FeynRules
Feynman rules made easy

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In collaboration with:
N.D. Christensen, M. Herquet, S. Schumann

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Tools08, Munich

- Why yet another tool..?
- FeynRules
- Example: QCD

  ⟶ For BSM physcis, see N. Christensen's talk
- Conclusion

Why yet another tool..?

- In general, a new model is given by a Lagrangian, containing all the particles and their mutual interactions.
- At some point, one would like to compare the model with experiment.

⇒ Needs in general some hard calculations:

- cross-sections
- decay rates
- radiative corrections

Why yet another tool..?

- Fortunately, several tools are available to do the calculations
  - MC generators (MadGraph, CalcHep, CompHEP, AMEGIC++)
  - FeynArts,...

New model

(Lagrangian, new particles,...)

Existing tools

(Programming language, files containing the new particles and interactions,...)

FeynRules

- Mathematica® based package that calculates Feynman rules from a Lagrangian.
- No special requirements on the form of the Lagrangian.
- Particle types supported so far: scalars, fermions (Dirac and Majorana), vectors, spin-2, ghosts.

FeynRules

- The FR model file contains all the information about the model:
  - Particles & fields
  - Parameters (masses, coupling constants,...)
  - mixing matrices
  - etc.

- Feynman rules are calculated by Mathematica using the information from the model-file and the Lagrangian.

- The vertices can be exported into a TeX-file.

FeynRules

- Example of how Feynman rules are calculated:

$$\mathcal{L}_{\bar{\psi}\psi A} = -e\bar{\psi}\gamma^{\mu}\psi A_{\mu} = -e\bar{\psi}_{s,f}\gamma^{\mu}_{ss'}\psi_{s',f}A_{\mu}$$

$\longrightarrow$ Use canonical quantisation to calculate

$$-e\langle 0|\bar{\psi}\gamma^{\mu}\psi A_{\mu}a^{\dagger}_{\bar{\psi}}a^{\dagger}_{\psi}a^{\dagger}_{A}|0\rangle$$

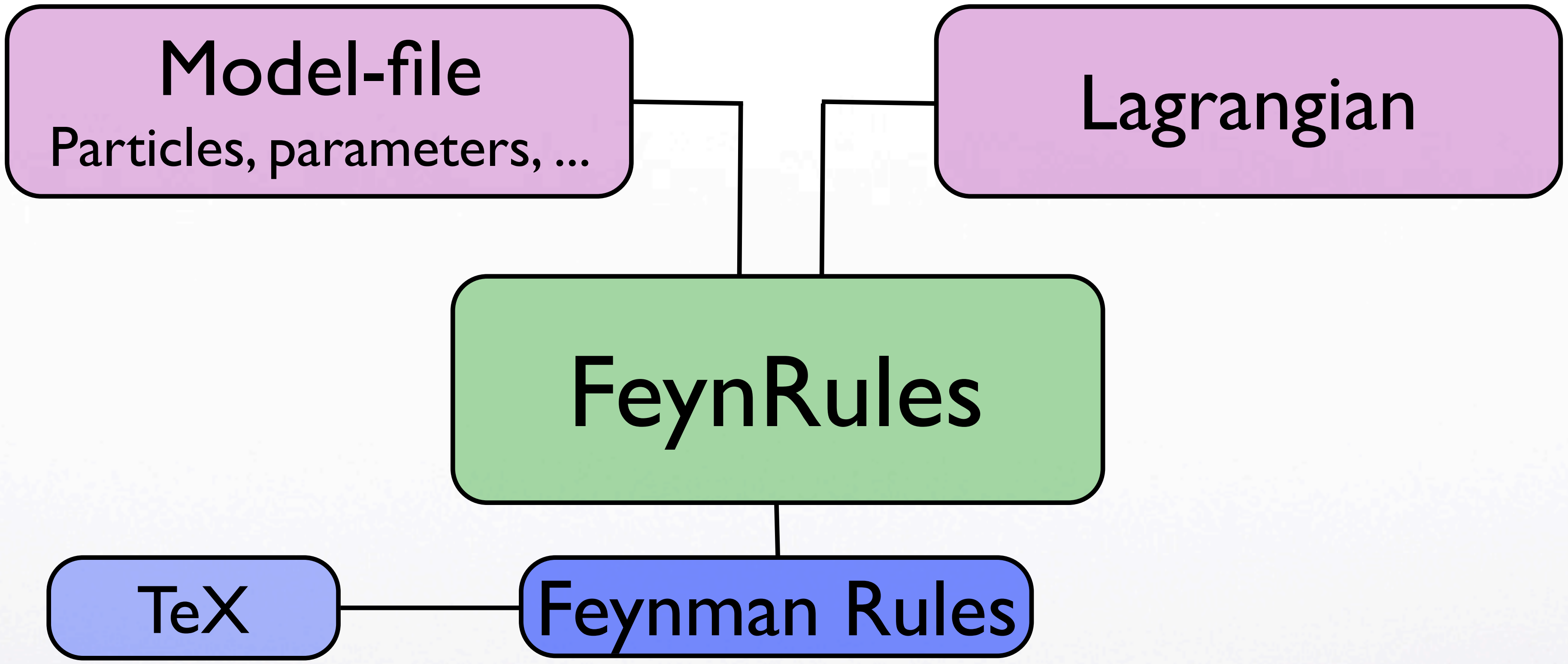
- After applying standard QFT commutation rules, we get

$$-e\delta_{ff'}\gamma^{\mu}_{ss'}\bar{u}_{sf}(p_1)u_{s'f'}(p_2)\epsilon_{\mu}(p_3)e^{-\imath(p_1+p_2+p_3)x}$$

- Dropping the wave functions we are left with the vertex

$$-ie\delta_{ff'}\gamma^{\mu}_{ss'}$$

FeynRules


Model-file
Particles, parameters, ...
Lagrangian
FeynRules
TeX
Feynman Rules

FeynRules

- Model file:

  - particles
  - parameters (masses, couplings,...)

- Lagrangian:

  - Interaction vertices (calculated by FeynRules)

FeynRules

- Model file:

  - particles
  - parameters (masses, couplings,...)

- Lagrangian:

  - Interaction vertices (calculated by FeynRules)

Generic model:
Suitable to be translated to any other code.

⟶ See N. Christensen's talk

FeynRules

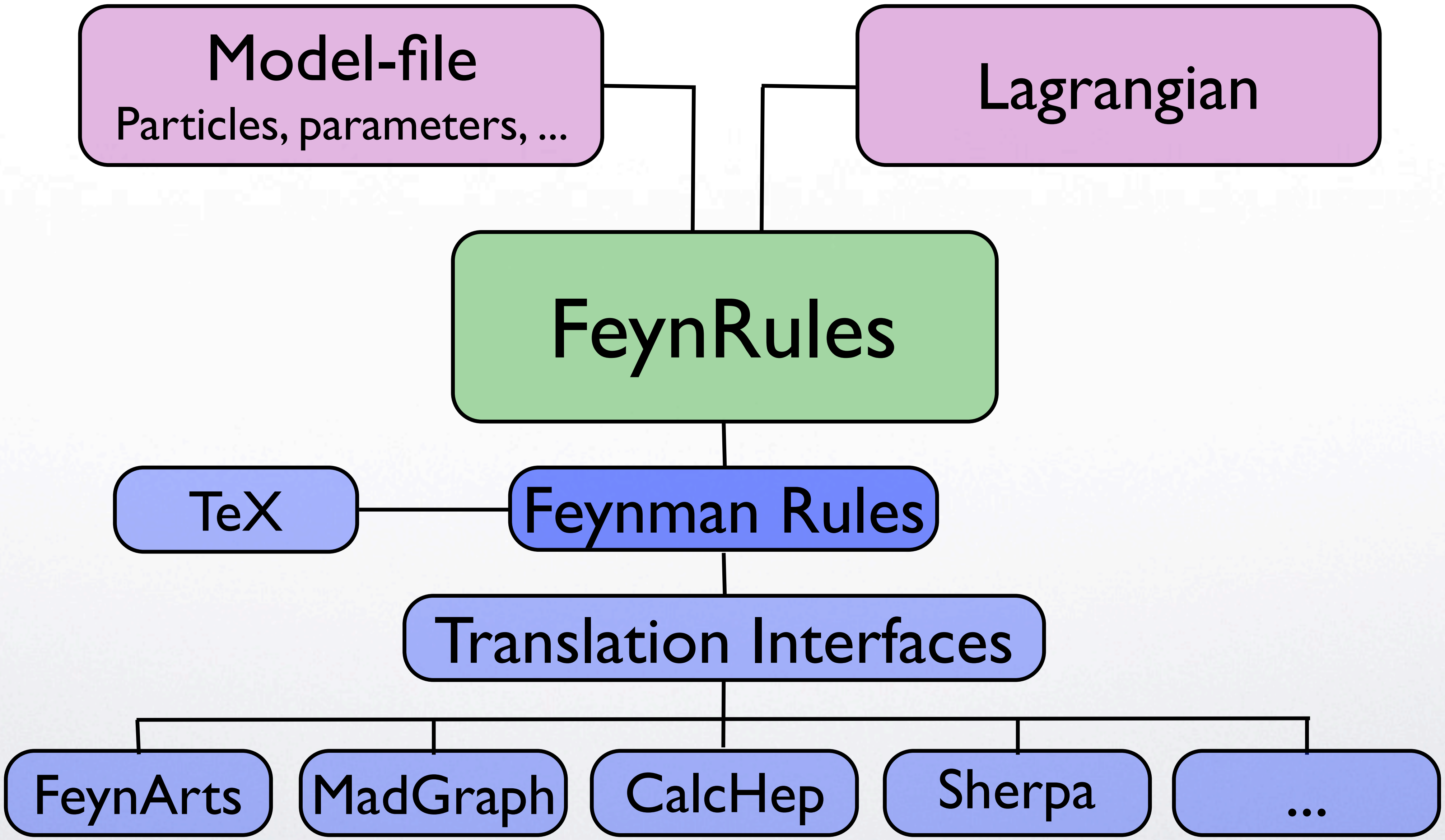
- FR includes a set of 'translation interfaces' which allow to translate the generic model information into any other format.

- FR creates all files needed to run the new model just by knowing the FR model-file and the Lagrangian.

- Translation interfaces available so far
  - FeynArts/FormCalc
  - MadGraph/MadEvent (CD, M. Herquet)
  - CalcHep/CompHep (CD, N. Christensen)
  - Sherpa (CD, S. Schumann)

We would like to see many more!

FeynRules


Model-file
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Feynman Rules
Translation Interfaces
FeynArts
MadGraph
CalcHep
Sherpa
...

FeynRules

- The philosophy:

  - Provide a 'theorist-friendly' environment to develop new models.

    → Use Mathematica based program.

  - Fill the gap between model building and collider simulation.

    → Automatic way to go from the Lagrangian to Monte Carlo simulation.

  - Exploit the strength of the different Feynman diagram calculators on the market, and avoid separate implementations of the same model into different programs.

    → Provide translation interfaces to more than one program.

Example: QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{q}_f (i \not{D} - m_f) q_f - \bar{\eta}^a \partial_\mu D^\mu \eta^a$$

- Four step implementation:
  - Step I: Define your parameters
  - Step II: Define your gauge group
  - Step III: Define your particles
  - Step IV: Write your lagrangian

(Steps I–III: Model file)

Example: QCD

- **Step 1:** Define your parameters:

```
\[Alpha]S == {
    ParameterType -> External,
    Value -> 0.118,
    ParameterName -> aS,
    BlockName -> SMINPUTS,
    InteractionOrder -> {QCD, 2}
 },
gs == {
    ParameterType -> Internal,
    Value -> Sqrt[4 Pi \[Alpha]S],
    ParameterName -> G,
    InteractionOrder -> {QCD, 1}
 }
```

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}_f (i \not{D} - m_f) q_f - \bar{\eta}^a \partial_\mu D^\mu \eta^a$$

Example: QCD

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 },
gs == {
    ParameterType -> Internal,
    Value -> Sqrt[4 Pi \[Alpha]S],
    ParameterName -> G,
    InteractionOrder -> {QCD, 1}
 }
```

Additional information needed by Monte Carlo programs to do the numerical integration.

Example: QCD

- **Step II:** Define your gauge group:

```
SU3C == {
    Abelian -> False,
    GaugeBoson -> G,
    StructureConstant -> f,
    Representations -> {T, Colour},
    CouplingConstant -> gs
}
```

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

$$+ \bar{q}_f (i \not{D} - m_f) q_f$$

$$- \bar{\eta}^a \partial_\mu D^\mu \eta^a$$

Example: QCD

- Step II: Define your gauge group:

```
SU3C == {
   Abelian -> False,
   GaugeBoson -> G,
   StructureConstant -> f,
   Representations -> {T, Colour},
   CouplingConstant -> gs
}
```

- This automatically defines the gluon field strength tensor:

```
FS[G,mu,nu,a]
```

- To be added in the future: Automatic definition of the covariant derivative.

Example: QCD

- **Step III:** Define your particles:

```
F[1] == {
  ClassName -> q,
  ClassMembers -> {d, u, s, c, b, t},

  SelfConjugate -> False,
  Indices -> {Index[Flavor], Index[Colour]},
  FlavorIndex -> Flavor,

  Mass -> {MQ, {MD, 0}, {MU, 0}, {MS, 0},
       {MC, 1.25}, {MB, 4.5}, {MT, 174}},
  Width -> {WQ, {WD, 0}, {WU, 0}, {WS, 0},
       {WC, 0}, {WB, 0}, {WT, 1.6}},
  PDG -> {1,2,3,4,5,6}
},
```

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

$$+ \bar{q}_f (i\not{D} - m_f) q_f$$

$$- \bar{\eta}^a \partial_\mu D^\mu \eta^a$$

Example: QCD

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  FlavorIndex -> Flavor,

  Mass -> {MQ, {MD, 0}, {MU, 0}, {MS, 0},
       {MC, 1.25}, {MB, 4.5}, {MT, 174}},
  Width -> {WQ, {WD, 0}, {WU, 0}, {WS, 0},
       {WC, 0}, {WB, 0}, {WT, 1.6}},
  PDG -> {1,2,3,4,5,6}
},
```

- Grouping particles allows to write very compact terms in the Lagrangian.

Example: QCD

- Step IV: Write your Lagrangian:

```
LQCD =
  -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a]

  +I qbar.Ga[mu].del[q, mu]
  + gs qbar.Ga[mu].T[a].q G[mu,a]
  - MQ[f]qbar[s,f,c].q[s,f,c]

  - gs \[Eta]bar[a].del[dBRSTG[mu,a],mu];

dBRSTG[mu_,a_] := 1/gs ( del[\[Eta][a], mu]
+ gs f[a,a2,a3] G[mu,a2] \[Phi][a3] );
```

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

$$+ \bar{q}_f (i \not{D} - m_f) q_f$$

$$- \bar{\eta}^a \partial_\mu D^\mu \eta^a$$

Example: QCD

- Step IV: Write your Lagrangian:

```
LQCD =
   -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a]

    +I qbar.Ga[mu].del[q, mu]
    + gs qbar.Ga[mu].T[a].q G[mu,a]
    - MQ[f]qbar[s,f,c].q[s,f,c]

    - gs \[Eta]bar[a].del[dBRSTG[mu,a],mu];

dBRSTG[mu_,a_] := 1/gs ( del[\[Eta][a], mu]
+ gs f[a,a2,a3] G[mu,a2] \[Phi][a3] );
```

$$i\, q^\dagger.\gamma^{\mathrm{mu}}.\partial_{\mathrm{mu}}(q)$$
$$+\,\mathrm{gs}\, q^\dagger.\gamma^{\mathrm{mu}}\, T^a.q\, G_{\mathrm{mu},a}$$
$$-\, {q^\dagger}_{s,f,c}.q_{s,f,c}\, \mathrm{MQ}(f)$$

- Not every class flavor must be written explicitly.

- It is enough to write the ‘quark class’.

Example: QCD

- Computation of the Feynman rules:

```
FeynmanRules[LQCD]
```

Vertex 1

Particle 1 : Vector , G

Particle 2 : Dirac , $q^\dagger$

Particle 3 : Dirac , q

Vertex:

$$i\, \mathrm{gs}\, \gamma^{\mu_1}{}_{s_2,s_3}\, \delta_{f_2,f_3}\, T^{a_1}{}_{i_2,i_3}$$

Feynman rules

- Computation of the Feynman rules:

```
FeynmanRules[LQCD, FlavorExpand -> True]
```

Vertex 1

Particle 1 : Vector , G

Particle 2 : Dirac , $b^\dagger$

Particle 3 : Dirac , b

Vertex:

$i\,\mathrm{gs}\,\gamma^{\mu_1}{}_{s_2,s_3}\,T^{a_1}{}_{i_2,i_3}$

Vertex 2

Particle 1 : Vector , G

Particle 2 : Dirac , $c^\dagger$

Particle 3 : Dirac , c

Vertex:

$i\,\mathrm{gs}\,\gamma^{\mu_1}{}_{s_2,s_3}\,T^{a_1}{}_{i_2,i_3}$

Translation interfaces

- Once a model is implemented into FeynRules, it can be used with any Feynman diagram calculator for which a translation interface is available.

`WriteFeynArtsOutput[LQCD]`	⟶	Creates a FeynArts model.
`WriteMGOutput[LQCD]`	⟶	Creates a MadGraph model.
`WriteCHOutput[LQCD]`	⟶	Creates a CalcHEP/CompHEP model.
`WriteSHOutput[LQCD]`	⟶	Creates a Sherpa model. (coming soon)

Translation interfaces

- Let's try a simple example : gg $\longrightarrow$ gg

```
WriteFeynArtsOutput[LQCD]

WriteMGOutput[LQCD]

WriteCHOutput[LQCD]

WriteSHOutput[LQCD]
```

MadGraph: 116600pb
Stock: 116510pb

CalcHEP: 116490pb
Stock: 116490pb

CompHEP: 116490pb
Stock: 116490pb

Translation interfaces

- Standard model: 29 key-processes tested against the stock version in MadGraph/MadEvent and CalcHEP/CompHEP.

Process	CalcHEP Stock	CalcHEP Feynman	CalcHEP Unitary	CompHEP Feynman	MadGraph Stock	MadGraph
gg->gg	116490.	116490.	116490.	116490.	116600.	116510.
$u\bar{u}$->gg	199.95	199.95	199.95	199.94	199.95	200.12
$t\bar{t}$->gg	64.595	64.595	64.595	64.592	64.549	64.652
e^+e^-->$\mu^+\mu^-$	0.37195	0.37195	0.37195	0.37194	0.3722	0.37187
$t\bar{t}$->$u\bar{u}$	16.018	16.018	16.018	16.018	16.05	16.028
$u\bar{u}$->$s\bar{s}$	9.6103	9.6102	9.6103	9.6097	9.6146	9.6284
$t\bar{t}$->W^+W^-	17.265	17.265	17.265	17.265	17.237	17.199
$t\bar{t}$->ZZ	1.2686	1.2686	1.2686	1.2686	1.2722	1.2704
ZZ->ZZ	1.9672	1.9672	1.9672	1.9672	1.9685	1.9666
W^+W^-->ZZ	290.85	290.85	290.85	290.85	291.15	290.67
hh->hh	1.94	1.94	1.94	1.94	-	1.9399
hh->ZZ	65.801	65.801	65.801	65.801	65.947	65.927
hh->W^+W^-	100.49	100.49	100.49	100.49	100.81	100.8

Conclusion

- FeynRules is a Mathematica®-based package to extract Feynman rules from a lagrangian.
- The output of FeynRules is completely generic and can be easily interfaced to other available codes.
- Available interfaces:
  - FeynArts/FormCalc
  - MadGraph/MadEvent
  - CalcHep/CompHep
  - Sherpa
  - ...
- The code can be downloaded from http://feynrules.ucl.ac.be

Backups

Getting Feynman rules

Kaluza-Klein States from Large Extra Dimensions

Tao Han[a], Joseph D. Lykken[b] and Ren-Jie Zhang[a]

[a] *Department of Physics, University of Wisconsin, Madison, WI 53706*

[b] *Theory Group, Fermi National Accelerator Laboratory, Batavia, IL 60510*

[hep-ph/9811350]

Particle content:

- Spin 2 graviton, KK-scalars
- Fermions
- Scalars
- Gauge bosons

Getting Feynman rules

- Lagrangian coupling the fermions to the graviton and the KK-scalar:

$$
\begin{aligned}
\kappa^{-1}\mathcal{L}_{\mathrm{F}}^{\vec{n}}(\kappa) &= \frac{1}{2}\Big[(\tilde{h}^{\vec{n}}\eta^{\mu\nu} - \tilde{h}^{\mu\nu,\vec{n}})\overline{\psi}i\gamma_\mu D_\nu\psi - m_\psi\tilde{h}^{\vec{n}}\overline{\psi}\psi + \frac{1}{2}\overline{\psi}i\gamma^\mu(\partial_\mu\tilde{h}^{\vec{n}} - \partial^\nu\tilde{h}^{\vec{n}}_{\mu\nu})\psi\Big] \\
&\quad + \frac{3\omega}{2}\tilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu D_\mu\psi - 2\omega m_\psi\tilde{\phi}^{\vec{n}}\overline{\psi}\psi + \frac{3\omega}{4}\partial_\mu\tilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu\psi \ . \qquad (44)
\end{aligned}
$$

- Very complicated structure as far as Feynman rules are concerned, but we are only a few steps away from the Feynman rules...

Getting Feynman rules

- Step 1: Add all the parameters in the lagrangian to the model file:

```
M$Parameters = {

   g, k, om, ...

}
```

$$\kappa^{-1}\mathcal{L}_{\rm F}^{\vec{n}}(\kappa) = \frac{1}{2}\Big[(\tilde{h}^{\vec{n}}\eta^{\mu\nu} - \tilde{h}^{\mu\nu,\vec{n}})\overline{\psi}i\gamma_\mu D_\nu\psi - m_\psi\tilde{h}^{\vec{n}}\overline{\psi}\psi + \frac{1}{2}\overline{\psi}i\gamma^\mu(\partial_\mu\tilde{h}^{\vec{n}} - \partial^\nu\tilde{h}^{\vec{n}}_{\mu\nu})\psi\Big] + \frac{3\omega}{2}\tilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu D_\mu\psi - 2\omega m_\psi\tilde{\phi}^{\vec{n}}\overline{\psi}\psi + \frac{3\omega}{4}\partial_\mu\tilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu\psi$$

Getting Feynman rules

- **Step II**: Add all the particles in the lagrangian to the model file:

```
M$ClassesDescription = {

  Sp2[1] == {
     ClassName -> h,
     SelfConjugate -> True,
     Symmetric -> True}

  ...

}
```

$$\kappa^{-1}\mathcal{L}_{\mathrm{F}}^{\vec{n}}(\kappa) = \frac{1}{2}\Big[(\tilde{h}^{\vec{n}}\eta^{\mu\nu} - \tilde{h}^{\mu\nu,\vec{n}})\overline{\psi}i\gamma_{\mu}D_{\nu}\psi - m_{\psi}\tilde{h}^{\vec{n}}\overline{\psi}\psi + \frac{1}{2}\overline{\psi}i\gamma^{\mu}(\partial_{\mu}\tilde{h}^{\vec{n}} - \partial^{\nu}\tilde{h}^{\vec{n}}_{\mu\nu})\psi\Big] + \frac{3\omega}{2}\tilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^{\mu}D_{\mu}\psi - 2\omega m_{\psi}\tilde{\phi}^{\vec{n}}\overline{\psi}\psi + \frac{3\omega}{4}\partial_{\mu}\tilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^{\mu}\psi$$

Getting Feynman rules

- **Step III:** The lagrangian

```
LF = k (1 / 2
 ((h[ρ, ρ] ME[μ, ν] - h[μ, ν])
    (I HC[ψ]. Ga[μ].del[ψ, ν] - g G[ν, a] HC[ψ].Ga[μ].T[a].ψ) -
   mpsi h[μ, μ] HC[ψ].ψ + I / 2 HC[ψ]. Ga[μ].ψ
    (del[h[ν, ν], μ] - del[h[μ, ν], ν])) +
3 om / 2 φ (I HC[ψ]. Ga[μ].del[ψ, μ] - g G[μ, a] HC[ψ].Ga[μ].T[a].ψ) -
2 om mpsi φ HC[ψ].ψ + 3 om / 4 del[φ, μ] I HC[ψ]. Ga[μ].ψ)
```

$$k\Big(-2\,\mathrm{mpsi}\,\mathrm{om}\,\phi\,\psi^\dagger.\psi + \frac{3}{4}\,i\,\mathrm{om}\,\partial_\mu(\phi)\,\psi^\dagger.\gamma^\mu.\psi + \frac{3}{2}\,\mathrm{om}\,\phi\left(i\,\psi^\dagger.\gamma^\mu.\partial_\mu(\psi) - g\,\psi^\dagger.\gamma^\mu.T^a.\psi\,G_{\mu,a}\right) + \frac{1}{2}\Big(\frac{1}{2}\,i\big(\partial_\mu(h_{\nu,\nu}) - \partial_\nu(h_{\mu,\nu})\big)\,\psi^\dagger.\gamma^\mu.\psi - \mathrm{mpsi}\,\psi^\dagger.\psi\,h_{\mu,\mu} + \left(i\,\psi^\dagger.\gamma^\mu.\partial_\nu(\psi) - g\,\psi^\dagger.\gamma^\mu.T^a.\psi\,G_{\nu,a}\right)\left(h_{\rho,\rho}\,\eta_{\mu,\nu} - h_{\mu,\nu}\right)\Big)\Big)$$

$$\begin{aligned}\kappa^{-1}\mathcal{L}_{\mathrm{F}}^{\vec{n}}(\kappa) &= \frac{1}{2}\Big[(\tilde{h}^{\vec{n}}\eta^{\mu\nu} - \tilde{h}^{\mu\nu,\vec{n}})\overline{\psi}i\gamma_\mu D_\nu\psi \\ &\quad - m_\psi\tilde{h}^{\vec{n}}\overline{\psi}\psi + \frac{1}{2}\overline{\psi}i\gamma^\mu(\partial_\mu\tilde{h}^{\vec{n}} - \partial^\nu\tilde{h}^{\vec{n}}_{\mu\nu})\psi\Big] \\ &\quad + \frac{3\omega}{2}\tilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu D_\mu\psi - 2\omega m_\psi\tilde{\phi}^{\vec{n}}\overline{\psi}\psi \\ &\quad + \frac{3\omega}{4}\partial_\mu\tilde{\phi}^{\vec{n}}\overline{\psi}i\gamma^\mu\psi\end{aligned}$$

Getting Feynman rules

- Step IV: The FeynmanRules

```
FeynmanRules[LF]
```

Calculating vertices...

4 vertices obtained.

(*****************************)

Vertex 1

Particle 1 : Scalar , ϕ

Particle 2 : Dirac , ψ

Particle 3 : Dirac , $\psi^\dagger$

Vertex:

$$\frac{1}{4}\, i\, k\, \text{om}\, \delta_{i_2,i_3} \left(3\, p_1^{\alpha 2}\, {\gamma^{\alpha 2}}_{s_3,s_2} + 6\, p_2^{\alpha 2}\, {\gamma^{\alpha 2}}_{s_3,s_2} - 8\, \text{mpsi}\, \delta_{s_2,s_3}\right)$$

Getting Feynman rules

- Step IV: The FeynmanRules

(* *)

Vertex 2

Particle 1 : Spin 2 , h

Particle 2 : Dirac , ψ

Particle 3 : Dirac , $\psi^\dagger$

Vertex:

$$-\frac{1}{8}\, i\, k\, \delta_{i_2,i_3}$$

$$\left(p_1^\beta\, \gamma^\alpha{}_{s_3,s_2} + 2\, p_2^\beta\, \gamma^\alpha{}_{s_3,s_2} + p_1^\alpha\, \gamma^\beta{}_{s_3,s_2} + 2\, p_2^\alpha\, \gamma^\beta{}_{s_3,s_2} - 2\, p_1^{\alpha 2}\, \gamma^{\alpha 2}{}_{s_3,s_2}\, \eta_{\alpha,\beta}\right.$$

$$\left. - 4\, p_2^{\gamma 2}\, \gamma^{\gamma 2}{}_{s_3,s_2}\, \eta_{\alpha,\beta} + 4\, \mathrm{mpsi}\, \delta_{s_2,s_3}\, \eta_{\alpha,\beta}\right)$$

Getting Feynman rules

- Step IV: The FeynmanRules

(* *)

Vertex 3

Particle 1 : Scalar , ϕ

Particle 2 : Vector , G

Particle 3 : Dirac , ψ

Particle 4 : Dirac , $\psi^\dagger$

Vertex:

$$-\frac{3}{2}\, i\, g\, k\,\mathrm{om}\, \gamma^{\mu_2}{}_{s_4,s_3}\, T^{a_2}{}_{i_4,i_3}$$

Getting Feynman rules

- Step IV: The FeynmanRules

(* *)

Vertex 4

Particle 1 : Vector , G

Particle 2 : Spin 2 , h

Particle 3 : Dirac , ψ

Particle 4 : Dirac , $\psi^\dagger$

Vertex:

$$\frac{1}{4}\, i\, g\, k \left({\gamma^{\beta}}_{s_4,s_3}\, \eta_{\mu_1,\alpha} + {\gamma^{\alpha}}_{s_4,s_3}\, \eta_{\mu_1,\beta} - 2\, {\gamma^{\mu_1}}_{s_4,s_3}\, \eta_{\alpha,\beta} \right) {T^{a_1}}_{i_4,i_3}$$