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LHC phenomenology with MadGraph: Exercises

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1 Introduction

This is a collection of tests, exercises, web applications and simulations useful for a first approach to understanding QCD and its role in collider physics. Some useful information on the exercises and the lectures can be found at

`http://cp3wks05.fynu.ucl.ac.be/twiki/bin/view/Physics/Milano2010`

and include

- Slides of the lectures.
- Exercises.
- Links to MadGraph/MadEvent with additional information

The basic references where most of the exercises and examples were taken from are:

- "QCD and Collider Physics" by R.K. Ellis, J.W. Stirling, B.R. Webber (Cambridge Monographs, 1996). In addition some of the exercises reproposed here are from lectures given at CERN by B.R. Webber.

Tests are collections of very easy questions on the content of the lectures, which sometimes imply a very short calculation.

The simulations can be performed with the help of the `MadGraph/MadEvent` MonteCarlo, directly from the web or by downloading the main code at `http://madgraph.phys.ucl.ac.be`.

The available servers are three:

- UCL : **http://madgraph.phys.ucl.ac.be**
- IT : **http://madgraph.roma2.infn.it**
- US : **http://madgraph.hep.uiuc.edu**

To generate events on the clusters higher clearance has to be obtained from the lecturer. For the tutorial we have activated a temporary account (User: Angels, Password: Demons) where web event generation is allowed.

2 Basic kinematics

The rapidity y and pseudo-rapidity η are defined as:

$$y = \frac{1}{2}\log\left(\frac{E+p_z}{E-p_z}\right) \tag{1}$$

$$\eta = -\log\left(\tan\left(\frac{\theta}{2}\right)\right), \tag{2}$$

where the z direction is that of the colliding beams.

(a) Verify that for a particle of mass m

$$E = \sqrt{m^2+p_T^2}\quad \cosh y \tag{3}$$

$$p_z = \sqrt{m^2+p_T^2}\quad \sinh y \tag{4}$$

$$p_T^2 = p_x^2+p_y^2\,. \tag{5}$$

(b) Prove that $\tanh\eta = \cos\theta$.

(c) Consider a set of particles produced uniformly in longitudinal phase space

$$dN = C\frac{dp_z}{E}\,. \tag{6}$$

Find the distribution in η.

(d) Prove that rapidity equals pseudo-rapidity, $\eta = y$ for a relativistic particle $E \gg m$.

(e) Prove that for Lorentz transformation (boost) in the beam (z) directions, the rapidity y of every particle is shifted by a constant y_0, related to the boost velocity. Find the relation between β and y_0 for a generic boost:

$$E' = \gamma(E-\beta p_z) \tag{7}$$

$$p_z' = \gamma(p_z-\beta E) \tag{8}$$

$$p_x' = p_x \tag{9}$$

$$p_y' = p_y \tag{10}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{11}$$

(f) Consider a generic particle X of mass M (such as a Z boson or a Higgs) produced on shell at the LHC , with zero transverse momentum, $pp \to X$. Find the relevant values of x_1, x_2 of the initial partons that can be accessed by producing such a particle. Compare your results with that of Fig. 1, considering the scale $Q = M$.

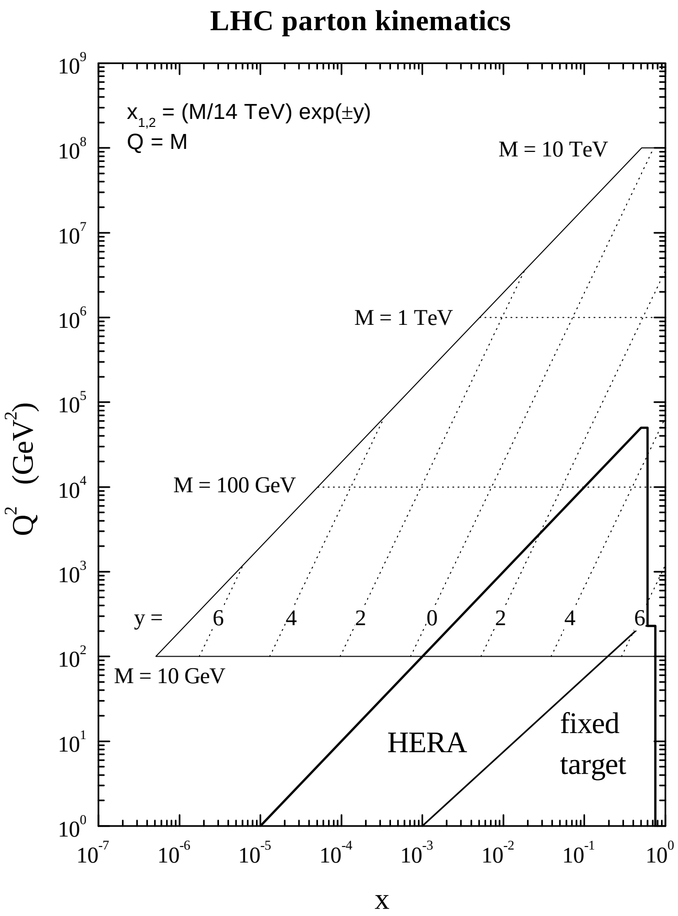


Figure 1: Range in x, Q accessible at the LHC.

3 Jet kinematics

At the LHC, partons in the incoming beams (beam energy E_b=7 TeV) collide with a momementum fraction $x_{1,2}$ and produce two jets with negligible mass, transverse momentum p_T and rapidities $y_{3,4}$.

(a) Show that

$$x_1 = \frac{p_T}{\sqrt{s}}(e^{y_3} + e^{y_4}), \qquad x_2 = \frac{p_T}{\sqrt{s}}(e^{-y_3} + e^{-y_4})\,. \tag{12}$$

(b) Show that the invariant mass of the jet-jet system is

$$M_{JJ} = 2p_T \cosh\left(\frac{y_3 - y_4}{2}\right)\,, \tag{13}$$

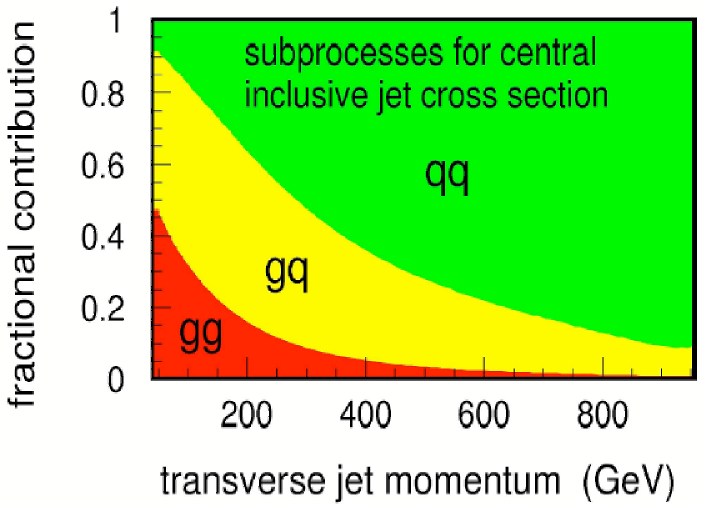


Figure 2: Plot showing the fraction of the jet E_T distribution initiated by different parton combinations.

and the centre-of-mass scattering angle is given by

$$\cos\theta^* = \tanh\left(\frac{y_3 - y_4}{2}\right) . \tag{14}$$

(c) Discuss the regions of $x_{1,2}, M_{JJ}$ and θ^* that can be studied at the LHC with a jet trigger of $p_T > 35$ GeV and $|y_{3,4}| < 3$.

3.1 MC simulations

3.1.1 Jet fraction from different parton combinations

Use `MadGraph/MadEvent` to obtain the relative contribution of $gg, qg + \bar{q}g, qq + q\bar{q}$ initial states to the jet E_T distribution as function of the E_T ($10 < E_T < E_{\rm max}/4$) at the Tevatron Run II ($p\bar{p}$ collisions at 1.96 TeV) and the LHC (pp collisions at 14 TeV). Compare with the results at the Tevatron, Run I shown in Fig. 2

3.1.2 Multijet production at the Tevatron$^{(*)}$

Use `MadGraph/MadEvent` to obtain the distributions of x_3, x_4 where $x_i = 2E_i/M_{3j}$ are the energy fraction of the jets normalized as $x_3 + x_4 + x_5 = 2$, with $x_3 > x_4 > x_5$, in three-jet events. Consider $p\bar{p}$ collisions at 1.8 TeV of c.m.s. Set the minimum p_T for the jets to 15 GeV, the maximum rapidity of 3.5 and the $\Delta R = 0.8$. Compare your results with the experimental data from CDF, Run I, shown in Fig. 3.

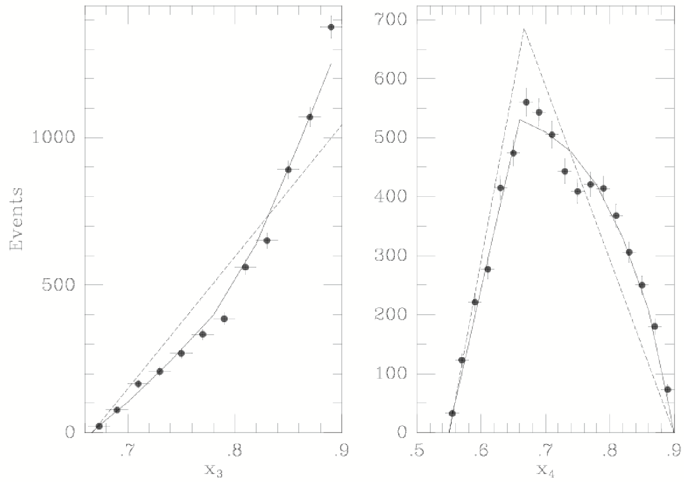
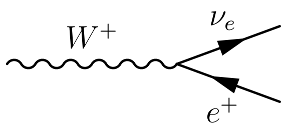


Figure 3: Distributions in the variables x_3 and x_4 in a sample of three jet events as measured by the CDF collaboration (Run I data), $p\bar{p}$ collisions at 1.8 TeV. The solid and dashed lines are the predicitons from QCD and phase space respectively.

4 Drell-Yan production at hadron colliders

4.1 The decay rate of the W



(a) Calculate analytically the tree-level decay rate of the W boson to leptons. The formula for the decay rate is given by

$$\Gamma = \frac{1}{2m} \int \mathrm{d}\Phi_2 \, |\mathcal{M}|^2, \tag{15}$$

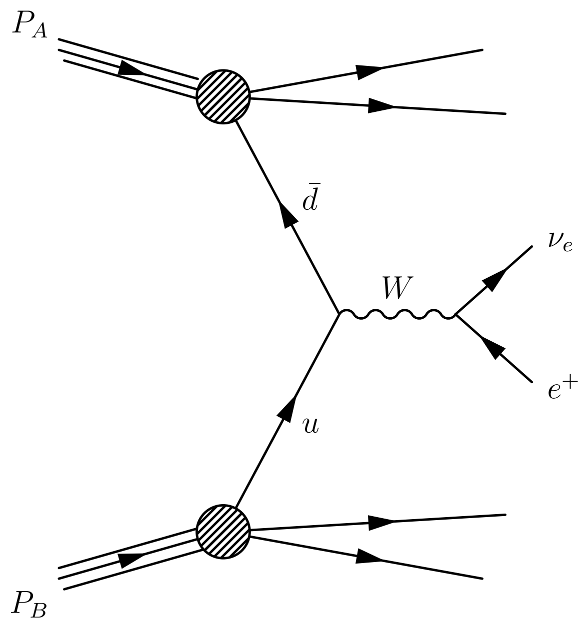
where $\mathcal{M}$ denotes the matrix element describing the decay, m is the mass of the decaying particle and $\mathrm{d}\Phi_2$ is the two-particle phase space measure.
Answer:

$$\Gamma(W \to \ell\nu) = \frac{G_F m_W^3}{6\sqrt{2}\pi} \tag{16}$$

(b) How does the decay rate get modified if the W decays to quarks?

(c) Give an estimate for the total width of the W.

4.2 The Drell-Yan cross-section in the narrow width approximation



(a) The partonic cross-section near the resonance is described by the Breit-Wigner formula:

$$\hat{\sigma}(u\bar{d} \to W^+ \to \ell^+\nu_\ell) \approx \frac{4\pi}{3} \frac{\Gamma_{\ell\nu_\ell}\Gamma_{u\bar{d}}}{(\hat{s} - m_W^2)^2 - \Gamma^2 m_W^2}, \tag{17}$$

where $\Gamma_{\ell\nu}$, $\Gamma_{u\bar{d}}$ and Γ denote the partial and total decay rates of the W (See Exercise 4.1), and $\hat{s}$ denotes the partonic center of mass energy.
In the limit where $m_W \gg \Gamma$, we can use the narrow width approximation for the cross-section. Use

$$\delta(x) = \lim_{\epsilon \to 0} \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2} \tag{18}$$

to derive the expression of the cross-section in the narrow width approximation.
Answer:

$$\hat{\sigma}(u\bar{d} \to W^+ \to \ell^+\nu_\ell) \approx \hat{\sigma}(\hat{s} = m_W^2)\delta(\hat{s} - m_W^2)\, m_w\, \Gamma. \tag{19}$$

(b) Fold the partonic cross-section with PDF's to obtain the full cross-section for Drell-Yan production at Tevatron,

$$\sigma(p\bar{p} \to W^+ \to \ell^+\nu_\ell) = \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2\, u(x_1)\, \bar{d}(x_2)\, \hat{\sigma}(u\bar{d} \to W^+ \to \ell^+\nu_\ell), \tag{20}$$

where $u(x)$ and $d(x)$ denote the PDF's of the u and d quarks inside the proton. For this exercise we choose

$$u(x) = 6(1-x)^2, \qquad d(x) = 3(1-x)^2. \tag{21}$$

4.3 Drell-Yan at the Tevatron and the LHC

Use Madgraph/MadEvent to generate $pp \to W^\pm \to e^\pm \nu_e$ at the Tevatron and the LHC. Compare the cross sections and indentify the qualititative differences.

4.4 W rapidity asymmetry at the Tevatron

The rapidity asymmetry $A_W(y)$ for $W^\pm$ production at a $p\bar{p}$ collider is defined as:

$$A_W(y) = \frac{d\sigma(W^+)/dy - d\sigma(W^-)/dy}{d\sigma(W^+)/dy + d\sigma(W^-)/dy}. \qquad (22)$$

(a) Give an estimate of such asymmetry and show that it is proportional to the slope of $d(x)/u(x)$ evaluated at $x = M_W/\sqrt{s}$.

(b) Use the web interface of `MadGraph/MadEvent` to plot the rapidity distributions of the the charged leptons coming from $W^\pm$ decays at the Tevatron.

(c) Can such an asymmetry be defined for the LHC as well?

5 Higgs Phenomenology

5.1 Decay modes of the Higgs boson

(a) A light SM Higgs boson mostly decays into $b\bar{b}$. Compute the decay rate for $H \to b\bar{b}$.
Answer:

$$\Gamma(H \to b\bar{b}) = \frac{G_F m_H m_b^2}{4\sqrt{2}\pi} \left(1 - 4\frac{m_b^2}{m_H^2}\right)^{3/2}. \qquad (23)$$

(b) For a Higgs mass of $\sim 160\,\text{GeV}$, the main decay mode is $H \to WW$. Compute the decay rate.
Answer:

$$\Gamma(H \to WW) = \frac{G_F m_H^3}{8\sqrt{2}\pi} \sqrt{1-4x^2}\left(1 - 4x^2 + 12x^4\right), \qquad (24)$$

where $x = m_W/m_H$.

The branching ratios of the Higgs boson can be found here:
`http://cp3wks05.fynu.ucl.ac.be/twiki/pub/Physics/DiscoverTheHiggs/h-br.pdf`

5.2 Gluon fusion

The primary production mechanism for a Higgs boson in hadronic collisions is through gluon fusion, $gg \to H$, which is shown in Fig. 4. The loop contains all massive colored particles in the model. Consider only the top quark. To evaluate the diagram of Fig. 4 (there are actually two diagrams, the one shown and another one with the gluons exchanged. They give the same contribution so we'll just multiply our final result by two), use use dimensional regularization in $D = 4 - 2\epsilon$ dimensions.

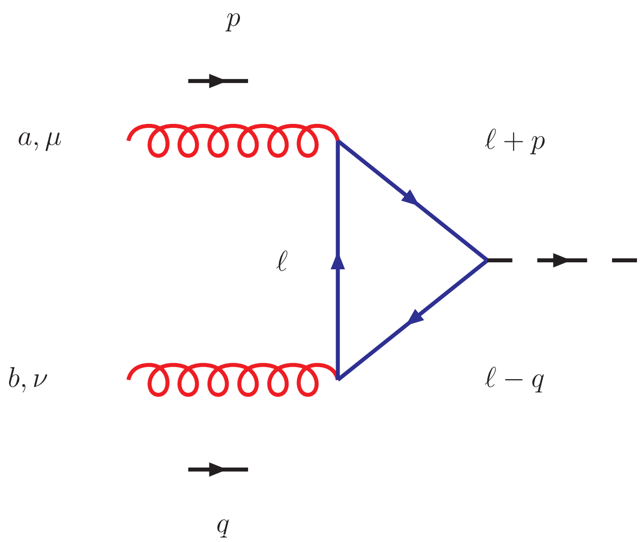


Figure 4: Representative Feynman diagram for the process $gg \to H$. Another diagram, the one with the gluons exchanged, contributes to the total amplitude.

(a) Using the QCD Feynman rules write the expression for the amplitude corresponding to the diagram of Fig. 4:

$$i\mathcal{A} = -(-ig_s)^2 \text{Tr}(t^a t^b) \left(\frac{-im_t}{v} \right) \int \frac{d^d \ell}{(2\pi)^n} \frac{T^{\mu\nu}}{\text{Den}} (i)^3 \epsilon_\mu(p) \epsilon_\nu(q) \tag{25}$$

where the overall minus sign is due to the closed fermion loop.[1] The denominator is $\text{Den} = (\ell^2 - m_t^2)[(\ell + p)^2 - m_t^2][(\ell - q)^2 - m_t^2]$.

(b) Use the usual Feynman parametrization method to combine the denominators of the loop integral into one, using the following:

$$\frac{1}{ABC} = 2 \int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1 - x - y)]^3} \tag{26}$$

and so the denominator becomes,

$$\frac{1}{\text{Den}} = 2 \int dx\, dy \frac{1}{[\ell^2 - m_t^2 + 2\ell \cdot (px - qy)]^3}. \tag{27}$$

(c) Shift the integration momenta to $\ell' = \ell + px - qy$ so the denominator takes the form

$$\frac{1}{\text{Den}} \to 2 \int dx\, dy \frac{1}{[\ell'^{\,2} - m_t^2 + M_H^2 xy]^3}. \tag{28}$$

[1] $\epsilon_\mu(p)$ are the transverse gluon polarizations.

(d) Evaluate the numerator of the loop integral in the shifted loop momentum:

$$\begin{aligned} T^{\mu\nu} &= \mathrm{Tr}\Big[(\not\ell + m_t)\gamma^\mu(\not\ell + \not p + m_t)(\not\ell - \not q + m_t)\gamma^\nu)\Big] \\ &= 4m_t\Big[g^{\mu\nu}(m_t^2 - \ell^2 - \frac{M_H^2}{2}) + 4\ell^\mu\ell^\nu + p^\nu q^\mu\Big] \end{aligned} \tag{29}$$

Use the fact that that for transverse gluons, $\epsilon(p)\cdot p = 0$ and so terms proportional to the external momenta, p_μ or q_ν, can be dropped. You should find that the trace is proportional to the quark mass. This can be easily understood as an effect of the spin-flip coupling of the Higgs. Gluons or photons do not change the spin of the fermion, while the Higgs does. If the quark circulating in the loop is massless then the trace vanishes due to helicity conservation. This is the reason why even when the Yukawa coupling of the light quark and the Higgs is enhanced (such as in SUSY or 2HDM with large $\tan(\beta)$), the contribution is anyway suppressed by the kimatical mass.

(e) Shift momenta in the numerator, drop terms linear in ℓ' and use the relation

$$\int d^dk \frac{k^\mu k^\nu}{(k^2 - C)^m} = \frac{1}{d} g^{\mu\nu} \int d^dk \frac{k^2}{(k^2 - C)^m} \tag{30}$$

to write the amplitude in the form

$$\begin{aligned} i\mathcal{A} &= -\frac{2g_s^2 m_t^2}{v}\delta^{ab}\int \frac{d^d\ell'}{(2\pi)^d}\int dxdy\Big\{g^{\mu\nu}\Big[m^2 + \ell'^2\Big(\frac{4-d}{d}\Big) + M_H^2(xy - \frac{1}{2})\Big] \\ &+ p^\nu q^\mu(1 - 4xy)\Big\}\frac{2dxdy}{(\ell'^2 - m_t^2 + M_H^2 xy)^3}\epsilon_\mu(p)\epsilon_\nu(q). \end{aligned} \tag{31}$$

(f) Compute the integral of Eq. 31 by using the well known formulas of dimensional regularization

$$\begin{aligned} \int \frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^2 - C)^3} &= \frac{i}{32\pi^2}(4\pi)^\epsilon\frac{\Gamma(1+\epsilon)}{\epsilon}(2 - \epsilon)C^{-\epsilon} \\ \int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2 - C)^3} &= -\frac{i}{32\pi^2}(4\pi)^\epsilon\Gamma(1+\epsilon)C^{-1-\epsilon}. \end{aligned} \tag{32}$$

You should find that your result is finite.

(g) Compare your result with the known result:

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\Big(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\Big)\int dxdy\Big(\frac{1 - 4xy}{m_t^2 - m_H^2 xy}\Big)\epsilon_\mu(p)\epsilon_\nu(q). \tag{33}$$

(Note that we have multiplied by 2 in Eq. (33) to include the diagram where the gluon legs are crossed.) The Feynman integral of Eq. (33) can easily be performed to find an analytic result if desired. Note that the tensor structure could have been predicted from the start by using the fact that $p^\mu \mathcal{A}^{\mu\nu} = q^\nu \mathcal{A}^{\mu\nu} = 0$.

(h) Define $I(a)$ as

$$I(a) \equiv \int_0^1 dx \int_0^{1-x} dy \frac{1-4xy}{1-axy}. \tag{34}$$

and express the amplitude in terms of such an expression. Plot the function $I(a)$ and verify that it goes quickly to its limiting values when $a \to 0$ and $a \to \infty$. Numerically, the heavy fermion mass limit is an extremely good approximation even for $m \sim M_H$. From this plot we can also see that the contribution of light quarks to gluon fusion of the Higgs boson is irrelevant. In fact we have,

$$I(a) \longrightarrow_{a\to\infty} \sim -\frac{1}{2a}\log^2(a). \tag{35}$$

Therefore, for the Standard Model, only the top quark is numerically important when computing Higgs boson production from gluon fusion.

(i) It is particularly interesting to consider the case when the fermion in the loop is much more massive than the Higgs boson, $M_H \ll m_t$. In this case we find,

$$\mathcal{A}(gg \to H) \longrightarrow_{m \gg M_H} -\frac{\alpha_S}{3\pi v}\delta^{ab}\Big(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\Big)\epsilon_\mu(p)\epsilon_\nu(q). \tag{36}$$

We see that the production process $gg \to H$ is independent of the mass of the heavy fermion in the loop in the limit $M_H \ll m_t$. Hence it counts the number of heavy generations and is a window into new physics at scales much above the energy being probed. This is a contradiction of our intuition that heavy particles should decouple and not affect the physics at lower energy. The reason the heavy fermions do not decouple is, of course, because the Higgs boson couples to the fermion mass.

(l) Cross section at the LHC. Resonant production of a heavy Higgs can be found from the standard formula:

$$\begin{aligned}\hat{\sigma} &= \frac{1}{2s}\overline{|\mathcal{A}|^2}\frac{d^3P}{(2\pi)^3 2E_H}(2\pi)^4\delta^4(p+q-P) \\ &= \frac{1}{2s}\overline{|\mathcal{A}|^2}2\pi\delta(s-m_H^2),\end{aligned} \tag{37}$$

using

$$\delta^{ab}\delta^{ab} = N_c^2 - 1$$

$$\Big(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\Big)^2 = \frac{m_H^4}{2} \tag{38}$$

$$\overline{|\mathcal{A}|^2} = \frac{1}{4}\frac{1}{(N_c^2-1)^2}|\mathcal{A}|^2\,. \tag{39}$$

Verify that the result is

$$\hat{\sigma}(gg \to H) = \frac{\alpha_S^2}{64\pi v^2} \mid I\Big(\frac{M_H^2}{m^2}\Big) \mid^2 \tau_0\delta(\tau - \tau_0)$$

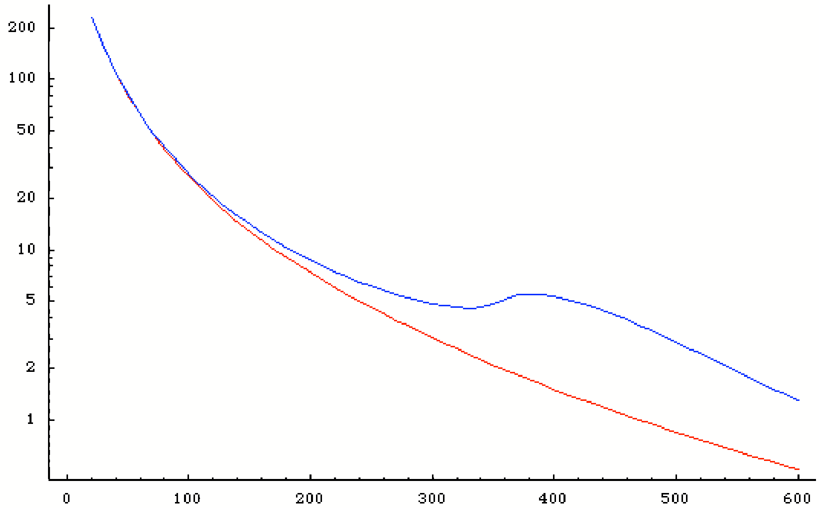


Figure 5: LO cross section for $pp \to H$ at LO at the LHC (pb) as a function of the Higgs mass (GeV).The red (lower) curve is the large top-mass limit, while the blue (upper) curve is the exact result.

where $s = x_1 x_2 S \equiv \tau S$ is the parton-parton energy squared, we have defined

$$z \equiv \frac{M_H^2}{s} = \frac{M_H^2}{\tau S} = \frac{\tau_0}{\tau} \tag{40}$$

with $\tau_0 = M_H^2/S$ and the integral I is defined by Eq. (34).

(m) To find the physical cross section we must integrate with the distribution of gluons in a proton,

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 g(x_1) g(x_2) \hat{\sigma}(gg \to H), \tag{41}$$

where $g(x)$ is the distribution of gluons in the proton. Perform the change of variables $x_1 \equiv \sqrt{\tau} e^y$, $x_2 \equiv \sqrt{\tau} e^{-y}$, and $\tau = x_1 x_2$. Find the Jacobian and the change of the integration limits and show that the result can be written as:

$$\sigma(pp \to H) \;=\; \frac{\alpha_S^2}{64\pi v^2} \mid I\left(\frac{M_H^2}{m^2}\right) \mid^2 \tau_0 \int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy g(\sqrt{\tau_0} e^y) g(\sqrt{\tau_0} e^{-y}) \tag{42}$$

Often the above integral over the parton distribution is given the name of gluon-gluon parton luminosity.

(n) Using the pdf's from the CTEQ collaboration, CTEQ5L (Fortran,C or Mathematica) compute the gluon-gluon luminosity and the LO Higgs cross section at the LHC. Compare with the results shown in Fig. 5.

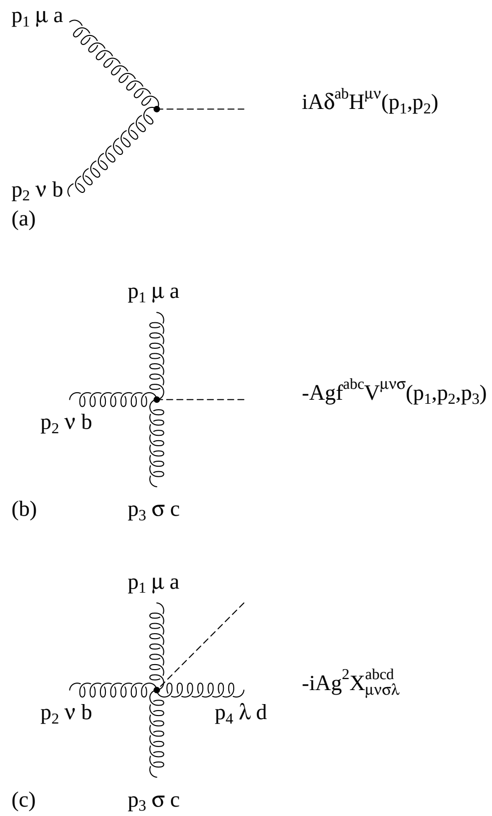


Figure 6: Feynman rules in the EFT where the top is integrated out. Gluon momenta are outgoing.

(n) Higgs Effective field theory.

A striking feature of our result for Higgs boson production from gluon fusion is that it is independent of the heavy quark mass for a light Higgs boson. In fact Eq. (36) can be derived from the effective vertex,

$$\begin{aligned}\mathcal{L}_{\rm eff} &= \frac{\alpha_S}{12\pi} G^A_{\mu\nu} G^{A\ \mu\nu} \left(\frac{H}{v}\right) \\ &= \frac{\beta_F}{g_s} G^A_{\mu\nu} G^{A\ \mu\nu} \left(\frac{H}{2v}\right)(1-\delta),\end{aligned}$$

where

$$\beta_F = \frac{g_s^3 N_H}{24\pi^2} \tag{43}$$

is the contribution of heavy fermion loops to the $SU(3)$ beta function and $\delta = 2\alpha_S/\pi$.[2]

[2]The $(1-\delta)$ term arises from a subtlety in the use of the low energy theorem. Since the Higgs coupling to the heavy fermions is $M_f(1+\frac{H}{v})\overline{f}f$, the counterterm for the Higgs Yukawa coupling is fixed in terms of the renormalization of the fermion mass and wavefunction. The beta function, on the other hand, is evaluated at $q^2=0$. The $1-\delta$ term corrects for this mismatch.

(N_H is the number of heavy fermions with $m >> M_H$.) The effective Lagrangian of Eq. (43) gives ggH, $gggH$ and $ggggH$ vertices and can be used to compute the radiative corrections of $\mathcal{O}(\alpha_S^3)$ to gluon production. The correction in principle involve 2-loop diagrams. However, using the effective vertices from Eq. (43), the $\mathcal{O}(\alpha_S^3)$ corrections can be found from a 1-loop calculation. To fix the notation we shall use

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} A H G^A_{\mu\nu} G^{A\;\mu\nu}, \tag{44}$$

where $G^A_{\mu\nu}$ is the field strength of the SU(3) color gluon field and H is the Higgs-boson field. The effective coupling A is given by

$$A = \frac{\alpha_S}{3\pi v}\left(1 + \frac{11}{4}\frac{\alpha_S}{\pi}\right), \tag{45}$$

where v is the vacuum expectation value parameter, $v^2 = (G_F\sqrt{2})^{-1} = (246)^2$ GeV^2 and the α_S correction is included, as discussed above. The effective Lagrangian generates vertices involving the Higgs boson and two, three or four gluons. The associated Feynman rules are displayed in Fig. 6 The two-gluon–Higgs-boson vertex is proportional to the tensor

$$H^{\mu\nu}(p_1, p_2) = g^{\mu\nu} p_1 \cdot p_2 - p_1^\nu p_2^\mu. \tag{46}$$

The vertices involving three and four gluons and the Higgs boson are proportional to their counterparts from pure QCD:

$$V^{\mu\nu\rho}(p_1, p_2, p_3) = (p_1 - p_2)^\rho g^{\mu\nu} + (p_2 - p_3)^\mu g^{\nu\rho} + (p_3 - p_1)^\nu g^{\rho\mu}, \tag{47}$$

and

$$\begin{aligned} X^{\mu\nu\rho\sigma}_{abcd} &= f_{abe} f_{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f_{ace} f_{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ &+ f_{ade} f_{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}). \end{aligned} \tag{48}$$

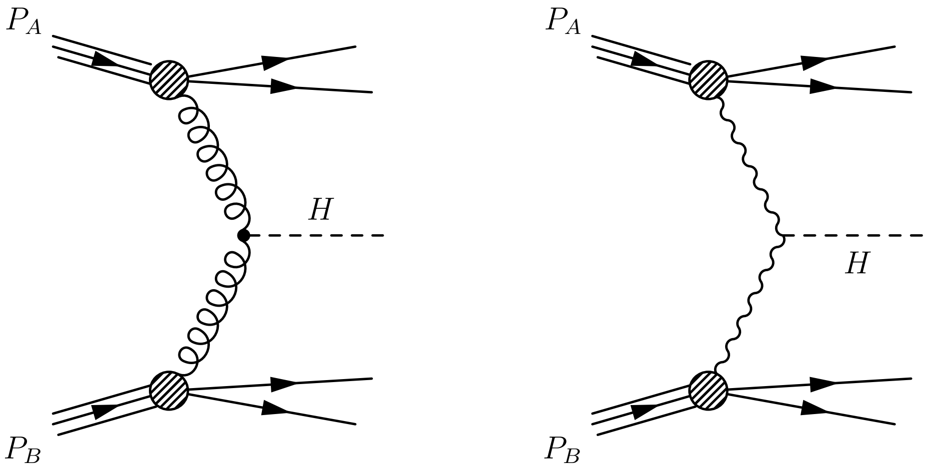
5.3 Discovering the Higgs boson

Go to the web page `http://cp3wks05.fynu.ucl.ac.be/twiki/bin/view/Physics/TAE2008` and choose a channel and investigate signal and background for the main Higgs production channels at the LHC:

1. The 4 lepton final state: $pp \to H \to ZZ \to e^+e^-\mu^+\mu^-$.
2. The 2 lepton + missing Et final state: $pp \to H \to W^+W^- \to e^-\bar{\nu}_e\mu^+\nu_\mu$.
3. Top associated production $pp \to t\bar{t}H$ with $H \to b\bar{b}$.

5.4 Gluon fusion vs. weak boson fusion

(a) Use the web interface of `MadGraph/MadEvent` to generate the cross-sections for $H + 2jets$ at the LHC via gluon fusion and weak boson fusion.
Hint: For the gluon fusion, you need to use the *Higgs efective field theory* (`heft`) in `MadGraph/MadEvent`.



(b) Plot the rapidities of the two jets for boson production channels.

(c) What do you observe? What do you conclude?

(d) What happens to this picture when a third jet is added?

6 The decay of the top quark

The top quark mainly decays into a botton quark and an on-shell W boson. Compute the decay width (You may neglect the mass of the bottom quark).
Answer:

$$\Gamma(t \to bW) = \frac{G_F^2 m_t^3}{8\pi\sqrt{2}} \, |V_{tb}|^2 \, (1 + 2x^2) \, (1 - x^2)^2, \tag{49}$$

where $x = m_W / m_t$.

7 $t\bar{t}$ production: Tevatron vs. LHC

$t\bar{t}$ production at hadron collider come from both $q\bar{q}$ annhilation and gg fusion.

(a) Use the web interface of `MadGraph/MadEvent` to find the LO cross sections for $t\bar{t}$ production at Tevatron and LHC. Which initial parton contributions are dominating in the two cases?

(b) Use the web interface of `MadGraph/MadEvent` to find the cross sections for $t\bar{t} + 1j$ production at the LHC. Select events for which the jet has $p_T > 20$ GeV and $|\eta| < 4$ (Is a ΔR cut needed to have a finite cross section?). Estimate the cross section and compare it with the LO result for $t\bar{t}$. Is the result reasonable? What's going on? Explain.

8 $t'\bar{t}'$ vs $t'j$ production at LHC

Consider an heavy partner of the top, with the same spin, t'.

(a) Use the web interface of `MadGraph/MadEvent` to find the LO cross sections for $t'\bar{t}'$ and single t' production in the t channel at the LHC for different masses. Assume a diagonal CKM matrix also for the t'. How do the cross sections change with the mass? Why?

(b) Consider the full CKM matrix and the corresponding production mechanisms. How do the previous results change?