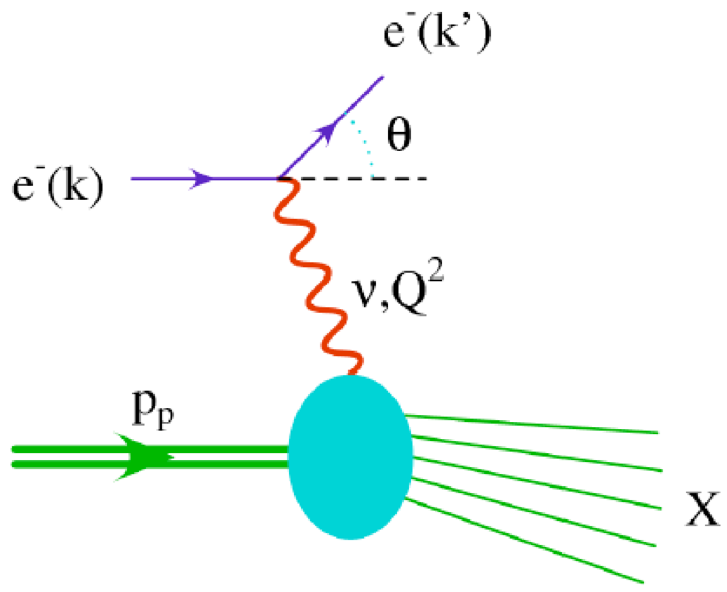
Lectures

1. Intro and QCD fundamentals
2. QCD in the final state
3. QCD in the initial state
4. From accurate QCD to useful QCD

QCD IN THE INITIAL STATE

1. DIS and the parton model
2. DIS with pQCD
3. The idea of factorization
4. Q^2 Evolution and PDF's
5. pp collisions

Deep-inelastic scattering: towards the parton model



$$s = (P+k)^2$$ cms energy^2

$$Q^2 = -(k-k')^2$$ momentum transfer^2

$$x = Q^2/2(P\cdot q)$$ scaling variable

$$\nu = (P\cdot q)/M = E - E'$$ energy loss

$$y = (P\cdot q)/(P\cdot k) = 1 - E'/E$$ rel. energy loss

$$W^2 = (P+q)^2 = M^2 + \frac{1-x}{x}Q^2$$ recoil mass

"deep inelastic" : $Q^2 >> 1$ GeV^2

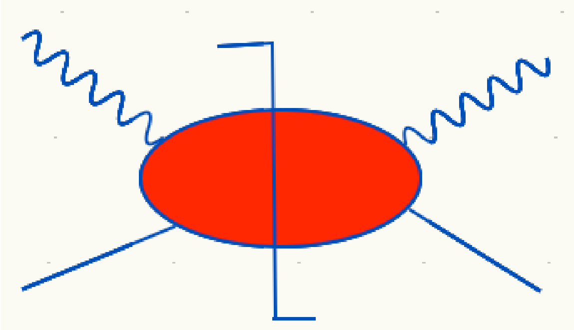
"scaling limit": $Q^2 \to \infty$, x fixed

The idea is that by measuring all the kinematics variables of the outgoing electron one can study the structure of the proton in terms of the probe characteristics, Q2,x,y...Very inclusive measurement from the QCD point of view.

Deep-inelastic scattering: towards the parton model

* Divide phase-space factor into a leptonic and a hadronic part:

$$d\Phi = \frac{d^3k'}{(2\pi)^3 2E'} d\Phi_X = \frac{ME}{8\pi^2} y\, dy\, dx\, d\Phi_X$$



* Separate also the square of the Feynman amplitude, by defining:

$$\frac{1}{4}\sum |\mathcal{M}|^2 = \frac{e^4}{Q^4} L^{\mu\nu} h_{X\mu\nu}$$

* The leptonic tensor can be calculated explicitly:

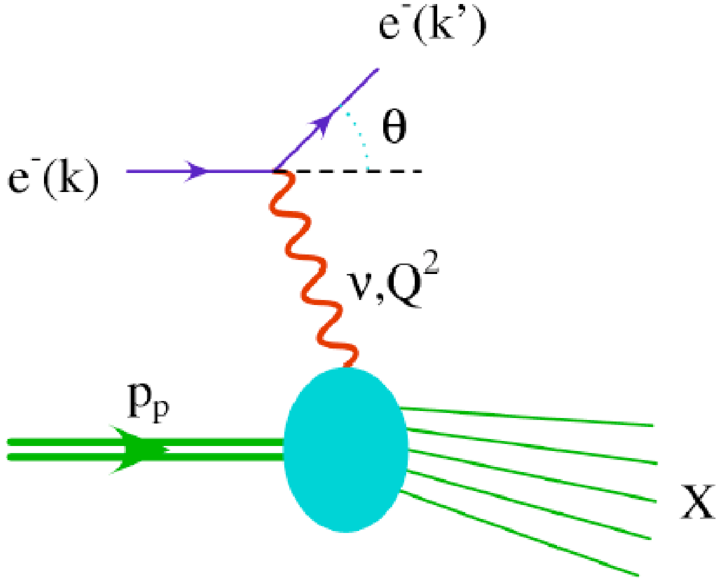
$$L^{\mu\nu} = \frac{1}{4}\mathrm{tr}[\not{k}\gamma^\mu \not{k}'\gamma^\nu] = k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu} k\cdot k'$$

* Combine the hadronic part of the amplitude and phase space into "hadronic tensor" and use just Lorentz symmetry and gauge invariance to write

$$W^{\mu\nu} = \sum_X \int d\Phi_X h_{X\mu\nu}$$

$$W_{\mu\nu}(p,q) = \left(-g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right)\frac{1}{p\cdot q} F_2(x,Q^2)$$

DEEP-INELASTIC SCATTERING: TOWARDS THE PARTON MODEL



$$\sigma^{ep\to eX} = \sum_X \frac{1}{4ME} \int d\Phi \, \frac{1}{4} \sum_{\rm spin} |\mathcal{M}|^2$$

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left\{ [1+(1-y)^2] F_1(x,Q^2) + \frac{1-y}{x} \left[F_2(x,Q^2) - 2x F_1(x,Q^2) \right] \right\}$$

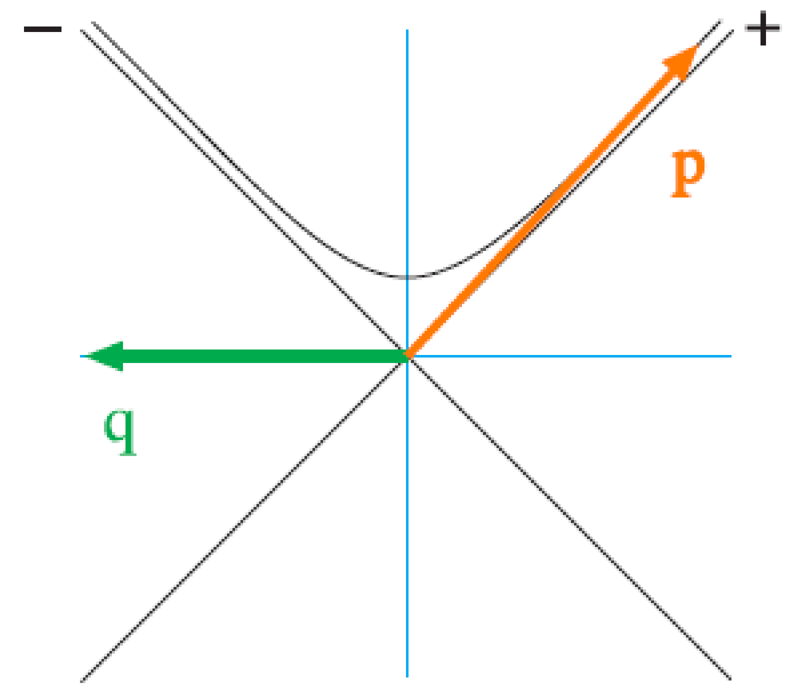
Comments:

* Different y dependence can differentiate between F_1 and F_2
* The first term represents the absorption of a transversely polarized photon, the second of a longitudinal one.
* Bjorken scaling ⇒ F_1 and F_2 obey scaling themselves, i.e. they do not depend on Q.

Deep-inelastic scattering: towards the parton model

We want to "watch" the scattering from a frame where the physics is clear. Feynman suggested that what happens can be best understood in a reference frame where the proton moves very fast and $Q \gg m_h$ is large.

4-vector	hadron rest frame	Breit frame
$(p^+, p^-, \vec{p}_T)$	$\frac{1}{\sqrt{2}}(m_h, m_h, \vec{0})$	$\frac{1}{\sqrt{2}}(\frac{Q}{x}, \frac{x m_h^2}{Q}, \vec{0})$
$(q^+, q^-, \vec{q}_T)$	$\frac{1}{\sqrt{2}}(-m_h x, \frac{Q^2}{m_h x}, \vec{0})$	$\frac{1}{\sqrt{2}}(-Q, Q, \vec{0})$

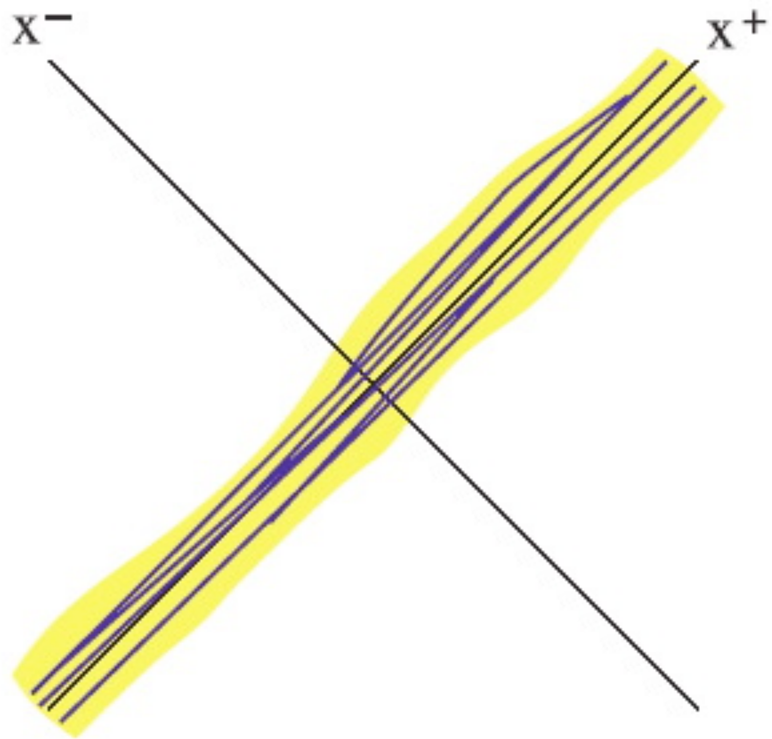


You can check that a Lorentz transformation acts on a light-cone formulation of the four-momentum:

$$(a^+, a^-, \vec{a}) \to (e^{\omega} a^+, e^{-\omega} a^-, \vec{a}) \quad \text{with} \quad e^{\omega} = Q/(x m_h)$$

Deep-inelastic scattering: towards the parton model

Now let's see how the proton looks in this frame, and in the light-cone space coordinates (suitable for describing relativistic particles).



Lorentz transformation divides out the interactions. Hadron at rest has separation of order:

$\Delta x^{+} \sim \Delta x^{-} \sim 1/m$,

while in the moving hadron has:

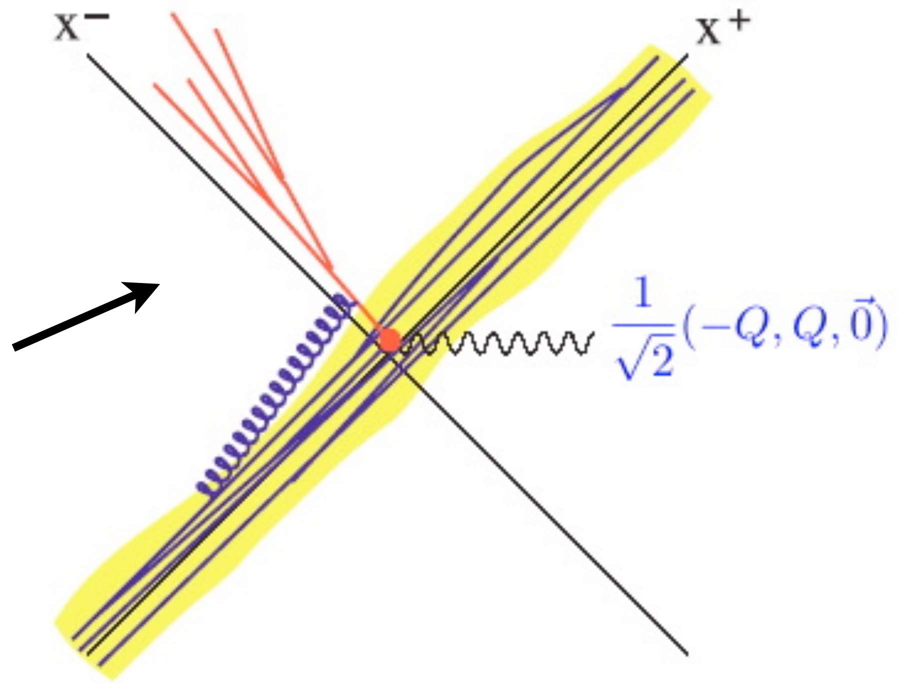
$\Delta x^{+} \sim 1/m \times Q/m = Q/m^2$ LARGE

$\Delta x^{-} \sim 1/m \times m/Q = 1/Q$, SMALL

Deep-inelastic scattering: towards the parton model

And now let the virtual photon hit the fast moving hadron:

Struck quark kicked into the x-direction



Moving hadron has:

$\Delta x^+ \sim Q/m^2$,

interaction with photon $q^- \sim Q$ is localized within

$\Delta x^+ \sim 1/Q$,

thus quarks and gluons are like partons and effectively free.

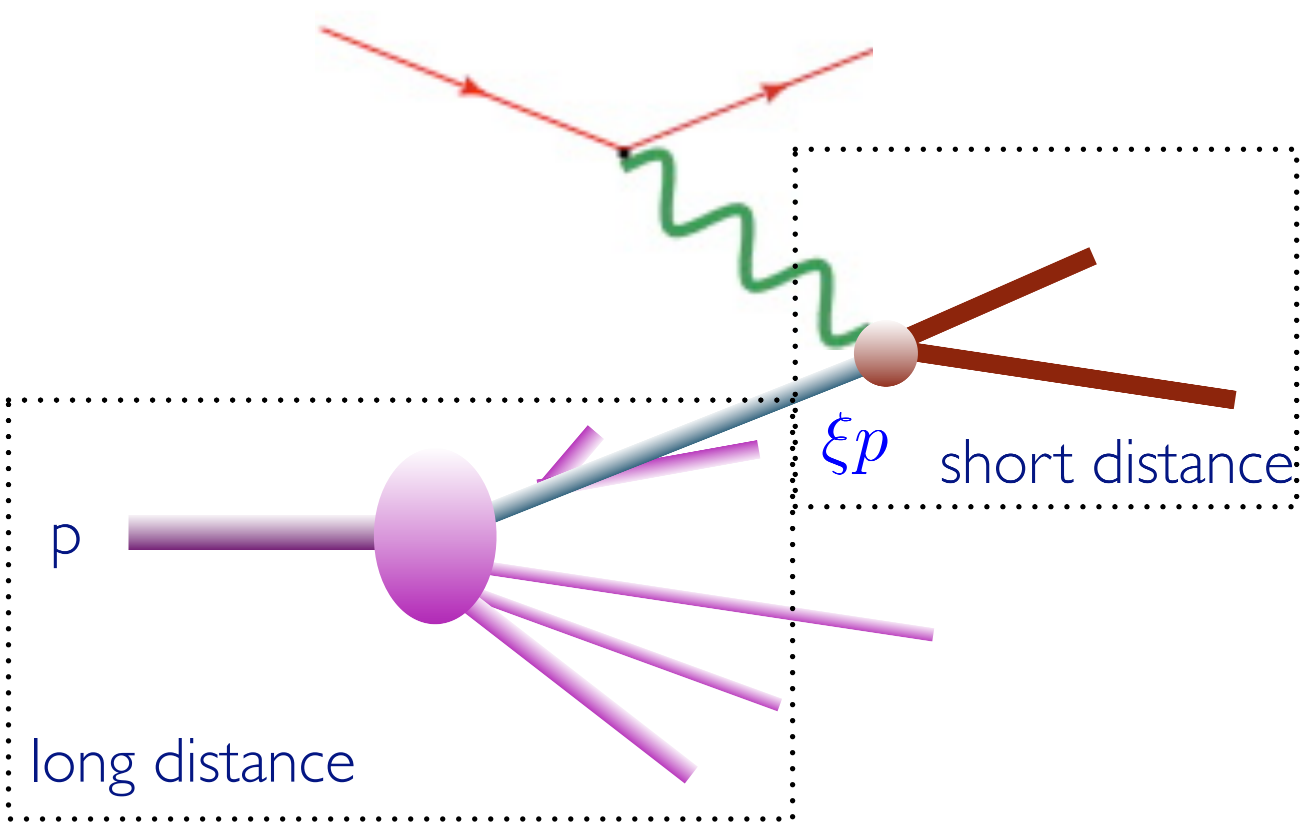
In this frame the time scale of a typical parton-parton interaction is much larger than the hard interaction time.

So we can picture the hadron as an incoherent flux of partons $(p^+, p^-, p^\perp)_i$, each carrying a fraction $0 < \xi_i = p_i^+/p^+ < 1$ of the total available momentum.

Deep-inelastic scattering: towards the parton model

The space-time picture suggests the possibility of separating short- and long-distance physics ⇒ factorization! Turned into the language of Feynman diagrams DIS looks like:

$$\frac{d^2\sigma}{dx dQ^2} = \int_0^1 \frac{d\xi}{\xi} \sum_i f_i(\xi) \frac{d^2\sigma}{d\hat{x} dQ^2}\left(\frac{x}{\xi}, Q^2\right)$$



where

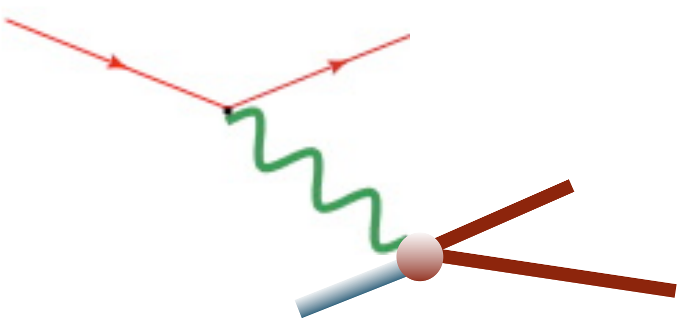
$f_{i/h}(\xi)$ is the probability to find a parton with flavor i in an hadron h carrying a light-cone momentum ξp+

$\frac{d^2\hat{\sigma}}{d\hat{x} dQ^2}$ is the cross section for electron-parton scattering

Deep-inelastic scattering: towards the parton model

We can now explain scaling within the parton model:

Let's take the LO computation we performed for e+e- → qq, cross it (which also mean to be careful with color), and use it the DIS variables to express the differential cross section in dQ^2



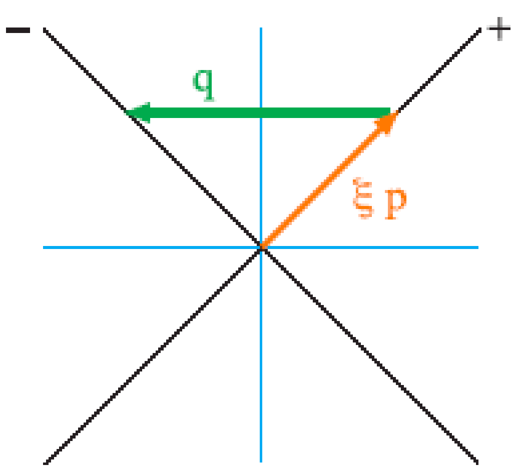
$$\frac{d\hat{\sigma}}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}\left[1 + (1-y)^2\right]$$

Notice that the outgoing quark is on its mass shell:

$$\xi p^+ + q^+ = 0$$

$$p^+ = Q/(x\sqrt{2})$$

$$q^+ = -Q/\sqrt{2}$$



$$\frac{d^2\hat{\sigma}}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4}\frac{1}{2}\left[1 + (1-y)^2\right]\delta(x-\xi)$$

This implies that $\xi = x$ at LO!

Deep-inelastic scattering: towards the parton model

We can now compare with our "inclusive" description of DIS in terms of structure functions (which, BTW, are physical measurable quantities),

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{[1+(1-y)^2]F_1(x,Q^2) + \frac{1-y}{x}\left[F_2(x,Q^2) - 2xF_1(x,Q^2)\right]\right\}$$

with our parton model formulas:

$$\frac{d^2\sigma}{dx dQ^2} = \int_0^1 \frac{d\xi}{\xi}\sum_i f_i(\xi)\frac{d^2\sigma}{d\hat{x}dQ^2}\left(\frac{x}{\xi},Q^2\right) \quad \text{with} \quad \frac{d^2\hat{\sigma}}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4}\frac{1}{2}\left[1+(1-y)^2\right]e_q^2\,\delta(x-\xi)$$

we find (be careful to distinguish x and ξ)

$$F_2(x) = 2xF_1 = \sum_{i=q,\bar{q}}\int_0^1 d\xi\, f_i(\xi)\, x e_q^2 \delta(x-\xi) = \sum_{i=q,\bar{q}} e_q^2\, x f_i(x)$$

* So we find the scaling is true: no dependence on Q^2.
* q and qbar enter together : no way to distinguish them with NC. Charged currents are needed.
* $F_L(x) = F_2(x) - 2 F_1(x)$ vanishes at LO (Callan-Gross relation), which is a test that quarks are spin 1/2 particles! In fact if the quarks where scalars we would have had $F_1(x) = 0$ and $F_2=F_L$.

Deep-inelastic scattering: towards the parton model

Probed at scale Q, sea contains all quarks flavours with mq less than Q.
For Q ~1 we expect

$$\begin{aligned} u(x) &= u_V(x) + \bar{u}(x) \\ d(x) &= d_V(x) + \bar{d}(x) \\ s(x) &= \bar{s}(x) \end{aligned}$$

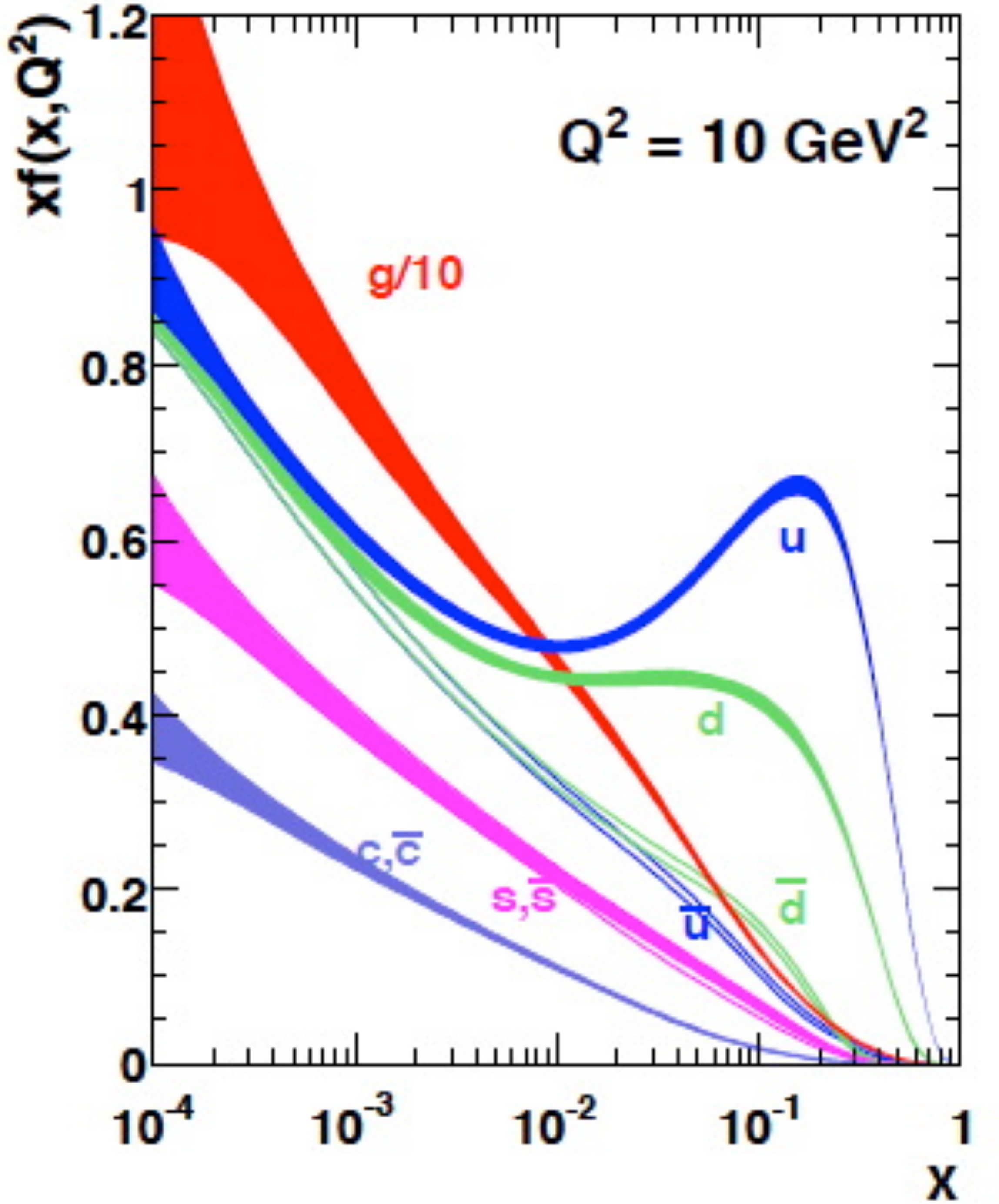
$$\int_0^1 dx\, u_V(x) = 2\,, \quad \int_0^1 dx\, d_V(x) = 1\,.$$

And experimentally one finds

$$\sum_q \int_0^1 dx\, x[q(x) + \bar{q}(x)] \simeq 0.5\,.$$

Thus quarks carry only about 50% of proton's momentum. The rest is carried by gluons. Although not directly measured in DIS, gluons participate in other hard scattering processes such as large-pt and prompt photon production.

Quark and gluon distribution functions



Comments:

The sea is NOT SU(3) flavor symmetric.

The gluon is huge at small x

There is an asymmetry between the ubar and dbar quarks in the sea.

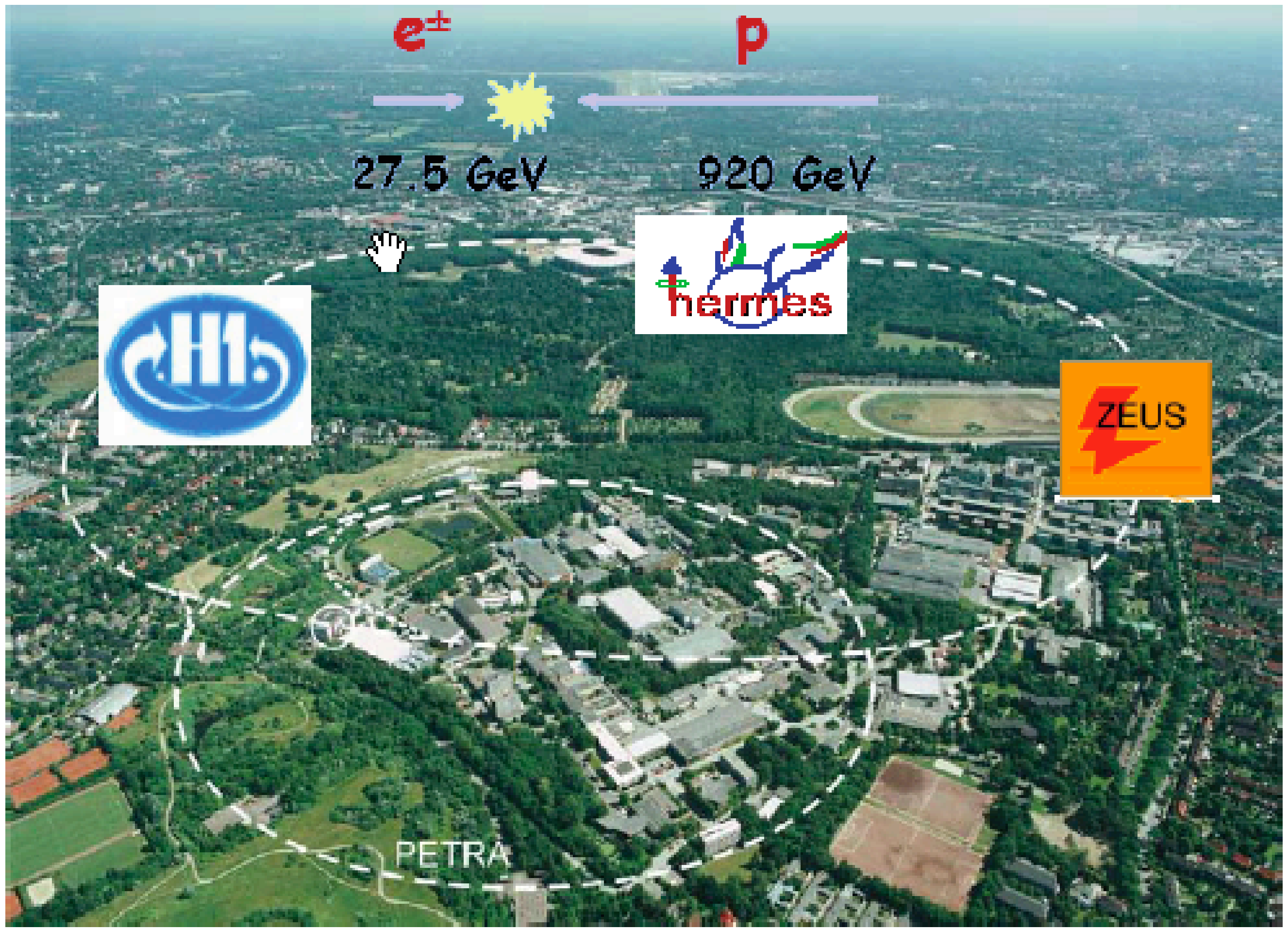
Note that there are uncertainty bands!!

Questions:

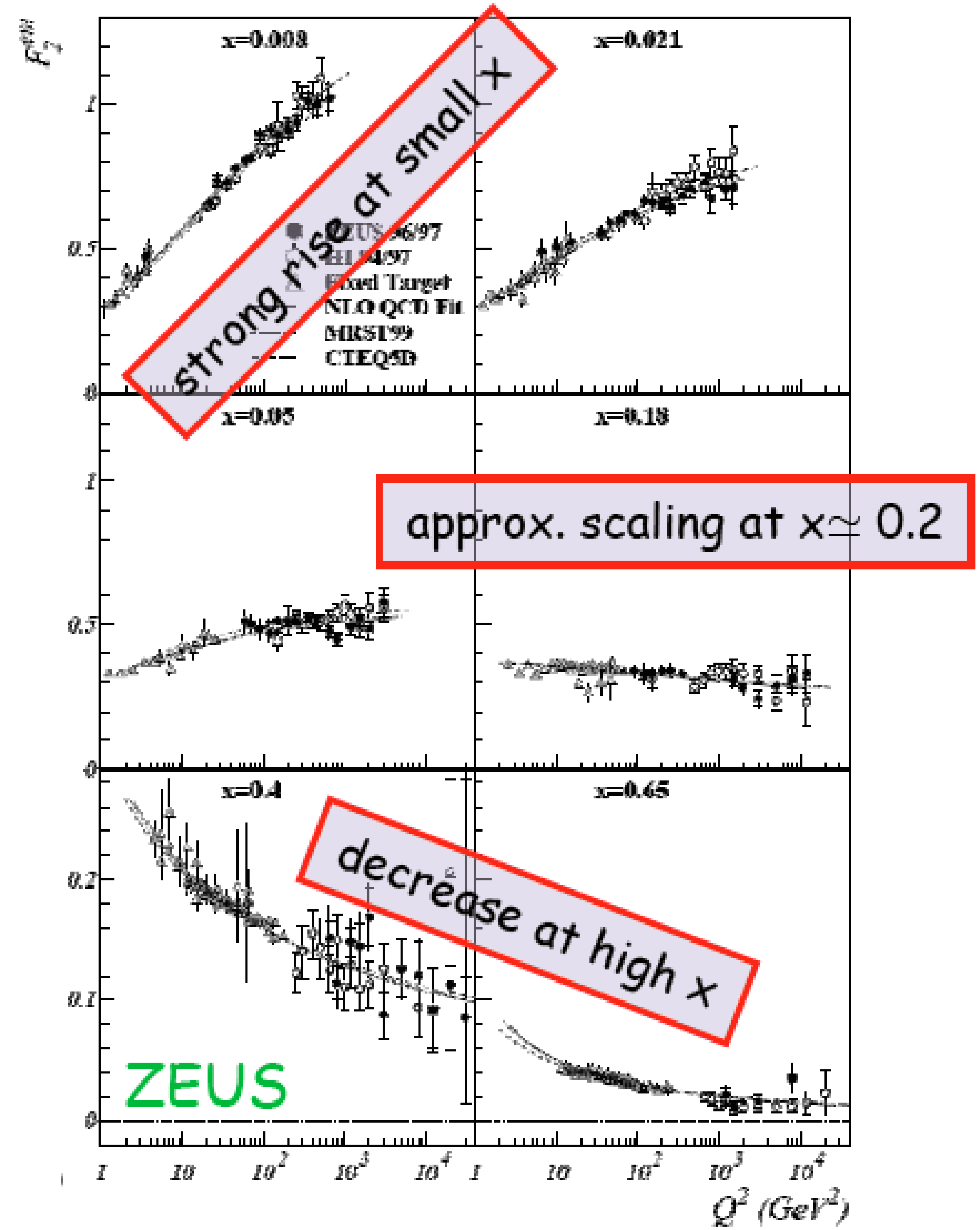
1. What has QCD to say about the naïve parton model?
2. Is the picture unchanged when higher order corrections are included?
3. Is scaling exact?

Scaling violations

first ep collider

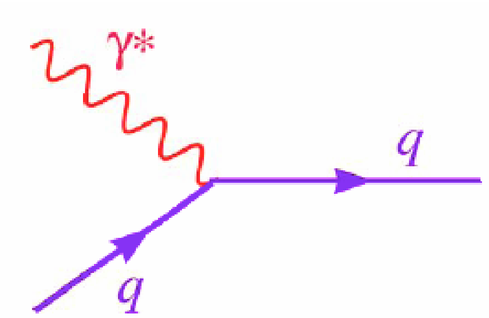


At HERA scaling violations were observed!



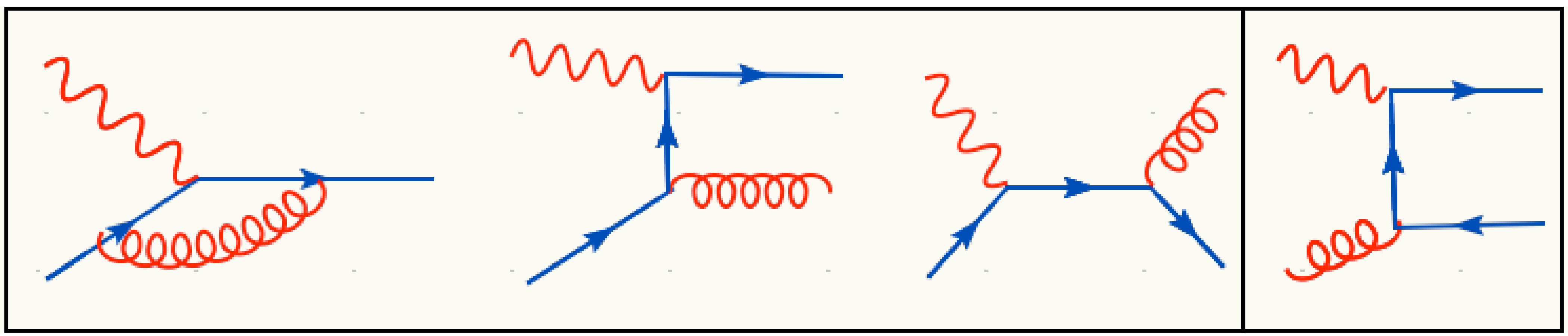
Deep-inelastic scattering in QCD

We got a long way without even invoking QCD. Let's do it now.



The first diagram to consider is the same as in the parton model:

At NLO we find again both real and virtual corrections:



α_S corrections to the LO process photon-gluon fusion

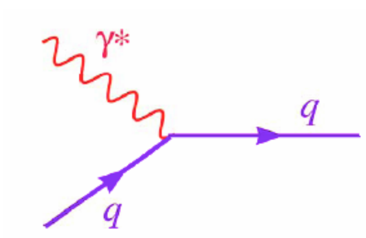
Our experience so far: have to expect IR divergences!
In order to make the intermediate steps of the calculation finite, we introduce a regulator, which will be removed at the end.

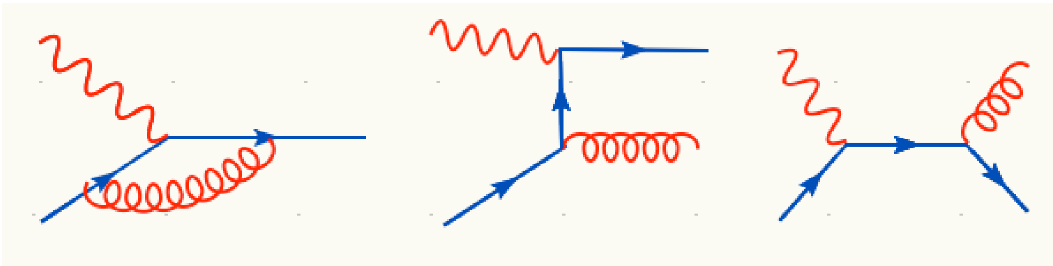
Dimensional regularization is the best choice to perform serious calculations.
However for illustrative purposes other regulators (that cannot be easily used beyond NLO) are better suited. We'll use here a small quark/gluon mass.

Deep-inelastic scattering in QCD

Once we compute the diagrams we indeed find that UV and soft divergences all cancel, but for a collinear divergence arising when the emitted gluon becomes collinear to the incoming quark:

$$\frac{d^2\hat{\sigma}}{dx\,dQ^2}\Big|_{F_2} \equiv \hat{F}_2^q$$



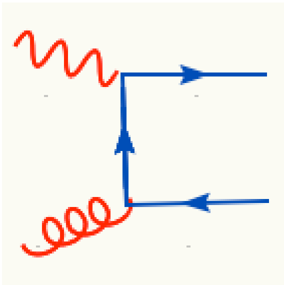


$$= e_q^2 x \left[\delta(1-x) + \frac{\alpha_S}{4\pi}\left[P_{qq}(x)\log\frac{Q^2}{m_g^2} + C_2^q(x)\right]\right]$$

$$\frac{d^2\hat{\sigma}}{dx\,dQ^2}\Big|_{F_2} \equiv \hat{F}_2^g$$

$$= \sum_q e_q^2 x \left[0 + \frac{\alpha_S}{4\pi}\left[P_{qg}(x)\log\frac{Q^2}{m_q^2} + C_2^g(x)\right]\right]$$

IR cutoff



The presence of large logs is a clear sign that we have a residual infrared sensitivity that we have to deal with!

Deep-inelastic scattering in QCD

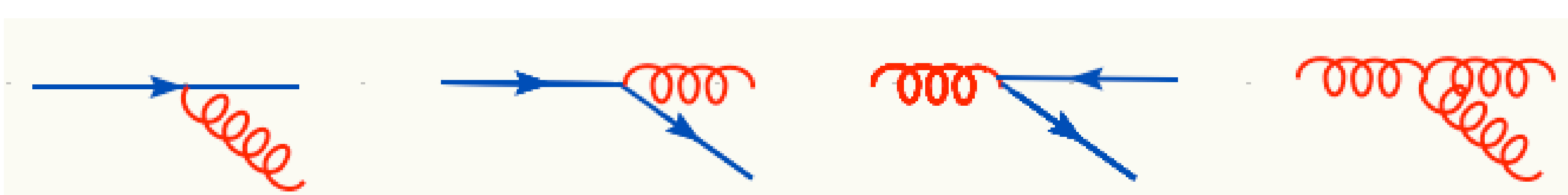
Important observations:

1. Large logarithms of Q^2/m^2 or ($1/\varepsilon$ in dim reg) incorporate ALL the RESIDUAL long-distance physics left after summing over all real and virtual diagram. This terms are of a collinear nature.

2. The coefficients $P_{ij}(x)$ that multiply the log's are UNIVERSAL and calculable in perturbative QCD.

They are called SPLITTING FUNCTIONS and their physical meaning is easy to give:

$P_{ij}(x)$ give the probability that a parton j splits collinearly into a parton i + something else carrying a momentum fraction x of the original parton j.



Pqq(x) Pgq(x) Pqg(x) Pgg(x)

Deep-inelastic scattering in QCD

So the natural question is: what is it that is going wrong? Do we have IR sensitiviness in a physical observable? Well not yet!!

To obtain the physical cross section we have to convolute our partonic results with the parton densities, as we have learned from the parton model.

For instance:

$$F_2^q(x,Q^2) = x\sum_{i=q,\bar{q}} e_q^2 \left[f_{i,0}(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} f_{i,0}(\xi)\left[P_{qq}(\frac{x}{\xi})\log\frac{Q^2}{m_g^2} + C_2^q(\frac{x}{\xi})\right]\right]$$

And now comes the magic: as long as the divergences are universal and do not depend on the hard scattering functions but only on the partons involved in the splitting, we can reabsorb the dependence on the IR cutoff (once for all!) into $f_{q,0}(x)$:

$$f_q(x,\mu_f) \equiv f_{q,0}(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) P_{qq}(\frac{x}{\xi})\log\frac{\mu_f^2}{m_g^2} + z_{qq}$$

"Renormalized" parton densities: we have factorized the IR collinear physics into a quantity that we cannot calculate but it is universal. So how does the final result looks like?

Deep-inelastic scattering in QCD

The structure function is a MEASURABLE object, therefore, at all orders, it cannot depend on the choice of scales.
This will lead exactly to the same concepts of renormalization group invariance that we found in the UV.

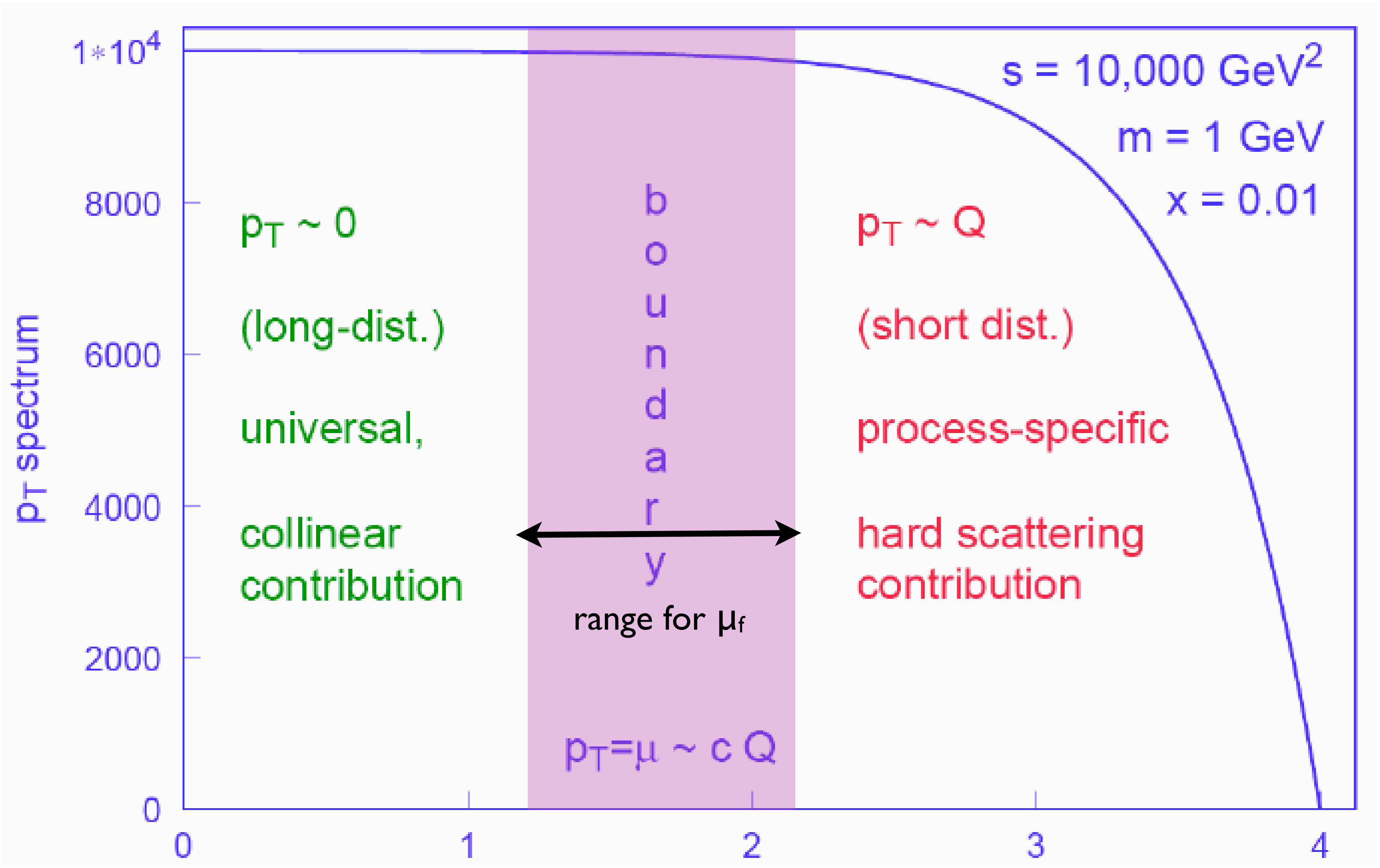
The final result depends of course also on α_S and therefore to the choice of the renormalization scale.

$$F_2^q(x,Q^2) = x \sum_{i=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_i(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_S(\mu_r)}{2\pi} \left[P_{qq}(\frac{x}{\xi}) \log\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})(\frac{x}{\xi}) \right] \right]$$

Long distance physics is universally factorized into the parton distribution functions. These cannot be calculated in pQCD. They depend on μ_f in the exact way so as to cancel the overall dependence at all orders.

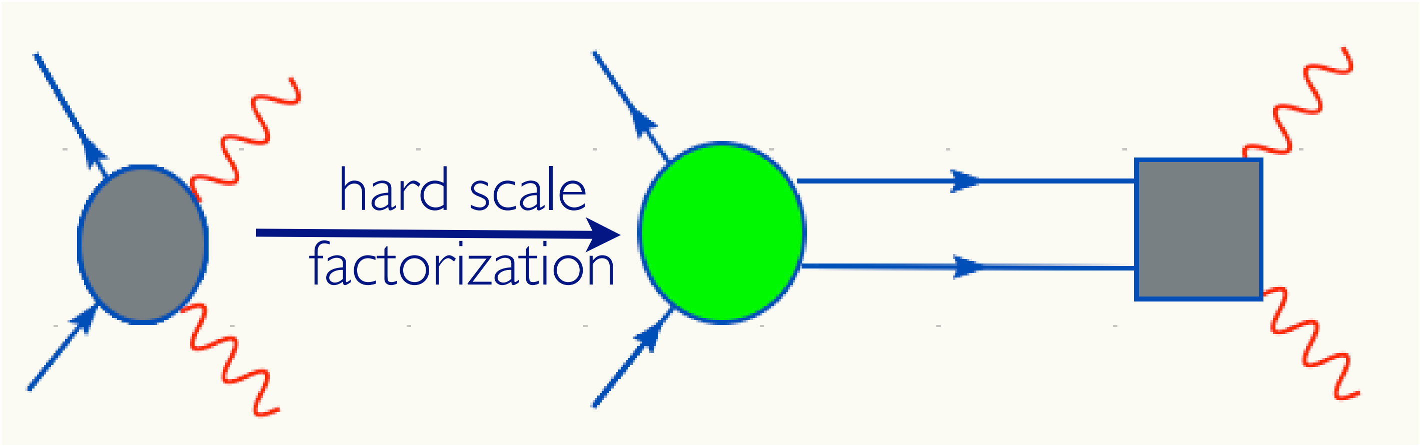
Short-distance (Wilson coefficient), perturbative calculable and finite. It depends on the factorization scale. It also depends on finite terms which define the factorization scheme.

Deep-inelastic scattering in QCD



$$\log(p_T^2 + m^2)/m^2$$

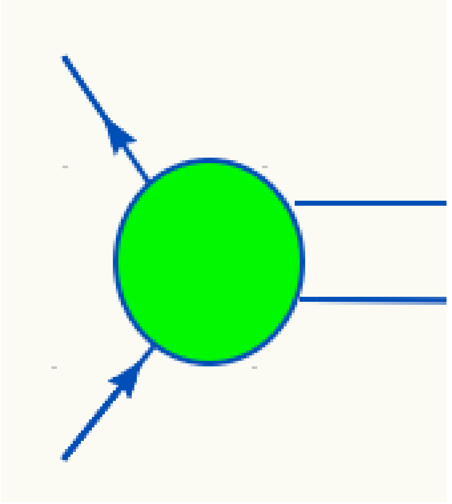
Deep-inelastic scattering in QCD

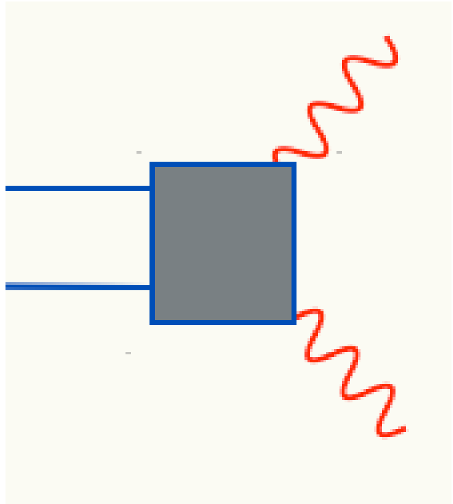


$$e.g. \quad F_2(x,Q^2) \sim \sum_i f_i(x,\mu_f) \otimes \hat{F}_2(x, \frac{Q}{\mu_f})$$

The separation between long- and short-distance physics is not unique.

long-distance
parton density





short-distance
Wilson coefficient

1. choice of μ_f : defines the borderline between long/short distances (see in a moment...)
2. choice of scheme: conventional. It reshuffle finite pieces. Nowadays, for pdf, we all use $\overline{\mathrm{MS}}$ scheme. The essential thing to keep in mind is that the same scheme has to be used for both the short and long-distance quantities.

Factorization in a nutshell

$$\frac{\sigma}{\sigma_0} = C(Q/\mu) \otimes f(\mu) + E = \sum_{n=0}^{N} \alpha_S^n(\mu) C^{(n)} \otimes f + D_N + E$$

Here C is the coefficient function (short-distance), f the parton densities (long-distance matrix element), Q is a kinematic variable for the hardness of the process, while $\boldsymbol{\sigma}_0$ is some convenient normalization to make the right-hand side dimensionless, and E represent the power corrections.
D_N represent the truncation error.

The power of factorization holds in the following statements:

* We don't know how to perform an exact calculation of the physical cross section from QCD

* But we can calculate finite order approximations to the coefficient functions and to the evolution kernels of the parton densities.

* However, in practice only rather low-order calculations can be actually done. Hence we only make approximate predictions, using truncated coefficient functions and evolution kernels.

* Although we do not know how to calculate pdfs from QCD, universality enables predictions to be made: pdfs are measured in a set of experiments and then used in other measurements.

Factorization

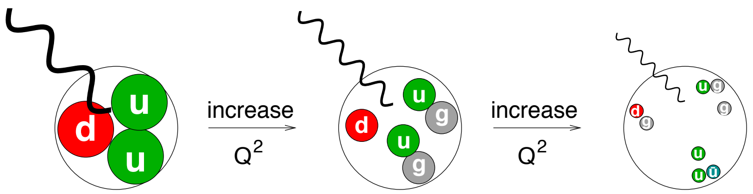
$$F_2^q(x,Q^2) = x \sum_{i=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_i(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_S(\mu_r)}{2\pi} \left[P_{qq}(\frac{x}{\xi}) \log\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})(\frac{x}{\xi}) \right] \right]$$

Questions:

1. Can we exploit the fact that physical quantities have to be scale independent to gain information on the pdfs?

2. What exactly have we gained in hiding the large logs in the redefined pdf's? Aren't we just hiding the problem?

Evolution



$$F_2(x,Q^2) \sim \sum_i f_i(x,\mu_f) \otimes \hat{F}_2(x,\frac{Q}{\mu_f})$$

As a first step it is very convenient to transform the nasty convolution into a simple product. This can be done with the help of a Mellin transform:

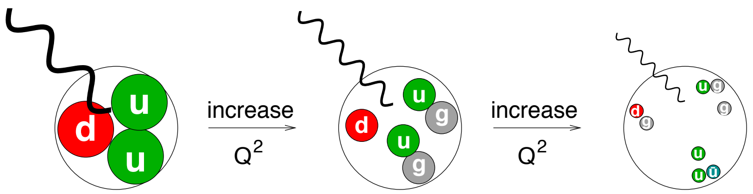
$$f(N) \equiv \int_0^1 dx x^{N-1} f(x)$$

Let us show that a Mellin transform turns a convolution into a simple product:

$$\int_0^1 dx x^{N-1}\left[\int_x^1 \frac{dy}{y} f(y) g(\frac{x}{y})\right] = \int_0^1 dx x^{N-1} \int_0^1 dy \int_0^1 dz \delta(x-zy) f(y) g(z)$$

$$= \int_0^1 dy \int_0^1 dz (zy)^{N-1} f(y) g(z) = f(N) g(N)$$

Evolution



$$F_2(x,Q^2) \sim \sum_i f_i(x,\mu_f) \otimes \hat{F}_2(x,\frac{Q}{\mu_f})$$

Let's now apply it to F_2

$$\frac{dF_2(x,Q^2)}{d\log\mu_f} = 0$$

we get:

$$\frac{dq(N,\mu_f)}{d\log\mu_f}\hat{F}_2(N,\frac{\mu_f}{Q}) + q(N,\mu_f)\frac{d\hat{F}_2(N,\frac{\mu_f}{Q})}{d\log\mu_f} = 0$$

$$\frac{d\log\hat{F}_2(N,\frac{Q}{\mu_f})}{d\log\frac{Q}{\mu_f}} = \frac{d\log q(N,\mu_f)}{d\log\mu_f} = -\gamma_{qq}(N)$$

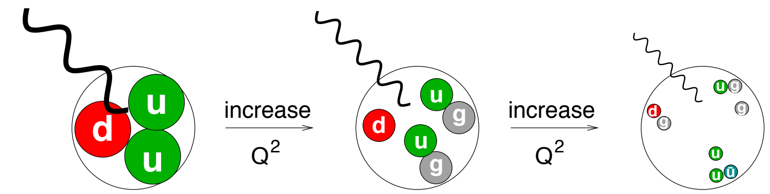
These are called anomalous dimensions and are just the Mellin transform of the corresponding splitting function

whose solution is:

$$q(N,\mu) = q(N,\mu_0)e^{-\gamma_{qq}(N)\log(\frac{\mu_f}{\mu_0})}$$

The pdf "evolves" with the scale!

Scaling violations



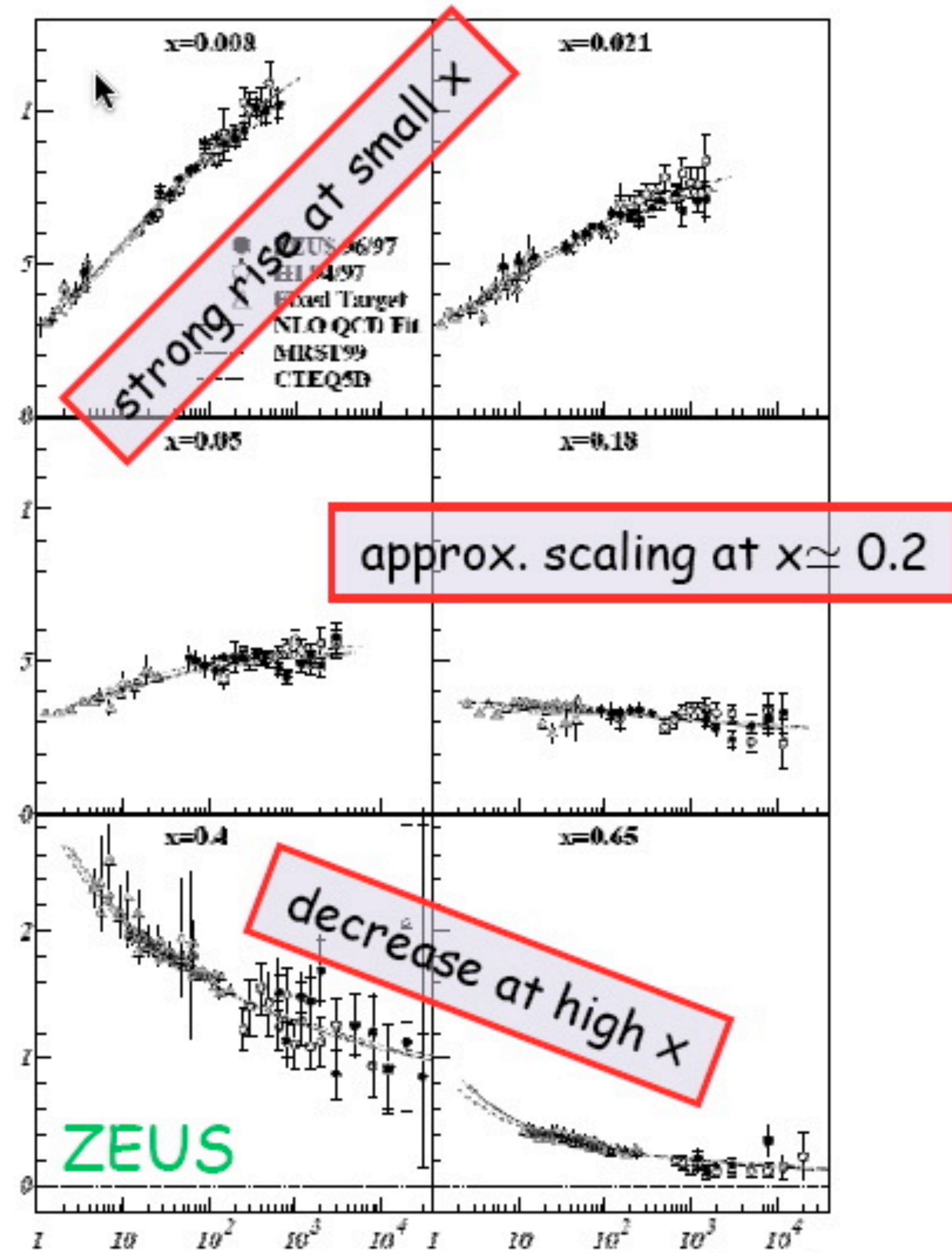
The solution for V can be rewritten in terms of t and α_S as follows:

$$\tilde{V}(N,t) = \tilde{V}(N,t)\left(\frac{\alpha_S(t_0)}{\alpha_S(t)}\right)^{d_{qq}(N)}$$

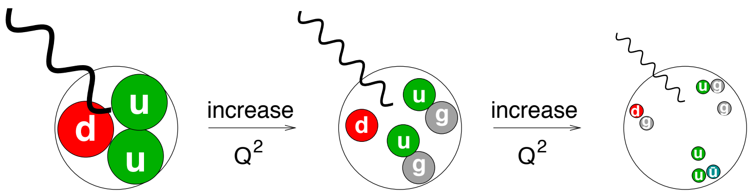
where

$$d_{qq}(N) = \gamma_q q^{(0)}/2\pi b_0$$

Now $d_{qq}(1)=0$ and $d_{qq}(N) <0$ for N>1. Thus as t increases V decreases at large x and increases at small x. Physically this is due to an increase in the phase space for gluon emission by quarks as t increases, leading to a loss of momentum.



Evolution



In fact the equations are a bit more complicated as quarks and gluons do mix.
It is convenient to introduce two linear combinations, the singlet **Σ** and the non-singlet q^{NS} to separate the piece that mixes with that that does not:

$$\Sigma(x, Q^2) = \sum_{i=1}^{n_f} (q_i(x, Q^2) + \bar{q}_i(x, Q^2))$$

this is coupled to the gluon

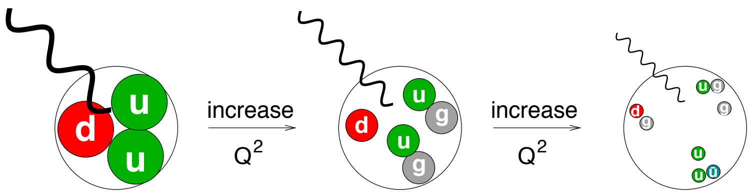
$$q^{\mathrm{NS}}(x, Q^2) = q_i(x, Q^2) - \bar{q}_j(x, Q^2)$$

these evolve independently

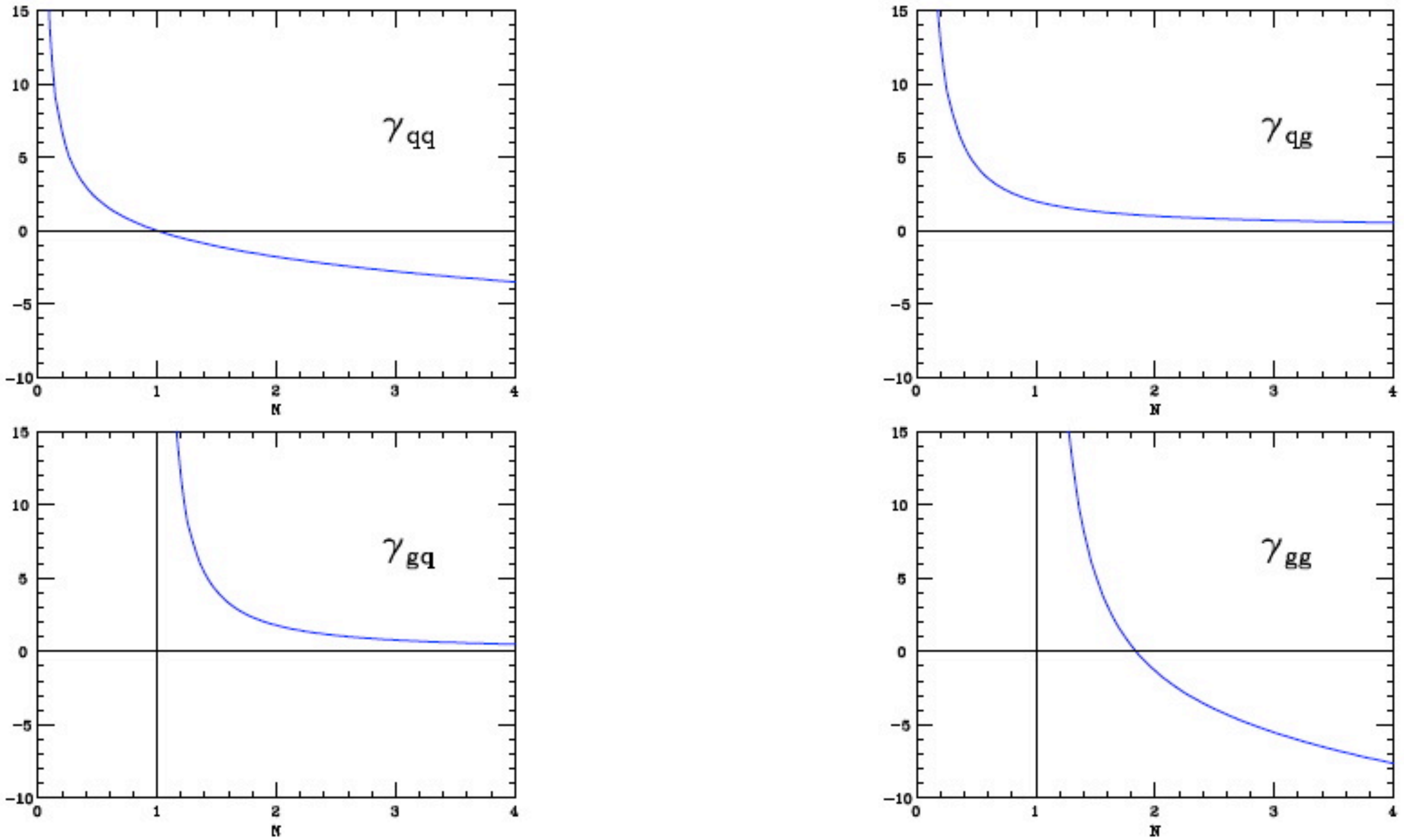
The complete evolution equations (in Mellin space) to solve are:

$$\frac{d}{dt} \Delta q^{\mathrm{NS}}(N, Q^2) = \frac{\alpha_S(t)}{2\pi} \gamma_{qq}^{\mathrm{NS}}(N, \alpha_S(t)) \Delta q^{\mathrm{NS}}(N, Q^2)$$

$$\frac{d}{dt} \begin{pmatrix} \Delta\Sigma(N, Q^2) \\ \Delta g(N, Q^2) \end{pmatrix} = \frac{\alpha_S(t)}{2\pi} \begin{pmatrix} \gamma_{qq}^{\mathrm{S}} & 2n_f \gamma_{qg}^{\mathrm{S}} \\ \gamma_{gq}^{\mathrm{S}} & \gamma_{gg}^{\mathrm{S}} \end{pmatrix} \begin{pmatrix} \Delta\Sigma(N, Q^2) \\ \Delta g(N, Q^2) \end{pmatrix}$$

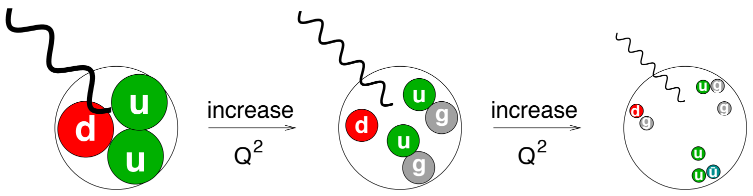


Evolution

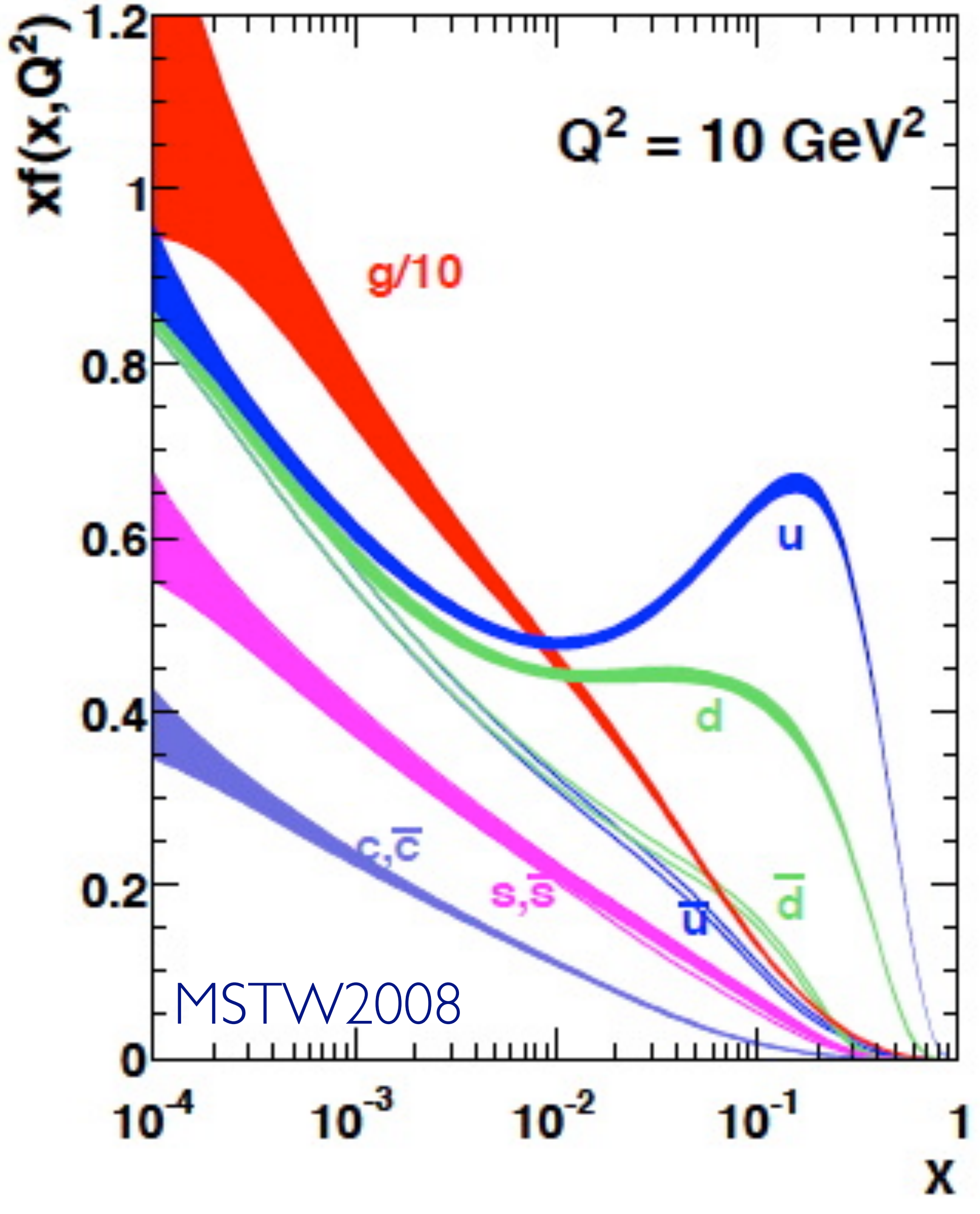


- As Q^2 increases, pdf's decrease at large x and increase at small x due to radiation and momentum loss.
- Gluon singularity at N=1 ⇒ it grows more at small x.
- $\gamma_{qq}(1)=0$ ⇒ number of quarks conserved.

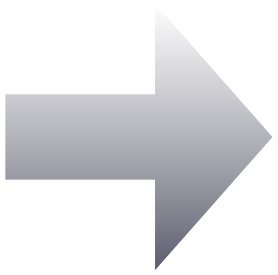
Evolution

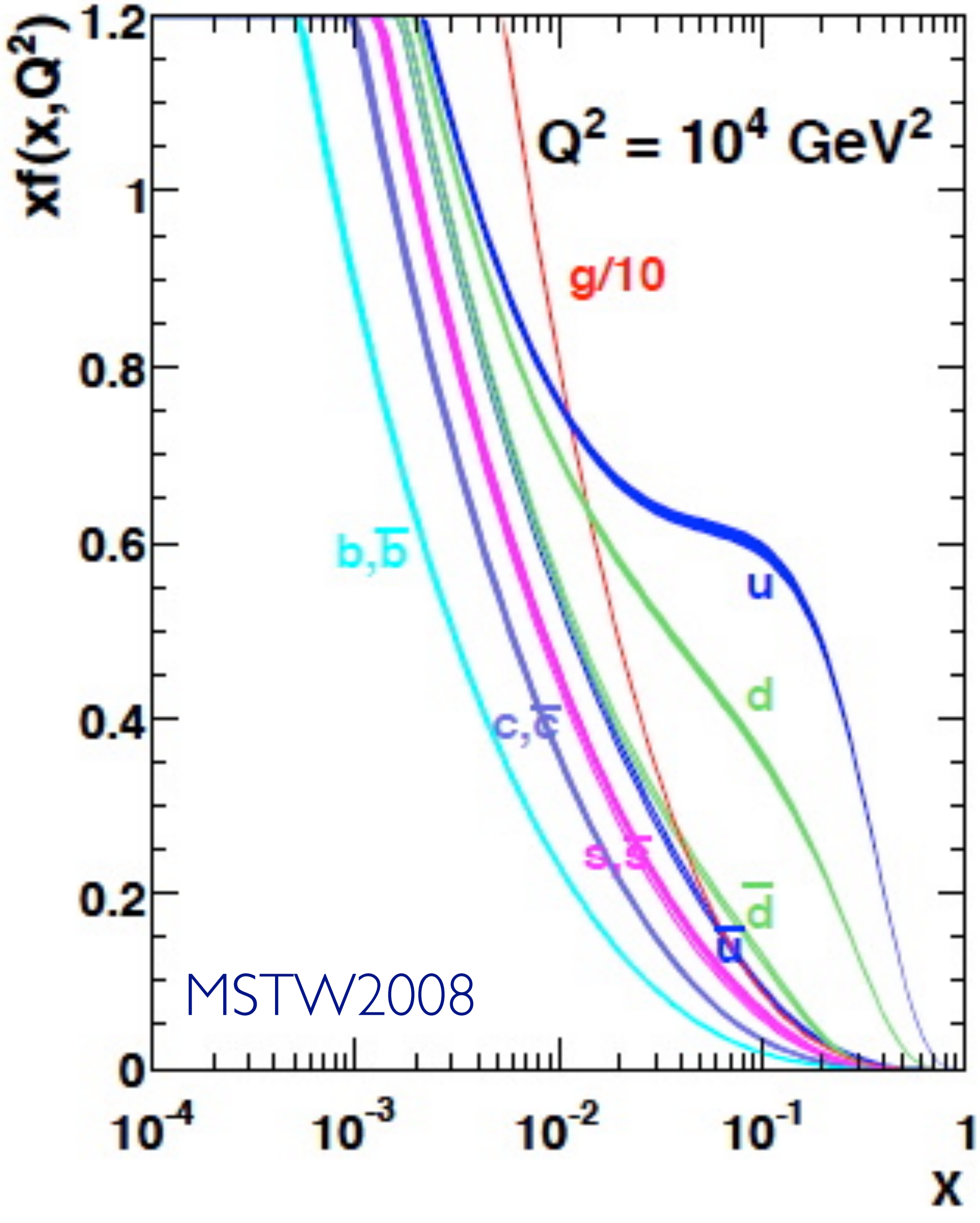


increase
Q2
increase
Q2



xf(x,Q2)
Q2 = 10 GeV2
g/10
u
d
c,c̄
s,s̄
ū
d̄
MSTW2008
x

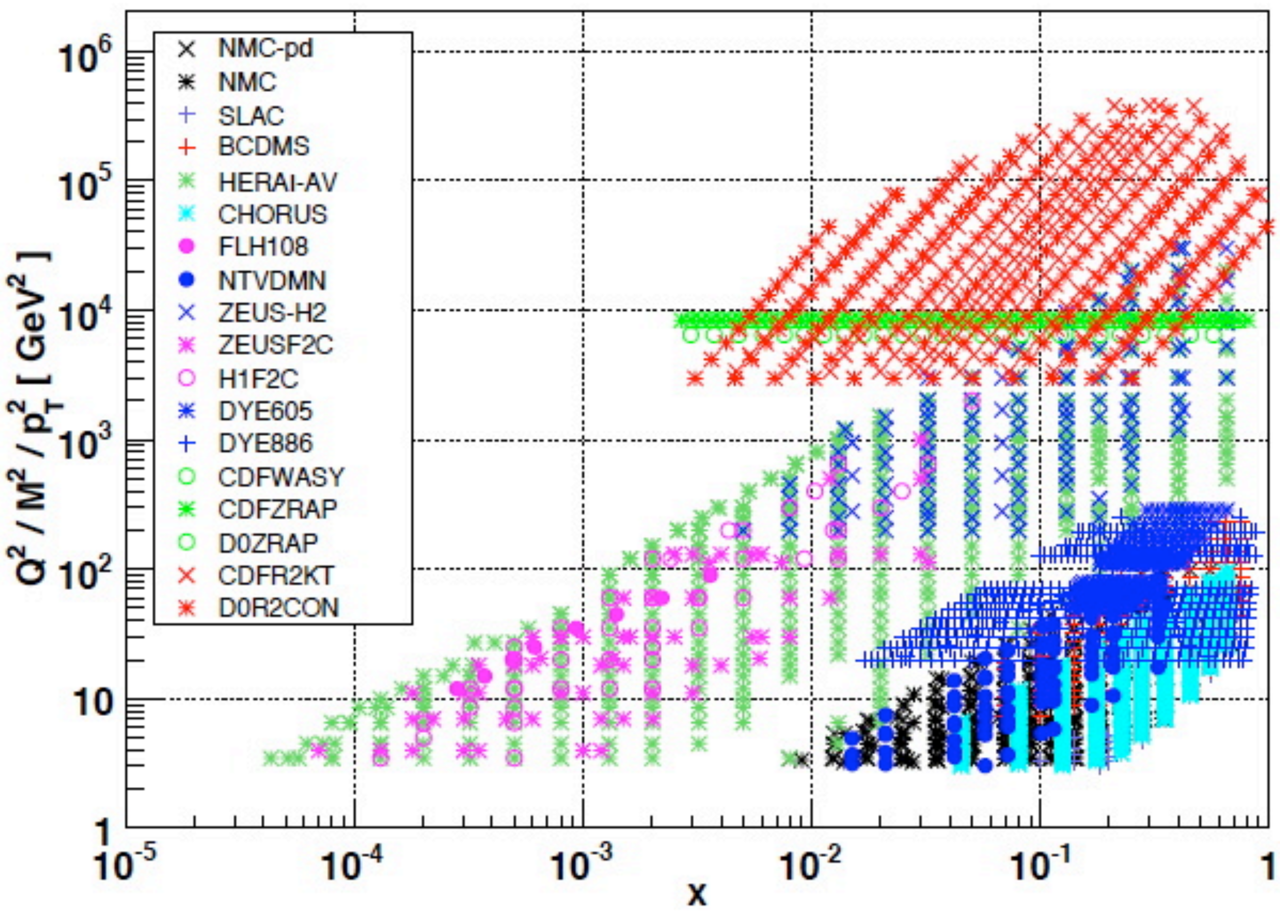




xf(x,Q2)
Q2 = 104 GeV2
g/10
b,b̄
u
d
c,c̄
s,s̄
ū
d̄
MSTW2008
x

Modern PDF sets

NNPDF2.1 NNLO dataset



There are now several collaborations providing PDF sets via a common interface (LHAPDF).

Three of them are global fits.

They provide uncertainties (be careful different procedures for each set!)

Several of them are now at NNLO and include HQ matched.

CTEQ6.6: GLOBAL, NLO, VFN, several α_s
MSTW08: GLOBAL, NNLO, VFN, several α_s
NNPDF2.1: GLOBAL, NNLO, VFN, several α_s

Plus other sets: Alekhin, HERAPDF, GRV/GJR...

Final strategy for QCD predictions

We now have a strategy to get a reliable result in perturbation theory:

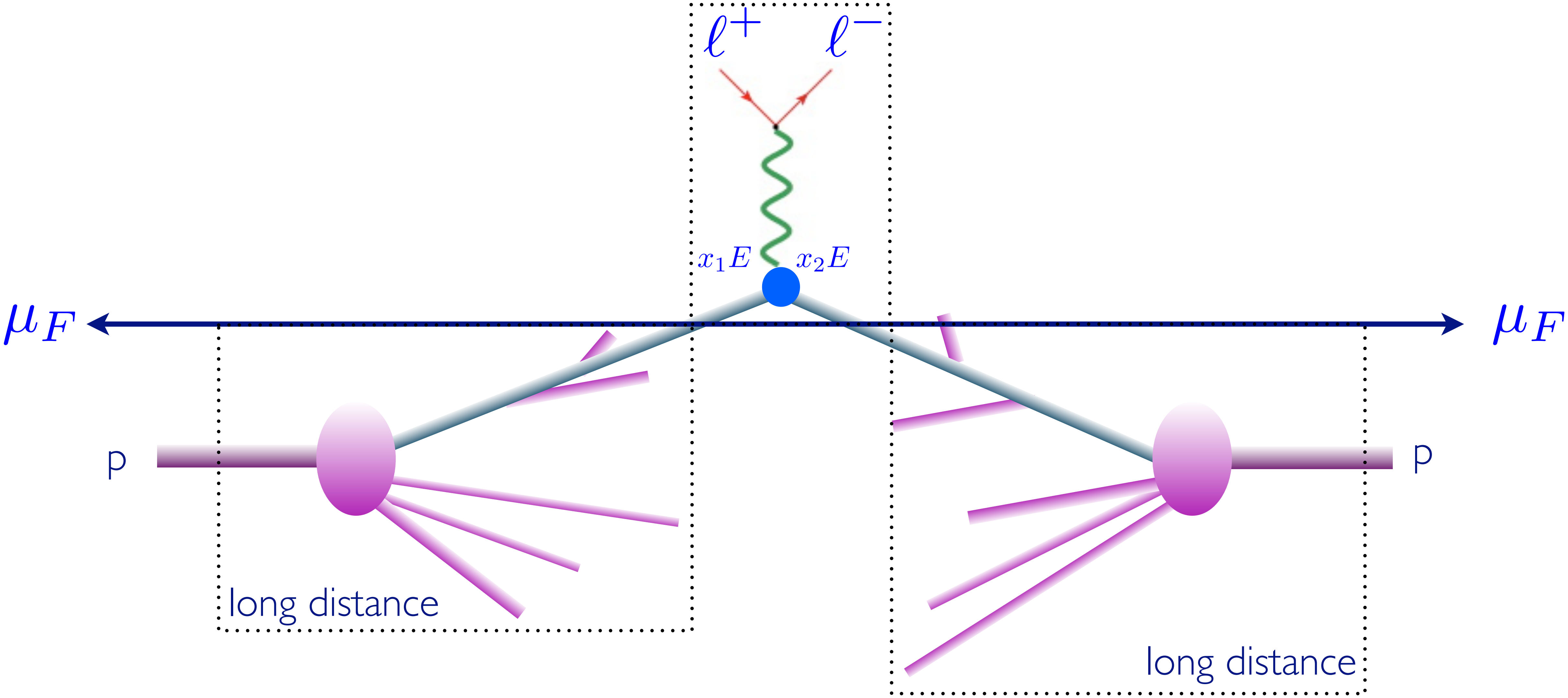
1. Calculate the short distance coefficient in pQCD corresponding to an observable. All divergences will cancel except those due to the collinear splitting of initial partons.

2. Re-absorbe such divergences in the pdf's and introduce a factorization scale.

3. Extract from experiment the initial condition for the pdf's at a given reference scale.

4. Evolve the pdf's at the scale of the process we are interested it. In so doing all large logs of the factorization scale over a small scale are resummed.

LHC master formula



$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}\left(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2}\right)$$

Remark on our master formula

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

- By calculating the short distance coefficient at tree-level we obtain the first estimate of rates for inclusive final states.

- Even at LO extra radiation is included: it is described by the PDF's in the initial state and by the definition of a final state parton, which at LO represents all possible final state evolutions.

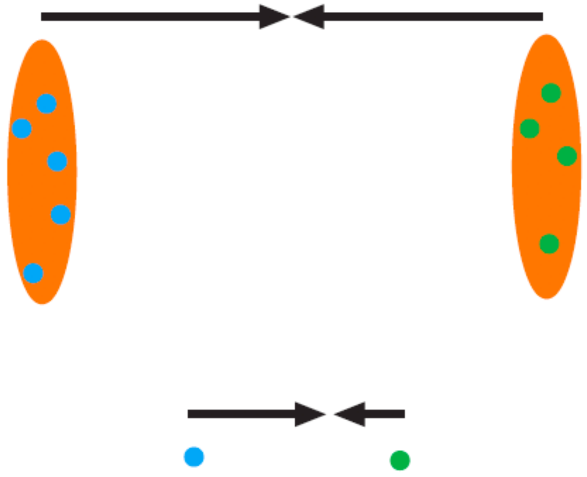
- Due to the above approximations a cross section at LO can strongly depend on the factorization and renormalization scales.

- Predictions can be systematically improved, at NLO and NNLO, by including higher order corrections in the short distance and in the evolution of the PDF's.

PP KINEMATICS

We describe the collision in terms of parton energies

$E_1 = x_1$ Ebeam
$E_2 = x_2$ Ebeam

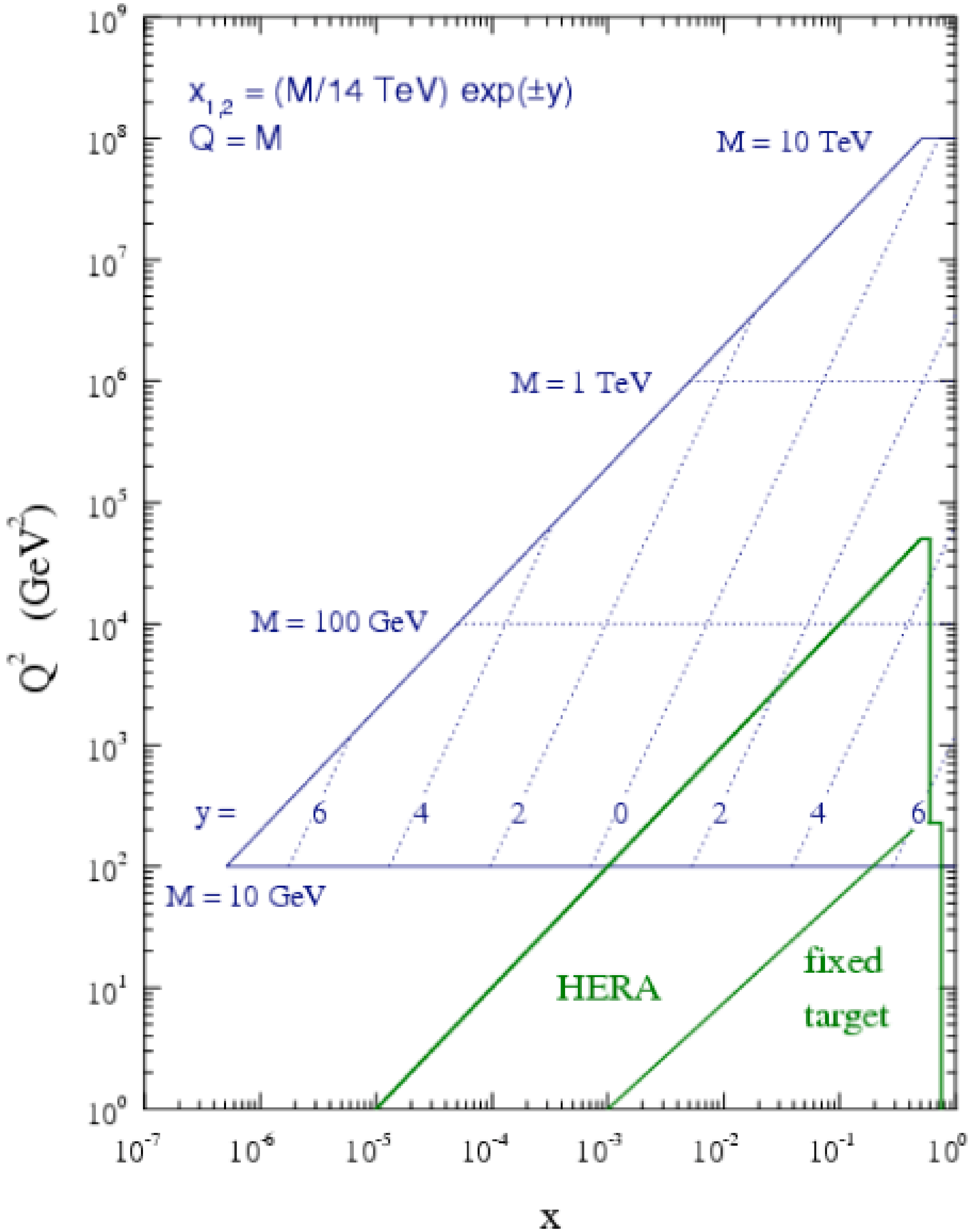


Obviously the partonic c.m.s. frame will be in general boosted. Let us say that the two partons annihilate into a particle of mass M.

$$M^2 = x_1 x_2 S = x_1 x_2 4E^2_{\rm beam}$$

$$y = \frac{1}{2}\log\frac{x_1}{x_2}$$

$$x_1 = \frac{M}{\sqrt{S}}e^{y} \quad x_2 = \frac{M}{\sqrt{S}}e^{-y}$$



QCD IN THE INITIAL STATE

1. We have introduced the physics of Deep Inelastic Scattering and the associated kinematics. We interpreted scaling in the parton model framework, trying to give a description of the physics involved by choosing a suitable frame.

2. We have shown that the parton model survives to QCD corrections, which affect the scaling picture only with logarithmic corrections.

3. In order to make prediction in pQCD, we have introduced the idea of factorization, which stands as a pillar for all interesting applications of pQCD.

4. The idea is to separate short-distance physics from long-distance one.
The first is calculable in pQCD. The second in non-perturbative and therefore not calculable but universal. So it can be measured in one experiment and used in another.

5. We have introduced the DGLAP equations that regulate the evolution of the pdf with the scale and allow the resummation of large logs.

6. We are now ready for pp collisions...!!

Lectures

1. Intro and QCD fundamentals
2. QCD in the final state
3. QCD in the initial state

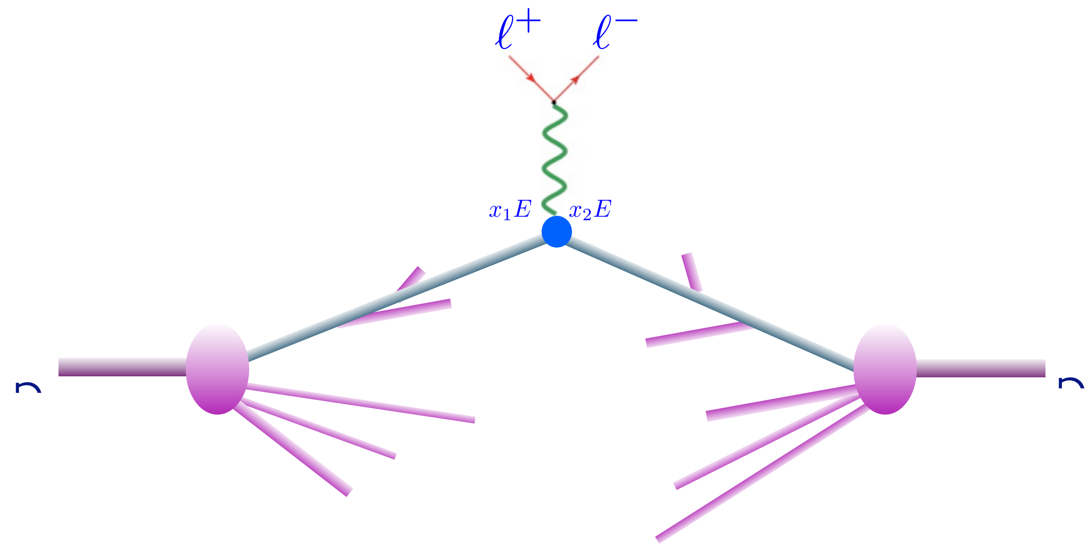
hard work

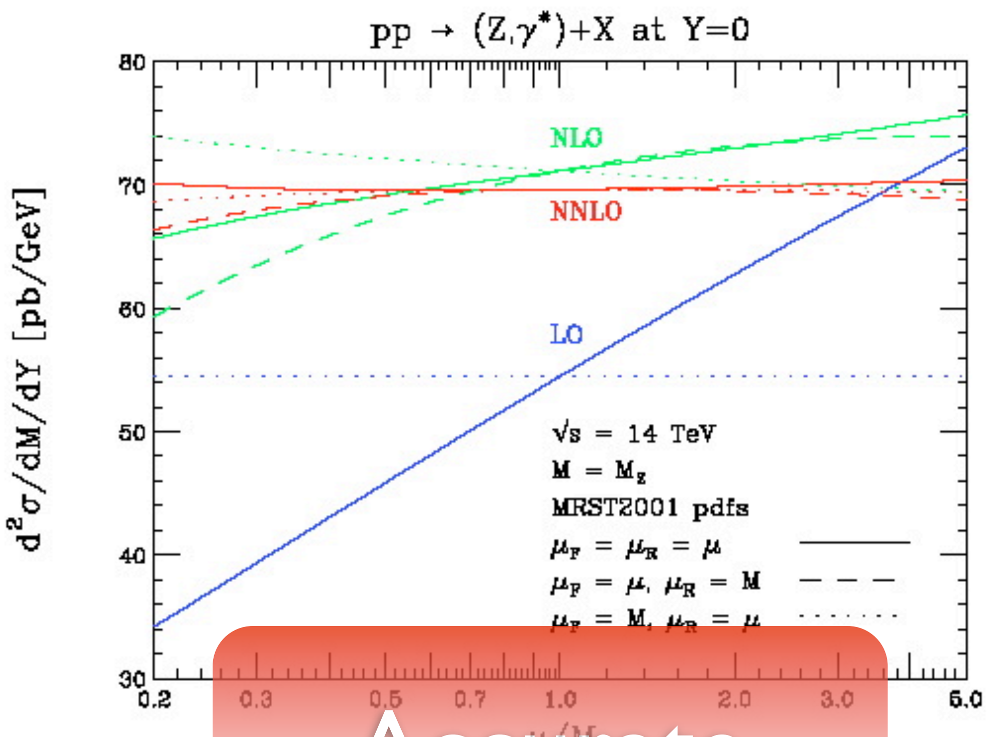
μ_F ⟷ μ_F

4. From accurate QCD to useful QCD

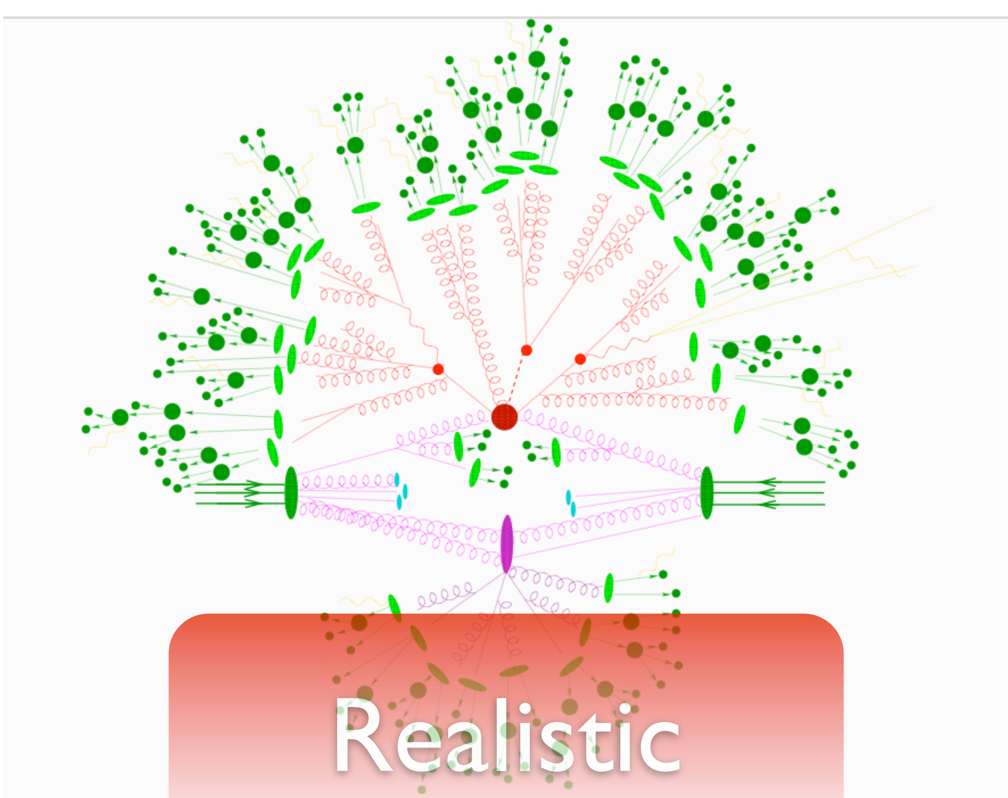
fun work

Tomorrow: we learn how to make predictions for the LHC!





Accurate



Realistic