

Supersymmetry with FEYNRULES

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Outline.

- 1 Why a superspace module in FEYNRULES?
- 2 Surfing in the superspace.
- 3 Supersymmetric Lagrangians.
- 4 Using the superspace module: supersymmetric models implementation.
- 5 Future (s)plan(ino)s in FEYNRULES
- 6 Summary.

Fields and superfields (1).

● Supported fields.

- * Scalar fields.
- * Weyl, Dirac and Majorana fermions.
- * Vector (and ghost) fields.
- * **Spin 3/2, being currently developed** (*).
- * Spin two fields (*).

Is this relevant / enough for the implementation of supersymmetric theories.

Yes, but ... let us investigate two short examples.

(*) not considered here, but might be relevant for supergravity or gauge-mediated supersymmetry breaking.

Fields and superfields (2).

- **Kinetic terms and gauge interactions.**

- * Terribly expressed in terms of **components fields**: *i.e.*, scalars, Dirac and Majorana fermions, vector fields (**13 terms**):

$$\mathcal{L}_{\text{kin}} \supset \dots \quad [\text{Censored: too ugly to appear on a slide}].$$

- * Not very nicely expressed in terms of **components fields**, *i. e.* scalars, Weyl fermions, vector fields (**13 terms**):

$$\begin{aligned} \mathcal{L}_{\text{kin}} \supset & D_\mu \tilde{Q}_i^\dagger D^\mu \tilde{Q}^i + \frac{i}{2} (\chi_Q^i \sigma^\mu D_\mu \bar{\chi}_{Qi} - D_\mu \chi_Q^i \sigma^\mu \bar{\chi}_{Qi}) + F_Q^\dagger F_Q \\ & + i\sqrt{2} \left[\frac{1}{6} g' \tilde{Q}^i \tilde{B} \cdot \bar{\chi}_{Qi} + g \overline{\tilde{W}}^k \cdot \bar{\chi}_{Qi} \frac{\sigma^k}{2} \tilde{Q}^i + g_s \overline{\tilde{G}}^a \cdot \bar{\chi}_{Qi} \frac{T^a}{2} \tilde{Q}^i + \text{h. c.} \right] \\ & - g' D_B \tilde{Q}_i^\dagger \tilde{Q}^i - g D_{W^k} \tilde{Q}_i^\dagger \frac{\sigma^k}{2} \tilde{Q}^i - g_s D_{G^a} \tilde{Q}_i^\dagger \frac{T^a}{2} \tilde{Q}^i \end{aligned}$$

- * Naturally expressed in terms of **superfields (1 terms)**:

$$\mathcal{L}_{\text{kin}} \supset \left[Q_i^\dagger e^{-2\frac{1}{6}g' \mathbf{V}_B} e^{-2g \mathbf{V}_{W^k} \frac{\sigma^k}{2}} e^{-2g_s \mathbf{V}_{G^a} \frac{T^a}{2}} Q^i \right] \Big|_{\theta \cdot \theta \bar{\theta} \cdot \bar{\theta}}$$

Fields and superfields (3).

● Kinetic terms and gauge interactions.

- * Not very nicely expressed in terms of **components fields**,
i. e. scalars, Weyl fermions, vector fields (**13 terms**):

$$\begin{aligned} \mathcal{L}_{\text{kin}} \supset & D_\mu \tilde{Q}_i^\dagger D^\mu \tilde{Q}^i + \frac{i}{2} (\chi_Q^j \sigma^\mu D_\mu \bar{\chi}_{Qj} - D_\mu \chi_Q^j \sigma^\mu \bar{\chi}_{Qj}) + F_Q^\dagger F_Q \\ & + i\sqrt{2} \left[\frac{1}{6} g' \tilde{Q}^i \tilde{B} \cdot \bar{\chi}_{Qj} + g \overline{\tilde{W}}^k \cdot \bar{\chi}_{Qj} \frac{\sigma^k}{2} \tilde{Q}^i + g_s \overline{\tilde{G}}^a \cdot \bar{\chi}_{Qj} \frac{T^a}{2} \tilde{Q}^i + \text{h. c.} \right] \\ & - g' D_B \tilde{Q}_i^\dagger \tilde{Q}^i - g D_{W^k} \tilde{Q}_i^\dagger \frac{\sigma^k}{2} \tilde{Q}^i - g_s D_{G^a} \tilde{Q}_i^\dagger \frac{T^a}{2} \tilde{Q}^i \end{aligned}$$

- * Are all **relative signs and factors of i** correct (especially in the non-gauge-like interactions)?
- * **Four-component fermions...** (They are a pain, but required for MCs).
- * **The superfield formalism is more convenient...**

$$\mathcal{L}_{\text{kin}} \supset \left[Q_i^\dagger e^{-2\frac{1}{6}g'V_B} e^{-2gV_{W^k} \frac{\sigma^k}{2}} e^{-2g_s V_{G^a} \frac{T^a}{2}} Q^i \right] \Big|_{\theta \cdot \theta \bar{\theta} \cdot \bar{\theta}}$$

Motivation and plans.

Motivation for the superspace module in FEYNRULES

- * **Natural** to implement any supersymmetric theory.
 - * **Zero probability** to introduce wrong signs, i factors,...
 - * Could be a **useful tool** for model building.
(not only a Lagrangian translator).
 - * **Convenient** for many possible extensions (RGEs, ...).
-
- * **Available and validated!**
vs. an exercise textbook [BenjF, Rausch de Traubenberg (Ed. Ellipse, 2011)].
 - * Scheduled extensions (2011).
Automated spectrum generator [Alloul, BenjF, Rausch de Traubenberg].
Gravitino/Goldstino in FEYNRULES [Les Houches'11].

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Superspace.

- **Superspace: adapted space to write down SUSY transformations naturally.**
- **Basic objects and their FEYNRULES (hardcoded) implementation.**
 - * The **Majorana spinor** $(\theta, \bar{\theta}) \Rightarrow$ a superspace point $\equiv G(x, \theta, \bar{\theta})$.
 - * Transformation parameters, the **Majorana spinors** $(\varepsilon_1, \bar{\varepsilon}_1), (\varepsilon_2, \bar{\varepsilon}_2), \dots$

```
W[1000] == {
  ClassName      -> theta,
  Chirality      -> Left,
  SelfConjugate  -> False}
```

```
W[2000] == {
  ClassName      -> eps1,
  Chirality      -> Left,
  SelfConjugate  -> False}
```

- * **The supercharges (Q, \bar{Q}) :** action to the left $\equiv G(0, \varepsilon, \bar{\varepsilon})G(x, \theta, \bar{\theta})$.
- * **The superderivatives (D, \bar{D}) :** action to the right $\equiv G(x, \theta, \bar{\theta})G(0, \varepsilon, \bar{\varepsilon})$.

$$Q_\alpha = -i(\partial_\alpha + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu) \quad \text{and} \quad \bar{Q}_{\dot{\alpha}} = i(\bar{\partial}_{\dot{\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu),$$

$$D_\alpha = \partial_\alpha - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \quad \text{and} \quad \bar{D}_{\dot{\alpha}} = \bar{\partial}_{\dot{\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu.$$

$Q_\alpha(\text{exp})$ and $\bar{Q}_{\dot{\alpha}}(\text{exp})$

```
QSUSY [exp_, alpha_]
QSUSYBar [exp_, alphasdot_]
```

$D_\alpha(\text{exp})$ and $\bar{D}_{\dot{\alpha}}(\text{exp})$

```
DSUSY [exp_, alpha_]
DSUSYBar [exp_, alphasdot_]
```


Superfields: chiral superfields (1).

- **Most general expansion in the $\theta, \bar{\theta}$ variables satisfying $\bar{D}_{\dot{\alpha}} \Phi(x, \theta, \bar{\theta}) = 0$.**

$$\Phi(x, \theta, \bar{\theta}) = \phi(y) + \sqrt{2}\theta \cdot \psi(y) - \theta \cdot \theta F(y) \text{ where } y^{\mu} = x^{\mu} - i\theta\sigma^{\mu}\bar{\theta} .$$

- * Describes **matter multiplet**.
- * One scalar field ϕ , one Weyl fermion χ , one auxiliary field F .

Chiral superfield - up-type Higgs doublet

```
CSF[1] == {
  ClassName      -> HU,
  Chirality      -> Left,
  Weyl           -> huw,
  Scalar         -> hus,
  QuantumNumbers -> {Y->1/2},
  Indices        -> {Index[SU2D]},
  FlavorIndex    -> SU2D}
```

- * The scalar and Weyl fermionic fields must be declared properly.
- * **The auxiliary field will be automatically generated, if not present.**

Superfields: chiral superfields (2).

- **Expansion in superspace with FEYNRULES:** $\Phi(y, \theta) = \phi(y) + \sqrt{2} \theta \cdot \psi(y) - \theta \cdot \theta F(y)$.

```
In[7]:= GrassmannExpand[HU]
```

$$\text{Out[7]} = \text{hus} + \sqrt{2} \theta_{sp\$1} \cdot \text{huw}_{sp\$1} - \text{FTerm4} \theta_{sp\$1} \cdot \theta_{sp\$1} - \frac{1}{4} \partial_{\mu\$1} [\partial_{\mu\$1} [\text{hus}]] \theta_{sp\$1} \cdot \theta_{sp\$1} \bar{\theta}_{sp\$1\text{dot}} \cdot \bar{\theta}_{sp\$1\text{dot}} -$$

$$i \partial_{\mu\$1} [\text{hus}] \theta_{sp\$1} \cdot \bar{\theta}_{sp\$1\text{dot}} (\sigma^{\mu\$1})_{sp\$1, sp\$1\text{dot}} + \frac{i \partial_{\mu\$1} [\text{huw}_{sp\$1}] \cdot \bar{\theta}_{sp\$1\text{dot}} \theta_{sp\$2} \cdot \theta_{sp\$2} (\sigma^{\mu\$1})_{sp\$1, sp\$1\text{dot}}}{\sqrt{2}}$$

- * FTerm4 was automatically generated.
- * Automatic y -expansion.
- * Straightforward extraction of the coefficients in θ .

```
In[10]:= GetScalarComponent[HU]
```

```
Out[10]= hus
```

```
In[9]:= GetThetaComponent[HU]
```

```
Out[9]= \sqrt{2} huw_{alpha\$2383}
```

```
In[8]:= GetTheta2Component[HU]
```

```
Out[8]= -FTerm4
```

Superfields: chiral superfields (3).

● SUSY transformation laws:

* In terms of **superfields**: $\delta_\varepsilon \Phi(x, \theta, \bar{\theta}) = i(\varepsilon \cdot Q + \bar{Q} \cdot \bar{\varepsilon}) \cdot \Phi(x, \theta, \bar{\theta})$.

* In terms of **component fields** (depending on y , not x):

$$\delta_\varepsilon \phi = \sqrt{2} \varepsilon \cdot \psi, \quad \delta_\varepsilon \psi = -i\sqrt{2} \sigma^\mu \bar{\varepsilon} \partial_\mu \phi - \sqrt{2} F \varepsilon, \quad \delta_\varepsilon F = -i\sqrt{2} \partial_\mu \psi \sigma^\mu \bar{\varepsilon}.$$

* With FEYNRULES:

```
In[15]= GetScalarComponent[ToNC[DeltaSUSY[HU, eps1]]]
Out[15]=  $\sqrt{2}$  huwsp$1.eps1sp$1

In[20]= Expand[GetThetaComponent[ToNC[DeltaSUSY[HU, eps1]]] / Sqrt[2]]
Out[20]=  $-\sqrt{2}$  FTerm4 eps1alpha$10235 - i  $\sqrt{2}$   $\partial_{\mu$1}$ [hus] eps1sp$1dot1 ( $\sigma^{\mu$1}$ )alpha$10235, sp$1dot

In[19]= -GetTheta2Component[ToNC[DeltaSUSY[HU, eps1]]]
Out[19]= -i  $\sqrt{2}$   $\partial_{\mu$1}$ [huwsp$1].eps1sp$1dot1 ( $\sigma^{\mu$1}$ )sp$1, sp$1dot
```

* ToNC breaks dot products and the NC structure keeps fermion ordering.

Superfields: vector superfields (1).

- Expansion in the $\theta, \bar{\theta}$ variables satisfying $\Phi = \Phi^\dagger$ in the Wess-Zumino gauge.

$$\Phi_{W.Z.}(x, \theta, \bar{\theta}) = \theta\sigma^\mu\bar{\theta}v_\mu + i\theta\cdot\theta\bar{\theta}\cdot\bar{\lambda} - i\bar{\theta}\cdot\bar{\theta}\theta\cdot\lambda + \frac{1}{2}\theta\cdot\theta\bar{\theta}\cdot\bar{\theta}D.$$

- * Describes **gauge supermultiplets**.
- * One Majorana fermion $(\lambda, \bar{\lambda})$, one gauge boson v , one auxiliary field D .

Vector superfield for $SU(2)_L$

```
VSF[1] == {
  ClassName      -> WSF,
  GaugeBoson     -> Wi,
  Gaugino        -> wow,
  Indices        -> {Index[SU2W]},
  FlavorIndex    -> SU2W}
```

Associated gauge group

```
SU2L == {
  Abelian          -> False,
  CouplingConstant -> gw,
  SF               -> WSF,
  StructureConstant -> ep,
  Representations  -> {...},
  Definitions      -> {...}}
```

- * The Weyl fermionic and vectorial fields must be declared properly.
- * **The auxiliary field will be automatically generated, if not present.**
- * Vector superfields can be **associated to a gauge group**.

Superfields: vector superfields (2).

- **Properties of vector superfields:**

$$\Phi_{W.Z.}^2 = \frac{1}{2} \theta \cdot \theta \bar{\theta} \cdot \bar{\theta} v^\mu v_\mu, \quad \Phi_{W.Z.}^3 = 0.$$

```
In[24]:= GrassmannExpand[WSF[aa] WSF[bb]]
Out[24]=  $\frac{1}{2} \theta_{sp\$1} \cdot \theta_{sp\$1} \bar{\theta}_{sp\$\dot{1}} \cdot \bar{\theta}_{sp\$\dot{1}} Wi_{mu\$1,aa} Wi_{mu\$1,b}$ 
In[25]:= GrassmannExpand[WSF[aa] WSF[bb] WSF[cc]]
Out[25]= 0
```

- **The superfield strength tensor is built from associated spinorial superfields:**

$$W_\alpha = -\frac{1}{4} \bar{D} \cdot \bar{D} e^{2gV} D_\alpha e^{-2gV}, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4} D \cdot D e^{-2gV} \bar{D}_{\dot{\alpha}} e^{2gV}.$$

$W_\alpha, (W_\alpha)_{ij}, W_\alpha^a, \bar{W}_{\dot{\alpha}}, \bar{W}_{\dot{\alpha}}^a, (\bar{W}_{\dot{\alpha}})_{ij}$

```
SuperfieldStrengthL[ SF, lower spin index ]
SuperfieldStrengthL[ SF, spin index, gauge index/indices ]
SuperfieldStrengthR[ SF, lower spin index ]
SuperfieldStrengthR[ SF, spin index, gauge index/indices ]
```

Superfields: more general stuff.

- **Ex. 1: most general, reducible, expansion in the $\theta, \bar{\theta}$ variables:**

$$\Phi(x, \theta, \bar{\theta}) = z(x) + \theta \cdot \xi(x) + \bar{\theta} \cdot \bar{\zeta}(x) + \theta \cdot \theta f(x) + \bar{\theta} \cdot \bar{\theta} g(x) + \theta \sigma^\mu \bar{\theta} v_\mu(x) + \bar{\theta} \cdot \bar{\theta} \theta \cdot \omega(x) + \theta \cdot \theta \bar{\theta} \cdot \bar{\rho}(x) + \theta \cdot \theta \bar{\theta} \cdot \bar{\theta} d(x).$$

scalars: z, f, g, d , Weyl fermions: ξ, ζ, ω, ρ , vector: v .

- * **Can be added easily**, as any expression in superspace.

`z + NC[theta[sp], xi[sp2]] Ueps[sp2, sp] + ...`

- **Ex. 2: the most general Kähler potential** [BenjF, Rausch de Traubenberg (in prep)].

$$\begin{aligned} \mathcal{K} = & K(\phi, \phi^\dagger) + \sqrt{2}\theta \cdot [K_i \psi^i] + \sqrt{2}\bar{\theta} \cdot [K^{\dagger i} \bar{\psi}_i] - \theta \cdot \theta [K_i F^i + \frac{1}{2} K_{ij} \psi^i \cdot \psi^j] - \bar{\theta} \cdot \bar{\theta} [K^{\dagger i} F_i^\dagger + \frac{1}{2} K^{\dagger ij} \bar{\psi}_i \cdot \bar{\psi}_j] + \\ & \theta \sigma^\mu \bar{\theta} [i K^{\dagger i} D_\mu \phi_i^\dagger - i K_i D_\mu \phi^i + K^{\dagger i} \psi^i \sigma_\mu \bar{\psi}_i] + \\ & \theta \cdot \theta \bar{\theta} \cdot [-\sqrt{2} K^{\dagger i} F_i^\dagger \bar{\psi}_i - \frac{1}{\sqrt{2}} K^{\dagger k} \Gamma_i^k \psi^i \cdot \psi^j \bar{\psi}_i + \frac{i}{\sqrt{2}} K^{\dagger i} \sigma^\mu \psi^i D_\mu \phi_i^\dagger - \\ & \quad \frac{i}{\sqrt{2}} \bar{\sigma}^\mu (K_i D_\mu + D_i K_j D_\mu \phi^j) \psi^i - ig K^{\dagger i} \lambda^a (\phi^\dagger T_a)_i - ig K_i \lambda^a (T_a \phi)^i] + \\ & \bar{\theta} \cdot \bar{\theta} \theta \cdot [-\sqrt{2} K^{\dagger i} F_i^\dagger \psi^i - \frac{1}{\sqrt{2}} K^{\dagger i} \Gamma_i^k \psi^i \cdot \bar{\psi}_j \cdot \psi^j + \frac{i}{\sqrt{2}} K^{\dagger i} \sigma^\mu \bar{\psi}_i D_\mu \phi_i^\dagger - \\ & \quad \frac{i}{\sqrt{2}} \bar{\sigma}^\mu (K^{\dagger i} D_\mu + D^{\dagger i} K^j D_\mu \phi_j^\dagger) \bar{\psi}_i + ig K^{\dagger i} \lambda^a (\phi^\dagger T_a)_i + ig K_i \lambda^a (T_a \phi)^i] + \\ & \theta \cdot \theta \bar{\theta} \cdot \bar{\theta} [-\frac{1}{4} \partial_\mu (K_i D_\mu \phi^i + K^{\dagger i} D_\mu \phi_i^\dagger) + K^{\dagger j} D_\mu \phi^i D_\mu \phi_j^\dagger + K^{\dagger i} F^i F_i^\dagger + \frac{1}{4} K^{\dagger ij} \psi^i \cdot \psi^j \bar{\psi}_i \cdot \bar{\psi}_j + \\ & \quad \frac{i}{2} (K^{\dagger i} \psi^i \sigma^\mu D_\mu \bar{\psi}_i - K^{\dagger i} D_\mu \psi^i \sigma^\mu \bar{\psi}_i) + \frac{1}{2} K^{\dagger i} \Gamma_i^j \psi^i \cdot \psi^k F^j \bar{\psi}_k + \frac{1}{2} K^{\dagger i} \Gamma_j^i \psi^j \cdot \psi^k F^i \bar{\psi}_k - \\ & \quad - \frac{g}{2} D^a (\phi_i^\dagger T_a^i \psi^i + K^{\dagger j} T_a^j \phi^j) - ig \sqrt{2} (\phi^\dagger T_a)_i K^{\dagger i} \psi^i \cdot \lambda^a + ig \sqrt{2} \lambda^a \cdot \bar{\psi}_i K^{\dagger i} (T_a \phi)^i], \end{aligned}$$

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Vector Lagrangians (1).

- **Lagrangian associated to the vector superfield content of the theory.**

- * Contains gauge interactions and kinetic terms for **vector superfields**.
- * Is entirely **fixed by SUSY and gauge invariance**
- * **Abelian groups**.

$$\begin{aligned}\mathcal{L} &= \frac{1}{4} W^\alpha W_\alpha|_{\theta\theta} + \frac{1}{4} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}} \\ &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda + \frac{1}{2} D^2.\end{aligned}$$

- * **Non-abelian groups**.

$$\begin{aligned}\mathcal{L} &= \frac{1}{16g^2\tau_{\mathcal{R}}} \text{Tr}(W^\alpha W_\alpha)|_{\theta\theta} + \frac{1}{16g^2\tau_{\mathcal{R}}} \text{Tr}(\bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}})|_{\bar{\theta}\bar{\theta}} \\ &= -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + i\bar{\lambda}_a\bar{\sigma}^\mu D_\mu\lambda^a + \frac{1}{2} D_a D^a\end{aligned}$$

- **Automatic extraction of the vector Lagrangian of a model:**

```
GetVSFKineticTerms []
GetVSFKineticTerms [ WSF ]
```


Vector Lagrangians (2).

- **Example: Abelian superfield strengths:**

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\lambda}\bar{\sigma}^{\mu}\partial_{\mu}\lambda + \frac{1}{2}D^2.$$

```

GetTheta2Component[GetVSFKineticTerms[BSP]] +
GetThetabar2Component[GetVSFKineticTerms[BSP]]

Out[7]=  $\frac{D\text{Term}^2}{2} - \frac{1}{2}\partial_{\mu\$2}[B_{\mu\$1}]^2 + \frac{1}{2}\partial_{\mu\$2}[B_{\mu\$1}]\partial_{\mu\$1}[B_{\mu\$2}] +$ 
 $\frac{1}{2}i\text{bow}_{sp\$1}.\partial_{\mu\$1}[\text{bow}^{\dagger}_{sp\$1\text{dot}}](\sigma^{\mu\$1})_{sp\$1,sp\$1\text{dot}} - \frac{1}{2}i\partial_{\mu\$1}[\text{bow}_{sp\$1}].\text{bow}^{\dagger}_{sp\$1\text{dot}}(\sigma^{\mu\$1})_{sp\$1,sp\$1\text{dot}}$ 

```

- **The Minimal Supersymmetric Standard Model.**

Vector Lagrangian for the MSSM

```

In[8]:= GetVSFKineticTerms[]

Out[8]=  $\frac{1}{4}\text{GetVSFKineticTerms}[BSP, \text{True}] +$ 
 $\frac{\text{GetVSFKineticTerms}[GSF, \text{False}]}{16g_s^2} + \frac{\text{GetVSFKineticTerms}[WSF, \text{False}]}{16g_w^2}$ 

```

Matter Lagrangians (1).

- **Lagrangian associated to the chiral superfield content of the theory.**
 - * Contains gauge interactions and kinetic terms for **chiral superfields**.
 - * Is entirely **fixed by SUSY and gauge invariance**
 - * **Example for $SU(3)_c \times SU(2)_L \times U(1)_Y$.**

$$\mathcal{L} = \left[\Phi^\dagger(x, \theta, \bar{\theta}) e^{-2y_\Phi g' \mathbf{V}_B} e^{-2g \mathbf{V}_W} e^{-2g_s \mathbf{V}_G} \Phi(x, \theta, \bar{\theta}) \right] \Big|_{\theta \cdot \theta \bar{\theta} \cdot \bar{\theta}}$$

(Non-abelian vector superfields contains group representation matrices.)

- **Automatic extraction of the matter Lagrangian of a model:**

```
GetCSFKineticTerms []
GetCSFKineticTerms [ ER ]
```

Matter Lagrangians (2).

- **Right electron kinetic Lagrangian:**

$$\mathcal{L} = D_\mu \phi^\dagger D^\mu \phi - \frac{i}{2} (D_\mu \bar{\psi} \bar{\sigma}^\mu \psi - \bar{\psi} \bar{\sigma}^\mu D_\mu \psi) + i\sqrt{2}g\tilde{\lambda}^a \cdot \bar{\psi} T_a \phi - i\sqrt{2}g\phi^\dagger T_a \psi \cdot \lambda^a + FF^\dagger - gD^a \phi^\dagger T^a \phi .$$

```
In[10]:= GetTheta2Thetabar2Component[GetCSFKineticTerms[ER]]
Out[10]=  $\frac{1}{2} \partial_{\mu\$\{1\}} [ER_{\text{GEN}\$\{1\}}] \partial_{\mu\$\{1\}} [ER_{\text{GEN}\$\{1\}}^\dagger] - \frac{1}{4} \partial_{\mu\$\{1\}} [\partial_{\mu\$\{1\}} [ER_{\text{GEN}\$\{1\}}^\dagger]] ER_{\text{GEN}\$\{1\}} -$ 
 $i g' B_{\mu\$\{1\}} \partial_{\mu\$\{1\}} [ER_{\text{GEN}\$\{1\}}^\dagger] ER_{\text{GEN}\$\{1\}} + i \sqrt{2} g' \text{bow}_{\text{sp}\$\{1\}\text{dot}, \text{GEN}\$\{1\}}^\dagger \cdot ER_{\text{sp}\$\{1\}\text{dot}, \text{GEN}\$\{1\}}^\dagger ER_{\text{GEN}\$\{1\}} -$ 
 $\frac{1}{4} \partial_{\mu\$\{1\}} [\partial_{\mu\$\{1\}} [ER_{\text{GEN}\$\{1\}}]] ER_{\text{GEN}\$\{1\}}^\dagger + i g' B_{\mu\$\{1\}} \partial_{\mu\$\{1\}} [ER_{\text{GEN}\$\{1\}}] ER_{\text{GEN}\$\{1\}}^\dagger -$ 
 $i \sqrt{2} g' ER_{\text{sp}\$\{1\}, \text{GEN}\$\{1\}} \cdot \text{bow}_{\text{sp}\$\{1\}} ER_{\text{GEN}\$\{1\}}^\dagger - D\text{Term}1 g' ER_{\text{GEN}\$\{1\}} ER_{\text{GEN}\$\{1\}}^\dagger + (g')^2 B_{\mu\$\{1\}}^2 ER_{\text{GEN}\$\{1\}} ER_{\text{GEN}\$\{1\}}^\dagger +$ 
 $F\text{Term}7_{\text{GEN}\$\{1\}} F\text{Term}7_{\text{GEN}\$\{1\}}^\dagger - \frac{1}{2} i \partial_{\mu\$\{1\}} [ER_{\text{sp}\$\{1\}, \text{GEN}\$\{1\}}] \cdot ER_{\text{sp}\$\{1\}\text{dot}, \text{GEN}\$\{1\}}^\dagger (\sigma^{\mu\$\{1\}})_{\text{sp}\$\{1\}, \text{sp}\$\{1\}\text{dot}} +$ 
 $\frac{1}{2} i ER_{\text{sp}\$\{1\}, \text{GEN}\$\{1\}} \cdot \partial_{\mu\$\{1\}} [ER_{\text{sp}\$\{1\}\text{dot}, \text{GEN}\$\{1\}}^\dagger] (\sigma^{\mu\$\{1\}})_{\text{sp}\$\{1\}, \text{sp}\$\{1\}\text{dot}} -$ 
 $g' B_{\mu\$\{1\}} ER_{\text{sp}\$\{1\}, \text{GEN}\$\{1\}} \cdot ER_{\text{sp}\$\{1\}\text{dot}, \text{GEN}\$\{1\}}^\dagger (\sigma^{\mu\$\{1\}})_{\text{sp}\$\{1\}, \text{sp}\$\{1\}\text{dot}}$ 
```

- **The Minimal Supersymmetric Standard Model.**

Matter Lagrangian for the MSSM

```
In[11]:= GetCSFKineticTerms[]
Out[11]:= GetCSFKineticTerms[DR] + GetCSFKineticTerms[ER] +
GetCSFKineticTerms[HD] + GetCSFKineticTerms[HU] + GetCSFKineticTerms[LL] +
GetCSFKineticTerms[QL] + GetCSFKineticTerms[UR] + GetCSFKineticTerms[VR]
```

Full SUSY Lagrangian (1).

- Complete Lagrangian for a model.

$$\mathcal{L} = \Phi^\dagger e^{-2gV} \Phi|_{\theta^2\bar{\theta}^2} + \frac{1}{16g^2\tau_{\mathcal{R}}} \text{Tr}(\mathbf{W}^\alpha \mathbf{W}_\alpha)|_{\theta^2} + \frac{1}{16g^2\tau_{\mathcal{R}}} \text{Tr}(\bar{\mathbf{W}}_{\dot{\alpha}} \bar{\mathbf{W}}^{\dot{\alpha}})|_{\bar{\theta}^2} \\ + \mathbf{W}(\Phi)|_{\theta^2} + \mathbf{W}^*(\Phi^\dagger)|_{\bar{\theta}^2} + \mathcal{L}_{\text{soft}}$$

- * **Chiral superfield** kinetic terms: automatic.
- * **Vector superfield** kinetic terms: automatic.
- * **Superpotential**: model dependent.
- * **Soft SUSY-breaking Lagrangian**: model dependent (and often not related to the superspace).

Any FEYNRULES implementation of a SUSY Lagrangian

```
GetTheta2Thetabar2Component[ GetCSFKineticTerms[] ] +
GetTheta2Component[ GetVSFKineticTerms[] + SuperPot ] +
GetThetabar2Component[ GetVSFKineticTerms[] + HC[SuperPot] ] +
LSoft
```

- * LSoft and SuperPot are the **only** pieces provided by the user.

Full SUSY Lagrangian (2).

- **Solution of the equation of motions.**

- * Get rid of the auxiliary D -fields and F -fields.

Equations of motion

```
lagr = SolveEqMotionD[ lagr ] ;  
lagr = SolveEqMotionF[ lagr ] ;
```

- **Back to four-component fermions.**

- * Usual FEYNRULES routine.

Four-component fermions

```
lagr = WeylToDirac[ lagr ] ;
```

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FEYNRULES supersymmetric model database (1).

- <http://feynrules.phys.ucl.ac.be/wiki/SusyModels>.

The screenshot shows a Mozilla Firefox browser window titled "SusyModels - FeynRules - Mozilla Firefox". The address bar contains the URL "http://feynrules.phys.ucl.ac.be/wiki/SusyModels". The page content is titled "FeynRules model database: Supersymmetric models".

This page contains a collection of supersymmetric models that are already implemented in FeynRules. For each model, a complete model-file is available, containing all the information that is needed, as well as the Lagrangian, as well as the references to the papers where this Lagrangian was taken from. All model-files can be freely downloaded and changed, serving like this as the starting point for building new models. A TeX-file for each model containing a summary of the Feynman Rules produced by FeynRules is also available.

Model	Short Description	Contact	Status
MSSM	The Minimal Supersymmetric extension of the SM.	B. Fuks	Available
NMSSM	The Next-to-Minimal Supersymmetric Standard Model.	B. Fuks	Available
RPV-MSSM	The Minimal Supersymmetric extension of the SM including R-parity violation (trilinear RPV interactions only).	B. Fuks	Available
R-MSSM	A R-symmetric supersymmetric extension of the SM.	B. Fuks	Soon available

Back to the [FeynRules model database](#).

Done

FEYNRULES supersymmetric model database (2).

- <http://feynrules.phys.ucl.ac.be/wiki/SusyModels>.
- **Most general possible versions of the models.**
 - * Any simpler limit easily taken.
 - * All SUSY CP phases included.
 - * All possible additional flavor violation sources included.
 - * **Simple extension requiring a generalized MSSM: the MSSM-CKM.**
- **Model parameters.**
 - * Follow the **SLHA conventions** (if existing).
- **Available on the web for each model.**
 - * A parameter file for one **specific benchmark scenario**.
 - * **All Monte Carlo model files** to be downloaded for that scenario.

FEYNRULES supersymmetric model database (3).

● The MSSM.

- * **Validated old implementation** in component fields.
[Christensen, de Aquino, Degrande, Duhr, BenjF, Herquet, Maltoni, Schumann (2011)]
- * **Public**: validated against the old implementation [Duhr, BenjF (in press)].

```
FeynmanRules[new-old] = {};
```

- * **Private**: (N)MFV benchmark points [Alwall, Duhr, BenjF (in prep.)].

● The NMSSM

- * **Validated old implementation** in components. [Braam, BenjF, Reuter @ LH'09]
- * **Public**: validated against the old implementation.

● The RPV MSSM

- * **Public**: only contains the trilinear RPV interactions.
- * Used and **under validation** by CMS-Strasbourg [Agram, Andrea, Conte, BenjF].

● The R-symmetric MSSM.

- * **Private**: not validated, no benchmark.
- * In progress [Alwall, Duhr, BenjF (in prep.)]

FEYNRULES supersymmetric model database (4).

Is any model missing?

- * Contact us for help with the implementation!
- * Any model can be made public on the FEYNRULES website.

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Automated spectrum generator (1) [Alloul, BenjF, Rausch de Traubenberg].

- **SUSY Lagrangians at low energy.**

- * Contains a **huge number** of free parameters.
- * Example: 105 for the MSSM.
- * Loss of **predictivity**.

- **SUSY Lagrangians at high energy.**

- * Assumption: **universal boundary conditions**.
⇒ A reduced number of free parameters.
- * Must be **evolved to low energy**.
- * Examples: gauge coupling constant and Yukawa interaction.

$$Q \frac{dg}{dQ} = - \frac{g^3}{16\pi^2} [3C(G) - T(R)] ,$$
$$Q \frac{df_{abc}}{dQ} = - \frac{1}{32\pi^2} [4g^2[C(A) + C(B) + C(C)]f_{abc} - (X_a^{a'} f_{a'bc} + X_b^{b'} f_{ab'c} + X_c^{c'} f_{abc'})]$$

- **The RGEs can be extracted automatically for any model.**

Automated spectrum generator (2) [Alloul, BenjF, Rausch de Traubenberg].

- **Done: analytical extraction of the RGEs and numerical checks.**

- * Independent MATHEMATICA routine: INSURGE.
- * **Validated on** SPS1a and on the left-right symmetric MSSM.
- * **Generic** \Rightarrow easily linked to any SUSY-breaking (mSUGRA, GSMB, ...).

Top quark Yukawa: INSURGE output

$$\frac{d f_{\{Q,AQU,Hu\}}}{dt} = - \frac{G[1, 3]^2 f_{\{Q,AQU,Hu\}}}{3 \pi^2} - \frac{3 G[2, 2]^2 f_{\{Q,AQU,Hu\}}}{16 \pi^2} - \frac{13 G[3, 1]^2 f_{\{Q,AQU,Hu\}}}{144 \pi^2} + \frac{f_{\{Q,AQD,Hd\}} f_{\{Q,AQU,Hu\}} f_{\{Q,AQD,Hd\}}}{16 \pi^2} + \frac{3 f_{\{Q,AQU,Hu\}}^2 f_{\{Q,AQU,Hu\}}}{8 \pi^2}$$

- **To-do list.**

- * Full embedding in FEYNRULES (@ LesHouches'11).
- * Automated extraction of the mass spectrum and its diagonalization.
- * **GOAL:**
Automated spectrum generator for any SUSY model (@ one-loop).

Other (on-going) SUSY perspectives with FEYNRULES.

- **Implementation of the gravitation supermultiplet.**

- * Spin3/2 in four- and two-component notations.
- * Lagrangian.
- * Auxiliary fields.

- **Extraction of the Goldstino Lagrangian.**

- * Conserved supercurrent.
- * SUSY transformation law using the conserved charge.

- **SUSY @ NLO.**

- * Automated renormalization.
- * Counterterms.
- * Use of FEYNARTS.

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Summary.

- **Superfield module of FEYNRULES.**

- * Achieved and validated.
- * More than just a Lagrangian translator.
- * Validated against textbook results.

- **Models.**

- * **All main models are public (MSSM, NMSSM, RPV).**
- * Other implementations are ongoing (RMSSM, left-right SUSY, SUSY see-saw, ...).

Is your favorite model missing? Please shout!

- **Important on-going projects**

- * Automatic renormalization for any SUSY theory.
- * RGEs and automated spectrum generator.
- * Gravitation supermultiplet.
- * **Physics** (new and original models \Leftrightarrow unusual phenomenology).