Collider Phenomenology: the SM of EW interactions

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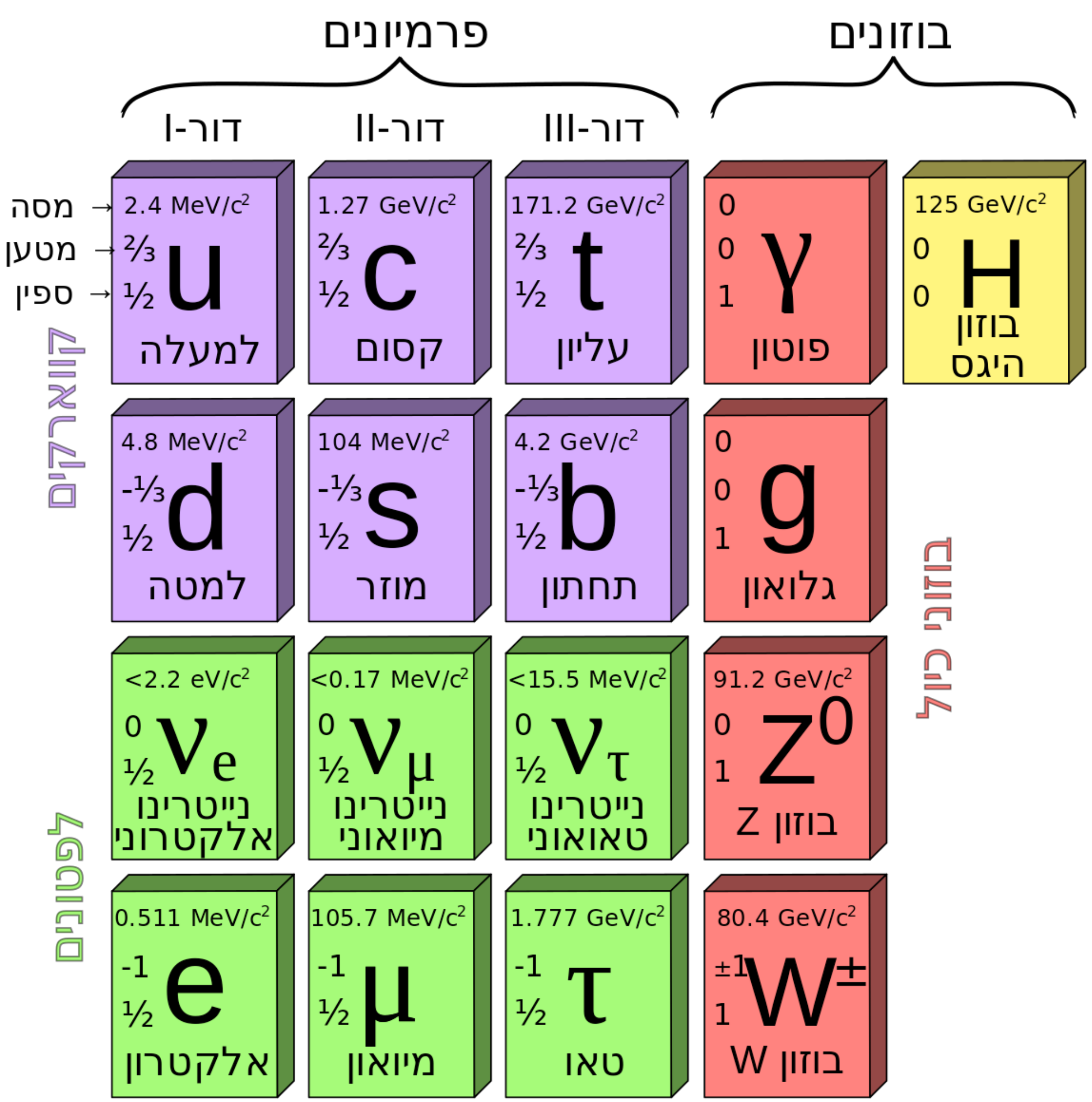
The SM in a nutshell



- $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetries.
- Matter is organised in chiral multiplets of the fundamental representation of the gauge groups.
- The $SU(2) \times U(1)$ symmetry is spontaneously broken to EM.
- Yukawa interactions are present that lead to fermion masses and CP violation.
- Neutrino masses can be accommodated in two distinct ways.
- Anomaly free.
- Renormalisable = valid to “arbitrary” high scales.

SU(2)$_{\mathrm{L}}$ x U(1)$_{\mathrm{Y}}$

Experimental evidence, such as charged weak currents couple only with left-handed fermions, the existence of a massless photon and a neutral $Z \ldots$, the electroweak group is chosen to be $SU(2)_L \times U(1)_Y$.

$$\psi_L \equiv \frac{1}{2}(1-\gamma_5)\psi \qquad \psi_R \equiv \frac{1}{2}(1+\gamma_5)\psi \qquad \psi = \psi_L + \psi_R$$

$$L_L \equiv \frac{1}{2}(1-\gamma_5)\left(\begin{array}{c}\nu_e\\ e\end{array}\right) = \left(\begin{array}{c}\nu_{eL}\\ e_L\end{array}\right) \qquad e_R \equiv \frac{1}{2}(1+\gamma_5)e$$

- $SU(2)_L$: weak isospin group. Three generators $\Longrightarrow$ three gauge bosons: W^1, W^2 and W^3, with gauge coupling g. The generators for doublets are $T^a = \sigma^a/2$, where σ^a are the 3 Pauli matrices (when acting on the gauge singlet e_R and ν_R, $T^a \equiv 0$).
- $U(1)_Y$: weak hypercharge Y. One gauge boson B with gauge coupling g'.
  One generator (charge) $Y(\psi)$, whose value depends on the corresponding field.

$SU(2)_L \times U(1)_Y$

Following the gauging recipe (for one generation of leptons. Quarks work the same way)

$$\mathcal{L}_\psi = i\,\bar{L}_L\,\not{D}\,L_L + i\,\bar{\nu}_{eR}\,\not{D}\,\nu_{eR} + i\,\bar{e}_R\,\not{D}\,e_R$$

where

$$D^\mu = \partial^\mu - igW_i^\mu T^i - ig'\frac{Y(\psi)}{2}B^\mu \qquad T^i = \frac{\sigma^i}{2} \quad \text{or} \quad T^i = 0 \qquad i = 1,2,3$$

$$\mathcal{L}_\psi \equiv \mathcal{L}_{kin} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

$$\begin{aligned}
\mathcal{L}_{kin} &= i\,\bar{L}_L\,\not{\partial}\,L_L + i\,\bar{\nu}_{eR}\,\not{\partial}\,\nu_{eR} + i\,\bar{e}_R\,\not{\partial}\,e_R \\
\mathcal{L}_{CC} &= g\,W_\mu^1\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_1}{2}\,L_L + g\,W_\mu^2\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_2}{2}\,L_L = \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{L}_L\,\gamma^\mu\,\sigma^+\,L_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{L}_L\,\gamma^\mu\,\sigma^-\,L_L \\
&= \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{\nu}_L\,\gamma^\mu\,e_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{e}_L\,\gamma^\mu\,\nu_L \\
\mathcal{L}_{NC} &= \frac{g}{2}\,W_\mu^3\,[\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} - \bar{e}_L\,\gamma^\mu\,e_L] + \frac{g'}{2}\,B_\mu\Big[Y(L)\,(\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} + \bar{e}_L\,\gamma^\mu\,e_L) \\
&\quad + Y(\nu_{eR})\,\bar{\nu}_{eR}\,\gamma^\mu\,\nu_{eR} + Y(e_R)\,\bar{e}_R\,\gamma^\mu\,e_R\Big]
\end{aligned}$$

with

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\,(W_\mu^1 \mp iW_\mu^2) \qquad \sigma^\pm = \frac{1}{2}\,(\sigma^1 \pm i\sigma^2)$$

$SU(2)_L \times U(1)_Y$

$$
\begin{aligned}
\mathcal{L}_{NC} \;=\; & \frac{g}{2} W^3_\mu \left[\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} - \bar{e}_L\,\gamma^\mu\,e_L\right] + \frac{g'}{2} B_\mu \Big[Y(L)\left(\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} + \bar{e}_L\,\gamma^\mu\,e_L\right) \\
& + Y(\nu_{eR})\,\bar{\nu}_{eR}\,\gamma^\mu\,\nu_{eR} + Y(e_R)\,\bar{e}_R\,\gamma^\mu\,e_R \Big]
\end{aligned}
$$

Neither W^3_μ nor B_μ can be interpreted as the photon field A_μ, since they couple to neutral fields.

$$
\Psi \equiv \begin{pmatrix} \nu_{eL} \\ e_L \\ \nu_{eR} \\ e_R \end{pmatrix}
\qquad
\mathcal{T}_3 \equiv \begin{pmatrix} 1/2 & 0 & & \\ 0 & -1/2 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}
\qquad
\mathcal{Y} \equiv \begin{pmatrix} Y(L) & & & \\ & Y(L) & & \\ & & Y(\nu_{eR}) & \\ & & & Y(e_R) \end{pmatrix}
$$

$$
\mathcal{L}_{NC} = g\,\bar{\Psi}\,\gamma^\mu\,\mathcal{T}_3\,\Psi\,W^3_\mu + g'\,\bar{\Psi}\,\gamma^\mu\,\frac{\mathcal{Y}}{2}\,\Psi\,B_\mu
$$

$SU(2)_L \times U(1)_Y$

We perform a rotation of an angle θ_W, the Weinberg angle, in the space of the two neutral gauge fields (W^3_μ and B_μ). We use an orthogonal transformation in order to keep the kinetic terms diagonal in the vector fields

$$
\begin{aligned}
B_\mu &= A_\mu \cos\theta_W - Z_\mu \sin\theta_W \\
W^3_\mu &= A_\mu \sin\theta_W + Z_\mu \cos\theta_W
\end{aligned}
$$

so that

$$\mathcal{L}_{NC} = \bar\Psi \gamma^\mu \left[g\ \sin\theta_W\ \mathcal{T}_3 + g'\ \cos\theta_W\ \frac{\mathcal{Y}}{2} \right] \Psi\, A_\mu + \bar\Psi \gamma^\mu \left[g\ \cos\theta_W\ \mathcal{T}_3 - g'\ \sin\theta_W\ \frac{\mathcal{Y}}{2} \right] \Psi\, Z_\mu$$

We can identify A_μ with the photon field provided

$$e\mathcal{Q} = g\ \sin\theta_W\ \mathcal{T}_3 + g'\ \cos\theta_W\ \frac{\mathcal{Y}}{2} \qquad\qquad \mathcal{Q} = \text{electromagnetic charge}$$

The weak hypercharges $\mathcal{Y}$ appear only through the combination $g'\,\mathcal{Y}$. We use this freedom to fix

$$Y(L) = -1$$

$SU(2)_L \times U(1)_Y$

With this choice, the doublet of left-handed leptons gives $\left(e\mathcal{Q} = g\,\sin\theta_W\,\mathcal{T}_3 + g'\,\cos\theta_W\,\frac{\mathcal{Y}}{2}\right)$

$$
\begin{aligned}
0 &= \frac{g}{2}\sin\theta_W - \frac{g'}{2}\cos\theta_W \\
-e &= -\frac{g}{2}\sin\theta_W - \frac{g'}{2}\cos\theta_W
\end{aligned}
$$

so that

$$g\sin\theta_W = g'\cos\theta_W = e$$

and

$$\mathcal{Q} = \mathcal{T}_3 + \frac{\mathcal{Y}}{2}$$ Gell-Mann–Nishijima formula.

From this formula we have $Y(\nu_{eR}) = 0$ and $Y(e_R) = -2$.

Notice that the right-handed neutrino has zero charge, zero hypercharge and it is in a SU(2) singlet: it does not take part in electroweak interactions.

$SU(2)_L \times U(1)_Y$

$$
\begin{aligned}
\mathcal{L}_{NC} &= \bar\Psi\gamma^\mu \left[g\,\sin\theta_W\,\mathcal{T}_3 + g'\,\cos\theta_W\,\frac{\mathcal{Y}}{2}\right]\Psi\,A_\mu + \bar\Psi\gamma^\mu\left[g\,\cos\theta_W\,\mathcal{T}_3 - g'\,\sin\theta_W\,\frac{\mathcal{Y}}{2}\right]\Psi\,Z_\mu \\
&= e\,\bar\Psi\gamma^\mu\mathcal{Q}\Psi\,A_\mu + \bar\Psi\gamma^\mu\mathcal{Q}_Z\Psi\,Z_\mu
\end{aligned}
$$

where $\mathcal{Q}_Z$ is a diagonal matrix given by

$$
\mathcal{Q}_Z = \frac{e}{\cos\theta_W\sin\theta_W}\left(\mathcal{T}_3 - \mathcal{Q}\sin^2\theta_W\right)
$$

We can proceed, in a similar way, with quarks (see more later)

$$
Q_L^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad \begin{aligned} u_R^i &= u_R, c_R, t_R \\ d_R^i &= d_R, s_R, b_R \end{aligned}
$$

SM charge assignments

Good Homework

				$SU(3)$	$SU(2)$	$U(1)_Y$	$Q = T_3 + \frac{Y}{2}$
$Q^i_L =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{3}$	$\frac{2}{3}$ $-\frac{1}{3}$
$u^i_R =$	u_R	c_R	t_R	3	1	$\frac{4}{3}$	$\frac{2}{3}$
$d^i_R =$	d_R	s_R	b_R	3	1	$-\frac{2}{3}$	$-\frac{1}{3}$
$L^i_L =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	-1	0 -1
$e^i_R =$	e_R	μ_R	τ_R	1	1	-2	-1
$\nu^i_R =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0	0

Masses

Gauge invariance and renormalizability completely determine the kinetic terms for the gauge bosons

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu}$$

$$\begin{aligned} B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\epsilon^{abc}\, W_{b,\mu}\, W_{c,\nu} \end{aligned}$$

The gauge symmetry does NOT allow any mass terms for $W^\pm$ and Z.

Mass terms for gauge bosons

$$\mathcal{L}_{mass} = \frac{1}{2}\, m_A^2\, A_\mu\, A^\mu$$

are not invariant under a gauge transformation

$$A^\mu \;\rightarrow\; U(x) \left(A^\mu + \frac{i}{g}\, \partial^\mu \right) U^{-1}(x)$$

However, the gauge bosons of weak interactions are massive (short range of weak interactions).

Two Subtleties...

Actually, the story is bit more subtle than this...

- For U(1) the apparent gauge violation of the mass term is irrelevant. The basic reason is that quantization implies a gauge fixing. This is can be easily seen by taking the limit of the $e \to 0$, $\lambda \to 0$, $v \to \infty$, with $\lambda v^2 = M^2$ and $ev = m$ fixed, of the Abelian Higgs model, which then becomes a free theory of two massive scalars and one massive vector boson. This vector boson can then be coupled to fermionic matter. This is called the Stuckelberg mechanism. However, for SU(N) this does not work since the selfcoupling of the field $g \to 0$.

Two Subtleties...

Actually, the story is bit more subtle than this...

- One can still realise the gauge symmetry in a non-linear way, as a gauged non-linear sigma model. In this case one groups the goldstone bosons into a triplet π whose interactions are described by

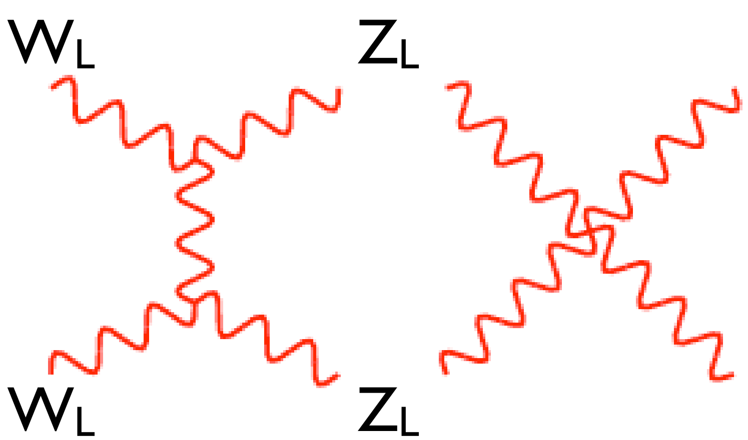
$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(D^\mu \Sigma)^\dagger D_\mu \Sigma$$

with $D^\mu \Sigma = \partial^\mu \Sigma + i(g/2)\sigma \cdot W^\mu \Sigma - i(g'/2)\Sigma \sigma^3 B^\mu$ and $\Sigma = \exp(i\sigma \cdot \pi / v)$

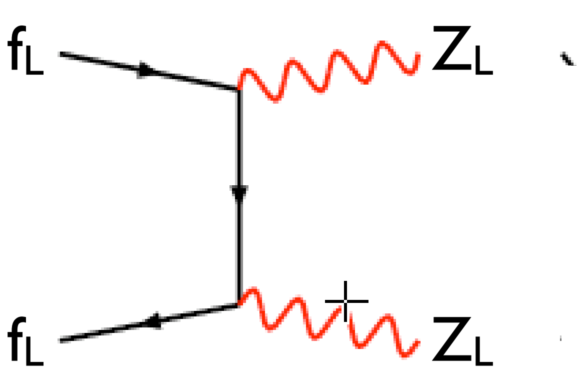
For the fermions one writes $\mathcal{L} = -m_f \bar{F}_L \Sigma \begin{pmatrix} 0 \\ 1 \end{pmatrix} f_R + \mathrm{H.c.}$

The unitarity bound

[Chanowitz, Gallard.1985]
[Appelquist, Chanowitz,1989]



$$a_0 \sim \frac{s}{v^2}$$



$$a_0 \sim \frac{\sqrt{s} m_f}{v^2}$$

Inelastic tree-level amplitudes for longitudinal W and Z and fermions violate unitarity at a scale:

$$\Lambda_{EWSB} = \sqrt{8\pi} v$$

Our effective description contains information on where it is going to fail.

Only case we know of where unknown physics has to appear below 1 TeV.

Spontaneous Symmetry Breaking

A symmetry is said to be spontaneously broken when the vacuum state is not invariant

$$\exp\left(i\,\delta\theta^{\,a}\,t^a\right)|0\rangle \neq |0\rangle \qquad \Longrightarrow \qquad Q^a|0\rangle \neq 0$$

This condition is equivalent to the existence of some set of fields operators ϕ_k with non-trivial transformation property under that symmetry transformation, and non-vanishing vacuum expectation values

$$\langle 0|\phi_k|0\rangle = v_k \neq 0$$

Proof

If the set of fields ϕ_j transforms non-trivially

$$\phi_j \to \left(e^{i\,\delta\theta^{\,a}\,t^a}\right)_{jk}\phi_k \sim \phi_j + \underbrace{i\,\delta\theta^{\,a}\,t^a_{jk}\,\phi_k}_{\delta\phi_j} = \phi_j + i\,\delta\theta^a\,[Q^a,\phi_j]$$

Taking the expectation value on the vacuum

$$t^a_{jk}\,\langle 0|\phi_k|0\rangle = \langle 0|\,[Q^a,\phi_j]\,|0\rangle \neq 0 \qquad \Longleftrightarrow \qquad Q^a|0\rangle \neq 0$$

BEH mechanism

We give mass to the gauge bosons through the Brout-Englert-Higgs mechanism: generate mass terms from the kinetic energy term of a scalar doublet field Φ that undergoes a broken-symmetry process.

Introduce a complex scalar doublet: four scalar real fields

$$
\begin{aligned}
\Phi &= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad\qquad Y(\Phi) = 1 \\
\mathcal{L}_{\text{Higgs}} &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V\left(\Phi^\dagger \Phi\right) \\
D^\mu &= \partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig' \frac{Y(\Phi)}{2} B^\mu \\
V\left(\Phi^\dagger \Phi\right) &= -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2, \qquad\qquad \mu^2, \lambda > 0
\end{aligned}
$$

- The reason why $Y(\Phi) = 1$ is not to break electric-charge conservation.
- Charge assignment for the Higgs doublet through $Q = T_3 + Y/2$. The potential has a minimum in correspondence of

$$|\Phi|^2 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

v is called the vacuum expectation value (VEV) of the neutral component of the Higgs doublet.

BEH mechanism

Expanding Φ around the minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left[v + H(x) + i\chi(x)\right] \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\left[\frac{i\sigma_i \theta^i(x)}{v}\right] \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We can rotate away the fields $\theta^i(x)$ by an $\mathrm{SU}(2)_L$ gauge transformation

$$\Phi(x) \to \Phi'(x) = U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where $U(x) = \exp\left[-\frac{i\sigma_i\theta^i(x)}{v}\right]$.

This gauge choice is called unitary gauge, and is equivalent to absorbing the Goldstone modes $\theta^i(x)$. Three would-be Goldstone bosons "eaten up" by three vector bosons $(W^\pm, Z)$ that acquire mass. This is why we introduced a complex scalar doublet (four elementary fields).

The vacuum state can be chosen to correspond to the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

BEH mechanism

We can easily verify that the vacuum state breaks the gauge symmetry.

A state $\tilde{\Phi}$ is invariant under a symmetry operation $\exp(igT^a\theta_a)$ if

$$\exp(igT^a\theta_a)\tilde{\Phi} = \tilde{\Phi}$$

This means that a state is invariant if (just expand the exponent)

$$T^a\tilde{\Phi} = 0$$

For the SU$(2)_L\times$ U$(1)_Y$ case we have

$$\sigma_1\Phi_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \qquad \text{broken}$$

$$\sigma_2\Phi_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \qquad \text{broken}$$

BEH mechanism

$$\sigma_3 \Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \qquad \text{broken}$$

$$Y \Phi_0 = Y(\Phi) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = +1 \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \qquad \text{broken}$$

But, if we examine the effect of the electric charge operator $\hat{Q} = Y/2 + T_3$ on the (electrically neutral) vacuum state, we have ($Y(\Phi) = 1$)

$$\hat{Q}\Phi_0 = \frac{1}{2}(\sigma_3 + Y)\,\Phi_0 = \frac{1}{2}\begin{pmatrix} Y(\Phi)+1 & 0 \\ 0 & Y(\Phi)-1 \end{pmatrix}\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

the electric charge symmetry is unbroken!

The Higgs potential

The scalar potential

$$V\left(\Phi^\dagger\Phi\right) = -\mu^2\Phi^\dagger\Phi \; + \; \lambda\left(\Phi^\dagger\Phi\right)^2$$

expanded around the vacuum state

$$\Phi(x) = \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v + H(x) \end{array}\right)$$

becomes

$$V \;=\; \frac{1}{2}\left(2\lambda v^2\right)H^2 \;+\; \lambda v H^3 \;+\; \frac{\lambda}{4}H^4 + \text{const}$$

- the scalar field H gets a mass

$$m_H^2 = 2\lambda v^2 \qquad\qquad v^2 = \mu^2/\lambda$$

- there is a term of cubic and quartic self-coupling.

Note: this means that $\lambda_3 = \lambda_4 = \lambda$ in the SM. To have (independent) deviations of the trilinear or quadrilinear, one needs to deform the potential with a BSM hypothesis.

Vector boson masses

$$\begin{aligned}
D^\mu \Phi &= \left(\partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig'\frac{1}{2}B^\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2\sqrt{2}}\left[g\begin{pmatrix} W_3^\mu & W_1^\mu - iW_2^\mu \\ W_1^\mu + iW_2^\mu & -W_3^\mu \end{pmatrix} + g' B^\mu\right]\begin{pmatrix} 0 \\ v + H \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}(v+H)\begin{pmatrix} g\,(W_1^\mu - iW_2^\mu) \\ -gW_3^\mu + g' B^\mu \end{pmatrix}\right] \\
&= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}\left(1 + \frac{H}{v}\right)\begin{pmatrix} gvW^{\mu+} \\ -v\sqrt{(g^2 + g'^2)/2}\, Z^\mu \end{pmatrix}\right]
\end{aligned}$$

Note: this means that the mass and the Higgs interactions are uniquely linked.

$$(D^\mu \Phi)^\dagger D_\mu \Phi = \frac{1}{2}\partial^\mu H \partial_\mu H + \left[\left(\frac{gv}{2}\right)^2 W^{\mu+}W_\mu^- + \frac{1}{2}\frac{(g^2 + g'^2)\, v^2}{4} Z^\mu Z_\mu\right]\left(1 + \frac{H}{v}\right)^2$$

Vector boson couplings

$$(D^\mu\Phi)^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu H \partial_\mu H + \left[\left(\frac{gv}{2}\right)^2 W^{\mu+}W^-_\mu + \frac{1}{2}\frac{\left(g^2+g'^2\right)v^2}{4} Z^\mu Z_\mu\right]\left(1+\frac{H}{v}\right)^2$$

- The W and Z gauge bosons have acquired masses

$$m_W^2 = \frac{g^2v^2}{4} \qquad m_Z^2 = \frac{\left(g^2+g'^2\right)v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

From the measured value of the Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{m_W^2} \qquad \Longrightarrow \qquad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22 \text{ GeV}$$

- the photon stays massless

- HWW and HZZ couplings from $2H/v$ term (and $HHWW$ and $HHZZ$ couplings from H^2/v^2 term)

$$\mathcal{L}_{HVV} = \frac{2m_W^2}{v} W^+_\mu W^{-\mu} H + \frac{m_Z^2}{v} Z^\mu Z_\mu H \equiv g m_W W^+_\mu W^{-\mu} H + \frac{1}{2}\frac{g m_Z}{\cos\theta_W} Z^\mu Z_\mu H$$

Fermion masses and couplings

A direct mass term is not invariant under $\mathrm{SU}(2)_L$ or $\mathrm{U}(1)_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} \quad = \quad & -\Gamma_d^{ij} \bar{Q}_L'^i \Phi d_R'^j - \Gamma_d^{ij*} \bar{d}_R'^i \Phi^\dagger Q_L'^j \\ & -\Gamma_u^{ij} \bar{Q}_L'^i \Phi_c u_R'^j + \text{h.c.} \\ & -\Gamma_e^{ij} \bar{L}_L^i \Phi e_R^j + \text{h.c.} \end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q', u' and d' are quark fields that are generic linear combination of the mass eigenstates u and d and Γ_u, Γ_d and Γ_e are 3×3 complex matrices in generation space, spanned by the indices i and j.

$$M^{ij} = \Gamma^{ij} \frac{v}{\sqrt{2}}$$

Fermion masses

In the unitary gauge we have

$$
\begin{aligned}
\bar{Q}_L'^{\,i}\,\Phi\,d_R'^{\,j} &= \left(\bar{u}_L'^{\,i} \quad \bar{d}_L'^{\,i}\right)\begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} d_R'^{\,j} = \frac{v+H}{\sqrt{2}}\,\bar{d}_L'^{\,i}\,d_R'^{\,j} \\
\bar{Q}_L'^{\,i}\,\Phi_c\,u_R'^{\,j} &= \left(\bar{u}_L'^{\,i} \quad \bar{d}_L'^{\,i}\right)\begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix} u_R'^{\,j} = \frac{v+H}{\sqrt{2}}\,\bar{u}_L'^{\,i}\,u_R'^{\,j}
\end{aligned}
$$

and we obtain

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij}\,\frac{v+H}{\sqrt{2}}\,\bar{d}_L'^{\,i}\,d_R'^{\,j} - \Gamma_u^{ij}\,\frac{v+H}{\sqrt{2}}\,\bar{u}_L'^{\,i}\,u_R'^{\,j} - \Gamma_e^{ij}\,\frac{v+H}{\sqrt{2}}\,\bar{e}_L^{i}\,e_R^{j} + \text{h.c.} \\
&= -\left[M_u^{ij}\,\bar{u}_L'^{\,i}\,u_R'^{\,j} + M_d^{ij}\,\bar{d}_L'^{\,i}\,d_R'^{\,j} + M_e^{ij}\,\bar{e}_L^{i}\,e_R^{j} + \text{h.c.}\right]\left(1 + \frac{H}{v}\right)
\end{aligned}
$$

$$
M^{ij} = \Gamma^{ij}\,\frac{v}{\sqrt{2}}
$$

Fermion masses

Theorem: For any generic complex squared matrix C, there exist two unitary matrices U, V such that

$$D = U^\dagger \, C \, V$$

is diagonal with real positive entries

We can now diagonalize the matrix M_f $(f = u, d, e)$ with the help of two unitary matrices, U_L^f and U_R^f

$$\left(U_L^f\right)^\dagger M_f \, U_R^f = \text{diagonal with real positive entries}$$

For example:

$$(U_L^u)^\dagger M_u \, U_R^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \qquad (U_L^d)^\dagger M_d \, U_R^d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

Fermion masses and couplings

We can make the following change of fermionic fields

$$f'_{Li} = \left(U_L^f\right)_{ij} f_{Lj} \qquad\qquad f'_{Ri} = \left(U_R^f\right)_{ij} f_{Rj}$$

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\sum_{f',i,j} \bar{f}'^{\,i}_L \, M_f^{ij} \, f'^{\,j}_R \left(1 + \frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_{f,i,j} \bar{f}^i_L \left[\left(U_L^f\right)^\dagger M_f \, U_R^f\right]_{ij} f^j_R \left(1 + \frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_{f} m_f \left(\bar{f}_L f_R + \bar{f}_R f_L\right)\left(1 + \frac{H}{v}\right)
\end{aligned}$$

Note: this means that the mass and the Yukawa are linked.

- We succeed in producing fermion masses and we got a fermion-antifermion-Higgs coupling proportional to the fermion mass.
- Notice that the fermionic field redefinition preserves the form of the kinetic terms in the Lagrangian ($\bar{\psi}\,\partial\!\!\!/\,\psi = \bar{\psi}_R\,\partial\!\!\!/\,\psi_R + \bar{\psi}_L\,\partial\!\!\!/\,\psi_L$ invariant for left and right field unitary transformation).
- The Higgs Yukawa couplings are flavor diagonal: no flavor changing Higgs interactions.

Mixing

The charged current interaction is given by

$$\frac{e}{\sqrt{2}\sin\theta_W}\bar{u}'^{\,i}_L\, \not{W}^+\, d'^{\,i}_L + \text{h.c.}$$

After the mass diagonalization described previously, this term becomes

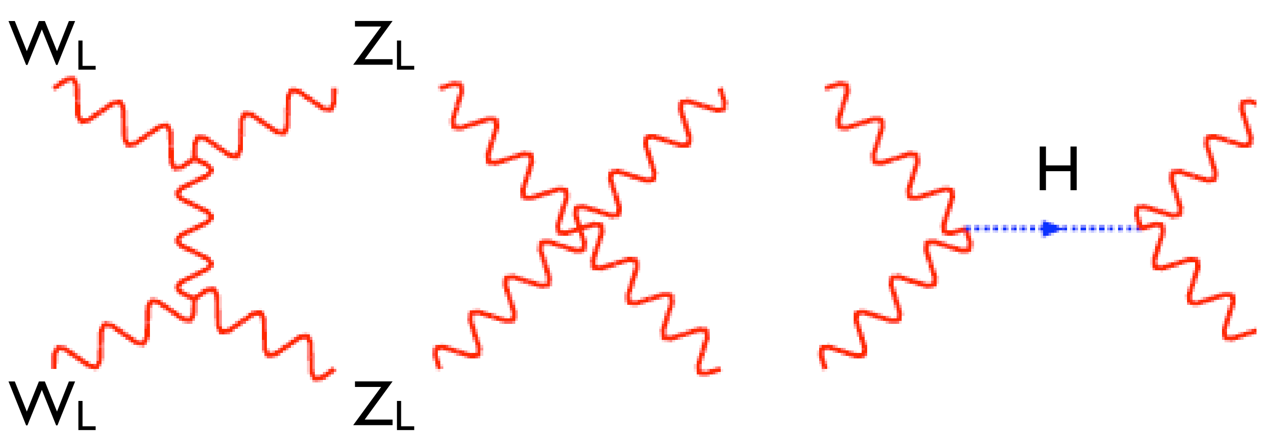
$$\frac{e}{\sqrt{2}\sin\theta_W}\bar{u}^i_L \left[(U^u_L)^\dagger\, U^d_L\right]_{ij} \not{W}^+ d^j_L + \text{h.c.}$$

and we define the Cabibbo-Kobayashi-Maskawa matrix V_{CKM}

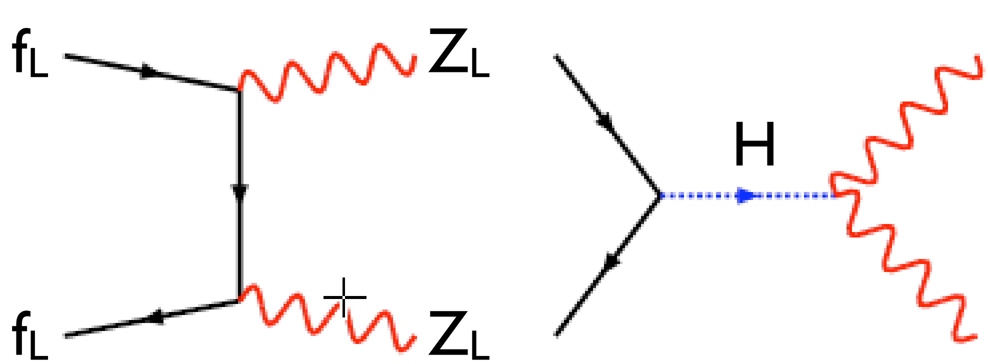
$$V_{CKM} = (U^u_L)^\dagger\, U^d_L$$

- V_{CKM} is a complex not diagonal matrix and then it mixes the flavors of the different quarks.
- For N flavour families, V_{CKM} depends on $(N-1)^2$ parameters. $(N-1)(N-2)/2$ of them are complex phases. For $N=3$ there is one complex phase and this implies violation of the **CP** symmetry (first observed in the K^0-$\bar{K}^0$ system in 1964).
- It is a unitary matrix and the values of its entries must be determined from experiments.

The Higgs restores unitarity



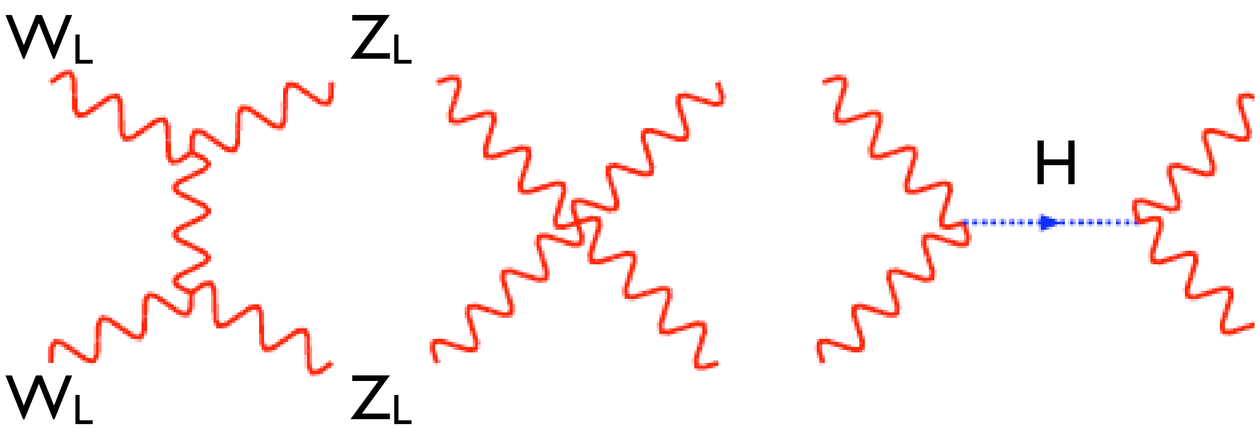
$$a_0 \sim \frac{s}{v^2} - \frac{s}{v^2} \sim \frac{m_H^2}{v^2}$$



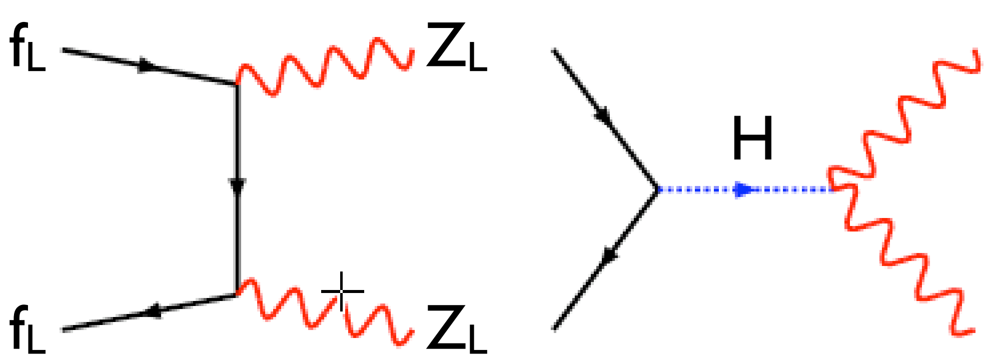
$$a_0 \sim \frac{\sqrt{s} m_f}{v^2} - \frac{\sqrt{s} m_f}{v^2} \sim \frac{m_f^2}{v^2}$$

SM is a linearly realised gauge theory which valid up to arbitrary high scales (if m_H<<1 TeV).

The Higgs restores unitarity



$$a_0 \sim \frac{s}{v^2} - \frac{s}{v^2} \sim \frac{m_H^2}{v^2}$$



$$a_0 \sim \frac{\sqrt{s} m_f}{v^2} - \frac{\sqrt{s} m_f}{v^2} \sim \frac{m_f^2}{v^2}$$

SM is a linearly realised gauge theory which valid up to arbitrary high scales (if m_H<<1 TeV).

Vacuum stability

The one-loop renormalization group equation (RGE) for $\lambda(\mu)$ is

$$\frac{d\lambda(\mu)}{d\log\mu^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + \frac{3}{8}g^4 + \frac{3}{16}\left(g^2+g'^2\right)^2 - 3h_t^4 - 3\lambda g^2 - \frac{3}{2}\lambda\left(g^2+g'^2\right) + 6\lambda h_t^2\right]$$

where

$$m_t = \frac{h_t v}{\sqrt{2}} \qquad\qquad m_H^2 = 2\lambda v^2$$

This equation must be solved together with the one-loop RGEs for the gauge and Yukawa couplings, which, in the Standard Model, are given by

$$\begin{aligned}
\frac{dg(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left(-\frac{19}{6}g^3\right)\\
\frac{dg'(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\frac{41}{6}g'^3\\
\frac{dg_s(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left(-7g_s^3\right)\\
\frac{dh_t(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left[\frac{9}{2}h_t^3 - \left(8g_s^2 + \frac{9}{4}g^2 + \frac{17}{12}g'^2\right)h_t\right]
\end{aligned}$$

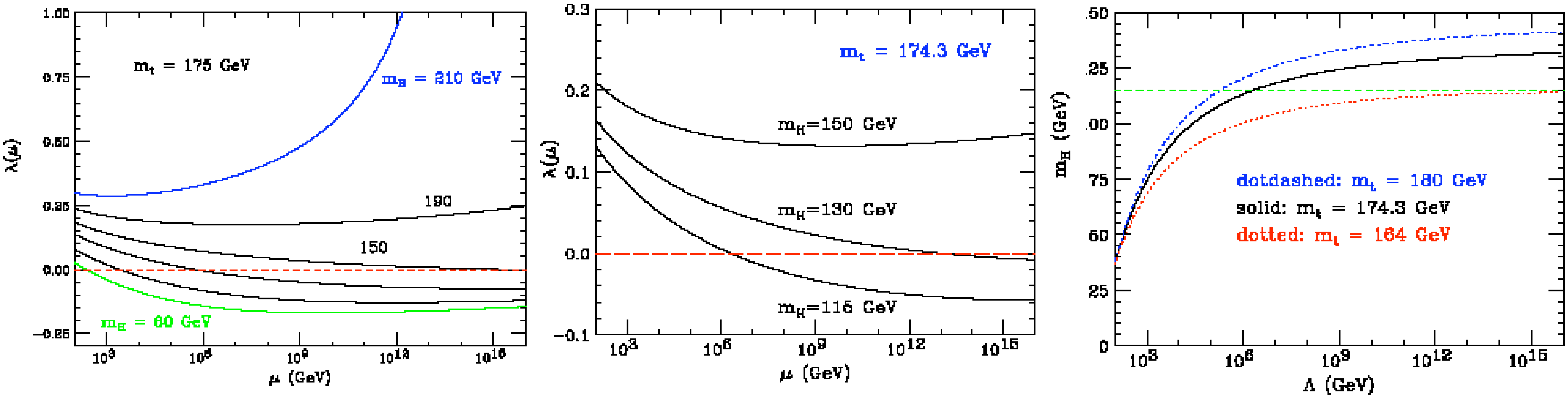
here g_s is the strong interaction coupling constant, and the $\overline{\text{MS}}$ scheme is adopted.

Solving this system of coupled equations with the initial condition

$$\lambda\left(m_H\right) = \frac{m_H^2}{2v^2}$$

Vacuum stability

It can be shown that the requirement that the Higgs potential be bounded from below, even after the inclusion of radiative corrections, is fulfilled if $\lambda(\mu)$ stays positive, at least up to a certain scale $\mu \approx \Lambda$, the maximum energy scale at which the theory can be considered reliable (use effective action).



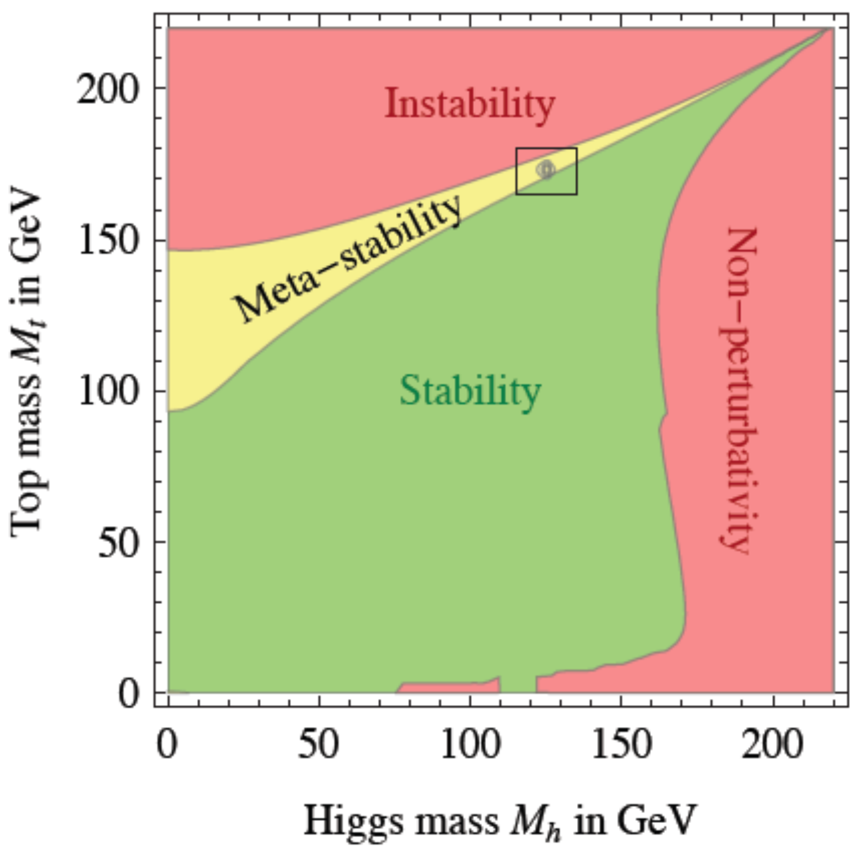
✗ This limit is extremely sensitive to the top-quark mass.

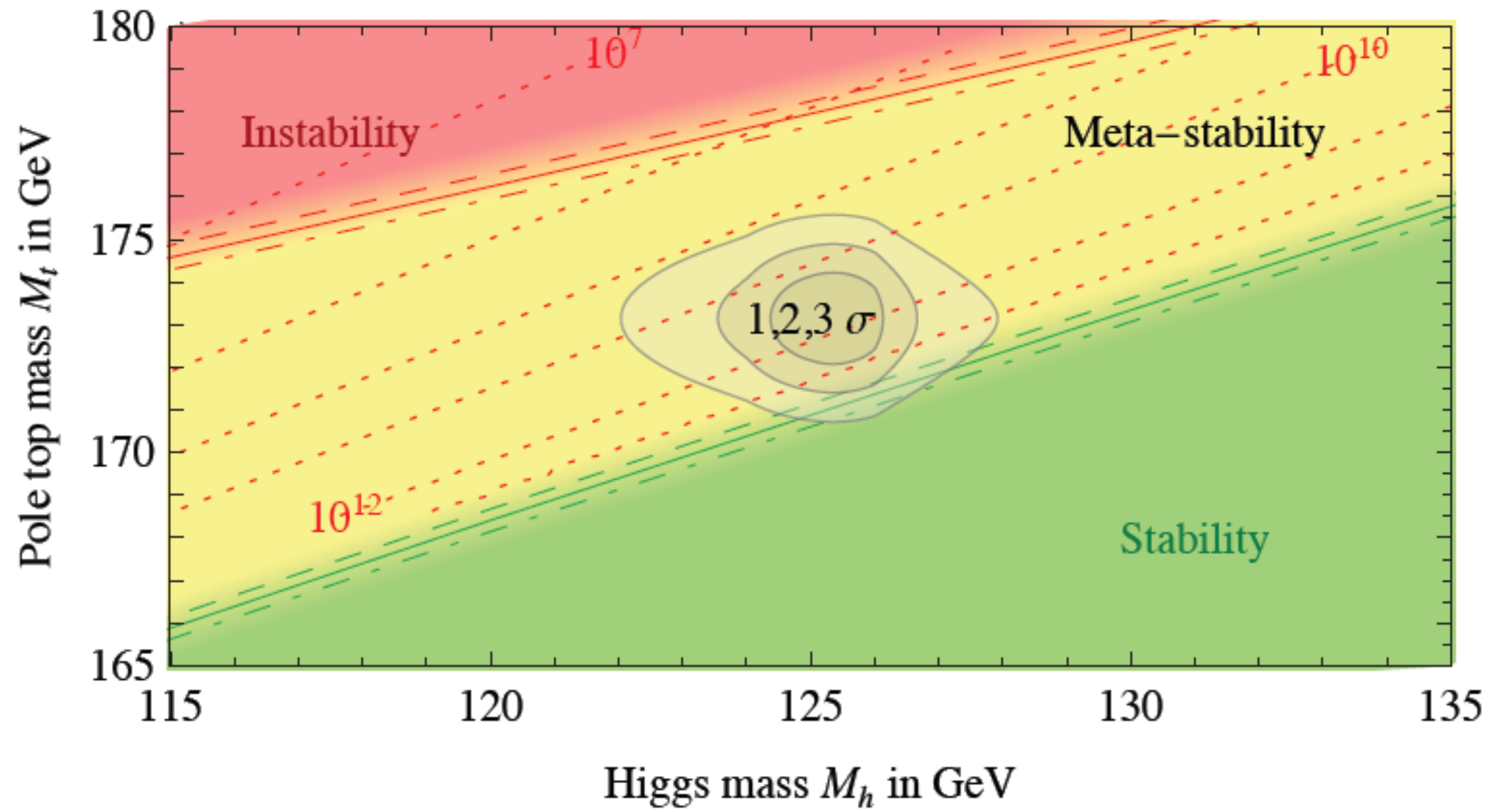
✓ The stability lower bound can be relaxed by allowing metastability

The future of the Universe

The fate of the Universe depends on 1GeV in m_t

[Degrassi, et al. '12]





$$y_t(M_t) = 0.93587 + 0.00557\left(\frac{M_t}{\text{GeV}} - 173.15\right)\ldots \pm 0.00200_{\text{th}}$$

It's the Yukawa that enters in this calculation.

Naturalness

Apart from the considerations made up to now, the SM must be considered as an effective low-energy theory: at very high energy new phenomena take place that are not described by the SM (gravitation is an obvious example) $\Longrightarrow$ other scales have to be considered.

Why the weak scale ($\sim 10^2$ GeV) is much smaller than other relevant scales, such as the Planck mass ($\approx 10^{19}$ GeV) or the unification scale ($\approx 10^{16}$ GeV) (or why the Planck scale is so high with respect to the weak scale $\Longrightarrow$ extra dimensions)?

This is the **hierarchy problem**.

And this problem is especially difficult to solve in the SM because of the un-naturalness of the Higgs boson mass.

As we have seen and as the experimental data suggest, the Higgs boson mass is of the same order of the weak scale. However, it's not naturally small, in the sense that there is no approximate symmetry that prevent it from receiving large radiative corrections.

As a consequence, it naturally tends to become as heavy as the heaviest degree of freedom in the underlying theory (Planck mass, unification scale).

Naturalness: example

Two scalars interacting through the potential

$$V(\varphi, \Phi) = \frac{m^2}{2}\varphi^2 + \frac{M^2}{2}\Phi^2 + \frac{\lambda}{4!}\varphi^4 + \frac{\sigma}{4!}\Phi^4 + \frac{\delta}{4}\varphi^2\Phi^2$$

which is the most general renormalizable potential, if we require the symmetry under $\varphi \to -\varphi$ and $\Phi \to -\Phi$. We assume that $M^2 \gg m^2$. Let's check if this hierarchy is conserved at the quantum level. Compute the one-loop radiative corrections to the pole mass m^2

$$m^2_{\text{pole}} = m^2(\mu^2) + \frac{\lambda m^2}{32\pi^2}\left(\log\frac{m^2}{\mu^2} - 1\right) + \frac{\delta M^2}{32\pi^2}\left(\log\frac{M^2}{\mu^2} - 1\right)$$

where the running mass $m^2(\mu^2)$ obeys the RGE

$$\frac{dm^2(\mu^2)}{d\log\mu^2} = \frac{1}{32\pi^2}\left(\lambda m^2 + \delta M^2\right)$$

Corrections to m^2 proportional to M^2 appear at one loop. One can choose $\mu^2 \approx M^2$ to get rid of them, but they reappear through the running of $m^2(\mu^2)$.

Naturalness: example

The only way to preserve the hierarchy $m^2 \ll M^2$ is carefully choosing the coupling constants

$$\lambda m^2 \approx \delta M^2$$

and this requires fixing the renormalized coupling constants with and unnaturally high accuracy

$$\frac{\lambda}{\delta} \approx \frac{M^2}{m^2}$$

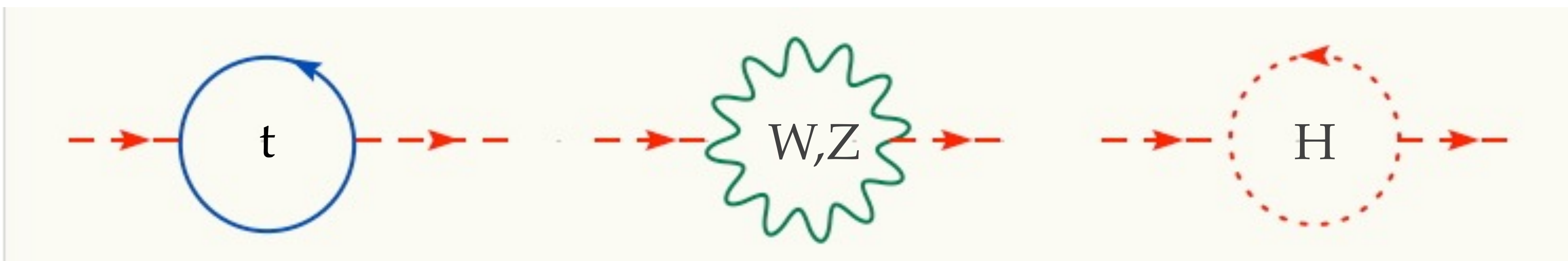
This is what is usually called the fine tuning of the parameters.

The same happens if the theory is spontaneously broken $(m^2 < 0,\, M^2 \gg |m^2| > 0)$.

Therefore, without a suitable fine tuning of the parameters, the mass of the scalar Higgs boson naturally becomes as large as the largest energy scale in the theory. This is related to the fact that no extra symmetry is recovered when the scalar masses vanish, in contrast to what happens to fermions, where the chiral symmetry prevents the dependence from powers of higher scales, and gives a typical logarithmic dependence.

Naturalness in the SM

The Higgs mass is renormalised additively. Using a cutoff the regularization :



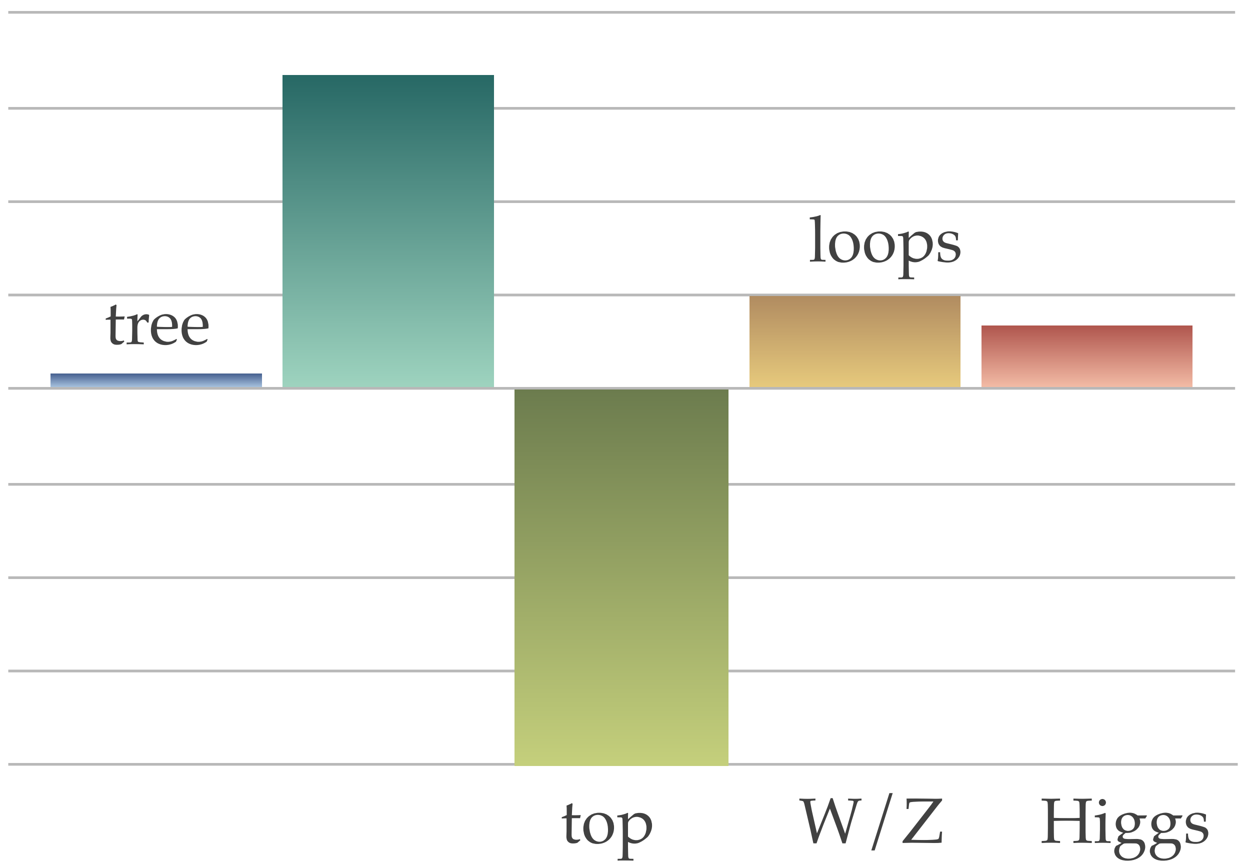
$$m_H^2 = m_{H0}^2 \quad - \frac{3}{8\pi^2} y_t \Lambda^2 \quad + \frac{1}{16\pi^2} g^2 \Lambda^2 \quad + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

Putting numbers, one gets:

$$(125\,\mathrm{GeV})^2 = m_{H0}^2 + \left[-(2\,\mathrm{TeV})^2 + (700\,\mathrm{GeV})^2 + (500\,\mathrm{GeV})^2\right] \left(\frac{\Lambda}{10\mathrm{TeV}}\right)^2$$

Naturalness in the SM

$m_H^2 \sim (125\ \text{GeV})^2$



$$(125\,\text{GeV})^2 = m_{H0}^2 + \left[-(2\,\text{TeV})^2 + (700\,\text{GeV})^2 + (500\,\text{GeV})^2\right]\left(\frac{\Lambda}{10\text{TeV}}\right)^2$$

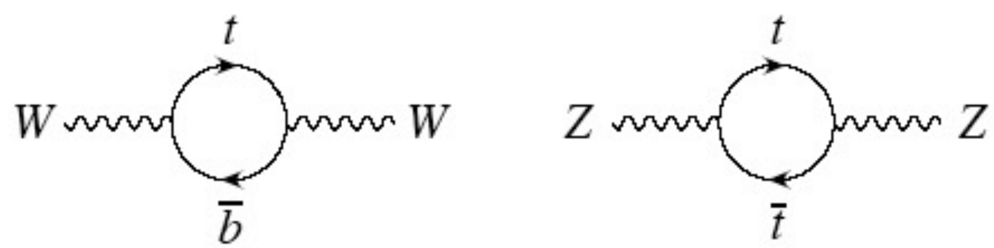
Definition of naturalness: less than 90% cancellation:

$\Lambda_t < 3\,\text{TeV}$ $\Rightarrow$ top partners must be "light"

Loop effects in the SM

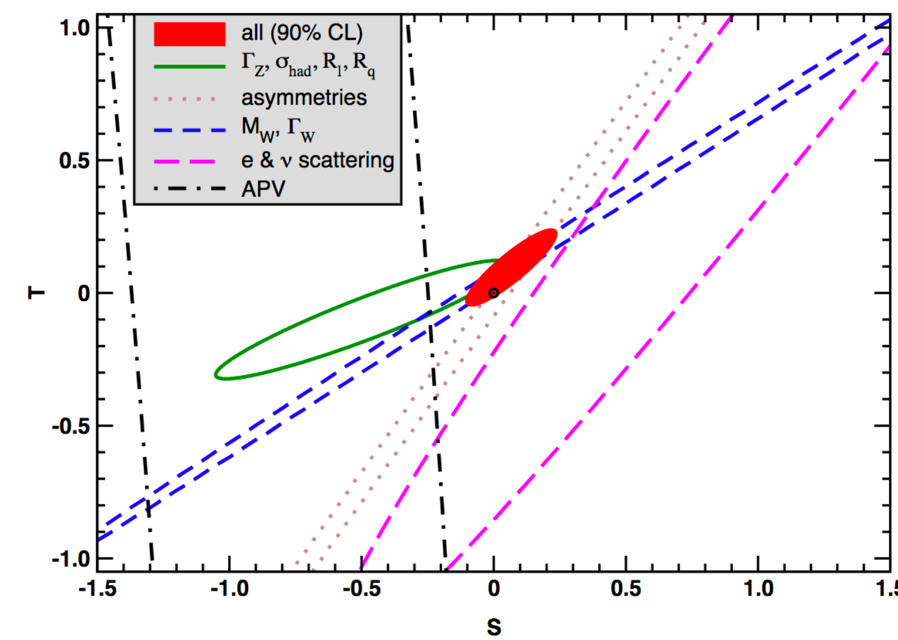
Indirect evidence for the existence of particles not yet detected can be inferred from quantum corrections. At tree level mW=mZ cos θW. At one loop:

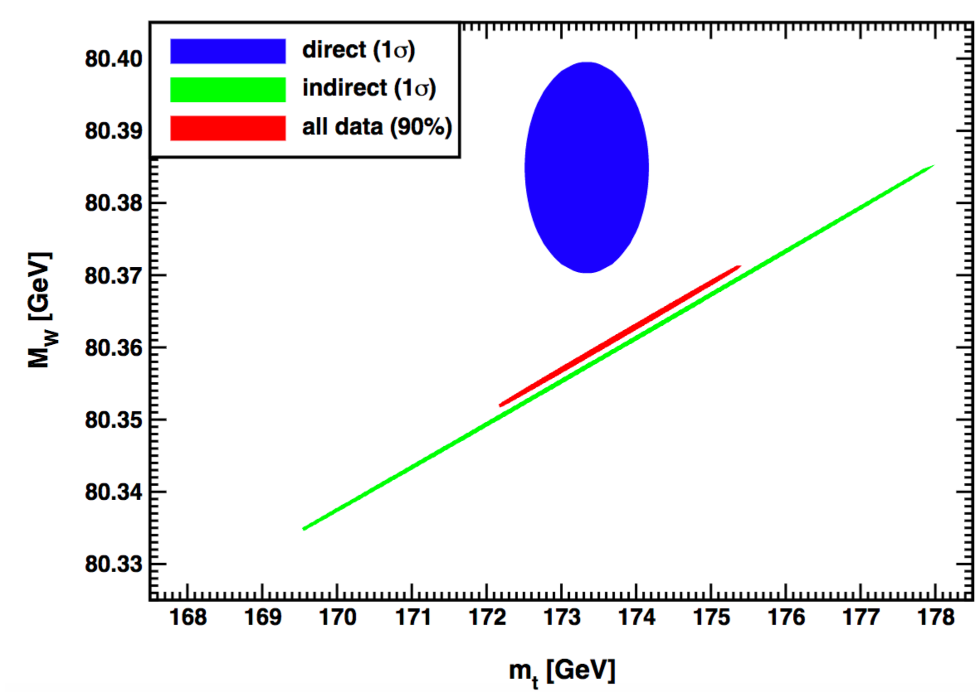
$$m_W^2\left(1-\frac{m_W^2}{m_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r)$$



$$\Delta r_{\rm top}=-\frac{3\alpha}{16\pi}\frac{\cos^2\theta_W}{\sin^4\theta_W}\frac{m_t^2}{m_W^2}$$

$$\Delta r_{\rm Higgs}=+\frac{11\alpha}{48\pi\sin^2\theta_W}\log\frac{m_H^2}{m_W^2}$$





Review questions: SM

1. What are the hypercharge assignments of the fermions in the SM? Can you explain in an elevator ride the anomaly cancellation mechanism in the SM? And its implications?

2. It is often said that a mass term for a gauge boson violates the gauge symmetry. What is the usual argument? Is this really true for an abelian gauge group? Is this true for non-abelian gauge group? Why?

3. Can I write a "SM" for which is SU(2)xU(1) invariant, yet does not contain the Higgs field? If so, how? Is it unitary?

4. If a mass term for the fermions is introduced that does not respect the EW gauge symmetry, at which scale the model will end to be valid?

5. What is the mass of the Goldstones in the SM? What is a shift symmetry? Can you describe the mysterious analogy of the SM EW sector with QCD at low-energy?

7. List the options that exist to give mass to neutrinos in a renormalizable way and by adding higher-dimensional operators.

8. Define as a “SM portal” a combination of SM fields which is a gauge singlet and has dimension less than four. How many of such portals do exist?