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RENAFAE

CERN

Basics of QCD for the LHC

Lecture II

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Lectures

1. Intro and QCD fundamentals
2. QCD in the final state
3. QCD in the initial state
4. From accurate QCD to useful QCD

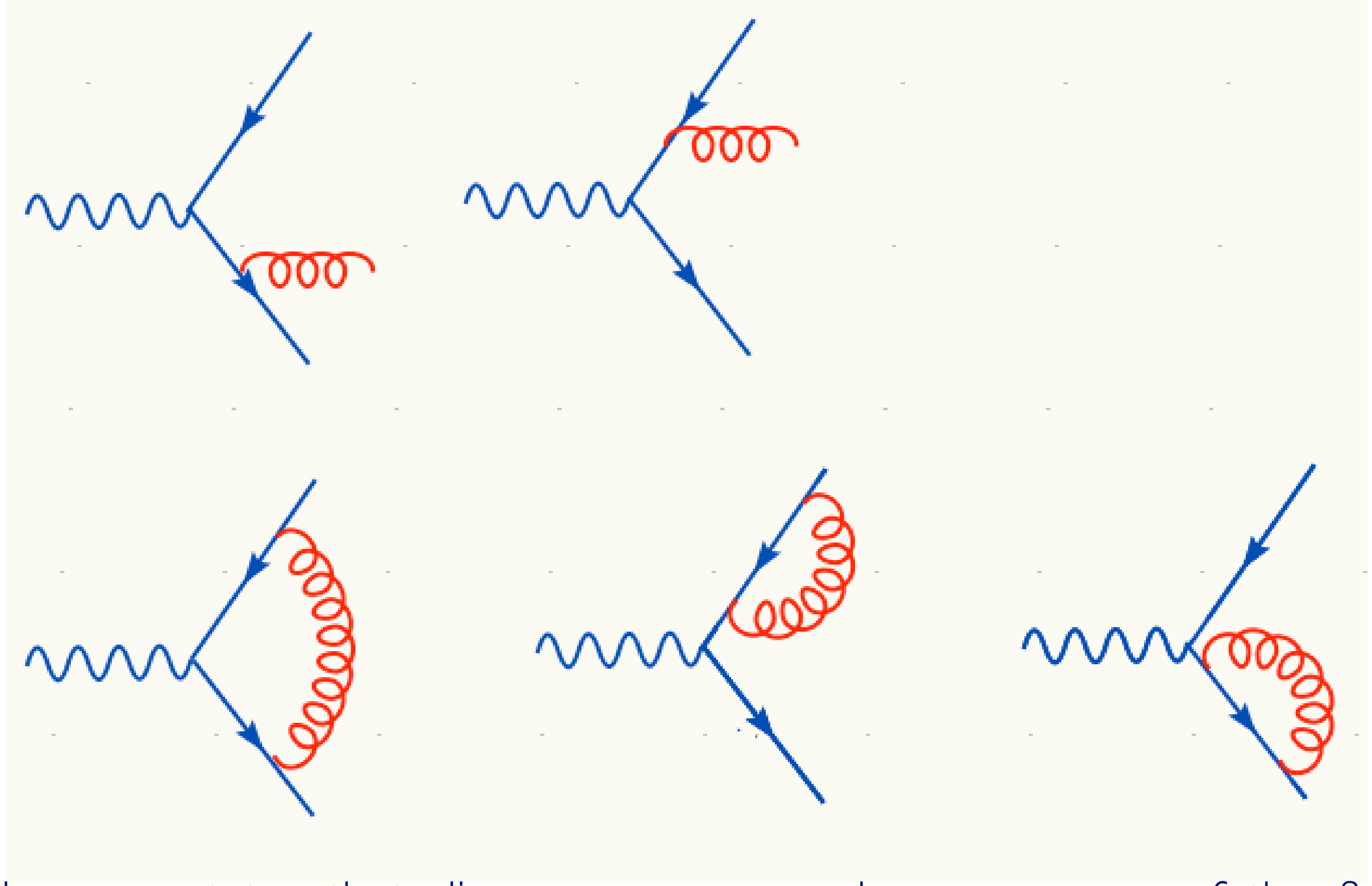
New set of questions

1. How can we identify a cross sections for producing quarks and gluons with a cross section for producing hadrons?

2. Given the fact that free quarks are not observed, why is the computed Born cross section so good?

3. Are there other calculable, i.e., that do not depend on the non-perturbative dynamics (like hadronization), quantities besides the total cross section?

ANATOMY OF A NLO CALCULATION



Real

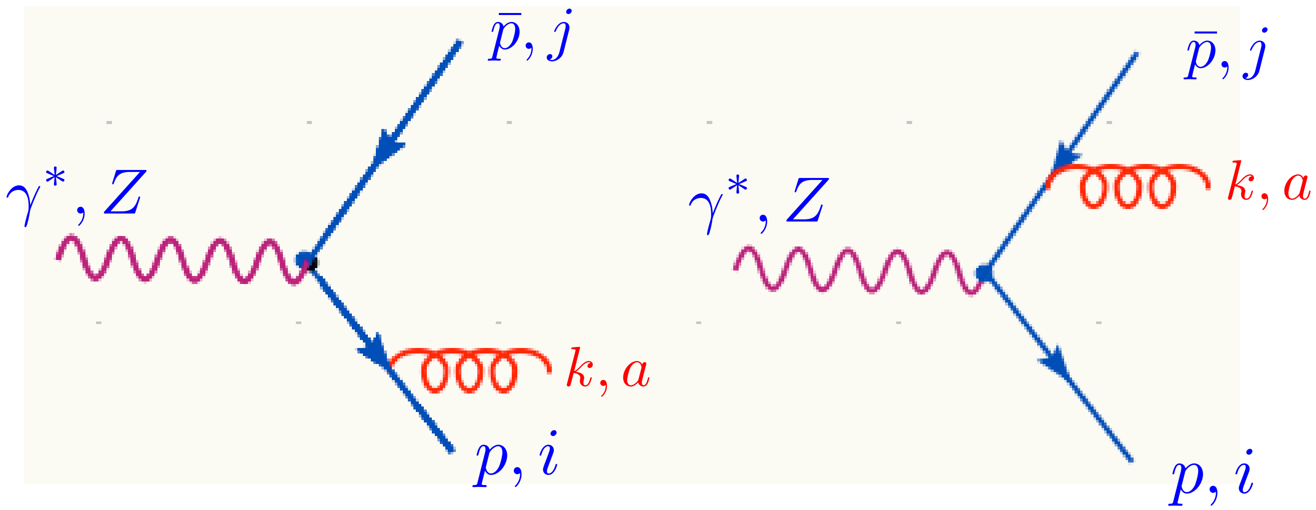
Virtual

The KLN theorem states that divergences appear because some of the final state are physically degenerate but we treated them as different. A final state with a soft gluon is nearly degenerate with a final state with no gluon at all (virtual).

$$\int \frac{d^d k}{(2\pi)^d} \cdots$$

$$\sigma^{\text{NLO}} = \int_R |M_{real}|^2 d\Phi_3 + \int_V 2Re\left(M_0 M^*_{virt}\right) d\Phi_2 = \text{finite!}$$

Anatomy of a NLO calculation



Let's consider the real gluon emission corrections to the process e+e- →qq. The full calculation is a little bit tedious, but since we in any case interested in the issues arising in the infra-red, we already start in that approximation.

$$A = \bar{u}(p)\not\epsilon(-ig_s)\frac{-i}{\not p + \not k}\Gamma^\mu v(\bar{p})t^a + \bar{u}(p)\Gamma^\mu \frac{i}{\not{\bar{p}} + \not k}(-ig_s)\not\epsilon v(\bar{p})t^a$$

$$= -g_s\left[\frac{\bar{u}(p)\not\epsilon(\not p + \not k)\Gamma^\mu v(\bar{p})}{2p\cdot k} - \frac{\bar{u}(p)\Gamma^\mu(\not{\bar{p}} + \not k)\not\epsilon v(\bar{p})}{2\bar{p}\cdot k}\right]t^a$$

The denominators $2p\cdot k = p_0 k_0(1-\cos\theta)$ give singularities for collinear (cos θ →1) or soft (k_0 →0) emission. By neglecting k in the numerators and using the Dirac equation, the amplitude simplifies and factorizes over the Born amplitude:

$$A_{soft} = -g_s t^a\left(\frac{p\cdot\epsilon}{p\cdot k} - \frac{\bar{p}\cdot\epsilon}{\bar{p}\cdot k}\right)A_{Born} \qquad A_{Born} = \bar{u}(p)\Gamma^\mu v(\bar{p})$$

Factorization: Independence of long-wavelength (soft) emission form the hard (short-distance) process. Soft emission is universal!!

Anatomy of a NLO calculation

By squaring the amplitude we obtain:

$$\sigma^{\rm REAL}_{q\bar{q}g} = C_F g_s^2 \sigma^{\rm Born}_{q\bar{q}} \int \frac{d^3k}{2k^0(2\pi)^3} 2 \frac{p\cdot\bar{p}}{(p\cdot k)(\bar{p}\cdot k)}$$

$$= C_F \frac{\alpha_S}{2\pi} \sigma^{\rm Born}_{q\bar{q}} \int d\cos\theta \, \frac{dk^0}{k^0} \frac{4}{(1-\cos\theta)(1+\cos\theta)}$$

Two collinear divergences and a soft one. Very often you find the integration over phase space expressed in terms of x_1 and x_2, the fraction of energies of the quark and anti-quark:

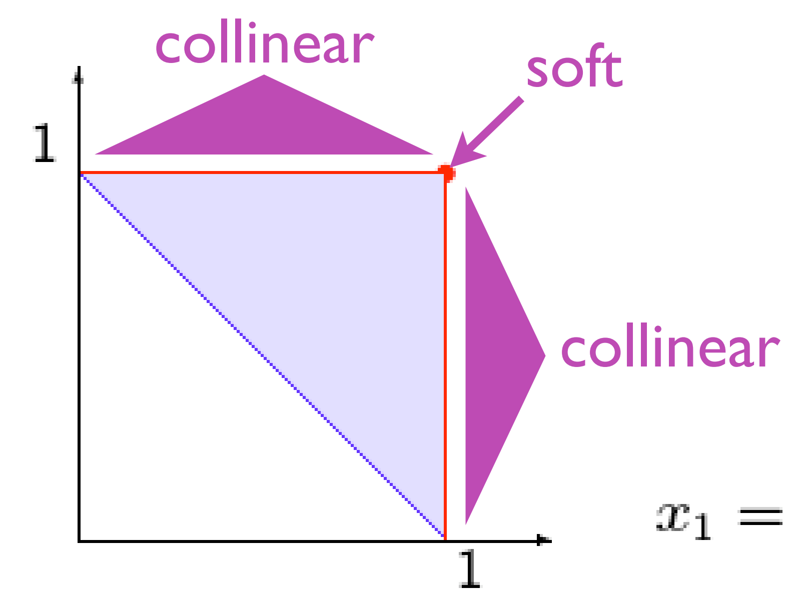
$$x_1 = 1 - x_2 x_3 (1-\cos\theta_{23})/2$$

$$x_2 = 1 - x_1 x_3 (1-\cos\theta_{13})/2$$

$$x_1 + x_2 + x_3 = 2$$

$$0 \le x_1, x_2 \le 1, \quad \text{and} \quad x_1 + x_2 \ge 1$$

$$x_2 = \frac{2E_{\bar{q}}}{\sqrt{s}}$$



$$x_1 = \frac{2E_q}{\sqrt{s}}$$

So we can now predict the divergent part of the virtual contribution, while for the finite part an explicit calculation is necessary:

$$\sigma^{\rm VIRT}_{q\bar{q}} = -\sigma^{\rm Born}_{q\bar{q}} C_F \frac{\alpha_S}{2\pi} \int d\cos\theta' \, \frac{dk_0'}{k_0'} \frac{1}{1-\cos^2\theta} 2\delta(k_0')[\delta(1-\cos\theta') + \delta(1+\cos\theta')] + \ldots$$

Anatomy of a NLO calculation

Summary:

$$\sigma^{\rm REAL} + \sigma^{\rm VIRT} = \infty - \infty = ?$$

Solution: regularize the "intermediate" divergences, by giving a gluon a mass (see later) or going to d=4-2**ε** dimensions.

$$\int^1 \frac{1}{1-x} dx = -\log 0 \xrightarrow{\text{regularization}} \int^1 \frac{(1-x)^{-2\epsilon}}{1-x} dx = -\frac{1}{2\epsilon}$$

This gives:

$$\sigma^{\rm REAL} = \sigma^{\rm Born} C_F \frac{\alpha_S}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 \right)$$

$$\sigma^{\rm VIRT} = \sigma^{\rm Born} C_F \frac{\alpha_S}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 \right)$$

$$\lim_{\epsilon \to 0} (\sigma^{\rm REAL} + \sigma^{\rm VIRT}) = C_F \frac{3}{4} \frac{\alpha_S}{\pi} \sigma^{\rm Born}$$

$$R_1 = R_0 \left(1 + \frac{\alpha_S}{\pi} \right)$$ as presented before

New set of questions

1. How can we identify a cross sections for producing (few) quarks and gluons with a cross section for producing (many) hadrons?

2. Given the fact that free quarks are not observed, why is the computed Born cross section so good?

Answers:

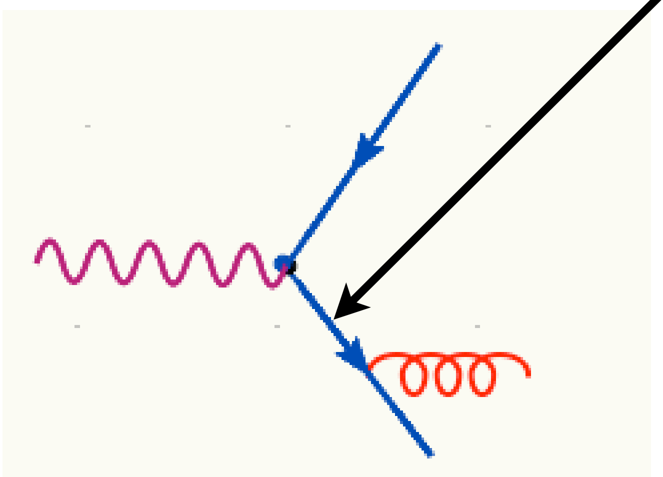
The Born cross section IS NOT the cross section for producing q qbar, since the coefficients of the perturbative expansion are infinite! But this is not a problem since we don't observe q qbar and nothing else. So there is no contradiction here.

On the other hand the cross section for producing hadrons is finite order by order and its lowest order approximation IS the Born.

A further insight can be gained by thinking of what happens in QED and what is different there. For instance soft and collinear divergence are also there. In QED one can prove that cross section for producing "only two muons" is zero...

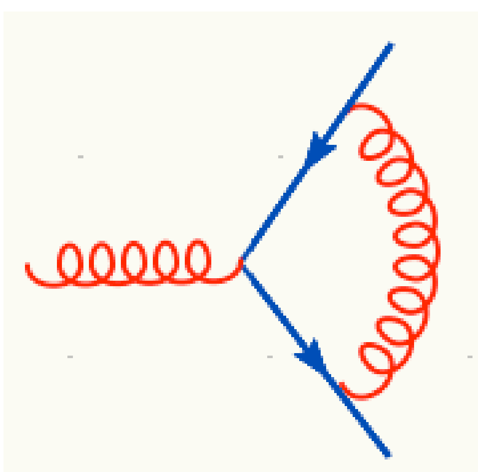
Infrared divergences

$$A_{soft} = -g_s t^a \left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right) A_{Born}$$



Even in high-energy, short-distance regime, long-distance aspects of QCD cannot be ignored.

This is because there are configurations in phase space for gluons and quarks, i.e. when gluons are soft and/or when are pairs of partons are collinear.

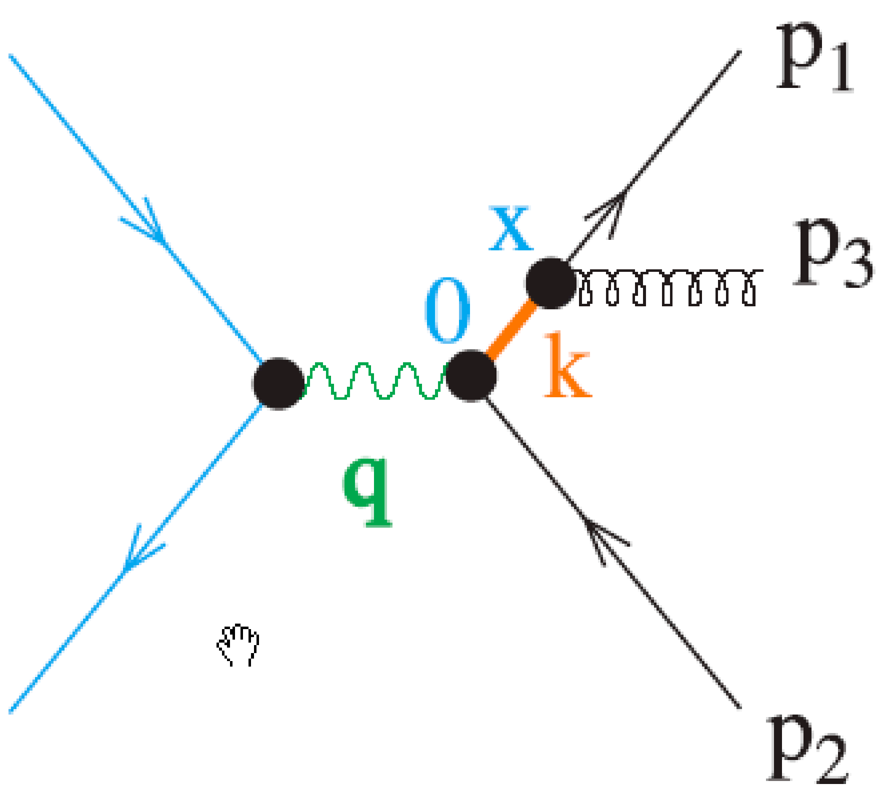


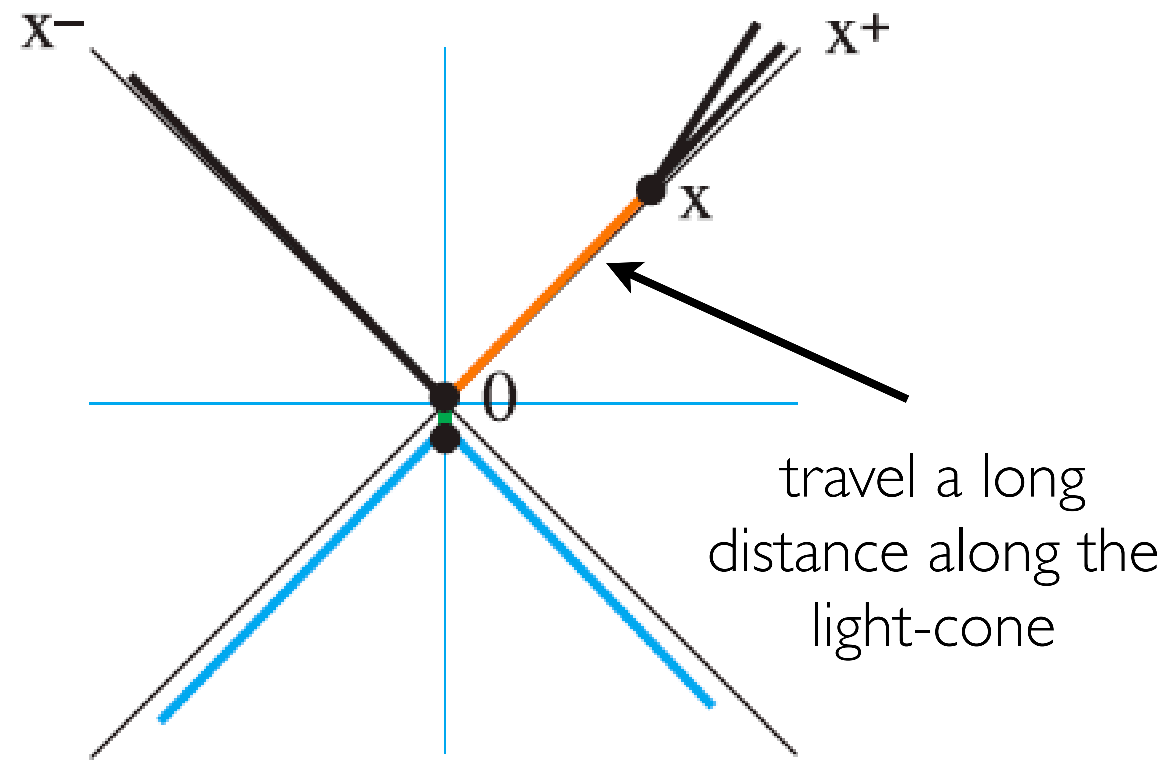
$$\Rightarrow \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k+p)^2 (k-\bar{p})^2}$$

also for soft and collinear or collinear configurations associated to the virtual partons with the region of integration of the loop momenta.

Space-time picture of IR singularities

The singularities can be understood in terms of light-cone coordinates. Take $p^\mu=(p^0, p^1, p^2, p^3)$ and define $p^\pm=(p^0 \pm p^3)/\sqrt{2}$. Then choose the direction of the + axis as the one of the largest between + and - . A particle with small virtuality travels for a long time along the x^+ direction.





$$k^+ \simeq \sqrt{s}/2$$ large

$$k^- \simeq (k^T + 2k^+k^-)\sqrt{s}/2$$ small

$$x^+ \simeq 1/k^-$$ large

$$x^- \simeq 1/k^+$$ small

Infrared divergences

Infrared divergences arise from interactions that happen a long time after the creation of the quark/antiquark pair.

When distances become comparable to the hadron size of ~1 Fermi, quasi-free partons of the perturbative calculation are confined/hadronized non-perturbatively.

We have seen that in total cross sections such divergences cancel. But what about for other quantities?

Well, obviously the only possibility is to try to use the pQCD calculations for quantities that are not sensitive to the to the long-distance physics.

Can we formulate a criterium that is valid in general?

YES! It is called INFRARED SAFETY

Infrared-safe quantities

DEFINITION: quantities are that are insensitive to soft and collinear branching.

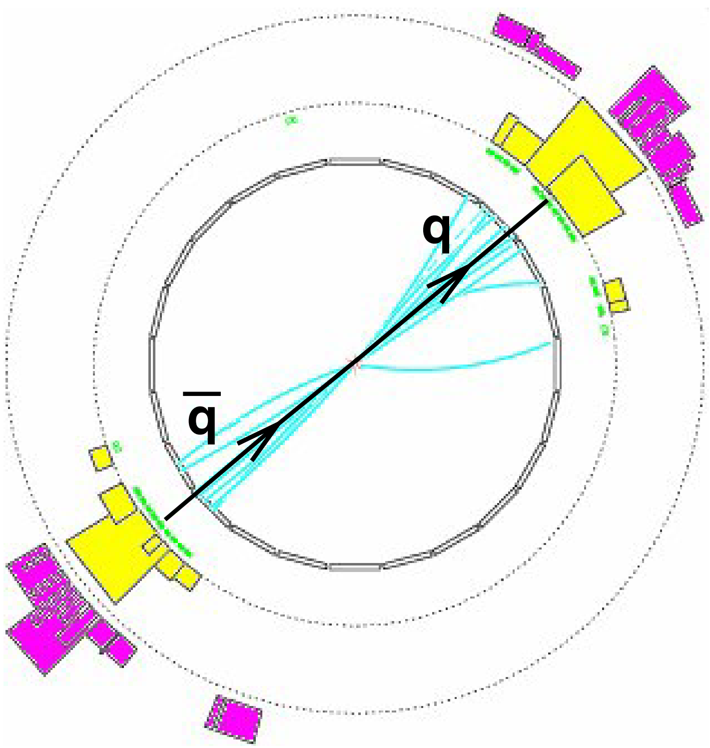
For these quantities, an extension of the general theorem (KLN) exists which proves that infrared divergences cancel between real and virtual or are simply removed by kinematic factors.

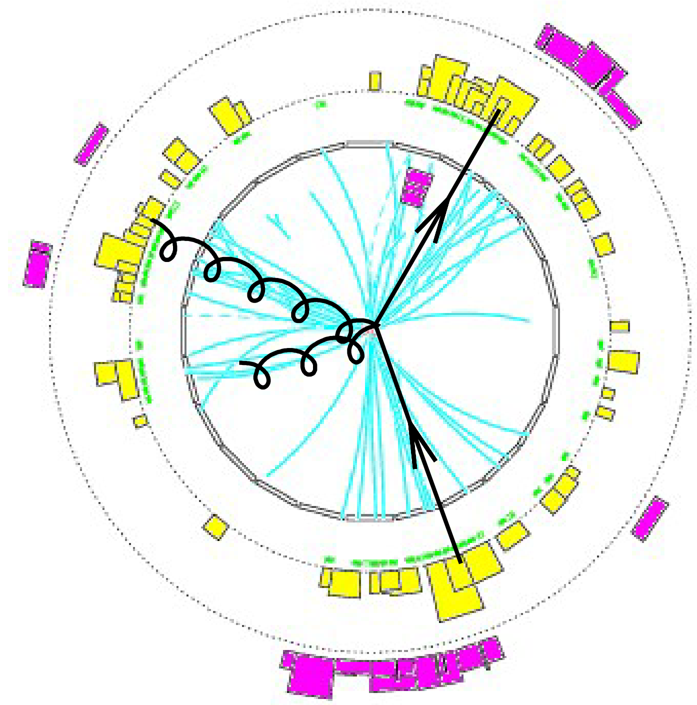
Such quantities are determined primarily by hard, short-distance physics. Long-distance effects give power corrections, suppressed by the inverse power of a large momentum scale (which must be present in the first place to justify the use of PT).

Examples:

1. Multiplicity of gluons is not IRC safe
2. Energy of hardest particle is not IRC safe
3. Energy flow into a cone is IRC safe

Event shape variables





pencil-like → spherical

Event shape variables

The idea is to give more information than just total cross section by defining "shapes" of an hadronic event (pencil-like, planar, spherical, etc..)

In order to be comparable with theory it MUST be IR-safe, that means that the quantity should not change if one of the parton "branches" $p_k \to p_i + p_j$

Examples are: Thrust, Spherocity, C-parameters,...

Similar quantities exist for hadron collider too, but they much less used.

Name of Observable	Definition	Typical Value for: (2-jet)	(3-jet)	(spherical)	QCD calculation
Thrust	$T = \max_{\vec{n}} \left(\frac{\Sigma_i \lvert \vec{p}_i \vec{n} \rvert}{\Sigma_i \lvert \vec{p}_i \rvert} \right)$	1	$\geq 2/3$	$\geq 1/2$	(resummed) $O(\alpha_s^2)$
Thrust major	Like T, however T_{maj} and $\vec{n}_{maj}$ in plane $\perp \vec{n}_T$	0	$\leq 1/3$	$\leq 1/\sqrt{2}$	$O(\alpha_s^2)$
Thrust minor	Like T, however T_{min} and $\vec{n}_{min}$ in direction $\perp$ to $\vec{n}_T$ and $\vec{n}_{maj}$	0	0	$\leq 1/2$	$O(\alpha_s^2)$
Oblateness	$O = T_{maj} - T_{min}$	0	$\leq 1/3$	0	$O(\alpha_s^2)$
Sphericity	$S = 1.5\,(Q_1 + Q_2)$; $Q_1 \leq \ldots \leq Q_3$ are Eigenvalues of $S^{\alpha\beta} = \frac{\Sigma_i p_i^\alpha p_i^\beta}{\Sigma_i p_i^2}$	0	$\leq 3/4$	≤ 1	none (not infrared safe)
Aplanarity	$A = 1.5\,Q_1$	0	0	$\leq 1/2$	none (not infrared safe)
Jet (Hemisphere) masses	$M_\pm^2 = (\Sigma_i E_i^2 - \Sigma_i \vec{p}_i^{\,2})_{i \in S_\pm}$ ($S_\pm$: Hemispheres $\perp$ to $\vec{n}_T$) $M_H^2 = \max(M_+^2, M_-^2)$ $M_D^2 = \lvert M_+^2 - M_-^2 \rvert$	0 0	$\leq 1/3$ $\leq 1/3$	$\leq 1/2$ 0	(resummed) $O(\alpha_s^2)$
Jet broadening	$B_\pm = \frac{\Sigma_{i \in S_\pm} \lvert \vec{p}_i \times \vec{n}_T \rvert}{2\,\Sigma_i \lvert \vec{p}_i \rvert}$; $B_T = B_+ + B_-$ $B_w = \max(B_+, B_-)$	0 0	$\leq 1/(2\sqrt{3})$ $\leq 1/(2\sqrt{3})$	$\leq 1/(2\sqrt{2})$ $\leq 1/(2\sqrt{3})$	(resummed) $O(\alpha_s^2)$
Energy-Energy Correlations	$EEC(\chi) = \sum_{events} \sum_{i,j} \frac{E_i E_j}{E_{vis}^2} \int_{\chi - \frac{\Delta\chi}{2}}^{\chi + \frac{\Delta\chi}{2}} \delta(\chi - \chi_{ij})$	0 π	0 π	0 π	(resummed) $O(\alpha_s^2)$
Asymmetry of EEC	$AEEC(\chi) = EEC(\pi - \chi) - EEC(\chi)$	0 π/2	0 π/2	0 π/2	$O(\alpha_s^2)$
Differential 2-jet rate	$D_2(y) = \frac{R_2(y - \Delta y) - R_2(y)}{\Delta y}$				(resummed) $O(\alpha_s^2)$

Is the thrust IR safe?

$$T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i \vec{p}_i}$$

Contribution from a particle with momentum going to zero drops out.

Replacing one particle with two collinear ones does not change the thrust:

$$|(1-\lambda)\vec{p}_k \cdot \vec{u}| + |\lambda \vec{p}_k \cdot \vec{u}| = |\vec{p}_k \cdot \vec{u}|$$

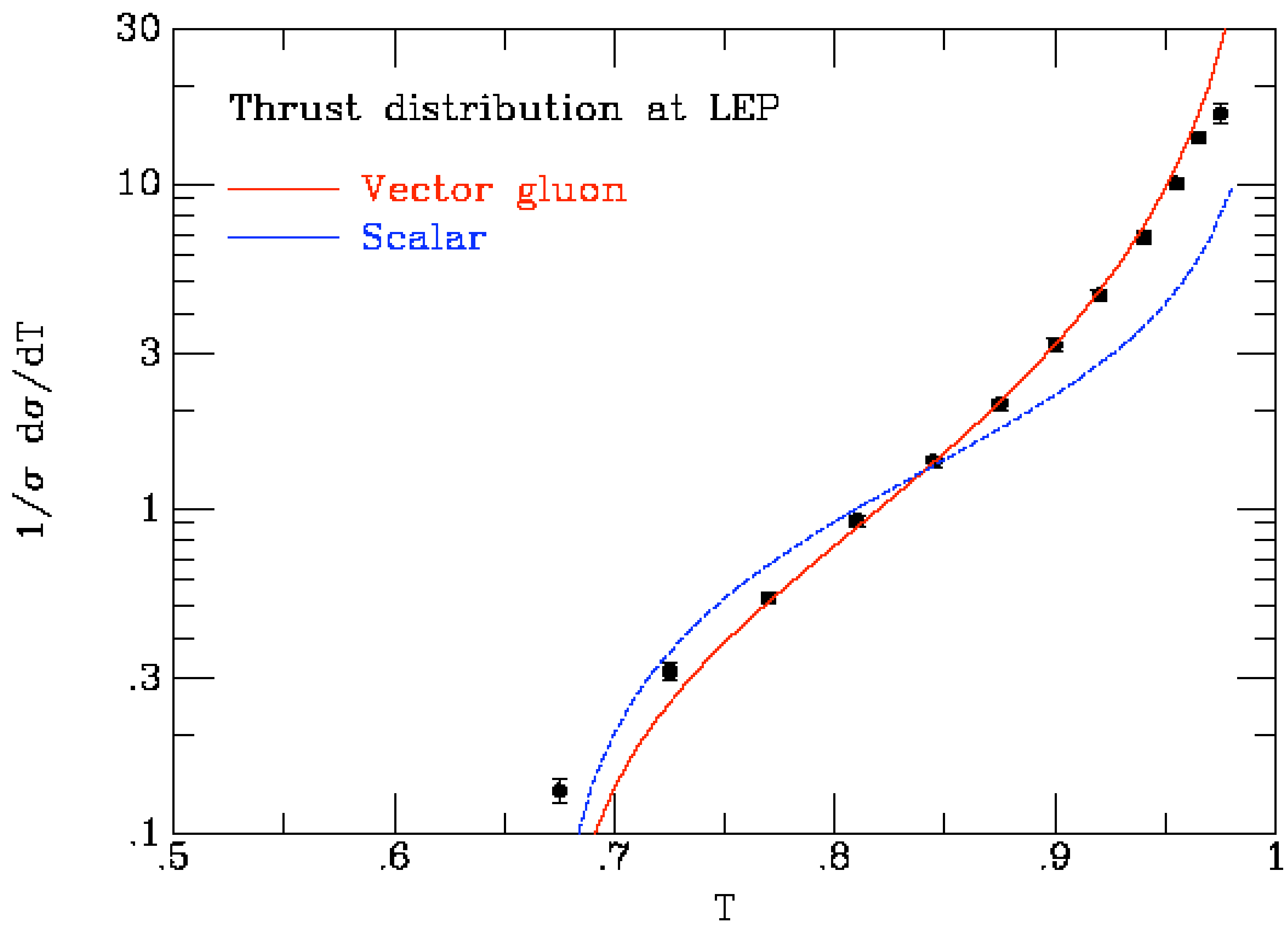
and

$$|(1-\lambda)\vec{p}_k| + |\lambda \vec{p}_k| = |\vec{p}_k|$$

Calculation of event shape variables: Thrust

The values of the different event-shape variables for different topologies are

$$\frac{1}{\sigma}\frac{d\sigma}{dT} = C_F \frac{\alpha_S}{2\pi}\left[\frac{2(3T^2-3T+2)}{T(1-T)}\log\left(\frac{2T-1}{1-T}\right) - \frac{3(3T-2)(2-T)}{1-T}\right].$$



$O(\alpha_S^2)$ corrections (NLO) are also known. Comparison with data provide test of QCD matrix elements, through shape distribution and measurement of α_S from overall rate. Care must be taken around T=1 where

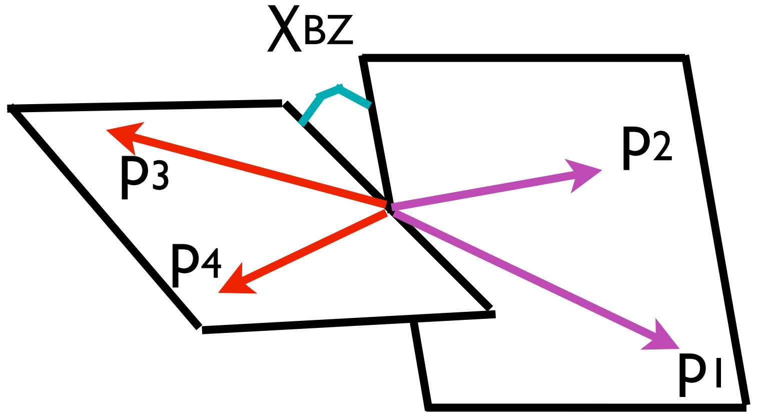
(a) hadronization effects become large and

(b) large higher order terms of the form $\alpha_S^N\,[\log^{2N-1}(1-T)]/(1-T)$ need to be resummed.

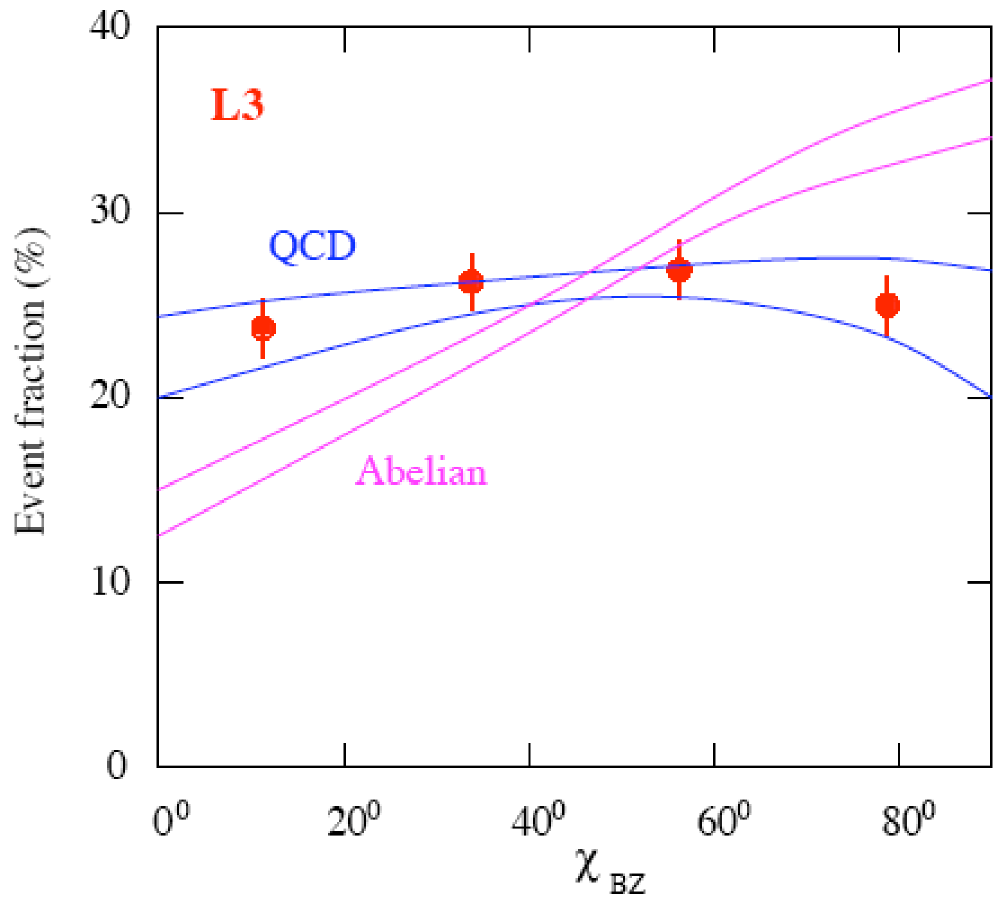
At lower T multi-jet matrix element become important.

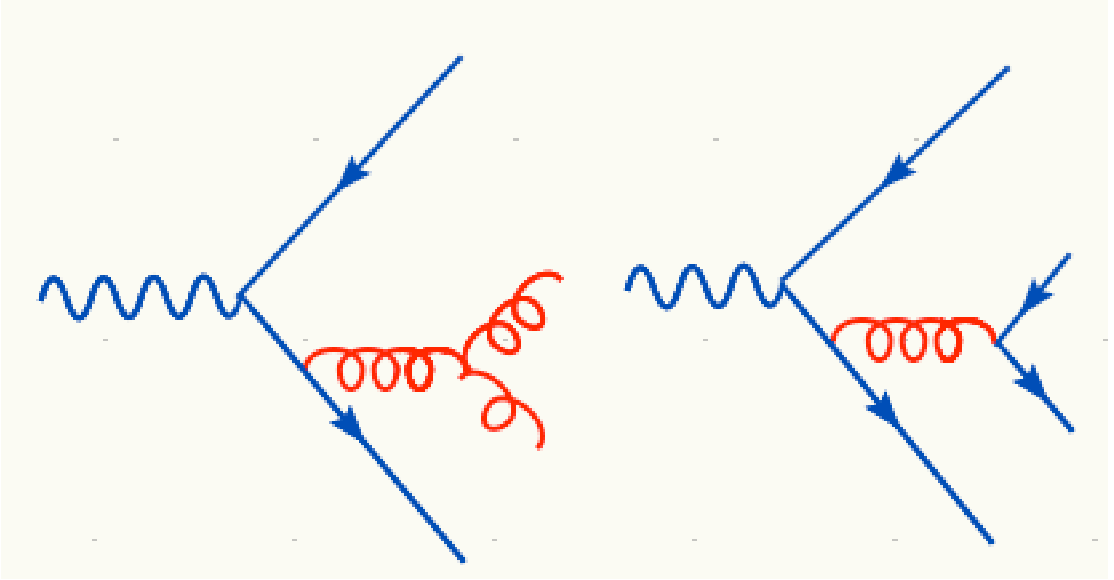
Intermezzo: how did we "see" the 3g vertex?

Angular correlations also provide interesting information about the properties of the matrix elements in QCD. One of these quantities is the so-called Bengtsson-Zerwas angle. It is the angle between planes of the two lowest and the two highest energy jets.



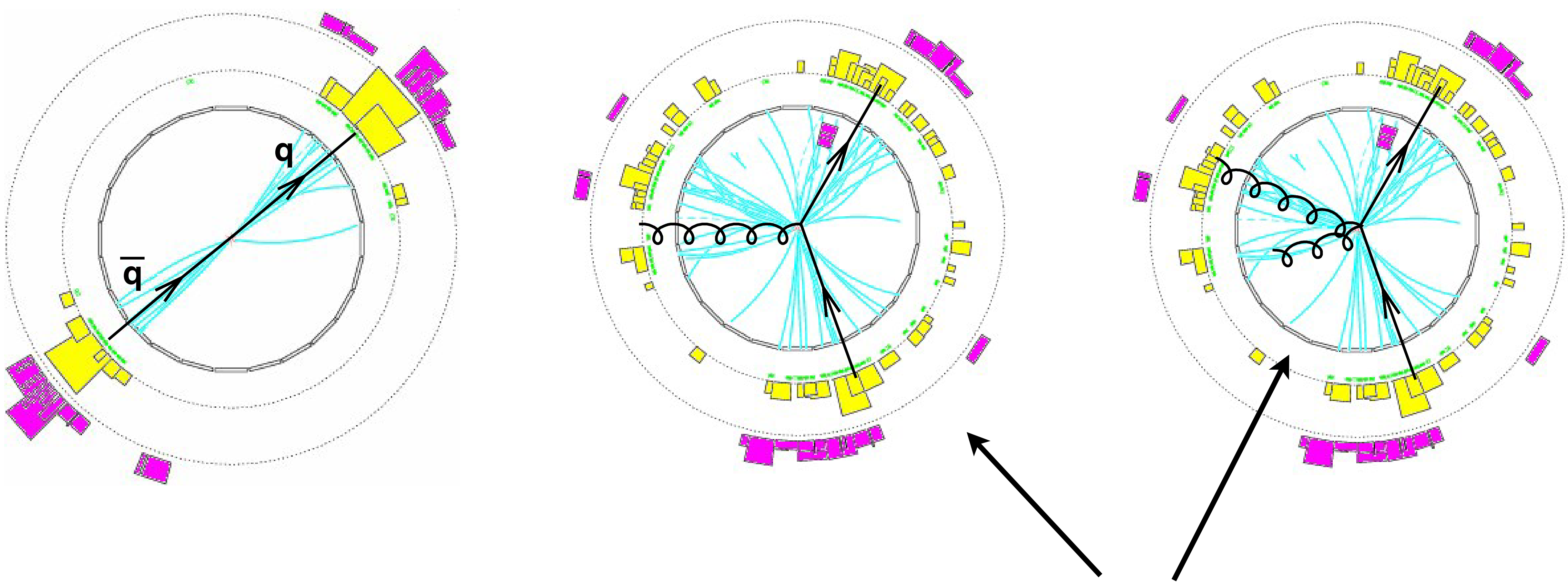
$$\cos \chi_{BZ} = \frac{(\boldsymbol{p}_1 \times \boldsymbol{p}_2) \cdot (\boldsymbol{p}_3 \times \boldsymbol{p}_4)}{|\boldsymbol{p}_1 \times \boldsymbol{p}_2|\ |\boldsymbol{p}_3 \times \boldsymbol{p}_4|} .$$





This quantity gives information on the presence and characteristics of the three-gluon vertex.

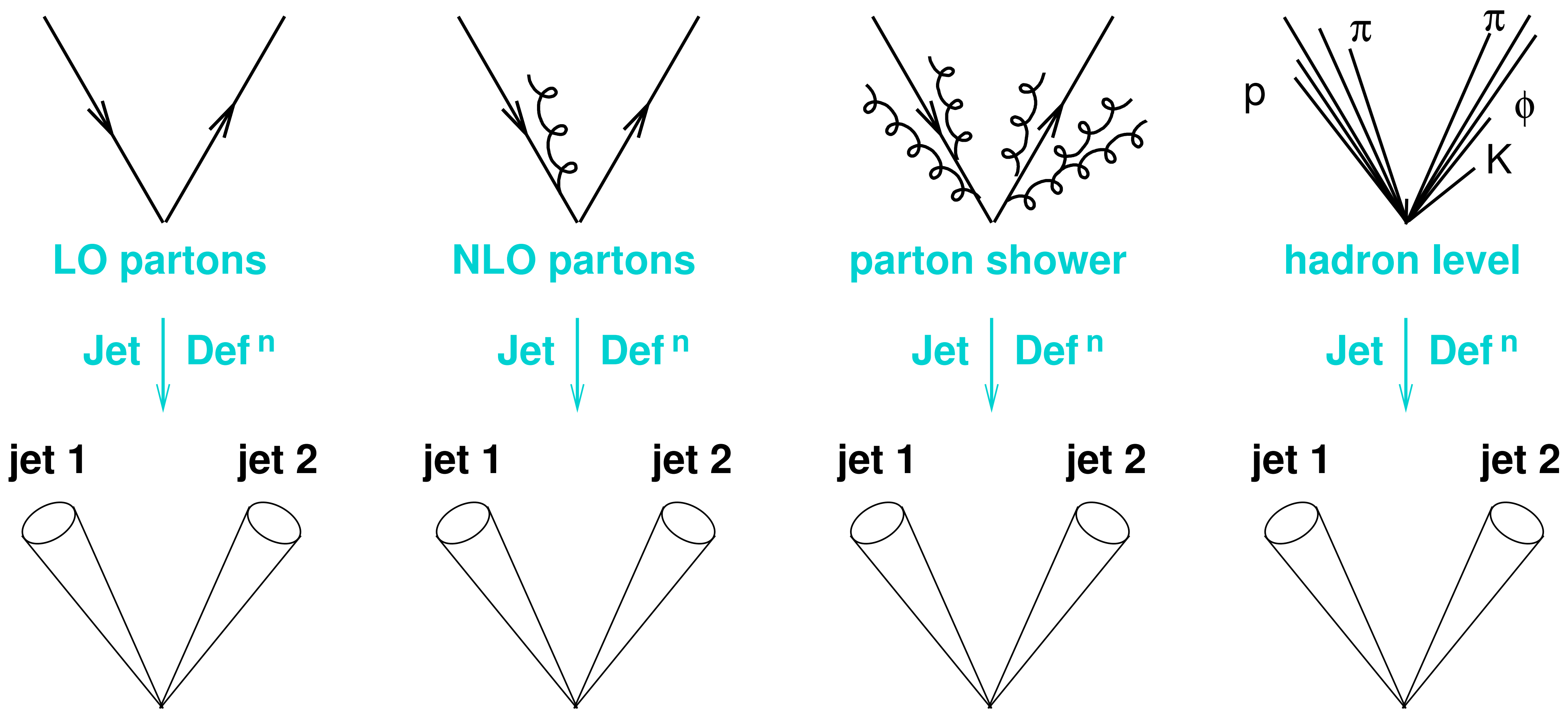
Jet algorithms



2-jets 3-jets same event!! 4-jets

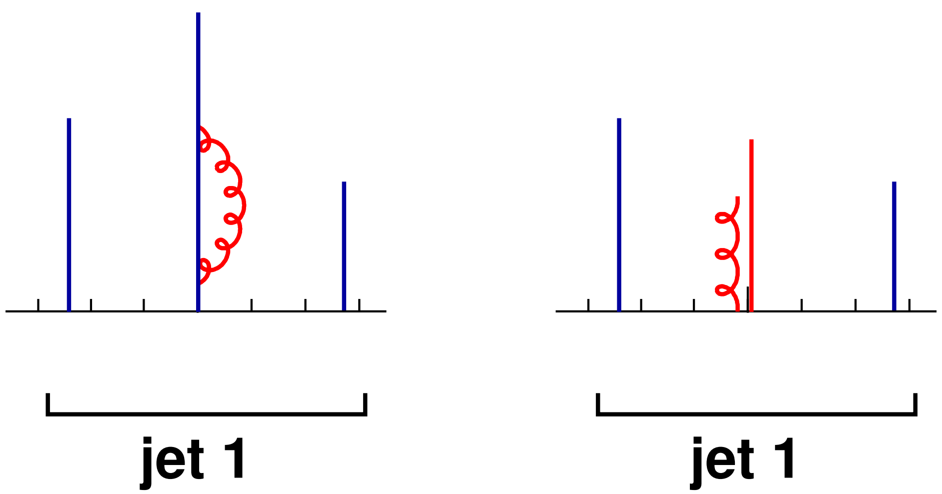
Jets are in the eye of the beholder!

Jet algorithms



Infrared safety and jet algo's

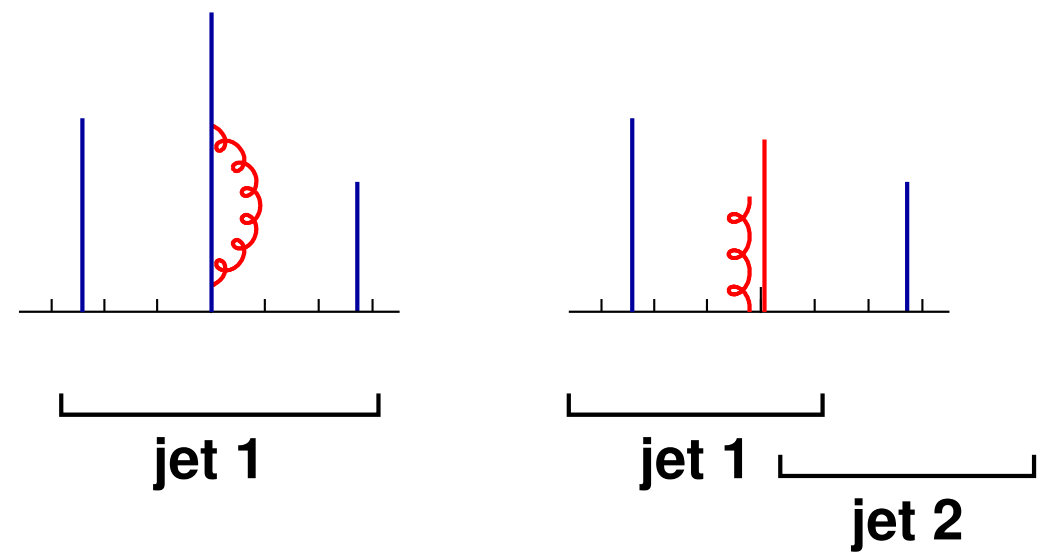
Collinear Safe



$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

Infinities cancel

Collinear Unsafe



$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

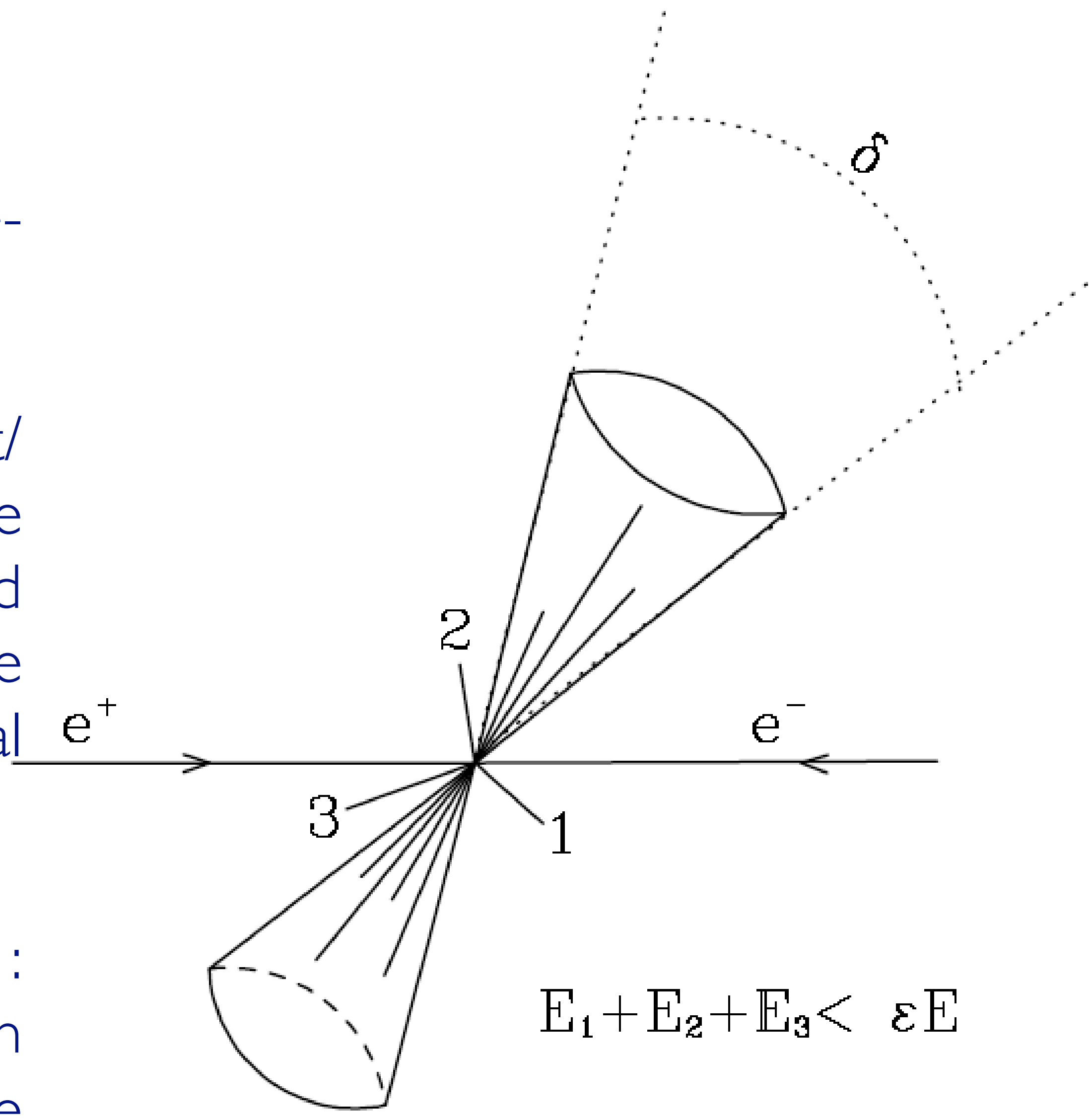
Infinities do not cancel

Jet algorithms

- The precise definition of a procedure how to cut be three-jet (and multi-jet) events is called "jet algorithm".

- Which jet algorithm to use for a given measurement/experiment needs to be found out. Different algorithms have very different behaviors both experimentally and theoretically. Of course, it is important that a complete information is given on the jet algorithm when experimental data are to be compared with theory predictions!

- Weinberg-Sterman jets (intuitive definition): "An event is identified as a 2-jets if one can find 2 cones with opening angle δ that contain all but a small fraction εE of the total energy E".



Jets (top-down) at e-e+

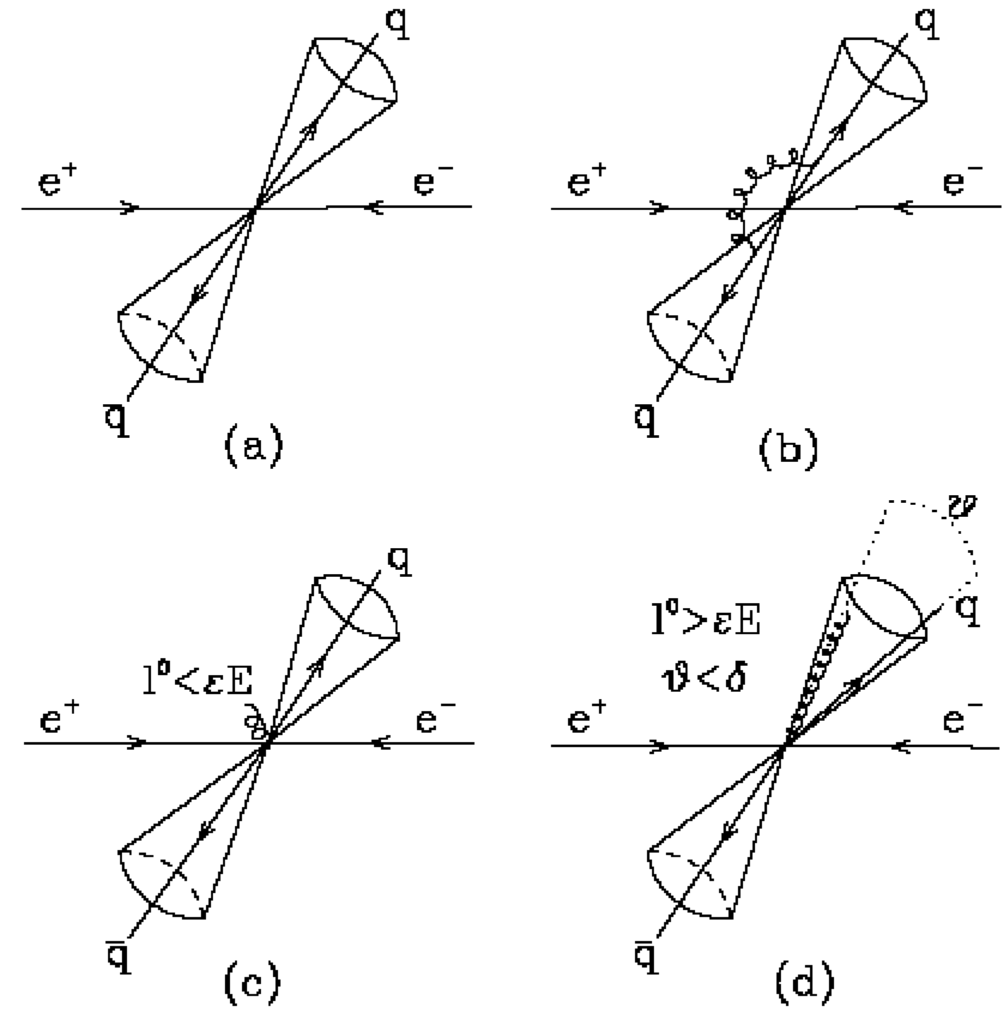
Let's see when the various contributions add up to the Sterman-Weinberg 2-jet cross section:

✷ The Born cross section contributes to the 2-jet cross section, INDEPENDENTLY of **ε** and **δ**.

✷The SAME as above for the virtual corrections.

✷The real corrections when k^0<**ε**E (soft).

✷The real corrections when k^0>**ε**E AND **θ**<**δ** (collinear).



$$\text{Born} + \text{Virtual} + \text{Real (a)} + \text{Real (b)} = \sigma^{\text{Born}} - \sigma^{\text{Born}} \frac{4\alpha_S C_F}{2\pi} \int_{\epsilon E}^{E} \frac{dk^0}{k^0} \int_{\delta}^{\pi-\delta} \frac{d\cos\theta}{1-\cos^2\theta}$$

$$= \sigma^{\text{Born}} \left(1 - \frac{4\alpha_S C_F}{2\pi} \log\epsilon \log\delta\right)$$

As long as **δ** and **ε** are not too small, we find that the fraction of 2-jet cross section is almost 1! At high energy most of the events are two-jet events. As the energy increases the jets become thinner.

A very simple jet iterative algorithm (bottom-up)

1. Consider $e^+e^- \to N$ partons
2. Consider all pairs i and j and calculate

   IF

   min $(p_i + p_j)^2 < y_{cut}\, s$

   THEN

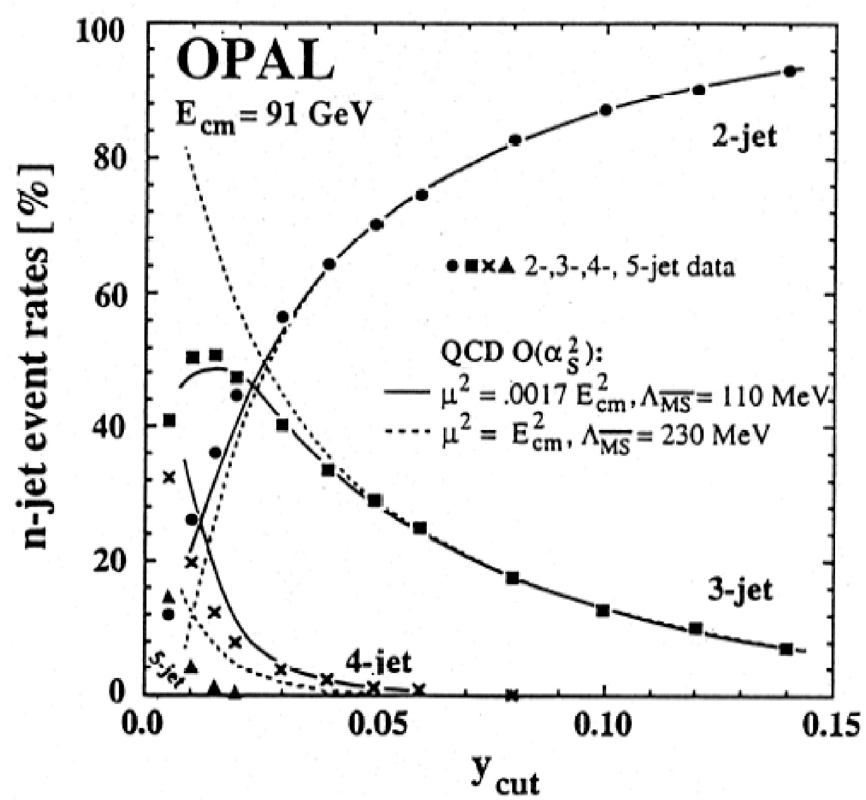
   replace the two partons i,j by $p_{ij} = p_i + p_j$ and decrease N → N-1
3. IF N=1 THEN stop ELSE goto 2.
4. N = number of jets in the event using the "scale" y.

In our example
$0<x_1,x_2<1-y$, $x_1+x_2>1+y$
$y<1/3$

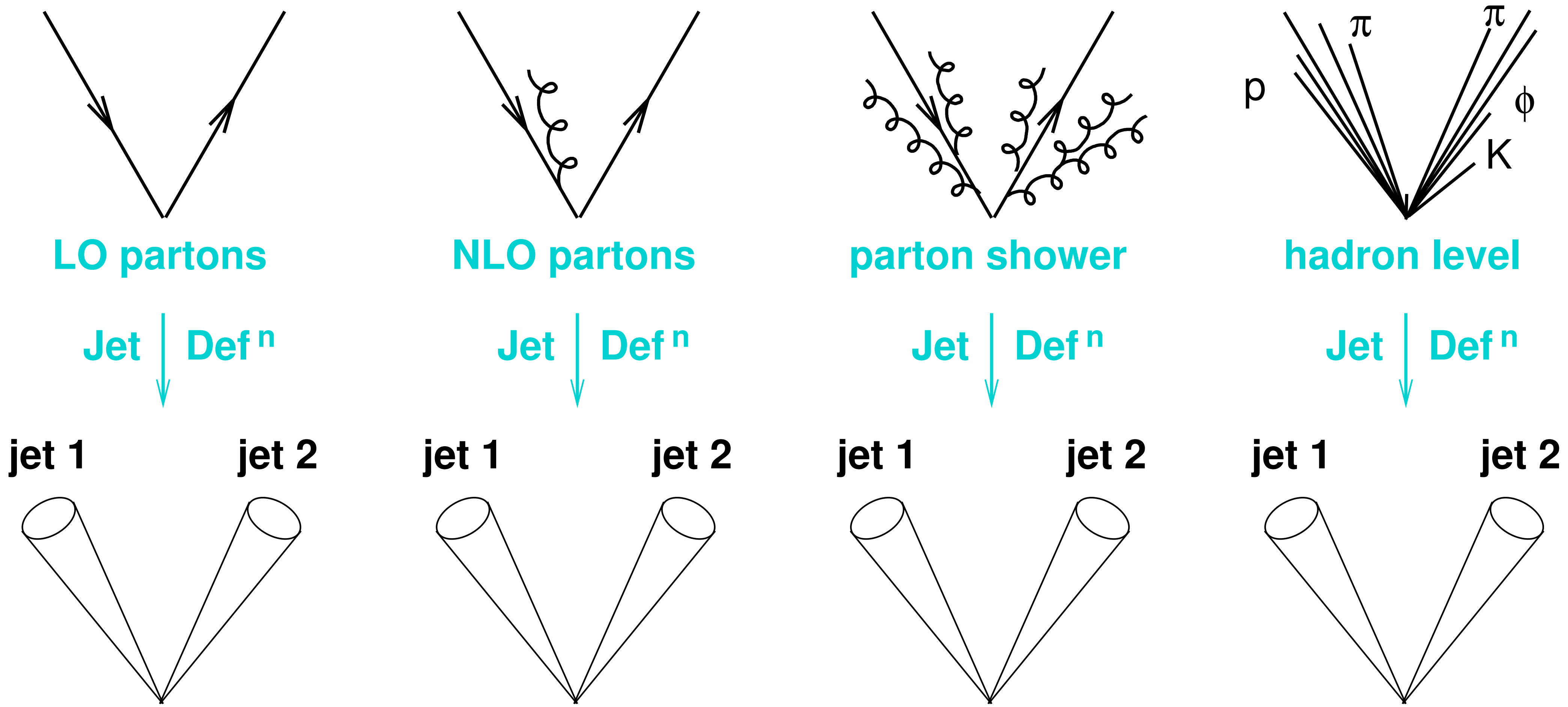
The result of the algo can be calculated analytically at NLO:

$$\sigma_{2j} = \sigma^{\text{Born}} \left(1 - \frac{\alpha_S C_F}{\pi} \log^2 y + \ldots \right)$$

$$\sigma_{3j} = \sigma^{\text{Born}} \frac{\alpha_S C_F}{\pi} \log^2 y + \ldots$$



Jet algorithms

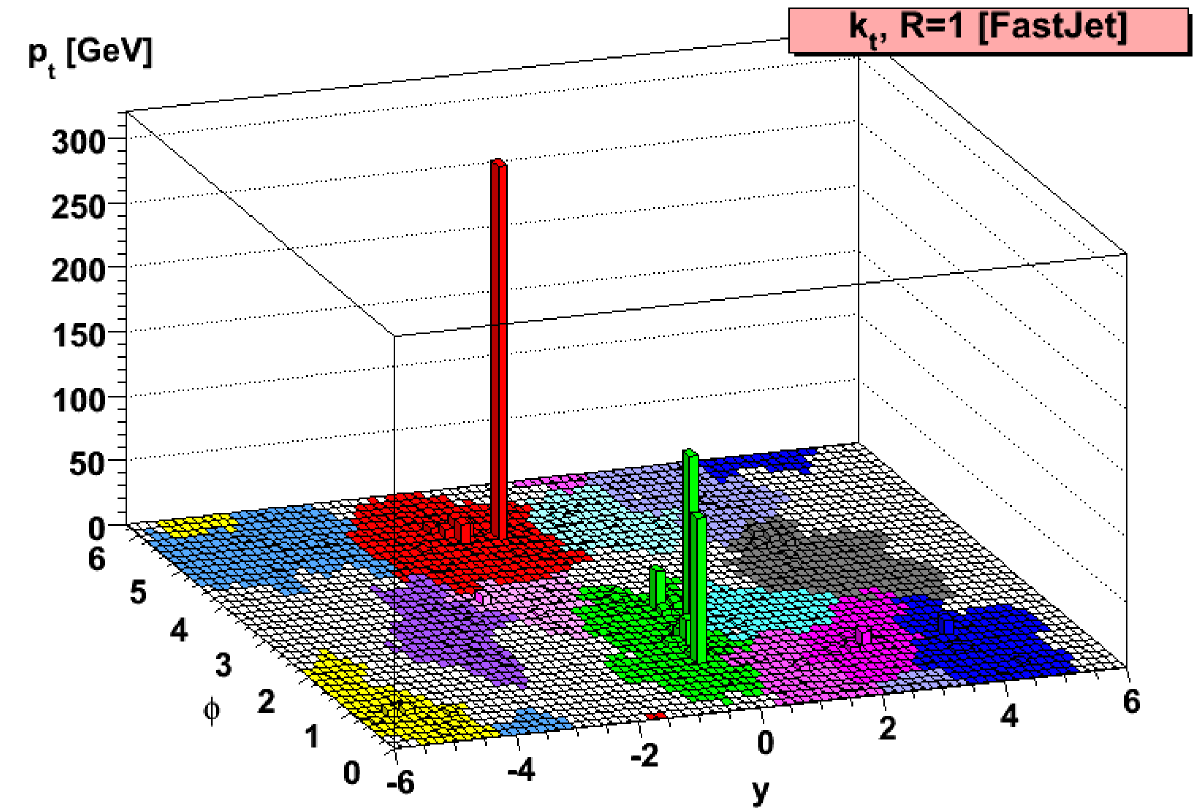


kT algorithm at hadron colliders

Measure (dimensionful):

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2)\frac{\Delta R_{ij}^2}{R^2}$$

$$d_{iB} = p_{ti}^2$$



The algorithm proceeds by searching for the smallest of the d_{ij} and the d_{iB}.

If it is a then d_{ij} particles i and j are recombined* into a single new particle.

If it is a d_{iB} then i is removed from the list of particles, and called a jet.

This is repeated until no particles remain.

kT algorigthm "undoes" the QCD shower

*a 4-momenta recombination scheme is needed (E-scheme)

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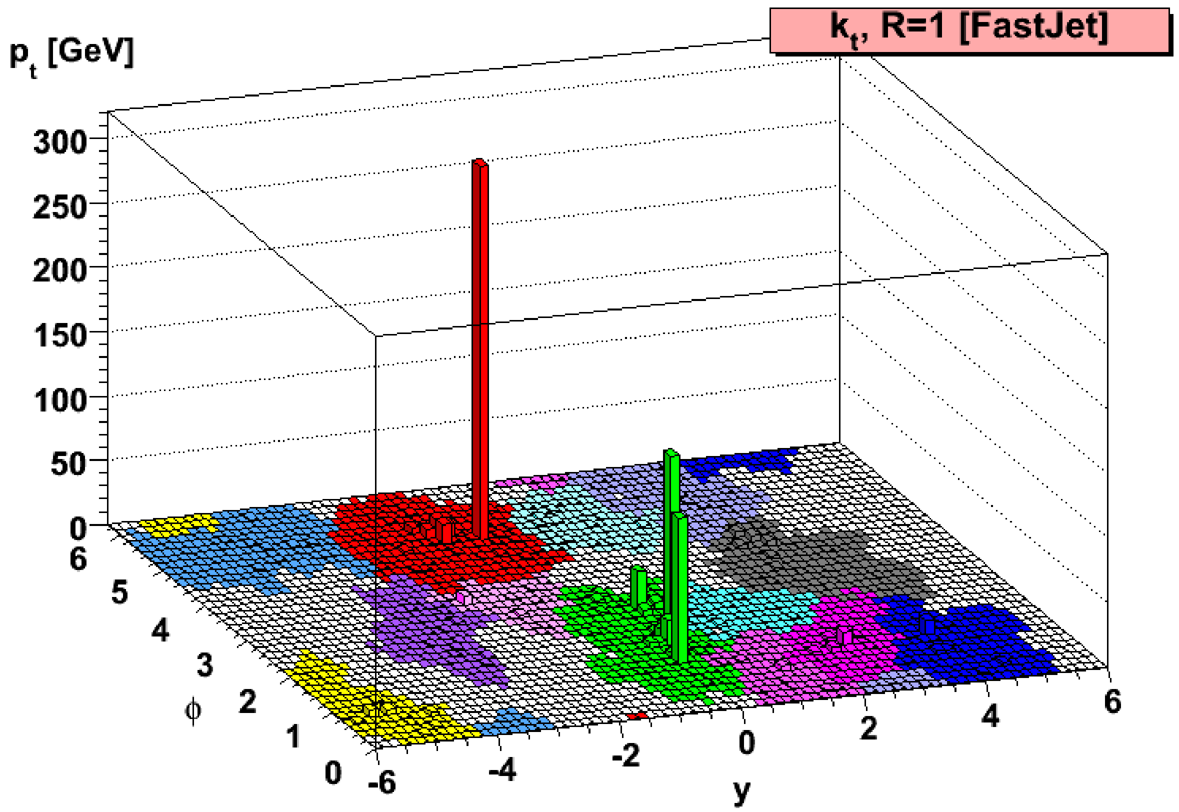
kT algorithm at hadron colliders



Comments:

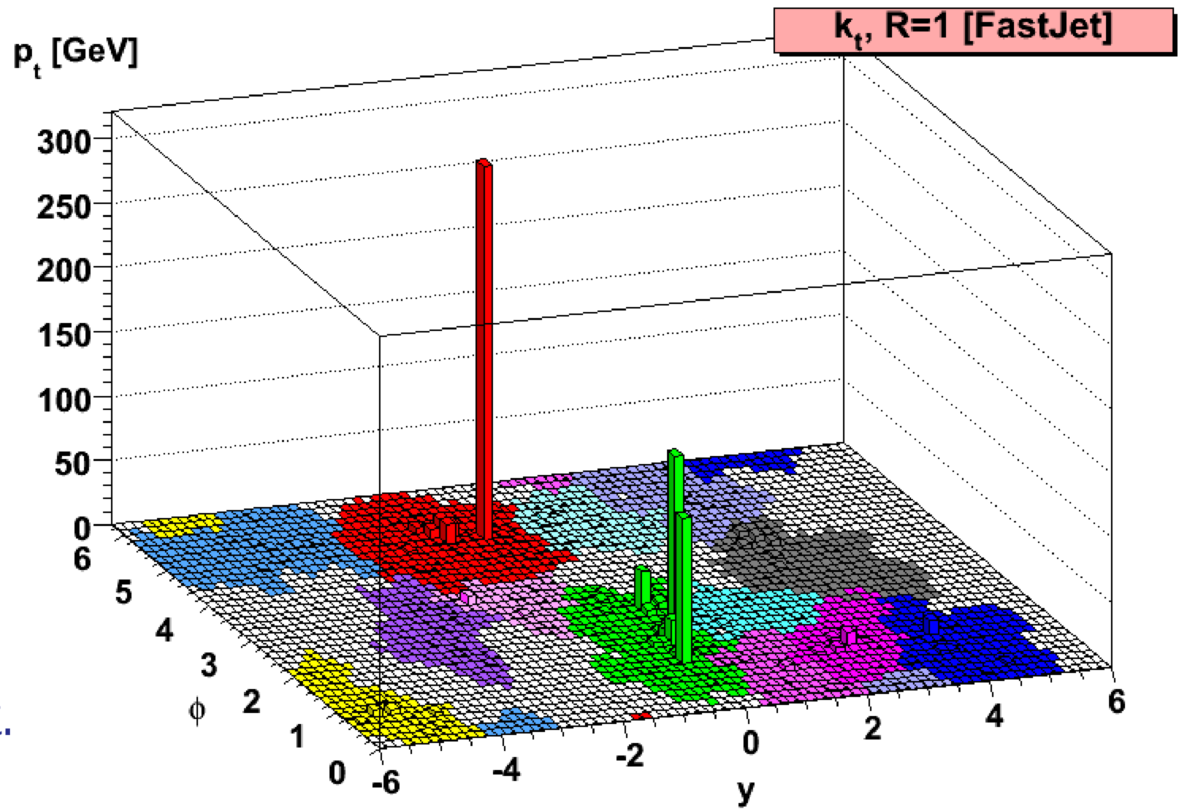
As with cone algorithms, arbitrarily soft particles can form jets. It is therefore standard to place a p_T^{MIN} cutoff on the jets one uses for `hard' physics.

R in the kT algorithm plays a similar role to R in cone algorithms: if two particles i and j are within R of each other, i.e., $\Delta R_{ij} < R$, then $d_{ij} < d_{iB}, d_{jB}$ and so i and j will prefer to recombine rather than forming separate jets.

If a particle i is separated by more than R from all other particles in the event then it will have $d_{iB} < d_{ij}$ for all j and so it will form a jet on its own.

For the kT algorithm, the jets have irregular edges, because many of the soft particles cluster together early in the recombination sequence

kT ALGORITHM AT HADRON COLLIDERS



Comments:

Irregular jets are an undesired feature kT algorithm.

1. Acceptance corrections are harder to calculate.
2. Underlying event corrections depend on the area.
3. Non-linear dependence on soft particles.

Energy calibrations for the jet algorithm are more difficult.

The kT algorithm that is attractive because it assigns a clustering sequence to the particles within the jet.
One can therefore "undo" the clustering and look inside the jet.

This has been exploited in a range of QCD studies and also in discussions of searches of hadronic decays of boosted massive particles such as W, H, or Z bosons, top quarks, or new particles.

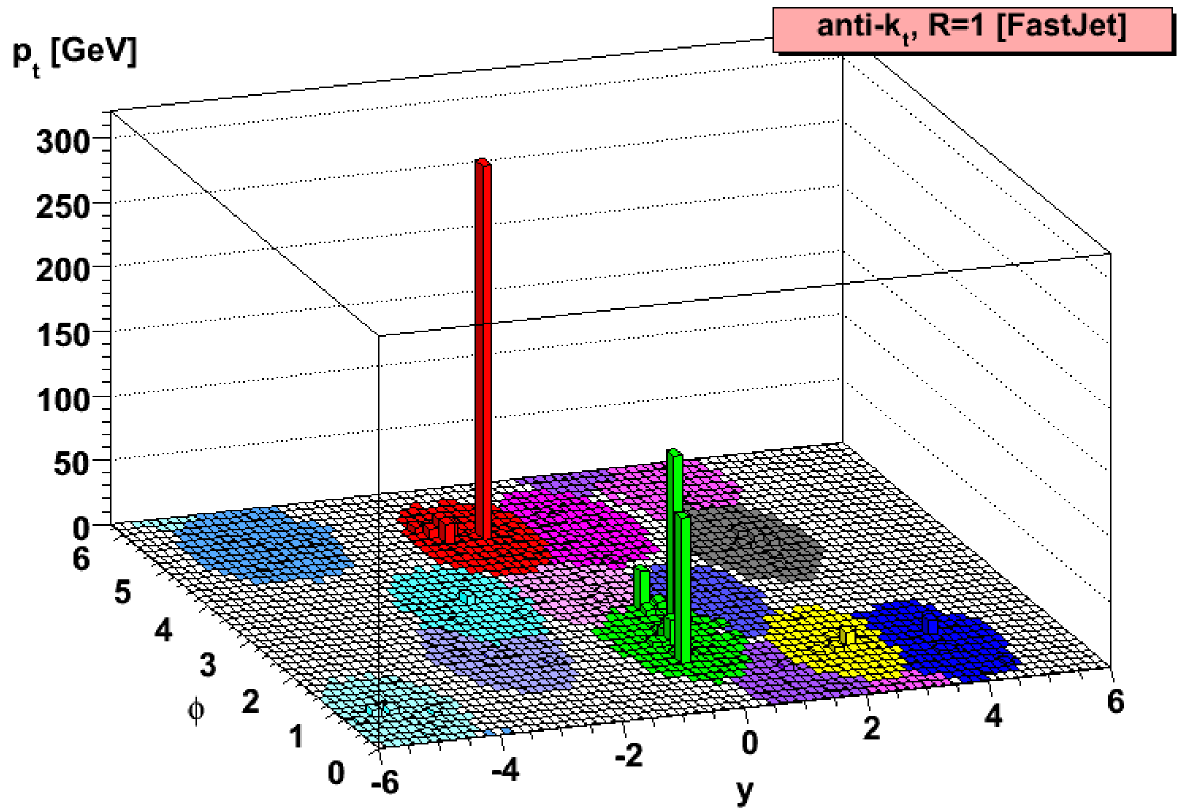
Jet substructure studies are also often carried out with the Cambridge/Aachen (C/A) algorithm which is like the kT but with $pT_j=1$

ANTI-KT ALGORITHM

Measure (dimensionful):

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}$$

$$d_{iB} = \frac{1}{p_{ti}^2}$$



Objects that are close in angle prefer to cluster early, but that clustering tends to occur with a hard particle (rather than necessarily involving soft particles). This means that jets `grow' in concentric circles out from a hard core, until they reach a radius R, giving circular jets.

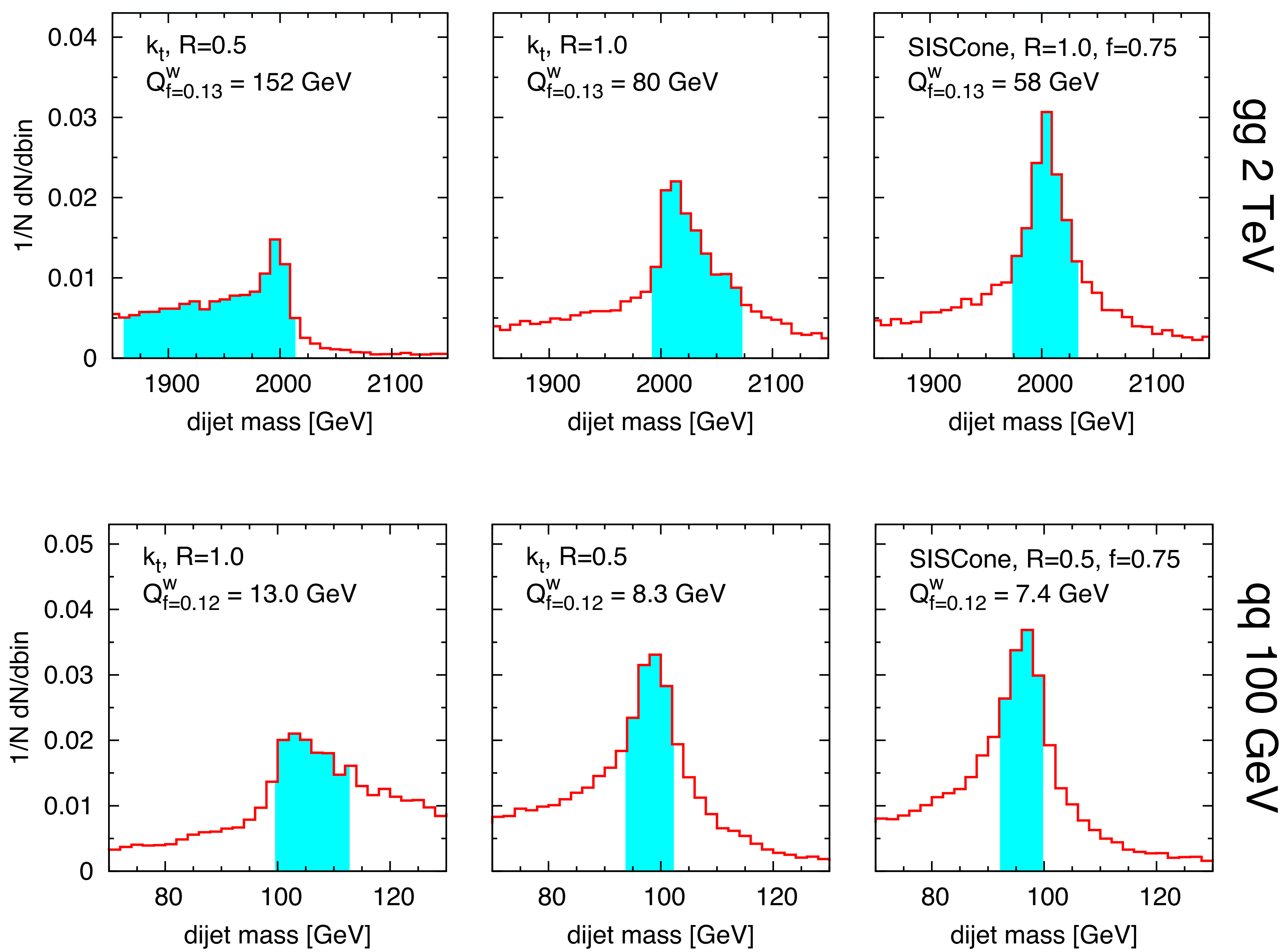
Unlike cone algorithms the `anti-kT' algorithm is collinear (and infrared) safe.

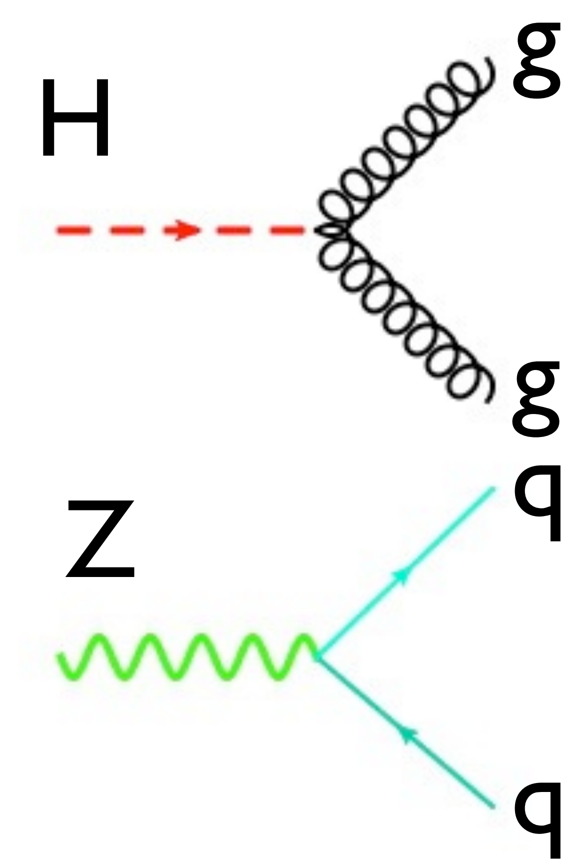
This, (and the fact that it has been implemented efficiently in FastJet, has led to be the default jet algorithm at the LHC.

It's a handy algorithm but it does not provide internal structure information.

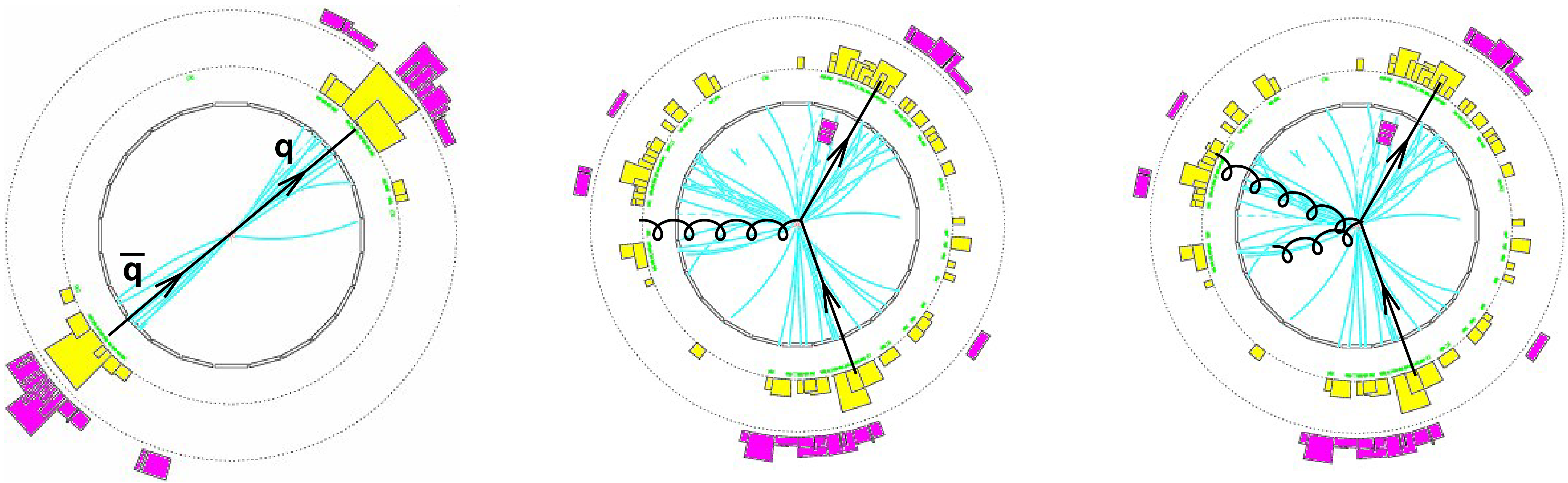
WHAT JET ALGO SHOULD I CHOOSE?

It depends on what are you looking (Singlet or colored, resonance decaying to gg, qq, bb) for and which observable you want to accurately measure : see a sharp peak or measure the position of the peak...

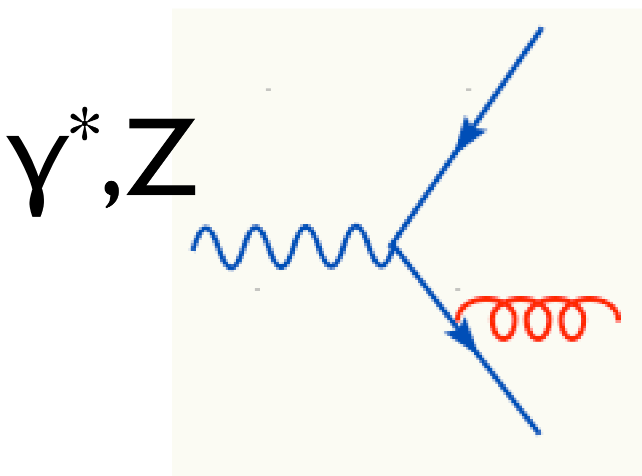


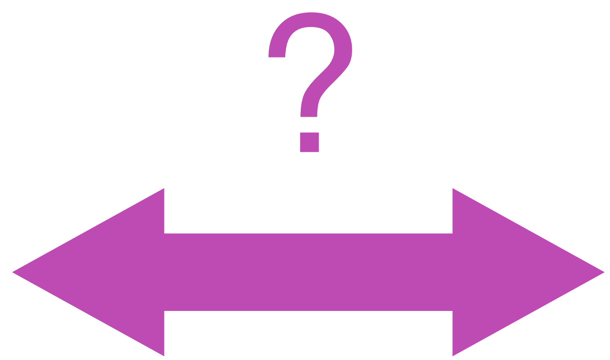


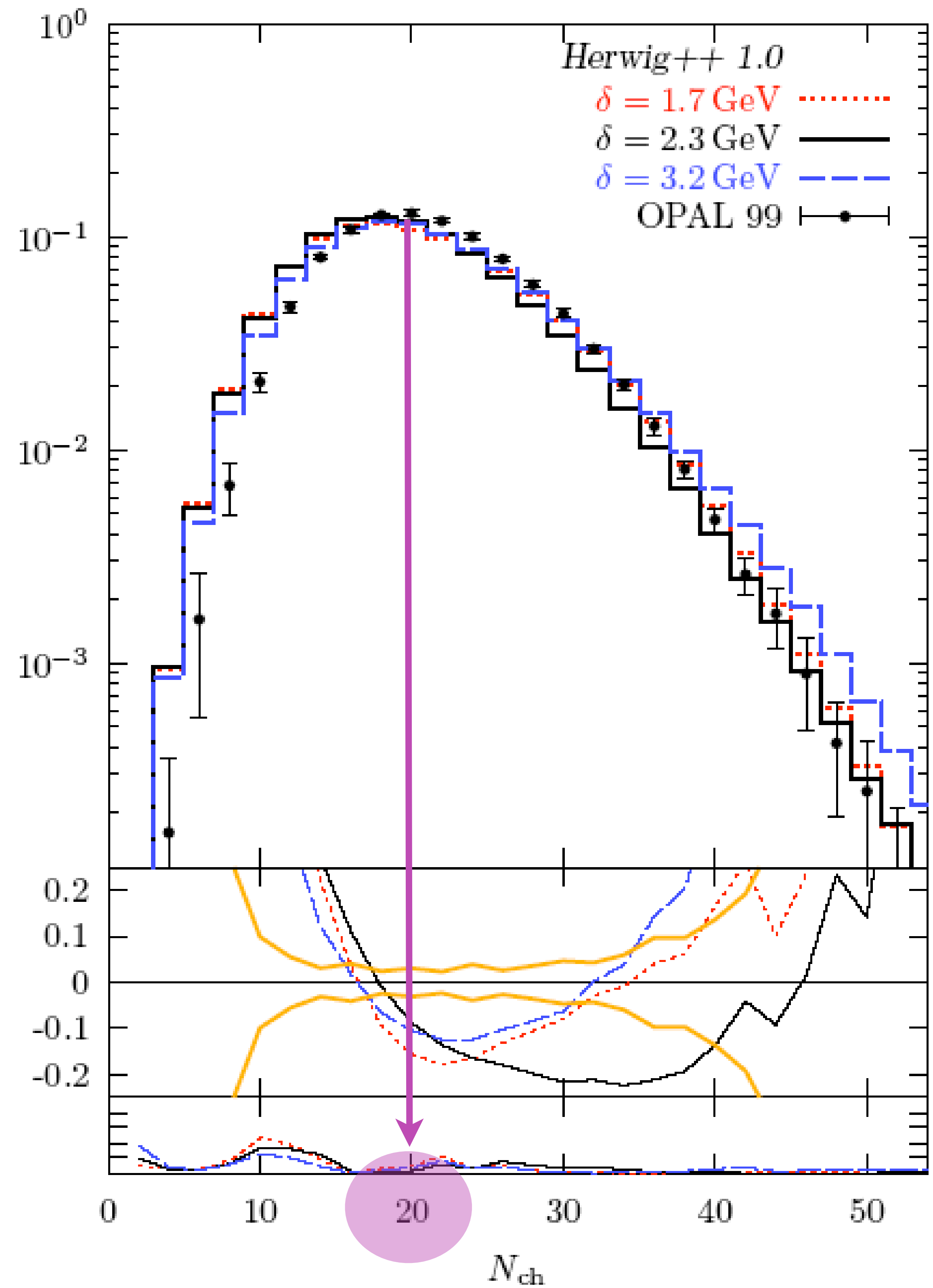
More exclusive quantities



Number of particles in the final state?
Number of particles per jet?
Jet mass?


γ*,Z


?


Herwig++ 1.0
δ = 1.7 GeV
δ = 2.3 GeV
δ = 3.2 GeV
OPAL 99
10^0
10^{-1}
10^{-2}
10^{-3}
0.2
0.1
0
-0.1
-0.2
0
10
20
30
40
50
N_{ch}

More exclusive quantities (AKA, the power of exponentation)

Assuming "abelian" gluons one finds that something magic happens at higher orders:

$$\sigma_{2j} = \sigma^{\rm Born}\left[1 - \frac{\alpha_S C_F}{\pi}\log^2 y + \frac{1}{2!}\left(\frac{\alpha_S C_F}{\pi}\log^2 y\right)^2 + \ldots\right] = \sigma^{\rm Born} e^{-\frac{\alpha_S C_F}{\pi}\log^2 y}$$

$$\sigma_{3j} = \sigma^{\rm Born}\frac{\alpha_S C_F}{\pi}\log^2 y\, e^{-\frac{\alpha_S C_F}{\pi}\log^2 y}$$

$$\vdots$$

$$\sigma_{nj} = \sigma^{\rm Born}\frac{1}{n!}\left(\frac{\alpha_S C_F}{\pi}\log^2 y\right)^n e^{-\frac{\alpha_S C_F}{\pi}\log^2 y}$$

The number of jets is distributed as a Poisson with average (and the full QCD result):

$$< n_j >= 2 + \frac{\alpha_S C_F}{\pi}\log^2 y$$

$$< n_j >_{\rm QCD} = \frac{C_F}{C_A}\exp\sqrt{\frac{\alpha_S C_A}{2\pi}\log^2\frac{1}{y}}$$

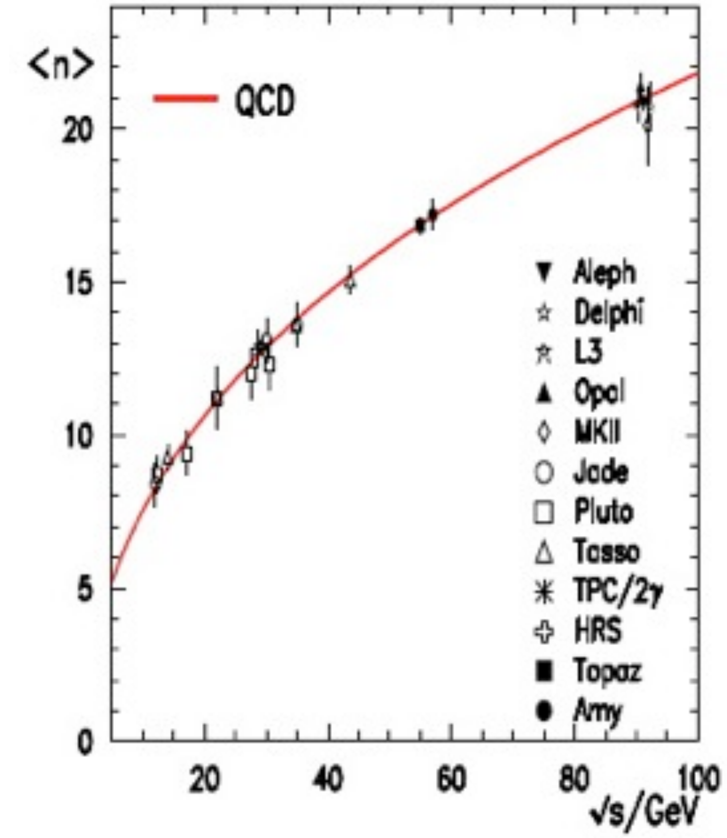
More exclusive quantities (AKA, the power of exponentation)

Identifying one particle with one jet at resolution scale of Λ_s one obtains an estimate for the average number of particles in an event (multiplicity):

$$< n_p >= \frac{\alpha_S C_F}{\pi} \log^2 \frac{s}{\Lambda_s^2} = \frac{C_F}{\pi b_0} \log \frac{s}{\Lambda_s^2}$$

$$< n_p >_{\mathrm{QCD}} = \exp \sqrt{\frac{2C_A}{\pi b_0} \log \frac{s}{\Lambda_s}}$$

ie. the multiplicity grows with the log of the com energy.



Finally the jet mass can also be easily estimated by integrating the cross sections over two emispheres identified by the thrust axis:

$$< m_j^2 >= \frac{1}{2\sigma_{\mathrm{Born}}} \left[\int_{(I)} (q+k)^2 d\sigma_g + \int_{(II)} (q+k)^2 d\sigma_g \right] = \frac{\alpha_S C_F}{\pi} s$$

This result gives the correct scaling of the jet mass, $m_j \sim \sqrt{\alpha_s} E_j$, which is also valid at hadron colliders (replacing E with p_t)!

SUMMARY

1. We have studied the problem of infrared divergences in the calculation of the fully inclusive cross section, with the help of the soft limit.

2. We have introduced the concept of an Infrared Safe quantity, i.e. an observable which is both computable at all orders in pQCD and has a well defined counterpart at the experimental level.

3. We have discussed more exclusive quantities, from shape functions to fully exclusive quantities and compared them with e+ e- data. We have introduced the method of exponentiation.

4. We have introduced the idea of jet algorithms (top-down and bottom-up) and discussed the most recent algorithms.