

# FeynRules Implementation of the 3-Site Model

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## Abstract

We describe in detail the aspects of the Minimal Higgsless Model or 3-Site Model that are relevant for implementation into the FeynRules package.

In this note, we describe the minimal Higgsless model in detail. We describe the sectors of the model in the following order: 1) the gauge sector, 2) the gauge fixing sector, 3) the ghost sector and 4) the fermion sector. In each section, we give the Lagrangian, work out the eigenstates and determine the relationship of the dependent parameters in terms of the independent parameters.

## 1 Gauge Sector

The gauge group of the Three Site Model is

$$G = SU(3)_{QCD} \times SU(2)_0 \times SU(2)_1 \times U(1)_2 \quad (1)$$

where  $SU(2)_0$  is represented by the leftmost circle in Figure 1 and has coupling  $g$ ,  $SU(2)_1$  is represented by the center circle in Figure 1 and has coupling  $\tilde{g}$  and  $U(1)_2$  is represented by the rightmost dashed circle in Figure 1 and has coupling  $g'$ . We define

$$x = \frac{g}{\tilde{g}} \quad \text{and} \quad t = \frac{g'}{g} = \frac{s}{c} \quad (2)$$

where  $s^2 + c^2 = 1$ .

The kinetic and self interaction terms for the gauge bosons is given by the usual gauge invariant terms:

$$\mathcal{L}_{F^2} = -\frac{1}{4}F_{QCD}^2 - \frac{1}{4}F_0^2 - \frac{1}{4}F_1^2 - \frac{1}{4}F_2^2 \quad (3)$$

The horizontal bars in Figure 1 represent nonlinear sigma models  $\Sigma_j$  which come from unspecified physics at a higher scale and which give mass to the 6 gauge bosons other than the photon. This is encoded in the leading order effective Lagrangian term

$$\mathcal{L}_{D\Sigma} = \frac{f^2}{4} \text{Tr} \left[ (D_\mu \Sigma_0)^\dagger D^\mu \Sigma_0 + (D_\mu \Sigma_1)^\dagger D^\mu \Sigma_1 \right] \quad (4)$$

where

$$D_\mu \Sigma_0 = \partial_\mu \Sigma_0 + igW_{0\mu} \Sigma_0 - i\tilde{g}\Sigma_0 W_{1,\mu} \quad (5)$$

$$D_\mu \Sigma_1 = \partial_\mu \Sigma_1 + i\tilde{g}W_{1\mu} \Sigma_1 - ig'\Sigma_1 W_{2,\mu} \quad (6)$$

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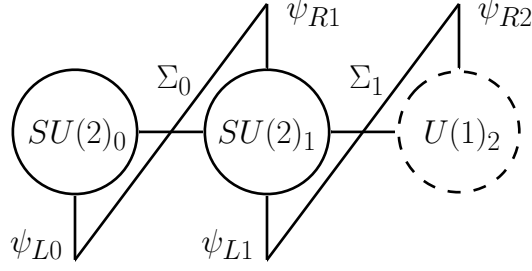


Figure 1: A schematic “moose” diagram of the 3-Site model. The circles represent gauge groups. The two circles on the left are  $SU(2)$  gauge groups while the one on the right is a  $U(1)$  gauge group. The horizontal lines represent the goldstone bosons eaten by the massive gauge bosons. They are in the form of nonlinear sigma fields. The physical vector bosons (mass eigenstates) are linear combinations of the gauge bosons at each site. The  $W$  gauge boson is a linear combination of the charged gauge bosons at sites 0 and 1. It lives mostly at site 0 with a small leakage into site 1. The  $W'$  boson is the orthogonal linear combination of charged gauge bosons at the first two sites. It lives mostly at site 1. The photon and  $Z$  are linear combinations of the neutral gauge bosons at all three sites, but live mostly at the edges with a small leakage into site 1. The  $Z'$  lives mostly at site 1. The vertical lines represent the fermions in the theory. The vertical lines below the circles represent the left chiral fields while those above the circles represent the right chiral fermions. The diagonal lines represent the yukawa interactions connecting the fermions and the nonlinear sigma fields. Each fermion is a fundamental representation of the gauge group to which it is attached and a singlet under all the other gauge groups except  $U(1)_2$ . The charges under  $U(1)_2$  are as follows: If the fermion is attached to an  $SU(2)$  then its charge is  $1/6$  for quarks and  $-1/2$  for leptons. If the fermion is attached to  $U(1)_2$  its charge is the same as its electromagnetic charge:  $0$  for neutrinos,  $-1$  for charged leptons,  $2/3$  for up type quarks and  $-1/3$  for down type quarks.

The nonlinear sigma models can be written in exponential form

$$\Sigma_j = e^{i2\pi_j/f} \quad (7)$$

which exposes the Goldstone bosons which become the longitudinal components of the massive gauge bosons.  $\pi_j$  and  $W_j$  are written in matrix form and are

$$\pi_j = \begin{pmatrix} \frac{1}{2}\pi_j^0 & \frac{1}{\sqrt{2}}\pi_j^+ \\ \frac{1}{\sqrt{2}}\pi_j^- & -\frac{1}{2}\pi_j^0 \end{pmatrix}, \quad W_j = \begin{pmatrix} \frac{1}{2}W_j^0 & \frac{1}{\sqrt{2}}W_j^+ \\ \frac{1}{\sqrt{2}}W_j^- & -\frac{1}{2}W_j^0 \end{pmatrix} \quad \text{and} \quad W_2 = \begin{pmatrix} \frac{1}{2}W_2^0 & 0 \\ 0 & -\frac{1}{2}W_2^0 \end{pmatrix} \quad (8)$$

where  $j$  is 0 or 1.

The mass matrices of the gauge bosons can be obtained by going to unitary gauge ( $\Sigma_j \rightarrow 1$ ) and are

$$M_{\pm}^2 = \frac{M_G^2}{2} \begin{pmatrix} x^2 & -x \\ -x & 2 \end{pmatrix} \quad \text{and} \quad M_n^2 = \frac{M_G^2}{2} \begin{pmatrix} x^2 & -x & 0 \\ -x & 2 & -xt \\ 0 & -xt & x^2t^2 \end{pmatrix} \quad (9)$$

for the charged and neutral gauge bosons respectively where

$$M_G^2 = \frac{\tilde{g}^2 f^2}{2}. \quad (10)$$

The photon is massless and given by the wavefunction

$$v_\gamma = e \left( \frac{1}{g}, \frac{1}{\tilde{g}}, \frac{1}{g'} \right) \quad (11)$$

where

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{\tilde{g}^2} + \frac{1}{g'^2}. \quad (12)$$

After diagonalizing the gauge boson mass matrices, we find that the other masses and wavefunctions are given by

$$M_W = \frac{M_G}{2} \sqrt{2 + x^2 - \sqrt{4 + x^4}} \quad (13)$$

$$v_W = \frac{1}{N_W} \left\{ 1, \frac{2x}{2 - x^2 + \sqrt{4 + x^4}} \right\} \quad (14)$$

$$M_{W'} = \frac{M_G}{2} \sqrt{2 + x^2 + \sqrt{4 + x^4}} \quad (15)$$

$$v_{W'} = \frac{1}{N_{W'}} \left\{ \frac{2 - x^2 - \sqrt{4 + x^4}}{2x}, 1 \right\} \quad (16)$$

for the charged gauge bosons (where  $N_W$  and  $N_{W'}$  are normalization constants) and

$$M_Z = \frac{M_G}{2} \sqrt{2 + x^2(1 + t^2) - A} \quad (17)$$

$$v_Z = \frac{1}{N_Z} \left\{ x^2 t + \frac{1}{t} (x^2 - A), \frac{-2 - x^2(1 - t^2) + A}{xt}, 2 \right\} \quad (18)$$

$$M_{Z'} = \frac{M_G}{2} \sqrt{2 + x^2(1 + t^2) + A} \quad (19)$$

$$v_{Z'} = \frac{1}{N_{Z'}} \left\{ -x^2 t + \frac{1}{t} (x^2 + A), \frac{-2 - x^2(1 - t^2) - A}{xt}, 2 \right\} \quad (20)$$

where

$$A = \sqrt{4 + x^4(1 - t^2)^2} \quad (21)$$

for the neutral gauge bosons (where  $N_Z$  and  $N_{Z'}$  are normalization constants). We note that  $x$  can be obtained from the ratio of the charged gauge boson masses

$$R_M^2 = \left( \frac{M_W}{M_{W'}} \right)^2 = \frac{2 + x^2 - \sqrt{4 + x^4}}{2 + x^2 + \sqrt{4 + x^4}} \quad (22)$$

which can be inverted to give

$$x = \frac{1 + R_M^2 - \sqrt{1 - 6 R_M^2 + R_M^4}}{2 R_M}. \quad (23)$$

The parameter  $t$  can be obtained from the ratio of the  $W$  and  $Z$  masses

$$c_M^2 = \frac{M_W^2}{M_Z^2} = \frac{2 + x^2 - \sqrt{4 + x^4}}{2 + x^2(1 + t^2) - \sqrt{4 + x^4(1 - t^2)^2}} \quad (24)$$

which can be inverted to give

$$t^2 = \frac{(1 - c_M^2) (x^2 (2 + x^2 - \sqrt{4 + x^4}) + B c_M^2 + 2 (x^2 + x^4) c_M^4)}{2 x^2 c_M^2 (x^2 - (2 + 3 x^2 + x^4) c_M^2 + (1 + x^2)^2 c_M^4)} \quad (25)$$

where

$$B = \left( -x^6 + x^2 (2 + x^2) (-4 + \sqrt{4 + x^4}) + 2 (-2 + \sqrt{4 + x^4}) \right) \quad (26)$$

In the following we also use  $s_M = \sqrt{1 - c_M^2}$  and  $t_M = s_M/c_M$ .

The couplings can be determined in terms of the electric charge  $e$ ,  $x$  and  $t$ .

$$g^2 = e^2 \left( 1 + x^2 + \frac{1}{t^2} \right) \quad (27)$$

$$\tilde{g}^2 = e^2 \left( 1 + \frac{1}{x^2} + \frac{1}{x^2 t^2} \right) \quad (28)$$

$$(g')^2 = e^2 (1 + t^2 + x^2 t^2) \quad (29)$$

## 2 Gauge Fixing Sector

As we mentioned previously, the horizontal lines in Figure 1 represent nonlinear sigma fields. Although tree level calculations can be done in unitary gauge, there are times when a different gauge is useful. Many calculations with gauge bosons in the external states can be computed more simply using the equivalence theorem and replacing the massive gauge bosons with the goldstone bosons that they eat. Another case that another gauge is advantageous is in CALCHEP, where the time of computation of processes is dramatically decreased in Feynman gauge. For this reason, we have implemented this model in both Feynman and unitary gauge. In this section, we outline the gauge fixing terms, while in the next we determine the ghost terms.

We must first determine the goldstone bosons that are eaten by the gauge bosons. We do this using the lagrangian of equation 4. Expanding the nonlinear sigma field, we obtain the mixing terms between the gauge bosons and the goldstone bosons. After inserting the eigenwave functions of these fields, we obtain

$$\mathcal{L}_{\pi W} = \frac{1}{2}\tilde{g}f\left(v_{\pi}^0(xv_W^0 - v_W^1) + v_{\pi}^1(v_W^1 - \delta xtv_W^2)\right)\{\partial_{\mu}\pi, W^{\mu}\} \quad (30)$$

$$+ \frac{1}{2}\tilde{g}f\left(v_{\pi'}^0(xv_{W'}^0 - v_{W'}^1) + v_{\pi'}^1(v_{W'}^1 - \delta xtv_{W'}^2)\right)\{\partial_{\mu}\pi', W'^{\mu}\} \quad (31)$$

where  $\delta$  is 1 if the gauge boson is neutral but 0 otherwise. Since  $\pi'$  does not mix with  $W$ , we obtain

$$v_{\pi'}^0(xv_W^0 - v_W^1) + v_{\pi'}^1(v_W^1 - \delta xtv_W^2) = 0 \quad (32)$$

or

$$v_{\pi'} = \frac{1}{N_{\pi}}\{-v_W^1 + \delta xtv_W^2, xv_W^0 - v_W^1\} \quad (33)$$

and since  $v_{\pi}$  is orthornormal to  $v_{\pi'}$  we get

$$v_{\pi} = \frac{1}{N_{\pi}}\{xv_W^0 - v_W^1, v_W^1 - \delta xtv_W^2\}. \quad (34)$$

The gauge fixing function is constructed to fixe the gauge and cancel the mixing of the goldstone bosons and gauge bosons. For each site, the gauge fixing term is

$$G_0 = \partial \cdot W_0 - \frac{\xi}{2}gf(\pi_0) \quad (35)$$

$$G_1 = \partial \cdot W_1 - \frac{\xi}{2}\tilde{g}f(\pi_1 - \pi_0) \quad (36)$$

$$G_2 = \partial \cdot W_2 - \frac{\xi}{2}g'f(-\pi_1^{ns}) \quad (37)$$

where by  $\pi_1^{ns}$  we mean just the neutral sector of  $\pi_1$ , namely

$$\pi_1^{ns} = \frac{1}{2}\pi_1^0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (38)$$

With this definition, the gauge fixing Lagrangian is

$$\mathcal{L}_{GF} = -\frac{1}{\xi}\text{Tr}(G_0^2 + G_1^2 + G_2^2) \quad (39)$$

where  $\xi = 1$  corresponds to the Feynman gauge and  $\xi \rightarrow \infty$  corresponds with unitary gauge.

## 3 Ghost Sector

The ghost lagrangian terms are obtained by multiplying the BRST transformation of the gauge fixing term on the left with the antighost. To do this, we must find the BRST transformations of the gauge fixing terms. We begin by writing the infinitesimal BRST transformation of the fields in the gauge fixing term.

$$\delta_{BRST}W_{\mu j} = -\left(\partial_{\mu}c_j + ig_j[W_{\mu j}, c_j]\right) \quad (40)$$

for the gauge bosons where  $c_j$  is the ghost for site  $j$  in matrix notation

$$c_j = \begin{pmatrix} \frac{1}{2}c_j^0 & \frac{1}{\sqrt{2}}c_j^+ \\ \frac{1}{\sqrt{2}}c_j^- & -\frac{1}{2}c_j^0 \end{pmatrix} \quad \text{and} \quad c_2 = \begin{pmatrix} \frac{1}{2}c_2^0 & 0 \\ 0 & -\frac{1}{2}c_2^0 \end{pmatrix} \quad (41)$$

where  $j$  is 0 or 1. The BRST transformation to quadratic order in the goldstone bosons is

$$\delta_{BRST}\pi_j = +\frac{1}{2}f(g_jc_j - g_{j+1}c_{j+1}) + \frac{i}{2}[g_jc_j + g_{j+1}c_{j+1}, \pi_j] - \frac{1}{6f}[\pi_j, [\pi_j, g_jc_j - g_{j+1}c_{j+1}]] \quad (42)$$

so that

$$\delta_{BRST}G_0 = \partial \cdot \delta_{BRST}W_0 - \frac{\xi}{2}gf(\delta_{BRST}\pi_0) \quad (43)$$

$$\delta_{BRST}G_1 = \partial \cdot \delta_{BRST}W_1 - \frac{\xi}{2}\tilde{g}f(\delta_{BRST}\pi_1 - \delta_{BRST}\pi_0) \quad (44)$$

$$\delta_{BRST}G_2 = \partial \cdot \delta_{BRST}W_2 - \frac{\xi}{2}g'f(-\delta_{BRST}\pi_1^{ns}) \quad (45)$$

The ghost Lagrangian is

$$\mathcal{L}_{ghost} = -\text{Tr}(\bar{c}_0\delta_{BRST}G_0 + \bar{c}_1\delta_{BRST}G_1 + \bar{c}_2\delta_{BRST}G_2) + h.c. \quad (46)$$

## 4 Fermion Sector

The vertical lines in Figure 1 represent the fermionic fields in the theory. The vertical lines on the bottom of the circles represent the left chiral fermions while the vertical lines attached to the tops of the circles are the right chiral fermions. Each fermion is a fundamental representation of the gauge group to which it is attached and a singlet under all the other gauge groups except  $U(1)_2$ . The charges under  $U(1)_2$  are as follows: If the fermion is attached to an  $SU(2)$  then its charge is  $1/6$  for quarks and  $-1/2$  for leptons. If the fermion is attached to  $U(1)_2$  its charge is the same as its electromagnetic charge:  $0$  for neutrinos,  $-1$  for charged leptons,  $2/3$  for up type quarks and  $-1/3$  for down type quarks. The usual gauge invariant kinetic terms are used:

$$\mathcal{L}_\psi = i\bar{\psi}_{L0}\not{D}\psi_{L0} + i\bar{\psi}_{L1}\not{D}\psi_{L1} + i\bar{\psi}_{R1}\not{D}\psi_{R1} + i\bar{\psi}_{R2}\not{D}\psi_{R2}. \quad (47)$$

The fermions attached to the internal site ( $SU(2)_1$ ) are vectorially coupled and are thus allowed Dirac masses. We have taken these masses to be  $M_F$ . The symmetries also allow various linkings of fermions via the nonlinear sigma fields. We have assumed a very simple form, inspired by an extra dimension and represented by the diagonal lines in Figure 1. The left chiral field at site  $j$  is linked to the right chiral field at site  $j+1$  through the nonlinear sigma field at link  $j$ . The mass parameter for these diagonal links is taken to be  $\epsilon_L M_F$  and  $\epsilon_R M_F$  for the left and right links respectively. All together, the masses of the fermions and the leading order interactions of the fermions and nonlinear sigma fields is given by

$$\mathcal{L}_{\psi\Sigma} = -M_F[\epsilon_L\bar{\psi}_{L0}\Sigma_0\psi_{R1} + \bar{\psi}_{L1}\psi_{R1} + \bar{\psi}_{L1}\epsilon_R\Sigma_1\psi_{R2}] \quad (48)$$

where  $\epsilon_L$  is the same for all fermions but  $\epsilon_R$  is a diagonal matrix which distinguishes flavors. For example, for the top and bottom quarks we have

$$\epsilon_R = \begin{pmatrix} \epsilon_{Rt} & 0 \\ 0 & \epsilon_{Rb} \end{pmatrix}. \quad (49)$$

The mass matrix can be obtained by going to unitary gauge and diagonalized by a biunitary transformation. By doing this, we find the following masses and wavefunctions

$$M_{f0} = \frac{M_F}{\sqrt{2}}\sqrt{1 + \epsilon_L^2 + \epsilon_R^2 - C} \quad (50)$$

$$v_{Lf0} = \frac{1}{N_{Lf0}} \left\{ -1, \frac{2\epsilon_L}{1 - \epsilon_L^2 + \epsilon_R^2 + C} \right\} \quad (51)$$

$$v_{Rf0} = \frac{1}{N_{Rf0}} \left\{ \frac{1 + \epsilon_L^2 - \epsilon_R^2 - C}{2\epsilon_R}, 1 \right\} \quad (52)$$

$$M_{f1} = \frac{M_F}{\sqrt{2}} \sqrt{1 + \epsilon_L^2 + \epsilon_R^2 + C} \quad (53)$$

$$v_{Lf1} = \frac{1}{N_{Lf1}} \left\{ \frac{1 - \epsilon_L^2 + \epsilon_R^2 - C}{2\epsilon_L}, -1 \right\} \quad (54)$$

$$v_{Rf1} = \frac{-1}{N_{Rf1}} \left\{ 1, \frac{2\epsilon_R}{1 + \epsilon_L^2 - \epsilon_R^2 + C} \right\} \quad (55)$$

where

$$C = \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2\epsilon_R^2} \quad (56)$$

Precision electroweak measurements can be satisfied at tree level in this theory. Custodial symmetry protects the value of  $T$  while  $S$  requires tuning of  $\epsilon_L$ . Although this theory accomodates a small  $S$  parameter, it does not explain it. The explanation lies in the UV completion.  $S$  can be made small by “ideally delocalizing” the left chiral fermions in the theory [?, ?, ?] by setting

$$g_i (v_L^i)^2 \propto v_W^i. \quad (57)$$

We take the ratio of this equation at sites 0 and 1 to cancel the normalization constant

$$\frac{g_0 (v_L^0)^2}{g_1 (v_L^1)^2} = \frac{v_W^0}{v_W^1}. \quad (58)$$

This relation gives

$$\epsilon_L^2 = \frac{2x^2}{2 - x^2 + \sqrt{4 + x^4}} \quad (59)$$

where we have dropped the small  $\epsilon_R$ .

Finally, the parameter  $\epsilon_{Rf}$  can be determined by taking the ratio of the masses of the light fermion and the massive partner.

$$R_{Mf}^2 = \frac{M_{f0}^2}{M_{f1}^2} = \frac{1 + \epsilon_L^2 + \epsilon_R^2 - \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2\epsilon_R^2}}{1 + \epsilon_L^2 + \epsilon_R^2 + \sqrt{(1 + \epsilon_L^2 + \epsilon_R^2)^2 - 4\epsilon_L^2\epsilon_R^2}} \quad (60)$$

from which we get

$$\epsilon_R^2 = \frac{\epsilon_L^2(1 + R_{Mf}^4) - 2R_{Mf}^2 - \epsilon_L(1 + R_{Mf}^2)\sqrt{\epsilon_L^2(1 - R_{Mf}^2)^2 - 4R_{Mf}^2}}{2R_{Mf}^2} \quad (61)$$

## 5 FEYNRULES implementation

The FEYNRULES implementation was initially based on a L<sup>A</sup>NHEP implementation. It was translated to FEYNRULES syntax and slightly modified to fit the requirements of the FEYNRULES package and interfaces. All symmetries, fields and parameters were implemented according to the definitions of the last section. The independent variables that the user can adjust are the electromagnetic and strong couplings, the masses of the  $Z$ ,  $W$ ,  $W'$  and SM fermions (where not set explicitly to 0) and the scale of the heavy fermions  $M_F$ . The scale of the  $Z$  pole ( $Z_M$ ) and Fermi constant ( $G_f$ ) are also implemented as required by some Monte Carlo codes, but they are not used directly in this model. A change in these last two parameters will not affect the value of the other parameters in this model.

Two gauges were implemented in this model file. A MATHEMATICA variable `FeynmanGauge` was created to switch between the two. When the switch is set to `True`, Feynman gauge is chosen and the Lagrangians contain the Goldstone bosons eaten by the gauge bosons and the ghosts. If, on the other hand, it is set to `False`, all Goldstone and ghost terms are set to zero in the Lagrangian.