Supersymmetry with FeynRules
From model building to ... MadGraph

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Outline.

Fields and superfields (1).

- **Supported fields**.
  * Scalar fields.
  * Dirac and Majorana fermions [**Weyl fermions privately available**].
  * Vector (and ghost) fields.
  * **No spin 3/2** (*).
  * Spin two fields (*).

Is this relevant / enough for the implementation of supersymmetric theories.

Yes, but ... let us investigate two short examples.

(*) not considered to date, but might be relevant for supergravity or gauge-mediated supersymmetry breaking.

Fields and superfields (2).

- **Superpotential**.

  * Terribly expressed in terms of **components fields**, i. e. scalars, Dirac and Majorana fermions, vector fields (**12 terms)**:

$$\mathcal{L}_{\rm W} \supset (\mathbf{y^e})_{ij} \Big[\widetilde{E}_R{}^i \widetilde{L}^j \cdot F_{H_D} + \widetilde{E}_R{}^i H_D \cdot F_L{}^j + \widetilde{L}^j \cdot H_D F_E^i + \widetilde{E}_R{}^i (\bar\psi_L^{cj} P_L \psi_{H_D}) + \widetilde{L}^j \cdot (\bar\psi_{H_D} P_L \psi_e{}^i) + (\bar\psi_e{}^i P_L \psi_L{}^j) \cdot H_D \Big]$$

  * Not very nicely expressed in terms of **components fields**, i. e. scalars, Weyl fermions, vector fields (**12 terms)**:

$$\mathcal{L}_{\rm W} \supset (\mathbf{y^e})_{ij} \Big[\widetilde{E}_R{}^i \widetilde{L}^j \cdot F_{H_D} + \widetilde{E}_R{}^i H_D \cdot F_L{}^j + \widetilde{L}^j \cdot H_D F_E^i + \widetilde{E}_R{}^i (\chi_L{}^j \cdot \widetilde{H}_D) + \widetilde{L}^j \cdot (\widetilde{H}_D \cdot \chi_E{}^i) + (\chi_E{}^i \cdot \chi_L{}^j) \cdot H_D \Big]$$

  * Naturally expressed in terms of **superfields (1 terms)**:

$$\mathcal{L}_{\rm W} \supset \Big[-(\mathbf{y^e})_{ij}\ E^i (L^j \cdot \mathbf{H_D}) \Big] \Big|_{\theta\cdot\theta}$$

Fields and superfields (3).

- **Superpotential**.

  * Terribly expressed in terms of **components fields**,
    i. e. scalars, Dirac and Majorana fermions, vector fields (**12 terms)**:

$$\mathcal{L}_{\rm W} \supset (\mathbf{y^e})_{ij} \Big[\widetilde{E}_R{}^i \widetilde{L}^j \cdot F_{H_D} + \widetilde{E}_R{}^i H_D \cdot F_L{}^j + \widetilde{L}^j \cdot H_D F_E^i + \widetilde{E}_R{}^i (\bar{\psi}_L^{cj} P_L \psi_{H_D}) + \widetilde{L}^j \cdot (\bar{\psi}_{H_D} P_L \psi_e{}^i) + (\bar{\psi}_e{}^i P_L \psi_L{}^j) \cdot H_D \Big]$$

* Are the **charge conjugated fields** correct?
* Are the signs in the **fermion flows** correct?
* **The superfield formalism seems more convenient...**

$$\mathcal{L}_{\rm W} \supset \Big[- (\mathbf{y^e})_{ij} \; E^i (L^j \cdot \mathbf{H_D}) \Big] \Big|_{\theta\cdot\theta}$$

Fields and superfields (4).

- **Kinetic terms and gauge interactions**.
  - * Terribly expressed in terms of **components fields**: i. e. scalars, Dirac and Majorana fermions, vector fields (**13 terms)**:

$$\mathcal{L}_{\rm kin} \supset \ldots \qquad \text{[Censured: too ugly to appear on a slide].}$$

  - * Not very nicely expressed in terms of **components fields**, i. e. scalars, Weyl fermions, vector fields (**13 terms)**:

$$\begin{aligned}\mathcal{L}_{\rm kin} \supset\ & D_\mu \widetilde{Q}^\dagger_i D^\mu \widetilde{Q}^i + \frac{i}{2}\left(\chi^i_Q \sigma^\mu D_\mu \bar{\chi}_{Q\,i} - D_\mu \chi^i_Q \sigma^\mu \bar{\chi}_{Q\,i}\right) + F^\dagger_{Q\,i} F^i_Q \\ & + i\sqrt{2}\Big[\frac{1}{6} g' \widetilde{Q}^i \bar{\widetilde{B}} \cdot \bar{\chi}_{Q\,i} + g \bar{\widetilde{W}}^k \cdot \bar{\chi}_{Q\,i} \frac{\sigma^k}{2} \widetilde{Q}^i + g_s \bar{\widetilde{G}}^a \cdot \bar{\chi}_{Q\,i} \frac{T^a}{2} \widetilde{Q}^i + \text{h. c.}\Big] \\ & - g' D_B \widetilde{Q}^\dagger_i \widetilde{Q}^i - g D_{W^k} \widetilde{Q}^\dagger_i \frac{\sigma^k}{2} \widetilde{Q}^i - g_s D_{G^a} \widetilde{Q}^\dagger_i \frac{T^a}{2} \widetilde{Q}^i\end{aligned}$$

  - * Naturally expressed in terms of **superfields (1 terms)**:

$$\mathcal{L}_{\rm kin} \supset \left[Q^\dagger_i e^{-2\frac{1}{6} g' \mathbf{V_B}} e^{-2g \mathbf{V_{W^k}} \frac{\sigma^k}{2}} e^{-2 g_s \mathbf{V_{G^a}} \frac{T^a}{2}} Q^i \right]\Big|_{\theta\cdot\theta\bar\theta\cdot\bar\theta}$$

Fields and superfields (5).

- **Kinetic terms and gauge interactions**.
  - Not very nicely expressed in terms of **components fields**, i. e. scalars, Weyl fermions, vector fields (**13 terms)**:

$$\mathcal{L}_{\rm kin} \supset D_\mu \widetilde{Q}^\dagger_i D^\mu \widetilde{Q}^i + \frac{i}{2}(\chi^i_Q \sigma^\mu D_\mu \bar\chi_{Qi} - D_\mu \chi^i_Q \sigma^\mu \bar\chi_{Qi}) + F^\dagger_{Qi} F^i_Q$$
$$+ i\sqrt{2}\Big[\frac{1}{6} g' \widetilde{Q}^i \bar{\widetilde{B}} \cdot \bar\chi_{Qi} + g \bar{\widetilde{W}}^k \cdot \bar\chi_{Qi} \frac{\sigma^k}{2} \widetilde{Q}^i + g_s \bar{\widetilde{G}}^a \cdot \bar\chi_{Qi} \frac{T^a}{2} \widetilde{Q}^i + \text{h. c.}\Big]$$
$$- g' D_B \widetilde{Q}^\dagger_i \widetilde{Q}^i - g D_{W^k} \widetilde{Q}^\dagger_i \frac{\sigma^k}{2} \widetilde{Q}^i - g_s D_{G^a} \widetilde{Q}^\dagger_i \frac{T^a}{2} \widetilde{Q}^i$$

* Are all **relative signs and factors of i** correct (especially in the non-gauge-like interactions)?
* **Four-component fermions**... (They are a pain, but required for MCs).
* **The superfield formalism seems more convenient...**

$$\mathcal{L}_{\rm kin} \supset \Big[Q^\dagger_i e^{-2\frac{1}{6} g' \mathbf{V_B}} e^{-2g \mathbf{V_{W^k}} \frac{\sigma^k}{2}} e^{-2g_s \mathbf{V_{G^a}} \frac{T^a}{2}} Q^i \Big]\Big|_{\theta\cdot\theta\bar\theta\cdot\bar\theta}$$

Motivation and plans.

Motivation for a superfield module in FeynRules

* **Natural** to implement any supersymmetric theory.
* **Zero probability** to introduce wrong signs, *i* factors,...
* Could be **a useful tool** for model building.
  (not only a Lagrangian translator).
* **Convenient** for many possible extensions (RGEs, ...).

---

* **Available** in the development version for ... yesterday.
* **Validated!** vs. an exercise textbook [BenjF, Rausch (in press)].
* Will be publicly available in early 2011.
  Manual.
  First pheno applications: FR-SF > MG5.
* Already two scheduled extensions (2011).
  RGEs [Alloul, BenjF, Rausch].
  Gravitino/Goldstino in FeynRules [BenjF].

Outline.

Superspace.

- **Superspace: adapted space to write down SUSY transformations naturally.**
- **Basic objects and their FeynRules (hardcoded) implementation.**
  - * The **Majorana spinor** $(\theta, \bar\theta)$ $\Rightarrow$ a superspace point $\equiv G(x, \theta, \bar\theta)$.
  - * The transformation parameters, the **Majorana spinor** $(\varepsilon_{\rm SUSY}, \bar\varepsilon_{\rm SUSY})$.

```
W[1000] == {
   ClassName      -> theta,
   Chirality      -> Left,
   SelfConjugate  -> False}
```

```
W[2000] == {
   ClassName      -> epsSUSY,
   Chirality      -> Left,
   SelfConjugate  -> False}
```

  - * **The supercharges $(\mathbf{Q}, \bar{\mathbf{Q}})$**: action to the left $\equiv G(0, \varepsilon, \bar\varepsilon) G(x, \theta, \bar\theta)$.
  - * **The superderivatives $(\mathbf{D}, \bar{\mathbf{D}})$**: action to the right $\equiv G(x, \theta, \bar\theta) G(0, \varepsilon, \bar\varepsilon)$.

$$Q_\alpha = -i(\partial_\alpha + i\sigma^\mu{}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu) \quad \text{and} \quad \bar Q_{\dot\alpha} = i(\bar\partial_{\dot\alpha} + i\theta^\alpha\sigma^\mu{}_{\alpha\dot\alpha}\partial_\mu) \ ,$$

$$D_\alpha = \partial_\alpha - i\sigma^\mu{}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \quad \text{and} \quad \bar D_{\dot\alpha} = \bar\partial_{\dot\alpha} - i\theta^\alpha\sigma^\mu{}_{\alpha\dot\alpha}\partial_\mu \ .$$

$Q_\alpha(\exp)$ and $\bar Q_{\dot\alpha}(\exp)$

```
QSUSY    [exp_, alpha_]
QSUSYBar[exp_, alphadot_]
```

$D_\alpha(\exp)$ and $\bar D_{\dot\alpha}(\exp)$

```
Dtheta    [exp_, alpha_]
Dthetabar[exp_, alphadot_]
```

Superfields.

- $N = 1$ **SUSY: three types of multiplets with complex transformation laws.**
  - * **Matter:** one scalar field ϕ, one Weyl fermion χ, one auxiliary field F.

$$\begin{aligned} \delta_\varepsilon \phi &= \sqrt{2}\varepsilon \cdot \psi \ , \\ \delta_\varepsilon \psi &= -i\sqrt{2}\sigma^\mu \bar\varepsilon \partial_\mu \phi - \sqrt{2} F \varepsilon \ , \\ \delta_\varepsilon F &= -i\sqrt{2}\partial_\mu \psi \sigma^\mu \bar\varepsilon \ , \end{aligned}$$

  - * **Gauge:** one Majorana $(\lambda, \bar\lambda)$, one gauge boson v_μ, one auxiliary field D.

$$\begin{aligned} \delta_\varepsilon v_\mu &= i(\varepsilon \sigma_\mu \bar\lambda - \lambda \sigma_\mu \bar\varepsilon) \ , \\ \delta_\varepsilon \lambda &= iD\varepsilon + \frac{1}{2}\sigma^\mu \bar\sigma^\nu \varepsilon F_{\mu\nu} \ , \\ \delta_\varepsilon D &= \varepsilon \sigma^\mu \partial_\mu \bar\lambda + \partial_\mu \lambda \sigma^\mu \bar\varepsilon \ , \end{aligned}$$

$v_\mu = v_\mu^a T_a, \ldots,$ T_a being some representation of some Lie algebra.

  - * **Gravity**: one Rarita-Schwinger, one spin-two and some auxiliary fields. [see later in this talk].

Superfields: the chiral superfield (1).

- **Most general expansion in the $\theta, \bar\theta$ variables satisfying $\bar D_{\dot\alpha} \Phi(x,\theta,\bar\theta) = 0$.**

$$\Phi(x,\theta,\bar\theta) = \phi(y) + \sqrt{2}\theta\cdot\psi(y) - \theta\cdot\theta F(y) \text{ where } y^\mu = x^\mu - i\theta\sigma^\mu\bar\theta \ .$$

  * Describes the **matter multiplet**.
  * One scalar field ϕ, one Weyl fermion χ, one auxiliary field F.

Chiral superfield - up-type Higgs doublet

```
CSF[1] == {
  ClassName          -> HU,
  Chirality          -> Left,
  Weyl               -> huw,
  Scalar             -> hus,
  QuantumNumbers     -> {Y->1/2},
  Indices            -> {Index[SU2D]},
  FlavorIndex        -> SU2D}
```

  * The scalar and Weyl fermionic fields must be declared properly.
  * **The auxiliary field will be automatically generated, if not present.**

Superfields: the chiral superfield (2).

- **Expansion in superspace with FeynRules:** $\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\cdot\psi(y) - \theta\cdot\theta F(y)$.

In[12]:= **SF2Components[HU][[1]]**

Out[12]= $\text{hus} + \sqrt{2}\,\theta_{sp\$1}.\text{huw}_{sp\$1} - \text{FTerm4}\,\theta_{sp\$1}.\theta_{sp\$1} - \frac{1}{4}\partial_{mu\$1}[\partial_{mu\$1}[\text{hus}]]\,\theta_{sp\$1}.\theta_{sp\$1}\,\bar{\theta}_{sp\$1dot}.\bar{\theta}_{sp\$1dot} - i\,\partial_{mu\$1}[\text{hus}]\,\theta_{sp\$1}.\bar{\theta}_{sp\$1dot}\,(\sigma^{mu\$1})_{sp\$1,sp\$1dot} + \frac{i\,\partial_{mu\$1}[\text{huw}_{sp\$1}].\bar{\theta}_{sp\$1dot}\,\theta_{sp\$2}.\theta_{sp\$2}\,(\sigma^{mu\$1})_{sp\$1,sp\$1dot}}{\sqrt{2}}$

In[18]:= **Do[If[SF2Components[HU][[2, i]] =!= 0, Print[i, "::", SF2Components[HU][[2, i]]]], {i, 1, 9}]**

1::hus

2::$\sqrt{2}\,\text{huw}_{alpha\$14236}$

4::$-i\,\partial_{mu\$14875}[\text{hus}]$

5::-FTerm4

7::$\frac{i\,\partial_{mu\$1}[\text{huw}_{sp\$1}]\,(\sigma^{mu\$1})_{sp\$1,alphadot\$15940}}{\sqrt{2}}$

9::$-\frac{1}{4}\partial_{mu\$1}[\partial_{mu\$1}[\text{hus}]]$

* $FTerm_4$ was automatically generated.
* Automatic y-expansion.

Superfields: the chiral superfield (3).

- **SUSY transformation laws**:
  - In terms of **superfields**: $\delta_\varepsilon \Phi(x,\theta,\bar\theta) = i(\varepsilon\cdot Q + \bar Q\cdot\bar\varepsilon)\cdot\Phi(x,\theta,\bar\theta)$.
  - In terms of **component fields** (depending on y, not x):

$$\delta_\varepsilon\phi = \sqrt{2}\varepsilon\cdot\psi\ ,\quad \delta_\varepsilon\psi = -i\sqrt{2}\sigma^\mu\bar\varepsilon\partial_\mu\phi - \sqrt{2}F\varepsilon\ ,\quad \delta_\varepsilon F = -i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar\varepsilon\ .$$

  - With FeynRules:

```
In[6]:= tmp = SF2Components[HU][[1]];
        tmp2 = Expand[I * NC[epssusy[sp3b], Expand[QSUSY[ToNC[tmp], sp3]]] Ueps[sp3, sp3b]];
        SF2Components[tmp2][[2, 1]]
        SF2Components[tmp2][[2, 2]] / Sqrt[2]
        SF2Components[tmp2][[2, 5]] / (-1)

Out[8]= √2 huw_sp$1 . epssusy_sp$1

Out[9]= -√2 FTerm4 epssusy_alpha$2251

Out[10]= 0
```

  - ToNC breaks dot products and the NC structure keeps fermion ordering.
  - Ueps corresponds to the antisymmetric tensor with upper indices.

Superfields: the chiral superfield (3).

- **SUSY transformation laws**:
  - In terms of **superfields**: $\delta_\varepsilon \Phi(x,\theta,\bar\theta) = i(\varepsilon \cdot Q + \bar{Q} \cdot \bar\varepsilon) \cdot \Phi(x,\theta,\bar\theta)$.
  - In terms of **component fields** (depending on y, not x):

$$\delta_\varepsilon \phi = \sqrt{2}\varepsilon \cdot \psi \ , \quad \delta_\varepsilon \psi = -i\sqrt{2}\sigma^\mu \bar\varepsilon \partial_\mu \phi - \sqrt{2} F \varepsilon \ , \quad \delta_\varepsilon F = -i\sqrt{2}\partial_\mu \psi \sigma^\mu \bar\varepsilon \ .$$

  - With FeynRules:

```
In[15]:= tmp = SF2Components[HU][[1]];
         tmp2 = Expand[I * NC[Expand[QSUSYBar[ToNC[tmp], adot]], epssusybar[bdot]] Ueps[adot, bdot]];
         SF2Components[tmp2][[2, 1]]
         SF2Components[tmp2][[2, 2]] / Sqrt[2]
         SF2Components[tmp2][[2, 5]] / (-1)

Out[17]= 0

Out[18]= -i √2 ∂_mu$1[hus] epssusy†_sp$1dot (σ^mu$1)_alpha$4408,sp$1dot

Out[19]= -i √2 ∂_mu$1[huw_sp$1].epssusy†_sp$1dot (σ^mu$1)_sp$1,sp$1dot
```

  - `ToNC` breaks dot products and the `NC` structure keeps fermion ordering.
  - `Ueps` corresponds to the antisymmetric tensor with upper indices.

Superfields: the vector superfield (1).

- **Expansion in the $\theta, \bar{\theta}$ variables satisfying $\Phi = \Phi^\dagger$ in the Wess-Zumino gauge**.

$$\Phi_{W.Z.}(x,\theta,\bar{\theta}) = \theta\sigma^\mu\bar{\theta}v_\mu + i\theta\cdot\theta\bar{\theta}\cdot\bar{\lambda} - i\bar{\theta}\cdot\bar{\theta}\theta\cdot\lambda + \frac{1}{2}\theta\cdot\theta\ \bar{\theta}\cdot\bar{\theta}D\ .$$

  - * Describes the **gauge multiplet**.
  - * One Majorana fermion $(\lambda, \bar{\lambda})$, one gauge boson v, one auxiliary field D.

Vector superfield for $SU(2)_L$

```
VSF[1] == {
  ClassName        -> WSF,
  GaugeBoson       -> Wi,
  Gaugino          -> wow,
  Indices          -> {Index[SU2W]},
  FlavorIndex      -> SU2W}
```

Associated gauge group

```
SU2L == {
  Abelian            -> False,
  GaugeBoson         -> Wi,
  CouplingConstant   -> gw,
  SF                 -> WSF,
  StructureConstant  -> ep,
  Representations    -> {...},
  Definitions        -> {...}}
```

  - * The Weyl fermionic and vectorial fields must be declared properly.
  - * **The auxiliary field will be automatically generated, if not present.**

Superfields: the vector superfield (2).

- **Expansion in superspace with FeynRules:**
  $\Phi = \theta\sigma^\mu\bar\theta v_\mu + i\theta\cdot\theta\bar\theta\cdot\bar\lambda - i\bar\theta\cdot\bar\theta\theta\cdot\lambda + \frac{1}{2}\theta\cdot\theta\;\bar\theta\cdot\bar\theta D$.

```
In[21]:= tmp = SF2Components[WSF[ii]][[1]]

Out[21]= -i θsp$1.wowsp$1,SU2W$1001 θ̄sp$1dot.θ̄sp$1dot + i θsp$1.θsp$1 wow†sp$1dot,SU2W$1001.θ̄sp$1dot +
         1/2 θsp$1.θsp$1 θ̄sp$1dot.θ̄sp$1dot DTerm2ii + θsp$1.θ̄sp$1dot (σmu$1)sp$1,sp$1dot Wimu$1,SU2W$1001

In[22]:= Do[If[SF2Components[WSF[ii]][[2, i]] =!= 0,
          Print[ i, "::", SF2Components[WSF[ii]][[2, i]]]], {i, 1, 9}]

4::Wimu$5793,SU2W$1001

7::i wow†alphadot$6369,SU2W$1001

8::-i wowalpha$6657,SU2W$1001

9::DTerm2ii/2
```

* $DTerm_2$ was automatically generated.
* The index naming scheme is optimized, for readibility.

Superfields: the vector superfield (3).

- **Properties of the vector superfield**:

$$\Phi^2_{W.Z.} = \frac{1}{2}\theta\cdot\theta\bar\theta\cdot\bar\theta v^\mu v_\mu, \quad \Phi^3_{W.Z.} = 0.$$

```
        SF2Components[WSF[ii] WSF[jj]][[1]]
Out[23]= 1/2 θsp$1.θsp$1 θ̄sp$1dot.θ̄sp$1dot Wimu$1,SU2W$1001 Wimu$1,SU2W$1002
In[24]:= SF2Components[WSF[ii] WSF[jj] WSF[kk]][[1]]
Out[24]= 0
```

- **The superfield strength tensor is built from associated spinorial superfields**:

$$W_\alpha = -\frac{1}{4}\bar D\cdot\bar D\ e^{2gV} D_\alpha e^{-2gV}, \quad \bar W_{\dot\alpha} = -\frac{1}{4} D\cdot D\ e^{-2gV}\bar D_{\dot\alpha} e^{2gV}.$$

W_α, $(W_\alpha)_{ij}$, $\bar W_{\dot\alpha}$ and $(\bar W_{\dot\alpha})_{ij}$

```
Sca2SpinL[ superfield, lower spin index ]
Sca2SpinL[ superfield, spin index, gauge index, gauge index ]
Sca2SpinR[ superfield, lower spin index ]
Sca2SpinR[ superfield, spin index, gauge index, gauge index ]
```

Superfields: the vector superfield (4).

- **Spinorial superfields**:

$$W_\alpha(y,\theta) = -2g\left(-i\lambda_\alpha + \left[-\frac{i}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F_{\mu\nu} + \theta_\alpha D\right] - \theta\cdot\theta(\sigma^\mu D_\mu\bar\lambda)_\alpha\right) .$$

```
In[6]:= tmp = SF2Components[Sca2SpinL[WSF, alpha, ii, jj]];
        Print["1 :: ", tmp[[2, 1]]];
        Print["2 :: ", tmp[[2, 2]]];
        Print["4 :: ", tmp[[2, 4]]];

1 :: 2 i g_w NC[wow_{alpha,SU2W$1}] Tadj[SU2W$1, ii, jj]

2 :: -2 g_w DTerm2_{SU2W$1} Tadj[SU2W$1, ii, jj] - i g_w ∂_{mu$2}[Wi_{mu$1,SU2W$1}]
    NC[TensDot2[(σ^{mu$1})_{alpha,sp$1dot}, (σ̄^{mu$2})_{sp$1dot,alpha$1586}][{down, Left, alpha}, {up, Left, alpha$1586}]]
    Tadj[SU2W$1, ii, jj] + i g_w ∂_{mu$2}[Wi_{mu$1,SU2W$1}]
    NC[TensDot2[(σ^{mu$2})_{alpha,sp$1dot}, (σ̄^{mu$1})_{sp$1dot,alpha$1586}][{down, Left, alpha}, {up, Left, alpha$1586}]]
    Tadj[SU2W$1, ii, jj] + i g_w^2 ep_{SU2W$1,SU2W$2,SU2W$3}
    NC[TensDot2[(σ^{mu$1})_{alpha,sp$1dot}, (σ̄^{mu$2})_{sp$1dot,alpha$1586}][{down, Left, alpha}, {up, Left, alpha$1586}]]
    Tadj[SU2W$3, ii, jj] Wi_{mu$1,SU2W$1} Wi_{mu$2,SU2W$2}

4 :: 2 g_w NC[∂_{mu$1586}[wow_{alpha,SU2W$1}]] Tadj[SU2W$1, ii, jj]
```

- * FeynRules has performed the ***y*-expansion**.
- * Spinors with **non-lower spin index** are embedded in a `TensDot2` structure.
- * `Tadj` matrices automatically added: $D = D^a T^a, \ldots$

Superfields: the most general superfield.

- **Most general, reducible, expansion in the $\theta, \bar\theta$ variables**:

$$\Phi(x,\theta,\bar\theta) = z(x) + \theta\cdot\xi(x) + \bar\theta\cdot\bar\zeta(x) + \theta\cdot\theta f(x) + \bar\theta\cdot\bar\theta g(x) + \theta\sigma^\mu\bar\theta\ v_\mu(x) + \bar\theta\cdot\bar\theta\theta\cdot\omega(x) + \theta\cdot\theta\cdot\bar\theta\cdot\bar\rho(x) + \theta\cdot\theta\bar\theta\cdot\bar\theta d(x).$$

scalars: $z, f, g, d,$ Weyl fermions: $\xi, \zeta, \omega, \rho,$ vector: v.

* **Not implemented by default**.
* **Can be added easily** with the use of the NC environment.
* All the fields must be declared explicitly.

```
z + NC[theta[sp],xi[sp2]] Ueps[sp2,sp] + ...
```

Superfield strength tensors - SFSTs (1).

- **Each vector superfield is attached to one gauge group**.
- **Vector superfields interactions are obtained by calculating SFSTs**.
  - * **Abelian groups**.

$$\mathcal{L} = \frac{1}{4} W^\alpha W_{\alpha}{}_{|\theta\theta} + \frac{1}{4} \bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha}{}_{|\bar\theta\bar\theta}$$
$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar\lambda \bar\sigma^\mu \partial_\mu \lambda + \frac{1}{2} D^2 \ .$$

  - * **Non-abelian groups**.

$$\mathcal{L} = \frac{1}{16 g^2 \tau_{\mathcal{R}}} \mathrm{Tr}(W^\alpha W_\alpha)_{|\theta\theta} + \frac{1}{16 g^2 \tau_{\mathcal{R}}} \mathrm{Tr}(\bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha})_{|\bar\theta\bar\theta}$$
$$= -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + i\, \bar\lambda_a \, \bar\sigma^\mu \, D_\mu \lambda^a + \frac{1}{2} D_a D^a$$

$\Rightarrow$ Interactions between gauge-bosons and gauginos.

- **Automatic extraction of the SFSTs of a model:**

```
GetSuperFS[]
```

Superfield strength tensors - SFSTs (2).

- **Abelian superfield strengths**:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\lambda}\bar{\sigma}^{\mu}\partial_{\mu}\lambda + \frac{1}{2} D^2 \ .$$

In[7]:= `GetSuperFS[]`

Out[7]= $\frac{1}{4}$ SuperFS[BSF, True] + $\frac{\text{SuperFS[WSF, False]}}{16\ g_w^2\ \tau_{SU2L}}$ + $\frac{\text{SuperFS[GSF, False]}}{16\ g_s^2\ \tau_{SU3C}}$

In[18]:=
```
tmp = SF2Components[%7[[1]]];
tmp[[2, 5]] + tmp[[2, 6]]
```

Out[19]= $\frac{\text{DTerm1}^2}{2} - \frac{1}{2}\partial_{mu\$2}[B_{mu\$1}]^2 + \frac{1}{2}\partial_{mu\$2}[B_{mu\$1}]\ \partial_{mu\$1}[B_{mu\$2}] + \frac{1}{2}\, i\ bow_{sp\$1}.\partial_{mu\$1}[bow^{\dagger}_{sp\$1dot}]\ (\sigma^{mu\$1})_{sp\$1,sp\$1dot} - \frac{1}{2}\, i\ \partial_{mu\$1}[bow_{sp\$1}].bow^{\dagger}_{sp\$1dot}\ (\sigma^{mu\$1})_{sp\$1,sp\$1dot}$

Superfield strength tensors - SFSTs (3).

- **Non-abelian superfield strengths**:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + i\,\bar\lambda_a \bar\sigma^\mu D_\mu \lambda^a + \frac{1}{2} D_a D^a \ .$$

```
In[16]:= tmp = SF2Components[%7[[2]]];
         tmp[[2, 5]] + tmp[[2, 6]]
```

Out[17]=

$$-\frac{1}{2}\partial_{mu\$2}[Wi_{mu\$1,SU2W\$1}]^2 + \frac{1}{2}\partial_{mu\$2}[Wi_{mu\$1,SU2W\$1}]\,\partial_{mu\$1}[Wi_{mu\$2,SU2W\$1}] +$$
$$\frac{DTerm2^2_{SU2W\$1}}{2} - \frac{1}{2}\, i\, \partial_{mu\$1}[wow_{sp\$1,SU2W\$1}].wow^\dagger_{sp\$1dot,SU2W\$1}\,(\sigma^{mu\$1})_{sp\$1,sp\$1dot} +$$
$$\frac{1}{2}\, i\, wow_{sp\$2,SU2W\$1}.\partial_{mu\$1}[wow^\dagger_{sp\$1dot,SU2W\$1}]\,(\sigma^{mu\$1})_{sp\$2,sp\$1dot} -$$
$$\frac{1}{2}\, i\, g_w\, wow_{sp\$1,SU2W\$1}.wow^\dagger_{sp\$2dot,SU2W\$2}\, ep_{SU2W\$1,SU2W\$2,SU2W\$3}\,(\sigma^{mu\$1})_{sp\$1,sp\$2dot}\, Wi_{mu\$1,SU2W\$3} -$$
$$\frac{1}{2}\, i\, g_w\, wow_{sp\$2,SU2W\$1}.wow^\dagger_{sp\$1dot,SU2W\$2}\, ep_{SU2W\$1,SU2W\$2,SU2W\$3}\,(\sigma^{mu\$1})_{sp\$2,sp\$1dot}\, Wi_{mu\$1,SU2W\$3} +$$
$$g_w\,\partial_{mu\$2}[Wi_{mu\$1,SU2W\$1}]\, ep_{SU2W\$1,SU2W\$2,SU2W\$3}\, Wi_{mu\$1,SU2W\$2}\, Wi_{mu\$2,SU2W\$3} -$$
$$\frac{1}{4} g_w^2\, ep_{SU2W\$1,SU2W\$2,SU2W\$3}\, ep_{SU2W\$4,SU2W\$5,SU2W\$3}\, Wi_{mu\$1,SU2W\$1}\, Wi_{mu\$1,SU2W\$4}\, Wi_{mu\$2,SU2W\$2}\, Wi_{mu\$2,SU2W\$5}$$

Kähler potential (1).

- **The Kähler potential contains kinetic terms for the chiral superfields.**

$$\mathcal{L} = \left[\Phi^\dagger(x,\theta,\bar\theta) e^{-2y_\Phi g' \mathbf{V_B}} e^{-2g\mathbf{V_W}} e^{-2g_s \mathbf{V_G}} \Phi(x,\theta,\bar\theta)\right]\Big|_{\theta\cdot\theta\bar\theta\cdot\bar\theta}$$

$\Rightarrow$ Gauge interactions.

* The vector superfields contains the proper representation matrices.

- **Automatic extraction of the Kähler potential of a model:**

```
In[20]:= Getkins[]

Out[20]= GetSFKineticTerms[DR] + GetSFKineticTerms[ER] + GetSFKineticTerms[HD] + GetSFKineticTerms[HU] +
          GetSFKineticTerms[LL] + GetSFKineticTerms[QL] + GetSFKineticTerms[UR] + GetSFKineticTerms[VR]
```

Kähler potential (2).

- **Right electron kinetic Lagrangian**:

$$\mathcal{L} = D_\mu\phi^\dagger D^\mu\phi - \frac{i}{2}\left(D_\mu\bar\psi\bar\sigma^\mu\psi - \bar\psi\bar\sigma^\mu D_\mu\psi\right) + i\sqrt{2}g\bar\lambda^a\cdot\bar\psi T_a\phi - i\sqrt{2}g\phi^\dagger T_a\psi\cdot\lambda^a$$
$$+ FF^\dagger - gD^a\phi^\dagger T^a\phi \ .$$

```
In[20]:= Getkins[]

Out[20]= GetSFKineticTerms[DR] + GetSFKineticTerms[ER] + GetSFKineticTerms[HD] + GetSFKineticTerms[HU] +
  GetSFKineticTerms[LL] + GetSFKineticTerms[QL] + GetSFKineticTerms[UR] + GetSFKineticTerms[VR]

In[23]:= SF2Components[Getkins[][[2]]][[2, -1]]
```

Out[23]=
$$\frac{1}{2}\partial_{mu\$1}[ERs_{GEN\$1}]\,\partial_{mu\$1}[ERs^\dagger_{GEN\$1}] - \frac{1}{4}\partial_{mu\$1}[\partial_{mu\$1}[ERs^\dagger_{GEN\$1}]]\,ERs_{GEN\$1} -$$
$$i\,g'\,B_{mu\$1}\,\partial_{mu\$1}[ERs^\dagger_{GEN\$1}]\,ERs_{GEN\$1} + i\sqrt{2}\,g'\,bow^\dagger_{sp\$1dot}.ERw^\dagger_{sp\$1dot,GEN\$1}\,ERs_{GEN\$1} -$$
$$\frac{1}{4}\partial_{mu\$1}[\partial_{mu\$1}[ERs_{GEN\$1}]]\,ERs^\dagger_{GEN\$1} + i\,g'\,B_{mu\$1}\,\partial_{mu\$1}[ERs_{GEN\$1}]\,ERs^\dagger_{GEN\$1} -$$
$$i\sqrt{2}\,g'\,ERw_{sp\$1,GEN\$1}.bow_{sp\$1}\,ERs^\dagger_{GEN\$1} - DTerm1\,g'\,ERs_{GEN\$1}\,ERs^\dagger_{GEN\$1} + (g')^2\,B^2_{mu\$1}\,ERs_{GEN\$1}\,ERs^\dagger_{GEN\$1} +$$
$$FTerm7_{GEN\$1}\,FTerm7^\dagger_{GEN\$1} - \frac{1}{2}\,i\,\partial_{mu\$1}[ERw_{sp\$1,GEN\$1}].ERw^\dagger_{sp\$1dot,GEN\$1}\,(\sigma^{mu\$1})_{sp\$1,sp\$1dot} +$$
$$\frac{1}{2}\,i\,ERw_{sp\$1,GEN\$1}.\partial_{mu\$1}[ERw^\dagger_{sp\$1dot,GEN\$1}]\,(\sigma^{mu\$1})_{sp\$1,sp\$1dot} -$$
$$g'\,B_{mu\$1}\,ERw_{sp\$1,GEN\$1}.ERw^\dagger_{sp\$1dot,GEN\$1}\,(\sigma^{mu\$1})_{sp\$1,sp\$1dot}$$

Full SUSY Lagrangian (1).

- **Complete Lagrangian for a model**.

$$\mathcal{L} = \Phi^\dagger e^{-2gV}\Phi_{|_{\theta^2\bar\theta^2}} + \frac{1}{16g^2\tau_\mathcal{R}}\mathrm{Tr}(W^\alpha W_\alpha)_{|_{\theta^2}} + \frac{1}{16g^2\tau_\mathcal{R}}\mathrm{Tr}(\bar W_{\dot\alpha}\bar W^{\dot\alpha})_{|_{\bar\theta^2}}$$
$$+ W(\Phi)_{|_{\theta^2}} + W^\star(\Phi^\dagger)_{|_{\bar\theta^2}} + \mathcal{L}_{\rm soft}$$

  * **Chiral superfields kinetic terms: automatic**.
  * **Vector superfield strengths: automatic**.
  * **Superpotential: model dependent.**
  * **Soft SUSY-breaking Lagrangian model dependent.**

SUSY Lagrangian

```
SF2Components[Getkins[]][[2,9]] +
SF2Components[GetSuperFS[] + SuperPot][[2,5]] +
SF2Components[GetSuperFS[] + HC[SuperPot]][[2,6]] +
LSoft
```

Full SUSY Lagrangian (2).

- **Solution of the equation of motions**.
  - Get rid of the auxiliary D- and F-fields.

Equations of motion

```
lagr = SolveEqMotionD[ lagr ] ;
lagr = SolveEqMotionF[ lagr ] ;
```

- **Back to four-component fermions**.
  - Usual FeynRules routine.

Four-component fermions

```
lagr = WeylToDirac[ lagr ] ;
```

Full MSSM Lagrangian.

```
In[6]:= lagr = Lag;
Generation of the superfield strengths...
 ** BSF superfield strength generated in 3.91 s.
 ** WSF superfield strength generated in 41.06 s.
 ** GSF superfield strength generated in 38.46 s.
... Achieved!
Generation of the Kaehler potential...
 ** DR kinetic terms generated in 5.11 s.
 ** ER kinetic terms generated in 2.17 s.
 ** HD kinetic terms generated in 5.07 s.
 ** HU kinetic terms generated in 5.06 s.
 ** LL kinetic terms generated in 5.18 s.
 ** QL kinetic terms generated in 9.83 s.
 ** UR kinetic terms generated in 5.36 s.
 ** VR kinetic terms generated in 0.45 s.
... Achieved!
Generation of the superpotential achieved in 15.43 s.
Generation of the soft SUSY-breaking Lagrangian achieved in 0.s.
D-equations of motion solved in 2.48 s.
F-equations of motion solved in 65.09 s.
Transformation to Dirac fermions...
 ** Removal of color antifundamentals in 0. s.
 ** First optimization of the index naming scheme done in 0.87 s.
 ** Expansion of the SU2L indices done in 93.6 s.
 ** Second optimization of the index naming scheme done in 8.88 s.
 ** Transforming Weyls into Diracs done in 285.72 s.
 ** Final optimization of the index naming scheme done in 9.59 s.
... Achieved!
```

Outline.

Automatized extraction of the RGEs (1).

RGEs [Alloul, BenjF, Rausch].

- **SUSY Lagrangians at low energy**.
  - * Contains **free parameters**.
  - * Example: 105 for the MSSM.
  - * Loss of **predictivity**.

- **SUSY Lagrangians at high energy**.
  - * Can assume **universal boundary conditions**.
  - * Must be **evolved to low energy**.
  - * Example: gauge coupling constant and Yukawa interaction.

$$
\begin{aligned}
Q\frac{\mathrm{d}g}{\mathrm{d}Q} &= -\frac{g^3}{16\pi^2}\Big[3C(G) - T(R)\Big] \ , \\
Q\frac{\mathrm{d}f_{abc}}{\mathrm{d}Q} &= -\frac{1}{32\pi^2}\Big[4g^2[C(A)+C(B)+C(C)]f_{abc} \\
&\quad - (X_a^{a'} f_{a'bc} + X_b^{b'} f_{ab'c} + X_c^{c'} f_{abc'})\Big]
\end{aligned}
$$

Automatized extraction of the RGEs (2).

[Alloul, BenjF, Rausch].

- **Done: analytical extraction of the RGEs for generic SUSY Lagrangian**.
  * Independent Mathematica routine: InSurGe.
  * Validated on SPS1a.
  * Generic enough for any SUSY-breaking model (mSUGRA, GSMB, ...).

Top quark Yukawa: InSurGe output

$$\frac{d\, f_{\{Q,AQU,Hu\}}}{dt} == -\frac{G[1,3]^2 f_{\{Q,AQU,Hu\}}}{3\pi^2} - \frac{3\, G[2,2]^2 f_{\{Q,AQU,Hu\}}}{16\pi^2} - \frac{13\, G[3,1]^2 f_{\{Q,AQU,Hu\}}}{144\pi^2} + \frac{f_{\{Q,AQD,Hd\}}\, f_{\{Q,AQU,Hu\}}\, f^{\{Q,AQD,Hd\}}}{16\pi^2} + \frac{3\, f^2_{\{Q,AQU,Hu\}}\, f^{\{Q,AQU,Hu\}}}{8\pi^2}$$

- **To-do list**.
  * Extraction of the mass spectrum (numerically).
  * Full embedding in FeynRules.
  * **GOAL:** Automatized spectrum generator for any SUSY model (one-loop).

Gravitino - project not started (1).

- FeynRules-**superfield can be used to extract the Goldstino Lagrangian**.
  - **Variation** of the SUSY Lagrangian (using FeynRules):

$$\delta_\varepsilon \mathcal{L} = \partial_\mu K^\mu \quad \text{using the supercharges,}$$

$$\delta_\varepsilon \mathcal{L} = \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu X)} \delta_\varepsilon X \right] \quad \text{using Nœther theorem .}$$

  - Extraction of the **conserved supercurrent**.

$$\varepsilon \cdot J^\mu + \bar{J}^\mu \cdot \bar{\varepsilon} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu X)} \delta_\varepsilon X - K^\mu \ .$$

  - Result to be reproduced:

$$J^\mu = \sqrt{2} D_\nu \phi_i^\dagger \sigma^\nu \bar{\sigma}^\mu \psi^i + i\sqrt{2} \sigma^\mu \bar{\psi}_i W^{\star i} - \frac{i}{2} \sigma^\rho \bar{\sigma}^\nu \sigma^\mu \bar{\lambda}_a F^a_{\nu\rho} + g \phi_i^\dagger \sigma^\mu \bar{\lambda} \phi^i \ .$$

  - **One of the auxiliary field gets a vev** (*e.g.*, F^1)

$$J^\mu_\alpha = i\sqrt{2} (\sigma^\mu \bar{\psi}_1)_\alpha F^1 + \tilde{J}^\mu_\alpha$$

  - Supercurrent conservation ⇒**Goldstino (ψ_1) interaction Lagrangian**:

$$\mathcal{L} = \frac{1}{2\sqrt{2}} \frac{1}{\langle F^1 \rangle} \psi^1 \cdot \partial_\mu \tilde{J} + \frac{1}{2\sqrt{2}} \frac{1}{\langle F^\dagger_1 \rangle} \bar{\psi}_1 \cdot \partial_\mu \bar{\tilde{J}} \ .$$

Gravitino - project not started (2).

- **Gravitino Lagrangian**.
  - * Implementation and validation of the spin 3/2 fields in FeynRules.
  - * Rarita-Schwinger Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu\gamma_5\gamma_\nu\partial_\rho\Psi_\sigma \ .$$

  - * Is the **coupling to the graviton relevant**?

    If yes, we need to introduce
    - a curved superspace,
    - a decent covariant derivative,
    - a spin 2 Lagrangian,
    - the gravity superfield,
    - ...
  - * **If not, then all the ingredients are (almost) there!**
    - Nœther procedure and local supersymmetry invariance $\Rightarrow$ interactions between the gravitino and the other fields.
    - **Could be derived with** FeynRules.

Outline.

General supersymmetric models

- **General versions** $\Rightarrow$ any usual limit easily taken.
  - * 6×6 CP and flavour violating scalar mixing matrices.
    e.g.
    $$\left(\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{u}_4, \tilde{u}_5, \tilde{u}_6\right)^T = R^{\tilde{u}}\left(\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R\right)^T,$$
    $$\left(\tilde{d}_1, \tilde{d}_2, \tilde{d}_3, \tilde{d}_4, \tilde{d}_5, \tilde{d}_6\right)^T = R^{\tilde{d}}\left(\tilde{d}_L, \tilde{s}_L, \tilde{b}_L, \tilde{d}_R, \tilde{s}_R, \tilde{b}_R\right)^T.$$
  - * **Simple extension requiring a generalized MSSM: the MSSM-CKM.**

$$M^2_{\tilde{U}} = \begin{pmatrix} V_{CKM}\left(\mathbf{m}^2_{\tilde{\mathbf{Q}}_\mathbf{i}}\delta_{ij}\right)V^\dagger_{CKM} & \left(\mathbf{T}^{\mathbf{u}\,*}_{\mathbf{i}}\delta_{ij}\right) \\ \left(\mathbf{T}^{\mathbf{u}}_{\mathbf{i}}\delta_{ij}\right) & \left(\mathbf{m}^2_{\tilde{\mathbf{U}}_\mathbf{i}}\delta_{ij}\right) \end{pmatrix} \text{ and } M^2_{\tilde{D}} = \begin{pmatrix} \left(\mathbf{m}^2_{\tilde{\mathbf{Q}}_\mathbf{i}}\delta_{ij}\right) & \left(\mathbf{T}^{\mathbf{d}\,*}_{\mathbf{i}}\delta_{ij}\right) \\ \left(\mathbf{T}^{\mathbf{d}}_{\mathbf{i}}\delta_{ij}\right) & \left(\mathbf{m}^2_{\tilde{\mathbf{D}}_\mathbf{i}}\delta_{ij}\right) \end{pmatrix}.$$

  - * **More general versions are needed!**

- **A lot of model parameters.**
  - * The **SLHA-FR format** (SLHA2-like format).
  - * C++ translator SLHA1/2 $\Leftrightarrow$ SLHA-FR .

Supersymmetric models in FeynRules.

- **The MSSM** [BenjF].
  - * **Public**: validated implementation with four-component fermions.
  - * **Private**: validated implementation with Weyl fermions.
  - * **Super-Private**: validated superfield implementation

```
FeynmanRules[new-old] = {};
```

- **The RPV MSSM** [BenjF].
  - * **Private**: partially validated implementation with Weyl fermions.
  - * MC blocking: ε_{ijk} in color space $\Rightarrow$ UFO, ALOHA, MG5.
  - * Already used by CMS-Strasbourg [Andrea, Conte].
- **The NMSSM** [Braam, BenjF, Reuter].
  - * **Private**: validated implementation with Weyl fermions.
  - * **Will be public in a month**.
- **The R-symmetric MSSM** [BenjF@LesHouches'09].
  - * **Private**: implementation with Weyl fermions.
  - * Never validated.
  - * Very interesting: scalar adjoints, ...

Outline.

Summary.

- **Superfield module of FeynRules.**
  * Achieved and validated.
  * More than just a Lagrangian translator.
  * Validated against textbook results.
  * **Public version expected for early 2011 (together with a publication)**.

- **Models**.
  * Still need to be translated to their superfield version.
  * **All main models will be public in early 2011 (NMSSM, RPV, RMSSM)**.
  * Other implementations are ongoing (numuSSM, left-right SUSY, ...). Maybe your favourite one (Please ask!)

- **To-do list**
  * RGEs and automatized spectrum generator.
  * Gravitino-Goldstino.
  * Ongoing physics projects (cool models means unusual phenomenology).