SMEFTatNLO UFO model

Operator definition & normalisation

The table below defines the list of SMEFT operators from the Warsaw basis [1] consistent with a

$$U(2)_q \times U(2)_u \times U(2)_d \times [U(1)_l \times U(1)_e]^3$$

flavor symmetry in the fermion sector. Coefficient names in the model are given in the `UFO` column. Grey cells denote operators not consistent with the restricted,

$$U(2)_q \times U(2)_u \times U(3)_d \times [U(1)_l \times U(1)_e]^3$$

flavor symmetry assumed in basic implementation, `SMEFTatNLO_U2_2_U3_3_cG_4F_LO_UFO`. Their Wilson coefficients are set to the corresponding, light-generation fermion flavor component, *e.g.*, `cpb`→`cpd` if present, otherwise they are set to zero.
The model focuses primarily interactions involving a top, in the quark sector, retaining only two-fermion operators involving solely the first two generations. See [2] for more details on conventions and the flavor symmetry implementation. The model contains a general cutoff parameter, Λ (`Lambda`), which normalises all operators in the Lagrangian as $\frac{c_i}{\Lambda^2}\mathcal{O}_i$.

Bosonic			SLHA Block: DIM6		
$\mathcal{O}_i$	UFO	Definition	$\mathcal{O}_i$	UFO	Definition
$\mathcal{O}_G$	cG	$g_S f_{ABC}\, G^A_{\mu\nu}\, G^{B,\nu\rho}\, G^{C,\mu}_{\ \ \ \ \rho}$	$\mathcal{O}_W$	cWWW	$\varepsilon_{IJK}\, W^I_{\mu\nu}\, W^{J,\nu\rho}\, W^{K,\mu}_{\ \ \ \ \rho}$
$\mathcal{O}_{\varphi G}$	cpG	$\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) G^{\mu\nu}_A\, G^A_{\mu\nu}$	$\mathcal{O}_{\varphi W}$	cpW	$\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) W^{\mu\nu}_I\, W^I_{\mu\nu}$
$\mathcal{O}_{\varphi B}$	cpBB	$\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) B^{\mu\nu}\, B_{\mu\nu}$	$\mathcal{O}_{\varphi WB}$	cpWB	$(\varphi^\dagger\tau_I\varphi)\, B^{\mu\nu} W^I_{\mu\nu}$
$\mathcal{O}_{\varphi}$	cp	$\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right)^3$	$\mathcal{O}_{\varphi d}$	cdp	$\partial_\mu(\varphi^\dagger\varphi)\partial^\mu(\varphi^\dagger\varphi)$
$\mathcal{O}_{\varphi D}$	cpDC	$(\varphi^\dagger D^\mu\varphi)^\dagger(\varphi^\dagger D_\mu\varphi)$			

2 fermion (chiral flip)			SLHA Block: DIM62F		
$\mathcal{O}_i$	UFO	Definition	$\mathcal{O}_i$	UFO	Definition
$\mathcal{O}_{t\varphi}$	ctp	$\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) \bar{Q}\, t\, \tilde{\varphi} + \text{h.c.}$	$\mathcal{O}_{tW}$	-	$i\big(\bar{Q}\tau^{\mu\nu}\, \tau_I\, t\big)\, \tilde{\varphi}\, W^I_{\mu\nu} + \text{h.c.}$
$\mathcal{O}_{tG}$	ctG	$ig_S\, \big(\bar{Q}\tau^{\mu\nu}\, T_A\, t\big)\, \tilde{\varphi}\, G^A_{\mu\nu} + \text{h.c.}$	$\mathcal{O}_{tB}$	-	$i\big(\bar{Q}\tau^{\mu\nu}\, t\big)\, \tilde{\varphi}\, B_{\mu\nu} + \text{h.c.}$
			-	ctW	C_{tW}
			-	ctZ	$-\sin\theta_W C_{tB} + \cos\theta_W C_{tW}$
$\mathcal{O}_{b\varphi}$	cbp	$\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) \bar{Q}\, b\, \varphi + \text{h.c.}$	$\mathcal{O}_{bW}$	cbW	$i\big(\bar{Q}\tau^{\mu\nu}\, \tau_I\, b\big)\, \varphi\, W^I_{\mu\nu} + \text{h.c.}$
$\mathcal{O}_{bG}$	cbG	$ig_S\, \big(\bar{Q}\tau^{\mu\nu}\, T_A\, b\big)\, \varphi\, G^A_{\mu\nu} + \text{h.c.}$	$\mathcal{O}_{bB}$	cbB	$i\big(\bar{Q}\tau^{\mu\nu}\, b\big)\, \varphi\, B_{\mu\nu} + \text{h.c.}$
$\mathcal{O}_{\tau\varphi}$	ctap	$\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right) \bar{Q}\, \tau\, \tilde{\varphi} + \text{h.c.}$	$\mathcal{O}_{\tau W}$	ctaW	$i\big(\bar{Q}\tau^{\mu\nu}\, \tau_I\, \tau\big)\, \tilde{\varphi}\, W^I_{\mu\nu} + \text{h.c.}$
			$\mathcal{O}_{\tau B}$	ctaB	$i\big(\bar{Q}\tau^{\mu\nu}\, \tau\big)\, \tilde{\varphi}\, B_{\mu\nu} + \text{h.c.}$

2 fermion (current)			SLHA Block:	DIM62F	
$\mathcal{O}_i$	UFO	Definition	$\mathcal{O}_i$	UFO	Definition
$\mathcal{O}_{\varphi l_1}^{(1)}$	cpl1	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{l}_1 \gamma^\mu l_1\big)$	$\mathcal{O}_{\varphi l_1}^{(3)}$	c3pl1	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \tau_I \varphi\big)\big(\bar{l}_1 \gamma^\mu \tau^I l_1\big)$
$\mathcal{O}_{\varphi l_2}^{(1)}$	cpl2	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{l}_2 \gamma^\mu l_2\big)$	$\mathcal{O}_{\varphi l_2}^{(3)}$	c3pl2	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \tau_I \varphi\big)\big(\bar{l}_2 \gamma^\mu \tau^I l_2\big)$
$\mathcal{O}_{\varphi l_3}^{(1)}$	cpl3	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{l}_3 \gamma^\mu l_3\big)$	$\mathcal{O}_{\varphi l_3}^{(3)}$	c3pl3	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \tau_I \varphi\big)\big(\bar{l}_3 \gamma^\mu \tau^I l_3\big)$
$\mathcal{O}_{\varphi e}$	cpe	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{e} \gamma^\mu e\big)$	$\mathcal{O}_{\varphi \mu}$	cpmu	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{\mu} \gamma^\mu \mu\big)$
$\mathcal{O}_{\varphi \tau}$	cpta	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{\tau} \gamma^\mu \tau\big)$	$\mathcal{O}_{\varphi tb}$	cptb	$i\big(\tilde{\varphi}\, D_\mu \varphi\big)\big(\bar{t} \gamma^\mu b\big) + \text{h.c.}$
$\mathcal{O}_{\varphi q_i}^{(1)}$	-	$\sum_{i=1,2} i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{q}_i \gamma^\mu q_i\big)$	$\mathcal{O}_{\varphi Q}^{(1)}$	-	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{Q} \gamma^\mu Q\big)$
$\mathcal{O}_{\varphi q_i}^{(3)}$	-	$\sum_{i=1,2} i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \tau_I \varphi\big)\big(\bar{q}_i \gamma^\mu \tau^I q_i\big)$	$\mathcal{O}_{\varphi Q}^{(3)}$	-	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \tau_I \varphi\big)\big(\bar{Q} \gamma^\mu \tau^I Q\big)$
-	cpq3i	$C_{\varphi q_i}^{(3)}$	-	cpQ3	$C_{\varphi Q}^{(3)}$
-	cpqMi	$C_{\varphi q_i}^{(1)} - C_{\varphi q_i}^{(3)}$	-	cpQM	$C_{\varphi Q}^{(1)} - C_{\varphi Q}^{(3)}$
$\mathcal{O}_{\varphi u_i}$	cpu	$\sum_{i=1,2} i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{u}_i \gamma^\mu u_i\big)$	$\mathcal{O}_{\varphi d_i}$	cpd	$\sum_{i=1,2(,3)} i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{d}_i \gamma^\mu d_i\big)$
$\mathcal{O}_{\varphi t}$	cpt	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{t} \gamma^\mu t\big)$	$\mathcal{O}_{\varphi b}$	cpb	$i\big(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi\big)\big(\bar{b} \gamma^\mu b\big)$

2 quark 2 lepton			SLHA Block:	DIM64F2L	
$\mathcal{O}_i$	UFO	Definition	$\mathcal{O}_i$	UFO	Definition
$\mathcal{O}_{Ql}^{-(1)}$	cQlM1	$[C_{lq}^1]^{1133} - [C_{lq}^3]^{1133}$	$\mathcal{O}_{Ql}^{3(1)}$	cQl31	$[C_{lq}^3]^{1133}$
$\mathcal{O}_{Ql}^{-(2)}$	cQlM2	$[C_{lq}^1]^{2233} - [C_{lq}^3]^{2233}$	$\mathcal{O}_{Ql}^{3(1)}$	cQl32	$[C_{lq}^3]^{2233}$
$\mathcal{O}_{Ql}^{-(3)}$	cQlM3	$[C_{lq}^1]^{3333} - [C_{lq}^3]^{3333}$	$\mathcal{O}_{Ql}^{3(1)}$	cQl33	$[C_{lq}^3]^{3333}$
$\mathcal{O}_{tl}^{(1)}$	ctl1	$[C_{lu}]^{1133}$	$\mathcal{O}_{bl}^{(1)}$	cbl1	$[C_{ld}]^{1133}$
$\mathcal{O}_{tl}^{(2)}$	ctl2	$[C_{lu}]^{2233}$	$\mathcal{O}_{bl}^{(2)}$	cbl2	$[C_{ld}]^{2233}$
$\mathcal{O}_{tl}^{(3)}$	ctl3	$[C_{lu}]^{3333}$	$\mathcal{O}_{bl}^{(3)}$	cbl3	$[C_{ld}]^{3333}$
$\mathcal{O}_{te}^{(1)}$	cte	$[C_{eu}]^{1133}$	$\mathcal{O}_{be}^{(1)}$	cbe	$[C_{ed}]^{1133}$
$\mathcal{O}_{te}^{(2)}$	ctmu	$[C_{eu}]^{2233}$	$\mathcal{O}_{be}^{(2)}$	cbmu	$[C_{ed}]^{2233}$
$\mathcal{O}_{te}^{(3)}$	ctta	$[C_{eu}]^{3333}$	$\mathcal{O}_{be}^{(3)}$	cbta	$[C_{ed}]^{3333}$
$\mathcal{O}_{Qe}^{(1)}$	cQe	$[C_{eQ}]^{1133}$	$\mathcal{O}_{t}^{S(3)}$	ctlS	$[C_{lequ}^{(1)}]^{3333}$
$\mathcal{O}_{Qe}^{(2)}$	cQmu	$[C_{eQ}]^{2233}$	$\mathcal{O}_{t}^{T(3)}$	ctlT	$[C_{lequ}^{(3)}]^{3333}$
$\mathcal{O}_{Qe}^{(3)}$	cQta	$[C_{eQ}]^{3333}$	$\mathcal{O}_{b}^{S(3)}$	cblS	$[C_{ledq}]^{3333}$

4 quark (2 heavy 2 light)			SLHA Block: DIM64F		
$\mathcal{O}_i$	`UFO`	Definition $(i=1,2)$	$\mathcal{O}_i$	`UFO`	Definition $(i=1,2)$
$\mathcal{O}_{Qq}^{1,1}$	`cQq11`	$[C_{qq}^{(1)}]^{ii33}+\frac{1}{6}[C_{qq}^{(1)}]^{i33i}+\frac{1}{2}[C_{qq}^{(3)}]^{i33i}$	$\mathcal{O}_{Qq}^{1,8}$	`cQq18`	$[C_{qq}^{(1)}]^{i33i}+3[C_{qq}^{(3)}]^{i33i}$
$\mathcal{O}_{Qq}^{3,1}$	`cQq31`	$[C_{qq}^{(3)}]^{ii33}+\frac{1}{6}[C_{qq}^{(1)}]^{i33i}-\frac{1}{6}[C_{qq}^{(3)}]^{i33i}$	$\mathcal{O}_{Qq}^{3,8}$	`cQq38`	$[C_{qq}^{(1)}]^{i33i}-[C_{qq}^{(3)}]^{i33i}$
$\mathcal{O}_{tu}^{1}$	`ctu1`	$[C_{uu}]^{ii33}+\frac{1}{3}[C_{uu}]^{i33i}$	$\mathcal{O}_{tu}^{8}$	`ctu8`	$2[C_{uu}]^{i33i}$
$\mathcal{O}_{td}^{1}$	`ctd1`	$[C_{ud}^{(1)}]^{33ii}$	$\mathcal{O}_{td}^{8}$	`ctd8`	$[C_{ud}^{(8)}]^{33ii}$
$\mathcal{O}_{ub}^{1}$	`cub1`	$[C_{ud}^{(1)}]^{ii33}$	$\mathcal{O}_{ub}^{8}$	`cub8`	$[C_{ud}^{(8)}]^{ii33}$
$\mathcal{O}_{tq}^{1}$	`ctq1`	$[C_{qu}^{(1)}]^{ii33}$	$\mathcal{O}_{tq}^{8}$	`ctq8`	$[C_{qu}^{(8)}]^{ii33}$
$\mathcal{O}_{Qu}^{1}$	`cQu1`	$[C_{qu}^{(1)}]^{33ii}$	$\mathcal{O}_{Qu}^{8}$	`cQu8`	$[C_{qu}^{(8)}]^{33ii}$
$\mathcal{O}_{bq}^{1}$	`cbq1`	$[C_{qd}^{(1)}]^{ii33}$	$\mathcal{O}_{bq}^{8}$	`cbq8`	$[C_{qd}^{(8)}]^{ii33}$
$\mathcal{O}_{Qd}^{1}$	`cQd1`	$[C_{qd}^{(1)}]^{33ii}$	$\mathcal{O}_{Qd}^{8}$	`cQd8`	$[C_{qd}^{(8)}]^{33ii}$

4 quark (4 heavy)			SLHA Block: DIM64F		
$\mathcal{O}_i$	`UFO`	Definition	$\mathcal{O}_i$	`UFO`	Definition
$\mathcal{O}_{QQ}^{1}$	`cQQ1`	$2[C_{qq}^{(1)}]^{3333}-\frac{2}{3}[C_{qq}^{(3)}]^{3333}$	$\mathcal{O}_{QQ}^{8}$	`cQQ8`	$8[C_{qq}^{(3)}]^{3333}$
$\mathcal{O}_{Qt}^{1}$	`cQt1`	$[C_{qu}^{(1)}]^{3333}$	$\mathcal{O}_{Qt}^{8}$	`cQt8`	$[C_{qu}^{(8)}]^{3333}$
$\mathcal{O}_{Qb}^{1}$	`cQb1`	$[C_{qd}^{(1)}]^{3333}$	$\mathcal{O}_{Qb}^{8}$	`cQb8`	$[C_{qd}^{(8)}]^{3333}$
$\mathcal{O}_{tb}^{1}$	`ctb1`	$[C_{ud}^{(1)}]^{3333}$	$\mathcal{O}_{tb}^{8}$	`ctb8`	$[C_{ud}^{(8)}]^{3333}$
$\mathcal{O}_{tt}$	`ctt`	$[C_{uu}^{(1)}]^{3333}$			
$\mathcal{O}_{QtQb}^{1}$	`cQtQb1`	$\mathrm{Re}[C_{quqd}^{(1)}]^{3333}$	$\mathcal{O}_{QtQb}^{8}$	`cQtQb8`	$\mathrm{Re}[C_{quqd}^{(8)}]^{3333}$

Definitions

$$\varphi^\dagger \overleftrightarrow{D}_\mu \varphi = \varphi^\dagger D^\mu \varphi - (D_\mu\varphi)^\dagger \varphi \tag{1}$$
$$\varphi^\dagger \tau_K \overleftrightarrow{D}^\mu \varphi = \varphi^\dagger \tau_K D^\mu \varphi - (D^\mu\varphi)^\dagger \tau_K \varphi \tag{2}$$
$$W_{\mu\nu}^K = \partial_\mu W_\nu^K - \partial_\nu W_\mu^K + g\epsilon_{IJ}{}^{K}\; W_\mu^I W_\nu^J \tag{3}$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{4}$$
$$D_\rho W_{\mu\nu}^K = \partial_\rho W_{\mu\nu}^K + g\epsilon_{IJ}^K W_\rho^I W_{\mu\nu}^J \tag{5}$$
$$D_\mu \varphi = \left(\partial_\mu - i\frac{g}{2}\tau_K W_\mu^K - i\frac{1}{2}g' B_\mu\right)\varphi \tag{6}$$
$$\tau^{\mu\nu} = \frac{1}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\mu\gamma^\mu\right) \tag{7}$$

where τ_I are the Pauli sigma matrices.

1 Field redefinitions & inputs

After Electroweak symmetry breaking, certain operators in the SMEFT lead to modifications of the SM Lagrangian. They necessarily involve the presence of a Higgs field, φ, which, upon taking its vacuum-expectation-value, v, generates effective dimension-4 operators proportional to $\frac{v^2}{\Lambda}$. In general, this leads to a non-canonical Lagrangian that can be returned to canonical form by appropriate Wilson coefficient dependent field redefintions. Some of the effects can be completely absorbed and do not lead to physical consequences. These are usually for operators that look like SM terms multiplied by the Higgs bilinear, $\varphi^\dagger\varphi$, such as $\mathcal{O}_\varphi$, $\mathcal{O}_{\varphi G}$, $\mathcal{O}_{\varphi W}$ and $\mathcal{O}_{\varphi B}$. Since combinations of field redefinitions and coupling shifts remove all traces of these operators at dimension-4, we choose the economical method of redefining the operators themselves, with the replacement

$$\varphi^\dagger\varphi \to \varphi^\dagger\varphi - \frac{v^2}{2}, \tag{8}$$

such that these modifications are not generated in the first place. We stress that this is equivalent to making the aforementioned field and coupling redefintions.

Other modifications of the dimension-4 Lagrangian cannot be fully absorbed and manifest themselves as mass and coupling shifts with respect to the SM. An important consequence of this is that the determination of the parameters of the SM from certain input measurements is modified. Besides the indirect effects from field redefinitions, SMEFT effects can enter directly by contributing to the matrix elements of the processes that are used determine the input parameters themselves. In general, the relationships between the input parameters and other, dependent, parameters of the model now also depend on the Wilson coefficients, which ultimately are additional input parameters of the SMEFT.

This occurs in the determination of the Fermi constant, G_F, from the muon decay lifetime that is ultimately used to set the value of Higgs vacuum expectation-value, v. This process is affected by operators that modify the W-e-ν_e and W-μ-ν_μ interactions, $\mathcal{O}^{(3)}_{\varphi l}$, as well as the four fermion operator, $\mathcal{O}_{ll}$, which contains the Fermi operator. Matching the new muon decay amplitude to the Fermi operator below the EW scale,

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}\left[1+\frac{v^2}{\Lambda^2}\left[C^{(3)}_{\varphi l}\right]^{11}\right]\left[1+\frac{v^2}{\Lambda^2}\left[C^{(3)}_{\varphi l}\right]^{22}\right] - \frac{1}{4}\frac{1}{\Lambda^2}\left(2\left[C_{ll}\right]^{1212} + \left[C_{ll}\right]^{1221} + \left[C_{ll}\right]^{2112}\right), \tag{9}$$

leads to a new expression of the Higgs vev in terms of G_F:

$$v^2 = v_0^2\left(1+\frac{1}{2}\left(\left[C^{(3)}_{\varphi l}\right]^{11} + \left[C^{(3)}_{\varphi l}\right]^{22} - \left[C_{ll}\right]^{1212} - \left[C_{ll}\right]^{1221}\right)\frac{v_0^2}{\Lambda^2}\right) \tag{10}$$

$$= v_0^2(1+\delta_v), \quad v_0^2 = \frac{1}{\sqrt{2}G_F}. \tag{11}$$

The $[C_{ll}]$ coefficients have a permutation symmetry equating $[C_{ll}]^{ijkl}$ and $[C_{ll}]^{klij}$ which has been used in the last equation.

In the Higgs sector, the operators $\mathcal{O}_{\varphi d}$ and $\mathcal{O}_{\varphi D}$ both shift the kinetic term for the Higgs field. Canonical form is restored by field redefinitions for the dynamical Higgs and neutral Goldstone that read at linear order,

$$h \to h\left(1+\left(C_{\varphi d} - \frac{C_{\varphi D}}{4}\right)\frac{v_0^2}{\Lambda^2}\right), \qquad G_0 \to G_0\left(1-\frac{C_{\varphi D}}{4}\frac{v_0^2}{\Lambda^2}\right). \tag{12}$$

This modifies the relationship between the m_H input parameter and the quadratic and quartic parameters of the Higgs potential, μ^2 and λ, which are now given by

$$\mu \to \frac{m_h}{\sqrt{2}}\left(1-\left(C_{\varphi d} - \frac{C_{\varphi D}}{4}\right)\frac{v_0^2}{\Lambda^2}\right); \qquad \lambda \to \frac{m_h^2}{2v^2}\left(1-\left(2C_{\varphi d} - \frac{C_{\varphi D}}{2}\right)\frac{v_0^2}{\Lambda^2}\right) \tag{13}$$

In the gauge sector, the $\mathcal{O}_{\varphi WB}$ induces a kinetic mixing term between the neutral $SU(2)$ and Hypercharge gauge fields,

$$-\frac{C_{\varphi WB}}{2}\frac{v_0^2}{\Lambda^2}B^{\mu\nu}W^3_{\mu\nu} \equiv -\frac{\delta_{WB}}{2}B^{\mu\nu}W^3_{\mu\nu},, \tag{14}$$

and $\mathcal{O}_{\varphi D}$ generates an explicit, custodial symmetry violating mass term for the Z boson field. Rotating away the kinetic mixing and diagonalising the modified mass matrix leads to a new Weinberg angle

$$c_W = \frac{g}{\sqrt{g^2+g'^2}}\left(1 - \frac{g\,g'}{g^2+g'^2}\delta_{WB}\right) \qquad s_W = \frac{g'}{\sqrt{g^2+g'^2}}\left(1 + \left(\frac{g}{g'} - \frac{gg'}{g^2+g'^2}\right)\delta_{WB}\right) \tag{15}$$

and a shifted Z-boson mass.

$$m_Z^2 = \frac{(g^2+g'^2)v^2}{4}\left(1 + \frac{1}{2}\frac{v_0^2}{\Lambda^2}\left(C_{\varphi D} + 4\frac{gg'}{g^2+g'^2}C_{\varphi WB}\right)\right). \tag{16}$$

$$= \frac{(g^2+g'^2)v_0^2}{4}\left(1 + \delta_v + 2\frac{gg'}{g^2+g'^2}\delta_{WB} + \delta T\right); \quad \delta T = \frac{1}{2}\frac{v_0^2}{\Lambda^2}C_{\varphi D}. \tag{17}$$

The W-mass is only shifted by δ_v,

$$m_W^2 = \frac{g^2 v^2}{4} = \frac{g^2 v_0^2}{4}(1+\delta_v). \tag{18}$$

The gauge boson interactions are generically modified, and notably, the photon interaction strength becomes

$$e = \frac{gg'}{\sqrt{g^2+g'^2}}\left(1 - \frac{gg'}{g^2+g'^2}\delta_{WB}\right) \tag{19}$$

Choosing a specific EW input parameter scheme, any effects that modify parameters or interactions that affect the input measurement must then be propagated into the dependent parameters. That is to say, we take the measured input values as they are and assume that they contain said contributions from SMEFT parameters, leading to consequences for the other, derived EW parameters of the SM.

1.1 m_Z, G_F, m_W scheme

The derived EW parameters starting from these three inputs are as follows:

Weinberg angle

The input combination that yields c_W in the SM,

$$\frac{m_W^2}{m_Z^2} = \frac{g}{\sqrt{g^2+g'^2}}\left(1 - 2\frac{gg'}{g^2+g'^2}\delta_{WB} - \delta T\right), \tag{20}$$

leads to the derived value of the Weinberg angle

$$c_W = c_{W,0}\left(1 + \frac{\delta_T}{2}\right), \qquad s_W = s_{W,0}\left(1 - \frac{c_{W,0}^2}{s_{W,0}^2}\frac{\delta_T}{2}\right),$$
$$c_{W,0} = \frac{m_W}{m_Z}. \qquad s_{W,0} = \sqrt{1 - \frac{m_W^2}{m_Z^2}}. \tag{21}$$

Gauge couplings

$$g = \frac{2m_W}{v} \qquad g' = \frac{2m_Z s_W}{v}\left(1 - \frac{g}{g'}\delta_{WB} - \frac{\delta_T}{2}\right)$$
$$= \frac{2m_W}{v_0}\left(1 - \frac{\delta_v}{2}\right) \qquad = \frac{2m_Z s_{W,0}}{v_0}\left(1 - \frac{\delta_v}{2} - \frac{c_{W,0}}{s_{W,0}}\delta_{WB} - \frac{\delta_T}{2s_{W,0}^2}\right) \tag{22}$$

$$e = \frac{2m_W s_W}{v}\left(1 - \frac{g}{g'}\delta_{WB}\right)$$
$$= \frac{2m_W s_{W,0}}{v_0}\left(1 - \frac{\delta_v}{2} - \frac{c_{W,0}}{s_{W,0}}\delta_{WB} - \frac{c_{W,0}^2}{s_{W,0}^2}\frac{\delta_T}{2}\right) \tag{23}$$

1.2 m_Z, G_F, α_{EW} scheme

α_{EW} is extracted from some physical process involving the photon interactions, such as Thomson scattering at zero momentum transfer. In this case, the value of e is identified with

$$e \equiv \sqrt{4\pi\alpha_{EW}} \tag{24}$$

and the remaining EW parameters should be expressed in terms of v_0, m_Z and e.

Weinberg angle

m_Z can be written in terms of a combination of input parameters

$$m_Z^2 = \left(\frac{ev_0}{2s_W c_W}\right)^2 \left(1+\delta_{m_Z^2}\right); \qquad \delta_{m_Z^2} = \delta_T + \delta_v + 2\frac{c_{W,0}}{s_{W,0}}\delta_{WB} \tag{25}$$

and solved to linear order for the Weiberg angle

$$s_W = s_{W,0}\left(1 + \frac{c_{W,0}^2}{c_{W,0}^2 - s_{W,0}^2}\frac{\delta_{m_Z^2}}{2}\right), \qquad c_W = c_{W,0}\left(1 - \frac{s_{W,0}^2}{c_{W,0}^2 - s_{W,0}^2}\frac{\delta_{m_Z^2}}{2}\right), \tag{26}$$

where the parameters $s_{W,0}$ and $c_{W,0}$ take their values as a function of the inputs in the SM limit,

$$s_{W,0} = \frac{1}{\sqrt{2}}\left(1 - \sqrt{1-\left(\frac{ev_0}{m_Z}\right)^2}\right)^{-\frac{1}{2}} \qquad c_{W,0} = \frac{1}{\sqrt{2}}\left(1 + \sqrt{1-\left(\frac{ev_0}{m_Z}\right)^2}\right)^{-\frac{1}{2}}. \tag{27}$$

Gauge couplings

$$\begin{aligned} g &= \frac{e}{s_W}\left(1+\frac{g}{g'}\delta_{WB}\right) & g' &= \frac{e}{c_W} \\ &= \frac{e}{s_{W,0}}\left(1 + \frac{c_{W,0}}{s_{W,0}}\delta_{WB} - \frac{c_{W,0}^2}{c_{W,0}^2 - s_{W,0}^2}\frac{\delta_{m_Z^2}}{2}\right), & &= \frac{e}{c_{W,0}}\left(1 + \frac{s_{W,0}^2}{c_{W,0}^2 - s_{W,0}^2}\frac{\delta_{m_Z^2}}{2}\right). \end{aligned} \tag{28}$$

W mass

$$m_W^2 = \left(\frac{ev_0}{2s_{W,0}}\right)^2 \left(1 + \delta_v + 2\frac{c_{W,0}}{s_{W,0}}\delta_{WB} - \frac{c_{W,0}^2}{c_{W,0}^2 - s_{W,0}^2}\delta_{m_Z^2}\right). \tag{29}$$

References

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